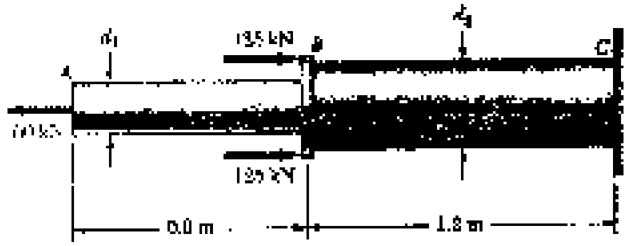
CHAPTER 1

PROBLEM 1.1

1.1 Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that $d_1 = 30$ mm and $d_2 = 50$ mm, find the average normal stress in the mid section of (a) rod AB, (b) rod BC.



SOLUTION

(a) rod AB   Force: $P = 60 \times 10^3$ N tension

Area: $A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30 \times 10^{-3})^2 = 706.86 \times 10^{-6}$ m^2

Normal stress: $\sigma_{AB} = \frac{P}{A} = \frac{60 \times 10^3}{706.86 \times 10^{-6}} = 84.88 \times 10^6$ Pa

$\sigma_{AB} = 84.9$ MPa ◄

(b) rod BC

Force: $P = 60 \times 10^3 - (2)(125 \times 10^3) = -190 \times 10^3$ N

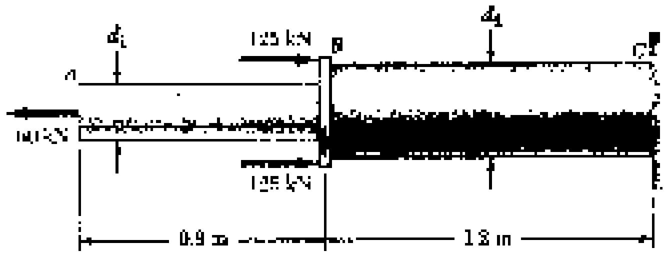
Area: $A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(50 \times 10^{-3})^2 = 1.9635 \times 10^{-3}$ m^2

Normal stress: $\sigma_{BC} = \frac{P}{A} = \frac{-190 \times 10^3}{1.9635 \times 10^{-3}} = -96.77 \times 10^6$ Pa

$\sigma_{BC} = -96.8$ MPa ◄

PROBLEM 1.2

1.2 Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that the average normal stress must not exceed 150 MPa in either rod, determine the smallest allowable values of the diameters d_1 and d_2.



SOLUTION

rod AB

Force: $P = 60 \times 10^3$ N Stress: $\sigma_{AB} = 150 \times 10^6$ Pa

Area: $A = \frac{\pi}{4} d_1^2$

$$\sigma_{AB} = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma_{AB}}$$

$$\frac{\pi}{4} d_1^2 = \frac{P}{\sigma_{AB}}$$

$$d_1^2 = \frac{4P}{\pi \sigma_{AB}} = \frac{(4)(60 \times 10^3)}{\pi (150 \times 10^6)} = 509.3 \times 10^{-6} \text{ m}^2$$

$$d_1 = 22.56 \times 10^{-3} \text{ m} \qquad d_1 = 22.6 \text{ mm}$$

rod BC

Force $P = 60 \times 10^3 - (2)(125 \times 10^3) = -190 \times 10^3$ N

Stress: $\sigma_{BC} = -150 \times 10^6$ Pa Area: $A = \frac{\pi}{4} d_2^2$

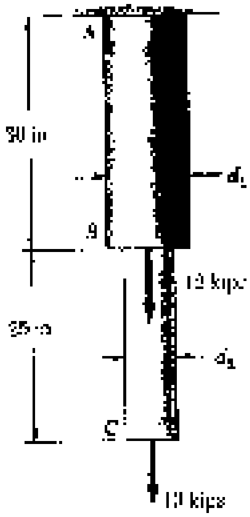
$$\sigma_{BC} = \frac{P}{A} = \frac{4P}{\pi d_2^2}$$

$$d_2^2 = \frac{4P}{\pi \sigma_{BC}} = \frac{(4)(-190 \times 10^3)}{\pi (-150 \times 10^6)} = 1.6128 \times 10^{-3} \text{ m}^2$$

$$d_2 = 40.16 \times 10^{-3} \text{ m} \qquad d_2 = 40.2 \text{ mm}$$

PROBLEM 1.3

1.3 Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that $d_1 = 1.25$ in. and $d_2 = 0.75$ in., find the normal stress at the midpoint of (a) rod AB, (b) rod BC.



SOLUTION

(a) rod AB

$$P = 12 + 10 = 22 \text{ kips}$$

$$A = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(1.25)^2 = 1.2272 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{22}{1.2272} = 17.93 \text{ ksi}$$

(b) rod BC

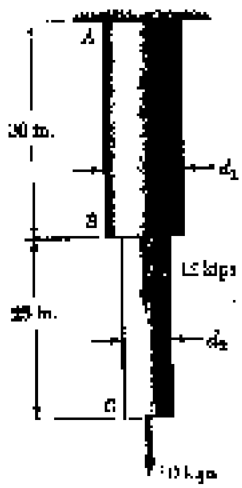
$$P = 10 \text{ kips}$$

$$A = \frac{\pi}{4} d_2^2 = \frac{\pi}{4}(0.75)^2 = 0.4418 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A} = \frac{10}{0.4418} = 22.6 \text{ ksi}$$

PROBLEM 1.4

1.4 Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Knowing that the normal stress must not exceed 25 ksi in either rod, determine the smallest allowable values of the diameters d_1 and d_2.



SOLUTION

rod AB:

$$P = 12 + 10 = 22 \text{ kips}$$

$$\sigma_{AB} = 25 \text{ ksi} \qquad A_{AB} = \frac{\pi}{4} d_1^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{4P}{\pi d_1^2}$$

$$d_1^2 = \frac{4P}{\pi \sigma_{AB}} = \frac{(4)(22)}{\pi(25)} = 1.1205 \text{ in}^2$$

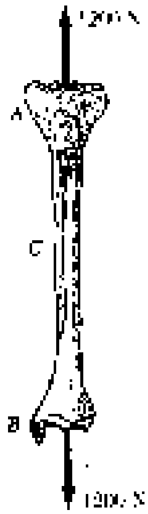
$$d_1 = 1.059 \text{ in}$$

rod BC: $P = 10$ kips $\quad \sigma_{BC} = 25$ ksi $\quad A_{BC} = \frac{\pi}{4} d_2^2$

$$d_2^2 = \frac{4P}{\pi \sigma_{BC}} = \frac{(4)(10)}{\pi(25)} = 0.5093 \text{ in}^2 \qquad d_2 = 0.714 \text{ in.}$$

PROBLEM 1.5

1.5 A strain gage located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at C to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at C.



SOLUTION

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

Geometry: $A = \frac{\pi}{4}(d_1^2 - d_2^2)$

$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

$$d_2^2 = (25\times10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80\times10^6)}$$

$$= 222.9\times10^{-6}\ \text{m}^2$$

$d_1 = 14.93\times10^{-3}$ m $\qquad\qquad d_1 = 14.93$ mm ◄

PROBLEM 1.6

1.6 Two steel plates are to be held together by means of 1/4-in.-diameter high-strength steel bolts fitting snugly inside cylindrical brass spacers. Knowing that the average normal stress must not exceed 30 ksi in the bolts and 18 ksi in the spacers, determine the outer diameter of the spacers which yields the most economical and safe design.

SOLUTION

At each bolt location the upper plate is pulled down by the tensile force P_b of the bolt. At the same time the spacer pushes that plate upward with a compressive force P_s. In order to maintain equilibrium

$$P_b = P_s$$

For the bolt $\quad \sigma_b = \dfrac{P_b}{A_b} = \dfrac{4P_b}{\pi d_b^2} \quad$ or $\quad P_b = \dfrac{\pi}{4}\sigma_b d_b^2$

For the spacer $\quad \sigma_s = \dfrac{P_s}{A_s} = \dfrac{4P_s}{\pi(d_s^2 - d_b^2)} \quad$ or $\quad P_s = \dfrac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$

Equating P_b and P_s

$$\frac{\pi}{4}\sigma_b d_b^2 = \frac{\pi}{4}\sigma_s(d_s^2 - d_b^2)$$

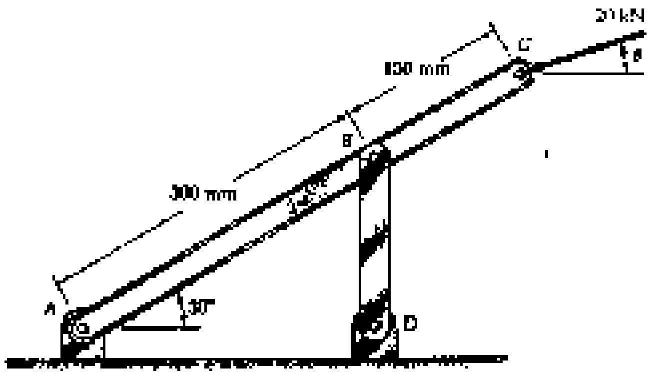
$$d_s^2 = d_b^2 + \frac{\sigma_b}{\sigma_s}d_b^2 = \left(1 + \frac{\sigma_b}{\sigma_s}\right)d_b^2$$

$$d_s = \left(1 + \frac{30}{18}\right)\left(\frac{1}{4}\right)^2 = 0.16667 \text{ in}^2$$

$$d_s = 0.408 \text{ in.}$$

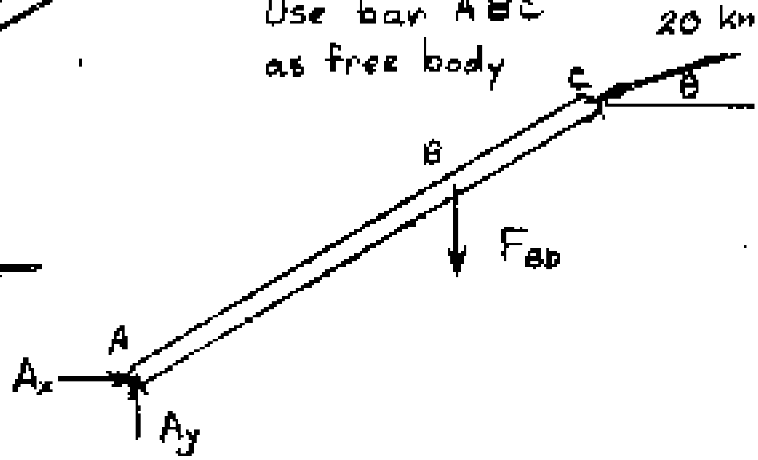
PROBLEM 1.7

1.7 Link BD consists of a single bar 30 mm wide and 12 mm thick. Knowing that each pin has a 10-mm diameter, determine the maximum value of the average normal stress in link BD if (a) $\theta = 0$, (b) $\theta = 90°$.



SOLUTION

Use bar ABC as free body



$\Sigma M_A = 0$

(a) $\theta = 0°$ $\quad (0.450 \sin 30°)(20\times10^3) - (0.300 \cos 30°) F_{BD} = 0$

$F_{BD} = 17.32 \times 10^3$ N

(b) $\theta = 90°$ $\quad (0.450 \cos 30°)(20\times10^3) - (0.300 \cos 30°) F_{BD} = 0$

$F_{BD} = -30 \times 10^3$ N

Areas

(a) tension loading $\quad A = (0.030 - 0.010)(0.012) = 240 \times 10^{-6}\ \mathrm{m}^2$

(b) compression $\quad A = (0.030)(0.012) = 360 \times 10^{-6}\ \mathrm{m}^2$

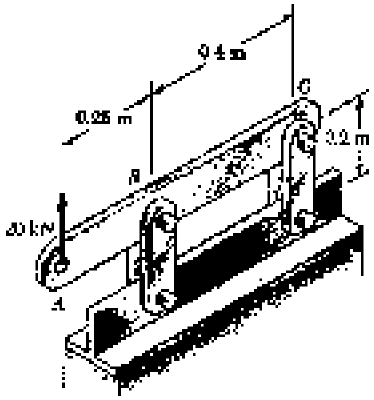
Stresses

(a) $\sigma = \dfrac{F_{BD}}{A} = \dfrac{17.32\times10^3}{240\times10^{-6}} = 72.2\times10^6$ $\quad$ 72.2 MPa

(b) $\sigma = \dfrac{F_{BD}}{A} = \dfrac{-30\times10^3}{360\times10^{-6}} = -83.3\times10^6$ $\quad$ −83.3 MPa

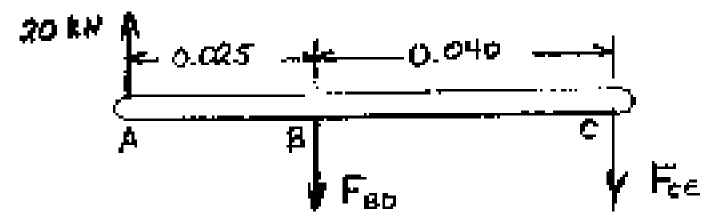
PROBLEM 1.8

1.8 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting (*a*) points *B* and *D*, (*b*) points *C* and *E*.



SOLUTION

Use bar ABC as a free body.



$\Sigma M_C = 0 \qquad (0.040)F_{BD} - (0.025 + 0.040)(20\times10^3) = 0$

$F_{BD} = 32.5\times10^3$ N $\qquad$ Link BD is in tension

$\Sigma M_B = 0 \qquad -(0.040)F_{CE} - (0.025)(20\times10^3) = 0$

$F_{CE} = -12.5\times10^3$ N $\qquad$ Link CE is in compression.

Net area of one link for tension $= (0.008)(0.036 - 0.016)$

$= 160\times10^{-6}$ m². For two parallel links $A_{net} = 320\times10^{-6}$ m²

Tensile stress in link BD

(a) $\qquad \sigma_{BD} = \dfrac{F_{BD}}{A_{net}} = \dfrac{32.5\times10^3}{320\times10^{-6}} = 101.56\times10^6$ or 101.6 MPa ◄

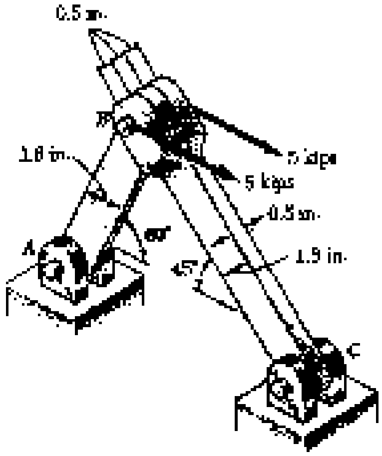
Area for one link in compression $= (0.008)(0.036)$

$= 288\times10^{-6}$ m². For two parallel links $A = 576\times10^{-6}$ m²

(b) $\qquad \sigma_{CE} = \dfrac{F_{CE}}{A} = \dfrac{-12.5\times10^3}{576\times10^{-6}} = -21.70\times10^6$ or -21.7 MPa ◄

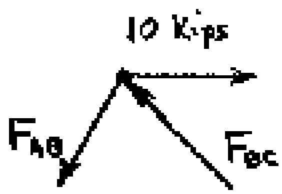
PROBLEM 1.9

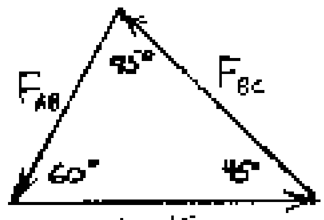
1.9 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (a) in link AB, (b) in link BC.



SOLUTION

Use joint B as free body.





Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ}$$

$F_{AB} = 7.3205$ kips $\qquad F_{BC} = 8.9658$ kips.

Link AB is a tension member

Minimum section at pin $\quad A_{net} = (1.8 - 0.8)(0.5) = 0.5 \text{ in}^2$

(a) Stress in AB $\quad \sigma_{AB} = \dfrac{F_{AB}}{A_{net}} = \dfrac{7.3205}{0.5} = 14.64$ ksi

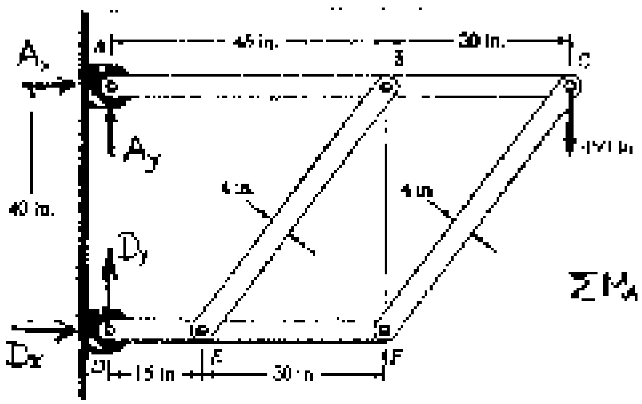
Link BC is a compression member

Cross sectional area is $\quad A = (1.8)(0.5) = 0.9 \text{ in}^2$

(b) Stress in BC $\quad \sigma_{BC} = \dfrac{-F_{BC}}{A} = \dfrac{-8.9658}{0.9} = -9.96$ ksi

PROBLEM 1.10

1.10 The frame shown consists of four wooden members, *ABC*, *DEF*, *BE*, and *CF*. Knowing that each member has a 2 × 4-in. rectangular cross section and that each pin has a ½-in. diameter, determine the maximum value of the average normal stress (*a*) in member *BE*, (*b*) in member *CF*.



SOLUTION

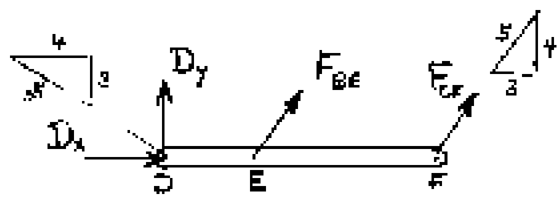
Add support reactions to figure as shown.

Using entire frame as free body

$\Sigma M_A = 0 \qquad 40\,D_x - (45+30)(480) = 0$

$D_x = 900$ lb.

Use member DEF as free body



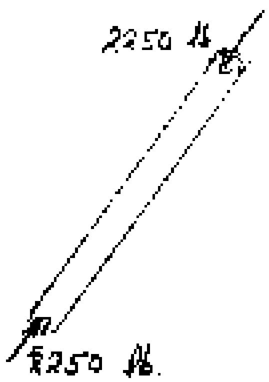
$\Sigma F \nwarrow = 0$

$\frac{3}{5}D_y - \frac{4}{5}D_x = 0$

$D_y = \frac{4}{3}D_x = 1200$ lb.

$\Sigma M_F = 0 \qquad -(30)(\frac{4}{5}F_{BE}) - (30+15)D_y = 0 \qquad F_{BE} = -2250$ lb.

$\Sigma M_E = 0 \qquad (30)(\frac{4}{5}F_{CF}) - (15)D_y = 0 \qquad F_{CF} = 750$ lb.



Stress in compression member BE

Area $\quad A = 2\text{ in} \times 4\text{ in} = 8\text{ in}^2$

(a) $\sigma_{BE} = \dfrac{F_{BE}}{A} = \dfrac{-2250}{8} = -281$ psi ◀

Stress in tension member CF

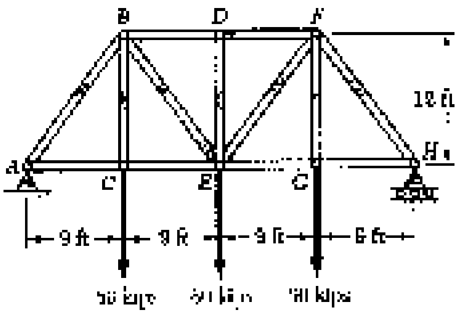
Minimum section area occurs at pin.

$A_{min} = (2)(4.0 - 0.5) = 7.0\text{ in}^2$

(b) $\sigma_{CF} = \dfrac{F_{CF}}{A_{min}} = \dfrac{750}{7.0} = 107.1$ psi ◀

PROBLEM 1.11

1.11 For the Pratt bridge truss and loading shown, determine the average normal stress in member *BE*, knowing that the cross-sectional area of that member is 5.87 in².



SOLUTION

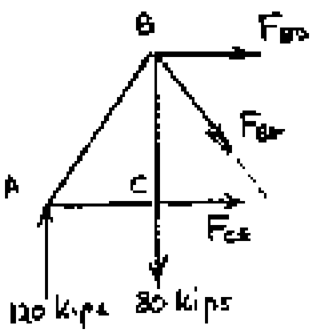
Use entire truss as free body

$\Sigma M_H = 0$

$(9)(80) + (18)(80) + (27)(80) - 36\, A_y = 0$

$A_y = 120$ kips

Use portion of truss to the left of a section cutting members BD, BE, and CE.



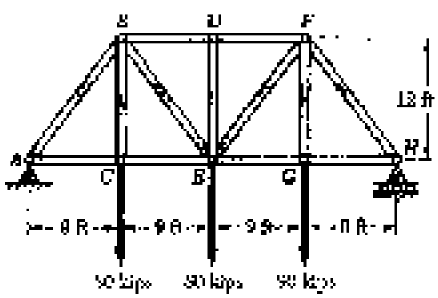
$+\uparrow\ \Sigma F_y = 0$

$120 - 80 - \frac{12}{15} F_{BE} = 0 \quad \therefore \quad F_{BE} = 50$ kips

$\sigma_{BE} = \frac{F_{BE}}{A} = \frac{50}{5.87} = 8.52$ ksi

PROBLEM 1.12

1.12 Knowing that the average normal stress in member *CE* of the Pratt bridge truss shown must not exceed 21 ksi for the given loading, determine the cross-sectional area of that member which will yield the most economical and safe design. Assume that both ends of the member will be adequately reinforced.



SOLUTION

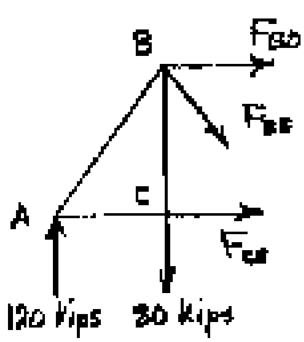
Use entire truss as free body

$\circlearrowleft \Sigma M_H = 0$

$(9)(80) + (18)(80) + (27)(80) - 36\, A_y = 0$

$A_y = 120$ kips

Use portion of truss to the left of a section cutting members BD, BE, and CE



$\circlearrowleft \Sigma M_B = 0$

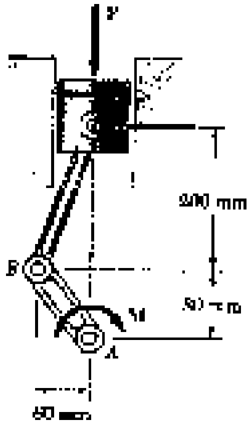
$12\, F_{CE} - (9)(120) = 0 \quad \therefore \quad F_{CE} = 90$ kips

$\sigma_{CE} = \frac{F_{CE}}{A_{CE}}$

$A_{CE} = \frac{F_{CE}}{\sigma_{CE}} = \frac{90}{21} = 4.29\ \text{in}^2$

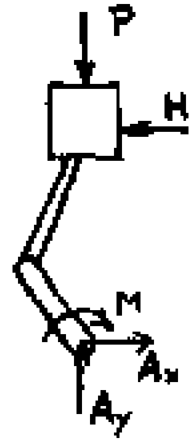
PROBLEM 1.13

1.13 A couple **M** of magnitude 1500 N·m is applied to the crank of an engine. For the position shown, determine (*a*) the force **P** required to hold the engine system in equilibrium, (*b*) the average normal stress in the connecting rod *BC*, which has a 450-mm² uniform cross section.



SOLUTION

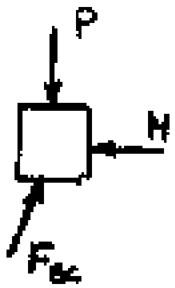
Use piston, rod, and crank together as free body. Add wall reaction H and bearing reactions A_x and A_y.

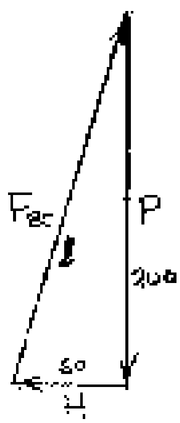


$$\circlearrowleft \Sigma M_A = 0$$

$$(0.280 \text{ m})H - 1500 \text{ N·m} = 0$$

$$H = 5.3571 \times 10^3 \text{ N}$$





Use piston alone as free body. Note that rod is a two-force member; hence the direction of force F_{BC} is known. Draw the force triangle and solve for P and F_{BC} by proportions.

$$l = \sqrt{200^2 + 60^2} = 208.81 \text{ mm}$$

$$\frac{P}{H} = \frac{200}{60} \quad \therefore \quad P = 17.86 \times 10^3 \text{ N}$$

(a) $P = 17.86$ kN ◄

$$\frac{F_{BC}}{H} = \frac{208.81}{60} \quad \therefore \quad F_{BC} = 18.643 \times 10^3 \text{ N}$$

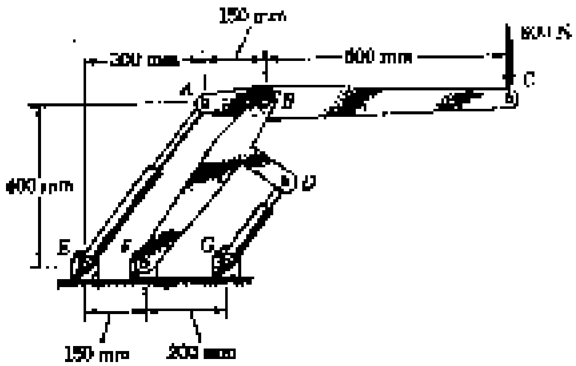
Rod BC is a compression member. Its area is $450 \text{ mm}^2 = 450 \times 10^{-6} \text{ m}^2$

Stress $\quad \sigma_{BC} = \dfrac{-F_{BC}}{A} = \dfrac{-18.643 \times 10^3}{450 \times 10^{-6}} = -41.4 \times 10^6$ Pa

(b) $\sigma_{BC} = -41.4$ MPa ◄

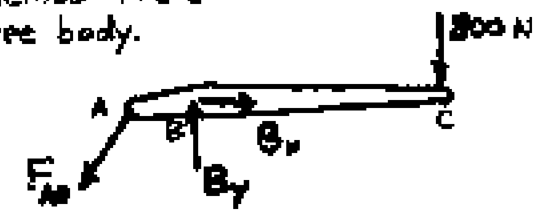
PROBLEM 1.14

1.14 Two hydraulic cylinders are used to control the position of the robotic arm *ABC*. Knowing that the control rods attached at *A* and *D* each have a 20-mm diameter and happen to be parallel in the position shown, determine the average normal stress in (*a*) member *AE*, (*b*) member *DG*.



SOLUTION

Use member ABC as free body.

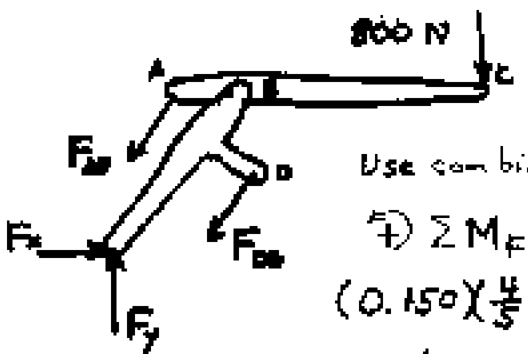


$$\circlearrowleft \Sigma M_B = 0 \qquad (0.150)\tfrac{4}{5}F_{AE} - (0.600)(800) = 0 \qquad F_{AE} = 4\times10^3\ \text{N}$$

Area of rod in member AE is $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20\times10^{-3})^2 = 314.16\times10^{-6}\ \text{m}^2$

Stress in rod AE: $\sigma_{AE} = \dfrac{F_{AE}}{A} = \dfrac{4\times10^3}{314.16\times10^{-6}} = 12.73\times10^6\ \text{Pa}$

(a) $\sigma_{AE} = 12.73\ \text{MPa}$



Use combined members ABC and BFD as free body.

$$\circlearrowleft \Sigma M_F = 0$$

$$(0.150)\left(\tfrac{4}{5}F_{AE}\right) - (0.200)\left(\tfrac{4}{5}F_{DG}\right) - (1.050 - 0.350)(800) = 0 \qquad F_{DG} = -1500\ \text{N}$$

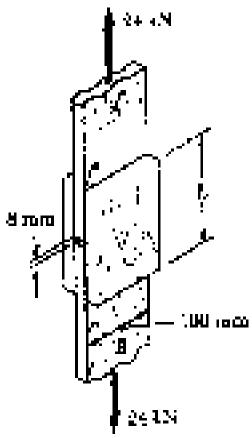
Area in rod DG is $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(20\times10^{-3})^2 = 314.16\times10^{-6}\ \text{m}^2$

Stress in rod DG: $\sigma_{DG} = \dfrac{F_{DG}}{A} = \dfrac{-1500}{3.1416\times10^{-4}} = -4.77\times10^6\ \text{Pa}$

(b) $\sigma_{DG} = -4.77\ \text{MPa}$

PROBLEM 1.15

1.15 The wooden members A and B are to be joined by plywood splice plates which will be fully glued on the surfaces in contact. As part of the design of the joint and knowing that the clearance between the ends of the members is to be 8 mm, determine the smallest allowable length L if the average shearing stress in the glue is not to exceed 800 kPa.



SOLUTION

There are four separate areas of glue. Each area must transmit half of the 24 kN load.

Therefore $F = 12\text{ kN} = 12 \times 10^3\text{ N}$

Shearing stress in glue $\tau = 800 \times 10^3\text{ Pa}$

$$\tau = \frac{F}{A} \quad \therefore \quad A = \frac{F}{\tau} = \frac{12 \times 10^3}{800 \times 10^3} = 15 \times 10^{-3}\text{ m}^2$$

Let l = length of glue area and w = width = 100 mm = 0.1 m

$$A = lw \quad \therefore \quad l = \frac{A}{w} = \frac{15 \times 10^{-3}}{0.1} = 150 \times 10^{-3}\text{ m} = 150\text{ mm}$$

$$L = 2l + \text{gap} = (2)(150) + 8 = 308\text{ mm}$$

PROBLEM 1.16

1.16 Determine the diameter of the largest circular hole which can be punched into a sheet of polystyrene 6-mm thick, knowing that the force exerted by the punch is 45 kN and that a 55-MPa average shearing stress is required to cause the material to fail.

SOLUTION

$A = \pi d t$ for cylindrical failure surface

Shearing stress $\tau = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\tau}$

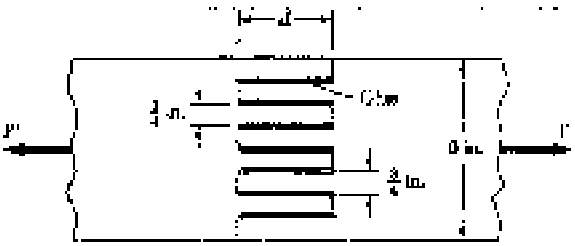
Equating A's $\pi d t = \frac{P}{\tau}$

Solving for d: $d = \frac{P}{\pi t \tau} = \frac{45 \times 10^3}{\pi (0.006)(55 \times 10^6)} = 43.4 \times 10^{-3}\text{ m}$

$d = 43.4\text{ mm}$

PROBLEM 1.17

1.17 Two wooden planks, each 7/8 - in. thick and 6 in. wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 120 psi, determine the smallest allowable length d of the cuts if the joint is to withstand an axial load of magnitude $P = 1200$ lb.



SOLUTION

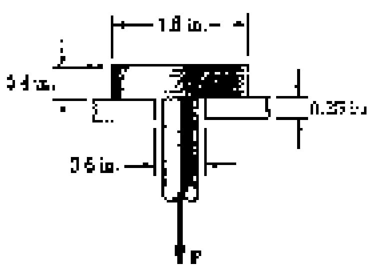
Seven surfaces carry the total load $P = 1200$ lb.

Area $A = (7)\left(\frac{7}{8}\right) d = \frac{49}{8} d$

$\tau = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\tau} \qquad \frac{49}{8} d = \frac{1200}{120} \qquad d = 1.633 \text{ in}$

PROBLEM 1.18

1.18 A load **P** is applied to a steel rod supported as shown by an aluminum plate into which a 0.6-in.-diameter hole has been drilled. Knowing that the shearing stress must not exceed 18 ksi in the steel rod and 10 ksi in the aluminum plate, determine the largest load **P** which may be applied to the rod.



SOLUTION

For steel $\quad A_1 = \pi d t = \pi(0.6)(0.4) = 0.7540 \text{ in}^2$

$\tau_1 = \frac{P}{A_1} \quad \therefore \quad P = A_1 \tau_1 = (0.7540)(18) = 13.57 \text{ kips}$

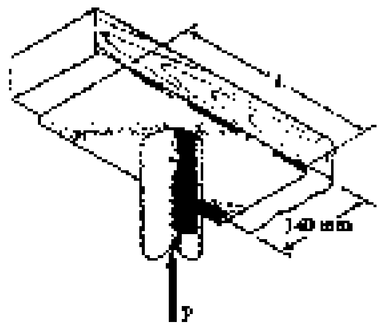
For aluminum $\quad A_2 = \pi d t = \pi(1.6)(0.25) = 1.2566 \text{ in}^2$

$\tau_2 = \frac{P}{A_2} \quad \therefore \quad P = A_2 \tau_2 = (1.2566)(10) = 12.57 \text{ kips}$

Limiting value of P is the smaller value $\therefore$ $P = 12.57$ kips

PROBLEM 1.19

1.19 The axial force in the column supporting the timber beam shown is $P = 75$ kN. Determine the smallest allowable length L of the bearing plate if the bearing stress in the timber is not to exceed 3.0 MPa.



SOLUTION

$$\sigma_b = \frac{P}{A} = \frac{P}{Lw}$$

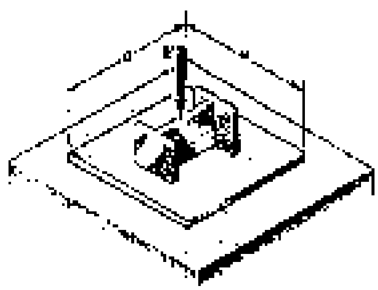
Solving for L: $L = \frac{P}{\sigma_b w} = \frac{75 \times 10^3}{(3.0 \times 10^6)(0.140)}$

$= 178.6 \times 10^{-3}$ m

$L = 178.6$ mm ◄

PROBLEM 1.20

1.20 An axial load **P** is supported by a short W250 × 67 column of cross-sectional area $A = 8580$ mm² and is distributed to a concrete foundation by a square plate as shown. Knowing that the average normal stress in the column must not exceed 150 MPa and that the bearing stress on the concrete foundation must not exceed 12.5 MPa, determine the side a of the plate which will provide the most economical and safe design.



SOLUTION

Area of column: $A = 8580\ \text{mm}^2 = 8580 \times 10^{-6}\ \text{m}^2$

Normal stress in column: $\sigma = 150 \times 10^6$ Pa

$\sigma = \frac{P}{A}$ ∴ $P = A\sigma = (8580 \times 10^{-6})(150 \times 10^6)$

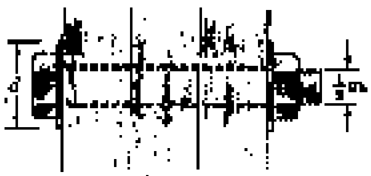
$= 1.287 \times 10^6$ N

Bearing plate: $\sigma_b = \frac{P}{A_b}$ and $A_b = a^2$ for square plate.

$A_b = a^2 = \frac{P}{\sigma_b} = \frac{1.287 \times 10^6}{12.5 \times 10^6} = 321 \times 10^{-3}$ m or 321 mm ◄

PROBLEM 1.21

1.21 Three wooden planks are fastened together by a series of bolts to form a column. The diameter of each bolt is ½ in. and the inner diameter of each washer is ⅝ in., which is slightly larger than the diameter of the holes in the planks. Determine the smallest allowable outer diameter d of the washers, knowing that the average normal stress in the bolts is 5 ksi and that the bearing stress between the washers and the planks must not exceed 1.2 ksi.



SOLUTION

Bolt: $A_{bolt} = \frac{\pi}{4} d_b^2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$

$\sigma_b = \frac{P}{A}$ ∴ Tensile force in bolt $P = \sigma_b A = (5)(0.19635) = 0.98175$ kips

Washer: inside diameter $= d_i = \frac{5}{8}$ in, outside diameter $= d_o$

Bearing area $A_w = \frac{\pi}{4}(d_o^2 - d_i^2)$ and $A_w = \frac{P}{\sigma_b}$.

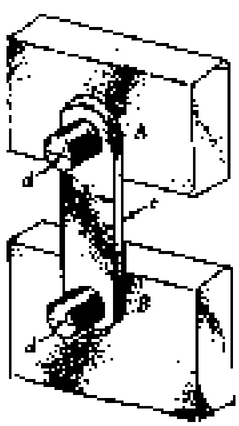
Equating $\frac{\pi}{4}(d_o^2 - d_i^2) = \frac{P}{\sigma_b}$

$$d_o^2 = d_i^2 + \frac{4P}{\pi\sigma_b} = \left(\frac{5}{8}\right)^2 + \frac{(4)(0.98175)}{\pi(1.2)} = 1.4323 \text{ in}^2$$

$$d_o = 1.197 \text{ in}$$ ◄

PROBLEM 1.22

1.22 Link AB, of width $b = 2$ in. and thickness $t = \frac{1}{4}$ in., is used to support the end of a horizontal beam. Knowing that the average normal stress in the link is −20 ksi and that the average shearing stress in each of the two pins is 12 ksi, determine (*a*) the diameter d of the pins, (*b*) the average bearing stress in the link.



SOLUTION

Rod AB is in compression.

$A = bt$ where $b = 2$ in and $t = \frac{1}{4}$ in

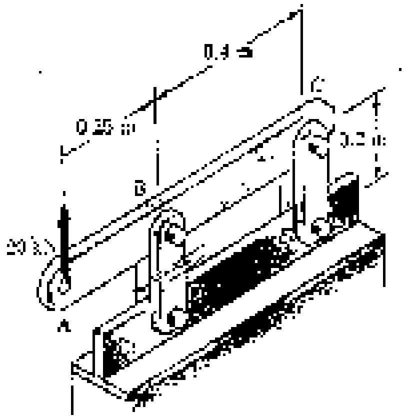
$P = -\sigma A = -(-20)(2)(\frac{1}{4}) = 10$ kips

Pin: $\tau_p = \frac{P}{A_p}$ and $A_p = \frac{\pi}{4} d^2$

(a) $d = \sqrt{\frac{4A_p}{\pi}} = \sqrt{\frac{4P}{\pi\tau_p}} = \sqrt{\frac{(4)(10)}{\pi(12)}} = 1.030$ in ◄

(b) $\sigma_b = \frac{P}{dt} = \frac{10}{(1.030)(0.25)} = 38.8$ ksi ◄

PROBLEM 1.23

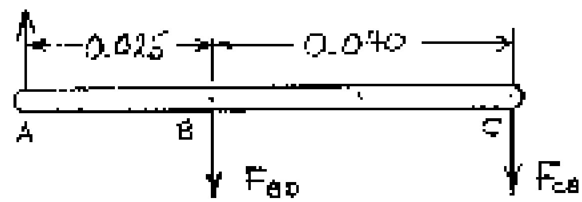


1.8 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter.

1.23 For the assembly and loading of Prob. 1.8, determine (a) the average shearing stress in the pin at B, (b) the average bearing stress at B in link BD, (c) the average bearing stress at B in member ABC, knowing that this member has a 10 × 50-mm uniform rectangular cross section.

SOLUTION

Use bar ABC as a free body



$$\Sigma M_C = 0 \qquad (0.040)F_{BD} - (0.025 + 0.040)(20 \times 10^3) = 0$$

$$F_{BD} = 32.5 \times 10^3 \text{ N}$$

(a) Shear pin at B $\qquad \tau = \dfrac{F_{BD}}{2A}$ for double shear

where $A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6} \text{ m}^2$

$$\tau = \frac{32.5 \times 10^3}{(2)(201.06 \times 10^{-6})} = 80.8 \times 10^6 \qquad 80.8 \text{ MPa}$$

(b) Bearing: link BD $\qquad A = dt = (0.016)(0.008) = 128 \times 10^{-6} \text{ m}^2$

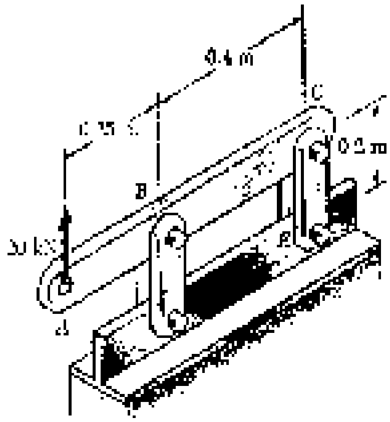
$$\sigma_b = \frac{\frac{1}{2}F_{BD}}{A} = \frac{(0.5)(32.5 \times 10^3)}{128 \times 10^{-6}} = 126.95 \times 10^6 \qquad 127.0 \text{ MPa}$$

(c) Bearing in ABC at B

$$A = dt = (0.016)(0.010) = 160 \times 10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{F_{BD}}{A} = \frac{32.5 \times 10^3}{160 \times 10^{-6}} = 203 \times 10^6 \qquad 203 \text{ MPa}$$

PROBLEM 1.24

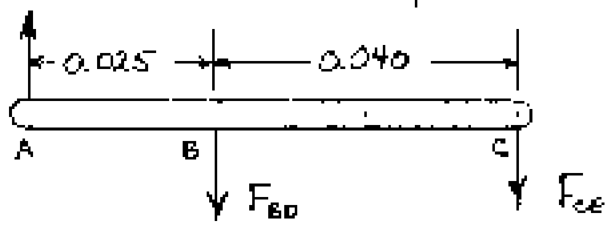


1.8 Each of the four vertical links has an 8 × 36-mm uniform rectangular cross section and each of the four pins has a 16-mm diameter.

1.24 For the assembly and loading of Prob. 1.8, determine (*a*) the average shearing stress in the pin at *C*, (*b*) the average bearing stress at *C* in link *CE*, (*c*) the average bearing stress at *C* in member *ABC*, knowing that this member has a 10 × 50-mm uniform rectangular cross section.

SOLUTION

Use bar ABC as a free body



$$\sum M_B = 0 \quad -(0.040)F_{CE} - (0.025)(20\times10^3) = 0 \qquad F_{CE} = -12.5\times10^3$$

(a) Shear in pin at C $\quad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.016)^2 = 201.06\times10^{-6}\ m^2$

Double shear $\quad \tau = \frac{F_{CE}}{2A} = \frac{12.5\times10^3}{(2)(201.06\times10^{-6})} = 31.1\times10^{6}$

31.1 MPa ◄

(b) Bearing in link CE at C

$$A = dt = (0.016)(0.008) = 128\times10^{-6}\ m^2$$

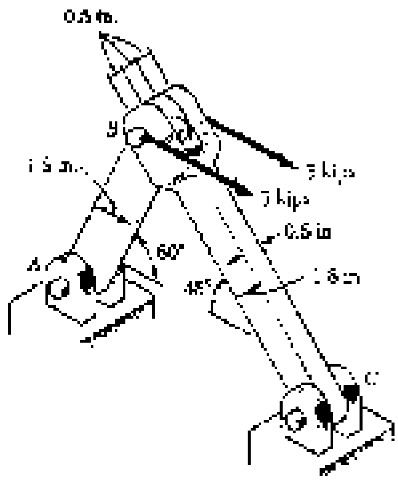
$$\sigma_b = \frac{\frac{1}{2}F_{CE}}{A} = \frac{(0.5)(12.5\times10^3)}{128\times10^{-6}} = 48.8\times10^6 \qquad 48.8\ MPa$$ ◄

(c) Bearing in ABC at C

$$A = dt = (0.016)(0.010) = 160\times10^{-6}\ m^2$$

$$\sigma_b = \frac{F_{CE}}{A} = \frac{12.5\times10^3}{160\times10^{-6}} = 78.1\times10^6 \qquad 78.1\ MPa$$ ◄

PROBLEM 1.25

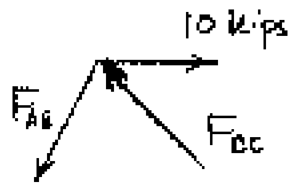


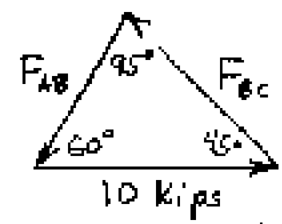
1.9 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (a) in link AB, (b) in link BC.

1.25 For the assembly and loading of Prob. 1.9, determine (a) the average shearing stress in the pin at A, (b) the average bearing stress at A in member AB.

SOLUTION

Use joint B as free body.





Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ} \qquad F_{AB} = 7.3205 \text{ kips}$$

(a) Shearing stress in pin at A $\qquad \tau = \dfrac{F_{AB}}{2A_P}$

where $A_P = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.8)^2 = 0.5026 \text{ in}^2$

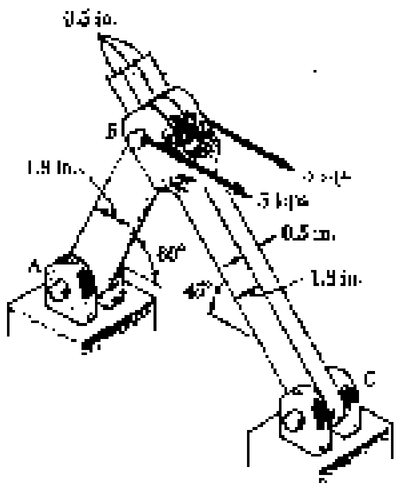
$$\tau = \frac{7.3205}{(2)(0.5026)} = 7.28 \qquad 7.28 \text{ ksi} \blacktriangleleft$$

(b) Bearing stress at A in member AB

$$A_b = t\,d = (0.5)(0.8) = 0.4 \text{ in}^2$$

$$\sigma_b = \frac{F_{AB}}{A_b} = \frac{7.3205}{0.4} = 18.30 \qquad 18.30 \text{ ksi} \blacktriangleleft$$

PROBLEM 1.26

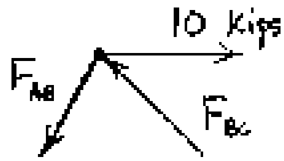


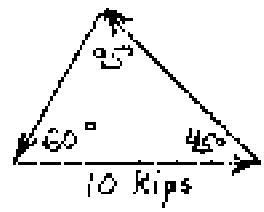
1.9 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Knowing that a pin of 0.8-in. diameter is used at each connection, determine the maximum value of the average normal stress (a) in link AB, (b) in link BC.

1.26 For the assembly and loading of Prob. 1.9, determine (a) the average shearing stress in the pin at C, (b) the average bearing stress at C in member BC, (c) the average bearing stress at B in member BC.

SOLUTION

Use joint B as free body





Force triangle

Law of Sines

$$\frac{F_{AB}}{\sin 45^\circ} = \frac{F_{BC}}{\sin 60^\circ} = \frac{10}{\sin 95^\circ} \qquad F_{BC} = 8.9658 \text{ kips}$$

(a) Shearing stress in pin at C $\qquad \tau = \frac{F_{BC}}{2A_P}$

$$A_P = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.8)^2 = 0.5026 \text{ in}^2$$

$$\tau = \frac{8.9658}{(2)(0.5026)} = 8.92 \qquad 8.92 \text{ ksi}$$

(b) Bearing stress at C in member BC $\qquad \sigma_b = \frac{F_{BC}}{A}$

$$A = t\,d = (0.5)(0.8) = 0.4 \text{ in}^2$$

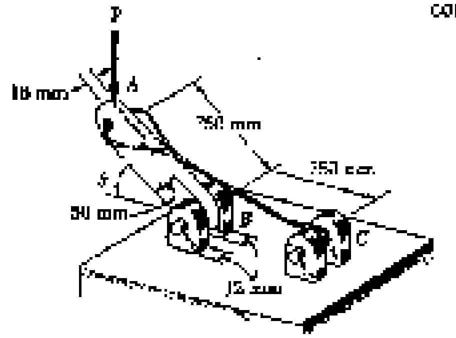
$$\sigma_b = \frac{8.9658}{0.4} = 22.4 \qquad 22.4 \text{ ksi}$$

(c) Bearing stress at B in member BC $\qquad \sigma_b = \frac{F_{BC}}{A}$

$$A = 2t\,d = 2(0.5)(0.8) = 0.8 \text{ in}^2$$

$$\sigma_b = \frac{8.9658}{0.8} = 11.21 \qquad 11.21 \text{ ksi}$$

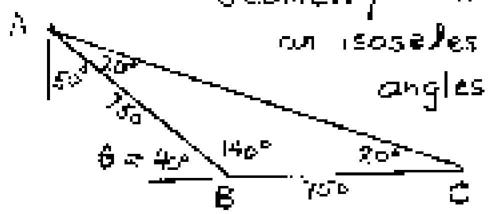
PROBLEM 1.27



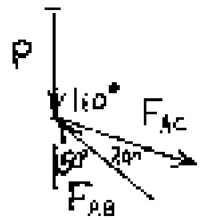
1.27 Knowing that $\theta = 40°$ and $P = 9$ kN, determine (a) the smallest allowable diameter of the pin at B if the average shearing stress in the pin is not to exceed 120 MPa, (b) the corresponding average bearing stress in member AB at B, (c) the corresponding average bearing stress in each of the support brackets at B.

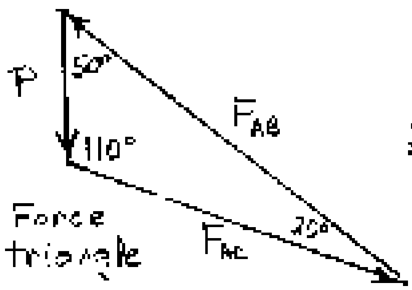
SOLUTION

Geometry: Triangle ABC is an isoseles triangle with angles shown here.



Use joint A as a free body.





Force triangle

Law of Sines applied to force triangle

$$\frac{P}{\sin 20°} = \frac{F_{AB}}{\sin 110°} = \frac{F_{AC}}{\sin 50°}$$

$$F_{AB} = \frac{P \sin 110°}{\sin 20°} = \frac{(9)\sin 110°}{\sin 20°} = 24.73 \text{ kN}$$

(a) Allowable pin diameter

$$\tau = \frac{F_{AB}}{2A_P} = \frac{F_{AB}}{2\frac{\pi}{4}d^2} = \frac{2F_{AB}}{\pi d^2} \qquad \text{where } F_{AB} = 24.73\times10^3 \text{ N}$$

$$d^2 = \frac{2F_{AB}}{\pi\tau} = \frac{(2)(24.73\times10^3)}{\pi(120\times10^6)} = 131.18\times10^{-6} \text{ m}^2$$

$$d = 11.45\times10^{-3} \text{ m} \qquad 11.45 \text{ mm} \blacktriangleleft$$

(b) Bearing stress in AB at A.

$$A_b = td = (0.016)(11.45\times10^{-3}) = 183.26\times10^{-6} \text{ m}^2$$

$$\sigma_b = \frac{F_{AB}}{A_b} = \frac{24.73\times10^3}{183.26\times10^{-6}} = 134.9\times10^6 \qquad 134.9 \text{ MPa} \blacktriangleleft$$

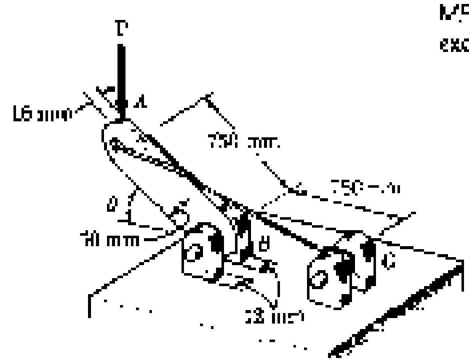
(c) Bearing stress in support brackets at B

$$A = td = (0.012)(11.45\times10^{-3}) = 137.4\times10^{-6} \text{ m}^2$$

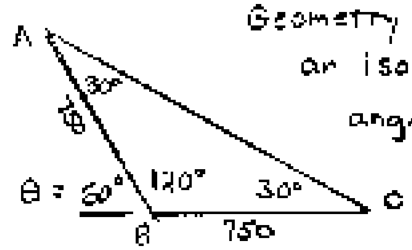
$$\sigma_b = \frac{\frac{1}{2}F_{AB}}{A} = \frac{(0.5)(24.73\times10^3)}{137.4\times10^{-6}} = 90.0\times10^6 \qquad 90.0 \text{ MPa} \blacktriangleleft$$

PROBLEM 1.28

1.28 Determine the largest load P which may be applied at A when $\theta = 60°$, knowing that the average shearing stress in the 10-mm-diameter pin at B must not exceed 120 MPa and that the average bearing stress in member AB and in the bracket at B must not exceed 90 MPa.

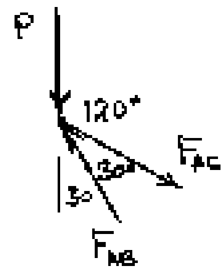


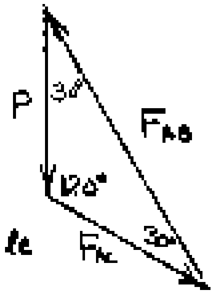
SOLUTION



Geometry: Triangle ABC is an isosceles triangle with angles shown here

Use joint A as free body





Force triangle

Law of sines applied to force triangle

$$\frac{P}{\sin 30°} = \frac{F_{AB}}{\sin 120°} = \frac{F_{AC}}{\sin 30°}$$

$$P = \frac{F_{AB} \sin 30°}{\sin 120°} = 0.57735\, F_{AB}$$

$$P = \frac{F_{AC} \sin 30°}{\sin 30°} = F_{AC}$$

If shearing stress in pin at B is critical

$$A_P = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\ \text{m}^2$$

$$F_{AB} = 2A\tau = (2)(78.54 \times 10^{-6})(120 \times 10^6) = 18.850 \times 10^3\ \text{N}$$

If bearing stress in member AB at bracket at A is critical

$$A_b = td = (0.016)(0.010) = 160 \times 10^{-6}\ \text{m}^2$$

$$F_{AB} = A_b \sigma_b = (160 \times 10^{-6})(90 \times 10^6) = 14.40 \times 10^3\ \text{N}$$

If bearing stress in the bracket at B is critical

$$A_b = 2td = (2)(0.012)(0.010) = 240 \times 10^{-6}\ \text{m}^2$$

$$F_{AB} = A_b \sigma_b = (240 \times 10^{-6})(90 \times 10^6) = 21.6 \times 10^3\ \text{N}$$

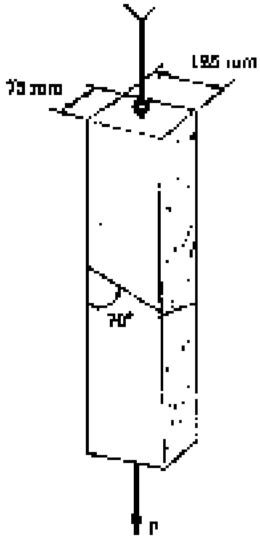
Allowable F_{AB} is the smallest, i.e. 14.40×10^3 N

Then, from Statics $P_{allow} = (0.57735)(14.40 \times 10^3)$

$= 8.31 \times 10^3$ N 8.31 kN

PROBLEM 1.29

1.29 The 6- kN load **P** is supported by two wooden members of 75 × 125- mm uniform rectangular cross section which are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.



SOLUTION

$P = 6 \times 10^3$ N $\qquad \theta = 90° - 70° = 20°$

$A_o = (0.075)(0.125) = 9.375 \times 10^{-3}$ m²

$$\sigma = \frac{P}{A_o}\cos^2\theta = \frac{(6\times10^3)\cos^2 20°}{9.375\times10^{-3}} = 565\times10^3$$

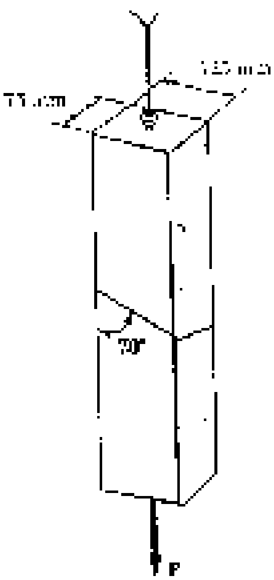
$\sigma = 565$ kPa

$$\tau = \frac{P}{2A_o}\sin 2\theta = \frac{(6\times10^3)\sin 40°}{(2)(9.375\times10^{-3})} = 206\times10^3$$

$\tau = 206$ kPa

PROBLEM 1.30

1.30 Two wooden members of 75 × 125- mm uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 500 kPa, determine (*a*) the largest load P which can be safely supported, (*b*) the corresponding shearing stress in the splice.



SOLUTION

$A_o = (0.075)(0.125) = 9.375 \times 10^{-3}$ m²

$\theta = 90° - 70° = 20° \qquad \sigma = 500 \times 10^3$ Pa

$$\sigma = \frac{P}{A_o}\cos^2\theta$$

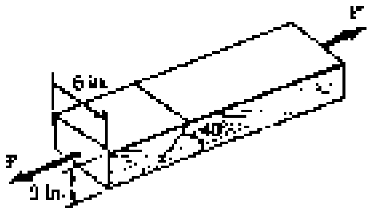
$$P = \frac{A_o\sigma}{\cos^2\theta} = \frac{(9.375\times10^{-3})(500\times10^3)}{\cos^2 20°} = 5.3085\times10^3$$

(a) $\qquad P = 5.31$ kN

$$\tau = \frac{P\sin 2\theta}{2A_o} = \frac{(5.3085\times10^3)\sin 40°}{(2)(9.375\times10^{-3})} = 181.99\times10^3$$

(b) $\qquad \tau = 182.0$ kPa

PROBLEM 1.31



1.31 Two wooden members of 3 × 6-in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 90 psi, determine (a) the largest load P which can be safely applied, (b) the corresponding tensile stress in the splice.

SOLUTION

$\theta = 90° - 40° = 50°$

$A_o = (3)(6) = 18 \text{ in}^2$

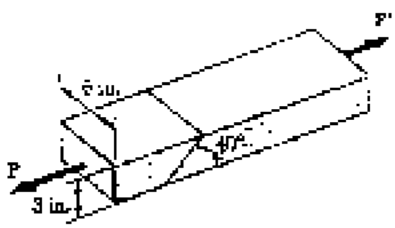
$\tau = \frac{P}{2A} \sin 2\theta$

$P = \frac{2A\tau}{\sin 2\theta} = \frac{(2)(18)(90)}{\sin 100°} = 3290$

(a) $P = 3290$ lb. ◄

(b) $\sigma = \frac{P \cos^2\theta}{A_o} = \frac{3290 \cos^2 50°}{18} = 75.5$ $\sigma = 75.5$ psi ◄

PROBLEM 1.32



1.32 Two wooden members of 3 × 6-in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that $P = 2400$ lb, determine the normal and shearing stresses in the glued splice.

SOLUTION

$\theta = 90° - 40° = 50°$ $P = 2400$ lb.

$A_o = (3)(6) = 18 \text{ in}^2$

$\sigma = \frac{P}{A_o} \cos^2\theta = \frac{(2400)\cos^2 50°}{18} = 55.1$

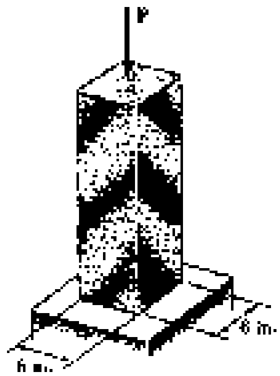
$\sigma = 55.1$ psi ◄

$\tau = \frac{P}{2A} \sin 2\theta = \frac{(2400)\sin 100°}{(2)(18)} = 65.7$

$\tau = 65.7$ psi ◄

PROBLEM 1.33

1.33 A centric load **P** is applied to the granite block shown. Knowing that the resulting maximum value of the shearing stress in the block is 2.5 ksi, determine (*a*) the magnitude of P, (*b*) the orientation of the surface on which the maximum shearing stress occurs, (*c*) the normal stress exerted on that surface, (*d*) the maximum value of the normal stress in the block.



SOLUTION

$A_o = (6)(6) = 36 \text{ in}^2 \qquad \tau_{max} = 2.5 \text{ ksi}$

$\theta = 45°$ For plane of τ_{max}

(a) $\tau_{max} = \dfrac{|P|}{2A_o} \therefore |P| = 2A_o \tau_{max} = (2)(36)(2.5)$

$= 180$ kips

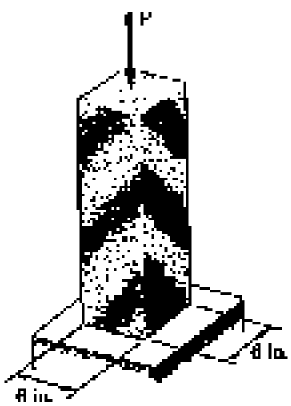
(b) $\sin 2\theta = 1 \quad 2\theta = 90° \quad \theta = 45°$

(c) $\sigma_{45} = \dfrac{P}{A_o}\cos^2 45° = \dfrac{P}{2A_o} = \dfrac{-180}{(2)(36)} = -2.5 \text{ ksi}$

(d) $\sigma_{max} = \dfrac{P}{A_o} = \dfrac{-180}{36} = -5 \text{ ksi}$

PROBLEM 1.34

1.34 A 240-kip load **P** is applied to the granite block shown. Determine the resulting maximum value of (*a*) the normal stress, (*b*) the shearing stress. Specify the orientation of the plane on which each of these maximum values occurs.



SOLUTION

$A_o = (6)(6) = 36 \text{ in}^2$

$\sigma = \dfrac{P}{A_o}\cos^2\theta = \dfrac{-240}{36}\cos^2\theta = -6.67\cos^2\theta$

(a) max. tensile stress $= 0$ at $\theta = 90°$

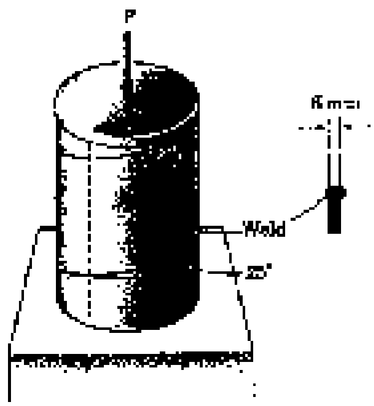
max. compressive stress $= 6.67$ ksi

at $\theta = 0°$

(b) $\tau_{max} = \dfrac{P}{2A_o} = \dfrac{240}{(2)(36)} = 3.33$ ksi

at $\theta = 45°$

PROBLEM 1.35



1.35 A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix which forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that a 250-kN axial force P is applied to the pipe, determine the normal and shearing stresses in directions respectively normal and tangential to the weld.

SOLUTION

$d_o = 0.300 \text{ m} \qquad r_o = \frac{1}{2} d_o = 0.150 \text{ m}$

$r_i = r_o - t = 0.150 - 0.006 = 0.144 \text{ m}$

$A_o = \pi(r_o^2 - r_i^2) = \pi(0.150^2 - 0.144^2)$

$= 5.54 \times 10^{-3} \text{ m}^2$

$\theta = 25°$

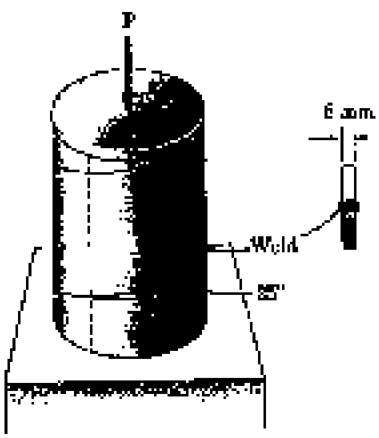
$$\sigma = \frac{P}{A_o} \cos^2\theta = \frac{-250 \times 10^3 \cos^2 25°}{5.54 \times 10^{-3}}$$

$= -37.1 \times 10^6 \qquad \sigma = -37.1 \text{ MPa}$ ◀

$$\tau = \frac{P}{2A_o} \sin 2\theta = \frac{-250 \times 10^3 \sin 50°}{(2)(5.54 \times 10^{-3})}$$

$= -17.28 \times 10^6 \qquad \tau = 17.28 \text{ MPa}$ ◀

PROBLEM 1.36



1.36 A steel pipe of 300-mm outer diameter is fabricated from 6-mm-thick plate by welding along a helix which forms an angle of 25° with a plane perpendicular to the axis of the pipe. Knowing that the maximum allowable normal and shearing stresses in directions respectively normal and tangential to the weld are $\sigma = 50$ MPa and $\tau = 30$ MPa, determine the magnitude P of the largest axial force that can be applied to the pipe.

SOLUTION

$d_o = 0.300 \text{ m} \qquad r_o = \frac{1}{2} d_o = 0.150 \text{ m}$

$r_i = r_o - t = 0.150 - 0.006 = 0.144 \text{ m}$

$A_o = \pi(r_o^2 - r_i^2) = \pi(0.150^2 - 0.144^2)$

$= 5.54 \times 10^{-3} \text{ m}^2$

$\theta = 25°$

Based on $|\sigma| = 50$ MPa: $\qquad \sigma = \frac{P}{A_o} \cos^2\theta$

$$P = \frac{A_o \sigma}{\cos^2\theta} = \frac{(5.54 \times 10^{-3})(50 \times 10^6)}{\cos^2 25°} = 337 \times 10^3$$

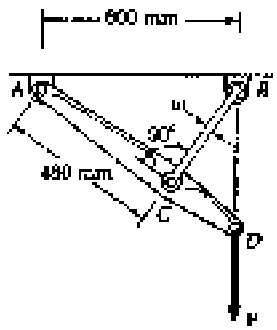
Based on $|\tau| = 30$ MPa $\qquad \tau = \frac{P}{2A_o} \sin 2\theta$

$$P = \frac{2A_o \tau}{\sin 2\theta} = \frac{(2)(5.54 \times 10^{-3})(30 \times 10^6)}{\sin 50°} = 434 \times 10^3$$

Smaller value is the allowable value of P $\therefore$ $P = 337$ kN ◀

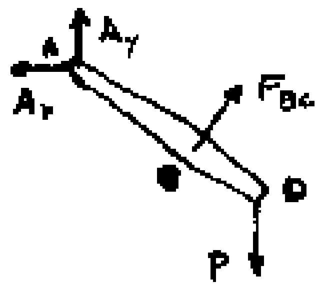
PROBLEM 1.37

1.37 Link BC is 6 mm thick, has a width $w = 25$ mm, and is made of a steel with a 480-MPa ultimate strength in tension. What was the safety factor used if the structure shown was designed to support a 16 kN load $\mathbf{P}$?



SOLUTION

Use bar ACD as a free body and note that member BD is a two-force member



$$\Sigma M_A = 0$$

$$(480)\, F_{BC} - (600) P = 0$$

$$F_{BC} = \frac{600}{480} P = \frac{(600)(16 \times 10^3)}{480} = 20 \times 10^3 \text{ N}$$

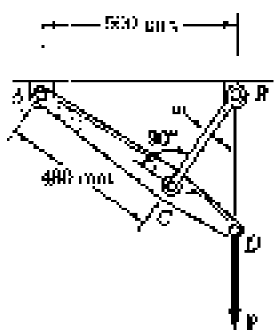
Ultimate load for member BC $\quad F_U = \sigma_U A$

$$F_U = (480 \times 10^6)(0.006)(0.025) = 72 \times 10^3 \text{ N}$$

Factor of safety $\quad F.S. = \frac{F_U}{F_{BC}} = \frac{72 \times 10^3}{20 \times 10^3} = 3.60$

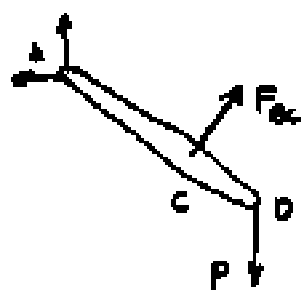
PROBLEM 1.38

1.38 Link BC is 6 mm thick and is made of a steel with a 450-MPa ultimate strength in tension. What should be its width w if the structure shown is being designed to support a 20-kN load $\mathbf{P}$ with a factor of safety of 3?



SOLUTION

Use bar ACD as a free body and note that member BC is a two-force member.



$$\Sigma M_A = 0$$

$$480\, F_{BC} - 600\, P = 0$$

$$F_{BC} = \frac{600\, P}{480} = \frac{(600)(20 \times 10^3)}{480} = 25 \times 10^3 \text{ N}$$

For a factor of safety $F.S. = 3$, the ultimate load of member BC

$$F_U = (F.S.)(F_{BC}) = (3)(25 \times 10^3) = 75 \times 10^3 \text{ N}$$

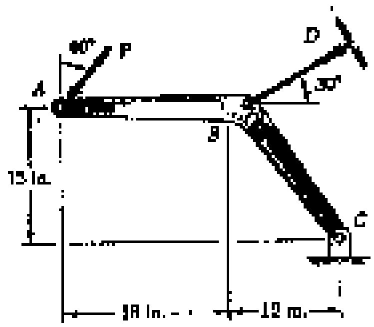
But $F_U = \sigma_U A \quad \therefore \quad A = \frac{F_U}{\sigma_U} = \frac{75 \times 10^3}{450 \times 10^6} = 166.67 \times 10^{-6} \text{ m}^2$

For a rectangular section $\quad A = wt \quad$ or $\quad w = \frac{A}{t} = \frac{166.67 \times 10^{-6}}{0.006}$

$$w = 27.8 \times 10^{-3} \text{ m} \quad \text{or} \quad 27.8 \text{ mm}$$

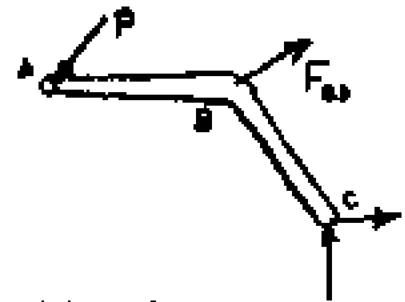
PROBLEM 1.39

1.39 Member ABC, which is supported by a pin and bracket at C and a cable BD, was designed to support the 4-kip load **P** as shown. Knowing that the ultimate load for cable BD is 25 kips, determine the factor of safety with respect to cable failure.



SOLUTION

Use member ABC as a free body and note that member BD is a two-force member.



$$\Sigma M_C = 0$$

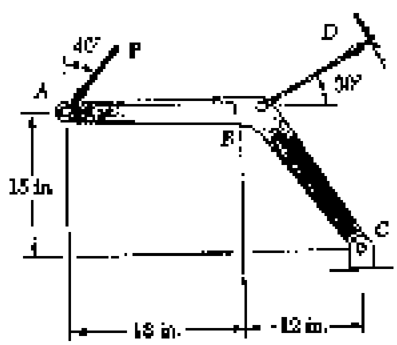
$$(P\cos 40^\circ)(30\text{ in}) + (P\sin 40^\circ)(15\text{ in.}) - (F_{BD}\cos 30^\circ)(15\text{ in})$$
$$-(F_{BD}\sin 30^\circ)(12\text{ in}) = 0$$

$$F_{BD} = \frac{32.623}{18.990}P = \frac{(32.623)(4)}{18.990} = 6.8715\text{ kips}$$

Factor of safety for cable BD $\quad F.S. = \frac{F_{ult}}{F_{BD}} = \frac{25}{6.8715} = 3.64$

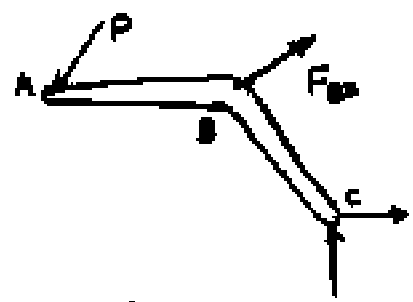
PROBLEM 1.40

1.40 Knowing that the ultimate load for cable BD is 25 kips and that a factor of safety of 3.2 with respect to cable failure is required, determine the magnitude of the largest force **P** which can be safely applied as shown to member ABC.



SOLUTION

Use member ABC as a free body and note that member BD is a two-force member.



$$\Sigma M_C = 0$$

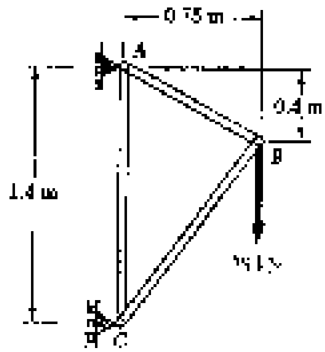
$$(P\cos 40^\circ)(30\text{ in.}) + (P\sin 40^\circ)(15\text{ in}) - (F_{BD}\cos 30^\circ)(15\text{ in})$$
$$-(F_{BD}\sin 30^\circ)(12\text{ in}) = 0$$

$$P = \frac{18.990}{32.623}F_{BD} = 0.58216\,F_{BD}$$

Allowable load for member BD is $F_{BD} = \frac{F_{ult}}{F.S.} = \frac{25}{3.2} = 7.8125\text{ kips}$

Allowable load $P = (0.58216)(7.8125) = 4.55\text{ kips}$

PROBLEM 1.41



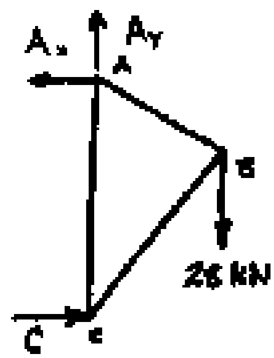
1.41 Members AB and AC of the truss shown consist of bars of square cross section made of the same alloy. It is known that a 20-mm-square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If bar AB has a 15-mm-square cross section, determine (*a*) the factor of safety for bar AB, (*b*) the dimensions of the cross section of bar AC if it is to have the same factor of safety as bar AB.

SOLUTION

Length of member AB

$$\ell_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

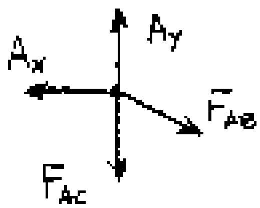
Use entire truss as a free body



$\circlearrowleft \Sigma M_C = 0 \qquad 1.4 A_x - (0.75)(28) = 0 \qquad A_x = 15 \text{ kN}$

$+\uparrow \Sigma F_y = 0 \qquad A_y - 28 = 0 \qquad A_y = 28 \text{ kN}$

Use joint A as free body



$\xrightarrow{+} \Sigma F_x = 0 \qquad \frac{0.75}{0.85} F_{AB} - A_x = 0$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$+\uparrow \Sigma F_y = 0 \qquad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar $\qquad A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \qquad P_U = 120 \times 10^3 \text{ N}$

For the material $\qquad \sigma_U = \frac{P_U}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

(a) For bar AB $\qquad \text{F.S.} = \frac{F_U}{F_{AB}} = \frac{\sigma_U A}{F_{AB}} = \frac{(300 \times 10^6)(0.015)^2}{17 \times 10^3}$

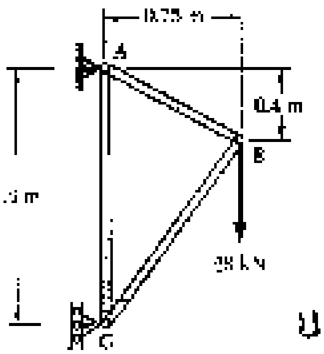
$$= 3.97$$

(b) For bar AC $\qquad \text{F.S.} = \frac{F_U}{F_{AC}} = \frac{\sigma_U A}{F_{AC}} = \frac{\sigma_U a^2}{F_{AC}}$

$$a^2 = \frac{(\text{F.S.}) F_{AC}}{\sigma_U} = \frac{(3.97)(20 \times 10^3)}{300 \times 10^6} = 264.7 \times 10^{-6} \text{ m}^2$$

$$a = 16.27 \times 10^{-3} \text{ m} \qquad 16.27 \text{ mm}$$

PROBLEM 1.42



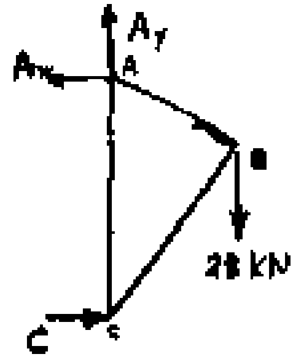
1.42 Members AB and AC of the truss shown consist of bars of square cross section made of the same alloy. It is known that a 20-mm-square bar of the same alloy was tested to failure and that an ultimate load of 120 kN was recorded. If a factor of safety of 3.2 is to be achieved for both bars, determine the required dimensions of the cross section of (a) bar AB, (b) bar AC.

SOLUTION

Length of member AB

$$l_{AB} = \sqrt{0.75^2 + 0.4^2} = 0.85 \text{ m}$$

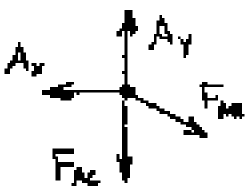
Use entire truss as a free body



$\circlearrowleft \Sigma M_C = 0 \qquad 1.4 A_x - (0.75)(28) = 0 \qquad A_x = 15 \text{ kN}$

$+\uparrow \Sigma F_y = 0 \qquad A_y - 28 = 0 \qquad A_y = 28 \text{ kN}$

Use joint A as free body



$\xrightarrow{+} \Sigma F_x = 0 \qquad \frac{0.75}{0.85} F_{AB} - A_x = 0$

$$F_{AB} = \frac{(0.85)(15)}{0.75} = 17 \text{ kN}$$

$+\uparrow \Sigma F_y = 0 \qquad A_y - F_{AC} - \frac{0.4}{0.85} F_{AB} = 0$

$$F_{AC} = 28 - \frac{(0.4)(17)}{0.85} = 20 \text{ kN}$$

For the test bar $\qquad A = (0.020)^2 = 400 \times 10^{-6} \text{ m}^2 \qquad P_U = 120 \times 10^3 \text{ N}$

For the material $\qquad \sigma_U = \frac{P_U}{A} = \frac{120 \times 10^3}{400 \times 10^{-6}} = 300 \times 10^6 \text{ Pa}$

(a) For member AB $\qquad F.S. = \frac{P_U}{F_{AB}} = \frac{\sigma_U A}{F_{AB}} = \frac{\sigma_U a^2}{F_{AB}}$

$$a^2 = \frac{(F.S.) F_{AB}}{\sigma_U} = \frac{(3.2)(17 \times 10^3)}{300 \times 10^6} = 181.33 \times 10^{-6} \text{ m}^2$$

$$a = 13.47 \times 10^{-3} \text{ m}$$

13.47 mm ◄

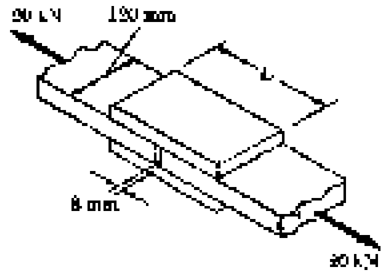
(b) For member AC $\qquad F.S. = \frac{P_U}{F_{AC}} = \frac{\sigma_U A}{F_{AC}} = \frac{\sigma_U b^2}{F_{AC}}$

$$b^2 = \frac{(F.S.) F_{AC}}{\sigma_U} = \frac{(3.2)(20 \times 10^3)}{300 \times 10^6} = 213.33 \times 10^{-6} \text{ m}^2$$

$$b = 14.61 \times 10^{-3} \text{ m}$$

14.61 mm ◄

PROBLEM 1.43



1.43 The two wooden members shown, which support a 20-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.8 MPa and the clearance between the members is 8 mm. Determine the factor of safety, knowing that the length of each splice is $L = 200$ mm.

SOLUTION

There are 4 separate areas of glue. Each glue area must transmit 10 kN of shear load.

$$P = 10 \times 10^3 \text{ N}$$

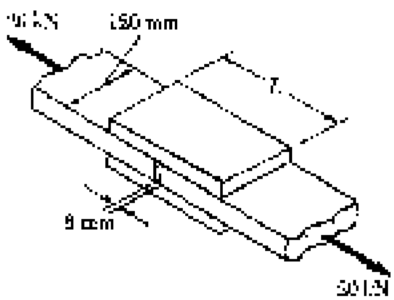
Length of splice $L = 2\ell + c$ where ℓ = length of glue and c = clearance. $\ell = \frac{1}{2}(L - c) = \frac{1}{2}(0.200 - 0.008) = 0.096$ m.

Area of glue $A = \ell w = (0.096)(0.120) = 11.52 \times 10^{-3}$ m^2

Ultimate load $P_U = \tau_U A = (2.8 \times 10^6)(11.52 \times 10^{-3}) = 32.256 \times 10^3$ N

Factor of safety $F.S. = \dfrac{P_U}{P} = \dfrac{32.256 \times 10^3}{10 \times 10^3} = 3.23$

PROBLEM 1.44



1.43 The two wooden members shown, which support a 20-kN load, are joined by plywood splices fully glued on the surfaces in contact. The ultimate shearing stress in the glue is 2.8 MPa and the clearance between the members is 8 mm.

1.44 For the joint and loading of Prob. 1.43, determine the required length L of each splice if a factor of safety of 3.5 is to be achieved.

SOLUTION

There are 4 separate areas of glue. Each glue area must transmit 10 kN of shear load.

$$P = 10 \times 10^3 \text{ N}$$

Required ultimate load $P_U = (F.S.)(P) = (3.5)(10 \times 10^3) = 35 \times 10^3$ N

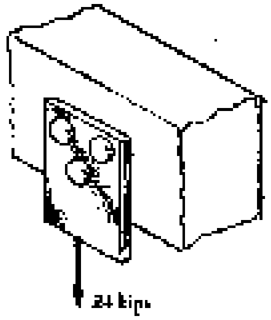
Required length ℓ of each glue area

$P_U = \tau_U A = \tau_U \ell w$ $\qquad \ell = \dfrac{P_U}{\tau_U w} = \dfrac{35 \times 10^3}{(2.8 \times 10^6)(0.120)} = 104.17 \times 10^{-3}$ m

Length of splice $L = 2\ell + c = (2)(104.17 \times 10^{-3}) + 0.008$

$= 216.3 \times 10^{-3}$ m $\qquad$ 216 mm

PROBLEM 1.45



1.45 Three $\frac{3}{4}$-in.-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 24-kip load and that the ultimate shearing stress for the steel used is 52 ksi, determine the factor of safety for this design.

SOLUTION

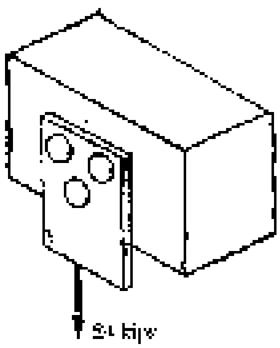
For each bolt $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$

$P_U = A\tau_U = (0.4418)(52) = 22.97 \text{ kips}$

Per bolt $P = \frac{24}{3} = 8 \text{ kips}$

$F.S. = \frac{P_U}{P} = \frac{22.97}{8} = 2.87$

PROBLEM 1.46



1.46 Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 24-kip load, that the ultimate shearing stress for the steel used is 52 ksi, and that a factor of safety of 3.37 is desired, determine the required diameter of the bolts.

SOLUTION

For each bolt $P = \frac{24}{3} = 8 \text{ kips}$

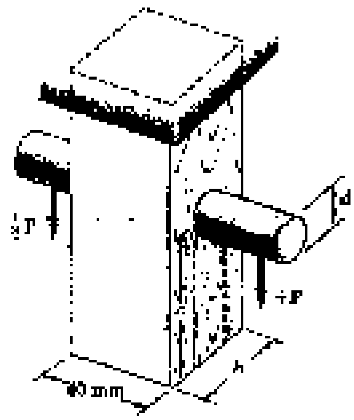
Required $P_U = (F.S.)P = (3.37)(8) = 26.96 \text{ kips}$

$\tau_U = \frac{P_U}{A} \therefore A = \frac{P_U}{\tau_U} = \frac{26.96}{52} = 0.51846 \text{ in}^2$

$A = \frac{\pi}{4}d^2 \therefore d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.51846)}{\pi}}$

$= 0.8125 \text{ in.}$

PROBLEM 1.47



1.47 A load **P** is supported as shown by a steel pin which has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 150 MPa in shear. Knowing that the diameter of the pin is d = 16 mm and that the magnitude of the load is P = 20 kN, determine (a) the factor of safety for the pin, (b) the required values of b and c if the factor of safety for the wooden member is to be the same as that found in part a for the pin.

SOLUTION

$$P = 20\text{ kN} = 20 \times 10^3\text{ N}$$

(a) Pin: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.016)^2 = 201.06 \times 10^{-6}\text{ m}^2$

Double shear $\tau = \frac{P}{2A}$ $\quad \tau_U = \frac{P_U}{2A}$

$$P_U = 2A\tau_U = (2)(201.16 \times 10^{-6})(150 \times 10^6)$$
$$= 60.319 \times 10^3\text{ N}$$

$$F.S. = \frac{P_U}{P} = \frac{60.319 \times 10^3}{20 \times 10^3} = 3.02$$

(b) Tension in wood $\quad P_U = 60.319 \times 10^3$ N for same F.S.

$$\sigma_U = \frac{P_U}{A} = \frac{P_U}{w(b-d)} \quad \text{where } w = 40\text{ mm} = 0.040\text{ m}$$

$$b = d + \frac{P_U}{w\sigma_U} = 0.016 + \frac{60.319 \times 10^3}{(0.040)(60 \times 10^6)} = 41.1 \times 10^{-3}\text{ m}$$

$$b = 41.1\text{ mm}$$

Shear in wood $\quad P_U = 60.319 \times 10^3$ N for same F.S.

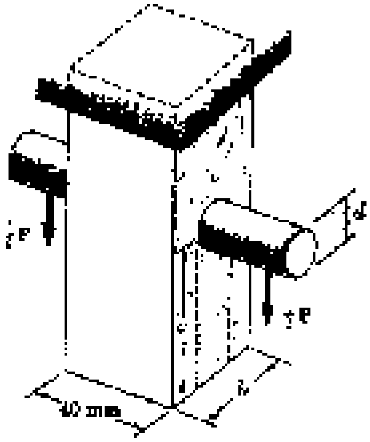
Double shear; each area is $A = wc$

$$\tau_U = \frac{P_U}{2A} = \frac{P_U}{2wc}$$

$$c = \frac{P_U}{2w\tau_U} = \frac{60.319 \times 10^3}{(2)(0.040)(7.5 \times 10^6)} = 100.5 \times 10^{-3}\text{ m}$$

$$c = 100.5\text{ mm}$$

PROBLEM 1.48



1.47 A load **P** is supported as shown by a steel pin which has been inserted in a short wooden member hanging from the ceiling. The ultimate strength of the wood used is 60 MPa in tension and 7.5 MPa in shear, while the ultimate strength of the steel is 150 MPa in shear.

1.48 For the support of Prob. 1.47, knowing that $b = 40$ mm, $c = 55$ mm and $d = 12$ mm, determine the allowable load **P** if an overall factor of safety of 3.2 is desired.

SOLUTION

Based on double shear in pin

$$P_U = 2A\tau_U = 2\frac{\pi}{4}d^2\tau_U$$

$$= \frac{\pi}{4}(2)(0.012)^2(150\times10^6) = 33.93\times10^3\ N$$

Based on tension in wood

$$P_U = A\sigma_U = w(b-d)\sigma_U$$

$$= (0.040)(0.040-0.012)(60\times10^6)$$

$$= 67.2\times10^3\ N$$

Based on double shear in the wood

$$P_U = 2A\tau_U = 2wc\tau_U = (2)(0.040)(0.055)(7.5\times10^6)$$

$$= 33.0\times10^3\ N$$

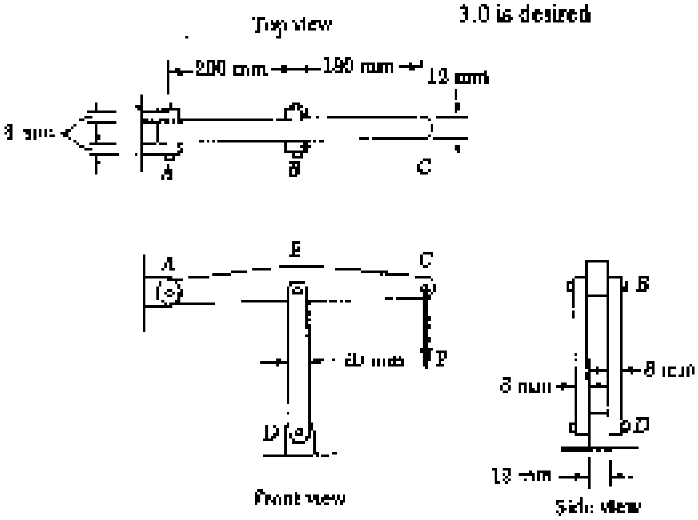
Use smallest $P_U = 33.0\times10^3\ N$

Allowable $P = \frac{P_U}{F.S.} = \frac{33.0\times10^3}{3.2} = 10.31\times10^3\ N$

10.31 kN

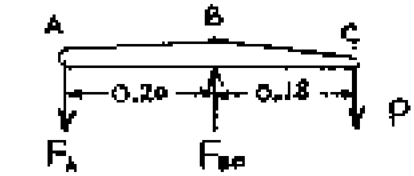
PROBLEM 1.49

1.49 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.



SOLUTION

Statics: Use ABC as free body.



$\Sigma M_B = 0 \qquad 0.20\, F_A - 0.18\, P = 0$

$P = \frac{10}{9} F_A$

$\Sigma M_A = 0 \qquad 0.20\, F_{BD} - 0.38\, P = 0$

$P = \frac{10}{19} F_{BD}$

Based on double shear in pin A

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.008)^2 = 50.266 \times 10^{-6}\ m^2$

$F_A = \frac{2\tau_u A}{F.S.} = \frac{(2)(100 \times 10^6)(50.266 \times 10^{-6})}{3.0} = 3.351 \times 10^3\ N$

$P = \frac{10}{9} F_A = 3.72 \times 10^3\ N$

Based on double shear in pins at B and D

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\ m^2$

$F_{BD} = \frac{2\tau_u A}{F.S.} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3\ N$

$P = \frac{10}{19} F_{BD} = 3.97 \times 10^3\ N$

Based on compression in links BD

For one link $\quad A = (0.020)(0.008) = 160 \times 10^{-6}\ m^2$

$F_{BD} = \frac{2\sigma_u A}{F.S.} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3\ N$

$P = \frac{10}{19} F_{BD} = 14.04 \times 10^3\ N$

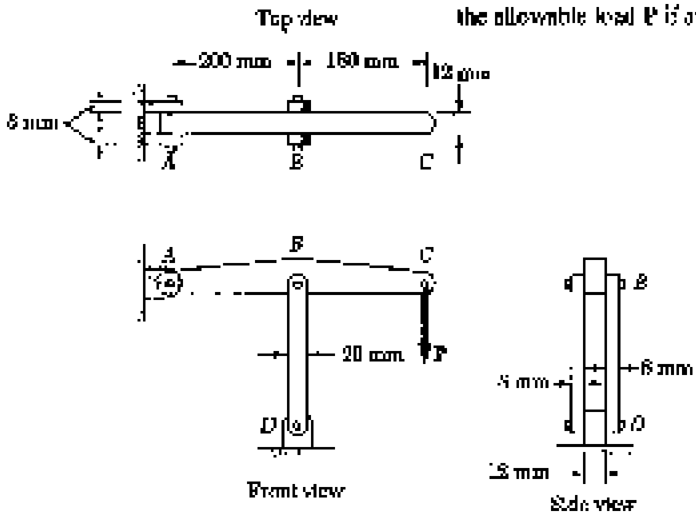
Allowable value of P is smallest $\therefore \quad P = 3.72 \times 10^3\ N$

3.72 kN

PROBLEM 1.50

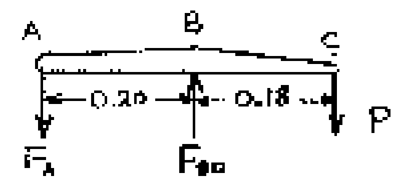
1.49 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

1.50 In an alternative design for the structure of Prob. 1.49, a pin of 10-mm-diameter is to be used at A. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.



SOLUTION

Statics: Use ABC as free body.



$\Sigma M_B = 0 \qquad 0.20\,F_A - 0.18\,P = 0$

$P = \frac{10}{9} F_A$

$\Sigma M_A = 0 \qquad 0.20\,F_{BD} - 0.38\,P = 0$

$P = \frac{10}{19} F_{BD}$

Based on double shear in pin A

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.010)^2 = 78.54 \times 10^{-6}\ m^2$

$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times10^6)(78.54\times10^{-6})}{3.0} = 5.236\times10^3\ N$

$P = \frac{10}{9} F_A = 5.82\times10^3\ N$

Based on double shear in pins at B and D

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.012)^2 = 113.10\times10^{-6}\ m^2$

$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times10^6)(113.10\times10^{-6})}{3.0} = 7.54\times10^3\ N$

$P = \frac{10}{19} F_{BD} = 3.97\times10^3\ N$

Based on compression in links BD

For one link $\quad A = (0.020)(0.008) = 160\times10^{-6}\ m^2$

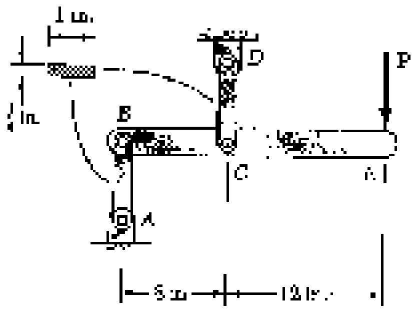
$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250\times10^6)(160\times10^{-6})}{3.0} = 26.7\times10^3\ N$

$P = \frac{10}{19} F_{BD} = 14.04\times10^3\ N$

Allowable value of P is smallest $\therefore \quad P = 3.97\times10^3\ N$

3.97 kN

PROBLEM 1.51



1.51 Each of the steel links AB and CD is connected to a support and to member BCE by $\frac{1}{2}$-in.-diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 24 ksi for the steel used in the pins and that the ultimate normal stress is 60 ksi for the steel used in the links, determine the allowable load P if an overall factor of safety of 3.2 is desired. (Note that the links are not reinforced around the pin holes.)

SOLUTION

Use BCE as free body

$\Sigma M_B = 0$

$8 F_{CD} - 20 P = 0$

$P = \frac{2}{5} F_{CD}$

$\Sigma M_C = 0 \qquad 8 F_{AB} - 12 P = 0 \qquad P = \frac{2}{3} F_{AB}$

Both links have the same area and same pin diameter; hence, being of the same material, they will have the same ultimate load.

Based on pin in single shear

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$

$F_U = \tau_U A = (24)(0.19635) = 4.7124 \text{ kips}$

Based on tension in link

$A = (b - d)t = \left(1 - \frac{1}{2}\right)\left(\frac{1}{4}\right) = 0.125 \text{ in}^2$

$F_U = S_U A = (60)(0.125) = 7.50 \text{ kips}$

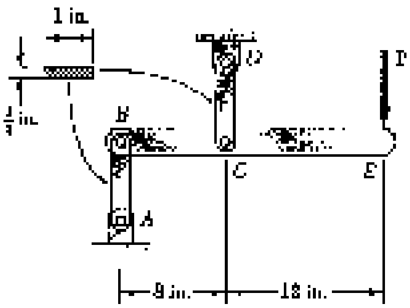
Ultimate load for link is smallest: $\quad F_U = 4.7124$ kips.

Allowable load for link $\quad F = \frac{F_U}{F.S.} = \frac{4.7124}{3.2} = 1.4726$ kips

Allowable load for structure $\quad P = \frac{2}{5} F = 0.589$ kips

$F = 589$ lb

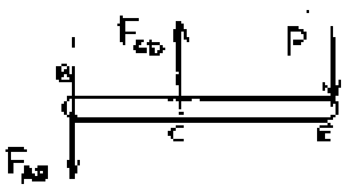
PROBLEM 1.52



1.51 Each of the steel links AB and CD is connected to a support and to member BCE by $\frac{1}{2}$-in.-diameter steel pins acting in single shear. Knowing that the ultimate shearing stress is 24 ksi for the steel used in the pins and that the ultimate normal stress is 60 ksi for the steel used in the links, determine the allowable load **P** if an overall factor of safety of 3.2 is desired. (Note that the links are not reinforced around the pin holes.)

1.52 An alternative design is being considered to support member BCE of Prob. 1.51 in which link CD will be replaced by two links, each of $\frac{1}{8} \times 1$-in. cross section, causing the pins at C and D to be in double shear. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.2 is desired.

SOLUTION



Use member BCE as free body

$\Sigma M_B = 0 \quad 8F_{CD} - 20P = 0 \qquad P = \frac{2}{5} F_{CD}$

$\Sigma M_C = 0 \quad 8F_{AB} - 12P = 0 \qquad P = \frac{2}{3} F_{AB}$

Based on pin A in single shear

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$

$F_U = \tau_U A = (24)(0.19635) = 4.7124$ kips

Based on tension in link AB

$A = (b-d)t = (1-\frac{1}{2})(\frac{1}{4}) = 0.125 \text{ in}^2$

$F_U = \sigma_U A = (60)(0.125) = 7.50$ kips

Ultimate load for link AB is smallest, i.e. $F_U = 4.7124$ kips

Corresponding ultimate load for structure: $P_U = \frac{2}{3} F_U = 3.1416$ kips

Based on pins at C and D in double shear

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.19635 \text{ in}^2$

$F_U = 2\tau_U A = (2)(24)(0.19635) = 9.4248$ kips

Based on tension in links BC

$A = (b-d)t = (1-\frac{1}{2})(\frac{1}{8}) = 0.0625 \text{ in}^2$ (one link)

$F_U = 2\sigma_U A = (2)(60)(0.0625) = 7.50$ kips (total, both links)

Ultimate load for links BC is smallest, i.e. $F_U = 7.50$ kips

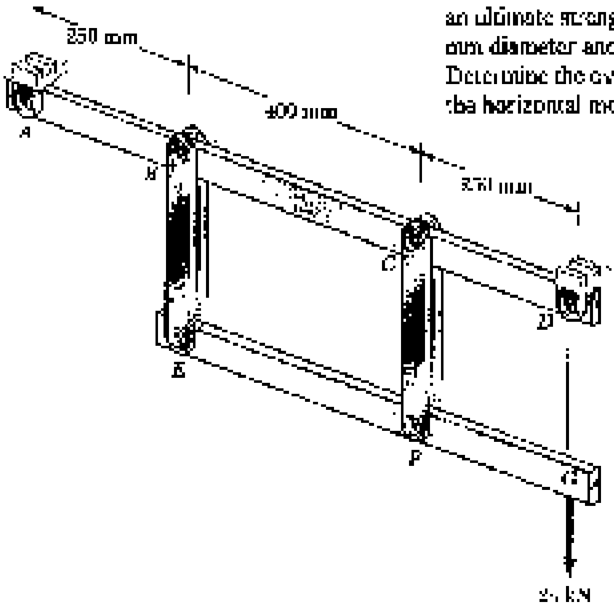
Corresponding ultimate load for structure $P_U = \frac{2}{5} F_U = 3.00$ kips.

Actual ultimate load is smallest, i.e. $P_U = 3.00$ kips

Allowable load for structure $P = \frac{P_U}{F.S.} = \frac{3.00}{3.2} = 0.938$ kip

$P = 938$ lb.

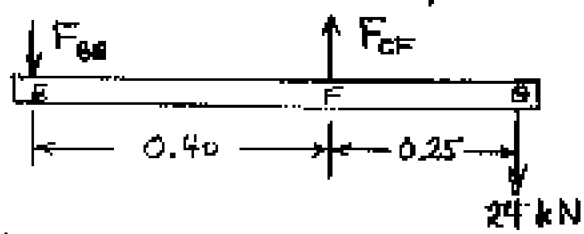
PROBLEM 1.53



1.53 Each of the two vertical links CF connecting the two horizontal members AD and EG has a 10 × 40-mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at C and F has a 20-mm diameter and is made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links CF and the pins connecting them to the horizontal members.

SOLUTION

Use member EFG as free body.



$\circlearrowright \Sigma M_E = 0$

$0.40\, F_{CF} - (0.65)(24 \times 10^3) = 0$

$F_{CF} = 39 \times 10^3$ N

Based on tension in links CF

$A = (b - d)\,t = (0.040 - 0.02)(0.010) = 200 \times 10^{-6}\ \text{m}^2$ (one link)

$F_U = 2\sigma_U A = (2)(400 \times 10^6)(200 \times 10^{-6}) = 160.0 \times 10^3$ N

Based on double shear in pins

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\ \text{m}^2$

$F_U = 2\tau_U A = (2)(150 \times 10^6)(314.16 \times 10^{-6}) = 94.248 \times 10^3$ N

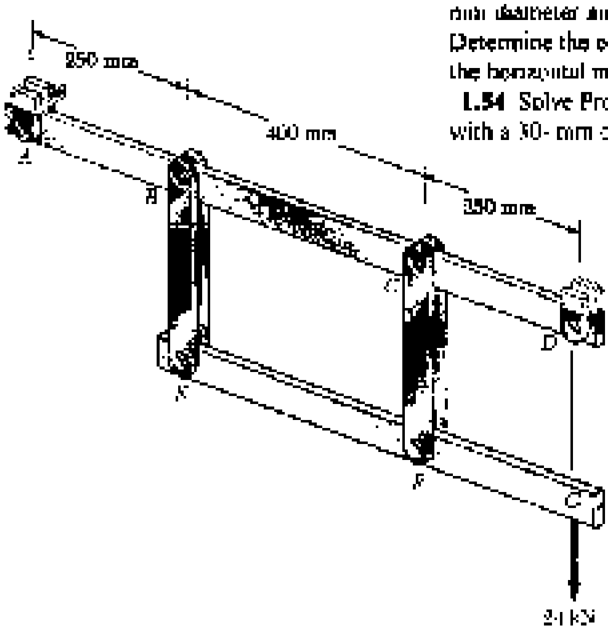
Actual F_U is smaller value, i.e. $F_U = 94.248 \times 10^3$ N

Factor of safety $\quad F.S. = \dfrac{F_U}{F_{CF}} = \dfrac{94.248 \times 10^3}{39 \times 10^3} = 2.42$

PROBLEM 1.54

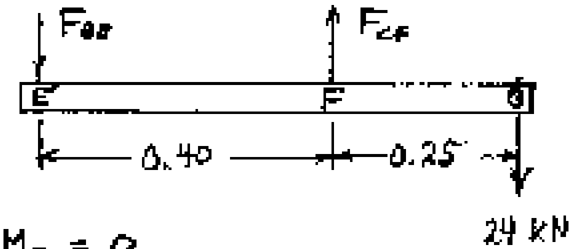
1.53 Each of the two vertical links CF connecting the two horizontal members AD and EG has a 10 × 40-mm uniform rectangular cross section and is made of a steel with an ultimate strength in tension of 400 MPa, while each of the pins at C and F has a 20-mm diameter and is made of a steel with an ultimate strength in shear of 150 MPa. Determine the overall factor of safety for the links CF and the pins connecting them to the horizontal members.

1.54 Solve Prob. 1.53, assuming that the pins at C and F have been replaced by pins with a 30-mm diameter.



SOLUTION

Use member EFG as free body.



$$+\circlearrowleft \Sigma M_E = 0$$

$$0.40\, F_{CF} - (0.65)(24 \times 10^3) = 0$$

$$F_{CF} = 39 \times 10^3 \text{ N}$$

Based on tension in links CF

$$A = (b - d)t = (0.040 - 0.030)(0.010) = 100 \times 10^{-6} \text{ m}^2 \quad \text{(one link)}$$

$$F_U = 2\sigma_U A = (2)(400 \times 10^6)(100 \times 10^{-6}) = 80.0 \times 10^3 \text{ N}$$

Based on double shear in pins

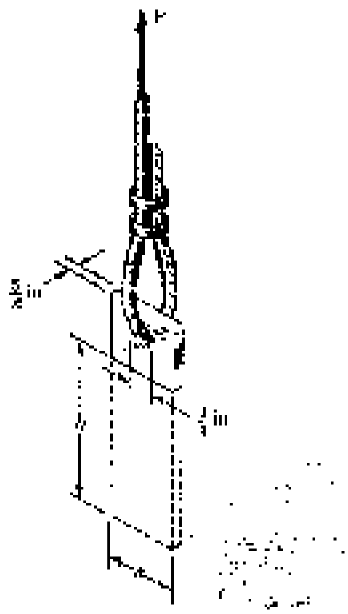
$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.030)^2 = 706.86 \times 10^{-6} \text{ m}^2$$

$$F_U = 2\tau_U A = (2)(150 \times 10^6)(706.86 \times 10^{-6}) = 212.06 \times 10^3 \text{ N}$$

Actual F_U is smaller value, i.e. $F_U = 80.0 \times 10^3$ N

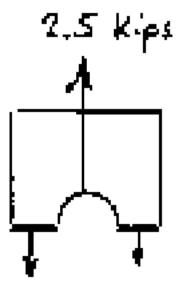
Factor of safety $\quad F.S. = \dfrac{F_U}{F_{CF}} = \dfrac{80.0 \times 10^3}{39 \times 10^3} = 2.05$

PROBLEM 1.55



1.55 A steel plate $\frac{5}{16}$-in. thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$-in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi. Knowing that a factor of safety of 3.60 is desired when $P = 2.5$ kips, determine (*a*) the required width *a* of the plate, (*b*) the minimum depth *b* to which a plate of that width should be embedded in the concrete slab. (Neglect the normal stresses between the concrete and the lower end of the plate.)

SOLUTION



Based on tension in plate

$$A = (a - d)\,t$$

$$P_U = \sigma_U A$$

$$F.S. = \frac{P_U}{P} = \frac{\sigma_U (a - d) t}{P}$$

Solving for b

$$a = d + \frac{(F.S.)P}{\sigma_U t} = \frac{3}{4} + \frac{(3.60)(2.5)}{(36)(\frac{5}{16})}$$

$$a = 1.550 \text{ in.}$$

Based on shear between plate and concrete slab

$$A = \text{perimeter} \times \text{depth} = 2(a + t)\,b \qquad \tau_U = 0.300 \text{ ksi}$$

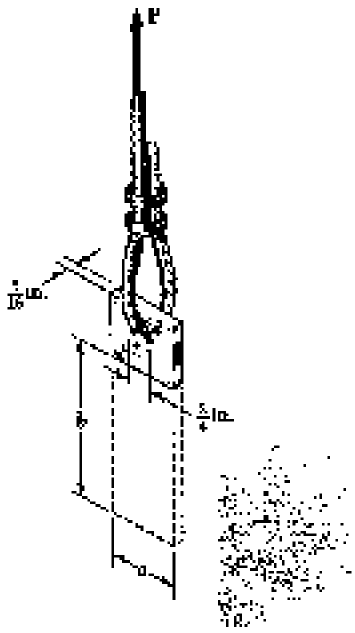
$$P_U = \tau_U A = 2\tau_U (a + t)\,b \qquad F.S. = \frac{P_U}{P}$$

Solving for b

$$b = \frac{(F.S.)P}{2(a+t)\tau_U} = \frac{(3.6)(2.5)}{(2)(1.550 + \frac{5}{16})(0.300)}$$

$$b = 8.05 \text{ in}$$

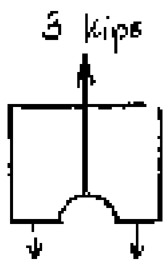
PROBLEM 1.56



1.55 A steel plate $\frac{5}{16}$-in. thick is embedded in a horizontal concrete slab and is used to anchor a high-strength vertical cable as shown. The diameter of the hole in the plate is $\frac{3}{4}$-in., the ultimate strength of the steel used is 36 ksi, and the ultimate bonding stress between plate and concrete is 300 psi.

1.56 Determine the factor of safety for the cable anchor of Prob. 1.55 when $P = 3$ kips, knowing that $a = 2$ in. and $b = 7.5$ in.

SOLUTION



Based on tension in plate

$$A = (a - d)t$$
$$= (2 - \tfrac{3}{4})(\tfrac{5}{16}) = 0.3906 \text{ in}^2$$
$$P_U = \sigma_U A$$
$$= (36)(0.3906) = 14.06 \text{ kips}$$
$$F.S. = \frac{P_U}{P} = \frac{14.06}{3} = 4.69$$

Based on shear between plate and concrete slab

$$A = \text{perimeter} \times \text{depth} = 2(a+t)b = 2(2+\tfrac{5}{16})(7.5)$$

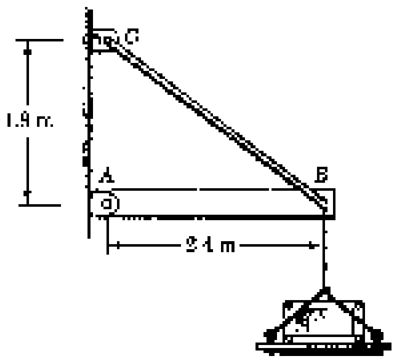
$$A = 34.69 \text{ in}^2 \qquad \tau_U = 0.300 \text{ ksi}$$

$$P_U = \tau_U A = (0.300)(34.69) = 10.41 \text{ kips}$$

$$F.S. = \frac{P_U}{P} = \frac{10.41}{3} = 3.47$$

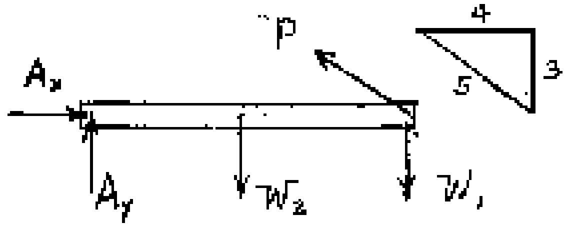
Actual factor of safety is the smaller value $\quad F.S. = 3.47$

PROBLEM 1.57



*1.57 A 40-kg platform is attached to the end B of a 50-kg wooden beam AB, which is supported as shown by a pin at A and by a slender steel rod BC with a 12-kN ultimate load. (*a*) Using the Load and Resistance Factor Design method with a resistance factor $\phi = 0.90$ and load factors $\gamma_D = 1.25$ and $\gamma_L = 1.6$, determine the largest load which can be safely placed on the platform. (*b*) What is the corresponding conventional factor of safety for rod BC?

SOLUTION



$$+\circlearrowleft \Sigma M_A = 0 \qquad (2.4)\tfrac{3}{5}P - 2.4\,W_1 - 1.2\,W_2 \qquad \therefore\ P = \tfrac{5}{3}W_1 + \tfrac{5}{6}W_2$$

For dead loading $\quad W_1 = (40)(9.81) = 392.4 \text{ N}$

$$W_2 = (50)(9.81) = 490.5 \text{ N}$$

$$P_D = (\tfrac{5}{3})(392.4) + (\tfrac{5}{6})(490.5) = 1.0628 \times 10^3 \text{ N}$$

For live loading $\quad W_1 = mg \qquad W_2 = 0$

$$P_L = \tfrac{5}{3}mg \qquad \text{from which} \qquad m = \tfrac{3}{5}\frac{P_L}{g}$$

Design criterion

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

$$P_L = \frac{\phi P_U - \gamma_D P_D}{\gamma_L} = \frac{(0.90)(12\times10^3) - (1.25)(1.0628\times10^{-3})}{1.6}$$

$$= 5.920\times10^3 \text{ N}$$

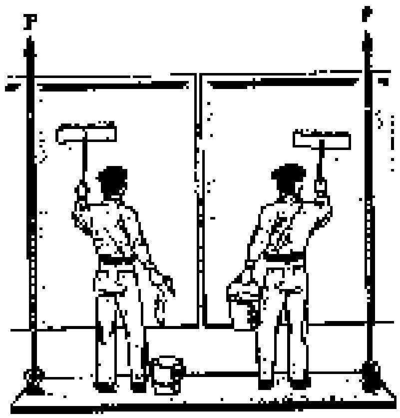
Allowable load $\quad m = \tfrac{3}{5}\,\frac{5.92\times10^3}{9.81} = 362 \text{ kg}$ ◄

Conventional factor safety

$$P = P_D + P_L = 1.0628\times10^3 + 5.920\times10^3 = 6.983\times10^3 \text{ N}$$

$$\text{F.S.} = \frac{P_U}{P} = \frac{12\times10^3}{6.983\times10^3} = 1.718$$ ◄

PROBLEM 1.58



*1.58 The Load and Resistance Factor Design method is to be used to select the two cables which will raise and lower a platform supporting two window washers. The platform weighs 160 lb and each of the window washers is assumed to weigh 195 lb with his equipment. Since these workers are free to move on the platform, 75% of their total weight and of the weight of their equipment will be used as the design live load of each cable. (*a*) Assuming a resistance factor $\phi = 0.85$ and load factors $\gamma_D = 1.2$ and $\gamma_L = 1.5$, determine the required minimum ultimate load of one cable. (*b*) What is the conventional factor of safety for the selected cables?

SOLUTION

$$\gamma_D P_D + \gamma_L P_L = \phi P_U$$

$$P_U = \frac{\gamma_D P_D + \gamma_L P_L}{\phi}$$

$$= \frac{(1.2)(\frac{1}{2} \times 160) + (1.5)(\frac{3}{4} \times 2 \times 195)}{0.85}$$

$$= 629 \text{ lb.}$$

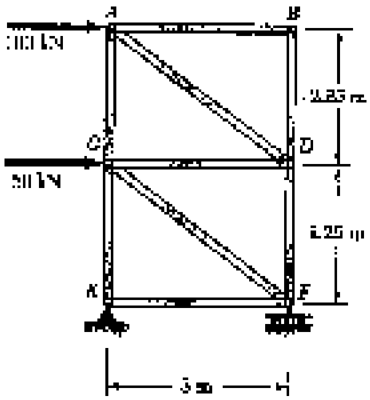
Conventional factor of safety

$$P = P_D + P_L = \frac{1}{2} \times 80 + 0.75 \times 2 \times 195 = 372.5 \text{ lb}$$

$$F.S. = \frac{P_U}{P} = \frac{629}{372.5} = 1.689$$

PROBLEM 1.59

1.59 For the truss and loading shown, determine the average normal stress in member *DF* knowing that the cross-sectional area of that member is 2500 mm².

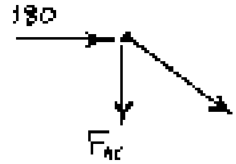


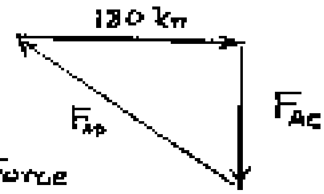
SOLUTION

Using method of joints to find member forces

Joint B: AB and BD are zero force members.

Joint A: $l_{AD} = \sqrt{3^2 + 2.25^2} = 3.75$ m





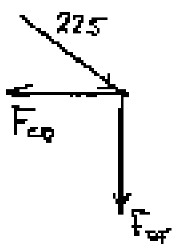
Force Triangle

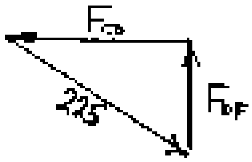
By similar triangles

$$\frac{F_{AD}}{3.75} = \frac{180}{3} \quad \therefore \quad F_{AD} = 225 \text{ lb.}$$

(compression)

Joint D





By similar triangles

$$\frac{F_{DF}}{2.25} = \frac{225}{3.75}$$

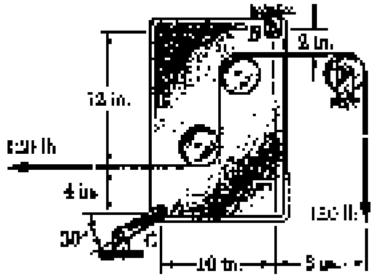
$$F_{DF} = 135 \text{ kN (comp)}$$
$$= 135 \times 10^3 \text{ N}$$

Area: $A_{DF} = 2500 \text{ mm}^2 = 2500 \times 10^{-6} \text{ m}^2$

Stress: $\sigma_{DF} = -\dfrac{135 \times 10^3}{2500 \times 10^{-6}} = -54 \times 10^6 \text{ Pa} = -54.0 \text{ MPa}$

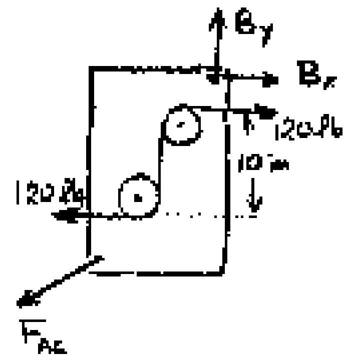
PROBLEM 1.60

1.60 Link AC has a uniform rectangular cross section $\frac{1}{8}$ in. thick and 1 in. wide. Determine the normal stress in the central portion of that link.



SOLUTION

Use the plate together with two pulleys as a free body. Note that the cable tension causes at 1200 lb-in clockwise couple to act on the body.



$$\circlearrowleft \Sigma M_B = 0$$

$$-(12+4)(F_{AC}\cos 30°) + (10)(F_{AC}\sin 30°) - 1200 = 0$$

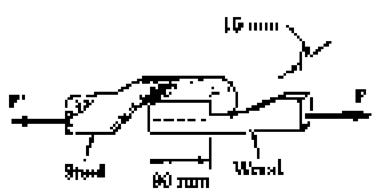
$$F_{AC} = -\frac{1200}{16\cos 30° - 10\sin 30°} = -135.50 \text{ lb.}$$

Area of link AC: $A = 1 \text{ in} \times \frac{1}{8} \text{ in} = 0.125 \text{ in}^2$

Stress in link AC: $\sigma_{AC} = \frac{F_{AC}}{A} = -\frac{135.50}{0.125} = 1084 \text{ psi} = 1.084 \text{ ksi}$ ◄

PROBLEM 1.61

1.61 When the force **P** reached 8 kN, the wooden specimen shown failed in shear along the surface indicated by the dashed line. Determine the average shearing stress along that surface at the time of failure.



Area being sheared
$A = 90 \text{ mm} \times 15 \text{ mm} = 1350 \text{ mm}^2 = 1350 \times 10^{-6} \text{ m}^2$

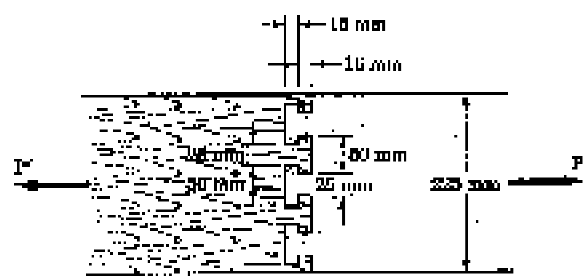
Force $P = 8 \times 10^3$ N

Shearing stress $\tau = \frac{P}{A} = \frac{8 \times 10^3}{1350 \times 10^{-6}} = 5.93 \times 10^6 \text{ Pa} = 5.93 \text{ MPa}$ ◄

PROBLEM 1.62

1.62 Two wooden planks, each 12 mm thick and 225 mm wide, are joined by the dry mortise joint shown. Knowing that the wood used shears off along its grain when the average shearing stress reaches 8 MPa, determine the magnitude P of the axial load which will cause the joint to fail.

SOLUTION



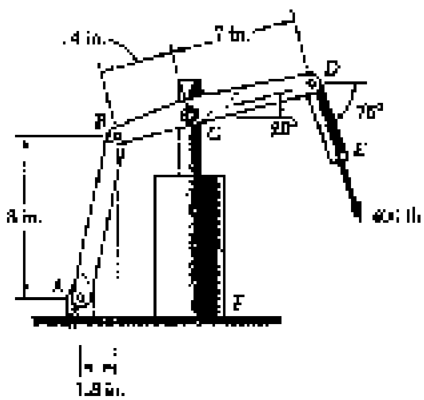
Six areas, each 16 mm × 12 mm are being sheared.

Area: $A = (6)(16)(12) = 1152 \text{ mm}^2$
$= 1152 \times 10^{-6} \text{ m}^2$

$$\tau = \frac{P}{A}$$

$$P = \tau A = (8 \times 10^6)(1152 \times 10^{-6}) = 9.22 \times 10^3 \text{ N} = 9.22 \text{ kN}$$ ◄

PROBLEM 1.63

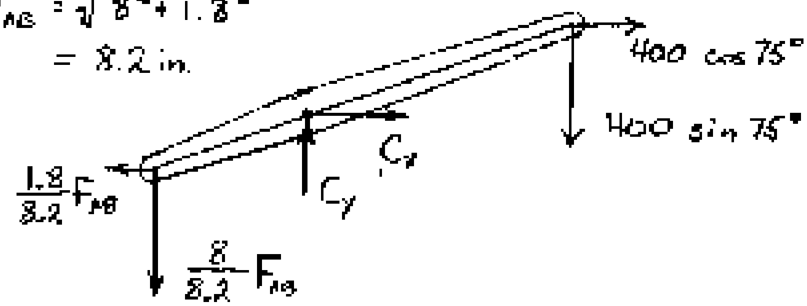


1.63 The hydraulic cylinder CF, which partially controls the position of rod DE, has been locked in the position shown. Member BD is $\frac{5}{8}$ in. thick and is connected to the vertical rod by a $\frac{3}{8}$-in.-diameter bolt. Determine (*a*) the average shearing stress in the bolt, (*b*) the bearing stress at C in member BD.

SOLUTION

Use member BCD as a free body, and note that AB is a two force member.

$$\ell_{AB} = \sqrt{8^2 + 1.8^2} = 8.2 \text{ in.}$$



$$\circlearrowleft \Sigma M_C = 0 \qquad (4\cos 20^\circ)\left(\frac{8}{8.2} F_{AB}\right) - (4\sin 20^\circ)\left(\frac{1.8}{8.2} F_{AB}\right)$$
$$- (7\cos 20^\circ)(400 \sin 75^\circ) - (7\sin 20^\circ)(400 \cos 75^\circ) = 0$$

$$3.36678\, F_{AB} - 2789.35 = 0 \qquad \therefore F_{AB} = 828.49 \text{ lb.}$$

$$\xrightarrow{+} \Sigma F_x = 0 \qquad -\frac{1.8}{8.2} F_{AB} + C_x + 400 \cos 75^\circ = 0$$

$$C_x = \frac{(1.8)(828.49)}{8.2} - 400 \cos 75^\circ = 78.34 \text{ lb.}$$

$$+\!\uparrow \Sigma F_y = 0 \qquad -\frac{8}{8.2} F_{AB} + C_y - 400 \sin 75^\circ = 0$$

$$C_y = \frac{(8)(828.49)}{8.2} + 400 \sin 75^\circ = 1194.65 \text{ lb.}$$

$$C = \sqrt{C_x^2 + C_y^2} = 1197.2 \text{ lb.}$$

(a) Shearing stress in the bolt: $\qquad P = 1197.2$ lb.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 0.11045 \text{ in}^2$$

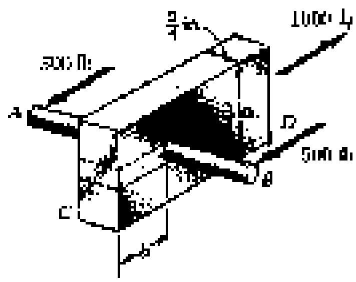
$$\tau = \frac{P}{A} = \frac{1197.2}{0.11045} = 10.84 \times 10^3 \text{ psi} = 10.84 \text{ ksi} \quad ◄$$

(b) Bearing stress at C in member BCD: $\qquad P = 1197.2$ lb

$$A_b = d\,t = \left(\frac{3}{8}\right)\left(\frac{5}{8}\right) = 0.234375 \text{ in}^2$$

$$\sigma_b = \frac{P}{A_b} = \frac{1197.2}{0.234375} = 5.11 \times 10^3 \text{ psi} = 5.11 \text{ ksi} \quad ◄$$

PROBLEM 1.64



1.64 A $\frac{1}{2}$-in.-diameter steel rod AB is fitted to a round hole near end C of the wooden member CD. For the loading shown, determine (*a*) the maximum average normal stress in the wood, (*b*) the distance b for which the average shearing stress is 90 psi on the surfaces indicated by the dashed lines, (*c*) the average bearing stress on the wood.

SOLUTION

(a) Maximum normal stress in the wood

$$A_{net} = \frac{3}{4}\left(3 - \frac{1}{2}\right) = 1.875 \text{ in}^2$$

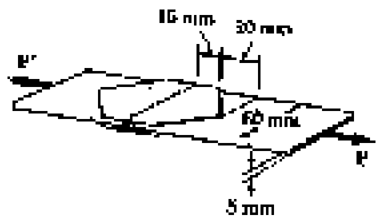
$$\sigma = \frac{P}{A_{net}} = \frac{1000}{1.875} = 533 \text{ psi}$$

(b) $$\tau = \frac{P}{A} = \frac{P}{2bt}$$

$$b = \frac{P}{2t\tau} = \frac{1000}{(2)(\frac{3}{4})(90)} = 7.41 \text{ in.}$$

(c) $$\sigma_b = \frac{P}{A_b} = \frac{P}{dt} = \frac{1000}{(\frac{1}{2})(\frac{3}{4})} = 2667 \text{ psi}$$

PROBLEM 1.65

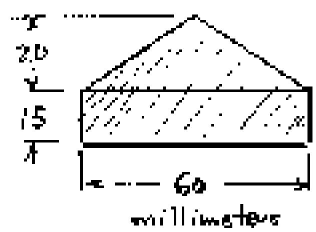


1.65 Two plates, each 3-mm thick, are used to splice a plastic strip as shown. Knowing that the ultimate shearing stress of the bonding between the surfaces is 900 kPa, determine the factor of safety with respect to shear when $P = 1500$ N.

SOLUTION

Bond area: (See figure)

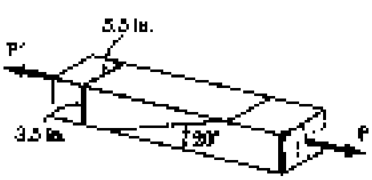
$$A = \frac{1}{2}(60)(30) + (15)(60)$$
$$= 1500 \text{ mm}^2 = 1500 \times 10^{-6} \text{ m}^2$$



$$P_U = 2A\tau_U = (2)(1500 \times 10^{-6})(900 \times 10^3) = 2700 \text{ N}$$

$$F.S. = \frac{P_U}{P} = \frac{2700}{1500} = 1.800$$

PROBLEM 1.66



1.66 Two wooden members of 3.5 × 5.5-in. uniform rectangular cross section are joined by the simple glued scarf splice shown. Knowing that the maximum allowable shearing stress in the glued splice is 75 psi, determine the largest axial load **P** which can be safely applied.

SOLUTION

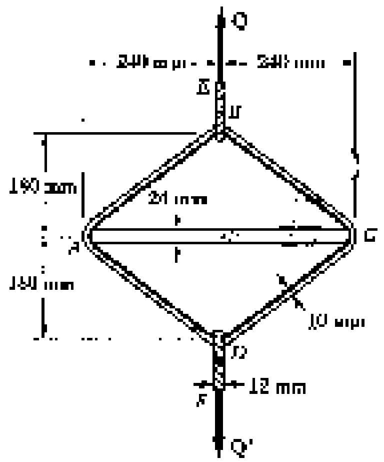
$$A_o = (3.5)(5.5) = 19.25 \text{ in}^2$$

$$\theta = 90° - 20° = 70°$$

$$\tau = \frac{P}{A_o}\sin\theta\cos\theta = \frac{P}{2A_o}\sin 2\theta$$

$$P = \frac{2A\tau}{\sin 2\theta} = \frac{(2)(19.25)(75)}{\sin 140°} = 4492 \text{ lb} = 4.49 \text{ kips}$$

PROBLEM 1.67



1.67 A steel loop $ABCD$ of length 1.2 m and of 10-mm diameter is placed as shown around a 24-mm-diameter aluminum rod AC. Cables BE and DF, each of 12-mm diameter, are used to apply the load Q. Knowing that the ultimate strength of the aluminum used for the rod is 260 MPa and that the ultimate strength of the steel used for the loop and the cables is 480 MPa, determine the largest load Q which can be applied if an overall factor of safety of 3 is desired.

SOLUTION

Using joint B as a free body and considering symmetry

$$2 \cdot \frac{3}{5} F_{AB} - Q = 0$$

$$Q = \frac{6}{5} F_{AB}$$

Using joint A as a free body and considering symmetry

$$2 \cdot \frac{4}{5} F_{AB} - F_{AC} = 0$$

$$\frac{8}{5} \cdot \frac{5}{6} Q - F_{AC} = 0 \quad \therefore \quad Q = \frac{3}{4} F_{AC}$$

Based on strength of cable BE

$$Q_U = \sigma_U A = \sigma_U \frac{\pi}{4} d^2 = (480 \times 10^6) \frac{\pi}{4} (0.012)^2 = 54.29 \times 10^3 \text{ N}$$

Based on strength of steel loop

$$Q_U = \frac{6}{5} F_{AB,U} = \frac{6}{5} \sigma_U A = \frac{6}{5} \sigma_U \frac{\pi}{4} d^2$$

$$= \frac{6}{5} (480 \times 10^6) \frac{\pi}{4} (0.010)^2 = 45.24 \times 10^3 \text{ N}$$

Based on strength of rod AC

$$Q_U = \frac{3}{4} F_{AC,U} = \frac{3}{4} \sigma_U A = \frac{3}{4} \sigma_U \frac{\pi}{4} d^2$$

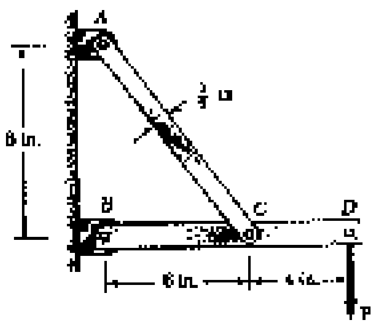
$$= \frac{3}{4} (260 \times 10^6) \frac{\pi}{4} (0.024)^2 = 88.22 \times 10^3 \text{ N}$$

Actual ultimate load Q_U is the smallest $\therefore \quad Q_U = 45.24 \times 10^3$ N

Allowable load $\quad Q = \frac{Q_U}{F.S.} = \frac{45.24 \times 10^3}{3} = 15.08 \times 10^3$ N

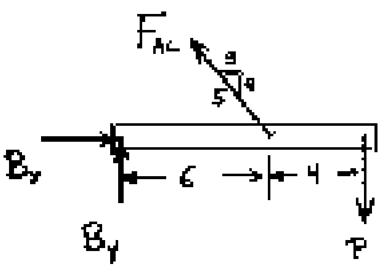
$= 15.08$ kN

PROBLEM 1.68



1.68 Link AC has a uniform $\frac{1}{4} \times \frac{1}{2}$-in. uniform rectangular cross section and is made of a steel with a 60-ksi ultimate normal stress. It is connected to a support at A and to member BCD at C by $\frac{3}{8}$-in.-diameter pins, while member BCD is connected to a support at B by a $\frac{5}{16}$-in.-diameter pin. All of the pins are in single shear and are made of a steel with a 25-ksi ultimate shearing stress. Knowing that an overall factor of safety of 3.25 is desired, determine the largest load $\mathbf{P}$ which can be safely applied at D. Note that link AC is not reinforced around the pin holes.

SOLUTION



$\circlearrowleft \Sigma M_B = 0 \qquad (6)(\frac{4}{5}F_{AC}) - 10P = 0$

$F_{AC} = 2.0833\,P \qquad P = 0.480\,F_{AC}$

$\xrightarrow{+} \Sigma F_x = 0 \qquad B_x - \frac{3}{5}F_{AC} = 0$

$B_x = \frac{3}{5}F_{AC} = (\frac{3}{5})(2.0833\,P) = 1.25\,P$

$+\uparrow \Sigma F_y = 0 \qquad B_y + \frac{4}{5}F_{AC} - P = 0$

$B_y = P - \frac{4}{5}(2.0833\,P) = -0.66667\,P$

$B = \sqrt{B_x^2 + B_y^2} = 1.41667\,P, \qquad P = 0.70588\,B$

Based on strength of link AC: $\quad \sigma_U = 60$ ksi

$A_{net} = (\frac{1}{4})(\frac{1}{2} - \frac{3}{8}) = 0.03125\text{ in}^2 \qquad F_{AC,U} = \sigma_U A_{net} = (60)(0.03125) = 1.875\text{ kip}$

$P_U = (0.480)(1.875) = 0.900$ kip.

Based on strength of pin at C: $\quad A_{pin} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(\frac{3}{8})^2 = 0.11045\text{ in}^2$

$\tau_U = 25$ ksi $\qquad F_{AC,U} = \tau_U A_{pin} = (25)(0.11045) = 2.761$ kip

$P_U = (0.480)(2.761) = 1.325$ kip.

Based on strength of pin at B: $\quad A_{pin} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(\frac{5}{16})^2 = 0.07670\text{ in}^2$

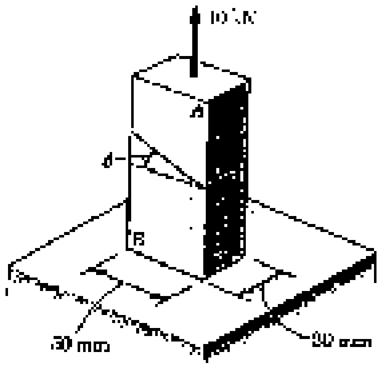
$B_U = \tau_U A_{pin} = (25)(0.07670) = 1.9175$ kip.

$P_U = (0.70588)(1.9175) = 1.3535$ kip

Actual P_U is the smallest $\therefore \qquad P_U = 0.900$ kip

Allowable value for P: $\quad P = \frac{P_U}{F.S.} = \frac{0.900}{3.25} = 0.277\text{ kip} = 277\text{ lb.}$

PROBLEM 1.69



1.69 The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine the range of values of θ for which the factor of safety of the member is at least 3.0.

SOLUTION

$$A_0 = (0.030)(0.050) = 1.50\times10^{-3}\ m^2$$

$$P = 10\times10^3\ N \qquad P_u = (F.S)P = 30\times10^3\ N$$

Based on tensile stress

$$\sigma_u = \frac{P_u}{A_0}\cos^2\theta$$

$$\cos^2\theta = \frac{\sigma_u A_0}{P_u} = \frac{(17\times10^6)(1.50\times10^{-3})}{30\times10^3} = 0.85$$

$$\cos\theta = 0.92195 \qquad \theta = 22.79° \qquad \theta \geq 22.79°$$

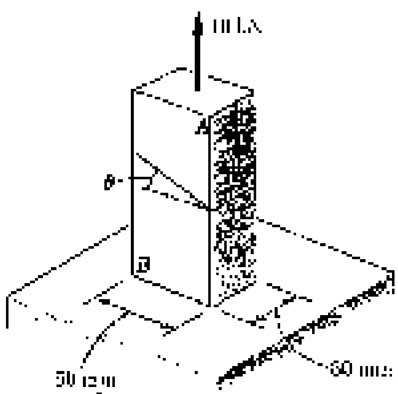
Based on shearing stress $\quad \tau_u = \frac{P_u}{A_0}\sin\theta\cos\theta = \frac{P_u}{2A_0}\sin 2\theta$

$$\sin 2\theta = \frac{2A_0\tau_u}{P_u} = \frac{(2)(1.50\times10^{-3})(9\times10^6)}{30\times10^3} = 0.900$$

$$2\theta = 64.16° \qquad \theta = 32.08° \qquad \theta \leq 32.08°$$

Hence $\quad 22.79° \leq \theta \leq 32.08°$

PROBLEM 1.70



1.70 The two portions of member AB are glued together along a plane forming an angle θ with the horizontal. Knowing that the ultimate stress for the glued joint is 17 MPa in tension and 9 MPa in shear, determine (a) the value of θ for which the factor of safety of the member is maximum, (b) the corresponding value of the factor of safety. (*Hint:* Equate the expressions obtained for the factors of safety with respect to normal stress and shear.)

SOLUTION

$$A_0 = (0.030)(0.050) = 1.50\times10^{-3}\ m^2$$

At the optimum angle $\quad (F.S.)_\sigma = (F.S.)_\tau$

Normal stress: $\sigma = \frac{P}{A_0}\cos^2\theta \quad \therefore \quad P_{u,\sigma} = \frac{\sigma_u A_0}{\cos^2\theta}$

$$(F.S.)_\sigma = \frac{P_{u,\sigma}}{P} = \frac{\sigma_u A_0}{P\cos^2\theta}$$

Shearing stress: $\tau = \frac{P}{A_0}\sin\theta\cos\theta \quad \therefore \quad P_{u,\tau} = \frac{\tau_u A_0}{\sin\theta\cos\theta}$

$$(F.S.)_\tau = \frac{P_{u,\tau}}{P} = \frac{\tau_u A_0}{P\sin\theta\cos\theta}$$

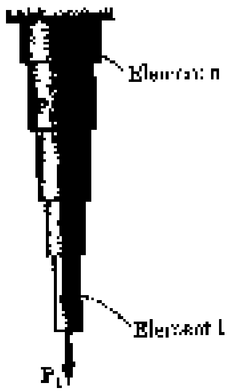
Equating: $\quad \frac{\sigma_u A_0}{P\cos^2\theta} = \frac{\tau_u A_0}{P\sin\theta\cos\theta}$

Solving: $\quad \frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{\tau_u}{\sigma_u} = \frac{9}{17} = 0.52941; \qquad \theta_{opt} = 27.9°$

Then $\quad P_u = \frac{\sigma_u A_0}{\cos^2\theta} = \frac{(17\times10^6)(1.50\times10^{-3})}{\cos^2 27.9°} = 32.65\times10^3;$

$$F.S. = \frac{P_u}{P} = \frac{32.65\times10^3}{10\times10^3} = 3.26$$

PROBLEM 1.C1



1.C1 A solid steel rod consisting of n cylindrical elements welded together is subjected to the loading shown. The diameter of element i is denoted by d_i and the load applied to its lower end by $\mathbf{P}_i$, with the magnitude P_i of this load being assumed positive if $\mathbf{P}_i$ is directed downward as shown and negative otherwise. (*a*) Write a computer program which can be used with either SI or U.S. customary units to determine the average stress in each element of the rod. (*b*) Use this program to solve Probs. 1.1 and 1.3.

SOLUTION

FORCE IN ELEMENT i:

It is the sum of the forces applied to that element and all lower ones:

$$F_i = \sum_{k=1}^{i} P_k$$

AVERAGE STRESS IN ELEMENT i:

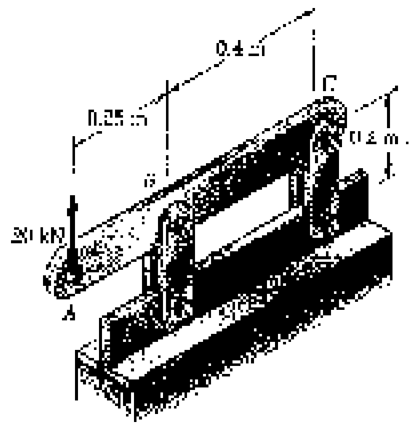
Area $= A_i = \frac{1}{4}\pi d_i^2$ Ave stress $= \dfrac{F_i}{A_i}$

PROGRAM OUTPUTS

```
     Problem    1.1
 Element   Stress (MPa)

    1        84.883
    2       -96.766
```

```
     Problem    1.3
 Element   Stress (ksi)

    1        22.635
    2        17.927
```

PROBLEM 1.C2

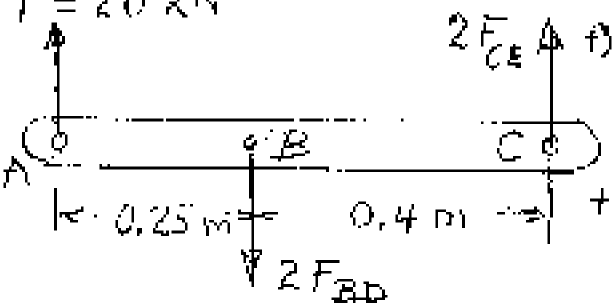


1.C2 A 20-kN force is applied as shown to the horizontal member *ABC*. Member *ABC* has a 10 × 50-mm uniform rectangular cross section and is supported by four vertical links, each of 8 × 36-mm uniform rectangular cross section. Each of the four pins at *A*, *B*, *C*, and *D* has the same diameter *d* and is in double shear. (*a*) Write a computer program to calculate for values of *d* from 10 to 30 mm, using 1-mm increments, (1) the maximum value of the average normal stress in the links connecting pins *B* and *D*, (2) the average normal stress in the links connecting pins *C* and *E*, (3) the average shearing stress in pin *B*, (4) the average shearing stress in pin *C*, (5) the average bearing stress at *B* in member *ABC*, (6) the average bearing stress at *C* in member *ABC*. (*b*) Check your program by comparing the values obtained for *d* = 16 mm with the answers given for Probs. 1.8, 1.23, and 1.24. (*c*) Use this program to find the permissible values of the diameter *d* of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 150 MPa, 90 MPa, and 230 MPa. (*d*) Solve part *c*, assuming that the thickness of member *ABC* has been reduced from 10 to 8 mm.

SOLUTION

FORCES IN LINKS

F. B. DIAGRAM OF ABC:



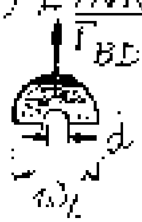
$+\circlearrowleft \Sigma M_C = 0: \; 2F_{BD}(BC) - P(AC) = 0$

$F_{BD} = P(AC)/2(BC)$ (TENSION)

$+\circlearrowleft \Sigma M_B = 0: \; 2F_{CE}(BC) - P(AB) = 0$

$F_{CE} = P(AB)/2(BC)$ (COMP.)

(1) LINK BD

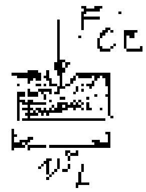


Thickness $= t_L$

$A_{BD} = t_L(w_L - d)$

$\sigma_{BD} = +F_{BD}/A_{BD}$

(2) LINK CE



Thickness $= t_L$

$A_{CE} = t_L w_L$

$\sigma_{CE} = -F_{CE}/A_{CE}$

(3) PIN B

$\tau_B = F_{BD}/(\pi d^2/4)$

(4) PIN C

$\tau_C = F_{CE}/(\pi d^2/4)$

(5) BEARING STRESS AT B

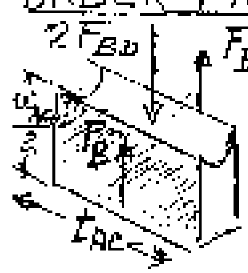
Thickness of member $AC = t_{AC}$

Sig Bear B $= F_{BD}/(d\,t_{AC})$

(6) BEARING STRESS AT C

Sig Bear C $= F_{CE}/(d\,t_{AC})$

SHEARING STRESS IN ABC UNDER PIN B



$F_B = \tau_{AC}\, t_{AC}(w_{AC}/2)$

$\Sigma F_y = 0: \; 2F_B = 2F_{BD}$

$\tau_{AC} = \dfrac{2F_{BD}}{t_{AC}\, w_{AC}}$

(CONTINUED)

PROGRAM OUTPUTS

INPUT DATA FOR PARTS (a), (b), (c): P = 20 kN, AB = 0.25 m, BC = 0.40 m, AC = 0.65 m, TL = 8 mm, WL = 36 mm, TAC = 10 mm, WAC = 50 mm

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.12	-21.70	~~206.90~~	79.58	~~325.00~~	125.00
11.00	81.25	-21.70	~~170.99~~	65.77	~~295.45~~	113.64
12.00	84.64	-21.70	~~143.68~~	55.26	~~270.83~~	104.17
13.00	88.32	-21.70	~~122.43~~	47.09	~~250.00~~	96.15
14.00	92.33	-21.70	~~105.56~~	40.60	~~232.14~~	89.29
15.00	96.73	-21.70	~~91.96~~	35.37	216.67	83.33
16.00	101.56	-21.70	80.82	31.08	203.12	78.13 (b)
17.00	106.91	-21.70	71.59	27.54	191.18	73.53
18.00	112.85	-21.70	63.86	24.56	180.56	69.44
19.00	119.49	-21.70	57.31	22.04	171.05	65.79
20.00	126.95	-21.70	51.73	19.89	162.50	62.50
21.00	135.42	-21.70	46.92	18.04	154.76	59.52
22.00	145.09	-21.70	42.75	16.44	147.73	56.82
23.00	~~156.25~~	-21.70	39.11	15.04	141.30	54.35
24.00	~~169.27~~	-21.70	35.92	13.82	135.42	52.08
25.00	~~184.66~~	-21.70	33.10	12.73	130.00	50.00
26.00	~~203.13~~	-21.70	30.61	11.77	125.00	48.08
27.00	~~225.69~~	-21.70	28.38	10.92	120.37	46.30
28.00	~~253.91~~	-21.70	26.39	10.15	116.07	44.64
29.00	~~290.18~~	-21.70	24.60	9.46	112.07	43.10
30.00	~~338.54~~	-21.70	22.99	8.84	108.33	41.67

(c) ANSWER : 16 mm ≤ d ≤ 22 mm (c)

CHECK For d = 22 mm, Tau AC = 65 MPa < 90 MPa O.K.

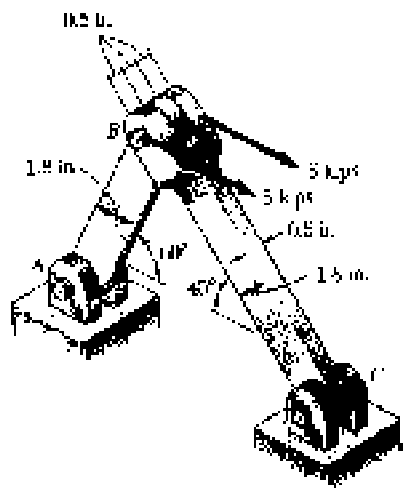
INPUT DATA FOR PART (d): P = 20 kN, AB = 0.25 m, BC = 0.40 m, AC = 0.65 m, TL = 8 mm, WL = 36 mm, TAC = 8 mm, WAC = 50 mm

d	Sigma BD	Sigma CE	Tau B	Tau C	SigBear B	SigBear C
10.00	78.12	-21.70	~~206.90~~	79.58	~~406.25~~	156.25
11.00	81.25	-21.70	~~170.99~~	65.77	~~369.32~~	142.05
12.00	84.64	-21.70	~~143.68~~	55.26	~~338.54~~	130.21
13.00	88.32	-21.70	~~122.43~~	47.09	~~312.50~~	120.19
14.00	92.33	-21.70	~~105.56~~	40.60	~~290.18~~	111.61
15.00	96.73	-21.70	~~91.96~~	35.37	~~270.83~~	104.17
16.00	101.56	-21.70	80.82	31.08	~~253.91~~	97.66
17.00	106.91	-21.70	71.59	27.54	~~238.97~~	91.91
18.00	112.85	-21.70	63.86	24.56	225.69	86.81
19.00	119.49	-21.70	57.31	22.04	213.82	82.24
20.00	126.95	-21.70	51.73	19.89	203.12	78.13
21.00	135.42	-21.70	46.92	18.04	193.45	74.40
22.00	145.09	-21.70	42.75	16.44	184.66	71.02
23.00	~~156.25~~	-21.70	39.11	15.04	176.63	67.93
24.00	~~169.27~~	-21.70	35.92	13.82	169.27	65.10
25.00	~~184.66~~	-21.70	33.10	12.73	162.50	62.50
26.00	~~203.13~~	-21.70	30.61	11.77	156.25	60.10
27.00	~~225.69~~	-21.70	28.38	10.92	150.46	57.87
28.00	~~253.91~~	-21.70	26.39	10.15	145.09	55.80
29.00	~~290.18~~	-21.70	24.60	9.46	140.09	53.88
30.00	~~338.54~~	-21.70	22.99	8.84	135.42	52.08

(d) ANSWER : 18 mm ≤ d ≤ 22 mm (d)

CHECK: For d = 22 mm, Tau AC = 81.25 MPa < 90 MPa O.K.

PROBLEM 1.C3

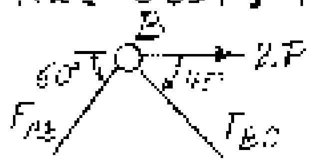


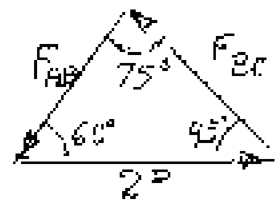
1.C3 Two horizontal 5-kip forces are applied to pin B of the assembly shown. Each of the three pins at A, B, and C has the same diameter d and is in double shear. (*a*) Write a computer program to calculate for values of d from 0.50 to 1.50 in., using 0.05-in. increments, (1) the maximum value of the average normal stress in member AB, (2) the average normal stress in member BC, (3) the average shearing stress in pin A, (4) the average shearing stress in pin C, (5) the average bearing stress at A in member AB, (6) the average bearing stress at C in member BC, (7) the average bearing stress at B in member BC. (*b*) Check your program by comparing the values obtained for $d = 0.8$ in. with the answers given for Probs. 1.9, 1.25, and 1.26. (*c*) Use this program to find the permissible values of the diameter d of the pins, knowing that the allowable values of the normal, shearing, and bearing stresses for the steel used are, respectively, 22 ksi, 13 ksi, and 36 ksi. (*d*) Solve part *c*, assuming that a new design is being investigated, in which the thickness and width of the two members are changed, respectively, from 0.5 to 0.3 in. and from 1.8 in. to 2.4 in.

SOLUTION

FORCES IN MEMBERS AB AND BC

FREE BODY: PIN B





FROM FORCE TRIANGLE:

$$\frac{F_{AB}}{\sin 45°} = \frac{F_{BC}}{\sin 60°} = \frac{2P}{\sin 75°}$$

$$F_{AB} = 2P(\sin 45°/\sin 75°)$$

$$F_{BC} = 2P(\sin 60°/\sin 75°)$$

(1) MAX. AVE. STRESS IN AB

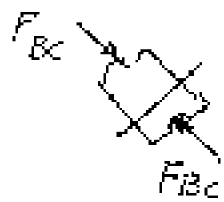
Width $= w$

Thickness $= t$

$$A_{AB} = (w - d)\,t$$

$$\sigma_{AB} = F_{AB}/A_{AB}$$

(2) AVE. STRESS IN BC



$$A_{BC} = wt$$

$$\sigma_{BC} = F_{BC}/A_{BC}$$

(3) PIN A

$$\tau_A = (F_{AB}/2)/(\pi d^2/4)$$

(4) PIN C

$$\tau_C = (F_{BC}/2)/(\pi d^2/4)$$

(5) BEARING STRESS AT A

Sig Bear A $= F_{AB}/dt$

(6) BEARING STRESS AT C

Sig Bear C $= F_{BC}/dt$

(7) BEARING STRESS AT B IN MEMBER BC

Sig Bear B $= F_{BC}/2dt$

(CONTINUED)

PROBLEM 1.C3 CONTINUED **PROGRAM OUTPUTS**

INPUT DATA FOR PARTS (a), (b), (c): P = 5 kips, w = 1.8 in., t = 0.5 in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	11.262	-9.962	18.642	22.911	29.282	35.863	17.932
0.550	11.717	-9.962	15.408	18.869	26.620	32.603	16.301
0.600	12.201	-9.962	12.945	16.055	24.402	29.885	14.943
0.650	12.731	-9.962	11.030	13.510	22.525	27.587	13.793
0.700	13.310	-9.962	9.511	11.649	20.916	25.616	12.808
0.750	13.944	-9.962	8.285	10.147	19.521	23.909	11.954
0.800	14.641	-9.962	7.282	8.918	18.301	22.414	11.207 (b)
0.850	15.412	-9.962	6.450	7.900	17.225	21.096	10.548
0.900	16.268	-9.962	5.754	7.047	16.268	19.924	9.962
0.950	17.225	-9.962	5.164	6.324	15.412	18.875	9.438
1.000	18.301	-9.962	4.660	5.708	14.641	17.932	8.966
1.050	19.521	9.962	4.227	5.177	13.944	17.078	8.539
1.100	20.916	-9.962	3.852	4.717	13.310	16.307	8.151
1.150	22.825	9.962	3.524	4.316	12.731	15.593	7.796
1.200	24.402	9.962	3.236	3.964	12.201	14.943	7.471
1.250	26.620	-9.962	2.983	3.653	11.713	14.345	7.173
1.300	29.282	-9.962	2.758	3.377	11.262	13.751	6.897
1.350	32.836	-9.962	2.557	3.132	10.845	13.283	6.541
1.400	36.603	-9.962	2.378	2.912	10.458	12.808	6.404
1.450	41.931	9.962	2.217	2.715	10.097	12.367	6.183
1.500	48.822	9.962	2.071	2.537	9.761	11.954	5.977

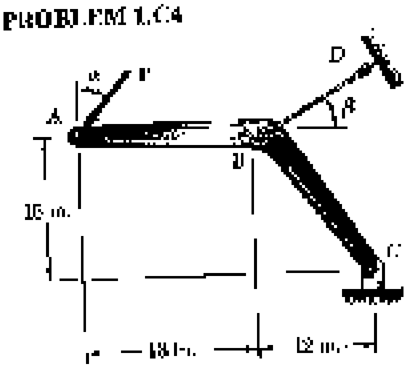
(c) ANSWER: 0.70 in. ≤ d ≤ 1.10 in. (c)

INPUT DATA FOR PART (d): P = 5 kips, w = 2.4 in., t = 0.3 in.

D in.	SIGAB ksi	SIGBC ksi	TAUA ksi	TAUC ksi	SIGBRGA ksi	SIGBRGC ksi	SIGBRGB ksi
0.500	12.842	-12.452	29.643	27.835	48.803	59.772	29.886
0.550	13.190	-12.452	25.405	18.869	44.367	54.338	27.169
0.600	13.556	-12.452	12.945	15.885	40.669	49.810	24.905
0.650	13.944	12.452	11.030	13.510	37.541	45.978	22.989
0.700	14.354	12.452	9.511	11.649	34.860	42.694	21.347
0.750	14.789	-12.452	8.285	10.147	32.536	39.848	19.924
0.800	15.251	-12.452	7.282	8.918	30.502	37.357	18.679
0.850	15.743	-12.452	6.450	7.900	28.708	35.160	17.580
0.900	16.268	-12.452	5.754	7.047	27.113	33.206	16.603
0.950	16.629	-12.452	5.164	6.324	25.686	31.459	15.729
1.000	17.430	12.452	4.660	5.708	24.402	29.886	14.943
1.050	18.075	12.452	4.227	5.177	23.240	28.463	14.231
1.100	18.771	-12.452	3.852	4.717	22.183	27.163	13.584
1.150	19.521	-12.452	3.524	4.316	21.219	25.988	12.994
1.200	20.335	-12.452	3.236	3.964	20.335	24.905	12.452
1.250	21.219	-12.452	2.983	3.653	19.521	23.909	11.954
1.300	22.154	12.452	2.758	3.377	18.771	22.989	11.495
1.350	23.240	12.452	2.557	3.132	18.075	22.138	11.069
1.400	24.402	12.452	2.378	2.912	17.430	21.247	10.674
1.450	25.686	-12.452	2.217	2.715	16.829	20.611	10.303
1.500	27.113	-12.452	2.071	2.537	16.268	19.934	9.967

(d) ANSWER: 0.95 in. ≤ d ≤ 1.25 in. (d)

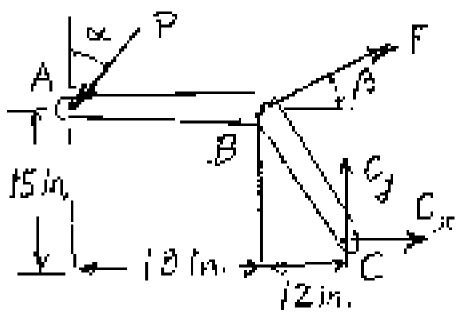
PROBLEM 1.C4



1.C4 A 4-kip force **P** forming an angle α with the vertical is applied as shown to member *ABC*, which is supported by a pin and bracket at *C* and by a cable *BD* forming an angle β with the horizontal. (*a*) Knowing that the ultimate load of the cable is 25 kips, write a computer program to construct a table of the values of the factor of safety of the cable for values of α and β from 0 to 45°, using increments in α and β corresponding to 0.1 increments in tan α and tan β. (*b*) Check that for any given value of α the maximum value of the factor of safety is obtained for β = 38.66° and explain why. (*c*) Determine the smallest possible value of the factor of safety for β = 38.66°, as well as the corresponding value of α, and explain the result obtained.

SOLUTION

(a) DRAW F.B. DIAGRAM OF ABC:



$$+\circlearrowleft \Sigma M_C = 0: (P\sin\alpha)(15\text{ in.}) + (P\cos\alpha)(30\text{ in.}) - (F\cos\beta)(15\text{ in.}) - (F\sin\beta)(12\text{ in.}) = 0$$

$$F = P\,\frac{15\sin\alpha + 30\cos\alpha}{15\cos\beta + 12\sin\beta}$$

$$F.S. = F_{ult}/F$$

OUTPUT FOR P = 11 kips AND F_{ult} = 20 kips

VALUES OF FS

BETA

ALPHA	0	5.71	11.31	16.70	21.80	26.57	30.96	34.99	38.66	41.99	45.00
0.000	3.130	3.354	3.555	3.719	3.830	3.918	3.966	3.994	4.002	3.985	3.977
5.711	2.991	3.214	3.402	3.552	3.666	3.745	3.796	3.823	3.833	3.824	3.807
11.310	2.887	3.103	3.290	3.441	3.557	3.626	3.677	3.703	3.710	3.704	3.687
16.699	2.827	3.049	3.227	3.370	3.477	3.553	3.600	3.626	3.633	3.627	3.611
21.801	2.805	3.014	3.190	3.331	3.439	3.512	3.560	3.585	3.592	3.586	3.570
26.565	2.795	3.004	3.179	3.320	3.425	3.500	3.547	3.572	3.579	3.573	3.556
30.964	2.803	3.013	3.189	3.330	3.436	3.510	3.558	3.583	3.590	3.584	3.568
34.992	2.826	3.036	3.214	3.355	3.463	3.538	3.586	3.611	3.619	3.612	3.596
38.660	2.859	3.072	3.252	3.395	3.503	3.579	3.628	3.653	3.661	3.655	3.639
41.987	2.899	3.116	3.298	3.444	3.554	3.631	3.680	3.706	3.713	3.707	3.690
45.000	2.946	3.165	3.351	3.499	3.611	3.689	3.739	3.765	3.773	3.767	3.750

↑(b)

(b) When β = 38.66°, tan β = 0.8 and cable BD is perpendicular to the lever arm BC.

(c) F.S. = 3.579 for α = 26.6°; P is perpendicular to the lever arm AC.

NOTE:

The value F.S. = 3.579 is the smallest of the values of F.S. corresponding to β = 38.66° and the largest of those corresponding to α = 26.6°. The point α = 26.6°, β = 38.66° is a "saddle point", or "minimax" of the function F.S.(α, β).

PROBLEM 1.C5

1.C5 A load **P** is supported as shown by two wooden members of uniform rectangular cross section which are joined by a simple glued scarf splice. (*a*) Denoting by σ_U and τ_U respectively, the ultimate strength of the joint in tension and in shear, write a computer program which, for given values of a, b, P, σ_U and τ_U, expressed in either SI or U.S. customary units, and for values of α from 5 to 85° at 5° intervals, can be used to calculate (1) the normal stress in the joint, (2) the shearing stress in the joint, (3) the factor of safety relative to failure in tension, (4) the factor of safety relative to failure in shear, (5) the overall factor of safety for the glued joint. (*b*) Apply this program, using the dimensions and loading of the members of Probs. 1.29 and 1.32, knowing that $\sigma_U = 1.26$ MPa and $\tau_U = 1.50$ MPa for the glue used in Prob. 1.29, and that $\sigma_U = 150$ psi and $\tau_U = 214$ psi for the glue used in Prob. 1.32. (*c*) Verify in each of these two cases that the shearing stress is maximum for $\alpha = 45°$.

SOLUTION

(1) and (2) Draw the F.B. diagram of lower member:

$+\searrow \Sigma F_x = 0: \quad -V + P\cos\alpha = 0 \qquad V = P\cos\alpha$

$+\nearrow \Sigma F_y = 0: \quad F - P\sin\alpha = 0 \qquad F = P\sin\alpha$

Area $= ab/\sin\alpha$

Normal stress:

$$\sigma = \frac{F}{\text{Area}} = (P/ab)\sin^2\alpha$$

Shearing stress: $\tau = \dfrac{V}{\text{Area}} = (P/ab)\sin\alpha\cos\alpha$

(3) F.S. for tension (normal stresses)

$$FSN = \sigma_U/\sigma$$

(4) F.S. for shear:

$$FSS = \tau_U/\tau$$

(5) OVERALL F.S.:

FS = The <u>smaller</u> of FSN and FSS.

(CONTINUED)

PROBLEM 1.C5 CONTINUED

PROGRAM OUTPUTS

For Prob. 1.29: $P = 6$ kN

$a = 125$ mm, $b = 75$ mm, $\alpha = 70°$, $\sigma_U = 1.26$ MPa, $\tau_U = 1.50$ MPa.

ALPHA	SIG(MPa)	TAU(MPa)	FSN	FSS	FS	
5.0000	0.0049	0.0556	259.1782	26.9942	26.9942	
10.0000	0.0193	0.1094	65.2905	13.7053	13.7053	
15.0000	0.0429	0.1600	29.3893	9.3750	9.3750	
20.0000	0.0749	0.2057	16.8301	7.2925	7.2925	
25.0000	0.1143	0.2451	11.0229	6.1191	6.1191	
30.0000	0.1600	0.2771	7.8750	5.4127	5.4127	
35.0000	0.2106	0.3007	5.9842	4.9883	4.9883	
40.0000	0.2644	0.3151	4.7649	4.7598	4.7598	
45.0000	0.3200	0.3200	3.9375	4.6875	3.9375	◄ (c)
50.0000	0.3756	0.3151	3.3549	4.7598	3.3549	
55.0000	0.4294	0.3007	2.9340	4.9883	2.9340	
60.0000	0.4800	0.2771	2.6250	5.4127	2.6250	
65.0000	0.5257	0.2451	2.3968	6.1191	2.3968	
70.0000	0.5651	0.2057	2.2296	7.2925	2.2296	◄ (b)
75.0000	0.5971	0.1600	2.1101	9.3750	2.1101	
80.0000	0.6207	0.1094	2.0300	13.7053	2.0300	
85.0000	0.6351	0.0556	1.9838	26.9942	1.9838	

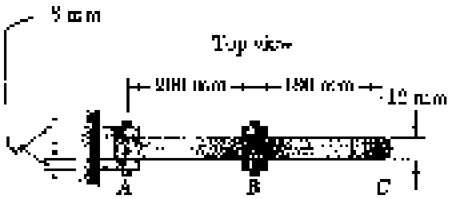
For Prob. 1.22: $P = 2400$ lb

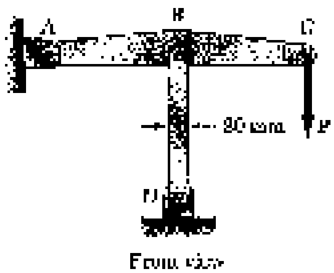
$a = 6$ in., $b = 3$ in., $\alpha = 40°$, $\sigma_U = 150$ psi, $\tau_U = 214$ psi.

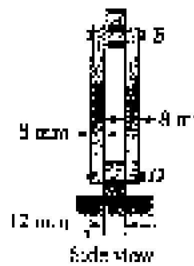
ALPHA	SIG(psi)	TAU(psi)	FSN	FSS	FS	
5.0000	1.0128	11.5765	148.1016	18.4857	18.4857	
10.0000	4.0205	22.8013	37.3089	9.3854	9.3854	
15.0000	8.9316	33.3333	16.7942	6.4200	6.4200	
20.0000	15.5970	42.8525	9.6172	4.9939	4.9939	
25.0000	23.8142	51.0696	6.2988	4.1904	4.1904	
30.0000	33.3333	57.7350	4.5000	3.7066	3.7066	
35.0000	43.8653	62.6462	3.4196	3.4160	3.4160	
40.0000	55.0901	65.6538	2.7228	3.2595	2.7228	◄ (b)
45.0000	66.6667	66.6667	2.2500	3.2100	2.2500	◄ (c)
50.0000	78.2432	65.6538	1.9171	3.2595	1.9171	
55.0000	89.4680	62.6462	1.6766	3.4160	1.6766	
60.0000	100.0000	57.7350	1.5000	3.7066	1.5000	
65.0000	109.5192	51.0696	1.3696	4.1904	1.3696	
70.0000	117.7363	42.8525	1.2740	4.9939	1.2740	
75.0000	124.4017	33.3333	1.2058	6.4200	1.2058	
80.0000	129.3128	22.8013	1.1600	9.3854	1.1600	
85.0000	132.3205	11.5765	1.1336	18.4857	1.1336	

PROBLEM 1.C6

1.C6 Member ABC is supported by a pin and bracket at A and by two links which are pin-connected to the member at B and to a fixed support at D. (*a*) Write a computer program to calculate the allowable load P_{all} for any given values of (1) the diameter d_1 of the pin at A, (2) the common diameter d_2 of the pins at B and D, (3) the ultimate normal stress σ_U in each of the two links, (4) the ultimate shearing stress τ_U in each of the three pins, (5) the desired overall factor of safety $F.S.$ Your program should also indicate which of the following three stresses is critical: the normal stress in the links, the shearing stress in the pin at A, or the shearing stress in the pins at B and D. (*b* and *c*) Check your program by using the data of Probs. 1.49 and 1.50, respectively, and comparing the answers obtained for P_{all} with those given in the text. (*d*) Use your program to determine the allowable load P_{all}, as well as which of the stresses is critical, when $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa for aluminum links, $\tau_U = 100$ MPa for steel pins, and $F.S. = 3.2$.

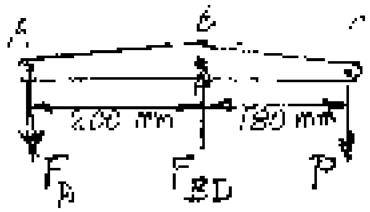






SOLUTION

(a) F.B. DIAGRAM OF ABC:



$\sum M_A = 0$: $P = \frac{200}{380} F_{BD}$

$\sum M_B = 0$: $P = \frac{200}{180} F_A$

(1) For given d_1 of pin A: $F_A = 2(\tau_U/FS)(\pi d_1^2/4)$, $P_1 = \frac{200}{180} F_A$

(2) For given d_2 of pins B and D: $F_{BD} = 2(\tau_U/FS)(\pi d_2^2/4)$, $P_2 = \frac{200}{380} F_{BD}$

(3) For ultimate stress in links BD: $F_{BD} = 2(\sigma_U/FS)(0.02)(0.008)$, $P_3 = \frac{200}{380} F_{BD}$

(4) For ult. shearing stress in pins: P_4 is the smaller of P_1 and P_2

(5) For desired overall F.S.: P_5 is the smaller of P_3 and P_4

If $P_3 < P_4$, stress is critical in links

If $P_4 < P_3$ and $P_1 < P_2$, stress is critical in pin A

If $P_4 < P_3$ and $P_2 < P_1$, stress is critical in pins B and D

PROGRAM OUTPUTS

(b) Prob. 1.49. DATA: $d_1 = 8$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.0$

$P_{all} = 3.72$ kN. Stress in pin A is critical ◄

(c) Prob. 1.50. DATA: $d_1 = 10$ mm, $d_2 = 12$ mm, $\sigma_U = 250$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.0$

$P_{all} = 5.97$ kN. Stress in pins B and D is critical ◄

(d) DATA: $d_1 = d_2 = 15$ mm, $\sigma_U = 110$ MPa, $\tau_U = 100$ MPa, $F.S. = 3.2$

$P_{all} = 5.79$ kN. Stress in links is critical ◄

CHAPTER 2

PROBLEM 2.1

2.1 A steel rod is 2.2 m long and must not stretch more than 1.2 mm when a 8.5 kN load is applied to it. Knowing that $E = 200$ GPa, determine (*a*) the smallest diameter rod which should be used, (*b*) the corresponding normal stress caused by the load.

SOLUTION

(a) $\delta = \dfrac{PL}{AE} \quad \therefore \quad A = \dfrac{PL}{E\delta} = \dfrac{(8.5\times10^3)(2.2)}{(200\times10^9)(1.2\times10^{-3})} = 77.92\times10^{-6}\ \text{m}^2$

$A = \dfrac{\pi}{4}d^2 \quad \therefore \quad d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(77.92\times10^{-6})}{\pi}} = 9.96\times10^{-3}\ \text{m}$

$= 9.96\ \text{mm}$ ◄

(b) $\sigma = \dfrac{P}{A} = \dfrac{8.5\times10^3}{77.92\times10^{-6}} = 109.1\times10^6\ \text{Pa} = 109.1\ \text{MPa}$ ◄

PROBLEM 2.2

2.2 A 4.8-ft-long steel wire of $\frac{1}{4}$-in. diameter steel wire is subjected to a 750-lb tensile load. Knowing that $E = 29\times10^6$ psi, determine (*a*) the elongation of the wire, (*b*) the corresponding normal stress.

SOLUTION

(a) $L = 4.8\ \text{ft} = 57.6\ \text{in.} \qquad A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}\left(\dfrac{1}{4}\right)^2 = 49.087\times10^{-3}\ \text{in}^2$

$\delta = \dfrac{PL}{AE} = \dfrac{(750)(57.6)}{(49.087\times10^{-3})(29\times10^6)} = 30.3\times10^{-3}\ \text{in} = 0.0303\ \text{in}$ ◄

(b) $\sigma = \dfrac{P}{A} = \dfrac{750}{49.087\times10^{-3}} = 15.28\times10^3\ \text{psi} = 15.28\ \text{ksi}$ ◄

PROBLEM 2.3

2.3 Two gage marks are placed exactly 10 inches apart on a $\frac{1}{2}$-in.-diameter aluminum rod with $E = 10.1\times10^6$ psi and an ultimate strength of 16 ksi. Knowing that the distance between the gage marks is 10.009 in. after a load is applied, determine (*a*) the stress in the rod, (*b*) the factor of safety.

SOLUTION

(a) $\delta = 10.009 - 10.000 = 0.009\ \text{in.}$

$\dfrac{\delta}{L} = \dfrac{\sigma}{E} \quad \therefore \quad \sigma = \dfrac{E\delta}{L} = \dfrac{(10.1\times10^6)(0.009)}{10} = 9.09\times10^3\ \text{psi}$

$= 9.09\ \text{ksi}$ ◄

(b) $F.S. = \dfrac{\sigma_U}{\sigma} = \dfrac{16}{9.09} = 1.760$ ◄

PROBLEM 2.4

2.4 A control rod made of yellow brass must not stretch more than 3 mm when the tension in the wire is 4 kN. Knowing that $E = 105$ GPa and that the maximum allowable normal stress is 180 MPa, determine (*a*) the smallest diameter that can be selected for the rod, (*b*) the corresponding maximum length of the rod.

SOLUTION

(a) $\sigma = \dfrac{P}{A} \quad \therefore \quad A = \dfrac{P}{\sigma} = \dfrac{4\times10^3}{180\times10^6} = 22.222\times10^{-6}\ \text{m}^2$

$A = \dfrac{\pi}{4}d^2 \quad \therefore \quad d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(22.222\times10^{-6})}{\pi}} = 5.32\times10^{-3}\ \text{m}$

$= 5.32\ \text{mm}$ ◄

(b) $\delta = \dfrac{PL}{AE} \quad \therefore \quad L = \dfrac{AE\delta}{P} = \dfrac{(22.222\times10^{-6})(105\times10^9)(3\times10^{-3})}{4\times10^3}$

$= 1.750\ \text{m}$ ◄

PROBLEM 2.5

2.5 A 9-m length of 6-mm-diameter steel wire is to be used in a hanger. It is noted that the wire stretches 18 mm when a tensile force **P** is applied. Knowing that $E = 200$ GPa, determine (*a*) the magnitude of the force **P**, (*b*) the corresponding normal stress in the wire.

SOLUTION

(a) $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.006)^2 = 28.274 \times 10^{-6}\ m^2$

$\delta = \frac{PL}{AE} \therefore P = \frac{AE\delta}{L} = \frac{(28.274\times10^{-6})(200\times10^9)(18\times10^{-3})}{9}$

$= 11.31 \times 10^3$ N $= 11.31$ kN

(b) $\sigma = \frac{P}{A} = \frac{11.31\times10^3}{28.274\times10^{-6}} = 400\times10^6$ Pa $= 400$ MPa

PROBLEM 2.6

2.6 A 4.5-ft aluminum pipe should not stretch more than 0.05 in. when it is subjected to a tensile load. Knowing that $E = 10.1 \times 10^6$ psi and that the allowable tensile strength is 14 ksi, determine (*a*) the maximum allowable length of the pipe, (*b*) the required area of the pipe if the tensile load is 127.5 kips.

SOLUTION

(a) $\delta = \frac{PL}{AE} \therefore L = \frac{EA\delta}{P} = \frac{E\delta}{\sigma} = \frac{(10.1\times10^6)(0.05)}{14\times10^3} = 36.1$ in

(b) $\sigma = \frac{P}{A} \therefore A = \frac{P}{\sigma} = \frac{127.5\times10^3}{14\times10^3} = 9.11$ in^2

PROBLEM 2.7

2.7 A nylon thread is subjected to a 8.5-N tension force. Knowing that $E = 3.3$ GPa and that the length of the thread increases by 1.1 %, determine (*a*) the diameter of the thread, (*b*) the stress in the thread.

SOLUTION

(a) $\frac{\delta}{L} = \frac{1.1}{100} \therefore \frac{L}{\delta} = 90.909$

$\delta = \frac{PL}{AE} \therefore A = \frac{PL}{E\delta} = \frac{(8.5)(90.909)}{3.3\times10^9} = 234.16\times10^{-9}\ m^2$

$A = \frac{\pi}{4}d^2 \therefore d = \sqrt{\frac{4A}{\pi}} = 0.546\times10^{-3}$ m $= 0.546$ mm

(b) $\sigma = \frac{P}{A} = \frac{8.5}{234.16\times10^{-9}} = 36.3\times10^6$ Pa $= 36.3$ MPa

PROBLEM 2.8

2.8 A cast-iron tube is used to support a compressive load. Knowing that $E = 10 \times 10^6$ psi and that the maximum allowable change in length is 0.025 percent, determine (*a*) the maximum normal stress in the tube, (*b*) the minimum wall thickness for a load of 1600 lb if the outside diameter of the tube is 2.0 in.

SOLUTION

(a) $\frac{\delta}{L} = \frac{0.025}{100} = 0.00025$

$\sigma = \frac{E\delta}{L} = (10\times10^6)(0.00025) = 2.5\times10^3$ psi $= 2.5$ ksi

(b) $\sigma = \frac{P}{A} \therefore A = \frac{P}{\sigma} = \frac{1600}{2.5\times10^3} = 0.640$ in^2

$A = \frac{\pi}{4}(d_o^2 - d_i^2)$

$d_i^2 = d_o^2 - \frac{4A}{\pi} = 2.0^2 - \frac{(4)(0.64)}{\pi} = 3.1851$ in^2 $\therefore d_i = 1.7847$ in

$t = \frac{1}{2}(d_o - d_i) = \frac{1}{2}(2.0 - 1.7847) = 0.1077$ in.

PROBLEM 2.9

2.9 A block of 10-in. length and 1.8×1.6 in. cross section is to support a centric compressive load P. The material to be used is a bronze for which $E = 14 \times 10^6$ psi. Determine the largest load which can be applied, knowing that the normal stress must not exceed 18 ksi and that the decrease in length of the block should be at most 0.12 percent of its original length.

SOLUTION

Considering allowable stress $\sigma = 18$ ksi $= 18 \times 10^3$ psi

$A = (1.8)(1.6) = 2.88$ in^2 $\quad \sigma = \frac{P}{A}$

$P = \sigma A = (18 \times 10^3)(2.88) = 51.8 \times 10^3$ lb

Considering allowable deformation $\frac{\delta}{L} = \frac{0.12}{100} = 0.0012$

$\delta = \frac{PL}{AE} \quad \therefore \quad P = AE\frac{\delta}{L} = (2.88)(14 \times 10^6)(0.0012) = 48.4 \times 10^3$ lb

Smaller value governs $\quad P = 48.4 \times 10^3$ lb $= 48.4$ kips

PROBLEM 2.10

2.10 A 9-kN tensile load will be applied to a 50-m length of steel wire with $E = 200$ GPa. Determine the smallest diameter wire which can be used, knowing that the normal stress must not exceed 150 MPa and that the increase in the length of the wire should be at most 25 mm.

SOLUTION

Considering allowable stress $\sigma = 150 \times 10^6$ Pa

$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{9 \times 10^3}{150 \times 10^6} = 60 \times 10^{-6}$ m^2

Considering allowable elongation $\delta = 25 \times 10^{-3}$ m

$\delta = \frac{PL}{AE} \quad \therefore \quad A = \frac{PL}{E\delta} = \frac{(9 \times 10^3)(50)}{(200 \times 10^9)(25 \times 10^{-3})} = 90 \times 10^{-6}$ m^2

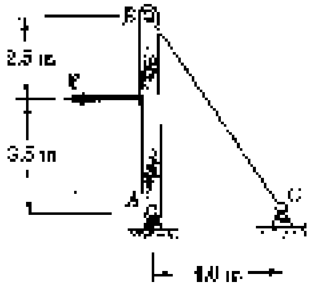
Larger area governs $\quad A = 90 \times 10^{-6}$ m^2

$A = \frac{\pi}{4}d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(90 \times 10^{-6})}{\pi}} = 10.70 \times 10^{-3}$ m

$= 10.70$ mm

PROBLEM 2.11

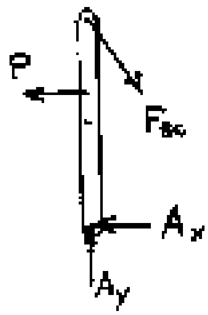
2.11 The 4-mm-diameter cable BC is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load P that can be applied as shown.



SOLUTION

$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111$ m

Use bar AB as a free body



$\circlearrowleft \Sigma M_A = 0 \qquad 3.5P - (6)\left(\frac{4}{7.2111} F_{BC}\right) = 0$

$P = 0.9509\ F_{BC}$

Considering allowable stress $\sigma = 190 \times 10^6$ Pa

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6}$ m^2

$\sigma = \frac{F_{BC}}{A} \therefore F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3$ N

Considering allowable elongation $\delta = 6 \times 10^{-3}$ m

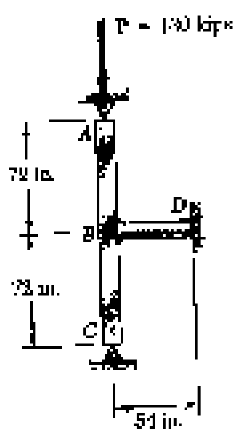
$\delta = \frac{F_{BC} L_{BC}}{AE} \therefore F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3$ N

Smaller value governs $\qquad F_{BC} = 2.091 \times 10^3$ N

$P = 0.9509\ F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3\text{ N} = 1.988$ kN

PROBLEM 2.12

2.12 Rod BD is made of steel ($E = 29 \times 10^6$ psi) and is used to brace the axially compressed member ABC. The maximum force that can be developed in member BD is $0.02P$. If the stress must not exceed 18 ksi and the maximum change in length of BD must not exceed 0.001 times the length of ABC, determine the smallest diameter rod that can be used for member BD.



SOLUTION

$F_{BD} = 0.02\ P = (0.02)(130) = 2.6$ kips $= 2.6 \times 10^3$ lb

Considering stress $\sigma = 18$ ksi $= 18 \times 10^3$ psi

$\sigma = \frac{F_{BD}}{A} \therefore A = \frac{F_{BD}}{\sigma} = \frac{2.6}{18} = 0.14444$ in^2

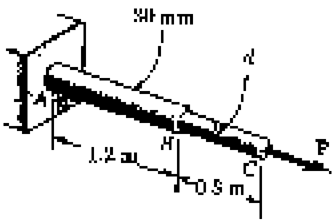
Considering deformation $\delta = (0.001)(144) = 0.144$ in.

$\delta = \frac{F_{BD} L_{BD}}{AE} \therefore A = \frac{F_{BD} L_{BD}}{E\delta} = \frac{(2.6 \times 10^3)(54)}{(29 \times 10^6)(0.144)} = 0.03362$ in^2

Larger area governs $\qquad A = 0.14444$ in^2

$A = \frac{\pi}{4} d^2 \therefore d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.14444)}{\pi}} = 0.429$ in.

PROBLEM 2.13



2.13 A single axial load of magnitude $P = 58$ kN is applied at end C of the brass rod ABC. Knowing that $E = 105$ GPa, determine the diameter d of portion BC for which the deflection of point C will be 3 mm.

SOLUTION

$$\delta_C = \sum \frac{P_i L_i}{A_i E} = \frac{P}{E}\left\{\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right\}$$

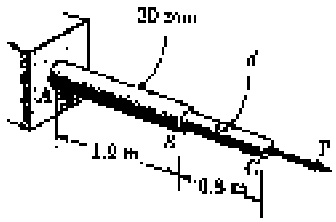
$$\frac{L_{BC}}{A_{BC}} = \frac{E\delta_C}{P} - \frac{L_{AB}}{A_{AB}} = \frac{(105\times10^9)(3\times10^{-3})}{58\times10^3} - \frac{1.2}{\frac{\pi}{4}(0.030)^2} = 3.7334\times10^3 \ m^{-1}$$

$$A_{BC} = \frac{L_{BC}}{3.7334\times10^3} = \frac{0.8}{3.7334\times10^3} = 214.28\times10^{-6}\ m^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 \quad \therefore \quad d_{BC} = \sqrt{\frac{4A_{BC}}{\pi}} = \sqrt{\frac{(4)(214.28\times10^{-6})}{\pi}} = 16.52\times10^{-3}\ m$$

$$= 16.52\ mm$$

PROBLEM 2.14



2.14 Both portions of the rod ABC are made of an aluminum for which $E = 73$ GPa. Knowing that the diameter of portion BC is $d = 20$ mm, determine the largest force $\mathbf{P}$ that can be applied if $\sigma_{all} = 160$ MPa and the corresponding deflection at point C is not to exceed 4 mm.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(0.030)^2 = 706.86\times10^{-6}\ m^2$$

$$A_{BC} = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}\ m^2$$

Considering allowable stress $\sigma = 160\times10^6$ Pa

$$\sigma = \frac{P}{A} \quad \therefore \quad P = A\sigma$$

Portion AB $\quad P = (706.86\times10^{-6})(160\times10^6) = 113.1\times10^3$ N

Portion BC $\quad P = (314.16\times10^{-6})(160\times10^6) = 50.3\times10^3$ N

Considering allowable deflection $\delta_C = 4\times10^{-3}$ m

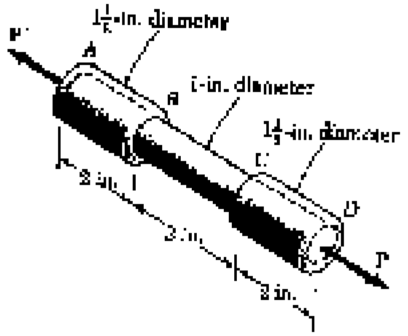
$$\delta_C = \sum\frac{PL_i}{A_i E} = \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$$

$$P = E\delta_C\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)^{-1} = (73\times10^9)(4\times10^{-3})\left(\frac{1.2}{706.86\times10^{-6}} + \frac{0.8}{314.16\times10^{-6}}\right)^{-1}$$

$$= 68.8\times10^3\ N$$

Smallest value for P governs $\quad P = 50.3\times10^3\ N = 50.3\ kN$

PROBLEM 2.15



2.15 The specimen shown is made from a 1-in.-diameter cylindrical steel rod with two 1.5-in.-outer-diameter sleeves bonded to the rod as shown. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the load **P** so that the total deformation is 0.002 in., (*b*) the corresponding deformation of the central portion *BC*.

SOLUTION

(a) $\delta = \sum \frac{P_i L_i}{A_i E_i} = \frac{P}{E} \sum \frac{L_i}{A_i}$

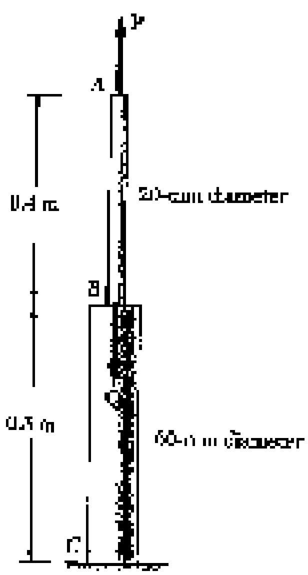
$P = E\delta \left(\sum \frac{L_i}{A_i} \right)^{-1}$ $\qquad A_i = \frac{\pi}{4} d_i^2$

	L, in.	d, in	A, in^2	L/A, in^{-1}
AB	2	1.5	1.7671	1.1318
BC	3	1.0	0.7854	3.8197
CD	2	1.5	1.7671	1.1318
				6.083 ← sum

$P = (29\times10^6)(0.002)(6.083)^{-1} = 9.535\times10^3$ lb. $= 9.53$ kips

(b) $\delta_{BC} = \frac{P L_{BC}}{A_{BC} E} = \frac{P}{E}\frac{L_{BC}}{A_{BC}} = \frac{9.535\times10^3}{29\times10^6}(3.8197) = 1.254\times10^{-3}$ in

PROBLEM 2.16



2.16 Both portions of the rod *ABC* are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at *A* is zero, (*b*) the corresponding deflection of *B*.

SOLUTION

(a) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}$ m^2

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274\times10^{-3}$ m^2

Force in member AB is P tension

Elongation $\delta_{AB} = \frac{P L_{AB}}{E A_{AB}} = \frac{(4\times10^3)(0.4)}{(70\times10^9)(314.16\times10^{-6})}$

$= 72.756\times10^{-6}$ m

Force in member BC is Q − P compression

Shortening $\delta_{BC} = \frac{(Q-P)L_{BC}}{E A_{BC}} = \frac{(Q-P)(0.5)}{(70\times10^9)(2.8274\times10^{-3})}$

$= 2.5263\times10^{-9}(Q-P)$

For zero deflection at A $\quad \delta_{BC} = \delta_{AB}$

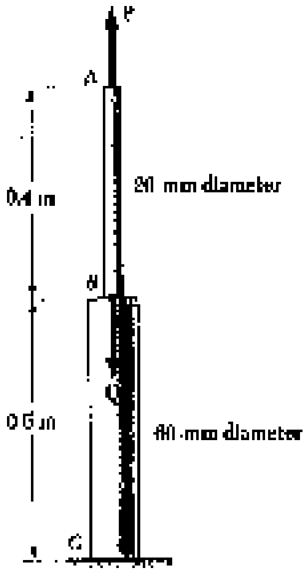
$2.5263\times10^{-9}(Q-P) = 72.756\times10^{-6} \quad \therefore \quad Q - P = 28.8\times10^3$ N

$Q = 28.3\times10^3 + 4\times10^3 = 32.8\times10^3$ N $= 32.8$ kN

(b) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756\times10^{-6}$ m $= 0.0728$ mm

PROBLEM 2.17

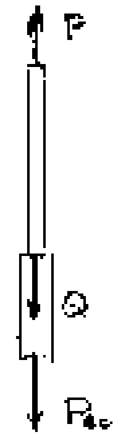
2.17 The rod ABC is made of an aluminum for which $E = 70$ GPa. Knowing that $P = 6$ kN and $Q = 42$ kN, determine the deflection of (a) point A, (b) point B.



SOLUTION

(a) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\ m^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\ m^2$



$P_{AB} = P = 6 \times 10^3$ N

$P_{BC} = P - Q = 6 \times 10^3 - 42 \times 10^3 = -36 \times 10^3$ N

$L_{AB} = 0.4$ m $\qquad L_{BC} = 0.5$ m

$\delta_{AB} = \frac{P_{AB} L_{AB}}{A_{AB} E_A} = \frac{(6 \times 10^3)(0.4)}{(314.16 \times 10^{-6})(70 \times 10^9)}$

$= 109.135 \times 10^{-6}$ m

$\delta_{BC} = \frac{P_{BC} L_{BC}}{A_{BC} E} = \frac{(-36 \times 10^3)(0.5)}{(2.8274 \times 10^{-3})(70 \times 10^9)}$

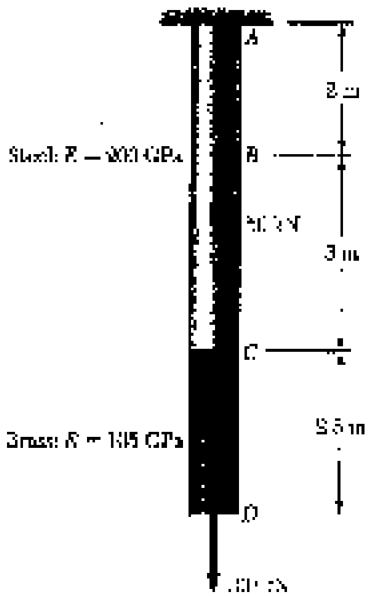
$= -90.947 \times 10^{-6}$ m

$\delta_A = \delta_{AB} + \delta_{BC} = 109.135 \times 10^{-6} - 90.947 \times 10^{-6}\,m = 18.19 \times 10^{-6}$ m

$= 0.01819$ mm ◄

(b) $\delta_B = \delta_{BC} = -90.9 \times 10^{-6}$ m $= -0.0909$ mm ◄

PROBLEM 2.18



2.18 The 36-mm-diameter steel rod ABC and a brass rod CD of the same diameter are joined at point C to form the 7.5-m rod $ABCD$. For the loading shown, and neglecting the weight of the rod, determine the deflection of (*a*) point C, (*b*) point D.

SOLUTION

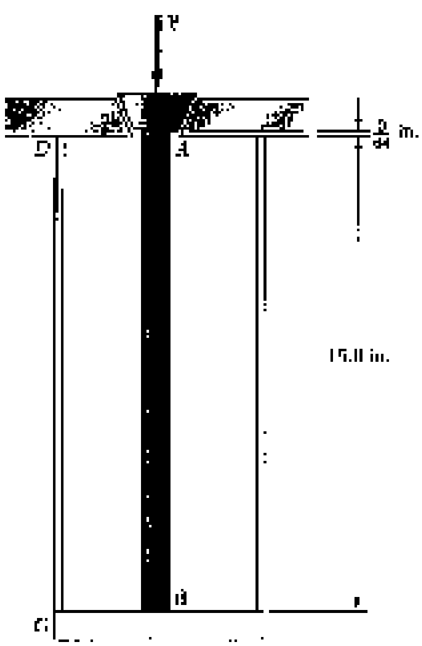
$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.036)^2 = 1.01787 \times 10^{-3} \text{ m}^2$$

Portion	P_i	L_i	E_i	$P_i L_i / A E_i$
AB	150 kN	2 m	200 GPa	1.474×10^{-3} m
BC	100 kN	3 m	200 GPa	1.474×10^{-3} m
CD	100 kN	2.5 m	105 GPa	2.339×10^{-3} m

(a) $\delta_C = \delta_{AB} + \delta_{BC} = 1.474 \times 10^{-3} + 1.474 \times 10^{-3}$
$= 2.948 \times 10^{-3}$ m $= 2.95$ mm ◄

(b) $\delta_D = \delta_C + \delta_{CD} = 2.948 \times 10^{-3} + 2.339 \times 10^{-3}$
$= 5.287 \times 10^{-3}$ m $= 5.29$ mm ◄

PROBLEM 2.19



2.19 The brass tube AB ($E = 15 \times 10^6$ psi) has a cross-sectional area of 0.22 in² and is fitted with a plug at A. The tube is attached at B to a rigid plate which is itself attached at C to the bottom of an aluminum cylinder ($E = 10.4 \times 10^6$ psi) with a cross-sectional area of 0.40 in². The cylinder is then hung from a support at D. In order to close the cylinder, the plug must move down through $\frac{3}{64}$ in. Determine the force **P** that must be applied to the cylinder.

Shortening of brass tube AB

$L_{AB} = 15 + \frac{3}{64} = 15.047$ in $\qquad A_{AB} = 0.22$ in²

$E_{AB} = 15 \times 10^6$ psi

$$\delta_{AB} = \frac{P L_{AB}}{E_{AB} A_{AB}} = \frac{P(15.047)}{(15 \times 10^6)(0.22)} = 4.5597 \times 10^{-6} P$$

Lengthening of aluminum cylinder CD

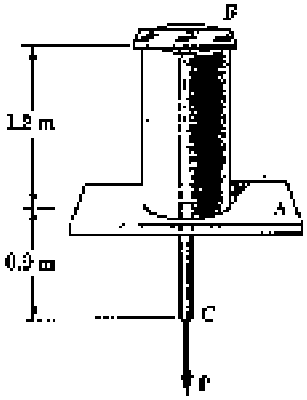
$L_{CD} = 15$ in, $\quad A_{CD} = 0.40$ in², $\quad E_{CD} = 10.4 \times 10^6$ psi

$$\delta_{CD} = \frac{P L_{CD}}{E_{CD} A_{CD}} = \frac{P(15)}{(10.4 \times 10^6)(0.40)} = 3.6058 \times 10^{-6} P$$

Total deflection $\qquad \delta_A = \delta_{AB} + \delta_{CD}$

$\frac{3}{64} = (4.5597 \times 10^{-6} + 3.6058 \times 10^{-6}) P \quad \therefore \quad P = 5.74 \times 10^3$ lb.
$= 5.74$ kips ◄

PROBLEM 2.20



2.20 A 1.2-m section of aluminum pipe of cross-sectional area 1100 mm² rests on a fixed support at A. The 15-mm-diameter steel rod BC hangs from a rigid bar that rests on the top of the pipe at B. Knowing that the modulus of elasticity is 200 GPa for steel and 72 GPa for aluminum, determine the deflection of point C when a 60-kN force is applied at C.

SOLUTION

Rod BC $\quad L_{BC} = 2.1\ \text{m}, \quad E_{BC} = 200\times10^{9}\ \text{Pa}$

$$A_{BC} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.015)^2 = 176.715\times10^{-6}\ \text{m}^2$$

$$\delta_{C/B} = \frac{PL_{BC}}{E_{BC}A_{BC}} = \frac{(60\times10^3)(2.1)}{(200\times10^9)(176.715\times10^{-6})}$$

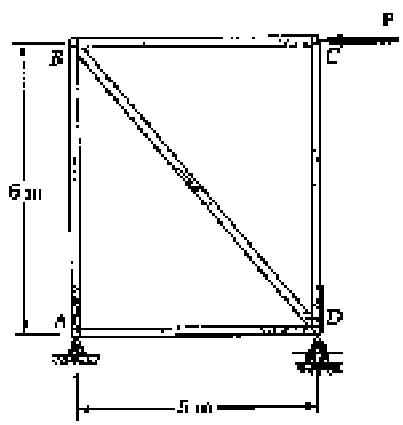
$$= 3.565\times10^{-3}\ \text{m}$$

Pipe AB: $\quad L_{AB} = 1.2\ \text{m}, \quad E_{AB} = 72\times10^9\ \text{Pa}, \quad A_{AB} = 1100\ \text{mm}^2 = 1100\times10^{-6}\ \text{m}^2$

$$\delta_{B/A} = \frac{PL_{AB}}{E_{AB}A_{AB}} = \frac{(60\times10^3)(1.2)}{(72\times10^9)(1100\times10^{-6})} = 909.1\times10^{-6}\ \text{m}^2$$

$$\delta_C = \delta_{B/A} + \delta_{C/B} = 909.1\times10^{-6} + 3.565\times10^{-3} = 4.47\times10^{-3}\ \text{m} = 4.47\ \text{mm}$$

PROBLEM 2.21



2.21 The steel frame ($E = 200$ GPa) shown has a diagonal brace BD with an area of 1920 mm². Determine the largest allowable load P if the change in length of member BD is not to exceed 1.6 mm.

SOLUTION

$\delta_{BD} = 1.6\times10^{-3}\ \text{m}, \quad A_{BD} = 1920\ \text{mm}^2 = 1920\times10^{-6}\ \text{m}^2$

$L_{BD} = \sqrt{5^2+6^2} = 7.810\ \text{m}, \quad E_{BD} = 200\times10^9\ \text{Pa}$

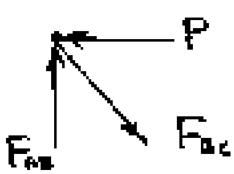
$$\delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}}$$

$$F_{BD} = \frac{E_{BD}A_{BD}\delta_{BD}}{L_{BD}} = \frac{(200\times10^9)(1920\times10^{-6})(1.6\times10^{-3})}{7.81}$$

$$= 78.67\times10^3\ \text{N}$$

Use joint B as a free body: $\xrightarrow{+}\Sigma F_x = 0$

$$\frac{5}{7.810}F_{BD} - P = 0$$

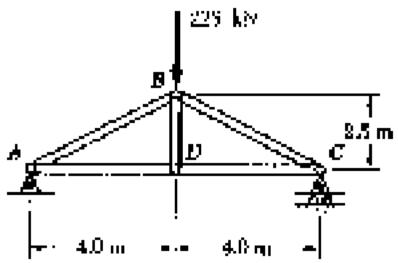


$$P = \frac{5}{7.810}F_{BD} = \frac{(5)(78.67\times10^3)}{7.810}$$

$$= 50.4\times10^3\ \text{N} = 50.4\ \text{kN}$$

PROBLEM 2.22

2.22 For the steel truss ($E = 200$ GPa) and loading shown, determine the deformations of members AB and AD, knowing that their cross-sectional areas are 2400 mm² and 1800 mm², respectively.



SOLUTION

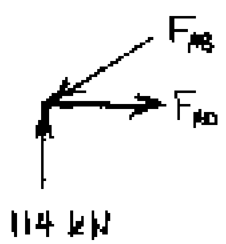
Statics: Reactions are 114 kN upward at A & C.

Member BD is a zero force member

$L_{AB} = \sqrt{4.0^2 + 2.5^2} = 4.717$ m

Use joint A as a free body: $+\uparrow \Sigma F_y = 0$ $\quad 114 - \frac{2.5}{4.717} F_{AB} = 0$

$F_{AB} = 215.10$ kN



$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad F_{AD} - \frac{4}{4.717} F_{AB} = 0$

$F_{AD} = \frac{(4)(215.10)}{4.717} = 182.4$ kN

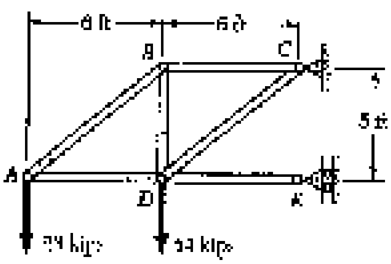
Member AB: $\quad \delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(215.10 \times 10^3)(4.717)}{(200 \times 10^9)(2400 \times 10^{-6})}$

$= 2.11 \times 10^{-3}$ m $= 2.11$ mm ◄

$\delta_{AD} = \frac{F_{AD} L_{AD}}{E A_{AD}} = \frac{(182.4 \times 10^3)(4.0)}{(200 \times 10^9)(1800 \times 10^{-6})} = 2.03 \times 10^{-3}$ m $= 2.03$ mm ◄

PROBLEM 2.23

2.23 Members AB and BC are made of steel ($E = 29 \times 10^6$ psi) with cross-sectional areas of 0.80 in² and 0.64 in², respectively. For the loading shown, determine the elongation of (a) member AB, (b) member BC.

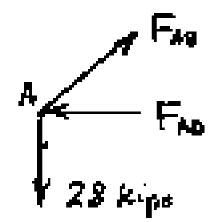


SOLUTION

(a) $\qquad L_{AB} = \sqrt{6^2 + 5^2} = 7.810$ ft $= 93.72$ in

Use joint A as a free body

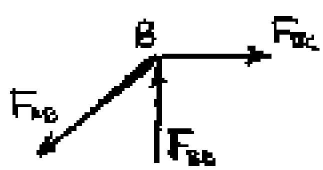
$+\uparrow \Sigma F_y = 0 \qquad \frac{5}{7.810} F_{AB} - 28 = 0$



$F_{AB} = 43.74$ kip $= 43.74 \times 10^3$ lb.

$\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(43.74 \times 10^3)(93.72)}{(29 \times 10^6)(0.80)} = 0.1767$ in. ◄

(b) Use joint B as a free body

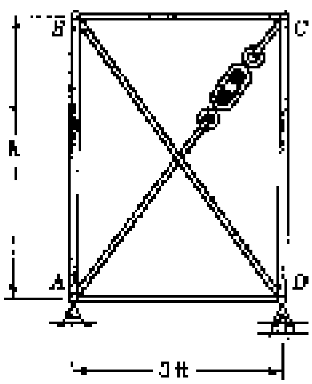


$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad F_{BC} - \frac{6}{7.810} F_{AB} = 0$

$F_{BC} = \frac{(6)(43.74)}{7.810} = 33.60$ kip $= 33.60 \times 10^3$ lb.

$\delta_{BC} = \frac{F_{BC} L_{BC}}{E A_{BC}} = \frac{(33.60 \times 10^3)(72)}{(29 \times 10^6)(0.64)} = 0.1304$ in. ◄

PROBLEM 2.24



2.24 Members AB and CD are $1\frac{1}{8}$-in.-diameter steel rods, and members BC and AD are $\frac{7}{8}$-in.-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that $E = 29 \times 10^6$ psi and $h = 4$ ft, determine the largest allowable tension in AC so that the deformations in members AB and CD do not exceed 0.04 in.

SOLUTION

$\delta_{AB} = \delta_{CD} = 0.04$ in $\qquad h = 4$ ft $= 48$ in. $= L_{CD}$

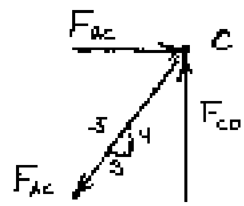
$A_{CD} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1.125)^2 = 0.99402 \text{ in}^2$

$\delta_{CD} = \dfrac{F_{CD} L_{CD}}{E A_{CD}}$

$F_{CD} = \dfrac{E A_{CD} \delta_{CD}}{L_{CD}} = \dfrac{(29 \times 10^6)(0.99402)(0.04)}{48}$

$= 24.022 \times 10^3$ lb.

Use joint C as a free body

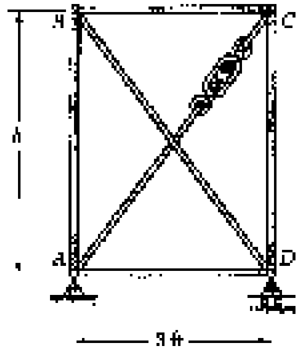


$+\uparrow \Sigma F_y = 0: \quad F_{CD} - \frac{4}{5} F_{AC} = 0 \qquad \therefore F_{AC} = \frac{5}{4} F_{CD}$

$F_{AC} = \frac{5}{4}(24.022 \times 10^3) = 30.0 \times 10^3$ lb.

30.0 kips ◄

PROBLEM 2.25

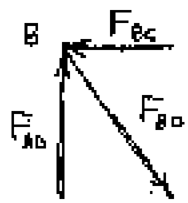


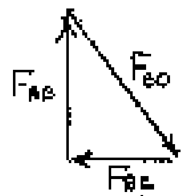
2.24 Members *AB* and *CD* are $1\frac{1}{8}$-in.-diameter steel rods, and members *BC* and *AD* are $\frac{7}{8}$-in.-diameter steel rods. When the turnbuckle is tightened, the diagonal member *AC* is put in tension. Knowing that $E = 29 \times 10^6$ psi and $h = 4$ ft, determine the largest allowable tension in *AC* so that the deformations in members *AB* and *CD* do not exceed 0.04 in.

2.25 For the structure in Prob. of 2.24, determine (*a*) the distance *h* so that the deformations in members *AB*, *BC*, *CD* and *AD* are all equal to 0.04 in., (*b*) the corresponding tension in member *AC*.

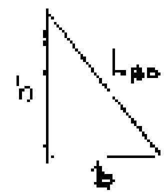
SOLUTION

(a) Statics: Use joint B as a free body





Force Triangle



Geometry

From similar triangles

$$\frac{F_{AB}}{h} = \frac{F_{BC}}{b} = \frac{F_{BD}}{L_{BD}}$$

$$F_{AB} = \frac{h}{b} F_{BC}$$

For equal deformations

$$\delta_{AB} = \delta_{BC} \quad \therefore \quad \frac{F_{AB}\, h}{E A_{AB}} = \frac{F_{BC}\, b}{E A_{BC}} \qquad F_{AB} = \frac{b}{h} \cdot \frac{A_{AB}}{A_{BC}} F_{BC}$$

Equating expressions for F_{AB}

$$\frac{h}{b} F_{BC} = \frac{b}{h} \frac{A_{AB}}{A_{BC}} F_{BC} \qquad \frac{h^2}{b^2} = \frac{A_{AB}}{A_{BC}} = \frac{\frac{\pi}{4} d_{AB}^2}{\frac{\pi}{4} d_{BC}^2} = \frac{d_{AB}^2}{d_{BC}^2}$$

$$\frac{h}{b} = \frac{d_{AB}}{d_{BC}} = \frac{9/8}{7/8} = \frac{9}{7} \qquad b = 3 \text{ ft} = 36 \text{ in.}$$

$$h = \frac{9}{7} b = \frac{9}{7}(3) = 3.86 \text{ ft} = 46.3 \text{ in.}$$

(b) Setting $\delta_{AB} = \delta_{BC} = 0.04$ in.

$$\delta_{BC} = \frac{F_{BC}\, b}{E A_{BC}} \quad \therefore \quad F_{BC} = \frac{E A_{BC} \delta_{BC}}{b} = \frac{(29 \times 10^6) \frac{\pi}{4} \left(\frac{7}{8}\right)^2 (0.04)}{36}$$

$$= 19.376 \times 10^3 \text{ lb.}$$

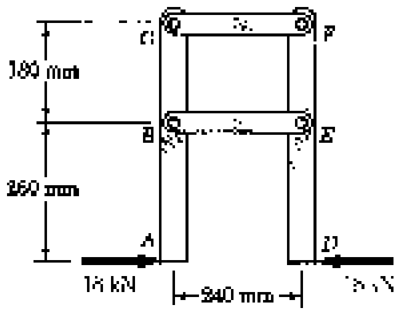
$$F_{AB} = \frac{h}{b} F_{BC} = \frac{9}{7}(19.376 \times 10^3) = 24.912 \times 10^3 \text{ lb}$$

From the force triangle

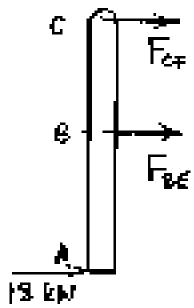
$$F_{BD} = F_{AC} = \sqrt{F_{BC}^2 + F_{AB}^2} = 31.6 \times 10^3 \text{ lb.}$$

PROBLEM 2.26

2.26 Members *ABC* and *DEF* are joined with steel links ($E = 200$ GPa). Each of the links is made of a pair of 25×35-mm plates. Determine the change in length of (*a*) member *BE*, (*b*) member *CF*.



SOLUTION



Use member ABC as a free body

$$\circlearrowleft \Sigma M_B = 0$$

$$(0.260)(18 \times 10^3) - (0.180)F_{CF} = 0$$

$$F_{CF} = \frac{(0.260)(18 \times 10^3)}{0.180} = 26 \times 10^3 \text{ N}$$

$$\circlearrowleft \Sigma M_C = 0 \qquad (0.440)(18 \times 10^3) + (0.180)F_{BE} = 0$$

$$F_{BE} = -\frac{(0.440)(18 \times 10^3)}{0.180} = -44 \times 10^3 \text{ N}$$

Area for link made of two plates

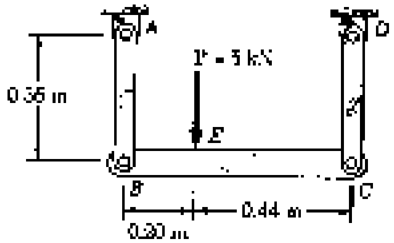
$$A = (2)(0.025)(0.035) = 1.75 \times 10^{-3} \text{ m}^2$$

(a) $$\delta_{BE} = \frac{F_{BE}L_{BE}}{EA} = \frac{(-44 \times 10^3)(0.240)}{(200 \times 10^9)(1.75 \times 10^{-3})} = -30.2 \times 10^{-6} \text{ m} = -0.0302 \text{ mm}$$

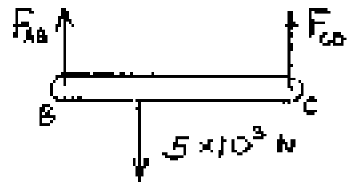
(b) $$\delta_{CF} = \frac{F_{CF}L_{CF}}{EA} = \frac{(26 \times 10^3)(0.240)}{(200 \times 10^9)(1.75 \times 10^{-3})} = 17.83 \times 10^{-6} \text{ m} = 0.01783 \text{ mm}$$

PROBLEM 2.27

2.27 Each of the links AB and CD is made of aluminum ($E = 75$ GPa) and has a cross-sectional area of 125 mm². Knowing that they support the rigid member BC, determine the deflection of point E.



SOLUTION



Use member BC as a free body

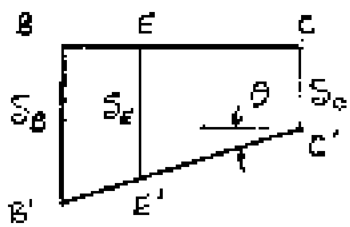
$\circlearrowleft \Sigma M_C = 0 \qquad -(0.64) F_{AB} + (0.44)(5\times10^3) = 0 \qquad F_{AB} = 3.4375 \times 10^3$ N

$\circlearrowleft \Sigma M_B = 0 \qquad (0.64)\, F_{CD} - (0.20)(5\times10^3) = 0 \qquad F_{CD} = 1.5625 \times 10^3$ N

For links AB and CD $\qquad A = 125$ mm² $= 125 \times 10^{-6}$ m²

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{EA} = \frac{(3.4325\times10^3)(0.36)}{(75\times10^9)(125\times10^{-6})} = 132.00 \times 10^{-6} \text{ m} = \delta_A$$

$$\delta_{CD} = \frac{F_{CD} L_{CD}}{EA} = \frac{(1.5625\times10^3)(0.36)}{(75\times10^9)(125\times10^{-6})} = 60.00 \times 10^{-6} \text{ m} = \delta_C$$



Deformation diagram

$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{l_{BC}} = \frac{72.00 \times 10^{-6}}{0.64}$$

$$= 112.5 \times 10^{-6} \text{ rad}$$

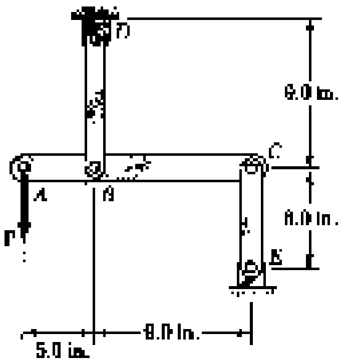
$$\delta_E = \delta_C + l_{EC}\,\theta$$

$$= 60.00\times10^{-6} + (0.44)(112.5 \times 10^{-6})$$

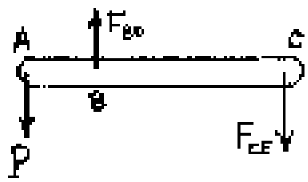
$$= 109.5 \times 10^{-6} \text{ m} = 0.1095 \text{ mm}$$

PROBLEM 2.28

2.28 Link BD is made of brass ($E = 15 \times 10^6$ psi) and has a cross-sectional area of 0.40 in². Link CE is made of aluminum ($E = 10.4 \times 10^6$ psi) and has a cross-sectional area of 0.50 in². Determine the maximum force **P** that can be applied vertically at point A if the deflection of A is not to exceed 0.014 in.



SOLUTION



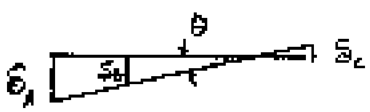
Use member ABC as a free body

$\circlearrowright \Sigma M_C = 0, \quad 14P - 9F_{BD} = 0, \qquad F_{BD} = 1.5556\ P$

$\circlearrowright \Sigma M_B = 0 \quad 5P - 9F_{CE} = 0, \qquad F_{CE} = 0.5556\ P$

$$\delta_B = \delta_{BD} = \frac{F_{BD} L_{BD}}{E_{BD} A_{BD}} = \frac{(1.5556\ P)(9.0)}{(15\times10^6)(0.40)} = 2.3333\times10^{-6}\ P \downarrow$$

$$\delta_C = \delta_{CE} = \frac{F_{CE} L_{CE}}{E_{CE} A_{CE}} = \frac{(0.5556\ P)(6.0)}{(10.4\times10^6)(0.50)} = 0.6410\times10^{-6}\ P \uparrow$$



Deformation Diagram

From the deformation diagram

$$\text{Slope } \theta = \frac{\delta_B + \delta_C}{l_{BC}} = \frac{2.9743\times10^{-6}\ P}{9}$$

$$= 0.3305\times10^{-6}\ P$$

$$\delta_A = \delta_B + l_{AB}\,\theta$$

$$= 2.3333\times10^{-6}P + (5)(0.3305\times10^{-6})P$$

$$= 3.9858\times10^{-6}\ P$$

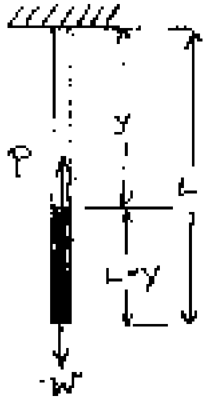
Apply displacement limit $\delta_A = 0.014$ in $= 3.9858\times10^{-6}\ P$

$$P = \frac{0.014}{3.9858\times10^{-6}} = 3.51\times10^3\ \text{lb} = 3.51\ \text{kips}$$

PROBLEM 2.29

2.29 A homogeneous cable of length L and uniform cross section is suspended from one end. (*a*) Denoting by ρ the density (mass per unit volume) of the cable and by E its modulus of elasticity, determine the elongation of the cable due to its own weight. (*b*) Assuming now the cable to be horizontal, determine the force that should be applied to each end of the cable to obtain the same elongation as in part *a*.

SOLUTION



(a) For element at point identified by coordinate y

P = weight of portion below the point

$$= \rho g A (L - y)$$

$$d\delta = \frac{P\,dy}{EA} = \frac{\rho g A (L-y)\,dy}{EA} = \frac{\rho g (L-y)}{E}\,dy$$

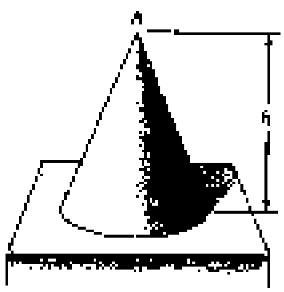
$$\delta = \int_0^L \frac{\rho g (L-y)}{E}\,dy = \frac{\rho g}{E}\left(Ly - \frac{1}{2}y^2\right)\Big|_0^L$$

$$= \frac{\rho g}{E}\left(L^2 - \frac{L^2}{2}\right) = \frac{1}{2}\frac{\rho g L^2}{E}$$

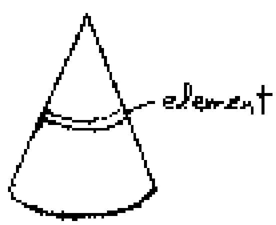
(b) For $\delta = \frac{PL}{EA}$ $\qquad P = \frac{EA\delta}{L} = \frac{EA}{L}\frac{\rho g L^2}{2E} = \frac{1}{2}\rho g L = \frac{1}{2}W$

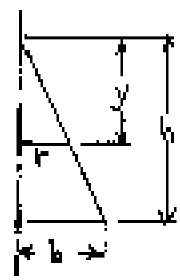
PROBLEM 2.30

2.30 Determine the deflection of the apex A of a homogeneous circular cone of height h, density ρ, and modulus of elasticity E, due to its own weight.



SOLUTION





Let b = radius of the base and r = radius at section with coordinate y.

$$r = \frac{b}{h}y$$

Volume of portion above element $\quad V = \frac{1}{3}\pi r^2 y = \frac{1}{3}\pi\frac{b^2}{h^2}y^3$

$$P = \rho g V = \frac{\pi \rho g b^2 y^3}{3h^2} \qquad A = \pi r^2 = \frac{\pi b^2}{h^2}y^2$$

$$\delta = \sum \frac{P\,\Delta y}{EA} = \int_0^h \frac{P\,dy}{EA} = \int_0^h \frac{\pi \rho g b^2 y^3}{3h^2}\cdot\frac{h^2}{E\pi b^2 y^2}\,dy = \int_0^h \frac{\rho g y}{3E}\,dy$$

$$= \frac{\rho g}{3E}\frac{y^2}{2}\Big|_0^h = \frac{\rho g h^2}{6E}$$

PROBLEM 2.31

2.31 The volume of a tensile specimen is essentially constant while plastic deformation occurs. If the initial diameter of the specimen is d_1, show that when the diameter is d, the true strain is $\epsilon_t = 2\ln(d_1/d)$.

SOLUTION

If the volume is constant $\quad \frac{\pi}{4} d^2 L = \frac{\pi}{4} d_1^2 L_0$

$$\frac{L}{L_0} = \frac{d_1^2}{d^2} = \left(\frac{d_1}{d}\right)^2$$

$$\varepsilon_t = \ln \frac{L}{L_0} = \ln \left(\frac{d_1}{d}\right)^2 = 2 \ln \frac{d_1}{d}$$

PROBLEM 2.32

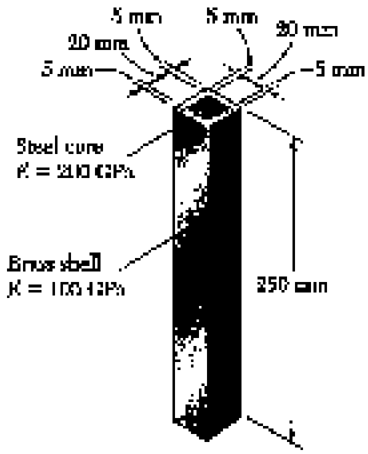
2.32 Denoting by ε the "engineering strain" in a tensile specimen, show that the true strain is $\epsilon_t = \ln(1 + \varepsilon)$.

SOLUTION

$$\varepsilon_t = \ln \frac{L}{L_0} = \ln \frac{L_0 + \delta}{L_0} = \ln \left(1 + \frac{\delta}{L_0}\right) = \ln(1 + \varepsilon)$$

Thus $\quad \varepsilon_t = \ln(1 + \varepsilon)$

PROBLEM 2.33



2.33 An axial force of 60 kN is applied to the assembly shown by means of rigid end plates. Determine (*a*) the normal stress in the brass shell, (*b*) the corresponding deformation of the assembly.

SOLUTION

Let P_b = portion of axial force carried by brass shell

P_s = portion of axial force carried by steel core

$$\delta = \frac{P_b L}{A_b E_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_s L}{A_s E_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_b + P_s = (E_b A_b + E_s A_s)\frac{\delta}{L}$$

$$\frac{\delta}{L} = \varepsilon = \frac{P}{E_b A_b + E_s A_s}$$

$$A_s = (0.020)(0.020) = 400 \times 10^{-6}\ \text{m}^2$$

$$A_b = (0.030)(0.030) - (0.020)(0.020) = 500 \times 10^{-6}\ \text{m}^2$$

$$\frac{\delta}{L} = \varepsilon = \frac{60 \times 10^3}{(105 \times 10^9)(500 \times 10^{-6}) + (200 \times 10^9)(400 \times 10^{-6})} = 452.83 \times 10^{-6}$$

(a) $\sigma_b = E_b \varepsilon = (105 \times 10^9)(452.83 \times 10^{-6}) = 47.5 \times 10^6\ \text{Pa}$

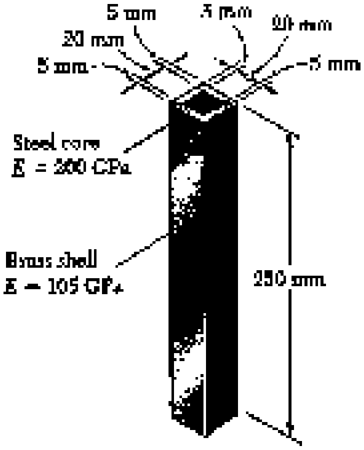
$= 47.5\ \text{MPa}$

(b) $\delta = L\varepsilon = (250 \times 10^{-3})(452.83 \times 10^{-6}) = 113.2 \times 10^{-6}\ \text{m}$

$= 0.1132 \times 10^{-3}\ \text{m}$

$= 0.1132\ \text{mm}$

PROBLEM 2.34



2.34 The length of the assembly decreases by 0.15 mm when an axial force is applied by means of rigid end plates. Determine (a) the magnitude of the applied force, (b) the corresponding stress in the steel core.

SOLUTION

Let P_b = portion of axial force carried by brass shell.

P_s = portion of axial force carried by steel core.

$$\delta = \frac{P_b L}{A_b E_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_s L}{A_s E_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_b + P_s = (E_b A_b + E_s A_s)\frac{\delta}{L}$$

$$A_s = (0.020)(0.020) = 400 \times 10^{-6}\ \text{m}^2$$

$$A_b = (0.030)(0.030) - (0.020)(0.020) = 500 \times 10^{-6}\ \text{m}^2$$

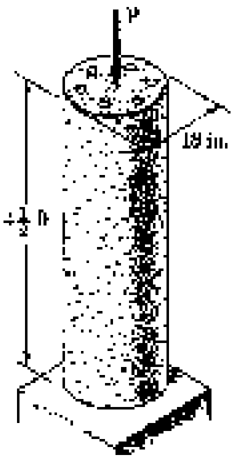
(a) $$P = [(105 \times 10^9)(500 \times 10^{-6}) + (200 \times 10^9)(400 \times 10^{-6})]\frac{0.15 \times 10^{-3}}{250 \times 10^{-3}}$$

$$= 79.5 \times 10^3\ \text{N} \qquad = 79.5\ \text{kN}$$

(b) $$\sigma_s = E_s \varepsilon = \frac{E_s \delta}{L} = \frac{(200 \times 10^9)(0.15 \times 10^{-3})}{250 \times 10^{-3}} = 120 \times 10^6\ \text{Pa}$$

$$= 120\ \text{MPa}$$

PROBLEM 3.35



3.35 The 4.5-ft concrete post is reinforced with six steel bars, each with a $1\frac{1}{8}$-in. diameter. Knowing that $E_s = 29 \times 10^6$ psi and $E_c = 4.2 \times 10^6$ psi, determine the normal stresses in the steel and in the concrete when a 350-kip axial centric force **P** is applied to the post.

SOLUTION

Let P_c = portion of axial force carried by concrete

P_s = portion carried by the six steel rods

$$\delta = \frac{P_c L}{E_c A_c} \qquad P_c = \frac{E_c A_c \delta}{L}$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_c + P_s = (E_c A_c + E_s A_s)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{P}{E_c A_c + E_s A_s}$$

$$A_s = 6\frac{\pi}{4}d_s^2 = \frac{6\pi}{4}(1.125)^2 = 5.964 \text{ in}^2$$

$$A_c = \frac{\pi}{4}d_c^2 - A_s = \frac{\pi}{4}(18)^2 - 5.964 = 248.5 \text{ in}^2$$

$$L = 4.5 \text{ ft} = 54 \text{ in}$$

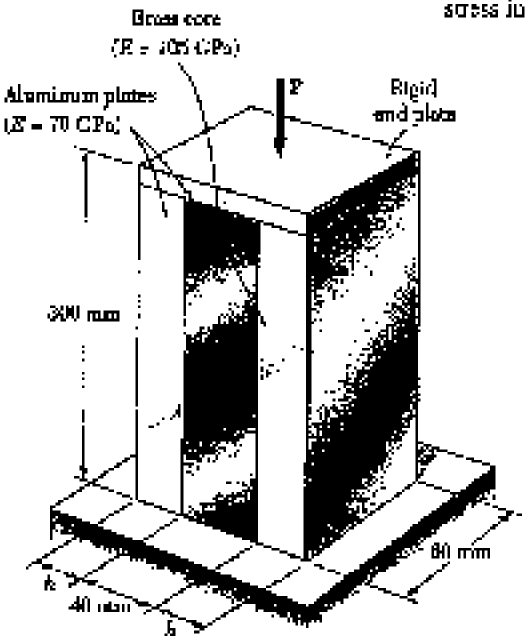
$$\varepsilon = -\frac{-350 \times 10^3}{(4.2\times10^6)(248.5) + (29\times10^6)(5.964)} = -287.67 \times 10^{-6}$$

$$\sigma_s = E_s \varepsilon = (29\times10^6)(-287.67\times10^{-6}) = -8.34\times10^3 \text{ psi} = -8.34 \text{ ksi}$$

$$\sigma_c = E_c \varepsilon = (4.2\times10^6)(-287.67\times10^{-6}) = -1.208\times10^3 \text{ psi} = -1.208 \text{ ksi}$$

PROBLEM 2.36

2.36 An axial centric force of magnitude $P = 450$ kN is applied to the composite block shown by means of a rigid end plate. Knowing that $h = 10$ mm, determine the normal stress in (*a*) the brass core, (*b*) the aluminum plates.



SOLUTION

Let P_b = portion of axial force carried by brass core

P_a = portion carried by two aluminum plates

$$\delta = \frac{P_b L}{E_b A_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_a L}{E_a A_a} \qquad P_a = \frac{E_a A_a \delta}{L}$$

$$P = P_b + P_a = (E_b A_b + E_a A_a)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{P}{E_b A_b + E_a A_a}$$

$$A_b = (60)(40) = 2400 \text{ mm}^2 = 2400\times10^{-6} \text{ m}^2$$

$$A_a = (2)(60)(10) = 1200 \text{ mm}^2 = 1200\times10^{-6} \text{ m}^2$$

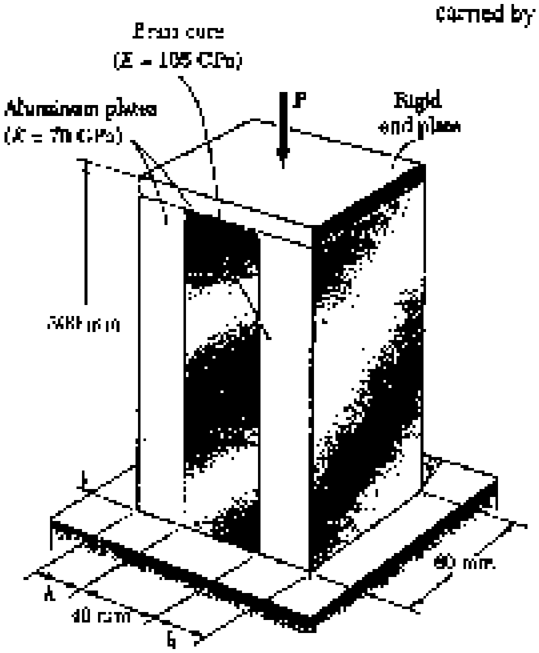
$$\varepsilon = \frac{450\times10^3}{(105\times10^9)(2400\times10^{-6}) + (70\times10^9)(1200\times10^{-6})} = 1.3393\times10^{-3}$$

(a) $\sigma_b = E_b \varepsilon = (105\times10^9)(1.3393\times10^{-3}) = 140.6\times10^6 \text{ Pa} = 140.6 \text{ MPa}$ ◄

(b) $\sigma_a = E_a \varepsilon = (70\times10^9)(1.3393\times10^{-3}) = 93.75\times10^6 \text{ Pa} = 93.75 \text{ MPa}$ ◄

PROBLEM 2.37

2.37 For the composite block shown in Prob. 2.36, determine (*a*) the value of *h* if the portion of the load carried by the aluminum plates is half the portion of the load carried by the brass core, (*b*) the total load if the stress in the brass is 80 MPa.



SOLUTION

Let P_b = portion of axial force carried by brass core

P_a = portion carried by the two aluminum plates

$$\delta = \frac{P_b L}{E_b A_b} \qquad P_b = \frac{E_b A_b \delta}{L}$$

$$\delta = \frac{P_a L}{E_a A_a} \qquad P_a = \frac{E_a A_a \delta}{L}$$

(a) Given $P_a = \frac{1}{2} P_b$

$$\frac{E_a A_a \delta}{L} = \frac{1}{2}\frac{E_b A_b \delta}{L}$$

$$A_a = \frac{1}{2}\frac{E_b}{E_a} A_b$$

$$A_b = (40)(60) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$$

$$A_a = \frac{1}{2}\frac{105 \times 10^9}{70 \times 10^9}\, 2400 = 1800 \text{ mm}^2 = (2)(60)\,h$$

$$h = \frac{1800}{(2)(60)} = 15 \text{ mm}$$

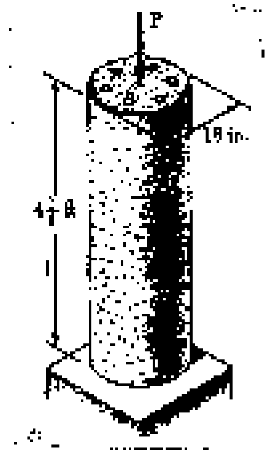
(b) $\sigma_b = \dfrac{P_b}{A_b}$

$$P_b = A_b \sigma_b = (2400 \times 10^{-6})(80 \times 10^6) = 192 \times 10^3 \text{ N}$$

$$P_a = \tfrac{1}{2} P_b = 96 \times 10^3 \text{ N}$$

$$P = P_b + P_a = 288 \times 10^3 \text{ N} = 288 \text{ kN}$$

PROBLEM 2.38



2.35 The 4.5-ft concrete post is reinforced with six steel bars, each with a $1\frac{1}{8}$-in. diameter. Knowing that $E_s = 29 \times 10^6$ psi and $E_c = 4.2 \times 10^6$ psi, determine the normal stresses in the steel and in the concrete when a 350-kip axial centric force **P** is applied to the post.

2.38 For the post of Prob. 2.35, determine the maximum centric force which may be applied if the allowable normal stress is 20 ksi in the steel and 2.4 ksi in the concrete.

SOLUTION

Determine allowable strain in each material

Steel: $\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{20 \times 10^3}{29 \times 10^6} = 689.97 \times 10^{-6}$

Concrete: $\varepsilon_c = \frac{\sigma_c}{E_c} = \frac{2.4 \times 10^3}{4.2 \times 10^6} = 571.43 \times 10^{-6}$

Smaller value governs $\quad \varepsilon = \frac{\delta}{L} = 571.43 \times 10^{-6}$

Let P_c = portion of load carried by concrete

P_s = portion carried by six steel rods

$\delta = \frac{P_c L}{E_c A_c}, \quad P_c = E_c A_c \frac{\delta}{L} = E_c A_c \varepsilon$

$\delta = \frac{P_s L}{E_s A_s}, \quad P_s = E_s A_s \frac{\delta}{L} = E_s A_s \varepsilon$

$P = P_c + P_s = (E_c A_c + E_s A_s)\varepsilon$

$A_s = 6 \frac{\pi}{4} d_s^2 = \frac{6\pi}{4}(1.125)^2 = 5.964 \text{ in}^2$

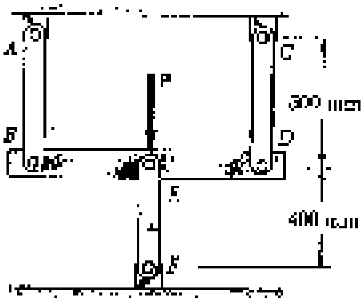
$A_c = \frac{\pi}{4} d_c^2 - A_s = \frac{\pi}{4}(18)^2 - 5.964 = 248.5 \text{ in}^2$

$P = [(4.2 \times 10^6)(248.5) + (29 \times 10^6)(5.964)](571.43 \times 10^{-6})$

$= 695 \times 10^3 \text{ lb} = 695 \text{ kips}$

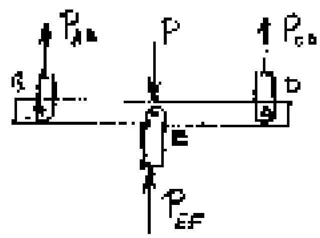
PROBLEM 2.39

2.39 Three steel rods (E = 200 GPa) support a 36-kN load P. Each of the rods AB and CD has a 200-mm² cross-sectional area and rod EF has a 625-mm² cross-sectional area. Determine the (a) the change in length of rod EF (b) the stress in each rod.



SOLUTION

Use member BED as a free body



By symmetry, or by $\Sigma M_E = 0$

$$P_{CD} = P_{AB}$$

$$\Sigma F_y = 0$$

$$P_{AB} + P_{CD} + P_{EF} - P = 0$$

$$P = 2P_{AB} + P_{EF}$$

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{E A_{AB}}, \quad \delta_{CD} = \frac{P_{CD} L_{CD}}{E A_{CD}}, \quad \delta_{EF} = \frac{P_{EF} L_{EF}}{E A_{EF}}$$

Since $L_{AB} = L_{CD}$ and $A_{AB} = A_{CD}$, $\delta_{AB} = \delta_{CD}$

Since points A, C, and E are fixed $\delta_B = \delta_{AB}$, $\delta_D = \delta_{CD}$, $\delta_E = \delta_{EF}$

Since member BED is rigid $\delta_E = \delta_B = \delta_C$

$$\frac{P_{AB} L_{AB}}{E A_{AB}} = \frac{P_{EF} L_{EF}}{E A_{EF}} \quad \therefore \quad P_{AB} = \frac{A_{AB}}{A_{EF}} \cdot \frac{L_{EF}}{L_{AB}} P_{EF} = \frac{200}{625} \cdot \frac{400}{500} P_{EF}$$

$$= 0.256\, P_{EF}$$

$$P = 2P_{AB} + P_{EF} = (2)(0.256)P_{EF} + P_{EF} = 1.512\, P_{EF}$$

$$P_{EF} = \frac{P}{1.512} = \frac{36 \times 10^3}{1.512} = 23.810 \times 10^3 \text{ N}$$

$$P_{AB} = P_{CD} = (0.256)(23.810 \times 10^3) = 6.095 \times 10^3 \text{ N}$$

(a) $$\delta = \delta_{EF} = \frac{(23.810 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(625 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m}$$

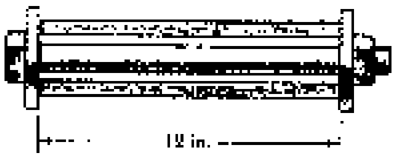
$$= 0.0762 \text{ mm}$$

or $$\delta = \delta_{AB} = \frac{(6.095 \times 10^3)(500 \times 10^{-3})}{(200 \times 10^9)(200 \times 10^{-6})} = 76.2 \times 10^{-6} \text{ m}$$

(b) $$\sigma_{AB} = \sigma_{CD} = \frac{P_{AB}}{A_{AB}} = \frac{6.095 \times 10^3}{200 \times 10^{-6}} = 30.5 \times 10^6 \text{ Pa} = 30.5 \text{ MPa}$$

$$\sigma_{EF} = -\frac{P_{EF}}{A_{EF}} = -\frac{23.810 \times 10^3}{625 \times 10^{-6}} = -38.1 \times 10^6 \text{ Pa} = -38.1 \text{ MPa}$$

PROBLEM 2.40



2.40 A brass bolt ($E_b = 15 \times 10^6$ psi) with a $\frac{3}{8}$-in. diameter is fitted inside a steel tube ($E_s = 29 \times 10^6$ psi) with a $\frac{7}{8}$-in. outer diameter and $\frac{1}{8}$-in. wall thickness. After the nut has been fit snugly, it is tightened one quarter of a full turn. Knowing that the bolt is single-threaded with a 0.1-in. pitch, determine the normal stress (a) in the bolt, (b) in the tube.

SOLUTION

The movement of the nut along the bolt after a quarter turn is equal to $\frac{1}{4} \times$ pitch.

$$\delta = (\tfrac{1}{4})(0.1) = 0.025 \text{ in}$$

Also $\delta = \delta_{bolt} + \delta_{tube}$ where δ_{bolt} = elongation of the bolt and δ_{tube} = shortening of the tube

Let P_{bolt} = axial tensile force in the bolt
P_{tube} = axial compressive force in the tube

For equilibrium of each end plate $P_{bolt} = P_{tube} = P$

$$A_{bolt} = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 0.11045 \text{ in}^2$$

$$A_{tube} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}\left(\left(\frac{7}{8}\right)^2 - \left(\frac{5}{8}\right)^2\right) = 0.29452 \text{ in}^2$$

$$\delta_{bolt} = \frac{P_{bolt} L}{E A_{bolt}} = \frac{(P)(12)}{(15\times10^6)(0.11045)} = 7.2431\times10^{-6}\, P$$

$$\delta_{tube} = \frac{P_{tube} L}{E A_{tube}} = \frac{(P)(12)}{(29\times10^6)(0.29452)} = 1.4050\times10^{-6}\, P$$

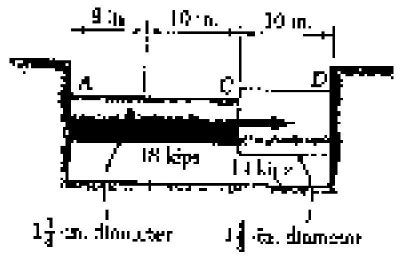
$$0.025 = 7.2431\times10^{-6} P + 1.4050\times10^{-6} P = 8.6481\times10^{-6} P$$

$$P = 2.8908\times10^3 \text{ lb.}$$

(a) $$\sigma_{bolt} = \frac{P}{A_{bolt}} = \frac{2.8908\times10^3}{0.11045} = 26.2\times10^3 \text{ psi} = 26.2 \text{ ksi}$$

(b) $$\sigma_{tube} = -\frac{P}{A_{tube}} = -\frac{2.8908\times10^3}{0.29452} = -9.82\times10^3 \text{ psi} = -9.82 \text{ ksi}$$

PROBLEM 2.41



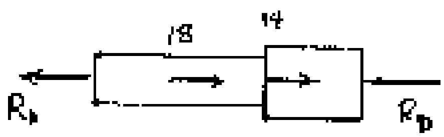
2.41 Two cylindrical rods, CD made of steel ($E = 29 \times 10^6$ psi) and AC made of aluminum ($E = 10.4 \times 10^6$ psi), are joined at B and restrained by rigid supports at A and D. Determine (*a*) the reactions at A and D, (*b*) the deflection of point C.

SOLUTION

AB: $\quad P = R_A, \quad L_{AB} = 8$ in

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.125)^2 = 0.99402 \text{ in}^2$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(8)}{(10.4\times10^6)(0.99402)} = 0.77386\times10^{-6}\, R_A$$



BC: $\quad P = R_A - 18\times10^3, \quad L = 10$ in, $\quad A = 0.99402$ in^2

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 18\times10^3)(10)}{(10.4\times10^6)(0.99402)} = 0.96732\times10^{-6} R_A - 17.412\times10^{-3}$$

CD: $\quad P = R_A - 18\times10^3 - 14\times10^3 = R_A - 32\times10^3$

$L = 10$ in $\qquad A = \frac{\pi}{4} d_{CD}^2 = \frac{\pi}{4}(1.625)^2 = 2.0739$ in^2

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 32\times10^3)(10)}{(29\times10^6)(2.0739)} = 0.16627\times10^{-6} R_A - 5.321\times10^{-3}$$

$$\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 1.9076\times10^{-6} R_A - 22.733\times10^{-3}$$

Since point D cannot move relative to A $\quad \delta_{AD} = 0$

(a) $\quad 1.9075\times10^{-6} R_A - 22.735\times10^{-3} = 0 \qquad R_A = 11.92\times10^3$ lb $\leftarrow$

$$R_D = 32\times10^3 - R_A = 20.08\times10^3 \text{ lb} \leftarrow$$

(b) $\quad \delta_C = \delta_{AB} + \delta_{BC}$

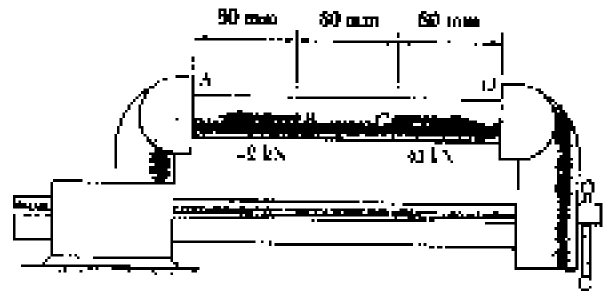
$$= 1.7412\times10^{-6} R_A - 17.412\times10^{-3}$$

$$= (1.7412\times10^{-6})(11.92\times10^3) - 17.412\times10^{-3} = 3.34\times10^{-3} \text{ in}$$

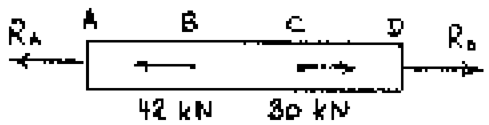
or $\quad \delta_C = \dfrac{R_D L_{CD}}{E_{CD} A_{CD}} = \dfrac{(20.08\times10^3)(10)}{(29\times10^6)(2.0739)} = 3.34\times10^{-3}$ in.

PROBLEM 2.42

2.42 A steel tube ($E = 200$ GPa) with a 32-mm outer diameter and a 4-mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine (*a*) the forces exerted by the vise on the tube at *A* and *D*, (*b*) the change in length of the portion *BC* of the tube.



SOLUTION



For the tube $d_i = d_o - 2t$
$= 32 - (2)(4) = 24$ mm

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(32^2 - 24^2)$$
$$= 351.86 \text{ mm}^2 = 351.86 \times 10^{-6} \text{ m}^2$$

AB: $P = R_A$, $L = 0.080$ m

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.080)}{(200\times10^9)(351.86\times10^{-6})} = 1.1368\times10^{-9} R_A$$

BC: $P = R_A + 42\times10^3$, $L = 0.080$ m

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A + 42\times10^3)(10)}{(200\times10^9)(351.86\times10^{-6})} = 1.1368\times10^{-9} R_A + 47.746\times10^{-6}$$

CD: $P = R_A + 12\times10^3$, $L = 0.080$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A + 12\times10^3)(0.080)}{(200\times10^9)(351.86\times10^{-6})} = 1.1368\times10^{-9} R_A + 13.642\times10^{-6}$$

Total: $\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 3.4104\times10^{-9} R_A + 61.388\times10^{-6}$

Given jaw movement $\delta_{AD} = -0.2 \text{ mm} = 0.2\times10^{-3}$ m

(a) $-0.2\times10^{-3} = 3.4104\times10^{-9} R_A + 61.388\times10^{-6}$ $\therefore$ $R_A = -76.6\times10^3$ N
$= -76.6$ kN

$R_D = R_A + 12\times10^3$ $R_D = -64.6\times10^3$ N
$= -64.6$ kN

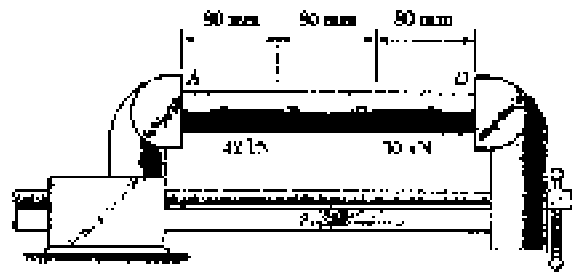
(b) $\delta_{BC} = (1.1368\times10^{-9})(-76.6\times10^3) + 47.746\times10^{-6} = -39.4\times10^{-6}$ m
$= -0.0394$ mm

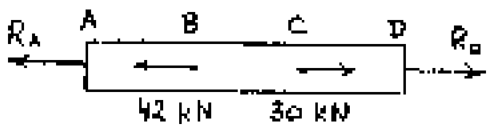
PROBLEM 2.43

SOLUTION

2.42 A steel tube ($E = 200$ GPa) with a 32-mm outer diameter and a 4-mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine (*a*) the forces exerted by the vise on the tube at *A* and *D*, (*b*) the change in length of the portion *BC* of the tube.

2.43 Solve Prob. 2.42, assuming that after the forces have been applied, the vise is adjusted to decrease the distance between its jaws by 0.1 mm.





For the tube $d_i = d_o - 2t$
$= 32 - (2)(4) = 24$ mm

$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi}{4}\left(32^2 - 24^2\right)$$
$$= 351.86 \text{ mm}^2 = 351.86 \times 10^{-6} \text{ m}^2$$

AB: $P = R_A$, $L = 0.080$ m

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.080)}{(200\times10^9)(351.86\times10^{-6})} = 1.1368\times10^{-9}\,R_A$$

BC: $P = R_A + 42\times10^3$, $L = 0.080$ m

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A + 42\times10^3)(0.080)}{(200\times10^9)(351.86\times10^{-6})} = 1.1368\times10^{-9}\,R_A + 47.746\times10^{-6}$$

CD: $P = R_A + 12\times10^3$, $L = 0.080$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A + 12\times10^3)(0.080)}{(200\times10^9)(351.86\times10^{-6})} = 1.1368\times10^{-9}\,R_A + 13.642\times10^{-6}$$

Total: $\delta_{AD} = \delta_{AB} + \delta_{BC} + \delta_{CD} = 3.4104\times10^{-9}\,R_A + 61.388\times10^{-6}$

Due to the movement of the jaws $\delta_{AD} = -0.1$ mm $= -0.1\times10^{-3}$ m

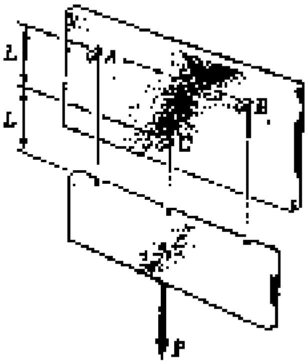
(a) $-0.1\times10^{-3} = 3.4104\times10^{-9}\,R_A + 61.388\times10^{-6}$ $\qquad R_A = -47.322\times10^3$ N
$= -47.3$ kN ◀

$R_D = R_A + 12\times10^3$ $\qquad = -35.3\times10^3$ N
$= -35.3$ kN ◀

(b) $\delta_{BC} = (1.1368\times10^{-9})(-47.322\times10^3) + 47.746\times10^{-6} = -6.05\times10^{-6}$ m
$= -0.00605$ mm ◀

PROBLEM 2.44

2.44 Three wires are used to suspend the plate shown. Aluminum wires are used at A and B with a diameter of $\frac{1}{8}$ in. and a steel wire is used at C with a diameter of $\frac{1}{12}$ in. Knowing that the allowable stress for aluminum ($E = 10.4 \times 10^6$ psi) is 14 ksi and that the allowable stress for steel ($E = 29 \times 10^6$ psi) is 18 ksi, determine the maximum load P that may be applied.



SOLUTION

By symmetry $P_A = P_B$, and $\delta_A = \delta_B$

Also, $\delta_C = \delta_A = \delta_B = \delta$

Strain in each wire

$$\varepsilon_A = \varepsilon_B = \frac{\delta}{2L}, \qquad \varepsilon_C = \frac{\delta}{L} = 2\varepsilon_A$$

Determine allowable strain

A & B $\qquad \varepsilon_A = \frac{\sigma_A}{E_A} = \frac{14 \times 10^3}{10.4 \times 10^6} = 1.3462 \times 10^{-3}$

$$\varepsilon_C = 2\varepsilon_A = 2.6924 \times 10^{-3}$$

C $\qquad \varepsilon_C = \frac{\sigma_C}{E_C} = \frac{18 \times 10^3}{29 \times 10^6} = 0.6207 \times 10^{-3}$

$$\varepsilon_A = \varepsilon_B = \tfrac{1}{2}\varepsilon_C = 0.3103 \times 10^{-3}$$

Allowable strain for wire C governs $\therefore$ $\sigma_C = 18 \times 10^3$ psi

$\sigma_A = E_A \varepsilon_A \qquad P_A = A_A E_A \varepsilon_A = \frac{\pi}{4}\left(\frac{1}{8}\right)^2 (10.4 \times 10^6)(0.3103 \times 10^{-3})$

$$= 39.61 \text{ lb}$$

$$P_B = 39.61 \text{ lb}$$

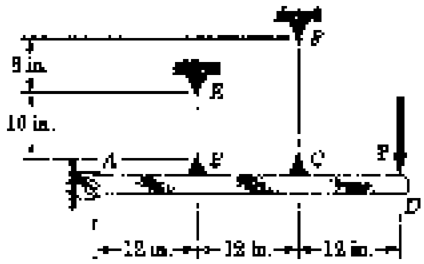
$\sigma_C = E_C \varepsilon_C \qquad P_C = A_C \sigma_C = \frac{\pi}{4}\left(\frac{1}{12}\right)^2 (18 \times 10^3) = 98.17$ lb.

For equilibrium of the plate

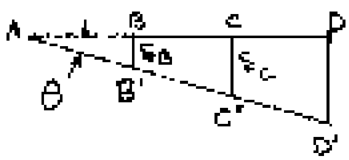
$$P = P_A + P_B + P_C = 177.4 \text{ lb}$$

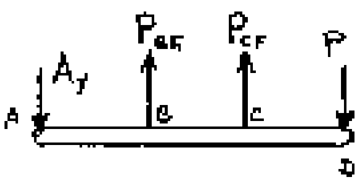
PROBLEM 2.45

2.45 The rigid bar AD is supported by two steel wires of $\frac{1}{16}$-in. diameter ($E = 29 \times 10^6$ psi) and a pin and bracket at D. Knowing that the wires were initially taught, determine (a) the additional tension in each wire when a 220-lb load **P** is applied at D, (b) the corresponding deflection of point D.



SOLUTION





Let θ be the rotation of bar ABCD

Then $\delta_B = 12\theta$

$\delta_C = 24\theta$

$$\delta_B = \frac{P_{BE} L_{BE}}{AE}$$

$$P_{BE} = \frac{EA\delta_{BE}}{L_{BE}} = \frac{(29 \times 10^6)\frac{\pi}{4}(\frac{1}{16})^2(12\theta)}{10}$$

$$= 106.77 \times 10^3\ \theta$$

$$\delta_C = \frac{P_{CF} L_{CF}}{EA}$$

$$P_{CF} = \frac{EA\delta_{CF}}{L_{CF}} = \frac{(29 \times 10^6)\frac{\pi}{4}(\frac{1}{16})^2(24\theta)}{18}$$

$$= 118.63 \times 10^3\ \theta$$

Using free body ABCD

$\circlearrowright \Sigma M_A = 0 \qquad 12 P_{BE} + 24 P_{CF} - 36 P = 0$

$$(12)(106.77 \times 10^3\ \theta) + (24)(118.63 \times 10^3\ \theta) - (36)(220) = 0$$

$$4.1283 \times 10^6\ \theta = (36)(220)$$

$$\theta = 1.9185 \times 10^{-3} \text{ rad}$$

(a) $P_{BE} = (106.77 \times 10^3)(1.9185 \times 10^{-3}) = 204.8$ lb ◄

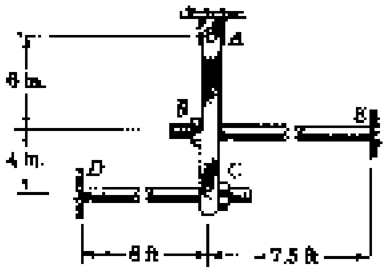
$P_C = (118.63 \times 10^3)(1.9185 \times 10^{-3}) = 227.6$ lb ◄

(b) $\delta_D = 36\ \theta = (36)(1.9185 \times 10^{-3}) = 69.1 \times 10^{-3}$ in

$= 0.0691$ in. ◄

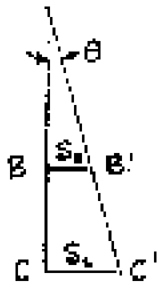
PROBLEM 2.46

2.46 The steel rods BE and CD each have a diameter of $\frac{5}{8}$ in. ($E = 29 \times 10^6$ psi). The ends are threaded with a pitch of 0.1 in. Knowing that after being snugly fit, the nut at B is tightened one full turn, determine (*a*) the tension in rod CD, (*b*) the deflection of point C of the rigid member ABC.



SOLUTION

Let θ be the rotation of bar ABC as shown.



Then, $\delta_B = 6\theta$ and $\delta_C = 10\theta$

But $\delta_B = \delta_{turn} - \dfrac{P_{BE} L_{BE}}{E_{BE} A_{BE}}$

$$P_{BE} = (E_{BE} A_{BE})(\delta_{turn} - \delta_B)/L_{BE}$$

$L_{BE} = 7.5 \text{ ft} = 90 \text{ in.}, \quad \delta_{turn} = 0.1 \text{ in}$

$$A_{BE} = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{5}{8}\right)^2 = 0.3068 \text{ in}^2$$

$$P_{BE} = \frac{(29\times10^6)(0.3068)(0.1 - 6\theta)}{90}$$

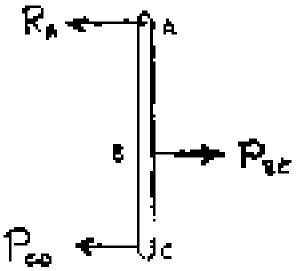
$$= 9.886 \times 10^3 - 593.15 \times 10^3\, \theta$$

$$\delta_C = \frac{P_{CD} L_{CD}}{E A_{CD}} \quad \therefore \quad P_{CD} = \frac{E A \delta_C}{L_{CD}}$$

$L_{CD} = 6 \text{ ft} = 72 \text{ in}, \quad A_{CD} = 0.3068 \text{ in}^2$

$$P_{CD} = \frac{(29\times10^6)(0.3068)(10\theta)}{72}$$

$$= 1.23572 \times 10^6\, \theta$$



$\circlearrowleft \Sigma M_A = 0 \qquad 6P_{BE} - 10P_{CD} = 0$

$$(6)(9.886\times10^3 - 593.16\times10^3\,\theta) - (10)(1.23572\times10^6)\,\theta = 0$$

$$59.316\times10^3 - 15.916\times10^6\,\theta = 0 \qquad \theta = 3.7268\times10^{-3} \text{ rad}$$

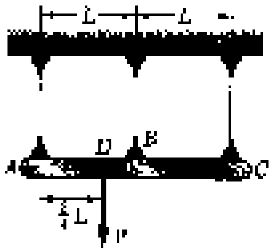
(a) $P_{CD} = (1.23572\times10^6)(3.7268\times10^{-3}) = 4.61\times10^3$

$= 4.61$ kips ◄

(b) $\delta_C = 10\,\theta = (10)(3.7268\times10^{-3}) = 37.3\times10^{-3}$ in

$= 0.0373$ in ◄

PROBLEM 2.47



2.47 The rigid rod $ABCD$ is suspended from three wires of the same material. The cross-sectional area of the wire at B is equal to half of the cross-sectional area of the wires A and C. Determine the tension in each wire caused by the load **P**.

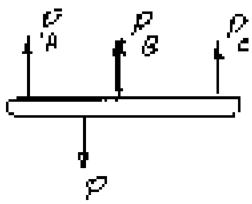
SOLUTION

$$+\circlearrowleft \Sigma M_A = 0 \qquad 2LP_C + LP_B - \frac{3}{4}LP = 0$$

$$P_C = \frac{3}{8}P - \frac{1}{2}P_B$$

$$+\circlearrowleft M_C = 0 \qquad -2LP_A - LP_B + \frac{5}{4}LP = 0$$

$$P_A = \frac{5}{8}P - \frac{1}{2}P_B$$

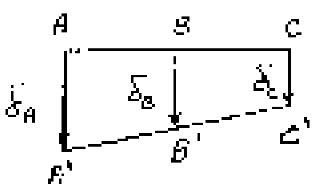


LET ℓ BE THE LENGTH OF THE WIRES

$$\delta_A = \frac{P_A \ell}{EA} = \frac{\ell}{EA}\left(\frac{5}{8}P - \frac{1}{2}P_B\right)$$

$$\delta_B = \frac{P_B \ell}{E(A/2)} = \frac{2\ell}{EA}P_B$$

$$\delta_C = \frac{P_C \ell}{EA} = \frac{\ell}{EA}\left(\frac{3}{8}P - \frac{1}{2}P_B\right)$$



FROM THE DEFORMATION DIAGRAM

$$\delta_A - \delta_B = \delta_B - \delta_C$$

OR $\quad \delta_B = \frac{1}{2}(\delta_A + \delta_C)$

$$\frac{\ell}{E(A/2)}P_B = \frac{1}{2}\frac{\ell}{EA}\left(\frac{5}{8}P - \frac{1}{2}P_B + \frac{3}{8}P - \frac{1}{2}P_B\right)$$

$\frac{5}{2}P_B = \frac{1}{2}P \; ; \quad P_B = \frac{1}{5}P \qquad P_B = 0.200P$

$P_A = \frac{5}{8}P - \frac{1}{2}\left(\frac{P}{5}\right) = \frac{21}{40}P \qquad P_A = 0.525P$

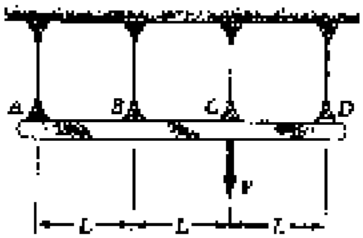
$P_C = \frac{3}{8}P - \frac{1}{2}\left(\frac{P}{5}\right) = \frac{11}{40}P \qquad P_C = 0.275P$

CHECK:

$P_A + P_B + P_C = 1.000P \quad$ OK

PROBLEM 2.48

2.48 The rigid bar *ABCD* is suspended from four identical wires. Determine the tension in each wire caused by the load **P**.



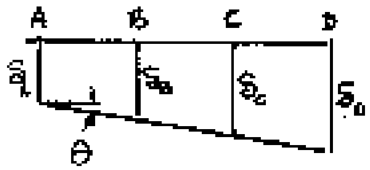
SOLUTION

Let θ be the slope of bar ABCD after deformation

$$\delta_B = \delta_A + L\theta$$

$$\delta_C = \delta_A + 2L\theta$$

$$\delta_D = \delta_A + 3L\theta$$

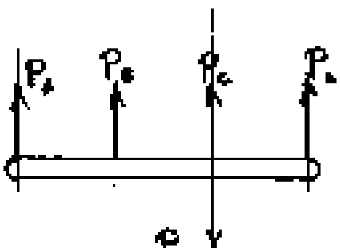


$$P_A = \frac{EA}{\ell}\delta_A$$

$$P_B = \frac{EA}{\ell}\delta_B = \frac{EA}{\ell}\delta_A + \frac{EAL}{\ell}\theta$$

$$P_C = \frac{EA}{\ell}\delta_C = \frac{EA}{\ell}\delta_A + \frac{2EAL}{\ell}\theta$$

$$P_D = \frac{EA}{\ell}\delta_D = \frac{EA}{\ell}\delta_A + \frac{3EAL}{\ell}\theta$$



$+\uparrow \Sigma F_y = 0$

$$P_A + P_B + P_C + P_D - P = 0$$

$$\frac{4EA}{\ell}\delta_A + \frac{6EA}{\ell}L\theta = P$$

$$4\delta_A + 6L\theta = \frac{P\ell}{EA} \qquad (1)$$

$\circlearrowleft \Sigma M_A = 0$

$$LP_B + 2LP_C + 3LP_D - 2LP = 0$$

$$\frac{6EAL}{\ell}\delta_A + \frac{14EAL}{\ell}L\theta = 2LP$$

$$6\delta_A + 14L\theta = \frac{2P\ell}{EA} \qquad (2)$$

Solving (1) and (2) simultaneously

$$L\theta = \frac{1}{10}\frac{P\ell}{EA}$$

$$\delta_A = \frac{1}{10}\frac{P\ell}{EA}$$

$$P_A = \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} = \frac{1}{10}P$$

$$P_B = \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} + \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} = \frac{1}{5}P$$

$$P_C = \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} + 2\frac{EA}{\ell}\frac{1}{10}\frac{P\ell}{EA} = \frac{3}{10}P$$

$$P_D = \frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} + 3\frac{EA}{\ell}\cdot\frac{1}{10}\frac{P\ell}{EA} = \frac{2}{5}P$$

PROBLEM 2.49

2.49 A steel railroad track ($E = 200$ GPa, $\alpha = 11.7 \times 10^{-6}/^\circ C$) was laid out at a temperature of 6°C. Determine the normal stress in the rails when the temperature reaches 48 °C, assuming that the rails (*a*) are welded to form a continuous track, (*b*) are 10 m long with 3-mm gaps between them.

SOLUTION

(a) $\delta_T = \alpha(\Delta T)L = (11.7\times10^{-6})(48-6)(10) = 4.914\times10^{-3}$ m

$$\delta_P = \frac{PL}{AE} = \frac{L\sigma}{E} = \frac{(10)\sigma}{200\times10^{9}} = 50\times10^{-12}\,\sigma$$

$$\delta = \delta_T + \delta_P = 4.914\times10^{-3} + 50\times10^{-12}\,\sigma = 0 \quad \therefore \; \sigma = -98.3\times10^{6}\ \text{Pa}$$

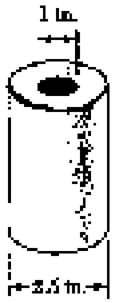
$$= -98.3\ \text{MPa} \blacktriangleleft$$

(b) $\delta = \delta_T + \delta_P = 4.914\times10^{-3} + 50\times10^{-12}\,\sigma = 3\times10^{-3}$

$$\sigma = \frac{3\times10^{-3} - 4.914\times10^{-3}}{50\times10^{-12}} = -38.3\times10^{6}\ \text{Pa} = -38.3\ \text{MPa} \blacktriangleleft$$

PROBLEM 2.50

2.50 The aluminum shell is fully bonded to the brass core, and the assembly is unstressed at a temperature of 78°F. Considering only axial deformations, determine the stress when the temperature reaches 180 °F (*a*) in the brass core, (*b*) in the aluminum shell.



Brass core
$E = 15 \times 10^6$ psi
$\alpha = 11.6 \times 10^{-6}/°F$

Aluminum shell
$E = 10.6 \times 10^6$ psi
$\alpha = 12.9 \times 10^{-6}/°F$

SOLUTION

$\Delta T = 180 - 78 = 102$ F

Let P_b be the tensile force developed in the brass core

For equilibrium with zero total force, the compressive force in the aluminum shell is P_b

Strains $\quad \varepsilon_b = \dfrac{P_b}{E_b A_b} + \alpha_b(\Delta T), \qquad \varepsilon_a = -\dfrac{P_b}{E_a A_a} + \alpha_a(\Delta T)$

Matching $\quad \varepsilon_b = \varepsilon_a \qquad \dfrac{P_b}{E_b A_b} + \alpha_b(\Delta T) = -\dfrac{P_b}{E_a A_a} + \alpha_a(\Delta T)$

$$\left(\frac{1}{E_b A_b} + \frac{1}{E_a A_a}\right) P_b = (\alpha_a - \alpha_b)(\Delta T)$$

$A_b = \frac{\pi}{4} d^2 = \frac{\pi}{4}(1)^2 = 0.7854 \text{ in}^2$

$A_a = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(2.5^2 - 1.0^2) = 4.1233 \text{ in}^2$

$\alpha_a - \alpha_b = 1.3 \times 10^{-6} /°F$

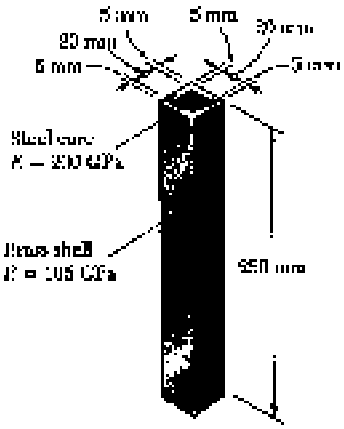
$$\left[\frac{1}{(15 \times 10^6)(0.7854)} + \frac{1}{(10.6 \times 10^6)(4.1233)}\right] P_b = (1.3 \times 10^{-6})(102)$$

$P_b = 1.2305 \times 10^3$ lb

$\sigma_b = \dfrac{P_b}{A_b} = \dfrac{1.2305 \times 10^3}{0.7854} = 1.567 \times 10^3 \text{ psi} = 1.567 \text{ ksi}$ ◄

$\sigma_a = -\dfrac{P_b}{A_a} = -\dfrac{1.2305 \times 10^3}{4.1233} = -0.298 \times 10^3 \text{ psi} = -0.298 \text{ ksi}$ ◄

PROBLEM 2.51



2.51 The brass shell ($\alpha_b = 20.9 \times 10^{-6}/°C$) is fully bonded to the steel core ($\alpha_s = 11.7 \times 10^{-6}/°C$). Determine the largest allowable increase in temperature if the stress in the steel core is not to exceed 55 MPa.

SOLUTION

Let P_s = axial force developed in the steel core

For equilibrium with zero total force, the compressive force in the brass shell is P_s.

Strains
$$\varepsilon_s = \frac{P_s}{E_s A_s} + \alpha_s(\Delta T)$$
$$\varepsilon_b = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

Matching $\varepsilon_s = \varepsilon_b$

$$\frac{P_s}{E_s A_s} + \alpha_s(\Delta T) = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

$$\left(\frac{1}{E_s A_s} + \frac{1}{E_b A_b}\right) P_s = (\alpha_b - \alpha_s)(\Delta T)$$

$$A_s = (0.020)(0.020) = 400 \times 10^{-6} \text{ m}^2$$

$$A_b = (0.030)(0.030) - (0.020)(0.020) = 500 \times 10^{-6} \text{ m}^2$$

$$\alpha_b - \alpha_s = 9.2 \times 10^{-6} /°C$$

$$P_s = \sigma_s A_s = (55 \times 10^6)(400 \times 10^{-6}) = 22 \times 10^3 \text{ N}$$

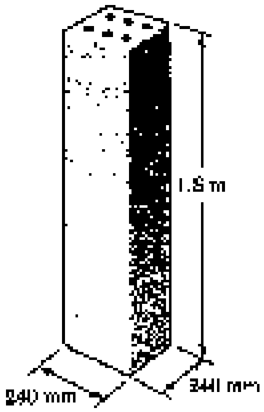
$$\frac{1}{E_s A_s} + \frac{1}{E_b A_b} = \frac{1}{(200 \times 10^9)(400 \times 10^{-6})} + \frac{1}{(105 \times 10^9)(500 \times 10^{-6})} = 31.55 \times 10^{-9} \text{ N}^{-1}$$

$$(31.55 \times 10^{-9})(22 \times 10^3) = (9.2 \times 10^{-6})(\Delta T)$$

$$\Delta T = 75.4 \text{ °C}$$

PROBLEM 2.52

2.52 The concrete post ($E_c = 25$ GPa and $\alpha_c = 9.9 \times 10^{-6}/°C$) is reinforced with six steel bars, each of 22-mm diameter ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 35°C.



SOLUTION

$$A_s = 6\frac{\pi}{4}d^2 = 6\cdot\frac{\pi}{4}(22)^2 = 2.2808\times10^3\ \text{mm}^2 = 2.2808\times10^{-3}\ \text{m}^2$$

$$A_c = 240^2 - A_s = 240^2 - 2.2808\times10^3 = 55.32\times10^3\ \text{mm}^2 = 55.32\times10^{-3}\ \text{m}^2$$

Let P_c = tensile force developed in the concrete

For equilibrium with zero total force, the compressive force in the six steel rods is P_c

Strains: $\varepsilon_s = -\dfrac{P_c}{E_s A_s} + \alpha_s(\Delta T)$; $\varepsilon_c = \dfrac{P_c}{E_c A_c} + \alpha_c(\Delta T)$

Matching: $\varepsilon_c = \varepsilon_s$ $\quad \dfrac{P_c}{E_c A_c} + \alpha_c(\Delta T) = -\dfrac{P_c}{E_s A_s} + \alpha_s(\Delta T)$

$$\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s}\right)P_c = (\alpha_s - \alpha_c)(\Delta T)$$

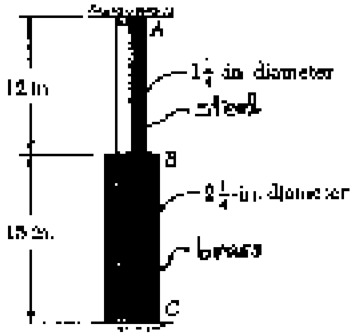
$$\left[\frac{1}{(25\times10^9)(55.32\times10^{-3})} + \frac{1}{(200\times10^9)(2.2808\times10^{-3})}\right]P_c = (1.8\times10^{-6})(35)$$

$$P_c = 21.61\times10^3\ \text{N}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{21.61\times10^3}{55.32\times10^{-3}} = 0.391\times10^6\ \text{Pa} = 0.391\ \text{MPa}$$

$$\sigma_s = -\frac{P_c}{A_s} = \frac{21.61\times10^3}{2.2808\times10^{-3}} = -9.47\times10^6\ \text{Pa} = -9.47\ \text{MPa}$$

PROBLEM 2.53



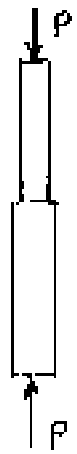
2.53 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°F$) and portion BC is made of brass ($E_b = 15 \times 10^6$ psi, $\alpha_b = 10.4 \times 10^{-6}/°F$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions AB and BC by a temperature rise of 65°F, (*b*) the corresponding deflection of point B.

SOLUTION

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.25)^2 = 1.2272 \text{ in}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2$$

Free thermal expansion



$$\delta_T = L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)$$

$$= (12)(6.5\times10^{-6})(65) + (15)(10.4\times10^{-6})(65)$$

$$= 15.21\times10^{-3} \text{ in}$$

Shortening due to induced compressive force P

$$\delta_P = \frac{PL_{AB}}{E_s A_{AB}} + \frac{PL_{BC}}{E_b A_{BC}}$$

$$= \frac{12\,P}{(29\times10^6)(1.2272)} + \frac{15\,P}{(15\times10^6)(3.9761)} = 588.69\times10^{-9}\,P$$

For zero net deflection $\quad \delta_P = \delta_T$

$$(588.69\times10^{-9})P = 15.21\times10^{-3}$$

$$P = 25.84\times10^3 \text{ lb}$$

(a) $$\sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{25.84\times10^3}{1.2272} = -21.1\times10^3 \text{ psi} = -21.1 \text{ ksi}$$

$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{25.84\times10^3}{3.9761} = -6.50\times10^3 \text{ psi} = -6.50 \text{ ksi}$$

(b) $$\delta_B = +\frac{PL_{AB}}{E_s A_{AB}} - L_{AB}\alpha_s(\Delta T)$$

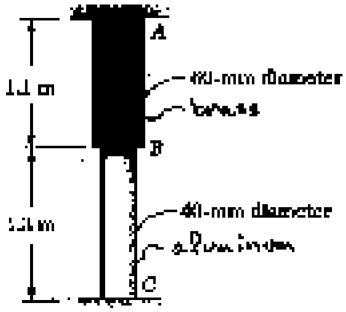
$$= +\frac{(25.84\times10^3)(12)}{(29\times10^6)(1.2272)} - (12)(6.5\times10^{-6})(65) = +3.64\times10^{-3} \text{ in} \uparrow$$

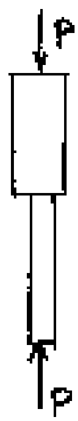
i.e. 3.64×10^{-3} in $\uparrow$

$= 0.00364$ in $\uparrow$

PROBLEM 2.54

2.54 A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of brass ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) and portion *BC* is made of aluminum ($E_a = 72$ GPa, $\alpha_a = 23.9 \times 10^{-6}/°C$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions *AB* and *BC* by a temperature rise of 42°C, (*b*) the corresponding deflection of point *B*.





SOLUTION

$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(60)^2 = 2.8274 \times 10^3 \text{ mm}^2 = 2.8274 \times 10^{-3} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(40)^2 = 1.2566 \times 10^3 \text{ mm}^2 = 1.2566 \times 10^{-3} \text{ m}^2$$

Free thermal expansion

$$\delta_T = L_{AB}\alpha_b(\Delta T) + L_{BC}\alpha_a(\Delta T)$$
$$= (1.1)(20.9 \times 10^{-6})(42) + (1.3)(23.9 \times 10^{-6})(42)$$
$$= 2.2705 \times 10^{-3} \text{ m}$$

Shortening due to induced compressive force

$$\delta_P = \frac{P L_{AB}}{E_b A_{AB}} + \frac{P L_{BC}}{E_a A_{BC}}$$
$$= \frac{1.1\, P}{(105 \times 10^9)(2.8274 \times 10^{-3})} + \frac{1.3\, P}{(72 \times 10^9)(1.2566 \times 10^{-3})}$$
$$= 18.074 \times 10^{-9}\, P$$

For zero net deflection $\quad \delta_P = \delta_T$

$$18.074 \times 10^{-9}\, P = 2.2705 \times 10^{-3}$$
$$P = 125.62 \times 10^3 \text{ N}$$

(a) $$\sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{125.62 \times 10^3}{2.8274 \times 10^{-3}} = -44.4 \times 10^6 \text{ Pa} = -44.4 \text{ MPa}$$

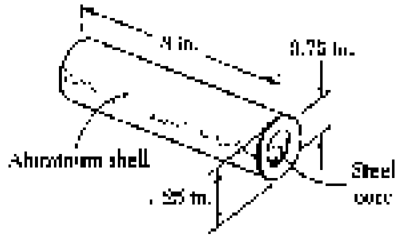
$$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{125.62 \times 10^3}{1.2566 \times 10^{-3}} = -100.0 \times 10^6 \text{ Pa} = -100.0 \text{ MPa}$$

(b) $$\delta_B = +\frac{P L_{AB}}{E_b A_{AB}} - L_{AB}\alpha_b(\Delta T)$$
$$= \frac{(125.62 \times 10^3)(1.1)}{(105 \times 10^9)(2.8274 \times 10^{-3})} - (1.1)(20.9 \times 10^{-6})(42)$$
$$= -500 \times 10^{-6} \text{ m} = -0.500 \text{ mm}$$

i.e. 0.500 mm ↓

PROBLEM 2.55

2.55 The assembly shown consists of an aluminum shell ($E_a = 10.6 \times 10^6$ psi, $\alpha_a = 12.9 \times 10^{-6}/°\text{F}$) fully bonded to a steel core ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F}$) and is unstressed. Determine (*a*) the largest allowable change in temperature if the stress in the aluminum shell is not to exceed 6 ksi, (*b*) the corresponding change in length of the assembly.



SOLUTION

Since $\alpha_a > \alpha_s$, the shell is in compression for a positive temperature rise

Let $\sigma_a = -6$ ksi $= -6 \times 10^3$ psi

$$A_a = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(1.25^2 - 0.75^2) = 0.78540 \text{ in}^2$$

$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2$$

$$P = -\sigma_a A_a = \sigma_s A_s$$ where P is the tensile force in the steel core.

$$\sigma_s = -\frac{\sigma_a A_a}{A_s} = \frac{(6 \times 10^3)(0.78540)}{0.44179} = 10.667 \times 10^3 \text{ psi}$$

$$\varepsilon = \frac{\sigma_s}{E_s} + \alpha_s(\Delta T) = \frac{\sigma_a}{E_a} + \alpha_a(\Delta T)$$

$$(\alpha_a - \alpha_s)(\Delta T) = \frac{\sigma_s}{E_s} - \frac{\sigma_a}{E_a}$$

$$(6.4 \times 10^{-6})(\Delta T) = \frac{10.667 \times 10^3}{29 \times 10^6} + \frac{6 \times 10^3}{10.6 \times 10^6} = 0.93385 \times 10^{-3}$$

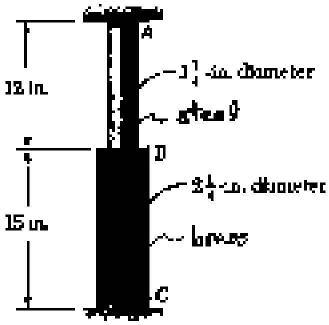
(a) $$\Delta T = 145.91\ °\text{F}$$

(b) $$\varepsilon = \frac{10.667 \times 10^3}{29 \times 10^6} + (6.5 \times 10^{-6})(145.91) = 1.3163 \times 10^{-3}$$

or $$\varepsilon = \frac{-6 \times 10^3}{10.6 \times 10^6} + (12.9 \times 10^{-6})(145.91) = 1.3163 \times 10^{-3}$$

$$\delta = L\varepsilon = (8.0)(1.3163 \times 10^{-3}) = 0.01053 \text{ in.}$$

PROBLEM 2.56



2.55 A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°F$) and portion BC is made of brass ($E_b = 15 \times 10^6$ psi, $\alpha_b = 10.4 \times 10^{-6}/°F$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions AB and BC by a temperature rise of 65°F, (*b*) the corresponding deflection of point B.

2.56 For the rod of Prob. 2.55, determine the maximum allowable temperature change if the stress in the steel portion AB is not to exceed 18 ksi and if the stress in the brass portion CB is not to exceed 7 ksi.

SOLUTION

Allowable force in each portion

AB: $\sigma_{AB} = -18 \text{ ksi} = -18 \times 10^3 \text{ psi}, \quad A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(1.25)^2 = 1.2272 \text{ in}^2$

$P = \sigma_{AB} A_{AB} = (-18 \times 10^3)(1.2272) = -22.090 \times 10^3 \text{ lb}$

BC: $\sigma_{BC} = -7 \text{ ksi} = -7 \times 10^3 \text{ psi}, \quad A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2$

$P = \sigma_{BC} A_{BC} = (-7 \times 10^3)(3.9761) = -27.833 \times 10^3 \text{ lb}$

Smaller absolute value governs $\therefore$ $P = -22.090 \times 10^3$ lb.

Deformation due to P

$$\delta_P = \frac{P L_{AB}}{E_{AB} A_{AB}} + \frac{P L_{BC}}{E_{BC} A_{BC}} = -\frac{(22.090 \times 10^3)(12)}{(29 \times 10^6)(1.2272)} - \frac{(22.090 \times 10^3)(15)}{(15 \times 10^6)(3.9761)}$$

$$= -13.004 \times 10^{-3} \text{ in}$$

Free thermal expansion

$$\delta_T = L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T) = (12)(6.5 \times 10^{-6})(\Delta T) + (15)(10.4 \times 10^{-6})(\Delta T)$$

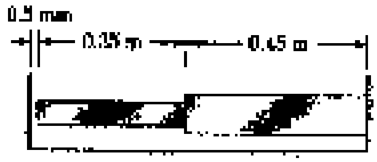
$$= (234 \times 10^{-6})(\Delta T)$$

Total deformation is zero

$$\delta_T + \delta_P = (234 \times 10^{-6})(\Delta T) - 13.004 \times 10^{-3} = 0$$

$$\Delta T = 55.6\ °F$$

PROBLEM 2.57



Bronze
$A = 1500\ mm^2$
$E = 105$ GPa
$\alpha = 21.6 \times 10^{-6}/°C$

Aluminum
$A = 1800\ mm^2$
$E = 73$ GPa
$\alpha = 23.2 \times 10^{-6}/°C$

2.57 Determine (*a*) the compressive force in the bars shown after a temperature rise of 96 °C, (*b*) the corresponding change in length of the bronze bar.

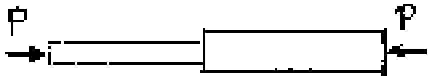
SOLUTION

Calculate free thermal expansion

$$\delta_T = L_b \alpha_b (\Delta T) + L_a \alpha_a \Delta T$$

$$= (0.35)(21.6 \times 10^{-6})(96) + (0.45)(23.2 \times 10^{-6})(96)$$

$$= 1.728 \times 10^{-3}\ m$$



Constrained expansion

$$\delta = 0.5\ mm = 0.500 \times 10^{-3}\ m$$

Shortening due to induced compressive force P

$$\delta_P = 1.728 \times 10^{-3} - 0.500 \times 10^{-3} = 1.228 \times 10^{-3}\ m$$

But, in terms of P

$$\delta_P = \frac{P L_b}{A_b E_b} + \frac{P L_a}{A_a E_a} = \left(\frac{L_b}{A_b E_b} + \frac{L_a}{A_a E_a} \right) P$$

$$= \left(\frac{0.35}{(1500 \times 10^{-6})(105 \times 10^9)} + \frac{0.45}{(1800 \times 10^{-6})(73 \times 10^9)} \right) P$$

$$= 5.6496 \times 10^{-9}\ P$$

(a) Equating $\quad 5.6496 \times 10^{-9}\ P = 1.228 \times 10^{-3} \quad \therefore P = 217.46 \times 10^3\ N$

$$= 217\ kN$$

(b) $$\delta_b = L_b \alpha_b (\Delta T) - \frac{P L_b}{A_b E_b}$$

$$= (0.35)(21.6 \times 10^{-6})(96) - \frac{(217.46 \times 10^3)(0.35)}{(1500 \times 10^{-6})(105 \times 10^9)}$$

$$= 725.76 \times 10^{-6} - 483.24 \times 10^{-6} = 242.5 \times 10^{-6}\ m$$

$$= 0.2425\ mm$$

PROBLEM 2.58

Bronze
$A = 1500\ mm^2$
$E = 105$ GPa
$\alpha = 21.6 \times 10^{-6}/°C$

Aluminum
$A = 1800\ mm^2$
$E = 73$ GPa
$\alpha = 23.2 \times 10^{-6}/°C$

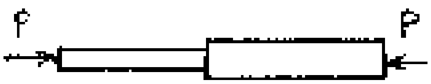
2.58 Knowing that a 0.5-mm gap exists when the temperature is 20 °C, determine (*a*) the temperature at which the normal stress in the aluminum bar will be equal to −90 MPa, (*b*) the corresponding exact length of the aluminum bar.

SOLUTION

$\sigma_a = -90 \times 10^6$ Pa $\qquad A_a = 1800 \times 10^{-6}\ m^2$

$P = -\sigma_a A_a = (90 \times 10^6)(1800 \times 10^{-6}) = 162 \times 10^3$ N

Shortening due to P



$$\delta_P = \frac{P L_b}{E_b A_b} + \frac{P L_a}{E_a A_a}$$

$$= \frac{(162 \times 10^3)(0.35)}{(105 \times 10^9)(1500 \times 10^{-6})} + \frac{(162 \times 10^3)(0.45)}{(73 \times 10^9)(1800 \times 10^{-6})}$$

$$= 914.79 \times 10^{-6}\ m = 0.91479\ mm$$

Available length for thermal expansion

$$\delta_T = 0.5\ mm + 0.91479\ mm = 1.41479\ mm = 1.41479 \times 10^{-3}\ m$$

But $\quad \delta_T = L_b \alpha_b (\Delta T) + L_a \alpha_a (\Delta T)$

$$= (0.35)(21.6 \times 10^{-6})\Delta T + (0.45)(23.2 \times 10^{-6})\Delta T$$

$$= 18.00 \times 10^{-6}(\Delta T)$$

Equating $\quad 18.00 \times 10^{-6}(\Delta T) = 1.41479 \times 10^{-3} \quad \therefore \quad \Delta T = 78.6\ °C$

(a) $\quad T_{hot} = T_{cold} + \Delta T$

$$= 20 + 78.6 = 98.6\ °C$$

(b) $\quad \delta_a = L_a \alpha_a (\Delta T) - \dfrac{P L_a}{E_a A_a}$

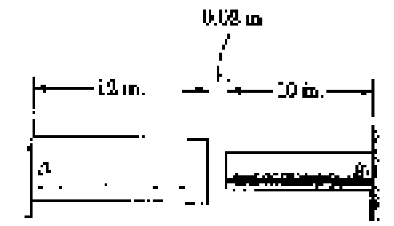
$$= (0.45)(23.2 \times 10^{-6})(78.6) - \frac{(162 \times 10^3)(0.45)}{(73 \times 10^9)(1800 \times 10^{-6})}$$

$$= 820.58 \times 10^{-6} - 554.79 \times 10^{-6} = 265.78 \times 10^{-6}\ m$$

$$L_{exact} = L_a + \delta_a = 0.45\ m + 265.78 \times 10^{-6}\ m$$

$$= 0.450266\ m = 450.0266\ mm$$

PROBLEM 2.59



Aluminum
$A = 2.8\ \text{in}^2$
$E = 10.4 \times 10^6$ psi
$\alpha = 13.3 \times 10^{-6}/°\text{F}$

Stainless steel
$A = 1.2\ \text{in}^2$
$E = 28.0 \times 10^6$ psi
$\alpha = 9.6 \times 10^{-6}/°\text{F}$

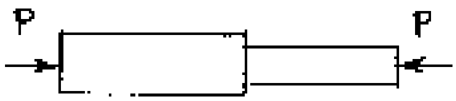
2.59 At room temperature (70°F) a 0.02-in. gap exists between the ends of the rods shown. At a later time when the temperature has reached 320°F, determine (*a*) the normal stress in the aluminum rod, (*b*) the change in length of the aluminum rod.

SOLUTION

$$\Delta T = 320 - 70 = 250\ °\text{F}$$

Free thermal expansion

$$\delta_T = L_a\alpha_a(\Delta T) + L_s\alpha_s(\Delta T)$$
$$= (12)(13.3\times10^{-6})(250) + (10)(9.6\times10^{-6})(250)$$
$$= 63.9\times10^{-3}\ \text{in} = 0.0639\ \text{in.}$$



Shortening due to P to meet constraint

$$\delta_P = 0.0639 - 0.02 = 0.0439\ \text{in.}$$

$$\delta_P = \frac{PL_a}{A_aE_a} + \frac{PL_s}{A_sE_s} = \left(\frac{L_a}{A_aE_a} + \frac{L_s}{A_sE_s}\right)P$$

$$= \left(\frac{12}{(2.8)(10.4\times10^6)} + \frac{10}{(1.2)(28.0\times10^6)}\right)P = 709.71\times10^{-9}\,P$$

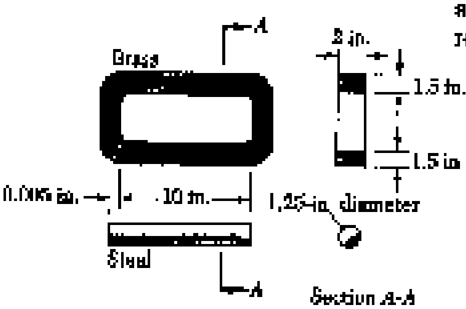
Equating $\quad 709.71\times10^{-9}\,P = 0.0439 \qquad P = 61.857\times10^3$ lb

(a) $$\sigma_a = -\frac{P}{A_a} = -\frac{61.857\times10^3}{2.8} = -22.09\times10^3\ \text{psi}$$
$$= -22.09\ \text{ksi}$$

(b) $$\delta_a = L_a\alpha_a(\Delta T) - \frac{PL_a}{A_aE_a}$$
$$= (12)(13.3\times10^{-6})(250) - \frac{(61.857\times10^3)(12)}{(2.8)(10.4\times10^6)}$$
$$= 39.90\times10^{-3} - 25.49\times10^{-3} = 14.41\times10^{-3}\ \text{in}$$
$$= 0.01441\ \text{in}$$

PROBLEM 2.60

2.60 A brass link ($E_b = 15 \times 10^6$ psi, $\alpha_b = 10.4 \times 10^{-6}/°\text{F}$) and a steel rod ($E_s = 29 \times 10^6$ ksi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F}$) have the dimensions shown at a temperature of 65°F. The steel rod is cooled until it fits freely into the link. The temperature of the whole assembly is then raised to 100°F. Determine (a) the final normal stress in the steel rod, (b) the final length of the steel rod.



SOLUTION

ΔT associated with difference between final and initial dimensions

$$\Delta T = 100 - 65 = 35\ °\text{F}$$

Free thermal expansion of each part

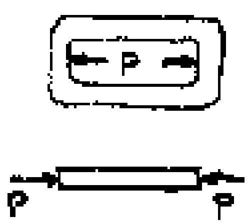
Brass link $\quad (\delta_T)_b = \alpha_b(\Delta T)(L) = (10.4\times10^{-6})(35)(10) = 3.64\times10^{-3}$ in

Steel rod $\quad (\delta_T)_s = \alpha_s(\Delta T)(L) = (6.5\times10^{-6})(35)(10) = 2.275\times10^{-3}$ in

At the final temperature the free length of the steel rod

$$0.005 + 2.275\times10^{-3} - 3.64\times10^{-3} = 3.635\times10^{-3} \text{ in}$$

longer than the brass link



Add equal but opposite forces P to elongate the brass link and contract the steel rod.

Brass link $\quad (\delta_P)_b = \dfrac{PL}{AE} = \dfrac{P(10)}{(2)(1.5)(2)(15\times10^6)}$

$$= 111.11\times10^{-9}\,P$$

Steel rod $\quad (\delta_P)_s = \dfrac{PL}{AE} = \dfrac{P(10)}{\frac{\pi}{4}(1.25)^2(29\times10^6)}$

$$= 280.99\times10^{-9}\,P$$

$$(\delta_P)_b + (\delta_P)_s = 3.635\times10^{-3}$$

$$(392.10\times10^{-9})P = 3.635\times10^{-3} \qquad P = 9.2705\times10^3 \text{ lb}$$

(a) Final stress in steel rod $\quad \sigma_s = -\dfrac{P}{A_s} = -\dfrac{9.2705\times10^3}{\frac{\pi}{4}(1.25)^2}$

$$= -7.55\times10^3 \text{ psi} = -7.55 \text{ ksi}$$

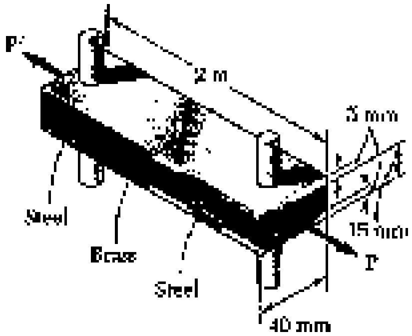
(b) Final length of steel rod

$$L_f = 10.000 + 0.005 + (\delta_T)_s - (\delta_P)_s$$

$$= 10.005 + 2.275\times10^{-3} - (280.99\times10^{-9})(9.2705\times10^3)$$

$$= 10.00467 \text{ in.}$$

PROBLEM 2.61



2.61 Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) which is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (a) the increase in temperature that was required to fit the steel bars on the pins, (b) the stress in the brass bar after the load is applied to it.

SOLUTION

(a) Required temperature change for fabrication

$\delta_T = 0.5$ mm $= 0.5 \times 10^{-3}$ m

(a) Temperature change required to expand steel bar by this amount

$\delta_T = L\alpha_s \Delta T \qquad 0.5 \times 10^{-3} = (2.00)(11.7 \times 10^{-6})(\Delta T), \quad \Delta T =$

$$0.5 \times 10^{-3} = (2)(11.7 \times 10^{-6})(\Delta T)$$
$$\Delta T = 21.368\ °C \qquad 21.4\ °C$$

(b) Once assembled, a tensile force P^* develops in the steel and a compressive force P^* develops in the brass, in order to elongate the steel and contract the brass.

Elongation of steel: $\quad A_s = (2)(5)(40) = 400\ mm^2 = 400 \times 10^{-6}\ m^2$

$$(\delta_P)_s = \frac{FL}{A_s E_s} = \frac{P(2.00)}{(400 \times 10^{-6})(200 \times 10^{9})} = 25 \times 10^{-9}\ P^*$$

Contraction of brass: $\quad A_b = (40)(15) = 600\ mm^2 = 600 \times 10^{-6}\ m^2$

$$(\delta_P)_b = \frac{P^* L}{A_b E_b} = \frac{P^*(2.00)}{(600 \times 10^{-6})(105 \times 10^{9})} = 31.746 \times 10^{-9} P^*$$

But $(\delta_P)_s + (\delta_P)_b$ is equal to the initial amount of misfit

$$(\delta_P)_s + (\delta_P)_b = 0.5 \times 10^{-3}, \quad 56.746 \times 10^{-9} P^* = 0.5 \times 10^{-3}$$

$$P^* = 8.811 \times 10^{3}\ N$$

Stresses due to fabrication

Steel: $\quad \sigma_s^* = \frac{P^*}{A_s} = \frac{8.811 \times 10^{3}}{400 \times 10^{-6}} = 22.03 \times 10^{6}\ Pa = 22.03\ MPa$

Brass: $\quad \sigma_b^* = -\frac{P^*}{A_b} = -\frac{8.811 \times 10^{3}}{600 \times 10^{-6}} = -14.68 \times 10^{6}\ Pa = -14.68\ MPa$

To these stresses must be added the stresses due to the 25 kN load.

continued

Problem 2.61 continued

For the added load, the additional deformation is the same for both the steel and the brass. Let δ' be the additional displacement. Also, let P_s and P_b be the additional forces developed in the steel and brass, respectively.

$$\delta' = \frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$$

$$P_s = \frac{A_s E_s}{L}\delta' = \frac{(400\times10^{-6})(200\times10^{9})}{2.00}\delta' = 40\times10^{6}\,\delta'$$

$$P_b = \frac{A_b E_b}{L}\delta' = \frac{(600\times10^{-6})(105\times10^{9})}{2.00}\delta' = 31.5\times10^{6}\,\delta'$$

Total $\quad P = P_s + P_b = 25\times10^{3}$ N

$$40\times10^{6}\,\delta' + 31.5\times10^{6}\,\delta' = 25\times10^{3} \qquad \delta' = 349.65\times10^{-6}\text{ m}$$

$$P_s = (40\times10^{6})(349.65\times10^{-6}) = 13.986\times10^{3}\text{ N}$$

$$P_b = (31.5\times10^{6})(349.65\times10^{-6}) = 11.140\times10^{3}\text{ N}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{13.986\times10^{3}}{400\times10^{-6}} = 34.97\times10^{6}\text{ Pa}$$

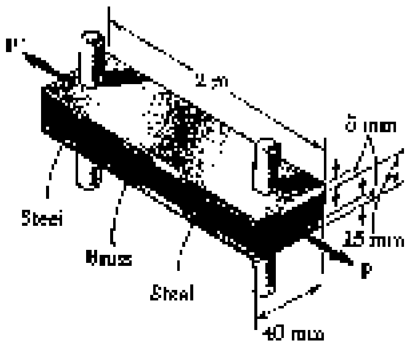
$$\sigma_b = \frac{P_b}{A_b} = \frac{11.140\times10^{3}}{600\times10^{-6}} = 18.56\times10^{6}\text{ Pa}$$

Add stress due to fabrication

$$\sigma_s = 34.97\times10^{6} + 22.03\times10^{6} = 57.0\times10^{6}\text{ Pa} = 57.0\text{ MPa}$$

$$\sigma_b = 18.56\times10^{6} - 14.68\times10^{6} = 3.88\times10^{6}\text{ Pa} = 3.88\text{ MPa}$$

PROBLEM 2.62



2.61 Two steel bars (E_s = 200 GPa and $\alpha_s = 11.7 \times 10^{-6}/°C$) are used to reinforce a brass bar (E_b = 105 GPa, $\alpha_b = 20.9 \times 10^{-6}/°C$) which is subjected to a load P = 25 kN. When the steel bars were fabricated, the distance between the centers of the holes which were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (a) the increase in temperature that was required to fit the steel bars on the pins, (b) the stress in the brass bar after the load is applied to it.

2.62 Determine the maximum load P that may be applied to the brass bar of Prob. 2.61 if the allowable stress in the steel bars is 30 MPa and the allowable stress in the brass bar is 25 MPa.

SOLUTION

See solution to PROBLEM 3.61 to obtain the fabrication stresses

$\sigma_s^* = 22.03$ MPa $\qquad \sigma_b^* = -14.68$ MPa

Allowable stresses: $\sigma_{s,all} = 30$ MPa , $\sigma_{b,all} = 25$ MPa

Available stress increase from load

$$\sigma_s = 30 - 22.03 = 7.97 \text{ MPa}$$
$$\sigma_b = 25 + 14.68 = 39.68 \text{ MPa}$$

Corresponding available strains

$$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{7.97 \times 10^6}{200 \times 10^9} = 39.85 \times 10^{-6}$$
$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{39.68 \times 10^6}{105 \times 10^9} = 377.9 \times 10^{-6}$$

Smaller value governs $\therefore \quad \varepsilon = 39.85 \times 10^{-6}$

Areas: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{ m}^2$
$A_b = (15)(40) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$

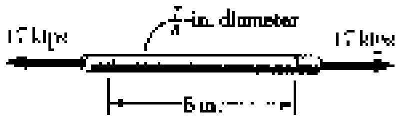
$$P_s = E_s A_s \varepsilon = (200 \times 10^9)(400 \times 10^{-6})(39.85 \times 10^{-6}) = 3.188 \times 10^3 \text{ N}$$

$$P_b = E_b A_b \varepsilon = (105 \times 10^9)(600 \times 10^{-6})(39.85 \times 10^{-6}) = 2.511 \times 10^3 \text{ N}$$

Total allowable additional force

$$P = P_s + P_b = 3.188 \times 10^3 + 2.511 \times 10^3 = 5.70 \times 10^3 \text{ N}$$
$$= 5.70 \text{ kN}$$

PROBLEM 2.63



2.63 In a standard tensile test a steel rod of $\frac{7}{8}$-in. diameter is subjected to a tension force of 17 kips. Knowing that $\nu = 0.3$ and $E = 29 \times 10^6$ psi, determine (*a*) the elongation of the rod in an 8-in. gage length, (*b*) the change in diameter of the rod.

SOLUTION

$P = 17$ kip $= 17 \times 10^3$ lb. $\qquad A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 0.60132$ in^2

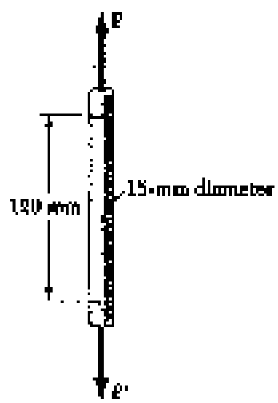
$\sigma = \frac{P}{A} = \frac{17 \times 10^3}{0.60132} = 28.27 \times 10^3$ psi $\qquad \varepsilon_x = \frac{\sigma}{E} = \frac{28.27 \times 10^3}{29 \times 10^6} = 974.9 \times 10^{-6}$

$\delta_x = L\varepsilon_x = (8.0)(974.9 \times 10^{-6}) = 7.80 \times 10^{-3}$ in $= 0.00780$ in. ◄

$\varepsilon_y = -\nu\varepsilon_x = -(0.3)(974.9 \times 10^{-6}) = -292.5 \times 10^{-6}$

$\delta_y = d\varepsilon_y = \left(\frac{7}{8}\right)(-292.5 \times 10^{-6}) = -256 \times 10^{-6}$ in $= -0.000256$ in. ◄

PROBLEM 2.64



2.64 A standard tension test is used to determine the properties of an experimental plastic. The test specimen is a 15-mm-diameter rod and it is subjected to a 3.5 kN tensile force. Knowing that an elongation of 11 mm and a decrease in diameter of 0.62 mm are observed in a 120 mm gage length, determine the modulus of elasticity, the modulus of rigidity, and Poisson's ratio of the material.

SOLUTION

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(15)^2 = 176.715$ mm^2 $= 176.715 \times 10^{-6}$ m^2

$P = 3.5 \times 10^3$ N

$\sigma = \frac{P}{A} = \frac{3.5 \times 10^3}{176.715 \times 10^{-6}} = 19.806 \times 10^6$ Pa

$\varepsilon_x = \frac{\delta_x}{L} = \frac{11}{120} = 91.667 \times 10^{-3}$

$E = \frac{\sigma}{\varepsilon_x} = \frac{19.806 \times 10^6}{91.667 \times 10^{-3}} = 216 \times 10^6$ Pa $= 216$ MPa ◄

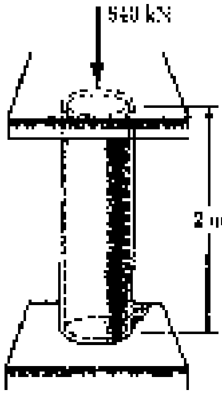
$\delta_y = -0.62$ mm

$\varepsilon_y = \frac{\delta_y}{d} = -\frac{0.62}{15} = -41.333 \times 10^{-3}$

$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = \frac{41.333 \times 10^{-3}}{91.667 \times 10^{-3}} = 0.4509$ ◄

$G = \frac{E}{2(1+\nu)} = \frac{216 \times 10^6}{2(1+0.4509)} = 74.5 \times 10^6$ Pa $= 74.5$ MPa ◄

PROBLEM 2.65



2.65 A 2-m length of an aluminum pipe of 240-mm outer diameter and 10-mm wall thickness is used as a short column and carries a centric axial load of 640 kN. Knowing that $E = 73$ GPa and $\nu = 0.33$, determine (*a*) the change in length of the pipe, (*b*) the change in its outer diameter, (*c*) the change in its wall thickness.

SOLUTION

$d_o = 240$ mm $\quad t = 10$ mm $\quad d_i = d_o - 2t = 220$ mm

$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(240^2 - 220^2) = 7.2257 \times 10^3$ mm^2

$= 7.2257 \times 10^{-3}$ m^2

$P = 640 \times 10^3$ N

(a) $\delta = -\frac{PL}{AE} = -\frac{(640 \times 10^3)(2.00)}{(7.2257 \times 10^{-3})(73 \times 10^9)} = -2.427 \times 10^{-3}$ m

$= -2.43$ mm

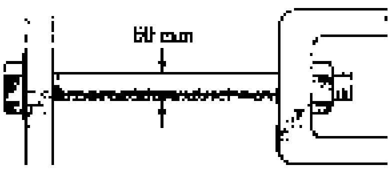
$\varepsilon = \frac{\delta}{L} = -\frac{2.427 \times 10^{-3}}{2.00} = -1.2133 \times 10^{-3}$

$\varepsilon_{lat} = -\nu\varepsilon = -(0.33)(-1.2133 \times 10^{-3}) = 400.4 \times 10^{-6}$

(b) $\Delta d_o = d_o \varepsilon_{lat} = (240)(400.4 \times 10^{-6}) = 0.0961$ mm

(c) $\Delta t = t\,\varepsilon_{lat} = (10)(400.4 \times 10^{-6}) = 0.00400$ mm

PROBLEM 2.66



2.66 The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that $E = 200$ GPa and $\nu = 0.29$, determine the internal force in the bolt, if the diameter is observed to decrease by 13 μm.

SOLUTION

$\delta_y = -13 \times 10^{-6}$ m $\qquad d = 60 \times 10^{-3}$ m

$\varepsilon_y = \frac{\delta_y}{d} = -\frac{13 \times 10^{-6}}{60 \times 10^{-3}} = -216.67 \times 10^{-6}$

$\nu = -\frac{\varepsilon_y}{\varepsilon_x} \quad \therefore \quad \varepsilon_x = -\frac{\varepsilon_y}{\nu} = \frac{216.67 \times 10^{-6}}{0.29} = 747.13 \times 10^{-6}$

$\sigma_x = E\varepsilon_x = (200 \times 10^9)(747.13 \times 10^{-6}) = 149.43 \times 10^6$ Pa

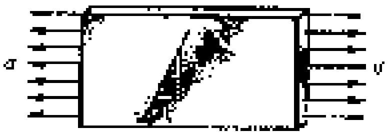
$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(60)^2 = 2.827 \times 10^3$ mm^2 $= 2.827 \times 10^{-3}$ m^2

$F = \sigma_x A = (149.43 \times 10^6)(2.827 \times 10^{-3}) = 422 \times 10^3$ N

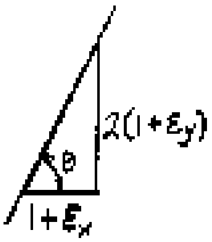
$= 422$ kN

PROBLEM 2.67

2.67 An aluminum plate (E = 74 GPa, ν = 0.33) plate is subjected to a centric axial load which causes a normal stress σ. Knowing that before loading, a line of slope 2:1 is scribed on the plate, determine the slope of the line when σ = 125 MPa.



SOLUTION



The slope after deformation is $\tan\theta = \dfrac{2(1+\varepsilon_y)}{1+\varepsilon_x}$

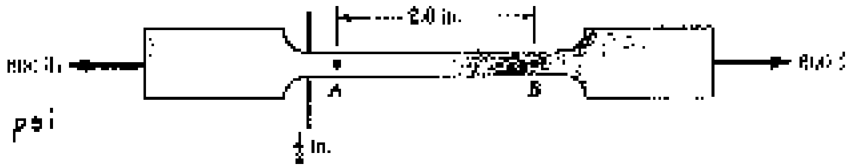
$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{125\times10^6}{74\times10^9} = 1.6892\times10^{-3}$$

$$\varepsilon_y = -\nu\varepsilon_x = -(0.33)(1.6892\times10^{-3}) = 0.5574\times10^{-3}$$

$$\tan\theta = \frac{2(1-0.0005574)}{1+0.0016892} = 1.99551$$

PROBLEM 2.68

2.68 A 600 lb tensile load is applied to a test coupon made from $\frac{1}{16}$ in. flat steel plate ($E = 29\times10^6$ psi, ν = 0.30). Determine the resulting change (a) in the 2.00-in. gage length, (b) in the width of portion AB of the test coupon, (c) in the thickness of portion AB, (d) in the cross-sectional area of portion AB.



SOLUTION

$$A = \left(\tfrac{1}{2}\right)\left(\tfrac{1}{16}\right) = 0.03125 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{600}{0.03125} = 19.2\times10^3 \text{ psi}$$

$$\varepsilon_x = \frac{\sigma}{E} = \frac{19.2\times10^3}{29\times10^6} = 662.07\times10^{-6}$$

(a) $\delta_x = L_0\varepsilon_x = (2.0)(662.07\times10^{-6}) = 1.324\times10^{-3}$ in.

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -(0.30)(662.07\times10^{-6}) = -198.62\times10^{-6}$$

(b) $\delta_{width} = w_0\varepsilon_y = \left(\tfrac{1}{2}\right)(-198.62\times10^{-6}) = -99.3\times10^{-6}$ in.

(c) $\delta_{thickness} = t_0\varepsilon_z = \left(\tfrac{1}{16}\right)(-198.62\times10^{-6}) = -12.41\times10^{-6}$ in.

(d) $A = wt = w_0(1+\varepsilon_y)t_0(1+\varepsilon_z)$

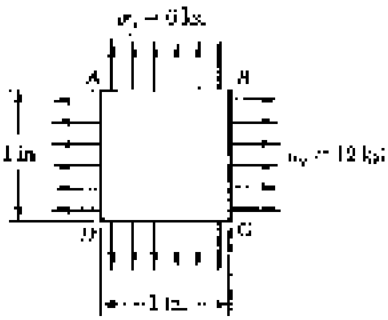
$$= w_0t_0(1+\varepsilon_y+\varepsilon_z+\varepsilon_y\varepsilon_z)$$

$$\Delta A = A - A_0 = w_0t_0(\varepsilon_y+\varepsilon_z+\varepsilon_y\varepsilon_z)$$

$$= \left(\tfrac{1}{2}\right)\left(\tfrac{1}{16}\right)(-198.62\times10^{-6} - 198.62\times10^{-6} + \text{negligible term})$$

$$= -12.41\times10^{-6} \text{ in}^2$$

PROBLEM 2.69



2.69 A 1-in. square is scribed on the side of a large steel pressure vessel. After pressurization the biaxial stress condition at the square is as shown. Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the change in length of (*a*) side AB, (*b*) side BC, (*c*) diagonal AC.

SOLUTION

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{29 \times 10^6}\left[12 \times 10^3 - (0.30)(6 \times 10^3)\right]$$
$$= 351.72 \times 10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{29 \times 10^6}\left[6 \times 10^3 - (0.30)(12 \times 10^3)\right]$$
$$= 82.76 \times 10^{-6}$$

(a) $\delta_{AB} = (\overline{AB})_0\varepsilon_x = (1.00)(351.72 \times 10^{-6}) = 351.7 \times 10^{-6}$ in.

(b) $\delta_{BC} = (\overline{BC})_0\varepsilon_y = (1.00)(82.76 \times 10^{-6}) = 82.8 \times 10^{-6}$ in.

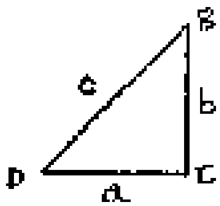
(c) $(\overline{AC}) = \sqrt{(\overline{AB})^2 + (\overline{BC})^2} = \sqrt{(\overline{AB}_0 + \delta_x)^2 + (\overline{BC}_0 + \delta_y)^2}$

$$= \sqrt{(1 + 352 \times 10^{-6})^2 + (1 + 82.8 \times 10^{-6})^2}$$

$$= 1.41452$$

$(\overline{AC})_0 = \sqrt{2}$ $\qquad \overline{AC} - (\overline{AC})_0 = 307 \times 10^{-6}$ in.

or use calculus as follows:



Lable sides using a, b, and c as shown

$$c^2 = a^2 + b^2$$

Obtain differentials $\quad 2c\,dc = 2a\,da + 2b\,db$

from which $\quad dc = \frac{a}{c}\,da + \frac{b}{c}\,db$

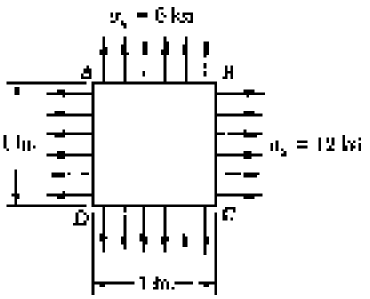
But $\quad a = 1.00$ in, $\quad b = 1.00$ in, $\quad c = \sqrt{2}$ in.

$da = \delta_{AB} = 351.72 \times 10^{-6}$ in, $\quad db = \delta_{BC} = 82.8 \times 10^{-6}$ in

$$\delta_{AC} = dc = \frac{1.00}{\sqrt{2}}(351.7 \times 10^{-6}) + \frac{1.00}{\sqrt{2}}(82.8 \times 10^{-6})$$

$$= 307 \times 10^{-6} \text{ in.}$$

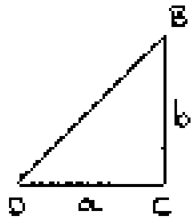
PROBLEM 2.70



2.69 A 1-in. square is scribed on the side of a large steel pressure vessel. After pressurization the biaxial stress condition at the square is as shown. Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the change in length of (*a*) side AB, (*b*) side BC, (*c*) diagonal AC.

2.70 For the square of Prob. 2.69, determine the percent change in the slope of diagonal DB due to the pressurization of the vessel.

SOLUTION



Label sides a and b as shown.

The slope is $s = \frac{b}{a}$

The change in slope is calculated from differential calculus

$$ds = \frac{a\,db - b\,da}{a^2} \quad \text{or} \quad \frac{ds}{s} = \frac{a}{b}\,\frac{a\,db - b\,da}{a^2} = \frac{db}{b} - \frac{da}{a}$$

$$\%\text{ change in slope} = \frac{ds}{s} \times 100\,\%$$

$$\frac{da}{a} = \varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{29 \times 10^6}\left[12 \times 10^3 - (0.29)(6 \times 10^3)\right]$$

$$= 351.72 \times 10^{-6}$$

$$\frac{db}{b} = \varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{29 \times 10^6}\left[6 \times 10^3 - (0.29)(12 \times 10^3)\right]$$

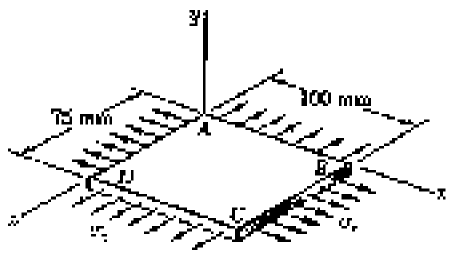
$$= 82.76 \times 10^{-6}$$

$$\frac{ds}{s} = 351.72 \times 10^{-6} - 82.76 \times 10^{-6} = 268.96 \times 10^{-6}$$

$$\%\text{ change in slope} = 268.96 \times 10^{-4}\,\%$$

$$= 0.0269\,\%$$

PROBLEM 2.71



2.71 A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 120$ MPa and $\sigma_z = 160$ MPa. Knowing that the properties of the fabric can be approximated as $E = 87$ GPa and $\nu = 0.34$, determine the change in length of (*a*) side AB, (*b*) side BC, (*c*) diagonal AC.

SOLUTION

$$\sigma_x = 120 \times 10^6 \text{ Pa}, \quad \sigma_y = 0, \quad \sigma_z = 160 \times 10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{87 \times 10^9}\left[120 \times 10^6 - (0.34)(160 \times 10^6)\right]$$

$$= 754.02 \times 10^{-6}$$

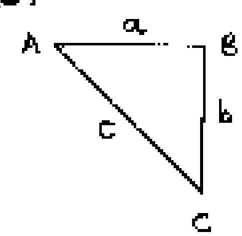
$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{87 \times 10^9}\left[-(0.34)(120 \times 10^6) + 160 \times 10^6\right]$$

$$= 1.3701 \times 10^{-3}$$

(a) $\delta_{AB} = (\overline{AB})\varepsilon_x = (100 \text{ mm})(754.02 \times 10^{-6}) = 0.0754$ mm ◄

(b) $\delta_{BC} = (\overline{BC})\varepsilon_z = (75 \text{ mm})(1.3701 \times 10^{-6}) = 0.1028$ mm ◄

(c)



Label sides of right triangle ABC as a, b, and c

$$c^2 = a^2 + b^2$$

Obtain differentials by calculus

$$2c\,dc = 2a\,da + 2b\,db$$

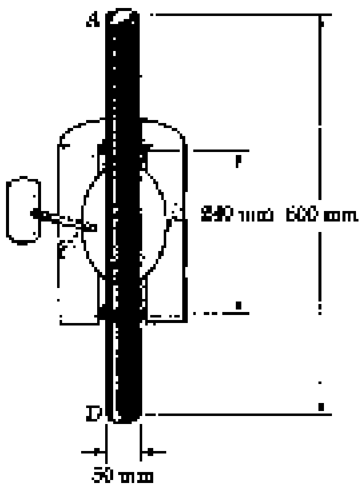
$$dc = \frac{a}{c}da + \frac{b}{c}db$$

But $a = 100$ mm, $b = 100$ mm $\quad c = \sqrt{100^2 + 75^2} = 125$ mm

$da = \delta_{AB} = 0.0754$ mm $\qquad db = \delta_{BC} = 0.1370$ mm

$$\delta_{AC} = dc = \frac{100}{125}(0.0754) + \frac{75}{125}(0.1028) = 0.1220 \text{ mm}$$ ◄

PROBLEM 2.72



2.72 The brass rod AD is fitted with a jacket that is used to apply a hydrostatic pressure of 48 MPa to the 240-mm portion BC of the rod. Knowing that $E = 105$ GPa, and $\nu = 0.33$, determine (*a*) the change in the total length AD, (*b*) the change in diameter of portion BC of the rod.

SOLUTION

$$\sigma_x = \sigma_z = -p = -48 \times 10^6 \text{ Pa}, \quad \sigma_y = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$= \frac{1}{105 \times 10^9}\left[-48 \times 10^6 - (0.33)(0) - (0.33)(-48 \times 10^6)\right]$$

$$= -306.29 \times 10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$

$$= \frac{1}{105 \times 10^9}\left[-(0.33)(-48 \times 10^6) + 0 - (0.33)(-48 \times 10^6)\right]$$

$$= 301.71 \times 10^{-6}$$

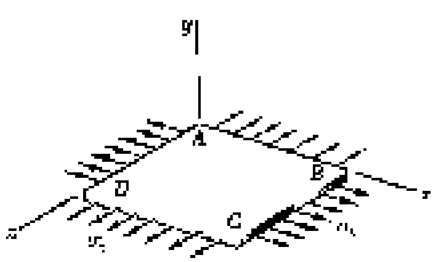
(a) Change in length: Only portion BC is strained. $L = 240$ mm

$$\delta_y = L\varepsilon_y = (240)(301.71 \times 10^{-6}) = 0.0724 \text{ mm}$$

(b) Change in diameter: $d = 50$ mm

$$\delta_x = \delta_z = d\,\varepsilon_x = (50)(-306.29 \times 10^{-6}) = -0.01531 \text{ mm}$$

PROBLEM 2.73



2.73 The homogeneous plate $ABCD$ is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the x direction must be zero, that is, $\varepsilon_x = 0$. Denoting by E the modulus of elasticity and by ν Poisson's ratio, determine (*a*) the required magnitude of σ_x, (*b*) the ratio σ_0 / ε_z.

$$\sigma_z = \sigma_0, \quad \sigma_y = 0, \quad \varepsilon_x = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{E}(\sigma_x - \nu\sigma_0)$$

(a) $\sigma_x = \nu\sigma_0$

(b) $\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{E}(-\nu^2\sigma_0 - 0 + \sigma_0) = \frac{1-\nu^2}{E}\sigma_0$

$$\frac{\sigma_0}{\varepsilon_z} = \frac{E}{1-\nu^2}$$

PROBLEM 2.74

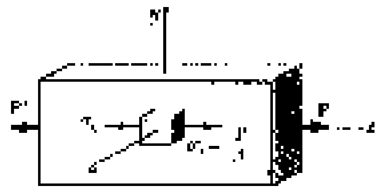


Fig 1.40 (a)

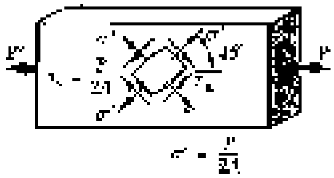


Fig 1.40 (b)

2.74 For a member under axial loading, express the normal strain ε' in a direction forming an angle of 45° with the axis of the load in terms of the axial strain ε_x by (*a*) comparing the hypothenuses of the triangles shown in Fig. 2.54, which represent respectively an element before and after deformation, (*b*) using the values of the corresponding stresses σ' and σ_x shown in Fig. 1.40, and the generalized Hooke's law.

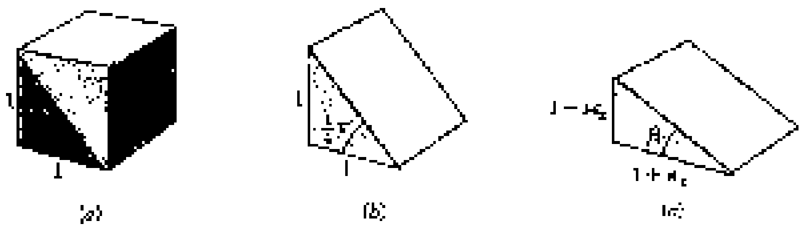
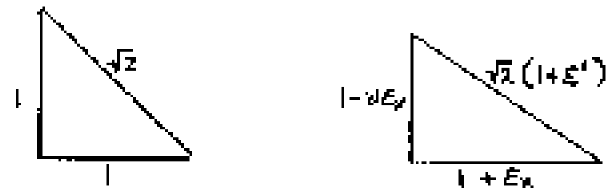


Fig 2.54

SOLUTION

(a)



Before deformation　　　After deformation

$$\left[\sqrt{2}\,(1+\varepsilon')\right]^2 = (1+\varepsilon_x)^2 + (1-\nu\varepsilon_x)^2$$

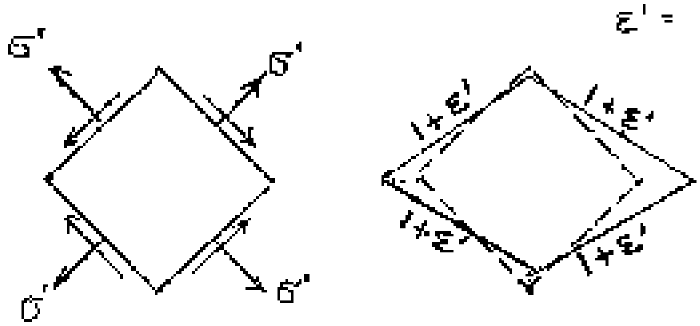
$$2(1+2\varepsilon'+\varepsilon'^2) = 1 + 2\varepsilon_x + \varepsilon_x^2 + 1 - 2\nu\varepsilon_x + \nu^2\varepsilon_x^2$$

$$4\varepsilon' + 2\varepsilon'^2 = 2\varepsilon_x + \varepsilon_x^2 - 2\nu\varepsilon_x + \nu^2\varepsilon_x^2$$

Neglect squares as small　　$4\varepsilon' = 2\varepsilon_x - 2\nu\varepsilon_x$

$$\varepsilon' = \frac{1-\nu}{2}\varepsilon_x$$

(b)



$$\varepsilon' = \frac{\sigma'}{E} - \frac{\nu\sigma'}{E}$$

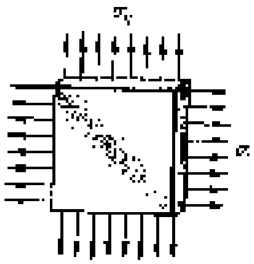
$$= \frac{1-\nu}{E}\cdot\frac{P}{2A}$$

$$= \frac{1-\nu}{2E}\sigma_x$$

$$= \frac{1-\nu}{2}\varepsilon_x$$

PROBLEM 2.75

2.75 In many situations it is known that the normal stress in a given direction is zero, for example $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as *plane stress*, show that if the strains ε_x and ε_y have been determined experimentally, we can express σ_x, σ_y and ε_z as follows:



$$\sigma_x = E\frac{\varepsilon_x + \nu\varepsilon_y}{1-\nu^2} \qquad \sigma_y = E\frac{\varepsilon_y + \nu\varepsilon_x}{1-\nu^2} \qquad \varepsilon_z = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

SOLUTION

$$\sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \quad (1) \qquad \varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y) \quad (2)$$

Multiplying (2) by ν and adding to (1)

$$\varepsilon_x + \nu\varepsilon_y = \frac{1-\nu^2}{E}\sigma_x \quad \text{or} \quad \sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y)$$

Multiplying (1) by ν and adding to (2)

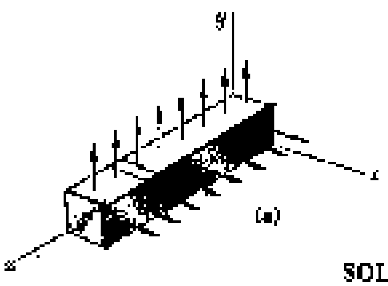
$$\varepsilon_y + \nu\varepsilon_x = \frac{1-\nu^2}{E}\sigma_y \quad \text{or} \quad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) = -\frac{\nu}{E}\cdot\frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y + \varepsilon_y + \nu\varepsilon_x)$$

$$= -\frac{\nu(1+\nu)}{1-\nu^2}(\varepsilon_x + \varepsilon_y) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y)$$

PROBLEM 2.76

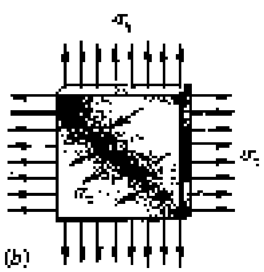
2.76 In many situations physical constraints prevent strain from occurring in a given direction, for example $\varepsilon_z = 0$ in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for this situation, which is known as *plane strain*, we can express σ_z, ε_x and ε_y as follows:



$$\sigma_z = \nu(\sigma_x + \sigma_y)$$

$$\varepsilon_x = \frac{1}{E}[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y] \qquad \varepsilon_y = \frac{1}{E}[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x]$$

SOLUTION



$$\varepsilon_z = 0 = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) \quad \text{or} \quad \sigma_z = \nu(\sigma_x + \sigma_y)$$

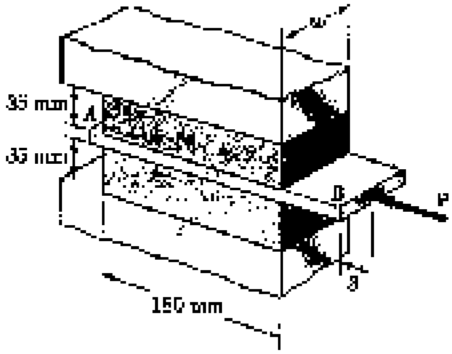
$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{E}[\sigma_x - \nu\sigma_y - \nu^2(\sigma_x + \sigma_y)]$$

$$= \frac{1}{E}[(1-\nu^2)\sigma_x - \nu(1+\nu)\sigma_y]$$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{1}{E}[-\nu\sigma_x + \sigma_y - \nu^2(\sigma_x + \sigma_y)]$$

$$= \frac{1}{E}[(1-\nu^2)\sigma_y - \nu(1+\nu)\sigma_x]$$

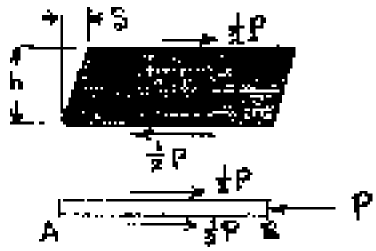
PROBLEM 2.77



2.77 Two blocks of rubber, each of width $w = 60$ mm, are bonded to rigid supports and to the movable plate AB. Knowing that a force of magnitude $P = 19$ kN causes a deflection $\delta = 3$ mm, determine the modulus of rigidity of the rubber used.

SOLUTION

Consider upper block of rubber. The force carried is $\frac{1}{2}P$.



The shearing stress is

$$\tau = \frac{\frac{1}{2}P}{A}$$

where $A = (180\text{ mm})(60\text{ mm}) = 10.8 \times 10^3\text{ mm}^2 = 10.8 \times 10^{-3}\text{ m}^2$

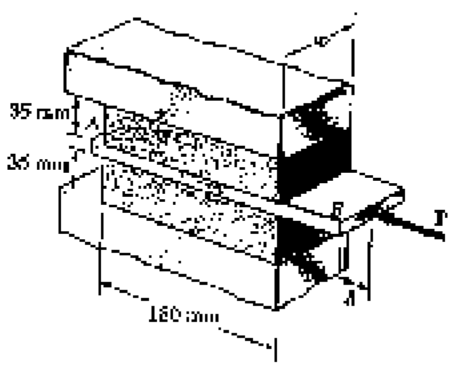
$P = 19 \times 10^3$ N

$$\tau = \frac{(\frac{1}{2})(19 \times 10^3)}{10.8 \times 10^{-3}} = 0.87963 \times 10^6\text{ Pa}$$

$$\gamma = \frac{\delta}{h} = \frac{3\text{ mm}}{35\text{ mm}} = 0.085714$$

$$G = \frac{\tau}{\gamma} = \frac{0.87963 \times 10^6}{0.085714} = 10.26 \times 10^6\text{ Pa} = 10.26\text{ MPa}$$

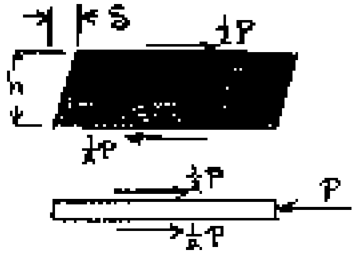
PROBLEM 2.78



2.78 Two blocks of rubber, for which $G = 7.5$ MPa, are bonded to rigid supports and to the movable plate AB. Knowing that the width of each block is $w = 80$ mm, determine the effective spring constant, $k = P/\delta$, of the system.

SOLUTION

Consider the upper block of rubber. The force carried is $\frac{1}{2}P$.



The shearing stress is

$$\tau = \frac{\frac{1}{2}P}{A}$$

from which

$$P = 2A\tau$$

The shearing strain is $\gamma = \frac{\delta}{h}$ from which $\delta = h\gamma$

Effective spring constant $k = \frac{P}{\delta} = \frac{2A\tau}{h\gamma}$

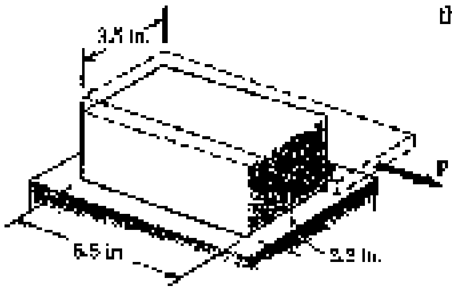
Noting that $\tau = G\gamma$,

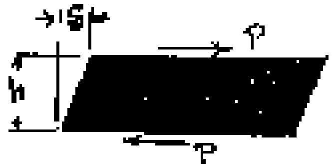
$$k = \frac{2AG}{h} = \frac{(2)(0.180)(0.080)(7.5 \times 10^6)}{0.035} = 6.17 \times 10^6\text{ N/m}$$

6.17×10^3 kN/m

PROBLEM 2.79

2.79 The plastic block shown is bonded to a fixed base and to a horizontal rigid plate to which a force **P** is applied. Knowing that for the plastic used $G = 55$ ksi, determine the deflection of the plate when $P = 9$ kips.





Consider the plastic block. The shearing force carried is $P = 9 \times 10^3$ lb.

The area is $A = (3.5)(5.5) = 19.25$ in^2

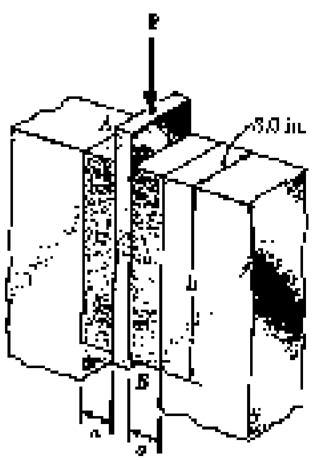
Shearing stress $\tau = \dfrac{P}{A} = \dfrac{9 \times 10^3}{19.25} = 467.52$ psi

Shearing strain $\gamma = \dfrac{\tau}{G} = \dfrac{467.52}{55 \times 10^3} = 0.0085006$

But $\gamma = \dfrac{\delta}{h}$ $\therefore$ $\delta = h\gamma = (2.2)(0.0085006) = 0.0187$ in.

PROBLEM 2.80

2.80 A vibration isolation unit consists of two blocks of hard rubber bonded to plate AB and to rigid supports as shown. For the type and grade of rubber used $\tau_{all} = 220$ psi and $G = 1800$ psi. Knowing that a centric vertical force of magnitude $P = 3.2$ kips must cause a 0.1 in. vertical deflection of the plate AB, determine the smallest allowable dimensions a and b of the block.



SOLUTION

Consider the rubber block on the right. It carries a shearing force equal to $\frac{1}{2}P$.

The shearing stress is $\tau = \dfrac{\frac{1}{2}P}{A}$

or required $A = \dfrac{P}{2\tau} = \dfrac{3.2 \times 10^3}{(2)(220)} = 7.2727$ in^2

But $A = (3.0)\, b$

Hence $b = \dfrac{A}{3.0} = 2.42$ in.

Use $b = 2.42$ in and $\tau = 220$ psi

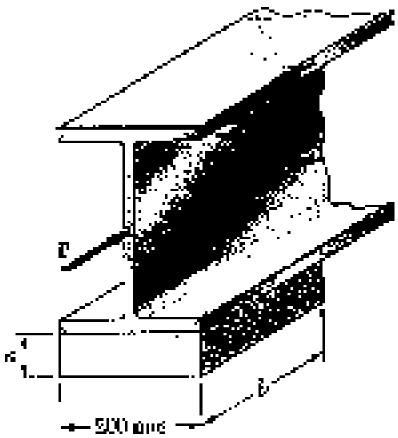
Shearing strain $\gamma = \dfrac{\tau}{G} = \dfrac{220}{1800} = 0.12222$

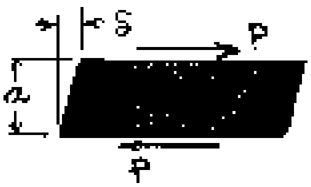
But $\gamma = \dfrac{\delta}{a}$

Hence $a = \dfrac{\delta}{\gamma} = \dfrac{0.1}{0.12222} = 0.818$ in.

PROBLEM 2.81

2.81 An elastomeric bearing ($G = 0.9$ MPa) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than 10 mm when a 22 kN lateral load is applied as shown. Determine (*a*) the smallest allowable dimension b, (*b*) the smallest required thickness a if the maximum allowable shearing stress is 420 kPa.





SOLUTION

Shearing force $P = 22 \times 10^3$ N

Shearing stress $\tau = 420 \times 10^3$ Pa

$$\tau = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\tau} = \frac{22 \times 10^3}{420 \times 10^3} = 52.381 \times 10^{-3}\ \text{m}^2$$

$$= 52.381 \times 10^3\ \text{mm}^2$$

$$A = (200\ \text{mm})(b)$$

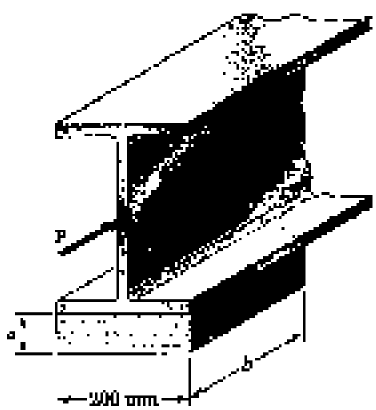
$$b = \frac{A}{200} = \frac{52.381 \times 10^3}{200} = 262\ \text{mm}$$

$$\gamma = \frac{\tau}{G} = \frac{420 \times 10^3}{0.9 \times 10^6} = 466.67 \times 10^{-3}$$

But $\gamma = \dfrac{\delta}{a}$ $\therefore$ $a = \dfrac{\delta}{\gamma} = \dfrac{10\ \text{mm}}{466.67 \times 10^{-3}} = 21.4$ mm

PROBLEM 2.82

2.82 For the elastomeric bearing in Prob. 2.81 with $b = 220$ mm and $a = 30$ mm, determine the shearing modulus G and the shear stress τ for a maximum lateral load $P = 19$ kN and a maximum displacement $\delta = 12$ mm.



SOLUTION

Shearing force $P = 19 \times 10^3$ N

Area $A = (200\ \text{mm})(220\ \text{mm}) = 44 \times 10^3\ \text{mm}^2$

$= 44 \times 10^{-3}\ \text{m}^2$

$$\tau = \frac{P}{A} = \frac{19 \times 10^3}{44 \times 10^{-3}} = 431.81 \times 10^3\ \text{Pa}$$

$$= 431\ \text{kPa}$$

Shearing strain $\gamma = \dfrac{\delta}{a} = \dfrac{12\ \text{mm}}{30\ \text{mm}} = 0.400$

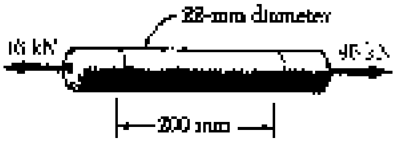
Shearing modulus

$$G = \frac{\tau}{\gamma} = \frac{431.81 \times 10^3}{0.4} = 1.080 \times 10^6\ \text{Pa}$$

$$= 1.080\ \text{MPa}$$

PROBLEM 2.53

*2.53 Determine the dilatation e and the change in volume of the 200-mm length of the rod shown if (*a*) the rod is made of steel with $E = 200$ GPa and $\nu = 0.30$; (*b*) the rod is made of aluminum with $E = 70$ GPa and $\nu = 0.35$.



SOLUTION

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(22)^2 = 380.13 \text{ mm}^2 = 380.13 \times 10^{-6} \text{ m}^2$$

$$P = 46 \times 10^3 \text{ N} \qquad \sigma_x = \frac{P}{A} = 121.01 \times 10^6 \text{ Pa} \qquad \sigma_y = \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x}{E} \qquad \varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -\nu\frac{\sigma_x}{E}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1}{E}(\sigma_x - \nu\sigma_z - \nu\sigma_y) = \frac{(1-2\nu)\sigma_x}{E}$$

Volume $\quad \mathcal{V} = AL = (380.13 \text{ mm}^2)(200 \text{ mm}) = 76.026 \times 10^3 \text{ mm}^3$

$$\Delta\mathcal{V} = \mathcal{V}e$$

(a) steel: $\quad e = \dfrac{(0.4)(121.01 \times 10^6)}{200 \times 10^9} = 242 \times 10^{-6}$ ◄

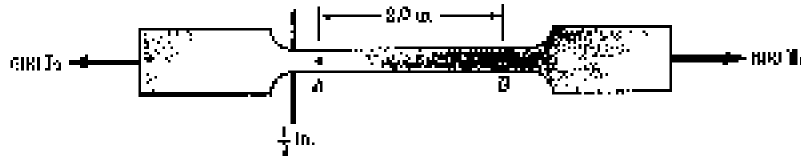
$$\Delta\mathcal{V} = (76.026 \times 10^3)(242 \times 10^{-6}) = 18.40 \text{ mm}^3$$ ◄

(b) aluminum: $\quad e = \dfrac{(0.3)(121.01 \times 10^6)}{70 \times 10^9} = 519 \times 10^{-6}$ ◄

$$\Delta\mathcal{V} = (76.026 \times 10^3)(519 \times 10^{-6}) = 39.4 \text{ mm}^3$$ ◄

PROBLEM 2.84

*2.84 Determine the change in volume of the 2-in. gage length segment AB in Prob. 2.68 (a) by computing the dilatation of the material, (b) by subtracting the original volume of portion AB from its final volume.



From PROBLEM 2.68

thickness $= \frac{1}{16}$ in

$E = 29 \times 10^6$ psi

$\nu = 0.30$

SOLUTION

(a) $A = (\frac{1}{2})(\frac{1}{16}) = 0.03125 \text{ in}^2$

Volume: $V_0 = AL_0 = (0.03125)(2.00) = 0.0625 \text{ in}^3$

$$\sigma_x = \frac{P}{A} = \frac{600}{0.03125} = 19.2 \times 10^3 \text{ psi} \qquad \sigma_y = \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{\sigma_x}{E} = \frac{19.2 \times 10^3}{29 \times 10^6} = 662.07 \times 10^{-6}$$

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -(0.30)(662.07 \times 10^{-6}) = -198.62 \times 10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 264.83 \times 10^{-6}$$

$$\Delta V = V_0 e = (0.0625)(264.83 \times 10^{-6}) = 16.55 \times 10^{-6} \text{ in}^3$$

(b) From the solution to PROBLEM 2.68

$$\delta_x = 1.324 \times 10^{-3} \text{ in}, \quad \delta_y = -99.3 \times 10^{-6} \text{ in} \quad \delta_z = -12.41 \times 10^{-6} \text{ in}$$

The dimensions when under a 600 lb tensile load are:

length $\quad L = L_0 + \delta_x = 2 + 1.324 \times 10^{-3} = 2.001324$ in.

width $\quad w = w_0 + \delta_y = \frac{1}{2} - 99.3 \times 10^{-6} = 0.4999007$ in.

thickness $\quad t = t_0 + \delta_z = \frac{1}{16} - 12.41 \times 10^{-6} = 0.06248759$ in

volume $\quad V = Lwt = 0.062516539 \text{ in}^3$

$$\Delta V = V - V_0 = 0.062516539 - 0.0625 = 16.54 \times 10^{-6} \text{ in}^3$$

PROBLEM 2.85

*2.85 A 6-in. diameter solid steel sphere is lowered into the ocean to a point where the pressure is 7.1 ksi (about 3 miles below the surface). Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine (*a*) the decrease in diameter of the sphere, (*b*) the decrease in volume of the sphere, (*c*) the percent increase in the density of the sphere.

SOLUTION

For a solid sphere $\quad V_0 = \frac{\pi}{6} d_0^3 = \frac{\pi}{6}(6.00)^3 = 113.097 \text{ in}^3$

$$\sigma_x = \sigma_y = \sigma_z = -p = -7.1 \times 10^3 \text{ psi}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = -\frac{(1-2\nu)p}{E} = -\frac{(0.4)(7.1 \times 10^3)}{29 \times 10^6} = -97.93 \times 10^{-6}$$

Likewise $\quad \varepsilon_y = \varepsilon_z = -97.93 \times 10^{-6}$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = -293.79 \times 10^{-6}$$

(a) $-\Delta d = -d_0\varepsilon_x = -(6.00)(-97.93 \times 10^{-6}) = 588 \times 10^{-6}$ in. ◄

(b) $-\Delta V = -V_0 e = -(113.097)(-293.79 \times 10^{-6}) = 33.2 \times 10^{-3}$ in^3 ◄

(c) Let m = mass of sphere $\qquad m$ = constant.

$$m = \rho_0 V_0 = \rho V = \rho V_0(1+e)$$

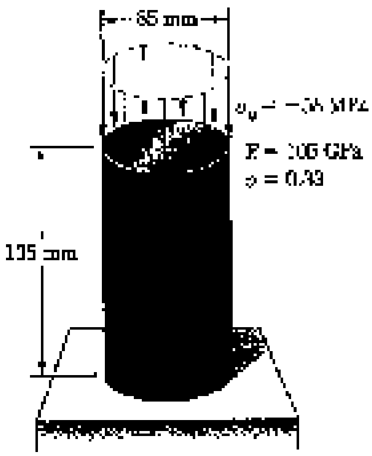
$$\frac{\rho - \rho_0}{\rho_0} = \frac{\rho}{\rho_0} - 1 = \frac{m}{V_0(1+e)} \cdot \frac{V_0}{m} - 1 = \frac{1}{1+e} - 1$$

$$= (1 - e + e^2 - e^3 + \dots) - 1 = -e + e^2 - e^3 + \dots$$

$$\approx -e = 293.79 \times 10^{-6}$$

$$\frac{\rho - \rho_0}{\rho_0} \times 100\% = (293.79 \times 10^{-6})(100\%) = 0.0294\%$$ ◄

PROBLEM 2.86



*2.86 (*a*) For the axial loading shown, determine the change in height and the change in volume of the brass cylinder shown. (*b*) Solve part *a* assuming that the loading is hydrostatic with $\sigma_x = \sigma_y = \sigma_z = -70$ MPa.

SOLUTION

$$h_o = 135 \text{ mm} = 0.135 \text{ m}$$

$$A_o = \frac{\pi}{4} d_o^2 = \frac{\pi}{4}(85)^2 = 5.6745 \times 10^3 \text{ mm}^2 = 5.6745 \times 10^{-3} \text{ m}^2$$

$$V_o = A_o h_o = 766.06 \times 10^3 \text{ mm}^3 = 766.06 \times 10^{-6} \text{ m}^3$$

(a) $\sigma_x = 0, \quad \sigma_y = -58 \times 10^6 \text{ Pa}, \quad \sigma_z = 0$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{\sigma_y}{E}$$

$$= -\frac{58 \times 10^6}{105 \times 10^9} = -552.38 \times 10^{-6}$$

$$\Delta h = h_o \varepsilon_y = (135 \text{ mm})(-552.38 \times 10^{-6}) = -0.0746 \text{ mm}$$

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(1-2\nu)\sigma_y}{E} = \frac{(0.34)(-58 \times 10^6)}{105 \times 10^9}$$

$$= -187.81 \times 10^{-6}$$

$$\Delta V = V_o e = (766.06 \times 10^3 \text{ mm}^3)(-187.81 \times 10^{-6}) = -143.9 \text{ mm}^3$$

(b) $\sigma_x = \sigma_y = \sigma_z = -70 \times 10^6 \text{ Pa} \qquad \sigma_x + \sigma_y + \sigma_z = -210 \times 10^6 \text{ Pa}$

$$\varepsilon_y = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z) = \frac{1-2\nu}{E}\sigma_y$$

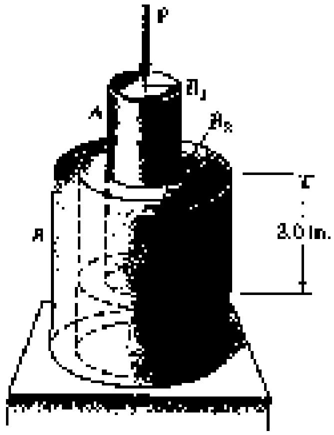
$$= \frac{(0.34)(-70 \times 10^6)}{105 \times 10^9} = -226.67 \times 10^{-6}$$

$$\Delta h = h_o \varepsilon_y = (135 \text{ mm})(-226.67 \times 10^{-6}) = -0.0306 \text{ mm}$$

$$e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(0.34)(-210 \times 10^6)}{105 \times 10^9} = -680 \times 10^{-6}$$

$$\Delta V = V_o e = (766.06 \times 10^3 \text{ mm}^3)(-680 \times 10^{-6}) = -521 \text{ mm}^3$$

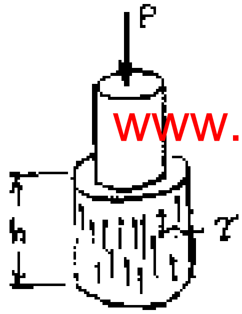
PROBLEM 2.87



*2.87 A vibration isolation support consists of a rod A of radius $R_1 = \frac{3}{8}$ in. and a tube B of inner radius $R_2 = 1$ in. bonded to a 3-in.-long hollow rubber cylinder with a modulus of rigidity $G = 1.8$ ksi. Determine the largest allowable force **P** which may be applied to rod A if its deflection is not to exceed 0.1 in.

SOLUTION

Let r be a radial coordinate. Over the hollow rubber cylinder $R_1 \le r \le R_2$

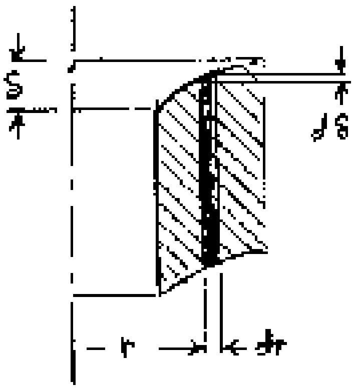


Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$



Shearing deformation over radial length dr

$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr = \frac{P}{2\pi G h}\frac{dr}{r}$$

Total deformation

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi G h}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi G h}\ln r\Big|_{R_1}^{R_2} = \frac{P}{2\pi G h}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi G h}\ln\frac{R_2}{R_1} \quad \text{or} \quad P = \frac{2\pi G h \delta}{\ln(R_2/R_1)}$$

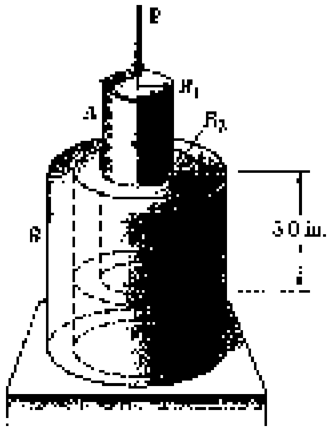
Data: $R_1 = \frac{3}{8}$ in $= 0.375$ in., $R_2 = 1.0$ in., $h = 3.0$ in

$G = 1.8\times10^3$ psi, $\delta = 0.1$ in

$$P = \frac{(2\pi)(1.8\times10^3)(3.0)(0.1)}{\ln(1.0/0.375)} = 3.46\times10^3 \text{ lb} = 3.46 \text{ kips}$$

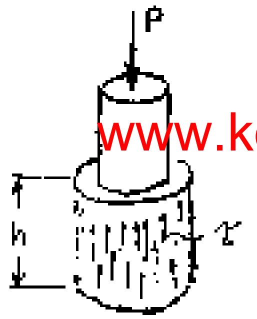
PROBLEM 2.88

*2.88 A vibration isolation support consists of a rod A of radius R_1 and a tube B of inner radius R_2 bonded to a 3-in.-long hollow rubber cylinder with a modulus of rigidity $G = 1.6$ ksi. Determine the required value of the ratio R_2/R_1 if a 2-kip force P is to cause a 0.12-in. deflection of rod A.



SOLUTION

Let r be a radial coordinate. Over the hollow rubber cylinder $R_1 \le r \le R_2$

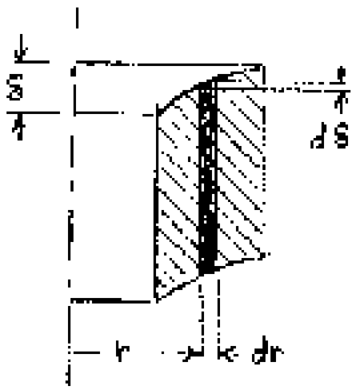


Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi r h}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi G h r}$$



Shearing deformation over radial length dr

$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr = \frac{P}{2\pi G h}\frac{dr}{r}$$

Total deformation

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi G h}\int_{R_1}^{R_2}\frac{dr}{r}$$

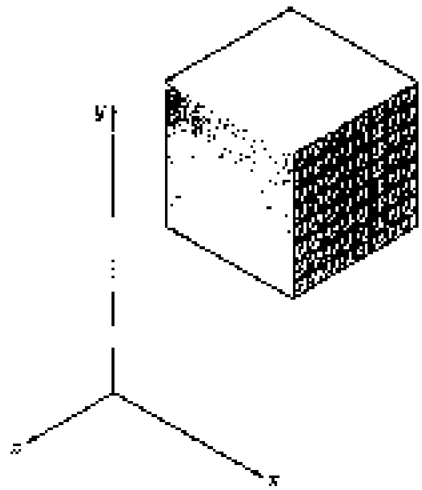
$$= \frac{P}{2\pi G h}\ln r\Big|_{R_1}^{R_2} = \frac{P}{2\pi G h}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi G h}\ln\frac{R_2}{R_1}$$

$$\ln\frac{R_2}{R_1} = \frac{2\pi G h \delta}{P} = \frac{(2\pi)(1.6\times10^3)(3.0)(0.12)}{2\times10^3} = 1.8096$$

$$\frac{R_2}{R_1} = \exp(1.8096) = 6.11$$

PROBLEM 2.89



*2.89 A composite cube with 40-mm sides and the properties shown is made with glass polymer fibers aligned in the x direction. The cube is constrained against deformations in the y and z directions and is subjected to a tensile load of 65 kN in the x direction. Determine (a) the change in the length of the cube in the x direction, (b) the stresses σ_x, σ_y, and σ_z.

$E_x = 50$ GPa $\quad \nu_{xz} = 0.254$
$E_y = 15.2$ GPa $\quad \nu_{xy} = 0.254$
$E_z = 15.2$ GPa $\quad \nu_{zy} = 0.428$

SOLUTION

Stress-to-strain equations are

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \qquad (1) \qquad\qquad \frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \qquad (4)$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \qquad (2) \qquad\qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \qquad (5)$$

$$\varepsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \qquad (3) \qquad\qquad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \qquad (6)$$

The constraint conditions are $\varepsilon_y = 0$ and $\varepsilon_z = 0$.

Using (2) and (3) with the constraint conditions gives

$$\frac{1}{E_y}\sigma_y - \frac{\nu_{zy}}{E_z}\sigma_z = \frac{\nu_{xy}}{E_x}\sigma_x \qquad (7)$$

$$-\frac{\nu_{yz}}{E_y}\sigma_y + \frac{1}{E_z}\sigma_z = \frac{\nu_{xz}}{E_x}\sigma_x \qquad (8)$$

$$\frac{1}{15.2}\sigma_y - \frac{0.428}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad \sigma_y - 0.428\,\sigma_z = 0.077216\,\sigma_x$$

$$-\frac{0.428}{15.2}\sigma_y + \frac{1}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad -0.428\,\sigma_y + \sigma_z = 0.077216\,\sigma_x$$

Solving simultaneously $\quad \sigma_y = \sigma_z = 0.134993\,\sigma_x$

Using (4) and (6) in (1) $\quad \varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{xy}}{E_x}\sigma_y - \frac{\nu_{xz}}{E}\sigma_z$

$$\varepsilon_x = \frac{1}{E_x}\left[1 - (0.254)(0.134993) - (0.254)(0.134993)\right]\sigma_x$$

$$= \frac{0.93142\,\sigma_x}{E_x}$$

$$A = (40)(40) = 1600\ \text{mm}^2 = 1600\times10^{-6}\ \text{m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{65\times10^3}{1600\times10^{-6}} = 40.625\times10^6\ \text{Pa}$$

continued

Problem 2.89 continued

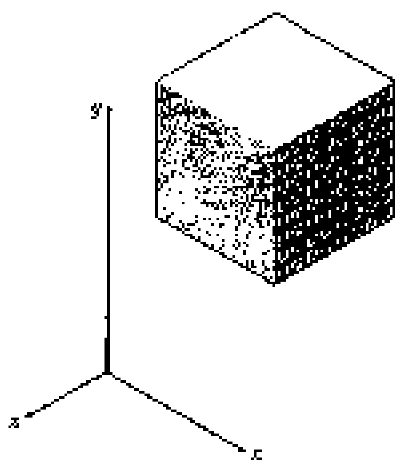
$$\varepsilon_x = \frac{(0.93142)(40.625\times10^6)}{50\times10^9} = 756.78\times10^{-6}$$

(a) $\delta_x = L_x\varepsilon_x = (40\text{ mm})(756.78\times10^{-6}) = 0.0303\text{ mm}$ ◄

(b) $\sigma_x = 40.625\times10^6\text{ Pa} = 40.6\text{ MPa}$ ◄

$\sigma_y = \sigma_z = (0.134993)(40.625\times10^6) = 5.48\times10^6\text{ Pa}$
$= 5.48\text{ MPa}$ ◄

PROBLEM 2.90



*2.90 The composite cube of Prob. 2.89 is constrained against deformation in the z direction and elongated in the x direction by 0.035 mm due to a tensile load in the x direction. Determine (a) the stresses σ_x, σ_y, and σ_z, (b) the change in the dimension in the y direction.

$E_x = 50$ GPa $\quad \nu_{xz} = 0.254$
$E_y = 15.2$ GPa $\quad \nu_{xy} = 0.254$
$E_z = 15.2$ GPa $\quad \nu_{zy} = 0.428$

SOLUTION

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} - \frac{\nu_{zx}\sigma_z}{E_z} \qquad (1) \qquad\qquad \frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \qquad (4)$$

$$\varepsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{\nu_{zy}\sigma_z}{E_z} \qquad (2) \qquad\qquad \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z} \qquad (5)$$

$$\varepsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \qquad (3) \qquad\qquad \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x} \qquad (6)$$

Constraint condition $\quad \varepsilon_z = 0$

Load condition $\quad \sigma_y = 0$

From equation (3) $\quad 0 = -\frac{\nu_{xz}}{E_x}\sigma_x + \frac{1}{E_z}\sigma_z$

$$\sigma_z = \frac{\nu_{xz}E_z}{E_x}\sigma_x = \frac{(0.254)(15.2)}{50} = 0.077216\,\sigma_x$$

continued

Problem 2.90 continued

From equation (1) with $\sigma_y = 0$

$$\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{\nu_{zx}}{E_z}\sigma_z = \frac{1}{E_x}\sigma_x - \frac{\nu_{xz}}{E_x}\sigma_z$$

$$= \frac{1}{E_x}\left[\sigma_x - 0.254\,\sigma_z\right] = \frac{1}{E_x}\left[1 - (0.254)(0.077216)\right]\sigma_x$$

$$= \frac{0.98089}{E_x}\sigma_x$$

$$\sigma_x = \frac{E_x\,\varepsilon_x}{0.98039}$$

But $\varepsilon_x = \frac{\delta_x}{L_x} = \frac{0.035 \text{ mm}}{40 \text{ mm}} = 875 \times 10^{-6}$

(a) $\sigma_x = \frac{(50 \times 10^9)(875 \times 10^{-6})}{0.98089} = 44.625 \times 10^6 \text{ Pa} = 44.6 \text{ MPa}$

$\sigma_y = 0$

$\sigma_z = (0.077216)(44.625 \times 10^6) = 3.446 \times 10^6 \text{ Pa} = 3.45 \text{ MPa}$

From (2) $\varepsilon_y = -\frac{\nu_{xy}}{E_x}\sigma_x + \frac{1}{E_y}\sigma_y - \frac{\nu_{zy}}{E_z}\sigma_z$

$$= -\frac{(0.254)(44.625 \times 10^6)}{50 \times 10^9} + 0 - \frac{(0.428)(3.446 \times 10^6)}{15.2 \times 10^9}$$

$$= -323.73 \times 10^{-6}$$

$\delta_y = L_y\varepsilon_y = (40 \text{ mm})(-323.73 \times 10^{-6}) = -0.0129 \text{ mm}$

PROBLEM 2.91

*2.91 Show that for any given material, the ratio G/E of the modulus of rigidity over the modulus of elasticity is always less than $\frac{1}{2}$ but more than $\frac{1}{3}$. [*Hint:* Refer to Eq. (2.43) and to Sec. 2.13.]

SOLUTION

$$G = \frac{E}{2(1+\nu)} \quad \text{or} \quad \frac{E}{G} = 2(1+\nu)$$

Assume $\nu \geq 0$ for almost all materials and $\nu < \frac{1}{2}$ for a positive bulk modulus

Applying the bounds $\quad 2 \leq \frac{E}{G} < 2\left(1+\frac{1}{2}\right) = 3$

Taking the reciprocals $\quad \frac{1}{2} \geq \frac{G}{E} > \frac{1}{3}$

$$\text{or} \quad \frac{1}{3} \leq \frac{G}{E} \leq \frac{1}{2}$$

PROBLEM 2.92

*2.92 The material constants E, G, k, and ν are related by Eqs. (2.33) and (2.43). Show that any one of these constants may be expressed in terms of any other two constants. For example, show that (*a*) $k = GE/(9G - 3E)$ and (*b*) $\nu = (3k - 2G)/(6k + 2G)$.

SOLUTION

$$k = \frac{E}{3(1-2\nu)} \quad \text{and} \quad G = \frac{E}{2(1+\nu)}$$

(a) $\quad 1+\nu = \frac{E}{2G} \quad \text{or} \quad \nu = \frac{E}{2G} - 1$

$$k = \frac{E}{3\left[1 - 2\left(\frac{E}{2G} - 1\right)\right]} = \frac{2EG}{3[2G - 2E + 4G]} = \frac{2EG}{18G - 6E}$$

$$= \frac{EG}{9G - 6E}$$

(b) $\quad \frac{k}{G} = \frac{2(1+\nu)}{3(1-2\nu)}$

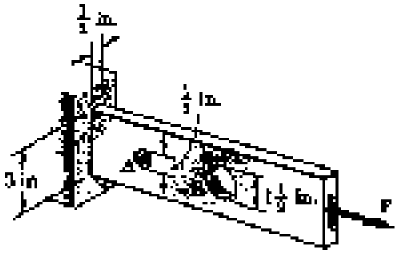
$$3k - 6k\nu = 2G + 2G\nu$$

$$3k - 2G = 2G + 6k$$

$$\nu = \frac{3k - 2G}{6k + 2G}$$

PROBLEM 2.93

2.93 Two holes have been drilled through a long steel bar that is subjected to a centric axial load as shown. For $P = 6.5$ kips, determine the maximum value of the stress (*a*) at *A*, (*b*) at *B*.



SOLUTION

(a) At hole A $\quad r = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$ in

$d = 3 - \frac{1}{2} = 2.50$ in

$A_{net} = dt = (2.50)(\frac{1}{2}) = 1.25$ in^2

$\sigma_{nom} = \frac{P}{A_{net}} = \frac{6.5}{1.25} = 5.2$ ksi

$\frac{r}{d} = \frac{1/4}{2.50} = 0.10$ $\quad$ From Fig 2.64 a $\quad K = 2.70$

$\sigma_{max} = K\sigma_{nom} = (2.70)(5.2) = 14.04$ ksi

(b) At hole B $\quad r = \frac{1}{2}(1.5) = 0.75$, $\quad d = 3 - 1.5 = 1.5$ in

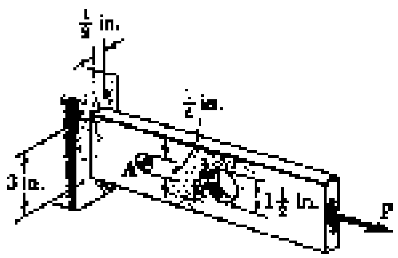
$A_{net} = dt = (1.5)(\frac{1}{2}) = 0.75$ in^2, $\quad \sigma_{nom} = \frac{P}{A_{net}} = \frac{6.5}{0.75} = 8.667$ ksi

$\frac{r}{d} = \frac{0.75}{1.5} = 0.5$ $\quad$ From Fig 2.64 a $\quad K = 2.10$

$\sigma_{max} = K\sigma_{nom} = (2.10)(8.667) = 18.2$ ksi

PROBLEM 2.94

2.94 Knowing that $\sigma_{all} = 16$ ksi, determine the maximum allowable value of the centric axial load P.



SOLUTION

At hole A $\quad r = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$ in

$d = 3 - \frac{1}{2} = 2.50$ in

$A_{net} = dt = (2.50)(\frac{1}{2}) = 1.25$ in^2

$\frac{r}{d} = \frac{1/4}{2.50} = 0.10$ $\quad$ From Fig 2.64 a, $\quad K = 2.70$

$\sigma_{max} = \frac{KP}{A_{net}} \therefore P = \frac{A_{net}\sigma_{max}}{K} = \frac{(1.25)(16)}{2.70} = 7.41$ kips

At hole B $\quad r = \frac{1}{2}(1.5) = 0.75$ in, $\quad d = 3 - 1.5 = 1.5$ in.

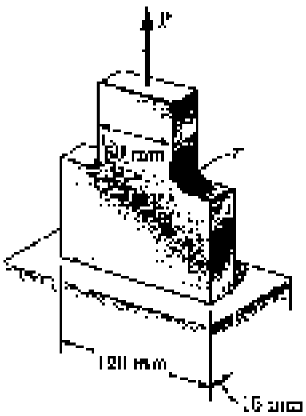
$A_{net} = dt = (1.5)(\frac{1}{2}) = 0.75$ in^2,

$\frac{r}{d} = \frac{0.75}{1.5} = 0.5$ $\quad$ From Fig 2.64 a $\quad K = 2.10$

$P = \frac{A_{net}\sigma_{max}}{K} = \frac{(0.75)(16)}{2.10} = 5.71$ kips

Smaller value for P controls $\quad P = 5.71$ kips

PROBLEM 2.95



2.95 Knowing that, for the plate shown, the allowable stress is 125 MPa, determine the maximum allowable value of P when (*a*) $r = 12$ mm, (*b*) $r = 18$ mm.

SOLUTION

$$A = (60)(15) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

$$\frac{D}{d} = \frac{120 \text{ mm}}{60 \text{ mm}} = 2.00$$

(a) $r = 12$ mm $\quad \frac{r}{d} = \frac{12 \text{ mm}}{60 \text{ mm}} = 0.2$

From Fig. 2.64 b $\quad K = 1.92 \quad \sigma_{max} = K\frac{P}{A}$

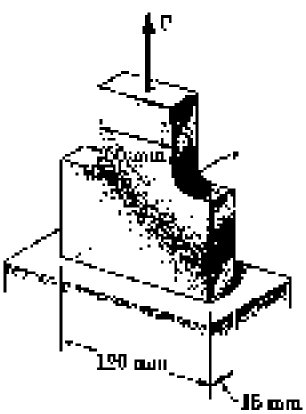
$$P = \frac{A\sigma_{max}}{K} = \frac{(900\times10^{-6})(125\times10^{6})}{1.92} = 58.6\times10^{3} \text{ N}$$

$$= 58.3 \text{ kN}$$

(b) $r = 18$ mm, $\quad \frac{r}{d} = \frac{18 \text{ mm}}{60 \text{ mm}} = 0.30$, From Fig 2.64 b $\quad K = 1.75$

$$P = \frac{A\sigma_{max}}{K} = \frac{(900\times10^{-6})(125\times10^{6})}{1.75} = 64.3\times10^{3} \text{ N} = 64.3 \text{ kN}$$

PROBLEM 2.96



2.96 Knowing that $P = 38$ kN, determine the maximum stress when (*a*) $r = 10$ mm, (*b*) $r = 16$ mm, (*c*) $r = 18$ mm.

SOLUTION

$$A = (60)(15) = 900 \text{ mm}^2 = 900\times10^{-6} \text{ m}^2$$

$$\frac{D}{d} = \frac{120 \text{ mm}}{60 \text{ mm}} = 2.00$$

(a) $r = 10$ mm $\quad \frac{r}{d} = \frac{10 \text{ mm}}{60 \text{ mm}} = 0.1667$

From Fig 2.64 b $\quad K = 2.06 \quad \sigma_{max} = \frac{KP}{A}$

$$\sigma_{max} = \frac{(2.06)(38\times10^{3})}{900\times10^{-6}} = 87.0\times10^{6} \text{ Pa} = 87.0 \text{ MPa}$$

(b) $r = 16$ mm $\quad \frac{r}{d} = \frac{16 \text{ mm}}{60 \text{ mm}} = 0.2667$

From Fig 2.64 b $\quad K = 1.78$

$$\sigma_{max} = \frac{(1.78)(38\times10^{3})}{900\times10^{-6}} = 75.2\times10^{6} \text{ Pa} = 75.2 \text{ MPa}$$

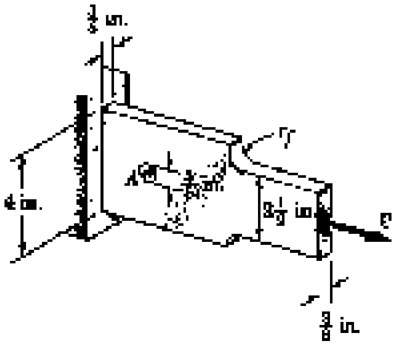
(c) $r = 18$ mm $\quad \frac{r}{d} = \frac{18 \text{ mm}}{60 \text{ mm}} = 0.30$

From Fig 2.64 b $\quad K = 1.75$

$$\sigma_{max} = \frac{(1.75)(38\times10^{3})}{900\times10^{-6}} = 73.9\times10^{6} \text{ Pa} = 73.9 \text{ MPa}$$

PROBLEM 2.97

2.97 Knowing that the hole has a diameter of $\frac{3}{8}$-in., determine (a) the radius r_f of the fillets for which the same maximum stress occurs at the hole A and at the fillets, (b) the corresponding maximum allowable load P if the allowable stress is 15 ksi.



SOLUTION

For the circular hole $\quad r = (\frac{1}{2})(\frac{3}{8}) = 0.1875$ in

$d = 4 - \frac{3}{8} = 3.625$ in $\qquad \frac{r}{d} = \frac{0.1875}{3.625} = 0.0517$

$A_{net} = dt = (3.625)(\frac{3}{8}) = 1.3594 \text{ in}^2$

From Fig 2.64 a $\qquad K_{hole} = 2.82$

$$\sigma_{max} = \frac{K_{hole} P}{A_{net}}$$

(b) $\quad P = \frac{A_{net}\sigma_{max}}{K_{hole}} = \frac{(1.3594)(15)}{2.82} = 7.23$ kips

(a) For fillet $\quad D = 4$ in, $\quad d = 2.5$ in $\qquad \frac{D}{d} = \frac{4.0}{2.5} = 1.60$

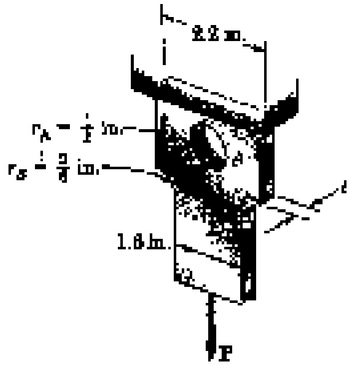
$A_{min} = dt = (2.5)(\frac{3}{8}) = 0.9375 \text{ in}^2$

$\sigma_{max} = \frac{K_{fillet} P}{A_{min}} \quad \therefore \quad K_{fillet} = \frac{A_{min}\sigma_{max}}{P} = \frac{(0.9375)(15)}{7.23} = 1.945$

From Fig 2.64 b $\quad \frac{r_f}{d} \approx 0.19 \quad \therefore \quad r_f \approx 0.17d = (0.17)(2.5) = 0.43$ in

PROBLEM 2.98

2.98 For $P = 8.5$ kips, determine the minimum plate thickness t required if the allowable stress is 18 ksi.



SOLUTION

At the hole: $\quad r_A = \frac{1}{2}$ in $\qquad d_A = 2.2 - 1.0 = 1.2$ in

$\frac{r_A}{d_A} = \frac{1/2}{1.2} = 0.417$

From Fig 2.64 a $\qquad K = 2.22$

$\sigma_{max} = \frac{KP}{A_{net}} = \frac{KP}{d_A t} \quad \therefore \quad t = \frac{KP}{d_A \sigma_{max}}$

$t = \frac{(2.22)(8.5)}{(1.2)(18)} = 0.87$ in.

At the fillet $\quad D = 2.2$ in, $\quad d_B = 1.6$ in $\qquad \frac{D}{d_B} = \frac{2.2}{1.6} = 1.375$

$r_B = \frac{3}{8} = 0.375$ in $\qquad \frac{r_B}{d_B} = \frac{0.375}{1.6} = 0.2344$

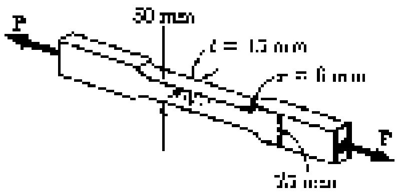
From Fig 2.64 b $\qquad K = 1.70 \qquad \sigma_{max} = \frac{KP}{A_{min}} = \frac{KP}{d_B t}$

$t = \frac{KP}{d_B \sigma_{max}} = \frac{(1.70)(8.5)}{(1.6)(18)} = 0.50$ in

The larger value is the required minimum plate thickness

$t = 0.87$ in.

PROBLEM 2.99



2.99 (*a*) Knowing that the allowable stress is 140 MPa, determine the maximum allowable magnitude of the centric load **P**. (*b*) Determine the percent change in the maximum allowable magnitude of **P** if the raised portions are removed at the ends of the specimen.

SOLUTION

$$\frac{D}{d} = \frac{75 \text{ mm}}{50 \text{ mm}} = 1.50, \quad \frac{r}{d} = \frac{6 \text{ mm}}{50 \text{ mm}} = 0.12$$

From Fig 2.64 b $\quad K = 2.10$

$$A_{min} = t\,d = (15)(50) = 750 \text{ mm}^2 = 750 \times 10^{-6} \text{ m}^2$$

(a) $\sigma_{max} = \dfrac{KP}{A_{min}} \quad \therefore \quad P = \dfrac{A_{min}\sigma_{max}}{K} = \dfrac{(750 \times 10^{-6})(140 \times 10^{6})}{2.10} = 50 \times 10^{3} \text{ N}$

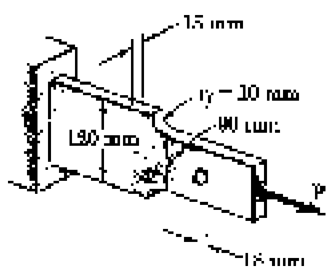
$= 50 \text{ kN}$

(b) Without raised section $\quad K = 1.00$

$$P = A_{min}\sigma_{max} = (750 \times 10^{-6})(140 \times 10^{6}) = 105 \times 10^{3} = 105 \text{ kN}$$

$$\% \text{ change} = \left(\frac{105 - 50}{50}\right) \times 100\% = 110\%$$

PROBLEM 2.100



2.100 A centric axial force is applied to the steel bar shown. Knowing that σ_{all} is 135 MPa, determine the maximum allowable load P.

SOLUTION

At the hole: $\quad r = 9 \text{ mm} \quad d = 90 - 18 = 72 \text{ mm}$

$\frac{r}{d} = 0.125 \quad$ From Fig 2.64 a $\quad K = 2.65$

$$A_{net} = t\,d = (15)(72) = 1.08 \times 10^{3} \text{ mm}^2 = 1.08 \times 10^{-3} \text{ m}^2$$

$$\sigma_{max} = \frac{KP}{A_{net}}$$

$$P = \frac{A_{net}\sigma_{max}}{K} = \frac{(1.08 \times 10^{-3})(135 \times 10^{6})}{2.65} = 55 \times 10^{3} \text{ N} = 55 \text{ kN}$$

At the fillet $\quad D = 120 \text{ mm}, \quad d = 90 \text{ mm}, \quad \frac{D}{d} = \frac{120}{90} = 1.333$

$r = 10 \text{ mm} \quad \frac{r}{d} = \frac{10}{90} = 0.1111 \quad$ From Fig 2.64 b $\quad K = 2.02$

$$A_{min} = t\,d = (15)(90) = 1.35 \times 10^{3} \text{ mm}^2 = 1.35 \times 10^{-3} \text{ m}^2$$

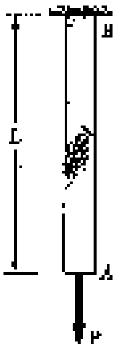
$$\sigma_{max} = \frac{KP}{A_{min}}$$

$$P = \frac{A_{min}\sigma_{max}}{K} = \frac{(1.35 \times 10^{-3})(135 \times 10^{6})}{2.02} = 90 \times 10^{3} \text{ N} = 90 \text{ kN}$$

Smaller value for P controls $\quad P = 55 \text{ kN}$

PROBLEM 2.101

2.101 The 30-mm square bar AB has a length $L = 2.2$ m; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force $\mathbf{P}$ is applied to the bar until end A has moved down by an amount δ_m. Determine the maximum value of the force $\mathbf{P}$ and the permanent set of the bar after the force has been removed, knowing that (a) $\delta_m = 4.5$ mm, (b) $\delta_m = 8$ mm.



SOLUTION

$$A = (30)(30) = 900\ \text{mm}^2 = 900\times10^{-6}\ \text{m}^2$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(2.2)(345\times10^6)}{200\times10^9} = 3.795\times10^{-3} = 3.795\ \text{mm}$$

If $\delta_m \geq \delta_Y$ $\quad P_m = A\sigma_Y = (900\times10^{-6})(345\times10^6) = 310.5\times10^3\ \text{N}$

Unloading $\quad \delta' = \dfrac{P_m L}{AE} = \dfrac{\sigma_Y L}{E} = \delta_Y = 3.795\ \text{mm}$

$$\delta_p = \delta_m - \delta'$$

(a) $\delta_m = 4.5\ \text{mm} > \delta_Y \qquad P_m = 310.5\times10^3\ \text{N} = 310.5\ \text{kN}$ ◄

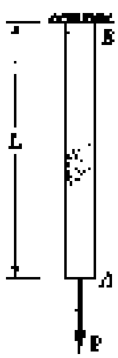
$\delta_{perm} = 4.5\ \text{mm} - 3.795\ \text{mm} = 0.705\ \text{mm}$ ◄

(b) $\delta_m = 8\ \text{mm} > \delta_Y \qquad P_m = 310.5\times10^3\ \text{N} = 310.5\ \text{kN}$ ◄

$\delta_p = 8.0\ \text{mm} - 3.795\ \text{mm} = 4.205\ \text{mm}$ ◄

PROBLEM 2.102

2.102 The 30-mm square bar AB has a length $L = 2.5$ m; it is made of mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force $\mathbf{P}$ is applied to the bar and then removed to give it a permanent set δ_p. Determine the maximum value of the force $\mathbf{P}$ and the maximum amount δ_m by which the bar should be stretched if the desired value of δ_p is (a) 3.5 mm, (b) 6.5 mm.



SOLUTION

$$A = (30)(30) = 900\ \text{mm}^2 = 900\times10^{-6}\ \text{m}^2$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(2.5)(345\times10^6)}{200\times10^9} = 4.3125\times10^{-3}\ \text{m} = 4.3125\ \text{mm}$$

When δ_m exceeds δ_Y, thus producing a permanent stretch of δ_p, the maximum force is

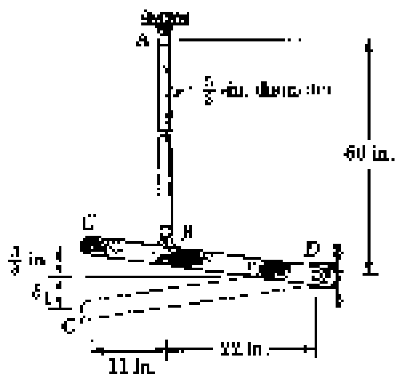
$$P_m = A\sigma_Y = (900\times10^{-6})(345\times10^6) = 310.5\times10^3\ \text{N} = 310.5\ \text{kN}$$ ◄

$$\delta_p = \delta_m - \delta' = \delta_m - \delta_Y \quad \therefore \quad \delta_m = \delta_p + \delta_Y$$

(a) $\delta_p = 3.5\ \text{mm} \qquad \delta_m = 3.5\ \text{mm} + 4.3125\ \text{mm} = 7.81\ \text{mm}$ ◄

(b) $\delta_p = 6.5\ \text{mm} \qquad \delta_m = 6.5\ \text{mm} + 4.3125\ \text{mm} = 10.81\ \text{mm}$ ◄

PROBLEM 2.103

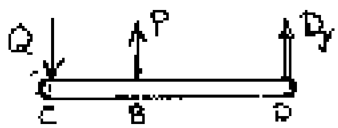


2.103 Rod AB is made of a mild steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ ksi and $\sigma_Y = 36$ ksi. After the rod has been attached to a rigid lever CD, it is found that end C is $\frac{3}{8}$-in. too high. A vertical force **Q** is then applied at C until this point has moved to position C'. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

SOLUTION

Since the rod AB is to be stretched permanently, the peak force in the rod is $P = P_Y$, where

$$P_Y = A\sigma_Y = \frac{\pi}{4}\left(\frac{3}{8}\right)^2(36) = 3.976 \text{ kips}$$



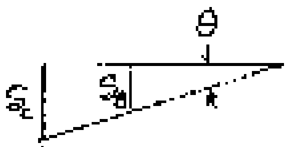
Referring the free body diagram of lever CD

$$\Sigma M_D = 0 \qquad 33\,Q - 22\,P = 0$$

$$Q = \frac{22}{33}P = \frac{(22)(3.976)}{33} = 2.65 \text{ kips}$$

During unloading, the spring back at B is

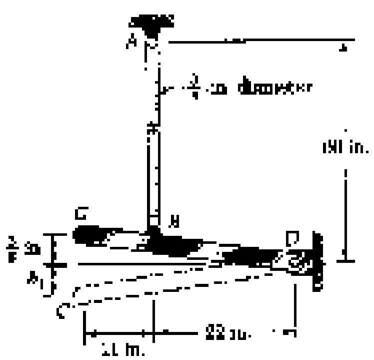
$$\delta_B = L_{AB}\varepsilon_Y = \frac{L_{AB}\sigma_Y}{E} = \frac{(60)(36\times10^3)}{29\times10^6} = 0.0745 \text{ in}$$



From the deformation diagram

$$\text{Slope } \theta = \frac{\delta_B}{22} = \frac{\delta_C}{33} \quad \therefore \quad \delta_C = \frac{33}{22}\delta_B = 0.1117 \text{ in}$$

PROBLEM 2.104



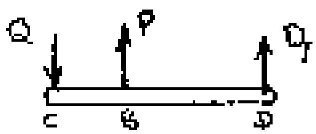
2.103 Rod AB is made of a mild steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ ksi and $\sigma_Y = 36$ ksi. After the rod has been attached to a rigid lever CD, it is found that end C is $\frac{3}{8}$-in. too high. A vertical force **Q** is then applied at C until this point has moved to position C'. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

2.104 Solve Prob. 2.103, assuming that the yield point of the mild steel used is 50 ksi.

SOLUTION

Since the rod AB is to be stretched permanently, the peak force in the rod is $P = P_Y$, where

$$P_Y = A\sigma_Y = \frac{\pi}{4}\left(\frac{3}{8}\right)^2(50) = 5.522 \text{ kips.}$$



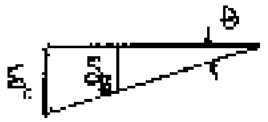
Referring to the free body diagram of lever CD

$$\Sigma M_D = 0 \qquad 33Q - 22\,P = 0$$

$$Q = \frac{22}{33}P = \frac{(22)(5.522)}{33} = 3.68 \text{ kips}$$

During unloading, the spring back at B is

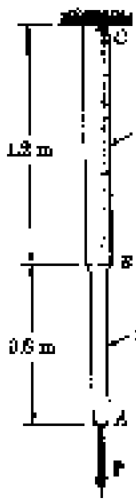
$$\delta_B = L_{AB}\varepsilon_Y = \frac{L_{AB}\sigma_Y}{E} = \frac{(60)(50\times10^3)}{29\times10^6} = 0.1034 \text{ in}$$



From the deformation diagram

$$\text{Slope } \theta = \frac{\delta_B}{22} = \frac{\delta_C}{33} \quad \therefore \quad \delta_C = \frac{33}{22}\delta_B = 0.1552 \text{ in}$$

PROBLEM 2.105



2.105 Rods AB and BC are made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. The rods are stretched until end has moved down 9 mm. Neglecting stress concentrations, determine (*a*) the maximum value of the force **P**, (*b*) the permanent set measured at points A and B after the force has been removed.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(0.035)^2 = 962.1\times10^{-6}\ \text{m}^2; \quad A_{BC} = \frac{\pi}{4}(0.045)^2 = 1.5904\times10^{-3}\ \text{m}^2$$

(a) $P_{max} = A_{min}\sigma_Y = (962.1\times10^{-6})(345\times10^{6})$

$= 331.93\times10^{3}$ N $= 332$ kN

(b) Spring back $\quad \delta' = \dfrac{PL_{AB}}{EA_{AB}} + \dfrac{PL_{BC}}{EA_{BC}} = \dfrac{P}{E}\left(\dfrac{L_{AB}}{A_{AB}} + \dfrac{L_{BC}}{A_{BC}}\right)$

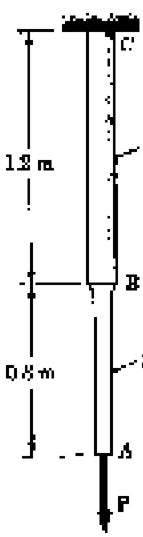
$$\delta' = \frac{331.93\times10^{3}}{200\times10^{9}}\left(\frac{0.8}{962.1\times10^{-6}} + \frac{1.2}{1.5904\times10^{-3}}\right)$$

$= 2.63\times10^{-3}$ m $= 2.63$ mm.

At point A $\quad \delta_p = \delta_m - \delta' = 9$ mm $- 2.63$ mm $= 6.37$ mm

At point B: No yielding in BC; hence $\delta_p = 0$

PROBLEM 2.106



2.105 Rods AB and BC are made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. The rods are stretched until end has moved down 9 mm. Neglecting stress concentrations, determine (*a*) the maximum value of the force **P**, (*b*) the permanent set measured at points A and B after the force has been removed.

2.106 Solve Prob. 2.105, assuming that the yield point of the mild steel used is 250 MPa.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(0.035)^2 = 962.1\times10^{-6}\ \text{m}^2, \quad A_{BC} = \frac{\pi}{4}(0.045)^2 = 1.5904\times10^{-3}\ \text{m}^2$$

(a) $P_{max} = A_{min}\sigma_Y = (962.1\times10^{-6})(250\times10^{6})$

$= 240.53\times10^{3}$ N $= 241$ kN

(b) Spring back $\quad \delta' = \dfrac{PL_{AB}}{EA_{AB}} + \dfrac{PL_{BC}}{EA_{BC}} = \dfrac{P}{E}\left(\dfrac{L_{AB}}{A_{AB}} + \dfrac{L_{BC}}{A_{BC}}\right)$

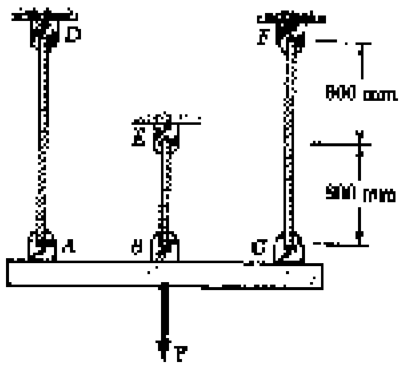
$$\delta' = \frac{240.53\times10^{3}}{200\times10^{9}}\left(\frac{0.8}{962.1\times10^{-6}} + \frac{1.2}{1.5904\times10^{-3}}\right)$$

$= 1.908\times10^{-3}$ m $= 1.908$ mm.

At point A $\quad \delta_p = \delta_m - \delta' = 9$ mm $- 1.908$ mm $= 7.09$ mm

At point B, no yielding in BC; hence $\delta_p = 0$

PROBLEM 2.107



2.107 Each of the three 6-mm-diameter steel cables is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **P** is applied to the rigid bar ABC until the bar has moved downward a distance $\delta = 2$ mm. Knowing that the cables were initially taut, determine (*a*) the maximum value of **P**, (*b*) the maximum stress that occurs in cable AD, (*c*) the final displacement of the bar after the load is removed. (*Hint:* In part *c*, cable BE is not taut.)

SOLUTION

For each cable $A = \frac{\pi}{4}(0.006)^2 = 28.274\times10^{-6}\ \text{m}^2$

Strain at initial yielding

$$\varepsilon_Y = \frac{\sigma_Y}{E} = \frac{345\times10^6}{200\times10^9} = 1.725\times10^{-3}$$

Strain in cables AD and CF: $\varepsilon_{AD} = \varepsilon_{CF} = \frac{\delta}{L_{AD}} = \frac{2\text{ mm}}{1600\text{ mm}} = 1.25\times10^{-3}$

Strain in cable BE: $\varepsilon_{BE} = \frac{\delta}{L_{BE}} = \frac{2\text{ mm}}{800\text{ mm}} = 2.50\times10^{-3}$

Since $\varepsilon_{AD} < \varepsilon_Y$, $\sigma_{AD} = E\varepsilon_{AD} = (200\times10^9)(1.25\times10^{-3}) = 250\times10^6$ Pa

Since $\varepsilon_{BE} > \varepsilon_Y$, $\sigma_{BE} = \sigma_Y = 345\times10^6$ Pa

Forces: $P_{AD} = P_{CF} = A\sigma_{AD} = (28.274\times10^{-6})(250\times10^6) = 7.0685\times10^3$ N

$P_{BE} = A\sigma_{BE} = (28.274\times10^{-6})(345\times10^6) = 9.7545\times10^3$ N

For equilibrium of bar ABC $\quad P_{AD} + P_{BE} + P_{CF} - P = 0$

(a) $P = P_{AD} + P_{BE} + P_{CF} = (7.0685 + 9.7545 + 7.0685)\times10^3$ N

$= 23.9\times10^3$ N $= 23.9$ kN ◄

(b) $\sigma_{AD} = 250\times10^6$ Pa $= 250$ MPa ◄

After unloading $\quad P = 0$

Cable BE is not taut $\quad P_{BE} = 0$

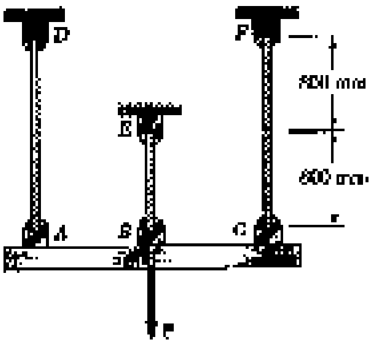
By symmetry $\quad P_{AD} = P_{CF}$

For equilibrium $\quad P_{AD} + P_{CF} = 0$

(c) Final displacement δ is controlled by the final lengths of cables AD and CF. Since these cables were never permanently deformed, the final displacement is

$$\delta = \delta_{AD} = \delta_{CF} = 0$$ ◄

PROBLEM 2.108



2.107 Each of the three 6-mm-diameter steel cables is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **P** is applied to the rigid bar ABC until the bar has moved downward a distance $\delta = 2$ mm. Knowing that the cables were initially taut, determine (*a*) the maximum value of **P**, (*b*) the maximum stress that occurs in cable AD, (*c*) the final displacement of the bar after the load is removed. (*Hint:* In part *c*, cable BE is not taut.)

2.108 Solve Prob. 2.107, assuming that the cables are replaced by rods of the same cross-sectional area and material. Further assume that the rods are braced so that they can carry compressive forces.

SOLUTION

For each rod $A = \frac{\pi}{4}(0.006)^2 = 28.274\times10^{-6}\ \text{m}^2$

Strain at initial yielding $\varepsilon_Y = \frac{\sigma_Y}{E} = \frac{345\times10^6}{200\times10^9} = 1.725\times10^{-3}$

Strain in rods AD and CF: $\varepsilon_{AD} = \varepsilon_{CF} = \frac{\delta}{L_{AD}} = \frac{2\ \text{mm}}{1600\ \text{mm}} = 1.25\times10^{-3}$

Strain in rod BE: $\varepsilon_{BE} = \frac{\delta}{L_{BE}} = \frac{2\ \text{mm}}{800\ \text{mm}} = 2.50\times10^{-3}$

Since $\varepsilon_{AD} < \varepsilon_Y$, $\sigma_{AD} = E\varepsilon_{AD} = (200\times10^9)(1.25\times10^{-3}) = 250\times10^6$ Pa

Since $\varepsilon_{BE} > \varepsilon_Y$, $\sigma_{BE} = \sigma_Y = 345\times10^6$ Pa

Forces: $P_{AD} = P_{CF} = A\sigma_{AD} = (28.274\times10^{-6})(250\times10^6) = 7.0685\times10^3$ N

$P_{BE} = A\sigma_{BE} = (28.274\times10^{-6})(345\times10^6) = 9.7545\times10^3$ N

For equilibrium of bar ABC $\quad P_{AD} + P_{BE} + P_{CF} - P = 0$

(a) $P = P_{AD} + P_{BE} + P_{CF} = (7.0685 + 9.7545 + 7.0685)\times10^3\ \text{N} = 23.9\ \text{kN}$ ◄

(b) $\sigma_{AD} = 250\times10^6\ \text{Pa} = 250\ \text{MPa}$ ◄

Let δ' = change in displacement during unloading

$P'_{AD} = \frac{EA}{L_{AD}}\delta' = \frac{(200\times10^9)(28.274\times10^{-6})}{1600\times10^{-3}}\delta' = 3.534\times10^6\ \delta' = P'_{CF}$

$P'_{BE} = \frac{EA}{L_{BE}}\delta' = \frac{(200\times10^9)(28.274\times10^{-6})}{800\times10^{-3}}\delta' = 7.0685\times10^6\ \delta'$

For equilibrium $\quad P' = P'_{AD} + P'_{BE} + P'_{CF} = 14.137\times10^6\ \delta'$

But $\quad P - P' = 0 \qquad P' = P = 23.89\times10^3$ N

$$\delta' = \frac{23.89\times10^3}{14.137\times10^6} = 1.690\times10^{-3}\ \text{m}$$

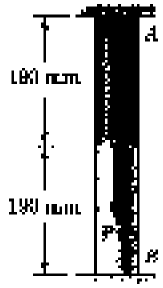
Permanent displacement of bar

$\delta_{final} = \delta_{max} - \delta' = 2\times10^{-3} - 1.690\times10^{-3} = 0.310\times10^{-3}$ m

$= 0.310$ mm ◄

PROBLEM 2.109

2.109 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 1750 mm². Portion AC is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion CB is made of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A load $\mathbf{P}$ is applied at C as shown. Assuming both steels to be elastoplastic, determine (a) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (b) the maximum stress in each portion of the rod, (c) the permanent deflection of C.



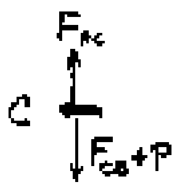
SOLUTION

Displacement at C to cause yielding of AC

$$\delta_{C,Y} = L_{AC}\,\varepsilon_{Y,AC} = \frac{L_{AC}\,\sigma_{Y,AC}}{E} = \frac{(0.190)(250\times10^6)}{200\times10^9} = 0.2375\times10^{-3}\ \text{m}$$

Corresponding force $\quad F_{AC} = A\,\sigma_{Y,AC} = (1750\times10^{-6})(250\times10^6)$
$= 437.5\times10^3$ N

$$F_{CB} = -\frac{EA\delta_C}{L_{CB}} = -\frac{(200\times10^9)(1750\times10^{-6})(0.2375\times10^{-3})}{0.190} = -437.5\times10^3\ \text{N}$$



For equilibrium of element at C

$$F_{AC} - (F_{CB} + P_Y) = 0 \qquad P_Y = F_{AC} - F_{CB} = 875\times10^3\ \text{N}$$

Since applied load $P = 975\times10^3$ N $> 875\times10^3$ N, portion AC yields.

$$F_{CB} = F_{AC} - P = 437.5\times10^3 - 975\times10^3\ \text{N} = -537.5\times10^3\ \text{N}$$

(a) $$\delta_C = -\frac{F_{CB}L_{CB}}{EA} = \frac{(537.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.29179\times10^{-3}\ \text{m}$$
$= 0.292$ mm ◄

(b) Maximum stresses $\quad \sigma_{AC} = \sigma_{Y,AC} = 250$ MPa ◄

$$\sigma_{BC} = \frac{F_{BC}}{A} = -\frac{537.5\times10^3}{1750\times10^{-6}} = -307.14\times10^6\ \text{Pa} = -307\ \text{MPa}$$ ◄

(c) Deflection and forces for unloading

$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore\ P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{CB}} = -P'_{AC}$$

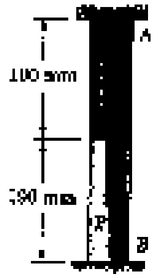
$$P' = 975\times10^3 = P'_{AC} - P'_{CB} = 2P'_{AC} \qquad P'_{AC} = 487.5\times10^3\ \text{N}$$

$$\delta' = \frac{(487.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.26464\times10^{-3}\ \text{m}$$

$$\delta_p = \delta_m - \delta' = 0.29179\times10^{-3} - 0.26464\times10^{-3} = 0.02715\times10^{-3}\ \text{m}$$

$= 0.027$ mm ◄

PROBLEM 2.110



2.109 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 1750 mm². Portion AC is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion BC is made of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C.

2.110 For the composite rod of Prob. 2.109, if P is gradually increased from zero until the deflection of point C reaches a maximum value of $\delta_m = 0.3$ mm and then decreased back to zero, determine, (*a*) the maximum value of P, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C after the load is removed.

SOLUTION

Displacement at C is $\delta_m = 0.30$ mm. The corresponding strains are

$$\varepsilon_{AC} = \frac{\delta_m}{L_{AC}} = \frac{0.30 \text{ mm}}{190 \text{ mm}} = 1.5789 \times 10^{-3}$$

$$\varepsilon_{CB} = -\frac{\delta_m}{L_{CB}} = -\frac{0.30 \text{ mm}}{190 \text{ mm}} = -1.5789 \times 10^{-3}$$

Strains at initial yielding

$$\varepsilon_{Y,AC} = \frac{\sigma_{Y,AC}}{E} = \frac{250 \times 10^6}{200 \times 10^9} = 1.25 \times 10^{-3} \qquad \text{(yielding)}$$

$$\varepsilon_{Y,CB} = -\frac{\sigma_{Y,CB}}{E} = -\frac{345 \times 10^6}{200 \times 10^9} = -1.725 \times 10^{-3} \qquad \text{(elastic)}$$

(a) Forces: $F_{AC} = A\sigma_Y = (1750 \times 10^{-6})(250 \times 10^6) = 437.5 \times 10^{3}$ N

$$F_{CB} = EA\varepsilon_{CB} = (200 \times 10^9)(1750 \times 10^{-6})(-1.5789 \times 10^{-3}) = -552.6 \times 10^3 \text{ N}$$

For equilibrium of element at C $\qquad F_{AC} - F_{CB} - P = 0$

$$P = F_{AC} - F_{CB} = 437.5 \times 10^3 + 552.6 \times 10^3 = 990.1 \times 10^3 \text{ N} = 990 \text{ kN}$$

(b) Stresses: AC $\quad \sigma_{AC} = \sigma_{Y,AC} = 250$ MPa

CB $\quad \sigma_{CB} = \dfrac{F_{CB}}{A} = -\dfrac{552.6 \times 10^3}{1750 \times 10^{-6}} = -316 \times 10^6$ Pa $= -316$ MPa

(c) Deflection and forces for unloading

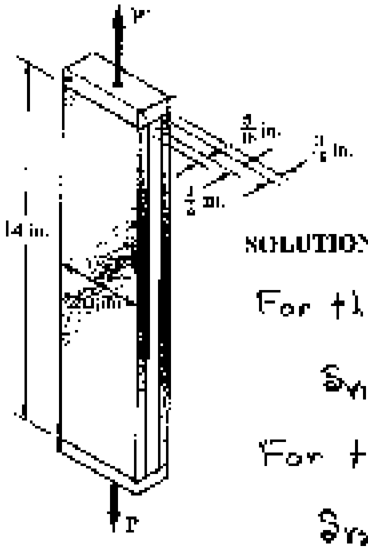
$$\delta' = \frac{P'_{AC} L_{AC}}{EA} = -\frac{P'_{CB} L_{CB}}{EA} \quad \therefore \quad P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{AB}} = -P_{AC}$$

$$P' = P'_{AC} - P'_{CB} = 2P'_{AC} = 990.1 \times 10^3 \text{ N} \quad \therefore \quad P'_{AC} = 495.05 \times 10^3 \text{ N}$$

$$\delta' = \frac{(495.05 \times 10^3)(0.190)}{(200 \times 10^9)(1750 \times 10^{-6})} = 0.26874 \times 10^{-3} \text{ m} = 0.26874 \text{ mm}$$

$$\delta_p = \delta_m - \delta' = 0.30 \text{ mm} - 0.26874 \text{ mm} = 0.031 \text{ mm}$$

PROBLEM 2.111



2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

For the mild steel $\quad A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$

$$\delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(14)(50 \times 10^3)}{29 \times 10^6} = 0.024138 \text{ in.}$$

For the tempered steel $\quad A_2 = 2(\frac{3}{16})(2) = 0.75 \text{ in}^2$

$$\delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(14)(100 \times 10^3)}{29 \times 10^3} = 0.048276 \text{ in.}$$

Total area: $\quad A = A_1 + A_2 = 1.75 \text{ in}^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$ $\quad$ The mild steel yields. Tempered steel is elastic.

(a) Forces $\quad P_1 = A_1\sigma_{Y1} = (1.00)(50 \times 10^3) = 50 \times 10^3$ lb.

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29 \times 10^6)(0.75)(0.04)}{14} = 62.14 \times 10^3 \text{ lb}$$

$$P = P_1 + P_2 = 112.14 \times 10^3 \text{ lb} = 112.1 \text{ kips}$$

(b) Stresses $\quad \sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 50 \times 10^3 \text{ psi} = 50 \text{ ksi}$

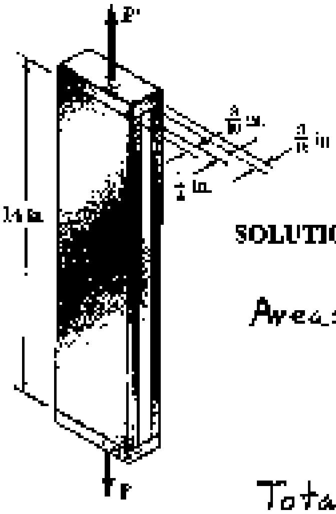
$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14 \times 10^3}{0.75} = 82.86 \times 10^3 \text{ psi} = 82.86 \text{ ksi}$$

Unloading $\quad \delta' = \frac{PL}{EA} = \frac{(112.14 \times 10^3)(14)}{(29 \times 10^6)(1.75)} = 0.03094 \text{ in.}$

(c) Permanent set $\quad \delta_p = \delta_m - \delta' = 0.04 - 0.03094$

$$= 0.00906 \text{ in.}$$

PROBLEM 2.112



2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

2.112 For the composite bar of Prob. 2.111, if P is gradually increased from zero to 98 kips and then decreased back to zero, determine (*a*) the maximum deformation of the bar, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

Areas: Mild steel $A_1 = (\frac{1}{2})(2) = 1.00\ in^2$

Tempered steel $A_2 = 2(\frac{3}{16})(2) = 0.75\ in^2$

Total: $A = A_1 + A_2 = 1.75\ in^2$

Total force to yield the mild steel

$$\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1.75)(50\times10^3) = 87.50\times10^3\ lb.$$

$P > P_Y$, therefore mild steel yields.

Let P_1 = force carried by mild steel

P_2 = force carried by tempered steel

$$P_1 = A_1\sigma_1 = (1.00)(50\times10^3) = 50\times10^3\ lb.$$

$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98\times10^3 - 50\times10^3 = 48\times10^3\ lb.$$

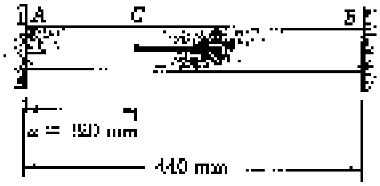
(a) $$\delta_m = \frac{P_2 L}{EA_2} = \frac{(48\times10^3)(14)}{(29\times10^6)(0.75)} = 0.03090\ in.$$

(b) $$\sigma_2 = \frac{P_2}{A_2} = \frac{48\times10^3}{0.75} = 64\times10^3\ psi = 64\ ksi$$

Unloading $$\delta' = \frac{PL}{EA} = \frac{(98\times10^3)(14)}{(29\times10^6)(1.75)} = 0.02703\ in$$

(c) $$\delta_p = \delta_m - \delta' = 0.03090 - 0.02703 = 0.00387\ in.$$

PROBLEM 2.113



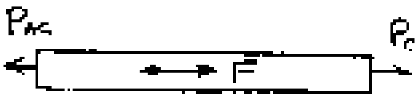
2.113 Bar AB has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point C, (*b*) the residual stress in the bar.

SOLUTION

$$A = 1200 \text{ mm}^2 = 1200 \times 10^{-6} \text{ m}^2$$

Force to yield portion AC: $\quad P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^6)$

$$= 300 \times 10^3 \text{ N}$$



For equilibrium $\quad F + P_{CB} - P_{AC} = 0$

$$P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^3$$
$$= -220 \times 10^3 \text{ N}$$

$$\delta_C = -\frac{P_{CB} L_{CB}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.120)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.293333 \times 10^{-3} \text{ m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = \frac{220 \times 10^3}{1200 \times 10^{-6}} = -183.333 \times 10^6 \text{ Pa}$$

Unloading

$$\delta_C' = \frac{P_{AC}' L_{AC}}{EA} = -\frac{P_{CB}' L_{CB}}{EA} = \frac{(F - P_{AC}') L_{CB}}{EA}$$

$$P_{AC}' \left(\frac{L_{AC}}{EA} + \frac{L_{AC}}{EA} \right) = \frac{F L_{CB}}{EA}$$

$$P_{AC}' = \frac{F L_{CB}}{L_{AC} + L_{CB}} = \frac{(520 \times 10^3)(0.440 - 0.120)}{0.440} = 378.182 \times 10^3 \text{ N}$$

$$P_{CB}' = P_{AC}' - F = 378.182 \times 10^3 - 520 \times 10^3 = -141.818 \times 10^3 \text{ N}$$

$$\sigma_{AC}' = \frac{P_{AC}'}{A} = \frac{378.182 \times 10^3}{1200 \times 10^{-6}} = 315.152 \times 10^6 \text{ Pa}$$

$$\sigma_{CB}' = \frac{P_{CB}'}{A} = -\frac{141.818 \times 10^3}{1200 \times 10^{-6}} = -118.182 \times 10^6 \text{ Pa}$$

$$\delta_C' = \frac{(378.182)(0.120)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.189091 \times 10^{-3} \text{ m}$$

(a) $\quad \delta_{C_p} = \delta_C - \delta_C' = 0.293333 \times 10^{-3} - 0.189091 \times 10^{-3} = 0.1042 \times 10^{-3}$ m

$= 0.1042$ mm ◄

(b) $\quad \sigma_{AC,res} = \sigma_Y - \sigma_{AC}' = 250 \times 10^6 - 315.152 \times 10^6 = -65.2 \times 10^6$ Pa

$= -65.2$ MPa ◄

$\sigma_{CB,res} = \sigma_{CB} - \sigma_{CB}' = -183.333 \times 10^6 + 118.182 \times 10^6 = -65.2 \times 10^6$ Pa

$= -65.2$ MPa ◄

PROBLEM 2.114

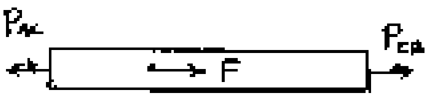
2.113 Bar AB has a cross-sectional area of 1200 mm² and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point C, (*b*) the residual stress in the bar.

2.114 Solve Prob. 2.113, assuming that $a = 180$ mm.

SOLUTION

$A = 1200\ \text{mm}^2 = 1200 \times 10^{-6}\ \text{m}^2$

Force to yield portion AC: $P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^6)$
$= 300 \times 10^3$ N



For equilibrium $F + P_{CB} - P_{AC} = 0$

$P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^3$
$= -220 \times 10^3$ N

$$\delta_C = -\frac{P_{CB} L_{CB}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.180)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.238333 \times 10^{-3}\ \text{m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = -\frac{220 \times 10^3}{1200 \times 10^{-6}} = -183.333 \times 10^6\ \text{Pa}$$

Unloading

$$\delta_C' = \frac{P_{AC}' L_{AC}}{EA} = -\frac{P_{CB}' L_{CB}}{EA} = \frac{(F - P_{AC}')L_{CB}}{EA} \qquad P_{AC}'\left(\frac{L_{AC}}{EA} + \frac{L_{CB}}{EA}\right) = \frac{F L_{CB}}{EA}$$

$$P_{AC}' = \frac{F L_{CB}}{L_{AC} + L_{CB}} = \frac{(520 \times 10^3)(0.440 - 0.180)}{0.440} = 307.273 \times 10^3\ \text{N}$$

$$P_{CB}' = P_{AC}' - F = 307.273 \times 10^3 - 520 \times 10^3 = -212.727 \times 10^3\ \text{N}$$

$$\delta_C' = \frac{(307.273 \times 10^3)(0.180)}{(200 \times 10^9)(1200 \times 10^{-6})} = 0.230455 \times 10^{-3}\ \text{m}$$

$$\sigma_{AC}' = \frac{P_{AC}'}{A} = \frac{307.273 \times 10^3}{1200 \times 10^{-6}} = 256.061 \times 10^6\ \text{Pa}$$

$$\sigma_{CB}' = \frac{P_{CB}'}{A} = \frac{-212.727 \times 10^3}{1200 \times 10^{-6}} = -177.273 \times 10^6\ \text{P}$$

(a) $\delta_{C,p} = \delta_C - \delta_C' = 0.238333 \times 10^{-3} - 0.230455 \times 10^{-3} = 0.00788 \times 10^{-3}$ m
$= 0.00788$ mm

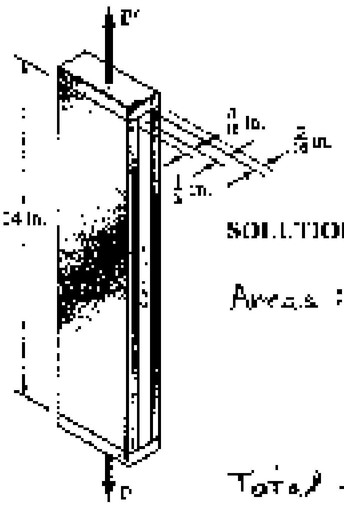
(b) $\sigma_{AC,res} = \sigma_{AC} - \sigma_{AC}' = 250 \times 10^6 - 256.061 \times 10^6 = -6.06 \times 10^6$ Pa
$= -6.06$ MPa

$\sigma_{CB,res} = \sigma_{CB} - \sigma_{CB}' = -183.333 \times 10^6 + 177.273 \times 10^6 = -6.06 \times 10^6$ Pa
$= -6.06$ MPa

PROBLEM 2.115

2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

***2.115** For the composite bar of Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero to 98 kips and then decreased back to zero.



SOLUTION

Areas: Mild steel $\quad A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$

Tempered steel $\quad A_2 = (2)(\frac{3}{16})(2) = 0.75 \text{ in}^2$

Total: $\quad A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel

$$\sigma_Y = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_Y = (1.75)(50 \times 10^3) = 87.50 \times 10^3 \text{ lb.}$$

$P > P_Y$; therefore mild steel yields

Let P_1 = force carried by mild steel
P_2 = force carried by tempered steel

$$P_1 = A_1 \sigma_{Y_1} = (1.00)(50 \times 10^3) = 50 \times 10^3 \text{ lb.}$$

$$P_1 + P_2 = P, \qquad P_2 = P - P_1 = 98 \times 10^3 - 50 \times 10^3 = 48 \times 10^3 \text{ lb.}$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{48 \times 10^3}{0.75} = 64 \times 10^3 \text{ psi}$$

Unloading $\quad \sigma' = \dfrac{P}{A} = \dfrac{98 \times 10^3}{1.75} = 56 \times 10^3 \text{ psi}$

Residual stresses

mild steel $\quad \sigma_{1,\text{res}} = \sigma_1 - \sigma' = 50 \times 10^3 - 56 \times 10^3 = -6 \times 10^3 \text{ psi}$

$= -6$ ksi

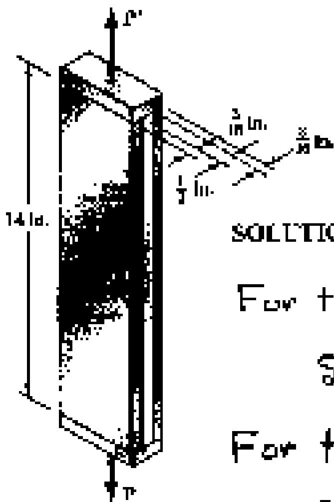
tempered steel $\quad \sigma_{2,\text{res}} = \sigma_2 - \sigma_1 = 64 \times 10^3 - 56 \times 10^3$

$= 8 \times 10^3 \text{ psi} = 8$ ksi

PROBLEM 2.116

2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29 \times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

*2.116 For the composite bar in Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and is then decreased back to zero.



SOLUTION

For the mild steel $\quad A_1 = (\frac{1}{2})(2) = 1.00 \text{ in}^2$

$$\delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(14)(50 \times 10^3)}{29 \times 10^6} = 0.024138 \text{ in}$$

For the tempered steel $\quad A_2 = 2(\frac{3}{16})(2) = 0.75 \text{ in}^2$

$$\delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(14)(100 \times 10^3)}{29 \times 10^6} = 0.048276 \text{ in}$$

Total area: $\quad A = A_1 + A_2 = 1.75 \text{ in}^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$ The mild steel yields. Tempered steel is elastic.

Forces $\quad P_1 = A_1\sigma_{Y1} = (1.00)(50 \times 10^3) = 50 \times 10^3$ lb

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29 \times 10^6)(0.75)(0.04)}{14} = 62.14 \times 10^3 \text{ lb}$$

Stresses $\quad \sigma_1 = \dfrac{P_1}{A_1} = \sigma_{Y1} = 50 \times 10^3$ psi

$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14 \times 10^3}{0.75} = 82.86 \times 10^3 \text{ psi}$$

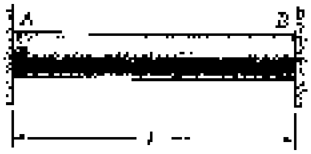
Unloading $\quad \sigma' = \dfrac{P}{A} = \dfrac{112.14}{1.75} = 64.08 \times 10^3$ psi

Residual stresses

$$\sigma_{1,res} = \sigma_1 - \sigma' = 50 \times 10^3 - 64.08 \times 10^3 = -14.08 \times 10^3 \text{ psi}$$
$$= -14.08 \text{ ksi}$$

$$\sigma_{2,res} = \sigma_2 - \sigma' = 82.86 \times 10^3 - 64.08 \times 10^3 = 18.78 \times 10^3 \text{ psi}$$
$$= 18.78 \text{ ksi}$$

PROBLEM 2.117



2.117 A uniform steel rod of cross-sectional area A is attached to rigid supports and is unstressed at a temperature of 8°C. The steel is assumed to be elastoplastic with σ_Y = 250 MPa and E = 200 GPa. Knowing that $\alpha = 11.7 \times 10^{-6}/°C$, determine the stress in the bar (*a*) when the temperature is raised to 165°C, (*b*) after the temperature has returned to 8°C.

SOLUTION

Determine temperature change to cause yielding

$$\delta = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_Y L}{E} + L\alpha(\Delta T)_Y = 0$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E\alpha} = \frac{250 \times 10^6}{(200 \times 10^9)(11.7 \times 10^{-6})} = 106.838\ °C$$

But $\Delta T = 165 - 8 = 157\ °C$

(a) Yielding occurs = $\sigma = -\sigma_Y = -250$ MPa ◀

Cooling $(\Delta T)' = 157\ °C$

$$\delta' = \delta'_P + \delta'_T = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0$$

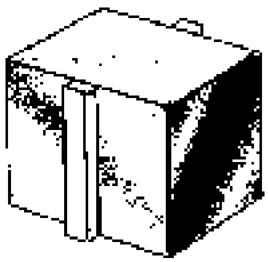
$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)'$$

$$= -(200 \times 10^9)(11.7 \times 10^{-6})(157) = -367.38 \times 10^6 \text{ Pa}$$

(b) $\sigma_{res} = -\sigma_Y - \sigma' = -250 \times 10^6 + 367.38 \times 10^6 = 117.38 \times 10^6$ Pa

$= 117.4$ MPa ◀

PROBLEM 2.118



2.118 A narrow bar of aluminum is bonded to the side of a thick steel plate as shown. Initially, at $T_1 = 20°C$, all stresses are zero. Knowing that the temperature will be slowly raised to T_2 and then reduced to T_1, determine (*a*) the highest temperature T_2 that does not result in residual stresses, (*b*) the temperature T_2 that will result in a residual stress in the aluminum equal to 100 MPa. Assume $\alpha_a = 23.6 \times 10^{-6}/°C$ for the aluminum and $\alpha_s = 11.7 \times 10^{-6}/°C$ for the steel. Further assume that the aluminum is elastoplastic, with $E = 70$ GPa and $\sigma_Y = 100$ MPa. (*Hint:* Neglect the small stresses in the plate.)

SOLUTION

Determine temperature change to cause yielding

$$\delta = \frac{PL}{EA} + L\alpha_a(\Delta T)_Y = L\alpha_s(\Delta T)_Y$$

$$\frac{P}{A} = \sigma = -E(\alpha_a - \alpha_s)(\Delta T)_Y = -\sigma_Y$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{100 \times 10^6}{(70 \times 10^9)(23.6 - 11.7)(10^{-6})} = 120.04\,°C$$

(a) $T_{2Y} = T_1 + (\Delta T)_Y = 20 + 120.04 = 140.04\,°C$

After yielding

$$\delta = \frac{\sigma_Y L}{E} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)$$

Cooling

$$\delta' = \frac{P'L}{AE} + L\alpha_a(\Delta T)' = L\alpha_s(\Delta T)'$$

The residual stress is

$$\sigma_{res} = \sigma_Y - \frac{P'}{A} = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

Set $\sigma_{res} = -\sigma_Y$

$$-\sigma_Y = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

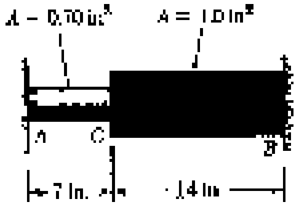
$$\Delta T = \frac{2\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{(2)(100 \times 10^6)}{(70 \times 10^9)(23.6 - 11.7)(10^{-6})} = 240.1\,°C$$

(b) $T_2 = T_1 + \Delta T = 20 + 240.1 = 260.1\,°C$

If $T_2 > 260.1\,°C$, the aluminum bar will most likely yield in compression.

PROBLEM 2.119

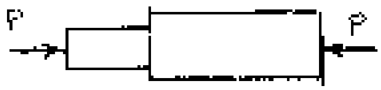
2.119 The steel rod ABC is attached to rigid supports and is unstressed at a temperature of 38°F. The steel is assumed elastoplastic, with $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. The temperature of both portions of the rod is then raised to 250 °F. Knowing that $\alpha = 6.5 \times 10^{-6}/°F$, determine (*a*) the stress in portion AC, (*b*) the deflection of point C.



SOLUTION

$$\delta_{B/A} = \delta_{B/A,P} + \delta_{B/A,T} = 0 \qquad \text{(constraint)}$$

Determine ΔT to cause yielding in AC.



$$-\frac{P L_{AC}}{E A_{AC}} - \frac{P L_{CB}}{E A_{CB}} + L_{AB}\,\alpha(\Delta T) = 0$$

$$(\Delta T) = \frac{P}{L_{AB} E \alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

At yielding $\quad P = A_{AC}\sigma_Y$

$$(\Delta T)_Y = \frac{A_{AC}\sigma_Y}{L_{AB} E \alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right) = \frac{(0.70)(36\times10^3)}{(21)(29\times10^6)(6.5\times10^{-6})}\left(\frac{7}{0.70} + \frac{14}{1.0}\right)$$

$$= 152.785\,°F$$

Actual $\Delta T = 250 - 38 = 212\,°F > (\Delta T)_Y \quad \therefore$ yielding occurs

$$\sigma_{AC} = -\sigma_Y = -36 \text{ ksi}$$

$$P = \sigma_Y A_{AC} = (36\times10^3)(0.70) = 25.2\times10^3 \text{ lb}$$

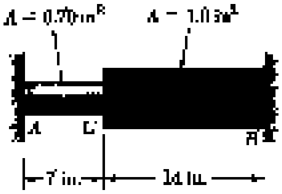
$$\delta_C = -\delta_{C/B} = \frac{P L_{CB}}{E A_{CB}} - L_{CB}\,\alpha(\Delta T)$$

$$= \frac{(25.2\times10^3)(14)}{(29\times10^6)(1.0)} - (14)(6.5\times10^{-6})(212)$$

$$= 0.012176 - 0.019292 \qquad = -0.007116 \text{ in}$$

$$\delta_C = 0.00712 \text{ in} \leftarrow$$

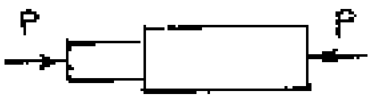
PROBLEM 2.120



2.119 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 38°F. The steel is assumed elastoplastic, with $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. The temperature of both portions of the rod is then raised to 250 °F. Knowing that $\alpha = 6.5 \times 10^{-6}/°F$, determine (*a*) the stress in portion *AC*, (*b*) the deflection of point *C*.

***2.120** Solve Prob. 2.119, assuming that the temperature of the rod is raised to 250 °F and then returned to 38 °F.

SOLUTION



$$\delta_{B/A} = \delta_{B/A,P} + \delta_{B/A,T} = 0 \qquad \text{(constraint)}$$

Determine ΔT to cause yielding in AC.

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} + L_{AB}\alpha(\Delta T) = 0$$

$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right) = \frac{P}{(21)(29\times10^6)(6.5\times10^{-6})}\left(\frac{7}{0.70} + \frac{14}{1.0}\right)$$

$= 6.0629 \times 10^{-3}\, P$ At yielding $P_Y = \sigma_Y A_{AC} = (36\times10^3)(0.7) = 25.2\times10^3$ lb

$$(\Delta T)_Y = (6.0629\times10^{-3})(25.2\times10^3) = 152.785\ °F$$

Actual $\Delta T = 250 - 38 = 212\ °F > (\Delta T)_Y$ $\therefore$ yielding occurs.

$$\sigma_{AC} = -\sigma_Y = -36\times10^3 \text{ psi}$$

$$\delta_C = -\delta_{B/C} = \frac{PL_{CB}}{EA_{CB}} - L_{BC}\alpha(\Delta T) = \frac{(25.2\times10^3)(14)}{(29\times10^6)(1.0)} - (14)(6.5\times10^{-6})(212)$$

$= 0.012176 - 0.019292 = -0.007116$ in

Cooling $\Delta T' = 212\ °F$ $P' = \dfrac{\Delta T}{6.0629\times10^{-3}} = \dfrac{212}{6.0629\times10^{-3}}$

$$P' = \frac{\Delta T'}{6.0629\times10^{-3}} = \frac{212}{6.0629\times10^{-3}} = 34.967\times10^3 \text{ lb.}$$

(a) Residual stress in AC

$$\sigma_{AC,res} = -\sigma_Y + \frac{P'}{A_{AC}} = -36\times10^3 + \frac{34.967\times10^3}{0.7} = 13.95\times10^3 \text{ psi}$$

$= 13.95$ ksi ◄

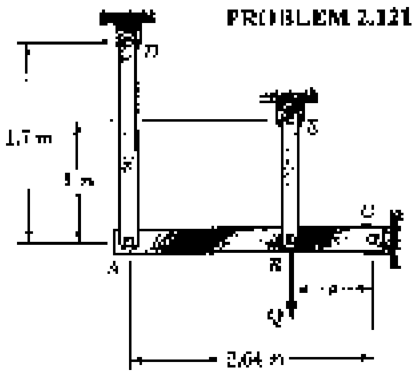
$$\delta_C' = -\delta_{B/C}' = -\frac{P'L_{CB}}{EA_{CB}} + L_{CB}\alpha(\Delta T)$$

$$= -\frac{(34.967\times10^3)(14)}{(29\times10^6)(1.0)} + (14)(6.5\times10^{-6})(212)$$

$= -0.016881 + 0.019292 = 0.002411$ in

$$\delta_{C,F} = \delta_C + \delta_C' = -0.007116 + 0.002411 = -0.00471 \text{ in}$$

0.00471 in ← ◄



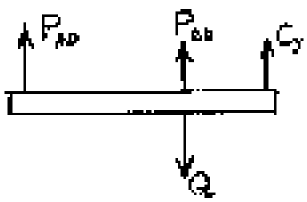
PROBLEM 2.121

2.121 The rigid bar *ABC* is supported by two links, *AD* and *BE*, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with E = 200 GPa and σ_Y = 250 MPa. The magnitude of the force **Q** applied at *B* is gradually increased from zero to 260 kN. Knowing that a = 0.640 m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point *B*.

SOLUTION

Statics: $\Sigma M_c = 0 \quad 0.640(Q - P_{BE}) - 2.64\,P_{AD} = 0$

Deformation: $\delta_A = 2.64\,\theta, \quad \delta_B = a\theta = 0.640\,\theta$

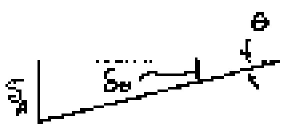


Elastic Analysis:

$A = (37.5)(6) = 225\ \text{mm}^2 = 225\times10^{-6}\ \text{m}^2$

$P_{AD} = \dfrac{EA}{L_{AD}}\,\delta_A = \dfrac{(200\times10^9)(225\times10^{-6})}{1.7}\,\delta_A = 26.47\times10^6\,\delta_A$

$= (26.47\times10^6)(2.64\,\theta) = 69.88\times10^6\,\theta$



$\sigma_{AD} = \dfrac{P_{AD}}{A} = 310.6\times10^9\,\theta$

$P_{BE} = \dfrac{EA}{L_{BE}}\,\delta_B = \dfrac{(200\times10^9)(225\times10^{-6})}{1.0}\,\delta_B = 45\times10^6\,\delta_B$

$= (45\times10^6)(0.640\,\theta) = 28.80\times10^6\,\theta$

$\sigma_{BE} = \dfrac{P_{BE}}{A} = 128\times10^9\,\theta$

From Statics $\quad Q = P_{BE} + \dfrac{2.64}{0.640}P_{AD} = P_{BE} + 4.125\,P_{AD}$

$= [28.80\times10^6 + (4.125)(69.88\times10^6)]\,\theta = 317.06\times10^6\,\theta$

θ_Y at yielding of link AD $\qquad \sigma_{AD} = \sigma_Y = 250\times10^6 = 310.6\times10^9\,\theta$

$\theta_Y = 804.89\times10^{-6}$

$Q_Y = (317.06\times10^6)(804.89\times10^{-6}) = 255.2\times10^3\ N$

Since $Q = 260\times10^3 > Q_Y$, link AD yields. $\qquad \sigma_{AD} = 250\ \text{MPa}$ ◄

$P_{AD} = A\,\sigma_Y = (225\times10^{-6})(250\times10^6) = 56.25\times10^3\ N$

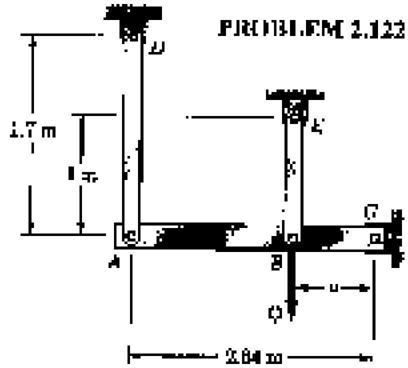
From Statics $\quad P_{BE} = Q - 4.125\,P_{AD} = 260\times10^3 - (4.125)(56.25\times10^3)$

$P_{BE} = 27.97\times10^3\ N \qquad \sigma_{BE} = \dfrac{P_{BE}}{A} = \dfrac{27.97\times10^3}{225\times10^{-6}} = 124.3\times10^6\ Pa$

$= 124.3\ \text{MPa}$ ◄

$\delta_B = \dfrac{P_{BE}L_{BE}}{EA} = \dfrac{(27.97\times10^3)(1.0)}{(200\times10^9)(225\times10^{-6})} = 621.63\times10^{-6}\ m$

$= 0.622\ \text{mm}$ ◄

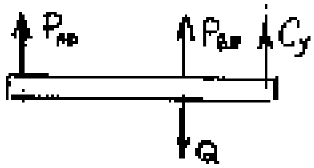


PROBLEM 2.122

2.121 The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point B.

2.122 Solve Prob. 2.121, knowing that $a = 1.76$ m and that the magnitude of the force **Q** applied at B is gradually increased from zero to 135 kN.

SOLUTION



Statics: $\Sigma M_C = 0 \quad 1.76(Q - P_{BE}) - 2.64\,P_{AD} = 0$

Deformation: $\delta_A = 2.64\,\theta, \quad \delta_B = 1.76\,\theta$

Elastic Analysis

$A = (37.5)(6) = 225\ \text{mm}^2 = 225\times10^{-6}\ \text{m}^2$

$P_{AD} = \frac{EA}{L_{AD}}\delta_A = \frac{(200\times10^9)(225\times10^{-6})}{1.7}\delta_A = 26.47\times10^6\ \delta_A$

$= (26.47\times10^6)(2.64\,\theta) = 69.88\times10^6\,\theta$

$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6\times10^9\,\theta$

$P_{BE} = \frac{EA}{L_{BE}}\delta_B = \frac{(200\times10^9)(225\times10^{-6})}{1.0}\delta_B = 45\times10^6\,\delta_B = (45\times10^6)(1.76\,\theta)$

$= 79.2\times10^6\,\theta \qquad \sigma_{BE} = \frac{P_{BE}}{A} = 352\times10^9\,\theta$

From Statics $\quad Q = P_{BE} + \frac{2.64}{1.76}P_{AD} = P_{BE} + 1.500\,P_{AD}$

$= [79.2\times10^6 + (1.500)(69.88\times10^6)]\,\theta = 178.62\times10^6\,\theta$

θ_Y at yielding of link BE $\quad \sigma_{BE} = \sigma_Y = 250\times10^6 = 352\times10^9\,\theta_Y$

$\theta_Y = 710.23\times10^{-6}$

$Q_Y = (178.62\times10^6)(710.23\times10^{-6}) = 126.86\times10^3$ N

Since $Q = 135\times10^3\,\text{N} > Q_Y$, link BE yields $\sigma_{BE} = \sigma_Y = 250$ MPa ◄

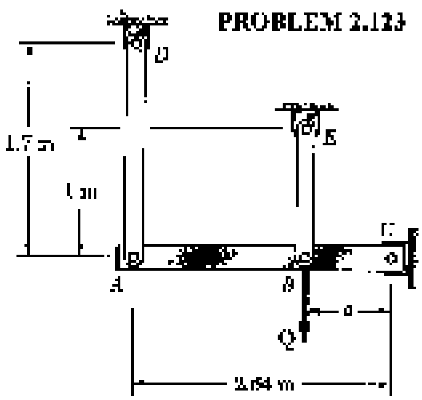
$P_{BE} = A\,\sigma_Y = (225\times10^{-6})(250\times10^6) = 56.25\times10^3$ N

From Statics $\quad P_{AD} = \frac{1}{1.500}(Q - P_{BE}) = 52.5\times10^3$ N

$\sigma_{AD} = \frac{P_{AD}}{A} = \frac{52.5\times10^3}{225\times10^{-6}} = 233.3\times10^6 = 233$ MPa ◄

From elastic analysis of AD $\quad \theta = \frac{P_{AD}}{69.88\times10^6} = 751.29\times10^{-6}$ rad

$\delta_B = 1.76\,\theta = 1.322\times10^{-3}$ m $= 1.322$ mm ◄



PROBLEM 2.123

2.121 The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (a) the value of the normal stress in each link, (b) the ~~maximum~~ final deflection of point B.

*2.123 Solve Prob. 2.121, assuming that the magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN and then decreased back to zero. Knowing that $a = 0.640$ m, determine (a) the residual stress in each link, (b) the final deflection of point B the residual stress in each link. Assume that the links are braced so that they can carry compressive forces without buckling.

SOLUTION

See solution to PROBLEM 2.121 for the normal stresses in each link and the deflection of point B after loading

$$\sigma_{AD} = 250\times10^6\ \text{Pa} \qquad \sigma_{BE} = 124.8\times10^6\ \text{Pa}$$

$$\delta_B = 621.53\times10^{-6}\ \text{m}$$

The elastic analysis given in the solution to PROBLEM 2.121 applies to the unloading

$$Q = 317.06\times10^6\ \theta'$$

$$\theta' = \frac{Q}{317.06\times10^6} = \frac{260\times10^3}{317.06\times10^6} = 820.03\times10^{-6}$$

$$\sigma'_{AD} = 310.6\times10^9\ \theta = (310.6\times10^9)(820.03\times10^{-6}) = 254.70\times10^6\ \text{Pa}$$

$$\sigma'_{BE} = 128\times10^9\ \theta = (128\times10^9)(820.03\times10^{-6}) = 104.96\times10^6\ \text{Pa}$$

$$\delta'_B = 0.640\ \theta' = 524.82\times10^{-6}\ \text{m}$$

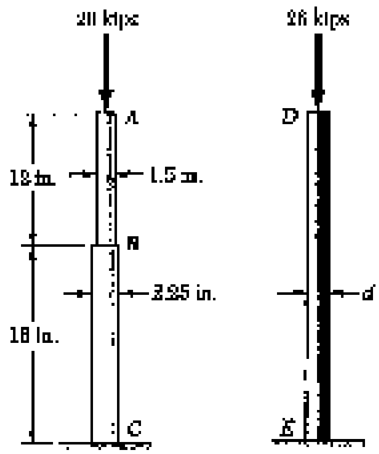
(a) Residual stresses

$$\sigma_{AD,res} = \sigma_{AD} - \sigma'_{AD} = 250\times10^6 - 254.70\times10^6 = -4.70\times10^6\ \text{Pa} = -4.70\ \text{MPa}$$

$$\sigma_{BE,res} = \sigma_{BE} - \sigma'_{BE} = 124.8\times10^6 - 104.96\times10^6 = 19.84\times10^6\ \text{Pa} = 19.84\ \text{MPa}$$

(b) $$\delta_{B,P} = \delta_B - \delta'_B = 621.53\times10^{-6} - 524.82\times10^{-6} = 96.71\times10^{-6}\ \text{m} = 0.0967\ \text{mm}$$

PROBLEM 2.124



2.124 The aluminum rod ABC ($E = 10.1 \times 10^6$ psi), which consists of two cylindrical portions AB and BC, is to be replaced with a cylindrical steel rod DE ($E = 29 \times 10^6$ psi) of the same overall length. Determine the minimum required diameter d of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 24 ksi.

SOLUTION

Deformation of aluminum rod

$$\delta_A = \frac{PL_{AB}}{A_{AB}E} + \frac{PL_{BC}}{A_{BC}E} = \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$$

$$= \frac{28 \times 10^3}{10.1 \times 10^6}\left(\frac{12}{\frac{\pi}{4}(1.5)^2} + \frac{18}{\frac{\pi}{4}(2.25)^2}\right) = 0.031376 \text{ in}$$

Steel rod $\qquad \delta = 0.031376$ in

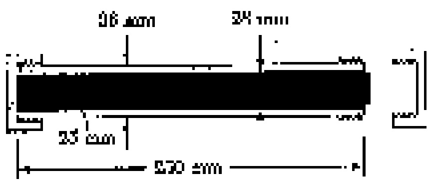
$$\delta = \frac{PL}{EA} \quad \therefore \quad A = \frac{PL}{E\delta} = \frac{(28 \times 10^3)(30)}{(29 \times 10^6)(0.031376)} = 0.92317 \text{ in}^2$$

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{28 \times 10^3}{24 \times 10^3} = 1.1667 \text{ in}^2$$

Required area is the larger value $\qquad A = 1.1667 \text{ in}^2$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(1.6667)}{\pi}} = 1.219 \text{ in.}$$

PROBLEM 2.125



2.125 A 250-mm-long aluminum tube ($E = 70$ GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod ($E = 105$ GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

SOLUTION

$$A_{tube} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12 \times 10^{-6} \text{ m}^2$$

$$A_{rod} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$\delta_{tube} = \frac{PL}{E_{tube}A_{tube}} = \frac{P(0.250)}{(70 \times 10^9)(402.12 \times 10^{-6})} = 8.8815 \times 10^{-9} P$$

$$\delta_{rod} = -\frac{PL}{E_{rod}A_{rod}} = \frac{P(0.250)}{(105 \times 10^9)(490.87 \times 10^{-6})} = -4.8525 \times 10^{-9} P$$

$$\delta^* = \frac{1}{4} \text{ turn} \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375 \times 10^{-6} \text{ m}$$

$$\delta_{tube} = \delta^* + \delta_{rod} \quad \text{or} \quad \delta_{tube} - \delta_{rod} = \delta^*$$

$$8.8815 \times 10^{-9} P + 4.8505 \times 10^{-9} P = 375 \times 10^{-6}$$

$$P = \frac{0.375 \times 10^{-3}}{(8.8815 + 4.8505)(10^{-9})} = 27.308 \times 10^3 \text{ N}$$

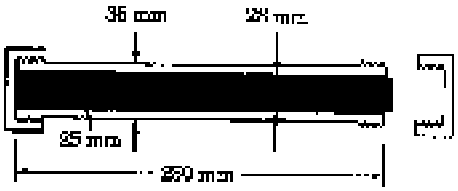
(a) $$\sigma_{tube} = \frac{P}{A_{tube}} = \frac{27.308 \times 10^3}{402.12 \times 10^{-6}} = 67.9 \times 10^6 \text{ Pa} = 67.9 \text{ MPa}$$

$$\sigma_{rod} = -\frac{P}{A_{rod}} = -\frac{27.308 \times 10^3}{490.87 \times 10^{-6}} = -55.6 \times 10^6 \text{ Pa} = -55.6 \text{ MPa}$$

(b) $$\delta_{tube} = (8.8815 \times 10^{-9})(27.308 \times 10^3) = 242.5 \times 10^{-6} \text{ m} = 0.2425 \text{ mm}$$

$$\delta_{rod} = -(4.8505 \times 10^{-9})(27.308 \times 10^3) = -132.5 \times 10^{-6} \text{ m} = -0.1325 \text{ mm}$$

PROBLEM 2.126



2.125 A 250-mm-long aluminum tube (E = 70 GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod (E = 105 GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

2.126 In Prob. 2.125, determine the average normal stress in the tube and the rod, assuming that the temperature was 15° C when the nuts were snugly fitted and that the final temperature is 55° C. (For aluminum, $\alpha = 23.6 \times 10^{-6}/°C$; for brass, $\alpha = 20.9 \times 10^{-6}/°C$.)

SOLUTION

$$A_{tube} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12 \times 10^{-6} \text{ m}^2$$

$$A_{rod} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87 \times 10^{-6} \text{ m}^2$$

$$\Delta T = 55 - 15 = 40\ °C$$

$$\delta_{tube} = \frac{PL}{E_{tube} A_{tube}} + L\alpha_{tube}(\Delta T) = \frac{P(0.250)}{(70\times 10^9)(402.12\times 10^{-6})} + (0.250)(23.6\times 10^{-6})(40)$$

$$= 8.8815 \times 10^{-9} P + 236 \times 10^{-6}$$

$$\delta_{rod} = -\frac{PL}{E_{rod} A_{rod}} + L\alpha_{rod}(\Delta T) = -\frac{P(0.250)}{(105\times 10^9)(490.87\times 10^{-6})} + (0.250)(20.9\times 10^{-6})(40)$$

$$= -4.8505 \times 10^{-9} P + 209 \times 10^{-6}$$

$$\delta^* = \frac{1}{4} \text{ turn} \times 1.5 \text{ mm} = 0.375 \text{ mm} = 375 \times 10^{-6} \text{ m}$$

$$\delta_{tube} = \delta_{rod} + \delta^*$$

$$8.8815 \times 10^{-9} P + 236 \times 10^{-6} = -4.8505 \times 10^{-9} P + 209 \times 10^{-6} + 375 \times 10^{-6}$$

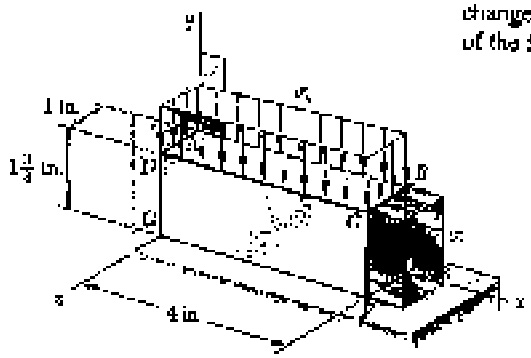
$$13.732 \times 10^{-9} P = 348 \times 10^{-6} \qquad P = 25.342 \times 10^{3} \text{ N}$$

$$\sigma_{tube} = \frac{P}{A_{tube}} = \frac{25.342 \times 10^3}{402.12 \times 10^{-6}} = 63.0 \times 10^6 \text{ Pa} = 63.0 \text{ MPa} \blacktriangleleft$$

$$\sigma_{rod} = -\frac{P}{A_{rod}} = -\frac{25.342 \times 10^3}{490.87 \times 10^{-6}} = -51.6 \times 10^6 \text{ Pa} = -51.6 \text{ MPa} \blacktriangleleft$$

PROBLEM 2.127

2.127 The block shown is made of a magnesium alloy for which $E = 6.5 \times 10^6$ psi and $\nu = 0.35$. Knowing that $\sigma_x = -20$ ksi, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.



SOLUTION

$$\sigma_y = 0 \qquad \varepsilon_y = 0$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 0$$

(a) $\sigma_y = \nu\sigma_x = (0.35)(-20 \times 10^3)$

$$= -7 \times 10^3 \text{ psi} = -7 \text{ ksi}$$

(b) $\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y) = -\frac{\nu(\sigma_x + \sigma_y)}{E}$

$$= \frac{(0.35)(-20 \times 10^3 - 7 \times 10^3)}{6.5 \times 10^6} = 1.4538 \times 10^{-3}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{-20 \times 10^3 - (0.35)(-7 \times 10^3)}{6.5 \times 10^6}$$

$$= -2.7 \times 10^{-3}$$

$$A_0 + \Delta A = L_x(1 + \varepsilon_x)L_z(1 + \varepsilon_z) = L_x L_z(1 + \varepsilon_x + \varepsilon_z + \varepsilon_x\varepsilon_z)$$

But $A_0 = L_x L_z$

$$\Delta A = L_x L_z(\varepsilon_x + \varepsilon_z + \varepsilon_x\varepsilon_z)$$

$$= (4.0)(1.0)(1.4538 \times 10^{-3} - 2.7 \times 10^{-3} + \text{small term})$$

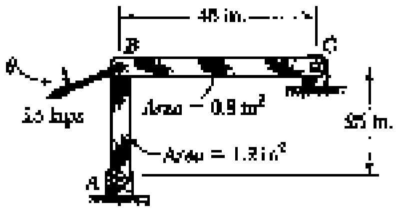
$$= -4.98 \times 10^{-3} \text{ in}^2 = -0.00498 \text{ in}^2$$

(c) Since L_y is constant

$$\Delta V = L_y(\Delta A) = (1.375)(-4.98 \times 10^{-3}) = -6.85 \times 10^{-3} \text{ in}^3$$

$$= -0.00685 \text{ in}^3$$

PROBLEM 2.128

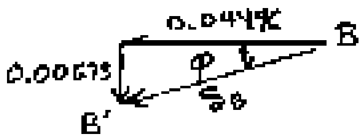


2.128 The uniform rods *AB* and *BC* are made of steel and are loaded as shown. Knowing that $E = 29 \times 10^6$ psi, determine the magnitude and direction of the deflection of point *B* when $\theta = 22°$.

SOLUTION

$$P_{BC} = P\cos\theta = (25\times10^3)\cos 22° = 23.18\times10^3\ \text{lb}$$

$$\delta_{BC} = \frac{P_{BC}L_{BC}}{EA_{BC}} = \frac{(23.18\times10^3)(45)}{(29\times10^6)(0.8)} = 0.04496\ \text{in}$$



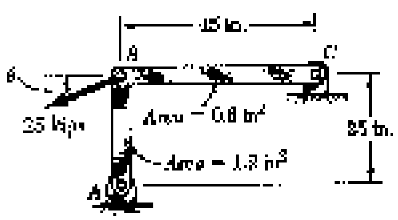
$$P_{AB} = P\sin\theta = (25\times10^3)\sin 22° = 9.365\times10^3\ \text{lb}$$

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{EA_{AB}} = \frac{(9.365\times10^3)(25)}{(29\times10^6)(1.2)} = 0.00673\ \text{in.}$$

$$\tan\varphi = \frac{0.00673}{0.04496} = 0.1496 \qquad \varphi = 8.51° \quad ◄$$

$$\delta = \sqrt{0.04496^2 + 0.00673^2} = 0.0455\ \text{in.} \quad ◄$$

PROBLEM 2.129



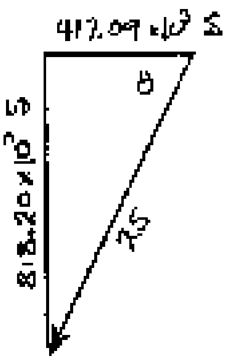
2.129 Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the value of θ for which the deflection of point *B* is down and to the left along a line forming an angle of 36° with the horizontal, (*b*) the corresponding magnitude of the deflection of *B*.

SOLUTION

$$\delta_{BC} = \delta\cos 36°$$

$$P_{BC} = \frac{EA_{BC}\delta_{BC}}{L_{BC}} = \frac{(29\times10^6)(0.8)\,\delta\cos 36°}{45}$$

$$= 417.09\times10^3\ \delta$$



$$\delta_{AB} = \delta\sin 36°$$

$$P_{AB} = \frac{EA_{AB}\delta_{AB}}{L_{AB}} = \frac{(29\times10^6)(1.2)\,\delta\sin 36°}{25}$$

$$= 818.20\times10^3\ \delta$$

$$\tan\theta = \frac{818.20\times10^3\ \delta}{417.09\times10^3\ \delta} = 1.9617 \qquad \theta = 63.0° \quad ◄$$

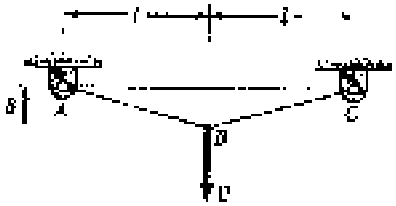
$$P = 25\times10^3 = \sqrt{(417.09\times10^3\ \delta)^2 + (818.20\times10^3\ \delta)^2} = 918.38\times10^3\ \delta$$

$$\delta = \frac{25\times10^3}{918.38\times10^3} = 0.0272\ \text{in.} \quad ◄$$

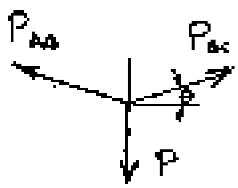
PROBLEM 2.130

2.130 The uniform wire ABC, of unstretched length $2l$, is attached to the supports shown and a vertical load $\mathbf{P}$ is applied at the midpoint B. Denoting by A the cross-sectional area of the wire and by E the modulus of elasticity, show that, for $\delta \ll l$, the deflection at the midpoint B is

$$\delta = l\sqrt[3]{\frac{P}{AE}}$$



SOLUTION



Use approximation

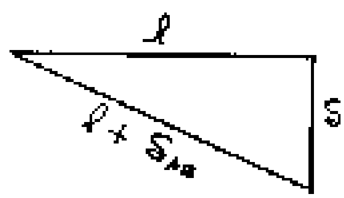
$$\sin\theta \approx \tan\theta \approx \frac{\delta}{l}$$

Statics $\quad \Sigma F_y = 0 \qquad 2P_{AB}\sin\theta - P = 0$

$$P_{AB} = \frac{P}{2\sin\theta} \approx \frac{Pl}{2\delta}$$

Elongation $\quad \delta_{AB} = \frac{P_{AB}\,l}{AE} = \frac{Pl^2}{2AE\delta}$

Deflection



From the right triangle

$$(l + \delta_{AB})^2 = l^2 + \delta^2$$

$$\delta^2 = \cancel{l^2} + 2l\,\delta_{AB} + \delta_{AB}^2 - \cancel{l^2}$$

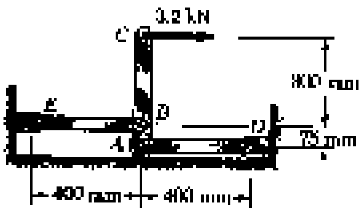
$$= 2l\,\delta_{AB}\left(1 + \frac{1}{2}\frac{\delta_{AB}}{l}\right) \approx 2l\,\delta_{AB}$$

$$\approx \frac{Pl^3}{AE\delta}$$

$$\delta^3 \approx \frac{Pl^3}{AE} \quad \therefore \quad \delta \approx l\sqrt[3]{\frac{P}{AE}}$$

PROBLEM 2.131

2.131 The steel bars BE and AD each have a 6 × 18-mm cross section. Knowing that E = 200 GPa, determine the deflections of points A, B, and C of the rigid bar ABC.



SOLUTION

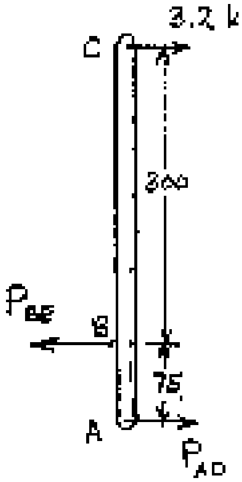
Use rigid bar ABC as a free body

$\circlearrowleft \Sigma M_B = 0 \quad (75)P_{AD} - (300)(3.2) = 0$

$P_{AD} = 12.8 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0 \quad -P_{BE} + 3.2 + P_{AD} = 0$

$P_{BE} = 16 \text{ kN}$



Deformations

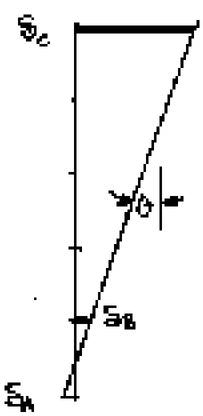
$A = (6)(18) = 108 \text{ mm}^2 = 108 \times 10^{-6} \text{ m}^2$

$\leftarrow \delta_A = \delta_{AD} = \dfrac{P_{AD} L_{AD}}{EA} = \dfrac{(12.8 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(108 \times 10^{-6})}$

$= 237.04 \times 10^{-6} \text{ m} = 0.237 \text{ mm} \leftarrow$

$\leftarrow \delta_B = \delta_{BE} = \dfrac{P_{BE} L_{BE}}{EA} = \dfrac{(16 \times 10^3)(400 \times 10^{-3})}{(200 \times 10^9)(108 \times 10^{-6})}$

$= 296.30 \times 10^{-6} \text{ m} = 0.296 \text{ mm} \rightarrow$



$\theta = \dfrac{\delta_A + \delta_B}{L_{AB}} = \dfrac{(237.04 + 296.30)(10^{-6})}{75 \times 10^{-3}}$

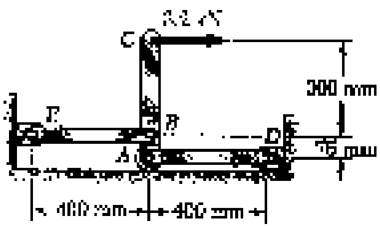
$= 7.1112 \times 10^{-3}$

$\delta_C = \delta_B + L_{BC}\,\theta$

$= 296.30 \times 10^{-6} + (300 \times 10^{-3})(7.1112 \times 10^{-3})$

$= 2.4297 \times 10^{-3} \text{ m} = 2.43 \text{ mm} \rightarrow$

PROBLEM 2.132

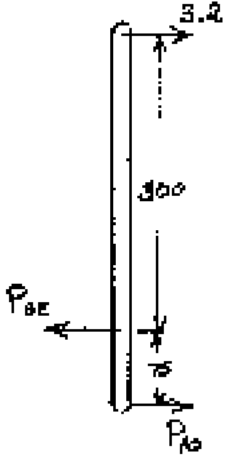


2.131 The steel bars *BE* and *AD* each have a 6×18-mm cross section. Knowing that $E = 200$ GPa, determine the deflections of points *A*, *B*, and *C* of the rigid bar *ABC*.

2.132 In Prob. 2.131, the 3.2-kN force caused point *C* to deflect to the right. Using $\alpha = 11.7 \times 10^{-6}/°C$, determine the (*a*) the overall change in temperature that causes point *C* to return to its original position, (*b*) the corresponding total deflection of points *A* and *B*.

SOLUTION

Use rigid ABC as a free body



$\circlearrowleft \Sigma M_B = 0 \qquad 75\, P_{AD} - (300)(3.2) = 0$

$P_{AD} = 12.8$ kN

$\xrightarrow{+} \Sigma F_x = 0 \qquad -P_{BE} + 3.2 + P_{AD} = 0$

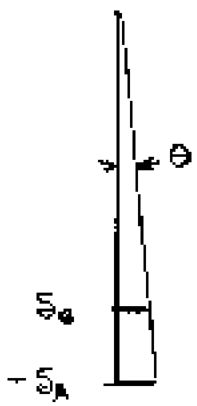
$P_{BE} = 16$ kN

Deformations:

$$\leftarrow \delta_A = \delta_{AD} = \frac{P_{AD} L_{AD}}{EA} + L_{AD}\alpha(\Delta T)$$

$$= \frac{(12.8\times10^3)(400\times10^{-3})}{(200\times10^9)(108\times10^{-6})} + (400\times10^{-3})(11.7\times10^{-6})(\Delta T)$$

$$= 237.04\times10^{-6} + 4.68\times10^{-6}(\Delta T)$$



$$\rightarrow \delta_B = \delta_{BE} = \frac{P_{BE} L_{BE}}{EA} + L_{BE}\alpha(\Delta T)$$

$$= \frac{(16\times10^3)(400\times10^{-3})}{(200\times10^9)(108\times10^{-6})} + (400\times10^{-3})(11.7\times10^{-6})(\Delta T)$$

$$= 296.30\times10^{-6} + 4.68\times10^{-6}(\Delta T)$$

$\delta_C = 0 \qquad \delta_B = 0.300\,\theta \qquad -\delta_A = 0.375\,\theta$

$-\delta_A = \dfrac{0.375}{0.300}\,\delta_B = 1.25\,\delta_B$

$$-(237.04\times10^{-6} + 4.68\times10^{-6}(\Delta T)) = (1.25)[296.30\times10^{-6} + 4.68\times10^{-6}(\Delta T)]$$

$-10.53\times10^{-6}(\Delta T) = 607.415\times10^{-6} \quad \ldots \quad \Delta T = -57.684$ °C

$= -57.7$ °C

$\delta_A = 237.04\times10^{-6} - (4.68\times10^{-6})(57.684) = -32.92\times10^{-6}$ m

$\delta_A = 0.0329$ mm $\rightarrow$

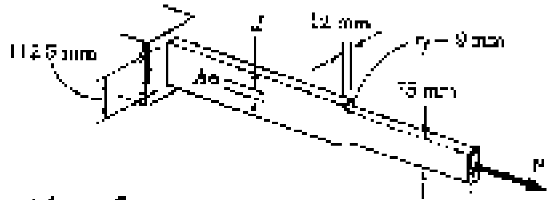
$\delta_B = 296.30\times10^{-6} - (4.68\times10^{-6})(57.684) = +26.34\times10^{-6}$ m

$\delta_B = 0.0263$ mm $\rightarrow$

PROBLEM 2.133

2.133 A hole is to be drilled in the plate at A. The diameters of the bits available to drill the hole range from 9 to 27 mm in 6-mm increments. (*a*) Determine the diameter d of the largest bit that can be used if the allowable load at the hole is not to exceed that at the fillets. (*b*) If the allowable stress in the plate is 145 MPa, what is the corresponding allowable load P?

SOLUTION



At the fillets, $r = 9$ mm $\quad d = 75$ mm

$D = 112.5$ mm $\quad \frac{D}{d} = \frac{112.5}{75} = 1.5$

$\frac{r}{d} = \frac{9}{75} = 0.12 \quad$ From Fig 2.64 b $\quad K = 2.10$

$A_{min} = (75)(12) = 900 \text{ mm}^2 = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$

$$\sigma_{max} = K \frac{P_{all}}{A_{min}} = \sigma_{all} \quad \therefore \quad P_{all} = \frac{A_{min}\sigma_{all}}{K} = \frac{(900 \times 10^{-6})(145 \times 10^6)}{2.10}$$

$$= 62.1 \times 10^3 \text{ N} = 62.1 \text{ kN}$$

At the hole: $\quad A_{net} = (D - 2r)t, \quad \frac{r}{d} = \frac{r}{D-2r}$

where $\quad D = 112.5$ mm $\quad r =$ radius of circle $\quad t = 12$ mm

K is taken from Fig 2.64 a

$$\sigma_{max} = K \frac{P}{A_{net}} = \sigma_{all} \quad \therefore \quad P_{all} = \frac{A_{net}\sigma_{all}}{K}$$

Hole diam	r	$d = D - 2r$	r/d	K	A_{net}	P_{all}
9 mm	4.5 mm	103.5 mm	0.0435	2.87	1242×10^{-6} m^2	62.7×10^3 N
15 mm	7.5 mm	97.5 mm	0.077	2.75	1170×10^{-6} m^2	61.7×10^3 N
21 mm	10.5 mm	91.5 mm	0.115	2.67	1098×10^{-6} m^2	59.6×10^3 N
27 mm	13.5 mm	85.5 mm	0.158	2.57	1026×10^{-6} m^2	57.9×10^3 N

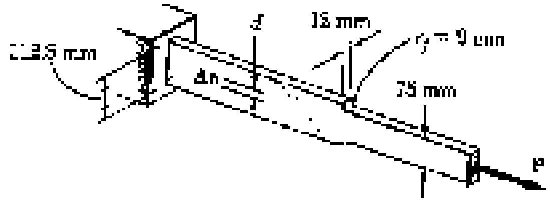
Largest hole with $P_{all} > 62$ kN is the 9 mm diameter hole.

Allowable force $\quad P_{all} = 62$ kN

PROBLEM 2.134

2.134 (*a*) For $P = 58$ kN and $d = 12$ mm, determine the maximum stress in the plate shown. (*b*) Solve part *a*, assuming that the hole at *A* is not drilled.

SOLUTION



Maximum stress at hole

Use Fig. 2.64 a for values of K

$$\frac{r}{d} = \frac{6}{112.5 - 12} = 0.0597, \qquad K = 2.80$$

$$A_{net} = (12)(112.5 - 12) = 1206 \text{ mm}^2 = 1206 \times 10^{-6} \text{ m}^2$$

$$\sigma_{max} = K\frac{P}{A_{net}} = \frac{(2.80)(58 \times 10^3)}{1206 \times 10^{-6}} = 134.7 \times 10^6 \text{ Pa}$$

Maximum stress at fillets

Use Fig. 2.64 b

$$\frac{r}{d} = \frac{9}{75} = 0.12, \qquad \frac{D}{d} = \frac{112.5}{75} = 1.50, \qquad K = 2.10$$

$$A_{min} = (12)(75) = 900 \text{ mm}^2 = 900 \times 10^{-6} \text{ m}^2$$

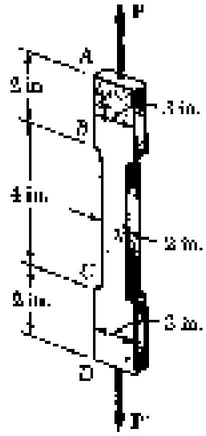
$$\sigma_{max} = K\frac{P}{A_{min}} = \frac{(2.10)(58 \times 10^3)}{900 \times 10^{-6}} = 135.3 \times 10^6 \text{ Pa}$$

(a) With hole and fillets $\sigma_{max} = 134.7$ MPa ◄

(b) Without hole $\sigma_{max} = 135.3$ MPa ◄

PROBLEM 2.135

2.135 The steel tensile specimen $ABCD$ ($E = 29 \times 10^6$ psi and $\sigma_Y = 50$ ksi) is loaded in tension until the maximum tensile strain is $\epsilon = 0.0025$. (*a*) Neglecting the effect of the fillets on the change in length of the specimen, determine the resulting overall length AD of the specimen after the load is removed. (*b*) Following the removal of the load in part (*a*), a compressive load is applied until the maximum compressive strain is $\epsilon = 0.0020$. Determine the resulting overall length AD after the load is removed.



SOLUTION

(a) $\varepsilon_Y = \dfrac{\sigma_Y}{E} = \dfrac{50 \times 10^3}{29 \times 10^6} = 0.001724$

$\varepsilon_{max} = 0.0025 > \varepsilon_Y$ Yielding occurs in portion BC

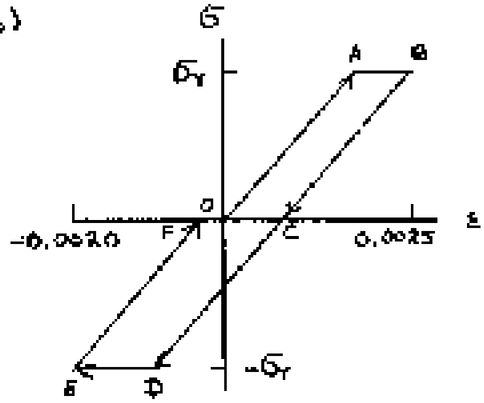
$\sigma_{BC} = \sigma_Y = 50 \times 10^3$ psi

Permanent strain in BC

$\varepsilon_{BC} = \varepsilon_{max} - \varepsilon_Y = 0.0025 - 0.001724 = 0.000776$

$\delta_{BC} = L_{BC}\,\varepsilon_{B} = (4)(0.000776) = 0.00310$ in. ◀

(b)



In reversed loading, at point E on stress-strain plot

$\varepsilon = -0.0020$

as given. During removal of the reversed load, the change in strain is $\sigma_Y/E = 0.001724$.

The permanent strain in BC is

$\varepsilon_{BC} = -0.0020 + 0.001724 = 0.000276$

$\delta_{BC} = L\,\varepsilon_{BC} = (4)(-0.000276) = -0.001104$ in. ◀

Note that portions AB and CD are always elastic, thus their deformations during loading and unloading do not contribute to any permanent deformation.

PROBLEM 2.C1

2.C1 A rod consisting of n elements, each of which is homogeneous and of uniform cross section, is subjected to the loading shown. The length of element i is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and the load applied to its right end by $\mathbf{P}_i$, the magnitude P_i of this load being assumed to be positive if $\mathbf{P}_i$ is directed to the right and negative otherwise. (*a*) Write a computer program that can be used to determine the average normal stress in each element, the deformation of each element, and the total deformation of the rod. (*b*) Use this program to solve Probs. 2.17 and 2.18.

SOLUTION

FOR EACH ELEMENT, ENTER

$L_i,\ A_i,\ E_i$

COMPUTE DEFORMATION

UPDATE AXIAL LOAD $\quad P = P + P_i$

COMPUTE FOR EACH ELEMENT

$$\sigma_i = P/A_i$$

$$\delta_i = PL_i/A_iE_i$$

TOTAL DEFORMATION:

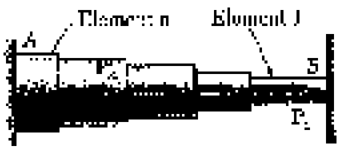
UPDATE THROUGH n ELEMENTS

$$\delta = \delta + \delta_i$$

PROGRAM OUTPUT

```
Problem 2.17
  Element   Stress (MPa)   Deformation (mm)

     1        19.0986          .1091
     2       -12.7324         -.0909
  Total Deformation =         .0182 mm

Problem 2.18
  Element   Stress (MPa)   Deformation (mm)

     1        98.2438         2.3391
     2        98.2438         1.4737
     3       147.3657         1.4737
  Total Deformation =        5.2865 mm
```

PROBLEM 2.C2



2.C2 Rod AB is horizontal with both ends fixed; it consists of n elements, each of which is homogeneous and of uniform cross section, and is subjected to the loading shown. The length of element i is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and the load applied to its right end by P_i, the magnitude P_i of this load being assumed to be positive if P_i is directed to the right and negative otherwise. (Note that $P_1 = 0$.) (*a*) Write a computer program that can be used to determine the reactions at A and B, the average normal stress in each element, and the deformation of each element. (*b*) Use this program to solve Prob. 2.41.

SOLUTION

WE CONSIDER THE REACTION AT B REDUNDANT AND RELEASE THE ROD AT B

COMPUTE δ_B WITH $R_B = 0$

FOR EACH ELEMENT, ENTER

L_i, A_i, E_i

UPDATE AXIAL LOAD

$P = P + P_i$

COMPUTE FOR EACH ELEMENT

$\sigma_i = P/A_i$

$\delta_i = PL_i/A_i E_i$

UPDATE TOTAL DEFORMATION

$\delta_B = \delta_B + \delta_i$

COMPUTE δ_B DUE TO UNIT LOAD AT B

UNIT $\sigma_i = 1/A_i$

UNIT $\delta_i = L_i/A_i E_i$

UPDATE TOTAL UNIT DEFORMATION

UNIT δ_B = UNIT δ_B + UNIT δ_i

SUPERPOSITION

FOR TOTAL DISPLACEMENT AT B = ZERO

$\delta_B + R_B$ UNIT $\delta_B = 0$

SOLVING:

$R_B = -\delta_B /$ UNIT δ_B

THEN:

$R_A = \Sigma P_i + R_B$

CONTINUED

FOR EACH ELEMENT

$$\sigma = \sigma_i + R_B \text{ UNIT } \sigma_i$$

$$\delta = \delta_i + R_B \text{ UNIT } \delta_i$$

PROGRAM OUTPUT

```
Problem 2.41
      RA =   -11.909 kips
      RB =    20.09[illegible] kips
   Element   Stress (ksi)   Deformation (in.)

      1          12.032            -.00523
      2          -6.128            -.00589
      3           9.[illegible]            -.00334
```

PROBLEM 2.C3

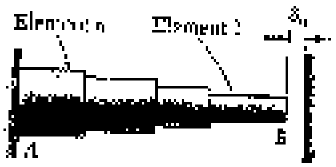


Fig. P2.C3

2.C3 Rod AB consists of n elements, each of which is homogeneous and of uniform cross section. End A is fixed, while initially there is a gap δ_0 between end B and the fixed vertical surface on the right. The length of element i is denoted by L_i, its cross-sectional area by A_i, its modulus of elasticity by E_i, and its coefficient of thermal expansion by α_i. After the temperature of the rod has been increased by ΔT, the gap at B is closed and the vertical surfaces exert equal and opposite forces on the rod. (a) Write a computer program that can be used to determine the magnitude of the reactions at A and B, the normal stress in each element, and the deformation of each element. (b) Use this program to solve Probs. 2.53, 2.54, 2.57, and 2.59.

SOLUTION

WE COMPUTE THE DISPLACEMENTS AT B ASSUMING THERE IS NO SUPPORT AT B:

ENTER L_i, A_i, E_i, α_i

ENTER TEMPERATURE CHANGE T

COMPUTE FOR EACH ELEMENT

$$\delta_i = \alpha_i L_i T$$

UPDATE TOTAL DEFORMATION

$$\delta_B = \delta_B + \delta_i$$

COMPUTE δ_B DUE TO UNIT LOAD AT B

$$\text{UNIT } \delta_i = L_i / A_i E_i$$

UPDATE TOTAL UNIT DEFORMATION

$$\text{UNIT } \delta_B = \text{UNIT } \delta_B + \text{UNIT } \delta_i$$

COMPUTE REACTIONS

FROM SUPERPOSITION

$$R_B = (\delta_B - \delta_0) / \text{UNIT } \delta_B$$

THEN

$$R_A = -R_B$$

FOR EACH ELEMENT

$$\sigma_i = -R_B / A_i$$

$$\delta_i = \alpha_i L_i T + R_B L_i / A_i E_i$$

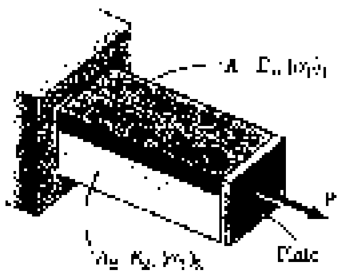
CONTINUED

PROBLEM 2.63 CONTINUED

PROGRAM OUTPUT

```
Problem  2.53
 R =       25.837 kips

  Element      Stress (ksi)    Deform.(10**-3 in.)

     1           -21.054         -3.642
     2            -6.498          3.642

Problem  2.54
 R =      125.820 kN

  Element      Stress (MPa)    Deform.(microm)

     1           -44.432        500.104
     2           -99.972       -500.104

Problem  2.57
 R =      217.465 kN

  Element      Stress (MPa)    Deform.(microm)

     1          -144.577        242.004
     2          -120.314        251.496

Problem  2.59
 R =       61.857 kips

  Element      Stress (ksi)    Deform.(10**-3 in.)

     1            22.092         14.410
     2           -51.547          5.590
```

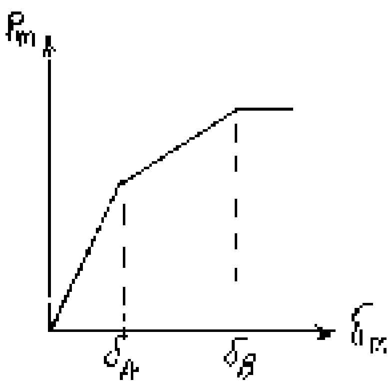
PROBLEM 2.C4



2.C4 Bar AB has a length L and is made of two different materials of given cross-sectional area, modulus of elasticity, and yield strength. The bar is subjected as shown to a load **P** which is gradually increased from zero until the deformation of the bar has reached a maximum value δ_m and then decreased back to zero. (a) Write a computer program that, for each of 25 values of δ_m equally spaced over a range extending from 0 to a value equal to 120% of the deformation causing both materials to yield, can be used to determine the maximum value P_m of the load, the maximum normal stress in each material, the permanent deformation δ_p of the bar, and the residual stress in each material. (b) Use this program to solve Probs. 2.109, 2.111, and 2.112.

SOLUTION

NOTE: THE FOLLOWING ASSUMES $(\sigma_Y)_1 < (\sigma_Y)_2$



DISPLACEMENT INCREMENT

$$\delta_m = 0.05\,(\sigma_Y)_2\,L/E_2$$

DISPLACEMENTS AT YIELDING

$$\delta_A = (\sigma_Y)_1\,L/E_1 \qquad \delta_B = (\sigma_Y)_2\,L/E_2$$

FOR EACH DISPLACEMENT

IF $\delta_m < \delta_A$:

$$\sigma_1 = \delta_m E_1/L$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = (\delta_m/L)(A_1E_1 + A_2E_2)$$

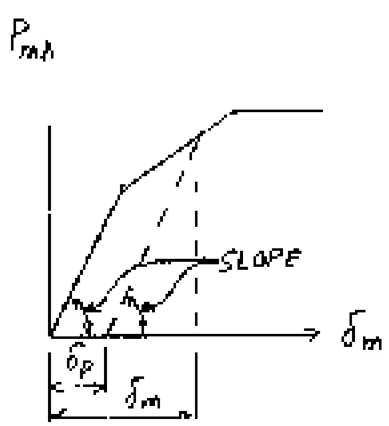
IF $\delta_A < \delta_m < \delta_B$:

$$\sigma_1 = (\sigma_Y)_1$$
$$\sigma_2 = \delta_m E_2/L$$
$$P_m = A_1\sigma_1 + (\delta_m/L)\,A_2E_2$$

IF $\delta_m > \delta_B$:

$$\sigma_1 = (\sigma_Y)_1 \qquad \sigma_2 = (\sigma_Y)_2$$
$$P_m = A_1\sigma_1 + A_2\sigma_2$$

PERMANENT DEFORMATIONS, RESIDUAL STRESSES



SLOPE OF FIRST (ELASTIC) SEGMENT

$$\text{SLOPE} = (A_1E_1 + A_2E_2)/L$$
$$\delta_p = \delta_m - (P_m/\text{SLOPE})$$
$$(\sigma_1)_{res} = \sigma_1 - (E_1P_m/(L\ \text{SLOPE}))$$
$$(\sigma_2)_{res} = \sigma_2 - (E_2P_m/(L\ \text{SLOPE}))$$

CONTINUED

PROBLEM 2.C4 CONTINUED

PROGRAM OUTPUT

Problem 2.109

DM KDm	EM kN	SIGM(1) MPa	SIGM(2) MPa	DF MUm	SIGR(1) MPa	SIG(2) MPa
.000	.000	.000	.000	.000	.000	.000
16.387	60.375	17.250	17.250	.000	.000	.000
32.775	120.750	34.500	34.500	.000	.000	.000
49.162	181.125	51.750	51.750	.000	.000	.000
65.550	241.500	69.000	69.000	.000	.000	.000
81.938	301.875	86.250	86.250	.000	.000	.000
98.325	362.250	103.500	103.500	.000	.000	.000
114.712	422.625	120.750	120.750	.000	.000	.000
131.100	483.000	138.000	138.000	.000	.000	.000
147.487	543.375	155.250	155.250	.000	.000	.000
163.875	603.750	172.500	172.500	.000	.000	.000
180.262	664.125	189.750	189.750	.000	.000	.000
196.650	724.500	207.000	207.000	.000	.000	.000
213.037	784.875	224.250	224.250	.000	.000	.000
229.425	845.250	241.500	241.500	.000	.000	.000
245.812	890.312	250.000	258.750	4.136	-4.375	4.375
262.200	920.500	250.000	276.000	12.350	-13.000	13.000
278.587	950.687	250.000	293.250	20.544	-21.625	21.625
294.975	980.875	250.000	310.500	28.737	-30.250	30.250
311.362	1011.062	250.000	327.750	36.931	-38.875	38.875
327.750	1041.250	250.000	345.000	45.125	-47.500	47.500
344.137	1041.250	250.000	345.000	61.512	-47.500	47.500
360.525	1041.250	250.000	345.000	77.900	-47.500	47.500
376.912	1041.250	250.000	345.000	94.287	-47.500	47.500
393.300	1041.250	250.000	345.000	110.675	-47.500	47.500

Problems 2.111 and 2.112

DM 10**-3 in.	EM kips	SIGM(1) ksi	SIGM(2) ksi	DE 10**-3 in.	SIGR(1) ksi	SIG(2) ksi
.000	.000	.000	.000	.000	.000	.000
2.414	8.750	5.000	5.000	.000	.000	.000
4.828	17.500	10.000	10.000	.000	.000	.000
7.241	26.250	15.000	15.000	.000	.000	.000
9.655	35.000	20.000	20.000	.000	.000	.000
12.069	43.750	25.000	25.000	.000	.000	.000
14.483	52.500	30.000	30.000	.000	.000	.000
16.897	61.250	35.000	35.000	.000	.000	.000
19.310	70.000	40.000	40.000	.000	.000	.000
21.724	78.750	45.000	45.000	.000	.000	.000
24.138	87.500	50.000	50.000	.000	.000	.000
26.552	91.250	50.000	55.000	1.379	-2.143	2.857
28.966	95.000	50.000	60.000	2.759	-4.286	5.714
31.379	98.750	50.000	65.000	4.138	-6.429	8.571
33.793	102.500	50.000	70.000	5.517	-8.571	11.429
36.207	106.250	50.000	75.000	6.897	-10.714	14.286
38.621	110.000	50.000	80.000	8.276	-12.857	17.143
41.034	113.750	50.000	85.000	9.655	-15.000	20.000
43.448	117.500	50.000	90.000	11.034	-17.143	22.857
45.862	121.250	50.000	95.000	12.414	-19.286	25.714
48.276	125.000	50.000	100.000	13.793	-21.429	28.571
50.690	125.000	50.000	100.000	16.207	-21.429	28.571
53.103	125.000	50.000	100.000	18.621	-21.429	28.571
55.517	125.000	50.000	100.000	21.034	21.429	28.571
57.931	125.000	50.000	100.000	23.448	-21.429	28.571

PROBLEM 2.C5

2.C5 The stress concentration factor for a flat bar with a centric hole under axial loading can be expressed as:

$$K = 3.00 - 3.13\left(\frac{2r}{D}\right) + 3.66\left(\frac{2r}{D}\right)^2 - 1.53\left(\frac{2r}{D}\right)^3$$

where r is the radius of the hole and D is the width of the bar. (a) Write a computer program that can be used to determine the allowable load P for given values of r, D, the thickness t of the bar, and the allowable stress σ_{all} of the material. (b) Use this program to solve Prob. 2.94.

SOLUTION

ENTER

r, D, t, σ_{all}

COMPUTE K

$RD = 2.0\, r/D$

$K = 3.00 - 3.13\,RD + 3.66\,RD^2 - 1.53\,RD^3$

COMPUTE AVERAGE STRESS

$\sigma_{ave} = \sigma_{all}/K$

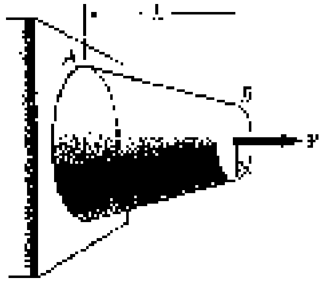
ALLOWABLE LOAD

$P_{all} = \sigma_{ave}\,(D - 2.0\,r)\,t$

PROGRAM OUTPUT

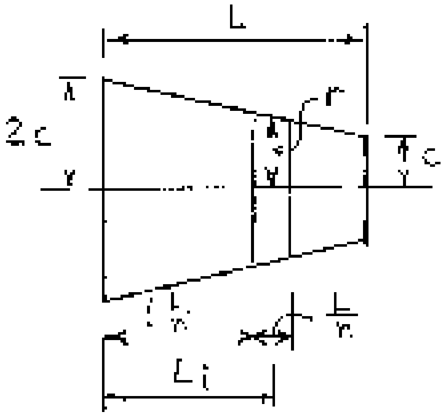
```
Problem 2.94

    Hole at A:

            K =    2.[illegible]     P =     7.[illegible] Kips

    Hole at B:

            K =    2.259     P =     5.550 Kips
```

PROBLEM 2.C6



2.C6 A solid truncated cone is subjected to an axial force P as shown. Write a computer program that can be used to obtain an approximation of the elongation of the cone by replacing it by n circular cylinders of equal thickness and of radius equal to the mean radius of the portion of cone they replace. Knowing that the exact value of the elongation of the cone is $(PL)/(2\pi c^2E)$ and using for P, L, c and E values of your choice, determine the percentage error involved when the program is used with (*a*) $n = 6$, (*b*) $n = 12$, (*c*) $n = 60$.

SOLUTION



FOR $i = 1$ TO n:

$$L_i = (i + 0.5)(L/n)$$

$$r_i = 2c - c(L_i/L)$$

AREA:

$$A = \pi r_i^2$$

DISPLACEMENT:

$$\delta = \delta + P(L/n)/(AE)$$

EXACT DISPLACEMENT:

$$\delta_{EXACT} = PL/(2.0\pi c^2 E)$$

PERCENTAGE ERROR:

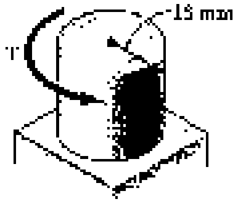
$$PERCENT = 100(\delta - \delta_{EXACT})/\delta_{EXACT}$$

<u>PROGRAM OUTPUT</u>

n	Approximate	Exact	Percent
6	0.15852	0.15915	-.40083
12	0.15899	0.15915	-.10100
60	0.15915	0.15915	-.00405

CHAPTER 3

PROBLEM 3.1



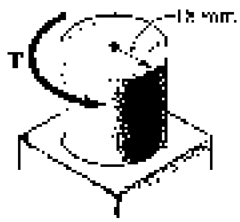
3.1 Determine the torque T which causes a maximum shearing stress of 70 MPa in the steel cylindrical shaft shown.

SOLUTION

$$\tau_{max} = \frac{Tc}{J} \qquad J = \frac{\pi}{2}c^4$$

$$T = \frac{\pi}{2}c^3\tau_{max} = \frac{\pi}{2}(0.018)^3(70\times10^6) = 641 \text{ N}\cdot\text{m}$$

PROBLEM 3.2



3.2 Determine the maximum shearing stress caused by a torque of magnitude T = 800 N·m.

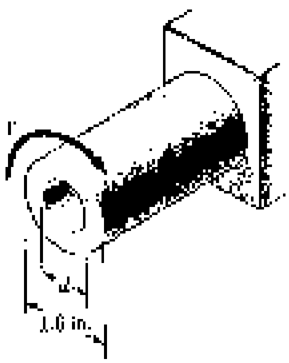
SOLUTION

$$\tau_{max} = \frac{Tc}{J} \qquad J = \frac{\pi}{2}c^4$$

$$\tau_{max} = \frac{2T}{\pi c^3} = \frac{(2)(800)}{\pi(0.018)^3} = 87.3\times10^6 \text{ Pa}$$

$$87.3 \text{ MPa}$$

PROBLEM 3.3



3.3 Knowing that the internal diameter of the hollow shaft shown is $d = 0.9$ in., determine the maximum shearing stress caused by a torque of magnitude $T = 9$ kip·in.

SOLUTION

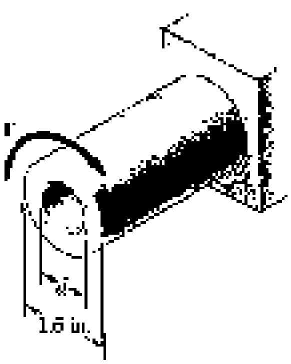
$$c_2 = \frac{1}{2}d_2 = \left(\frac{1}{2}\right)(1.6) = 0.8 \text{ in} \qquad c = 0.8 \text{ in.}$$

$$c_1 = \frac{1}{2}d_1 = \left(\frac{1}{2}\right)(0.9) = 0.45 \text{ in}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.8^4 - 0.45^4) = 0.5790 \text{ in}^4$$

$$\tau_{max} = \frac{Tc}{J} = \frac{(9)(0.8)}{0.5790} = 12.44 \text{ ksi}$$

PROBLEM 3.4



3.4 Knowing that $d = 1.2$ in., determine the torque T which causes a maximum shearing stress of 7.5 ksi in the hollow shaft shown.

SOLUTION

$$c_2 = \frac{1}{2}d_2 = \left(\frac{1}{2}\right)(1.6) = 0.8 \text{ in} \qquad c = 0.8 \text{ in}$$

$$c_1 = \frac{1}{2}d_1 = \left(\frac{1}{2}\right)(1.2) = 0.6 \text{ in}$$

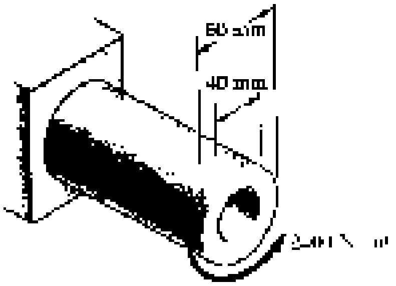
$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.8^4 - 0.6^4) = 0.4398 \text{ in}^4$$

$$\tau_{max} = \frac{Tc}{J}$$

$$T = \frac{J\tau_{max}}{c} = \frac{(0.4398)(7.5)}{0.8} = 4.12 \text{ kip}\cdot\text{in}$$

PROBLEM 3.5

3.5 (*a*) For the hollow shaft and loading shown, determine the maximum shearing stress. (*b*) Determine the diameter of a solid shaft for which the maximum shearing stress is the same as in part *a*.



SOLUTION

$c_1 = \frac{1}{2}d_1 = (\frac{1}{2})(0.040) = 0.020\ m$

$c_2 = \frac{1}{2}d_2 = (\frac{1}{2})(0.060) = 0.030\ m \qquad c = 0.030\ m$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.030^4 - 0.020^4)$

$= 1.0210 \times 10^{-6}\ m^4$

(a) $\tau_{max} = \frac{Tc}{J} = \frac{(2400)(0.03)}{1.0210 \times 10^{-6}} = 70.52 \times 10^6\ Pa$

$70.5\ MPa$ ◄

(b) $\tau = \frac{Tc_3}{J}, \quad J = \frac{\pi}{2}c_3^4 \qquad \tau = \frac{2T}{\pi c_3^3}$

$c_3^3 = \frac{2T}{\pi \tau} = \frac{(2)(2400)}{\pi(70.52 \times 10^6)} = 21.67 \times 10^{-6}\ m^3$

$c_3 = 27.88 \times 10^{-3}\ m \qquad d_3 = 2c_3 = 55.8 \times 10^{-3}\ m \qquad 55.8\ mm$ ◄

PROBLEM 3.6

3.6 (*a*) Determine the torque which may be applied to a solid shaft of 90-mm outer diameter without exceeding an allowable shearing stress of 75 MPa. (*b*) Solve part *a*, assuming that the solid shaft is replaced by a hollow shaft of the same mass and of 90-mm inner diameter.

SOLUTION

(a) For the solid shaft $\quad c = \frac{1}{2}d = (\frac{1}{2})(0.090) = 0.045\ m$

$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\pi}{2}(0.045)^3 = 143.14 \times 10^{-6}\ m^3$

$\tau_{max} = \frac{Tc}{J} \quad \therefore \quad T = \frac{\tau_{max} J}{c} = (75 \times 10^6)(143.14 \times 10^{-6}) = 10.74 \times 10^3\ N \cdot m$

$10.74\ kN \cdot m$ ◄

(b) Hollow shaft $\qquad c_1 = \frac{1}{2}d_1 = (\frac{1}{2})(0.090) = 0.045\ m$

For equal masses the cross sectional areas must be equal

$A = \pi c^2 = \pi(c_2^2 - c_1^2) \quad \text{or} \quad c_2 = \sqrt{c_1^2 + c^2}$

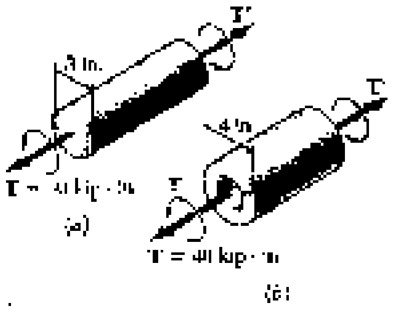
$c_2 = \sqrt{0.045^2 + 0.045^2} = 0.0636396\ m$

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 19.3237 \times 10^{-6}\ m^4$

$T = \frac{\tau_{max} J}{c_2} = \frac{(75 \times 10^6)(19.3237 \times 10^{-6})}{0.0636396} = 22.77 \times 10^3\ N \cdot m$

$22.8\ kN \cdot m$ ◄

PROBLEM 3.7



3.7 (*a*) For the 3-in.-diameter solid cylinder and loading shown, determine the maximum shearing stress. (*b*) Determine the inner diameter of the hollow cylinder, of 4-in. outer diameter, for which the maximum stress is the same as in part *a*.

SOLUTION

(a) Solid shaft $\quad c = \frac{1}{2}d = \frac{1}{2}(3.0) = 1.5$ in

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(40)}{\pi(1.5)^3} = 7.545 \text{ ksi}$$

(b) Hollow shaft $\quad c_2 = \frac{1}{2}d = \frac{1}{2}(4.0) = 2.0$ in.

$$\frac{J}{c_2} = \frac{\frac{\pi}{2}(c_2^4 - c_1^4)}{c_2} = \frac{T}{\tau_{max}}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi \tau_{max}} = 2.0^4 - \frac{(2)(40)(2.0)}{\pi(7.545)} = 9.25 \text{ in}^4$$

$$c_1 = 1.74395 \text{ in} \qquad d_1 = 2c_1 = 3.49 \text{ in}$$

PROBLEM 3.8

3.8 (*a*) Determine the torque which may be applied to a solid shaft of 0.75-in. diameter without exceeding an allowable shearing stress of 10 ksi. (*b*) Solve part *a*, assuming that the solid shaft has been replaced by a hollow shaft of the same cross-sectional area and with an inner diameter equal to half its outer diameter.

SOLUTION

(a) Solid shaft: $\quad c = \frac{1}{2} = (\frac{1}{2})(0.75) = 0.375$ in.

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.375)^4 = 0.031063 \text{ in}^4 \qquad \tau_{max} = 10 \text{ ksi}$$

$$T = \frac{J\tau_{max}}{c} = \frac{(0.031063)(10)}{0.375} = 0.828 \text{ kip}\cdot\text{in} \text{ or } 828 \text{ lb}\cdot\text{in}$$

(b) Hollow shaft

For the same area as the solid shaft

$$A = \pi(c_2^2 - c_1^2) = \pi\left[c_2^2 - (\tfrac{1}{2}c_2)^2\right] = \tfrac{3}{4}\pi c_2^2 = \pi c^2$$

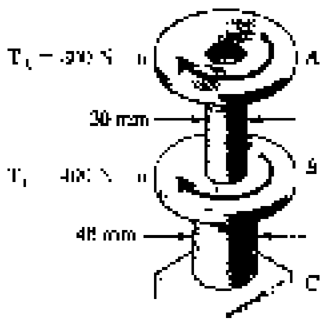
$$c_2 = \frac{2}{\sqrt{3}}c = \frac{2}{\sqrt{3}}(0.375) = 0.433013 \text{ in}$$

$$c_1 = \tfrac{1}{2}c_2 = 0.216506$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.433013^4 - 0.216506^4) = 0.051772 \text{ in}^4$$

$$T = \frac{\tau_{max}J}{c_2} = \frac{(10)(0.051772)}{0.433013} = 1.196 \text{ kip}\cdot\text{in} \text{ or } 1196 \text{ in}\cdot\text{lb}$$

PROBLEM 3.9



3.9 The torques shown are exerted on pulleys *A* and *B*. Knowing that each shaft is solid, determine the maximum shearing stress (*a*) in shaft *AB*, (*b*) in shaft *BC*.

SOLUTION

Shaft AB: $T_{AB} = 300$ N·m, $d = 0.030$ m, $c = 0.015$ m

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi(0.015)^3}$$

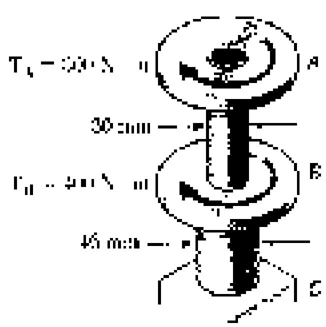
$$= 56.588 \times 10^6 \text{ Pa} = 56.6 \text{ MPa}$$

Shaft BC: $T_{BC} = 300 + 400 = 700$ N·m

$d = 0.046$ m, $c = 0.023$ m $\quad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi(0.023)^3}$

$$= 36.626 \times 10^6 \text{ Pa} \qquad 36.6 \text{ MPa}$$

PROBLEM 3.10



3.10 The torques shown are exerted on pulleys *A* and *B* which are attached to solid circular shafts *AB* and *BC*. In order to reduce the total mass of the assembly, determine the smallest diameter of shaft *BC* for which the largest shearing stress in the assembly is not increased.

SOLUTION

Shaft AB: $T_{AB} = 300$ N·m, $d = 0.030$ m, $c = 0.015$ m

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi(0.015)^3}$$

$$= 56.583 \times 10^6 \text{ Pa} = 56.6 \text{ MPa}$$

Shaft BC: $T_{BC} = 300 + 400 = 700$ N·m

$d = 0.046$ m, $c = 0.23$ m $\quad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi(0.023)^3}$

$$= 36.626 \times 10^6 \text{ Pa} = 36.6 \text{ MPa}$$

The largest stress (56.588×10^6 Pa) occurs in portion AB

Reduce the diameter of BC to provide the same stress.

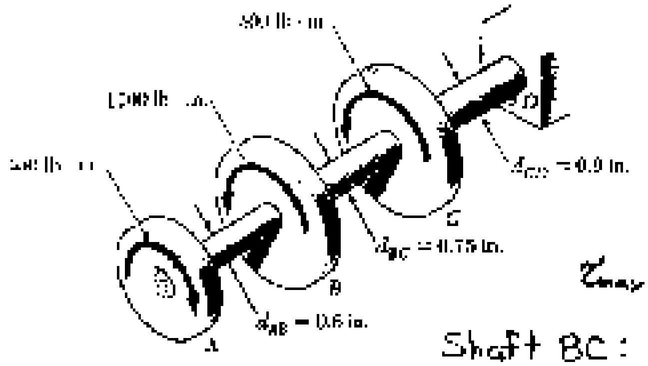
$T_{BC} = 700$ N·m $\qquad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$$c^3 = \frac{2T}{\pi \tau_{max}} = \frac{(2)(700)}{\pi(56.588 \times 10^6)} = 7.875 \times 10^{-6} \text{ m}^3$$

$c = 19.895 \times 10^{-3}$ m $\qquad d = 2c = 39.79 \times 10^{-3}$ m $\qquad$ 39.8 mm

PROBLEM 3.11

3.11 Knowing that each portion of the shaft *AD* consists of a solid circular rod, determine (*a*) the portion of the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.



SOLUTION

Shaft AB : $T = 400$ lb·in

$c = \frac{1}{2}d = 0.30$ in

$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$\tau_{max} = \frac{(2)(400)}{\pi(0.30)^3} = 9431$ psi

Shaft BC : $T = -400 + 1200 = 800$ lb·in

$c = \frac{1}{2}d = 0.375$ in $\qquad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(800)}{\pi(0.375)^3} = 9658$ psi

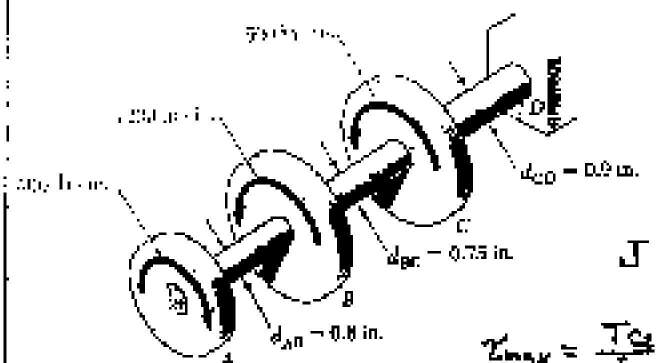
Shaft CD: $T = -400 + 1200 + 500 = 1300$ lb·in

$c = \frac{1}{2}d = 0.45$ in $\qquad \tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1300)}{\pi(0.45)^3} = 9082$ psi

Answers: (a) shaft BC $\qquad$ (b) 9.66 ksi

PROBLEM 3.12

3.12 Knowing that a 0.30-in.-diameter hole has been drilled through each portion of shaft *AD*, determine (*a*) the portion of the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.



SOLUTION

Hole: $c_1 = \frac{1}{2}d_1 = 0.15$ in

Shaft AB: $T = 400$ lb·in

$c_2 = \frac{1}{2}d_2 = 0.30$ in

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.30^4 - 0.15^4)$

$= 0.011928$ in^4

$\tau_{max} = \frac{Tc_2}{J} = \frac{(400)(0.30)}{0.011928} = 10\,060$ psi

Shaft BC: $T = -400 + 1200 = 800$ lb·in $\qquad c_2 = \frac{1}{2}d_2 = 0.375$ in

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.375^4 - 0.15^4) = 0.030268$ in^4

$\tau_{max} = \frac{Tc_2}{J} = \frac{(800)(0.375)}{0.030268} = 9911$ psi

Shaft CD: $T = -400 + 1200 + 500 = 1300$ lb·in $\qquad c_2 = \frac{1}{2}d_2 = 0.45$ in

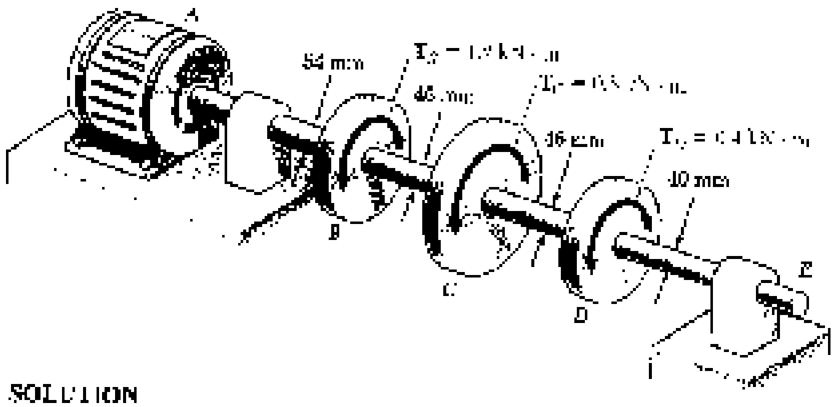
$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.45^4 - 0.15^4) = 0.063617$ in^4

$\tau_{max} = \frac{Tc_2}{J} = \frac{(1300)(0.45)}{0.063617} = 9196$ psi

Answers: (a) shaft AB $\qquad$ (b) 10.06 ksi

PROBLEM 3.13

3.13 Under normal operating conditions, the electric motor exerts a torque of 2.4 kN·m at A. Knowing that each shaft is solid, determine the maximum shearing stress (*a*) in shaft AB, (*b*) in shaft BC, (*c*) in shaft CD.



SOLUTION

Shaft AB: $T_{AB} = 2.4 \times 10^3$ N·m, $c = \frac{1}{2}d = 0.027$ m

$$\tau_{AB} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2.4\times10^3)}{\pi(0.027)^3} = 77.625\times10^6 \text{ Pa} \qquad 77.6 \text{ MPa}$$

Shaft BC: $T_{BC} = 2.4$ kN·m $-$ 1.2 kN·m $=$ 1.2 kN·m, $c = \frac{1}{2}d = 0.023$ m

$$\tau_{BC} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1.2\times10^3)}{\pi(0.023)^3} = 62.788\times10^6 \text{ Pa} \qquad 62.8 \text{ MPa}$$

Shaft CD: $T_{CD} = 0.4\times10^3$ N·m $\quad c = \frac{1}{2}d = 0.023$ m

$$\tau_{CD} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(0.4\times10^3)}{\pi(0.023)^3} = 20.929\times10^6 \text{ Pa} \qquad 20.9 \text{ MPa}$$

PROBLEM 3.14

3.14 Under normal operating conditions, the electric motor exerts a torque of 2.4 kN·m at A. In order to reduce the mass of the assembly, determine the smallest diameter of shaft BC for which the largest shearing stress in the assembly is not increased.

SOLUTION

See solution to problem 3.14 for figure and for maximum shearing stresses in portions AB, BC, and CD of the shaft. The largest value is $\tau_{max} = 77.625\times10^6$ Pa occuring in AB.

Adjust diameter of BC to obtain the same value of stress

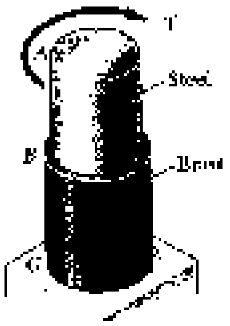
$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi\tau} = \frac{(2)(1.2\times10^3)}{\pi(77.625\times10^6)} = 9.8415\times10^{-6} \text{ m}^3$$

$$c = 21.43\times10^{-3} \text{ m} \qquad d = 2c = 42.8\times10^{-3} \text{ m} \qquad 42.8 \text{ mm}$$

PROBLEM 3.15

3.15 The allowable stress is 15 ksi in the 1.5-in.-diameter rod AB and 8 ksi in the 1.8-in.-diameter rod BC. Neglecting the effect of stress concentrations, determine the largest torque that may be applied at A.



SOLUTION

$$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}c^4, \quad T = \frac{\pi}{2}c^3\tau_{max}$$

Shaft AB: $\tau_{max} = 15$ ksi $\quad c = \frac{1}{2}d = 0.75$ in

$$T = \frac{\pi}{2}(0.75)^3(15) = 9.94 \text{ kip-in}$$

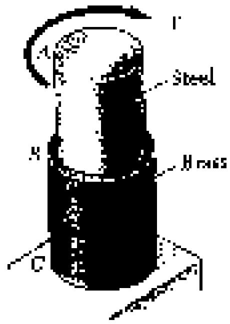
Shaft BC: $\tau_{max} = 8$ ksi $\quad c = \frac{1}{2}d = 0.90$ in

$$T = \frac{\pi}{2}(0.90)^3(8) = 9.16 \text{ kip-in.}$$

The allowable torque is the smaller value $\quad T = 9.16$ kip-in ◄

PROBLEM 3.16

3.16 The allowable stress is 15 ksi in the steel rod AB and 8 ksi in the brass rod BC. Knowing that a torque $T = 10$ kip·in. is applied at A, determine the required diameter of (a) rod AB, (b) for BC.



SOLUTION

$$\tau_{max} = \frac{Tc}{J}, \quad J = \frac{\pi}{2}, \quad c^3 = \frac{2T}{\pi\tau_{max}}$$

Shaft AB: $T = 10$ kip-in $\quad \tau_{max} = 15$ ksi

$$c^3 = \frac{(2)(10)}{\pi(15)} = 0.4244 \text{ in}^3$$

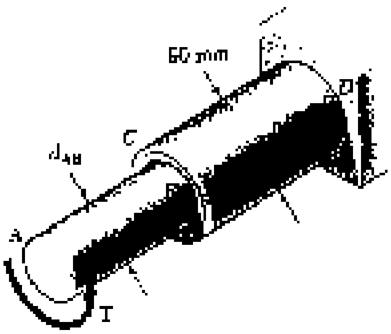
$c = 0.7515$ in $\quad d = 2c = 1.503$ in. ◄

Shaft BC: $T = 10$ kip-in $\quad \tau_{max} = 8$ ksi

$$c^3 = \frac{(2)(10)}{\pi(8)} = 0.79577 \text{ in}$$

$c = 0.4267$ in $\quad d = 2c = 1.853$ in. ◄

PROBLEM 3.17



3.17 The solid rod AB has a diameter d_{AB} = 60 mm. The pipe CD has an outer diameter of 90 mm and a wall thickness of 6 mm. Knowing that both the rod and the pipe are made of a steel for which the allowable shearing stress is 75 MPa, determine the largest torque T which may be applied at A.

SOLUTION

$\tau_{all} = 75 \times 10^6$ Pa $\qquad T_{all} = \dfrac{J\,\tau_{all}}{c}$

Rod AB: $c = \frac{1}{2}d = 0.030$ m, $\quad J = \frac{\pi}{2}c^4$

$T_{all} = \frac{\pi}{2}c^3\tau_{all} = \frac{\pi}{2}(0.030)^3(75\times10^6)$

$= 3.181 \times 10^3$ N·m

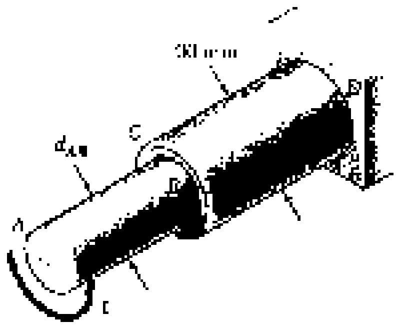
Pipe CD: $c_2 = \frac{1}{2}d_2 = 0.045$ m $\qquad c_1 = c_2 - t = 0.045 - 0.006 = 0.039$ m

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.045^4 - 0.039^4) = 2.8073 \times 10^{-6}$ m^4

$T_{all} = \dfrac{(2.8073\times10^{-6})(75\times10^6)}{0.045} = 4.679 \times 10^3$ N·m

Allowable torque is the smaller value $(3.18 \times 10^3$ N·m$)$ 3.18 kN·m ◀

PROBLEM 3.18



3.18 The solid rod AB has a diameter d_s = 60 mm and is made of a steel for which the allowable shearing stress is 85 MPa. The pipe CD has an outer diameter of 90 mm and a wall thickness of 6 mm; it is made of an aluminum for which the allowable shearing stress is 54 MPa. Determine the largest torque **T** which may be applied at A.

SOLUTION

Rod AB: $\tau_{all} = 85 \times 10^6$ Pa, $\quad c = \frac{1}{2}d = 0.030$ m

$T_{all} = \dfrac{J\,\tau_{all}}{c} = \frac{\pi}{2}c^3\tau_{all}$

$= \frac{\pi}{2}(0.030)^3(85\times10^6) = 3.605 \times 10^3$ N·m

Pipe CD: $\tau_{all} = 54 \times 10^6$ Pa $\qquad c_2 = \frac{1}{2}d_2 = 0.045$ m

$c_1 = c_2 - t = 0.045 - 0.006 = 0.039$ m

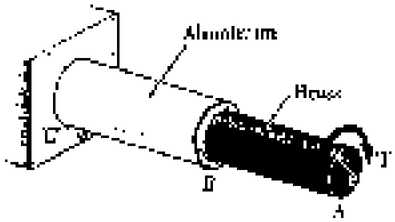
$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.045^4 - 0.039^4) = 2.8073 \times 10^{-6}$ m^4

$T_{all} = \dfrac{J\,\tau_{all}}{c_2} = \dfrac{(2.8073\times10^{-6})(54\times10^6)}{0.045} = 3.369 \times 10^3$ N·m

Allowable torque is smaller value $T_{all} = 3.369 \times 10^3$ N·m

3.37 kN·m ◀

PROBLEM 3.19



3.19 The allowable stress is 50 MPa in the brass rod *AB* and 25 MPa in the aluminum rod *BC*. Knowing that a torque $T = 1250$ N·m is applied at *A*, determine the required diameter of (*a*) rod *AB*, (*b*) rod *BC*.

SOLUTION

$$\tau_{max} = \frac{Tc}{J} \qquad J = \frac{\pi}{2}c^4 \qquad c^3 = \frac{2T}{\pi \tau_{max}}$$

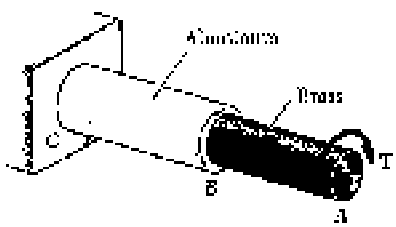
Rod AB: $c^3 = \dfrac{(2)(1250)}{\pi(50\times 10^6)} = 15.915\times 10^{-6}\ \text{m}^3$

$c = 25.15\times 10^{-3}\ \text{m} = 25.15\ \text{mm} \qquad d_{AB} = 2c = 50.3\ \text{mm}$ ◄

Rod BC: $c^3 = \dfrac{(2)(1250)}{\pi(25\times 10^6)} = 31.831\times 10^{-6}\ \text{m}^3$

$c = 31.69\times 10^{-3}\ \text{m} = 31.69\ \text{mm} \qquad d_{BC} = 2c = 63.4\ \text{mm}$ ◄

PROBLEM 3.20



3.20 The *solid* rod *BC* has a diameter of 30 mm and is made an aluminum for which the allowable shearing stress is 25 MPa. Rod *AB* is *hollow* and has an outer diameter of 25 mm; it is made a brass for which the allowable shearing stress is 50 MPa. Determine (*a*) the largest inner diameter of rod *AB* for which the factor of safety is the same for each rod, (*b*) the largest torque that may be applied at *A*.

SOLUTION

Solid rod BC: $\tau = \dfrac{Tc}{J} \qquad J = \dfrac{\pi}{2}c^4$

$\tau_{all} = 25\times 10^6\ \text{Pa} \qquad c = \frac{1}{2}d = 0.015\ \text{m}$

$T_{all} = \frac{\pi}{2}c^3\tau_{all} = \frac{\pi}{2}(0.015)^3(25\times 10^6) = 132.536\ \text{N·m}$

Hollow rod AB: $\tau_{all} = 50\times 10^6\ \text{Pa} \qquad T_{all} = 132.536\ \text{N·m}$

$c_2 = \frac{1}{2}d_2 = \frac{1}{2}(0.025) = 0.0125\ \text{m} \qquad c_1 = ?$

$T_{all} = \dfrac{J\tau_{all}}{c_2} = \dfrac{\pi}{2}(c_2^4 - c_1^4)\dfrac{\tau_{all}}{c_2}$

$c_1^4 = c_2^4 - \dfrac{2T_{all}c_2}{\pi\tau_{all}} = 0.0125^4 - \dfrac{(2)(132.536)(0.0125)}{\pi(50\times 10^6)} = 3.3203\times 10^{-9}\ \text{m}^4$

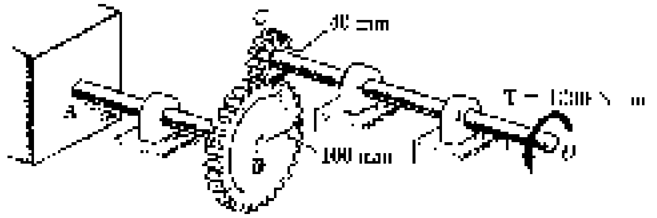
$c_1 = 7.59\times 10^{-3}\ \text{m} = 7.59\ \text{mm} \qquad d_1 = 2c_1 = 15.18\ \text{mm}$ ◄

Allowable torque $\quad T_{all} = 132.5\ \text{N·m}$ ◄

PROBLEM 3.21

3.21 A torque of magnitude $T = 1000$ N·m is applied at D as shown. Knowing that the diameter of shaft AB is 56 mm and the diameter of shaft CD is 42 mm, determine the maximum shearing stress in (a) shaft AB, (b) shaft CD.

SOLUTION



$T_{CD} = 1000$ N·m

$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500$ N·m

Shaft AB: $c = \frac{1}{2}d = 0.028$ m

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2500)}{\pi(0.028)^3} = 72.50 \times 10^6$ 72.5 MPa ◄

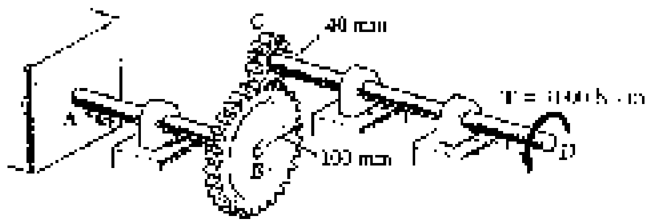
Shaft BC: $c = \frac{1}{2}d = 0.020$ m

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1000)}{\pi(0.020)^3} = 68.7 \times 10^6$ 68.7 MPa ◄

PROBLEM 3.22

3.22 A torque of magnitude $T = 1000$ N·m is applied at D as shown. Knowing that the allowable shearing stress is 60 MPa in each shaft, determine the required diameter of (a) shaft AB, (b) shaft CD.

SOLUTION



$T_{CD} = 1000$ N·m

$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500$ N·m

Shaft AB: $\tau_{all} = 60 \times 10^6$ Pa

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$ $\quad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(2500)}{\pi(60 \times 10^6)} = 26.526 \times 10^{-6}$ m^3

$c = 29.82 \times 10^{-3} = 29.82$ mm $\quad d = 2c = 59.6$ mm ◄

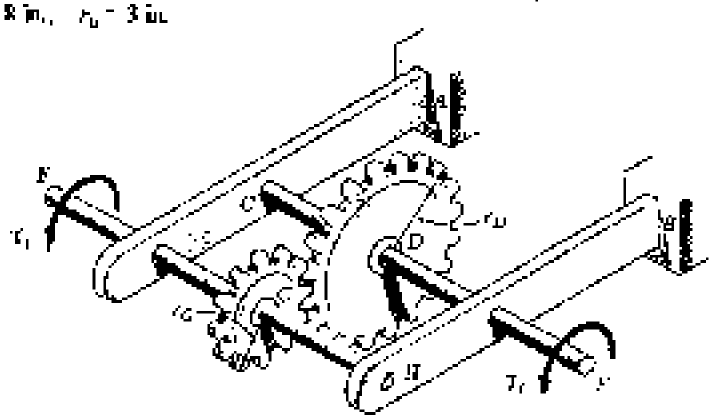
Shaft CD: $\tau_{all} = 60 \times 10^6$ Pa

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$ $\quad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(1000)}{\pi(60 \times 10^6)} = 10.610 \times 10^{-6}$ m^3

$c = 21.97 \times 10^{-3}$ m $= 21.97$ mm $\quad d = 2c = 43.9$ mm ◄

PROBLEM 3.23

3.23 and 3.24 Under normal operating conditions a motor exerts a torque of magnitude $T_F = 1200$ lb·in. at F. Knowing that the allowable shearing stress is 10.5 ksi in each shaft, for the given data, determine the required diameter of (*a*) shaft *CDE*, (*b*) shaft *FGH*.
3.23 $r_D = 8$ in., $r_G = 3$ in.



SOLUTION

$T_F = 1200$ lb·in

$T_E = \frac{r_D}{r_G} T_F = \frac{8}{3}(1200) = 3200$ lb·in

$\tau_{all} = 10.5$ ksi $= 10500$ psi

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}, \quad c^3 = \frac{2T}{\pi \tau}$

(a) Shaft CDE

$c^3 = \frac{(2)(3200)}{\pi(10500)} = 0.194012$ in^3

$c = 0.5789 \qquad d_{DE} = 2c = 1.158$ in.

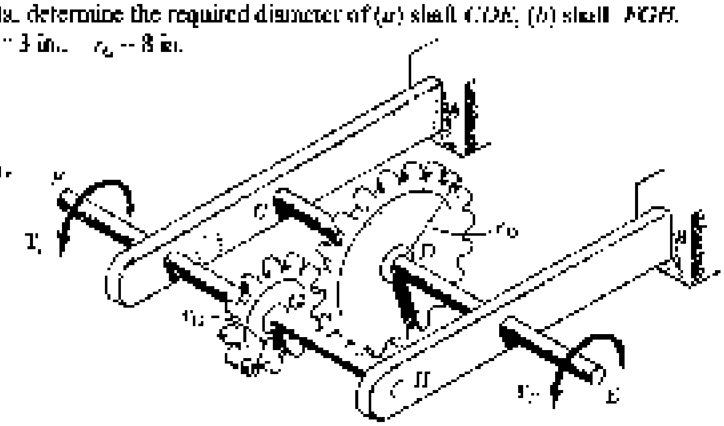
(b) Shaft FGH

$c^3 = \frac{(2)(1200)}{\pi(10500)} = 0.072757$ in^3

$c = 0.4174$ in $\qquad d_{FG} = 2c = 0.835$ in

PROBLEM 3.24

3.23 and 3.24 Under normal operating conditions a motor exerts a torque of magnitude $T_F = 1200$ lb·in. at F. Knowing that the allowable shearing stress is 10.5 ksi in each shaft, for the given data, determine the required diameter of (*a*) shaft *CDE*, (*b*) shaft *FGH*.
3.24 $r_D = 3$ in., $r_G = 8$ in.



SOLUTION

$T_F = 1200$ lb·in

$T_E = \frac{r_D}{r_G} T_F = \frac{3}{8}(1200) = 450$ lb·in.

$\tau_{all} = 10.5$ ksi $= 10500$ psi

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi \tau}$

Shaft CDE

$c^3 = \frac{(2)(450)}{\pi(10500)} = 0.027284$ in^3

$c = 0.30105$ in $\qquad d_{DE} = 2c = 0.602$ in

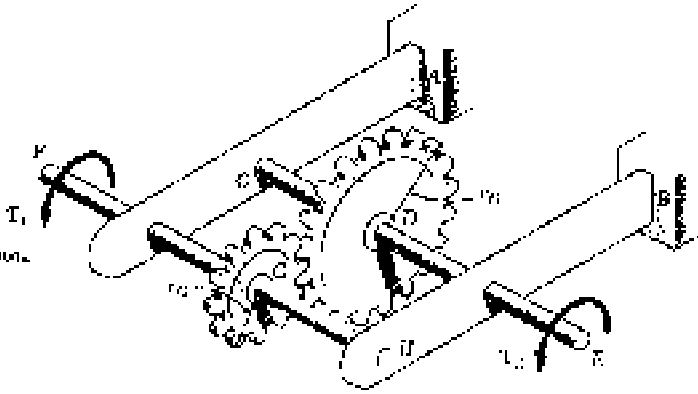
Shaft FGH

$c^3 = \frac{(2)(1200)}{\pi(10500)} = 0.072757$ in^3

$c = 0.4174$ in. $\qquad d_{FG} = 2c = 0.835$ in

PROBLEM 3.25

3.25 Under normal operating conditions a motor exerts a torque of magnitude T_F at F. The shafts are made of a steel for which the allowable shearing stress is 12 ksi and have diameters of d_{CDE} = 0.900 in. and d_{FGH} = 0.800 in. Knowing that r_D = 6.5 in. and r_G = 4.5 in., determine the largest torque $\mathbf{T}_F$ which may be exerted at F.



SOLUTION

$\tau_{all} = 12$ ksi

Shaft FG: $c = \frac{1}{2}d = 0.400$ in

$$T_{F,all} = \frac{J\tau_{all}}{c} = \frac{\pi}{2}c^3\tau_{all}$$

$$= \frac{\pi}{2}(0.400)^3(12) = 1.206 \text{ kip}\cdot\text{in}$$

Shaft DE: $c = \frac{1}{2}d = 0.450$ in

$$T_{E,all} = \frac{\pi}{2}c^3\tau_{all}$$

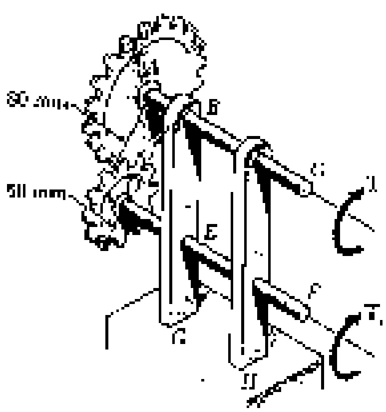
$$= \frac{\pi}{2}(0.450)^3(12) = 1.7177 \text{ kip}\cdot\text{in}$$

$T_F = \frac{r_G}{r_D}T_E \qquad T_{F,all} = \frac{4.5}{6.5}(1.7177) = 1.189$ kip·in

Allowable value of T_F is the smaller $\qquad T_{F,all} = 1.189$ kip·in

PROBLEM 3.26

3.26 The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 60 MPa. Knowing that a 600 N·m torque $\mathbf{T}_C$ is applied at C, determine the required diameter of (a) shaft BC, (b) shaft EF.



SOLUTION

Shaft BC: $T_C = 600$ N·m, $\quad \tau_{all} = 60\times10^6$ Pa

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi\tau} = \frac{(2)(600)}{\pi(60\times10^6)} = 6.3662\times10^{-6} \text{ m}^3$$

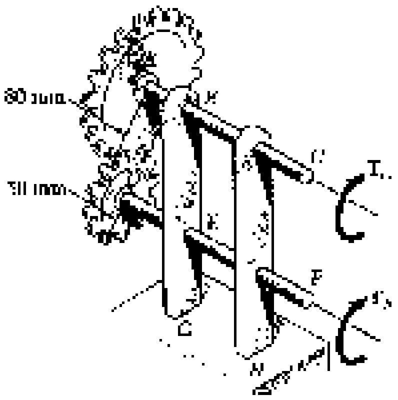
$c = 18.53\times10^{-3}$ m $= 18.53$ mm, $\quad d_{BC} = 2c = 37.1$ mm

Shaft EF: $T_F = \frac{r_B}{r_A}T_C = \frac{50}{80}(600) = 375$ N·m

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi\tau} = \frac{(2)(375)}{\pi(60\times10^6)} = 3.9787\times10^{-6} \text{ m}^3$$

$c = 15.85\times10^{-3}$ m $= 15.85$ mm, $\quad d_{EF} = 2c = 31.7$ mm

PROBLEM 3.27



3.27 The two solid shafts are connected by gears as shown and are made of a steel for which the allowable shearing stress is 50 MPa. Knowing that the diameters of the two shafts are, respectively, d_{BC} = 40 mm and d_{EF} = 32 mm, determine the largest torque T_C which may be applied at C.

SOLUTION

Shaft AC: $\tau_{max} = 50\times10^6$ Pa, $c = \frac{1}{2}d = 0.020$ m

$$T_C = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.020)^3(50\times10^6)$$

$$= 628.3 \text{ N·m}$$

Shaft DF: $\tau_{max} = 50\times10^6$ Pa, $c = \frac{1}{2}d = 0.016$ m

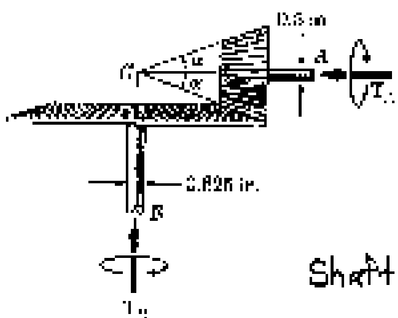
$$T_F = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.016)^3(50\times10^6)$$

$$= 321.7 \text{ N·m}$$

From Statics: $T_C = \frac{r_C}{r_D}T_F = \frac{80}{50}(321.7) = 514.7$ N·m

Allowable value of T_C is the smaller, i.e. $T_F = 515$ N·m

PROBLEM 3.28



3.28 In the bevel-gear system shown α = 18.43°. Knowing that the allowable shearing stress is 8 ksi in each shaft, determine the largest torque T_A which may be applied at A.

SOLUTION

Shaft A: $\tau = 8$ ksi $\quad c = \frac{1}{2}d = 0.25$ in

$$T_A = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.25)^3(8) = 0.19635 \text{ kip·in}$$

Shaft B: $\tau = 8$ ksi $\quad c = \frac{1}{2}d = 0.3125$ in

$$T_B = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.3125)^3(8) = 0.3835 \text{ kip·in}$$

From Statics: $T_A = \frac{r_A}{r_B}T_B = (\tan\alpha)T_B = (\tan 18.43°)(0.3835)$

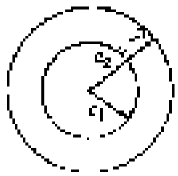
$$= 0.12779 \text{ kip·in}$$

Allowable value of T_A is the smaller

$$T_A = 0.1278 \text{ kip·in} = 127.8 \text{ lb·in}$$

PROBLEM 3.29

3.29 (a) For a given allowable stress, determine the ratio T/w of the maximum allowable torque T and the weight per unit length w for the hollow shaft shown. (b) Denoting by $(T/w)_0$ the value of this ratio computed for a solid shaft of the same radius c_2, express the ratio T/w for the hollow shaft in terms of $(T/w)_0$ and c_1/c_2.



SOLUTION

w = weight per unit length , γ = specific weight

W = total weight , L = length

$$w = \frac{W}{L} = \frac{\gamma L A}{L} = \gamma A = \gamma\pi(c_2^4 - c_1^4)$$

$$T = \frac{J\tau_{al}}{c_2} = \frac{\pi}{2}\,\frac{c_2^4 - c_1^4}{c_2}\,\tau_{al} = \frac{\pi}{2}\,\frac{(c_2^2+c_1^2)(c_2^2-c_1^2)}{c_2}\,\tau_{al}$$

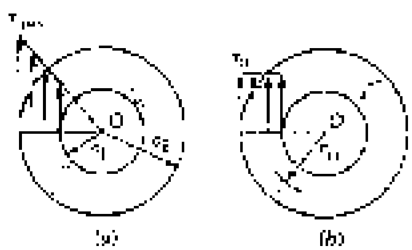
$$\left(\frac{T}{w}\right)_h = \frac{(c_2^2+c_1^2)\,\tau_{al}}{2\gamma c_2} = \frac{c_2\tau_{al}}{2\gamma}\left(1 + \frac{c_1^2}{c_2^2}\right) \qquad \text{(hollow shaft)}$$

$c_1 = 0$ for solid shaft $\quad \left(\dfrac{T}{w}\right)_0 = \dfrac{c_2\tau_{al}}{2\gamma} \qquad$ (solid shaft)

$$\frac{(T/w)_h}{(T/w)_0} = 1 + \frac{c_1^2}{c_2^2} \qquad \left(\frac{T}{w}\right)_h = \left(\frac{T}{w}\right)_0\left(1 + \frac{c_1^2}{c_2^2}\right)$$

PROBLEM 3.30

3.30 While the exact distribution of the shearing stresses in a hollow cylinder shaft is as shown in Fig. (1), an approximate value may be obtained for τ_{max} by assuming the stresses to be uniformly distributed over the area A of the cross section, as shown in Fig. (2), and then further assuming that all the elementary shearing forces act a distance from O equal to the mean radius $r_m = \frac{1}{2}(c_1 + c_2)$ of the cross section. This approximate value is $\tau_0 = T/Ar_m$, where T is the applied torque. Determine the ratio τ_{max}/τ_0 of the true value of the maximum shearing stress and its approximate value τ_0 for values of c_1/c_2 respectively equal to 1.00, 0.95, 0.75, 0.50, and 0.



SOLUTION

For a hollow shaft: $\quad \tau_{max} = \dfrac{Tc_2}{J} = \dfrac{2Tc_2}{\pi(c_2^4 - c_1^4)} = \dfrac{2Tc_2}{\pi(c_2^2 - c_1^2)(c_2^2 + c_1^2)}$

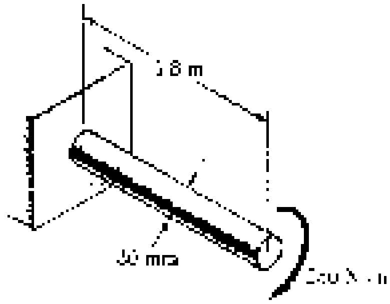
$$= \frac{2Tc_2}{A(c_2^2 + c_1^2)}$$

By definition $\quad \tau_0 = \dfrac{T}{A r_m} = \dfrac{2T}{A(c_2 + c_1)}$

Dividing $\quad \dfrac{\tau_{max}}{\tau_0} = \dfrac{c_2(c_2 + c_1)}{c_2^2 + c_1^2} = \dfrac{1 + (c_1/c_2)}{1 + (c_1/c_2)^2}$

c_1/c_2	1.0	0.95	0.75	0.5	0.0
τ_{max}/τ_0	1.0	1.025	1.120	1.200	1.0

PROBLEM 3.31



3.31 (a) For the solid steel shaft shown ($G = 77$ GPa), determine the angle of twist at A. (b) Solve part a, assuming that the steel shaft is hollow with a 30-mm outer diameter and a 20-mm inner diameter.

SOLUTION

(a) $c = \frac{1}{2}d = 0.015$ m, $J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4$

$J = 79.522 \times 10^{-9}$ m^4, $L = 1.8$ m, $G = 77 \times 10^9$ Pa

$T = 250$ N·m $\qquad \varphi = \frac{TL}{GJ}$

$$\varphi = \frac{(250)(1.8)}{(77 \times 10^9)(79.522 \times 10^{-9})} = 73.49 \times 10^{-3} \text{ rad}$$

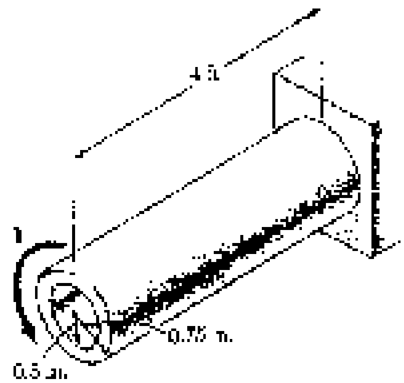
$$\varphi = \frac{(73.49 \times 10^{-3})180}{\pi} = 4.21^\circ$$

(b) $c_2 = 0.015$ m, $c_1 = \frac{1}{2}d_1 = 0.010$ m, $J = \frac{\pi}{2}(c_2^4 - c_1^4)$

$J = \frac{\pi}{2}(0.015^4 - 0.010^4) = 63.814 \times 10^{-9}$ m^4 $\qquad \varphi = \frac{TL}{GJ}$

$$\varphi = \frac{(250)(1.8)}{(77 \times 10^9)(63.814 \times 10^{-9})} = 91.58 \times 10^{-3} \text{ rad} = \frac{180}{\pi}(91.58 \times 10^{-3}) = 5.25^\circ$$

PROBLEM 3.32



3.32 For the aluminum shaft shown ($G = 3.9 \times 10^6$ psi), determine (a) the torque T which causes an angle of twist of 5°, (b) the angle of twist caused by the same torque T in a solid cylindrical shaft of the same length and cross-sectional area.

SOLUTION

(a) $\varphi = \frac{TL}{GJ}$, $\quad T = \frac{GJ\varphi}{L}$

$\varphi = 5^\circ = 87.266 \times 10^{-3}$ rad, $\quad L = 4$ ft $= 48$ in

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.75^4 - 0.5^4) = 0.39884$ in^4

$$T = \frac{(3.9 \times 10^6)(0.39884)(87.266 \times 10^{-3})}{48}$$

$= 2.8279 \times 10^3$ lb·in $= 2.83$ kip·in

(b) Hollow shaft $\quad A = \pi(c_2^2 - c_1^2) \qquad$ Solid shaft $\quad A = \pi c^2$

Matching areas $\qquad c^2 = c_2^2 - c_1^2 = 0.75^2 - 0.5^2 = 0.3125$ in^2

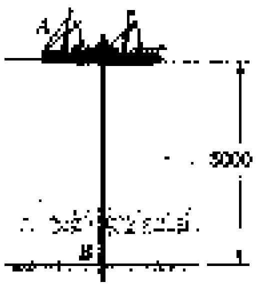
$c = 0.5590$ in, $\quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.5590)^4 = 153.398 \times 10^{-3}$ in^4

$$\varphi = \frac{TL}{GJ} = \frac{(2.8279 \times 10^3)(48)}{(3.9 \times 10^6)(153.398 \times 10^{-3})} = 226.89 \times 10^{-3} \text{ rad}$$

$= 13.00^\circ$

PROBLEM 3.33

3.33 The ship at A has just started to drill for oil on the ocean floor at a depth of 5000 ft. Knowing that the top of the 8-in.-diameter steel drill pipe ($G = 11.2 \times 10^6$ psi) rotates through two complete revolutions before the drill bit at B starts to operate, determine the maximum shearing stress caused in the pipe by torsion.



SOLUTION

$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L}$$

$$\tau = \frac{Tc}{J} = \frac{GJ\varphi c}{JL} = \frac{G\varphi c}{L}$$

$\varphi = 2 \text{ rev} = (2)(2\pi) = 12.566 \text{ rad}, \qquad c = \frac{1}{2}d = 4.0 \text{ in}$

$L = 5000 \text{ ft} = 60000 \text{ in} \qquad \tau = \dfrac{(11.2\times10^6)(12.566)(4.0)}{60000}$

$= 9.3826 \times 10^3 \text{ psi} = 9.38 \text{ ksi}$

PROBLEM 3.34

3.34 Determine the largest allowable diameter of a 3-m-long steel rod ($G = 77$ GPa) if the rod is to be twisted through 30° without exceeding a shearing stress of 80 MPa.

SOLUTION

$L = 3 \text{ m}, \quad \varphi = \dfrac{30\pi}{180} = 523.6 \times 10^{-3} \text{ rad}, \quad \tau = 80 \times 10^6 \text{ Pa}$

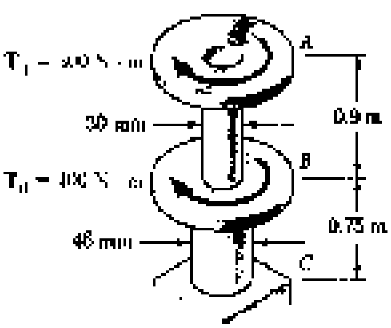
$\varphi = \dfrac{TL}{GJ}, \quad T = \dfrac{GJ\varphi}{L}, \quad \tau = \dfrac{Tc}{J} = \dfrac{GJ\varphi c}{JL} = \dfrac{G\varphi c}{L}, \quad c = \dfrac{\tau L}{G\varphi}$

$c = \dfrac{(80\times10^6)(3.0)}{(77\times10^9)(523.6\times10^{-3})} = 5.953 \times 10^{-3} \text{ m} = 5.953 \text{ mm}$

$d = 2c = 11.91 \text{ mm}$

PROBLEM 3.35

3.35 The torques shown are exerted on pulleys A and B. Knowing that the shafts are solid and made of aluminum ($G = 77$ GPa), determine the angle of twist between (a) A and B, (b) A and C.



SOLUTION

(a) $T_{AB} = 300 \text{ N}\cdot\text{m}, \quad L_{AB} = 0.9 \text{ m}, \quad c_{AB} = \frac{1}{2}d = 0.015 \text{ m}$

$J_{AB} = \frac{\pi}{2}(0.015)^4 = 79.522 \times 10^{-9} \text{ m}^4$

$\varphi_{AB} = \dfrac{T_{AB}L_{AB}}{GJ} = \dfrac{(300)(0.9)}{(77\times10^9)(79.522\times10^{-9})} = 44.095 \times 10^{-3} \text{ rad}$

$\varphi_{AB} = 2.53°$

(b) $T_{BC} = 300 + 400 = 700 \text{ N}\cdot\text{m}, \quad L_{BC} = 0.75 \text{ m}, \quad c_{BC} = \frac{1}{2}d = 0.023 \text{ m}$

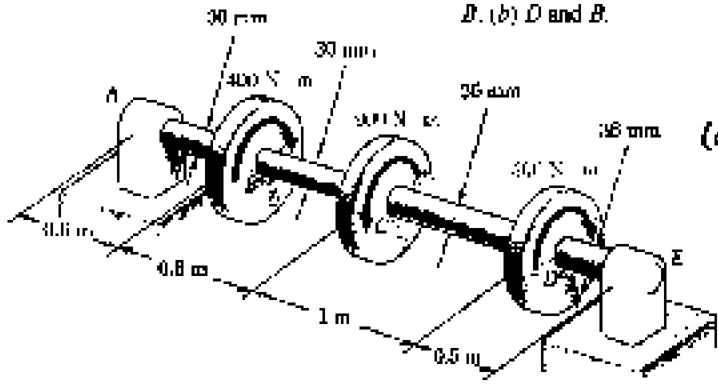
$J_{BC} = \frac{\pi}{2}(0.023)^4 = 439.573 \times 10^{-9} \text{ m}^4$

$\varphi_{BC} = \dfrac{T_{BC}L_{BC}}{GJ_{BC}} = \dfrac{(700)(0.75)}{(77\times10^9)(439.573\times10^{-9})} = 15.511 \times 10^{-3} \text{ rad}$

$\varphi_{AC} = \varphi_{AB} + \varphi_{BC} = 59.606 \times 10^{-3} \text{ rad} = 3.42°$

PROBLEM 3.36

3.36 The torques shown are exerted on pulleys B, C and D. Knowing that the entire shaft is made of steel (G = 27 GPa), determine the angle of twist between (a) C and B, (b) D and B.



SOLUTION

(a) Shaft BC: $c = \frac{1}{2}d = 0.015$ m

$J_{BC} = \frac{\pi}{2}c^4 = 79.522 \times 10^{-9}$ m^4

$L_{BC} = 0.8$ m, $G = 27 \times 10^9$ Pa

$$\varphi_{BC} = \frac{TL}{GJ} = \frac{(400)(0.8)}{(27\times10^9)(79.522\times10^{-9})}$$

$= 0.149904$ rad $= 8.59°$ ◀

(b) Shaft CD: $c = \frac{1}{2}d = 0.018$ m $\qquad J_{CD} = \frac{\pi}{2}c^4 = 164.896 \times 10^{-9}$ m^4

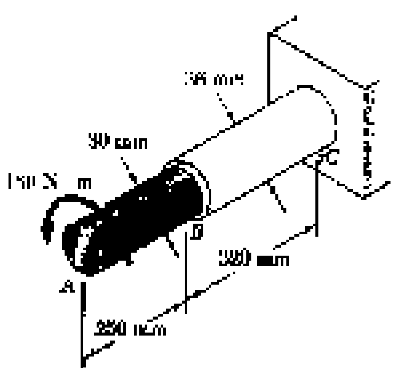
$L_{CD} = 1.0$ m $\qquad T_{CD} = 400 - 900 = -500$ N·m

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(-500)(1.0)}{(27\times10^9)(164.896\times10^{-9})} = -0.11230 \text{ rad}$$

$\varphi_{BD} = \varphi_{BC} + \varphi_{CD} = 0.14904 - 0.11230 = 0.03674$ rad $= 2.11°$ ◀

PROBLEM 3.37

3.37 The solid brass rod AB (G = 39 GPa) is bonded to the solid aluminum rod BC (G = 27 GPa). Determine the angle of twist (a) at B, (b) at A.



SOLUTION

Shaft AB: $c = \frac{1}{2}d = 0.015$ m $\qquad L = 0.250$ m

$G = 39 \times 10^9$ Pa $\qquad T = 180$ N·m

$J = \frac{\pi}{2}c^4 = 79.522 \times 10^{-9}$ m^4

$$\varphi_{AB} = \frac{TL}{GJ} = \frac{(180)(0.250)}{(39\times10^9)(79.522\times10^{-9})} = 14.510\times10^{-3} \text{ rad}$$

Shaft BC: $c = \frac{1}{2}d = 0.018$ m, $L = 0.320$ m

$G = 27 \times 10^9$ Pa, $T = 180$ N·m

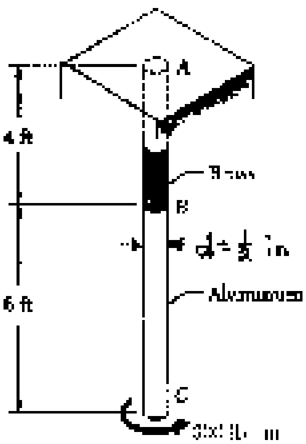
$J = \frac{\pi}{2}c^4 = 164.896 \times 10^{-9}$ m^4

$$\varphi_{BC} = \frac{(180)(0.320)}{(27\times10^9)(164.896\times10^{-9})} = 12.937\times10^{-3} \text{ rad}$$

Answers: (a) $\varphi_B = \varphi_{BC} = 12.937 \times 10^{-3}$ rad $= 0.741°$ ◀

(b) $\varphi_A = \varphi_{BC} + \varphi_{AB} = 27.447 \times 10^{-3}$ rad $= 1.573°$ ◀

PROBLEM 3.38



3.38 The brass rod AB ($G = 5.6 \times 10^6$ psi) is bonded to the aluminum rod BC ($G = 3.9 \times 10^6$ psi). Knowing that each rod is solid, determine the angle of twist (*a*) at B, (*b*) at C.

SOLUTION

Both portions $c = \frac{1}{2}d = 0.25$ in

$J = \frac{\pi}{2}c^4 = 6.1359 \times 10^{-3}$ in^4 $\quad T = 300$ lb·in

Shaft AB: $G_{AB} = 5.6 \times 10^6$ psi $\quad L_{AB} = 4$ ft $= 48$ in

$$\varphi_B = \varphi_{AB} = \frac{TL_{AB}}{G_{AB}J} = \frac{(300)(48)}{(5.6 \times 10^6)(6.1359 \times 10^{-3})}$$

$$= 0.419 \text{ rad} = 24.0°$$

Shaft BC: $G = 3.9 \times 10^6$ psi $\quad L_{BC} = 6$ ft $= 72$ in

$$\varphi_{BC} = \frac{TL_{BC}}{G_{BC}J} = \frac{(300)(72)}{(3.9 \times 10^6)(6.1359 \times 10^{-3})}$$

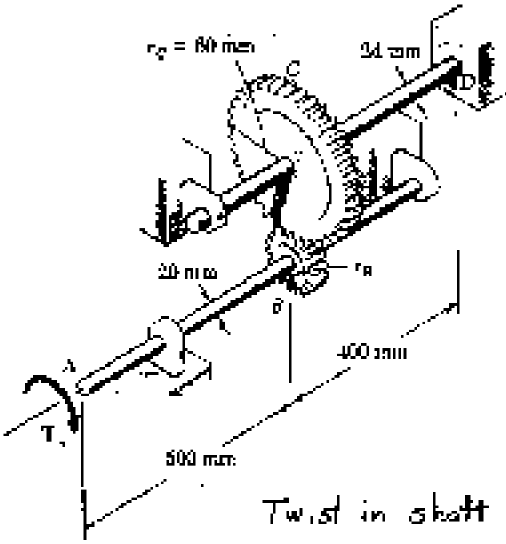
$$= 0.903 \text{ rad} = 51.7°$$

$$\varphi_C = \varphi_B + \varphi_{BC}$$

$$= 0.417 + 0.903 = 1.320 \text{ rad} = 75.6°$$

PROBLEM 3.39

3.39 Two solid steel shafts ($G = 77$ GPa) are connected by the gears shown. Knowing that the radius of gear B is $r_B = 20$ mm, determine the angle through which end A rotates when $T_A = 75$ N·m.



SOLUTION

Calculation of torques

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \quad \therefore \quad T_{CD} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = T_A = 75 \text{ N·m}$$

$$T_{CD} = \frac{0.060}{0.020}(75) = 225 \text{ N·m}$$

Twist in shaft CD

$$J_{CD} = \frac{\pi}{2} c_{CD}^4 = \frac{\pi}{2}(0.012)^4 = 32.572 \times 10^{-9} \text{ m}^4, \qquad L_{CD} = 0.400 \text{ m}$$

$$G = 77 \times 10^9 \text{ Pa} \qquad \varphi_{CD} = \frac{TL}{GJ} = \frac{(225)(0.400)}{(77\times10^9)(32.572\times10^{-9})} = 35.885 \times 10^{-3} \text{ rad}$$

Rotation angle at C $\qquad \varphi_C = \varphi_{CD} = 35.885 \times 10^{-3}$ rad

Circumferential displacement at contact points of gears B and C

$$\delta = r_C \varphi_C = r_B \varphi_B$$

Rotation angle at B: $\qquad \varphi_B = \frac{r_C}{r_B}\varphi_C = \frac{0.060}{0.020}(35.885 \times 10^{-3}) = 107.654 \times 10^{-3}$ rad

Twist in shaft AB:

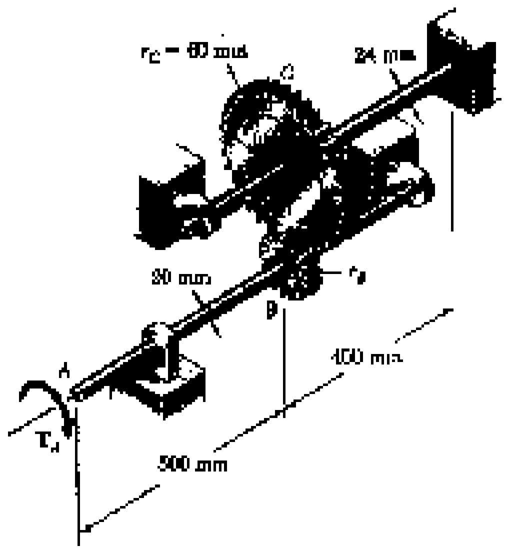
$$J_{AB} = \frac{\pi}{2} c_{AB}^4 = \frac{\pi}{2}(0.010)^4 = 15.708 \times 10^{-9} \text{ m}^4, \qquad L_{AB} = 0.500 \text{ m}$$

$$G = 77 \times 10^9 \text{ Pa}, \quad \varphi_{AB} = \frac{TL}{GJ} = \frac{(75)(0.500)}{(77\times10^9)(15.708\times10^{-9})} = 31.004 \times 10^{-3} \text{ rad}$$

Rotation at A $\qquad \varphi_A = \varphi_B + \varphi_{AB} = 138.7 \times 10^{-3} \text{ rad} = 7.94°$ ⟵

PROBLEM 3.40

3.40 Solve Prob. 3.39, assuming that a change in design of the assembly resulted in the radius of gear B being increased to 30 mm.



$G = 77$ GPa, $r_B = 30$ mm, $T_A = 75$ N·m

Determine the angle through which end A rotates.

SOLUTION

Calculation of torques

Circumferential contact force between gears B and C

$$F = \frac{T_{AB}}{r_B} = \frac{T_{CD}}{r_C} \quad \therefore \quad T_{CD} = \frac{r_C}{r_B} T_{AB}$$

$$T_{AB} = T_A = 75 \text{ N·m}$$

$$T_{CD} = \frac{0.060}{0.030}(75) = 150 \text{ N·m}$$

Twist in shaft CD

$$J_{CD} = \frac{\pi}{2} c_{CD}^4 = \frac{\pi}{2}(0.012)^4 = 32.572 \times 10^{-9} \text{ m}^4, \qquad L_{CD} = 0.400 \text{ m}$$

$$G = 77 \times 10^9 \text{ Pa}, \qquad \varphi_{CD} = \frac{TL}{GJ} = \frac{(150)(0.400)}{(77\times10^9)(32.572\times10^{-9})} = 23.923\times10^{-3} \text{ rad}$$

Rotation angle at C $\quad \varphi_C = \varphi_{CD} = 23.923\times10^{-3}$ rad.

Circumferential displacement at contact points of gears B and C.

$$\delta = r_C \varphi_C = r_B \varphi_B$$

Rotation angle at B $\quad \varphi_B = \frac{r_C}{r_B}\varphi_C = \frac{0.060}{0.030}(23.923\times10^{-3}) = 47.846\times10^{-3}$ rad

Twist in shaft AB

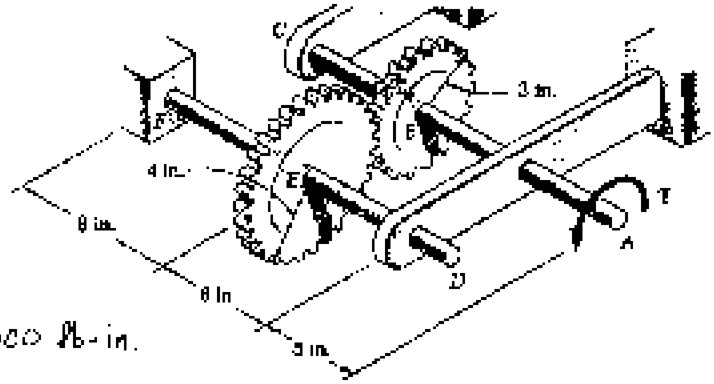
$$J_{AB} = \frac{\pi}{2} c_{AB}^4 = \frac{\pi}{2}(0.010)^4 = 15.708\times10^{-9} \text{ m}^4, \qquad L_{AB} = 0.500 \text{ m}$$

$$G = 77\times10^9 \text{ Pa}, \qquad \varphi_{AB} = \frac{TL}{GJ} = \frac{(75)(0.500)}{(77\times10^9)(15.708\times10^{-9})} = 31.004\times10^{-3} \text{ rad}$$

Rotation at A $\quad \varphi_A = \varphi_B + \varphi_{AB} = 78.85\times10^{-3}$ rad $= 4.52°$ ◄

PROBLEM 3.41

3.41 Two shafts, each of $\frac{3}{4}$ - in. diameter, are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi and that the shaft at F is fixed, determine the angle through which end A rotates when a 750 lb·in. torque is applied at A.



SOLUTION

Calculation of torques

Circumferential contact force between gears B and E

$$F = \frac{T_{AB}}{r_B} = \frac{T_{EF}}{r_E}$$

$$\therefore \quad T_{EF} = \frac{r_E}{r_B} T_{AB} = \frac{4}{3}(750) = 1000 \text{ lb·in.}$$

Twist in shaft FE

$$L_{FE} = 8 \text{ in}, \quad J_{FE} = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.375)^4 = 31.063 \times 10^{-3} \text{ in}^4, \quad G = 11.2 \times 10^6 \text{ psi}$$

$$\varphi_{EF} = \frac{T_{EF} L_{FE}}{G J_{FE}} = \frac{(1000)(8)}{(11.2 \times 10^6)(31.063 \times 10^{-3})} = 22.995 \times 10^{-3} \text{ rad}$$

Rotation at E $\qquad \varphi_E = 22.995 \times 10^{-3}$ rad

Tangential displacement at gear circle $\quad s = r_E \varphi_E = r_B \varphi_B$

Rotation at B $\qquad \varphi_B = \frac{r_E}{r_B} \varphi_E = \frac{4}{3}(22.995 \times 10^{-3}) = 30.660 \times 10^{-3}$ rad

Twist in shaft BA

$$L_{BA} = 6 + 5 = 11 \text{ in}, \quad J_{BA} = 31.063 \times 10^{-3} \text{ in}^4$$

$$\varphi_{AB} = \frac{T_{AB} L_{BA}}{G J_{AB}} = \frac{(750)(11)}{(11.2 \times 10^6)(31.063 \times 10^{-3})} = 23.713 \times 10^{-3} \text{ rad}$$

Rotation at A

$$\varphi_A = \varphi_B + \varphi_{AB} = 30.660 \times 10^{-3} + 23.713 \times 10^{-3} = 54.373 \times 10^{-3} \text{ rad}$$

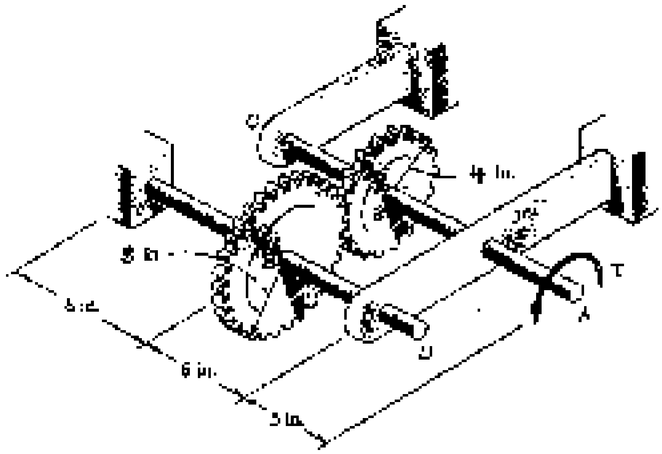
$$= 3.12^\circ$$

PROBLEM 3.42

3.41 Two shafts, each of $\frac{3}{4}$ - in. diameter, are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi and that the shaft at F is fixed, determine the angle through which end A rotates when a 750 lb·in. torque is applied at A.

3.42 Solve Prob. 3.41, assuming that after a design change the radius of gear B is 4 in. and the radius of gear E is 3 in.

SOLUTION



Calculation of torques

Tangential contact force between gears B and E

$$F = \frac{T_{AB}}{r_B} = \frac{T_{EF}}{r_E}$$

$$T_{EF} = \frac{r_E}{r_B} T_{AB} = \frac{3}{4}(750)$$

$$= 562.5 \text{ lb}\cdot\text{in}$$

Twist in shaft FE

$L_{FE} = 8 \text{ in},\quad J_{FE} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.375)^4 = 31.063 \times 10^{-3} \text{ in}^4 \quad G = 11.2 \times 10^6 \text{ psi}$

$$\varphi_{FE} = \frac{T_{FE}L_{FE}}{GJ_{FE}} = \frac{(562.5)(8)}{(11.2\times10^6)(31.063\times10^{-3})} = 12.935 \times 10^{-3} \text{ rad}$$

Rotation at E $\qquad \varphi_E = 12.935 \times 10^{-3}$ rad.

Tangential displacement at gear circle $\qquad \delta = r_E\varphi_E = r_B\varphi_B$

Rotation at B $\qquad \varphi_B = \frac{r_E}{r_B}\varphi_E = \frac{3}{4}(12.935\times10^{-3}) = 9.70 \times 10^{-3}$ rad

Twist shaft AB

$L_{AB} = 6 + 5 = 11 \text{ in.},\quad J_{AB} = 31.063 \times 10^{-3} \text{ in}^4,$

$$\varphi_{AB} = \frac{T_{AB}L_{AB}}{GJ_{AB}} = \frac{(750)(11)}{(11.2\times10^6)(31.063\times10^{-3})} = 23.713\times10^{-3} \text{ rad}$$

Rotation at A $\qquad \varphi_A = \varphi_B + \varphi_{AB}$

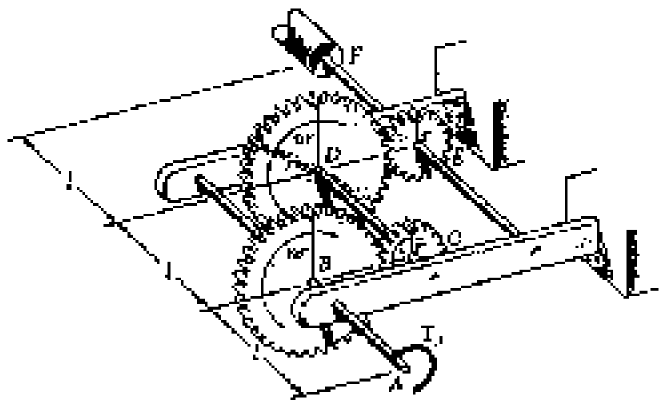
$$= 9.701\times10^{-3} + 23.713\times10^{-3} = 33.414\times10^{-3} \text{ rad}$$

$$= 1.914^\circ$$

PROBLEM 3.43

3.43 A coder F, used to record in digital form the rotation of shaft A, is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter d. Two of the gears have a radius r and the other two a radius nr. If the rotation of the coder F is prevented, determine in terms of T, l, G, J, and n the angle through which end A rotates.

SOLUTION



$$T_{AB} = T_A$$

$$T_{CD} = \frac{r_C}{r_B} T_{AB} = \frac{T_{AB}}{n} = \frac{T_A}{n}$$

$$T_{EF} = \frac{r_E}{r_D} T_{CD} = \frac{T_{CD}}{n} = \frac{T_A}{n^2}$$

$$\varphi_E = \varphi_{EF} = \frac{T_{EF} l_{EF}}{GJ} = \frac{T_A l}{n^2 GJ}$$

$$\varphi_D = \frac{r_E}{r_D} \varphi_E = \frac{\varphi_E}{n} = \frac{T_A l}{n^3 GJ}$$

$$\varphi_{CD} = \frac{T_{CD} l_{CD}}{GJ} = \frac{T_A l}{n GJ}$$

$$\varphi_C = \varphi_D + \varphi_{CD} = \frac{T_A l}{n^3 GJ} + \frac{T_A l}{n GJ} = \frac{T_A l}{GJ}\left(\frac{1}{n^3} + \frac{1}{n}\right)$$

$$\varphi_B = \frac{r_C}{r_B} \varphi_C = \frac{\varphi_C}{n} = \frac{Tl}{GJ}\left(\frac{1}{n^4} + \frac{1}{n^2}\right)$$

$$\varphi_{AB} = \frac{T_{AB} l_{AB}}{GJ} = \frac{T_A l}{GJ}$$

$$\varphi_A = \varphi_B + \varphi_{AB} = \frac{T_A l}{GJ}\left(\frac{1}{n^4} + \frac{1}{n^2} + 1\right)$$

PROBLEM 3.44

3.43 A coder F, used to record in digital form the rotation of shaft A, is connected to the shaft by means of the gear train shown, which consists of four gears and three solid steel shafts each of diameter d. Two of the gears have a radius r and the other two a radius nr. If the rotation of the coder F is prevented, determine in terms of T, l, G, J, and n the angle through which end A rotates.

3.44 For the gear train described in Prob. 3.43, determine the angle through which end A rotates when $T = 0.75$ N·m, $l = 60$ mm, $d = 4$ mm, $G = 77$ GPa, and $n = 2$.

SOLUTION

See solution to PROBLEM 3.43 for development of equation for φ_A

$$\varphi_A = \frac{Tl}{GJ}\left(1 + \frac{1}{n^2} + \frac{1}{n^4}\right)$$

Data: $T = 0.75$ N·m, $l = 0.060$ m, $c = \frac{1}{2}d = 0.002$ m, $G = 77 \times 10^9$ Pa

$n = 2$, $J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.002)^4 = 25.133 \times 10^{-12}$ m^4

$$\varphi_A = \frac{(0.75)(0.060)}{(77 \times 10^9)(25.133 \times 10^{-12})}\left(1 + \frac{1}{4} + \frac{1}{16}\right) = 30.52 \times 10^{-3} \text{ rad}$$

$$= 1.749°$$

PROBLEM 3.45

3.45 The design specifications of a 2-m-long solid circular transmission shaft require that the angle of twist of the shaft not exceed 3° when a torque of 9 kN·m is applied. Determine the required diameter of the shaft, knowing that the shaft is made of (*a*) a steel with an allowable shearing stress of 90 MPa and a modulus of rigidity of 77 GPa, (*b*) a bronze with an allowable shearing stress of 35 MPa and a modulus of rigidity of 42 GPa.

SOLUTION

$\varphi = 3° = 52.360\times10^{-3}$ rad, $\quad T = 9\times10^{3}$ N·m $\quad L = 2.0$ m

$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi c^4 G} \quad \therefore \quad c^4 = \frac{2TL}{\pi G\varphi}$ based on twist angle

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi\tau}$ based shearing stress

(a) Steel shaft: $\tau = 90\times10^{6}$ Pa, $\quad G = 77\times10^{9}$ Pa

Based on twist angle $\quad c^4 = \frac{(2)(9\times10^3)(2.0)}{\pi(77\times10^9)(52.360\times10^{-3})} = 2.842\times10^{-6}$ m^4

$c = 41.06\times10^{-3}$ m $= 41.06$ mm $\qquad d = 2c = 82.1$ mm

Based on shearing stress $\quad c^3 = \frac{(2)(9\times10^3)}{\pi(90\times10^6)} = 63.662\times10^{-6}$ m^3

$c = 39.93\times10^{-3}$ m $= 39.93$ mm $\qquad d = 2c = 79.9$ mm

Required value of d is the larger: $\qquad d = 82.1$ mm ➡

(b) Bronze shaft: $\tau = 35\times10^{6}$ Pa, $\quad G = 42\times10^{9}$ Pa

Based on twist angle $\quad c^4 = \frac{(2)(9\times10^3)(2.0)}{\pi(42\times10^9)(52.360\times10^{-3})} = 5.2103\times10^{-6}$ m^4

$c = 47.78\times10^{-3}$ m $= 47.78$ mm $\qquad d = 2c = 95.6$ mm

Based on shearing stress $\quad c^3 = \frac{(2)(9\times10^3)}{\pi(35\times10^6)} = 163.702\times10^{-6}$ m^3

$c = 54.70\times10^{-3}$ m $= 54.70$ mm $\qquad d = 2c = 109.4$ mm

Required value of d is the larger $\qquad d = 109.4$ mm ➡

PROBLEM 3.46

3.46 The design specifications of a 4-ft-long solid circular transmission shaft require that the angle of twist of the shaft not exceed 4° when a torque of 6 kip·in. is applied. Determine the required diameter of the shaft, knowing that the shaft is made of a steel with an allowable shearing stress of 12 ksi and a modulus of rigidity of 11.2×10^6 psi.

SOLUTION

Based on twist angle $\quad \varphi = 4° = 69.81 \times 10^{-3}$ rad, $\quad L = 4$ ft $= 48$ in.

$T = 6$ kip·in $= 6000$ lb·in, $\quad G = 11.2 \times 10^6$ psi

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4} \qquad c^4 = \frac{2TL}{\pi G \varphi} = \frac{(2)(6000)(48)}{\pi(11.2\times10^6)(69.81\times10^{-3})} = 0.2345 \text{ in}^4$$

$c = 0.696$ in $\qquad d = 2c = 1.392$ in.

Based on shearing stress $\quad \tau = 12$ ksi $= 12000$ psi

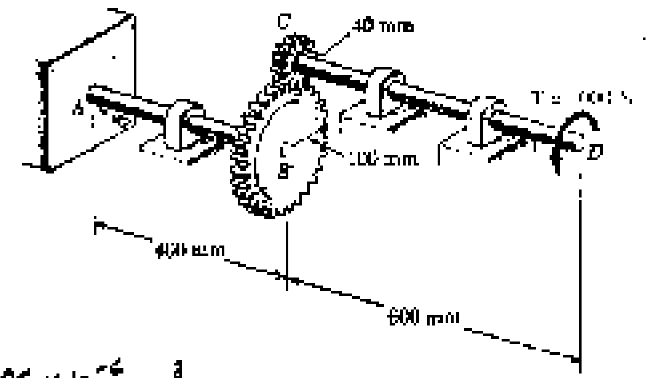
$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(6000)}{\pi(12000)} = 0.3183 \text{ in}^3$$

$c = 0.683$ in $\qquad d = 2c = 1.366$ in.

Required diameter is the larger $\qquad d = 1.392$ in. ◄

PROBLEM 3.47

3.47 The design of the gear-and-shaft system shown requires that steel shafts of the same diameter be used for both AB and CD. It is further required that $\tau_{max} \le 60$ MPa and that the angle ϕ_D through which end D of shaft CD rotates not exceed 1.5°. Knowing that $G = 77$ GPa, determine the required diameter of the shafts.



SOLUTION

$T_{CD} = T_D = 1000$ N·m

$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500$ N·m

For design based on stress, use larger torque $T_{AB} = 2500$ N·m

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$

$c^3 = \frac{2T}{\pi \tau} = \frac{(2)(2500)}{\pi(60 \times 10^6)} = 26.526 \times 10^{-6}$ m³

$c = 29.82 \times 10^{-3}$ m $= 29.82$ mm, $\quad d = 2c = 59.6$ mm

Design based on rotation angle $\varphi_D = 1.5° = 26.18 \times 10^{-3}$ rad

Shaft AB: $T_{AB} = 2500$ N·m, $L = 0.4$ m

$\varphi_{AB} = \frac{TL}{GJ} = \frac{(2500)(0.4)}{GJ} = \frac{1000}{GJ}$

Gears $\begin{cases} \varphi_B = \varphi_{AB} = \frac{1000}{GJ} \\ \varphi_C = \frac{r_B}{r_C}\varphi_B = \frac{100}{40} \cdot \frac{1000}{GJ} = \frac{2500}{GJ} \end{cases}$

Shaft CD $\quad T_{CD} = 1000$ N·m, $L = 0.6$ m

$\varphi_{CD} = \frac{TL}{GJ} = \frac{(1000)(0.6)}{GJ} = \frac{600}{GJ}$

$\varphi_D = \varphi_C + \varphi_{CD} = \frac{2500}{GJ} + \frac{600}{GJ} = \frac{3100}{GJ} = \frac{3100}{G\frac{\pi}{2}c^4}$

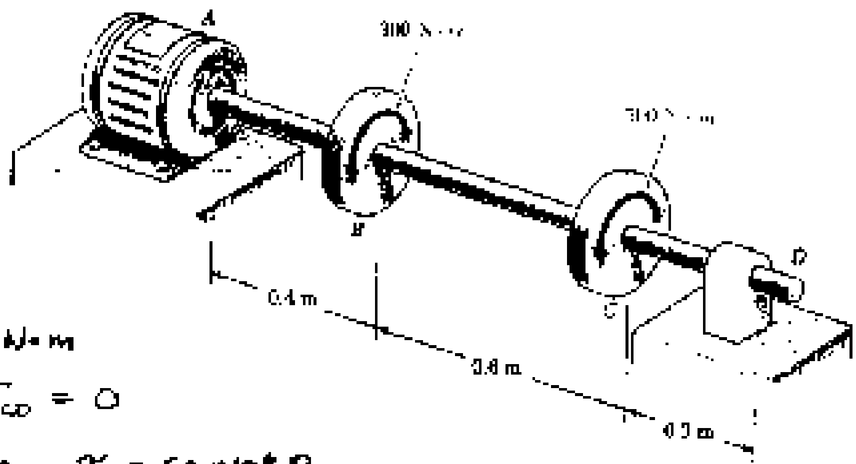
$c^4 = \frac{(2)(3100)}{\pi G \varphi_D} = \frac{(2)(3100)}{\pi(77 \times 10^9)(26.18 \times 10^{-3})} = 979.06 \times 10^{-9}$ m⁴

$c = 31.46 \times 10^{-3}$ m $= 31.46$ mm, $\quad d = 2c = 62.9$ mm

Design must use larger value for d $\quad d = 62.9$ mm

PROBLEM 3.48

3.48 The electric motor exerts a torque of 800 N·m on the steel shaft *ABCD* when it is rotating at constant speed. Design specifications require that the diameter of the shaft be uniform from *A* to *D* and that the angle of twist between *A* and *D* not exceed 1.5°. Knowing that $\tau_{max} \le 60$ MPa and $G = 77$ GPa, determine the minimum diameter shaft that may be used.



SOLUTION

Torques

$T_{AB} = 300 + 500 = 800$ N·m

$T_{BC} = 500$ N·m, $\quad T_{CD} = 0$

Design based on stress $\quad \tau = 60 \times 10^6$ Pa

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \qquad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(800)}{\pi(60 \times 10^6)} = 8.488 \times 10^{-6} \text{ m}^3$$

$c = 20.40 \times 10^{-3}$ m $= 20.40$ mm, $\quad d = 2c = 40.8$ mm

Design based on deformation $\quad \varphi_{D/A} = 1.5° = 26.18 \times 10^{-3}$ rad

$$\varphi_{D/C} = 0$$

$$\varphi_{C/B} = \frac{T_{BC} L_{BC}}{GJ} = \frac{(500)(0.6)}{GJ} = \frac{600}{GJ}$$

$$\varphi_{B/A} = \frac{T_{AB} L_{AB}}{GJ} = \frac{(800)(0.4)}{GJ} = \frac{320}{GJ}$$

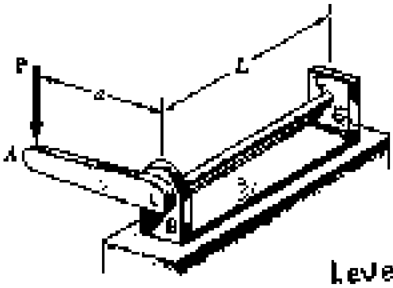
$$\varphi_{D/A} = \varphi_{D/C} + \varphi_{C/B} + \varphi_{B/A} = \frac{620}{GJ} = \frac{620}{G \frac{\pi}{2} c^4} = \frac{(2)(620)}{\pi G c^4}$$

$$c^4 = \frac{(2)(620)}{\pi G \varphi_{D/A}} = \frac{(2)(620)}{\pi (77 \times 10^9)(26.18 \times 10^{-3})} = 195.80 \times 10^{-9} \text{ m}^4$$

$c = 21.04 \times 10^{-3}$ m $= 21.04$ mm, $\quad d = 2c = 42.1$ mm

Design must use larger value of d $\qquad d = 42.1$ mm

PROBLEM 3.49

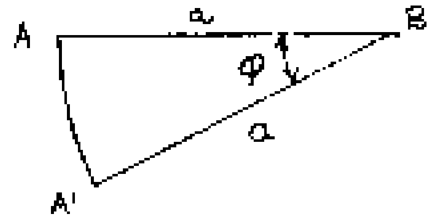


3.49 The solid cylindrical rod BC is attached to the rigid lever AB and to the fixed support at C. The vertical force $\mathbf{P}$ applied at A causes a small displacement Δ at point A. Show that the corresponding maximum shearing stress in the rod is

$$\tau = \frac{Gd}{2La}\Delta$$

where d is the diameter of the rod and G its modulus of rigidity.

SOLUTION



Lever AB turns through angle φ to position $A'B$ as shown in the auxiliary figure.

Vertical displacement is $\Delta = a \sin\varphi$

from which $\varphi = \arcsin \frac{\Delta}{a}$

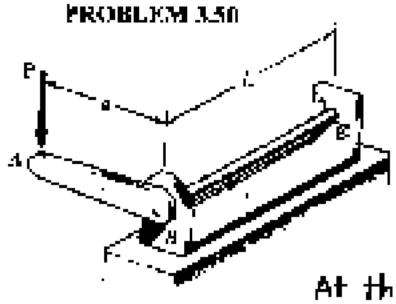
The maximum shearing stress in rod BC is

$$\tau_{max} = G\gamma_{max} = G\frac{c\varphi}{L} = G\frac{d\varphi}{2L} = \frac{Gd}{2L}\arcsin\frac{\Delta}{a}$$

For small $\frac{\Delta}{a}$, $\arcsin\frac{\Delta}{a} \approx \frac{\Delta}{a}$

$$\tau_{max} = \frac{Gd\Delta}{2La}$$

PROBLEM 3.50



3.50 and 3.51 The solid cylindrical rod BC of length $L = 24$ in. is attached to the rigid lever AB of length $a = 15$ in. and to the support at C. When a 100-lb force **P** is applied at A, design specifications require that the displacement of A not exceed 1 in. when a 100-lb force **P** is applied at A. For the material indicated determine the required diameter of the rod.

3.50 Steel: $\tau_{all} = 15$ ksi, $G = 11.2 \times 10^6$ psi.

SOLUTION

At the allowable twist angle $\quad \sin\varphi = \frac{\Delta}{a} = \frac{1}{15} = 0.06667$

$\varphi = 3.8226° = 0.066716$ rad.

$T = Pa\cos\varphi = (100)(15)\cos 3.8226° = 1496.7$ lb·in

Based on twist $\quad \varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4} \quad \therefore \quad c^4 = \frac{2TL}{\pi G \varphi}$

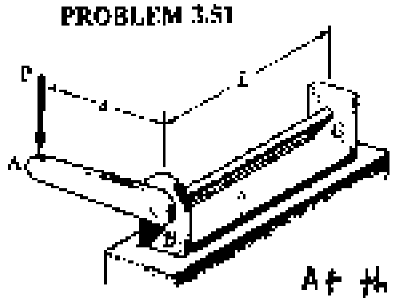
$c^4 = \frac{(2)(1496.7)(24)}{\pi(11.2\times10^6)(0.066716)} = 30.608\times10^{-3}$ in^4 $\qquad c = 0.418$ in.

Based on stress $\quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi\tau} \qquad (\tau = 15000$ psi$)$

$c^3 = \frac{2(1496.7)}{\pi(15000)} = 63.522\times10^{-3}$ in^3 $\qquad c = 0.399$ in.

Use larger value for design $\quad c = 0.399$ in $\quad d = 2c = 0.837$ in ◄

PROBLEM 3.51



3.50 and 3.51 The solid cylindrical rod BC of length $L = 24$ in. is attached to the rigid lever AB of length $a = 15$ in. and to the support at C. When a 100-lb force **P** is applied at A, design specifications require that the displacement of A not exceed 1 in. when a 100-lb force **P** is applied at A. For the material indicated determine the required diameter of the rod.

3.51 Aluminum: $\tau_{all} = 10$ ksi, $G = 3.9 \times 10^6$ psi.

SOLUTION

At the allowable twist angle $\quad \sin\varphi = \frac{\Delta}{a} = \frac{1}{16} = 0.06667$

$\varphi = 3.8226° = 0.066716$ rad.

$T = Pa\cos\varphi = (100)(15)\cos 3.8226° = 1496.7$ lb·in

Based on twist $\quad \varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4} \quad \therefore \quad c^4 = \frac{2TL}{\pi G \varphi}$

$c^4 = \frac{(2)(1496.7)(24)}{\pi(3.9\times10^6)(0.066716)} = 87.888\times10^{-3}$ in^4 $\qquad c = 0.544$ in.

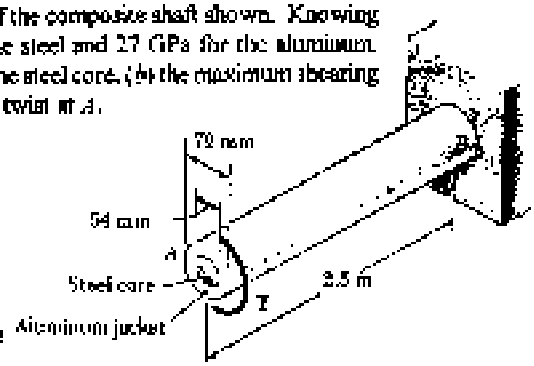
Based on stress $\quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi\tau} \qquad (\tau = 10000$ psi$)$

$c^3 = \frac{(2)(1496.7)}{\pi(10000)} = 95.283\times10^{-3}$ in^3 $\qquad c = 0.457$ in

Use larger value for design $\quad c = 0.544$ in $\quad d = 2c = 1.089$ in ◄

PROBLEM 3.52

3.52 A 4-kN·m torque **T** is applied at end A of the composite shaft shown. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine (*a*) the maximum shearing stress in the steel core, (*b*) the maximum shearing stress in the aluminum jacket, (*c*) the angle of twist at A.



SOLUTION

Steel core: $c_1 = \frac{1}{2} d_1 = 0.027$ m

$J_1 = \frac{\pi}{2} c_1^4 = \frac{\pi}{2}(0.027)^4 = 834.79 \times 10^{-9}$

$G_1 J_1 = (77\times10^9)(834.79\times10^{-9}) = 64.28\times10^3$ N·m²

Torque carried by steel core $\quad T_1 = G_1 J_1 \varphi / L$

Aluminum jacket: $c_1 = \frac{1}{2} d_1 = 0.027$ m, $\quad c_2 = \frac{1}{2} d_2 = 0.036$ m

$J_2 = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.036^4 - 0.027^4) = 1.80355\times10^{-6}$ m⁴

$G_2 J_2 = (27\times10^9)(1.80355\times10^{-6}) = 48.70\times10^3$ N·m²

Torque carried by aluminum jacket $\quad T_2 = G_2 J_2 \varphi / L$

Total torque $\quad T = T_1 + T_2 = (G_1J_1 + G_2J_2)\varphi/L$

$$\frac{\varphi}{L} = \frac{T}{G_1J_1 + G_2J_2} = \frac{4\times10^3}{64.28\times10^3 + 48.70\times10^3} = 35.406\times10^{-3} \text{ rad/m}$$

(a) Maximum shearing stress in steel core

$\tau = G_1\gamma = G_1 c_1 \frac{\varphi}{L} = (77\times10^9)(0.027)(35.406\times10^{-3})$

$= 73.6\times10^6$ Pa $\qquad$ 73.6 MPa ◄

(b) Maximum shearing stress in aluminum jacket

$\tau = G_2\gamma = G_2 c_2 \frac{\varphi}{L} = (27\times10^9)(0.036)(35.406\times10^{-3})$

$= 34.4\times10^6$ Pa $\qquad$ 34.4 MPa ◄

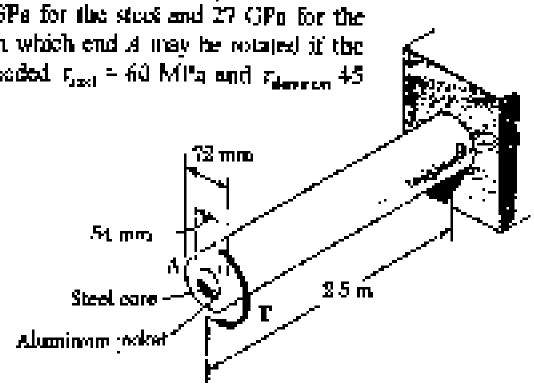
(c) Angle of twist

$\varphi = L\frac{\varphi}{L} = (2.5)(35.406\times10^{-3}) = 88.5\times10^{-3}$ rad

$= 5.07°$ ◄

PROBLEM 3.53

3.53 The composite shaft shown is to be twisted by applying a torque **T** at end *A*. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine the largest angle through which end *A* may be rotated if the following allowable stresses are not to be exceeded: $\tau_{steel} = 60$ MPa and $\tau_{aluminum} = 45$ MPa.



SOLUTION

$$\tau_{max} = G\gamma_{max} = G\,c_{max}\frac{\varphi}{L}$$

$$\frac{\varphi_{all}}{L} = \frac{\tau_{all}}{G\,c_{max}} \quad \text{for each material}$$

Steel core: $\tau_{all} = 60\times10^6$ Pa, $c_{max} = \frac{1}{2}d = 0.027$ m, $G = 77\times10^9$ Pa

$$\frac{\varphi_{all}}{L} = \frac{60\times10^6}{(77\times10^9)(0.027)} = 28.860\times10^{-3} \text{ rad/m}$$

Aluminum jacket: $\tau_{all} = 45\times10^6$ Pa, $c_{max} = \frac{1}{2}d = 0.036$ m, $G = 27\times10^9$ Pa

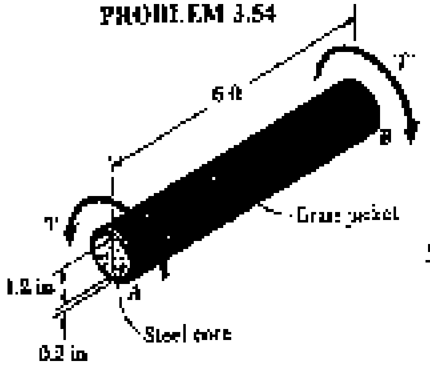
$$\frac{\varphi_{all}}{L} = \frac{45\times10^6}{(27\times10^9)(0.036)} = 46.296\times10^{-3} \text{ rad/m}$$

Smaller value governs $\quad \frac{\varphi_{all}}{L} = 28.860\times10^{-3}$ rad/m

Allowable angle of twist $\quad \varphi_{all} = L\frac{\varphi_{all}}{L} = (2.5)(28.860\times10^{-3})$

$$= 72.15\times10^{-3} \text{ rad} = 4.13°$$

PROBLEM 3.54



3.54 The composite shaft shown consists of a 0.2-in.-thick brass jacket ($G = 5.6 \times 10^6$ psi) bonded to a 1.2-in.-diameter steel core ($G = 11.2 \times 10^6$ psi). Knowing that the shaft is subjected to 5-kip·in. torques, determine (*a*) the maximum shearing stress in the brass jacket, (*b*) the maximum shearing stress in the steel core, (*c*) the angle of twist of end *B* relative to end *A*.

SOLUTION

Steel core: $c_1 = \frac{1}{2}d = 0.6$ in

$J_1 = \frac{\pi}{2}c_1^4 = \frac{\pi}{2}(0.6)^4 = 0.203575\ \text{in}^4$

$G_1J_1 = (11.2\times10^6)(0.203575) = 2.2800\times10^6\ \text{lb}\cdot\text{in}^2$

Torque carried by steel core $\quad T_1 = G_1J_1\frac{\varphi}{L}$

Brass jacket: $c_2 = c_1 + t = 0.6 + 0.2 = 0.8$ in

$J_2 = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.8^4 - 0.6^4) = 0.439823\ \text{in}^4$

$G_2J_2 = (5.6\times10^6)(0.439823) = 2.4630\times10^6\ \text{lb}\cdot\text{in}^2$

Torque carried by brass jacket $\quad T_2 = G_2J_2\frac{\varphi}{L}$

Total torque $\quad T = T_1 + T_2 = (G_1J_1 + G_2J_2)\frac{\varphi}{L}$

$$\frac{\varphi}{L} = \frac{T}{G_1J_1 + G_2J_2} = \frac{5\times10^3}{2.2800\times10^6 + 2.4630\times10^6} = 1.0542\times10^{-3}\ \text{rad/in}$$

(a) Maximum shearing stress in brass jacket

$$\tau_{max} = G_2\gamma_{max} = G_2c_2\frac{\varphi}{L} = (5.6\times10^6)(0.8)(1.0542\times10^{-3})$$

$$= 4.72\times10^3\ \text{psi} \qquad 4.72\ \text{ksi}$$

(b) Maximum shearing stress in steel core

$$\tau_{max} = G_1\gamma_{max} = G_1c_1\frac{\varphi}{L} = (11.2\times10^6)(0.6)(1.0542\times10^{-3})$$

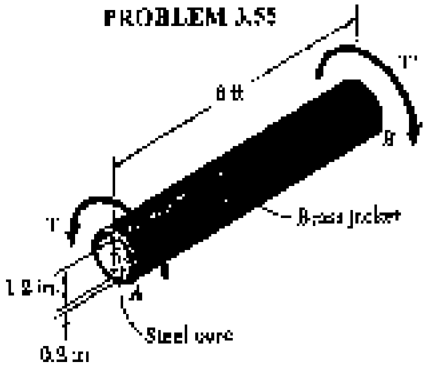
$$= 7.08\times10^3\ \text{psi} \qquad 7.08\ \text{ksi}$$

(c) Angle of twist $\qquad (L = 6\ \text{ft} = 72\ \text{in})$

$$\varphi = L\frac{\varphi}{L} = (72)(1.0542\times10^{-3}) = 75.9\times10^{-3}\ \text{rad}$$

$$= 4.35^\circ$$

PROBLEM 3.55



3.55 The composite shaft shown is to be twisted by applying the torques shown. Knowing that the modulus of rigidity is 11.2×10^6 psi for the steel and 5.6×10^6 psi for the brass, determine the largest angle of twist of end B relative to end A if the following allowable stresses are not to be exceeded $\tau_{steel} = 15$ ksi and $\tau_{brass} = 8$ ksi.

SOLUTION

$$\tau_{max} = G\gamma_{max} = G\,c_{max}\frac{\varphi}{L}$$

$$\frac{\varphi_{all}}{L} = \frac{\tau_{all}}{G\,c_{max}} \quad \text{for each material}$$

Steel core: $\tau_{all} = 15$ ksi $= 15000$ psi, $c_{max} = \frac{1}{2}d = 0.6$ in

$$\frac{\varphi_{all}}{L} = \frac{15000}{(11.2\times10^6)(0.6)} = 2.2321\times10^{-3} \text{ rad/in}$$

Brass jacket: $\tau_{all} = 8$ ksi $= 8000$ psi, $c_{max} = 0.6 + 0.2 = 0.8$ in

$$\frac{\varphi_{all}}{L} = \frac{8000}{(5.6\times10^6)(0.8)} = 1.7857\times10^{-3} \text{ rad/in.}$$

Smaller value governs $\quad \frac{\varphi_{all}}{L} = 1.7857\times10^{-3}$ rad/in

Allowable angle of twist

$L = 6$ ft $= 72$ in

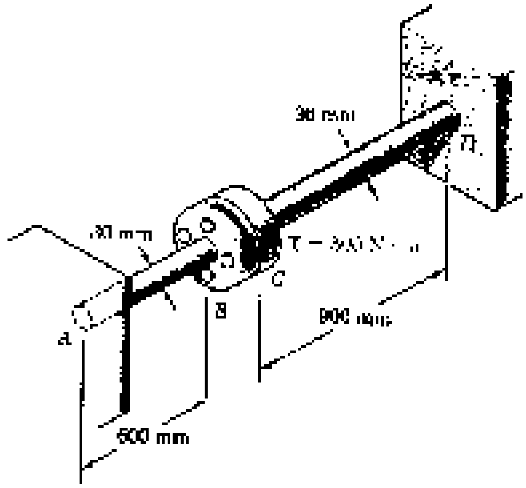
$$\varphi_{all} = L\frac{\varphi_{all}}{L} = (72)(1.7857\times10^{-3}) = 128.57\times10^{-3} \text{ rad}$$

$$= 7.37^\circ$$

PROBLEM 3.56

3.56 Two solid steel shafts are fitted with flanges which are then connected by fitted bolts so that there is no relative rotation between the flanges. Knowing that $G = 77$ GPa, determine the maximum shearing stress in each shaft when a 500 N·m torque is applied to flange B.

SOLUTION



Shaft AB

$T = T_{AB}$, $L_{AB} = 0.6$ m, $c = \frac{1}{2}d = 0.015$ m

$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4 = 79.52 \times 10^{-9}$ m^4

$$\varphi_B = \frac{T_{AB}L_{AB}}{G_{AB}J_{AB}}$$

$$T_{AB} = \frac{G_{AB}J_{AB}}{L_{AB}}\varphi_B = \frac{(77\times10^9)(79.52\times10^{-9})}{0.6}\varphi_B$$

$$= 10.205\times10^3\ \varphi_B$$

Shaft CD

$T = T_{CD}$, $L_{CD} = 0.9$ m, $c = \frac{1}{2}d = 0.018$ m, $J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4$

$J_{CD} = 164.896 \times 10^{-9}$ m^4

$$T_{CD} = \frac{G_{CD}J_{CD}}{L_{CD}}\varphi_C = \frac{(77\times10^9)(164.896\times10^{-9})}{0.9}\varphi_C = 14.108\times10^3\ \varphi_C$$

Matching rotation at the flanges $\quad \varphi_B = \varphi_C = \varphi$

Total torque on flanges $\quad T = T_{AB} + T_{CD} = 500$ N-m

$$500 = (10.205\times10^3 + 14.108\times10^3)\varphi \qquad \therefore \varphi = 20.565\times10^{-3}\ \text{rad}$$

$$T_{AB} = (10.205\times10^3)(20.565\times10^{-3}) = 209.87\ \text{N-m}$$

$$T_{CD} = (14.108\times10^3)(20.565\times10^{-3}) = 290.13\ \text{N-m}$$

Maximum shearing stress in AB

$$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(209.87)(0.015)}{79.52\times10^{-9}} = 39.59\times10^6\ \text{Pa} \qquad 39.6\ \text{MPa}$$

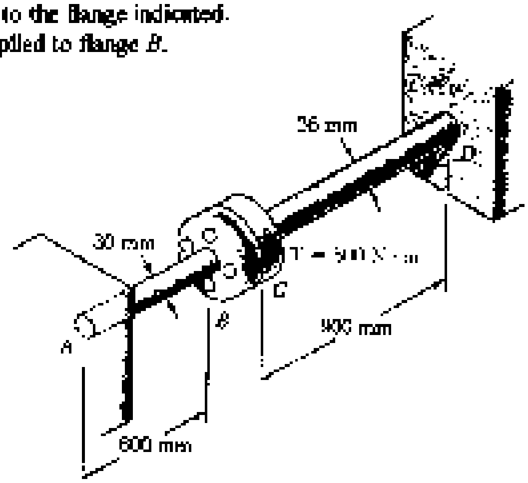
Maximum shearing stress in CD

$$\tau_{CD} = \frac{T_{CD}c}{J_{CD}} = \frac{(290.13)(0.018)}{164.896\times10^{-9}} = 31.67\times10^6\ \text{Pa} \qquad 31.7\ \text{MPa}$$

PROBLEM 3.57

3.57 and 3.58 Two solid steel shafts are fitted with flanges which are then connected by bolts as shown. The bolts are slightly undersized and permit a 1.5° rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that $G = 77$ GPa, determine the maximum shearing stress in each shaft when a 500 N·m torque **T** is applied to the flange indicated.

3.57 The torque **T** is applied to flange B.



SOLUTION

Shaft AB

$T = T_{AB}, \quad L = 0.6 \text{ m}, \quad c = \frac{1}{2}d = 0.015 \text{ m}$

$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4 = 79.52 \times 10^{-9} \text{ m}^4$

$\varphi_B = \frac{T_{AB} L_{AB}}{G_{AB} J_{AB}}$

$T_{AB} = \frac{G_{AB} J_{AB}}{L}\varphi_B = \frac{(77 \times 10^9)(79.52 \times 10^{-9})}{0.6}\varphi_B$

$= 10.205 \times 10^3 \ \varphi_B$

Shaft CD

$T = T_{CD}, \quad L_{CD} = 0.9 \text{ m}, \quad c = \frac{1}{2}d = 0.018 \text{ m}, \quad J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4$

$J_{CD} = 164.896 \times 10^{-9} \text{ m}^4$

$T_{CD} = \frac{G_{CD} J_{CD}}{L_{CD}}\varphi_C = \frac{(77 \times 10^9)(164.896 \times 10^{-9})}{0.9}\varphi_C = 14.108 \times 10^3 \ \varphi_C$

Clearance rotation for flange B $\quad \varphi_B' = 1.5° = 26.18 \times 10^{-3}$ rad

Torque to remove clearance: $T_{AB}' = (10.205 \times 10^{3})(26.18 \times 10^{-3}) = 267.17$ N·m

Torque T'' to cause additional rotation φ'': $T'' = 500 - 267.17 = 232.83$ N·m

$T'' = T_{AB}'' + T_{CD}''$

$232.83 = (10.205 \times 10^3 + 14.108 \times 10^3)\varphi'' \quad \therefore \quad \varphi'' = 9.5765 \times 10^{-3}$ rad

$T_{AB}'' = (10.205 \times 10^3)(9.5765 \times 10^{-3}) = 97.73$ N·m

$T_{CD}'' = (14.108 \times 10^3)(9.5765 \times 10^{-3}) = 135.10$ N·m

Maximum shearing stress in AB

$\tau_{AB} = \frac{T_{AB} c}{J_{AB}} = \frac{(267.17 + 97.73)(0.015)}{79.52 \times 10^{-9}} = 68.8 \times 10^6 \text{ Pa} \qquad 68.8 \text{ MPa}$ ◄

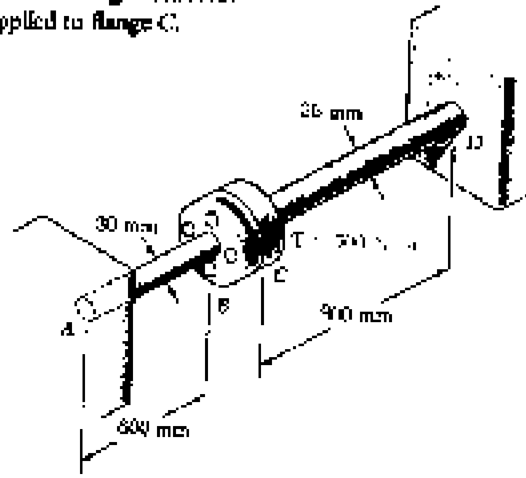
Maximum shearing stress in CD

$\tau_{CD} = \frac{T_{CD} c}{J_{CD}} = \frac{(135.10)(0.018)}{164.896 \times 10^{-9}} = 14.75 \times 10^6 \text{ Pa} \qquad 14.75 \text{ MPa}$ ◄

PROBLEM 3.58

3.57 and 3.58 Two solid steel shafts are fitted with flanges which are then connected by bolts as shown. The bolts are slightly undersized and permit a 1.5° rotation of one flange with respect to the other before the flanges begin to rotate as a single unit. Knowing that $G = 77$ GPa, determine the maximum shearing stress in each shaft when a 500 N·m torque **T** is applied to the flange indicated.

3.58 The torque **T** is applied to flange C.



SOLUTION

Shaft AB

$T = T_{AB}$, $L_{AB} = 0.6$ m, $c = \frac{1}{2}d = 0.015$ m

$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4 = 79.52 \times 10^{-9}$ m^4

$$\varphi_B = \frac{T_{AB}L_{AB}}{G_{AB}J_{AB}}$$

$$T_{AB} = \frac{G_{AB}J_{AB}}{L_{AB}}\varphi_B = \frac{(77\times10^9)(79.52\times10^{-9})}{0.6}\varphi_B$$

$$= 10.205\times10^3\ \varphi_B$$

Shaft CD

$T = T_{CD}$, $L_{CD} = 0.9$ m, $c = \frac{1}{2}d = 0.018$ m, $J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4$

$J_{CD} = 164.896\times10^{-9}$ m^4

$$T_{CD} = \frac{G_{CD}J_{CD}}{L_{CD}}\varphi_C = \frac{(77\times10^9)(164.896\times10^{-9})}{0.9}\varphi_C = 14.108\times10^3\ \varphi_C$$

Clearance rotation for flange C $\quad \varphi_C' = 1.5° = 26.18\times10^{-3}$ rad

Torque to remove clearance: $T_{CD}' = (14.108\times10^3)(26.18\times10^{-3}) = 369.35$ N·m

Torque T'' to cause additional rotation φ'': $\quad T'' = 500 - 369.35 = 130.65$ N·m

$$T'' = T_{AB}'' + T_{CD}''$$

$$130.65 = (10.205\times10^3 + 14.108\times10^3)\varphi'' \qquad \varphi'' = 5.3737\times10^{-3}\ \text{rad}$$

$$T_{AB}'' = (10.205\times10^3)(5.3737\times10^{-3}) = 54.84\ \text{N·m}$$

$$T_{CD}'' = (14.108\times10^3)(5.3737\times10^{-3}) = 75.81\ \text{N·m}$$

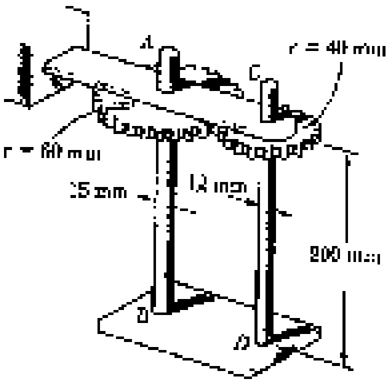
Maximum shearing stress in AB

$$\tau_{AB} = \frac{T_{AB}c}{J_{AB}} = \frac{(54.84)(0.015)}{79.52\times10^{-9}} = 10.34\times10^6\ \text{Pa} \qquad 10.34\ \text{MPa}$$

Maximum shearing stress in CD

$$\tau_{CD} = \frac{T_{CD}c}{J_{CD}} = \frac{(369.35 + 75.81)(0.018)}{164.896\times10^{-9}} = 48.6\times10^6\ \text{Pa} = 48.6\ \text{MPa}$$

PROBLEM 3.59



3.59 At a time when rotation is prevented at the lower end of each shaft, a 50-N·m torque is applied to end A of shaft AB. Knowing that $G = 77$ GPa for both shafts, determine (a) the maximum shearing stress in shaft CD, (b) the angle of rotation at A.

SOLUTION

Let T_A = torque applied at A = 50 N·m

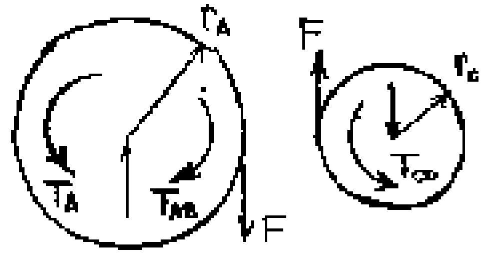
T_{AB} = torque in shaft AB

T_{CD} = torque in shaft CD

Statics

$$T_A - T_{AB} - Fr_A = 0$$

$$T_{CD} - Fr_C = 0$$



Gear A Gear C

$$T_{CD} = \frac{r_C}{r_A}(T_A - T_{AB}) = \frac{2}{3}(T_A - T_{AB})$$

Kinematics: $r_A\varphi_A = r_C\varphi_C$ $\qquad \varphi_A = \frac{r_C}{r_A}\varphi_C = \frac{2}{3}\varphi_C$

Angles of twist $\qquad \varphi_A = \frac{T_{AB}L}{GJ_{AB}} \qquad \varphi_C = \frac{T_{CD}L}{GJ_{CD}} = \frac{2}{3}\frac{(T_A - T_{AB})L}{GJ_{CD}}$

$$\frac{T_{AB}L}{GJ_{AB}} = \frac{2}{3}\cdot\frac{2}{3}\frac{(T_A - T_{AB})L}{GJ_{CD}}$$

$$\left(\frac{4}{9} + \frac{J_{CD}}{J_{AB}}\right)T_{AB} = \left(\frac{4}{9} + \left(\frac{12}{15}\right)^4\right)T_{AB} = \frac{4}{9}T_A$$

$$T_{AB} = 0.5204\,T_A = (0.5204)(50) = 26.02 \text{ N·m}$$

$$T_{CD} = \frac{2}{3}(50 - 26.02) = 15.99 \text{ N·m}$$

(a) Maximum shearing stress in shaft CD

$$\tau_{CD} = \frac{T_{CD}c}{J_{CD}} = \frac{2T_{CD}}{\pi c^3} = \frac{(2)(15.99)}{\pi(0.006)^3} = 47.1\times10^6 \text{ Pa} = 47.1 \text{ MPa}$$

(b) Angle of rotation at A

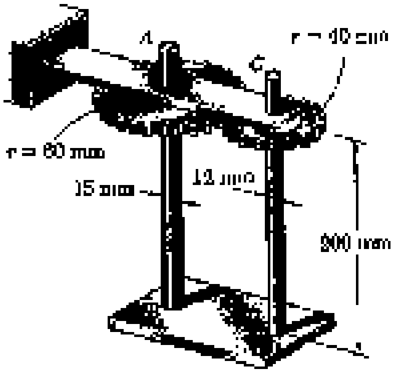
$$\varphi_A = \frac{T_{AB}L}{GJ_{AB}} = \frac{2T_{AB}L}{\pi G c_{AB}^4} = \frac{(2)(26.02)(0.200)}{\pi(77\times10^9)(0.0075)^4} = 13.598\times10^{-3} \text{ rad}$$

$$= 0.779°$$

PROBLEM 3.60

3.59 At a time when rotation is prevented at the lower end of each shaft, a 50-N·m torque is applied to end A of shaft AB. Knowing that $G = 77$ GPa for both shafts, determine (*a*) the maximum shearing stress in shaft CD, (*b*) the angle of rotation at A.

3.60 Solve Prob. 3.59, assuming that the 50-N·m torque is applied to end C of shaft CD.



SOLUTION

Let T_C = torque applied at C = 50 N·m

T_{CD} = torque in shaft CD

T_{AB} = torque in shaft AB

Statics.

$$T_{AB} - r_A F = 0$$

$$T_C - T_{CD} - r_C F = 0$$

$$T_{AB} = \frac{r_A}{r_C}(T_C - T_{CD}) = \frac{3}{2}(T_C - T_{CD})$$

Kinematics: $\quad r_A \varphi_A = r_C \varphi_C \qquad \varphi_C = \frac{r_A}{r_C}\varphi_A = \frac{3}{2}\varphi_A$

Angles of twist $\quad \varphi_C = \frac{T_{CD} L}{G J_{CD}} \qquad \varphi_A = \frac{T_{AB} L}{G J_{AB}} = \frac{3}{2}\frac{(T_C - T_{CD})L}{G J_{AB}}$

$$\frac{T_{CD} L}{G J_{CD}} = \frac{3}{2}\cdot\frac{3}{2}\,\frac{T_C - T_{CD}}{G J_{AB}}$$

$$\left(\frac{J_{AB}}{J_{CD}} + \frac{9}{4}\right)T_{CD} = \left(\left(\frac{15}{12}\right)^4 + \frac{9}{4}\right)T_{CD} = \frac{9}{4}T_C$$

$$T_{CD} = 0.4796\,T_C = (0.4796)(50) = 23.98 \text{ N·m}$$

$$T_{AB} = \frac{3}{2}(50 - 23.98) = 39.03 \text{ N·m}$$

(a) Maximum shearing stress in shaft CD

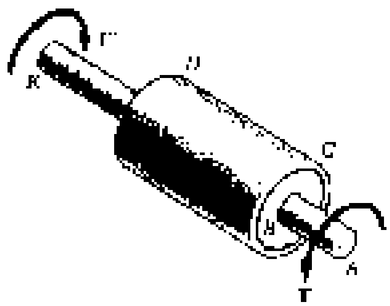
$$\tau_{CD} = \frac{T_{CD}c}{J} = \frac{2T_{CD}}{\pi c^3} = \frac{(2)(23.98)}{\pi(0.006)^3} = 70.7\times10^6 \text{ Pa} = 70.7 \text{ MPa}$$

(b) Angle of rotation at A

$$\varphi_A = \frac{T_{AB}L}{GJ_{AB}} = \frac{2T_{AB}L}{\pi G c_{AB}^4} = \frac{(2)(39.03)(0.200)}{\pi(77\times10^9)(0.0075)^4} = 20.379\times10^{-3} \text{ rad}$$

$$= 1.169°$$

PROBLEM 3.61



3.61 The steel jacket CD has been attached to the 40-mm-diameter steel shaft AE by means of *rigid* flanges welded to the jacket and to the rod. The outer diameter of the jacket is 80 mm and its wall thickness is 4 mm. If 500-N·m torques are applied as shown, determine the maximum shearing stress in the jacket.

SOLUTION

Solid shaft: $c = \frac{1}{2}d = 0.020$ m

$$J_S = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.020)^4 = 251.33 \times 10^{-9} \text{ m}^4$$

Jacket: $c_2 = \frac{1}{2}d = 0.040$ m

$$c_1 = c_2 - t = 0.040 - 0.004 = 0.036 \text{ m}$$

$$J_J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.040^4 - 0.036^4) = 1.3829 \times 10^{-6} \text{ m}^4$$

Torque carried by shaft $\quad T_S = GJ_S\varphi/L$

Torque carried by jacket $\quad T_J = GJ_J\varphi/L$

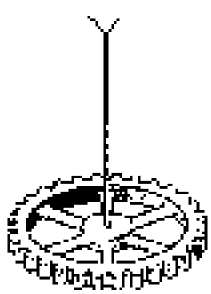
Total torque $\quad T = T_S + T_J = (J_S + J_J)G\varphi/L \quad \therefore \quad \frac{G\varphi}{L} = \frac{T}{J_S + J_J}$

$$T_J = \frac{J_J}{J_S + J_J}T = \frac{(1.3829\times10^{-6})(500)}{1.3829\times10^{-6} + 251.33\times10^{-9}} = 423.1 \text{ N·m}$$

Maximum shearing stress in jacket

$$\tau = \frac{T_J c}{J_J} = \frac{(423.1)(0.040)}{1.3829\times10^{-6}} = 12.24\times10^{6} \text{ Pa} \qquad 12.24 \text{ MPa} \blacktriangleleft$$

PROBLEM 3.62



3.62 The mass moment of inertia of a gear is to be determined experimentally by using a torsional pendulum consisting of a 6-ft steel wire. Knowing that $G = 11.2 \times 10^6$ psi, determine the diameter of the wire for which the torsional spring constant will be 4.27 lb·ft/rad.

SOLUTION

Torsion spring constant $K = 4.27$ lb·ft/rad $= 51.24$ lb·in/rad

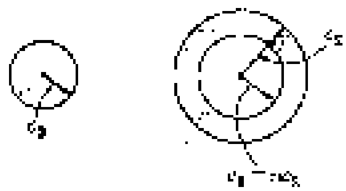
$$K = \frac{T}{\varphi} = \frac{T}{TL/GJ} = \frac{GJ}{L} = \frac{\pi G c^4}{2L}$$

$$c^4 = \frac{2LK}{\pi G} = \frac{(2)(72)(51.24)}{\pi(11.2\times10^6)} = 209.7\times10^{-6} \text{ in}^4$$

$c = 0.1203$ in. $\qquad d = 2c = 0.241$ in. $\blacktriangleleft$

PROBLEM 3.63

3.63 A solid shaft and a hollow shaft are made of the same material and are of the same weight and length. Denoting by n the ratio c_1/c_2, show that the ratio T_s/T_h of the torque T_s in the solid shaft to the torque T_h in the hollow shaft is (a) $\sqrt{1-n^2}/(1+n^2)$ if the maximum shearing stress is the same in each shaft. (b) $(1-n^2)/(1+n^2)$ if the angle of twist is the same for each shaft.



SOLUTION

For equal weight and length, the areas are equal

$$\pi c_0^2 = \pi(c_2^2 - c_1^2) = \pi c_2^2(1-n^2) \quad \therefore \quad c_0 = c_2\sqrt{1-n^2}$$

$$J_s = \frac{\pi}{2}c_0^4 = \frac{\pi}{2}c_2^4(1-n^2)^2 \qquad J_h = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}c_2^4(1-n^4)$$

(a) For equal stresses

$$\tau = \frac{T_s c_0}{J_s} = \frac{T_h c_2}{J_h}$$

$$\frac{T_s}{T_h} = \frac{J_s c_2}{J_h c_0} = \frac{\frac{\pi}{2}c_2^4(1-n^2)^2 c_2}{\frac{\pi}{2}c_2^4(1-n^4)c_2\sqrt{1-n^2}} = \frac{1-n^2}{(1+n^2)\sqrt{1-n^2}} = \frac{\sqrt{1-n^2}}{1+n^2}$$

(b) For equal angles of twist

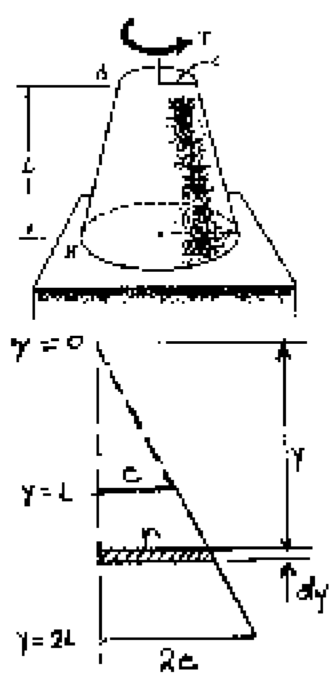
$$\varphi = \frac{T_s L}{G J_s} = \frac{T_h L}{G J_h}$$

$$\frac{T_s}{T_h} = \frac{J_s}{J_h} = \frac{\frac{\pi}{2}c_2^4(1-n^2)^2}{\frac{\pi}{2}c_2^4(1-n^4)} = \frac{(1-n^2)^2}{1-n^4} = \frac{1-n^2}{1+n^2}$$

PROBLEM 3.64

3.64 A torque $\mathbf{T}$ is applied as shown to a solid tapered shaft AB. Show by integration that the angle of twist at A is

$$\phi = \frac{7TL}{12\pi Gc^4}$$



SOLUTION

Introduce coordinate y as shown.

$$r = \frac{cy}{L}$$

Twist in length dy

$$d\varphi = \frac{T\,dy}{GJ} = \frac{T\,dy}{G\frac{\pi}{2}r^4} = \frac{2TL^4\,dy}{\pi G c^4 y^4}$$

$$\varphi = \int_L^{2L}\frac{2TL^4}{\pi Gc^4}\frac{dy}{y^4} = \frac{2TL^4}{\pi Gc^4}\int_L^{2L}\frac{dy}{y^4}$$

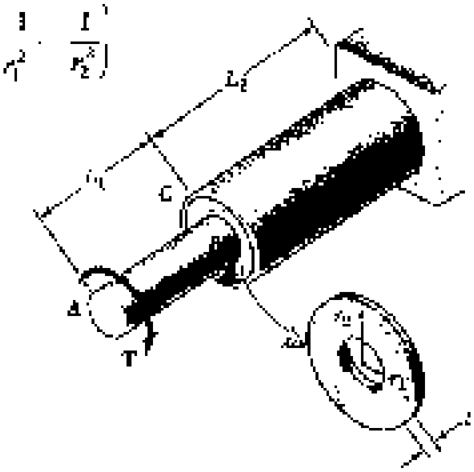
$$= \frac{2TL^4}{\pi Gc^4}\left\{-\frac{1}{3y^3}\right\}_L^{2L} = \frac{2TL^4}{\pi Gc^4}\left\{-\frac{1}{24L^3} + \frac{1}{3L^3}\right\}$$

$$= \frac{2TL^4}{\pi Gc^4}\left\{\frac{7}{24L^3}\right\} = \frac{7TL}{12\pi Gc^4}$$

PROBLEM 3.65

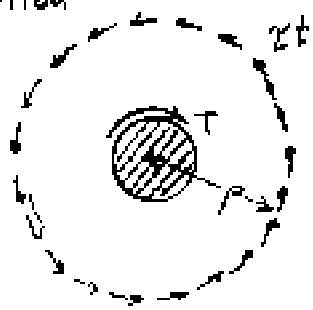
3.65 An annular plate of thickness t and modulus of rigidity G is used to connect shaft AB of radius r_1 to tube CD of inner radius r_2. Knowing that a torque **T** is applied to end A of shaft AB and that end D of tube CD is fixed, (a) determine the magnitude and location of the maximum shearing stress in the annular plate, (b) show that the angle through which end B of the shaft rotates with respect to end C of the tube is

$$\phi_{B/C} = \frac{T}{4\pi G t}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right)$$



SOLUTION

Use a free body consisting of shaft AB and an inner portion of the plate BC, the outer radius of this portion being ρ



The force per unit length of circumference is τt.

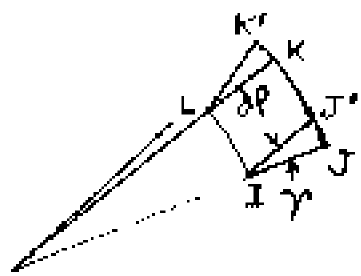
$\Sigma M = 0$

$$\tau t(2\pi\rho)\rho - T = 0$$

$$\tau = \frac{T}{2\pi t\rho^2}$$

(a) Maximum shearing stress occurs at $\rho = r_1$ $\qquad \tau_{max} = \frac{T}{2\pi t r_1^2}$ $\qquad$ (1)

Shearing strain $\qquad \gamma = \frac{\tau}{G} = \frac{T}{2\pi G t\rho^2}$



The relative circumferential displacement in radial length $d\rho$ is

$$dS = \gamma\, d\rho = \rho\, d\varphi$$

$$d\varphi = \gamma \frac{d\rho}{\rho}$$

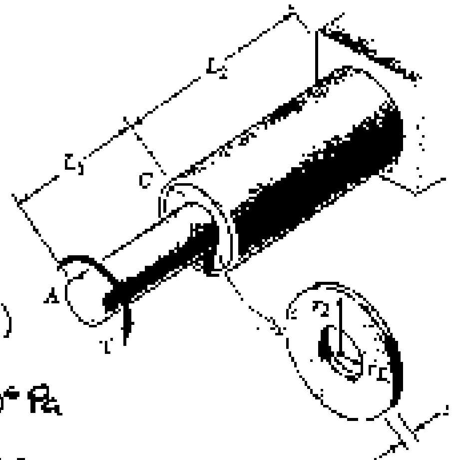
$$d\varphi = \frac{T}{2\pi G t\rho^2}\frac{d\rho}{\rho} = \frac{T\, d\rho}{2\pi G t\rho^3}$$

$$\varphi_{B/C} = \int_{r_1}^{r_2} \frac{T}{2\pi G t}\frac{d\rho}{\rho^3} = \frac{T}{2\pi G t}\int_{r_1}^{r_2}\frac{d\rho}{\rho^3} = \frac{T}{2\pi G t}\left\{-\frac{1}{2\rho^2}\right\}\Bigg|_{r_1}^{r_2}$$

$$= \frac{T}{2\pi G t}\left\{-\frac{1}{2r_2^2} + \frac{1}{2r_1^2}\right\} = \frac{T}{4\pi G t}\left\{\frac{1}{r_1^2} - \frac{1}{r_2^2}\right\}$$

PROBLEM 3.66

3.66 An annular aluminum plate ($G = 27$ GPa), of thickness $t = 6$ mm, is used to connect the aluminum shaft AB, of length $L_1 = 90$ mm and radius $r_1 = 30$ mm, to the aluminum tube CD, of length $L_2 = 150$ mm, inner radius $r_2 = 75$ mm and 4 mm thickness. Knowing that a 2500-N·m torque **T** is applied to end A of shaft AB and that end D of tube CD is fixed, determine (*a*) the maximum shearing stress in the shaft-plate-tube system, (*b*) the angle through which end A rotates. (*Hint:* Use the formula derived in Prob. 3.65 to solve part *b*.)



SOLUTION

Shaft AB

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2T}{\pi r_1^3}$$

$$= \frac{(2)(2500)}{\pi(0.030)^3} = 58.9\times10^6 \text{ Pa}$$

$$58.9 \text{ MPa}$$

Plate BC (See PROBLEM 3.65 for derivation)

$$\tau = \frac{T}{2\pi t r_1^2} = \frac{2500}{2\pi(0.006)(0.030)^2} = 73.7\times10^6 \text{ Pa}$$

$$73.7 \text{ MPa}$$

Shaft CD $\quad c_1 = r_2 = 0.075$ m, $\quad c_2 = r_2 + t = 0.075 + 0.004 = 0.079$ m

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.079^4 - 0.075^4) = 11.482\times10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc_2}{J} = \frac{(2500)(0.079)}{11.482\times10^{-6}} = 17.20\times10^6 \text{ Pa} = 17.20 \text{ MPa}$$

(a) Largest stress $\qquad \tau = 73.7$ MPa ◄

Shaft AB $\qquad \varphi_{AB} = \frac{T L_{AB}}{GJ} = \frac{2TL_{AB}}{\pi G c^4} = \frac{(2)(2500)(0.090)}{\pi(27\times10^9)(0.030)^4}$

$$= 6.550\times10^{-3} \text{ rad}$$

Plate BC (See PROBLEM 3.65 for derivation)

$$\varphi_{BC} = \frac{T}{4\pi G t}\left\{\frac{1}{r_1^2} - \frac{1}{r_2^2}\right\} = \frac{2500}{4\pi(27\times10^9)(0.006)}\left\{\frac{1}{0.030^2} - \frac{1}{0.075^2}\right\}$$

$$= 1.146\times10^{-3} \text{ rad}$$

Shaft CD $\qquad \varphi_{CD} = \frac{TL_{CD}}{GJ} = \frac{(2500)(0.150)}{(27\times10^9)(11.482\times10^{-6})} = 1.210\times10^{-3}$ rad

Total rotation angle $\qquad \varphi = \varphi_{AB} + \varphi_{BC} + \varphi_{CD} = 8.91\times10^{-3}$ rad

$$= 0.510° \quad ◄$$

PROBLEM 3.67

3.67 Using an allowable stress of 55 MPa, design a solid steel shaft to transmit 10 kW at a frequency of 15 Hz.

SOLUTION

$\tau_{all} = 55 \times 10^6$ Pa $\quad P = 10 \times 10^3$ W, $\quad f = 15$ Hz

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{2\pi(15)} = 106.10 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(106.10)}{\pi(55 \times 10^6)} = 1.2281 \times 10^{-6} \text{ m}^3$$

$c = 10.71 \times 10^{-3}$ m $= 10.71$ mm $\qquad d = 2c = 21.4$ mm ◄

PROBLEM 3.68

3.68 Using an allowable stress of 5 ksi, design a solid steel shaft to transmit $\frac{1}{2}$ hp at a speed of 1725 rpm.

SOLUTION

$\tau_{all} = 5$ ksi $= 5000$ psi $\qquad P = \frac{1}{2}$ hp $= \frac{1}{2}(6600) = 3300$ lb·in/s

$$f = \frac{1725}{60} = 28.75 \text{ Hz} \qquad T = \frac{P}{2\pi f} = \frac{3300}{2\pi(28.75)} = 18.268 \text{ lb}\cdot\text{in}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(18.268)}{\pi(5000)} = 2.3260 \times 10^{-3} \text{ in}^3$$

$c = 0.1325$ in $\qquad d = 2c = 0.265$ in ◄

PROBLEM 3.69

3.69 Design a solid steel shaft to transmit 100 hp at a speed of 1200 rpm, if the maximum shearing stress is not to exceed 7500 psi.

SOLUTION

$\tau_{all} = 7500$ psi $\qquad P = 100$ hp $= 660 \times 10^3$ lb·in/s

$$f = \frac{1200}{60} = 20 \text{ Hz} \qquad T = \frac{P}{2\pi f} = \frac{660 \times 10^3}{2\pi(20)} = 5.2521 \times 10^3 \text{ lb}\cdot\text{in}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(5.2521 \times 10^3)}{\pi(7500)} = 0.4458 \text{ in}^3$$

$c = 0.7639$ in $\qquad d = 2c = 1.528$ in. ◄

PROBLEM 3.70

3.70 Design a solid steel shaft to transmit 0.375 kW at a frequency of 29 Hz, if the shearing stress in the shaft is not to exceed 35 MPa.

SOLUTION

$\tau_{all} = 35 \times 10^6$ Pa $\qquad P = 0.375 \times 10^3$ W $\qquad f = 29$ Hz

$$T = \frac{P}{2\pi f} = \frac{0.375 \times 10^3}{2\pi(29)} = 2.0580 \text{ N}\cdot\text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(2.0580)}{\pi(35 \times 10^6)} = 37.43 \times 10^{-9} \text{ m}^3$$

$c = 3.345 \times 10^{-3}$ m $= 3.345$ mm $\qquad d = 2c = 6.69$ mm ◄

PROBLEM 3.71

3.71 A hollow shaft is to transmit 250 kW at a frequency of 30 Hz. Knowing that the shearing stress must not exceed 50 MPa, design a shaft for which the ratio of the inner diameter to the outer diameter is 0.75.

SOLUTION

$\tau_{all} = 50 \times 10^6$ Pa $\qquad P = 250 \times 10^3$ W $\qquad f = 30$ Hz

$$T = \frac{P}{2\pi f} = \frac{250 \times 10^3}{2\pi(30)} = 1326.3 \text{ N·m}$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}\left(1 - \left(\tfrac{3}{4}\right)^4\right)c_2^4 = 1.0738\, c_2^4$$

$$\tau = \frac{Tc_2}{J} = \frac{Tc_2}{1.0738\, c_2^4} \quad \therefore \quad c_2^3 = \frac{T}{1.0738\,\tau} = \frac{1326.3}{(1.0738)(50 \times 10^6)}$$

$$c_2^3 = 24.70 \times 10^{-6} \text{ m}^3$$

$$c_2 = 29.12 \times 10^{-3} \text{ m} = 29.12 \text{ mm} \qquad d_2 = 2c_2 = 58.2 \text{ mm}$$

PROBLEM 3.72

3.72 One of two hollow drive shafts of an ocean liner is 125 ft long, and its outer and inner diameters are 16 in. and 8 in., respectively. The shaft is made of a steel for which $\tau_{all} = 8500$ psi and $G = 11.2 \times 10^6$ psi. Knowing that the maximum speed of rotation of the shaft is 165 rpm, determine (*a*) the maximum power that can be transmitted by the one shaft to its propeller, (*b*) the corresponding angle of twist of the shaft.

SOLUTION

$c_2 = \frac{1}{2}d_2 = 8$ in

$c_1 = \frac{1}{2}d_1 = 4$ in $\qquad J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(8^4 - 4^4) = 6031.8 \text{ in}^4$

$$\tau = \frac{Tc_2}{J} \quad \therefore \quad T = \frac{J\tau}{c_2} = \frac{(6031.8)(8500)}{8} = 6.4088 \times 10^6 \text{ lb·in}$$

$$f = \frac{165}{60} = 2.75 \text{ Hz}$$

(a) $$P = 2\pi f T = 2\pi(2.75)(6.4088 \times 10^6) = 110.74 \times 10^6 \text{ lb·in/s}$$

$$\frac{110.74 \times 10^6 \text{ lb·in/s}}{6600 \text{ lb·in/s/hp}} = 16.78 \times 10^3 \text{ hp}$$

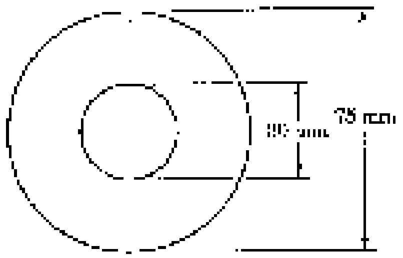
$L = 125$ ft $= 1500$ in

(b) $$\varphi = \frac{TL}{GJ} = \frac{(6.4088 \times 10^6)(1500)}{(11.2 \times 10^6)(6031.8)} = 0.1423 \text{ rad}$$

$$= 8.15°$$

PROBLEM 3.73

3.73 While a steel shaft of the cross section shown rotates at 120 rpm, a stroboscopic measurement indicates that the angle of twist is 2° in a 4-m length. Using $G = 77$ GPa, determine the power being transmitted.



SOLUTION

Twist angle $\varphi = 2° = 34.907 \times 10^{-3}$ rad

$c_1 = \frac{1}{2} d_1 = 0.015$ m, $\quad c_2 = \frac{1}{2} d_2 = 0.0375$ m

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.0375^4 - 0.015^4)$

$J = 3.0268 \times 10^{-6}$ m^4, $\qquad L = 4$ m

$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L} = \frac{(77\times10^9)(3.0268\times10^{-6})(34.907\times10^{-3})}{4}$$

$T = 2.0339 \times 10^3$ N·m $\qquad f = 120$ rpm $= \frac{120}{60}$ Hz $= 2$ Hz

$P = (2\pi f)T = 2\pi(2)(2.0339\times10^3) = 25.6 \times 10^3$ W $= 25.6$ kW ◄

PROBLEM 3.74

3.74 Determine the required thickness of the 50-mm tubular shaft of Example 3.07, if it is to transmit the same power while rotating at a frequency of 40 Hz.

SOLUTION

From Example 3.07 $\qquad P = 100$ kW $= 100 \times 10^3$ W

$\tau_{all} = 60$ MPa $= 60 \times 10^6$ Pa $\qquad c_2 = \frac{1}{2} d = 0.025$ m

Given $f = 40$ Hz

$$T = \frac{P}{2\pi f} = \frac{100\times10^3}{2\pi(40)} = 397.89 \text{ N·m}$$

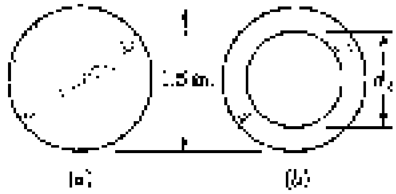
$$J = \frac{\pi}{2}(c_2^4 - c_1^4) \qquad \tau = \frac{Tc_2}{J} = \frac{Tc_2}{\frac{\pi}{2}(c_2^4 - c_1^4)} \quad \therefore \quad c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau} = 0.025^4 - \frac{(2)(397.89)(0.025)}{\pi(60\times10^6)} = 285.081 \times 10^{-9} \text{ m}^4$$

$c_1 = 23.11 \times 10^{-3}$ m $\qquad t = c_2 - c_1 = 1.89 \times 10^{-3}$ m $= 1.89$ mm ◄

PROBLEM 3.75

3.75 The design of a machine element calls for a 1.5-in.-outer-diameter shaft to transmit 60 hp. (*a*) If the speed of rotation is 720 rpm, determine the maximum shearing stress in shaft *a*. (*b*) If the speed of rotation can be increased 50% to 1080 rpm, determine the largest inner diameter of shaft *b* for which the maximum shearing stress will be the same in each shaft.



SOLUTION

$$P = 60 \text{ hp} = (60)(6600) = 396 \times 10^3 \text{ lb}\cdot\text{in/s}$$

$$f = \frac{720}{60} = 12 \text{ Hz} \qquad c = \tfrac{1}{2}d = 0.75 \text{ in}$$

$$T = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi(12)} = 5.2521 \times 10^3 \text{ lb}\cdot\text{in}$$

$$\text{(a)}\ \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(5.2521 \times 10^3)}{\pi(0.75)^3} = 7.9256 \times 10^3 \text{ psi}$$

$$7.93 \text{ ksi}$$

$$\text{(b)}\quad f = \frac{1080}{60} = 18 \text{ Hz} \qquad c_2 = \tfrac{1}{2}d_2 = 0.75 \text{ in}$$

$$T = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi(18)} = 3.5014 \times 10^3 \text{ lb}\cdot\text{in}$$

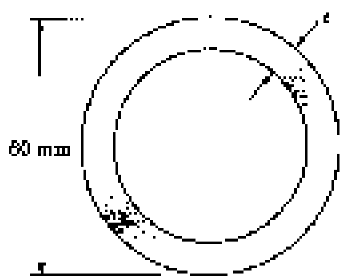
$$\tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau} = 0.75^4 - \frac{(2)(3.5014 \times 10^3)(0.75)}{\pi(7.9256 \times 10^3)} = 0.10547 \text{ in}^4$$

$$c_1 = 0.5699 \text{ in} \qquad d_1 = 2c_1 = 1.140 \text{ in}$$

PROBLEM 3.76

3.76 A steel pipe of 60-mm outer diameter is to be used to transmit a torque of 350 N·m without exceeding an allowable shearing stress of 12 MPa. A series of 60-mm-outer-diameter pipes is available for use. Knowing that the wall thickness of the available pipes varies from 4 mm to 10 mm in 2-mm increments, choose the lightest pipe that can be used.



SOLUTION

$$\tau_{all} = 60 \times 10^6 \text{ Pa} \qquad c_2 = \tfrac{1}{2}d_2 = 0.030 \text{ m}$$

$$\tau = \frac{Tc_2}{J} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$$

$$c_1^4 = c_2^4 - \frac{2Tc_2}{\pi\tau} = 0.030^4 - \frac{(2)(350)(0.030)}{\pi(60 \times 10^6)} = 252.96 \times 10^{-9} \text{ m}^4$$

$$c_1 = 22.43 \times 10^{-3} \qquad t = c_2 - c_1 = 30 \text{ mm} - 22.43 \text{ mm} = 7.57 \text{ mm}$$

Required thickness $t > 7.57$ mm. Available size $t = 8$ mm

PROBLEM 3.77

3.77 A steel drive shaft is 6 ft long and its outer and inner diameters are respectively equal to 2.25 in. and 1.75 in. (*a*) Knowing that the shaft transmits 240 hp while rotating at 1800 rpm, determine the maximum shearing stress. (*b*) Using $G = 11.2 \times 10^6$ psi, determine the corresponding angle of twist of the shaft.

SOLUTION

$c_1 = \frac{1}{2} d_1 = 0.875$ in, $\quad c_2 = \frac{1}{2} d_2 = 1.125$ in, $\quad L = 6 \text{ ft} = 72$ in

$P = 240 \text{ hp} = (240)(6600) = 1.584 \times 10^6$ lb·in/s

$f = \frac{1800}{60} = 30$ Hz

$T = \frac{P}{2\pi f} = \frac{1.584 \times 10^6}{2\pi(30)} = 8.4034 \times 10^3$ lb·in

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(1.125^4 - 0.875^4) = 1.59530$ in^4

(a) $\quad \tau = \frac{Tc_2}{J} = \frac{(8.4034 \times 10^3)(1.125)}{1.59530} = 5926$ psi ◄

(b) $\quad \varphi = \frac{TL}{GJ} = \frac{(8.4034 \times 10^3)(72)}{(11.2 \times 10^6)(1.59530)} = 33.86 \times 10^{-3}$ rad $= 1.940°$ ◄

PROBLEM 3.78

3.78 Knowing that the allowable shearing stress of the steel to be used is 7500 psi, determine (*a*) the smallest permissible diameter of a shaft which must transmit 15 hp while rotating at 2000 rpm, (*b*) the corresponding angle of twist in a 4-ft length of the shaft ($G = 11.2 \times 10^6$ psi).

SOLUTION

$\tau_{all} = 7500$ psi, $\quad f = \frac{2000}{60} = 33.333$ Hz

$P = 15 \text{ hp} = (15)(6600) = 99 \times 10^3$ lb·in/s

$T = \frac{P}{2\pi f} = \frac{99 \times 10^3}{2\pi(33.333)} = 472.69$ lb·in

(a) $\quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(472.69)}{\pi(7500)} = 40.123 \times 10^{-3}$ in^3

$c = 0.3425$ in $\qquad d = 2c = 0.685$ in. ◄

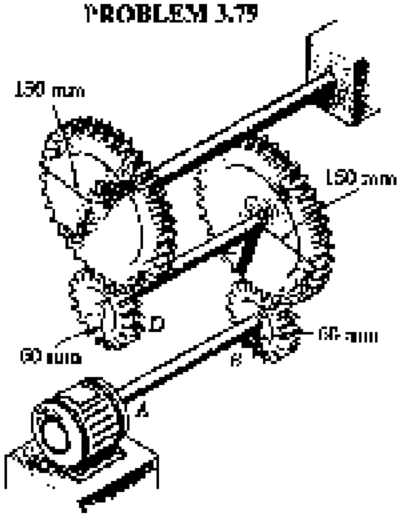
(b) $\quad J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.3425)^4 = 21.516 \times 10^{-3}$ in^4

$L = 4 \text{ ft} = 48$ in

$\varphi = \frac{TL}{GJ} = \frac{(472.69)(48)}{(11.2 \times 10^6)(21.516 \times 10^{-3})} = 93.89 \times 10^{-3}$ rad

$= 5.38°$ ◄

PROBLEM 3.79



3.79 Three shafts and four gears are used to form a gear train which will transmit 7.5 kW from the motor at A to a machine tool at F. (Bearing for the shafts are omitted in the sketch.) Knowing that the frequency of the motor is 30 Hz and that the allowable stress for each shaft is 60 MPa, determine the required diameter of each shaft.

SOLUTION

$$P = 7.5 \text{ kW} = 7.5\times10^3 \text{ W}$$
$$\tau_{all} = 60 \text{ MPa} = 60\times10^6 \text{ Pa}$$

Shaft AB: $f_{AB} = 30$ Hz

$$T_{AB} = \frac{P}{2\pi f_{AB}} = \frac{7.5\times10^3}{2\pi(30)} = 39.789 \text{ N}\cdot\text{m}$$

$$\tau = \frac{T c_{AB}}{J_{AB}} = \frac{2T}{\pi c_{AB}^3} \quad \therefore \quad c_{AB}^3 = \frac{2T}{\pi\tau}$$

$$c_{AB}^3 = \frac{(2)(39.789)}{\pi(60\times10^6)} = 422.17\times10^{-9} \text{ m}^3$$

$$c_{AB} = 7.50\times10^{-3} \text{ m} = 7.50 \text{ mm}$$

$$d_{AB} = 2c_{AB} = 15.00 \text{ mm}$$

Shaft CD:

$$f_{CD} = \frac{r_B}{r_C} f_{AB} = \frac{60}{150}(30) = 12 \text{ Hz}$$

$$T_{CD} = \frac{P}{2\pi f_{CD}} = \frac{7.5\times10^3}{2\pi(12)} = 99.472 \text{ N}\cdot\text{m}$$

$$\tau_{all} = \frac{T c_{CD}}{J_{CD}} = \frac{2T}{\pi c_{CD}^3} \quad \therefore \quad c_{CD}^3 = \frac{2T_{CD}}{\pi\tau} = \frac{2(99.472)}{\pi(60\times10^6)} = 1.05543\times10^{-6} \text{ m}^3$$

$$c_{CD} = 10.18\times10^{-3} \text{ m} = 10.18 \text{ mm} \qquad d_{CD} = 2c_{CD} = 20.4 \text{ mm}$$

Shaft EF:

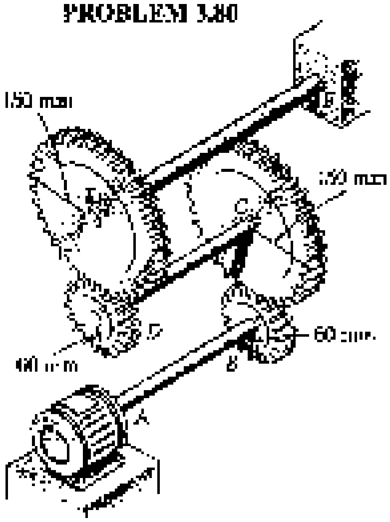
$$f_{EF} = \frac{r_D}{r_E} f_{CD} = \frac{60}{150}(12) = 4.8 \text{ Hz}$$

$$T_{EF} = \frac{P}{2\pi f_{EF}} = \frac{7.5\times10^3}{2\pi(4.8)} = 248.68 \text{ N}\cdot\text{m}$$

$$\tau = \frac{T c_{EF}}{J_{EF}} = \frac{2T}{\pi c_{EF}^3} \quad \therefore \quad c_{EF}^3 = \frac{(2)(248.68)}{\pi(60\times10^6)} = 2.6386\times10^{-6} \text{ m}^3$$

$$c_{EF} = 13.82\times10^{-3} = 13.82 \text{ mm} \qquad d_{EF} = 2c_{EF} = 27.6 \text{ mm}$$

PROBLEM 3.80



3.80 Three shafts and four gears are used to form a gear train which will transmit power from the motor at A to a machine tool at F. (Bearing for the shafts are omitted in the sketch.) The diameter of each shaft is as follows: $d_{AB} = 16$ mm, $d_{CD} = 20$ mm, $d_{EF} = 28$ mm. Knowing that the frequency of the motor is 24 Hz and that the allowable shearing stress for each shaft is 75 MPa, determine the maximum power that can be transmitted.

SOLUTION

$$\tau_{all} = 75 \text{ MPa} = 75 \times 10^6 \text{ Pa}$$

Shaft AB: $c_{AB} = \frac{1}{2} d_{AB} = 0.008$ m

$$\tau = \frac{T c_{AB}}{J_{AB}} = \frac{2T}{\pi c_{AB}^3}$$

$$T_{all} = \frac{\pi}{2} c_{AB}^3 \tau_{all} = \frac{\pi}{2}(0.008)^3(75\times10^6) = 60.319 \text{ N}\cdot\text{m}$$

$f_{AB} = 24$ Hz $\quad P_{all} = 2\pi f_{AB} T_{all} = 2\pi(24)(60.319)$

$$= 9.10 \times 10^3 \text{ W}$$

Shaft CD: $c_{CD} = \frac{1}{2} d_{CD} = 0.010$ m

$$\tau = \frac{T c_{CD}}{J_{CD}} = \frac{2T}{\pi c_{CD}^3} \quad \therefore \quad T_{all} = \frac{\pi}{2} c_{CD}^3 \tau_{all} = \frac{\pi}{2}(0.010)^3(75\times10^6) = 117.81 \text{ N}\cdot\text{m}$$

$$f_{CD} = \frac{r_B}{r_C} f_{AB} = \frac{60}{150}(24) = 9.6 \text{ Hz}$$

$$P_{all} = 2\pi f_{CD} T_{all} = 2\pi(9.6)(117.81) = 7.11 \times 10^3 \text{ W}$$

Shaft EF: $c_{EF} = \frac{1}{2} d_{EF} = 0.014$ m

$$T_{all} = \frac{\pi}{2} c_{EF}^3 \tau_{all} = \frac{\pi}{2}(0.014)^3(75\times10^6) = 323.27 \text{ N}\cdot\text{m}$$

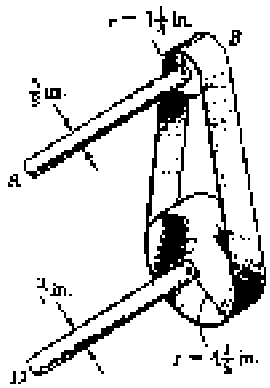
$$f_{EF} = \frac{r_D}{r_E} f_{CD} = \frac{60}{150}(9.6) = 3.84 \text{ Hz}$$

$$P_{all} = 2\pi f_{EF} T_{all} = 2\pi(3.84)(323.27) = 7.80 \times 10^3 \text{ W}$$

Maximum allowable power is the smaller

$$P_{all} = 7.11\times10^3 \text{ W} = 7.11 \text{ kW}$$

PROBLEM 3.81



3.81 The shaft-disk-belt arrangement shown is used to transmit 3 hp from point A to point D. (a) Using an allowable shearing stress of 9500 psi, determine the required speed of shaft AB. (b) Solve part a, assuming that the diameters of shafts AB and CD are respectively 0.75 in. and 0.625 in.

SOLUTION

$\tau = 9500$ psi, $P = 3$ hp $= (3)(6600) = 19800$ lb·in/s

$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$ $\qquad T = \frac{\pi}{2} c^3 \tau$

Allowable torques

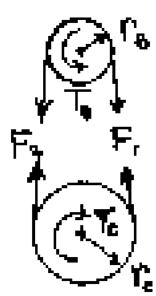
$\frac{5}{8}$ in. diameter shaft

$c = \frac{5}{16}$ in, $T_{all} = \frac{\pi}{2}\left(\frac{5}{16}\right)^3(9500) = 455.4$ lb·in

$\frac{3}{4}$ in diameter shaft

$c = \frac{3}{8}$ in, $T_{all} = \frac{\pi}{2}\left(\frac{3}{8}\right)^3(9500) = 786.9$ lb·in

Statics:



$T_B = r_B(F_1 - F_2) \qquad T_C = r_C(F_1 - F_2)$

$T_B = \frac{r_B}{r_C} T_C = \frac{1.125}{4.5} T_C = 0.25\,T_C$

(a) Allowable torques $\qquad T_{B,all} = 455.4$ lb·in, $T_{C,all} = 786.9$ lb·in

Assume $T_C = 786.9$ lb·in. Then $T_B = (0.25)(786.9) = 196.73$ lb·in < 455.4 lb·in (okay)

$P = 2\pi f T \qquad f_{AB} = \frac{P}{2\pi T_B} = \frac{19800}{2\pi(196.73)} = 16.02$ Hz ◄

(b) Allowable torques $\qquad T_{B,all} = 786.9$ lb·in, $T_{C,all} = 455.4$ lb·in.

Assume $T_C = 455.4$ lb·in Then $T_B = (0.25)(455.4) = 113.85$ lb·in < 786.9 lb·in

$P = 2\pi f T \qquad f_{AB} = \frac{P}{2\pi T_B} = \frac{19800}{2\pi(113.85)} = 27.7$ Hz ◄

PROBLEM 3.82

3.82 A steel shaft must transmit 150 kW at a speed of 360 rpm. Knowing that $G = 77$ GPa, design a solid shaft so that the maximum stress will not exceed 50 MPa and the angle of twist in a 2.5-m length will not exceed 3°.

SOLUTION

$P = 150 \times 10^3$ W $\qquad f = \frac{360}{60} = 6$ Hz

$$T = \frac{P}{2\pi f} = \frac{150 \times 10^3}{2\pi(6)} = 3.9789 \times 10^3 \text{ N}\cdot\text{m}$$

Design for stress limit $\quad \tau = 50$ MPa $= 50 \times 10^6$ Pa

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad c^3 = \frac{2T}{\pi \tau} = \frac{(2)(3.9789 \times 10^3)}{\pi(50 \times 10^6)} = 50.661 \times 10^{-6} \text{ m}^3$$

$$c = 37.00 \times 10^{-3} \text{ m}$$

Design for angle of twist limit $\quad \varphi = 3° = 52.36 \times 10^{-3}$ rad

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4} \quad \therefore \quad c^4 = \frac{2TL}{\pi G \varphi} = \frac{(2)(3.9789 \times 10^3)(2.5)}{\pi(77 \times 10^9)(52.36 \times 10^{-3})} = 1.5707 \times 10^{-6} \text{ m}^4$$

$$c = 35.40 \times 10^{-3} \text{ m}$$

Use larger value $\qquad c = 37.00 \times 10^{-3}$ m $= 37.0$ mm ; $d = 2c = 74.0$ mm ◄

PROBLEM 3.83

3.83 A steel shaft of 1.5-m length and 48-mm diameter is to be used to transmit 36 kW between a motor and a machine tool. Knowing that $G = 77$ GPa, determine the lowest speed of rotation of the shaft at which the maximum stress will not exceed 60 MPa and the angle of twist will not exceed 2.5°.

SOLUTION

$P = 36 \times 10^3$ W, $\quad c = \frac{1}{2}d = 0.024$ m, $\quad L = 1.5$ m, $\quad G = 77 \times 10^9$ Pa

Torque based on maximum stress $\quad \tau = 60$ MPa $= 60 \times 10^6$ Pa

$$\tau = \frac{Tc}{J} \quad T = \frac{J\tau}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.024)^3(60 \times 10^6) = 1.30288 \times 10^3 \text{ N}\cdot\text{m}$$

Torque based on twist angle $\quad \varphi = 2.5° = 43.633 \times 10^{-3}$ rad

$$\varphi = \frac{TL}{GJ} \quad \therefore \quad T = \frac{GJ\varphi}{L} = \frac{\pi c^4 G\varphi}{2L} = \frac{\pi(0.024)^4(77 \times 10^9)(43.633 \times 10^{-3})}{(2)(1.5)}$$

$$= 1.16730 \times 10^3 \text{ N}\cdot\text{m}$$

Smaller torque governs $\qquad T = 1.16730 \times 10^3$ N·m

$$P = 2\pi f T \quad \therefore \quad f = \frac{P}{2\pi T} = \frac{36 \times 10^3}{2\pi(1.16730 \times 10^3)} = 4.91 \text{ Hz}$$ ◄

PROBLEM 3.84

3.84 A 1.5-in.-diameter steel shaft of length 4 ft will be used to transmit 60 hp between a motor and a pump. Knowing that $G = 11.2 \times 10^6$ psi, determine the lowest speed of rotation at which the shearing stress will not exceed 8500 psi and the angle of twist will not exceed 2°.

SOLUTION

$c = \frac{1}{2}d = 0.75$ in , $L = 4$ ft $= 48$ in.

Torque based on maximum shearing stress limit $\tau = 8500$ psi

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} \quad \therefore \quad T = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.75)^3(8500) = 5.633 \times 10^3 \text{ lb}\cdot\text{in}$$

Torque based on twist angle limit $\varphi = 2° = 34.907 \times 10^{-3}$ rad

$$\varphi = \frac{TL}{GJ} \quad \therefore \quad T = \frac{GJ\varphi}{L} = \frac{\pi c^4 G\varphi}{2L} = \frac{\pi(0.75)^4(11.2\times10^6)(34.907\times10^{-3})}{(2)(48)}$$

$$= 4.048 \times 10^3 \text{ lb}\cdot\text{in}$$

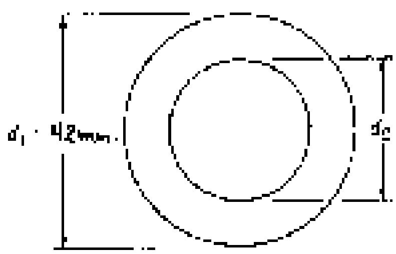
Smaller torque governs $\quad T = 4.048 \times 10^3$ lb·in

$P = 2\pi f T \quad$ where $\quad P = 60$ hp $= (60)(6600) = 396 \times 10^3$ lb·in/s

$$f = \frac{P}{2\pi T} = \frac{396\times10^3}{2\pi(4.048\times10^3)} = 15.57 \text{ Hz} = 934 \text{ rpm}$$

PROBLEM 3.85

3.85 A 1.6-m-long tubular steel shaft of 42-mm outer diameter d_1 having the cross section shown is to be made of a steel for which $\tau_{all} = 75$ MPa and $G = 77$ GPa. Knowing that the angle of twist of the shaft must not exceed 4° when the shaft is subjected to a torque of 900 N·m, determine the largest inner diameter d_2 which can be specified in the design.



SOLUTION

$c_1 = \frac{1}{2}d_1 = 0.021$ m $\quad L = 1.6$ m

Based on stress limit $\tau = 75$ MPa $= 75 \times 10^6$ Pa

$$\tau = \frac{Tc_1}{J} \quad \therefore \quad J = \frac{Tc_1}{\tau} = \frac{(900)(0.021)}{75\times10^6} = 252 \times 10^{-9} \text{ m}^4$$

Based on angle of twist limit $\varphi = 4° = 69.813 \times 10^{-3}$ rad

$$\varphi = \frac{TL}{GJ} \quad \therefore \quad J = \frac{TL}{G\varphi} = \frac{(900)(1.6)}{(77\times10^9)(69.813\times10^{-3})} = 267.88 \times 10^{-9} \text{ m}^4$$

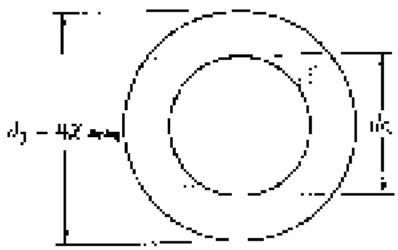
Larger value for J governs $\quad J = 267.88 \times 10^{-9}$ m^4

$$J = \frac{\pi}{2}(c_1^4 - c_2^4)$$

$$c_2^4 = c_1^4 - \frac{2J}{\pi} = 0.021^4 - \frac{(2)(267.88\times10^{-9})}{\pi} = 23.943 \times 10^{-9} \text{ m}^4$$

$c_2 = 12.44 \times 10^{-3}$ m $= 12.44$ mm $\quad d_2 = 2c_2 = 24.9$ mm

PROBLEM 3.86



3.86 A 1.6-m-long tubular steel shaft ($G = 77$ GPa) of 42-mm outer diameter d_1 and 30-mm inner diameter d_2 is to transmit 120 kW between a turbine and a generator. Knowing that the allowable shearing stress is 65 MPa and that the angle of twist must not exceed 3°, determine the minimum frequency at which the shaft may rotate.

SOLUTION

$c_1 = \frac{1}{2}d_1 = 0.021$ m, $\quad c_2 = \frac{1}{2}d_2 = 0.015$ m

$J = \frac{\pi}{2}(c_1^4 - c_2^4) = \frac{\pi}{2}(0.021^4 - 0.015^4) = 225.97\times10^{-9}$ m^4

Based on stress limit $\quad \tau = 65$ MPa $= 65\times10^6$ Pa

$$\tau = \frac{Tc_1}{J} \quad \therefore \quad T = \frac{J\tau}{c_1} = \frac{(225.97\times10^{-9})(65\times10^6)}{0.021} = 699.43 \text{ N}\cdot\text{m}$$

Based on angle of twist limit $\quad \varphi = 3° = 52.36\times10^{-3}$ rad

$$\varphi = \frac{TL}{GJ} \quad \therefore \quad T = \frac{GJ\varphi}{L} = \frac{(77\times10^9)(225.97\times10^{-9})(52.36\times10^{-3})}{1.6}$$

$$= 569.40 \text{ N}\cdot\text{m}$$

Smaller torque governs $\quad T = 569.40$ N·m

$P = 120$ kW $= 120\times10^3$ W

$$P = 2\pi f T \quad \therefore \quad f = \frac{P}{2\pi T} = \frac{120\times10^3}{2\pi(569.40)} = 33.54 \text{ Hz}$$

$$= 2012 \text{ rpm}$$

PROBLEM 3.87

3.87 The stepped shaft shown rotates at 450 rpm. Knowing that $r = 10$ mm, determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 45 MPa.

SOLUTION

$d = 100$ mm, $D = 120$ mm, $r = 10$ mm

$\frac{D}{d} = \frac{120}{100} = 1.2$, $\frac{r}{d} = \frac{10}{100} = 0.10$. From Fig. 3.32 $K = 1.33$

For smaller shaft $c = \frac{1}{2}d = 0.050$ m $\qquad \tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.050)^3(45\times10^6)}{(2)(1.33)} = 6.643\times10^3 \text{ N}\cdot\text{m}$$

$f = 450$ rpm $= 7.5$ Hz

Power $P = 2\pi f T = 2\pi(7.5)(6.643\times10^3) = 313\times10^3$ W $= 313$ kW

PROBLEM 3.88

3.88 The stepped shaft shown rotates at 450 rpm. Knowing that $r = 4$ mm, determine the maximum power that can be transmitted without exceeding an allowable shearing stress of 45 MPa.

SOLUTION

$d = 100$ mm, $D = 120$ mm, $r = 4$ mm

$\frac{D}{d} = \frac{120}{100} = 1.2$, $\frac{r}{d} = \frac{4}{100} = 0.04$. From Fig. 3.32 $K = 1.55$

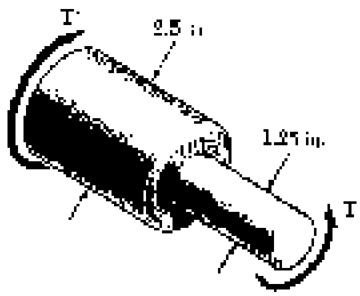
For smaller shaft $c = \frac{1}{2}d = 0.050$ m $\qquad \tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.050)^3(45\times10^6)}{(2)(1.55)} = 5.70\times10^3 \text{ N}\cdot\text{m}$$

$f = 450$ rpm $= 7.5$ Hz

Power $P = 2\pi f T = 2\pi(7.5)(5.70\times10^3) = 268\times10^3$ W $= 268$ kW

PROBLEM 3.89



3.89 Knowing that the stepped shaft shown must transmit 60 hp at a speed of 2100 rpm, determine the minimum radius r of the fillet if an allowable stress of 6000 psi is not to be exceeded.

SOLUTION

$$f = \frac{2100}{60} = 35 \text{ Hz}$$

$$P = 60 \text{ hp} = (60)(6600) = 396 \times 10^3 \text{ lb}\cdot\text{in/s}$$

$$T = \frac{P}{2\pi f} = \frac{396 \times 10^3}{2\pi(35)} = 1.8007 \times 10^3 \text{ lb}\cdot\text{in}$$

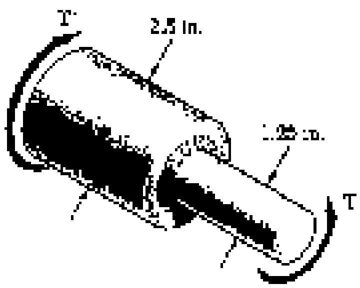
For smaller shaft $c = \frac{1}{2}d = 0.625$ in $\qquad \tau = K\frac{Tc}{J} = \frac{2KT}{\pi c^3}$

$$K = \frac{\pi c^3 \tau}{2T} = \frac{\pi(0.625)^3(6000)}{(2)(1.8007 \times 10^3)} = 1.28$$

$\frac{D}{d} = \frac{2.5}{1.25} = 2 \qquad$ From Fig 3.32 $\qquad \frac{r}{d} = 0.18$

$$r = 0.18\,d = (0.050)(1.25 \text{ in}) = 0.225 \text{ in.}$$

PROBLEM 3.90



3.90 The stepped shaft shown must transmit 60 hp. Knowing that the allowable shearing stress in the shaft is 6000 psi and that the radius of the fillet is $r = 0.25$ in., determine the smallest permissible speed of the shaft.

SOLUTION

$\frac{r}{d} = \frac{0.25}{1.25} = 0.200, \qquad \frac{D}{d} = \frac{2.5}{1.25} = 2.00$

From Fig. 3.32 $\quad K = 1.26$

For smaller shaft $\quad c = \frac{1}{2}d = 0.625$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.625)^3(6000)}{(2)(1.26)} = 1.826 \times 10^3 \text{ lb}\cdot\text{in}$$

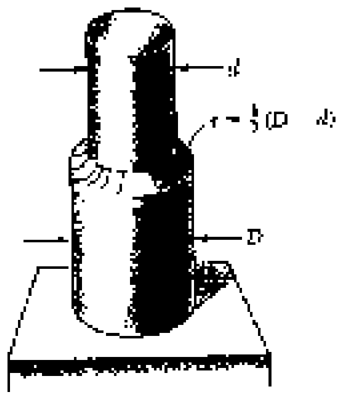
$$P = 60 \text{ hp} = (60)(6600) = 396 \times 10^3 \text{ lb}\cdot\text{in/s}$$

$$P = 2\pi f T \quad \therefore \quad f = \frac{P}{2\pi T} = \frac{396 \times 10^3}{2\pi(1.826 \times 10^3)} = 34.9 \text{ Hz}$$

$$= 2076 \text{ rpm}$$

PROBLEM 3.91

3.91 A 25-N·m torque is applied to the stepped shaft shown which has a full quarter-circular fillet. Knowing that $D = 24$ mm, determine the maximum shearing stress in the shaft when (a) $d = 20$ mm, (b) $d = 21.6$ mm.



Full quarter-circular fillet extends to edge of larger shaft

SOLUTION

(a) $\frac{D}{d} = \frac{24}{20} = 1.20$

$r = \frac{1}{2}(D - d) = \frac{1}{2}(24 - 20) = 2$ mm

$\frac{r}{d} = \frac{2}{20} = 0.10$

From Fig 3.32 $\quad K = 1.34$

For smaller shaft $\quad c = \frac{1}{2}d = 0.010$ m

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3} = \frac{(2)(1.34)(25)}{\pi(0.010)^3} = 21.3 \times 10^6 \text{ Pa} = 21.6 \text{ MPa} \blacktriangleleft$$

(b) $\frac{D}{d} = \frac{24}{21.6} = 1.111$

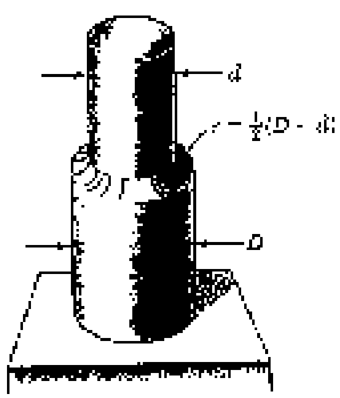
$r = \frac{1}{2}(D - d) = \frac{1}{2}(24 - 21.6) = 1.2$ mm $\qquad \frac{r}{d} = \frac{1.2}{21.6} = 0.0556$

From Fig 3.32 $\quad K = 1.42$. $\qquad$ For smaller shaft $\quad c = \frac{1}{2}d = 0.0108$ m.

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3} = \frac{(2)(1.42)(25)}{\pi(0.0108)^3} = 17.9 \times 10^6 \text{ Pa} = 17.9 \text{ MPa} \blacktriangleleft$$

PROBLEM 3.92

3.92 In the stepped shaft shown, which has a full quarter-circular fillet, $D = 1.5$ in. and $d = 1.2$ in. Knowing that the speed of the shaft is 1800 rpm and that the allowable shearing stress is 8000 psi, determine the maximum power that may be transmitted by the shaft.



Full quarter-circular fillet extends to edge of larger shaft

SOLUTION

$\frac{D}{d} = \frac{1.5}{1.2} = 1.25 \qquad r = \frac{1}{2}(D - d) = \frac{1}{2}(1.5 - 1.2) = 0.15$ in

$\frac{r}{d} = \frac{0.15}{1.2} = 0.125$

From Fig. 3.32 $\quad K = 1.31$

For smaller shaft $\quad c = \frac{1}{2}d = 0.6$ in

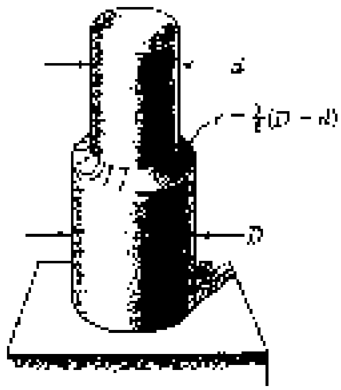
$\tau = \frac{KTc}{J} \qquad T = \frac{J\tau}{Kc} = \frac{\pi c^3 \tau}{2K}$

$T = \frac{\pi(0.6)^3(8000)}{(2)(1.31)} = 2072$ lb·in $\qquad f = 1800$ rpm $= 30$ Hz

Power $\quad P = 2\pi f T = 2\pi(30)(2072) = 390.6 \times 10^3$ lb·in/s

$$\frac{390.6 \times 10^3 \text{ lb·in/s}}{6600 \text{ lb·in/s/hp}} = 59.2 \text{ hp} \blacktriangleleft$$

PROBLEM 3.93



Full quarter-circular fillet extends to edge of larger shaft

3.93 In the stepped shaft shown, which has a full quarter-circular fillet, the allowable shearing stress is 12 ksi. Knowing that $D = 1.25$ in., determine the largest allowable torque that may be applied to the shaft if (*a*) $d = 1.1$ in., (*b*) $d = 1.0$ in.

SOLUTION

$$\tau_{all} = 12 \text{ ksi} = 12000 \text{ psi}$$

(a) $D = 1.25 \quad d = 1.1$ in $\qquad \frac{D}{d} = \frac{1.25}{1.1} = 1.09$

$$r = \tfrac{1}{2}(D-d) = \tfrac{1}{2}(1.25 - 1.1) = 0.075 \text{ in}$$

$$\frac{r}{d} = \frac{0.075}{1.1} = 0.0682$$

From Fig 3.32 $\quad K = 1.40$

For smaller shaft $\quad c = \frac{1}{2}d = 0.55$ in

$$\tau = \frac{KTc}{J} \quad \therefore T = \frac{J\tau}{Kc} = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.55)^3 (12000)}{(2)(1.40)}$$

$$= 2240 \text{ lb-in}$$

(b) $D = 1.25$ in. $\quad d = 1.0$ in $\qquad r = \frac{1}{2}(D-d) = \frac{1}{2}(1.25 - 1.0) = 0.125$ in

$\frac{D}{d} = \frac{1.25}{1.0} = 1.25, \quad \frac{r}{d} = \frac{0.125}{1.0} = 0.125 \quad$ From Fig 3.32 $\quad K = 1.31$

For smaller shaft $\quad c = \frac{1}{2}d = 0.50$ in

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi (0.50)^3 (12000)}{(2)(1.31)} = 1798 \text{ lb-in}$$

PROBLEM 3.94

3.94 A 54-mm-diameter solid shaft is made of mild steel which is assumed to be elastoplastic with $\tau_Y = 145$ MPa. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (*a*) 4 kN·m, (*b*) 5 kN·m.

SOLUTION

$c = \frac{1}{2} d = 0.027$ m $\qquad \tau_Y = 145 \times 10^6$ Pa

Compute T_Y $\qquad T_Y = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2}(0.027)^3(145 \times 10^6) = 4.483 \times 10^3$ N·m

(a) $T = 4.0 \times 10^3$ N·m $< T_Y$ $\qquad$ elastic $\qquad \rho = c = 27$ mm

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(4.0 \times 10^3)}{\pi (0.027)^3} = 129.4 \times 10^6 \text{ Pa} = 129.4 \text{ MPa}$$

(b) $T = 5.0 \times 10^3$ N·m $> T_Y$ $\qquad$ plastic region with elastic core.

The maximum shearing stress is $\tau_{max} = \tau_Y = 145$ MPa

$$T = \frac{4}{3} T_Y \left(1 - \frac{\rho_Y^3}{c^3}\right)$$

$$\frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(5 \times 10^3)}{4.483 \times 10^3} = 0.6540 \qquad \frac{\rho_Y}{c} = 0.8680$$

$$\rho_Y = 0.8680\, c = (0.8680)(0.027) = 0.02344 \text{ m} = 23.4 \text{ mm}$$

PROBLEM 3.95

3.95 A 1.5-in.-diameter solid shaft is made of mild steel which is assumed to be elastoplastic with $\tau_Y = 21$ ksi. Determine the maximum shearing stress and the radius of the elastic core caused by the application of a torque of magnitude (*a*) 12 kip·in., (*b*) 18 kip·in.

SOLUTION

$c = \frac{1}{2} d = 0.75$ in $\qquad \tau_Y = 21$ ksi

Compute T_Y $\qquad T_Y = \frac{J \tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2}(0.75)^3(21) = 13.916$ kip·in

(a) $T = 12$ kip·in $< T_Y$ $\qquad$ elastic $\qquad \rho = c = 0.75$ in.

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(12)}{\pi (0.75)^3} = 18.11 \text{ ksi}$$

(b) $T = 18$ kip·in $> T_Y$ $\qquad$ plastic region with elastic core

The maximum shearing stress is $\tau_{max} = \tau_Y = 21$ ksi

$$T = \frac{4}{3} T_Y \left(1 - \frac{\rho_Y^3}{c^3}\right)$$

$$\frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(18)}{13.916} = 0.11957 \qquad \frac{\rho_Y}{c} = 0.49265$$

$$\rho_Y = 0.49265\, c = (0.49265)(0.75) = 0.369 \text{ in.}$$

PROBLEM 3.96

3.96 A 30-mm-diameter solid rod is made of an elastoplastic material with $\tau_Y = 3.5$ MPa. Knowing that the elastic core of the rod is 25 mm in diameter, determine the magnitude of the torque applied to the rod.

SOLUTION

$\tau_Y = 3.5 \times 10^6$ Pa $\qquad c = \frac{1}{2}d = 0.015$ m $\qquad \rho_Y = \frac{1}{2}d_Y = 0.0125$ m

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.015)^3(3.5\times10^6) = 18.555 \text{ N}\cdot\text{m}$$

$$\frac{\rho_Y}{c} = \frac{0.0125}{0.015} = 0.83333$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(18.555)\left[1 - \frac{1}{4}(0.83333)^3\right] = 21.2 \text{ N}\cdot\text{m}$$

PROBLEM 3.97

3.97 It is observed that a straightened steel paper clip can be twisted through several revolutions by the application of a torque of approximately 0.8 lb·in. Knowing that the diameter of the wire used to form the paper clip is 0.04 in., determine the approximate value of the yield stress of the steel.

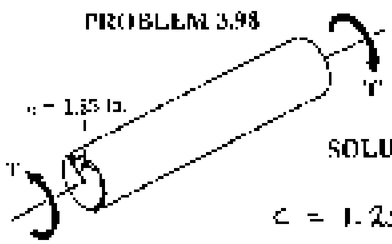
SOLUTION

$c = \frac{1}{2}d = 0.02$ in. $\qquad T_P = 0.8$ lb·in

$$T_P = \frac{4}{3}T_Y = \frac{4}{3}\frac{J\tau_Y}{c} = \frac{4}{3}\cdot\frac{\pi}{2}c^3\tau_Y = \frac{2\pi}{3}c^3\tau_Y$$

$$\tau_Y = \frac{3T_P}{2\pi c^3} = \frac{(3)(0.8)}{2\pi(0.02)^3} = 47.7 \text{ ksi}$$

PROBLEM 3.98



3.98 The solid circular shaft shown is made of a steel which is assumed to be elastoplastic with $\tau_Y = 21$ ksi. Determine the magnitude T of the applied torque when the plastic zone is (*a*) 0.6 in. deep, (*b*) 1 in. deep.

SOLUTION

$c = 1.25$ in $\qquad \tau_Y = 21$ ksi

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(1.25)^3(21) = 64.427 \text{ kip}\cdot\text{in}$$

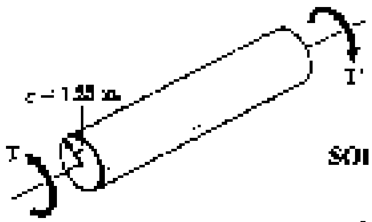
(a) $t_Y = 0.6$ in, $\quad \rho_Y = c - t_Y = 1.25 - 0.60 = 0.65$ in, $\quad \frac{\rho_Y}{c} = \frac{0.65}{1.25} = 0.520$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(64.427)\left[1 - \frac{1}{4}(0.520)^3\right] = 82.9 \text{ kip}\cdot\text{in}$$

(b) $t_Y = 1.0$ in, $\quad \rho_Y = c - t_Y = 1.25 - 1.0 = 0.25$ in $\quad \frac{\rho_Y}{c} = \frac{0.25}{1.25} = 0.200$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(64.427)\left[1 - \frac{1}{4}(0.200)^3\right] = 85.7 \text{ kip}\cdot\text{in}$$

PROBLEM 3.99



3.98 The solid circular shaft shown is made of a steel which is assumed to be elastoplastic with $\tau_Y = 21$ ksi. Determine the magnitude T of the applied torque when the plastic zone is (*a*) 0.6 in. deep, (*b*) 1 in. deep.

3.99 For the shaft and loading of Prob. 3.98, determine the angle of twist in a 4-ft length of shaft.

SOLUTION

$c = 1.25$ in, $\tau_Y = 21$ ksi $= 21\times10^3$ psi, $G = 11.2\times10^6$ psi

$L = 4$ ft $= 48$ in

$$\gamma = \frac{\rho\varphi}{L} \qquad \gamma_Y = \frac{c\varphi_Y}{L} \quad \therefore \quad \varphi_Y = \frac{L\gamma_Y}{c} = \frac{L\tau_Y}{cG} = \frac{(48)(21\times10^3)}{(1.25)(11.2\times10^6)} = 72.00\times10^{-3}\ \text{rad}$$

$$\frac{\rho_Y}{c} = \frac{\varphi_Y}{\varphi} \quad \therefore \quad \varphi = \frac{c}{\rho_Y}\varphi_Y$$

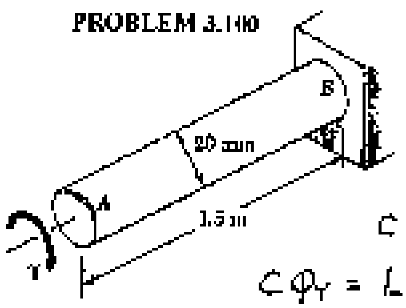
(a) $t_Y = 0.6$ in $\quad \rho_Y = c - t_Y = 1.25 - 0.6 = 0.65 \quad \frac{\rho_Y}{c} = \frac{0.65}{1.25} = 0.520$

$$\varphi = \frac{\varphi_Y}{\rho_Y/c} = \frac{72.00\times10^{-3}}{0.520} = 138.46\times10^{-3}\ \text{rad} = 7.93^\circ$$

(b) $t_Y = 1.0$ in $\quad \rho_Y = c - t_Y = 1.25 - 1.0 = 0.25 \quad \frac{\rho_Y}{c} = \frac{0.25}{1.25} = 0.200$

$$\varphi = \frac{\varphi_Y}{\rho_Y/c} = \frac{72.00\times10^{-3}}{0.20} = 360\times10^{-3}\ \text{rad} = 20.6^\circ$$

PROBLEM 3.100



3.100 A torque **T** is applied to the 20-mm-diameter steel rod AB. Assuming the steel to be elastoplastic with $G = 77$ GPa and $\tau_Y = 145$ MPa, determine (*a*) the torque **T** when the angle of twist at A is 25°, (*b*) the corresponding diameter of the elastic core of the shaft.

SOLUTION

$c = \frac{1}{2}d = 0.010$ m, $L = 1.5$ m, $G = 77\times10^9$ Pa

$$c\varphi_Y = L\gamma_Y = \frac{L\tau_Y}{G} \qquad \varphi_Y = \frac{L\tau_Y}{Gc}$$

$$\varphi_Y = \frac{(1.5)(145\times10^6)}{(77\times10^9)(0.010)} = 282.47\times10^{-3}\ \text{rad}$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.010)^3(145\times10^6) = 227.77\ \text{N}\cdot\text{m}$$

$$\varphi = 25^\circ = 436.33\times10^{-3}\ \text{rad} > \varphi_Y \qquad \frac{\varphi_Y}{\varphi} = \frac{282.47\times10^{-3}}{436.33\times10^{-3}} = 0.64737$$

(a) $T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\varphi_Y^3}{\varphi^3}\right) = \frac{4}{3}(227.77)\left[1 - \frac{1}{4}(0.64737)^3\right] = 283\ \text{N}\cdot\text{m}$

(b) $\frac{\rho_Y}{c} = \frac{\varphi_Y}{\varphi} = 0.64737 \qquad \rho_Y = 0.64737\,c = (0.64737)(0.010)$

$\rho_Y = 6.4737\times10^{-3}\ \text{m} = 6.4737\ \text{mm} \qquad d_Y = 2\rho_Y = 12.95\ \text{mm}$

PROBLEM 3.101

3.101 A 18-mm-diameter solid circular shaft is made of a material which is assumed to be elastoplastic with $G = 77$ GPa and $\tau_Y = 145$ MPa. For a 1.2-m length, determine the maximum shearing stress and the angle of twist caused by a 200 N·m torque.

SOLUTION

$\tau_Y = 145 \times 10^6$ Pa, $\quad c = \frac{1}{2}d = 0.009$ m, $\quad L = 1.2$ m, $\quad T = 200$ N·m

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.009)^3(145\times10^6) = 166.04 \text{ N·m}$$

$T > T_Y$ plastic region with elastic core $\quad \tau_{max} = \tau_Y = 145$ MPa ◀

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(166.04)(1.2)}{\pi(0.009)^4(77\times10^9)} = 251.08\times10^{-3} \text{ rad}$$

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\varphi_Y^3}{\varphi^3}\right)$$

$$\left(\frac{\varphi_Y}{\varphi}\right)^3 = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(200)}{166.04} = 0.38641 \qquad \frac{\varphi_Y}{\varphi} = 0.72837$$

$$\varphi = \frac{\varphi_Y}{0.72837} = \frac{251.08\times10^{-3}}{0.72837} = 344.7\times10^{-3} \text{ rad} = 19.75^\circ$$ ◀

PROBLEM 3.102

3.102 A solid circular rod is made of a material which is assumed to be elastoplastic. Denoting by T_Y and ϕ_Y respectively, the torque and angle of twist at the onset of yield, determine the angle of twist if the torque is increased to (a) $T = 1.1\,T_Y$, (b) $T = 1.25\,T_Y$, (c) $T = 1.3\,T_Y$.

SOLUTION

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\varphi_Y^3}{\varphi^3}\right)$$

$$\frac{\varphi_Y}{\varphi} = \sqrt[3]{4 - \frac{3T}{T_Y}} \quad \text{or} \quad \frac{\varphi}{\varphi_Y} = \frac{1}{\sqrt[3]{4 - \frac{3T}{T_Y}}}$$

(a) $\dfrac{T}{T_Y} = 1.10 \qquad \dfrac{\varphi}{\varphi_Y} = \dfrac{1}{\sqrt[3]{4-(3)(1.10)}} = 1.126 \qquad \varphi = 1.126\,\varphi_Y$ ◀

(b) $\dfrac{T}{T_Y} = 1.25 \qquad \dfrac{\varphi}{\varphi_Y} = \dfrac{1}{\sqrt[3]{4-(3)(1.25)}} = 1.587 \qquad \varphi = 1.587\,\varphi_Y$ ◀

(c) $\dfrac{T}{T_Y} = 1.3 \qquad \dfrac{\varphi}{\varphi_Y} = \dfrac{1}{\sqrt[3]{4-(3)(1.3)}} = 2.15 \qquad \varphi = 2.15\,\varphi_Y$ ◀

PROBLEM 3.103

SOLUTION

3.103 A 0.75-in-diameter solid circular shaft is made of a material which is assumed to be elastoplastic with $G = 11.2 \times 10^6$ psi and $\tau_Y = 21$ ksi. For a 5-ft length of the shaft, determine the maximum shearing stress and the angle of twist caused by a 2-kip·in. torque.

$c = \frac{1}{2}d = 0.375$ in, $G = 11.2\times10^6$ psi, $\tau_Y = 21$ ksi $= 21000$ psi

$L = 5$ ft $= 60$ in $\qquad T = 2$ kip·in $= 2\times10^3$ lb·in

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(0.375)^3(21000) = 1.7395\times10^3 \text{ lb}\cdot\text{in}$$

$T > T_Y$ plastic region with elastic core $\therefore$ $\tau_{max} = \tau_Y = 21$ ksi

$$\gamma_Y = \frac{c\varphi_Y}{L} \quad \therefore \varphi_Y = \frac{L\gamma_Y}{c} = \frac{L\tau_Y}{cG} = \frac{(60)(21000)}{(0.375)(11.2\times10^6)} = 300\times10^{-3} \text{ rad}$$

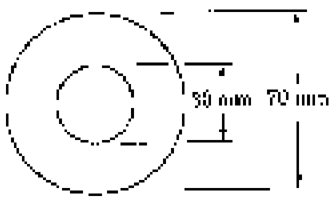
$$T = T_Y\left(1 - \frac{1}{4}\frac{\varphi_Y^3}{\varphi^3}\right) \therefore$$

$$\frac{\varphi}{\varphi_Y} = \frac{1}{\sqrt[3]{4 - \frac{3T}{T_Y}}} = \frac{1}{\sqrt[3]{4 - \frac{(3)(2\times10^3)}{1.7395\times10^3}}} = 1.220$$

$$\varphi = 1.220\,\varphi_Y = (1.220)(300\times10^{-3}) = 366\times10^{-3} \text{ rad} = 21.0°$$

PROBLEM 3.104

3.104 A hollow steel shaft is 0.9 m long and has the cross section shown. The steel is assumed to be elastoplastic with $\tau_Y = 180$ MPa and $G = 77$ GPa. Determine the applied torque and the corresponding angle of twist (*a*) at the onset of yield, (*b*) when the plastic zone is 10 mm deep.



SOLUTION

(a) At the onset of yield, the stress distribution is the elastic distribution with $\tau_{max} = \tau_Y$

$$c_2 = \tfrac{1}{2} d_2 = 0.035 \text{ m}, \quad c_1 = \tfrac{1}{2} d_1 = 0.015 \text{ m}.$$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.035^4 - 0.015^4) = 2.2777 \times 10^{-6} \text{ m}$$

$$\tau_{max} = \tau_Y = \frac{T_Y c_2}{J} \quad \therefore \quad T_Y = \frac{J \tau_Y}{c_2} = \frac{(2.2777 \times 10^{-6})(180 \times 10^6)}{0.035} = 11.714 \times 10^3 \text{ N·m}$$

$$= 11.71 \text{ kN·m}$$

$$\varphi_Y = \frac{T_Y L}{GJ} = \frac{(11.714 \times 10^3)(0.9)}{(77 \times 10^9)(2.2777 \times 10^{-6})} = 60.11 \times 10^{-3} \text{ rad} = 3.44°$$

(b) $t = 0.010$ m $\qquad \rho_Y = c_2 - t = 0.035 - 0.010 = 0.025$ m

$$\gamma = \frac{\rho \varphi}{L} = \frac{\rho_Y \varphi}{L} = \gamma_Y = \frac{\tau_Y}{G}$$

$$\varphi = \frac{\tau_Y L}{G \rho_Y} = \frac{(180 \times 10^6)(0.9)}{(77 \times 10^9)(0.025)} = 84.156 \times 10^{-3} \text{ rad} = 4.82°$$

Torque T_1 carried by elastic portion $c_1 \le \rho \le \rho_Y$

$\tau = \tau_Y$ at $\rho = \rho_Y$. $\qquad \tau_Y = \dfrac{T_1 \rho_Y}{J_1}$ $\qquad$ where $J_1 = \dfrac{\pi}{2}(\rho_Y^4 - c_1^4)$

$$J_1 = \frac{\pi}{2}(0.025^4 - 0.015^4) = 534.07 \times 10^{-9} \text{ m}^4$$

$$T_1 = \frac{J_1 \tau_Y}{\rho_Y} = \frac{(534.07 \times 10^{-9})(180 \times 10^6)}{0.025} = 3.845 \times 10^3 \text{ N·m}$$

Torque T_2 carried by plastic portion

$$T_2 = 2\pi \int_{\rho_Y}^{c_2} \tau_Y \rho^2 \, d\rho = 2\pi \tau_Y \frac{\rho^3}{3}\Big|_{\rho_Y}^{c_2} = \frac{2\pi}{3} \tau_Y (c_2^3 - \rho_Y^3)$$

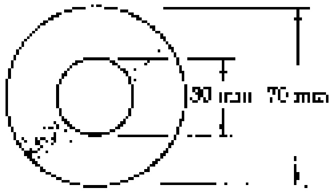
$$= \frac{2\pi}{3}(180 \times 10^6)(0.035^3 - 0.025^3) = 10.273 \times 10^3 \text{ N·m}$$

Total torque

$$T = T_1 + T_2 = 3.845 \times 10^3 + 10.273 \times 10^3 = 14.12 \times 10^3 \text{ N·m}$$

$$14.12 \text{ kN·m}$$

PROBLEM 3.105



3.105 A hollow steel shaft is 0.9 m long and has the cross section shown. The steel is assumed to be elastoplastic with $\tau_Y = 180$ MPa and $G = 77$ GPa. Determine (*a*) the angle of twist at which the section first becomes fully plastic, (*b*) the corresponding magnitude of the applied torque.

SOLUTION

$$c_1 = \tfrac{1}{2} d_1 = 0.015 \text{ m} \qquad c_2 = \tfrac{1}{2} d_2 = 0.035 \text{ m}$$

(a) For onset of fully plastic yielding, $\rho_Y = c_1$

$$\tau = \tau_Y \quad \therefore \quad \gamma = \frac{\tau_Y}{G} = \frac{\rho_Y \varphi}{L} = \frac{c_1 \varphi}{L}$$

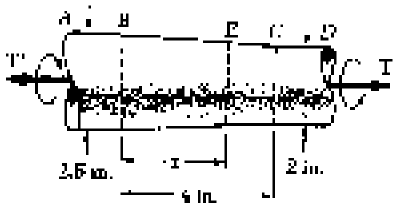
$$\varphi = \frac{L \tau_Y}{c_1 G} = \frac{(0.9)(180 \times 10^6)}{(0.015)(77 \times 10^9)} = 140.26 \times 10^{-3} \text{ rad} = 8.04°$$

(b) $$T_P = 2\pi \int_{c_1}^{c_2} \tau_Y \rho^2 d\rho = 2\pi \tau_Y \frac{\rho^3}{3} \Big|_{c_1}^{c_2} = \frac{2\pi}{3} \tau_Y (c_2^3 - c_1^3)$$

$$= \frac{2\pi}{3} (180 \times 10^6)(0.035^3 - 0.015^3) = 14.89 \times 10^3 \text{ N·m}$$

$$= 14.89 \text{ kN·m}$$

PROBLEM 3.106



3.106 A shaft of mild steel is machined to the shape shown and then twisted by torques of magnitude 45 kip·in. Assuming the steel to be elastoplastic with $\tau_Y = 21$ ksi, determine (*a*) the thickness of the plastic zone in portion *CD* of the shaft, (*b*) the length of the portion *BE* which remains fully elastic.

SOLUTION

(a) In portion CD $\quad c = \tfrac{1}{2} d = 1.00$ in

$$T_Y = \frac{J_{CD} \tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2}(1.00)^3(21) = 32.987 \text{ kip·in}$$

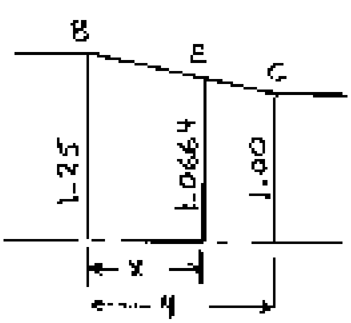
$$T = \frac{4}{3} T_Y \left(1 - \frac{\rho_Y^3}{c^3}\right) \quad \therefore \quad \frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(40)}{32.987} = 0.36620$$

$$\frac{\rho_Y}{c} = 0.71283, \quad \rho_Y = (0.71283)(1.00) = 0.713 \text{ in}, \quad t_Y = c - \rho_Y = 0.287 \text{ in}$$

(b) For yielding at point E $\quad \tau = \tau_Y, \; c = c_E, \; T = 40$ kip·in

$$\tau_Y = \frac{T c_E}{J_E} = \frac{2T}{\pi c_E^3} \quad \therefore \quad c_E^3 = \frac{2T}{\pi \tau_Y} = \frac{(2)(40)}{\pi (21)} = 1.2126 \text{ in}^3$$

$$c_E = 1.0664 \text{ in.}$$



Using proportions from the sketch

$$\frac{1.25 - 1.0664}{1.25 - 1.00} = \frac{x}{4}$$

$$x = 2.94 \text{ in}$$

PROBLEM 3.107

3.107 The magnitude of the torque T applied to the tapered shaft of Prob. 3.106 is slowly increased. Determine (a) the largest torque which may be applied to the shaft, (b) the length of portion *BE* which remains fully elastic.

SOLUTION

(a) The largest torque which may be applied to the shaft makes portion CD fully plastic.

In portion CD $\quad c = \frac{1}{2}d = 1.00$ in

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau_Y = \frac{\pi}{2}(1.00)^3(21) = 32.987 \text{ kip}\cdot\text{in.}$$

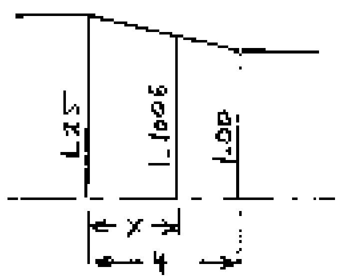
For fully plastic shaft $\quad \rho_Y = 0$

$$T = \frac{4}{3}T_Y\left(1 - \frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}T_Y = \frac{4}{3}(32.987) = 43.982 \text{ kip}\cdot\text{in} \approx 44.0 \text{ kip}\cdot\text{in}$$

(b) For yielding at point E, $\tau = \tau_Y$, $c = c_E$, $T = 43.982$ kip·in.

$$\tau_Y = \frac{Tc_E}{J_E} = \frac{2T}{\pi c_E^3}, \qquad c_E^3 = \frac{2T}{\pi\tau_Y} = \frac{(2)(43.982)}{\pi(21)} = 1.33333 \text{ in}^3$$

$c_E = 1.1006$ in



Using proportions from the sketch

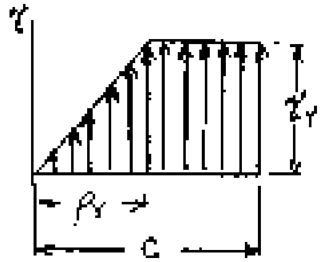
$$\frac{1.25 - 1.1006}{1.25 - 1.00} = \frac{x}{4}$$

$$x = 2.39 \text{ in}$$

PROBLEM 3.108

3.108 Considering the partially plastic shaft of Fig. 3.38c, derive Eq. (3.32) by recalling that the integral in Eq. (3.26) represents the second moment about the τ axis of the area under the τ-ρ curve.

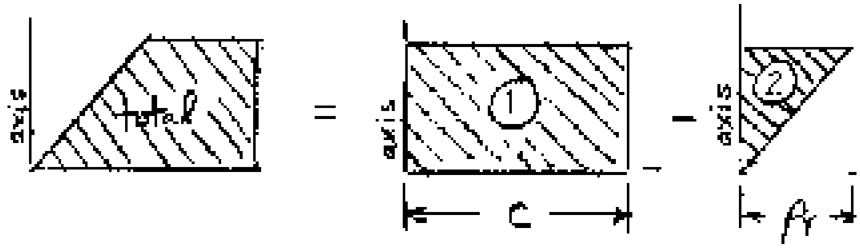
SOLUTION



The stress is that shown on the left.

$$T = 2\pi \int_0^c \rho^2 \tau \, d\rho = 2\pi \int \rho^2 \, dA = 2\pi I$$

where $dA = \tau \, d\rho$ and I = 2nd moment about the τ-axis.



$$I = I_1 - I_2$$

$$= \tfrac{1}{3}\tau_Y c^3 - \left\{ \tfrac{1}{36}\tau_Y \rho_Y^3 + \tfrac{1}{2}\tau_Y \rho_Y \left(\tfrac{1}{3}\rho_Y\right)^2 \right\}$$

$$= \tfrac{1}{3}\tau_Y c^3 - \tfrac{1}{12}\tau_Y \rho_Y^3 = \tfrac{1}{3}\tau_Y c^3 \left(1 - \tfrac{1}{4}\tfrac{\rho_Y^3}{c^3}\right)$$

$$T = 2\pi I = \tfrac{2\pi}{3}\tau_Y c^3 \left(1 - \tfrac{1}{4}\tfrac{\rho_Y^3}{c^3}\right)$$

Recall that $T_Y = \dfrac{J\tau_Y}{c} = \dfrac{\pi}{2} c^3 \tau_Y \quad \therefore \quad \dfrac{2\pi}{3}\tau_Y c^3 = \dfrac{4}{3} T_Y$

Hence $\quad T = \dfrac{4}{3} T_Y \left(1 - \dfrac{1}{4}\dfrac{\rho_Y^3}{c^3}\right)$

PROBLEM 3.109

3.109 Using the stress-strain diagram shown, determine (*a*) the torque which causes a maximum shearing stress of 15 ksi in a 0.8-diameter solid rod, (*b*) the corresponding angle of twist in a 20-in. length of the rod.

SOLUTION

(a) $\tau_{max} = 15$ ksi $\qquad c = \frac{1}{2}d = 0.400$ in

From the stress-strain diagram $\gamma_{max} = 0.008$

Let $z = \frac{\gamma}{\gamma_{max}} = \frac{\rho}{c}$

$$T = 2\pi \int_0^c \rho^2 \tau \, d\rho = 2\pi c^3 \int_0^1 z^2 \tau \, dz = 2\pi c^3 I$$

where the integral I is given by $\quad I = \int_0^1 z^2 \tau \, dz$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w \, z^2 \tau$$

where w is a weighting factor. Using $\Delta z = 0.25$, we get the values given in the table below.

z	γ	τ, ksi	$z^2\tau$, ksi	w	$wz^2\tau$, ksi
0	0.000	0	0.000	1	0.00
0.25	0.002	8	0.500	4	2.00
0.5	0.004	12	3.000	2	6.00
0.75	0.006	14	7.875	4	31.50
1.0	0.008	15	15.000	1	15.00
					54.50 ← $\sum wz^2\tau$

$$I = \frac{(0.25)(54.50)}{3} = 4.54 \text{ ksi}$$

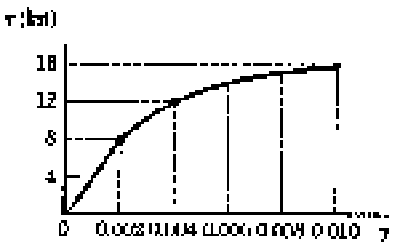
(a) $T = 2\pi c^3 I = 2\pi (0.400)^3 (4.54) = 1.826$ kip-in ◄

(b) $\gamma_{max} = \frac{c\varphi}{L}$

$$\varphi = \frac{L\gamma_m}{c} = \frac{(20)(0.008)}{0.400} = 400 \times 10^{-3} \text{ rad} = 22.9°$$ ◄

Note: Answers may differ slightly due to differences of opinion in reading the stress-strain curve.

PROBLEM 3.110



3.110 A hollow shaft of outer and inner diameters respectively equal to 0.6 in. and 0.2 in. is fabricated from an aluminum alloy for which the stress-strain diagram is given in the sketch. Determine the torque required to twist a 9-in. length of the shaft through 10°.

SOLUTION

$$\varphi = 10° = 174.53 \times 10^{-3} \text{ rad}$$

$$c_1 = \tfrac{1}{2} d_1 = 0.100 \text{ in}, \quad c_2 = \tfrac{1}{2} d_2 = 0.300 \text{ in}$$

$$\gamma_{max} = \frac{c_2 \varphi}{L} = \frac{(0.300)(174.53 \times 10^{-3})}{9} = 0.00582$$

$$\gamma_{min} = \frac{c_1 \varphi}{L} = \frac{(0.100)(174.53 \times 10^{-3})}{9} = 0.00194$$

Let $z = \dfrac{\gamma}{\gamma_{max}} = \dfrac{\rho}{c_2}$ $\qquad z_1 = \dfrac{c_1}{c_2} = \dfrac{1}{3}$

$$T = 2\pi \int_{c_1}^{c_2} \rho^2 \tau \, d\rho = 2\pi c_2^3 \int_{z_1}^{1} z^2 \tau \, dz = 2\pi c_2^3 I$$

where the integral I is given by $\quad I = \int_{1/3}^{1} z^2 \tau \, dz$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = \frac{1}{6}$ we get the values given in the table below.

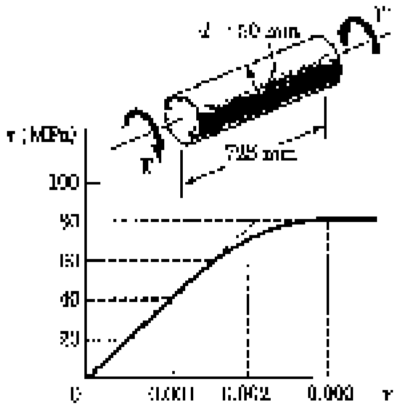
z	γ	τ, ksi	$z^2\tau$, ksi	w	$wz^2\tau$, ksi
1/3	0.00194	8.0	0.89	1	0.89
1/2	0.00291	10.0	2.50	4	10.00
2/3	0.00383	11.5	5.11	2	10.22
5/6	0.00485	13.0	9.03	4	36.11
1	0.00582	14.0	14.0	1	14.00
					71.22 ← $\sum wz^2\tau$

$$I = \frac{(1/6)(71.22)}{3} = 3.96 \text{ ksi}$$

$$T = 2\pi c_2^3 I = 2\pi (0.300)^3 (3.96) = 0.671 \text{ kip} \cdot \text{in} = 671 \text{ lb} \cdot \text{in}$$

Note: Answer may differ slightly due to differences of opinion in reading the stress-strain curve.

PROBLEM 3.111



3.111 A 50-mm-diameter cylinder is made of a brass for which the stress-strain diagram is as shown. Knowing that the angle of twist is 5° in a 725-mm length, determine by approximate means the magnitude T of the torque applied to the shaft.

SOLUTION

$$\varphi = 5° = 87.266 \times 10^{-3} \text{ rad}$$

$$c = \tfrac{1}{2} d = 0.025 \text{ m}, \quad L = 0.725 \text{ m}$$

$$\gamma_{max} = \frac{c\varphi}{L} = \frac{(0.025)(87.266\times10^{-3})}{0.725} = 0.00301$$

$$\text{Let } z = \frac{\gamma}{\gamma_{max}} = \frac{\rho}{c}$$

$$T = 2\pi \int_0^c \rho^2 \tau \, d\rho = 2\pi c^3 \int_0^1 z^2 \tau \, dz = 2\pi c^3 I$$

where the integral I is given by $I = \int_0^1 z^2 \tau \, dz$

Evaluate I using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$I = \frac{\Delta z}{3} \sum w z^2 \tau$$

where w is a weighting factor. Using $\Delta z = 0.25$, we get the values given in the table below.

z	γ	τ, MPa	$z^2\tau$, MPa	w	$wz^2\tau$, MPa
0	0	0	0	1	0
0.25	0.00075	30	1.875	4	7.5
0.5	0.0015	55	13.75	2	27.5
0.75	0.00226	75	42.19	4	168.75
1.0	0.00301	80	80.	1	80.
					283.75

$\leftarrow \sum w z^2 \tau = 283.75 \times 10^6$ Pa

$$I = \frac{(0.25)(283.75\times10^6)}{3} = 23.65\times10^6 \text{ Pa}$$

$$T = 2\pi c^3 I = 2\pi(0.025)^3(23.65\times10^6) = 2.32\times10^3 \text{ N}\cdot\text{m}$$

$$= 2.32 \text{ kN}\cdot\text{m}$$

PROBLEM 3.112

3.111 A 50-mm-diameter cylinder is made of a brass for which the stress-strain diagram is as shown. Knowing that the angle of twist is 5° in a 725-mm length, determine by approximate means the magnitude T of the torque applied to the shaft.

3.112 Three points on the nonlinear stress-strain diagram used in Prob. 3.111 are (0,0), (0.0015,55MPa), and (0.003,80MPa). By fitting the polynomial $\tau = A + B\gamma + C\gamma^2$ through these points the following approximate relation has been obtained.

$$\tau = 46.7 \times 10^9\,\gamma - 6.67 \times 10^{12}\,\gamma^2$$

Solve Prob. 3.111 using the relation, Eq. (3.2) and Eq. (3.26).

SOLUTION

$\varphi = 5° = 87.266 \times 10^{-3}$ rad, $\quad c = \frac{1}{2}d = 0.025$ m, $\quad L = 0.725$ m

$$\gamma_{max} = \frac{c\varphi}{L} = \frac{(0.025)(87.266\times10^{-3})}{0.725} = 3.009 \times 10^{-3}$$

Let $z = \frac{\gamma}{\gamma_{max}} = \frac{\rho}{c}$

$$T = 2\pi \int_0^c \rho^2 \tau\, d\rho = 2\pi c^3 \int_0^1 z^2 \tau\, dz$$

The given stress strain curve is

$$\tau = A + B\gamma + C\gamma^2 = A + B\gamma_{max} z + C\gamma_{max}^2 z^2$$

$$T = 2\pi c^3 \int_0^1 z^2 (A + B\gamma_{max} z + C\gamma_{max}^2 z^2)\, dz$$

$$= 2\pi c^3 \left\{ A\int_0^1 z^2 dz + B\gamma_{max}\int_0^1 z^3 dz + C\gamma_{max}^2 \int_0^1 z^4 dz \right\}$$

$$= 2\pi c^3 \left\{ \tfrac{1}{3}A + \tfrac{1}{4}B\gamma_{max} + \tfrac{1}{5}C\gamma_{max}^2 \right\}$$

Data: $A = 0$, $\quad B = 46.7\times10^9$, $\quad C = -6.67\times10^{12}$

$\frac{1}{3}A = 0$, $\quad \frac{1}{4}B\gamma_{max} = \frac{1}{4}(46.7\times10^9)(3.009\times10^{-3}) = 35.13\times10^6$

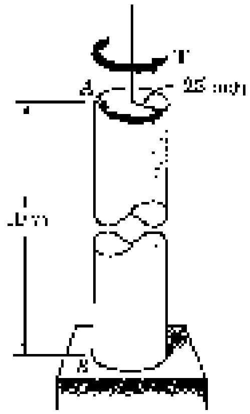
$\frac{1}{5}C\gamma_{max}^2 = -\frac{1}{5}(6.67\times10^{12})(3.009\times10^{-3})^2 = -12.08\times10^3$

$$T = 2\pi(0.025)^3\{0 + 35.13\times10^3 - 12.08\times10^3\} = 2.26\times10^3 \text{ N·m}$$

$$= 2.26 \text{ kN·m}$$

PROBLEM 3.113

3.113 The solid circular drill rod AB is made of a steel which is assumed to be elastoplastic with $\tau_Y = 160$ MPa and $G = 77$ GPa. Knowing that a torque $T = 5$ kN·m is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.



SOLUTION

$c = 0.025$ m

$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.025)^4 = 613.59 \times 10^{-9}$ m^4

$T_Y = \frac{J \tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2}(0.032)^3 (160 \times 10^6) = 3.927 \times 10^3$ N·m

Loading: $T = 5 \times 10^3$ N·m

$T = \frac{4}{3} T_Y \left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right)$

$\frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(5 \times 10^3)}{3.927 \times 10^3} = 0.18029$

$\frac{\rho_Y}{c} = 0.5649$, $\quad \rho_Y = 0.5649\,c = 0.014123 \times 10^{-3}$ m $= 14.123$ mm

Unloading: $\quad \tau' = \frac{T\rho}{J}$ where $T = 5 \times 10^3$ kN·m

At $\rho = c$ $\quad \tau' = \frac{(5)(0.025)}{613.59 \times 10^{-9}} = 203.72 \times 10^6$ Pa

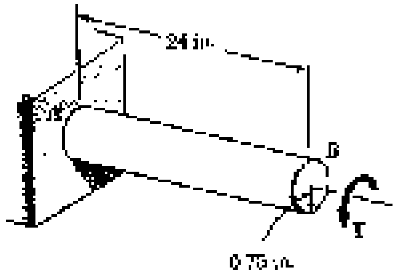
At $\rho = \rho_Y$ $\quad \tau' = \frac{(5)(0.014123)}{613.59 \times 10^{-9}} = 115.08 \times 10^6$ Pa

Residual: $\quad \tau_{res} = \tau_{load} - \tau'$

At $\rho = c$ $\quad \tau_{res} = 160 \times 10^6 - 203.72 \times 10^6 = -43.72 \times 10^6$ Pa $= -43.7$ MPa

At $\rho = \rho_Y$ $\quad \tau_{res} = 160 \times 10^6 - 115.08 \times 10^6 = 44.92 \times 10^6$ Pa $= 44.9$ MPa

PROBLEM 3.114



3.114 The solid circular shaft AB is made of a steel which is assumed to be elastoplastic with $G = 11.2 \times 10^6$ psi and $\tau_Y = 21$ ksi. The torque **T** is increased until the radius of the elastic core is 0.25 in. Determine the maximum residual shearing stress in the shaft after the torque T has been removed.

SOLUTION

$c = 0.75$ in. $\quad \rho_Y = 0.25$ in. $\quad \frac{\rho_Y}{c} = \frac{0.25}{0.75} = \frac{1}{3}$

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.75)^4 = 0.49701\ \text{in}^4$

$T_Y = \frac{J\tau_Y}{c} = \frac{(0.49701)(21)}{0.75} = 13.916$ kip·in

At end of loading: $\quad T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(13.916)\left[1 - \frac{1}{4}\left(\frac{1}{3}\right)^3\right]$

$= 18.383$ kip·in

The stresses are $\quad \tau_{load} = 0$ at $\rho = 0$

$\tau_{load} = \tau_Y = 21$ ksi at $\rho = 0.25$ in.

$\tau_{load} = \tau_Y = 21$ ksi at $\rho = 0.75$ in.

Torque change during unloading $\quad T = -18.383$ kip·in.

Stress changes during unloading $\quad \tau' = \frac{T\rho}{J}$ (elastic)

At $\rho = 0 \qquad \tau' = 0$

At $\rho = 0.25$ in $\qquad \tau' = \frac{(18.383)(0.25)}{0.49701} = 9.25$ ksi

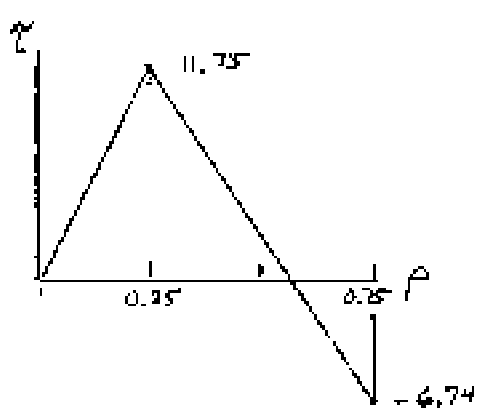
At $\rho = 0.75$ in $\qquad \tau' = \frac{(18.383)(0.75)}{0.49701} = 27.74$ ksi

Residual stresses are found by adding $\quad \tau_{res} = \tau_{load} - \tau'$

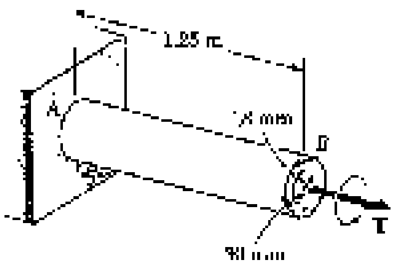
At $\rho = 0 \qquad \tau_{res} = 0$

At $\rho = 0.25$in $\qquad \tau_{res} = 21 - 9.75 = 11.75$ ksi

At $\rho = 0.75$ in $\qquad \tau_{res} = 21 - 27.74 = -6.74$ ksi



PROBLEM 3.115



3.115 The hollow shaft AB is made of a steel which is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77$ GPa. The magnitude T of the torque is slowly increased until the plastic zone first reaches the inner surface; the torque is then removed. Determine (a) the maximum residual shearing stress, (b) the permanent angle of twist of the shaft.

SOLUTION

inner radius $c_1 = 0.018$ m, outer radius $c_2 = 0.030$ m

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.030^4 - 0.018^4)$$
$$= 1.10745 \times 10^{-6} \text{ m}^4$$

Loading: When ρ_Y reaches inner surface, then $\tau = \tau_Y$

$$T_{load} = 2\pi\int_{c_1}^{c_2} \rho^2 \tau_Y \, d\rho = 2\pi\tau_Y \frac{\rho^3}{3}\Big|_{c_1}^{c_2} = \frac{2\pi}{3}\tau_Y(c_2^3 - c_1^3)$$
$$= \frac{2\pi}{3}(145\times10^6)(0.030^3 - 0.018^3) = 6.4285\times10^3 \text{ N}\cdot\text{m}$$

$$\gamma = \frac{\tau_Y}{G} \text{ at } \rho = c_1 \qquad \text{Also } \gamma = \frac{\rho\varphi}{L} \quad \therefore \varphi = \frac{L\gamma}{\rho} = \frac{L\tau_Y}{c_1 G}$$

$$\varphi_{load} = \frac{(1.25)(145\times10^6)}{(0.018)(77\times10^9)} = 130.77\times10^{-3} \text{ rad} = 7.493^\circ$$

Unloading: $T = 6.4285\times10^3$ N·m (elastic) $\quad \Delta\tau = \frac{\Delta T\rho}{J}$

At $\rho = c_2$ $\quad \tau' = \frac{(6.4285)(0.030)}{1.10745\times10^{-6}} = 174.14\times10^6$ Pa

At $\rho = c_1$ $\quad \tau' = \frac{(6.4285)(0.018)}{1.10745\times10^{-6}} = 104.49\times10^6$ Pa

$$\varphi' = \frac{(\Delta T)L}{GJ} = \frac{(6.4285)(1.25)}{(77\times10^9)(1.10745\times10^{-6})} = 94.23\times10^{-3}\text{ rad} = -5.399^\circ$$

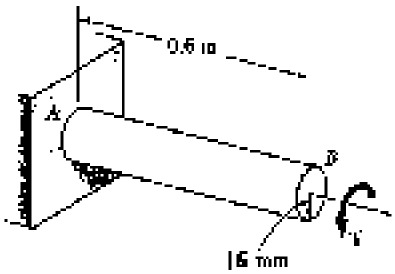
Residual. $\quad \tau_{res} = \tau_{load} - \tau' \qquad \varphi_{perm} = \varphi_{load} - \varphi'$

(a) At $\rho = c_2$ $\quad \tau_{res} = 145\times10^6 - 174.14\times10^6 = -29.14\times10^6$ Pa
$= -29.1$ MPa ◀

At $\rho = c_1$ $\quad \tau_{res} = 145\times10^6 - 104.49\times10^6 = 40.51\times10^6$ Pa
$= 40.5$ MPa ◀

(b) $\varphi_{perm} = 130.77\times10^{-3} - 94.23\times10^{-3} = 36.54\times10^{-3}$ rad
$= 2.09^\circ$ ◀

PROBLEM 3.116



3.116 The solid shaft shown is made of a steel which is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77$ GPa. The torque **T** is increased in magnitude until the shaft has been twisted through 6° and then removed. Determine (*a*) the magnitude and location of the maximum residual shearing stress, (*b*) the permanent angle of twist of the shaft.

SOLUTION

$c = 0.016$ m $\qquad \varphi = 6° = 104.72 \times 10^{-3}$ rad

$$\gamma_{max} = \frac{c\varphi}{L} = \frac{(0.016)(104.72 \times 10^{-3})}{0.6} = 0.0027925$$

$$\gamma_Y = \frac{\tau_Y}{G} = \frac{145 \times 10^6}{77 \times 10^9} = 0.0018831$$

$$\frac{\rho_Y}{c} = \frac{\gamma_Y}{\gamma_{max}} = \frac{0.0018831}{0.0027925} = 0.67433$$

$$J = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.016)^4 = 102.944 \times 10^{-9} \text{ m}^4$$

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2} c^3 \tau_Y = \frac{\pi}{2}(0.016)^3(145 \times 10^6) = 932.93 \text{ N}\cdot\text{m}$$

At end of loading $\quad T_{load} = \frac{4}{3} T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) = \frac{4}{3}(932.93)\left[1 - \frac{1}{4}(0.67433)^3\right]$

$$= 1.14855 \times 10^3 \text{ N}\cdot\text{m}$$

Unloading: $\qquad$ elastic $\qquad T' = -1.14855 \times 10^3$ N·m

At $\rho = c \qquad \tau' = \frac{T'c}{J} = \frac{(-1.14855 \times 10^3)(0.016)}{102.944 \times 10^{-9}} = 178.52 \times 10^6$ Pa

At $\rho = \rho_Y \qquad \tau' = \frac{T'c}{J}\frac{\rho_Y}{c} = (-178.52 \times 10^6)(0.67433) = 120.38 \times 10^6$ Pa

$$\varphi' = \frac{T'L}{GJ} = \frac{(1.14855 \times 10^3)(0.6)}{(77 \times 10^9)(102.944 \times 10^{-9})} = 86.94 \times 10^{-3} \text{ rad}$$

Residual: $\qquad \tau_{res} = \tau_{load} - \tau' \qquad \varphi_p = \varphi_{load} - \varphi'$

At $\rho = c \qquad \tau_{res} = 145 \times 10^6 - 178.52 \times 10^6 = -33.52 \times 10^6$ Pa

$= -33.5$ MPa

At $\rho = \rho_Y \qquad \tau_{res} = 145 \times 10^6 - 120.38 \times 10^6 = 24.62 \times 10^6$ Pa

$= 24.6$ MPa

$$\varphi_{perm} = 104.72 \times 10^{-3} - 86.94 \times 10^{-3} = 17.78 \times 10^{-3} \text{ rad} = 1.019°$$

PROBLEM 3.117

3.113 The solid circular drill rod AB is made of a steel which is assumed to be elastoplastic with $\tau_Y = 160$ MPa and $G = 77$ GPa. Knowing that a torque $T = 5$ kN·m is applied to the rod and then removed, determine the maximum residual shearing stress in the rod.

3.117 In Prob. 3.113, determine the permanent angle of twist of the rod.

SOLUTION

From the solution to PROBLEM 3.113

$c = 0.025$ m, $J = 613.59 \times 10^{-9}$ m^4, $\frac{\rho_Y}{c} = 0.5649$, $\rho_Y = 0.014123$ m

After loading $\gamma = \frac{\rho\varphi}{L}$; $\varphi = \frac{L\gamma}{\rho} = \frac{L\gamma_Y}{\rho_Y} = \frac{L\tau_Y}{\rho_Y G}$

$$\varphi_{load} = \frac{(10)(160\times10^6)}{(0.014123)(77\times10^9)} = 1.4713 \text{ rad} = 84.30°$$

During unloading $\varphi' = \frac{TL}{GJ}$ (elastic) $T = 5\times10^3$ N·m

$$\varphi' = \frac{(5\times10^3)(10)}{(77\times10^9)(613.59\times10^{-9})} = 1.0583 \text{ rad} = 60.64°$$

Permanent twist angle

$$\varphi_{perm} = \varphi_{load} - \varphi' = 1.4713 - 1.0583 = 0.4130 \text{ rad} = 23.7°$$

PROBLEM 3.118

3.114 The solid circular shaft AB is made of a steel which is assumed to be elastoplastic with $G = 11.2 \times 10^6$ psi and $\tau_Y = 21$ ksi. The torque $\mathbf{T}$ is increased until the radius of the elastic core is 0.25 in. Determine the maximum residual shearing stress in the shaft after the torque $\mathbf{T}$ has been removed.

3.118 In Prob. 3.114, determine the permanent angle of twist of the shaft.

SOLUTION

From the solution to PROBLEM 3.114, $c = 0.75$ in, $J = 0.49701$ in^4

After loading $T = 18.383$ kip·in, $\rho_Y = 0.25$ in,

$$\gamma = \frac{\rho\varphi}{L} \quad \therefore \quad \varphi = \frac{L\gamma}{\rho} = \frac{L\tau_Y}{\rho_Y G}$$

where $L = 24$ in, $\tau_Y = 21$ ksi, $G = 11.2\times10^6$ psi $= 11.2\times10^3$ ksi

$$\varphi_{load} = \frac{(24)(21)}{(0.25)(11.2\times10^3)} = 180\times10^{-3} \text{ rad} = 10.31°$$

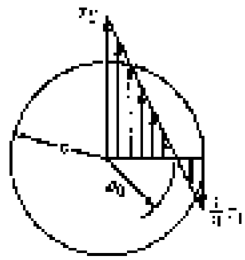
Unloading $T = 18.383$ kip-in

$$\varphi' = \frac{TL}{GJ} = \frac{(-18.383)(24)}{(11.2\times10^3)(0.49701)} = 79.26\times10^{-3} \text{ rad} = 4.54°$$

Permanent angle of twist $\varphi_{perm} = \varphi_{load} - \varphi'$

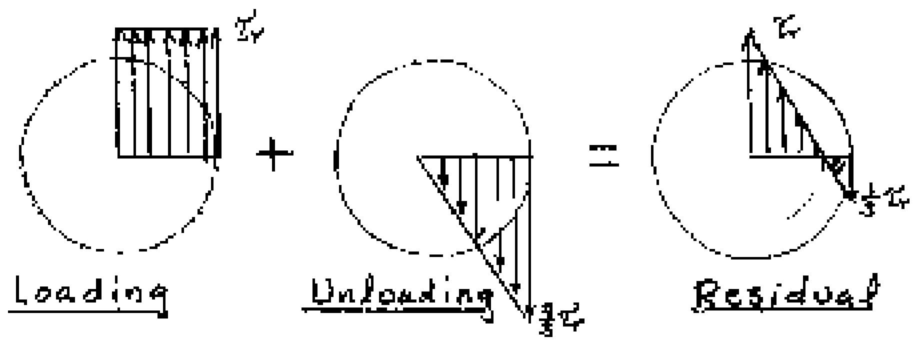
$$\varphi_{perm} = 180\times10^{-3} - 79.26\times10^{-3} = 100.74\times10^{-3} \text{ rad} = 5.77°$$

PROBLEM 3.119



3.119 A torque **T** applied to a solid rod made of an elastoplastic material is increased until the rod becomes fully plastic and then is removed. (*a*) Show that the distribution of residual stresses is as represented in the figure. (*b*) Determine the magnitude of the torque due the stresses acting on the portion of the rod located within a circle of radius c_0.

SOLUTION



(a)

After loading $\quad \rho_Y = 0, \quad T_{load} = \frac{4}{3}T_Y = \frac{4}{3}\frac{\pi}{2}c^3\tau_Y = \frac{2\pi}{3}c^3\tau_Y$

Unloading $\quad \tau' = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(T_{load})}{\pi c^3} = \frac{4}{3}\tau_Y$ at $\rho = c$

$$\tau' = \frac{4}{3}\tau_Y\frac{\rho}{c}$$

Residual $\quad \tau_{res} = \tau_Y - \frac{4}{3}\tau_Y\frac{\rho}{c} = \tau_Y\left(1 - \frac{4\rho}{3c}\right)$

To find c_0 set $\tau_{res} = 0$ and $\rho = c_0$

$$0 = 1 - \frac{4c_0}{3c} \quad \therefore \quad c_0 = \frac{3}{4}c$$

(b) $T = 2\pi\int_0^{c_0}\rho^2\tau\,d\rho = 2\pi\int_0^{\frac{3}{4}c}\rho^2\tau_Y\left(1 - \frac{4}{3}\frac{\rho}{c}\right)d\rho$

$$= 2\pi\tau_Y\left(\frac{\rho^3}{3} - \frac{4}{3}\frac{\rho^4}{4c}\right)\Bigg|_0^{\frac{3}{4}c} = 2\pi\tau_Y c^3\left\{\frac{1}{3}\left(\frac{3}{4}\right)^3 - \left(\frac{4}{3}\right)\frac{1}{4}\left(\frac{3}{4}\right)^4\right\}$$

$$= 2\pi\tau_Y c^3\left\{\frac{9}{64} - \frac{27}{256}\right\} = \frac{9\pi}{128}\tau_Y c^3 = 0.2209\,\tau_Y c^3$$

PROBLEM 3.120

3.116 The solid shaft shown is made of a steel which is assumed to be elastoplastic with $\tau_Y = 145$ MPa and $G = 77$ GPa. The torque T is increased in magnitude until the shaft has been twisted through 6° and then removed. Determine (*a*) the magnitude and location of the maximum residual shearing stress, (*b*) the permanent angle of twist of the shaft.

3.120 After the solid shaft of Prob. 3.116 has been loaded and unloaded as described in that problem, a torque $\mathbf{T}_1$ of sense opposite to the original torque **T** is applied to the shaft. Assuming no change in the value of ϕ_p, determine the angle of twist ϕ_1 for which yield is initiated in this second loading and compare it with the angle ϕ_Y for which the shaft started to yield in the original loading.

SOLUTION

From the solution to PROBLEM 3.116 $\quad c = 0.016$ m, $L = 0.6$ m

$\tau_Y = 145 \times 10^6$ Pa, $\quad J = 102.944 \times 10^{-9}$ m^4

The residual stress at $\rho = c$ is $\quad \tau_{res} = 33.5$ MPa

For loading in the opposite sense, the change in stress to produce reversed yielding is

$$\tau_1 = \tau_Y - \tau_{res} = 145 \times 10^6 - 33.5 \times 10^6 = 111.5 \times 10^6 \text{ Pa}$$

$$\tau_1 = \frac{T_1 c}{J} \quad \therefore \quad T_1 = \frac{J\tau_1}{c} = \frac{(102.944 \times 10^{-9})(111.5 \times 10^6)}{0.016}$$

$$= 717 \text{ N}\cdot\text{m}$$

Angle of twist

$$\phi_1 = \frac{T_1 L}{GJ} = \frac{(717 \times 10^3)(0.6)}{(77 \times 10^9)(102.944 \times 10^{-9})} = 54.3 \times 10^{-3} \text{ rad}$$

$$= 3.11°$$

PROBLEM 3.121

3.114 The solid circular shaft AB is made of a steel which is assumed to be elastoplastic with $G = 11.2 \times 10^6$ psi and $\tau_Y = 21$ ksi. The torque **T** is increased until the radius of the elastic core is 0.25 in. Determine the maximum residual shearing stress in the shaft after the torque **T** has been removed.

3.121 After the solid shaft of Prob. 3.114 has been loaded and unloaded as described in that problem, a torque $\mathbf{T}_1$ of sense opposite to the original torque **T** is applied to the shaft. Assuming no change in the value of τ_Y, determine the magnitude T_1 of the torque $\mathbf{T}_1$ required to initiate yield in this second loading and compare it with the magnitude T_Y of the torque **T** which caused the shaft to yield in the original loading.

SOLUTION

From the solution to PROBLEM 3.114 $\quad c = 0.75$ in, $\quad L = 24$ in.

$\tau_Y = 21$ ksi, $\quad J = 0.49701\ \text{in}^4$

The residual stress at $\rho = c$ is $\quad \tau_{res} = 6.74$ ksi

For loading in the opposite sense, the change in stress to produce reversed yielding is

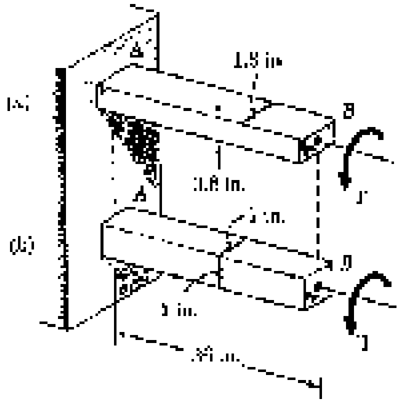
$$\tau_1 = \tau_Y - \tau_{res} = 21 - 6.74 = 14.26 \text{ ksi}$$

$$\tau_1 = \frac{T_1 c}{J} \quad \therefore \quad T_1 = \frac{J\tau_1}{c} = \frac{(0.49701)(14.26)}{0.75} = 9.45 \text{ kip}\cdot\text{in}$$

$$\therefore \quad T_Y = \frac{J\tau_Y}{c} = \frac{(0.49701)(21)}{0.75} = 13.92 \text{ kip}\cdot\text{in}$$

PROBLEM 3.122

3.122 Knowing that the magnitude of the torque **T** is 1800 lb·in., determine for each of the aluminum bars shown the maximum shearing stress and the angle of twist at end *B*. Use $G = 3.9 \times 10^6$ psi



SOLUTION

$T = 1800$ lb·in, $L = 36$ in

(a) $a = 1.8$ in, $b = 0.6$ in $\quad \frac{a}{b} = \frac{1.8}{0.6} = 3$

From Table 3.1 $\quad c_1 = 0.267$, $c_2 = 0.263$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{1800}{(0.267)(1.8)(0.6)^2}$$

$$= 10.40 \times 10^3 \text{ psi} = 10.40 \text{ ksi}$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(1800)(36)}{(0.263)(1.8)(0.6)^3(3.9 \times 10^6)} = 162.5 \times 10^{-3} \text{ rad} = 9.31°$$

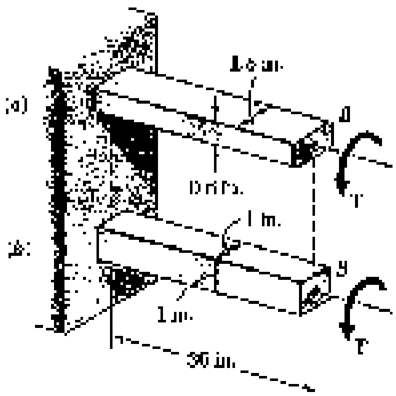
(b) $a = 1.0$ in, $b = 1.0$ in, $\frac{a}{b} = 1.00$, $c_1 = 0.208$, $c_2 = 0.1406$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{1800}{(0.208)(1.0)(1.0)^2} = 8.65 \times 10^3 \text{ psi} = 8.65 \text{ ksi}$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(1800)(36)}{(0.1406)(1.0)(1.0)^3(3.9 \times 10^6)} = 118.2 \times 10^{-3} \text{ rad} = 6.77°$$

PROBLEM 3.123

3.123 Using $\tau_{all} = 10$ ksi, determine for each of the aluminum bars shown the largest torque **T** which may be applied and the corresponding angle of twist. Use $G = 3.9 \times 10^6$ psi.



SOLUTION

$\tau_{all} = 10$ ksi, $G = 3.9 \times 10^6$ psi $= 3.9 \times 10^3$ ksi

(a) $a = 1.8$ in, $b = 0.6$ in, $\frac{a}{b} = \frac{1.8}{0.6} = 3$

From Table 3.1 $c_1 = 0.267$, $c_2 = 0.263$

$\tau_{max} = \frac{T}{c_1 a b^2}$ $\therefore$ $T = c_1 a b^2 \tau_{max}$

$T = (0.267)(1.8)(0.6)^2(10) = 1.730$ kip·in

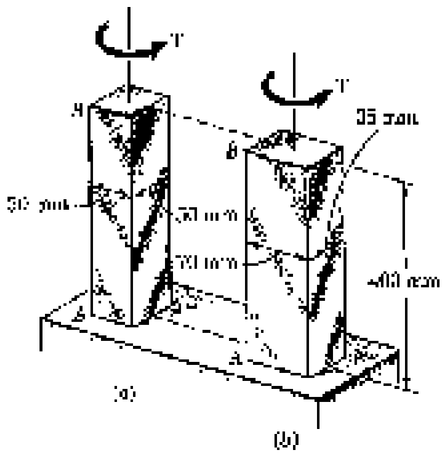
$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(1.730)(36)}{(0.263)(1.8)(0.6)^3(3.9\times10^3)} = 156.2\times10^{-3} \text{ rad}$$
$$= 8.95^\circ$$

(b) $a = 1.0$ in, $b = 1.0$ in. $\frac{a}{b} = 1.00$ $c_1 = 0.208$, $c_2 = 0.1406$

$T = c_1 a b^2 \tau_{max} = (0.208)(1.0)(1.0)^2(10) = 2.08$ kip·in

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(2.08)(36)}{(0.1406)(1.0)(1.0)^3(3.9\times10^3)} = 136.6\times10^{-3} \text{ rad}$$
$$= 7.82^\circ$$

PROBLEM 3.124



3.124 Knowing that $T = 800$ N·m, determine for each of the cold-rolled yellow brass bars shown the maximum shearing stress and the angle of twist at end B. Use $G = 39$ GPa.

SOLUTION

$T = 800$ N·m $\quad L = 400$ mm $= 0.400$ m

$G = 39 \times 10^9$ Pa

(a) $\quad a = 50$ mm $= 0.050$ m, $\quad b = 50$ mm $= 0.050$ m

$$\frac{a}{b} = \frac{50}{50} = 1.00$$

From Table 3.1, $c_1 = 0.208$, $c_2 = 0.1406$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{800}{(0.208)(0.050)(0.050)^2} = 30.8 \times 10^6 \text{ Pa} = 30.8 \text{ MPa}$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(800)(0.400)}{(0.1406)(0.05)(0.05)^3(39 \times 10^9)} = 9.33 \times 10^{-3} \text{ rad}$$

$$= 0.535°$$

(b) $\quad a = 70$ mm $= 0.070$ m, $\quad b = 35$ m $= 0.035$ m, $\quad \frac{a}{b} = \frac{70}{35} = 2.0$

$c_1 = 0.246$, $\quad c_2 = 0.229$

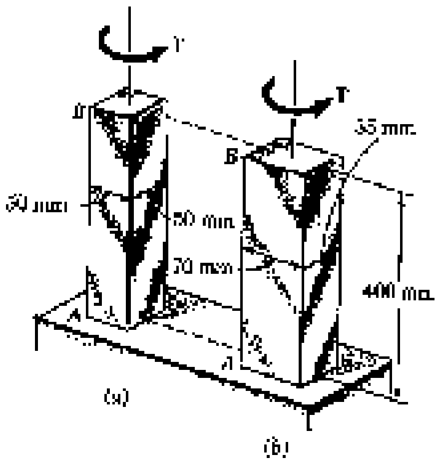
$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{800}{(0.246)(0.070)(0.035)^2} = 37.9 \times 10^6 \text{ Pa} = 37.9 \text{ MPa}$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(800)(0.400)}{(0.229)(0.070)(0.035)^3(39 \times 10^9)} = 11.94 \times 10^{-3} \text{ rad}$$

$$= 0.684°$$

PROBLEM 3.125

3.125 Using $\tau_{all} = 50$ MPa, determine for each of the cold-rolled yellow brass bars shown the largest torque T which may be applied and the corresponding angle of twist. Use $G = 39$ GPa.



SOLUTION

$\tau_{all} = 50 \times 10^6$ Pa, $\quad L = 400$ mm $= 0.400$ m

$G = 39 \times 10^9$ Pa

(a) $\quad a = 50$ mm $= 0.050$ m, $\quad b = 50$ mm $= 0.050$ m

$\frac{a}{b} = 1.00$

From Table 3.1 $\quad c_1 = 0.208, \quad c_2 = 0.1406$

$$\tau_{max} = \frac{T}{c_1 a b^2}$$

$$T = c_1 a b^2 \tau_{max} = (0.208)(0.050)(0.050)^2(50 \times 10^6) = 1300 \text{ N}\cdot\text{m}$$

$$= 1.300 \text{ kN}\cdot\text{m}$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(1300)(0.400)}{(0.1406)(0.050)(0.050)^3(39 \times 10^9)} = 15.17 \times 10^{-3} \text{ rad}$$

$$= 0.869°$$

(b) $\quad a = 70$ mm $= 0.070$ m, $\quad b = 35$ mm $= 0.035$ m, $\quad \frac{a}{b} = \frac{70}{35} = 2.0$

From Table 3.1 $\quad c_1 = 0.246, \quad c_2 = 0.229$

$$T = c_1 a b^2 \tau_{max} = (0.246)(0.070)(0.035)^2(50 \times 10^6) = 1055 \text{ N}\cdot\text{m}$$

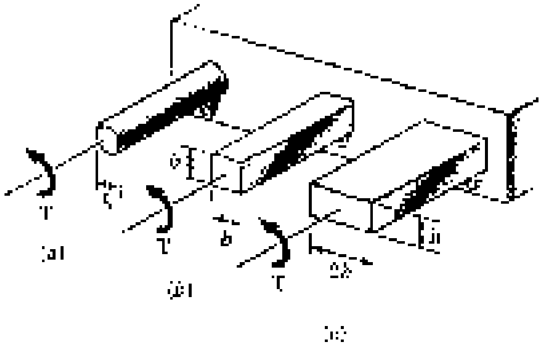
$$= 1.055 \text{ kN}\cdot\text{m}$$

$$\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(1055)(0.400)}{(0.229)(0.070)(0.035)^3(39 \times 10^9)} = 15.74 \times 10^{-3} \text{ rad}$$

$$= 0.902°$$

PROBLEM 3.126

3.126 A 2-kip-in. torque **T** is applied to each of the steel bars shown. Knowing that $\tau_{all} = 6$ ksi, determine the required dimension b for each bar.



SOLUTION

$T = 2$ kip·in $\quad \tau_{max} = 6$ ksi

(a) circle: $\quad c = \frac{1}{2} b$

$$\tau_m = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{16\,T}{\pi b^3}$$

$$b^3 = \frac{16\,T}{\pi \tau_{max}} = \frac{(16)(2)}{\pi (6)} = 1.698 \text{ in}^3$$

$$b = 1.193 \text{ in} \quad \blacktriangleleft$$

(b) square: $a = b$, $\frac{a}{b} = 1.0$. From Table 3.1 $\quad c_1 = 0.208$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{T}{c_1 b^3} \quad \therefore \quad b^3 = \frac{T}{c_1 \tau_{max}} = \frac{2}{(0.208)(6)} = 1.603 \text{ in}^3$$

$$b = 1.170 \text{ in} \quad \blacktriangleleft$$

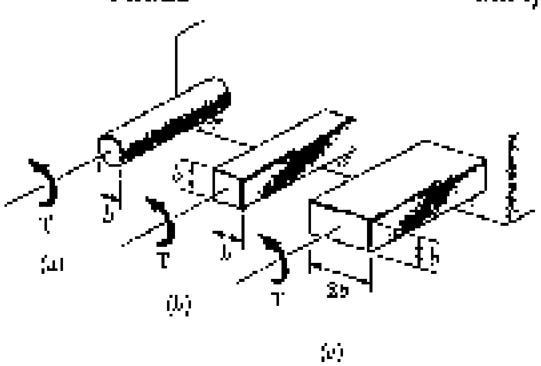
(c) rectangle: $a = 2b$, $\frac{a}{b} = 2.0$, $\quad c_1 = 0.246$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{T}{2c_1 b^3} \quad \therefore \quad b^3 = \frac{T}{2c_1 \tau_{max}} = \frac{2}{(2)(0.246)(6)} = 0.678 \text{ in}^3$$

$$b = 0.878 \text{ in} \quad \blacktriangleleft$$

PROBLEM 3.127

3.127 A 300-N·m torque T is applied to each of the aluminum bars shown. Knowing that $\tau_{all} = 60$ MPa, determine the required dimension b for each bar.



SOLUTION

$T = 300$ N·m $\quad \tau_{max} = 60 \times 10^6$ Pa

(a) circle: $\quad c = \frac{1}{2} b$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{16\,T}{\pi b^3}$$

$$b^3 = \frac{16\,T}{\pi \tau_{max}} = \frac{(16)(300)}{\pi (60 \times 10^6)} = 25.46 \times 10^{-6} \text{ m}^3$$

$$b = 29.4 \times 10^{-3} \text{ m} = 29.4 \text{ mm} \quad \blacktriangleleft$$

(b) square: $a = b$, $\frac{a}{b} = 1.0$. From Table 3.1 $\quad c_1 = 0.208$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{T}{c_1 b^3} \quad \therefore \quad b^3 = \frac{T}{c_1 \tau_{max}} = \frac{300}{(0.208)(60 \times 10^6)} = 24.04 \times 10^{-6} \text{ m}^3$$

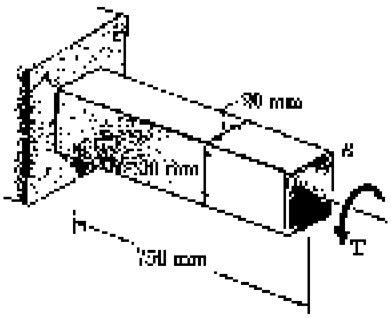
$$b = 28.9 \times 10^{-3} \text{ m} = 28.9 \text{ mm} \quad \blacktriangleleft$$

(c) rectangle: $a = 2b$, $\frac{a}{b} = 2.0$, $\quad c_1 = 0.246$

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{T}{2c_1 b^3} \quad \therefore \quad b^3 = \frac{T}{2c_1 \tau_{max}} = \frac{300}{(2)(0.246)(60 \times 10^6)} = 10.16 \times 10^{-6} \text{ m}^3$$

$$b = 21.7 \times 10^{-3} \text{ m} = 21.7 \text{ mm} \quad \blacktriangleleft$$

PROBLEM 3.128



3.128 The torque **T** causes a rotation of 2° at end *B* of the stainless steel bar shown. Knowing that *G* = 77 GPa, determine the maximum shearing stress in the bar.

SOLUTION

$a = 30\text{ mm} = 0.030\text{ m}, \quad b = 20\text{ mm} = 0.020\text{ m}$

$\varphi = 2° = 34.907\times10^{-3}\text{ rad}$

$$\varphi = \frac{TL}{c_2 a b^3 G} \quad \therefore \quad T = \frac{c_2 a b^3 G\varphi}{L}$$

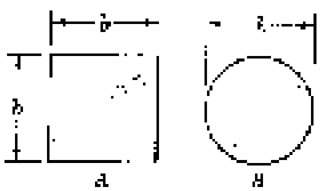
$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{c_2 a b^3 G\varphi}{c_1 a b^2 L} = \frac{c_2 b G \varphi}{c_1 L}$$

$\frac{a}{b} = \frac{30}{20} = 1.5 \qquad$ From Table 3.1 $\quad c_1 = 0.231 \qquad c_2 = 0.1958$

$$\tau_{max} = \frac{(0.1958)(0.020)(77\times10^9)(34.907\times10^{-3})}{(0.231)(0.750)} = 60.8\times10^6\text{ Pa}$$

$$= 60.8\text{ MPa}$$

PROBLEM 3.129



3.129 Two shafts are made of the same material. The cross section of shaft *A* is a square of side *b* and that of shaft *B* is a circle of diameter *b*. Knowing that the shafts are subjected to the same torque, determine the ratio of the maximum shearing stresses occurring in the shafts.

A. square $\quad \frac{a}{b} = 1, \quad c_1 = 0.208$ (Table 3.1)

$$\tau_A = \frac{T}{c_1 a b^2} = \frac{T}{0.208\, b^3}$$

B. circle $\quad c = \frac{1}{2}b \qquad \tau_B = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{16T}{\pi b^3}$

Ratio $\quad \frac{\tau_A}{\tau_B} = \frac{1}{0.208}\cdot\frac{\pi}{16} = 0.3005\,\pi = 0.944$

PROBLEM 3.130

3.130 Determine the largest allowable square cross section of a steel shaft of length 4 m if the maximum shearing stress is not to exceed 100 MPa when the shaft is twisted through one complete revolution. Use *G* = 77 GPa.

SOLUTION

$\varphi = 2\pi\text{ rad}, \qquad L = 4\text{ m}, \qquad \tau_{max} = 100\times10^6\text{ Pa} \qquad G = 77\times10^9\text{ Pa}$

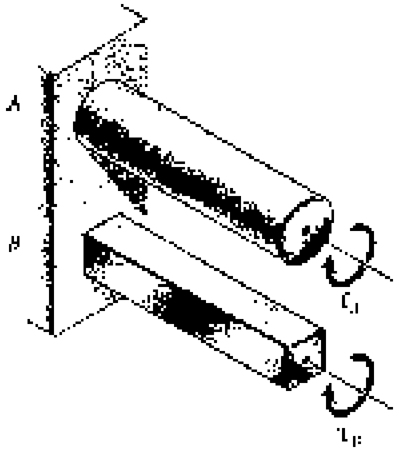
square section : $\quad c_1 = 0.208$ and $c_2 = 0.1406$ from Table 3.1

$$\tau_{max} = \frac{T}{c_1 a b^2} = \frac{T}{c_1 b^3}, \qquad \varphi = \frac{TL}{c_2 a b^3 G} = \frac{TL}{c_2 b^4 G}$$

$$\frac{\tau_{max}}{\varphi} = \frac{T}{c_1 b^3}\cdot\frac{c_2 b^4 G}{TL} \quad \therefore \quad b = \frac{c_1 \tau_{max} L}{c_2 G \varphi} = \frac{(0.208)(100\times10^6)(4.0)}{(0.1406)(77\times10^9)(2\pi)}$$

$$= 1.223\times10^{-3}\text{ m} = 1.223\text{ mm}$$

PROBLEM 3.131



3.131 Shafts A and B are made of the same material and have the same cross sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the maximum torques T_A and T_B which may be safely applied to A and B, respectively.

SOLUTION

Let c = radius of circular section A and b = side of square section B.

For equal areas $\pi c^2 = b^2$

$$c = \frac{b}{\sqrt{\pi}}$$

Circle: $\tau_A = \frac{T_A c}{J} = \frac{2T_A}{\pi c^3} \quad \therefore \quad T_A = \frac{\pi}{2} c^3 \tau_A$

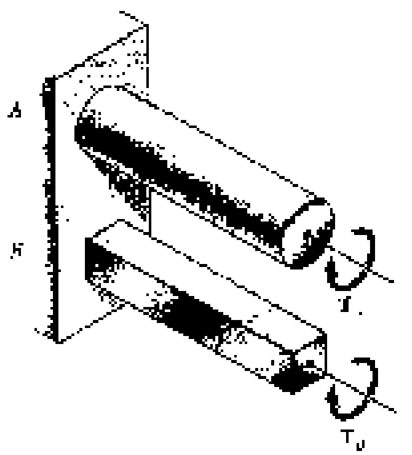
Square: $c_1 = 0.208$ from Table 3.1

$$\tau_B = \frac{T_B}{c_1 a b^2} = \frac{T_B}{c_1 b^3} \quad \therefore \quad T_B = c_1 b^3 \tau_B$$

Ratio: $\frac{T_A}{T_B} = \frac{\frac{\pi}{2} c^3 \tau_A}{c_1 b^3 \tau_B} = \frac{\frac{\pi}{2} \cdot \frac{b^3}{\pi^{3/2}} \tau_A}{c_1 b^3 \tau_B} = \frac{1}{2 c_1 \sqrt{\pi}} \frac{\tau_A}{\tau_B}$

For the same stresses $\tau_B = \tau_A \quad \therefore \quad \frac{T_A}{T_B} = \frac{1}{(2)(0.208)\sqrt{\pi}} = 1.356$ ◄

PROBLEM 3.132



3.132 Shafts A and B are made of the same material and have the same length and cross sectional area, but A has a circular cross section and B has a square cross section. Determine the ratio of the maximum values of the angles ϕ_A and ϕ_B through which shafts A and B, respectively, may be twisted.

SOLUTION

Let c = radius of circular section A and b = side of square section B.

For equal areas $\pi c^2 = b^2 \quad \therefore \quad b = \sqrt{\pi}\, c$

Circle: $\tau_{max} = \frac{\tau_A}{G} = \frac{c\varphi_A}{L} \quad \therefore \quad \varphi_A = \frac{L\tau_A}{cG}$

Square: Table 3.1 $c_1 = 0.208$, $c_2 = 0.1406$

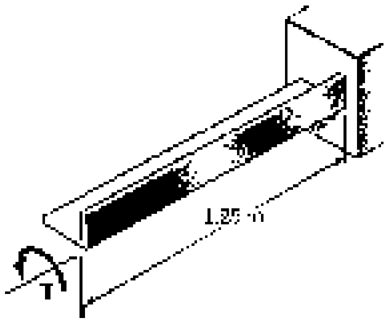
$$\tau_B = \frac{T_B}{c_1 a b^2} = \frac{T_B}{0.208\, b^3} \quad \therefore \quad T_B = 0.208\, b^3 \tau_B$$

$$\varphi_B = \frac{T_B L}{c_2 a b^3 G} = \frac{0.208\, b^3 \tau_B L}{0.1406\, b^4 G} = \frac{1.4794\, L\tau_B}{bG}$$

Ratio $\frac{\varphi_A}{\varphi_B} = \frac{L\tau_A}{cG} \cdot \frac{bG}{1.4794\, L\tau_B} = 0.676 \frac{b\tau_A}{c\tau_B} = 0.676\sqrt{\pi}\frac{\tau_A}{\tau_B}$

For equal stresses $\tau_A = \tau_B$ $\quad \frac{\varphi_B}{\varphi_A} = 0.676\sqrt{\pi} = 1.198$ ◄

PROBLEM 3.133



3.133 A 1.25-m-long steel angle has an L $127 \times 76 \times 6.4$ cross section. From Appendix C we find that the thickness of the section is 6.4 mm and that its area is 1252 mm^2. Knowing that $\tau_{all} = 60$ MPa, $G = 77$ GPa, and ignoring the effect of stress concentrations, determine (*a*) the largest torque **T** which may be applied, (*b*) the corresponding angle of twist.

SOLUTION

$A = 1252 \text{ mm}^2 \qquad b = 6.4 \text{ mm} = 0.0064 \text{ m}$

$a = \frac{A}{b} = \frac{1252}{6.4} = 195.6 \text{ mm} = 0.1956 \text{ m}$

$\frac{a}{b} = \frac{195.6}{6.4} = 30.56$

$c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3265$

$$\tau_{max} = \frac{T}{c_1 a b^2} \quad \therefore \quad T = c_1 a b^2 \tau_{max}$$

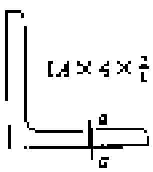
(a) $T = (0.3265)(0.1956)(0.0064)^2(60 \times 10^6) = 156.95 \times 10^3 \text{ N}\cdot\text{m}$
$= 157.0 \text{ kN}\cdot\text{m}$ ◄

(b) $\varphi = \frac{TL}{c_2 a b^3 G} = \frac{c_1 a b^2 \tau_{max} L}{c_2 a b^3 G} = \frac{c_1 \tau_{max} L}{c_2 b G} = \frac{\tau_{max} L}{bG}$

$$\varphi = \frac{(60 \times 10^6)(1.25)}{(0.0064)(77 \times 10^9)} = 152.19 \times 10^{-3} \text{ rad} = 8.72^\circ$$ ◄

PROBLEM 3.134

3.134 A 3000 lb·in. torque is applied to a 6-ft-long steel angle with a L $4 \times 4 \times \frac{3}{8}$ cross section. From Appendix C we find that the thickness of the section is $\frac{3}{8}$ in. and that its area is 2.86 in^2. Knowing that $G = 11.2 \times 10^6$ psi, determine (*a*) the maximum shearing stress along line *a-a*, (*b*) the angle of twist.



SOLUTION

$A = 2.86 \text{ in}^2, \quad b = \frac{3}{8} \text{ in} = 0.375 \text{ in}, \quad a = \frac{A}{a} = \frac{2.86}{0.375} = 7.627 \text{ in.}$

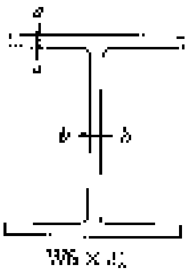
$\frac{a}{b} = \frac{7.627}{0.375} = 20.34 \qquad c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3230$

(a) $\tau_{max} = \frac{T}{c_1 a b^2} = \frac{3000}{(0.3230)(7.627)(0.375)^2} = 8.66 \times 10^3 \text{ psi} = 8.66 \text{ ksi}$ ◄

(b) $\varphi = \frac{TL}{c_2 a b^3 G} = \frac{(3000)(72)}{(0.3230)(7.627)(0.375)^3(11.2 \times 10^6)} = 148.45 \times 10^{-3} \text{ rad}$
$= 8.51^\circ$ ◄

Note: $L = 6 \text{ ft.} = 72 \text{ in}$

PROBLEM 3.135



3.135 An 8-ft-long steel member with a W 8 × 31 cross section is subjected to a 5 kip·in. torque. From Appendix C we find that the thickness of the section is $\frac{3}{8}$ in. and that its area is 2.86 in^2. Knowing that $G = 11.2 \times 10^6$ psi, determine (a) the maximum shearing stress along line *a-a*, (b) the maximum shearing stress along line *b-b*, (c) the angle of twist. (*Hint:* Consider the web and flanges separately and obtain a relation between the torques exerted on the web and a flange, respectively, by expressing that the resulting angles of twist are equal.)

SOLUTION

Flange: $a = 7.995$ in, $b = 0.435$, $\frac{a}{b} = \frac{7.995}{0.435} = 18.38$

$c_1 = c_2 = \frac{1}{3}(1 - 0.630\frac{b}{a}) = 0.3219$ $\qquad \varphi_F = \frac{T_F L}{c_2 a b^3 G}$

$T_F = c_2 a b^3 \frac{G\varphi_F}{L} = K_F \frac{G\varphi}{L}$ $\qquad$ where $K_F = c_2 a b^3$

$K_F = (0.3219)(7.995)(0.435)^3 = 0.2138 \text{ in}^4$

Web: $a = 8.0 - (2)(0.435) = 7.13$ in, $b = 0.285$ in, $\frac{a}{b} = \frac{7.13}{0.285} = 25.02$

$c_1 = c_2 = \frac{1}{3}(1 - 0.630\frac{b}{a}) = 0.3249$ $\qquad \varphi_W = \frac{T_W L}{c_2 a b^3 G}$

$T_W = c_2 a b^3 \frac{G\varphi_W}{L} = K_W \frac{G\varphi}{L}$ $\qquad$ where $K_W = c_2 a b^3$

$K_W = (0.3249)(7.13)(0.285)^3 = 0.0563 \text{ in}^4$

For matching twist angles $\qquad \varphi_F = \varphi_W = \varphi$

Total torque $\qquad T = 2T_F + T_W = (2K_F + K_W)\frac{G\varphi}{L}$

$$\frac{G\varphi}{L} = \frac{T}{2K_F + K_W}, \quad T_F = \frac{K_F T}{2K_F + K_W}, \quad T_W = \frac{K_W T}{2K_W + K_W}$$

$$T_F = \frac{(0.2138)(5000)}{(2)(0.2138) + 0.0563} = 2211 \text{ lb·in}; \quad T_W = \frac{(0.0563)(5000)}{(2)(0.2138) + 0.0563} = 557 \text{ lb·in}$$

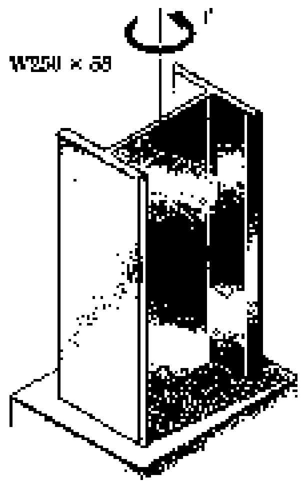
(a) $\quad \tau_F = \frac{T_F}{c_1 a b^2} = \frac{2221}{(0.3219)(7.995)(0.435)^2} = 4570 \text{ psi} = 4.57 \text{ ksi}$ ◄

(b) $\quad \tau_W = \frac{T_W}{c_1 a b^2} = \frac{557}{(0.3249)(7.13)(0.285)^2} = 2960 \text{ psi} = 2.96 \text{ ksi}$ ◄

(c) $\quad \frac{G\varphi}{L} = \frac{T}{2K_F + K_W} \quad \therefore \quad \varphi = \frac{TL}{G(2K_F + K_W)}$ $\qquad$ where $L = 8 \text{ ft} = 96$ in.

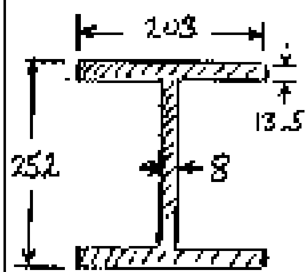
$$\varphi = \frac{(5000)(96)}{(11.2 \times 10^6)[(2)(0.2138) + 0.563]} = 88.6 \times 10^{-3} \text{ rad} = 5.08°$$ ◄

PROBLEM 3.136



3.136 A 3-m-long steel member has an W 250 × 58 cross section. Knowing that G = 77 GPa and that the allowable shearing stress is 35 MPa, determine (*a*) the largest torque **T** which may be applied, (*b*) the corresponding angle of twist. Refer to Appendix C for the dimensions of the cross section and neglect the effect of stress concentrations. (See hint of Prob. 3.135.)

SOLUTION



Flange: $a = 203$ mm, $b = 13.5$ mm, $\frac{a}{b} = 15.04$

$$C_1 = C_2 = \frac{1}{3}\left(1 - 0.630\frac{b}{a}\right) = 0.3194$$

$$\varphi_F = \frac{T_F L}{C_2 a b^3 G} \quad \therefore \quad T_F = C_1 a b^3 \frac{G\varphi}{L} = K_F \frac{G\varphi}{L}$$

$$K_F = (0.3194)(0.203)(0.0135)^3 = 159.53 \times 10^{-9} \ \text{m}^4$$

Web: $a = 252 - (2)(13.5) = 225$ mm, $b = 8$ mm

$$\frac{a}{b} = 28.13, \quad C_1 = C_2 = \frac{1}{3}\left(1 - 0.63\frac{b}{a}\right) = 0.3259$$

$$\varphi_w = \frac{T_w L}{C_2 a b^3 G} \quad \therefore \quad T_w = C_2 a b^3 \frac{G\varphi}{L} = K_w \frac{G\varphi}{L}$$

$$K_w = (0.3259)(0.225)(0.008)^3 = 37.54 \times 10^{-9} \ \text{m}^4$$

For matching twist angles $\varphi_F = \varphi_w = \varphi$

Total torque: $T = 2T_F + T_w = (2K_F + K_w)\frac{G\varphi}{L}$

$$\frac{G\varphi}{L} = \frac{T}{2K_F + K_w}, \quad T_F = \frac{K_F T}{2K_F + K_w} \quad \therefore \quad T = \frac{2K_F + K_w}{K_F} T_F$$

$$T_w = \frac{K_w T}{2K_F + K_w} \quad \therefore \quad T = \frac{2K_F + K_w}{K_w} T_w$$

Allowable value for T based on allowable value for T_F

$$T_F = c_1 a b^2 \tau = (0.3194)(0.203)(0.0135)^2(35 \times 10^6) = 413.6 \ \text{N}\cdot\text{m}$$

$$T = \frac{(2)(159.53) + (37.54)}{159.53}(413.6) = 924.5 \ \text{N}\cdot\text{m}$$

Allowable value for T base on allowable value for T_w

$$T_w = c_1 a b^2 \tau = (0.3259)(0.225)(0.008)^2(35 \times 10^6) = 164.25 \ \text{N}\cdot\text{m}$$

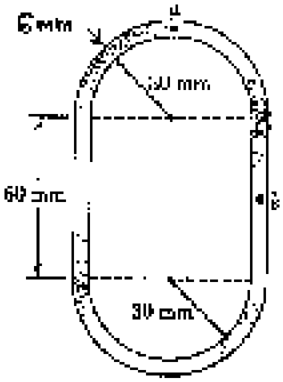
$$T = \frac{(2)(159.53) + 37.54}{37.54}(164.25) = 1560 \ \text{N}\cdot\text{m}$$

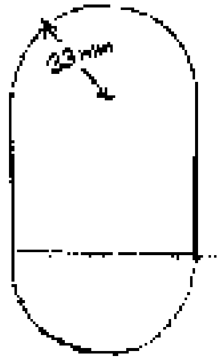
Choose smaller value $\qquad T = 925$ N-m ◄

$$\varphi = \frac{TL}{(2K_F + K_w)G} = \frac{(924.5)(3.00)}{(356.6 \times 10^{-9})(77 \times 10^9)} = 101.0 \times 10^{-3} \ \text{rad} = 5.79° \quad ◄$$

PROBLEM 3.137

3.137 and 3.138 A 750-N·m torque **T** is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentration, determine the shearing stress at points *a* and *b*. Thickness = 6 mm.





SOLUTION

Area bounded by center line

$$a = 2\,\frac{\pi}{2}(33)^2 + (60)(66) = 7381\ \text{mm}^2$$

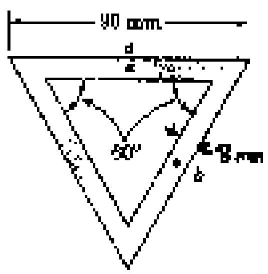
$$= 7381 \times 10^{-6}\ \text{m}^2$$

$t = 0.006$ m at both a and b.

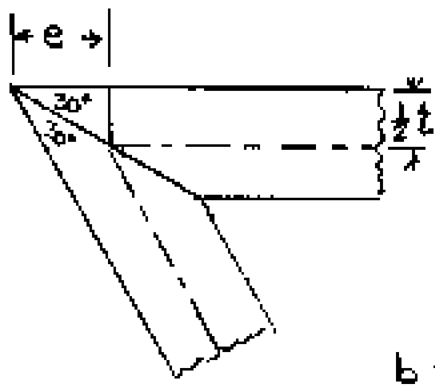
$$\tau = \frac{T}{2ta} = \frac{750}{(2)(0.006)(7381\times10^{-6})} = 8.47\times10^{6}\ \text{Pa} = 8.47\ \text{MPa}$$

PROBLEM 3.138

3.137 and 3.138 A 750-N·m torque **T** is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentration, determine the shearing stress at points *a* and *b*. Thickness = 8 mm.



SOLUTION

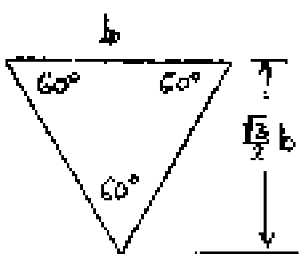


Detail of corner

$$\tfrac{1}{2}t = e\tan 30°$$

$$e = \frac{t}{2\tan 30°}$$

$$= \frac{8}{2\tan 30°} = 6.928\ \text{mm}$$



$$b = 90 - 2e = 76.144\ \text{mm}$$

Area bounded by center line

$$a = \tfrac{1}{2}\,b\,\frac{\sqrt{3}}{2}\,b = \frac{\sqrt{3}}{4}b^2 = \frac{\sqrt{3}}{4}(76.144)^2$$

$$= 2510.6\ \text{mm}^2 = 2510.6\times10^{-6}\ \text{m}^2$$

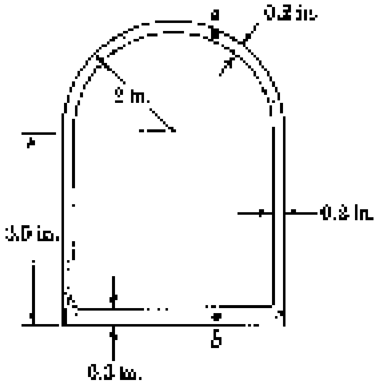
$$t = 0.008\ \text{m}$$

$$\tau = \frac{T}{2ta} = \frac{750}{(2)(0.008)(2510.6\times10^{-6})} = 18.67\times10^{6}\ \text{Pa}$$

$$= 18.67\ \text{MPa}$$

PROBLEM 3.139

3.139 and 3.140 A 50-kip·in. torque **T** is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentration, determine the shearing stress at points *a* and *b*.

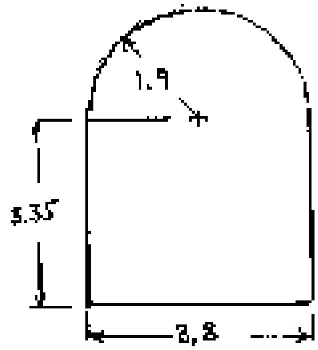


SOLUTION

Area bounded by center line.

$$\mathcal{Q} = \frac{\pi}{2}(1.9)^2 + (3.35)(3.8)$$

$$= 18.40 \text{ in}^2$$



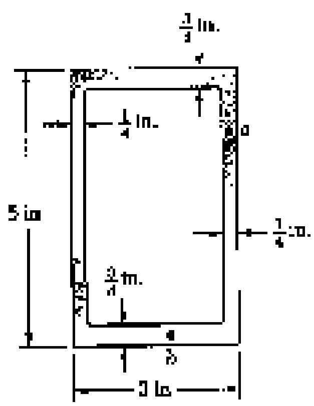
At point a $\quad t = 0.2$ in

$$\tau = \frac{T}{2t\mathcal{Q}} = \frac{50}{(2)(0.2)(18.40)} = 6.79 \text{ ksi}$$

At point b $\quad t = 0.3$ in. $\quad \tau = \frac{T}{2t\mathcal{Q}} = \frac{50}{(2)(0.3)(18.40)} = 4.53$ ksi

PROBLEM 3.140

3.139 and 3.140 A 50-kip·in. torque **T** is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentration, determine the shearing stress at points *a* and *b*.

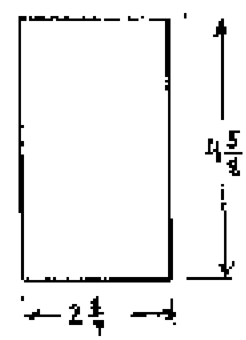


SOLUTION

Area bounded by centerline

$$\mathcal{Q} = (2\tfrac{3}{4})(4\tfrac{5}{8}) = 12\tfrac{23}{32}$$

$$= 12.719 \text{ in}^2$$



At point a $\quad t = \frac{1}{4}$ in $= 0.25$ in

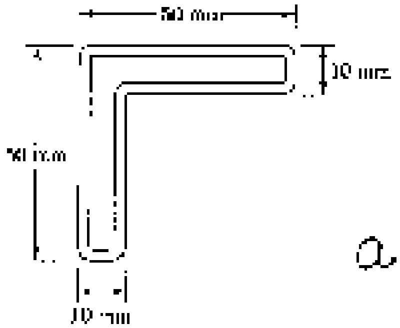
$$\tau = \frac{T}{2t\mathcal{Q}} = \frac{50}{(2)(0.25)(12.719)} = 7.86 \text{ ksi}$$

At point b $\quad t = \frac{3}{8}$ in. $= 0.375$ in

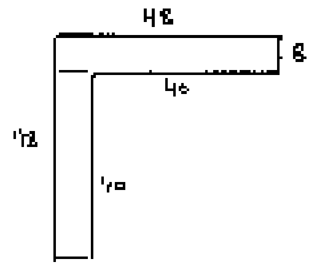
$$\tau = \frac{T}{2t\mathcal{Q}} = \frac{50}{(2)(0.375)(12.719)} = 5.24 \text{ ksi}$$

PROBLEM 3.141

3.141 and 3.142 A hollow member having the cross section shown is formed from sheet metal of 2-mm thickness. Knowing that the shearing stress must not exceed 3 MPa, determine the largest torque which may be applied to the member.



SOLUTION



Area bounded by centerline

$\mathcal{a} = (48)(8) + (40)(8)$

$= 704 \text{ mm}^2 = 704 \times 10^{-6} \text{ m}^2$

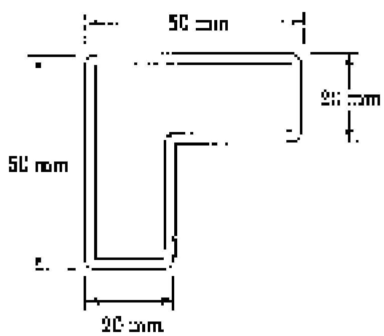
$t = 0.002 \text{ m}$

$\tau = \dfrac{T}{2t\mathcal{a}} \quad \therefore \quad T = 2t\mathcal{a}\tau = (2)(0.002)(704 \times 10^{-6})(3 \times 10^{6})$

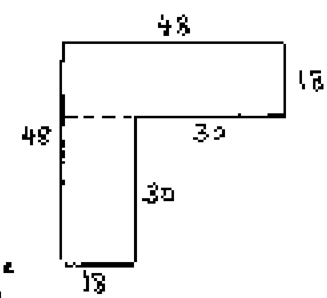
$= 8.45 \text{ N}\cdot\text{m}$ ◀

PROBLEM 3.142

3.141 and 3.142 A hollow member having the cross section shown is formed from sheet metal of 2-mm thickness. Knowing that the shearing stress must not exceed 3 MPa, determine the largest torque which may be applied to the member.



SOLUTION



Area bounded by centerline

$\mathcal{a} = (48)(18) + (30)(18)$

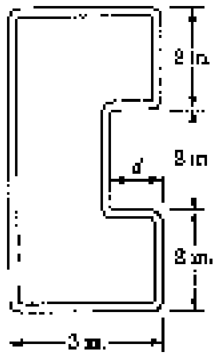
$= 1404 \text{ mm}^2 = 1404 \times 10^{-6} \text{ m}^2$

$t = 0.002 \text{ m}$

$\tau = \dfrac{T}{2t\mathcal{a}} \quad \therefore \quad T = 2t\mathcal{a}\tau = (2)(0.002)(1404 \times 10^{-6})(3 \times 10^{6})$

$= 16.85 \text{ N}\cdot\text{m}$ ◀

PROBLEM 3.143



3.143 and 3.144 A hollow member having the cross section shown is to be formed from sheet metal of 0.06 in. thickness. Knowing that a 1250 lb·in. torque will be applied to the member, determine the smallest dimension d which may be used if the shearing stress is not to exceed 750 psi.

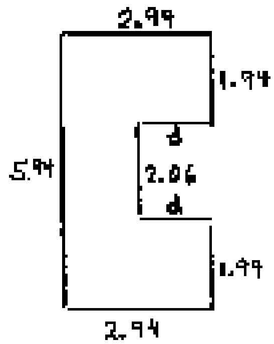
SOLUTION

Area bounded by center line

$$a = (5.94)(2.94) - 2.06d = 17.4636 - 2.06d$$

$t = 0.06$ in, $\tau = 750$ psi, $T = 1250$ lb·in

$$\tau = \frac{T}{2ta}$$

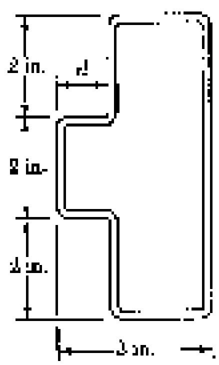


$$a = \frac{T}{2t\tau}$$

$$17.4636 - 2.06d = \frac{1250}{(2)(0.06)(750)} = 13.8889$$

$$d = \frac{3.5747}{2.06} = 1.735 \text{ in}$$

PROBLEM 3.144



3.143 and 3.144 A hollow member having the cross section shown is to be formed from sheet metal of 0.06 in. thickness. Knowing that a 1250 lb·in. torque will be applied to the member, determine the smallest dimension d which may be used if the shearing stress is not to exceed 750 psi.

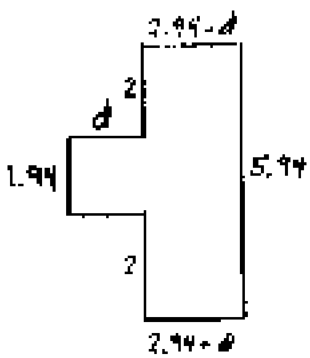
SOLUTION

Area bounded by center

$$a = (5.94)(2.94 - d) + 1.94d = 17.4636 - 4.00d$$

$t = 0.06$ in, $\tau = 750$ psi, $T = 1250$ lb·in

$$\tau = \frac{T}{2ta}$$

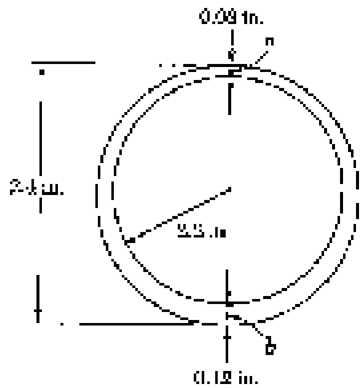


$$a = \frac{T}{2t\tau}$$

$$17.4636 - 4.00d = \frac{1250}{(2)(0.06)(750)} = 13.8889$$

$$d = \frac{3.5747}{4.00} = 0.894 \text{ in}$$

PROBLEM 3.145



3.145 A hollow cylindrical shaft was designed to have a uniform wall thickness of 0.1 in. Defective fabrication, however, resulted in the shaft having the cross section shown. Knowing that a 15-kip·in. torque **T** is applied to the shaft, determine the shearing stress at points a and b.

SOLUTION

Radius of outer circle = 1.2 in
Radius of inner circle = 1.1 in

Mean radius = 1.15 in

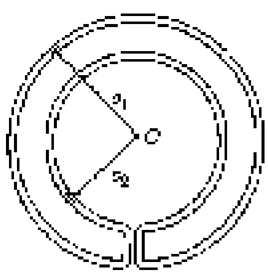
Area bounded by centerline

$\mathcal{Q} = \pi r_m^2 = \pi(1.15)^2 = 4.155 \text{ in}^2$

At point a $\quad t = 0.08$ in $\quad \tau = \frac{T}{2t\mathcal{Q}} = \frac{15}{(2)(0.08)(4.155)} = 22.6$ ksi

At point b $\quad t = 0.12$ in $\quad \tau = \frac{T}{2t\mathcal{Q}} = \frac{15}{(2)(0.12)(4.155)} = 15.04$ ksi

PROBLEM 3.146



3.146 A cooling tube having the cross section shown is formed from a sheet of stainless steel of 3 mm thickness. The radii $c_1 = 150$ mm and $c_2 = 100$ mm are measured to the centerline of the sheet metal. Knowing that a torque of magnitude $T = 3$ kN·m is applied to the tube, determine (a) the maximum shearing stress in the tube, (b) the magnitude of the torque carried by the outer circular shell. Neglect the dimension of the small opening where the outer and inner shells are connected.

SOLUTION

Area bounded by centerline

$\mathcal{Q} = \pi(c_1^2 - c_2^2) = \pi(150^2 - 100^2) = 39.27 \times 10^3 \text{ mm}^2$

$= 39.27 \times 10^{-3} \text{ m}^2$

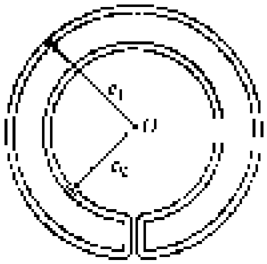
$t = 0.003$ m

(a) $\tau = \frac{T}{2t\mathcal{Q}} = \frac{3 \times 10^3}{(2)(0.003)(39.27 \times 10^{-3})} = 12.73 \times 10^6 \text{ Pa} = 12.73 \text{ MPa}$

(b) $T_1 = (2\pi c_1 t \tau c_1) = 2\pi c_1^2 t \tau$

$= 2\pi(0.150)^2(0.003)(12.73 \times 10^6) = 5.40 \times 10^3 \text{ N·m}$
$= 5.40 \text{ kN·m}$

PROBLEM 3.147



3.147 A cooling tube having the cross section as shown is formed from a sheet of stainless steel of thickness t. The radii c_1 and c_2 are measured to the centerline of the sheet metal. Knowing that a torque **T** is applied to the tube, determine in terms of T, c_1, c_2, and t the maximum shearing stress in the tube.

SOLUTION

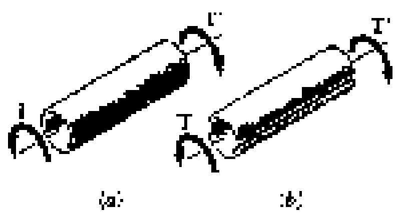
Area bounded by centerline

$$a = \pi(c_1^2 - c_2^2)$$

Shearing stress

$$\tau = \frac{T}{2ta} = \frac{T}{2\pi t(c_1^2 - c_2^2)}$$

PROBLEM 3.148



3.148 Equal torques are applied to thin-walled tubes of the same length L, same thickness t, and same radius c. One of the tubes has been slit lengthwise as shown. Determine (*a*) the ratio τ_b/τ_a of the maximum shearing stresses in the tubes, (*b*) the ratio φ_b/φ_a of the angles of twist of the shafts.

SOLUTION

Without slit

Area bounded by centerline: $a = \pi c^2$

$$\tau_a = \frac{T}{2ta} = \frac{T}{2\pi c^2 t}$$

$$J \approx 2\pi c^3 t \qquad \varphi_a = \frac{TL}{GJ} = \frac{TL}{2\pi c^3 t G}$$

With slit: $\quad a = 2\pi c, \quad b = t, \quad \frac{a}{b} = \frac{2\pi c}{t} \gg 1$

$$c_1 = c_2 = \frac{1}{3}$$

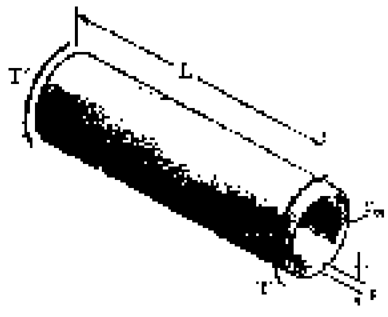
$$\tau_b = \frac{T}{c_1 a b^2} = \frac{3T}{2\pi c t^2}$$

$$\varphi_b = \frac{T}{c_2 a b^3 G} = \frac{3TL}{2\pi c t^3 G}$$

Stress ratio: $\quad \frac{\tau_b}{\tau_a} = \frac{3T}{2\pi c t^2} \cdot \frac{2\pi c^2 t}{T} = \frac{3c}{t}$

Twist ratio: $\quad \frac{\varphi_b}{\varphi_a} = \frac{3TL}{2\pi c t^3 G} \cdot \frac{2\pi c^3 t G}{TL} = \frac{3c^2}{t^2}$

PROBLEM 3.149



3.149 A hollow cylindrical shaft of length L, mean radius c_m and uniform thickness t is subjected to torques of magnitude T. Consider, on the one hand, the values of the average shearing stress τ_{ave} and the angle of twist ϕ obtained from the elastic torsion formulas developed in Secs. 3.4 and 3.5 and, on the other hand, the corresponding values obtained from the formulas developed in Sec. 3.13 for thin-walled hollow shafts. (*a*) Show that the relative error introduced by using the thin-wall-shaft formulas rather than the elastic torsion formulas is the same for τ_{ave} and ϕ and that the relative error is positive and proportional to the square of the ratio t/c_m. (*b*) Compare the percent error corresponding to values of the ratio t/c_m equal 0.1, 0.2 and 0.4.

SOLUTION

Let c_2 = outer radius $= c_m + \frac{1}{2}t$ and c_1 = inner radius $= c_m - \frac{1}{2}t$

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(c_2^2 + c_1^2)(c_2 + c_1)(c_2 - c_1)$$

$$= \frac{\pi}{2}(c_m^2 + c_m t + \tfrac{1}{4}t^2 + c_m^2 - c_m t + \tfrac{1}{4}t^2)(2c_m)\,t$$

$$= 2\pi(c_m^2 + \tfrac{1}{4}t^2)\,c_m t$$

$$\tau_m = \frac{Tc_m}{J} = \frac{T}{2\pi(c_m^2 + \frac{1}{4}t^2)t}$$

$$\varphi_1 = \frac{TL}{JG} = \frac{TL}{2\pi(c_m^2 + \frac{1}{4}t^2)c_m t\,G}$$

Area bounded by centerline $\quad \mathcal{Q} = \pi c_m^2$

$$\tau_{ave} = \frac{T}{2t\mathcal{Q}} = \frac{T}{2\pi c_m^2 t}$$

$$\varphi_2 = \frac{TL}{4\mathcal{Q}^2 G}\oint\frac{ds}{t} = \frac{TL(2\pi c_m/t)}{4(\pi c_m^2)^2 G} = \frac{TL}{2\pi c_m^3 t\,G}$$

Ratios:
$$\frac{\tau_{ave}}{\tau_m} = \frac{T}{2\pi c_m^2 t}\cdot\frac{2\pi(c_m^2 + \frac{1}{4}t^2)t}{T} = 1 + \frac{1}{4}\frac{t^2}{c_m^2}$$

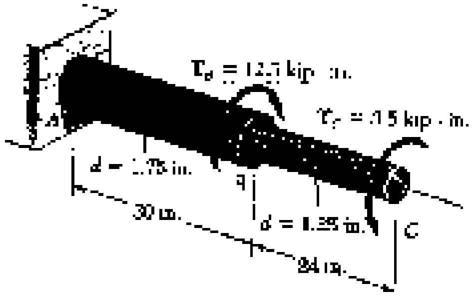
$$\frac{\varphi_2}{\varphi_1} = \frac{TL}{2\pi c_m^3 t G}\cdot\frac{2\pi(c_m^2 + \frac{1}{4}t^2)c_m t G}{TL} = 1 + \frac{1}{4}\frac{t^2}{c_m^2}$$

$$\frac{\tau_{ave}}{\tau_m} - 1 = \frac{\varphi_2}{\varphi_1} - 1 = \frac{1}{4}\frac{t^2}{c_m^2}$$

$\frac{t}{c_m}$	0.1	0.2	0.4
$\frac{1}{4}\frac{t^2}{c_m^2}$	0.0025	0.01	0.04
	0.25%	1%	4%

PROBLEM 3.150

3.150 For the solid brass shaft shown, determine the maximum shearing stress in (*a*) portion *AB*, (*b*) portion *BC*.



SOLUTION

AB : $T = 12.5 - 3.5 = 9$ kip·in , $c = \frac{1}{2}d = 0.875$ in

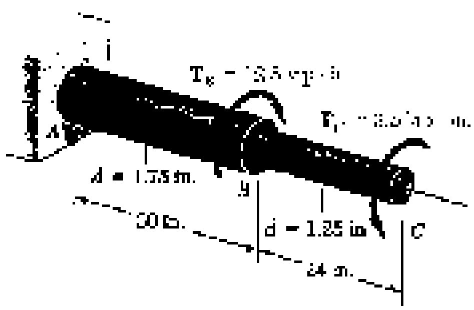
$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(9)}{\pi(0.875)^3} = 8.55 \text{ ksi}$$

BC : $T = 3.5$ kip·in , $c = \frac{1}{2}d = 0.625$ in

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(3.5)}{\pi(0.625)^3} = 9.13 \text{ ksi}$$

PROBLEM 3.151

3.151 Knowing that $G = 5.6 \times 10^6$ psi for the solid brass shaft shown, determine the angle of twist at point *C*.



SOLUTION

AB : $T = 12.5 - 3.5 = 9$ kip·in , $c = \frac{1}{2}d = 0.875$ in

$G = 5.6 \times 10^6$ psi $= 5.6 \times 10^3$ ksi

$J_{AB} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.875)^4 = 0.92077 \text{ in}^4$

$$\varphi_{AB} = \frac{T_{AB}L_{AB}}{GJ_{AB}} = \frac{(9)(30)}{(5.6\times10^3)(0.92077)} = 52.36 \times 10^{-3} \text{ rad}$$

BC : $T = 3.5$ kip·in , $c = \frac{1}{2}d = 0.625$ in

$J_{BC} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.625)^4 = 0.23968 \text{ in}^4$

$$\varphi_{BC} = \frac{T_{BC}L_{BC}}{GJ} = \frac{(3.5)(24)}{(5.6\times10^3)(0.23968)} = 62.60 \times 10^{-3} \text{ rad}$$

Net angle of twist $\varphi_{BC} - \varphi_{AB} = 10.24 \times 10^{-3}$ rad $= 0.587°$

PROBLEM 3.152

3.152 The stepped shaft shown rotates at 900 rpm. Knowing that $\tau_{all} = 42$ MPa, determine the maximum power which can be transmitted if the radius r of the fillet is (a) 12 mm, (b) 20 mm.

SOLUTION

frequency $f = \frac{900 \text{ rpm}}{60 \text{ rpm/Hz}} = 15 \text{ Hz}$

$D = 160$ mm, $d = 80$ mm, $\tau_{all} = 42 \text{ MPa} = 42 \times 10^6$ Pa

$c = \frac{1}{2}d = 40 \text{ mm} = 0.040$ m $\qquad \frac{D}{d} = \frac{160}{80} = 2.0$

(a) $r = 12$ mm, $\frac{r}{d} = \frac{12}{80} = 0.15 \qquad K = 1.33$

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.040)^3(42\times10^6)}{(2)(1.33)} = 3.175\times10^3 \text{ N}\cdot\text{m}$$

$$P = 2\pi f T = (2\pi)(15)(3.175\times10^3) = 300\times10^3 \text{ W} = 300 \text{ kW}$$

(b) $r = 20$ mm, $\frac{r}{d} = \frac{20}{80} = 0.25 \qquad K = 1.20$

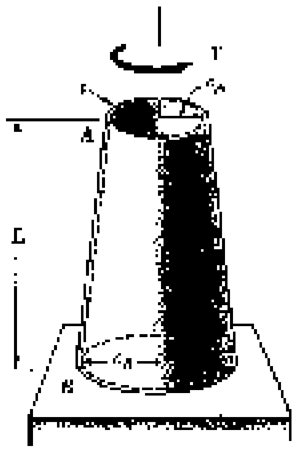
$$T = \frac{\pi c^3 \tau}{2K} = \frac{(\pi)(0.040)^3(42\times10^6)}{(2)(1.20)} = 3.520\times10^3 \text{ N}\cdot\text{m}$$

$$P = 2\pi f T = (2\pi)(15)(3.520\times10^3) = 332\times10^3 \text{ W} = 332 \text{ kW}$$

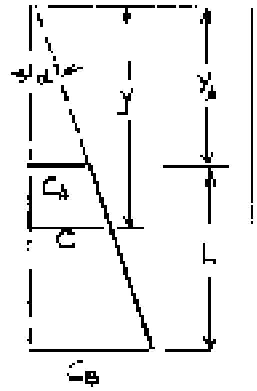
PROBLEM 3.153

3.153 The long, hollow, tapered shaft AB has a uniform thickness t. Denoting by G the modulus of rigidity, show that the angle of twist at end A is

$$\phi_A = \frac{TL}{4\pi Gt}\frac{c_A + c_B}{c_A^2 c_B^2}$$



SOLUTION



From geometry

$$\tan\alpha = \frac{c_B - c_A}{L}$$

$$c = y\tan\alpha = \frac{c_B - c_A}{L}y$$

$$y_A = \frac{c_A}{\tan\alpha} = \frac{c_A L}{c_B - c_A}$$

$$y_B = \frac{c_B}{\tan\alpha} = \frac{c_B L}{c_B - c_A}$$

$$J = 2\pi c^3 t = 2\pi\frac{(c_B - c_A)^3}{L^3}y^3 t$$

$$\varphi = \int_{y_A}^{y_B}\frac{T\,dy}{GJ} = \frac{TL^3}{2\pi(c_B - c_A)^3 tG}\int_{y_A}^{y_B}\frac{dy}{y^3} = \frac{TL^3}{2\pi(c_B - c_A)^3 tG}\left(-\frac{1}{2y^2}\Big|_{y_A}^{y_B}\right)$$

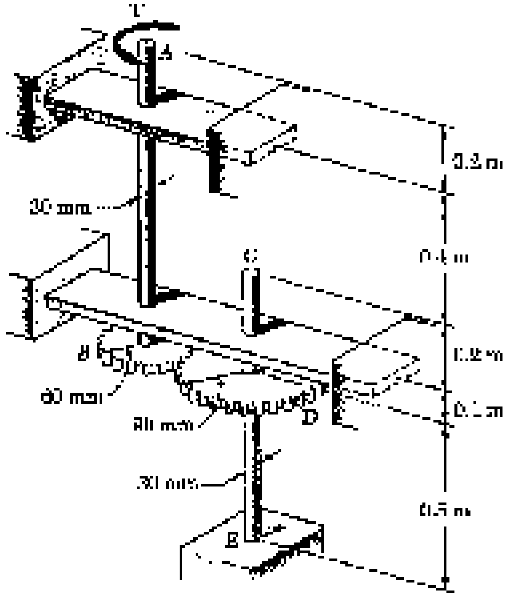
$$= \frac{TL^3}{4\pi(c_B - c_A)^3 tG}\left\{\frac{1}{y_A^2} - \frac{1}{y_B^2}\right\} = \frac{TL^3}{4\pi(c_B - c_A)^3 tG}\left\{\frac{(c_B - c_A)^2}{L^2 c_A^2} - \frac{(c_B - c_A)^2}{L^2 c_B^2}\right\}$$

$$= \frac{TL}{4\pi(c_B - c_A)tG}\left\{\frac{1}{c_A^2} - \frac{1}{c_B^2}\right\} = \frac{TL\,(c_B^2 - c_A^2)}{4\pi(c_B - c_A)tGc_A^2 c_B^2}$$

$$= \frac{TL\,(c_B + c_A)}{4\pi Gt\,c_A^2 c_B^2}$$

PROBLEM 3.154

3.154 Two solid steel shafts, each of 30-mm diameter, are connected by the gears shown. Knowing that $G = 77$ GPa, determine the angle through which end A rotates when a 200-N·m torque $\mathbf{T}$ is applied at A.



SOLUTION

Calculation of torques

Circumferential contact force between gears B and D

$$F = \frac{T_{AB}}{r_B} = \frac{T_{DE}}{r_D}$$

$$T_{DE} = \frac{r_D}{r_B} T_{AB} = \frac{90}{60}(200) = 300 \text{ N·m}$$

Twist in shaft DE

$$J_{DE} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4 = 79.522 \times 10^{-9} \text{ m}^4$$

$$L_{DE} = 0.5 \text{ m}$$

$$\varphi_{DE} = \frac{T_{DE}L_{DE}}{GJ_{DE}} = \frac{(300)(0.5)}{(77\times10^9)(79.522\times10^{-9})} = 24.497\times10^{-3} \text{ rad}$$

Rotation at D $\qquad \varphi_D = \varphi_{DE} = 24.497\times10^{-3}$ rad

Circumferential displacement at gear circles $\qquad \delta = r_D\varphi_D = r_B\varphi_B$

Rotation at B $\qquad \varphi_B = \frac{r_D}{r_B}\varphi_D = \frac{90}{60}(24.497\times10^{-3}) = 36.745\times10^{-3}$ rad

Twist in shaft AB

$$L_{AB} = 0.1 + 0.2 + 0.4 + 0.2 = 0.9 \text{ m}, \quad J_{AB} = 79.522\times10^{-9} \text{ m}^4$$

$$\varphi_{AB} = \frac{T_{AB}L_{AB}}{GJ_{AB}} = \frac{(200)(0.9)}{(77\times10^9)(79.522\times10^{-9})} = 29.396\times10^{-3} \text{ rad}$$

Rotation at A $\qquad \varphi_A = \varphi_B + \varphi_{AB}$

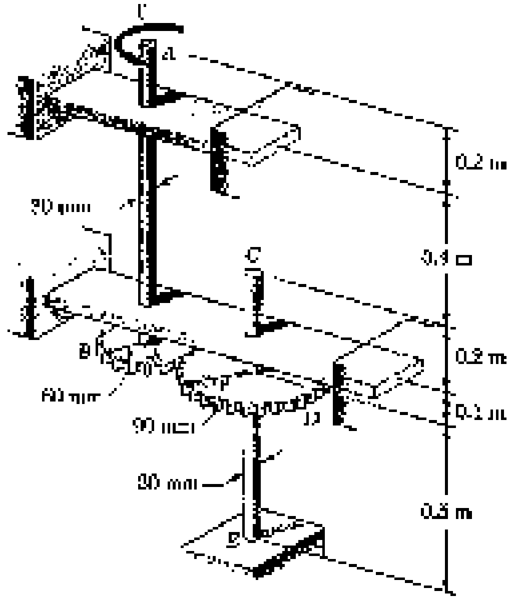
$$= 36.745\times10^{-3} + 29.396\times10^{-3} \text{ rad} = 66.14\times10^{-3} \text{ rad}$$

$$= 3.79° \quad \blacktriangleleft$$

PROBLEM 3.155

3.155 The angle of rotation of end A of the gear-and-shaft system shown must not exceed 4°. Knowing that the shafts are made of a steel for which $\tau_{all} = 65$ MPa and $G = 77$ GPa, determine the largest torque T which can be safely applied at end A.

SOLUTION



Calculation of torque ratio

Contact force F

$$F = \frac{T_{AB}}{r_B} = \frac{T_{DE}}{r_D}$$

$$T_{DE} = \frac{r_D}{r_B} T_{AB} = \frac{90}{60} T = 1.5\,T$$

Since larger torque occurs in shaft, we find the torque limit based on stress in shaft DE

$$\tau = \frac{T_{DE}\,c}{J} = \frac{2\,T_{DE}}{\pi c^3} = \frac{(2)(1.5)\,T}{\pi c^3}$$

$$T = \frac{\pi}{(2)(1.5)} c^3 \tau = \frac{\pi}{3}(0.015)^3(65\times10^6) = 229.7 \text{ N·m}$$

Twist in shaft DE

$J_{DE} = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.015)^4 = 79.522\times10^{-9}$ m⁴ , $L_{DE} = 0.5$ m , $T_{DE} = 1.5\,T$

$$\varphi_{DE} = \frac{T_{DE} L_{DE}}{G J_{DE}} = \frac{(1.5\,T)(0.5)}{(77\times10^9)(79.522\times10^{-9})} = 122.485\times10^{-6}\,T$$

Rotation at D $\qquad \varphi_D = 122.485\times10^{-6}\,T$

Circumferential displacement at gear circles $\quad s = r_D\varphi_D = r_B\varphi_B$

Rotation at B $\qquad \varphi_B = \frac{r_D}{r_B}\varphi_D = \frac{90}{60}(122.485\times10^{-6}\,T) = 183.728\times10^{-6}\,T$

Twist in shaft AB : $\quad L_{AB} = 0.1 + 0.2 + 0.4 + 0.2 = 0.9$ m

$$\varphi_{AB} = \frac{T_{AB} L_{AB}}{G J_{AB}} = \frac{T\,(0.9)}{(77\times10^9)(79.522\times10^{-9})} = 146.982\times10^{-6}\,T$$

Rotation at A $\qquad \varphi_A = \varphi_B + \varphi_{AB} = 183.728\times10^{-6}\,T + 146.982\times10^{-6}\,T$

$$= 330.71\times10^{-6}\,T$$

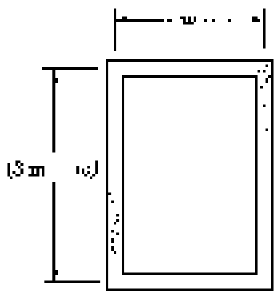
Rotation limit $\qquad \varphi_A = 4° = 69.813\times10^{-3}$ rad.

Equating $\quad 330.71\times10^{-6}\,T = 69.813\times10^{-3} \;;\; T = 211.1$ N·m

Limiting torque is the smaller value $\qquad T = 211$ N·m

PROBLEM 3.156

3.156 A sheet metal strip of width 6 in. and 0.12 in. thickness is to be formed into a tube of rectangular cross section. Knowing that $\tau_{all} = 4$ ksi, determine the largest torque that may be applied to the tube when (a) $w = 1.5$ in., (b) $w = 1.2$ in., (c) $w = 1$ in.



SOLUTION

perimeter $\quad p = 6 \text{ in} = 2w + 2d$

depth $\quad d = \frac{p}{2} - w$

Area bounded by centerline $\quad a = wd = w\left(\frac{p}{2} - w\right)$

$$\tau = \frac{T}{2ta} \quad \therefore \quad T = 2ta\tau = 2tw\left(\frac{p}{2} - w\right)\tau$$

Data: $t = 0.12$ in, $p = 6$ in, $\tau = 4$ ksi

(a) $w = 1.5 \qquad T = (2)(0.12)(1.5)(1.5)(4) = 2.16$ kip·in ◄

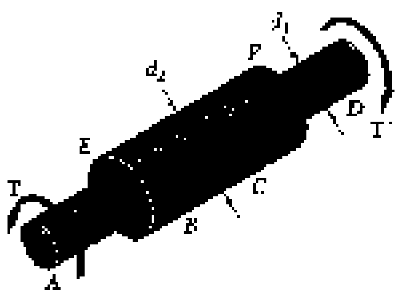
(b) $w = 1.2 \qquad T = (2)(0.12)(1.2)(1.8)(4) = 2.07$ kip·in ◄

(c) $w = 1.0 \qquad T = (2)(0.12)(1.0)(2.0)(4) = 1.92$ kip·in ◄

The largest allowable torque corresponds to a square section.

PROBLEM 3.157

3.157 Two solid brass rods AB and CD are brazed to a brass sleeve EF. Determine the ratio d_2/d_1 for which the same maximum shearing stress occurs in the rods and in the sleeve.



SOLUTION

Let $c_1 = \frac{1}{2}d_1$ and $c_2 = \frac{1}{2}d_2$

Shaft AB $\qquad \tau_1 = \frac{Tc_1}{J_1} = \frac{2T}{\pi c_1^3}$

Sleeve EF $\qquad \tau_2 = \frac{Tc_2}{J_2} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$

For equal stresses $\qquad \frac{2T}{\pi c_1^3} = \frac{2Tc_2}{\pi(c_2^4 - c_1^4)}$

$$c_2^4 - c_1^4 = c_1^3 c_2$$

Let $x = \frac{c_2}{c_1} \qquad x^4 - 1 = x \quad$ or $\quad x = \sqrt[4]{1 + x}$

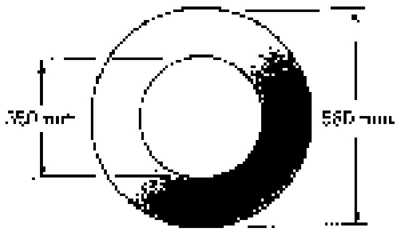
Solve by successive approximations starting with $x_0 = 1.0$

$x_1 = \sqrt[4]{2} = 1.189, \quad x_2 = \sqrt[4]{2.189} = 1.216 \quad x_3 = \sqrt[4]{2.216} = 1.220$

$x_4 = \sqrt[4]{2.220} = 1.221 \quad x_5 = \sqrt[4]{2.221} = 1.221$ (converged)

$x = 1.221 \qquad \frac{c_2}{c_1} = \frac{d_2}{d_1} = 1.221$ ◄

PROBLEM 3.158



3.158 One of the two hollow steel drive shafts of an ocean liner is 75 m long and has the cross section shown. Knowing that $G = 77$ GPa and that the shaft transmits 44 MW to its propeller when rotating at 144 rpm, determine (*a*) the maximum shearing stress in the shaft, (*b*) the angle of twist of the shaft.

SOLUTION

$L = 75\ \text{m}; \qquad f = 144\ \text{rpm} = \frac{144}{60} = 2.4\ \text{Hz}$

$P = 44\ \text{MW} = 44 \times 10^6\ \text{W}$

$P = 2\pi f T \quad \therefore \quad T = \frac{P}{2\pi f} = \frac{44\times10^6}{2\pi(2.4)} = 2.9178\times10^6\ \text{N}\cdot\text{m}$

$c_1 = \frac{d_1}{2} = \frac{320}{2} = 160\ \text{mm} = 0.160\ \text{m}$

$c_2 = \frac{d_2}{2} = \frac{580}{2} = 290\ \text{mm} = 0.290\ \text{m}$

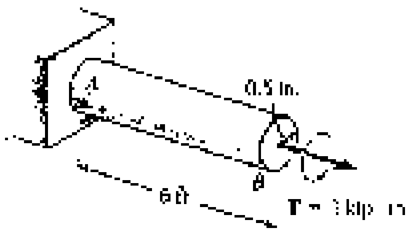
$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}(0.290^4 - 0.160^4) = 10.08\times10^{-3}\ \text{m}$

(a) $\quad \tau = \frac{Tc_2}{J} = \frac{(2.9178\times10^6)(0.290)}{10.08\times10^{-3}} = 83.9\times10^6\ \text{Pa} = 83.9\ \text{MPa}$

(b) $\quad \varphi = \frac{TL}{GJ} = \frac{(2.9178\times10^6)(75)}{(77\times10^9)(10.08\times10^{-3})} = 281.9\times10^{-3}\ \text{rad} = 16.15°$

PROBLEM 3.159

3.159 The shaft AB is made of a material which is elastoplastic with $\tau_Y = 12.5$ ksi and $G = 4 \times 10^6$ psi. For the loading shown, determine (a) the radius of the elastic core of the shaft, (b) the angle of twist at end B.



SOLUTION

$c = 0.5$ in

$$T_Y = \frac{J\tau_Y}{c} = \frac{\pi}{2}c^3\tau = \frac{\pi}{2}(0.5)^3(12.5) = 2.454 \text{ kip}\cdot\text{in}$$

$T = 3$ kip·in $> T_Y$ plastic region with elastic core

$$T = \frac{4}{3}T_Y\left(1 - \frac{1}{4}\frac{\rho_Y^3}{c^3}\right) \therefore \frac{\rho_Y^3}{c^3} = 4 - \frac{3T}{T_Y} = 4 - \frac{(3)(2.454)}{3} = 0.33307$$

$$\frac{\rho_Y}{c} = 0.69318 \qquad \rho_Y = (0.69318)(0.5) = 0.347 \text{ in.}$$

$L = 6$ ft $= 72$ in. $\qquad G = 4 \times 10^6$ psi $= 4 \times 10^3$ ksi

$$\varphi_Y = \frac{T_Y L}{JG} = \frac{2T_Y L}{\pi c^4 G} = \frac{(2)(2.454)(72)}{\pi(0.5)^4(4\times10^3)} = 0.4499 \text{ rad}$$

$$\frac{\varphi_Y}{\varphi} = \frac{\rho_Y}{c} \quad \therefore \quad \varphi = \frac{\varphi_Y}{\rho_Y/c} = \frac{0.4499}{0.69318} = 0.64904 \text{ rad} = 37.2°$$

PROBLEM 3.160

3.160 If the 3 kip·in. torque applied to the shaft of Prob. 3.159 is removed, determine (a) the magnitude and location of the maximum residual shearing stress in the shaft, (b) the permanent angle of twist of the shaft.

SOLUTION

From the solution of PROBLEM 3.159, at the end of loading $T = 3$ kip·in

$$\frac{\rho_Y}{c} = 0.69318, \qquad \varphi_{load} = 0.64904 \text{ rad}$$

Stresses $\quad \tau = 0$ at $\rho = 0$, $\quad \tau = 12.5$ ksi at $\rho = \rho_Y$, $\quad \tau = 12.5$ ksi at $\rho = c$.

Unloading $\quad T = -3$ kip·in $\qquad G = 4\times10^6$ psi $= 4\times10^3$ ksi

$$\Delta\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(-3)}{\pi(0.5)^3} = -15.28 \text{ ksi at } \rho = c$$

$$\Delta\tau = (0.69318)(-15.28) = -10.59 \text{ ksi at } \rho = \rho_Y.$$

$$\Delta\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi c^4 G} = \frac{(2)(-3)(72)}{\pi(0.5)^4(4\times10^3)} = -0.55004 \text{ rad.}$$

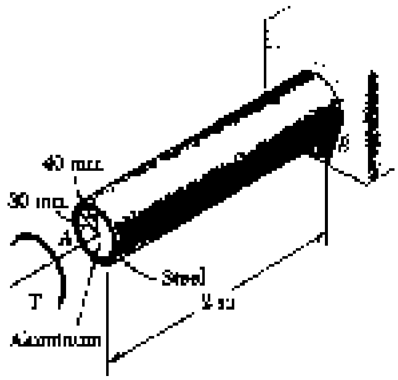
Residual $\quad \tau_{res} = \tau_{load} + \Delta\tau$

At $\rho = c \quad \tau_{res} = 12.5 - 15.28 = -2.78$ ksi

At $\rho = \rho_Y \quad \tau_{res} = 12.5 - 10.59 = 1.91$ ksi

$$\varphi_{perm} = \varphi_{load} + \Delta\varphi = 0.64904 - 0.55004 = 0.099 \text{ rad} = 5.67°$$

PROBLEM 3.161



3.161 The composite shaft shown is twisted by applying a torque T at end A. Knowing that the maximum shearing stress in the steel shell is 150 MPa, determine the corresponding maximum shearing stress in the aluminum core. Use $G = 77$ GPa for steel and $G = 27$ GPa for aluminum.

SOLUTION

Let G_1, J_1, and τ_1 refer to the aluminum core.
and G_2, J_2, and τ_2 refer to the steel shell

At the outer surface on the steel shell

$$\gamma_2 = \frac{c_2\varphi}{L} \quad \therefore \quad \frac{\varphi}{L} = \frac{\gamma_2}{c_2} = \frac{\tau_2}{c_2 G_2}$$

At the outer surface of the aluminum core

$$\gamma_1 = \frac{c_1\varphi}{L} \quad \therefore \quad \frac{\varphi}{L} = \frac{\gamma_1}{c_1} = \frac{\tau_1}{c_1 G}$$

Matching $\frac{\varphi}{L}$ for both components

$$\frac{\tau_2}{c_2 G_2} = \frac{\tau_1}{c_1 G_1}$$

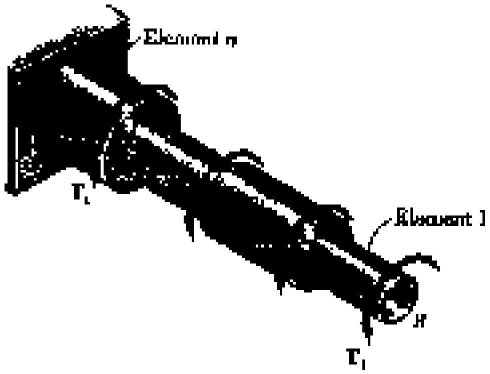
Solving for τ_2

$$\tau_1 = \frac{c_1}{c_2} \cdot \frac{G_1}{G_2}\,\tau_2$$

$$= \frac{0.030}{0.040} \cdot \frac{27\times10^9}{77\times10^9} \cdot 150\times10^6$$

$$= 39.4\times10^6 \text{ Pa} \qquad = 39.4 \text{ MPa}$$

PROBLEM 3.C1



3.C1 Shaft AB consists of n homogeneous cylindrical elements, which can be solid or hollow. Its end A is fixed, while its end B is free, and it is subjected to the loading shown. The length of element i is denoted by L_i, its outer diameter by OD_i, its inner diameter by ID_i, its modulus of rigidity by G_i, and the torque applied to its right end by $\mathbf{T}_i$, the magnitude T_i of this torque being assumed to be positive if $\mathbf{T}_i$ is observed as counterclockwise from end B and negative otherwise. (Note that $ID_i = 0$ if the element is solid.) (*a*) Write a computer program that can be used to determine the maximum shearing stress in each element, the angle of twist of each element, and the angle of twist of the entire shaft. (*b*) Use this program to solve Probs. 3.9, 3.35, 3.37, 3.150, and 3.151.

SOLUTION

FOR EACH CYLINDRICAL ELEMENT, ENTER

$L_i\,,\ OD_i\,,\ ID_i\,,\ G_i\,,\ T_i$

AND COMPUTE

$$J_i = (\pi/32)(OD_i^4 - ID_i^4)$$

OUTLINE OF PROGRAM

UPDATE TORQUE $\quad T = T + T_i$

AND COMPUTE

$$TAU_i = T\,(OD_i/2)/J_i$$

$$PHI_i = T L_i / G_i J_i$$

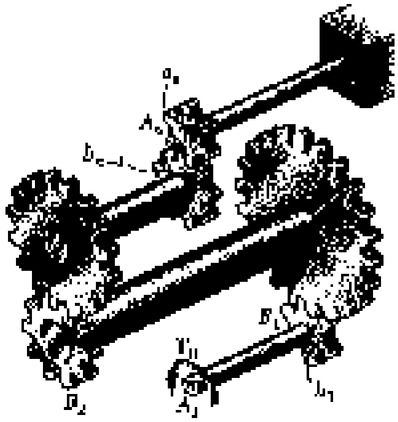
ANGLE OF TWIST OF ENTIRE SHAFT, STARTING WITH $\Theta = 0$, UPDATE THROUGH n^{th} ELEMENT

$$\Theta = \Theta + PHI_i$$

PROGRAM OUTPUT

```
   Problem   3.9 and 3.35
  Element   Maximum Stress   Angle of Twist
               (MPa)           (degrees)

   1.0000      56.5884         2.5265
   2.0000      36.6264         0.8887
Angle of twist for entire shaft =   3.4152 °

   Problem   3.37
  Element   Maximum Stress   Angle of Twist
               (MPa)           (degrees)

   1.0000      33.9531         0.8314
   2.0000      19.6488         0.7413
Angle of twist for entire shaft =   1.5726 °

   Problem   3.150 and 3.151
  Element   Maximum Stress   Angle of Twist
               (ksi)           (degrees)

   1.0000       9.1266         3.5857
   2.0000      -8.5826        -3.0002
Angle of twist for entire shaft =   0.5855 °
```

PROBLEM 3.C2



3.C2 The assembly shown consists of n cylindrical shafts, which can be solid or hollow, connected by gears and supported by brackets (not shown). End A_1 of the first shaft is free and is subjected to a torque $\mathbf{T}_0$, while end B_n of the last shaft is fixed. The length of shaft A_iB_i is denoted by L_i, its outer diameter by OD_i, its inner diameter by ID_i, and its modulus of rigidity by G_i. (Note that $ID_i = 0$ if the element is solid.) The radius of gear A_i is denoted by a_i, and the radius of gear B_i by b_i. (a) Write a computer program that can be used to determine the maximum shearing stress in each shaft, the angle of twist of each shaft, and the angle through which end A_1 rotates. (b) Use this program to solve Probs. 3.21, 3.39, 3.41, 3.42, and 3.154.

SOLUTION

TORQUE IN SHAFTS. ENTER $T_1 = T_0$

$$T_{i+1} = T_i\,(a_{i+1}/b_i)$$

FOR EACH SHAFT, ENTER

$L_i \quad OD_i \quad ID_i \quad G_i$

COMPUTE: $J_i = (\pi/32)(OD_i^4 - ID_i^4)$

$$TAU_i = T_i\,(OD_i/2)/J_i$$

$$PHI_i = T_i\,L_i/G_i\,J_i$$

ANGLE OF ROTATION AT END A_1

COMPUTE ROTATION AT THE "A" END OF EACH SHAFT

START WITH ANGLE = PHI_n AND UPDATE

FROM n TO 1, AND ADD PHI_i

$$ANGLE = ANGLE\,(a_i)/b_{i-1} + PHI_{i-1}$$

PROGRAM OUTPUT

Problem 3.21

Shaft No.	Max.Stress (MPa)	Twist Angle (degrees)
1.0000	68.7420	1.4615
2.0000	72.5013	0.7707

Angle through which A1 rotates = 3.388 °

Problem 3.39

Shaft No.	Max.Stress (MPa)	Twist Angle (degrees)
1.0000	47.7465	1.7764
2.0000	82.8932	2.0560

Angle through which A1 rotates = 7.945 °

Problem 3.41

Shaft No.	Max.Stress (ksi)	Twist Angle (degrees)
1.0000	9.0541	1.3587
2.0000	13.0732	1.3175

Angle through which A1 rotates = 3.115 °

Problem 3.42

Shaft No.	Max.Stress (ksi)	Twist Angle (degrees)
1.0000	9.0541	1.3587
2.0000	6.7906	0.7411

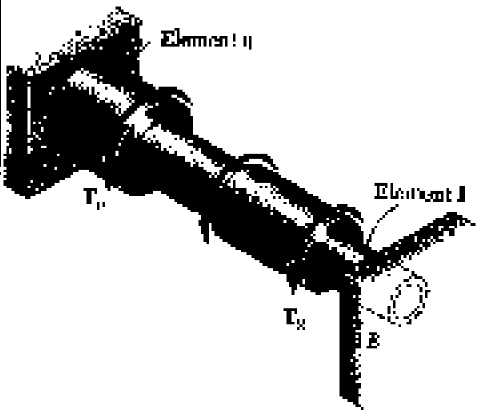
Angle through which A1 rotates = 1.914 °

Problem 3.154

Shaft No.	Max.Stress (MPa)	Twist Angle (degrees)
1.0000	37.7256	1.6843
2.0000	56.5884	1.4036

Angle through which A1 rotates = 3.790 °

PROBLEM 3.C3



3.C3 Shaft AB consists of n homogeneous cylindrical elements, which can be solid or hollow. Both of its ends are fixed, and it is subjected to the loading shown. The length of element i is denoted by L_i, its outer diameter by OD_i, its inner diameter by ID_i, its modulus of rigidity by G_i, and the torque applied to its right end by $\mathbf{T}_i$, the magnitude T_i of this torque being assumed to be positive if $\mathbf{T}_i$ is observed as counterclockwise from end B and negative otherwise. Note that $ID_i = 0$ if the element is solid and also that $T_1 = 0$. Write a computer program that can be used to determine the reactions at A and B, the maximum shearing stress in each element, and the angle of twist of each element. Use this program (*a*) to solve Prob. 3.56, (*b*) to determine the maximum shearing stress in the shaft of Example 3.05.

SOLUTION WE CONSIDER THE REACTION AT B AS REDUNDANT AND RELEASE THE SHAFT AT B.

COMPUTE θ_B WITH $T_B = 0$:

FOR EACH ELEMENT ENTER

L_i , OD_i , ID_i , G_i , T_i (NOTE $T_1 = -T_B = 0$)

COMPUTE

$J_i = (\pi/32)(OD_i^4 - ID_i^4)$

UPDATE TORQUE

$T = T + T_i$

AND COMPUTE FOR EACH ELEMENT

$TAU_i = T(OD_i/2)/J_i$

$PHI_i = T L_i / G_i J_i$

COMPUTE θ_B: STARTING WITH $\theta = 0$ AND UPDATING THROUGH n ELEMENTS:

$\theta_i = \theta_i + PHI_i$; $\theta_B = \theta_n$

COMPUTE θ_B DUE TO UNIT TORQUE AT B

UNIT $TAU_i = OD_i/2J_i$

UNIT $PHI_i = L_i/G_i J_i$

FOR n ELEMENTS:

UNIT $\theta_B(i)$ = UNIT $\theta_B(i)$ + UNIT PHI_i

SUPERPOSITION

FOR TOTAL ANGLE AT B TO BE ZERO. $\theta_B + T_B(\text{UNIT } \theta_B(n)) = 0$

$T_B = -\theta_B / (\text{UNIT } \theta_B(n))$

THEN $T_A = \Sigma T(i) + T_B$

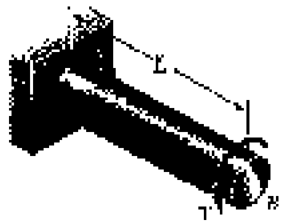
FOR EACH ELEMENT : MAX STRESS: TOTAL $TAU_i = TAU_i + T_B(\text{UNIT } TAU_i)$

ANGLE OF TWIST: TOTAL $PHI_i = PHI_i + T_B(\text{UNIT } PHI_i)$

PROGRAM OUTPUT

```
Problem 3.56        TA =   -0.290 kN*m
                    TB =   -0.210 kN*m
Element    tau max (MPa)    Angle of Twist (degrees)
  1          -39.588              -1.178
  2           31.670               1.178


Problem 3.05        TA =   -51.733 lb*ft
                    TB =   -38.267 lb*ft
```

PROBLEM 3.C4



3.C4 The homogeneous, solid cylindrical shaft AB has a length L, a diameter d, a modulus of rigidity G, and a yield strength τ_Y. It is subjected to a torque T that is gradually increased from zero until the angle of twist of the shaft has reached a maximum value ϕ_m and then decreased back to zero. (a) Write a computer program that, for each of 16 values of ϕ_m equally spaced over a range extending from 0 to a value 3 times as large as the angle of twist at the onset of yield, can be used to determine the maximum value T_m of the torque, the radius of the elastic core, the maximum shearing stress, the permanent twist, and the residual shearing stress both at the surface of the shaft and at the interface of the elastic core and the plastic region. (b) Use this program to obtain approximate answers to Probs. 3.95, 3.113, 3.159, and 3.160.

SOLUTION

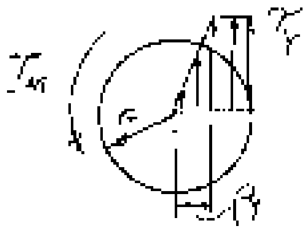
AT ONSET OF YIELD

T_Y

$$T_Y = \tau_Y \frac{J}{c} = \frac{\pi}{2}\tau_Y c^3$$

$$\phi_Y = \frac{T_Y L}{GJ} = \left(\frac{\tau_Y J}{c}\right)\frac{L}{GJ} = \frac{\tau_Y L}{cG}$$

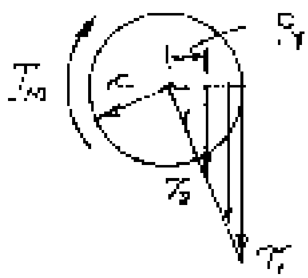
LOADING: $T_M > T_Y$



$$T_M = \frac{4}{3}T_Y\left[1 - \frac{1}{4}\left(\frac{\phi_Y}{\phi_M}\right)^3\right] \qquad \text{EQ. (1)}$$

$$\rho_Y = c\,\frac{\phi_Y}{\phi_M} \qquad \text{EQ. (2)}$$

UNLOADING (ELASTIC)



$\phi_U = \dfrac{T_M L}{GJ}$ $\qquad \phi_U$ = ANGLE OF TWIST FOR UNLOADING

$\tau_1 = T_M \dfrac{c}{J}$ $\qquad \tau_1$ = TAU AT $\rho = c$

$\tau_2 = \tau_1 \cdot \dfrac{\rho_Y}{c}$ $\qquad \tau_2$ = TAU AT $\rho = \rho_Y$

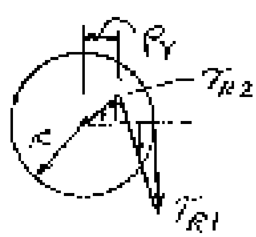
SUPERPOSE LOADING AND UNLOADING

FOR $\phi = 0$ TO $\phi = 3\phi_Y$ USING $0.2\phi_Y$ INCREMENTS

WHEN $\phi < \phi_Y$: $\quad T_M = T_Y \dfrac{\phi}{\phi_Y} \quad \rho_Y = \frac{1}{2}d \quad \tau_M = \tau_Y \dfrac{\phi}{\phi_Y}$

WHEN $\phi > \phi_Y$: $\quad T_M$ USE EQ(1) $\quad \rho_Y$ USE EQ(2)

RESIDUAL: $\quad \phi_P = \phi_M - \phi_1 \quad \tau_{R1} = \tau_1 - \tau_Y \quad \tau_{R2} = \tau_2 - \tau_Y$



CONTINUED

PROBLEM 3.C4 - CONTINUED

INTERPOLATE BETWEEN VALUES TO VALUE AT T_M INDICATED.

Problem 3.95

PHIM deg	TM kip*in.	RY in.	TAUM ksi	PHIP deg	TAUR1 ksi	TAUR2 ksi	
0.000	0.000	0.750	0.000	0.000	0.000	0.000	
0.417	2.783	0.750	4.200	0.000	0.000	0.000	
0.833	5.567	0.750	8.400	0.000	0.000	0.000	
1.250	8.350	0.750	12.600	0.000	0.000	0.000	
1.667	11.133	0.750	16.800	0.000	0.000	0.000	
2.083	13.916	0.750	21.000	0.000	0.000	0.000	← $T_M = 12$ kip·in.
2.500	15.871	0.625	21.000	0.124	1.042	-2.949	
2.917	16.865	0.536	21.000	0.392	2.822	-4.449	
3.334	17.423	0.469	21.000	0.725	4.560	-5.291	
3.750	17.760	0.417	21.000	1.091	6.111	-5.800	
4.167	17.975	0.375	21.000	1.476	7.438	-6.125	← $T_M = 18$ kip·in.
4.584	18.119	0.341	21.000	1.871	8.572	-6.343	
5.000	18.219	0.313	21.000	2.273	9.544	-6.494	
5.417	18.291	0.288	21.000	2.679	10.384	-6.602	
5.834	18.344	0.268	21.000	3.087	11.114	-6.681	
6.250	18.383	0.250	21.000	3.498	11.753	-6.741	

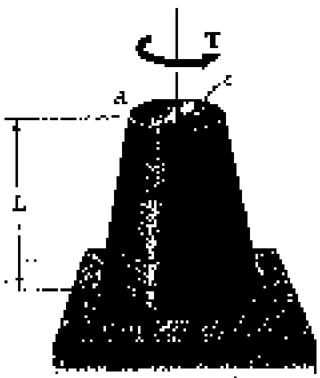
Problem 3.113

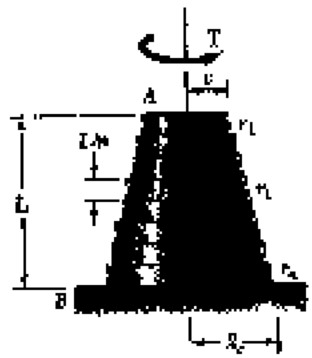
PHIM deg	TM kN*m	RY mm	TAUM MPa	PHIP deg	TAUR1 MPa	TAUR2 MPa	
0.000	0.000	25.000	0.000	0.000	0.000	0.000	
9.524	0.785	25.000	32.000	0.000	0.000	0.000	
19.049	1.571	25.000	64.000	0.000	0.000	0.000	
28.573	2.356	25.000	96.000	0.000	0.000	0.000	
38.098	3.142	25.000	128.000	0.000	0.000	0.000	
47.622	3.927	25.000	160.000	0.000	0.000	0.000	
57.147	4.478	20.833	160.000	2.837	7.942	-22.469	
66.671	4.759	17.857	160.000	8.960	21.502	-33.897	
76.196	4.916	15.625	160.000	16.575	34.805	-40.323	
85.720	5.012	13.889	160.000	24.946	46.562	-44.188	← $T_M = 5$ kN·m
95.245	5.072	12.500	160.000	33.723	56.667	-46.667	
104.769	5.113	11.364	160.000	42.764	65.307	-48.325	
114.294	5.141	10.417	160.000	51.946	72.719	-49.475	
123.818	5.162	9.615	160.000	61.225	79.116	-50.299	
133.343	5.176	8.929	160.000	70.569	84.677	-50.904	
142.867	5.188	8.333	160.000	79.959	89.547	-51.359	

Problems 3.159 and 3.160

PHIM deg	TM kip*in.	RY in.	TAUM ksi	PHIP deg	TAUR1 ksi	TAUR2 ksi	
0.000	0.000	0.500	0.000	0.000	0.000	0.000	
5.157	0.491	0.500	2.500	0.000	0.000	0.000	
10.313	0.982	0.500	5.000	0.000	0.000	0.000	
15.470	1.473	0.500	7.500	0.000	0.000	0.000	
20.626	1.963	0.500	10.000	0.000	0.000	0.000	
25.783	2.454	0.500	12.500	0.000	0.000	0.000	
30.940	2.799	0.417	12.500	1.536	0.620	-1.755	
36.096	2.974	0.357	12.500	4.251	1.680	-2.648	← $T_M = 3$ kip·in.
41.253	3.079	0.313	12.500	8.974	2.719	-3.149	
46.410	3.132	0.278	12.500	13.506	3.638	-3.452	
51.566	3.170	0.250	12.500	18.263	4.427	-3.646	
56.723	3.196	0.227	12.500	23.192	5.102	-3.775	
61.879	3.213	0.208	12.500	28.124	5.687	-3.865	
67.036	3.226	0.192	12.500	33.148	6.181	-3.930	
72.193	3.235	0.179	12.500	38.207	6.615	-3.977	
77.349	3.242	0.167	12.500	43.290	6.996	-4.012	

PROBLEM 3.C5





3.C5 The exact expression is given in Prob. 3.64 for the angle of twist of the solid tapered shaft AB when a torque **T** is applied as shown. Derive an approximate expression for the angle of twist by replacing the tapered shaft by n cylindrical shafts of equal length and of radius $r_i = (n + i - \frac{1}{2})(c/n)$, where $i = 1, 2, \ldots, n$. Using for T, L, G, and c values of your choice, determine the percentage error in the approximate expression when (a) $n = 4$, (b) $n = 8$, (c) $n = 20$, (d) $n = 100$.

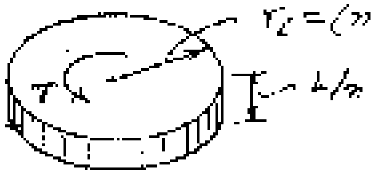
SOLUTION

FROM PROB. 3.64 EXACT EXPRESSION:

$$\phi = \frac{7TL}{12\pi G c^4}$$

$$\text{OR,}\quad \phi = \left(\frac{7}{12\pi}\right)\frac{TL}{Gc^4} = 0.18568\,\frac{TL}{Gc^4}$$

CONSIDER TYPICAL ith SHAFT



$$r_i = (n + i - \tfrac{1}{2})(c/n)$$

$$J_i = \frac{\pi}{2}(r_i)^4$$

$$\Delta\phi = \frac{T(L/n)}{G J_i}$$

ENTER UNIT VALUES OF T, L, G, AND c.
(NOTE: SPECIFIC VALUES CAN BE ENTERED)

ENTER INITIAL VALUE OF ZERO FOR ϕ
ENTER n = NUMBER CYLINDRICAL SHAFTS

FOR $i = 1$ TO n, UPDATE ϕ

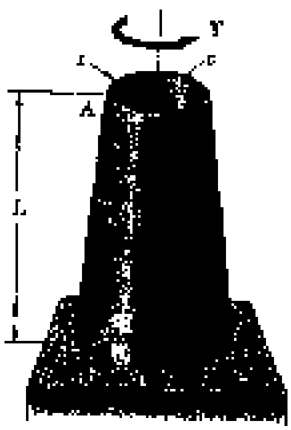
$$\phi = \phi + \Delta\phi$$

OUTPUT OF PROGRAM

```
COEFFICIENT of TL/GC^4

Exact coefficient from Prob. 3.64 is 0.18568
Number of elemental disks = n
```

n	approximate	exact	percent error
4	0.17959	0.18568	-3.28185
8	0.18410	0.18568	-0.85311
20	0.18542	0.18568	-0.13810
100	0.18567	0.18568	-0.00554

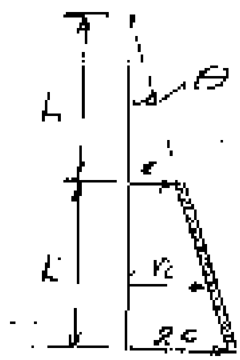
PROBLEM 3.C6



3.C6 A torque **T** is applied as shown to the long, hollow, tapered shaft *AB* of uniform thickness *t*. The exact expression for the angle of twist of the shaft can be obtained from the expression given in Prob. 3.153. Derive an approximate expression for the angle of twist by replacing the tapered shaft by *n* cylindrical rings of equal length and of radius $r_i = (n + i - \frac{1}{2})(c/n)$, where $i = 1, 2, \ldots, n$. Using for *T*, *L*, *G*, *c* and *t* values of your choice, determine the percentage error in the approximate expression when (*a*) $n = 4$, (*b*) $n = 8$, (*c*) $n = 20$, (*d*) $n = 100$.

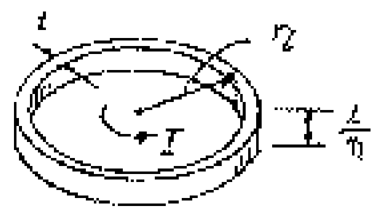
SOLUTION

SINCE THE SHAFT IS LONG $c \ll L$, THE ANGLE θ IS SMALL AND WE CAN USE t AS THE THICKNESS OF THE r_i CYLINDRICAL RINGS.



FOR $c \ll L$

$$\theta \approx \tan\theta = \frac{2c - c}{L} = \frac{c}{L}$$



$$r_i = (n + i - \tfrac{1}{2})\left(\frac{c}{n}\right)$$

$$J_i \approx (\text{AREA})\, r_i^2 = (2\pi r_i t)\, r_i^2 = 2\pi t\, r_i^3$$

$$\Delta\phi = \frac{T(L/n)}{G J_i}$$

ENTER UNIT VALUES FOR T, L, G, t, AND c

(NOTE: SPECIFIC VALUES CAN BE ENTERED IF DESIRED)

ENTER INITIAL VALUE OF ZERO FOR ϕ

ENTER n = NUMBER OF CYLINDRICAL RINGS

FOR $i = 1$ TO n, UPDATE ϕ

$$\phi = \phi + \Delta\phi$$

OUTPUT OF PROGRAM

COEFFICIENT of TL/Gtc^3

Exact coefficient from Prob. 3.153 is 0.05968

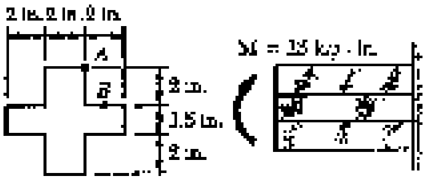
Number of elemental disks = n

n	approximate	exact	percent error
4	0.058559	0.059683	-1.883078
8	0.059394	0.059683	-0.483688
20	0.059637	0.059683	-0.078022
100	0.059681	0.059683	-0.003127

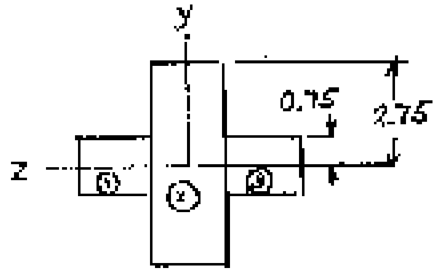
CHAPTER 4

PROBLEM 4.1

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION



For rectangle $I = \frac{1}{12} b h^3$

For cross sectional area

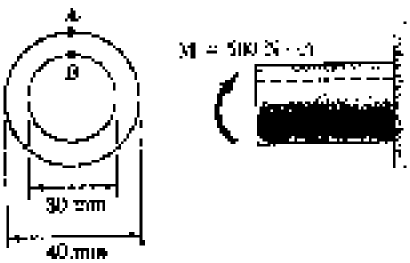
$$I = I_1 + I_2 + I_3 = \frac{1}{12}(2)(1.5)^3 + \frac{1}{12}(2)(5.5)^3 + \frac{1}{12}(2)(1.5)^3 = 28.854 \text{ in}^4$$

(a) $y_A = 2.75 \text{ in}$ $\quad \sigma_A = -\frac{M y_A}{I} = -\frac{(25)(2.75)}{28.854} = -2.38 \text{ ksi}$

(b) $y_B = 0.75 \text{ in.}$ $\quad \sigma_B = -\frac{M y_B}{I} = -\frac{(25)(0.75)}{28.854} = -0.650 \text{ ksi}$

PROBLEM 4.2

4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

$r_i = \frac{1}{2} d_i = 15 \text{ mm}$ $\quad r_o = \frac{1}{2} d_o = 20 \text{ mm}$

$$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(20^4 - 15^4)$$

$$= 85.903 \times 10^3 \text{ mm}^4 = 85.903 \times 10^{-9} \text{ m}^4$$

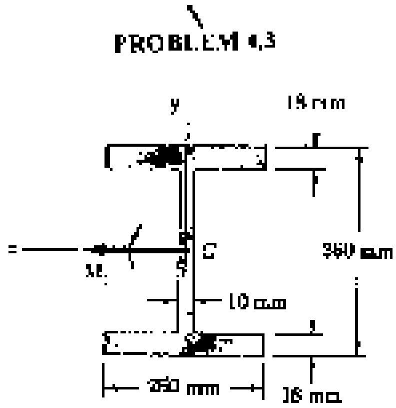
(a) $y_A = 20 \text{ mm} = 0.020 \text{ m}$ $\quad \sigma_A = -\frac{M y_A}{I} = -\frac{(500)(0.020)}{85.903 \times 10^{-9}}$

$$= -116.4 \times 10^6 \text{ Pa} = -116.4 \text{ MPa}$$

(b) $y_B = 15 \text{ mm} = 0.015 \text{ m}$ $\quad \sigma_B = -\frac{M y_B}{I} = -\frac{(500)(0.015)}{85.903 \times 10^{-9}}$

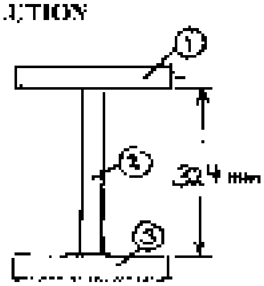
$$= -87.3 \times 10^6 \text{ Pa} = -87.3 \text{ MPa}$$

PROBLEM 4.3



4.3 The wide-flange beam shown is made of a high-strength, low-alloy steel for which $\sigma_Y = 345$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.0, determine the largest couple that can be applied to the beam when it is bent about the z axis. Neglect the effect of fillets.

SOLUTION



$$I_1 = \frac{1}{12} b h^3 + A d^2$$

$$= \frac{1}{12}(250)(18^3) + (250)(18)(171)^2$$

$$= 131.706 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(10)(324)^3 = 28.344 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 131.706 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 291.76 \times 10^6 \text{ mm}^4 = 291.76 \times 10^{-6} \text{ m}^4$$

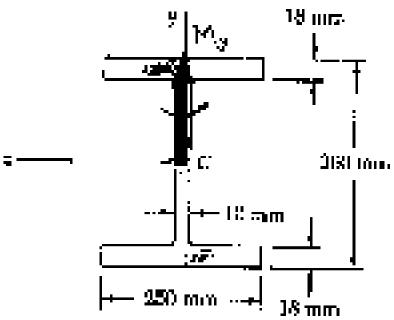
$$\sigma = \frac{Mc}{I} \quad \text{where} \quad c = \frac{360}{2} = 180 \text{ mm} = 0.180 \text{ m}$$

$$\sigma_{all} = \frac{\sigma_U}{FS} = \frac{450 \times 10^6}{3.0} = 150 \times 10^6 \text{ Pa}$$

$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(150 \times 10^6)(291.76 \times 10^{-6})}{0.180} = 243 \times 10^3 \text{ N}\cdot\text{m}$$

$$= 243 \text{ kN}\cdot\text{m}$$

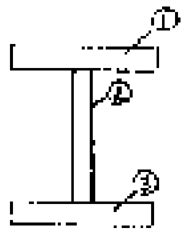
PROBLEM 4.4



4.3 The wide-flange beam shown is made of a high-strength, low-alloy steel for which $\sigma_Y = 345$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.0, determine the largest couple that can be applied to the beam when it is bent about the z axis. Neglect the effect of fillets.

4.4 Solve Prob. 4.3, assuming that is bent about the y axis.

SOLUTION



$I_1 = \frac{1}{12}(18)(250)^3$

$= 23.438 \times 10^6 \ \text{mm}^4$

$I_2 = \frac{1}{12}(324)(10)^3$

$= 27 \times 10^3 \ \text{mm}^4$

$I_3 = I_1 = 23.438 \ \text{mm}^4$

$I_y = I_1 + I_2 + I_3 = 46.903 \times 10^6 \ \text{mm}^4 = 46.903 \times 10^{-6} \ \text{m}^4$

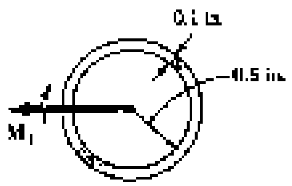
$c = \frac{250}{2} \ \text{mm} = 125 \ \text{mm} = 0.125 \ \text{m}$

$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{450 \times 10^6}{3.0} = 150 \times 10^6 \ \text{Pa}$

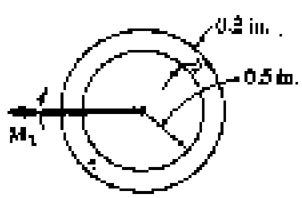
$\sigma = \frac{Mc}{I} \qquad M_y = \frac{\sigma_{all} I}{c} = \frac{(150 \times 10^6)(46.903 \times 10^{-6})}{0.125}$

$= 56.3 \times 10^3 \ \text{N·m} = 56.3 \ \text{kN·m}$ ◄

PROBLEM 4.5



(a)



(b)

4.5 Using an allowable stress of 16 ksi, determine the largest that can be applied to each pipe.

SOLUTION

(a) $I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(0.6^4 - 0.5^4) = 52.7 \times 10^{-3} \ \text{in}^4$

$c = 0.6 \ \text{in}$

$\sigma = \frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma I}{c} = \frac{(16)(52.7 \times 10^{-3})}{0.6}$

$= 1.405 \ \text{kip·in.}$ ◄

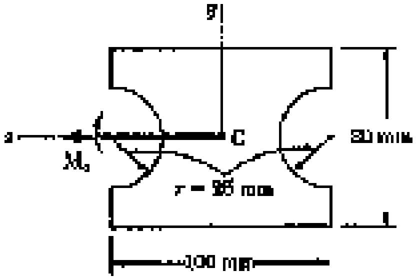
(b) $I = \frac{\pi}{4}(0.7^4 - 0.5^4) = 139.49 \times 10^{-3} \ \text{in}^4$

$c = 0.7 \ \text{in}$

$\sigma = \frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma I}{c} = \frac{(16)(139.49 \times 10^{-3})}{0.7}$

$= 3.19 \ \text{kip·in}$ ◄

PROBLEM 4.6



4.6 A nylon spacing bar has the cross section shown. Knowing that the allowable stress for the grade of nylon used is 24 MPa, determine the largest couple M_z that can be applied to the bar.

SOLUTION

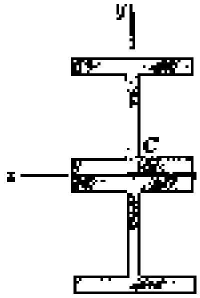
$$I = I_{rect} - I_{circle}$$
$$= \frac{1}{12} b h^3 - \frac{\pi}{4} r^4$$
$$= \frac{1}{12}(100)(80)^3 - \frac{\pi}{4}(25)^4 = 3.9599 \times 10^6 \text{ mm}^4$$
$$= 3.9599 \times 10^{-6} \text{ m}^4$$

$$c = \frac{80}{2} = 40 \text{ mm} = 0.040 \text{ m}$$

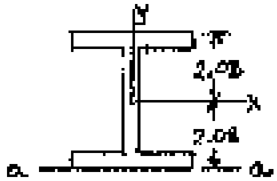
$$\sigma = \frac{Mc}{I} \qquad M = \frac{\sigma I}{c} = \frac{(24 \times 10^6)(3.9599 \times 10^{-6})}{0.040} = 2.38 \times 10^3 \text{ N}\cdot\text{m}$$
$$= 2.38 \text{ kN}\cdot\text{m}$$

PROBLEM 4.7

4.7 and 4.8 Two W 4 × 13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.



SOLUTION



Properties of W 4×13 rolled section
See Appendix B

Area = 3.83 in² Depth = 4.16 in
I_x = 11.3 in⁴

For one rolled section, moment of inertia about axis a-a is

$$I_a = I_x + Ad^2 = 11.3 + (3.83)(2.08)^2 = 27.87 \text{ in}^4$$

For both sections $I_z = 2I_a = 55.74 \text{ in}^4$

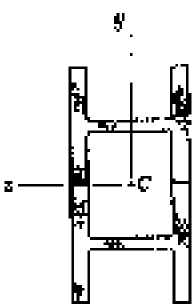
c = depth = 4.16 in

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

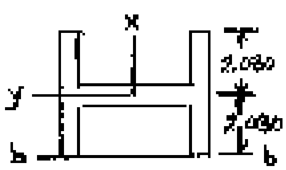
$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(19.333)(55.74)}{4.16} = 259 \text{ kip}\cdot\text{in}$$

PROBLEM 4.8

4.7 and 4.8 Two W 4 × 13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.



SOLUTION



Properties of W 4×13 rolled section
See Appendix B

Area = 3.83 in² Width = 4.060 in
I_y = 3.86 in⁴

For one rolled section, moment of inertia about axis b-b is

$$I_b = I_y + Ad^2 = 3.86 + (3.83)(2.030)^2 = 19.643 \text{ in}^4$$

For both sections $I_z = 2I_b = 39.286 \text{ in}^4$

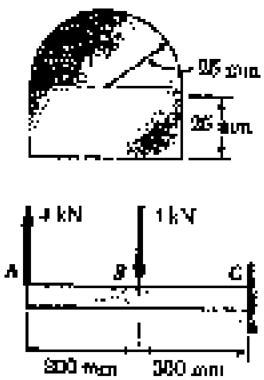
c = width = 4.060 in

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

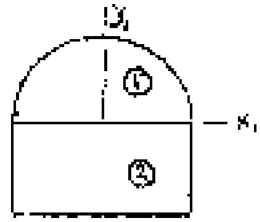
$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(19.333)(39.286)}{4.060} = 187.1 \text{ kip}\cdot\text{in}$$

PROBLEM 4.9

4.9 through 4.11 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.



SOLUTION



$A_1 = \frac{\pi}{2}r^2 = \frac{\pi}{2}(25)^2 = 981.7 \text{ mm}^2$

$\bar{y}_1 = \frac{4r}{3\pi} = \frac{(4)(25)}{3\pi} = 10.610 \text{ mm}$

$A_2 = bh = (50)(25) = 1250 \text{ mm}^2$

$\bar{y}_2 = -\frac{h}{2} = -\frac{25}{2} = -12.5 \text{ mm}$

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2} = \frac{(981.7)(10.610) + (1250)(-12.5)}{981.7 + 1250}$$

$= -2.334 \text{ mm}$

$\bar{I}_1 = I_{x_1} - A_1\bar{y}_1^2 = \frac{\pi}{8}r^4 - A_1\bar{y}_1^2 = \frac{\pi}{8}(25)^4 - (981.7)(10.610)^2 = 42.886\times10^3 \text{ mm}^4$

$d_1 = \bar{y}_1 - \bar{y} = 10.610 - (-2.334) = 12.944 \text{ mm}$

$I_1 = \bar{I}_1 + A_1d_1^2 = 42.866\times10^3 + (981.7)(12.944)^2 = 207.35\times10^3 \text{ mm}^4$

$\bar{I}_2 = \frac{1}{12}bh^3 = \frac{1}{12}(50)(25)^3 = 65.104\times10^3 \text{ mm}^4$

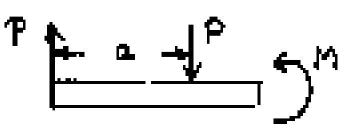
$d_2 = |\bar{y}_2 - \bar{y}| = |-12.5-(-2.334)| = 10.166 \text{ mm}$

$I_2 = \bar{I}_2 + A_2d_2^2 = 65.104\times10^3 + (1250)(10.166)^2 = 194.288\times10^3 \text{ mm}^4$

$I = I_1 + I_2 = 401.16\times10^3 \text{ mm}^4 = 401.16\times10^{-9} \text{ m}^4$

$y_{top} = 25 + 2.334 = 27.334 \text{ mm} = 0.027334 \text{ m}$

$y_{bot} = -25 + 2.334 = -22.666 \text{ mm} = -0.022666 \text{ m}$



$M - Pa = 0 \qquad M = Pa = (4\times10^3)(300\times10^{-3})$
$= 1200 \text{ N·m}$

$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(1200)(0.027334)}{401.16\times10^{-9}} = -81.76\times10^6 \text{ Pa}$$

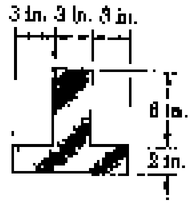
$= -81.8 \text{ MPa}$ ◀

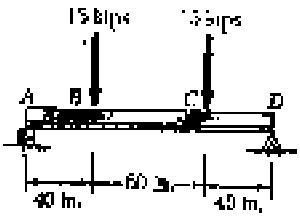
$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(1200)(-0.022666)}{401.16\times10^{-9}} = 67.80\times10^6 \text{ Pa}$$

$= 67.8 \text{ MPa}$ ◀

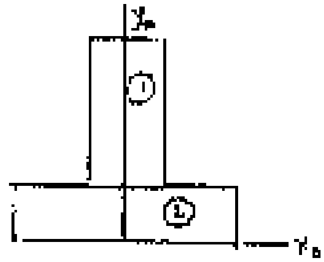
PROBLEM 4.10

4.9 through 4.11 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.





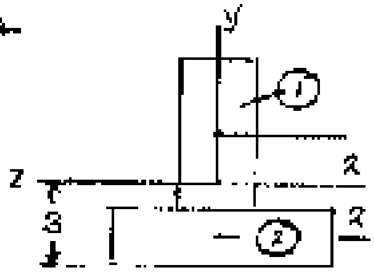
SOLUTION



	A	$\bar{y}_o$	$A\bar{y}_o$
①	18	5	90
②	18	1	18
Σ	36		108

$$\bar{Y}_o = \frac{108}{36} = 3 \text{ in}$$

Neutral axis lies 3 in. above the base.

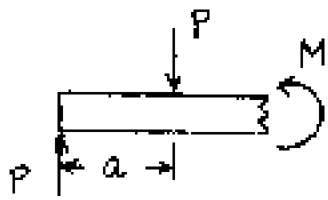


$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(6)^3 + (18)(2)^2 = 126 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(9)(2)^3 + (18)(2)^2 = 78 \text{ in}^4$$

$$I = I_1 + I_2 = 126 + 78 = 204 \text{ in}^4$$

$$y_{top} = 5 \text{ in} \qquad y_{bot} = -3 \text{ in.}$$



$$M - Pa = 0$$

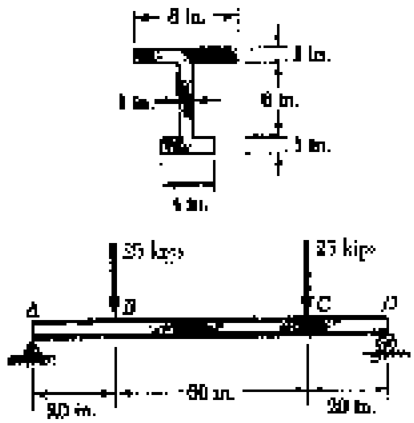
$$M = Pa = (15)(40) = 600 \text{ kip}\cdot\text{in.}$$

$$\sigma_{top} = -\frac{M y_{top}}{I} = -\frac{(600)(5)}{204} = -14.71 \text{ ksi}$$

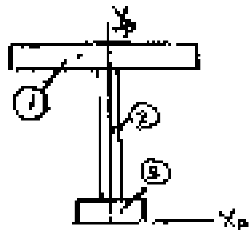
$$\sigma_{bot} = -\frac{M y_{bot}}{I} = -\frac{(600)(-3)}{204} = 8.82 \text{ ksi}$$

4.9 through 4.11 Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

PROBLEM 4.11



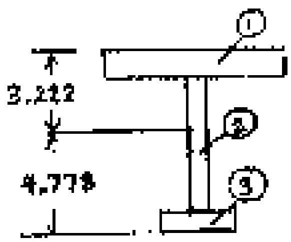
SOLUTION



	A	$\bar{y}_0$	$A\bar{y}_0$
①	8	7.5	60
②	6	4	24
③	4	0.5	2
Σ	18		86

$$\bar{Y}_0 = \frac{86}{18} = 4.778 \text{ in}$$

Neutral axis lies 4.778 in above the base.



$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(8)(1)^3 + (8)(2.722)^2$$

$$= 59.94 \text{ in}^4$$

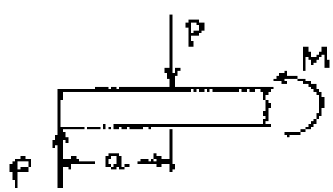
$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(1)(6)^3 + (6)(0.778)^2$$

$$= 21.68 \text{ in}^4$$

$$I_3 = \frac{1}{12} b_3 h_3^3 + A_3 d_3^2 = \frac{1}{12}(4)(1)^3 + (4)(4.278)^2 = 73.54 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 59.94 + 21.68 + 73.54 = 155.16 \text{ in}^4$$

$y_{top} = 3.222$ in $\qquad y_{bot} = -4.778$ in



$$M - Pa = 0$$

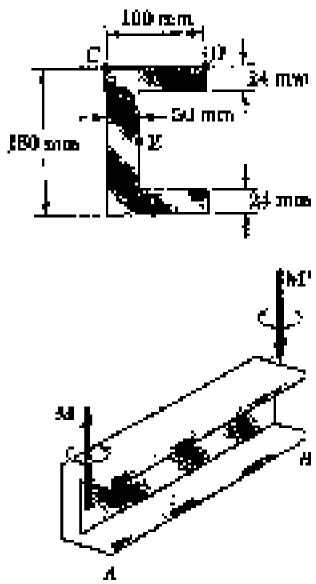
$$M = Pa = (25)(20) = 500 \text{ kip}\cdot\text{in}$$

$$\sigma_{top} = -\frac{M y_{top}}{I} = -\frac{(500)(3.222)}{155.16} = -10.38 \text{ ksi}$$

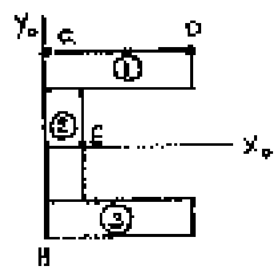
$$\sigma_{bot} = -\frac{M y_{bot}}{I} = -\frac{(500)(-4.778)}{155.16} = 15.40 \text{ ksi}$$

PROBLEM 4.12

4.12 Two equal and opposite couples of magnitude of $M = 15$ kN·m are applied to the channel-shaped beam AB. Observing that the couples cause the beam to bend in a horizontal plane, determine the stress (*a*) at point C, (*b*) at point D, (*c*) at point E.

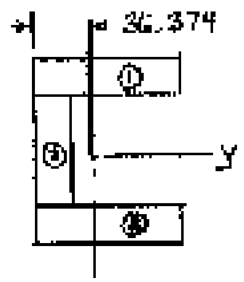


SOLUTION



	A, mm²	$\bar{x}_o$, mm	$A\bar{x}_o$, mm³
①	2400	50	120×10^3
②	3060	15	45.9×10^3
③	2400	50	120×10^3
Σ	7860		285.9×10^3

$$\bar{X} = \frac{285.9 \times 10^3}{7860} = 36.374 \text{ mm}$$



$y_C = -36.374$ mm $= -0.036374$ m

$y_D = 100 - 36.374 = 63.626$ mm $= 0.063626$ m

$y_E = 30 - 36.374 = -6.374$ mm $= -0.006374$ m

$d_1 = 50 - 36.374 = 13.626$ mm

$d_2 = 36.374 - 15 = 21.374$ mm

$d_3 = d_1$

$$I_1 = I_3 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(24)(100)^3 + (2400)(13.626)^2 = 2.4456 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(102)(30)^3 + (3060)(21.374)^2 = 1.6275 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 6.5187 \times 10^6 \text{ mm}^4 = 6.5187 \times 10^{-6} \text{ m}^4$$

$$M = 15 \times 10^3 \text{ N·m}$$

(a) Point C: $\sigma_C = -\dfrac{My_C}{I} = -\dfrac{(15 \times 10^3)(-0.036374)}{6.5187 \times 10^{-6}} = 83.7 \times 10^6$ Pa

$= 83.7$ MPa

(b) Point D: $\sigma_D = -\dfrac{My_D}{I} = -\dfrac{(15 \times 10^3)(0.063626)}{6.5187 \times 10^{-6}} = -146.4 \times 10^6$ Pa

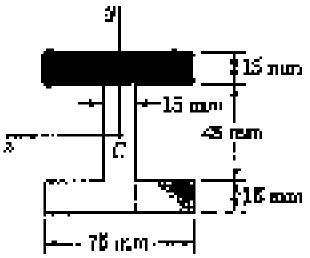
$= -146.4$ MPa

(c) Point E: $\sigma_E = -\dfrac{My_E}{I} = -\dfrac{(15 \times 10^3)(-0.006374)}{6.5187 \times 10^{-6}} = 14.67 \times 10^6$ Pa

$= 14.67$ MPa

PROBLEM 4.13

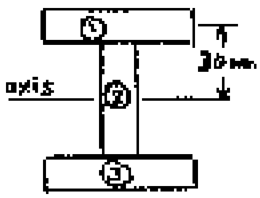
4.13 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 8 kN · m, determine the total force acting on the top flange.



SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$



where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x \, dA = -\frac{My}{I}\, dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I}\, dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

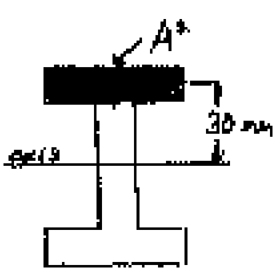
where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(75)(15)^3 + (75)(15)(30)^2 = 1.0336 \times 10^6 \ \text{mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(15)(45)^3 = 0.1139 \times 10^6 \ \text{mm}^4$$

$$I_3 = I_1 = 1.0336 \times 10^6 \ \text{mm}^4$$

$$I = I_1 + I_2 + I_3 = 2.1811 \times 10^6 \ \text{mm}^4 = 2.1811 \times 10^{-6} \ \text{m}^4$$



$$A^* = (75)(15) = 1125 \ \text{mm}^2 = 1125 \times 10^{-6} \ \text{m}^2$$

$$\bar{y}^* = 30 \ \text{mm} = 0.030 \ \text{m}$$

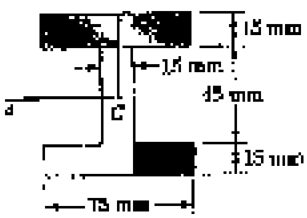
$$F = -\frac{M\bar{y}^* A}{I} = -\frac{(8\times 10^3)(0.030)(1125\times 10^{-6})}{2.1811 \times 10^{-6}}$$

$$= -123.8 \times 10^3 \ \text{N} = -123.8 \ \text{kN}$$

PROBLEM 4.14

91

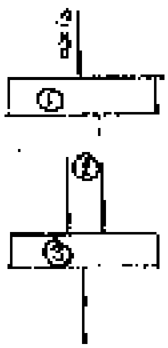
4.14 Knowing that a beam of the cross section shown is bent about a vertical axis and that the bending moment is 4 kN · m, determine the total force acting on the shaded portion of the lower flange.



SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$



where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x\, dA = -\frac{My}{I}\, dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

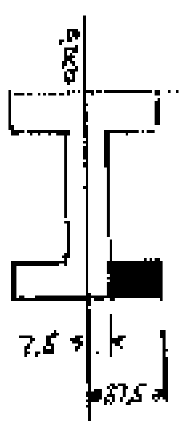
where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$I_1 = \frac{1}{12} b_1 h_1^3 = \frac{1}{12}(15)(75)^3 = 0.52734 \times 10^6 \ mm^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(45)(15)^3 = 0.01256 \times 10^6 \ mm^4$$

$$I_3 = I_1 = 0.5273 \times 10^{-6}$$

$$I = I_1 + I_2 + I_3 = 1.0672 \times 10^6 \ mm^4 = 1.0672 \times 10^{-6} \ m^4$$



$$A^* = (37.5 - 7.5)(15) = 450 \ mm^2 = 450 \times 10^{-6} \ m^2$$

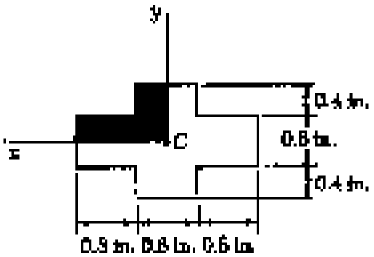
$$\bar{y}^* = \frac{1}{2}(37.5 + 7.5) = 22.5 \ mm = 0.0225 \ m$$

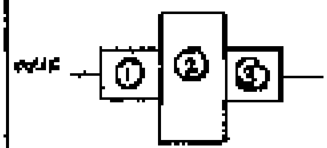
$$F = \frac{M\bar{y}^* A^*}{I} = \frac{(4\times 10^3)(0.0225)(450\times 10^{-6})}{1.0672\times 10^{-6}}$$

$$= 37.9 \times 10^3 \ N = 37.9 \ kN$$

PROBLEM 4.15

4.15 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 3.5 kip·in., determine the total force acting on the shaded portion of the beam.





SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x\, dA = -\frac{My}{I}\, dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I}\, dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

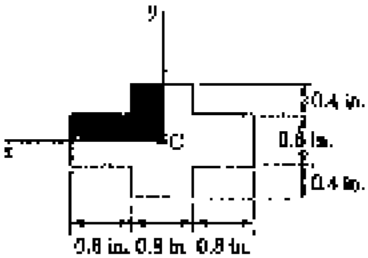
where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$I = I_1 + I_2 + I_3 = \tfrac{1}{12} b_1 h_1^3 + \tfrac{1}{12} b_2 h_2^3 + \tfrac{1}{12} b_3 h_3^3$$

$$= \tfrac{1}{12}(0.8)(0.6)^3 + \tfrac{1}{12}(0.8)(1.4)^3 + \tfrac{1}{12}(0.8)(0.6)^3 = 0.21173 \text{ in}^4$$

(b)

PROBLEM 4.16



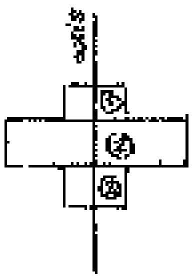
4.15 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 3.5 kip·in., determine the total force acting on the shaded portion of the beam.

4.16 Solve Prob. 4.15, assuming that the beam is bent about a vertical axis and that the bending moment is 6 kip·in.

SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$



where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x\, dA = -\frac{My}{I}\, dA$$

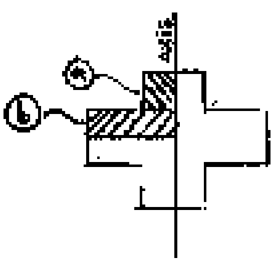
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I}\, dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$I = I_1 + I_2 + I_3 = \frac{1}{12} b_1 h_1^3 + \frac{1}{12} b_2 h_2^3 + \frac{1}{12} b_3 h_3^3$$

$$= \frac{1}{12}(0.4)(0.8)^3 + \frac{1}{12}(0.8)(2.4)^3 + \frac{1}{12}(0.4)(0.8)^3 = 0.7253 \text{ in}^4$$



$$\bar{y}^* A^* = \bar{y}_a A_a + \bar{y}_b A_b$$

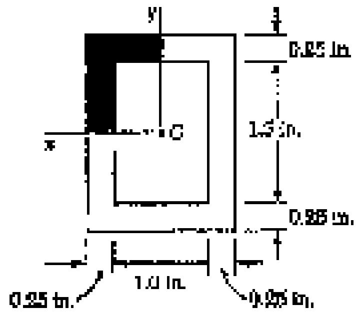
$$= (0.2)(0.4)(0.4) + (0.6)(0.8)(1.2)$$

$$= 0.248 \text{ in}^3$$

$$F = \frac{M\bar{y}^* A^*}{I} = \frac{(6)(0.248)}{0.7253} = 2.05 \text{ kips}$$

PROBLEM 4.17

4.17 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kip·in., determine the total force acting on the shaded portion of the beam.



SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

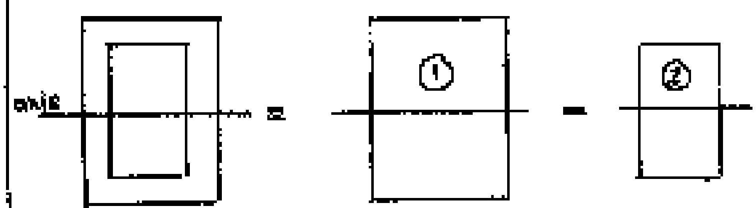
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x \, dA = -\frac{My}{I} dA$$

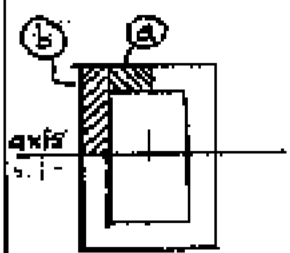
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.



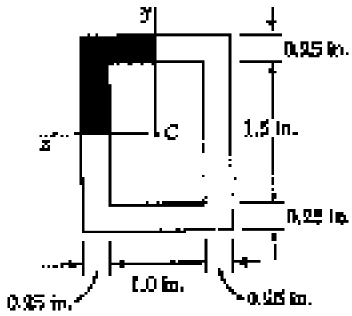
$$I = I_1 - I_2$$
$$= \frac{1}{12}b_1 h_1^3 - \frac{1}{12}b_2 h_2^3$$
$$= \frac{1}{12}(1.5)(2.0)^3 - \frac{1}{12}(1.0)(1.5)^3$$
$$= 0.71875 \text{ in}^4$$



$$\bar{y}^* A^* = \bar{y}_a A_a + \bar{y}_b A_b$$
$$= (0.875)(0.5)(0.25) + (0.5)(0.25)(1.0) = 0.23438 \text{ in}^3$$

$$F = \frac{M\bar{y}^* A^*}{I} = \frac{(6)(0.23438)}{0.71875} = 1.957 \text{ kips}$$

PROBLEM 4.18



4.17 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kip·in., determine the total force acting on the shaded portion of the beam.

4.18 Solve Prob. 4.17, assuming that the beam is bent about a vertical axis and that the bending moment is 6 kip·in.

SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

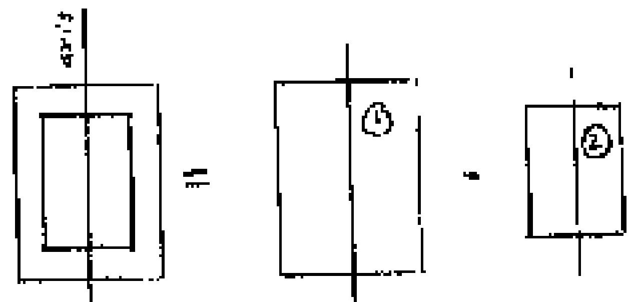
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x\, dA = -\frac{My}{I}\, dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I}\, dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

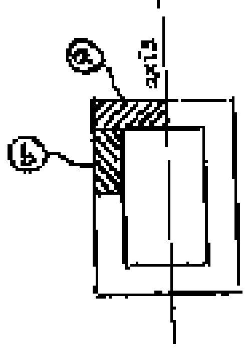


$$I = I_1 - I_2$$

$$= \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3$$

$$= \frac{1}{12}(2)(1.5)^3 - \frac{1}{12}(1.5)(1.0)^3$$

$$= 0.4375 \text{ in}^4$$



$$\bar{y}^* A^* = \bar{y}_a A_a + \bar{y}_b A_b$$

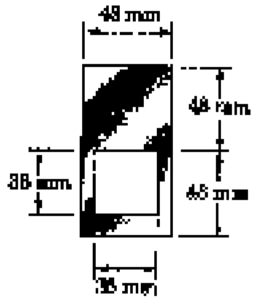
$$= (0.375)(0.25)(0.75) + (0.625)(0.75)(0.25)$$

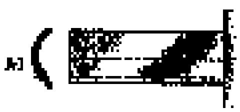
$$= 0.1875 \text{ in}^3$$

$$F = \frac{M\bar{y}^* A^*}{I} = \frac{(6)(0.1875)}{0.4375} = 2.57 \text{ kips.}$$

PROBLEM 4.19

4.19 and 4.20 Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple M that can be applied.





SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
① solid rectangle	4608	48	221184
② square cutout	-1296	30	-38880
Σ	3312		182304

$$\bar{Y} = \frac{182304}{3312} = 55.04 \text{ mm}$$

Neutral axis lies 55.04 mm above bottom.

$$y_{top} = 96 - 55.04 = 40.96 \text{ mm} = 0.04096 \text{ m}$$

$$y_{bot} = -55.04 \text{ mm} = -0.05504 \text{ m}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(48)(96)^3 + (48)(96)(7.04)^2 = 3.7673 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(36)(36)^3 + (36)(36)(25.04)^2 = 0.9526 \times 10^6 \text{ mm}^4$$

$$I = I_1 - I_2 = 2.8147 \times 10^6 \text{ mm}^4 = 2.8147 \times 10^{-6} \text{ m}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \quad \therefore \quad M = \pm\left|\frac{\sigma I}{y}\right|$$

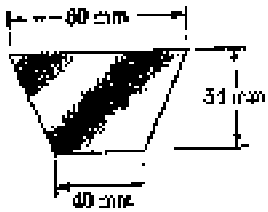
Top: tension side $\quad M = \dfrac{(120 \times 10^6)(2.8147 \times 10^{-6})}{0.04096} = 8.25 \times 10^3 \text{ N}\cdot\text{m}$

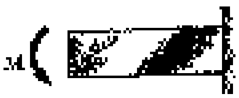
Bottom: compression $\quad M = \dfrac{(150 \times 10^6)(2.8147 \times 10^{-6})}{0.05504} = 7.67 \times 10^3 \text{ N}\cdot\text{m}$

M_{all} is the smaller value $\quad M = 7.67 \times 10^3 \text{ N}\cdot\text{m} = 7.67 \text{ kN}\cdot\text{m}$ ◄

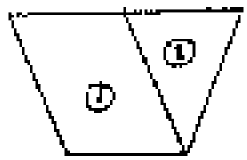
PROBLEM 4.20

4.19 and 4.20 Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.





SOLUTION



	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	2160	27	58320
②	1080	36	38880
Σ	3240		97200

$$\bar{Y} = \frac{97200}{3240} = 30 \text{ mm}$$

The neutral axis lies 30 mm above the bottom.

$$y_{top} = 54 - 30 = 24 \text{ mm} = 0.024 \text{ m}$$
$$y_{bot} = -30 \text{ mm} = -0.030 \text{ m}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(40)(54)^3 + (40)(54)(3)^2 \simeq 544.32 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{36} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{36}(40)(54)^3 + \frac{1}{2}(40)(54)(6)^2 = 213.84 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 758.16 \times 10^3 \text{ mm}^4 = 758.16 \times 10^{-9} \text{ m}^4$$

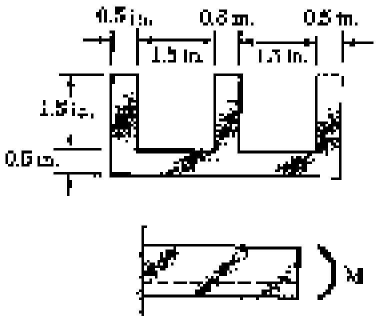
$$|\sigma| = \left|\frac{My}{I}\right| \qquad |M| = \left|\frac{\sigma I}{y}\right|$$

top: tension side $\quad M = \dfrac{(120 \times 10^6)(758.16 \times 10^{-9})}{0.024} = 3.7908 \times 10^3 \text{ N·m}$

bottom: compression $\quad M = \dfrac{(150 \times 10^6)(758.16 \times 10^{-9})}{0.030} = 3.7908 \times 10^3 \text{ N·m}$

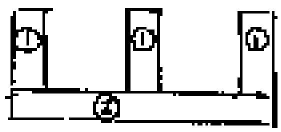
Choose the smaller as M_{all} $\qquad M_{all} = 3.7908 \times 10^3$ N·m $= 3.79$ kN·m

PROBLEM 4.21



4.21 Knowing that for the extruded beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple M that can be applied.

SOLUTION



	A	$\bar{y}_0$	$A\bar{y}_0$
①	2.25	1.25	2.8125
②	2.25	0.25	0.5625
	4.50		3.375

$$\bar{Y} = \frac{3.375}{4.50} = 0.75 \text{ in}$$

The neutral axis lies 0.75 in. above bottom.

$y_{top} = 2.0 - 0.75 = 1.25$ in, $\quad y_{bot} = -0.75$ in.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(1.5)(1.5)^3 + (2.25)(0.5)^2 = 0.984375 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(4.5)(0.5)^3 + (2.25)(0.5)^2 = 0.609375 \text{ in}^4$$

$$I = I_1 + I_2 = 1.59375 \text{ in}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

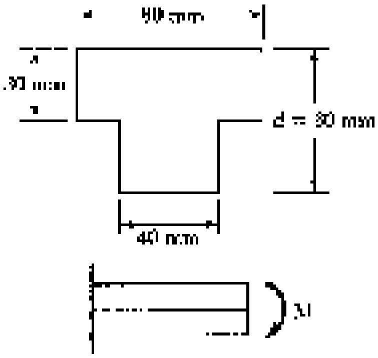
Top: compression $\quad M = \dfrac{(16)(1.59375)}{1.25} = 20.4$ kip-in

Bottom: tension $\quad M = \dfrac{(12)(1.59375)}{0.75} = 25.5$ kip-in

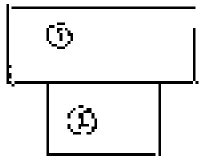
Choose the smaller as M_{all} $\qquad M_{all} = 20.4$ kip-in.

PROBLEM 4.22

4.22 The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple M that can be applied to the beam.



SOLUTION



	A, mm²	$\bar{y}_0$, mm	$A\bar{y}_0$, mm³
①	2400	45	108 000
②	1200	15	18 000
Σ	3600		126 000

$$\bar{Y}_0 = \frac{126000}{3600} = 35 \text{ mm}$$

The neutral axis lies 35 mm above the bottom.

$y_{top} = 60 - 35 = 25 \text{ mm} = 0.025 \text{ m}$; $y_{bot} = -35 \text{ mm} = -0.035 \text{ m}$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(80)(30)^3 + (2400)(10)^2 = 420 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(40)(30)^3 + (1200)(20)^2 = 570 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 990 \times 10^3 \text{ mm}^4 = 990 \times 10^{-9} \text{ m}^4$$

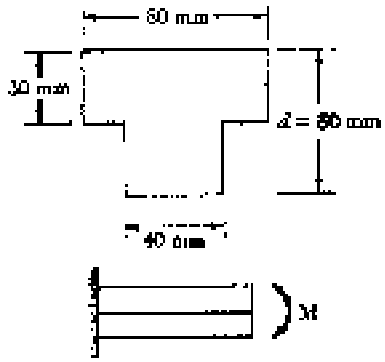
$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: tension side $\quad M = \dfrac{(24 \times 10^6)(990 \times 10^{-9})}{0.025} = 950 \text{ N}\cdot\text{m}$

Bottom: compression $\quad M = \dfrac{(30 \times 10^6)(990 \times 10^{-9})}{0.035} = 849 \text{ N}\cdot\text{m}$

Choose smaller value $\qquad M = 849 \text{ N}\cdot\text{m}$

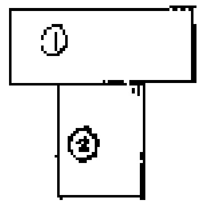
PROBLEM 4.23



4.22 The beam shown is made of a nylon for which the allowable stress 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.

4.23 Solve Prob. 4.22, assuming that $d = 80$ mm.

SOLUTION



	A, mm²	$\bar{y}_o$, mm	$A\bar{y}_o$, mm³
①	2400	65	156000
②	2000	25	50000
Σ	4400		206000

$$\bar{Y}_o = \frac{206000}{4400} = 46.82 \text{ mm}$$

The neutral axis lies 46.82 mm above the bottom.

$y_{top} = 80 - 46.82 = 33.18 \text{ mm} = 0.03318 \text{ m}$

$y_{bot} = -46.82 \text{ mm} = -0.04682 \text{ m}$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(80)(30)^3 + (2400)(18.18)^2 = 0.97323 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(40)(50)^3 + (2000)(21.82)^2 = 1.36889 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 2.342 \times 10^6 \text{ mm}^4 = 2.342 \times 10^{-6} \text{ m}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

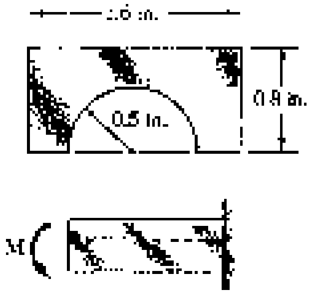
Top: tension side $\quad M = \dfrac{(24 \times 10^6)(2.342 \times 10^{-6})}{0.03318} = 1.694 \times 10^3 \text{ N·m}$

Bottom: compression $\quad M = \dfrac{(30 \times 10^6)(2.342 \times 10^{-6})}{0.04682} = 1.501 \times 10^3 \text{ N·m}$

Choose smaller value $\quad M = 1.501 \times 10^3 \text{ N·m} = 1.501 \text{ kN·m}$

PROBLEM 4.24

4.24 Knowing that for the beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple M that can be applied.



SOLUTION

① = rectangle   ② = semi-circular cutout

$A_1 = (1.6)(0.8) = 1.28 \text{ in}^2$

$A_2 = \frac{\pi}{2}(0.5)^2 = 0.3927 \text{ in}^2$

$A = 1.28 - 0.3927 = 0.8873 \text{ in}^2$

$\bar{y}_1 = 0.4 \text{ in}$ $\bar{y}_2 = \frac{4r}{3\pi} = \frac{(4)(0.5)}{3\pi} = 0.2122 \text{ in}$

$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{(1.28)(0.4) - (0.3927)(0.2122)}{0.8873} = 0.4831 \text{ in.}$

Neutral axis lies 0.4831 in above the bottom

Moment of inertia about the base

$I_b = \frac{1}{3}bh^3 - \frac{\pi}{8}r^4 = \frac{1}{3}(1.6)(0.8)^3 - \frac{\pi}{8}(0.5)^4 = 0.24852 \text{ in}^4$

Centroidal moment of inertia

$\bar{I} = I_b - A\bar{Y}^2 = 0.24852 - (0.8873)(0.4831)^2$

$= 0.04144 \text{ in}^4$

$y_{top} = 0.8 - 0.4831 = 0.3169 \text{ in}$, $y_{bot} = -0.4831 \text{ in}^4$

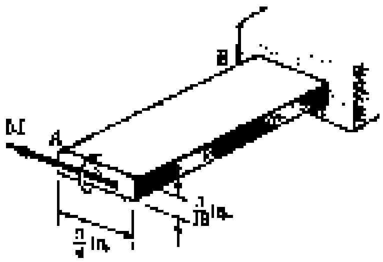
$|\sigma| = \left|\frac{My}{I}\right|$ $M = \left|\frac{\sigma I}{y}\right|$

Top: tension side $M = \frac{(12)(0.04144)}{0.3169} = 1.569 \text{ kip} \cdot \text{in}$

Bottom: compression $M = \frac{(16)(0.04144)}{0.4831} = 1.372 \text{ kip} \cdot \text{in}$

Choose the smaller value $M = 1.372 \text{ kip} \cdot \text{in}$

PROBLEM 4.25



4.25 Knowing that $\sigma_a = 24$ ksi for the steel strip AB, determine (a) the largest couple M that can be applied, (b) the corresponding radius of curvature. Use $E = 29 \times 10^6$ psi.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}\left(\frac{3}{4}\right)\left(\frac{3}{16}\right)^3 = 412.0 \times 10^{-6} \text{ in}^4$$

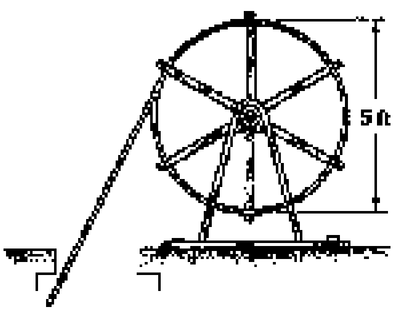
$$\sigma = \frac{Mc}{I} \qquad c = \frac{1}{2}\left(\frac{3}{16}\right) = 0.09375 \text{ in}$$

(a) $$M = \frac{\sigma I}{c} = \frac{(24 \times 10^3)(412.0 \times 10^{-6})}{0.09375}$$

$$= 105.5 \text{ lb}\cdot\text{in}$$

(b) $$\frac{c}{\rho} = \frac{\sigma_{max}}{E} \qquad \rho = \frac{Ec}{\sigma_{max}} = \frac{(29 \times 10^6)(0.09375)}{24 \times 10^3} = 113.3 \text{ in}$$

PROBLEM 4.26



4.26 Straight rods of 0.30-in. diameter and 200-ft length are sometimes used to clear underground conduits of obstructions or to thread wires through a new conduit. The rods are made of high-strength steel and, for storage and transportation, are wrapped on spools of 5-ft diameter. Assuming that the yield strength is not exceeded, determine (a) the maximum stress in a rod, when the rod, which was initially straight, is wrapped on a spool, (b) the corresponding bending moment in the rod. Use $E = 29 \times 10^6$ psi.

SOLUTION

$$r = \frac{1}{2}d = \frac{1}{2}(0.30) = 0.15 \text{ in}$$

$$I = \frac{\pi}{4}r^4 = \frac{\pi}{4}(0.15)^4 = 397.61 \times 10^{-6} \text{ in}^4$$

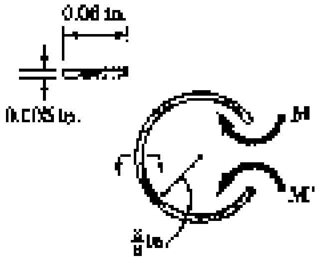
$$D = 5 \text{ ft} = 60 \text{ in} \qquad \rho = \frac{1}{2}D = 30 \text{ in}$$

$$c = r = 0.15 \text{ in.}$$

(a) $$\sigma_{max} = \frac{Ec}{\rho} = \frac{(29 \times 10^6)(0.15)}{30} = 145 \times 10^3 \text{ psi} = 145 \text{ ksi}$$

(b) $$M = \frac{EI}{\rho} = \frac{(29 \times 10^6)(397.61 \times 10^{-6})}{30} = 384 \text{ lb}\cdot\text{in}$$

PROBLEM 4.27



4.27 It is observed that a thin steel strip of 0.06-in. width can be bent into a circle of $\frac{3}{4}$-in. diameter without any resulting permanent deformation. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the maximum stress in the bent strip, (*b*) the magnitude of the couple required to bend the strip.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(0.06)(0.005)^3 = 625 \times 10^{-12} \text{ in}^4$$

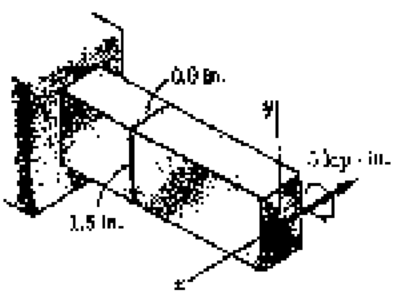
$$\rho = \frac{1}{2}D = \frac{1}{2}\left(\frac{3}{4}\right) = 0.375 \text{ in}$$

$$c = \frac{1}{2}h = 0.0025 \text{ in}$$

(a) $$\sigma_{max} = \frac{Ec}{\rho} = \frac{(29\times10^6)(0.0025)}{0.375} = 193.3\times10^3 \text{ psi} = 193.3 \text{ ksi}$$

(b) $$M = \frac{EI}{\rho} = \frac{(29\times10^6)(625\times10^{-12})}{0.375} = 0.0483 \text{ lb}\cdot\text{in.}$$

PROBLEM 4.28



4.28 A 3 kip·in. couple is applied to the steel bar shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the bar. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use $E = 29 \times 10^6$ psi.

SOLUTION

(a) Bending about z-axis

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(0.9)(1.5)^3 = 0.25313 \text{ in}^4$$

$$c = \frac{1}{2}h = \frac{1}{2}(1.5) = 0.75 \text{ in}$$

$$\sigma = \frac{Mc}{I} = \frac{(3\times10^3)(0.75)}{0.25313} = 8.89\times10^3 \text{ psi}$$
$$= 8.89 \text{ ksi}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{3\times10^3}{(29\times10^6)(0.25313)} = 409\times10^{-6} \text{ in}^{-1}$$

$$\rho = 2450 \text{ in} = 204 \text{ ft}$$

(b) Bending about y-axis

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(1.5)(0.9)^3 = 0.091125 \text{ in}^4$$

$$c = \frac{1}{2}h = \frac{1}{2}(0.9) = 0.45 \text{ in}$$

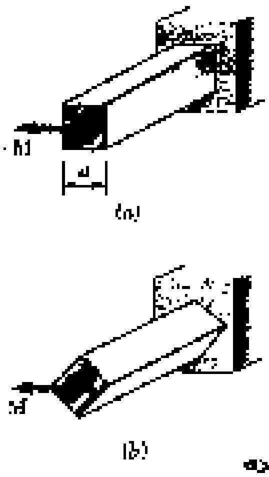
$$\sigma = \frac{Mc}{I} = \frac{(3\times10^3)(0.45)}{0.091125} = 14.81\times10^3 \text{ psi} = 14.81 \text{ ksi}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{3\times10^3}{(29\times10^6)(0.091125)} = 1.135\times10^{-3} \text{ in}^{-1}$$

$$\rho = 881 \text{ in} = 73.4 \text{ ft}$$

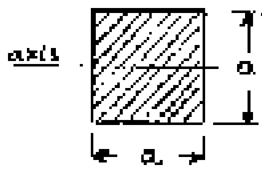
PROBLEM 4.29

4.29 A couple of magnitude M is applied to a square bar of side a. For each of the orientations shown, determine the maximum stress and the curvature of the bar.



SOLUTION

(a)



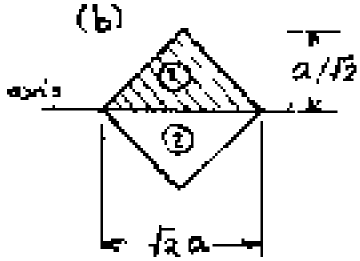
$$I = \frac{1}{12} b h^3 = \frac{1}{12} a a^3 = \frac{a^4}{12}$$

$$c = \frac{a}{2}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{M\frac{a}{2}}{\frac{a^4}{12}} = \frac{6M}{a^3}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E\frac{a^4}{12}} = \frac{12M}{Ea^4}$$

(b)



For one triangle the moment of inertia about its base is

$$I_1 = \frac{1}{12} b h^3 = \frac{1}{12}(\sqrt{2}a)\left(\frac{a}{\sqrt{2}}\right)^3 = \frac{a^4}{24}$$

$$I_2 = I_1 = \frac{a^4}{24}$$

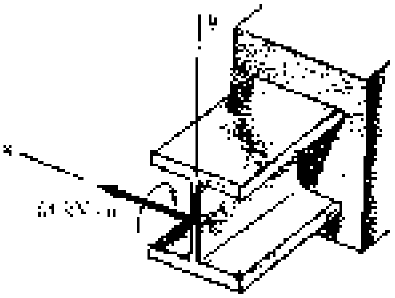
$$I = I_1 + I_2 = \frac{a^4}{12}$$

$$c = \frac{a}{\sqrt{2}}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{M\,a/\sqrt{2}}{a^4/12} = \frac{6\sqrt{2}M}{a^3}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E\frac{a^4}{12}} = \frac{12M}{Ea^4}$$

PROBLEM 4.30



4.30 A 24 kN·m couple is applied to the W200 × 46.1 rolled-steel beam shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the beam. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use: $E = 200$ GPa.

SOLUTION

For W 200 × 46.1 rolled steel section

$I_x = 45.5 \times 10^6 \ mm^4 = 45.5 \times 10^{-6} \ m^4$

$S_x = 448 \times 10^3 \ mm^3 = 448 \times 10^{-6} \ m^3$

$I_y = 15.3 \times 10^6 \ mm^4 = 15.3 \times 10^{-6} \ m^4$

$S_y = 151 \times 10^3 \ mm^3 = 151 \times 10^{-6} \ m^3$

(a) $M_z = 24 \ kN \cdot m = 24 \times 10^3 \ N \cdot m$

$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{448 \times 10^{-6}} = 53.6 \times 10^6 \ Pa = 53.6 \ MPa$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(45.5 \times 10^{-6})} = 2.637 \times 10^{-3} \ m^{-1}$$

$$\rho = 379 \ m$$

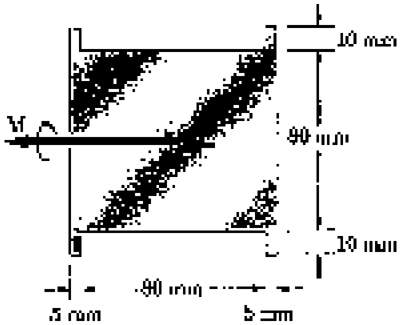
(b) $M_y = 24 \ kN \cdot m = 24 \times 10^3 \ N \cdot m$

$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{151 \times 10^{-6}} = 158.9 \times 10^6 \ Pa = 158.9 \ MPa$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(15.3 \times 10^{-6})} = 7.84 \times 10^{-3} \ m^{-1}$$

$$\rho = 127.5 \ m$$

PROBLEM 4.31



4.31 (*a*) Using an allowable stress of 120 MPa, determine the largest couple **M** that can be applied to a beam of the cross section shown. (*b*) Solve part *a*, assuming that the cross section of the beam is an 80-mm square.

SOLUTION

(a) $I = I_1 + 4I_2$, where I_1 is the moment of inertia of an 80-mm square and I_2 is the moment of inertia of one of the 4 protruding ears.

$$I_1 = \frac{1}{12}bh^3 = \frac{1}{12}(80)(80)^3 = 3.4133\times10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(5)(10)^3 + (5)(10)(45)^2 = 101.667\times10^3 \text{ mm}^4$$

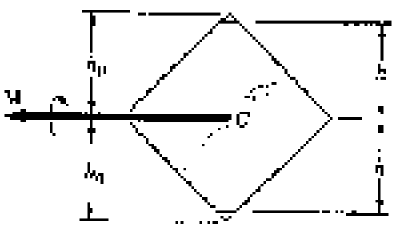
$$I = I_1 + 4I_2 = 3.82\times10^6 \text{ mm}^4 = 3.82\times10^{-6} \text{ m}^4, \qquad c = 50 \text{ mm} = 0.050 \text{ m}$$

$$\sigma = \frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma I}{c} = \frac{(120\times10^6)(3.82\times10^{-6})}{0.050} = 9.168\times10^3 \text{ N}\cdot\text{m} = 9.17 \text{ kN}\cdot\text{m}$$

(b) Without the ears $I = I_1 = 3.4133\times10^{-6} \text{ m}^4$, $c = 40 \text{ mm} = 0.040 \text{ m}$

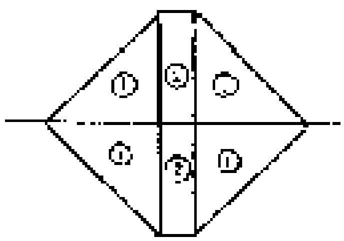
$$M = \frac{\sigma I}{c} = \frac{(120\times10^6)(3.4133\times10^{-6})}{0.040} = 10.24\times10^3 \text{ N}\cdot\text{m} = 10.24 \text{ kN}\cdot\text{m}$$

PROBLEM 4.32



4.32 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal diagonal by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$, where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.

SOLUTION



$$I = 4I_1 + 2I_2$$

$$= (4)\left(\tfrac{1}{12}\right)h\,h^3 + (2)\left(\tfrac{1}{3}\right)(2h_0 - 2h)(h^3)$$

$$= \tfrac{1}{3}h^4 + \tfrac{4}{3}h_0 h^3 - \tfrac{4}{3}h\,h^3 = \tfrac{4}{3}h_0 h^3 - h^4$$

$$c = h$$

$$\sigma = \frac{Mc}{I} = \frac{Mh}{\tfrac{4}{3}h_0 h^3 - h^4} = \frac{3M}{(4h_0 - 3h)h^3}$$

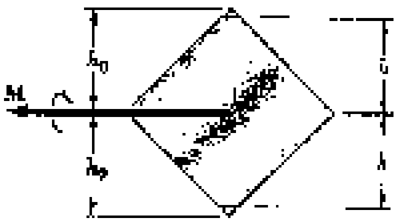
For the original square $\quad h = h_0,\ c = h_0$

$$\sigma_0 = \frac{3M}{(4h_0 - 3h_0)h_0^2} = \frac{3M}{h_0^3}$$

$$\frac{\sigma}{\sigma_0} = \frac{h_0^3}{(4h_0 - 3h)h^2} = \frac{h_0^3}{(4h_0 - (3)(0.9)h_0)(0.9h_0^2)} = 0.950$$

$\sigma = 0.950\,\sigma_0 \qquad\qquad k = 0.950$

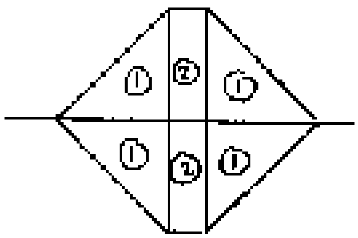
PROBLEM 4.33



4.32 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal diagonal by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$, where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.

4.33 In Prob. 4.32, determine (*a*) the value of h for which the maximum stress σ_m is as small as possible, (*b*) the corresponding value of k.

SOLUTION



$$I = 4I_1 + 2I_2$$
$$= (4)\left(\tfrac{1}{12}\right)h h^3 + (2)\left(\tfrac{1}{3}\right)(2h_0 - 2h)h^3$$
$$= \tfrac{1}{3}h^4 - \tfrac{4}{3}h_0 h^3 - \tfrac{4}{3}h^3 \quad = \tfrac{4}{3}h_0 h^3 - h^4$$

$$c = h \qquad \frac{I}{c} = \tfrac{4}{3}h_0 h^2 - h^3$$

$\frac{I}{c}$ is maximum at $\frac{d}{dh}\left[\tfrac{4}{3}h_0 h^2 - h^3\right] = 0$

$$\tfrac{8}{3}h_0 h - 3h^2 = 0 \qquad h = \tfrac{8}{9}h_0$$

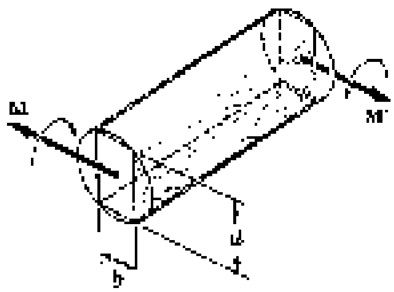
$$\frac{I}{c} = \tfrac{4}{3}h_0\left(\tfrac{8}{9}h_0\right)^2 - \left(\tfrac{8}{9}h_0\right)^3 = \frac{256}{729}h_0^3 \qquad \sigma = \frac{Mc}{I} = \frac{729}{256}\frac{M}{h_0^3}$$

For the original square $h = h_0$, $c = h_0$, $\frac{I_0}{c_0} = \frac{1}{3}h_0^3$

$$\sigma_0 = \frac{Mc_0}{I_0} = \frac{3M}{h_0^3}$$

$$\frac{\sigma}{\sigma_0} = \frac{729}{256}\cdot\frac{1}{3} = \frac{729}{768} = 0.949 \qquad k = 0.949$$

PROBLEM 4.34



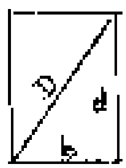
4.34 A couple **M** will be applied to a beam of rectangular cross section which is to be sawed from a log of circular cross section. Determine the ratio d/b, for which (*a*) the maximum stress σ_m will be as small as possible, (*b*) the radius of curvature of the beam will be maximum.

SOLUTION

Let D be the diameter of the log.

$$D^2 = b^2 + d^2 \qquad d^2 = D^2 - b^2$$

$$I = \frac{1}{12} b d^3 \qquad c = \frac{1}{2} d \qquad \frac{I}{c} = \frac{1}{6} b d^2$$



(a) σ_m is minimum when $\frac{I}{c}$ is maximum

$$\frac{I}{c} = \frac{1}{6} b (D^2 - b^2) = \frac{1}{6} D^2 b - \frac{1}{6} b^3$$

$$\frac{d}{db}\left(\frac{I}{c}\right) = \frac{1}{6} D^2 - \frac{3}{6} b^2 = 0 \qquad b = \frac{1}{\sqrt{3}} D$$

$$d = \sqrt{D^2 - \tfrac{1}{3} D^2} = \sqrt{\tfrac{2}{3}}\, D \qquad \frac{d}{b} = \sqrt{2}$$

$$\rho = \frac{EI}{M} \qquad \rho \text{ is maximum when } I \text{ is maximum.}$$

$\frac{1}{12} b d^3$ is maximum or $b^2 d^6$ is maximum

$(D^2 - d^2) d^6$ is maximum.

$$6D^2 d^5 - 8 d^7 = 0 \qquad d = \frac{\sqrt{3}}{2} D$$

$$b = \sqrt{D^2 - \tfrac{3}{4} D^2} = \tfrac{1}{2} D \qquad \frac{d}{b} = \sqrt{3}$$

PROBLEM 4.35

4.35 For the bar and loading of Example 4.01, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section, (*c*) the angle between the sides of the bar which were originally vertical. Use $E = 29 \times 10^6$ psi and $\nu = 0.29$.

SOLUTION

From Example 4.01 $\quad M = 30$ kip·in, $\quad I = 1.042$ in^4

(a) $$\frac{1}{\rho} = \frac{M}{EI} = \frac{(30\times10^3)}{(29\times10^6)(1.042)} = 993\times10^{-6}\ \text{in}^{-1} \qquad \rho = 1007\ \text{in.}$$

(b) $$\varepsilon' = \nu\varepsilon = \frac{\nu c}{\rho} = \nu\frac{c}{\rho'}$$

$$\frac{1}{\rho'} = \nu\frac{1}{\rho} = (0.29)(993\times10^{-6})\ \text{in}^{-1} = 288\ \text{in}^{-1} \qquad \rho' = 3470\ \text{in.}$$

(c) $$\theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{0.8}{3470} = 230\times10^{-6}\ \text{rad} = 0.01320^\circ$$

PROBLEM 4.36

4.36 For the aluminum bar and loading of Sample Prob. 4.1, determine (*a*) the radius of curvature ρ' of a transverse cross section, (*b*) the angle between the sides of the bar which were originally vertical. Use $E = 10.6 \times 10^6$ psi and $\nu = 0.33$.

SOLUTION

From Sample Problem 4.1 $\quad I = 12.97$ in^4 $\quad M = 103.8$ kip·in

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{103.8\times10^3}{(10.6\times10^6)(12.97)} = 755\times10^{-6}\ \text{in}^{-1}$$

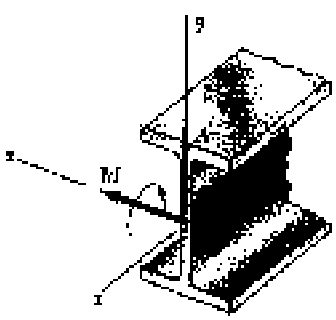
(a) $$\frac{1}{\rho'} = \nu\frac{1}{\rho} = (0.33)(755\times10^{-6}) = 249\times10^{-6}\ \text{in}^{-1}$$

$$\rho' = 4010\ \text{in} = 334\ \text{ft}$$

(b) $$\theta = \frac{\text{length of arc}}{\text{radius}} = \frac{b}{\rho'} = \frac{3.25}{4010} = 810\times10^{-6}\ \text{rad} = 0.0464^\circ$$

PROBLEM 4.37

4.37 A W 200 × 31.3 rolled-steel beam is subjected to a couple **M** of moment 45 kN·m. Knowing that $E = 200$ GPa, $\nu = 0.29$, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section.



SOLUTION

For W 200×31.3 rolled steel section

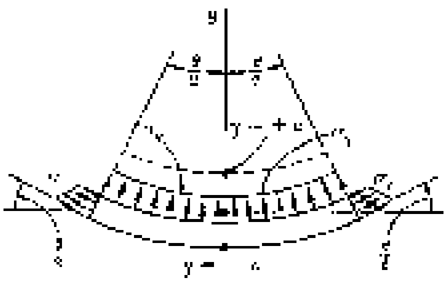
$$I = 31.4\times10^6\ \text{mm}^4 = 31.4\times10^{-6}\ \text{m}^4$$

(a) $$\frac{1}{\rho} = \frac{M}{EI} = \frac{45\times10^3}{(200\times10^9)(31.4\times10^{-6})} = 7.17\times10^{-3}\ \text{m}^{-1}$$

$$\rho = 139.6\ \text{m}$$

(b) $$\frac{1}{\rho'} = \nu\frac{1}{\rho} = (0.29)(7.17\times10^{-3}) = 2.07\times10^{-3}\ \text{m}^{-1} \qquad \rho' = 481\ \text{m}$$

PROBLEM 4.38



4.38 It was assumed in Sec. 4.3 that the normal stresses σ_y in a member in pure bending are negligible. For an initially straight elastic member of rectangular cross section, (*a*) derive an approximate expression for σ_y as a function of y, (*b*) show that $(\sigma_y)_{max} = -(c/2\rho)(\sigma_x)_{max}$ and, thus, that σ_y can be neglected in all practical situations. (*Hint:* Consider the free-body diagram of the portion of beam located below the surface of ordinate y and assume the distribution of the stress σ_x is still linear.)

SOLUTION

Denote the width of the beam by b and the length by L.

$$\theta = \frac{L}{\rho}$$

Using the free body diagram above, with $\cos\frac{\theta}{2} \approx 1$

$$\Sigma F_y = 0 \qquad \sigma_y b L + 2\int_{-c}^{y} \sigma_x \, b\,dy \, \sin\frac{\theta}{2} = 0$$

$$\sigma_y = -\frac{2}{L}\sin\frac{\theta}{2}\int_{-c}^{y} \sigma_x\,dy \approx -\frac{\theta}{L}\int_{-c}^{y} \sigma_x\,dy = -\frac{1}{\rho}\int_{-c}^{y} \sigma_x\,dy$$

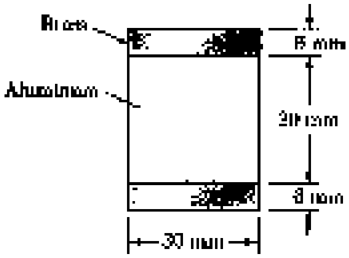
But $\quad \sigma_x = -(\sigma_x)_{max}\dfrac{y}{c}$

$$\sigma_y = \frac{(\sigma_x)_{max}}{\rho c}\int_{-c}^{y} y\,dy = \frac{(\sigma_x)_{max}}{\rho c}\,\frac{y^2}{2}\Bigg|_{-c}^{y} = \frac{(\sigma_x)_{max}}{2\rho c}\left(y^2 - c^2\right)$$

The maximum value σ_y occurs at $y = 0$

$$(\sigma_y)_{max} = -\frac{(\sigma_x)_{max}c^2}{2\rho c} = -\frac{(\sigma_x)_{max}c}{2\rho}$$

PROBLEM 4.39



4.39 and 4.40 Two brass strips are securely bonded to an aluminum bar of 30 × 30-mm square cross section. Using the data given below, determine the largest permissible bending moment when the composite member is bent about a horizontal axis.

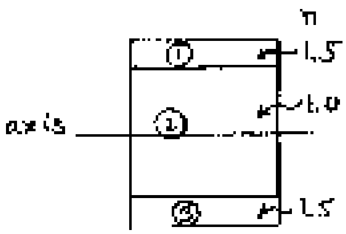
	Aluminum	Brass
Modulus of elasticity:	70 GPa	105 GPa
Allowable stress:	100 MPa	160 MPa

SOLUTION

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = E_b/E_a = 105/70 = 1.5$ in brass



For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{1.5}{12}(30)(6)^3 + (1.5)(30)(6)(18)^2 = 88.29 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.0}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4, \qquad I_3 = I_1 = 88.29 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 244.08 \times 10^3 \text{ mm}^4 = 244.08 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \frac{\sigma I}{ny}$$

Aluminum: $n = 1.0$, $y = 15 \text{ mm} = 0.015 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

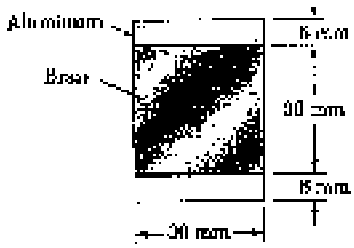
$$M = \frac{(100 \times 10^6)(244.08 \times 10^{-9})}{(1.0)(0.015)} = 1.627 \times 10^3 \text{ N·m}$$

Brass: $n = 1.5$, $y = 21 \text{ mm} = 0.021 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

$$M = \frac{(160 \times 10^6)(244.08 \times 10^{-9})}{(1.5)(0.021)} = 1.240 \times 10^3 \text{ N·m}$$

Choose the smaller value $M = 1.240 \times 10^3 \text{ N·m} = 1.240 \text{ kN·m}$ ◄

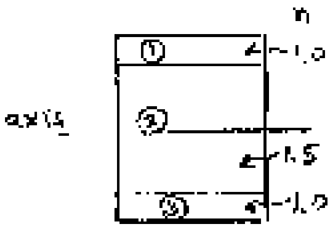
PROBLEM 4.40



4.39 and 4.40 Two ~~brass~~ aluminum strips are securely bonded to ~~an aluminum~~ brass bar of 30 × 30-mm square cross section. Using the data given below, determine the largest permissible bending moment when the composite member is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity:	70 GPa	105 GPa
Allowable stress:	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = E_b/E_a = 105/70 = 1.5$ in brass

For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{1.0}{12}(30)(6)^3 + (1.0)(30)(6)(18)^2 = 58.86 \times 10^3 \text{ mm}^4$$

$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12}(30)(30)^3 = 101.25 \times 10^3 \text{ mm}^4$, $\quad I_3 = I_1 = 58.86 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 + I_3 = 218.97 \times 10^3 \text{ mm}^4 = 218.97 \times 10^{-9} \text{ m}^4$

$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$

Aluminum: $n = 1.0$, $y = 21 \text{ mm} = 0.021 \text{ m}$ $\quad \sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(218.97 \times 10^{-9})}{(1.0)(0.021)} = 1.043 \times 10^3 \text{ N}\cdot\text{m}$$

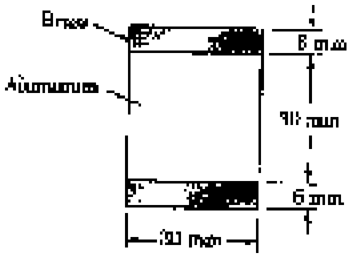
Brass: $n = 1.5$, $y = 15 \text{ mm} = 0.015 \text{ m}$, $\sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(218.97 \times 10^{-9})}{(1.5)(0.015)} = 1.557 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value $\quad M = 1.043 \times 10^3 \text{ N}\cdot\text{m} = 1.043 \text{ kN}\cdot\text{m}$

PROBLEM 4.41

4.41 and 4.42 For the composite bar indicated, determine the permissible bending moment when the bar is bent about a vertical axis.
4.41 Bar of Prob. 4.39



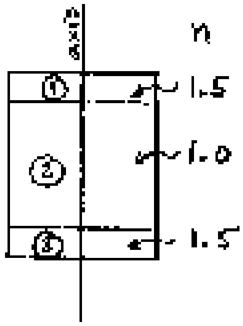
SOLUTION

Use aluminum as reference material

$n = 1.0$ in aluminum

$n = E_b / E_a = 105/70 = 1.5$ in brass

For the transformed section



$$I_1 = \frac{n_1}{12} b_1 h_1^3$$

$$= \frac{1.5}{12}(6)(30)^3 = 20.25 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3$$

$$= \frac{1.0}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 20.25 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 108 \times 10^3 \text{ mm}^4 = 108 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Aluminum: $n = 1.0$, $y = 15 \text{ mm} = 0.015 \text{ m}$, $\sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(108 \times 10^{-9})}{(1.0)(0.015)} = 720 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $y = 15 \text{ mm} = 0.015 \text{ m}$, $\sigma = 160 \times 10^6 \text{ Pa}$

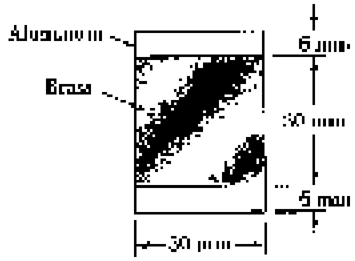
$$M = \frac{(160 \times 10^6)(108 \times 10^{-9})}{(1.5)(0.015)} = 768 \text{ N}\cdot\text{m}$$

Choose the smaller value $M = 720 \text{ N}\cdot\text{m}$

PROBLEM 4.42

4.41 and 4.42 For the composite bar indicated, determine the permissible bending moment when the bar is bent about a vertical axis.

4.42 Bar of Prob. 4.40



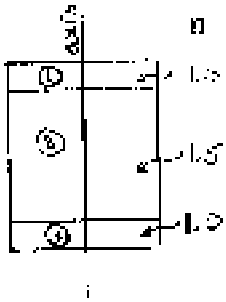
SOLUTION

Use aluminum as reference material

$n = 1.0$ in aluminum

$n = E_b / E_a = 105/70 = 1.5$ in brass

For the transformed section



$$I_1 = \frac{n_1}{12} b_1 h_1^3$$

$$= \frac{1.0}{12}(6)(30)^3 = 13.5 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12}(30)(30)^3 = 101.25 \text{ mm}^4$$

$$I_3 = I_1 = 13.5 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 128.25 \times 10^3 \text{ mm}^4 = 128.25 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{n M y}{I} \right| \qquad M = \frac{\sigma I}{n y}$$

Aluminum: $n = 1.0$, $y = 15 \text{ mm} = 0.015 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

$$M = \frac{(100 \times 10^6)(128.25 \times 10^{-9})}{(1.0)(0.015)} = 855 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $y = 15 \text{ mm} = 0.015 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

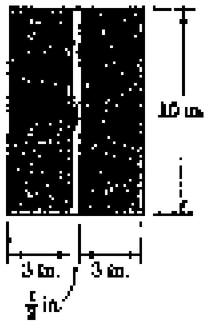
$$M = \frac{(160 \times 10^6)(128.25 \times 10^{-9})}{(1.5)(0.015)} = 912 \text{ N}\cdot\text{m}$$

Choose the smaller value $\qquad M = 855 \text{ N}\cdot\text{m}$

PROBLEM 4.43

4.43 and 4.44 Wooden beams and steel plates are securely bolted together to form the composite members shown. Using the data given below, determine the largest permissible bending moment when the composite beam is bent about a horizontal axis.

	Wood	Steel
Modulus of elasticity:	2×10^6 psi	30×10^6 psi
Allowable stress:	2000 psi	22 ksi



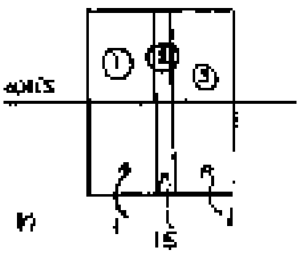
SOLUTION

Use wood as the reference material

$n = 1.0$ in wood

$n = E_s/E_w = 30/2 = 15$ in steel

For the transformed section



$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12}(3)(10)^3 = 250 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{15}{12}\left(\frac{1}{2}\right)(10)^3 = 625 \text{ in}^4$$

$$I_3 = I_1 = 250 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 1125 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Wood: $n = 1.0$, $y = 5$ in, $\sigma = 2000$ psi

$$M = \frac{(2000)(1125)}{(1.0)(5)} = 450 \times 10^3 \text{ lb}\cdot\text{in}$$

Steel: $n = 15$, $y = 5$ in, $\sigma = 22$ ksi $= 22 \times 10^3$ psi

$$M = \frac{(22 \times 10^3)(1125)}{(15)(5)} = 330 \times 10^3 \text{ lb}\cdot\text{in}$$

Choose the smaller value $\quad M = 330 \times 10^3 \text{ lb}\cdot\text{in} = 330 \text{ kip}\cdot\text{in}$

PROBLEM 4.44

4.43 and 4.44 Wooden beams and steel plates are securely bolted together to form the composite members shown. Using the data given below, determine the largest permissible bending moment when the composite beam is bent about a horizontal axis.

	Wood	Steel
Modulus of elasticity:	2×10^6 psi	30×10^6 psi
Allowable stress:	2000 psi	22 ksi

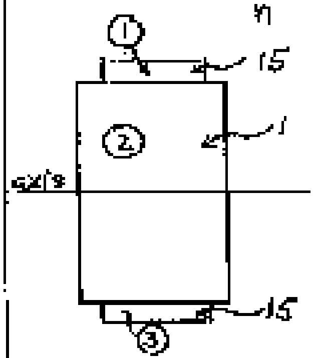
SOLUTION

Use wood as the reference material

$n = 1.0$ in wood

$n = E_s / E_w = 30/2 = 15$ in steel

For the transformed section



$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{15}{12}(5)\left(\tfrac{1}{2}\right)^3 + (15)(5)\left(\tfrac{1}{2}\right)(5.25)^2 = 1034.4 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.0}{12}(6)(10)^3 = 500 \text{ in}^4$$

$$I_3 = I_1 = 1034.4 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 2569 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad \therefore \qquad M = \frac{\sigma I}{ny}$$

Wood: $n = 1.0, \quad y = 5 \text{ in}, \quad \sigma = 2000$ psi

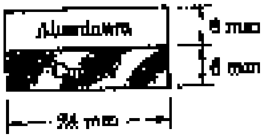
$$M = \frac{(2000)(2569)}{(1.0)(5)} = 1.028 \times 10^6 \text{ lb}\cdot\text{in}$$

Steel $n = 15, \quad y = 5.5 \text{ in}, \quad \sigma = 22 \text{ ksi} = 22 \times 10^3$ psi

$$M = \frac{(22 \times 10^3)(2569)}{(15)(5.5)} = 685 \times 10^3 \text{ lb}\cdot\text{in}$$

Choose the smaller value $\qquad M = 685 \times 10^3$ lb·in $= 685$ kip·in

PROBLEM 4.45



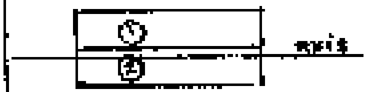
4.45 and 4.46 A copper strip (E_c = 105 GPa) and an aluminum strip (E_a = 75 GPa) are bonded together to form the composite bar shown. Knowing that the bar is bent about a horizontal axis by a couple of moment 35 N·m, determine the maximum stress in (*a*) the aluminum strip, (*b*) the copper strip.

SOLUTION

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = E_c/E_a = 105/75 = 1.4$ in copper



Transformed section

	A, mm^2	nA, mm^2	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm^3
1	144	144	9	1296
2	144	201.6	3	604.8
Σ		345.6		1900.8

$$\bar{Y}_o = \frac{1900.8}{345.6} = 5.50 \text{ mm}$$

The neutral axis lies 5.50 mm above the bottom

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(24)(6)^3 + (1.0)(24)(6)(3.5)^2 = 2196 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.4}{12}(24)(6)^3 + (1.4)(24)(6)(2.5)^2 = 1864.8 \text{ mm}^4$$

$$I = I_1 + I_2 = 4060.8 \text{ mm}^4 = 4.0608 \times 10^{-9} \text{ m}^4$$

(a) Aluminum $\quad n = 1.0 \quad y = 12 - 5.5 = 6.5 \text{ mm} = 0.0065 \text{ m}$

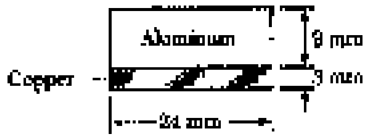
$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(35)(0.0065)}{4.0608 \times 10^{-9}} = -56.0 \times 10^6 \text{ Pa} = -56.0 \text{ MPa}$$

(b) Copper $\quad n = (1.4) \quad y = -5.5 \text{ mm} = -0.0055 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.4)(35)(-0.0055)}{4.0608 \times 10^{-9}} = 66.4 \times 10^6 \text{ Pa} = 66.4 \text{ MPa}$$

4.45 and 4.46 A copper strip (E_c = 105 GPa) and an aluminum strip (E_a = 75 GPa) are bonded together to form the composite bar shown. Knowing that the bar is bent about a horizontal axis by a couple of moment 35 N·m, determine the maximum stress in (*a*) the aluminum strip, (*b*) the copper strip.

PROBLEM 4.46

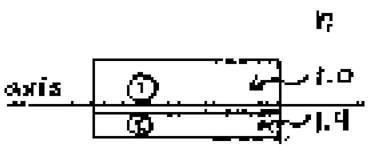


SOLUTION

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = E_c / E_a = 105/75 = 1.4$ in copper



Transformed section

	A, mm²	nA, mm²	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm³
①	216	216	7.5	1620
②	72	100.8	1.5	151.8
Σ		316.8		1771.2

$$\bar{Y}_o = \frac{1771.2}{316.8} = 5.5909 \text{ mm}$$

The neutral axis lies 5.5909 mm above the bottom.

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(24)(9)^3 + (1.0)(24)(9)(1.9091)^2 = 2245.2 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.4}{12}(24)(3)^3 + (1.4)(24)(3)(4.0909)^2 = 1762.5 \text{ mm}^4$$

$$I = I_1 + I_2 = 4008 \text{ mm}^4 = 4.008 \times 10^{-9} \text{ m}^4$$

(a) Aluminum: $n = 1.0$ $\quad y = 12 - 5.5909 = 6.4091 \text{ mm} = 0.0064091$

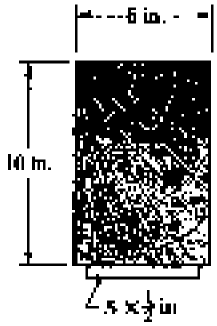
$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(35)(0.0064091)}{4.008 \times 10^{-9}} = -56.0 \times 10^6 \text{ Pa} = -56.0 \text{ MPa} \blacktriangleleft$$

(b) Copper: $n = 1.4$, $\quad y = -5.5909 \text{ mm} = -0.0055909$ m

$$\sigma = -\frac{nMy}{I} = -\frac{(1.4)(35)(-0.0055909)}{4.008 \times 10^{-9}} = 68.4 \times 10^6 \text{ Pa} = 68.4 \text{ MPa} \blacktriangleleft$$

PROBLEM 4.47

4.47 and 4.48 A 6 × 10-in. timber beam has been strengthened by bolting to it the steel straps shown. The modulus of elasticity is 1.5×10^6 psi for the wood and 30×10^6 psi for the steel. Knowing that the beam is bent about a horizontal axis by a couple of moment 200 kip·in., determine the maximum stress in (*a*) the wood, (*b*) the steel.



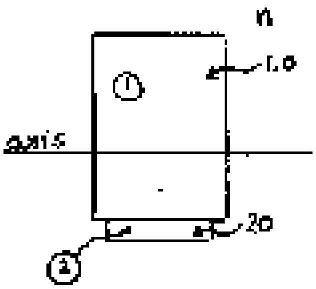
SOLUTION

Use wood as the reference material.

$n = 1.0$ in wood

$n = E_s/E_w = 30/1.5 = 20$ for steel

Transformed section



	A	nA	$\bar{y}_o$	$nA\bar{y}_o$
①	60	60	5.5	330
②	2.5	50	0.25	12.5
Σ		110		342.5

$$\bar{Y}_o = \frac{342.5}{110} = 3.114 \text{ in}$$

The neutral axis lies 3.114 in above the bottom.

$$I_1 = \frac{n_1}{12}b_1h_1^3 + n_1A_1d_1^2 = \frac{1.0}{12}(6)(10)^3 + (1.0)(60)(2.386)^2 = 841.6 \text{ in}^4$$

$$I_2 = \frac{n_2}{12}b_2h_2^3 + n_2A_2d_2^2 = \frac{20}{12}(5)(\tfrac{1}{2})^3 + (20)(2.5)(2.864)^2 = 411.2 \text{ in}^4$$

$$I = I_1 + I_2 = 1252.8 \text{ in}^4$$

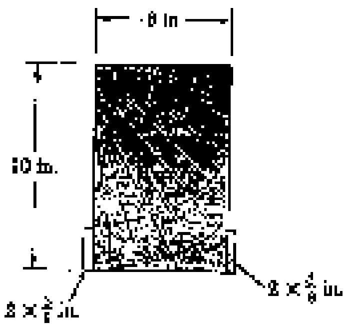
(a) Wood: $n = 1.0 \qquad y = 10.5 - 3.114 = 7.386$ in

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(200)(7.386)}{1252.8} = -1.179 \text{ ksi}$$

(b) Steel: $n = 20 \qquad y = -3.114$ in

$$\sigma = -\frac{nMy}{I} = -\frac{(20)(200)(-3.114)}{1252.8} = 9.94 \text{ ksi}$$

PROBLEM 4.48



4.47 and 4.48 A 6 × 10-in. timber beam has been strengthened by bolting to it the steel straps shown. The modulus of elasticity is 1.5×10^6 psi for the wood and 30×10^6 psi for the steel. Knowing that the beam is bent about a horizontal axis by a couple of moment 200 kip·in., determine the maximum stress in (*a*) the wood, (*b*) the steel.

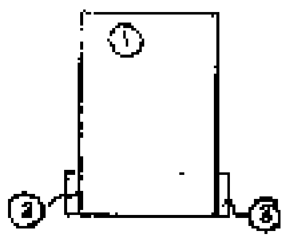
SOLUTION

Use wood as the reference material

$n = 1.0$ in wood

$n = E_s / E_w = 30/1.5 = 20$ in steel

Transformed section



	A	nA	$\bar{y}_0$	$nA\bar{y}_0$
1	60	60	5	300
2	0.75	15	1	15
3	0.75	15	1	15
Σ		90		330

$$\bar{Y}_0 = \frac{330}{90} = 3.667 \text{ in}$$

The neutral axis lies 3.667 in. above the bottom

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(6)(10)^3 + (60)(1.333)^2 = 606.7 \text{ in}^4$$

$$I_2 = I_3 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{20}{12}\left(\tfrac{3}{8}\right)(2)^3 + (15)(2.667)^2 = 111.7 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 830 \text{ in}^4$$

(a) Wood: $n = 1.0$, $y = 10 - 3.667 = 6.333$ in

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(200)(6.333)}{830} = -1.526 \text{ ksi}$$

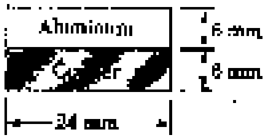
(b) Steel: $n = 20$ $y = -3.667$ in

$$\sigma = -\frac{nMy}{I} = -\frac{(20)(200)(-3.667)}{830} = 17.67 \text{ ksi}$$

PROBLEM 4.49

4.49 and 4.50 For the composite bar indicated, determine the radius of curvature caused by the couple of moment 35 N·m.

4.49 Bar of Prob. 4.45



SOLUTION

See solution to PROBLEM 4.45 for the calculation of I.

$$\frac{1}{\rho} = \frac{M}{E_a I} = \frac{35}{(75\times 10^9)(4.0608\times 10^{-9})} = 0.1149\ \text{m}^{-1}, \qquad \rho = 8.70\ \text{m}$$

PROBLEM 4.50

4.49 and 4.50 For the composite bar indicated, determine the radius of curvature caused by the couple of moment 35 N·m.

4.50 Bar of Prob. 4.46

SOLUTION

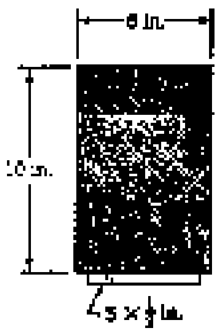
See solution to PROBLEM 4.46 for calculation of I.

$$\frac{1}{\rho} = \frac{M}{E_a I} = \frac{35}{(75\times 10^9)(4.008\times 10^{-9})} = 0.1164\ \text{m}^{-1}, \qquad \rho = 8.59\ \text{m}$$

PROBLEM 4.51

4.51 and 4.52 For the composite beam indicated, determine the radius of curvature caused by the couple of moment 200 kip·in.

4.51 Beam of Prob. 4.47



SOLUTION

See solution to PROBLEM 4.47 for calculation of I.

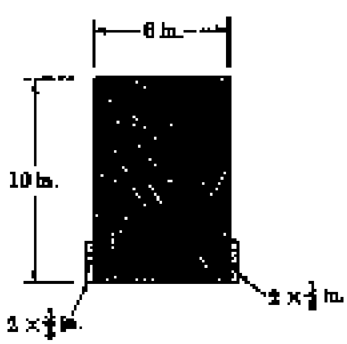
$$\frac{1}{\rho} = \frac{M}{E_w I} = \frac{200\times 10^3}{(1.5\times 10^6)(1252.8)} = 106.4\times 10^{-6}\ \text{in}^{-1}$$

$$\rho = 9396\ \text{in} = 783\ \text{ft.}$$

PROBLEM 4.52

4.51 and 4.52 For the composite beam indicated, determine the radius of curvature caused by the couple of moment 200 kip·in.

4.52 Beam of Prob. 4.48



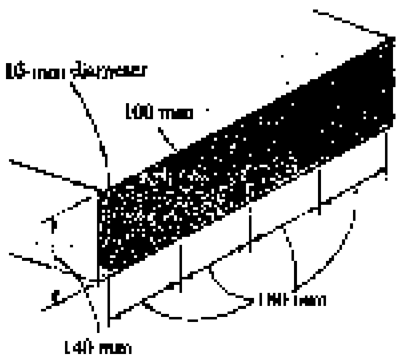
SOLUTION

See solution to PROBLEM 4.48 for calculation of I.

$$\frac{1}{\rho} = \frac{M}{E_w I} = \frac{200\times 10^3}{(1.5\times 10^6)(830)} = 160.6\times 10^{-6}\ \text{in}^{-1}$$

$$\rho = 6225\ \text{in} = 519\ \text{ft.}$$

PROBLEM 4.53



4.53 A concrete slab is reinforced by 16-mm-diameter steel rods placed on 180-mm centers as shown. The modulus of elasticity is 20 GPa for concrete and 200 GPA for steel. Using an allowable stress of 9 MPa for the concrete and of 120 MPA for the steel, determine the largest allowable positive bending moment in a portion of slab 1 m wide.

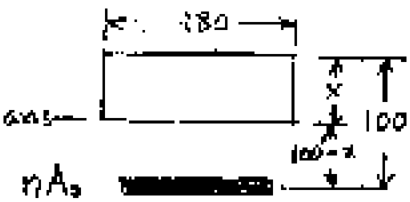
SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{20 \text{ GPa}} = 10$$

Consider a section 180 mm wide with one steel rod.

$$A_s = \frac{\pi}{4} d^2 = \frac{\pi}{4}(16)^2 = 201.06 \text{ mm}^2$$

$$nA_s = 2.0106 \times 10^3 \text{ mm}^2$$



Locate the neutral axis

$$180\, x \frac{x}{2} - (100 - x)(2.0106 \times 10^3) = 0$$

$$90x^2 + 2.0106 \times 10^3 x - 201.06 \times 10^3 = 0$$

Solving for x $\quad x = \dfrac{-2.0106 \times 10^3 + \sqrt{(2.0106 \times 10^3)^2 + (4)(90)(201.06 \times 10^3)}}{(2)(90)}$

$x = 37.397$ mm, $\qquad 100 - x = 62.603$ mm

$$I = \tfrac{1}{3}(180)x^3 + (2.0106 \times 10^3)(100 - x)^2$$

$$= \tfrac{1}{3}(180)(37.397)^3 + (2.0106 \times 10^3)(62.603)^2$$

$$= 11.018 \times 10^6 \text{ mm}^4 = 11.018 \times 10^{-6} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $\quad n = 1, \quad y = 37.397 \text{ mm} = 0.037397 \text{ m}, \quad \sigma = 9 \times 10^6 \text{ Pa}$

$$M = \frac{(9 \times 10^6)(11.018 \times 10^{-6})}{(1.0)(0.037397)} = 2.6516 \times 10^3 \text{ N}\cdot\text{m}$$

Steel: $\quad n = 10, \quad y = 62.603 \text{ mm} = 0.062603 \text{ m}, \quad \sigma = 120 \times 10^6 \text{ Pa}$

$$M = \frac{(120 \times 10^6)(11.018 \times 10^{-6})}{(10)(0.062603)} = 2.1120 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value $\quad M = 2.1120 \times 10^3$ N·m

The above is the allowable positive moment for a 180 mm wide section.

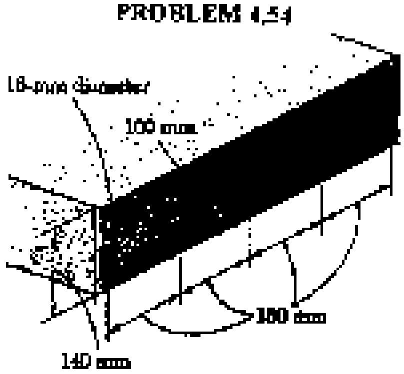
For a 1 m = 1000 mm width, multiply by $\frac{1000}{180} = 5.556$

$$M = (5.556)(2.1120 \times 10^3) = 11.73 \times 10^3 \text{ Nm} = 11.73 \text{ kN}\cdot\text{m}$$

4.53 A concrete slab is reinforced by 16-mm-diameter steel rods placed on 180-mm centers as shown. The modulus of elasticity is 20 GPa for concrete and 200 GPA for steel. Using an allowable stress of 9 MPa for the concrete and of 120 MPA for the steel, determine the largest allowable positive bending moment in a portion of slab 1 m wide.

4.54 Solve Prob. 4.53, assuming that the spacing of the 16-mm-diameter rods is increased to 225 mm on centers.

PROBLEM 4.54



SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200\text{ GPa}}{20\text{ GPa}} = 10$$

Consider a section 225 mm wide with one steel rod.

$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(16)^2 = 201.06\text{ mm}^2$$

$$nA_s = 2.0106 \times 10^3\text{ mm}^2$$

Locate the neutral axis

$$225x\frac{x}{2} - (100 - x)(2.0106\times10^3) = 0$$

$$112.5x^2 + 2.0106x - 201.06\times10^3 = 0$$

Solving for x $\quad x = \dfrac{-2.0106\times10^3 + \sqrt{(2.0106\times10^3)^2 + (4)(112.5)(201.06\times10^3)}}{(2)(112.5)}$

$x = 34.273$ mm $\qquad 100 - x = 65.727$

$$I = \frac{1}{3}(225)x^3 + 2.0106\times10^3(100-x)^2$$

$$= \frac{1}{3}(225)(34.273)^3 + (2.0106\times10^3)(65.727)^2$$

$$= 11.705\times10^6\text{ mm}^4 = 11.705\times10^{-6}\text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $n = 1$, $y = 34.273$ mm $= 0.034273$ m, $\sigma = 9\times10^6$ Pa

$$M = \frac{(9\times10^6)(11.705\times10^{-6})}{(1)(0.034273)} = 3.0738\times10^3\text{ N}\cdot\text{m}$$

Steel: $n = 10$, $y = 65.727$ m $= 0.065727$ m, $\sigma = 120\times10^6$ Pa

$$M = \frac{(120\times10^6)(11.705\times10^{-6})}{(10)(0.065727)} = 2.1370\times10^3\text{ N}\cdot\text{m}$$

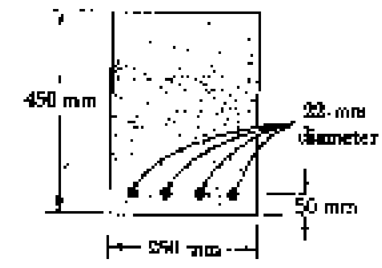
Choose the smaller value $\quad M = 2.1370\times10^3$ N·m

The above is the allowable positive moment for a 225 mm wide section.

For a 1 m = 1000 mm section, multiply by $\frac{1000}{225} = 4.4444$

$$M = (4.4444)(2.1370\times10^3) = 9.50\times10^3\text{ N}\cdot\text{m} = 9.50\text{ kN}\cdot\text{m}$$

PROBLEM 4.55



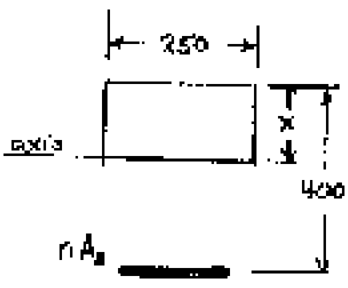
4.55 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN·m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200\text{ GPa}}{25\text{ GPa}} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4)\left(\frac{\pi}{4}\right)(22)^2 = 1.5205 \times 10^3 \text{ mm}^2$$

$$nA_s = 12.164 \times 10^3 \text{ mm}^2$$



Locate the neutral axis

$$250\, x \frac{x}{2} - (12.164 \times 10^3)(400 - x) = 0$$

$$125 x^2 + 12.164 \times 10^3 x - 4.8657 \times 10^6 = 0$$

Solving for x
$$x = \frac{-12.164 \times 10^3 + \sqrt{(12.164 \times 10^3)^2 + (4)(125)(4.8657 \times 10^6)}}{(2)(125)}$$

$$x = 154.55 \text{ mm}, \qquad 400 - x = 245.45 \text{ mm}$$

$$I = \frac{1}{3} 250\, x^3 + (12.164 \times 10^3)(400 - x)^2$$

$$= \frac{1}{3}(250)(154.55)^3 + (12.164 \times 10^3)(245.45)^2$$

$$= 1.0404 \times 10^9 \text{ mm}^4 = 1.0404 \times 10^{-3} \text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

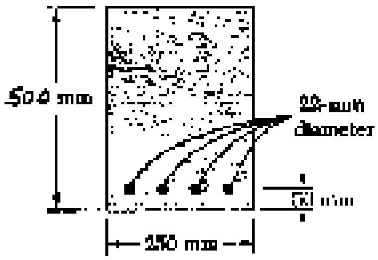
(a) Steel: $y = -245.45$ mm $= -0.24545$ m

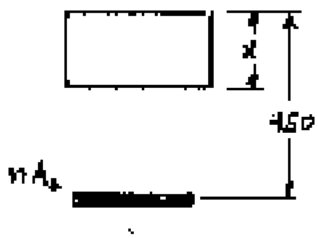
$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.24545)}{1.0404 \times 10^{-3}} = 330 \times 10^6 \text{ Pa} = 330 \text{ MPa} \quad \blacktriangleleft$$

(b) Concrete: $y = 154.55$ mm $= 0.15455$ m

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.15455)}{1.0404 \times 10^{-3}} = -26.0 \times 10^6 \text{ Pa} = -26.0 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 4.56





4.55 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN·m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.

4.56 Solve Prob. 4.55 assuming that the 450-mm depth of the beam is increased to 500 mm.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200\ GPa}{25\ GPa} = 8.0$$

$$A_s = 4\frac{\pi}{4}d^2 = (4)(\frac{\pi}{4})(22)^2 = 1.5205\times10^3\ mm^2$$

$$nA_s = 12.164\times10^3\ mm^2$$

Locate the neutral axis

$$250\ x\frac{x}{2} - (12.164\times10^3)(450-x) = 0$$

$$125\ x^2 + 12.164\times10^3\ x - 5.4738\times10^6 = 0$$

Solving for x
$$x = \frac{-12.164\times10^3 + \sqrt{(12.164\times10^3)^2 + (4)(125)(5.4738\times10^6)}}{(2)(125)}$$

$$x = 166.19\ mm, \qquad 450 - x = 283.81\ mm$$

$$I = \frac{1}{3}(250)x^3 + (12.164\times10^3)(450-x)^2$$

$$= \frac{1}{3}(250)(166.19)^3 + (12.164\times10^3)(283.81)^2$$

$$= 1.3623\times10^9\ mm^4 = 1.3623\times10^{-3}\ m^4$$

$$\sigma = -\frac{nMy}{I}$$

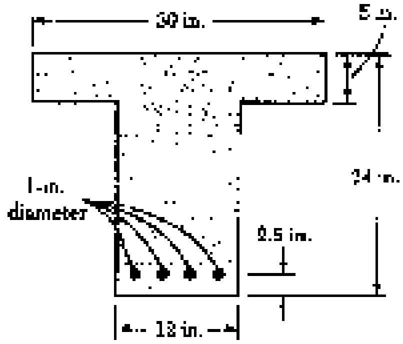
(a) Steel: $y = -283.81\ mm = -0.28381\ m$

$$\sigma = -\frac{(8.0)(175\times10^3)(-0.28381)}{1.3623\times10^{-3}} = 292\times10^6\ Pa = 292\ MPa$$

(b) Concrete: $y = 166.19\ mm = 0.16619\ m$

$$\sigma = -\frac{(1.0)(175\times10^3)(0.16619)}{1.3623\times10^{-3}} = -21.3\times10^6\ Pa = -21.3\ MPa$$

PROBLEM 4.57



4.57 Knowing that the bending moment in the reinforced concrete beam shown is +150 kip·ft and that the modulus of elasticity is 3.75 × 10⁶ psi for the concrete and 30 × 10⁶ psi for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.

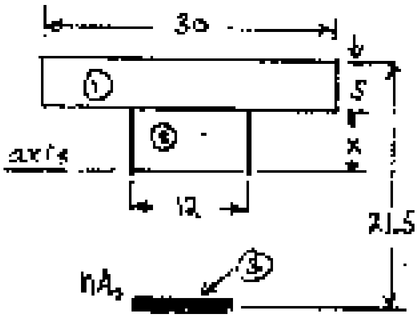
SOLUTION

$$n = \frac{E_s}{E_c} = \frac{30\times10^6}{3.75\times10^6} = 8.0$$

$$A_s = 4\frac{\pi}{4}d^2 = 4\left(\frac{\pi}{4}\right)(1)^2 = 3.1416 \text{ in}^2$$

$$nA_s = 25.133 \text{ in}^2$$

Locate the neutral axis



$$(30)(5)(x+2.5) + 12x\frac{x}{2} - (25.133)(16.5-x) = 0$$

$$150x + 375 + 6x^2 - 414.69 + 25.133x = 0$$

$$6x^2 + 175.133x - 39.69 = 0$$

Solve for x $\quad x = \dfrac{-175.133 + \sqrt{(175.133)^2 + (4)(6)(39.69)}}{(2)(6)} = 0.225$ in.

$$16.5 - x = 16.275 \text{ in.}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(30)(5)^3 + (30)(5)(2.725)^2 = 1426.3 \text{ in}^4$$

$$I_2 = \frac{1}{3}b_2x^3 = \frac{1}{3}(12)(0.225)^3 = 0.1 \text{ in}^4$$

$$I_3 = nA_sd_3^2 = (25.133)(16.275)^2 = 6657.1 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 8083.5 \text{ in}^4$$

$$\sigma = -\frac{nMy}{I} \quad \text{where} \quad M = 150 \text{ kip}\cdot\text{ft} = 1800 \text{ kip}\cdot\text{in.}$$

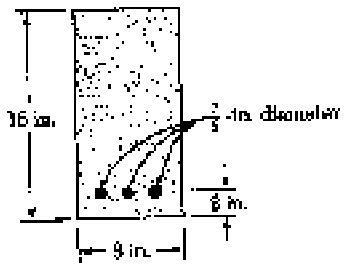
(a) Steel $\quad n = 8.0, \quad y = -16.275$ in

$$\sigma = -\frac{(8.0)(1800)(-16.275)}{8083.5} = 29.0 \text{ ksi}$$

(b) Concrete $\quad n = 1.0, \quad y = 5.225$ in

$$\sigma = -\frac{(1.0)(1800)(5.225)}{8083.5} = -1.163 \text{ ksi}$$

PROBLEM 4.58



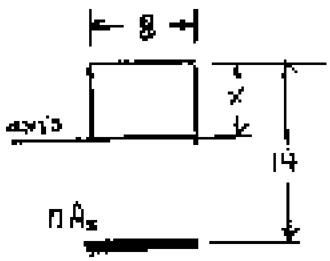
4.58 A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 30×10^6 psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{30 \times 10^6}{3 \times 10^6} = 10$$

$$A_s = 3\,\frac{\pi}{4} d^2 = 3\left(\frac{\pi}{4}\right)\left(\frac{7}{8}\right)^2 = 1.8040 \text{ in}^2$$

$$nA_s = 18.040 \text{ in}^2$$



Locate neutral axis.

$$8x\frac{x}{2} - (18.040)(14 - x) = 0$$

$$4x^2 + 18.040x - 252.56 = 0$$

Solve for x $\quad x = \frac{-18.040 + \sqrt{18.040^2 + (4)(4)(252.56)}}{(2)(4)} = 6.005 \text{ in.}$

$$14 - x = 7.995 \text{ in}$$

$$I = \frac{1}{3}\,8\,x^3 + nA_s(14 - x)^2 = \frac{1}{3}(8)(6.005)^3 + (18.040)(7.995)^2$$

$$= 1730.4 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $\quad n = 1.0, \quad |y| = 6.005 \text{ in}, \quad |\sigma| = 1350 \text{ psi}$

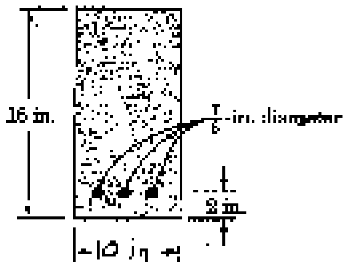
$$M = \frac{(1350)(1730.5)}{(1.0)(6.005)} = 389 \times 10^3 \text{ lb}\cdot\text{in} = 389 \text{ kip}\cdot\text{in}$$

Steel: $\quad n = 10, \quad |y| = 7.995, \quad \sigma = 20 \times 10^3 \text{ psi}$

$$M = \frac{(20 \times 10^3)(1730.5)}{(10)(7.995)} = 433 \times 10^3 \text{ lb}\cdot\text{in} = 433 \text{ kip}\cdot\text{in}$$

Choose the smaller value $\quad M = 389 \text{ kip}\cdot\text{in} = 32.4 \text{ kip}\cdot\text{ft}$

PROBLEM 4.59



4.58 A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 30×10^6 psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.

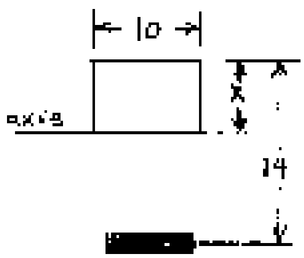
4.59 Solve Prob. 4.58, assuming that the width of the concrete beam is increased to 10 in.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{30\times10^6}{3\times10^6} = 10$$

$$A_s = 3\,\frac{\pi}{4} d^2 = 3\left(\frac{\pi}{4}\right)\left(\frac{7}{8}\right)^2 = 1.8040 \text{ in}^2$$

$$nA_s = 18.040 \text{ in}^2$$



Locate the neutral axis

$$10\,x\,\frac{x}{2} - (18.040)(14-x) = 0$$

$$5x^2 + 18.040\,x - 252.56 = 0$$

Solve for x $\qquad x = \dfrac{-18.040 + \sqrt{(18.040)^2 + (4)(5)(252.56)}}{(2)(5)} = 5.529$ in

$$14 - x = 8.471 \text{ in}$$

$$I = \frac{1}{3}(10)x^3 + nA_s(14-x)^2 = \frac{1}{3}(10)(5.529)^3 + (18.040)(8.471)^2$$

$$= 1857.9 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $\quad n = 1.0 \qquad |y| = 5.529$ in $\qquad |\sigma| = 1350$ psi

$$M = \frac{(1350)(1857.9)}{(1.0)(5.529)} = 453.6\times10^3 \text{ lb}\cdot\text{in}$$

Steel: $\quad n = 10 \qquad |y| = 8.471$ in $\qquad |\sigma| = 20\times10^3$ psi

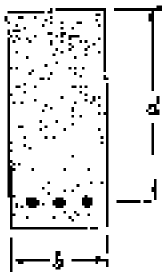
$$M = \frac{(20\times10^3)(1857.9)}{(10)(8.471)} = 438.6\times10^3 \text{ lb}\cdot\text{in}$$

Choose the smaller value $\qquad M = 438.6\times10^3$ lb·in

$= 438.6$ kip·in

$= 36.6$ kip·ft ◄

PROBLEM 4.60

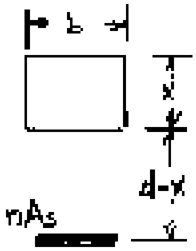


4.60 The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c. Show that to achieve a balanced design the distance x from the top of the beam to the neutral axis must be

$$x = \frac{d}{1 + \dfrac{\sigma_s E_c}{\sigma_c E_s}}$$

where E_c and E_s are the moduli of elasticity of concrete and steel, respectively, and d is the distance from the top of the beam to the reinforcing steel.

SOLUTION



$$\sigma_s = \frac{n M (d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{E_c\,\sigma_s}{E_s\,\sigma_c}$$

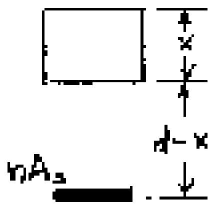
$$x = \frac{d}{1 + \dfrac{E_c\,\sigma_s}{E_s\,\sigma_c}}$$

PROBLEM 4.61

4.60 The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c.

4.61 For the concrete beam shown, the modulus of elasticity is 3.5×10^6 psi for the concrete and 29×10^6 psi for the steel. Knowing that $b = 8$ in. and $d = 22$ in., and using an allowable stress of 1800 psi for the concrete and 20 ksi for the steel, determine (*a*) the required area A_s of the steel reinforcement if the design of the beam is to be balanced, (*b*) the largest allowable bending moment. (See Prob. 4.60 for definition of a balanced beam.)

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3.5 \times 10^6} = 8.2857$$

$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.2857} \cdot \frac{20 \times 10^3}{1800} = 2.3410$$

$$x = 0.42717\, d = (0.42717)(22) = 9.398 \text{ in}$$

$$d - x = 22 - 9.398 = 12.602 \text{ in}$$

Locate neutral axis

$$bx\frac{x}{2} - nA_s(d-x) = 0$$

(a) $$A_s = \frac{bx^2}{2n(d-x)} = \frac{(8)(9.398)^2}{(2)(8.2857)(12.602)} = 3.3835 \text{ in}^2$$

$$I = \frac{1}{3}bx^3 + nA_s(d-x)^2 = \frac{1}{3}(8)(9.398)^3 + (8.2857)(3.3835)(12.602)^2$$

$$= 6665.6 \text{ in}^4$$

$$\sigma = \frac{nMy}{I} \qquad M = \frac{\sigma I}{ny}$$

Concrete: $n = 1.0 \quad y = 9.398$ in $\quad \sigma = 1800$ psi

$$M = \frac{(1800)(6665.6)}{(1.0)(9.398)} = 1.277 \times 10^6 \text{ lb}\cdot\text{in}$$

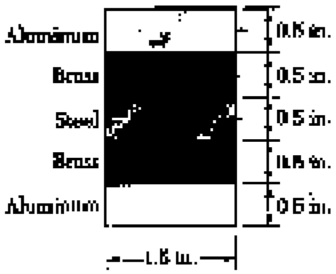
Steel: $n = 8.2857 \quad |y| = 12.602$ in $\quad \sigma = 20 \times 10^3$ psi

$$M = \frac{(20 \times 10^3)(6665.6)}{(8.2857)(12.602)} = 1.277 \times 10^6 \text{ lb}\cdot\text{in}$$

Note that both values are the same for balanced design

$$M = 1.277 \times 10^3 \text{ kip}\cdot\text{in} = 106.4 \text{ kip}\cdot\text{ft}$$

PROBLEM 4.62



4.62 and 4.63 Five metal strips, each of 0.5 × 1.5-in. cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 30×10^6 psi for the steel, 15×10^6 psi for the brass, and 10×10^6 psi for the aluminum. Knowing that the beam is bent about a horizontal axis by couples of moment 12 kip·in., determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

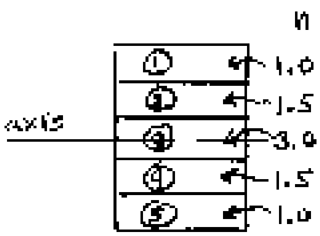
SOLUTION

Use aluminum as the reference material

$$n = \frac{E_s}{E_a} = \frac{30\times10^6}{10\times10^6} = 3.0 \text{ in steel}$$

$$n = \frac{E_b}{E_a} = \frac{15\times10^6}{10\times10^6} = 1.5 \text{ in brass}$$

$$n = 1.0 \text{ in aluminum.}$$



For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(1.5)(0.5)^3 + (0.75)(1.0)^2$$

$$= 0.7656 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.5}{12}(1.5)(0.5)^3 + (1.5)(0.75)(0.5)^2 = 0.3047 \text{ in}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{3.0}{12}(1.5)(0.5)^3 = 0.0469 \text{ in}^4$$

$$I_4 = I_2 = 0.3047 \text{ in}^4, \qquad I_5 = I_1 = 0.7656 \text{ in}^4$$

$$I = \sum_1^5 I_i = 2.1875 \text{ in}^4$$

(a) Aluminum: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.0)(12)(1.25)}{2.1875} = 6.86$ ksi ◄

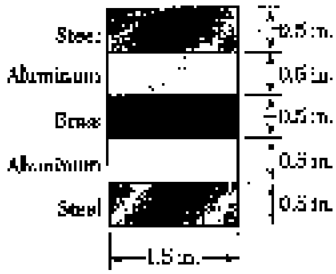
Brass: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.5)(12)(0.75)}{2.1875} = 6.17$ ksi ◄

Steel: $\sigma = \dfrac{nMy}{I} = \dfrac{(3.0)(12)(0.25)}{2.1875} = 4.11$ ksi ◄

(b) $\dfrac{1}{\rho} = \dfrac{M}{E_a I} = \dfrac{12\times10^3}{(10\times10^6)(2.1875)} = 548.57\times10^{-6} \text{ in}^{-1}$

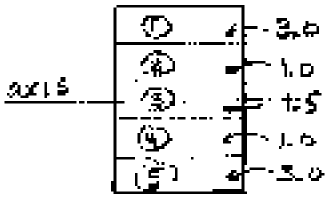
$\rho = 1823$ in $= 151.9$ ft ◄

PROBLEM 4.63



4.62 and 4.63 Five metal strips, each of 0.5 × 1.5-in. cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 30×10^6 psi for the steel, 15×10^6 psi for the brass, and 10×10^6 psi for the aluminum. Knowing that the beam is bent about a horizontal axis by couples of moment 12 kip·in., determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

SOLUTION



Use aluminum as the reference material

$$n = \frac{E_s}{E_a} = \frac{30 \times 10^6}{10 \times 10^6} = 3.0 \quad \text{in steel}$$

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \quad \text{in brass}$$

$$n = 1.0 \quad \text{in aluminum}$$

For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{3.0}{12}(1.5)(0.5)^3 + (3.0)(0.75)(1.0)^2 = 2.2969 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.0}{12}(1.5)(0.5)^3 + (1.0)(0.75)(0.5)^2 = 0.2031 \text{ in}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{1.5}{12}(1.5)(0.5)^3 = 0.0234 \text{ in}^4$$

$$I_4 = I_2 = 0.2031 \text{ in}^4, \qquad I_5 = I_1 = 2.2969 \text{ in}^4$$

$$I = \sum_1^5 I_i = 5.0234 \text{ in}^4$$

(a) Steel: $\sigma = \dfrac{nMy}{I} = \dfrac{(3.0)(12)(1.25)}{5.0234} = 8.96 \text{ ksi}$ ◄

Aluminum: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.0)(12)(0.75)}{5.0234} = 1.792 \text{ ksi}$ ◄

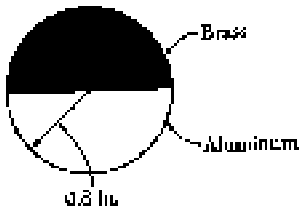
Brass: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.5)(12)(0.25)}{5.0234} = 0.896 \text{ ksi}$ ◄

(b) $\dfrac{1}{\rho} = \dfrac{M}{E_a I} = \dfrac{12 \times 10^3}{(10 \times 10^6)(5.0234)} = 238.89 \times 10^{-6} \text{ in}^{-1}$

$\rho = 4186 \text{ in.} = 349 \text{ ft}$ ◄

PROBLEM 4.64

4.64 The composite beam shown is formed by bonding together a brass rod and an aluminum rod of semicircular cross sections. The modulus of elasticity is 15×10^6 psi for the brass and 10×10^6 psi for the aluminum. Knowing that the composite beam is bent about a horizontal axis by couples of moment 8 kip·in., determine the maximum stress (*a*) in the brass, (*b*) in the aluminum.



SOLUTION

For each semicircle $r = 0.8$ in,

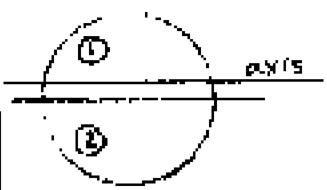
$$A = \frac{\pi}{2} r^2 = 1.00531 \text{ in}^2, \quad \bar{y}_o = \frac{4r}{3\pi} = \frac{(4)(0.8)}{3\pi} = 0.33953 \text{ in}$$

$$I_{base} = \frac{\pi}{8} r^4 = 0.160850 \text{ in}^4$$

$$\bar{I} = I_{base} - A\bar{y}_o^2 = 0.160850 - (1.00531)(0.33953)^2$$

$$= 0.044957 \text{ in}^4$$

Use aluminum as the reference material



$n = 1.0$ in aluminum

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \text{ in brass}$$

Locate neutral axis

	A, in^2	nA, in^2	$\bar{y}_o$, in	$nA\bar{y}_o$, in^3
①	1.00531	1.50796	0.33953	0.51200
②	1.0053	1.00531	−0.33953	−0.34133
Σ		2.51327		0.17067

$$\bar{Y}_o = \frac{0.17067}{2.51327} = 0.06791 \text{ in}$$

The neutral axis lies 0.06791 in above the material interface.

$$d_1 = 0.33953 - 0.06791 = 0.27162 \text{ in}, \quad d_2 = 0.33953 + 0.06791 = 0.40744 \text{ in}$$

$$I_1 = n_1\bar{I} + n_1 A d_1^2 = (1.5)(0.044957) + (1.5)(1.00531)(0.27162)^2 = 0.17869 \text{ in}^4$$

$$I_2 = n_2\bar{I} + n_2 A d_2^2 = (1.0)(0.044957) + (1.0)(1.00531)(0.40744)^2 = 0.21185 \text{ in}^4$$

$$I = I_1 + I_2 = 0.39054 \text{ in}^4$$

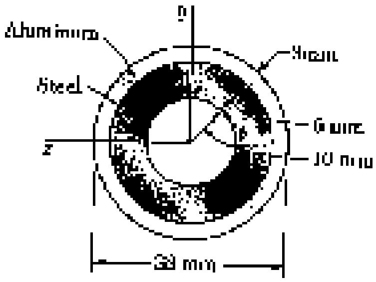
(a) Brass: $n = 1.5$, $y = 0.8 - 0.06791 = 0.73209$ in

$$\sigma = -\frac{nMy}{I} = -\frac{(1.5)(8)(0.73209)}{0.39054} = -22.5 \text{ ksi}$$

(b) Aluminum: $n = 1.0$ $y = -0.8 - 0.06791 = -0.86791$ in

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(8)(-0.86791)}{0.39054} = 17.78 \text{ ksi}$$

PROBLEM 4.65



4.65 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couples of moment 500 N·m, determine the maximum stress (*a*) in the aluminum, (*b*) in the steel.

SOLUTION

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0$ in steel

Steel: $I_1 = n_1 \frac{\pi}{4}(r_o^4 - r_i^4) = (3.0)\frac{\pi}{4}(16^4 - 10^4) = 130.85 \times 10^3 \ mm^4$

Aluminum: $I_2 = n_2 \frac{\pi}{4}(r_o^4 - r_i^4) = (1.0)\frac{\pi}{4}(19^4 - 16^4) = 50.88 \times 10^3 \ mm^4$

$I = I_1 + I_2 = 181.73 \times 10^3 \ mm^4 = 181.73 \times 10^{-9} \ m^4$

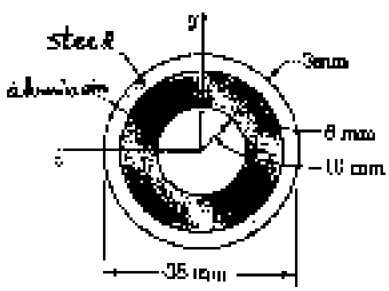
(a) Aluminum: $c = 19 \ mm = 0.019 \ m$

$$\sigma = \frac{nMc}{I} = \frac{(1.0)(500)(0.019)}{181.73 \times 10^{-9}} = 52.3 \times 10^6 \ Pa = 52.3 \ MPa \quad \blacktriangleleft$$

(b) Steel: $c = 16 \ mm = 0.016 \ m$

$$\frac{nMc}{I} = \frac{(3.0)(500)(0.016)}{181.73 \times 10^{-9}} = 132.1 \times 10^6 \ Pa = 132.1 \ MPa \quad \blacktriangleleft$$

PROBLEM 4.66



4.65 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couples of moment 500 N·m, determine the maximum stress (*a*) in the aluminum, (*b*) in the steel.

4.66 Solve Prob. 4.65, assuming that the 6-mm-thick inner pipe is made of aluminum and that the 3-mm-thick outer pipe is made of steel.

SOLUTION

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0$ in steel

Steel: $I_1 = n_1 \frac{\pi}{4}(r_o^4 - r_i^4) = (3.0)\frac{\pi}{4}(19^4 - 16^4) = 152.65 \times 10^3 \ mm^4$

Aluminum: $I_2 = n_2 \frac{\pi}{4}(r_o^4 - r_i^4) = (1.0)\frac{\pi}{4}(16^4 - 10^4) = 43.62 \times 10^3 \ mm^4$

$I = I_1 + I_2 = 196.27 \times 10^3 \ mm^4 = 196.27 \times 10^{-9} \ m^4$

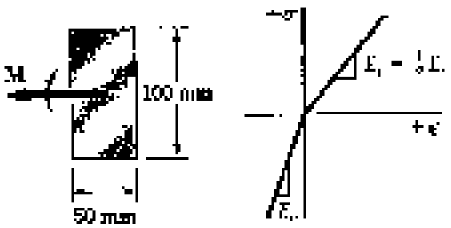
(a) Aluminum: $c = 16 \ mm = 0.016 \ m$

$$\sigma = \frac{nMc}{I} = \frac{(1.0)(500)(0.016)}{196.27 \times 10^{-9}} = 40.8 \times 10^6 \ Pa = 40.8 \ MPa \quad \blacktriangleleft$$

(b) Steel: $c = 19 \ mm = 0.019 \ m$

$$\sigma = \frac{nMc}{I} = \frac{(3.0)(500)(0.019)}{196.27 \times 10^{-9}} = 145.2 \times 10^6 \ Pa = 145.2 \ MPa \quad \blacktriangleleft$$

PROBLEM 4.67

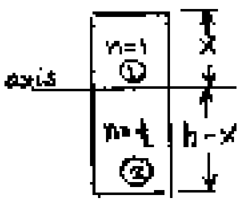


4.67 The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one half of its value in compression. For a bending moment $M = 600$ N·m, determine the maximum (*a*) tensile stress, (*b*) compressive stress.

SOLUTION

$n = \frac{1}{2}$ on the tension side of neutral axis

$n = 1$ on the compression side



Locate neutral axis.

$$n_1 b x \frac{x}{2} - n_2 b(h-x)\frac{h-x}{2} = 0$$

$$\frac{1}{2}bx^2 - \frac{1}{4}b(h-x)^2 = 0$$

$$x^2 = \frac{1}{2}(h-x)^2 \qquad x = \frac{1}{\sqrt{2}}(h-x)$$

$$x = \frac{1}{\sqrt{2}+1}h = 0.41421\,h = 41.421 \text{ mm}$$

$$h - x = 58.579 \text{ mm}$$

$$I_1 = n_1 \frac{1}{3}bx^3 = (1)\left(\frac{1}{3}\right)(50)(41.421)^3 = 1.1844\times10^6 \text{ mm}^4$$

$$I_2 = n_2\frac{1}{3}b(h-x)^3 = \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(50)(58.579)^3 = 1.6751\times10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 2.8595\times10^6 \text{ mm}^4 = 2.8595\times10^{-6} \text{ m}^4$$

(a) tensile stress: $\quad n = \frac{1}{2}, \quad y = -58.579 \text{ mm} = -0.058579 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(0.5)(600)(-0.058579)}{2.8595\times10^{-6}} = 6.15\times10^6 \text{ Pa}$$

$$= 6.15 \text{ MPa}$$

(b) compressive stress: $\quad n = 1, \quad y = 41.421 \text{ mm} = 0.041421 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(600)(0.041421)}{2.8595\times10^{-6}} = -8.69\times10^6 \text{ Pa}$$

$$= -8.69 \text{ MPa}$$

PROBLEM 4.68

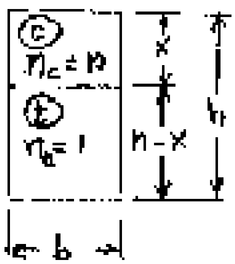
*4.68 A rectangular beam is made of a material for which the modulus of elasticity is E_t in tension and E_c in compression. Show that the curvature of the beam in pure bending is

$$\frac{1}{\rho} = \frac{M}{E_r I}$$

where

$$E_r = \frac{4E_t E_c}{(\sqrt{E_t} + \sqrt{E_c})^2}$$

SOLUTION



Use E_t as the reference modulus.

Then $E_c = nE_t$

Locate neutral axis

$$nbx\frac{x}{2} - b(h-x)\frac{h-x}{2} = 0$$

$$nx^2 - (h-x)^2 = 0 \qquad \sqrt{n}\,x = (h-x)$$

$$x = \frac{h}{\sqrt{n}+1} \qquad h - x = \frac{\sqrt{n}\,h}{\sqrt{n}+1}$$

$$I_{trans} = \frac{n}{3}bx^3 + \frac{1}{3}b(h-x)^3 = \left[\frac{n}{3}\left(\frac{1}{\sqrt{n}+1}\right)^3 + \left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)^3\right] bh^3$$

$$= \frac{1}{3}\frac{n + n^{3/2}}{(\sqrt{n}+1)^3}bh^3 = \frac{1}{3}\frac{n(1+\sqrt{n})}{(\sqrt{n}+1)^3}bh^3 = \frac{1}{3}\cdot\frac{n}{(\sqrt{n}+1)^2}bh^3$$

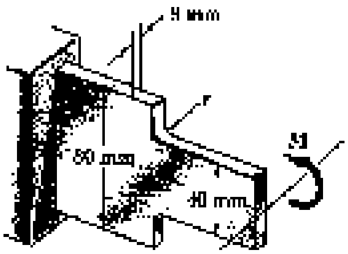
$$\frac{1}{\rho} = \frac{M}{E_t I_{trans}} = \frac{M}{E_r I} \qquad \text{where } I = \frac{1}{12}bh^3$$

$$E_r I = E_t I_{trans}$$

$$E_r = \frac{E_t I_{trans}}{I} = \frac{12}{bh^3}\cdot E_t \cdot \frac{n}{3(\sqrt{n}+1)^2}bh^3$$

$$= \frac{4E_t E_c/E_t}{(\sqrt{E_c/E_t}+1)^2} = \frac{4E_t E_c}{(\sqrt{E_c}+\sqrt{E_t})^2}$$

PROBLEM 4.69



4.69 Knowing that $M = 250$ N·m, determine the maximum stress in the beam shown when the radius r of the fillets is (*a*) 4 mm, (*b*) 8 mm.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667\times10^3\ \text{mm}^4 = 42.667\times10^{-9}\ \text{m}^4$$

$$c = 20\ \text{mm} = 0.020\ \text{m}$$

$$\frac{D}{d} = \frac{80\ \text{mm}}{40\ \text{mm}} = 2.00$$

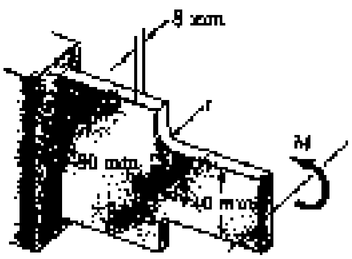
(a) $\frac{r}{d} = \frac{4\ \text{mm}}{40\ \text{mm}} = 0.10$ From Fig. 4.31 $K = 1.87$

$$\sigma_{max} = K\frac{Mc}{I} = \frac{(1.87)(250)(0.020)}{42.667\times10^{-9}} = 219\times10^6\ \text{Pa} = 219\ \text{MPa}$$

(b) $\frac{r}{d} = \frac{8\ \text{mm}}{40\ \text{mm}} = 0.20$ From Fig. 4.31 $K = 1.50$

$$\sigma_{max} = K\frac{Mc}{I} = \frac{(1.50)(250)(0.020)}{42.667\times10^{-9}} = 176\times10^6\ \text{Pa} = 176\ \text{MPa}$$

PROBLEM 4.70



4.70 Knowing that the allowable stress for the beam shown is 90 MPa, determine the allowable bending moment M when the radius r of the fillets is (*a*) 8 mm, (*b*) 12 mm.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667\times10^3\ \text{mm}^4 = 42.667\times10^{-9}\ \text{m}^4$$

$$c = 20\ \text{mm} = 0.020\ \text{m}$$

$$\frac{D}{d} = \frac{80\ \text{mm}}{40\ \text{mm}} = 2.00$$

(a) $\frac{r}{d} = \frac{8\ \text{mm}}{40\ \text{mm}} = 0.2$ From Fig. 4.31 $K = 1.50$

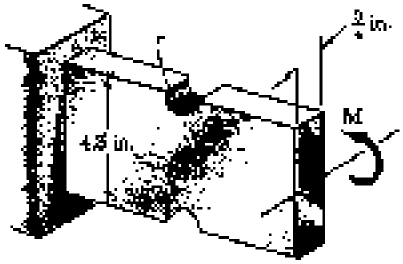
$$\sigma_{max} = K\frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma_{max} I}{Kc} = \frac{(90\times10^6)(42.667\times10^{-9})}{(1.50)(0.020)}$$

$$= 128\ \text{N·m}$$

(b) $\frac{r}{d} = \frac{12\ \text{mm}}{40\ \text{mm}} = 0.3$ From Fig 4.31 $K = 1.35$

$$M = \frac{(90\times10^6)(42.667\times10^{-9})}{(1.35)(0.020)} = 142\ \text{N·m}$$

PROBLEM 4.71



4.71 Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Using an allowable stress of 8 ksi, determine the largest bending moment that can be applied to the member when the radius r of the semicircular grooves is (a) $\frac{3}{8}$ in., (b) $\frac{3}{4}$ in.

SOLUTION

(a) $d = D - 2r = 4.5 - (2)(\frac{3}{8}) = 3.75$ in

$\frac{D}{d} = \frac{4.5}{3.75} = 1.20, \quad \frac{r}{d} = \frac{0.375}{3.75} = 0.10$

From Fig. 4.32 $\quad K = 2.07$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.75)^3 = 3.296 \text{ in}^4, \qquad c = \frac{1}{2} = 1.875 \text{ in}$

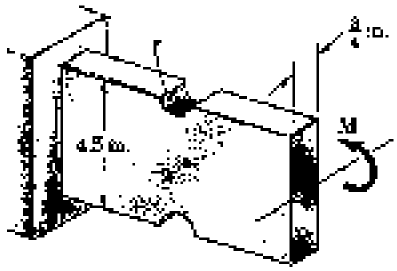
$\sigma = K\frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma I}{Kc} = \frac{(8)(3.296)}{(2.07)(1.875)} = 6.79 \text{ kip}\cdot\text{in}$

(b) $d = D - 2r = 4.5 - (2)(\frac{3}{4}) = 3.0, \quad \frac{D}{d} = \frac{4.5}{3.0} = 1.5, \quad \frac{r}{d} = \frac{0.75}{3.0} = 0.25$

From Fig. 4.32 $\quad K = 1.61, \qquad I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.0)^3 = 1.6875 \text{ in}^4$

$c = \frac{1}{2}d = 1.5 \text{ in.} \qquad M = \frac{\sigma I}{Kc} = \frac{(8)(1.6875)}{(1.61)(1.5)} = 5.59 \text{ kip}\cdot\text{in.}$

PROBLEM 4.72



4.72 Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Knowing that $M = 4$ kip·in., determine the maximum stress in the member when (a) $r = \frac{3}{8}$ in., (b) $r = \frac{3}{4}$ in.

SOLUTION

(a) $d = D - 2r = 4.5 - (2)(\frac{3}{8}) = 3.75$ in.

$\frac{D}{d} = \frac{4.5}{3.75} = 1.20, \quad \frac{r}{d} = \frac{0.375}{3.75} = 0.10$

From Fig. 4.32 $\quad K = 2.07$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.75)^3 = 3.296 \text{ in}^4, \qquad c = \frac{1}{2}d = 1.875 \text{ in.}$

$\sigma = K\frac{Mc}{I} = \frac{(2.07)(4)(1.875)}{3.296} = 4.71 \text{ ksi}$

(b) $d = D - 2r = 4.5 - (2)(\frac{3}{4}) = 3.00 \text{ in}, \quad \frac{D}{d} = \frac{4.5}{3.00} = 1.50, \quad \frac{r}{d} = \frac{0.75}{3.00} = 0.25$

From Fig 4.32 $\quad K = 1.61$

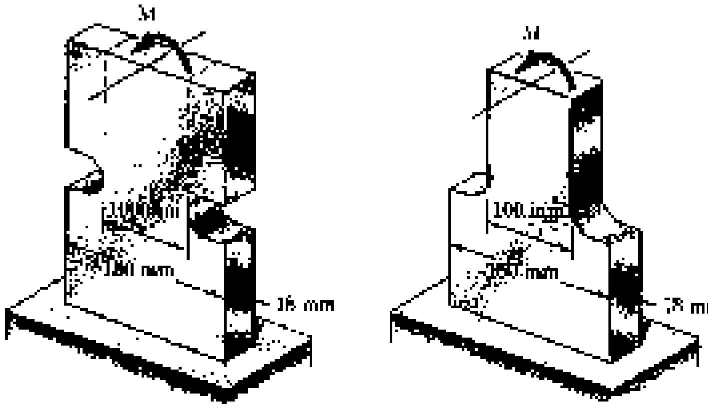
$I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.00)^3 = 1.6875 \text{ in}^4, \qquad c = \frac{1}{2}d = 1.5 \text{ in}$

$\sigma = K\frac{Mc}{I} = \frac{(1.61)(4)(1.5)}{1.6875} = 5.72 \text{ ksi}$

PROBLEM 4.73

SOLUTION

4.73 The allowable stress used in the design of a steel bar is 80 MPa. Determine the largest couple **M** that can be applied to the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = 15$ mm, as shown in Fig. *a*, (*b*) if the bar is redesigned by removing the material above the grooves as shown in Fig. *b*.



(a) (b)

For both configurations

$D = 150$ mm, $d = 100$ mm,

$r = 15$ mm.

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{15}{100} = 0.15$$

For configuration (a), Fig 4.32 gives $K_a = 1.92$.

For configuration (b) Fig 4.31 gives $K_b = 1.57$.

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2} d = 50 \text{ mm} = 0.050 \text{ m}$$

(a) $\sigma = \dfrac{KMc}{I} \quad \therefore \quad M = \dfrac{\sigma I}{Kc} = \dfrac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.92)(0.05)} = 1.25 \times 10^3$ N·m

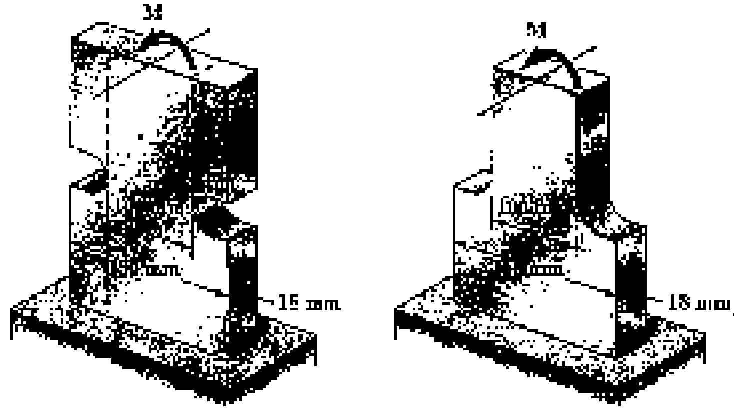
$= 1.25$ kN·m

(b) $M = \dfrac{\sigma I}{Kc} = \dfrac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.57)(0.050)} = 1.53 \times 10^3$ N·m $= 1.53$ kN·m

PROBLEM 4.74

4.74 A couple of moment $M = 2$ kN·m is to be applied to the end of a steel bar. Determine the maximum stress in the bar (a) if the bar is designed with grooves having semicircular portions of radius $r = 10$ mm, as shown in Fig. a. (b) If the bar is redesigned by removing the material above the grooves as shown in Fig. b.

SOLUTION



(a) (b)

For both configurations

$D = 150$ mm, $d = 100$ mm

$r = 10$ mm.

$\frac{D}{d} = \frac{150}{100} = 1.50$

$\frac{r}{d} = \frac{10}{100} = 0.10$

For configuration (a),

Fig 4.32 give $K_a = 2.21$

For configuration (b), Fig. 4.31 gives $K_b = 1.79$

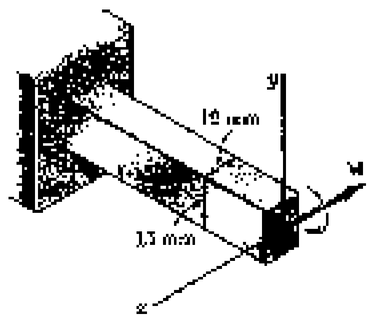
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(100)^3 = 1.5\times10^6 \text{ mm}^4 = 1.5\times10^{-6} \text{ m}^4$$

$$c = \frac{1}{2}d = 50 \text{ mm} = 0.05 \text{ m}$$

$$\text{(a)} \quad \sigma = \frac{KMc}{I} = \frac{(2.21)(2\times10^3)(0.05)}{1.5\times10^{-6}} = 147\times10^6 \text{ Pa} = 147 \text{ MPa}$$

$$\text{(b)} \quad \sigma = \frac{KMc}{I} = \frac{(1.79)(2\times10^3)(0.05)}{1.5\times10^{-6}} = 119.\times10^6 \text{ Pa} = 119. \text{ MPa}$$

PROBLEM 4.75



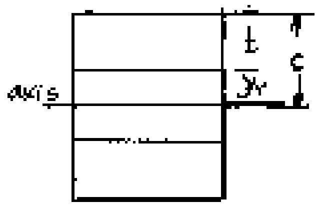
4.75 A bar of rectangular cross section, made of a steel assumed to be elastoplastic with $\sigma_Y = 320$ MPa, is subjected to a couple **M** parallel to the z axis. Determine the moment M of the couple for which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 5 mm thick.

SOLUTION

(a) $I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(15)^3 = 3.375\times10^3\ \text{mm}^4 = 3.375\times10^{-9}\ \text{m}^4$

$c = \frac{1}{2}h = 7.5\ \text{mm} = 0.0075\ \text{m}$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(320\times10^6)(3.375\times10^{-9})}{0.0075} = 144\ \text{N}\cdot\text{m}$$

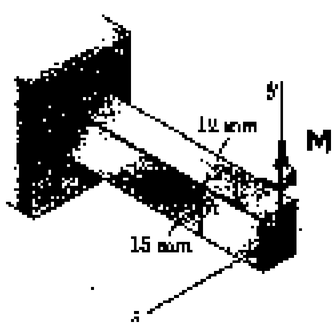


(b) $t = 5\ \text{mm}$ $\quad y_Y = c - t = 7.5 - 5\ \text{mm} = 2.5\ \text{mm} = 0.0025\ \text{m}$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$$

$$= \frac{3}{2}(144)\left[1 - \frac{1}{3}\left(\frac{2.5}{7.5}\right)^2\right] = 208\ \text{N}\cdot\text{m}$$

PROBLEM 4.76



4.75 A bar of rectangular cross section, made of a steel assumed to be elastoplastic with $\sigma_Y = 320$ MPa, is subjected to a couple **M** parallel to the z axis. Determine the moment M of the couple for which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 5 mm thick.

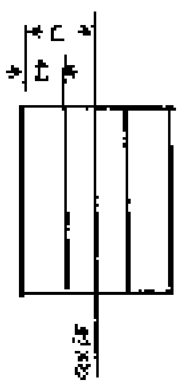
4.76 Solve Prob. 4.75, assuming that the couple **M** is parallel to the y axis.

SOLUTION

(a) $I = \frac{1}{12}bh^3 = \frac{1}{12}(15)(12)^3 = 2.16\times10^3\ \text{mm}^4 = 2.16\times10^{-9}\ \text{m}^4$

$c = \frac{1}{2}h = 6\ \text{mm} = 0.006\ \text{m}$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(320\times10^6)(2.16\times10^{-9})}{0.006} = 115.2\ \text{N}\cdot\text{m}$$

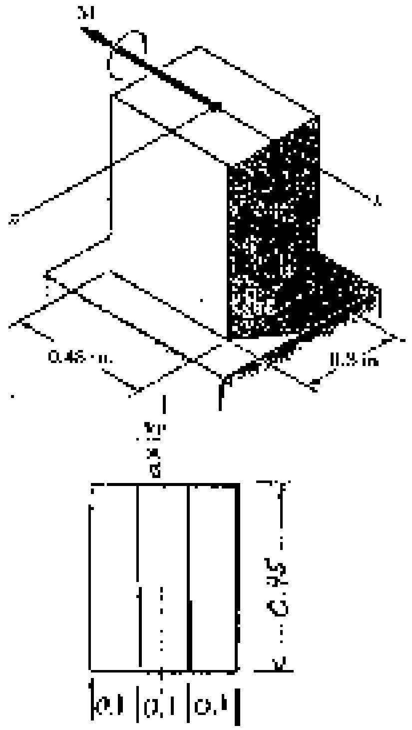


(b) $t = 5\ \text{mm}$ $\quad z_Y = c - t = 6 - 5 = 1\ \text{mm}$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{z_Y}{c}\right)^2\right]$$

$$\frac{3}{2}(115.2)\left[1 - \frac{1}{3}\left(\frac{1}{6}\right)^2\right] = 171.2\ \text{N}\cdot\text{m}$$

PROBLEM 4.77



4.77 The prismatic bar shown, made of a steel assumed to be elastoplastic with $\sigma_Y = 42$ ksi, is subjected to a couple **M** parallel to the x axis. Determine the moment M of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 0.1 in. thick.

SOLUTION

(a) $I = \frac{1}{12} b h^3 = \frac{1}{12}(0.45)(0.3)^3 = 1.0125 \times 10^{-3}$ in^4

$c = \frac{1}{2} h = 0.15$ in

$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(1.0125 \times 10^{-3})}{0.15} = 0.2835$ kip·in

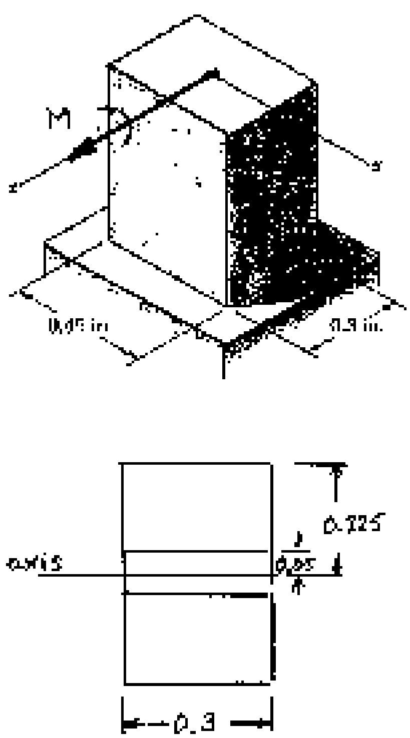
$= 283.5$ lb·in ◀

(b) $z_Y = \frac{1}{2} t_x = \frac{1}{2}(0.1) = 0.05$ in

$M_p = \frac{3}{2} M_Y \left[1 - \frac{1}{3}\left(\frac{z_Y}{c}\right)^2 \right]$

$= \frac{3}{2}(283.5)\left[1 - \frac{1}{3}\left(\frac{0.05}{0.15}\right)^2 \right] = 409.5$ lb·in ◀

PROBLEM 4.78



4.77 The prismatic bar shown, made of a steel assumed to be elastoplastic with $\sigma_Y = 42$ ksi, is subjected to a couple **M** parallel to the x axis. Determine the moment M of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 0.1 in. thick.

4.78 Solve Prob. 4.77, assuming that the couple **M** is parallel to the z axis.

SOLUTION

(a) $I = \frac{1}{12} b h^3 = \frac{1}{12}(0.3)(0.45)^3 = 2.2781 \times 10^{-3}$ in^4

$c = \frac{1}{2} h = 0.225$ in

$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(2.2781 \times 10^{-3})}{0.225} = 0.425$ kip·in

425 lb·in ◀

(b) $x_Y = \frac{1}{2} t_x = \frac{1}{2}(0.1) = 0.05$ in.

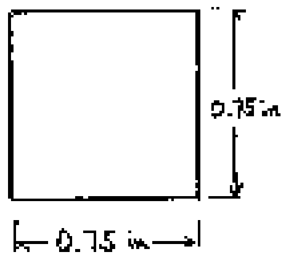
$M_p = \frac{3}{2} M_Y \left[1 - \frac{1}{3}\left(\frac{x_Y}{c}\right)^2 \right]$

$= \frac{3}{2}(425)\left[1 - \frac{1}{3}\left(\frac{0.05}{0.225}\right)^2 \right] = 627$ lb·in ◀

PROBLEM 4.79

4.79 A solid square rod of side 0.75 in. is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 40$ ksi. Determine the maximum stress and the radius of curvature caused by a 4 kip·in. couple applied and maintained about an axis parallel to a side of the cross section.

SOLUTION



$$I = \tfrac{1}{12} b h^3 = \tfrac{1}{12}(0.75)(0.75)^3 = 0.026367 \text{ in}^4$$

$$c = \tfrac{1}{2} h = 0.375 \text{ in}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(40)(0.026367)}{0.375} = 2.8125 \text{ kip·in}$$

$$M = \tfrac{3}{2} M_Y \left(1 - \tfrac{1}{3}\frac{y_Y^2}{c^2}\right) \quad \text{or} \quad \frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}}$$

$$\frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}} = \sqrt{3 - \frac{(2)(4)}{2.8125}} = 0.39441$$

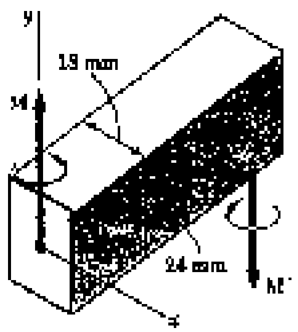
$$\frac{c}{\rho_Y} = \varepsilon_Y = \frac{\sigma_Y}{E} \quad \therefore \quad \rho_Y = \frac{Ec}{\sigma_Y} = \frac{(29\times10^6)(0.375)}{40\times10^3} = 271.88 \text{ in}$$

$$\frac{\rho}{\rho_Y} = \frac{y_Y}{c} \quad \therefore \quad \rho = \rho_Y \frac{y_Y}{c} = (271.88)(0.39441) = 107.2 \text{ in}$$

$$= 8.94 \text{ ft}$$

PROBLEM 4.80

4.80 The prismatic rod shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 280$ MPa. Knowing that couples **M** and **M'** of moment 525 N·m are applied and maintained about axes parallel to the y axis, determine (a) the thickness of the elastic core, (b) the radius of curvature of the bar.



SOLUTION

$$I = \tfrac{1}{12} b h^3 = \tfrac{1}{12}(24)(18)^3 = 11.664\times10^3 \text{ mm}^4 = 11.664\times10^{-9} \text{ m}^4$$

$$c = \tfrac{1}{2} h = 9 \text{ mm} = 0.009 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(280\times10^6)(11.664\times10^{-9})}{0.009} = 362.88 \text{ N·m}$$

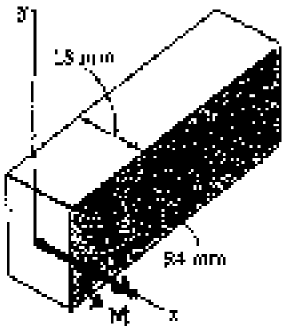
$$M = \tfrac{3}{2} M_Y \left(1 - \tfrac{1}{3}\frac{y_Y^2}{c^2}\right) \quad \text{or} \quad \frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}}$$

$$\frac{y_Y}{c} = \sqrt{3 - \frac{(2)(525)}{362.88}} = 0.32632, \quad y_Y = 0.32632\, c = 2.9368 \text{ mm}$$

(a) $$t_{core} = 2y_Y = 5.87 \text{ mm}$$

(b) $$\varepsilon_Y = \frac{y_Y}{\rho} = \frac{\sigma_Y}{E} \quad \therefore \quad \rho = \frac{E y_Y}{\sigma_Y} = \frac{(200\times10^9)(2.9368\times10^{-3})}{280\times10^6} = 2.09 \text{ m}$$

PROBLEM 4.81



4.80 The prismatic rod shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 280$ MPa. Knowing that couples **M** and **M'** of moment 525 N·m are applied and maintained about axes parallel to the y axis, determine (*a*) the thickness of the elastic core, (*b*) the radius of curvature of the bar.

4.81 Solve Prob. 4.80, assuming that the couples **M** and **M'** are applied and maintained about axes parallel to the x axis.

SOLUTION

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(18)(24)^3 = 20.736 \times 10^3 \text{ mm}^4 = 20.736 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2} h = 12 \text{ mm} = 0.012 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(280 \times 10^6)(20.736 \times 10^{-9})}{0.012} = 483.84 \text{ N·m}$$

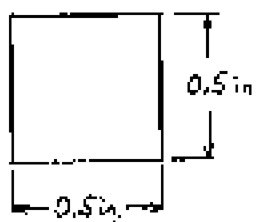
$$M = \frac{3}{2} M_Y \left(1 - \frac{1}{3}\frac{y_Y^2}{c^2}\right) \quad \text{or} \quad \frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}}$$

$$\frac{y_Y}{c} = \sqrt{3 - \frac{(2)(525)}{483.84}} = 0.91097, \qquad y_Y = 0.91097\,c = 10.932 \text{ mm}$$

(a) $$t_{core} = 2 y_Y = 21.9 \text{ mm}$$

(b) $$\varepsilon_Y = \frac{y_Y}{\rho} = \frac{\sigma_Y}{E} \quad \therefore \quad \rho = \frac{E y_Y}{\sigma_Y} = \frac{(200 \times 10^9)(10.932 \times 10^{-3})}{280 \times 10^6} = 7.81 \text{ m}$$

PROBLEM 4.82



4.82 A solid square rod of side 0.5 in. is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi. Knowing that a couple **M** is applied and maintained about an axis parallel to a side of the cross section, determine the moment M of the couple for which the radius of curvature is (*a*) 5 ft, (*b*) 2 ft.

SOLUTION

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(0.5)(0.5)^3 = 5.2083 \times 10^{-3} \text{ in}^4$$

$$c = \frac{1}{2} h = 0.25 \text{ in.}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42 \times 10^3)(5.2083 \times 10^{-3})}{0.25} = 875 \text{ lb·in}$$

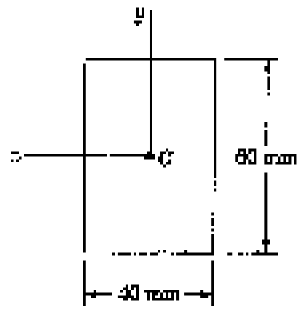
$$\varepsilon_Y = \frac{c}{\rho_Y} = \frac{\sigma_Y}{E} \quad \therefore \quad \rho_Y = \frac{Ec}{\sigma_Y} = \frac{(29 \times 10^6)(0.25)}{42 \times 10^3} = 172.62 \text{ in.}$$

$$M = \frac{3}{2} M_Y \left[1 - \frac{1}{3}\left(\frac{\rho}{\rho_Y}\right)^2\right]$$

(a) $\rho = 5 \text{ ft} = 60 \text{ in.}$ $$M = \frac{3}{2}(875)\left[1 - \frac{1}{3}\left(\frac{60}{172.62}\right)^2\right] = 1260 \text{ lb·in}$$

(b) $\rho = 2 \text{ ft} = 24 \text{ in.}$ $$M = \frac{3}{2}(875)\left[1 - \frac{1}{3}\left(\frac{24}{172.62}\right)^2\right] = 1304 \text{ lb·in}$$

PROBLEM 4.83



4.83 and 4.84 A bar of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 20 mm thick.

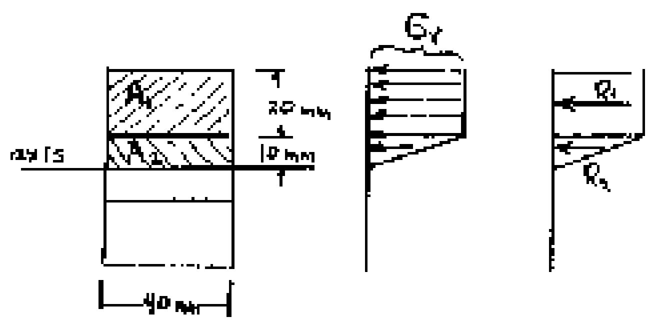
SOLUTION

(a) $I = \frac{1}{12} b h^3 = \frac{1}{12}(40)(60)^3 = 720 \times 10^3 \text{ mm}^4 = 720 \times 10^{-9} \text{ m}^4$

$c = \frac{1}{2} h = 30 \text{ mm} = 0.030 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(720 \times 10^{-9})}{0.030} = 5.76 \times 10^3 \text{ N·m}$

$= 5.76 \text{ kN·m}$



$R_1 = \sigma_Y A_1 = (240 \times 10^6)(0.040)(0.020)$

$= 192 \times 10^3 \text{ N}$

$y_1 = 10 \text{ mm} + 10 \text{ mm} = 0.020 \text{ m}$

$R_2 = \frac{1}{2} \sigma_Y A_2 = (\frac{1}{2})(240 \times 10^6)(0.040)(0.010)$

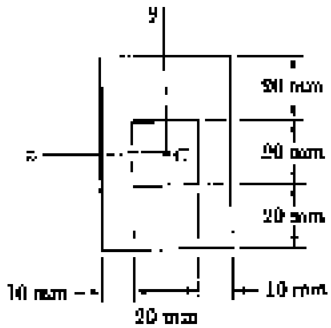
$= 48 \times 10^3 \text{ N}$

$y_2 = \frac{2}{3}(10 \text{ mm}) = 6.667 \text{ mm} = 0.006667 \text{ m}$

(b) $M = 2(R_1 y_1 + R_2 y_2) = 2[(192 \times 10^3)(0.020) + (48 \times 10^3)(0.006667)]$

$= 8.32 \times 10^3 \text{ N·m} = 8.32 \text{ kN·m}$

PROBLEM 4.84



4.83 and 4.84 A bar of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 20 mm thick.

SOLUTION

(a) $I_{rect} = \frac{1}{12}bh^3 = \frac{1}{12}(40)(60)^3 = 720 \times 10^3 \text{ mm}^4$

$I_{cutout} = \frac{1}{12}bh^3 = \frac{1}{12}(20)(20)^3 = 13.33 \times 10^3 \text{ mm}^4$

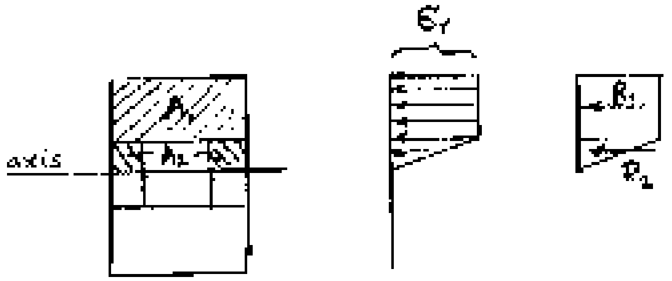
$I = 720 \times 10^3 - 13.33 \times 10^3 = 706.67 \times 10^3 \text{ mm}^4$

$= 706.67 \times 10^{-9} \text{ m}^4$

$c = \frac{1}{2}h = 30 \text{ mm} = 0.030 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(706.67 \times 10^{-9})}{0.030}$

$= 5.6533 \times 10^3 \text{ N}\cdot\text{m} = 5.65 \text{ kN}\cdot\text{m}$ ◀



$R_1 = \sigma_Y A_1 = (240 \times 10^6)(0.040)(0.020) = 192 \times 10^3 \text{ N}$

$y_1 = 10 \text{ mm} + 10 \text{ mm} = 20 \text{ mm} = 0.020 \text{ m}$

$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(240 \times 10^6)(0.020)(0.010) = 24 \times 10^3 \text{ N}$

$y_2 = \frac{2}{3}(10 \text{ mm}) = 6.667 \text{ mm} = 0.006667 \text{ m}$

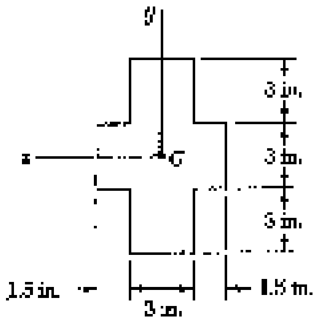
(b) $M = 2(R_1 y_1 + R_2 y_2)$

$= 2[(192 \times 10^3)(0.020) + (24 \times 10^3)(0.006667)]$

$= 8.00 \times 10^3 \text{ N}\cdot\text{m} = 8.00 \text{ kN}\cdot\text{m}$ ◀

4.85 and 4.86 A bar of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic moment if the top and bottom of the bar are 3 in. thick.

PROBLEM 4.85



SOLUTION

(a) $I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(3)(3)^3 + (3)(3)(3)^2 = 87.75 \text{ in}^4$

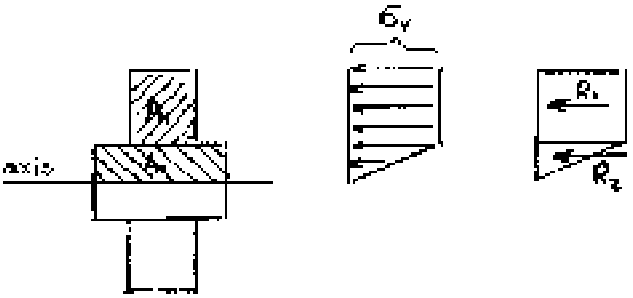
$I_2 = \frac{1}{12}b_2h_2^3 = \frac{1}{12}(6)(3)^3 = 13.5 \text{ in}^4$

$I_3 = I_1 = 87.75 \text{ in}^4$

$I = I_1 + I_2 + I_3 = 188.5 \text{ in}^4$

$c = 4.5$ in

$M_y = \frac{\sigma_Y I}{c} = \frac{(42)(188.5)}{4.5} = 1759. \text{ kip}\cdot\text{in}$ ◄



$R_1 = \sigma_Y A_1 = (42)(3)(3) = 378$ kip

$y_1 = 1.5 + 1.5 = 3.0$ in

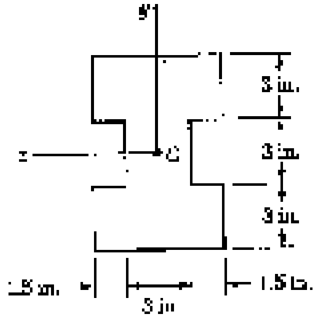
$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(6)(1.5)$

$= 189$ kip

$y_2 = \frac{2}{3}(1.5) = 1.0$ in

(b) $M = 2(R_1y_1 + R_2y_2) = 2[(378)(3.0) + (189)(1.0)] = 2646 \text{ kip}\cdot\text{in}$ ◄

PROBLEM 4.86



4.85 and 4.86 A bar of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi. For bending about the z axis, determine the bending moment at which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 3 in. thick.

SOLUTION

(a) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(6)(3)^3 + (6)(3)(3)^2 = 175.5 \text{ in}^4$

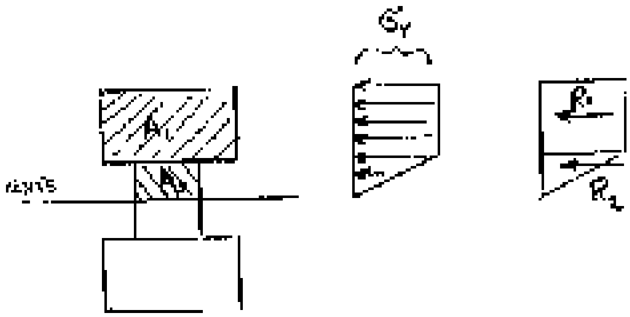
$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(3)(3)^3 = 6.75 \text{ in}^4$

$I_3 = I_1 = 175.5 \text{ in}^4$

$I = I_1 + I_2 + I_3 = 357.75 \text{ in}^4$

$c = 4.5$ in.

$M_Y = \dfrac{\sigma_Y I}{c} = \dfrac{(42)(357.75)}{4.5} = 3339$ kip·in ◄



$R_1 = \sigma_Y A_1 = (42)(6)(3) = 756$ kip

$y_1 = 1.5 + 1.5 = 3$ in

$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(3)(1.5)$

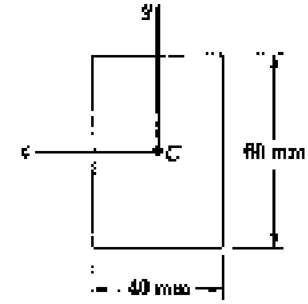
$= 94.5$ kip

$y_2 = \frac{2}{3}(1.5) = 1.0$ in.

(b) $M = 2(R_1 y_1 + R_2 y_2) = 2[(756)(3) + (94.5)(1.0)] = 4725$ kip·in ◄

PROBLEM 4.87

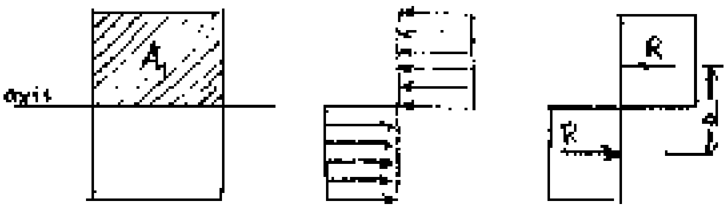
4.87 through 4.90 For the bar indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.
4.87 Bar of Prob. 4.83



SOLUTION

From PROBLEM 4.83 $E = 200$ GPa and $\sigma_Y = 240$ MPa.

$$A_1 = (40)(30) = 1200 \text{ mm}^2$$
$$= 1200 \times 10^{-6} \text{ m}^2$$



$$R = \sigma_Y A_1$$
$$= (240\times10^6)(1200\times10^{-6})$$
$$= 288 \times 10^3 \text{ N}$$
$$d = 30 \text{ mm} = 0.030 \text{ m}$$

(a) $M_p = R d = (288\times10^3)(0.030) = 8.64\times10^3 \text{ N}\cdot\text{m} = 8.64 \text{ kN}\cdot\text{m}$

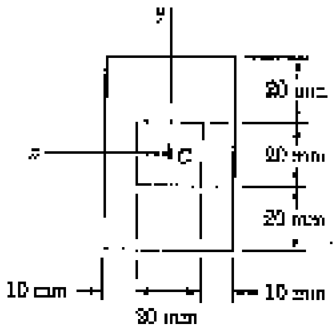
(b) $I = \frac{1}{12} bh^3 = \frac{1}{12}(40)(60)^3 = 720\times10^3 \text{ mm}^4 = 720\times10^{-9} \text{ m}^4$

$$c = 30 \text{ mm} = 0.030 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240\times10^6)(720\times10^{-9})}{0.030} = 5.76 \text{ kN}\cdot\text{m}$$

$$k = \frac{M_P}{M_Y} = \frac{8.64}{5.76} = 1.5$$

PROBLEM 4.88

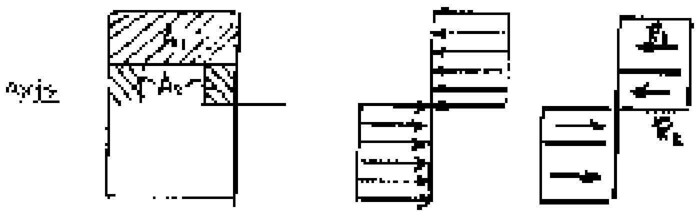


4.87 through 4.90 For the bar indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

4.88 Bar of Prob. 4.84

SOLUTION

From PROBLEM 4.84 $E = 200$ GPa and $\sigma_Y = 240$ MPa.



$$R_1 = \sigma_Y A_1$$
$$= (240 \times 10^6)(0.040)(0.020)$$
$$= 192 \times 10^3 \text{ N}$$

$$y_1 = 10 \text{ mm} + 10 \text{ mm} = 20 \text{ mm}$$
$$= 0.020 \text{ m}$$

$$R_2 = \sigma_Y A_2$$
$$= (240 \times 10^6)(0.020)(0.010)$$
$$= 48 \times 10^3 \text{ N}$$

$$y_2 = \tfrac{1}{2}(10) = 5 \text{ mm} = 0.005 \text{ m}$$

$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(192 \times 10^3)(0.020) + (48 \times 10^3)(0.005)]$$
$$= 8.16 \times 10^3 \text{ N}\cdot\text{m} = 8.16 \text{ kN}\cdot\text{m}$$ ◄

(b) $I_{rect} = \frac{1}{12} b h^3 = \frac{1}{12}(40)(60)^3 = 720 \times 10^3 \text{ mm}^4$

$I_{cutout} = \frac{1}{12} b h^3 = \frac{1}{12}(20)(20)^3 = 13.33 \times 10^3 \text{ mm}^4$

$I = I_{rect} - I_{cutout} = 720 \times 10^3 - 13.33 \times 10^3 = 706.67 \times 10^3 \text{ mm}^4$
$= 706.67 \times 10^{-9} \text{ m}^4$

$c = \frac{1}{2} h = 30 \text{ mm} = 0.030 \text{ m}$

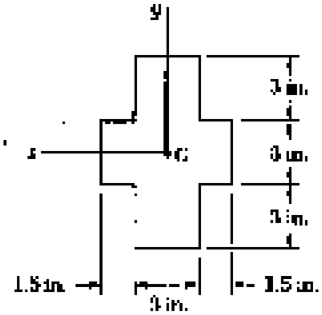
$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(706.67 \times 10^{-9})}{0.030} = 5.6533 \text{ N}\cdot\text{m}$$

$$k = \frac{M_p}{M_Y} = \frac{8.16}{5.6533} = 1.443$$ ◄

PROBLEM 4.89

4.87 through 4.90 For the bar indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

4.89 Bar of Prob. 4.83

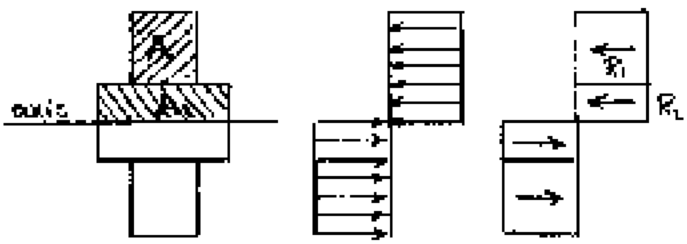


SOLUTION

From PROBLEM 4.85 $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi.

$R_1 = \sigma_Y A_1 = (42)(3)(3) = 378$ kip

$y_1 = 1.5 + 1.5 = 3.0$ in



$R_2 = \sigma_Y A_2 = (42)(6)(1.5) = 378$ kip.

$y_2 = \frac{1}{2}(1.5) = 0.75$ in

$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(378)(3.0) + (378)(0.75)] = 2835 \text{ kip}\cdot\text{in}$$

(b) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(3)^3 + (3)(3)(3)^2 = 87.75 \text{ in}^4$

$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(6)(3)^3 = 13.5 \text{ in}^4$

$I_3 = I_1 = 87.75 \text{ in}^4$

$I = I_1 + I_2 + I_3 = 188.5 \text{ in}^4$

$c = 4.5$ in.

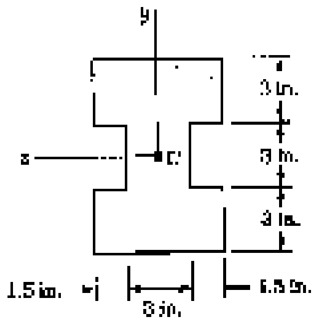
$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(188.5)}{4.5} = 1759.3 \text{ kip}\cdot\text{in.}$$

$$k = \frac{M_p}{M_Y} = \frac{2835}{1759.3} = 1.611$$

PROBLEM 4.90

4.87 through 4.90 For the bar indicated, determine (a) the fully plastic moment M_p, (b) the shape factor of the cross section.

4.90 Bar of Prob. 4.86

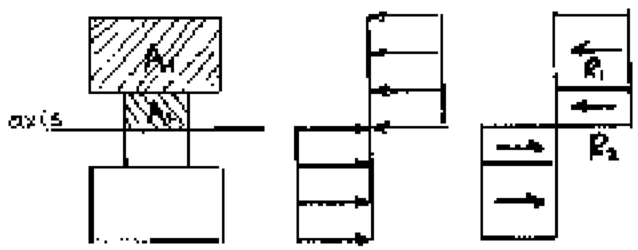


SOLUTION

From PROBLEM 4.86 $E = 29\times10^6$ and $\sigma_Y = 42$ ksi.

$R_1 = \sigma_Y A_1 = (42)(6)(3) = 756$ kip

$y_1 = 1.5 + 1.5 = 3.0$ in



$R_2 = \sigma_Y A_2 = (42)(3)(1.5) = 189$ kip

$y_2 = \frac{1}{2}(1.5) = 0.75$ in.

$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(756)(3.0) + (189)(0.75)] = 4819.5 \text{ kip}\cdot\text{in}$$

(b) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(6)(3)^3 + (6)(3)(3)^2 = 175.5 \text{ in}^4$

$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(3)(3)^3 = 6.75 \text{ in}^4$

$I_3 = I_1 = 175.5 \text{ in}^4$

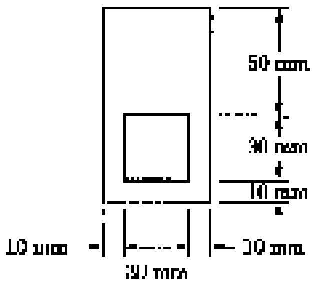
$I = I_1 + I_2 + I_3 = 357.75 \text{ in}^4$

$c = 4.5$ in

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(357.75)}{4.5} = 3339 \text{ kip}\cdot\text{in}$$

$$k = \frac{M_p}{M_Y} = \frac{4819.5}{3339} = 1.443$$

PROBLEM 4.91



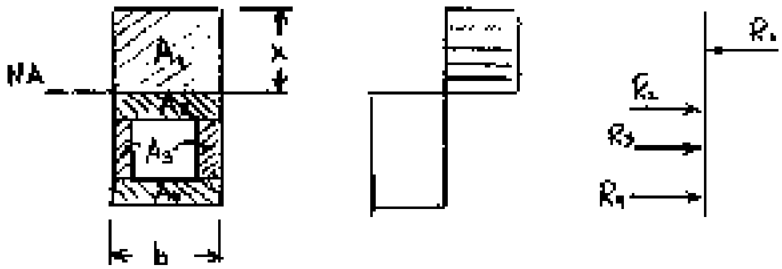
4.91 and 4.92 Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION

Total area $A = (50)(90) - (30)(30) = 3600 \text{ mm}^2$

$\frac{1}{2}A = 1800 \text{ mm}^2$

$x = \frac{\frac{1}{2}A}{b} = \frac{1800}{50} = 36 \text{ mm}$



$A_1 = (50)(36) = 1800 \text{ mm}^2, \quad \bar{y}_1 = 18 \text{ mm} \qquad A_1\bar{y}_1 = 32.4 \times 10^3 \text{ mm}^3$

$A_2 = (50)(14) = 700 \text{ mm}^2, \quad \bar{y}_2 = 7 \text{ mm} \qquad A_2\bar{y}_2 = 4.9 \times 10^3 \text{ mm}^3$

$A_3 = (20)(30) = 600 \text{ mm}^2, \quad \bar{y}_3 = 29 \text{ mm} \qquad A_3\bar{y}_3 = 17.4 \times 10^3 \text{ mm}^3$

$A_4 = (50)(10) = 500 \text{ mm}^2, \quad \bar{y}_4 = 49 \text{ mm} \qquad A_4\bar{y}_4 = 24.5 \times 10^3 \text{ mm}^3$

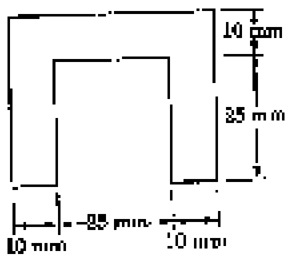
$A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4 = 79.2 \times 10^3 \text{ mm}^3 = 79.2 \times 10^{-6} \text{ m}^3$

$M_p = \sigma_Y \Sigma A_i\bar{y}_i = (240 \times 10^6)(79.2 \times 10^{-6}) = 19.008 \times 10^3 \text{ N}\cdot\text{m}$

$= 19.01 \text{ kN}\cdot\text{m}$

PROBLEM 4.92

4.91 and 4.92 Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

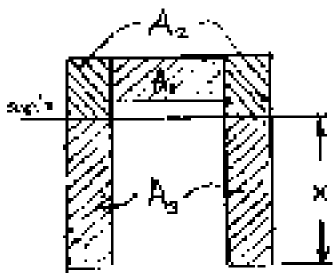


SOLUTION

Total area $A = (25)(10) + (2)(10)(35) = 950 \text{ mm}^2$

$\frac{1}{2}A = 475 \text{ mm}^2$

$x = \frac{\frac{1}{2}A}{2b} = \frac{475}{20} = 23.75 \text{ mm} = 0.02375 \text{ m}$



$R_1 = \sigma_Y A_1 = (240\times10^6)(0.025)(0.010) = 60\times10^3 \quad N$

$\bar{y}_1 = 30 - 23.75 = 6.25 \text{ mm} = 0.00625 \text{ m}$

$R_2 = \sigma_Y A_2 = (240\times10^6)(0.020)(0.01125) = 54\times10^3 \quad N$

$\bar{y}_2 = \frac{1}{2}(0.01125) = 0.005625 \text{ m}$

$R_3 = \sigma_Y A_3 = (240\times10^6)(0.020)(0.02375) = 114\times10^3 \quad N$

$\bar{y}_3 = \frac{1}{2}x = 0.011875 \text{ m}$

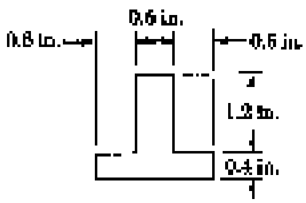
$M_p = R_1\bar{y}_1 + R_2\bar{y}_2 + R_3\bar{y}_3$

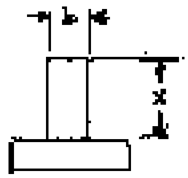
$= (60\times10^3)(0.00625) + (54\times10^3)(0.005625) + (114\times10^3)(0.011875)$

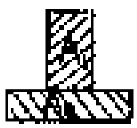
$= 2.0375\times10^3 \text{ N}\cdot\text{m} = 2.08 \text{ kN}\cdot\text{m}$

PROBLEM 4.93

4.93 and 4.94 Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 36 ksi.







SOLUTION

Total area $A = (1.8)(0.4) + (0.6)(1.2) = 1.44 \text{ in}^2$

$\frac{1}{2}A = 0.72 \text{ in}^2$

$x = \frac{\frac{1}{2}A}{b} = \frac{0.72}{0.6} = 1.2 \text{ in.}$

Neutral axis lies 1.2 in below the top

$A_1 = \frac{1}{2}A = 0.72 \text{ in}^2, \quad \bar{y}_1 = \frac{1}{2}(1.2) = 0.6 \text{ in}$

$A_2 = \frac{1}{2}A = 0.72 \text{ in}^2, \quad \bar{y}_2 = \frac{1}{2}(0.4) = 0.2 \text{ in}$

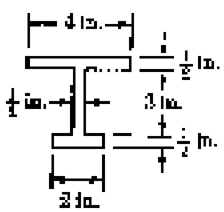
$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2)$

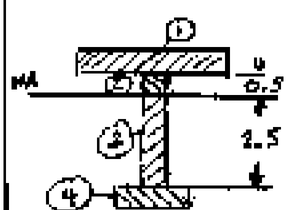
$= (36)[(0.72)(0.6) + (0.72)(0.2)]$

$= 20.7 \text{ kip}\cdot\text{in}$

PROBLEM 4.94

4.93 and 4.94 Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 36 ksi.





SOLUTION

Total area: $A = (4)(\frac{1}{2}) + (\frac{1}{2})(3) + (2)(\frac{1}{2}) = 4.5 \text{ in}^2$

$\frac{1}{2}A = 2.25 \text{ in}^2$

$A_1 = 2.00 \text{ in}^2, \quad \bar{y}_1 = 0.75, \quad A_1\bar{y}_1 = 1.50 \text{ in}^3$

$A_2 = 0.25 \text{ in}^2, \quad \bar{y}_2 = 0.25, \quad A_2\bar{y}_2 = 0.0625 \text{ in}^3$

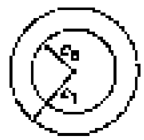
$A_3 = 1.25 \text{ in}^2, \quad \bar{y}_3 = 1.25, \quad A_3\bar{y}_3 = 1.5625 \text{ in}^3$

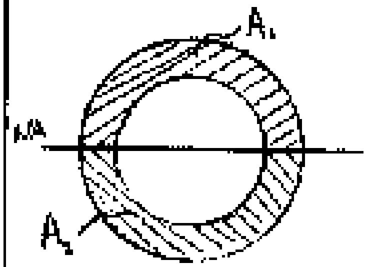
$A_4 = 1.00 \text{ in}^2, \quad \bar{y}_4 = 2.75, \quad A_4\bar{y}_4 = 2.75 \text{ in}^3$

$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4)$

$= (36)(1.50 + 0.0625 + 1.5625 + 2.75) = 211.5 \text{ Kip}\cdot\text{in}$

PROBLEM 4.95





4.95 A thick-walled pipe of the cross section shown is made of a steel that is assumed to be elastoplastic with a yield strength σ_Y. Derive an expression for the plastic moment M_p of the pipe in terms of c_1, c_2, and σ_Y.

SOLUTION

$$A_1\bar{y}_1 = A_a\bar{y}_a - A_b\bar{y}_b$$

$$= \left(\frac{\pi}{2}c_1^2\right)\left(\frac{4c_1}{3\pi}\right) - \left(\frac{\pi}{2}c_2^2\right)\left(\frac{4c_2}{3\pi}\right)$$

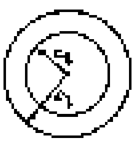
$$= \frac{2}{3}\left(c_1^3 - c_2^3\right)$$

$$A_2\bar{y}_2 = A_1\bar{y}_1 = \frac{2}{3}\left(c_1^3 - c_2^3\right)$$

$$M_p = \sigma_Y\left(A_1\bar{y}_1 + A_2\bar{y}_2\right) = \frac{4}{3}\sigma_Y\left(c_1^3 - c_2^3\right)$$

PROBLEM 4.96

4.96 . Determine the plastic moment M_p of a thick-walled pipe of the cross section shown, knowing that $c_1 = 60$mm, $c_2 = 40$ mm, and $\sigma_Y = 240$ MPa.



SOLUTION

See the solution to PROBLEM 4.95 for derivation of the following expression for M_p.

$$M_p = \frac{4}{3}\sigma_Y\left(c_1^3 - c_2^3\right)$$

Data: $\sigma_Y = 240 \text{ MPa} = 240\times10^6 \text{ Pa}$

$c_1 = 60 \text{ mm} = 0.060 \text{ m}$

$c_2 = 40 \text{ mm} = 0.040 \text{ m}$

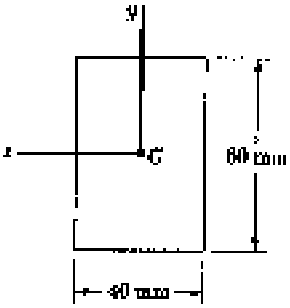
$$M_p = \frac{4}{3}\left(240\times10^6\right)\left(0.060^3 - 0.040^3\right) = 48.64\times10^3 \text{ N}\cdot\text{m}$$

$$= 48.6 \text{ kN}\cdot\text{m}$$

PROBLEM 4.97

4.97 and 4.98 For the beam indicated a couple of moment equal to the fully plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 30$ mm.

4.97 Beam of Prob. 4.83

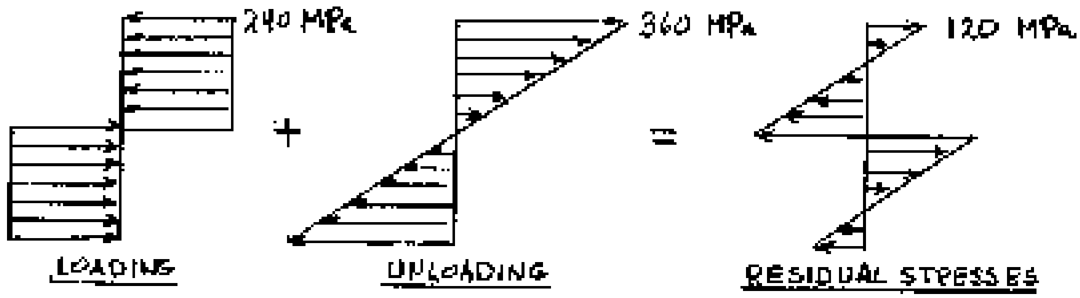


SOLUTION

$M_p = 8.64$ kN·m (See solution to problem 4.87)

$I = 720 \times 10^{-9}\ m^4$, $c = 0.030$ m

$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I}$ at $y = c = 30$ mm



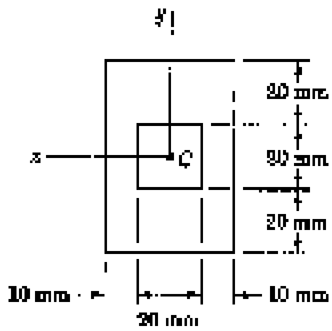
$\sigma' = \frac{(8.64 \times 10^3)(0.030)}{720 \times 10^{-9}} = 360 \times 10^6$ Pa

$\sigma_{res} = \sigma' - \sigma_Y = 360 \times 10^6 - 240 \times 10^6 = 120 \times 10^6$ Pa $= 120$ MPa

PROBLEM 4.98

4.97 and 4.98 For the beam indicated a couple of moment equal to the fully plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 30$ mm.

4.98 Beam of Prob. 4.84



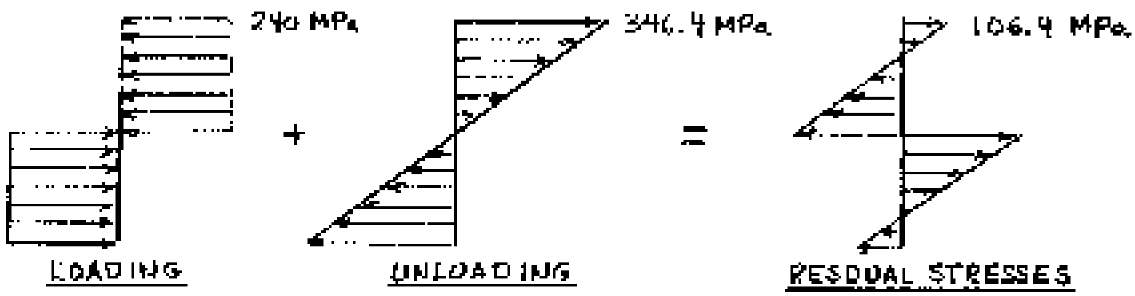
SOLUTION

$M_p = 8.16$ kN-m (See SOLUTION for PROBLEM 4.88)

$I = 706.67 \times 10^{-9}\ m^4$, $c = 0.080$ m

$\sigma' = \frac{M_{max}\, y}{I} = \frac{M_p c}{I}$ at $y = c$.

$\sigma' = \frac{(8.16 \times 10^3)(0.080)}{706.67 \times 10^{-9}} = 346.4 \times 10^6$ Pa

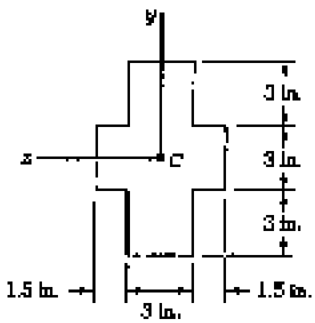


$\sigma_{res} = \sigma' - \sigma_Y = 346.4 \times 10^6 - 240 \times 10^6 = 106.4 \times 10^6$ Pa $= 106.4$ MPa

4.99 and 4.100 For the beam indicated a couple of moment equal to the fully plastic moment M_p is applied and then removed. Using a yield strength of 42 ksi, determine the residual stress at $y = 4.5$ in.

4.99 Beam of Prob. 4.85

PROBLEM 4.99



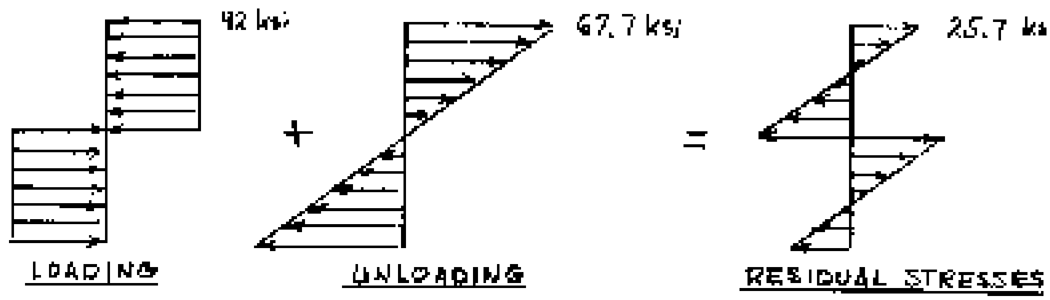
SOLUTION

$M_p = 2835$ kip·in (See solution to problem 4.89)

$I = 188.5\ \text{in}^4, \qquad c = 4.5$ in

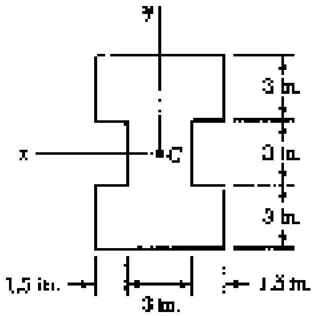
$$\sigma' = \frac{M_{max}\, y}{I} = \frac{M_p c}{I} \quad \text{at } y = c.$$

$$\sigma' = \frac{(2835)(4.5)}{188.5} = 67.7 \text{ ksi}$$



$\sigma_{res} = \sigma' - \sigma_Y = 67.7 - 42 = 25.7$ ksi

PROBLEM 4.100



4.99 and 4.100 For the beam indicated a couple of moment equal to the fully plastic moment M_p is applied and then removed. Using a yield strength of 42 ksi, determine the residual stress at $y = 4.5$ in.

4.100 Beam of Prob. 4.86

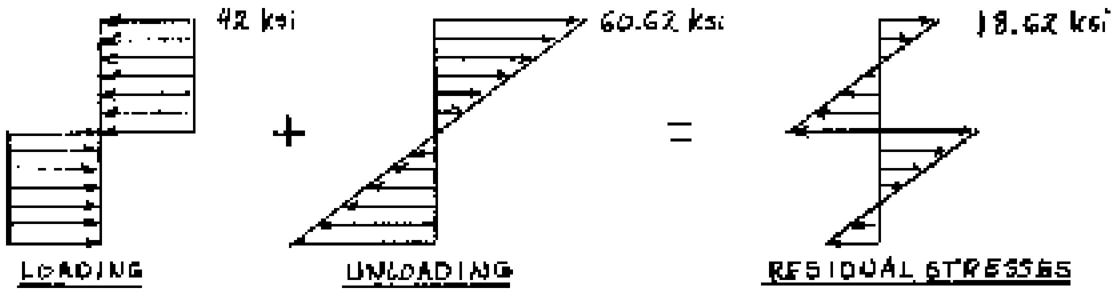
SOLUTION

$M_p = 4819.5$ kip·in (See solution to problem 4.90)

$I = 357.75 \text{ in}^4, \qquad c = 4.5$ in.

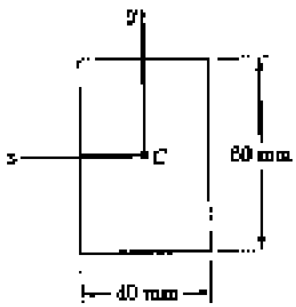
$$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I} \quad \text{for } y = c$$

$$\sigma' = \frac{(4819.5)(4.5)}{357.75} = 60.62 \text{ ksi}$$



$$\sigma_{res} = \sigma' - \sigma_Y = 60.62 - 42 = 18.62 \text{ ksi}$$

PROBLEM 4.101



4.101 and 4.102 A bending couple is applied to the bar indicated, causing plastic zones 20-mm thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 30$ mm, (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

4.101 Bar of Prob. 4.83

SOLUTION

See solution to problem 4.83 for bending couple and stress distribution during loading.

$M = 8.32$ kN $\qquad y_Y = 10$ mm $= 0.010$ m

$E = 200$ GPa $\qquad \sigma_Y = 240$ MPa

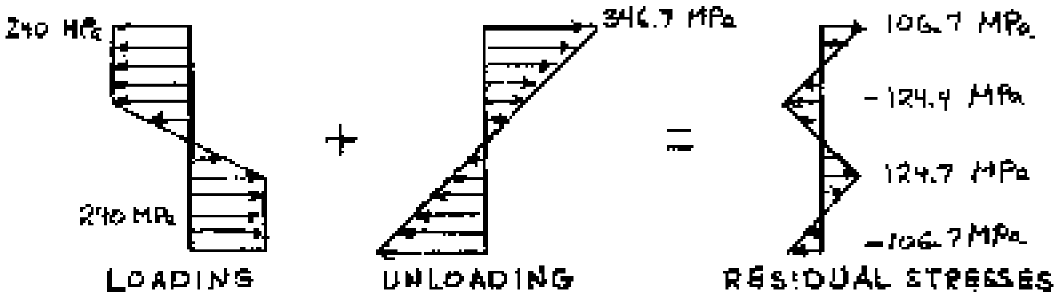
$I = 720 \times 10^{-9}$ m^4 $\qquad c = 0.030$ m

(a) $\sigma' = \dfrac{Mc}{I} = \dfrac{(8.32 \times 10^3)(0.030)}{720 \times 10^{-9}} = 346.7 \times 10^6$ Pa $= 346.7$ MPa

$\sigma'' = \dfrac{My_Y}{I} = \dfrac{(8.32 \times 10^3)(0.010)}{720 \times 10^{-9}} = 115.6 \times 10^6$ Pa $= 115.6$ MPa

At $y = c$ $\qquad \sigma_{res} = \sigma' - \sigma_Y = 346.7 - 240 = 106.7$ MPa

At $y = y_Y$ $\qquad \sigma_{res} = \sigma'' - \sigma_Y = 115.6 - 240 = -124.4$ MPa



(b) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_0}{I} - \sigma_Y = 0$

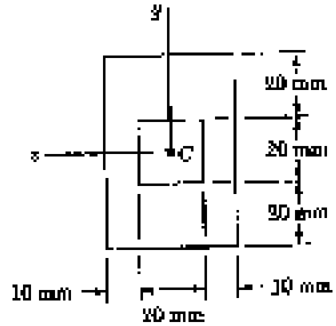
$y_0 = \dfrac{I\sigma_Y}{M} = \dfrac{(720 \times 10^{-9})(240 \times 10^6)}{8.32 \times 10^3} = 20.77 \times 10^{-3}$ m $= 20.77$ mm

ans. $y_0 = -20.77$ mm, 0, 20.77 mm

(c) At $y = y_Y$, $\quad \sigma_{res} = -124.4 \times 10^6$ Pa

$\sigma = -\dfrac{Ey}{\rho} \quad \therefore \quad \rho = -\dfrac{Ey}{\sigma} = \dfrac{(200 \times 10^9)(0.010)}{-124.4 \times 10^6} = 16.08$ m

PROBLEM 4.102



4.101 and 4.102 A bending couple is applied to the bar indicated, causing plastic zones 20-mm thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 30$ mm, (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

4.102 Bar of Prob. 4.84.

SOLUTION

See SOLUTION to PROBLEM 4.84 for bending couple and stress distribution during loading.

$M = 8.00\ \text{kN}\cdot\text{m} \qquad y_Y = 10\ \text{mm} = 0.010\ \text{m}$

$E = 200\ \text{GPa} \qquad \sigma_Y = 240\ \text{MPa}$

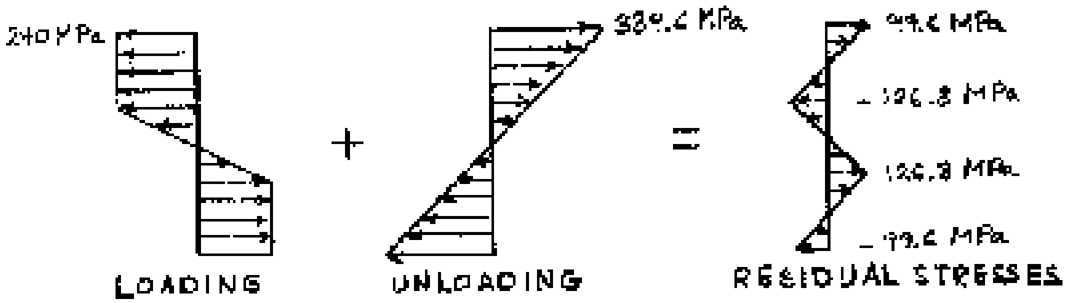
$I = 706.67 \times 10^{-9}\ \text{m}^4 \qquad c = 0.030\ \text{m}$

(a) $\sigma' = \dfrac{Mc}{I} = \dfrac{(8.00\times10^3)(0.030)}{706.67\times10^{-9}} = 339.6\times10^6\ \text{Pa} = 339.6\ \text{MPa}$

$\sigma'' = \dfrac{My_Y}{I} = \dfrac{(8.00\times10^3)(0.010)}{706.67\times10^{-9}} = 113.2\times10^6\ \text{Pa} = 113.2\ \text{MPa}$

At $y = c$ $\quad \sigma_{res} = \sigma' - \sigma_Y = 339.6 - 240 = 99.6\ \text{MPa}$

At $y = y_Y$ $\quad \sigma_{res} = \sigma'' - \sigma_Y = 113.2 - 240 = -126.8\ \text{MPa}$



(b) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_o}{I} - \sigma_Y = 0$

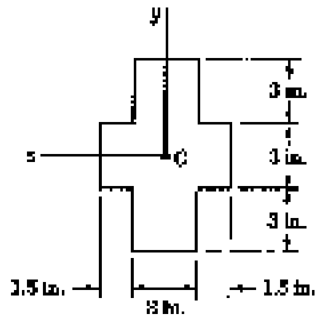
$y_o = \dfrac{I\sigma_Y}{M} = \dfrac{(706.67\times10^{-9})(240\times10^6)}{8.00\times10^3} = 21.2\times10^{-3}\ \text{m} = 21.2\ \text{mm}$

ans. $\quad y_o = -21.2\ \text{mm},\ 0,\ 21.2\ \text{mm}$

(c) At $y = y_Y$ $\quad \sigma_{res} = -126.8\times10^6\ \text{Pa}$

$\sigma = -\dfrac{Ey}{\rho} \quad \therefore \quad \rho = -\dfrac{Ey}{\sigma} = \dfrac{(200\times10^9)(0.010)}{126.8\times10^6} = 15.77\ \text{m}$

PROBLEM 4.103



4.103 and 4.104 A bending couple is applied to the bar indicated, causing plastic zones 3-in. thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 4.5$ in., (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

4.103 Bar of Prob. 4.85

SOLUTION

See SOLUTION to PROBLEM 4.85 for bending couple and stress distribution during loading

$M = 2646$ kip · in $\qquad y_Y = 1.5$ in

$E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi $\qquad \sigma_Y = 42$ ksi

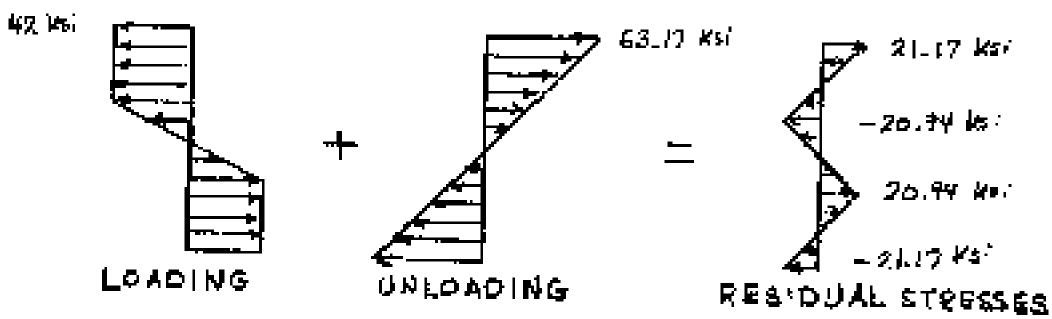
$I = 188.5$ in^4 $\qquad c = 4.5$ in.

(a) $\sigma' = \dfrac{Mc}{I} = \dfrac{(2646)(4.5)}{188.5} = 63.17$ ksi

$\sigma'' = \dfrac{My_Y}{I} = \dfrac{(2646)(1.5)}{188.5} = 21.06$ ksi

At $y = c$ $\qquad \sigma_{res} = \sigma' - \sigma_Y = 63.17 - 42 = 21.17$ ksi

At $y = y_Y$ $\qquad \sigma_{res} = \sigma'' - \sigma_Y = 21.06 - 42 = -20.94$ ksi



(b) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_0}{I} = \sigma_Y$

$y_0 = \dfrac{I\sigma_Y}{M} = \dfrac{(188.5)(42)}{2646} = 2.992$ in

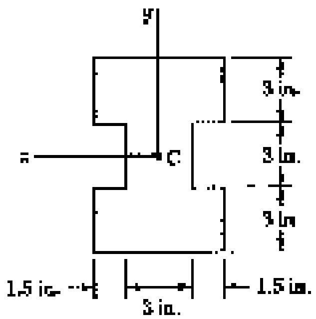
ans. $\qquad y_0 = -2.992$ in., 0, 2.992 in

(c) At $y = y_Y$, $\quad \sigma_{res} = -20.94$ ksi

$\sigma = -\dfrac{Ey}{\rho} \quad \therefore \quad \rho = -\dfrac{Ey}{\sigma} = \dfrac{(29 \times 10^3)(1.5)}{20.94} = 2077$ in

$= 173.1$ ft

PROBLEM 4.104



4.103 and 4.104 A bending couple is applied to the bar indicated, causing plastic zones 3-in. thick to develop at the top and bottom of the bar. After the couple has been removed, determine (a) the residual stress at $y = 4.5$ in., (b) the points where the residual stress is zero, (c) the radius of curvature corresponding to the permanent deformation of the bar.

4.104 Bar of Prob. 4.86

SOLUTION

See SOLUTION to PROBLEM 4.86 for bending couple and stress distribution

$M = 4725$ kip·in $\qquad y_Y = 1.5$ in.

$E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi $\qquad \sigma_Y = 42$ ksi

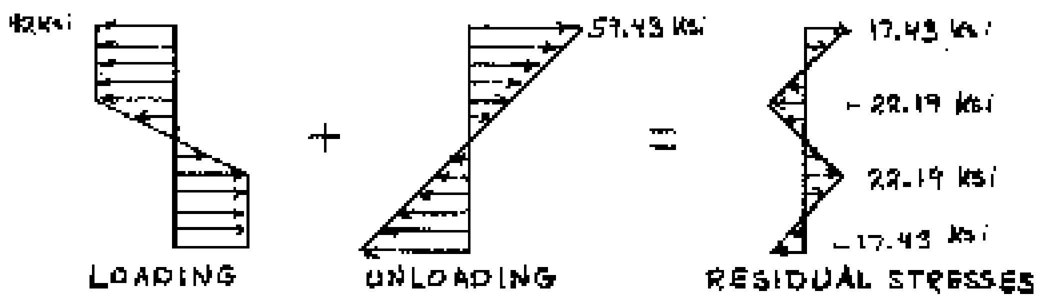
$I = 357.75$ in^4 $\qquad c = 4.5$ in.

(a) $\sigma' = \dfrac{Mc}{I} = \dfrac{(4725)(4.5)}{357.75} = 59.43$ ksi

$\sigma'' = \dfrac{My_Y}{I} = \dfrac{(4725)(1.5)}{357.75} = 19.81$ ksi

At $y = c$ $\quad \sigma_{res} = \sigma' - \sigma_Y = 59.43 - 42 = 17.43$ ksi

At $y = y_Y$ $\quad \sigma_{res} = \sigma'' - \sigma_Y = 19.81 - 42 = -22.19$ ksi



(b) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_0}{I} - \sigma_Y = 0$

$y_0 = \dfrac{I\sigma_Y}{M} = \dfrac{(357.75)(42)}{4725} = 3.18$ in

ans. $y_0 = -3.18$ in, 0, 3.18 in.

(c) At $y = y_Y$, $\quad \sigma_{res} = -22.19$ ksi

$\sigma = -\dfrac{Ey}{\rho} \quad \therefore \quad \rho = -\dfrac{Ey}{\sigma} = \dfrac{(29 \times 10^3)(1.5)}{22.19} = 1960$ in

$= 163.4$ ft.

PROBLEM 4.105

*4.105 A rectangular bar that is straight and unstressed is bent into an arc of circle of radius ρ by two couples of moment M. After the couples are removed, it is observed that the radius of curvature of the bar is ρ_R. Denoting by ρ_Y the radius of curvature of the bar at the onset of yield, show that the radii of curvature satisfy the following relation

$$\frac{1}{\rho_R} = \frac{1}{\rho}\left\{1 - \frac{3}{2}\frac{\rho}{\rho_Y}\left[1 - \frac{1}{3}\left(\frac{\rho}{\rho_Y}\right)^2\right]\right\}$$

SOLUTION

$$\frac{1}{\rho_Y} = \frac{M_Y}{EI}, \qquad M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \text{Let } m \text{ denote } \frac{M}{M_Y}$$

$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{\rho^2}{\rho_Y^2}\right) \qquad \therefore \qquad \frac{\rho^2}{\rho_Y^2} = 3 - 2m$$

$$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y}$$

$$= \frac{1}{\rho}\left\{1 - \frac{\rho}{\rho_Y}m\right\} = \frac{1}{\rho}\left\{1 - \frac{3}{2}\frac{\rho}{\rho_Y}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)\right\}$$

PROBLEM 4.106

4.106 A solid bar of rectangular cross section is made of a material that is assumed to be elastoplastic. Denoting by M_Y and ρ_Y, respectively, the bending moment and radius of curvature at the onset of yield, determine (*a*) the radius of curvature when a couple of moment $M = 1.25M_Y$ is applied to the bar, (*b*) the radius of curvature after the couple is removed. Check the results obtained by using the relation derived in Prob. 4.105.

SOLUTION

(a) $$\frac{1}{\rho_Y} = \frac{M_Y}{EI}, \qquad M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \text{Let } m = \frac{M}{M_Y} = 1.25$$

$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \frac{\rho}{\rho_Y} = \sqrt{3 - 2m} = 0.70711$$

$$\rho = 0.70711\,\rho_Y$$

(b) $$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y} = \frac{1}{0.70711\,\rho_Y} - \frac{1.25}{\rho_Y}$$

$$= \frac{0.16421}{\rho_Y} \qquad \therefore \qquad \rho_R = 6.09\,\rho_Y$$

PROBLEM 4.107

4.106 A solid bar of rectangular cross section is made of a material that is assumed to be elastoplastic. Denoting by M_Y and ρ_Y, respectively, the bending moment and radius of curvature at the onset of yield, determine (a) the radius of curvature when a couple of moment $M = 1.25 M_Y$ is applied to the bar, (b) the radius of curvature after the couple is removed. Check the results obtained by using the relation derived in Prob. 4.105.

4.107 Solve Prob. 4.106, assuming that the moment of the couple applied to the bar is $1.40 M_Y$.

SOLUTION

(a) $\frac{1}{\rho_Y} = \frac{M_Y}{EI}$, $\quad M = \frac{3}{2} M_Y \left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)$ $\qquad$ Let $m = \frac{M}{M_Y} = 1.40$

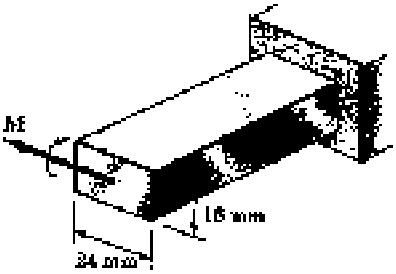
$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \frac{\rho}{\rho_Y} = \sqrt{3 - 2m} = 0.44721$$

$$\rho = 0.44721\, \rho_Y$$

(b) $\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{m M_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y} = \frac{1}{0.44721\, \rho_Y} - \frac{1.40}{\rho_Y}$

$= \frac{0.83607}{\rho_Y} \qquad \therefore \qquad \rho_R = 1.196\, \rho_Y$

PROBLEM 4.108



4.108 The prismatic bar shown is made of a steel that is assumed to be elastoplastic and for which $E = 200$ GPa. Knowing that the radius of curvature of the bar is 2.4 m when a couple of moment $M = 420$ N·m is applied as shown, determine (*a*) the yield strength σ_Y of the steel, (*b*) the thickness of the elastic core of the bar.

SOLUTION

$$M = \frac{3}{2} M_Y \left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)$$
$$= \frac{3}{2}\frac{\sigma_Y I}{c}\left(1 - \frac{1}{3}\frac{\rho^2 \sigma_Y^2}{E^2 c^2}\right)$$
$$= \frac{3}{2}\frac{\sigma_Y b(2c)^3}{12c}\left(1 - \frac{1}{3}\frac{\rho^2 \sigma_Y^2}{E^2 c^2}\right)$$
$$= \sigma_Y bc^2 \left(1 - \frac{1}{3}\frac{\rho^2 \sigma_Y^2}{E^2 c^2}\right)$$

(a) $bc^2\sigma_Y\left(1 - \frac{\rho^2 \sigma_Y^2}{3E^2c^2}\right) = M$ Cubic equation for σ_Y

Data: $E = 200 \times 10^9$ Pa, $M = 420$ N·m $\rho = 2.4$ m

$b = 24$ mm $= 0.024$ m $c = \frac{1}{2}h = 8$ mm $= 0.008$ m

$$(1.536 \times 10^{-6})\sigma_Y \left[1 - 750 \times 10^{-21}\sigma_Y^2\right] = 420$$

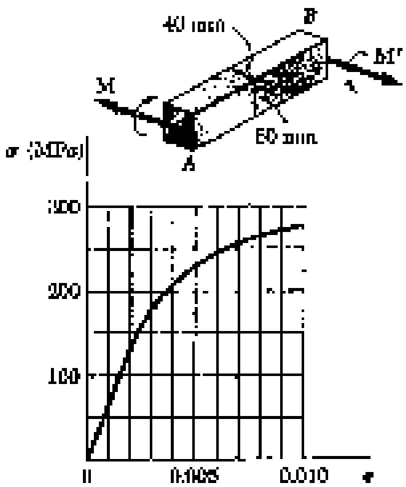
$$\sigma_Y \left[1 - 750 \times 10^{-21}\sigma_Y^2\right] = 273.44 \times 10^6$$

Solving by trial $\sigma_Y = 292 \times 10^6$ Pa $= 292$ MPa ◄

(b) $y_Y = \frac{\sigma_Y \rho}{E} = \frac{(292 \times 10^6)(2.4)}{200 \times 10^9} = 3.504 \times 10^{-3}$ m $= 3.504$ mm

thickness of elastic core $= 2y_Y = 7.01$ mm ◄

PROBLEM 4.109



4.109 The prismatic bar AB is made of an aluminum alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ-ε diagram is the same in compression as in tension, determine (*a*) the radius of curvature of the bar when the maximum stress is 250 MPa, (*b*) the corresponding value of the bending moment. (*Hint:* For part *b*, plot σ versus y and use an approximate method of integration.)

SOLUTION

(a) $\sigma_m = 250 \text{ MPa} = 250 \times 10^6 \text{ Pa}$

$\varepsilon_m = 0.0064$ from curve

$c = \frac{1}{2}h = 30 \text{ mm} = 0.030 \text{ m}$

$b = 40 \text{ mm} = 0.040 \text{ m}$

$$\frac{1}{\rho} = \frac{\varepsilon_m}{c} = \frac{0.0064}{0.030} = 0.21333 \text{ m}^{-1}$$

$\rho = 4.69 \text{ m}$

(b) Strain distribution $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u$ where $u = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 u|\sigma|\,du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u|\sigma|\,du$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\sum w u|\sigma|$$

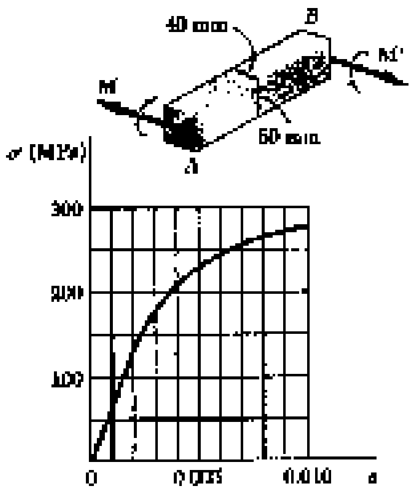
where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, (MPa)	$u\lvert\sigma\rvert$, (MPa)	w	$wu\lvert\sigma\rvert$, (MPa)
0	0	0	0	1	0
0.25	0.0016	110	27.5	4	110
0.5	0.0032	180	90	2	180
0.75	0.0048	225	168.75	4	675
1.00	0.0064	250	250	1	250
					1215 ← $\sum wu\lvert\sigma\rvert$

$$J = \frac{(0.25)(1215)}{3} = 101.25 \text{ MPa} = 101.25 \times 10^6 \text{ Pa}$$

$$M = (2)(0.040)(0.030)^2(101.25\times 10^6) = 7.29 \times 10^3 \text{ N·m} = 7.29 \text{ kN·m}$$

PROBLEM 4.110



4.110 For the bar of Prob. 4.109, determine (*a*) the maximum stress when the radius of curvature of the bar is 3 m, (*b*) the corresponding value of the bending moment. (See hint given in Prob. 4.109.)

SOLUTION

(a) $\rho = 3$ m , $c = 0.030$ mm $= 0.030$ m

$b = 40$ mm $= 0.040$ m

$$\varepsilon_m = \frac{c}{\rho} = \frac{0.030}{3} = 0.010$$

From curve $\sigma_m = 275$ MPa ◄

(b) Strain distribution $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u$ where $u = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 u|\sigma|\,du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u|\sigma|\,du$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\sum w u|\sigma|$$

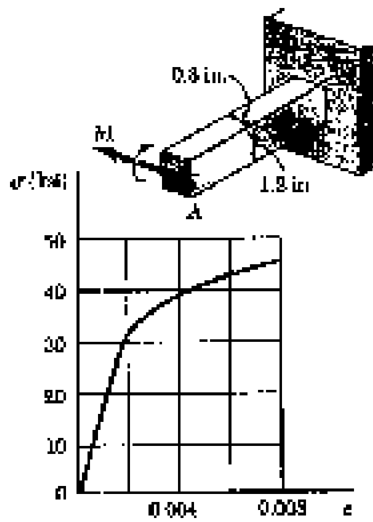
where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$ (MPa)	$u\lvert\sigma\rvert$ (MPa)	w	$wu\lvert\sigma\rvert$ (MPa)
0	0	0	0	1	0
0.25	0.0025	160	40	4	160
0.5	0.0050	254	127	2	254
0.75	0.0075	266	199.5	4	798
1.00	0.0100	275	275	1	275
					1487 ← $\Sigma wu\lvert\sigma\rvert$

$$J = \frac{(0.25)(1487)}{3} = 123.9 \text{ MPa} = 123.9\times10^6 \text{ Pa}$$

$$M = (2)(0.040)(0.030)^2(123.9\times10^6) = 8.92\times10^3 \text{ N}\cdot\text{m} = 8.92 \text{ kN}\cdot\text{m}$$ ◄

PROBLEM 4.111



4.111 The prismatic bar AB is made of a bronze alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ-ϵ diagram is the same in compression as in tension, determine (*a*) the maximum stress in the bar when the radius of curvature of the bar is 100 in., (*b*) the corresponding value of the bending moment. (See hint given in Prob. 4.109.)

SOLUTION

(a) $\rho = 100$ in, $b = 0.8$ in, $c = 0.6$ in.

$$\varepsilon_m = \frac{c}{\rho} = \frac{0.6}{100} = 0.006$$

From the curve $\sigma_m = 43$ ksi

(b) Strain distribution $\quad \varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u \quad$ where $u = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 u|\sigma|\,du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u|\sigma|\,du$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\sum w u|\sigma|$$

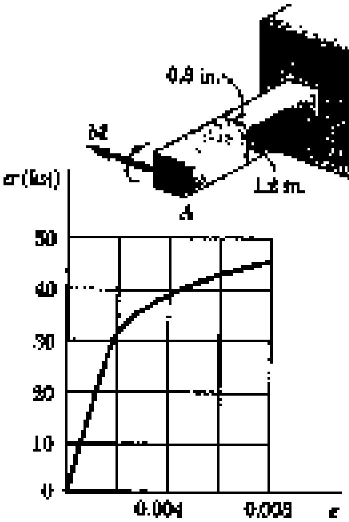
where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, ksi	$u\lvert\sigma\rvert$, ksi	w	$wu\lvert\sigma\rvert$, ksi
0	0	0	0	1	0
0.25	0.0015	25	6.25	4	25
0.5	0.003	36	18	2	36
0.75	0.0045	40	30	4	120
1.00	0.006	43	43	1	43
					224 ← $\Sigma wu\lvert\sigma\rvert$

$$J = \frac{(0.25)(224)}{3} = 18.67 \text{ ksi}$$

$$M = (2)(0.8)(0.6)^2(18.67) = 10.75 \text{ kip}\cdot\text{in}$$

PROBLEM 4.112



4.112 For the bar of Prob. 4.111, determine (*a*) the radius of curvature of the bar when the maximum stress is 45 ksi; (*b*) the corresponding value of the bending moment. (See hint given in Prob. 4.109.)

SOLUTION

(a) $b = 0.8$ in $\quad c = 0.6$ in

$\sigma_m = 45$ ksi

From the curve $\quad \varepsilon_m = 0.008$

$$\frac{1}{\rho} = \frac{\varepsilon_m}{c} = \frac{0.008}{0.6} = 0.013333 \text{ in}^{-1}$$

$$\rho = 75 \text{ in.}$$

(b) Strain distribution $\quad \varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u \quad$ where $u = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 u|\sigma|\,du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u|\sigma|\,du$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\sum w u|\sigma|$$

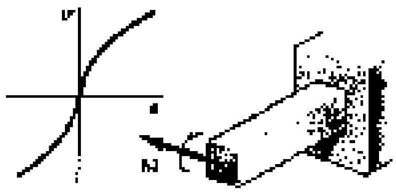
where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, ksi	$u\lvert\sigma\rvert$, ksi	w	$wu\lvert\sigma\rvert$, ksi
0	0.	0	0	1	0
0.25	0.002	32	8.	4	32
0.5	0.004	38	19	2	38
0.75	0.006	43	32.25	4	129
1.0	0.008	45	45	1	45
					244 ← $\sum wu\lvert\sigma\rvert$

$$J = \frac{(0.25)(244)}{3} = 20.33 \text{ ksi}$$

$$M = (2)(0.8)(0.6)^2(20.33) = 11.7 \text{ kip-in}$$

PROBLEM 4.113



4.113 A prismatic bar of rectangular cross section is made of an alloy for which the stress-strain diagram can be represented by the relation, $\epsilon = k\sigma^n$ for $\sigma > 0$, and $\epsilon = -|k\sigma^n|$ for $\sigma < 0$. If a couple **M** is applied to the bar, show that the maximum stress is

$$\sigma_m = \frac{1+2n}{3n}\frac{Mc}{I}$$

SOLUTION

Strain distribution $\quad \epsilon = -\epsilon_m \frac{y}{c} = -\epsilon_m u \quad$ where $\quad u = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma\, b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 \frac{y}{c}|\sigma|\frac{dy}{c}$$

$$= 2bc^2\int_0^1 u|\sigma|\,du$$

For $\quad \epsilon = K\sigma^n, \qquad \epsilon_m = K\sigma_m^n$

$$\frac{\epsilon}{\epsilon_m} = u = \left(\frac{\sigma}{\sigma_m}\right)^n \qquad \therefore\ |\sigma| = \sigma_m u^{\frac{1}{n}}$$

Then $\quad M = 2bc^2\int_0^1 u\,\sigma_m u^{\frac{1}{n}}\,du = 2bc^2\sigma_m\int_0^1 u^{1+\frac{1}{n}}\,du$

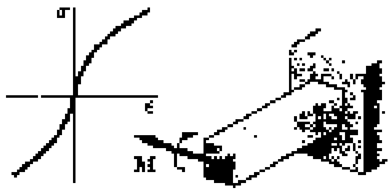
$$= 2bc^2\sigma_m \left.\frac{u^{2+\frac{1}{n}}}{2+\frac{1}{n}}\right|_0^1 = \frac{2n}{2n+1}bc^2\sigma_m$$

$$\sigma_m = \frac{2n+1}{2}\frac{M}{bc^2}$$

Recall $\quad \frac{I}{c} = \frac{1}{12}\frac{b(2c)^3}{c} = \frac{2}{3}bc^2 \qquad \therefore\ \frac{1}{bc^2} = \frac{2}{3}\frac{c}{I}$

$$\sigma_m = \frac{2n+1}{3n}\frac{Mc}{I}$$

PROBLEM 4.114



4.114 A prismatic bar of rectangular cross section is made of an alloy for which the stress-strain diagram can be represented by the relation $\epsilon = k\sigma^3$. If a couple **M** is applied to the bar, show that the maximum stress is

$$\sigma_m = \frac{7}{9}\frac{Mc}{I}$$

SOLUTION

Strain distribution $\quad \epsilon = -\epsilon_m \frac{y}{c} = -\epsilon_m \upsilon \quad$ where $\quad \upsilon = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 \frac{y}{c}|\sigma|\frac{dy}{c}$$

$$= 2bc^2\int_0^1 \upsilon|\sigma|\,d\upsilon$$

For $\quad \epsilon = K\sigma^n, \qquad \epsilon_m = K\sigma_m$

$$\frac{\epsilon}{\epsilon_m} = \upsilon = \left(\frac{\sigma}{\sigma_m}\right)^n \qquad \therefore \quad |\sigma| = \sigma_m \upsilon^{\frac{1}{n}}$$

Then $\quad M = 2bc^2\int_0^1 \upsilon\sigma_m\upsilon^{\frac{1}{n}}\,d\upsilon = 2bc^2\sigma_m\int_0^1 \upsilon^{1+\frac{1}{n}}\,d\upsilon$

$$= 2bc^2\sigma_m \left.\frac{\upsilon^{2+\frac{1}{n}}}{2+\frac{1}{n}}\right|_0^1 = \frac{2n}{2n+1}bc^2\sigma_m$$

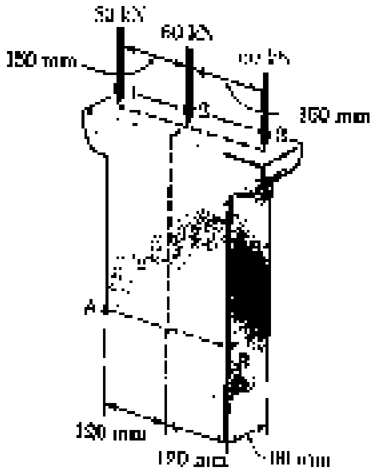
$$\sigma_m = \frac{2n+1}{2}\frac{M}{bc^2}$$

Recall $\quad \frac{I}{c} = \frac{1}{12}\frac{b(2c)^3}{c} = \frac{2}{3}bc^2 \qquad \therefore \quad \frac{1}{bc^2} = \frac{2}{3}\frac{c}{I}$

$$\sigma_m = \frac{2n+1}{3n}\frac{Mc}{I}$$

With $\quad n = 3 \qquad \sigma_m = \frac{(2)(3)+1}{(3)(3)}\frac{Mc}{I} = \frac{7}{9}\frac{Mc}{I}$

PROBLEM 4.115



4.115 Determine the stress at points A and B ... 60-kN loads are applied at points 1 and 2 only.

SOLUTION

(a) Loading is centric.

$$P = 180 \text{ kN} = 180\times10^3 \text{ N}$$

$$A = (90)(240) = 21.6\times10^3 \text{ mm}^2 = 21.6\times10^{-3} \text{ m}^2$$

At A and B $\quad \sigma = -\frac{P}{A} = -\frac{180\times10^3}{21.6\times10^{-3}} = -8.33\times10^6 \text{ Pa}$

$$= -8.33 \text{ MPa}$$

(b) Eccentric loading

$$P = 120 \text{ kN} = 120\times10^3$$

$$M = (60\times10^3)(150\times10^{-3}) = 9.0\times10^3 \text{ N}\cdot\text{m}$$

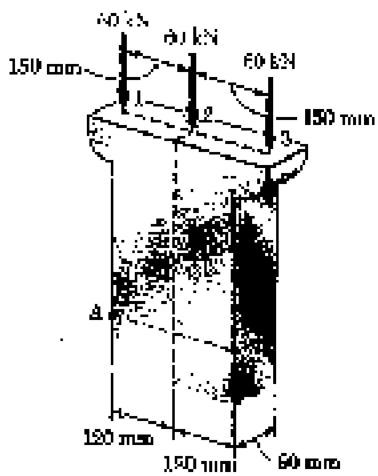
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(90)(240)^3 = 103.68\times10^6 \text{ mm}^4 = 103.68\times10^{-6} \text{ m}^4$$

$$c = 120 \text{ mm} = 0.120 \text{ m}$$

At A $\quad \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{120\times10^3}{21.6\times10^{-3}} - \frac{(9.0\times10^3)(0.120)}{103.68\times10^{-6}} = -15.97\times10^6 \text{ Pa} = -15.97 \text{ MPa}$

At B $\quad \sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{120\times10^3}{21.6\times10^{-3}} + \frac{(9.0\times10^3)(0.120)}{103.68\times10^{-6}} = 4.86\times10^6 \text{ Pa} = 4.86 \text{ MPa}$

PROBLEM 4.116



4.116 Determine the stress at points A and B, (*a*) for the loading shown, (*b*) if the 60-kN loads applied at points 2 and 3 are removed.

SOLUTION

(a) Loading is centric

$$P = 180 \text{ kN} = 180\times10^3 \text{ N}$$

$$A = (90)(240) = 21.6\times10^3 \text{ mm}^2 = 21.6\times10^{-3} \text{ m}^2$$

At A and B $\quad \sigma = -\frac{P}{A} = -\frac{180\times10^3}{21.6\times10^{-3}} = -8.33\times10^6 \text{ Pa}$

$$= -8.33 \text{ MPa}$$

(b) Eccentric loading

$$P = 60 \text{ kN} = 60\times10^3 \text{ N}$$

$$M = (60\times10^3)(150\times10^{-3}) = 9.0\times10^3 \text{ N}\cdot\text{m}$$

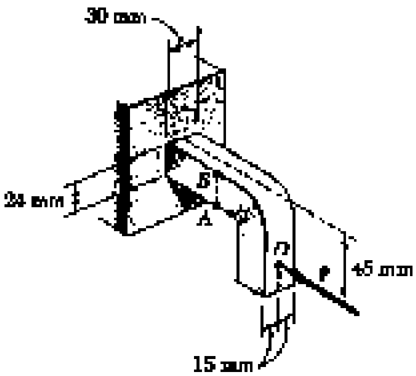
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(90)(240)^3 = 103.68\times10^6 \text{ mm}^4 = 103.68\times10^{-6} \text{ m}^4$$

$$c = 120 \text{ mm} = 0.120 \text{ m}$$

At A $\quad \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{60\times10^3}{21.6\times10^{-3}} - \frac{(9.0\times10^3)(0.120)}{103.68\times10^{-6}} = -13.19\times10^6 \text{ Pa} = -13.19 \text{ MPa}$

At B $\quad \sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{60\times10^3}{21.6\times10^{-3}} + \frac{(9.0\times10^3)(0.120)}{103.68\times10^{-6}} = 7.64\times10^6 \text{ Pa} = 7.64 \text{ MPa}$

PROBLEM 4.117



4.117 Knowing that the magnitude of the horizontal force **P** is 8 kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$A = (30)(24) = 720 \text{ mm}^2 = 720 \times 10^{-6} \text{ m}^2$

$e = 45 - 12 = 33 \text{ mm} = 0.033 \text{ m}$

$I = \frac{1}{12} b h^3 = \frac{1}{12}(30)(24)^3 = 34.56 \times 10^3 \text{ mm}^4 = 34.56 \times 10^{-9} \text{ m}^4$

$c = 24 \text{ mm} = 0.12 \text{ m} \qquad P = 8 \times 10^3 \text{ N}$

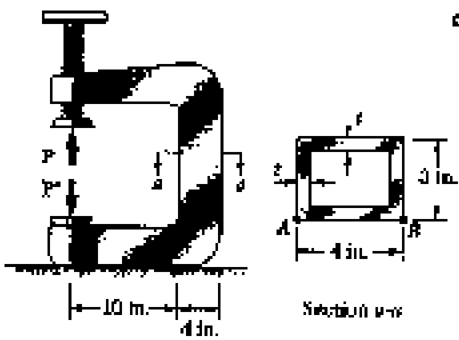
$M = Pe = (8 \times 10^3)(0.033) = 264 \text{ N}\cdot\text{m}$

At A $\quad \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} - \frac{(264)(0.12)}{34.56 \times 10^{-9}}$

$= -102.8 \times 10^6 \text{ Pa} = -102.8 \text{ MPa}$

At B $\quad \sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} + \frac{(264)(0.12)}{34.56 \times 10^{-9}} = 80.6 \times 10^6 \text{ Pa} = 80.6 \text{ MPa}$

PROBLEM 4.118



4.118 The vertical portion of the press shown consists of a rectangular tube having a wall thickness $t = \frac{1}{2}$ in. Knowing that the press has been tightened until $P = 6$ kips, determine the stress (*a*) at point *A*, (*b*) at point *B*.

SOLUTION

$t = \frac{1}{2} \text{ in.} \qquad P = 6 \text{ kips}$

$A = (3)(4) - (2)(3) = 6 \text{ in}^2$

$I = \frac{1}{12}(3)(4)^3 - \frac{1}{12}(2)(3)^3 = 11.5 \text{ in}^4$

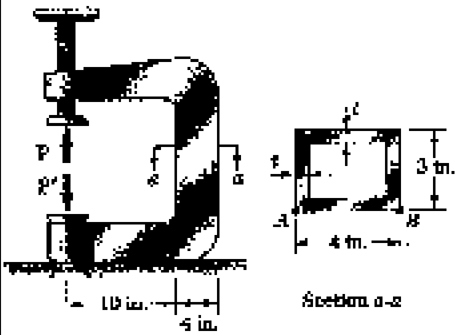
$c = 2 \text{ in.}, \qquad e = 10 + 2 = 12 \text{ in.}$

$M = Pe = (6)(12) = 72 \text{ kip}\cdot\text{in.}$

(a) $\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{6}{6} + \frac{(72)(2)}{11.5} = 13.52 \text{ ksi}$

(b) $\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{6}{6} - \frac{(72)(2)}{11.5} = -11.52 \text{ ksi}$

PROBLEM 4.119



4.118 The vertical portion of the press shown consists of a rectangular tube having a wall thickness $t = \frac{1}{2}$ in. Knowing that the press has been tightened until $P = 6$ kips, determine the stress (*a*) at point *A*, (*b*) at point *B*.

4.119 Solve Prob. 4.118, assuming that the wall thickness of the vertical portion of the press is $t = \frac{3}{8}$ in.

SOLUTION

Rectangular cutout is $2\frac{1}{4}$ in. × $3\frac{1}{4}$ in.

$$A = (3)(4) - (2.25)(3.25) = 4.6875 \text{ in}^2$$

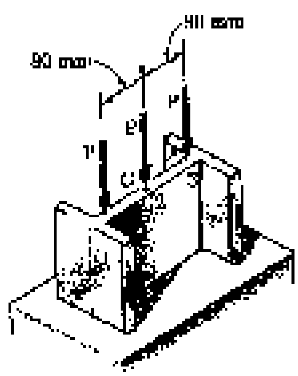
$$I = \frac{1}{12}(3)(4)^3 - \frac{1}{12}(2.25)(3.25)^3 = 9.5635 \text{ in}^4$$

$$c = 2 \text{ in.}, \quad e = 10 + 2 = 12 \text{ in.}, \quad M = Pe = (6)(12) = 72 \text{ kip}\cdot\text{in}$$

$$(a) \quad \sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{6}{4.6875} + \frac{(72)(2)}{9.5635} = 16.34 \text{ ksi}$$

$$(b) \quad \sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{6}{4.6875} - \frac{(72)(2)}{9.5635} = -13.78 \text{ ksi}$$

PROBLEM 4.120



4.120 As many as three axial loads each of magnitude $P = 50$ kN can be applied to the end of a ~~[illegible]~~ rolled-steel shape. Determine the stress at point *A*, (*a*) for the loading shown, (*b*) if loads are applied at points 1 and 2 only.

W 200 × 31.3

SOLUTION

For W 200 × 31.3 rolled steel shape

$$A = 4000 \text{ mm}^2 = 4.000 \times 10^{-3} \text{ m}^2$$

$$c = \frac{1}{2}d = \frac{1}{2}(210) = 105 \text{ mm} = 0.105 \text{ m}$$

$$I = 31.4 \times 10^6 \text{ mm}^4 = 31.4 \times 10^{-6} \text{ m}^4$$

(a) Centric load

$$3P = 50 + 50 + 50 = 150 \text{ kN} = 150 \times 10^3 \text{ N}$$

$$\sigma = -\frac{3P}{A} = -\frac{150 \times 10^3}{4.0 \times 10^{-3}} = -37.5 \times 10^6 \text{ Pa} = -37.5 \text{ MPa}$$

(b) Eccentric loading $\quad e = 80 \text{ mm} = 0.080 \text{ m}$

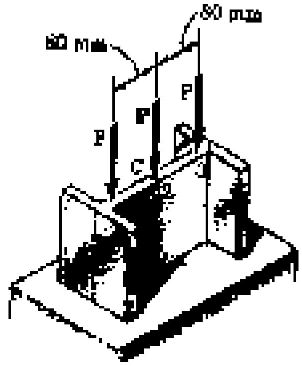
$$2P = 50 + 50 = 100 \text{ kN} = 100 \times 10^3 \text{ N}$$

$$M = Pe = (50 \times 10^3)(0.080) = 4.0 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_A = -\frac{2P}{A} - \frac{Mc}{I} = -\frac{100 \times 10^3}{4.0 \times 10^{-3}} - \frac{(4.0 \times 10^3)(0.105)}{31.4 \times 10^{-6}} = -38.4 \times 10^6 \text{ Pa}$$

$$= -38.4 \text{ MPa}$$

PROBLEM 4.121



4.121 As many as three axial loads, each of magnitude $P = 50$ kN, can be applied to the end of a ~~W8×21~~ rolled-steel shape. Determine the stress at point A, (a) for the loading shown, (b) if loads are applied at points 2 and 3 only.

SOLUTION W 200 × 31.3

For a W 200 × 31.3 rolled steel shape

$A = 4000 \text{ mm}^2 = 4.00 \times 10^{-3} \text{ m}^2$

$c = \frac{1}{2} d = \frac{1}{2}(210) = 105 \text{ mm} = 0.105 \text{ m}$

$I = 31.4 \times 10^6 \text{ mm}^4 = 31.4 \times 10^{-6} \text{ m}^4$

(a) Centric loading

$3P = 50 + 50 + 50 = 150 \text{ kN} = 150 \times 10^3 \text{ N}$

$$\sigma = -\frac{3P}{A} = -\frac{150 \times 10^3}{4.0 \times 10^{-3}} = -37.5 \times 10^6 \text{ Pa} = -37.5 \text{ MPa}$$

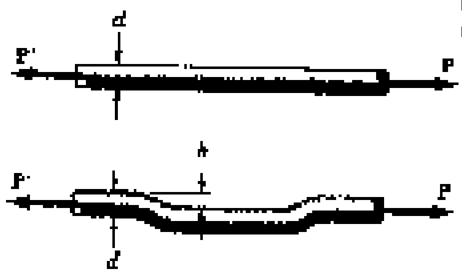
(b) Eccentric loading $e = 80 \text{ mm} = 0.080 \text{ m}$

$$M = Pe = (50 \times 10^3)(0.080) = 4.0 \times 10^3 \text{ N}\cdot\text{m}$$

$$\sigma_A = -\frac{3P}{A} + \frac{Mc}{I} = -\frac{100 \times 10^3}{4.0 \times 10^{-3}} + \frac{(4.0 \times 10^3)(0.105)}{31.4 \times 10^{-6}} = -11.62 \times 10^6 \text{ Pa}$$

$$= -11.62 \text{ MPa}$$

PROBLEM 4.122



4.122 An offset h must be introduced into a solid circular rod of diameter d. Knowing that the maximum stress after the offset is introduced must not exceed four times the stress in the rod when it was straight, determine the largest offset that can be used.

SOLUTION

For centric loading $\sigma_c = \frac{P}{A}$

For eccentric loading $\sigma_e = \frac{P}{A} + \frac{Phc}{I}$

Given $\sigma_e = 4\sigma_c$

$$\frac{P}{A} + \frac{Phc}{I} = 4\frac{P}{A}$$

$$\frac{Phc}{I} = 3\frac{P}{A} \quad \therefore \quad h = \frac{3I}{cA} = \frac{(3)(\frac{\pi}{64} d^4)}{(\frac{d}{2})(\frac{\pi}{4} d^2)} = \frac{3}{8} d = 0.375\, d$$

PROBLEM 4.123

4.123 An offset h must be introduced into a metal tube of 18-mm outer diameter and 2-mm wall thickness. Knowing that the maximum stress after the offset is introduced must not exceed four times the stress in the rod when it was straight, determine the largest offset that can be used.

SOLUTION

$d_i = d_o - 2t = 18 - (2)(2) = 14$ mm $\qquad c = \frac{1}{2} d_o = 9$ mm

$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(18^2 - 14^2) = 100.53$ mm²

$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(18^4 - 14^4) = 3.2673 \times 10^3$ mm⁴

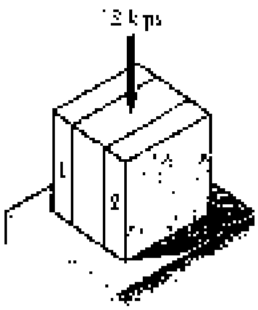
For centric loading $\sigma_c = \frac{P}{A}$; For eccentric loading $\sigma_e = \frac{P}{A} + \frac{Phc}{I}$

Given $\sigma_e = 4\sigma_c = \frac{P}{A} + \frac{Phc}{I} = 4\frac{P}{A} \quad \therefore \quad \frac{Phc}{I} = 3\frac{P}{A}$

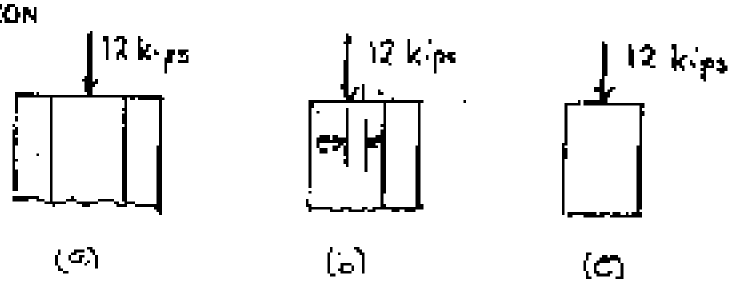
$$h = \frac{3I}{cA} = \frac{(3)(3.2673 \times 10^3)}{(9)(100.53)} = 10.83 \text{ mm}$$

PROBLEM 4.124

4.124 A short column is made by nailing two 1 × 4-in. planks to a 2 × 4-in. timber. Determine the largest compressive stress created in the column by a 12-kip load applied as shown at the center of the top section of the timber if (*a*) the column is as described, (*b*) plank 1 is removed, (*c*) both planks are removed.



SOLUTION



(a) Centric loading: 4 in × 4 in cross section $\qquad A = (4)(4) = 16$ in²

$$\sigma = -\frac{P}{A} = -\frac{12}{16} = -0.75 \text{ ksi}$$

(b) Eccentric loading: 4 in × 3 in cross section $\qquad A = (4)(3) = 12$ in²

$c = (\frac{1}{2})(3) = 1.5$ in $\qquad e = 1.5 - 1.0 = 0.5$ in

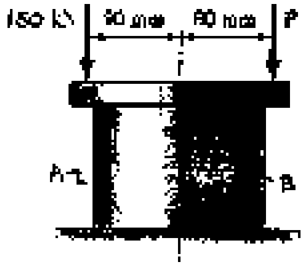
$I = \frac{1}{12}bh^3 = \frac{1}{12}(4)(3)^3 = 9$ in⁴

$$\sigma = -\frac{P}{A} - \frac{Pec}{I} = -\frac{12}{12} - \frac{(12)(0.5)(1.5)}{9} = -2.00 \text{ ksi}$$

(c) Centric loading: 4 in. × 2 in cross section $\qquad A = (4)(2) = 8$ in²

$$\sigma = -\frac{P}{A} = -\frac{12}{8} = -1.50 \text{ ksi}$$

PROBLEM 4.125



4.125 The two forces shown are applied to a rigid plate supported by a steel pipe of 140-mm outer diameter and 120-mm inner diameter. Knowing that the allowable compressive stress is 100 MPa, determine the range of allowable values of P.

SOLUTION

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(140^2 - 120^2) = 4.084\times10^3\ \text{mm}^2 = 4.084\times10^{-3}\ \text{m}^2$$

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(140^4 - 120^4) = 8.679\times10^6\ \text{mm}^4 = 8.679\times10^{-6}\ \text{m}^4$$

$$c = \tfrac{1}{2}d_o = 70\ \text{mm} = 0.070\ \text{m}$$

$$F = 150\times10^3 + P, \qquad M = (0.090)(150\times10^3) - (0.090)P = 13.5\times10^3 - 0.09P$$

At A $$\sigma_A = -\frac{F}{A} - \frac{Mc}{I} = -\frac{(150\times10^3) + P}{4.084\times10^{-3}} - \frac{(13.5\times10^3 - 0.09P)(0.070)}{8.679\times10^{-6}}$$

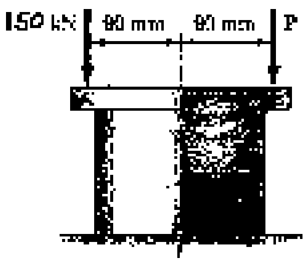
$$= -145.61\times10^6 + 481.08P = -100\times10^6 \quad \therefore \quad P = 94.8\times10^3\ \text{N}$$

At B $$\sigma_B = -\frac{F}{A} + \frac{Mc}{I} = -\frac{(150\times10^3) + P}{4.084\times10^{-3}} + \frac{(13.5\times10^3 - 0.09P)(0.070)}{8.679\times10^{-6}}$$

$$= 72.155\times10^6 - 970.75P = -100\times10^6 \quad \therefore \quad P = 177.3\times10^3\ \text{N}$$

$$94.8\ \text{kN} < P < 177.3\ \text{kN}$$

PROBLEM 4.126



4.126 The two forces shown are applied to a rigid plate supported by a steel pipe of 140-mm outer diameter and 120-mm inner diameter. Determine the range of allowable values of P for which all stresses in the pipe are compressive and less than 100 MPa.

SOLUTION

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(140^2 - 120^2) = 4.084\times10^3\ \text{mm}^2 = 4.084\times10^{-3}\ \text{m}^2$$

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(140^4 - 120^4) = 8.679\times10^6\ \text{mm}^4 = 8.679\times10^{-6}\ \text{m}^4$$

$$c = \tfrac{1}{2}d_o = 70\ \text{mm} = 0.070\ \text{m}$$

$$F = 150\times10^3 + P, \qquad M = (0.090)(150\times10^3) - 0.090P = 13.5\times10^3 - 0.09P$$

At A $$\sigma_A = -\frac{F}{A} - \frac{Mc}{I} = -\frac{(150\times10^3) + P}{4.084\times10^{-3}} - \frac{(13.5\times10^3 - 0.09P)(0.070)}{8.679\times10^{-6}}$$

$$= -145.61\times10^6 + 481.08P = -100\times10^6 \quad \therefore \quad P = 94.8\times10^3\ \text{N}$$

$$\sigma_A = -145.61\times10^6 + 481.08P = 0 \qquad P = 303\times10^3\ \text{N}$$

Based on stress limits at A $\quad 94.8\ \text{kN} \le P \le 303\ \text{kN}$

At B $$\sigma_B = -\frac{F}{A} + \frac{Mc}{I} = -\frac{(150\times10^3) + P}{4.084\times10^{-3}} + \frac{(13.5\times10^3 - 0.09P)(0.070)}{8.679\times10^{-6}}$$

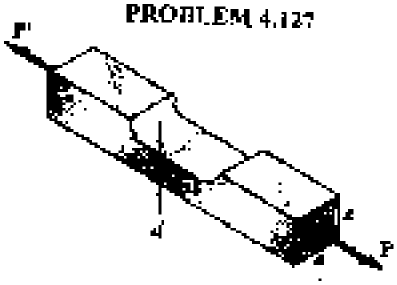
$$= 72.155\times10^6 - 970.75P = -100\times10^6 \qquad P = 177.3\times10^3\ \text{N}$$

$$\sigma_B = 72.155\times10^6 - 970.75P = 0 \qquad P = 74.3\times10^3\ \text{N}$$

Based on stress limits at B $\quad 74.3\ \text{kN} \le P \le 177.3\ \text{kN}$

Based on both limits $\quad 94.8\ \text{kN} \le P \le 177.3\ \text{kN}$

PROBLEM 4.127



4.127 A milling operation was used to remove a portion of a solid bar of square cross section. Knowing that $a = 1.2$ in., $d = 0.8$ in., and $\sigma_{all} = 8$ ksi, determine the largest magnitude P of the forces that can be safely applied at the centers of the ends of the bar.

SOLUTION

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

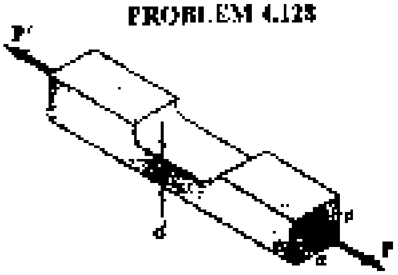
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{6Ped}{ad^3}$$

$$\sigma = \frac{P}{ad} + \frac{3P(a-d)}{ad^2} = KP$$

$$K = \frac{1}{ad} + \frac{3(a-d)}{ad^2} = \frac{1}{(1.2)(0.8)} + \frac{(3)(1.2-0.8)}{(1.2)(0.8)^2} = 2.604 \text{ in}^{-1}$$

$$P = \frac{\sigma}{K} = \frac{8}{2.604} = 3.07 \text{ kips}$$

PROBLEM 4.128



4.128 A milling operation was used to remove a portion of a solid bar of square cross section. Forces of magnitude $P = 4$ kips are applied at the centers of the ends of the bar. Knowing that $a = 1.2$ in. and $\sigma_{all} = 8$ ksi, determine the smallest allowable depth d of the milled portion of the rod.

SOLUTION

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

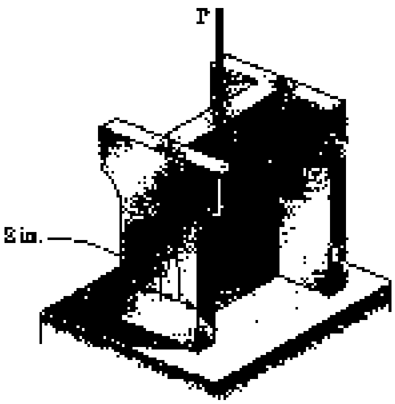
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{Pec}{I} = \frac{P}{ad} + \frac{P\frac{1}{2}(a-d)\frac{1}{2}d}{\frac{1}{12}ad^3} = \frac{P}{ad} + \frac{3P(a-d)}{ad^2}$$

$$\sigma = \frac{3P}{d^2} - \frac{2P}{ad} \qquad \text{or} \qquad \sigma d^2 + \frac{2P}{a}d - 3P = 0$$

Solving for d $\quad d = \frac{1}{2\sigma}\left\{\sqrt{\left(\frac{2P}{a}\right)^2 + 12P\sigma} - \frac{2P}{a}\right\}$

$$d = \frac{1}{(2)(8)}\left\{\sqrt{\left[\frac{(2)(4)}{1.2}\right]^2 + (12)(4)(8)} - \frac{(2)(4)}{1.2}\right\} = 0.877 \text{ in.}$$

PROBLEM 4.129



4.129 Three steel plates, each of 1 × 6-in. cross section, are welded together to form a short H-shaped column. Later, for architectural reasons, a 1-in. strip is removed from each side of one of the flanges. Knowing that the load remains centric with respect to the original cross section, and that the allowable stress is 15 ksi, determine the largest force **P**, (*a*) which could be applied to the original column, (*b*) which can be applied to the modified column.

SOLUTION

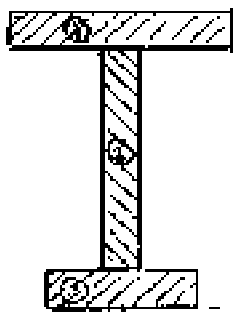
(a) Centric loading

$A = (3)(1)(6) = 18 \text{ in}^2$

$\sigma = -\dfrac{P}{A} \quad \therefore \quad P = -\sigma A = -(-15)(18) = 270 \text{ kips}$

(b) Eccentric loading

Reduced cross section



	A, in^2	$\bar{y}_0, \text{in}$	$A\bar{y}_0, \text{in}^3$
①	6	3.5	21.0
②	6	0	0
③	4	-3.5	-14.0
Σ	16		7.0

$\bar{Y}_0 = \dfrac{\Sigma A\bar{y}_0}{\Sigma A} = \dfrac{7.0}{16} = 0.4375 \text{ in}$

The centroid lies 0.4375 in from the midpoint of the web.

$I_1 = \frac{1}{12}(6)(1)^3 + (6)(3.0625)^2 = 56.773 \text{ in}^4$

$I_2 = \frac{1}{12}(1)(6)^3 + (6)(0.4375)^2 = 19.148 \text{ in}^4$

$I_3 = \frac{1}{12}(4)(1)^3 + (4)(3.9375)^2 = 62.349 \text{ in}^4$

$I = I_1 + I_2 + I_3 = 138.27 \text{ in}^4, \qquad c = 4.4375 \text{ in}$

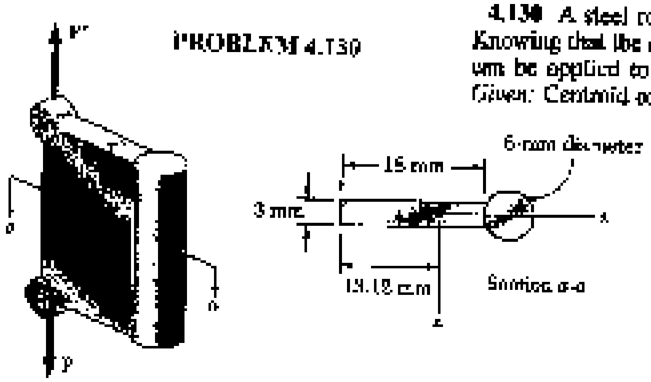
$M = Pe \quad \text{where} \quad e = 0.4375 \text{ in}$

$\sigma = -\dfrac{P}{A} - \dfrac{Mc}{I} = -\dfrac{P}{A} - \dfrac{Pec}{I} = -KP$

$K = \dfrac{1}{A} + \dfrac{ec}{I} = \dfrac{1}{16} + \dfrac{(0.4375)(4.4375)}{138.27} = 0.076541 \text{ in}^{-2}$

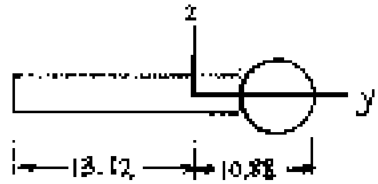
$P = -\dfrac{\sigma}{K} = -\dfrac{-15}{0.076541} = 196.0 \text{ kips}$

PROBLEM 4.130



4.130 A steel rod is welded to a steel plate to form the machine element shown. Knowing that the allowable stress is 135 MPa, determine (*a*) the largest force **P** that can be applied to the element, (*b*) the corresponding location of the neutral axis. Given: Centroid of the cross section is at *C* and $I_z = 4195\ \text{mm}^4$.

SOLUTION



(a) $A = (3)(18) + \frac{\pi}{4}(6)^2 = 82.27\ \text{mm}^2 = 82.27 \times 10^{-6}\ \text{m}^2$

$I = 4195\ \text{mm}^4 = 4195 \times 10^{-12}\ \text{m}^4$

$e = 13.12\ \text{mm} = 0.01312\ \text{m}$

Based on tensile stress at $y = -13.12\ \text{mm} = -0.01312\ \text{m}$

$$\sigma = \frac{P}{A} + \frac{Pec}{I} = \left(\frac{1}{A} + \frac{ec}{I}\right)P = KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{82.27 \times 10^{-6}} + \frac{(0.01312)(0.01312)}{4195 \times 10^{-12}} = 53.188 \times 10^3\ \text{m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{135 \times 10^6}{53.188 \times 10^3} = 2.538 \times 10^3\ \text{N} = 2.54\ \text{kN}$$

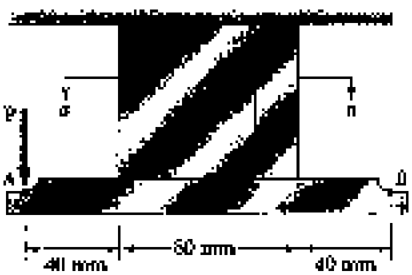
(b) Location neutral axis. $\sigma = 0$

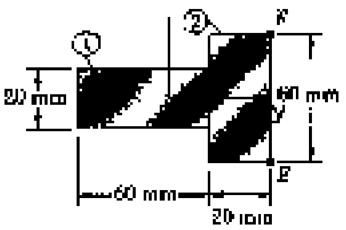
$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{4195 \times 10^{-12}}{(82.27 \times 10^{-6})(0.01312)} = 3.89 \times 10^{-3}\ \text{m} = 3.89\ \text{mm}$$

The neutral axis lies 3.89 mm to the right of the centroid or 17.01 mm to the right of the line of action of the loads.

PROBLEM 4.131





Section a–a

4.131 Knowing that the allowable stress is 150 MPa in section *a-a* of the hanger shown, determine (*a*) the largest vertical force **P** that can be applied at point *A*, (*b*) the corresponding location of the neutral axis of section *a-a*.

SOLUTION

Locate centroid

	A, mm²	$\bar{y}_0$, mm	$A\bar{y}_0$, mm³
①	1200	30	36×10^3
②	1200	70	84×10^3
Σ	2400		120×10^3

$$\bar{Y}_0 = \frac{\Sigma A\bar{y}_0}{\Sigma A} = \frac{120 \times 10^3}{2400} = 50 \text{ mm}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple $M = Pe$

$$e = 40 + 50 = 90 \text{ mm} = 0.090 \text{ m}$$

$$I_1 = \frac{1}{12}(20)(60)^3 + (1200)(20)^2 = 840 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(60)(20)^3 + (1200)(20)^2 = 520 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1.360 \times 10^6 \text{ mm}^4 = 1.360 \times 10^{-6} \text{ m}^4, \qquad A = 2400 \times 10^{-6} \text{ m}^2$$

(a) Based on tensile stress at left edge: $y = -50 \text{ mm} = -0.050 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = KP$$

$$K = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400 \times 10^{-6}} - \frac{(0.090)(-0.050)}{1.360 \times 10^{-6}} = 3.7255 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{150 \times 10^6}{3.7255 \times 10^3} = 40.3 \times 10^3 \text{ N} = 40.3 \text{ kN}$$

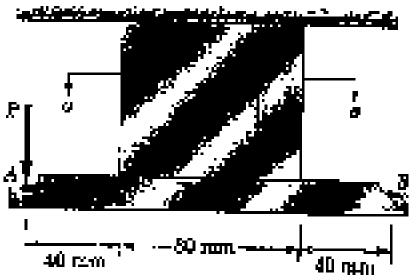
(b) Location of neutral axis: $\sigma = 0$

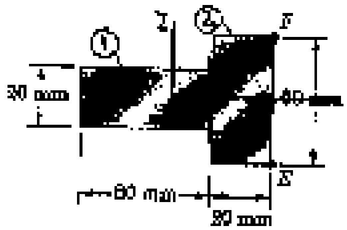
$$\sigma = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{1.360 \times 10^{-6}}{(2400 \times 10^{-6})(0.090)} = 6.30 \times 10^{-3} \text{ m} = 6.30 \text{ mm}$$

The neutral axis lies 6.30 mm to the right of the centroid or 56.30 mm from the left edge

PROBLEM 4.132





Section *a-a*

4.131 Knowing that the allowable stress is 150 MPa in section *a-a* of the hanger shown, determine (*a*) the largest vertical force **P** that can be applied at point *A*, (*b*) the corresponding location of the neutral axis of section *a-a*.

4.132 Solve Prob. 4.131, assuming that the vertical force **P** is applied at point *B*.

SOLUTION

Locate centroid

	A, mm²	$\bar{y}_o$, mm	$A\bar{y}_o$, mm³
①	1200	30	36×10^3
②	1200	70	84×10^3
Σ	2400		120×10^3

$$\bar{Y}_o = \frac{\Sigma A\bar{y}_o}{\Sigma A} = \frac{120\times10^3}{2400} = 50 \text{ mm}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple $M = Pe$

$e = 50 - 120 = -70$ mm $= -0.070$ m

$$I_1 = \tfrac{1}{12}(20)(60)^3 + (1200)(20)^2 = 840\times10^3 \text{ mm}^4$$

$$I_2 = \tfrac{1}{12}(60)(20)^3 + (1200)(20)^2 = 520\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1.360\times10^6 \text{ mm}^4 = 1.360\times10^{-6} \text{ m}^4, \quad A = 2400\times10^{-6} \text{ m}^2$$

(a) Based on stress at left edge of section: $y = -50$ mm $= -0.050$ m

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = K_L P$$

$$K_L = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400\times10^{-6}} - \frac{(-0.070)(-0.050)}{1.360\times10^{-6}} = -2.1569\times10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K_L} = \frac{-150\times10^6}{-2.1569\times10^3} = 69.6\times10^3 \text{ N}$$

Based on stress at right edge of section: $y = 30$ mm $= 0.030$ m

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = K_R P$$

$$K_R = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400\times10^{-6}} - \frac{(-0.070)(0.030)}{1.360\times10^{-6}} = 1.9608\times10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{150\times10^6}{1.9608\times10^3} = 76.5\times10^3 \text{ N}$$

Choose the smaller value $P = 69.6\times10^3$ N $= 69.6$ kN ◄

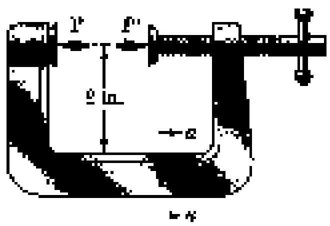
(b) Location of neutral axis: $\sigma = 0$

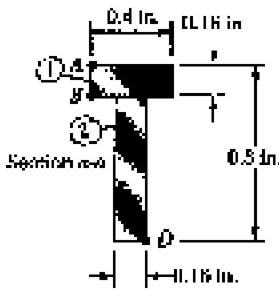
$$\sigma = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{1.360\times10^{-6}}{(2400\times10^{-6})(-0.070)} = -8.10\times10^{-3} \text{ m} = -8.10 \text{ mm}$$

Neutral axis lies $50 - 8.10 = 41.9$ mm from left edge. ◄

PROBLEM 4.133





4.133 Knowing that the clamp shown has been tightened until $P = 75$ lb, determine in section a-a (*a*) the stress at point A, (*b*) the stress at point , (*c*) the location of the neutral axis.

SOLUTION

Locate centroid

Part	A, in^2	$\bar{y}_o$, in	$A\bar{y}_o$, in^3
①	0.064	0.72	0.04608
②	0.1024	0.32	0.03277
Σ	0.1664		0.07885

$$\bar{Y}_o = \frac{\Sigma A\bar{y}_o}{\Sigma A} = \frac{0.07885}{0.1664} = 0.4739 \text{ in.}$$

The centroid lies 0.4739 in. above point D.

Bending couple $\quad M = Pe$

$$e = -(2 + 0.8 - 0.4739) = -2.3261 \text{ in}$$

$$I_1 = \tfrac{1}{12}(0.4)(0.16)^3 + (0.064)(0.72 - 0.4739)^2 = 4.013 \times 10^{-3} \text{ in}^4$$

$$I_2 = \tfrac{1}{12}(0.16)(0.64)^3 + (0.1024)(0.4739 - 0.32)^2 = 5.921 \times 10^{-3} \text{ in}^4$$

$$I = I_1 + I_2 = 9.934 \times 10^{-3} \text{ in}^4$$

(a) Stress at point A: $\quad y = 0.8 - 0.4739 = 0.3261$ in

$$\sigma_A = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = \frac{75}{0.1664} - \frac{(75)(-2.3261)(0.3261)}{9.934 \times 10^{-3}}$$

$$= 6.18 \times 10^3 \text{ psi} = 6.18 \text{ ksi}$$

(b) Stress at point D: $\quad y = -0.4789$ in. $= 0.1661$ in

$$\sigma_D = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = \frac{75}{0.1664} - \frac{(75)(-2.3261)(-0.4739)}{9.934 \times 10^{-3}}$$

$$= -7.87 \times 10^3 \text{ psi} = -7.87 \text{ ksi}$$

(c) Location of neutral axis $\qquad \sigma = 0$

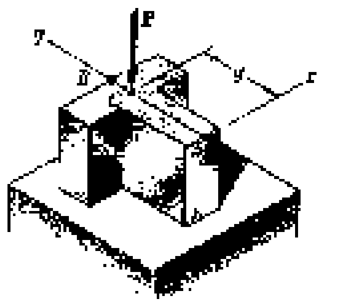
$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

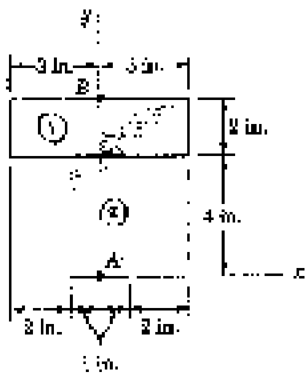
$$y = \frac{I}{Ae} = \frac{9.934 \times 10^{-4}}{(0.1664)(-2.3261)} = -0.0257 \text{ in}$$

The neutral axis lies $0.4789 - 0.0257 = 0.448$ in. above point D.

PROBLEM 4.134

4.134 A vertical force **P** of magnitude 20 kips is applied at a point C located on the line of symmetry of the cross section of a short column. Knowing that $y = 5$ in., determine (*a*) the stress at point A, (*b*) the stress at point B, (*c*) the location of the neutral axis.





(*b*)

SOLUTION

Locate centroid

Part	A, in²	$\bar{y}$, in	$A\bar{y}$, in³
①	12	5	60
②	8	2	16
Σ	20		76

$$\bar{y} = \frac{\Sigma A\bar{y}_i}{\Sigma A_i} = \frac{76}{20} = 3.8 \text{ in}$$

Eccentricity of load $e = 5 - 3.8 = 1.2$ in.

$$I_1 = \tfrac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4$$

$$I_2 = \tfrac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

(a) Stress at A $\quad c_A = 3.8$ in

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = -\frac{20}{20} + \frac{(20)(1.2)(3.8)}{57.867} = 0.576 \text{ ksi}$$

(b) Stress at B $\quad c_B = 6 - 3.8 = 2.2$ in

$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = -\frac{20}{20} - \frac{(20)(1.2)(2.2)}{57.867} = -1.912 \text{ ksi}$$

(c) Location of neutral axis: $\quad \sigma = 0$

$$\sigma = -\frac{P}{A} + \frac{Pea}{I} = 0 \quad \therefore \quad \frac{ea}{I} = \frac{1}{A}$$

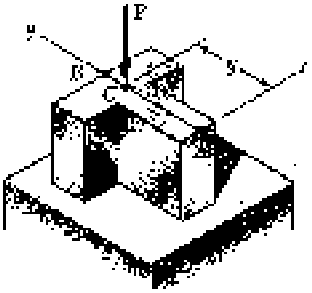
$$a = \frac{I}{Ae} = \frac{57.867}{(20)(1.2)} = 2.411 \text{ in}$$

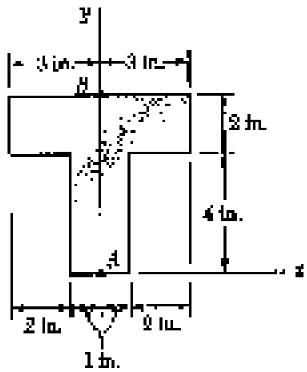
Neutral axis lies 2.411 in. below centroid or $3.8 - 2.411$
$= 1.389$ in above point A.

Answer 1.389 in from point A

PROBLEM 4.135

4.135 A vertical force **P** is applied at a point C located on the line of symmetry of the cross section of a short column. Determine the range of values of y for which tensile stresses do not occur in the column.





SOLUTION

Locate centroid

	A, in^2	$\bar{y}$, in	$A\bar{y}$, in^3
①	12	5	60
②	8	2	16
Σ	20		76

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{76}{20} = 3.8 \text{ in}$$

Eccentricity of load $\quad e = y - 3.8$ in.

$$y = e + 3.8 \text{ in.}$$

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4$$

$$I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

If stress at A equals zero. $\quad c_A = 3.8$ in

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = 0 \quad \therefore \quad \frac{ec_A}{I} = \frac{1}{A}$$

$$e = \frac{I}{Ac_A} = \frac{57.867}{(20)(3.8)} = 0.761 \text{ in} \qquad y = 0.761 + 3.8 = 4.561 \text{ in.}$$

If stress at B equals zero. $\quad c_B = 6 - 3.8 = 2.2$ in

$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = 0 \quad \therefore \quad \frac{ec_B}{I} = -\frac{1}{A}$$

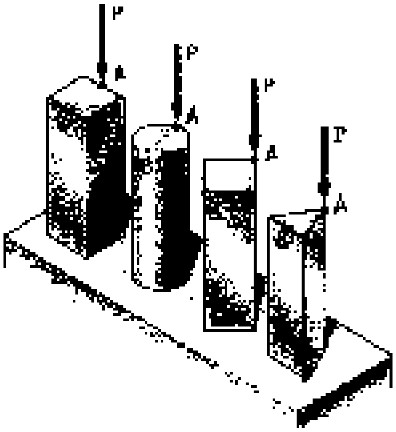
$$e = -\frac{I}{Ac_B} = -\frac{57.867}{(20)(2.2)} = -1.315 \text{ in}$$

$$y = -1.315 + 3.8 = 2.485 \text{ in.}$$

Answer: $2.485 \text{ in} < y < 4.561 \text{ in.}$

PROBLEM 4.136

4.136 The four bars shown have the same cross-sectional area. For the given loadings, show that (*a*) the maximum compressive stresses are in the ratio 4:5:7:9, (*b*) the maximum tensile stresses are in the ratio 2:3:5:3. (*Note:* the cross section of the triangular bar is an equilateral triangle.)



SOLUTION

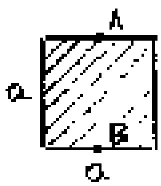
Stresses

At A $\quad \sigma_A = -\frac{P}{A} - \frac{Pec_A}{I}$

$= -\frac{P}{A}\left(1 + \frac{Aec_A}{I}\right)$

At B $\quad \sigma_B = -\frac{P}{A} + \frac{Pec_B}{I}$

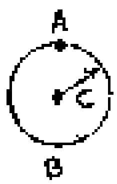
$= \frac{P}{A}\left(\frac{Aec_B}{I} - 1\right)$



$A_1 = a^2, \quad I_1 = \frac{1}{12}a^4, \quad c_A = c_B = \frac{1}{2}a, \quad e = \frac{1}{2}a$

$\sigma_A = -\frac{P}{A}\left(1 + \frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12}a^4}\right) = -4\frac{P}{A_1}$

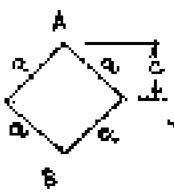
$\sigma_B = \frac{P}{A}\left(\frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12}a^4} - 1\right) = 2\frac{P}{A_1}$



$A_2 = \pi c^2 = a^2 \quad \therefore c = \frac{a}{\sqrt{\pi}}, \quad I_2 = \frac{\pi}{4}c^4$

$\sigma_A = -\frac{P}{A_2}\left(1 + \frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4}\right) = -5\frac{P}{A_2}$

$\sigma_B = \frac{P}{A_2}\left(\frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4} - 1\right) = 3\frac{P}{A_2}$



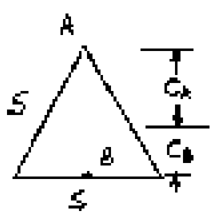
$A_3 = a^2 \qquad c = \frac{\sqrt{2}}{2}a \qquad I_3 = \frac{1}{12}a^4 \qquad e = c$

$\sigma_A = -\frac{P}{A_3}\left(1 + \frac{(a^2)(\frac{\sqrt{2}}{2}a)(\frac{\sqrt{2}}{2}a)}{\frac{1}{12}a^4}\right) = -7\frac{P}{A_3}$

$\sigma_B = \frac{P}{A_3}\left(\frac{(a^2)(\frac{\sqrt{2}}{2}a)(\frac{\sqrt{2}}{2}a)}{\frac{1}{12}a^4} - 1\right) = 5\frac{P}{A_3}$

$A_4 = \frac{1}{2}(s)(\frac{\sqrt{3}}{2}s) = \frac{\sqrt{3}}{4}s^2 \qquad I_4 = \frac{1}{36}s(\frac{\sqrt{3}}{2}s)^3 = \frac{\sqrt{3}}{96}s^4$

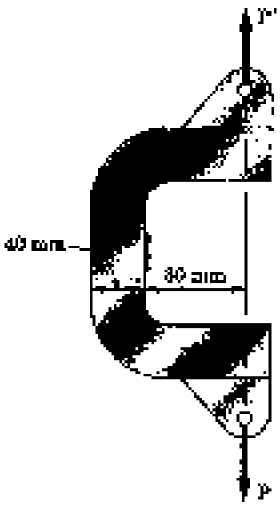
$c_A = \frac{2}{3}\frac{\sqrt{3}}{2}s = \frac{s}{\sqrt{3}} = e \qquad c_B = \frac{s}{2\sqrt{3}}$



$\sigma_A = -\frac{P}{A_4}\left(1 + \frac{(\frac{\sqrt{3}}{4}s^2)(\frac{s}{\sqrt{3}})(\frac{s}{\sqrt{3}})}{\frac{\sqrt{3}}{96}s^4}\right) = -9\frac{P}{A_4}$

$\sigma_B = \frac{P}{A_4}\left(\frac{(\frac{\sqrt{3}}{4}s^2)(\frac{s}{\sqrt{3}})(\frac{s}{2\sqrt{3}})}{\frac{\sqrt{3}}{96}s^4} - 1\right) = 3\frac{P}{A_4}$

PROBLEM 4.137



4.137 The C-shaped steel bar is used as a dynamometer to determine the magnitude P of the forces shown. Knowing that the cross section of the bar is a square of side 40 mm and that strain on the inner edge was measured and found to be 450 μ, determine the magnitude P of the forces. Use $E = 200$ GPa.

SOLUTION

At the strain gage location

$$\sigma = E\varepsilon = (200\times10^{9})(450\times10^{-6}) = 90\times10^{6}$$

$$A = (40)(40) = 1600\ \text{mm}^2 = 1600\times10^{-6}\ \text{m}^2$$

$$I = \frac{1}{12}(40)(40)^3 = 213.33\times10^{3}\ \text{mm}^4 = 213.33\times10^{-9}\ \text{m}^4$$

$$e = 80 + 20 = 100\ \text{mm} = 0.100\ \text{m}$$

$$c = 20\ \text{mm} = 0.020\ \text{m}$$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{1600\times10^{-6}} + \frac{(0.100)(0.020)}{213.33\times10^{-9}} = 10.00\times10^{3}\ \text{m}^{-2}$$

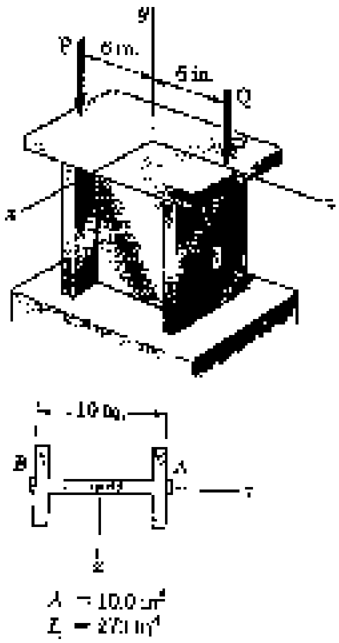
$$P = \frac{\sigma}{K} = \frac{90\times10^{6}}{10.00\times10^{3}} = 9.00\times10^{3}\ \text{N} = 9.00\ \text{kN}$$

PROBLEM 4.138

4.138 A short length of a rolled-steel column supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points *A* and *B* on the center lines of the outer faces of the flanges have been measured and found to be

$\varepsilon_A = -400 \times 10^{-6}$ in./in. $\qquad \varepsilon_B = -300 \times 10^{-6}$ in./in.

Knowing that $E = 29 \times 10^6$ psi, determine the magnitude of each load.



SOLUTION

Stresses at A and B from strain gages

$$\sigma_A = E\varepsilon_A = (29 \times 10^6)(-400 \times 10^{-6}) = -11.6 \times 10^3 \text{ psi}$$

$$\sigma_B = E\varepsilon_B = (29 \times 10^6)(-300 \times 10^{-6}) = -8.7 \times 10^3 \text{ psi}$$

Centric force $\quad F = P + Q$

Bending couple $\quad M = 6P - 6Q$

$c = 5$ in.

$$\sigma_A = -\frac{F}{A} + \frac{Mc}{I} = -\frac{P+Q}{10.0} + \frac{(6P - 6Q)(5)}{273}$$

$$-11.6 \times 10^3 = +0.00989\,P - 0.20989\,Q \qquad (1)$$

$$\sigma_B = -\frac{F}{A} - \frac{Mc}{I} = -\frac{P+Q}{10.0} - \frac{(6P - 6Q)(5)}{273}$$

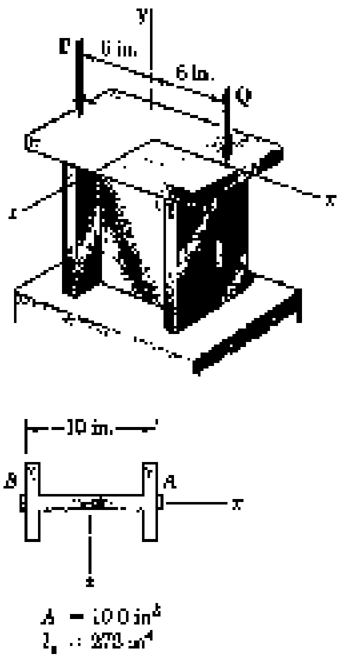
$$-8.7 \times 10^3 = -0.20989\,P + 0.00989\,Q \qquad (2)$$

Solving (1) and (2) simultaneously

$$P = 44.2 \times 10^3 \text{ lb} = 44.2 \text{ kips}$$

$$Q = 57.3 \times 10^3 \text{ lb} = 57.3 \text{ kips}$$

PROBLEM 4.139



4.138 A short length of a rolled-steel column supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points A and B on the center lines of the outer faces of the flanges have been measured and found to be

$\epsilon_A = -400 \times 10^{-6}$ in./in. $\qquad \epsilon_B = -300 \times 10^{-6}$ in./in.

Knowing that $E = 29 \times 10^6$ psi, determine the magnitude of each load.

4.139 Solve Prob. 4.138, assuming that the measured strains are

$\epsilon_A = -350 \times 10^{-6}$ in./in. $\qquad \epsilon_B = -50 \times 10^{-6}$ in./in.

SOLUTION

Stresses at A and B from strain gages

$$\sigma_A = E\epsilon_A = (29\times10^6)(-350\times10^{-6}) = -10.15\times10^3 \text{ psi}$$

$$\sigma_B = E\epsilon_B = (29\times10^6)(-50\times10^{-6}) = -1.45\times10^3 \text{ psi}$$

Centric force $\quad F = P + Q$

Bending couple $\quad M = 6P - 6Q$

$c = 5$ in

$$\sigma_A = -\frac{F}{A} + \frac{Mc}{I} = -\frac{P+Q}{10.0} + \frac{(6P-6Q)(5)}{273}$$

$$-10.15\times10^3 = 0.00989\,P - 0.20989\,Q \qquad (1)$$

$$\sigma_B = -\frac{F}{A} - \frac{Mc}{I} = -\frac{P+Q}{10.0} - \frac{(6P-6Q)(5)}{273}$$

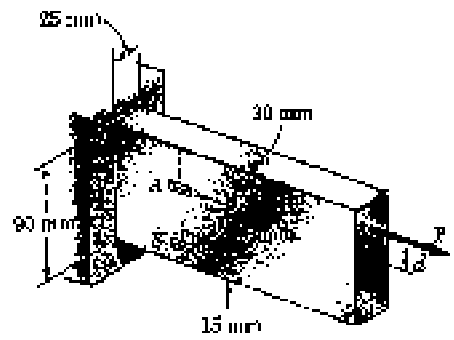
$$-1.45\times10^3 = -0.20989\,P + 0.00989\,Q \qquad (2)$$

Solving (1) and (2) simultaneously

$P = 9.21\times10^3$ lb $= 9.21$ kips ◄

$Q = 48.8\times10^3$ lb. $= 48.8$ kips ◄

PROBLEM 4.140



4.140 An eccentric axial force **P** is applied as shown to a steel bar of 25 × 90-mm cross section. The strains at A and B have been measured and found to be

$$\epsilon_A = +350\ \mu \qquad \epsilon_B = -70\ \mu$$

Knowing that $E = 200$ GPa, determine (*a*) the distance d, (*b*) the magnitude of the force **P**.

SOLUTION

$h = 15 + 45 + 30 = 90$ mm

$b = 25$ mm $\qquad c = \frac{1}{2}h = 45\text{ mm} = 0.045$ m

$A = bh = (25)(90) = 2.25\times10^3\ \text{mm}^2 = 2.25\times10^{-3}\ \text{m}^2$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875\times10^6\ \text{mm}^4$

$= 1.51875\times10^{-6}\ \text{m}^4$

$y_A = 60 - 45 = 15\text{ mm} = 0.015\text{ m}, \quad y_B = 15 - 45 = -30\text{ mm} = -0.030\text{ m}$

Stresses from strain gages at A and B

$\sigma_A = E\epsilon_A = (200\times10^9)(350\times10^{-6}) = 70\times10^6$ Pa

$\sigma_B = E\epsilon_B = (200\times10^9)(-70\times10^{-6}) = -14\times10^6$ Pa

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \qquad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \qquad (2)$$

Subtracting $\qquad \sigma_A - \sigma_B = -\frac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875\times10^{-6})(84\times10^6)}{0.045} = -2835\ \text{N}\cdot\text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting

$$y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\frac{P}{A}$$

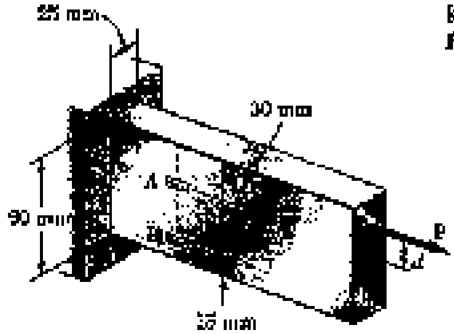
$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25\times10^{-3})[(0.015)(-14\times10^6) - (-0.030)(70\times10^6)]}{0.045}$$

$= 94.5\times10^3$ N

(a) $\quad M = -Pd \quad \therefore \quad d = -\frac{M}{P} = -\frac{-2835}{94.5\times10^3} = 0.030\text{ m} = 30\text{ mm}$

(b) $\qquad P = 94.5$ kN.m

PROBLEM 4.141



4.140 An eccentric axial force **P** is applied as shown to a steel bar of 25 × 90-mm cross section. The strains at *A* and *B* have been measured and found to be

$\epsilon_A = +350\ \mu \qquad \epsilon_B = -70\ \mu$

Knowing that $E = 200$ GPa, determine (*a*) the distance *d*, (*b*) the magnitude of the force **P**.

4.141 Solve Prob. 4.140, assuming that the measured strains are

$\epsilon_A = +600\ \mu \qquad \epsilon_B = +420\ \mu$

SOLUTION

$$h = 15 + 45 + 30 = 90 \text{ mm}$$

$$b = 25 \text{ mm} \qquad c = \tfrac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh = (25)(90) = 2.25\times10^3 \text{ mm}^2 = 2.25\times10^{-3} \text{ m}^2$$

$$I = \tfrac{1}{12}bh^3 = \tfrac{1}{12}(25)(90)^3 = 1.51875\times10^6 \text{ mm}^4 = 1.51875\times10^{-6} \text{ m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m}, \qquad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gages at A and B

$$\sigma_A = E\varepsilon_A = (200\times10^9)(600\times10^{-6}) = 120\times10^6 \text{ Pa}$$

$$\sigma_B = E\varepsilon_B = (200\times10^9)(420\times10^{-6}) = 84\times10^6 \text{ Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \qquad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \qquad (2)$$

Subtracting $\qquad \sigma_A - \sigma_B = -\dfrac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875\times10^{-6})(36\times10^6)}{0.045} = -1215 \text{ N}\cdot\text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting

$$y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\frac{P}{A}$$

$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25\times10^{-3})[(0.015)(84\times10^6) - (-0.030)(120\times10^6)]}{0.045}$$

$$= 243\times10^3 \text{ N}$$

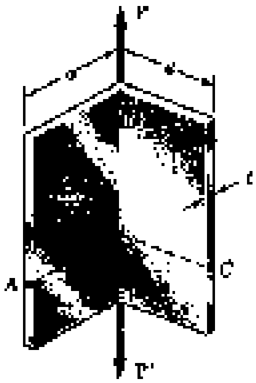
$$M = -Pd$$

(a) $\qquad \therefore\ d = -\dfrac{M}{P} = -\dfrac{-1215}{243\times10^3} = 5\times10^{-3} \text{ m} = 5 \text{ mm}$ ◄

(b) $\qquad P = 243 \text{ kN}$ ◄

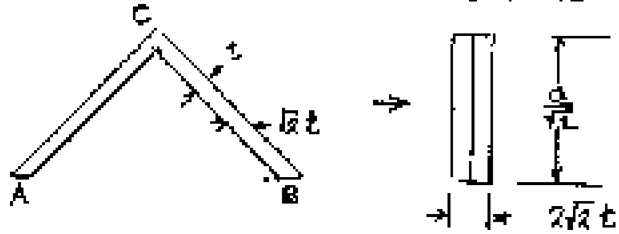
PROBLEM 4.142

4.142 The shape shown was formed by bending a thin steel plate. Assuming that the thickness t is small compared to the length a of a side of the shape, determine the stress (*a*) at A, (*b*) at B, (*c*) at C.



SOLUTION

Moment of inertia about centroid



$$I = \frac{1}{12}(2\sqrt{2}t)\left(\frac{a}{\sqrt{2}}\right)^3 = \frac{1}{12}ta^3$$

Area $\quad A = (2\sqrt{2}t)\left(\frac{a}{\sqrt{2}}\right) = 2at \qquad c = \frac{a}{2\sqrt{2}}$

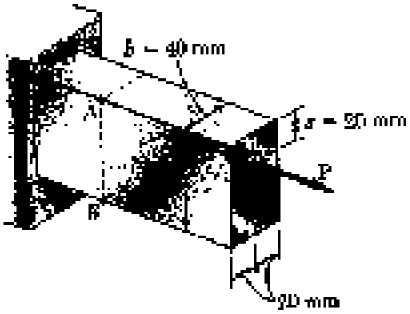
(a) $\quad \sigma_A = \frac{P}{A} - \frac{Pec}{I} = \frac{P}{2at} - \frac{P\left(\frac{a}{2\sqrt{2}}\right)\left(\frac{a}{2\sqrt{2}}\right)}{\frac{1}{12}ta^3} = -\frac{P}{2at}$

(b) $\quad \sigma_B = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{2at} + \frac{P\left(\frac{a}{2\sqrt{2}}\right)\left(\frac{a}{2\sqrt{2}}\right)}{\frac{1}{12}ta^3} = \frac{2P}{at}$

(c) $\quad \sigma_C = \sigma_A = -\frac{P}{2at}$

PROBLEM 4.143

4.143 The eccentric axial force **P** acts at point *D*, which must be located 25 mm below the top surface of the steel bar shown. For $P = 60$ kN, determine (*a*) the depth *d* of the bar for which the tensile stress at point *A* is maximum, (*b*) the corresponding stress at point *A*.



SOLUTION

$$A = bd \qquad I = \tfrac{1}{12} b d^3$$

$$c = \tfrac{1}{2} d \qquad e = \tfrac{1}{2} d - a$$

$$\sigma_A = \frac{P}{A} + \frac{Pec}{I}$$

$$\sigma_A = \frac{P}{b}\left\{\frac{1}{d} + \frac{12\left(\frac{1}{2}d - a\right)\left(\frac{1}{2}d\right)}{d^3}\right\} = \frac{P}{b}\left\{\frac{4}{d} - \frac{6a}{d^2}\right\}$$

(a) Depth d for maximum σ_A. Differentiate with respect to d.

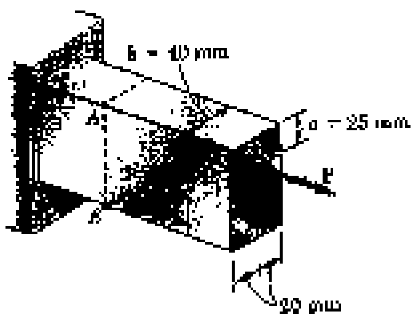
$$\frac{d\sigma_A}{dd} = \frac{P}{b}\left\{-\frac{4}{d^2} + \frac{12a}{d^3}\right\} = 0 \qquad d = 3a = 75 \text{ mm}$$

(b) $$\sigma_A = \frac{60\times10^3}{40\times10^{-3}}\left\{\frac{4}{75\times10^{-3}} - \frac{(6)(25\times10^{-3})}{(75\times10^{-3})^2}\right\} = 40\times10^6 \text{ Pa} = 40 \text{ MPa}$$

PROBLEM 4.144

4.143 The eccentric axial force **P** acts at point *D*, which must be located 25 mm below the top surface of the steel bar shown. For $P = 60$ kN, determine (*a*) the depth *d* of the bar for which the tensile stress at point *A* is maximum, (*b*) the corresponding stress at point *A*.

4.144 For the bar and loading of Prob. 4.143, determine (*a*) the depth *d* of the bar for which the compressive stress at point *B* is maximum, (*b*) the corresponding stress at point *B*.



SOLUTION

$$A = bd \qquad I = \tfrac{1}{12} b d^3$$

$$c = \tfrac{1}{2} d \qquad e = \tfrac{1}{2} d - a$$

$$\sigma_B = \frac{P}{A} - \frac{Pec}{I}$$

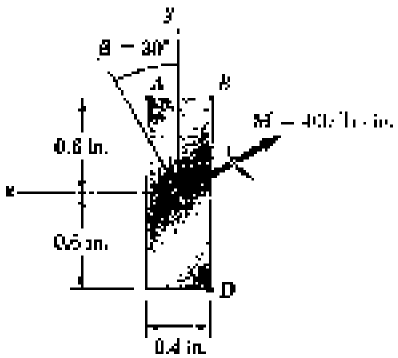
$$\sigma_B = \frac{P}{b}\left\{\frac{1}{d} - \frac{(12)\left(\frac{1}{2}d - a\right)\left(\frac{1}{2}d\right)}{d^3}\right\} = \frac{P}{b}\left\{-\frac{2}{d} + \frac{6a}{d^2}\right\}$$

(a) Depth d for maximum σ_B: Differentiate with respect to d

$$\frac{d\sigma_B}{dd} = \frac{P}{b}\left\{\frac{2}{d^2} - \frac{12a}{d^3}\right\} = 0 \qquad d = 6a = 150 \text{ mm}$$

(b) $$\sigma_B = \frac{60\times10^3}{40\times10^{-3}}\left\{-\frac{2}{150\times10^{-3}} + \frac{(6)(25\times10^{-3})}{(150\times10^{-3})^2}\right\} = -10\times10^6 \text{ Pa} = -10 \text{ MPa}$$

PROBLEM 4.145



4.145 through 4.147 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.

SOLUTION

$$I_z = \frac{1}{12}(0.4)(1.2)^3 = 57.6\times10^{-3}\ \text{in}^4$$

$$I_y = \frac{1}{12}(1.2)(0.4)^3 = 6.40\times10^{-3}\ \text{in}^4$$

$$y_A = y_B = -y_D = 0.6\ \text{in}$$

$$z_A = -z_B = -z_D = \left(\tfrac{1}{2}\right)(0.4) = 0.2\ \text{in.}$$

$$M_y = 400\cos 60^\circ = 200\ \text{lb·in}, \quad M_z = -400\sin 60^\circ = -346.41\ \text{lb·in}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-346.41)(0.6)}{57.6\times10^{-3}} + \frac{(200)(0.2)}{6.40\times10^{-3}}$$

$$= 9.86\times10^3\ \text{psi} = 9.86\ \text{ksi}$$

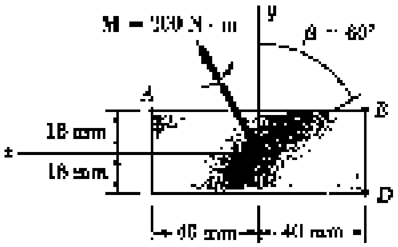
(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-346.41)(0.6)}{57.6\times10^{-3}} + \frac{(200)(-0.2)}{6.4\times10^{-3}}$$

$$= -2.64\times10^3\ \text{psi} = -2.64\ \text{ksi}$$

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-346.41)(-0.6)}{57.6\times10^{-3}} + \frac{(200)(-0.2)}{6.40\times10^{-3}}$$

$$= -9.86\times10^3\ \text{psi} = -9.86\ \text{ksi}$$

PROBLEM 4.146



4.145 through 4.147 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point D.

SOLUTION

$$I_z = \frac{1}{12}(80)(32)^3 = 218.45\times10^3\ \text{mm}^4 = 218.45\times10^{-9}\ \text{m}^4$$

$$I_y = \frac{1}{12}(32)(80)^3 = 1.36533\times10^6\ \text{mm}^4 = 1.36533\times10^{-6}\ \text{m}^4$$

$$y_A = y_B = -y_D = 16\ \text{mm}$$

$$z_A = -z_B = -z_D = 40\ \text{mm}$$

$$M_y = 300\cos 30° = 259.81\ \text{N}\cdot\text{m},\quad M_z = 300\sin 30° = 150\ \text{N}\cdot\text{m}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(150)(16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(40\times10^{-3})}{1.36533\times10^{-6}}$$

$$= -3.37\times10^6\ \text{Pa} = -3.37\ \text{MPa}$$

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(150)(16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(-40\times10^{-3})}{1.36533\times10^{-6}}$$

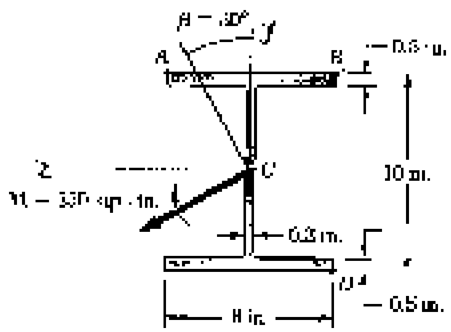
$$= -18.60\times10^6\ \text{Pa} = -18.60\ \text{MPa}$$

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(150)(-16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(-40\times10^{-3})}{1.36533\times10^{-6}}$$

$$= 3.37\times10^6\ \text{Pa} = 3.37\ \text{MPa}$$

PROBLEM 4.147

4.145 through 4.147 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point D.



SOLUTION

Flange: $I_z = \frac{1}{12}(8)(0.5)^3 + (8)(0.5)(4.75)^2$

$= 90.333 \text{ in}^4$

$I_y = \frac{1}{12}(0.5)(8)^3 = 21.333 \text{ in}^4$

Web: $I_z = \frac{1}{12}(0.3)(9)^3 = 18.225 \text{ in}^4$

$I_y = \frac{1}{12}(9)(0.3)^3 = 0.02025 \text{ in}^4$

Total: $I_z = (2)(90.333) + 18.225 = 198.89 \text{ in}^4$

$I_y = (2)(21.333) + 0.02025 = 42.687 \text{ in}^4$

$y_A = y_B = -y_D = 5 \text{ in.} \quad ; \quad z_A = -z_B = -z_D = 4 \text{ in.}$

$M_z = 250 \cos 30° = 216.51 \text{ kip-in}$

$M_y = -250 \sin 30° = -125 \text{ kip-in}$

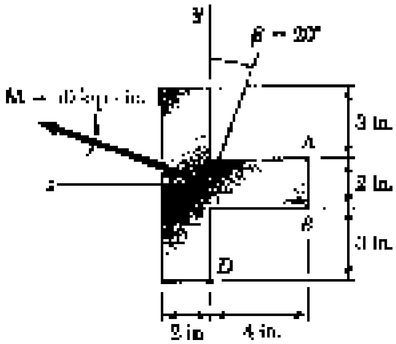
(a) $\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(216.51)(5)}{198.89} + \frac{(-125)(4)}{42.687} = -17.16 \text{ ksi}$ ◄

(b) $\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(216.51)(5)}{198.89} + \frac{(-125)(-4)}{42.687} = 6.27 \text{ ksi}$ ◄

(c) $\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(216.51)(-5)}{198.89} + \frac{(-125)(4)}{42.687} = 17.16 \text{ ksi}$ ◄

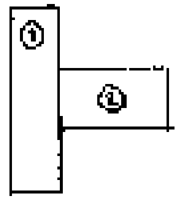
PROBLEM 4.148

4.148 through 4.150 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point *D*.



SOLUTION

Locate centroid



	A, in^2	$\bar{z}, \text{in}$	$A\bar{z}, \text{in}^3$
①	16	−1	−16
②	8	2	16
Σ	24		0

The centroid lies at point C

$$I_z = \frac{1}{12}(2)(8)^3 + \frac{1}{12}(4)(2)^3 = 88 \text{ in}^4$$

$$I_y = \frac{1}{3}(8)(2)^3 + \frac{1}{3}(2)(4)^3 = 64 \text{ in}^4$$

$$y_A = -y_B = 1 \text{ in}, \qquad y_D = -4 \text{ in}$$

$$z_A = z_B = -4 \text{ in}, \qquad z_D = 0$$

$$M_z = 10 \cos 20° = 9.3969 \text{ kip-in}$$

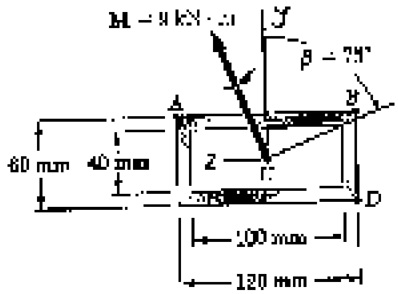
$$M_y = 10 \sin 20° = 3.4202 \text{ kip-in.}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(9.3969)(1)}{88} + \frac{(3.4202)(4)}{64} = 0.321 \text{ ksi}$$ ◄

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(9.3969)(-1)}{88} + \frac{(3.4202)(4)}{64} = -0.107 \text{ ksi}$$ ◄

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(9.3969)(-4)}{88} + \frac{(3.4202)(0)}{64} = 0.427 \text{ ksi}$$ ◄

PROBLEM 4.149



4.148 through 4.150 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point D.

SOLUTION

$$I_z = \frac{1}{12}(120)(60)^3 - \frac{1}{12}(100)(40)^3 = 1.62667 \times 10^6 \text{ mm}^4$$
$$= 1.62667 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(60)(120)^3 - \frac{1}{12}(40)(100)^3 = 5.3067 \times 10^6 \text{ mm}^4$$
$$= 5.3067 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = 30 \text{ mm}$$

$$z_A = -z_B = -z_D = 60 \text{ mm}$$

$$M_z = (9 \times 10^3) \sin 15^\circ = 2.3294 \times 10^3 \text{ N}\cdot\text{m}$$

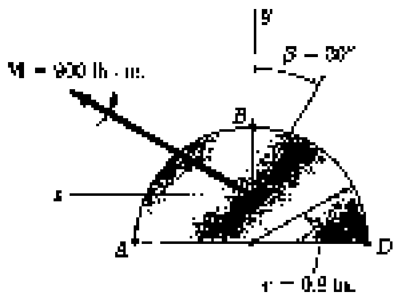
$$M_y = (9 \times 10^3) \cos 15^\circ = 8.6933 \times 10^3 \text{ N}\cdot\text{m}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(2.3294 \times 10^3)(30 \times 10^{-3})}{1.62667 \times 10^{-6}} + \frac{(8.6933 \times 10^3)(60 \times 10^{-3})}{5.3067 \times 10^{-6}}$$
$$= 55.3 \times 10^6 \text{ Pa} = 55.3 \text{ MPa}$$

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(2.3294 \times 10^3)(30 \times 10^{-3})}{1.62667 \times 10^{-6}} + \frac{(8.6933 \times 10^3)(-60 \times 10^{-3})}{5.3067 \times 10^{-6}}$$
$$= -141.2 \times 10^6 \text{ Pa} = -141.2 \text{ MPa}$$

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(2.3294 \times 10^3)(-30 \times 10^{-3})}{1.62667 \times 10^{-6}} + \frac{(8.6933 \times 10^3)(-60 \times 10^{-3})}{5.3067 \times 10^{-6}}$$
$$= -55.3 \times 10^6 \text{ Pa} = -55.3 \text{ MPa}$$

PROBLEM 4.150



4.148 through 4.150 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (a) point A, (b) point B, (c) point D.

SOLUTION

$$I_z = \frac{\pi}{8}r^4 - \left(\frac{\pi}{2}r^2\right)\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$$

$$= (0.109757)(0.8)^4 = 44.956\times10^{-3}\ \text{in}^4$$

$$I_y = \frac{\pi}{8}r^4 = \frac{\pi}{8}(0.8)^4 = 160.85\times10^{-3}\ \text{in}^4$$

$$y_A = y_D = -\frac{4r}{3\pi} = -\frac{(4)(0.8)}{3\pi} = -0.33953\ \text{in}$$

$$y_B = 0.8 - 0.33953 = 0.46047\ \text{in}$$

$$z_A = -z_D = 0.8\ \text{in.}, \qquad z_B = 0$$

$$M_y = 900\sin 30^\circ = 450\ \text{lb}\cdot\text{in}$$

$$M_z = 900\cos 30^\circ = 779.42\ \text{lb}\cdot\text{in.}$$

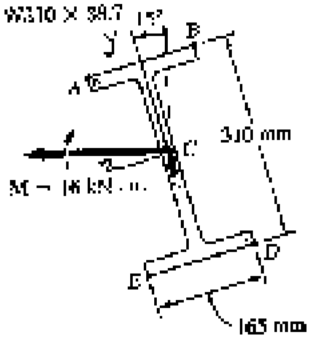
(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(779.42)(-0.33953)}{44.956\times10^{-3}} + \frac{(450)(0.8)}{160.85\times10^{-3}} = 8.12\times10^3\ \text{psi}$$
$$= 8.12\ \text{ksi}$$

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(779.42)(0.46047)}{44.956\times10^{-3}} + \frac{(450)(0)}{160.85\times10^{-3}} = -7.98\times10^3\ \text{psi}$$
$$= -7.98\ \text{ksi}$$

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(779.42)(-0.33953)}{44.956\times10^{-3}} + \frac{(450)(-0.8)}{160.85\times10^{-3}} = 3.65\times10^3\ \text{psi}$$
$$= 3.65\ \text{ksi}$$

PROBLEM 4.151

4.151 through 4.153 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum tensile stress in the beam.



SOLUTION

For W 310 × 38.7 rolled steel shape

$$I_z = 85.1 \times 10^6 \text{ mm}^4 = 85.1 \times 10^{-6} \text{ m}^4$$

$$I_y = 7.27 \times 10^6 \text{ mm}^4 = 7.27 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = -y_E = (\tfrac{1}{2})(310) = 155 \text{ mm}$$

$$z_A = z_E = -z_B = -z_D = (\tfrac{1}{2})(165) = 82.5 \text{ mm}$$

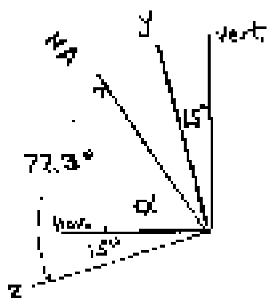
$$M_z = (16 \times 10^3) \cos 15° = 15.455 \times 10^3 \text{ N}\cdot\text{m}$$

$$M_y = (16 \times 10^3) \sin 15° = 4.1411 \times 10^3 \text{ N}\cdot\text{m}$$

(a) $$\tan \varphi = \frac{I_z}{I_y} \tan \theta = \frac{85.1 \times 10^{-6}}{7.27 \times 10^{-6}} \tan 15° = 3.1365$$

$$\varphi = 72.3°$$

$$\alpha = 72.3 - 15 = 57.3°$$



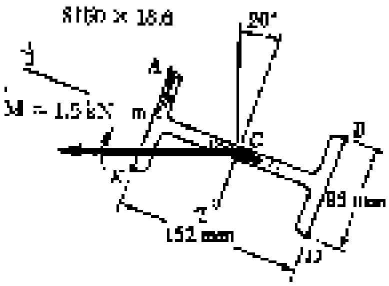
(b) Maximum tensile stress occurs at point E

$$\sigma_E = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(15.455 \times 10^3)(-155 \times 10^{-3})}{85.1 \times 10^{-6}} + \frac{(4.1411 \times 10^3)(82.5 \times 10^{-3})}{7.27 \times 10^{-6}}$$

$$= 75.1 \times 10^6 \text{ Pa} = 75.1 \text{ MPa}$$

PROBLEM 4.152

4.151 through 4.153 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum tensile stress in the beam.



SOLUTION

For S150 × 18.6 rolled steel shape

$I_z = 9.11 \times 10^6 \text{ mm}^4 = 9.11 \times 10^{-6} \text{ m}^4$

$I_y = 0.782 \times 10^6 \text{ mm}^4 = 0.782 \times 10^{-6} \text{ m}^4$

$z_E = -z_A = -z_B = z_D = (\frac{1}{2})(85) = 42.5 \text{ mm}$

$y_A = y_B = -y_D = -y_E = (\frac{1}{2})(152) = 76 \text{ mm}$

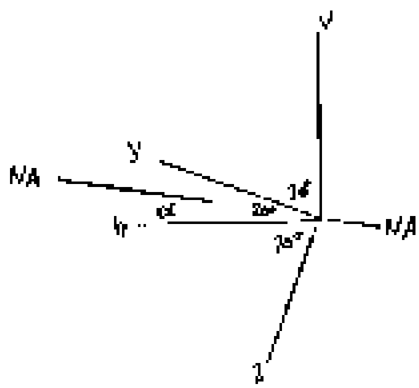
$M_z = (1.5 \times 10^3) \sin 20° = 0.51303 \times 10^3 \text{ N·m}$

$M_y = (1.5 \times 10^3) \cos 20° = 1.4095 \times 10^3 \text{ N·m}$

(a) $\tan \varphi = \frac{I_z}{I_y} \tan \theta = \frac{9.11 \times 10^{-6}}{0.782 \times 10^{-6}} \tan (90° - 20°) = 32.007$

$\varphi = 88.21$

$\alpha = 88.21 - 70° = 18.21°$



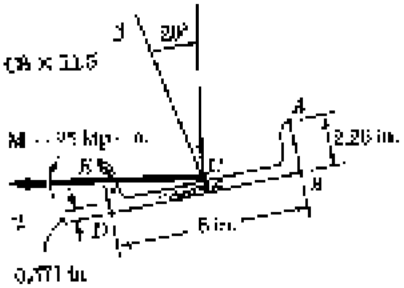
(b) Maximum tensile stress occurs at point D

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(0.51343 \times 10^3)(-76 \times 10^{-3})}{9.11 \times 10^{-6}} + \frac{(1.4095 \times 10^3)(42.5 \times 10^{-3})}{0.782 \times 10^{-6}}$$

$$= 80.9 \times 10^6 \text{ Pa} = 80.9 \text{ MPa}$$

PROBLEM 4.153

4.151 through 4.153 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine (a) the angle that the neutral axis forms with the horizontal plane, (b) the maximum tensile stress in the beam.



SOLUTION

For C8×11.5 rolled steel shape

$I_z = 1.32 \text{ in}^4 \quad , \quad I_y = 32.6 \text{ in}^4$

$z_E = z_D = 0.4 \text{ in} \quad , \quad z_B = z_A = -0.4 \text{ in}$

$y_D = y_B = -0.571 \text{ in.}$

$y_E = y_A = 2.26 - 0.571 = 1.689 \text{ in.}$

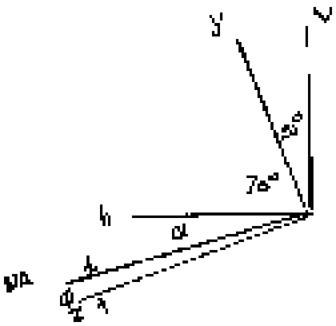
$M_y = 25 \sin 20° = 8.5505 \text{ kip·in}$

$M_z = 25 \cos 20° = 23.492 \text{ kip·in.}$

(a) $\tan \varphi = \dfrac{I_z}{I_y} \tan \theta = \dfrac{1.32}{32.6} \tan 20° = 0.014737$

$\varphi = 0.844°$

$\alpha = 20 - 0.844 = 19.16°$

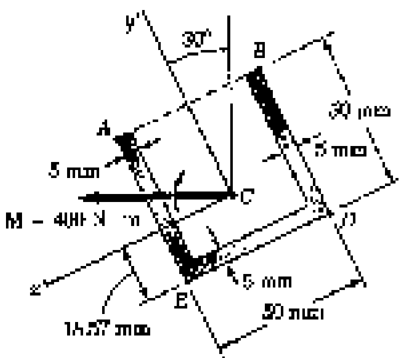


(b) Maximum tensile stress occurs at point D

$\sigma_D = -\dfrac{M_z y_D}{I_z} + \dfrac{M_y z_D}{I_y} = -\dfrac{(23.492)(-0.571)}{1.32} + \dfrac{(8.5505)(4)}{32.6}$

$= 10.162 + 1.049 = 11.21 \text{ ksi}$

PROBLEM 4.154



$I_{y'} = 281 \times 10^3\ mm^4$
$I_{z'} = 176.9 \times 10^3\ mm^4$

4.154 through 4.156 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum tensile stress in the beam.

SOLUTION

$I_{z'} = 176.9 \times 10^3\ mm^4 = 176.9 \times 10^{-9}\ m^4$

$I_{y'} = 281 \times 10^3\ mm^4 = 281 \times 10^{-9}\ m^4$

$y_E' = -18.57\ mm, \quad z_E = 25\ mm$

$M_{z'} = 400 \cos 30° = 346.41\ N \cdot m$

$M_{y'} = 400 \sin 30° = 200\ N \cdot m$

(a) $\tan \varphi = \dfrac{I_{z'}}{I_{y'}} \tan \theta = \dfrac{176.9 \times 10^{-9}}{281 \times 10^{-9}} \cdot \tan 30° = 0.36346$

$\varphi = 19.97°$

$\alpha = 30° - 19.97° = 10.03°$

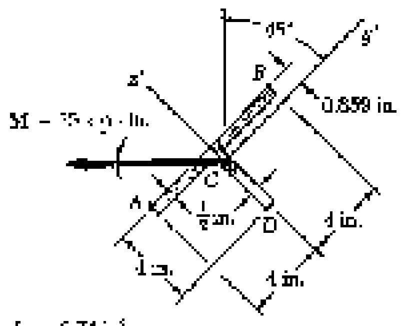
(b) Maximum tensile stress occurs at point E

$$\sigma_b = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(346.41)(-18.57 \times 10^{-3})}{176.9 \times 10^{-9}} + \frac{(200)(25 \times 10^{-3})}{281 \times 10^{-9}}$$

$= 36.36 \times 10^6 + 17.79 \times 10^6 = 54.2 \times 10^6\ Pa$

$= 54.2\ MPa$

PROBLEM 4.155



$I_y = 6.74$ in^4
$I_z = 21.4$ in^4

4.154 through 4.156 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum tensile stress in the beam.

SOLUTION

$I_{z'} = 21.4 \text{ in}^4, \quad I_{y'} = 6.74 \text{ in}^4$

$z'_A = z'_B = 0.859 \text{ in}$

$z_C = -4 + 0.859 \text{ in} = -3.141 \text{ in}$

$y_A = -4 \text{ in}, \quad y_B = 4 \text{ in}, \quad y_D = -0.25 \text{ in}$

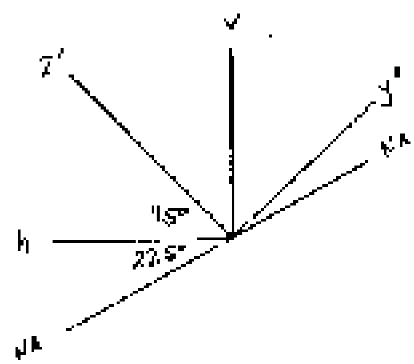
$M_{y'} = -25 \sin 45° = -17.678 \text{ kip}\cdot\text{in}$

$M_{z'} = 25 \cos 45° = 17.678 \text{ kip}\cdot\text{in}$

(a) $\tan\varphi = \frac{I_{z'}}{I_{y'}} \tan\theta = \frac{21.4}{6.74} \tan(-45°) = 3.1751$

$\varphi = -72.5°$

$\alpha = 72.5° - 45° = 27.5°$



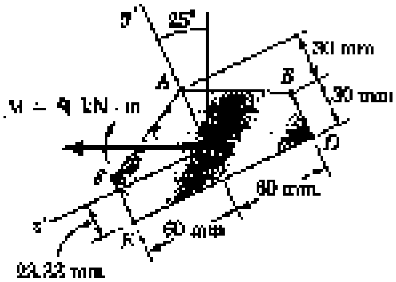
(b) Maximum tensile stress occurs at point D.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(17.678)(-0.25)}{21.4} + \frac{(-17.678)(-3.141)}{6.74}$$

$$= 0.2065 + 8.238 = 8.44 \text{ ksi}$$

PROBLEM 4.156

4.154 through 4.156 The couple M acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum tensile stress in the beam.



SOLUTION

$$I_z = \frac{1}{36}(120)(30)^3 + \left(\frac{1}{2}\right)(120)(30)(40-23.33)^2$$
$$+ \frac{1}{12}(120)(30)^3 + (120)(30)(23.33-15)^2$$
$$= 1.11\times10^6\ \text{mm}^4 = 1.11\times10^{-6}\ \text{m}^4$$

$$I_y = 2\left\{\frac{1}{12}(30)(20)^3 + \frac{1}{3}(30)(20)^3\right\} = 5.40\times10^6\ \text{mm}^4 = 5.40\times10^{-6}\ \text{m}^4$$

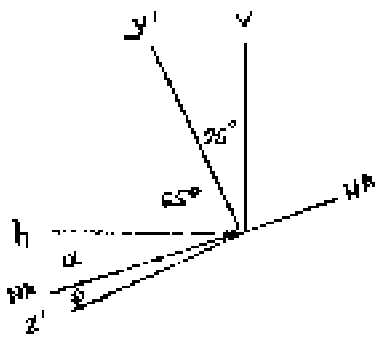
$y_E = -23.33$ mm $\qquad z_E = 60$ mm

$$M_{z'} = (4\times10^3)\cos25^\circ = 3.6252\times10^3\ \text{N}\cdot\text{m}$$
$$M_{y'} = (4\times10^3)\sin25^\circ = 1.6905\times10^3\ \text{N}\cdot\text{m}$$

(a) $\tan\varphi = \frac{I_{z'}}{I_{y'}}\tan\theta = \frac{1.11\times10^{-6}}{5.40\times10^{-6}}\tan25^\circ = 0.095822$

$\varphi = 5.475^\circ$

$\alpha = 25^\circ - 5.475^\circ = 19.52^\circ$



(b) Maximum tensile stress occurs at point E

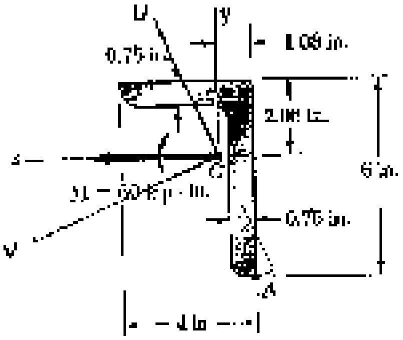
$$\sigma_E = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(3.6252\times10^3)(-23.33\times10^{-3})}{1.11\times10^{-6}} + \frac{(1.6905\times10^3)(60\times10^{-3})}{5.40\times10^{-6}}$$

$$= 76.195\times10^6 + 18.783\times10^6 = 95.0\times10^6\ \text{Pa}$$

$= 95.0$ MPa

PROBLEM 4.157

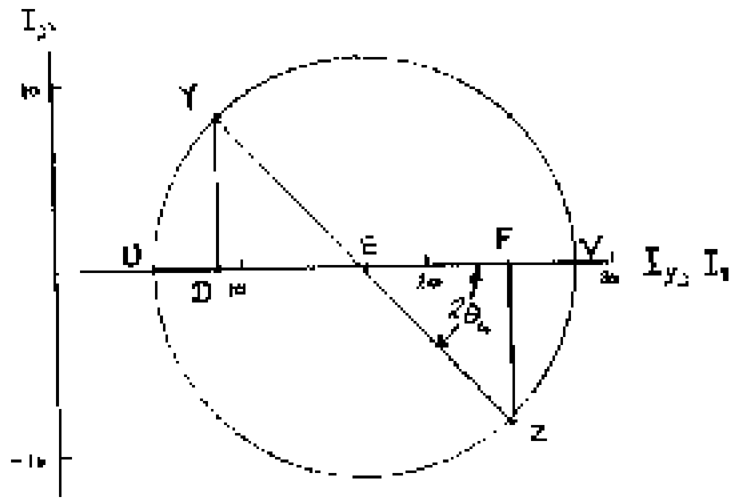
*4.157 and 4.158 The couple M acts in a vertical plane and is applied to a beam of the cross section shown. Determine the stress at point A.



$I_y = 8.7$ in^4
$I_z = 24.5$ in^4
$I_{yz} = 8.3$ in^4

SOLUTION

Using Mohr's circle, determine the principal axes and principal moments of inertia.



Y (8.7, 8.3) in^4
Z (24.5, −8.3) in^4
E (16.6, 0) in^4

$EF = 7.9$ in^4
$FZ = 8.3$ in^4

$R = \sqrt{7.9^2 + 8.3^2} = 11.46$ in^4 $\qquad \tan 2\theta_m = \frac{FZ}{EF} = \frac{8.3}{7.9} = 1.0506$

$\theta_m = 23.2°$ $\qquad I_u = 16.6 - 11.46 = 5.14$ in^4, $\quad I_v = 16.6 + 11.46 = 28.06$ in^4

$M_u = M \sin\theta_m = (60)\sin 23.2° = 23.64$ kip·in

$M_v = M\cos\theta_m = (60)\cos 23.2° = 55.15$ kip·in

$u_A = y_A\cos\theta_m + z_A\sin\theta_m = -3.92\cos 23.2° + 1.08\sin 23.2° = -4.03$ in.

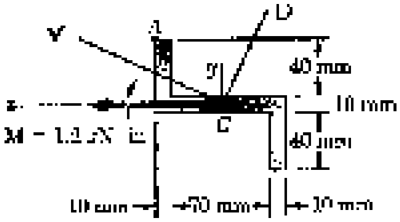
$v_A = z_A\cos\theta_m - y_A\sin\theta_m = -1.08\cos 23.2° + 3.92\sin 23.2° = 0.552$ in.

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(55.15)(-4.03)}{28.06} + \frac{(23.64)(0.552)}{5.14}$$

$= 10.46$ ksi

PROBLEM 4.158

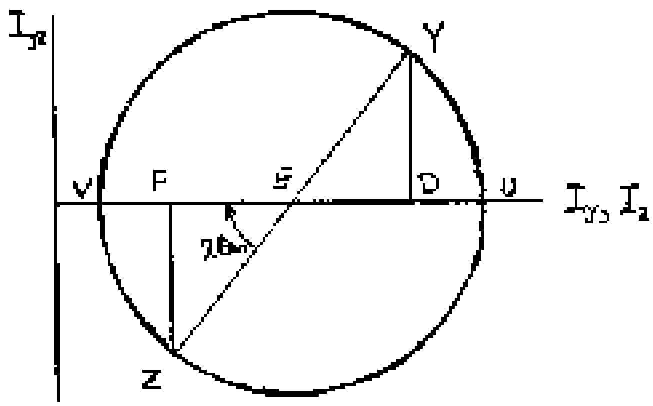
*4.157 and 4.158 The couple **M** acts in a vertical plane and is applied to a beam of the cross section shown. Determine the stress at point A.



$I_y = 1.894 \times 10^6\ \text{mm}^4$
$I_z = 0.614 \times 10^6\ \text{mm}^4$
$I_{yz} = -0.800 \times 10^6\ \text{mm}^4$

SOLUTION

Using Mohr's circle, determine the principal axes and the principal moments of inertia.



$Y\ (1.894,\ 0.800) \times 10^6\ \text{mm}^4$
$Z\ (0.614,\ 0.800) \times 10^6\ \text{mm}^4$
$E\ (1.254,\ 0\ ) \times 10^6\ \text{mm}^4$

$$R = \sqrt{\overline{EF}^2 + \overline{FZ}^2} = \sqrt{0.640^2 + 0.800^2} \times 10^{-6} = 1.0245 \times 10^6\ \text{mm}^4$$

$$I_v = (1.254 - 1.0245) \times 10^6\ \text{mm}^4 = 0.2295 \times 10^6\ \text{mm}^4 = 0.2295 \times 10^{-6}\ \text{m}^4$$

$$I_u = (1.254 + 1.0245) \times 10^6\ \text{mm}^4 = 2.2785 \times 10^6\ \text{mm}^4 = 2.2785 \times 10^{-6}\ \text{m}^4$$

$$\tan 2\theta_m = \frac{FZ}{FE} = \frac{0.800 \times 10^6}{0.640 \times 10^6} = 1.25 \qquad \theta_m = 25.67°$$

$$M_v = M \cos\theta_m = (1.2 \times 10^3)\cos 25.67° = 1.0816 \times 10^3\ \text{N}\cdot\text{m}$$

$$M_u = -M \sin\theta_m = -(1.2 \times 10^3)\sin 25.67° = -0.5198 \times 10^3\ \text{N}\cdot\text{m}$$

$$u_A = y_A \cos\theta_m - z_A \sin\theta_m = 45 \cos 25.67° - 45 \sin 25.67° = 21.07\ \text{mm}$$

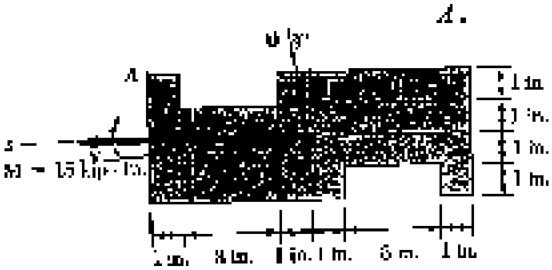
$$v_A = z_A \cos\theta_m + y_A \sin\theta_m = 45 \cos 25.67° + 45 \sin 25.67° = 60.05\ \text{mm}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(1.0816 \times 10^3)(21.07 \times 10^{-3})}{0.2295 \times 10^{-6}} + \frac{(-0.5198 \times 10^3)(60.05 \times 10^{-3})}{2.2785 \times 10^{-6}}$$

$$= 113.0 \times 10^6\ \text{Pa} = 113.0\ \text{MPa}$$

PROBLEM 4.159

*4.159 A 4 × 10-in. timber has been trimmed to form a beam of the cross section shown. Knowing that the couple **M** acts in a vertical plane, determine the stress at point *A*.



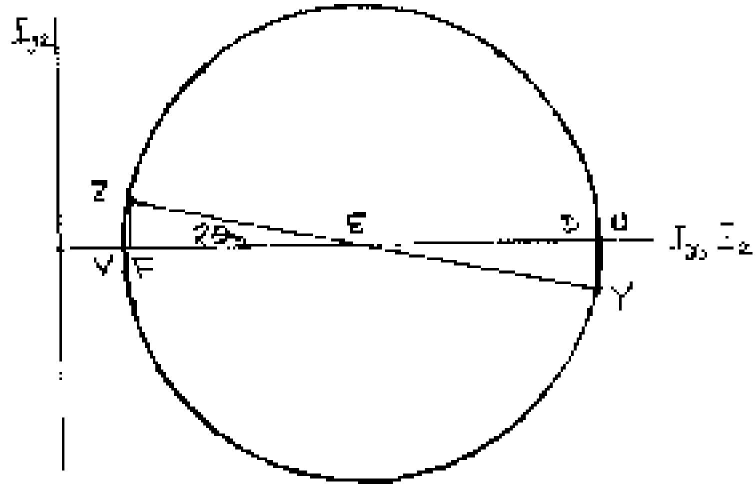
$I_y = 291\ \text{in}^4$

$I_z = 39.3\ \text{in}^4$

$I_{yz} = -22.5\ \text{in}^4$

SOLUTION

Using Mohr's circle determine the principal axes and principal moments of inertia.



Y $(291, -22.5)$ in⁴

Z $(39.3, 22.5)$ in⁴

E $(165.15, 0)$ in⁴

$$\tan 2\theta_m = \frac{FZ}{EF} = \frac{22.5}{125.85} = 0.17878$$

$$\theta_m = 5.07°$$

$$R = \sqrt{EF^2 + FZ^2} = \sqrt{125.85^2 + 22.5^2} = 127.85\ \text{in}^4$$

$$I_v = 165.15 - 127.85 = 37.30\ \text{in}^4$$

$$I_u = 165.15 + 127.85 = 293.0\ \text{in}^4$$

$$u_A = y_A \cos\theta_m + z_A \sin\theta_m = 2\cos 5.07° + 5\sin 5.07° = 2.434\ \text{in}$$

$$v_A = z_A \cos\theta_m - y_A \sin\theta_m = 5\cos 5.07° - 2\sin 5.07° = 4.804\ \text{in}$$

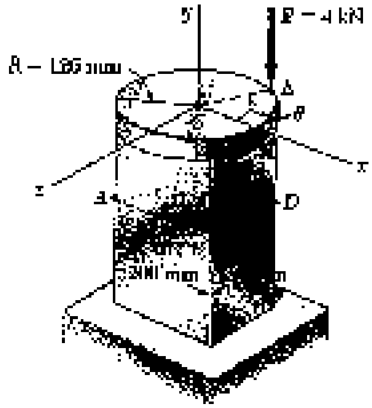
$$M_v = 15\cos 5.07° = 14.94\ \text{kip}\cdot\text{in}$$

$$M_u = 15\sin 5.07° = 1.326\ \text{kip}\cdot\text{in}$$

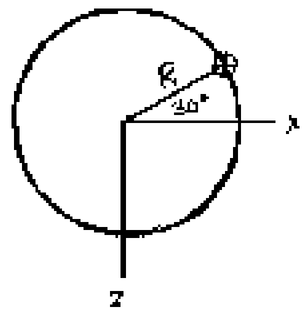
$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(14.94)(2.434)}{37.30} + \frac{(1.326)(4.804)}{293.0} = -0.953\ \text{ksi}$$

PROBLEM 4.160

4.160 A rigid plate of 125-mm diameter is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at *E* with $\theta = 30°$, determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the point where the neutral axis intersects line *ABD*.



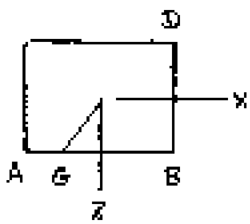
SOLUTION



$P = 4 \times 10^3$ N (compression)

$M_x = -PR \sin 30°$
$= -(4\times10^3)(125\times10^{-3}) \sin 30°$
$= -250$ N·m

$M_z = -PR \cos 30°$
$= -(4\times10^3)(125\times10^{-3}) \cos 30°$
$= -433$ N·m



$I_x = \frac{1}{12}(200)(150)^3 = 56.25\times10^6 \text{ mm}^4 = 56.25\times10^{-6} \text{ m}^4$

$I_z = \frac{1}{12}(150)(200)^3 = 100\times10^6 \text{ mm}^4 = 100\times10^{-6} \text{ m}^4$

$-x_A = x_B = 100$ mm

$z_A = z_B = 75$ mm

$A = (200)(150) = 30\times10^3 \text{ mm}^2 = 30\times10^{-3} \text{ m}^2$

(a) $\sigma_A = -\frac{P}{A} + \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4\times10^3}{30\times10^{-3}} - \frac{(-250)(75\times10^{-3})}{56.25\times10^{-6}} + \frac{(-433)(-100\times10^{-3})}{100\times10^{-6}}$

$= 633\times10^3$ Pa $= 633$ kPa ◄

(b) $\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -\frac{4\times10^3}{30\times10^{-3}} - \frac{(-250)(75\times10^{-3})}{56.25\times10^{-6}} + \frac{(-433)(100\times10^{-3})}{100\times10^{-6}}$

$= -233\times10^3$ Pa $= -233$ kPa ◄

(c) Let G be the point on AB where neutral axis intersects.

$\sigma_G = 0 \qquad z_G = 75 \text{ mm} \qquad x_G = ?$

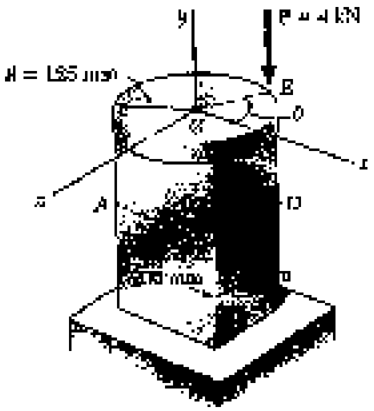
$\sigma_G = -\frac{P}{A} - \frac{M_x z_G}{I_x} + \frac{M_z x_G}{I_z} = 0$

$x_G = \frac{I_z}{M_z}\left\{\frac{P}{A} + \frac{M_x z_G}{I_x}\right\} = \frac{100\times10^{-6}}{-433}\left\{\frac{4\times10^3}{30\times10^{-3}} + \frac{(-250)(75\times10^{-3})}{56.25\times10^{-6}}\right\}$

$= 46.2\times10^{-3}$ m $= 46.2$ mm

Point G lies 146.2 mm from point A ◄

PROBLEM 4.161

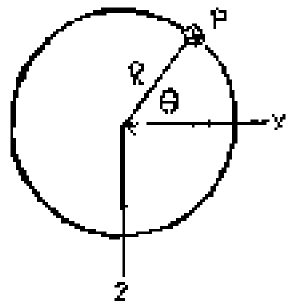


4.160 A rigid plate of 125-mm diameter is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at *E* with θ = 30°, determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the point where the neutral axis intersects line *ABD*.

4.161 In Prob. 4.160, determine (*a*) the value of θ for which the stress at *D* reaches its largest value, (*b*) the corresponding values of the stress at *A*, *B*, *C*, and *D*.

SOLUTION

(a)

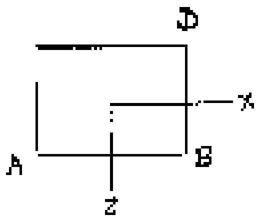


$P = 4\times10^3$ N

$PR = (4\times10^3)(125\times10^{-3}) = 500$ N·m

$M_x = -PR\sin\theta = -500\sin\theta$

$M_z = -PR\cos\theta = -500\cos\theta$



$$I_x = \frac{1}{12}(200)(150)^3 = 56.25\times10^6\ \text{mm}^4 = 56.25\times10^{-6}\ \text{m}^4$$

$$I_z = \frac{1}{12}(150)(200)^3 = 100\times10^6\ \text{mm}^4 = 100\times10^{-6}\ \text{m}^4$$

$$x_D = 100\ \text{mm} \qquad z_D = -75\ \text{mm}$$

$$A = (200)(150) = 30\times10^3\ \text{mm}^2 = 30\times10^{-3}\ \text{m}^2$$

$$\sigma = -\frac{P}{A} - \frac{M_x z}{I_x} + \frac{M_z x}{I_z} = -P\left\{\frac{1}{A} - \frac{Rz\sin\theta}{I_x} + \frac{Rx\cos\theta}{I_z}\right\}$$

For σ to be a maximum $\frac{d\sigma}{d\theta} = 0$ with $z = z_D$, $x = x_D$

$$\frac{d\sigma_D}{d\theta} = -P\left\{0 + \frac{Rz_D\cos\theta}{I_x} + \frac{Rx_D\sin\theta}{I_z}\right\} = 0$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = -\frac{I_z z_D}{I_x x_D} = -\frac{(100\times10^{-6})(-75\times10^{-3})}{(56.25\times10^{-6})(100\times10^{-3})} = \frac{4}{3}$$

$$\sin\theta = 0.8, \qquad \cos\theta = 0.6, \qquad \theta = 53.1°$$

(b) $$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4\times10^3}{30\times10^{-3}} + \frac{(500)(0.8)(75\times10^{-3})}{56.25\times10^{-6}} - \frac{(500)(0.6)(-100\times10^{-3})}{100\times10^{-6}}$$

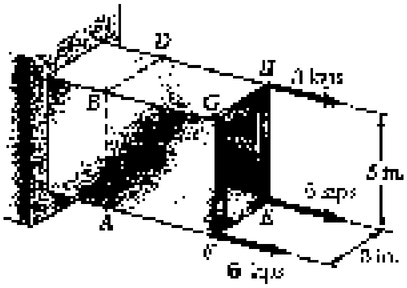
$$= (-0.13333 + 0.53333 + 0.300)\times10^6\ \text{Pa} = 0.700\times10^6\ \text{Pa} = 700\ \text{kPa}$$

$$\sigma_B = (-0.13333 + 0.53333 - 0.300)\times10^6\ \text{Pa} = 0.100\times10^6\ \text{Pa} = 100\ \text{kPa}$$

$$\sigma_C = (-0.13333 + 0 + 0)\times10^6\ \text{Pa} = -133.3\ \text{kPa}$$

$$\sigma_D = (-0.13333 - 0.53333 - 0.300)\times10^6\ \text{Pa} = -0.967\times10^6\ \text{Pa} = -967\ \text{kPa}$$

PROBLEM 4.162



4.162 The tube shown has a uniform wall thickness of 0.5 in. For the given loading, determine (*a*) the stress at points *A* and *B*, (*b*) the point where the neutral axis intersects line *ABD*.

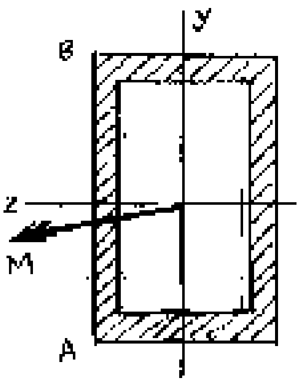
SOLUTION

Add y- and z- axes as shown

$$I_z = \frac{1}{12}(3)(5)^3 - \frac{1}{12}(2)(4)^3 = 20.583 \text{ in}^4$$

$$I_y = \frac{1}{12}(5)(3)^3 - \frac{1}{12}(4)(2)^3 = 8.5833 \text{ in}^4$$

$$A = (3)(5) - (2)(4) = 7.0 \text{ in}^2$$



Resultant force and bending couples

$$P = 3 + 6 + 6 = 15 \text{ kips}$$

$$M_z = -(2.5)(3) + (2.5)(6) + (2.5)(6) = 22.5 \text{ kip}\cdot\text{in}$$

$$M_y = -(1.5)(3) - (1.5)(6) + (1.5)(6) = -4.5 \text{ kip}\cdot\text{in}$$

(a) $$\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{15}{7} - \frac{(22.5)(-2.5)}{20.583} + \frac{(-4.5)(1.5)}{8.5833} = 4.09 \text{ ksi}$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{15}{7} - \frac{(22.5)(2.5)}{20.583} + \frac{(-4.5)(1.5)}{8.5833} = -1.376 \text{ ksi}$$

(b) Let point H be the point where the neutral axis intersects AB.

$$z_H = 1.5, \quad y_H = ?, \quad \sigma_H = 0$$

$$0 = \frac{P}{A} - \frac{M_z y_H}{I_z} + \frac{M_y z_H}{I_y}$$

$$y_H = \frac{I_z}{M_z}\left(\frac{P}{A} + \frac{M_y z_H}{I_y}\right) = \frac{20.583}{22.5}\left\{\frac{15}{7} + \frac{(-4.5)(1.5)}{8.5833}\right\} = 1.241 \text{ in.}$$

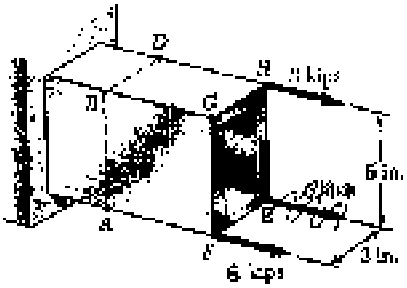
$$2.5 + 1.241 = 3.741 \text{ in.}$$

Answer: 3.741 in. above point A.

PROBLEM 4.163

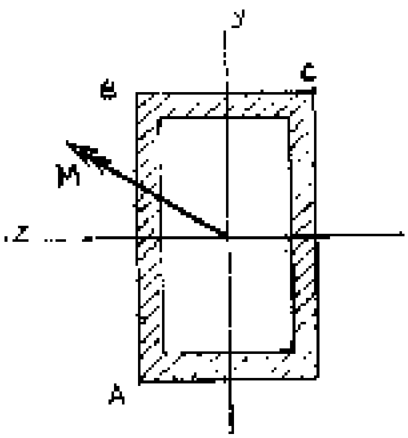
4.162 The tube shown has a uniform wall thickness of 0.5 in. For the given loading, determine (*a*) the stress at points *A* and *B*, (*b*) the point where the neutral axis intersects line *ABD*.

4.163 Solve Prob. 4.162, assuming that the 6-kip force at point *E* is removed.



SOLUTION

Add y- and z-axes as shown.



$$I_z = \frac{1}{12}(3)(5)^3 - \frac{1}{12}(2)(4)^3 = 20.583 \text{ in}^4$$

$$I_y = \frac{1}{12}(5)(3)^3 - \frac{1}{12}(4)(2)^3 = 8.5833 \text{ in}^4$$

$$A = (3)(5) - (2)(4) = 7 \text{ in}^2$$

Resultant force and couples

$$P = 3 + 6 = 9 \text{ kips}$$

$$M_z = -(2.5)(3) + (2.5)(6) = 7.5 \text{ kip}\cdot\text{in.}$$

$$M_y = -(1.5)(3) + (1.5)(6) = 4.5 \text{ kip}\cdot\text{in.}$$

(a) $$\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{9}{7} - \frac{(7.5)(-2.5)}{20.583} + \frac{(4.5)(1.5)}{8.5833} = 2.98 \text{ ksi}$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{9}{7} - \frac{(7.5)(2.5)}{20.583} + \frac{(4.5)(1.5)}{8.5833} = 1.161 \text{ ksi}$$

(b) Let point K be the point where the neutral axis intersects BC.

$$y_K = 2.5 \text{ in}, \quad z_K = ?, \quad \sigma_K = 0$$

$$0 = \frac{P}{A} - \frac{M_z y_K}{I_z} + \frac{M_y z_K}{I_y}$$

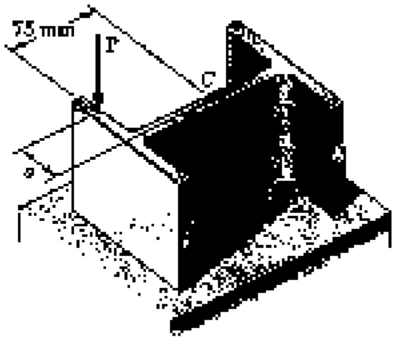
$$z_K = \frac{I_y}{M_y}\left(\frac{M_z y_K}{I_z} - \frac{P}{A}\right) = \frac{8.5833}{4.5}\left\{\frac{(7.5)(2.5)}{20.583} - \frac{9}{7}\right\} = -0.715 \text{ in.}$$

$$1.5 + 0.715 = 2.215 \text{ in}$$

Answer: 2.215 in. to the right of point B.

PROBLEM 4.164

4.164 An axial load **P** of magnitude 50 kN is applied as shown to a short section of a W 150 × 24 rolled-steel member. Determine the largest distance a for which the maximum compressive stress does not exceed 90 MPa.



SOLUTION

Add y- and z- axes.

For W 150 × 24 rolled-steel section

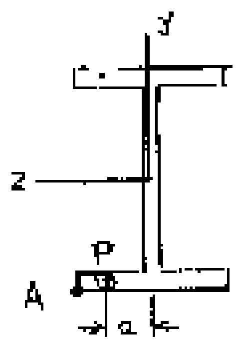
$A = 3060\ mm^2 = 3060 \times 10^{-6}\ m^2$

$I_z = 13.4 \times 10^6\ mm^4 = 13.4 \times 10^{-6}\ m^4$

$I_y = 1.83 \times 10^6\ mm^4 = 1.83 \times 10^{-6}\ m^4$

$d = 160\ mm, \quad b_f = 102\ mm$

$y_A = -\frac{d}{2} = -80\ mm, \quad z_A = \frac{b_f}{2} = 51\ mm$



$P = 50 \times 10^3\ N$

$M_z = -(50 \times 10^3)(75 \times 10^{-3}) = -3.75 \times 10^3\ N\cdot m$

$M_y = -Pa$

$$\sigma_A = -\frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} \qquad \sigma_A = -90 \times 10^6\ Pa$$

$$M_y = \frac{I_y}{z_A}\left\{\frac{M_z y_A}{I_z} + \frac{P}{A} + \sigma_A\right\}$$

$$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}}\left\{\frac{(-3.75 \times 10^3)(-80 \times 10^{-3})}{13.4 \times 10^{-6}} + \frac{50 \times 10^3}{3060 \times 10^{-6}} + (-90 \times 10^6)\right\}$$

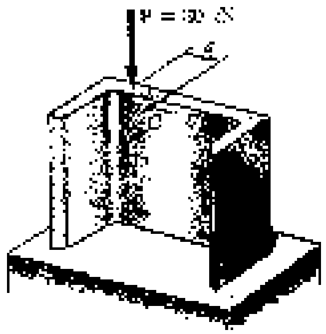
$$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}}\left\{+22.388 + 16.340 - 90\right\} \times 10^6$$

$$= -1.8398 \times 10^3\ N\cdot m$$

$$a = -\frac{M_y}{P} = -\frac{-1.8398 \times 10^3}{50 \times 10^3} = 36.8 \times 10^{-3}\ m = 36.8\ mm$$

PROBLEM 4.165

4.165 An axial load **P** of magnitude 30 kN is applied as shown to a short section of a C 150 × 12.2 rolled-steel channel. Determine the largest distance a for which the maximum compressive stress is 60 MPa.



SOLUTION

Add y- and z- axes as shown

For C 150 × 12.2 rolled steel section

$A = 1540\ mm^2 = 1540\times10^{-6}\ m^2$

$d = 152\ mm$

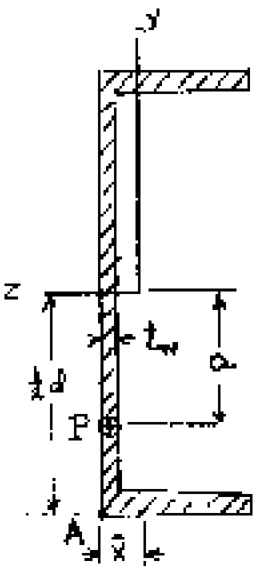
$b_f = 48\ mm$

$t_w = 5.1\ mm$

$I_z = 5.35\times10^6\ mm^4 = 5.35\times10^{-6}\ m^4$

$I_y = 0.276\times10^6\ mm^4 = 0.276\times10^{-6}\ m^4$

$\bar{x} = 12.7\ mm$



Line of action of force P

$y_P = -a \qquad z_P = \bar{x} - \frac{1}{2}t_w = 10.15\ mm$

$P = 30\times10^3\ N$

$M_y = -Pz_P = -(30\times10^3)(10.15\times10^{-3}) = -304.5\ N\cdot m$

$M_z = -Pa \qquad \sigma_A = -60\times10^6\ Pa$

$y_A = -\frac{1}{2}d = -76\ mm \qquad z_A = \bar{x} = 12.7\ mm$

$$\sigma_A = -\frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$M_z = \frac{I_z}{y_A}\left\{\frac{M_y z_A}{I_y} - \frac{P}{A} - \sigma_A\right\}$$

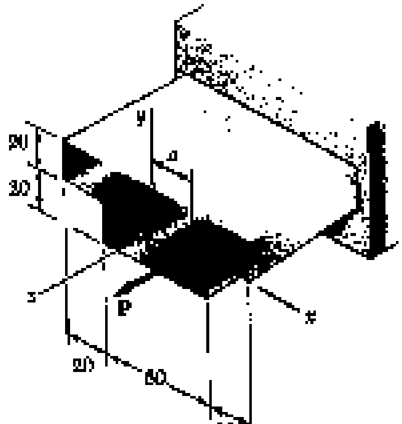
$$= \frac{5.35\times10^{-6}}{-76\times10^{-3}}\left\{\frac{(-304.5)(12.7\times10^{-3})}{0.276\times10^{-6}} - \frac{30\times10^3}{1540\times10^{-6}} + 60\times10^6\right\}$$

$$= \frac{5.35\times10^{-6}}{76\times10^{-3}}\left\{-14.011 - 19.481 + 60\right\}\times10^6 = -1.866\times10^3\ N\cdot m$$

$$a = -\frac{M_z}{P} = -\frac{(-1.866\times10^3)}{30\times10^3} = 62.2\times10^{-3}\ m = 62.2\ mm$$

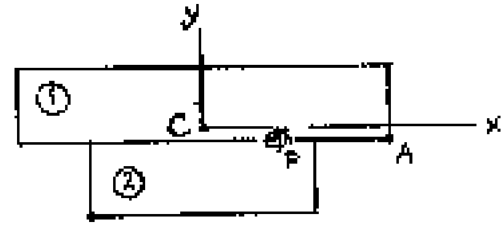
PROBLEM 4.166

4.166 A horizontal load **P** is applied to the beam shown. Knowing that $a = 20$ mm and that the tensile stress in the beam is not to exceed 75 MPa, determine the largest permissible load **P**.



SOLUTION

Locate the centroid



	A, mm^2	$\bar{y}$, mm	$A\bar{y}$, mm^3
①	2000	10	20×10^3
②	1200	−10	-12×10^3
Σ	3200		8×10^3

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8 \times 10^3}{3200} = 2.5 \text{ mm}$$

Move coordinate origin to the centroid.

Coordinates of load point: $x_P = a$, $y_P = -2.5$ mm

Bending couples $\quad M_x = y_P P \qquad M_y = -aP$

$$I_x = \tfrac{1}{12}(100)(20)^3 + (2000)(7.5)^2 + \tfrac{1}{12}(60)(20)^3 + (1200)(12.5)^2 = 0.40667 \times 10^6 \text{ mm}^4$$
$$= 0.40667 \times 10^{-6} \text{ m}^4$$

$$I_y = \tfrac{1}{12}(20)(100)^3 + \tfrac{1}{12}(20)(60)^3 = 2.0267 \times 10^6 \text{ mm}^4 = 2.0267 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = P\left\{\frac{1}{A} + \frac{y_P y}{I_x} + \frac{a x}{I_y}\right\} = KP$$

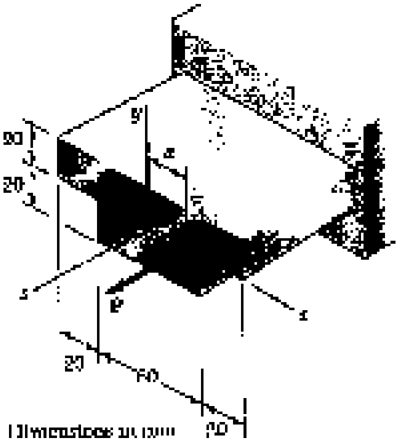
For point A $\quad K_A = \left\{\frac{1}{3200 \times 10^{-6}} + \frac{(-2.5 \times 10^{-3})(-2.5 \times 10^{-3})}{0.40667 \times 10^{-6}} + \frac{(20 \times 10^{-3})(50 \times 10^{-3})}{2.0267 \times 10^{-6}}\right\}$

$$= 821.28 \text{ m}^{-2}$$

$$P = \frac{\sigma_A}{K_A} = \frac{75 \times 10^6}{821.28} = 91.3 \times 10^3 \text{ N} = 91.3 \text{ kN}$$

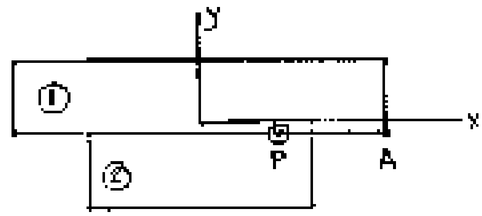
PROBLEM 4.167

4.167 A horizontal load **P** of magnitude 100 kN is applied to the beam shown. Determine the largest distance *a* for which the maximum tensile stress in the beam does not exceed 75 MPa.



SOLUTION

Locate the centroid



	A, mm²	$\bar{y}$, mm	$A\bar{y}$, mm³
①	2000	10	20×10^3
②	1200	−10	-12×10^3
Σ	3200		8×10^3

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8\times10^3}{3200} = 2.5 \text{ mm}$$

Move coordinate origin to the centroid

Coordinates of load point: $x_P = a$, $y_P = -2.5$ mm

Bending couples $M_x = y_P P$ $M_y = -aP$

$$I_x = \frac{1}{12}(100)(20)^3 + (2000)(7.5)^2 + \frac{1}{12}(60)(20)^3 + (1200)(12.5)^2 = 0.40667\times10^6 \text{ mm}^4 = 0.40667\times10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(20)(100)^3 + \frac{1}{12}(20)(60)^3 = 2.0267\times10^6 \text{ mm}^4 = 2.0267\times10^{-6} \text{ m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} - \frac{M_y x}{I_y} \qquad \sigma_A = 75\times10^6 \text{ Pa}, \quad P = 100\times10^3 \text{ N}$$

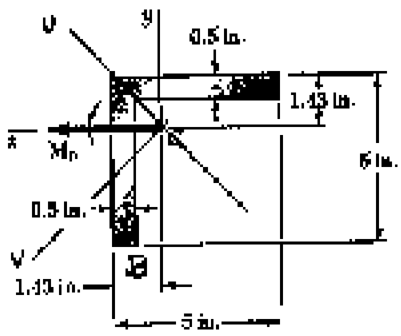
$$M_y = \frac{I_y}{x}\left\{\frac{P}{A} + \frac{M_x y}{I_x} - \sigma\right\} \qquad \text{For point A } x = 50 \text{ mm}, y = -2.5 \text{ mm}$$

$$M_y = \frac{2.0267\times10^{-6}}{50\times10^{-3}}\left\{\frac{100\times10^3}{3200\times10^{-6}} + \frac{(-2.5)(100\times10^3)(-2.5\times10^{-3})}{0.40667\times10^{-6}} - 75\times10^6\right\}$$

$$= \frac{2.0267\times10^{-6}}{50\times10^{-3}}\left\{31.25 + 1.537 - 75\right\}\times10^6 = -1.711\times10^3 \text{ N}\cdot\text{m}$$

$$a = -\frac{M_y}{P} = -\frac{(-1.711\times10^3)}{100\times10^3} = 17.11\times10^{-3} \text{ m} = 17.11 \text{ mm}$$

PROBLEM 4.168



4.168 A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ which acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 12 ksi. *Given:* $I_y = I_z = 11.3$ in^4, $A = 4.75$ in^2, $k_{min} = 0.983$ in. (*Hint:* By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{min} = Ak_{min}^2$ and $I_{min} + I_{max} = I_y + I_z$.)

SOLUTION

$$M_u = M_0 \sin 45° = 0.7071\, M_0$$

$$M_v = M_0 \cos 45° = 0.7071\, M_0$$

$$I_{min} = A k_{min}^2 = (4.75)(0.983)^2 = 4.59 \text{ in}^4$$

$$I_{max} = I_y + I_z - I_{min} = 11.3 + 11.3 - 4.59 = 18.01 \text{ in}^4$$

$$u_B = y_B \cos 45° + z_B \sin 45° = -3.57 \cos 45° + 0.93 \sin 45° = -1.866 \text{ in.}$$

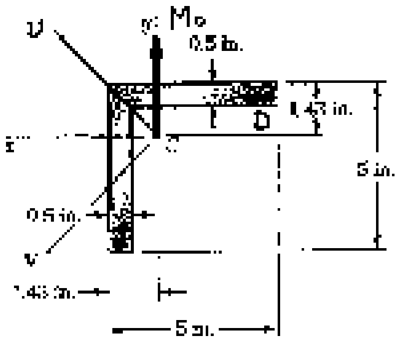
$$v_B = z_B \cos 45° - y_B \sin 45° = -0.93 \cos 45° - (-3.57) \sin 45° = 3.182 \text{ in}$$

$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u} = 0.7071\, M_0 \left[-\frac{u_B}{I_{min}} + \frac{v_B}{I_{max}} \right]$$

$$= 0.7071\, M_0 \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124\, M_0$$

$$M_0 = \frac{\sigma_B}{0.4124} = \frac{12}{0.4124} = 29.1 \text{ kip·in}$$

PROBLEM 4.169



4.168 A beam having the cross section shown is subjected to a couple M_0 which acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 12 ksi. *Given*: $I_y = I_z = 11.3$ in^4, $A = 4.75$ in^2, $k_{min} = 0.983$ in. (*Hint*: By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{min} = Ak_{min}^2$ and $I_{min} + I_{max} = I_y + I_z$.)

4.169 Solve Prob. 4.168, assuming that the couple M_0 acts in a horizontal plane.

SOLUTION

$$M_u = M_0 \cos 45° = 0.70711\, M_0$$

$$M_v = -M_0 \sin 45° = -0.70711\, M_0$$

$$I_{min} = A k_{min}^2 = (4.75)(0.983)^2 = 4.59 \text{ in}^4$$

$$I_{max} = I_y + I_z - I_{min} = 11.3 + 11.3 - 4.59 = 18.01 \text{ in}^4$$

$$u_D = y_D \cos 45° + z_D \sin 45° = 0.93 \cos 45° + (-3.57 \sin 45°) = -1.866 \text{ in.}$$

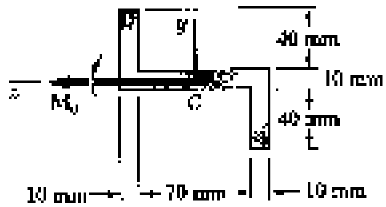
$$v_D = z_D \cos 45° - y_D \sin 45° = (-3.57)\cos 45° - (0.93) \sin 45° = 3.182 \text{ in.}$$

$$\sigma_D = -\frac{M_v u_D}{I_v} + \frac{M_u v_D}{I_u} = 0.70711\, M_0 \left[-\frac{u_D}{I_{min}} + \frac{v_D}{I_{max}}\right]$$

$$= 0.70711\, M_0 \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01}\right] = 0.4124\, M_0$$

$$M_0 = \frac{\sigma_D}{0.4124} = \frac{12}{0.4124} = 29.1 \text{ kip}\cdot\text{in}$$

PROBLEM 4.170

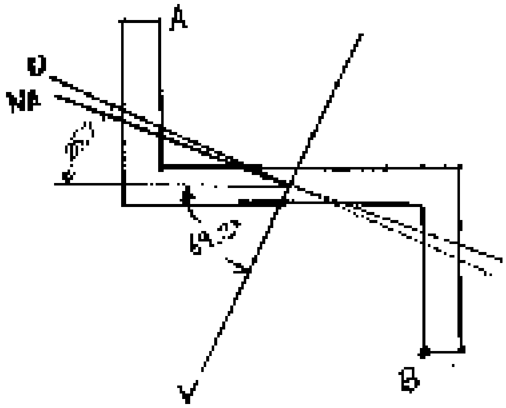


4.170 The Z section shown is subjected to a couple M_0 acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given:* $I_{max} = 2.28 \times 10^6$ mm^4, $I_{min} = 0.23 \times 10^6$ mm^4, principal axes 25.7° and 64.3°.

SOLUTION

$$I_v = I_{max} = 2.28 \times 10^6 \text{ mm}^4 = 2.28 \times 10^{-6} \text{ m}^4$$

$$I_u = I_{min} = 0.23 \times 10^6 \text{ mm}^4 = 0.23 \times 10^{-6} \text{ m}^4$$



$$M_v = M_0 \cos 64.3°$$

$$M_u = M_0 \sin 64.3°$$

$$\theta = 64.3°$$

$$\tan \varphi = \frac{I_v}{I_u} \tan \theta$$

$$= \frac{2.28 \times 10^{-6}}{0.23 \times 10^{-6}} \tan 64.3° = 20.597$$

$$\varphi = 87.22°$$

Points A and B are farthest from the neutral axis.

$$u_B = y_B \cos 64.3° + z_B \sin 64.3° = (-45)\cos 64.3° + (-35)\sin 64.3°$$

$$= -51.05 \text{ mm}$$

$$v_B = z_B \cos 64.3° - y_B \sin 64.3° = (-35)\cos 64.3° - (-45)\sin 64.3°$$

$$= +25.37 \text{ mm}$$

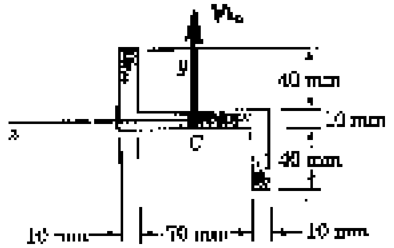
$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u}$$

$$80 \times 10^6 = -\frac{(M_0 \cos 64.3°)(-51.05 \times 10^{-3})}{2.28 \times 10^{-6}} + \frac{(M_0 \sin 64.3°)(25.37 \times 10^{-3})}{0.23 \times 10^{-6}}$$

$$= 73.81 \times 10^3 \quad M_0$$

$$M_0 = \frac{80 \times 10^6}{109.1 \times 10^3} = 733 \quad \text{N} \cdot \text{m}$$

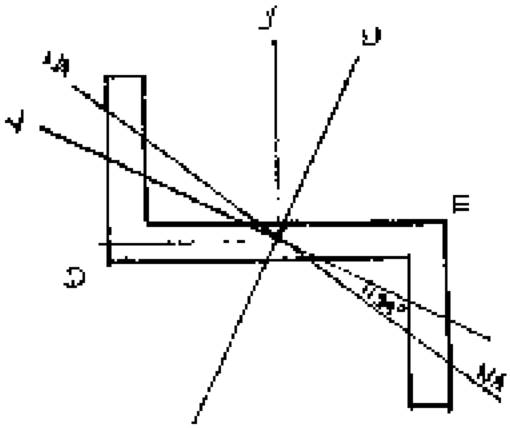
PROBLEM 4.171



4.170 The Z section shown is subjected to a couple M_0 acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given*: $I_{max} = 2.28 \times 10^6$ mm^4, $I_{min} = 0.23 \times 10^6$ mm^4, principal axes 25.7° ↘ and 64.3° ↗.

4.171 Solve Prob. 4.170, assuming that the couple M_0 acts in a horizontal plane.

SOLUTION



$$I_v = I_{min} = 0.23 \times 10^6 \text{ mm}^4 = 0.23 \times 10^{-6} \text{ m}^4$$

$$I_u = I_{max} = 2.23 \times 10^6 \text{ mm}^4 = 2.23 \times 10^{-6} \text{ m}^4$$

$$M_v = M_0 \cos 64.3°$$

$$M_u = M_0 \sin 64.3°$$

$$\theta = 64.3°$$

$$\tan \varphi = \frac{I_v}{I_u} \tan \theta$$

$$= \frac{0.23 \times 10^{-6}}{2.28 \times 10^{-6}} \tan 64.3° = 0.2096$$

$$\varphi = 11.84°$$

Points D and E are farthest from the neutral axis.

$$v_D = y_D \cos 25.7° - z_D \sin 25.7° = (-5)\cos 25.7° - 45 \sin 25.7°$$
$$= -24.02 \text{ mm}$$

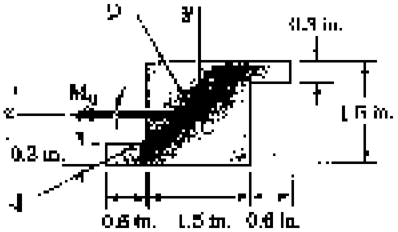
$$u_D = z_D \cos 25.7° + y_D \sin 25.7° = 45 \cos 25.7° + (-5) \sin 25.7°$$
$$= 38.38 \text{ mm}$$

$$\sigma_D = -\frac{M_v v_D}{I_v} + \frac{M_u u_D}{I_u} = -\frac{(M_0 \cos 64.3°)(-24.02 \times 10^{-3})}{0.23 \times 10^{-6}} + \frac{(M_0 \sin 64.3°)(38.38 \times 10^{-3})}{2.28 \times 10^{-6}}$$

$$80 \times 10^6 = 60.48 \times 10^3 \; M_0$$

$$M_0 = 1.323 \times 10^3 \text{ N·m} = 1.323 \text{ kN·m}$$

PROBLEM 4.172



4.172 An extruded aluminum member having the cross section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 12 ksi. *Given:* $I_{max} = 0.957\ \text{in}^4$, $I_{min} = 0.427\ \text{in}^4$, principal axes 29.4° and 60.6°.

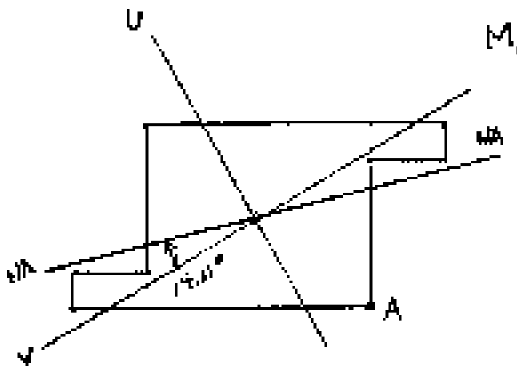
SOLUTION

$$I_u = I_{max} = 0.957\ \text{in}^4$$

$$I_v = I_{min} = 0.427\ \text{in}^4$$

$$M_v = M_0 \sin 29.4°, \quad M_u = M_0 \cos 29.4°$$

$$\theta = 29.4°$$



$$\tan\varphi = \frac{I_v}{I_u}\tan\theta = \frac{0.427}{0.957}\tan 29.4°$$

$$= 0.2514 \qquad \varphi = 14.11°$$

Point A is farthest from the neutral axis.

$$y_A = -0.75\ \text{in}, \quad z_A = -0.75\ \text{in.}$$

$$u_A = y_A \cos 29.4° + z_A \sin 29.4° = -1.0216\ \text{in}$$

$$v_A = z_A \cos 29.4° - y_A \sin 29.4° = -0.2852\ \text{in.}$$

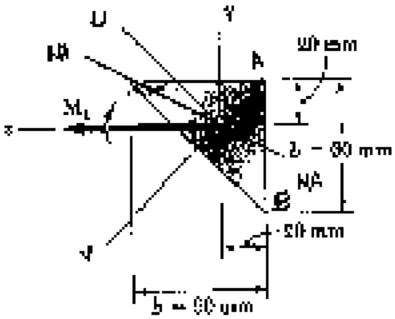
$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(M_0 \cos 29.4°)(-1.0216)}{0.427} + \frac{(M_0 \sin 29.4°)(-0.2852)}{0.957}$$

$$= 1.9381\ M_0$$

$$M_0 = \frac{\sigma_{A\,max}}{1.9381} = \frac{12}{1.9381} = 6.19\ \text{ksi}$$

PROBLEM 4.173

4.173 A beam having the cross section shown is subjected to a couple M_0 acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 100 MPa. Given: $I_y = I_z = b^4/36$ and $I_{yz} = b^4/72$.



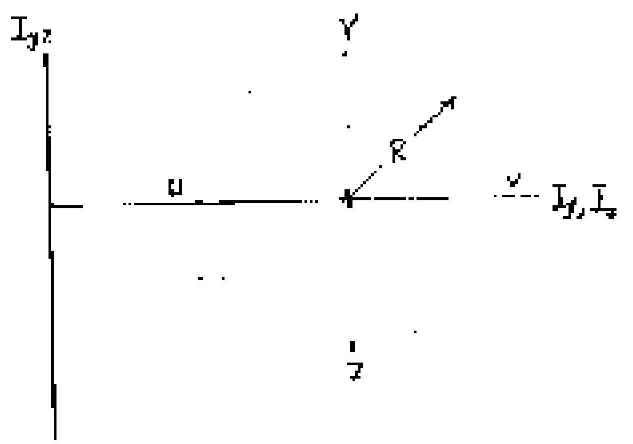
SOLUTION

$$I_y = I_z = \frac{b^4}{36} = \frac{60^4}{36} = 0.360 \times 10^6 \text{ mm}^4$$

$$I_{yz} = \frac{b^4}{72} = \frac{60^4}{72} = 0.180 \times 10^6 \text{ mm}^4$$

Principal axes are symmetry axes.

Using Mohr's circle determine the principal moments of inertia



$$R = |I_{yz}| = 0.180 \times 10^6 \text{ mm}^4$$

$$I_v = \frac{I_y + I_z}{2} + R$$

$$= 0.540 \times 10^6 \text{ mm}^4 = 0.540 \times 10^{-6} \text{ m}^4$$

$$I_u = \frac{I_y + I_z}{2} - R$$

$$= 0.180 \times 10^6 \text{ mm}^4 = 0.180 \times 10^{-6} \text{ m}^4$$

$$M_u = M_0 \sin 45^\circ = 0.7071\, M_0, \qquad M_v = M_0 \cos 45^\circ = 0.7071\, M_0$$

$$\theta = 45^\circ \qquad \tan\varphi = \frac{I_v}{I_u}\tan\theta = \frac{0.540 \times 10^{-6}}{0.180 \times 10^{-6}}\tan 45^\circ = 3$$

$$\varphi = 71.56^\circ$$

Point A: $u_A = 0$, $v_A = -20\sqrt{2}$ mm

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = 0 + \frac{(0.7071\, M_0)(-20\sqrt{2} \times 10^{-3})}{0.180 \times 10^{-6}} = -111.11 \times 10^3\, M_0$$

$$M_0 = -\frac{\sigma_A}{111.11 \times 10^3} = -\frac{-100 \times 10^6}{111.11 \times 10^3} = 900 \text{ N}\cdot\text{m}$$

Point B: $u_B = -\frac{60}{\sqrt{2}}$ mm, $v_B = \frac{20}{\sqrt{2}}$ mm

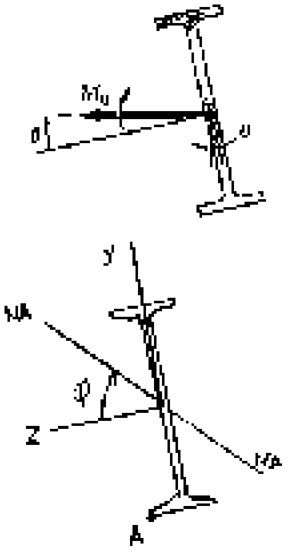
$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u} = -\frac{(0.7071\, M_0)(-\frac{60}{\sqrt{2}} \times 10^{-3})}{0.540 \times 10^{-6}} + \frac{(0.7071\, M_0)(\frac{20}{\sqrt{2}} \times 10^{-3})}{0.180 \times 10^{-6}}$$

$$= 111.11 \times 10^3\, M_0$$

$$M_0 = \frac{\sigma_B}{111.11 \times 10^3} = \frac{100 \times 10^6}{111.11 \times 10^3} = 900 \text{ N}\cdot\text{m}$$

PROBLEM 4.174

4.174 A couple M_0 acting in a vertical plane is applied to a W 12 × 16 rolled-steel beam, whose web forms an angle θ with the vertical. Denoting by σ_0 the maximum stress in the beam when $\theta = 0$, determine the angle of inclination θ of the beam for which the maximum stress is $2\sigma_0$.



SOLUTION

For W 12 × 16 rolled steel section

$I_z = 103 \text{ in}^4 \qquad I_y = 2.82 \text{ in}^4$

$d = 11.99 \text{ in} \qquad b_f = 3.990 \text{ in.}$

$y_A = -\frac{d}{2} \qquad z_A = \frac{b_f}{2}$

$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{103}{2.82}\tan\theta = 36.52\tan\theta$

Point A is farthest from the neutral axis.

$M_y = M_0 \sin\theta \qquad M_z = M_0\cos\theta$

$$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{M_0 d}{2I_z}\cos\theta + \frac{M_0 b_f}{2I_y}\sin\theta$$

$$= \frac{M_0 d}{2I_z}\left(1 + \frac{I_z b_f}{I_y d}\tan\theta\right)$$

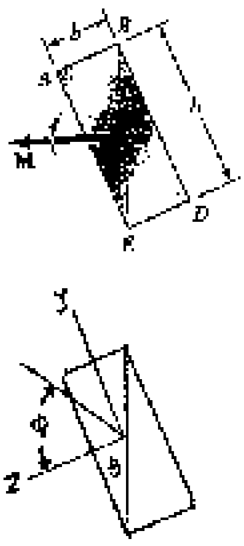
For $\theta = 0 \qquad \sigma_0 = \frac{M_0 d}{2I_z}$.

$$\sigma_A = \sigma_0\left(1 + \frac{I_z b_f}{I_y d}\tan\theta\right) = 2\sigma_0$$

$$\tan\theta = \frac{I_y d}{I_z b_f} = \frac{(2.82)(11.99)}{(103)(3.990)} = 0.08273 \qquad \theta = 4.70° \blacktriangleleft$$

PROBLEM 4.175

4.175 Show that, if a solid rectangular beam is bent by a couple applied in a plane containing one diagonal of the rectangular cross section, the neutral axis will lie along the other diagonal.



SOLUTION

$\tan\theta = \frac{b}{h}$

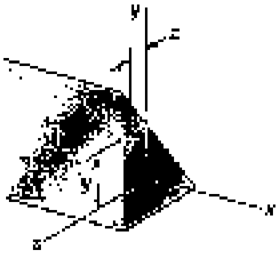
$M_z = M\cos\theta, \qquad M_y = M\sin\theta$

$I_z = \frac{1}{12}bh^3 \qquad I_y = \frac{1}{12}hb^3$

$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{\frac{1}{12}bh^3}{\frac{1}{12}hb^3}\cdot\frac{b}{h} = \frac{h}{b}$$

Thus neutral axis passes through corner A $\blacktriangleleft$

PROBLEM 4.176



4.176 A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_z$ acting in the vertical xy plane. Show that the stress at point A, of coordinates y and z, is

$$\sigma_A = -\frac{yI_y - zI_{yz}}{I_y I_z - I_{yz}^2} M_z$$

where I_y, I_z and I_{yz} denote the moments and product of inertia of the cross section with respect to centroidal axes, and M_z the moment of the couple.

SOLUTION

The stress σ_A varies linearly with the coordinates y and z. Since the axial force is zero, the y- and z-axes are centroidal axes.

$\sigma_A = C_1 y + C_2 z$ where C_1 and C_2 are constants.

$$M_y = \int z\sigma_A\,dA = C_1 \int yz\,dA + C_2 \int z^2\,dA$$

$$= I_{yz} C_1 + I_y C_2 = 0$$

$$C_2 = -\frac{I_{yz}}{I_y} C_1$$

$$M_z = -\int y\sigma_A\,dA = -C_1 \int y^2\,dA + C_2 \int yz\,dA$$

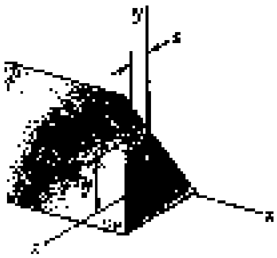
$$= -I_z C_1 - I_{yz} \frac{I_{yz}}{I_y} C_1$$

$$I_y M_z = -(I_y I_z - I_{yz}^2)\, C_1$$

$$C_1 = -\frac{I_y M_z}{I_y I_z - I_{yz}^2} \qquad C_2 = +\frac{I_{yz} M_z}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = -\frac{I_y z - I_{yz} y}{I_y I_z - I_{yz}^2} M_z$$

PROBLEM 4.177



4.177 A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_y$ acting in the horizontal xz plane. Show that the stress at point A is

$$\sigma_A = -\frac{zI_z - yI_{yz}}{I_y I_z - I_{yz}^2} M_y$$

where I_y, I_z and I_{yz} denote the moments and product of inertia of the cross section with respect to centroidal axes, and M_y the moment of the couple.

SOLUTION

The stress σ_A varies linearly with the coordinates y and z. Since the axial force is zero, the y- and z-axes are centroidal axes.

$\sigma_A = C_1 y + C_2 z$ where C_1 and C_2 are constants.

$$M_z = -\int y\sigma_A\, dA = -C_1\int y^2\, dA - C_2 \int yz\, dA$$

$$= -I_z C_1 - I_{yz} C_2 = 0$$

$$C_1 = -\frac{I_{yz}}{I_z} C_2$$

$$M_y = \int z\sigma_A\, dA = C_1 \int yz\, dA + C_2\int z^2\, dA$$

$$= I_{yz} C_1 + I_y C_2$$

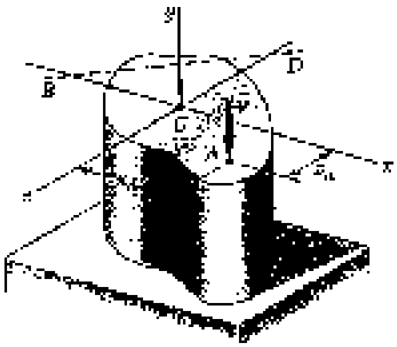
$$= -I_{yz}\frac{I_{yz}}{I_z} C_2 + I_y C_2$$

$$I_z M_y = (I_y I_z - I_{yz}^2)\, C_2$$

$$C_2 = \frac{I_z M_y}{I_y I_z - I_{yz}^2} \qquad C_1 = -\frac{I_{yz} M_y}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = \frac{I_z y - I_{yz} z}{I_y I_z - I_{yz}^2} M_y$$

PROBLEM 4.178



4.178 (a) Show that, if a vertical force **P** is applied at point A of the section shown, the equation of the neutral axis BD is

$$\left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1$$

where k_z and k_x denote the radius of gyration of the cross section with respect to the z axis and the x axis, respectively. (b) Further show that, if a vertical force **Q** is applied at any point located on line BD, the stress at point A will be zero.

SOLUTION

Definitions $\quad k_x^2 = \frac{I_x}{A}, \quad k_z^2 = \frac{I_z}{A}$

(a) $\quad M_x = P z_A \qquad M_z = -P x_A$

$$\sigma_E = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_E}{I_x} = -\frac{P}{A} - \frac{P x_A x_E}{A k_z^2} - \frac{P z_A z_E}{A k_x^2}$$

$$= -\frac{P}{A}\left[1 + \left(\frac{x_A}{k_z^2}\right)x_E + \left(\frac{z_A}{k_x^2}\right)z_E\right] = 0 \quad \text{if } E \text{ lies on neutral axis.}$$

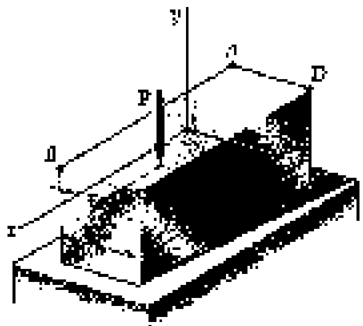
$$1 + \left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = 0, \qquad \left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1$$

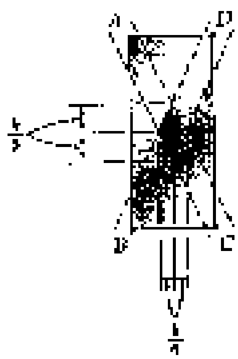
(b) $\quad M_x = P z_E \qquad M_z = -P x_E$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x} = -\frac{P}{A} - \frac{P x_E x_A}{A k_z^2} - \frac{P z_E z_A}{A k_x^2}$$

$= 0$ by equation from Part (a)

PROBLEM 4.179





4.179 (*a*) Show that the stress at corner *A* of the prismatic member shown in Fig. (*a*) will be zero if the vertical force **P** is applied at a point located on the line

$$\frac{x}{b/6}+\frac{z}{h/6}=1$$

(*b*) Further show that, if no tensile stress is to occur in the member, the force **P** must be applied at a point located within the area bounded by the line found in part *a* and the three similar lines corresponding to the condition of zero stress at *B*, *C*, and *D*, respectively. This area, shown in Fig.(*b*), is known as the *kern* of the cross section.

SOLUTION

$I_z = \frac{1}{12}hb^3 \qquad I_x = \frac{1}{12}bh^3 \qquad A = bh$

$z_A = -\frac{h}{2} \qquad x_A = -\frac{b}{2}$

Let P be the load point

$M_z = -Px_P \qquad M_x = Pz_P$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x}$$

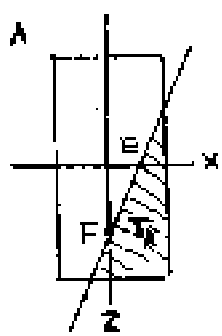
$$= -\frac{P}{bh} + \frac{(-Px_P)(-\frac{b}{2})}{\frac{1}{12}hb^3} - \frac{Pz_P(-\frac{h}{2})}{\frac{1}{12}bh^3}$$

$$= -\frac{P}{bh}\left[1 - \frac{x_P}{b/6} - \frac{z_P}{h/6}\right]$$

For $\sigma_A = 0$ $\qquad 1 - \frac{x}{b/6} - \frac{z}{h/6} = 0, \qquad \frac{x}{b/6} + \frac{z}{h/6} = 1$

At point E $\qquad z = 0 \qquad \therefore\ x_E = b/6$

At point F $\qquad x = 0 \qquad \therefore\ z_F = h/6$

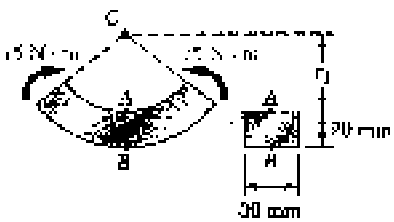


If the line of action (x_P, z_P) lies within the portion marked T_A, a tensile will occur at corner A.

By considering $\sigma_B = 0$, $\sigma_C = 0$, and $\sigma_D = 0$, the other portions producing tensile stresses are identified.

PROBLEM 4.180

4.180 For the curved bar and loading shown, determine the stress at point A when (a) $r_1 = 30$ mm, (b) $r_1 = 50$ mm.



SOLUTION

(a) $r_1 = 30$ mm $\quad r_2 = 30 + 20 = 50$ mm

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{20}{\ln \frac{50}{30}} = 39.1523 \text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 40 \text{ mm}$$

$$e = \bar{r} - R = 0.8477 \text{ mm}$$

$$A = (20)(30) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$$

$$y_A = 39.1523 - 30 = 9.1523 \text{ mm}$$

$$\sigma_A = -\frac{My}{Aer} = -\frac{(75)(9.1523 \times 10^{-3})}{(600 \times 10^{-6})(0.8477 \times 10^{-3})(30 \times 10^{-3})} = -45.0 \times 10^6 \text{ Pa}$$

$$= -45.0 \text{ MPa}$$

(b) $r_1 = 50$ mm, $\quad r_2 = 50 + 20 = 70$ mm

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{20}{\ln \frac{70}{50}} = 59.44027$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 60 \text{ mm}$$

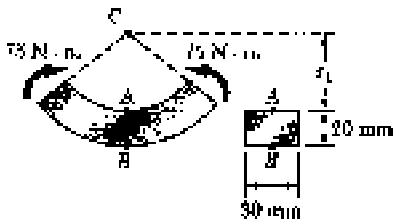
$$e = \bar{r} - R = 0.55973 \text{ mm}$$

$$y_A = 59.44027 - 50 = 9.44027 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(75)(9.44027)}{(600 \times 10^{-6})(0.55973 \times 10^{-3})(50 \times 10^{-3})} = -42.2 \times 10^6$$

$$= -42.2 \text{ MPa}$$

PROBLEM 4.181



4.181 For the curved bar and loading shown, determine the stress at points A and B when $r_1 = 40$ mm.

SOLUTION

$h = 20$ mm $\quad r_1 = 40$ mm $\quad r_2 = 40 + 20 = 60$ mm

$A = (30)(20) = 600\ \text{mm}^2 = 600 \times 10^{-6}\ \text{m}^2$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{20}{\ln \frac{60}{40}} = 49.3261\ \text{mm}$$

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 50$ mm

$e = \bar{r} - R = 0.6739$ mm

$y_A = 49.3261 - 40 = 9.3261$ mm $\qquad r_A = 40$ mm

$$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(75)(9.3261 \times 10^{-3})}{(600 \times 10^{-6})(0.6739 \times 10^{-3})(40 \times 10^{-3})} = -43.2 \times 10^6\ \text{Pa}$$

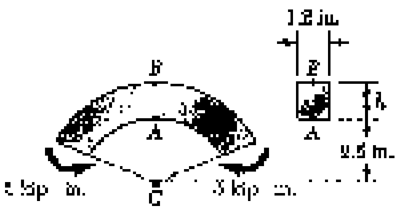
$$= -43.2\ \text{MPa}$$

$y_B = 49.3261 - 60 = -10.6739$ mm

$$\sigma_B = -\frac{M y_B}{A e r_B} = -\frac{(75)(-10.6739 \times 10^{-3})}{(600 \times 10^{-6})(0.6739 \times 10^{-3})(60 \times 10^{-3})} = 33.0 \times 10^6\ \text{Pa}$$

$$= 33.0\ \text{MPa}$$

PROBLEM 4.182



4.182 For the curved bar and loading shown, determine the stress at point A when (a) $h = 2.5$ in., (b) $h = 3$ in.

SOLUTION

(a) $h = 2.5$ in., $r_1 = 2.5$ in. $r_2 = 5$ in.

$A = (1.2)(2.5) = 3.00$ in^2, $M = 5$ kip · in.

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{2.5}{\ln \frac{5}{2.5}} = 3.6067$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 3.75$$

$$e = \bar{r} - R = 0.1433 \text{ in.}$$

$y_A = 3.6067 - 2.5 = 1.1067$ in $r_A = 2.5$ in.

$$\sigma_A = -\frac{My_A}{Ae\,r_A} = -\frac{(5)(1.1067)}{(3.00)(0.1433)(2.5)} = -5.15 \text{ ksi}$$

(b) $h = 3$ in., $r_1 = 2.5$ in $r_2 = 5.5$ in $A = (1.2)(3) = 3.6$ in^2

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{3}{\ln \frac{5.5}{2.5}} = 3.8049 \text{ in}$$

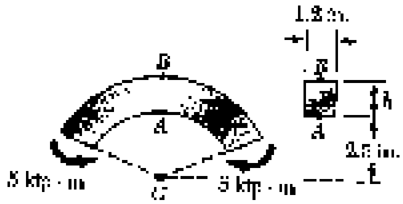
$\bar{r} = \frac{1}{2}(r_1 + r_2) = 4.0000$ in, $e = \bar{r} - R = 0.1951$ in.

$y_A = 3.8049 - 2.5 = 1.3049$ in $r_A = 2.5$ in

$$\sigma_A = -\frac{My_A}{Ae\,r_A} = -\frac{(5)(1.3049)}{(3.6)(0.1951)(2.5)} = -3.72 \text{ ksi}$$

PROBLEM 4.193

4.193 For the curved bar and loading shown, determine the stress at points A and B when $h = 2.75$ in.



SOLUTION

$$h = 2.75 \text{ in} \qquad r_1 = 2.5 \text{ in}, \qquad r_2 = 5.25 \text{ in}$$

$$A = (1.2)(2.75) = 3.30 \text{ in}^2, \qquad M = 5 \text{ kip}\cdot\text{in}$$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{2.75}{\ln \frac{5.25}{2.5}} = 3.7065 \text{ in}$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 3.875 \text{ in.} \qquad e = \bar{r} - R = 0.1685 \text{ in.}$$

$$y_A = 3.7065 - 2.5 = 1.2065 \text{ in.} \qquad r_A = 2.5 \text{ in.}$$

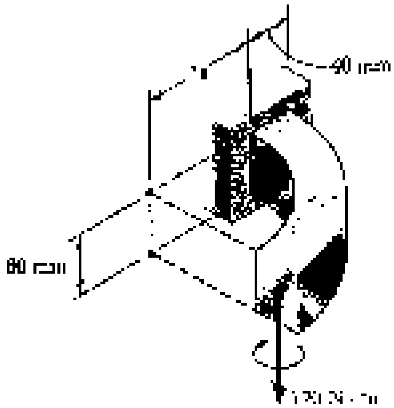
$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(5)(1.2065)}{(3.30)(0.1685)(2.5)} = -4.34 \text{ ksi}$$

$$y_B = 3.7065 - 5.25 = -1.5435 \text{ in.} \qquad r_B = 5.25 \text{ in}$$

$$\sigma_B = -\frac{My_B}{Aer_B} = -\frac{(5)(-1.5435)}{(3.30)(0.1685)(5.25)} = 2.64 \text{ ksi}$$

PROBLEM 4.184

4.184 The curved bar shown has a cross section of 40 × 60 mm and an inner radius $r_1 = 15$ mm. For the loading shown determine the largest tensile and compressive stresses.



SOLUTION

$h = 40$ mm, $\quad r_1 = 15$ mm, $\quad r_2 = 55$ mm

$A = (60)(40) = 2400\ \text{mm}^2 = 2400 \times 10^{-6}\ \text{m}^2$

$R = \dfrac{h}{\ln \frac{r_2}{r_1}} = \dfrac{40}{\ln \frac{55}{15}} = 30.786$ mm

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 35$ mm

$e = \bar{r} - R = 4.214$ mm $\qquad \sigma = -\dfrac{My}{Aer}$

At $r = 15$ mm $\qquad y = 30.786 - 15 = 15.786$ mm

$$\sigma = -\frac{(120)(15.786 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(15 \times 10^{-3})} = -12.49 \times 10^{6}\ \text{Pa}$$

$= -12.49$ MPa ◀

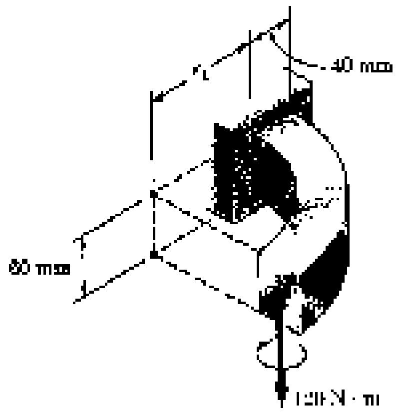
At $r = 55$ mm $\qquad y = 30.786 - 55 = -24.214$ mm

$$\sigma = -\frac{(120)(-24.214 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(55 \times 10^{-3})} = 5.22 \times 10^{6}\ \text{Pa}$$

$= 5.22$ MPa ◀

PROBLEM 4.185

4.185 For the curved bar and loading shown, determine the percent error introduced in the computation of the maximum stress by assuming that the bar is straight. Consider the case when (*a*) $r_1 = 20$ mm, (*b*) $r_1 = 200$ mm, (*c*) $r_1 = 2$ m.



SOLUTION

$h = 40$ mm, $A = (60)(40) = 2400\ mm^2 = 2400 \times 10^{-6}\ m^2$

$M = 120$ N·m

$I = \frac{1}{12}bh^3 = \frac{1}{12}(60)(40)^3 = 0.32 \times 10^6\ mm^4$
$= 0.32 \times 10^{-6}\ m^4$

$c = \frac{1}{2}h = 20$ mm

Assuming that the bar is straight

$$\sigma_s = -\frac{Mc}{I} = -\frac{(120)(20 \times 10^{-3})}{0.32 \times 10^{-6}} = 7.5 \times 10^6\ Pa = 7.5\ MPa$$

(a) $r_1 = 20$ mm $\quad r_2 = 60$ mm

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{40}{\ln \frac{60}{20}} = 36.4096\ mm \qquad r_1 - R = -16.4096\ mm$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 40\ mm \qquad e = \bar{r} - R = 3.5904\ mm$$

$$\sigma_a = \frac{M(r_1 - R)}{Aer_1} = \frac{(120)(-16.4096 \times 10^{-3})}{(2400 \times 10^{-6})(3.5904 \times 10^{-3})(20 \times 10^{-3})} = -11.426 \times 10^6\ Pa$$
$$= -11.426\ MPa$$

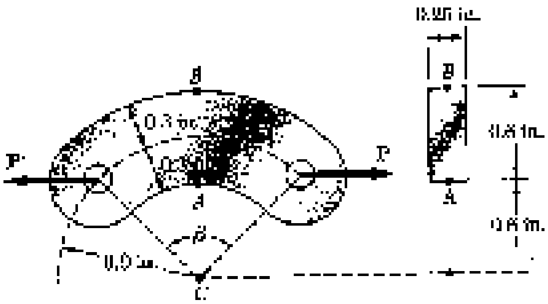
$$\%\ error = \frac{-11.426 - (-7.5)}{-11.426} \times 100\% = -34.4\%$$

For parts (b) and (c) we get the values in the table below:

	r_1, mm	r_2, mm	R, mm	$\bar{r}$, mm	e, mm	σ, MPa	% error
(a)	20	60	36.4096	40	3.5904	−11.426	−34.4 %
(b)	200	240	219.3926	220	0.6074	−7.982	6.0 %
(c)	2000	2040	2019.9340	2020	0.0660	−7.546	0.6 %

PROBLEM 4.186

4.186 Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 15 ksi, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.

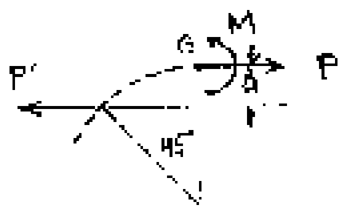


SOLUTION

Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$

The bending couple is $M = -Pa$



For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln\frac{r_2}{r_1}}. \qquad \text{Also} \quad e = \bar{r} - R$$

At point A the tensile stress is

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{Pay_A}{Aer_1} = \frac{P}{A}\left(1 + \frac{ay_A}{er_1}\right) = K\frac{P}{A}$$

where $K = 1 + \frac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 0.9$ in, $r_1 = 0.6$ in, $r_2 = 1.2$ in, $h = 0.6$ in, $b = 0.25$ in

$A = (0.25)(0.6) = 0.15 \text{ in}^2$, $\quad R = \frac{0.6}{\ln\frac{1.2}{0.6}} = 0.86562$ in

$e = 0.9 - 0.86562 = 0.03438$ in., $\quad y_A = 0.86562 - 0.6 = 0.26562$ in

$a = 0.9(1 - \cos 45°) = 0.26360$ in

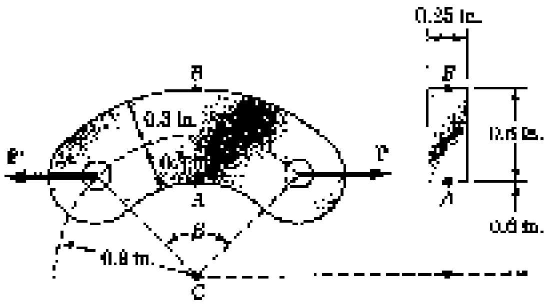
$$K = 1 + \frac{(0.26360)(0.26562)}{(0.03438)(0.6)} = 4.3943$$

$$P = \frac{(0.15)(15)}{4.3943} = 0.512 \text{ kips} = 512 \text{ lb}$$

PROBLEM 4.187

4.186 Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 15 ksi, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.

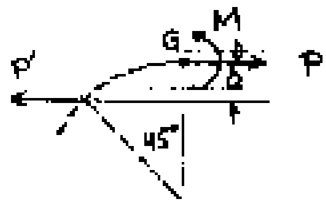
4.187 Solve Prob. 4.186, assuming that $\beta = 60°$.



SOLUTION

Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$



The bending couple is: $M = -Pa$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln\frac{r_2}{r_1}}. \qquad \text{Also} \quad e = \bar{r} - R$$

At point A the tensile stress is

$$\sigma_A = \frac{P}{A} + \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{Pay_A}{Aer_1} = \frac{P}{A}\left(1 + \frac{ay_A}{er_1}\right) = K\frac{P}{A}$$

where $K = 1 + \dfrac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 0.9$ in, $r_1 = 0.6$ in, $r_2 = 1.2$ in, $h = 0.6$ in, $b = 0.25$

$A = (0.25)(0.6) = 0.15 \text{ in}^2$, $\quad R = \dfrac{0.6}{\ln\frac{1.2}{0.6}} = 0.86562$ in.

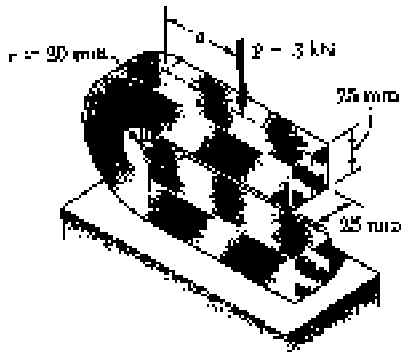
$e = 0.9 - 0.86562 = 0.03438$ in., $\quad y_A = 0.86562 - 0.6 = 0.26562$

$a = 0.9(1 - \cos 30°) = 0.12058$ in.

$$K = 1 + \frac{(0.12058)(0.26562)}{(0.03438)(0.6)} = 2.5526$$

$$P = \frac{(0.15)(15)}{2.5526} = 0.881 \text{ kips} = 881 \text{ lb}$$

PROBLEM 4.188

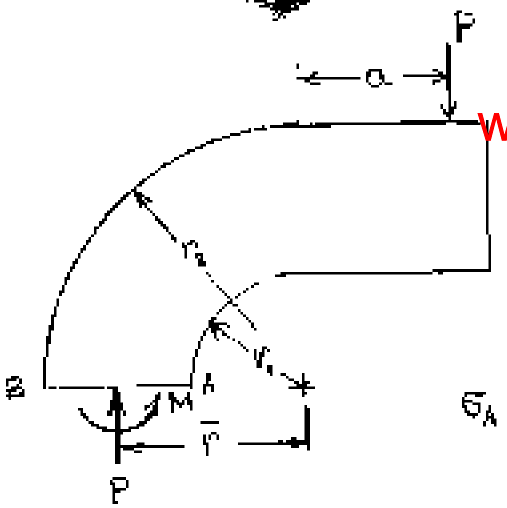


4.188 The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the line of action of the 3-kN force is located at a distance $a = 60$ mm from the vertical plane containing the center of curvature of the bar, determine the largest compressive stress in the bar.

SOLUTION

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$



For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}. \qquad \text{Also} \quad e = \bar{r} - R$$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a+\bar{r})y_A}{Aer_1}$$

$$= -K\frac{P}{A} \qquad \text{with} \quad y_A = R - r_1$$

$$\text{Thus,} \quad K = 1 + \frac{(a+\bar{r})(R - r_1)}{er_1}$$

Data: $h = 25$ mm, $r_1 = 20$ mm, $r_2 = 45$ mm, $\bar{r} = 32.5$ mm

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$$b = 25 \text{ mm}, \quad A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{ m}^2$$

$$a = 60 \text{ mm}, \quad a + \bar{r} = 92.5 \text{ mm}, \quad R - r_1 = 10.8288 \text{ mm}$$

$$K = 1 + \frac{(92.5)(10.8288)}{(1.6712)(20)} = 30.968$$

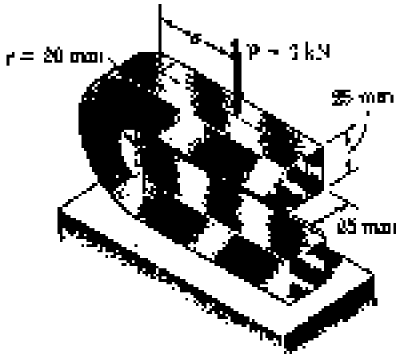
$$P = 3 \times 10^3 \text{ N}$$

$$\sigma_A = -\frac{KP}{A} = -\frac{(30.968)(3\times10^3)}{625\times10^{-6}} = -148.6 \times 10^6 \text{ Pa}$$

$$= -148.6 \text{ MPa}$$

PROBLEM 4.189

4.189 Knowing that the allowable stress in the bar is 150 MPa, determine the largest permissible distance a from the line of action of the 3-kN force to the vertical plane containing the center of curvature of the bar.



SOLUTION

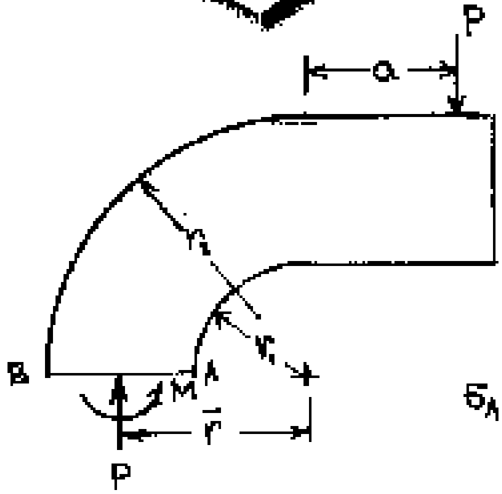
Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$

For the rectangular section, the neutral axis for bending couple only lies

$$R = \frac{h}{\ln \frac{r_2}{r_1}} \qquad \text{Also} \quad e = \bar{r} - R$$

The maximum compressive stress occurs at point A. It is given by



$$\sigma_A = -\frac{P}{A} - \frac{M y_A}{A e r_1} = -\frac{P}{A} - \frac{P(a+\bar{r})y_A}{A e r_1}$$

$$= -K\frac{P}{A} \qquad \text{with} \quad y_A = R - r_1$$

Thus, $\quad K = 1 + \frac{(a+\bar{r})(R - r_1)}{e r_1}$

Data: $h = 25$ mm, $r_1 = 20$ mm, $r_2 = 45$ mm, $\bar{r} = 32.5$ mm

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$$b = 25 \text{ mm}, \quad A = bh = (25)(25) = 625 \text{ mm}^2 = 625\times10^{-6} \text{ m}^2$$

$$R - r_1 = 10.8288 \text{ mm}$$

$$P = 3\times10^3 \text{ N}\cdot\text{m} \qquad \sigma_A = -150\times10^6 \text{ Pa}$$

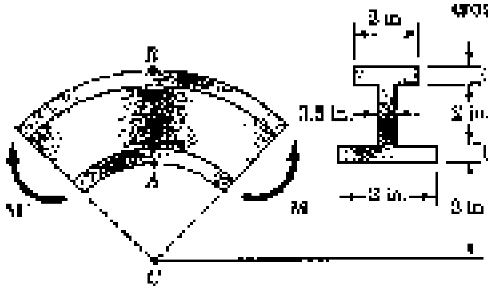
$$K = -\frac{\sigma_A A}{P} = -\frac{(-150\times10^6)(625\times10^{-6})}{3\times10^3} = 31.25$$

$$a + \bar{r} = \frac{(K-1)e r_1}{R - r_1} = \frac{(30.25)(1.6712)(20)}{10.8288} = 93.37 \text{ mm}$$

$$a = 93.37 - 32.5 = 60.9 \text{ mm}$$

PROBLEM 4.190

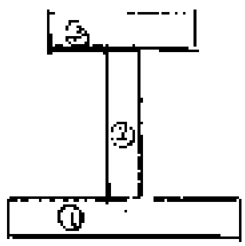
4.190 Three plates are welded together to form the curved beam shown. For the given loading, determine the distance e between the neutral axis and the centroid of the cross section.



SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



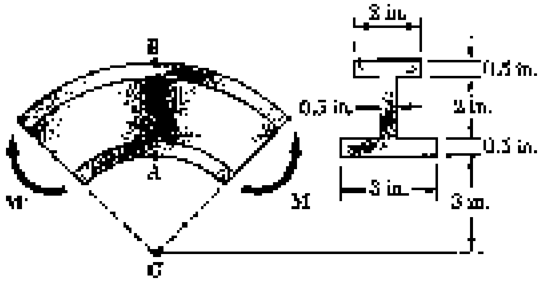
r	part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A\bar{r}$
3	①	3	0.5	1.5	0.462452	3.25	4.875
3.5	②	0.5	2	1.0	0.225993	4.5	4.5
5.5	③	2	0.5	1.0	0.174023	5.75	5.75
6	Σ			3.5	0.862468		15.125

$R = \frac{3.5}{0.862468} = 4.05812$ in., $\quad \bar{r} = \frac{15.125}{3.5} = 4.32143$ in.

$e = \bar{r} - R = 0.26331$ in

PROBLEM 4.191

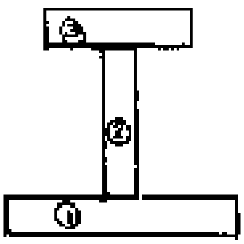
4.191 Three plates are welded together to form the curved beam shown. For $M = 8$ kip·in., determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) the centroid of the cross section.



SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



r	part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A\bar{r}$
3	①	3	0.5	1.5	0.462452	3.25	4.875
3.5	②	0.5	2	1.0	0.225993	4.5	4.5
5.5	③	2	0.5	1.0	0.174023	5.75	5.75
6	Σ			3.5	0.862468		15.125

$$R = \frac{3.5}{0.862468} = 4.05812 \text{ in.}, \quad \bar{r} = \frac{15.125}{3.5} = 4.32143 \text{ in.}$$

$$e = \bar{r} - R = 0.26331 \text{ in.} \qquad M = -8 \text{ kip·in}$$

(a) $y_A = R - r_1 = 4.05812 - 3 = 1.05812$ in

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-8)(1.05812)}{(3.5)(0.26331)(3)} = 3.06 \text{ ksi}$$

(b) $y_B = R - r_2 = 4.05812 - 6 = -1.94188$ in.

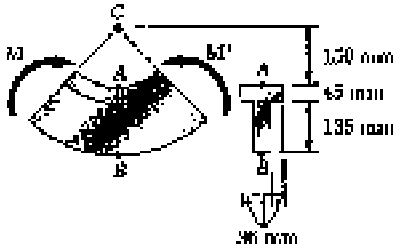
$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-8)(-1.94188)}{(3.5)(0.26331)(6)} = -2.81 \text{ ksi}$$

(c) $y_C = R - \bar{r} = -e$

$$\sigma_C = -\frac{My_C}{Ae\bar{r}} = -\frac{Me}{Ae\bar{r}} = -\frac{M}{A\bar{r}} = -\frac{-8}{(3.5)(4.32143)} = 0.529 \text{ ksi}$$

PROBLEM 4.193

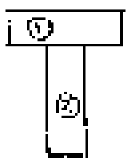
4.192 and 4.193 Knowing that $M = 20$ kN·m, determine the stress at (a) point A, (b) point B.



SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A_i \bar{r}_i}{\Sigma A_i}$$



r, mm	Part	b, mm	h, mm	A, mm²	$b_i \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}$, mm	$A\bar{r}$, mm³
150							
	①	105	45	4860	28.9253	172.5	838.35×10^3
195							
	②	36	135	4860	18.9394	262.5	1275.75×10^3
330							
	Σ			9720	47.2747		2114.1×10^3

$$R = \frac{9720}{47.2747} = 205.606 \text{ mm} \qquad \bar{r} = \frac{2114.1 \times 10^3}{9720} = 217.5 \text{ mm}$$

$$e = \bar{r} - R = 11.894 \text{ mm.} \qquad M = 20 \times 10^3 \text{ N·m}$$

(a) $y_A = R - r_1 = 205.606 - 150 = 55.606$ mm

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(20 \times 10^3)(55.606 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(150 \times 10^{-3})}$$

$$= -64.1 \times 10^6 \text{ Pa} = -64.1 \text{ MPa}$$

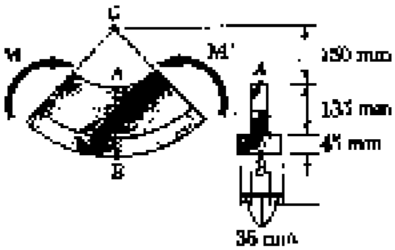
(b) $y_B = R - r_2 = 205.606 - 330 = -124.394$ mm

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(20 \times 10^3)(-124.394 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(330 \times 10^{-3})}$$

$$= 65.2 \times 10^6 \text{ Pa} = 65.2 \text{ MPa}$$

PROBLEM 4.193

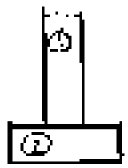
4.192 and 4.193 Knowing that $M = 20$ kN·m, determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A_i \bar{r}_i}{\Sigma A_i}$$



r		b, mm	h, mm	A, mm^2	$b \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}$, mm	$A\bar{r}$, mm^3
150	①	36	135	4860	23.1067	217.5	1.05705×10^6
285	②	108	45	4860	15.8332	307.5	1.49445×10^6
330	Σ			9720	38.9399		2.5515×10^6

$$R = \frac{9720}{38.9399} = 249.615 \text{ mm}, \qquad \bar{r} = \frac{2.5515 \times 10^6}{9720} = 262.5 \text{ mm}$$

$$e = \bar{r} - R = 12.885 \text{ mm}, \qquad M = 20 \times 10^3 \text{ N}\cdot\text{m}$$

(a) $y_A = R - r_1 = 249.615 - 150 = 99.615$ mm

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(20 \times 10^3)(99.615 \times 10^{-3})}{(9720 \times 10^{-6})(12.885 \times 10^{-3})(150 \times 10^{-3})}$$

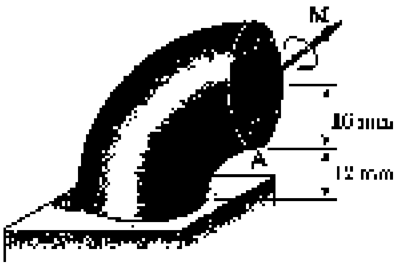
$$= -106.1 \times 10^6 \text{ Pa} = -106.1 \text{ MPa}$$

(b) $y_B = R - r_2 = 249.615 - 330 = -80.385$ mm

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(20 \times 10^3)(-80.385 \times 10^{-3})}{(9720 \times 10^{-6})(12.885 \times 10^{-3})(330 \times 10^{-3})}$$

$$= 38.9 \times 10^6 \text{ Pa} = 38.9 \text{ MPa}$$

PROBLEM 4.194



4.194 The curved bar shown has a circular cross section of 32-mm diameter. Determine the largest couple **M** that can be applied to the bar about a horizontal axis if the maximum stress is not to exceed 60 MPa.

SOLUTION

$c = 16\text{ mm}$ $\qquad \bar{r} = 12 + 16 = 28\text{ mm}$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right]$$

$$= \frac{1}{2}\left[28 + \sqrt{28^2 - 16^2}\right] = 25.4891\text{ mm}$$

$$e = \bar{r} - R = 28 - 25.4891 = 2.5109\text{ mm}$$

σ_{max} occurs at A is given by $|\sigma_{max}| = \left|\dfrac{M y_A}{A e r_1}\right|$ from which

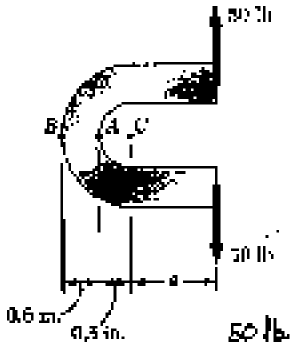
$$M = \frac{A e r_1 |\sigma_{max}|}{y_A} \qquad \text{Also} \quad A = \pi c^2 = \pi(16)^2 = 804.25\text{ mm}^2$$

Data: $y_A = R - r_1 = 25.4891 - 12 = 13.4891\text{ mm}$

$$M = \frac{(804.25\times10^{-6})(2.5109\times10^{-3})(12\times10^{-3})(60\times10^{6})}{13.4891\times10^{-3}} = 107.8\text{ N}\cdot\text{m}$$

PROBLEM 4.195

4.195 The bar shown has a circular cross section of 0.6-in. diameter. Knowing that $a = 1.2$ in., determine the stress at (*a*) point *A*. (*b*) point *B*.



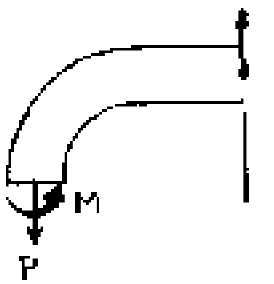
SOLUTION

$c = \frac{1}{2}d = 0.3$ in. $\quad \bar{r} = 0.5 + 0.3 = 0.8$ in.

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[0.8 + \sqrt{0.8^2 - 0.3^2}\right]$$

$$= 0.77081 \text{ in}$$

$e = \bar{r} - R = 0.02919$ in.



$A = \pi c^2 = \pi(0.3)^2 = 0.28274 \text{ in}^2$

$M = -P(a + \bar{r}) = -50(1.2 + 0.8) = -100$ lb·in

$y_A = R - r_1 = 0.77081 - 0.5 = 0.27081$ in.

$y_B = R - r_2 = 0.77081 - 1.1 = -0.32919$ in.

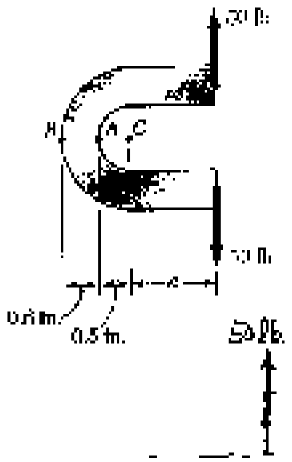
(a) $\sigma_A = \dfrac{P}{A} - \dfrac{My_A}{Aer_1} = \dfrac{50}{0.28274} - \dfrac{(-100)(0.27081)}{(0.28274)(0.02919)(0.5)} = 6.74 \times 10^3$ psi

$= 6.74$ ksi ◄

(b) $\sigma_B = \dfrac{P}{A} - \dfrac{My_B}{Aer_2} = \dfrac{50}{0.28274} - \dfrac{(-100)(-0.32919)}{(0.28274)(0.02919)(1.1)} = -3.45 \times 10^3$ psi

$= -3.45$ ksi ◄

PROBLEM 4.196



4.196 The bar shown has a circular cross section of 0.6-in. diameter. Knowing that the allowable tensile stress is 8 ksi, determine the largest permissible distance a from the line of action of the 50-lb forces to the plane containing the center of curvature of the bar.

SOLUTION

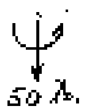
$$c = \tfrac{1}{2}d = 0.3 \text{ in}, \qquad \bar{r} = 0.5 + 0.3 = 0.8 \text{ in}$$

$$R = \tfrac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \tfrac{1}{2}\left[0.8 + \sqrt{0.8^2 - 0.3^2}\right]$$

$$= 0.77081 \text{ in.} \qquad e = \bar{r} - R = 0.02919 \text{ in.}$$

$$A = \pi c^2 = \pi(0.3)^2 = 0.28274 \text{ in}^2$$

$$M = -P(a + \bar{r})$$



$$y_A = R - r_1 = 0.77081 - 0.5 = 0.27081 \text{ in.}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{P(a+\bar{r})y_A}{Aer_1} = \frac{P}{A}\left[1 + \frac{(a+\bar{r})y_A}{er_1}\right]$$

$$= \frac{KP}{A} \quad \text{where} \quad K = 1 + \frac{(a+\bar{r})y_A}{er_1}$$

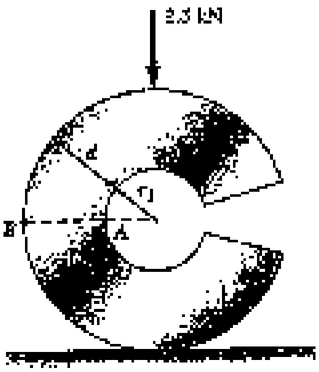
$$K = \frac{\sigma_A A}{P} = \frac{(8\times10^3)(0.28274)}{50} = 45.238$$

$$a + \bar{r} = \frac{(K-1)er_1}{y_A} = \frac{(44.238)(0.02919)(0.5)}{0.27081} = 2.384 \text{ in}$$

$$a = 2.384 - 0.8 = 1.584 \text{ in}$$

PROBLEM 4.197

4.197 The split ring shown has an inner radius $r_1 = 20$ mm and a circular cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

$c = \frac{1}{2}d = 16$ mm $\quad r_1 = 20$ mm, $\quad r_2 = r_1 + d = 52$ mm

$\bar{r} = r_1 + c = 36$ mm

$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[36 + \sqrt{36^2 - 16^2}\right]$

$= 34.1245$ mm

$e = \bar{r} - R = 1.8755$ mm

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25$ mm^2 $= 804.25 \times 10^{-6}$ m^2

$P = 2.5 \times 10^3$ N $\qquad M = P\bar{r} = (2.5 \times 10^3)(36 \times 10^{-3}) = 90$ N·m

(a) Point A: $\quad y_A = R - r_1 = 34.1245 - 20 = 14.1245$ mm

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(14.1245 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(20 \times 10^{-3})}$$

$= -45.2 \times 10^6$ Pa $= -45.2$ MPa

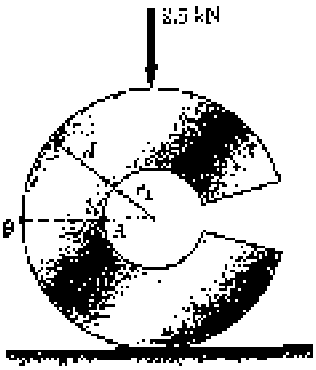
(b) Point B: $\quad y_B = R - r_2 = 34.1245 - 52 = -17.8755$ mm

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(-17.8755 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(52 \times 10^{-3})}$$

$= 17.40 \times 10^6$ Pa $= 17.40$ MPa

PROBLEM 4.198

4.198 The split ring shown has an inner radius $r_1 = 16$ mm and a circular cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

$$c = \tfrac{1}{2}d = 16 \text{ mm}, \quad r_1 = 16 \text{ mm}, \quad r_2 = r_1 + d = 48 \text{ mm}$$

$$\bar{r} = r_1 + c = 32 \text{ mm}$$

$$R = \tfrac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \tfrac{1}{2}\left[32 + \sqrt{32^2 - 16^2}\right]$$

$$= 29.8564 \text{ mm}$$

$$e = \bar{r} - R = 2.1436 \text{ mm}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25 \text{ mm}^2 = 804.25 \times 10^{-6} \text{ m}^2$$

$$P = 2.5 \times 10^3 \text{ N} \qquad M = P\bar{r} = (2.5 \times 10^3)(32 \times 10^{-3}) = 80 \text{ N}\cdot\text{m}$$

(a) Point A: $y_A = R - r_1 = 29.8564 - 16 = 13.8564$ mm

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(80)(13.8564 \times 10^{-3})}{(804.25 \times 10^{-6})(2.1436 \times 10^{-3})(16 \times 10^{-3})}$$

$$= -43.3 \times 10^6 \text{ Pa} = -43.3 \text{ MPa}$$

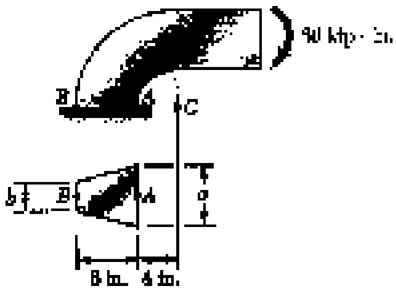
(b) Point B: $y_B = R - r_2 = 29.8564 - 48 = -18.1436$ mm

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(80)(-18.1436 \times 10^{-3})}{(804.25 \times 10^{-6})(2.1436 \times 10^{-3})(48 \times 10^{-3})}$$

$$= 14.43 \times 10^6 \text{ Pa} = 14.43 \text{ MPa}$$

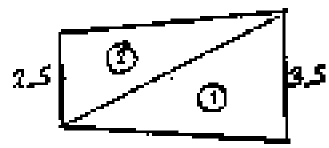
PROBLEM 4.199

4.199 Knowing that the machine component shown has a trapezoidal cross section with $a = 3.5$ in. and $b = 2.5$ in., determine the stress at (a) point A, (b) point B.



SOLUTION

Locate centroid



	A, in²	$\bar{r}$, in	$A\bar{r}$, in³
①	10.5	6	63
②	7.5	8	60
Σ	18		123

$$\bar{r} = \frac{123}{18} = 6.8333 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1+b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(3.5+2.5)}{[(3.5)(10)-(2.5)(4)]\ln\frac{10}{4} - (6)(3.5-2.5)} = 6.3878 \text{ in}$$

$$e = \bar{r} - R = 0.4452 \text{ in} \qquad M = 80 \text{ kip·in.}$$

(a) $y_A = R - r_1 = 6.3878 - 4 = 2.3878$ in.

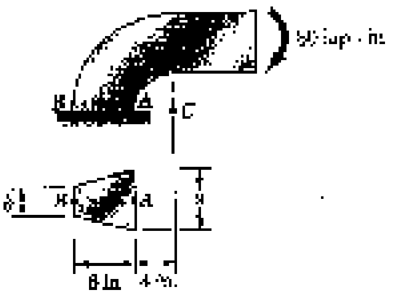
$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(80)(2.3878)}{(18)(0.4452)(4)} = -5.96 \text{ ksi}$$

(b) $y_B = R - r_2 = 6.3878 - 10 = -3.6122$ in.

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(80)(-3.6122)}{(18)(0.4452)(10)} = 3.61 \text{ ksi}$$

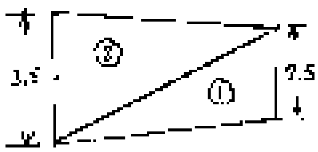
PROBLEM 4.200

4.200 Knowing that the machine component shown has a trapezoidal cross section with $a = 2.5$ in. and $b = 3.5$ in., determine the stress at (a) point A, (b) point B.



SOLUTION

Locate centroid



	A, in²	$\bar{r}$, in.	$A\bar{r}$, in³
1	7.5	6	45
2	10.5	8	84
Σ	18		129

$$\bar{r} = \frac{129}{18} = 7.1667 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1+b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(2.5+3.5)}{[(2.5)(10)-(3.5)(4)]\ln\frac{10}{4} - (6)(2.5-3.5)} = 6.7168 \text{ in.}$$

$$e = \bar{r} - R = 0.4499 \text{ in.} \qquad M = 80 \text{ kip}\cdot\text{in.}$$

(a) $y_A = R - r_1 = 2.7168$ in.

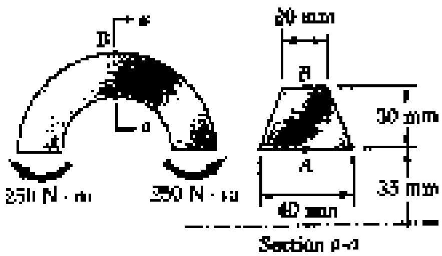
$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(80)(2.7168)}{(18)(0.4499)(4)} = -6.71 \text{ ksi}$$

(b) $y_B = R - r_2 = -3.2832$ in.

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(80)(-3.2832)}{(18)(0.4499)(10)} = 3.24 \text{ ksi}$$

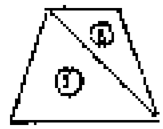
PROBLEM 4.201

4.201 For the curved beam and loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

Locate centroid



	A, mm²	$\bar{r}$, mm	$A\bar{r}$, mm³
1	600	45	27×10^3
2	300	55	16.5×10^3
Σ	900		43.5×10^3

$$\bar{r} = \frac{43.5 \times 10^3}{900} = 48.333 \text{ mm}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_2 - b_1)}$$

$$= \frac{(0.5)(30)^2(40 + 20)}{[(40)(65) - (20)(35)]\ln\frac{65}{35} - (30)(40 - 20)} = 46.8608 \text{ mm}$$

$$e = \bar{r} - R = 1.4725 \text{ mm} \qquad M = -250 \text{ N}\cdot\text{m}$$

(a) $y_A = R - r_1 = 11.8608$ mm.

$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(-250)(11.8608 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(35 \times 10^{-3})} = 63.9 \times 10^6 \text{ Pa}$$

$$= 63.9 \text{ MPa}$$

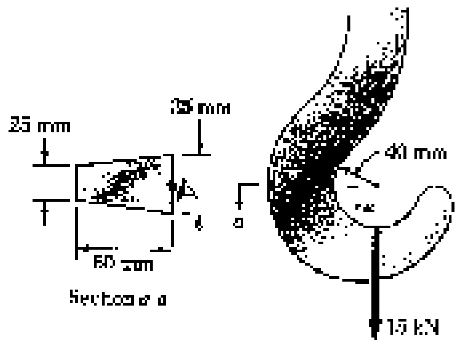
(b) $y_B = R - r_2 = -18.1392$ mm

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(-250)(-18.1392 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(65 \times 10^{-3})} = -52.6 \times 10^6 \text{ Pa}$$

$$= -52.6 \text{ MPa}$$

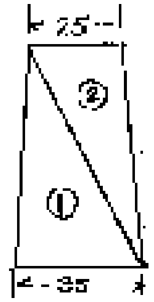
PROBLEM 4.202

4.202 For the crane hook shown, determine the largest tensile stress in section *a-a*.



SOLUTION

Locate centroid.



	A, mm²	$\bar{r}$, mm	$A\bar{r}$, mm³
1	1050	60	63×10^3
2	750	80	60×10^3
Σ	1800		103×10^3

$$\bar{r} = \frac{103 \times 10^3}{1800} = 68.333 \text{ mm}$$

Force-couple system at centroid: $P = 15 \times 10^3$ N

$$M = -P\bar{r} = -(15 \times 10^3)(68.333 \times 10^{-3}) = -1.025 \times 10^3 \text{ N}\cdot\text{m}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(60)^2(35 + 25)}{[(35)(100) - (25)(40)]\ln\frac{100}{40} - (60)(35 + 25)} = 63.878 \text{ mm}$$

$$e = \bar{r} - R = 4.452 \text{ mm}$$

Maximum tensile stress occurs at point A

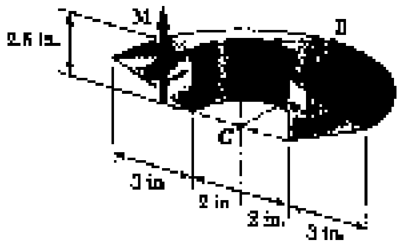
$$y_A = R - r_1 = 23.878 \text{ mm}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{15 \times 10^3}{1800 \times 10^{-6}} - \frac{-(1.025 \times 10^3)(23.878 \times 10^{-3})}{(1800 \times 10^{-6})(4.452 \times 10^{-3})(40 \times 10^{-3})}$$

$$= 84.7 \times 10^6 \text{ Pa} = 84.7 \text{ MPa}$$

PROBLEM 4.203

4.203 and 4.204 Knowing that $M = 5$ kip·in., determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

$$A = \frac{1}{2}bh = \frac{1}{2}(2.5)(3) = 3.75\ \text{in}^2$$

$$\bar{r} = 2 + 1 = 3.00000\ \text{in}$$

$$b_1 = 2.5\ \text{in.},\quad r_1 = 2\ \text{in},\quad b_2 = 0,\quad r_2 = 5\ \text{in.}$$

Use formula for trapezoid

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(2.5+0)}{[(2.5)(5) - (0)(2)]\ln\frac{5}{2} - (3)(2.5 - 0)} = 2.84548\ \text{in.}$$

$$e = \bar{r} - R = 0.15452\ \text{in.} \qquad M = 5\ \text{kip·in}$$

(a) $y_A = R - r_1 = 0.84548\ \text{in.}$

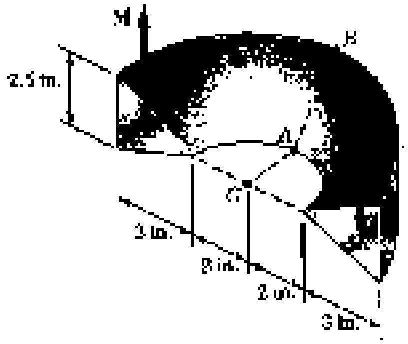
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(5)(0.84548)}{(3.75)(0.15452)(2)} = -3.65\ \text{ksi}$$

(b) $y_B = R - r_2 = -2.15452\ \text{in.}$

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(5)(-2.15452)}{(3.75)(0.15452)(5)} = 3.72\ \text{ksi}$$

PROBLEM 4.204

4.203 and 4.204 Knowing that $M = 5$ kip·in., determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

$A = \frac{1}{2}(2.5)(3) = 3.75 \text{ in}^2$

$\bar{r} = 2 + 2 = 4.00000 \text{ in}$

$b_1 = 0, \quad r_1 = 2 \text{ in.}, \quad b_2 = 2.5 \text{ in.} \quad r_2 = 5 \text{ in.}$

Use formula for trapezoid.

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(0 + 2.5)}{[(0)(5) - (2.5)(2)]\ln\frac{5}{2} - (3)(0 - 2.5)} = 3.85466 \text{ in}$$

$e = \bar{r} - R = 0.14534 \text{ in.} \qquad M = 5 \text{ kip}\cdot\text{in.}$

(a) $y_A = R - r_1 = 1.85466 \text{ in}$

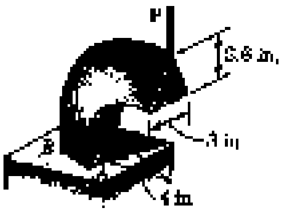
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(5)(1.85466)}{(3.75)(0.14534)(2)} = -8.51 \text{ ksi}$$

(b) $y_B = R - r_2 = -1.14534 \text{ in.}$

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(5)(-1.14534)}{(3.75)(0.14534)(5)} = 2.10 \text{ ksi}$$

PROBLEM 4.205

4.205 Knowing that $P = 3.5$ kips, determine the stress at (a) point A, (b) point B.



SOLUTION

$b = 3$ in, $h = 3.6$ in, $r_1 = 4$ in

$A = \frac{1}{2}bh = \frac{1}{2}(3)(3.6) = 5.4$ in^2

$r_2 = r_1 + h = 7.6$ in. $\qquad \bar{r} = r_1 + \frac{1}{3}h = 5.2$ in

Reduce section forces to a force-couple system at the centroid

$P = 3.5$ kips $\qquad M = P\bar{r} = (3.5)(5.2) = 18.2$ kip-in

For a triangular section $R = \dfrac{\frac{1}{2}h}{\frac{r_2}{h}\ln\frac{r_2}{r_1} - 1}$

$$= \frac{(0.5)(3.6)}{\frac{7.6}{3.6}\ln\frac{7.6}{4} - 1} = 5.07007 \text{ in}$$

$e = \bar{r} - R = 0.12993$ in.

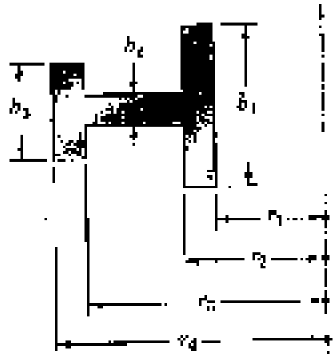
(a) $y_A = R - r_1 = 1.07007$ in.

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{3.5}{5.4} - \frac{(18.2)(1.07007)}{(5.4)(0.12993)(4)} = -7.59 \text{ ksi}$$

(b) $y_B = R - r_2 = 2.52993$ in

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{3.5}{5.4} - \frac{(18.2)(-2.52993)}{(5.4)(0.12993)(7.6)} = 7.99 \text{ ksi}$$

PROBLEM 4.206



4.206 Show that if the cross section of a curved beam consists of two or more rectangles, the radius R of the neutral surface can be expressed as

$$R = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]}$$

where A is the total area of the cross section.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r}\,dA} = \frac{A}{\Sigma\, b_i \ln \frac{r_{i+1}}{r_i}}$$

$$= \frac{A}{\Sigma \ln\left(\frac{r_{i+1}}{r_i}\right)^{b_i}} = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]}$$

Note that for each rectangle $\int \frac{1}{r}\,dA = \int_{r_i}^{r_{i+1}} b_i \frac{dr}{r}$

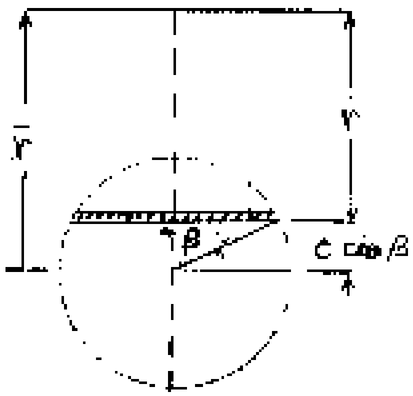
$$= b_i \int_{r_i}^{r_{i+1}} \frac{dr}{r} = b_i \ln \frac{r_{i+1}}{r_i}$$

PROBLEM 4.207

4.207 through 4.209 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

*4.207 A circular cross section

SOLUTION



Use polar coordinate β as shown

width $w = 2c\sin\beta$

$r = \bar{r} - c\cos\beta$

$dr = c\sin\beta\, d\beta$

$dA = w\,dr = 2c^2\sin^2\beta\, d\beta$

$$\int\frac{dA}{r} = \int_0^\pi \frac{2c^2\sin^2\beta}{\bar{r} - c\cos\beta}\,d\beta$$

$$\int\frac{dA}{r} = \int_0^\pi \frac{c^2(1-\cos^2\beta)}{\bar{r}-c\cos\beta}\,d\beta = 2\int_0^\pi \frac{\bar{r}^2 - c^2\cos^2\beta - (\bar{r}^2 - c^2)}{\bar{r} - c\cos\beta}\,d\beta$$

$$= 2\int_0^\pi (\bar{r} + c\cos\beta)\,d\beta - 2(\bar{r}^2 - c^2)\int_0^\pi \frac{dr}{\bar{r} - c\cos\beta}$$

$$= 2\bar{r}\beta\Big|_0^\pi + 2c\sin\beta\Big]_0^\pi$$

$$- 2(\bar{r}^2 - c^2)\frac{2}{\sqrt{\bar{r}^2 - c^2}}\tan^{-1}\frac{\sqrt{\bar{r}^2 - c^2}\tan\frac{1}{2}\beta}{\bar{r}+c}\Big]_0^\pi$$

$$= 2\bar{r}(\pi - 0) + 2c(0-0) - 4\sqrt{\bar{r}^2 - c^2}\cdot\left(\frac{\pi}{2} - 0\right)$$

$$2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2 - c^2}$$

$$A = \pi c^2$$

$$R = \frac{A}{\int\frac{dA}{r}} = \frac{\pi c^2}{2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2 - c^2}}$$

$$= \frac{1}{2}\,\frac{c^2}{\bar{r} - \sqrt{\bar{r}^2 - c^2}}\cdot\frac{\bar{r} + \sqrt{\bar{r}^2 - c^2}}{\bar{r} + \sqrt{\bar{r}^2 - c^2}}$$

$$= \frac{1}{2}\,\frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{\bar{r}^2 - (\bar{r}^2 - c^2)} = \frac{1}{2}\,\frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{c^2}$$

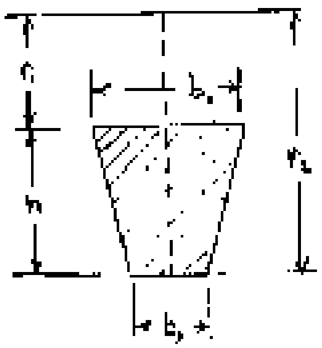
$$= \frac{1}{2}\left(\bar{r} + \sqrt{\bar{r}^2 - c^2}\right)$$

PROBLEM 4.208

4.207 through 4.209 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

4.208 A trapezoidal section

SOLUTION



The section width w varies linearly with r

$$w = c_0 + c_1 r$$

$$w = b_1 \text{ at } r = r_1 \quad \text{and} \quad w = b_2 \text{ at } r = r_2$$

$$b_1 = c_0 + c_1 r_1$$

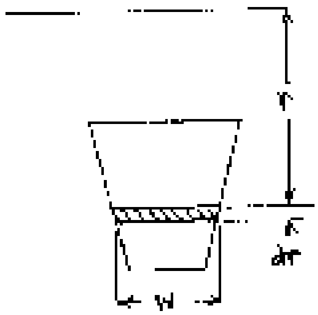
$$b_2 = c_0 + c_1 r_2$$

$$b_1 - b_2 = c_1(r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b_1 - b_2}{h}$$

$$r_2 b_1 - r_1 b_2 = (r_2 - r_1) c_0 = h c_0$$

$$c_0 = \frac{r_2 b_1 - r_1 b_2}{h}$$



$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r}\,dr = \int_{r_1}^{r_2} \frac{c_0 + c_1 r}{r}\,dr$$

$$= c_0 \ln r \Big|_{r_1}^{r_2} + c_1 r \Big|_{r_1}^{r_2}$$

$$= c_0 \ln \frac{r_2}{r_1} + c_1 (r_2 - r_1)$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - \frac{b_1 - b_2}{h} h$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - (b_1 - b_2)$$

$$A = \frac{1}{2}(b_1 + b_2) h$$

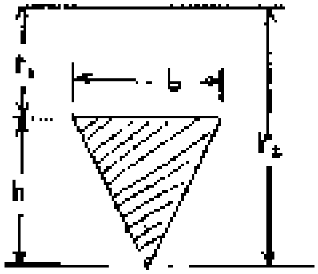
$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(r_2 b_1 - r_1 b_2) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

PROBLEM 4.209

4.207 through 4.209 Using Eq. (4.66), derive the expression for R given in Fig. 4.79 for

4.209 A triangular cross section

SOLUTION



The section width w varies linearly with r.

$$w = c_0 + c_1 r$$

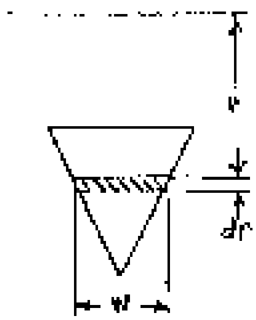
$$w = b \text{ at } r = r_1 \quad \text{and} \quad w = 0 \text{ at } r = r_2$$

$$b = c_0 + c_1 r_1$$

$$0 = c_0 + c_1 r_2$$

$$b = c_1(r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b}{h} \quad \text{and} \quad c_0 = -c_1 r_2 = \frac{b r_2}{h}$$



$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r}\,dr = \int_{r_1}^{r_2} \frac{c_0 + c_1 r}{r}\,dr$$

$$= c_0 \ln r \Big|_{r_1}^{r_2} + c_1 r \Big|_{r_1}^{r_2}$$

$$= c_0 \ln \frac{r_2}{r_1} + c_1 (r_2 - r_1)$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - \frac{b}{h} h$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - b = b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1\right)$$

$$A = \frac{1}{2} b h$$

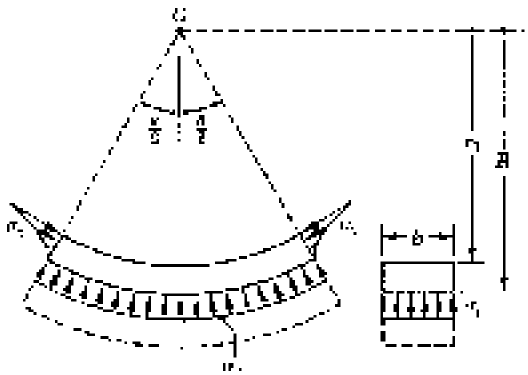
$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} b h}{b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1\right)} = \frac{\frac{1}{2} h}{\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1}$$

PROBLEM 4.210

*4.210 For a curved bar of rectangular cross section subjected to a bending couple M, show that the radial stress at the neutral surface is

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

and compute the value of σ_r for the curved bar of Examples 4.10 and 4.11. (*Hint:* consider the free-body diagram of the portion of the beam located above the neutral surface.)



SOLUTION

At radial distance r

$$\sigma_\theta = \frac{M(r-R)}{Aer} = \frac{M}{Ae} - \frac{MR}{Aer}$$

For portion above the neutral axis, the resultant force is

$$H = \int \sigma_\theta \, dA = \int_{r_1}^{R} \sigma_\theta \, b \, dr$$

$$= \frac{Mb}{Ae}\int_{r_1}^{R} dr - \frac{MRb}{Ae}\int_{r_1}^{R}\frac{dr}{r}$$

$$= \frac{Mb}{Ae}(R - r_1) - \frac{MRb}{Ae}\ln\frac{R}{r_1} = \frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

Resultant of σ_r

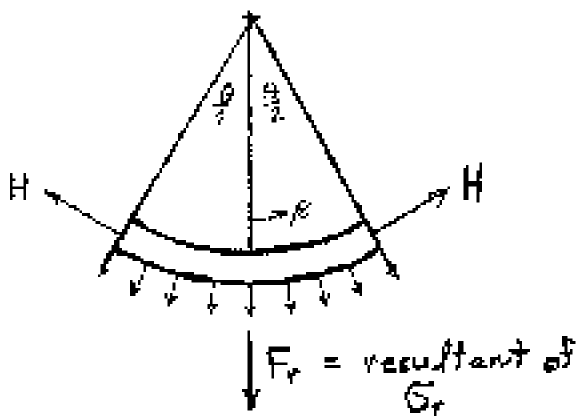
$$F_r = \int \sigma_r \cos\beta \, dA$$

$$= \int_{-\frac{\phi}{2}}^{\frac{\phi}{2}} \sigma_r \cos\beta \; b\,R\,d\beta$$

$$= \sigma_r bR \int_{-\frac{\phi}{2}}^{\frac{\phi}{2}} \cos\beta \, d\beta$$

$$= \sigma_r bR \sin\beta \Big|_{-\frac{\phi}{2}}^{\frac{\phi}{2}}$$

$$= 2\sigma_r bR \sin\frac{\phi}{2}$$

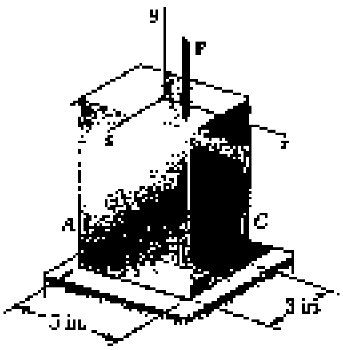


For equilibrium $\quad F_r - 2H\sin\frac{\phi}{2} = 0$

$$2\sigma_r bR\sin\frac{\phi}{2} - 2\frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right) = 0$$

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

PROBLEM 4.211



4.211 A single vertical force **P** is applied to a short steel post as shown. Gages located at A, B, and C indicate the following strains:

$\epsilon_A = -500\ \mu$ $\qquad$ $\epsilon_B = -1000\ \mu$ $\qquad$ $\epsilon_C = -200\ \mu$

Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the magnitude of **P**, (*b*) the line of action of **P**, (*c*) the corresponding strain at the hidden edge of the post, where $x = -2.5$ in. and $z = -1.5$ in.

SOLUTION

$$I_x = \frac{1}{12}(5)(3)^3 = 11.25 \text{ in}^4$$

$$I_z = \frac{1}{12}(3)(5)^3 = 31.25 \text{ in}^4$$

$$A = (5)(3) = 15 \text{ in}^2$$

$$M_x = Pz \qquad M_z = -Px$$

$$x_A = -2.5 \text{ in}, \quad x_B = 2.5 \text{ in}, \quad x_C = 2.5 \text{ in}, \quad x_D = -2.5 \text{ in}$$

$$z_A = 1.5 \text{ in}, \quad z_B = 1.5 \text{ in}, \quad z_C = -1.5 \text{ in}, \quad z_D = -1.5 \text{ in}$$

$$\sigma_A = E\epsilon_A = (29\times10^6)(-500\times10^{-6}) = -14500 \text{ psi} = -14.5 \text{ ksi}$$

$$\sigma_B = E\epsilon_B = (29\times10^6)(-1000\times10^{-6}) = -29000 \text{ psi} = -29 \text{ ksi}$$

$$\sigma_C = E\epsilon_C = (29\times10^6)(-200\times10^{-6}) = -5800 \text{ psi} = -5.8 \text{ ksi}$$

$$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -0.06667P - 0.13333M_x - 0.08M_z \qquad (1)$$

$$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -0.06667P - 0.13333M_x + 0.08M_z \qquad (2)$$

$$\sigma_C = -\frac{P}{A} - \frac{M_x z_C}{I_x} + \frac{M_z x_C}{I_z} = -0.06667P + 0.13333M_x + 0.08M_z \qquad (3)$$

Substituting the values for σ_A, σ_B, and σ_C into (1), (2), and (3) and solving the simultaneous equations gives

$$M_x = 87 \text{ kip}\cdot\text{in}, \quad M_z = -90.625 \text{ kip}\cdot\text{in}, \quad P = 152.25 \text{ kips}$$

$$x = -\frac{M_z}{P} = -\frac{-90.625}{152.25} = 0.595 \text{ in.}$$

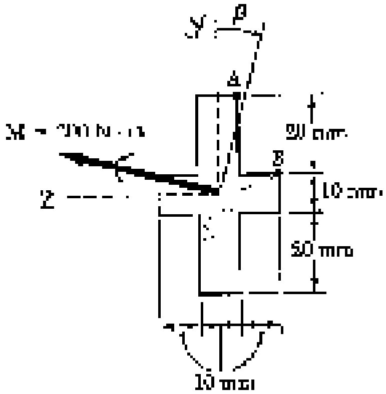
$$z = \frac{M_x}{P} = \frac{87}{152.25} = 0.571 \text{ in.}$$

$$\sigma_D = -\frac{P}{A} - \frac{M_x z_D}{I_x} + \frac{M_z x_D}{I_z} = -0.06667P + 0.13333M_x - 0.08M_z$$

$$= -(0.06667)(152.25) + (0.13333)(87) - (0.08)(-90.625)$$

$$= 8.70 \text{ ksi}$$

PROBLEM 4.212



4.212 The couple **M**, which acts in a vertical plane ($\beta = 0$), is applied to an aluminum beam of the cross section shown. Determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the radius of curvature of the beam. Use $E = 72$ GPa.

SOLUTION

Label axes y and z as shown on the sketch.

$$I_z = \frac{1}{12}(10)(50)^3 + 2 \cdot \frac{1}{12}(10)(10)^3$$

$$= 0.105833 \times 10^{6}\ \text{mm}^4 = 0.105833 \times 10^{-6}\ \text{m}^4$$

$$M_z = 300\ \text{N} \cdot \text{m} \qquad M_y = 0$$

(a) $y_A = 25$ mm $\qquad \sigma_A = -\frac{M_z y_A}{I_z} = -\frac{(300)(25 \times 10^{-3})}{0.105833 \times 10^{-6}} = -70.9 \times 10^{6}$ Pa

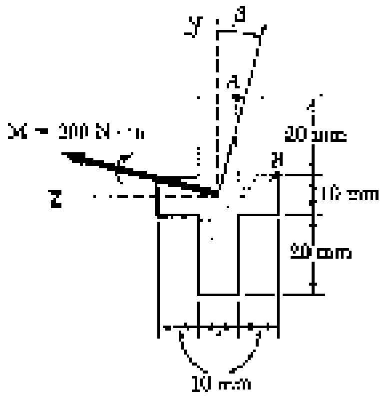
$= -70.9$ MPa ◄

(b) $y_B = 5$ mm $\qquad \sigma_B = -\frac{M_z y_B}{I_z} = -\frac{(300)(5 \times 10^{-3})}{0.105833 \times 10^{-6}} = -14.17 \times 10^{6}$ Pa

$= -14.17$ MPa ◄

(c) $\frac{1}{\rho} = \frac{M_z}{E I_z} \quad \therefore \quad \rho = \frac{E I_z}{M_z} = \frac{(72 \times 10^{9})(0.105833 \times 10^{-6})}{300} = 25.4$ m ◄

PROBLEM 4.213



4.213 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle $\beta = 15°$ with the vertical. Determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the angle that the neutral axis forms with the horizontal.

SOLUTION

Label axes y and z as shown on the sketch

$$I_z = \frac{1}{12}(10)(50)^3 + 2\cdot\frac{1}{12}(10)(10)^3$$

$$= 0.105833\times10^6\ \text{mm}^4 = 0.105833\times10^{-6}\ \text{m}^4$$

$$I_y = \frac{1}{12}(10)(30)^3 + 2\cdot\frac{1}{12}(20)(10)^3$$

$$= 0.025833\times10^6\ \text{mm}^4 = 0.025833\times10^{-6}\ \text{m}^4$$

For $\beta = 15°$ $\qquad M_z = 300\cos 15° = 289.78\ \text{N·m}$

$$M_y = 300\sin 15° = 77.65\ \text{N·m}$$

(a) $\quad y_A = 25\ \text{mm}, \quad z_A = -5\ \text{mm}$

$$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(289.78)(25\times10^{-3})}{0.105833\times10^{-6}} + \frac{(77.65)(-5\times10^{-3})}{0.025833\times10^{-6}}$$

$$= -83.5\times10^6\ \text{Pa} = -83.5\ \text{MPa}$$

(b) $\quad y_B = 5\ \text{mm}, \quad z_B = -15\ \text{mm}$

$$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(289.78)(5\times10^{-3})}{0.105833\times10^{-6}} + \frac{(77.65)(-15\times10^{-3})}{0.025833\times10^{-6}}$$

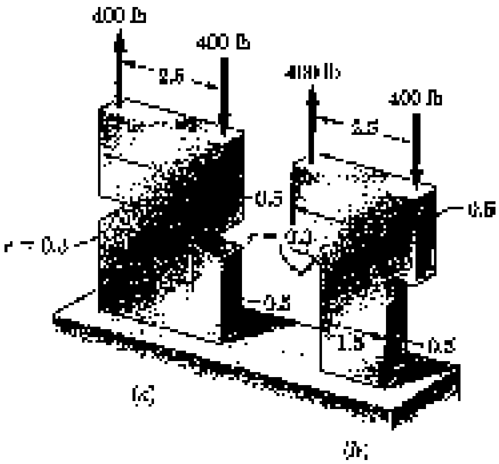
$$= -58.8\times10^6\ \text{Pa} = -58.8\ \text{MPa}$$

(c) $$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{0.105833\times10^{-6}}{0.025833\times10^{-6}}\tan 15° = 1.0977$$

$$\varphi = 47.7°$$

PROBLEM 4.214

4.214 Determine the maximum stress in each of the two machine elements shown.



All dimensions in inches

SOLUTION

For each case $M = (400)(2.5) = 1000$ lb·in

At the minimum section

$I = \frac{1}{12}(0.5)(1.5)^3 = 0.140625$ in^4

$c = 0.75$ in.

(a) $D/d = 3/1.5 = 2$

$r/d = 0.3/1.5 = 0.2$

From Fig 4.32 $K = 1.75$

$\sigma_{max} = \frac{KMc}{I} = \frac{(1.75)(1000)(0.75)}{0.140625} = 9.33 \times 10^3$ psi $= 9.33$ ksi ◄

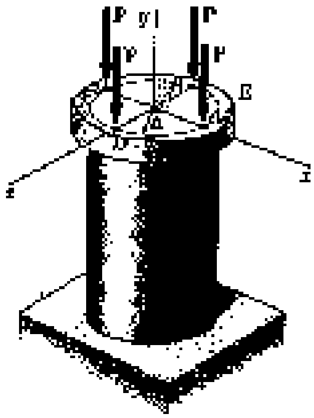
(b) $D/d = 3/1.5 = 2$ $r/d = 0.3/1.5 = 0.2$

From Fig 4.31 $K = 1.60$

$\sigma_{max} = \frac{KMc}{I} = \frac{(1.60)(1000)(0.75)}{0.140625} = 8.00 \times 10^3$ psi $= 8.00$ ksi ◄

PROBLEM 4.215

4.215 The four forces shown are applied to a rigid plate supported by a solid steel post of radius a. Determine the maximum stress in the post when (a) all four forces are applied, (b) the force at D is removed, (c) the forces at C and D are removed.



SOLUTION

For a solid circular section of radius a

$A = \pi a^2 \qquad I = \frac{\pi}{4} a^4$

(a) Centric force $\quad F = 4P, \quad M_x = M_z = 0$

$$\sigma = -\frac{F}{A} = -\frac{4P}{\pi a^2} = -1.273\ P/a^2$$

(b) Force at D is removed.

$F = 3P, \quad M_x = -Pa, \quad M_z = 0$

$$\sigma = -\frac{F}{A} - \frac{M_x z}{I} = -\frac{3P}{\pi a^2} - \frac{(-Pa)(-a)}{\frac{\pi}{4} a^4} = -\frac{7P}{\pi a^2} = -2.228\ P/a^2$$

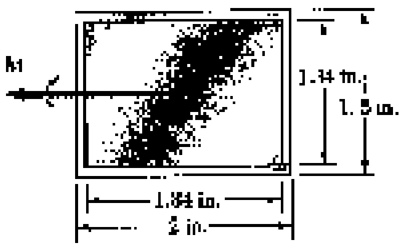
(c) Forces at C and D are removed

$F = 2P \qquad M_x = -Pa, \qquad M_z = -Pa$

Resultant bending couple $\quad M = \sqrt{M_x^2 + M_z^2} = \sqrt{2}\, Pa$

$$\sigma = -\frac{F}{A} - \frac{Mc}{I} = -\frac{2P}{\pi a^2} - \frac{\sqrt{2}\, Pa\, a}{\frac{\pi}{4} a^4} = -\frac{2+4\sqrt{2}}{\pi}\frac{P}{a^2} = -2.437\ P/a^2$$

PROBLEM 4.216



4.216 In order to increase corrosion resistance, a 0.08-in.-thick cladding of aluminum has been added to a steel bar as shown. The modulus of elasticity is 29×10^6 psi for steel and 10.4×10^6 psi for aluminum. For a bending moment of 12 kip-in., determine (*a*) the maximum stress in the steel, (*b*) the maximum stress in the aluminum, (*c*) the radius of curvature of the bar.

SOLUTION

Use steel as the reference material

$$n_{steel} = 1 \qquad n_{alum} = \frac{E_a}{E_s} = \frac{10.4}{29} = 0.3586$$

$$I_{trans} = I_{steel} + n_{alum} I_{alum}$$

$$= \frac{1}{12}(1.84)(1.34)^3 + 0.3586 \cdot \frac{1}{12}\left[(2)(1.5)^3 - (1.84)(1.34)^3\right] = 0.43835 \text{ in}^4$$

(a) $y_s = \frac{1.34}{2} = 0.67$ in

$$\sigma_s = \frac{M y_s}{I_{trans}} = \frac{(12)(0.67)}{0.43835} = 18.35 \text{ ksi}$$

(b) $y_a = \frac{1.5}{2} = 0.75$ in

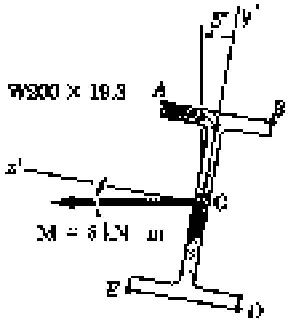
$$\sigma_a = n_a \frac{M y_a}{I} = 0.3586 \frac{(12)(0.75)}{0.43835} = 7.36 \text{ ksi}$$

(c) $$\frac{1}{\rho} = \frac{M}{E_{st} I_{trans}} = \frac{12 \times 10^3}{(29 \times 10^6)(0.43835)} = 944 \times 10^{-6} \text{ in}^{-1}$$

$$\rho = 1059 \text{ in} = 88.3 \text{ ft.}$$

PROBLEM 4.217

4.217 A couple **M** of moment 8 kN·m acting in a vertical plane is applied to a W 200 × 19.3 rolled-steel beam as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum stress in the beam.



SOLUTION

For W 200 × 19.3 rolled steel section

$$I_{z'} = 16.6 \times 10^6 \ \text{mm}^4 = 16.6 \times 10^{-6} \ \text{m}^4$$

$$I_{y'} = 1.15 \times 10^6 \ \text{mm}^4 = 1.15 \times 10^{-6} \ \text{m}^4$$

$$y_A = y_B = -y_D = -y_E = \frac{203}{2} = 101.5 \ \text{mm}$$

$$z_A = -z_B = -z_D = z_E = \frac{102}{2} = 51 \ \text{mm}$$

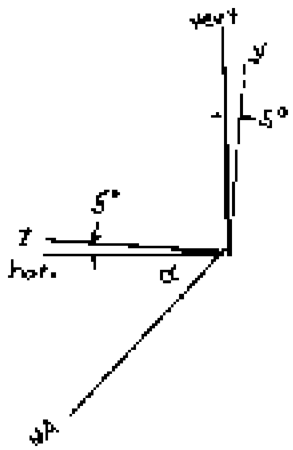
$$M_z = (8 \times 10^3)\cos 5^\circ = 7.9696 \times 10^3 \ \text{N}\cdot\text{m}$$

$$M_y = -(8 \times 10^3)\sin 5^\circ = -0.6972 \times 10^3 \ \text{N}\cdot\text{m}$$

(a) $$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{16.6 \times 10^{-6}}{1.15 \times 10^{-6}}\tan(-5^\circ) = -1.2629$$

$$\varphi = -51.6^\circ$$

$$\alpha = 51.6^\circ - 5^\circ = 46.6^\circ$$



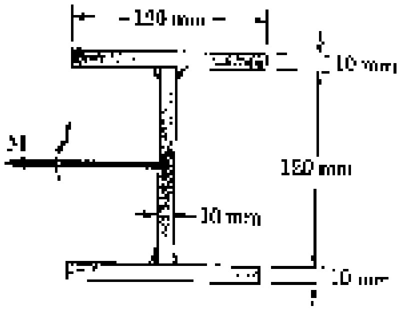
(b) Maximum tensile stress occurs at point D

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(7.9696 \times 10^3)(-101.5 \times 10^{-3})}{16.6 \times 10^{-6}} + \frac{(0.6972 \times 10^3)(51 \times 10^{-3})}{1.15 \times 10^{-6}}$$

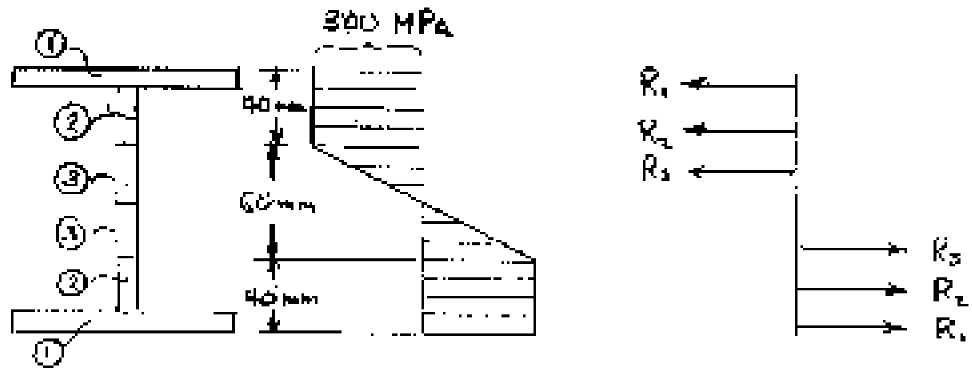
$$= 79.6 \times 10^6 \ \text{Pa} = 79.6 \ \text{MPa}$$

PROBLEM 4.218

4.218 Three 120 × 10-mm steel plates have been welded together to form the beam shown. Assuming that the steel is elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa, determine (*a*) the bending moment for which the plastic zones at the top and bottom of the beam are 40 mm thick, (*b*) the corresponding radius of curvature of the beam.



SOLUTION



$A_1 = (120)(10) = 1200\ mm^2 \qquad R_1 = \sigma_Y A_1 = (300\times10^6)(1200\times10^{-6}) = 360\times10^3\ N$

$A_2 = (30)(10) = 300\ mm^2 \qquad R_2 = \sigma_Y A_2 = (300\times10^6)(300\times10^{-6}) = 90\times10^3\ N$

$A_3 = (30)(10) = 300\ mm^2 \qquad R_3 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(300\times10^6)(300\times10^{-6})$

$= 45\times10^3\ N$

$y_1 = 65\ mm = 65\times10^{-3}\ m$

$y_2 = 45\ mm = 45\times10^{-3}\ m$

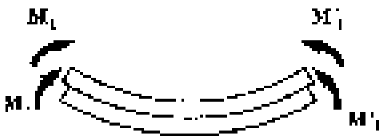
$y_3 = 20\ mm = 20\times10^{-3}\ m$

(a) $M = 2(R_1y_1 + R_2y_2 + R_3y_3) = 2\{(360)(65) + (90)(45) + (45)(20)\}$

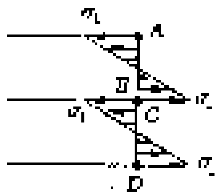
$= 56.7\times10^3\ N\cdot m = 56.7\ kN\cdot m$

(b) $\dfrac{y_Y}{\rho} = \dfrac{\sigma_Y}{E} \qquad \rho = \dfrac{Ey_Y}{\sigma_Y} = \dfrac{(200\times10^9)(30\times10^{-3})}{300\times10^6} = 20\ m$

PROBLEM 4.219



4.219 Two thin strips of the same material and same cross section are bent by couples of the same magnitude and glued together. After the two surfaces of contact have been securely bonded, the couples are removed. Denoting by σ_1 the maximum stress and by ρ_1 the radius of curvature of each strip while the couples were applied, determine (*a*) the final stresses at points *A*, *B*, *C*, and *D*, (*b*) the final radius of curvature.



SOLUTION

Let b = width and t = thickness of one strip.

Loading one strip $\quad M = M_1$

$$I_1 = \frac{1}{12} b t^3, \qquad c = \frac{1}{2} t$$

$$\sigma_1 = \frac{M_1 c}{I} = \frac{6 M_1}{b t^2}$$

$$\frac{1}{\rho_1} = \frac{M_1}{E I_1} = \frac{12 M_1}{E t^3}$$

After M_1 is applied to each of the strips, the stresses are those given in the sketch above. They are

$$\sigma_A = -\sigma_1, \quad \sigma_B = \sigma_1, \quad \sigma_C = -\sigma_1, \quad \sigma_D = \sigma_1$$

The total bending couple is $2M_1$.

After gluing, this couple is removed.

$$M' = 2M_1, \qquad I' = \frac{1}{12} b (2t)^3 = \frac{2}{3} b t^3$$

$c = t$. The stresses removed are

$$\sigma' = -\frac{M'y}{I'} = -\frac{2M_1 y}{\frac{2}{3} b t^3} = -\frac{3 M_1 y}{b t^3}$$

$$\sigma'_A = -\frac{3M_1}{bt^2} = -\frac{1}{2}\sigma_1, \quad \sigma'_B = \sigma'_C = 0, \quad \sigma'_D = \frac{3M_1}{bt^2} = \frac{1}{2}\sigma_1$$

(a) Final stresses: $\quad \sigma_A = -\sigma_1 - (-\frac{1}{2}\sigma_1) = -\frac{1}{2}\sigma_1$

$$\sigma_B = \sigma_1$$

$$\sigma_C = -\sigma_1$$

$$\sigma_D = \sigma_1 - \frac{1}{2}\sigma_1 = \frac{1}{2}\sigma_1$$

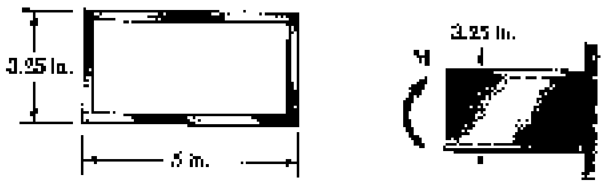
$$\frac{1}{\rho'} = \frac{M'}{EI'} = \frac{2M_1}{E\frac{2}{3}bt^3} = \frac{3M_1}{Et^3} = \frac{1}{4}\frac{1}{\rho_1}$$

(b) Final radius $\quad \frac{1}{\rho} = \frac{1}{\rho_1} - \frac{1}{\rho'} = \frac{1}{\rho_1} - \frac{1}{4}\frac{1}{\rho_1} = \frac{3}{4}\frac{1}{\rho_1}$

$$\rho = \frac{4}{3}\rho_1$$

PROBLEM 4.220

4.220 Knowing that the hollow beam shown has a uniform wall thickness of 0.25 in. determine (*a*) the largest couple that can be applied without exceeding the allowable stress of 20 ksi, (*b*) the corresponding radius of curvature of the beam.



SOLUTION

$E = 10.6 \times 10^6$ psi

$$I = \frac{1}{12} b_o h_o^3 - \frac{1}{12} b_i h_i^3 = \frac{1}{12}(5)(3.25)^3 - \frac{1}{12}(4.5)(2.75)^3 = 6.5046 \text{ in}^4$$

$$c = \frac{3.25}{2} = 1.625 \text{ in.}$$

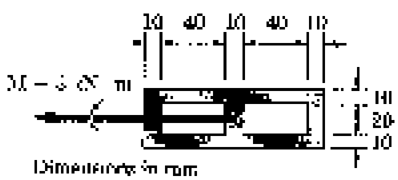
(a) $\sigma_{max} = \frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma_{max} I}{c} = \frac{(20)(6.5046)}{1.625} = 80.1 \text{ kip·in.}$

(b) $\varepsilon_{max} = \frac{c}{\rho} = \frac{\sigma_{max}}{E} \quad \therefore \quad \rho = \frac{Ec}{\sigma_{max}} = \frac{(10.6 \times 10^6)(1.625)}{20 \times 10^3}$

$$= 861 \text{ in} = 71.8 \text{ ft}$$

PROBLEM 4.221

4.221 A beam of the cross section shown is extruded from an aluminum alloy for which $E = 72$ GPa. Knowing that the couple shown acts in a vertical plane, determine (*a*) the maximum stress in the beam, (*b*) the corresponding radius of curvature.



SOLUTION

For outer rectangle: $b = 110$ mm, $h = 40$ mm

$$I_1 = \frac{1}{12} bh^3 = \frac{1}{12}(110)(40)^3 = 0.58667 \times 10^6 \text{ mm}^4$$

For one cutout rectangle: $b = 40$ mm, $h = 20$ mm

$$I_2 = \frac{1}{12} bh^3 = \frac{1}{12}(40)(20)^3 = 0.02667 \times 10^6 \text{ mm}^4$$

$$I = I_1 - 2I_2 = 0.53333 \times 10^6 \text{ mm}^4 = 0.53333 \times 10^{-6} \text{ m}^4$$

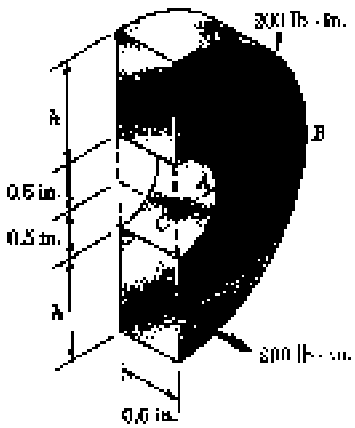
$M = 3 \times 10^3$ N·m $\qquad c = 20$ mm $= 20 \times 10^{-3}$ m

(a) $\sigma = \frac{Mc}{I} = \frac{(3 \times 10^3)(20 \times 10^{-3})}{0.53333 \times 10^{-6}} = 112.5 \times 10^6 \text{ Pa} = 112.5 \text{ MPa}$

(b) $\frac{1}{\rho} = \frac{M}{EI} \quad \therefore \quad \rho = \frac{EI}{M} = \frac{(72 \times 10^9)(0.53333 \times 10^{-6})}{3 \times 10^3} = 12.80 \text{ m}$

PROBLEM 4.222

4.222 For the machine element and loading shown, determine the stress at point A, knowing that (a) $h = 0.9$ in., (b) $h = 1.5$ in.



SOLUTION

(a) $h = 0.9$ in, $r_1 = 0.5$ in, $r_2 = 1.4$ in

$A = (0.6)(0.9) = 0.54 \text{ in}^2$

$R = \dfrac{h}{\ln \frac{r_2}{r_1}} = \dfrac{0.9}{\ln \frac{1.4}{0.5}} = 0.87411$ in

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 0.95$ in

$e = \bar{r} - R = 0.07589$ in

$M = -200$ lb·in , $y_A = R - r_1 = 0.37411$

$\sigma_A = -\dfrac{M y_A}{A e r_1} = -\dfrac{(-200)(0.37411)}{(0.54)(0.07589)(0.5)} = 3.65 \times 10^3$ psi $= 3.65$ ksi

(b) $h = 1.5$ in, $r_1 = 0.5$ in, $r_2 = 2.0$ in.

$A = (0.6)(1.5) = 0.90 \text{ in}^2$

$R = \dfrac{h}{\ln \frac{r_2}{r_1}} = \dfrac{1.5}{\ln \frac{2.0}{0.5}} = 1.08202$ in

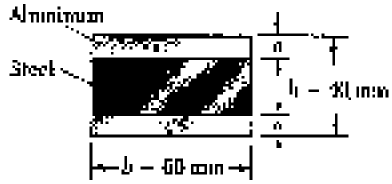
$\bar{r} = \frac{1}{2}(r_1 + r_2) = 1.25$ in.

$e = \bar{r} - R = 0.16798$ in.

$M = -200$ lb·in. $y_A = R - r_1 = 0.58202$ in

$\sigma_A = -\dfrac{M y_A}{A e r} = -\dfrac{(-200)(0.58202)}{(0.90)(0.16798)(0.5)} = 1.540 \times 10^3$ psi $= 1.540$ ksi

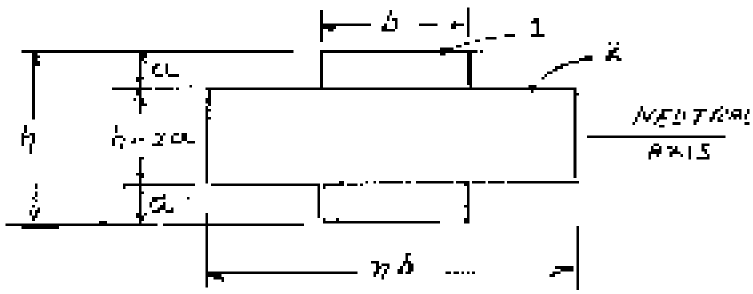
PROBLEM 4.C1

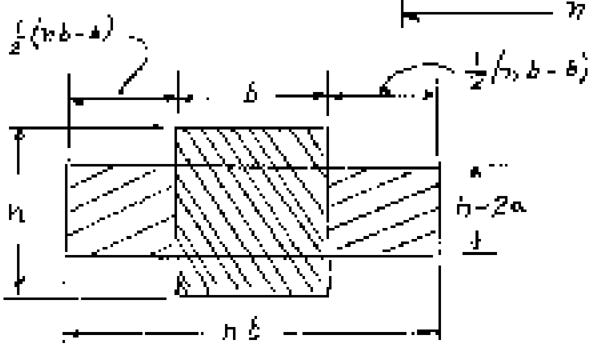


4.C1 Two aluminum strips and a steel strip are to be bonded together to form a composite member of width $b = 60$ mm and depth $h = 40$ mm. The modulus of elasticity is 200 GPa for the steel and 75 GPa for the aluminum. Knowing that $M = 1500$ N · m, write a computer program to calculate the maximum stress in the aluminum and in the steel for values of a from 0 to 20 mm using 2-mm increments. Using appropriate smaller increments, determine (*a*) the largest stress that can occur in the steel, (*b*) the corresponding value of a.

SOLUTION

TRANSFORMED SECTION (ALL STEEL) $n = \dfrac{E_{STEEL}}{E_{ALUM}}$





$$\bar{I} = \frac{1}{12} b h^3 + \frac{1}{12}\left[2\left(\frac{1}{2}\right)(nb-b)\right](h-2a)^3$$

AT POINT 1: $\sigma_{ALUM} = \dfrac{M\left(\frac{h}{2}\right)}{\bar{I}}$

AT POINT 2: $\sigma_{STEEL} = n\,\dfrac{M\left(\frac{h}{2}-a\right)}{\bar{I}}$

FOR a = 0 TO 20 mm USING 2-mm INTERVALS COMPUTE: n, $\bar{I}$, σ_{ALUM}, σ_{STEEL}.

b = 60 mm   h = 40 mm   M = 1500 N.m
Moduli of elasticity: Steel = 200 GPa   Aluminum = 75 GPa

PROGRAM OUTPUT

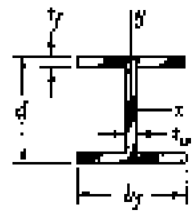
a mm	I m^4/10^6	sigma aluminum MPa	sigma steel MPa
0.000	0.8533	35.156	93.750
2.000	0.7088	42.325	101.580
4.000	0.5931	50.585	107.914
6.000	0.5029	59.650	111.347
8.000	0.4352	68.934	110.294
10.000	0.3867	77.586	103.448
12.000	0.3541	84.714	90.361
14.000	0.3344	89.713	71.770
16.000	0.3243	92.516	49.342
18.000	0.3205	93.594	24.958
20.000	0.3200	93.750	0.000

Find 'a' for max steel stress
and the corresponding aluminum stress

6.600	0.4804	62.447	111.572083
6.610	0.4800	62.494	111.572159
6.620	0.4797	62.540	111.572113

Max Steel Stress = 111.6 MPa occurs when a = 6.61 mm
Corresponding Aluminum stress = 62.5 MPa

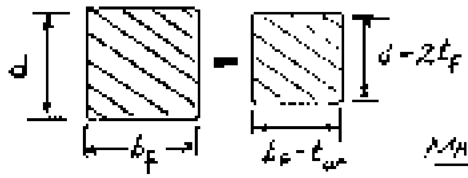
PROBLEM 4.C2



4.C2 A beam of the cross section shown, made of a steel that is assumed to be elastoplastic with a yield strength σ_Y and a modulus of elasticity E, is bent about the x axis. (*a*) Denoting by y_Y the half thickness of the elastic core, write a computer program to calculate the bending moment M and the radius of curvature ρ for values of y_Y from $\frac{1}{2}d$ to $\frac{1}{6}d$ using decrements equal to $\frac{1}{2}t_f$. Neglect the effect of fillets. (*b*) Use this program to solve Prob. 4.218.

SOLUTION

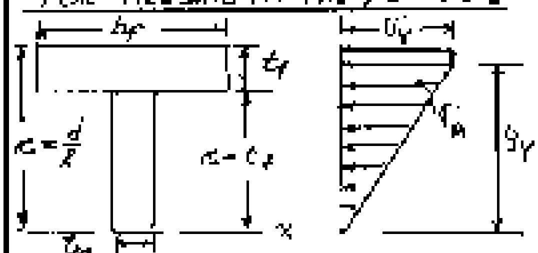
COMPUTE MOMENT OF INERTIA I_x



$$I_x = \frac{1}{12} b_f d^3 - \frac{1}{12}(b_f - t_w)(d-2t_f)^3$$

MAXIMUM ELASTIC MOMENT: $M_Y = \sigma_Y \frac{I_x}{(d/2)}$

FOR YIELDING IN THE FLANGES: (CONSIDER UPPER HALF OF CROSS SECTION) $c = \frac{d}{2}$

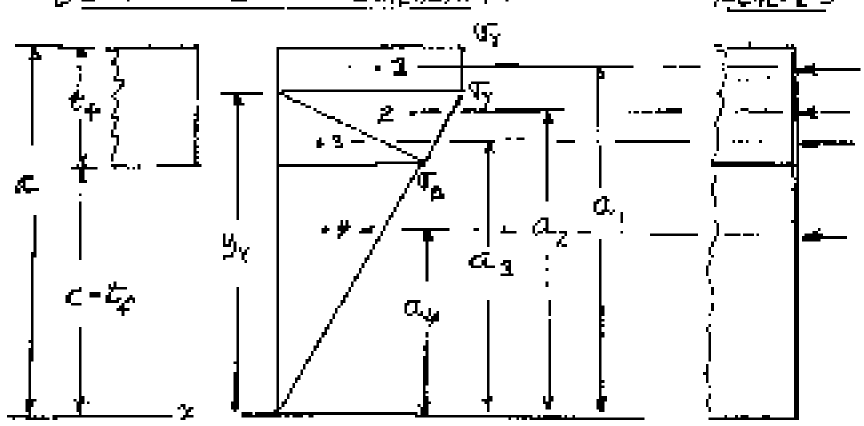


STRESS AT JUNCTION OF WEB AND FLANGE

$$\sigma_A = \frac{(d/2) - t_f}{y_Y}\sigma_Y$$

DETAIL OF STRESS DIAGRAM.

RESULTANT FORCES



$R_1 = y_Y\left[b_f(c - y_Y)\right]$

$R_2 = \frac{1}{2}\sigma_Y\, b_f\left[y_Y - (c - t_f)\right]$

$R_3 = \frac{1}{2}\sigma_A\, b_f\left[y_Y - (c - t_f)\right]$

$R_4 = \frac{1}{2}\sigma_A\, t_w\,(c - t_f)$

$a_1 = \frac{1}{2}(c + y_Y)$

$a_2 = y_Y - \frac{1}{3}\left[y_Y - (c - t_f)\right]$

$a_3 = y_Y - \frac{2}{3}\left[y_Y - (c - t_f)\right]$

$a_4 = \frac{2}{3}(c - t_f)$

BENDING MOMENT

$$M = 2\sum_{n=1}^{4} R_n a_n$$

RADIUS OF CURVATURE

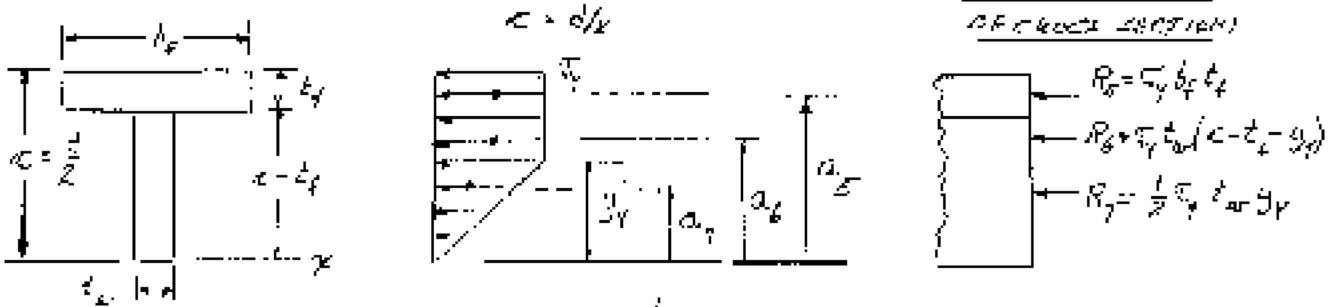
$$y_Y = \epsilon_Y \rho = \frac{\sigma_Y}{E}\rho \;; \quad \rho = \frac{y_Y E}{\sigma_Y}$$

CONTINUED

PROBLEM 4.C2 - CONTINUED

FOR YIELDING IN THE WEB (CONSIDER UPPER HALF OF CROSS SECTION)

$c = d/2$



$$a_5 = c - \tfrac{1}{2} t_f$$

$$a_6 = \tfrac{1}{2}[y_Y + (c - t_f)]$$

$$a_7 = \tfrac{2}{3} y_Y$$

BENDING MOMENT $\quad M = 2 \sum_{n=5}^{7} R_n a_n$

RADIUS OF CURVATURE $\quad y_Y = \epsilon_Y \rho = \frac{\sigma_Y}{E} \rho \qquad \rho = \frac{y_Y E}{\sigma_Y}$

PROGRAM: KEY IN EXPRESSIONS FOR a_n AND R_n FOR $n = 1$ TO 7.

FOR $y_Y = c$ TO $(c - t_f)$ AT $-t_f/2$ DECREMENTS

COMPUTE $M = 2 \sum R_n a_n$ FOR $n = 1$ TO 4 AND $\rho = \frac{y_Y E}{\sigma_Y}$, THEN PRINT

FOR $y_Y = (c - t_w)$ TO $c/2$ AT $-t_f/2$ DECREMENTS

COMPUTE $M = 2 \sum R_n a_n$ FOR $n = 5$ TO 7 AND $\rho = \frac{y_Y E}{\sigma_Y}$, THEN PRINT

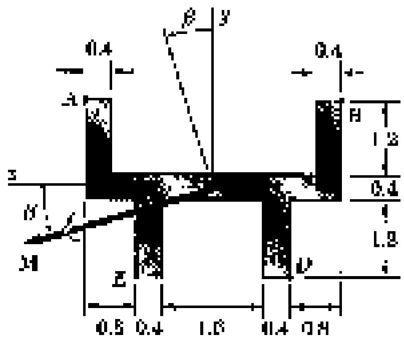
INPUT NUMERICAL VALUES AND RUN PROGRAM

PROGRAM OUTPUT

```
For a beam of Prob 4.218
Depth d = 140.00 mm                      Width of flange bf = 120.00 mm
Thickness of flange tf =  10.00 mm       Thickness of web tw =  10.00 mm

I = 0.000011600 m to the 4th
Yield strength of Steel sigmaY = 300 MPa
Yield Moment MY = 49.71 kip.in.
```

yY (mm)	M (kN.m)	rho(m)
For yielding still in the flange.		
70.000	49.71	46.67
65.000	52.59	43.33
60.000	54.00	40.00
For yielding in the web		
60.000	54.00	40.00
55.000	54.58	36.67
50.000	55.10	33.33
45.000	55.58	30.00
40.000	56.00	26.67
35.000	56.38	23.33
30.000	56.70	20.00
25.000	56.97	16.67

PROBLEM 4.C3



Dimensions in inches

4.C3 An 8 kip · in. couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Noting that the centroid of the cross section is located at C and that the y and z axes are principal axes, write a computer program to calculate the stress at A, B, C, and D for values of β from 0 to 180° using 10° increments. (*Given:* $I_y = 6.23\ \text{in}^4$ and $I_z = 1.481\ \text{in}^4$.)

SOLUTION

INPUT COORDINATES OF A, B, C, D

$z_A = z(1) = 2$ $\quad y_A = y(1) = 1.4$

$z_B = z(2) = -2$ $\quad y_B = y(2) = 1.4$

$z_C = z(3) = -1$ $\quad y_C = y(3) = -1.4$

$z_D = z(4) = 1$ $\quad y_D = y(4) = -1.4$

COMPONENTS OF M:

$M_y = -M \sin\beta \qquad M_z = M\cos\beta$

Eq 4.55 page 273: $\sigma(n) = -\dfrac{M_z\, y(n)}{I_z} + \dfrac{M_y\, z(n)}{I_y}$

PROGRAM: FOR $\beta = 0$ TO 180° USING 10° INCREMENTS

FOR $n = 1$ TO 4 USING UNIT INCREMENTS

EVALUATE EQ 4.55 AND PRINT STRESS

RETURN

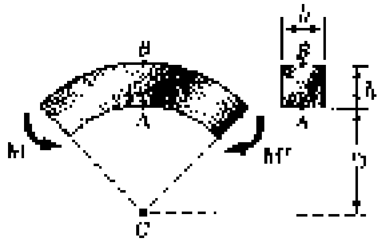
RETURN

PROGRAM OUTPUT

```
Moment of couple M =  8.00 kip·in.
Moments of inertia: Iy = 6.23  in^4      Iz  = 1.481 in^4

          Coordinates of points A, B, D, and E
        Point A:  z(1) =  2:      y(1) =  1.4
        Point B:  z(2) = -2:      y(2) =  1.4
        Point D:  z(3) = -1:      y(3) = -1.4
        Point E:  z(4) =  1:      y(4) = -1.4
              - - - Stress at Points - - -
```

beta °	A ksi	B ksi	D ksi	E ksi
0	-7.565	7.565	7.565	7.565
10	-7.896	-7.004	7.673	7.227
20	-7.987	-6.230	7.548	6.669
30	-7.836	-5.267	7.193	5.909
40	-7.446	-4.144	6.621	4.970
50	-6.830	-2.895	5.846	3.879
60	-6.007	-1.558	4.895	2.670
70	-5.001	-0.174	3.794	1.381
80	-3.843	1.216	2.578	0.049
90	-2.569	2.569	1.284	-1.284
100	-1.216	3.843	-0.049	-2.578
110	0.174	5.001	-1.381	-3.794
120	1.558	6.007	-2.670	-4.895
130	2.895	6.830	-3.879	5.846
140	4.144	7.446	-4.970	-6.621
150	5.267	7.836	-5.909	-7.193
160	6.230	7.987	-6.669	-7.548
170	7.004	7.896	7.227	7.673
180	7.565	7.565	-7.565	-7.565

PROBLEM 4.C4



4.C4 Couples of moment $M = 2$ kN · m are applied as shown to a curved bar having a rectangular cross section with $h = 100$ mm and $b = 25$ mm. Write a computer program and use it to calculate the stresses at points A and B for values of the ratio r_1/h from 10 to 1 using decrements of 1, and from 1 to 0.1 using decrements of 0.1. Using appropriate smaller increments, determine the ratio r_1/h for which the maximum stress in the curved bar is 50 percent larger than the maximum stress in a straight bar of the same cross section.

SOLUTION INPUT: $h = 100$ mm, $b = 25$ mm, $M = 2$ kN·m

FOR STRAIGHT BAR: $\sigma_{STRAIGHT} = \frac{M}{S} = \frac{6M}{h^2 b} = 48$ MPa

FOLLOWING NOTATION OF SEC. 4.15, KEY IN THE FOLLOWING:

$r_2 = h + r_1$; $R = h/\ln(r_2/r_1)$; $\bar{r} = r_1 + r_2$; $e = \bar{r} - R$; $A = bh = 2500$ (I)

STRESSES: $\sigma_A = \sigma_1 = M(r_1 - R)/(Aer_1)$ $\sigma_B = \sigma_2 = M(r_2 - R)/(Aer_2)$ (II)

SINCE $h = 100$ mm, FOR $r_1/h = 10$, $r_1 = 1000$ mm, ALSO $r_1/h = 1.0$, $r_1 = 100$

PROGRAM: FOR $r_1 = 1000$ TO 100 AT -100 DECREMENTS

USING EQUATIONS OF LINES I AND II EVALUATE r_2, R, $\bar{r}$, e, σ_1 AND σ_2

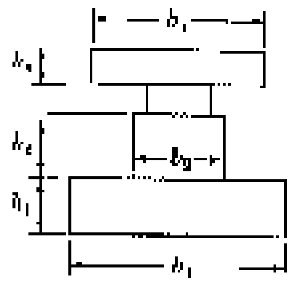
ALSO EVALUATE: ratio $= \sigma_1/\sigma_{STRAIGHT}$

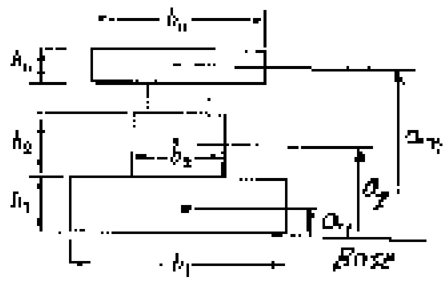
RETURN AND REPEAT FOR $r_1 = 100$ TO 10 AT -10 DECREMENT

PROGRAM OUTPUT

```
M = Bending Moment = 2.  kN.m     h = 100.000 in.     A = 2500.00 mm^2
Stress in straight beam  =  48.00 MPa

  r1       rbar      R        e         sigma1     sigma2     r1/h      ratio
  mm       mm        mm       mm        MPa        MPa        -         -
 1000      1050     1049     0.794     -49.57      46.51     10.000    -1.033
  900       950      949     0.878     -49.74      46.36      9.000    -1.036
  800       850      849     0.981     -49.95      46.18      8.000    -1.041
  700       750      749     1.112     -50.22      45.95      7.000    -1.046
  600       650      649     1.284     -50.59      45.64      6.000    -1.054
  500       550      548     1.518     -51.30      45.24      5.000    -1.064
  400       450      448     1.858     -51.82      44.66      4.000    -1.080
  300       350      348     2.394     -53.03      43.77      3.000    -1.105
  200       250      247     3.370     -55.35      42.24      2.000    -1.153
  100       150      144     5.730     -61.80      38.90      1.000    -1.288
      ---------------------------------------------------------
  100       150      144     5.730     -61.80      38.90      1.000    -1.288
   90       140      134     6.170     -63.15      38.33      0.900    -1.316
   80       130      123     6.685     -64.80      37.69      0.800    -1.350
   70       120      113     7.299     -66.86      36.94      0.700    -1.393
   60       110      102     8.045     -69.53      36.07      0.600    -1.449
   50       100       93     8.976     -73.13      35.04      0.500    -1.523
   40        90       80    10.176     -78.27      33.79      0.400    -1.631
   30        80       68    11.803     -86.30      32.22      0.300    -1.798
   20        70       56    14.189    -100.95      30.16      0.200    -2.103
   10        60       42    18.297    -138.62      27.15      0.100    -2.888
      ======================================================================
 Find r1/h for (sigma max)/(sigma straight) = 1.5
   52.70   103       94     8.703     -72.036     35.34      0.527    -1.501
   52.80   103       94     8.693     -71.998     35.35      0.528    -1.500
   52.90   103       94     8.683     -71.959     35.36      0.529    -1.499
Ratio of stresses is 1.5 for r1 = 52.8 mm or r1/h = 0.529

   [ Note: The desired ratio r1/h is valid for any beam
           having a rectangular cross section. ]
```

PROBLEM 4.C5





4.C5 The couple **M** is applied to a beam of the cross section shown. (*a*) Write a computer program that, for loads expressed in either SI or U.S. customary units, can be used to calculate the maximum tensile and compressive stresses in the beam. (*b*) Use this program to solve Probs. 4.1, 4.10, and 4.11.

SOLUTION

INPUT: BENDING MOMENT M

FOR n = 1 TO n: ENTER b_n AND h_n

$\Delta AREA = b_n h_n$ (PRINT)

$a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$

[MOMENT OF RECTANGLE ABOUT BASE]

$\Delta m = (\Delta AREA)\, a_n$

[FOR WHOLE CROSS SECTION]

$m = m + \Delta m$; $AREA = AREA + \Delta AREA$

LOCATION OF CENTROID ABOVE BASE

$\bar{y} = m/AREA$ (PRINT)

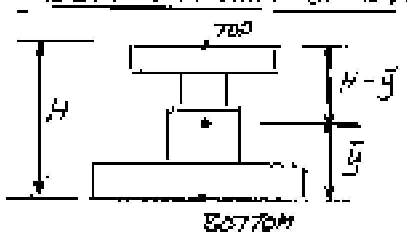
MOMENT OF INERTIA ABOUT HORIZONTAL CENTROIDAL AXIS

FOR n = 1 TO n: $a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$

$\Delta I = b_n h_n^3/12 + (b_n h_n)(\bar{y} - a_n)^2$

$I = I + \Delta I$ (PRINT)

COMPUTATION OF STRESSES



TOTAL HEIGHT: FOR n = 1 TO n

$H = H + h_n$

STRESS AT TOP

$M_{TOP} = -M \dfrac{H-\bar{y}}{I}$ (PRINT)

STRESS AT BOTTOM

$M_{BOTTOM} = M \dfrac{\bar{y}}{I}$ (PRINT)

SEE NEXT PAGE FOR PRINTOUTS FOR PROBLEMS 4.1, 4.10, 4.11

CONTINUED

PROBLEM 4.C5 - CONTINUED

<u>Problem 4.1</u>

Summary of Cross Section Dimensions

Width (in.)	Height (in.)
2.00	2.00
6.00	1.50
2.00	2.00

Bending Moment = 25.000 kip.in.
Centroid is 2.750 in. above lower edge
Centroidal Moment of Inertia is 28.854 in^4

Stress at top of beam = -2.383 ksi
Stress at bottom of beam = 2.383 ksi

<u>Problem 4.13</u>

Summary of Cross Section Dimensions

Width (in.)	Height (in.)
9.00	2.00
3.00	6.00

Bending Moment = 600.000 kip.in.
Centroid is 3.000 in. above lower edge
Centroidal Moment of Inertia is 204.000 in^4

Stress at top of beam = -14.706 ksi
Stress at bottom of beam = 8.824 ksi

<u>Problem 4.21</u>

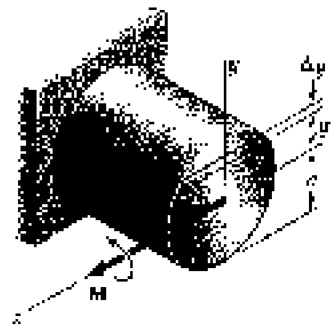
Summary of Cross Section Dimensions

Width (in.)	Height (in.)
4.00	1.00
1.00	6.00
8.00	1.00

Bending Moment = 500.000 kip.in.
Centroid is 4.778 in. above lower edge
Centroidal Moment of Inertia is 155.111 in^4

Stress at top of beam = -10.387 ksi
Stress at bottom of beam = 15.401 ksi

PROBLEM 4.C6



4.C6 A solid rod of radius $c = 1.2$ in. is made of a steel that is assumed to be elastoplastic with $E = 29{,}000$ ksi and $\sigma_Y = 42$ ksi. The rod is subjected to a couple of moment M that increases from zero to the maximum elastic moment M_Y and then to the plastic moment M_p. Denoting by y_Y the half thickness of the elastic core, write a computer program and use it to calculate the bending moment M and the radius of curvature ρ for values of y_Y from 1.2 in. to 0 using 0.2-in. decrements. (*Hint*: Divide the cross section into 80 horizontal elements of 0.03-in. height.)

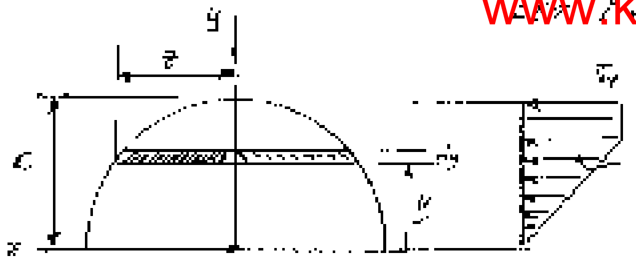
SOLUTION $M_Y = \sigma_Y \frac{\pi}{4} c^3 = (42\text{ ksi})\frac{\pi}{4}(1.2\text{ in})^3 = 57$ kip·in

$M_p = \sigma_Y \frac{4}{3} c^3 = (42\text{ ksi})\frac{4}{3}(1.2\text{ in})^3 = 96.8$ kip·in

CONSIDER TOP HALF OF ROD

LET i = NUMBER OF ELEMENTS IN TOP HALF

Δh = HEIGHT OF EACH ELEMENT: $\Delta h = \frac{c}{i}$



σ_E = STRESS IN ELEMENT

FOR $n = 0$ TO $i-1$ STEP 1

$y = n(\Delta h)$

$z = \left[c^2 - \{(n+0.5)\Delta h\}^2\right]^{1/2}$ ← z AT MIDPOINT OF ELEMENT

IF $y \geq y_Y$ GO TO 100

$\sigma_E = \sigma_Y \frac{(n+0.5)\Delta h}{y_Y}$ ← STRESS IN ELASTIC CORE

GO TO 200

100 $\sigma_E = \sigma_Y$ ← STRESS IN PLASTIC ZONE

200 $\text{AREA} = 2z(\Delta h)$

$\Delta\text{FORCE} = \sigma_E(\Delta\text{AREA})$

$\Delta\text{MOMENT} = \Delta\text{FORCE}\,(n+.5)\Delta h$

$M = M + \Delta\text{MOMENT}$

$\rho = y_Y E/\sigma_Y$

PRINT y_Y, M, AND ρ.

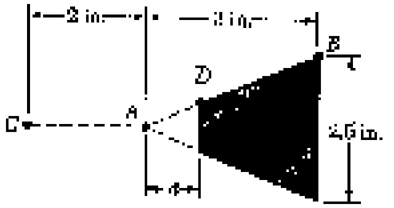
NEXT

REPEAT FOR $y_Y = 1.2$ in. TO $y_Y = 0$ AT -0.2-in. DECREMENTS

PROGRAM OUTPUT

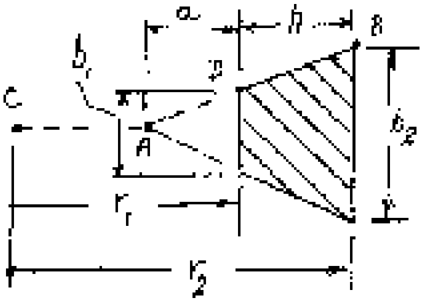
```
Radius of rod = 1.2 in.
Yield point of steel = 42 ksi
Yield moment = 57.0  kip·in          Plastic moment = 96.8  kip.in.
Number of elements in half of the rod = 40

For yY =   1.20 in.     M = 57.1 kip·in.     Radius of curvature = 828.57 in.
For yY =   1.00 in.     M = 67.2 kip·in.     Radius of curvature = 690.48 in.
For yY =   0.80 in.     M = 76.9 kip·in.     Radius of curvature = 552.38 in.
For yY =   0.60 in.     M = 85.2 kip·in.     Radius of curvature = 414.29 in.
For yY =   0.40 in.     M = 91.6 kip·in.     Radius of curvature = 276.19 in.
For yY =   0.20 in.     M = 95.5 kip·in.     Radius of curvature = 138.10 in.
For yY =   0.00 in.     M = infinite         Radius of curvature = zero
```

PROBLEM 4.C7



4.C7 The machine element of Prob. 4.204 is to be redesigned by removing part of the triangular cross section. It is believed that the removal of a small triangular area of width a will lower the maximum stress in the element. In order to verify this design concept, write a computer program to calculate the maximum stress in the element for values of a from 0 to 1 in. using 0.1-in. increments. Using appropriate smaller increments, determine the distance a for which the maximum stress is as small as possible and the corresponding value of the maximum stress.

SOLUTION SEE FIG 4.79 PAGE 289



$M = 5$ kip·in. $r_2 = 5$ in. $b_2 = 2.5$ in.

For a = 0 to 1 at 0.1 INTERVALS

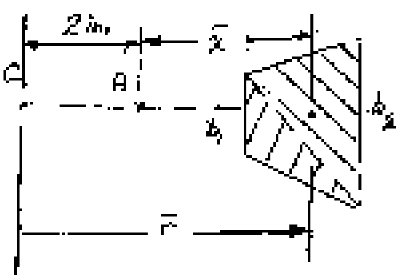
$h = 3 - a$

$r_1 = 2 + a$

$b_1 = b_2\,(a/(h+a))$

$AREA = (b_1 + b_2)(h/2)$

$\bar{x} = a + \left[\frac{1}{2} b_1 h (h/3) + \frac{1}{2} b_2 h (2h/3)\right] / AREA$



$\bar{r} = r_2 - (h - \bar{x})$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$e = \bar{r} - R$

$\sigma_D = M(r_1 - R) / [AREA\,(e)(r_1)]$

$\sigma_B = M(r_2 - R) / [AREA\,(e)(r_2)]$

PRINT AND RETURN

PROGRAM OUTPUT

a in.	R in.	sigmaD ksi	sigmaB ksi	b1	rbar	e
0.00	3.856	-8.5071	2.1014	0.00	4.00	0.145
0.10	3.858	-7.7736	2.1197	0.08	4.00	0.144
0.20	3.869	-7.2700	2.1689	0.17	4.01	0.140
0.30	3.884	-6.9260	2.3438	0.25	4.02	0.134
0.40	3.904	-6.7004	2.3423	0.33	4.03	0.127
0.50	3.928	-6.5683	2.4641	0.42	4.05	0.119
0.60	3.956	-6.5143	2.6102	0.50	4.07	0.111
0.70	3.985	-6.5296	2.7828	0.58	4.09	0.103
0.80	4.018	-6.6098	2.9852	0.67	4.11	0.094
0.90	4.052	-6.7541	3.2220	0.75	4.14	0.086
1.00	4.089	-6.9647	3.4992	0.83	4.17	0.078

Determination of the maximum compressive stress that is as small as possible

a in.	R in.	sigmaD ksi	sigmaB ksi	b1	rbar	e
0.620	3.961	-6.51198	2.6428	0.52	4.07	0.109
0.625	3.963	-6.51185	2.6507	0.52	4.07	0.109
0.630	3.964	-6.51198	2.6591	0.52	4.07	0.109

ANSWER: When a = 625 in. the compressive stress is 6.51 ksi

CHAPTER 5