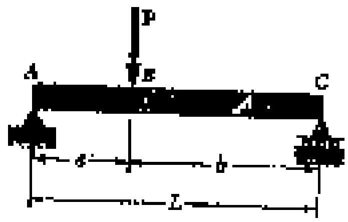
CHAPTER 5

PROBLEM 5.1

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

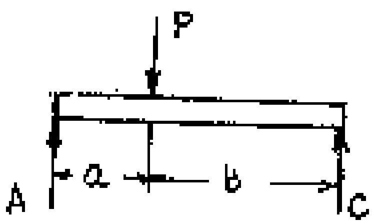


SOLUTION

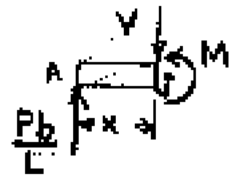
Reactions

$\Sigma M_C = 0 \qquad LA - bP = 0 \qquad A = \frac{Pb}{L}$

$\Sigma M_A = 0 \qquad LC - aP = 0 \qquad C = \frac{Pa}{L}$



From A to B $\qquad 0 < x < a$

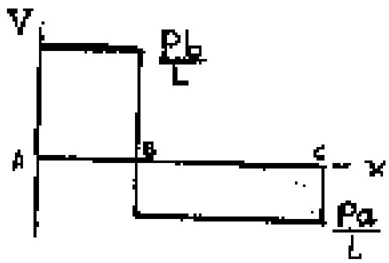


$+\uparrow \Sigma F_y = 0 \qquad \frac{Pb}{L} - V = 0$

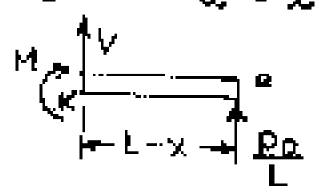
$V = \frac{Pb}{L}$

$\circlearrowleft \Sigma M_J = 0 \qquad M - \frac{Pb}{L}x = 0$

$M = \frac{Pbx}{L}$



From B to C $\qquad a < x < L$

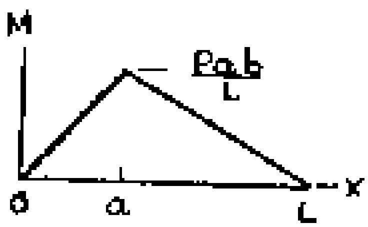


$+\uparrow \Sigma F_y = 0 \qquad V + \frac{Pa}{L} = 0$

$V = -\frac{Pa}{L}$

$\circlearrowleft \Sigma M_K = 0 \qquad -M + \frac{Pa}{L}(L-x) = 0$

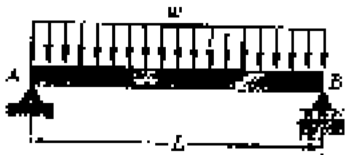
$M = \frac{Pa(L-x)}{L}$



At section B $\qquad M = \frac{Pab}{L}$

PROBLEM 5.2

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

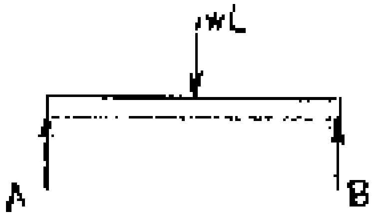


SOLUTION

Reactions

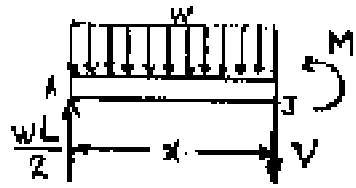
$\circlearrowleft \Sigma M_B = 0 \qquad -AL + wL\frac{L}{2} = 0 \qquad A = \frac{wL}{2}$

$\circlearrowleft \Sigma M_A = 0 \qquad BL - wL\cdot\frac{L}{2} = 0 \qquad B = \frac{wL}{2}$

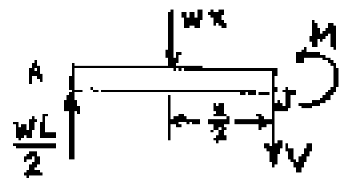


Free body diagram for determining reactions

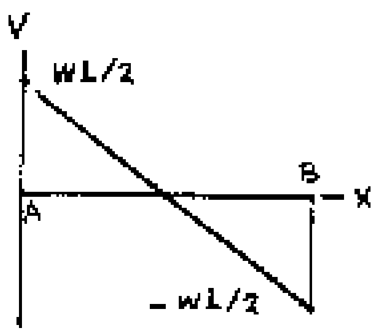
Over whole beam $\quad 0 < x < L$



Place section at x.



Replace distributed load by equivalent concentrated load.



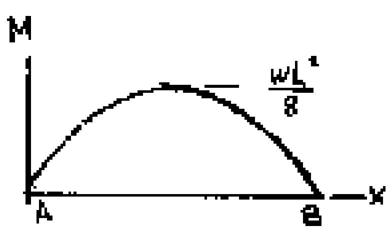
$+\uparrow \Sigma F_y = 0 \qquad \frac{wL}{2} - wx - V = 0$

$V = w\left(\frac{L}{2} - x\right)$ ◄

$\circlearrowleft \Sigma M_J = 0 \qquad -\frac{wL}{2}x + wx\frac{x}{2} + M = 0$

$M = \frac{w}{2}(Lx - x^2)$

$= \frac{w}{2}\,x(L - x)$ ◄



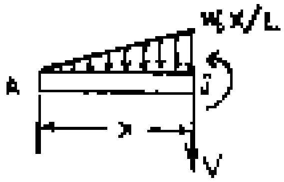
Maximum bending moment occurs at $x = \frac{L}{2}$.

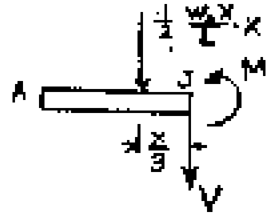
$M_{max} = \frac{wL^2}{8}$ ◄

PROBLEM 5.3

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION





$+\uparrow \Sigma F_y = 0 \qquad -\frac{1}{2}\frac{w_0 x}{L}\cdot x - V = 0$

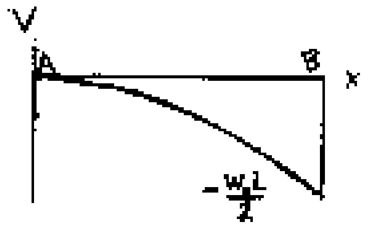
$$V = -\frac{w_0 x^2}{2L}$$

$\circlearrowleft \Sigma M_J = 0 \qquad \frac{1}{2}\frac{w_0 x}{L}\cdot x\cdot\frac{x}{3} + M = 0$

$$M = -\frac{w_0 x^3}{6L}$$

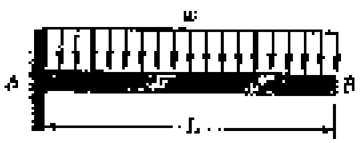
At $x = L$ $\qquad V = -\frac{w_0 L}{2} \qquad |V|_{max} = \frac{w_0 L}{2}$

$\qquad M = -\frac{w_0 L^2}{6} \qquad |M|_{max} = \frac{w_0 L^2}{6}$



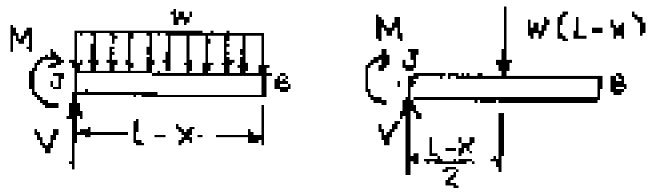
PROBLEM 5.4

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



SOLUTION

Use portion to the right of the section as the free body.



Replace distributed load by equivalent concentrated load.

$+\uparrow \Sigma F_y = 0 \qquad V - w(L-x) = 0$

$$V = w(L-x)$$

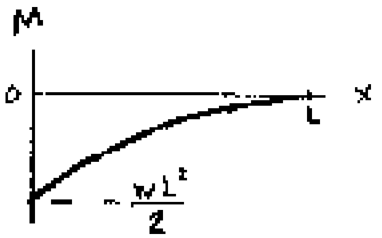
$\circlearrowright \Sigma M_J = 0 \qquad -M - w(L-x)\left(\frac{L-x}{2}\right) = 0$

$$M = -\frac{w}{2}(L-x)^2$$

Largest negative bending moment occurs at $x = 0$.

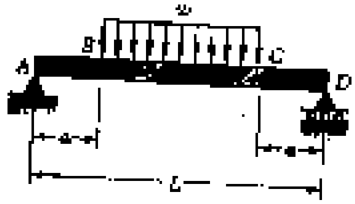
$$M_{min} = -\frac{wL^2}{2}$$

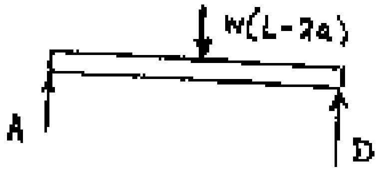
Thus, $\qquad |M|_{max} = \dfrac{wL^2}{2}$

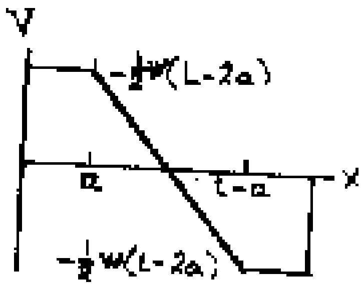


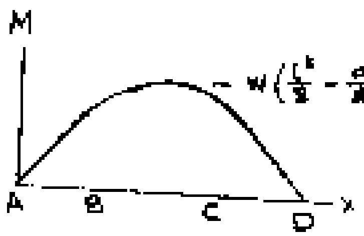
PROBLEM 5.5

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.









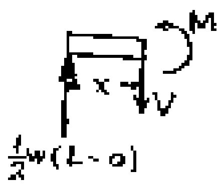
SOLUTION

Calculate reactions after replacing distributed load by an equivalent concentrated load.

Reactions are

$$A = D = \tfrac{1}{2}w(L-2a)$$

From A to B $\quad 0 < x < a$



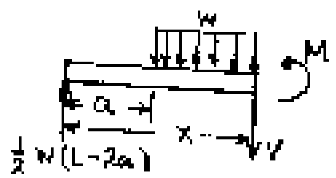
$+\uparrow \Sigma F_y = 0 \qquad \frac{1}{2}w(L-2a) - V = 0$

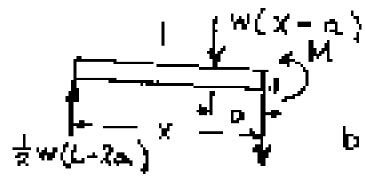
$$V = \tfrac{1}{2}w(L-2a)$$

$\circlearrowleft \Sigma M = 0 \qquad -\frac{1}{2}w(L-2a) + M = 0$

$$M = \tfrac{1}{2}w(L-2a)x$$

From B to C $\quad a < x < L-a$





$b = \frac{x-a}{2}$

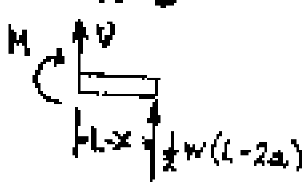
Place section cut at x. Replace distributed load by equiv. conc. load.

$+\uparrow \Sigma F_y = 0 \qquad \frac{1}{2}w(L-2a) - w(x-a) - V = 0 \qquad V = w\left(\frac{L}{2} - x\right)$

$\circlearrowleft M_J = 0 \qquad -\frac{1}{2}w(L-2a)x + w(x-a)\left(\frac{x-a}{2}\right) + M = 0$

$$M = \tfrac{1}{2}w\left[(L-2a)x - (x-a)^2\right]$$

From C to D $\quad L-a < x < L$



$+\uparrow \Sigma F_y = 0 \qquad V + \frac{1}{2}w(L-2a) = 0$

$$V = -\tfrac{w}{2}(L-2a)$$

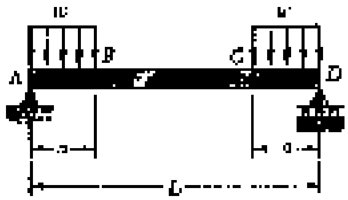
$\circlearrowleft \Sigma M_J = 0 \qquad -M + \frac{1}{2}w(L-2a)(L-x) = 0$

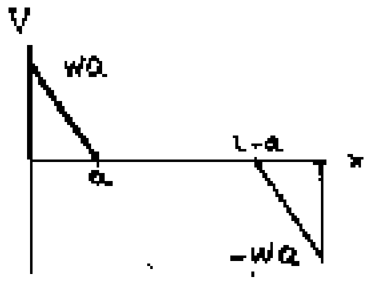
$$M = \tfrac{1}{2}w(L-2a)(L-x)$$

At $x = \frac{L}{2}$ $\qquad M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right)$

PROBLEM 5.6

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

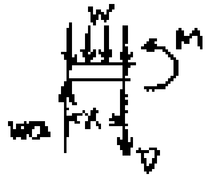


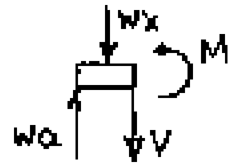


SOLUTION

Reactions: $A = D = wa$

From A to B $\quad 0 < x < a$





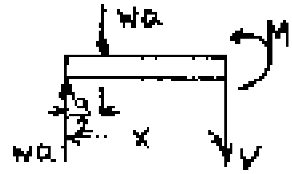
$+\uparrow \Sigma F_y = 0 \qquad wa - wx - V = 0$

$V = w(a - x)$

$\circlearrowleft \Sigma M_J = 0 \qquad -wax + (wx)\frac{x}{2} + M = 0$

$M = w\left(ax - \frac{x^2}{2}\right)$

From B to C $\quad a < x < L - a$



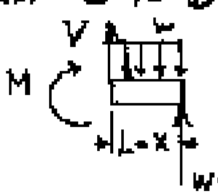
$\Sigma F_y = 0$

$wa - wa - V = 0$

$V = 0$

$\circlearrowleft \Sigma M_J = 0 \qquad -wax + wa\left(x - \frac{a}{2}\right) + M = 0 \qquad M = \frac{1}{2}wa^2$

From C to D $\quad L - a < x < L$

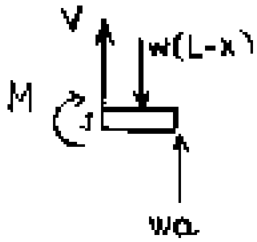


$+\uparrow \Sigma F_y = 0 \qquad V - w(L - x) + wa = 0$

$V = w(L - x - a)$

$\circlearrowleft \Sigma M_J = 0 \qquad -M - w(L-x)\left(\frac{L-x}{2}\right) + wa(L-x) = 0$

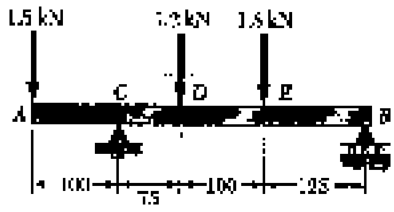
$M = w\left[a(L-x) - \frac{1}{2}(L-x)^2\right]$



PROBLEM 5.7	**5.7 through 5.12** Determine the equations of the shear and bending-moment curves for the beam and loading shown. (Place the origin at point *A*.)
SOLUTION	See PROBLEM 5.1
PROBLEM 5.8	**5.7 through 5.12** Determine the equations of the shear and bending-moment curves for the beam and loading shown. (Place the origin at point *A*.)
SOLUTION	See PROBLEM 5.2
PROBLEM 5.9	**5.7 through 5.12** Determine the equations of the shear and bending-moment curves for the beam and loading shown. (Place the origin at point *A*.)
SOLUTION	See PROBLEM 5.3
PROBLEM 5.10	**5.7 through 5.12** Determine the equations of the shear and bending-moment curves for the beam and loading shown. (Place the origin at point *A*.)
SOLUTION	See PROBLEM 5.4
PROBLEM 5.11	**5.7 through 5.12** Determine the equations of the shear and bending-moment curves for the beam and loading shown. (Place the origin at point *A*.)
SOLUTION	See PROBLEM 5.5
PROBLEM 5.12	**5.7 through 5.12** Determine the equations of the shear and bending-moment curves for the beam and loading shown. (Place the origin at point *A*.)
SOLUTION	See PROBLEM 5.6

PROBLEM 5.13

5.13 and 5.14 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the



Dimensions in mm

SOLUTION

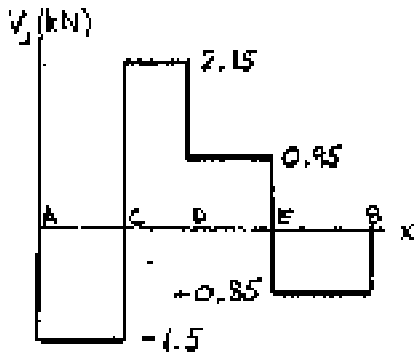
Calculate reactions

$\circlearrowleft \Sigma M_B = 0$

$(400)(1.5) - 300\,C + (225)(1.2) + (125)(1.8) = 0$

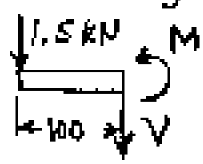
$C = 3.65$ kN

$\circlearrowleft \Sigma M_C = 0 \qquad B = 0.85$ kN



At A $\qquad V = -1.5$ kN, $\quad M = 0$

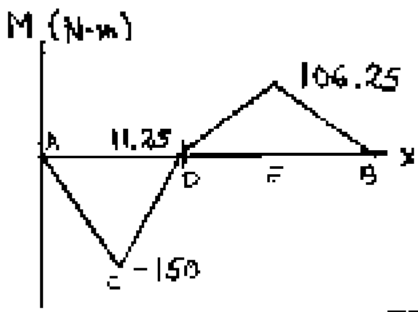
At C^-



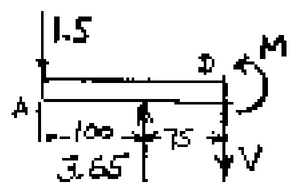
$\Sigma F_y = 0 \qquad -1.5 - V = 0 \qquad V = -1.5$ kN

$\Sigma M_C = 0 \qquad (100)(1.5) + M = 0$

$M = -150$ N·m ◀ (b)



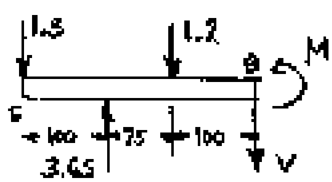
At D^-



$\Sigma F_y = 0 \qquad -1.5 + 3.65 - V = 0, \qquad V = 2.15$ kN ◀ (a)

$\circlearrowleft \Sigma M_D = 0, \qquad (175)(1.5) - (75)(3.65) + M = 0 \qquad M = 11.25$ N·m

At E^-



$\Sigma F_y = 0$

$-1.5 + 3.65 - 1.2 - V = 0$

$V = 0.95$ kN

$\circlearrowleft \Sigma M_E = 0 \qquad (275)(1.5) - (175)(3.65) + (100)(1.2) + M = 0$

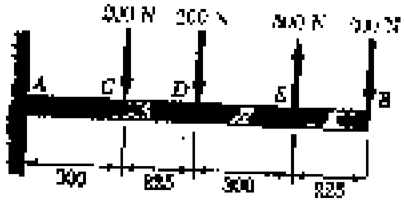
$M = 106.25$ N·m

At B $\qquad V = -B = -0.85$ kN

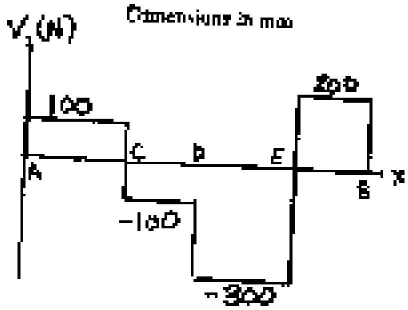
$M = 0$

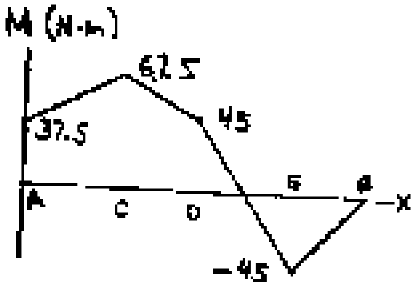
PROBLEM 5.14

5.13 and 5.14 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



Dimensions in mm

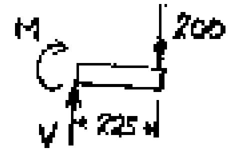




SOLUTION

At B $\qquad V = 200 \text{ N}, \quad M = 0$

At E^+

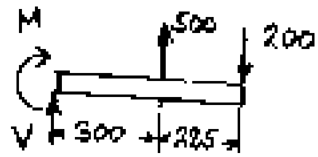


$+\uparrow \Sigma F_y = 0 \qquad V - 200 = 0 \qquad V = 200 \text{ N}$

$\circlearrowleft \Sigma M_E = 0 \qquad -M - (0.225)(200) = 0$

$M = -45 \text{ N}\cdot\text{m}$

At D^+



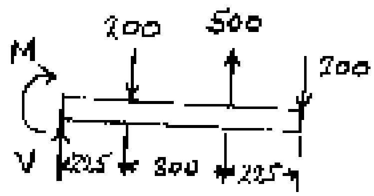
$+\uparrow \Sigma F_y = 0, \qquad V + 500 - 200 = 0$

$V = -300 \text{ N} \quad \longleftarrow \text{(a)}$

$\circlearrowleft \Sigma M_D = 0 \qquad -M + (0.3)(500) - (0.525)(200) = 0$

$M = 45 \text{ N}\cdot\text{m}$

At C^+

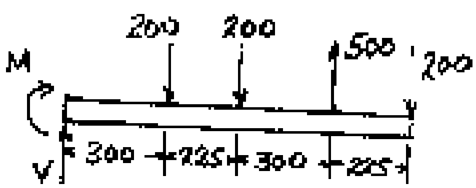


$+\uparrow \Sigma F_y = 0 \qquad V - 200 + 500 - 200 = 0 \qquad V = -100 \text{ N}$

$+\circlearrowleft \Sigma M_C = 0 \qquad -M - (0.225)(200) + (0.525)(500) - (0.75)(200) = 0$

$M = 67.5 \text{ N}\cdot\text{m} \quad \longleftarrow \text{(b)}$

At A



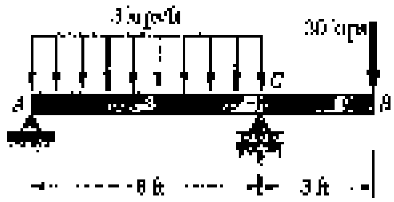
$+\uparrow \Sigma F_y = 0 \qquad V - 200 - 200 + 500 - 200 = 0 \qquad V = 100 \text{ N}$

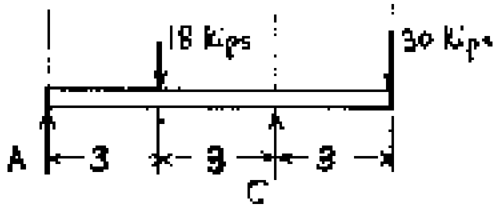
$+\circlearrowleft \Sigma M_A = 0 \qquad -M - (0.3)(200) - (0.525)(200) + (0.825)(500) - (1.05)(200) = 0$

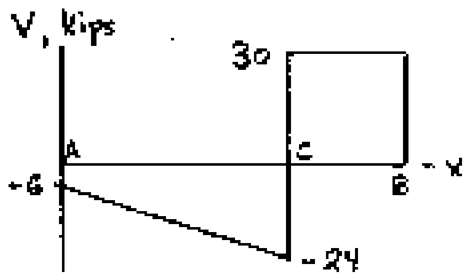
$M = 37.5 \text{ N}\cdot\text{m}$

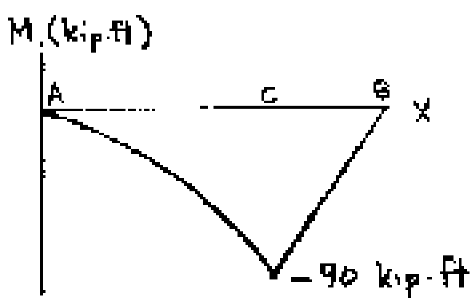
PROBLEM 5.15

5.15 and 5.16 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.









SOLUTION

Reactions

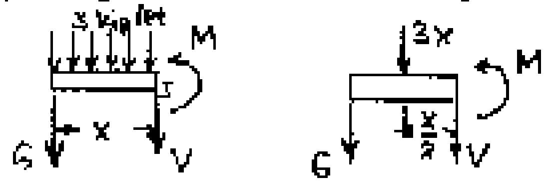
$\circlearrowleft \Sigma M_C = 0 \qquad -6A + (3)(18) - (3)(30) = 0$

$A = -6$ kips i.e. 6 kips ↓

$\circlearrowleft \Sigma M_A = 0 \qquad 6C - (3)(18) - (9)(30) = 0$

$C = 54$ kips ↑

A to C $0 < x < 6$ ft.



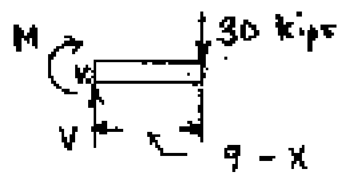
$+\uparrow \Sigma F_y = 0 \qquad -6 - 3x - V = 0$

$V = -6 - 3x$ kips

$\circlearrowleft \Sigma M_J = 0 \qquad -6x - (3x)\left(\frac{x}{2}\right) - M = 0$

$M = -6x - 1.5x^2$ kip·ft

C to B $6 \text{ ft} < x < 9 \text{ ft}$



$+\uparrow \Sigma F_y = 0 \qquad V - 30 = 0$

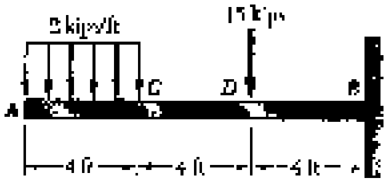
$V = 30$ kips ◄ (a)

$\circlearrowleft \Sigma M_K = 0 \qquad -M - (9 - x)(30) = 0$

$M = 30x - 270$ kip·ft

(b) $|M|_{max} = 90$ kip·ft ◄

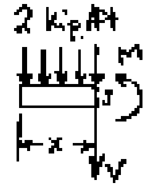
PROBLEM 5.16

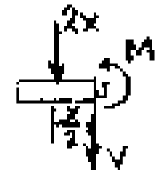


5.15 and 5.16 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

A to C $0 < x < 4$ ft





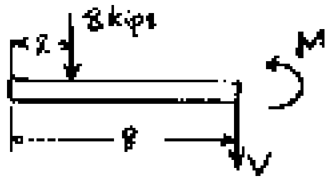
$+\uparrow \Sigma F_y = 0 \quad -V - 2x = 0 \qquad V = -2x$ kips

$\circlearrowleft \Sigma M_J = 0 \quad M + (2x)\left(\frac{x}{2}\right) = 0$

$M = -x$ kip ft

At C $\quad V = -8$ kips $\qquad M = -16$ kip ft

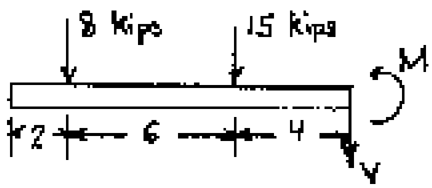
At D^-



$+\uparrow \Sigma F_y = 0 \quad -8 - V = 0 \qquad V = -8$ kips

$\circlearrowleft \Sigma M_D = 0 \quad (6)(8) - M = 0 \qquad M = -48$ kip ft

At B^-



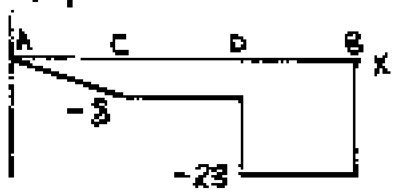
$+\uparrow \Sigma F_y = 0 \quad -8 - 15 - V = 0$

$V = -23$ kips ◄ (a)

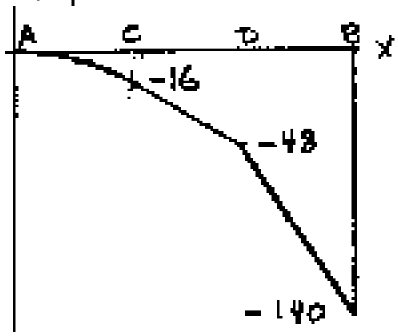
$\circlearrowleft \Sigma M_B = 0 \quad -(10)(8) - (4)(15) - M = 0$

$M = -140$ kip ft ◄ (b)

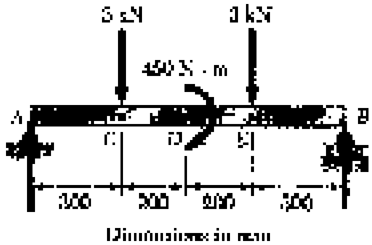
V (kips)



M (kip·ft)



PROBLEM 5.17



5.17 and 5.18 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

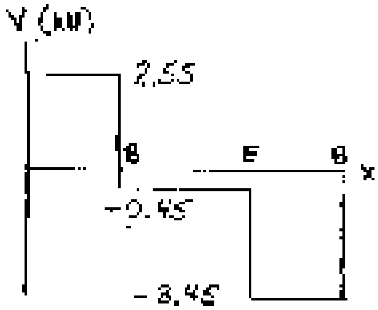
SOLUTION

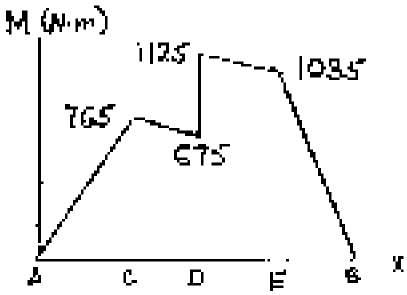
$\circlearrowleft \Sigma M_B = 0 \qquad (700)(3) - 450 + (300)(3) - 1000A = 0$

$A = 2.55 \text{ kN} \uparrow$

$\circlearrowleft \Sigma M_A = 0 \qquad -(300)(3) - 450 - (700)(3) + 1000B = 0$

$B = 3.45 \text{ kN} \uparrow$





At A $\qquad V = 2.55 \text{ kN} \qquad M = 0$

A to C $\qquad V = 2.55 \text{ kN}$

At C

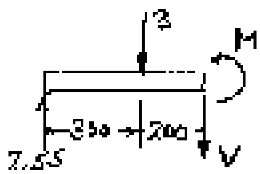
$\circlearrowleft \Sigma M_C = 0$

$-(300)(2.55) + M = 0$

$M = 765 \text{ N·m}$

C to E $\qquad V = -0.45 \text{ N·m}$

At D^-

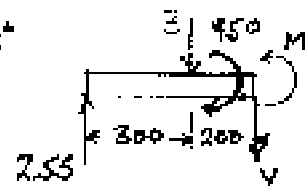


$\circlearrowleft \Sigma M_D = 0$

$-(500)(2.55) + (200)(3) + M = 0$

$M = 675 \text{ N·m}$

At D^+



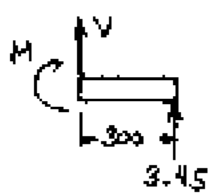
$\circlearrowleft \Sigma M_D = 0$

$-(500)(2.55) + (200)(3) - 450 + M = 0$

$M = 1125 \text{ N·m}$

E to B $\qquad V = -3.45 \text{ kN}$

At E



$\circlearrowleft \Sigma M_E = 0$

$-M + (300)(3.45) = 0$

$M = 1035 \text{ N·m}$

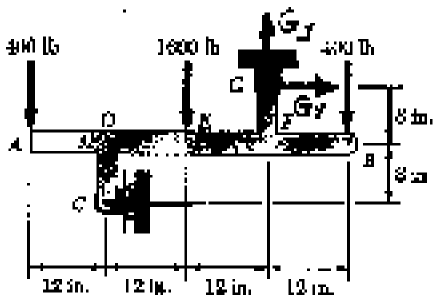
At B $\qquad V = 3.45 \text{ N·m}, \qquad M = 0$

Maximum $|V| = 3.45$ kN ◄

Maximum $|M| = 1125$ N·m ◄

PROBLEM 5.18

5.17 and 5.18 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



SOLUTION

$\circlearrowleft \Sigma M_G = 0$

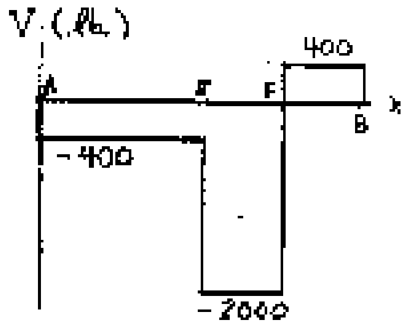
$-16\,C + (36)(400) + (12)(1600) - (12)(400) = 0$

$C = 1800\ lb.$

$\xrightarrow{+} \Sigma F_x = 0 \qquad -C + G_x = 0 \qquad G_x = 1800\ lb.$

$+\uparrow \Sigma F_y = 0 \qquad -400 - 1600 + G_y - 400 = 0$

$G_y = 2400\ lb.$



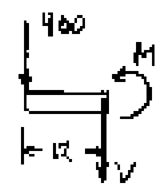
A to E $\qquad V = -400\ lb.$

E to F $\qquad V = -2000\ lb$

F to B $\qquad V = 400\ lb.$

At A and B $\qquad M = 0$

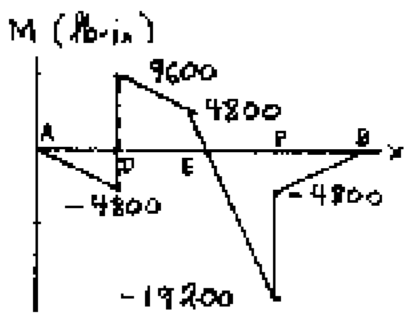
At D^-



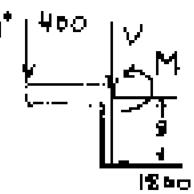
$\circlearrowleft \Sigma M_D = 0$

$(12)(400) + M = 0$

$M = -4800\ lb\cdot in.$



At D^+

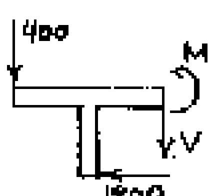


$\circlearrowleft \Sigma M_D = 0$

$(12)(400) - (8)(1800) + M = 0$

$M = 9600\ lb\cdot in.$

At E



$\circlearrowleft \Sigma M_E = 0$

$(24)(400) - (8)(1800) + M = 0$

$M = 4800\ lb\cdot in$

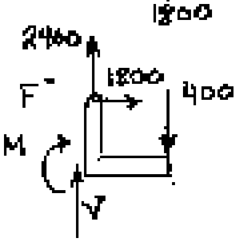
(a) Maximum $|V|$

$= 2000\ lb$ ◀

(b) Maximum $|M|$

$= 19200\ lb\cdot in.$ ◀

At F^-

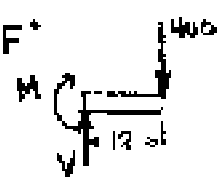


$\circlearrowleft \Sigma M_F = 0$

$-M - (8)(1800) - (12)(400) = 0$

$M = -19200\ lb\cdot in$

At F^+



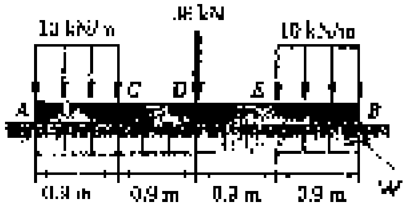
$\Sigma M_F = 0$

$-M - (12)(400) = 0$

$M = -4800\ lb\cdot in.$

PROBLEM 5.19

5.19 and 5.20 Assuming the upward reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum value (a) of the shear, (b) of the bending moment.



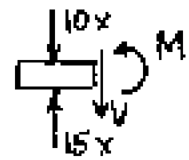
SOLUTION

Over whole beam $+\uparrow \Sigma F_y = 0$

$$3.6\,w - (0.9)(10) - 36 - (0.9)(10) = 0$$

$$w = 15 \text{ kN/m}$$

A to C $\qquad 0 < x < 0.9$ m



$+\uparrow \Sigma F_y = 0 \qquad 15x - 10x - V = 0$

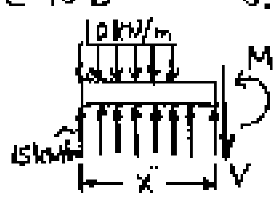
$$V = 5x$$

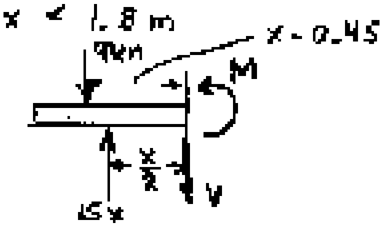
$\circlearrowleft \Sigma M_J = 0 \qquad -(15x)\frac{x}{2} + (10x)\frac{x}{2} + M = 0$

$$M = 2.5x^2$$

At $x = C$ $\qquad V = 4.5$ kN
$\qquad M = 2.025$ kN·m

C to D $\qquad 0.9 \text{ m} < x < 1.8 \text{ m}$





$+\uparrow \Sigma F_y = 0 \qquad 15x - 9 - V = 0$

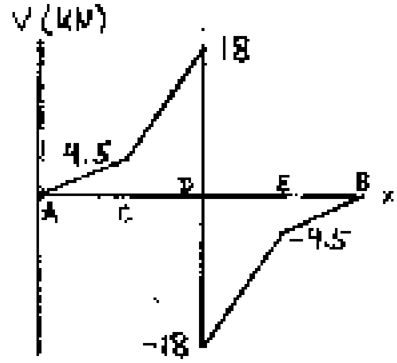
$$V = 15x - 9$$

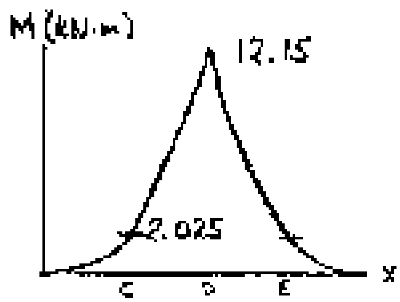
$\circlearrowleft \Sigma M_J = 0 \qquad -(15x)\left(\frac{x}{2}\right) + 9(x - 0.45) + M = 0$

$$M = 7.5x^2 - 9x + 4.05 = 0$$

At D^- $\qquad V = 18$ kN
$\qquad M = 12.15$ kN·m

D to B $\qquad$ Use symmetry to calculate the shear and bending moment.



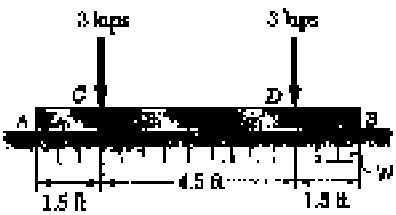


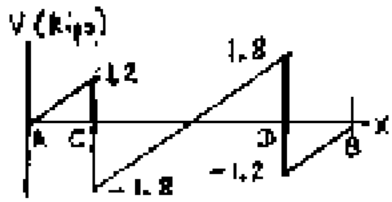
(a) Maximum $|V|$
$= 18$ kN ◄

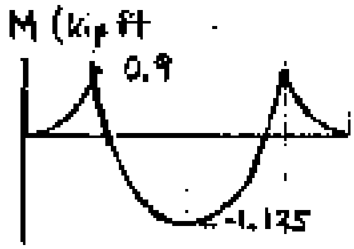
(b) Maximum $|M|$
$= 12.15$ kN·m ◄

PROBLEM 5.20

5.19 and 5.20 Assuming the upward reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum value (*a*) of the shear, (*b*) of the bending moment.







SOLUTION

Over the whole beam

$+\uparrow \Sigma F_y = 0 \qquad 1.5w - 3 - 3 = 0$

$w = 0.8 \text{ kip/ft}$

A to C $\qquad 0 < x < 1.5 \text{ ft}$

$+\uparrow \Sigma F_y = 0 \qquad 0.8x - V = 0$

$V = 0.8x$

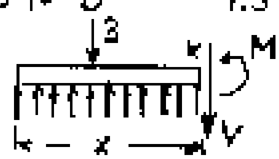
$\circlearrowleft \Sigma M_J = 0$

$-(0.8x)\left(\frac{x}{2}\right) + M = 0$

$M = 0.4x^2$

At C^- $\qquad V = 1.2 \text{ kips}, \qquad M = 0.9 \text{ kip·ft}$

C to D $\qquad 1.5 \text{ ft} < x < 6 \text{ ft}$



$+\uparrow \Sigma F_y = 0$

$0.8x - 3 - V = 0$

$V = 0.8x - 3$

$\circlearrowleft \Sigma M_K = 0 \qquad -(0.8x)\left(\frac{x}{2}\right) + 3(x - 1.5) + M = 0$

$M = 0.4x^2 - 3x + 4.5$

At the center of the beam $\qquad x = 3.75 \text{ ft}$

$V = 0, \qquad M = -1.125 \text{ kip ft}$

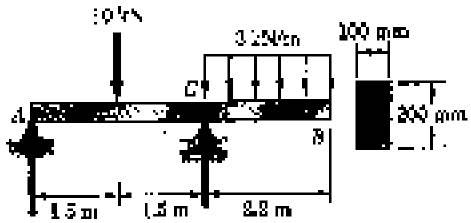
At C^+ $\qquad V = -1.8 \text{ kip} \qquad M = 0.9 \text{ kip ft}$

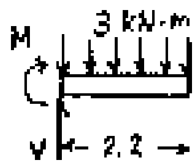
(a) Maximum $|V| = 1.8$ kips ◄

(b) Maximum $|M| = 1.125$ kip·ft ◄

PROBLEM 5.21

5.21 For the beam and loading shown, determine the maximum normal stress on a transverse section at C.





SOLUTION

Using CB as a free body

$\curvearrowleft \Sigma M_C = 0$

$-M + (2.2)(3\times10^3)(1.1) = 0$

$M = 7.26\times10^3$ N·m

Section modulus for rectangle

$S = \frac{1}{6}bh^2$

$= \frac{1}{6}(100)(200)^2 = 666.7\times10^3\ \text{mm}^3$

$= 666.7\times10^{-6}\ \text{m}^3$

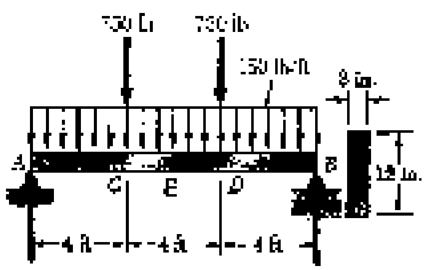
Normal stress

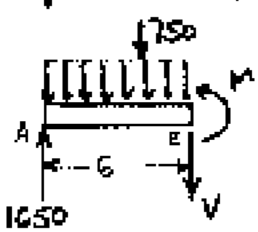
$\sigma = \frac{M}{S} = \frac{7.26\times10^3}{666.7\times10^{-6}} = 10.89\times10^6$ Pa

$\sigma = 10.89$ MPa

PROBLEM 5.22

5.22 For the beam and loading shown, determine the maximum normal stress on a transverse section at the center of the beam.





SOLUTION

Reactions: $C = A$ by symmetry

$+\uparrow \Sigma F_y = 0 \qquad A + C - (2)(750) - (12)(150) = 0$

$A = C = 1650$ lb.

Use left half of beam as free body

$\curvearrowleft \Sigma M_E = 0$

$-(1650)(6) + (750)(2) + (150)(6)(3) + M = 0$

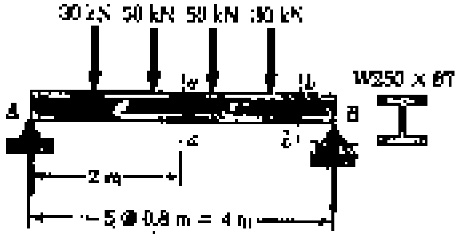
$M = 5700$ lb·ft $= 68.4\times10^3$ lb·in

Section modulus $S = \frac{1}{6}bh^2 = (\frac{1}{6})(3)(12)^2 = 72\ \text{in}^3$

Normal stress $\sigma = \frac{M}{S} = \frac{68.4\times10^3}{72} = 950$ psi

PROBLEM 5.23

5.23 For the beam and loading shown, determine the maximum normal stress on section *a-a*.

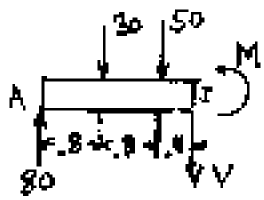


SOLUTION

Reactions: By symmetry $A = B$

$+\uparrow \Sigma F_y = 0 \qquad A = B = 80 \text{ kN}$

Using left half of beam as free body



$\circlearrowleft \Sigma M_J = 0$

$-(80)(2) + (30)(1.2) + (50)(0.4) + M = 0$

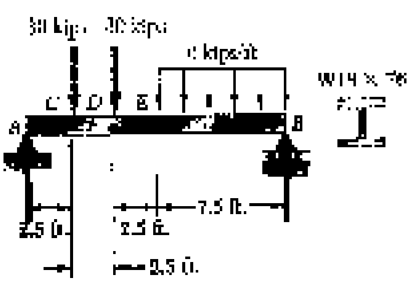
$M = 104 \text{ kN}\cdot\text{m} = 104 \times 10^3 \text{ N}\cdot\text{m}$

For W250 × 67 $\quad S = 809 \times 10^3 \text{ mm}^3 = 809 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{104 \times 10^3}{809 \times 10^{-6}} = 128.6 \times 10^6 \text{ Pa} = 128.6 \text{ MPa}$ ◄

PROBLEM 5.24

5.24 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.



SOLUTION

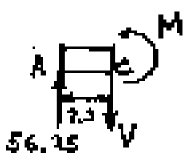
Use entire beam as free body

$\circlearrowleft \Sigma M_B = 0$

$-15A + (12.5)(30) + (10)(30) + (6)(7.5)(3.75) = 0$

$A = 56.25 \text{ kips}$

Use portion AC as free body



$\circlearrowleft \Sigma M_C = 0 \qquad -(56.25)(2.5) + M = 0$

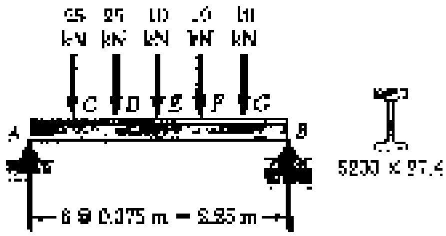
Bending moment at C $\quad M = 140.625 \text{ kip}\cdot\text{ft} = 1687.5 \text{ kip}\cdot\text{in}.$

For W18 × 76 $\quad S = 146 \text{ in}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{1687.5}{146} = 11.56 \text{ ksi}$ ◄

PROBLEM 5.25

5.25 and 5.26 For the beam and loading shown, determine the maximum normal stress on a transverse section at C.



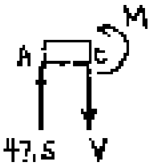
SOLUTION

Use entire beam as free body

$\circlearrowleft \Sigma M_B = 0$

$$2.25A - (1.875)(25) - (1.5)(25) - (1.125)(10) - (0.75)(10) - (0.375)(10) = 0$$

$$A = 47.5 \text{ kN}$$



Use portion AC as free body

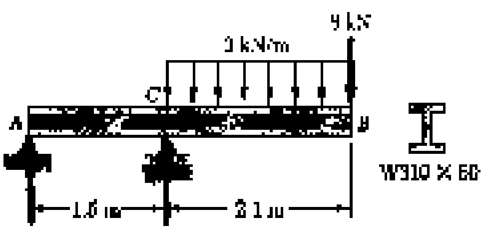
$$-(0.375)(47.5) + M = 0 \qquad M = 17.8125 \text{ kN}\cdot\text{m}$$

For $S\,200 \times 27.4$ $\quad S = 235 \times 10^3 \text{ mm}^3 = 235 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{17.8125 \times 10^3}{235 \times 10^{-6}} = 75.8 \times 10^6 \text{ Pa} = 75.8 \text{ MPa}$ ◄

PROBLEM 5.26

5.25 and 5.26 For the beam and loading shown, determine the maximum normal stress on a transverse section at C.



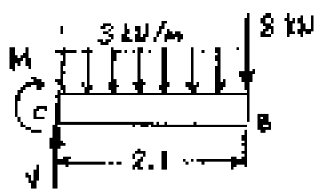
SOLUTION

Use portion CB as free body.

$\circlearrowleft \Sigma M_C = 0$

$$-M + (3)(2.1)(1.05) + (8)(2.1) = 0$$

$$M = -23.415 \text{ kN}\cdot\text{m}$$

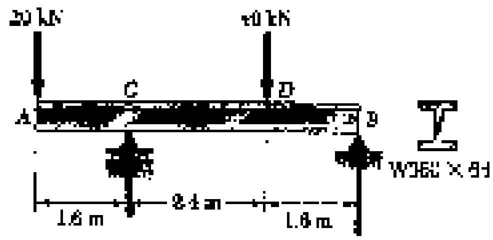


For $W\,310 \times 60$ $\quad S = 851 \times 10^3 \text{ mm}^3 = 851 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{23.415 \times 10^3}{851 \times 10^{-6}} = 27.5 \times 10^6 \text{ Pa} = 27.5 \text{ MPa}$ ◄

PROBLEM 5.27

5.27 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$\circlearrowleft \Sigma M_C = 0$

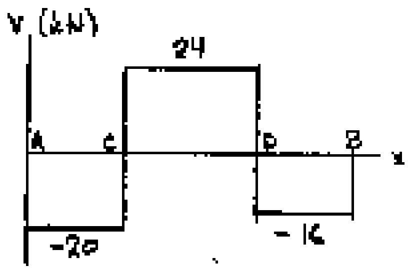
$(1.6)(20) - (2.4)(40) + (4.0)B = 0$

$B = 16$ kN

$\circlearrowleft \Sigma M_B = 0$

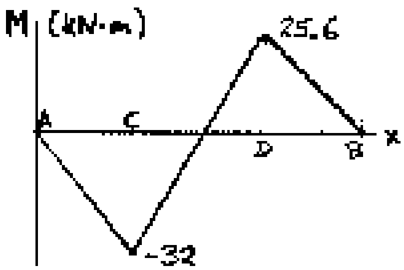
$(5.6)(20) - (4.0)C + (1.6)(40) = 0$

$C = 44$ kN



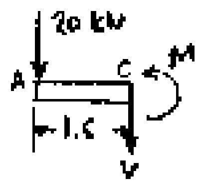
Shear

A to C^-	$V = -20$ kN
C^+ to D^-	$V = 24$ kN
D^+ to B	$V = -16$ kN



Bending moment

At C

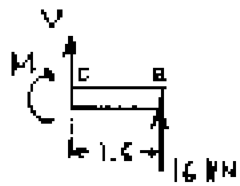


$\circlearrowleft \Sigma M_C = 0$

$(1.6)(20) + M = 0$

$M = -32$ kN·m

At D



$\circlearrowleft \Sigma M_D = 0$

$-M + (1.6)(16) = 0$

$M = 25.6$ kN·m

$\max |M| = 32$ kN·m $= 32 \times 10^3$ N·m ◄

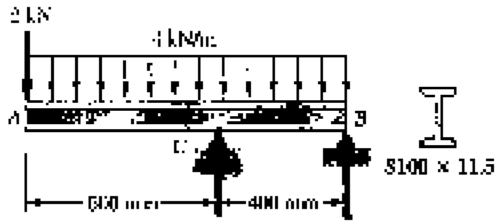
For rolled steel section W 360 × 64 $\quad S = 1030 \times 10^3$ mm^3 $= 1030 \times 10^{-6}$ m^3

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{32 \times 10^3}{1030 \times 10^{-6}} = 31.1 \times 10^6$ Pa

$= 31.1$ MPa ◄

PROBLEM 5.28

5.28 and 5.29 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$\circlearrowleft \Sigma M_C = 0$

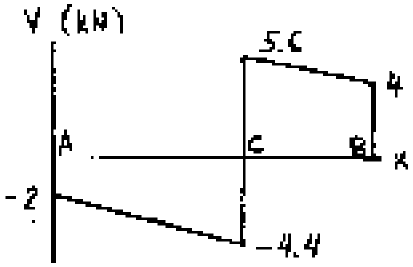
$(0.6)(2) + (0.1)(4) + (0.4)B = 0$

$B = -4$ kN

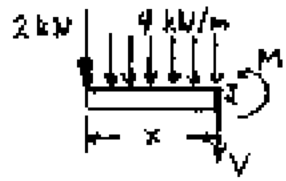
$\circlearrowleft \Sigma M_B = 0$

$(1.0)(2) + (0.5)(4) - (0.4)C = 0$

$C = 10$ kN



A to C $\quad 0 < x < 0.6$ m



$+\uparrow \Sigma F_y = 0$

$-2 - 4x - V = 0$

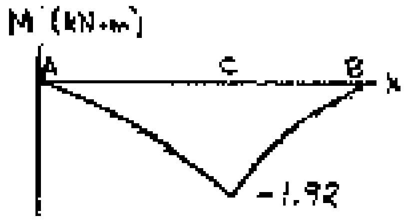
$V = -2 - 4x$

$\circlearrowleft \Sigma M_J = 0$

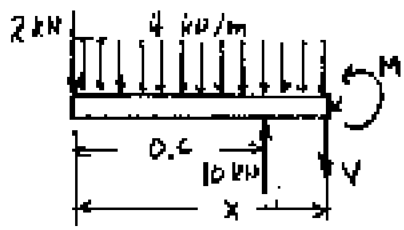
$2x + (4x)\left(\frac{x}{2}\right) + M = 0$

$M = -2x^2 - 2x$

At C $\quad M = -1.92$ kN·m



C to B $\quad 0.6$ m $< x < 1.0$ m



$+\uparrow \Sigma F_y = 0 \qquad -2 - 4x + 10 - V = 0$

$V = 8 - 4x$

$\circlearrowleft \Sigma M_K = 0 \qquad -2x + (4x)\left(\frac{x}{2}\right) - (10)(x - 0.6) + M = 0$

$$M = -2x^2 + 8x - 6$$

$\max |M| = 1.92$ kN·m $= 1.92 \times 10^3$ N·m

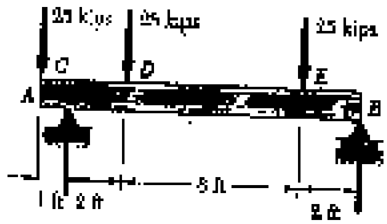
For rolled steel section S 100×11.5 $\qquad S = 49.6 \times 10^3$ mm^3 $= 49.6 \times 10^{-6}$ m^3

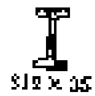
Maximum normal stress $\qquad \sigma = \dfrac{|M|}{S} = \dfrac{1.92 \times 10^3}{49.6 \times 10^{-6}} = 38.7 \times 10^6$ Pa

$= 38.7$ MPa ◄

PROBLEM 5.29

5.28 and 5.29 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.





SOLUTION

$\circlearrowleft \Sigma M_B = 0$

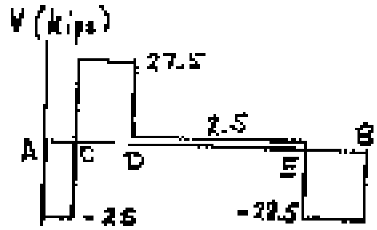
$(11)(25) - 10C + (8)(25) + (2)(25) = 0$

$C = 52.5$ kips

$\circlearrowleft \Sigma M_C = 0$

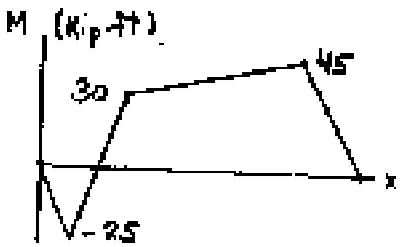
$(1)(25) - (2)(25) - (8)(25) + 10B = 0$

$B = 22.5$ kips



Shear

A to C^-	$V = -25$ kips
C^+ to D^-	$V = 27.5$ kips
D^+ to E^-	$V = 2.5$ kips
E^+ to B	$V = -22.5$ kips



Bending moments

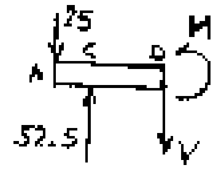
At C

$\circlearrowleft \Sigma M_C = 0$

$(1)(25) + M = 0$

$M = -25$ kip·ft

At D

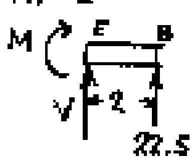


$\circlearrowleft \Sigma M_D = 0$

$(3)(25) - (2)(52.5) + M = 0$

$M = 30$ kip·ft

At E



$\circlearrowleft \Sigma M_E = 0$ $\quad -M + (2)(22.5) = 0 \quad M = 45$ kip·ft

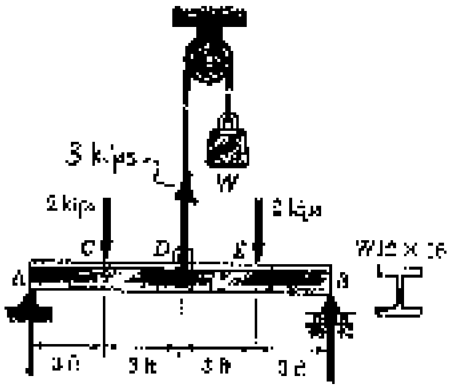
max $|M| = 45$ kip·ft $= 540$ kip·in

For S12 × 35 rolled steel section $\quad S = 38.2$ in^3

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{540}{38.2} = 14.14$ ksi

PROBLEM 5.30

5.30 Knowing that $W = 1$ kips, draw the shear and bending-moment diagrams for beam AB and determine the maximum normal stress due to bending.



SOLUTION

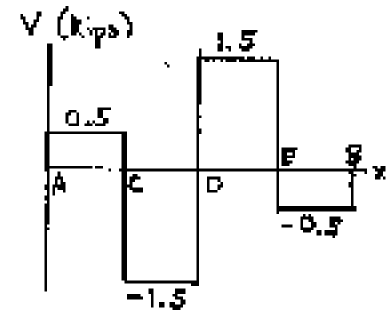
By symmetry $\quad A = B$

$+\uparrow \Sigma F_y = 0 \qquad A - 2 + 3 - 2 + B = 0$

$A = B = 0.5$ kip

Shear

A to C^-	$V = 0.5$ kips
C^+ to D^-	$V = -1.5$ kips
D^+ to E^-	$V = 1.5$ kips
E^+ to B	$V = -0.5$ kips



Bending moment

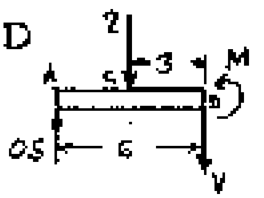
At C

$\circlearrowleft \Sigma M_C = 0$

$-(3)(0.5) + M = 0$

$M = 1.5$ kip·ft

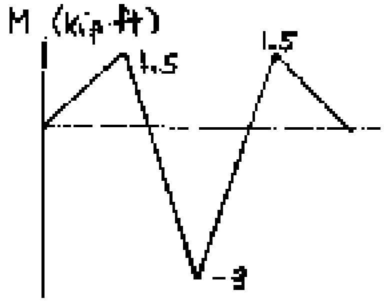
At D



$\circlearrowleft \Sigma M_D = 0$

$-(6)(0.5) + (3)(2) + M = 0$

$M = -3$ kip·ft



At E $\qquad M = 1.5$ kip·ft $\qquad$ by symmetry

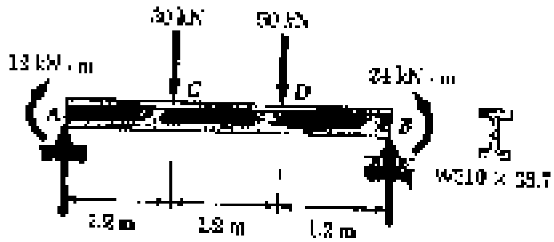
max $|M| = 3$ kip·ft $= 36$ kip·in

For rolled steel section $\quad$ W 12 × 16 $\quad S = 17.1\ \text{in}^3$

Normal stress $\qquad \sigma = \dfrac{|M|}{S} = \dfrac{36}{17.1} = 2.11$ ksi

PROBLEM 5.31

5.31 and 5.32 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$\circlearrowleft \Sigma M_B = 0$

$-12 - 3.6A + (2.4)(30) + (1.2)(50) + 24 = 0$

$A = 40 \text{ kN}$

$\circlearrowleft \Sigma M_A = 0$

$-12 - (1.2)(30) - (2.4)(50) + 24 + 3.6B = 0$

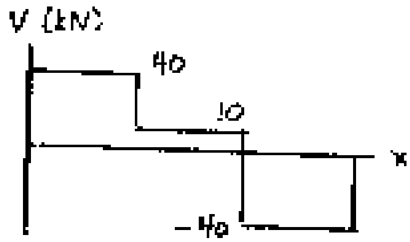
$B = 40 \text{ kN}$

Shear

A to C^- $\quad V = 40 \text{ kN}$

C^+ to D^- $\quad V = 10 \text{ kN}$

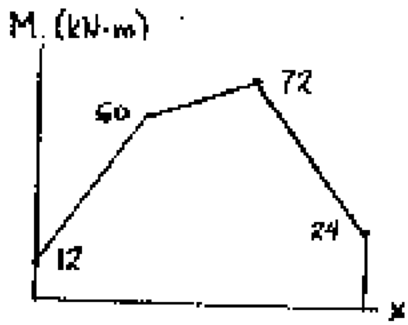
D^+ to B $\quad V = -50 \text{ kN}$



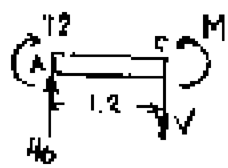
Bending moment

At A $\quad M = 12 \text{ kN·m}$

At B $\quad M = 24 \text{ kN·m}$



At C

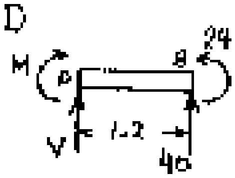


$\circlearrowleft \Sigma M_C = 0$

$-12 - (1.2)(40) + M = 0$

$M = 60 \text{ kN·m}$

At D



$\circlearrowleft M_D = 0$

$-M + 24 + (1.2)(40) = 0$

$M = 72 \text{ kN·m}$

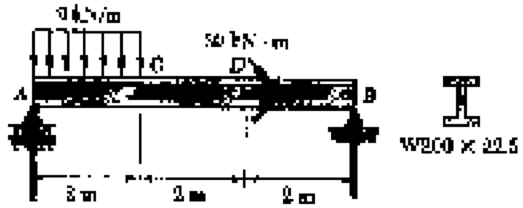
$\max|M| = 72 \text{ kN·m} = 72 \times 10^3 \text{ N·m}$

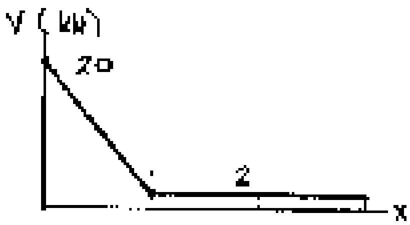
For rolled steel section W 310 × 38.7 $\quad S = 549 \times 10^3 \text{ mm}^3 = 549 \times 10^{-6} \text{ m}^3$

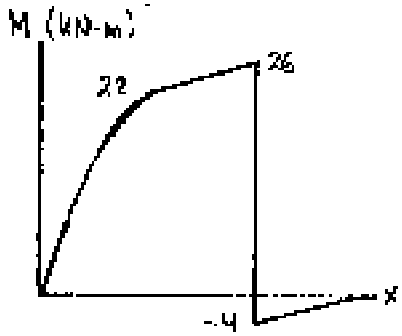
Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{72 \times 10^3}{549 \times 10^{-6}} = 131.1 \times 10^6 \text{ Pa} = 131.1 \text{ MPa}$ ◄

PROBLEM 5.31

5.31 and 5.32 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.







SOLUTION

$+\circlearrowleft \sum M_B = 0$

$-6A + (2)(9)(5) + 30 = 0$

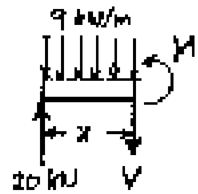
$A = 20 \text{ kN}$

$+\circlearrowleft \sum M_A = 0$

$-(2)(9)(1) + 30 + 6B = 0$

$B = -2 \text{ kN}$ i.e. $2 \text{ kN} \downarrow$

A to C $\quad 0 < x < 2 \text{ m}$



$+\uparrow \sum F_y = 0 \qquad 20 - 9x - V = 0$

$V = 20 - 9x$

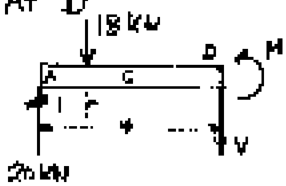
$+\circlearrowleft \sum M_J = 0$

$-20x + (9x)\frac{x}{2} + M = 0$

$M = 20x - 4.5x^2$

At C $\qquad V = 2 \text{ kN} \qquad M = 22 \text{ kN-m}$

At D^-



$+\uparrow \sum F_y = 0$

$20 - 18 - V = 0$

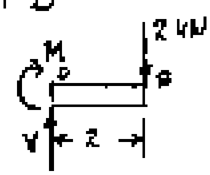
$V = 2 \text{ kN}$

$+\circlearrowleft \sum M_D = 0$

$-(4)(20) + (3)(18) + M = 0$

$M = 26 \text{ kN-m}$

At D^+



$+\uparrow \sum F_y = 0 \qquad V - 2 = 0$

$V = 2 \text{ kN}$

$+\circlearrowleft \sum M_D = 0$

$-M - (2)(2) = 0$

$M = -4 \text{ kN·m}$

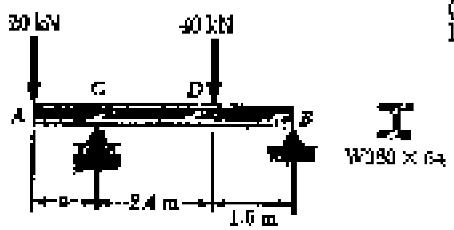
$\max |M| = 26 \text{ kN·m} = 26 \times 10^3 \text{ N·m}$

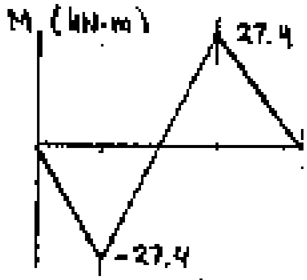
For rolled steel section $\quad$ W 200 × 22.5 $\qquad S = 194 \times 10^3 \text{ mm}^3 = 194 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{26 \times 10^3}{194 \times 10^{-6}} = 134.0 \times 10^6 \text{ Pa} = 134.0 \text{ MPa}$

PROBLEM 5.33

5.33 Determine (a) the distance a for which the maximum absolute value of the bending moment in the beam is as small as possible, (b) the corresponding maximum normal stress due to bending.
(*Hint:* Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)





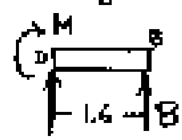
SOLUTION

Reaction at B $\quad \circlearrowleft \Sigma M_C = 0$

$$20a - (2.4)(40) + (4.0)B = 0$$

$$B = 24 - 5a$$

Bending moment at D

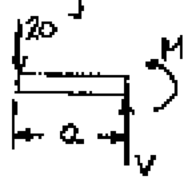


$$\circlearrowleft \Sigma M_D = 0$$

$$-M + 1.6B = 0$$

$$M_D = 1.6B = 38.4 - 8a$$

Bending moment at C



$$\circlearrowleft \Sigma M_C = 0$$

$$20a + M = 0$$

$$M_C = -20a$$

Equate $\quad -M_C = M_D$

$$20a = 38.4 - 8a$$

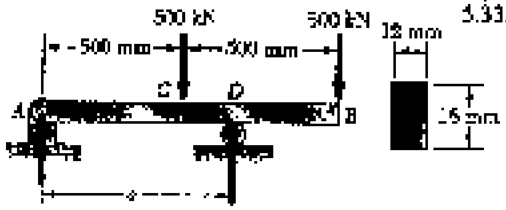
$$a = 1.3714 \text{ m}$$

$M_C = -27.429$ kN·m $\qquad M_D = 27.429$ kN·m

For W 360 × 64 rolled steel section $\quad S = 1030 \times 10^3 \text{ mm}^3 = 1030 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{27.429 \times 10^3}{1030 \times 10^{-6}} = 26.6 \times 10^6 \text{ Pa} = 26.6 \text{ MPa}$

PROBLEM 5.34



5.34 For the beam and loading shown, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.33.)

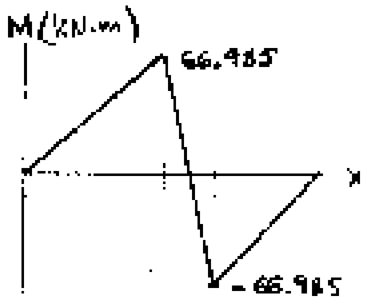
SOLUTION

Reaction at A $\quad \circlearrowleft \Sigma M_D = 0$

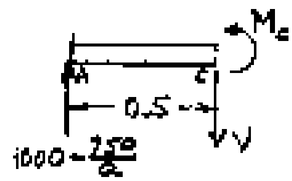
$$-Aa + (500)(a - 0.5) - 500(1 - a) = 0$$

$$Aa = 1000a - 750$$

$$A = 1000 - \frac{750}{a}$$



Bending moment at C $\quad \circlearrowleft \Sigma M_C = 0$



$$-(0.5)\left(1000 - \frac{750}{a}\right) + M_C = 0$$

$$M_C = 500 - \frac{375}{a}$$

Bending moment at D $\quad \circlearrowleft \Sigma M_D = 0$

$$-M_D - (500)(1-a) = 0$$

$$M_D = -500(1-a)$$

Equate $\quad -M_D = M_C \qquad 500(1-a) = 500 - \frac{375}{a}$

$$a = 0.86603 \text{ m} = 866.03 \text{ mm}$$

$A = 133.98$ kN $\qquad M_C = 66.985$ kN·m $\qquad M_D = -66.985$ kN·m

For rectangular cross section $\quad S = \frac{1}{6}bh^3 = \frac{1}{6}(12)(18)^3 = 11.664 \times 10^3 \text{ mm}^3$

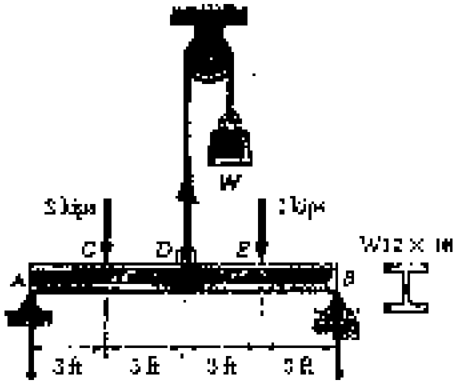
$$= 11.664 \times 10^{-6} \text{ m}^3$$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{66.985 \times 10^3}{11.664 \times 10^{-6}} = 5.74 \times 10^6 \text{ Pa}$

$$= 5.74 \text{ MPa}$$

PROBLEM 5.35

5.35 Determine (*a*) the magnitude of the counterweight W for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.33.)



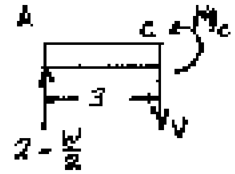
SOLUTION

By symmetry $\quad A = B$

$+\uparrow \Sigma F_y = 0 \qquad A - 2 + W - 2 + B = 0$

$$A = B = 2 - \frac{W}{2}$$

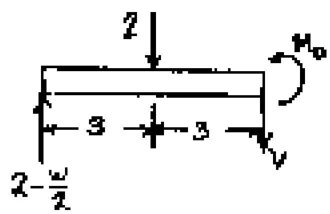
Bending moment at C $\quad \circlearrowleft \Sigma M_C = 0$



$-(3)(2 - \frac{W}{2}) + M_C = 0$

$M_C = 6 - 1.5\,W$

Bending moment at D $\quad \circlearrowleft \Sigma M_D = 0$



$-(6)(2 - \frac{W}{2}) + (3)(2)$
$+ M_D = 0$

$M_D = 6 - 3W$

Equate $\quad -M_D = M_C$

$3W - 6 = 6 - 1.5W \qquad\qquad W = 2.667$ kips ◄

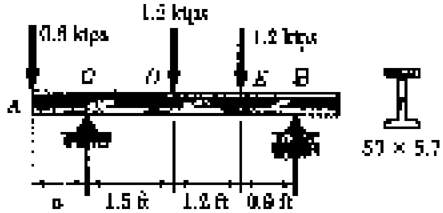
$M_C = 2.0$ kip·ft $\qquad\qquad M_D = -2.0$ kip·ft

max $|M| = 2.0$ kip·ft $= 24$ kip·in

For W 12×16 rolled steel section $\quad S = 17.1$ in³

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{24}{17.1} = 1.404$ ksi ◄

PROBLEM 5.36



5.36 For the beam and loading shown, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.33).

SOLUTION

$\circlearrowright \Sigma M_C = 0$

$0.8a - (1.5)(1.2) - (2.7)(1.2) + (3.6)B = 0$

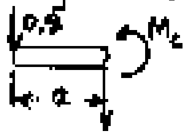
$B = 1.4 - 0.22222a$

$\circlearrowright \Sigma M_B = 0$

$(0.8)(3.6 + a) - 3.6C + (2.1)(1.2) + (0.9)(1.2) = 0$

$C = 1.8 + 0.22222a$

Bending moment at C

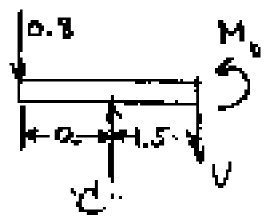


$\circlearrowright \Sigma M_C = 0$

$M_C + (0.8)(a) = 0$

$M_C = -0.8a$

Bending moment at D

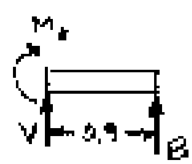


$\Sigma M_D = 0$

$M_D + (0.8)(a + 1.5) - 1.5C = 0$

$M_D = 1.5 - 0.46667a$

Bending moment at E



$\circlearrowright \Sigma M_E = 0$

$-M_E + 0.9B = 0$

$M_E = 1.26 - 0.2a$

Assume $-M_C = M_E$ $\quad 0.8a = 1.26 - 0.2a \quad\quad a = 1.26$ ft

$M_C = -1.008$ kip·ft $\quad M_E = 1.008$ kip·ft $\quad M_D = 0.912$ kip·ft

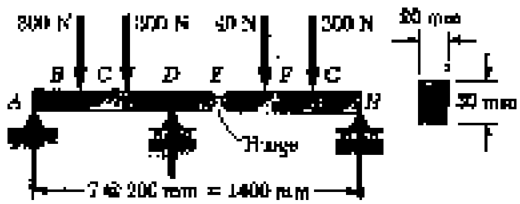
max $|M| = 1.008$ kip·ft $= 12.096$ kip·in

For rolled steel section S 3×5.7 $\quad S = 1.68\ \text{in}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{12.096}{1.68} = 7.20$ ksi

PROBLEM 5.37

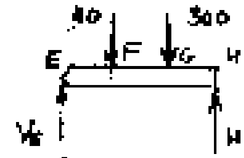
5.37 and 5.38 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

Free body EFGH

Note that $M_E = 0$ due to hinge.



$\curvearrowright \Sigma M_E = 0$

$0.6\,H - (0.2)(40) - (0.4)(300) = 0$

$H = 213.33$ N

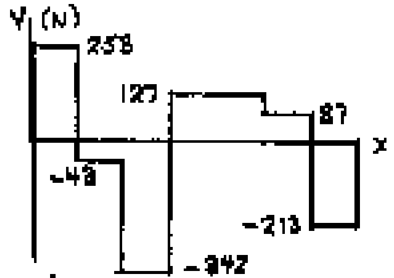
$+\uparrow \Sigma F_y = 0 \qquad V_E - 40 - 300 + 213.33 = 0$

$V_E = 126.67$ N

Shear: E to F $\quad V = 126.67$ N·m

F to G $\quad V = 86.67$ N·m

G to H $\quad V = -213.33$ N·m

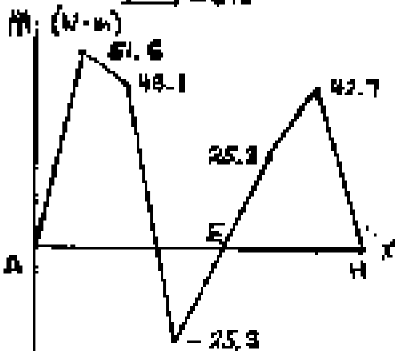


Bending moment at F

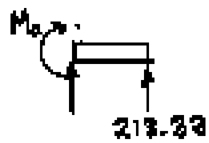
$\curvearrowright \Sigma M_F = 0$

$M_F - (0.2)(126.67) = 0$

$M_F = 25.33$ N·m



Bending moment at G

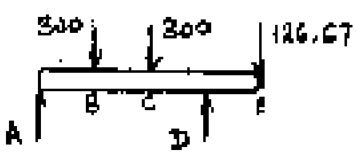


$\curvearrowright \Sigma M_G = 0$

$-M_G + (0.2)(213.33) = 0$

$M_G = 42.67$ N·m

Free body ABCDE



$+\curvearrowright \Sigma M_D = 0 \qquad -0.6\,A + (0.4)(300) + (0.2)(300) - (0.2)(126.67) = 0$

$A = 257.78$ N

$+\curvearrowright \Sigma M_A = 0 \qquad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6\,D = 0$

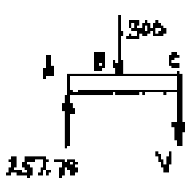
$D = 468.89$ N

Bending moment at B

$\curvearrowright \Sigma M_B = 0$

$-(0.2)(257.78) + M_B = 0$

$M_B = 51.56$ N·m



Bending moment at C

$+\curvearrowright \Sigma M_C = 0$

$-(0.4)(257.78) + (0.2)(300) + M_C = 0$

$M_C = 43.11$ N·m

Bending moment at D

$\curvearrowright \Sigma M_D = 0$

$-M_D - (0.2)(213.33) = 0$

$M_D = -25.33$ N·m

$\max |M| = 51.56$ N·m

$S = \frac{1}{6} b h^2 = \frac{1}{6}(20)(30)^2$

$= 3 \times 10^3\ \text{mm}^3 = 3 \times 10^{-6}\ \text{m}^3$

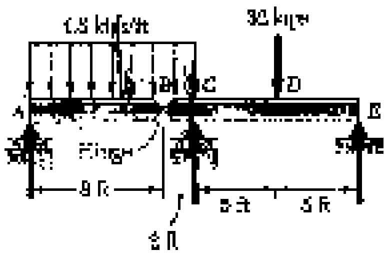
Normal stress

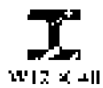
$\sigma = \dfrac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6$ Pa

$= 17.19$ MPa ◄

PROBLEM 5.38

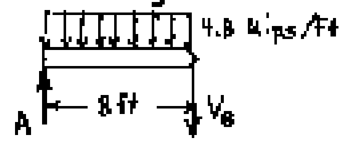
5.37 and 5.38 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.





SOLUTION

Free body AB



$\circlearrowleft \Sigma M_B = 0$

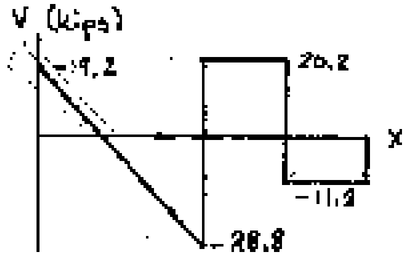
$(4.8)(8)(4) - 8A = 0$

$A = 19.2$ kips

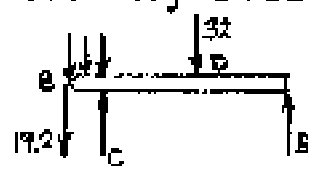
$\circlearrowleft M_A = 0$

$-(4.8)(8)(4) - 8V_B = 0$

$V_B = -19.2$ kips



Free body BCDE



$+\circlearrowleft \Sigma M_E = 0$

$(19.2)(12) + (4.8)(2)(11)$

$- 10C + (32)(5) = 0$

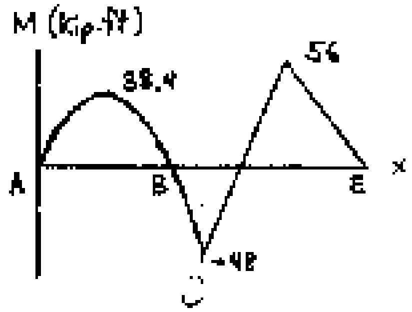
$C = 49.2$ kips

$+\circlearrowleft M_C = 0$

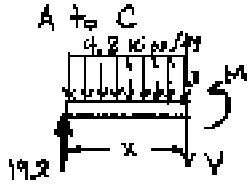
$(19.2)(2) + (4.8)(2)(1)$

$-(32)(5) + 10E = 0$

$E = 11.2$ kips



A to C



$0 < x < 10$ ft.

$+\uparrow \Sigma F_y = 0$

$19.2 - 4.8x - V = 0$

$V = 19.2 - 4.8x$ kips.

$\circlearrowleft M_J = 0$

$-19.2x + (4.8x)\frac{x}{2} + M = 0$

$M = 19.2x - 2.4x^2$ kip·ft

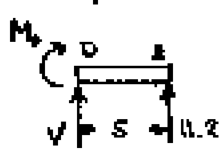
At C^- $x = 10$ $V = 19.2 - (4.8)(10) = -28.8$ kips

At C $x = 10$ $M_C = (19.2)(10) - (2.4)(10)^2 = -48$ kip·ft

C to D $V = 19.2 - (4.8)(10) + 49.6 = 20.8$ kips.

D to E $V = -11.2$ kips

Bending moment at D



$\circlearrowleft \Sigma M_D = 0$

$-M_D + (11.2)(5) = 0$

$M_D = 56$ kip·ft

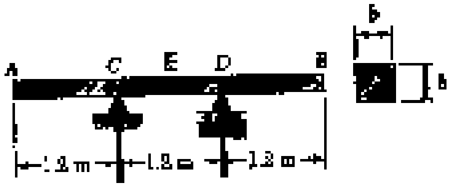
max $|M| = 56$ kip·ft $= 672$ kip·in

For W 12 × 40 rolled steel section $S = 51.9$ in^3

Normal stress $\sigma = \frac{|M|}{S} = \frac{672}{51.9} = 12.95$ ksi

PROBLEM 5.39

5.39 A solid steel bar has a square cross section of side b and is supported as shown. Knowing that for steel $\rho = 7860\ \mathrm{kg/m^3}$, determine the dimension b of the bar for which the maximum normal stress due to bending is (*a*) 10 MPa, (*b*) 50 MPa.



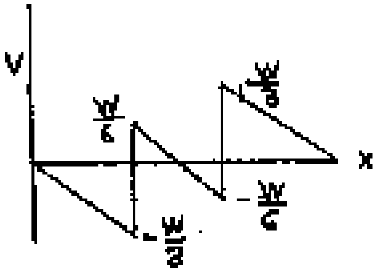
SOLUTION

Weight density $\gamma = \rho g$

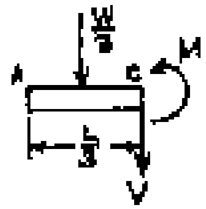
Let L = total length of beam

$$W = V\gamma = AL\rho g = b^2 L\rho g$$

Reactions at C and D $\qquad C = D = \frac{W}{2}$



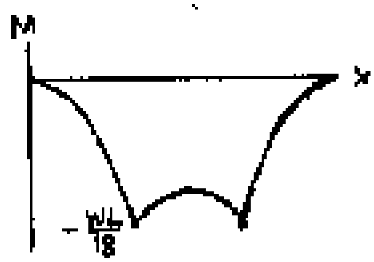
Bending moment at C



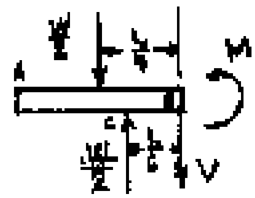
$$\circlearrowleft \Sigma M_C = 0$$

$$\left(\frac{L}{6}\right)\left(\frac{W}{3}\right) + M = 0$$

$$M = -\frac{WL}{18}$$



Bending moment at center of beam



$$\circlearrowleft \Sigma M_E = 0$$

$$\left(\frac{L}{4}\right)\left(\frac{W}{2}\right) - \left(\frac{L}{6}\right)\left(\frac{W}{2}\right) + M = 0$$

$$M = -\frac{WL}{24}$$

$$\max|M| = \frac{WL}{18} = \frac{b^2L^2\rho g}{18}$$

For a square section $\qquad S = \frac{1}{6}b^3$

Normal stress $\qquad \sigma = \frac{|M|}{S} = \frac{b^2L^2\rho g/18}{b^3/6} = \frac{L^2\rho g}{3b}$

Solve for $b \qquad b = \frac{L^2\rho g}{3\sigma}$

Data: $L = 3.6\ \mathrm{m} \qquad \rho = 7860\ \mathrm{kg/m^3} \qquad g = 9.81\ \mathrm{m/s^2}$

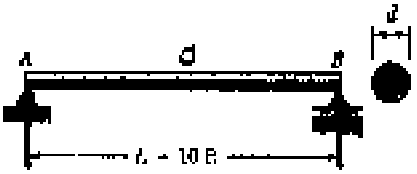
(a) $\sigma = 10\times10^6\ \mathrm{Pa} \qquad$ (b) $\sigma = 50\times10^6\ \mathrm{Pa}$

(a) $\quad b = \frac{(3.6)^2(7860)(9.81)}{(3)(10\times10^6)} = 33.3\times10^{-3}\ \mathrm{m} = 33.3\ \mathrm{mm}$ ◄

(b) $\quad b = \frac{(3.6)^2(7860)(9.81)}{(3)(50\times10^6)} = 6.66\times10^{-3}\ \mathrm{m} = 6.66\ \mathrm{mm}$ ◄

PROBLEM 5.40

5.40 A solid steel rod of diameter d is supported as shown. Knowing that for steel $\gamma = 490\ \text{lb/ft}^3$, determine the smallest diameter d which can be used if the normal stress due to bending is not to exceed 4 ksi.



SOLUTION

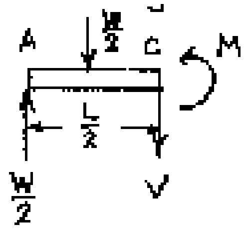
Let W = total weight

$$W = V\gamma = AL\gamma = \frac{\pi}{4} d^2 L \gamma$$

Reaction at A

$$A = \frac{1}{2} W$$

Bending moment at center of beam



$$\circlearrowleft \Sigma M_C = 0$$

$$-\left(\frac{W}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{W}{2}\right)\left(\frac{L}{4}\right) + M = 0$$

$$M = \frac{WL}{8} = \frac{\pi}{32} d^2 L^2 \gamma$$

For circular cross section $\left(c = \frac{1}{2} d\right)$

$$I = \frac{\pi}{4} c^4, \qquad S = \frac{I}{c} = \frac{\pi}{4} c^3 = \frac{\pi}{32} d^3$$

Normal stress

$$\sigma = \frac{M}{S} = \frac{\frac{\pi}{32} d^2 L^2 \gamma}{\frac{\pi}{32} d^3} = \frac{L^2 \gamma}{d}$$

Solving for d $\qquad d = \dfrac{L^2 \gamma}{\sigma}$

Data: $\quad L = 10\ \text{ft} = (12)(10) = 120\ \text{in}$

$$\gamma = 490\ \text{lb/ft}^3 = \frac{490}{12^3} = 0.28356\ \text{lb/in}^3$$

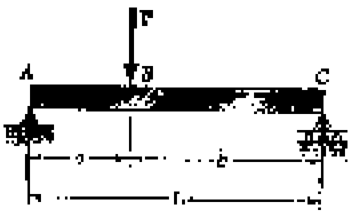
$$\sigma = 4\ \text{ksi} = 4000\ \text{lb/in}^2$$

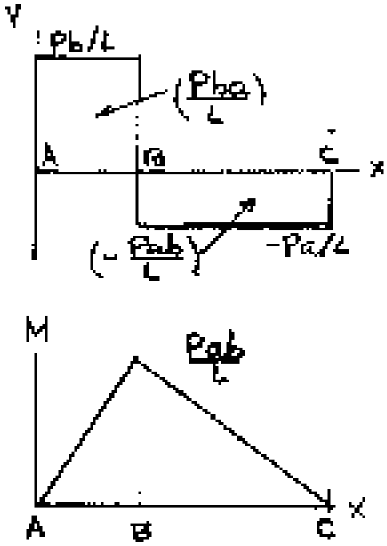
$$d = \frac{(120)^2 (0.28356)}{4000} = 1.021\ \text{in.}$$

PROBLEM 5.41

5.41 Using the methods of Sec. 5.3, solve Prob. 5.1.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.





SOLUTION

$\circlearrowright \Sigma M_C = 0 \qquad LA - bP = 0 \qquad A = \frac{Pb}{L}$

$\circlearrowright \Sigma M_A = 0 \qquad LC - aP = 0 \qquad C = \frac{Pa}{L}$

At A^+ $\qquad V = A = \frac{Pb}{L} \qquad M = 0$

A to B^- $\qquad 0 < x < a$

$$w = 0 \qquad \int_0^x w\,dx = 0$$

$$V_B - V_A = 0 \qquad V_B = \frac{Pb}{L}$$

$$M_B - M_A = \int_0^a V\,dx = \int_0^a \frac{Pb}{L}\,dx = \frac{Pba}{L}$$

$$M_B = \frac{Pba}{L}$$

At B^+ $\qquad V = A - P = \frac{Pb}{L} - P = -\frac{Pa}{L}$

B^+ to C $\qquad a < x < L$

$$w = 0 \qquad \int_a^x w\,dx = 0$$

$$V_C - V_B = 0 \qquad V = -\frac{Pa}{L}$$

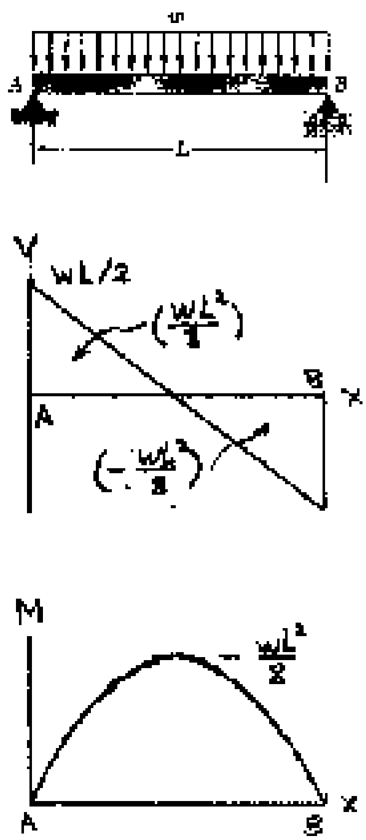
$$M_C - M_B = \int_a^L V\,dx = -\frac{Pa}{L}(L-a) = -\frac{Pab}{L}$$

$$M_C = M_B - \frac{Pab}{L} = \frac{Pba}{L} - \frac{Pab}{L} = 0$$

PROBLEM 5.42

5.42 Using the methods of Sec. 5.3, solve Prob. 5.2.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



SOLUTION

$\circlearrowleft \Sigma M_B = 0 \qquad -AL + wL\cdot\frac{L}{2} = 0 \qquad A = \frac{wL}{2}$

$\circlearrowleft \Sigma M_A = 0 \qquad BL - wL\cdot\frac{L}{2} = 0 \qquad B = \frac{wL}{2}$

$\frac{dV}{dx} = -w$

$V - V_A = -\int_0^x w\,dx = -wx$

$V = V_A - wx = A - wx = \frac{wL}{2} - wx$

$\frac{dM}{dx} = V$

$M - M_A = \int_0^x V\,dx = \int_0^x \left(\frac{wL}{2} - wx\right)dx$

$= \frac{wLx}{2} - \frac{wx^2}{2}$

$M = M_A + \frac{wLx}{2} - \frac{wx^2}{2} = \frac{w}{2}(Lx - x^2)$

Maximum M occurs at $x = \frac{L}{2}$ where

$V = \frac{dM}{dx} = 0$

$\text{Max } M = \frac{wL^2}{8}$

PROBLEM 5.43

5.43 Using the methods of Sec. 5.3, solve Prob. 5.3

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION

$$w = w_0 \frac{x}{L}$$

$$V_A = 0 \,, \qquad M_A = 0$$

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

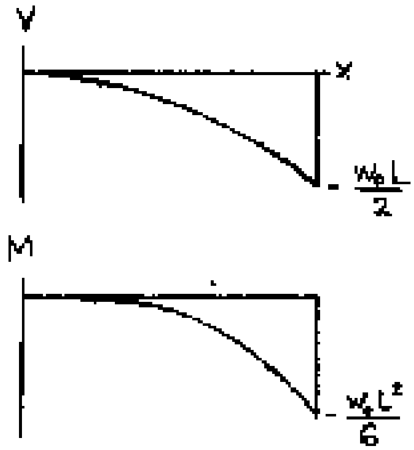
$$V - V_A = -\int_0^x \frac{w_0 x}{L} = -\frac{w_0 x^2}{2L}$$

$$V = -\frac{w_0 x^2}{2L}$$

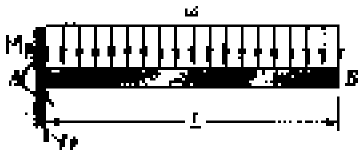
$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L}$$

$$M - M_A = \int_0^x V\,dx = -\int_0^x \frac{w_0 x^2}{2L}\,dx$$

$$= -\frac{w_0 x^3}{6L}$$



PROBLEM 5.44



5.44 Using the methods of Sec. 5.3, solve Prob. 5.4.

5.3 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION

$+\uparrow \Sigma F_y = 0 \qquad V_A - wL = 0 \qquad V_A = wL$

$\circlearrowleft \Sigma M_A = 0 \qquad -M - (wL)\left(\frac{L}{2}\right) = 0 \qquad M_A = -\frac{wL^2}{2}$

$$\frac{dV}{dx} = -w$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = wL - wx$$

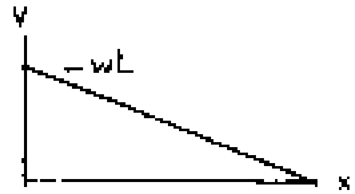
$$\frac{dM}{dx} = V = wL - wx$$

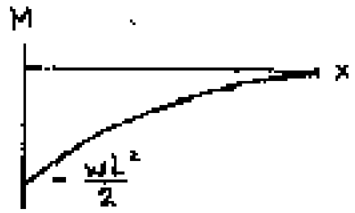
$$M - M_A = \int_0^x (wL - wx)\,dx = wLx - \frac{wx^2}{2}$$

$$M = -\frac{wL^2}{2} + wLx - \frac{wx^2}{2}$$

$\max |V| = wL$

$\max |M| = \frac{wL^2}{2}$

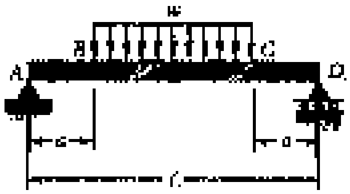




PROBLEM 5.45

5.45 Using the methods of Sec. 5.3, solve Prob. 5.5.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



SOLUTION

Reactions $\quad A = D = \frac{1}{2}w(L-2a)$

At A $\quad V_A = A = \frac{1}{2}w(L-2a), \quad M_A = 0$

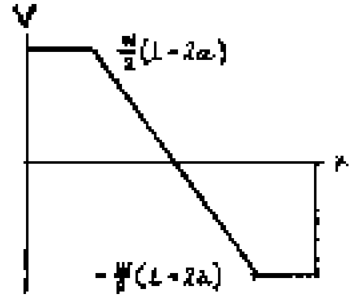
A to B $\quad 0 < x < a \qquad w = 0$

$$V_B - V_A = -\int_0^a w\,dx = 0$$

$$V_B = V_A = \frac{1}{2}w(L-2a)$$

$$M_B - M_A = \int_0^a V\,dx = \int_0^a \frac{1}{2}w(L-2a)\,dx$$

$$M_B = \frac{1}{2}w(L-2a)a$$



B to C $\quad a < x < L-a \qquad w = w$

$$V - V_B = -\int_a^x w\,dx = -w(x-a)$$

$$V = \frac{1}{2}w(L-2a) - w(x-a) = \frac{1}{2}w(L-2x)$$

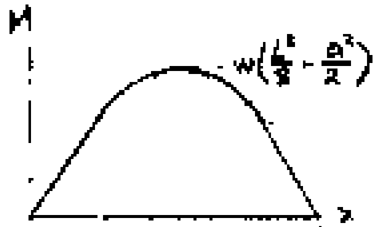
$$\frac{dM}{dx} = V = \frac{1}{2}w(L-2x)$$

$$M - M_B = \int_a^x V\,dx = \frac{1}{2}w(Lx - x^2)\Big|_a^x$$

$$\ldots = \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$M = \frac{1}{2}w(L-2a)a + \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$= \frac{1}{2}w(Lx - x^2 - a^2)$$



At C $\quad x = L-a \qquad V_C = -\frac{1}{2}w(L-2a) \qquad M_C = \frac{1}{2}(L-2a)a$

C to D $\qquad V = V_C = -\frac{1}{2}w(L-2a)$

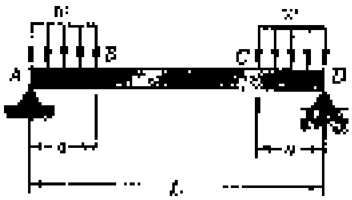
$\qquad M_D = 0$

At $x = \frac{L}{2} \qquad M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right)$

PROBLEM 5.46

5.46 Using the methods of Sec. 5.3, solve Prob. 5.6

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



SOLUTION

Reactions $\quad A = D = wa$

A to B $\quad 0 < x < a \quad w = w$

$$V_A = A = wa, \qquad M_A = 0$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

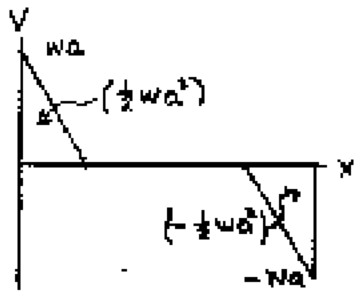
$$V = w(a - x) \qquad V_B = 0$$

$$\frac{dM}{dx} = V = wa - wx$$

$$M - M_A = \int_0^x V\,dx = \int_0^x (wa - wx)\,dx$$

$$= wax - \tfrac{1}{2}wx^2$$

$$M_B = \tfrac{1}{2}wa^2 \quad \text{at } x = a.$$

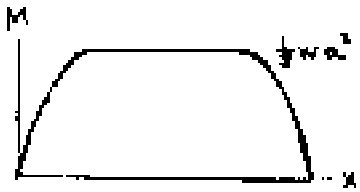


B to C $\quad a < x < L - a \quad V = 0$

$$\frac{dM}{dx} = V = 0$$

$$M - M_B = \int_a^x V\,dx = 0$$

$$M = M_B = \tfrac{1}{2}wa^2$$



C to D $\qquad V - V_C = -\int_{L-a}^{x} w\,dx = -w[x - (L-a)]$

$$V = -w[x - (L-a)]$$

$$M - M_C = \int_{L-a}^{x} V\,dx = \int_{L-a}^{x} -w[x - (L-a)]\,dx$$

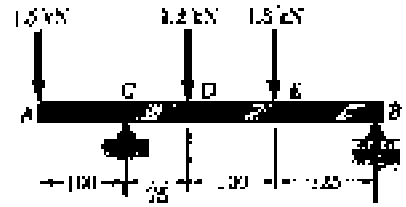
$$= -w\left[\frac{x^2}{2} - (L-a)x\right]\Big|_{L-a}^{x}$$

$$= -w\left[\frac{x^2}{2} - (L-a)x - \frac{(L-a)^2}{2} + (L-a)^2\right]$$

$$= -w\left[\frac{x^2}{2} - (L-a)x + \frac{(L-a)^2}{2}\right]$$

$$M = \tfrac{1}{2}wa^2 - w\left[\frac{x^2}{2} - (L-a)x + \frac{(L-a)^2}{2}\right]$$

PROBLEM 5.47



Dimensions in mm

5.47 Using the methods of Sec. 5.3, solve Prob. 5.13.

5.13 and 5.14 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

SOLUTION

$\circlearrowleft M_B = 0$

$(400)(1.5) - 300\,C + (225)(1.2) + (125)(1.8) = 0$

$C = 3.65$ kN

$\circlearrowleft M_C = 0 \qquad B = 0.85$ kN

Shear:

A to C $\qquad V = -1.5$ kN

C to D $\qquad V = -1.5 + 3.65 = 2.15$ kN

D to E $\qquad V = 2.15 - 1.2 = 0.95$ kN

E to B $\qquad V = 0.95 - 1.8 = -0.85$ kN

Areas of shear diagram

A to C $\qquad \int V dx = (-1.5)(100) = -150$ N·m

C to D $\qquad \int V dx = (2.15)(75) = 161.25$ N·m

D to E $\qquad \int V dx = (0.95)(100) = 95$ N·m

E to B $\qquad \int V dx = (-0.85)(125) = -106.25$ N·m

Bending moments

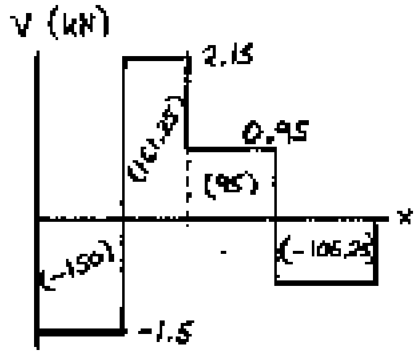
$M_A = 0$

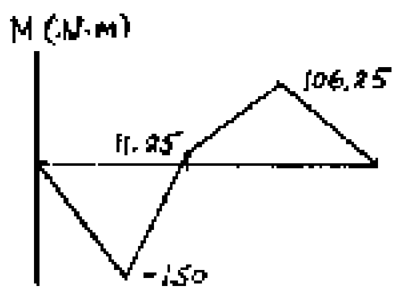
$M_C = M_A + \int_A^C V dx = 0 - 150 = -150$ N·m

$M_D = M_C + \int_C^D V dx = -150 + 161.25 = 11.25$ N·m

$M_E = M_D + \int_D^E V dx = 11.25 + 95 = 106.25$ N·m

$M_B = M_E + \int_E^B V dx = 106.25 - 106.25 = 0$





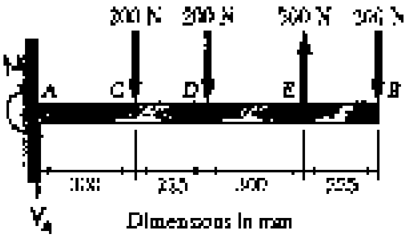
Maximum $|V| = 2.15$ kN ◄

Maximum $|M| = 150$ N·m ◄

PROBLEM 5.48

5.48 Using the methods of Sec. 5.3, solve Prob. 5.14.

5.13 and 5.14 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



SOLUTION

$$\circlearrowleft \Sigma M_A = 0$$

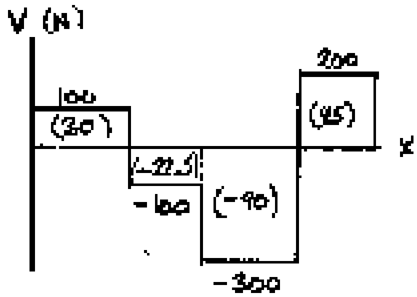
$$-M_A - (0.3)(200) - (0.525)(200) + (0.825)(500) - (1.05)(200) = 0$$

$$M_A = 37.5 \text{ N}\cdot\text{m}$$

$$+\uparrow \Sigma F_y = 0$$

$$V_A - 200 - 200 + 500 - 200 = 0$$

$$V_A = 100 \text{ N}$$



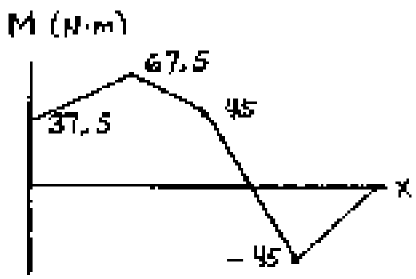
Shear

A to C $\quad V = 100$ N

C to D $\quad V = 100 - 200 = -100$ N

D to E $\quad V = -100 - 200 = -300$ N

E to B $\quad V = -300 + 500 = 200$ N



Areas under shear diagram

A to C $\quad \int V\,dx = (100)(0.3) = 30$ N·m

C to D $\quad \int V\,dx = (-100)(0.225) = -22.5$ N·m

D to E $\quad \int V\,dx = (-300)(0.3) = -90$ N·m

E to B $\quad \int V\,dx = (200)(0.225) = 45$ N·m

Bending moments

$$M_A = 37.5 \text{ N}\cdot\text{m}$$

$$M_C = M_A + \int_A^C V\,dx = 37.5 + 30 = 67.5 \text{ N}\cdot\text{m}$$

$$M_D = M_C + \int_C^D V\,dx = 67.5 - 22.5 = 45 \text{ N}\cdot\text{m}$$

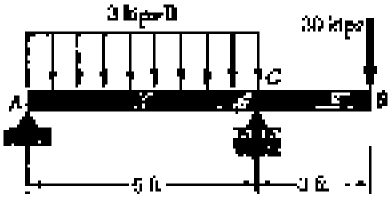
$$M_E = M_D + \int_D^E V\,dx = 45 - 90 = -45 \text{ N}\cdot\text{m}$$

$$M_B = M_E + \int_E^B V\,dx = -45 + 45 = 0$$

Maximum $|V| = 300$ N ◄

Maximum $|M| = 67.5$ N·m ◄

PROBLEM 5.49



5.49 Using the methods of Sec. 5.3, solve Prob. 5.15.

5.15 and 5.16 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

$+\circlearrowleft \Sigma M_C = 0 \quad -6A + (3)(18) - (3)(30) = 0$

$A = -6$ kips i.e. 6 kips ↓

$+\circlearrowleft \Sigma M_A = 0 \quad 6C - (3)(18) - (9)(30) = 0$

$C = 54$ kips ↑

Shear

$V_A = -6$ Kips

A to C $\quad 0 < x < 6$ ft. $\quad w = -3$ kips/ft

$V_B - V_A = -\int_0^6 w\,dx = -\int_0^6 3\,dx = -18$ Kips

$V_B = -6 - 18 = -24$ kips

C to B $\quad V = -24 + 54 = 30$ kips.

Areas under shear diagram

A to C $\quad \int V dx = (\frac{1}{2})(-6-24)(6)$

$= -90$ kip ft.

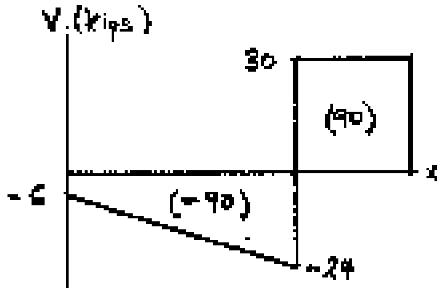
C to B $\quad \int V\,dx = (3)(30) = 90$ kip.ft

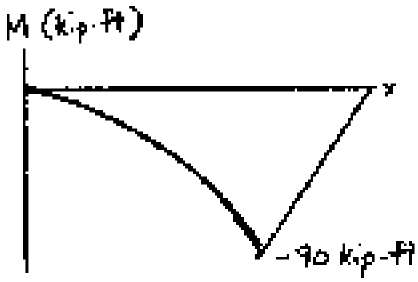
Bending moments

$M_A = 0$

$M_C = M_A + \int V dx = 0 - 90 = -90$ kip.ft

$M_B = M_C + \int V dx = -90 + 90 = 0$

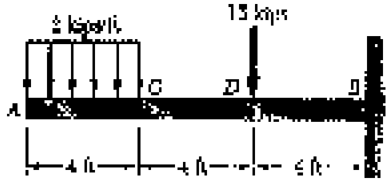




Maximum $|V| = 30$ Kips ◄

Maximum $|M| = 90$ Kip.ft ◄

PROBLEM 5.50



5.50 Using the methods of Sec. 5.3, solve Prob. 5.16.

5.15 and 5.16 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

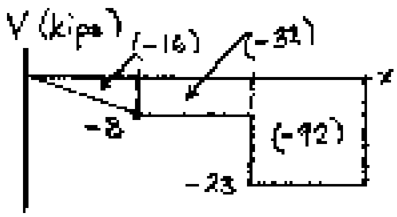
Shear

$V_A = 0$

$V_B = V_A - \int_A^B w\,dx = 0 - (4)(2) = -8$ kips

C to D $\quad V = -8$ kips

D to B $\quad V = -8 - 15 = -23$ kips

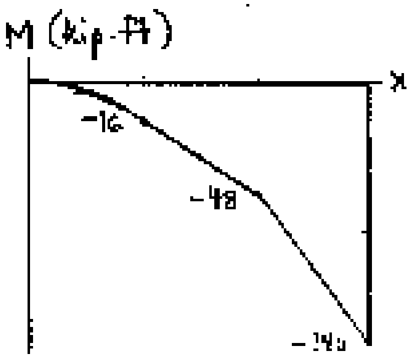


Areas under shear diagram

A to C $\quad \int V\,dx = (\frac{1}{2})(4)(-8) = -16$ kip·ft

C to D $\quad \int V\,dx = (4)(-8) = -32$ kip·ft

D to B $\quad \int V\,dx = (4)(-23) = -92$ kip·ft



Bending moments

$M_A = 0$

$M_C = M_A + \int V\,dx = 0 - 16 = -16$ kip·ft

$M_D = M_C + \int V\,dx = -16 - 32 = -48$ kip·ft

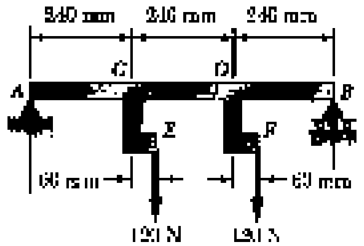
$M_B = M_D + \int V\,dx = -48 - 92 = -140$ kip·ft

Maximum $|V| = 23$ kips ◄

Maximum $|M| = 140$ kip·ft ◄

PROBLEM 5.51

5.51 and 5.52 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



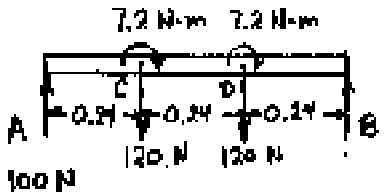
SOLUTION

$+\circlearrowleft \Sigma M_B = 0 \qquad -0.72A + (0.48)(120) + (0.24)(120) - 7.2 - 7.2 = 0$

$A = 100 \text{ N}$

$+\circlearrowleft \Sigma M_A = 0 \qquad -(0.24)(120) - (0.48)(120) - 7.2 - 7.2 + 0.72B = 0$

$B = 140 \text{ N}$



Shear

A to C $\quad V = 100 \text{ N}$

C to D $\quad V = 100 - 120 = -20 \text{ N}$

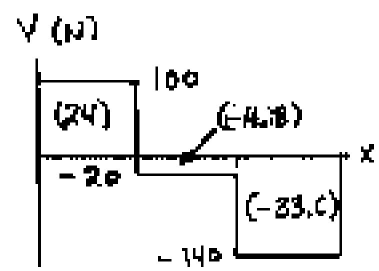
D to B $\quad V = -20 - 120 = -140 \text{ N}$

Areas under shear diagram

A to C $\quad \int V\,dx = (0.24)(100) = 24 \text{ N·m}$

C to D $\quad \int V\,dx = (0.24)(-20) = -4.8 \text{ N·m}$

D to B $\quad \int V\,dx = (0.24)(-140) = -33.6 \text{ N·m}$



Bending moments

$M_A = 0$

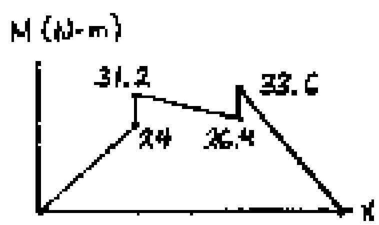
$M_C^- = 0 + 24 = 24 \text{ N·m}$

$M_C^+ = 24 + 7.2 = 31.2 \text{ N·m}$

$M_D^- = 31.2 - 4.8 = 26.4 \text{ N·m}$

$M_D^+ = 26.4 + 7.2 = 33.6 \text{ N·m}$

$M_B = 33.6 - 33.6 = 0$

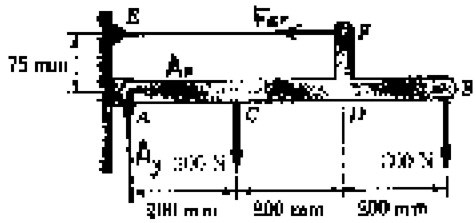


Maximum $|V| = 140 \text{ N}$ ◄

Maximum $|M| = 33.6 \text{ N·m}$ ◄

PROBLEM 5.52

5.51 and 5.52 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



SOLUTION

$\circlearrowleft \Sigma M_A = 0$

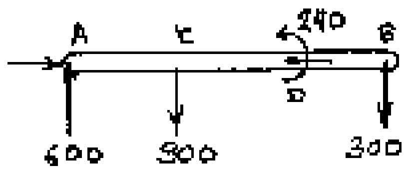
$0.075\, F_{EF} - (0.2)(300) - (0.6)(300) = 0$

$F_{EF} = 3.2 \times 10^3$ N

$\xrightarrow{+} \Sigma F_x = 0 \qquad A_x - F_{EF} = 0 \qquad A_x = 3.2 \times 10^3$ N

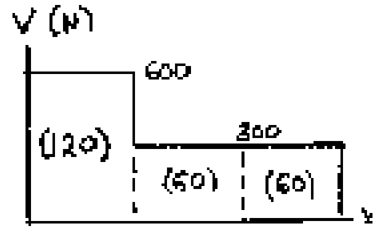
$+\uparrow \Sigma F_y = 0 \qquad A_y - 300 - 300 = 0$

$A_y = 600$ N



Couple at D $\quad M_D = (0.075)(3.2 \times 10^3)$

$= 240$ N·m



Shear

A to C $\qquad V = 600$ N

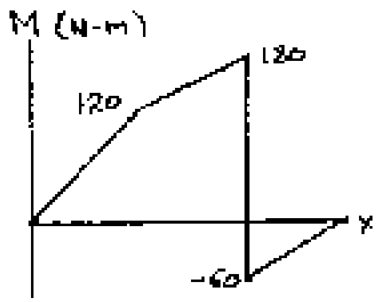
C to B $\qquad V = 600 - 300 = 300$ N

Areas under shear diagram

A to C $\qquad \int V dx = (0.2)(600) = 120$ N·m

C to D $\qquad \int V dx = (0.2)(300) = 60$ N·m

D to B $\qquad \int V dx = (0.2)(300) = 60$ N·m



Bending moments

$M_A = 0$

$M_C = 0 + 120 = 120$ N·m

$M_D^- = 120 + 60 = 180$ N·m

$M_D^+ = 180 - 240 = -60$ N·m

$M_B = -60 + 60 = 0$

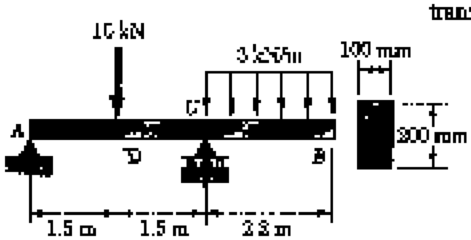
Maximum $|V| = 600$ N ◄

Maximum $|M| = 180$ N·m ◄

PROBLEM 5.53

5.53 Using the methods of Sec. 5.3, solve Prob. 5.21.

5.21 For the beam and loading shown, determine the maximum normal stress on a transverse section at C.



SOLUTION

$\circlearrowleft \Sigma M_C = 0$

$-3A + (1.5)(10) - (1.1)(2.2)(3) = 0$

$A = 2.58 \text{ kN}$

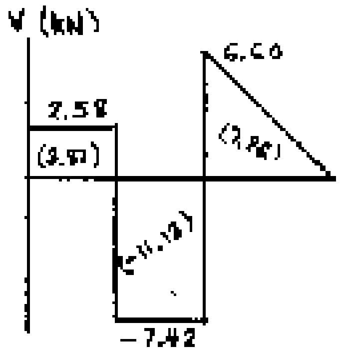
$\circlearrowleft \Sigma M_A = 0$

$-(1.5)(10) + 3C - (4.1)(2.2)(3) = 0$

$C = 14.02 \text{ kN}$

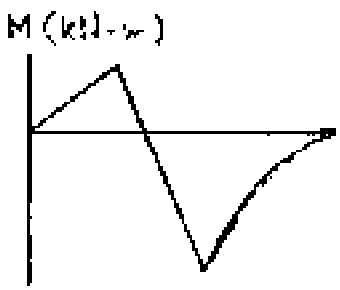
Shear

A to D^-	$V = 2.58$ kN
D^+ to C^-	$V = 2.58 - 10 = -7.42$ kN
C^+	$V = -7.42 + 14.02 = 6.60$ kN
B	$V = 6.60 - (2.2)(3) = 0$



Areas under shear diagram

A to D	$\int V dx = (1.5)(2.58) = 3.87$ kN·m
D to C	$\int V dx = (1.5)(-7.42) = -11.13$ kN·m
C to B	$\int V dx = (\frac{1}{2})(2.2)(6.60) = 7.26$ kN·m



Bending moments

$M_A = 0$

$M_D = 0 + 3.87 = 3.87$ kN·m

$M_C = 3.87 - 11.13 = -7.26$ kN·m

$M_B = 7.26 - 7.26 = 0$

$|M_C| = 7.26 \text{ kN}\cdot\text{m} = 7.26 \times 10^3 \text{ N}\cdot\text{m}$

For rectangular cross section $S = \frac{1}{6}bh^2 = (\frac{1}{6})(100)(200)^2$

$= 666.67 \times 10^3 \text{ mm}^3 = 666.67 \times 10^{-6} \text{ m}^3$

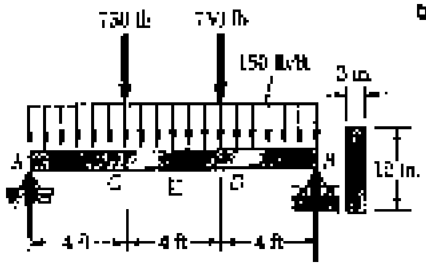
Normal stress $\sigma = \dfrac{|M_C|}{S} = \dfrac{7.26 \times 10^3}{666.67 \times 10^{-6}} = 10.89 \times 10^6$ Pa

$= 10.89$ MPa ◄

PROBLEM 5.54

5.54 Using the methods of Sec. 5.3, solve Prob. 5.22.

5.22 For the beam and loading shown, determine the maximum normal stress on a transverse section at the center of the beam.

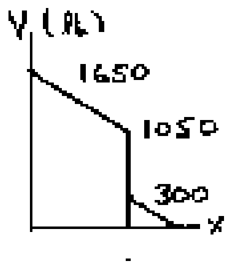


SOLUTION

Reactions : $C = A$ by symmetry

$+\uparrow \Sigma F_y = 0 \quad A + C - (2)(750) - (12)(150) = 0$

$A = C = 1650$ lb.



Shear :

$V_A = 1650$ lb

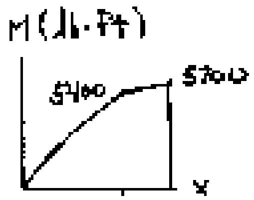
$V_{C^-} = 1650 - (4)(150) = 1050$ lb

$V_{C^+} = 1050 - 750 = 300$ lb

$V_E = 300 - (2)(150) = 0$

Areas under shear diagram

A to C $\quad \int V dx = (\frac{1}{2})(1650 + 1050)(4)$
$= 5400$ lb·ft



C to E $\quad \int V dx = (\frac{1}{2})(300)(2) = 300$ lb·ft

Bending moments

$M_A = 0$

$M_C = 0 + 5400 = 5400$ lb·ft

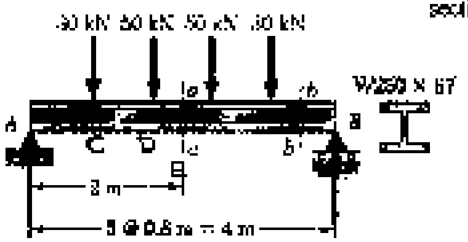
$M_E = 5400 + 300 = 5700$ lb·ft

$M_E = 5700$ lb·ft $= 68.4 \times 10^3$ lb·in.

For rectangular cross section $\quad S = \frac{1}{6} bh^2 = (\frac{1}{6})(3)(12)^2 = 72$ in^3

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{68.4 \times 10^3}{72} = 950$ psi

PROBLEM 5.55



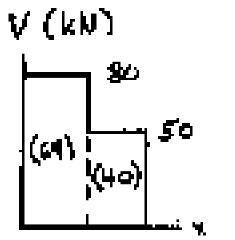
5.55 Using the methods of Sec. 5.3, solve Prob. 5.23.

5.23 For the beam and loading shown, determine the maximum normal stress on section a-a.

SOLUTION

Reactions: By symmetry $A = B$

$+\uparrow \Sigma F_y = 0 \qquad A = B = 80 \text{ kN}$



Shear

A to C $\quad V = 80 \text{ kN}$

C to D $\quad V = 80 - 30 = 50 \text{ kN}$

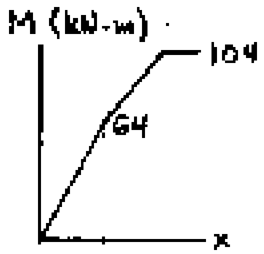
D to E $\quad V = 50 - 50 = 0$

Areas under shear diagram

A to C $\quad \int V dx = (80)(0.8) = 64 \text{ kN·m}$

C to D $\quad \int V dx = (50)(0.8) = 40 \text{ kN·m}$

D to E $\quad \int V dx = 0$



Bending moments

$M_A = 0$

$M_C = 0 + 64 = 64 \text{ kN·m}$

$M_D = 64 + 40 = 104 \text{ kN·m}$

$M_E = 104 + 0 = 104 \text{ kN·m}$

$M_E = 104 \text{ kN·m} = 104 \times 10^3 \text{ N·m}$

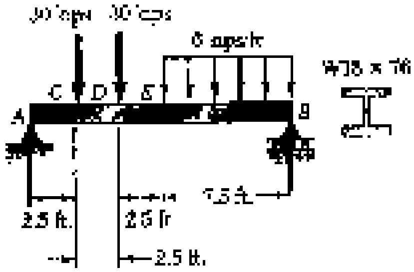
For W 250 × 67 $\quad S = 809 \times 10^3 \text{ mm}^3 = 809 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{104 \times 10^3}{809 \times 10^{-6}} = 128.6 \times 10^6 \text{ Pa} = 128.6 \text{ MPa}$ ◀

PROBLEM 5.56

5.56 Using the methods of Sec. 5.3, solve Prob. 5.24

5.24 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



SOLUTION

$\circlearrowleft \Sigma M_B = 0$

$-15A + (12.5)(30) + (10)(30) + (6)(7.5)(3.75) = 0$

$A = 56.25$ kips

Shear A to C $V = 56.25$ kips

Area under shear curve A to C. $\int V\,dx = (56.25)(2.5)$
$= 140.625$ kip·ft

$M_A = 0$

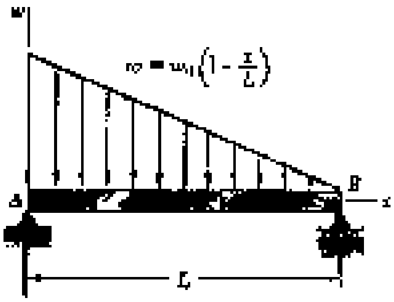
$M_C = 0 + 140.625 = 140.625$ kip·ft $= 1687.5$ kip·in

For W 18×76 rolled steel section $S = 146$ in³

Normal stress $\sigma = \dfrac{M}{S} = \dfrac{1687.5}{146} = 11.56$ ksi

PROBLEM 5.57

5.57 and 5.58 Determine (*a*) the equations of the shear and bending-moment curves for the given beam and loading, (*b*) the maximum absolute value of the bending moment in the beam.



SOLUTION

$$w = w_0\left(1 - \frac{x}{L}\right)$$

$$\frac{dV}{dx} = -w = -w_0 + \frac{w_0 x}{L}$$

$$V = -w_0 x + \frac{w_0 x^2}{2L} + C_1 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L} + C_1 x + C_2$$

$M = 0$ at $x = 0$ $\qquad C_2 = 0$

$M = 0$ at $x = L$ $\qquad 0 = -\frac{w_0 L^2}{2} + \frac{w_0 L^2}{6} + C_1 L \quad \therefore \quad C_1 = \frac{w_0 L}{3}$

$$V = -w_0 x + \frac{w_0 x^2}{2L} + \frac{w_0 L}{3}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L} + \frac{w_0 L x}{3}$$

M is maximum where $\frac{dM}{dx} = V = 0$

$$0 = -w_0 x_m + \frac{w_0 x_m^2}{2L} + \frac{w_0 L}{3}$$

$$\frac{1}{2}x_m^2 - L x_m + \frac{1}{3}L^2 = 0 \qquad x_m = \frac{L \pm \sqrt{L^2 - (4)(\frac{1}{2})(\frac{1}{3}L^2)}}{(2)(\frac{1}{2})}$$

$$= \left(1 \pm \frac{\sqrt{3}}{3}\right)L$$

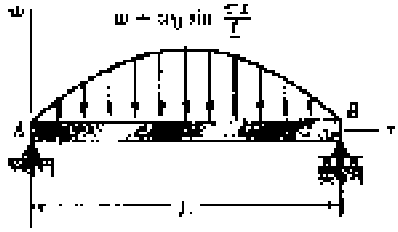
$$= \cancel{1.57735\,L},\ 0.42265\,L$$

$$M_{max} = \frac{-w_0(0.42265\,L)^2}{2} + \frac{w_0(0.42265\,L)^3}{6L} + \frac{w_0 L(0.42265\,L)}{3}$$

$$= 0.06415\ w_0 L^2$$

PROBLEM 5.58

5.57 and 5.58 Determine (a) the equations of the shear and bending-moment curves for the given beam and loading, (b) the maximum absolute value of the bending moment in the beam.



SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{L}$$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$M = 0$ at $x = 0$ $\qquad C_2 = 0$

$M = 0$ at $x = L$ $\qquad 0 = 0 + C_1 L + 0 \qquad C_1 = 0$

$$V = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L}$$

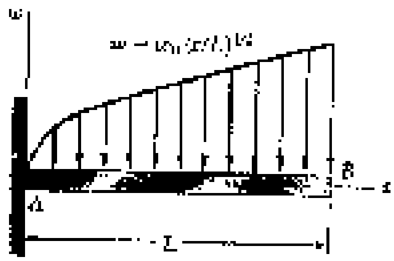
$$M = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L}$$

$$\frac{dM}{dx} = V = 0 \quad \text{at} \quad x = \frac{L}{2}$$

$$M_{max} = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi}{2} = \frac{w_0 L^2}{\pi^2}$$

PROBLEM 5.59

5.59 Determine (*a*) the equations of the shear and bending-moment curves for the given beam and loading, (*b*) the maximum absolute value of the bending moment in the beam.



SOLUTION

$$\frac{dV}{dx} = -w = -w_0\left(\frac{x}{L}\right)^{\frac{1}{2}} = -\frac{w_0 x^{1/2}}{L^{1/2}}$$

$$V = -\frac{2}{3}\frac{w_0 x^{3/2}}{L^{1/2}} + C_1$$

$V = 0$ at $x = L$

$$0 = -\frac{2}{3}w_0 L + C_1 \qquad C_1 = \frac{2}{3}w_0 L$$

$$V = \frac{2}{3}w_0 L - \frac{2}{3}\frac{w_0 x^{3/2}}{L^{1/2}}$$

$$\frac{dM}{dx} = V \qquad M = C_2 + \frac{2}{3}w_0 L x - \frac{2}{3}\cdot\frac{2}{5}\frac{w_0 x^{5/2}}{L^{1/2}}$$

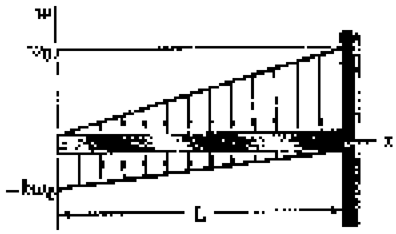
$M = 0$ at $x = L$ $\qquad 0 = C + \frac{2}{3}w_0 L^2 - \frac{4}{15}w_0 L^2 \qquad C_2 = -\frac{2}{5}w_0 L^2$

$$M = \frac{2}{3}w_0 L x - \frac{4}{15}\frac{w_0 x^{5/2}}{L^{1/2}} - \frac{2}{5}w_0 L^2$$

$|M|_{max}$ occurs at $x = 0$ $\qquad |M|_{max} = \frac{2}{5}w_0 L^2$

PROBLEM 5.60

5.60 For the beam and loading shown, determine the equations of the shear and bending-moment curves and the maximum absolute value of the bending moment in the beam, knowing that (a) $k = 1$, (b) $k = 0.5$.



SOLUTION

$$w = \frac{w_0 x}{L} - \frac{k w_0 (L - x)}{L} = (1+k)\frac{w_0 x}{L} - k w_0$$

$$\frac{dV}{dx} = -w = k w_0 - (1+k)\frac{w_0 x}{L}$$

$$V = k w_0 x - (1+k)\frac{w_0 x^2}{2L} + C_1$$

$V = 0$ at $x = 0$ $\qquad C_1 = 0$

$$\frac{dM}{dx} = V = k w_0 x - (1+k)\frac{w_0 x^2}{2L}$$

$$M = \frac{k w_0 x^2}{2} - (1+k)\frac{w_0 x^3}{6L} + C_2$$

$M = 0$ at $x = 0$ $\qquad C_2 = 0$

$$M = \frac{k w_0 x^2}{2} - \frac{(1+k) w_0 x^3}{6L}$$

(a) $k = 1$ $\qquad V = w_0 x - \dfrac{w_0 x^2}{L}$

$$M = \frac{w_0 x^2}{2} - \frac{w_0 x^3}{3L}$$

Maximum M occurs at $x = L$ $\qquad |M|_{max} = \dfrac{w_0 L^2}{6}$

(b) $k = \frac{1}{2}$ $\qquad V = \dfrac{w_0 x}{2} - \dfrac{3 w_0 x^2}{4L}$

$$M = \frac{w_0 x^2}{4} - \frac{w_0 x^3}{4L}$$

$V = 0$ at $x = \frac{2}{3}L$

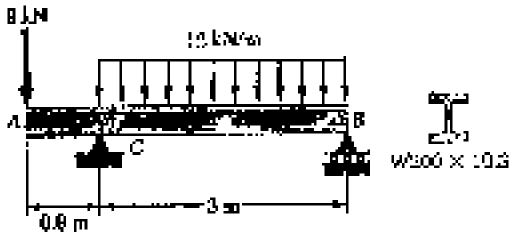
At $x = \frac{2}{3}L$ $\qquad M = \dfrac{w_0(\frac{2}{3}L)^2}{4} - \dfrac{w_0(\frac{2}{3}L)^3}{4L} = \dfrac{w_0 L^2}{27} = 0.03704\, w_0 L^2$

At $x = L$ $\qquad M = 0$

$$|M|_{max} = \frac{w_0 L^2}{27}$$

PROBLEM 5.61

5.61 and 5.62 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$\circlearrowleft \Sigma M_C = 0$

$(0.9)(9) - (1.5)(3)(12) + 3B = 0$

$B = 15.3$ kN

$\circlearrowleft \Sigma M_B = 0$

$(3.9)(9) - 3C + (1.5)(3)(12) = 0$

$C = 29.7$ kN

Shear: A to C $V = -9$ kN

C^+ $V = -9 + 29.7 = 20.7$ kN

B $V = 20.7 - (3)(12) = -15.3$ kN

Locate point E where $V = 0$

$\frac{e}{20.7} = \frac{3-e}{15.3}$ $36e = (20.7)(3)$

$e = 1.725$ ft $3 - e = 1.275$ ft

Areas under shear diagram

A to C $\int V dx = (0.9)(9) = 8.1$ kN·m

C to E $\int V dx = (\frac{1}{2})(1.725)(20.7) = 17.85375$ kN·m

E to B $\int V dx = (\frac{1}{2})(-1.275)(15.3) = -9.75375$ kN·m

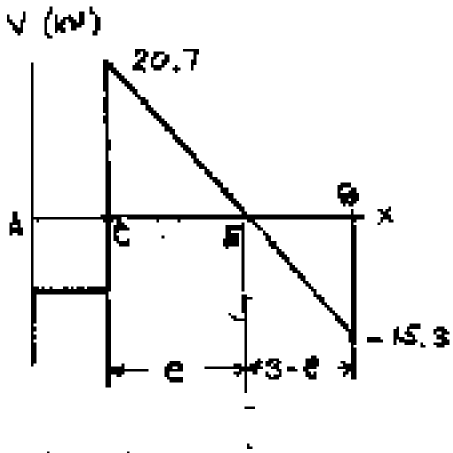
Bending moments

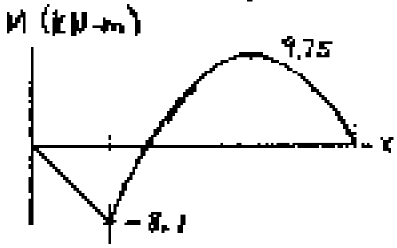
$M_A = 0$

$M_C = 0 - 8.1 = -8.1$ kN·m

$M_E = -8.1 + 17.85375 = 9.75375$ kN·m

$M_B = 9.75375 - 9.75375 = 0$





Maximum $|M| = 9.75375 \times 10^3$ N·m at point E

For W 200×19.3 rolled steel section $S = 164 \times 10^3 \text{ mm}^3 = 164 \times 10^{-6} \text{ m}^3$

Normal stress $\sigma = \frac{|M|}{S} = \frac{9.75375 \times 10^3}{164 \times 10^{-6}} = 59.5 \times 10^6$ Pa $= 59.5$ MPa

PROBLEM 5.62

5.61 and 5.62 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

$+\circlearrowleft \Sigma M_B = 0$

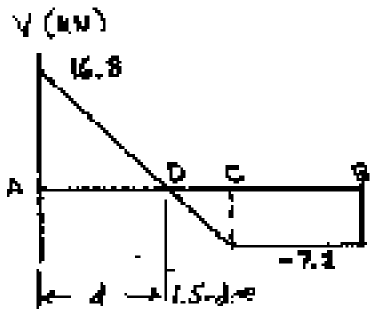
$-2.5A + (1.75)(1.5)(16) = 0$

$A = 16.8 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0$

$-(0.75)(1.5)(16) + 2.5B = 0$

$B = 7.2 \text{ kN}$



Shear

$V_A = 16.8 \text{ kN}$

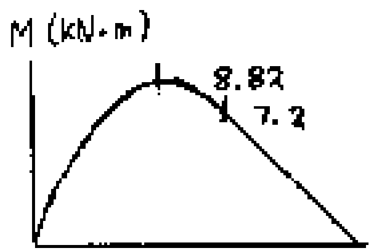
$V_C = 16.8 - (1.5)(16) = -7.2 \text{ kN}$

$V_B = -7.2 \text{ kN}$

Locate point D where $V = 0$

$\frac{d}{16.8} = \frac{1.5 - d}{7.2}$ $\qquad 24d = 25.2$

$d = 1.05 \text{ m}$ $\qquad 1.5 - d = 0.45 \text{ m}$



Areas under shear diagram

A to D $\quad \int V\,dx = (\frac{1}{2})(1.05)(16.8) = 8.82 \text{ kN}\cdot\text{m}$

D to C $\quad \int V\,dx = (\frac{1}{2})(0.45)(-7.2) = -1.62 \text{ kN}\cdot\text{m}$

C to B $\quad \int V\,dx = (1)(-7.2) = -7.2 \text{ kN}\cdot\text{m}$

Bending moments

$M_A = 0$

$M_D = 0 + 8.82 = 8.82 \text{ kN}\cdot\text{m}$

$M_C = 8.82 - 1.62 = 7.2 \text{ kN}\cdot\text{m}$

$M_B = 7.2 - 7.2 = 0$

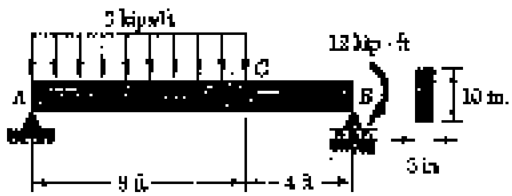
Maximum $|M| = 8.82 \text{ kN}\cdot\text{m} = 8.82 \times 10^3 \text{ N}\cdot\text{m}$

For S 150 × 18.6 rolled steel section $\quad S = 120 \times 10^3 \text{ mm}^3 = 120 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{8.82 \times 10^3}{120 \times 10^{-6}} = 73.5 \times 10^6 \text{ Pa} = 73.5 \text{ MPa}$ ◄

PROBLEM 5.63

5.63 and 5.64 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$+\circlearrowleft \Sigma M_B = 0$

$-12A + (3)(8)(8) - 12 = 0$

$A = 15 \text{ kips}$

$\circlearrowleft \Sigma M_A = 0$

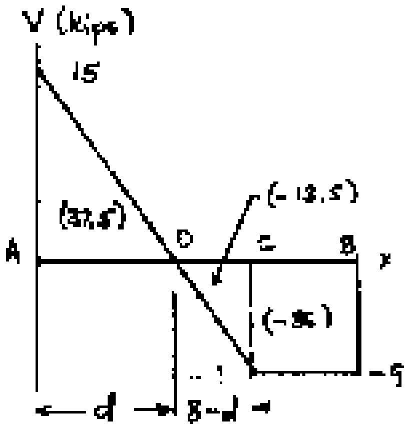
$-(4)(3)(8) + 12B - 12 = 0$

$B = 9 \text{ kips}$

Shear: $V_A = 15 \text{ kips}$

$V_C = 15 - (3)(8) = -9 \text{ kips}$

C to B $\quad V = -9 \text{ kips}$



Locate point D where $V = 0$

$$\frac{d}{15} = \frac{8-d}{9} \qquad 24d = 120$$

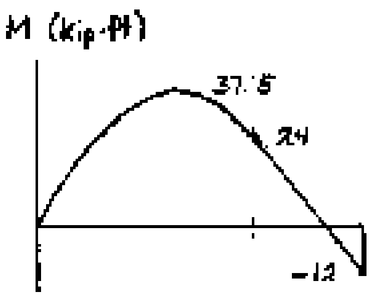
$$d = 5 \text{ ft} \qquad 8 - d = 3 \text{ ft}$$

Areas under shear diagram

A to D $\quad \int V\,dx = (\tfrac{1}{2})(5)(15) = 37.5 \text{ kip·ft}$

D to C $\quad \int V\,dx = (\tfrac{1}{2})(3)(-9) = -13.5 \text{ kip·ft}$

C to B $\quad \int V\,dx = (4)(-9) = -36 \text{ kip·ft}$



Bending moments: $\quad M_A = 0$

$M_D = 0 + 37.5 = 37.5 \text{ kip·ft}$

$M_C = 37.5 - 13.5 = 24 \text{ kip·ft}$

$M_B = 24 - 36 = -12 \text{ kip·ft}$

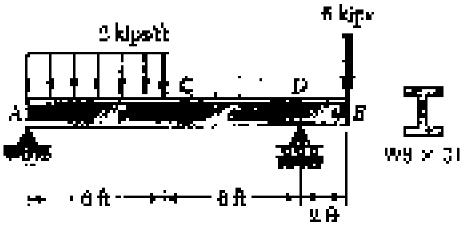
Maximum $|M| = 37.5 \text{ kip·ft} = 450 \text{ kip·in}$

For rectangular cross section $\quad S = \frac{1}{6}bh^2 = (\frac{1}{6})(3)(10)^2 = 50 \text{ in}^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{450}{50} = 9 \text{ ksi}$

PROBLEM 5.64

5.63 and 5.64 Draw the shear and bending moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$+\circlearrowleft M_D = 0$

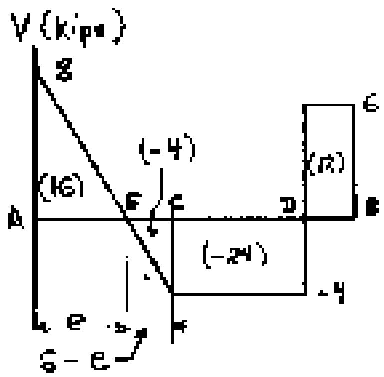
$-12A + (9)(6)(2) - (2)(6) = 0$

$A = 8$ kips

$+\circlearrowleft M_A = 0$

$-(3)(6)(2) + 12D - (14)(6) = 0$

$D = 10$ kips



Shear: $V_A = 8$ kips

$V_C = 8 - (6)(2) = -4$ kips

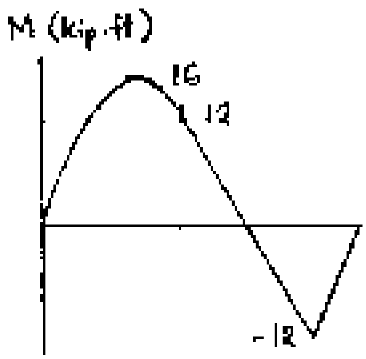
C to D $V = -4$ kips

D to B $V = -4 + 10 = 6$ kips

Locate point E where $V = 0$

$\frac{e}{8} = \frac{6-e}{4}$ $12e = 48$

$e = 4$ ft $6 - e = 2$ ft



Areas under shear diagram

A to E $\int V\,dx = (\frac{1}{2})(4)(8) = 16$ kip·ft

E to C $\int V\,dx = (\frac{1}{2})(2)(-4) = -4$ kip·ft

C to D $\int V\,dx = (6)(-4) = -24$ kip·ft

D to B $\int V\,dx = (2)(6) = 12$ kip·ft

Bending moments: $M_A = 0$

$M_E = 0 + 16 = 16$ kip·ft

$M_C = 16 - 4 = 12$ kip·ft

$M_D = 12 - 24 = -12$ kip·ft

$M_B = -12 + 12 = 0$

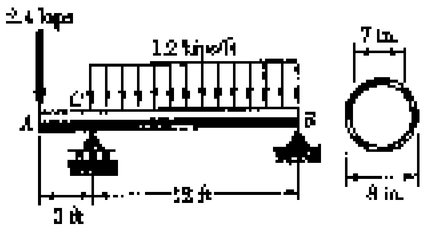
Maximum $|M| = 16$ kip·ft $= 192$ kip·in

For W 8×31 rolled steel section $S = 27.5\ \text{in}^3$

Normal stress $\sigma = \frac{|M|}{S} = \frac{192}{27.5} = 6.98$ ksi

PROBLEM 5.65

5.65 and 5.66 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$+\circlearrowleft \Sigma M_C = 0$

$(3)(2.4) - (6)(12)(1.2) + 12B = 0$

$B = 6.6$ kips

$+\circlearrowleft \Sigma M_B = 0$

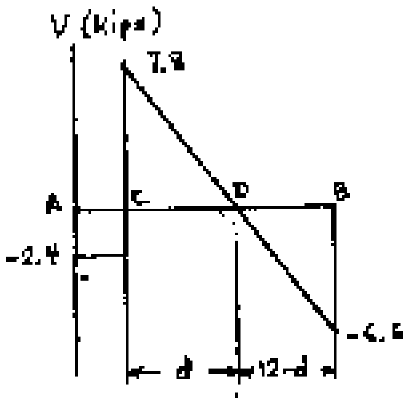
$(15)(2.4) + (6)(12)(1.2) - 12C = 0$

$C = 10.2$ kips

Shear: A to C $\quad V = -2.4$ kips

$C^+ \quad V = -2.4 + 10.2 = 7.8$ kips

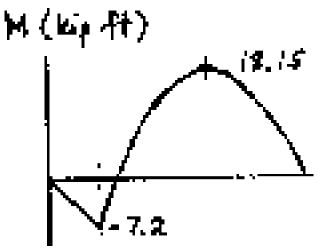
B $\quad V_B = 7.8 - (12)(1.2) = -6.6$ kips



Locate point D where $V = 0$

$\frac{d}{7.8} = \frac{12-d}{6.6} \qquad 14.4\,d = 93.6$

$d = 6.5$ ft $\qquad 12 - d = 5.5$ ft



Areas under shear diagram

A to C $\quad \int V\,dx = (3)(-2.4) = -7.2$ kip·ft

C to D $\quad \int V\,dx = (\frac{1}{2})(6.5)(7.8) = 25.35$ kip·ft

D to B $\quad \int V\,dx = (\frac{1}{2})(5.5)(-6.6) = -18.15$ kip·ft

Bending moments $\quad M_A = 0$

$M_C = 0 - 7.2 = -7.2$ kip·ft

$M_D = -7.2 + 25.35 = 18.15$ kip·ft

$M_B = 18.15 - 18.15 = 0$

Maximum $|M| = 18.15$ kip·ft $= 217.8$ kip·in

For pipe $\quad c_o = \frac{d_o}{2} = \frac{8}{2} = 4$ in $\qquad c_i = \frac{d_i}{2} = \frac{7}{2} = 3.5$ in

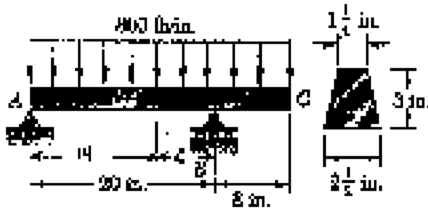
$I = \frac{\pi}{4}(c_o^4 - c_i^4) = \frac{\pi}{4}(4^4 - 3.5^4) = 82.20$ in^4

$S = \frac{I}{c_o} = \frac{82.20}{4} = 20.80$ in^3

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{217.8}{20.80} = 10.47$ ksi

PROBLEM 5.66

5.65 and 5.66 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



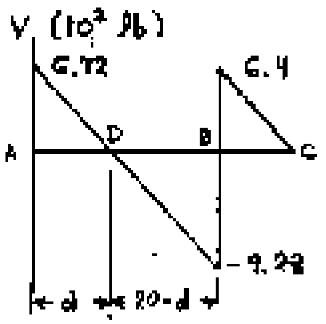
SOLUTION

$\circlearrowleft \Sigma M_B = 0 \qquad -20A + (6)(28)(800) = 0$

$A = 6.72 \times 10^3$ lb.

$\circlearrowleft \Sigma M_A = 0 \qquad 20B - (14)(28)(800) = 0$

$B = 15.68 \times 10^3$ lb.



Shear: $V_A = 6.72 \times 10^3$ lb

B^- $\quad V_{B^-} = 6.72\times10^3 - (20)(800) = -9.28 \times 10^3$ lb

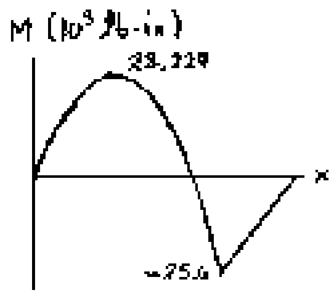
B^+ $\quad V_{B^+} = -9.28\times10^3 + (15.68\times10^3) = 6.4 \times 10^3$ lb

C $\quad V_C = 6.4\times10^3 - (8)(800) = 0$

Locate point D where $V = 0$

$$\frac{d}{6.72} = \frac{20-d}{9.28} \qquad 16\,d = 134.4$$

$d = 8.4$ in $\qquad 20 - d = 11.6$ in



Areas under shear diagram

A to D $\quad \int V dx = (\frac{1}{2})(8.4)(6.72\times10^3) = 28.224\times10^3$ lb·in

D to B $\quad \int V dx = (\frac{1}{2})(11.6)(-9.28\times10^3) = -53.824\times10^3$ lb·in

B to C $\quad \int V dx = (\frac{1}{2})(8)(6.4\times10^3) = 25.6\times10^3$ lb·in

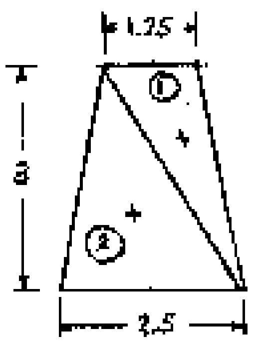
Bending moments: $\quad M_A = 0$

$M_D = 0 + 28.224\times10^3 = 28.224\times10^3$ lb·in

$M_B = 28.224\times10^3 - 53.824\times10^3 = -25.6\times10^3$ lb·in

$M_C = -25.6\times10^3 + 25.6\times10^3 = 0$

Maximum $|M| = 28.224\times10^3$ lb·in



Locate centroid of cross section

$$\bar{Y} = \frac{7.5}{5.625} = 1.3333 \text{ in} \quad \text{from bottom}$$

For each triangle $\bar{I} = \frac{1}{36} b h^3$

Part	A, in²	$\bar{y}$, in	$A\bar{y}$, in³	d, in	Ad^2, in⁴	$\bar{I}$, in⁴
1	1.875	2	3.75	0.6667	0.8333	0.9375
2	3.75	1	3.75	0.3333	0.4167	1.875
Σ	5.625		7.5		1.25	2.8125

Moment of inertia

$I = \Sigma\bar{I} + \Sigma Ad^2$

$= 1.25 + 2.8125 = 4.0625$ in⁴

Normal stress

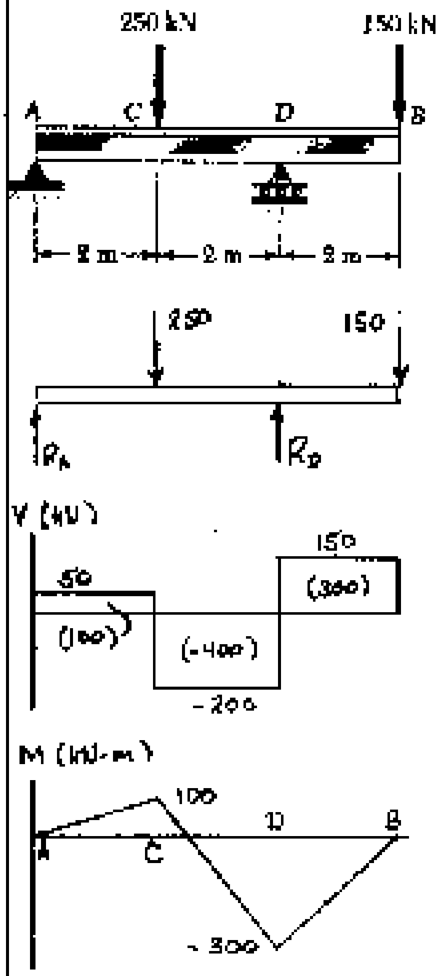
$$\sigma = \frac{Mc}{I} = \frac{(28.224\times10^3)(1.6667)}{4.0625}$$

$= 11.58\times10^3$ psi

$= 11.58$ ksi

PROBLEM 5.67

5.67 and 5.68 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



SOLUTION

$w = 0$

$\circlearrowleft \Sigma M_D = 0$

$-4R_A + (2)(250) - (2)(150) = 0$

$R_A = 50 \text{ kN} \uparrow$

$\circlearrowleft \Sigma M_A = 0$

$4R_D - (2)(250) - (6)(150) = 0$

$R_D = 350 \text{ kN} \uparrow$

Shear: $V_A = 50$ kN

A to C $\quad V = 50$ kN

C to D $\quad V = 50 - 250 = -200$ kN

D to B $\quad V = -200 + 350 = 150$ kN

Areas of shear diagram

A to C $\quad \int V dx = (50)(2) = 100$ kN·m

C to D $\quad \int V dx = (-200)(2) = -400$ kN·m

D to B $\quad \int V dx = (150)(2) = 300$ kN·m

Bending moments: $\quad M_A = 0$

$M_C = M_A + \int V dx = 0 + 100 = 100$ kN·m

$M_D = M_C + \int V dx = 100 - 400 = -300$ kN·m

$M_B = M_D + \int V dx = -300 + 300 = 0$

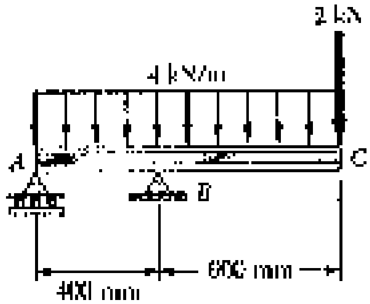
maximum $|M| = 300$ kN·m $= 300 \times 10^3$ N·m

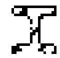
For W 410 × 114 rolled steel section $\quad S_x = 2200 \times 10^3 \text{ mm}^3 = 2200 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{300 \times 10^3}{2200 \times 10^{-6}} = 136.4 \times 10^6 \text{ Pa} = 136.4 \text{ MPa}$$

PROBLEM 5.68

5.67 and 5.68 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.





S100 × 11.5

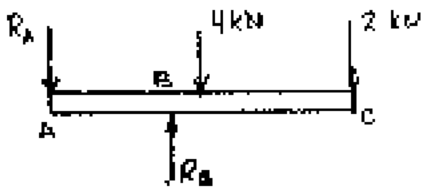
SOLUTION

$\circlearrowleft \Sigma M_B = 0 \quad (0.4)(R_A) - (0.1)(4) - (0.6)(2) = 0$

$R_A = 4 \text{ kN} \downarrow$

$\circlearrowleft \Sigma M_A = 0 \quad (0.4)(R_B) - (0.5)(4) - (1)(2) = 0$

$R_B = 10 \text{ kN} \uparrow$



A to B $\quad 0 < x < 0.4$ m

$\frac{dV}{dx} = -w = -4$ kN/m

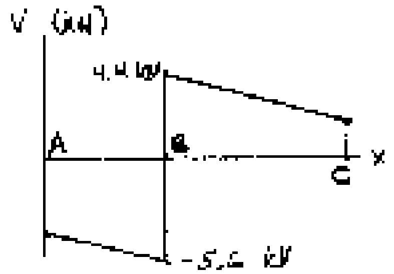
$V = -4x - 4$ kN

At $x = 0.4$ m $\quad V_B = -5.6$ kN

$\frac{dM}{dx} = -4x - 4$

$M = M_A - 2x^2 - 4x = 0 - 2x^2 - 4x$

At $x = 0.4$ m $\quad M_B = 1.92$ kN·m



B to C $\quad 0.4 \text{ m} < x < 1.0$ m

$\frac{dV}{dx} = -w = -4$ kN/m

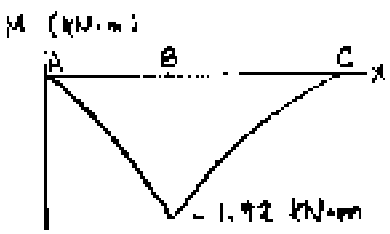
$V = 4.4 - 4(x - 0.4) = 6 - 4x$ kN

$\frac{dM}{dx} = 6 - 4x$

$M = 6x - 2x^2 + C_1$ kN·m

$M = 0$ at $x = 1 \quad \therefore \quad C_1 = 4$ kN·m

$M = 4 + 6x - 2x^2$ kN·m



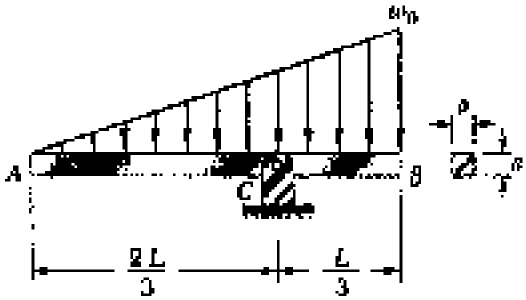
$|M|_{max} = 1.92 \text{ kN·m} = 1.92 \times 10^3 \text{ N·m}$

For S100 × 11.5 rolled steel section $\quad S_x = 49.6 \times 10^3 \text{ mm}^3 = 49.6 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{1.92 \times 10^3}{49.6 \times 10^{-6}} = 38.7 \times 10^6 \text{ Pa} = 38.7 \text{ MPa}$$

PROBLEM 5.69

5.69 Beam AB, of length L and square cross section of side a, is supported by a pivot at C and loaded as shown. (a) Check that the beam is in equilibrium. (b) Show that the maximum normal stress due to bending occurs at C and is equal to $w_0L^2/(1.5a)^3$.

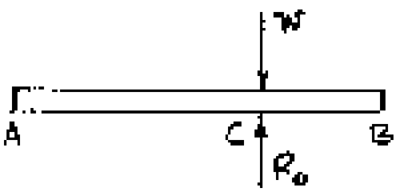


SOLUTION

Replace distributed load by equivalent concentrated load at the centroid of the area of the load diagram.

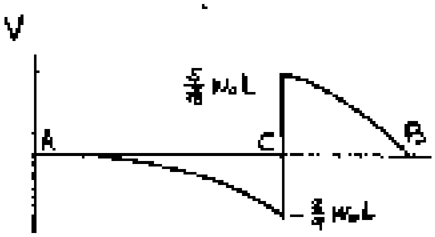
For the triangular distribution the centroid lies at $x = \frac{2L}{3}$.

$$W = \frac{1}{2} w_0 L$$



$\Sigma F_y = 0 \qquad R_0 - W = 0 \qquad R_0 = \frac{1}{2} w_0 L$

$\Sigma M_c = 0 \qquad 0 = 0 \qquad$ equilibrium



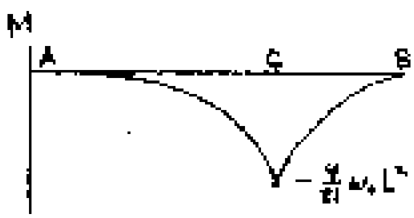
$V = 0, \quad M = 0 \quad$ at $x = 0$

$0 < x < \frac{2L}{3}$

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L} + C_1 = -\frac{w_0 x^2}{2L}$$

$$M = -\frac{w_0 x^3}{6L} + C_2 = -\frac{w_0 x^3}{6L}$$



Just to the left of C

$$V = -\frac{w_0 (2L/3)^2}{2L} = -\frac{2}{9} w_0 L$$

Just to the right of C

$$V = -\frac{2}{9} w_0 L + R_0 = \frac{5}{18} w_0 L$$

Note sign change. Maximum $|M|$ occurs at C.

$$M_c = -\frac{w_0 (2L/3)^3}{6L} = -\frac{4}{81} w_0 L^2$$

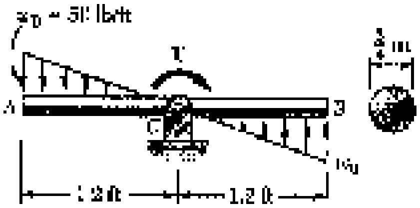
Maximum $|M| = \frac{4}{81} w_0 L^2$

For square cross section $\qquad I = \frac{1}{12} a^4 \qquad c = \frac{1}{2} a$

$$\sigma_m = \frac{|M|_{max} c}{I} = \frac{4}{81} \frac{w_0 L^2 \, 6}{a^3} = \frac{8}{27} \frac{w_0 L^2}{a^3} = \left(\frac{2}{3}\right)^3 \frac{w_0 L^2}{a^3} = \frac{w_0 L^2}{(1.5\,a)^3}$$

PROBLEM 5.70

5.70 Knowing that rod *AB* is in equilibrium under the loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.



SOLUTION

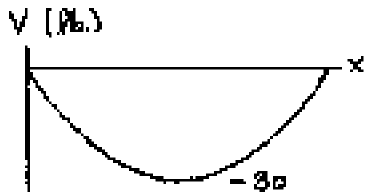
A to C $\quad 0 < x < 1.2$ ft

$w = 50\left(1 - \frac{x}{1.2}\right) = 50 - 41.667x$

$\frac{dV}{dx} = -w = 41.667x - 50$

$V = V_A + \int_0^x (41.667x - 50)\,dx$

$= 0 + 20.833x^2 - 50x = \frac{dM}{dx}$

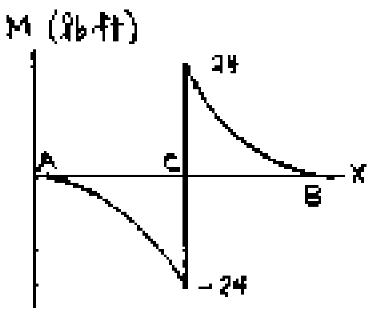


$M = M_A + \int_0^x V\,dx$

$= 0 + \int_0^x (20.833x^2 - 50x)\,dx$

$= 6.944x^3 - 25x^2$

At $x = 1.2$ ft, $\quad V = -30$ lb,
$M = -24$ lb·in.



C to B $\quad$ Use symmetry conditions.

Maximum $|M| = 24$ lb·ft $= 288$ lb·in.

Cross section $\quad c = \frac{d}{2} = \left(\frac{1}{2}\right)(0.75) = 0.375$ in

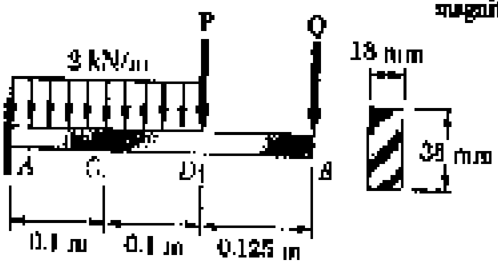
$I = \frac{\pi}{4}c^4 = \left(\frac{\pi}{4}\right)(0.375) = 15.532 \times 10^{-3}$ in^4

Normal stress $\quad \sigma = \frac{|M|c}{I} = \frac{(288)(0.375)}{15.532 \times 10^{-3}} = 6.95 \times 10^3$ psi

$= 6.95$ ksi

PROBLEM 5.71

*5.71 Beam *AB* supports a uniformly distributed load of 2 kN/m and two concentrated loads **P** and **Q**. It has been experimentally determined that the normal stress due to bending on the bottom edge of the beam is −56.9 MPa at *A* and −29.9 MPa at *C*. Draw the shear and bending-moment diagrams for the beam and determine the magnitudes of the loads **P** and **Q**.

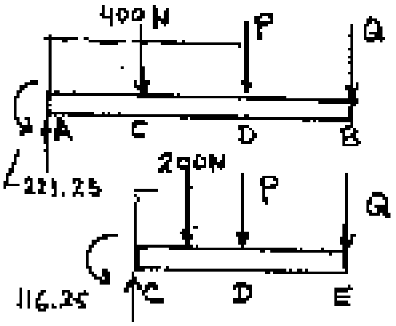


SOLUTION

$$I = \frac{1}{12}(18)(36)^3 = 69.984\times10^3\ \text{mm}^4$$

$$c = \frac{1}{2}d = 18\ \text{mm}$$

$$S = \frac{I}{c} = 3.888\times10^3\ \text{mm}^3 = 3.888\times10^{-6}\ \text{m}^3$$



At A $\quad M_A = S\sigma_A$

$$M_A = (3.888\times10^{-6})(-56.9) = -221.25\ \text{N·m}$$

At C $\quad M_C = S\sigma_C$

$$M_C = (3.888\times10^{-6})(-29.9) = -116.25\ \text{N·m}$$

$$\circlearrowleft \Sigma M_A = 0$$

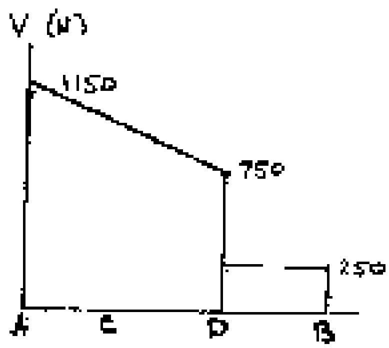
$$221.25 - (0.1)(400) - 0.2P - 0.325Q = 0$$

$$0.2P + 0.325Q = 181.25 \qquad (1)$$

$$+\circlearrowleft M_C = 0$$

$$116.25 - (0.05)(200) - 0.1P - 0.225Q = 0$$

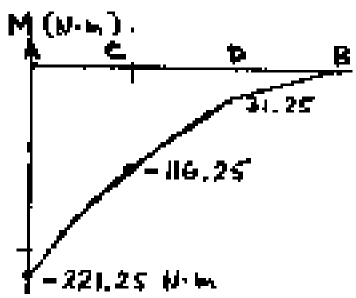
$$0.1P + 0.225Q = 106.25 \qquad (2)$$



Solving (1) and (2) simultaneously

$P = 500\ \text{N}$ ⟶

$Q = 250\ \text{N}$ ⟶



Reaction force at A

$$R_A - 400 - 500 - 250 = 0$$

$$R_A = 1150\ \text{N·m}$$

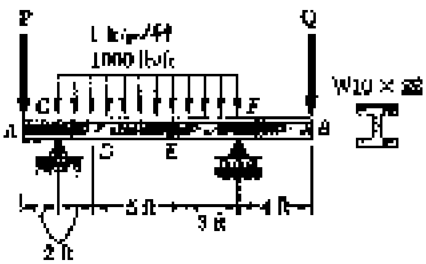
$V_A = 1150\ \text{N} \qquad V_D = 250$

$M_A = -221.25\ \text{N·m}$

$M_C = -116.25\ \text{N·m}$

$M_D = -31.25\ \text{N·m}$

PROBLEM 5.72



*5.72 Beam *AB* supports a uniformly distributed load of 1000 lb/ft and two concentrated loads P and Q. It has been experimentally determined that the normal stress due to bending on the bottom edge of the lower flange of the W 10 × 22 rolled-steel beam is +2.07 ksi at *D* and +0.776 ksi at *E*. (*a*) Draw the shear and bending-moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending which occurs in the beam.

SOLUTION

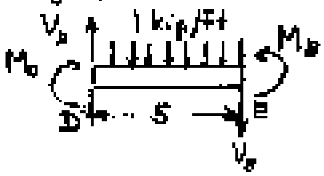
For W10 × 22 rolled steel section $S = 23.2\ \text{in}^3$

Bending moments at D and E $\quad M = S\sigma$

$M_D = (23.2)(2.07) = 48.0\ \text{kip}\cdot\text{in} = 4.00\ \text{kip}\cdot\text{ft}$

$M_E = (23.2)(0.776) = 18.0\ \text{kip}\cdot\text{in} = 1.50\ \text{kip}\cdot\text{ft}$

Using portion DE as a free body



$+\circlearrowleft \Sigma M_p = 0 \qquad -M_D + M_E - 5V_D + (2.5)(5)(1) = 0$

$V_D = 2\ \text{kips}$

$+\circlearrowleft \Sigma M_D = \qquad -M_D + M_E - 5V_E - (2.5)(5)(1) = 0$

$V_E = -3\ \text{kips}$

Using portion ACD as a free body

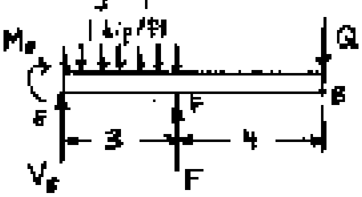
$\circlearrowleft \Sigma M_C = 0 \qquad 2P + (1)(2)(1) + M_D - 2V_D = 0$

$P = 1\ \text{kip}$

$+\uparrow \Sigma F_y = 0 \qquad -P + C - (2)(1) - V_D = 0$

$C = 5\ \text{kips}.$

Using portion EFB as a free body



$+\circlearrowleft \Sigma M_F = 0 \qquad -4Q + (1.5)(3)(1) - 3V_E - M_E = 0$

$Q = 3\ \text{kips}$

$+\uparrow \Sigma F_y = 0 \qquad F + V_E - (3)(1) - Q = 0$

$F = 9\ \text{kips}$

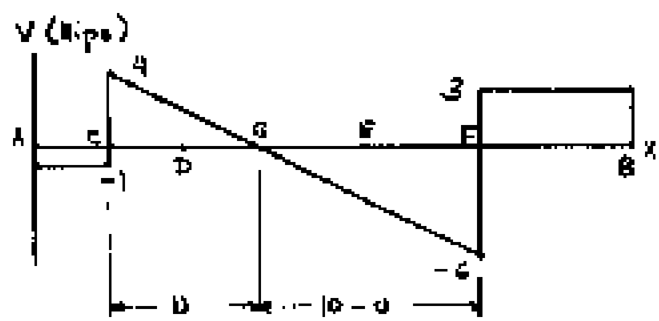
Shear: A to C $\quad V = -1\ \text{kips}$

$C^+ \quad V = -1 + 5 = 4\ \text{kips}$

$F^- \quad V = 4 - (10)(1) = -6\ \text{kips}$

$F^+ \quad V = -6 + 9 = 3\ \text{kips}$

F to B $\quad V = 3\ \text{kips}.$



Locate point G where V = 0

$\dfrac{u}{4} = \dfrac{10 - u}{6} \qquad 10u = 40$

$u = 4\ \text{ft} \qquad 10 - u = 6\ \text{ft}$

continued

Problem 5.72 continued

Areas under shear diagram

A to C $\quad \int V dx = (2)(-1) = -2$ kip·ft

C to G $\quad \int V dx = (\frac{1}{2})(4)(4) = 8$ kip·ft

G to F $\quad \int V dx = (\frac{1}{2})(6)(-6) = -18$ kip·ft

F to B $\quad \int V dx = (4)(3) = 12$ kip·ft

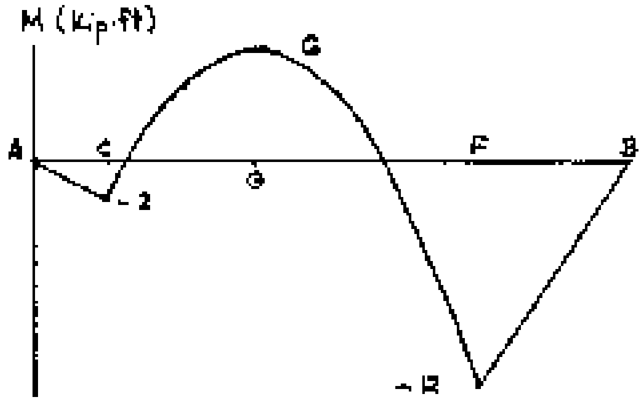
Bending moments $\quad M_A = 0$

$M_C = 0 - 2 = -2$ kip·ft

$M_G = -2 + 8 = 6$ kip·ft

$M_F = 6 - 18 = -12$ kip·ft

$M_B = -12 + 12 = 0$



Maximum $|M| = 12$ kip·ft
$= 144$ kip·in.

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{144}{23.2} = 6.21$ ksi

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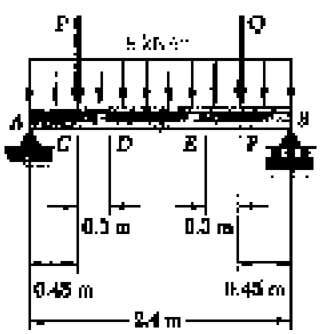
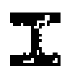
PROBLEM 5.73

*5.73 Beam AB supports a uniformly distributed load of 8 kN/m and two concentrated loads P and Q. It has been experimentally determined that the normal stress due to bending on the bottom edge of the lower flange of the W 200 × 52 rolled-steel beam is 100 MPa at D and 70 MPa at E. (*a*) Draw the shear and bending-moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending which occurs in the beam.





W200 × 52

SOLUTION

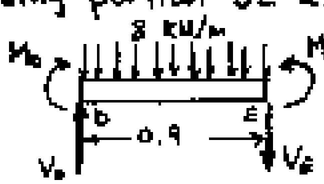
For W 200 × 52 rolled steel section
$S = 512 \times 10^3$ mm^3 $= 512 \times 10^{-6}$ m^3

Bending moments at D and E $\qquad M = S\sigma$

$M_D = (512 \times 10^{-6})(100 \times 10^6) = 51.2 \times 10^3$ N·m

$M_E = (512 \times 10^{-6})(70 \times 10^6) = 35.84 \times 10^3$ N·m

Using portion DE as a free body



$+\circlearrowleft \Sigma M_E = 0 \quad -0.9 V_D - M_D + M_E + (0.45)(0.9)(8) = 0$

$V_D = -13.467$ kN

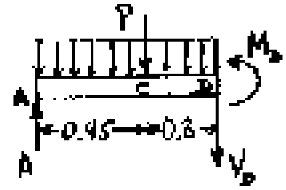
$+\uparrow \Sigma F_y = 0 \quad V_D - V_E - (0.9)(8) = 0$

$V_E = -20.667$ kN

continued

Problem 5.73 continued

Using portion ACD as a free body



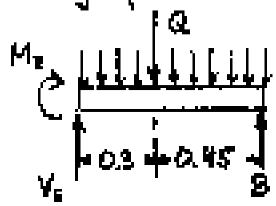
$+\circlearrowleft \Sigma M_A = 0 \qquad -0.45P - (0.375)(0.75)(8) - 0.75V_D + M_D = 0$

$P = 131.222$ kN

$+\Sigma F_y = 0 \qquad A - P - V_D - (0.75)(8) = 0$

$A = 123.756$ kN

Using portion EFB as a free body



$+\Sigma M_B = 0 \qquad 0.45Q + (0.375)(0.75)(8) - 0.75V_E - M_E = 0$

$Q = 40.2$ kN

$+\uparrow\Sigma F_y = 0 \qquad V_E - Q - (0.75)(8) + B = 0$

$B = 66.867$ kN

Shear: $V_A = 123.756$ kN

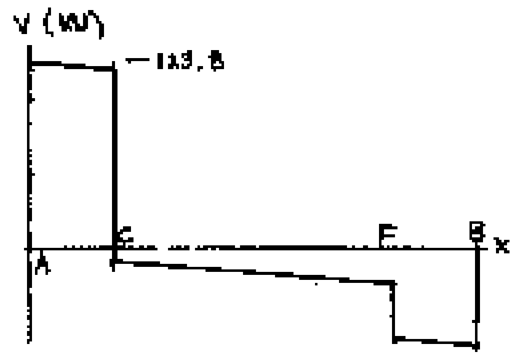
$V_{C^-} = 123.756 - (0.45)(8) = 120.155$ kN

$V_{C^+} = 120.155 - 131.222 = -11.067$ kN

$V_{F^-} = -11.067 - (1.5)(8) = -23.067$ kN

$V_{F^+} = -23.067 - 40.2 = -63.267$ kN

$V_B = -63.276 - (0.45)(8) = -66.867$ kN

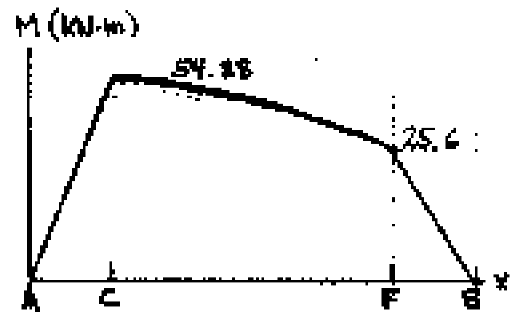


Areas under shear diagram

A to C: $\frac{1}{2}(0.45)(123.756 + 120.155) = 54.88$ kN·m

C to F: $\frac{1}{2}(1.5)(-11.067 - 23.067) = -25.6$ kN·m

F to B: $\frac{1}{2}(0.45)(-63.267 - 66.867) = -29.28$ kN·m



Bending moments: $M_A = 0$

$M_C = 0 + 54.88 = 54.88$ kN·m

$M_F = 54.88 - 25.6 = 29.28$ kN·m

$M_B = 29.28 - 29.28 = 0$

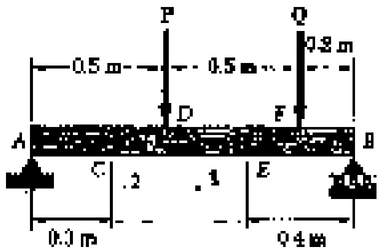
Maximum $|M| = 54.88$ kN·m $= 54.88\times10^3$ N·m

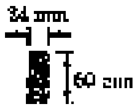
Normal stress $\qquad \sigma = \dfrac{|M|}{S} = \dfrac{54.88\times10^3}{512\times10^{-6}} = 107.2\times10^6$ Pa

$= 107.2$ MPa

PROBLEM 5.74

*5.74 Beam *AB* supports two concentrated loads **P** and **Q**. It has been experimentally determined that the normal stress due to bending on the bottom edge of the beam is +15 MPa at *C* and +22 MPa at *E*. (*a*) Draw the shear and bending-moment diagrams for the beam. (*b*) Determine the maximum normal stress due to bending which occurs in the beam.





SOLUTION

For rectangular cross section $S = \frac{1}{6} b h^2$

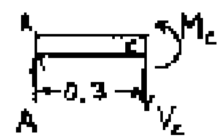
$S = (\frac{1}{6})(24)(60)^2 = 14.4 \times 10^3 \text{ mm}^3 = 14.4 \times 10^{-6} \text{ m}^3$

Bending moments at C and E $\quad M = S\sigma$

$M_C = (14.4 \times 10^{-6})(15 \times 10^6) = 216 \text{ N·m}$

$M_E = (14.4 \times 10^{-6})(22 \times 10^6) = 316.8 \text{ N·m}$

Using portion AC as a free body



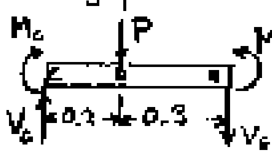
$\circlearrowleft \Sigma M_C = 0 \quad -0.3A + M_C = 0$

$A = 720 \text{ N}$

$+\uparrow \Sigma F_y = 0 \quad A - V_C = 0$

$V_C = 720 \text{ N}$

Using portion CDE as a free body



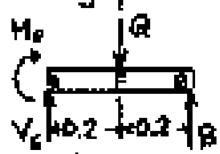
$\circlearrowleft \Sigma M_E = 0$

$0.3P - 0.5V_C - M_C + M_E = 0$

$P = 864 \text{ N}$

$+\uparrow \Sigma F_y = 0, \quad V_C - P - V_E = 0, \quad V_E = -144 \text{ N}$

Using portion EFB as a free body

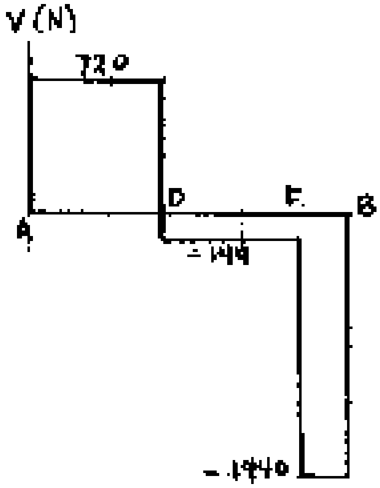


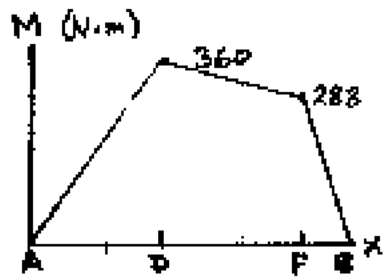
$\circlearrowleft \Sigma M_B = 0$

$0.2Q - 0.4V_E - M_E = 0$

$Q = 1296 \text{ N}$

$+\uparrow \Sigma F_y = 0, \quad V_E - Q + B = 0, \quad B = 1440 \text{ N}$





Areas under shear diagram

A to D $\quad (0.5)(720) = 360 \text{ N·m}$

D to F $\quad (0.5)(-144) = -72 \text{ N·m}$

F to B $\quad (0.2)(-1440) = -288 \text{ N·m}$

Bending moments: $M_A = 0$

$M_D = 0 + 360 = 360 \text{ N·m}$

$M_F = 360 - 72 = 288 \text{ N·m}$

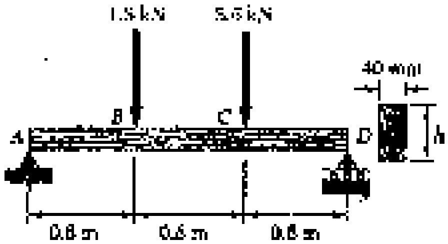
$M_B = 288 - 288 = 0$

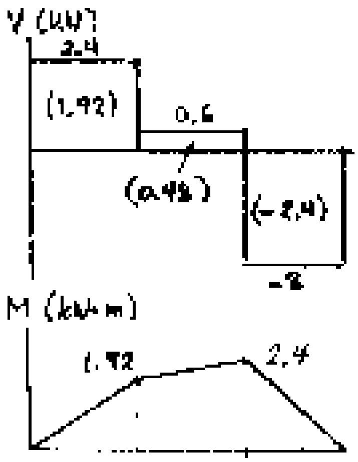
Maximum $|M| = 360 \text{ N·m}$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{360}{14.4 \times 10^{-6}} = 25 \times 10^6 \text{ Pa} = 25 \text{ MPa}$ ◄

PROBLEM 5.75

5.75 and 5.76 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.





SOLUTION

$+\circlearrowleft \Sigma M_D = 0$

$-2.4A + (1.6)(1.8) + (0.8)(3.6) = 0$

$A = 2.4 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0$

$-(0.8)(1.8) - (1.6)(3.6) + 2.4D = 0$

$D = 3 \text{ kN}$

Construct shear and bending moment diagrams

$|M|_{max} = 2.4 \text{ kN}\cdot\text{m} = 2.4 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$

$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{2.4 \times 10^3}{12 \times 10^6} = 200 \times 10^{-6} \text{ m}^3$

$= 200 \times 10^3 \text{ mm}^3$

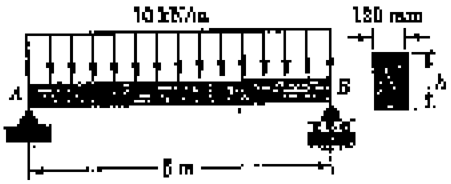
$S = \frac{1}{6} bh^2 = \frac{1}{6}(40)h^2 = 200 \times 10^3$

$h^2 = \frac{(6)(200 \times 10^3)}{40} = 30 \times 10^3 \text{ mm}^2$

$h = 173.2 \text{ mm}$

PROBLEM 5.76

5.75 and 5.76 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.

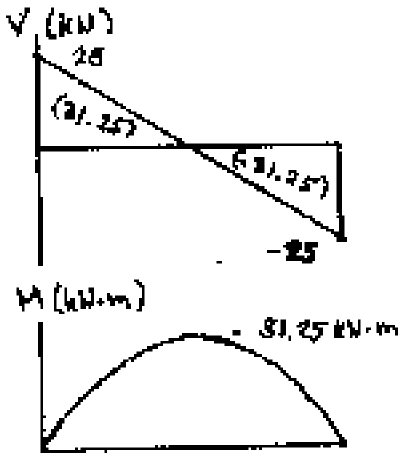


SOLUTION

Reactions: $A = B$ by symmetry

$+\uparrow \Sigma F_y = 0 \qquad A + B - (5)(10) = 0$

$A = B = 25 \text{ kN}$



From bending moment diagram

$|M|_{max} = 31.25 \text{ kN·m} = 31.25 \times 10^3 \text{ N·m}$

$\sigma_{all} = 12 \text{ MPa} = 12 \times 10^6 \text{ Pa}$

$S_{min} = \dfrac{|M|_{max}}{\sigma_{all}} = \dfrac{31.25 \times 10^3}{12 \times 10^6} = 2.604 \times 10^{-3} \text{ m}^3$

$= 2.604 \times 10^6 \text{ mm}^3$

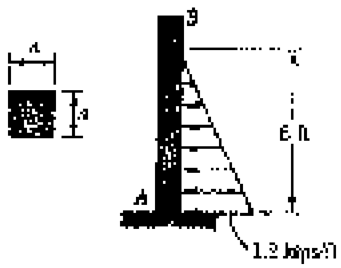
$S = \frac{1}{6} b h^2 = (\frac{1}{6})(120) h^2 = 2.604 \times 10^6$

$h^2 = \dfrac{(6)(2.069 \times 10^6)}{120} = 130.21 \times 10^3 \text{ mm}^2$

$h = 361 \text{ mm}$

PROBLEM 5.77

5.77 and 5.78 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1.75 ksi.



SOLUTION

Equivalent concentrated load

$P = (\frac{1}{2})(6)(1.2) = 3.6$ kips

Bending moment at A

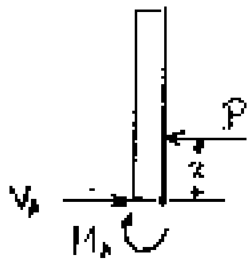
$M_A = (2)(3.6) = 7.2$ kip·ft $= 86.4$ kip·in

$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{86.4}{1.75} = 49.37$ in^3

For a square section $S = \frac{1}{6}a^3$

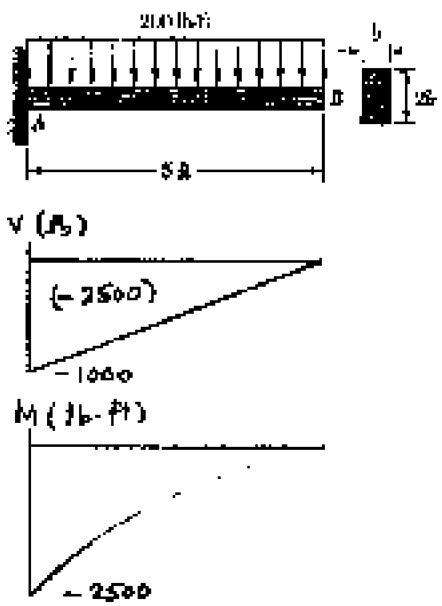
$a = \sqrt[3]{6\ S}$

$a_{min} = \sqrt[3]{(6)(49.37)} = 6.67$ in. ◄



PROBLEM 5.78

5.77 and 5.78 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1.75 ksi.



SOLUTION

Construct shear and bending moment curves.

$|M|_{max} = 2500$ lb·ft $= 2.5$ kip·ft

$= 30$ kip·in.

$\sigma_{all} = 1.75$ ksi

$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{30}{1.75} = 17.143$ in^3

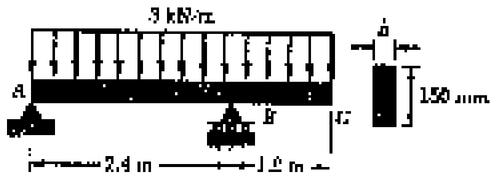
$S = \frac{1}{6}bh^2 = \frac{1}{6}b(2b)^2 = \frac{2}{3}b^3 = 17.143$

$b^3 = \frac{(3)(17.143)}{2} = 25.7$ in^3,

$b = 2.95$ in. ◄

PROBLEM 5.79

5.79 and 5.80 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 Mpa.



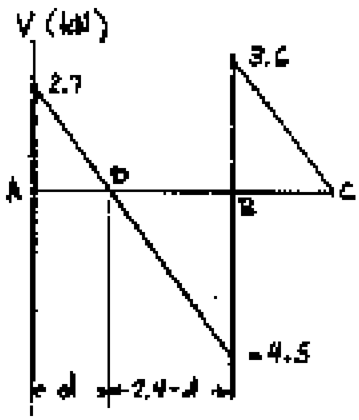
SOLUTION

$+\circlearrowleft M_B = 0$

$-2.4A + (0.6)(3.6)(3) = 0 \qquad A = 2.7 \text{ kN}$

$+\circlearrowleft M_A = 0$

$-(1.8)(3.6)(3) + 2.4B = 0 \qquad B = 8.1 \text{ kN}$



Shear: $V_A = 2.7$ kN

$V_B^- = 2.7 - (2.4)(3) = -4.5$ kN

$V_B^+ = -4.5 + 8.1 = 3.6$ kN

$V_C = 3.6 - (1.2)(3) = 0$

Locate point D where $V = 0$

$$\frac{d}{2.7} = \frac{2.4-d}{4.5} \qquad 7.2\,d = 6.48$$

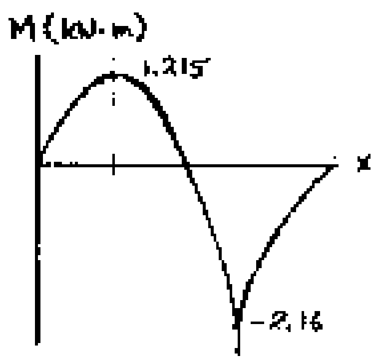
$d = 0.9$ m $\qquad 2.4 - d = 1.5$ m

Areas under shear curve

A to D $\quad \int V\,dx = (\tfrac{1}{2})(0.9)(2.7) = 1.215$ kN·m

D to B $\quad \int V\,dx = (\tfrac{1}{2})(1.5)(-4.5) = -3.375$ kN·m

B to C $\quad \int V\,dx = (\tfrac{1}{2})(1.2)(3.6) = 2.16$ kN·m



Bending moments: $\quad M_A = 0$

$M_D = 0 + 1.215 = 1.215$ kN·m

$M_B = 1.215 - 3.375 = -2.16$ kN·m

$M_C = -2.16 + 2.16 = 0$

Maximum $|M| = 2.16$ kN·m $= 2.16 \times 10^3$ N·m

$\sigma_{all} = 12$ MPa $= 12 \times 10^6$ Pa

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{2.16 \times 10^3}{12 \times 10^6} = 180 \times 10^{-6} \text{ m}^3 = 180 \times 10^3 \text{ mm}^3$$

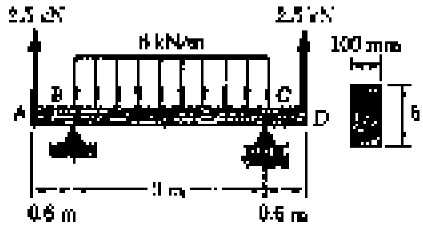
For rectangular section

$$S = \tfrac{1}{6} b h^2 = \tfrac{1}{6} b (150)^2 = 180 \times 10^3$$

$$b = \frac{(6)(180 \times 10^3)}{150^2} = 48 \text{ mm}$$

PROBLEM 5.80

5.79 and 5.80 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.



SOLUTION

By symmetry $B = C$

$+\uparrow \Sigma F_y = 0 \qquad B + C + 2.5 + 2.5 - (3)(6) = 0$

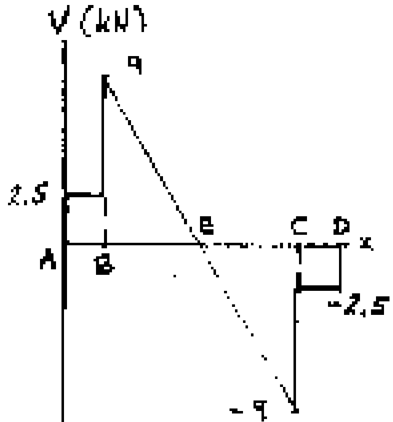
$B = C = 6.5$ kN

Shear: A to B $\quad V = 2.5$ kN

$V_{B^+} = 2.5 + 6.5 = 9$ kN

$V_{C^-} = 9 - (3)(6) = -9$ kN

C to D $\quad V = -9 + 6.5 = -2.5$ kN



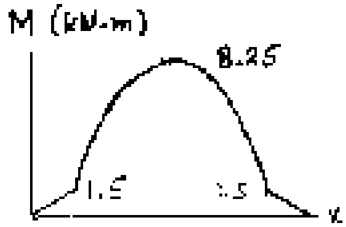
Areas under shear diagram

A to B $\quad \int V\,dx = (0.6)(2.5) = 1.5$ kN·m

B to E $\quad \int V\,dx = (\frac{1}{2})(1.5)(9) = 6.75$ kN·m

E to C $\quad \int V\,dx = -6.75$ kN·m

C to D $\quad \int V\,dx = -1.5$ kN·m



Bending moments $\quad M_A = 0$

$M_B = 0 + 1.5 = 1.5$ kN·m

$M_E = 1.5 + 6.75 = 8.25$ kN·m

$M_C = 8.25 - 6.75 = 1.5$ kN·m

$M_D = 1.5 - 1.5 = 0$

Maximum $|M| = 8.25$ kN·m $= 8.25 \times 10^3$ N·m

$\sigma_{all} = 12$ MPa $= 12 \times 10^6$ Pa

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{8.25 \times 10^3}{12 \times 10^6} = 687.5 \times 10^{-6}\ \text{m}^3 = 687.5 \times 10^3\ \text{mm}^3$$

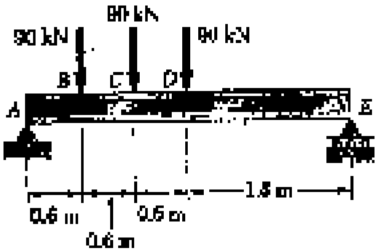
For a rectangular section $\quad S = \frac{1}{6} b h^2$

$$687.5 \times 10^3 = (\tfrac{1}{6})(100)\,h^2 \qquad h^2 = \frac{(6)(687.5 \times 10^3)}{100} = 41.25 \times 10^3\ \text{mm}^2$$

$h = 203$ mm

5.81 and 5.82 Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical metric wide-flange beam to support the loading shown.

PROBLEM 5.81



SOLUTION

$\circlearrowleft \Sigma M_E = 0 \qquad -3.6A + (3)(90) + (2.4)(90) + (1.8)(90) = 0$

$A = 180 \text{ kN}$

$\circlearrowleft \Sigma M_A = 0 \qquad 3.6E - (1.8)(90) - (1.2)(90) - (0.6)(90) = 0$

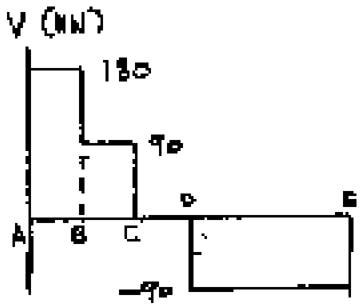
$E = 90 \text{ kN}$

Shear: A to B $\quad V = 180 \text{ kN}$

B to C $\quad V = 180 - 90 = 90 \text{ kN}$

C to D $\quad V = 90 - 90 = 0$

D to E $\quad V = 0 - 90 = -90 \text{ kN}$



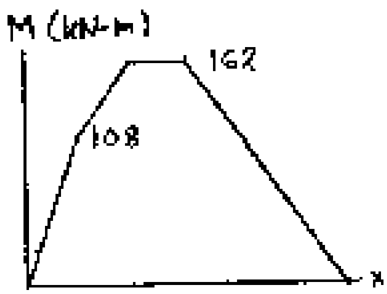
Areas under shear diagram

A to B $\quad \int V\,dx = (0.6)(180) = 108 \text{ kN}\cdot\text{m}$

B to C $\quad \int V\,dx = (0.6)(90) = 54 \text{ kN}\cdot\text{m}$

C to D $\quad \int V\,dx = 0$

D to E $\quad \int V\,dx = (1.8)(-90) = -162 \text{ kN}\cdot\text{m}$



Bending moments $\quad M_A = 0$

$M_B = 0 + 108 = 108 \text{ kN}\cdot\text{m}$

$M_C = 108 + 54 = 162 \text{ kN}\cdot\text{m}$

$M_D = 162 + 0 = 162 \text{ kN}\cdot\text{m}$

$M_E = 162 - 162 = 0$

Maximum $|M| = 162 \text{ kN}\cdot\text{m} = 162 \times 10^3 \text{ N}\cdot\text{m}$

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{162 \times 10^3}{160 \times 10^6} = 1.0125 \times 10^{-3} \text{ m}^3 = 1012.5 \times 10^3 \text{ mm}^3$$

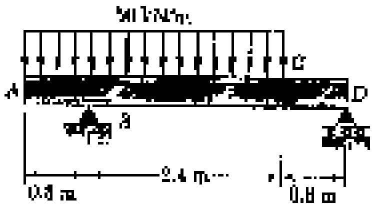
Shape	S (10^3 mm^3)
W 460 × 74	1460
W 410 × 60	1060
W 360 × 64	1030
W 310 × 74	1060

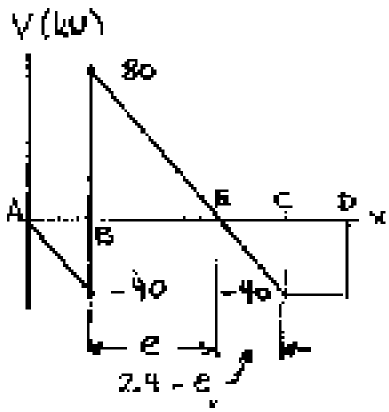
Lightest wide flange beam

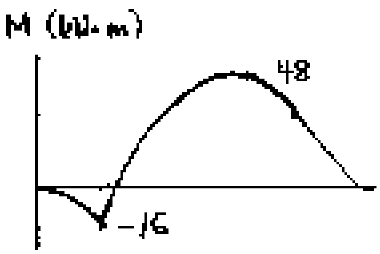
W 410 × 60 @ 60 kg/m

PROBLEM 5.82

5.81 and 5.82 Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical metric wide-flange beam to support the loading shown.







SOLUTION

$+\circlearrowleft \Sigma M_D = 0 \quad -3.2B + (2.4)(3.2)(50) = 0$

$B = 120 \text{ kN}$

$+\circlearrowleft \Sigma M_B = 0 \quad 3.2D - (0.8)(3.2)(50) = 0$

$D = 40 \text{ kN}$

Shear: $V_A = 0$

$V_{B^-} = 0 - (0.8)(50) = -40 \text{ kN}$

$V_{B^+} = -40 + 120 = 80 \text{ kN}$

$V_C = 80 - (2.4)(50) = -40 \text{ kN}$

$V_D = -40 + 0 = -40 \text{ kN}$

Locate point E where $V = 0$

$\frac{e}{80} = \frac{2.4 - e}{40} \qquad 120e = 192$

$e = 1.6 \text{ m} \qquad 2.4 - e = 0.8 \text{ m}$

Areas: A to B, $\int V dx = (\frac{1}{2})(0.8)(-40) = -16 \text{ kN·m}$

B to E $\int V dx = (\frac{1}{2})(1.6)(80) = 64 \text{ kN·m}$

E to C $\int V dx = (\frac{1}{2})(0.8)(-40) = -16 \text{ kN·m}$

C to D $\int V dx = (0.8)(-40) = -32 \text{ kN·m}$

Bending moments: $M_A = 0$

$M_B = 0 - 16 = -16 \text{ kN·m}$

$M_E = -16 + 64 = 48 \text{ kN·m}$

$M_C = 48 - 16 = 32 \text{ kN·m}$

$M_D = 32 - 32 = 0$

Maximum $|M| = 48 \text{ kN·m} = 48 \times 10^3 \text{ N·m}$

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{48 \times 10^3}{160 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^3 \text{ mm}^3$

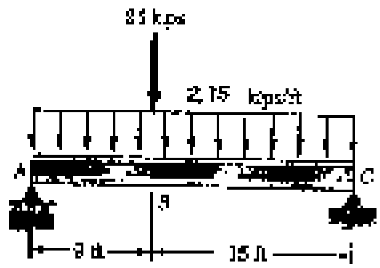
Shape	S (10^3 mm^3)
W 310 × 32.7	415
W 250 × 28.4	308
W 200 × 35.9	342

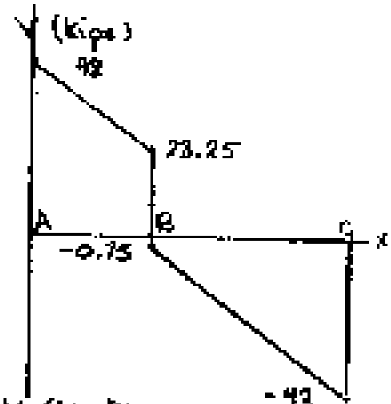
Lightest wide flange beam

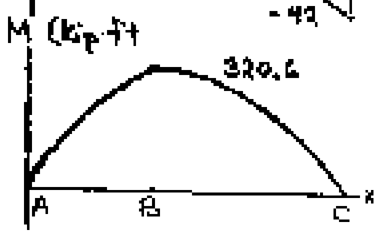
W 250 × 28.4 @ 28.4 kg/m

PROBLEM 5.83

5.83 and 5.84 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.







SOLUTION

$+\circlearrowleft \Sigma M_C = 0 \qquad -24A + (12)(24)(2.75) + (15)(24) = 0$

$A = 48$ kips

$+\circlearrowleft \Sigma M_A = 0 \qquad 24C - (12)(24)(2.75) - (9)(24) = 0$

$C = 42$ kips

Shear: $V_A = 48$

$V_{B^-} = 48 - (9)(2.75) = 23.25$ kips

$V_{B^+} = 23.25 - 24 = -0.75$ kips

$V_C = -0.75 - (15)(2.75) = -42$ kips

Areas under shear diagram

A to B $\int V dx = (\frac{1}{2})(9)(48 + 23.25) = 320.6$ kip·ft

B to C $\int V dx = (\frac{1}{2})(15)(-0.75 - 42) = -320.6$ kip·ft

Bending moments: $M_A = 0$

$M_B = 0 + 320.6 = 320.6$ kip·ft

$M_C = 320.6 - 320.6 = 0$

Maximum $|M| = 320.6$ kip·ft $= 3848$ kip·in

$\sigma_{all} = 24$ ksi

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3848}{24} = 160.3 \text{ in}^3$$

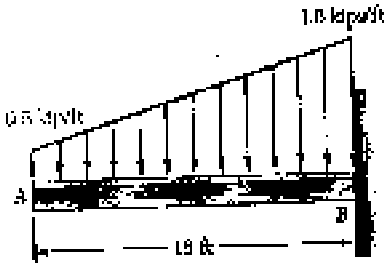
Shape	S (in³)
W 30 × 99	269
W 27 × 84	213
W 24 × 104	258
W 21 × 101	227
W 18 × 106	204

Lightest wide flange beam

W 27 × 84 @ 84 lb/ft

PROBLEM 5.84

5.83 and 5.84 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.



SOLUTION

$$w = 0.5 + \frac{(1.5 - 0.5)x}{18} = 0.5 + 0.0556\ x$$

$$\frac{dV}{dx} = -w = -0.5 + 0.05556\ x$$

$$V = 0 - 0.5x - 0.02778\ x^2 = \frac{dM}{dx}$$

$$M = 0 - 0.25\ x^2 - 0.009259\ x^3$$

Maximum $|M|$ occurs at $x = 18$ ft.

$$|M|_{max} = (0.25)(18)^2 + (0.009259)(18)^3 = 135 \text{ kip}\cdot\text{ft} = 1620 \text{ kip}\cdot\text{in}$$

$\sigma_{all} = 24$ ksi $\qquad S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{1620}{24} = 67.5 \text{ in}^3$

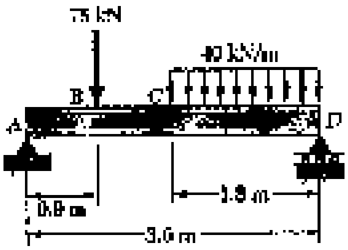
Shape	S (in³)
W 21 × 44	81.6
W 18 × 50	88.9
W 16 × 57	92.2
W 14 × 53	77.8
W 12 × 72	97.4
W 10 × 68	75.7

Lightest wide flange beam

W 18 × 50 @ 50 lb/ft

PROBLEM 5.85

5.85 and 5.86 Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical metric S-shape beam to support the loading shown.



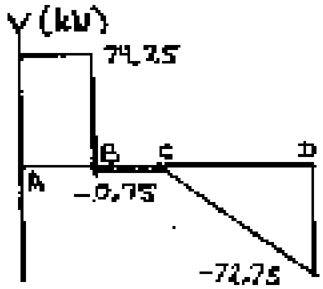
SOLUTION

$+\circlearrowleft \Sigma M_D = 0 \quad -3.6A + (2.7)(75) + (0.9)(1.8)(40) = 0$

$A = 74.25 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0 \quad 3.6D - (0.9)(75) - (2.7)(1.8)(40) = 0$

$D = 72.75 \text{ kN}$



Shear: A to B $\quad V = 74.25 \text{ kN}$

B to C $\quad V = 74.25 - 75 = -0.75 \text{ kN}$

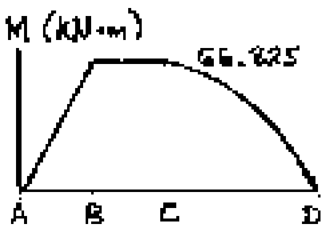
$V_D = -0.75 - (1.8)(40) = -72.75 \text{ kN}$

Areas under shear diagram

A to B $\quad \int V dx = (0.9)(74.25) = 66.825 \text{ kN·m}$

B to C $\quad \int V dx = (0.9)(-0.75) = -0.675 \text{ kN·m}$

C to D $\quad (\frac{1}{2})(1.8)(-0.75 - 72.75) = -66.15 \text{ kN·m}$



Bending moments: $\quad M_A = 0$

$M_B = 0 + 66.825 = 66.825 \text{ kN·m}$

$M_C = 66.825 - 0.675 = 66.15 \text{ kN·m}$

$M_D = 66.15 - 66.15 = 0$

Maximum $|M| = 66.825 \text{ kN·m} = 66.825 \times 10^3 \text{ N·m}$

$\sigma_{all} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{66.825 \times 10^3}{160 \times 10^6} = 417.7 \times 10^{-6} \text{ m}^3 = 417.7 \times 10^3 \text{ mm}^3$$

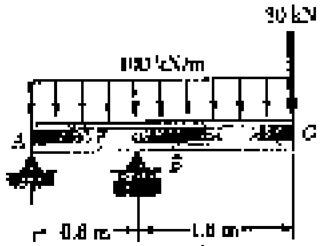
Shape	S (10^3 mm^3)
S 380 × 64	971
S 310 × 47.3	593
S 250 × 52	482

Lightest S-section

S 310 × 47.3 @ 47.3 kg/m

PROBLEM 5.86

5.85 and 5.86 Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical metric S-shape beam to support the loading shown.



SOLUTION

$+\circlearrowleft \Sigma M_B = 0 \qquad 0.8A - (0.4)(2.4)(100) - (1.6)(80) = 0$

$A = 280 \text{ kN} \downarrow$

$+\circlearrowleft \Sigma M_A = 0 \qquad 0.8B - (1.2)(2.4)(100) - (2.4)(80) = 0$

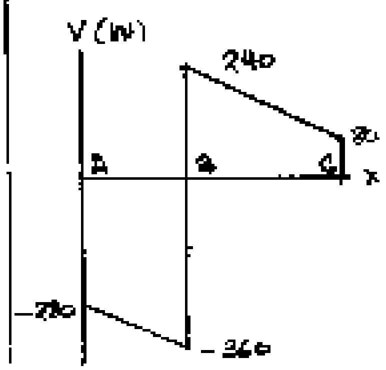
$B = 600 \text{ kN} \uparrow$

Shear: $V_A = -280$ kN

$V_{B^-} = -280 - (0.8)(100) = -360$ kN

$V_{B^+} = -360 + 600 = 240$ kN

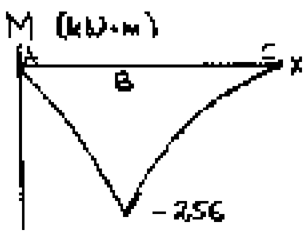
$V_C = 240 - (1.6)(100) = 80$ kN



Areas under shear diagram

A to B $\quad (\frac{1}{2})(0.8)(-280 - 360) = -256$ kN·m

B to C $\quad (\frac{1}{2})(1.6)(240 + 80) = 256$ kN·m



Bending moments: $\quad M_A = 0$

$M_B = 0 - 256 = -256$ kN·m

$M_C = -256 + 256 = 0$

Maximum $|M| = 256$ kN·m $= 256 \times 10^3$ N·m

$\sigma_{all} = 160$ MPa $= 160 \times 10^6$ Pa

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{256 \times 10^3}{160 \times 10^6} = 1.6 \times 10^{-3} \text{ m}^3 = 1600 \times 10^3 \text{ mm}^3$$

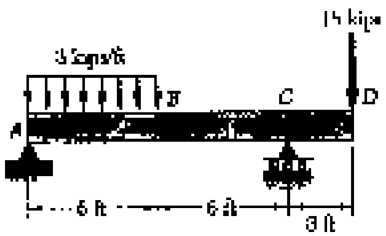
Shape	$S\ (10^3 \text{ mm}^3)$
S 510 × 98.3	1960
S 460 × 104	1685

Lightest S-section

S 510 × 98.3 ◄

PROBLEM 5.87

5.87 and 5.88 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.



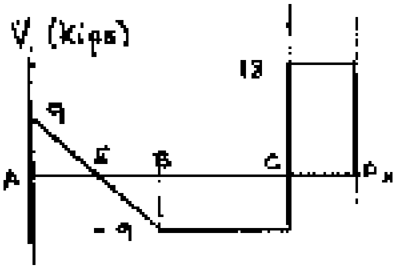
SOLUTION

$\circlearrowleft \Sigma M_C = 0 \qquad -12A + (9)(6)(3) - (3)(18) = 0$

$A = 9 \text{ kips}$

$+\circlearrowright \Sigma M_A = 0 \qquad 12C - (3)(6)(3) - (15)(18) = 0$

$C = 27 \text{ kips}$



Shear: $V_A = 9$ kips

B to C $\quad V = 9 - (6)(3) = -9$ kips

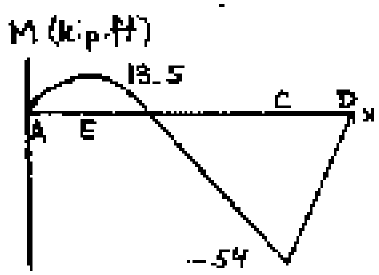
C to D $\quad V = -9 + 27 = 18$ kips

Areas: A to E $\quad (\frac{1}{2})(3)(9) = 13.5$ kip·ft

E to B $\quad (\frac{1}{2})(3)(-9) = -13.5$ kip·ft

B to C $\quad (6)(-9) = -54$ kip·ft

C to D $\quad (3)(18) = 54$ kip·ft



Bending moments: $M_A = 0$

$M_E = 0 + 13.5 = 13.5$ kip·ft

$M_B = 13.5 - 13.5 = 0$

$M_C = 0 + 54 = 54$ kip·ft

$M_D = 54 - 54 = 0$

Maximum $|M| = 54$ kip·ft $= 648$ kip·in

$\sigma_{all} = 24$ ksi

$S_{min} = \dfrac{648}{24} = 27 \text{ in}^3$

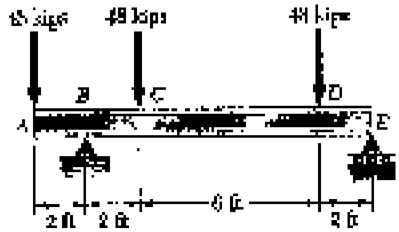
Shape	$S(in^3)$
S 12×31.8	36.4
S 10×35	29.4

Lightest S-shaped beam

S 12×31.8

PROBLEM 5.88

5.87 and 5.88 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.



SOLUTION

$+\sum M_E = 0 \quad (12)(48) - 10B + (8)(48) + (2)(48) = 0$

$B = 105.6$ kips

$+\sum M_B = 0 \quad (2)(48) - (2)(48) - (8)(48) + 10E = 0$

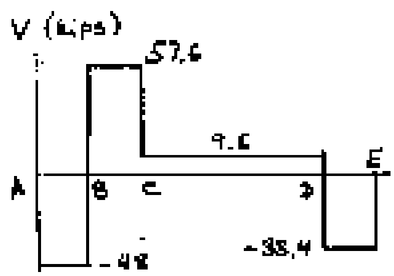
$E = 38.4$ kips

Shear: A to B $\quad V = -48$ kips

B to C $\quad V = -48 + 105.6 = 57.6$ kips

C to D $\quad V = 57.6 - 48 = 9.6$ kips

D to E $\quad V = 9.6 - 48 = -38.4$ kips

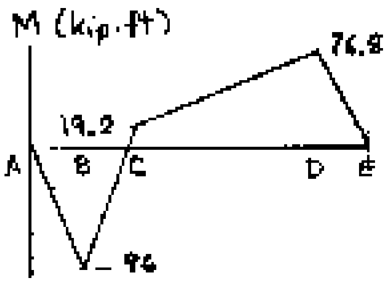


Areas: A to B $\quad (2)(-48) = -96$ kip·ft

B to C $\quad (2)(57.6) = 115.2$ kip·ft

C to D $\quad (6)(9.6) = 57.6$ kip·ft

D to E $\quad (2)(-38.4) = 76.8$ kip·ft



Bending moments: $\quad M_A = 0$

$M_B = 0 - 96 = -96$ kip·ft

$M_C = -96 + 115.2 = 19.2$ kip·ft

$M_D = 19.2 + 57.2 = 76.8$ kip·ft

$M_E = 76.8 - 76.8 = 0$

Maximum $|M| = 96$ kip·ft $= 1152$ kip·in

$\sigma_{all} = 24$ ksi

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{1152}{24} = 48 \text{ in}^3$$

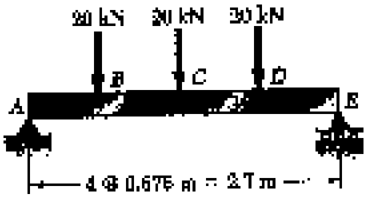
Shape	S (in³)
S 15 × 42.9	59.4
S 12 × 50	50.8

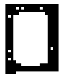
Lightest S-shaped beam

S 15 × 42.9

PROBLEM 5.89

5.89 Two metric rolled-steel channels are to be welded along their edges and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 150 MPa, determine the most economical channels that can be used.



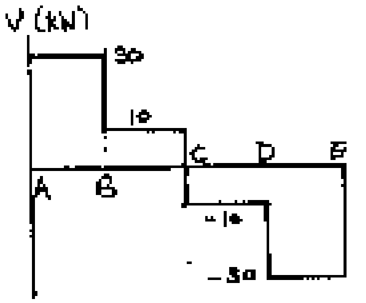


SOLUTION

By symmetry $A = E$

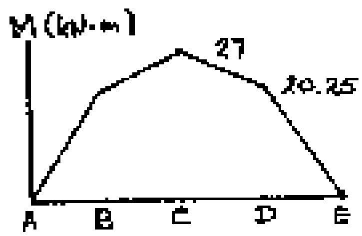
$+\uparrow \Sigma F_y = 0 \quad A + E - 20 - 20 - 20 = 0$

$A = E = 30$ kN



Shear: A to B $V = 30$ kN
B to C $V = 30 - 20 = 10$ kN
C to D $V = 10 - 20 = -10$ kN
D to E $V = -10 - 20 = -30$ kN

Areas: A to B $(0.675)(30) = 20.25$ kN·m
B to C $(0.675)(10) = 6.75$ kN·m
C to D $(0.675)(-10) = -6.75$ kN·m
D to E $(0.675)(-30) = -20.25$ kN·m



Bending moments: $M_A = 0$
$M_B = 0 + 20.25 = 20.25$ kN·m
$M_C = 20.25 + 6.75 = 27$ kN·m
$M_D = 27 - 6.75 = 20.25$ kN·m
$M_E = 20.25 - 20.25 = 0$

Maximum $|M| = 27$ kN·m $= 27 \times 10^3$ N·m

$\sigma_{all} = 150$ MPa $= 150 \times 10^6$ Pa

For a section consisting of two channels

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{27 \times 10^3}{150 \times 10^6} = 180 \times 10^{-6}\ \text{m}^3 = 180 \times 10^3\ \text{mm}^3$$

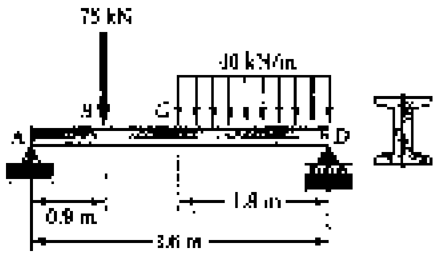
For each channel $S_{min} = (\tfrac{1}{2})(180 \times 10^3) = 90 \times 10^3$ mm^3

Shape	S (10^3 mm^3)
C 180 × 14.6	99.2
C 150 × 19.3	93.6

Lightest channel section
C 180 × 14.6

PROBLEM 5.90

5.90 Two metric rolled-steel channels are to be welded back to back and used to support the loading shown. Knowing that the allowable normal stress for the steel used is 190 MPa, determine the most economical channels that can be used.



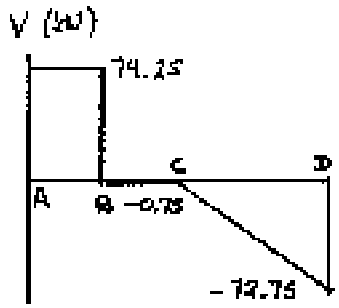
SOLUTION

$+\circlearrowleft \Sigma M_D = 0 \quad -3.6A + (2.7)(75) + (0.9)(1.8)(40) = 0$

$A = 74.25$ kN

$+\circlearrowleft \Sigma M_A = 0 \quad 3.6D - (0.9)(75) - (2.7)(1.8)(40) = 0$

$D = 72.75$ kN



Shear: A to B $\quad V = 74.25$ kN

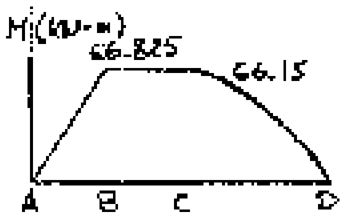
B to C $\quad V = 74.25 - 75 = -0.75$ kN

$V_D = -0.75 - (1.8)(40) = -72.75$ kN

Areas: A to B $\quad (0.9)(74.25) = 66.825$ kN·m

B to C $\quad (0.9)(-0.75) = -0.675$ kN·m

C to D $\quad (\frac{1}{2})(1.8)(-0.75 - 72.75) = -66.15$



Bending moments: $\quad M_A = 0$

$M_B = 0 + 66.825 = 66.825$ kN·m

$M_C = 66.825 - 0.675 = 66.15$ kN·m

$M_D = 66.15 - 66.15 = 0$

Maximum $|M| = 66.825$ kN·m $= 66.825 \times 10^3$ N·m

$\sigma_{all} = 190$ MPa $= 190 \times 10^6$ Pa

For double channel $\quad S_{min} = \frac{|M|}{\sigma_{all}} = \frac{66.825 \times 10^3}{190 \times 10^6} = 351.7 \times 10^{-6}\ m^3$

$= 351.7 \times 10^3\ mm^3$

For each channel $\quad S_{min} = (\frac{1}{2})(351.7 \times 10^3) = 175.9 \times 10^3\ mm^3$

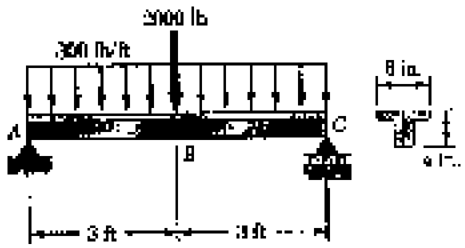
Shape	$S(10^3 mm^3)$
C 230 × 22	185
C 200 × 27.9	179

lightest channel section

C 230 × 22

PROBLEM 5.91

5.91 Two L 4 × 3 rolled-steel angles are bolted together to support the loading shown. Knowing that the allowable normal stress for the steel used is 24 ksi, determine the minimum angle thickness that can be used.



SOLUTION

By symmetry $A = C$

$+\Sigma F_y = 0 \qquad A + C - 2000 - (6)(300) = 0$

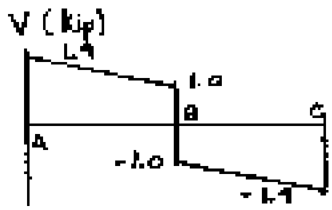
$A = C = 1900$ lb.

Shear: $V_A = 1900$ lb $= 1.9$ kips

$V_{B^-} = 1900 - (3)(300) = 1000$ lb $= 1$ kip

$V_{B^+} = 1000 - 2000 = -1000$ lb $= -1$ kip

$V_C = -1000 - (3)(300) = -1900$ lb $= -1.9$ kip



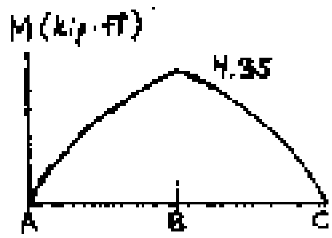
Areas: A to B $(\frac{1}{2})(3)(1.9 + 1) = 4.35$ kip·ft

B to C $(\frac{1}{2})(3)(-1 - 1.9) = -4.35$ kip·ft

Bending moments: $M_A = 0$

$M_B = 0 + 4.35 = 4.35$ kip·ft

$M_C = 4.35 - 4.35 = 0$



Maximum $|M| = 4.35$ kip·ft $= 52.2$ kip·in

$\sigma_{all} = 24$ ksi

For section consisting of two angles $S_{min} = \dfrac{|M|}{\sigma_{all}} = \dfrac{52.2}{24} = 2.175$ in^3

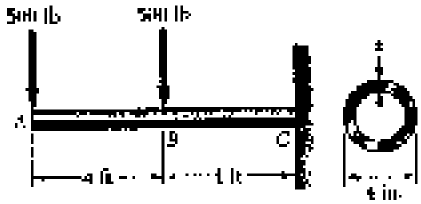
For each angle $S_{min} = (\frac{1}{2})(2.175) = 1.0875$ in^3

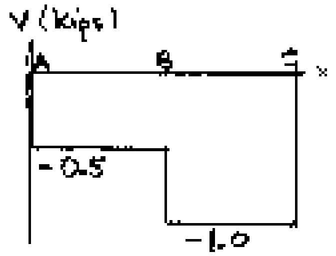
Angle section	S (in^3)
L 4×3×$\frac{1}{2}$	1.89
L 4×3×$\frac{3}{8}$	1.46
L 4×3×$\frac{1}{4}$	1.00

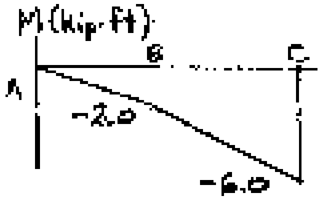
Smallest allowable thickness

$t = \frac{3}{8}$ in ◀

PROBLEM 5.92







5.92 A steel pipe of 4-in. diameter is to support the loading shown. Knowing that the stock of pipes available have thicknesses varying from $\frac{1}{4}$ in. to 1 in. in $\frac{1}{8}$-in. increments, and that the allowable normal stress for the steel used is 24 ksi, determine the minimum wall thickness t that can be used.

SOLUTION

Shear: A to B $\quad V = -500 \text{ lb} = -0.5 \text{ kip}$

B to C $\quad V = -500 - 500 = -1000 \text{ lb} = -1.0 \text{ kip}$

Areas: A to B $\quad (4)(-0.5) = -2.0 \text{ kip·ft}$

B to C $\quad (4)(-1.0) = -4.0 \text{ kip·ft}$

Bending moments: $\quad M_A = 0$

$M_B = 0 - 2.0 = -2.0 \text{ kip·ft}$

$M_C = -2.0 - 4.0 = -6.0 \text{ kip·ft}$

Maximum $|M| = 6.0 \text{ kip·ft} = 72 \text{ kip·in}$

$\sigma_{all} = 24 \text{ ksi}$

$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{72}{24} = 3 \text{ in}^3$$

$I = \frac{\pi}{4}(c_2^4 - c_1^4) \qquad c = c_2 \qquad c_2 = \frac{1}{2}d = 2.0 \text{ in}$

$$S = \frac{I}{c} = \frac{\pi}{4}\frac{c_2^4 - c_1^4}{c_2} = \frac{\pi}{4}\frac{2^4 - c_1^4}{2} = 3 \text{ in}^3$$

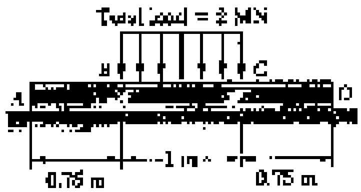
$c_1^4 = 2^4 - \frac{(4)(2)(3)}{\pi} = 8.3606 \text{ in}^4 \qquad c_1 = 1.7004 \text{ in.}$

$t_{min} = c_2 - c_1 = 2.0 - 1.7004 = 0.2996 \text{ in}$

Using $\frac{1}{8}$ in. increments for design $\quad t = \frac{3}{8}$ in. ⟶

PROBLEM 5.93

5.93 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 170 MPa, select the most economical metric wide-flange beam to support the loading shown.

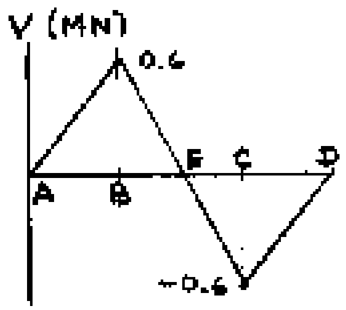


SOLUTION

Downward distributed load $w = \frac{2}{1.0} = 2$ MN/m

Upward distributed reaction $q = \frac{2}{2.5} = 0.8$ MN/m

Net distributed load over BC 1.2 MN/m



Shear: $V_A = 0$

$V_B = 0 + (0.75)(0.8) = 0.6$ MN

$V_C = 0.6 - (1.0)(1.2) = -0.6$ MN

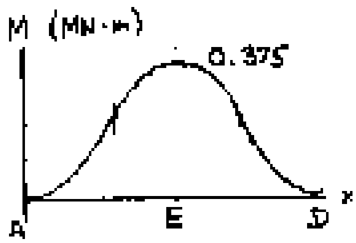
$V_D = -0.6 + (0.75)(0.8) = 0$

Areas: A to B $(\frac{1}{2})(0.75)(0.6) = 0.225$ MN·m

B to E $(\frac{1}{2})(0.5)(0.6) = 0.150$ MN·m

E to C $(\frac{1}{2})(0.5)(-0.6) = -0.150$ MN·m

C to D $(\frac{1}{2})(0.75)(-0.6) = -0.225$ MN·m



Bending moments: $M_A = 0$

$M_B = 0 + 0.225 = 0.225$ MN·m

$M_E = 0.225 + 0.150 = 0.375$ MN·m

$M_C = 0.375 - 0.150 = 0.225$ MN·m

$M_D = 0.225 - 0.225 = 0$

Maximum $|M| = 0.375$ MN·m $= 375 \times 10^3$ N·m

$\sigma_{all} = 170$ MPa $= 170 \times 10^6$ Pa

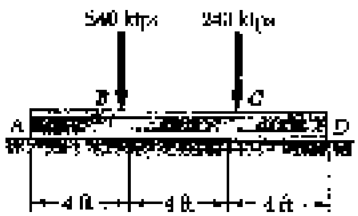
$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{375 \times 10^3}{170 \times 10^6} = 2.206 \times 10^{-3}\ m^3 = 2206 \times 10^3\ mm^3$$

Shape	$S(10^3 mm^3)$
W 690 × 125	3510
W 610 × 101	2530
W 530 × 150	3720
W 460 × 113	2400

Lightest wide flange section

W 610 × 101

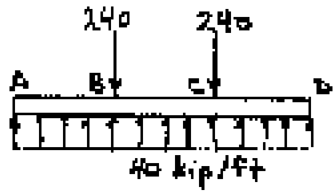
PROBLEM 5.94



5.94 Assuming the upward reaction of the ground to be uniformly distributed and knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical S-shape beam to support the loading shown.

SOLUTION

Distributed reaction $q = \frac{480}{12} = 40 \text{ kip/ft}$



Shear: $V_A = 0$

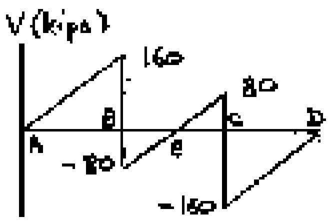
$V_B^- = 0 + (4)(40) = 160 \text{ kips}$

$V_B^+ = 160 - 240 = -80 \text{ kips}$

$V_C^- = -80 + (4)(40) = 80 \text{ kips}$

$V_C^+ = 80 - 240 = -160 \text{ kips}$

$V_D = -160 + (4)(40) = 0$

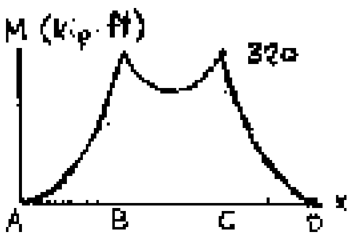


Areas: A to B $(\frac{1}{2})(4)(160) = 320 \text{ kip}\cdot\text{ft}$

B to E $(\frac{1}{2})(2)(-80) = -80 \text{ kip}\cdot\text{ft}$

E to C $(\frac{1}{2})(2)(80) = 80 \text{ kip}\cdot\text{ft}$

C to D $(\frac{1}{2})(4)(-160) = -320 \text{ kip}\cdot\text{ft}$



Bending moments: $M_A = 0$

$M_B = 0 + 320 = 320 \text{ kip}\cdot\text{ft}$

$M_E = 320 - 80 = 240 \text{ kip}\cdot\text{ft}$

$M_C = 240 + 80 = 320 \text{ kip}\cdot\text{ft}$

$M_D = 320 - 320 = 0$

Maximum $|M| = 320 \text{ kip}\cdot\text{ft} = 3840 \text{ kip}\cdot\text{in}$

$\sigma_{all} = 24 \text{ ksi}$ $\qquad S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3840}{24} = 160 \text{ in}^3$

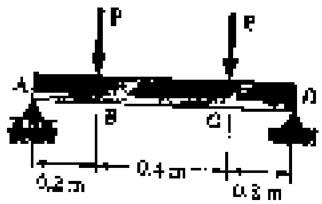
Shape	S (in³)
S 24 × 80	175
S 20 × 96	165

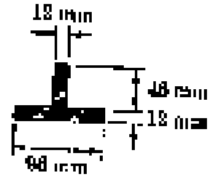
Lightest S-shaped section

S 24 × 80

PROBLEM 5.95

5.95 and 5.96 Determine the largest permissible value of P for the beam and loading shown, knowing that the allowable normal stress is +80 MPa in tension and −140 MPa in compression.





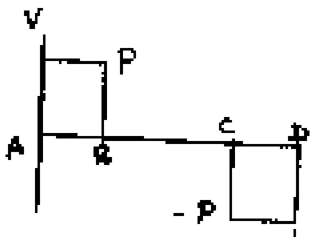
SOLUTION

Reactions: $A = D = P$

Shear: A to B $V = P$

B to C $V = P - P = 0$

C to D $V = 0 - P = -P$



Areas: A to B $0.2P$

B to C 0

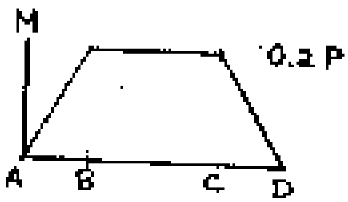
C to D $-0.2P$

Bending moments: $M_A = 0$

$M_B = 0 + 0.2P = 0.2P$

$M_C = 0.2P + 0 = 0.2P$

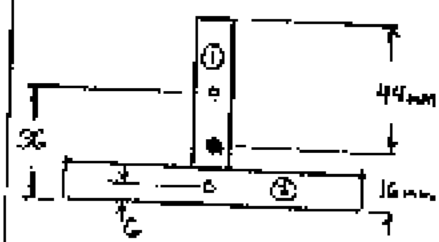
$M_D = 0.2P - 0.2P = 0$



Largest positive bending moment: $0.2P$

Largest negative bending moment: 0

Centroid and moment of inertia



Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm³)	d (mm)	Ad^2 (mm⁴)	$\bar{I}$ mm⁴
1	576	36	20736	20	230.4×10^3	110.6×10^3
2	1152	6	6912	10	115.2×10^3	13.8×10^3
Σ	1728		27648		345.6×10^3	124.4×10^3

$$\bar{Y} = \frac{27648}{1728} = 16 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 470.0\times10^3 \text{ mm}^4$$

Top $c = 44$ mm $\quad S = \frac{I}{c} = \frac{470.0\times10^3}{44} = 10.68\times10^3 \text{ mm}^3 = 10.68\times10^{-6} \text{ m}^3$

Allowable pos. M. $\quad M = |\sigma|S = (140\times10^6)(10.68\times10^{-6}) = 1495$ N·m

Bot. $c = 16$ mm $\quad S = \frac{I}{c} = \frac{470.0\times10^3}{16} = 29.38\times10^3 \text{ mm}^3 = 29.38\times10^{-6} \text{ m}^3$

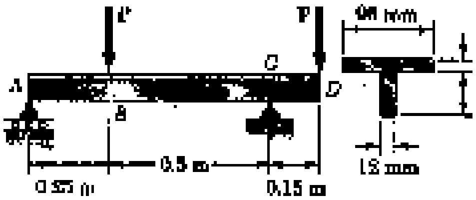
Allowable pos. M $\quad M = |\sigma_{all}|S = (80\times10^6)(29.38\times10^{-6}) = 2350$ N·m

Smaller value $\quad M = 1495$ N·m

Allowable value of P $\quad 0.2P = 1495 \quad P = 7475$ N $= 7.48$ kN

PROBLEM 5.96

5.95 and 5.96 Determine the largest permissible value of P for the beam and loading shown, knowing that the allowable normal stress is +80 MPa in tension and −140 MPa in compression.



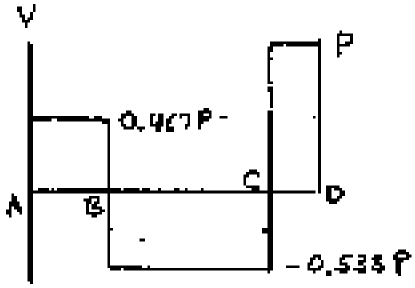
SOLUTION

$\circlearrowleft \Sigma M_C = 0 \quad -0.75A + 0.5P - 0.15P = 0$

$A = 0.46667\ P$

$\circlearrowleft \Sigma M_A = 0 \quad 0.75C - 0.25P - 0.9P = 0$

$C = 1.53333\ P$



Shear: A to B $\quad V = 0.46667\ P$

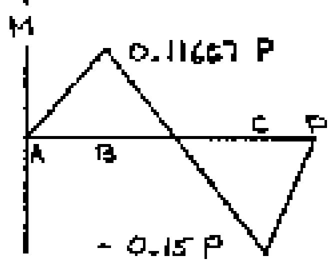
B to C $\quad V = 0.46667P - P = -0.53333\ P$

C to D $\quad V = -0.53333P + 1.53333P = P$

Areas: A to B $\quad (.25)(0.46667P) = 0.11667\ P$

B to C $\quad (0.5)(-0.53333P) = -0.26667\ P$

C to D $\quad (0.15)P = 0.15P$



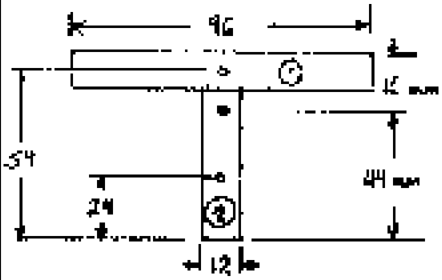
Bending moments: $\quad M_A = 0$

$M_B = 0 + 0.11667P = 0.11667\ P$

$M_C = 0.11667P - 0.26667P = -0.15\ P$

$M_D = -0.15P + 0.15P = 0$

Centroid and moment of inertia.



Part	$A\,(mm^2)$	$\bar{y}\,(mm)$	$A\bar{y}\,(mm^3)$	$d\,(mm)$	$Ad^2\,(mm^4)$	$\bar{I}\,(mm^4)$
①	1152	54	62208	10	115200	13824
②	576	24	13824	20	230400	110592
Σ	1728		76032		345600	124416

$\bar{Y} = \dfrac{76032}{1728} = 44\ mm$

$I = \Sigma Ad^2 + \Sigma \bar{I} = 470016\ mm^4$

Top: $\quad y = 16\ mm \qquad \dfrac{I}{y} = \dfrac{470016}{16} = 29.376\times10^3\ mm^3 = 29.376\times10^{-6}\ m^3$

Bottom: $\quad y = -44\ mm \qquad \dfrac{I}{y} = \dfrac{470016}{44} = 10.682\times10^3\ mm^3 = -10.682\times10^{-6}\ m^3$

Bending moment limits: $\quad M = -\dfrac{I\sigma}{y}$

Tension at B $\quad = (-10.682\times10^{-6})(80\times10^6) = 854.56\ N\cdot m \quad \lhd B$

Comp. at B $\quad -(29.376\times10^{-6})(-140\times10^6) = 4.1126\times10^3\ N\cdot m$

Tension at C $\quad -(29.376\times10^{-6})(80\times10^6) = -2.35\times10^3\ N\cdot m$

Comp. at C $\quad -(-10.682\times10^{-6})(-140\times10^6) = -1.4955\times10^3\ N\cdot m \quad \lhd C$

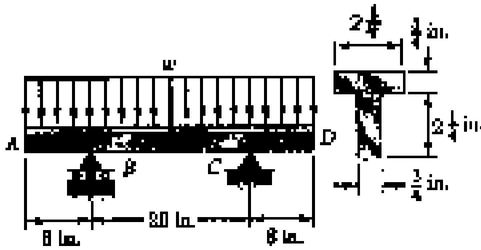
Allowable load: $\quad 0.11667P = 854.56 \qquad P = 7.32\times10^3\ N$

$-0.15P = -1.4955\times10^3 \qquad P = 9.97\times10^3\ N$

The smaller value is $\quad P = 7.32\ kN$

PROBLEM 5.97

5.97 Determine the largest permissible uniformly distributed load w for the beam shown, knowing that the allowable normal stress is +12 ksi in tension and −19.5 ksi in compression.



SOLUTION

Reactions: $B + C - 36w = 0 \qquad B = C = 18w$

Shear: $V_A = 0$

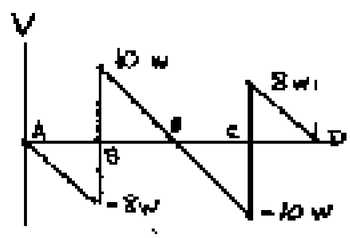
$V_{B^-} = 0 - 8w = -8w$

$V_{B^+} = -8w + 18w = 10w$

$V_{C^-} = 10w - 20w = -10w$

$V_{C^+} = -10w + 18w = 8w$

$V_D = 8w - 8w = 0$



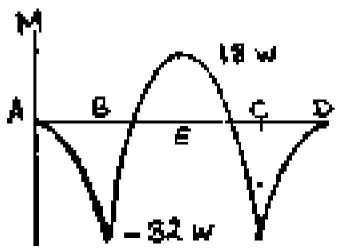
Areas: A to B $(\frac{1}{2})(8)(-8w) = -32w$

B to E $(\frac{1}{2})(10)(10w) = 50w$

Bending moments: $M_A = 0$

$M_B = 0 - 32w = -32w$

$M_E = -32w + 50w = 18w$

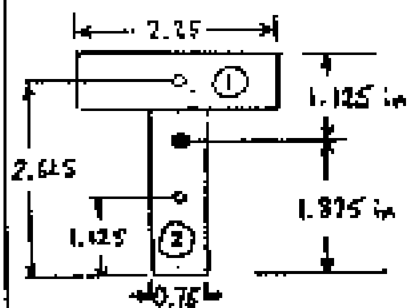


Centroid and moment of inertia

Part	$A\ (in^2)$	$\bar{y}\ (in)$	$A\bar{y}\ (in^3)$	d in	$Ad^2\ (in^4)$	$\bar{I}\ (in^4)$
1	1.6875	2.625	4.4297	0.75	0.9492	0.0791
2	1.6875	1.125	1.8984	0.75	0.9492	0.7119
Σ	3.375		6.3281		1.8984	0.7910

$\bar{Y} = \frac{6.3281}{3.375} = 1.875$ in

$I = \Sigma Ad^2 + \Sigma \bar{I} = 2.6894\ in^4$



Top: $y = 1.125 \qquad I/y = 2.3906\ in^3$

Bottom: $y = -1.875 \qquad I/y = -1.4343\ in^3$

Bending moment limits $\qquad M = -\sigma I/y$

Tension at B and C	$-(12)(2.3906) =$	-28.687 kip·in
Comp. at B and C	$-(-19.5)(-1.4343) =$	-27.969 kip·in
Tension at E	$-(12)(-1.4343) =$	17.212 kip·in
Compression at E	$-(-19.5)(2.3906) =$	46.6 kip·in

Allowable load w

B & C $\qquad -32w = -27.969 \qquad w = 0.874$ kip/in

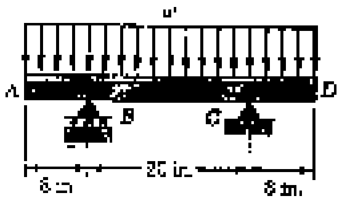
E $\qquad 18w = 17.212 \qquad w = 0.956$ kip/in

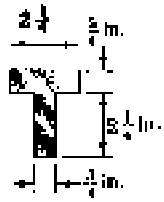
Smallest $\qquad w = 0.874$ kip/in $= 10.49$ kip/ft.

PROBLEM 5.98

5.97 Determine the largest permissible uniformly distributed load w for the beam shown, knowing that the allowable normal stress is +12 ksi in tension and −19.5 ksi in compression.

5.98 Solve Prob. 5.97, assuming that the cross section of the beam is reversed, with the flange of the beam resting on the supports at B and C.





SOLUTION

Reactions $B + C - 36w = 0 \qquad B = C = 18w$

Shear: $V_A = 0$

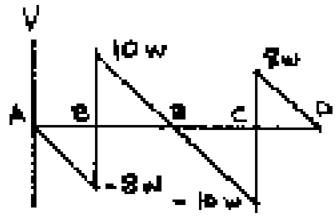
$V_{B^-} = 0 - 8w = -8w$

$V_{B^+} = -8w + 18w = 10w$

$V_{C^-} = 10w - 20w = -10w$

$V_{C^+} = -10w + 18w = 8w$

$V_D = 8w - 8w = 0$



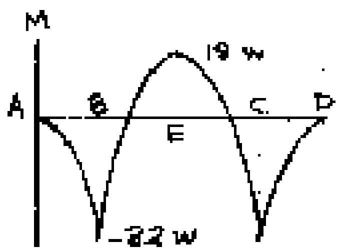
Areas: A to B $(\frac{1}{2})(8)(-8w) = -32w$

B to E $(\frac{1}{2})(10)(10w) = 50w$

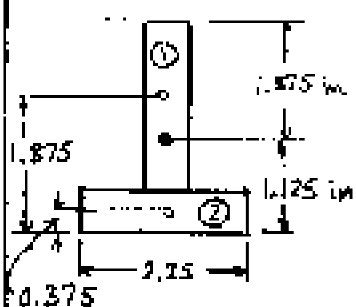
Bending moments: $M_A = 0$

$M_B = 0 - 32w = -32w$

$M_E = -32w + 50w = 18w$



Centroid and moment of inertia



Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)	d (in)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
1	1.6875	1.875	3.1641	0.75	0.9492	0.7119
2	1.6875	0.375	0.6328	0.75	0.9492	0.0791
Σ	3.375		3.7969		1.8984	0.7910

$$\bar{Y} = \frac{3.7969}{3.375} = 1.125 \text{ in.}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 2.6894 \text{ in}^4$$

Top: $y = 1.875$ in $\qquad I/y = 1.4343$ in³

Bottom $y = -1.125 \qquad -I/y = -2.3906$ in³

Bending moment limits $\qquad M = -\sigma I/y$

Tension at B and C $\quad -(12)(1.4343) = -17.212$ kip-in ←

Comp. at B and C $\quad -(-19.5)(-2.3906) = -46.6$ kip-in

Tension at E $\quad -(12)(-2.3906) = 28.687$ kip-in

Compression at E $\quad -(-19.5)(1.4343) = 27.969$ kip-in ←

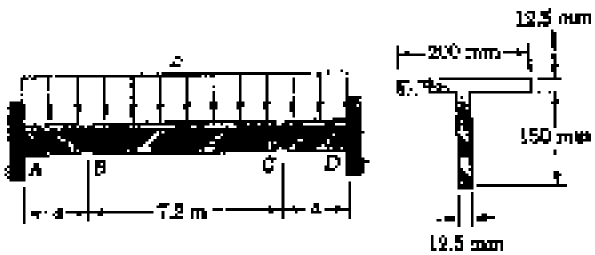
Allowable load w: $\quad$ B & C $\quad -32w = -17.212 \qquad w = 0.538$ kip/in

E $\quad 18w = 27.969 \qquad w = 1.554$ kip/in

Smallest $\qquad w = 0.538$ kip/in $= 6.45$ kip/ft ←

PROBLEM 5.99

5.99 Beams AB, BC, and CD have the cross section shown and are pin-connected at B and C. Knowing that the allowable normal stress is +110 MPa in tension and −150 MPa in compression, determine (*a*) the largest permissible value of w if beam BC is not to be overstressed, (*b*) the corresponding maximum distance a for which the cantilever beams AB and CD are not overstressed.



SOLUTION

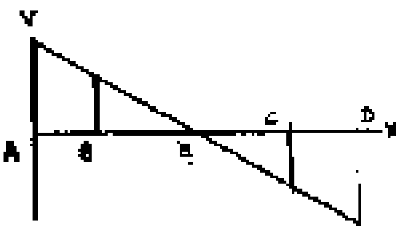
(a) $M_B = M_C = 0$

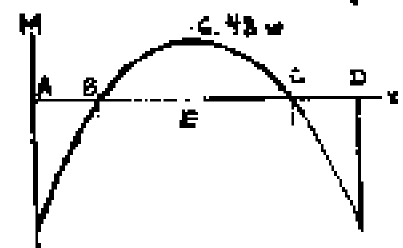
$V_B = -V_C = (\frac{1}{2})(7.2)w = 3.6\,w$

Area B to E of shear diagram

$(\frac{1}{2})(3.6)(3.6\,w) = 6.48\,w$

$M_E = 0 + 6.48\,w = 6.48\,w$



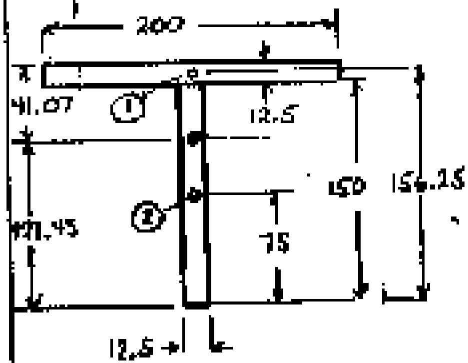


Centroid and moment of inertia

Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (mm³)	d (mm)	Ad^2 (mm⁴)	$\bar{I}$ (mm⁴)
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6

$\bar{Y} = \frac{531250}{4375} = 121.43$ mm

$I = \Sigma Ad^2 + \Sigma\bar{I} = 10.621\times10^6\ \text{mm}^4$



Location	y (mm)	I/y (10^3 mm³)
top	41.07	258.6
bottom	−121.43	−87.47

← also (10^{-6} m³)

Bending moment limits $\quad M = -\sigma I/y$

Tension at E: $-(110\times10^6)(-87.47\times10^{-6}) = 9.622\times10^3$ N·m

Comp. at E: $-(-150\times10^6)(258.6\times10^{-6}) = 38.8\times10^3$ N·m

Tension at A+D: $-(110\times10^6)(258.6\times10^{-6}) = -28.45\times10^3$ N·m

Comp. at A+D: $-(-150\times10^6)(-87.47\times10^{-6}) = -13.121\times10^3$ N·m

Allowable load w $\qquad 6.48\,w = 9.622\times10^3 \qquad w = 1.485\times10^3$ N/m

$= 1.485$ kN/m ◄

Shear at A $\qquad V_A = (a + 3.6)\,w$

Area A to B of shear diagram $\qquad \frac{1}{2}a(V_A + V_B) = \frac{1}{2}a(a + 7.2)\,w$

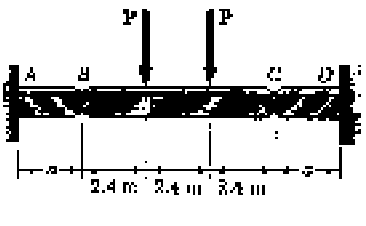
Bending moment at A (also D) $\qquad M_A = -\frac{1}{2}a(a + 7.2)\,w$

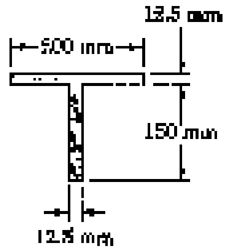
$-\frac{1}{2}a(a + 7.2)(1.485\times10^3) = -13.121\times10^3$

$\frac{1}{2}a^2 + 3.6\,a - 8.837 = 0 \qquad a = 1.935$ m ◄

PROBLEM 5.100

5.100 Beams AB, BC, and CD have the cross section shown and are pin-connected at B and C. Knowing that the allowable normal stress is +110 MPa in tension and −150 MPa in compression, determine (*a*) the largest permissible value of **P** if beam BC is not to be overstressed, (*b*) the corresponding maximum distance a for which the cantilever beams AB and CD are not overstressed.





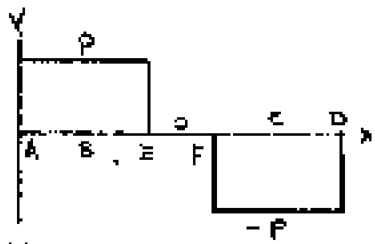
SOLUTION

(a) $M_B = M_C = 0$

$V_B = -V_C = P$

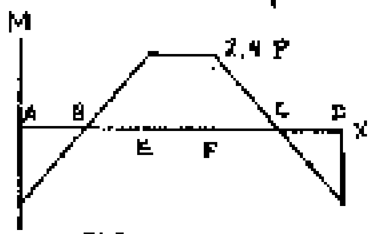
Area B to E of shear diagram, $2.4\ P$

$M_E = 0 + 2.4\ P = 2.4\ P = M_F$



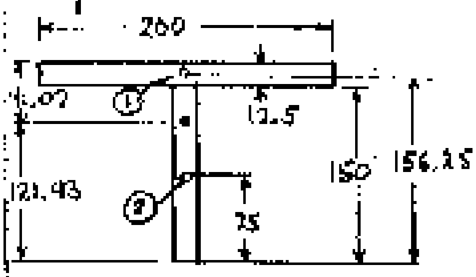
Centroid and moment of inertia

Part	$A\ (mm^2)$	$\bar{y}\ (mm)$	$A\bar{y}\ (mm^3)$	$d\ (mm)$	$Ad^2\ (mm^4)$	$\bar{I}\ (mm^4)$
①	2500	156.25	390625	34.82	3.031×10^6	0.0326×10^6
②	1875	75	140625	46.43	4.042×10^6	3.516×10^6
Σ	4375		531250		7.073×10^6	3.548×10^6



$$\bar{Y} = \frac{531250}{4375} = 121.43\ mm$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 10.621\times10^6\ mm^4$$



Location	y (mm)	I/y $(10^3\ mm^3)$ ← also $(10^{-6}\ m^3)$
top	41.07	258.6
bottom	−121.43	−87.47

Bending moment limits $M = -\sigma I/y$

Tension at E & F: $-(110\times10^6)(-87.47\times10^{-6}) = 9.622\times10^3$ N·m

Comp. at E & F: $-(-150\times10^6)(258.6\times10^{-6}) = 38.8\times10^3$ N·m

Tension at A & D: $-(110\times10^6)(258.6\times10^{-6}) = -28.45\times10^3$ N·m

Comp. at A & D: $-(-150\times10^6)(-87.47\times10^{-6}) = -13.121\times10^3$ N·m

Allowable load P $\quad 2.4\ P = 9.622\times10^3 \quad P = 4.01\times10^3\ N$

$= 4.01\ kN$ ◀

Shear at A $\quad V_A = P$

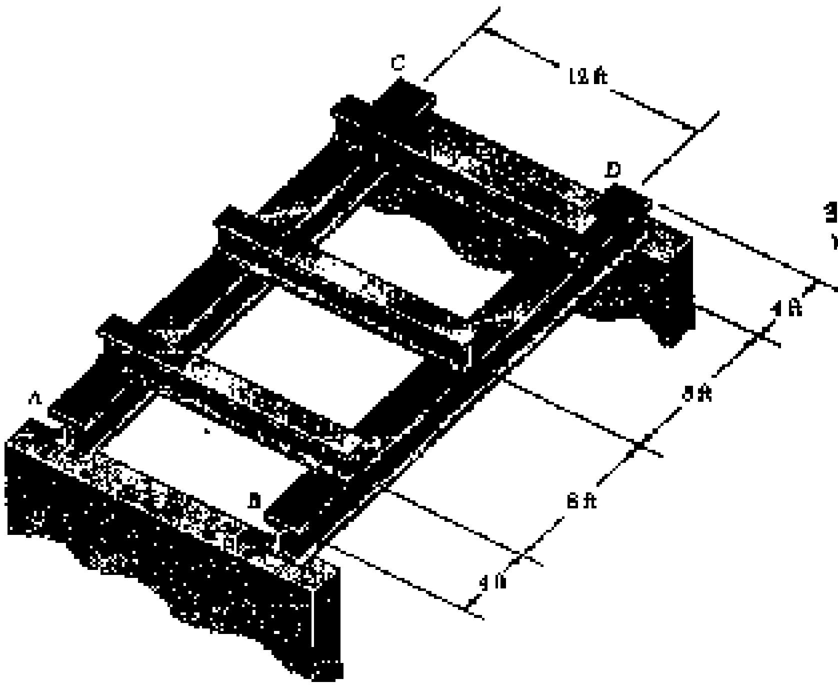
Area A to B of shear diagram $\quad aV_A = aP$

Bending moment at A $\quad M_A = -aP = -4.01\times10^3\ a$

Distance a $\quad -4.01\times10^3\ a = -13.121\times10^3 \quad a = 3.27\ m$ ◀

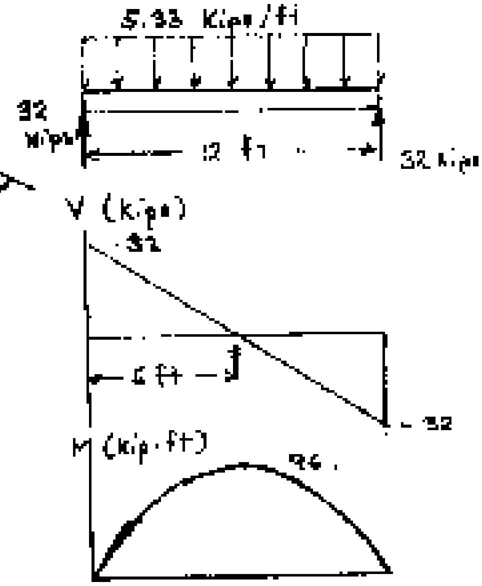
PROBLEM 5.101

5.101 Each of the three rolled-steel beams shown (numbered 1, 2, and 3) is to carry a 64-kip load uniformly distributed over the beam. Each of these beams has a 12-ft span and is to be supported by the two 24-ft rolled-steel girders AC and BD. Knowing that the allowable normal stress for the steel used is 24 ksi, select (a) the most economical S-shape for the three beams, (b) the most economical W-shape for the two girders.



SOLUTION

Beams 1, 2, and 3



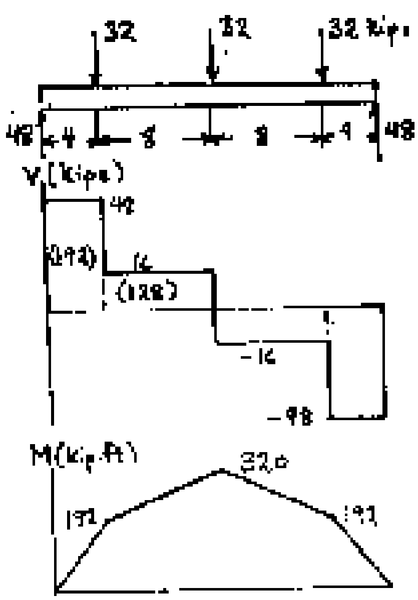
Maximum $M = (\frac{1}{2})(6)(32) = 96$ kip·ft $= 1152$ kip·in

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{1152}{24} = 48$ in³

Shape	S (in³)
S 15 × 42.9	59.6
S 12 × 50	50.8

(a) Use S 15 × 42.9

Beams AC and BD



Areas under shear diagram

$(4)(48) = 192$ kip·ft

$(8)(16) = 128$ kip·ft

Maximum $M = 192 + 128 = 320$ kip·ft $= 3840$ kip·in

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3840}{24} = 160$ in³

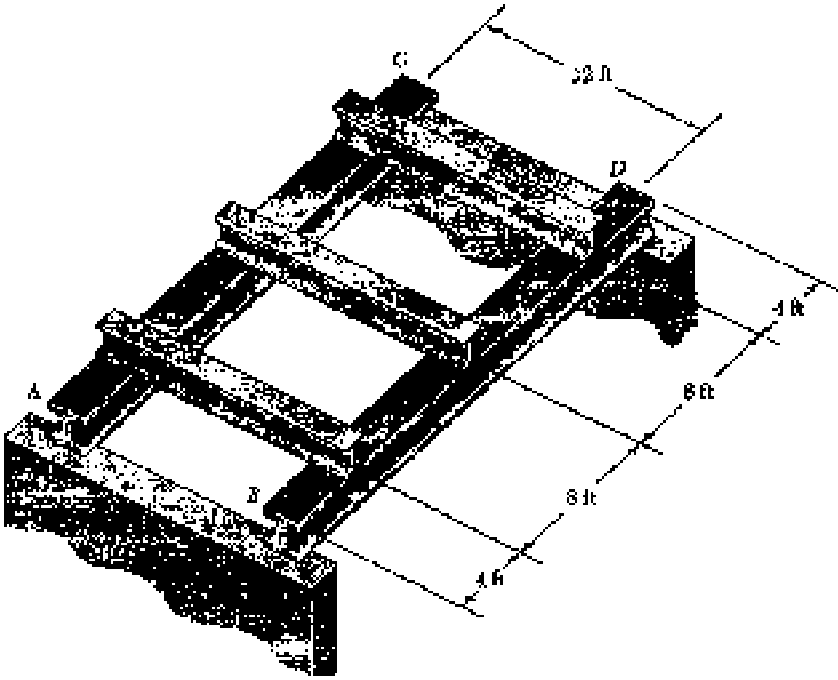
Shape	S (in³)
W 30 × 99	269
W 27 × 84	213
W 24 × 104	258
W 21 × 101	227
W 18 × 106	204

(b) Use W 27 × 84

PROBLEM 5.102

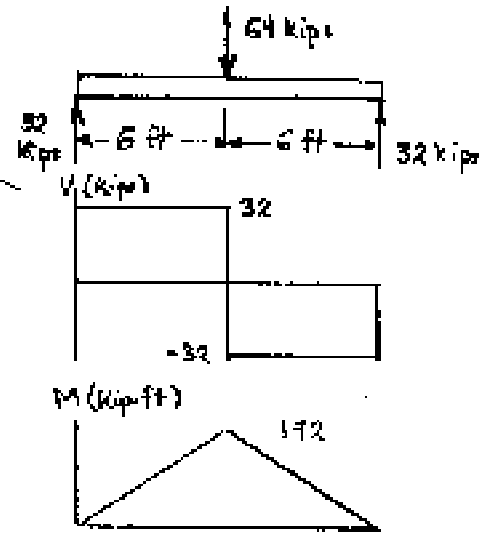
5.101 Each of the three rolled-steel beams shown (numbered 1, 2, and 3) is to carry a 64-kip load uniformly distributed over the beam. Each of these beams has a 12-ft span and is to be supported by the two 24-ft rolled-steel girders *AC* and *BD*. Knowing that the allowable normal stress for the steel used is 24 ksi, select (*a*) the most economical S-shape for the three beams, (*b*) the most economical W-shape for the two girders.

5.102 Solve Prob. 5.101, assuming that the 64-kip distributed loads are replaced by 64-kip concentrated loads applied at the midpoints of the three beams.



SOLUTION

Beams 1, 2, and 3



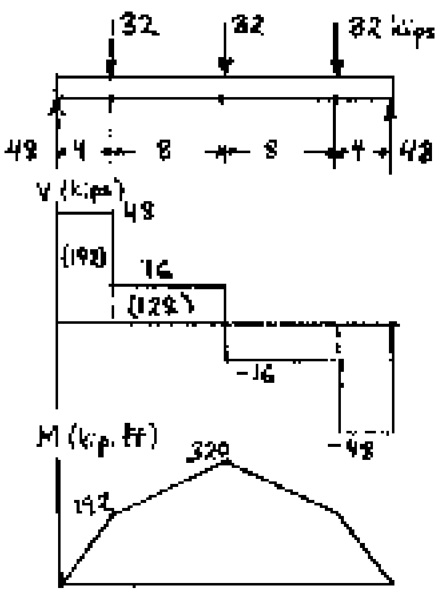
Maximum $M = (6)(32) = 192$ kip·ft $= 2304$ kip·in

$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{2304}{24} = 96$ in^3

Shape	S (in^3)
S20×66	119
S18×70	103

(a) Use S20×66 ◄

Beams AC and BD



Areas under shear diagram

$(4)(48) = 192$ kip·ft

$(8)(16) = 128$ kip·ft

Maximum $M = 192 + 128 = 320$ kip·ft $= 3840$ kip·in

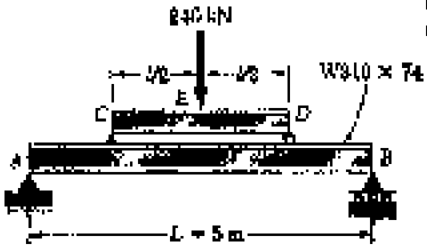
$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{3840}{24} = 160$ in^3

Shape	S (in^3)
W 30×99	269
W 27×84	213
W 24×104	258
W 21×101	227
W 18×106	204

(b) Use W 27×84 ◄

PROBLEM 5.103

5.103 A 240-kN load is to be supported at the center of the 5-m span shown. Knowing that the allowable normal stress for the steel used is 165 MPa, determine (*a*) the smallest allowable length l of beam *CD* if the W 310 × 74 beam *AB* is not to be overstressed, (*b*) the W shape which should be used for beam *CD*. Neglect the weight of both beams.



SOLUTION

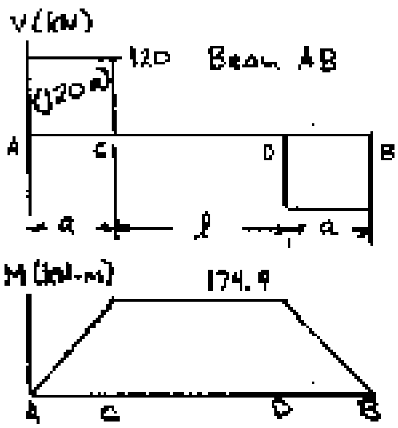
For rolled steel section W 310 × 74 of beam AB

$S = 1060 \times 10^3\ mm^3 = 1060 \times 10^{-6}\ m^3$

$\sigma_{all} = 165\ MPa = 165 \times 10^6\ Pa$

Allowable bending moment

$$M_{all} = S\sigma_{all} = (1060 \times 10^{-6})(165 \times 10^6) = 174.9 \times 10^3\ N\cdot m$$
$$= 174.9\ kN\cdot m$$



(a) Beam AB

Area A to C of shear diagram = 120 a

Bending moment at C 120 a

$120\,a = 174.9 \quad 1.4575\ m$

Geometry: $2a + l = 5 \qquad l = 5 - 2a = 2.085\ m$ ⟵

(b) Beam CD (midpoint E)

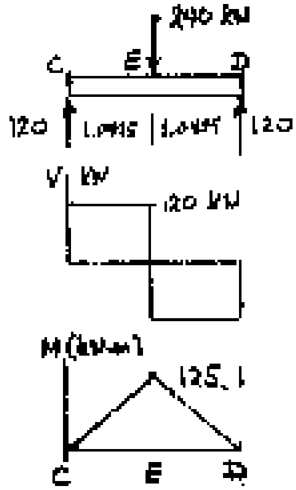
Area C to E of shear diagram

$= (1.0425)(120) = 125.1\ kN\cdot m$

Bending moment at E

$M = 125.1\ kN\cdot m = 125.1 \times 10^3\ N\cdot m$

$$S_{min} = \frac{M}{\sigma_{all}} = \frac{125.1 \times 10^3}{165 \times 10^6} = 758.2 \times 10^{-6}\ m^3$$
$$= 758.2 \times 10^3\ mm^3$$

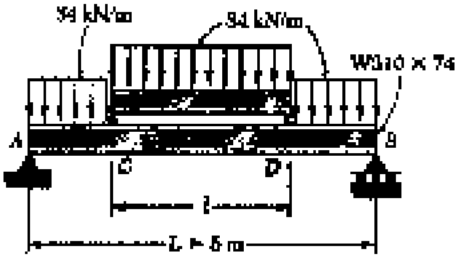


Shape	S ($10^3\ mm^3$)
W 460 × 52	942
W 410 × 46.1	774 ⟵
W 360 × 57.8	899
W 310 × 60	851
W 250 × 67	809
W 200 × 86	853

Answer W 410 × 46.1 ⟵

PROBLEM 5.104

5.104 A uniformly distributed load of 84 kN/m is to be supported over the 5-m span shown. Knowing that the allowable normal stress for the steel used is 165 MPa, determine (*a*) the smallest allowable length *l* of beam *CD* if the W 310 × 74 beam *AB* is not to be overstressed, (*b*) the W shape which should be used for beam *CD*. Neglect the weight of both beams.



SOLUTION

For rolled steel section W310×74 of beam AB

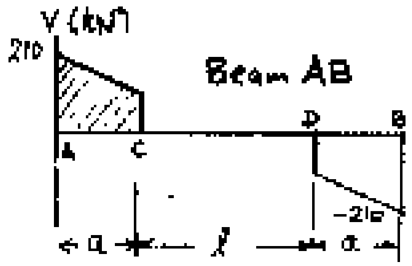
$S = 1060 \times 10^3\ mm^3 = 1060 \times 10^{-6}\ m^3$

$\sigma_{all} = 165\ MPa = 165 \times 10^6\ Pa$

Allowable bending moment

$M_{all} = S\sigma_{all} = (1060 \times 10^{-6})(165 \times 10^6) = 174.9 \times 10^3\ N\cdot m$

$= 174.9\ kN\cdot m$



By symmetry reactions A and B are equal.

$+\uparrow \Sigma F_y = 0 \quad A + B - (5)(84) = 0$

$A = B = 210\ kN$

By symmetry, reaction at C and D are equal.

$+\uparrow \Sigma F_y = 0 \quad C + D - 84\ell = 0$

$C = D = 42\ell$

Geometry $\quad a = \frac{1}{2}(5 - \ell)$

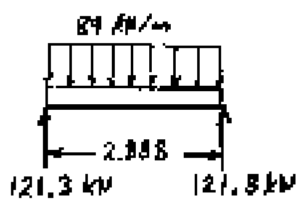
Beam AB: Area A to C of shear diagram

$\frac{1}{2}(a)(A + C) = \frac{1}{2}\cdot\frac{1}{2}(5 - \ell)(210 + 42\ell)$

$= \frac{1}{4}(1050 - 42\ell^2)$

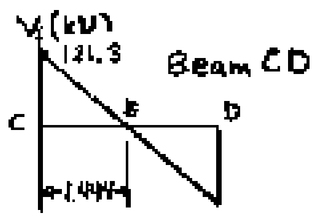
Bending moment at C $\quad \frac{1}{4}(1050 - 42\ell^2)$

$\frac{1}{4}(1050 - 42\ell^2) = 174.9 \qquad \ell^2 = 8.3429\ m^2$



(a) $\quad \ell = 2.888\ m$ ◄

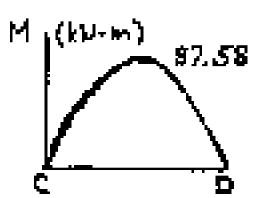
$C = D = 42\ell = 121.3\ kN$



Beam CD (midpoint E)

Area C to E of shear diagram

$\frac{1}{2}(1.444)(121.3) = 87.58\ kN\cdot m$



Bending moment at E $\qquad M = 87.58\ kN\cdot m = 87.58 \times 10^3\ N\cdot m$

$\sigma_{all} = 165 \times 10^6\ Pa$

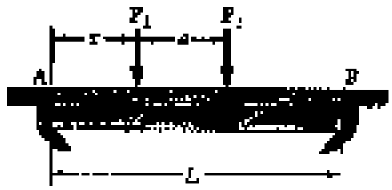
$S_{min} = \frac{M}{\sigma_{all}} = \frac{87.58 \times 10^3}{165 \times 10^6} = 531 \times 10^{-6}\ m^3 = 531 \times 10^3\ mm^3$

Shape	$S\ (10^3\ mm^3)$
W 460 × 52	942
W 410 × 38.8	637
W 360 × 39	578
W 310 × 38.7	549
W 250 × 44.8	535
W 200 × 59	582

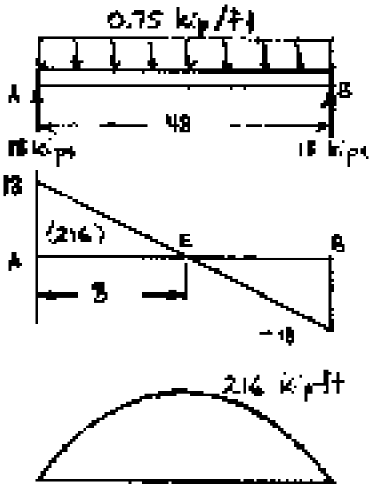
(b) Use W 310 × 38.7 ◄

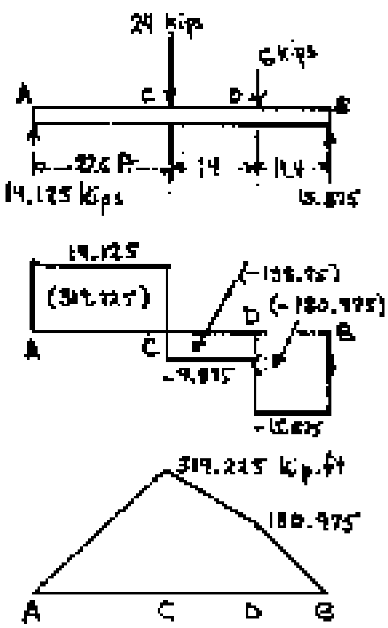
PROBLEM 5.105

*5.105 A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength $\sigma_U = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kip/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads P_1 and P_2 exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using *LRFD* with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint:* It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/2(P_1 + P_2)$.]



$L = 48$ ft
$a = 14$ ft
$P_1 = 24$ kips
$P_2 = 6$ kips
$w = 0.75$ kip/ft





SOLUTION

Dead load: $R_A = R_B = (\frac{1}{2})(48)(0.75) = 18$ kips

Area A to E of shear diagram $(\frac{1}{2})(8)(18) = 216$ kip·ft

$M_{max} = 216$ kip·ft $= 2592$ kip·in. at point E

Live load: $u = \dfrac{aP_2}{2(P_1+P_2)} = \dfrac{(14)(6)}{(2)(30)} = 1.4$ ft

$x = \frac{L}{2} - u = 24 - 1.4 = 22.6$ ft
$x + a = 22.6 + 14 = 36.6$ ft
$L - x - a = 48 - 36.6 = 11.4$ ft

$\Sigma M_B = 0 \quad -48 R_A + (25.4)(24) + (11.4)(6) = 0$
$R_A = 14.125$ kips

Shear: A to C $\quad V = 14.125$ kips
C to D $\quad V = 14.125 - 24 = -9.875$ kips
D to B $\quad V = -15.875$ kips

Area A to C $\quad (22.6)(14.125) = 319.225$ kip·ft

Bending moment: $M_L = 319.225$ kip·ft $= 3831$ kip·in.

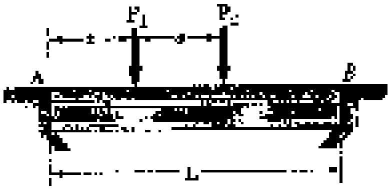
Design: $\gamma_D M_D + \gamma_L M_L = \phi M_U = \phi \sigma_U S_{min}$

$$S_{min} = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_U} = \frac{(1.25)(2592) + (1.75)(3831)}{(0.9)(60)}$$
$$= 184.2 \text{ in}^3$$

Shape	S (in³)
W 30 × 99	269
W 27 × 84	213 ←
W 24 × 104	258
W 21 × 101	227
W 18 × 106	204

W27 × 84 ◄

PROBLEM 5.106



$L = 48$ ft
$a = 14$ ft
$P_1 = 24$ kips
$P_2 = 6$ kips
$w = 0.75$ kip/ft

*5.105 A bridge of length $L = 48$ ft is to be built on a secondary road whose access to trucks is limited to two-axle vehicles of medium weight. It will consist of a concrete slab and of simply supported steel beams with an ultimate strength $\sigma_U = 60$ ksi. The combined weight of the slab and beams can be approximated by a uniformly distributed load $w = 0.75$ kip/ft on each beam. For the purpose of the design, it is assumed that a truck with axles located at a distance $a = 14$ ft from each other will be driven across the bridge and that the resulting concentrated loads $\mathbf{P}_1$ and $\mathbf{P}_2$ exerted on each beam could be as large as 24 kips and 6 kips, respectively. Determine the most economical wide-flange shape for the beams, using *LRFD* with the load factors $\gamma_D = 1.25$, $\gamma_L = 1.75$ and the resistance factor $\phi = 0.9$. [*Hint*: It can be shown that the maximum value of $|M_L|$ occurs under the larger load when that load is located to the left of the center of the beam at a distance equal to $aP_2/2(P_1 + P_2)$.]

*5.106 Assuming that the front and rear axle loads remain in the same ratio as for the truck of Prob. 5.105, determine how much heavier a truck could safely cross the bridge designed in that problem.

SOLUTION

See solution to PROBLEM 5.105 for calculation of the following:

$M_D = 2592$ kip·in $\qquad M_L = 3831$ kip·in.

For rolled steel section $\quad$ W 27 × 84 $\quad S = 213$ in³

Allowable live load moment M_L^*

$$\gamma_D M_D + \gamma_L M_L^* = \phi M_U = \phi \sigma_U S$$

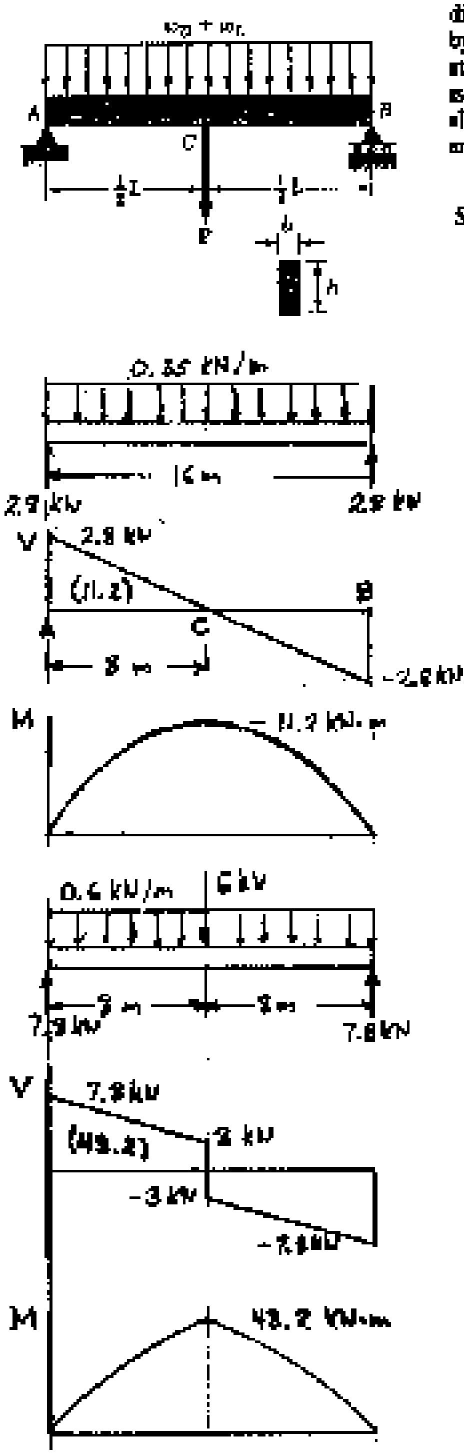
$$M_L^* = \frac{\phi \sigma_U S - \gamma_D M_D}{\gamma_L} = \frac{(0.9)(60)(213) - (1.25)(2592)}{1.75} = 4721 \text{ kip·in}$$

Ratio $\quad \dfrac{M_L^*}{M_L} = \dfrac{4721}{3831} = 1.232 = 1 + 0.232$

Increase $\qquad$ 23.2 % ◄

PROBLEM 5.107

*5.107 A roof structure consisting of plywood and roofing material is supported by several timber beams of length $L = 16$ m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load $w_D = 350$ N/m. The live loads consist of the snow load, represented by a uniformly distributed load $w_L = 600$ N/m, and a 6-kN concentrated load **P** applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is $\sigma_U = 50$ MPa and that the width of the beams is $b = 75$ mm, determine the minimum allowable depth h of the beams, using LRFD with the load factors $\gamma_D = 1.2$, $\gamma_L = 1.6$ and the resistance factor $\phi = 0.9$.



SOLUTION

$L = 16$ m, $w_D = 350$ N/m $= 0.35$ kN/m

$w_L = 600$ N/m $= 0.6$ kN/m, $P = 6$ kN

Dead load: $R_A = (\frac{1}{2})(16)(0.35) = 2.8$ kN

Area A to C of shear diagram

$(\frac{1}{2})(8)(2.8) = 11.2$ kN·m

Bending moment at C: 11.2 kN·m $= 11.2 \times 10^3$ N·m

Live load: $R_A = \frac{1}{2}[(16)(0.6) + 6] = 7.8$ kN

Shear at C^- $V = 7.8 - (8)(0.6) = 3$ kN

Area A to C of shear diagram

$(\frac{1}{2})(8)(7.8 + 3) = 43.2$ kN·m

Bending moment at C: 43.2 kN·m $= 43.2 \times 10^3$ N·m

Design $\gamma_D M_D + \gamma_L M_L = \phi M_U = \phi \sigma_U S$

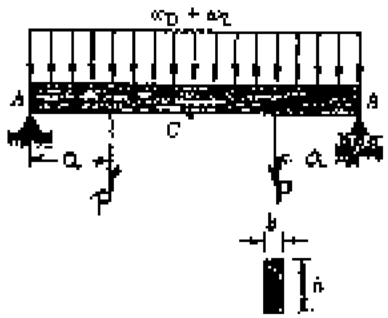
$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_U} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(43.2 \times 10^3)}{(0.9)(50 \times 10^6)}$$

$$= 1.8347 \times 10^{-3}\ \text{m}^3 = 1.8347 \times 10^6\ \text{mm}^3$$

For a rectangular section $S = \frac{1}{6} b h^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.8347 \times 10^6)}{75}} = 383\ \text{mm}$$

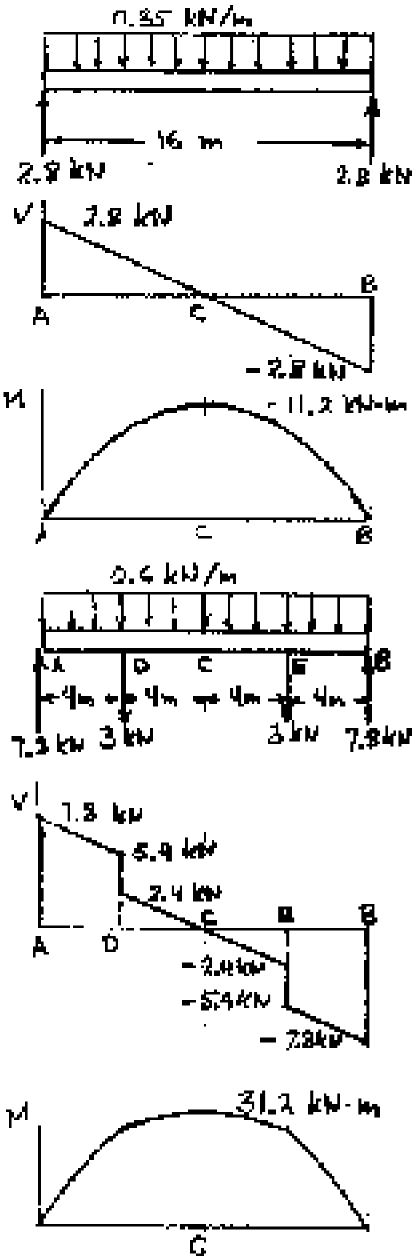
PROBLEM 5.108



*5.107 A roof structure consisting of plywood and roofing material is supported by several timber beams of length L = 16 m. The dead load carried by each beam, including the estimated weight of the beam, can be represented by a uniformly distributed load w_D = 350 N/m. The live loads consist of the snow load, represented by a uniformly distributed load w_L = 600 N/m, and a 6-kN concentrated load **P** applied at the midpoint C of each beam. Knowing that the ultimate strength for the timber used is σ_U = 50 MPa and that the width of the beams is b = 75 mm, determine the minimum allowable depth h of the beams, using *LRFD* with the load factors γ_D = 1.2, γ_L = 1.6 and the resistance factor ϕ = 0.9.

*5.108 Solve Prob. 5.107, assuming that the 6-kN concentrated loads are replaced by 3-kN concentrated loads P_1 and P_2 applied at a distance of 4 m from each end of the beams.

SOLUTION



$L = 16$ m, $a = 4$ m, $w_D = 350$ N/m $= 0.35$ kN/m

$w_L = 600$ N/m $= 0.6$ kN/m $P = 3$ kN

Dead load: $R_A = (\frac{1}{2})(16)(0.35) = 2.8$ kN

Area A to C of shear diagram

$(\frac{1}{2})(8)(2.8) = 11.2$ kN·m

Bending moment at C: 11.2 kN·m $= 11.2 \times 10^3$ N·m

Live load: $R_A = \frac{1}{2}[(16)(0.6) + 3 + 3] = 7.8$ kN

Shear at D^- $\quad 7.8 - (4)(0.6) = 5.4$ kN

Shear at D^+ $\quad 5.4 - 3 = 2.4$ kN

Area A to D $\quad (\frac{1}{2})(4)(7.8 + 5.4) = 26.4$ kN·m

Area D to C $\quad (\frac{1}{2})(4)(2.4) = 4.8$ kN·m

Bending moment at C $= 26.4 + 4.8 = 31.2$ kN·m $= 31.2 \times 10^3$ N·m

Design $\quad \gamma_D M_D + \gamma_L M_L = \phi M_U = \phi \sigma_U S$

$$S = \frac{\gamma_D M_D + \gamma_L M_L}{\phi \sigma_U} = \frac{(1.2)(11.2 \times 10^3) + (1.6)(31.2 \times 10^3)}{(0.9)(50 \times 10^6)}$$

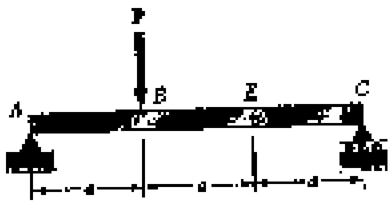
$$= 1.408 \times 10^{-3}\ \text{m}^3 = 1.408 \times 10^6\ \text{mm}^3$$

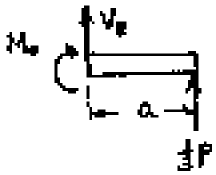
For a rectangular section $\quad S = \frac{1}{6} b h^2$

$$h = \sqrt{\frac{6S}{b}} = \sqrt{\frac{(6)(1.408 \times 10^6)}{75}} = 336\ \text{mm}$$

PROBLEM 5.109

5.109 through 5.111 *(a)* Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. *(b)* Use the equation obtained for M to determine the bending moment at point E and check your answer by drawing the free-body diagram of the portion of beam to the right of E.





SOLUTION

$\circlearrowleft \Sigma M_C = 0 \qquad -3aA + 2aP = 0 \qquad A = \frac{2}{3}P$

$V = \frac{2}{3}P - P\langle x-a\rangle^0$

$M = \frac{2}{3}Px - P\langle x-a\rangle^1$

At point E $\quad x = 2a$

$M_E = \frac{2}{3}P(2a) - Pa = \frac{1}{3}Pa$

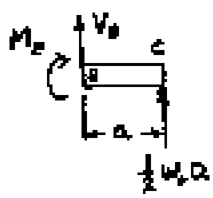
$\Sigma M_A = 0 \qquad 3aC - aP = 0 \qquad C = \frac{1}{3}P$

$\circlearrowleft \Sigma M_E = 0 \qquad -M_E + (a)(\frac{1}{3}P) = 0$

$M_E = \frac{1}{3}Pa$

PROBLEM 5.110

5.109 through 5.111 *(a)* Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. *(b)* Use the equation obtained for M to determine the bending moment at point E and check your answer by drawing the free-body diagram of the portion of beam to the right of E.



SOLUTION

$\circlearrowleft \Sigma M_C = 0 \qquad -4aA + (3a)(2a w_0) = 0 \qquad A = \frac{3}{2}w_0 a$

$w = w_0 - w_0\langle x-2a\rangle^0 = -\frac{dV}{dx}$

$V = -w_0 x + w_0\langle x-2a\rangle^1 + \frac{3}{2}w_0 a = \frac{dM}{dx}$

$M = -\frac{1}{2}w_0 x^2 + \frac{1}{2}w_0\langle x-2a\rangle^2 + \frac{3}{2}w_0 a x + 0$

At point E $\quad x = 3a$

$M_E = -\frac{1}{2}w_0(3a)^2 + \frac{1}{2}w_0 a^2 + \frac{3}{2}w_0 a(3a)$
$= \frac{1}{2}w_0 a^2$

$\circlearrowleft \Sigma M_A = 0 \qquad 4aC - (a)(2a w_0) = 0 \qquad C = \frac{1}{2}w_0 a$

$\circlearrowleft \Sigma M_E = 0 \qquad -M_E + (a)(\frac{1}{2}w_0 a) = 0$

$M_E = \frac{1}{2}w_0 a^2$

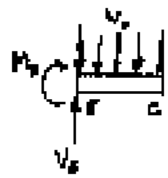
PROBLEM 5.111

5.109 through 5.111 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *E* and check your answer by drawing the free-body diagram of the portion of beam to the right of *E*.

SOLUTION

$$\circlearrowleft \Sigma M_C = 0 \qquad -2aA - \left(\frac{a}{2}\right)(3a\,w_0) = 0 \qquad A = -\frac{3}{4}w_0 a$$

$$\circlearrowleft \Sigma M_A = 0 \qquad 2aC - \left(\frac{5a}{2}\right)(3a\,w_0) = 0 \qquad C = \frac{15}{4}w_0 a$$



$$w = w_0\langle x-a\rangle^0 = -\frac{dV}{dx}$$

$$V = -w_0\langle x-a\rangle^1 - \frac{3}{4}w_0 a + \frac{15}{4}w_0 a\langle x-2a\rangle^0 = \frac{dM}{dx}$$

$$M = -\frac{1}{2}w_0\langle x-a\rangle^2 - \frac{3}{4}w_0 a x + \frac{15}{4}w_0 a\langle x-2a\rangle^1 + 0$$

At point E $\quad x = 3a$

$$M_E = -\frac{1}{2}w_0(2a)^2 - \frac{3}{4}w_0 a(3a) + \frac{15}{4}w_0 a(a)$$

$$= -\frac{1}{2}w_0 a^2$$

Check: $+\Sigma M_E = 0 \qquad -M_E - \frac{a}{2}(w_0 a) = 0$

$$M_E = -\frac{1}{2}w_0 a^2$$

PROBLEM 5.112

5.112 through 5.114 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *C* and check your answer by drawing the free-body diagram of the entire beam.

SOLUTION

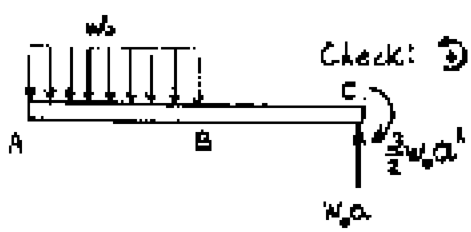
$$w = w_0 - w_0\langle x-a\rangle^0 = -\frac{dV}{dx}$$

$$V = -w_0 x + w_0\langle x-a\rangle^1 = \frac{dM}{dx}$$

$$M = -\frac{1}{2}w_0 x^2 + \frac{1}{2}w_0\langle x-a\rangle^2$$

At point C $\quad x = 2a$

$$M_C = -\frac{1}{2}w_0(2a)^2 + \frac{1}{2}w_0 a^2 = -\frac{3}{2}w_0 a^2$$

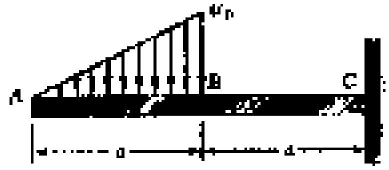


Check: $\circlearrowleft \Sigma M_C = 0 \qquad \left(\frac{3a}{2}\right)(w_0 a) + M_C = 0$

$$M_C = -\frac{3}{2}w_0 a^2$$

PROBLEM 5.113

5.112 through 5.114 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *C* and check your answer by drawing the free-body diagram of the entire beam.



SOLUTION

$$w = \frac{w_0 x}{a} - w_0\langle x-a\rangle^0 - \frac{w_0}{a}\langle x-a\rangle^1 = -\frac{dV}{dx}$$

$$V = -\frac{w_0 x^2}{2a} + w_0\langle x-a\rangle^1 + \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$

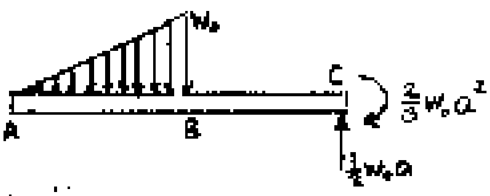
$$M = -\frac{w_0 x^3}{6a} + \frac{w_0}{2}\langle x-a\rangle^2 + \frac{w_0}{6a}\langle x-a\rangle^3$$

At point C $\quad x = 2a$

$$M_C = -\frac{w_0(2a)^3}{6a} + \frac{w_0 a^2}{2} + \frac{w_0 a^3}{6a} = -\frac{2}{3}w_0 a^2$$

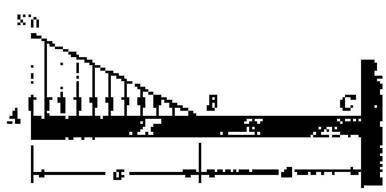
Check: $+\circlearrowleft \Sigma M_C = 0 \qquad \left(\frac{4a}{3}\right)\left(\frac{1}{2}w_0 a\right) + M_C = 0$

$$M_C = -\frac{2}{3}w_0 a^2$$



PROBLEM 5.114

5.112 through 5.114 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment at point *C* and check your answer by drawing the free-body diagram of the entire beam.



SOLUTION

$$w = w_0 - \frac{w_0 x}{a} + \frac{w_0}{a}\langle x-a\rangle^1 = -\frac{dV}{dx}$$

$$V = -w_0 x + \frac{w_0 x^2}{2a} - \frac{w_0}{2a}\langle x-a\rangle^2 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6a} - \frac{w_0}{6a}\langle x-a\rangle^3$$

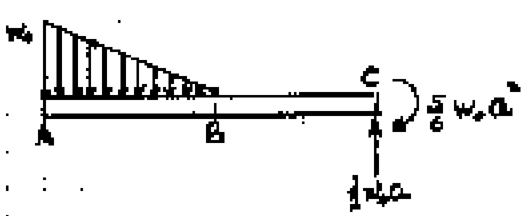
At point C $\quad x = 2a$

$$M_C = -\frac{w_0(2a)^2}{2} + \frac{w_0(2a)^3}{6a} - \frac{w_0 a^3}{6a} = -\frac{5}{6}w_0 a^2$$

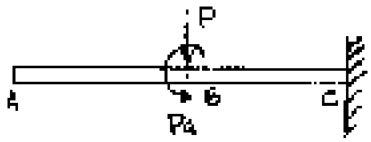
Check:

$+\circlearrowleft \Sigma M_C = 0 \qquad \left(\frac{5}{3}a\right)\left(\frac{1}{2}w_0 a\right) + M_C = 0$

$$M_C = -\frac{5}{6}w_0 a^2$$



PROBLEM 5.115



5.115 and 5.116 (*a*) Using singularity functions, write the equations defining the shear and bending moment for beam *ABC* under the loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment just to the right of point *B*.

SOLUTION

$$V = -P\langle x-a\rangle^0$$

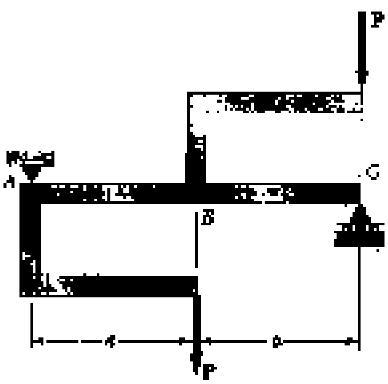
$$\frac{dM}{dx} = -P\langle x-a\rangle^0$$

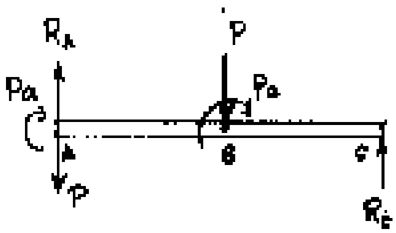
$$M = -P\langle x-a\rangle^1 - Pa\langle x-a\rangle^0$$

Just to the right of B $\quad x = a^+$

$$M = -0 - Pa = -Pa$$

PROBLEM 5.116





5.115 and 5.116 (*a*) Using singularity functions, write the equations defining the shear and bending moment for beam *ABC* under the loading shown. (*b*) Use the equation obtained for *M* to determine the bending moment just to the right of point *B*.

SOLUTION

$$\Sigma M_C = 0 \qquad (2a)P + aP - 2(Pa) - 2aR_A = 0$$

$$R_A = \tfrac{1}{2}P$$

$$V = (R_A - P) - P\langle x-a\rangle^0$$

$$= -\tfrac{1}{2}P - P\langle x-a\rangle^0$$

$$\frac{dM}{dx} = -\tfrac{1}{2}P - P\langle x-a\rangle^0$$

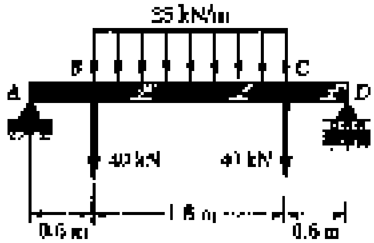
$$M = -\tfrac{1}{2}Px - P\langle x-a\rangle^1 + Pa + Pa\langle x-a\rangle^0$$

Just to the right of point B $\quad x = a^+$

$$M = -\tfrac{1}{2}Pa - 0 + Pa + Pa = \tfrac{3}{2}Pa$$

PROBLEM 5.117

5.117 through 5.120 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.



SOLUTION

By symmetry $R_A = R_D$

$+\uparrow \Sigma F_y = 0 \qquad R_A + R_D - 40 - (1.8)(25) - 40 = 0$

$R_A = R_D = 62.5 \text{ kN}$

$w = 25\langle x-0.6\rangle^0 - 25\langle x-2.4\rangle^0 = -\frac{dV}{dx}$

$V = 62.5 - 25\langle x-0.6\rangle^1 + 25\langle x-2.4\rangle^1 - 40\langle x-0.6\rangle^0 - 40\langle x-2.4\rangle^0 \text{ kN}$

$M = 62.5x - 12.5\langle x-0.6\rangle^2 + 12.5\langle x-2.4\rangle^2 - 40\langle x-0.6\rangle^1 - 40\langle x-2.4\rangle^1 \text{ kN}\cdot\text{m}$

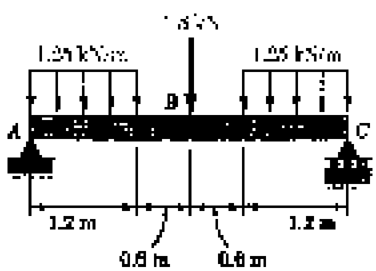
Locate point where $V = 0$. Assume $0.6 < x^* < 1.8$

$0 = 62.5 - 25(x^* - 0.6) + 0 - 40 - 0 \qquad x^* = 1.5 \text{ m}$

$M = (62.5)(1.5) - (12.5)(0.9)^2 + 0 - (40)(0.9) - 0 = 47.625 \text{ kN}\cdot\text{m}$

PROBLEM 5.118

5.117 through 5.120 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.



SOLUTION

By symmetry $R_A = R_C$

$+\uparrow \Sigma F_y = 0 \qquad R_A + R_C - (1.2)(1.25) - 1.8 - (1.2)(1.25) = 0$

$R_A = R_C = 2.4 \text{ kN}$

$w = 1.25 - 1.25\langle x-1.2\rangle^0 + 1.25\langle x-2.4\rangle^0 = -\frac{dV}{dx}$

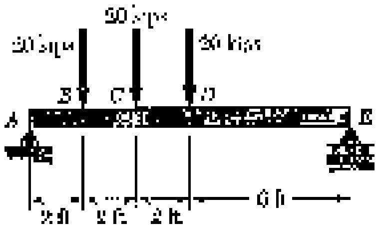
$V = -1.25x + 1.25\langle x-1.2\rangle^1 - 1.25\langle x-2.4\rangle^1 + 2.4 - 1.8\langle x-1.8\rangle^0 \text{ kN}$

$M = -0.625x^2 + 0.625\langle x-1.2\rangle^2 - 0.625\langle x-2.4\rangle^2 + 2.4x - 1.8\langle x-1.8\rangle^1 \text{ kN}\cdot\text{m}$

M_{max} occurs at $x = 1.8$ m

$M_{max} = -(0.625)(1.8)^2 + (0.625)(0.6)^2 + 0 + (2.4)(1.8) - 0 = 2.52 \text{ kN}\cdot\text{m}$

PROBLEM 5.119



5.117 through 5.120 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$+\sum M_E = 0 \qquad -12A + (10)(20) + (8)(20) + (6)(20) = 0$

$A = 40 \text{ kips}$

$V = 40 - 20\langle x-2\rangle^0 - 20\langle x-4\rangle^0 - 20\langle x-6\rangle^0 \quad \text{kips}$

$M = 40x - 20\langle x-2\rangle^1 - 20\langle x-4\rangle^1 - 20\langle x-6\rangle^1 \quad \text{kip}\cdot\text{ft}$

Values of V

A to B $\quad V = 40$ kip

B to C $\quad V = 40 - 20 = 20$ kips

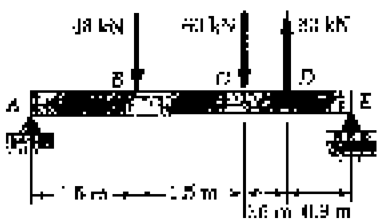
C to D $\quad V = 40 - 20 - 20 = 0$

D to E $\quad V = 40 - 20 - 20 - 20 = -20$ kip

Bending moment is constant and maximum over C to D.

At C $\quad x = 4$ ft $\quad M = (40)(4) - (20)(2) - 0 - 0 = 120 \text{ kip}\cdot\text{ft}$

PROBLEM 5.120



5.117 through 5.120 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$+\circlearrowleft \sum M_E = 0 \qquad -4.5 R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$

$R_A = 40 \text{ kN}$

$V = 40 - 48\langle x-1.5\rangle^0 - 60\langle x-3.0\rangle^0 + 60\langle x-3.6\rangle^0 \quad \text{kN}$

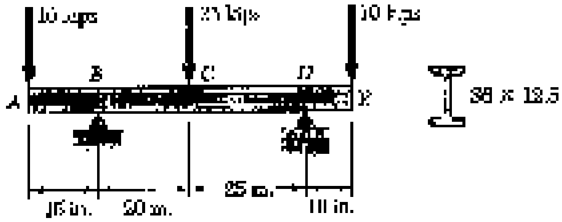
$M = 40x - 48\langle x-1.5\rangle^1 - 60\langle x-3.0\rangle^1 + 60\langle x-3.6\rangle^1 \quad \text{kN}\cdot\text{m}$

Pt.	x (m)	M (kN·m)
A	0	0
B	1.5	(40)(1.5) = 60 kN·m
C	3.0	(40)(3.0) − (48)(1.5) = 48 kN·m
D	3.6	(40)(3.6) − (48)(2.1) − (60)(0.6) = 7.2 kN·m
E	4.5	(40)(4.5) − (48)(3.0) − (60)(1.5) + (60)(0.9) = 0

$M_{max} = 60 \text{ kN}\cdot\text{m}$

PROBLEM 5.121

5.121 and 5.122 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



SOLUTION

$+\circlearrowleft \Sigma M_D = 0$

$(60)(10) + 45 R_B + (25)(25) - (10)(10) = 0$

$R_B = 25$ kips

$+\circlearrowleft \Sigma M_B = 0$

$(15)(10) - (20)(25) + 45 R_D - (55)(10) = 0$

$R_D = 20$ kips

$$V = -10 + 25\langle x-15\rangle^0 - 25\langle x-35\rangle^0 + 20\langle x-60\rangle^0 \quad \text{kips}$$

$$M = -10x + 25\langle x-15\rangle^1 - 25\langle x-35\rangle^1 + 20\langle x-60\rangle^1 \quad \text{kip}\cdot\text{in}$$

Pt	x (ft)	M (kip·in)
B	15	$-(10)(15) = -150$ kip·in
C	35	$-(10)(35) + (25)(20) = 150$ kip·in
D	60	$-(10)(60) + (25)(45) - (25)(25) = -100$ kip·in
E	70	$-(10)(70) + (25)(55) - (25)(35) + (20)(10) = 0$ checks

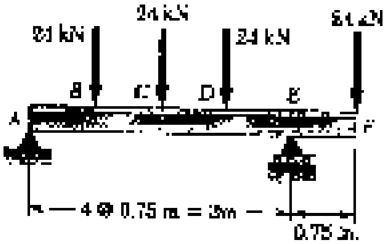
Maximum $|M| = 150$ kip·in

For S6×12.5 rolled steel section $S = 7.37\ \text{in}^3$

Normal stress $\sigma = \dfrac{|M|}{S} = \dfrac{150}{7.37} = 20.35$ ksi

PROBLEM 5.122

5.121 and 5.122 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



SOLUTION

$+\circlearrowleft \Sigma M_E = 0$

$-3R_A + (2.25)(24) - (1.5)(24) - (0.75)(24) + (0.75)(24) = 0$

$R_A = 30$ kips

$\circlearrowleft \Sigma M_A = 0 \quad -(0.75)(24) - (1.5)(24) - (2.25)(24) + 3R_E - (3.75)(24) = 0$

$R_D = 66$ kips

$V = 30 - 24\langle x-0.75\rangle^0 - 24\langle x-1.5\rangle^0 - 24\langle x-2.25\rangle^0 + 66\langle x-3\rangle^0$ kN

$M = 30x - 24\langle x-0.75\rangle^1 - 24\langle x-1.5\rangle^1 - 24\langle x-2.25\rangle^1 + 66\langle x-3\rangle^1$ kN·m

Pt	x (m)	M (kN·m)
B	0.75	$(30)(0.75) = 22.5$ kN·m
C	1.5	$(30)(1.5) - (24)(0.75) = 27$ kN·m
D	2.25	$(30)(2.25) - (24)(1.5) - (24)(0.75) = 13.5$ kN·m
E	3.0	$(30)(3.0) - (24)(2.25) - (24)(1.5) - (24)(0.75) = -18$ kN·m
F	3.75	$(30)(3.75) - (24)(3.0) - (24)(2.25) - (24)(1.5) + (66)(0.75) = 0$ ✓

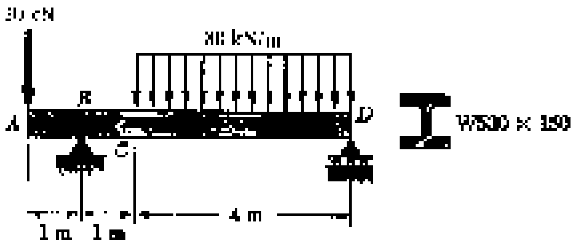
Maximum $|M| = 27$ kN·m $= 27\times10^3$ N·m

For rolled steel section W250 × 28.4 $\quad S = 308\times10^3\ \text{mm}^3 = 308\times10^{-6}\ \text{m}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{27\times10^3}{308\times10^{-6}} = 87.7\times10^6\ \text{Pa} = 87.7\ \text{MPa}$

PROBLEM 5.123

5.123 and 5.124 (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



SOLUTION

$+\circlearrowleft M_D = 0$

$(6)(10) - 5R_B + (2)(4)(80) = 0$

$R_B = 140$ kN

$w = 80\langle x-2\rangle^0$ kN/m $= -dV/dx$

$V = -10 + 140\langle x-1\rangle^0 - 80\langle x-2\rangle^1$ kN

A to B $\quad V = -10$ kN

B to C $\quad V = -10 + 140 = 130$ kN

D $\quad (x = 6) \quad V = -10 + 140 - 80(4) = -190$ kN

V changes sign at B and at point E $(x = x_E)$ between C and D.

$V = 0 = -10 + 140\langle x_E - 1\rangle^0 - 80\langle x_E - 2\rangle^1$

$= -10 + 140 - 80(x_E - 2) \qquad x_E = 3.625$ m

$M = -10x + 140\langle x-1\rangle^1 - 40\langle x-2\rangle^2$ kN·m

At pt. B $\quad x = 1 \qquad M_B = -(10)(1) = -10$ kN·m

At pt. E $\quad x = 3.625$

$M_E = -(10)(3.625) + (140)(2.625) - (40)(1.625)^2 = 225.6$ kN·m

$|M|_{max} = 225.6$ kN·m at $x = 3.625$ m

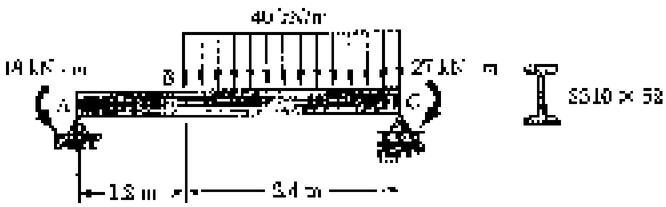
For W 530 × 150 $\quad S = 3720 \times 10^3$ mm^3 $= 3720 \times 10^{-6}$ m^3

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{225.6 \times 10^3}{3720 \times 10^{-6}} = 60.6 \times 10^6$ Pa

$= 60.6$ MPa

PROBLEM 5.124

5.123 and 5.124 (*a*) Using singularity functions, find the magnitude and location of the maximum bending moment for the beam and loading shown. (*b*) Determine the maximum normal stress due to bending.



SOLUTION

$\circlearrowleft \Sigma M_C = 0$

$18 - 3.6 R_A + (1.2)(2.4)(40) - 27 = 0$

$R_A = 29.5 \text{ kN}$

$V = 29.5 - 40 \langle x - 1.2 \rangle^1 \text{ kN}$

Point D $\quad V = 0 \quad 29.5 - 40(x_D - 1.2) = 0$

$x_D = 1.9375 \text{ m}$

$M = -18 + 29.5x - 20\langle x - 1.2 \rangle^2 \text{ kN·m}$

$M_A = -18 \text{ kN·m}$

$M_D = -18 + (29.5)(1.9375) - (20)(0.7375)^2 = 28.278 \text{ kN·m}$

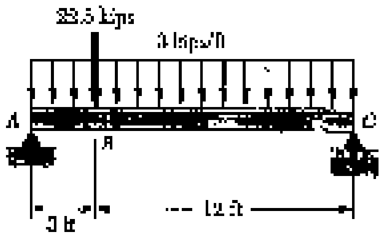
$M_E = -18 + (29.5)(3.6) - (20)(2.4)^2 = -27 \text{ kN·m}$

Maximum $|M| = 28.278 \text{ kN·m}$ at $x = 1.9375 \text{ m}$ ◄ ◄

For S310 × 52 rolled steel section $\quad S = 625 \times 10^3 \text{ mm}^3 = 625 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{28.278 \times 10^3}{625 \times 10^{-6}} = 45.2 \times 10^6 \text{ Pa} = 45.2 \text{ MPa}$ ◄

PROBLEM 5.125



5.125 and 5.126 A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, determine the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the allowable stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that should be selected.

SOLUTION

$$\circlearrowleft \Sigma M_C = 0 \quad -15 R_A + (7.5)(15)(3) + (12)(22.5) = 0$$

$$R_A = 40.5 \text{ kips}$$

$$w = 3 \text{ kips/ft} = -\frac{dV}{dx}$$

$$V = 40.5 - 3x - 22.5\langle x-3\rangle^0 \quad \text{kips}$$

Location of point D where $V = 0$. Assume $3 < x_D < 12$

$$0 = 40.5 - 3x_D - 22.5 \qquad x_D = 6 \text{ ft}$$

$$M = 40.5x - 1.5x^2 - 22.5\langle x-3\rangle^1 \quad \text{kip}\cdot\text{ft}$$

At point D $(x = 6 \text{ ft})$ $\quad M = (40.5)(6) - (1.5)(6)^2 - (22.5)(3)$

$$= 121.5 \text{ kip}\cdot\text{ft} = 1458 \text{ kip}\cdot\text{in.}$$

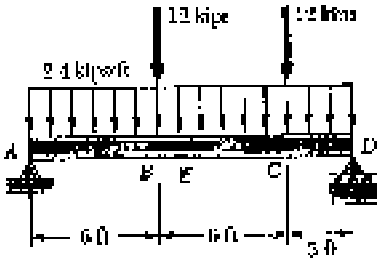
Maximum $|M| = 121.5$ kip·ft at $x = 6$ ft ◄

$$S_{min} = \frac{M}{\sigma_{all}} = \frac{1458}{24} = 60.75 \text{ in}^3$$

Shape	S (in³)
W 21 × 44	81.6
W 18 × 50	88.9
W 16 × 40	64.7 ←
W 14 × 43	62.7
W 12 × 50	64.7
W 10 × 68	75.7

Answer W 16 × 40 ◄

PROBLEM 5.126



5.125 and 5.126 A beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, determine the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the allowable stress for the steel to be used is 24 ksi, find the most economical wide-flange shape that should be selected.

SOLUTION

$$\circlearrowleft \Sigma M_D = 0$$

$$-15 R_A + (7.5)(15)(2.4) - (9)(12) - (3)(12) = 0$$

$$R_A = 27.6 \text{ kips}$$

$$w = 2.4 \text{ kips/ft} = -\frac{dV}{dx}$$

$$V = 27.6 - 2.4x - 12\langle x-6\rangle^0 - 12\langle x-12\rangle^0 \quad \text{kips}$$

$$V_{B^-} = 27.6 - (2.4)(6) = 13.2 \text{ kips}$$
$$V_{B^+} = 27.6 - (2.4)(6) - 12 = 1.2 \text{ kips}$$
$$V_{C^-} = 27.6 - (2.4)(12) - 12 = -13.2 \text{ kips}$$

Point where $V = 0$ lies between B and C.

Locate point E where $V = 0$

$$0 = 27.6 - 2.4x_E - 12 - 0 \qquad x_E = 6.5 \text{ ft}$$

$$M = 27.6x - 1.2x^2 - 12\langle x-6\rangle^1 - 12\langle x-12\rangle^1 \quad \text{kip·ft}$$

At point E $(x = 6.5 \text{ ft})$ $\quad M = (27.6)(6.5) - (1.2)(6.5)^2 - (12)(0.5) - 0$

$$= 122.7 \text{ kip·ft} = 1472.4 \text{ kip·in.}$$

Maximum $|M|$ = 122.7 kip·ft at $x = 6.5$ ft

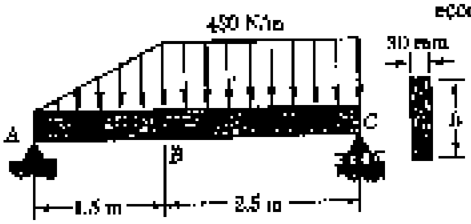
$$S_{min} = \frac{|M|}{\sigma_{all}} = \frac{1472.4}{24} = 61.35 \text{ in}^3$$

Shape	S (in^3)
W21 × 44	81.6
W18 × 50	88.9
W16 × 40	64.7 ←
W14 × 43	62.7
W12 × 50	64.7
W10 × 68	75.7

Answer: W16 × 40

PROBLEM 5.127

5.127 and 5.128 A timber beam is being designed to be supported and loaded as shown. (*a*) Using singularity functions, determine the magnitude and location of the maximum bending moment in the beam. (*b*) Knowing that the available stock consists of beams with a 12-MPa allowable stress and a rectangular cross section of 30-mm width and depth *h* varying from 80 to 160 mm in 10-mm increments, determine the most economical cross section that can be used.



SOLUTION

$480 \text{ N/m} = 0.48 \text{ kN/m}$

$+\circlearrowright \Sigma M_C = 0$

$-4R_A + (3)(\tfrac{1}{2})(1.5)(0.48) + (1.25)(2.5)(0.48) = 0$

$R_A = 0.645 \text{ kN}$

$w = \frac{0.48}{1.5}x - \frac{0.48}{1.5}\langle x - 1.5\rangle^1 = 0.32x - 0.32\langle x-1.5\rangle^1 \text{ kN/m} = -\frac{dV}{dx}$

$V = 0.645 - 0.16x^2 + 0.16\langle x - 1.5\rangle^2 \quad \text{kN}$

Locate point D where $V = 0$. Assume $1.5 \text{ m} < x_D < 4 \text{ m}$

$0 = 0.645 - 0.16x_D^2 + 0.16(x_D - 1.5)^2$

$= 0.645 - \cancel{0.16x_D^2} + \cancel{0.16x_D^2} - 0.48x_D + 0.36$

$x_D = 2.09375 \text{ m}$

$M = 0.645x - 0.05333x^3 + 0.05333\langle x-1.5\rangle^3 \quad \text{kN·m}$

At point D $\quad M_D = (0.645)(2.09375) - (0.05333)(2.09375)^3 + (0.05333)(0.59375)^3$

$= 0.87211 \text{ kN·m}$

$S_{min} = \frac{M_D}{\sigma_{all}} = \frac{0.87211 \times 10^3}{12 \times 10^6} = 72.6758 \times 10^{-6} \text{ m}^3 = 72.6758 \times 10^3 \text{ mm}^3$

For a rectangular cross section $\quad S = \frac{1}{6}bh^2 \qquad h = \sqrt{\frac{6S}{b}}$

$h_{min} = \sqrt{\frac{(6)(72.6758 \times 10^3)}{30}} = 120.56 \text{ mm}$

At next larger 10-mm increment $\qquad h = 130 \text{ mm}$

PROBLEM 5.128

5.127 and 5.128 A timber beam is being designed to be supported and loaded as shown. (a) Using singularity functions, determine the magnitude and location of the maximum bending moment in the beam. (b) Knowing that the available stock consists of beams with a 12-MPa allowable stress and a rectangular cross section of 30-mm width and depth h varying from 80 to 160 mm in 10-mm increments, determine the most economical cross section that can be used.

SOLUTION

$500 \text{ N/m} = 0.5 \text{ kN/m}$

$+\circlearrowleft \Sigma M_C = 0$

$-4R_A + (3.2)(1.6)(0.5) + (1.6)(\frac{1}{2})(2.4)(0.5) = 0$

$R_A = 0.880 \text{ kN}$

$w = 0.5 - \frac{0.5}{2.4}\langle x-1.6\rangle^1 = 0.5 - 0.20833\langle x-1.6\rangle^1 \text{ kN/m} = -\frac{dV}{dx}$

$V = 0.880 - 0.5x + 0.104167\langle x-1.6\rangle^2 \text{ kN}$

$V_A = 0.880 \text{ kN}$

$V_B = 0.880 - (0.5)(1.6) = 0.080 \text{ kN}$ } Sign change

$V_C = 0.880 - (0.5)(4) + (0.104167)(2.4)^2 = -0.520 \text{ kN}$ }

Locate point D where $V = 0$

$0 = 0.880 - 0.5x_D + 0.104167(x_D - 1.6)^2$

$0.104167x_D^2 - 0.83333x_D + 1.14667 = 0$

$$x_D = \frac{0.83333 \pm \sqrt{(0.83333)^2 - (4)(0.104167)(1.14667)}}{(2)(0.104167)}$$

$= 4.0 \pm 2.2342 = \cancel{6.2342},\ 1.7658 \text{ m}$

$M = 0.880x - 0.25x^2 + 0.347222\langle x-1.6\rangle^3 \text{ kN·m}$

$M_D = (0.880)(1.7658) - (0.25)(1.7658)^2 + (0.34722)(0.1658)^3 = 0.776 \text{ kN·m}$

$M_{max} = 0.776 \text{ kN·m}$ at $x = 1.7658 \text{ m}$

$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{0.776 \times 10^3}{12 \times 10^6} = 64.66 \times 10^{-6} \text{ m}^3 = 64.66 \times 10^3 \text{ mm}^3$

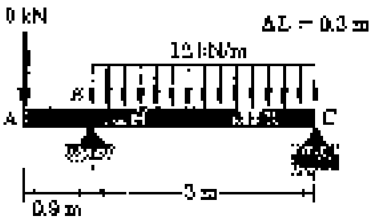
For a rectangular cross section $S = \frac{1}{6}bh^2 \qquad h = \frac{6S}{b}$

$h_{min} = \sqrt{\frac{(6)(64.66 \times 10^3)}{30}} = 113.7 \text{ mm}$

At next higher 10-mm increment $\quad h = 120 \text{ mm}$

PROBLEM 5.129

5.129 through 5.132 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increments ΔL, starting at point A and ending at the right-hand support.



SOLUTION

$$+\circlearrowleft M_C = 0 \qquad (3.9)(9) - 3R_B + (1.5)(3.0)(12) = 0$$

$$R_B = 29.7 \text{ kN}$$

$$w = 12\langle x - 0.9\rangle^0$$

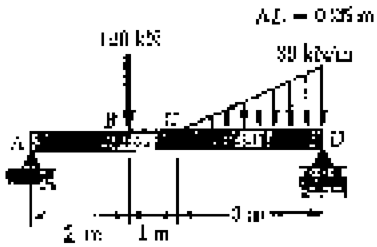
$$V = -9 + 29.7\langle x-0.9\rangle^0 - 12\langle x-0.9\rangle^1 \text{ kN}$$

$$M = -9x + 29.7\langle x-0.9\rangle^1 - 6\langle x-0.9\rangle^2 \text{ kN}\cdot\text{m}$$

x m	V kN	M kN·m
0.0	-9.0	0.00
0.3	-9.0	-2.70
0.6	-9.0	-5.40
0.9	20.7	-8.10
1.2	17.1	-2.43
1.5	13.5	2.16
1.8	9.9	5.67
2.1	6.3	8.10
2.4	2.7	9.45
2.7	-0.9	9.72
3.0	-4.5	8.91
3.3	-8.1	7.02
3.6	-11.7	4.05
3.9	-15.3	0.00

PROBLEM 5.130

5.129 through 5.132 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increments ΔL, starting at point A and ending at the right-hand support.



SOLUTION

$$+\circlearrowleft \Sigma M_D = 0$$

$$-6R_A + (4)(120) + (1)(\tfrac{1}{2})(3)(36) = 0$$

$$R_A = 89 \text{ kN}$$

$$w = \frac{36}{3}\langle x-3\rangle^1 = 12\langle x-3\rangle^1$$

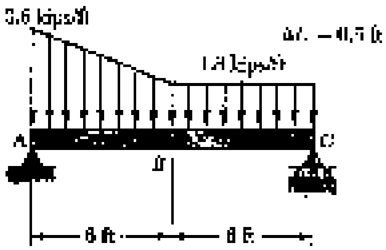
$$V = 89 - 120\langle x-2\rangle^0 - 6\langle x-3\rangle^2 \text{ kN}$$

$$M = 89x - 120\langle x-2\rangle^1 - 2\langle x-3\rangle^3 \text{ kN}\cdot\text{m}$$

x m	V kN	M kN·m
0.0	89.0	0.0
0.3	89.0	22.3
0.5	89.0	44.5
0.8	89.0	66.8
1.0	89.0	89.0
1.3	89.0	111.3
1.5	89.0	133.5
1.8	89.0	155.8
2.0	-31.0	178.0
2.3	-31.0	170.3
2.5	-31.0	162.5
2.8	-31.0	154.8
3.0	-31.0	147.0
3.3	-31.4	139.2
3.5	-32.5	131.3
3.8	-34.4	122.9
4.0	-37.0	114.0
4.3	-40.4	104.3
4.5	-44.5	93.8
4.8	-49.4	82.0
5.0	-55.0	69.0
5.3	-61.4	54.5
5.5	-68.5	38.3
5.8	-76.4	20.2
6.0	-85.0	-0.0

PROBLEM 5.131

5.129 through 5.132 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increments ΔL, starting at point A and ending at the right-hand support.



SOLUTION

$+\circlearrowleft \Sigma M_C = 0$

$-12 R_A + (6)(12)(1.8) + (10)(\tfrac{1}{2})(6)(1.8) = 0$

$R_A = 15.3$ kips

$w = 3.6 - \frac{1.8}{6} x + \frac{1.8}{6} \langle x-6 \rangle^1$

$= 3.6 - 0.3x + 0.3 \langle x-6 \rangle^1$

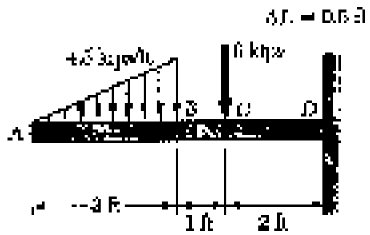
$V = 15.3 - 3.6x + 0.15x^2 - 0.15\langle x-6 \rangle^2$ kips ◄

$M = 15.3x - 1.8x^2 + 0.05x^3 - 0.05\langle x-6 \rangle^3$ kip·ft ◄

x ft	V kips	M kip·ft
0.0	15.30	0.0
0.5	13.54	7.2
1.0	11.85	13.6
1.5	10.24	19.1
2.0	8.70	23.8
2.5	7.24	27.8
3.0	5.85	31.1
3.5	4.54	33.6
4.0	3.30	35.6
4.5	2.14	37.0
5.0	1.05	37.8
5.5	0.04	38.0
6.0	-0.90	37.8
6.5	-1.80	37.1
7.0	-2.70	36.0
7.5	-3.60	34.4
8.0	-4.50	32.4
8.5	-5.40	29.9
9.0	-6.30	27.0
9.5	-7.20	23.6
10.0	-8.10	19.8
10.5	-9.00	15.5
11.0	-9.90	10.8
11.5	-10.80	5.6
12.0	-11.70	0.0

PROBLEM 5.132

5.129 through 5.132 Using a computer and step functions, calculate the shear and bending moment for the beam and loading shown. Use the specified increment ΔL, starting at point A and ending at the right-hand support.



SOLUTION

$$w = \frac{4.5}{3}x - \frac{4.5}{3}\langle x-3\rangle^1 - 4.5\langle x-3\rangle^0$$

$$= 1.5x - 1.5\langle x-3\rangle^1 - 4.5\langle x-3\rangle^0$$

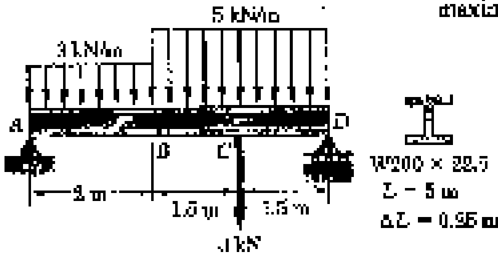
$$V = -0.75x^2 + 0.75\langle x-3\rangle^2 + 4.5\langle x-3\rangle^1 - 6\langle x-4\rangle^0 \quad \text{kips}$$

$$M = -0.25x^3 + 0.25\langle x-3\rangle^3 + 2.25\langle x-3\rangle^2 - 6\langle x-4\rangle^1 \quad \text{kip}\cdot\text{ft}$$

x ft	V kips	M kip·ft
0.0	0.00	0.00
0.5	-0.19	-0.03
1.0	-0.75	-0.25
1.5	-1.69	-0.84
2.0	-3.00	-2.00
2.5	-4.69	-3.91
3.0	-6.75	-6.75
3.5	-6.75	-10.13
4.0	-12.75	-13.50
4.5	-12.75	-19.88
5.0	-12.75	-26.25
5.5	-12.75	-32.63
6.0	-12.75	-39.00

PROBLEM 5.133

5.133 and 5.134 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2-percent accuracy the maximum normal stress in the beam.



SOLUTION

$\circlearrowleft \Sigma M_D = 0$

$-5R_A + (4.0)(2.0)(3) + (1.5)(2)(5) + (1.5)(3) = 0$

$R_A = 10.2 \text{ kN}$

$w = 3 + 2\langle x-2\rangle^0 \text{ kN/m} = -\frac{dV}{dx}$

$V = 10.2 - 3x - 2\langle x-2\rangle^1 - 3\langle x-3.5\rangle^0 \text{ kN}$

$M = 10.2x - 1.5x^2 - \langle x-2\rangle^2 - 3\langle x-3.5\rangle^1 \text{ kN·m}$

x m	V kN	M kN·m	sigma MPa
0.00	10.20	0.00	0.0
0.25	9.45	2.46	12.7
0.50	8.70	4.72	24.4
0.75	7.95	6.81	35.1
1.00	7.20	8.70	44.8
1.25	6.45	10.41	53.6
1.50	5.70	11.92	61.5
1.75	4.95	13.26	68.3
2.00	4.20	14.40	74.2
2.25	2.95	15.29	78.8
2.50	1.70	15.88	81.8
2.75	0.45	16.14	83.2
3.00	-0.80	16.10	83.0
3.25	-2.05	15.74	81.2
3.50	-6.30	15.07	77.7
3.75	-7.55	13.34	68.8
4.00	-8.80	11.30	58.2
4.25	-10.05	8.94	46.1
4.50	-11.30	6.27	32.3
4.75	-12.55	3.29	17.0
5.00	-13.80	-0.00	-0.0
2.83	0.05	16.164	83.3
2.84	0.00	16.164	83.3
2.85	-0.05	16.164	83.3

For rolled steel section W 200 × 22.5

$S = 194 \times 10^3 \text{ mm}^3$

$S = 194 \times 10^3 \text{ mm}^3 = 194 \times 10^{-6} \text{ m}^3$

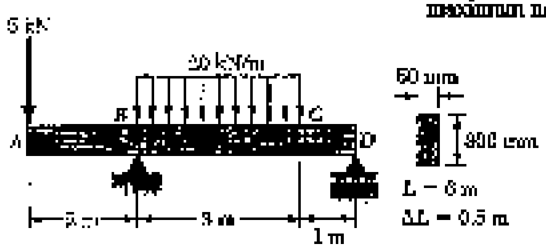
$\sigma_{max} = \frac{M_{max}}{S} = \frac{16.164 \times 10^3}{194 \times 10^{-6}}$

$= 83.3 \times 10^6 \text{ Pa}$

$= 83.3 \text{ MPa}$

PROBLEM 5.134

5.133 and 5.134 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2-percent accuracy the maximum normal stress in the beam.



SOLUTION

$+\circlearrowleft \Sigma M_D = 0$

$-4R_B + (6)(5) + (2.5)(3)(20) = 0$

$R_B = 45 \text{ kN}$

$w = 20\langle x-2\rangle^0 - 20\langle x-5\rangle^0 \text{ kN/m} = -\frac{dV}{dx}$

$V = -5 + 45\langle x-2\rangle^0 - 20\langle x-2\rangle^1 + 20\langle x-5\rangle^1 \text{ kN}$

$M = -5x + 45\langle x-2\rangle^1 - 10\langle x-2\rangle^2 + 10\langle x-5\rangle^2 \text{ kN·m}$

x m	V kN	M kN·m	sigma MPa
0.00	-5	0.00	0.0
0.50	-5	-2.50	-3.3
1.00	-5	-5.00	-6.7
1.50	-5	-7.50	-10.0
2.00	40	-10.00	-13.3
2.50	30	7.50	10.0
3.00	20	20.00	26.7
3.50	10	27.50	36.7
4.00	0	30.00	40.0
4.50	-10	27.50	36.7
5.00	-20	20.00	26.7
5.50	-20	10.00	13.3
6.00	-20	0.00	0.0

Maximum $|M| = 30 \text{ kN·m}$ at $x = 4.0 \text{ m}$

For rectangular cross section

$S = \frac{1}{6}bh^2 = (\frac{1}{6})(50)(300)^2$

$= 750 \times 10^3 \text{ mm}^3$

$= 750 \times 10^{-6} \text{ m}^3$

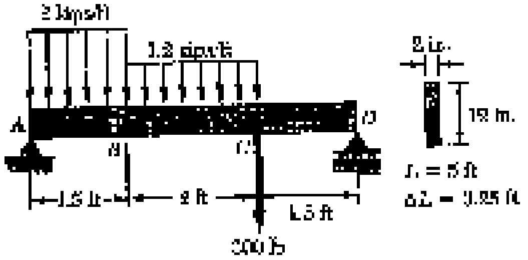
$\sigma_{max} = \frac{M_{max}}{S} = \frac{30 \times 10^3}{750 \times 10^{-6}}$

$= 40 \times 10^6 \text{ Pa} = 40 \text{ MPa}$

$\sigma = \frac{|M|}{S}$

PROBLEM 5.135

5.135 and 5.136 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2-percent accuracy the maximum normal stress in the beam.



SOLUTION

$300\ lb = 0.3\ kips$

$+\circlearrowleft \Sigma M_D = 0$

$-5R_A + (4.25)(1.5)(2) + (2.5)(2)(1.2) + (1.5)(0.3) = 0$

$R_A = 3.84$ kips

$w = 2 - 0.8\langle x-1.5\rangle^0 - 1.2\langle x-3.5\rangle^0$ kip/ft

$V = 3.84 - 2x + 0.8\langle x-1.5\rangle^1 + 1.2\langle x-3.5\rangle^1 - 0.3\langle x-3.5\rangle^0$ kips

$M = 3.84x - x^2 + 0.4\langle x-1.5\rangle^2 + 0.6\langle x-3.5\rangle^2 - 0.3\langle x-3.5\rangle^1$ kip·ft

x ft	V kips	M kip·ft	sigma ksi
0.00	3.84	0.00	0.000
0.25	3.34	0.90	0.224
0.50	2.84	1.67	0.417
0.75	2.34	2.32	0.579
1.00	1.84	2.84	0.710
1.25	1.34	3.24	0.809
1.50	0.84	3.51	0.877
1.75	0.54	3.68	0.921
2.00	0.24	3.78	0.945
2.25	-0.06	3.80	0.951
2.50	-0.36	3.75	0.937
2.75	-0.66	3.62	0.906
3.00	-0.96	3.42	0.855
3.25	-1.26	3.14	0.786
3.50	-1.86	2.79	0.697
3.75	-1.86	2.32	0.581
4.00	-1.86	1.86	0.465
4.25	-1.86	1.39	0.349
4.50	-1.86	0.93	0.232
4.75	-1.86	0.46	0.116
5.00	-1.86	-0.00	-.000
2.10	0.12	3.80	0.949
2.20	0.00	3.80	0.951
2.30	-0.12	3.80	0.949

Maximum $|M| = 3.804$ kip·ft
$= 45.648$ kip·in
at $x = 2.20$ ft

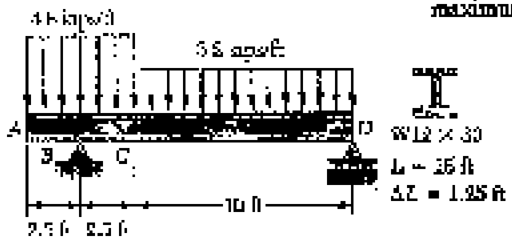
Rectangular section
2 in × 12 in.

$S = \frac{1}{6}bh^2 = (\frac{1}{6})(2)(12)^2$
$= 48\ in^3$

$\sigma = \frac{M}{S} = \frac{45.648}{48}$
$= 0.951$ ksi

PROBLEM 5.136

5.135 and 5.136 For the beam and loading shown, and using a computer and step functions, (*a*) tabulate the shear, bending moment, and maximum normal stress in sections of the beam from $x = 0$ to $x = L$, using the increments ΔL indicated, (*b*) using smaller increments if necessary, determine with a 2-percent accuracy the maximum normal stress in the beam.



SOLUTION

$\circlearrowleft \Sigma M_D = 0$

$-12.5 R_B + (12.5)(5.0)(4.8) + (5)(10)(3.2) = 0$

$R_B = 36.8$ kips

$$w = 4.8 - 1.6\langle x-5\rangle^0 \text{ kips/ft}$$

$$V = -4.8x + 36.8\langle x-2.5\rangle^0 + 1.6\langle x-5\rangle^1 \text{ kips}$$

$$M = -2.4x^2 + 36.8\langle x-2.5\rangle^1 + 0.8\langle x-5\rangle^2 \text{ kip·ft}$$

x ft	V kips	M kip·ft	sigma ksi
0.00	0.0	0.00	0.00
1.25	-6.0	-3.75	-1.17
2.50	24.8	-15.00	-4.66
3.75	18.8	12.25	3.81
5.00	12.8	32.00	9.95
6.25	8.8	45.50	14.15
7.50	4.8	54.00	16.79
8.75	0.8	57.50	17.88
10.00	-3.2	56.00	17.41
11.25	-7.2	49.50	15.39
12.50	-11.2	38.00	11.81
13.75	-15.2	21.50	6.68
15.00	-19.2	0.00	0.00
8.90	0.32	57.58	17.90
9.00	-0.00	57.60	17.91 ←
9.10	-0.32	57.58	17.90

Maximum $M = 57.6$ kip·ft
$= 691.2$ kip·in
at $x = 9.0$ ft.

For rolled steel section W12×30

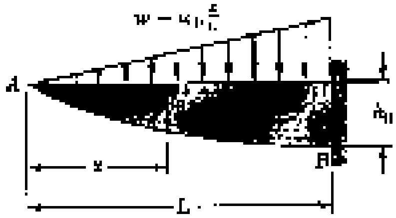
$S = 38.6 \text{ in}^3$

Maximum normal stress

$$\sigma = \frac{M}{S} = \frac{691.2}{38.6} = 17.91 \text{ ksi}$$

PROBLEM 5.137

5.137 and 5.138 The cantilever beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the distributed load $w(x)$ shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (b) Determine the smallest allowable value of h_0 if $L = 750$ mm, $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{all} = 250$ MPa.



SOLUTION

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

$$V = -\frac{w_0 x^2}{2L} = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^3}{6L} \qquad |M| = \frac{w_0 x^3}{6L}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{w_0 x^3}{6L\sigma_{all}}$$

For a rectangular cross section $\quad S = \frac{1}{6} b h^2$

Equating $\quad \frac{1}{6} b h^2 = \frac{w_0 x^3}{6L\sigma_{all}} \qquad h = \sqrt{\frac{w_0 x^3}{\sigma_{all} b L}}$

At $x = L \qquad h = h_0 = \sqrt{\frac{w_0 L^2}{\sigma_{all} b}} \qquad \therefore \quad h = h_0 \left(\frac{x}{L}\right)^{3/2}$

Data: $L = 750$ mm $= 0.75$ m, $\quad b = 30$ mm $= 0.030$ m
$w_0 = 300$ kN/m $= 300 \times 10^3$ N/m, $\quad \sigma_{all} = 250$ MPa $= 250 \times 10^6$ Pa

$$h_0 = \sqrt{\frac{(300 \times 10^3)(0.75)^2}{(250 \times 10^6)(0.030)}} = 150 \times 10^{-3} \text{ m} = 150 \text{ mm}$$

PROBLEM 5.138

5.137 and 5.138 The cantilever beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the distributed load $w(x)$ shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (*b*) Determine the smallest allowable value of h_0 if $L = 750$ mm, $b = 30$ mm, $w_0 = 300$ kN/m, and $\sigma_{all} = 250$ Mpa.

SOLUTION

$$\frac{dV}{dx} = -w = -w_0 \sin \frac{\pi x}{2L}$$

$$V = \frac{2w_0 L}{\pi} \cos \frac{\pi x}{2L} + C_1$$

$$V = 0 \quad \text{at} \quad x = 0 \quad \longrightarrow \quad C_1 = -\frac{2w_0 L}{\pi}$$

$$\frac{dM}{dx} = V = -\frac{2w_0 L}{\pi}\left(1 - \cos \frac{\pi x}{2L}\right)$$

$$M = -\frac{2w_0 L}{\pi}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right) \qquad |M| = \frac{2w_0 L}{\pi}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{L}\right)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{2w_0 L}{\pi \sigma_{all}}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$$

For a rectangular cross section $\quad S = \frac{1}{6} b h^2$

Equating $\quad \frac{1}{6} b h^2 = \frac{2w_0 L}{\pi \sigma_{all}}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)$

$$h = \left\{\frac{12 w_0 L}{\pi \sigma_{all} b}\left(x - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right)\right\}^{1/2}$$

At $x = L \qquad h = h_0 = \left\{\frac{12 w_0 L^2}{\pi \sigma_{all} b}\left(1 - \frac{2}{\pi}\right)\right\}^{1/2} = 1.178\sqrt{\frac{w_0 L^2}{\sigma_{all} b}}$

(a) $\quad h = h_0\left[\left(\frac{x}{L} - \frac{2L}{\pi} \sin \frac{\pi x}{2L}\right) / \left(1 - \frac{2}{\pi}\right)\right]^{1/2}$ ◄

Data: $\quad L = 750\ \text{mm} = 0.75\ \text{m}, \quad b = 30\ \text{mm} = 0.030\ \text{m}$

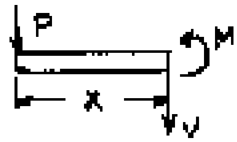
$w_0 = 300\ \text{kN/m} = 300 \times 10^3\ \text{N/m}, \quad \sigma_{all} = 250\ \text{MPa} = 250 \times 10^6\ \text{Pa}$

(b) $\quad h_0 = 1.178\sqrt{\frac{(300 \times 10^3)(0.75)^2}{(250 \times 10^6)(0.030)}} = 176.7 \times 10^{-3}\ \text{m} = 176.7\ \text{mm}$ ◄

PROBLEM 5.139

5.139 and 5.140 The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (b) Determine the maximum allowable load if $L = 36$ in., $h_0 = 12$ in., $b = 1.25$ in., and $\sigma_{all} = 36$ ksi.

SOLUTION



$V = -P$

$M = -Px \qquad |M| = Px$

$S = \dfrac{|M|}{\sigma_{all}} = \dfrac{P}{\sigma_{all}}x$

For a rectangular cross section $\quad S = \frac{1}{6}bh^2$

Equating $\quad \frac{1}{6}bh^2 = \dfrac{Px}{\sigma_{all}} \qquad h = \left(\dfrac{6Px}{\sigma_{all}b}\right)^{\frac{1}{2}}$

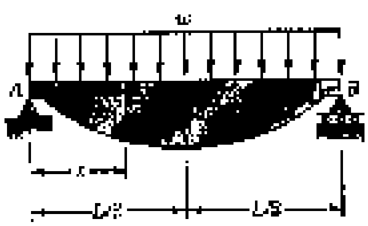
At $x = L \qquad h = h_0 = \left\{\dfrac{6PL}{\sigma_{all}b}\right\}^{1/2}$

$h = h_0\sqrt{\dfrac{x}{L}}$

Solving for P $\qquad P = \dfrac{\sigma_{all}bh_0^2}{6L} = \dfrac{(36)(1.25)(12)^2}{(6)(36)} = 30$ kips

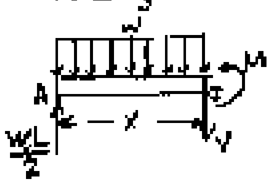
PROBLEM 5.140

5.139 and 5.140 The beam AB, consisting of a cast-iron plate of uniform thickness b and length L, is to support the load shown. (a) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0. (b) Determine the maximum allowable load if $L = 36$ in., $h_0 = 12$ in., $b = 1.25$ in., and $\sigma_{all} = 36$ ksi.



SOLUTION

$+\uparrow \Sigma F_y = 0 \qquad R_A + R_B - wL = 0 \qquad R_A = R_B = \dfrac{wL}{2}$



$+\circlearrowleft \Sigma M_J = 0$

$\dfrac{wL}{2}x - wx\dfrac{x}{2} + M = 0$

$M = \dfrac{w}{2}x(L - x)$

$S = \dfrac{|M|}{\sigma_{all}} = \dfrac{wx(L-x)}{2\sigma_{all}}$

For a rectangular cross section $\quad S = \frac{1}{6}bh^2$

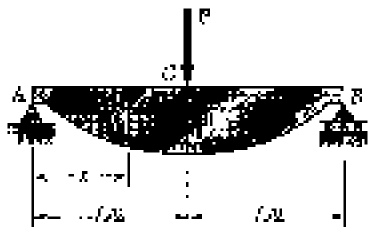
Equating $\quad \frac{1}{6}bh^2 = \dfrac{wx(L-x)}{2\sigma_{all}} \qquad h = \left\{\dfrac{3wx(L-x)}{\sigma_{all}b}\right\}^{1/2}$

At $x = \dfrac{L}{2} \qquad h = h_0 = \left\{\dfrac{3wL^2}{4\sigma_{all}b}\right\}^{1/2} \qquad h = h_0\left[\dfrac{x}{L}\left(1 - \dfrac{x}{L}\right)\right]^{\frac{1}{2}}$

Solving for w $\qquad w = \dfrac{4\sigma_{all}bh_0^2}{3L^2} = \dfrac{(4)(36)(1.25)(12)^2}{(3)(36)^2} = 6.67$ kip/in

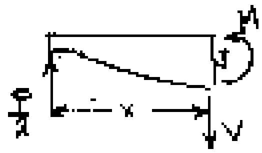
PROBLEM 5.141

5.141 and 5.142 The beam *AB*, consisting of a cast-aluminum plate of uniform thickness *b* and length *L*, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express *h* in terms of *x*, *L*, and h_0 for portion *AC* of the beam. (*b*) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ MPa.



SOLUTION

$R_A = R_B = \frac{P}{2}$



$\circlearrowleft \Sigma M_J = 0$

$-\frac{P}{2}x + M = 0$

$M = \frac{Px}{2} \qquad (0 < x < \frac{L}{2})$

$S = \frac{M}{\sigma_{all}} = \frac{Px}{2\sigma_{all}}$

For a rectangular cross section $S = \frac{1}{6}bh^2$

Equating $\frac{1}{6}bh^2 = \frac{Px}{2\sigma_{all}} \qquad h = \sqrt{\frac{3Px}{\sigma_{all} b}}$

(a) At $x = \frac{L}{2} \qquad h = h_0 = \sqrt{\frac{3PL}{2\sigma_{all} b}} \qquad h = h_0\sqrt{\frac{2x}{L}}, \quad 0 < x < \frac{L}{2}$ ◄

For $x > \frac{L}{2}$ replace x by $L - x$

(b) Solving for P $\quad P = \frac{2\sigma_{all} b h_0^2}{3L} = \frac{(2)(72 \times 10^6)(0.025)(0.200)^2}{(3)(0.8)} = 60 \times 10^3$ N

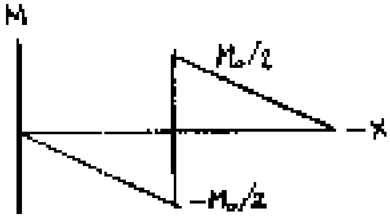
$= 60$ kN ◄

PROBLEM 5.142

5.141 and 5.142 The beam AB, consisting of a cast-aluminum plate of uniform thickness b and length L, is to support the load shown. (*a*) Knowing that the beam is to be of constant strength, express h in terms of x, L, and h_0 for portion AC of the beam. (*b*) Determine the maximum allowable load if $L = 800$ mm, $h_0 = 200$ mm, $b = 25$ mm, and $\sigma_{all} = 72$ Mpa.

SOLUTION

$R_A = M_0/L \downarrow \qquad R_B = M_0/L \uparrow$



$\circlearrowright \Sigma M_J = 0$

$\frac{M_0}{L}x + M = 0$

$M = -\frac{M_0 x}{L} \quad (0 < x < \frac{L}{2})$

For $x > \frac{L}{2}$ $\qquad M = \frac{M_0(L-x)}{L} \quad (\frac{L}{2} < x < L)$

$S = \frac{|M|}{\sigma_{all}} = \frac{M_0 x}{\sigma_{all} L}$ for $(0 < x < \frac{L}{2})$ For $x > \frac{L}{2}$ replace x by $L - x$.

For a rectangular cross section $S = \frac{1}{6}bh^2$

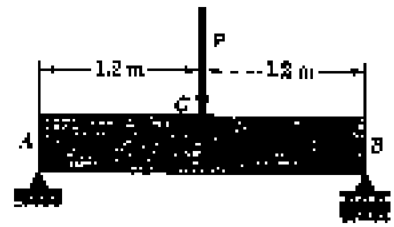
Equating $\frac{1}{6}bh^2 = \frac{M_0 x}{\sigma_{all} L} \qquad h = \sqrt{\frac{6M_0 x}{\sigma_{all} bL}}$

At $x = \frac{L}{2}$ $\quad h = h_0 = \sqrt{\frac{3M_0}{\sigma_{all} b}} \qquad h = h_0\sqrt{2x/L}$

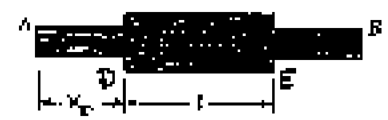
Solving for M_0 $\quad M_0 = \frac{\sigma_{all} b h_0^2}{3} = \frac{(72\times10^6)(0.025)(0.200)^2}{3} = 24\times10^3$ N·m

$= 24$ kN·m

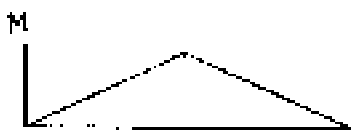
PROBLEM 5.143



(a)



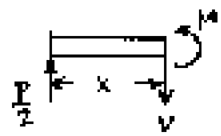
(b)



5.143 and 5.144 A preliminary design based on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50 × 50-mm cross section. Determine the length *l* of the two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$R_A = R_B = \frac{P}{2}$$



$0 < x < \frac{L}{2}$

$\circlearrowright \Sigma M_J = 0 \qquad -\frac{P}{2}x + M = 0$

$M = \frac{Px}{2} \quad$ or $\quad M = \frac{M_{max}\,x}{1.2}$

Bending moment diagram is two straight lines.

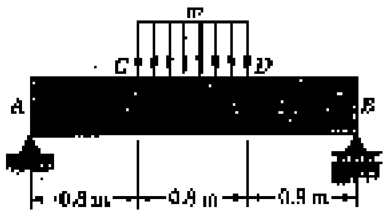
At C $\qquad S_C = \frac{1}{6} b h_C^2 \qquad M_C = M_{max}$

At D $\qquad S_D = \frac{1}{6} b h_D^2 \qquad M_D = \frac{M_{max}\,x_D}{1.2}$

$$\frac{S_D}{S_C} = \frac{h_D^2}{h_C^2} = \left(\frac{100\text{ mm}}{200\text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_D}{M_C} = \frac{x_D}{1.2} \qquad x_D = 0.3\text{ m}$$

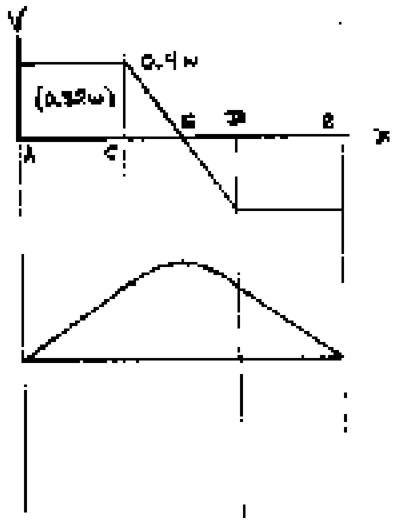
$$\frac{l}{2} = 1.2 - x_D = 0.9 \qquad l = 1.800\text{ m}$$

PROBLEM 5.144



(a)

(b)



5.143 and 5.144 A preliminary design based on the use of a simply supported prismatic timber beam indicated that a beam with a rectangular cross section 50 mm wide and 200 mm deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, four pieces of the same timber as the original beam and of 50 × 50-mm cross section. Determine the length *l* of the two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION

$$R_A = R_B = \frac{0.8\,w}{2} = 0.4\,w$$

Shear: A to C $\quad V = 0.4\,w$

D to B $\quad V = -0.4\,w$

Areas: A to C $\quad (0.8)(0.4)\,w = 0.32\,w$

C to E $\quad (\tfrac{1}{2})(0.4)(0.4)\,w = 0.08\,w$

Bending moments.

At C $\quad M_C = 0.40\,w$

A to C $\quad M = 0.40\,w\,x$

At C $\quad S_c = \frac{1}{6} b h_c^2 \qquad M_c = M_{max} = 0.40\,w$

At F $\quad S_F = \frac{1}{6} b h_F^2 \qquad M_F = 0.40\,w\,x_F$

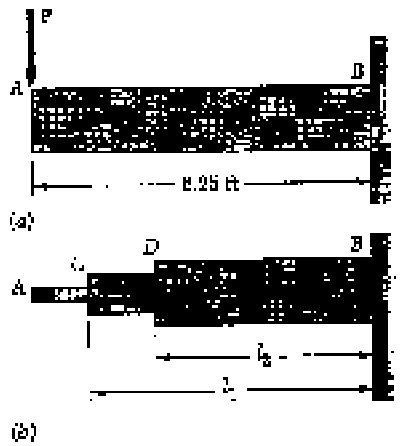
$$\frac{S_F}{S_c} = \frac{h_F^2}{h_c^2} = \left(\frac{100\text{ mm}}{200\text{ mm}}\right)^2 = \frac{1}{4} = \frac{M_F}{M_c} = \frac{0.40\,w\,x_F}{0.40\,w}$$

$x_F = 0.25\text{ m} \qquad \frac{l}{2} = 1.2 - x_F = 0.95\text{ m}$

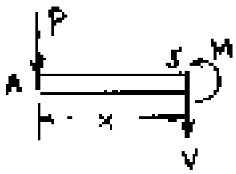
$$l = 1.900\text{ m}$$

PROBLEM 5.145

5.145 and 5.146 A preliminary design based on the use of a cantilever prismatic beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2 × 2-in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and two outer pieces of timber that will yield the same factor of safety as the original design.



SOLUTION



$+\circlearrowleft \Sigma M_J = 0$

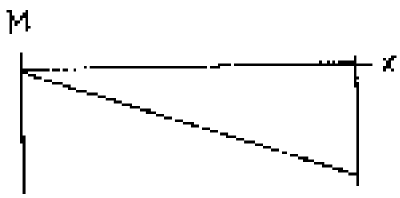
$Px + M = 0 \qquad M = -Px$

$|M| = Px$

At B $\quad |M|_B = M_{max}$

At C $\quad |M|_C = M_{max}\, x_C / 6.25$

At D $\quad |M|_D = M_{max}\, x_D / 6.25$



$S_B = \frac{1}{6} b h^2 = \frac{1}{6} \cdot b(5b)^2 = \frac{25}{6} b^3$

A to C $\quad S_C = \frac{1}{6} \cdot b(b)^2 = \frac{1}{6} b^3$

C to D $\quad S_D = \frac{1}{6} b(3b)^2 = \frac{9}{6} b^3$

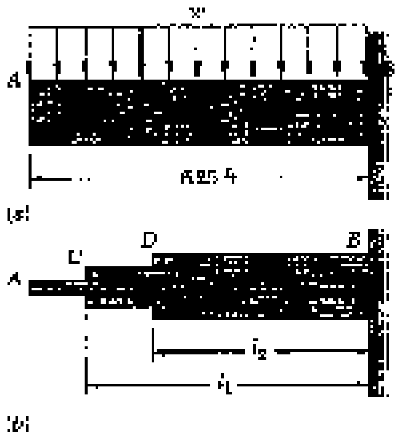
$\frac{|M|_C}{|M|_B} = \frac{x_C}{6.25} = \frac{S_C}{S_B} = \frac{1}{25} \qquad x_C = \frac{(1)(6.25)}{25} = 0.25 \text{ ft}$

$l_1 = 6.25 - 0.25 = 6.00 \text{ ft}$ ◄

$\frac{|M|_D}{|M|_B} = \frac{x_D}{6.25} = \frac{S_D}{S_B} = \frac{9}{25} \qquad x_D = \frac{(9)(6.25)}{25} = 2.25 \text{ ft}$

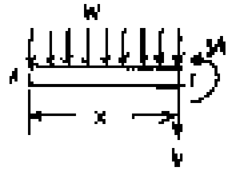
$l_2 = 6.25 - 2.25 = 4.00 \text{ ft}$ ◄

PROBLEM 5.146



5.145 and 5.146 A preliminary design based on the use of a cantilever prismatic beam indicated that a beam with a rectangular cross section 2 in. wide and 10 in. deep would be required to safely support the load shown in part *a* of the figure. It was then decided to replace that beam with a built-up beam obtained by gluing together, as shown in part *b* of the figure, five pieces of the same timber as the original beam and of 2 × 2-in. cross section. Determine the respective lengths l_1 and l_2 of the two inner and two outer pieces of timber that will yield the same factor of safety as the original design.

SOLUTION



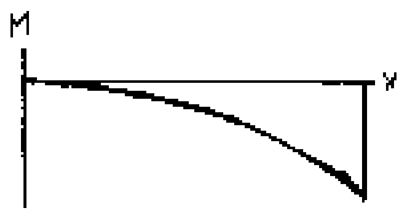
$+\circlearrowleft \Sigma M_J = 0 \qquad wx\frac{x}{2} + M = 0$

$M = -\frac{wx^2}{2} \qquad |M| = \frac{wx^2}{2}$

At B $\quad |M|_B = |M|_{max}$

At C $\quad |M|_C = |M|_{max}(x_C/6.25)^2$

At D $\quad |M|_D = |M|_{max}(x_D/6.25)^2$



At B $\quad S_B = \frac{1}{6}bh^2 = \frac{1}{6}b(5b)^2 = \frac{25}{6}b^3$

A to C $\quad S_C = \frac{1}{6}bh^2 = \frac{1}{6}b(b)^2 = \frac{1}{6}b^3$

C to D $\quad S_D = \frac{1}{6}bh^2 = \frac{1}{6}b(3b)^2 = \frac{9}{6}b^3$

$\frac{|M|_C}{|M|_B} = \left(\frac{x_C}{6.25}\right)^2 = \frac{S_C}{S_B} = \frac{1}{25} \qquad x_C = \frac{6.25}{\sqrt{25}} = 1.25 \text{ ft}$

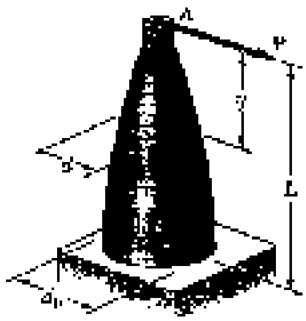
$l_1 = 6.25 - 1.25 \text{ ft} = 5.00 \text{ ft}$ ◀

$\frac{|M|_D}{|M|_B} = \left(\frac{x_D}{6.25}\right)^2 = \frac{S_D}{S_B} = \frac{9}{25} \qquad x_D = \frac{6.25\sqrt{9}}{\sqrt{25}} = 3.75 \text{ ft}$

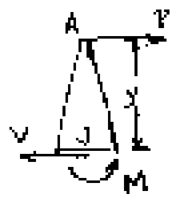
$l_2 = 6.25 - 3.75 \text{ ft} = 2.50 \text{ ft}$ ◀

PROBLEM 5.147

5.147 A cantilevered machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support a horizontal concentrated load **P** as shown. (*a*) Knowing that the machine element is to be of constant strength, express d in terms of y, L, and d_0. (*b*) Determine the maximum allowable value of P if $L = 300$ mm, $d_0 = 60$ mm, and $\sigma_{all} = 72$ Mpa.



SOLUTION



$+\circlearrowleft \Sigma M_J = 0 \qquad M - Py = 0$

$M = Py$

$S = \dfrac{|M|}{\sigma_{all}} = \dfrac{Py}{\sigma_{all}}$

For a solid circular cross section $\quad c = \dfrac{d}{2} \qquad I = \dfrac{\pi}{4}c^4$

$$S = \frac{I}{c} = \frac{\pi}{4}c^3 = \frac{\pi d^3}{32}$$

Equating $\quad \dfrac{\pi d^3}{32} = \dfrac{Py}{\sigma_{all}} \qquad d = \left(\dfrac{32\,Py}{\pi\sigma_{all}}\right)^{1/3}$

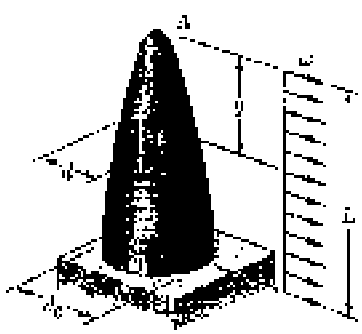
At $y = L \qquad d = d_0 = \left(\dfrac{32\,PL}{\pi\sigma_{all}}\right)^{1/3} \qquad \dfrac{d}{d_0} = \left(\dfrac{y}{L}\right)^{1/3}$ ◄

Solving for $P \qquad P = \dfrac{\pi d_0^3\sigma_{all}}{32\,L} = \dfrac{\pi(0.060)^3(72\times10^6)}{(32)(0.300)} = 5.09\times10^3$ N

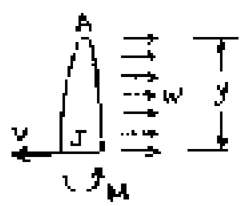
$= 5.09$ kN ◄

PROBLEM 5.148

5.148 A cantilevered machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support a horizontal distributed load w as shown. (*a*) Knowing that the machine element is to be of constant strength, express d in terms of y, L, and d_0. (*b*) Determine the smallest allowable value of d_0 if $L = 300$ mm, $w = 20$ kN/m, and $\sigma_{all} = 72$ MPa.



SOLUTION



$+\circlearrowleft \Sigma M_J = 0$

$M - \dfrac{y}{2}wy = 0 \qquad M = \dfrac{wy^2}{2}$

$S = \dfrac{|M|}{\sigma_{all}} = \dfrac{wy^2}{2\sigma_{all}}$

For a solid circular cross section $\quad c = \dfrac{d}{2}$

$I = \dfrac{\pi}{4}c^4 \qquad S = \dfrac{I}{c} = \dfrac{\pi c^3}{4} = \dfrac{\pi d^3}{32}$

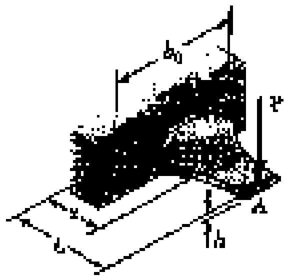
Equating $\quad \dfrac{\pi d^3}{32} = \dfrac{wy^2}{2\sigma_{all}} \qquad d = \left(\dfrac{16\,wy^2}{\pi\sigma_{all}}\right)^{1/3}$

At $x = L \qquad d = d_0 = \left(\dfrac{16\,wL^2}{\pi\sigma_{all}}\right)^{1/3} \qquad d = d_0\left(\dfrac{y}{L}\right)^{2/3}$ ◄

Using the data $\quad d_0 = \left\{\dfrac{(16)(20\times10^3)(0.300)^2}{\pi(72\times10^6)}\right\}^{1/3} = 50.3\times10^{-3}$ m

$= 50.3$ mm ◄

PROBLEM 5.149



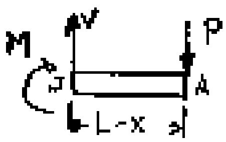
5.149 A cantilever beam AB consisting of a steel plate of uniform depth h and variable width b is to support a concentrated load **P** at point A. (a) Knowing that the beam is to be of constant strength, express b in terms of x, L, and b_0. (b) Determine the smallest allowable value of h if $L = 12$ in., $b_0 = 15$ in., $P = 3.2$ kips, and $\sigma_{all} = 24$ ksi.

SOLUTION

$$+\circlearrowleft \Sigma M_J = 0 \qquad -M - P(L-x) = 0 \qquad M = -P(L-x)$$

$$|M| = P(L-x)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{P(L-x)}{\sigma_{all}}$$



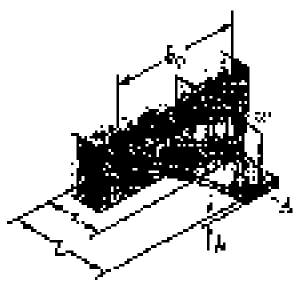
For a rectangular cross section $S = \frac{1}{6}bh^2$

Equating $\frac{1}{6}bh^2 = \frac{P(L-x)}{\sigma_{all}}$ $\qquad b = \frac{6P(L-x)}{\sigma_{all}h^2}$

At $x = 0$ $\qquad b = b_0 = \frac{6PL}{\sigma_{all}h^2}$ $\qquad b = b_0\left(1 - \frac{x}{L}\right)$

Solving for h $\qquad h = \sqrt{\frac{6PL}{\sigma_{all}b_0}} = \sqrt{\frac{(6)(3.2)(12)}{(24)(15)}} = 0.800$ in

PROBLEM 5.150



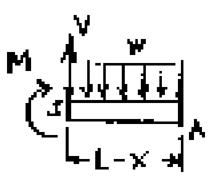
5.150 A cantilever beam AB consisting of a steel plate of uniform depth h and variable width b is to support a distributed load w along its center line AB. (a) Knowing that the beam is to be of constant strength, express b in terms of x, L, and b_0. (b) Determine the maximum allowable value of w if $L = 15$ in., $b_0 = 18$ in., $h = 0.75$ in., and $\sigma_{all} = 24$ ksi.

SOLUTION

$$+\circlearrowleft \Sigma M_J = 0 \qquad -M - w(L-x)\frac{L-x}{2} = 0$$

$$M = -\frac{w(L-x)^2}{2} \qquad |M| = \frac{w(L-x)^2}{2}$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{w(L-x)^2}{2\sigma_{all}}$$



For a rectangular cross section $S = \frac{1}{6}bh^2$

$\frac{1}{6}bh^2 = \frac{w(L-x)^2}{2\sigma_{all}}$ $\qquad b = \frac{3w(L-x)^2}{\sigma_{all}h^2}$

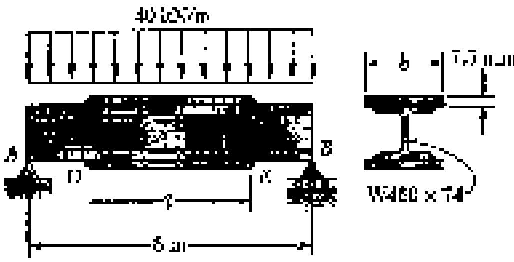
At $x = 0$ $\qquad b = b_0 = \frac{3wL^2}{\sigma_{all}h^2}$ $\qquad b = b_0\left(1 - \frac{x}{L}\right)^2$

Solving for w $\qquad w = \frac{\sigma_{all}b_0h^2}{3L^2} = \frac{(24)(18)(0.75)^2}{(3)(15)^2} = 0.360$ kip/in

$= 360$ lb/in

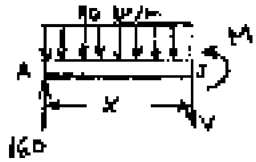
PROBLEM 5.151

5.151 Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that $l = 5$ m and $b = 200$ mm, determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.



SOLUTION

$$R_A = R_B = 160 \text{ kN}$$



$$+\circlearrowleft \Sigma M_J = 0$$

$$-160x + (40x)\frac{x}{2} + M = 0$$

$$M = 160x - 20x^2 \quad \text{kN}\cdot\text{m}$$

At center of beam $x = 4$ m $\quad M_C = 320 \text{ kN}\cdot\text{m}$

At D $\quad x = \frac{1}{2}(8 - l) = 1.5 \text{ m} \quad M_D = 195 \text{ kN}\cdot\text{m}$

At center of beam $\quad I = I_{beam} + 2I_{plate}$

$$= 333\times10^6 + 2\left\{(200)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(200)(7.5)^3\right\}$$

$$= 494.8 \times 10^6 \text{ mm}^4$$

$$c = \frac{457}{2} + 7.5 = 236 \text{ mm} \qquad S = \frac{I}{c} = 2097\times10^3 \text{ mm}^3 = 2097\times10^{-6} \text{ m}^3$$

Normal stress $\quad \sigma = \frac{M}{S} = \frac{320\times10^3}{2097\times10^{-6}} = 152.6\times10^6 \text{ Pa}$

$$= 152.6 \text{ MPa}$$

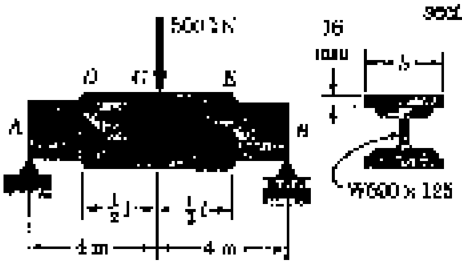
At D $\quad S = 1460\times10^3 \text{ mm}^3 = 1460\times10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \frac{M}{S} = \frac{195\times10^3}{1460\times10^{-6}} = 133.6\times10^6 \text{ Pa}$

$$= 133.6 \text{ MPa}$$

PROBLEM 5.152

5.152 Assuming that the length and width of the cover plates used with the beam of Sample Prob. 5.12 are, respectively, $l = 4$ m and $b = 285$ mm, and recalling that the thickness of each plate is 16 mm, determine the maximum normal stress on a transverse section (a) through the center of the beam, (b) just to the left of D.



SOLUTION

$R_A = R_B = 250$ kN

$\Sigma M_J = 0$

$-250x + M = 0$

$M = 250x$ kN·m

At center of beam $x = 4$ m $\quad M_C = (250)(4) = 1000$ kN·m

At D $\quad x = \frac{1}{2}(8 - l) = \frac{1}{2}(8-4) = 2$ m $\quad M_D = 500$ kN·m

At center of beam $\quad I = I_{beam} + 2I_{plate}$

$= 1190 \times 10^6 + 2\left\{(285)(16)\left(\frac{678}{2} + \frac{16}{2}\right)^2 + \frac{1}{12}(285)(16)^3\right\}$

$= 2288 \times 10^6$ mm^4

$c = \frac{678}{2} + 16 = 355$ mm $\qquad S = \frac{I}{c} = 6445 \times 10^3$ mm^3

$= 6445 \times 10^{-6}$ m^3

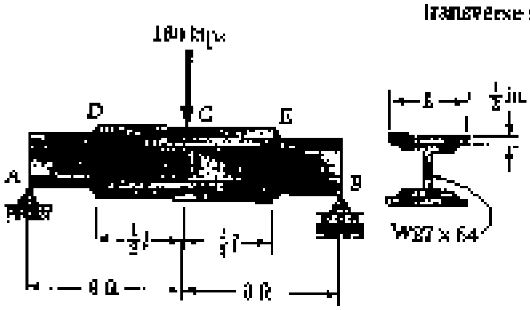
Normal stress $\quad \sigma = \frac{M}{S} = \frac{1000 \times 10^3}{6445 \times 10^{-6}} = 155.2 \times 10^6$ Pa $= 155.2$ MPa

At D $\quad S = 3510 \times 10^3$ mm^3 $= 3510 \times 10^{-6}$ m^3

Normal stress $\quad \sigma = \frac{M}{S} = \frac{500 \times 10^3}{3510 \times 10^{-6}} = 142.4 \times 10^6$ Pa $= 142.4$ MPa

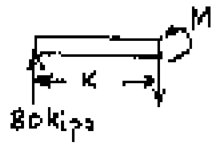
PROBLEM 5.153

5.153 Two cover plates, each $\frac{1}{2}$-in. thick, are welded to a W27 × 84 beam as shown. Knowing that $l = 10$ ft and $b = 10.5$ in., determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.



SOLUTION

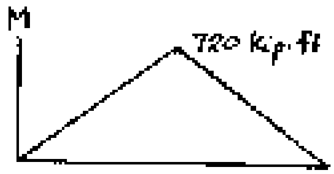
$R_A = R_B = 80$ kips



$+\circlearrowleft \Sigma M_J = 0$

$-80x + M = 0$

$M = 80x$ kip·ft



At C $\quad x = 9$ ft $\quad M_C = 720$ kip·ft $= 8640$ kip·in

At D $\quad x = 9 - 5 = 4$ ft

$M_D = (80)(4) = 320$ kip·ft $= 3840$ kip·in.

At center of beam $\quad I = I_{beam} + 2I_{plate}$

$I = 2850 + 2\left\{(10.5)(0.500)\left(\frac{26.71}{2} + \frac{0.500}{2}\right)^2 + \frac{1}{12}(10.5)(0.500)^3\right\}$

$= 4794 \text{ in}^4$

$c = \frac{26.71}{2} + 0.500 = 13.855$ in

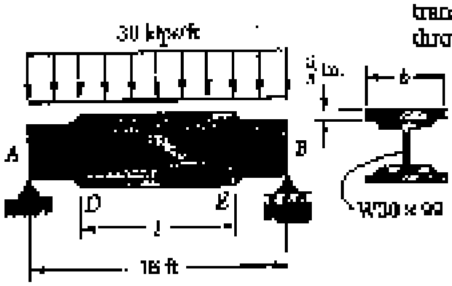
Normal stress $\quad \sigma = \frac{Mc}{I} = \frac{(8640)(13.855)}{4794} = 25.0$ ksi ◄

At point D $\quad S = 213 \text{ in}^3$

Normal stress $\quad \sigma = \frac{M}{S} = \frac{3840}{213} = 18.03$ ksi ◄

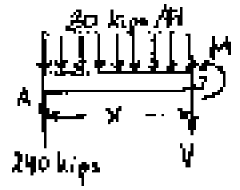
PROBLEM 5.154

5.154 Two cover plates, each $\frac{5}{8}$-in. thick, are welded to a W30 × 99 beam as shown. Knowing that l = 9 ft and b = 12 in., determine the maximum normal stress on a transverse section (*a*) through the center of the beam, (*b*) just to the left of *D*.



SOLUTION

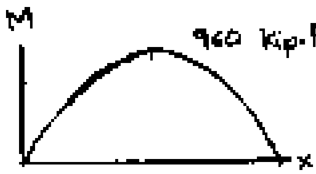
$R_A = R_B = 240$ kips



$+\circlearrowleft \Sigma M_J = 0$

$-240x + 30x\frac{x}{2} + M = 0$

$M = 240x - 15x^2$ kip·ft



At center of beam $x = 8$ ft

$M_c = 960$ kip·ft $= 11520$ kip·in

At point D, $x = \frac{1}{2}(16 - 9) = 3.5$ ft

$M_D = 656.25$ kip·ft $= 7875$ kip·in.

At center of beam $I = I_{beam} + 2I_{plate}$

$$I = 3990 + 2\left\{(12)(0.625)\left(\frac{29.65}{2} + \frac{0.625}{2}\right)^2 + \frac{1}{12}(12)(0.625)^3\right\} = 7428 \text{ in}^4$$

$$c = \frac{29.65}{2} + 0.625 = 15.45 \text{ in}$$

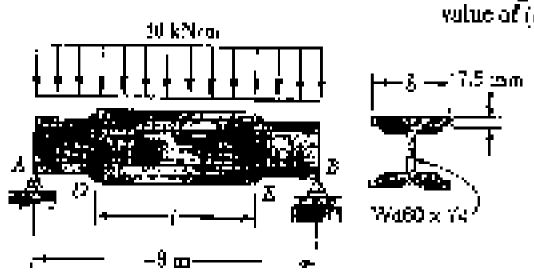
Normal stress $\sigma = \frac{Mc}{I} = \frac{(11520)(15.45)}{7428} = 24.0$ ksi ◄

At point D $S = 269$ in³

Normal stress $\sigma = \frac{M}{S} = \frac{7875}{269} = 29.3$ ksi ◄

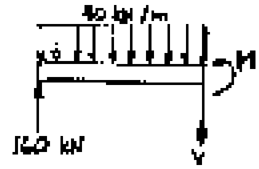
PROBLEM 5.155

5.155 Two cover plates, each 7.5 mm thick, are welded to a W460 × 74 beam as shown. Knowing that $\sigma_{all} = 150$ MPa for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.



SOLUTION

$R_A = R_B = 160$ kN



$+\circlearrowleft \Sigma M_J = 0$

$-160x + (40x)\left(\frac{x}{2}\right) + M = 0$

$M = 160x - 20x^2$ kN·m

For W 460 x 74 rolled steel beam

$S = 1460 \times 10^3\ \text{mm}^3 = 1460 \times 10^{-6}\ \text{m}^3$

Allowable bending moment

$M_{all} = \sigma_{all} S = (150 \times 10^6)(1460 \times 10^{-6})$

$= 219 \times 10^3$ N·m $= 219$ kN·m

To locate points D and E, set $M = M_{all}$

$160x - 20x^2 = 219$ $\qquad 20x^2 - 160x + 219 = 0$

$$x = \frac{160 \pm \sqrt{160^2 - (4)(20)(219)}}{(2)(20)} = \begin{matrix} 1.753\ \text{m} \\ 6.247\ \text{m} \end{matrix}$$

$x_D = 1.753$ ft. $\qquad x_E = 6.247$ ft $\qquad l = x_E - x_D = 4.49$ m

At center of beam $\quad M = 320$ kN·m $= 320 \times 10^3$ N·m

$S = \dfrac{M}{\sigma_{all}} = \dfrac{320 \times 10^3}{150 \times 10^6} = 2133 \times 10^{-6}\ \text{m}^3 = 2133 \times 10^3\ \text{mm}^3$

$c = \dfrac{457}{2} + 7.5 = 236$ mm⁴

Required moment of inertia $\quad I = Sc = 503.4 \times 10^6\ \text{mm}^4$

But $\quad I = I_{beam} + 2I_{plate}$

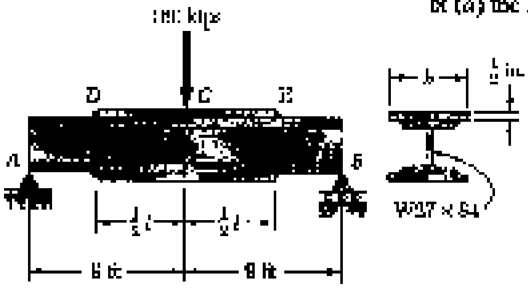
$503.4 \times 10^6 = 333 \times 10^6 + 2\left\{(b)(7.5)\left(\frac{457}{2} + \frac{7.5}{2}\right)^2 + \frac{1}{12}(b)(7.5)^3\right\}$

$= 333 \times 10^6 + 809.2 \times 10^3\ b$

$b = 211$ mm

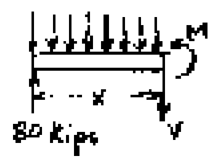
PROBLEM 5.156

5.156 Two cover plates, each $\frac{1}{2}$-in. thick, are welded to a W27 × 84 beam as shown. Knowing that $\sigma_{all} = 24$ ksi for both the beam and the plates, determine the required value of (*a*) the length of the plates, (*b*) the width of the plates.



SOLUTION

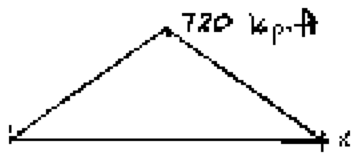
$R_A = R_B = 80$ kips



$+\circlearrowleft \Sigma M_J = 0$

$-80x + M = 0$

$M = 80x \quad$ kip·ft



At D $\qquad S = 213$ in^3

Allowable bending moment

$M_{all} = \sigma_{all} S = (24)(213) = 5112$ kip·in

$= 426$ kip

Set $\quad M_D = M_{all} \qquad 80\, x_D = 426 \qquad x_D = 5.325$ ft.

$l = 18 - 2x_D = 7.35$ ft

At center of beam $\qquad M = (80)(9) = 720$ kip·ft $= 8640$ kip·in.

$$S = \frac{M}{\sigma_{all}} = \frac{8640}{24} = 360 \text{ in}^3$$

$$c = \frac{26.71}{2} + 0.500 = 13.855 \text{ in}$$

Required moment of inertia $\qquad I = Sc = 4987.8$ in^4

But $\quad I = I_{beam} + 2 I_{plate}$

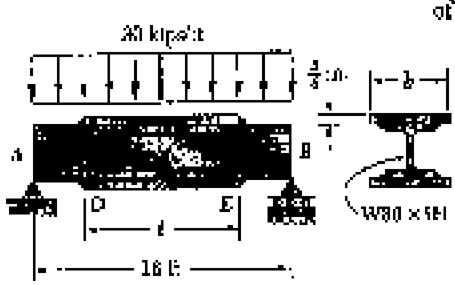
$$4987.8 = 2850 + 2\left\{(b)(0.500)\left(\frac{26.71}{2} + \frac{0.500}{2}\right)^2 + \frac{1}{12}(b)(0.500)^3\right\}$$

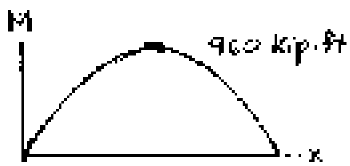
$$= 2850 + 185.12\, b$$

$b = 11.55$ in.

PROBLEM 5.157

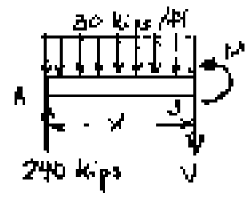
5.157 Two cover plates, each $\frac{5}{8}$-in. thick, are welded to a W30 × 99 beam as shown. Knowing that $\sigma_{all} = 22$ ksi for both the beam and the plates, determine the required value of (a) the length of the plates, (b) the width of the plates.





SOLUTION

$R_A = R_B = 240$ kips



$+\circlearrowleft \Sigma M_J = 0$

$-240x + 30x\frac{x}{2} + M = 0$

$M = 240x - 15x^2$ kip·ft

For W 30 × 99 rolled steel section

$S = 269$ in^3

Allowable bending moment

$M_{all} = \sigma_{all} S = (22)(269) = 5918$ kip·in

$= 493.167$ kip·ft

To locate points D and E, set $M = M_{all}$

$240x - 15x^2 = 493.167 \qquad 15x^2 - 240x + 493.167 = 0$

$$x = \frac{240 \pm \sqrt{(240)^2 - (4)(15)(493.167)}}{(2)(15)} = 2.42 \text{ ft}, \; 13.58 \text{ ft}$$

$l = x_E - x_D = 13.58 - 2.42 = 11.16$ ft

Center of beam $\qquad M = 960$ kip·ft $= 11520$ kip·in

$$S = \frac{M}{\sigma_{all}} = \frac{11520}{22} = 523.64 \text{ in}^3$$

$$c = \frac{29.65}{2} + 0.625 = 15.45 \text{ in}$$

Required moment of inertia $\qquad I = Sc = 8090$ in^4

But $\quad I = I_{beam} + 2I_{plate}$

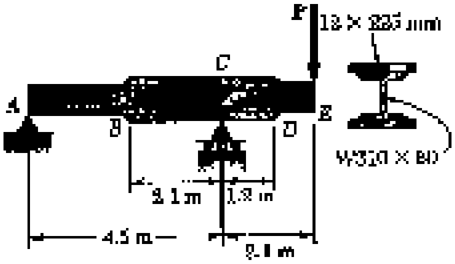
$$8090 = 3990 + 2\left\{(b)(0.625)\left(\frac{29.65}{2} + \frac{0.625}{2}\right)^2 + \frac{1}{12}(b)(0.625)^3\right\}$$

$$= 3990 + 286.47\, b$$

$b = 14.31$ in.

PROBLEM 5.158

be applied at end E of the beam shown. Neglect the weights of the beam and of the plates.



SOLUTION

$\circlearrowleft \Sigma M_C = 0 \quad -4.5 R_A - 2.1 P = 0$

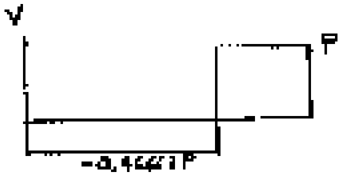
$R_A = -0.46667 P \quad$ i.e. $0.46667 P \downarrow$

$+\circlearrowleft \Sigma M_A = 0 \quad 4.5 R_C - 6.6 P = 0$

$R_C = 1.46667 P$

Shear: A to C $\quad V = -0.46667 P$

C to E $\quad V = P$

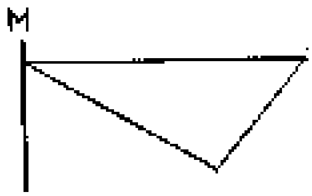


Bending moments:

$M_C = -(4.5)(0.46667 P) = -2.10 P \ \text{kN·m}$

$M_B = \frac{2.4}{4.5} M_C = -1.12 P \ \text{kN·m}$

$M_D = \frac{0.9}{2.1} M_C = -0.9 P \ \text{kN·m}$



At B and D $\quad S = 851 \times 10^3 \ \text{mm}^3 = 851 \times 10^{-6} \ \text{m}^3$

$$\sigma_{all} = \frac{|M|}{S} = \frac{1.120 P_{all}}{851 \times 10^{-6}} = 165 \times 10^6 \quad \text{at } B$$

$P_{all} = 125.4 \ \text{kN}$

At C $\quad I = I_{beam} + 2 I_{plate}$

$$= 129 \times 10^6 + 2\left\{(225)(12)\left(\frac{310}{2} + \frac{12}{2}\right)^2 + \frac{1}{12}(225)(12)^3\right\}$$

$$= 269 \times 10^6 \ \text{mm}^4$$

$c = \frac{310}{2} + 12 = 167 \ \text{mm} \qquad S = \frac{I}{c} = 1611 \times 10^3 \ \text{mm}^3 = 1611 \times 10^{-6} \ \text{m}^3$

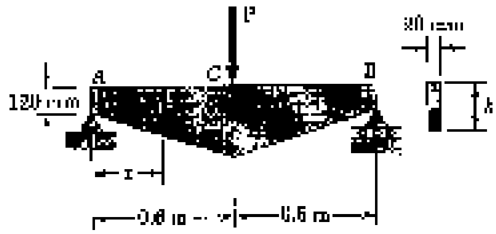
$$\sigma_{all} = \frac{|M|}{S} = \frac{2.10 P}{1611 \times 10^{-6}} = 165 \times 10^6$$

$P_{all} = 126.6 \ \text{kN}$

Allowable load is the smaller value $\quad P = 125.4 \ \text{kN}$

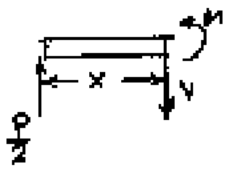
PROBLEM 5.159

5.159 For the tapered beam shown, and knowing that $P = 150$ kN, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the corresponding value of the normal stress.



SOLUTION

$R_A = R_B = \frac{P}{2}$



$\circlearrowleft \Sigma M_J = 0 \qquad -\frac{Px}{2} + M = 0$

$M = \frac{Px}{2} \qquad (0 < x < \frac{L}{2})$

For a tapered rectangular beam $\qquad h = a + kx \qquad (0 < x < \frac{L}{2})$

$S = \frac{1}{6} b h^2 = \frac{1}{6} b (a + kx)^2$

Bending stress $\qquad \sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3P}{b}\frac{d}{dx}\left\{\frac{x}{(a+kx)^2}\right\} = \frac{3P}{b}\,\frac{(a+kx)^2 - x \cdot 2(a+kx)k}{(a+kx)^4}$$

$$= \frac{3P}{b} \cdot \frac{a - kx}{(a+kx)^3} = 0 \qquad x_m = \frac{a}{k}$$

Data: $\quad a = 120$ mm, $\quad k = \frac{300 - 120}{0.6} = 300$ mm/m

$$x_m = \frac{120}{300} = 0.400 \text{ m}$$

$$M_m = \frac{Px_m}{2} = \frac{(150)(0.400)}{2} = 30 \text{ kN}\cdot\text{m}$$

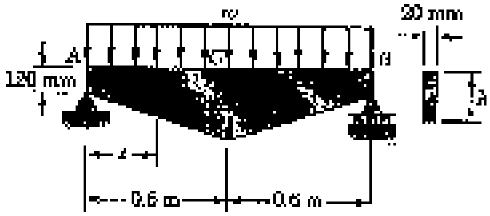
$$h_m = a + kx_m = 120 + (300)(0.400) = 240 \text{ mm}$$

$$S_m = \frac{1}{6} b h_m^2 = (\tfrac{1}{6})(20)(240)^2 = 192 \times 10^3 \text{ mm}^3 = 192 \times 10^{-6} \text{ m}^3$$

$$\sigma_m = \frac{M_m}{S_m} = \frac{30 \times 10^3}{192 \times 10^{-6}} = 156.3 \times 10^6 \text{ Pa} = 156.3 \text{ MPa}$$

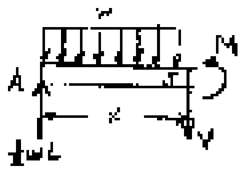
PROBLEM 5.160

5.160 For the tapered beam shown, and knowing that $w = 160$ kN/m, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the corresponding value of the normal stress.



SOLUTION

$R_A = R_B = \frac{1}{2} wL$

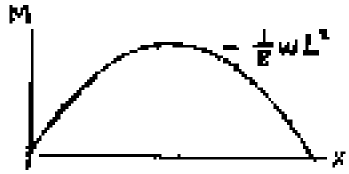


$+\circlearrowleft \Sigma M_J = 0$

$-\frac{1}{2}wLx + wx\frac{x}{2} + M = 0$

$M = \frac{w}{2}(Lx - x^2)$

$= \frac{w}{2}x(L - x)$



where $w = 160$ kN/m and $L = 1.2$ m.

For the tapered beam $h = a + kx$

$a = 120$ mm $\qquad k = \frac{300 - 120}{0.6} = 300$ mm/m

For a rectangular cross section $S = \frac{1}{6}bh^2 = \frac{1}{6}b(a + kx)^2$

Bending stress $\sigma = \frac{M}{S} = \frac{3w}{b} \cdot \frac{Lx - x^2}{(a + kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a + kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a+kx)^2(L-2x) - (Lx - x^2)2(a+kx)k}{(a + kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - 2ax - kLx}{(a + kx)^3}\right\} = 0$$

$$x_m = \frac{aL}{2a + kL} = \frac{(120)(1.2)}{(2)(120) + (300)(1.2)} = 0.24 \text{ m.}$$

$h_m = a + kx_m = 120 + (300)(0.24) = 192$ mm

$S_m = \frac{1}{6}bh_m^2 = \frac{1}{6}(20)(192)^2 = 122.88 \times 10^3\ \text{mm}^3 = 122.88 \times 10^{-6}\ \text{m}^3$

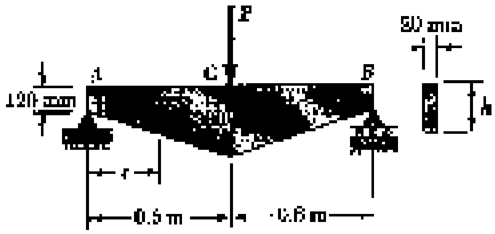
$M_m = \frac{w}{2}x_m(L - x_m) = \frac{160 \times 10^3}{2}(0.24)(0.96) = 18.432 \times 10^3$ N·m

Maximum bending stress $\sigma_m = \frac{M_m}{S_m} = \frac{18.432 \times 10^3}{122.88 \times 10^{-6}} = 150 \times 10^6$ Pa

$= 150$ MPa

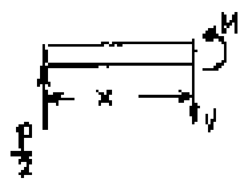
PROBLEM 5.161

5.161 For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest concentrated load **P** that can be applied, knowing that $\sigma_{all} = 140$ MPa.



SOLUTION

$$R_A = R_B = \frac{P}{2}$$



$$\circlearrowleft \Sigma M_J = 0$$

$$-\frac{Px}{2} + M = 0$$

$$M = \frac{Px}{2} \qquad \left(0 < x < \frac{L}{2}\right)$$

For a tapered beam $\quad h = a + kx$

For rectangular cross section $\quad S = \frac{1}{6} b h^2 = \frac{1}{6} b (a + kx)^2$

Bending stress $\quad \sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3P}{b}\frac{d}{dx}\left\{\frac{x}{(a+kx)^2}\right\} = \frac{3P}{b} \cdot \frac{(a+kx)^2 - x \cdot 2(a+kx)k}{(a+kx)^4}$$

$$= \frac{3P}{b}\frac{a-kx}{(a+kx)^3} = 0 \qquad x_m = \frac{a}{k}$$

Then $\quad M_m = \frac{Px_m}{2} = \frac{Pa}{2k}$

$$h_m = a + kx_m = 2a$$

$$S_m = \frac{1}{6} b h_m^2 = \frac{2}{3} b a^2$$

Data: $\quad a = 120$ mm $\qquad k = \frac{300 - 120}{0.6} = 300$ mm/m $\qquad b = 20$ mm

$$x_m = \frac{120 \text{ mm}}{300 \text{ mm/m}} = 0.400 \text{ m} \blacktriangleleft$$

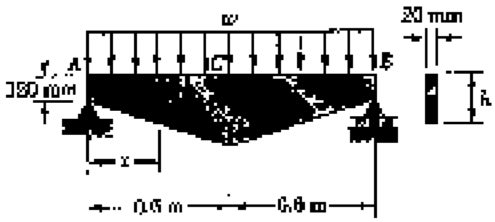
$$S_m = \frac{2}{3}(20)(120)^2 = 192 \times 10^3 \text{ mm}^3 = 192 \times 10^{-6} \text{ m}^3$$

$$M_m = \sigma_{all} S_m = (140 \times 10^6)(192 \times 10^{-6}) = 26.88 \times 10^3 \text{ N}\cdot\text{m}$$

$$P = \frac{2M_m}{x_m} = \frac{(2)(26.88 \times 10^3)}{0.400} = 134.4 \times 10^3 \text{ N} = 134.4 \text{ kN} \blacktriangleleft$$

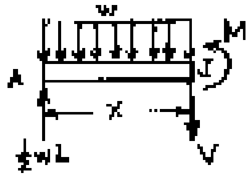
PROBLEM 5.162

5.162 For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest distributed load w that can be applied, knowing that $\sigma_{all} = 140$ MPa.



SOLUTION

$R_A = R_B = \frac{1}{2}wL \qquad L = 1.2 \text{ m}$

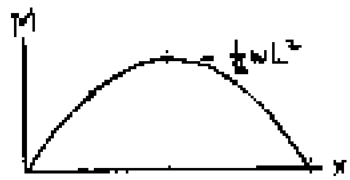


$\circlearrowleft \Sigma M_J = 0$

$-\frac{1}{2}wL + wx\frac{x}{2} + M = 0$

$M = \frac{w}{2}(Lx - x^2)$

$= \frac{w}{2}x(L - x)$



For the tapered beam $\quad h = a + kx$

$a = 120 \text{ mm} \qquad k = \frac{300 - 120}{0.6} = 300 \text{ mm/m}$

For rectangular cross section $\quad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a + kx)^2$

Bending stress $\quad \sigma = \frac{M}{S} = \frac{3w}{b}\frac{Lx - x^2}{(a + kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a + kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a + kx)^2(L - 2x) - (Lx - x^2)2(a + kx)k}{(a + kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a + kx)(L - 2x) - 2k(Lx - x^2)}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax \cancel{- 2kx^2} - 2kLx \cancel{+ 2kx^2}}{(a + kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a + kL)x}{(a + kx)^3}\right\} = 0$$

$$x_m = \frac{aL}{2a + kL} = \frac{(120)(1.2)}{(2)(120) + (300)(1.2)} = 0.24 \text{ m}$$

$$h_m = a + kx_m = 120 + (300)(0.24) = 192 \text{ mm}$$

$$S_m = \frac{1}{6}bh_m^2 = \frac{1}{6}(20)(192)^2 = 122.88 \times 10^3 \text{ mm}^3 = 122.88 \times 10^{-6} \text{ m}^3$$

Allowable value of $M_m \qquad M_m = S_m\sigma_{all} = (122.88 \times 10^{-6})(140 \times 10^6)$

$= 17.2032 \times 10^3 \text{ N}\cdot\text{m}$

Allowable value of $w \qquad w = \frac{2M_m}{x_m(L - x_m)} = \frac{(2)(17.2032 \times 10^3)}{(0.24)(0.96)}$

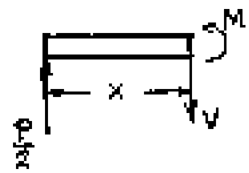
$= 149.3 \times 10^3 \text{ N/m} = 149.3 \text{ kN/m}$

PROBLEM 5.163

5.163 For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest concentrated load **P** that can be applied, knowing that $\sigma_{all} = 24$ ksi.

SOLUTION

$$R_A = R_B = \frac{P}{2}$$



$$+\circlearrowleft \Sigma M_J = 0$$

$$-\frac{Px}{2} + M = 0$$

$$M = \frac{Px}{2} \qquad (0 < x < \tfrac{L}{2})$$

For a tapered beam $\qquad h = a + kx$

For a rectangular cross section $\qquad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress $\qquad \sigma = \frac{M}{S} = \frac{3Px}{b(a+kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3P}{b}\frac{d}{dx}\left\{\frac{x}{(a+kx)^2}\right\} = \frac{3P}{b}\,\frac{(a+kx)^2 - x\cdot 2(a+kx)k}{(a+kx)^4}$$

$$= \frac{3P}{b}\,\frac{a-kx}{(a+kx)^3} = 0 \qquad\qquad x_m = \frac{a}{k}$$

Data: $\quad a = 4$ in., $\quad k = \frac{8-4}{30} = 0.13333$ in/in

$$x_m = \frac{4}{0.13333} = 30 \text{ in.}$$

$$h_m = a + kx_m = 8 \text{ in}$$

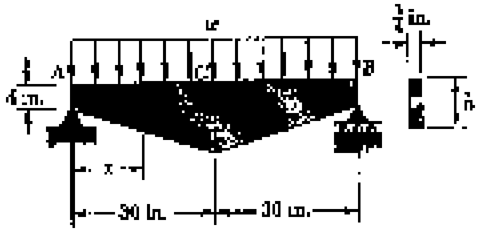
$$S_m = \frac{1}{6}bh_m^2 = \left(\frac{1}{6}\right)\left(\frac{3}{4}\right)(8)^2 = 8 \text{ in}^3$$

$$M_m = \sigma_{all} S_m = (24)(8) = 192 \text{ kip}\cdot\text{in}$$

$$P = \frac{2M_m}{x_m} = \frac{(2)(192)}{30} = 12.8 \text{ kips}$$

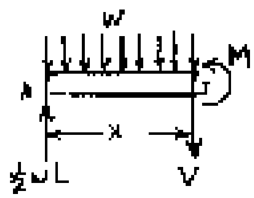
PROBLEM 5.164

5.164 For the tapered beam shown, determine (*a*) the transverse section in which the maximum normal stress occurs, (*b*) the largest distributed load w that can be applied, knowing that $\sigma_{all} = 24$ ksi.



SOLUTION

$R_A = R_B = \frac{1}{2}wL \qquad L = 60$ in.

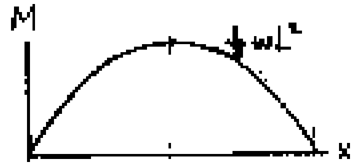


$\Sigma M_J = 0$

$-\frac{1}{2}wLx + wx\frac{x}{2} + M = 0$

$M = \frac{w}{2}(Lx - x^2)$

$= \frac{w}{2}x(L-x)$



For the tapered beam $\quad h = a + kx$

$a = 4$ in $\qquad k = \frac{8-4}{30} = \frac{2}{15}$ in/in.

For a rectangular cross section $\quad S = \frac{1}{6}bh^2 = \frac{1}{6}b(a+kx)^2$

Bending stress $\quad \sigma = \frac{M}{S} = \frac{3w}{b}\cdot\frac{Lx - x^2}{(a+kx)^2}$

To find location of maximum bending stress set $\frac{d\sigma}{dx} = 0$

$$\frac{d\sigma}{dx} = \frac{3w}{b}\frac{d}{dx}\left\{\frac{Lx - x^2}{(a+kx)^2}\right\} = \frac{3w}{b}\left\{\frac{(a+kx)^2(L-2x) - (Lx-x^2)2(a+kx)k}{(a+kx)^4}\right\}$$

$$= \frac{3w}{b}\left\{\frac{(a+kx)(L-2x) - 2k(Lx-x^2)}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL + kLx - 2ax - 2kx^2 - 2kLx + 2kx^2}{(a+kx)^3}\right\}$$

$$= \frac{3w}{b}\left\{\frac{aL - (2a+kL)x}{(a+kx)^3}\right\} = 0$$

$$x_m = \frac{aL}{2a+kL} = \frac{(4)(60)}{(2)(4) + (\frac{2}{15})(60)} = 15 \text{ in.}$$

$h_m = a + kx_m = 4 + (\frac{2}{15})(15) = 6.00$ in.

$S_m = \frac{1}{6}bh_m^2 = (\frac{1}{6})(\frac{3}{4})(6.00)^2 = 4.50$ in^3

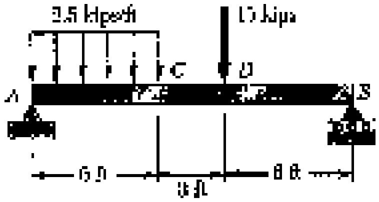
Allowable value of $M_m = S_m\sigma_{all} = (4.50)(24) = 180.0$ kip-in

Allowable value of $w \qquad w = \frac{2M_m}{x_m(L-x_m)} = \frac{(2)(108.0)}{(15)(45)} = 0.320$ kip/in

$= 320$ lb/in

PROBLEM 5.165

5.165 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.



SOLUTION

$+\circlearrowleft \Sigma M_B = 0$

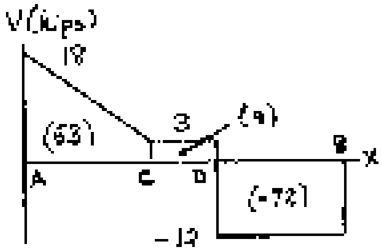
$-15 R_A + (12)(6)(2.5) + (6)(15) = 0$

$R_A = 18$ kips

$+\circlearrowleft \Sigma M_A = 0$

$15 R_B - (3)(6)(2.5) - (9)(15) = 0$

$R_B = 12$ kips



Shear: $V_A = 18$ kips

$V_C = 18 - (6)(2.5) = 3$ kips

C to D $V = 3$ kips

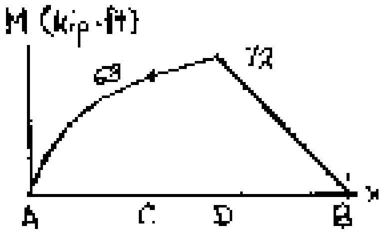
D to B $V = 3 - 15 = -12$ kips

Areas under shear diagram

A to C $\int V dx = (\frac{1}{2})(6)(18 + 3) = 63$ kip·ft

C to D $\int V dx = (3)(3) = 9$ kip·ft

D to B $\int V dx = (6)(-12) = -72$ kip·ft



Bending moments: $M_A = 0$

$M_C = 0 + 63 = 63$ kip·ft

$M_D = 63 + 9 = 72$ kip·ft

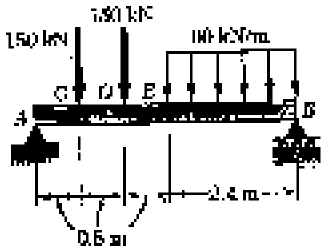
$M_B = 72 - 72 = 0$

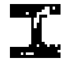
$|V|_{max} = 18$ kips

$|M|_{max} = 72$ kip·ft

PROBLEM 5.166

5.166 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at D.





W460 × 113

SOLUTION

$\circlearrowleft \Sigma M_B = 0$

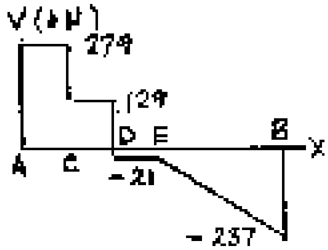
$-4.8 R_A + (4.0)(150) + (3.2)(150) + (1.2)(2.4)(90) = 0$

$R_A = 279 \text{ kN}$

$\circlearrowleft \Sigma M_A = 0$

$4.8 R_B - (0.8)(150) - (1.6)(150) - (3.6)(2.4)(90) = 0$

$R_B = 237 \text{ kN}$



Shear:

A to C $\quad V = 279$

C to D $\quad V = 279 - 150 = 129 \text{ kN}$

D to E $\quad V = 129 - 150 = -21 \text{ kN}$

$V_E = -21 \text{ kN}$

$V_B = -21 - (2.4)(90) = -237 \text{ kN}$

Areas under shear diagram

A to C $\quad \int V dx = (0.8)(279) = 223.2 \text{ kN·m}$

C to D $\quad \int V dx = (0.8)(129) = 103.2 \text{ kN·m}$

Maximum bending moment occurs at point D where V changes sign.

$$M_D = M_A + \int_A^D V dx$$

$$= 0 + 223.2 + 103.2 = 326.4 \text{ kN·m}$$

For rolled steel section W 460 × 113

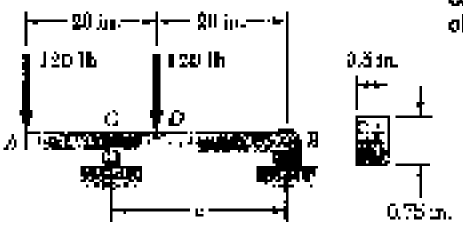
$S = 2400 \times 10^3 \text{ mm}^3 = 2400 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{326.4 \times 10^3}{2400 \times 10^{-6}} = 136.0 \times 10^6 \text{ Pa}$

$= 136.0 \text{ MPa}$

PROBLEM 5.167

5.167 Determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (*Hint:* Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)



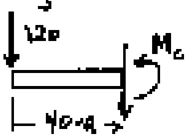
SOLUTION

$+\circlearrowleft \Sigma M_B = 0$

$-R_C\, a + (40)(120) + (20)(120) = 0$

$R_C = \dfrac{7200}{a}$

Bending moment at C

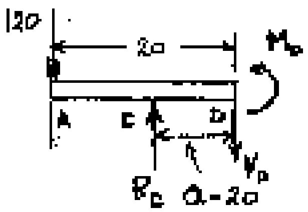


$\circlearrowright \Sigma M_C = 0$

$M_C + 120\,(40 - a) = 0$

$M_C = -4800 + 120\,a$

Bending moment at D



$+\circlearrowleft \Sigma M_D = 0$

$M_D + (20)(120) - R_C(a - 20) = 0$

$M_D = R_C(a-20) - 2400$

$= R_C a - 20\,R_C - 2400$

$= 7200 - \dfrac{(20)(7200)}{a} - 2400$

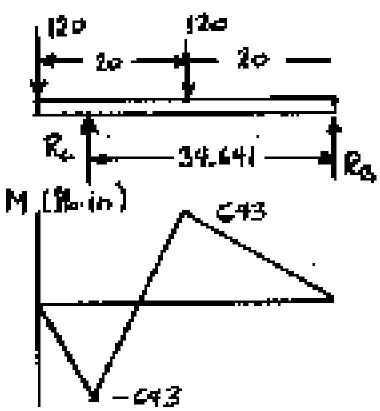
$= 4800 - \dfrac{144000}{a}$

Equate $-M_C = M_D$ $\qquad 4800 - 120\,a = 4800 - \dfrac{144000}{a}$

$120\,a^2 = 144000. \qquad a = 34.641 \text{ in.}$ ◄

$M_C = -4800 + (120)(34.641) = -643.08 \text{ lb}\cdot\text{in}$

$M_D = 4800 - \dfrac{144000}{34.641} = 643.08 \text{ lb}\cdot\text{in}$ ✓



For rectangular section $\quad S = \frac{1}{6}bh^2$

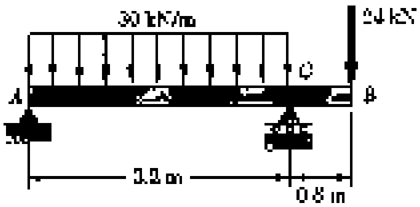
$S = \left(\frac{1}{6}\right)(0.5)(0.75)^2 = 0.046875 \text{ in}^3$

Maximum normal stress

$\sigma_{max} = \dfrac{M_{max}}{S} = \dfrac{643.08}{0.046875} = 13.72 \times 10^3 \text{ psi}$

$= 13.72 \text{ ksi}$ ◄

PROBLEM 5.168



5.168 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

SOLUTION

$+\circlearrowleft \Sigma M_C = 0$

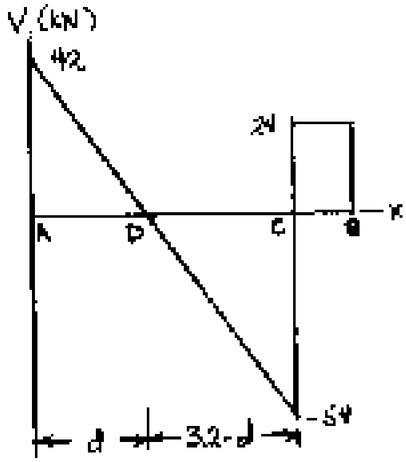
$-3.2 R_A + (1.6)(3.2)(30) - (0.8)(24) = 0$

$R_A = 42 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0$

$3.2 R_C - (1.6)(3.2)(30) - (4.0)(24) = 0$

$R_C = 78 \text{ kN}$



Shear: $V_A = 42 \text{ kN}$

$V_{C^-} = 42 - (3.2)(30) = -54 \text{ kN}$

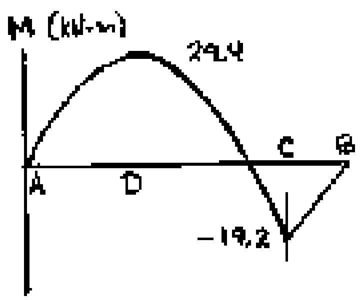
$V_{C^+} = -54 + 78 = 24 \text{ kN}$

C to D $\quad V = 24 \text{ kN}.$

Locate point D where $V = 0$

$$\frac{d}{42} = \frac{3.2-d}{54} \qquad 96d = 134.4$$

$d = 1.4 \text{ m} \qquad 3.2 - d = 1.8 \text{ m}$



Areas under shear diagram

A to D $\quad \int V dx = (\frac{1}{2})(1.4)(42) = 29.4 \text{ kN}\cdot\text{m}$

D to C $\quad \int V dx = (\frac{1}{2})(1.8)(54) = -48.6 \text{ kN}\cdot\text{m}$

C to B $\quad \int V dx = (0.8)(24) = 19.2 \text{ kN}\cdot\text{m}$

Bending moments: $\quad M_A = 0$

$M_D = 0 + 29.4 = 29.4 \text{ kN}\cdot\text{m}$

$M_C = 29.4 - 48.6 = -19.2 \text{ kN}\cdot\text{m}$

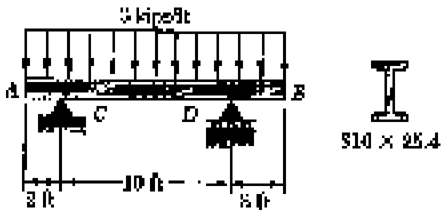
$M_B = 19.2 - 19.2 = 0 \quad$ checks

Maximum $|V| = 54 \text{ kN}$ ◄

Maximum $|M| = 29.4 \text{ kN}\cdot\text{m}$ ◄

PROBLEM 5.169

5.169 For the beam and loading shown, determine (*a*) the maximum value of the bending moment, (*b*) the maximum normal stress due to bending.



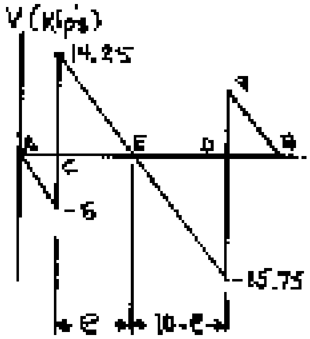
SOLUTION

$+\circlearrowleft \Sigma M_D = 0 \qquad -10 R_C + (4.5)(15)(3) = 0$

$R_C = 20.25$ kips

$+\circlearrowleft \Sigma M_C = 0 \qquad 10 R_D - (5.5)(15)(3) = 0$

$R_D = 24.75$ kips



Shear: $V_A = 0$

$V_{C^-} = 0 - (2)(3) = -6$ kips

$V_{C^+} = -6 + 20.25 = 14.25$ kips

$V_{D^-} = 14.25 - (10)(3) = -15.75$ kips

$V_{D^+} = -15.75 + 24.75 = 9$ kips

$V_B = 9 - (3)(3) = 0$ checks

Locate point E where $V = 0$

$\dfrac{e}{14.25} = \dfrac{10-e}{15.75} \qquad 30e = 142.5$

$e = 4.75$ ft $\qquad 10 - e = 5.25$ ft

Areas under shear diagram

A to C $\quad \int V dx = (\frac{1}{2})(2)(-6) = -6$ kip·ft

C to E $\quad \int V dx = (\frac{1}{2})(4.75)(14.25) = 33.84$ kip·ft

E to D $\quad \int V dx = (\frac{1}{2})(5.25)(15.75) = -41.34$ kip·ft

D to B $\quad \int V dx = (\frac{1}{2})(3)(9) = 13.5$ kip·ft

Bending moments: $M_A = 0$

$M_C = 0 - 6 = -6$ kip·ft

$M_E = -6 + 33.84 = 27.84$ kip·ft

$M_D = 27.84 - 41.34 = -13.5$ kip·ft

$M_B = -13.5 + 13.5 = 0$ checks

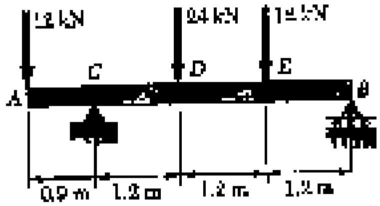
Maximum $|M| = 27.84$ kip·ft ◄

$= 334.1$ kip·in

For rolled steel section S10 × 25.4 $\qquad S = 24.7 \text{ in}^3$

Maximum normal stress $\qquad \sigma_{max} = \dfrac{M_{max}}{S} = \dfrac{334.1}{24.7} = 13.53$ ksi ◄

PROBLEM 5.170



5.170 (*a*) Using singularity functions, write the equations defining the shear and bending moment for the beam and loading shown. (*b*) Determine the maximum value of the bending moment in the beam.

SOLUTION

$+\circlearrowleft \Sigma M_B = 0$

$(4.5)(12) - 3.6 R_C + (2.4)(24) + (1.2)(12) = 0$

$R_C = 35 \text{ kN}$

$V = -12 + 35\langle x - 0.9\rangle^0 - 24\langle x - 2.1\rangle^0 - 12\langle x - 3.3\rangle^0 \text{ kN}$

$M = -12x + 35\langle x - 0.9\rangle^1 - 24\langle x - 2.1\rangle^1 - 12\langle x - 3.3\rangle^1 \text{ kN·m}$

Pt. C $(x = 0.9 \text{ m})$ $M_C = -(12)(0.9) = -10.8 \text{ kN·m}$

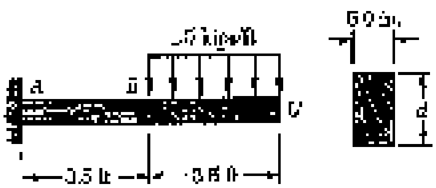
Pt D $(x = 2.1 \text{ m})$ $M_D = -(12)(2.1) + (35)(1.2) = 16.8 \text{ kN·m}$

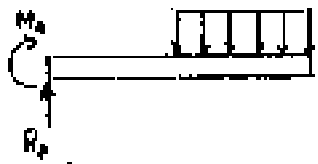
Pt E $(x = 3.3 \text{ m})$ $M_E = -(12)(3.3) + (35)(2.4) - (24)(1.2)$
$= 15.6 \text{ kN·m}$

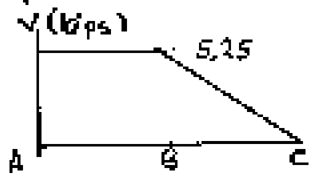
Maximum $|M| = 16.8 \text{ kN·m}$

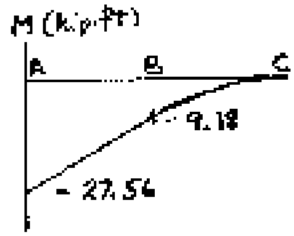
PROBLEM 5.171

5.171 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1.75 ksi.









SOLUTION

$+\uparrow \Sigma F_y = 0 \qquad R_A - (3.5)(1.5) = 0$

$R_A = 5.25$ kN

$+\circlearrowleft \Sigma M_A = 0 \qquad -M_A - (5.25)(3.5)(1.5) = 0$

$M_A = -27.56$ kN

Shear: A to B $\qquad V = 5.25$ kips

$V_C = 5.25 - (3.5)(1.5) = 0$ checks

Areas of shear diagram

A to B $\qquad \int V dx = (3.5)(5.25) = 18.38$ kip·ft

B to C $\qquad \int V dx = (\frac{1}{2})(3.5)(5.25) = 9.19$ kip·ft

Bending moments $\qquad M_A = -27.56$ kip·ft

$M_B = -27.56 + 18.38 = -9.18$

$M_C = -9.18 + 9.19 \approx 0$ checks.

Maximum $|M| = 27.56$ kip·ft $= 330.7$ kip·in

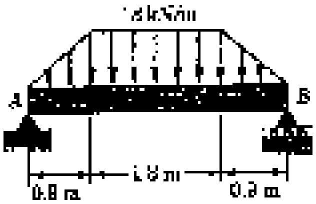
$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{330.7}{1.75} = 189.0 \text{ in}^3$$

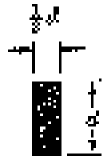
For a rectangular cross section $\qquad S = \frac{1}{6} b d^2 \qquad d = \sqrt{\frac{6S}{b}}$

$$d = \sqrt{\frac{(6)(189.0)}{5}} = 15.06 \text{ in.}$$

PROBLEM 5.172

5.172 For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 12 MPa.





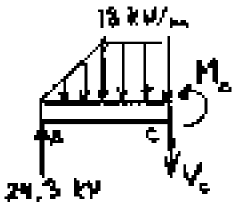
SOLUTION

By symmetry $R_A = R_B$ $\qquad +\uparrow \Sigma F_y = 0$

$$R_A - (\tfrac{1}{2})(0.9)(18) - (1.8)(18) - (\tfrac{1}{2})(0.9)(18) + R_B = 0$$

$$R_A = R_B = 24.3 \text{ kN}$$

Maximum bending moment occurs at the center



$$\circlearrowleft \Sigma M_C = 0$$

$$-(1.8)(24.3) + (1.2)(\tfrac{1}{2})(0.9)(18) + (0.45)(0.9)(18) + M_C = 0$$

$$M_C = 26.73 \text{ kN}\cdot\text{m} = 26.73\times10^3 \text{ N/m}$$

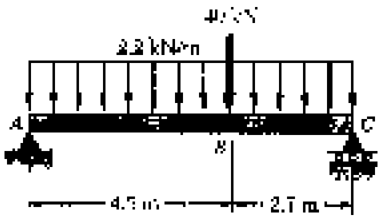
$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{26.73\times10^3}{12\times10^6} = 2.2275\times10^{-3} \text{ m}^3 = 2227.5\times10^3 \text{ mm}^3$$

For a rectangular section $\quad S = \frac{1}{6}bd^2 = \frac{1}{6}(\frac{1}{3}d)d^2 = \frac{1}{18}d^3$

$$\frac{1}{18}d^3 = 2227.5\times10^3 \qquad d^3 = 40.09\times10^6 \qquad d = 342 \text{ mm}$$

PROBLEM 5.173

5.173 Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical metric wide-flange beam to support the loading shown.



SOLUTION

$\circlearrowleft \Sigma M_C = 0$

$-7.2 R_A + (3.6)(7.2)(2.2) + (2.7)(40) = 0$

$R_A = 22.92$ kN

Shear: $V_A = 22.92$ kN

$V_{B^-} = 22.92 - (4.5)(2.2) = 13.02$ kN

$V_{B^+} = 13.02 - 40 = -26.98$ kN

$V_C = -26.98 - (2.7)(2.2) = -32.92$ kN

Areas under shear diagram

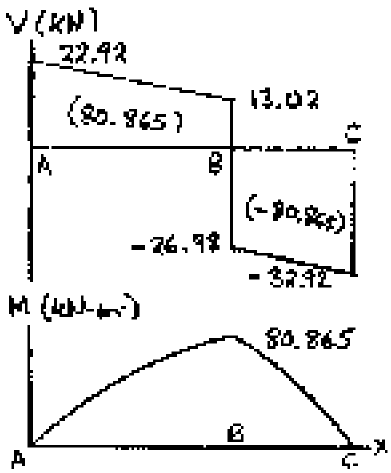
A to B $\int V\,dx = (\frac{1}{2})(4.5)(22.92 + 13.02) = 80.865$ kN·m

B to C $\int V\,dx = (\frac{1}{2})(2.7)(-26.98 - 32.92) = -80.865$ kN·m

Bending moments: $M_A = 0$

$M_B = 0 + 80.865 = 80.865$ kN-m

$M_C = 80.865 - 80.865 = 0$ checks



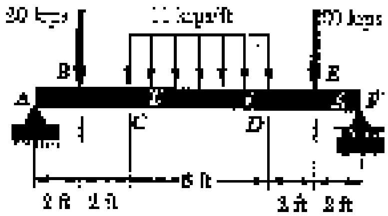
$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{80.865 \times 10^3}{160 \times 10^6} = 505 \times 10^{-6}\ m^3 = 505 \times 10^3\ mm^3$$

Shape	S ($10^3\ mm^3$)
W 410 × 38.8	637
W 360 × 39	578
W 310 × 38.7	549 —
W 250 × 44.8	535
W 200 × 52	512

Use W 310 × 38.7 ◄

PROBLEM 5.174

5.174 Knowing that the allowable normal stress for the steel used is 24 ksi, select the most economical wide-flange beam to support the loading shown.



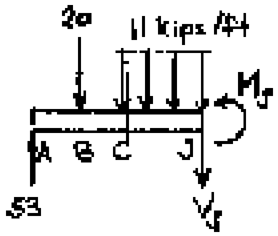
SOLUTION

By symmetry $R_A = R_F$

$+\uparrow \Sigma F_y = 0 \qquad R_A - 20 - (6)(11) - 20 + R_F = 0$

$R_A = R_F = 50$ kips.

Maximum bending moment occurs at center of beam.



$\circlearrowleft \Sigma M_J = 0 \qquad -(7)(53) + (5)(20) + (1.5)(3)(11) + M_J = 0$

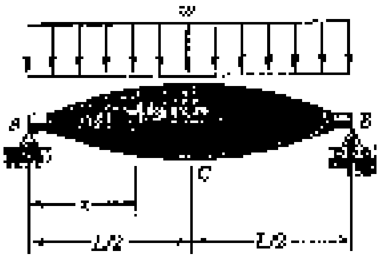
$M_J = 221.5$ kip·ft $= 2658$ kip·in

$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{2658}{24} = 110.75 \text{ in}^3$$

Shape	S (in³)
W 24 × 68	154
W 21 × 62	127
W 18 × 76	146
W 16 × 77	134
W 14 × 82	123
W 12 × 96	131

Use W 21 × 62

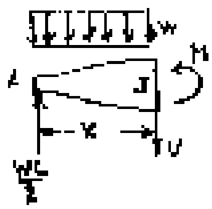
PROBLEM 5.175



5.175 A machine element of cast aluminum and in the shape of a solid of revolution of variable diameter d is being designed to support a distributed load w as shown. (a) Knowing that the machine element is to be of constant strength, express d in terms of x, L, and d_0. (b) Determine the smallest allowable value of d_0 if $L = 250$ mm, $w = 30$ kN/m, and $\sigma_{all} = 72$ MPa.

SOLUTION

$$R_A = R_B = \frac{wL}{2}$$



$$+\circlearrowleft \Sigma M_J = 0$$

$$-\frac{wL}{2}x + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}x(L-x)$$

$$S = \frac{|M|}{\sigma_{all}} = \frac{wx(L-x)}{2\sigma_{all}}$$

For a solid circular cross section $\quad c = \frac{d}{2} \quad I = \frac{\pi}{4}c^3 \quad S = \frac{I}{c} = \frac{\pi d^3}{32}$

Equating $\quad \frac{\pi d^3}{32} = \frac{wx(L-x)}{2\sigma_{all}} \qquad d = \left\{\frac{16\,wx(L-x)}{\pi\sigma_{all}}\right\}^{1/3}$

At $x = \frac{L}{2} \qquad d = d_0 = \left\{\frac{4wL^2}{\pi\sigma_{all}}\right\}^{1/3} \qquad \therefore d = d_0\left\{4\frac{x}{L}\left(1-\frac{x}{L}\right)\right\}^{1/3}$ ◄

Using the data $\quad d_0 = \frac{(4)(30\times10^3)(0.250)^2}{\pi(72\times10^6)} = 32.1\times10^{-3}\text{ m} = 32.1\text{ mm}$ ◄

PROBLEM 5.C1

5.C1 Several concentrated loads P_i ($i = 1, 2, \ldots, n$) can be applied to a beam as shown. Write a computer program that can be used to calculate the shear, bending moment, and normal stress at any point of the beam for a given loading of the beam and a given value of its section modulus. Use this program to solve Probs. 5.23, 5.27, and 5.29. (*Hint:* Maximum values will occur at a support or under a load.)

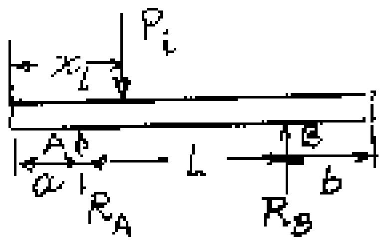
SOLUTION

REACTIONS AT A AND B

$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - \sum_i P_i (x_i - a)$$

$$R_B = (1/L) \sum_i P_i (x_i - a)$$

$$R_A = \sum_i P_i - R_B$$



WE USE STEP FUNCTIONS (See bottom of page 348 of text.)

WE DEFINE: IF $x \geq a$ THEN STPA=1 ELSE STPA = 0

IF $x \geq a + L$ THEN STPB=1 ELSE STPB = 0

IF $x \geq x_i$ THEN STP(I)=1 ELSE STP(I)=0

$$V = R_A \, STPA + R_B \, STPB - \sum P_i \, STP(I)$$

$$M = R_A (x-a) STPA + R_B (x-a-L) STPB - \sum_i P_i (x - x_i) STP(I)$$

$\sigma = M/S$, where S is obtained from Appendix C.

PROGRAM OUTPUTS

Prob. 5.23

RA=80.0 kN RB=80.0 kN

X m	V kN	M kN.m	Sigma MPa
2.00	0.00	104.00	128.55

Prob. 5.27

R1 = 40.0 kN R2 = 16.0 kN

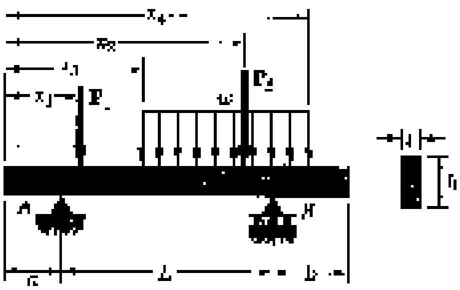
X m	V kN	M kN.m	Sigma MPa
0.00	-20.00	0.00	0.00
1.60	24.00	-32.00	-21.27
4.00	16.00	25.60	24.55
5.60	-16.00	0.00	0.00

Prob. 5.29

R1 = 52.5 kips R2 = 22.5 kips

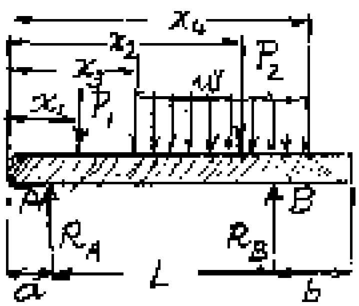
X ft	V kips	M kip.ft	Sigma ksi
0.00	-25.00	0.00	0.00
1.00	27.50	-25.00	-7.85
3.00	2.50	30.00	9.42
9.00	-22.50	45.00	14.14
11.00	0.00	0.00	0.00

PROBLEM 5.C2



5.C2 A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress in the beam will not exceed a given allowable value σ_{all}. Write a computer program that can be used to calculate at given intervals ΔL the shear, the bending moment, and the smallest acceptable value of the unknown dimension. Apply this program to solve the following problems, using the intervals ΔL indicated: (*a*) Prob. 5.75 ($\Delta L = 0.1$ m), (*b*) Prob. 5.79 ($\Delta L = 0.2$ m), (*c*) Prob. 5.80 ($\Delta L = 0.3$ m).

SOLUTION



REACTIONS AT A AND B

$$+\circlearrowleft \sum M_A = 0: \quad R_B L - P_1(x_1 - a) - P_2(x_2 - a) - w(x_4 - x_3)\left(\frac{x_4 + x_3}{2} - a\right) = 0$$

$$R_B = \frac{1}{L}\left[P_1(x_1 - a) + P_2(x_2 - a) + \tfrac{1}{2} w(x_4 - x_3)(x_4 + x_3 - 2a)\right]$$

$$R_A = P_1 + P_2 + w(x_4 - x_3) - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text)

SET $n = (a + b + L)/\Delta L$

FOR $i = 0$ TO n: $x = (\Delta L)\, i$

WE DEFINE: IF $x \geq a$ THEN STPA = 1 ELSE STPA = 0

IF $x \geq a + L$ THEN STPB = 1 ELSE STPB = 0

IF $x \geq x_1$ THEN STP1 = 1 ELSE STP1 = 0

IF $x \geq x_2$ THEN STP2 = 1 ELSE STP2 = 0

IF $x \geq x_3$ THEN STP3 = 1 ELSE STP3 = 0

IF $x \geq x_4$ THEN STP4 = 1 ELSE STP4 = 0

$$V = R_A\,\text{STPA} + R_B\,\text{STPB} - P_1\,\text{STP1} - P_2\,\text{STP2} - w(x - x_3)\,\text{STP3} + w(x - x_4)\,\text{STP4}$$

$$M = R_A(x - a)\,\text{STPA} + R_B(x - a - L)\,\text{STPB} - P_1(x - x_1)\,\text{STP1} - P_2(x - x_2)\,\text{STP2} - \tfrac{1}{2} w(x - x_3)^2\,\text{STP3} + \tfrac{1}{2} w(x - x_4)^2\,\text{STP4}$$

$$S_{min} = |M| / \sigma_{all}$$

IF UNKNOWN DIMENSION IS h:

From $S = \frac{1}{6} t h^2$, we have $h = \sqrt{6S/t}$

IF UNKNOWN DIMENSION IS t:

From $S = \frac{1}{6} t h^2$, we have $t = 6S/h^2$

(CONTINUED)

PROBLEM 5.C2 CONTINUED

PROGRAM OUTPUTS

Prob. 5.75

RA = 2.40 kN RB = 3.00 kN

X m	V kN	M kN.m	H mm
0.00	2.40	0.000	0.00
0.10	2.40	0.240	54.77
0.20	2.40	0.480	77.46
0.30	2.40	0.720	94.87
0.40	2.40	0.960	109.54
0.50	2.40	1.200	122.47
0.60	2.40	1.440	134.16
0.70	2.40	1.680	144.91
0.80	0.60	1.920	154.92
0.90	0.60	1.980	157.32
1.00	0.60	2.040	159.69
1.10	0.60	2.100	162.02
1.20	0.60	2.160	164.32
1.30	0.60	2.220	166.58
1.40	0.60	2.280	168.82
1.50	0.60	2.340	171.03
1.60	-3.00	2.400	173.21
1.70	-3.00	2.100	162.02
1.80	-3.00	1.800	150.00
1.90	-3.00	1.500	136.93
2.00	-3.00	1.200	122.47
2.10	-3.00	0.900	106.07
2.20	-3.00	0.600	86.60
2.30	-3.00	0.300	61.24
2.40	0.00	0.000	0.05

Prob. 5.79

RA = 2.70 kN RB = 6.10 kN

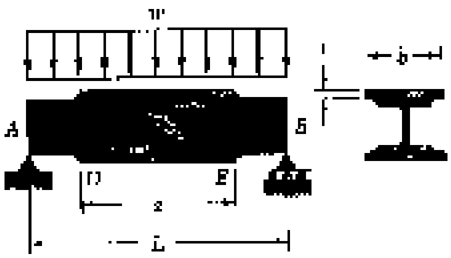
X m	V kN	M kN.m	T mm
0.00	2.70	0.000	0.00
0.20	2.10	0.480	10.67
0.40	1.50	0.840	18.67
0.60	0.90	1.080	24.00
0.80	0.30	1.200	26.67
1.00	-0.30	1.200	26.67
1.20	-0.90	1.080	24.00
1.40	-1.50	0.840	18.67
1.60	-2.10	0.480	10.67
1.80	-2.70	0.000	0.00
2.00	-3.30	-0.600	13.33
2.20	-3.90	-1.320	29.33
2.40	3.60	-2.160	48.00
2.60	3.00	-1.500	33.33
2.80	2.40	-0.960	21.33
3.00	1.80	-0.540	12.00
3.20	1.20	-0.240	5.33
3.40	0.60	-0.060	1.33
3.60	0.00	-0.000	0.00

Prob. 5.80

RA - 6.50 kN RB = 6.50 kN

X m	V kN	M kN.m	H mm
0.00	2.50	0.000	0.00
0.30	2.50	0.750	61.24
0.60	9.00	1.500	86.60
0.90	7.20	3.930	140.18
1.20	5.40	5.820	170.59
1.50	3.60	7.170	189.34
1.80	1.80	7.980	199.75
2.10	-0.00	8.250	203.10
2.40	-1.80	7.980	199.75
2.70	-3.60	7.170	189.34
3.00	-5.40	5.820	170.59
3.30	-7.20	3.930	140.18
3.60	-2.50	1.500	86.60
3.90	-2.50	0.750	61.24
4.20	0.00	0.000	0.06

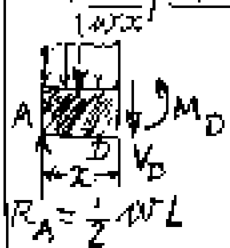
PROBLEM 5.C3



5.C3 Two cover plates, each of thickness t, are to be welded to a wide-flange beam of length L, which is to support a uniformly distributed load w. Denoting by σ_{all} the allowable normal stress in the beam and in the plates, by d the depth of the beam, and by I_b and S_b, respectively, the moment of inertia and the section modulus of the cross section of the unreinforced beam about a horizontal centroidal axis, write a computer program that can be used to calculate the required value of (a) the length a of the plates, (b) the width b of the plates. Use this program to solve Probs. 5.155 and 5.157.

SOLUTION

(a) Required Length of Plates



$FB = AD$: $+\circlearrowleft \Sigma M_D = 0$: $M_D + wx\left(\frac{x}{2}\right) - R_A x = 0$

But: $R_A = \frac{1}{2}wL$ and $M_D = S\sigma_{all}$. Divide by $\frac{1}{2}w$:

$$x^2 - Lx + (2S\sigma_{all}/w) = 0, \quad \text{Set } k = 2S\sigma_{all}/w$$

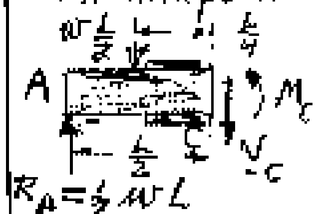
$$x^2 - Lx + k = 0$$

Solving the quadratic, $x = \dfrac{L - \sqrt{L^2 - 4k}}{2}$

Compute x and $a = L - 2x$

(b) Required Width of Plates

At midpoint C of beam:

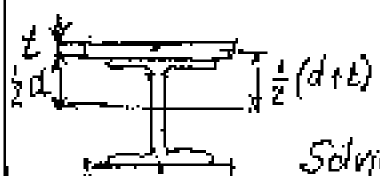


$FB = AC$: $+\circlearrowleft \Sigma M_C = 0$: $M_C + \frac{wL}{2}\frac{L}{4} - \frac{wL}{2}\frac{L}{2} = 0$

Compute $M_C = \frac{1}{8}wL^2$

Compute $c = t + \frac{1}{2}d$

From $\sigma_{all} = \frac{M_C c}{I}$ compute $I = \frac{M_C c}{\sigma_{all}}$



But $I = I_{beam} + I_{plates} = I_b + 2\left[\frac{1}{12}bt^3 + bt\left(\frac{d+t}{2}\right)^2\right]$

Solving for b: $b = \dfrac{6(I - I_b)}{t\left[t^2 + 3(d+t)^2\right]}$

PROGRAM OUTPUTS

PROB. 5.155: W460×74, $\sigma_{all} = 150$ MPa
$w = 40$ kN/m, $L = 8$ m, $t = 7.5$ mm
$d = 457$ mm, $I_b = 333\times10^6$ mm⁴, $S = 1460\times10^3$ mm³

Prob. 5.155

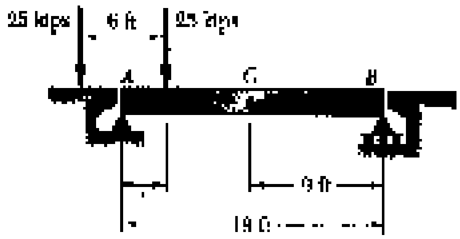
a = 4.49 m
b = 211 mm

PROB. 5.157: W30×99, $\sigma_{all} = 22$ ksi
$w = 30$ kips/ft, $L = 16$ ft, $t = 5/8$ in.
$d = 29.65$ in., $I_b = 3990$ in⁴, $S = 269$ in³

Prob. 5.157

a = 11.16 ft
b = 14.31 in.

PROBLEM 5.C4



5.C4 Two 25-kip loads are maintained 6 ft apart as they are moved slowly across the 18-ft beam AB. Write a computer program and use it to calculate the bending moment under each load and at the midpoint C of the beam for values of x from 0 to 24 ft at intervals $\Delta x = 1.5$ ft.

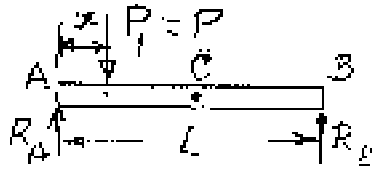
SOLUTION

NOTATION: Length of beam $= L = 18$ ft

Loads: $P_1 = P_2 = P = 25$ kips

Distance between loads $= d = 6$ ft

We note that $d < L/2$

(1) 

(1) FROM $x = 0$ TO $x = d$:

$+\circlearrowleft \sum M_B = 0$: $P(L-x) - R_A L = 0$

$$R_A = P(L-x)/L$$

Under P_1: $M_1 = R_A x$

At C: $M_C = R_A\left(\frac{L}{2}\right) - P\left(\frac{L}{2} - x\right)$

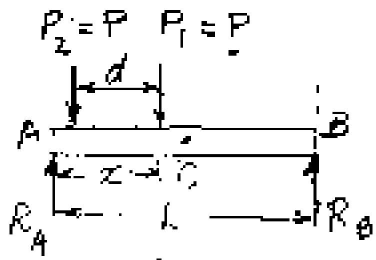
(2) FROM $x = d$ TO $x = L$:

$+\circlearrowleft \sum M_B = 0$: $P(L-x) + P(L-x+d) - R_A L = 0$

$$R_A = P(2L - 2x + d)/L$$

Under P_1: $M_1 = R_A x - Pd$

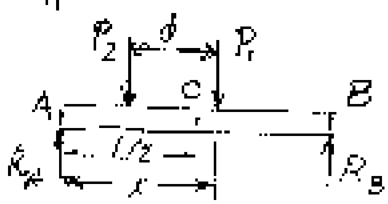
Under P_2: $M_2 = R_A(x-d)$

(2A) 

(2A) FROM $x = d$ TO $x = L/2$:

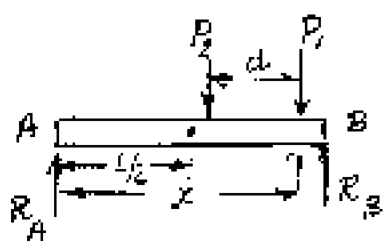
$M_C = R_A\left(\frac{L}{2}\right) - P\left(\frac{L}{2} - x\right) - P\left(\frac{L}{2} - x + d\right)$

$= R_A(L/2) - P(L - 2x + d)$

(2B) 

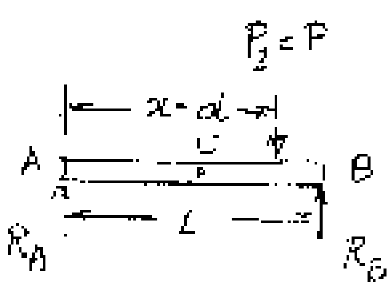
(2B) FROM $x = L/2$ TO $x = L/2 + d$:

$M_C = R_A(L/2) - P\left(\frac{L}{2} - x + d\right)$

(2C) 

(2C) FROM $x = L/2 + d$ TO $x = L$:

$M_C = R_A L/2$

(3) 

(3) FROM $x = L$ TO $x = L + d$:

$+\circlearrowleft \sum M_B = 0$: $P(L - x + d) - R_A L = 0$

$$R_A = P(L - x + d)/L$$

Under P_2: $M_2 = R_A(x-d)$

At C: $M_C = R_A(L/2)$

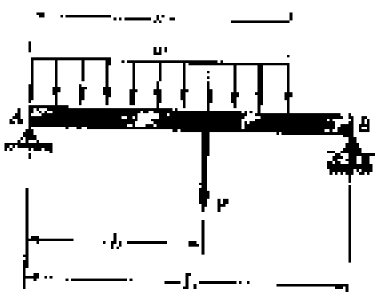
(CONTINUED)

PROGRAM OUTPUT

P = 25 kips, L = 18 ft, D = 6 ft

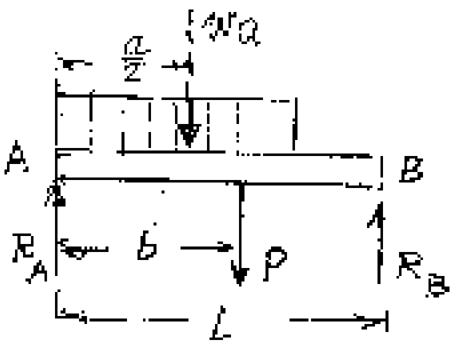
x ft	MC kip.ft	M1 kip.ft	M2 kip.ft
0.0	0.00	0.00	0.00
1.5	18.75	34.38	0.00
3.0	37.50	62.50	0.00
4.5	56.25	84.38	0.00
6.0	75.00	100.00	0.00
7.5	112.50	131.25	56.25
9.0	150.00	150.00	100.00
10.5	150.00	156.25	131.25
12.0	150.00	150.00	150.00
13.5	150.00	131.25	156.25
15.0	150.00	100.00	150.00
16.5	112.50	56.25	131.25
18.0	75.00	0.00	100.00
19.5	56.25	0.00	84.38
21.0	37.50	0.00	62.50
22.5	18.75	0.00	34.38
24.0	0.00	0.00	0.00

PROBLEM 5.C6



5.C6 Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval $\Delta L = 0.2$ ft to the beam and loading of (*a*) Prob. 5.83, (*b*) Prob. 5.125.

SOLUTION



REACTIONS AT A AND B

USING FB DIAGRAM OF BEAM:

$$+\circlearrowleft \Sigma M_A = 0: \quad R_B L - Pb - wa(a/2) = 0$$

$$R_B = (1/L)\left(Pb + \tfrac{1}{2} wa^2\right)$$

$$R_A = P + wa - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 348 of text).

SET $n = L/\Delta L$. FOR $i = 0$ TO n: $x = (\Delta L)\, i$

WE DEFINE: IF $x \geq a$ THEN STPA=1 ELSE STPA=0

IF $x \geq b$ THEN STPB=1 ELSE STPB=0

$$V = R_A - wx + w(x-a)\,STPA - P\,STPB$$

$$M = R_A x - \tfrac{1}{2} w x^2 + \tfrac{1}{2} w (x-a)^2\, STPA - P(x-b)\,STPB$$

LOCATE AND PRINT (x, V) AND (x, M)

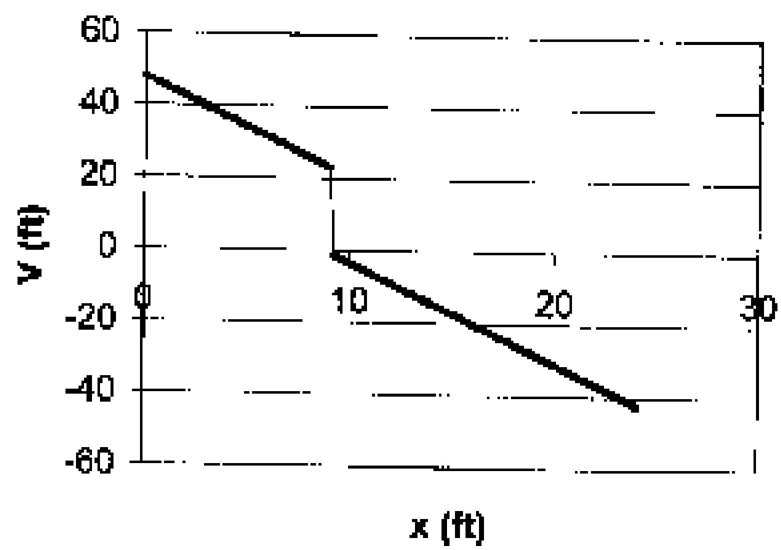
SEE NEXT PAGES FOR PROGRAM OUTPUTS

(CONTINUED)

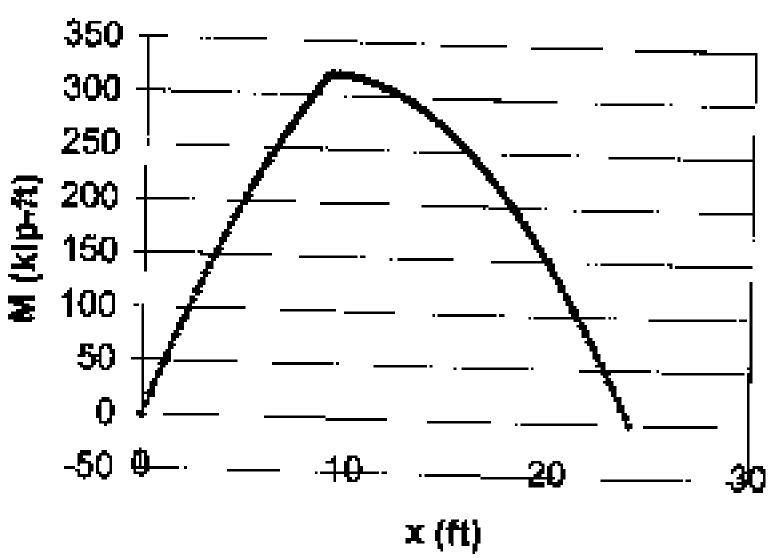
PROGRAM OUTPUT FOR P5.83

```
PROBLEM 5.83

RA =    48.00 kips   RB =     42.00 kips
```

Shear Diagram



Moment Diagram



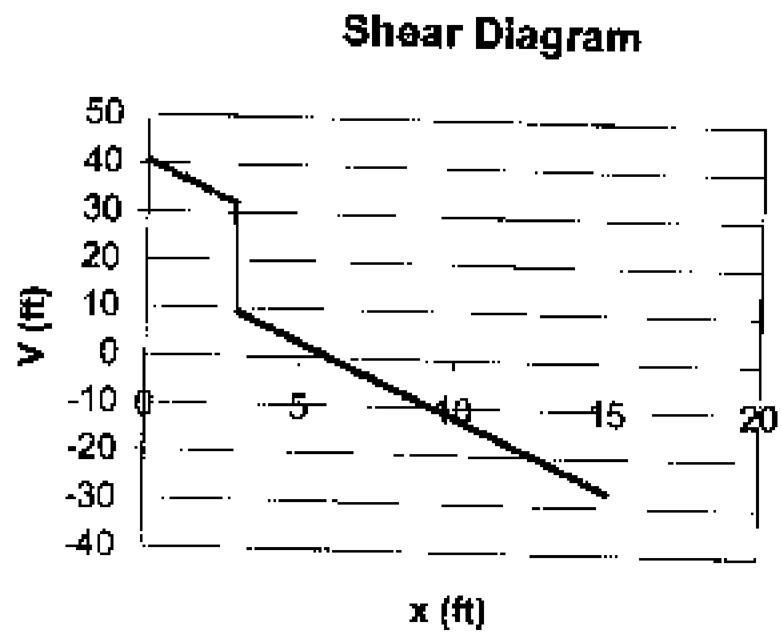
(CONTINUED)

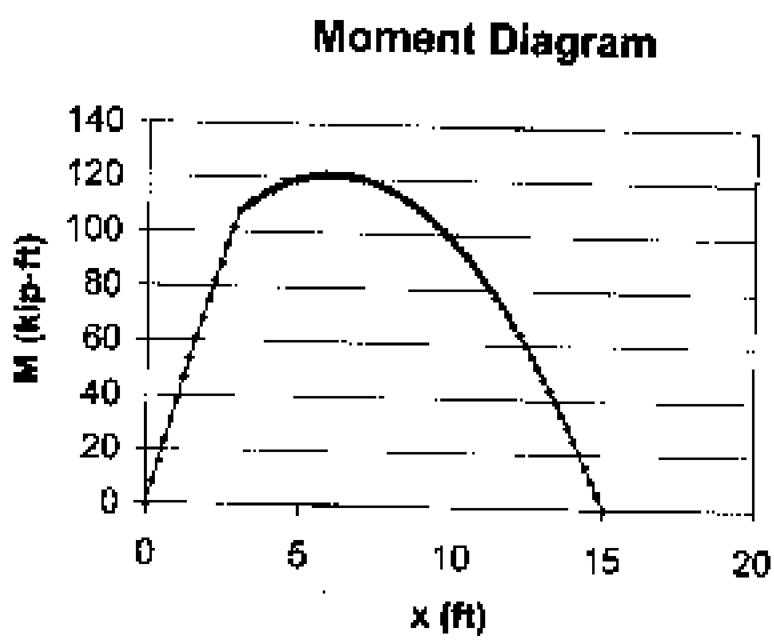
PROBLEM 5.C5 CONTINUED

PROGRAM OUTPUT FOR P5.125

PROBLEM 5.125

RA = 40.50 kips RB = 27.00 kips

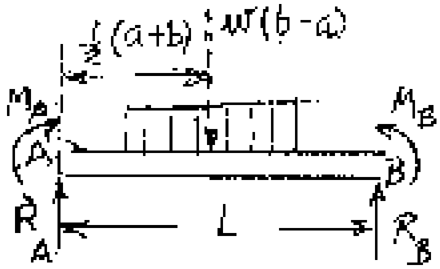




PROBLEM 5.C6

5.C6 Write a computer program that can be used to plot the shear and bending-moment diagrams for the beam and loading shown. Apply this program with a plotting interval $\Delta L = 0.025$ m to the beam and loading of Prob. 5.124.

SOLUTION



REACTIONS AT A AND B

$+\circlearrowleft \sum M_A = 0:$

$$R_B L + M_B - M_A - w(b-a)\tfrac{1}{2}(a+b) = 0$$

$$R_B = (1/L)\left[M_A - M_B + \tfrac{1}{2}w(b^2 - a^2)\right]$$

$$R_A = w(b-a) - R_B$$

WE USE STEP FUNCTIONS (See bottom of page 340 of text)

SET $n = L/\Delta L$, FOR $i = 0$ TO n: $x = (\Delta L)i$

WE DEFINE: IF $x \geq a$ THEN STPA = 1 ELSE STPA = 0

IF $x \geq b$ THEN STPB = 1 ELSE STPB = 0

$$V = R_A - w(x-a)\,STPA + w(x-b)\,STPB$$

$$M = M_A + R_A x - \tfrac{1}{2}w(x-a)^2\,STPA + \tfrac{1}{2}w(x-b)^2\,STPB$$

LOCATE AND PRINT (x, V) AND (x, M)

PROGRAM OUTPUT ON NEXT PAGE

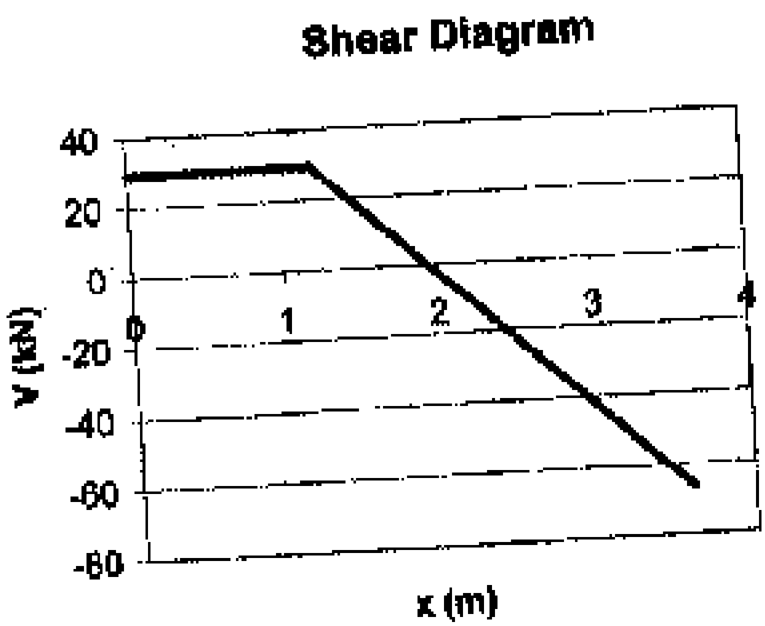
(CONTINUED)

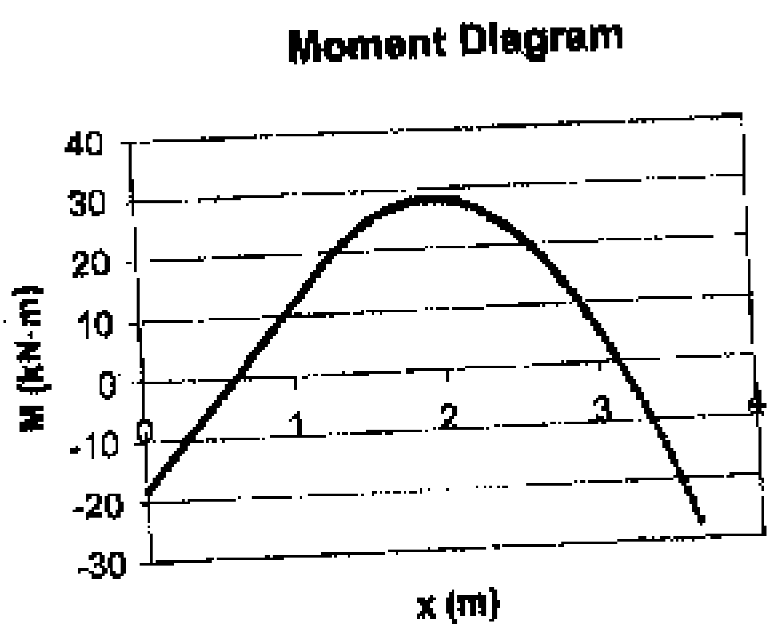
PROBLEM 5.C6 CONTINUED

PROGRAM OUTPUT

PROBLEM 5.124

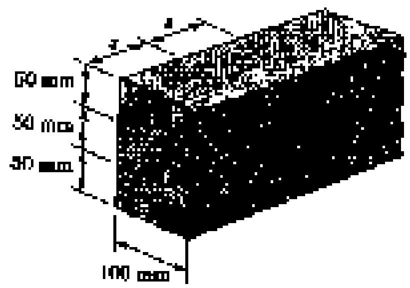
RA = 29.50 kips RB = 66.50 kips





CHAPTER 6

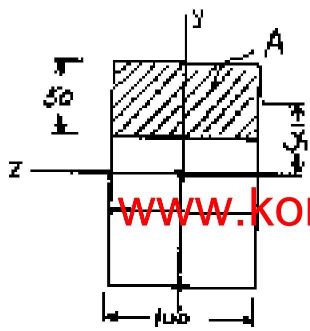
PROBLEM 6.1



6.1 Three full-size 50 × 100-mm boards are nailed together to form a beam that is subjected to a vertical shear of 1500 N. Knowing that the allowable shearing force in each nail is 400 N, determine the largest longitudinal spacing s that can be used between each pair of nails.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(100)(150)^3 = 28.125\times10^{6}\ \text{mm}^4$$
$$= 28.125\times10^{-6}\ \text{m}^4$$



$$A = (100)(50) = 5000\ \text{mm}^2$$

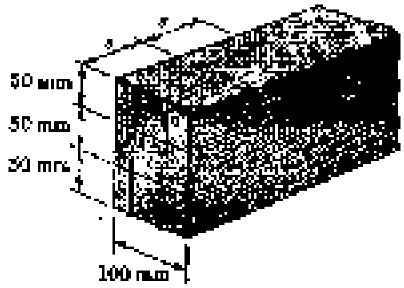
$$\bar{y}_1 = 50\ \text{mm}$$

$$Q = A\bar{y}_1 = 250\times10^{3}\ \text{mm}^3$$
$$= 250\times10^{-6}\ \text{m}^3$$

$$q = \frac{VQ}{I} = \frac{(1500)(250\times10^{-6})}{28.125\times10^{-6}} = 13.333\times10^{3}\ \text{N/m}$$

$$qs = 2F_{nail} \qquad s = \frac{2F_{nail}}{q} = \frac{(2)(400)}{13.333\times10^{3}} = 60\times10^{-3}\ \text{m}$$
$$= 60\ \text{mm}$$

PROBLEM 6.2

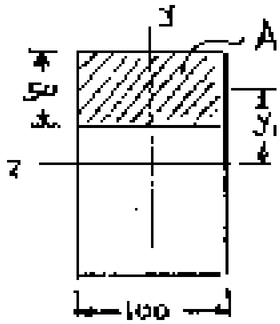


6.1 Three full-size 50 × 100-mm boards are nailed together to form a beam that is subjected to a vertical shear of 1500 N. Knowing that the allowable shearing force in each nail is 400 N, determine the largest longitudinal spacing s that can be used between each pair of nails.

6.2 For the built-up beam of Prob. 6.1, determine the allowable shear if the spacing between each pair of nails is $s = 45$ mm.

SOLUTION

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(100)(150)^3 = 28.125 \times 10^6 \text{ mm}^4$$
$$= 28.125 \times 10^{-6} \text{ m}^4$$



$$A = (100)(50) = 5000 \text{ mm}^2$$

$$\bar{y}_1 = 50 \text{ mm}$$

$$Q = A\bar{y}_1 = 250 \times 10^3 \text{ mm}^3 = 250 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} \qquad qs = 2F_{nail}$$

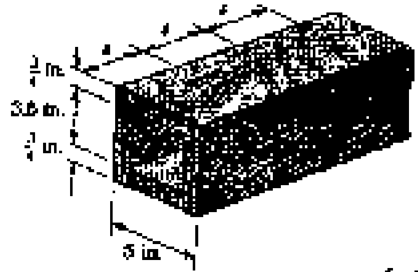
Eliminating q $\qquad \frac{VQ}{I} = \frac{2F_{nail}}{s}$

Solving for V $\qquad V = \frac{2IF_{nail}}{Qs} = \frac{(2)(28.125\times10^{-6})(400)}{(250\times10^{-6})(45\times10^{-3})}$

$$= 2 \times 10^3 \text{ N} = 2 \text{ kN}$$

PROBLEM 6.3

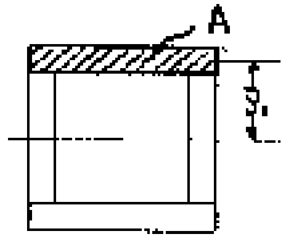
6.3 A square box beam is made of two $\frac{3}{4} \times 3.5$-in. planks and two $\frac{3}{4} \times 5$-in. planks nailed together as shown. Knowing that the spacing between nails is $s = 1.25$ in. and that the vertical shear in the beam is $V = 250$ lb, determine (*a*) the shearing force in each nail, (*b*) the maximum shearing stress in the beam.



SOLUTION

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(5)(5)^3 - \frac{1}{12}(3.5)(3.5)^3 = 39.578 \text{ in}^4$$



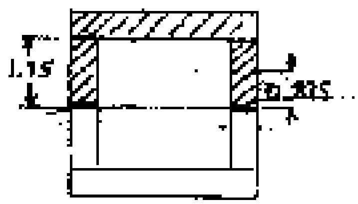
(a) $A = (5)(\frac{3}{4}) = 3.75 \text{ in}^2$

$$\bar{y}_1 = 2.5 - \frac{3}{8} = 2.125 \text{ in}$$

$$Q_1 = A\bar{y}_1 = (3.75)(2.125) = 7.969 \text{ in}^3$$

$$q = \frac{VQ_1}{I} = \frac{(250)(7.969)}{39.578} = 50.34 \text{ lb/in}$$

$$qs = 2F_{nail} \qquad F_{nail} = \frac{qs}{2} = \frac{(50.34)(1.25)}{2} = 31.5 \text{ lb.}$$



(b) $Q_2 = Q_1 + (2)(1.75)(\frac{3}{4})(0.875)$

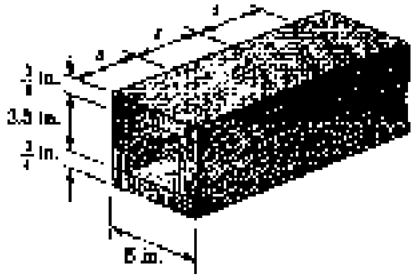
$$= 7.969 + 2.297 = 10.266 \text{ in}^3$$

$$t = (2)(\tfrac{3}{4}) = 1.5 \text{ in.}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(250)(10.266)}{(39.578)(1.5)} = 43.2 \text{ psi}$$

PROBLEM 6.4

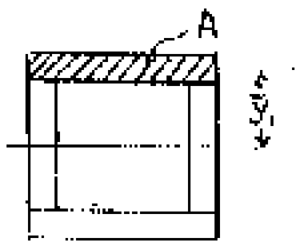
6.4 A square box beam is made of two $\frac{3}{4} \times 3.5$-in. planks and two $\frac{3}{4} \times 5$-in. planks nailed together as shown. Knowing that the spacing between nails is $s = 2$ in. and that the allowable shearing force in each nail is 75 lb, determine (*a*) the largest allowable vertical shear in the beam, (*b*) the corresponding maximum shearing stress in the beam.



SOLUTION

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(5)(5)^3 - \frac{1}{12}(3.5)(3.5)^3 = 39.578 \text{ in}^4$$



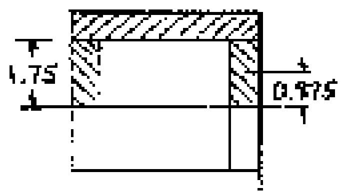
(a) $A = (5)\left(\frac{3}{4}\right) = 3.75 \text{ in}^2$

$$\bar{y}_1 = 2.5 - \frac{3}{8} = 2.125 \text{ in.}$$

$$Q_1 = A\bar{y}_1 = (3.75)(2.125) = 7.969 \text{ in}^3$$

$$q_{all} = \frac{2F_{nail}}{s} = \frac{(2)(75)}{2} = 75 \text{ lb/in}$$

$$q = \frac{VQ}{I} \qquad V_{all} = \frac{I q_{all}}{Q_1} = \frac{(39.578)(75)}{7.969} = 372 \text{ lb}$$



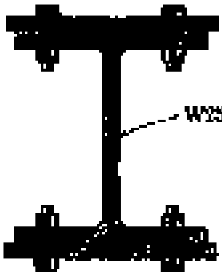
(b) $Q = Q_1 + (2)(1.75)\left(\frac{3}{4}\right)(0.875)$

$$= 7.969 + 2.297 = 10.266 \text{ in}^3$$

$$t = (2)\left(\frac{3}{4}\right) = 1.5 \text{ in}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(372)(10.266)}{(39.578)(1.5)} = 64.4 \text{ psi}$$

PROBLEM 6.5



6.5 The beam shown has been reinforced by attaching to it two 12 × 175-mm plates, using bolts of 18-mm diameter spaced longitudinally every 125 mm. Knowing that the average allowable shearing stress in the bolts is 85 MPa, determine the largest permissible vertical shearing force.

SOLUTION

Calculate moment of inertia

Part	A (mm²)	d (mm)	Ad^2 (10^6 mm⁴)	$\bar{I}$ (10^6 mm⁴)
Top plate	2100	* 139	40.574	0.025
W250 × 44.8	5720	0	0	71.1
Bot. plate	2100	* 139	40.574	0.025
Σ			81.148	71.15

$$* \quad d = \frac{266}{2} + \frac{12}{2} = 139 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 152.30 \times 10^6 \text{ mm}^4 = 152.30 \times 10^{-6} \text{ m}^4$$

$$Q = A_{plate} d_{plate} = (2100)(139) = 291.9 \times 10^3 \text{ mm}^3 = 291.9 \times 10^{-6} \text{ m}^3$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

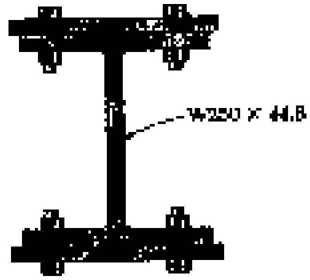
$$F_{bolt} = \tau_{bolt} A_{bolt} = (85 \times 10^6)(254.47 \times 10^{-6}) = 21.63 \times 10^3 \text{ N}$$

$$q = \frac{2F_{bolt}}{s} = \frac{(2)(21.63 \times 10^3)}{125 \times 10^{-3}} = 346.1 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(152.30 \times 10^{-6})(346.1 \times 10^3)}{291.9 \times 10^{-6}} = 180.6 \times 10^3 \text{ N}$$

$$= 180.6 \text{ kN}$$

PROBLEM 6.6



6.5 The beam shown has been reinforced by attaching to it two 12 × 175-mm plates, using bolts of 18-mm diameter spaced longitudinally every 125 mm. Knowing that the average allowable shearing stress in the bolts is 85 MPa, determine the largest permissible vertical shearing force.

6.6 Solve Prob. 6.5, assuming that the reinforcing plates are only 9 mm thick.

SOLUTION

Calculate moment of inertia.

Part	A (mm²)	d (mm)	Ad^2 (10^6 mm⁴)	$\bar{I}$ (10^6 mm⁴)
Top plate	1575	* 137.5	29.777	0.011
W250×44.8	5720	0	0	71.1
Bot. plate	1575	137.5	29.777	0.011
Σ			59.555	71.121

$$* \quad d = \frac{266}{2} + \frac{9}{2} = 137.5 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 130.68 \times 10^6 \text{ mm}^4 = 130.68 \times 10^{-6} \text{ m}^4$$

$$Q = A_{plate}\, d_{plate} = (1575)(137.5) = 216.56 \times 10^3 \text{ mm}^3 = 216.56 \times 10^{-6} \text{ m}^3$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

$$F_{bolt} = \tau_{all} A_{bolt} = (85 \times 10^6)(254.47 \times 10^{-6}) = 21.63 \times 10^3 \text{ N}$$

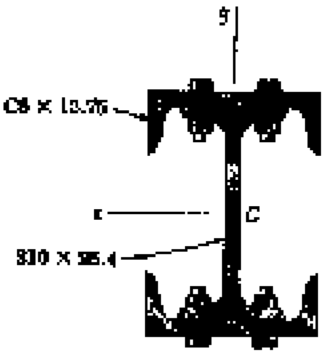
$$q = \frac{2F_{bolt}}{s} = \frac{(2)(21.63 \times 10^3)}{125 \times 10^{-3}} = 346.1 \times 10^3 \text{ N/m}$$

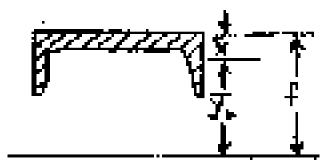
$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(130.68 \times 10^{-6})(346.1 \times 10^3)}{216.56 \times 10^{-6}} = 209 \times 10^3 \text{ N}$$

$$= 209 \text{ kN}$$

PROBLEM 6.7

6.7 and 6.8 A column is fabricated by connecting the rolled-steel members shown by bolts of $\frac{3}{4}$-in. diameter spaced longitudinally every 5 in. Determine the average shearing stress in the bolts caused by a shearing force of 30 kips parallel to the y axis.





SOLUTION

Geometry

$$f = \left(\frac{d}{2}\right)_s + (t_w)_c$$

$$= \frac{10}{2} + 0.303 = 5.303 \text{ in}$$

$$\bar{x} = 0.533 \text{ in}$$

$$\bar{y}_1 = f - \bar{x} = 5.303 - 0.533 = 4.770 \text{ in.}$$

Determine moment of inertia.

Part	A (in²)	d (in)	Ad² (in⁴)	$\bar{I}$ (in⁴)
C8 × 13.75	4.04	4.770	91.92	1.53
S10 × 25.4	7.46	0	0	124
C8 × 13.75	4.04	4.770	91.92	1.53
Σ			183.84	127.06

$$I = \Sigma A d^2 + \Sigma \bar{I} = 183.84 + 127.06 = 310.9 \text{ in}^4$$

$$Q = A\bar{y}_1 = (4.04)(4.770) = 19.271 \text{ in}^3$$

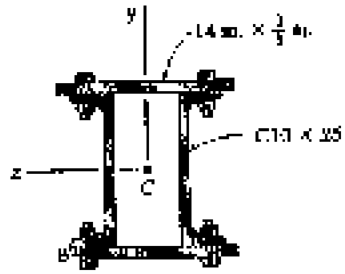
$$q = \frac{VQ}{I} = \frac{(30)(19.271)}{310.9} = 1.8595 \text{ kip/in}$$

$$F_{bolt} = \frac{1}{2} q s = \left(\frac{1}{2}\right)(1.8595)(5) = 4.649 \text{ kip}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{4.649}{0.4418} = 10.52 \text{ ksi}$$

PROBLEM 6.8



6.7 and 6.8 A column is fabricated by connecting the rolled-steel members shown by bolts of $\frac{3}{4}$-in. diameter spaced longitudinally every 5 in. Determine the average shearing stress in the bolts caused by a shearing force of 30 kips parallel to the y axis.

SOLUTION

Calculate moment of inertia

Part	$A\ (in^2)$	$d\ (in)$	$Ad^2\ (in^4)$	$\bar{I}\ (in^4)$
Top plate	5.25	*5.1875	141.28	0.06
C 10 × 25	7.35	0		91.2
C 10 × 25	7.35	0		91.2
Bot. plate	5.25	*5.1875	141.28	0.06
Σ			282.56	182.52

$$* \ d = \frac{10}{2} + \frac{1}{2}\left(\frac{3}{8}\right) = 5.1875 \text{ in} = \bar{y}_1$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 282.56 + 182.52 = 465.08 \text{ in}^4$$

$$Q = A_{plate}\,\bar{y}_1 = (5.25)(5.1875) = 27.234 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(30)(27.234)}{465.08} = 1.7567 \text{ kips/in}$$

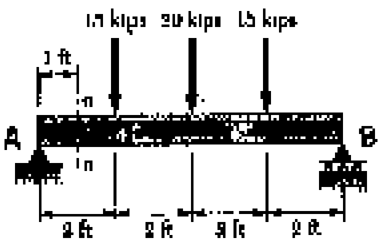
$$F_{bolt} = \frac{1}{2} q s = \left(\frac{1}{2}\right)(1.7567)(5) = 4.392 \text{ kips}$$

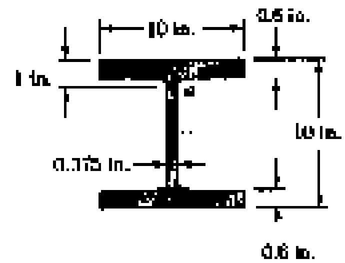
$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}\left(\frac{3}{4}\right)^2 = 0.4418 \text{ in}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{4.392}{0.4418} = 9.94 \text{ ksi}$$

PROBLEM 6.9

6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.





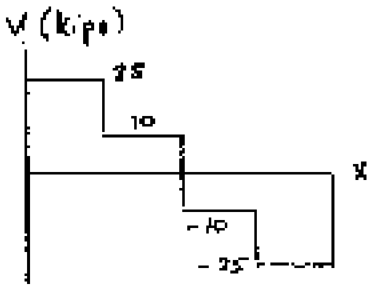
SOLUTION

By symmetry $R_A = R_B$

$+\uparrow \Sigma F_y = 0$

$R_A + R_B - 15 - 20 - 15 = 0$

$R_A = R_B = 25$ kips



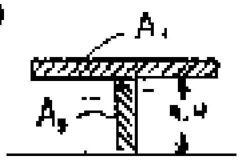
From shear diagram $V = 30$ kips at n-n.

Determine moment of inertia.

Part	A (in²)	d (in)	Ad² (in⁴)	Ī (in⁴)
Top Flng	6	4.7	132.54	0.18
Web	3.30	0	0	21.30
Bot. Flng	6	4.7	132.54	0.18
Σ			265.08	21.66

$$I = \Sigma A d^2 + \Sigma \bar{I} = 286.74 \text{ in}^4$$

(a)



Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)
①	6	4.7	28.2
②	1.65	2.2	3.63
Σ			31.83

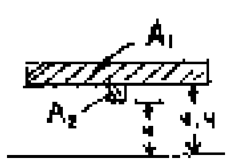
$Q = \Sigma A\bar{y}$

$= 31.83 \text{ in}^3$

$t = 0.375$ in

$$\tau_{max} = \frac{V Q_{max}}{I t} = \frac{(25)(31.83)}{(286.74)(0.375)} = 7.40 \text{ ksi}$$

(b)



Part	A(in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)
①	6	4.7	28.2
②	0.15	4.2	0.63
Σ			28.83

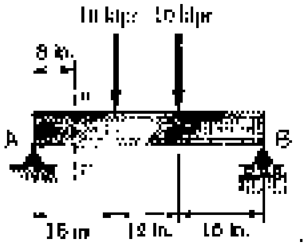
$Q = \Sigma A\bar{y} = 28.83 \text{ in}^3$

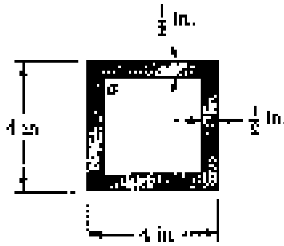
$t = 0.375$ in

$$\tau = \frac{VQ}{It} = \frac{(25)(28.83)}{(286.74)(0.375)} = 6.70 \text{ ksi}$$

PROBLEM 6.10

6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.



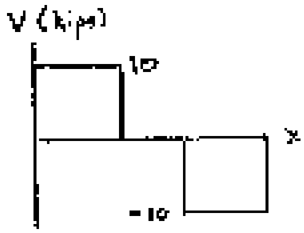


SOLUTION

By symmetry $R_A = R_B$

$+\uparrow \Sigma F_y = 0 \qquad R_A + R_B - 10 - 10 = 0$

$R_A = R_B = 10$ kips

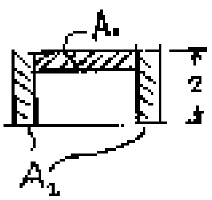


From the shear diagram $V = 10$ kips at n-n.

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(4)(4)^3 - \frac{1}{12}(3)(3)^3 = 14.583 \text{ in}^4$$

(a)



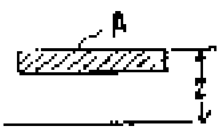
$$Q = A_1 \bar{y}_1 + A_2 \bar{y}_2 = (3)(\tfrac{1}{2})(1.75) + (2)(\tfrac{1}{2})(2)(1)$$

$$= 4.625 \text{ in}^3$$

$$t = \tfrac{1}{2} + \tfrac{1}{2} = 1 \text{ in.}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(10)(4.625)}{(14.583)(1)} = 3.17 \text{ ksi}$$

(b)



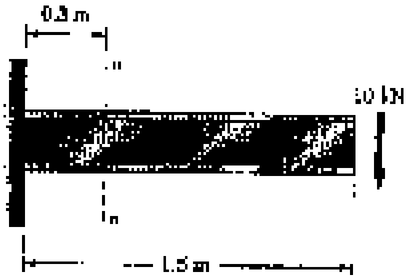
$$Q = A\bar{y} = (4)(\tfrac{1}{2})(1.75) = 3.5 \text{ in}^3$$

$$t = \tfrac{1}{2} + \tfrac{1}{2} = 1 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(10)(3.5)}{(14.583)(1)} = 2.40 \text{ ksi}$$

PROBLEM 6.11

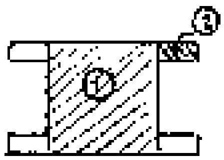
6.9 through 6.12 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.



SOLUTION

At section n-n

$V = 10 \text{ kN}$



$$I = I_1 + 4 I_2$$

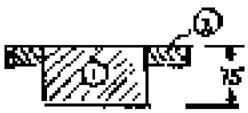
$$= \tfrac{1}{12} b_1 h_1^3 + 4\left(\tfrac{1}{12} b_2 h_2^3 + A_2 d_2^2\right)$$

$$= \tfrac{1}{12}(100)(150)^3 + 4\left[(\tfrac{1}{12})(50)(12)^3 + (50)(12)(69)^2\right]$$

$$= 28.125 \times 10^6 + 4\left[0.0072 \times 10^6 + 2.8566 \times 10^6\right]$$

$$= 39.58 \times 10^6 \text{ mm}^4 = 39.58 \times 10^{-6} \text{ m}^4$$

(a)



$$Q = A_1 \bar{y}_1 + 2 A_2 \bar{y}_2$$

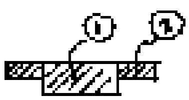
$$= (100)(75)(37.5) + (2)(50)(12)(69)$$

$$= 364.05 \times 10^3 \text{ mm}^3 = 364.05 \times 10^{-6} \text{ m}^3$$

$$t = 100 \text{ mm} = 0.100 \text{ m}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(10 \times 10^3)(364.05 \times 10^{-6})}{(39.58 \times 10^{-6})(0.100)} = 920 \times 10^3 \text{ Pa} = 920 \text{ kPa}$$

(b)



$$Q = A_1 \bar{y}_1 + 2 A_2 \bar{y}_2$$

$$= (100)(40)(55) + (2)(50)(12)(69)$$

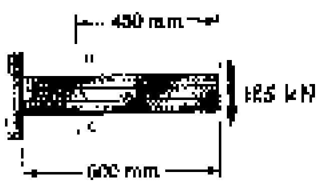
$$= 302.8 \times 10^3 \text{ mm}^3 = 302.8 \times 10^{-6} \text{ m}^3$$

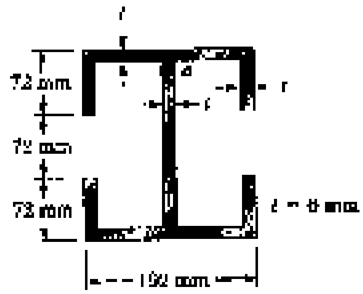
$$t = 100 \text{ mm} = 0.100 \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(10 \times 10^3)(302.8 \times 10^{-6})}{(39.58 \times 10^{-6})(0.100)} = 765 \times 10^3 \text{ Pa} = 765 \text{ kPa}$$

PROBLEM 6.12

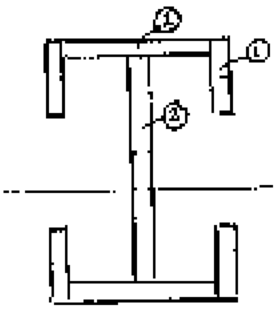
6.9 through 6.12 For the beam and loading shown, consider section n-n and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a.





SOLUTION

At section n-n $\quad V = 125$ kN



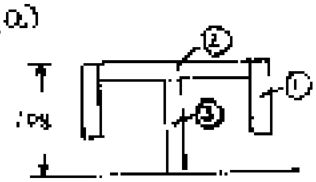
$$I_1 = \frac{1}{12}(6)(72)^3 + (6)(72)(72)^2 = 2.4261 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(180)(6)^3 + (180)(6)(105)^2 = 11.910 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12}(6)(204)^3 = 4.2448 \times 10^6 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 + I_3 = 37.77 \times 10^6 \text{ mm}^4 = 37.77 \times 10^{-6} \text{ m}^4$$

(a)



$$Q = 2A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3$$

$$= (2)(6)(72)(72) + (180)(6)(105) + (6)(102)(51)$$

$$= 206.82 \times 10^3 \text{ mm}^3 = 206.82 \times 10^{-6} \text{ m}^4$$

$$t = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(125 \times 10^3)(206.82 \times 10^{-6})}{(37.77 \times 10^{-6})(6 \times 10^{-3})} = 114.1 \times 10^6 \text{ Pa} = 114.1 \text{ MPa}$$

(b)

$$Q = 2A_1\bar{y}_1 + A_2\bar{y}_2$$

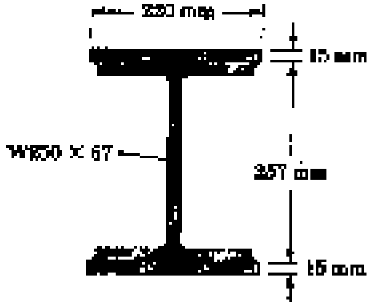
$$= (2)(6)(72)(72) + (180)(6)(105) = 175.61 \times 10^3 \text{ mm}^3 = 175.61 \times 10^{-6} \text{ m}^3$$

$$t = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$\tau = \frac{VQ}{It} = \frac{(125 \times 10^3)(175.61 \times 10^{-6})}{(37.77 \times 10^{-6})(6 \times 10^{-3})} = 96.9 \times 10^6 \text{ Pa} = 96.9 \text{ MPa}$$

PROBLEM 6.13

6.13 Two steel plates of 15 × 220-mm rectangular cross section are welded to the W250 × 67 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 100 MPa.



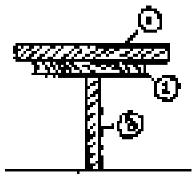
SOLUTION

Calculate moment of inertia

Part	A (mm²)	d (mm)	Ad^2 (10^6 mm⁴)	$\bar{I}$ (10^6 mm⁴)
Top plate	3300	* 136	61.036	0.062
W 250 × 67		0	0	104
Bot. plate	3300	136	61.036	0.062
Σ			122.072	104.124

$* \ d = \frac{257}{2} + \frac{15}{2} = 136$ mm

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 226.2 \times 10^6 \text{ mm}^4 = 226.2 \times 10^{-6} \text{ m}^4$$



Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (10^3 mm³)
① Top plate	3300	136	448.8
② Top flange	3203	120.65	386.4
③ Half web	1004	56.40	56.6
Σ			891.8

$$Q = \Sigma A\bar{y} = 891.8 \times 10^3 \text{ mm}^3 = 891.8 \times 10^{-6} \text{ m}^3$$

$$t = t_w = 8.9 \text{ mm} = 8.9 \times 10^{-3} \text{ m}$$

$$\tau_{max} = \frac{VQ}{It}$$

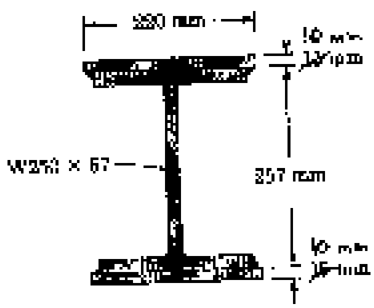
$$V = \frac{It\tau_{max}}{Q} = \frac{(226.2 \times 10^{-6})(8.9 \times 10^{-3})(100 \times 10^6)}{891.8 \times 10^{-6}} = 226 \times 10^3 \text{ N}$$

$$= 226 \text{ kN}$$

PROBLEM 6.14

6.13 Two steel plates of 15 × 220-mm rectangular cross section are welded to the W250 × 67 beam as shown. Determine the largest allowable vertical shear if the shearing stress in the beam is not to exceed 100 MPa.

6.14 Solve Prob. 6.13, assuming that the two steel plates are (*a*) replaced by steel plates of 10 × 220-mm rectangular cross section, (*b*) removed.



SOLUTION

Calculate moment of inertia for part (a)

Part	A (mm²)	d (mm)	Ad^2 (10^6 mm⁴)	$\bar{I}$ (10^6 mm⁴)
Top plate	2200	*133.5	39.209	0.018
W 250 × 67		0	0	104
Bot. plate	2200	*133.5	39.209	0.018
Σ			78.42	104.04

$$* \; d = \frac{257}{2} + \frac{10}{2} = 133.5 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 182.46 \times 10^6 \text{ mm}^4 = 182.46 \times 10^{-6} \text{ m}^4$$

Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (10^3 mm³)
① Top plate	2200	133.5	293.7
② Top flange	3203	120.65	386.4
③ Half web	1004	56.40	56.6
Σ			736.7

$$Q = \Sigma A\bar{y} = 736.7 \times 10^3 \text{ mm}^3 = 736.7 \times 10^{-6} \text{ m}^3$$

$$t = t_w = 8.9 \text{ mm} = 8.9 \times 10^{-3} \text{ m} \qquad \tau_{max} = \frac{VQ}{It}$$

$$V = \frac{I t \tau_{max}}{Q} = \frac{(182.46 \times 10^{-6})(8.9 \times 10^{-3})(100 \times 10^6)}{736.7 \times 10^{-6}} = 220 \times 10^3 \text{ N} = 220 \text{ kN}$$

(b) $I = 104 \times 10^6 \text{ mm}^4 = 104 \times 10^{-6} \text{ m}^4$

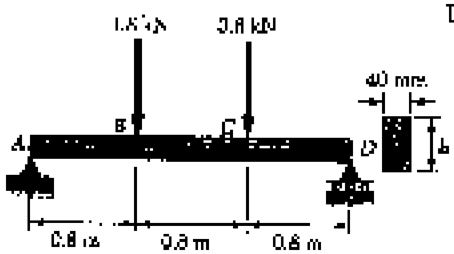
Consider Q for top flange and half web

$$Q = A_2\bar{y}_2 + A_3\bar{y}_3 = 386.4 \times 10^3 + 56.6 \times 10^3 = 443 \times 10^3 \text{ mm}^3 = 443 \times 10^{-6} \text{ m}^3$$

$$V = \frac{I t \tau_{max}}{Q} = \frac{(104 \times 10^{-6})(8.9 \times 10^{-3})(100 \times 10^6)}{443 \times 10^{-6}} = 209 \times 10^3 \text{ N} = 209 \text{ kN}$$

PROBLEM 6.15

6.15 Knowing that the allowable shearing stress for the timber used is 825 kPa, check whether the design obtained for the beam indicated is acceptable and, if not, redesign the cross section of the beam. Consider the beam of (*a*) Prob. 5.75, (*b*) Prob. 5.76.



(a) SOLUTION

From solution to PROBLEM 5.75

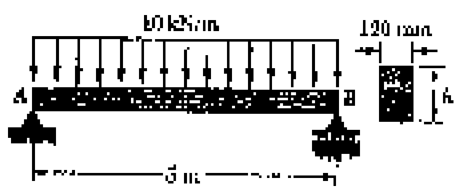
$|V|_{max} = 2.4 \text{ kN} \qquad h = 173.2 \text{ mm}$

$A = bh = (40)(173.2) = 6928 \text{ mm}^2$
$= 6928 \times 10^{-6} \text{ m}^2$

For a rectangular cross section $\quad \tau_{max} = \frac{3}{2}\frac{|V|_{max}}{A}$

$$\tau_{max} = \frac{3}{2}\frac{2.4 \times 10^3}{6928 \times 10^{-6}} = 520 \times 10^3 \text{ Pa} = 520 \text{ kPa} < 825 \text{ kPa}$$

Design is acceptable.



(b) SOLUTION

From solution to PROBLEM 5.76

$|V|_{max} = 25 \text{ kN} \qquad h = 361 \text{ mm}$

$A = bh = (120)(361) = 43.32 \times 10^3 \text{ mm}^2$
$= 43.32 \times 10^{-3} \text{ m}^2$

For a rectangular cross section $\quad \tau_{max} = \frac{3}{2}\frac{|V|_{max}}{A}$

$$\tau_{max} = \frac{3}{2}\frac{25 \times 10^3}{43.32 \times 10^{-3}} = 865 \times 10^3 \text{ Pa} = 865 \text{ kPa} > 825 \text{ kPa}$$

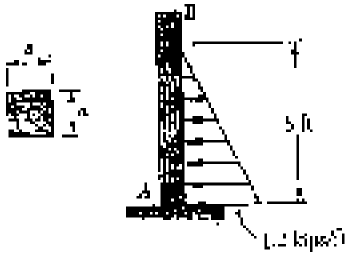
Design is not acceptable.

Resign $\quad A = \frac{3}{2}\frac{|V|_{max}}{\tau_{all}} = \frac{3}{2}\frac{25 \times 10^3}{825 \times 10^3} = 45.45 \times 10^{-3} \text{ m}^2$

$= 45.45 \times 10^3 \text{ mm}^2$

$h = \frac{A}{b} = \frac{45.45 \times 10^3}{120} = 379 \text{ mm} \qquad h = 379 \text{ mm}$

PROBLEM 6.16



6.16 Knowing that the allowable shearing stress for the timber used is 130 psi, check whether the design obtained for the beam indicated is acceptable and, if not, redesign the cross section of the beam. Consider the beam of (a) Prob. 5.77, (b) Prob. 5.78.

(a) SOLUTION

$V_{max} = \frac{1}{2}(6)(1.2) = 3.6$ kips

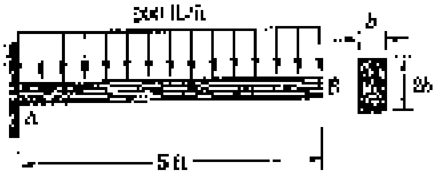
From solution to PROBLEM 5.77

$a = 6.67$ in $\qquad A = a^2 = 44.45$ in^2

For a rectangular section $\quad \tau_{max} = \frac{3}{2}\frac{V_{max}}{A}$

$\tau_{max} = \frac{3}{2}\frac{3.6}{44.45} = 0.1215$ ksi $= 121.5$ psi < 130 psi

Design is acceptable.



(b) SOLUTION

From solution to PROBLEM 5.78

$|V|_{max} = 1000$ lb $\qquad b = 2.95$ in

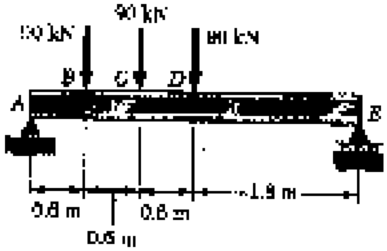
$A = (b)(2b) = 2b^2 = 17.40$ in^2

For a rectangular cross section $\quad \tau_{max} = \frac{3}{2}\frac{|V|_{max}}{A}$

$\tau_{max} = \frac{3}{2}\frac{1000}{17.40} = 86.2$ psi < 130 psi

Design is acceptable.

PROBLEM 6.17



6.17 Determine the average shearing stress in the web of the beam indicated and check whether the design obtained earlier for that beam is acceptable, knowing that the allowable shearing stress for the steel used is 100 MPa. Consider the beam of (*a*) Prob. 5.81, (*b*) Prob. 5.82.

(a) SOLUTION

From the solution to PROBLEM 5.81

$|V|_{max} = 180$ kN

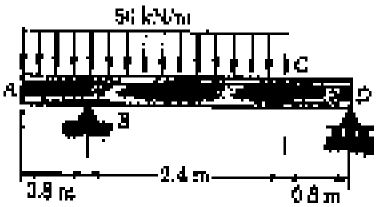
The selected section is W 410 × 60

For that section $t_w = 7.7$ mm $d = 407$ mm

$$A_{web} = t_w d = 3.13 \times 10^3 \text{ mm}^2 = 3.13 \times 10^{-3} \text{ m}^2$$

$$\tau_{ave} = \frac{|V|_{max}}{A_{web}} = \frac{180 \times 10^3}{3.13 \times 10^{-3}} = 57.4 \times 10^6 \text{ Pa} = 57.4 \text{ MPa} < 100 \text{ MPa}$$

Design is acceptable.



(b) SOLUTION

From the solution to PROBLEM 5.82

$|V|_{max} = 80$ kN

The selected section is W 250 × 28.4

For that section $t_w = 6.4$ mm $d = 260$ mm

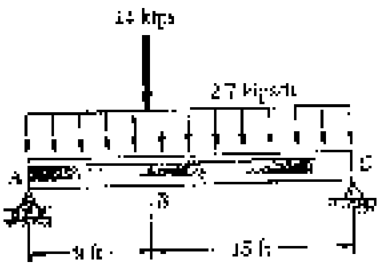
$$A_{web} = t_w d = (6.4)(260) = 1664 \text{ mm}^2 = 1664 \times 10^{-6} \text{ m}^2$$

$$\tau_{ave} = \frac{|V|_{max}}{A_{web}} = \frac{80 \times 10^3}{1664 \times 10^{-6}} = 48.1 \times 10^6 \text{ Pa} = 48.1 \text{ MPa} < 100 \text{ MPa}$$

Design is acceptable.

PROBLEM 6.18

6.18 Determine the average shearing stress in the web of the beam indicated and check whether the design obtained earlier for that beam is acceptable, knowing that the allowable shearing stress for the steel used is 14.5 ksi. Consider the beam of (a) Prob. 5.83, (b) Prob. 5.84.



(a) SOLUTION

From the solution to PROBLEM 5.83

$|V|_{max} = 48$ kips

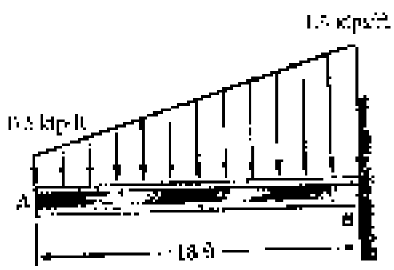
The selected section is W 27 × 84

For that section $t_w = 0.460$ in. $d = 26.71$ in.

$$A_{web} = t_w d = (0.460)(26.71) = 12.29 \text{ in}^2$$

$$\tau_{ave} = \frac{|V|_{max}}{A_{web}} = \frac{48}{12.29} = 3.91 \text{ ksi} < 14.5 \text{ ksi}$$

Design is acceptable.



(b) SOLUTION

From the solution to PROBLEM 5.84

$|V|_{max} = \frac{1}{2}(18)(0.5 + 1.5) = 18$ kips

The selected section is W 18 × 50

For that section $t_w = 0.355$ in. $d = 17.99$ in.

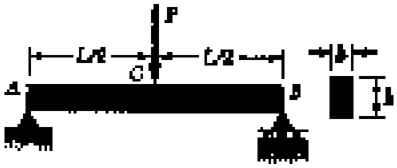
$$A_{web} = t_w d = (0.355)(17.99) = 6.39 \text{ in}^2$$

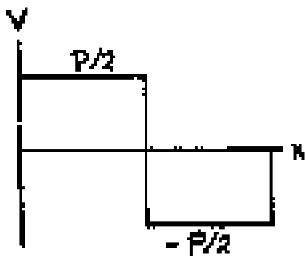
$$\tau_{ave} = \frac{|V|_{max}}{A_{web}} = \frac{18}{6.39} = 2.82 \text{ ksi} < 14.5 \text{ ksi}$$

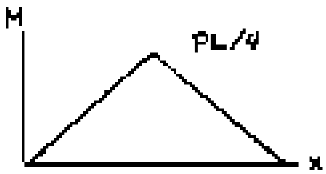
Design is acceptable.

PROBLEM 6.19

6.19 A simply supported timber beam AB of rectangular cross section carries a single concentrated load **P** at its midpoint C. (*a*) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $h/2L$, where h and L are, respectively, the depth and the length of the beam. (*b*) Determine the depth h and width b of the beam, knowing that $L = 2$ m, $P = 40$ kN, $\tau_m = 960$ kPa, and $\sigma_m = 12$ MPa.







SOLUTION

Reactions $\quad R_A = R_B = P/2$

(1) $V_{max} = R_A = \frac{P}{2}$

(2) $A = bh$ for rectangular section

(3) $\tau_m = \frac{3}{2}\frac{V_{max}}{A} = \frac{3P}{4bh}$ for rectangular section

(4) $M_{max} = \frac{PL}{4}$

(5) $S = \frac{1}{6}bh^2$ for rectangular section

(6) $\sigma_m = \frac{M_{max}}{S} = \frac{3PL}{2bh^2}$

(a) $\frac{\tau_m}{\sigma_m} = \frac{h}{2L}$

(b) Solving for h: $\quad h = \frac{2L\tau_m}{\sigma_m} = \frac{(2)(2)(960\times10^3)}{12\times10^6} = 320\times10^{-3}$ m

$= 320$ mm

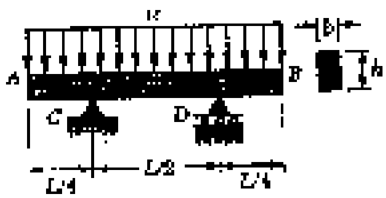
Solving equation (3) for b

$b = \frac{3P}{4h\tau_m} = \frac{(3)(40\times10^3)}{(4)(320\times10^{-3})(960\times10^3)} = 97.7\times10^{-3}$ m

$= 97.7$ mm

PROBLEM 6.20

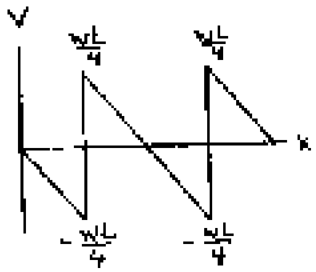
6.20 A timber beam AB of length L and rectangular cross section carries a uniformly distributed load w and is supported as shown. (*a*) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where h and L are, respectively, the depth and the length of the beam. (*b*) Determine the depth h and width b of the beam, knowing that $L = 5$ m, $w = 8$ kN/m, $\tau_m = 1.08$ MPa, and $\sigma_m = 12$ MPa.



SOLUTION

$$R_A = R_B = \frac{wL}{2}$$

From shear diagram $|V|_m = \frac{wL}{4}$ (1)

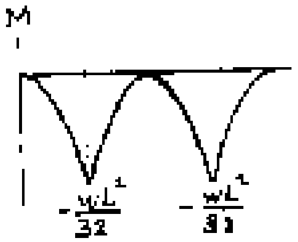


For rectangular section $A = bh$ (2)

$$\tau_m = \frac{3}{2}\frac{V_m}{A} = \frac{3wL}{8bh} \qquad (3)$$

From bending moment diagram

$$|M|_m = \frac{wL^2}{32} \qquad (4)$$



For a rectangular cross section

$$S = \frac{1}{6}bh^2 \qquad (5)$$

$$\sigma_m = \frac{|M|_m}{S} = \frac{3wL^2}{16bh^2} \qquad (6)$$

(a) Dividing eq. (3) by eq. (6) $\quad \frac{\tau_m}{\sigma_m} = \frac{2h}{L}$

(b) Solving for h

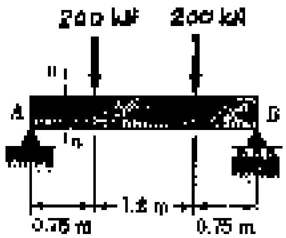
$$h = \frac{L\tau_m}{2\sigma_m} = \frac{(5)(1.08\times10^6)}{(2)(12\times10^6)} = 225\times10^{-3}\text{ m} = 225\text{ mm}$$

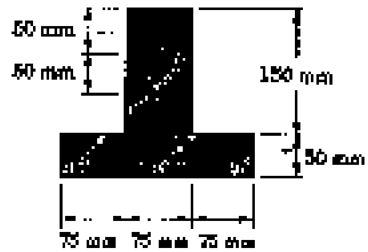
Solving eq. (3) for b

$$b = \frac{3wL}{8h\tau_m} = \frac{(3)(8\times10^3)(5)}{(8)(225\times10^{-3})(1.08\times10^6)} = 61.7\times10^{-3}\text{ m} = 61.7\text{ mm}$$

PROBLEM 6.21

6.21 and 6.22 For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.



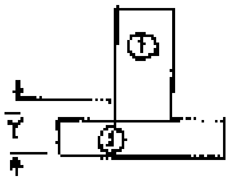


SOLUTION

$R_A = R_B = 200$ kN

At section n-n $V = 200$ kN

Locate centroid and calculate moment of inertia.

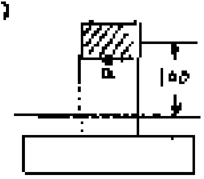


Part	A (mm²)	$\bar{y}$ (mm)	$A\bar{y}$ (10^3 mm³)	d (mm)	Ad^2 (10^6 mm⁴)	$\bar{I}$ (10^6 mm⁴)
①	11250	125	1406.25	50	28.125	21.094
②	11250	25	281.25	50	28.125	2.344
Σ	22500		1687.5		56.25	23.438

$$\bar{Y} = \frac{1687.5 \times 10^3}{22500} = 75 \text{ mm}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 79.688 \times 10^6 \text{ mm}^4 = 79.688 \times 10^{-6} \text{ m}^4$$

(a)



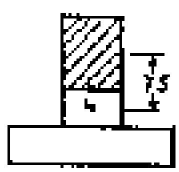
$$Q_a = A\bar{y} = (75)(50)(100) = 375 \times 10^3 \text{ mm}^3 = 375 \times 10^{-6} \text{ m}^3$$

$$t = 75 \text{ mm} = 75 \times 10^{-3} \text{ m}$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(200 \times 10^3)(375 \times 10^{-6})}{(79.768 \times 10^{-6})(75 \times 10^{-3})} = 12.55 \times 10^6 \text{ Pa}$$

$$= 12.55 \text{ MPa}$$

(b)



$$Q_b = A\bar{y} = (75)(100)(75) = 562.5 \times 10^3 \text{ mm}^3 = 562.5 \times 10^{-6} \text{ m}^3$$

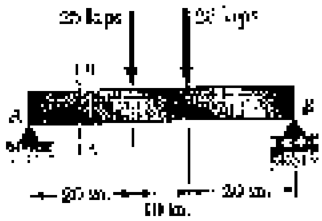
$$t = 75 \text{ mm} = 75 \times 10^{-3} \text{ m}$$

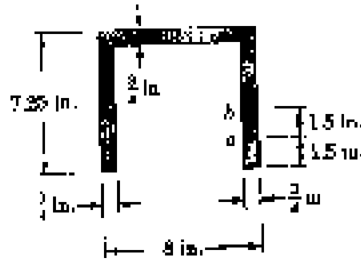
$$\tau_b = \frac{VQ_b}{It} = \frac{(200 \times 10^3)(562.5 \times 10^{-6})}{(79.688 \times 10^{-6})(75 \times 10^{-3})} = 18.82 \times 10^6 \text{ Pa}$$

$$= 18.82 \text{ MPa}$$

PROBLEM 6.22

6.21 and 6.22 For the beam and loading shown, consider section n-n and determine the shearing stress at (a) point a, (b) point b.



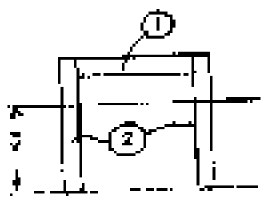


SOLUTION

$R_A = R_B = 25$ kips

At section n-n $V = 25$ kips.

Locate centroid and calculate moment of inertia.

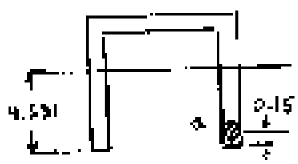


Part	$A\ (in^2)$	$\bar{y}\ (in)$	$A\bar{y}\ (in^3)$	$d\ (in)$	$Ad^2\ (in^4)$	$\bar{I}\ (in^4)$
1	4.875	6.875	33.52	2.244	24.55	0.23
2	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.86

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 35.56 + 47.86 = 83.42 \text{ in}^4$$

(a)

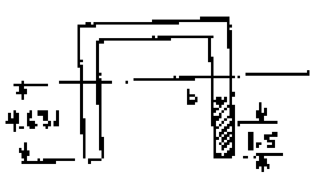


$$Q_a = A\bar{y} = \left(\tfrac{3}{4}\right)(1.5)(4.631 - 0.75) = 4.366 \text{ in}^3$$

$$t = \tfrac{3}{4} = 0.75 \text{ in}$$

$$\tau_a = \frac{VQ}{It} = \frac{(25)(4.366)}{(83.42)(0.75)} = 1.745 \text{ ksi}$$

(b)



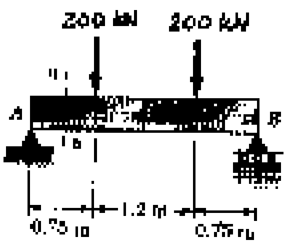
$$Q_b = A\bar{y} = \left(\tfrac{3}{4}\right)(3)(4.631 - 1.5) = 7.045 \text{ in}^3$$

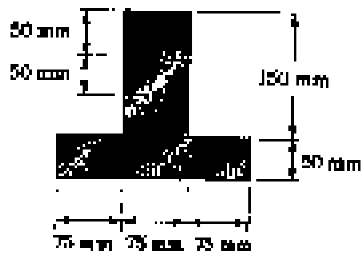
$$t = 0.75 \text{ in.}$$

$$\tau_b = \frac{VQ}{It} = \frac{(25)(7.045)}{(83.42)(0.75)} = 2.82 \text{ ksi}$$

PROBLEM 6.23

6.23 and 6.24 For the beam and loading shown, determine the largest shearing stress in section n-n.



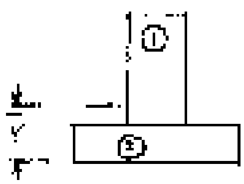


SOLUTION

$R_A = R_B = 200$ kN

At section n-n $\quad V = 200$ kN

Locate centroid and calculate moment of inertia.

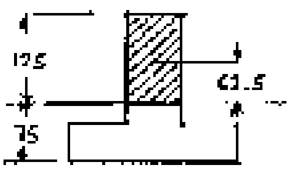


Part	$A\ (mm^2)$	$\bar{y}\ (mm)$	$A\bar{y}\ (10^3 mm^3)$	$d\ (mm)$	$Ad^2\ (10^6 mm^4)$	$\bar{I}\ (10^6 mm^4)$
1	11250	125	1406.25	50	28.125	21.094
2	11250	25	281.25	50	28.125	2.344
Σ	22500		1687.5		56.25	23.438

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{1687.5 \times 10^3}{22500} = 75 \text{ mm}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 79.688 \times 10^6 \text{ mm}^4 = 79.688 \times 10^{-6} \text{ m}^4$$

Largest shearing stress occurs on section through centroid of entire cross section.



$$Q = A\bar{y} = (75)(125)(62.5) = 585.94 \times 10^3 \text{ mm}^3$$

$$= 585.94 \times 10^{-6} \text{ m}^3$$

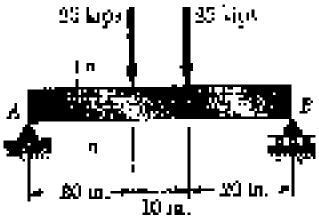
$$t = 75 \text{ mm} = 75 \times 10^{-3} \text{ m}$$

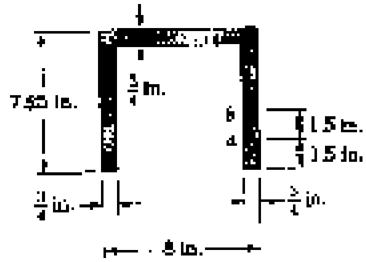
$$\tau = \frac{VQ}{It} = \frac{(200 \times 10^3)(585.94 \times 10^{-6})}{(79.688 \times 10^{-6})(75 \times 10^{-3})} = 19.61 \times 10^6 \text{ Pa}$$

$$= 19.61 \text{ MPa}$$

PROBLEM 6.24

6.23 and 6.24 For the beam and loading shown, determine the largest shearing stress in section n-n.



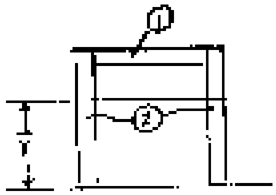


SOLUTION

$R_A = R_B = 25$ kips

At section n-n $\quad V = 25$ kips

Locate centroid and calculate moment of inertia

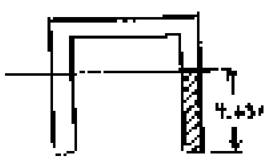


Part	A (in^2)	$\bar{y}$ (in)	$A\bar{y}$ (in^3)	d (in)	Ad^2 (in^4)	$\bar{I}$ (in^4)
①	4.875	6.875	33.52	2.244	24.55	0.23
②	10.875	3.625	39.42	1.006	11.01	47.68
Σ	15.75		72.94		35.56	47.86

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{72.94}{15.75} = 4.631 \text{ in}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 35.56 + 47.86 = 83.42 \text{ in}^4$$

Largest shearing stress occurs on section through centroid of entire cross section.



$$Q = A\bar{y} = \left(\tfrac{3}{4}\right)(4.631)\left(\frac{4.631}{2}\right) = 8.042 \text{ in}^3$$

$$t = \tfrac{3}{4} = 0.75 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(25)(8.042)}{(83.42)(0.75)} = 3.21 \text{ ksi}$$

PROBLEM 6.25

6.25 Two W200 × 46.1 rolled steel sections are to be welded at A and B in either of the two ways shown to form a composite beam. Knowing that for each weld the allowable horizontal shearing force is 500 kN per meter of weld, determine the maximum allowable shear in the composite beam for each of the two arrangements shown.

(a) (b)

SOLUTION

For rolled steel section W 200 × 46.1

$A = 5860\ \text{mm}^2 \qquad d = 203\ \text{mm} \qquad b_f = 203\ \text{mm}$

$I_x = 45.5 \times 10^6\ \text{mm}^4 \qquad I_y = 15.3 \times 10^6\ \text{mm}^4$

(a) $I = 2\left[I_x + A\left(\frac{d}{2}\right)^2\right] = 2\left[45.5\times10^6 + (5860)\left(\frac{203}{2}\right)^2\right] = 211.7\times10^6\ \text{mm}^4$
$= 211.7\times10^{-6}\ \text{m}^4$

$Q = A\frac{d}{2} = (5860)\left(\frac{203}{2}\right) = 594.8\times10^3\ \text{mm}^3 = 594.8\times10^{-6}\ \text{m}^3$

$q = 500\ \text{kN/m}$ for one weld. For 2 welds $q_{all} = 1000\ \text{kN/m}$

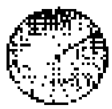
$q_{all} = \frac{VQ}{I} \qquad V_{all} = \frac{I q_{all}}{Q} = \frac{(211.7\times10^{-6})(1000\times10^3)}{594.8\times10^{-6}} = 356\times10^3\ \text{N}$
$= 356\ \text{kN}$ ⟵

(b) $I = 2\left[I_y + A\left(\frac{b_f}{2}\right)^2\right] = 2\left[15.3\times10^6 + 5860\left(\frac{203}{2}\right)^2\right] = 151.34\times10^6\ \text{mm}^4$
$= 151.34\times10^{-6}\ \text{m}^4$

$Q = A\frac{b_f}{2} = (5860)\left(\frac{203}{2}\right) = 594.8\times10^3\ \text{mm}^3 = 594.8\times10^{-6}\ \text{m}^3$

$V_{all} = \frac{I q_{all}}{Q} = \frac{(151.34\times10^{-6})(1000\times10^3)}{594.8\times10^{-6}} = 254\times10^3\ \text{N} = 254\ \text{kN}$ ⟵

PROBLEM 6.26



6.26 through 6.28 A beam having the cross section shown is subjected to a vertical shear V. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

$$\tau_{max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

SOLUTION

$I = \frac{\pi}{4}c^4$ and $A = \pi c^2$

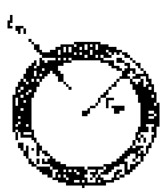
For semicircle $A_s = \frac{\pi}{2}c^2$ $\quad \bar{y} = \frac{4c}{3\pi}$

$Q = A\bar{y} = \frac{\pi}{2}c^2 \cdot \frac{4c}{3\pi} = \frac{2}{3}c^3$

τ_{max} occurs at center where $t = 2c$

$$\tau_{max} = \frac{VQ}{It} = \frac{V \cdot \frac{2}{3}c^3}{\frac{\pi}{4}c^4 \cdot 2c} = \frac{4V}{3\pi c^2} = \frac{4}{3}\frac{V}{A} \qquad k = \frac{4}{3} = 1.333$$

PROBLEM 6.27



6.26 through 6.28 A beam having the cross section shown is subjected to a vertical shear V. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

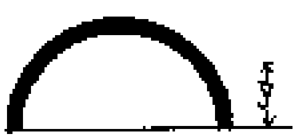
$$\tau_{max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.

SOLUTION

For a thin walled circular section $\quad A = 2\pi r_m t_m$

$J = Ar_m^2 = 2\pi r_m^3 t_m \qquad I = \frac{1}{2}J = \pi r_m^3 t_m$



For a semicircular arc $\quad \bar{y} = \frac{2r_m}{\pi}$

$A_s = \pi r_m t_m \qquad Q = A_s\bar{y} = \pi r_m t_m \frac{2r_m}{\pi} = 2r_m^2 t_m$

$t = 2t_m$

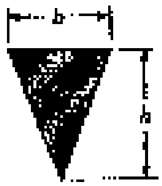
$$\tau_{max} = \frac{VQ}{It} = \frac{V(2r_m^2 t_m)}{(\pi r_m^3 t_m)(2t_m)} = \frac{V}{\pi r_m t_m} = \frac{2V}{A} \qquad k = 2.00$$

PROBLEM 6.28

6.26 through 6.28 A beam having the cross section shown is subjected to a vertical shear V. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant *k* in the following expression for the maximum shearing stress

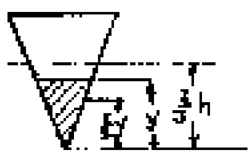
$$\tau_{max} = k\frac{V}{A}$$

where *A* is the cross-sectional area of the beam.



SOLUTION

$$A = \tfrac{1}{2}bh \qquad I = \tfrac{1}{36}bh^3$$



For a cut at location y

$$A(y) = \tfrac{1}{2}\left(\tfrac{by}{h}\right)y = \frac{by^2}{2h}$$

$$\bar{y}(y) = \tfrac{2}{3}h - \tfrac{2}{3}y$$

$$Q(y) = A\bar{y} = \frac{by^2}{3}(h-y)$$

$$t(y) = \frac{by}{h}$$

$$\tau(y) = \frac{VQ}{It} = \frac{V\frac{by^2}{3}(h-y)}{\left(\frac{1}{36}bh^3\right)\frac{by}{h}} = \frac{12Vy(h-y)}{bh^3} = \frac{12V}{bh^3}(hy-y^2)$$

To find location of maximum of τ, set $\frac{d\tau}{dy} = 0$

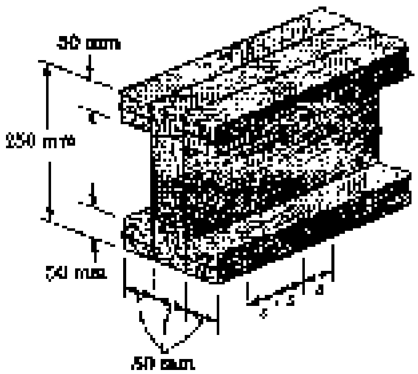
$$\frac{d\tau}{dy} = \frac{12V}{bh^3}(h-2y_m) = 0 \qquad y_m = \tfrac{1}{2}h$$

$$\tau_m = \frac{12V}{bh^3}(hy_m - y_m^2) = \frac{12V}{bh^3}\left[\tfrac{1}{2}h^2 - \left(\tfrac{1}{2}h\right)^2\right] = \frac{3V}{bh^2} = \frac{3}{2}\frac{V}{A}$$

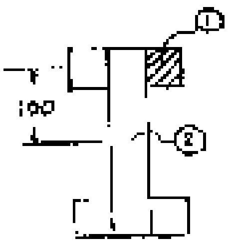
$$k = \tfrac{3}{2} = 1.500$$

PROBLEM 6.29

6.29 The built-up wooden beam shown is subjected to a vertical shear of 5 kN. Knowing that the longitudinal spacing of the nails is $s = 45$ mm and that each nail is 90 mm long, determine the shearing force in each nail.



SOLUTION



$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$$

$$= \frac{1}{12}(50)(50)^3 + (50)(50)(100)^2 = 25.52 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(50)(250)^3 = 65.10 \times 10^6 \text{ mm}^4$$

$$I = 4I_1 + I_2 = 167.18 \times 10^6 \text{ mm}^4 = 167.18 \times 10^{-6} \text{ m}^4$$

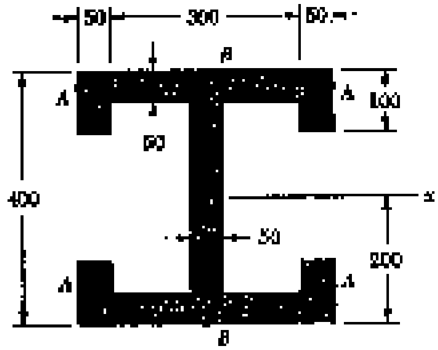
$$Q = Q_1 = A_1 \bar{y}_1 = (50)(50)(100) = 250 \times 10^3 \text{ mm}^3$$
$$= 250 \times 10^{-6} \text{ m}^3$$

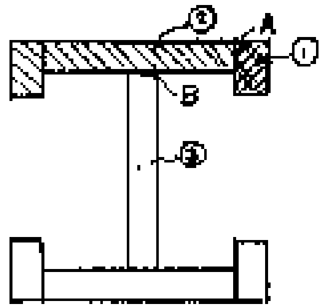
$$q = \frac{VQ}{I} = \frac{(5 \times 10^3)(250 \times 10^{-6})}{167.18 \times 10^{-6}} = 7.477 \times 10^3 \text{ N/m}$$

$$F_{nail} = qs = (7.477 \times 10^3)(45 \times 10^{-3}) = 336 \text{ N}$$

PROBLEM 6.30

6.30 The built-up wooden beam shown is subjected to a vertical shear of 8 kN. Knowing that the nails are spaced longitudinally every 60 mm at A and every 25 mm at B, determine the shearing force in the nails (*a*) at A, (*b*) at B. (Given: $I_x = 1.504 \times 10^9\ \text{mm}^4$.)





SOLUTION

$$I_x = 1.504 \times 10^9\ \text{mm}^4 = 1504 \times 10^{-6}\ \text{m}^4$$

$$s_A = 60\ \text{mm} = 0.060\ \text{m}$$

$$s_B = 25\ \text{mm} = 0.025\ \text{m}$$

(a) $Q_A = Q_1 = A_1\bar{y}_1 = (50)(100)(150) = 750 \times 10^3\ \text{mm}^3$
$= 750 \times 10^{-6}\ \text{m}^3$

$$F_A = q_A s_A$$

$$= \frac{VQ_1 s_A}{I} = \frac{(8 \times 10^3)(750 \times 10^{-6})(0.060)}{1504 \times 10^{-6}}$$

$$= 239\ \text{N}$$

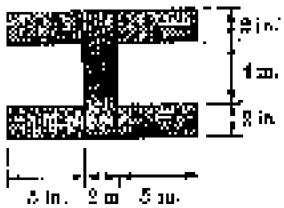
(b) $Q_2 = A_2\bar{y}_2 = (300)(50)(175) = 2625 \times 10^3\ \text{mm}^3$

$Q_B = 2Q_1 + Q_2 = 4125 \times 10^3\ \text{mm}^3$
$= 4125 \times 10^{-6}\ \text{m}^3$

$$F_B = q_B s_A = \frac{VQ_B s_B}{I} = \frac{(8 \times 10^3)(4125 \times 10^{-6})(0.025)}{1504 \times 10^{-6}} = 549\ \text{N}$$

PROBLEM 6.31

6.31 The built-up beam shown is made up by gluing together five planks. Knowing that the allowable average shearing stress in the glued joints is 60 psi, determine the largest permissible vertical shear in the beam.

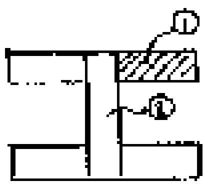


SOLUTION

$$I_1 = \tfrac{1}{12}(5)(2)^3 + (5)(2)(3)^2 = 93.33 \text{ in}^4$$

$$I_2 = \tfrac{1}{12}(2)(8)^3 = 85.33 \text{ in}^4$$

$$I = 4I_1 + I_2 = 458.66 \text{ in}^4$$



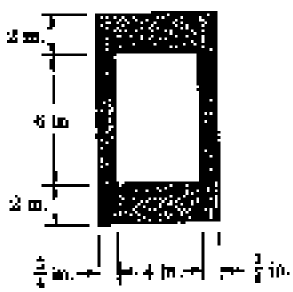
$$Q = A_1\bar{y}_1 = (5)(2)(3) = 30 \text{ in}^3$$

For each glued joint $t = 2$ in.

$$\tau = \frac{VQ}{It} \qquad V = \frac{It\tau}{Q} = \frac{(458.66)(2)(60)}{30} = 1835 \text{ lb.}$$

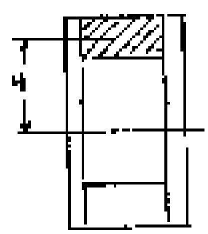
PROBLEM 6.32

6.32 The built-up beam shown is made up by gluing together two $\frac{3}{4}\times 10$-in. plywood strips and two 2×4-in. planks. Knowing that the allowable average shearing stress in the glued joints is 50 psi, determine the largest permissible vertical shear in the beam.



SOLUTION

$$I = \tfrac{1}{12}(5.5)(10)^3 - \tfrac{1}{12}(4)(6)^3 = 386.33 \text{ in}^4$$



$$Q = A\bar{y} = (4)(2)(4) = 32 \text{ in}^3$$

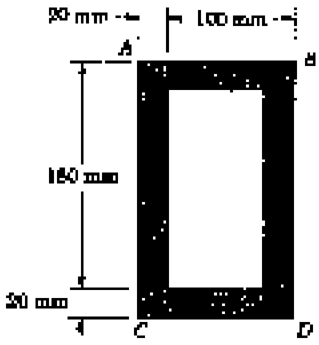
$$t = 2 + 2 = 4 \text{ in}$$

$$\tau = \frac{VQ}{It}$$

$$V = \frac{It\tau}{Q} = \frac{(386.33)(4)(50)}{32} = 2410 \text{ lb.}$$

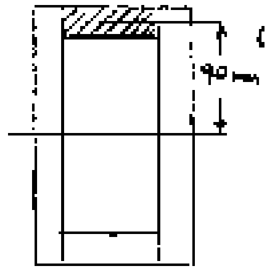
PROBLEM 6.33

6.33 Two 20 × 100-mm and two 20 × 180-mm boards are glued together as shown to form a 120 × 200-mm box beam. Knowing that the beam is subjected to a vertical shear of 3.5 kN, determine the average shearing stress in the glued joint (a) at A, (b) at B.



SOLUTION

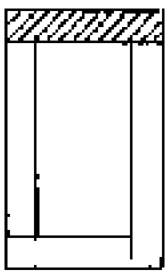
$$I = \frac{1}{12}(120)(200)^3 - \frac{1}{12}(80)(160)^3 = 52.693 \times 10^6 \text{ mm}^4$$
$$= 52.693 \times 10^{-6} \text{ m}^4$$



(a) $Q_A = (80)(20)(90) = 144 \times 10^3 \text{ mm}^3$
$= 144 \times 10^{-6} \text{ m}^3$

$t_A = (2)(20) = 40 \text{ mm} = 0.040 \text{ m}$

$$\tau_A = \frac{VQ_A}{It_A} = \frac{(3.5\times10^3)(144\times10^{-6})}{(52.693\times10^{-6})(0.040)}$$
$$= 239 \times 10^3 \text{ Pa} = 239 \text{ kPa}$$



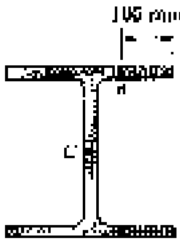
(b) $Q_B = (120)(20)(90) = 216 \times 10^3 \text{ mm}^3 = 216 \times 10^{-6} \text{ m}^3$

$t_B = (2)(20) = 40 \text{ mm} = 0.040 \text{ m}$

$$\tau_B = \frac{VQ_B}{It_B} = \frac{(3.5\times10^3)(216\times10^{-6})}{(52.693\times10^{-6})(0.040)} = 359 \times 10^3 \text{ Pa}$$
$$= 359 \text{ kPa}$$

PROBLEM 6.34

6.34 Knowing that a W360 × 122 rolled-steel beam is subjected to a 250-kN vertical shear, determine the shearing stress (*a*) at point *A*, (*b*) at the centroid *C* of the section.

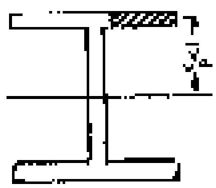


SOLUTION

For W360 × 122, $d = 363$ mm, $b_f = 257$ mm, $t_f = 21.70$ mm, $t_w = 13.0$ mm

$I = 365 \times 10^6 \text{ mm}^4 = 365 \times 10^{-6} \text{ m}^4$

(a)



$A_a = (105)(21.70) = 2278.5 \text{ mm}^2$

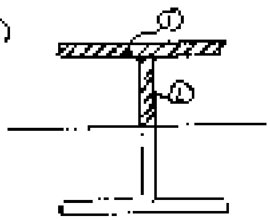
$\bar{y}_a = \frac{d}{2} - \frac{t_f}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65 \text{ mm}$

$Q_a = A_a \bar{y}_a = 388.8 \times 10^3 \text{ mm}^3 = 388.8 \times 10^{-6} \text{ m}^3$

$t_a = t_f = 21.70 \text{ mm} = 21.7 \times 10^{-3} \text{ m}$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(250 \times 10^3)(388.8 \times 10^{-6})}{(365 \times 10^{-6})(21.7 \times 10^{-3})} = 12.27 \times 10^6 \text{ Pa} = 12.27 \text{ MPa} \quad \blacktriangleleft$$

(b)



$A_1 = b_f t_f = (257)(21.70) = 5577 \text{ mm}^2$

$\bar{y}_1 = \frac{d}{2} - \frac{t_f}{2} = \frac{363}{2} - \frac{21.70}{2} = 170.65 \text{ mm}$

$A_2 = t_w\left(\frac{d}{2} - t_f\right) = (13.0)(159.8) = 2077 \text{ mm}^2$

$\bar{y}_2 = \frac{1}{2}\left(\frac{d}{2} - t_f\right) = 79.9 \text{ mm}$

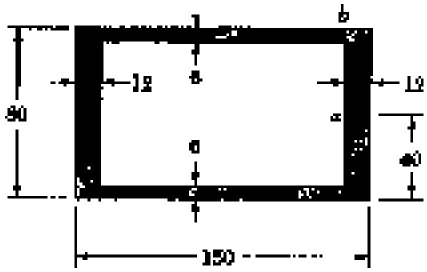
$Q_c = \Sigma A\bar{y} = (5577)(170.65) + (2077)(79.9) = 1117.7 \times 10^3 \text{ mm}^3 = 1117.7 \times 10^{-6} \text{ m}^3$

$t_c = t_w = 13.0 \text{ mm} = 13 \times 10^{-3} \text{ m}$

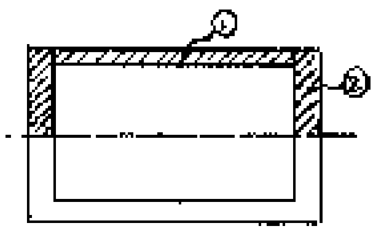
$$\tau_c = \frac{VQ_c}{It_c} = \frac{(250 \times 10^3)(1117.7 \times 10^{-6})}{(365 \times 10^{-6})(13 \times 10^{-3})} = 58.9 \times 10^6 \text{ Pa} = 58.9 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 6.35

6.35 and 6.36 An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 150 kN, determine the shearing stress at (*a*) point *a*, (*b*) point *b*.



Dimensions in mm



SOLUTION

$$I = \frac{1}{12}(150)(80)^3 - \frac{1}{12}(126)(68)^3$$
$$= 3.098 \times 10^6 \text{ mm}^4 = 3.098 \times 10^{-6} \text{ m}^4$$

(a) $Q_a = A_1\bar{y}_1 + 2A_2\bar{y}_2$

$$= (126)(6)(37) + (2)(12)(40)(20)$$
$$= 47.172 \times 10^3 \text{ mm}^3 = 47.172 \times 10^{-6} \text{ m}^3$$

$$t_a = (2)(12) = 24 \text{ mm} = 0.024 \text{ m}$$

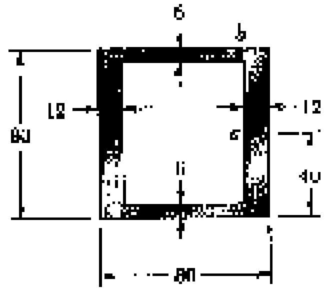
$$\tau_a = \frac{VQ_a}{It_a} = \frac{(150 \times 10^3)(47.172 \times 10^{-6})}{(3.098 \times 10^{-6})(0.024)}$$
$$= 95.2 \times 10^6 \text{ Pa} = 95.2 \text{ MPa}$$

(b) $Q_b = A_1\bar{y}_1 = (126)(6)(37) = 27.97 \times 10^3 \text{ mm}^3 = 27.97 \times 10^{-6} \text{ m}^3$

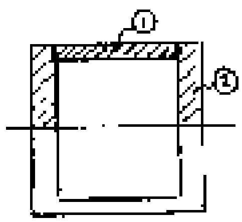
$$t_b = (2)(6) = 12 \text{ mm} = 0.012 \text{ m}$$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(150 \times 10^3)(27.97 \times 10^{-6})}{(3.098 \times 10^{-6})(0.012)} = 112.9 \times 10^6 \text{ Pa} = 112.9 \text{ MPa}$$

PROBLEM 6.36



Dimensions in mm



6.35 and 6.36 An extruded aluminum beam has the cross section shown. Knowing that the vertical shear in the beam is 150 kN, determine the shearing stress at (a) point a, (b) point b.

SOLUTION

$$I = \frac{1}{12}(80)(80)^3 - \frac{1}{12}(56)(68)^3 = 1.9460 \times 10^6 \text{ mm}^4 = 1.946 \times 10^{-6} \text{ m}^4$$

(a) $Q_a = A_1\bar{y}_1 + 2A_2\bar{y}_2$

$= (56)(6)(37) + (2)(12)(40)(20) = 31.632 \times 10^3 \text{ mm}^3 = 31.632 \times 10^{-6} \text{ m}^3$

$t_a = (2)(12) = 24 \text{ mm} = 0.024 \text{ m}$

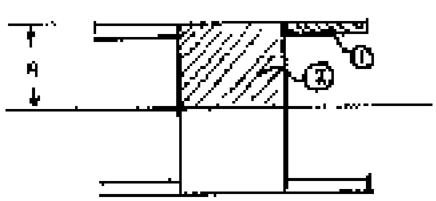
$$\tau_a = \frac{VQ_a}{It_a} = \frac{(150\times10^3)(31.632\times10^{-6})}{(1.946\times10^{-6})(0.024)} = 101.6 \times 10^6 \text{ Pa} = 101.6 \text{ MPa}$$

(b) $Q_b = A_1\bar{y}_1 = (56)(6)(37) = 12.432 \times 10^3 \text{ mm}^3 = 12.432 \times 10^{-6} \text{ m}^3$

$t_b = (2)(6) = 12 \text{ mm} = 0.012 \text{ m}$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(150\times10^3)(12.432\times10^{-6})}{(1.946\times10^{-6})(0.012)} = 79.9 \times 10^6 \text{ Pa} = 79.9 \text{ MPa}$$

PROBLEM 6.37



6.37 The vertical shear is 1200 lb in a beam having the cross section shown. Knowing that $d = 4$ in., determine the shearing stress (a) at point a, (b) at point b.

SOLUTION

$$I_1 = \frac{1}{12}(4)(0.5)^3 + (4)(0.5)(3.75)^2 = 28.167 \text{ in}^4$$

$$I_2 = \frac{1}{3}(5)(4)^3 = 106.67 \text{ in}^4$$

$$I = 4I_1 + 2I_2 = 326 \text{ in}^4$$

(a) $Q_a = 2A_1\bar{y}_1 + A_2\bar{y}_2$

$= (2)(4)(0.5)(3.75) + (5)(4)(2) = 55 \text{ in}^3$

$t_a = 5$ in.

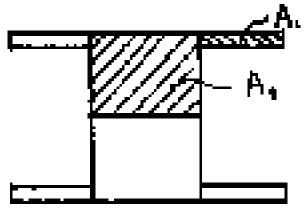
$$\tau_a = \frac{VQ_a}{It_a} = \frac{(1200)(55)}{(326)(5)} = 40.5 \text{ psi}$$

(b) $Q_b = A_1\bar{y}_1 = (4)(0.5)(3.75) = 7.5 \text{ in}^3$ $\quad t_b = 0.5$ in

$$\tau_b = \frac{VQ_b}{It_b} = \frac{(1200)(7.5)}{(326)(0.5)} = 55.2 \text{ psi}$$

PROBLEM 6.38

6.38 The vertical shear is 1200 lb in a beam having the cross section shown. Determine (a) the distance d for which $\tau_a = \tau_b$, (b) the corresponding shearing stress at points a and b.



SOLUTION

$A_1 = 0.5\,d\ \text{in}^2,\quad \bar{y}_1 = 3.75\ \text{in} \qquad t_b = 0.5\ \text{in.}$

$A_2 = (5)(4) = 20\ \text{in}^2,\quad \bar{y}_2 = 2\ \text{in} \qquad t_a = 5\ \text{in}$

$Q_b = A_1 y_1 = 1.875\,d\ \text{in}^3$

$$\tau_b = \frac{VQ_b}{It_b} = \frac{V}{I}\frac{1.875\,d}{0.5} = 3.75\frac{Vd}{I}$$

$$Q_a = A_2\bar{y}_2 + 2Q_b = (20)(2) + (2)(1.875\,d)$$
$$= 40 + 3.75\,d$$

$t_a = 5\ \text{in.}$

(a) $$\tau_a = \frac{VQ_a}{It_a} = \frac{V(40 + 3.75d)}{I\ (5)} = 8\frac{V}{I} + 0.75\frac{Vd}{I} = \tau_b = 3.75\frac{Vd}{I}$$

$$8 + 0.75d = 3.75\,d \qquad d = \frac{8}{3} = 2.667\ \text{in.}$$

(b) $$I_1 = \frac{1}{12}(2.667)(0.5)^3 + (2.667)(0.5)(3.75)^2 = 18.78\ \text{in}^4$$

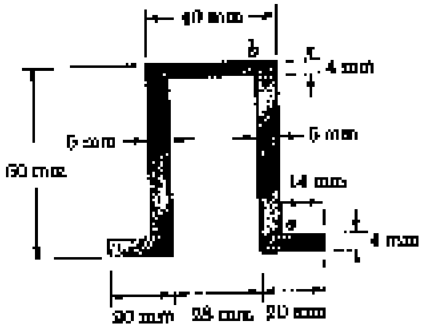
$$I_2 = \frac{1}{3}(5)(4)^3 = 106.67\ \text{in}^4$$

$$I = 4I_1 + 2I_2 = 288.45\ \text{in}^4$$

$$\tau_a = \tau_b = 3.75\frac{Vd}{I} = \frac{(3.75)(1200)(2.667)}{288.45} = 41.6\ \text{psi}$$

PROBLEM 6.39

6.39 Knowing that a given vertical shear V causes a maximum shearing stress of 75 MPa in the hat-shaped extrusion shown, determine the corresponding shearing stress (*a*) at point *a*, (*b*) at point *b*.

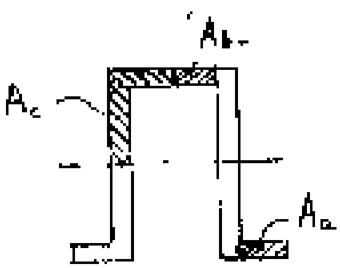


SOLUTION

Neutral axis lies 30 mm above bottom

$$\tau_c = \frac{VQ_c}{It} \qquad \tau_a = \frac{VQ_a}{It_a} \qquad \tau_b = \frac{VQ_b}{It_b}$$

$$\frac{\tau_a}{\tau_c} = \frac{Q_a t_c}{Q_c t_a} \qquad \frac{\tau_b}{\tau_c} = \frac{Q_b t_c}{Q_c t_b}$$



$Q_c = (6)(30)(15) + (14)(4)(28) = 4260\ \text{mm}^3$

$t_c = 6\ \text{mm}$

$Q_a = (14)(4)(28) = 1568\ \text{mm}^3$

$t_a = 4\ \text{mm}$

$Q_b = (14)(4)(28) = 1568\ \text{mm}^3$

$t_b = 4\ \text{mm}$

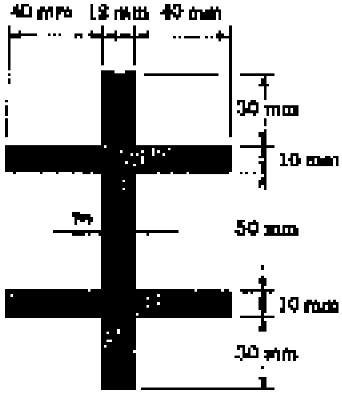
$$\tau_c = 75\ \text{MPa}$$

$$\tau_a = \frac{Q_a}{Q_c}\cdot\frac{t_c}{t_a}\,\tau_c = \frac{1568}{4260}\cdot\frac{6}{4}\cdot 75 = 41.4\ \text{MPa}$$

$$\tau_b = \frac{Q_b}{Q_c}\cdot\frac{t_c}{t_b}\,\tau_c = \frac{1568}{4260}\cdot\frac{6}{4}\cdot 75 = 41.4\ \text{MPa}$$

PROBLEM 6.40

6.40 Knowing that a given vertical shear **V** causes a maximum shearing stress of 50 MPa in a thin-walled member having the cross section shown, determine the corresponding shearing stress (*a*) at point *a*, (*b*) at point *b*, (*c*) at point *c*.



SOLUTION

$$Q_a = (12)(30)(25+10+15) = 18 \times 10^3 \text{ mm}^3$$

$$Q_b = (40)(10)(25+5) = 12 \times 10^3 \text{ mm}^3$$

$$Q_c = Q_a + 2Q_b + (12)(10)(25+5) = 45.6 \times 10^3 \text{ mm}^3$$

$$Q_m = Q_c + (12)(25)\left(\tfrac{25}{2}\right) = 49.35 \times 10^3 \text{ mm}^3$$

$$t_a = t_c = t_m = 12 \text{ mm}$$

$$t_b = 10 \text{ mm}$$

$$\tau_m = 50 \text{ MPa}$$

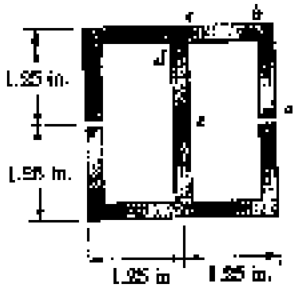
(a) $\dfrac{\tau_a}{\tau_m} = \dfrac{Q_a}{Q_m} \cdot \dfrac{t_m}{t_a} = \dfrac{18}{49.35} \cdot \dfrac{12}{12} = 0.3647$ $\qquad \tau_a = 18.23 \text{ MPa}$

(b) $\dfrac{\tau_b}{\tau_m} = \dfrac{Q_b}{Q_m} \cdot \dfrac{t_m}{t_b} = \dfrac{12}{49.35} \cdot \dfrac{12}{10} = 0.2918$ $\qquad \tau_b = 14.59 \text{ MPa}$

(c) $\dfrac{\tau_c}{\tau_m} = \dfrac{Q_c}{Q_m} \cdot \dfrac{t_m}{t_c} = \dfrac{45.6}{49.35} \cdot \dfrac{12}{12} = 0.9240$ $\qquad \tau_c = 46.2 \text{ MPa}$

PROBLEM 6.41

6.41 and *6.42* The extruded beam shown has a uniform wall thickness of $\frac{1}{8}$ in. Knowing that the vertical shear in the beam is 2 kips, determine the shearing stress at each of the five points indicated.



SOLUTION

$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{12}(2.125)(2.25)^3 = 1.2382 \text{ in}^4$

$t = 0.125$ in at all sections

$V = 2$ kips

$Q_a = 0 \qquad \tau_a = \frac{VQ_a}{It} = 0$

$Q_b = (0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.09766 \text{ in}^3$

$\tau_b = \frac{VQ_b}{It} = \frac{(2)(0.09766)}{(1.2382)(0.125)} = .26 \text{ ksi}$

$Q_c = Q_b + (1.0625)(0.125)(1.1875) = 0.25537 \text{ in}^3$

$\tau_c = \frac{VQ_c}{It} = \frac{(2)(0.25537)}{(1.2382)(0.125)} = 3.30 \text{ ksi}$

$Q_d = 2Q_c + (0.125)^2(1.1875) = 0.52929$

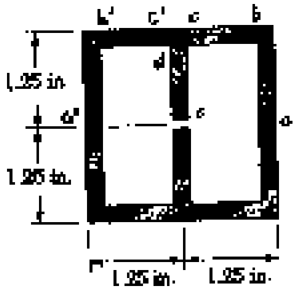
$\tau_d = \frac{VQ_d}{It} = \frac{(2)(0.52929)}{(1.2382)(0.125)} = 6.84 \text{ ksi}$

$Q_e = Q_d + (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.60839$

$\tau_e = \frac{VQ}{It} = \frac{(2)(0.60839)}{(1.2382)(0.125)} = 7.86 \text{ ksi}$

PROBLEM 6.42

6.41 and 6.42 The extruded beam shown has a uniform wall thickness of $\frac{1}{8}$ in. Knowing that the vertical shear in the beam is 2 kips, determine the shearing stress at each of the five points indicated.



SOLUTION

$$I = \frac{1}{12}(2.50)(2.50)^3 - \frac{1}{12}(2.125)(2.25)^3 = 1.2382 \text{ in}^4$$

Add symmetric points c', b', and a'.

$$Q_e = 0$$

$$Q_d = (0.125)(1.125)\left(\frac{1.125}{2}\right) = 0.07910 \text{ in}^3 \qquad t_d = 0.125 \text{ in}$$

$$Q_c = Q_e + (0.125)^2(1.1875) = 0.09765 \text{ in}^3 \qquad t_c = 0.25 \text{ in.}$$

$$Q_b = Q_c + (2)(1.0625)(0.125)(1.1875) = 0.41308 \text{ in}^3 \qquad t_b = 0.25 \text{ in.}$$

$$Q_a = Q_b + (2)(0.125)(1.25)\left(\frac{1.25}{2}\right) = 0.60889 \text{ in}^3 \qquad t_a = 0.25 \text{ in}$$

$$\tau_a = \frac{VQ_a}{It_a} = \frac{(2)(0.60889)}{(1.2382)(0.25)} = 3.93 \text{ ksi}$$ ◀

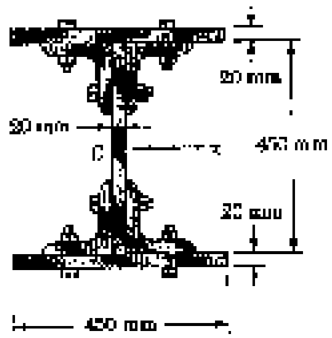
$$\tau_b = \frac{VQ_b}{It_b} = \frac{(2)(0.41308)}{(1.2382)(0.25)} = 2.67 \text{ ksi}$$ ◀

$$\tau_c = \frac{VQ_c}{It_c} = \frac{(2)(0.09765)}{(1.2382)(0.25)} = 0.63 \text{ ksi}$$ ◀

$$\tau_d = \frac{VQ_d}{It_d} = \frac{(2)(0.07910)}{(1.2382)(0.125)} = 1.02 \text{ ksi}$$ ◀

$$\tau_e = \frac{VQ_e}{It_e} = 0$$ ◀

PROBLEM 6.43



6.43 Three 20 × 450-mm steel plates are bolted to four L152 × 152 × 19.0 angles to form a beam with the cross section shown. The bolts have a 22-mm diameter and are spaced longitudinally every 125 mm. Knowing that the allowable average shearing stress in the bolts is 90 MPa, determine the largest permissible vertical shear in the beam. (Given: $I_x = 1896 \times 10^6\ \text{mm}^4$.)

SOLUTION

Flange: $I_f = \frac{1}{12}(450)(20)^3 + (450)(20)(235)^2 = 497.3 \times 10^6\ \text{mm}^4$

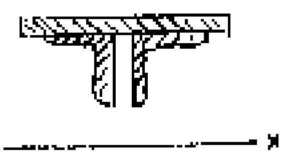
Web: $I_w = \frac{1}{12}(20)(450)^3 = 151.9 \times 10^6\ \text{mm}^4$

Angle: $\bar{I} = 11.6 \times 10^6\ \text{m}^4, \quad A = 5420\ \text{mm}^2$

$\bar{y} = 44.9\ \text{mm} \qquad d = 225 - 44.9 = 180.1\ \text{mm}$

$I_a = \bar{I} + Ad^2$
$= 11.6 \times 10^6 + (5420)(180.1)^2 = 187.4 \times 10^6\ \text{mm}^4$

$I = 2I_f + I_w + 4I_a = 1896 \times 10^6\ \text{mm}^4 = 1896 \times 10^{-6}\ \text{m}^4$



$Q_f = (450)(20)(235) = 2115 \times 10^3\ \text{mm}^3$

$Q_a = (5420)(180.1) = 976 \times 10^3\ \text{mm}^3$

$Q = Q_f + 2Q_a = 4067 \times 10^3\ \text{mm}^3 = 4067 \times 10^{-6}\ \text{m}^3$

$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(22)^2 = 380.1\ \text{mm}^2 = 380.1 \times 10^{-6}\ \text{m}^2$

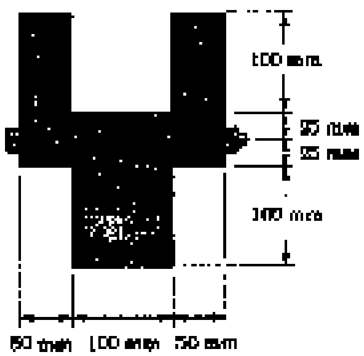
$F_{bolt} = 2\tau_{bolt} A_b = (2)(90 \times 10^6)(380.1 \times 10^{-6}) = 68.42 \times 10^3\ \text{N}$

$q_{all} = \frac{F_{bolt}}{s} = \frac{68.42 \times 10^3}{0.125} = 547.36 \times 10^3\ \text{N/m}$

$q = \frac{VQ}{I} \qquad V_{all} = \frac{I q_{all}}{Q} = \frac{(1896 \times 10^{-6})(547.36 \times 10^3)}{4067 \times 10^{-6}} = 255 \times 10^3\ \text{N}$

$= 255\ \text{kN}$

PROBLEM 6.44

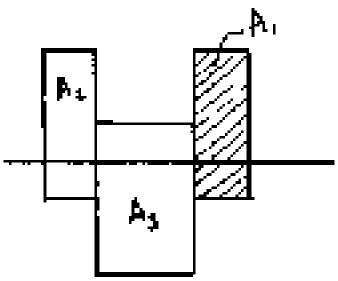


6.44 A beam consists of three planks connected by steel bolts with a longitudinal spacing of 225 mm. Knowing that the shear in the beam is vertical and equal to 6 kN and that the allowable average shearing stress in each bolt is 60 MPa, determine the smallest permissible bolt diameter that can be used.

SOLUTION

Part	$A (mm^2)$	$\bar{y}$ (mm)	$A\bar{y}^2$ ($10^6 mm^4$)	$\bar{I}$ ($10^6 mm^4$)
1	7500	50	18.75	14.06
2	7500	50	18.75	14.06
3	15000	-50	37.50	28.12
Σ			75.00	56.25

$$I = \Sigma A\bar{y}^2 + \Sigma \bar{I} = 131.25 \times 10^6\ mm^4 = 131.25 \times 10^{-6}\ m^4$$



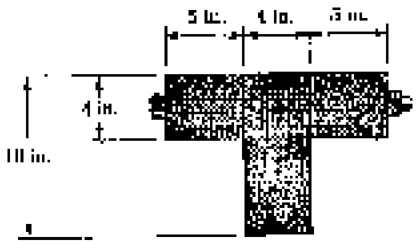
$$Q = A_1 \bar{y}_1 = (7500)(50) = 375 \times 10^3\ mm^3$$
$$= 375 \times 10^{-6}\ m^3$$

$$F_{bolt} = \tau_{bolt} A_{bolt} = q s = \frac{VQs}{I}$$

$$A_{bolt} = \frac{VQs}{\tau_{all} I} = \frac{(6 \times 10^3)(375 \times 10^{-6})(0.225)}{(60 \times 10^6)(131.25 \times 10^{-6})} = 64.286 \times 10^{-6}\ m^2$$
$$= 64.286\ mm^2$$

$$d_{bolt} = \sqrt{\frac{4 A_{bolt}}{\pi}} = \sqrt{\frac{(4)(64.286)}{\pi}} = 9.05\ mm$$

PROBLEM 6.45



6.45 and 6.46 Three planks are connected as shown by bolts of $\frac{3}{8}$-in. diameter spaced every 6 in. along the longitudinal axis of the beam. For a vertical shear of 2.5 kips, determine the average shearing stress in the bolts.

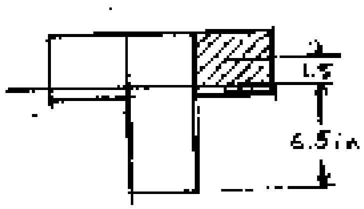
SOLUTION

Locate neutral axis.

$$\Sigma A = (2)(5)(4) + (4)(10) = 80 \text{ in}^2$$

$$\Sigma A\bar{y} = (2)(5)(4)(8) + (4)(10)(5) = 520 \text{ in}^3$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = 6.5 \text{ in}$$



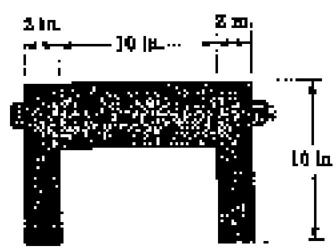
$$Q = (5)(4)(1.5) = 30 \text{ in}^3$$

$$I = 2\left[\tfrac{1}{12}(5)(4)^3 + (5)(4)(1.5)^2\right] + \tfrac{1}{12}(4)(10)^3 + (4)(10)(1.5)^2 = 566.7 \text{ in}^4$$

$$F = qs = \frac{VQs}{I} = \frac{(2.5)(30)(6)}{566.7} = 0.7941 \text{ kips}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 \quad 0.1104 \text{ in}^2 \qquad \tau_{bolt} = \frac{F}{A_{bolt}} = \frac{0.7941}{0.1104} = 7.19 \text{ ksi}$$

PROBLEM 6.46



6.45 and 6.46 Three planks are connected as shown by bolts of $\frac{3}{8}$-in. diameter spaced every 6 in. along the longitudinal axis of the beam. For a vertical shear of 2.5 kips, determine the average shearing stress in the bolts.

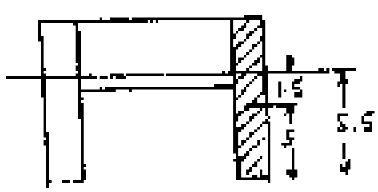
SOLUTION

Locate neutral axis

$$\Sigma A = (2)(2)(10) + (10)(4) = 80 \text{ in}^2$$

$$\Sigma A\bar{y} = (2)(2)(10)(5) + (10)(4)(8) = 520 \text{ in}^3$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{520}{80} = 6.5 \text{ in}$$



$$I = 2\left[\tfrac{1}{12}(2)(10)^3 + (2)(10)(1.5)^2\right] + \tfrac{1}{12}(10)(4)^3 + (10)(4)(1.5)^2 = 566.7 \text{ in}^4$$

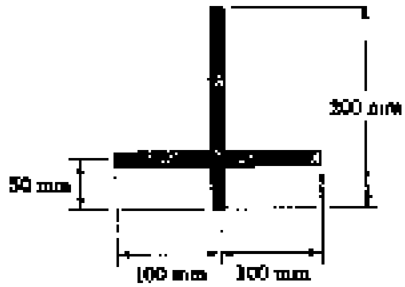
$$Q = (2)(10)(1.5) = 30 \text{ in}^3$$

$$F = qs = \frac{VQs}{I} = \frac{(2.5)(30)(6)}{566.7} = 0.7941 \text{ kips}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 0.1104 \text{ in}^2 \qquad \tau_{bolt} = \frac{F}{A_{bolt}} = \frac{0.7941}{0.1104} = 7.19 \text{ ksi}$$

PROBLEM 6.47

6.47 Three plates, each 12-mm thick, are welded together to form the section shown. For a vertical shear of 100 kN, determine the shear flow through the welded surfaces and sketch the shear flow in the cross section.



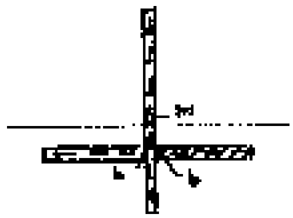
SOLUTION

Locate neutral axis

$$\Sigma A = (12)(200) + (2)(94)(12) = 4656 \text{ mm}^2$$

$$\Sigma A\bar{y} = (12)(200)(100) + (2)(94)(12)(50) = 352.8 \times 10^3 \text{ mm}^3$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = 75.77 \text{ mm}$$



$$I = \tfrac{1}{12}(12)(200)^3 + (12)(200)(24.23)^2 + 2\left[\tfrac{1}{12}(94)(12)^3 + (94)(12)(25.77)^2\right]$$

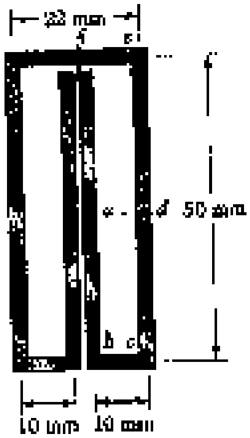
$$= 10.934 \times 10^6 \text{ mm}^4 = 10.934 \times 10^{-6} \text{ m}^4$$

$$Q = (94)(12)(25.77) = 29.07 \times 10^3 \text{ mm}^3 = 29.07 \times 10^{-6} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(100 \times 10^3)(29.07 \times 10^{-6})}{10.934 \times 10^{-6}} = 266 \times 10^3 \text{ N/m} = 266 \text{ kN/m}$$

PROBLEM 6.45

6.49 A plate of 2-mm thickness is bent as shown and then used as a beam. For a vertical shear of 5 kN, determine the shearing stress at the five points indicated and sketch the shear flow in the cross section.



SOLUTION

$$I = 2\left[\tfrac{1}{12}(2)(48)^3 + \tfrac{1}{12}(2)(52)^3 + \tfrac{1}{12}(20)(2)^3 + (20)(2)(25)^2\right]$$

$$= 133.76\times10^3\ \text{mm}^4 = 133.75\times10^{-9}\ \text{m}^3$$

$$Q_a = (2)(24)(12) = 576\ \text{mm}^3 = 576\times10^{-9}\ \text{m}^3$$

$$Q_b = 0$$

$$Q_c = Q_b - (12)(2)(25) = -600\ \text{mm}^3 = -600\times10^{-9}\ \text{m}^3$$

$$Q_d = Q_c - (2)(24)(12) = -1.176\times10^3\ \text{mm}^3 = -1.176\times10^{-6}\ \text{m}^3$$

$$Q_e = Q_d + (2)(26)(13) = -600\ \text{mm}^3 = -500\times10^{-9}\ \text{m}^3$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(5\times10^3)(576\times10^{-9})}{(133.75\times10^{-9})(2\times10^{-3})} = 10.77\times10^6\ \text{Pa} = 10.76\ \text{MPa}$$

$$\tau_b = \frac{VQ_b}{It} = 0$$

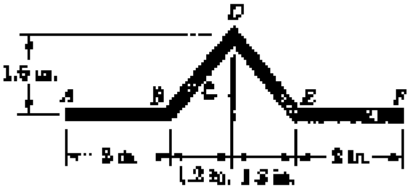
$$\tau_c = \frac{VQ_c}{It} = \frac{(5\times10^3)(600\times10^{-9})}{(133.75\times10^{-9})(2\times10^{-3})} = 11.21\times10^6\ \text{Pa} = 11.21\ \text{MPa}$$

$$\tau_d = \frac{VQ_d}{It} = \frac{(5\times10^3)(1.176\times10^{-6})}{(133.75\times10^{-9})(2\times10^{-3})} = 22.0\times10^6\ \text{Pa} = 22.0\ \text{MPa}$$

$$\tau_e = \frac{VQ_e}{It} = \frac{(5\times10^3)(500\times10^{-9})}{(133.75\times10^{-9})(2\times10^{-3})} = 9.35\times10^6\ \text{Pa} = 9.35\ \text{MPa}$$

PROBLEM 6.49

6.49 A plate of $\frac{1}{4}$-in. thickness is corrugated as shown and then used as a beam. For a vertical shear of 1.2 kips, determine (*a*) the maximum shearing stress in the section, (*b*) the shearing stress at point *B*. Also sketch the shear flow in the cross section.



SOLUTION

$$L_{BD} = \sqrt{(1.2)^2 + (1.6)^2} = 2.0 \text{ in}$$

$$A_{BD} = (0.25)(2.0) = 0.5 \text{ in}^2$$

Locate neutral axis and compute moment of inertia.

Part	A (in^2)	$\bar{y}$ (in)	$A\bar{y}$ (in^3)	d (in)	Ad^2 (in^4)	$\bar{I}$ (in^4)
AB	0.5	0	0	0.4	0.080	neglect
BD	0.5	0.8	0.4	0.4	0.080	*0.1067
DE	0.5	0.8	0.4	0.4	0.080	*0.1067
EF	0.5	0	0	0.4	0.080	neglect
Σ	2.0		0.8		0.320	0.2133

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{0.8}{2.0} = 0.4 \text{ in}$$

$$* \ \frac{1}{12} A_{BD} h^2 = \frac{1}{12}(0.5)(1.6)^2 = 0.1067 \text{ in}^4$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 0.5333 \text{ in}^4$$

(a)

$$Q_m = Q_{AB} + Q_{BC}$$

$$Q_{AB} = (2)(0.25)(0.4) = 0.2 \text{ in}^3$$

$$Q_{BC} = (0.5)(0.25)(0.2) = 0.025 \text{ in}^3$$

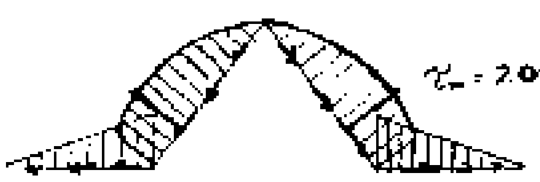
$$Q_m = 0.225 \text{ in}^3$$

$$\tau_m = \frac{VQ_m}{It} = \frac{(1.2)(0.225)}{(0.5333)(0.25)} = 2.025 \text{ ksi}$$

(b) $Q_B = Q_{AB} = 0.2 \text{ in}^3$

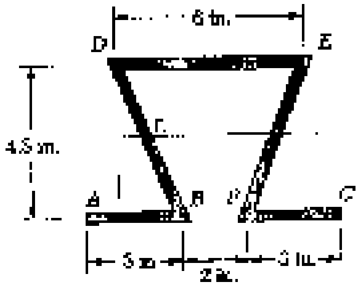
$$\tau_B = \frac{VQ_B}{It} = \frac{(1.2)(0.2)}{(0.5333)(0.25)} = 1.80 \text{ ksi}$$

$$\tau_A = 0$$



PROBLEM 6.50

6.50 A plate of thickness t is bent as shown and then used as a beam. For a vertical shear of 600 lb, determine (a) the thickness t for which the maximum shearing stress is 300 psi, (b) the corresponding shearing stress at point E. Also sketch the shear flow in the cross section.



SOLUTION

$$L_{BD} = L_{EF} = \sqrt{4.8^2 + 2^2} = 5.2 \text{ in.}$$

Neutral axis lies at 2.4 in. above AB

Calculate I

$$I_{AB} = (3t)(2.4)^2 = 17.28\ t$$

$$I_{BD} = \tfrac{1}{12}(5.2\ t)(4.8)^2 = 9.984\ t$$

$$I_{DE} = (6\ t)(2.4)^2 = 34.56\ t$$

$$I_{EF} = I_{DB} = 9.984\ t$$

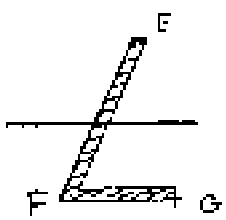
$$I_{FG} = I_{AB} = 17.28\ t$$

$$I = \Sigma I = 89.09\ t$$

(a) At point C $\quad Q_c = Q_{AB} + Q_{BC} = (3t)(2.4) + (2.6\ t)(1.2) = 10.32\ t$

$$\tau = \frac{VQ_c}{It} \quad ; \quad t = \frac{VQ}{\tau I} = \frac{(600)(10.32\ t)}{(300)(89.09\ t)} = 0.23168 \text{ in}$$

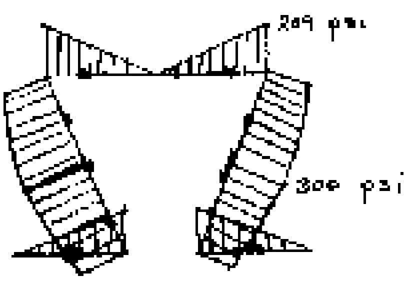
(b) $I = (89.09)(0.23168) = 20.64 \text{ in}^3$



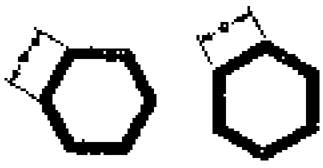
$$Q_E = Q_{EF} + Q_{FG}$$

$$= 0 + (3)(0.23168)(2.4) = 1.668 \text{ in}^3$$

$$\tau_E = \frac{VQ_E}{It} = \frac{(600)(1.668)}{(20.64)(0.23168)} = 209 \text{ psi}$$



PROBLEM 6.51

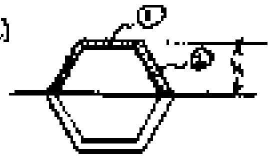


(a) (b)

6.51 and 6.52 An extruded beam has a uniform wall thickness t. Denoting by V the vertical shear and by A the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{max} = k(V/A)$ and determine the constant k for each of the two orientations shown.

SOLUTION

(a)



$h = \frac{\sqrt{3}}{2} a$

$A_1 = A_2 = at$

$I_1 = A_1 h^2 = a t h^2 = \frac{3}{4} a^3 t$

$I_2 = \frac{1}{3} A_2 h^2 = \frac{1}{3} a t \frac{3}{4} a^2 = \frac{1}{4} a^3 t$

$I = 2I_1 + 4I_2 = \frac{5}{2} a^3 t$

$Q_1 = A_1 h = \frac{\sqrt{3}}{2} a^2 t$

$Q_2 = A_2 \frac{h}{2} = \frac{\sqrt{3}}{4} a^2 t$

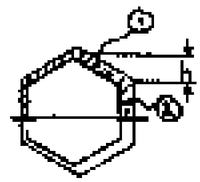
$Q_m = Q_1 + 2Q_2 = \sqrt{3} a^2 t$

$$\tau_m = \frac{VQ}{I(2t)} = \frac{V\sqrt{3} a^2 t}{(\frac{5}{2} a^3 t)(2t)} = \frac{\sqrt{3}}{5} \frac{V}{at}$$

$$= \frac{6\sqrt{3}}{5} \frac{V}{6at} = \frac{6\sqrt{3}}{5} \frac{V}{A} = k \frac{V}{A}$$

$k = \frac{6\sqrt{3}}{5} = 2.08$ ◄

(b)



$h = \frac{a}{2}$

$A_1 = at \quad A_2 = \frac{1}{2} at$

$I_1 = \bar{I}_1 + A_1 d^2$

$= \frac{1}{12} a t h^2 + a t \left(\frac{a}{2} + \frac{h}{2}\right)^2$

$= \frac{1}{48} a^3 t + \frac{9}{16} a^3 t = \frac{7}{12} a^3 t$

$I_2 = \frac{1}{3} t \left(\frac{a}{2}\right)^3 = \frac{1}{24} a^3 t$

$I = 4I_1 + 4I_2 = \frac{5}{2} a^3 t$

$Q_1 = at\left(\frac{a}{2} + \frac{h}{2}\right) = \frac{3}{4} a^2 t$

$Q_2 = \left(\frac{1}{2} at\right)\left(\frac{a}{4}\right) = \frac{1}{8} a^2 t$

$Q = 2Q_1 + 2Q_2 = \frac{7}{4} a^2 t$

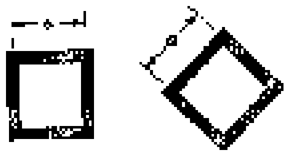
$$\tau_m = \frac{VQ}{I(2t)} = \frac{V \cdot \frac{7}{4} a^2 t}{(\frac{5}{2} a^3 t)(2t)}$$

$$= \frac{7}{20} \frac{V}{at} = \frac{42}{20} \frac{V}{6at} = \frac{21}{10} \frac{V}{A}$$

$= k \frac{V}{A} \qquad k = \frac{21}{10} = 2.10$ ◄

PROBLEM 6.52

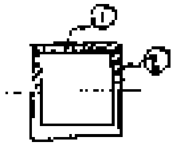
6.51 and 6.52 An extruded beam has a uniform wall thickness t. Denoting by V the vertical shear and by A the cross-sectional area of the beam, express the maximum shearing stress as $\tau_{max} = k(V/A)$ and determine the constant k for each of the two orientations shown.



(a) (b)

SOLUTION

(a)



$I_1 = (at)\left(\frac{a}{2}\right)^2 = \frac{1}{4}a^3 t$

$I_2 = \frac{1}{3}t\left(\frac{a}{2}\right)^3 = \frac{1}{24}a^3 t$

$I = 2I_1 + 4I_2 = \frac{2}{3}a^3 t$

$Q_1 = (at)\left(\frac{a}{2}\right) = \frac{1}{2}a^2 t$

$Q_2 = \left(\frac{1}{2}at\right)\left(\frac{a}{4}\right) = \frac{1}{8}a^2 t$

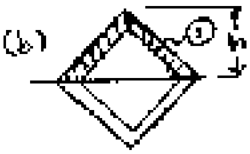
$Q = Q_1 + 2Q_2 = \frac{3}{4}a^2 t$

$\tau_{max} = \frac{VQ}{I(2t)} = \frac{V\left(\frac{3}{4}a^2 t\right)}{\left(\frac{2}{3}a^3 t\right)(2t)} =$

$= \frac{9}{16}\frac{V}{at} = \frac{9}{4}\frac{V}{4at} = \frac{9}{4}\frac{V}{A}$

$= k\frac{V}{A} \quad \therefore \quad k = \frac{9}{4} = 2.25$ ◄

(b)



$h = \frac{1}{2}\sqrt{2}\,a$

$I_1 = \frac{1}{3}A_1 h^2 = \left(\frac{1}{3}at\right)\left(\frac{\sqrt{2}}{2}a\right)^2$

$= \frac{1}{6}a^3 t$

$I = 4I_1 = \frac{2}{3}a^3 t$

$Q_1 = at\left(\frac{h}{2}\right) = \frac{1}{4}\sqrt{2}\,a^2 t$

$Q = 2Q_1 = \frac{1}{2}\sqrt{2}\,a^2 t$

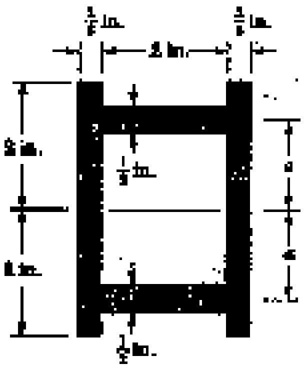
$\tau_{max} = \frac{VQ}{I(2t)} = \frac{V\left(\frac{1}{2}\sqrt{2}\,a^2 t\right)}{\left(\frac{2}{3}a^3 t\right)(2t)}$

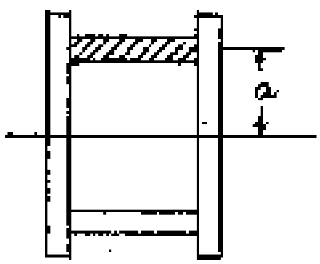
$= \frac{3\sqrt{2}}{8}\frac{V}{at} = \frac{3\sqrt{2}}{2}\frac{V}{4at}$

$= \frac{3\sqrt{2}}{2}\frac{V}{A} = k\frac{V}{A}$

$k = \frac{3\sqrt{2}}{2} = 2.12$ ◄

PROBLEM 6.53





6.53 The design of a beam calls for connecting two vertical rectangular $\frac{3}{8} \times 4$-in. plates by welding them to two horizontal $\frac{1}{2} \times 2$-in. plates as shown. For a vertical shear V, determine the dimension a for which the shear flow through the welded surfaces is maximum.

SOLUTION

$$I = (2)\left(\tfrac{1}{12}\right)\left(\tfrac{3}{8}\right)(4)^3 + (2)\left(\tfrac{1}{12}\right)(2)\left(\tfrac{1}{2}\right)^3 + (2)(2)\left(\tfrac{1}{2}\right)a^2$$

$$= 4.041667 + 2a^2 \quad \text{in}^4$$

$$Q = (2)\left(\tfrac{1}{2}\right)a = a \quad \text{in}^3$$

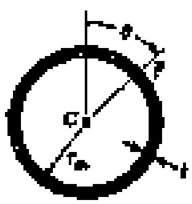
$$q = \frac{VQ}{I} = \frac{Va}{4.041667 + 2a^2} \qquad \text{Set } \frac{dq}{da} = 0$$

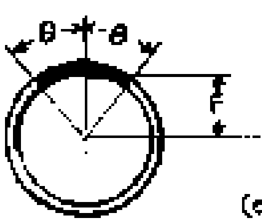
$$\frac{dq}{da} = \left[\frac{(4.041667 + 2a^2) - (a)(4a)}{(4.041667 + 2a^2)^2}\right]V = 0$$

$$2a^2 = 4.041667$$

$$a = 1.422 \text{ in.}$$

PROBLEM 6.54





6.54 (*a*) Determine the shearing stress at point *P* of a thin-walled pipe of the cross section shown caused by a vertical shear V. (*b*) Show that the maximum shearing stress occurs for $\theta = 90°$ and is equal to $2V/A$, where *A* is the cross-sectional area of the pipe.

SOLUTION

$$A = 2\pi r_m t \qquad J = A r_m^2 = 2\pi r_m^3 t \qquad I = \tfrac{1}{2}J = \pi r_m^3 t$$

$$\bar{r} = \frac{\sin\theta}{\theta} \quad \text{for a circular arc}$$

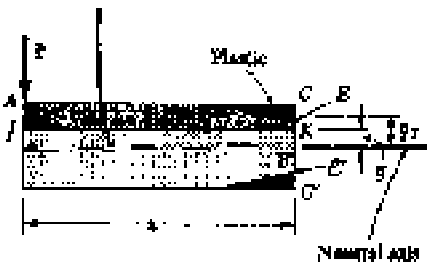
$$A_p = 2r\theta t$$

$$Q_p = A\bar{y} = 2r^2 t \sin\theta$$

(a) $$\tau_p = \frac{VQ_p}{I(2t)} = \frac{(V)(2r^2 t\sin\theta)}{(\pi r_m^3 t)(2t)} = \frac{V\sin\theta}{\pi r_m t}$$

(b) $$\tau_m = \frac{2V\sin\frac{\pi}{2}}{2\pi r_m t} = \frac{2V}{A}$$

PROBLEM 6.55



6.55 Consider the cantilever beam AB discussed in Sec. 6.8 and the portion $ACKJ$ of the beam that is located to the left of the transverse section CC' and above the horizontal plane JK, where K is a point at a distance $y < y_Y$ above the neutral axis (Fig P6.55). (*a*) Recalling that $\sigma_x = \sigma_Y$ between C and E and $\sigma_x = (\sigma_Y/y_Y)y$ between E and K, show that the magnitude of the horizontal shearing force **H** exerted on the lower face of the portion of beam $ACKJ$ is

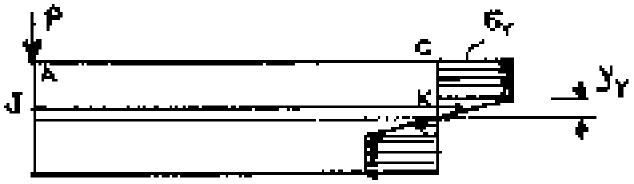
$$H = \frac{1}{2}b\sigma_Y\left(2c - y_Y - \frac{y^2}{y_Y}\right)$$

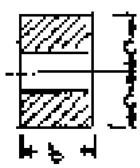
(*b*) Observing that the shearing stress at K is

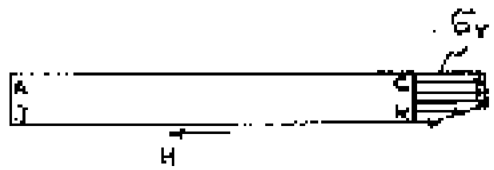
$$\tau_{xy} = \lim_{\Delta A \to 0}\frac{\Delta H}{\Delta A} = \lim_{\Delta x \to 0}\frac{1}{b}\frac{\Delta H}{\Delta x} = \frac{1}{b}\frac{\partial H}{\partial x}$$

and recalling that y_Y is a function of x defined by Eq. (6.14), derive Eq. (6.15).

SOLUTION







Point K is located a distance y above the neutral axis.

The stress distribution is given by

$$\sigma = \sigma_Y \frac{y}{y_Y} \quad \text{for } 0 \le y < y_Y \quad \text{and} \quad \sigma = \sigma_Y \quad \text{for } y_Y \le y \le c.$$

For equilibrium of horizontal forces acting on ACKJ

$$H = \int \sigma \, dA = \int_y^{y_Y} \frac{\sigma_Y y b}{y_Y} dy + \int_{y_Y}^{c} \sigma_Y b \, dy = \frac{\sigma_Y b}{y_Y}\left(\frac{y_Y^2 - y^2}{2}\right) + \sigma_Y b (c - y_Y)$$

$$= \frac{1}{2} b \sigma_Y \left(2c - y_Y - \frac{y^2}{y_Y}\right) \quad \longleftarrow \text{(a)}$$

Note that y_Y is a function of x

$$\tau_{xy} = \frac{1}{b}\frac{\partial H}{\partial x} = \frac{1}{2}\sigma_Y\left(-\frac{\partial y_Y}{\partial x} + \frac{y^2}{y_Y^2}\frac{dy_Y}{dx}\right) = -\frac{1}{2}\sigma_Y\left(1 - \frac{y^2}{y_Y^2}\right)\frac{dy_Y}{dx}$$

But $\quad M = Px = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{y_Y^2}{c^2}\right)$

Differentiating $\quad \frac{dM}{dx} = P = \frac{3}{2}M_Y\left(-\frac{2}{3}\frac{y_Y}{c^2}\frac{dy_Y}{dx}\right)$

$$\frac{dy_Y}{dx} = -\frac{Pc^2}{y_Y M_Y} = -\frac{Pc^2}{y_Y \frac{2}{3}\sigma_Y b c^2} = -\frac{3}{2}\frac{P}{\sigma_Y b y_Y}$$

Then $\quad \tau_{xy} = \frac{1}{2}\sigma_Y\left(1 - \frac{y^2}{y_Y^2}\right)\frac{3}{2}\frac{P}{\sigma_Y b y_Y} = \frac{3P}{4 b y_Y}\left(1 - \frac{y^2}{y_Y^2}\right) \quad \longleftarrow \text{(b)}$

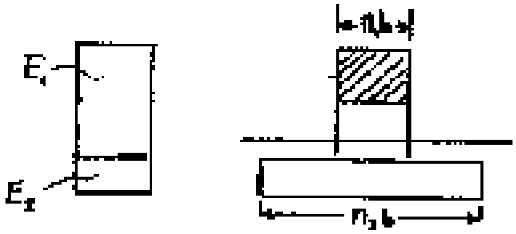
PROBLEM 6.56

6.56 For a beam made of two or more materials with different moduli of elasticity, show that Eq. (6.6)

$$\tau = \frac{VQ}{It}$$

remains valid provided that both Q and I are computed using the transformed section of the beam (see Sec. 4.6) and provided further that t is the actual width of the beam at the point where τ is computed.

SOLUTION



Actual Section

Transformed Section

Let E_{ref} be a reference modulus of elasticity

$$n_1 = \frac{E_1}{E_{ref}}, \quad n_2 = \frac{E_2}{E_{ref}}, \quad \text{etc.}$$

Widths b of actual section are multiplied by n's to obtain the transformed section. The bending stress distribution in the cross section is given by

$$\sigma_x = -\frac{nMy}{I}$$

where I is the moment of inertia of the transformed cross section and y is measured from the centroid of the transformed section.

The horizontal shearing force over length Δx is

$$\Delta H = -\int (\Delta\sigma_x)\, dA = \int \frac{n(\Delta M)y}{I}\, dA = \frac{(\Delta M)}{I}\int ny\, dA = \frac{Q(\Delta M)}{I}$$

$Q = \int ny\, dA$ = first moment of transformed section

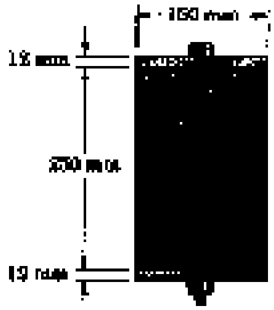
Shear Flow $\quad q = \dfrac{\Delta H}{\Delta x} = \dfrac{\Delta M}{\Delta x}\dfrac{Q}{I} = \dfrac{VQ}{I}$

q is distributed over actual width t, thus $\tau = \dfrac{q}{t}$

$$\tau = \frac{VQ}{It}$$

PROBLEM 6.57

6.57 and 6.58 A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. Determine the average shearing stress in the bolts caused by a vertical shearing force of 4 kN. (*Hint.* Use the method indicated in Prob. 6.56.)



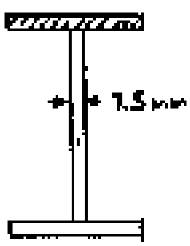
SOLUTION

Let $E_{ref} = E_s = 200$ GPa

$n_s = 1 \qquad n_w = \dfrac{E_w}{E_s} = \dfrac{10 \text{ GPa}}{200 \text{ GPa}} = \dfrac{1}{20}$

Widths of transformed section

$b_s = 150$ mm $\qquad b_w = (\frac{1}{20})(150) = 7.5$ mm



$$I = 2\left[\tfrac{1}{12}(150)(12)^3 + (150)(12)(125+6)^2\right] + \tfrac{1}{12}(7.5)(250)^3$$

$$= 2\left[0.0216\times10^6 + 30.890\times10^6\right] + 9.766\times10^6$$

$$71.589\times10^6 \text{ mm}^4 = 71.589\times10^{-6} \text{ m}^4$$

$$Q = (150)(12)(125+6) = 235.8\times10^3 \text{ mm}^3 = 235.8\times10^{-6} \text{ m}^3$$

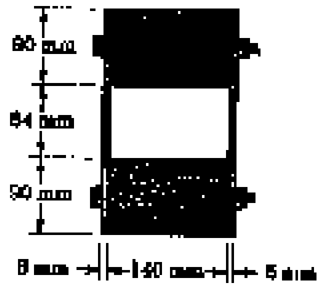
$$q = \frac{VQ}{I} = \frac{(4\times10^3)(235.8\times10^{-6})}{71.589\times10^{-6}} = 13.175\times10^3 \text{ N/m}$$

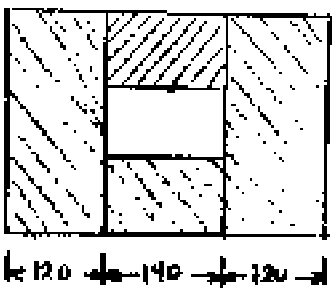
$$F_{bolt} = qs = (13.175\times10^3)(200\times10^{-3}) = 2.635\times10^3 \text{ N}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = (\tfrac{\pi}{4})(12)^2 = 113.1 \text{ mm}^2 = 113.1\times10^{-6} \text{ m}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{2.635\times10^3}{113.1\times10^{-6}} = 23.3\times10^6 \text{ Pa} = 23.3 \text{ MPa}$$

PROBLEM 6.58





6.57 and 6.58 A composite beam is made by attaching the timber and steel portions shown with bolts of 12-mm diameter spaced longitudinally every 200 mm. The modulus of elasticity is 10 GPa for the wood and 200 GPa for the steel. Determine the average shearing stress in the bolts caused by a vertical shearing force of 4 kN. (*Hint*. Use the method indicated in Prob. 6.56.)

SOLUTION

Let wood be the reference material

$$n_w = 1.0 \qquad n_s = \frac{E_s}{E_w} = \frac{200\text{ GPa}}{10\text{ GPa}} = 20$$

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(380)(264)^3 - \frac{1}{12}(140)(84)^3 = 575.7\times10^6 \text{ mm}^4$$

$$= 575.7\times10^{-6} \text{ m}^4$$

$$Q = (140)(90)(42+45) = 1.0962\times10^6 \text{ mm}^3$$

$$= 1.096\times10^{-3} \text{ m}^3$$

$$q = \frac{VQ}{I} = \frac{(4\times10^3)(1.096\times10^{-3})}{575.7\times10^{-6}} = 7.615\times10^3 \text{ N/m}$$

$$F_{bolt} = qs = (7.615\times10^3)(200\times10^{-3}) = 1.523\times10^3 \text{ N}$$

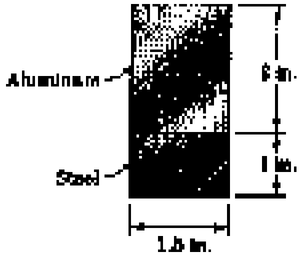
$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(12)^2 = 113.1 \text{ mm}^2 = 113.1\times10^{-6} \text{ m}^2$$

Double shear $\quad \tau_{bolt} = \dfrac{F_{bolt}}{2A_{bolt}} = \dfrac{1.523\times10^3}{(2)(113.1\times10^{-6})} = 6.73\times10^6 \text{ Pa}$

$$= 6.73 \text{ MPa}$$

PROBLEM 6.59

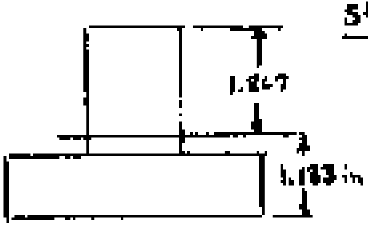
6.59 and 6.60 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum stress in the beam. (*Hint:* Use the method indicated in Prob. 6.56.)



SOLUTION

$n = 1$ in aluminum $\qquad n = \dfrac{29 \times 10^6 \text{ psi}}{10.6 \times 10^6 \text{ psi}} = 2.7358$ in steel

Part	nA (in²)	$\bar{y}$ (in)	$nA\bar{y}$ (in³)	d (in)	nAd^2 (in⁴)	$n\bar{I}$ (in⁴)
Alum.	3.0	2.0	6.0	0.8665	2.2525	1.0
Steel	4.1038	0.5	2.0519	0.6335	1.6469	0.3420
Σ	7.1038		8.0519		3.8994	1.3420



$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma nA} = \frac{8.0519}{7.1038} = 1.1335$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.2414 \text{ in}^4$$

(a) At the bonded surface $\qquad Q = (1.5)(2)(0.8665) = 2.5995 \text{ in}^3$

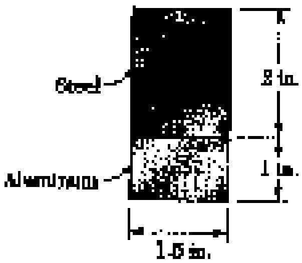
$$\tau = \frac{VQ}{It} = \frac{(4)(2.5995)}{(5.2414)(1.5)} = 1.322 \text{ ksi}$$

(b) At the neutral axis $\qquad Q = (1.5)(1.8665)\left(\frac{1.8665}{2}\right) = 2.6129 \text{ in}^3$

$$\tau_{max} = \frac{VQ}{It} = \frac{(4)(2.6129)}{(5.2414)(1.5)} = 1.329 \text{ ksi}$$

PROBLEM 6.60

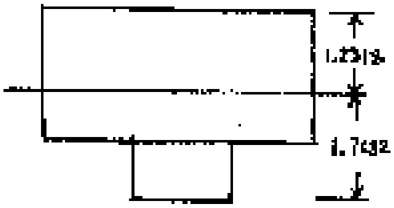
6.59 and 6.60 A steel bar and an aluminum bar are bonded together as shown to form a composite beam. Knowing that the vertical shear in the beam is 4 kips and that the modulus of elasticity is 29×10^6 psi for the steel and 10.6×10^6 psi for the aluminum, determine (*a*) the average stress at the bonded surface, (*b*) the maximum stress in the beam. (*Hint.* Use the method indicated in Prob. 6.56.)



SOLUTION

$n = 1$ in aluminum $\qquad n = \dfrac{29 \times 10^6 \text{ psi}}{10.6 \times 10^6 \text{ psi}} = 2.7358$ in steel

Part	nA (in²)	$\bar{y}$ (in)	$nA\bar{y}$ (in³)	d (in)	nAd^2 (in⁴)	$n\bar{I}$ (in⁴)
Steel	8.2074	2.0	16.4148	0.2318	0.4410	2.7358
Alum.	1.5	0.5	0.75	1.7682	2.4125	0.1250
Σ	9.7074		17.1648		2.8535	2.8608



$$\bar{Y} = \frac{\Sigma nA\bar{y}}{\Sigma A} = \frac{17.1648}{9.7074} = 1.7682 \text{ in}$$

$$I = \Sigma nAd^2 + \Sigma n\bar{I} = 5.7143 \text{ in}^4$$

(a) At the bonded surface $\qquad Q = (1.5)(1.2682) = 1.9023 \text{ in}^3$

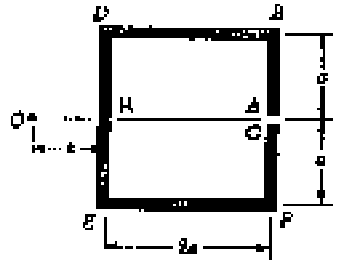
$$\tau = \frac{VQ}{It} = \frac{(4)(1.9023)}{(5.7143)(1.5)} = 0.888 \text{ ksi} \quad \blacktriangleleft$$

(b) At the neutral axis $\qquad Q = (2.7358)(1.5)(1.2318)\left(\frac{1.2318}{2}\right) = 3.1133 \text{ in}^3$

$$\tau_{max} = \frac{VQ}{It} = \frac{(4)(3.1133)}{(5.7143)(1.5)} = 1.453 \text{ ksi} \quad \blacktriangleleft$$

PROBLEM 6.61

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

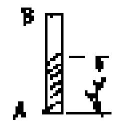


SOLUTION

$$I_{AB} = I_{FG} = \tfrac{1}{3} t a^3 \qquad I_{BD} = I_{EF} = 2ata^2 + \tfrac{1}{12} 2at t^3 \approx 2ta^3$$

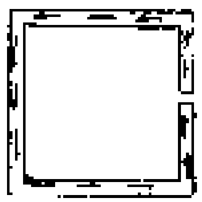
$$I_{DE} = \tfrac{1}{12} t (2a)^3 = \tfrac{2}{3} t a^3 \qquad I = \Sigma I = \tfrac{16}{3} t a^3$$

Part AB $\qquad A = ty \qquad \bar{y} = \frac{y}{2} \qquad Q = \frac{1}{2} t y^2$

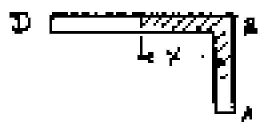


$$\tau = \frac{VQ}{It} = \frac{V \cdot \frac{1}{2} t y^2}{\frac{16}{3} t a^3 t} = \frac{3 V y^2}{32 a^3 t}$$

$$F_1 = \int \tau \, dA = \int_0^a \tau t \, dy = \frac{3V}{32 a^3} \int_0^a y^2 \, dy = \frac{1}{32} V$$



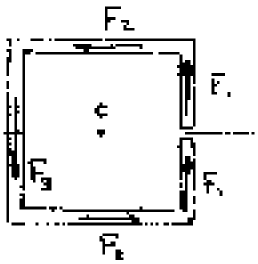
Part BD



$$Q = Q_B + txa = \tfrac{1}{2} t a^2 + tax$$

$$\tau = \frac{VQ}{It} = \frac{Vt}{\frac{16}{3} a^3 t} \left(\tfrac{1}{2} a^2 + ax \right)$$

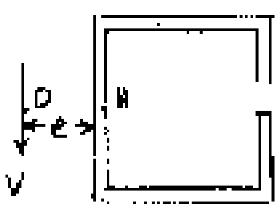
$$= \frac{3V}{32 a^2} (a + 2x)$$



$$F_2 = \int \tau \, dA = \int_0^{2a} \frac{3V}{32 a^2} (a + 2x) \, dx$$

$$= \frac{3V}{32 a^2} (ax + x^2) \Big|_0^{2a} = \frac{3V}{32 a^2} (2a^2 + 4a^2) = \frac{9}{16} V$$

$$\circlearrowleft \Sigma M_H = \circlearrowleft \Sigma M_H$$

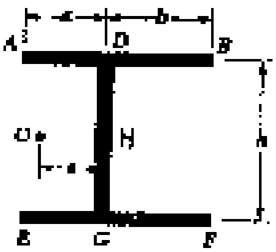


$$Ve = (2a)(2F_1) + (2a)(F_2)$$

$$= \tfrac{1}{8} Va + \tfrac{9}{8} Va = \tfrac{5}{4} Va \qquad e = \tfrac{5}{4} a$$

PROBLEM 6.62

6.61 through 6.64 Determine the location of the shear center *O* of a thin-walled beam of uniform thickness having the cross section shown.

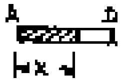


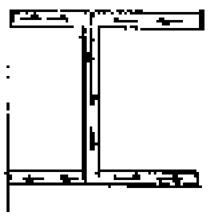
SOLUTION

$$I_{AB} = I_{EF} = (a+b)t\left(\frac{h}{2}\right)^2 + \frac{1}{12}(a+b)t^3 \approx \frac{1}{4}t(a+b)h^2$$

$$I_{DG} = \frac{1}{12}th^3 \qquad I = \Sigma I = \frac{1}{12}t(6a+6b+h)h^2$$

Part AD $\qquad Q = tx\frac{h}{2} = \frac{1}{2}thx$

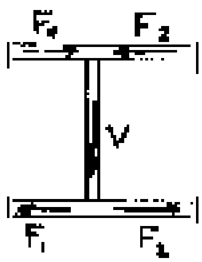




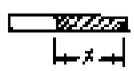
$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_1 = \int \tau\, dA = \int_0^a \frac{Vhx}{2I}t\,dx = \frac{Vht}{2I}\int_0^a x\,dx$$

$$= \frac{Vht}{2I}\left.\frac{x^2}{2}\right|_0^a = \frac{Vhta^2}{4I}$$



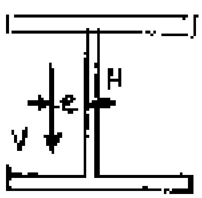
Part BD $\qquad Q = tx\frac{h}{2} = \frac{1}{2}thx$



$$\tau = \frac{VQ}{It} = \frac{Vhx}{2I}$$

$$F_2 = \int \tau\, dA = \int_0^b \frac{Vhx}{2I}t\,dx = \frac{Vht}{2I}\int_0^b x\,dx$$

$$= \frac{Vht}{2I}\left.\frac{x^2}{2}\right|_0^b = \frac{Vhtb^2}{4I}$$



$$\circlearrowleft \Sigma M_H = \circlearrowleft \Sigma M_H$$

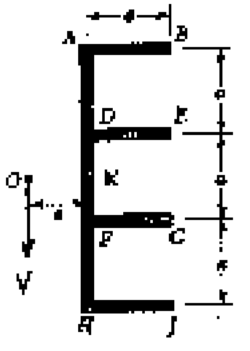
$$Ve = F_2h - F_1h = \frac{Vh^2t(b^2-a^2)}{4I}$$

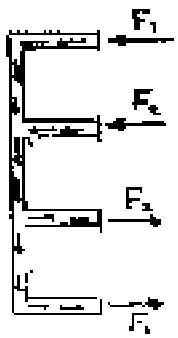
$$= \frac{Vh^2t(b^2-a^2)}{4\cdot\frac{1}{12}t(6a+6b+h)h^2} = \frac{3V(b^2-a^2)}{6a+6b+h}$$

$$e = \frac{3(b^2-a^2)}{6(a+b)+h}$$

PROBLEM 6.63

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.





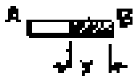
SOLUTION

$$I_{AB} = I_{HJ} = at\left(\frac{3a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{9}{4}ta^3$$

$$I_{DE} = I_{FG} = at\left(\frac{a}{2}\right)^2 + \frac{1}{12}at^3 \approx \frac{1}{4}ta^3$$

$$I_{AH} = \frac{1}{12}t(3a)^3 = \frac{9}{4}ta^3 \qquad I = \Sigma I = \frac{29}{4}ta^3$$

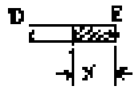
Part AB $\qquad A = tx \qquad \bar{y} = \frac{3a}{2} \qquad Q = \frac{3}{2}atx$



$$\tau = \frac{VQ}{It} = \frac{V\cdot\frac{3}{2}atx}{\frac{29}{4}ta^3t} = \frac{6Vx}{29a^3t}$$

$$F_1 = \int \tau\,dA = \int_0^a \frac{6Vx}{29a^3t}\,t\,dx = \frac{6V}{29a^3}\int_0^a x\,dx = \frac{3}{29}V$$

Part DE $\qquad A = tx \qquad \bar{y} = \frac{a}{2} \qquad Q = \frac{1}{2}atx$



$$\tau = \frac{VQ}{It} = \frac{V\cdot\frac{1}{2}atx}{\frac{29}{4}ta^3t} = \frac{2Vx}{29a^3t}$$

$$F_2 = \int \tau\,dA = \int_0^a \frac{2Vx}{29a^3t}\,t\,dx = \frac{2V}{29a^3}\int_0^a x\,dx = \frac{1}{29}V$$

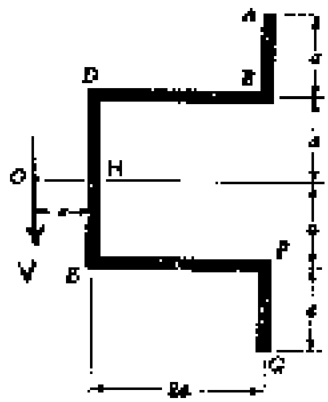
$$\circlearrowleft\Sigma M_K = \circlearrowleft\Sigma M_K$$

$$Ve = F_1(3a) + F_2(a) = \frac{9}{29}Va + \frac{1}{29}Va = \frac{10}{29}Va$$

$$e = \frac{10}{29}a$$

PROBLEM 6.64

6.61 through 6.64 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



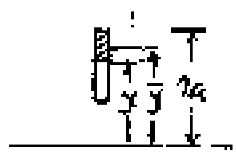
SOLUTION

$$I_{AB} = I_{FG} = \tfrac{1}{12} t a^3 + (ta)\left(\tfrac{3a}{2}\right)^2 = \tfrac{7}{3} t a^3$$

$$I_{DB} = I_{EF} = (2at)a^2 + \tfrac{1}{12}(2a)t^3 \approx 2a^3 t$$

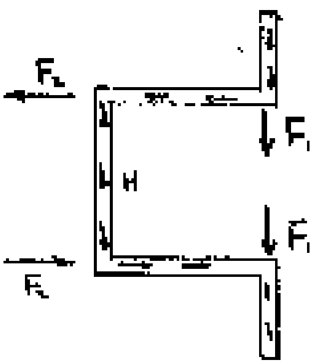
$$I_{DE} = \tfrac{1}{12} t (2a)^3 = \tfrac{2}{3} t a^3 \qquad I = \Sigma I = \tfrac{28}{3} t a^3$$

Part AB $\qquad A = t(2a-y), \quad \bar{y} = \dfrac{2a+y}{2}$



$$Q = A\bar{y} = \tfrac{1}{2} t (2a-y)(2a+y)$$
$$= \tfrac{1}{2} t (4a^2 - y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V}{2I}(4a^2 - y^2)$$

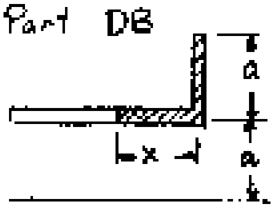


$$F_1 = \int \tau\, dA = \int_a^{2a} \frac{V}{2I}(4a^2 - y^2)\, t\, dy$$

$$= \frac{Vt}{2I}\left(4a^2 y - \frac{y^3}{3}\right)\Bigg|_a^{2a} = \frac{Vta^3}{2I}\left[(4)(2) - \frac{(2)^3}{3} - (4)(1) + \left(\frac{1}{3}\right)\right]$$

$$= \frac{5}{6}\,\frac{Vta^3}{I} = \frac{5}{56} V$$

Part DB



$$Q = (ta)\frac{3a}{2} + t x a$$
$$= ta\left(\frac{3a}{2} + x\right)$$

$$\tau = \frac{VQ}{It} = \frac{Va}{I}\left(\frac{3a}{2} + x\right)$$

$$F_2 = \int \tau\, dA = \int_0^{2a} \frac{Va}{I}\left(\frac{3a}{2} + x\right) t\, dx = \frac{Vta}{I}\int_0^{2a}\left(\frac{3a}{2} + x\right) dx$$

$$= \frac{Vta}{I}\left(\frac{3ax}{2} + \frac{x^2}{2}\right)\Bigg|_0^{2a} = \frac{Vta^3}{I}\left[\frac{(3)(2)}{2} + \frac{(2)^2}{2}\right]$$

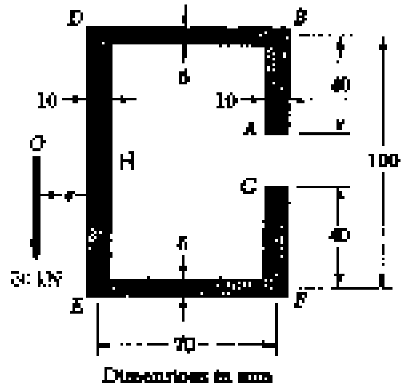
$$= 5\,\frac{Vta^3}{I} = \frac{15}{28} V$$

$$\circlearrowleft \Sigma M_H = \circlearrowleft \Sigma M_H:$$

$$Ve = F_2(2a) - 2F_1(2a) = \frac{30}{28} Va - \frac{20}{56} Va = \frac{5}{7} Va$$

$$e = \frac{5}{7} a$$

PROBLEM 6.65



6.65 and 6.66 An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by a 50-kN vertical shearing force applied at *O*.

SOLUTION

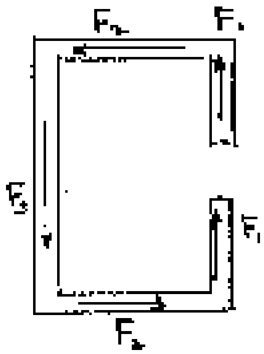
All quantities in mm, mm^2, etc.

$$I_{AB} = \tfrac{1}{12}(10)(40)^3 + (10)(40)(30)^2 = 0.41333 \times 10^6 \ mm^4$$

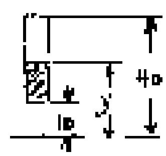
$$I_{DB} = (70)(6)(50)^2 + \tfrac{1}{12}(70)(6)^3 = 1.05126 \times 10^6 \ mm^4$$

$$I_{DE} = \tfrac{1}{12}(10)(100)^3 = 0.83333 \times 10^6 \ mm^4$$

$$I = \Sigma I = 3.7625 \times 10^6 \ mm^4$$



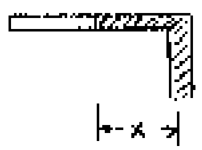
Part AB:



$$A = 10(y-10)$$
$$\bar{y} = \tfrac{1}{2}(y+10)$$
$$Q = A\bar{y} = 5(y-10)(y+10) = 5(y^2-100)$$
$$\tau = \frac{VQ}{It} \qquad Q_B = 5(50^2-100) = 12 \times 10^3 \ mm^3$$
$$F_1 = \int \tau\, dA = \int_{10}^{50} \frac{V\,5(y^2-100)}{I\,t}\, t\, dy$$

$$\frac{F_1}{V} = \frac{5}{I}\int_{10}^{50}(100-y^2)\,dy = \frac{5}{I}\cdot\left(\frac{y^3}{3} - 100y\right)\Big|_{10}^{50} = \frac{5}{I}\left[\frac{50^3}{3} - (100)(50) - \frac{10^3}{3} + (100)(10)\right]$$
$$= \frac{(5)(36.667\times10^3)}{3.7625\times10^6} = 0.048726$$

Part DB



$$Q = Q_B + (6x)(50) = 12\times10^3 + 300x$$

$$\tau = \frac{VQ}{It} \qquad Q_D = 12\times10^3 + (300)(70) = 33\times10^3 \ mm^3$$

$$F_2 = \int \tau\, dA = \int_0^{70} \frac{V(12\times10^3 + 300x)}{I\,t}\, t\, dx = \frac{V}{I}\int_0^{70}(12\times10^3 + 300x)\,dx$$

$$\frac{F_2}{V} = \frac{1}{I}\left[(12\times10^3)x + 300\frac{x^2}{2}\right]_0^{70} = \frac{(12\times10^3)(70) + (300)(70^2)/2}{3.7625\times10^6}$$
$$= 0.41860$$

$$\circlearrowright \Sigma M_H = \circlearrowright M_H \qquad Ve = 2F_1(70) + F_2(100) = (2)(0.048726V)(70) + (0.41860V)(100)$$
$$= 48.7\ V \qquad e = 48.7 \ mm$$

At point H $\qquad Q_H = Q_D + (10)(50)(25) = 33\times10^3 + 12.5\times10^3 = 45.5\times10^3 \ mm^3$

continued

PROBLEM 6.65 (continued)

$V = 50 \times 10^3\ N \qquad I = 3.7625 \times 10^6\ mm^4 = 3.7625 \times 10^{-6}\ m^4$

Part AB, Point A $\qquad Q = 0 \qquad \tau = 0$

Part AB, Point B $\qquad Q = Q_B = 12 \times 10^3\ mm^3 = 12 \times 10^{-6}\ m^3, \qquad t = 10 \times 10^{-3}\ m$

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(12 \times 10^{-6})}{(3.7625 \times 10^{-6})(10 \times 10^{-3})} = 15.95 \times 10^6\ Pa = 15.95\ MPa$$

Part BD, Point B $\qquad Q = 12 \times 10^{-6}\ m^3 \qquad t = 6 \times 10^{-3}\ m$

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(12 \times 10^{-6})}{(3.7625 \times 10^{-6})(6 \times 10^{-3})} = 26.6 \times 10^6\ Pa = 26.6\ MPa$$

Part BD, Point D $\qquad Q = 33 \times 10^{-6}\ m^3 \qquad t = 6 \times 10^{-3}\ m$

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(33 \times 10^{-6})}{(3.7625 \times 10^{-6})(6 \times 10^{-3})} = 73.1 \times 10^6\ Pa = 73.1\ MPa$$

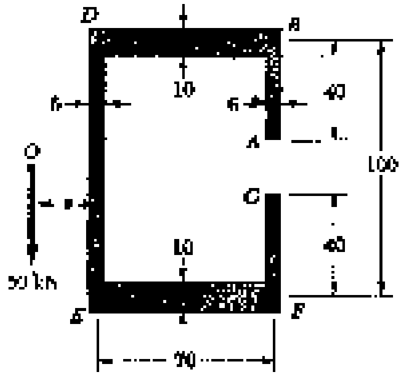
Part DE, Point D $\qquad Q = 33 \times 10^{-6}\ m^3 \qquad t = 10 \times 10^{-3}\ m$

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(33 \times 10^{-6})}{(3.7625 \times 10^{-6})(10 \times 10^{-3})} = 43.9 \times 10^6\ Pa = 43.9\ MPa$$

Point H $\qquad Q = 45.5 \times 10^{-6}\ m^3 \qquad t = 10 \times 10^{-3}$

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(45.5 \times 10^{-6})}{(3.7625 \times 10^{-6})(10 \times 10^{-3})} = 60.5 \times 10^6\ Pa = 60.5\ MPa$$

PROBLEM 6.66



Dimensions in mm

6.65 and 6.66 An extruded beam has the cross section shown. Determine (a) the location of the shear center O, (b) the distribution of the shearing stresses caused by a 50-kN vertical shearing force applied at O.

SOLUTION

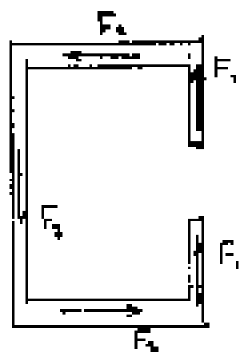
All quantities in mm, mm³, etc.

$$I_{AB} = \tfrac{1}{12}(6)(40)^3 + (6)(40)(30)^2 = 0.248\times10^6\ \text{mm}^4$$

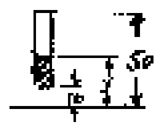
$$I_{BD} = (70)(10)(50)^2 + \tfrac{1}{12}(70)(10)^3 = 1.75583\times10^6\ \text{mm}^4$$

$$I_{DE} = \tfrac{1}{12}(6)(100)^3 = 0.500\times10^6\ \text{mm}^4$$

$$I = \Sigma I = 4.5076\times10^6\ \text{mm}^4$$



Part AB: $A = 6(y-10)$ $\quad \bar{y} = \tfrac{1}{2}(y+10)$



$$Q = A\bar{y} = 3(y-10)(y+10) = 3(y^2-100)$$

$$Q_B = 3(50^2-100) = 7.2\times10^3\ \text{mm}^3$$

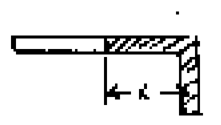
$$\tau = \frac{VQ}{It}$$

$$F_1 = \int \tau\, dA = \int_{10}^{50} \frac{V\,3(y^2-100)}{I\cancel{t}}\cancel{t}\,dy = \frac{3V}{I}\int_{10}^{50}(y^2-100)\,dy$$

$$\frac{F_1}{V} = \frac{3}{I}\int_{10}^{50}(y^2-100)\,dy = \frac{3}{I}\left(\frac{y^3}{3}-100y\right)\Big|_{10}^{50} = \frac{3}{I}\left[\frac{50^3}{3}-(100)(50)-\frac{10^3}{3}+(100)(10)\right]$$

$$= \frac{(3)(36.667\times10^3)}{4.5076\times10^6} = 0.02440$$

Part DB: $\quad Q = Q_B + (10x)(50) = 7.2\times10^3 + 500x$



$$\tau = \frac{VQ}{It} \qquad Q_D = 7.2\times10^3 + (500)(70) = 42.2\times10^3\ \text{mm}^3$$

$$F_2 = \int \tau\, dA = \int_0^{70} \frac{V\,(7.2\times10^3+500x)}{I\cancel{t}}\cancel{t}\,dx$$

$$\frac{F_2}{V} = \frac{1}{I}\int_0^{70}\left[(7.2\times10^3)+500x\right]dx = \frac{1}{I}\left[(7.2\times10^3)x + 500\frac{x^2}{2}\right] = \frac{1}{I}\left[(7.2\times10^3)(70)+(500)\frac{(70)^2}{2}\right]$$

$$= \frac{1.729\times10^6}{4.5076\times10^6} = 0.38357$$

$$\circlearrowright \Sigma M_H = \circlearrowright M_H \qquad Ve = (2F_1)(70)+(F_2)(100) = (2)(0.02440V)(70)+(0.38357V)(100)$$

$$= 41.8\ V \qquad\qquad e = 41.8\ \text{mm}$$

At point H $\quad Q_H = Q_D + (6)(50)(25) = 42.2\times10^3 + 7.5\times10^3 = 49.7\times10^3\ \text{mm}^3$

continued

PROBLEM 6.66 (continued)

$V = 50 \times 10^3$ N $\quad I = 4.5076 \times 10^6\ mm^4 = 4.5076 \times 10^{-6}\ m^4$

Point A $\quad Q = 0 \quad \tau = 0$

Part AB, Point B $\quad Q_B = 7.2 \times 10^{-6}\ m^3 \quad t = 6 \times 10^{-3}$ m

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(7.2 \times 10^{-6})}{(4.5076 \times 10^{-6})(6 \times 10^{-3})} = 13.31 \times 10^6\ Pa = 13.31\ MPa$$

Part BD, Point B $\quad Q = 7.2 \times 10^{-6}\ m^3 \quad t = 10 \times 10^{-3}$ m

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(7.2 \times 10^{-6})}{(4.5076 \times 10^{-6})(10 \times 10^{-3})} = 7.99 \times 10^6\ Pa = 7.99\ MPa$$

Part BD, Point D $\quad Q = 42.2 \times 10^{-6}\ m^3 \quad t = 10 \times 10^{-3}$ m

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(42.2 \times 10^{-6})}{(4.5076 \times 10^{-6})(10 \times 10^{-3})} = 46.8 \times 10^6\ Pa = 46.8\ MPa$$

Part DE, Point D $\quad Q = 42.2 \times 10^{-6}\ m^3 \quad t = 6 \times 10^{-3}$

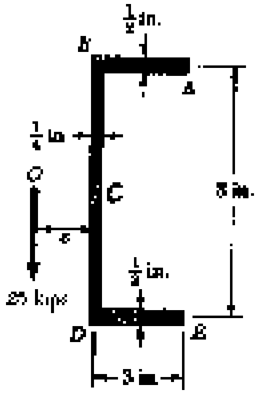
$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(42.2 \times 10^{-6})}{(4.5076 \times 10^{-6})(6 \times 10^{-3})} = 78.0 \times 10^6\ Pa = 78.0\ MPa$$

Point H $\quad Q = 49.7 \times 10^{-6}\ m^3 \quad t = 6 \times 10^{-3}$ m

$$\tau = \frac{VQ}{It} = \frac{(50 \times 10^3)(49.7 \times 10^{-6})}{(4.5076 \times 10^{-6})(6 \times 10^{-3})} = 91.9 \times 10^6\ Pa = 91.9\ MPa$$

PROBLEM 6.67

6.67 and 6.68 An extruded beam has the cross section shown. Determine (*a*) the location of the shear center *O*, (*b*) the distribution of the shearing stresses caused by a 25-kip vertical shearing force applied at *O*.



SOLUTION

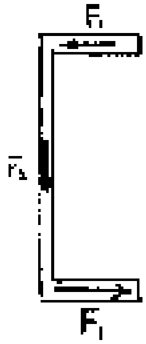
$$I = 2\left[\tfrac{1}{12}(3)(\tfrac{1}{2})^3 + (3)(\tfrac{1}{2})(4)^2\right] + \tfrac{1}{12}(\tfrac{1}{4})(8)^3 = 58.729 \text{ in}^4$$

Part AB $\quad A = \frac{1}{2}x, \quad \bar{y} = 4, \quad Q = A\bar{y} = 2x$

$$\tau = \frac{VQ}{It} = \frac{(25)(2x)}{(58.729)(\frac{1}{2})} = 1.7027x$$

Point A $\quad x = 0 \quad \tau = 0$

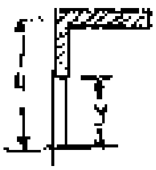
Point B $\quad x = 3 \quad \tau = 5.11$ ksi



$$F_1 = \int \tau\, dA = \int_0^3 1.7027x \cdot \tfrac{1}{2}\, dx = \frac{1.7027}{4}x^2\Big|_0^3$$
$$= \frac{(1.7027)(3)^2}{4} = 3.8311 \text{ kips}$$

$$\circlearrowleft \Sigma M_H = \circlearrowleft \Sigma M_H \qquad 25\,e = (F_1)(8)$$
$$e = \frac{(3.8311)(8)}{25} = 1.226 \text{ in}$$

Part BD $\quad Q = (2)(3) + (\tfrac{1}{4})(4-y)(\tfrac{4+y}{2})$



$$= 6 + \tfrac{1}{8}(16 - y^2) = 8 - \tfrac{1}{8}y^2$$

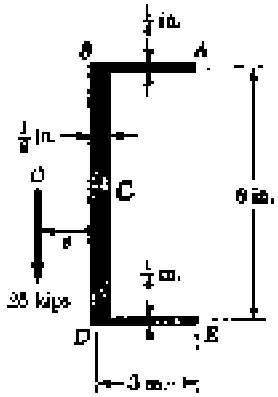
$$\tau = \frac{VQ}{It} = \frac{25(8 - \frac{1}{8}y^2)}{(58.729)(\frac{1}{4})}$$
$$= 13.622 - 0.2128\,y^2$$

Point B $\quad y = 4$ in $\quad \tau = 10.22$ ksi

Point C $\quad y = 0 \quad \tau = 13.62$ ksi

PROBLEM 6.68

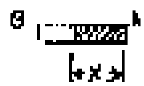
6.67 and 6.68 An extruded beam has the cross section shown. Determine (*a*) the location of the shear center O, (*b*) the distribution of the shearing stresses caused by a 25-kip vertical shearing force applied at O.



SOLUTION

$$I = 2\left[\tfrac{1}{12}(3)(\tfrac{1}{4})^3 + (3)(\tfrac{1}{4})(4)^2\right] + \tfrac{1}{12}(\tfrac{1}{2})(8)^3 = 45.341 \text{ in}^4$$

Part AB $\quad A = \frac{1}{4}x, \quad \bar{y} = 4 \quad Q = A\bar{y} = x$



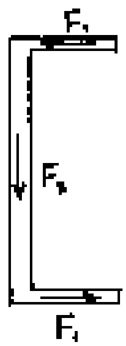
$$\tau = \frac{VQ}{It} = \frac{(25)(x)}{(45.341)(\frac{1}{4})} = 2.2055\,x$$

Point A $\quad x = 0 \quad \tau = 0$

Point B $\quad x = 3 \quad \tau = 6.62$ ksi

$$F_1 = \int \tau\, dA = \int_0^3 (2.2055\,x)\tfrac{1}{4}\,dx = \frac{2.2055}{4}\,\frac{x^2}{2}\Big|_0^3$$

$$= \frac{(2.2055)(3)^2}{(4)(2)} = 2.4812 \text{ kips}$$



$$+\circlearrowright M_H = +\circlearrowright M_H \qquad 25e = F_1(8)$$

$$e = 0.794 \text{ in.}$$

Part BD $\quad Q = 3 + \frac{1}{2}(4-y)\frac{4+y}{2}$

$$= 3 + \tfrac{1}{4}(16 - y^2) = 7 - \tfrac{1}{4}y^2$$

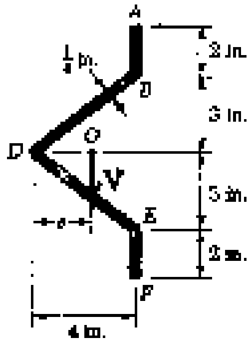
$$\tau = \frac{VQ}{It} = \frac{(25)(7 - \frac{1}{4}y^2)}{(45.341)(\frac{1}{2})} = 7.7193 - 0.2757\,y^2$$

Point B $\quad y = 4 \quad \tau = 3.31$ ksi

Point C $\quad y = 0 \quad \tau = 7.72$ ksi

PROBLEM 6.69

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



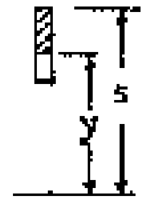
SOLUTION

$$L_{DB} = \sqrt{4^2 + 3^2} = 5 \text{ in.} \qquad A_{DB} = L_{DB}\, t = (5)(\tfrac{1}{4}) = 1.25 \text{ in}^2$$

$$I_{DB} = \tfrac{1}{3} A_{AB} h^2 = (\tfrac{1}{3})(1.25)(3)^2 = 3.75 \text{ in}^4$$

$$I_{AB} = \tfrac{1}{12}(\tfrac{1}{4})(2)^3 + (\tfrac{1}{4})(2)(4)^2 = 8.1667 \text{ in}^4$$

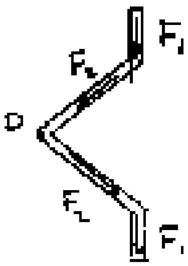
$$I = (2)(3.75) + (2)(8.1667) = 23.833 \text{ in}^4$$



Part AB: $A = \tfrac{1}{4}(5-y) \text{ in}^2, \quad \bar{y} = \tfrac{1}{2}(5+y) \text{ in}$

$$Q = A\bar{y} = \tfrac{1}{8}(5-y)(5+y) = \tfrac{1}{8}(25-y^2)$$

$$\tau = \frac{VQ}{It} = \frac{V(25-y^2)}{(8)(23.833)(0.25)} = \frac{V(25-y^2)}{47.667}$$



$$F_1 = \int \tau \, dA = \int_3^5 \frac{V(25-y^2)}{47.667} \cdot \frac{1}{4}\, dy$$

$$= \frac{V}{190.667}\left[25y - \tfrac{1}{3}y^3\right]_3^5 =$$

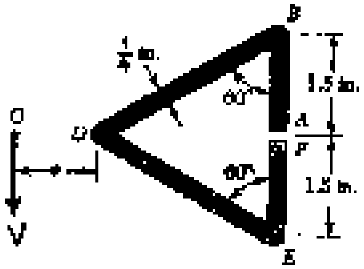
$$= \frac{V}{190.667}\left[(25)(5) - \tfrac{1}{3}(5)^3 - (25)(3) + \tfrac{1}{3}(3)^3\right] = 0.07091\, V$$

$$\circlearrowright M_D = \circlearrowright M_D \qquad -Ve = -2F_1(4) = -0.7273\, V$$

$$e = 0.727 \text{ in.}$$

PROBLEM 6.70

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



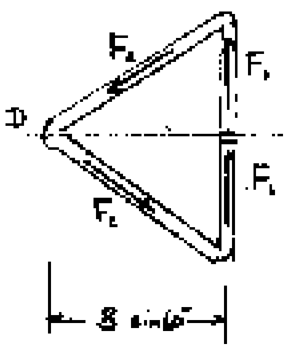
SOLUTION

$$I_{AB} = \frac{1}{3}\left(\frac{1}{4}\right)(1.5)^3 = 0.28125\ \text{in}^4$$

$$L_{BD} = 3\ \text{in} \qquad A_{BD} = (3)\left(\tfrac{1}{4}\right) = 0.75\ \text{in}^2$$

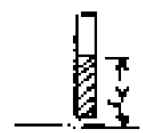
$$I_{BD} = \frac{1}{3}A_{BD}h^2 = \frac{1}{3}(0.75)(1.5)^2 = 0.5625\ \text{in}^4$$

$$I = (2)(0.28125) + (2)(0.5625) = 1.6875\ \text{in}^4$$



Part AB: $\quad A = \frac{1}{4}y \qquad \bar{y} = \frac{1}{2}y \qquad Q = A\bar{y} = \frac{1}{8}y^2$

$$\tau = \frac{VQ}{It} = \frac{V\,\frac{1}{8}y^2}{(8)(1.6875)(0.25)} = \frac{Vy^2}{3.375}$$



$$F_1 = \int \tau\, dA = \int_0^{1.5} \frac{Vy^2}{3.375}\cdot(0.25\,dy)$$

$$= \frac{(0.25)V}{3.375}\,\frac{y^3}{3}\Big|_0^{1.5} = \frac{(0.25)(1.5)^3}{(3.375)(3)}$$

$$= 0.08333\ V$$

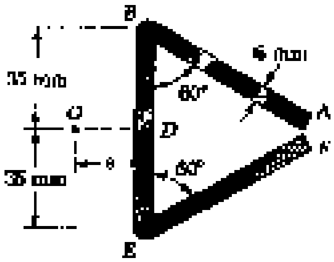
$$\circlearrowleft \Sigma M_D = \circlearrowleft \Sigma M_D \qquad Ve = 2F_1(3\sin 60°)$$

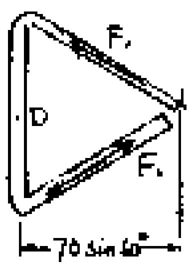
$$Ve = (2)(0.08333)V(3\sin 60°)$$

$$e = (2)(0.08333)(3\sin 60°) = 0.433\ \text{in.}$$

PROBLEM 6.71

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.





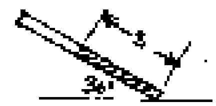
SOLUTION

$I_{DB} = \frac{1}{3}(6)(35)^3 = 85.75 \times 10^3 \text{ mm}^4$

$L_{AB} = 70 \text{ mm} \quad A_{AB} = (70)(6) = 420 \text{ mm}^2$

$I_{AB} = \frac{1}{3} A_{AB} h^2 = (\frac{1}{3})(420)(35)^2 = 171.5 \times 10^3 \text{ mm}^4$

$I = (2)(85.75 \times 10^3) + (2)(171.5 \times 10^3) = 514.5 \times 10^3 \text{ mm}^4$



Part AB $\quad A = ts = 6s$

$\bar{y} = \frac{1}{2} s \sin 30° = \frac{1}{4} s$

$Q = A\bar{y} = \frac{3}{2} s^2$

$$\tau = \frac{VQ}{It} = \frac{3Vs^2}{2It}$$

$$F_1 = \int \tau \, dA = \int_0^{70} \frac{3Vs^2}{2It} t \, ds = \frac{3V}{2I} \int_0^{70} s^2 \, ds$$

$$= \frac{(3)(70)^3}{(2)(3) I} V = \frac{1}{3} V$$

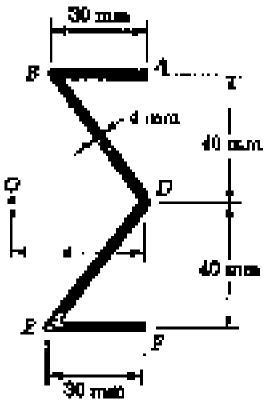
$+\circlearrowleft \Sigma M_D = +\circlearrowleft \Sigma M_D \qquad Ve = 2[F_1 \cos 60°](70 \sin 60°)]$

$= 20.2 \; V$

$e = 20.2 \text{ mm}$ ◄

PROBLEM 6.72

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.



SOLUTION

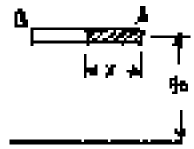
$$I_{AB} = (30)(4)(40)^2 = 192\times10^3\ \text{mm}^4$$

$$L_{BD} = \sqrt{30^2+40^2} = 50\ \text{mm} \qquad A_{BD} = (50)(4) = 200\ \text{mm}^2$$

$$I_{BD} = \tfrac{1}{3}A_{BD}h^2 = \tfrac{1}{3}(200)(40)^2 = 106.67\times10^3\ \text{mm}^4$$

$$I = 2I_{BD} + 2I_{AB} = 597.33\times10^3\ \text{mm}^4$$

Part AB



$$A = 4x \qquad \bar{y} = 40 \qquad Q = A\bar{y} = 160x$$

$$\tau = \frac{VQ}{It} = \frac{V(160x)}{I(4)} = \frac{40V}{I}x$$

$$F_1 = \int \tau\, dA = \int_0^{30}\frac{40V}{I}4\,dx = \frac{160V}{I}\int_0^{30}x\,dx$$

$$= \frac{160Vx^2}{2I}\Big|_0^{30} = \frac{(160)(30)^2}{(2)(597.33\times10^3)}V$$

$$= 0.12054\ V$$

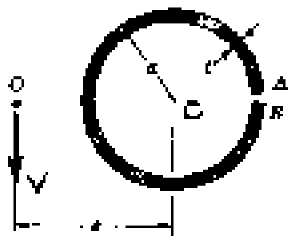
$$+\circlearrowleft M_D = +\circlearrowleft M_D \qquad Ve = (2)F_1(40) = 9.64\ V$$

$$e = 9.64\ \text{mm}$$

PROBLEM 6.73

6.69 through 6.74 Determine the location of the shear center *O* of a thin-walled beam of uniform thickness having the cross section shown.

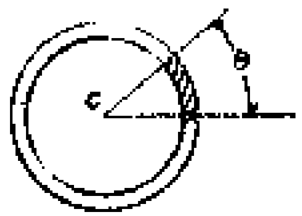
SOLUTION



For whole cross section $\quad A = 2\pi a t$

$J = A a^2 = 2\pi a^3 t \qquad I = \frac{1}{2} J = \pi a^3 t$

Use polar coordinate θ for partial cross section.



$A = st = a\theta t \qquad s = \text{arc length}$

$\bar{r} = a\,\frac{\sin\alpha}{\alpha} \quad \text{where} \quad \alpha = \frac{1}{2}\theta$

$\bar{y} = \bar{r}\sin\alpha = a\,\frac{\sin^2\alpha}{\alpha}$

$Q = A\bar{y} = a\theta t\, a\,\frac{\sin^2\alpha}{\alpha} = a^2 t\; 2\sin^2\alpha$

$= a^2 t\; 2\sin^2\frac{\theta}{2} = a^2 t\,(1 - \cos\theta)$

$\tau = \frac{VQ}{It} = \frac{Va^2}{I}(1-\cos\theta)$

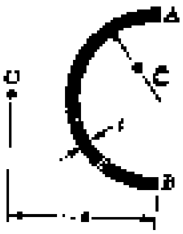
$M_c = \int a\tau\, dA = \int_0^{2\pi} \frac{Va^2}{I}(1-\cos\theta)\, t\, a\, d\theta = \frac{Va^3t}{I}(\theta - \sin\theta)\Big|_0^{2\pi}$

$= \frac{2\pi V a^3 t}{\pi a^3 t} = 2aV$

But $\quad M_c = Ve, \qquad$ hence $\qquad e = 2a$

PROBLEM 6.74

6.69 through 6.74 Determine the location of the shear center O of a thin-walled beam of uniform thickness having the cross section shown.

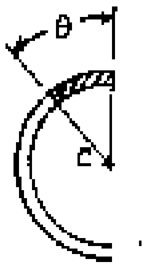


SOLUTION

For a thin-walled hollow circular cross section $A = 2\pi a t$

$J = a^2 A = 2\pi a^3 t \qquad I = \frac{1}{2} J = \pi a^3 t$

For the half-pipe section $I = \frac{\pi}{2} a^3 t$



Use polar coordinate θ for partial cross section

$A = st = a\theta t \qquad s =$ arc length

$\bar{r} = a \frac{\sin\alpha}{\alpha}$ where $\alpha = \frac{\theta}{2}$

$\bar{y} = \bar{r}\cos\alpha = a \frac{\sin\alpha\cos\alpha}{\alpha}$

$Q = A\bar{y} = a\theta t\, a \frac{\sin\alpha\cos\alpha}{\alpha} = a^2 t\,(2\sin\alpha\cos\alpha)$

$= a^2 t \sin 2\alpha = a^2 t \sin\theta$

$\tau = \frac{VQ}{It} = \frac{Va^2}{I}\sin\theta$

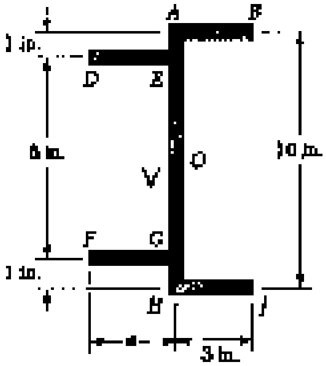
$M_c = \int a\tau\, dA = \int_0^{\pi} a \frac{Va^2}{I} \sin\theta\; t a\, d\theta = \frac{Va^4 t}{I} \left. -\cos\theta \right]_0^{\pi}$

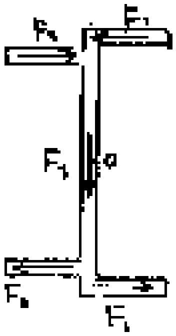
$= 2 \frac{Va^4 t}{I} = \frac{4}{\pi} Va$

But $M_c = Ve$, hence $e = \frac{4}{\pi} a = 1.273\, a$

PROBLEM 6.75

6.75 and 6.76 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension a for which the shear center O of the cross section is located at the point indicated.





SOLUTION

Part AB $\quad A = tx \quad \bar{y} = 5 \text{ in} \quad Q = A\bar{y} = 5tx$

$$\tau = \frac{VQ}{It} = \frac{V \cdot 5tx}{It} = \frac{5Vx}{I}$$

$$F_1 = \int \tau \, dA = \int_0^3 \frac{5Vx}{I} t \, dx = \frac{5Vt}{I} \int_0^3 x \, dx$$

$$= \frac{(5)(3)^2}{2} \frac{Vt}{I} = 22.5 \frac{Vt}{I}$$

Part DE $\quad A = tx \quad \bar{y} = 4 \text{ in} \quad Q = A\bar{y} = 4tx$

$$\tau = \frac{VQ}{It} = \frac{V \, 4tx}{It} = \frac{4Vx}{I}$$

$$F_2 = \int \tau \, dA = \int_0^a \frac{4Vx}{I} \cdot t \, dx = \frac{4Vt}{I} \int_0^a x \, dx$$

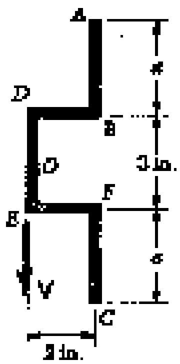
$$= \frac{2Vta^2}{I}$$

$$\circlearrowleft \Sigma M_O = \Sigma M_O \qquad 0 = (10)(22.5 \frac{Vt}{I}) - (8) \frac{2Vta^2}{I}$$

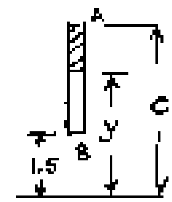
$$a^2 = \frac{(10)(22.5)}{(8)(2)} = 14.0625 \text{ in}^2 \qquad a = 3.75 \text{ in.}$$

PROBLEM 6.76

6.75 and 6.76 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension a for which the shear center O of the cross section is located at the point indicated.



SOLUTION



Part AB   Let $c = 1.5 + a$ as shown.

$$A = t(c-y) \qquad \bar{y} = \tfrac{1}{2}(c+y)$$

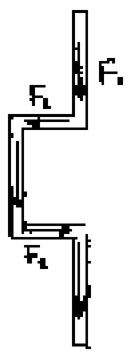
$$Q = A\bar{y} = \tfrac{1}{2}t(c-y)(c+y) = \tfrac{1}{2}t(c^2-y^2)$$

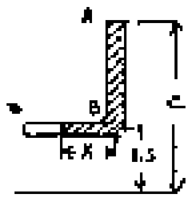
$$\tau = \frac{VQ}{It} = \frac{V(c^2-y^2)}{2I}$$

$$F_1 = \int \tau\, dA = \int_{1.5}^{c} \frac{V(c^2-y^2)}{2I}\, t\, dy = \frac{Vt}{2I}\int_{1.5}^{c}(c^2-y^2)\,dy$$

$$= \frac{Vt}{2I}\left(c^2y - \frac{y^3}{3}\right)\Big|_{1.5}^{c} = \frac{Vt}{2I}\left[c^3 - \frac{c^3}{3} - 1.5c^2 + \frac{(1.5)^3}{3}\right]$$

$$= \frac{Vt}{2I}\left[\tfrac{2}{3}c^3 - 1.5c^2 + 1.125\right]$$





Part BD $Q = Q_{AB} + t x \bar{y}_{BD}$

$$= \tfrac{1}{2}t(c^2 - 1.5^2) + t\,x\,(1.5)$$

$$\tau = \frac{VQ}{It} = \frac{V}{2I}(c^2 - 1.5^2 + 3x)$$

$$F_2 = \int \tau\, dA = \int_0^2 \frac{V}{2I}(c^2 - 1.5^2 + 3x)\,t\,dx$$

$$= \frac{Vt}{2I}\left[(c^2-1.5^2)x + 1.5x^2\right]\Big|_0^2 = \frac{Vt}{2I}\left[2c^2 - (2)(1.5)^2 + (1.5)(2)^2\right]$$

$$= \frac{Vt}{2I}\left[2c^2 + 1.5\right]$$

$$+\circlearrowleft \Sigma M_O = +\circlearrowleft \Sigma M_O \qquad 0 = 3F_2 - (2)(2)F_1$$

$$\frac{Vt}{2I}\left\{3(2c^2+1.5) - 4\left(\tfrac{2}{3}c^3 - 1.5c^2 + 1.125\right)\right\} = 0$$

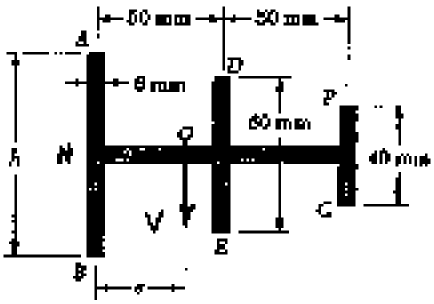
$$-\tfrac{8}{3}c^3 + 12c^2 = 0$$

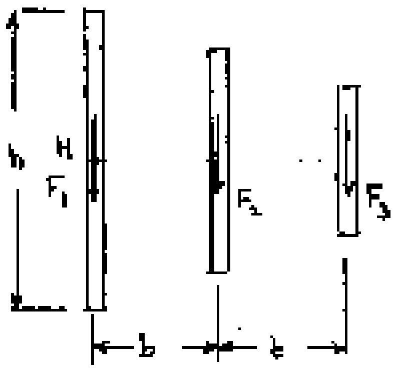
$$c = \frac{(12)(3)}{8} = 4.5 \text{ in}$$

$$a = 4.5 - 1.5 = 3.00 \text{ in.}$$

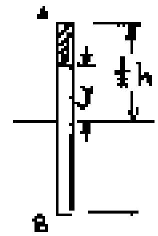
PROBLEM 6.77

6.77 A thin-walled beam of uniform thickness has the cross section shown. Determine the location of the shear center O of the cross section, knowing that $h = 80$ mm.





SOLUTION



Let $h_1 = \overline{AB} = h$, $h_2 = \overline{DE}$, $h_3 = \overline{FG}$

$$I = \frac{1}{12} t \left(h_1^3 + h_2^3 + h_3^3\right)$$

Part AB: $A = \left(\frac{1}{2}h_1 - y\right)t$

$$\bar{y} = \frac{1}{2}\left(\frac{1}{2}h_1 + y\right)$$

$$Q = A\bar{y} = \frac{1}{2}t\left(\frac{1}{2}h_1 - y\right)\left(\frac{1}{2}h_1 + y\right) = \frac{1}{2}t\left(\frac{1}{4}h_1^2 - y^2\right)$$

$$\tau = \frac{VQ}{It} = \frac{V}{2I}\left(\frac{1}{4}h_1^2 - y^2\right)$$

$$F_1 = \int \tau\, dA = \int_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1} \frac{V}{2I}\left(\frac{1}{4}h_1^2 - y^2\right) t\, dy$$

$$= \frac{Vt}{2I}\left(\frac{1}{4}h_1^2 y - \frac{y^3}{3}\right)\Bigg|_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1}$$

$$= \frac{Vt}{I}\left(\frac{1}{4}h_1^2 \cdot \frac{1}{2}h_1 - \frac{1}{3}\left(\frac{h_1}{2}\right)^3\right) = \frac{V t h_1^3}{12 I}$$

$$= \frac{h_1^3 V}{h_1^3 + h_2^3 + h_3^3}$$

Likewise, for Part DE $\qquad F_2 = \dfrac{h_2^3 V}{h_1^3 + h_2^3 + h_3^3}$

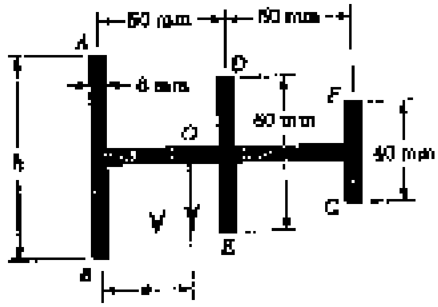
and for Part FG $\qquad F_3 = \dfrac{h_3^3 V}{h_1^3 + h_2^3 + h_3^3}$

$$+\circlearrowleft \Sigma M_H = +\circlearrowleft \Sigma M_H \qquad Ve = bF_2 + 2bF_3 = \frac{b h_2^3 + 2b h_3^3}{h_1^3 + h_2^3 + h_3^3} V$$

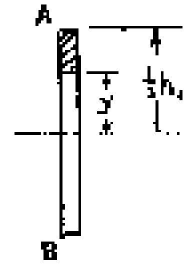
$$e = \frac{h_2^3 + 2h_3^3}{h_1^3 + h_2^3 + h_3^3}\, b = \frac{(60)^3 + (2)(40)^3}{(80)^3 + (60)^3 + (40)^3}(50) = 21.7 \text{ mm}$$

PROBLEM 6.78

6.78 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension h for which the shear center O of the cross section is located at a distance $e = 25$ mm from the center of the flange AB.



SOLUTION



Let $h_1 = \overline{AB} = h$, $h_2 = \overline{DE}$, $h_3 = \overline{FG}$

$$I = \frac{1}{12}t\left(h_1^3 + h_2^3 + h_3^3\right)$$

Part AB $\quad A = \left(\tfrac{1}{2}h_1 - y\right)t$

$$\bar{y} = \tfrac{1}{2}\left(\tfrac{1}{2}h_1 + y\right)$$

$$Q = A\bar{y} = \tfrac{1}{2}t\left(\tfrac{1}{2}h_1 - y\right)\left(\tfrac{1}{2}h_1 + y\right) = \tfrac{1}{2}t\left(\tfrac{1}{4}h_1^2 - y^2\right)$$

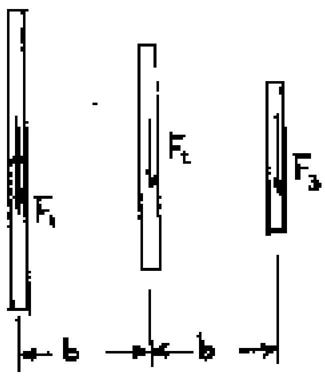
$$\tau = \frac{VQ}{It} = \frac{V}{2I}\left(\tfrac{1}{4}h_1^2 - y^2\right)$$

$$F_1 = \int \tau\, dA = \int_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1} \frac{V}{2I}\left(\tfrac{1}{4}h_1^2 - y^2\right)t\,dy$$

$$= \frac{Vt}{2I}\left(\tfrac{1}{4}h_1^2 y - \tfrac{1}{3}y^3\right)\Big|_{-\frac{1}{2}h_1}^{\frac{1}{2}h_1}$$

$$= \frac{Vt}{I}\left[\left(\tfrac{1}{4}h_1^2\right)\left(\tfrac{1}{2}h_1\right) - \tfrac{1}{3}\left(\tfrac{1}{2}h_1\right)^3\right] = \frac{Vth_1^3}{12\,I}$$

$$= \frac{h_1^3 V}{h_1^3 + h_2^3 + h_3^3}$$



Likewise, for Part DE $\qquad F_2 = \dfrac{h_2^3 V}{h_1^3 + h_2^3 + h_3^3}$

and for Part FG $\qquad F_3 = \dfrac{h_3^3 V}{h_1^3 + h_2^3 + h_3^3}$

$+\circlearrowleft \Sigma M_O = \circlearrowleft \Sigma M_O \qquad 0 = eF_1 - (b-e)F_2 - (2b-e)F_3$

$$\frac{eh_1^3 - (b-e)h_2^3 - (2b-e)h_3^3}{h_1^3 + h_2^3 + h_3^3}V = 0$$

$$h_1^3 = \frac{b-e}{e}h_2^3 + \frac{(2b-e)}{e}h_3^3 = \frac{(25)(60)^3}{25} + \frac{(75)(40)^3}{25} = 408\times10^3\ \text{mm}^3$$

$$h_1 = 74.2\ \text{mm}$$

PROBLEM 6.79

6.79 For the angle shape and loading of Sample Prob. 6.5, check that $\int q\,dz = 0$ along the horizontal leg of the angle and $\int q\,dy = P$ along its vertical leg.

SOLUTION

Referring to Sample Prob. 6.5

Along horizontal leg $\quad \tau_f = \dfrac{3P(a-z)(a-3z)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 - 4az + 3z^2)$

$$\int q\,dz = \int_0^a \tau_f t\,dz = \frac{3P}{4a^3}\int_0^a (a^2 - 4az + 3z^2)\,dz = \frac{3P}{4a^3}\left(a^2z - 4a\frac{z^2}{2} + 3\frac{z^3}{3}\right)\Big|_0^a$$

$$= \frac{3P}{4a^3}(a^3 - 2a^3 + a^3) = 0$$

Along vertical leg $\quad \tau_e = \dfrac{3P(a-y)(a+5y)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 + 4ay - 5y^2)$

$$\int q\,dy = \int_0^a \tau_e t\,dy = \frac{3P}{4a^3}\int_0^a (a^2 + 4ay - 5y^2)\,dy = \frac{3P}{4a^3}\left(a^2y + 4a\frac{y^2}{2} - 5\frac{y^3}{3}\right)\Big|_0^a$$

$$= \frac{3P}{4a^3}\left(a^3 + 2a^3 - \frac{5}{3}a^3\right) = \frac{3P}{4a^3}\cdot\frac{4}{3}a^3 = P$$

PROBLEM 6.80

6.80 For the angle shape and loading of Sample Prob. 6.5, (*a*) determine the points where the shearing stress is maximum and the corresponding values of the stress, (*b*) verify that the points obtained are located on the neutral axis corresponding to the given loading.

SOLUTION

Referring to Sample Prob. 6.5

(a) Along vertical leg $\tau_e = \dfrac{3P(a-y)(a+5y)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 + 4ay - 5y^2)$

$$\frac{d\tau_e}{dy} = \frac{3P}{4ta^3}(4a - 10y) = 0 \qquad y = \frac{2}{5}a$$

$$\tau_m = \frac{3P}{4ta^3}\left[a^2 + (4a)(\tfrac{2}{5}a) - (5)(\tfrac{2}{5}a)^2\right] = \frac{3P}{4ta^3}(\tfrac{9}{5}a^2) = \frac{27}{20}\frac{P}{ta}$$

Along horizontal leg $\tau_f = \dfrac{3P(a-z)(a-3z)}{4ta^3} = \dfrac{3P}{4ta^3}(a^2 - 4az + 3z^2)$

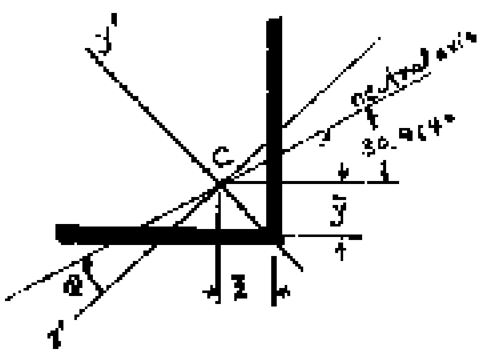
$$\frac{d\tau_f}{dz} = \frac{3P}{4ta^3}(-4a + 6z) = 0 \qquad z = \frac{2}{3}a$$

$$\tau_m = \frac{3P}{4ta^3}\left[a^2 - (4a)(\tfrac{2}{3}a) + (3)(\tfrac{2}{3}a)^2\right] = \frac{3P}{4ta^3}(-\tfrac{5}{8}a^2) = -\frac{1}{4}\frac{P}{ta}$$

At the corner $\quad y = 0, \; z = 0 \qquad \tau = \dfrac{3}{4}\dfrac{P}{ta}$

(b) $I_{y'} = \frac{1}{3}ta^3 \qquad I_{z'} = \frac{1}{12}ta^3 \qquad \theta = 45^\circ$

$$\tan\varphi = \frac{I_{z'}}{I_{y'}}\tan\theta = \frac{1}{4} \qquad \varphi = 14.036^\circ$$



$$\theta - \varphi = 45 - 14.036 = 30.964^\circ$$

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{at(a/2)}{2at} = \frac{1}{4}a$$

$$\bar{z} = \frac{\Sigma A\bar{z}}{\Sigma A} = \frac{at(a/2)}{2at} = \frac{1}{4}a$$

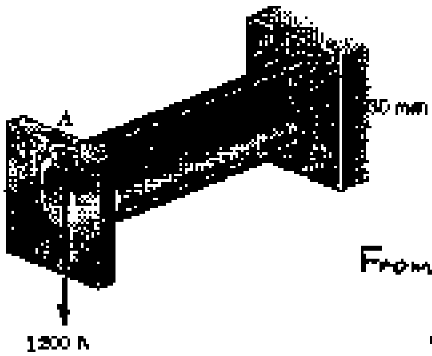
Neutral axis intersects vertical leg at

$$y = \bar{y} + \bar{z}\tan 30.964^\circ$$
$$= (\tfrac{1}{4} + \tfrac{1}{4}\tan 30.964^\circ)a = 0.400\,a$$
$$= \tfrac{2}{5}a$$

Neutral axis intersects horizontal leg at

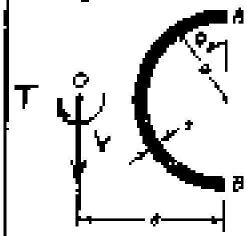
$$z = \bar{z} + \bar{y}\tan(45^\circ + \varphi)$$
$$= (\tfrac{1}{4} + \tfrac{1}{4}\tan 59.036^\circ)a = 0.6667\,a$$
$$= \tfrac{2}{3}a$$

PROBLEM 6.81



*6.81 A cantilever beam AB, consisting of half of a thin-walled pipe of 30-mm mean radius and 6-mm wall thickness, is subjected to a 1200-N vertical load. Knowing that the line of action of the load passes through the centroid C of the cross section of the beam, determine (a) the equivalent force-couple system at the shear center of the cross section, (b) the maximum stress in the beam. (*Hint*: The location of the shear center of the cross section was determined in Prob. 6.74.)

SOLUTION



From the solution to PROBLEM 6.74

$$e = \frac{4}{\pi}a \qquad I = \frac{\pi}{2}a^3 t$$

$$Q = a^2 t \sin\theta \qquad \tau = \frac{VQ}{It}$$

$$Q_{max} = a^2 t \quad \text{at} \quad \theta = 90°$$

Due to shearing force $\quad \tau_{90°} = \dfrac{V Q_{max}}{It}$

$V = 1200 \text{ N} \qquad t = 6\times10^{-3} \text{ m}$

$I = \frac{\pi}{2}(30)^3(6) = 254.47\times10^{3} \text{ mm}^4 = 254.47\times10^{-9} \text{ m}^4$

$Q_{max} = (30)^2(6) = 5.4\times10^{3} \text{ mm}^3 = 5.4\times10^{-6} \text{ m}^3$

$$\tau_{90°} = \frac{(1200)(5.4\times10^{-6})}{(254.47\times10^{-9})(6\times10^{-3})} = 4.24\times10^{6} \text{ Pa} = 4.24 \text{ MPa}$$

$$e = \frac{4}{\pi}a, \qquad \bar{x} = \frac{2}{\pi}a \qquad e - \bar{x} = \frac{2}{\pi}a$$

Torque $\quad T = (e - \bar{x})V = \frac{2}{\pi}(30\times10^{-3})(1200) = 22.92 \text{ N·m}$

$\ell = \pi a = \pi(30) = 94.248 \text{ mm} = 94.248\times10^{-3} \text{ m}$

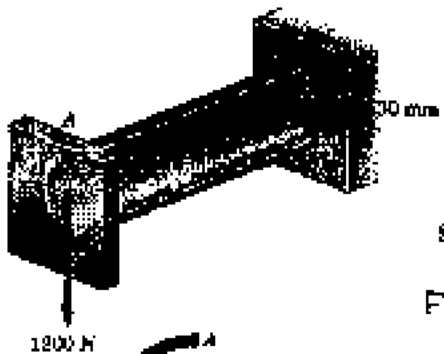
For torsion of a rectangular bar $\quad c_1 = c_2 = \frac{1}{3}\left(1 - 0.630\frac{t}{\ell}\right)$

$$= \frac{1}{3}\left(1 - \frac{(0.630)(6)}{94.248}\right) = 0.31996$$

$$\tau_{torsion} = \frac{T}{c_1 \ell t^2} = \frac{22.92}{(0.31996)(94.248\times10^{-3})(6\times10^{-3})^2} = 21.11\times10^{6} \text{ Pa} = 21.11 \text{ MPa}$$

By superposition $\quad \tau_{max} = 4.24 + 21.11 = 25.35 \text{ MPa}$

PROBLEM 6.82

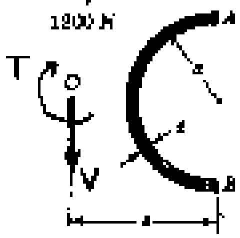


*6.81 A cantilever beam AB, consisting of half of a thin-walled pipe of 30-mm mean radius and 6-mm wall thickness, is subjected to a 1200-N vertical load. Knowing that the line of action of the load passes through the centroid C of the cross section of the beam, determine (a) the equivalent force-couple system at the shear center of the cross section, (b) the maximum stress in the beam. (*Hint:* The location of the shear center of the cross section was determined in Prob. 6.74.)

*6.82 Solve Prob. 6.81, assuming that the thickness of the beam is reduced to 5 mm.

SOLUTION

From the solution to PROBLEM 6.74



$$e = \frac{4}{\pi}a \qquad I = \frac{\pi}{2}a^3 t$$

$$Q = a^2 t \sin\theta \qquad \tau = \frac{VQ}{It} = \frac{Va^2}{I}$$

$$Q_{max} = a^2 t \quad \text{at } \theta = 90°$$

Due to shearing force $\quad \tau_{sv} = \dfrac{VQ_{max}}{It}$

$V = 1200$ N $\qquad t = 5 \times 10^{-3}$ m

$I = \frac{\pi}{2}(30)^3(5) = 212.06 \times 10^3$ mm^4 $= 212.06 \times 10^{-9}$ m^4

$Q_{max} = (30)^2(5) = 4.5 \times 10^3$ mm^3 $= 4.5 \times 10^{-6}$ m^3

$$\tau_{sv} = \frac{(1200)(4.5 \times 10^{-6})}{(212.06 \times 10^{-9})(5 \times 10^{-3})} = 5.09 \times 10^6 \text{ Pa} = 5.09 \text{ MPa}$$

$$e = \frac{4}{\pi}a \qquad \bar{x} = \frac{2}{\pi}a \qquad e - \bar{x} = \frac{2}{\pi}a$$

Torque $\quad T = (e - \bar{x})V = \frac{2}{\pi}(30 \times 10^{-3})(1200) = 22.92$ N·m

For torsion of a rectangular bar $\quad c_1 = c_2 = \frac{1}{3}\left[1 - 0.630\frac{t}{\ell}\right]$

$$= \frac{1}{3}\left[1 - \frac{(0.630)(5)}{94.248}\right] = 0.32219$$

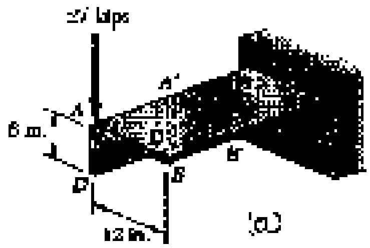
$$\tau_{torsion} = \frac{T}{c_1 \ell t^2} = \frac{22.92}{(0.32219)(94.248 \times 10^{-3})(5 \times 10^{-3})^2} = 30.19 \times 10^6 \text{ Pa}$$

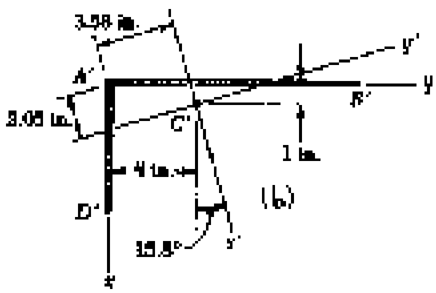
$$= 30.19 \text{ MPa}$$

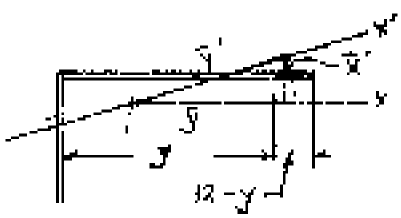
By superposition $\quad \tau_{max} = 5.09 + 30.19 = 35.3$ MPa

PROBLEM 6.83

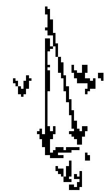
*6.83 The cantilever beam shown consists of an angle shape of $\frac{3}{8}$ - in. thickness. For the given loading, determine the location and magnitude of the largest shearing stress along line $A'B'$ in the horizontal leg of the angle shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 115.7\ \text{in}^4$ and $I_{y'} = 12.61\ \text{in}^4$.







SOLUTION



$V = 27$ kips $\qquad \beta = 15.8°$

$V_{x'} = V\cos\beta \qquad V_{y'} = V\sin\beta$

In the horizontal leg use coordinate y as shown.

$A = \frac{3}{8}(12 - y)\ \text{in}^2 \qquad t = \frac{3}{8}$ in

$\bar{y} = \frac{1}{2}(12 + y) - 4 = 2 + \frac{1}{2}y$ in.

$\bar{x} = 1$ in.

$\bar{x}' = \bar{x}\cos\beta - \bar{y}\sin\beta$

$\bar{y}' = \bar{y}\cos\beta + \bar{x}\sin\beta$

Due to $V_{x'}$: $\quad \tau_1 = \dfrac{V_{x'}A\bar{x}'}{I_{y'}t} = \dfrac{(V\cos\beta)(\frac{3}{8})(12-y)[(1)\cos\beta - (2+\frac{1}{2}y)\sin\beta]}{(12.61)(\frac{3}{8})}$

$= 2.0603(12 - y)(0.41765 - 0.13614\,y)$ ksi

Due to $V_{y'}$: $\quad \tau_2 = \dfrac{V_{y'}A\bar{y}'}{I_{x'}t} = \dfrac{(V\sin\beta)(\frac{3}{8})(12-y)[(2+\frac{1}{2}y)\cos\beta + (1)\sin\beta]}{(115.6)(\frac{3}{8})}$

$= 0.063595(12 - y)(2.19672 + 0.48111\,y)$ ksi

Total $\quad \tau_1 + \tau_2 = (12 - y)(1.000 - 0.250\,y)$ ksi

y (in)	0	2	4	6	8	10	12
τ (ksi)	12.00	5.00	0	−3.00	−4.00	−3.00	0

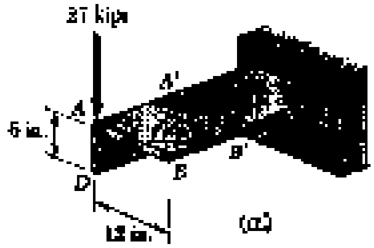
$\tau = 12$ ksi at corner

$\tau = -4$ ksi at $y = 8$ in

$\dfrac{d\tau}{dx} = -(0.25)(12 - y_m) + (1 - 0.25\,y_m)$

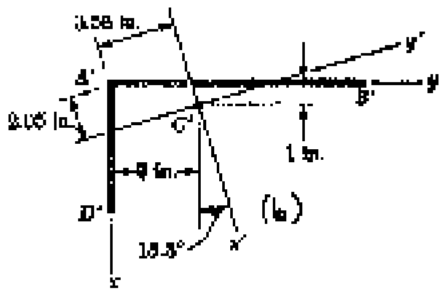
$= 0.5\,y_m - 4 = 0 \qquad y_m = 8$ in

PROBLEM 6.84

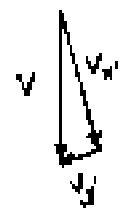


*6.83 The cantilever beam shown consists of an angle shape of $\frac{3}{8}$-in. thickness. For the given loading, determine the location and magnitude of the largest shearing stress along line $A'B'$ in the horizontal leg of the angle shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 115.7\ in^4$ and $I_{y'} = 12.61\ in^4$.

*6.84 For the cantilever beam and loading of Prob. 6.83, determine the location and magnitude of the largest shearing stress along line $A'D'$ in the vertical leg of the angle shape.



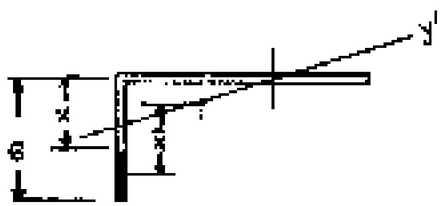
SOLUTION



$V = 27$ kips $\qquad \beta = 15.8°$

$V_{x'} = V\cos\beta \qquad V_{y'} = V\sin\beta$

In vertical leg use coordinate x as shown.



$A = \frac{3}{8}(6 - x)\ in^2 \qquad t = \frac{3}{8}$ in.

$\bar{y} = 4$ in

$\bar{x} = \frac{1}{2}(6 + x) - 1 = 2 + \frac{1}{2}x$

$\bar{x}' = \bar{x}'\cos\beta - \bar{y}\sin\beta$

$\bar{y}' = \bar{y}\cos\beta + \bar{x}\sin\beta$

Due to $V_{x'}$: $\quad \tau_1 = \dfrac{V_{x'}A\bar{x}'}{I_{y'}t} = \dfrac{(V\cos\beta)(\frac{3}{8})(6-x)\left[(2+\frac{1}{2}x)\cos\beta - 4\sin\beta\right]}{(12.61)(\frac{3}{8})}$

$= 2.0603\,(6 - x)(0.83531 + 0.48111\,x)$

Due to $V_{y'}$: $\quad \tau_2 = \dfrac{V_{y'}A\bar{y}'}{I_{x'}t} = \dfrac{(V\sin\beta)(\frac{3}{8})(6-x)\left[4\cos\beta + (2+\frac{1}{2}x)\sin\beta\right]}{(115.6)(\frac{3}{8})}$

$= 0.06359\,(6 - x)(4.3984 + 0.13614\,x)$

Total: $\tau_1 + \tau_2 = (6 - x)(2.000 + 1.000\,x)$

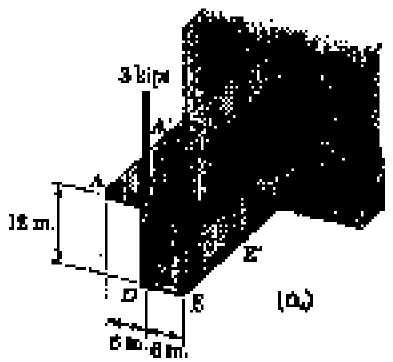
x (in)	0	1	2	3	4	5	6
τ (ksi)	12.00	15.00	16.00	15.00	12.00	7.00	0

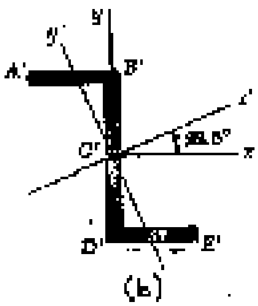
$\tau_{max} = 16$ ksi at $x = 2$ in. ◄

$\frac{d\tau}{dx} = (6 - x_m)(1) + (2 + x_m)(-1)$

$= 4 - 2x_m = 0 \qquad x_m = 2$ in

PROBLEM 6.87



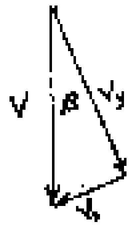


*6.87 The cantilever beam shown consists of a Z shape of $\frac{1}{4}$-in. thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 166.3\ \text{in}^4$ and $I_{y'} = 13.61\ \text{in}^4$.

SOLUTION

$V = 3$ kips $\qquad \beta = 22.5°$

$V_{x'} = V \sin\beta \qquad V_{y'} = V\cos\beta$



In upper horizontal leg use coordinate x $\quad (-6\ \text{in} \le x \le 0)$

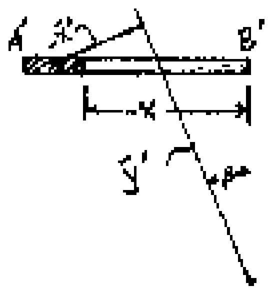
$A = \frac{1}{4}(6 + x)\ \text{in}$

$\bar{x} = \frac{1}{2}(-6 + x)\ \text{in}$

$\bar{y} = 6\ \text{in.}$

$\bar{x}' = \bar{x}\cos\beta + \bar{y}\sin\beta$

$\bar{y}' = \bar{y}\cos\beta - \bar{x}\sin\beta$



Due to $V_{x'}$ $\quad \tau_1 = \dfrac{V_{x'} A \bar{x}'}{I_{y'}\, t}$

$$\tau_1 = \frac{(V\sin\beta)(\frac{1}{4})(6+x)\left[\frac{1}{2}(-6+x)\cos\beta + 6\sin\beta\right]}{(13.61)(\frac{1}{4})}$$

$$= 0.084353\,(6+x)(-0.47554 + 0.46194\,x)$$

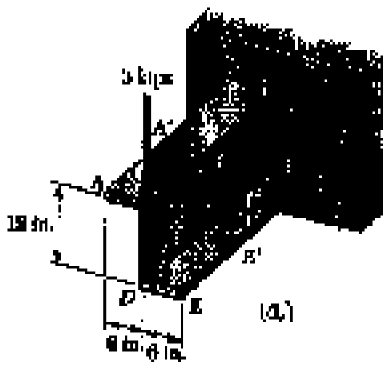
Due to $V_{y'}$ $\quad \tau_2 = \dfrac{V_{y'} A \bar{y}'}{I_{x'}\, t} = \dfrac{(V\cos\beta)(\frac{1}{4})(6+x)\left[6\cos\beta - \frac{1}{2}(-6+x)\sin\beta\right]}{(166.3)(\frac{1}{4})}$

$$= 0.0166665\,(6+x)\left[6.69132 - 0.19134\,x\right]$$

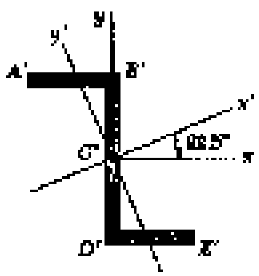
Total $\tau_1 + \tau_2 = (6+x)\left[-0.07141 + 0.085896\,x\right]$

x (in)	−6	−5	−4	−3	−2	−1	0
τ (ksi)	0	−0.105	−0.140	−0.104	0.003	0.180	0.428

PROBLEM 6.88



(a)



(b)

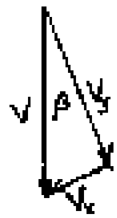
*6.87 The cantilever beam shown consists of a Z shape of $\frac{1}{4}$-in. thickness. For the given loading, determine the distribution of the shearing stresses along line $A'B'$ in the upper horizontal leg of the Z shape. The x' and y' axes are the principal centroidal axes of the cross section and the corresponding moments of inertia are $I_{x'} = 166.3\ \text{in}^4$ and $I_{y'} = 13.61\ \text{in}^4$.

*6.88 For the cantilever beam and loading of Prob. 6.87, determine the distribution of the shearing stresses along line $B'D'$ in the vertical web of the Z shape.

SOLUTION

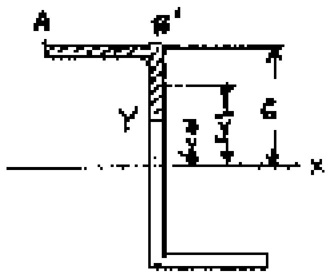
$V = 3\ \text{kips} \qquad \beta = 22.5°$

$V_{x'} = V\sin\beta \qquad V_{y'} = V\cos\beta$



For part AB': $A = (\frac{1}{4})(6) = 1.5\ \text{in}^2$, $\bar{x} = -3\ \text{in}$, $\bar{y} = 6\ \text{in}.$

For part $B'Y$: $A = \frac{1}{4}(6-y)$, $\bar{x} = 0$, $\bar{y} = \frac{1}{2}(6+y)$



$x' = x\cos\beta + y\sin\beta$

$y' = y\cos\beta - x\sin\beta$

Due to $V_{x'}$:

$$\tau_1 = \frac{V_{x'}(A_{AB}\bar{x}'_{AB} + A_{B'Y}\bar{x}'_{B'Y})}{I_{y'}\,t}$$

$$\tau_1 = \frac{(V\sin\beta)\left[(1.5)(-3\cos\beta + 6\sin\beta) + \frac{1}{4}(6-y)\frac{1}{2}(6+y)\sin\beta\right]}{(13.61)(\frac{1}{4})}$$

$$= \frac{(V\sin\beta)\left[-0.7133 + 1.7221 - 0.047835\,y^2\right]}{3.4025} = 0.3409 - 0.01614\,y^2$$

Due to $V_{y'}$:

$$\tau_2 = \frac{V_{y'}(A_{AB}\bar{y}'_{AB} + A_{B'Y}\bar{y}')}{I_{x'}\,t}$$

$$\tau_2 = \frac{(V\cos\beta)\left[(1.5)(6\cos\beta + 3\sin\beta) + \frac{1}{4}(6-y)\frac{1}{2}(6+y)\cos\beta\right]}{(166.3)(\frac{1}{4})}$$

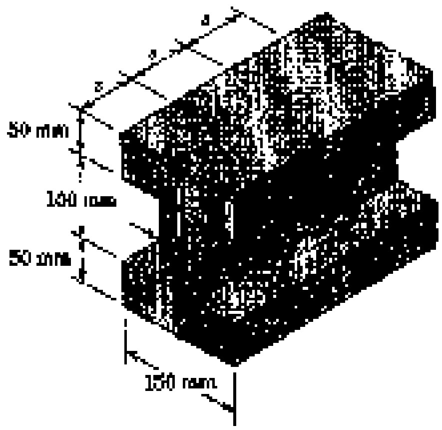
$$= \frac{(V\cos\beta)\left[10.037 + 4.1575 - 0.11548\,y^2\right]}{(166.3)(\frac{1}{4})} = 0.9463 - 0.00770\,y^2$$

Total $\tau_1 + \tau_2 = 1.2867 - 0.02384\,y^2$

y (in)	0	±2	±4	±6
τ (ksi)	1.287	1.191	0.905	0.428

PROBLEM 6.89

6.89 Three boards, each 50 mm thick, are nailed together to form a beam that is subjected to a 1200-N vertical shear. Knowing that the allowable shearing force in each nail is 600 N, determine the largest permissible spacing s between the nails.



SOLUTION

Calculate moment of inertia

Part	$A\ (\text{mm}^2)$	$d\ (\text{mm})$	$Ad^2\ (10^6\ \text{mm}^4)$	$\bar{I}\ (10^6\ \text{mm}^4)$
Top	7500	75	42.19	1.56
Middle	5000	0	0	4.17
Bottom	7500	75	42.19	1.56
Σ			84.38	7.29

$$I = \Sigma A d^2 + \Sigma \bar{I} = 91.67 \times 10^6\ \text{mm}^4 = 91.67 \times 10^{-6}\ \text{m}^4$$

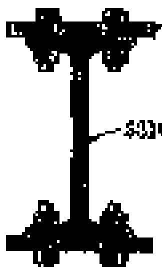
$$Q = A_{top} d_{top} = (7500)(75) = 562.5 \times 10^3\ \text{mm}^3 = 562.5 \times 10^{-6}\ \text{m}^3$$

$$q = \frac{VQ}{I} = \frac{(1200)(562.5 \times 10^{-6})}{91.67 \times 10^{-6}} = 7.363 \times 10^3\ \text{N/m}$$

$$F_{nail} = q s \qquad s = \frac{F_{nail}}{q} = \frac{600}{7.363 \times 10^3} = 81.5 \times 10^{-3}\ \text{m} = 81.5\ \text{mm}$$

PROBLEM 6.90

6.90 The American Standard rolled-steel beam shown has been reinforced by attaching to it two 16 × 200-mm plates, using bolts of 18-mm diameter spaced longitudinally every 120 mm. Knowing that the allowable average shearing stress in the bolts is 90 MPa, determine the largest permissible shearing force.



SOLUTION

Calculate moment of inertia

$$* \; d = \frac{305}{2} + \frac{16}{2} = 160.5 \text{ mm}$$

Part	A (mm²)	d (mm)	Ad² (10⁶ mm⁴)	$\bar{I}$ (10⁶ mm⁴)
Top plate	3200	*160.5	82.43	0.07
S310 × 52	6650	0		95.3
Bot. plate	3200	*160.5	82.43	0.07
Σ			164.86	95.44

$$I = \Sigma A d^2 + \Sigma \bar{I} = 260.3 \times 10^6 \text{ mm}^4 = 260.3 \times 10^{-6} \text{ m}^4$$

$$Q = A_{plate}\, d_{plate} = (3200)(160.5) = 513.6 \times 10^3 \text{ mm}^3 = 513.6 \times 10^{-6} \text{ m}^3$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}(18 \times 10^{-3})^2 = 254.47 \times 10^{-6} \text{ m}^2$$

$$F_{bolt} = \tau_{all} A_{bolt} = (90 \times 10^6)(254.47 \times 10^{-6}) = 22.90 \times 10^3 \text{ N}$$

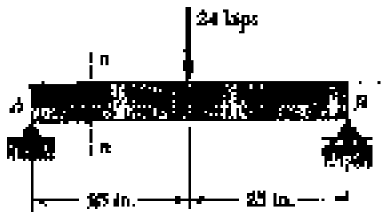
$$q\, s = 2 F_{bolt} \qquad q = \frac{2 F_{bolt}}{s} = \frac{(2)(22.90 \times 10^3)}{120 \times 10^{-3}} = 381.7 \times 10^3 \text{ N/m}$$

$$q = \frac{VQ}{I} \qquad V = \frac{Iq}{Q} = \frac{(260.3 \times 10^{-6})(381.7 \times 10^3)}{513.6 \times 10^{-6}} = 193.5 \times 10^3 \text{ N}$$

$$= 193.5 \text{ kN}$$

PROBLEM 6.91

6.91 For the beam and loading shown, consider section *n-n* and determine the shearing stress at (*a*) point *a*, (*b*) point *b*.

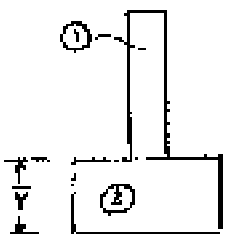


SOLUTION

$R_A = R_B = 12$ kips

At section n-n $\quad V = 12$ kips.

Locate centroid and calculate moment of inertia.

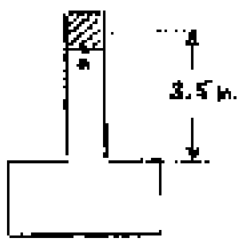


Part	A (in²)	$\bar{y}$ (in)	$A\bar{y}$ (in³)	d (in)	Ad^2 (in⁴)	$\bar{I}$ (in⁴)
①	4	4	16	2	16	5.33
②	8	1	8	1	8	2.67
Σ	12		24		24	8

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{24}{12} = 2 \text{ in}$$

$$I = \Sigma A d^2 + \Sigma \bar{I} = 24 + 8 = 32 \text{ in}^4$$

(a)

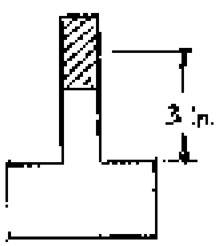


$$Q_a = A_a \bar{y}_a = (1)(1)(3.5) = 3.5 \text{ in}^3$$

$$t = 1 \text{ in}$$

$$\tau_a = \frac{VQ_a}{It} = \frac{(12)(3.5)}{(32)(1)} = 1.313 \text{ ksi}$$

(b)



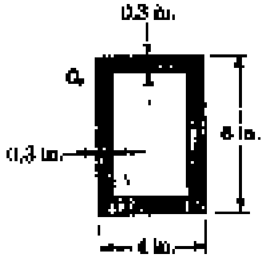
$$Q_b = A_b \bar{y}_b = (1)(2)(3) = 6 \text{ in}^3$$

$$t = 1 \text{ in.}$$

$$\tau_b = \frac{VQ}{It} = \frac{(12)(6)}{(32)(1)} = 2.25 \text{ ksi}$$

PROBLEM 6.92

6.92 For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.



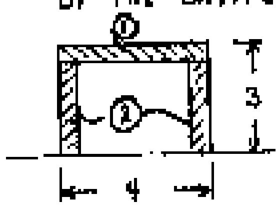
SOLUTION

At section n-n $V = 8$ kips

Moment of inertia

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$
$$= \frac{1}{12}(4)(6)^3 - \frac{1}{12}(3.4)(5.4)^3$$
$$= 27.3852 \text{ in}^4$$

(a) The largest shearing stress occurs on a section through the centroid of the entire cross section.

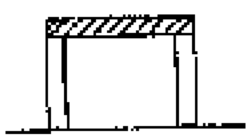


Part	A (in²)	ȳ (in)	Q = Aȳ (in³)
①	1.2	2.85	3.42
②	1.62	1.35	2.187
Σ	2.82		5.607

$Q_m = 4.5185 \text{ in}^3$ $t = (2)(0.3) = 0.6$ in.

$$\tau_m = \frac{VQ_m}{It} = \frac{(8)(5.607)}{(27.3852)(0.6)} = 2.73 \text{ ksi}$$

(b)



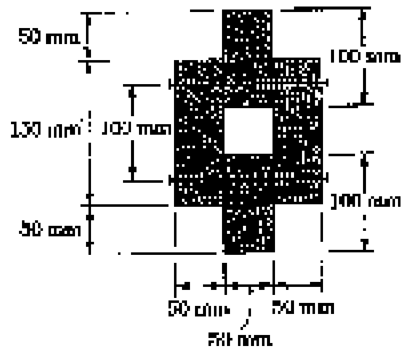
$Q_a = A_a \bar{y}_a = (1.2)(2.85) = 3.42 \text{ in}^3$

$t_a = (2)(0.3) = 0.6$ in.

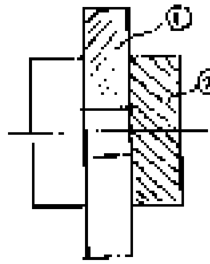
$$\tau_a = \frac{VQ_a}{It} = \frac{(8)(3.42)}{(27.3852)(0.6)} = 1.665 \text{ ksi}$$

PROBLEM 6.93

6.93 The built-up timber beam shown is subjected to a 6-kN vertical shear. Knowing that the longitudinal spacing of the nails is $s = 60$ mm and that each nail is 90 mm long, determine the shearing force in each nail.



SOLUTION



$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$$

$$= \frac{1}{12}(50)(100)^3 + (50)(100)(75)^2$$

$$= 32.292 \times 10^6 \ \text{mm}^4$$

$$I_2 = \frac{1}{12} b h^3 = \frac{1}{12}(50)(150)^3$$

$$= 14.0625 \times 10^6 \ \text{mm}^4$$

$$I = 2I_1 + 2I_2 = 92.71 \times 10^6 \ \text{mm}^4 = 92.71 \times 10^{-6} \ \text{m}^4$$

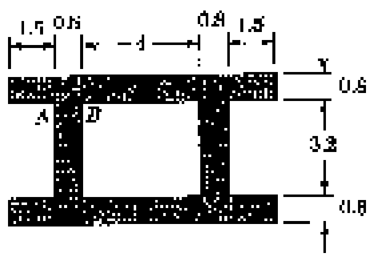
$$Q = Q_1 = A_1 \bar{y}_1 = (50)(100)(75) = 375 \times 10^3 \ \text{mm}^3 = 375 \times 10^{-6} \ \text{m}^3$$

$$q = \frac{VQ}{I} = \frac{(6 \times 10^3)(375 \times 10^{-6})}{92.71 \times 10^{-6}} = 24.27 \times 10^3 \ \text{N/m} \qquad s = 60 \ \text{mm} = 60 \times 10^{-3} \ \text{m}$$

$$2F_{nail} = qs \qquad F_{nail} = \frac{1}{2} qs = \frac{1}{2}(24.27 \times 10^3)(60 \times 10^{-3}) = 728 \ \text{N}$$

PROBLEM 6.94

6.94 The built-up beam shown was made by gluing together several wooden planks. Knowing that the beam is subjected to a 1200-lb vertical shear, determine the average shearing stress in the glued joint (*a*) at *A*, (*b*) at *B*.

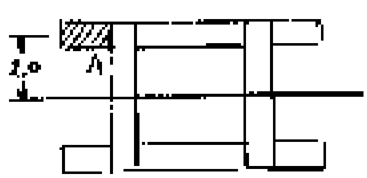


Dimensions in inches

SOLUTION

$$I = 2\left[\frac{1}{12}(0.8)(4.8)^3 + \frac{1}{12}(7)(0.8)^3 + (7)(0.8)(2.0)^2\right]$$

$$= 60.143 \ \text{in}^4$$

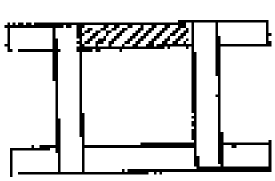


(a) $A_a = (1.5)(0.8) = 1.2 \ \text{in}^2 \qquad \bar{y}_a = 2.0 \ \text{in.}$

$Q_a = A_a \bar{y}_a = 2.4 \ \text{in}^3$

$t_a = 0.8 \ \text{in.}$

$$\tau_a = \frac{VQ_a}{I t_a} = \frac{(1200)(2.4)}{(60.143)(0.8)} = 59.9 \ \text{psi}$$



(b) $A_b = (4)(0.8) = 3.2 \ \text{in}^2 \qquad \bar{y}_b = 2.0 \ \text{in.}$

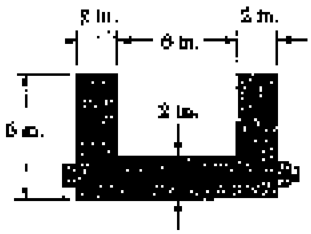
$Q_b = A_b \bar{y}_b = (3.2)(2.0) = 6.4 \ \text{in}^3$

$t_b = (2)(0.8) = 1.6 \ \text{in.}$

$$\tau_b = \frac{VQ_b}{I t_b} = \frac{(1200)(6.4)}{(60.143)(1.6)} = 79.8 \ \text{psi}$$

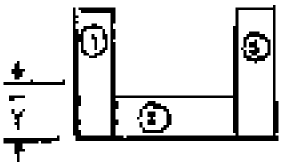
PROBLEM 6.95

6.95 A beam consists of three planks connected as shown by $\frac{3}{8}$-in.-diameter bolts spaced every 12 in. along the longitudinal axis of the beam. Knowing that the beam is subjected to a 2500-lb vertical shear, determine the maximum shearing stress in the bolts.



SOLUTION

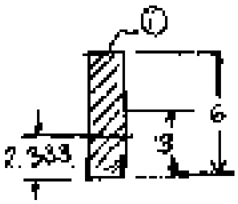
Locate neutral axis and compute moment of inertia.



Part	A (in^2)	$\bar{y}$ (in)	$A\bar{y}$ in^3	d (in)	Ad^2 (in^4)	$\bar{I}$ (in^4)
1	12	3	36	0.667	5.333	36
2	12	1	12	1.333	21.333	4
3	12	3	36	0.667	5.333	36
Σ	36		84		32	76

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{84}{36} = 2.333 \text{ in}$$

$$I = \Sigma Ad^2 + \Sigma \bar{I} = 108 \text{ in}^4$$



$$Q = A_1 \bar{y}_1 = (2)(6)(3 - 2.333) = 8 \text{ in}^3$$

$$q = \frac{VQ}{I} = \frac{(2500)(8)}{108} = 185.2 \text{ lb/in}$$

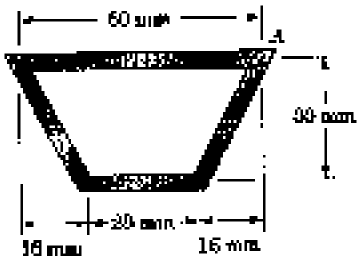
$$F_{bolt} = qs = (185.2)(12) = 2.222 \times 10^3 \text{ lb.}$$

$$A_{bolt} = \frac{\pi}{4} d_{bolt}^2 = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 = 0.1104 \text{ in}^2$$

$$\tau_{bolt} = \frac{F_{bolt}}{A_{bolt}} = \frac{2.222 \times 10^3}{0.1104} = 20.1 \times 10^3 \text{ psi} = 20.1 \text{ ksi}$$

PROBLEM 6.96

6.96 An extruded beam with the cross section shown and a 3-mm wall thickness is subjected to a 10-kN vertical shear. Determine (a) the shearing stress at point A, (b) the maximum shearing stress in the beam. Also sketch the shear flow in the cross section.

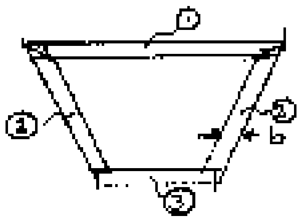


SOLUTION

For part ② height $h = 30$ mm

$b = \frac{\sqrt{16^2+30^2}}{30} t = (1.13333)(3) = 3.4$ mm

$\bar{I}_2 = \frac{1}{12}(2b)h^3 = \quad \times 10^3\ mm^4$



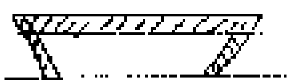
Part	A (mm²)	$\bar{y}$(mm)	$A\bar{y}$ mm³	d (mm)	$Ad^2 (10^3 mm^4)$	$\bar{I}\ (10^3 mm^4)$
①	180	30	5400	11.92	25.58	0.135
②	204	15	3060	3.08	1.94	15.3
③	84	0	0	18.08	27.46	0.063
	468		8460		54.98	15.50

$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8460}{468} = 18.08$ mm

$I = \Sigma Ad^2 + \Sigma\bar{I} = 70.48 \times 10^3\ mm^4 = 70.48 \times 10^{-9}\ m^4$

$Q_A = (60)(3)(11.92) = 2.146 \times 10^3\ mm^3$
$= 2.146 \times 10^{-6}\ m^3$

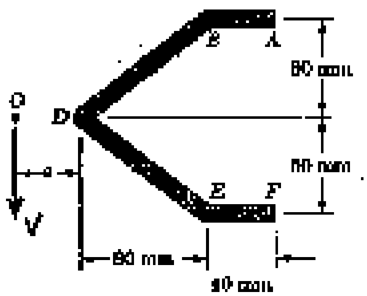
$\tau_A = \frac{VQ_A}{It} = \frac{(10\times10^3)(2.146\times10^{-6})}{(70.48\times10^{-9})(6\times10^{-3})} = 50.7 \times 10^6$ Pa
$= 50.7$ MPa ◄

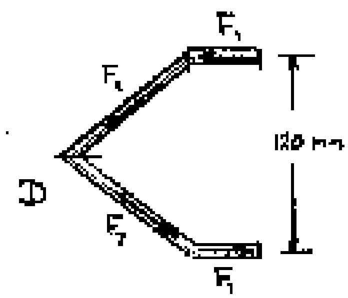


$Q_m = Q_A + (2)(3.4)(11.92)\frac{11.92}{2} = 2.629 \times 10^3\ mm^3$
$= 2.629 \times 10^{-6}\ m^3$

$\tau_m = \frac{VQ}{It} = \frac{(10\times10^3)(2.629\times10^{-6})}{(70.48\times10^{-9})(6\times10^{-3})} = 62.6 \times 10^6$ Pa
$= 62.6$ MPa ◄

PROBLEM 6.97





6.97 and 6.98 A thin-walled beam of uniform thickness has the cross section shown. Determine the location of the shear center O of the cross section.

SOLUTION

$I_{AB} = (40t)(60)^2 = 144\times10^3\, t$

$L_{DB} = \sqrt{80^2+60^2} = 100$ mm $\qquad A_{DB} = 100\, t$

$I_{DB} = \frac{1}{3} A_{DB} h^2 = \frac{1}{3}(100t)(60)^2 = 120\times10^3\, t$

$I = 2I_{AB} + 2I_{DB} = 528\times10^3\, t$

Part AB: $\quad A = tx \qquad \bar{y} = 60$ mm

A B
$\leftarrow x \rightarrow$

$Q = A\bar{y} = 60\, tx$ mm^3

$$\tau = \frac{VQ}{It} = \frac{V(60tx)}{It} = \frac{60Vx}{I}$$

$$F_1 = \int \tau\, dA = \int_0^{40} \frac{60Vx}{I}\, t\, dx = \frac{60Vt}{I}\int_0^{40} x\, dx$$

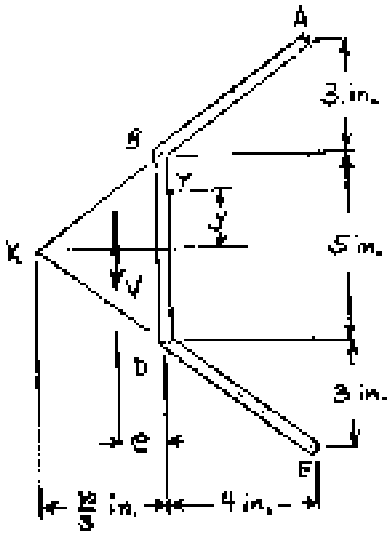
$$= \frac{60Vt}{I}\,\frac{x^2}{2}\Big|_0^{40} = \frac{(60)(40)^2\, Vt}{(2)(528\times10^3)t} = 0.051136\, V$$

$\circlearrowright \Sigma M_D = \circlearrowright \Sigma M_D \qquad Ve = (0.051136\, V)(120) \qquad e = (0.051136)(120) = 6.14$ mm

PROBLEM 6.98

6.97 and 6.98 A thin-walled beam of uniform thickness has the cross section shown. Determine the location of the shear center O of the cross section.

SOLUTION



$L_{AB} = \sqrt{4^2 + 3^2} = 5 \text{ in.} \qquad A_{AB} = 5t$

$I_{AB} = \frac{1}{12} A_{AB} h^2 + A_{AB} d^2 = \frac{1}{12}(5t)(3)^2 + (5t)(4)^2$

$= 83.75\ t \text{ in}^4$

$I_{BD} = \frac{1}{12}(t)(5)^3 = 10.417\ t \text{ in}^4$

$I = 2I_{AB} + I_{BD} = 177.917\ t \text{ in.}$

In part BD $\quad Q = Q_{AB} + Q_{BY}$

$Q = (5t)(4) + (2.5 - y)t(\frac{1}{2})(2.5 + y)$

$= 20t + 3.125t - \frac{1}{2}ty^2$

$= (23.125 - \frac{1}{2}y^2)t$

$\tau = \frac{VQ}{It} \qquad F_{BD} = \int \tau\, dA = \int_{-2.5}^{2.5} \frac{V(23.125 - \frac{1}{2}y^2)t}{It} \cdot t\, dy$

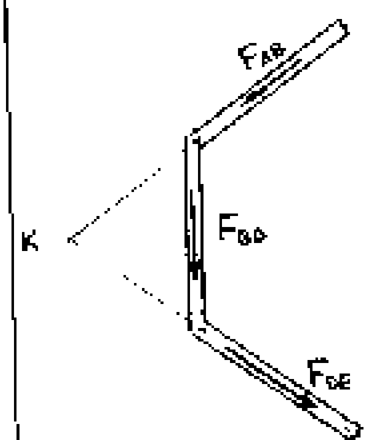
$= \frac{Vt}{I} \int_{-2.5}^{2.5} (23.125 - \frac{1}{2}y^2)dy = \frac{Vt}{I}\left[23.125y - \frac{1}{6}y^3\right]_{-2.5}^{2.5}$

$= \frac{Vt}{I} \cdot 2\left[(23.125)(2.5) - \frac{(2.5)^3}{6}\right] = \frac{Vt\,(110.417)}{177.917\ t}$

$= 0.62061\ V$

$\circlearrowright \Sigma M_K = \circlearrowright \Sigma M_K \qquad -V\left(\frac{10}{3} - e\right) = -\frac{10}{3}(0.62061\ V)$

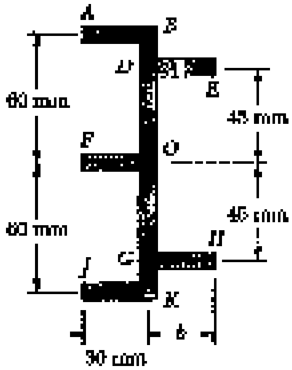
$e = \frac{10}{3}[1 - 0.62061] = 1.265 \text{ in.}$



Note that the lines of action of F_{AB} and F_{DE} pass through point K. Thus, these forces have zero moment about point K.

PROBLEM 6.99

6.99 A thin-walled beam of uniform thickness has the cross section shown. Determine the dimension b for which the shear center O of the cross section is located at the point indicated.



SOLUTION

Part AB: $A = tx \quad \bar{y} = 60 \text{ mm}$

$Q = A\bar{y} = 60\,t\,x \text{ mm}^3$

$\tau = \frac{VQ}{It} = \frac{60\,Vx}{I}$

$F_1 = \int \tau\, dA = \int_0^{30} \frac{60\,Vx}{I}\, t\, dx = \frac{60\,Vt}{I}\int_0^{30} x\, dx$

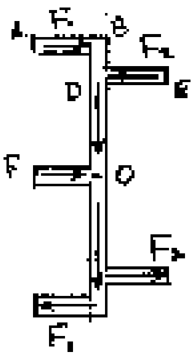
$= \frac{60\,Vt}{I}\, \frac{x^2}{2}\Big|_0^{30} = \frac{(60)(30)^2}{2}\frac{Vt}{I} = 27\times10^3\, \frac{Vt}{I}$

Part DE $\quad A = tx \quad \bar{y} = 45 \text{ mm}$

$Q = A\bar{y} = 45\,t\,x$

$\tau = \frac{VQ}{It} = \frac{45\,Vx}{I}$

$F_2 = \int \tau\, dA = \int_0^b \frac{45\,Vx}{I}\, t\, dx = \frac{45\,Vt}{I}\int_0^b x\, dx = \frac{45\,b^2\,Vt}{2I}$



$+\circlearrowleft \Sigma M_o = +\circlearrowleft \Sigma M_o \qquad 0 = (2)(45)F_2 - (2)(60)F_1$

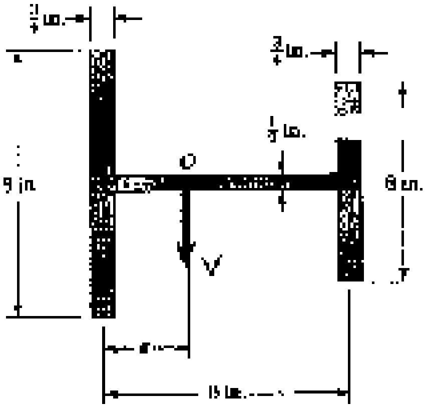
$\left[(45)^2\, b^2 - (2)(60)(27\times10^3)\right]\frac{Vt}{I} = 0$

$b^2 = \frac{(2)(60)(27\times10^3)}{45^2} = 1600 \text{ mm}^2 \qquad b = 40 \text{ mm}$

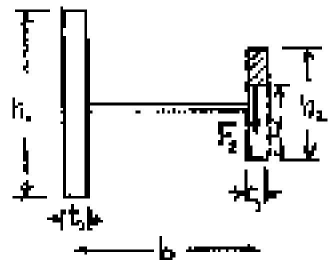
Note that the pair of F_1 forces form a couple.
Likewise, the pair of F_2 forces.
The lines of action of the forces in BDOGK pass through point O.

PROBLEM 6.100

6.100 A thin-walled beam has the cross section shown. Determine the location of the shear center O of the cross section.



SOLUTION



$$I = \frac{1}{12}t_1h_1^3 + \frac{1}{12}t_2h_2^3$$

Right Flange

$$A = \left(\tfrac{1}{2}h_2 - y\right)t_2$$

$$\bar{y} = \tfrac{1}{2}\left(\tfrac{1}{2}h_2 + y\right)t_2$$

$$Q = A\bar{y} = \tfrac{1}{2}\left(\tfrac{1}{2}h_2 - y\right)\left(\tfrac{1}{2}h_2 + y\right)t_2 = \tfrac{1}{2}\left(\tfrac{1}{4}h_2^2 - y^2\right)t_2$$

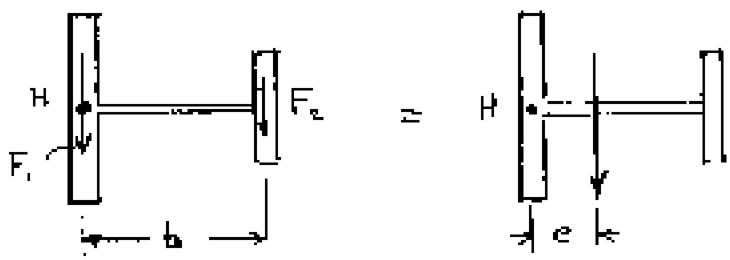
$$\tau = \frac{VQ}{It_2} = \frac{V}{2It_2}\left(\tfrac{1}{4}h_2^2 - y^2\right)t_2$$

$$F_2 = \int \tau\, dA = \int_{-h_2/2}^{h_2/2} \frac{Vt_2}{2It_2}\left(\tfrac{1}{4}h_2^2 - y^2\right)t_2\, dy = \frac{Vt_2}{2I}\left(\tfrac{1}{4}h_2^2 y - \frac{y^3}{3}\right)\Bigg|_{-h_2/2}^{h_2/2}$$

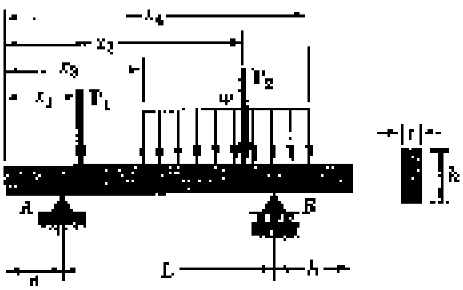
$$= \frac{Vt_2}{2I}\left\{\tfrac{1}{4}h_2^2\,\frac{h_2}{2} - \tfrac{1}{3}\left(\frac{h_2}{2}\right)^3 + \tfrac{1}{4}h_2^2\,\frac{h_2}{2} - \tfrac{1}{3}\left(\frac{h_2}{2}\right)^3\right\} = \frac{Vt_2h_2^3}{12I} = \frac{Vt_2h_2^3}{t_1h_1^3 + t_2h_2^3}$$

$$\circlearrowleft \Sigma M_H = \circlearrowleft \Sigma M_H \qquad -Ve = -F_2 b = -V\frac{t_2h_2^3 b}{t_1h_1^3 + t_2h_2^3}$$

$$e = \frac{t_2h_2^3 b}{t_1h_1^3 + t_2h_2^3} = \frac{(0.75)(6)^3(8)}{(0.75)(8)^3 + (0.75)(6)^3} = 2.37 \text{ in.}$$



PROBLEM 6.C1



6.C1 A timber beam is to be designed to support a distributed load and up to two concentrated loads as shown. One of the dimensions of its uniform rectangular cross section has been specified and the other is to be determined so that the maximum normal stress and the maximum shearing stress in the beam will not exceed given allowable values σ_{all} and τ_{all}. Measuring x from end A and using SI units, write a computer program to calculate for successive cross sections, from $x = 0$ to $x = L$ and using given increments Δx, the shear, the bending moment, and the smallest value of the unknown dimension that satisfies in that section (1) the allowable normal stress requirement, (2) the allowable shearing stress requirement. Use this program to design the beams of uniform cross section of the following problems, assuming $\sigma_{all} = 12$ MPa and $\tau_{all} = 825$ kPa, and using the increments indicated: (*a*) Prob. 5.75 ($\Delta x = 0.1$ m), (*b*) Prob. 5.76 ($\Delta x = 0.2$ m).

SOLUTION

See solution of P 5.C2 for the determination of R_A, R_B, $V(x)$, and $M(x)$

We recall that

$$V(x) = R_A\,STPA + R_B\,STPB - P_1\,STP1 - P_2\,STP2 - w(x - x_3)\,STP3 + w(x - x_4)\,STP4$$

$$M(x) = R_A(x-a)\,STPA + R_B(x-a-L)\,STPB - P_1(x-x_1)\,STP1 - P_2(x-x_2)\,STP2 - \tfrac{1}{2}w(x-x_3)^2\,STP3 + \tfrac{1}{2}w(x-x_4)^2\,STP4$$

where STPA, STPB, STP1, STP2, STP3, and STP4 are step functions defined in P 5.C2

(1) <u>TO SATISFY THE ALLOWABLE NORMAL STRESS REQUIREMENT:</u>

<u>If unknown dimension is h:</u>

$S_{min} = |M|/\sigma_{all}$. From $S = \frac{1}{6}th^2$, we have $h_\sigma = h = \sqrt{6S/t}$

<u>If unknown dimension is t:</u>

$S_{min} = |M|/\sigma_{all}$. From $S = \frac{1}{6}th^2$, we have $t_\sigma = t = 6S/h^2$

(2) <u>TO SATISFY THE ALLOWABLE SHEARING STRESS REQUIREMENT:</u>

We use Eq. (6.10), page 372: $\tau_{max} = \dfrac{3}{2}\dfrac{|V|}{A} = \dfrac{3}{2}\dfrac{M}{th}$

<u>If unknown dimension is h:</u> $h_\tau = h = \dfrac{3M}{2t\tau_{all}}$

<u>If unknown dimension is t:</u> $t_\tau = t = \dfrac{3M}{2h\tau_{all}}$

(CONTINUED)

PROBLEM 6.C1 CONTINUED

PROGRAM OUTPUTS

Prob. 5.75

RA = 2.40 kN   RB = 3.00 kN

X m	V kN	M kN.m	HSIG mm	HTAU mm
0.00	2.40	0.000	0.00	109.09
0.10	2.40	0.240	54.77	109.09
0.20	2.40	0.480	77.46	109.09
0.30	2.40	0.720	94.87	109.09
0.40	2.40	0.960	109.54	109.09
0.50	2.40	1.200	122.47	109.09
0.60	2.40	1.440	134.16	109.09
0.70	2.40	1.680	144.91	109.09
0.80	0.60	1.920	154.92	27.27
0.90	0.60	1.980	157.32	27.27
1.00	0.60	2.040	159.69	27.27
1.10	0.60	2.100	162.02	27.27
1.20	0.60	2.160	164.32	27.27
1.30	0.60	2.220	166.58	27.27
1.40	0.60	2.280	168.82	27.27
1.50	0.60	2.340	171.03	27.27
1.60	-3.00	2.400	173.21	136.36
1.70	-3.00	2.100	162.02	136.36
1.80	-3.00	1.800	150.00	136.36
1.90	-3.00	1.500	136.93	136.36
2.00	-3.00	1.200	122.47	136.36
2.10	-3.00	0.900	106.07	136.36
2.20	-3.00	0.600	86.60	136.36
2.30	-3.00	0.300	61.24	136.36
2.40	0.00	0.000	0.00	0.00

Prob. 5.76

RA = 25.00 kN   RB = 25.00 kN

X m	V kN	M kN.m	HSIG mm	HTAU mm
0.00	25.00	0.000	0.00	378.79
0.20	23.00	4.800	141.42	348.48
0.40	21.00	9.200	195.79	318.18
0.60	19.00	13.200	234.52	287.88
0.80	17.00	16.800	264.58	257.58
1.00	15.00	20.000	288.68	227.27
1.20	13.00	22.800	308.22	196.97
1.40	11.00	25.200	324.04	166.67
1.60	9.00	27.200	336.65	136.36
1.80	7.00	28.800	346.41	106.06
2.00	5.00	30.000	353.55	75.76
2.20	3.00	30.800	358.24	45.45
2.40	1.00	31.200	360.56	15.15
2.60	-1.00	31.200	360.56	15.15
2.80	-3.00	30.800	358.24	45.45
3.00	-5.00	30.000	353.55	75.76
3.20	-7.00	28.800	346.41	106.06
3.40	-9.00	27.200	336.65	136.36
3.60	-11.00	25.200	324.04	166.67
3.80	-13.00	22.800	308.22	196.97
4.00	-15.00	20.000	288.68	227.27
4.20	-17.00	16.800	264.58	257.58
4.40	-19.00	13.200	234.52	287.88
4.60	-21.00	9.200	195.79	318.18
4.80	-23.00	4.800	141.42	348.48
5.00	0.00	0.000	0.00	0.00

The smallest allowable value of h is the largest of the values shown in the last two columns.

For Prob. 5.75, $h = h_\sigma = 173.2$ mm.

For Prob 5.76, $h = h_\tau = 379$ mm

PROBLEM 6.C2

6.C2 A cantilever timber beam AB of length L and of the uniform rectangular section shown supports a concentrated load **P** at its free end and a uniformly distributed load w along its entire length. Write a computer program to determine the length L and the width b of the beam for which both the maximum normal stress and the maximum shearing stress in the beam reach their largest allowable values. Assuming $\sigma_{all} = 1.8$ ksi and $\tau_{all} = 120$ psi, use this program to determine the dimensions L and b when (a) $P = 1000$ lb and $w = 0$, (b) $P = 0$ and $w = 12.5$ lb/in., (c) $P = 500$ lb and $w = 12.5$ lb/in.

SOLUTION

Both the maximum shear and the maximum bending moment occur at A. We have

$$V_A = P + wL \qquad M_A = PL + \frac{1}{2} wL^2$$

TO SATISFY THE ALLOWABLE NORMAL STRESS REQUIREMENT:

$$\sigma_{all} = \frac{M_A}{S} = \frac{M_A}{\frac{1}{6} b(8b)^2} = \frac{3M_A}{32 b^3} \qquad b_\sigma = b = \left[\frac{3}{32}\frac{M_A}{\sigma_{all}}\right]^{1/3}$$

TO SATISFY THE ALLOWABLE SHEARING STRESS REQUIREMENT:

We use Eq. (6.10), page 378:

$$\tau_{all} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{V_A}{b(8b)} = \frac{3V_A}{16 b^2} \qquad b_\tau = b = \left[\frac{3}{16}\frac{V_A}{\tau_{all}}\right]^{1/2}$$

PROGRAM

For $L = 0$, $V_A = P$ and $b_\tau > 0$, while $M_A = 0$ and $b_\sigma = 0$.
Starting with $L = 0$ and using increments $\Delta L = 0.001$ in., we increase L until b_σ and b_τ become equal. We then print L and b.

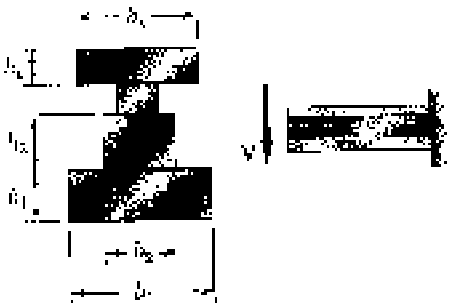
PROGRAM OUTPUTS

```
For P = 1000 lb, w =   0.0 lb/in.
Increment = 0.0010 in.
L = 37.5 in., b = 1.250 in.
```

```
For P =    0 lb, w = 12.5 lb/in.
Increment = 0.0010 in.
L = 70.3 in., b = 1.172 in.
```

```
For P =  500 lb, w = 12.5 lb/in.
Increment = 0.0010 in.
L = 59.8 in., b = 1.396 in.
```

PROBLEM 6.C3



6.C3 A beam having the cross section shown is subjected to a vertical shear **V**. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to calculate the shearing stress along the line between any two adjacent rectangular areas forming the cross section. Use this program to solve (*a*) Prob. 6.10, (*b*) Prob. 6.11, (*c*) Prob. 6.21, (*d*) Prob. 6.23.

SOLUTION

1. Enter V and the number n of rectangles.
2. For $i = 1$ to n, enter the dimensions b_i and h_i
3. Determine the area $A_i = b_i h_i$ of each rectangle.
4. Determine the elevation of the centroid of each rectangle

$$\bar{y}_i = \sum_{k=1}^{i} h_k - 0.5 h_i$$

and the elevation $\bar{y}$ of the centroid of the entire section

$$\bar{y} = \left(\sum_i A_i \bar{y}_i\right) / \left(\sum_i A_i\right)$$

5. Determine the centroidal moment of inertia of the entire section:

$$I = \sum_i \left[\frac{1}{12} b_i h_i^3 + A_i (\bar{y}_i - \bar{y})^2\right]$$

6. For each surface separating two rectangles i and $i+1$, determine Q_i of the area below that surface

$$Q_i = \sum_{k=1}^{i} A_k (\bar{y}_k - \bar{y})$$

7. Select for t_i the smaller of b_i and b_{i+1}.

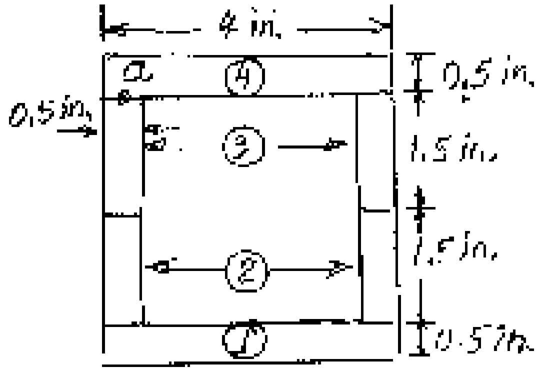
The shearing stress on the surface between the rectangles i and $i+1$ is

$$\tau_i = \frac{V Q_i}{I t_i}$$

(CONTINUED)

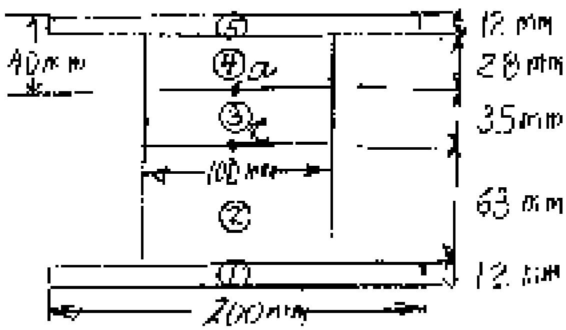
PROBLEM 6.C3 CONTINUED

PROGRAM OUTPUTS



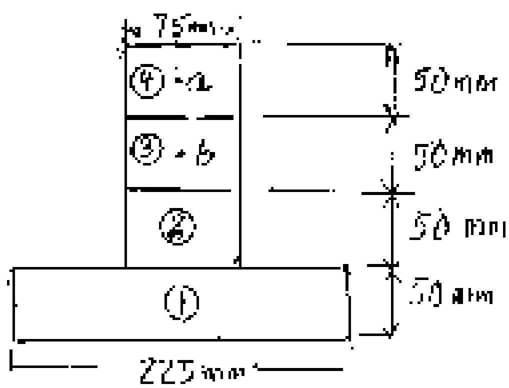
Problem 6.10

V=10.00 kips
YBAR of Section = 2.000 in.
I= 14.583 in^4
Between elements 1 and 2:
Tau = 2.400 ksi
Between elements 2 and 3: (a)
Tau = 3.171 ksi
Between elements 3 and 4: (b)
Tau = 2.400 ksi



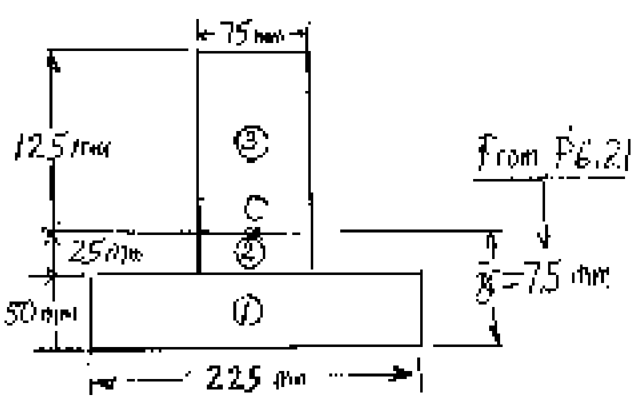
Problem 6.11

V= 10.00 kN
YBAR of Section = 75.50 mm
I= 39.580*10^-6 m^4
Between elements 1 and 2:
Tau = 418.39 kPa
Between elements 2 and 3: (a)
Tau = 919.78 kPa
Between elements 2 and 4: (b)
Tau = 765.23 kPa
Between elements 4 and 5:
Tau = 418.39 kPa



Problem 6.21

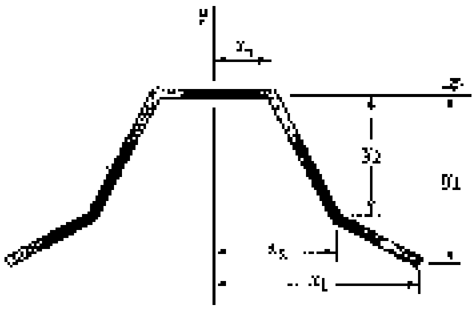
V=200.00 kN
YBAR of Section = 75.00 mm
I= 79.687*10^-6 m^4
Between elements 1 and 2:
Tau = 16.82 MPa
Between elements 2 and 3: (b)
Tau = 16.02 MPa
Between elements 3 and 4: (a)
Tau = 12.55 MPa



Problem 6.23

V=200.00 kN
YBAR of Section = 75.00 mm
I= 79.688*10^-6 m^4
Between elements 1 and 2:
Tau = 16.82 MPa
Between elements 2 and 3: (c)
Tau = 12.61 MPa

PROBLEM 6.C4



6.C4 A plate of uniform thickness t is bent as shown into a shape with a vertical plane of symmetry and is then used as a beam. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine the distribution of shearing stresses caused by a vertical shear V. Use this program (a) to solve Prob. 6.49, (b) to find the shearing stress at point E for the shape and load of Prob. 6.50, assuming a thickness $t = \frac{1}{4}$ in.

SOLUTION

For each element on the right-hand side, we compute (for $i = 1$ to n):

Length of element $= L_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$

Area of element $= A_i = t L_i$ where $t = \frac{1}{4}$ in.

Distance from x axis to centroid of element $= \bar{y}_i = \frac{1}{2}(y_i + y_{i+1})$

Distance from x axis to centroid of section:

$$\bar{y} = (\Sigma A_i \bar{y}_i)/\Sigma A_i$$

Note that $y_n = 0$ and that $x_{n+1} = y_{n+1} = 0$

Moment of inertia of section about centroidal axis:

$$\bar{I} = 2\Sigma A_i \left[\frac{1}{12}(y_i - y_{i+1})^2 + (\bar{y}_i - \bar{y})^2\right]$$

Computation of Q at point P where stress is desired

$Q = \Sigma A_i (\bar{y}_i - \bar{y})$ where sum extends to the areas located between one end of section and point P.

Shearing stress at P:

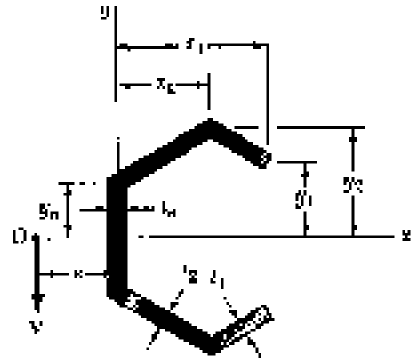
$$\tau = \frac{VQ}{It}$$

NOTE: τ_{max} occurs on neutral axis, i.e., for $y_P = \bar{y}$.

PROGRAM OUTPUTS

Part (a):	Part (b):
I = 0.5333 in^4	I = 22.27 in^4
Taumax = 2.02 ksi	TauE = 194.0 psi
TauB = 1.800 ksi	

PROBLEM 6.C5



6.C5 The cross section of an extruded beam is symmetric with respect to the x axis and consists of several straight segments as shown. Write a computer program that, for loads and dimensions expressed in either SI or U.S. customary units, can be used to determine (*a*) the location of the shear center O, (*b*) the distribution of shearing stresses caused by a vertical force applied at O. Use this program to solve Probs. 6.65, 6.68, 6.69, and 6.70.

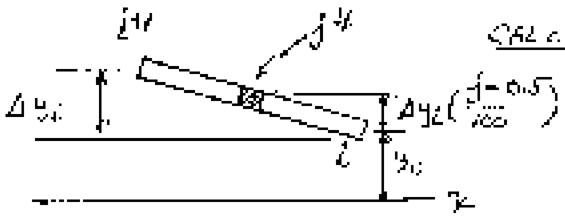
SOLUTION

SINCE SECTION IS SYMMETRIC WITH x AXIS,
COMPUTATIONS WILL BE DONE FOR TOP HALF.

FOR $i = 1$ TO $n+1$ (NOTE: $n+1$ IS THE ORIGIN)
ENTER t_i, x_i, y_i

COMPUTE LENGTH OF EACH SEGMENT
FOR $i = 1$ TO n

$$\Delta x_i = x_{i+1} - x_i$$
$$\Delta y_i = y_{i+1} - y_i$$
$$L = (\Delta x_i^2 + \Delta y_i^2)^{1/2}$$



CALCULATE MOMENT OF INERTIA I_x

CONSIDER EACH SEGMENT AS MADE OF 100 EQUAL PARTS

FOR $i = 1$ TO n

$\Delta AREA = L_i t_i / 100$

FOR $j = 1$ TO 100

$$y = y_i + \Delta y_i (j - 0.5)/100$$
$$\Delta I = (\Delta AREA)\, y^2$$
$$I_x = I_x + \Delta I$$

SINCE ONLY TOP HALF WAS USED

$$I_x = 2 I_x$$

CALCULATE SHEARING STRESS AT ENDS OF SEGMENTS AND SHEAR FORCES IN SEGMENTS

FOR $i = 1$ TO n

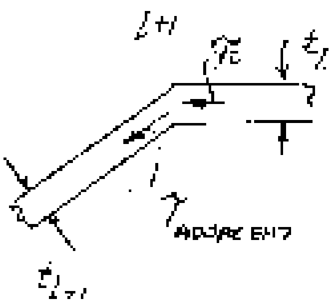
$\Delta AREA = L_i t_i / 100$, $\tau_{new} = \tau_{prev}$

FOR $j = 1$ TO 100

$$y = y_i + \Delta y_i (j - 0.5)/100$$
$$\Delta Q = (\Delta AREA)\, y$$
$$\tau_{old} = \tau_{new}, \quad Q = Q + \Delta Q$$
$$\tau_{new} = VQ / I_x t_i$$
$$\tau_{ave} = 0.5(\tau_{old} + \tau_{new})$$
$$\tau = \tau + \tau_{ave}$$

CONTINUED

PROBLEM 6.C5 - CONTINUED



$\text{FORCE}_i = \tau(\text{AREA})$

$\tau_i = VQ/I_x t_i$

$(\tau_{\text{ADJACENT}})_i = VQ/I_x t_{i+1}$

$Q_i = Q$

$\tau_{\text{MAX}} = (\tau_{\text{ADJACENT}})_i$

LOCATION OF SHEAR CENTER

CALCULATE MOMENT OF SHEAR FORCES ABOUT ORIGIN

FOR $i = 1$ TO n

$(F_x)_i = \text{FORCE}_i\,(\Delta x_i)/L_i$

$(F_y)_i = \text{FORCE}_i\,(\Delta y_i)/L_i$

$\text{MOMENT}_i = -(F_x)_i\, y_i + (F_y)_i\, x_i$

$\text{MOMENT} = \text{MOMENT} + \text{MOMENT}_i$

FOR WHOLE SECTION $\text{MOMENT} = \Sigma(\text{MOMENT}_i)$

SHEAR CENTER IS AT

$e = \text{MOMENT}/V$

PROGRAM OUTPUT

Prob. 6.65

	T(i) mm	X(i) mm	Y(i) mm	L(i) mm
1	10.00	70.00	10.00	40.000
2	6.00	70.00	50.00	70.000
3	10.00	0.00	50.00	50.000
4	10.00	0.00	0.00	

Moment of inertia: Ix = 3759956 mm^4 Shear = 50000 N

Junction of segments	Q mm^3	Tau Before MPa	Tau After MPa	Force in segment kN
1 and 2	12000.000	15.96	26.60	2482.37
2 and 3	33000.000	73.14	43.88	20888.54
3 and 4	45500.000	60.51	60.51	27372.75

Moment of shear forces about origin: M = 2436.386 N·m
+ counterclockwise

Distance from origin to shear center: e = 48.728 mm

CONTINUED

PROBLEM 6.C5 - PROGRAM PRINTOUTS CONTINUED

Prob. 6.68

i	T(i) in.	X(i) in.	Y(i) in.	L(i) in.
1	0.25	3.00	4.00	3.000
2	0.50	0.00	4.00	4.000
3	0.50	0.00	0.00	

Moment of inertia: Ix = 45.3328 in^4 Shear = 25.000 kips

Junction of segments	Q in^3	Tau Before ksi	Tau After ksi	Force in segment kips
1 and 2	3.000	6.52	3.31	2.48
2 and 3	7.000	7.72	7.72	12.47

Moment of shear forces about origin: M = 19.853 kip-in.
+ counterclockwise

Distance from origin to shear center: e = 0.7941 in.

Prob. 6.69

i	T(i) in.	X(i) in.	Y(i) in.	L(i) in.
1	0.25	4.00	5.00	2.000
2	0.25	4.00	3.00	5.000
3	0.25	0.00	0.00	

Moment of inertia: Ix = 23.8331 in^4 Shear = 10.000 kips

Junction of segments	Q in^3	Tau Before ksi	Tau After ksi	Force in segment kips
1 and 2	2.000	3.36	3.36	0.91
2 and 3	3.875	6.50	6.50	6.80

Moment of shear forces about origin: M = -7.273 kip-in.
+ counterclockwise

Distance from origin to shear center: e = -0.7273 in.

Prob. 6.70

i	T(i) in.	X(i) in.	Y(i) in.	L(i) in.
1	0.25	2.60	0.00	1.500
2	0.25	2.60	1.50	3.002
3	0.25	0.00	0.00	

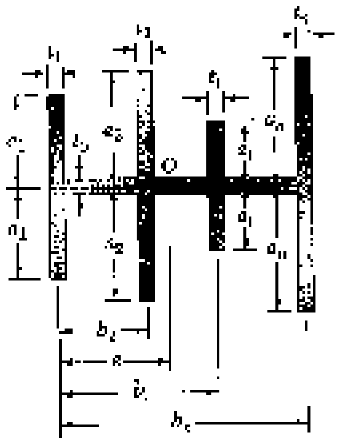
Moment of inertia: Ix = 1.6801 in^4 Shear = 10.000 kips

Junction of segments	Q in^3	Tau Before ksi	Tau After ksi	Force in segment kips
1 and 2	0.281	6.66	6.66	0.83
2 and 3	0.844	20.00	20.00	11.65

Moment of shear forces about origin: M = 4.332 kip-in.
+ counterclockwise

Distance from origin to shear center: e = 0.4332 in.

PROBLEM 6.C6



6.C6 A thin-walled beam has the cross section shown. Write a computer program that, for dimensions expressed in either SI or U.S. customary units, can be used to determine the location of the shear center O of the cross section. Use this program to solve Prob. 6.100.

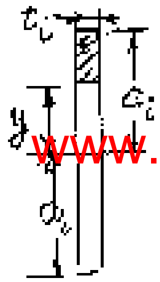
SOLUTION

Distribution of shearing stresses in element i

Let V = shear in cross section

I = Centroidal moment of inertia of section

We have for shaded area

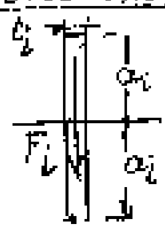


$$Q = A\bar{y} = t_i(a_i - y)\,\frac{a_i + y}{2}$$

$$= \frac{1}{2} t_i\,(a_i^2 - y^2)$$

$$\tau = \frac{QV}{I t_i} = \frac{V}{2I}(a_i^2 - y^2)$$

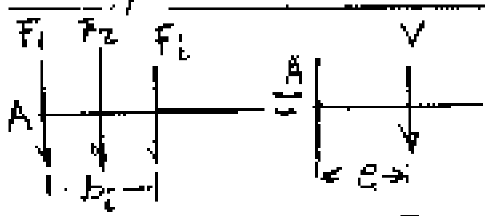
Force exerted on element i



$$F_i = \int_{-a_i}^{a_i} \tau\,(t_i\,dy) = \frac{Vt_i}{2I}\int_{-a_i}^{a_i}(a_i^2 - y^2)\,dy$$

$$= \frac{Vt_i}{I}\int_0^{a_i}(a_i^2 - y^2)\,dy = \frac{Vt_i}{I}\left(a_i^3 - \frac{1}{3}a_i^3\right) = \frac{2}{3}\frac{V}{I}t_i a_i^3$$

The system of the forces F_i must be equivalent to V at shear center.



$$\Sigma F = \Sigma F: \quad \frac{2}{3}\frac{V}{I}\Sigma t_i a_i^3 = V \qquad (1)$$

$$\Sigma M_A = \Sigma M_A: \quad \frac{2}{3}\frac{V}{I}\Sigma t_i a_i^3 b_i = eV \qquad (2)$$

Divide (2) by (1): $\quad e = \dfrac{\Sigma t_i a_i^3 b_i}{\Sigma t_i a_i^3}$ ◀

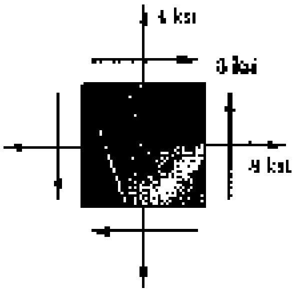
PROGRAM OUTPUT:

```
Prob. 6.100

For element 1:
t = 0.75 in., a = 4 in., b = 0

For element 2:
t = 0.75 in., a = 3 in., b= 8 in.

Answer: e = 2.37 in.
```
◀

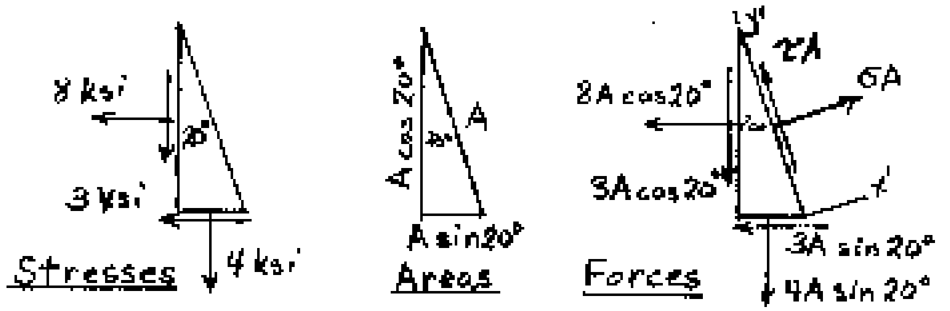
CHAPTER 7

PROBLEM 7.1

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



SOLUTION



$+\searrow \Sigma F = 0$

$\sigma A - 8A\cos 20°\cos 20° - 3A\cos 20°\sin 20° - 3A\sin 20°\cos 20° - 4A\sin 20°\sin 20° = 0$

$\sigma = 8\cos^2 20° + 3\cos 20°\sin 20° + 3\sin 20°\cos 20° + 4\sin^2 20° = 9.46\text{ ksi}$

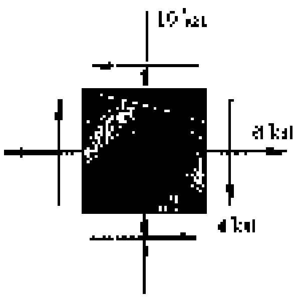
$+\nearrow \Sigma F = 0$

$\tau A + 8A\cos 20°\sin 20° - 3A\cos 20°\cos 20° + 3A\sin 20°\sin 20° - 4A\sin 20°\cos 20° = 0$

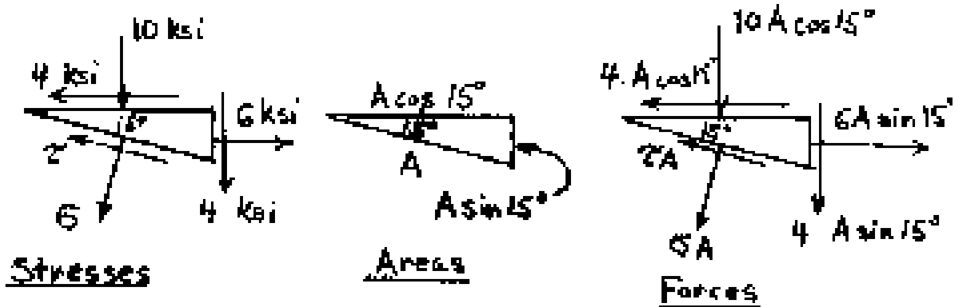
$\tau = -8\cos 20°\sin 20° + 3(\cos^2 20° - \sin^2 20°) + 4\sin 20°\cos 20° = 1.013\text{ ksi}$

PROBLEM 7.2

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



SOLUTION



$+\swarrow \Sigma F = 0$

$\sigma A + 4A\cos 15°\sin 15° + 10A\cos 15°\cos 15° - 6A\sin 15°\sin 15° + 4A\sin 15°\cos 15° = 0$

$\sigma = -4\cos 15°\sin 15° - 10\cos^2 15° + 6\sin^2 15° - 4\sin 15°\cos 15° = 10.93\text{ ksi}$

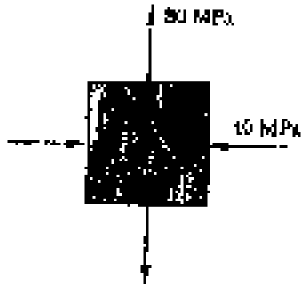
$+\nwarrow \Sigma F = 0$

$\tau A + 4A\cos 15°\cos 15° - 10A\cos 15°\sin 15° - 6A\sin 15°\cos 15° - 4A\sin 15°\sin 15° = 0$

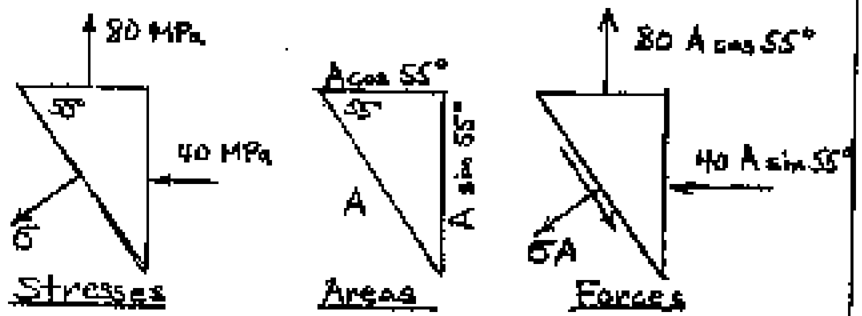
$\tau = -4(\cos^2 15° - \sin^2 15°) + (10+6)\cos 15°\sin 15° = 0.536\text{ ksi}$

PROBLEM 7.3

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



SOLUTION



$+\swarrow \Sigma F = 0$

$$\sigma A - 80 A \cos 55^\circ \cos 55^\circ + 40 A \sin 55^\circ \sin 55^\circ = 0$$

$$\sigma = 80 \cos^2 55^\circ - 40 \sin^2 55^\circ = -0.521 \text{ MPa}$$

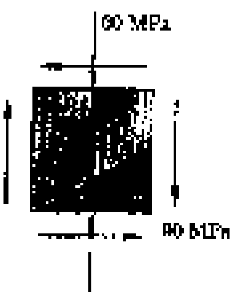
$+\searrow \Sigma F = 0$

$$\tau A - 80 A \cos 55^\circ \sin 55^\circ - 40 A \sin 55^\circ \cos 55^\circ$$

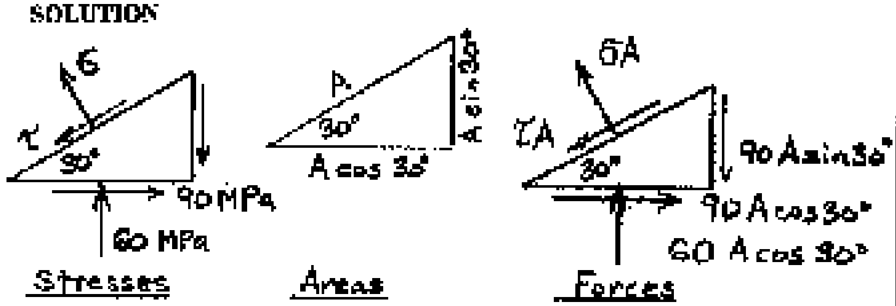
$$\tau = 120 \cos 55^\circ \sin 55^\circ = 56.4 \text{ MPa} \searrow$$

PROBLEM 7.4

7.1 through 7.4 For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.



SOLUTION



$+\nwarrow \Sigma F = 0$

$$\sigma A - 90 A \sin 30^\circ \cos 30^\circ - 90 A \cos 30^\circ \sin 30^\circ + 60 A \cos 30^\circ \cos 30^\circ = 0$$

$$\sigma = 180 \sin 30^\circ \cos 30^\circ - 60 \cos^2 30^\circ = 32.9 \text{ MPa}$$

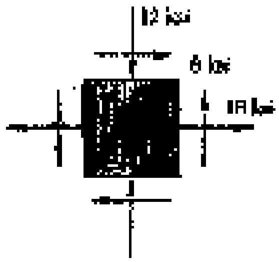
$+\swarrow \Sigma F = 0$

$$\tau A + 90 A \sin 30^\circ \sin 30^\circ - 90 A \cos 30^\circ \cos 30^\circ - 60 A \cos 30^\circ \sin 30^\circ = 0$$

$$\tau = 90(\cos^2 30^\circ - \sin^2 30^\circ) + 60 \cos 30^\circ \sin 30^\circ = 71.0 \text{ MPa} \swarrow$$

PROBLEM 7.5

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.



SOLUTION

$\sigma_x = 18$ ksi $\qquad \sigma_y = -12$ ksi $\qquad \tau_{xy} = 8$ ksi

(a) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(8)}{18+12} = 0.5333$$

$$2\theta_p = 28.07^\circ \qquad \theta_p = 14.04^\circ, \quad 104.04^\circ$$

(b) $$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{18-12}{2} \pm \sqrt{\left(\frac{18+12}{2}\right)^2 + (8)^2}$$

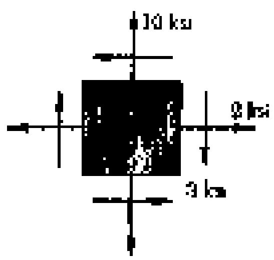
$$= 3 \pm 17 \text{ ksi}$$

$$\sigma_{max} = 20 \text{ ksi}$$

$$\sigma_{min} = -14 \text{ ksi}$$

PROBLEM 7.6

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.



SOLUTION

$\sigma_x = 2$ ksi $\qquad \sigma_y = 10$ ksi $\qquad \tau_{xy} = -3$ ksi

(a) $$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-3)}{2-10} = 0.750$$

$$2\theta_p = 36.87^\circ \qquad \theta_p = 18.43^\circ, \quad 108.43^\circ$$

(b) $$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{2+10}{2} \pm \sqrt{\left(\frac{2-10}{2}\right)^2 + (-3)^2}$$

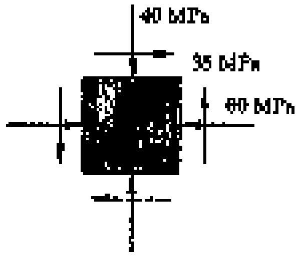
$$= 6 \pm 5 \text{ ksi}$$

$$\sigma_{max} = 11 \text{ ksi}$$

$$\sigma_{min} = 1 \text{ ksi}$$

PROBLEM 7.7

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.



SOLUTION

$\sigma_x = -60$ MPa $\qquad \sigma_y = -40$ MPa $\qquad \tau_{xy} = 35$ MPa

(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(35)}{-60+40} = -3.50$

$2\theta_p = -74.05°$ $\qquad \theta_p = -37.03°,\ 52.97°$ ◄

(b) $\sigma_{max,min} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \dfrac{-60-40}{2} \pm \sqrt{\left(\dfrac{-60+40}{2}\right)^2 + (35)^2}$

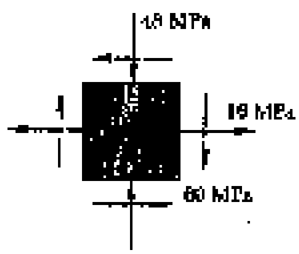
$= -50 \pm 36.4$ MPa

$\sigma_{max} = -13.60$ MPa ◄

$\sigma_{min} = -86.4$ MPa ◄

PROBLEM 7.8

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.



SOLUTION

$\sigma_x = 16$ MPa $\qquad \sigma_y = -48$ MPa $\qquad \tau_{xy} = -60$ MPa

(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(-60)}{16+48} = -1.875$

$2\theta_p = -61.93°$ $\qquad \theta_p = -30.96°,\ 59.04°$ ◄

(b) $\sigma_{max,min} = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

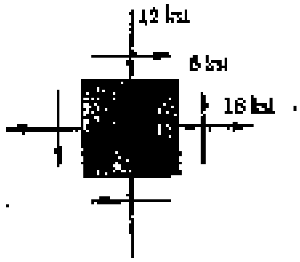
$= \dfrac{16-48}{2} \pm \sqrt{\left(\dfrac{16+48}{2}\right)^2 + (-60)^2}$

$= -16 \pm 68$

$\sigma_{max} = 52$ MPa ◄

$\sigma_{min} = -84$ MPa ◄

PROBLEM 7.9



7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$\sigma_x = 18 \text{ ksi} \qquad \sigma_y = -12 \text{ ksi} \qquad \tau_{xy} = 8 \text{ ksi}$

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{18+12}{(2)(8)} = -1.875$

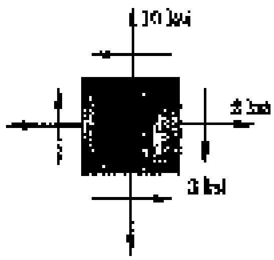
$2\theta_s = -61.93° \qquad \theta_s = -30.96°,\ 59.04°$ ◀

(b) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{18+12}{2}\right)^2 + (8)^2} = 17 \text{ ksi}$ ◀

(c) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{18-12}{2} = 3 \text{ ksi}$ ◀

PROBLEM 7.10



7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$\sigma_x = 2 \text{ ksi} \qquad \sigma_y = 10 \text{ ksi} \qquad \tau_{xy} = -3 \text{ ksi}$

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{2-10}{(2)(-3)} = -1.3333$

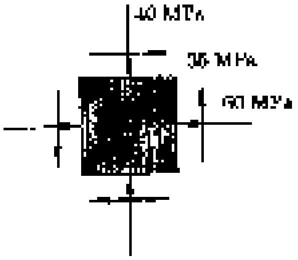
$2\theta_s = -53.13° \qquad \theta_s = -26.57°,\ 63.43°$ ◀

(b) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{2-10}{2}\right)^2 + (-3)^2} = 5 \text{ ksi}$ ◀

(c) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{2+10}{2} = 6 \text{ ksi}$ ◀

PROBLEM 7.11



7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$\sigma_x = -60$ MPa $\qquad \sigma_y = -40$ MPa $\qquad \tau_{xy} = 35$ MPa

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{-60+40}{(2)(35)} = 0.2857$

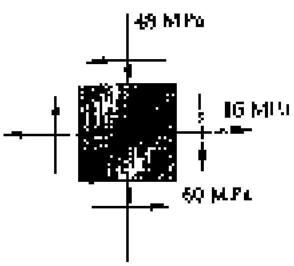
$2\theta_s = 15.95°$ $\qquad \theta_s = 7.97°,\ 97.97°$ ◄

(b) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{-60+40}{2}\right)^2 + (35)^2} = 36.4$ MPa ◄

(c) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{-60-40}{2} = -50$ MPa ◄

PROBLEM 7.12



7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$\sigma_x = 16$ MPa $\qquad \sigma_y = -48$ MPa $\qquad \tau_{xy} = -60$ MPa

(a) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{16+48}{(2)(-60)} = 0.5333$

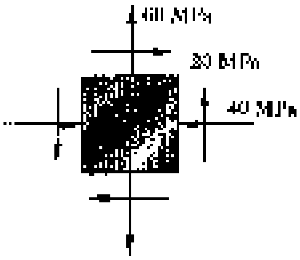
$2\theta_s = 28.07°$ $\qquad \theta_s = 14.04°,\ 104.04°$ ◄

(b) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{16+48}{2}\right)^2 + (-60)^2} = 68$ MPa ◄

(c) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{16-48}{2} = -16$ MPa ◄

PROBLEM 7.13



7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$\sigma_x = -40 \text{ MPa} \qquad \sigma_y = 60 \text{ MPa} \qquad \tau_{xy} = 20 \text{ MPa}$

$\dfrac{\sigma_x + \sigma_y}{2} = 10 \text{ MPa} \qquad \dfrac{\sigma_x - \sigma_y}{2} = -50 \text{ MPa}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(a) $\theta = -25^\circ \qquad 2\theta = -50^\circ$

$\sigma_{x'} = 10 - 50\cos(-50^\circ) + 20\sin(-50^\circ) = -37.5 \text{ MPa}$ ◄

$\tau_{x'y'} = +50\sin(-50^\circ) + 20\cos(-50^\circ) = -25.4 \text{ MPa}$ ◄

$\sigma_{y'} = 10 + 50\cos(-50^\circ) - 20\sin(-50^\circ) = 57.5 \text{ MPa}$ ◄

(b) $\theta = 10^\circ \qquad 2\theta = 20^\circ$

$\sigma_{x'} = 10 - 50\cos(20^\circ) + 20\sin(20^\circ) = -30.1 \text{ MPa}$ ◄

$\tau_{x'y'} = +50\sin(20^\circ) + 20\cos(20^\circ) = 35.9 \text{ MPa}$ ◄

$\sigma_{y'} = 10 + 50\cos(20^\circ) - 20\sin(20^\circ) = 50.1 \text{ MPa}$ ◄

PROBLEM 7.14

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$\sigma_x = 0 \qquad \sigma_y = -80 \text{ MPa} \qquad \tau_{xy} = -50 \text{ MPa}$

$\dfrac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa} \qquad \dfrac{\sigma_x - \sigma_y}{2} = 40 \text{ MPa}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(a) $\theta = -25° \qquad 2\theta = -50°$

$\sigma_{x'} = -40 + 40\cos(-50°) - 50\sin(-50°) = 24.0 \text{ MPa}$ ◄

$\tau_{x'y'} = -40\sin(-50°) - 50\cos(-50°) = -1.5 \text{ MPa}$ ◄

$\sigma_{y'} = -40 - 40\cos(-50°) + 50\sin(-50°) = -104.0 \text{ MP}$ ◄

(b) $\theta = 10° \qquad 2\theta = 20°$

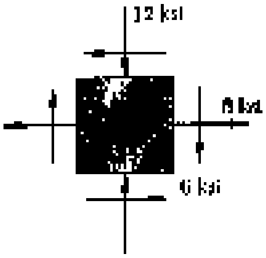
$\sigma_{x'} = -40 + 40\cos(20°) - 50\sin(20°) = -19.5 \text{ MPa}$ ◄

$\tau_{x'y'} = -40\sin(20°) - 50\cos(20°) = -60.7 \text{ MPa}$ ◄

$\sigma_{y'} = -40 - 40\cos(20°) + 50\sin(20°) = -60.5 \text{ MPa}$ ◄

PROBLEM 7.15

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



SOLUTION

$\sigma_x = 8$ ksi $\qquad \sigma_y = -12$ ksi $\qquad \tau_{xy} = -6$ ksi

$\dfrac{\sigma_x + \sigma_y}{2} = -2$ ksi $\qquad \dfrac{\sigma_x - \sigma_y}{2} = 10$ ksi

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(a) $\theta = -25°$ $\qquad 2\theta = -50°$

$\sigma_{x'} = -2 + 10\cos(-50°) - 6\sin(-50°) = 9.02$ ksi ◄

$\tau_{x'y'} = -10\sin(-50°) - 6\cos(-50°) = 3.80$ ksi ◄

$\sigma_{y'} = -2 - 10\cos(-50°) + 6\sin(-50°) = -13.02$ ksi ◄

(b) $\theta = 10°$ $\qquad 2\theta = 20°$

$\sigma_{x'} = -2 + 10\cos(20°) - 6\sin(20°) = 5.34$ ksi ◄

$\tau_{x'y'} = -10\sin(20°) - 6\cos(20°) = -9.06$ ksi ◄

$\sigma_{y'} = -2 - 10\cos(20°) + 6\sin(20°) = -9.34$ ksi ◄

PROBLEM 7.16

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$\sigma_x = 0 \qquad \sigma_y = 16 \text{ ksi} \qquad \tau_{xy} = 10 \text{ ksi}$

$\dfrac{\sigma_x + \sigma_y}{2} = 8 \text{ ksi} \qquad \dfrac{\sigma_x - \sigma_y}{2} = -8 \text{ ksi}$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

(a) $\theta = -25^\circ \qquad 2\theta = -50^\circ$

$\sigma_{x'} = 8 - 8\cos(-50^\circ) + 10\sin(-50^\circ) = -4.80 \text{ ksi}$ ◄

$\tau_{x'y'} = 8\sin(-50^\circ) + 10\cos(-50^\circ) = 0.30 \text{ ksi}$ ◄

$\sigma_{y'} = 8 + 8\cos(-50^\circ) - 10\sin(-50^\circ) = 20.80 \text{ ksi}$ ◄

(b) $\theta = 10^\circ \qquad 2\theta = 20^\circ$

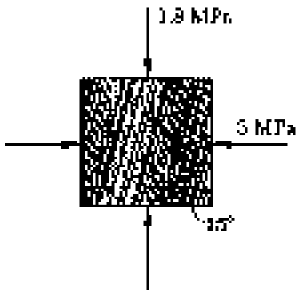
$\sigma_{x'} = 8 - 8\cos(20^\circ) + 10\sin(20^\circ) = 3.90 \text{ ksi}$ ◄

$\tau_{x'y'} = 8\sin(20^\circ) + 10\cos(20^\circ) = 12.13 \text{ ksi}$ ◄

$\sigma_{y'} = 8 + 8\cos(20^\circ) - 10\cos(20^\circ) = 12.10 \text{ ksi}$ ◄

PROBLEM 7.17

7.17 and *7.18* The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.



SOLUTION

$\sigma_x = -3 \text{ MPa} \qquad \sigma_y = -1.8 \text{ MPa} \qquad \tau_{xy} = 0$

$\theta = -15^\circ \qquad 2\theta = -30^\circ$

(a) $\tau_{x'y'} = -\dfrac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\sin 2\theta$

$= -\dfrac{-3+1.8}{2}\sin(-30^\circ) + 0$

$= -0.300 \text{ MPa}$ ◀

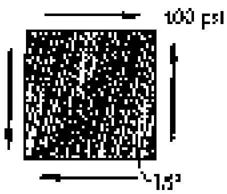
(b) $\sigma_{x'} = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

$= \dfrac{-3-1.8}{2} + \dfrac{-3+1.8}{2}\cos(-30^\circ) + 0$

$= -2.92 \text{ MPa}$ ◀

PROBLEM 7.18

7.17 and *7.18* The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.



SOLUTION

$\sigma_x = 0 \qquad \sigma_y = 0 \qquad \tau_{xy} = 400 \text{ psi}$

$\theta = -15^\circ \qquad 2\theta = -30^\circ$

(a) $\tau_{x'y'} = -\dfrac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

$= -0 + 400\cos(-30^\circ)$

$= 346 \text{ psi}$ ◀

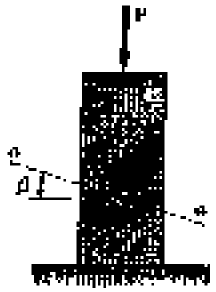
(b) $\sigma_{x'} = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

$= 0 + 0 + 400\sin(-30^\circ)$

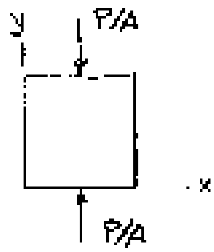
$= -200 \text{ psi}$ ◀

PROBLEM 7.19

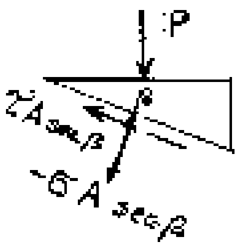
7.19 The centric force **P** is applied to a short post as shown. Knowing that the stresses on plane *a-a* are $\sigma = -15$ ksi and $\tau = 5$ ksi, determine (*a*) the angle β that plane *a-a* forms with the horizontal, (*b*) the maximum compressive stress in the post.



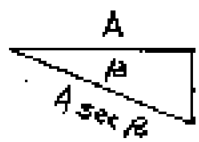
SOLUTION



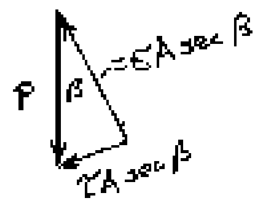
$$\sigma_x = 0$$
$$\tau_{xy} = 0$$
$$\sigma_y = \sigma_{max\ comp.} = -\frac{P}{A}$$



Forces



Areas



Force Triangle

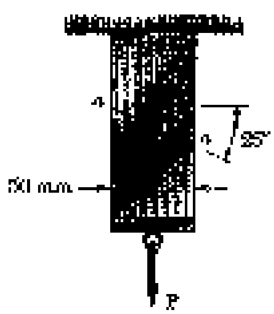
$$\tan\beta = \frac{\tau A \sec\beta}{-\sigma A \sec\beta} = \frac{5}{15} = \frac{1}{3}$$

(a) $\beta = \arctan\frac{1}{3} = 18.4°$ ◄

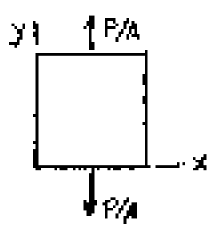
(b) $P = (-\sigma A \sec\beta)(\sec\beta)$ $\quad \dfrac{P}{A} = \dfrac{-\sigma}{\cos^2\beta} = \dfrac{15}{\cos^2 18.4°} = 16.67$ ksi ◄

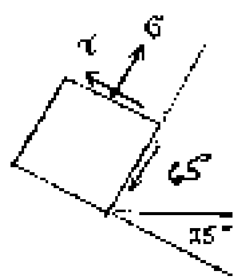
PROBLEM 7.20

7.20 Two members of uniform cross section 50 × 80 mm are glued together along plane *a-a*, which forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest axial load **P** that can be applied.



SOLUTION





For plane a-a $\theta = 65°$

$$\sigma_x = 0 \qquad \tau_{xy} = 0 \qquad \sigma_y = \frac{P}{A}$$

$$\sigma = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta = 0 + \frac{P}{A}\sin^2 65° + 0$$

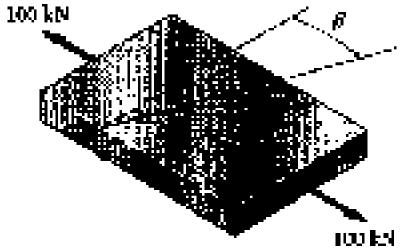
$$P = \frac{A\sigma}{\sin^2 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(800\times10^{3})}{\sin^2 65°} = 3.90\times10^{3}\ \text{N}$$

$$\tau = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) = \frac{P}{A}\sin 65°\cos 65° + 0$$

$$P = \frac{A\tau}{\sin 65°\cos 65°} = \frac{(50\times10^{-3})(80\times10^{-3})(600\times10^{3})}{\sin 65°\cos 65°} = 6.27\times10^{3}\ \text{N}$$

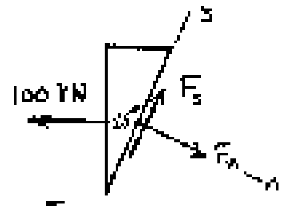
Allowable value of P is the smaller. $\quad P = 3.90\times10^{3}\ \text{N} = 3.90\ \text{kN}$ ◄

PROBLEM 7.21



7.21 Two steel plates of uniform cross section 10 × 80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that $\beta = 25°$, determine (*a*) the in-plane shearing stress parallel to the weld; (*b*) the normal stress perpendicular to the weld.

SOLUTION



Forces

Area of weld

$$A_w = \frac{(10\times10^{-3})(80\times10^{-3})}{\cos 25°} = 882.7 \quad 10^{-6}\ \text{m}^2$$

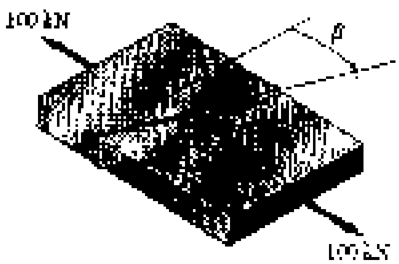
(a) $\Sigma F_s = 0 \qquad F_s - 100 \sin 25° = 0 \qquad F_s = 42.26\ \text{kN}$

$$\tau_w = \frac{F_s}{A_w} = \frac{42.26\times10^3}{882.7\times10^{-6}} = 47.9\times10^6\ \text{Pa} = 47.9\ \text{MPa}$$

(b) $\Sigma F_n = 0 \qquad F_n - 100 \cos 25° = 0 \qquad F_n = 90.63\ \text{kN}$

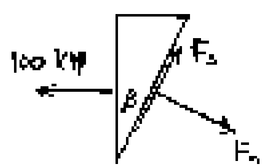
$$\sigma_w = \frac{F_n}{A_w} = \frac{90.63\times10^3}{882.7\times10^{-6}} = 102.7\times10^6\ \text{Pa} = 102.7\ \text{MPa}$$

PROBLEM 7.22



7.22 Two steel plates of uniform cross section 10 × 80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (*a*) the angle β; (*b*) the corresponding normal stress perpendicular to the weld.

SOLUTION



Forces

Area of weld

$$A_w = \frac{(10\times10^{-3})(80\times10^{-3})}{\cos\beta} = \frac{800\times10^{-6}}{\cos\beta}\ \text{m}^2$$

(a) $\Sigma F_s = 0 \qquad F_s - 100 \sin\beta = 0 \qquad F_s = 100 \sin\beta\ \text{kN} = 100\times10^3 \sin\beta\ \text{N}$

$$\tau_w = \frac{F_s}{A_w} \qquad 30\times10^6 = \frac{100\times10^3 \sin\beta}{800\times10^{-6}/\cos\beta} = 125\times10^6 \sin\beta\cos\beta$$

$$\sin\beta\cos\beta = \tfrac{1}{2}\sin 2\beta = \frac{30\times10^6}{125\times10^6} = 0.240 \qquad \beta = 14.34°$$

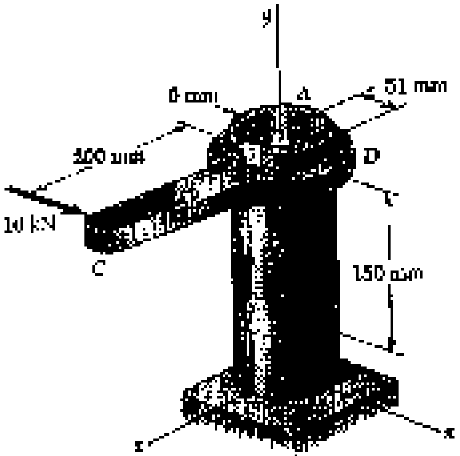
(b) $\Sigma F_n = 0 \qquad F_n - 100 \cos\beta = 0 \qquad F_n = 100 \cos 14.34° = 96.88\ \text{kN}$

$$A_w = \frac{800\times10^{-6}}{\cos 14.34} = 825.74\times10^{-6}\ \text{m}^2$$

$$\sigma = \frac{F_n}{A_w} = \frac{96.88\times10^3}{825.74\times10^{-6}} = 117.3\times10^6\ \text{Pa} = 117.3\ \text{MPa}$$

PROBLEM 7.23

7.23 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point H.



SOLUTION

$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$

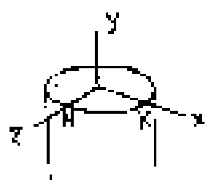
$I = \frac{1}{2}J = 2.0927 \text{ m}^4$

Force-couple system at center of tube in the plane containing points H and K.

$F_x = 10 \times 10^3 \text{ N}$

$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N}\cdot\text{m}$

$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N}\cdot\text{m}$

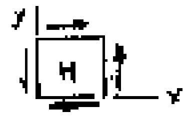


Torsion

$T = M_y = 2000 \text{ N}\cdot\text{m}$

$c = r_o = 51 \times 10^{-3} \text{ m}$

$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \times 10^6 \text{ Pa}$



Transverse Shear

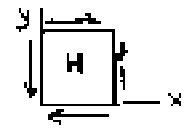
For semicircle $\quad A = \frac{\pi}{2}r^2 \quad \bar{y} = \frac{4}{3\pi}r$

$Q = A\bar{y} = \frac{2}{3}r^3$

$Q = Q_o - Q_i = \frac{2}{3}r_o^3 - \frac{2}{3}r_i^3 = 27.684 \times 10^3 \text{ mm}^3 = 27.684 \times 10^{-6} \text{ m}^3$

$V = F_x = 10 \times 10^3 \text{ N} \qquad t = (2)(6 \text{ mm}) = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$

$\tau_{xy} = \frac{VQ}{It} = \frac{(10 \times 10^3)(27.684 \times 10^{-6})}{(2.0927 \times 10^{-6})(12 \times 10^{-3})} = 11.02 \times 10^6 \text{ Pa}$



Bending: Point H lies on neutral axis. $\sigma_y = 0$

Total stresses at point H: $\sigma_x = 0, \ \sigma_y = 0$

$\tau_{xy} = 24.37 \times 10^6 + 11.02 \times 10^6 = 35.39 \times 10^6 \text{ Pa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 35.39 \times 10^6 \text{ Pa}$

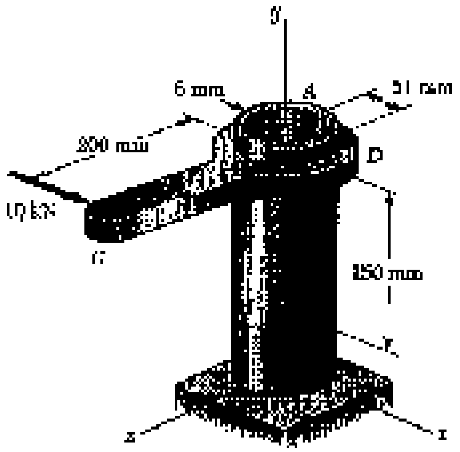
$\sigma_{max} = \sigma_{ave} + R = 35.39 \times 10^6 \text{ Pa} = 35.4 \text{ MPa}$

$\sigma_{min} = \sigma_{ave} - R = -35.39 \times 10^6 \text{ Pa} = -35.4 \text{ MPa}$

$\tau_{max} = R = 35.4 \text{ MPa}$

PROBLEM 7.24

7.24 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K.



SOLUTION

$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$

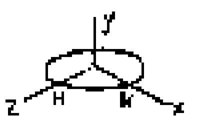
$I = \frac{1}{2}J = 2.0927 \times 10^{-6} \text{ m}^4$

Force-couple system at center of tube in the plane containing points H and K

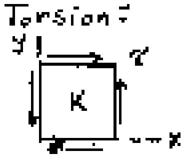
$F_x = 10 \text{ kN} = 10 \times 10^3 \text{ N}$

$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N·m}$

$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N·m}$



Torsion:



At point K, place local x-axis in negative global z-direction

$T = M_y = 2000 \text{ N·m} \qquad c = r_o = 51 \times 10^{-3} \text{ m}$

$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \times 10^6 \text{ Pa} = 24.37 \text{ MPa}$

Transverse Shear: Stress due to transverse shear $V = F_x$ is zero at pt. K.

Bending: $|\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-6}} = 36.56 \times 10^6 \text{ Pa} = 36.56 \text{ MPa}$

Point K lies on compression side of neutral axis: $\sigma_y = -36.56 \text{ MPa}$

Total stresses at point K $\qquad \sigma_x = 0 \qquad \sigma_y = -36.56 \text{ MPa}, \quad \tau_{xy} = 24.37 \text{ MPa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46 \text{ MPa}$

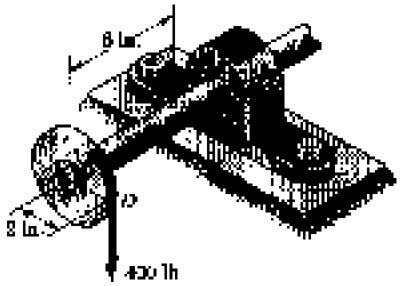
$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46 = +12.18 \text{ MPa}$

$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46 = -48.74 \text{ MPa}$

$\tau_{max} = R = 30.46 \text{ MPa}$

PROBLEM 7.25

7.25 A 400-lb vertical force is applied at D to a gear attached to the solid one-inch diameter shaft AB. Determine the principal stresses and the maximum shearing stress at point H located as shown on top of the shaft.



SOLUTION

Equivalent force-couple system at center of shaft in section at point H.

$V = 400$ lb, $\quad M = (400)(6) = 2400$ lb·in

$T = (400)(2) = 800$ lb·in.

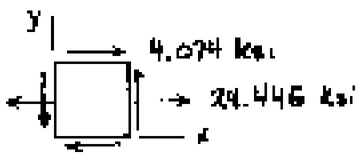
Shaft cross section. $\quad d = 1$ in $\quad c = \frac{1}{2}d = 0.5$ in

$J = \frac{\pi}{2}c^4 = 0.098175$ in^4 $\quad I = \frac{1}{2}J = 0.049087$ in^4

Torsion: $\quad \tau = \dfrac{Tc}{J} = \dfrac{(800)(0.5)}{0.098175} = 4.074 \times 10^3$ psi $= 4.074$ ksi

Bending: $\quad \sigma = \dfrac{Mc}{I} = \dfrac{(2400)(0.5)}{0.049087} = 24.446 \times 10^3$ psi $= 24.446$ ksi

Transverse shear: Stress at point H is zero.



$\sigma_x = 24.446$ ksi $\quad \sigma_y = 0 \quad \tau_{xy} = 4.074$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 12.223$ ksi

$R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(12.223)^2 + (4.074)^2}$

$= 12.884$ ksi

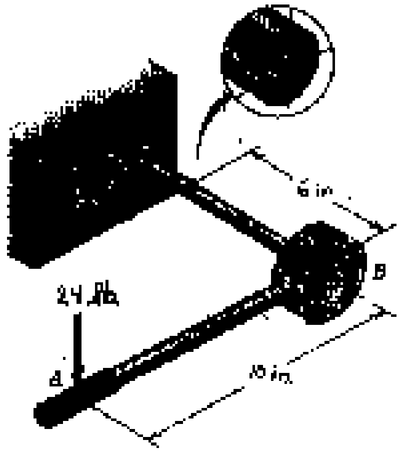
$\sigma_a = \sigma_{ave} + R = 25.107$ ksi

$\sigma_b = \sigma_{ave} - R = -0.661$ ksi

$\tau_{max} = R = 12.884$ ksi

PROBLEM 7.26

7.26 A mechanic uses a crowfoot wrench to loosen a bolt at *E*. Knowing that the mechanic applies a vertical 24-lb force at *A*, determine the principal stresses and the maximum shearing stress at point *H* located as shown on top of the $\frac{3}{4}$ - in. diameter shaft.



SOLUTION

Equivalent force-couple system at center of shaft in section at point H.

$V = 24$ lb. $\quad M = (24)(6) = 144$ lb·in

$T = (24)(10) = 240$ lb·in

Shaft cross section: $d = 0.75$ in, $\quad c = \frac{1}{2}d = 0.375$ in.

$J = \frac{\pi}{2}c^4 = 0.031063$ in^4 $\qquad I = \frac{1}{2}J = 0.015532$ in^4

Torsion: $\tau = \dfrac{Tc}{J} = \dfrac{(240)(0.375)}{0.031063} = 2.897 \times 10^3$ psi $= 2.897$ ksi

Bending: $\sigma = \dfrac{Mc}{I} = \dfrac{(144)(0.375)}{0.015532} = 3.477 \times 10^3$ psi $= 3.477$ ksi

Transverse Shear: At point H stress due to transverse shear is zero.

Resultant stresses: $\sigma_x = 3.477$ ksi, $\quad \sigma_y = 0$, $\quad \tau_{xy} = 2.897$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 1.738$

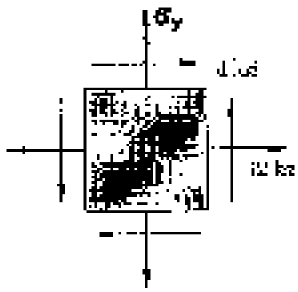
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{1.738^2 + 2.897^2} = 3.378$ ksi

$\sigma_a = \sigma_{ave} + R = 5.116$ ksi ◄

$\sigma_b = \sigma_{ave} - R = -1.640$ ksi ◄

$\tau_{max} = R = 3.378$ ksi ◄

PROBLEM 7.27



7.27 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 15 ksi.

SOLUTION

$\sigma_x = 12$ ksi, $\sigma_y = ?$, $\tau_{xy} = 4$ ksi

Let $u = \frac{\sigma_x - \sigma_y}{2}$ $\qquad \sigma_y = \sigma_x - 2u$

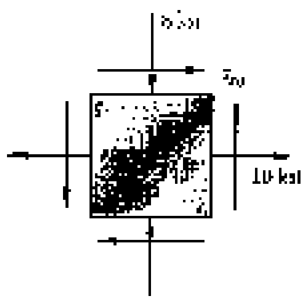
$R = \sqrt{u^2 + \tau_{xy}^2} = 15$ ksi

$u = \pm\sqrt{R^2 - \tau_{xy}^2} = \pm\sqrt{15^2 - 4^2} = \pm 14.457$ ksi

$\sigma_y = \sigma_x - 2u = 12 \mp (2)(14.457) = 40.9$ ksi, -16.91 ksi

Largest value for σ_y is required. $\sigma_y = 40.9$ ksi

PROBLEM 7.28



7.28 For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (*b*) the corresponding principal stresses.

SOLUTION

$\sigma_x = 10$ ksi, $\sigma_y = -8$ ksi, $\tau_{xy} = ?$

$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{10-(-8)}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{9^2 + \tau_{xy}^2} = 12$ ksi

(a) $\tau_{xy} = \sqrt{12^2 - 9^2} = 7.94$ ksi

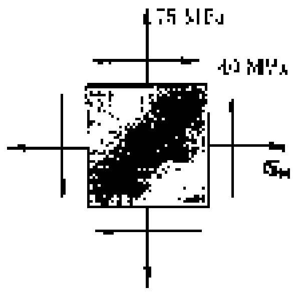
(b) $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 1$ ksi

$\sigma_a = \sigma_{ave} + R = 1 + 12 = 13$ ksi

$\sigma_b = \sigma_{ave} - R = 1 - 12 = -11$ ksi

PROBLEM 7.29

7.29 Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 50 MPa.



SOLUTION

$\sigma_x = ?$, $\sigma_y = 75$ MPa , $\tau_{xy} = 40$ MPa

Let $u = \dfrac{\sigma_x - \sigma_y}{2}$ $\qquad \sigma_x = \sigma_y + 2u$

$R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{max} = 50$ MPa

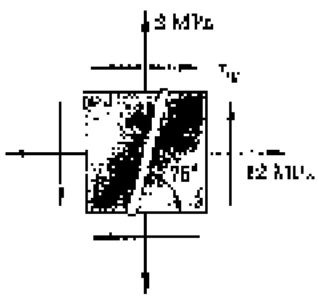
$u = \pm\sqrt{R^2 - \tau_{xy}^2} = \pm\sqrt{50^2 - 40^2} = \pm 30$ MPa

$\sigma_x = \sigma_y + 2u = 75 \pm (2)(30) = 135$ MPa , 15 MPa

Allowable range $\quad 15 \text{ MPa} \leq \sigma_x \leq 135 \text{ MPa}$ ◀

PROBLEM 7.30

7.30 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.



SOLUTION

$\sigma_x = 12$ MPa , $\sigma_y = 2$ MPa , $\tau_{xy} = ?$

Since $\tau_{x'y'} = 0$, x'-direction is a principal direction.

$\theta_p = -15°$

$\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

(a) $\tau_{xy} = \frac{1}{2}(\sigma_x - \sigma_y)\tan 2\theta_p = \frac{1}{2}(12-2)\tan(-30°) = -2.89$ MPa ◀

$R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{5^2 + 2.89^2} = 5.7735$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 7$ MPa

(b) $\sigma_a = \sigma_{ave} + R = 7 + 5.7735 = 12.77$ MPa ◀

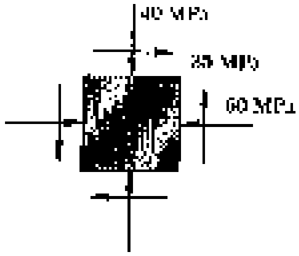
$\sigma_b = \sigma_{ave} - R = 7 - 5.7735 = 1.226$ MPa ◀

PROBLEM 7.31

7.31 Solve Probs. 7.7 and 7.11, using Mohr's circle.

7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.



SOLUTION

$\sigma_x = -60$ MPa $\sigma_y = -40$ MPa $\tau_{xy} = 35$ MPa

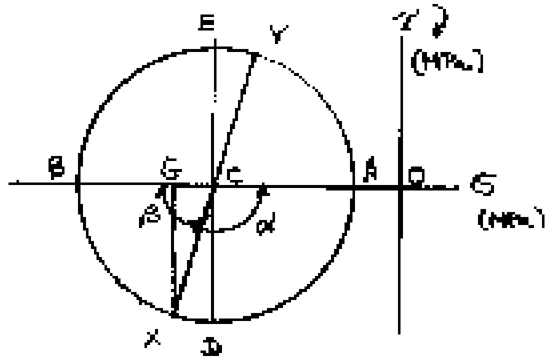
$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -50$ MPa

Points

$X: (\sigma_x, -\tau_{xy}) = (-60 \text{ MPa}, -35 \text{ MPa})$

$Y: (\sigma_y, \tau_{xy}) = (-40 \text{ MPa}, 35 \text{ MPa})$

$C: (\sigma_{ave}, 0) = (-50 \text{ MPa}, 0)$



$\tan\beta = \dfrac{GX}{CG} = \dfrac{35}{10} = 3.500$

$\beta = 74.05°$

$\theta_B = -\frac{1}{2}\beta = -37.03°$ ◄

$\alpha = 180° - \beta = 105.95°$

$\theta_A = \frac{1}{2}\alpha = 52.97°$ ◄

$R = \sqrt{\overline{CG}^2 + \overline{GX}^2} = \sqrt{10^2 + 35^2} = 36.4$ MPa

$\sigma_{min} = \sigma_{ave} - R = -50 - 36.4 = -86.4$ MPa ◄

$\sigma_{max} = \sigma_{ave} + R = -50 + 36.4 = -13.6$ MPa ◄

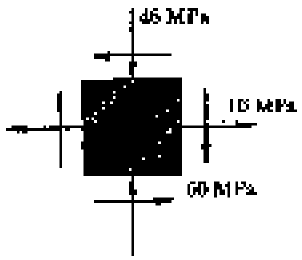
$\theta_D = \theta_B + 45° = 7.97°$ ◄

$\theta_E = \theta_A + 45° = 97.97°$ ◄

$\tau_{max} = R = 36.4$ MPa ◄

$\sigma' = \sigma_{ave} = -50$ MPa ◄

PROBLEM 7.32



7.32 Solve Probs. 7.5 and 7.12, using Mohr's circle.

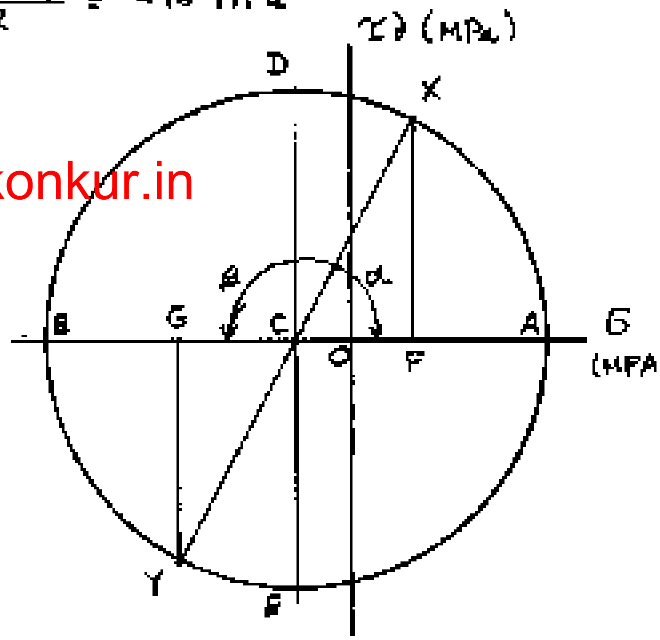
7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$\sigma_x = 16 \text{ MPa} \qquad \sigma_y = -48 \text{ MPa} \qquad \tau_{xy} = -60 \text{ MPa}$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -16 \text{ MPa}$$



Points:

X: $(\sigma_x, -\tau_{xy}) = (16 \text{ MPa}, 60 \text{ MPa})$

Y: $(\sigma_y, \tau_{xy}) = (-48 \text{ MPa}, -60 \text{ MPa})$

C: $(\sigma_{ave}, 0) = (-16 \text{ MPa}, 0)$

$$\tan\alpha = \frac{FX}{CF} = \frac{60}{32} = 1.875$$

$\alpha = 61.93°$

$\theta_A = -\frac{1}{2}\alpha = -30.96°$

$\beta = 180° - \alpha = 118.07°$

$\theta_B = \frac{1}{2}\beta = 59.04°$

$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{32^2 + 60^2} = 68 \text{ MPa}$

$\sigma_A = \sigma_{max} = \sigma_{ave} + R = -16 + 68 = 52 \text{ MPa}$

$\sigma_B = \sigma_{min} = \sigma_{ave} - R = -16 - 68 = -84 \text{ MPa}$

$\theta_D = \theta_A + 45° = 14.04°$

$\theta_E = \theta_B + 45° = 104.04°$

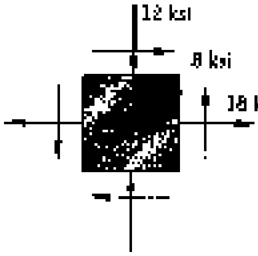
$\tau_{max} = R = 68 \text{ MPa}$

$\sigma' = \sigma_{ave} = -16 \text{ MPa}$

PROBLEM 7.33

7.33 Solve Prob. 7.9, using Mohr's circle.

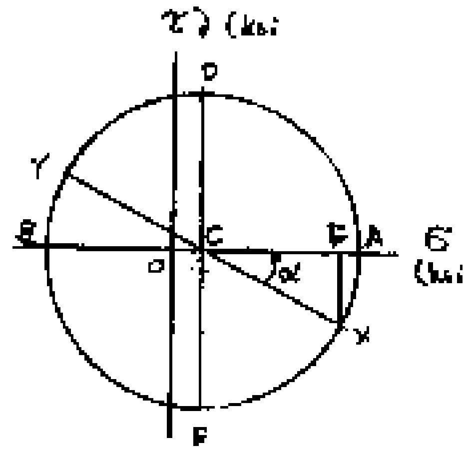
7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.



SOLUTION

$\sigma_x = 18$ ksi $\quad \sigma_y = -12$ ksi $\quad \tau_{xy} = 8$ ksi

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 3 \text{ ksi}$$



Points

X: $(\sigma_x, -\tau_{xy}) = (18 \text{ ksi}, -8 \text{ ksi})$

Y: $(\sigma_y, \tau_{xy}) = (-12 \text{ ksi}, 8 \text{ ksi})$

C: $(\sigma_{ave}, 0) = (3 \text{ ksi}, 0)$

$$\tan \alpha = \frac{FX}{CF} = \frac{8}{15} = 0.5333$$

$$\alpha = 28.07°$$

$$\theta_A = \tfrac{1}{2}\alpha = 14.04°$$

$$\theta_D = \theta_A + 45° = 59.04° \quad ◄$$

$$\theta_B = \theta_A - 45° = -30.96° \quad ◄$$

$$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{15^2 + 8^2} = 17 \text{ ksi}$$

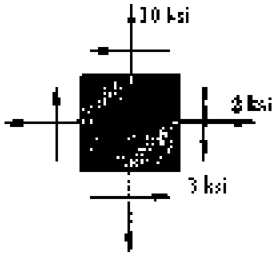
$$\tau_{max} = R = 17 \text{ ksi} \quad ◄$$

$$\sigma' = \sigma_{ave} = 3 \text{ ksi} \quad ◄$$

PROBLEM 7.34

7.34 Solve Prob. 7.10, using Mohr's circle.

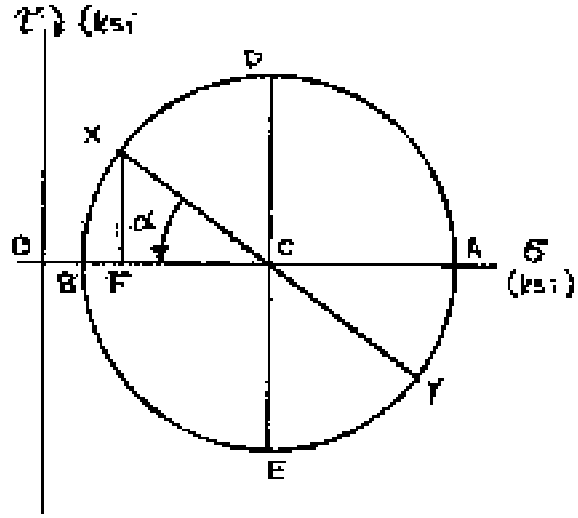
7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.



SOLUTION

$\sigma_x = 2$ ksi $\sigma_y = 10$ ksi $\tau_{xy} = -3$ ksi

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{2+10}{2} = 6 \text{ ksi}$$



Points

X: $(\sigma_x, -\tau_{xy}) = (2 \text{ ksi}, 3 \text{ ksi})$

Y: $(\sigma_y, \tau_{xy}) = (10 \text{ ksi}, -3 \text{ ksi})$

C: $(\sigma_{ave}, 0) = (6 \text{ ksi}, 0)$

$$\tan\alpha = \frac{FX}{FC} = \frac{3}{4} = 0.75$$

$$\alpha = 36.87°$$

$$\theta_B = \tfrac{1}{2}\alpha = 18.43°$$

$$\theta_D = \theta_B - 45° = -26.57° \quad \blacktriangleleft$$

$$\theta_E = \theta_B + 45° = 63.43° \quad \blacktriangleleft$$

$$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{4^2 + 3^2} = 5 \text{ ksi}$$

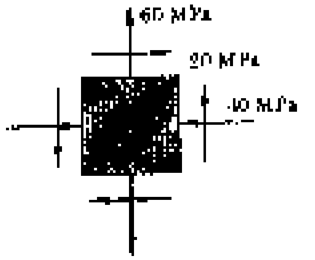
$$\tau_{max} = R = 5 \text{ ksi} \quad \blacktriangleleft$$

$$\sigma' = \sigma_{ave} = 6 \text{ ksi} \quad \blacktriangleleft$$

PROBLEM 7.35

7.35 Solve Prob. 7.13, using Mohr's circle.

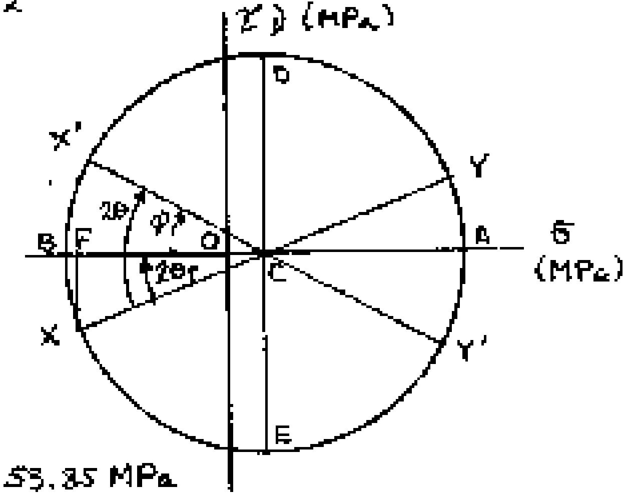
7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



SOLUTION

$\sigma_x = -40$ MPa $\quad \sigma_y = 60$ MPa $\quad \tau_{xy} = 20$ MPa

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 10 \text{ MPa}$$



Points

X: (−40 MPa, −20 MPa)

Y: (60 MPa, 20 MPa)

C: (10 MPa, 0)

$$\tan 2\theta_p = \frac{FX}{FC} = \frac{20}{50} = 0.4$$

$2\theta_p = 21.80°$ $\qquad \theta_p = 10.90°$

$$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{50^2 + 20^2} = 53.85 \text{ MPa}$$

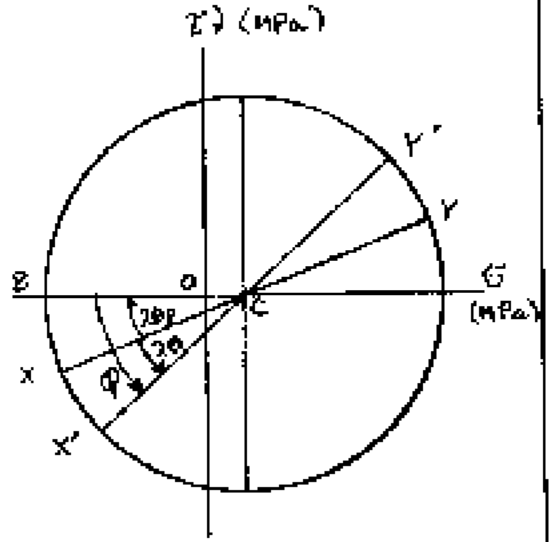
(a) $\theta = 25°$ $\qquad 2\theta = 50°$

$\varphi = 2\theta - 2\theta_p = 50° - 21.80° = 28.20°$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi = -37.5$ MPa ◀

$\tau_{x'y'} = -R\sin\varphi = -25.4$ MPa ◀

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi = 57.5$ MPa ◀



(b) $\theta = 10°$ $\qquad 2\theta = 20°$

$\varphi = 2\theta_p + 2\theta = 21.80° + 20° = 41.80°$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi = -30.1$ MPa ◀

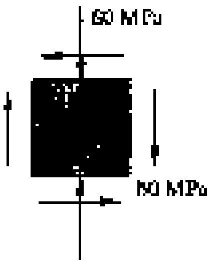
$\tau_{x'y'} = R\sin\varphi = 35.9$ MPa ◀

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi = 50.1$ MPa ◀

PROBLEM 7.36

7.36 Solve Prob. 7.14, using Mohr's circle.

7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



SOLUTION

$\sigma_x = 0 \qquad \sigma_y = -80 \text{ MPa} \qquad \tau_{xy} = -50 \text{ MPa}$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa}$

Points

X: (0, 50 MPa)

Y: (−80 MPa, −50 MPa)

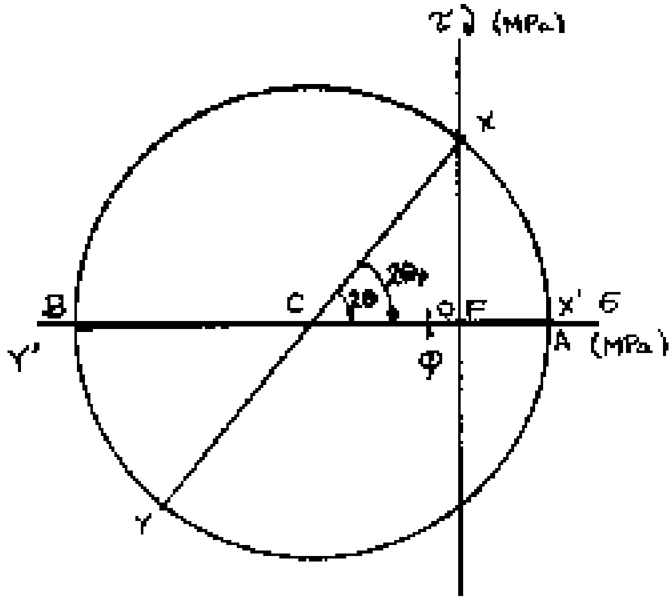
C: (−40 MPa, 0)

$\tan 2\theta_p = \dfrac{FX}{CF} = \dfrac{50}{40} = 1.25$

$2\theta_p = 51.34°$

$R = \sqrt{CF^2 + FX^2} = \sqrt{40^2 + 50^2}$

$= 64.03 \text{ MPa}$



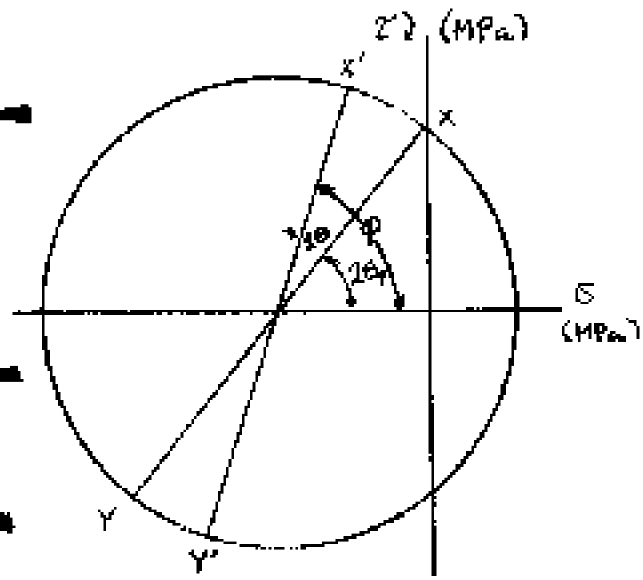
(a) $\theta = 25°$ $\qquad 2\theta = 50°$

$\varphi = 51.34° - 50° = 1.34°$

$\sigma_{x'} = \sigma_{ave} + R\cos\varphi = 24.0 \text{ MPa}$ ◄

$\tau_{x'y'} = -R\sin\varphi = -1.5 \text{ MPa}$ ◄

$\sigma_{y'} = \sigma_{ave} - R\cos\varphi = -104.0 \text{ MPa}$ ◄



(b) $\theta = 10°$ $\qquad 2\theta = 20°$

$\varphi = 51.34° + 20° = 71.34°$

$\sigma_{x'} = \sigma_{ave} + R\cos\varphi = -19.5 \text{ MPa}$ ◄

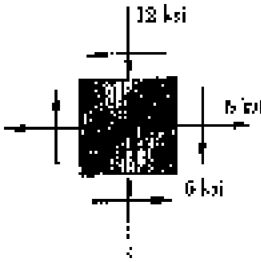
$\tau_{x'y'} = +R\sin\varphi = -60.7 \text{ MPa}$ ◄

$\sigma_{y'} = \sigma_{ave} - R\cos\varphi = -60.5 \text{ MPa}$ ◄

PROBLEM 7.37

7.37 Solve Prob. 7.15, using Mohr's circle.

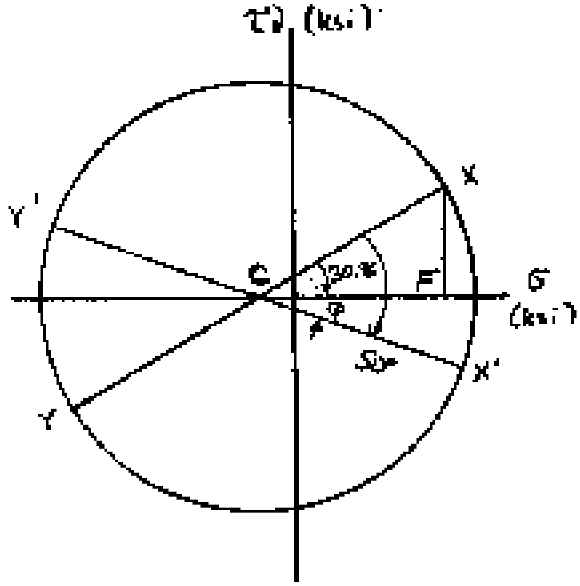
7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.



SOLUTION

$\sigma_x = 8$ ksi $\qquad \sigma_y = -12$ ksi $\qquad \tau_{xy} = -6$ ksi

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -2 \text{ ksi}$$



Points
X : (8 ksi, 6 ksi)
Y : (−12 ksi, −6 ksi)
C : (−2 ksi, 0)

$$\tan 2\theta_p = \frac{FX}{CF} = \frac{6}{10} = 0.6$$

$$2\theta_p = 30.96°$$

$$R = \sqrt{\overline{CF}^2 + \overline{FX}^2} = \sqrt{10^2 + 6^2} = 11.66 \text{ ksi}$$

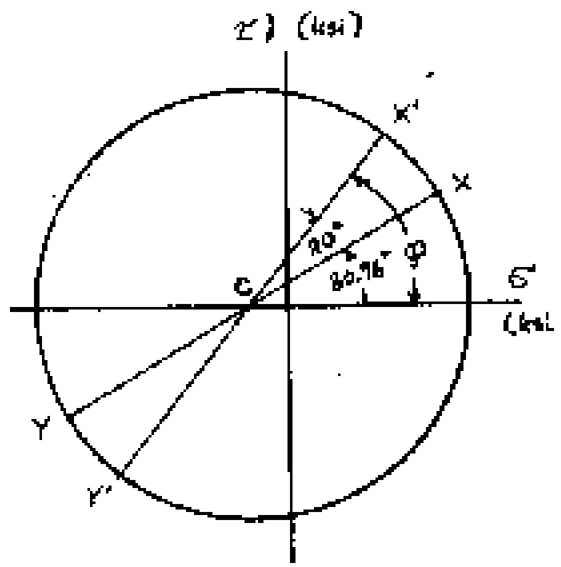
(a) $\theta = 25°$ ↻ $\qquad 2\theta = 50°$ ↻

$$\varphi = 50° - 30.96° = 19.04°$$

$$\sigma_{x'} = \sigma_{ave} + R\cos\varphi = 9.02 \text{ ksi} \blacktriangleleft$$

$$\tau_{x'y'} = R\sin\varphi = 3.80 \text{ ksi} \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{ave} - R\cos\varphi = -13.02 \text{ ksi} \blacktriangleleft$$



(b) $\theta = 10°$ ↺ $\qquad 2\theta = 20°$ ↺

$$\varphi = 30.96° + 20° = 50.96°$$

$$\sigma_{x'} = \sigma_{ave} + R\cos\varphi = 5.34 \text{ ksi} \blacktriangleleft$$

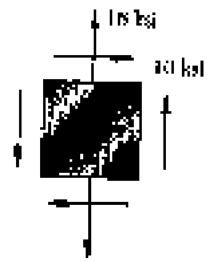
$$\tau_{x'y'} = -R\sin\varphi = -9.06 \text{ ksi} \blacktriangleleft$$

$$\sigma_{y'} = \sigma_{ave} - R\cos\varphi = -9.34 \text{ ksi} \blacktriangleleft$$

PROBLEM 7.38

7.38 Solve Prob. 7.16, using Mohr's circle.

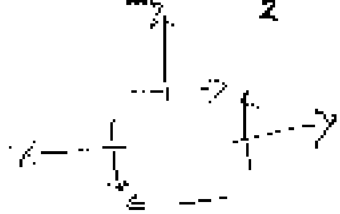
7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

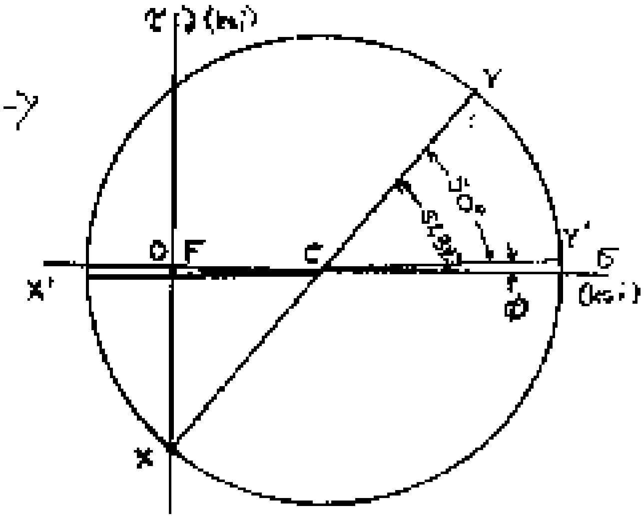


SOLUTION

$\sigma_x = 0 \qquad \sigma_y = 16\text{ ksi} \qquad \tau_{xy} = 10\text{ ksi}$

$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = 8\text{ ksi}$





Points:
X: (0, −10 ksi)
Y: (16 ksi, 10 ksi)
C: (8 ksi, 0)

$\tan 2\theta_p = \dfrac{FX}{FC} = \dfrac{10}{8} = 1.25$

$2\theta_p = 51.34°$

$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{8^2 + 10^2}$
$= 12.81\text{ ksi}$

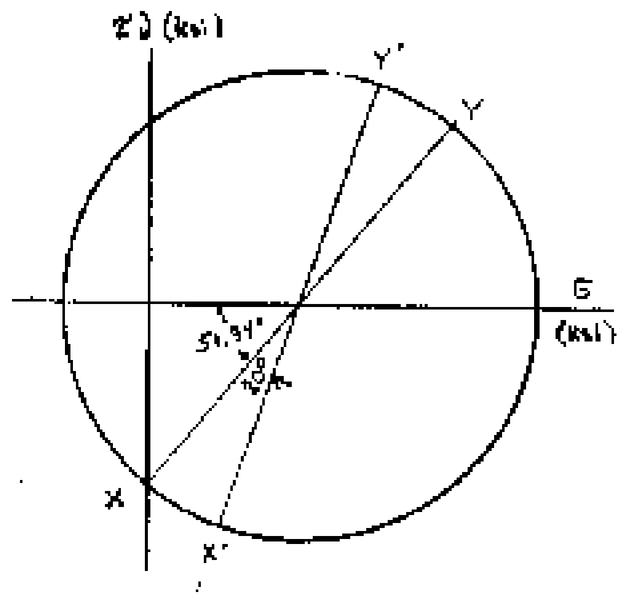
(a) $\theta = 25°$ ↻ $\qquad 2\theta = 50°$ ↻

$\varphi = 51.34° - 50° = 1.34°$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi = -4.81\text{ ksi}$ ◀

$\tau_{x'y'} = R\sin\varphi = 0.30\text{ ksi}$ ◀

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi = 20.81\text{ ksi}$ ◀



(b) $\theta = 10°$ ↺ $\qquad 2\theta = 20°$ ↺

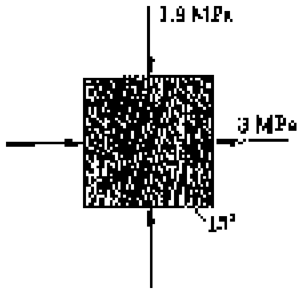
$\varphi = 51.34° + 20° = 71.34°$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi = 3.90\text{ ksi}$ ◀

$\tau_{x'y'} = R\sin\varphi = 12.14\text{ ksi}$ ◀

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi = 12.10\text{ ksi}$ ◀

PROBLEM 7.39



7.39 Solve Prob. 7.17, using Mohr's circle.

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$\sigma_x = -3\text{ MPa} \qquad \sigma_y = -1.8\text{ MPa} \qquad \tau_{xy} = 0$

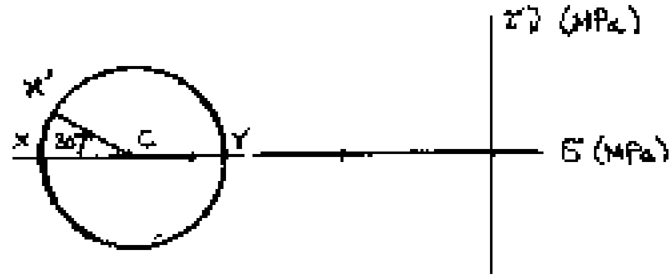
$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = -2.4\text{ MPa}$

Points

X: $(\sigma_x, -\tau_{xy}) = (-3\text{ MPa}, 0)$

Y: $(\sigma_y, \tau_{xy}) = (-1.8\text{ MPa}, 0)$

C: $(\sigma_{ave}, 0) = (-2.4\text{ MPa}, 0)$



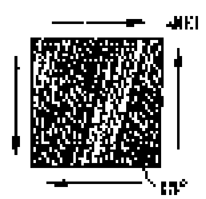
$\theta = -15° \qquad 2\theta = -30°$

$\overline{CX} = 0.6\text{ MPa} \qquad R = 0.6\text{ MPa}$

(a) $\tau_{x'y'} = -\overline{CX'}\sin 30 = -R\sin 30° = -0.6\sin 30° = -0.3\text{ MPa}$

(b) $\sigma_{x'} = \sigma_{ave} - \overline{CX'}\cos 30° = -2.4 - 0.6\cos 30° = -2.92\text{ MPa}$

PROBLEM 7.40



7.40 Solve Prob. 7.18, using Mohr's circle.

7.17 and 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$\sigma_x = \sigma_y = 0 \qquad \tau_{xy} = 400\text{ psi}$

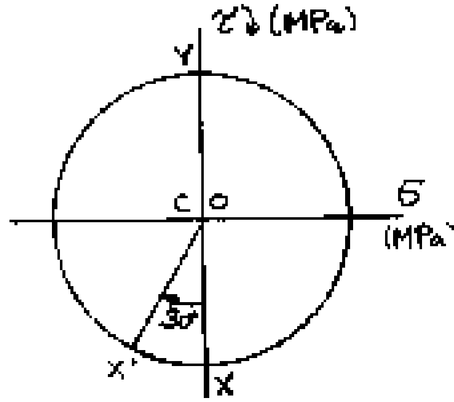
$\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = 0$

Points

X: $(\sigma_x, -\tau_{xy}) = (0, -400\text{ psi})$

Y: $(\sigma_y, \tau_{xy}) = (0, 400\text{ psi})$

C: $(\sigma_{ave}, 0) = (0, 0)$



$\theta = -15° \qquad 2\theta = -30°$

$\overline{CX} = R = 400\text{ psi}$

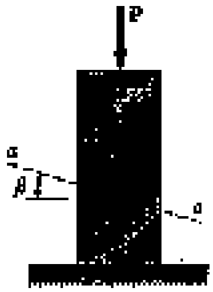
(a) $\tau_{x'y'} = R\cos 30° = 400\cos 30° = 346\text{ psi}$

(b) $\sigma_{x'} = \sigma_{ave} - R\sin 30° = -400\sin 30° = -200\text{ psi}$

PROBLEM 7.41

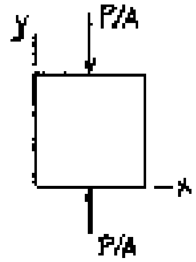
7.41 Solve Prob. 7.19, using Mohr's circle.

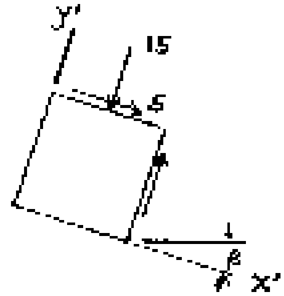
7.19 The centric force **P** is applied to a short post as shown. Knowing that the stresses on plane *a-a* are $\sigma = -15$ ksi and $\tau = 5$ ksi, determine (*a*) the angle β that plane *a-a* forms with the horizontal, (*b*) the maximum compressive stress in the post.

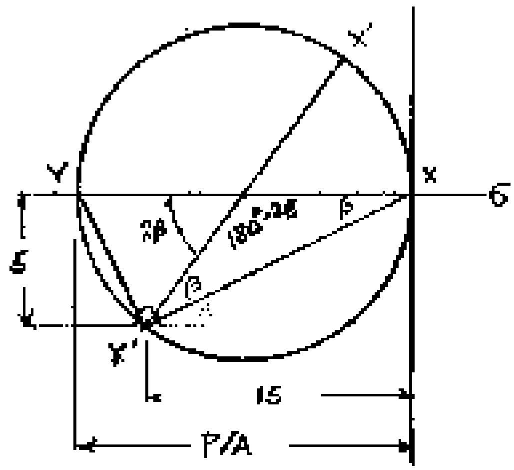


SOLUTION

$$\sigma_x = 0$$
$$\tau_{xy} = 0$$
$$\sigma_y = -P/A$$







From the Mohr's circle

$$\tan\beta = \frac{5}{15} = 0.3333 \qquad \beta = 18.4° \blacktriangleleft$$

$$-\sigma = \frac{P}{2A} + \frac{P}{2A}\cos 2\beta$$

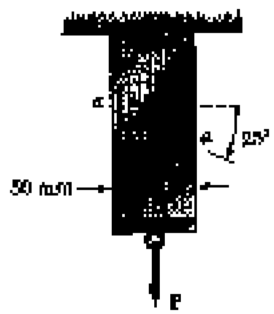
$$\frac{P}{A} = \frac{2(-\sigma)}{1+\cos 2\beta} = \frac{(2)(15)}{1+\cos 2\beta}$$

$$= 16.67 \text{ ksi} \blacktriangleleft$$

PROBLEM 7.42

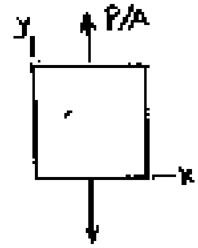
7.42 Solve Prob. 7.20, using Mohr's circle.

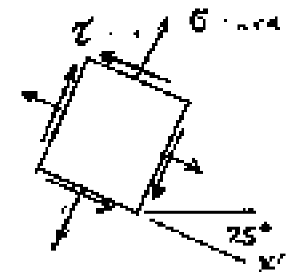
7.20 Two members of uniform cross section 50 × 80 mm are glued together along plane *a-a*, which forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest axial load **P** that can be applied.



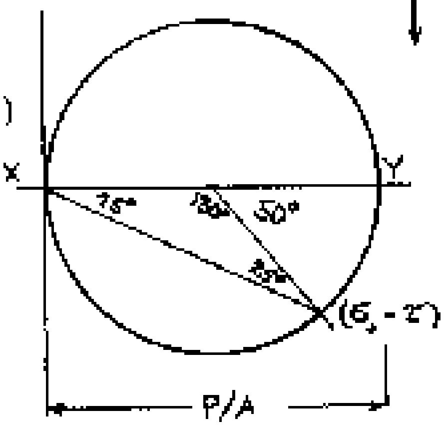
SOLUTION

$\sigma_x = 0$
$\tau_{xy} = 0$
$\sigma_y = P/A$





$A = (50\times10^{-3})(80\times10^{-3})$
$= 4\times10^{-3}\ m^2$



$\sigma = \frac{P}{2A}(1 + \cos 50°)$

$P = \frac{2A\sigma}{1 + \cos 50°}$

$P \le \frac{(2)(4\times10^{-3})(800\times10^{3})}{1 + \cos 50°}$

$P \le 3.90\times10^{3}$ N

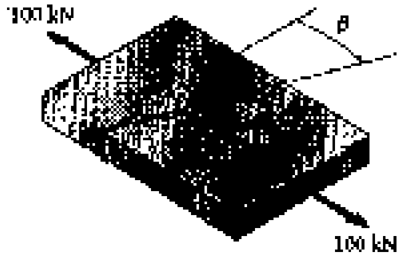
$\tau = \frac{P}{2A}\sin 50°$ $\qquad P = \frac{2A\tau}{\sin 50°} \le \frac{(2)(4\times10^{-3})(600\times10^{3})}{\sin 50°} = 6.27\times10^{3}$ N

Choosing the smaller value $\quad P \le 3.90\times10^{3}\ N = 3.90$ kN

PROBLEM 7.43

7.43 Solve Prob. 7.21, using Mohr's circle.

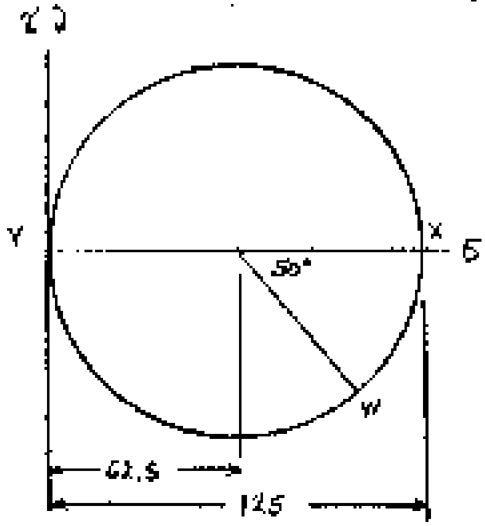
7.21 Two steel plates of uniform cross section 10 × 80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that $\beta = 25°$, determine (*a*) the in-plane shearing stress parallel to the weld, (*b*) the normal stress perpendicular to the weld.



SOLUTION

$$\sigma_x = \frac{P}{A} = \frac{100 \times 10^3}{(10\times10^{-3})(80\times10^{-3})} = 125\times10^6 \text{ Pa} = 125 \text{ MPa}$$

$$\sigma_y = 0 \qquad \tau_{xy} = 0$$

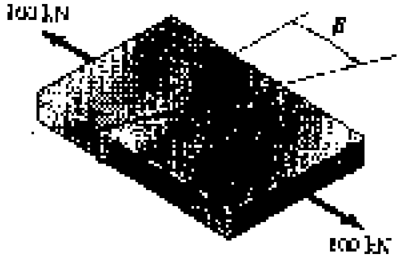


From Mohr's circle

(a) $\tau_w = 62.5 \sin 50° = 47.9 \text{ MPa}$ ◄

(b) $\sigma_w = 62.5 + 62.5 \cos 50°$
$= 102.7 \text{ MPa}$ ◄

PROBLEM 7.44



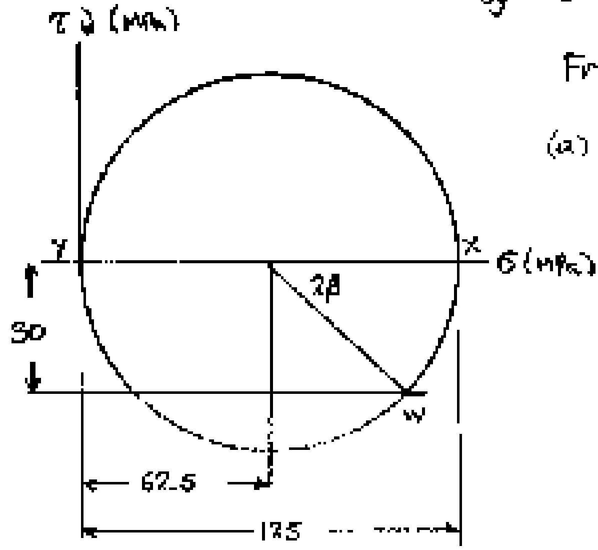
7.44 Solve Prob. 7.22, using Mohr's circle.

7.22 Two steel plates of uniform cross section 10 × 80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that the in-plane shearing stress parallel to the weld is 30 MPa, determine (*a*) the angle β, (*b*) the corresponding normal stress perpendicular to the weld.

SOLUTION

$$\sigma_x = \frac{P}{A} = \frac{100\times 10^3}{(10\times 10^{-3})(80\times 10^{-3})} = 125\times 10^6 \text{ Pa} = 125 \text{ MPa}$$

$$\sigma_y = 0 \qquad \tau_{xy} = 0$$



From Mohr's circle

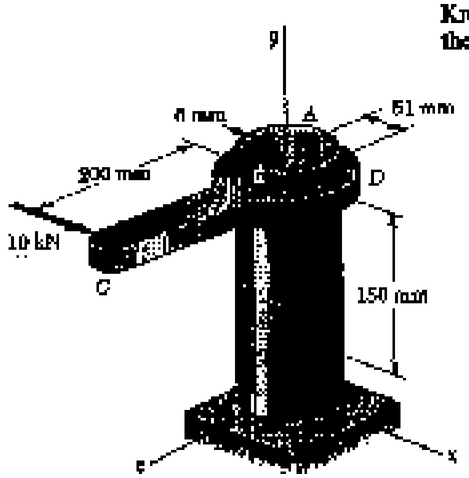
(a) $\sin 2\beta = \frac{30}{62.5} = 0.48 \qquad \beta = 14.3°$

(b) $\sigma = 62.5 + 62.5 \cos 2\beta$
$= 117.3 \text{ MPa}$

PROBLEM 7.45

7.45 Solve Prob. 7.23, using Mohr's circle.

7.23 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point H.



SOLUTION

$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855 \times 10^6 \text{ mm}^4 = 4.1855 \times 10^{-6} \text{ m}^4$

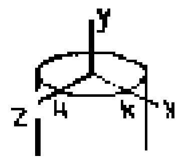
$I = \frac{1}{2} J = 2.0927 \times 10^{-6} \text{ m}^4$

Force-couple system at center of tube in the plane containing points H and K

$F_x = 10 \times 10^3 \text{ N}$

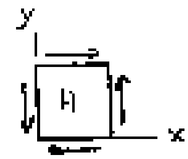
$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000 \text{ N·m}$

$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500 \text{ N·m}$

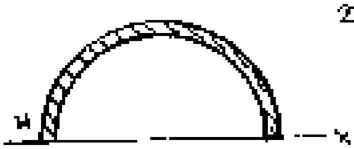


Torsion: $T = M_y = 2000 \text{ N·m}$

$c = r_o = 51 \times 10^{-3} \text{ m}$



$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37 \text{ MPa}$$



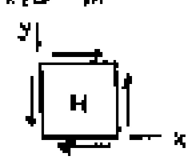
Transverse Shear: For semicircle $\quad A = \frac{\pi}{2} r^2 \qquad \bar{y} = \frac{4}{3\pi} r$

$Q = A\bar{y} = \frac{2}{3} r^3$

For pipe $\quad Q = Q_o - Q_i = \frac{2}{3} r_o^3 - \frac{2}{3} r_i^3 = 27.684 \times 10^3 \text{ mm}^3 = 27.684 \times 10^{-6} \text{ m}^3$

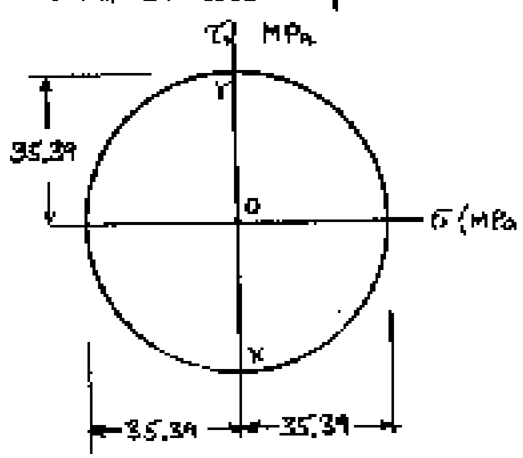
$V = F_x = 10 \times 10^3 \text{ N} \qquad t = (2)(6) = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(10 \times 10^3)(27.684 \times 10^{-6})}{(2.0927 \times 10^{-6})(12 \times 10^{-3})} = 11.02 \text{ MPa}$$



Bending: Point H lies on neutral axis $\qquad \sigma_y = 0$

Total stresses at point H $\qquad \sigma_x = 0, \ \sigma_y = 0 \qquad \tau_{xy} = 24.37 + 11.02 = 35.39 \text{ MPa}$



$\sigma_{ave} = 0$

$R = 35.39 \text{ MPa}$

$\sigma_{max} = \sigma_{ave} + R = 35.39 \text{ MPa}$

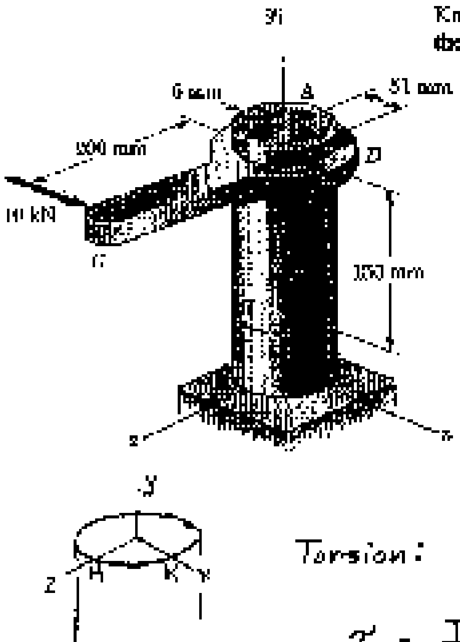
$\sigma_{min} = \sigma_{ave} - R = -35.39 \text{ MPa}$

$\tau_{max} = R = 35.39 \text{ MPa}$

PROBLEM 7.46

7.46 Solve Prob. 7.24, using Mohr's circle.

7.24 The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K.



SOLUTION

$r_o = \frac{d_o}{2} = \frac{102}{2} = 51$ mm $\quad r_i = r_o - t = 45$ mm

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 4.1855 \times 10^6$ mm^4 $= 4.1855 \times 10^{-6}$ m^4

$I = \frac{1}{2} J = 2.0927 \times 10^{-6}$ m^4

Force-couple system at center of tube in the plane containing points H and K

$F_x = 10 \times 10^3$ N

$M_y = (10 \times 10^3)(200 \times 10^{-3}) = 2000$ N·m

$M_z = -(10 \times 10^3)(150 \times 10^{-3}) = -1500$ N·m

Torsion: $T = M_y = 2000$ N·m

$c = r_o = 51 \times 10^{-3}$ m

$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^{-6}} = 24.37$ MPa

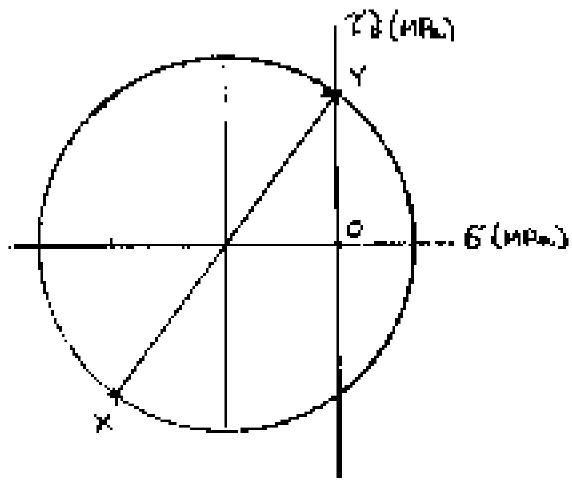
Note that local x-axis is taken along negative global z-direction.

Transverse Shear: Stress due to $V = F_x$ is zero at point K.

Bending: $|\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51 \times 10^{-3})}{2.0927 \times 10^{-6}} = 36.56$ MPa

Point K lies on compression side of neutral axis. $\sigma_y = -36.56$ MPa

Total stresses at point K $\quad \sigma_x = 0$, $\sigma_y = -36.56$ MPa, $\tau_{xy} = 24.37$ MPa



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46$ MPa

$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46$
$= 12.18$ MPa

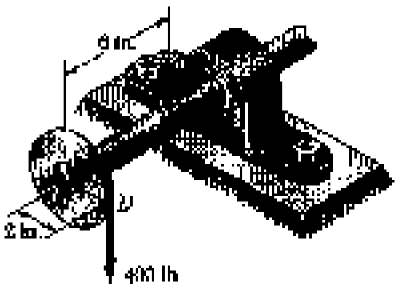
$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46$
$= -48.74$ MPa

$\tau_{max} = R = 30.46$ MPa

PROBLEM 7.47

7.47 Solve Prob. 7.25, using Mohr's circle.

7.25 A 400-lb vertical force is applied at D to a gear attached to the solid one-inch diameter shaft AB. Determine the principal stresses and the maximum shearing stress at point H located as shown on top of the shaft.



SOLUTION

Equivalent force-couple system at center of shaft in section at point H.

$V = 400$ lb. $\quad M = (400)(6) = 2400$ lb·in

$T = (400)(2) = 800$ lb·in.

Shaft cross section $\quad d = 1$ in $\quad c = \frac{1}{2}d = 0.5$ in.

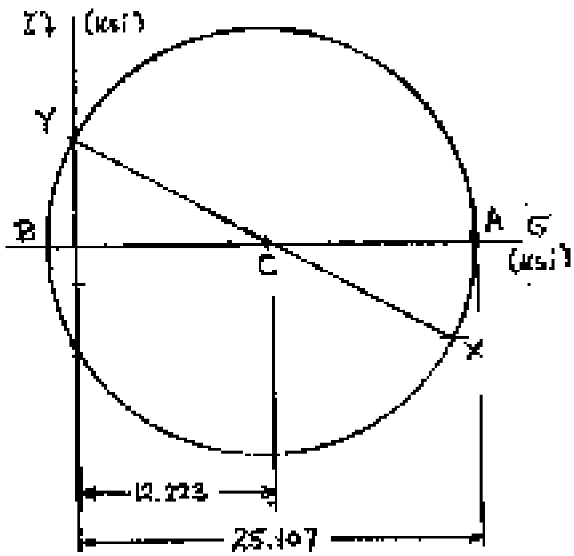
$J = \frac{\pi}{2}c^4 = 0.098175$ in^4 $\quad I = \frac{1}{2}J = 0.049087$ in^4

Torsion: $\quad \tau = \frac{Tc}{J} = \frac{(800)(0.5)}{0.098175} = 4.074 \times 10^3$ psi $= 4.074$ ksi

Bending: $\quad \sigma = \frac{Mc}{I} = \frac{(2400)(0.5)}{0.049087} = 24.446 \times 10^3$ psi $= 24.446$ ksi

Transverse Shear: $\quad$ Stress at point H is zero.

Resultant stresses: $\quad \sigma_x = 24.446$ ksi, $\sigma_y = 0$, $\tau_{xy} = 4.074$ ksi



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 12.223$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{(12.223)^2 + (4.074)^2} = 12.884$ ksi

$\sigma_a = \sigma_{ave} + R = 25.107$ ksi

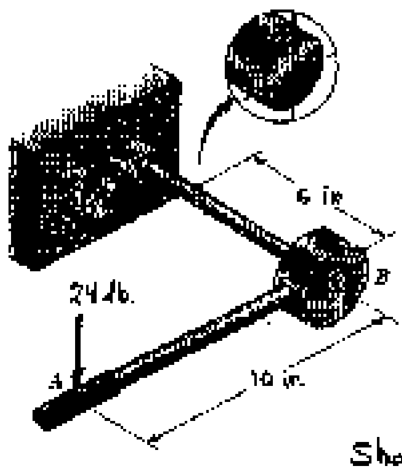
$\sigma_b = \sigma_{ave} - R = -0.661$ ksi

$\tau_{max} = R = 12.88$ ksi

PROBLEM 7.48

7.48 Solve Prob. 7.26, using Mohr's circle.

7.26 A mechanic uses a crowfoot wrench to loosen a bolt at E. Knowing that the mechanic applies a vertical 24-lb force at A, determine the principal stresses and the maximum shearing stress at point H located as shown on top of the $\frac{3}{4}$-in. diameter shaft.



SOLUTION

Equivalent force-couple system at center of shaft in section at point H.

$V = 24$ lb. $\quad M = (24)(6) = 144$ lb·in

$T = (24)(10) = 240$ lb·in

Shaft cross section: $\quad d = 0.75$ in. $\quad c = \frac{1}{2}d = 0.375$ in

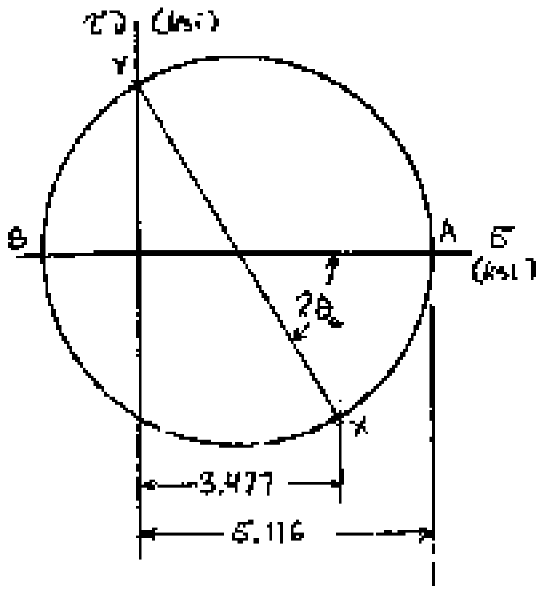
$J = \frac{\pi}{2}c^4 = 0.031063$ in^4 $\quad I = \frac{1}{2}J = 0.015532$ in^4

Torsion: $\tau = \frac{Tc}{J} = \frac{(240)(0.375)}{0.031063} = 2.897 \times 10^3$ psi $= 2.897$ ksi

Bending: $\sigma = \frac{Mc}{I} = \frac{(144)(0.375)}{0.015532} = 3.477 \times 10^3$ psi $= 3.477$ ksi

Transverse Shear: At point H stress due to transverse shear is zero.

Resultant stresses: $\sigma_x = 3.477$ ksi, $\sigma_y = 0$, $\tau_{xy} = 2.897$ ksi



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 1.738$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{1.738^2 + 2.897^2} = 3.378$ ksi

$\sigma_a = \sigma_{ave} + R = 5.116$ ksi

$\sigma_b = \sigma_{ave} - R = -1.640$ ksi

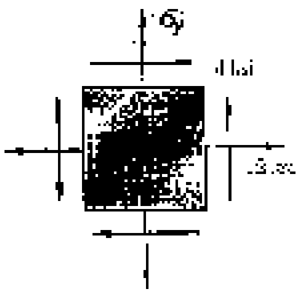
$\tau_{max} = R = 3.378$ ksi

The center of Mohr's circle lies at point C

PROBLEM 7.49

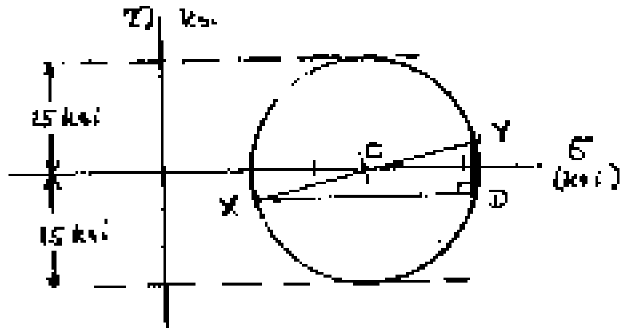
7.49 Solve Prob. 7.27, using Mohr's circle.

7.27 For the state of plane stress shown, determine the largest value of σ_y for which the maximum in-plane shearing stress is equal to or less than 15 ksi.



SOLUTION

$\sigma_x = 12$ ksi , $\sigma_y = ?$, $\tau_{xy} = 4$ ksi



Given: $\tau_{max} = R = 15$ ksi

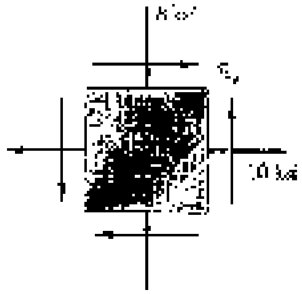
$\overline{XY} = 2R = 30$ ksi

$\overline{DY} = (2)(\tau_{xy}) = 8$ ksi

$\overline{XD} = \sqrt{\overline{XY}^2 - \overline{DY}^2} = \sqrt{30^2 - 8^2} = 28.9$ ksi

$\sigma_y = \sigma_x + \overline{XD} = 12 + 28.9 = 40.9$ ksi

PROBLEM 7.50

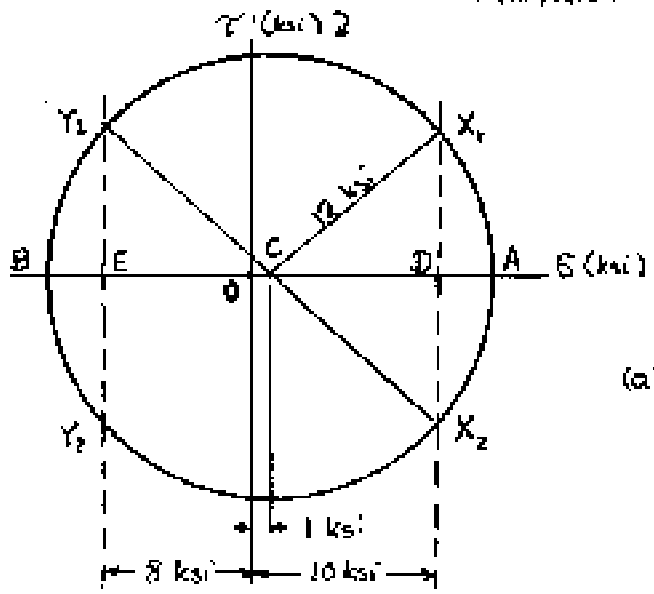


*7.50 Solve Prob. 7.28, using Mohr's circle.

7.28 For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (*b*) the corresponding principal stresses.

SOLUTION

The center of the Mohr's circle lies at point C with coordinates $\left(\frac{\sigma_x+\sigma_y}{2}, 0\right) = \left(\frac{10-8}{2}, 0\right)$ $= (1.0\ \text{ksi}, 0)$. The radius of the circle is $\tau_{max(in\text{-}plane)} = 12$ ksi.



The stress point $(\sigma_x, -\tau_{xy})$ lie along the line X_1X_2 of th Mohr circle diagram. The extreme points with $R \le 12$ ksi are X_1 and X_2.

(a) The largest allowable value of τ_{xy} is obtained from triangle CDX_1

$$\overline{DX_1}^2 = \overline{DX_2}^2 = \sqrt{\overline{CX_1}^2 - \overline{CD}^2}$$

$$\tau_{xy}^2 = \sqrt{12^2 - 9^2} = 7.94\ \text{ksi}$$

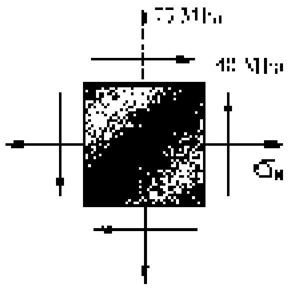
(b) The principal stresses are $\sigma_a = 1 + 12 = 13$ ksi

$\sigma_b = 1 - 12 = -11$ ksi

PROBLEM 7.51

7.51 Solve Prob. 7.29, using Mohr's circle.

7.29 Determine the range of values of σ_x for which the maximum in-plane shearing stress is equal to or less than 50 MPa.



SOLUTION

For the Mohr's circle, point Y lies at (75 MPa, 40 MPa).

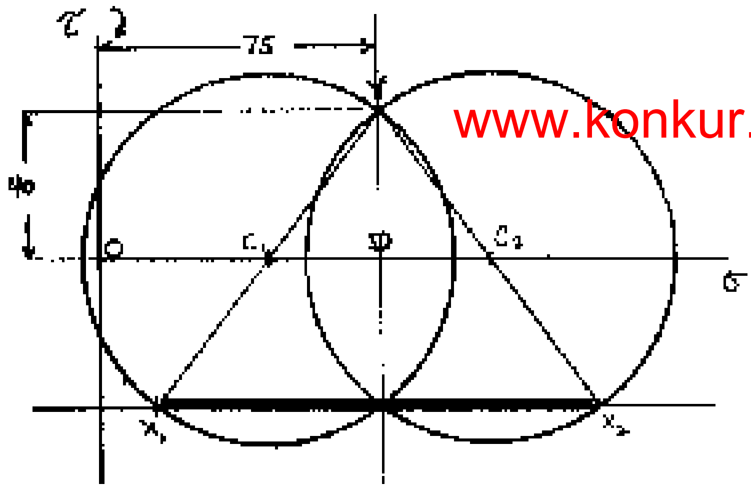
The radius of limiting circles is $R = 50$ MPa

Let C_1 be the location of the left most limiting circle and C_2 be that of the right most one.

$\overline{C_1Y} = 50$ MPa

$\overline{C_2Y} = 50$ MPa

Noting right triangles C_1DY and C_2DY



$\overline{C_1D}^2 + \overline{DY}^2 = \overline{C_1Y}^2 \qquad \overline{C_1D}^2 + 40^2 = 50^2 \qquad \overline{C_1D} = 30$

Coordinates of point C_1 are $(0, 75 - 30) = (0, 45 \text{ MPa})$

Likewise, coordinates of point C_2 are $= (0, 75 + 30) = (0, 105 \text{ MPa})$

Coordinates of point X_1 $(45 - 30, -40) = (15 \text{ MPa}, -40 \text{ MPa})$

Coordinates of point X_2 $(105 + 30, -40) = (135 \text{ MPa}, -40 \text{ MPa})$

The point $(\sigma_x, -\tau_{xy})$ must lie on the line X_1X_2

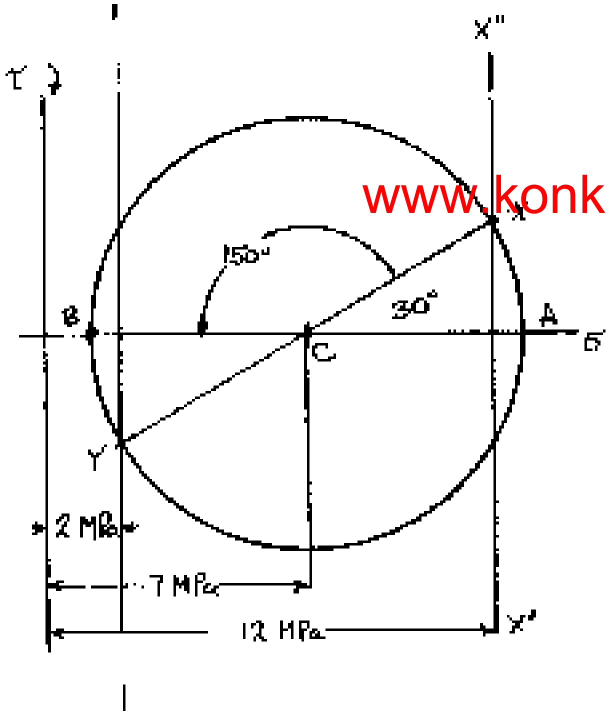
Thus $\qquad 15 \text{ MPa} \le \sigma_x \le 135 \text{ MPa}$

PROBLEM 7.52

7.52 Solve Prob. 7.30, using Mohr's circle.

7.30 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.

SOLUTION



Point X of Mohr's circle must lie on X'X'' so that $\sigma_x = 12$ MPa. Likewise, point Y lies on line Y'Y'' so that $\sigma_y = 2$ MPa. The coordinates of C are

$$\frac{2+12}{2},\ 0 = (7\ \text{MPa}, 0).$$

Counterclockwise rotation through 150° brings line CX to CB, where $\tau = 0$.

$$R = \frac{\sigma_x - \sigma_y}{2}\sec 30° = \frac{12-2}{2}\sec 30° = 5.77\ \text{MPa}$$

$$\tau_{xy} = -\frac{\sigma_x - \sigma_y}{2}\tan 30° = -\frac{12-2}{2}\tan 30° = -2.89\ \text{MPa}$$

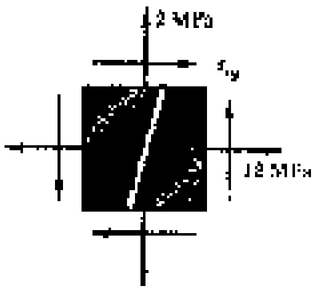
$$\sigma_a = \sigma_{ave} + R = 7 + 5.77 = 12.77\ \text{MPa}$$

$$\sigma_b = \sigma_{ave} - R = 7 - 5.77 = 1.23\ \text{MPa}$$

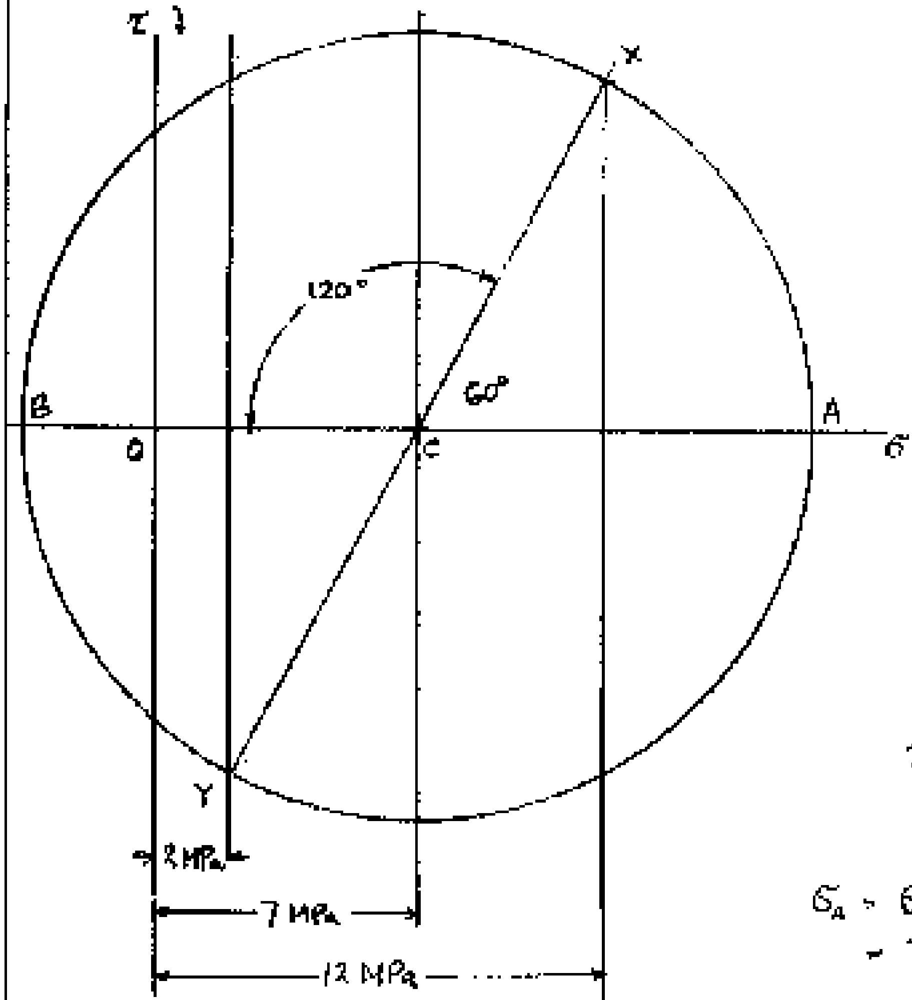
PROBLEM 7.53

7.53 Solve Prob. 7.30, using Mohr's circle and assuming that the weld forms an angle of 60° with the horizontal.

7.30 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.



SOLUTION



Locate point C at $\sigma = \frac{12+2}{2} = 7$ MPa with $\tau = 0$.

Angle XCB = 120°

$$\frac{\sigma_x - \sigma_y}{2} = \frac{12-2}{2} = 5 \text{ MPa}$$

$$R = 5 \sec 60° = 10 \text{ MPa}$$

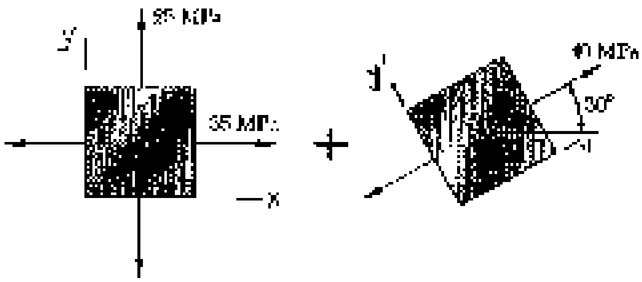
$$\tau_{xy} = -5 \tan 60° = -8.66 \text{ MPa}$$

$$\sigma_A = \sigma_{ave} + R = 7 + 10 = 17 \text{ MPa}$$

$$\sigma_B = \sigma_{ave} - R = 7 - 10 = -3 \text{ MPa}$$

PROBLEM 7.54

7.54 through 7.57 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.



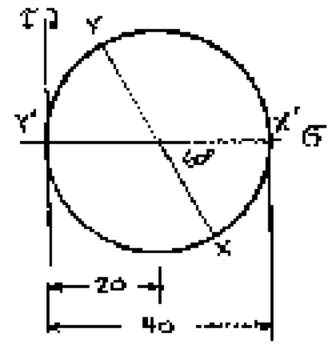
SOLUTION

Mohr's circle for 2nd stress state

$\sigma_x = 20 + 20\cos 60°$
$= 30$ MPa

$\sigma_y = 20 - 20\cos 60°$
$= 10$ MPa

$\tau_{xy} = 20\sin 60°$
$= 17.32$ MPa

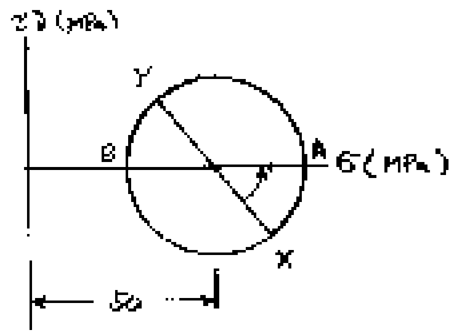


Resultant stresses

$\sigma_x = 35 + 30 = 65$ MPa

$\sigma_y = 25 + 10 = 35$ MPa

$\tau_{xy} = 0 + 17.32 = 17.32$ MPa



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(65 + 35) = 50$ MPa

$\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(17.32)}{65 - 35} = 1.1547$

$2\theta_p = 49.11°$ $\quad \theta_a = 24.6°$ $\quad \theta_b = 114.6°$

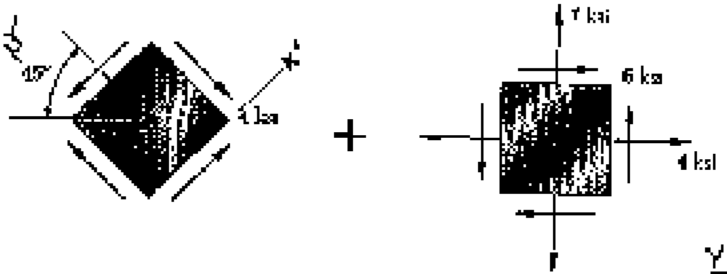
$R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 22.91$ MPa

$\sigma_a = \sigma_{ave} + R = 72.91$ MPa

$\sigma_b = \sigma_{ave} - R = 27.09$ MPa

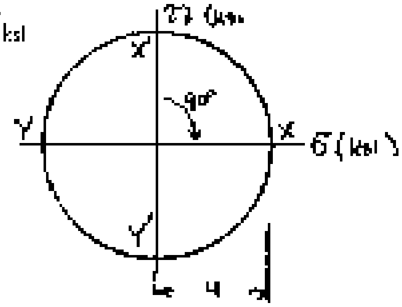
PROBLEM 7.55

7.54 through 7.57 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.



SOLUTION

Mohr's circle for 1st stress state.



$\sigma_x = 4$ ksi

$\sigma_y = -4$ ksi

$\tau_{xy} = 0$

Resultant stresses

$\sigma_x = 4 + 4 = 8$ ksi

$\sigma_y = -4 + 7 = 3$ ksi

$\tau_{xy} = 6 + 0 = 6$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 5.5$ ksi

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(6)}{5} = 2.4$

$2\theta_p = 67.38°$ $\quad \theta_a = 33.69°$

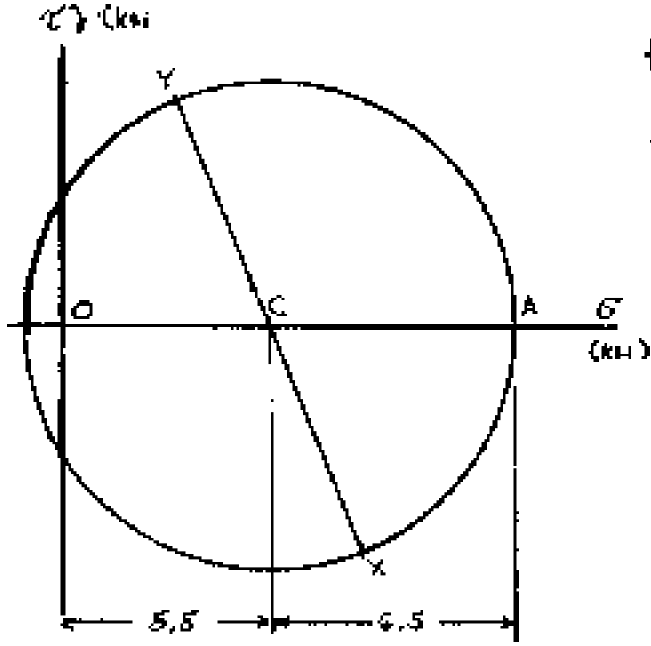
$\theta_b = 123.69°$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{2.5^2 + 6^2} = 6.5$ ksi

$\sigma_a = \sigma_{ave} + R = 12$ ksi

$\sigma_b = \sigma_{ave} - R = -1$ ksi

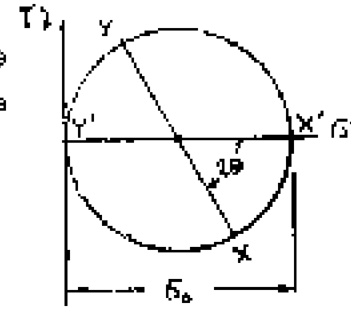


PROBLEM 7.56

7.54 through 7.57 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.

SOLUTION

Mohr's circle for 2nd stress state

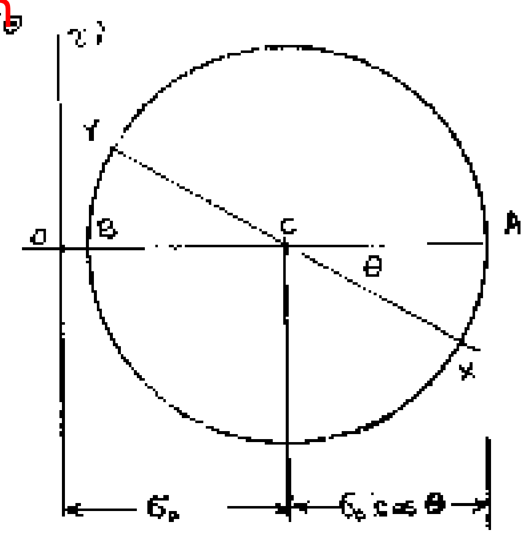


$$\sigma_x = \tfrac{1}{2}\sigma_0 + \tfrac{1}{2}\sigma_0\cos 2\theta$$
$$\sigma_y = \tfrac{1}{2}\sigma_0 - \tfrac{1}{2}\sigma_0\cos 2\theta$$
$$\tau_{xy} = \tfrac{1}{2}\sigma_0\sin 2\theta$$

Resultant stresses

$$\sigma_x = \sigma_0 + \tfrac{1}{2}\sigma_0 + \tfrac{1}{2}\sigma_0\cos 2\theta = \tfrac{3}{2}\sigma_0 + \tfrac{1}{2}\sigma_0\cos 2\theta$$
$$\sigma_y = 0 + \tfrac{1}{2}\sigma_0 - \tfrac{1}{2}\sigma_0\cos 2\theta = \tfrac{1}{2}\sigma_0 - \tfrac{1}{2}\sigma_0\cos 2\theta$$

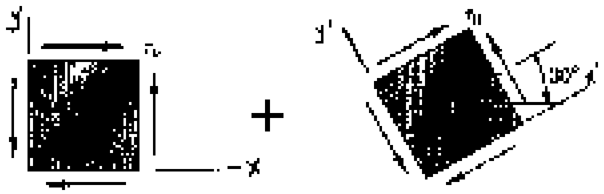
$$\tau_{xy} = 0 + \tfrac{1}{2}\sigma_0\sin 2\theta = \tfrac{1}{2}\sigma_0\sin 2\theta$$
$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = \sigma_0$$
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{\sigma_0\sin 2\theta}{\sigma_0 + \sigma_0\cos 2\theta}$$
$$= \frac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$$
$$\theta_p = \tfrac{1}{2}\theta$$
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} =$$
$$= \sqrt{\left(\tfrac{1}{2}\sigma_0 + \tfrac{1}{2}\sigma_0\cos 2\theta\right)^2 + \left(\tfrac{1}{2}\sigma_0\sin 2\theta\right)^2}$$
$$= \tfrac{1}{2}\sigma_0\sqrt{1 + 2\cos 2\theta + \cos^2 2\theta + \sin^2 2\theta}$$
$$= \frac{\sqrt{2}}{2}\sigma_0\sqrt{1 + \cos 2\theta} = \sigma_0|\cos\theta|$$



$$\sigma_a = \sigma_{ave} + R = \sigma_0 + \sigma_0\cos\theta \quad \blacktriangleleft$$
$$\sigma_b = \sigma_{ave} - R = \sigma_0 - \sigma_0\cos\theta \quad \blacktriangleleft$$

PROBLEM 7.57

7.54 through 7.57 Determine the principal planes and the principal stresses for the state of plane stress resulting from the superposition of the two states of stress shown.



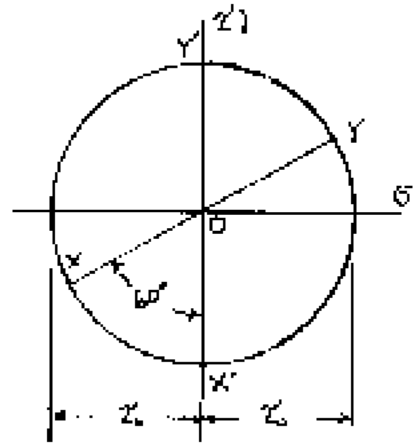
SOLUTION

Mohr's circle for 2nd state of stress

$\sigma_x' = 0$

$\sigma_y' = 0$

$\tau_{x'y'} = \tau_0$



$\sigma_x = -\tau_0 \sin 60° = -\frac{\sqrt{3}}{2}\tau_0$

$\sigma_y = \tau_0 \sin 60° = \frac{\sqrt{3}}{2}\tau_0$

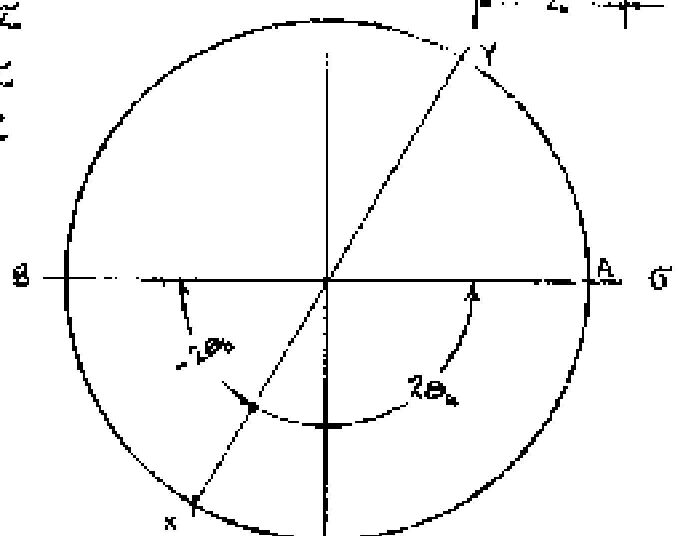
$\tau_{xy} = \tau_0 \cos 60° = \frac{1}{2}\tau_0$

Resultant stresses

$\sigma_x = 0 - \frac{\sqrt{3}}{2}\tau_0 = -\frac{\sqrt{3}}{2}\tau_0$

$\sigma_y = 0 + \frac{\sqrt{3}}{2}\tau_0 = \frac{\sqrt{3}}{2}\tau_0$

$\tau_{xy} = \tau_0 + \frac{1}{2}\tau_0 = \frac{3}{2}\tau_0$



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\frac{\sqrt{3}}{2}\tau_0\right)^2 + \left(\frac{3}{2}\tau_0\right)^2}$

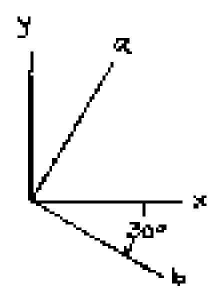
$= \sqrt{3}\,\tau_0$

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\cdot\frac{3}{2}}{-\sqrt{3}} = -\sqrt{3}$

$2\theta_p = -60°$ $\quad\theta_b = -30°$ $\quad\theta_a = 60°$ ◄

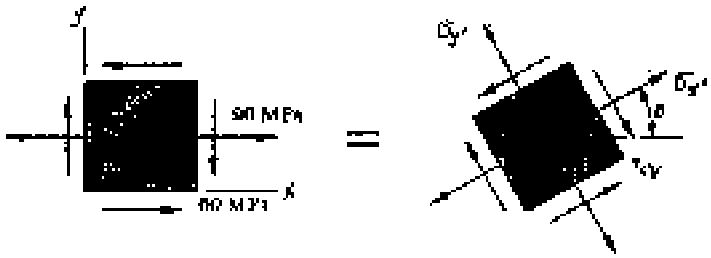
$\sigma_a = \sigma_{ave} + R = \sqrt{3}\,\tau_0$ ◄

$\sigma_b = \sigma_{ave} - R = -\sqrt{3}\,\tau_0$ ◄



PROBLEM 7.58

7.58 For the state of stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than 100 MPa.



SOLUTION

$\sigma_x = 90$ MPa , $\sigma_y = 0$

$\tau_{xy} = -60$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 45$ MPa

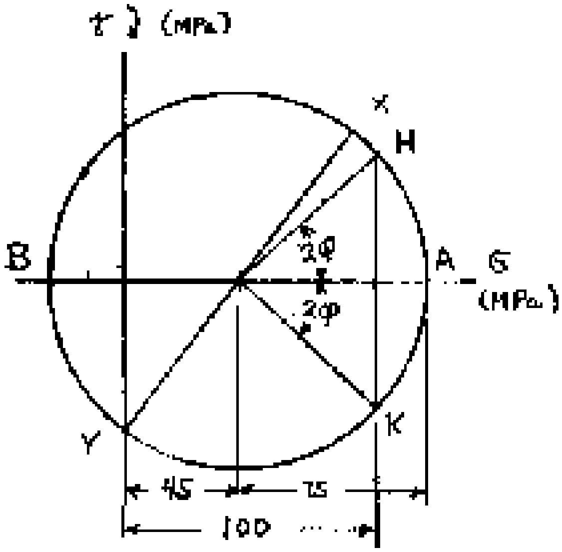
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-60)}{90} = -\frac{4}{3}$$

$2\theta_p = -53.13°$

$\theta_a = -26.565°$



$\sigma_{x'} \le 100$ MPa for states of stress corresponding to arc HBK of Mohr's circle. From the circle

$R \cos 2\varphi = 100 - 45 = 55$ MPa

$$\cos 2\varphi = \frac{55}{75} = 0.73333$$

$2\varphi = 42.833°$ $\qquad \varphi = 21.417°$

$\theta_H = \theta_a + \varphi = -26.565° + 21.417° = -5.15°$

$2\theta_K = 2\theta_H + 360° - 4\varphi = -10.297° + 360° - 85.666° = 264.037°$

$\theta_K = 132.02°$

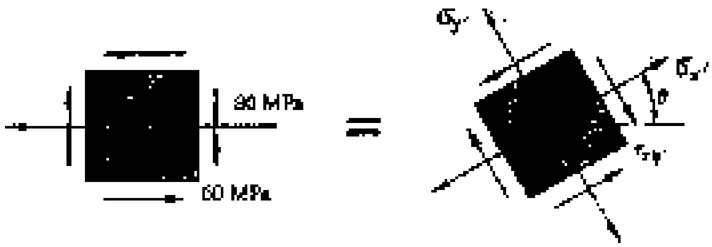
Permissible range of θ is $\theta_H \le \theta \le \theta_K$

$$-5.15° \le \theta \le 132.02°$$

Also $174.85° \le \theta \le 312.02°$

PROBLEM 7.59

7.59 For the state of stress shown, determine the range of values of θ for which the normal stress $\sigma_{x'}$ is equal to or less than 50 MPa.



SOLUTION

$\sigma_x = 90$ MPa, $\sigma_y = 0$

$\tau_{xy} = -60$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 45$ MPa

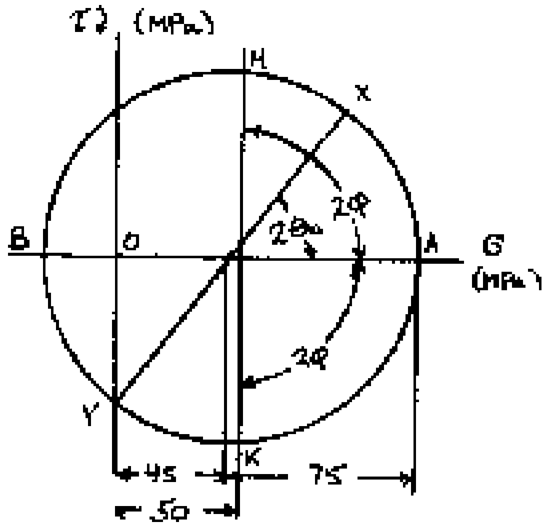
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{45^2 + 60^2} = 75$ MPa

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-60)}{90} = -\frac{4}{3}$

$2\theta_p = -53.13°$

$\theta_a = -26.565°$



$\sigma_{x'} \le 50$ MPa for states of stress corresponding to the arc HBK of Mohr's circle. From the circle

$R \cos 2\varphi = 50 - 45 = 5$ MPa

$\cos 2\varphi = \frac{5}{75} = 0.066667$

$2\varphi = 86.177°$ $\qquad \varphi = 43.089°$

$\theta_H = \theta_a + \varphi = -26.565° + 43.089° = 16.524°$

$2\theta_K = 2\theta_H + 360° - 4\varphi = 32.524° + 360° - 172.355° = 220.169°$

$\theta_K = 110.085°$

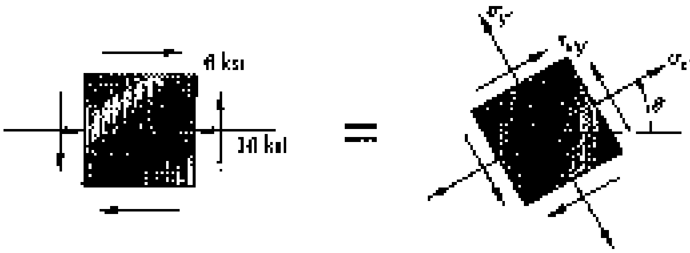
Permissible range of θ is $\theta_H \le \theta \le \theta_K$

$16.524° \le \theta \le 110.085°$

Also $\quad 196.524° \le \theta \le 290.085°$

PROBLEM 7.60

7.60 For the state of stress shown, determine the range of values of θ for which the magnitude of the shearing stress $\tau_{x'y'}$ is equal to or less than 8 ksi.



SOLUTION

$\sigma_x = -16$ ksi, $\sigma_y = 0$

$\tau_{xy} = 6$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -8$ ksi

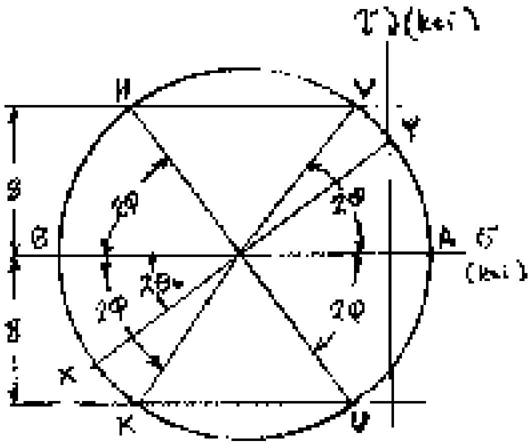
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{(-8)^2 + (6)^2} = 10$ ksi

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(6)}{-16} = -0.75$

$2\theta_p = -36.870°$

$\theta_b = -18.435°$



$|\tau_{xy}| \leq 8$ ksi for states of stress corresponding to arcs HBK and UAV of Mohr's circle. The angle φ is calculated from

$R \sin 2\varphi = 8$

$\sin 2\varphi = \frac{8}{10} = 0.8$

$2\varphi = 53.130°$ $\qquad \varphi = 26.565°$

$\theta_H = \theta_b - \varphi = -18.435° - 26.565° = -45°$

$\theta_K = \theta_b + \varphi = -18.435 + 26.565° = 8.13°$

$\theta_U = \theta_H + 90° = 45°$

$\theta_V = \theta_K + 90° = 98.13°$

Permissible ranges of θ $\qquad \theta_H \leq \theta \leq \theta_K$

$-45° \leq \theta \leq 8.13°$

$\theta_U \leq \theta \leq \theta_V$

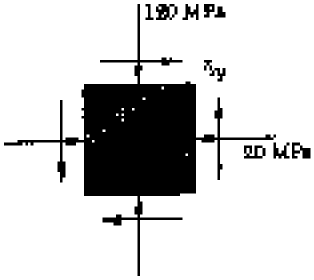
$45° \leq \theta \leq 98.13°$

Also $\qquad 135° \leq \theta \leq 188.13°$

$225° \leq \theta \leq 278.13°$

PROBLEM 7.61

7.61 For the element shown, determine the range of values of τ_{xy} for which the maximum tensile stress is equal to or less than 60 MPa.



SOLUTION

$\sigma_x = -20$ MPa $\quad \sigma_y = -120$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -70$ MPa

Set $\sigma_{max} = 60$ MPa $= \sigma_{ave} + R$

$R = \sigma_{max} - \sigma_{ave} = 130$ MPa

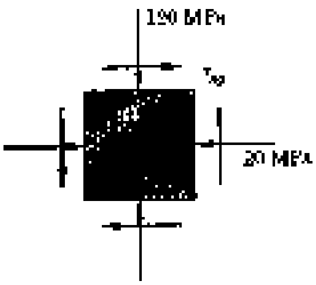
But $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{130^2 - 50^2} = 120$ MPa

Range of τ_{xy} $\quad -120 \text{ MPa} \leq \tau_{xy} \leq 120 \text{ MPa}$ ◄

PROBLEM 7.62

7.62 For the element shown, determine the range of values of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 150 MPa.



SOLUTION

$\sigma_x = -20$ MPa $\quad \sigma_y = -120$ MPa

$\frac{1}{2}(\sigma_x - \sigma_y) = 50$ MPa

Set $\tau_{max(in\text{-}plane)} = R = 150$ MPa

But $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$|\tau_{xy}| = \sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{150^2 - 50^2} = 141.4$ MPa

Range of τ_{xy} $\quad -141.4 \text{ MPa} \leq \tau_{xy} \leq 141.4 \text{ MPa}$ ◄

PROBLEM 7.63

7.63 For the state of stress shown it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 14$ ksi, $\sigma_y = 9$ ksi, and $\sigma_{min} = 5$ ksi. Determine (*a*) the orientation of the principal planes, (*b*) the principal stress σ_{max}, (*c*) the maximum in-plane shearing stress.

SOLUTION

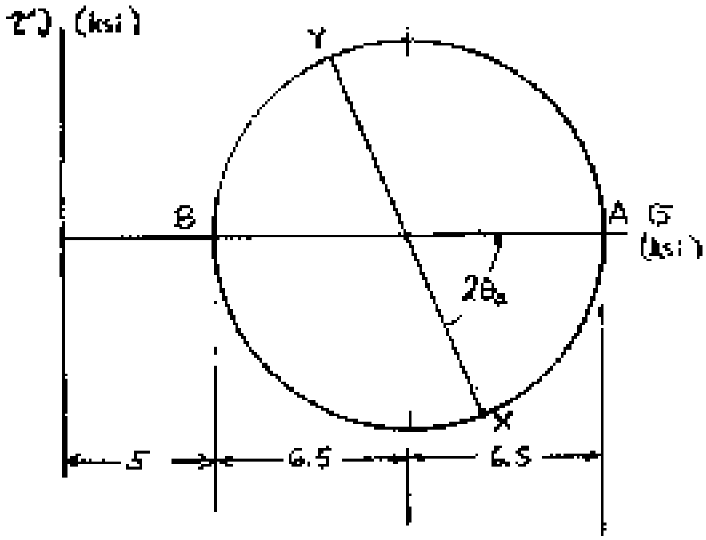
$\sigma_x = 14$ ksi, $\sigma_y = 9$ ksi, $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 11.5$ ksi

$\sigma_{min} = \sigma_{ave} - R \quad \therefore \quad R = \sigma_{ave} - \sigma_{min} = 11.5 - 5 = 6.5$ ksi

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm\sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \pm\sqrt{6.5^2 - 2.5^2} = \pm 6 \text{ ksi}$$

But it is given that τ_{xy} is positive, thus $\tau_{xy} = +6$ ksi



(a) $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(6)}{5} = 2.4$

$2\theta_p = 67.38°$

$\theta_a = 33.69°$

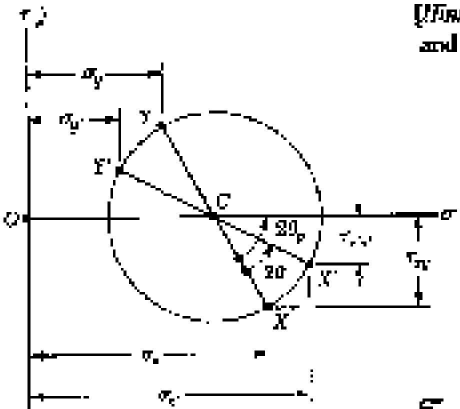
$\theta_b = 123.69°$

(b) $\sigma_{max} = \sigma_{ave} + R = 18$ ksi

(c) $\tau_{max(in\text{-}plane)} = R = 6.5$ ksi

PROBLEM 7.64

7.64 The Mohr circle shown corresponds to the state of stress given in Fig. xxa and b, page yyy. Noting that $\sigma_{x'} = \overline{OC} - (\overline{CX'})\cos(2\theta_p - 2\theta)$ and that $\tau_{x'y'} = (\overline{CX'})\sin(2\theta_p - 2\theta)$, derive the expressions for $\sigma_{x'}$ and $\tau_{x'y'}$ given in Eqs. (7.6) and (7.x), respectively. [*Hint:* Use $\sin(A - B) = \sin A\cos B - \cos A\sin B$ and $\cos(A + B) = \cos A\cos B + \sin A\sin B$.]



SOLUTION

$$\overline{OC} = \tfrac{1}{2}(\sigma_x + \sigma_y) \qquad \overline{CX'} = \overline{CX}$$

$$\overline{CX'}\cos 2\theta_p = \overline{CX}\cos 2\theta_p = \frac{\sigma_x - \sigma_y}{2}$$

$$\overline{CX'}\sin 2\theta_p = \overline{CX}\sin 2\theta_p = \tau_{xy}$$

$$\sigma_{x'} = \overline{OC} + \overline{CX'}\cos(2\theta_p - 2\theta)$$

$$= \overline{OC} + \overline{CX'}(\cos 2\theta_p\cos 2\theta + \sin 2\theta_p\sin 2\theta)$$

$$= \overline{OC} + \overline{CX'}\cos 2\theta_p\cos 2\theta + \overline{CX'}\sin 2\theta_p\sin 2\theta$$

$$= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = \overline{CX'}\sin(2\theta_p - 2\theta) = \overline{CX'}(\sin 2\theta_p\cos 2\theta - \cos 2\theta_p\sin 2\theta)$$

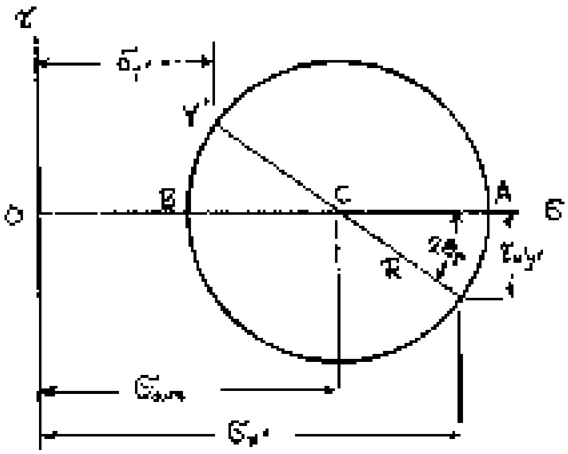
$$= \overline{CX'}\sin 2\theta_p\cos 2\theta - \overline{CX'}\cos 2\theta_p\sin 2\theta$$

$$= \tau_{xy}\cos 2\theta - \frac{\sigma_x - \sigma_y}{2}\sin 2\theta$$

PROBLEM 7.65

7.65 (a) Prove that the expression $\sigma_{x'}\sigma_{y'} - \tau^2_{x'y'}$, where $\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$ are components of stress along the rectangular axes x' and y', is independent of the orientation of these axes. Also, show that the given expression represents the square of the tangent drawn from the origin of the coordinates to Mohr's circle. (b) Using the invariance property established in part a, express the shearing stress τ_{xy} in terms of σ_x, σ_y, and the principal stresses σ_{max} and σ_{min}.

SOLUTION



(a) From Mohr's circle

$\tau_{x'y'} = R \sin 2\theta_p$

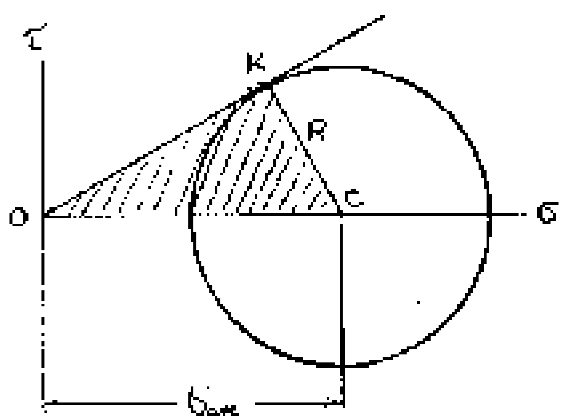
$\sigma_{x'} = \sigma_{ave} + R \cos 2\theta_p$

$\sigma_{y'} = \sigma_{ave} - R \cos 2\theta_p$

$\sigma_{x'}\sigma_{y'} - \tau_{x'y'}^2$

$= \sigma_{ave}^2 - R^2\cos^2 2\theta_p - R^2 \sin^2 2\theta_p$

$= \sigma_{ave}^2 - R^2$; independent of θ_p.



Draw line $\overline{OK}$ from origin tangent to the circle at K. Triangle OCK is a right triangle.

$\overline{OC}^2 = \overline{OK}^2 + \overline{CK}^2$

$\overline{OK}^2 = \overline{OC}^2 - \overline{CK}^2$

$= \sigma_{ave}^2 - R^2$

$= \sigma_{x'}\sigma_{y'} - \tau_{x'y'}$

(b) Applying above to σ_x, σ_y, and τ_{xy} and to σ_a, σ_b,

$$\sigma_x\sigma_y - \tau_{xy}^2 = \sigma_a\sigma_b - \tau_{ab}^2 = \sigma_{ave}^2 - R^2$$

But $\tau_{ab} = 0$, $\sigma_a = \sigma_{max}$, $\sigma_b = \sigma_{min}$

$$\sigma_x\sigma_y - \tau_{xy}^2 = \sigma_{max}\sigma_{min}$$

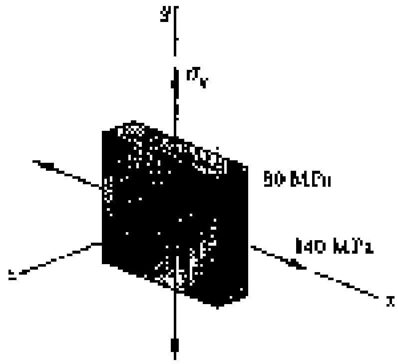
$$\tau_{xy}^2 = \sigma_x\sigma_y - \sigma_{max}\sigma_{min}$$

$$\tau_{xy} = \pm\sqrt{\sigma_x\sigma_y - \sigma_{max}\sigma_{min}}$$

The sign cannot be determined from above equations.

PROBLEM 7.66

7.66 For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 20$ MPa, (*b*) $\sigma_y = 140$ MPa. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)



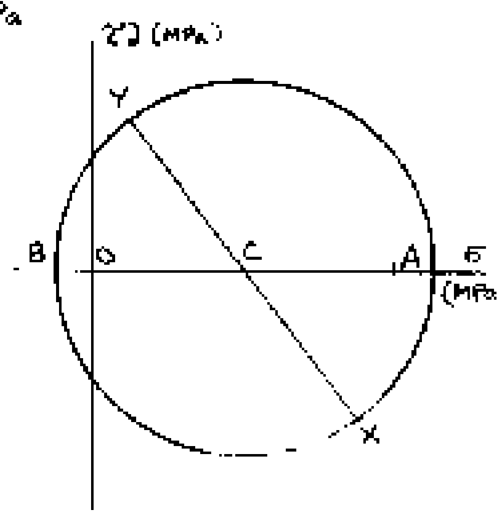
SOLUTION

(a) $\sigma_x = 140$ MPa, $\sigma_y = 20$ MPa

$\tau_{xy} = 80$ MPa

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 80 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{60^2 + 80^2} = 100 \text{ MPa}$$



$\sigma_a = \sigma_{ave} + R = 80 + 100 = 180$ MPa (max)

$\sigma_b = \sigma_{ave} - R = 80 - 100 = -20$ MPa (min)

$\sigma_c = 0$

$\tau_{max(in\text{-}plane)} = \tfrac{1}{2}(\sigma_a - \sigma_b) = 100$ MPa

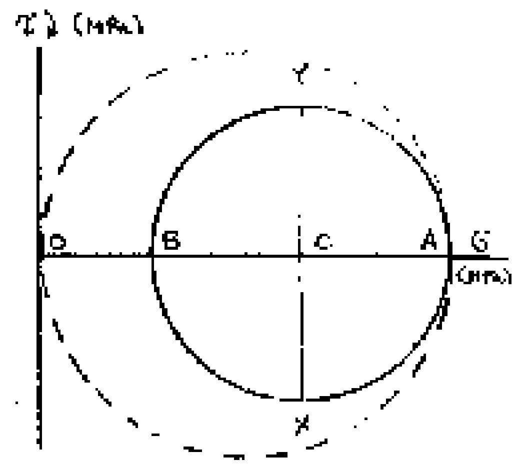
$\tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = 100$ MPa ◀

(b) $\sigma_x = 140$ MPa, $\sigma_y = 140$ MPa

$\tau_{xy} = 80$ MPa

$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 140$ MPa

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + 80^2} = 80 \text{ MPa}$$



$\sigma_a = \sigma_{ave} + R = 220$ MPa (max)

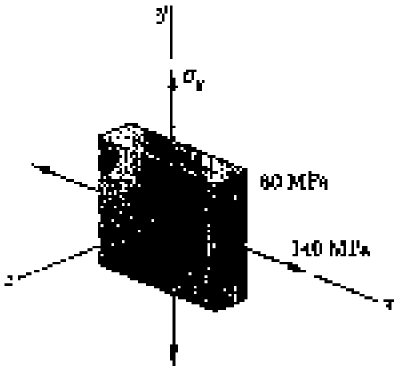
$\sigma_b = \sigma_{ave} - R = 60$ MPa

$\sigma_c = 0$ (min)

$\tau_{max(in\text{-}plane)} = \tfrac{1}{2}(\sigma_a - \sigma_b) = 80$ MPa

$\tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = 110$ MPa ◀

PROBLEM 7.67



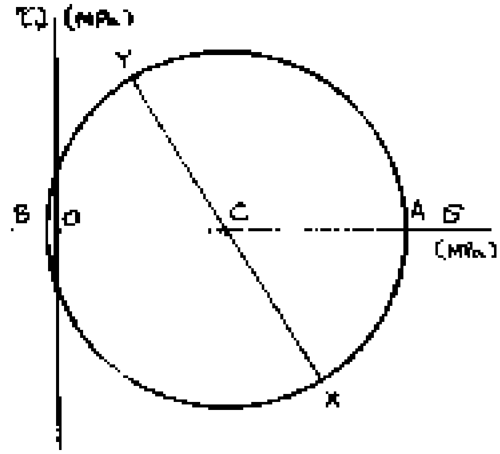
7.67 For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_y = 40$ MPa, (*b*) $\sigma_y = 120$ MPa. (*Hint*: Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(a) $\sigma_x = 140$ MPa $\quad \sigma_y = 40$ MPa $\quad \tau_{xy} = 80$ MPa

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 90 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{50^2 + 80^2} = 94.34 \text{ MPa}$$



$\sigma_a = \sigma_{ave} + R = 184.34$ MPa (max)

$\sigma_b = \sigma_{ave} - R = -4.34$ MPa (min)

$\sigma_c = 0$

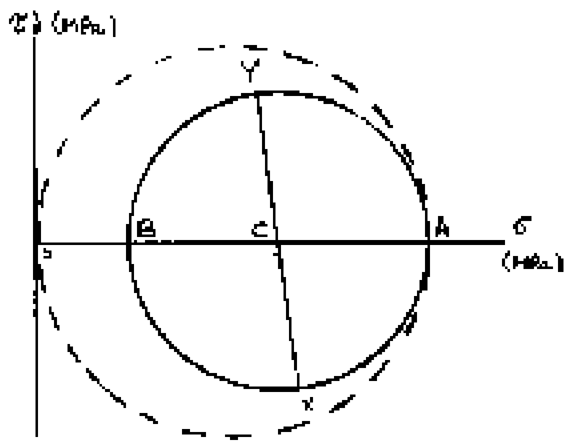
$$\tau_{max(in\text{-}plane)} = \frac{1}{2}(\sigma_a - \sigma_b) = R = 94.34 \text{ MPa}$$

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(\sigma_a - \sigma_b) = 94.34 \text{ MPa}$$

(b) $\sigma_x = 140$ MPa, $\sigma_y = 120$ MPa, $\tau_{xy} = 80$ MPa

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 130 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{10^2 + 80^2} = 80.62 \text{ MPa}$$



$\sigma_a = \sigma_{ave} + R = 210.62$ MPa (max)

$\sigma_b = \sigma_{ave} - R = 49.38$ MPa

$\sigma_c = 0$ (min)

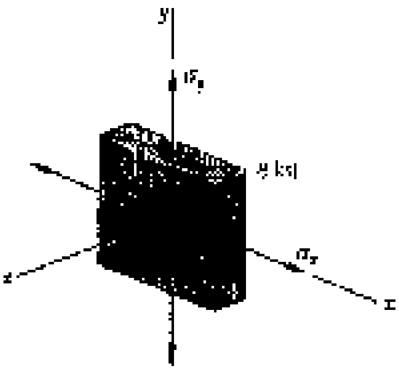
$\sigma_{max} = \sigma_a = 210.62$ MPa

$\sigma_{min} = \sigma_c = 0$

$\tau_{max(in\text{-}plane)} = R = 86.62$ MPa

$$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 105.31 \text{ MPa}$$

PROBLEM 7.68



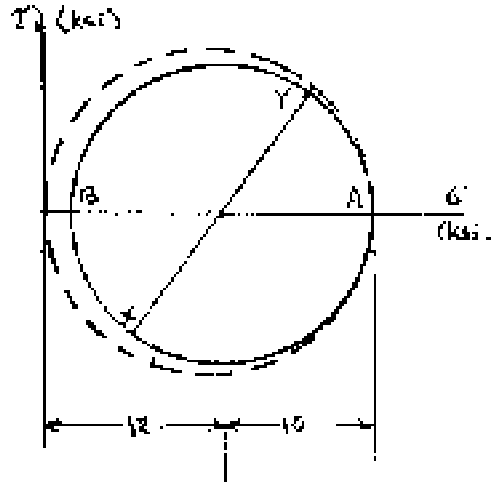
7.68 For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 6$ ksi and $\sigma_y = 18$ ksi, (*b*) $\sigma_x = 14$ ksi and $\sigma_y = 2$ ksi. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)

SOLUTION

(a) $\sigma_x = 6$ ksi $\quad \sigma_y = 18$ ksi $\quad \tau_{xy} = 8$ ksi

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 12 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{6^2 + 8^2} = 10 \text{ ksi}$$



$\sigma_a = \sigma_{ave} + R = 12 + 10 = 22$ ksi (max)

$\sigma_b = \sigma_{ave} - R = 12 - 10 = 2$ ksi

$\sigma_c = 0$ (min)

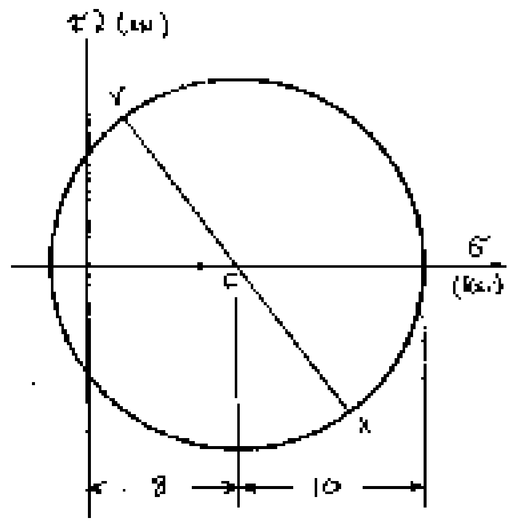
$\tau_{max(in\text{-}plane)} = R = 10$ ksi

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 11$ ksi ◀

(b) $\sigma_x = 14$ ksi $\quad \sigma_y = 2$ ksi $\quad \tau_{xy} = 8$ ksi

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 8 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{6^2 + 8^2} = 10 \text{ ksi}$$



$\sigma_a = \sigma_{ave} + R = 18$ ksi (max)

$\sigma_b = \sigma_{ave} - R = -2$ ksi (min)

$\sigma_c = 0$

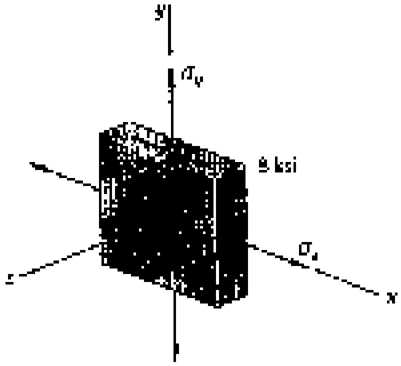
$\sigma_{max} = 18$ ksi

$\sigma_{min} = -2$ ksi

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 10$ ksi ◀

PROBLEM 7.69

7.69 For the state of plane stress shown, determine the maximum shearing stress when (*a*) $\sigma_x = 0$ and $\sigma_y = 12$ ksi, (*b*) $\sigma_x = 21$ ksi and $\sigma_y = 9$ ksi. (*Hint:* Consider both in-plane and out-of-plane shearing stresses.)



SOLUTION

(a) $\sigma_x = 0$, $\sigma_y = 12$ ksi, $\tau_{xy} = 8$ ksi

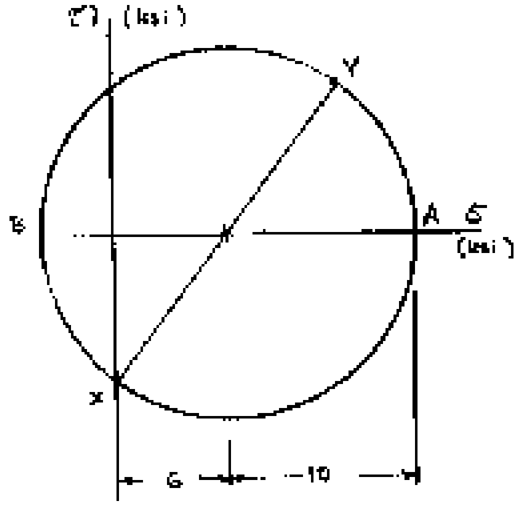
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$

$= 6$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{(-6)^2 + 8^2}$

$= 10$ ksi



$\sigma_a = \sigma_{ave} + R = 16$ ksi (max)

$\sigma_b = \sigma_{ave} - R = -4$ ksi (min)

$\sigma_c = 0$

$\sigma_{max} = 16$ ksi $\qquad \sigma_{min} = -4$ ksi

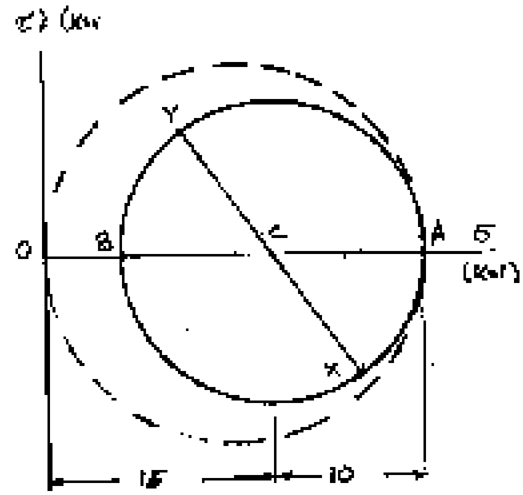
$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 10$ ksi ◄

(b) $\sigma_x = 21$ ksi $\quad \sigma_y = 9$ ksi $\quad \tau_{xy} = 8$ ksi

$\sigma_{ave} = 15$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{(-6)^2 + 8^2} = 10$ ksi



$\sigma_a = \sigma_{ave} + R = 25$ ksi (max)

$\sigma_b = \sigma_{ave} - R = 5$ ksi

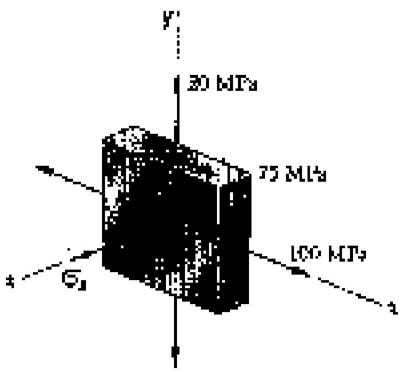
$\sigma_c = 0$ (min)

$\sigma_{max} = 25$ ksi, $\quad \sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 12.5$ ksi ◄

PROBLEM 7.70

7.70 and 7.71 For the state of stress shown, determine the maximum shearing stress when (*a*) $\sigma_z = 0$, (*b*) $\sigma_z = +45$ MPa, (*c*) $\sigma_z = -45$ MPa.



SOLUTION

$\sigma_x = 100$ MPa, $\sigma_y = 20$ MPa, $\tau_{xy} = 75$ MPa

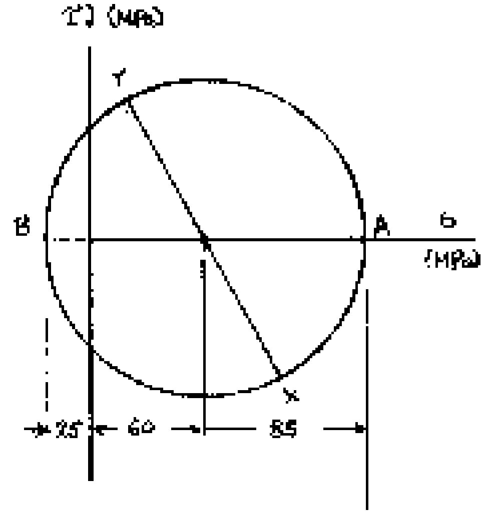
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$

$= 60$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{40^2 + 75^2}$

$= 85$ MPa



$\sigma_a = \sigma_{ave} + R = 145$ MPa

$\sigma_b = \sigma_{ave} - R = -25$ MPa

(a) $\sigma_z = 0$, $\sigma_a = 145$ MPa, $\sigma_b = -25$ MPa

$\sigma_{max} = 145$ MPa, $\sigma_{min} = -25$ MPa, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 85$ MPa ◄

(b) $\sigma_z = +45$ MPa, $\sigma_a = 145$ MPa, $\sigma_b = -25$ MPa

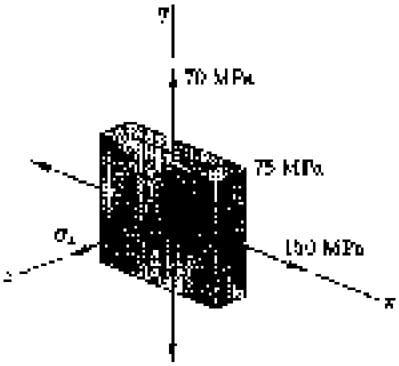
$\sigma_{max} = 145$ MPa, $\sigma_{min} = -25$ MPa, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 85$ MPa ◄

(c) $\sigma_z = -45$ MPa, $\sigma_a = 145$ MPa, $\sigma_b = -25$ MPa

$\sigma_{max} = 145$ MPa, $\sigma_{min} = -45$ MPa $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 95$ MPa ◄

PROBLEM 7.71

7.70 and 7.71 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_z = 0$, (b) $\sigma_z = +45$ MPa, (c) $\sigma_z = -45$ MPa.



SOLUTION

$\sigma_x = 150$ MPa, $\sigma_y = 70$ MPa, $\tau_{xy} = 75$ MPa

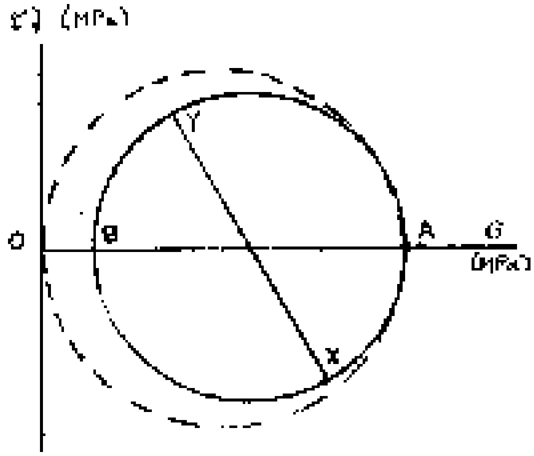
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$

$= 110$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{40^2 + 75^2}$

$= 85$ MPa



$\sigma_a = \sigma_{ave} + R = 195$ MPa

$\sigma_b = \sigma_{ave} - R = 25$ MPa

(a) $\sigma_z = 0$, $\sigma_a = 195$ MPa, $\sigma_b = 25$ MPa

$\sigma_{max} = 195$ MPa, $\sigma_{min} = 0$, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 97.5$ MPa ◄

(b) $\sigma_z = +45$ MPa, $\sigma_a = 195$ MPa, $\sigma_b = 25$ MPa

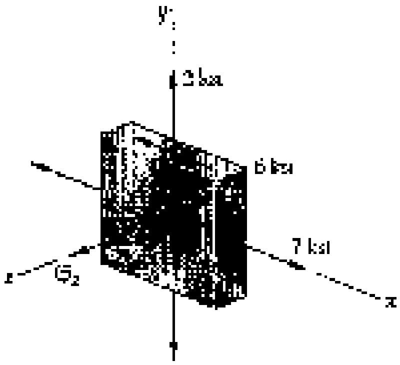
$\sigma_{max} = 195$ MPa, $\sigma_{min} = 25$ MPa, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 85$ MPa ◄

(c) $\sigma_z = -45$ MPa, $\sigma_a = 195$ MPa, $\sigma_b = 25$ MPa

$\sigma_{max} = 195$ MPa, $\sigma_{min} = -45$ MPa, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 120$ MPa ◄

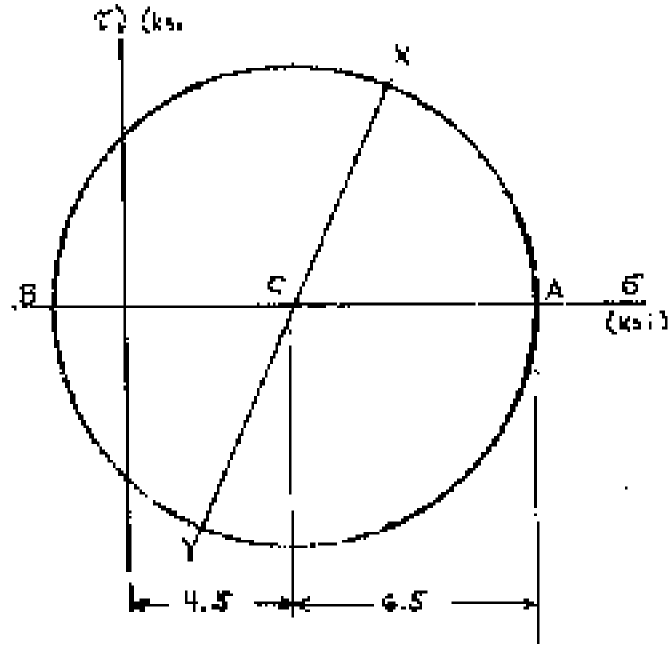
PROBLEM 7.72

7.72 and 7.73 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_z = +4$ ksi, (b) $\sigma_z = -4$ ksi, (c) $\sigma_z = 0$.



SOLUTION

$\sigma_x = 7$ ksi, $\sigma_y = 2$ ksi, $\tau_{xy} = -6$ ksi



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 4.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{2.5^2 + (-6)^2} = 6.5 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 11 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = -2 \text{ ksi}$$

(a) $\sigma_z = 4$ ksi, $\sigma_a = 11$ ksi, $\sigma_b = -2$ ksi

$\sigma_{max} = 11$ ksi, $\sigma_{min} = -2$ ksi, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.5$ ksi ◄

(b) $\sigma_z = -4$ ksi, $\sigma_a = 11$ ksi, $\sigma_b = -2$ ksi

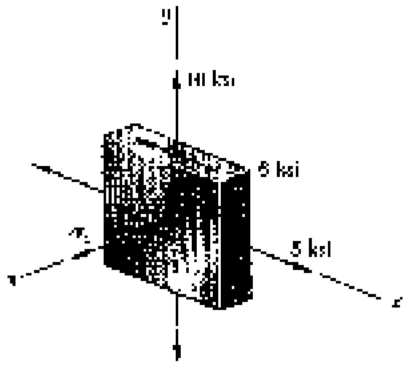
$\sigma_{max} = 11$ ksi, $\sigma_{min} = -4$ ksi, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 7.5$ ksi ◄

(c) $\sigma_z = 0$, $\sigma_a = 11$ ksi, $\sigma_b = -2$ ksi

$\sigma_{max} = 11$ ksi, $\sigma_{min} = -2$ ksi, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.5$ ksi ◄

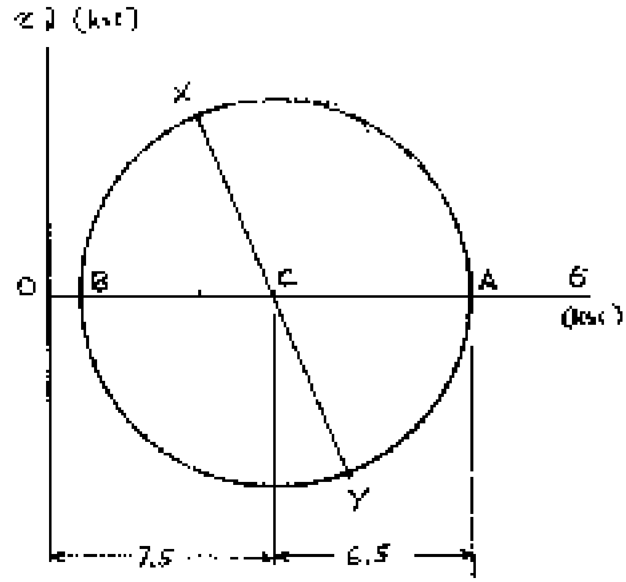
PROBLEM 7.73

7.72 and 7.73 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_z = +4$ ksi, (b) $\sigma_z = -4$ ksi, (c) $\sigma_z = 0$.



SOLUTION

$\sigma_x = 5$ ksi, $\sigma_y = 10$ ksi, $\tau_{xy} = -6$ ksi



$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 7.5 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{(-2.5)^2 + (-6)^2} = 6.5 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = 14 \text{ ksi}$$

$$\sigma_b = \sigma_{ave} - R = 1 \text{ ksi}$$

(a) $\sigma_z = +4$ ksi, $\quad \sigma_a = 14$ ksi, $\quad \sigma_b = 1$ ksi

$\sigma_{max} = 14$ ksi, $\quad \sigma_{min} = 1$ ksi, $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.5$ ksi ◄

(b) $\sigma_z = -4$ ksi, $\quad \sigma_a = 14$ ksi, $\quad \sigma_b = 1$ ksi

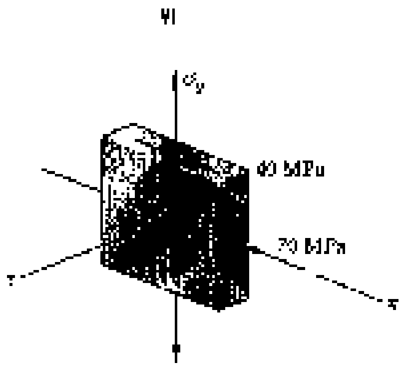
$\sigma_{max} = 14$ ksi, $\quad \sigma_{min} = -4$ ksi, $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 9$ ksi ◄

(c) $\sigma_z = 0$, $\quad \sigma_a = 14$ ksi, $\quad \sigma_b = 1$ ksi

$\sigma_{max} = 14$ ksi, $\quad \sigma_{min} = 0$, $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 7$ ksi ◄

PROBLEM 7.74

7.74 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 75 MPa.



SOLUTION

$\sigma_x = -70$ MPa, $\tau_{xy} = 40$ MPa

Let $u = \dfrac{\sigma_y - \sigma_x}{2}$ $\qquad \sigma_y = 2u + \sigma_x$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$

$R = \sqrt{u^2 + \tau_{xy}^2}$ $\qquad u = \pm\sqrt{R^2 - \tau_{xy}^2}$

Case 1 $\quad \tau_{max} = R = 75$ MPa, $\quad u = \pm\sqrt{75^2 - 40^2} = \pm 63.44$ MPa

(1a) $u = +63.44$ MPa $\qquad \sigma_y = 2u + \sigma_x = 56.88$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.56$ MPa

$\sigma_a = \sigma_{ave} + R = 68.44$ MPa, $\quad \sigma_b = \sigma_{ave} - R = -81.56$ MPa

$\sigma_c = 0 \qquad \sigma_{max} = 68.44$ MPa, $\quad \sigma_{min} = -81.56$ MPa $\qquad \tau_{max} = 75$ MPa

(1b) $u = -63.44$ MPa $\qquad \sigma_y = 2u + \sigma_x = -196.88$ MPa (reject)

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -133.44$ MPa $\qquad \sigma_a = \sigma_{ave} + R = -58.44$ MPa

$\sigma_b = \sigma_{ave} - R = -208.44$ MPa, $\quad \sigma_c = 0$, $\quad \sigma_{max} = 0$

$\sigma_{min} = -208.44$ MPa, $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 104.22$ MPa $\neq 75$ MPa

Case (2) $\quad$ Assume $\sigma_{max} = 0$, $\quad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 75$ MPa

$\sigma_{min} = -150$ MPa $= \sigma_b$

$\sigma_b = \sigma_{ave} - R = \sigma_x + u - \sqrt{u^2 + \tau_{xy}^2}$

$\sqrt{u^2 + \tau_{xy}^2} = \sigma_x + u - \sigma_b$

$\cancel{u^2} + \tau_{xy}^2 = (\sigma_x - \sigma_b)^2 + 2(\sigma_x - \sigma_b)u + \cancel{u^2}$

$$2u = \frac{\tau_{xy}^2 - (\sigma_x - \sigma_b)^2}{\sigma_x - \sigma_b} \qquad \frac{(40)^2 - (-70 + 150)^2}{-70 + 150} = -60 \text{ MPa}$$

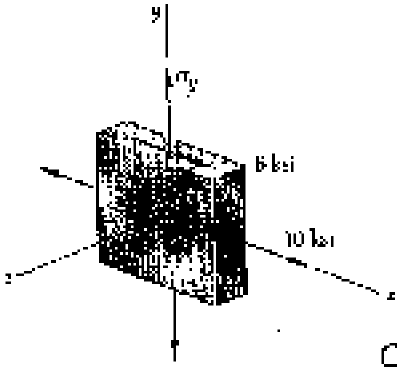
$u = -30$ MPa $\qquad \sigma_y = 2u + \sigma_x = -130$ MPa

$R = \sqrt{u^2 + \tau_{xy}^2} = 50$ MPa

$\sigma_a = \sigma_b + 2R = -150 + 100 = -50$ MPa $\qquad$ O.K.

PROBLEM 7.75

7.75 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 7.5 ksi.



SOLUTION

$\sigma_x = 10$ ksi, $\tau_{xy} = 6$ ksi, $\tau_{max} = 7.5$ ksi

Let $u = \dfrac{\sigma_y - \sigma_x}{2}$ $\qquad \sigma_y = 2u + \sigma_x$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \sigma_x + u$

$R = \sqrt{u^2 + \tau_{xy}^2}$ $\qquad u = \pm\sqrt{R^2 - \tau_{xy}^2}$

Case 1 $\quad \tau_{max} = R = 7.5$ ksi, $u = \pm 4.5$ ksi

(1a) $u = +4.5$ ksi $\qquad \sigma_y = 2u + \sigma_x = 19$ ksi $\qquad$ reject

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 14.5$ ksi, $\sigma_a = \sigma_{ave} + R = 22$ ksi, $\sigma_b = \sigma_{ave} - R = 7$ ksi

$\sigma_{max} = 22$ ksi, $\sigma_{min} = 0$, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 11$ ksi $\neq 7.5$ ksi

(1b) $u = -4.5$ ksi $\qquad \sigma_y = 2u + \sigma_x = 1$ ksi ◄

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 5.5$ ksi, $\sigma_a = \sigma_{ave} + R = 13$ ksi, $\sigma_b = \sigma_{ave} - R = -2$ ksi

$\sigma_{max} = 13$ ksi, $\sigma_{min} = -2$ ksi, $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 7.5$ ksi $\quad$ OK.

Case 2 $\quad$ Assume $\sigma_{min} = 0 \qquad \sigma_{max} = 2\tau_{max} = 15$ ksi $= \sigma_a$

$\sigma_a = \sigma_{ave} + R = \sigma_x + u + \sqrt{u^2 + \tau_{xy}^2}$

$\sigma_a - \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$

$(\sigma_a - \sigma_x - u)^2 = u^2 + \tau_{xy}^2$

$(\sigma_a - \sigma_x)^2 - 2(\sigma_a - \sigma_x)u + \cancel{u^2} = \cancel{u^2} + \tau_{xy}^2$

$2u = \dfrac{(\sigma_a - \sigma_x)^2 - \tau_{xy}^2}{\sigma_a - \sigma_x} = \dfrac{(15 - 10)^2 - 6^2}{15 - 10} = -2.2$ ksi

$u = -1.1$ ksi $\qquad \sigma_y = 2u + \sigma_x = 7.8$ ksi ◄

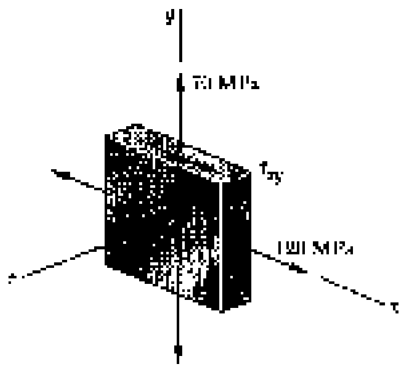
$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 8.9$ ksi $\qquad R = \sqrt{u^2 + \tau_{xy}^2} = 6.1$ ksi

$\sigma_a = \sigma_{ave} + R = 15$ ksi ✓ $\qquad \sigma_b = \sigma_{ave} - R = 2.8$ ksi

$\sigma_{max} = 15$ ksi, $\sigma_{min} = 0 \qquad \tau_{max} = 7.5$ ksi ✓

PROBLEM 7.76

7.76 For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is 80 MPa.



SOLUTION

$$\sigma_x = 120 \text{ MPa} \qquad \sigma_y = 70 \text{ MPa}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 95 \text{ MPa}$$

$$\frac{\sigma_x - \sigma_y}{2} = \frac{120 - 70}{2} = 25 \text{ MPa}$$

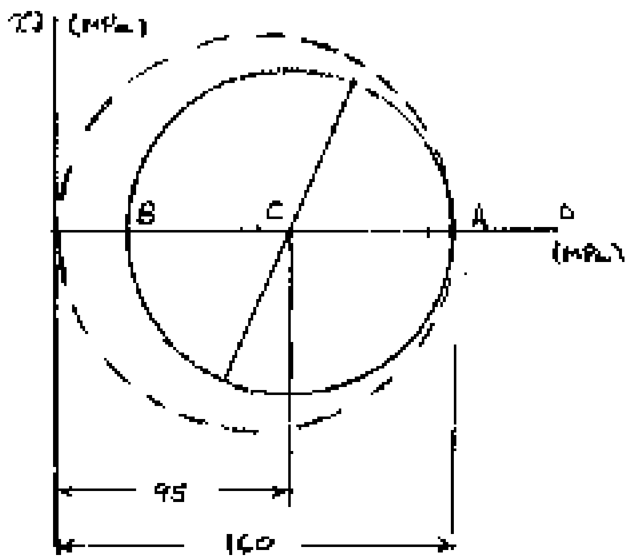
$$\text{Assume} \quad \sigma_{min} = 0 \qquad \sigma_{max} = 2\tau_{max} = 160 \text{ MPa}$$

$$\sigma_a = \sigma_{max} = \sigma_{ave} + R \qquad R = \sigma_{max} - \sigma_{ave} = 160 - 95 = 65 \text{ MPa}$$

$$R^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \qquad \tau_{xy}^2 = R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 = 65^2 - 25^2 = 60^2$$

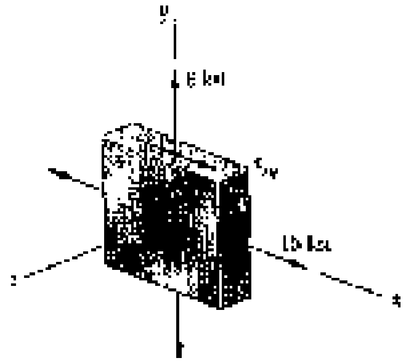
$$\tau_{xy} = \pm 60 \text{ ksi}$$

$$\sigma_b = \sigma_a - 2R = 160 - 130 = 30 \text{ MPa} \geq 0 \qquad \text{O.K.}$$



PROBLEM 7.77

7.77 For the state of stress shown, determine the value of τ_{xy} for which the maximum shearing stress is (*a*) 9 ksi, (*b*) 12 ksi.



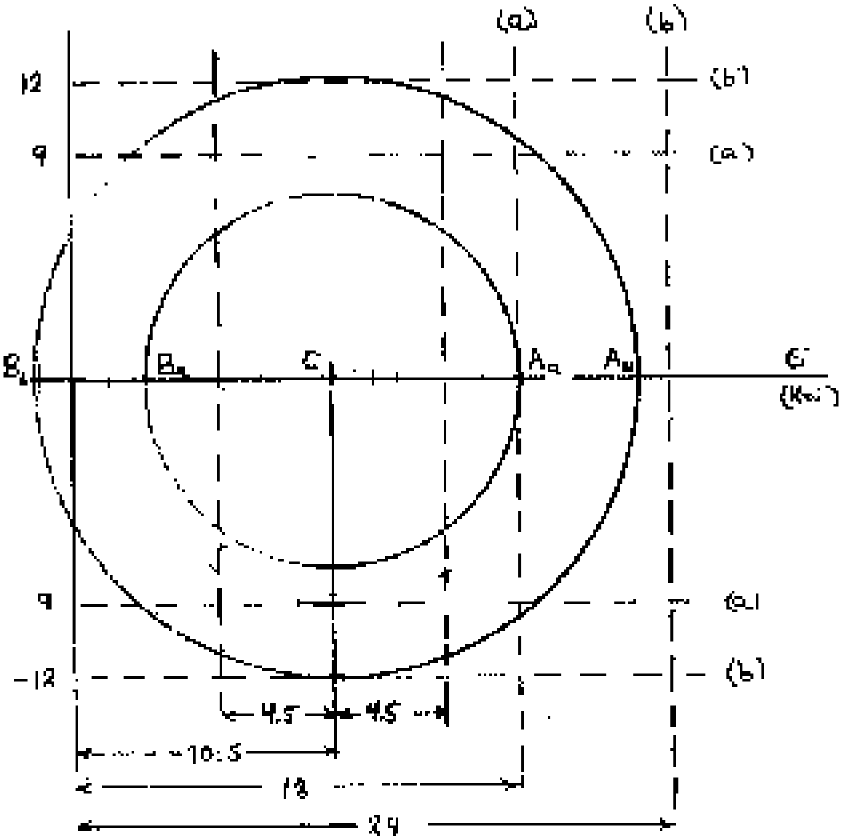
SOLUTION

$\sigma_x = 15$ ksi $\qquad \sigma_y = 6$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 10.5$ ksi

$D = \frac{\sigma_x - \sigma_y}{2} = 4.5$ ksi

τ (ksi)



(a) For $\tau_{max} = 9$ ksi

Center of Mohrs circle lies at point C. Lines marked (a) show the limits on τ_{max}. limit on σ_{max} is $\sigma_{max} = 2\tau_{max} = 18$ ksi. For the Mohr's circle $\sigma_a = \sigma_{max}$ corresponds to point A_a.

$R = \sigma_a - \sigma_{ave}$
$\quad = 18 - 10.5 = 7.5$ ksi

$R = \sqrt{D^2 + \tau_{xy}^2}$

$\tau_{xy} = \pm\sqrt{R^2 - D^2}$

$\quad = \pm\sqrt{7.5^2 - 4.5^2}$

$\quad = \pm 6$ ksi

(b) For $\tau_{max} = 12$ ksi.

Center of Mohr's circle lies at point C. $\qquad R = 12$ ksi

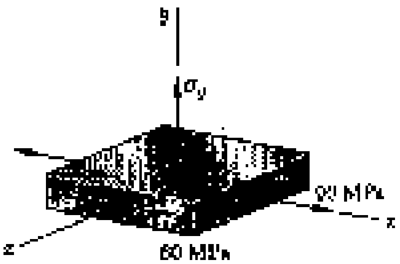
$\tau_{xy} = \pm\sqrt{R^2 - D^2} = \pm 11.24$ ksi

Checking $\quad \sigma_a = 10.5 + 12 = 22.5$ ksi $\qquad \sigma_b = 10.5 - 12 = -1.5$ ksi
$\qquad\qquad \sigma_c = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 12$ ksi $\quad$ O.K

PROBLEM 7.78

7.78 For the state of stress shown, determine two values of σ_y for which the maximum shearing stress is 80 MPa.



SOLUTION

$\sigma_x = 90$ MPa $\qquad \sigma_z = 0 \qquad \tau_{zx} = 60$ MPa

Mohr's circle for stresses in zx-plane

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_z + \sigma_x) = 45 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_z - \sigma_x}{2}\right)^2 + \tau_{zx}^2} = \sqrt{45^2 + 60^2} = 75 \text{ MPa}$$

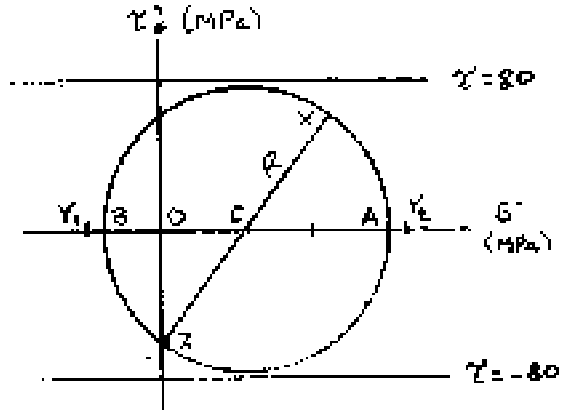
$\sigma_a = \sigma_{ave} + R = 120$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = -30$ MPa

Assume $\sigma_{max} = \sigma_a = 120$ MPa

$$\sigma_y = \sigma_{min} = \sigma_{max} - 2\tau_{max}$$
$$= 120 - (2)(80) = -40 \text{ MPa} \blacktriangleleft$$

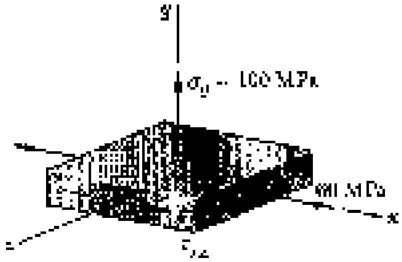
Assume $\sigma_{min} = \sigma_b = -30$ MPa

$$\sigma_y = \sigma_{max} = \sigma_{min} + 2\tau_{max}$$
$$= -30 + (2)(80) = 130 \text{ MPa} \blacktriangleleft$$



PROBLEM 7.79

7.79 For the state of stress shown, determine the range of values of τ_{xz} for which the maximum shearing stress is equal to or less than 60 MPa.



SOLUTION

$\sigma_x = 60$ MPa, $\sigma_z = 0$, $\sigma_y = 100$ MPa

For Mohr's circle of stresses in zx-plane

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_z) = 30$ MPa

$u = \frac{\sigma_x - \sigma_z}{2} = 30$

Assume $\sigma_{max} = \sigma_y = 100$ MPa

$\sigma_{min} = \sigma_b = \sigma_{max} - 2\tau_{max}$

$= 100 - (2)(60) = -20$ MPa

$R = \sigma_{ave} - \sigma_b$

$= 30 - (-20) = 50$ MPa

$\sigma_a = \sigma_{ave} + R$

$= 30 + 50 = 80 \text{ MPa} < \sigma_y$

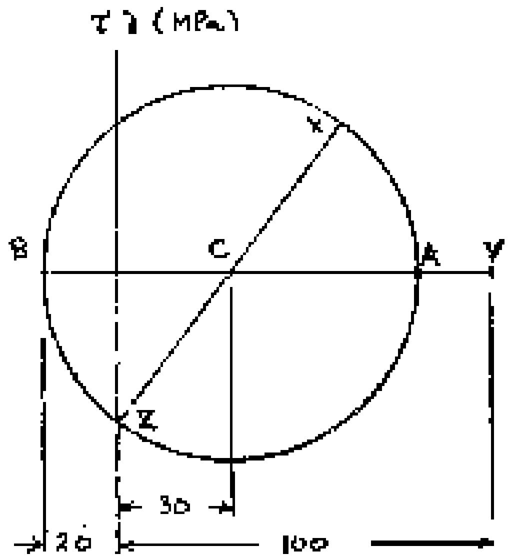
o.k.

$R = \sqrt{u^2 + \tau_{xz}^2}$

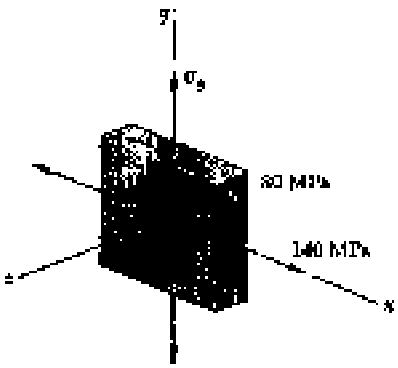
$\tau_{xz} = \pm\sqrt{R^2 - u^2}$

$= \pm\sqrt{50^2 - 30^2} = \pm 40$ MPa

$-40 \text{ MPa} \le \tau_{xz} \le 40 \text{ MPa}$



PROBLEM 7.80



*7.80 For the state of stress of Prob. 6.66, determine (*a*) the value of σ_y for which the maximum shearing stress is as small as possible, (*b*) the corresponding value of the shearing stress.

SOLUTION

Let $u = \frac{\sigma_x - \sigma_y}{2}$ $\quad \sigma_y = \sigma_x - 2u$

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = \sigma_x - u$$

$$R = \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{ave} - R = \sigma_x - u - \sqrt{u^2 + \tau_{xy}^2}$$

Assume τ_{max} is the in-plane shearing stress $\quad \tau_{max} = R$

Then $\tau_{max(in\text{-}plane)}$ is minimum if $u = 0$

$$\sigma_y = \sigma_x - 2u = \sigma_x = 140 \text{ MPa}, \quad \sigma_{ave} = \sigma_x - u = 140 \text{ MPa}$$

$$R = |\tau_{xy}| = 80 \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 140 + 80 = 220 \text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = 140 - 80 = 60 \text{ MPa}$$

$$\sigma_{max} = 220 \text{ MPa}, \quad \sigma_{min} = 0, \quad \tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = 110 \text{ MPa}$$

Assumption is incorrect.

Assume $\sigma_{max} = \sigma_a = \sigma_{ave} + R = \sigma_x - u + \sqrt{u^2 + \tau_{xy}^2}$

$$\sigma_{min} = 0 \qquad \tau_{max} = \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) = \tfrac{1}{2}\sigma_a$$

$$\frac{d\sigma_a}{du} = -1 + \frac{u}{\sqrt{u^2 + \tau_{xy}^2}} \neq 0 \qquad \text{(no minimum)}$$

Optimum value for u occurs when $\tau_{max(out\text{-}of\text{-}plane)} = \tau_{max(in\text{-}plane)}$

$$\therefore \tfrac{1}{2}(\sigma_a + R) = R \quad \text{or} \quad \sigma_a = R \quad \text{or} \quad \sigma_x - u = \sqrt{u^2 + \tau_{xy}^2}$$

$$(\sigma_x - u)^2 = \sigma_x^2 - 2u\sigma_x + \cancel{u^2} = \cancel{u^2} + \tau_{xy}^2$$

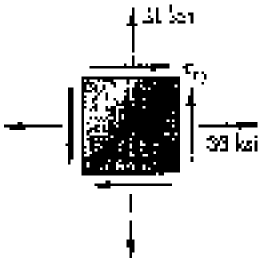
$$2u = \frac{\sigma_x^2 - \tau_{xy}^2}{\sigma_x} = \frac{140^2 - 80^2}{140} = 94.3 \text{ MPa} \qquad u = 47.14 \text{ MPa}$$

$$\sigma_y = \sigma_x - 2u = 140 - 94.3 = 45.7 \text{ MPa} \quad \blacktriangleleft$$

$$R = \sqrt{u^2 + \tau_{xy}^2} = \tau_{max} = 92.9 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 7.81

7.81 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 45$ ksi. Using the maximum-distortion-energy criterion, determine whether yield occurs when (*a*) $\tau_{xy} = 9$ ksi, (*b*) $\tau_{xy} = 18$ ksi, (*c*) $\tau_{xy} = 20$ ksi. If yield does not occur, determine the corresponding factor of safety.



SOLUTION

$\sigma_x = 36$ ksi $\qquad \sigma_y = 21$ ksi $\qquad \sigma_z = 0$

For stresses in xy-plane $\quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 28.5$ ksi

$\frac{\sigma_x - \sigma_y}{2} = 7.5$ ksi

(a) $\tau_{xy} = 9$ ksi $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (9)^2} = 11.715$ ksi

$\sigma_a = \sigma_{ave} + R = 40.215$ ksi, $\qquad \sigma_b = \sigma_{ave} - R = 16.875$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 34.977$ ksi < 45 ksi (No yielding)

$F.S. = \frac{45}{34.977} = 1.287$

(b) $\tau_{xy} = 18$ ksi $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (18)^2} = 19.5$ ksi

$\sigma_a = \sigma_{ave} + R = 48$ ksi, $\qquad \sigma_b = \sigma_{ave} - R = 9$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 44.193$ ksi < 45 ksi (No yielding)

$F.S. = \frac{45}{44.193} = 1.018$

(c) $\tau_{xy} = 20$ ksi $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7.5)^2 + (20)^2} = 21.36$ ksi

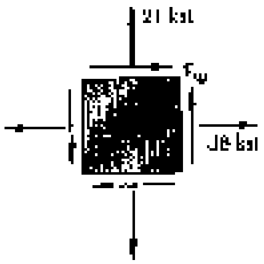
$\sigma_a = \sigma_{ave} + R = 49.86$, $\qquad \sigma_b = \sigma_{ave} - R = 7.14$ ksi

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 46.732$ ksi > 45 ksi (Yielding occurs)

PROBLEM 7.82

7.81 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 45$ ksi. Using the maximum-distortion-energy criterion, determine whether yield occurs when (*a*) $\tau_{xy} = 9$ ksi, (*b*) $\tau_{xy} = 18$ ksi, (*c*) $\tau_{xy} = 20$ ksi. If yield does not occur, determine the corresponding factor of safety.

7.82 Solve Prob. 7.81, using the maximum-shearing-stress criterion.



SOLUTION

$\sigma_x = 36$ ksi $\quad \sigma_y = 21$ ksi $\quad \sigma_z = 0$

For stresses in xy-plane $\quad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 28.5$ ksi

$$\frac{\sigma_x - \sigma_y}{2} = 7.5 \text{ ksi}$$

(a) $\tau_{xy} = 9$ ksi $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 11.715$ ksi

$\sigma_a = \sigma_{ave} + R = 40.215$ ksi, $\quad \sigma_b = \sigma_{ave} - R = 16.875$ ksi

$\sigma_{max} = 34.977$ ksi, $\quad \sigma_{min} = 0$

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 40.215$ ksi < 45 ksi $\qquad$ (No yielding)

$$F.S. = \frac{45}{40.215} = 1.119$$

(b) $\tau_{xy} = 18$ ksi $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 19.5$ ksi

$\sigma_a = \sigma_{ave} + R = 48$ ksi, $\quad \sigma_b = \sigma_{ave} - R = 9$ ksi

$\sigma_{max} = 48$ ksi $\quad \sigma_{min} = 0$

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 48$ ksi > 45 ksi $\qquad$ (Yielding occurs)

(c) $\tau_{xy} = 20$ ksi $\qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 21.36$ ksi

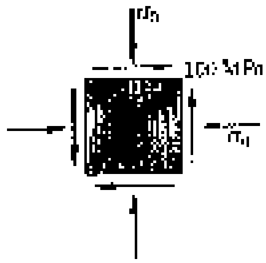
$\sigma_a = \sigma_{ave} + R = 49.86$ ksi $\qquad \sigma_b = \sigma_{ave} - R = 7.14$ ksi

$\sigma_{max} = 49.86$ ksi $\qquad \sigma_{min} = 0$

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 49.86$ ksi > 45 ksi $\qquad$ (Yielding occurs)

PROBLEM 7.83

7.83 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-shearing-stress criterion, determine whether yield occurs when (a) $\sigma_0 = 200$ MPa, (b) $\sigma_0 = 240$ MPa, (c) $\sigma_0 = 280$ MPa. If yield does not occur, determine the corresponding factor of safety.



SOLUTION

$$\sigma_{ave} = -\sigma_0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(a) $\sigma_0 = 200$ MPa, $\quad \sigma_{ave} = -200$ MPa

$\sigma_a = \sigma_{ave} + R = -100$ MPa $\qquad \sigma_b = \sigma_{ave} - R = -300$ MPa

$\sigma_{max} = 0$, $\quad \sigma_{min} = -300$ MPa

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 300 \text{ MPa} < 325$ MPa $\quad$ (No yielding)

$$\text{F.S.} = \frac{325}{300} = 1.083$$

(b) $\sigma_0 = 240$ MPa, $\quad \sigma_{ave} = -240$ MPa

$\sigma_a = \sigma_{ave} + R = -140$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = -340$ MPa

$\sigma_{max} = 0$, $\quad \sigma_{min} = -340$ MPa

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 340 \text{ MPa} > 325$ MPa $\quad$ (Yielding occurs)

(c) $\sigma_0 = 280$ MPa, $\quad \sigma_{ave} = -280$ MPa

$\sigma_a = \sigma_{ave} + R = -180$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = -380$ MPa

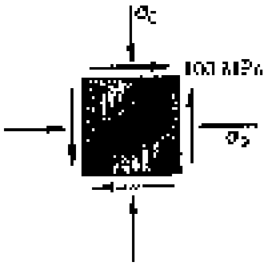
$\sigma_{max} = 0$, $\quad \sigma_{min} = -380$ MPa

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 380 \text{ MPa} > 325$ MPa $\quad$ (Yielding occurs)

PROBLEM 7.84

7.83 The state of plane stress shown occurs in a machine component made of a steel with $\sigma_Y = 325$ MPa. Using the maximum-shearing-stress criterion, determine whether yield occurs when (*a*) $\sigma_0 = 200$ MPa, (*b*) $\sigma_0 = 240$ MPa, (*c*) $\sigma_0 = 280$ MPa. If yield does not occur, determine the corresponding factor of safety.

7.84 Solve Prob. 7.83, using the maximum-distortion-energy criterion.



SOLUTION

$$\sigma_{ave} = -\sigma_0 \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 100 \text{ MPa}$$

(a) $\sigma_0 = 200$ MPa $\qquad \sigma_{ave} = -200$ MPa

$\sigma_a = \sigma_{ave} + R = -100$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = -300$ MPa

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 264.56$ MPa < 325 MPa (No yielding)

$$F.S. = \frac{325}{264.56} = 1.228$$

(b) $\sigma_0 = 240$ MPa $\qquad \sigma_{ave} = -240$ MPa

$\sigma_a = \sigma_{ave} + R = -140$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = -340$ MPa

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 295.97$ MPa < 325 MPa (No yielding)

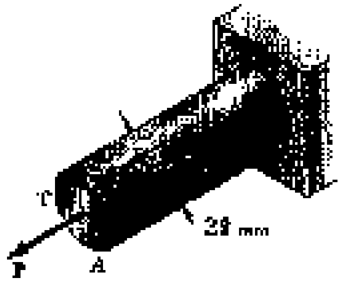
$$F.S. = \frac{325}{295.97} = 1.098$$

(c) $\sigma_0 = 280$ MPa $\qquad \sigma_{ave} = -280$ MPa

$\sigma_a = \sigma_{ave} + R = -180$ MPa, $\qquad \sigma_b = \sigma_{ave} - R = -380$ MPa

$\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 329.24$ MPa > 325 MPa (Yielding occurs)

PROBLEM 7.85



7.85 The 38-mm-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 250$ MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the torque T for which yield occurs when $P = 240$ kN.

SOLUTION

$P = 240 \times 10^3$ N

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(38)^2 = 1.1341 \times 10^3 \text{ mm}^2 = 1.1341 \times 10^{-3} \text{ m}^2$

$\sigma_x = \frac{P}{A} = \frac{240 \times 10^3}{1.1341 \times 10^{-3}} = 211.6 \times 10^6 \text{ Pa} = 211.6 \text{ MPa}$

$\sigma_y = 0 \qquad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$

$2\tau_{max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_Y$

$4\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2 \qquad \tau_{xy} = \frac{1}{2}\sqrt{\sigma_Y^2 - \sigma_x^2} = \frac{1}{2}\sqrt{250^2 - 211.6^2}$

$= 66.568 \text{ MPa} = 66.568 \times 10^6 \text{ Pa}$

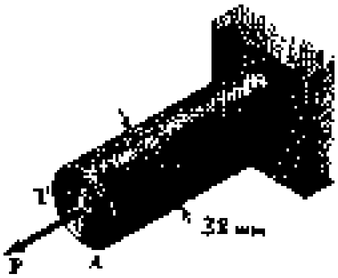
From torsion $\qquad \tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$

$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{38}{2}\right)^4 = 204.71 \times 10^3 \text{ mm}^4 = 204.71 \times 10^{-9} \text{ m}^4$

$c = \frac{1}{2}d = 19 \times 10^{-3}$ m

$T = \frac{(204.71 \times 10^{-9})(66.568 \times 10^6)}{19 \times 10^{-3}} = 717 \text{ N}\cdot\text{m}$

PROBLEM 7.86



7.85 The 38-mm-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 250$ MPa. Using the maximum-shearing-stress criterion, determine the magnitude of the torque **T** for which yield occurs when $P = 240$ kN.

7.86 Solve Prob. 7.85, using the maximum-distortion-energy criterion.

SOLUTION

$$P = 240 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(38)^2 = 1.1341 \times 10^3 \text{ mm}^2 = 1.1341 \times 10^{-3} \text{ m}^2$$

$$\sigma_x = \frac{P}{A} = \frac{240 \times 10^3}{1.1341 \times 10^{-3}} = 211.6 \times 10^6 \text{ Pa} = 211.6 \text{ MPa}$$

$$\sigma_y = 0 \qquad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R = \frac{1}{2}\sigma_x + \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\sigma_b = \sigma_{ave} - R = \frac{1}{2}\sigma_x - \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$$

$$\begin{aligned}\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b = {} & \frac{1}{4}\sigma_x^2 + \sigma_x\sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2} + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ & + \frac{1}{4}\sigma_x^2 - \sigma_x\sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2} + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ & - \frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \tau_{xy}^2 \\ = {} & \sigma_x^2 + 3\tau_{xy}^2 = \sigma_Y^2\end{aligned}$$

$$\tau_{xy}^2 = \frac{1}{3}(\sigma_Y^2 - \sigma_x^2)$$

$$\tau_{xy} = \frac{1}{\sqrt{3}}\sqrt{250^2 - 211.6^2} = 76.867 \text{ MPa} = 76.867 \times 10^6 \text{ Pa}$$

From torsion $\qquad \tau_{xy} = \dfrac{Tc}{J} \qquad T = \dfrac{J\tau_{xy}}{c}$

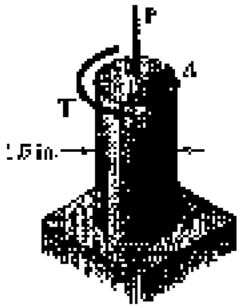
$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{38}{2}\right)^4 = 204.71 \times 10^3 \text{ mm}^4 = 204.71 \times 10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}d = 19 \times 10^{-3} \text{ m}$$

$$T = \frac{(204.71 \times 10^{-9})(76.876 \times 10^6)}{19 \times 10^{-3}} = 828 \text{ N}\cdot\text{m}$$

PROBLEM 7.87

7.87 The 1.5-in-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 42$ ksi. Using the maximum-shearing-stress criterion, determine the magnitude of the torque $\mathbf{T}$ for which yield occurs when $P = 60$ kips.



SOLUTION

$P = 60$ kips $\qquad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.5)^2 = 1.7671\ \text{in}^2$

$\sigma_x = -\frac{P}{A} = -\frac{60}{1.7671} = -33.953$ ksi

$\sigma_y = 0 \qquad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$

$2\tau_{max} = 2R = \sqrt{\sigma_x^2 + 4\tau_{xy}^2} = \sigma_Y$

$4\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2 \qquad \tau_{xy} = \frac{1}{2}\sqrt{\sigma_Y^2 - \sigma_x^2} = \frac{1}{2}\sqrt{42^2 - 33.953^2}$

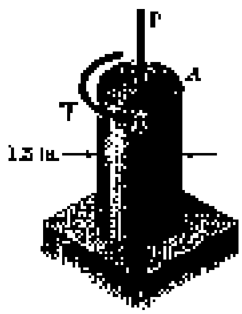
$= 12.361$ ksi

From torsion $\qquad \tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$

$c = \frac{1}{2}d = 0.75$ in $\qquad J = \frac{\pi}{2}c^4 = 0.49701\ \text{in}^4$

$T = \frac{(0.49701)(12.361)}{0.75} = 8.19$ kip·in

PROBLEM 7.88



7.87 The 1.5-in-diameter shaft AB is made of a grade of steel for which the yield strength is $\sigma_Y = 42$ ksi. Using the maximum-shearing-stress criterion, determine the magnitude of the torque T for which yield occurs when $P = 60$ kips.

7.88 Solve Prob. 7.87, using the maximum-distortion-energy criterion.

SOLUTION

$P = 60$ kips $\qquad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.5)^2 = 1.76715$ in^2

$\sigma_x = -\frac{P}{A} = -\frac{60}{1.76715} = -33.953$ ksi

$\sigma_y = 0 \qquad \sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}\sigma_x$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{1}{4}\sigma_x^2 + \tau_{xy}^2}$

$\sigma_a = \sigma_{ave} + R \qquad \sigma_b = \sigma_{ave} - R$

$\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b = (\sigma_{ave} + R)^2 + (\sigma_{ave} - R)^2 - (\sigma_{ave} + R)(\sigma_{ave} - R)$

$= \sigma_{ave}^2 + 2\sigma_{ave}R + R^2 + \sigma_{ave}^2 - 2\sigma_{ave}R + R^2 - \sigma_{ave}^2 + R^2$

$= \sigma_{ave}^2 + 3R^2$

$= \frac{1}{4}\sigma_x^2 + 3\left(\frac{1}{4}\sigma_x^2 + \tau_{xy}^2\right) = \sigma_x^2 + 3\tau_{xy}^2 = \sigma_Y^2$

$3\tau_{xy}^2 = \sigma_Y^2 - \sigma_x^2 \qquad \tau_{xy} = \frac{1}{\sqrt{3}}\left(\sigma_Y^2 - \sigma_x^2\right) = \frac{1}{\sqrt{3}}\sqrt{42^2 - 33.953^2}$

$= 14.273$ ksi

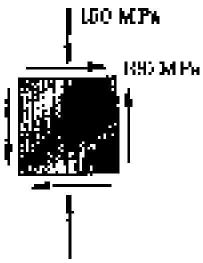
From torsion $\qquad \tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c}$

$c = \frac{1}{2}d = 0.75$ in. $\qquad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.75)^4 = 0.49701$ in^4

$T = \frac{(0.49701)(14.273)}{0.75} = 9.46$ kip·in

PROBLEM 7.89

7.89 and 7.90 The state of plane stress shown is expected to occur in a cast-iron machine base. Knowing that for the grade of cast iron used $\sigma_{UT} = 160$ MPa and $\sigma_{UC} = 320$ MPa and using Mohr's criterion, determine whether rupture of the component will occur.



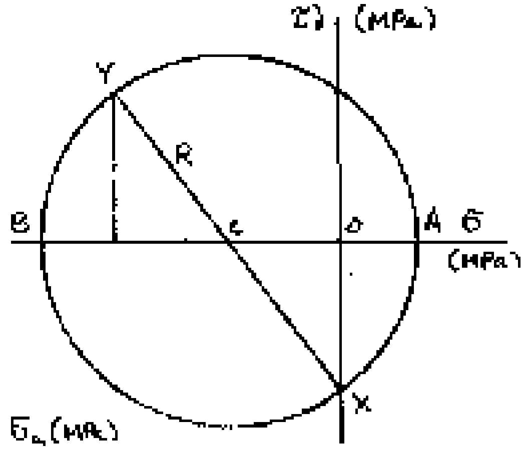
SOLUTION

$\sigma_x = 0 \qquad \sigma_y = -150 \text{ MPa} \qquad \tau_{xy} = 100 \text{ MPa}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -75 \text{ MPa}$

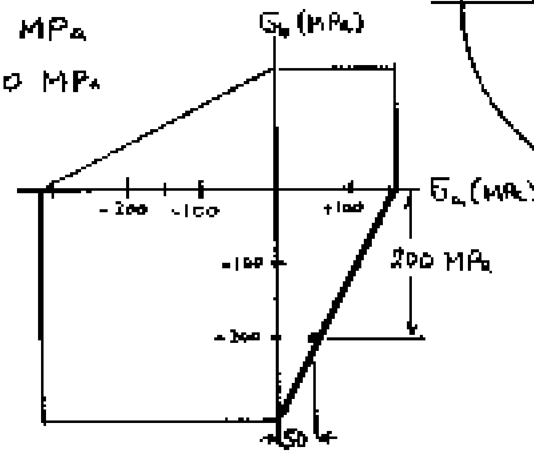
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{75^2 + 100^2} = 125 \text{ MPa}$



$\sigma_a = \sigma_{ave} + R = 50 \text{ MPa}$

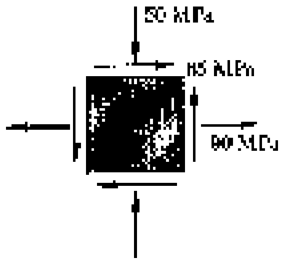
$\sigma_b = \sigma_{ave} - R = -200 \text{ MPa}$



Equation of the 4th quadrant boundary is $\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$

$\frac{50}{160} - \frac{(-200)}{320} = 0.9375 < 1$, No rupture.

PROBLEM 7.90



7.89 and 7.90 The state of plane stress shown is expected to occur in a cast-iron machine base. Knowing that for the grade of cast iron used $\sigma_{UT} = 160$ MPa and $\sigma_{UC} = 320$ MPa and using Mohr's criterion, determine whether rupture of the component will occur.

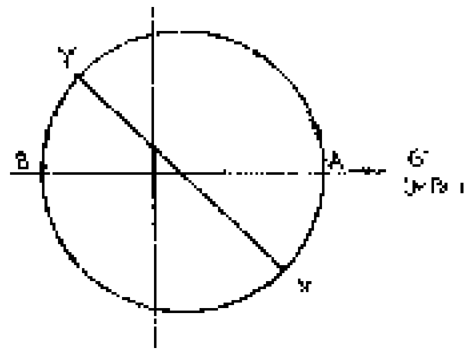
SOLUTION

$\sigma_x = 90$ MPa , $\sigma_y = -50$ MPa , $\tau_{xy} = 65$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 20$ MPa

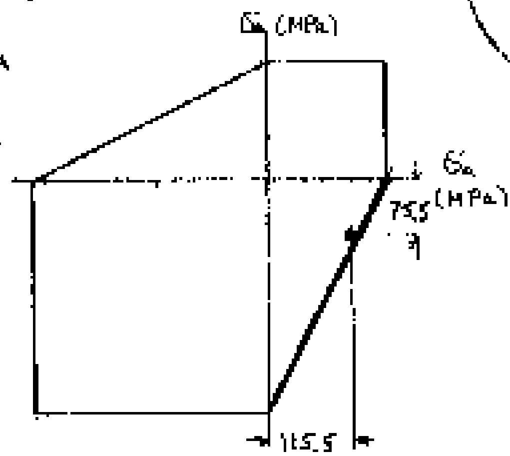
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{70^2 + 65^2} = 95.5 \text{ MPa}$$



$\sigma_a = \sigma_{ave} + R = 115.5$ MPa

$\sigma_b = \sigma_{ave} - R = -75.5$ MPa



Equation of 4th quadrant boundary $\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$

$$\frac{115.5}{160} - \frac{(-75.5)}{320} = 0.958 < 1$$

No rupture

PROBLEM 7.91

7.91 and 7.92 The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 10$ ksi and $\sigma_{UC} = 30$ ksi and using Mohr's criterion, determine whether rupture of the component will occur.

SOLUTION

$\sigma_x = -8$ ksi, $\quad \sigma_y = 0$, $\quad \tau_{xy} = 7$ ksi

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = -4 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{4^2 + 7^2} = 8.062 \text{ ksi}$$

$$\sigma_a = \sigma_{ave} + R = -4 + 8.062 = 4.062 \text{ ksi}$$

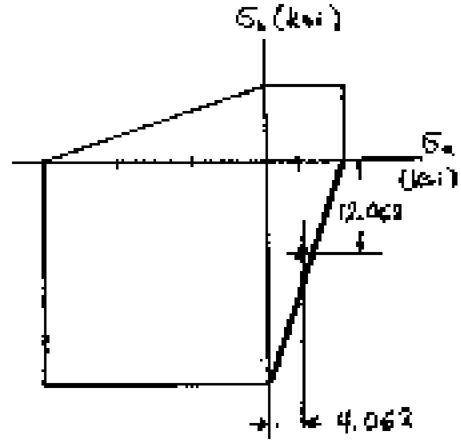
$$\sigma_b = \sigma_{ave} - R = -4 - 8.062 = -12.062 \text{ ksi}$$

Equation of 4th quadrant of boundary

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

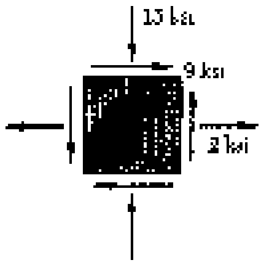
$$\frac{4.062}{10} - \frac{(-12.062)}{30} = 0.808 < 1$$

(No rupture)



PROBLEM 7.92

7.91 and 7.92 The state of plane stress shown is expected to occur in an aluminum casting. Knowing that for the aluminum alloy used $\sigma_{UT} = 10$ ksi and $\sigma_{UC} = 30$ ksi and using Mohr's criterion, determine whether rupture of the component will occur.



SOLUTION

$\sigma_x = 2$ ksi $\quad \sigma_y = -15$ ksi $\quad \tau_{xy} = 9$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -6.5$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{8.5^2 + 9^2} = 12.379$ ksi

$\sigma_a = \sigma_{ave} + R = 5.879$ ksi

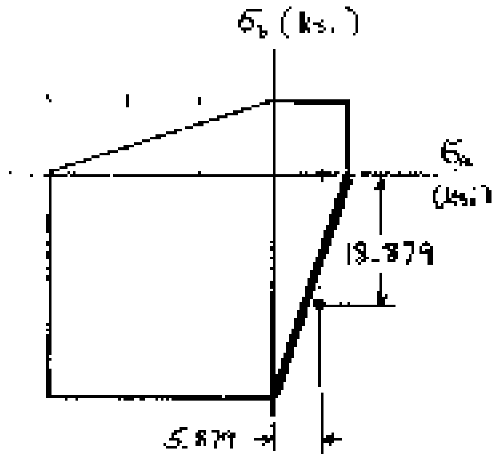
$\sigma_b = \sigma_{ave} - R = -18.879$ ksi

Equation of 4th quadrant of boundary

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{5.879}{10} - \frac{(-18.879)}{30} = 1.217 > 1$$

Rupture will occur.



PROBLEM 7.93

7.93 The state of plane stress shown will occur at a critical point in a cast pipe made of an aluminum alloy for which $\sigma_{UT} = 75$ MPa and $\sigma_{UC} = 150$ MPa. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.

SOLUTION

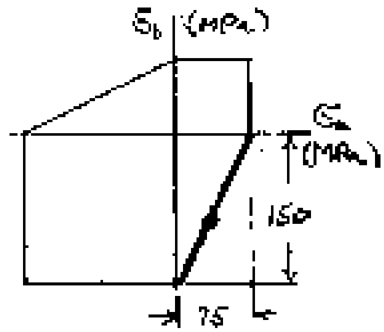
$$\sigma_x = -80 \text{ MPa} \qquad \sigma_y = 0 \qquad \tau_{xy} = -\tau_0$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -40 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{40^2 + \tau_0^2} \text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R, \qquad \sigma_b = \sigma_{ave} - R, \qquad \tau_0 = \pm\sqrt{R^2 - 40^2}$$

Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is



$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

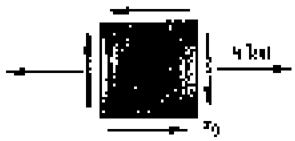
$$\frac{-40 + R}{75} - \frac{-40 - R}{150} = 1$$

$$\frac{R}{75} + \frac{R}{150} = 1 + \frac{40}{75} - \frac{40}{150} = 1.2667$$

$$R = 63.33 \text{ MPa}, \qquad \tau_0 = \pm\sqrt{63.33^2 - 40^2} = \pm 49.1 \text{ MPa}$$

PROBLEM 7.94

7.94 The state of plane stress shown will occur in an aluminum casting that is made of an alloy for which $\sigma_{UT} = 10$ ksi and $\sigma_{UC} = 25$ ksi. Using Mohr's criterion, determine the shearing stress τ_0 for which failure should be expected.



SOLUTION

$\sigma_x = 8$ ksi, $\sigma_y = 0$, $\tau_{xy} = \tau_0$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 4$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{4^2 + \tau_0^2}$, $\tau_0 = \pm\sqrt{R^2 - 4^2}$

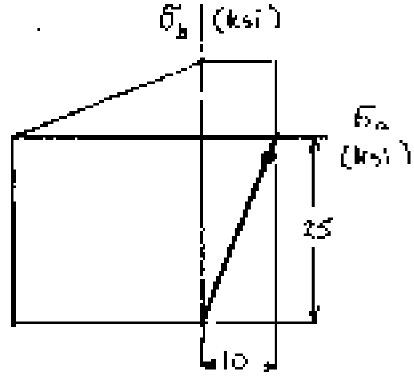
$\sigma_a = \sigma_{ave} + R = (4 + R)$ ksi $\quad \sigma_b = \sigma_{ave} - R = (4 - R)$ ksi

Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{4+R}{10} - \frac{4-R}{25} = 1$$

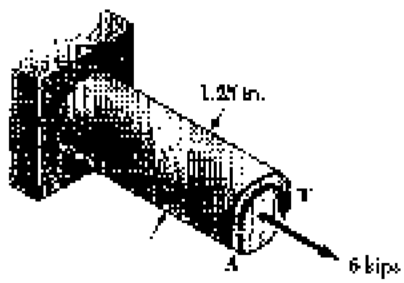
$$\left(\frac{1}{10} + \frac{1}{25}\right)R = 1 - \frac{4}{10} + \frac{4}{25}$$



$R = 5.429$ ksi $\quad \tau_0 = \pm\sqrt{5.429^2 - 4^2} = \pm 3.67$ ksi

PROBLEM 7.95

7.95 The cast-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 8$ ksi and $\sigma_{UC} = 16$ ksi. Using Mohr's criterion, determine the magnitude of the torque **T** for which rupture should be expected.



SOLUTION

$P = 6$ kips, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.25)^2 = 1.2272\ \text{in}^2$

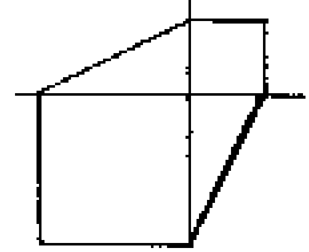
$\sigma_x = \frac{P}{A} = 4.889$ ksi, $\sigma_y = 0$, $\tau_{xy} = \frac{Tc}{J}$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 2.4446$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{5.976 + \tau_{xy}^2}$ ksi, $\quad \tau_{xy} = \pm\sqrt{R^2 - 5.976}$ ksi

$\sigma_a = \sigma_{ave} + R = 2.4446 + R$ ksi, $\quad \sigma_b = 2.4446 - R$ ksi

Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of 4th quadrant boundary is



$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{2.4446 + R}{8} - \frac{2.4446 - R}{16} = 1$$

$$\left(\frac{1}{8} + \frac{1}{16}\right)R = 1 - \frac{2.4446}{8} + \frac{2.4446}{16}$$

$R = 4.5185$ ksi $\quad \tau_{xy} = \pm\sqrt{4.5185^2 - 5.976} = 3.80$ ksi

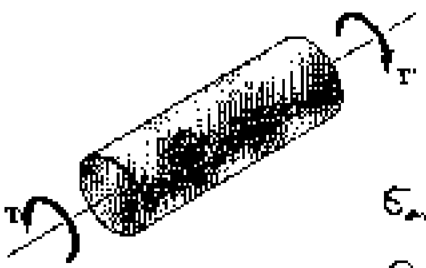
For torsion: $\quad c = \frac{1}{2}d = 0.625$ in

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.625)^4 = 0.23968\ \text{in}^4$$

$$T = \frac{J\tau_{xy}}{c} = \frac{(0.23968)(3.80)}{0.625} = 1.457\ \text{kip}\cdot\text{in}$$ ◀

PROBLEM 7.96

7.96 The cast-aluminum rod shown is made of an alloy for which $\sigma_{UT} = 70$ MPa and $\sigma_{UC} = 175$ MPa. Knowing that the magnitude T of the applied torques is slowly increased and using Mohr's criterion, determine the shearing stress τ_0 which should be expected at rupture.



SOLUTION

$\sigma_x = 0, \quad \sigma_y = 0 \qquad \tau_{xy} = -\tau_0$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 0$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{0 + \tau_{xy}^2} = |\tau_{xy}|$

$\sigma_a = \sigma_{ave} + R = R \qquad \sigma_b = \sigma_{ave} - R = -R$

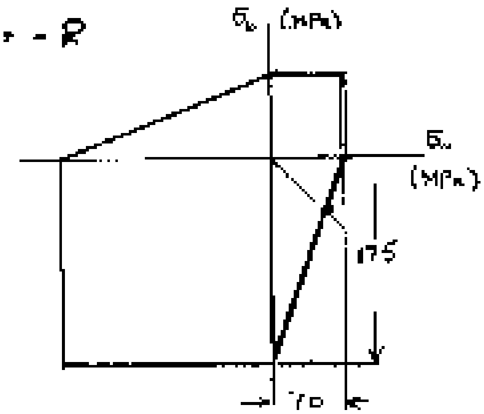
Since $|\sigma_{ave}| < R$, stress point lies in 4th quadrant. Equation of boundary of 4th quadrant is

$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$

$\frac{R}{70} - \frac{-R}{175} = 1$

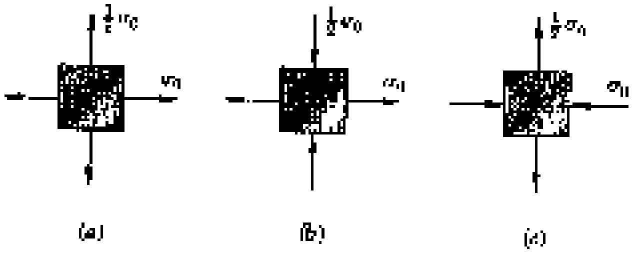
$\left(\frac{1}{70} + \frac{1}{175}\right) R = 1$

$R = 50$ MPa $\qquad |\tau_{xy}| = 50$ MPa



PROBLEM 7.97

7.97 A machine component is made of a grade of cast iron for which $\sigma_{UT} = 8$ ksi and $\sigma_{UC} = 20$ ksi. For each of the states of plane stress shown, and using Mohr's criterion, determine the normal stress σ_0 at which rupture of the component should be expected.

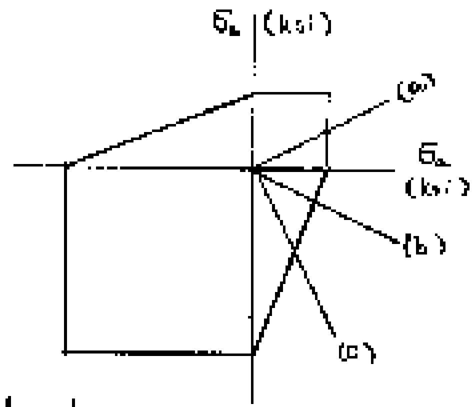


SOLUTION

(a) $\sigma_a = \sigma_0$, $\sigma_b = \frac{1}{2}\sigma_0$

Stress point lies in 1st quadrant.

$\sigma_a = \sigma_0 = \sigma_{UT} = 8$ ksi



(b) $\sigma_a = \sigma_0$, $\sigma_b = -\frac{1}{2}\sigma_0$

Stress point lies in 4th quadrant.
Equation of 4th quadrant boundary is

$$\frac{\sigma_a}{\sigma_{UT}} - \frac{\sigma_b}{\sigma_{UC}} = 1$$

$$\frac{\sigma_0}{8} - \frac{-\frac{1}{2}\sigma_0}{20} = 1 \qquad \sigma_0 = 6.67 \text{ ksi}$$

(c) $\sigma_a = \frac{1}{2}\sigma_0$, $\sigma_b = -\sigma_0$, 4th quadrant

$$\frac{\frac{1}{2}\sigma_0}{8} - \frac{-\sigma_0}{20} = 1 \qquad \sigma_0 = 8.89 \text{ ksi}$$

PROBLEM 7.98

7.98 Determine the normal stress in a basketball of 9.5-in. diameter and 0.125-in. wall thickness that is inflated to a gage pressure of 9 psi.

SOLUTION

$$r = \tfrac{1}{2}d - t = (\tfrac{1}{2})(9.5) - 0.125 = 4.625 \text{ in}$$

$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(9)(4.625)}{(2)(0.125)} = 166.5 \text{ psi}$$

PROBLEM 7.99

7.99 A spherical gas container made of steel has an 18-ft diameter and a wall thickness of $\frac{3}{8}$ in. Knowing that the internal pressure is 60 psi, determine the maximum normal stress and the maximum shearing stress in the container.

SOLUTION

$d = 18 \text{ ft} = 216 \text{ in} \qquad r = \frac{1}{2}d - t = 107.625 \text{ in}$

$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(60)(107.625)}{(2)(0.375)} = 8610 \text{ psi} = 8.61 \text{ ksi}$ ◄

$\tau_{max\,(out\text{-}of\text{-}plane)} = \frac{1}{2}\sigma_1 = 4.31 \text{ ksi}$ ◄

PROBLEM 7.100

7.100 The maximum gage pressure is known to be 8 MPa in a spherical steel pressure vessel having a 250-mm diameter and a 6-mm wall thickness. Knowing that the ultimate stress in the steel used is $\sigma_U = 400$ MPa, determine the factor of safety with respect to tensile failure.

SOLUTION

$p = 8 \text{ MPa} = 8 \times 10^6 \text{ Pa} \qquad t = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$

$r = \frac{1}{2}d - t = \frac{1}{2}(250) - 6 = 119 \text{ mm} = 0.119 \text{ m}$

$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(8 \times 10^6)(0.119)}{(2)(6 \times 10^{-3})} = 79.33 \times 10^6 \text{ Pa} = 79.33 \text{ MPa}$

$F.S. = \frac{\sigma_U}{\sigma_1} = \frac{400}{79.33} = 5.04$ ◄

PROBLEM 7.101

7.101 A spherical pressure vessel of 900-mm outside diameter is to be fabricated from a steel having an ultimate stress $\sigma_U = 400$ MPa. Knowing that a factor of safety of 4 is desired and that the gage pressure can reach 3.5 MPa, determine the smallest wall thickness that should be used.

SOLUTION

$p = 3.5 \text{ MPa}, \qquad r = \frac{1}{2}d - t = (\frac{1}{2})(900) - t = 450 - t \text{ mm}$

$\sigma_1 = \sigma_2 = \frac{\sigma_U}{F.S.} = \frac{400}{4} = 100 \text{ MPa}$

$\sigma_1 = \frac{pr}{2t} \quad \therefore \quad t = \frac{pr}{2\sigma_1} = \frac{(3.5)(450 - t)}{(2)(100)} = 7.875 - 0.0175t$

$1.0175t = 7.875 \qquad t = 7.74 \text{ mm}$ ◄

PROBLEM 7.102

7.102 A spherical gas container having a diameter of 5 m and a wall thickness of 24 mm is made of a steel for which $E = 200$ GPa and $\nu = 0.29$. Knowing that the gage pressure in the container is increased from zero to 1.8 MPa, determine (*a*) the maximum normal stress in the container, (*b*) the increase in the diameter of the container.

SOLUTION

$p = 1.8$ MPa $\qquad r = \frac{1}{2}d - t = \frac{1}{2}(5) - 24\times10^{-3} = 2.476$ m

(a) $\sigma_1 = \sigma_2 = \dfrac{pr}{2t} = \dfrac{(1.8)(2.476)}{(2)(24\times10^{-3})} = 92.85$ MPa ◄

$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{1-\nu}{E}\sigma_1 = \frac{1-0.29}{200\times10^9}(92.85\times10^6) = 329.6\ \mu$

$\Delta d = d\,\varepsilon_1 = (5)(329.6\times10^{-6}) = 1.648\times10^{-3}$ m $= 1.648$ mm ◄

PROBLEM 7.103

7.103 A spherical pressure vessel is 3 m in diameter and has a wall thickness of 12 mm. Knowing that for the steel used $\sigma_{all} = 80$ MPa, $E = 200$ GPa and $\nu = 0.29$, determine (*a*) the allowable gage pressure, (*b*) the corresponding increase in the diameter of the vessel.

SOLUTION

$r = \frac{1}{2}d - t = \frac{1}{2}(3000) - 12 = 1488$ mm

$\sigma_1 = \sigma_2 = \sigma_{all} = 8$ MPa

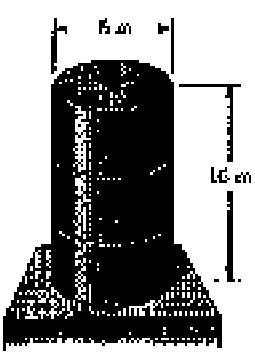
(a) $\sigma_1 = \sigma_2 = \dfrac{pr}{2t} \qquad p = \dfrac{2t\sigma_1}{r} = \dfrac{(2)(12)(80)}{1488} = 1.290$ MPa ◄

$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) = \frac{1-\nu}{E}\sigma_1 = \frac{1-0.29}{200\times10^9}(8\times10^6) = 28.4\ \mu$

(b) $\Delta d = d\,\varepsilon_1 = (3000)(28.4\times10^{-6}) = 85.2\times10^{-3}$ mm $= 0.0852$ mm ◄

PROBLEM 7.104

7.104 When filled to capacity, the unpressurized storage tank shown contains water to a height of 15.5 m above its base. Knowing that the lower portion of the tank has a wall thickness of 16 mm, determine the maximum normal stress and the maximum shearing stress in the tank. (Density of water = 1000 kg/m³.)



SOLUTION

$p = \rho g h = (1000)(9.81)(15.5) = 152.06\times10^3$ Pa

$r = \frac{1}{2}d - t = \frac{1}{2}(8) - 16\times10^{-3} = 3.984$ m

$\sigma_1 = \dfrac{pr}{t} = \dfrac{(152.06\times10^3)(3.984)}{16\times10^{-3}} = 37.9\times10^6$ Pa

$= 37.9$ MPa ◄

$\tau_{max\,(out\ of\ plane)} = \frac{1}{2}\sigma_1 = 18.93$ MPa ◄

PROBLEM 7.105

7.105 Determine the largest internal pressure that can be applied to a cylindrical tank of 5.5-ft diameter and $\frac{5}{8}$-in. wall thickness if the ultimate normal stress of the steel used is 65 ksi and a factor of safety of 5.0 is desired.

SOLUTION

$$\sigma_1 = \frac{\sigma_U}{F.S.} = \frac{65}{5.0} = 13 \text{ ksi} \qquad d = 5.5 \text{ ft} = 66 \text{ in}$$

$$r = \frac{1}{2}d - t = \frac{1}{2}(66) - 0.625 = 32.375 \text{ in}$$

$$\sigma_1 = \frac{pr}{t} \qquad p = \frac{\sigma_1 t}{r} = \frac{(13)(0.625)}{32.375} = 0.251 \text{ ksi} = 251 \text{ psi}$$

PROBLEM 7.106

7.106 The storage tank shown contains liquified propane under a pressure of 210 psi at a temperature of 100° F. Knowing that the tank has a diameter of 12.6 in. and a wall thickness of 0.11 in, determine the maximum normal stress and the maximum shearing stress in the tank.

SOLUTION

$$p = 210 \text{ psi}, \qquad r = \frac{1}{2}d - t = \frac{1}{2}(12.6) - 0.11 = 6.19 \text{ in.}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(210)(6.19)}{0.11} = 11.82 \times 10^3 \text{ psi} = 11.82 \text{ ksi}$$

$$\tau_{max\,(out\text{-}of\text{-}plane)} = \frac{1}{2}\sigma_1 = 5.91 \text{ ksi}$$

PROBLEM 7.107

7.107 The bulk storage tank shown in Fig. 7.49 has an outer diameter of 3.3 m and a wall thickness of 18 mm. At a time when the internal pressure of the tank is 1.5 MPa, determine the maximum normal stress and the maximum shearing stress in the tank.

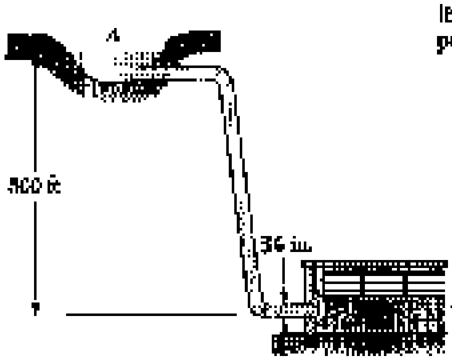
SOLUTION

$$d = 3.3 \text{ m}, \qquad t = 18 \times 10^{-3} \text{ m} \qquad r = \frac{1}{2}d - t = 1.632 \text{ m}$$

$$p = 1.5 \text{ MPa} \qquad \sigma_1 = \frac{pr}{t} = \frac{(1.5 \times 10^6)(1.632)}{18 \times 10^{-3}} = 136 \times 10^6 \text{ Pa} = 136 \text{ MPa}$$

$$\tau_{max\,(out\text{-}of\text{-}plane)} = \frac{1}{2}\sigma_1 = 68 \text{ MPa}$$

PROBLEM 7.108



7.108 A 36-in.-diameter penstock has a 0.5-in. wall thickness and connects a reservoir at A with a generating station at B. Knowing that the specific weight of water is 62.4 lb/ft³, determine the maximum normal stress and the maximum shearing stress in the penstock under static conditions.

SOLUTION

$$r = \tfrac{1}{2}d - t = \tfrac{1}{2}(36) - 0.5 = 17.5 \text{ in.}$$

$$p = \gamma h = (62.4 \text{ lb/ft}^3)(500 \text{ ft}) = 31.2\times10^3 \text{ lb/ft}^2$$

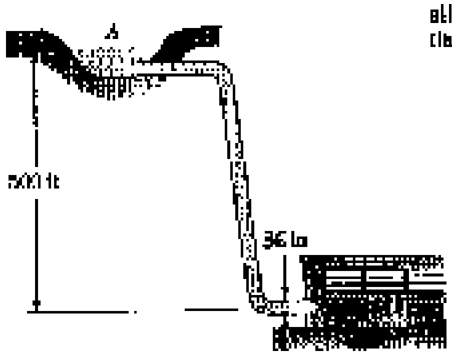
$$= 216.67 \text{ psi}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(216.67)(17.5)}{0.5} = 7583 \text{ psi}$$

$$= 7.58 \text{ ksi}$$

$$\tau_{max\,(\text{out-of-plane})} = \tfrac{1}{2}\sigma_1 = 3.79 \text{ ksi}$$

PROBLEM 7.109



7.109 A 36-in.-diameter steel penstock connects a reservoir at A with a generating station at B. Knowing that the specific weight of water is 62.4 lb/ft³ and that the allowable normal stress in the steel is 12.5 ksi, determine the smallest wall thickness that can be used for the penstock.

SOLUTION

$$p = \gamma h = (62.4 \text{ lb/ft}^3)(500 \text{ ft}) = 31.2\times10^3 \text{ lb/ft}^2$$

$$= 216.67 \text{ psi}$$

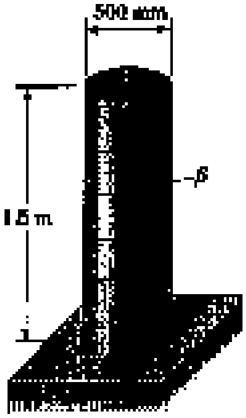
$$\sigma_1 = 12.5 \text{ ksi} = 12.5\times10^3 \text{ psi}$$

$$r = \tfrac{1}{2}d - t = 18 - t$$

$$\sigma_1 = \frac{pr}{t}, \qquad \frac{r}{t} = \frac{\sigma_1}{p}, \qquad \frac{18-t}{t} = \frac{12.5\times10^3}{216.67} = 57.692$$

$$\frac{18}{t} = 58.692 \qquad t = 0.307 \text{ in.}$$

PROBLEM 7.110

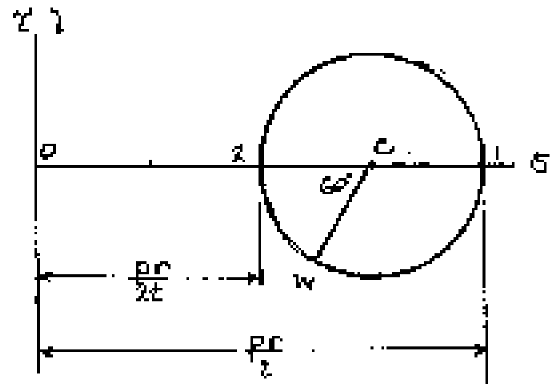


7.110 The cylindrical portion of the compressed air tank shown is fabricated of 6-mm-thick plate welded along a helix forming an angle $\beta = 30°$ with the horizontal. Knowing that the allowable stress normal to the weld is 75 MPa, determine the largest gage pressure that can be used in the tank.

SOLUTION

$$r = \tfrac{1}{2}d - t = \tfrac{1}{2}(500) - 6 = 244 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{1}{2}\frac{pr}{t}$$



$$\sigma_{ave} = \tfrac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

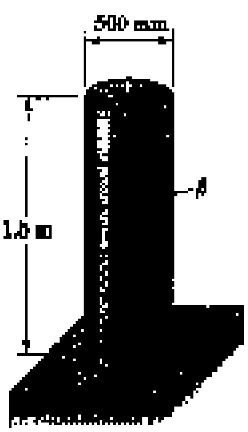
$$\sigma_w = \sigma_{ave} - R\cos 60°$$

$$= \frac{5}{8}\frac{pr}{t}$$

$$p = \frac{8}{5}\frac{\sigma_w t}{r}$$

$$p = \frac{8}{5}\frac{(75)(6)}{244} = 2.95 \text{ MPa}$$

PROBLEM 7.111

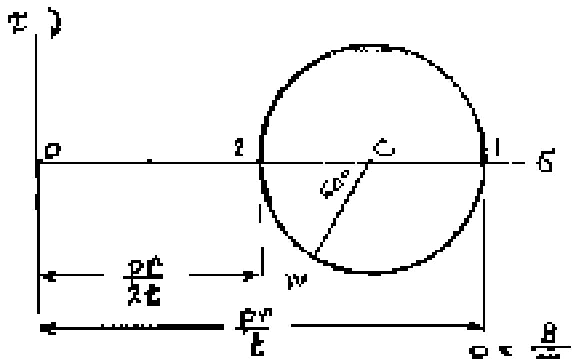


7.111 The cylindrical portion of the compressed air tank shown is fabricated of 6-mm-thick plate welded along a helix forming an angle $\beta = 30°$ with the horizontal. Determine the gage pressure that will cause a shearing stress parallel to the weld of 30 MPa.

SOLUTION

$$r = \tfrac{1}{2}d - t = \tfrac{1}{2}(500) - 6 = 244 \text{ mm}$$

$$\sigma_1 = \frac{pr}{t}, \qquad \sigma_2 = \frac{1}{2}\frac{pr}{t}$$



$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

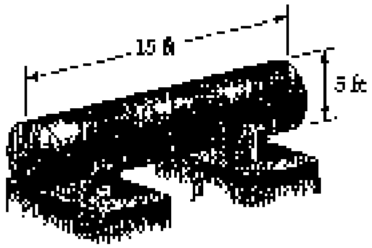
$$\tau_w = R\sin 60°$$

$$= \frac{\sqrt{3}}{8}\frac{pr}{t}$$

$$p = \frac{8}{\sqrt{3}}\frac{\tau_w t}{r}$$

$$p = \frac{8}{\sqrt{3}}\frac{(30)(6)}{244} = 3.41 \text{ MPa}$$

PROBLEM 7.112



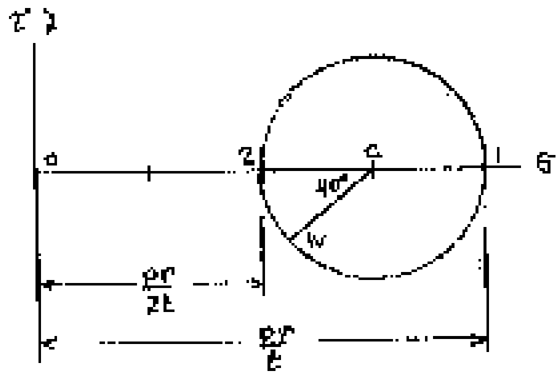
7.112 The pressure tank shown has a $\frac{3}{8}$-in. wall thickness and butt-welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 85 psi, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$d = 5\text{ ft} = 60\text{ in.} \qquad r = \frac{1}{2}d - t = 30 - \frac{3}{8} = 29.625\text{ in}$

$$\sigma_1 = \frac{pr}{t} = \frac{(85)(29.625)}{0.375} = 6715\text{ psi}$$

$$\sigma_2 = \frac{1}{2}\sigma_1 = 3357.5\text{ psi}$$



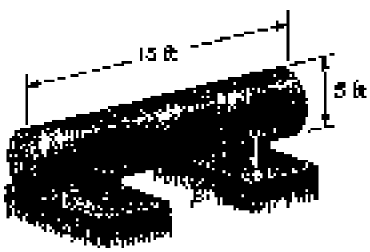
$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 5036.25\text{ psi}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = 1678.75\text{ psi}$$

(a) $\sigma_w = \sigma_{ave} - R\cos 40° = 3750\text{ psi}$

(b) $\tau_w = R\sin 40° = 1078\text{ psi}$

PROBLEM 7.113



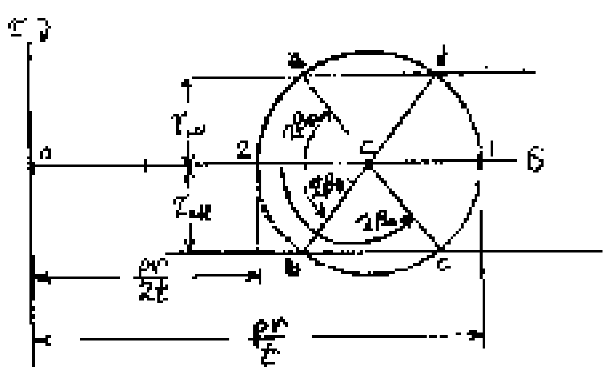
7.113 The pressure tank shown has a $\frac{3}{8}$-in. wall thickness and butt-welded seams forming an angle β with a transverse plane. Determine the range of values of β that can be used if the shearing stress parallel to the weld is not to exceed 1350 psi when the gage pressure is 85 psi.

SOLUTION

$d = 5\text{ ft} = 60\text{ in} \qquad r = \frac{1}{2}d - t = 30 - \frac{3}{8} = 29.625\text{ in.}$

$$\sigma_1 = \frac{pr}{t} = \frac{(85)(29.625)}{0.375} = 6715\text{ psi}$$

$$\sigma_2 = \frac{1}{2}\sigma_1 = 3357.5\text{ psi}$$



$$R = \frac{\sigma_1 - \sigma_2}{2} = 1678.75$$

$$\tau_w = R\sin 2\beta = \tau_w$$

$$\sin 2\beta_a = \frac{\tau_w}{R} = \frac{1350}{1678.75} = 0.80417$$

$2\beta_a = -53.53°$

$2\beta_b = +53.53°$

$2\beta_c = -53.53° + 180° = 126.47°$

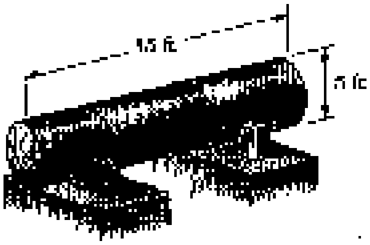
$2\beta_d = 53.53° + 180° = 233.53°$

$\left.\begin{array}{l}\beta_a = -26.8°\\ \beta_b = 26.8°\end{array}\right\} \quad -26.8° \le \beta \le 26.8°$

$\left.\begin{array}{l}\beta_c = 63.2°\\ \beta_d = 116.8°\end{array}\right\} \quad 63.2° \le \beta \le 116.8°$

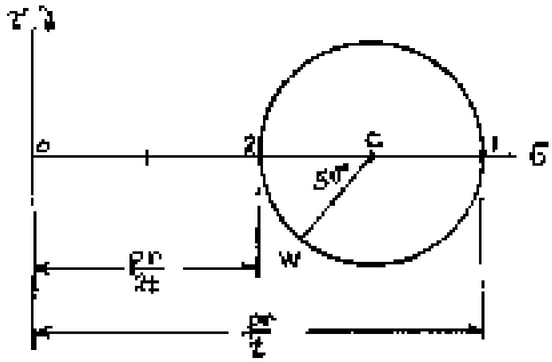
PROBLEM 7.114

7.114 The pressure tank shown has a $\frac{3}{8}$-in. wall thickness and butt-welded seams forming an angle $\beta = 25°$ with a transverse plane. Determine the largest allowable gage pressure, knowing that the allowable normal stress perpendicular to the weld is 18 ksi and the allowable shearing stress parallel to the weld is 10 ksi.



SOLUTION

$d = 5\text{ ft} = 60\text{ in.} \qquad r = \frac{1}{2}d - t = 30 - \frac{3}{8} = 29.625\text{ in}$



$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 = \frac{pr}{2t}$$

$$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{ave} - R\cos 50°$$

$$= \left(\frac{3}{4} - \frac{1}{4}\cos 50°\right)\frac{pr}{t}$$

$$= 0.5893\frac{pr}{t}$$

$$p = \frac{\sigma_w t}{0.5893\, r} = \frac{(18)(0.375)}{(0.5893)(29.625)} = 0.387\text{ ksi} = 387\text{ psi}$$

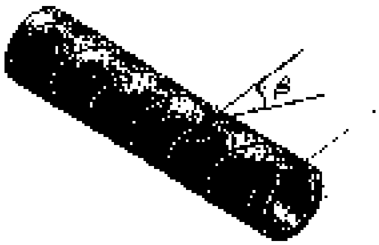
$$\tau_w = R\sin 50° = 0.19151\frac{pr}{t}$$

$$p = \frac{\tau_w t}{0.19151\, r} = \frac{(10)(0.375)}{(0.19151)(29.625)} = 0.661\text{ ksi} = 661\text{ psi}$$

Allowable gage pressure is the smaller value $\qquad p = 387\text{ psi}$

PROBLEM 7.115

7.115 The pipe shown was fabricated by welding strips of plate along a helix forming an angle β with a transverse plane. Determine the largest value of β that can be used if the normal stress perpendicular to the weld is not to be larger than 85 percent of the maximum stress in the pipe.



SOLUTION

$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$

$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$

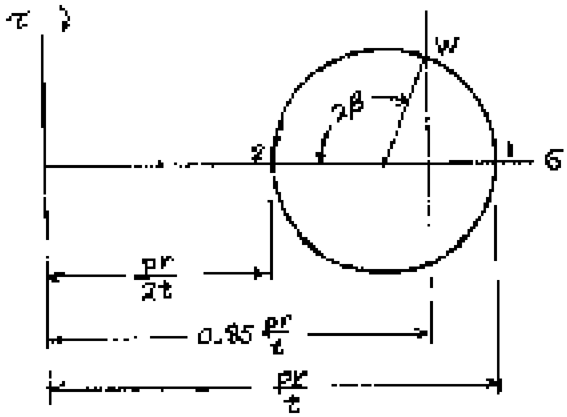
$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$

$\sigma_w = \sigma_{ave} - R\cos 2\beta$

$0.85\frac{pr}{t} = \left(\frac{3}{4} - \frac{1}{4}\cos 2\beta\right)\frac{pr}{t}$

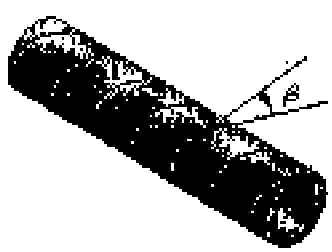
$\cos 2\beta = -4\left(0.85 - \frac{3}{4}\right) = -0.4$

$2\beta = 113.6° \qquad \beta = 56.8°$



PROBLEM 7.116

7.116 The pipe shown has a diameter of 600 mm and was fabricated by welding strips of 10-mm-thick plate along a helix forming an angle β = 25° with a transverse plane. Knowing that the ultimate normal stress perpendicular to the weld is 450 MPa and that a factor of safety of 6.0 is desired, determine the largest allowable gage pressure that can be used.



SOLUTION

$t = 10 \text{ mm} \qquad r = \frac{1}{2}d - t = 300 - 10 = 290 \text{ mm}$

$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$

$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$

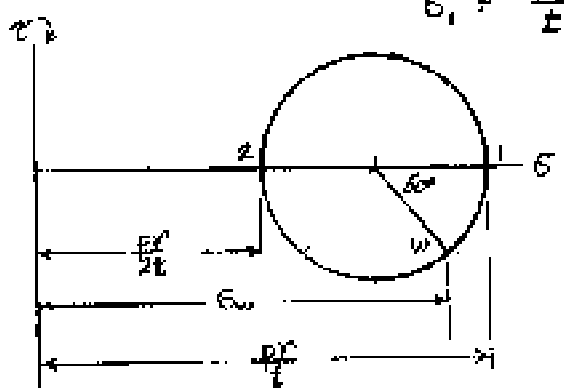
$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$

$\sigma_w = \sigma_{ave} + R\cos 50° = 0.9107\frac{pr}{t}$

$\sigma_{w,all} = \frac{\sigma_U}{F.S} = \frac{450}{6} = 75 \text{ MPa}$

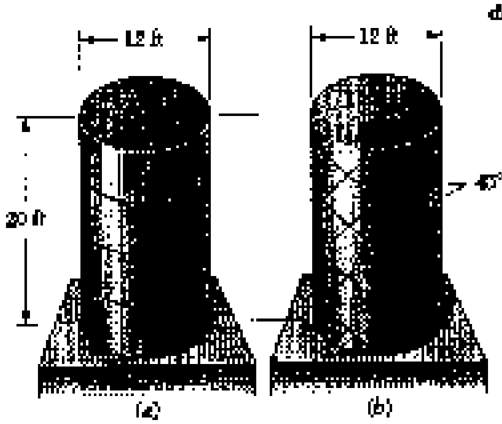
$0.9107\frac{pr}{t} = 75$

$p = \frac{(75)(10)}{(0.9107)(290)} = 2.84 \text{ MPa}$



PROBLEM 7.117

7.117 Square plates, each of 0.5-in. thickness, can be bent and welded together in either of the two ways shown to form the cylindrical portion of a compressed-air tank. Knowing that the allowable normal stress perpendicular to the weld is 12 ksi, determine the largest allowable gage pressure in each case.



SOLUTION

$d = 12 \text{ ft} = 144 \text{ in} \qquad r = \frac{1}{2}d - t = 71.5 \text{ in}$

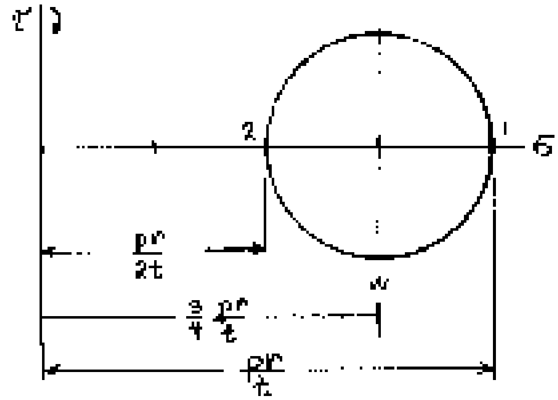
$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$

(a) $\sigma_1 = 12 \text{ ksi}$

$$p = \frac{\sigma_1 t}{r} = \frac{(12)(0.5)}{71.5} = 0.0839 \text{ ksi}$$

$$= 83.9 \text{ psi}$$

(b)



$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$

$R = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{4}\frac{pr}{t}$

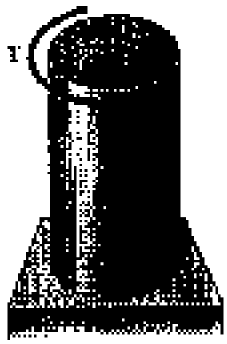
$\beta = \pm 45°$

$\sigma_w = \sigma_{ave} + R\cos\beta$

$= \frac{3}{4}\frac{pr}{t}$

$$p = \frac{4}{3}\frac{\sigma_w t}{r} = \frac{4}{3}\cdot\frac{(12)(0.5)}{71.5} = 0.1119 \text{ ksi} = 111.9 \text{ psi}$$

PROBLEM 7.118



7.118 A torque of magnitude $T = 12$ kN · m is applied to the end of a tank containing compressed air under a pressure of 8 MPa. Knowing that the tank has a 180-mm inside diameter and a 12-mm wall thickness, determine the maximum normal stress and the maximum shearing stress in the tank.

SOLUTION

$d = 180$ mm $\quad r = \frac{1}{2}d = 90$ mm $\quad t = 12$ mm

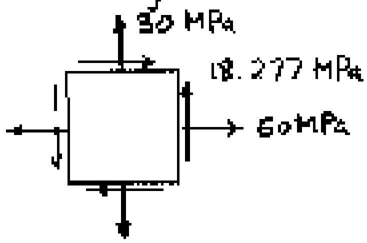
Torsion: $c_1 = 90$ mm $\quad c_2 = 90 + 12 = 102$ mm

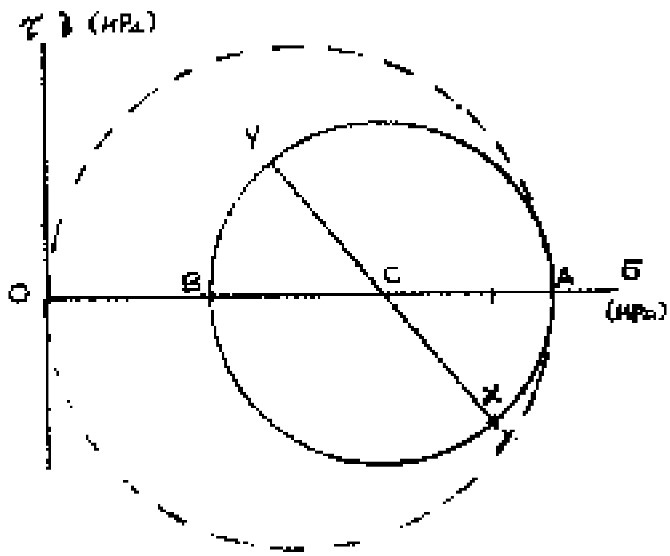
$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} = \frac{(12 \times 10^3)(102 \times 10^{-3})}{66.968 \times 10^{-6}} = 18.277 \text{ MPa}$$

Pressure: $\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60$ MPa $\qquad \sigma_2 = \frac{pr}{2t} = 30$ MPa

Summary of stresses $\qquad \sigma_x = 60$ MPa, $\sigma_y = 30$ MPa, $\tau_{xy} = 18.277$ MPa





$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 45$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 23.64$ MPa

$\sigma_a = \sigma_{ave} + R = 68.64$ MPa

$\sigma_b = \sigma_{ave} - R = 21.36$ MPa

$\sigma_c \approx 0$

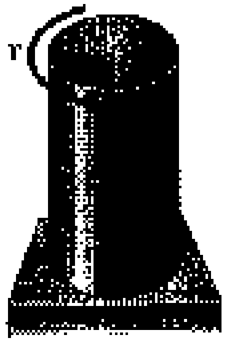
$\sigma_{max} = 68.64$ MPa ◀

$\sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 34.32$ MPa ◀

PROBLEM 7.119

7.119 The tank shown has a 180-mm inside diameter and a 12-mm wall thickness. Knowing that the tank contains compressed air under a pressure of 8 MPa, determine the magnitude T of the applied torque for which the maximum normal stress in the tank is 75 MPa.



SOLUTION

$r = \frac{1}{2}d = (\frac{1}{2})(180) = 90$ mm $\qquad t = 12$ mm

$\sigma_1 = \frac{pr}{t} = \frac{(8)(90)}{12} = 60$ MPa $\qquad \sigma_2 = \frac{pr}{2t} = 30$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 45$ MPa

$\sigma_{max} = 75$ MPa $\qquad R = \sigma_{max} - \sigma_{ave} = 30$ MPa

$R = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau_{xy}^2} = \sqrt{15^2 + \tau_{xy}^2}$

$\tau_{xy} = \sqrt{R^2 - 15^2} = \sqrt{30^2 - 15^2} = 25.98$ MPa

$= 25.98 \times 10^6$ Pa

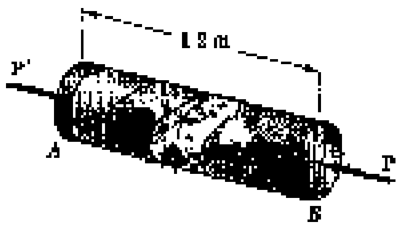
Torsion: $c_1 = 90$ mm $\qquad c_2 = 90 + 12 = 102$ mm

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 66.968 \times 10^6 \text{ mm}^4 = 66.968 \times 10^{-6} \text{ m}^4$

$\tau_{xy} = \frac{Tc}{J} \qquad T = \frac{J\tau_{xy}}{c} = \frac{(66.968 \times 10^{-6})(25.98 \times 10^6)}{102 \times 10^{-3}} = 17.06 \times 10^3$ N·m

$= 17.06$ kN·m

PROBLEM 7.120



7.120 A pressure vessel of 250-mm inside diameter and 6-mm wall thickness is fabricated from a 1.2-m section of spirally welded pipe *AB* and is with two rigid end plates. The gage pressure inside the vessel is 2 MPa and 45-kN centric axial forces **P** and **P'** are applied to the end plates. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$r = \frac{1}{2}d = 125$ mm $\qquad t = 6$ mm

$$\sigma_1 = \frac{pr}{t} = \frac{(2)(125)}{6} = 41.67 \text{ MPa}$$

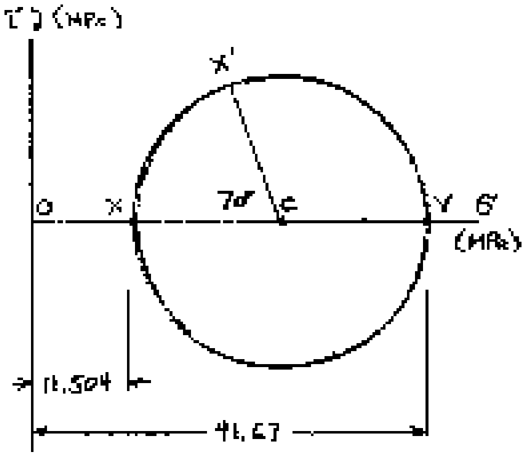
$$\sigma_2 = \frac{pr}{2t} = \frac{(2)(125)}{(2)(6)} = 20.83 \text{ MPa}$$

$r_o = r + t = 125 + 6 = 131$ mm $\qquad A = \pi(r_o^2 - r^2) = 4.825 \times 10^3 \text{ mm}^2 = 4.825 \times 10^{-3} \text{ m}^2$

$$\sigma = -\frac{P}{A} = -\frac{45 \times 10^3}{4.825 \times 10^{-3}} = -9.326 \times 10^6 \text{ Pa} = -9.326 \text{ MPa}$$

Total stresses: Longitudinal $\quad \sigma_x = 20.83 - 9.326 = 11.504$ MPa

Circumferential $\quad \sigma_y = 41.67$ MPa



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 26.585$ MPa

$R = \frac{\sigma_x - \sigma_y}{2} = 15.081$

(a) $\sigma_{x'} = \sigma_{ave} + R\cos 70°$

$= 26.585 - 15.081 \cos 70°$

$= 21.4$ MPa

(b) $\tau_{x'y'} = R\sin 70° = 15.081 \sin 70°$

$= 14.17$ MPa

PROBLEM 7.121

7.120 A pressure vessel of 250-mm inside diameter and 6-mm wall thickness is fabricated from a 1.2-m section of spirally welded pipe *AB* and is with two rigid end plates. The gage pressure inside the vessel is 2 MPa and 45-kN centric axial forces **P** and **P'** are applied to the end plates. Determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

7.121 Solve Prob. 7.120, assuming that the magnitude *P* of the two forces is increased to 120 kN.

SOLUTION

$r = \frac{1}{2}d = 125$ mm $\qquad t = 6$ mm

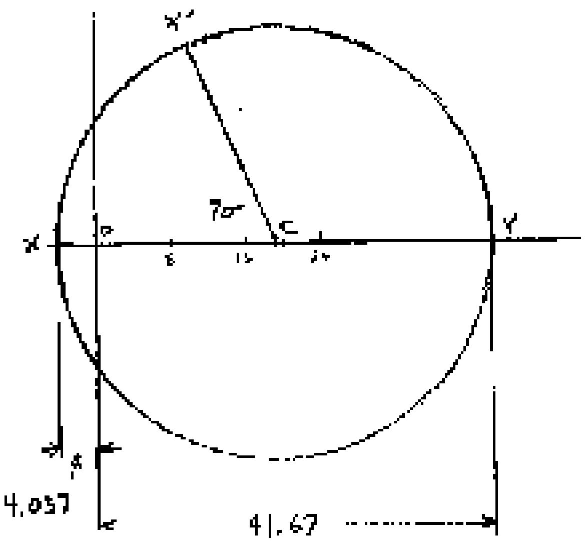
$\sigma_1 = \frac{pr}{t} = \frac{(2)(125)}{6} = 41.67$ MPa $\qquad \sigma_2 = \frac{pr}{2t} = 20.833$ MPa

$r_o = r + t = 125 + 6 = 131$ mm $\qquad A = \pi(r_o^2 - r^2) = 4.825 \times 10^3\ \text{mm}^2 = 4.825 \times 10^{-3}\ \text{m}^2$

$\sigma = -\frac{P}{A} = -\frac{120 \times 10^3}{4.825 \times 10^{-3}} = -24.870 \times 10^6$ Pa $= -24.870$ MPa

Total stresses: Longitudinal $\sigma_x = 20.833 - 24.870 = -4.037$ MPa

Circumferential $\sigma_y = 41.67$ MPa



$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 18.815$ MPa

$R = \left|\frac{\sigma_x - \sigma_y}{2}\right| = 22.852$ MPa

(a) $\sigma_{x'} = \sigma_{ave} - R\cos 70°$

$= 18.815 - 22.852 \cos 70°$

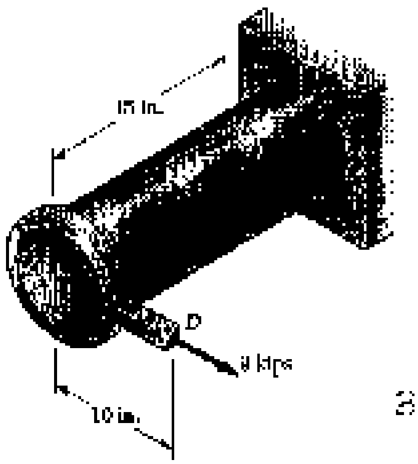
$= 11.00$ MPa ◄

(b) $\tau_{xy} = R \sin 70° = 22.852 \sin 70°$

$= 21.5$ MPa ◄

PROBLEM 7.122

7.122 The cylindrical tank *AB* has an 8-in. inside diameter and a 0.32-in. wall thickness. Knowing that the pressure inside the tank is 600 psi, determine the maximum normal stress and the maximum shearing stress at point *K*.



SOLUTION

$r_i = \frac{d}{2} = 4 \text{ in} \qquad r_o = r_i + t = 4.32 \text{ in.}$

$\sigma_1 = \frac{p r_i}{t} = \frac{(600)(4)}{0.32} = 7500 \text{ psi} = 7.50 \text{ ksi}$

$\sigma_2 = \frac{1}{2}\sigma_1 = 3.75 \text{ ksi}$

Torsion: No applied torque

Bending: Point K lies on neutral axis.

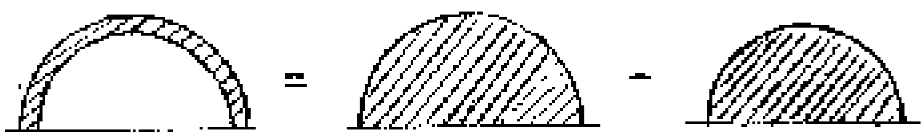
Transverse shear: $V = 9$ kips

For semicircle

$A = \frac{\pi}{2} r^2$

$\bar{y} = \frac{4r}{3\pi}$

$Q = \frac{2}{3} r^3$



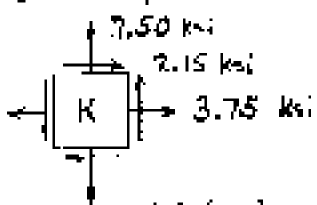
$Q = Q_o - Q_i = \frac{2}{3} r_o^3 - \frac{2}{3} r_i^3 = \frac{2}{3}(4.32^3 - 4^3) = 11.081 \text{ in}^3$

$t = (2)(0.32) = 0.64 \text{ in}$

$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(4.32^4 - 4^4) = 72.481 \text{ in}^4$

$\tau = \frac{VQ}{It} = \frac{(9)(11.081)}{(72.481)(0.64)} = 2.15 \text{ ksi}$

Summary of stresses:



Longitudinal $\sigma_x = \sigma_2 = 3.75$ ksi

Circumferential $\sigma_y = \sigma_1 = 7.50$ ksi

Shear $\tau_{xy} = 2.15$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 5.625 \text{ ksi}$

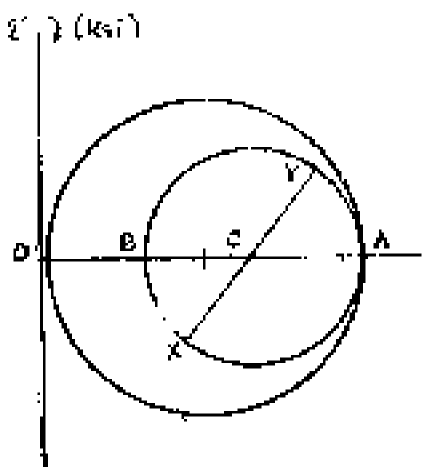
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= 2.853 \text{ ksi}$

$\sigma_a = \sigma_{ave} + R = 8.48 \text{ ksi}$

$\sigma_b = \sigma_{ave} - R = 2.77 \text{ ksi}$

$\sigma_z = 0$

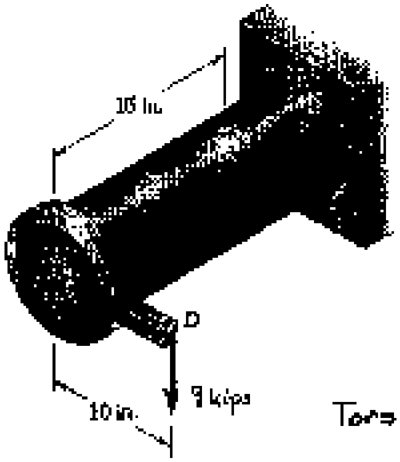


$\sigma_{max} = 8.48 \text{ ksi}$ ◄

$\sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 4.24 \text{ ksi}$ ◄

PROBLEM 7.123



7.122 The cylindrical tank *AB* has an 8-in. inside diameter and a 0.32-in. wall thickness. Knowing that the pressure inside the tank is 600 psi, determine the maximum normal stress and the maximum shearing stress at point *K*.

7.123 Solve Prob. 7.122, assuming that the 9-kip force applied at point *D* is directed vertically downward.

SOLUTION

$r_1 = \frac{d_1}{2} = 4 \text{ in.} \qquad r_2 = r_1 + t = 4.32 \text{ in}$

$\sigma_1 = \frac{p r_1}{t} = \frac{(600)(4)}{0.32} = 7500 \text{ psi} = 7.50 \text{ ksi}$

$\sigma_2 = \frac{1}{2}\sigma_1 = 3.75 \text{ ksi}$

Torsion: $J = \frac{\pi}{2}(r_2^4 - r_1^4) = 144.96 \text{ in}^4 \qquad c = r_2 = 4.32 \text{ in}$

$T = (9)(10) = 90 \text{ kip}\cdot\text{in}$

$\tau = \frac{Tc}{J} = \frac{(90)(4.32)}{144.96} = 2.68 \text{ ksi}$

Bending: $I = \frac{1}{2}J = 72.48 \text{ in}^4 \qquad c = r_2 = 4.32 \text{ in}$

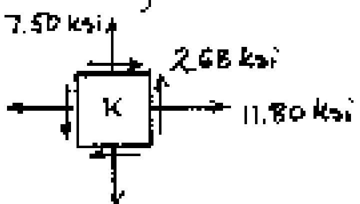
$M = (9)(15) = 135 \text{ kip}\cdot\text{in} \qquad \sigma_m = \frac{Mc}{I} = \frac{(135)(4.32)}{72.48} = 8.05 \text{ ksi}$

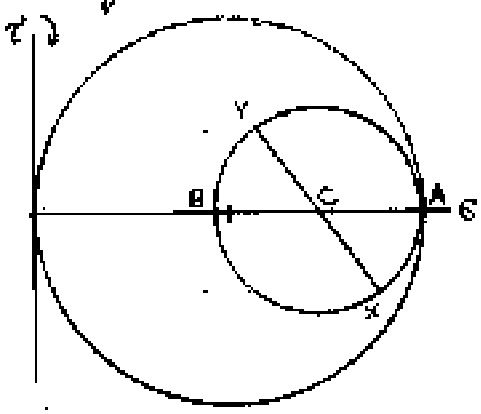
Transverse shear: At point K, $VQ/It = 0$

Summary of stresses: Longitudinal $\sigma_x = \sigma_1 = 3.75 + 8.05 = 11.80 \text{ ksi}$

Circumferential $\sigma_y = \sigma_2 = 7.50 \text{ ksi}$

Shear $\tau_{xy} = 2.68 \text{ ksi}$





$\sigma_{ave} = \frac{1}{2}(11.80 + 7.50) = 9.65 \text{ ksi}$

$R = \sqrt{\left(\frac{11.80 - 7.50}{2}\right)^2 + (2.68)^2} = 3.44 \text{ ksi}$

$\sigma_a = \sigma_{ave} + R = 13.09 \text{ ksi}$

$\sigma_b = \sigma_{ave} - R = 6.21 \text{ ksi}$

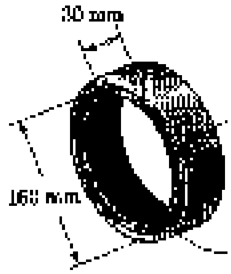
$\sigma_c = 0$

$\sigma_{max} = 13.09 \text{ ksi}$ ◄

$\sigma_{min} = 0$

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 6.54 \text{ ksi}$ ◄

PROBLEM 7.124



7.124 A brass ring of 160-mm outside diameter fits exactly inside a steel ring of 160-mm inside diameter when the temperature of both rings is 5° C. Knowing that the temperature of the rings is then raised to 55° C, determine (*a*) the tensile stress in the steel ring, (*b*) the corresponding pressure exerted by the brass ring on the steel ring.

SOLUTION

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring: Internal pressure p, $\quad \sigma_s = \dfrac{pr}{t_s} \qquad (1)$

Corresponding strain $\quad \varepsilon_{sp} = \dfrac{\sigma_s}{E_s} = \dfrac{pr}{E_s t_s}$

Strain due to temperature change $\quad \varepsilon_{sT} = \alpha_s \, \Delta T$

Total strain $\quad \varepsilon_s = \dfrac{pr}{E_s t_s} + \alpha_s \, \Delta T$

Change in length of circumference

$$\Delta L_s = 2\pi r \, \varepsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \, \Delta T \right)$$

Brass ring: External pressure p, $\quad \sigma_b = -\dfrac{pr}{t_b}$

Corresponding strains $\quad \varepsilon_{bp} = -\dfrac{pr}{E_b t_b}, \quad \varepsilon_{bT} = \alpha_b \, \Delta T$

Change in length of circumference

$$\Delta L_b = 2\pi r \, \varepsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \, \Delta T \right)$$

Equating ΔL_s to ΔL_b $\qquad \dfrac{pr}{E_s t_s} + \alpha_s \, \Delta T = -\dfrac{pr}{E_b t_b} + \alpha_b \, \Delta T$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s) \, \Delta T \qquad (2)$$

Data $\quad \Delta T = 55°C - 5° = 50°C$

$r = \frac{1}{2} d = 80$ mm

From eq. (2) $\quad \left\{ \dfrac{80 \times 10^{-3}}{(210 \times 10^9)(4 \times 10^{-3})} + \dfrac{80 \times 10^{-3}}{(105 \times 10^9)(4 \times 10^{-3})} \right\} p = (8 \times 10^{-6})(50)$

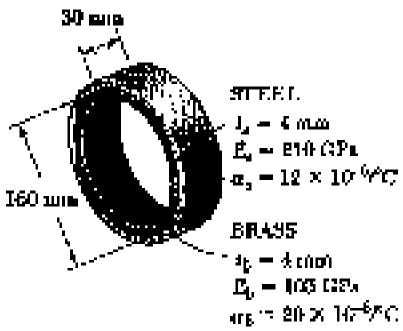
$285.71 \times 10^{-12} \, p = 400 \times 10^{-6}, \qquad p = 1.4 \times 10^6$ Pa

From eq. (1) $\quad \sigma_s = \dfrac{pr}{t_s} = \dfrac{(1.4 \times 10^6)(80 \times 10^{-3})}{4 \times 10^{-3}} = 28 \times 10^6$ Pa

(a) $\quad \sigma_s = 28.0$ MPa

(b) $\quad p = 1.400$ MPa

PROBLEM 7.125



7.124 A brass ring of 160-mm outside diameter fits exactly inside a steel ring of 160-mm inside diameter when the temperature of both rings is 5° C. Knowing that the temperature of the rings is then raised to 55° C, determine (*a*) the tensile stress in the steel ring, (*b*) the corresponding pressure exerted by the brass ring on the steel ring.

7.125 Solve Prob. 7.124, assuming that the thickness of the brass ring is t_b = 6 mm.

SOLUTION

Let p be the contact pressure between the rings. Subscript s refers to the steel ring. Subscript b refers to the brass ring.

Steel ring: Internal pressure p, $\quad \sigma_s = \dfrac{pr}{t_s} \qquad (1)$

Corresponding strain $\quad \varepsilon_{sp} = \dfrac{\sigma_s}{E_s} = \dfrac{pr}{E_s t_s}$

Strain due to temperature change $\quad \varepsilon_{sT} = \alpha_s \Delta T$

Total strain $\quad \varepsilon_s = \dfrac{pr}{E_s t_s} + \alpha_s \Delta T$

Change in length of circumference

$$\Delta L_s = 2\pi r \varepsilon_s = 2\pi r \left(\frac{pr}{E_s t_s} + \alpha_s \Delta T \right)$$

Brass ring: External pressure p, $\quad \sigma_b = -\dfrac{pr}{t_b}$

Corresponding strains $\quad \varepsilon_{bp} = -\dfrac{pr}{E_b t_b}, \quad \varepsilon_{bT} = \alpha_b \Delta T$

Change in length of circumference

$$\Delta L_b = 2\pi r \varepsilon_b = 2\pi r \left(-\frac{pr}{E_b t_b} + \alpha_b \Delta T \right)$$

Equating ΔL_s to ΔL_b $\qquad \dfrac{pr}{E_s t_s} + \alpha_s \Delta T = -\dfrac{pr}{E_b t_b} + \alpha_b \Delta T$

$$\left(\frac{r}{E_s t_s} + \frac{r}{E_b t_b} \right) p = (\alpha_b - \alpha_s) \Delta T \qquad (2)$$

Data: $\Delta T = 55°C - 5°C = 50°C \qquad t_b = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$

$r = \frac{1}{2} d = 80$ mm

From eq. (2) $\left\{ \dfrac{(80 \times 10^{-3})}{(210 \times 10^9)(4 \times 10^{-3})} + \dfrac{(80 \times 10^{-3})}{(105 \times 10^9)(6 \times 10^{-3})} \right\} p = (8 \times 10^{-6})(50)$

$222.22 \times 10^{-12}\, p = 400 \times 10^{-6} \qquad p = 1.8 \times 10^6$ Pa

From eq. (1) $\quad \sigma_s = \dfrac{pr}{t_s} = \dfrac{(1.8 \times 10^6)(80 \times 10^{-3})}{4 \times 10^{-3}} = 36 \times 10^6$ Pa

$\sigma_s = 36.0$ MPa ◄

$p = 1.800$ MPa ◄

PROBLEM 7.126

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ.

$\varepsilon_x = -720\mu$, $\varepsilon_y = 0$, $\gamma_{xy} = +360\mu$, $\theta = -30°$

SOLUTION

$$\frac{\varepsilon_x+\varepsilon_y}{2} = -360\mu \qquad \frac{\varepsilon_x-\varepsilon_y}{2} = -360\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{-360 - 360\cos(-60°) + \frac{300}{2}\sin(-60°)\right\}\mu = -670\mu$$

$$\varepsilon_{y'} = \frac{\varepsilon_x+\varepsilon_y}{2} - \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{-360 - (-360)\cos(-60°) - \frac{300}{2}\sin(-60°)\right\}\mu = -50\mu$$

$$\gamma_{x'y'} = -(\varepsilon_x-\varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= \left\{-(-720-0)\sin(-60°) + 300\cos(-60°)\right\}\mu = -474\mu$$

PROBLEM 7.127

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ.

$\varepsilon_x = 0$ $\quad \varepsilon_y = +320\mu$ $\quad \gamma_{xy} = -100\mu$ $\quad \theta = 30°$

SOLUTION

$$\frac{\varepsilon_x+\varepsilon_y}{2} = 160\mu \qquad \frac{\varepsilon_x-\varepsilon_y}{2} = -160\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x+\varepsilon_y}{2} + \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{160 - 160\cos 60° - \frac{100}{2}\sin 60°\right\}\mu = +36.7\mu$$

$$\varepsilon_{y'} = \frac{\varepsilon_x+\varepsilon_y}{2} - \frac{\varepsilon_x-\varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{160 + 160\cos 60° + \frac{100}{2}\sin 60°\right\}\mu = +283\mu$$

$$\gamma_{x'y'} = -(\varepsilon_x-\varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= \left\{-(0-320)\sin 60° - 100\cos 60°\right\}\mu = +227\mu$$

PROBLEM 7.128

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ.

$\varepsilon_x = -800\,\mu$, $\varepsilon_y = +450\,\mu$, $\gamma_{xy} = +200\,\mu$, $\theta = -25°$

SOLUTION

$$\frac{\varepsilon_x + \varepsilon_y}{2} = -175\,\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -625\,\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{-175 - 625\cos(-50°) + \frac{200}{2}\sin(-50°)\right\}\mu = -653\,\mu$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left\{-175 + 625\cos(-50°) - \frac{200}{2}\sin(-50°)\right\}\mu = +303\,\mu$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= \left\{-(-800 - 450)\sin(-50°) + 200\cos(-50°)\right\}\mu = -829\,\mu$$

PROBLEM 7.129

7.126 through 7.129 For the given state of plane strain, use the methods of Sec. 7.10 to determine the state of strain associated with axes x' and y' rotated through the given angle θ.

$\varepsilon_x = -500\,\mu$, $\varepsilon_y = +250\,\mu$, $\gamma_{xy} = 0$, $\theta = 15°$

SOLUTION

$$\frac{\varepsilon_x + \varepsilon_y}{2} = -125\,\mu \qquad \frac{\varepsilon_x - \varepsilon_y}{2} = -375\,\mu$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \{-125 - 375\cos 30° + 0\}\mu = -450\,\mu$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \{-125 + 375\cos 30° - 0\}\mu = +200\,\mu$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

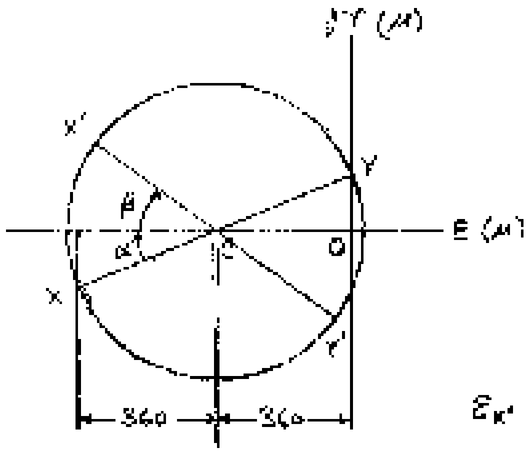
$$= \{-(-500 - 250)\sin 30° + 0\}\mu = +375\,\mu$$

PROBLEM 7.130

7.130 through 7.133 For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes x' and y' rotated through the angle θ.

$\varepsilon_x = -720\,\mu$, $\varepsilon_y = 0$, $\gamma_{xy} = +300\,\mu$, $\theta = -30°$

SOLUTION



Plotted points

X: $(-720\,\mu,\ -150\,\mu)$

Y: $(0,\ 150\,\mu)$

C: $(-360\,\mu,\ 0)$

$\tan\alpha = \dfrac{150\,\mu}{360\,\mu}$ $\qquad \alpha = 22.62°$

$R = \sqrt{(360\,\mu)^2 + (150\,\mu)^2} = 390\,\mu$

$\beta = 2\theta - \alpha = 60° - 22.62° = 37.38°$

$\varepsilon_{x'} = \varepsilon_{ave} - R\cos\beta = -360\,\mu - 390\,\mu\cos 37.38°$

$= -670\,\mu$

$\varepsilon_{y'} = \varepsilon_{ave} + R\cos\beta = -360\,\mu + 390\,\mu\cos 37.38°$

$= -50\,\mu$

$\dfrac{\gamma_{x'y'}}{2} = -R\sin\beta = -390\,\mu\sin 37.38°$

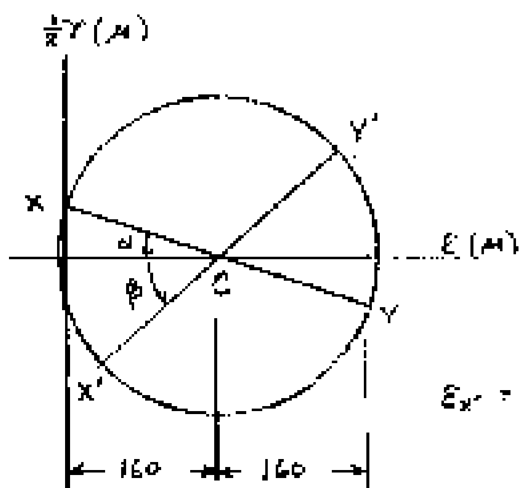
$\gamma_{x'y'} = -474\,\mu$

PROBLEM 7.131

7.130 through 7.133 For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes x' and y' rotated through the angle θ.

$\varepsilon_x = 0$ $\quad \varepsilon_y = +320\,\mu$ $\quad \gamma_{xy} = -100\,\mu$ $\quad \theta = 30°$

SOLUTION



Plotted points

X: $(0,\ 50\,\mu)$

Y: $(320\,\mu,\ -50\,\mu)$

C: $(160\,\mu,\ 0)$

$\tan\alpha = \dfrac{50}{160}$ $\qquad \alpha = 17.35°$

$R = \sqrt{(160\,\mu)^2 + (50\,\mu)^2} = 167.63\,\mu$

$\beta = 2\theta - \alpha = 60° - 17.35° = 42.65°$

$\varepsilon_{x'} = \varepsilon_{ave} - R\cos\beta = 160\,\mu - 167.63\,\mu\cos 42.65°$

$= -36.7\,\mu$

$\varepsilon_{y'} = \varepsilon_{ave} + R\cos\beta = 160\,\mu + 167.63\,\mu\cos 42.65°$

$= 283\,\mu$

$\dfrac{\gamma_{x'y'}}{2} = R\sin\beta = 167.63\,\mu\sin 42.65°$

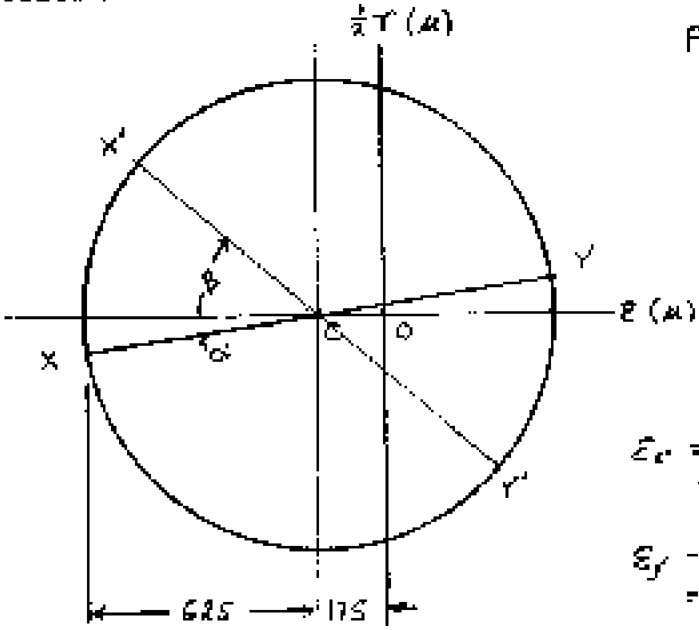
$\gamma_{x'y'} = 227\,\mu$

PROBLEM 7.132

7.130 through 7.133 For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes x' and y' rotated through the angle θ.

$\varepsilon_x = -800\,\mu \quad \varepsilon_y = 450\,\mu \quad \gamma_{xy} = +200\,\mu \quad \theta = -25°$

SOLUTION



Plotted points

X: $(-800\,\mu,\ -100\,\mu)$

Y: $(+450\,\mu,\ +100\,\mu)$

C: $(-175\,\mu,\ 0)$

$\tan\alpha = \dfrac{100}{625} \qquad \alpha = 9.09°$

$R = \sqrt{(625\,\mu)^2 + (100\,\mu)^2} = 632.95\,\mu$

$\beta = 2\theta - \alpha = 50° - 9.09° = 40.91°$

$\varepsilon_{x'} = \varepsilon_{ave} - R\cos\beta = -175\,\mu - 632.95\,\mu \cos 40.91°$
$= -653\,\mu$

$\varepsilon_{y'} = \varepsilon_{ave} + R\cos\beta = -175$
$= +303\,\mu$

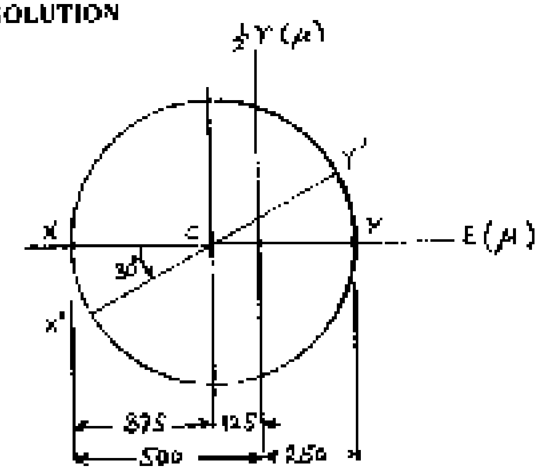
$\dfrac{\gamma_{x'y'}}{2} = -R\sin\beta = -632.95\,\mu \sin 40.91° \qquad \gamma_{x'y'} = -829\,\mu$

PROBLEM 7.133

7.130 through 7.133 For the given state of plane strain, use Mohr's circle to determine the state of strain associated with axes x' and y' rotated through the angle θ.

$\varepsilon_x = -500\,\mu, \quad \varepsilon_y = +250\,\mu, \quad \gamma_{xy} = 0, \quad \theta = 15°$

SOLUTION



Plotted points

X: $(-500\,\mu,\ 0)$

Y: $(+250\,\mu,\ 0)$

C: $(-125\,\mu,\ 0)$

$R = 375\,\mu$

$\varepsilon_{x'} = \varepsilon_{ave} - R\cos 2\theta = -125 - 375\cos 30°$
$= -450\,\mu$

$\varepsilon_{y'} = \varepsilon_{ave} + R\cos 2\theta = -125 + 375\cos 30°$
$= 200\,\mu$

$\frac{1}{2}\gamma_{xy} = R\sin 2\theta = 375\sin 30°$

$\gamma_{x'y'} = 375\,\mu$

PROBLEM 7.134

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$.)

$\varepsilon_x = +160\,\mu \qquad \varepsilon_y = -480\,\mu \qquad \gamma_{xy} = 600\,\mu$

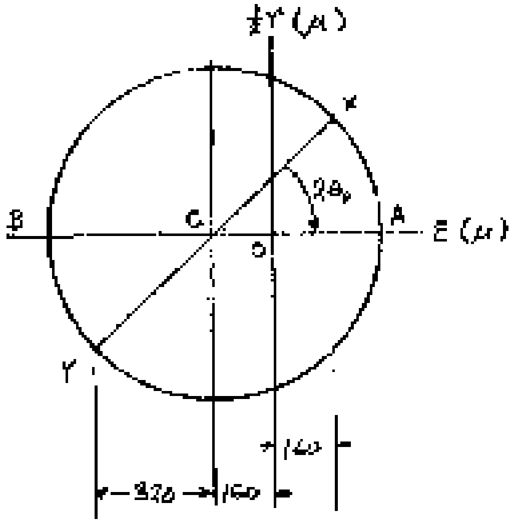
SOLUTION

(a) For Mohr's circle of strain, plot points

X: $(160\,\mu,\ 300\,\mu)$

Y: $(-480\,\mu,\ -300\,\mu)$

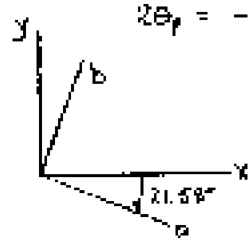
C: $(-160\,\mu,\ 0)$



(a) $\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{-300}{320} = -0.9375$

$2\theta_p = -43.15° \qquad \theta_p = -21.58°$

and $-21.58 + 90 = 68.42°$



$\theta_a = -21.58°$ ◄

$\theta_b = 68.42°$ ◄

$R = \sqrt{(320\,\mu)^2 + (300\,\mu)^2} = 438.6\,\mu$

$\varepsilon_a = \varepsilon_{ave} + R = -160\,\mu + 438.6\,\mu = +278.6\,\mu$ ◄

$\varepsilon_b = \varepsilon_{ave} - R = -160\,\mu - 438.6\,\mu = -598.6\,\mu$ ◄

(b) $\frac{1}{2}\gamma_{(max,\ in\text{-}plane)} = R \qquad \gamma_{(max,\ in\text{-}plane)} = 2R = 877\,\mu$ ◄

(c) $\varepsilon_c = -\dfrac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\dfrac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\dfrac{1/3}{2/3}(160\,\mu - 480\,\mu)$

$= 160\,\mu$

$\varepsilon_{max} = 278.6\,\mu \qquad \varepsilon_{min} = -598.6\,\mu$

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 278.6\,\mu + 598.6\,\mu = 877\,\mu$ ◄

PROBLEM 7.135

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

$\epsilon_x = -260\,\mu \qquad \epsilon_y = -60\,\mu \qquad \gamma_{xy} = 480\,\mu$

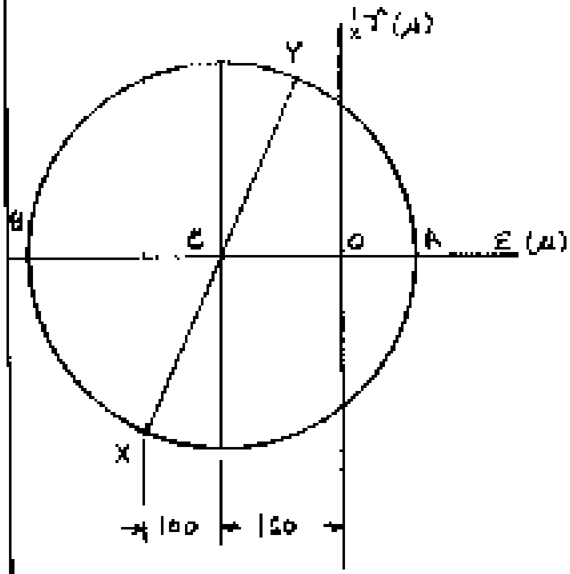
SOLUTION

For Mohr's circle of strain plot points

$X: (-260\,\mu,\ -240\,\mu)$

$Y: (-60\,\mu,\ 240\,\mu)$

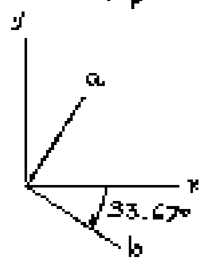
$C: (-160\,\mu,\ 0)$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = \frac{480}{-260+60} = -2.4$$

$2\theta_p = -67.38°$ $\qquad \theta_b = -33.67°$

$\theta_a = 56.31°$



$R = \sqrt{(100\,\mu)^2 + (240\,\mu)^2}$

$R = 260\,\mu$

(a) $\epsilon_a = \epsilon_{ave} + R = -160\,\mu + 260\,\mu = 100\,\mu$

$\epsilon_b = \epsilon_{ave} - R = -160\,\mu - 260\,\mu = -420\,\mu$

(b) $\frac{1}{2}\gamma_{max\,(in\text{-}plane)} = R \qquad \gamma_{max\,(in\text{-}plane)} = 2R = 520\,\mu$

$$\epsilon_c = -\frac{\nu}{1-\nu}(\epsilon_a + \epsilon_b) = -\frac{\nu}{1-\nu}(\epsilon_x + \epsilon_y) = -\frac{1/3}{2/3}(-260-60)$$
$$= 160\,\mu$$

$\epsilon_{max} = 160\,\mu \qquad \epsilon_{min} = -420\,\mu$

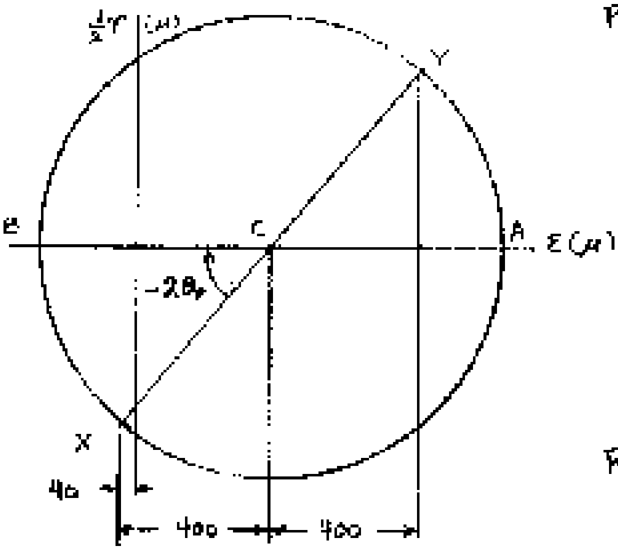
(c) $\gamma_{max} = \epsilon_{max} - \epsilon_{min} = 160\,\mu + 420\,\mu = 580\,\mu$

PROBLEM 7.136

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

$\varepsilon_x = -40\mu \qquad \varepsilon_y = 760\mu \qquad \gamma_{xy} = +960\mu$

SOLUTION



Plotted points

X: $(-40\mu, -480\mu)$

Y: $(760\mu, +480\mu)$

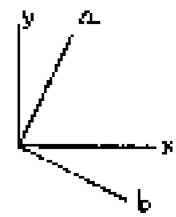
C: $(360\mu, 0)$

$\tan 2\theta_p = \frac{-480}{400} = -1.2$

$2\theta_p = -50.19°$

$\theta_b = -25.10°$

$\theta_a = 64.90°$



$R = \sqrt{(400\mu)^2 + (480\mu)^2}$

$= 624.8\,\mu$

(a) $\varepsilon_a = \varepsilon_{ave} + R = 360\mu + 624.8\mu = 985\,\mu$

$\varepsilon_b = \varepsilon_{ave} - R = 360\mu - 624.8\mu \quad -265\,\mu$

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 1250\,\mu$

$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(-40\mu + 760\mu)$

$= -360\,\mu$

$\varepsilon_{max} = 985\,\mu \qquad \varepsilon_{min} = -265\,\mu$

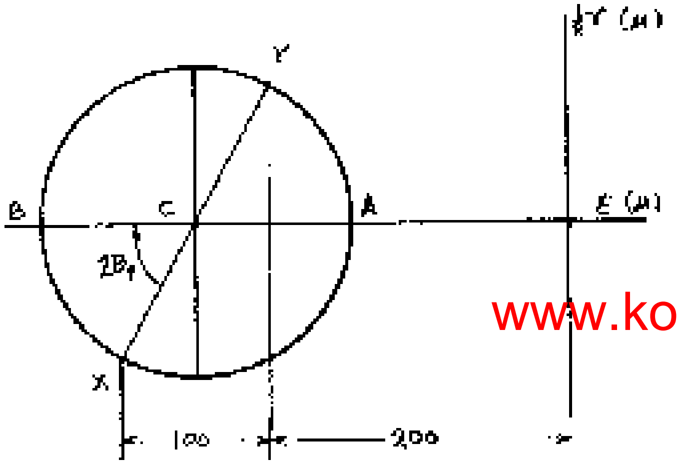
(c) $\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 985 + 360 = 1345\,\mu$

PROBLEM 7.137

7.134 through 7.137 The following state of strain has been measured on the surface of a thin plate. Knowing that the surface of the plate is unstressed, determine (*a*) the direction and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain. (Use $\nu = \frac{1}{3}$)

$\varepsilon_x = -300\,\mu \qquad \varepsilon_y = -200\,\mu \qquad \gamma_{xy} = +175\,\mu$

SOLUTION



Plotted points

X: $(-300\,\mu,\ -87.5\,\mu)$

Y: $(-200\,\mu,\ +87.5\,\mu)$

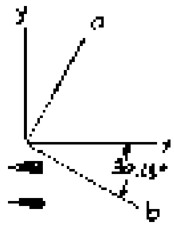
C: $(-250\,\mu,\ 0)$

$\tan 2\theta_p = -\frac{87.5}{50}$

$2\theta_p = -60.26$

$\theta_b = -30.13°$

$\theta_a = 59.87°$



$R = \sqrt{(50\,\mu)^2 + (87.5\,\mu)^2}$
$= 100.8$

(a) $\varepsilon_a = \varepsilon_{ave} + R = -250\,\mu + 100.8\,\mu = -149.2\,\mu$

$\varepsilon_b = \varepsilon_{ave} - R = -250\,\mu - 100.8\,\mu = -351\,\mu$

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 201.6\,\mu$

$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b) = -\frac{\nu}{1-\nu}(\varepsilon_x + \varepsilon_y) = -\frac{1/3}{2/3}(-300\,\mu - 200\,\mu)$
$= +250\,\mu$

$\varepsilon_{max} = 250\,\mu \qquad \varepsilon_{min} = -351\,\mu$

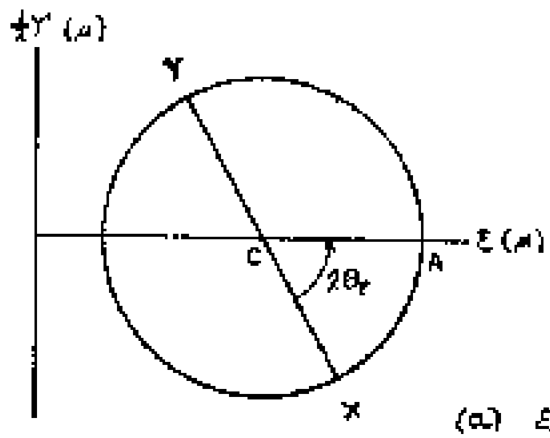
(c) $\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 250\,\mu + 351\,\mu = 601\,\mu$

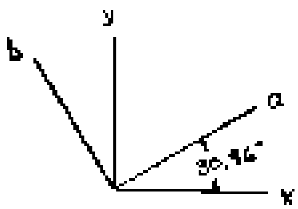
PROBLEM 7.140

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

SOLUTION

$\varepsilon_x = +400\,\mu \qquad \varepsilon_y = +200\,\mu \qquad \gamma_{xy} = +375\,\mu$





Plotted points

X: $(+400\mu,\ -187.5\mu)$

Y: $(+200\mu,\ +187.5\mu)$

C: $(+300\mu,\ 0)$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{375}{400-200} = 1.875$$

$2\theta_p = 61.93° \qquad \theta_a = 30.96° \qquad \theta_b = 120.96°$

$$R = \sqrt{(100\mu)^2 + (187.5\mu)^2} = 212.5\,\mu$$

(a) $\varepsilon_a = \varepsilon_{ave} + R = 300\mu + 212.5\mu = 512.5\,\mu$

$\varepsilon_b = \varepsilon_{ave} - R = 300\mu - 212.5\mu = 87.5\,\mu$

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 425\,\mu$

(c) $\varepsilon_c = 0 \qquad \varepsilon_{max} = 512.5\,\mu \qquad \varepsilon_{min} = 0$

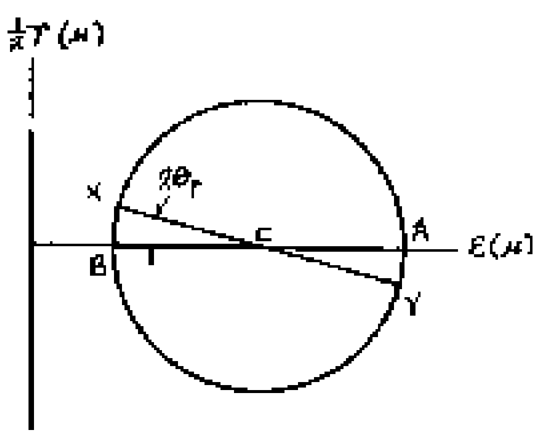
$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 512.5\,\mu$

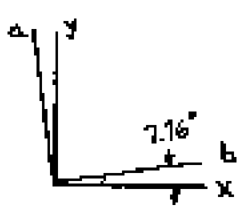
PROBLEM 7.141

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

SOLUTION

$\varepsilon_x = +60\,\mu \qquad \varepsilon_y = +240\,\mu \qquad \gamma_{xy} = -50\,\mu$





Plotted points X: $(60\mu,\ 25\mu)$

Y: $(240\mu,\ -25\mu)$ C: $(150\mu,\ 0)$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-50}{60-240} = 0.27778$$

$2\theta_p = 15.52° \qquad \theta_b = 7.76° \qquad \theta_a = 97.76°$

$$R = \sqrt{(90\mu)^2 + (25\mu)^2} = 93.4\,\mu$$

(a) $\varepsilon_a = \varepsilon_{ave} + R = 150\mu + 93.4\,\mu = 243.4\,\mu$

$\varepsilon_b = \varepsilon_{ave} - R = 150\,\mu - 93.4\,\mu = 56.6\,\mu$

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 186.8\,\mu$

(c) $\varepsilon_c = 0 \qquad \varepsilon_{max} = 243.4\,\mu \qquad \varepsilon_{min} = 0$

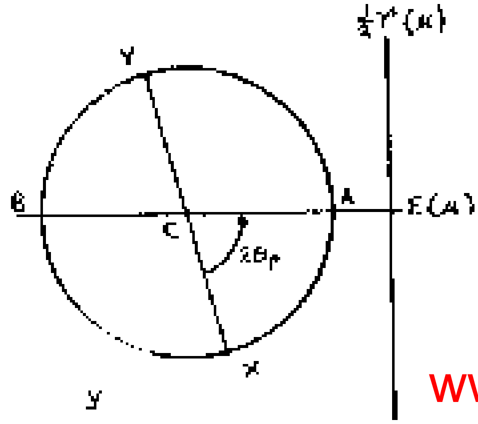
$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 243.4$

PROBLEM 7.138

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

$\varepsilon_x = -90\,\mu \qquad \varepsilon_y = -130\,\mu \qquad \gamma_{xy} = +150\,\mu$

SOLUTION



Plot points

X: $(-90\mu, -75\mu)$ Y: $(-130\mu, +75\mu)$

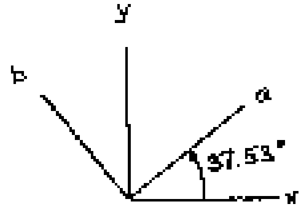
C: $(-110\mu, 0)$

(a) $\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{150}{40} = 3.75$

$2\theta_p = 75.07°$ $\qquad \theta_a = 37.53°$

$\theta_b = 127.53°$

$R = \sqrt{(20\mu)^2 + (75\mu)^2} = 77.6\,\mu$



$\varepsilon_a = \varepsilon_{ave} + R = -110\mu + 77.6\mu = -32.4\mu$

$\varepsilon_b = \varepsilon_{ave} - R = -110\mu - 77.6\mu = -187.6\mu$

(b) $\gamma_{max(in\text{-}plane)} = 2R = 155.2\,\mu$

(c) $\varepsilon_c = 0 \qquad \varepsilon_{max} = 0, \quad \varepsilon_{min} = -187.6\mu$

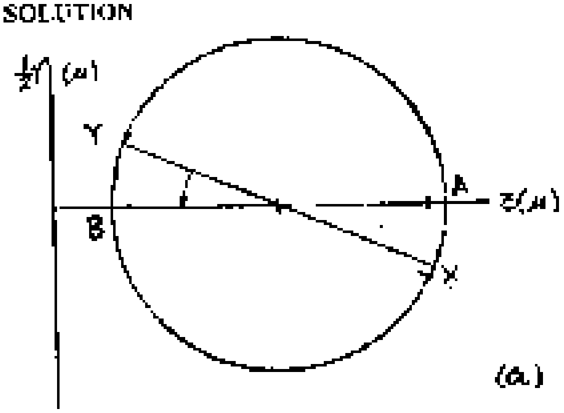
$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 0 + 187.6\mu = 187.6\mu$

PROBLEM 7.139

7.138 through 7.141 The for given state of plane strain, use Mohr's circle to determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane strain, (*c*) the maximum shearing strain.

$\varepsilon_x = +375\,\mu \qquad \varepsilon_y = +75\,\mu \qquad \gamma_{xy} = +125\,\mu$

SOLUTION



X: $(375\mu, -62.5\mu)$, Y: $(75\mu, 62.5\mu)$

C: $(225\mu, 0)$

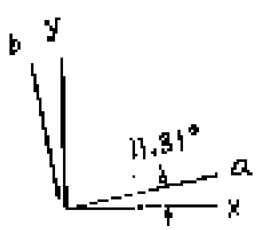
$\tan 2\theta_p = \dfrac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \dfrac{125}{375-75} = 22.62°$

$\theta_a = 11.31° \qquad \theta_b = 101.31°$

$R = \sqrt{(150\mu)^2 + (62.5\mu)^2} = 162.5\,\mu$

(a) $\varepsilon_a = \varepsilon_{ave} + R = 225\mu + 162.5\mu = 387.5\mu$

$\varepsilon_b = \varepsilon_{ave} - R = 225\mu - 162.5\mu = 62.5\mu$

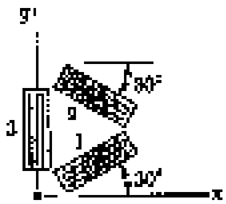


(b) $\gamma_{max(in\text{-}plane)} = 2R = 325\,\mu$

(c) $\varepsilon_c = 0 \qquad \varepsilon_{max} = 387.5\mu \qquad \varepsilon_{min} = 0$

$\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 387.5\mu$

PROBLEM 7.142



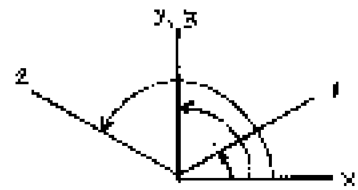
7.142 The strains determined by use of the rosette shown during the test of a rocker arm are:

$\epsilon_1 = +600\ \mu \qquad \epsilon_2 = +450\ \mu \qquad \epsilon_3 = -75\ \mu$

Determine (*a*) the in-plane principal strains, (*b*) the in-plane maximum shearing strain.

SOLUTION

$\theta_1 = 30°$
$\theta_2 = 150°$
$\theta_3 = 90°$



$$\epsilon_x \cos^2\theta_1 + \epsilon_y \sin^2\theta_1 + \gamma_{xy}\sin\theta_1\cos\theta_1 = \epsilon_1$$

$$0.75\,\epsilon_x + 0.25\,\epsilon_y + 0.43301\,\gamma_{xy} = 600\mu \quad (1)$$

$$\epsilon_x \cos^2\theta_2 + \epsilon_y \sin^2\theta_2 + \gamma_{xy}\sin\theta_2\cos\theta_2 = \epsilon_2$$

$$0.75\,\epsilon_x + 0.25\,\epsilon_y - 0.43301\,\gamma_{xy} = 450\mu \quad (2)$$

$$\epsilon_x \cos^2\theta_3 + \epsilon_y \sin^2\theta_3 + \gamma_{xy}\sin\theta_3\cos\theta_3 = \epsilon_3$$

$$0 \quad + \quad \epsilon_y \quad \quad 0 = -75\mu \quad (3)$$

Solving (1), (2), and (3) simultaneously

$$\epsilon_x = 725\ \mu, \quad \epsilon_y = -75\ \mu, \quad \gamma_{xy} = 173.21\ \mu$$

$$\epsilon_{ave} = \tfrac{1}{2}(\epsilon_x + \epsilon_y) = 325\ \mu$$

$$R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{725 + 75}{2}\right)^2 + \left(\frac{173.21}{2}\right)^2} = 409.3\ \mu$$

(a) $\epsilon_a = \epsilon_{ave} + R = 734\ \mu$

$\epsilon_b = \epsilon_{ave} - R = -84.3\ \mu$

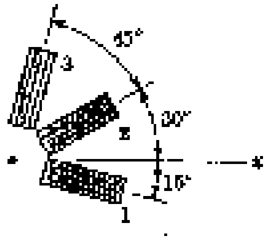
(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 819\ \mu$

PROBLEM 7.143

7.143 Determine the strain ϵ_x, knowing that the following strains have been determined by use of the rosette shown:

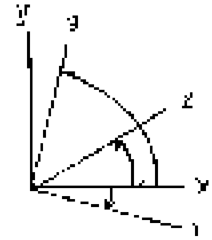
$\epsilon_1 = +720 \times 10^{-6}$ in./in. $\quad \epsilon_2 = -180 \times 10^{-6}$ in./in.

$\epsilon_3 = +120 \times 10^{-6}$ in./in.



SOLUTION

$\theta_1 = -15°$
$\theta_2 = 30°$
$\theta_3 = 75°$



$$\epsilon_x \cos^2\theta_1 + \epsilon_y \sin^2\theta_1 - \gamma_{xy} \sin\theta_1 \cos\theta_1 = \epsilon_1$$

$$0.9330\,\epsilon_x + 0.06699\,\epsilon_y - 0.25\,\gamma_{xy} = 720 \times 10^{-6} \qquad (1)$$

$$\epsilon_x \cos^2\theta_2 + \epsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2 \cos\theta_2 = \epsilon_2$$

$$0.75\,\epsilon_x + 0.25\,\epsilon_y + 0.4330\,\gamma_{xy} = -180 \times 10^{-6} \qquad (2)$$

$$\epsilon_x \cos^2\theta_3 + \epsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3 \cos\theta_3 = \epsilon_3$$

$$0.06699\,\epsilon_x + 0.9330\,\epsilon_y + 0.25\,\gamma_{xy} = 120 \times 10^{-6} \qquad (3)$$

Solving (1), (2), and (3) simultaneously

$\epsilon_x = 380 \times 10^{-6}$ in/in, $\quad \epsilon_y = 460 \times 10^{-6}$ in/in, $\quad \gamma_{xy} = -1339 \times 10^{-6}$ in/in

$$\epsilon_x = 380 \times 10^{-6} \text{ in/in.}$$

PROBLEM 7.144

7.144 The rosette shown has been used to determine the following strains at a point on the surface of a crane hook:

$\epsilon_1 = +420\ \mu \qquad \epsilon_2 = -45\ \mu \qquad \epsilon_4 = +165\ \mu$

(*a*) What should be the reading of gage 3? (*b*) Determine the principal strains and the maximum in-plane shearing strain.

SOLUTION

(a) Gages 2 and 4 are 90° apart $\qquad \varepsilon_{ave} = \frac{1}{2}(\varepsilon_2 + \varepsilon_4)$

$$\varepsilon_{ave} = \frac{1}{2}(-45\mu + 165\mu) = 60\mu$$

Gages 1 and 3 are also 90° apart $\qquad \varepsilon_{ave} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3)$

$$\varepsilon_3 = 2\varepsilon_{ave} - \varepsilon_1 = (2)(60\mu) - 420\mu = -300\mu$$

(b) $\varepsilon_x = \varepsilon_1 = 420\ \mu \qquad \varepsilon_y = \varepsilon_3 = -300\mu$

$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_x - \varepsilon_y = (2)(-45\mu) - 420\mu + 300\mu$$

$$= -210\mu$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{420\mu + 300\mu}{2}\right)^2 + \left(\frac{-210\mu}{2}\right)^2}$$

$$= 375\mu$$

$$\varepsilon_a = \varepsilon_{ave} + R = 60\mu + 375\mu = 435\mu$$

$$\varepsilon_b = \varepsilon_{ave} - R = 60\mu - 375\mu = -315\mu$$

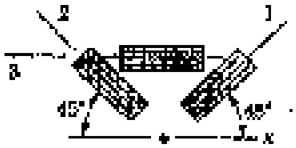
$$\gamma_{max\,(in\text{-}plane)} = 2R = 750\mu$$

PROBLEM 7.145

7.145 Determine the largest in-plane normal strain, knowing that the following strains have been obtained by use of the rosette shown:

$\epsilon_1 = -50 \times 10^{-6}$ in./in. $\epsilon_2 = +360 \times 10^{-6}$ in./in.

$\epsilon_3 = +315 \times 10^{-6}$ in./in.



SOLUTION

$\theta_1 = 45°$ $\theta_2 = -45°$ $\theta_3 = 0$

$$\epsilon_x \cos^2\theta_1 + \epsilon_y \sin^2\theta_1 + \gamma_{xy}\sin\theta_1\cos\theta_1 = \epsilon_1$$

$$0.5\,\epsilon_x + 0.5\,\epsilon_y + 0.5\,\gamma_{xy} = -50\times10^{-6} \qquad (1)$$

$$\epsilon_x \cos^2\theta_2 + \epsilon_y \sin^2\theta_2 + \gamma_{xy}\sin\theta_2\cos\theta_2 = \epsilon_2$$

$$0.5\,\epsilon_x + 0.5\,\epsilon_y - 0.5\,\gamma_{xy} = 360\times10^{-6} \qquad (2)$$

$$\epsilon_x \cos^2\theta_3 + \epsilon_y \sin^2\theta_3 + \gamma_{xy}\sin\theta_3\cos\theta_3 = \epsilon_3$$

$$\epsilon_x + 0 + 0 = 315\times10^{-6} \qquad (3)$$

From (3) $\epsilon_x = 315\times10^{-6}$ in./in.

Eq. (1) − Eq. (2) $\gamma_{xy} = -50\times10^{-6} - 360\times10^{-6} = -410\times10^{-6}$ in/in

Eq (1) + Eq (2) $\epsilon_x + \epsilon_y = \epsilon_1 + \epsilon_2$

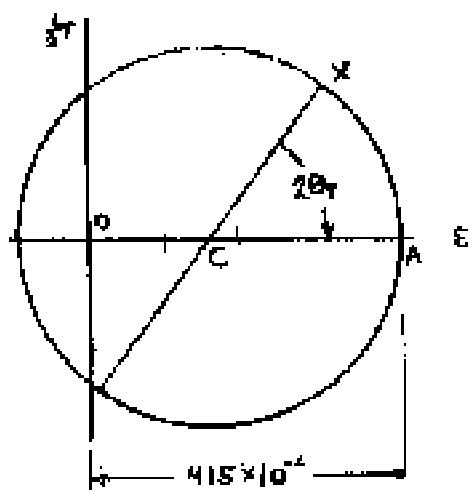
$$\epsilon_y = \epsilon_1 + \epsilon_2 - \epsilon_x = -50\times10^{-6} + 360\times10^{-6} - 315\times10^{-6} = -5\times10^{-6} \text{ in/in.}$$

$$\epsilon_{ave} = \tfrac{1}{2}(\epsilon_x + \epsilon_y) = 155\times10^{-6} \text{ in/in}$$

$$R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \sqrt{\left(\frac{315\times10^{-6} + 5\times10^{-6}}{2}\right)^2 + \left(\frac{-410\times10^{-6}}{2}\right)^2}$$

$$= 260\times10^{-6} \text{ in/in}$$

$$\epsilon_{max} = \epsilon_{ave} + R = 415\times10^{-6} \text{ in/in}$$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = -1.323$$

$$2\theta_p = -52.0°$$

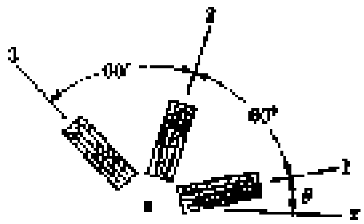
$$\theta_p = -26.0°$$

PROBLEM 7.144

7.144 Show that the sum of the three strain measurements made with a 60° rosette is independent of the orientation of the rosette and equal to

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_{ave}$$

where ε_{ave} is the abscissa of the center of the corresponding Mohr's circle for strain.



SOLUTION

$$\varepsilon_1 = \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (1)$$

$$\varepsilon_2 = \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 120°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 120°)$$

$$= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 120° \cos 2\theta - \sin 120° \sin 2\theta)$$

$$+ \frac{\gamma_{xy}}{2}(\cos 120° \sin 2\theta + \sin 120° \cos 2\theta)$$

$$= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\frac{1}{2}\cos 2\theta - \frac{\sqrt{3}}{2}\sin 2\theta\right)$$

$$+ \frac{\gamma_{xy}}{2}\left(-\frac{1}{2}\sin 2\theta + \frac{\sqrt{3}}{2}\cos 2\theta\right) \qquad (2)$$

$$\varepsilon_3 = \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos(2\theta + 240°) + \frac{\gamma_{xy}}{2}\sin(2\theta + 240°)$$

$$= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}(\cos 240° \cos 2\theta - \sin 240° \sin 2\theta)$$

$$+ \frac{\gamma_{xy}}{2}(\cos 240° \sin 2\theta + \sin 240° \cos 2\theta)$$

$$= \varepsilon_{ave} + \frac{\varepsilon_x - \varepsilon_y}{2}\left(-\frac{1}{2}\cos 2\theta + \frac{\sqrt{3}}{2}\sin 2\theta\right)$$

$$+ \frac{\gamma_{xy}}{2}\left(-\frac{1}{2}\sin 2\theta - \frac{\sqrt{3}}{2}\cos 2\theta\right) \qquad (3)$$

Adding (1), (2), and (3)

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3\varepsilon_{ave} + 0 + 0$$

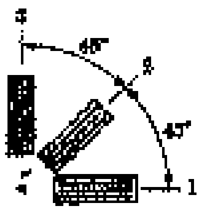
$$3\varepsilon_{ave} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

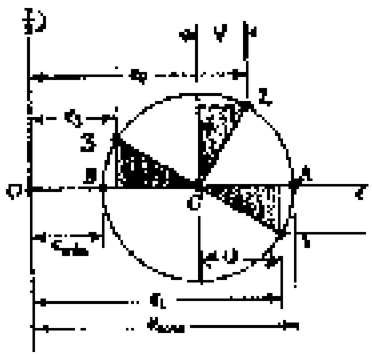
PROBLEM 7.147

7.147 Using a 45° rosette, the strains ε_1, ε_2, and ε_3 have been determined at a given point. Using Mohr's circle, show that the principal strains are

$$\varepsilon_{\max,\min} = \frac{1}{2}(\varepsilon_1 + \varepsilon_3) \pm \frac{1}{\sqrt{2}}\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2\right]^{1/2}$$

(*Hint:* The shaded triangles are congruent.)





SOLUTION

Since gage directions 1 and 3 are 90° apart

$$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

Let $u = \varepsilon_1 - \varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_1 - \varepsilon_3)$

$$v = \varepsilon_2 - \varepsilon_{ave} = \varepsilon_2 - \tfrac{1}{2}(\varepsilon_1 + \varepsilon_3)$$

$$R^2 = u^2 + v^2$$

$$= \tfrac{1}{4}(\varepsilon_1 - \varepsilon_3)^2 + \varepsilon_2^2 - \varepsilon_2(\varepsilon_1 + \varepsilon_3) + \tfrac{1}{4}(\varepsilon_1 + \varepsilon_3)^2$$

$$= \tfrac{1}{4}\varepsilon_1^2 - \tfrac{1}{2}\varepsilon_1\varepsilon_3 + \tfrac{1}{4}\varepsilon_3^2$$
$$+ \varepsilon_2^2 - \varepsilon_2\varepsilon_1 - \varepsilon_2\varepsilon_3$$
$$+ \tfrac{1}{4}\varepsilon_1^2 + \tfrac{1}{2}\varepsilon_1\varepsilon_3 + \tfrac{1}{4}\varepsilon_3^2$$

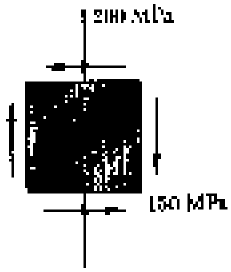
$$= \tfrac{1}{2}\varepsilon_1^2 - \varepsilon_2\varepsilon_1 + \varepsilon_2^2 - \varepsilon_2\varepsilon_3 + \tfrac{1}{2}\varepsilon_3^2$$

$$= \tfrac{1}{2}(\varepsilon_1 - \varepsilon_2)^2 + \tfrac{1}{2}(\varepsilon_2 - \varepsilon_3)^2$$

$$R = \frac{1}{\sqrt{2}}\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2\right]$$

$\varepsilon_{\max,\min} = \varepsilon_{ave} \pm R$ gives the required formula.

PROBLEM 7.148



7.148 The given state of plane stress is known to exist on the surface of a machine component. Knowing that $E = 200$ GPa and $G = 77$ GPa, determine the direction and magnitude of the three principal strains (a) by determining the corresponding state of strain [use Eq. (2.43), page 94, and Eq. (2.38) page 91] and then using Mohr's circle for strain, (b) by using Mohr's circle for stress to determine the principal planes and principal stresses and then determining the corresponding strains.

SOLUTION

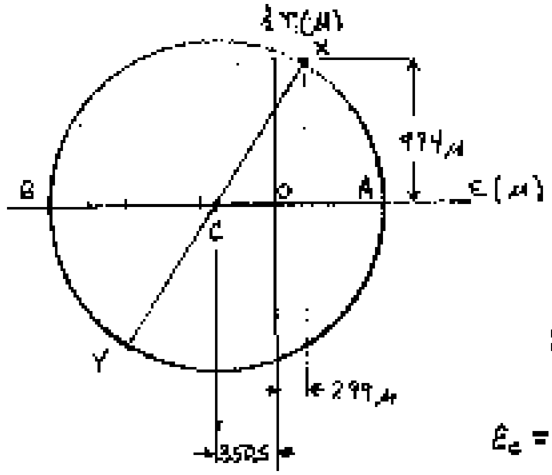
(a) $\sigma_x = 0, \quad \sigma_y = -200\times10^6$ Pa, $\quad \tau_{xy} = -150\times10^6$ Pa
$E = 200\times10^9$ Pa $\quad G = 77\times10^9$ Pa

$$G = \frac{E}{2(1+\nu)} \qquad \nu = \frac{E}{2G} - 1 = 0.2987$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{200\times10^9}\left[0 + (0.2987)(200\times10^6)\right] = 299\,\mu$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{200\times10^9}\left[(-200\times10^6) - 0\right] = -1000\,\mu$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{-150\times10^6}{77\times10^9} = -1948\,\mu \qquad \frac{\gamma_{xy}}{2} = -974\,\mu$$



$$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) = -350.5\,\mu$$

$$\varepsilon_x - \varepsilon_y = 1299\,\mu$$

$$\tan 2\theta_a = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-1948}{1299} = -1.4996$$

$$2\theta_a = -56.3^\circ \qquad \theta_a = -28.15^\circ$$

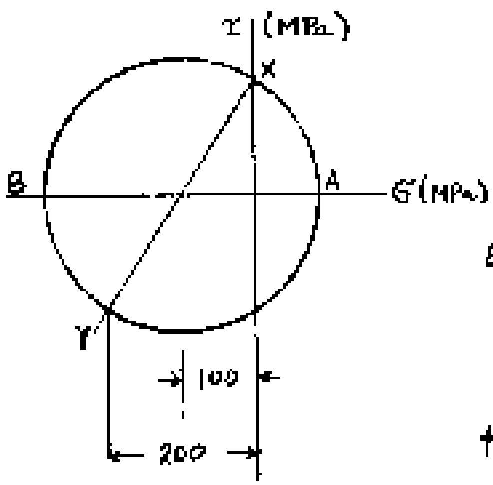
$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 1171\,\mu$$

$$\varepsilon_a = \varepsilon_{ave} + R = 820\,\mu$$

$$\varepsilon_b = \varepsilon_{ave} - R = -1521\,\mu$$

$$\varepsilon_c = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{(0.2987)(0 - 200\times10^6)}{200\times10^9} = -299\,\mu$$

(b)



$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = 100\text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{0+200}{2}\right)^2 + 150^2} = 180.28\text{ MPa}$$

$$\sigma_a = \sigma_{ave} + R = 80.3\text{ MPa}$$

$$\sigma_b = \sigma_{ave} - R = -280.3\text{ MPa}$$

$$\varepsilon_a = \frac{1}{E}(\sigma_a - \nu\sigma_b)$$

$$= \frac{1}{200\times10^9}\left[80.3\times10^6 - (0.2987)(-280.3\times10^6)\right]$$

$$= 820\times10^{-6} = 820\,\mu$$

$$\tan 2\theta_a = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -1.5 \qquad 2\theta_a = -56.3^\circ$$

$$\theta_a = -28.15^\circ$$

PROBLEM 7.149

7.149 The following state of strain has been determined on the surface of a cast-iron machine element:

$\varepsilon_1 = -720 \times 10^{-6}$ in./in. $\qquad \varepsilon_2 = -400 \times 10^{-6}$ in./in.

$\gamma = +660 \times 10^{-6}$ rad

Knowing that $E = 10 \times 10^6$ psi and $G = 4 \times 10^6$ psi, determine the principal planes and the principal stresses (*a*) by determining the corresponding state of plane stress [use Eq. 2.36, page 94 Eq. 2.43; page 97 ; and the first two equations of Prob. 2.75, page ww] and then using Mohr's circle for stress, (*b*) by using Mohr's circle for strain to determine the orientation and magnitude of the principal strains and then determining the corresponding stresses.

SOLUTION

$G = \dfrac{E}{2(1+\nu)} \qquad \nu = \dfrac{E}{2G} - 1 = \dfrac{10}{(2)(4)} - 1 = 0.25$

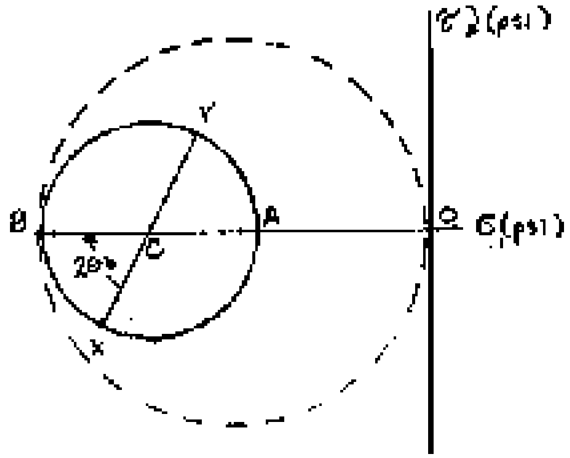
Note that the 3rd principal stress $\sigma_c = 0$

$\dfrac{E}{1-\nu^2} = \dfrac{10\times 10^6}{1-0.25^2} = 10.667 \times 10^6$ psi

(a) $\sigma_1 = \dfrac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2) = 10.667\times10^6\left[-720\times10^{-6} + (0.25)(-400\times10^{-6})\right]$
$= -8746.7$ psi

$\sigma_2 = \dfrac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1) = 10.667\times10^6\left[-400\times10^{-6} + (0.25)(-720\times10^{-6})\right]$
$= -6186.7$ psi

$\tau = G\gamma = (4\times10^6)(660\times10^{-6}) = 2640$ psi



$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = -7466.7$ psi

$\tan 2\theta_b = \dfrac{2\tau}{\sigma_1 - \sigma_2} = -2.0625$

$2\theta_b = -64.1° \qquad \theta_b = -32.1° \qquad \theta_a = 57.9°$

$R = \sqrt{\left(\dfrac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2} = 2934$ psi

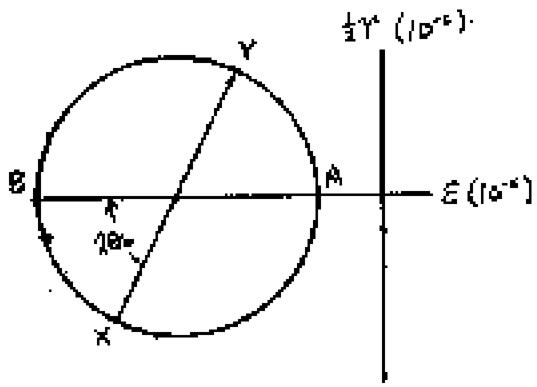
$\sigma_a = \sigma_{ave} + R = -4533$ psi

$\sigma_b = \sigma_{ave} - R = -10400$ psi

(b) $\varepsilon_{ave} = \frac{1}{2}(\varepsilon_1 + \varepsilon_2) = -560 \times 10^{-6}$

$\tan 2\theta_b = \dfrac{\gamma}{\varepsilon_1 - \varepsilon_2} = \dfrac{660}{-720+400} = -2.0625$

$2\theta_b = -64.1° \qquad \theta_b = -32.1° \qquad \theta_a = 57.9°$



$R = \sqrt{\left(\dfrac{\varepsilon_1 - \varepsilon_2}{2}\right)^2 + \left(\dfrac{\gamma}{2}\right)^2} = 366.74 \times 10^{-6}$

$\varepsilon_a = \varepsilon_{ave} + R = -193.26 \times 10^{-6}$

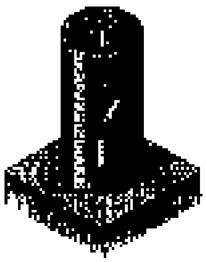
$\varepsilon_b = \varepsilon_{ave} - R = -926.74 \times 10^{-6}$

$\sigma_a = \dfrac{E}{1-\nu^2}(\varepsilon_a + \nu\varepsilon_b) = -4533$ psi

$\sigma_b = \dfrac{E}{1-\nu^2}(\varepsilon_b + \nu\varepsilon_a) = -10400$ psi

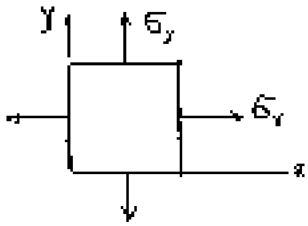
PROBLEM 7.150

7.150 A single strain gage forming an angle $\beta = 30^\circ$ with the vertical is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is $\frac{3}{8}$ in. thick, has a 36-in. inside diameter, and is made of a steel with $E = 29 \times 10^6$ psi and $\nu = 0.30$. Determine the pressure in the tank corresponding to a gage reading of 220×10^{-6} in./in.



SOLUTION

Stresses in the tank wall



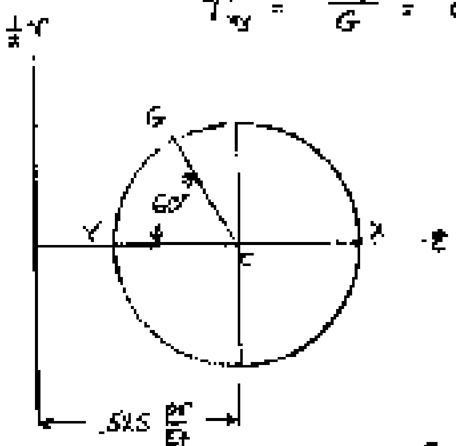
$$\sigma_x = \frac{pr}{t} \qquad \sigma_y = \frac{pr}{2t} \qquad \tau_{xy} = 0$$

Strains

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{E}\left(\frac{pr}{t} - \nu\frac{pr}{2t}\right) = \frac{pr}{Et}\left(1 - \frac{\nu}{2}\right) = 0.85\,\frac{pr}{Et}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{E}\left(\frac{pr}{2t} - \nu\frac{pr}{t}\right) = \frac{pr}{Et}\left(\frac{1}{2} - \nu\right) = 0.20\,\frac{pr}{Et}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$$



$$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0.525\,\frac{pr}{Et}$$

$$R = \frac{1}{2}(\varepsilon_x - \varepsilon_y) = 0.325$$

$$\varepsilon_g = \varepsilon_{ave} - R\cos 60^\circ$$

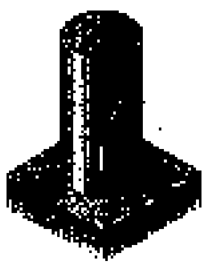
$$= .525\,\frac{pr}{Et} - 0.325\,\frac{pr}{Et}\cos 60^\circ$$

$$= 0.3625\,\frac{pr}{Et}$$

Solving for p $\qquad p = \dfrac{Et\,\varepsilon_g}{0.3625\,r}$

$$p = \frac{(29\times10^6)(\frac{3}{8})(220\times10^{-6})}{(0.3625)(36/2)} = 367 \text{ psi}$$

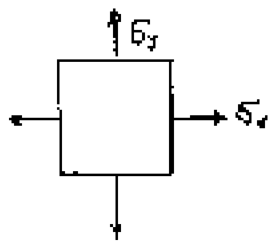
PROBLEM 7.151



7.150 A single strain gage forming an angle $\beta = 30°$ with the vertical is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is $\frac{3}{8}$ in. thick, has a 36-in. inside diameter, and is made of a steel with $E = 29 \times 10^6$ psi and $\nu = 0.30$. Determine the pressure in the tank corresponding to a gage reading of 220×10^{-6} in./in.

7.151 Solve Prob. 7.150, assuming that the gage forms an angle $\beta = 60°$ with the vertical.

SOLUTION



Stresses: $\sigma_x = \frac{pr}{t}$ $\quad \sigma_y = \frac{pr}{2t}$ $\quad \tau_{xy} = 0$

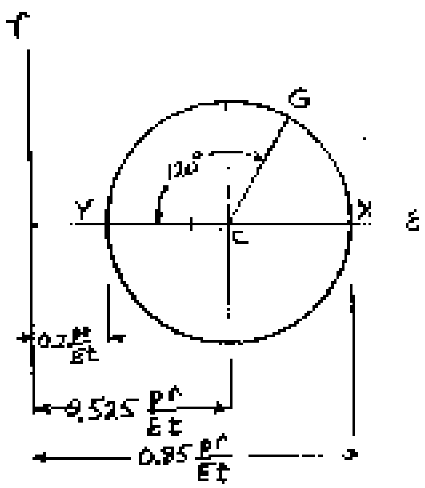
Strains: $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{E}\left(\frac{pr}{t} - \nu\frac{pr}{2t}\right)$

$= \frac{pr}{Et}\left(1 - \frac{\nu}{2}\right) = 0.85\frac{pr}{Et}$

$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{E}\left(\frac{pr}{2t} - \nu\frac{pr}{t}\right)$

$= \left(\frac{1}{2} - \nu\right)\frac{pr}{Et} = 0.20\frac{pr}{Et}$

$\gamma_{xy} = \frac{\tau_{xy}}{G} = 0$



$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0.525\frac{pr}{Et}$

$R = \frac{1}{2}(\varepsilon_x - \varepsilon_y) = 0.325\frac{pr}{Et}$

$\varepsilon_g = \varepsilon_{ave} + R\cos 60°$

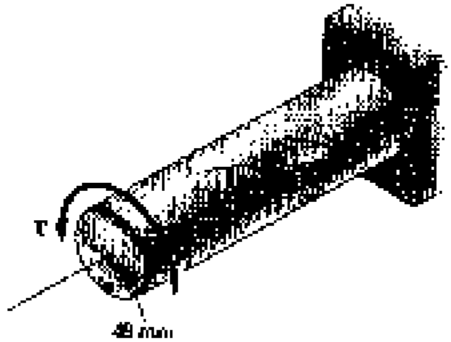
$= 0.525\frac{pr}{Et} + 0.325\frac{pr}{Et}\cos 60°$

$= 0.6875\frac{pr}{Et}$

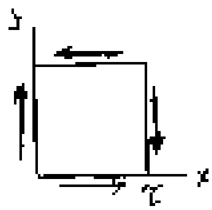
Solving for p $\quad p = \frac{Et\varepsilon_g}{0.6875\,r} = \frac{(29\times10^6)(\frac{3}{8})(220\times10^{-6})}{(0.6875)(36/2)} = 193.3$ psi

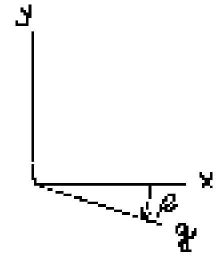
PROBLEM 7.152

7.152 A single strain gage is cemented to a solid 96-mm-diameter aluminum shaft at an angle $\beta = 20°$ with a line parallel to the axis of the shaft. Knowing that $G = 27$ GPa, determine the torque **T** corresponding to a gage reading of 400 μ.



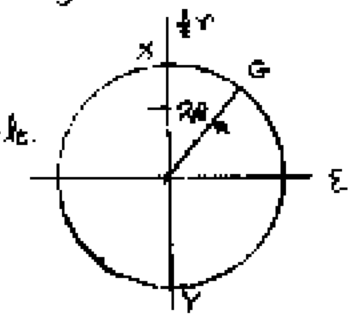
SOLUTION





$\tau = \dfrac{Tc}{J}$ $\qquad J = \dfrac{\pi}{2} c^4$ $\qquad \gamma = \dfrac{\tau}{G}$

$\sigma_x = \sigma_y = 0$ $\qquad \varepsilon_x = \varepsilon_y = 0$



Sketch Mohr's circle for strain.

Gage direction is β clockwise from x

Point G is 2β clockwise from X on Mohr's circle.

$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0$

$R = \frac{1}{2}\gamma_{xy}$

$\varepsilon_g = \varepsilon_{ave} + R \sin 2\beta = \frac{1}{2}\gamma_{xy} \sin 2\beta = \dfrac{\tau_{xy}}{2G} \sin 2\beta$

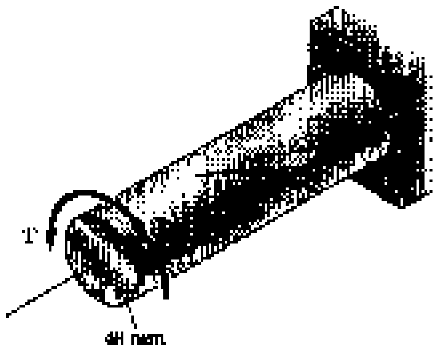
$= \dfrac{Tc}{2GJ} \sin 2\beta =$

Solving for T $\qquad T = \dfrac{2GJ\varepsilon_g}{c \sin 2\beta} = \dfrac{\pi G c^3 \varepsilon_g}{\sin 2\beta}$

$$T = \frac{\pi (27\times10^9)(48\times10^{-3})^3(400\times10^{-6})}{\sin 40°} = 5.84\times10^3 \text{ N}\cdot\text{m}$$

$= 5.84$ kN·m

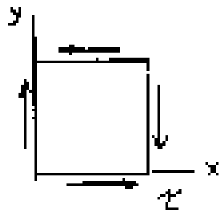
PROBLEM 7.153

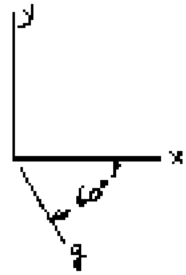


7.152 A single strain gage is cemented to a solid 96-mm-diameter aluminum shaft at an angle $\beta = 20°$ with a line parallel to the axis of the shaft. Knowing that $G = 27$ GPa, determine the torque **T** corresponding to a gage reading of 400 μ.

7.153 Solve Prob. 7.152, assuming that the gage forms an angle $\beta = 60°$ with a line parallel to the axis of the shaft.

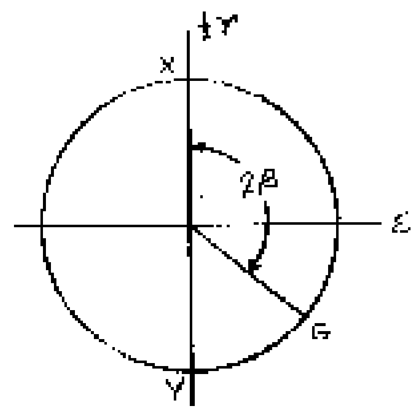
SOLUTION





$\tau = \dfrac{Tc}{J} \qquad J = \dfrac{\pi}{2}c^4 \qquad \gamma = \dfrac{\tau}{G}$

$\sigma_x = \sigma_y = 0 \qquad \varepsilon_x = \varepsilon_y = 0$



Sketch Mohr's circle for strain.
Gage direction g is $\beta = 60°$ clockwise from X.
Point G is $2\beta = 120°$ clockwise from point X on Mohr's circle.

$\varepsilon_{ave} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) = 0$

$R = \frac{1}{2}\gamma$

$\varepsilon_g = \varepsilon_{ave} + R\sin 2\beta = \frac{1}{2}\gamma \sin 2\beta = \dfrac{\tau}{2G}\sin 2\beta$

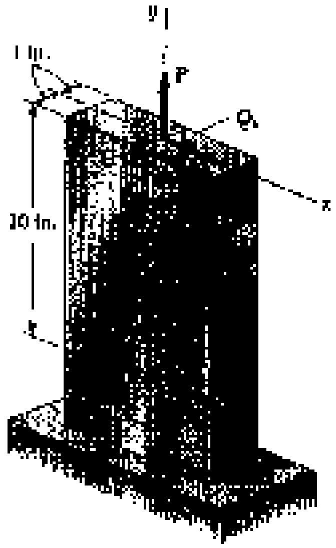
$= \dfrac{Tc}{2GJ}\sin 2\beta$

Solving for T $\qquad T = \dfrac{2GJ\varepsilon_g}{c\sin 2\beta} = \dfrac{\pi G c^3 \varepsilon_g}{\sin 2\beta}$

$T = \dfrac{\pi(27\times10^9)(48\times10^{-3})^3(400\times10^{-6})}{\sin 120°} = 4.33\times10^3\ \text{N}\cdot\text{m}$

$= 4.33\ \text{kN}\cdot\text{m}$

PROBLEM 7.154



7.154 A centric axial force **P** and a horizontal force $\mathbf{Q}_x$ are both applied at point C of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point A indicates the following strains:

$\epsilon_1 = -75 \times 10^{-6}$ in./in. $\quad \epsilon_2 = +300 \times 10^{-6}$ in./in. $\quad \epsilon_3 = +250 \times 10^{-6}$ in./in.

Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the magnitudes of **P** and $\mathbf{Q}_x$.

SOLUTION

$\varepsilon_x = \varepsilon_1 = -75 \times 10^{-6} \qquad \varepsilon_y = \varepsilon_3 = 250 \times 10^{-6}$

$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = 425 \times 10^{-6}$

$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{29}{1-0.3^2}\left[-75 + (0.3)(250)\right]$

$= 0$

$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{29}{1-0.3^2}\left[250 + (0.3)(-75)\right]$

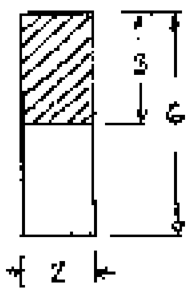
$= 7.25 \times 10^3$ psi

$\frac{P}{A} = \sigma_y \qquad P = A\sigma_y = (2)(6)(7.25 \times 10^3)$

$= 87.0 \times 10^3$ lb $= 87.0$ kips ◀

$G = \frac{E}{2(1+\nu)} = \frac{29 \times 10^6}{(2)(1.3)} = 11.154 \times 10^6$ psi

$\tau_{xy} = G\gamma_{xy} = (11.154)(425) = 4.740 \times 10^3$ psi



$I = \frac{1}{12} bh^3 = \frac{1}{12}(2)(6)^3 = 36$ in^4

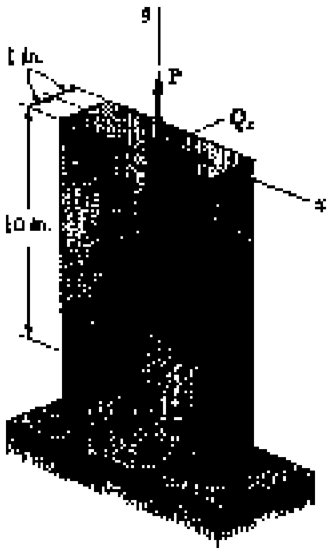
$Q = A\bar{y} = (2)(3)(1.5) = 9$ in$^3 \qquad t = 2$ in

$\tau_{xy} = \frac{VQ}{It}$

$V = \frac{It\tau_{xy}}{Q} = \frac{(36)(2)(4.74 \times 10^3)}{9} = 37.9 \times 10^3$ lb

$Q_x = V = 37.9 \times 10^3$ lb $= 37.9$ kips ◀

PROBLEM 7.155



7.154 A centric axial force **P** and a horizontal force $\mathbf{Q}_x$ are both applied at point *C* of the rectangular bar shown. A 45° strain rosette on the surface of the bar at point *A* indicates the following strains:

$\epsilon_1 = -75 \times 10^{-6}$ in./in. $\qquad \epsilon_2 = +300 \times 10^{-6}$ in./in. $\qquad \epsilon_3 = +250 \times 10^{-6}$ in./in.

Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the magnitudes of **P** and $\mathbf{Q}_x$.

7.155 Solve Prob. 7.154, assuming that the rosette at point *A* indicates the following strains:

$\epsilon_1 = -60 \times 10^{-6}$ in./in. $\qquad \epsilon_2 = -410 \times 10^{-6}$ in./in. $\qquad \epsilon_3 = +200 \times 10^{-6}$ in./in.

SOLUTION

$$\varepsilon_x = \varepsilon_1 = -60 \times 10^{-6} \qquad \varepsilon_y = \varepsilon_3 = 200 \times 10^{-6}$$

$$\gamma_{xy} = 2\varepsilon_2 - \varepsilon_1 - \varepsilon_3 = 680 \times 10^{-6}$$

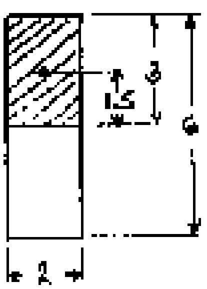
$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = \frac{29}{1-0.3^2}\left[-60 + (0.3)(200)\right]$$
$$= 0$$

$$\sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = \frac{29}{1-0.3^2}\left[200 + (0.3)(-60)\right]$$
$$= 5.800 \times 10^3 \text{ psi}$$

$$\frac{P}{A} = \sigma_y \qquad P = A\sigma_y = (2)(6)(5.800 \times 10^3)$$
$$= 69.6 \times 10^3 \text{ lb} = 69.6 \text{ kips} \quad \blacktriangleleft$$

$$G = \frac{E}{2(1+\nu)} = \frac{29 \times 10^6}{(2)(1.3)} = 11.154 \times 10^6 \text{ psi}$$

$$\tau_{xy} = G\gamma_{xy} = (11.154)(680) = 7.585 \times 10^3 \text{ psi}$$



$$I = \frac{1}{12}bh^3 = \frac{1}{12}(2)(6)^3 = 36 \text{ in}^4$$

$$Q = A\bar{y} = (2)(3)(1.5) = 9 \text{ in}^3 \qquad t = 2 \text{ in.}$$

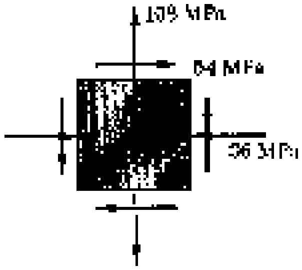
$$\tau_{xy} = \frac{VQ}{It}$$

$$V = \frac{It\tau_{xy}}{Q} = \frac{(36)(2)(7.585 \times 10^3)}{9} = 60.7 \times 10^3 \text{ lb.}$$

$$Q_x = V = 60.7 \times 10^3 \text{ lb.} = 60.7 \text{ kips} \quad \blacktriangleleft$$

PROBLEM 7.156

7.156 The state of stress shown occurs in a steel member made of a grade of steel with a tensile yield strength of 270 MPa. Determine the factor of safety with respect to yield strength, using (*a*) the maximum-shearing-stress criterion, (*b*) the maximum-distortion-energy criterion.



SOLUTION

$\sigma_x = -36$ MPa, $\quad \sigma_y = 108$ MPa, $\quad \tau_{xy} = 54$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 36$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 90$ MPa

$\sigma_a = \sigma_{ave} + R = 126$ MPa, $\quad \sigma_b = \sigma_{ave} - R = -54$ MPa, $\quad \sigma_z = 0$

(a) $\sigma_{max} = 126$ MPa, $\quad \sigma_{min} = -54$ MPa

$2\tau_{max} = \sigma_{max} - \sigma_{min} = 180$ MPa < 270 MPa (No yielding)

$F.S. = \frac{\sigma_Y}{2\tau_{max}} = \frac{270}{180} = 1.500$

(b) $\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b} = 159.99$ MPa < 270 MPa (No yielding)

$F.S. = \frac{\sigma_Y}{\sqrt{\sigma_a^2 + \sigma_b^2 - \sigma_a\sigma_b}} = \frac{270}{159.99} = 1.688$

PROBLEM 7.157

7.157 A spherical pressure tank has 1.2-m outer diameter and a uniform wall thickness of 10 mm. Knowing that the gage pressure is 1.25 MPa in the tank, determine (*a*) the maximum normal stress, (*b*) the maximum shearing stress, (*c*) the normal strain on the surface of the tank. (Use $E = 200$ GPa and $\nu = 0.30$.)

SOLUTION

$t = 10 \times 10^{-3}$ m, $\quad r = \frac{1}{2}d - t = \frac{1}{2}(1.2) - 10\times10^{-3} = 0.590$ m, $\quad p = 1.25$ MPa

For a spherical tank under internal pressure

$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{(1.25)(0.590)}{(2)(10\times10^{-3})} = 36.9$ MPa.

$\sigma_3 \approx 0$

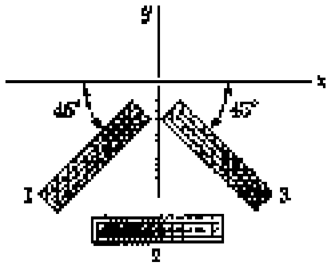
(a) $\sigma_{max} = 36.9$ MPa

(b) $\sigma_{min} = 0 \qquad \tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 18.44$ MPa

(c) $\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2 - \nu\sigma_3) = \frac{1}{200\times10^9}[36.9\times10^6 - (0.3)(36.9\times10^6) - 0]$

$= 129 \times 10^{-6} = 129\,\mu$

PROBLEM 7.158

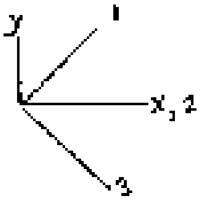


7.158 The strains determined by the use of a rosette attached as shown to the surface of a structural member are:

$\epsilon_1 = 220 \times 10^{-6}$ in./in. $\quad \epsilon_2 = 425 \times 10^{-6}$ in./in. $\quad \epsilon_3 = 480 \times 10^{-6}$ in./in.

Determine (*a*) the orientation and magnitude of the principal strains in the plane of the rosette, (*b*) the maximum in-plane shearing strain.

SOLUTION



$\theta_1 = 45°$

$\theta_2 = 0$

$\theta_3 = -45°$

$$\varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy} \sin\theta_1 \cos\theta_1 = \varepsilon_1$$

$$\tfrac{1}{2}\varepsilon_x + \tfrac{1}{2}\varepsilon_y + \tfrac{1}{2}\gamma_{xy} = 220 \times 10^{-6} \text{ in/in} \qquad (1)$$

$$\varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy} \sin\theta_2 \cos\theta_2 = \varepsilon_2$$

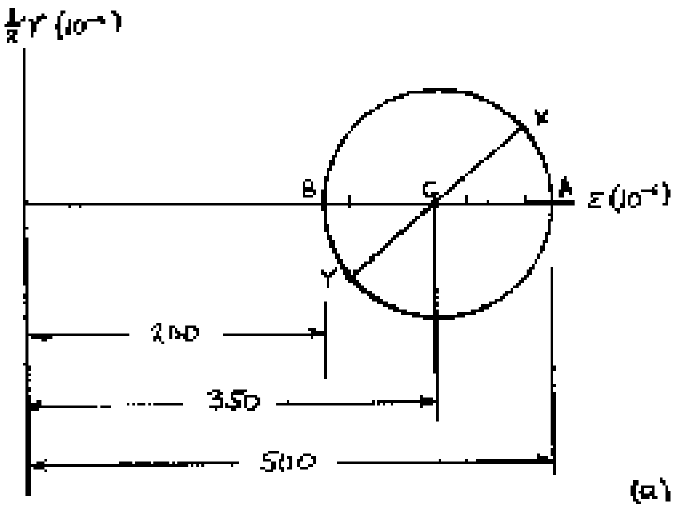
$$\varepsilon_x + 0 + 0 = 425 \times 10^{-6} \text{ in/in} \qquad (2)$$

$$\varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy} \sin\theta_3 \cos\theta_3 = \varepsilon_3$$

$$\tfrac{1}{2}\varepsilon_x + \tfrac{1}{2}\varepsilon_y - \tfrac{1}{2}\gamma_{xy} = 480 \times 10^{-6} \text{ in/in} \qquad (3)$$

Solving (1), (2) and (3) simultaneously gives

$$\varepsilon_x = 425 \times 10^{-6} \text{ in/in}, \quad \varepsilon_y = 275 \times 10^{-6} \text{ in/in}, \quad \gamma_{xy} = -260 \times 10^{-6} \text{ in/in}.$$



$$\varepsilon_{ave} = \tfrac{1}{2}(\varepsilon_x + \varepsilon_y) = 350 \times 10^{-6} \text{ in/in}$$

$$\tan 2\theta_a = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \frac{-260}{425 - 275} = -1.7333$$

$$2\theta_a = -60° \qquad \theta_a = -30° \qquad \theta_b = 60°$$

$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 150 \times 10^{-6} \text{ in/in}$$

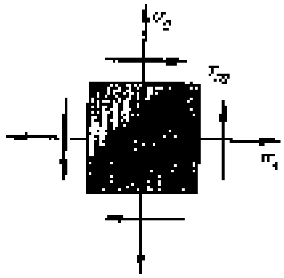
(a) $\varepsilon_a = \varepsilon_{ave} + R = 500 \times 10^{-6}$ in/in

$\varepsilon_b = \varepsilon_{ave} - R = 200 \times 10^{-6}$ in/in

(b) $\gamma_{max\,(in\text{-}plane)} = 2R = 300 \times 10^{-6}$ in/in

PROBLEM 7.159

7.159 For a state of plane stress it is known that the normal and shearing stresses are directed as shown and that $\sigma_x = 5$ ksi, $\sigma_y = 12$ ksi, and $\sigma_{min} = 18$ ksi. Determine (*a*) the orientation of the principal planes, (*b*) the maximum in-plane shearing stress.



SOLUTION

(a) $\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = \frac{1}{2}(5 + 12) = 8.5$ ksi

$R = \sigma_{max} - \sigma_{ave} = 18 - 8.5 = 9.5$ ksi

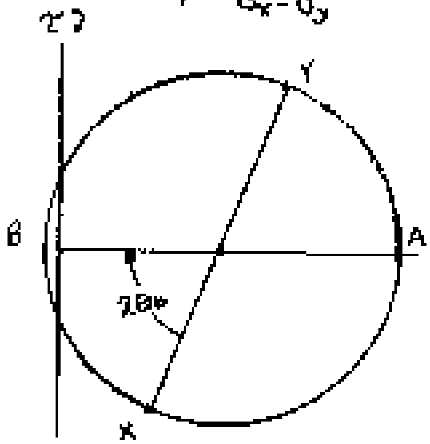
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{xy} = \pm\sqrt{R^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{9.5^2 - \left(\frac{5-12}{2}\right)^2}$$

$= \pm 8.83$ ksi

In the sketch τ_{xy} is shown positive; hence $\tau_{xy} = +8.83$ ksi

$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = -2.523 \qquad 2\theta_p = -68.4°$



$\theta_b = -34.2°, \quad \theta_a = 55.8°$

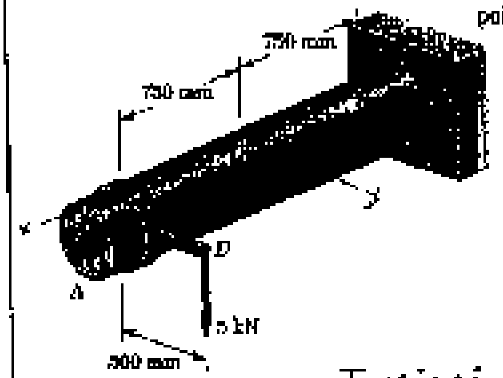
$\sigma_a = \sigma_{ave} + R = \sigma_{max} = 18$ ksi

$\sigma_b = \sigma_{ave} - R = \sigma_{min} = -1$ ksi

(b) $\tau_{max\,(in\text{-}plane)} = R = 9.5$ ksi

PROBLEM 7.160

7.160 The compressed-air tank AB has an inside diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure in the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at points a and b on the top of the tank.



SOLUTION

$r = \frac{1}{2} d = 225$ mm $\qquad t = 6$ mm

$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45$ MPa

$\sigma_2 = \frac{pr}{2t} = 22.5$ MPa

Torsion: $c_1 = 225$ mm , $\quad c_2 = 225 + 6 = 231$ mm

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 446.9 \times 10^6$ mm^4 $= 446.9 \times 10^{-6}$ m^4

$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500$ N·m

$\tau = \frac{Tc}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6$ Pa $= 1.292$ MPa

Transverse shear: $\tau = 0$ at points a and b.

Bending: $I = \frac{1}{2} J = 223.45 \times 10^{-6}$ m^4 , $\quad c = 231 \times 10^{-3}$ m.

Point a

$M = (5 \times 10^3)(750 \times 10^{-3}) = 3750$ N·m

$\sigma = \frac{Mc}{I} = \frac{(3750)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 3.88$ MPa

Total stresses (MPa)

longitudinal $\sigma_x = 22.5 + 3.88 = 26.38$

Circumferential $\sigma_y = 45$

Shear $\tau_{xy} = 1.292$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 35.69$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 9.40$ MPa

$\sigma_{max} = \sigma_{ave} + R = 45.1$ MPa ◄

$\tau_{max\,(in\text{-}plane)} = R = 9.40$ MPa ◄

Point b

$M = (5 \times 10^3)(2 \times 750 \times 10^{-3}) = 7500$ N·m

$\sigma = \frac{Mc}{I} = 7.75$ MPa

Total stresses (MPa)

$\sigma_x = 22.5 + 7.75 = 30.25$

$\sigma_y = 45$

$\tau_{xy} = 1.292$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 37.625$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7.487$ MPa

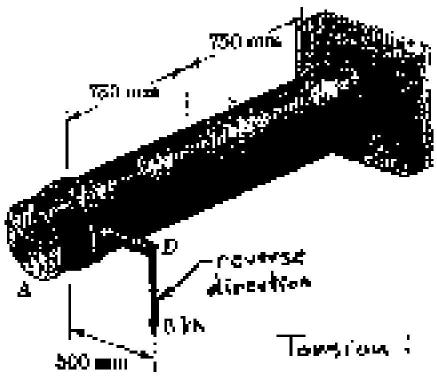
$\sigma_{max} = \sigma_{ave} + R = 45.1$ MPa ◄

$\tau_{max\,(in\text{-}plane)} = R = 7.49$ MPa ◄

PROBLEM 7.161

7.160 The compressed-air tank *AB* has an inside diameter of 450 mm and a uniform wall thickness of 6 mm. Knowing that the gage pressure in the tank is 1.2 MPa, determine the maximum normal stress and the maximum in-plane shearing stress at points *a* and *b* on the top of the tank.

7.161 Solve Prob. 7.160, assuming that the 5-kN force applied at *D* is directed vertically upward.



SOLUTION

$r = \frac{1}{2}d = 225$ mm $\qquad t = 6$ mm

$\sigma_1 = \frac{pr}{t} = \frac{(1.2)(225)}{6} = 45$ MPa

$\sigma_2 = \frac{pr}{2t} = 22.5$ MPa

Torsion: $c_1 = 225$ mm $\quad c_2 = 225 + 6 = 231$ mm

$J = \frac{\pi}{2}(c_2^4 - c_1^4) = 446.9 \times 10^6 \text{ mm}^4 = 446.9 \times 10^{-6} \text{ m}^4$

$T = (5 \times 10^3)(500 \times 10^{-3}) = 2500$ N·m

$\tau = \frac{Tc}{J} = \frac{(2500)(231 \times 10^{-3})}{446.9 \times 10^{-6}} = 1.292 \times 10^6$ Pa $= 1.292$ MPa

Transverse shear: $\tau = 0$ at points a and b.

Bending: $I = \frac{1}{2}J = 223.45 \times 10^{-6} \text{ m}^4$, $\quad c = 231 \times 10^{-3}$ m

Point a

$M = (5 \times 10^3)(750 \times 10^{-3}) = 3750$ N·m

$\sigma = \frac{Mc}{I} = \frac{(3750)(231 \times 10^{-3})}{223.45 \times 10^{-6}} = 3.88$ MPa

Total stresses (MPa)

Longitudinal $\sigma_x = 22.5 - 3.88 = 18.62$ MPa

Circumferential $\sigma_y = 45$ MPa

Shear $\tau_{xy} = -1.292$ MPa

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 31.81$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 13.25$ MPa

$\sigma_{max} = \sigma_{ave} + R = 45.1$ MPa ◄

$\tau_{max\,(in\text{-}plane)} = R = 13.25$ MPa ◄

Point b

$M = (5 \times 10^3)(2 \times 750 \times 10^{-3}) = 7500$ N·m

$\sigma = \frac{Mc}{I} = 7.75$ MPa

Total stresses (MPa)

$\sigma_x = 22.5 - 7.75 = 14.75$

$\sigma_y = 45$

$\tau_{xy} = -1.292$

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 29.875$ MPa

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 15.18$ MPa

$\sigma_{max} = \sigma_{ave} + R = 45.1$ MPa ◄

$\tau_{max\,(in\text{-}plane)} = R = 15.18$ MPa ◄

PROBLEM 7.162

7.162 The steel pressure tank shown has a 30-in. inside diameter and a $\frac{3}{8}$-in. wall thickness. Knowing that the butt-welded seams form an angle of 50° with the longitudinal axis of the tank and that the gage pressure in the tank is 200 psi, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

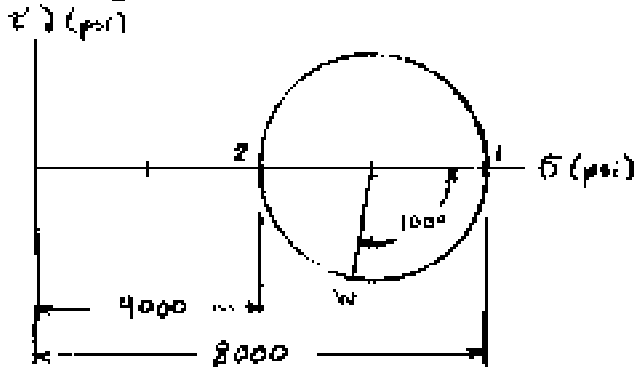
$r = \frac{1}{2}d = 15$ in

$\sigma_1 = \frac{pr}{t} = \frac{(200)(15)}{0.375} = 8000$ psi

$\sigma_2 = \frac{1}{2}\sigma_1 = 4000$ psi

$\sigma_{ave} = \frac{1}{2}(\sigma_1 + \sigma_2) = 6000$ psi

$R = \frac{\sigma_1 - \sigma_2}{2} = 2000$ psi



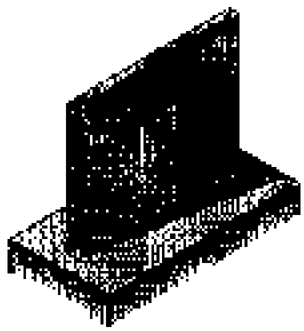
(a) $\sigma_w = \sigma_{ave} + R\cos 100°$

$= 5652$ psi

(b) $\tau_w = R\sin 100°$

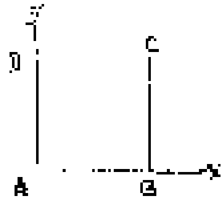
$= 1970$ psi

PROBLEM 7.163



7.163 A square *ABCD* of 2.4-in. side is scribed on the surface of a thin plate while the plate is unloaded. After the plate is loaded, the lengths of sides *AB* and *AD* are observed to have increased, respectively, by 540×10^{-6} in. and 900×10^{-6} in., while the angle *DAB* is observed to have decreased by 360×10^{-6} rad. Knowing that $\nu = \frac{1}{3}$, determine (*a*) the orientation and magnitude of the principal strains, (*b*) the maximum in-plane shearing strain, (*c*) the maximum shearing strain.

SOLUTION



$$\varepsilon_x = \frac{\Delta\delta_x}{\Delta x} = \frac{\Delta \overline{AB}}{\overline{AB}} = \frac{540 \times 10^{-6}}{2.4} = 225 \times 10^{-6}$$

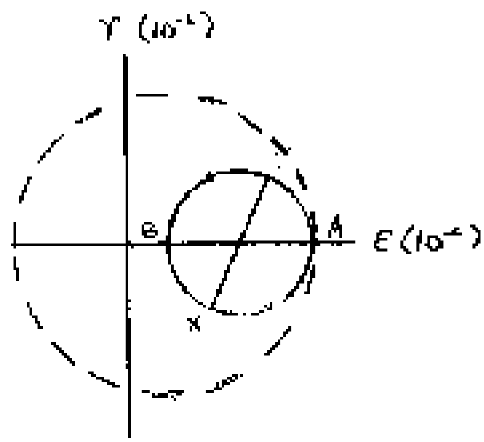
$$\varepsilon_y = \frac{\Delta\delta_y}{\Delta y} = \frac{\Delta \overline{AD}}{\overline{AD}} = \frac{900 \times 10^{-6}}{2.4} = 375 \times 10^{-6}$$

γ_{xy} = decrease in right angle DAB $= 360 \times 10^{-6}$ rad $= 360 \times 10^{-6}$

$$\varepsilon_{ave} = \tfrac{1}{2}\varepsilon_x + \varepsilon_y = 300 \times 10^{-6}$$

$$\tan 2\theta_p = -\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = -\frac{360}{225 - 375} = -2.4$$

$$2\theta_p = -67.38^\circ \qquad \theta_p = -33.7^\circ$$



$$R = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 195 \times 10^{-6}$$

$$\varepsilon_a = \varepsilon_{ave} + R = 495 \times 10^{-6}$$

$$\varepsilon_b = \varepsilon_{ave} - R = 105 \times 10^{-6}$$

(b) $\gamma_{max\,(in\text{-}plane)} = \varepsilon_a - \varepsilon_b = 390 \times 10^{-6}$

$$\varepsilon_c = -\frac{\nu}{1-\nu}(\varepsilon_a + \varepsilon_b)$$

$$= -\frac{(1/3)}{(2/3)}(495 \times 10^{-6} + 105 \times 10^{-6}) = -300 \times 10^{-6}$$

$$\varepsilon_{max} = 495 \times 10^{-6} \qquad \varepsilon_{min} = -300 \times 10^{-6}$$

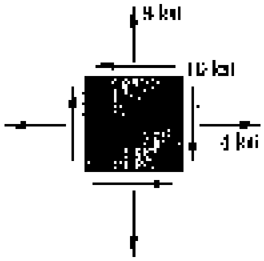
(c) $\gamma_{max} = \varepsilon_{max} - \varepsilon_{min} = 795 \times 10^{-6}$

$$\left.\begin{aligned} \varepsilon_{ave} &= \tfrac{1}{2}(\varepsilon_{max} + \varepsilon_{min}) = 97.5 \times 10^{-6} \\ R &= \tfrac{1}{2}\gamma_{max} = 397.5 \times 10^{-6} \end{aligned}\right\} \text{For dotted Mohr's circle}$$

PROBLEM 7.164

7.164 For the state of plane stress shown, determine (*a*) the principal planes, (*b*) the principal stresses, (*c*) the maximum shearing stress.

SOLUTION



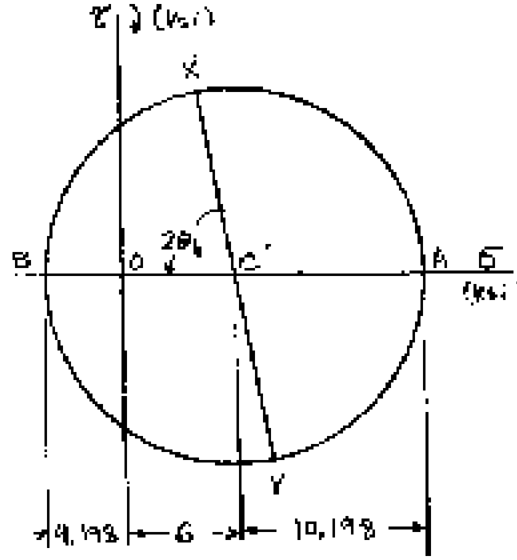
$\sigma_x = 4$ ksi, $\sigma_y = 8$ ksi, $\tau_{xy} = -10$ ksi

$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y)$

$= 6$ ksi

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= 10.198$ ksi



(a) $\tan 2\theta_b = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = 5.00$

$2\theta_b = 78.69°$ $\qquad \theta_b = 39.345°$ ◀

$\theta_a = \theta_b - 90° = -50.655°$ ◀

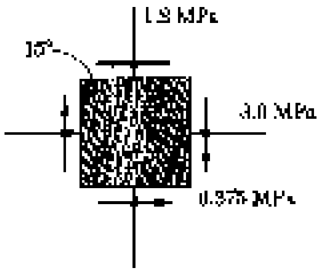
(b) $\sigma_a = \sigma_{ave} + R = 16.198$ ksi ◀

$\sigma_b = \sigma_{ave} - R = -4.198$ ksi ◀

(c) $\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}(\sigma_a - \sigma_b) = R = 10.198$ ksi ◀

PROBLEM 7.165

7.165 The grain a wooden member forms an angle of 15° with the vertical. For the state of plane stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.



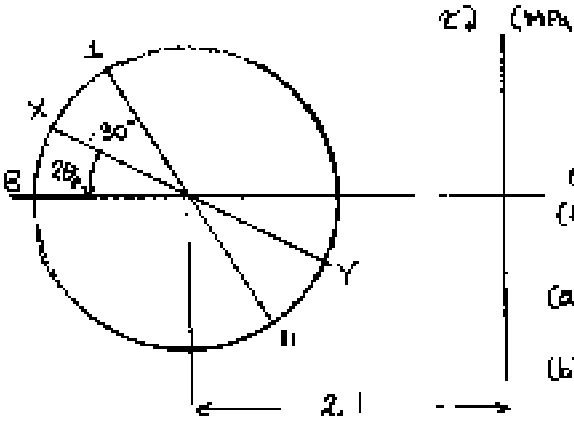
SOLUTION

$$\sigma_x = -3.0 \text{ MPa}, \quad \sigma_y = -1.2 \text{ MPa}$$

$$\tau_{xy} = -0.375 \text{ MPa}$$

$$\sigma_{ave} = \tfrac{1}{2}(\sigma_x + \sigma_y) = -2.10 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 0.975 \text{ MPa}$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 0.41667$$

$$2\theta_p = 22.62°$$

$$2\theta_p + 30° = 52.62°$$

(a) $\tau_{\perp(\text{in-plane})} = R \sin 52.5° = 0.775 \text{ MPa}$ ◀

(b) $\sigma_\perp = \sigma_{ave} - R \cos 52.62°$
$= -2.10 - 0.592 = -2.692 \text{ MPa}$ ◀

PROBLEM 7.166

7.166 A cylindrical steel pressure tank has a 26-in. inside diameter and a uniform $\frac{1}{4}$-in. wall thickness. Knowing that the ultimate stress of the steel used is 65 ksi, determine the maximum allowable gage pressure if a factor of safety of 5.0 must be maintained.

SOLUTION

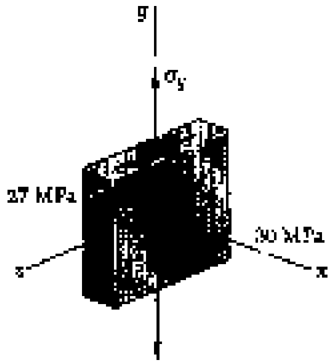
$$r = \tfrac{1}{2} d = \tfrac{1}{2}(26) = 13 \text{ in.} \qquad t = 0.25 \text{ in}$$

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{65}{5} = 13 \text{ ksi} \qquad \sigma_1 = \frac{pr}{t}$$

$$p = \frac{\sigma_1 t}{r} = \frac{(13)(0.25)}{13} = 0.25 \text{ ksi} = 250 \text{ psi}$$ ◀

PROBLEM 7.167

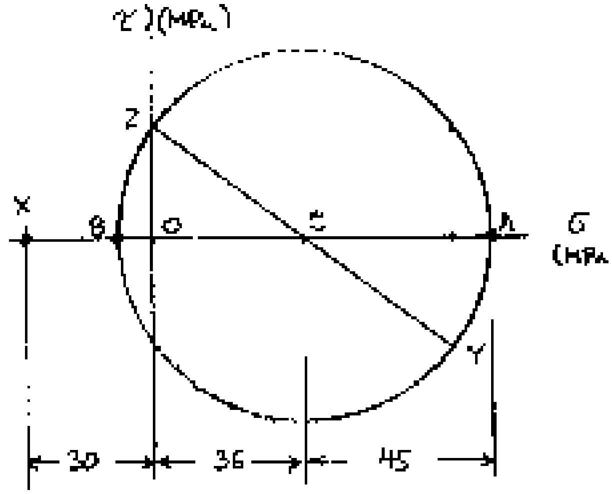
7.167 For the state of stress shown, determine the maximum shearing stress when (a) $\sigma_y = +72$ MPa, (b) $\sigma_y = -72$ MPa.



SOLUTION

$\sigma_x = -30$ MPa $\qquad \tau_{yz} = 27$ MPa , $\sigma_z = 0$

(a) $\sigma_y = +72$ MPa



$\sigma_{ave} = \frac{1}{2}(\sigma_y + \sigma_z) = 36$ MPa

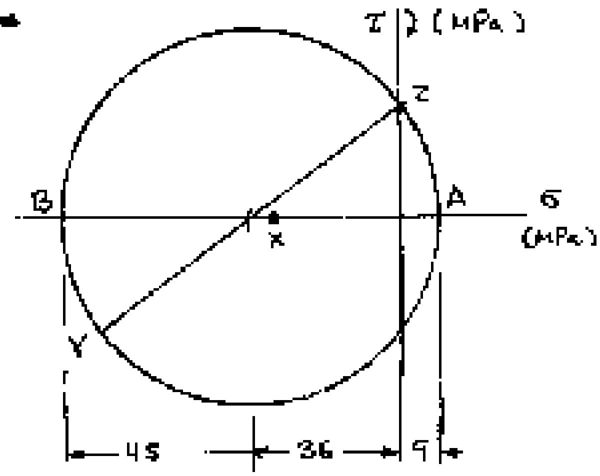
$R = \sqrt{\left(\frac{\sigma_y - \sigma_z}{2}\right)^2 + \tau_{yz}^2} = 45$ MPa

$\sigma_{max} = \sigma_a = \sigma_{ave} + R = 81$ MPa

$\sigma_{min} = \sigma_x = -30$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 55.5$ MPa ◄

(b) $\sigma_y = -72$ MPa



$\sigma_{ave} = \frac{1}{2}(\sigma_y + \sigma_z) = -36$ MPa

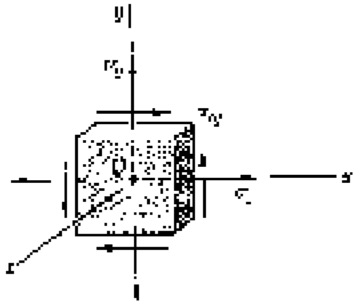
$R = \sqrt{\left(\frac{\sigma_y - \sigma_z}{2}\right)^2 + \tau_{yz}^2} = 45$ MPa

$\sigma_{max} = \sigma_a = \sigma_{ave} + R = 9$ MPa

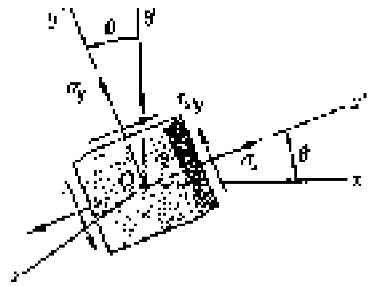
$\sigma_{min} = \sigma_b = \sigma_{ave} - R = -81$ MPa

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 45$ MPa ◄

PROBLEM 7.C1



(a)



(b)

7.C1 A state of plane stress is defined by the stress components σ_x, σ_y, and τ_{xy} associated with the element shown in Fig. P7.C1*a*. (*a*) Write a computer program that can be used to calculate the stress components $\sigma_{x'}$, $\sigma_{y'}$, and $\tau_{x'y'}$ associated with the element after it has rotated through an angle θ about the z axis (Fig. P7.C1*b*). (*b*) Use this program to solve Probs. 7.13 through 7.16.

SOLUTION PROGRAM FOLLOWING EQUATIONS

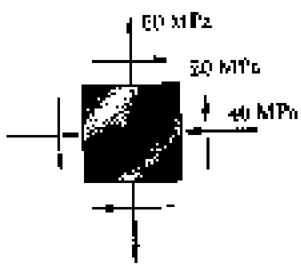
EQ (7.5), p. 427: $\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

EQ (7.7), p. 427: $\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

EQ (7.6), p. 427: $\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

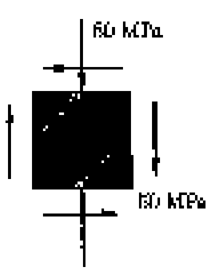
ENTER σ_x, σ_y, τ_{xy} AND θ

PRINT VALUES OBTAINED FOR $\sigma_{x'}$, $\sigma_{y'}$, AND $\tau_{x'y'}$




```
Problem 7.13a

  Sigma x  =  -40 MPa
  Sigma y  =   60 MPa
  Tau xy   =   20 MPa

Rotation of element
(+ counterclockwise)
  theta  =  -25 degrees

  Sigma x'  =  -37.46 MPa
  Sigma y'  =   57.46 MPa
  Tau x'y'  =  -25.45 MPa
```

```
Problem 7.13b

  Sigma x  =  -40 MPa
  Sigma y  =   60 MPa
  Tau xy   =   20 MPa

Rotation of element
(+ counterclockwise)
  theta  =   10 degrees

  Sigma x'  =  -30.14 MPa
  Sigma y'  =   50.14 MPa
  Tau x'y'  =   35.99 MPa
```



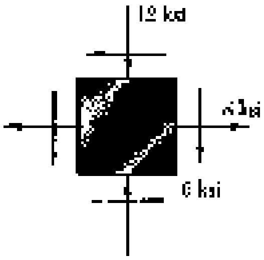

```
Problem 7.14a

  Sigma x  =    0 MPa
  Sigma y  =  -80 MPa
  Tau xy   =  -50 MPa

Rotation of element
(+ counterclockwise)
  theta  =  -25 degrees

  Sigma x'  =    24.01 MPa
  Sigma y'  =  -104.01 MPa
  Tau x'y'  =    -1.50 MPa
```

```
Problem 7.14b

  Sigma x  =    0 MPa
  Sigma y  =  -80 MPa
  Tau xy   =  -50 MPa

Rotation of element
(+ counterclockwise)
  theta  =   10 degrees

  Sigma x'  =  -19.51 MPa
  Sigma y'  =  -60.49 MPa
  Tau x'y'  =  -60.67 MPa
```

CONTINUED

PROBLEM 7.C1 - CONTINUED

Program Output

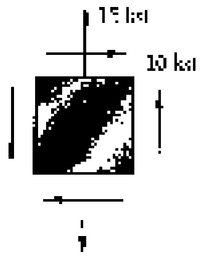


<u>Problem 7.15a</u>

```
  Sigma x  =    8 ksi
  Sigma y  = -12 ksi
  Tau xy   =   -6 ksi

Rotation of element
(+ counterclockwise)
  theta = -25 degrees

  Sigma x'  =   9.02 ksi
  Sigma y'  = -13.02 ksi
  Tau x'y'  =   3.80 ksi
```

<u>Problem 7.15b</u>

```
  Sigma x  =    8 ksi
  Sigma y  = -12 ksi
  Tau xy   =   -6 ksi

Rotation of element
(+ counterclockwise)
  theta =  10 degrees

  Sigma x'  =   5.34 ksi
  Sigma y'  =  -9.34 ksi
  Tau x'y'  =  -9.06 ksi
```



<u>Problem 7.16a</u>

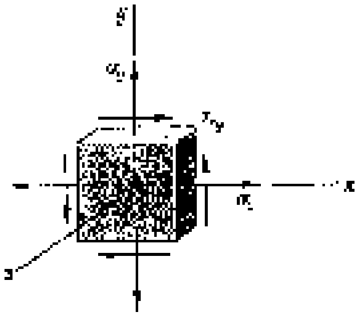
```
  Sigma x  =    0 ksi
  Sigma y  =   16 ksi
  Tau xy   =   10 ksi

Rotation of element
(+ counterclockwise)
  theta = -25 degrees

  Sigma x'  =  -4.80 ksi
  Sigma y'  =  20.80 ksi
  Tau x'y'  =   0.30 ksi
```

<u>Problem 7.16b</u>

```
  Sigma x  =    0 ksi
  Sigma y  =   16 ksi
  Tau xy   =   10 ksi

Rotation of element
(+ counterclockwise)
  theta =  10 degrees

  Sigma x'  =   3.90 ksi
  Sigma y'  =  12.10 ksi
  Tau x'y'  =  12.13 ksi
```

PROBLEM 7.C2

7.C2 A state of plane stress is defined by the stress components σ_x, σ_y, and τ_{xy} associated with the element shown in Fig. P7.C1*a*. (*a*) Write a computer program that can be used to determine the principal axes, the principal stresses, the maximum in-plane shearing stress, and the maximum shearing stress. (*b*) Use this program to solve Probs. 7.7, 7.11, 7.66, and 7.67.



SOLUTION Program Following Equations

Eq (7.10) $\quad \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2}$; $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

Eq (7.14) $\quad \sigma_{max} = \sigma_{ave} + R$

$\sigma_{min} = \sigma_{ave} - R$

Eq (7.12) $\quad \theta_p = \tan^{-1} \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

Eq (7.15) $\quad \theta_s = \tan^{-1} - \dfrac{\sigma_x - \sigma_y}{2\tau_{xy}}$

Shearing Stress If $\sigma_{max} > 0$ and $\sigma_{min} < 0$:

Then $\tau_{max\,(in\text{-}plane)} = R$; $\tau_{max\,(out\text{-}of\text{-}plane)} = R$

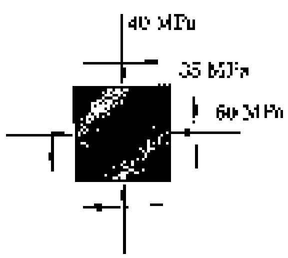
If $\sigma_{max} > 0$ and $\sigma_{min} > 0$:

Then $\tau_{max\,(in\text{-}plane)} = R$; $\tau_{max\,(out\text{-}of\text{-}plane)} = \frac{1}{2}\sigma_{max}$

If $\sigma_{max} < 0$ and $\sigma_{min} < 0$:

Then $\tau_{max\,(in\text{-}plane)} = R$; $\tau_{max\,(out\text{-}of\text{-}plane)} = \frac{1}{2}|\sigma_{min}|$

PROGRAM OUTPUT




```
Problems 7.7 AND 7.11

  Sigma x  =  -60.00 MPa
  Sigma y  =  -40.00 MPa
  Tau  xy  =   35.00 MPa

Angle between xy axes and principal axes
            ( + counterclockwise )
        Theta p  =  -37.03 deg.   and   52.97 deg.
  Sigma max  =  -13.60 MPa
  Sigma min  =  -86.40 MPa
Angle between xy axis and planes of maximum in-plane shearing stress
            ( + counterclockwise )
        Theta s  =    7.97 deg.   and   97.97 deg.
  Tau max  (in plane)  =   36.40 MPa
  Tau max      =   43.20 MPa
```

CONTINUED

PROBLEM 7.C2 - CONTINUED

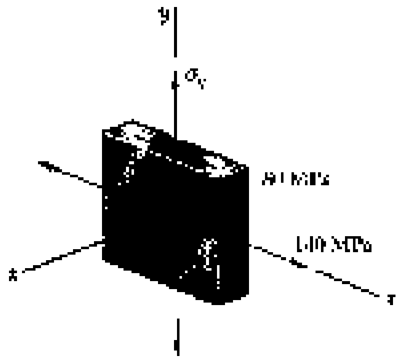


Fig. P7.66 and P7.67

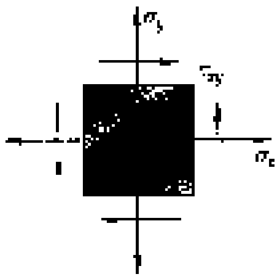
```
Problem 7.66a:  Sigma x  = 140.00 MPa
                Sigma y  =  20.00 MPa
                Tau xy   =  80.00 MPa

Angle between xy axes and principal axes
          ( + counterclockwise )
     Theta p =  26.57 deg.   and 116.57 deg.
  Sigma max = 180.00 MPa
  Sigma min = -20.00 MPa
Angle between xy axis and planes of maximum in-plane
     in-plane shearing stress ( + counterclockwise )
     Theta s =   71.57 deg.   and 161.57 deg.
  Tau max (in-plane)          = 100.00 MPa
  Tau max (out-of-plane)      = 100.00 MPa
```

```
Problem 7.66b:  Sigma x  = 140.00 MPa
                Sigma y  = 140.00 MPa
                Tau xy   =  80.00 MPa

Angle between xy axes and principal axes
          ( + counterclockwise )
     Theta p =  45.00 deg.   and 135.00 deg.
  Sigma max = 220.00 MPa
  Sigma min =  60.00 MPa
Angle between xy axis and planes of maximum in-plane
     in-plane shearing stress ( + counterclockwise )
     Theta s =  90.00 deg.   and 180.00 deg.
  Tau max (in-plane)          =  80.00 MPa
  Tau max (out-of-plane)      = 110.00 MPa
```

```
Problem 7.67a:  Sigma x  = 140.00 MPa
                Sigma y  =  40.00 MPa
                Tau xy   =  80.00 MPa

Angle between xy axes and principal axes
          ( + counterclockwise )
     Theta p =  29.00 deg.   and 119.00 deg.
  Sigma max = 184.34 MPa
  Sigma min =  -4.34 MPa
Angle between xy axis and planes of maximum in-plane
     in-plane shearing stress ( + counterclockwise )
     Theta s =  74.00 deg.   and 164.00 deg.
  Tau max (in-plane)          =  94.34 MPa
  Tau max (out-of-plane)      =  94.34 MPa
```

```
Problem 7.67b:  Sigma x  = 140.00 MPa
                Sigma y  = 120.00 MPa
                Tau xy   =  80.00 MPa

Angle between xy axes and principal axes
          ( + counterclockwise )
     Theta p =  41.44 deg.   and 131.44 deg.
  Sigma max = 210.62 MPa
  Sigma min =  49.38 MPa
Angle between xy axis and planes of maximum in-plane
     in-plane shearing stress ( + counterclockwise )
     Theta s =  86.44 deg.   and 176.44 deg.
  Tau max (in-plane)          =  80.62 MPa
  Tau max (out-of-plane)      = 105.31 MPa
```

PROBLEM 7.C3



7.C3 (*a*) Write a computer program that, for a given state of plane stress and a given yield strength of a ductile material, can be used to determine whether the material will yield. The program should use both the maximum-shearing-strength criterion and the maximum-distortion-energy criterion. It should also print the values of the principal stresses and, if the material does not yield, calculate the factor of safety. (*b*) Use this program to solve Probs. 7.81 through 7.84.

SOLUTION

PRINCIPAL STRESSES

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \quad ; \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R$$

$$\sigma_b = \sigma_{ave} - R$$

MAXIMUM-SHEARING-STRESS CRITERION $\quad \tau_Y = \frac{1}{2}\sigma_Y$

IF σ_a AND σ_b HAVE SAME SIGN, $\tau_{max} = \frac{1}{2}\sigma_a$

IF $\tau_{max} > \tau_Y$, YIELDING OCCURS

IF $\tau_{max} < \tau_Y$, NO YIELDING OCCURS, AND

$$\text{FACTOR OF SAFETY} = \frac{\tau_Y}{\tau_{max}}$$

MAXIMUM-DISTORTION-ENERGY CRITERION

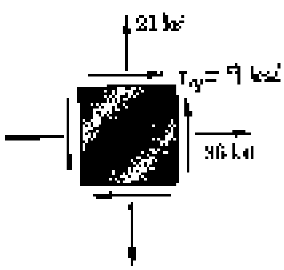
COMPUTE $\quad \text{RADICAL} = \sqrt{\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2}$

IF RADICAL $> \sigma_Y$, YIELDING OCCURS

IF RADICAL $< \sigma_Y$, NO YIELDING OCCURS, AND

$$\text{FACTOR OF SAFETY} = \frac{\sigma_Y}{\text{RADICAL}}$$

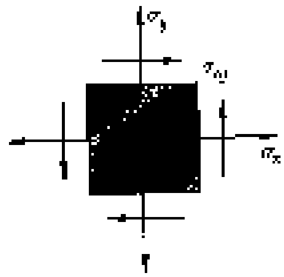
PROGRAM OUTPUT




```
Problems 7.81a and 7.82a     Sigma x =   36.00 ksi
                             Sigma y =   21.00 ksi
                             Tau xy  =    9.00 ksi
          Sigmax =   40.22 ksi
          Sigmin =   16.78 ksi
     Using the maximum-shearing-stress criterion:
       Material will not yield
       F.S. =  1.119
     Using the maximum-distortion-energy criterion:
       Material will not yield
       F.S. =  1.286
```

CONTINUED

PROBLEM 7.C4



7.C4 (*a*) Write a computer program based on Mohr's fracture criterion for brittle materials that, for a given state of plane stress and given values of the ultimate strength of the material in tension and in compression, can be used to determine whether rupture will occur. The program should also print the values of the principal stresses. (*b*) Use this program to solve Probs. 7.91 and 7.92 and to check the answers given for Probs. 7.93 and 7.94.

SOLUTION

PRINCIPAL STRESSES

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_a = \sigma_{ave} + R$$

$$\sigma_b = \sigma_{ave} - R$$

MOHR'S FRACTURE CRITERION

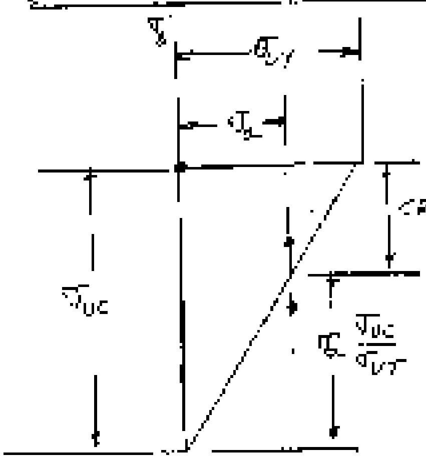
IF σ_a AND σ_b HAVE SAME SIGN, AND

$\sigma_a < \sigma_{UT}$ AND $\sigma_b < \sigma_{UC}$, NO FAILURE

$\sigma_a > \sigma_{UT}$ OR $\sigma_b > \sigma_{UC}$, FAILURE

IF $\sigma_a > 0$ AND $\sigma_b < 0$:

CONSIDER FOURTH QUADRANT OF FIG 7.47



$$\text{CRITERION} = \sigma_{UC} - \sigma_a \frac{\sigma_{UC}}{\sigma_{UT}}$$

FOR NO RUPTURE TO OCCUR POINT (σ_a, σ_b) MUST LIE WITHIN MOHR'S ENVELOPE (FIG 7.47)

IF σ_b > CRITERION, THEN RUPTURE OCCURS

IF σ_b < CRITERION, THEN NO RUPTURE OCCURS

PROGRAM OUTPUT

Fig. P7.91

```
Problem 7.91        Sigma x =   -8.00 ksi
                    Sigma y =    0.00 ksi
                     Tau xy =    7.00 ksi
Ultimate strength in tension       = 10 ksi
Ultimate strength in compression  = 30 ksi

   Sigma max = Sigma a     4.06 ksi
   Sigma min = Sigma b   -12.06 ksi
Rupture will not occur
```

CONTINUED

PROBLEM 7.C5

7.C5 A state of plane strain is defined by the strain components ϵ_x, ϵ_y, and γ_{xy} associated with the x and y axes. (*a*) Write a computer program that can be used to calculate the strain components $\epsilon_{x'}$, $\epsilon_{y'}$, and $\gamma_{x'y'}$ associated with the frame of reference $x'y'$ obtained by rotating the x and y axes through an angle θ. (*b*) Use this program to solve Probs. 7.126 through 7.129.

SOLUTION PROGRAM FOLLOWING EQUATIONS

EQ (7.44) $\epsilon_x' = \frac{\epsilon_x+\epsilon_y}{2} + \frac{\epsilon_x-\epsilon_y}{2}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta$

EQ (7.45) $\epsilon_y' = \frac{\epsilon_x+\epsilon_y}{2} - \frac{\epsilon_x-\epsilon_y}{2}\sin 2\theta - \frac{1}{2}\gamma_{xy}\cos 2\theta$

EQ. (7.46) $\gamma_{xy}' = -(\epsilon_x-\epsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$

ENTER ϵ_x, ϵ_y, γ_{xy}, AND θ

PRINT VALUES OBTAINED FOR ϵ_x', ϵ_y', AND γ_{xy}'

PROGRAM OUTPUT

```
Problem 7.126          Epsilon x  =  -720 micro meters
                       Epsilon y  =     0 micro meters
                       Gamma xy   =   300 micro radians
Rotation of element,   in degrees  ( + counterclockwise )
                       Theta =  -30 degrees

           Epsilon x'=  -669.90 micro meters
           Epsilon y'=   -50.10 micro meters
           Gamma x'y'=  -473.54 micro radians

Problem 7.127          Epsilon x  =     0 micro meters
                       Epsilon y  =   320 micro meters
                       Gamma xy   =  -100 micro radians
Rotation of element,   in degrees  ( + counterclockwise )
                       Theta =   30 degrees

           Epsilon x'=    36.70 micro meters
           Epsilon y'=   283.30 micro meters
           Gamma x'y'=   227.13 micro radians

Problem 7.128          Epsilon x  =  -800 micro meters
                       Epsilon y  =   450 micro meters
                       Gamma xy   =   200 micro radians
Rotation of element,   in degrees  ( + counterclockwise )
                       Theta =  -25 degrees

           Epsilon x'=  -653.35 micro meters
           Epsilon y'=   303.35 micro meters
           Gamma x'y'=  -829.00 micro radians

Problem 7.129          Epsilon x  =  -500 micro meters
                       Epsilon y  =   250 micro meters
                       Gamma xy   =     0 micro radians
Rotation of element,   in degrees  ( + counterclockwise )
                       Theta =   15 degrees

           Epsilon x'=  -449.76 micro meters
           Epsilon y'=   199.76 micro meters
           Gamma x'y'=   375.00 micro radians
```

PROBLEM 7.C6

7.C6 A state of strain is defined by the strain components ϵ_x, ϵ_y, and γ_{xy} associated with the x and y axes. (*a*) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (*b*) Use this program to solve Probs. 7.134 through 7.137.

SOLUTION PROGRAM FOLLOWING EQUATIONS

EQ (7.50) $\quad \epsilon_{ave} = \frac{\epsilon_x + \epsilon_y}{2} \qquad R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$

EQ (7.51) $\quad \epsilon_{max} = \epsilon_{ave} + R \qquad \epsilon_{min} = \epsilon_{ave} - R$

EQ (7.52) $\quad \theta_p = \tan^{-1} \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$\gamma_{max(in\text{-}plane)} = 2R$

CALCULATE OUT-OF-PLANE SHEARING STRAIN AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN

LET $\epsilon_a = \epsilon_{max}$

$\epsilon_b = \epsilon_{min}$

CALCULATE $\epsilon_c = -\frac{\nu}{1-\nu}(\epsilon_a + \epsilon_b)$

IF $\epsilon_a > \epsilon_b > \epsilon_c$; $\gamma_{out\text{-}of\text{-}plane} = \epsilon_a - \epsilon_c$

IF $\epsilon_a > \epsilon_c > \epsilon_b$; $\gamma_{out\text{-}of\text{-}plane} = \epsilon_a - \epsilon_b = 2R$

IF $\epsilon_c > \epsilon_a > \epsilon_b$; $\gamma_{out\text{-}of\text{-}plane} = \epsilon_c - \epsilon_b$

PROGRAM PRINTOUT

```
Problem 7.134      Epsilon x =  160 micro meters
                   Epsilon y = -480 micro meters
                   Gamma xy  = -600 micro radians
                   nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)
        Theta p   =  -21.58 degrees
        Epsilon a =  278.63 micro meters
        Epsilon b = -598.63 micro meters
        Epsilon c =  159.98 micro meters

        Gamma max (in plane) =  877.27 micro radians
        Gamma max =  877.27 micro radians
```

CONTINUED

PROBLEM 7.C6 - CONTINUED

```
Problem 7.135       Epsilon x = -260 micro meters
                    Epsilon y =  -60 micro meters
                    Gamma xy  =  480 micro radians
                    nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)
        Theta p     =  -33.69 degrees
        Epsilon a =   100.00 micro meters
        Epsilon b =  -420.00 micro meters
        Epsilon c =   159.98 micro meters

        Gamma max (in plane) =  520.00 micro radians
        Gamma max =  579.98 micro radians


Problem 7.136       Epsilon x =  -40 micro meters
                    Epsilon y =  760 micro meters
                    Gamma xy  =  960 micro radians
                    nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)
        Theta p     =  -25.10 degrees
        Epsilon a =   984.82 micro meters
        Epsilon b =  -264.82 micro meters
        Epsilon c =  -359.95 micro meters

        Gamma max (in plane) = 1249.64 micro radians
        Gamma max = 1344.77 micro radians


Problem 7.137       Epsilon x = -300 micro meters
                    Epsilon y = -200 micro meters
                    Gamma xy  =  175 micro radians
                    nu = 0.333

Angle between xy axes and principal axes (+ = counterclockwise)
        Theta p     =  -30.13 degrees
        Epsilon a =  -149.22 micro meters
        Epsilon b =  -350.78 micro meters
        Epsilon c =   250.00 micro meters

        Gamma max (in plane) =  201.56 micro radians
        Gamma max =  600.77 micro radians
```

PROBLEM 7.C7

7.C7 A state of plane strain is defined by the strain components ϵ_x, ϵ_y, and γ_{xy} measured at a point. (*a*) Write a computer program that can be used to determine the orientation and magnitude of the principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (*b*) Use this program to solve Probs. 7.138 through 7.141.

SOLUTION

PROGRAM FOLLOWING EQUATIONS

EQ(7.50) $\epsilon_{ave} = \dfrac{\epsilon_x + \epsilon_y}{2}$ $\qquad R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\dfrac{\gamma_{xy}}{2}\right)^2}$

EQ(7.51) $\epsilon_{max} = \epsilon_{ave} + R \qquad \epsilon_{min} = \epsilon_{ave} - R$

EQ(7.52) $\theta_p = \tan^{-1} \dfrac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$\gamma_{xy}(\text{in-plane}) = 2R$

CALCULATE OUT-OF-PLANE SHEARING STRAIN AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN

LET $\epsilon_a = \epsilon_{max}$

$\epsilon_b = \epsilon_{min}$

$\epsilon_c = 0$ (PLAIN STRAIN)

IF $\epsilon_a > \epsilon_b > \epsilon_c$: $\gamma_{\text{OUT-OF-PLANE}} = \epsilon_a - \epsilon_c$

IF $\epsilon_a > \epsilon_c > \epsilon_b$: $\gamma_{\text{OUT-OF-PLANE}} = \epsilon_a - \epsilon_b = 2R$

IF $\epsilon_c > \epsilon_a > \epsilon_b$: $\gamma_{\text{OUT-OF-PLANE}} = \epsilon_c - \epsilon_b$

PROGRAM PRINTOUT

```
Problem 7.138      Epsilon x =  -90
                   Epsilon y = -130
                   Gamma xy  =  150

Angle between xy axes and principal axes (+ = counterclockwise)
        Theta p   =    37.53 and -52.47 degrees
        Epsilon a =   -32.38 micro meters at   37.53 degrees
        Epsilon b =  -187.62 micro meters at  -52.47 degrees
        Epsilon c =     0.00 micro meters

        Gamma max (in plane) =  155.24 micro radians
        Gamma max =  187.62 micro radians
```

CONTINUED

PROBLEM 7.C7 - CONTINUED

```
Problem 7.139        Epsilon x  =  375
                     Epsilon y  =   75
                     Gamma xy   =  125

Angle between xy axes and principal axes (+ = counterclockwise)
        Theta p    =     11.31 and -78.69 degrees
        Epsilon a =    387.50 micro meters at    11.31 degrees
        Epsilon b =     62.50 micro meters at   -78.69 degrees
        Epsilon c =      0.00 micro meters

        Gamma max (in plane) =  325.00 micro radians
        Gamma max =  387.50 micro radians


Problem 7.140        Epsilon x  =  400
                     Epsilon y  =  200
                     Gamma xy   =  375

Angle between xy axes and principal axes (+ = counterclockwise)
        Theta p    =     30.96 and -59.04 degrees
        Epsilon a =    512.50 micro meters at    30.96 degrees
        Epsilon b =     87.50 micro meters at   -59.04 degrees
        Epsilon c =      0.00 micro meters

        Gamma max (in plane) =  425.00 micro radians
        Gamma max =  512.50 micro radians


Problem 7.141        Epsilon x  =   60
                     Epsilon y  =  240
                     Gamma xy   =  -50

Angle between xy axes and principal axes (+ = counterclockwise)
        Theta p    =      7.76 and -82.24 degrees
        Epsilon a =    243.41 micro meters at     7.76 degrees
        Epsilon b =     56.59 micro meters at    97.76 degrees
        Epsilon c =      0.00 micro meters

        Gamma max (in plane) =  186.82 micro radians
        Gamma max =  243.41 micro radians
```

PROBLEM 7.C8

7.C8 A rosette consisting of three gages forming, respectively, angles θ_1, θ_2, and θ_3 with the x axis is attached to the free surface of a machine component made of a material with a given Poisson's ratio ν. (*a*) Write a computer program that, for given readings ϵ_1, ϵ_2, and ϵ_3 of the gages, can be used to calculate the strain components associated with the x and y axes and to determine the orientation and magnitude of the three principal strains, the maximum in-plane shearing strain, and the maximum shearing strain. (*b*) Use this program to solve Probs. 7.142 through 7.145.

SOLUTION

FOR $n = 1$ TO 3, ENTER θ_n AND ϵ_n

ENTER $NU = \nu$

SOLVE EQS. (7.60) FOR ϵ_x, ϵ_y, AND γ_{xy} USING METHOD OF DETERMINANTS OR ANY OTHER METHOD.

ENTER $\epsilon_{ave} = \dfrac{\epsilon_x + \epsilon_y}{2}$; $R = \sqrt{\left(\dfrac{\epsilon_x - \epsilon_y}{2}\right)^2 + \gamma_{xy}^2}$

$$\epsilon_a = \epsilon_{max} = \epsilon_{ave} + R$$

$$\epsilon_b = \epsilon_{min} = \epsilon_{ave} - R$$

$$\epsilon_c = -\frac{\nu}{1-\nu}(\epsilon_a + \epsilon_b)$$

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

SHEARING STRAINS

MAXIMUM IN-PLANE SHEARING STRAIN

$$\gamma_{max\,(in-plane)} = 2R$$

CALCULATE OUT-OF-PLANE SHEARING STRAIN, AND CHECK WHETHER IT IS THE MAXIMUM SHEARING STRAIN,

IF $\epsilon_c < \epsilon_b$: $\gamma_{out-of-plane} = \epsilon_a - \epsilon_c$

IF $\epsilon_c > \epsilon_a$: $\gamma_{out-of-plane} = \epsilon_c - \epsilon_b$

OTHERWISE: $\gamma_{out-of-plane} = 2R$

PROGRAM OUTPUT

Problem 7.142

Gage	theta degrees	epsilon micro meters
1	30	600
2	-30	450
3	90	-75

```
Epsilon x  =   725.000 micro meters
Epsilon y  =   -75.000 micro meters
Gamma xy   =   173.205 micro radians

Epsilon a  =   734.268 micro meters
Epsilon b  =   -84.268 micro meters
Gamma max (in plane)  =  818.535 micro radians
```

CONTINUED

PROBLEM 7.C8 - CONTINUED

Problem 7.143

Gage	theta degrees	epsilon in./in.
1	-15	720
2	30	-180
3	75	120

```
Epsilon x =   379.808 in./in.
Epsilon y =   460.192 in./in.
Gamma xy  = -1339.230 micro radians

Epsilon a =  1090.820 in./in.
Epsilon b =  -250.820 in./in.
Gamma max (in plane) = 1341.641 micro radians
```

Problem 7.144

OBSERVE THAT GAGE 3 IS ORIENTED ALONG THE y AXIS. THEREFORE

ENTER ϵ_4 AND ϵ_4 AS ϵ_3 AND ϵ_3,

THE VALUE OF ϵ_y THAT IS OBTAINED IS ALSO THE EXPECTED READING OF GAGE 3.

Gage	theta degrees	epsilon micro meters
1	0	420
2	45	-45
4 → ~~3~~	135	165

```
Epsilon x =   420.000 micro meters
Epsilon y =  -300.000 micro meters
Gamma xy  =  -210.000 micro radians

Epsilon a =   435.000 micro meters
Epsilon b =  -315.000 micro meters
Gamma max (in plane) =   750.000 micro radians
```

Problem 7.145

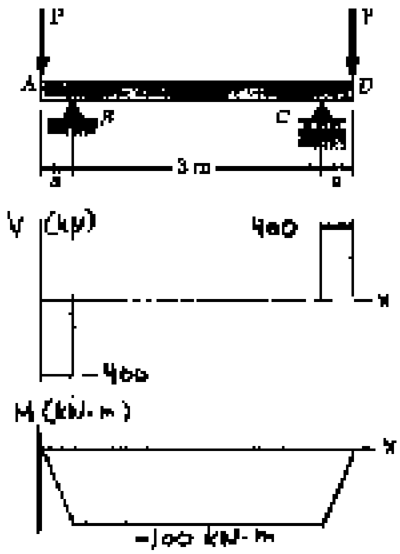
Gage	theta degrees	epsilon in./in.
1	45	-50
2	-45	360
3	0	315

```
Epsilon x =   315.000 in./in.
Epsilon y =    -5.000 in./in.
Gamma xy  =  -410.000 micro radians

Epsilon a =   415.048 in./in.
Epsilon b =  -105.048 in./in.
Gamma max (in plane) =   520.096 micro radians
```

CHAPTER 8

PROBLEM 8.1

8.1 An overhanging W250 × 58 rolled-steel beam supports two loads as shown. Knowing that $P = 400$ kN, $a = 0.25$ m, and $\sigma_{all} = 250$ MPa, determine (a) the maximum value of the normal stress σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web, (c) whether the specified shape is acceptable as far as these two stresses are concerned.



$$|V|_{max} = 400 \text{ kN} = 400 \times 10^3 \text{ N}$$

$$|M|_{max} = (400 \times 10^3)(0.25) = 100 \times 10^3 \text{ N·m}$$

For W 250 × 58 rolled steel section

$d = 252$ mm, $b_f = 203$ mm, $t_f = 13.5$ mm

$t_w = 8.0$ mm, $I_x = 87.3 \times 10^6$ mm^4, $S_x = 693 \times 10^3$ mm^3

$c = \frac{1}{2}d = 126$ mm, $y_b = c - t_f = 112.5$ mm

(a) $$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{100 \times 10^3}{693 \times 10^{-6}} = 144.3 \times 10^6 \text{ Pa} = 144.3 \text{ MPa}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \frac{112.5}{126}(144.3) = 128.84 \text{ MPa}$$

$$A_f = b_f t_f = (203)(13.5) = 2740.5 \text{ mm}^2$$

$$\bar{y}_f = \frac{1}{2}(c + y_b) = 119.25 \text{ mm}$$

$$Q_b = A_f \bar{y}_f = 326.80 \times 10^3 \text{ mm}^3 = 326.80 \times 10^{-6} \text{ m}^3$$

$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(400 \times 10^3)(326.80 \times 10^{-6})}{(87.3 \times 10^{-6})(8 \times 10^{-3})} = 187.2 \times 10^6 \text{ Pa} = 187.2 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 197.97 \text{ MPa}$$

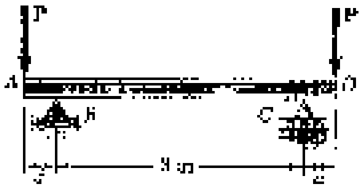
(b) $$\sigma_{max} = \frac{\sigma_b}{2} + R = 262 \text{ MPa}$$

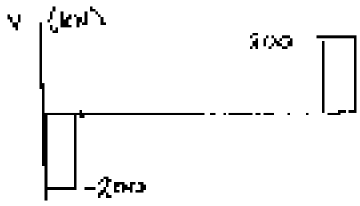
(c) Since $\sigma_{max} > 250$ MPa, W250 × 58 is <u>not acceptable</u>.

PROBLEM 8.2

8.1 An overhanging W250 × 58 rolled-steel beam supports two loads as shown. Knowing that $P = 400$ kN, $a = 0.25$ m, and $\sigma_{all} = 250$ MPa, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of the flange and the web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

8.2 Solve Prob. 8.1, assuming that $P = 200$ kN and $a = 0.5$ m.





$|V|_{max} = 200 \text{ kN} = 200 \times 10^3 \text{ N}$

$|M|_{max} = (200 \times 10^3)(0.5) = 100 \times 10^3 \text{ N}\cdot\text{m}$

For W250 × 58 rolled steel section

$d = 252$ mm $\qquad b_f = 203$ mm $\qquad t_f = 13.5$ mm

$t_w = 8.0$ mm $\qquad I_x = 87.3 \times 10^6 \text{ mm}^4 \qquad S_x = 693 \times 10^3 \text{ mm}^3$

$c = \frac{1}{2} d = 126$ mm $\qquad y_b = c - t_f = 112.5$ mm

(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{100 \times 10^3}{693 \times 10^{-6}} = 144.3 \times 10^6$ Pa
$= 144.3$ MPa ◄

$\sigma_b = \dfrac{y_b}{c} \sigma_m = \dfrac{112.5}{126}(144.3) = 128.84$ MPa

$A_f = b_f t_f = (203)(13.5) = 2740.5 \text{ mm}^2$

$\bar{y}_f = \frac{1}{2}(c + y_b) = 119.25$ mm

$Q_b = A_f \bar{y}_f = 326.80 \times 10^3 \text{ mm}^3 = 326.80 \times 10^{-6} \text{ m}^3$

$$\tau_{xy} = \frac{|V|_{max} Q_b}{I_x t_w} = \frac{(200 \times 10^3)(326.80 \times 10^{-6})}{(87.3 \times 10^{-6})(8 \times 10^{-3})} = 93.6 \times 10^6 \text{ Pa} = 93.6 \text{ MPa}$$

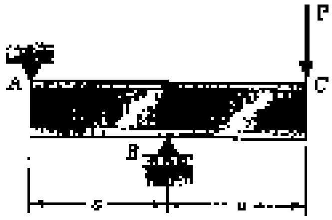
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = 113.63 \text{ MPa}$$

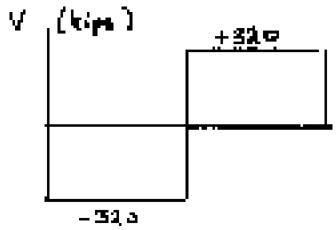
(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 178.0$ MPa ◄

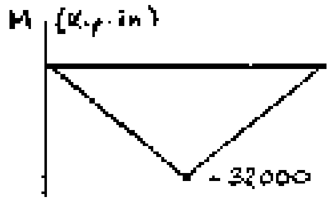
(c) Since $\sigma_{max} < 250$ MPa, W 250×58 is <u>acceptable</u>. ◄

PROBLEM 8.3

8.3 An overhanging W36 × 300 rolled steel beam supports a load **P** as shown. Knowing that $P = 320$ kips, $a = 100$ in., and $\sigma_{all} = 29$ ksi, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.







$|V|_{max} = 320$ kips

$|M|_{max} = (320)(100) = 32000$ kip·in.

For W 36 × 300 rolled steel beam

$d = 36.74$ in. $\quad b_f = 16.655$ in. $\quad t_f = 1.680$ in.

$t_w = 0.945$ in. $\quad I_x = 20300$ in^4 $\quad S_x = 1110$ in^3

$c = \frac{1}{2}d = 18.37$ in. $\quad y_b = c - t_f = 16.69$ in.

(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{32000}{1110} = 28.8$ ksi

$\sigma_b = \dfrac{y_b}{c}\sigma_m = \dfrac{(16.69)}{(18.37)}(28.8) = 26.2$ ksi

$A_f = b_f t_f = 27.98$ in^2

$\bar{y}_f = \frac{1}{2}(c + y_b) = 17.53$ in.

$Q_b = A_f \bar{y}_f = 490.49$ in^3

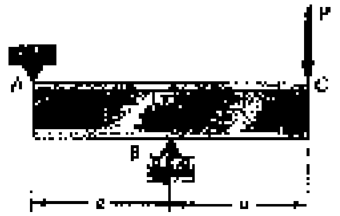
$\tau_{xy} = \dfrac{|V|_{max} Q_b}{I_x t_w} = \dfrac{(320)(490.49)}{(20300)(0.945)} = 8.18$ ksi

$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{26.2}{2}\right)^2 + (8.18)^2} = 15.44$ ksi

(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 28.5$ ksi

(c) Since 28.5 ksi $< \sigma_{all}$, W36 × 300 is <u>acceptable</u>.

PROBLEM 8.4



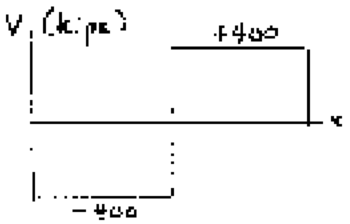
8.3 An overhanging W36 × 300 rolled-steel beam supports a load **P** as shown. Knowing that $P = 320$ kips, $a = 100$ in., and $\sigma_{all} = 29$ ksi, determine (*a*) the maximum value of the normal stress σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web, (*c*) whether the specified shape is acceptable as far as these two stresses are concerned.

8.4 Solve Prob. 8.3, assuming that $P = 400$ kips and $a = 80$ in.

$|V|_{max} = 400$ kips

$|M|_{max} = (400)(80) = 32000$ kip·in

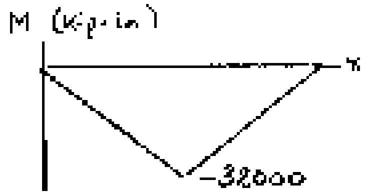
For W36 × 300 rolled steel section



$d = 36.74$ in $\qquad b_f = 16.655$ in $\qquad t_f = 1.680$ in

$t_w = 0.945$ in $\qquad I_x = 20300$ in^4 $\qquad S_x = 1110$ in^3

$c = \frac{1}{2}d = 18.37$ in $\qquad y_b = c - t_f = 16.69$ in.



(a) $\sigma_m = \dfrac{|M|_{max}}{S_x} = \dfrac{32000}{1110} = 28.8$ ksi

$\sigma_b = \dfrac{y_b}{c}\sigma_m = \dfrac{(16.69)}{(18.37)}(28.8) = 26.2$ ksi

$A_f = b_f t_f = 27.98$ in^2

$\bar{y}_f = \frac{1}{2}(c + y_b) = 17.53$ in.

$Q_b = A_f \bar{y}_f = 490.49$ in^3

$\tau_{xy} = \dfrac{|V|_{max} Q_b}{I_x t_w} = \dfrac{(400)(490.49)}{(20300)(0.945)} = 10.23$ ksi

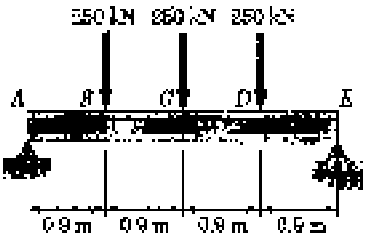
$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(13.1)^2 + (10.23)^2} = 16.62$ ksi

(b) $\sigma_{max} = \dfrac{\sigma_b}{2} + R = 29.7$ ksi

(c) Since 29.7 ksi $> \sigma_{all}$. W 36 × 300 is <u>not acceptable</u>

PROBLEM 8.5

8.5 and 8.6 (*a*) Knowing that $\sigma_{all} = 160$ MPa and $\tau_{all} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.



Reactions: $R_A = 375$ kN ↑, $R_E = 375$ kN ↑

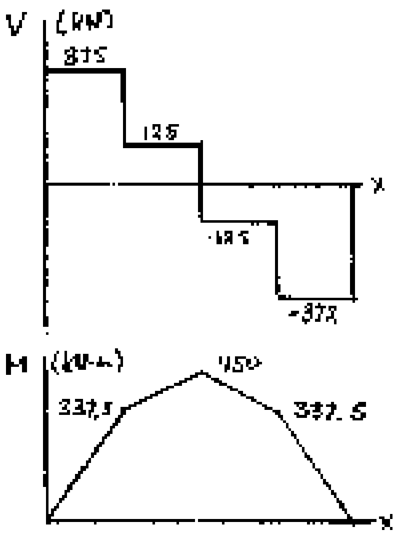
$|V|_{max} = 375$ kN

$|M|_{max} = 450$ kN·m

$|V|$ at point C   125 kN

$$S_{min} = \frac{M_{max}}{\sigma_{all}} = \frac{450\times10^3}{160\times10^6} = 2.8125\times10^{-3}\ m^3$$

$$= 2812.5\times10^3\ mm^3$$



Shape	S_x (10^3 mm^3)
W 840 × 176	5890
W 760 × 147	4410
→ W 690 × 125	3510
W 610 × 155	4220
W 530 × 150	3720
W 460 × 158	3340
W 360 × 216	3800

(a) Use W 690 × 125 ◄

$d = 678$ mm

$t_f = 16.30$ mm

$t_w = 11.7$ mm

$$\sigma_m = \frac{|M|_{max}}{S_x} = \frac{450\times10^3}{3510\times10^{-6}} = 128.2\times10^6\ Pa = 128.2\ MPa \quad ◄$$

$$\tau_m = \frac{|V|_{max}}{A_w} = \frac{|V|_{max}}{d\,t_w} = \frac{375\times10^3}{(678\times10^{-3})(11.7\times10^{-3})} = 47.3\times10^6\ Pa = 47.3\ MPa \quad ◄$$

At point C $\quad \tau_w = \dfrac{V}{A_w} = \dfrac{125\times10^3}{(678\times10^{-3})(11.7\times10^{-3})} = 15.76\times10^6\ Pa$

$= 15.76$ MPa

$c = \frac{1}{2}d = \frac{678}{2} = 339$ mm $\qquad y_b = c - t_f = 339 - 16.30 = 322.7$ mm

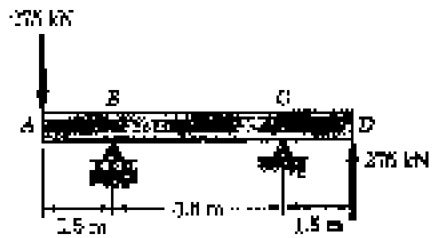
$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{322.7}{339}\right)(128.2) = 122.0\ MPa$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(61.0)^2 + (15.76)^2} = 63.0\ MPa$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 61.0 + 63.0 = 124.0\ MPa \quad ◄$$

PROBLEM 8.6

8.5 and 8.6 (a) Knowing that $\sigma_{all} = 160$ MPa and $\tau_{all} = 100$ MPa, select the most economical metric wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.

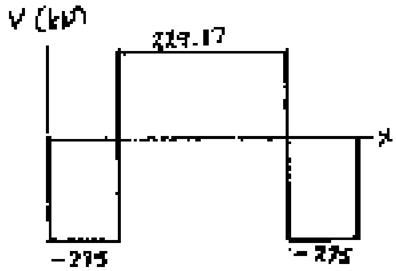


$R_B = 504.17$ kN ↑ $\quad R_C = 504.17$ ↓

$|V|_{max} = 275$ kN

$|M|_{max} = 412.5$ kN·m

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{412.5 \times 10^3}{160 \times 10^6} = 2578 \times 10^{-6}\ \text{m}^3 = 2578 \times 10^3\ \text{mm}^3$$



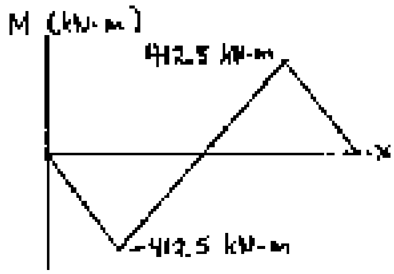
Shape	S_x (10^3 mm^3)
W 760 × 147	4410
W 690 × 125	3510
W 530 × 150	3720
W 460 × 158	3340
W 360 × 216	3800

(a) Use W 690 × 125

$d = 678$ mm

$t_f = 16.30$ mm

$t_w = 11.7$ mm



$$\sigma_m = \frac{|M|_{max}}{S} = \frac{412.5 \times 10^3}{3150 \times 10^{-6}} = 117.5 \times 10^6\ \text{Pa} = 117.5\ \text{MPa}$$

$$\tau_m = \frac{|V|_{max}}{A_w} = \frac{|V|_{max}}{d t_w} = \frac{275 \times 10^3}{(678 \times 10^{-3})(11.7 \times 10^{-3})} = 34.7 \times 10^6\ \text{Pa} = 34.7\ \text{MPa}$$

$c = \frac{1}{2}d = \frac{678}{2} = 339$ mm $\quad t_f = 16.30$ mm $\quad y_b = c - t_f = 339 - 16.30 = 322.7$ mm

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{322.7}{339}\right)(117.5) = 111.85\ \text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(55.925)^2 + (34.7)^2} = 65.815\ \text{MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 55.925 + 65.815 = 121.7\ \text{MPa}$$

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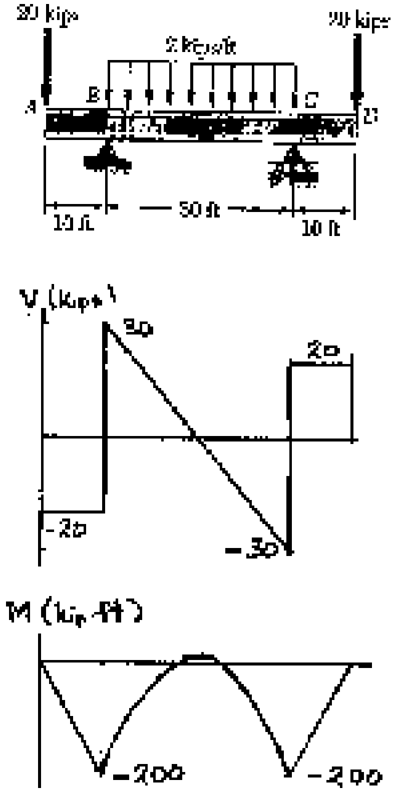
PROBLEM 8.7

8.7 and 8.8 (a) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.



$R_A = 50$ kips ↑ $\qquad R_D = 50$ kips ↑

$|V|_{max} = 30$ kips

$|M|_{max} = 200$ kip·ft $= 2400$ kip·in.

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{2400}{24} = 100 \text{ in}^3$$

Shape	S (in³)
W 24 × 68	154
→ W 21 × 62	127
W 18 × 76	146
W 16 × 77	134
W 12 × 96	103
W 10 × 112	131

(a) Use W 21 × 62

$d = 20.99$ in.

$t_f = 0.615$ in.

$t_w = 0.400$ in.

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{2400}{127} = 18.90 \text{ ksi}$$

$$\tau_m = \frac{|V|_{max}}{d\,t_w} = \frac{30}{(20.99)(0.400)} = 3.57 \text{ ksi}$$

$$c = \tfrac{1}{2}d = \frac{20.99}{2} = 10.495 \text{ in}$$

$$y_b = c - t_f = 10.495 - 0.615 = 9.88 \text{ in}$$

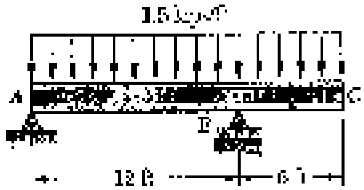
$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{9.88}{10.495}\right)(18.90) = 17.79 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{(8.896)^2 + (3.57)^2} = 9.586 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.896 + 9.586 = 18.48 \text{ ksi}$$

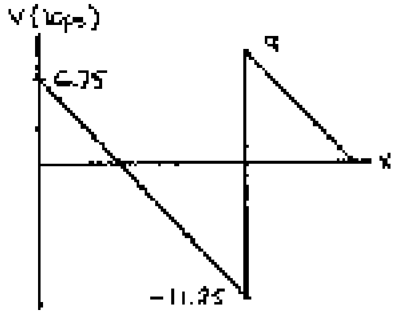
PROBLEM 8.8

8.7 and 8.8 (a) Knowing that $\sigma_{all} = 24$ ksi and $\tau_{all} = 14.5$ ksi, select the most economical wide-flange shape that should be used to support the loading shown. (b) Determine the values to be expected for σ_m, τ_m, and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.



$+\circlearrowleft \Sigma M_B = 0 \quad -12R_A + (1.5)(18)(3) = 0 \quad R_A = 6.75 \text{ kips} \uparrow$

$+\circlearrowleft \Sigma M_A = 0 \quad 12R_B - (1.5)(18)(9) = 0 \quad R_B = 20.25 \text{ kips} \uparrow$



$|V|_{max} = 11.25$ kips

$|M|_{max} = 27$ kip·ft $= 324$ kip·in

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{324}{24} = 13.5 \text{ in}^3$$

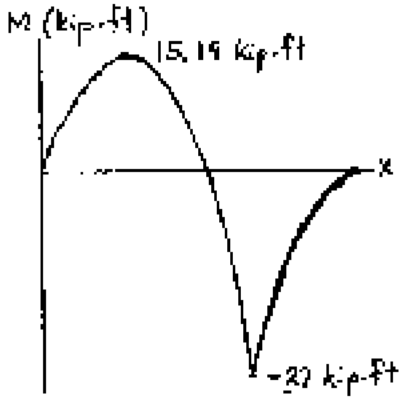
Shape	S (in³)
W 12 × 16	17.1
→ W 10 × 15	13.8
W 8 × 18	15.2
W 6 × 20	13.4

(a) Use W 10 × 15 ⟵

$d = 9.99$ in.

$t_f = 0.270$ in.

$t_w = 0.230$ in.



(b) $$\sigma_m = \frac{|M|_{max}}{S} = \frac{324}{13.8} = 23.5 \text{ ksi}$$ ⟵

$$\tau_m = \frac{|V|_{max}}{d\,t_w} = \frac{11.25}{(9.99)(0.230)} = 4.90 \text{ ksi}$$ ⟵

$$c = \tfrac{1}{2}d = \frac{9.99}{2} = 4.995 \text{ in.}$$

$$y_b = c - t_f = 4.995 - 0.270 = 4.725 \text{ in.}$$

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{4.725}{4.995}\right)(23.5) = 22.2 \text{ ksi}$$

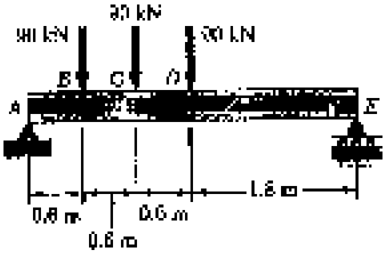
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_m^2} = \sqrt{\left(\frac{22.2}{2}\right)^2 + (4.90)^2} = 12.1 \text{ ksi}$$

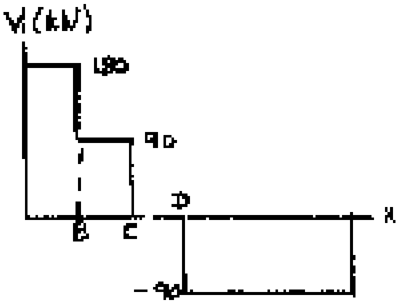
$$\sigma_{max} = \frac{\sigma_b}{2} + R = \frac{22.2}{2} + 12.1 = 23.2 \text{ ksi}$$ ⟵

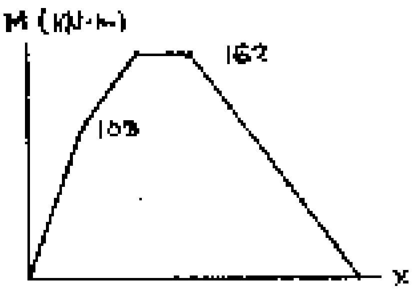
PROBLEM 8.9

8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.9 Loading of Prob. 5.81 and selected W410 × 60 shape.







From Problem 5.81 $\sigma_{all} = 160$ MPa

$|M|_{max} = 162$ kN·m at C and D

$|V| = 90$ kN at C and D

For W 410 × 60 rolled steel section

$d = 407$ mm, $b_f = 178$ mm, $t_f = 12.80$ mm

$t_w = 7.7$ mm, $I_z = 216 \times 10^6$ mm^4, $S_z = 1060 \times 10^3$ mm^3

$c = \frac{1}{2} d = 203.5$ mm

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{162 \times 10^3}{1060 \times 10^{-6}} = 152.8 \text{ MPa} \longrightarrow$$

$y_b = c - t_f = 190.7$ mm

$\sigma_b = \frac{y_b}{c}\sigma_m = 143.2$ MPa

$A_f = b_f t_f = 2278$ mm^2

$\bar{y} = \frac{1}{2}(c + y_b) = 197.1$ mm

$Q = A_f \bar{y} = (2278)(197.1) = 449 \times 10^3$ mm^3

$$\tau_b = \frac{VQ}{I t_w} = \frac{(90 \times 10^3)(449 \times 10^{-6})}{(216 \times 10^{-6})(7.7 \times 10^{-3})} = 24.3 \text{ MPa}$$

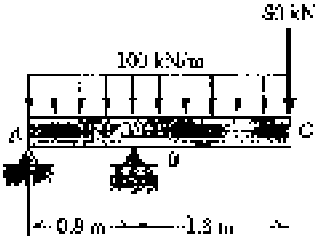
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{71.6^2 + 24.3^2} = 75.6 \text{ MPa}$$

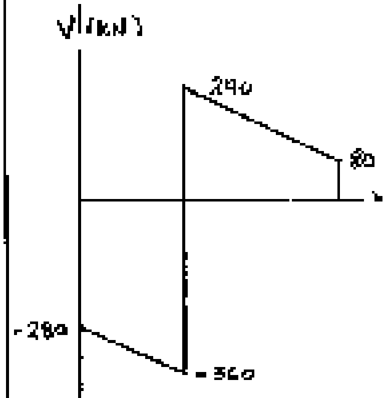
$$\sigma_{max} = \frac{\sigma_b}{2} + R = 71.6 + 75.6 = 147.2 \text{ MPa} \longrightarrow$$

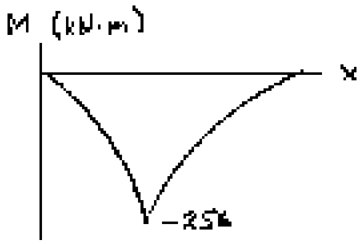
PROBLEM 8.10

8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.10 Loading of Prob. 5.86 and selected S 510 × 98.3 shape







From Problem 5.86 $\quad \sigma_{all} = 160$ MPa

$|M|_{max} = 256$ kN·m at point B

$|V| = 360$ kN at B

For S 510 × 98.3 rolled steel section

$d = 508$ mm, $b_f = 159$ mm, $t_f = 20.2$ mm

$t_w = 12.8$ mm, $I_z = 495 \times 10^6$ mm^4, $S_z = 1950 \times 10^3$ mm^3

$c = \frac{1}{2} d = 254$ mm

$$\sigma_m = \frac{|M|_{max}}{S_z} = \frac{256 \times 10^3}{1950 \times 10^{-6}} = 131.3 \text{ MPa}$$

$y_b = c - t_f = 233.8$

$\sigma_b = \frac{y_b}{c}\sigma_m = 120.9$ MPa $\qquad \frac{\sigma_b}{2} = 60.45$ MPa

$A_f = b_f t_f = 3212$ mm^2

$\bar{y} = \frac{1}{2}(c + y_b) = 243.9$ mm

$Q = A_f \bar{y} = 783.4 \times 10^3$ mm^3

$$\tau_b = \frac{VQ}{I t_w} = \frac{(360 \times 10^3)(783.4 \times 10^{-6})}{(495 \times 10^{-6})(12.8 \times 10^{-3})} = 44.5 \text{ MPa}$$

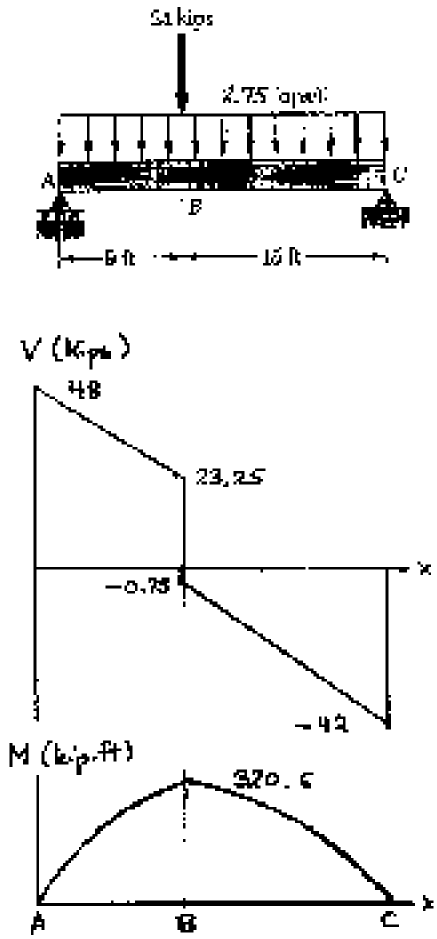
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{60.45^2 + 44.5^2} = 75.06 \text{ MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 60.45 + 75.06 = 135.5 \text{ MPa}$$

PROBLEM 8.11

8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (*a*) the actual value of σ_m in the beam, (*b*) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.11 Loading of Prob. 5.83 and selected W27 × 84 shape.



From Problem 5.83 $\sigma_{all} = 24$ ksi

$|M|_{max} = 320.6$ kip·ft $= 3847$ kip·in

At B− $|V| = 23.25$ kips

For W 27 × 84 rolled steel section

$d = 26.71$ in, $b_f = 9.960$ in, $t_f = 0.640$ in

$t_w = 0.460$ in, $I_x = 2850\ \text{in}^4$, $S_x = 213\ \text{in}^3$

$c = \frac{1}{2}d = 13.355$ in.

$$\sigma_m = \frac{|M|_{max}}{S} = \frac{3847}{213} = 18.06\ \text{ksi}$$

$y_b = c - t_f = 12.715$ in.

$\sigma_b = \frac{y_b}{c}\sigma_m = 17.20$ ksi $\qquad \frac{\sigma_b}{2} = 8.60$ ksi

$A_f = b_f t_f = (9.960)(0.640) = 6.3744\ \text{in}^2$

$\bar{y} = \frac{1}{2}(c + y_b) = 13.035$ in

$Q = A_f \bar{y} = 83.09\ \text{in}^3$

$$\tau_b = \frac{VQ}{I_x t_w} = \frac{(23.25)(83.09)}{(2850)(0.460)} = 1.47\ \text{ksi}$$

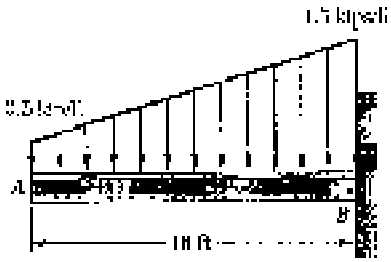
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{(8.60)^2 + (1.47)^2} = 8.72\ \text{ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.60 + 8.72\ \text{ksi} = 17.32\ \text{ksi}$$

PROBLEM 8.12

8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.12 Loading of Prob. 5.84 and selected W18 × 50 shape.



From Problem 5.84 $\sigma_{all} = 24$ ksi

$|M|_{max} = 135$ kip·ft $= 1620$ kip·in at B

$|V|_{max} = 18$ kips at B

For W18×50 shape $d = 17.99$ in, $b_f = 7.495$ in, $t_f = 0.570$ in

$t_w = 0.355$ in, $I_z = 800$ in^4, $S_z = 88.9$ in^3, $c = \frac{1}{2}d = 8.995$ in

$$\sigma_m = \frac{|M|_{max}}{S_z} = 18.22 \text{ ksi}$$

$y_b = c - t_f = 8.425$ in

$\sigma_b = \frac{y_b}{c}\sigma_m = 17.07$ ksi $\qquad \frac{\sigma_b}{2} = 8.535$ ksi

$A_f = b_f t_f = 4.272$ in^2

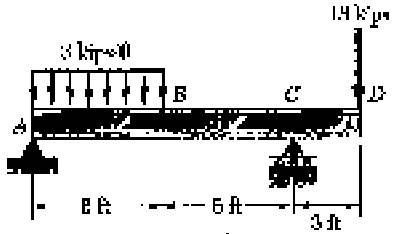
$\bar{y} = \frac{1}{2}(c + y_b) = 8.71$ in $\qquad Q = A_f \bar{y} = 37.21$ in^3

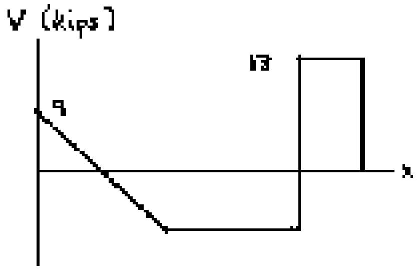
$$\tau_b = \frac{VQ}{I_z t_w} = \frac{(18)(37.21)}{(800)(0.355)} = 2.36 \text{ ksi}$$

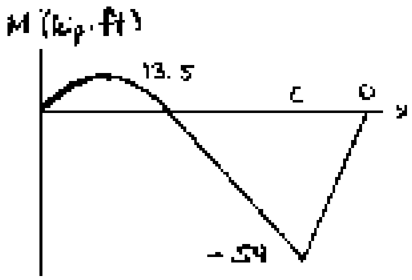
$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.535^2 + 2.36^2} = 8.855 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.535 + 8.855 = 17.39 \text{ ksi}$$

PROBLEM 8.13







8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.

8.13 Loading of Prob. 5.87 and selected S12 × 31.8 shape

From Problem 5.87 $\sigma_{all} = 24$ ksi

$|M|_{max} = 54$ kip·ft $= 648$ kip·in at C

At C $|V| = 18$ kips

For S12 × 31.8

$d = 12.00$ in, $b_f = 5.00$ in, $t_f = 0.544$ in

$t_w = 0.350$ in $I_z = 218$ in^4, $S_z = 36.4$ in^3

$c = \frac{1}{2}d = 6.00$ in

$$\sigma_m = \frac{|M|}{S_z} = \frac{648}{36.4} = 17.80 \text{ ksi}$$

$y_b = c - t_f = 5.456$ in

$$\sigma_b = \frac{y_b}{c}\sigma_m = 16.186 \text{ ksi} \qquad \frac{\sigma_b}{2} = 8.093 \text{ ksi}$$

$A_f = b_f t_f = 2.72$ in^2

$\bar{y} = \frac{1}{2}(c + y_b) = 5.728$ in

$Q = A_f \bar{y} = 15.58$ in^3

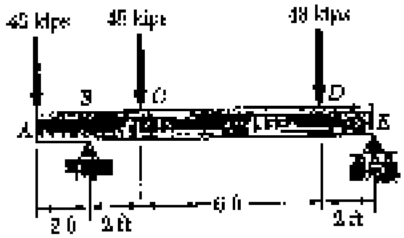
$$\tau_b = \frac{VQ}{I_z t_w} = \frac{(18)(15.58)}{(218)(0.350)} = 3.675 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.093^2 + 3.675^2} = 8.889 \text{ ksi}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.093 + 8.889 = 16.98 \text{ ksi}$$

PROBLEM 8.14

8.9 through 8.14 Each of the following problems refers to a rolled-steel shape selected in a problem of Chap. 5 to support a given loading at a minimal cost while satisfying the requirement $\sigma_m \le \sigma_{all}$. For the selected design, determine (a) the actual value of σ_m in the beam, (b) the maximum value of the principal stress σ_{max} at the junction of a flange and the web.



8.14 Loading of Prob. 5.88 and selected S15 × 42.9 shape.

From problem 5.88 $\quad \sigma_{all} = 24$ ksi

$|M|_{max} = 96$ kip·ft $= 1152$ kip·in at D

At D $\quad |V| = 38.4$ kips

For S15 × 42.9 shape

$d = 15.00$ in, $\quad b_f = 5.501$ in, $\quad t_f = 0.622$ in

$t_w = 0.411$ in, $\quad I_x = 447$ in^4, $\quad S_x = 59.6$ in^3

$c = \frac{1}{2} d = 7.5$ in

$\sigma_m = \frac{|M|}{S} = \frac{1152}{59.6} = 19.33$ ksi

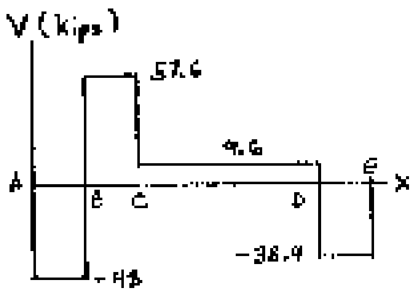
$y_b = c - t_f = 6.878$ in.

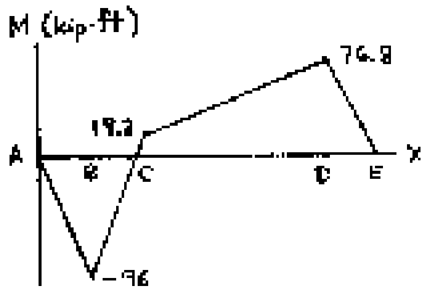
$\sigma_b = \frac{y_b}{c} \sigma_m = 17.73$ ksi $\qquad \frac{\sigma_b}{2} = 8.86$ ksi

$A_f = b_f t_f = 3.4216$ in^2

$\bar{y} = \frac{1}{2}(c + y_b) = 7.189$ in

$Q = A_f \bar{y} = 24.60$ in^3





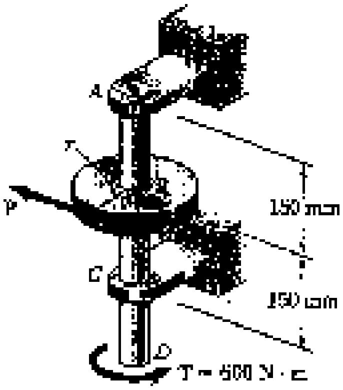
$\tau_b = \frac{VQ}{I_z t_w} = \frac{(57.6)(24.60)}{(447)(0.411)} = 7.71$ ksi

$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{8.86^2 + 7.71^2} = 11.74$ ksi

$\sigma_{max} = \frac{\sigma_b}{2} + R = 8.86 + 11.74 = 20.6$ ksi

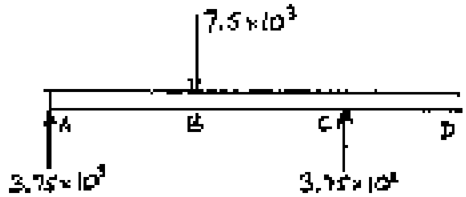
PROBLEM 8.15

8.15 Determine the smallest allowable diameter of the solid shaft ABCD, that τ_{all} = 60 MPa and that the radius of disk B is r = 80 mm.



SOLUTION

$\Sigma M_{axis} = 0 \qquad T - Pr = 0 \qquad P = \frac{T}{r} = \frac{600}{80\times10^{-3}} = 7.5\times10^{3}\ N$

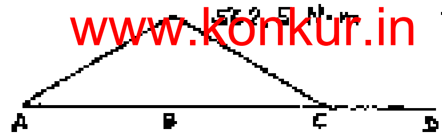


$R_A = R_C = \frac{1}{2}P$

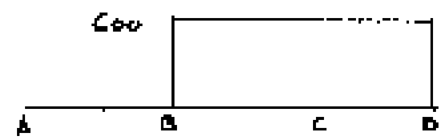
$= 3.75\times10^{3}\ N$

$M_B = (3.75\times10^{3})(150\times10^{-3})$

$= 562.5\ N\cdot m$



Bending moment



Torque

Critical section lies at point B $\qquad M = 562.5\ N\cdot m,\ T = 600\ N\cdot m$

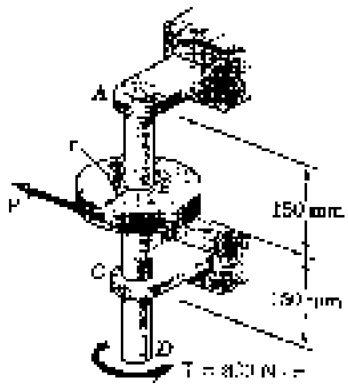
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M^2+T^2}\right)_{max}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{2}{\pi}\frac{\sqrt{(562.5)^2+(600)^2}}{60\times10^{6}} = 8.726\times10^{-6}\ m^3$$

$c = 20.58\times10^{-3}\ m \qquad d = 2c = 41.2\times10^{-3}\ m = 41.2\ mm$

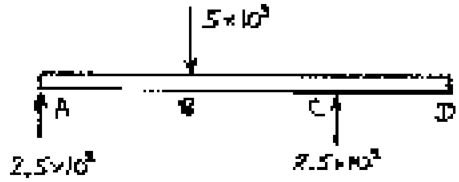
PROBLEM 8.16

8.16 Determine the smallest allowable diameter of the solid shaft $ABCD$, knowing that $\tau_{all} = 60$ MPa and that the radius of disk B is $r = 120$ mm.



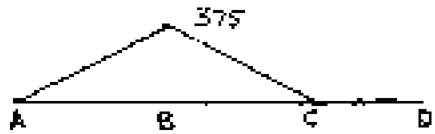
SOLUTION

$$\Sigma M_{AD} = 0 \qquad T - Pr = 0 \qquad P = \frac{T}{r} = \frac{600}{120\times10^{-3}} = 5\times10^{3}\ \text{N}$$

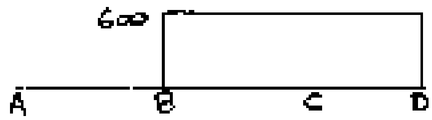


$$R_A = R_C = \tfrac{1}{2}P = 2.5\times10^{3}\ \text{N}$$

$$M_B = (2.5\times10^{3})(0.150\times10^{-3}) = 375\ \text{N·m}$$



Bending moment



Torque

Critical section lies at point B $\qquad M = 375$ N·m, $\quad T = 600$ N·m

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M^2+T^2}\right)_{max}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi}\frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{2}{\pi}\frac{\sqrt{375^2+600^2}}{60\times10^{6}} = 7.507\times10^{-6}\ \text{m}^3$$

$$c = 19.58\times10^{-3}\ \text{m} \qquad d = 2c = 39.2\times10^{-3}\ \text{m} = 39.2\ \text{mm}$$

PROBLEM 8.17

8.17 Using the notation of Sec. 8.3 and neglecting the effect of shearing stresses caused by transverse loads, show that the maximum normal stress in a cylindrical shaft can be expressed as

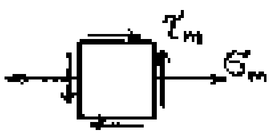
$$\sigma_{max} = \frac{c}{J}\left[\left(M_y^2 + M_z^2\right)^{\frac{1}{2}} + \left(M_y^2 + M_z^2 + T^2\right)^{\frac{1}{2}}\right]_{max}$$

SOLUTION

Maximum bending stress $\quad \sigma_m = \frac{|M|c}{I} = \frac{\sqrt{M_y^2 + M_z^2}\,c}{I}$

Maximum torsional stress $\quad \tau_m = \frac{Tc}{J}$

$$\frac{\sigma_m}{2} = \frac{\sqrt{M_y^2 + M_z^2}\,c}{2I} = \frac{c}{J}\sqrt{M_y^2 + M_z^2}$$



Using Mohr's circle

$$R = \sqrt{\left(\frac{\sigma_m}{2}\right)^2 + \tau_m^2} = \sqrt{\frac{c^2}{J^2}(M_y^2 + M_z^2) + \frac{T^2c^2}{J^2}}$$

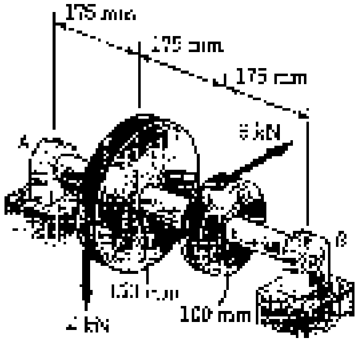
$$= \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

$$\sigma_{max} = \frac{\sigma_m}{2} + R = \frac{c}{J}\sqrt{M_y^2 + M_z^2} + \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

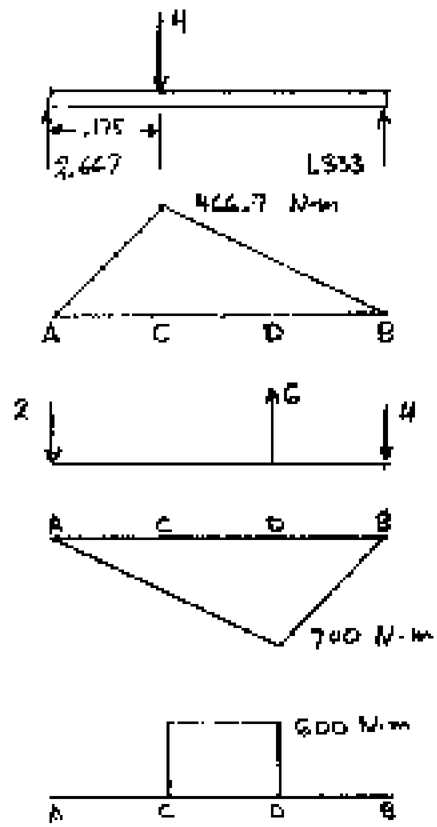
$$= \frac{c}{J}\left[(M_y^2 + M_z^2)^{\frac{1}{2}} + (M_y^2 + M_z^2 + T^2)^{\frac{1}{2}}\right]$$

PROBLEM 8.18

8.18 Use the expression given in Prob. 8.17 to determine the maximum normal stress in the solid shaft AB, knowing that its diameter is 36 mm.



SOLUTION



Vertical forces, kN

Bending moment M_z

$M_{zC} = (0.175)(2.667 \times 10^3)$
$= 466.7$ N-m

Horizontal forces, kN

Bending moment M_y

$M_{yD} = (0.175)(4 \times 10^3)$
$= 700$ N-m

Torque

$T = (6 \times 10^3)(100 \times 10^{-3})$
$= 600$ N-m

At point C $\quad \sqrt{M_y^2 + M_z^2} = \sqrt{350^2 + 466.7^2} = 583.3$ N-m

At point D $\quad \sqrt{M_y^2 + M_z^2} = \sqrt{700^2 + 233.3^2} = 737.9$ N-m

Point D is critical

$c = \frac{1}{2} d = 18 \text{ mm} = 18 \times 10^{-3}$ m

$J = \frac{\pi}{2} c^4 = 164.90 \times 10^3 \text{ mm}^4 = 164.90 \times 10^{-9} \text{ m}^4$

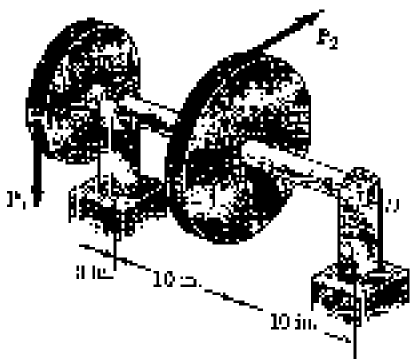
$$\sigma_{max} = \frac{c}{J}\left[\sqrt{M_y^2 + M_z^2} + \sqrt{M_y^2 + M_z^2 + T^2}\right]$$

$$= \frac{18 \times 10^{-3}}{164.90 \times 10^{-9}}\left[737.9 + \sqrt{737.9^2 + 600^2}\right] = 184.4 \times 10^6 \text{ Pa}$$

$$= 184.4 \text{ MPa}$$

PROBLEM 8.19

8.19 The vertical force $\mathbf{P}_1$ and the horizontal force $\mathbf{P}_2$ are applied as shown to disks welded to the solid shaft AD. Knowing that the diameter of the shaft is 1.75 in. and that $\tau_{all} = 8$ ksi, determine the largest permissible magnitude of the force $\mathbf{P}_2$.



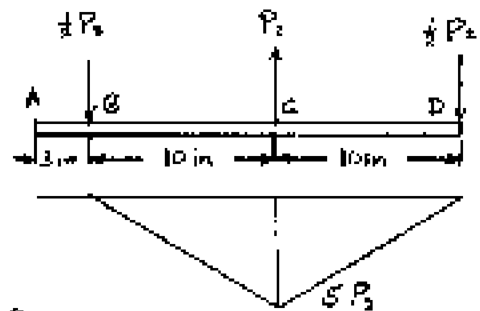
SOLUTION

Let P_2 be in kips.

$\Sigma M_{shaft} = 0 \qquad 6P_1 - 8P_2 = 0 \qquad P_1 = \frac{4}{3}P_2$

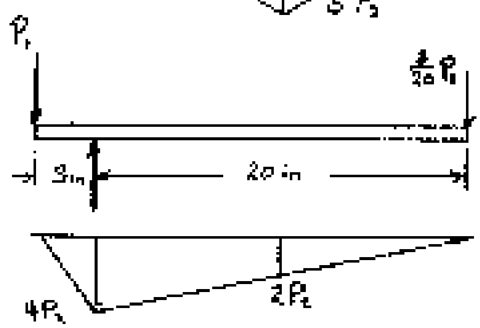
Torque over portion ABC $\qquad T = 8P_2$

Bending in horizontal plane



$M_{Cy} = 10 \cdot \frac{1}{2}P_2 = 5P_2$

Bending in vertical plane



$M_{Bz} = 3P_1 = 3 \cdot \frac{4}{3}P_2 = 4P_2$

Critical point is just to the left of point C.

$T = 8P_2 \qquad M_y = 5P_2 \qquad M_z = 2P_2$

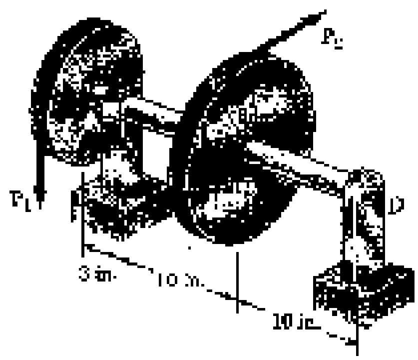
$d = 1.75$ in $\qquad c = \frac{1}{2}d = 0.875$ in $\qquad J = \frac{\pi}{2}(0.875)^4 = 0.92077$ in^4

$$\tau_{all} = \frac{c}{J}\sqrt{T^2 + M_y^2 + M_z^2}$$

$$8 = \frac{0.875}{0.92077}\sqrt{(8P_2)^2 + (5P_2)^2 + (2P_2)^2} = 9.164\,P_2$$

$P_2 = 0.873$ kips $= 873$ lb.

PROBLEM 8.20



8.19 The vertical force P_1 and the horizontal force P_2 are applied as shown to disks welded to the solid shaft AD. Knowing that the diameter of the shaft is 1.75 in. and that τ_{all} = 8 ksi, determine the largest permissible magnitude of the force P_2.

8.20 Solve Prob. 8.19, assuming that the solid shaft AD has been replaced by a hollow shaft of the same material and of inner diameter 1.50 in. and outer diameter 1.75 in.

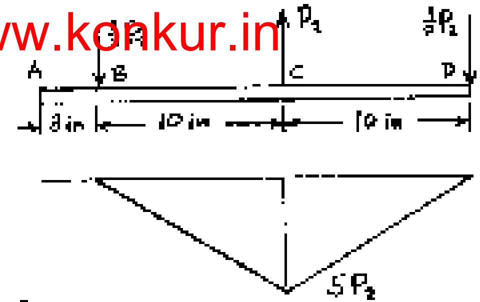
SOLUTION

Let P_2 be in kips

$\Sigma M_{shaft} = 0 \qquad 6P_1 - 8P_2 = 0 \qquad P_1 = \frac{4}{3}P_2$

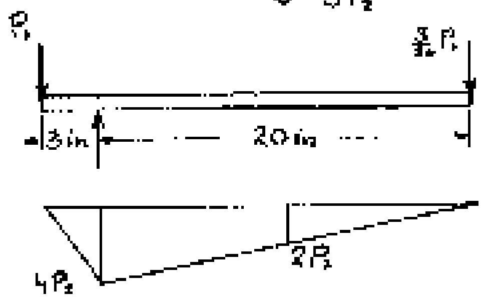
Torque over portion ABC $\qquad T = 8P_2$

Bending in horizontal plane.



$M_{Cy} = 10 \cdot \frac{1}{2}P_2$
$= 5P_2$

Bending in vertical plane



$M_{Bz} = 3P_1$
$= 3 \cdot \frac{4}{3}P_2$
$= 4P_2$

Critical point is just to the left of point C.

$T = 8P_2 \qquad M_y = 5P_2 \qquad M_z = 2P_2$

$c_o = \frac{1}{2}d_o = 0.875 \text{ in.} \qquad c_i = \frac{1}{2}d_i = 0.750 \text{ in}$

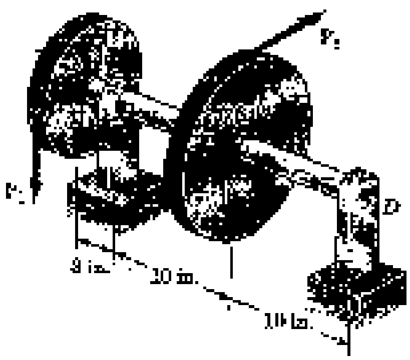
$J = \frac{\pi}{2}(c_o^4 - c_i^4) = 0.42376 \text{ in}^4$

$\tau_{all} = \frac{c_o}{J}\sqrt{T^2 + M_y^2 + M_z^2}$

$8 = \frac{0.875}{0.42376}\sqrt{(8P_2)^2 + (5P_2)^2 + (2P_2)^2} = 19.913\ P_2$

$P_2 = 0.402 \text{ kips} = 402 \text{ lb.}$

PROBLEM 8.22



8.22 Assuming that the magnitudes of the forces applied to disks A and C of Prob. 8.19 are, respectively, $P_1 = 1080$ lb and $P_2 = 810$ lb, and using the expressions given in Prob. 8.21, determine the values of τ_H and τ_K in a section (a) just to the left of B, (b) just to the left of C.

SOLUTION

From Prob. 8.19, shaft diameter = 1.75 in.

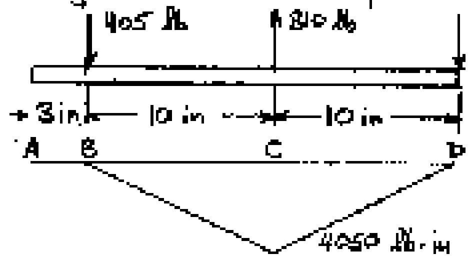
$c = \frac{1}{2}d = 0.875$ in

$J = \frac{\pi}{2}c^4 = 0.92077$

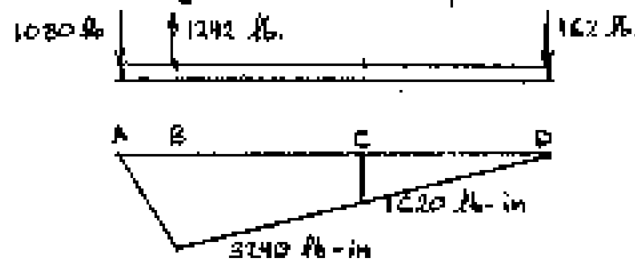
Torque over portion ABC

$T = (6)(1080) = (8)(810) = 6480$ lb-in

Bending in horizontal plane

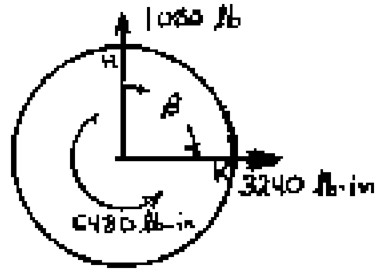


Bending in vertical plane



(a) Just to the left of point B $\quad V = 1080$ lb. $\quad M = 3240$ lb-in

$\beta = 90°$ $\quad T = 6480$ lb-in



$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2} = \frac{0.875}{0.92077}\sqrt{(3240)^2 + (6480)^2}$$
$$= 6880 \text{ psi}$$

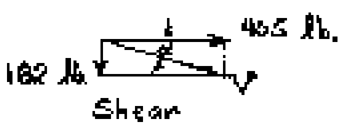
$$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + (\tfrac{2}{3}Vc + T)^2} = \frac{c}{J}\left[\tfrac{2}{3}Vc + T\right]$$
$$= \frac{0.875}{0.92077}\left[(\tfrac{2}{3})(1080)(0.875) + 6480\right] = 6760 \text{ psi}$$

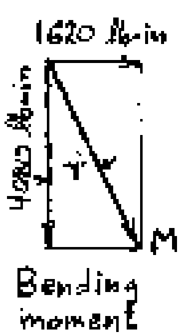
(b) Just to the left of point C $\quad V = \sqrt{(162)^2 + (405)^2} = 436.2$ lb.

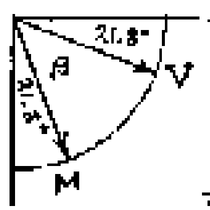
$\alpha = \tan^{-1}\frac{162}{405} = 21.80°$

$M = \sqrt{(1620)^2 + (4050)^2} = 4362$ lb-in

$\gamma = \tan^{-1}\frac{1620}{4050} = 21.80°$







$\beta = 90° - 21.8° - 21.8° = 46.4°$

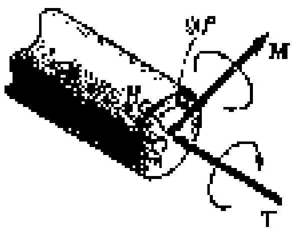
$$\tau_H = \frac{0.875}{0.92077}\sqrt{(6480)^2 + (4362)^2} = 7420 \text{ psi}$$

$\frac{2}{3}Vc + T = (\frac{2}{3})(436.2)(0.875) + 6480 = 6734$ lb-in

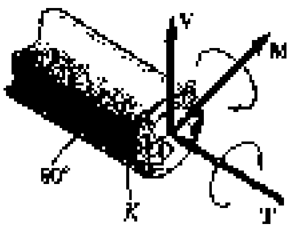
$M\cos\beta = 4362\cos 46.4° = 3008$ lb-in

$$\tau_K = \frac{0.875}{0.92077}\sqrt{(3008)^2 + (6734)^2} = 7010 \text{ psi}$$

PROBLEM 8.21



(a)



(b)

8.21 It was stated in Sec. 8.3 that the shearing stresses produced in a shaft by the transverse loads are usually much smaller than those produced by the torques. In the preceding problems their effect was ignored and it was assumed that the maximum shearing stress in a given section occurred at point H (Fig. P8.21a) and was equal to the expression obtained in Eq. (8.5), namely,

$$\tau_H = \frac{c}{J}\sqrt{M^2 + T^2}$$

Show that the maximum shearing stress at point K (Fig. P8.21b), where the effect of the shear V is greatest, can be expressed as

$$\tau_K = \frac{c}{J}\sqrt{(M\cos\beta)^2 + \left(\tfrac{2}{3}cV + T\right)^2}$$

where β is the angle between the vectors **V** and **M**. It is clear that the effect of the shear V cannot be ignored when $\tau_K \geq \tau_H$. (*Hint.* Only the component of **M** along **V** contributes to the shearing stress at K.)

SOLUTION

Shearing stress at point K

Due to V: For a semicircle $Q = \frac{2}{3}c^3$

For a circle cut across its diameter $t = d = 2c$

For a circular section $I = \frac{1}{2}J$

$$\tau_{xy} = \frac{VQ}{It} = \frac{(V)(\frac{2}{3}c^3)}{(\frac{1}{2}J)(2c)} = \frac{2}{3}\frac{Vc^2}{J}$$

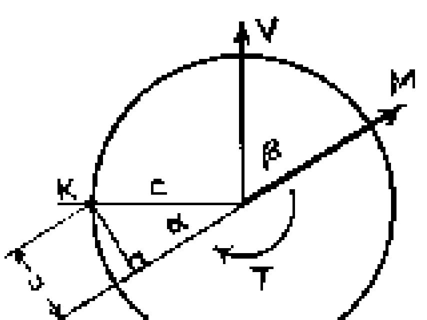
Due to T $\tau_{xy} = \frac{Tc}{J}$

Since these shearing stresses have the same orientation

$$\tau_{xy} = \frac{c}{J}\left(\tfrac{2}{3}Vc + T\right)$$

Bending stress at point K. $\sigma_x = \frac{Mu}{I} = \frac{2Mu}{J}$

where u is distance between point K and the neutral axis,



cross-section

$$u = c\sin\alpha = c\sin\left(\tfrac{\pi}{2} - \beta\right) = c\cos\beta$$

$$\sigma_x = \frac{2Mc\cos\beta}{J}$$

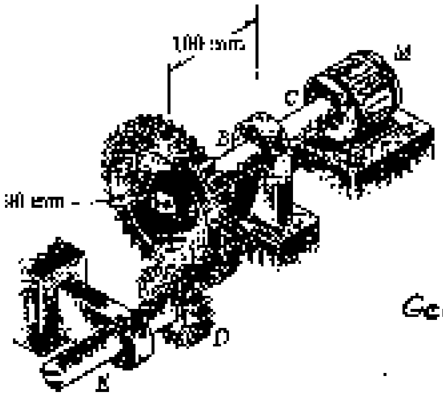
Using Mohr's circle

$$\tau_K = R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{c}{J}\sqrt{(M\cos\beta)^2 + \left(\tfrac{2}{3}Vc + T\right)^2}$$

PROBLEM 8.23

8.23 The solid shaft *ABC* and the gears shown are used to transmit 10 kW from the motor *M* to a machine tool connected to gear *D*. Knowing that the motor rotates at 240 rpm and that τ_{all} = 60 MPa, determine the smallest permissible diameter of shaft *ABC*.

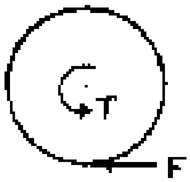


SOLUTION

$$f = \frac{240 \text{ rpm}}{60 \text{ sec/min}} = 4 \text{ Hz}$$

$$T = \frac{P}{2\pi f} = \frac{10 \times 10^3}{(2\pi)(4)} = 397.89 \text{ N}\cdot\text{m}$$

Gear A



$$F r_A - T = 0$$

$$F = \frac{T}{r_A} = \frac{397.89}{90 \times 10^{-3}} = 4421 \text{ N}$$

Bending moment at B $\qquad M_B = L_{AB} F = (100 \times 10^{-3})(4421) = 442.1 \text{ N}\cdot\text{m}$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

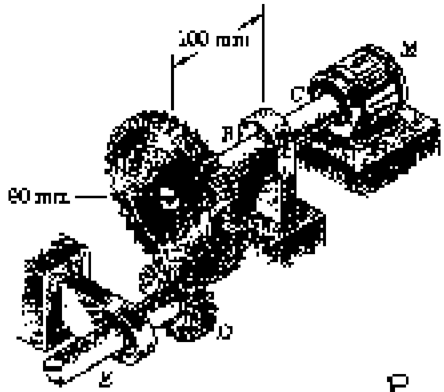
$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}}$$

$$c^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{(2)\sqrt{442.1^2 + 397.89^2}}{\pi (60 \times 10^6)} = 6.3108 \times 10^{-6} \text{ m}^3$$

$$c = 18.479 \times 10^{-3} \text{ m} \qquad d = 2c = 37.0 \times 10^{-3} \text{ m} = 37.0 \text{ mm}$$

PROBLEM 8.24

8.24 Assuming that shaft *ABC* of Prob. 8.23 is hollow and has an outer diameter of 50 mm, determine the largest permissible inner diameter of the shaft.



SOLUTION

From Prob. 8.23

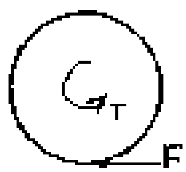
Power transmitted $P = 10 \text{ kW}$

Motor speed $= 240 \text{ rpm} = 4 \text{ Hz}$

$\tau_{all} = 60 \text{ MPa}$

$$T = \frac{P}{2\pi f} = \frac{10\times10^3}{(2\pi)(4)} = 397.89 \text{ N}\cdot\text{m}$$

Gear A



$$F r_A - T = 0$$

$$F = \frac{T}{r_A} = \frac{397.89}{90\times10^{-3}} = 4421 \text{ N}$$

Bending moment at B $\quad M_B = L_{AB} F = (100\times10^{-3})(4421) = 442.1 \text{ N}\cdot\text{m}$

$$\tau_{all} = \frac{c_o}{J}\sqrt{M^2+T^2} \qquad c_o = \tfrac{1}{2}d_o = 25\times10^{-3} \text{ m}$$

$$\frac{J}{c_o} = \frac{\pi}{2}\frac{(c_o^4 - c_i^4)}{c_o} = \frac{\sqrt{M^2+T^2}}{\tau_{all}}$$

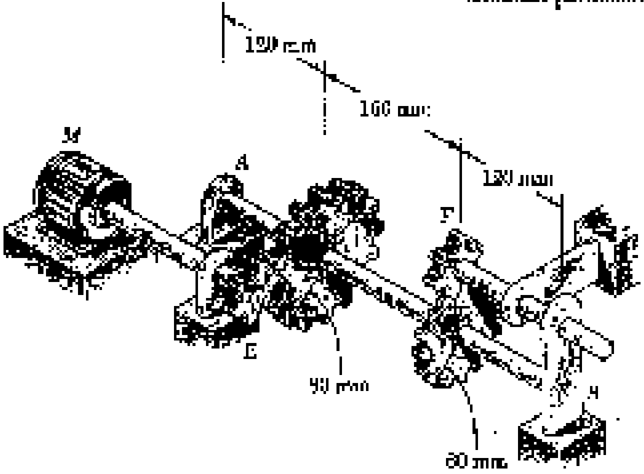
$$c_i^4 = c_o^4 - \frac{2c_o\sqrt{M^2+T^2}}{\pi\,\tau_{all}} = (25\times10^{-3})^4 - \frac{(2)(25\times10^{-3})\sqrt{442.1^2+397.89^2}}{\pi\,(60\times10^6)}$$

$$= 390.625\times10^{-9} - 157.772\times10^{-9} = 232.85\times10^{-9}$$

$$c_i = 21.967\times10^{-3} \text{ m} \qquad d_i = 2c_i = 43.93\times10^{-3} \text{ m} = 43.9 \text{ mm} \blacktriangleleft$$

PROBLEM 8.25

8.25 The solid shaft *AB* rotates at 600 rpm and transmits 80 kW from the motor *M* to a machine tool connected to gear *F*. Knowing that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft *AB*.



SOLUTION

$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

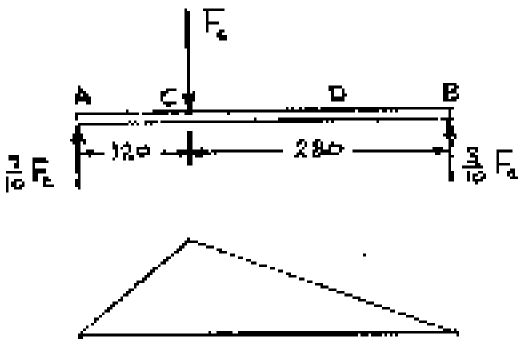
$$T = \frac{P}{2\pi f} = \frac{80\times10^3}{(2\pi)(10)} = 1273.2 \text{ N}\cdot\text{m}$$

Gear C $\qquad F_C = \frac{T}{r_C}$

$$F_C = \frac{1273.2}{80\times10^{-3}} = 15.913\times10^3 \text{ N}$$

Gear D $\qquad F_D = \frac{T}{r_D}$

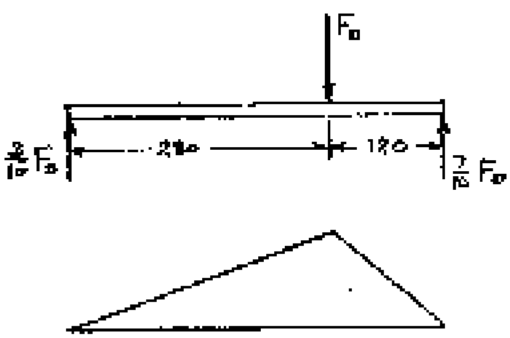
$$F_D = \frac{1273.2}{60\times10^{-3}} = 21.221\times10^3 \text{ N}$$



Forces in vertical plane

$$M_{Cz} = (120\times10^{-3})(\tfrac{7}{10}F_C) = 1336.7 \text{ N}\cdot\text{m}$$

$$M_{Dz} = \frac{120}{280} M_{Cz} = 572.9 \text{ N}\cdot\text{m}$$



Forces in horizontal plane

$$M_{Dy} = (120\times10^{-3})(\tfrac{7}{10}F_D) = 1782.6 \text{ N}\cdot\text{m}$$

$$M_{Cy} = \frac{120}{280} M_{Dy} = 764.0 \text{ N}\cdot\text{m}$$

At C: $\sqrt{M_y^2 + M_z^2 + T^2} = 1997.9 \text{ N}\cdot\text{m}$

At D: $\sqrt{M_y^2 + M_z^2 + T^2} = 2264.3 \text{ N}\cdot\text{m}$

$$\tau_{all} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}$$

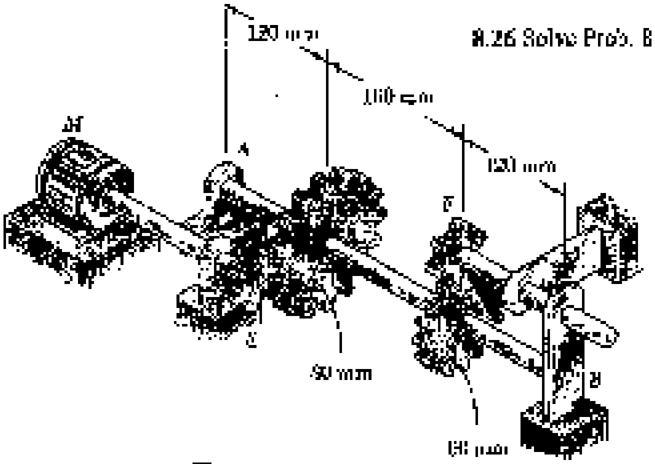
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}}{\tau_{all}} = \frac{2264.3}{60\times10^6} = 37.738\times10^{-6} \text{ m}^3$$

$c = 28.85\times10^{-3}$ m $\qquad d = 2c = 57.7\times10^{-3}$ m $= 57.7$ mm

PROBLEM 8.26

8.25 The solid shaft AB rotates at 600 rpm and transmits 80 kW from the motor M to a machine tool connected to gear F. Knowing that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft AB.

8.26 Solve Prob. 8.25, assuming that shaft AB rotates at 720 rpm.



SOLUTION

$$f = \frac{720 \text{ rpm}}{60 \text{ sec/min}} = 12 \text{ Hz}$$

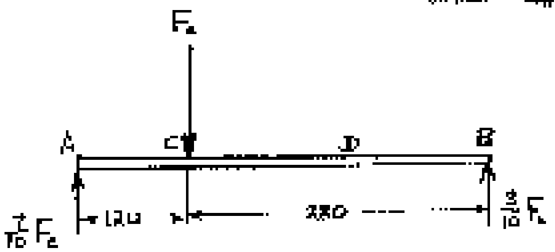
$$T = \frac{P}{2\pi f} = \frac{80 \times 10^3}{(2\pi)(12)} = 1061.0 \text{ N·m}$$

Gear C $\quad F_c = \dfrac{T}{r_c}$

$$F_c = \frac{1061.0}{80 \times 10^{-3}} = 13.262 \times 10^3 \text{ N}$$

Gear D $\quad F_D = \dfrac{T}{r_D}$

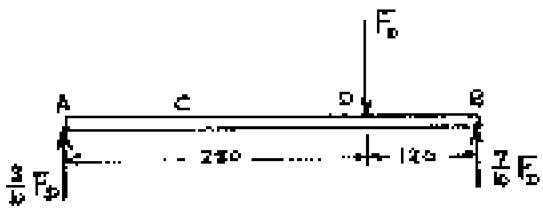
$$F_D = \frac{1061.0}{60 \times 10^{-3}} = 17.684 \times 10^3 \text{ N}$$



Forces in vertical plane

$$M_{cz} = (120 \times 10^{-3})(\tfrac{7}{10} F_c) = 1114.0 \text{ N·m}$$

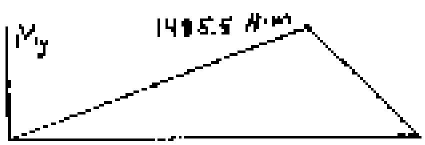
$$M_{Dz} = \frac{120}{280} M_{cz} = 477.4 \text{ N·m}$$



Forces in horizontal plane

$$M_{Dy} = (120 \times 10^{-3})(\tfrac{7}{10} F_D) = 1485.5 \text{ N·m}$$

$$M_{cy} = \frac{120}{280} M_{Dy} = 636.6 \text{ N·m}$$



At C: $\quad \sqrt{M_y^2 + M_z^2 + T^2} = 1664.9 \text{ N·m}$

At D: $\quad \sqrt{M_y^2 + M_z^2 + T^2} = 1886.9 \text{ N·m}$

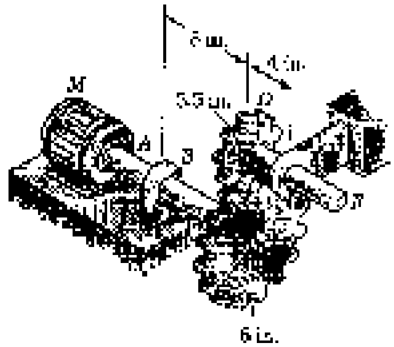
$$\tau_{all} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}}{\tau_{all}} = \frac{1886.9}{60 \times 10^6} = 31.448 \times 10^{-6} \text{ m}^3$$

$$c = 27.15 \times 10^{-3} \text{ m} \qquad d = 2c = 54.3 \times 10^{-3} \text{ m} = 54.3 \text{ mm}$$

PROBLEM 8.27

8.27 The solid shafts *ABC* and *DEF* and the gears shown are used to transmit 20 hp from the motor *M* to a machine tool connected to shaft *DEF*. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 7.5$ ksi, determine the smallest permissible diameter of (*a*) shaft *ABC*, (*b*) shaft *DEF*.



SOLUTION

$$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in}\cdot\text{lb/s}$$

$$240 \text{ rpm} = \frac{240}{60} = 4 \text{ Hz}$$

(a) Shaft ABC $\quad T = \frac{P}{2\pi f} = \frac{132 \times 10^3}{(2\pi)(4)} = 5252 \text{ in}\cdot\text{lb}$

Gear C $\quad F_{CD} = \frac{T}{r_C} = \frac{5252}{6} = 875.4 \text{ lb.}$

Bending moment at B $\quad M_B = (8)(875.4) = 7003 \text{ in}\cdot\text{lb.}$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{\sqrt{(5252)^2 + (7003)^2}}{7500} = 1.1671 \text{ in}^3$$

$c = 0.9057 \text{ in} \qquad d = 2c = 1.811 \text{ in.}$ ◄

(b) Shaft DEF $\quad T = r_D F_{CD} = (3.5)(875.4) = 3064 \text{ in}\cdot\text{lb.}$

Bending moment at E $\quad M_E = (4)(875.4) = 3502 \text{ in}\cdot\text{lb.}$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

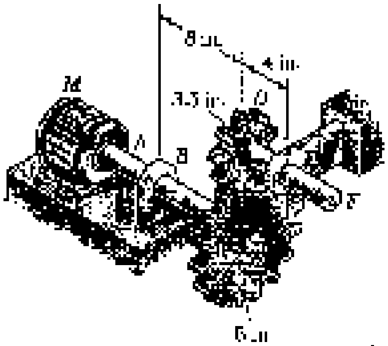
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2+T^2}}{\tau_{all}} = \frac{\sqrt{(3502)^2 + (3064)^2}}{7500} = 0.6204 \text{ in}^3$$

$c = 0.7337 \text{ in} \qquad d = 2c = 1.467 \text{ in}$ ◄

PROBLEM 8.28

8.27 The solid shafts ABC and DEF and the gears shown are used to transmit 20 hp from the motor M to a machine tool connected to shaft DEF. Knowing that the motor rotates at 240 rpm and that $\tau_{all} = 7.5$ ksi, determine the smallest permissible diameter of (a) shaft ABC, (b) shaft DEF.

8.28 Solve Prob. 8.27, assuming that the motor rotates at 360 rpm.



SOLUTION

$$20 \text{ hp} = (20)(6600) = 132 \times 10^3 \text{ in}\cdot\text{lb/s}$$

$$360 \text{ rpm} = \frac{360}{60} = 6 \text{ Hz}$$

(a) Shaft ABC $\quad T = \dfrac{P}{2\pi f} = \dfrac{132 \times 10^3}{(2\pi)(6)} = 3501 \text{ in}\cdot\text{lb}$

Gear C $\quad F_{CD} = \dfrac{T}{r_C} = \dfrac{3501}{6} = 583.6 \text{ lb}$

Bending moment at B $\quad M_B = (8)(583.6) = 4669 \text{ in}\cdot\text{lb}$

$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{4669^2 + 3501^2}}{7500} = 0.77806 \text{ in}^3$$

$c = 0.791$ in. $\qquad d = 2c = 1.582$ in. ◀

(b) Shaft DEF $\quad T = r_D F_{CD} = (3.5)(583.6) = 2043 \text{ in}\cdot\text{lb}$

Bending moment at E $\quad M_E = (4)(583.6) = 2334 \text{ in}\cdot\text{lb}$

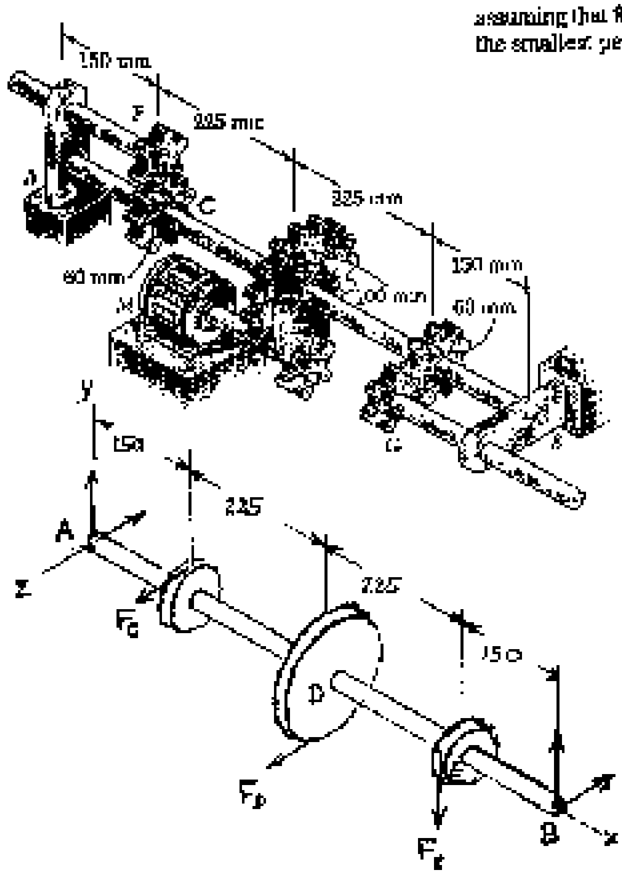
$$\tau_{all} = \frac{c}{J}\sqrt{M^2 + T^2}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{\sqrt{2334^2 + 2043^2}}{7500} = 0.41362 \text{ in}^3$$

$c = 0.6410$ in $\qquad d = 2c = 1.282$ in. ◀

PROBLEM 8.29

8.29 The solid shaft AB rotates at 450 rpm and transmits 20 kW from the motor M to machine tools connected to gears F and G. Knowing that $\tau_{all} = 55$ MPa and assuming that 8 kW is taken off at gear F and 12 kW is taken off at gear G, determine the smallest permissible diameter of shaft AB.



SOLUTION

$$f = \frac{450}{60} = 7.5 \text{ Hz}$$

Torque applied at D

$$T_D = \frac{P}{2\pi f} = \frac{20 \times 10^3}{(2\pi)(7.5)} = 424.41 \text{ N}\cdot\text{m}$$

Torques on gears C and E

$$T_C = \frac{8}{20} T_D = 169.76 \text{ N}\cdot\text{m}$$

$$T_E = \frac{12}{20} T_D = 254.65 \text{ N}\cdot\text{m}$$

Forces on gears

$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100 \times 10^{-3}} = 4244 \text{ N}$$

$$F_C = \frac{T_C}{r_C} = \frac{169.76}{60 \times 10^{-3}} = 2829 \text{ N}$$

$$F_E = \frac{T_E}{r_E} = \frac{254.65}{60 \times 10^{-3}} = 4244 \text{ N}$$

Torques in various parts

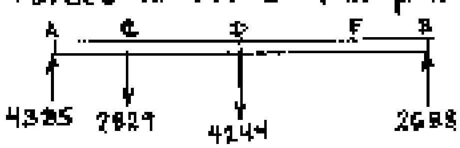
AC: $T = 0$

CD: $T = 169.76$ N·m

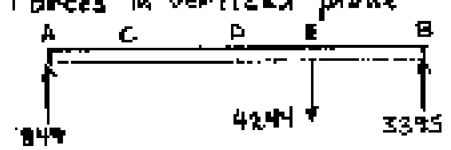
DE: $T = 254.65$ N·m

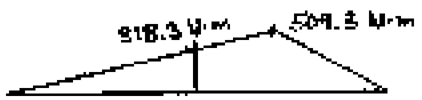
EB: $T = 0$

Forces in horizontal plane:



Forces in vertical plane





Critical point lies just the right of D

$T = 254.65$ N·m

$M_y = 1007.9$ N·m

$M_z = 318.5$ N·m

$$\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max} = 1087.2 \text{ N}\cdot\text{m}$$

$$\tau_{all} = \frac{c}{J}\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\left(\sqrt{M_y^2 + M_z^2 + T^2}\right)_{max}}{\tau_{all}} = \frac{1087.2}{55 \times 10^6}$$

$$= 19.767 \times 10^{-6} \text{ m}^3$$

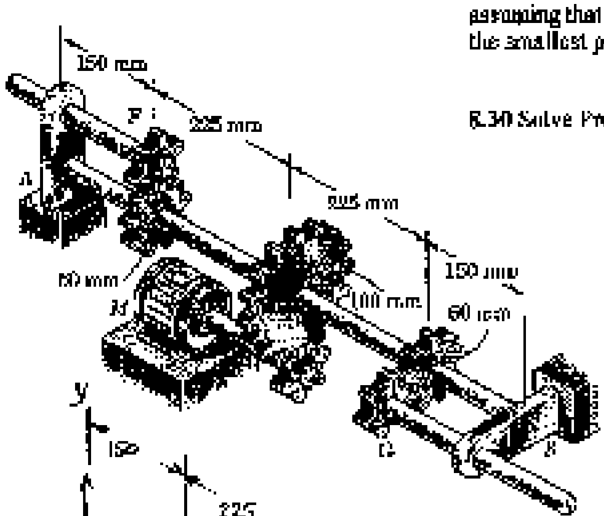
$$c = 23.26 \times 10^{-3} \text{ m}$$

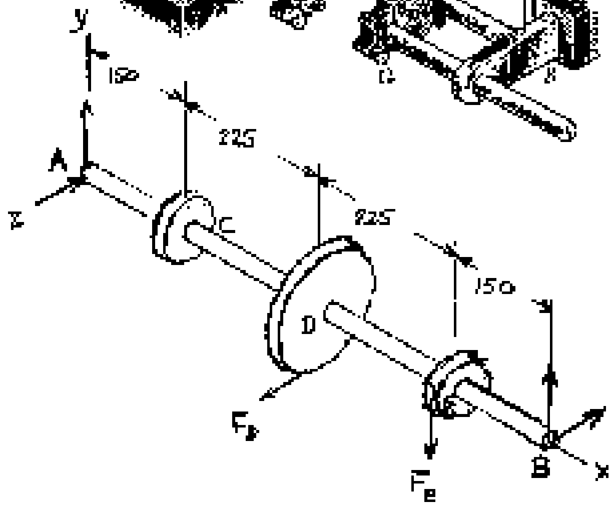
$$d = 2c = 46.5 \times 10^{-3} \text{ m} = 46.5 \text{ mm}$$

PROBLEM 8.30

8.29 The solid shaft AB rotates at 450 rpm and transmits 20 kW from the motor M to machine tools connected to gears F and G. Knowing that $\tau_{all} = 55$ MPa and assuming that 8 kW is taken off at gear F and 12 kW is taken off at gear G, determine the smallest permissible diameter of shaft AB.

8.30 Solve Prob. 8.29, assuming that the entire 20 kW is taken off at gear G.





SOLUTION

$$f = \frac{450}{60} = 7.5 \text{ Hz}$$

Torque applied at D

$$T_D = \frac{P}{2\pi f} = \frac{20 \times 10^3}{(2\pi)(7.5)} = 424.41 \text{ N}\cdot\text{m}$$

Torque on gear E

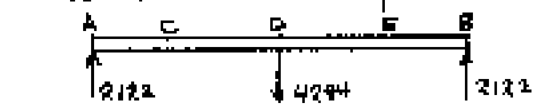
$$T_E = T_D = 424.41 \text{ N}\cdot\text{m}$$

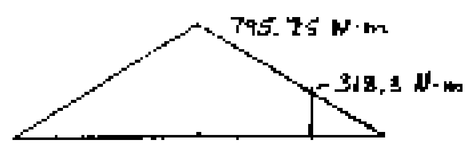
Forces on gears D and E

$$F_D = \frac{T_D}{r_D} = \frac{424.41}{100 \times 10^{-3}} = 4244 \text{ N}$$

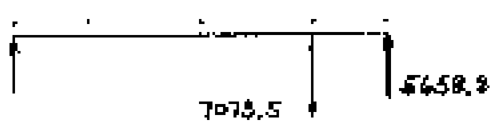
$$F_E = \frac{T_E}{r_E} = \frac{424.41}{60 \times 10^{-3}} = 7073.5 \text{ N}$$

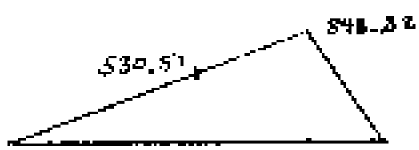
Forces in horizontal plane





Forces in vertical plane





Bending moments

$$M_D = \sqrt{530.51^2 + 795.75^2} = 956.4 \text{ N}\cdot\text{m}$$

$$M_E = \sqrt{848.82^2 + 318.3^2} = 906.5 \text{ N}\cdot\text{m}$$

$$\left(\sqrt{M^2 + T^2}\right)_{max} = \sqrt{956.4^2 + 424.41^2} = 1046.3 \text{ N}\cdot\text{m}$$

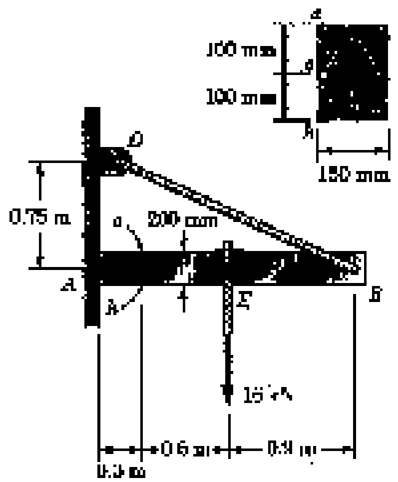
$$\tau_{all} = \frac{c}{J}\left(\sqrt{M^2 + T^2}\right)_{max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{all}} = \frac{1046.3}{55 \times 10^6} = 19.024 \times 10^{-6} \text{ m}^3$$

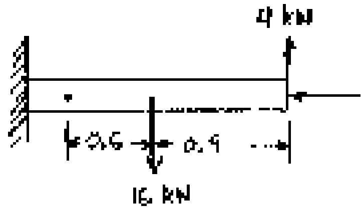
$$c = 22.96 \times 10^{-3} \text{ m} \qquad d = 2c = 45.9 \times 10^{-3} \text{ m} = 45.9 \text{ mm}$$

PROBLEM 8.31

8.31 The cantilever beam *AB* has a rectangular cross section of 150 × 200 mm. Knowing that the tension in cable *BD* is 10.4 kN and neglecting the weight of the beam, determine the normal and shearing stresses at the three points indicated.



SOLUTION



$\overline{DB} = \sqrt{.75^2 + 1.8^2}$
$= 1.95 \text{ m}$

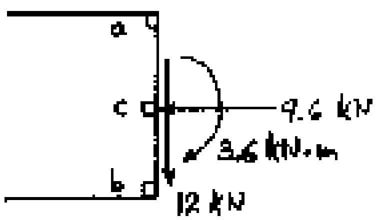
Vertical component of T_{DB}
$\left(\frac{0.75}{1.95}\right)(10.4) = 4 \text{ kN}$

Horizontal component of T_{DB} $\left(\frac{1.8}{1.95}\right)(10.4) = 9.6 \text{ kN}$

At section containing points a, b, and c

$P = -9.6 \text{ kN}$ $\qquad 16 - 4 = 12 \text{ kN}$

$M = (1.5)(4) - (0.6)(16) = -3.6 \text{ kN}\cdot\text{m}$



Section properties

$A = (0.150)(0.200) = 0.030 \text{ m}^2$

$I = \frac{1}{12}(0.150)(0.200)^3 = 100 \times 10^{-6} \text{ m}^4$

$c = 0.100 \text{ m}$

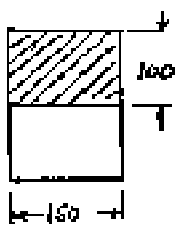
At point a $\qquad \sigma_x = -\frac{P}{A} + \frac{Mc}{I} = -\frac{9.6 \times 10^3}{0.030} + \frac{(3.6 \times 10^3)(0.100)}{100 \times 10^{-6}} = 3.28 \text{ MPa}$

$\tau_{xy} = 0$

At point b $\qquad \sigma_x = -\frac{P}{A} + \frac{Mc}{I} = -\frac{9.6 \times 10^3}{0.030} - \frac{(3.6 \times 10^3)(0.100)}{100 \times 10^{-6}} = -3.92 \text{ MPa}$

$\tau_{xy} = 0$

At point c $\qquad \sigma_x = -\frac{P}{A} = -\frac{9.6 \times 10^3}{0.030} = -0.320 \text{ MPa}$

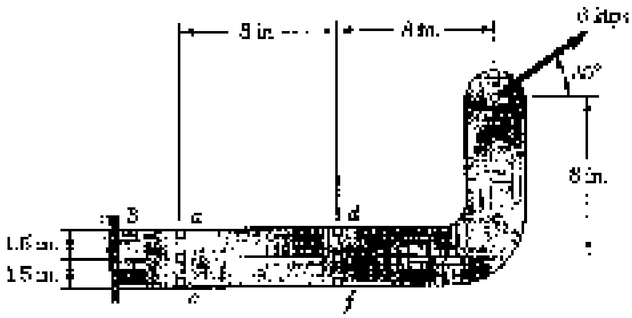


$Q = (150)(100)(50) = 750 \times 10^3 \text{ mm}^3 = 750 \times 10^{-6} \text{ m}^3$

$\tau_{xy} = -\frac{VQ}{It} = -\frac{(12 \times 10^3)(750 \times 10^{-6})}{(100 \times 10^{-6})(0.150)} = -0.600 \text{ MPa}$

PROBLEM 8.32

8.32 A 6-kip force is applied to the machine element *AB* as shown. Determine the normal and shearing stresses at (*a*) point *a*, (*b*) point *b*, (*c*) point *c*.



SOLUTION

thickness = 0.8 in.

At the section containing points a, b, and c

$P = 6\cos 35° = 4.9149 \text{ kips} \qquad V = 6 \sin 35° = 3.4415 \text{ kips}$

$M = (6\sin 35°)(16) - (6\cos 35°)(8) = 15.744 \text{ kip}\cdot\text{in.}$

$A = (0.8)(3.0) = 2.4 \text{ in}^2$

$I = \frac{1}{12}(0.8)(3.0)^3 = 1.80 \text{ in}^4$

(a) At point a $\qquad \sigma_x = \frac{P}{A} - \frac{Mc}{I} = \frac{4.9149}{2.4} - \frac{(15.744)(1.5)}{1.80} = -11.07 \text{ ksi}$

$\tau_{xy} = 0$

(b) At point b $\qquad \sigma_x = \frac{P}{A} = \frac{4.9149}{2.4} = 2.05 \text{ ksi}$

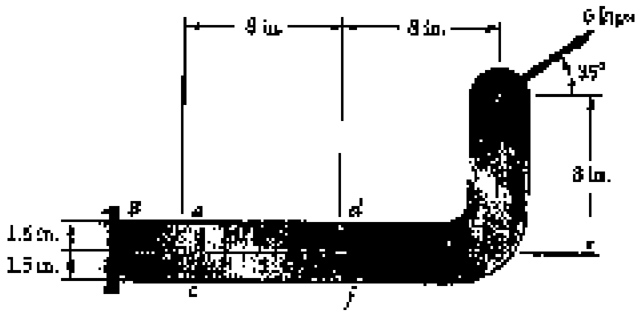
$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\cdot\frac{3.4415}{2.4} = 2.15 \text{ ksi}$

(c) At point c $\qquad \sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{4.9149}{2.4} + \frac{(15.744)(1.5)}{1.8} = 15.17 \text{ ksi}$

$\tau_{xy} = 0$

PROBLEM 8.33

8.33 A 6-kip force is applied to the machine element AB as shown. Determine the normal and shearing stresses at (*a*) point *d*, (*b*) point *e*, (*c*) point *f*.



SOLUTION

thickness = 0.8 in

At the section containing points *d*, *e*, and *f*

$P = 6\cos 35° = 4.9149$ kips $\qquad V = 6 \sin 35° = 3.4415$ kips

$M = (6 \sin 35°)(8) - (6 \cos 35°)(8) = -11.788$ kip·in.

$A = (0.8)(3.0) = 2.4 \text{ in}^2$

$I = \frac{1}{12}(0.8)(3.0)^3 = 1.80 \text{ in}^4$

(a) At point d $\qquad \sigma_x = \frac{P}{A} - \frac{Mc}{I} = \frac{4.9149}{2.4} + \frac{(11.788)(1.5)}{1.8} = 11.87$ ksi

$\tau_{xy} = 0$

(b) At point e $\qquad \sigma_x = \frac{P}{A} = \frac{4.9149}{2.4} = 2.05$ ksi

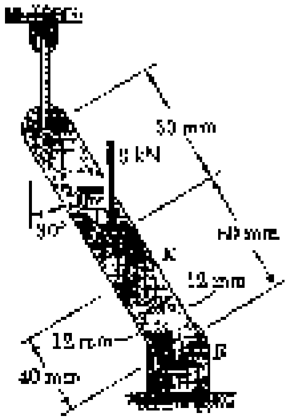
$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\cdot\frac{3.4415}{2.4} = 2.15$ ksi

(c) At point f $\qquad \sigma_x = \frac{P}{A} + \frac{Mc}{I} = \frac{4.9149}{2.4} - \frac{(11.788)(1.5)}{1.8} = -7.78$ ksi

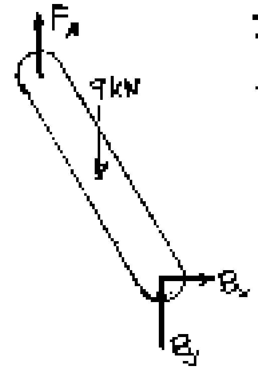
$\tau_{xy} = 0$

PROBLEM 8.34

8.34 through 8.36 Member AB has a uniform rectangular cross section of 10×24 mm. For the loading shown, determine the normal and shearing stresses at (*a*) point H, (*b*) point K.



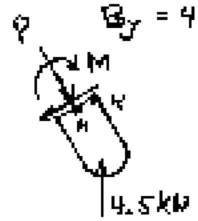
SOLUTION



$\Sigma F_x = 0 \qquad B_x = 0$

$\Sigma M_A = 0 \qquad B_y(120 \sin 30°) - 9(60 \sin 30°) = 0$

$B_y = 4.5 \text{ kN}$



At the section containing points H and K

$P = 4.5 \cos 30° = 3.897 \text{ kN} \qquad V = 4.5 \sin 30° = 2.25 \text{ kN}$

$M = (4.5 \times 10^3)(40 \times 10^{-3} \sin 30°) = 90 \text{ N}\cdot\text{m}$

$A = 10 \times 24 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$

$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$

(a) At point H $\qquad \sigma_x = -\dfrac{P}{A} = -\dfrac{3.897 \times 10^3}{240 \times 10^{-6}} = -16.24 \text{ MPa}$ ◄

$\tau_{xy} = \dfrac{3}{2}\dfrac{V}{A} = \dfrac{3}{2}\dfrac{2.25 \times 10^3}{240 \times 10^{-6}} = 14.06 \text{ MPa}$ ◄

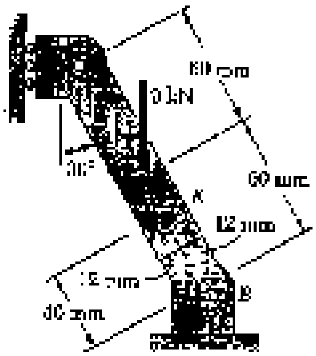
(b) At point K $\qquad \sigma_x = -\dfrac{P}{A} - \dfrac{Mc}{I} = -\dfrac{3.897 \times 10^3}{240 \times 10^{-6}} - \dfrac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$

$= -110.0 \text{ MPa}$ ◄

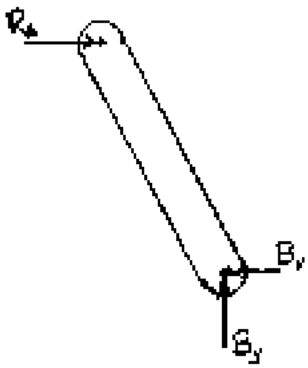
$\tau_{xy} = 0$ ◄

PROBLEM 8.35

8.34 through 8.36 Member *AB* has a uniform rectangular cross section of 10 × 24 mm. For the loading shown, determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.



SOLUTION



$\circlearrowleft \Sigma M_B = 0$

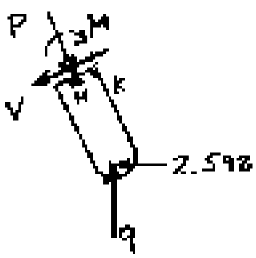
$(120 \cos 30°) R_A - (60 \sin 30°)(9) = 0$

$R_A = 2.598 \text{ kN}$

$+\uparrow \Sigma F_y = 0 \qquad B_y - 9 = 0 \qquad B_y = 9 \text{ kN}\uparrow$

$\stackrel{+}{\rightarrow} \Sigma F_x = 0 \qquad 2.598 - B_x = 0$

$B_x = 2.598 \text{ kN} \leftarrow$



At the section containing points H and K

$P = 9 \cos 30° + 2.598 \sin 30° = 9.093 \text{ kN}$

$V = 9 \sin 30° - 2.598 \cos 30° = 2.25 \text{ kN}$

$M = (9 \times 10^3)(40 \times 10^{-3} \sin 30°) - (2.598 \times 10^3)(40 \times 10^{-3} \cos 30°) = 90 \text{ N}\cdot\text{m}$

$A = 10 \times 240 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$

$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$

(a) At point H $\qquad \sigma_x = -\dfrac{P}{A} = -\dfrac{9.093 \times 10^3}{240 \times 10^{-6}} = -37.9 \text{ MPa}$

$\tau_{xy} = \dfrac{3}{2}\dfrac{V}{A} = \dfrac{3}{2}\dfrac{2.25 \times 10^3}{240 \times 10^{-6}} = 14.06 \text{ MPa}$

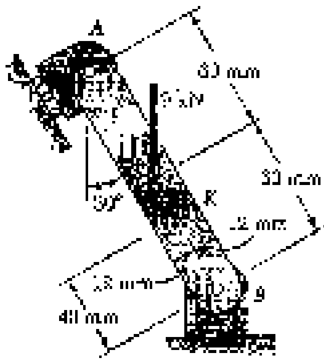
(b) At point K $\qquad \sigma_x = -\dfrac{P}{A} - \dfrac{Mc}{I} = -\dfrac{9.093 \times 10^3}{240 \times 10^{-6}} - \dfrac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$

$= -131.6 \text{ MPa}$

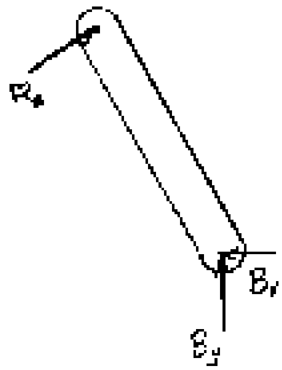
$\tau_{xy} = 0$

PROBLEM 8.36

8.34 through 8.36 Member AB has a uniform rectangular cross section of 10 × 24 mm. For the loading shown, determine the normal and shearing stresses at (*a*) point H, (*b*) point K.



SOLUTION



$$+\circlearrowleft \Sigma M_B = 0$$

$$(9)(60 \sin 30°) - 120 R_A = 0$$

$$R_A = 2.25 \text{ kN}$$

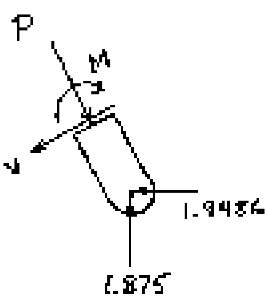
$$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad 2.25 \cos 30° - B_x = 0$$

$$B_x = 1.9486 \text{ kN} \leftarrow$$

$$+\uparrow \Sigma F_y = 0$$

$$2.25 \sin 30° - 9 + B_y = 0$$

$$B_y = 7.875 \text{ kN} \uparrow$$



At the section containing points H and K

$$P = 7.875 \cos 30° + 1.9486 \sin 30° = 7.794 \text{ kN}$$

$$V = 7.875 \sin 30° - 1.9486 \cos 30° = 2.25 \text{ kN}$$

$$M = (7.875 \times 10^3)(40 \times 10^{-3} \sin 30°) - (1.9486 \times 10^3)(40 \times 10^{-3} \cos 30°) = 90 \text{ N} \cdot \text{m}$$

$$A = 10 \times 24 = 240 \text{ mm}^2 = 240 \times 10^{-6} \text{ m}^2$$

$$I = \frac{1}{12}(10)(24)^3 = 11.52 \times 10^3 \text{ mm}^4 = 11.52 \times 10^{-9} \text{ m}^4$$

(a) At point H
$$\sigma_x = -\frac{P}{A} = -\frac{7.794 \times 10^3}{240 \times 10^{-6}} = -32.5 \text{ MPa}$$

$$\tau_{xy} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{2.25 \times 10^3}{240 \times 10^{-6}} = 14.06 \text{ MPa}$$

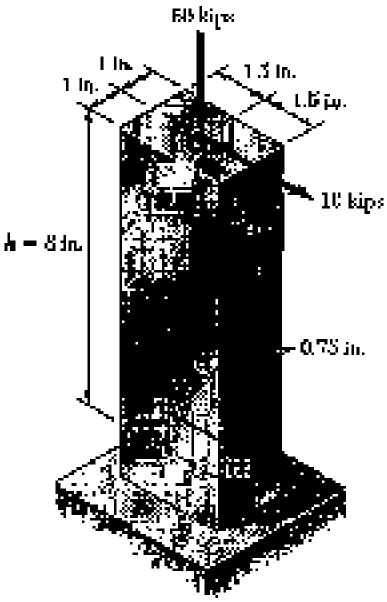
(b) At point K
$$\sigma_x = -\frac{P}{A} - \frac{Mc}{I} = -\frac{7.794 \times 10^3}{240 \times 10^{-6}} - \frac{(90)(12 \times 10^{-3})}{11.52 \times 10^{-9}}$$

$$= -126.2 \text{ MPa}$$

$$\tau_{xy} = 0$$

PROBLEM 8.37

8.37 Two forces are applied to the bar shown. At point *a*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.



SOLUTION

At the section containing point a and b.

$V = 10$ kips $\qquad P = 60$ kips (compression)

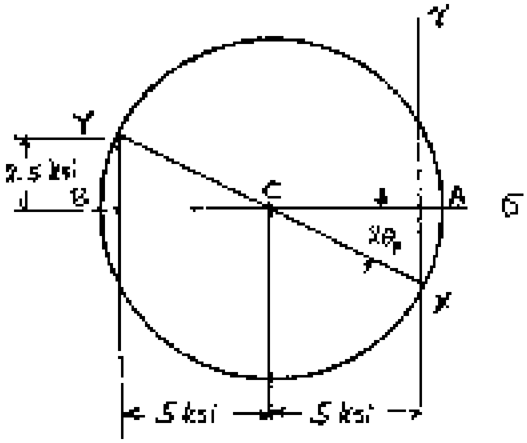
$M = (8)(10) = 80$ kip-in

$A = (2)(3) = 6\ in^2$

$I = \frac{1}{12}(2)(3)^3 = 4.5$ kip-in

At point a $\quad \sigma_y = -\frac{P}{A} = -\frac{60}{6} = -10$ ksi

$\tau = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\frac{(10)}{6} = 2.5$ ksi, $\sigma_x = 0$



Use Mohr's circle

$\sigma_c = -5$ ksi

$R = \sqrt{5^2 + 2.5^2} = 5.590$ ksi

$\sigma_a = \sigma_c + R = 0.590$ ksi

$\sigma_b = \sigma_c - R = -10.59$ ksi

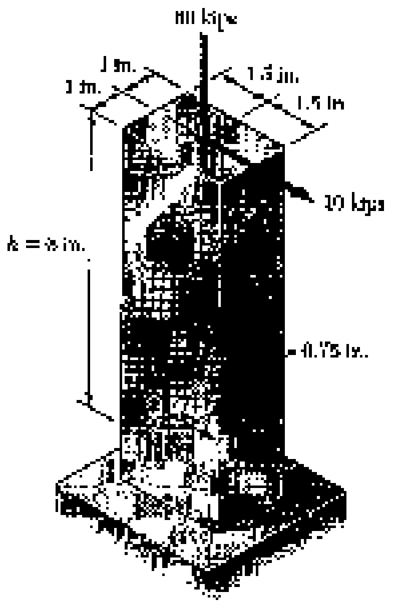
$\tan 2\theta_p = \frac{2.5}{5} = 0.5$

$\theta_p = 13.3°, 103.3°$

$\tau_{max} = R = 5.59$ ksi

PROBLEM 8.38

8.38 Two forces are applied to the bar shown. At point b, determine (a) the principal stresses and principal planes, (b) the maximum shearing stress.



SOLUTION

At the section containing points a and b

$V = 10$ kips, $\quad P = 60$ kips (compression)

$M = (8)(10) = 80$ kip·in.

$A = (2)(3) = 6 \text{ in}^2$

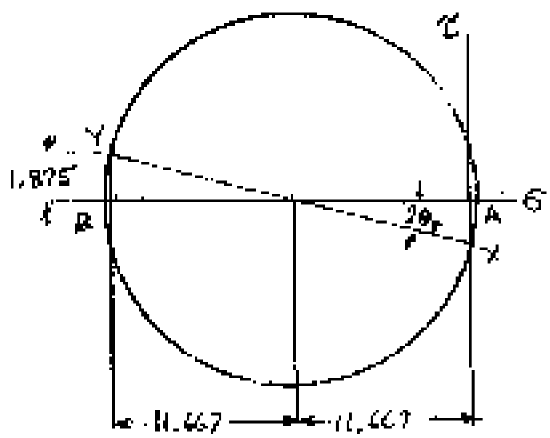
$I = \frac{1}{12}(2)(3)^3 = 4.5 \text{ in}^4$

At point b $\qquad \sigma_x = 0$

$$\sigma_y = -\frac{P}{A} - \frac{Mx}{I} = -\frac{60}{6} - \frac{(80)(0.75)}{4.5} = -23.33 \text{ ksi}$$

$$\tau = \frac{VQ}{It} = \frac{(10)(2)(0.75)(1.125)}{(4.5)(2)} = 1.875 \text{ ksi}$$

Use Mohr's circle



$\sigma_c = -11.667$ ksi

$R = \sqrt{11.667^2 + 1.875^2} = 11.8164$ ksi

$\sigma_a = \sigma_c + R = 0.150$ ksi

$\sigma_b = \sigma_c - R = -23.5$ ksi

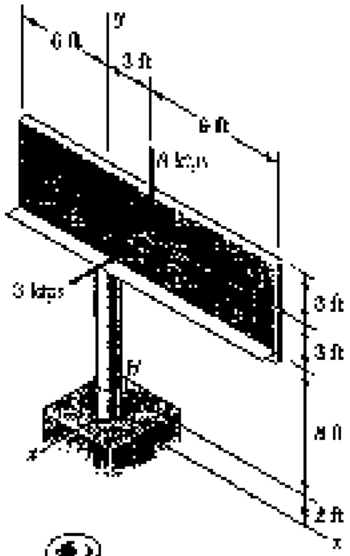
$\tan 2\theta_p = \frac{1.875}{11.667} = 0.16071$

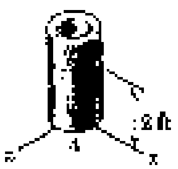
$\theta_p = 4.6°, \quad 94.6°$

$\tau_{max} = R = 11.82$ ksi

PROBLEM 8.39

8.39 The billboard shown weighs 8,000 lb and is supported by a structural tube that has a 15-in. outer diameter and a 0.5-in. wall thickness. At a time when the resultant of the wind pressure is 3 kips located at the center C of the billboard, determine the normal and shearing stresses at point H.





SOLUTION

At section containing point H

$P = 8$ kips (compression)

$T = (3)(3) = 9$ kip·ft $= 108$ kip·in

$M_x = -(11)(3) = -33$ kip·ft $= -396$ kip·in

$M_z = -(3)(8) = -24$ kip·ft $= -288$ kip·in

$V = 3$ kip

Section properties.

$d_o = 15$ in $\quad c_o = \frac{1}{2}d_o = 7.5$ in $\quad c_i = c_o - t = 7.0$ in

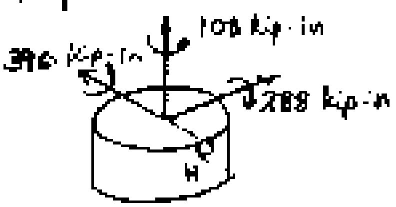
$A = \pi(c_o^2 - c_i^2) = 22.777$ in^2

$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 599.31$ in^4

$J = 2I = 1198.62$ in^4

$Q = \frac{2}{3}(c_o^3 - c_i^3) = 52.583$ in^3

couples

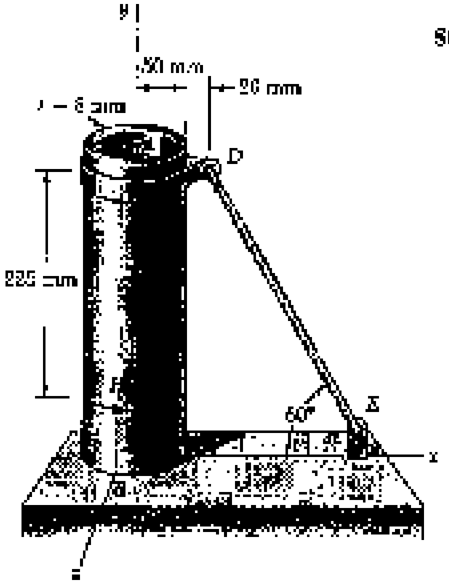


$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8}{22.777} - \frac{(288)(7.5)}{599.31} = -0.351 - 3.604 = -3.96 \text{ ksi}$$

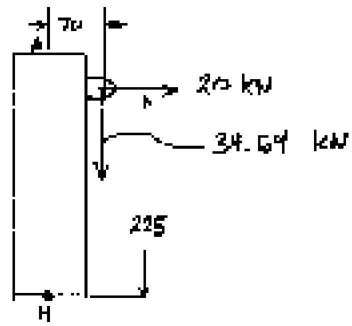
$$\tau = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(108)(7.5)}{1198.62} + \frac{(3)(52.583)}{(599.31)(1.0)} = 0.675 + 0.268 = 0.938 \text{ ksi}$$

PROBLEM 8.40

8.40 The steel pipe *AB* has a 100-mm outer diameter and an 8-mm wall thickness. Knowing that the tension in the cable is 40 kN, determine the normal and shearing stresses at point *H*.



SOLUTION



Vertical force
$40 \cos 30° = 34.64$ kN

Horizontal force
$40 \sin 30° = 20$ kN

Point H lies on neutral axis of bending

Section properties

$d_o = 100$ mm $\quad c_o = \frac{1}{2} d_o = 50$ mm $\quad c_i = c_o - t = 42$ mm

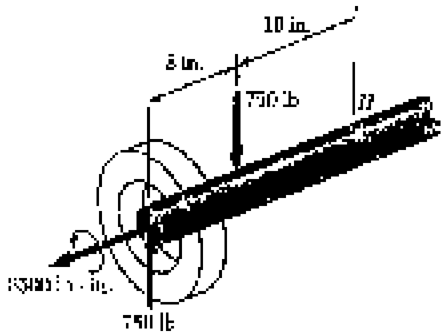
$A = \pi(c_o^2 - c_i^2) = 2.312 \times 10^3\ \text{mm}^2 = 2.312 \times 10^{-3}\ \text{m}^2$

$\sigma = -\frac{P}{A} = -\frac{34.64 \times 10^3}{2.312 \times 10^{-3}} = -14.98$ MPa ◄

For thin pipe $\quad \tau = 2\frac{V}{A} = \frac{(2)(20 \times 10^3)}{2.314 \times 10^{-3}} = 17.29$ MPa ◄

PROBLEM 8.41

8.41 The axle of a small truck is acted upon by the forces and couple shown. Knowing that the diameter of the axle is 1.42 in., determine the normal and shearing stresses at point *H* located on the top of the axle.



SOLUTION

The bending moment causing normal stress at point H is

$$M = (8)(750) = 6000 \text{ lb}\cdot\text{in}$$

$$c = \tfrac{1}{2}d = 0.71 \text{ in.}$$

$$I = \tfrac{\pi}{4}c^4 = 0.19958 \text{ in}^4, \qquad J = 2I = 0.39916 \text{ in}^4$$

Normal stress at H $\quad \sigma_H = -\frac{Mc}{I} = -\frac{(6000)(0.71)}{0.19958} = -21.3\times10^3 \text{ psi}$

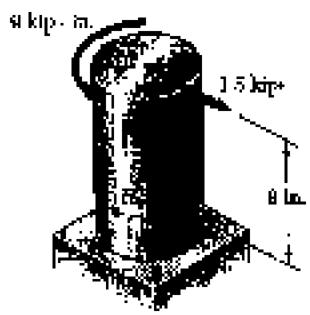
$$= -21.3 \text{ ksi}$$

At the section containing point H $\quad V = 0, \quad T = 3500 \text{ lb}\cdot\text{in}$

$$\tau_H = \frac{Tc}{J} = \frac{(3500)(0.71)}{0.39916} = 6.23 \text{ ksi}$$

PROBLEM 8.42

8.42 A 1.5-kip force and a 9-kip·in. couple are applied at the top of the cast-iron post shown. Determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.



SOLUTION $\quad$ diameter = 2.5 in.

At the section containing points H and K.

$$P = 0 \qquad V = 1.5 \text{ kips}$$

$$T = 9 \text{ kip-in} \qquad M = (1.5)(9) = 13.5 \text{ kip-in}$$

$$d = 2.5 \text{ in} \qquad c = \tfrac{1}{2}d = 1.25 \text{ in}$$

$$A = \pi c^2 = 4.909 \text{ in}^2 \qquad I = \tfrac{\pi}{4}c^4 = 1.9175 \text{ in}^4 \qquad J = 2I = 3.835 \text{ in}^4$$

For a semicircle $\quad Q = \tfrac{2}{3}c^3 = 1.3021 \text{ in}^3$

(a) At point H $\quad \sigma_H = 0$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(9)(1.25)}{3.835} + \frac{(1.5)(1.3021)}{(1.9175)(2.5)} = 2.934 + 0.407$$

$$= 3.34 \text{ ksi}$$

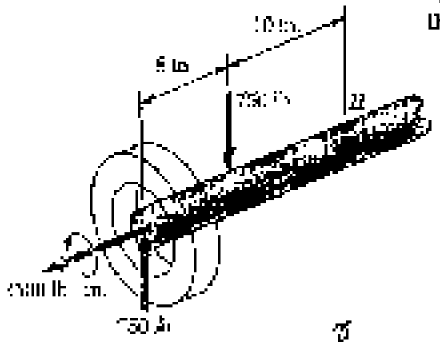
(b) At point K $\quad \sigma_K = -\frac{Mc}{I} = -\frac{(13.5)(1.25)}{1.9175} = -8.80 \text{ ksi}$

$$\tau_K = \frac{Tc}{J} = \frac{(9)(1.25)}{3.835} = 2.93 \text{ ksi}$$

PROBLEM 8.43

8.41 The axle of a small truck is acted upon by the forces and couple shown. Knowing that the diameter of the axle is 1.42 in., determine the normal and shearing stresses at point *H* located on the top of the axle.

8.43 For the truck axle and loading of Prob. 8.41, determine the principal stresses and the maximum shearing stress at point *H*.

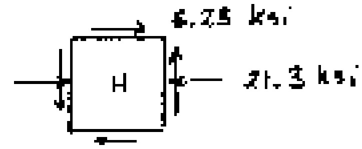


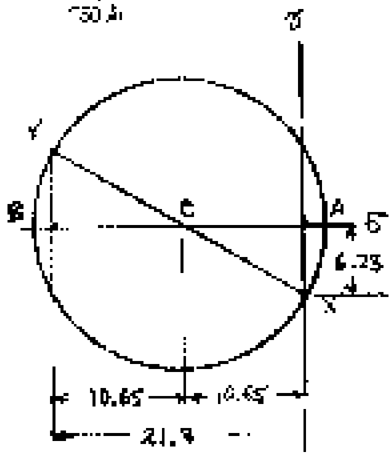
SOLUTION

From the solution of Prob. 8.41

$\sigma_H = -21.3$ ksi

$\tau_H = 6.23$ ksi





$$\sigma_c = -\frac{21.3}{2} = -10.65 \text{ ksi}$$

$$R = \sqrt{\left(\frac{21.3}{2}\right)^2 + (6.23)^2} = 12.34 \text{ ksi}$$

$$\sigma_a = \sigma_c + R = 1.69 \text{ ksi}$$

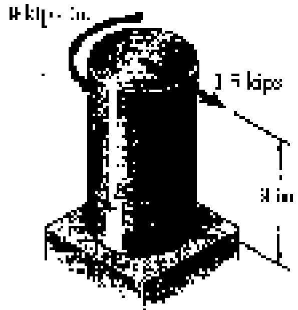
$$\sigma_b = \sigma_c - R = -23.0 \text{ ksi}$$

$$\tau_{max} = R = 12.34 \text{ ksi}$$

PROBLEM 8.44

8.42 A 1.5-kip force and a 9-kip·in. couple are applied at the top of the cast-iron post shown. Determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.

8.44 For the post and loading of Prob. 8.42, determine the principal stresses and the maximum shearing stress at (*a*) point *H*, (*b*) point *K*.



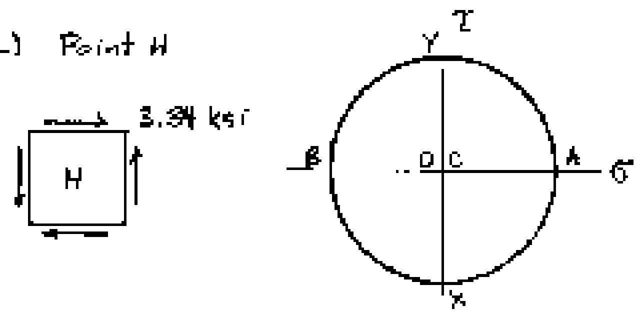
SOLUTION

From the solution of Prob. 8.42

(a) $\sigma_H = 0$, $\tau_H = 3.34$ ksi

(b) $\sigma_K = -8.80$ ksi, $\tau_K = 2.93$ ksi

(a) Point H



$\sigma_c = 0$

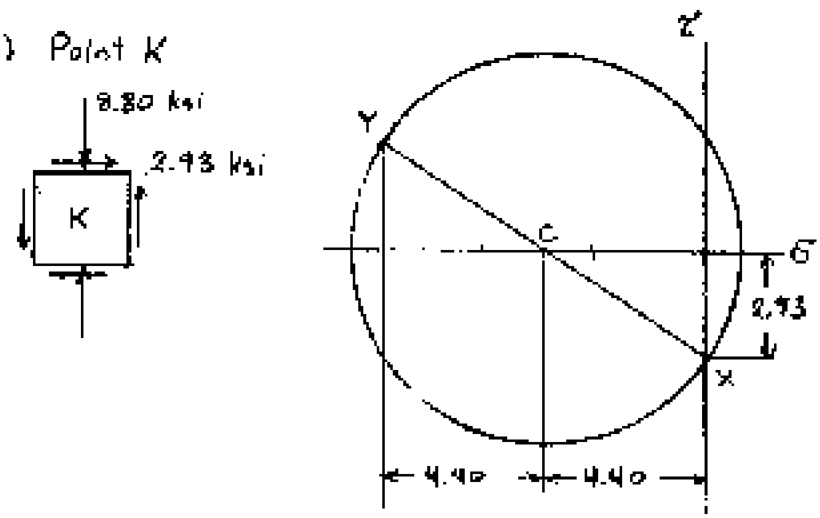
$R = 3.34$ ksi

$\sigma_a = \sigma_c + R = 3.34$ ksi

$\sigma_b = \sigma_c - R = -3.34$ ksi

$\tau_{max} = R = 3.34$ ksi

(b) Point K



$$\sigma_c = -\frac{8.80}{2} = -4.40 \text{ ksi}$$

$$R = \sqrt{\left(\frac{8.80}{2}\right)^2 + 2.93^2} = 5.29 \text{ ksi}$$

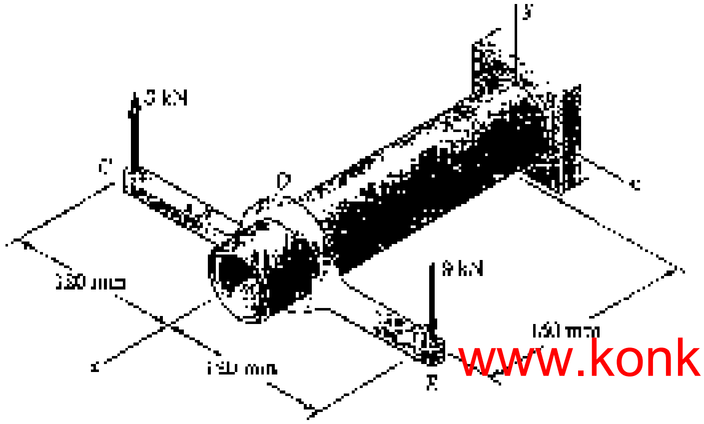
$\sigma_a = \sigma_c + R = 0.89$ ksi

$\sigma_b = \sigma_c - R = -9.69$ ksi

$\tau_{max} = R = 5.29$ ksi

PROBLEM 8.45

8.45 The steel pipe AB has a 72-mm outer diameter and a 5-mm wall thickness. Knowing that arm CDE is rigidly attached to the pipe, determine the principal stresses, principal planes, and maximum shearing stress at point H.

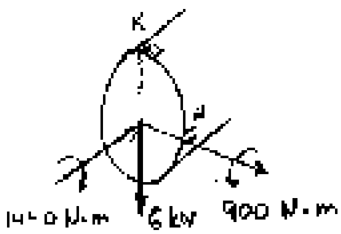


SOLUTION

Replace the forces at C and E by an equivalent force-couple system at D.

$F_D = 9 - 3 = 6 \text{ kN} \downarrow$

$T_D = (9\times10^3)(120\times10^{-3}) + (3\times10^3)(120\times10^{-3}) = 1440 \text{ N}\cdot\text{m}$



At the section containing points H and K

$P = 0, \quad V = 6 \text{ kN}, \quad T = 1440 \text{ N}\cdot\text{m}$

$M = (6\times10^3)(150\times10^{-3}) = 900 \text{ N}\cdot\text{m}$

Section properties: $d_o = 72$ mm $\quad c_o = \frac{1}{2}d_o = 36$ mm $\quad c_i = c_o - t = 31$ mm

$A = \pi(c_o^2 - c_i^2) = 1.0524\times10^3 \text{ mm}^2 = 1.0524\times10^{-3} \text{ m}^2$

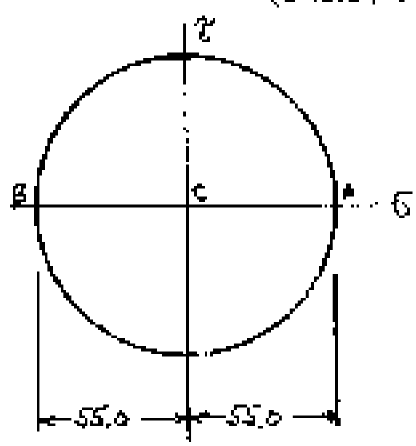
$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 593.84\times10^3 \text{ mm}^4 = 593.84\times10^{-9} \text{ m}^4$

$J = 2I = 1.1877\times10^{-6} \text{ m}^4$

For half-pipe $\quad Q = \frac{2}{3}(c_o^3 - c_i^3) = 11.243\times10^3 \text{ mm}^3 = 11.243\times10^{-6} \text{ m}^3$

At point H $\quad$ Point H lies on the neutral axis of bending. $\sigma_H = 0$.

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(1440)(36\times10^{-3})}{1.1877\times10^{-6}} + \frac{(6\times10^3)(11.243\times10^{-6})}{(593.84\times10^{-9})(10\times10^{-3})} = 55.0 \text{ MPa}$$



$\sigma_c = 0$

$R = 55.0 \text{ MPa}$

$\sigma_a = \sigma_c + R = 55.0 \text{ MPa}$

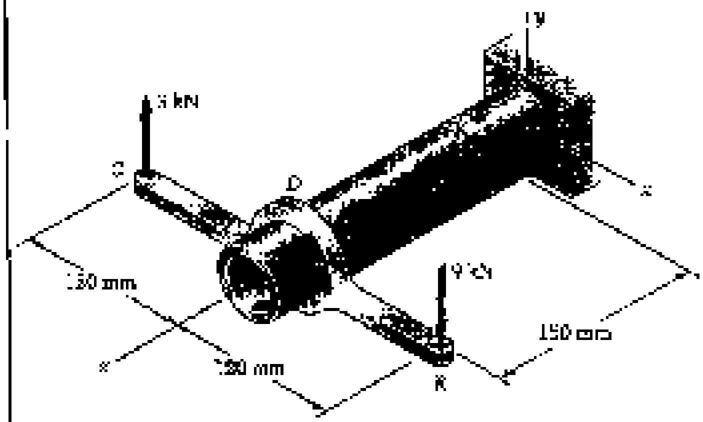
$\sigma_b = \sigma_c - R = -55.0 \text{ MPa}$

$\theta_a = -45^\circ, \quad \theta_b = +45^\circ$

$\tau_{max} = R = 55.0 \text{ MPa}$

PROBLEM 8.46

8.46 The steel pipe AB has a 72-mm outer diameter and a 5-mm wall thickness. Knowing that arm CDE is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K.



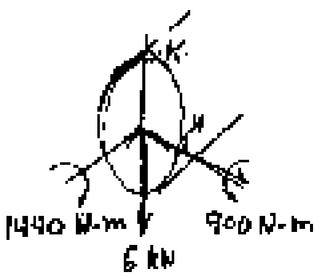
SOLUTION

Replace the forces at C and E by an equivalent force-couple system at D.

$F_D = 9 - 3 = 6$ kN ↓

$T_D = (9\times10^3)(120\times10^{-3}) + (3\times10^3)(120\times10^{-3}) = 1440$ N·m

At the section containing points H and K



$P = 0$, $V = 6$ kN $T = 1440$ N·m

$M = (6\times10^3)(150\times10^{-3}) = 900$ N·m

Section properties: $d_o = 72$ mm $c_o = \frac{1}{2}d_o = 36$ mm $c_i = c_o - t = 31$ mm

$A = \pi(c_o^2 - c_i^2) = 1.0524\times10^3$ mm^2 $= 1.0524\times10^{-3}$ m^3

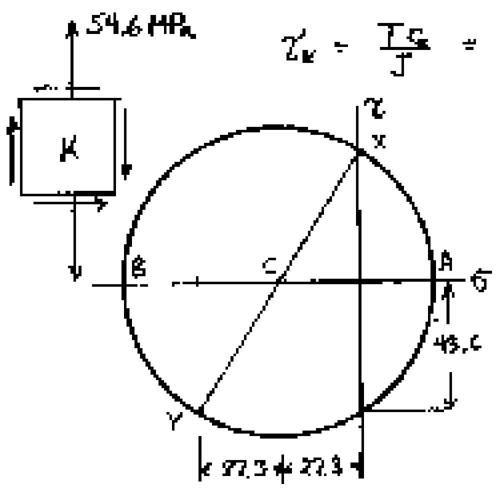
$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 593.84\times10^3$ mm^4 $= 593.84\times10^{-7}$ m^4

$J = 2I = 1.1877\times10^{-6}$ m^4

For half-pipe $Q = \frac{2}{3}(c_o^3 - c_i^3) = 11.243\times10^3$ mm^3 $= 11.243\times10^{-6}$ m^3

At point K $\sigma_K = \dfrac{Mc}{I} = \dfrac{(900)(36\times10^{-3})}{(593.84\times10^{-7})} = 54.6$ MPa

$\tau_K = \dfrac{Tc}{J} = \dfrac{(1440)(36\times10^{-3})}{1.1877\times10^{-6}} = 43.6$ MPa



$\sigma_c = -\dfrac{54.6}{2} = -27.3$ MPa

$R = \sqrt{\left(\frac{54.6}{2}\right)^2 + 43.6^2} = 51.4$ MPa

$\sigma_a = \sigma_c + R = 24.1$ MPa ◄

$\sigma_b = \sigma_c - R = -78.7$ MPa ◄

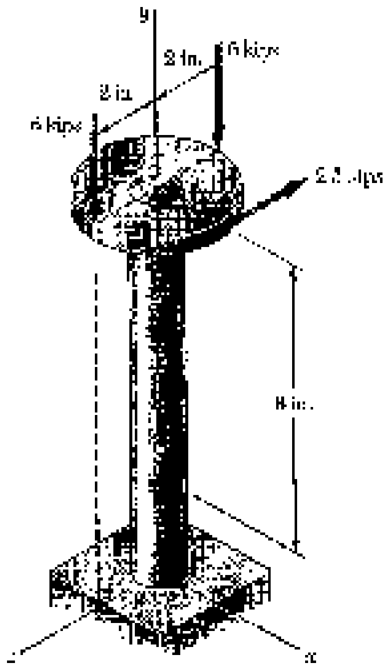
$\tan 2\theta_p = \dfrac{43.6}{27.3} = 1.597$

$\theta_a = 57.9°$, $\theta_b = -32.1°$

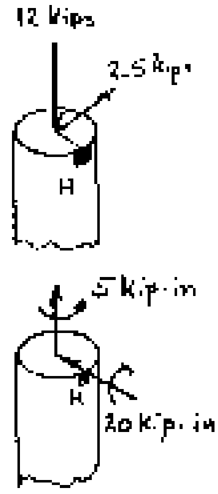
$\tau_{max} = R = 51.4$ MPa ◄

PROBLEM 8.47

8.47 Three forces are applied to 4-in.-diameter plate that is attached to the solid 1.8-in. diameter shaft *AB*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.



SOLUTION



At the section containing point H

$P = 12$ kips (compression)

$V = 2.5$ kips

$T = (2)(2.5) = 5$ kip·in

$M = (8)(2.5) = 20$ kip·in.

$d = 1.8$ in $\qquad c = \frac{1}{2}d = 0.9$ in.

$A = \pi c^2 = 2.545$ in^2

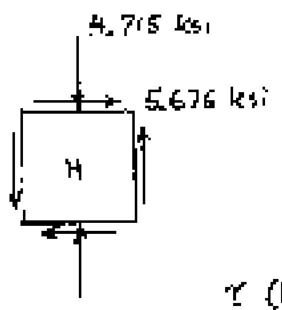
$I = \frac{\pi}{4}c^4 = 0.5153$ in^4

$J = 2I = 1.0306$ in^4

For a semicircle $\qquad Q = \frac{2}{3}c^3 = 0.486$ in^3

Point H lies on neutral axis of bending $\qquad \sigma_H = \frac{P}{A} = -\frac{12}{2.545} = -4.715$ ksi

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(5)(0.9)}{1.0306} + \frac{(2.5)(0.486)}{(0.5153)(1.8)} = 5.676 \text{ ksi}$$



$\sigma_c = \frac{1}{2}(-4.715) = -2.3575$ ksi

$R = \sqrt{\left(\frac{4.715}{2}\right)^2 + 5.676^2} = 6.1461$

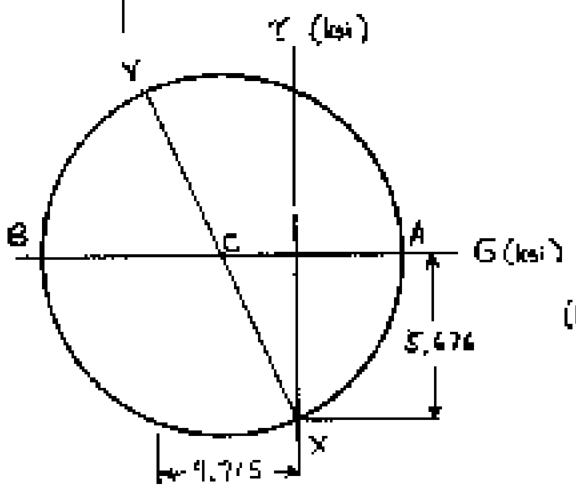
(a) $\sigma_a = \sigma_c + R = 3.79$ ksi

$\sigma_b = \sigma_c - R = -8.50$ ksi

$\tan 2\theta_p = \frac{(2)(5.676)}{4.715} = 2.408$

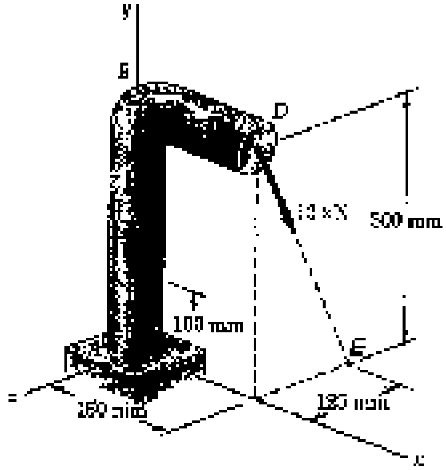
$\theta_a = 33.7°$ $\qquad \theta_b = 123.7°$

(b) $\tau_{max} = R = 6.15$ ksi



PROBLEM 8.48

8.48 A 13-kN force is applied as shown to the 60-mm-diameter cast-iron post *ABD*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.



SOLUTION

$$DE = \sqrt{125^2 + 300^2} = 325 \text{ mm}$$

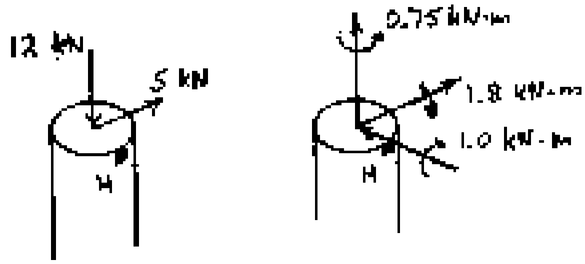
At point D $\qquad F_x = 0$

$$F_y = -\left(\frac{300}{325}\right)(13) = -12 \text{ kN}$$

$$F_z = -\left(\frac{125}{300}\right)(13) = -5 \text{ kN}$$

Moment of equivalent force-couple system at C, the centroid of the section containing point H

$$\vec{M} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.150 & 0.200 & 0 \\ 0 & -12 & -5 \end{vmatrix} = -1.00\,\mathbf{i} + 0.75\,\mathbf{j} - 1.8\,\mathbf{k} \quad \text{kN}\cdot\text{m}$$



Section properties

$d = 60$ mm $\qquad c = \frac{1}{2}d = 30$ mm

$A = \pi c^2 = 2.8274 \times 10^3 \text{ mm}^2$

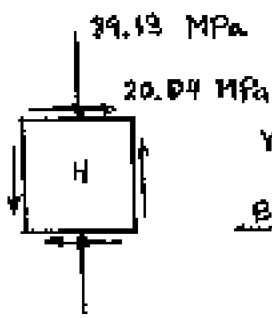
$I = \frac{\pi}{4}c^4 = 636.17 \times 10^3 \text{ mm}^4$

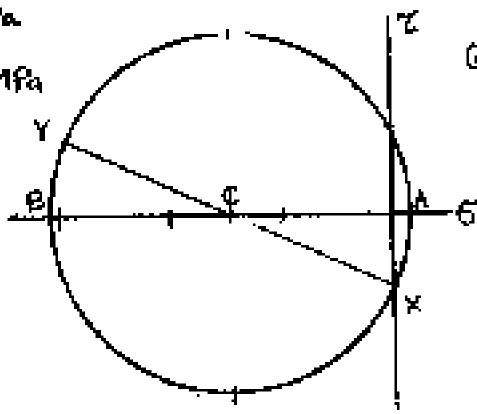
$J = 2I = 1.2723 \times 10^6 \text{ mm}^4$

For a semicircle $\qquad Q = \frac{2}{3}c^3 = 18.00 \times 10^3 \text{ mm}^3$

At point H $\qquad \sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{12 \times 10^3}{2.8274 \times 10^{-3}} - \frac{(1.8 \times 10^3)(30 \times 10^{-3})}{636.17 \times 10^{-9}} = -89.13 \text{ MPa}$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(0.75 \times 10^3)(30 \times 10^{-3})}{1.2723 \times 10^{-6}} + \frac{(5 \times 10^3)(18.00 \times 10^{-6})}{(636.17 \times 10^{-9})(60 \times 10^{-3})} = 20.04 \text{ MPa}$$





(a) $\qquad \sigma_c = \frac{\sigma_H}{2} = -44.565 \text{ MPa}$

$$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 48.865 \text{ MPa}$$

$\sigma_a = \sigma_c + R = 4.3 \text{ MPa}$

$\sigma_b = \sigma_c - R = -93.4 \text{ MPa}$

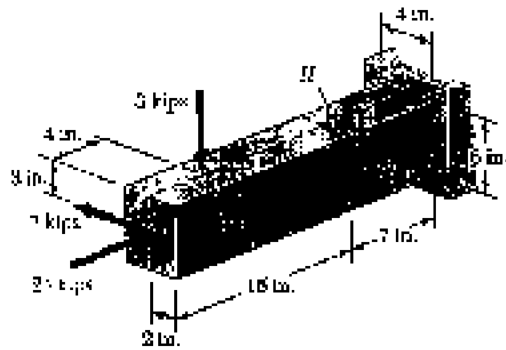
$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = 0.4497$

$\theta_a = 12.1°, \quad \theta_b = 102.1°$

(b) $\qquad \tau_{max} = R = 48.9 \text{ MPa}$

PROBLEM 8.49

8.49 Three forces are applied to the cantilever beam shown. Determine the normal and shearing stresses at point *H*.



SOLUTION

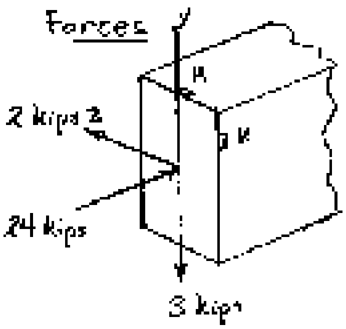
At the section containing points H and K, the axial and shearing forces are:

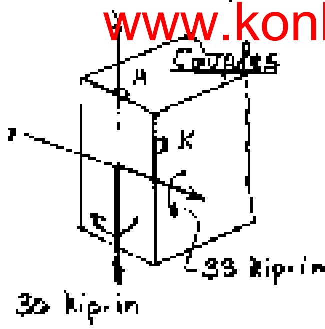
$$P = 24 \text{ kips}, \quad V = \begin{cases} 3 \text{ kips vertical} \\ 2 \text{ kips horizontal} \end{cases}$$

The bending moment components are:

about horizontal axis: $M = (15-4)(3) = 33$ kip·in

about vertical axis $M = (15)(2) = 30$ kip·in





Section properties

$A = (4)(6) = 24 \text{ in}^2$

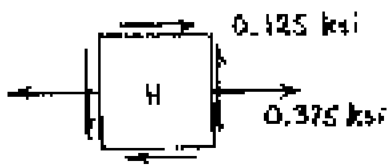
$I_z = \frac{1}{12}(4)(6)^3 = 72 \text{ in}^4$

$I_y = \frac{1}{12}(6)(4)^3 = 32 \text{ in}^4$

At point H

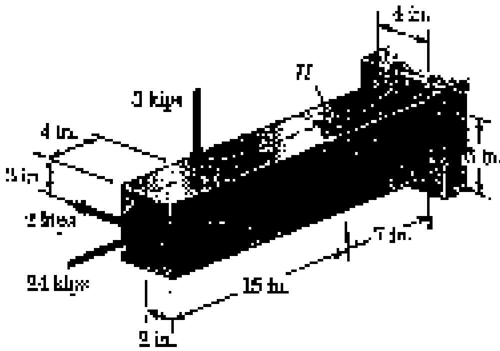
$$\sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{24}{24} + \frac{(33)(3)}{72} = 0.375 \text{ ksi}$$

$$\tau_H = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\,\frac{2}{24} = 0.125 \text{ ksi}$$



PROBLEM 8.50

8.50 Three forces are applied to the cantilever beam shown. Determine the normal and shearing stresses at point K.



SOLUTION

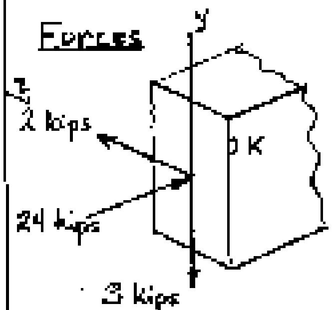
At the section containing points H and K the axial and shearing forces are

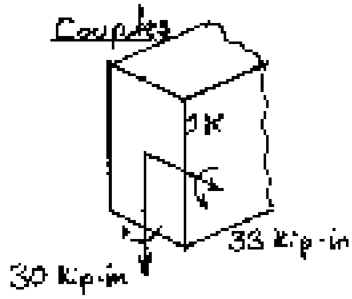
$P = 24$ kips, $V = $ 3 kips vertical, 2 kips horizontal

The bending moment components are

about horizontal axis: $M = (15-4)(3) = 33$ kip-in

about vertical axis: $M = (15)(2) = 30$ kip-in





Section properties

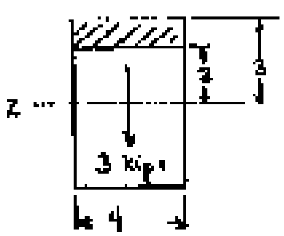
$A = (4)(6) = 24 \text{ in}^2$

$I_z = \frac{1}{12}(4)(6)^3 = 72 \text{ in}^4$

$I_y = \frac{1}{12}(6)(4)^3 = 32 \text{ in}^4$

At point K

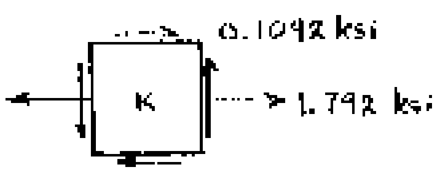
$$\sigma_K = -\frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} = -\frac{24}{24} - \frac{(-33)(2)}{72} + \frac{(-30)(-2)}{32} = 1.792 \text{ ksi}$$



$A^* = (1)(4) = 4 \text{ in}^2$ $\qquad \bar{y} = 2.5$ in.

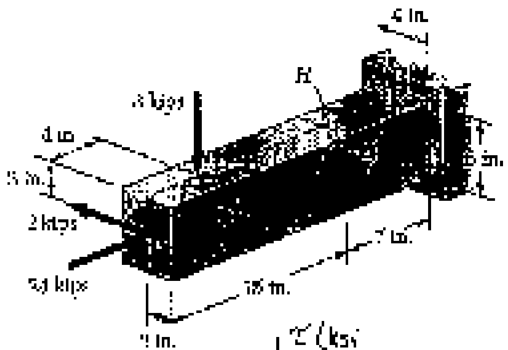
$Q = A^* \bar{y} = (4)(2.5) = 10 \text{ in}^3$

$$\tau_K = \frac{VQ}{It} = \frac{(3)(10)}{(72)(4)} = 0.1042 \text{ ksi}$$



PROBLEM 8.51

8.51 For the beam and loading of Prob. 8.49, determine the principal stresses and the maximum shearing stress at point H.

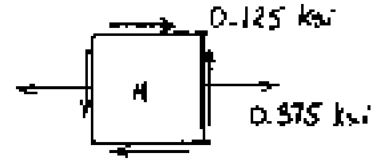


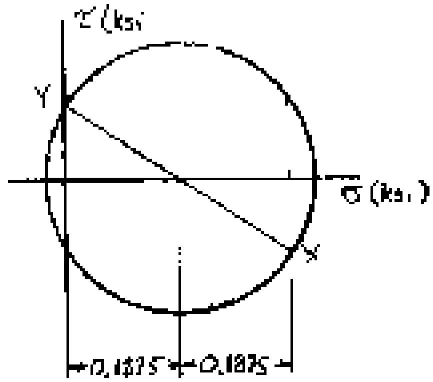
SOLUTION

From the solution of Prob. 8.49

$\sigma_H = 0.375$ ksi

$\tau_H = 0.125$ ksi





$$\sigma_c = \frac{0.375}{2} = 0.1875 \text{ ksi}$$

$$R = \sqrt{\left(\frac{0.375}{2}\right)^2 + (0.125)^2} = 0.2253 \text{ ksi}$$

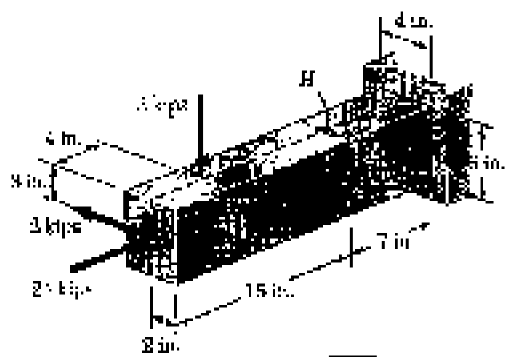
$$\sigma_a = \sigma_c + R = 0.413 \text{ ksi}$$

$$\sigma_b = \sigma_c - R = -0.0378 \text{ ksi}$$

$$\tau_{max} = R = 0.225 \text{ ksi}$$

PROBLEM 8.52

8.52 For the beam and loading of Prob. 8.50, determine the principal stresses and the maximum shearing stress at point K.

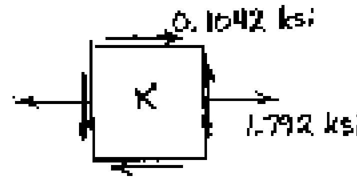


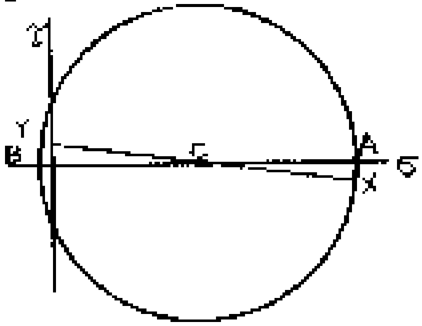
SOLUTION

From the solution of Prob. 8.50

$\sigma_H = 1.792$ ksi

$\tau_H = 0.1042$ ksi





$$\sigma_c = \frac{1.792}{2} = 0.896 \text{ ksi}$$

$$R = \sqrt{\left(\frac{1.792}{2}\right)^2 + (0.1042)^2} = 0.902 \text{ ksi}$$

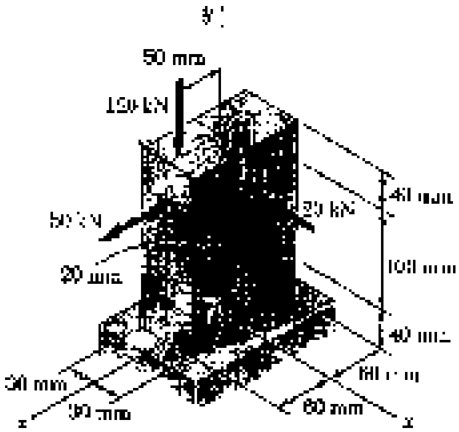
$$\sigma_a = \sigma_c + R = 1.798 \text{ ksi}$$

$$\sigma_b = \sigma_c - R = -0.006 \text{ ksi}$$

$$\tau_{max} = R = 0.902 \text{ ksi}$$

PROBLEM 8.53

8.53 Three forces are applied to a steel post as shown. Determine the normal and shearing stresses at point H.



SOLUTION

$$A = (120)(60) = 7.2\times10^{3}\ \text{mm}^2 = 7.2\times10^{-3}\ \text{m}^2$$

$$I_x = \tfrac{1}{12}(60)(120)^3 = 8.64\times10^{6}\ \text{mm}^4 = 8.64\times10^{-6}\ \text{m}^4$$

$$I_z = \tfrac{1}{12}(120)(60)^3 = 2.16\times10^{6}\ \text{mm}^4 = 2.16\times10^{-6}\ \text{m}^4$$

At the section containing points H and K.

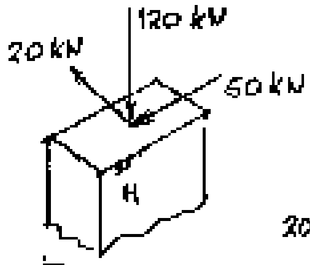
$$P = 120\ \text{kN (compression)}$$

$$V_x = -20\ \text{kN}$$

$$V_z = 50\ \text{kN}$$

$$M_z = (20\times10^{3})(100\times10^{-3}) = 2000\ \text{N·m}$$

$$M_x = (120\times10^{3})(50\times10^{-3}) + (50\times10^{3})(100\times10^{-3}) = 11000\ \text{N·m}$$



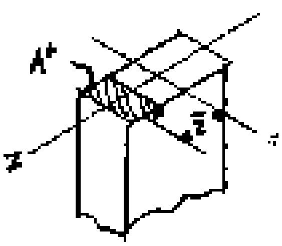
Forces

Couples

Stresses at point H

$$\sigma_H = -\frac{P}{A} - \frac{M_x z}{I_x} + \frac{M_z x}{I_z} = -\frac{120\times10^{3}}{7.2\times10^{-3}} - \frac{(11000)(20\times10^{-3})}{8.64\times10^{-6}} + \frac{(2000)(30\times10^{-3})}{2.16\times10^{-6}}$$

$$= -16.67\ \text{MPa} - 25.46\ \text{MPa} + 27.78\ \text{MPa} = -14.35\ \text{MPa}$$

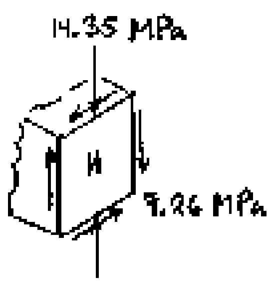


$$A^* = (60)(60-20) = 2.4\times10^{3}\ \text{mm}^2$$

$$\bar{z} = \left(20 + \frac{40}{2}\right) = 40\ \text{mm}$$

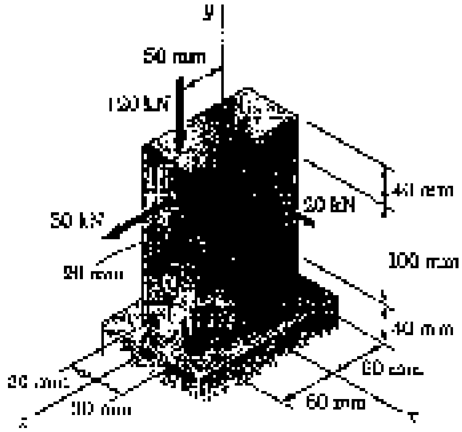
$$\bar{Q}_x = A^*\bar{z} = 96\times10^{3}\ \text{mm}^3 = 96\times10^{-6}\ \text{m}^3$$

$$\tau_H = \frac{V_z Q_x}{I_x t} = \frac{(50\times10^{3})(96\times10^{-6})}{(8.64\times10^{-6})(60\times10^{-3})} = 9.26\ \text{MPa}$$



PROBLEM 8.54

8.54 Three forces are applied to a steel post as shown. Determine the normal and shearing stresses at point K.



SOLUTION

$A = (120)(60) = 7.2\times10^{3}\ \text{mm}^2 = 7.2\times10^{-3}\ \text{m}^2$

$I_x = \frac{1}{12}(60)(120)^3 = 8.64\times10^{6}\ \text{mm}^4 = 8.64\times10^{-6}\ \text{m}^4$

$I_z = \frac{1}{12}(120)(60)^3 = 2.16\times10^{6}\ \text{mm}^4 = 2.16\times10^{-6}\ \text{m}^4$

At the section containing points H and K.

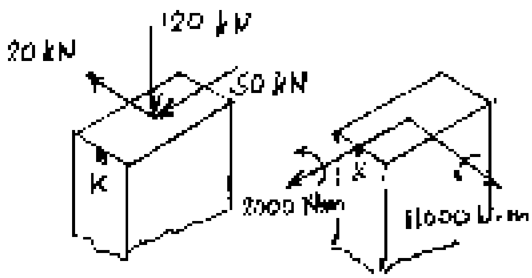
$P = 120$ kN (compression)

$V_x = -20$ kN

$V_z = 50$ kN

$M_z = (20\times10^3)(100\times10^{-3}) = 2000$ N-m

$M_x = (120\times10^3)(50\times10^{-3})$
$\quad + (50\times10^3)(100\times10^{-3})$
$\quad = 11000$ N·m

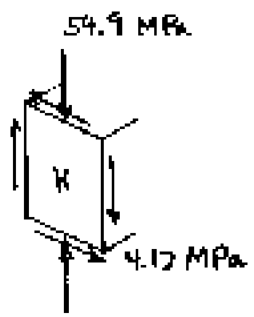


Stresses at point K

$$\sigma_K = -\frac{P}{A} - \frac{M_x z}{I_x} + \frac{M_z x}{I_z} = -\frac{120\times10^3}{7.2\times10^{-3}} - \frac{(11000)(30\times10^{-3})}{8.64\times10^{-6}} + 0$$

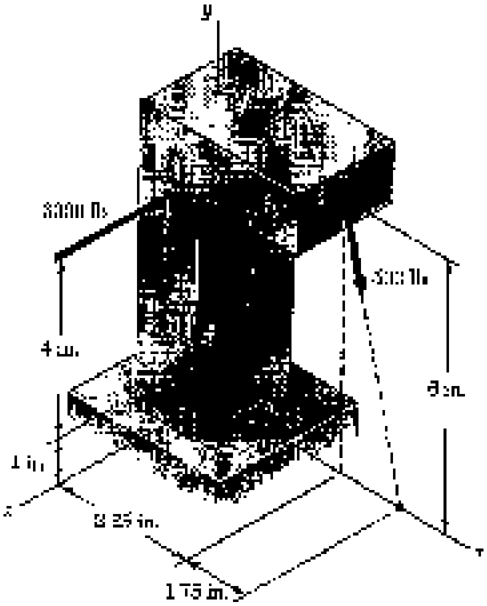
$$= -16.67\ \text{MPa} - 38.39\ \text{MPa} + 0 = -55.1\ \text{MPa}$$

$$\tau_K = \frac{3}{2}\frac{|V_x|}{A} = \frac{3}{2}\,\frac{20\times10^3}{7.2\times10^{-3}} = 4.17\ \text{MPa}$$



PROBLEM 8.55

8.55 Two forces are applied to the small post BD as shown. Knowing that the vertical portion of the post has a cross section of 1.5×2.4 in., determine the principal stresses, principal planes, and maximum shearing stress at point H.



SOLUTION

Components of 500 lb. force

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb.}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb.}$$

Moment arm of 500 lb. force

$$\vec{r} = 3.25\,\vec{i} + (6-1)\,\vec{j}$$

Moment of 500 lb. force

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260\,\vec{k} \text{ lb-in}$$

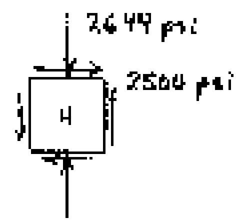
At the section containing point H: $\quad P = -480$ lb, $\quad V_x = 140$ lb.

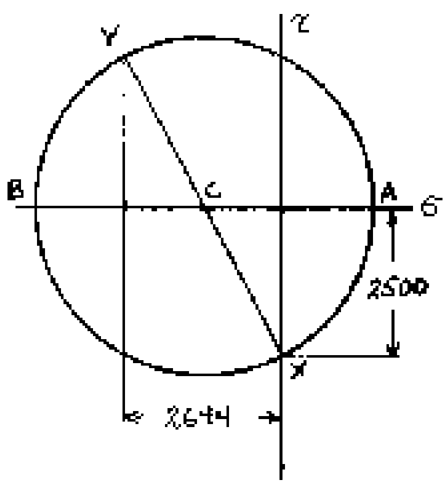
$V_z = -6000$ lb., $\quad M_z = -2260$ lb-in, $\quad M_x = -(4)(6000) = -24000$ lb-in.

$A = (1.5)(2.4) = 3.6 \text{ in}^2 \qquad I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$

$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{6000}{3.6} = 2500 \text{ psi}$$





$$\sigma_c = -\frac{2644}{2} = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (2500)^2} = 2828 \text{ psi}$$

$$\sigma_a = \sigma_c + R = 1506 \text{ psi} \quad \blacktriangleleft$$

$$\sigma_b = \sigma_c - R = -4150 \text{ psi} \quad \blacktriangleleft$$

$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(2500)}{2644} = 1.891$$

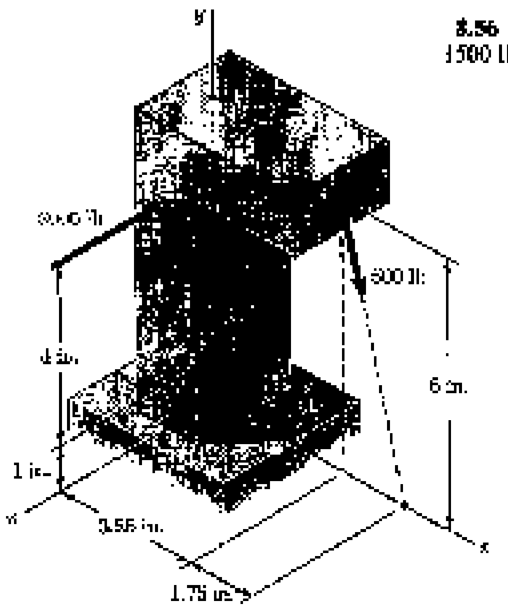
$$\theta_a = 31.1^\circ, \quad \theta_b = 121.1^\circ \quad \blacktriangleleft$$

$$\tau_{max} = R = 2828 \text{ psi} \quad \blacktriangleleft$$

PROBLEM 8.56

8.55 Two forces are applied to the small post BD as shown. Knowing that the vertical portion of the post has a cross section of 1.5 × 2.4 in., determine the principal stresses, principal planes, and maximum shearing stress at point H.

8.56 Solve Prob 8.55, assuming that the magnitude of the 6000-lb force is reduced to 1500 lb.



SOLUTION

Components of 500 lb. force

$$F_x = \frac{(500)(1.75)}{6.25} = 140 \text{ lb}$$

$$F_y = -\frac{(500)(6)}{6.25} = -480 \text{ lb.}$$

Moment arm of 500 lb. force

$$\vec{r} = 3.25\,\vec{i} + (6-1)\,\vec{j}$$

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3.25 & 5 & 0 \\ 140 & -480 & 0 \end{vmatrix} = -2260\,\vec{k} \text{ lb·in}$$

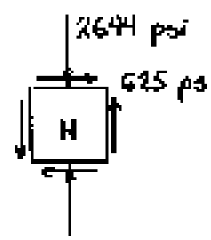
At the section containing point H: $P = -480$ lb $\quad V_x = 140$ lb.

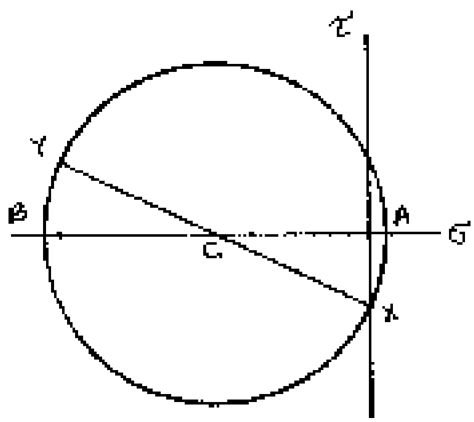
$V_z = -1500$ lb, $\quad M_z = -2260$ lb·in, $\quad M_x = -(4)(1500) = -6000$ lb·in.

$A = (1.5)(2.4) = 3.6 \text{ in}^2 \qquad I_z = \frac{1}{12}(2.4)(1.5)^3 = 0.675 \text{ in}^4$

$$\sigma_H = \frac{P}{A} + \frac{M_z x}{I_z} = -\frac{480}{3.6} + \frac{(-2260)(0.75)}{0.675} = -2644 \text{ psi}$$

$$\tau_H = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{1500}{3.6} = 625 \text{ psi}$$





$$\sigma_c = \tfrac{1}{2}\sigma_H = -1322 \text{ psi}$$

$$R = \sqrt{\left(\frac{2644}{2}\right)^2 + (625)^2} = 1462 \text{ psi}$$

$$\sigma_a = \sigma_c + R = 140 \text{ psi}$$

$$\sigma_b = \sigma_c - R = -2784 \text{ psi}$$

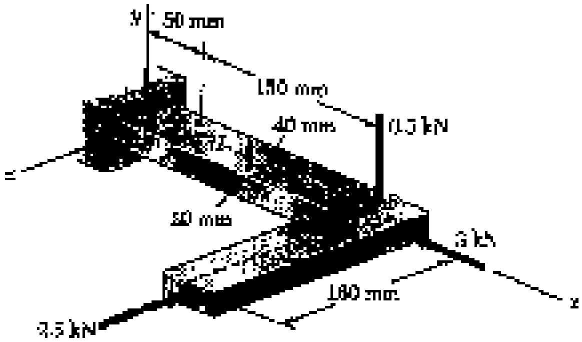
$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = \frac{(2)(625)}{2644} = 0.4728$$

$\theta_a = 12.7° \qquad \theta_b = 102.7°$

$$\tau_{max} = R = 1462 \text{ psi}$$

PROBLEM 8.57

8.57 Three forces are applied to the machine component *ABD* as shown. Knowing that the cross section containing point *H* is a 20 × 40-mm rectangle, determine the principal stresses and the maximum shearing stress at point *H*.



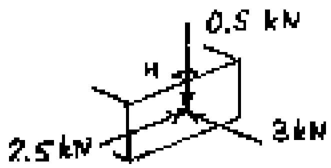
SOLUTION

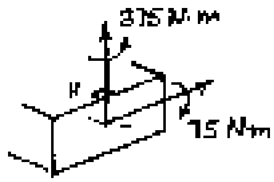
Equivalent force-couple system at section containing point H

$F_x = -3\text{ kN}, \quad F_y = -0.5\text{ kN}, \quad F_z = -2.5\text{ kN}$

$M_x = 0, \quad M_y = (0.150)(2500) = 375\text{ N}\cdot\text{m}$

$M_z = -(0.150)(500) = -75\text{ N}\cdot\text{m}$



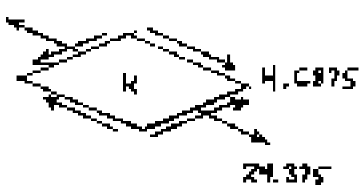


$A = (20)(40) = 800\text{ mm}^2 = 800\times10^{-6}\text{ m}^2$

$I_z = \frac{1}{12}(40)(20)^3 = 26.667\times10^3\text{ mm}^4 = 26.667\times10^{-9}\text{ m}^4$

$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z} = \frac{-3000}{800\times10^{-6}} - \frac{(-75)(10\times10^{-3})}{26.667\times10^{-9}} = 24.375\text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{|V_y|}{A} = \frac{3}{2}\cdot\frac{2500}{800\times10^{-6}} = 4.6875\text{ MPa}$$



$\sigma_c = \frac{1}{2}\sigma_H = 12.1875\text{ MPa}$

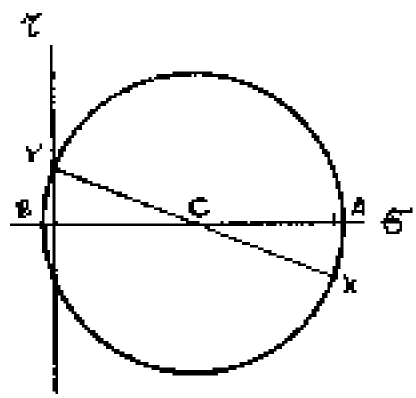
$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (4.6875)^2} = 13.0579\text{ MPa}$

$\sigma_a = \sigma_c + R = 25.2\text{ MPa}$ ◀

$\sigma_b = \sigma_c - R = -0.87\text{ MPa}$ ◀

$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(4.6875)}{24.375} = 0.3846$

$\theta_a = 10.5°, \quad \theta_b = 100.5°$

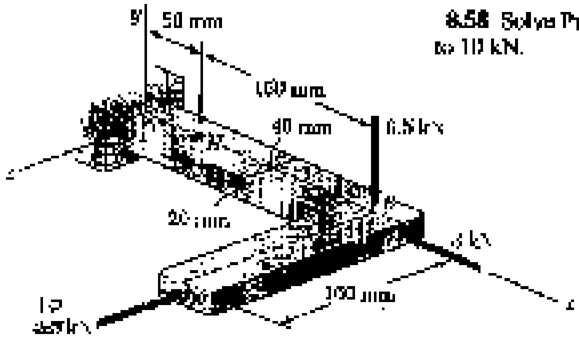


$\tau_{max} = R = 13.06\text{ MPa}$ ◀

PROBLEM 8.58

8.57 Three forces are applied to the machine component *ABD* as shown. Knowing that the cross section containing point *H* is a 20 × 40-mm rectangle, determine the principal stresses and the maximum shearing stress at point *H*.

8.58 Solve Prob. 8.57, assuming that the magnitude of the 2.5-kN force is increased to 10 kN.



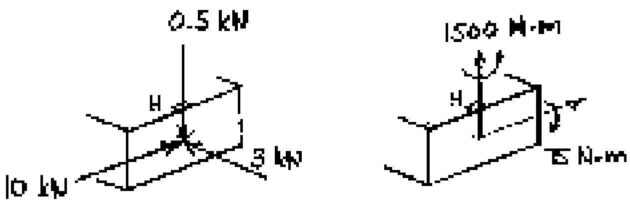
SOLUTION

Equivalent force-couple system at section containing point H.

$F_x = -3\ \text{kN}, \quad F_y = -0.5\ \text{kN}, \quad F_z = -10\ \text{kN}$

$M_x = 0, \quad M_y = (0.150)(10000) = 1500\ \text{N}\cdot\text{m}$

$M_z = -(0.150)(500) = -75\ \text{N}\cdot\text{m}$

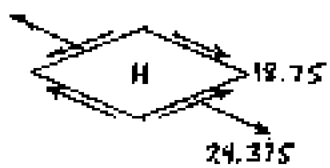


$A = (20)(40) = 800\ \text{mm}^2 = 800 \times 10^{-6}\ \text{m}^2$

$I_z = \frac{1}{12}(40)(20)^3 = 26.667 \times 10^3\ \text{mm}^4 = 26.667 \times 10^{-9}\ \text{m}^4$

$$\sigma_H = \frac{P}{A} - \frac{M_z y}{I_z} = \frac{-3000}{800 \times 10^{-6}} - \frac{(-75)(10 \times 10^{-3})}{26.667 \times 10^{-9}} = 24.375\ \text{MPa}$$

$$\tau_H = \frac{3}{2}\frac{|V_z|}{A} = \frac{3}{2} \cdot \frac{10000}{800 \times 10^{-6}} = 18.75\ \text{MPa}$$



$\sigma_c = \frac{1}{2}\sigma_H = 12.1875\ \text{MPa}$

$R = \sqrt{\left(\frac{24.375}{2}\right)^2 + (18.75)^2} = 22.363\ \text{MPa}$

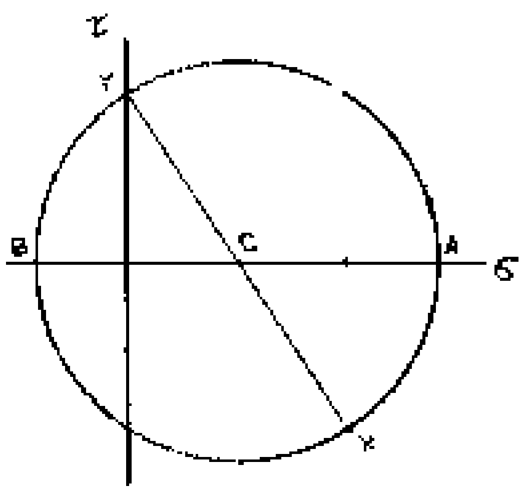
$\sigma_a = \sigma_c + R = 34.6\ \text{MPa}$

$\sigma_b = \sigma_c - R = -10.18\ \text{MPa}$

$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{(2)(18.75)}{24.375} = 1.5385$

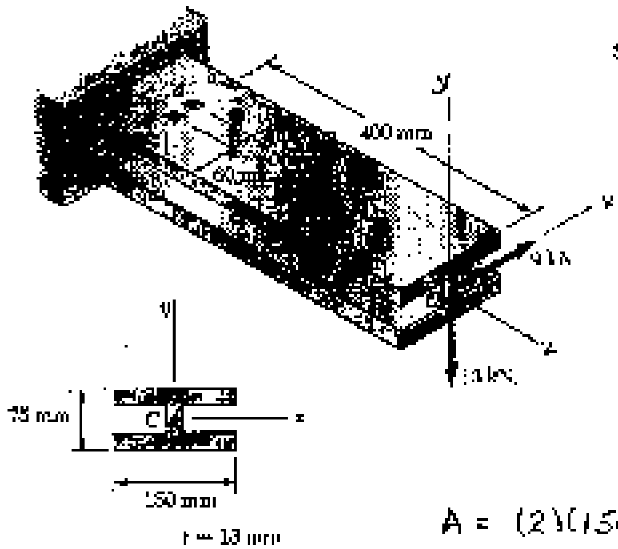
$\theta_a = 28.5°, \quad \theta_b = 118.5°$

$\tau_{max} = R = 22.4\ \text{MPa}$



PROBLEM 8.59

8.59 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points a and b.



SOLUTION

Equivalent force-couple system at section containing points a and b.

$F_x = 9\text{ kN}, \quad F_y = -13\text{ kN}, \quad F_z = 0$

$M_x = (0.400)(13\times10^3) = 5200\text{ N}\cdot\text{m}$

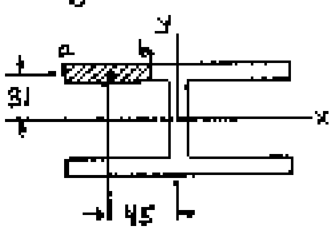
$M_y = 0.400(9\times10^3) = 3600\text{ N}\cdot\text{m}$

$M_z = 0$

$$A = (2)(150)(13) + (13)(75-26) = 4537\text{ mm}^2 = 4537\times10^{-6}\text{ m}^2$$

$$I_x = 2\left[\tfrac{1}{12}(150)(13)^3 + (150)(13)(37.5-6.5)^2\right] + \tfrac{1}{12}(13)(75-26)^3 = 3.9303\times10^6\text{ mm}^4 = 3.9303\times10^{-6}\text{ m}^4$$

$$I_y = 2\cdot\tfrac{1}{12}(13)(150)^3 + \tfrac{1}{12}(75-26)(13)^3 = 7.3215\times10^6\text{ mm}^4 = 7.3215\times10^{-6}\text{ m}^4$$



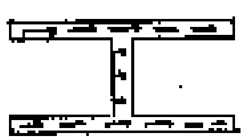
For point a $\quad Q_x = 0 \qquad Q_y = 0$

For point b $\quad A^* = (60)(13) = 780\text{ mm}^2$

$\bar{x} = -45\text{ mm} \qquad \bar{y} = 31\text{ mm}$

$Q_x = A^*\bar{y} = 24.18\times10^3\text{ mm}^3 = 24.18\times10^{-6}\text{ m}^3$

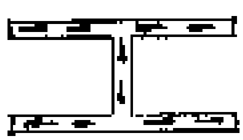
$Q_y = A^*\bar{x} = -35.1\times10^{3}\text{ mm}^3 = -35.1\times10^{-6}\text{ m}^3$



At point $\underline{a}$ $\quad \sigma_a = \dfrac{M_x y}{I_x} - \dfrac{M_y x}{I_y}$

$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(-75\times10^{-3})}{7.3215\times10^{-6}} = 86.5\text{ MPa}$$

$$\tau_a = 0$$



At point $\underline{b}$ $\quad \sigma_b = \dfrac{M_x y}{I_x} - \dfrac{M_y x}{I_y}$

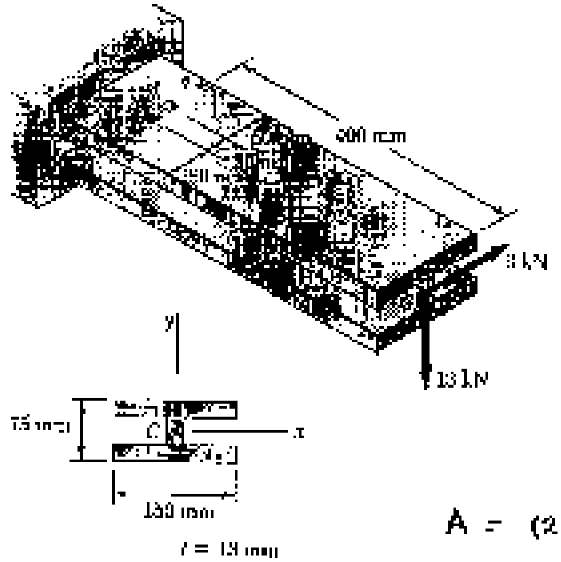
$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(-15\times10^{-3})}{7.3215\times10^{-6}} = 57.0\text{ MPa}$$

$$\tau_b = \frac{|V_x||Q_y|}{I_y t} + \frac{|V_y||Q_x|}{I_x t} = \frac{(9\times10^3)(35.1\times10^{-6})}{(7.3215\times10^{-6})(13\times10^{-3})} + \frac{(13\times10^3)(24.18\times10^{-6})}{(3.9303\times10^{-6})(13\times10^{-3})}$$

$$= 3.32\text{ MPa} + 6.15\text{ MPa} = 9.47\text{ MPa}$$

PROBLEM 8.60

8.60 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *d* and *e*.



SOLUTION

Equivalent force-couple system at section containing points a and b.

$F_x = 9 \text{ kN}, \quad F_y = -13 \text{ kN}, \quad F_z = 0$

$M_x = (0.400)(13\times10^3) = 5200 \text{ N}\cdot\text{m}$

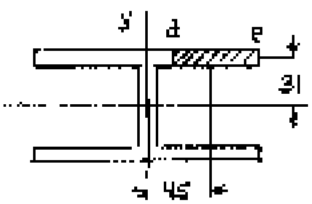
$M_y = (0.400)(9\times10^3) = 3600 \text{ N}\cdot\text{m}$

$M_z = 0$

$A = (2)(150)(13) + (13)(75-26) = 4537 \text{ mm}^2$
$\quad = 4537\times10^{-6} \text{ m}^2$

$I_x = 2\left[\frac{1}{12}(150)(13)^3 + (150)(13)(37.5-6.5)^2\right] + \frac{1}{12}(13)(75-26)^3 = 3.9303\times10^6 \text{ mm}^4$
$\quad = 3.9303\times10^{-6} \text{ m}^4$

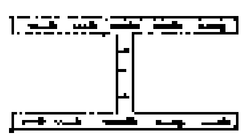
$I_y = 2\left[\frac{1}{12}(13)(150)^3\right] + \frac{1}{12}(75-26)(13)^3 = 7.3215\times10^6 \text{ mm}^4 = 7.3215\times10^{-6} \text{ m}^4$



For point d $\quad A^* = (60)(13) = 780 \text{ mm}^2$
$\bar{x} = 45 \text{ mm} \qquad \bar{y} = 31 \text{ mm}$

$Q_x = A^*\bar{y} = 24.18\times10^3 \text{ mm}^3 = 24.18\times10^{-6} \text{ m}^3$

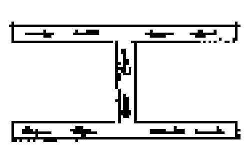
$Q_y = A^*\bar{x} = 35.1\times10^3 \text{ mm}^3 = 35.1\times10^{-6} \text{ m}^3$



For point e $\quad Q_x = 0, \quad Q_y = 0$

At point d $\quad \sigma_d = \dfrac{M_x y}{I_x} - \dfrac{M_y x}{I_y}$

$$= \frac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \frac{(3600)(15\times10^{-3})}{7.3215\times10^{-6}} = 42.2 \text{ MPa}$$



Due to V_x $\quad \tau_d = \dfrac{|V_x||Q_y|}{I_y t} = \dfrac{(9000)(35.1\times10^{-6})}{(7.3215\times10^{-6})(13\times10^{-3})} = 3.32 \text{ MPa}$

Due to V_y $\quad \tau_d = \dfrac{|V_y||Q_x|}{I_x t} = \dfrac{(13000)(24.18\times10^{-6})}{(3.9303\times10^{-6})(13\times10^{-3})} = 6.15 \text{ MPa}$

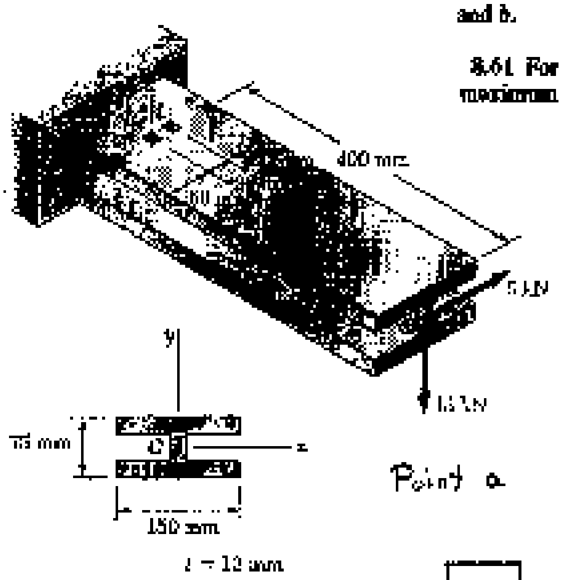
Net $\quad \tau_d = 2.83 \text{ MPa}$

At point e $\quad \sigma_e = \dfrac{M_x y}{I_x} - \dfrac{M_y x}{I_y} = \dfrac{(5200)(37.5\times10^{-3})}{3.9303\times10^{-6}} - \dfrac{(3600)(75\times10^{-3})}{7.3215\times10^{-6}}$

$$= 12.74 \text{ MPa}$$

$\tau_e = 0$

PROBLEM 8.61



8.59 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points *a* and *b*.

8.61 For the beam and loading of Prob. 8.59, determine the principal stresses and the maximum shearing stress at points *a* and *b*.

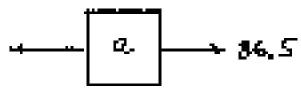
SOLUTION

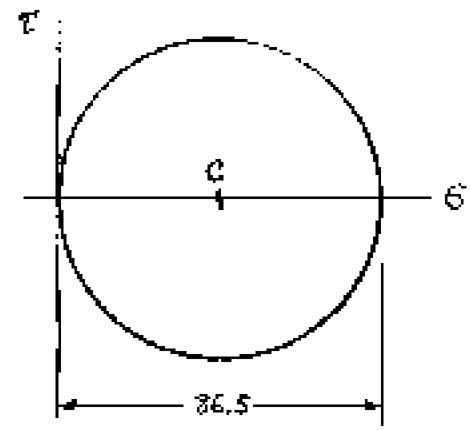
From the solution of Prob. 8.59

$\sigma_a = 86.5$ MPa $\qquad \tau_a = 0$

$\sigma_b = 57.0$ MPa $\qquad \tau_b = 9.47$ MPa

Point a





$\sigma_c = \frac{86.5}{2}$ MPa $= 43.25$ MPa

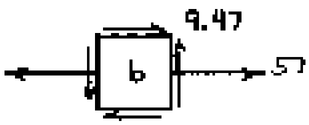
$R = \frac{86.5}{2}$ MPa $= 43.25$ MPa

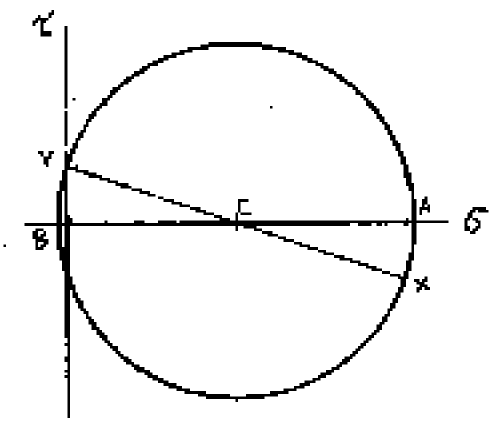
$\sigma_{max} = \sigma_c + R = 86.5$ MPa

$\sigma_{min} = \sigma_c - R = 0$

$\tau_{max} = R = 43.3$ MPa

Point b





$\sigma_c = \frac{57.0}{2} = 28.5$ MPa

$R = \sqrt{\left(\frac{57.0}{2}\right)^2 + (9.47)^2} = 30.03$ MPa

$\sigma_{max} = \sigma_c + R = 58.5$ MPa

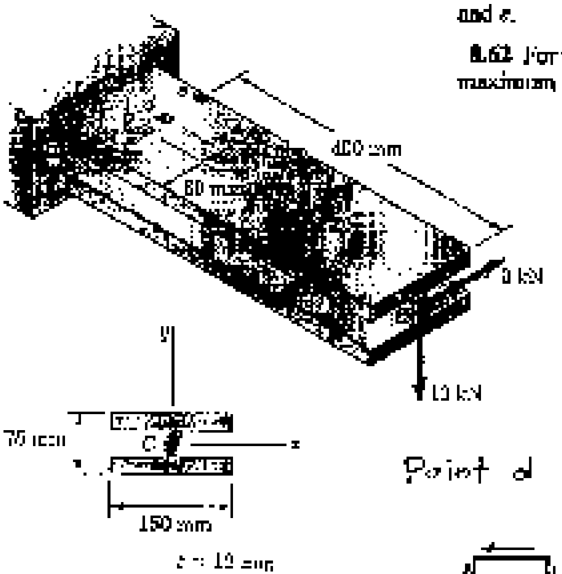
$\sigma_{min} = \sigma_c - R = -1.53$ MPa

$\tau_{max} = R = 30.0$ MPa

PROBLEM 8.62

8.60 Three steel plates, each 13 mm thick, are welded together to form a cantilever beam. For the loading shown, determine the normal and shearing stresses at points d and e.

8.62 For the beam and loading of Prob. 8.60, determine the principal stresses and the maximum shearing stress at points d and e.



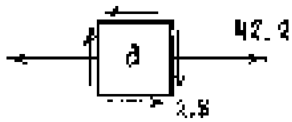
SOLUTION

From the solution of Prob. 8.60

$\sigma_d = 42.2$ MPa $\qquad \tau_d = 2.83$ MPa

$\sigma_e = 12.74$ MPa $\qquad \tau_e = 0$

Point d



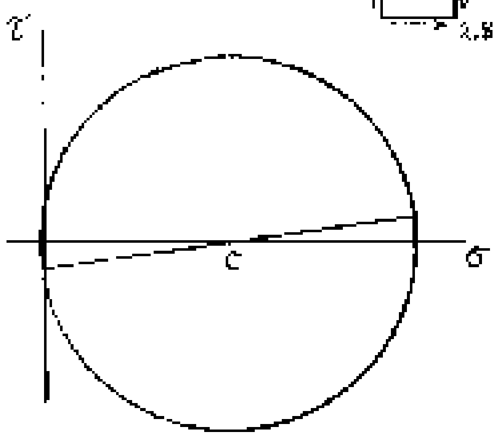
$$\sigma_c = \frac{42.2}{2} = 42.2 \text{ MPa}$$

$$R = \sqrt{\left(\frac{42.2}{2}\right)^2 + (2.83)^2} = 21.29 \text{ MPa}$$

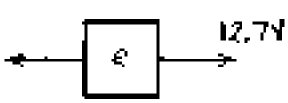
$\sigma_{max} = \sigma_c + R = 42.4$ MPa ◄

$\sigma_{min} = \sigma_c - R = -0.19$ MPa ◄

$\tau_{max} = R = 21.3$ MPa ◄



Point e



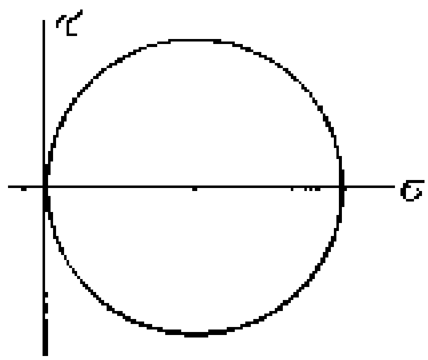
$$\sigma_c = \frac{12.74}{2} = 6.37 \text{ MPa}$$

$$R = \frac{12.74}{2} = 6.37 \text{ MPa}$$

$\sigma_{max} = \sigma_c + R = 12.74$ MPa ◄

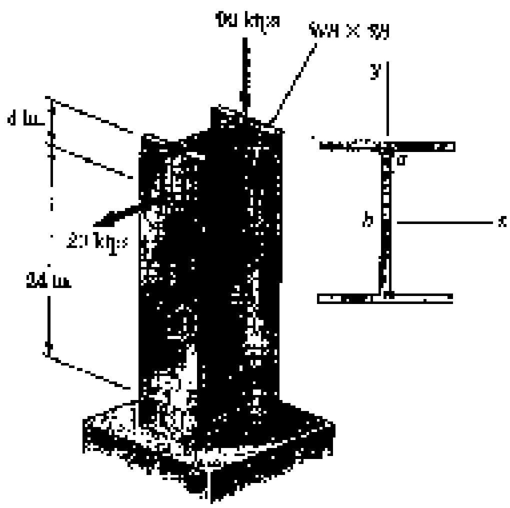
$\sigma_{min} = \sigma_c - R = 0$ ◄

$\tau_{max} = R = 6.37$ MPa ◄



PROBLEM 8.63

8.63 Two forces are applied to a W8 × 28 rolled-steel beam as shown. Determine the principal stresses, principal planes, and maximum shearing stress at point *a*.



SOLUTION

For W8 × 28 rolled steel section

$A = 8.25\ \text{in}^2,\quad d = 8.06\ \text{in},\quad b_f = 6.535\ \text{in}$

$t_f = 0.465\ \text{in},\quad t_w = 0.285\ \text{in},\quad I_x = 98.0\ \text{in}^4$

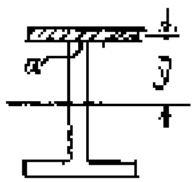
At the section containing points a and b.

$P = -90\ \text{kips},\qquad V = 20\ \text{kips}$

$M = (20)(24) - (4.03)(90) = 117.3\ \text{kip}\cdot\text{in}$

At point a $\qquad y = \frac{1}{2}d - t_f = 4.03 - 0.465 = 3.565\ \text{in}$

$$\sigma = \frac{P}{A} + \frac{My}{I} = -\frac{90}{8.25} - \frac{(-117.3)(3.565)}{98.0} = -6.642\ \text{ksi}$$

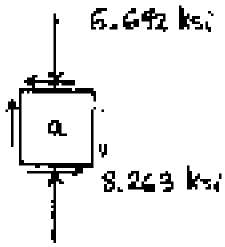


$\bar{y} = \frac{1}{2}d - \frac{1}{2}t_f = 4.03 - 0.2325 = 3.7975\ \text{in}$

$A_f = b_f t_f = (6.535)(0.465) = 3.0388\ \text{in}^2$

$Q_a = A_f\bar{y} = 11.540\ \text{in}^3$

$$\tau = \frac{VQ_a}{It_w} = \frac{(20)(11.540)}{(98.0)(0.285)} = 8.263\ \text{ksi}$$



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-8.263)}{0 + 6.642} = -2.4881$$

$\theta_a = -34.1°,\quad \theta_b = 55.9°$ ◄

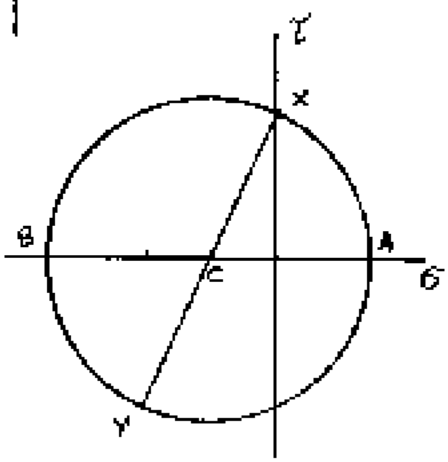
$\sigma_c = -\frac{6.642}{2} = -3.321\ \text{ksi}$

$R = \sqrt{\left(\frac{6.642}{2}\right)^2 + (8.263)^2} = 8.905\ \text{ksi}$

$\sigma_a = \sigma_c + R = 5.58\ \text{ksi}$ ◄

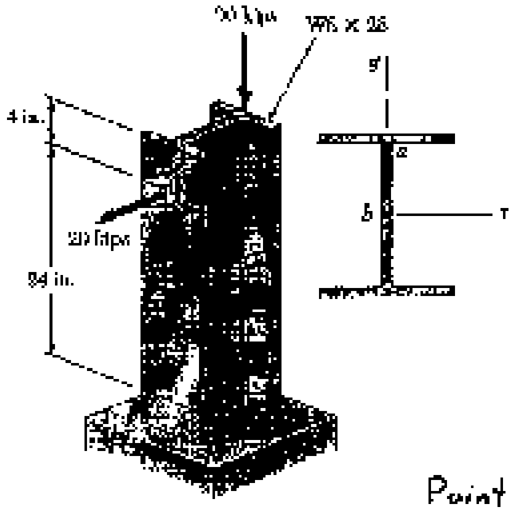
$\sigma_b = \sigma_c - R = -12.23\ \text{ksi}$ ◄

$\tau_{max} = R = 8.91\ \text{ksi}$ ◄



PROBLEM 8.64

8.64 Two forces are applied to a W8 × 28 rolled-steel beam as shown. Determine the principal stresses, principal planes, and maximum shearing stress at point *b*.



SOLUTION

For W 8 × 28 rolled steel section

$A = 8.25\ \text{in}^2,\quad d = 8.06\ \text{in},\quad b_f = 6.535\ \text{in}$

$t_f = 0.465\ \text{in},\quad t_w = 0.285\ \text{in},\quad I_x = 98.0\ \text{in}^4$

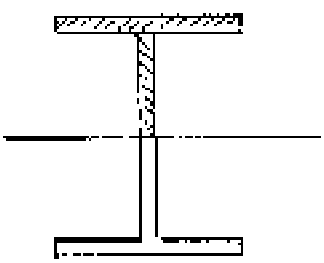
At the section containing points a and b.

$P = -90\ \text{kips},\qquad V = 20\ \text{kips}$

$M = (20)(24) - (4.03)(90) = -117.8\ \text{kip}\cdot\text{in.}$

Point b lies on the neutral axis of bending

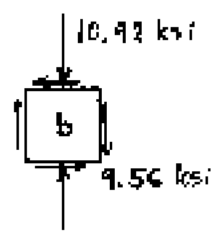
At point b $\qquad \sigma = \dfrac{P}{A} = \dfrac{-90}{8.25} = -10.92\ \text{ksi}$



Part	$A\ (\text{in}^2)$	$\bar{y}\ (\text{in})$	$A\bar{y}\ (\text{in}^3)$
Flange	3.0388	3.7975	11.540
Half-web	1.0161	1.7825	1.811
Σ			13.351

$Q_b = 13.351\ \text{in}^3$

$\tau = \dfrac{VQ_b}{It_w} = \dfrac{(20)(13.351)}{(98.0)(0.285)} = 9.56\ \text{ksi}$



$\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{(2)(-9.56)}{0 + 10.92} = -1.7509$

$\theta_a = -30.1^\circ \qquad \theta_b = 59.9^\circ$

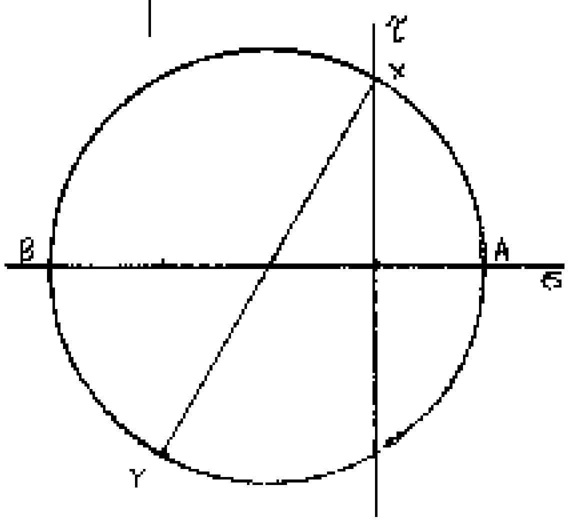
$\sigma_c = -\dfrac{10.92}{2} = -5.46\ \text{ksi}$

$R = \sqrt{\left(\dfrac{10.92}{2}\right)^2 + (9.56)^2} = 11.01\ \text{ksi}$

$\sigma_{max} = \sigma_c + R = 5.55\ \text{ksi}$

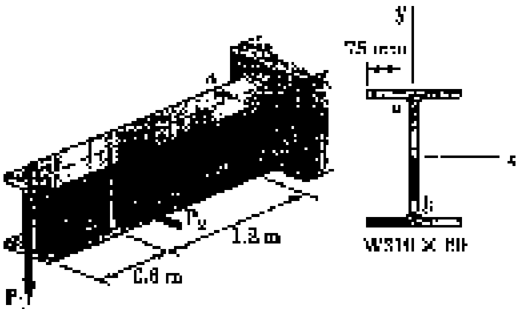
$\sigma_{min} = \sigma_c - R = -16.47\ \text{ksi}$

$\tau_{max} = R = 11.01\ \text{ksi}$



PROBLEM 8.65

8.65 Two forces P_1 and P_2 are applied as shown in directions perpendicular to the longitudinal axis of a W310 × 60 beam. Knowing that $P_1 = 25$ kN and $P_2 = 24$ kN, determine the principal stresses and the maximum shearing stress at point a.



SOLUTION

At the section containing points a and b

$$M_x = (1.8)(25) = 45 \text{ kN}\cdot\text{m}$$

$$M_y = -(1.2)(24) = -28.8 \text{ kN}\cdot\text{m}$$

$$V_x = -24 \text{ kN} \qquad V_y = -25 \text{ kN}$$

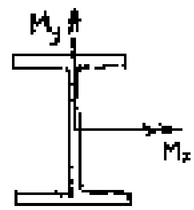
For W 310 × 60 rolled steel section

$d = 303$ mm, $b_f = 203$ mm, $t_f = 13.1$ mm, $t_w = 7.5$ mm

$I_x = 129 \times 10^6 \text{ mm}^4 = 129 \times 10^{-6} \text{ m}^4$, $I_y = 18.3 \times 10^6 \text{ mm}^4 = 18.3 \times 10^{-6} \text{ m}^4$

Normal stress at point a $\quad x = -\frac{b_f}{2} + 75 = -26.5$ mm

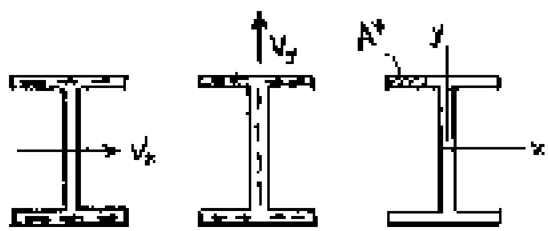
$$y = \tfrac{1}{2} d = 151.5 \text{ mm}$$



$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(45 \times 10^3)(151.5 \times 10^{-3})}{129 \times 10^{-6}} - \frac{(-28.8 \times 10^3)(-26.5 \times 10^{-3})}{18.3 \times 10^{-6}}$$

$$= 52.849 \text{ MPa} - 41.705 \text{ MPa} = 11.144 \text{ MPa}$$

Shearing stress at point a



$$\tau_{xz} = -\frac{V_x A^* \bar{x}}{I_y t_f} - \frac{V_y A^* \bar{y}}{I_x t_f}$$

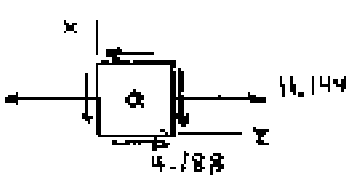
$$A^* = (75 \times 10^{-3})(13.1 \times 10^{-3}) = 982.5 \times 10^{-6} \text{ m}^2$$

$$\bar{x} = -\frac{b_f}{2} + \frac{75}{2} = -64 \text{ mm}$$

$$\bar{y} = \frac{d}{2} - \frac{t_f}{2} = 144.95 \text{ mm}$$

$$\tau_{xz} = -\frac{(-24 \times 10^3)(982.5 \times 10^{-6})(-64 \times 10^{-3})}{(18.3 \times 10^{-6})(13.1 \times 10^{-3})} - \frac{(-25 \times 10^3)(982.5 \times 10^{-6})(144.95 \times 10^{-3})}{(129 \times 10^{-6})(13.1 \times 10^{-3})}$$

$$= -6.295 \text{ MPa} + 2.107 \text{ MPa} = -4.188 \text{ MPa}$$



$$\sigma_{ave} = \frac{11.144}{2} = 5.572 \text{ MPa}$$

$$R = \sqrt{\left(\frac{11.144}{2}\right)^2 + (4.188)^2} = 6.970 \text{ MPa}$$

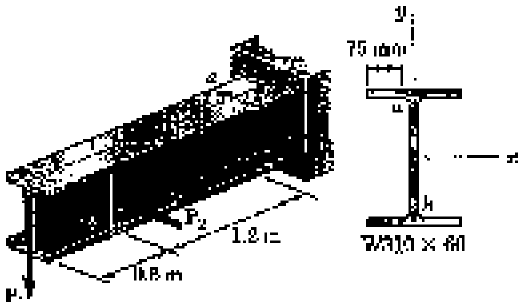
$$\sigma_{max} = \sigma_{ave} + R = 12.54 \text{ MPa} \quad \blacktriangleleft$$

$$\sigma_{min} = \sigma_{ave} - R = -1.40 \text{ MPa} \quad \blacktriangleleft$$

$$\tau_{max} = R = 6.97 \text{ MPa} \quad \blacktriangleleft$$

PROBLEM 8.66

8.66 Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$ are applied as shown in directions perpendicular to the longitudinal axis of a W310 × 60 beam. Knowing that $P_1 = 25$ kN and $P_2 = 24$ kN, determine the principal stresses and the maximum shearing stress at point b.



SOLUTION

At the section containing points a and b

$M_x = (1.8)(25) = 45$ kN·m

$M_y = -(1.2)(24) = -28.8$ kN·m

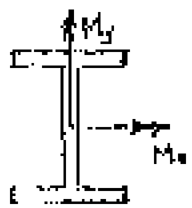
$V_x = -24$ kN, $\quad V_y = -25$ kN

For W310×60 rolled steel section

$d = 303$ mm, $\quad b_f = 203$ mm, $\quad t_f = 13.1$ mm, $\quad t_w = 7.5$ mm

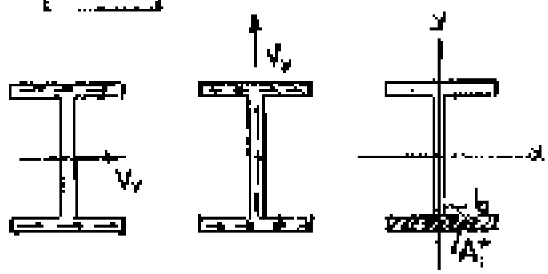
$I_x = 129\times10^6\ \text{mm}^4 = 129\times10^{-6}\ \text{m}^4$, $\quad I_y = 18.3\times10^6\ \text{mm}^4 = 18.3\times10^{-6}\ \text{m}^4$

Normal stress at point b $\qquad x = 0$, $\quad y = -\frac{1}{2}d + t_f = -138.4$ mm.



$$\sigma_z = \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = \frac{(45\times10^3)(-138.4\times10^{-3})}{129\times10^{-6}} - 0$$
$$= -48.28\ \text{MPa}$$

Shearing stress at point b.

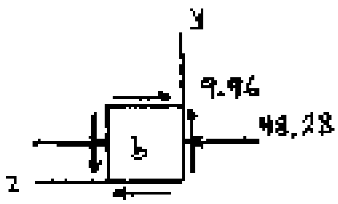


$$\tau_{yz} = -\frac{V_y A^* \bar{y}}{I_x t_w}$$

$A^* = A_f = b_f t_f = 2659\ \text{mm}^2$
$= 2659\times10^{-6}\ \text{m}^2$

$\bar{x} = 0$, $\quad \bar{y} = -\frac{1}{2}d + \frac{1}{2}t_f = -144.95$ mm

$$\tau_{yz} = -\frac{(-25\times10^3)(2659\times10^{-6})(-144.95\times10^{-3})}{(129\times10^{-6})(7.5\times10^{-3})} = -9.96\ \text{MPa}$$



$\sigma_{ave} = -\frac{48.28}{2} = -24.14$ MPa

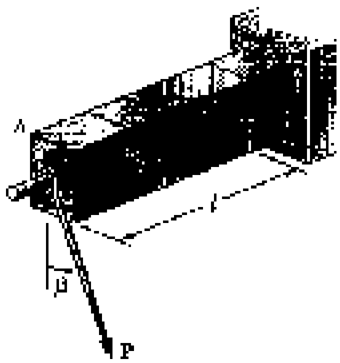
$R = \sqrt{\left(\frac{48.28}{2}\right)^2 + (9.96)^2} = 26.11$

$\sigma_{max} = \sigma_{ave} + R = 1.97$ MPa ◄

$\sigma_{min} = \sigma_{ave} - R = -50.3$ MPa ◄

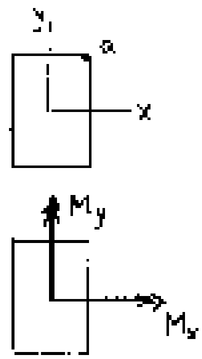
$\tau_{max} = R = 26.1$ MPa ◄

PROBLEM 8.67



8.67 A force **P** is applied to a cantilever beam by means of a cable attached to a bolt located at the center of the free end of the beam. Knowing that **P** acts in a direction perpendicular to the longitudinal axis of the beam, determine (*a*) the normal stress at point *a* in terms of *P*, *b*, *h*, *l*, and *β*, (*b*) the values of *β* for which the normal stress at *a* is zero.

SOLUTION



$$I_x = \frac{1}{12} b h^3 \qquad I_y = \frac{1}{12} h b^3$$

$$\sigma = \frac{M_x (h/2)}{I_x} - \frac{M_y (b/2)}{I_y} = \frac{6 M_x}{b h^2} - \frac{6 M_y}{h b^2}$$

$$\vec{P} = P \sin\beta\, \vec{i} - P \cos\beta\, \vec{j} \qquad \vec{r} = l\, \vec{k}$$

$$\vec{M} = \vec{r} \times \vec{P} = l\,\vec{k} \times (P\sin\beta\,\vec{i} - P\cos\beta\,\vec{j}) = Pl\cos\beta\,\vec{i} + Pl\sin\beta\,\vec{j}$$

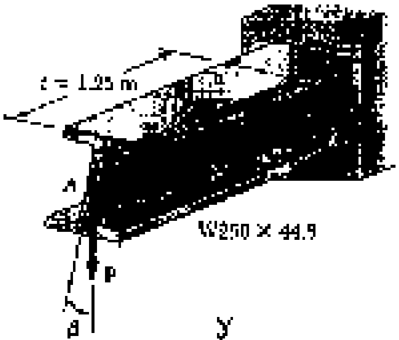
$$M_x = Pl\cos\beta \qquad M_y = Pl\sin\beta$$

(a) $$\sigma = \frac{6Pl\cos\beta}{b h^2} - \frac{6Pl\sin\beta}{h b^2} = \frac{6Pl}{bh}\left[\frac{\cos\beta}{h} - \frac{\sin\beta}{b}\right]$$ ◄

(b) $$\sigma = 0 \qquad \frac{\cos\beta}{h} - \frac{\sin\beta}{b} = 0 \qquad \tan\beta = \frac{b}{h}$$

$$\beta = \tan^{-1}\left(\frac{b}{h}\right)$$ ◄

PROBLEM 8.68



8.68 A vertical force **P** is applied at the center of the free end of cantilever beam AB. (*a*) If the beam is installed with the web vertical ($\beta = 0$) and with its longitudinal axis AB horizontal, determine the magnitude of the force **P** for which the normal stress at point a is +120 MPa. (*b*) Solve part *a*, assuming that the beam is installed with $\beta = 3°$.

SOLUTION

For W 250 × 44.8 rolled steel section

$S_x = 535 \times 10^3 \text{ mm}^3 = 535 \times 10^{-6} \text{ m}^3$

$S_y = 95.0 \times 10^3 \text{ mm}^3 = 95.0 \times 10^{-6} \text{ m}^3$

At the section containing point a

$M_x = Pl \cos\beta \quad , \quad M_y = Pl \sin\beta$

Stress at a

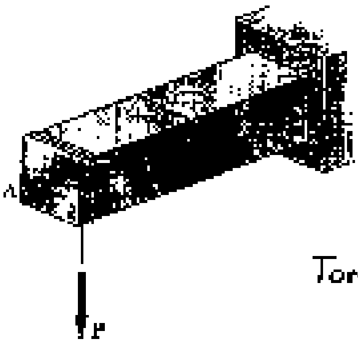
$$\sigma = \frac{M_x}{S_x} + \frac{M_y}{S_y} = \frac{Pl\cos\beta}{S_x} + \frac{Pl\sin\beta}{S_y}$$

Allowable load $\quad P_{all} = \dfrac{\sigma_{all}}{l}\left[\dfrac{\cos\beta}{S_x} + \dfrac{\sin\beta}{S_y}\right]^{-1}$

(a) $\beta = 0$ $\quad P_{all} = \dfrac{120 \times 10^6}{1.25}\left[\dfrac{1}{535 \times 10^{-6}} + 0\right]^{-1} = 51.4 \times 10^3 \text{ N} = 51.4 \text{ kN}$ ◄

(b) $\beta = 3°$ $\quad P_{all} = \dfrac{120 \times 10^6}{1.25}\left[\dfrac{\cos 3°}{535 \times 10^{-6}} + \dfrac{\sin 3°}{95.0 \times 10^{-6}}\right]^{-1} = 39.7 \text{ kN}$ ◄

PROBLEM 8.69



*8.69 A 500-lb force **P** is applied to a wire that is wrapped around the bar AB as shown. Knowing that the cross section of the bar is a square of side $d = 0.75$ in., determine the principal stresses and the maximum shearing stress at point a.

SOLUTION

Bending: Point a lies on the neutral axis.

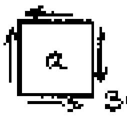
$$\sigma = 0$$

Torsion: $\tau = \frac{T}{c_1 a b^2}$ where $a = b = d$ and $c_1 = 0.208$ for a square section.

Since $T = \frac{Pd}{2}$ $\tau = \frac{P}{0.416\, d^2} = 2.404 \frac{P}{d^2}$

Transverse shear: $V = P$ $\tau = \frac{3}{2}\frac{V}{A} = 1.5 \frac{P}{d^2}$

Using superposition: $\tau = 3.904 \frac{P}{d^2} = 3.904 \frac{500}{(0.75)^2} = 3470$ psi



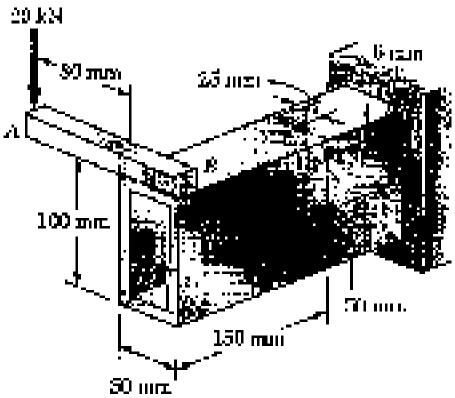
$\sigma_{max} = 3470$ psi ◄

$\sigma_{min} = -3470$ psi ◄

$\tau_{max} = 3470$ psi ◄

PROBLEM 8.70

*8.70 A vertical 20-kN force is applied to end A of the bar AB, which is welded to an extruded aluminum tube. Knowing that the tube has a uniform wall thickness of 6 mm, determine the shearing stress at points a, b, and c.

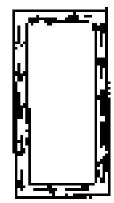


SOLUTION

$$I = \tfrac{1}{12}(50)(100)^3 - \tfrac{1}{12}(38)(88)^3 = 2.0087\times10^6\ \text{mm}^4 = 2.0087\times10^{-6}\ \text{m}^4$$

Torsion: $T = (20\times10^3)(80+25)(10^{-3}) = 2100\ \text{N}\cdot\text{m}$

$$Q = (44)(94) = 4.136\times10^3\ \text{mm}^2 = 4.136\times10^{-3}\ \text{m}^2$$

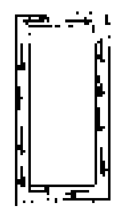


For points a, b, and c

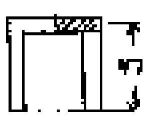
$$\tau = \frac{T}{2tQ} = \frac{2100}{(2)(6\times10^{-3})(4.136\times10^{-3})} = 42.31\ \text{MPa}$$

Transverse shear: $V = 20\times10^3\ \text{N}$

Point c – on symmetry axis $\tau = 0$



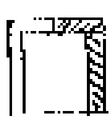
Point b



$$Q_b = (25)(6)(47) = 7.05\times10^3\ \text{mm}^3 = 7.05\times10^{-6}\ \text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(20\times10^3)(7.05\times10^{-6})}{(2.0087\times10^{-6})(6\times10^{-3})} = 11.70\ \text{MPa}$$

Point a



$$Q_a = Q_b + (6)(44)(22) = 12.858\times10^3\ \text{mm}^3 = 12.858\times10^{-6}\ \text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(20\times10^3)(12.858\times10^{-6})}{(2.0087\times10^{-6})(6\times10^{-3})} = 21.34\ \text{MPa}$$

Net shearing stress:

Point a $\quad \tau = 42.31 - 0 = 42.3\ \text{MPa}$

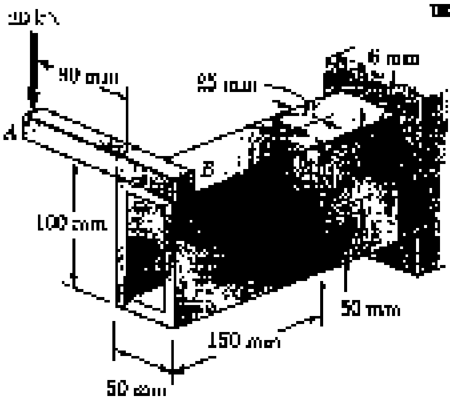
Point b $\quad \tau = 42.31 - 11.70 = 30.6\ \text{MPa}$

Point c $\quad \tau = 42.31 - 21.34 = 21.0\ \text{MPa}$

PROBLEM 8.71

*8.70 A vertical 20-kN force is applied to end A of the bar AB, which is welded to an extruded aluminum tube. Knowing that the tube has a uniform wall thickness of 6 mm, determine the shearing stress at points a, b, and c.

*8.71 For the tube and loading of Prob. 8.70, determine the principal stresses and the maximum shearing stress at point b.



SOLUTION

Bending: $M = (20\times10^3)(150\times10^{-3}) = 3000\ \text{N}\cdot\text{m}$

$$I = \tfrac{1}{12}(50)(100)^3 - \tfrac{1}{12}(38)(88)^3 = 2.0087\times10^6\ \text{mm}^4 = 2.0087\times10^{-6}\ \text{m}^4$$

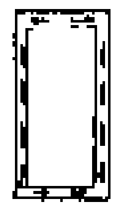
$$\sigma = \frac{My}{I} = \frac{(3000)(44\times10^{-3})}{2.0087\times10^{-6}} = 65.7\ \text{MPa}$$

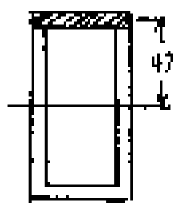
Torsion: $T = (20\times10^3)(80+25)(10^{-3}) = 2100\ \text{N}\cdot\text{m}$

$$a = (44)(94) = 4.136\times10^3\ \text{mm}^2 = 4.136\times10^{-3}\ \text{m}^2$$

$$\tau = \frac{T}{2ta} = \frac{2100}{(2)(6\times10^{-3})(4.136\times10^{-3})} = 42.31\ \text{MPa}$$

Transverse shear: $V = 20\times10^3\ \text{N}$

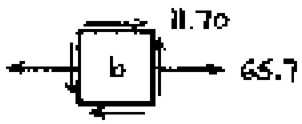




$$Q = (50)(6)(47) = 14.1\times10^3\ \text{mm}^3 = 14.1\times10^{-6}\ \text{m}^3$$

$$\tau = \frac{VQ}{It} = \frac{(20\times10^3)(14.1\times10^{-6})}{(2.0087\times10^{-6})(12\times10^{-3})} = 11.70\ \text{MPa}$$

Net shearing stress $\tau = 42.31 - 11.70 = 30.6\ \text{MPa}$



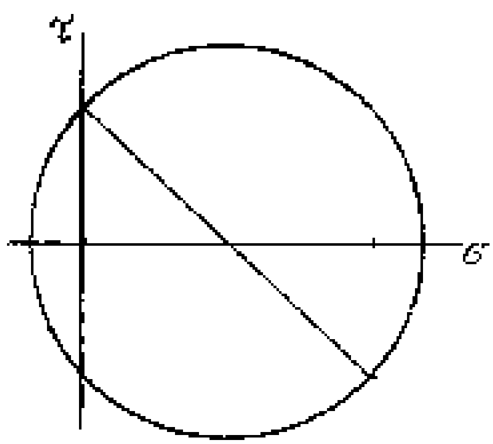
$$\sigma_c = \tfrac{1}{2}\sigma = 32.85\ \text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{65.7}{2}\right)^2 + (30.6)^2} = 44.89\ \text{MPa}$$

$$\sigma_{max} = \sigma_c + R = 77.7\ \text{MPa}$$

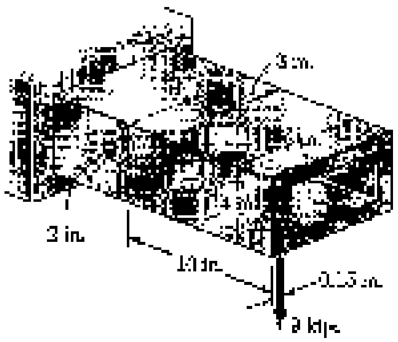
$$\sigma_{min} = \sigma_c - R = -12.04\ \text{MPa}$$

$$\tau_{max} = R = 44.9\ \text{MPa}$$



PROBLEM 8.72

*8.72 Knowing that the structural tube shown has a uniform wall thickness of 0.3 in., determine the principal stresses, principal planes, and maximum shearing stress at (a) point H, (b) point K.



SOLUTION

At the section containing points H and K

$V = 9$ kips $\qquad M = (9)(10) = 90$ kip·in.

$T = (9)(3 - 0.15) = 25.65$ kip·in

Torsion:

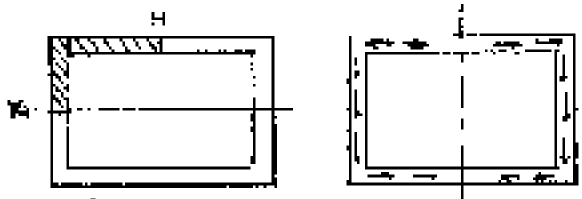
$\mathcal{Q} = (5.7)(3.7) = 21.09\ \text{in}^2$

$$\tau = \frac{T}{2t\mathcal{Q}} = \frac{25.65}{(2)(0.3)(21.09)} = 2.027\ \text{ksi}$$

Transverse shear:

$Q_H = 0$

$Q_K = (3)(2)(1) - (2.7)(1.7)(0.85)$
$= 2.0985\ \text{in}^3$

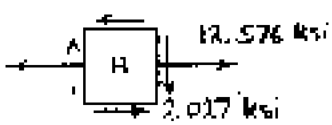


$I = \frac{1}{12}(6)(4)^3 - \frac{1}{12}(5.4)(3.4)^3 = 14.3132\ \text{in}^4$

$\tau_H = 0 \qquad \tau_K = \frac{VQ_K}{It} = \frac{(9)(2.0985)}{(14.3132)(0.3)} = 4.398$ ksi

Bending: $\quad \sigma_H = \frac{Mc}{I} = \frac{(90)(2)}{14.3132} = 12.576$ ksi , $\qquad \sigma_K = 0$

(a) Point H:



$\sigma_c = \frac{12.576}{2} = 6.288$ ksi

$R = \sqrt{\left(\frac{12.576}{2}\right)^2 + (2.027)^2} = 6.607$ ksi

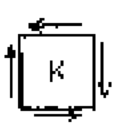
$\sigma_{max} = \sigma_c + R = 12.90$ ksi

$\sigma_{min} = \sigma_c - R = -0.32$ ksi

$\tan 2\theta_p = \frac{2\tau}{\sigma} = -0.3224 \qquad \theta_p = -8.9°, 81.1°$

$\tau_{max} = R = 6.61$ ksi

(b) Point K: $\qquad \sigma = 0 \qquad \tau = 2.027 + 4.398 = 6.425$ ksi



$\sigma_{max} = 6.43$ ksi

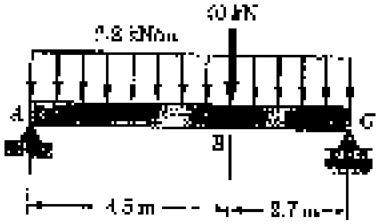
$\sigma_{min} = -6.43$ ksi

$\theta_p = \pm 45°$

$\tau_{max} = 6.43$ ksi

PROBLEM 8.73

8.73 (*a*) Knowing that $\sigma_{all} = 165$ MPa and $\tau_{all} = 100$ MPa, select the most economical wide-flange shape that should be used to support the loading shown. (*b*) Determine the values to be expected for σ_m, τ_m and the principal stress σ_{max} at the junction of a flange and the web of the selected beam.



$+\Sigma M_C = 0$

$-7.2\,R_A + (2.2)(7.2)(3.6) + (40)(2.7) = 0$

$R_A = 22.92$ kN

$V_A = R_A = 22.92$ kN

$V_B^- = 22.92 - (2.2)(4.5) = 13.02$ kN

$V_B^+ = 13.02 - 40 = -26.98$ kN

$V_C = -26.98 - (2.2)(2.7) = -32.92$ kN

$M_A = 0$

$M_B = 0 + \frac{1}{2}(22.92 + 13.02)(4.5) = 80.865$ kN·m

$M_C = 0$

$$S_{min} = \frac{|M|_{max}}{\sigma_{all}} = \frac{80.865\times10^3}{165\times10^6} = 490\times10^{-6}\ \text{m}^3 = 490\times10^3\ \text{mm}^3$$

Shape	S (10^3 mm^3)
W 360 × 39	578
W 310 × 38.7	549 ←
W 250 × 44.8	535
W 200 × 52	512

Try W 310 × 38.7 ⟵

$d = 310$ mm $\qquad t_f = 9.7$ mm

$t_w = 5.8$ mm

$$\sigma_m = \frac{M_B}{S} = \frac{80.865\times10^3}{549\times10^{-6}} = 147.3\ \text{MPa}$$ ⟵

$$\tau_m = \frac{|V|_{max}}{d\,t_w} = \frac{32.92\times10^3}{(310\times10^{-3})(5.8\times10^{-3})} = 18.31\ \text{MPa}$$ ⟵

$c = \frac{1}{2}d = 155$ mm $\qquad y_b = c - t_f = 155 - 9.7 = 145.3$ mm

$$\sigma_b = \frac{y_b}{c}\sigma_m = \left(\frac{145.3}{155}\right)(147.3) = 138.1\ \text{MPa}$$

At point B $\qquad \tau_w = \dfrac{V}{d\,t_w} = \dfrac{(26.98\times10^3)}{(310\times10^{-3})(5.8\times10^{-3})} = 15.0$ MPa

$$R = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau_w^2} = \sqrt{(69.05)^2 + (15.0)^2} = 70.66\ \text{MPa}$$

$$\sigma_{max} = \frac{\sigma_b}{2} + R = 69.05 + 70.66 = 139.7\ \text{MPa}$$ ⟵

PROBLEM 8.74

8.74 Knowing that the shear and bending moment in a given section of a W21 × 101 rolled-steel beam are, respectively, 120 kips and 300 kip · ft, determine the values in that section of (*a*) the maximum normal stress σ_m, (*b*) the principal stress σ_{max} at the junction of a flange and the web.

SOLUTION

$M = 300 \text{ kip}\cdot\text{ft} = 3600 \text{ kip}\cdot\text{in}$ $\qquad V = 120 \text{ kips}$

For W21 × 101 shape $\qquad d = 21.36 \text{ in} \qquad b_f = 12.290 \text{ in} \qquad t_f = 0.800 \text{ in.}$

$t_w = 0.500 \text{ in}, \quad I_z = 2420 \text{ in}^4, \quad S_z = 227 \text{ in}^3, \quad c = \frac{1}{2}d = 10.68 \text{ in}$

(a) $\sigma_m = \dfrac{M}{S} = \dfrac{3600}{227} = 15.86 \text{ ksi}$

(b) $y_b = c - t_f = 9.88 \text{ in}$

$\sigma_b = \dfrac{y_b}{c}\sigma_m =$ 14.67 ksi 7.336

$A_f = b_f t_f = 9.832 \text{ in}^2 \qquad \bar{y} = \frac{1}{2}(c + y_b) = 10.28 \text{ in.}$

$Q = A_f \bar{y} = 101.07 \text{ in}^3$

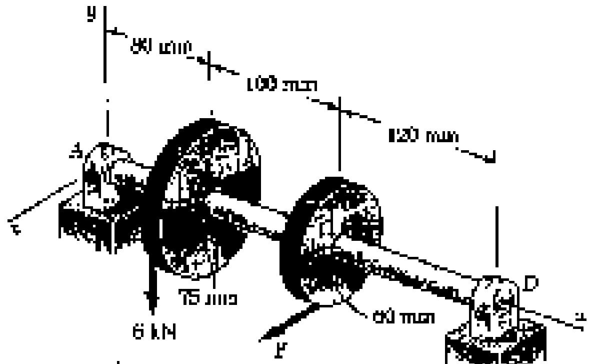
$\tau_b = \dfrac{VQ}{I_z t_w} = \dfrac{(120)(101.07)}{(2420)(0.500)} = 10.024 \text{ ksi}$

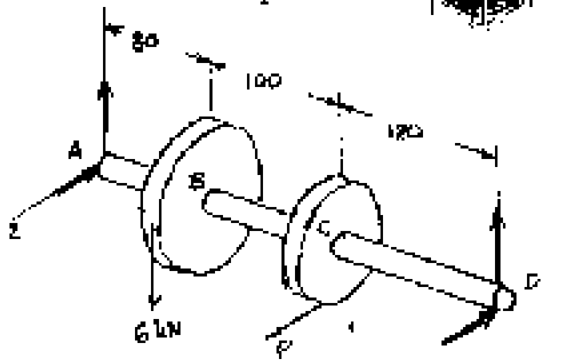
$R = \sqrt{\left(\dfrac{\sigma_b}{2}\right)^2 + \tau_b^2} = \sqrt{7.336^2 + 10.024^2} = 12.421 \text{ ksi}$

$\sigma_{max} = \dfrac{\sigma_b}{2} + R = 7.336 + 10.421 = 19.76 \text{ ksi}$

PROBLEM 8.75

8.75 The 6-kN force is vertical and the force **P** is parallel to the z axis. Knowing that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of the solid shaft AD.





SOLUTION

$\Sigma M_x = 0$

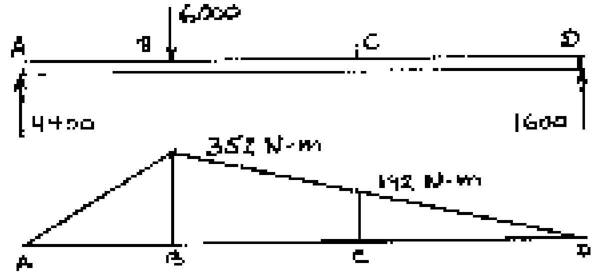
$(6\times10^3)(75\times10^{-3}) - (60\times10^{-3})P = 0$

$P = 7.5\times10^3$ N

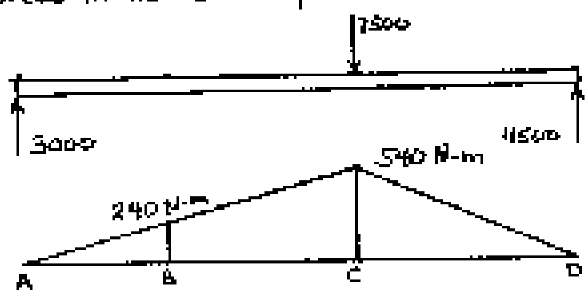
Over portion BC

$T = (6\times10^3)(75\times10^{-3}) = 450$ N·m

Forces in vertical plane



Forces in horizontal plane



Bending moments

At B $\quad M = \sqrt{352^2 + 240^2}$

$= 426.0$ N·m

At C $\quad M = \sqrt{540^2 + 192^2}$

$= 573.1$ N·m

Critical section is just to the left of gear C

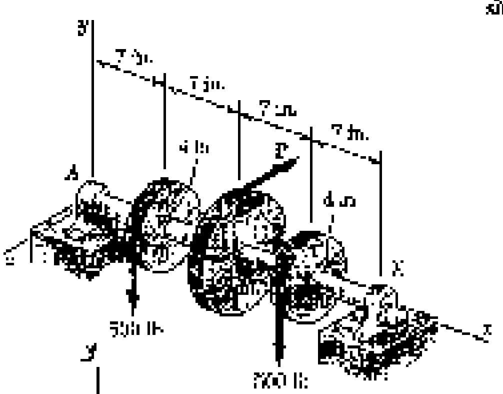
$M = 573.1$ N·m $\qquad T = 450$ N·m $\qquad \sqrt{M^2+T^2} = 728.67$ N·m

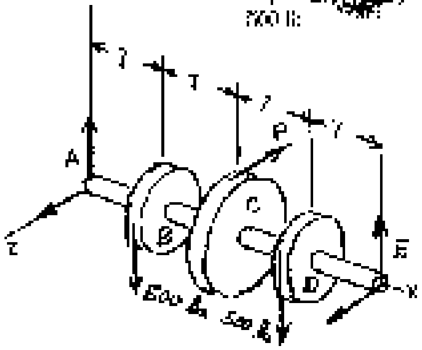
$$\tau_{all} = \frac{c}{J}\left(\sqrt{M^2+T^2}\right)_{max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M^2+T^2}\right)_{max}}{\tau_{all}} = \frac{728.67}{60\times10^6} = 12.145\times10^{-6}\ \text{m}^3$$

$c = 19.77\times10^{-3}$ m $\qquad d = 2c = 39.5\times10^{-3}$ m $= 39.5$ mm

PROBLEM 8.76





8.76 The two 500-lb forces are vertical and the force **P** is parallel to the z axis. Knowing that $\tau_{all} = 8$ ksi, determine the smallest permissible diameter of the solid shaft AE.

SOLUTION

$\Sigma M_x = 0 \qquad (4)(500) - 6P + (4)(500) = 0$

$P = 666.67$ lb.

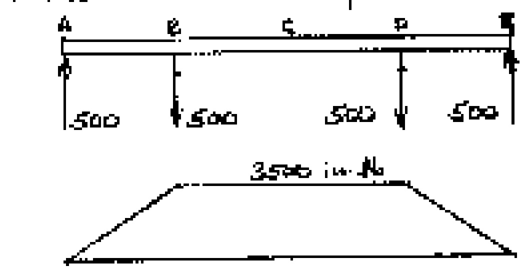
Torques: AB: $T = 0$

BC: $T = -(4)(500) = -2000$ in$\cdot$lb

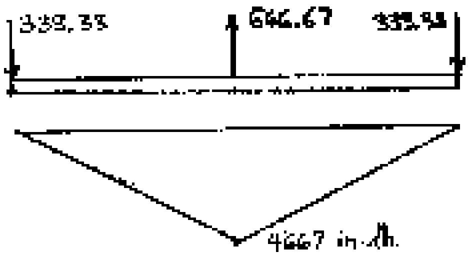
CD: $T = (4)(500) = 2000$ in$\cdot$lb

DE: $T = 0$

Forces in vertical plane



Forces in horizontal plane



Critical sections are either side of disk C

$T = 2000$ in$\cdot$lb. $\quad M_z = 3500$ in$\cdot$lb

$M_y = 4667$ in$\cdot$lb

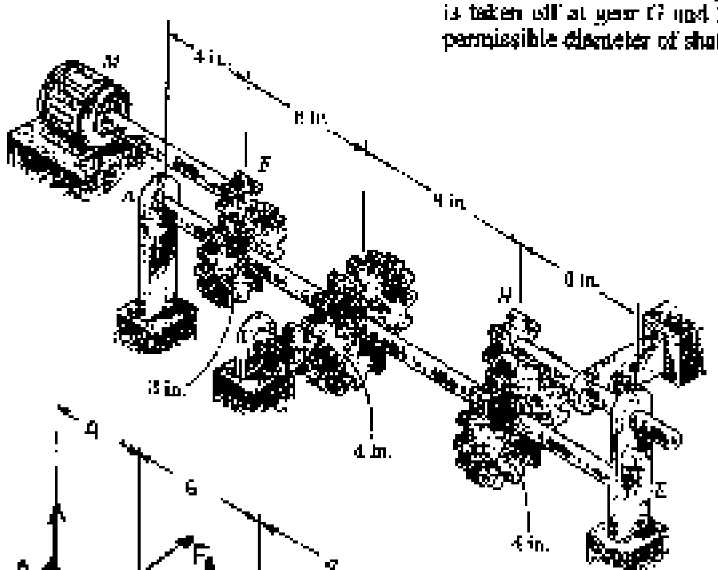
$$\tau_{all} = \frac{c}{J}\sqrt{M_y^2 + M_z^2 + T^2}$$

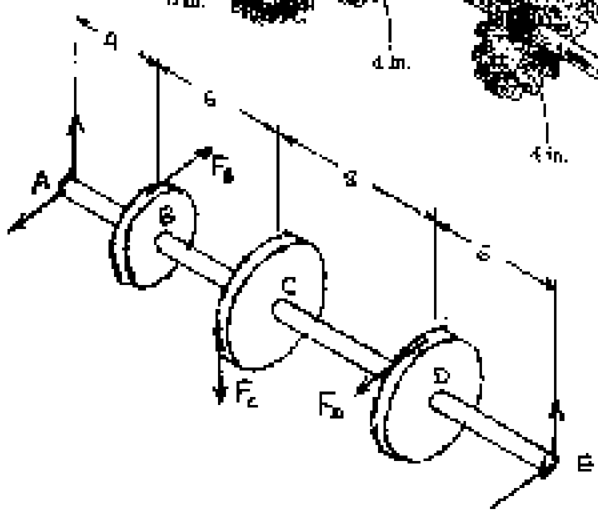
$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M_y^2 + M_z^2 + T^2}}{\tau_{all}} = \frac{\sqrt{4667^2 + 3500^2 + 2000^2}}{8 \times 10^3} = 0.77083 \text{ in}^3$$

$c = 0.789$ in. $\qquad d = 2c = 1.578$ in

PROBLEM 8.77

8.77 The solid shaft AE rotates at 600 rpm and transmits 60 hp from the motor M to machine tools connected to gears G and H. Knowing that $\tau_{all} = 8$ ksi and that 40 hp is taken off at gear G and 20 hp is taken off at gear H, determine the smallest permissible diameter of shaft AE.





SOLUTION

$$60 \text{ hp} = (60)(6600) = 396\times10^3 \text{ in}\cdot\text{lb/sec}$$

$$f = \frac{600 \text{ rpm}}{60 \text{ sec/min}} = 10 \text{ Hz}$$

Torque on gear B

$$T_B = \frac{P}{2\pi f} = \frac{396\times10^3}{2\pi(10)} = 6302.5 \text{ in}\cdot\text{lb}$$

Torques on gears C and D

$$T_C = \frac{40}{60} T_B = 4201.7 \text{ in}\cdot\text{lb}$$

$$T_D = \frac{20}{60} T_B = 2100.8 \text{ in}\cdot\text{lb}$$

Shaft torques

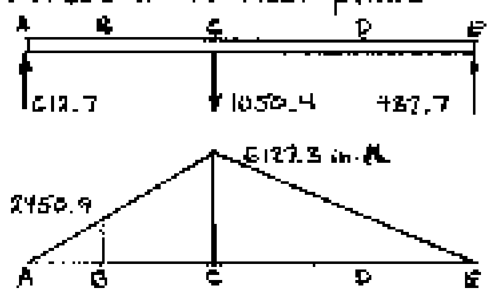
AB: $T_{AB} = 0$

BC: $T_{BC} = 6302.5$ in·lb

CD: $T_{CD} = 2100.8$ in·lb

DE: $T_{DE} = 0$

Forces in vertical plane



Gear forces

$$F_B = \frac{T_B}{r_B} = \frac{6302.5}{3} = 2100.8 \text{ lb.}$$

$$F_C = \frac{T_C}{r_C} = \frac{4201.7}{4} = 1050.4 \text{ lb.}$$

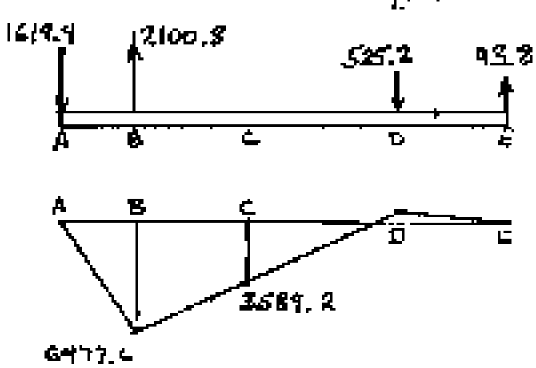
$$F_D = \frac{T_D}{r_D} = \frac{2100.8}{4} = 525.2 \text{ lb.}$$

At B^+: $\sqrt{M_z^2 + M_y^2 + T^2}$

$$= \sqrt{2450.9^2 + 6477.6^2 + 6302.5^2}$$

$$= 9364 \text{ in}\cdot\text{lb}$$

Forces in horizontal plane



At C^-: $\sqrt{M_z^2 + M_y^2 + T^2}$

$$= \sqrt{6127.3^2 + 3589.2^2 + 6302.5^2}$$

$$= 9495 \text{ in}\cdot\text{lb.} \quad \text{(maximum)}$$

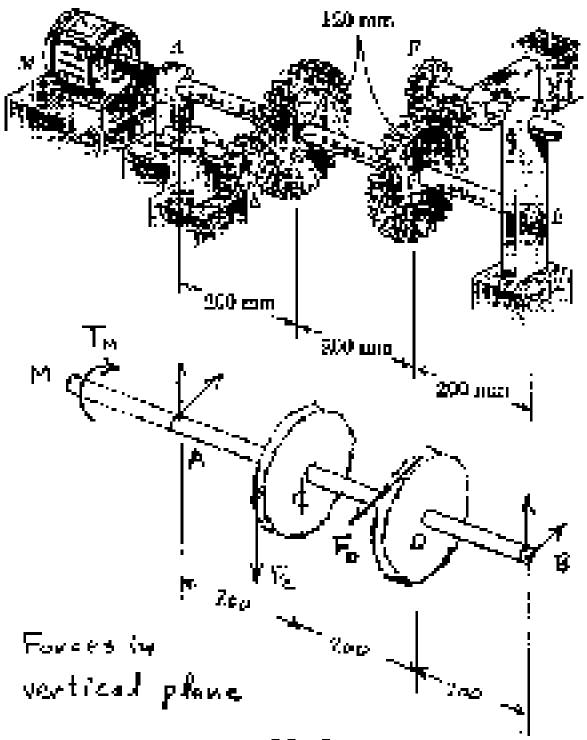
$$\tau_{all} = \frac{c}{J}\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)_{max}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\left(\sqrt{M_z^2 + M_y^2 + T^2}\right)}{\tau_{all}} = \frac{9495}{8\times10^3} = 1.1868 \text{ in}^3$$

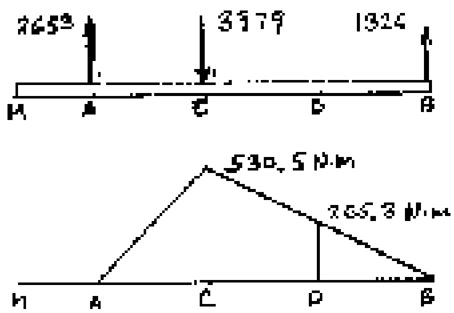
$c = 0.911$ in $\qquad d = 2c = 1.822$ in.

PROBLEM 3.78

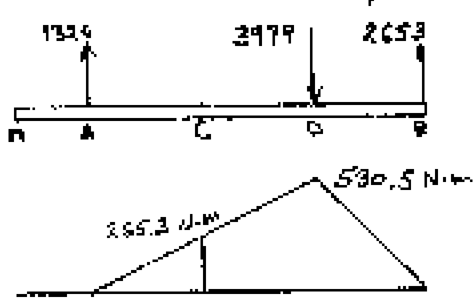
3.78 The motor M rotates at 300 rpm and transmits 30 kW to the solid shaft AB through a flexible connection. Half of this power is transferred to a machine tool connected to gear B and the other half to a machine tool connected to gear A. Knowing that $\tau_{all} = 60$ MPa, determine the smallest permissible diameter of shaft AB.



Forces in vertical plane



Forces in horizontal plane



SOLUTION

$300 \text{ rpm} = \frac{300}{60} = 5 \text{ Hz}$

$T_M = \frac{P}{2\pi f} = \frac{30\times10^3}{(2\pi)(5)} = 954.9 \text{ N·m}$

Torques on gears C and D

$T_C = T_D = \frac{1}{2}T_M = 477.5 \text{ N·m}$

Shaft torques.

MA: $T_{MA} = 954.9$ N·m

AC: $T_{AC} = 954.9$ N·m

CD: $T_{CD} = 477.5$ N·m

DB: $T_{DB} = 0$

Gear forces

$F_C = \frac{T_C}{r_C} = \frac{477.5}{120\times10^{-3}} = 3979 \text{ N}$

$F_D = \frac{T_D}{r_D} = \frac{477.5}{120\times10^{-3}} = 3979 \text{ N}$

Critical point is just to the left of gear C

$T_{AC} = 954.9$ N·m

$M_{Cz} = 530.5$ N·m

$M_{Cy} = 265.3$ N·m

$\sqrt{M_z^2 + M_y^2 + T^2} = 1124.1 \text{ N·m}$

$\tau_{all} = \frac{c}{J}\sqrt{M_z^2 + M_y^2 + T^2}$

$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{\sqrt{M_z^2 + M_y^2 + T^2}}{\tau_{all}}$

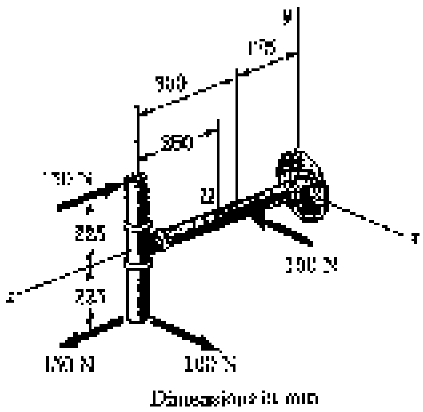
$= \frac{1124.1}{60\times10^6} = 18.735\times10^{-6} \text{ m}^3$

$c = 22.85\times10^{-3}$ m

$d = 2c = 45.7\times10^{-3} \text{ m} = 45.7 \text{ mm}$ ◄

PROBLEM 8.79

8.79 Several forces are applied to the pipe assembly shown. Knowing that each section of pipe has inner and outer diameters respectively equal to 36 mm and 42 mm, determine the normal and shearing stresses at point H located at the top of the outer surface of the pipe.



Dimensions in mm

SOLUTION

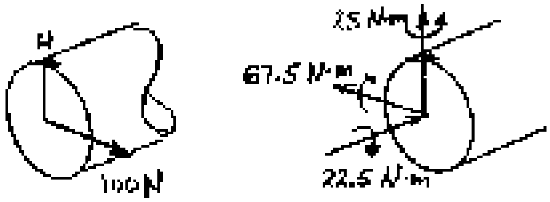
At the section containing point H

$P = 0, \quad V_x = 100\text{ N}, \quad V_y = 0$

$M_x = -(0.450)(150) = -67.5\text{ N}\cdot\text{m}$

$M_y = (0.250)(100) = 25\text{ N}\cdot\text{m}$

$M_z = -(0.225)(100) = -22.5\text{ N}\cdot\text{m}$



$d_o = 42\text{ mm} \qquad d_i = 32\text{ mm}$

$c_o = 21\text{ mm} \qquad c_i = 18\text{ mm}$

$t = c_o - c_i = 3\text{ mm}$

$A = \pi(c_o^2 - c_i^2) = 367.57\text{ mm}^2 = 367.57\times10^{-6}\text{ m}^2$

$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 70.30\times10^3\text{ mm}^4 = 70.30\times10^{-9}\text{ m}^4, \quad J = 2I = 140.59\times10^{-9}\text{ m}^4$

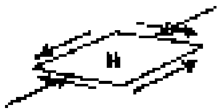
For half-pipe $Q = \frac{2}{3}(c_o^3 - c_i^3) = 2.286\times10^3\text{ mm}^3 = 2.286\times10^{-6}\text{ m}^3$

$\sigma_H = \dfrac{M_x y}{I_x} = \dfrac{(-67.5)(21\times10^{-3})}{70.30\times10^{-9}} = -20.2\text{ MPa}$

Due to torque $\quad (\tau_H)_T = \dfrac{Tc}{J} = \dfrac{(22.5)(21\times10^{-3})}{140.59\times10^{-9}} = 3.36\text{ MPa}$

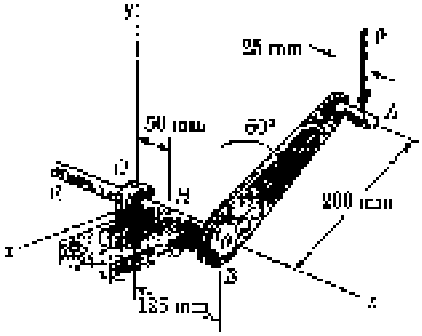
Due to shear $\quad (\tau_H)_V = \dfrac{VQ}{It} = \dfrac{(100)(2.286\times10^{-6})}{(70.30\times10^{-9})(6\times10^{-3})} = 0.54\text{ MPa}$

Net $\quad \tau_H = 3.36 - 0.54 = 2.82\text{ MPa}$



PROBLEM 8.80

8.80 A vertical force **P** of magnitude 250 N is applied to the crank at point *A*. Knowing that the shaft *BDE* has a diameter of 18 mm, determine the principal stresses and the maximum shearing stress at point *H* located at the top of the shaft, 50 mm to the right of support *D*.



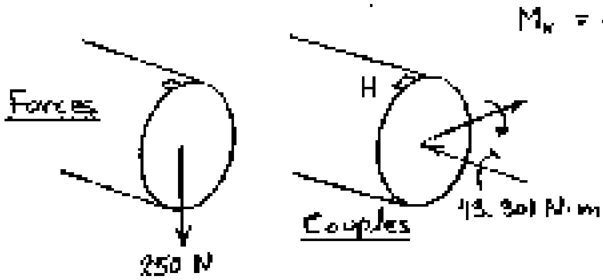
SOLUTION

Force-couple system at the centroid of the section containing point H.

$F_x = 0, \quad V_y = -250\ \text{N}, \quad V_z = 0$

$M_z = -(125 - 50 + 25)(10^{-3})(250) = -25\ \text{N·m}$

$M_x = -(200 \sin 60°)(10^{-3})(250) = -43.301\ \text{N·m}$



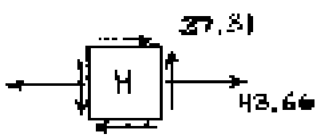
$d = 18\ \text{mm} \qquad c = \frac{1}{2}d = 9\ \text{mm}$

$I = \frac{\pi}{4}c^4 = 5.153 \times 10^3\ \text{mm}^4$
$= 5.153 \times 10^{-9}\ \text{m}^4$

$J = 2I = 10.306 \times 10^{-9}\ \text{m}^4$

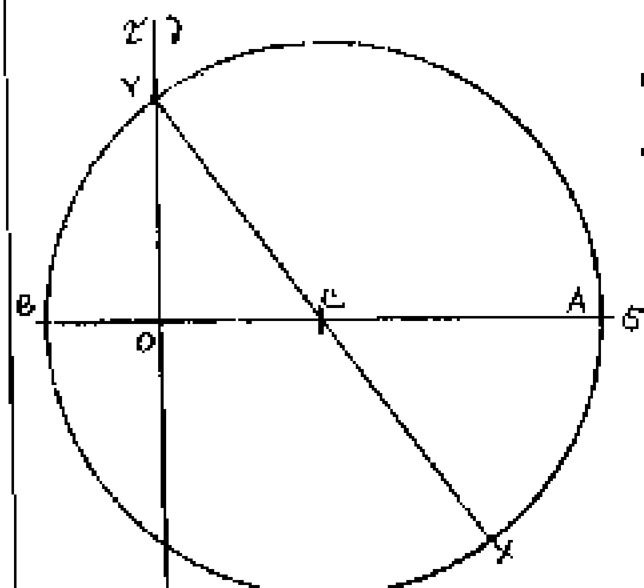
At point H $\quad \sigma_H = -\frac{M_z y}{I_z} = -\frac{(-25)(9 \times 10^{-3})}{5.153 \times 10^{-9}} = 43.66\ \text{MPa}$

$\tau_H = \frac{Tc}{J} = \frac{(43.301)(9 \times 10^{-3})}{10.306 \times 10^{-9}} = 37.81\ \text{MPa}$



$\sigma_c = \frac{1}{2}\sigma_H = 21.83\ \text{MPa}$

$R = \sqrt{\left(\frac{43.66}{2}\right)^2 + (37.81)^2} = 43.66\ \text{MPa}$



$\sigma_a = \sigma_c + R = 65.5\ \text{MPa}$

$\sigma_b = \sigma_c - R = -21.8\ \text{MPa}$

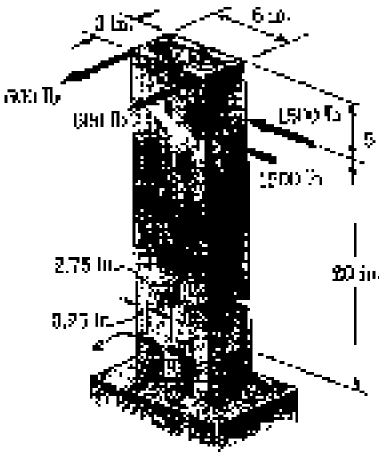
$\tan 2\theta_p = \frac{2\tau_H}{\sigma_H} = \frac{75.62}{43.66} = 1.7320$

$\theta_a = 30°, \qquad \theta_b = 120°$

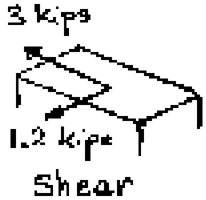
$\tau_{max} = R = 43.7\ \text{MPa}$

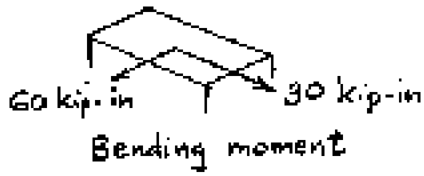
PROBLEM 8.81

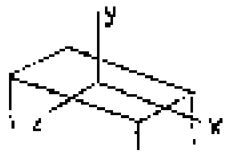
8.81 Knowing that the structural tube shown has a uniform wall thickness of 0.25 in., determine the normal and shearing stresses at the three points indicated.



SOLUTION







$b_o = 6$ in, $\quad b_i = b_o - 2t = 5.5$ in.
$h_o = 3$ in, $\quad h_i = h_o - 2t = 2.5$ in

$I_x = \frac{1}{12}(b_o h_o^3 - b_i h_i^3) = 6.8385\ \text{in}^4 \qquad I_z = \frac{1}{12}(h_o b_o^3 - h_i b_i^3) = 19.3385\ \text{in}^4$

Normal stresses

$$\sigma = \frac{M_z x}{I_z} - \frac{M_x z}{I_x}$$

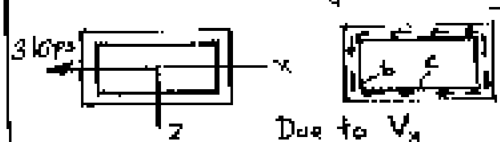
(a) $\dfrac{(60)(-3)}{19.3385} - \dfrac{(30)(1.5)}{6.8385} = -16.41$ ksi

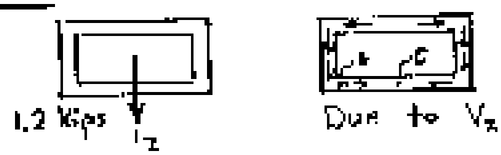
(b) $\dfrac{(60)(-2.75)}{19.3385} - \dfrac{(30)(1.5)}{6.8385} = -15.63$ ksi

(c) $\dfrac{(60)(0)}{19.3385} - \dfrac{(30)(1.5)}{6.8385} = -7.10$ ksi

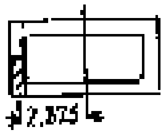
Shearing stresses (a) Point a is an outside corner; $\tau_a = 0$

Direction of shearing stresses





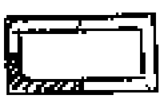
At point b



$Q_{zb} = (1.5)(0.25)(2.875) = 1.0781\ \text{in}^3$

$\tau_{b,V_x} = \dfrac{V_x Q_z}{I_z t} = \dfrac{(3)(1.0781)}{(19.3385)(0.25)} = 0.669$ ksi

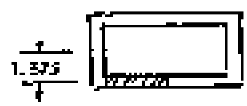
At point c



$Q_{zc} = Q_{zb} + (2.75)(0.25)\left(\frac{2.75}{2}\right)$
$= 2.0234\ \text{in}^3$

$\tau_{c,V_x} = \dfrac{V_x Q_z}{I_z t} = \dfrac{(3)(2.0234)}{(19.3385)(0.25)} = 1.256$ ksi

At point b



$Q_{xb} = (2.75)(0.25)(1.375) = 0.9453\ \text{in}^3$

$\tau_{b,V_z} = \dfrac{V_z Q_x}{I_x t} = \dfrac{(1.2)(0.9453)}{(6.8385)(0.25)} = 0.716$ ksi

At point c (symmetry axis)
$\tau_{c,V_z} = 0$

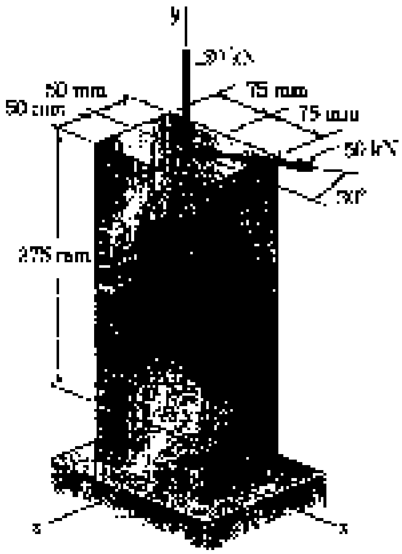
Net shearing stress at point b and c

$\tau_b = 0.716 - 0.669 = 0.047$ ksi

$\tau_c = 1.256$ ksi

PROBLEM 8.82

8.82 For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point *H*.



SOLUTION

Components of force at point C.

$F_x = 50 \cos 30° = 43.301$ kN

$F_z = -50 \sin 30° = -25$ kN, $\quad F_y = -120$ kN

Section forces and couples at the section containing points H and K.

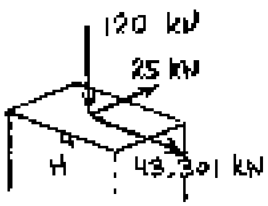
$P = 120$ kN (compression)

$V_x = 43.301$ kN, $\quad V_z = -25$ kN

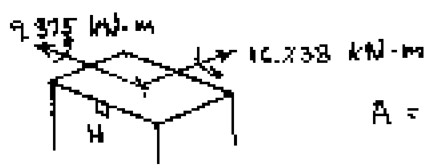
$M_x = -(25)(0.375) = -9.375$ kN·m

$M_y = 0$

$M_z = -(43.301)(0.375) = -16.238$ kN·m



Forces



Couples

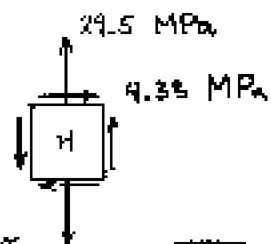
$A = (100)(150) = 15 \times 10^3 \text{ mm}^2 = 15 \times 10^{-3} \text{ m}^2$

$I_x = \frac{1}{12}(150)(100)^3 = 12.5 \times 10^6 \text{ mm}^4 = 12.5 \times 10^{-6} \text{ m}^4$

Stresses at point H

$$\sigma_H = -\frac{P}{A} - \frac{M_x z}{I_x} = -\frac{(120 \times 10^3)}{15 \times 10^{-3}} - \frac{(-9.375 \times 10^3)(50 \times 10^{-3})}{12.5 \times 10^{-6}} = 29.5 \text{ MPa}$$

$$\tau_H = \frac{3}{2}\frac{V_x}{A} = \frac{3}{2}\frac{43.301 \times 10^3}{15 \times 10^{-3}} = 4.33 \text{ MPa}$$



$\sigma_c = \frac{1}{2}\sigma_H = 14.75$ MPa

$R = \sqrt{\left(\frac{29.5}{2}\right)^2 + 4.33^2} = 15.37$ MPa

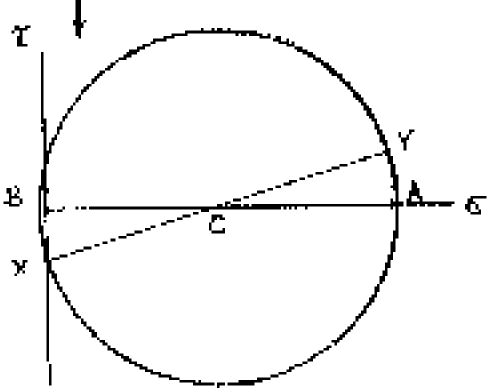
$\sigma_a = \sigma_c + R = 30.1$ MPa

$\sigma_b = \sigma_c - R = -0.62$ MPa

$\tan 2\theta_p = \frac{2\tau_H}{-\sigma_H} = -0.2936$

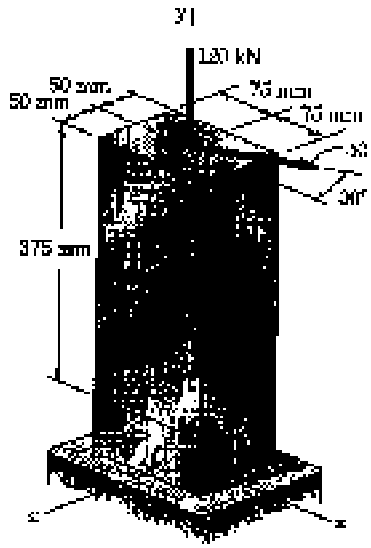
$\theta_a = -8.2°$ $\qquad \theta_b = 81.8°$

$\tau_{max} = R = 15.37$ MPa



PROBLEM 8.83

8.83 For the post and loading shown, determine the principal stresses, principal planes, and maximum shearing stress at point K.



SOLUTION

Components of force at point C

$F_x = 50\cos 30° = 43.301$ kN

$F_z = -50\sin 30° = -25$ kN $\qquad F_y = -120$ kN

Section forces and couples at the section containing points H and K.

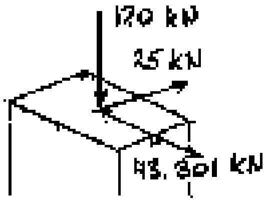
$P = 120$ kN (compression)

$V_x = 43.301$ kN, $\qquad V_z = -25$ kN

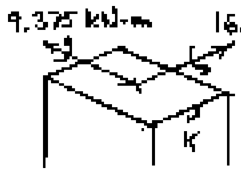
$M_x = -(25)(0.375) = -9.375$ kN·m

$M_y = 0$

$M_z = -(43.301)(0.375) = -16.238$ kN·m



Forces



Couples

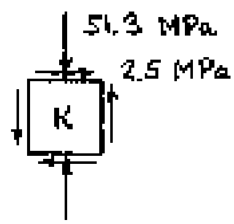
$A = (100)(150) = 15\times10^3$ mm^2
$= 15\times10^{-3}$ m^2

$I_z = \frac{1}{12}(100)(150)^3 = 28.125\times10^6$ mm^4
$= 28.125\times10^{-6}$ m^4

Stresses at point K

$$\sigma_K = -\frac{P}{A} + \frac{M_z x}{I_z} = -\frac{120\times10^3}{15\times10^{-3}} + \frac{(-16.238\times10^3)(75\times10^{-3})}{28.125\times10^{-6}} = -51.3 \text{ MPa}$$

$$\tau_K = \frac{3}{2}\frac{V_z}{A} = \frac{3}{2}\frac{25\times10^3}{15\times10^{-3}} = 2.5 \text{ MPa}$$



$\sigma_c = \frac{1}{2}\sigma_K = -25.65$ MPa

$R = \sqrt{\left(\frac{51.3}{2}\right)^2 + (2.5)^2} = 25.77$ MPa

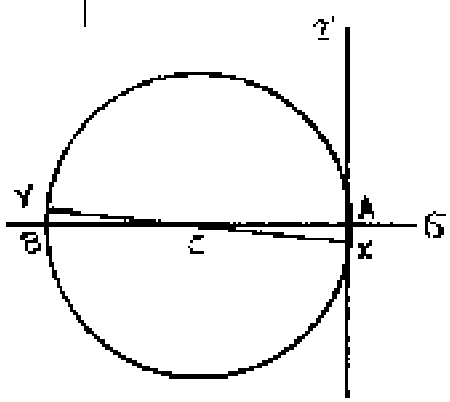
$\sigma_a = \sigma_c + R = 0.12$ MPa ◄

$\sigma_b = \sigma_c - R = -51.4$ MPa ◄

$\tan 2\theta_p = \frac{2\tau_K}{-\sigma_K} = 0.09747$

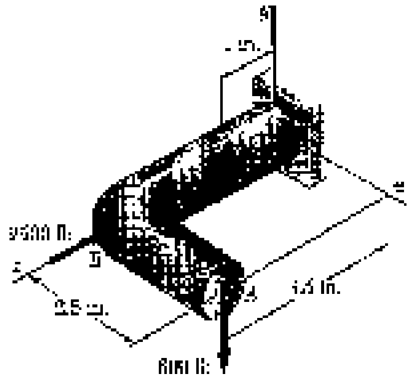
$\theta_a = 2.8°$ $\qquad \theta_b = 92.8°$ ◄

$\tau_{max} = R = 25.8$ MPa ◄



PROBLEM 8.84

8.84 Forces are applied at points A and B of the solid cast-iron bracket shown. Knowing that the bracket has a diameter of 0.8 in., determine the principal stresses and the maximum shearing stress (*a*) at point H, (*b*) at point K.



SOLUTION

At the section containing points H and K

$P = 2500$ lb (compression)

$V_y = -600$ lb $\qquad V_z = 0$

$M_x = (3.5 - 1)(600) = 1500$ lb·in

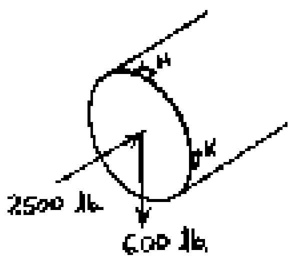
$M_y = 0 \qquad M_z = -(2.5)(600) = -1500$ lb·in

$c = \frac{1}{2}d = 0.4$ in

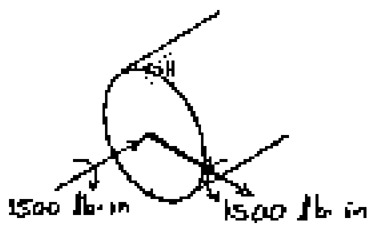
$A = \pi c^2 = 0.50265$ in^2

$I = \frac{\pi}{4}c^4 = 20.106 \times 10^{-3}$ in^4

$J = 2I = 40.212 \times 10^{-3}$ in^4



Forces

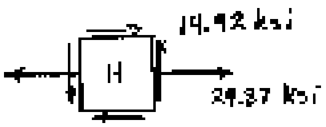


Couples

For semi circle $\quad Q = \frac{2}{3}c^3 = 42.667 \times 10^{-3}$ in^3

(a) At point H: $\quad \sigma_H = \frac{P}{A} + \frac{Mc}{I} = -\frac{2500}{0.50265} + \frac{(1500)(0.4)}{20.106 \times 10^{-3}} = 24.87 \times 10^3$ psi

$\tau_H = \frac{Tc}{J} = \frac{(1500)(0.4)}{40.212 \times 10^{-3}} = 14.92 \times 10^3$ psi



$\sigma_{ave} = \frac{24.87}{2} = 12.435$ ksi

$R = \sqrt{\left(\frac{24.87}{2}\right)^2 + (14.92)^2} = 19.423$ ksi

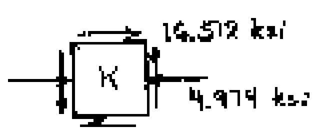
$\sigma_{max} = \sigma_{ave} + R = 31.9$ ksi

$\sigma_{min} = \sigma_{ave} - R = -6.99$ ksi

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 19.42$ ksi

(b) At point K: $\quad \sigma_K = \frac{P}{A} = -\frac{2500}{0.50265} = -4.974 \times 10^3$ psi

$\tau_K = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(1500)(0.4)}{40.212 \times 10^{-3}} + \frac{(600)(42.667 \times 10^{-3})}{(20.106 \times 10^{-3})(0.8)} = 16.512 \times 10^3$ psi



$\sigma_{ave} = -\frac{4.974}{2} = -2.487$ ksi

$R = \sqrt{\left(-\frac{4.974}{2}\right)^2 + (16.512)^2} = 16.698$ ksi

$\sigma_{max} = \sigma_{ave} + R = 14.21$ ksi

$\sigma_{min} = \sigma_{ave} - R = -19.18$ ksi

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = 16.70$ ksi

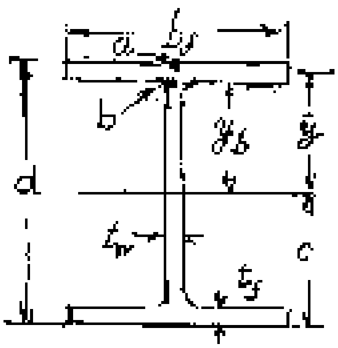
PROBLEM 8.C1

8.C1 Let us assume that the shear V and the bending moment M have been determined in a given section of a rolled-steel beam. Write a computer program to calculate in that section, from the data available in Appendix C, (*a*) the maximum normal stress σ_m, (*b*) the principal stress σ_{max} at the junction of a flange and the web. Use this program to solve parts *a* and *b* of the following problems:

(1) Prob. 8.1 (Use $V = 400$ kN and $M = 100$ kN · m)
(2) Prob. 8.2 (Use $V = 200$ kN and $M = 100$ kN · m)
(3) Prob. 8.3 (Use $V = 320$ kips and $M = 32 \times 10^3$ kip · in.)
(4) Prob. 8.74.

SOLUTION

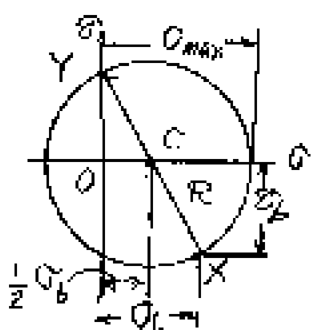
We enter the given values of V and M obtain from Appendix C the values of d, b_f, t_f, t_w, I, and S for the given WF shape.



We compute $c = d/2$, $y_b = c - t_f$

$\bar{y} = c - \frac{1}{2}t_f$, $\sigma_a = M/S$, $\sigma_b = \sigma_a(y_b/c)$

$Q = b_f t_f \bar{y}$, $\tau_b = \dfrac{VQ}{I t_w}$



From Mohr's circle:

$$\sigma_{max} = \frac{1}{2}\sigma_b + R$$

$$\sigma_{max} = \frac{1}{2}\sigma_b + \sqrt{\left(\frac{1}{2}\sigma_b\right)^2 + \tau_b^2}$$

PROGRAM OUTPUTS

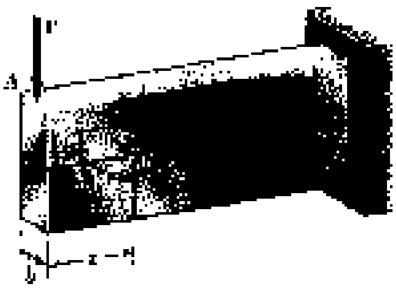
```
Prob. 8.1

Given Data:
V = 400 kN, M = 100 kN.m
d = 252 mm, bf = 203 mm
tf = 13.5 mm, tw = 8.0 mm
I = 87.30 (10^6 mm^4)
S = 693.0 (10^3 mm^3)
Answers:
(a) SIGA = 144.3 MPa
(b) SIGM = 250.1 MPa
```

```
Prob. 8.2

Given Data:
V = 200 kN, M = 100 kN.m
d = 252 mm, bf = 203 mm
tf = 13.5 mm, tw = 8.0 mm
I = 87.30 (10^6 mm^4)
S = 693.0 (10^3 mm^3)
Answers:
(a) SIGA = 144.3 MPa
(b) SIGM = 172.7 MPa
```

```
Prob. 8.3

Given Data:
V = 320 kips, M = 32000 kip.in.
d = 36.74 in., bf = 16.655 in.
tf = 1.680 in., tw = 0.945 in.
I = 20300 in^4, S = 1110 in^3
Answers:
(a) SIGA = 28.8 ksi
(b) SIGM = 28.5 ksi
```

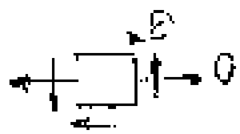
```
Prob. 8.74

Given Data:
V = 120 kips, M = 3600 kip.in.
d = 21.36 in., bf = 12.290 in
tf = 0.800 in., tw = 0.500 in.
I = 2420 in^4, S = 227 in^3
Answers:
(a) SIGA = 15.86 ksi
(b) SIGM = 19.76 ksi
```

PROBLEM 8.C2



8.C2 A cantilever beam AB with a rectangular cross section of width b and depth $2c$ supports a single concentrated load P at its end A. Write a computer program to calculate, for any values of x/c and y/c, (*a*) the ratios σ_{max}/σ_m and σ_{min}/σ_m, where σ_{max} and σ_{min} are the principal stresses at point $K(x, y)$ and σ_m the maximum normal stress in the same transverse section, (*b*) the angle θ_p that the principal planes at K form with a transverse and a horizontal plane through K. Use this program to check the values shown in Fig. 8.8 and to verify that σ_{max} exceeds σ_m if $x \le 0.544c$, as indicated in the second footnote on page 499.

SOLUTION



Since the distribution of the normal stresses is linear, we have $\sigma = \sigma_m (y/c)$ (1)

where $\sigma_m = \dfrac{Mc}{I} = \dfrac{Pxc}{I}$ (2)

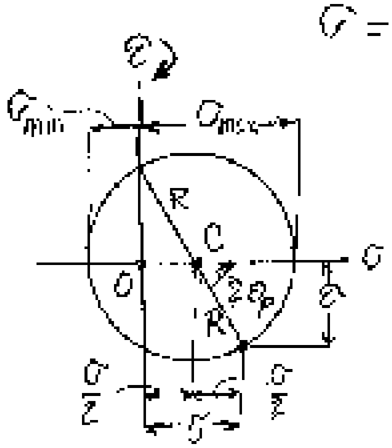
We use Eq. (8.4), page 498: $\tau = \dfrac{3}{2}\dfrac{P}{A}\left(1 - \dfrac{y^2}{c^2}\right)$ ↻ (3)

Dividing (3) by (2): $\dfrac{\tau}{\sigma_m} = \dfrac{3}{2}\dfrac{I}{A}\,\dfrac{1-(y/c)^2}{xc}$

or, since $\dfrac{I}{A} = \dfrac{\frac{1}{12}b(2c)^3}{b(2c)} = \dfrac{1}{3}c^2$; $\dfrac{\tau}{\sigma_m} = \dfrac{1}{2}\,\dfrac{1-(y/c)^2}{x/c}$ (4)

Letting $X = x/c$ and $Y = y/c$, Eqs. (1) and (4) yield

$$\sigma = \sigma_m Y \qquad \tau = \sigma_m \frac{1-Y^2}{2X} \;↻$$



Using Mohr's circle, we calculate

$$R = \sqrt{\left(\tfrac{1}{2}\sigma\right)^2 + \tau^2}$$

$$= \frac{1}{2}\sigma_m \sqrt{Y^2 + \left(\frac{1-Y^2}{X}\right)^2}$$

$$\frac{\sigma_{max}}{\sigma_m} = \frac{1}{2}Y + R \qquad \frac{\sigma_{min}}{\sigma_m} = \frac{1}{2}Y - R$$

$$\tan 2\theta_p = \frac{\tau}{\sigma/2} = \frac{1-Y^2}{2X(Y/2)} = \frac{1-Y^2}{XY} \qquad \theta_p = \frac{1}{2}\tan^{-1}\left(\frac{1-Y^2}{XY}\right) \;↻$$

<u>NOTE</u>

For $y > 0$, the angle θ_p is ↻, which is opposite to what was arbitrarily assumed in Fig. P8.C2.

(CONTINUED)

PROGRAM OUTPUTS

For x/c = 2 :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta°
1.0	0.000	1.000	0.00
0.8	0.010	0.810	6.34
0.6	-0.040	0.640	14.04
0.4	-0.090	0.490	23.20
0.2	-0.160	0.360	33.69
0.0	-0.250	0.250	45.00
-0.2	-0.360	0.160	33.69
-0.4	0.490	0.090	-23.20
-0.6	-0.640	0.040	-14.04
-0.8	0.810	0.010	-6.34
-1.0	-1.000	0.000	0.00

For x/c = 6 :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta°
1.0	0.000	1.006	0.00
0.8	-0.001	0.801	1.61
0.6	0.003	0.603	3.80
0.4	-0.007	0.407	7.35
0.2	-0.017	0.217	15.48
0.0	-0.062	0.063	45.00
-0.2	-0.217	0.017	-15.44
-0.4	-0.407	0.007	-7.15
-0.6	-0.603	0.003	-3.80
-0.8	-0.801	0.001	-1.61
-1.0	-1.000	0.000	-0.00

To check that $\sigma_{max} > \sigma_m$ if $x \leq 0.544c$, we run the program for $x/c = 0.544$ and for $x/c = 0.545$ and observe that σ_{max}/σ_m exceeds 1 for several values of y/c in the first case, but does not exceed 1 in the second case.

For x/c = 0.544 :

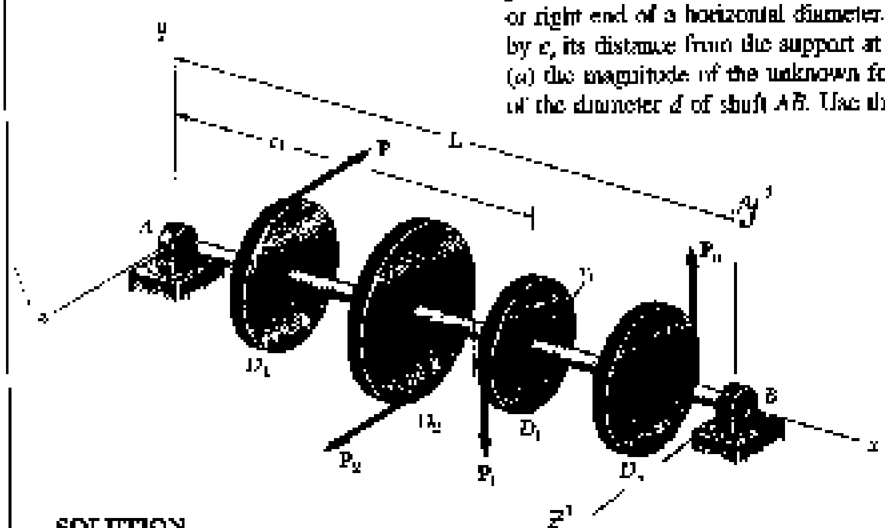
y/c	Sigmin/Sigm	Sigmax/Sigm	Theta°
0.30	0.703	0.9997	39.92
0.31	-0.693	1.0001	39.72
0.32	-0.682	1.0004	39.51
0.33	-0.672	1.0005	39.32
0.34	-0.662	1.0005	39.09
0.35	-0.655	1.0003	38.89
0.36	-0.640	1.0000	38.66
0.37	0.630	0.9996	38.44
0.38	-0.619	0.9990	38.21
0.39	-0.628	0.9983	37.98
0.40	-0.590	0.9973	37.74

For x/c = 0.545 :

y/c	Sigmin/Sigm	Sigmax/Sigm	Theta°
0.30	-0.698	0.9982	39.91
0.31	-0.689	0.9986	39.71
0.32	-0.679	0.9989	39.50
0.33	-0.669	0.9990	39.29
0.34	-0.659	0.9990	39.08
0.35	-0.649	0.9960	38.87
0.36	-0.639	0.9956	38.65
0.37	0.628	0.9982	38.42
0.38	-0.618	0.9976	38.20
0.39	-0.607	0.9970	37.96
0.40	-0.596	0.9962	37.73

PROBLEM 8.C3

8.C3 Disks $D_1, D_2, \ldots, D_n$ are attached as shown in Fig. P8.C3 to the solid shaft AB of length L, uniform diameter d, and allowable shearing stress τ_{all}. Forces $\mathbf{P}_1, \mathbf{P}_2, \ldots, \mathbf{P}_n$ of known magnitude (except for one of them) are applied to the disks, either at the top or bottom of a vertical diameter, or at the left or right end of a horizontal diameter. Denoting by r_i the radius of disk D_i and by c_i its distance from the support at A, write a computer program to calculate (*a*) the magnitude of the unknown force $\mathbf{P}_i$, (*b*) the smallest permissible value of the diameter d of shaft AB. Use this program to solve Probs. 8.75 and 8.76.



SOLUTION

1. Determine the unknown force P_i by equating to zero the sum of their torques T_i about the x axis.

2. Determine the components $(F_y)_i$ and $(F_z)_i$ of all forces.

3. Determine the components A_y and A_z of reaction at A by summing moments about axes $Bz' \parallel z$ and $By' \parallel y$:

$$\Sigma M_{z'} = 0: \quad -A_y L - \Sigma (F_y)_i (L - c_i) = 0, \qquad A_y = -\frac{1}{L} \Sigma (F_y)_i (L - c_i)$$

$$\Sigma M_{y'} = 0: \quad A_z L + \Sigma (F_z)_i (L - c_i) = 0, \qquad A_z = -\frac{1}{L} \Sigma (F_z)_i (L - c_i)$$

4. Determine $(M_y)_i$, $(M_z)_i$, and torque T_i just to the left of disk D_i:

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k < c_i - c_k >^1$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k < c_i - c_k >^1$$

$$T_i = \sum_k T_k < c_i - c_k >^0$$

where $< >$ indicates a singularity function.

5. The minimum diameter d required to the left of D_i is obtained by first computing $(J/c)_i$ from Eq. (8.7):

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{(M_y)_i^2 + (M_z)_i^2 + T_i^2}}{\tau_{all}}$$

(CONTINUED)

6. Recalling that $J = \frac{1}{2}\pi c^4$ and, thus, that $\left(\frac{J}{c}\right)_i = \frac{1}{2}\pi c_i^3$, we have $c_i = \left(\frac{2}{\pi}\left(\frac{J}{c}\right)_i\right)^{1/3}$ and $d_i = \left(\frac{4}{\pi}\left(\frac{J}{c}\right)_i\right)^{1/3}$

This is the required diameter just to the left of disk D_i

7. The required diameter just to the right of disk D_i is obtained by replacing T_i with T_{i+1} in the above computation.

8. The smallest permissible value of the diameter of the shaft is the largest of the values obtained for d_i

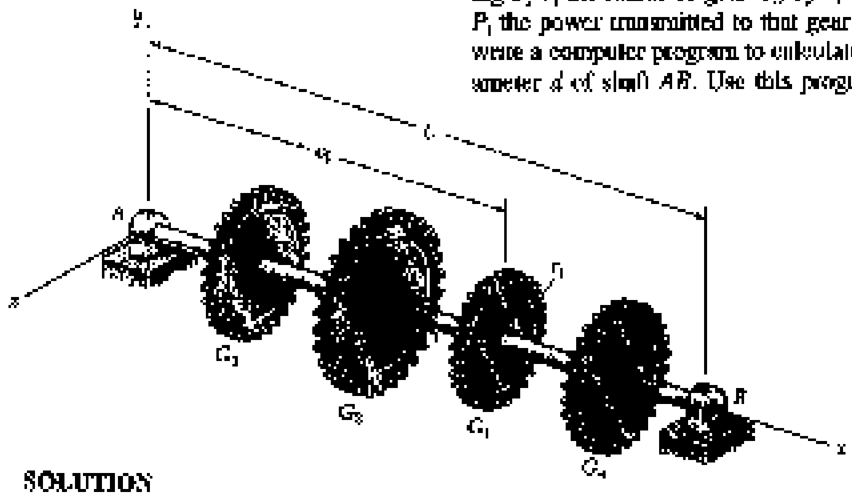
PROGRAM OUTPUTS

```
Prob. 8.75
Length of shaft = 500 mm
TAU = 60 MPa
For Disk 1
Force= 6.000 kN
Radius of disk = 75 mm
Distance from A in mm = 80
For Disk 2
Force= 0.000 kN
Radius of disk = 60 mm
Distance from A in mm = 180
Unknown force= -7.500 kN
AY= 4.400 kN, AZ= -3.000 kN
BY= 1.600 kN, BZ= 4.500 kN
Just to the left of Disk 1
MY=-240.00 Nm
MZ=-352.00 Nm
T=   0.00 Nm
Diameter must be at least  33.07 mm
Just to the right of Disk 1
T= 450.00 Nm
Diameter must be at least  37.42 mm
Just to the left of Disk 2
MY= 540.00 Nm
MZ=-192.00 Nm
T= 450.00 Nm
Diameter must be at least  39.55 mm
Just to the right of Disk 2
T=   0.00 Nm
Diameter must be at least  35.51 mm
```

```
Prob 8.76
Length of shaft=   28 in.
TAU (ksi)=      8
For Disk 1
Force =  0.500 kips
Radius of disk =   4.0 in.
Distance from A=   7.0 in.
For Disk 2
Force =  0.000 kips
Radius of disk =   6.0 in.
Distance from A=  14.0 in.
For Disk 3
Force =  0.500 kips
Radius of disk =   4.0 in.
Distance from A=  21.0 in.
Unknown force= -0.667 kips
AY=  0.500 kips, AZ=  0.333 kips
BY=  0.500 kips, BZ=  0.333 kips
Just to the left of Disk 1
MY=  2.3333 kip.in.
MZ=  -3.5000 kip.in.
T=   0.0000 kip.in.
Diameter must be at least  1.389 in.
Just to the right of Disk 1
T=   2.00 kip.in.
Diameter must be at least  1.437 in.
Just to the left of Disk 2
MY=  4.6667 kip.in.
MZ=  -3.5000 kip.in.
T=   2.0000 kip.in.
Diameter must be at least  1.578 in.
Just to the right of Disk 2
T=  -2.00 kip.in.
Diameter must be at least  1.578 in.
Just to the left of Disk 3
MY=  2.3333 kip.in.
MZ=  -3.5000 kip.in.
T=  -2.0000 kip.in.
Diameter must be at least  1.437 in.
Just to the right of Disk 3
T=   0.00 kip.in.
Diameter must be at least  1.389 in.
```

PROBLEM 8.C4

8.C4 The solid shaft AB of length L, uniform diameter d, and allowable shearing stress τ_{all} rotates at a given speed expressed in rpm (Fig. P8.C4). Gears $G_1, G_2, \ldots, G_n$ are attached to the shaft and each of these gears meshes with another gear (not shown), either at the top or bottom of its vertical diameter, or at the left or right end of its horizontal diameter. One of these other gears is connected to a motor and the rest of them to various machine tools. Denoting by r_i the radius of gear G_i, by c_i its distance from the support at A, and by P_i the power transmitted to that gear (+ sign) or taken off that gear (− sign), write a computer program to calculate the smallest permissible value of the diameter d of shaft AB. Use this program to solve Probs. 8.25, 8.29, and 8.77.



SOLUTION

1. Enter ω in rpm and determine frequency $f = \omega/60$.

2. For each gear, determine the torque $T_i = P_i/2\pi f$, where P_i is the power input (+) or output (−) at the gear.

3. For each gear, determine the force $F_i = T_i/r_i$ exerted on the gear and its components $(F_y)_i$ and $(F_z)_i$.

4. Determine the components A_y and A_z of reaction at A by summing moments about axes $Bz' \parallel z$ and $By' \parallel y$:

$$\Sigma M_{z'} = 0: \; -A_y L - \Sigma (F_y)_i (L - c_i) = 0, \quad A_y = -\frac{1}{L}\Sigma (F_y)_i (L - c_i)$$

$$\Sigma M_{y'} = 0: \; A_z L + \Sigma (F_z)_i (L - c_i) = 0, \quad A_z = -\frac{1}{L}\Sigma (F_z)_i (L - c_i)$$

5. Determine $(M_y)_i$, $(M_z)_i$, and torque T_i just to the left of gear G_i:

$$(M_y)_i = A_z c_i + \sum_k (F_z)_k \langle c_i - c_k \rangle^1$$

$$(M_z)_i = -A_y c_i - \sum_k (F_y)_k \langle c_i - c_k \rangle^1$$

$$T_i = \sum_k T_k \langle c_i - c_k \rangle^0$$

where $\langle\ \rangle$ indicates a singularity function.

(CONTINUED)

6. The minimum diameter d required to the left of G_i is obtained by first computing $(J/c)_i$ from Eq. (8.7):

$$\left(\frac{J}{c}\right)_i = \frac{\sqrt{(M_y)_i^2 + (M_z)_i^2 + T_i^2}}{\tau_{all}}$$

7. Recalling that $J = \frac{1}{2}\pi c^4$ and, thus, that $\left(\frac{J}{c}\right)_i = \frac{1}{2}\pi c_i^3$

we have $c_i = \frac{2}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$ and $d_i = \frac{4}{\pi}\left(\frac{J}{c}\right)_i^{1/3}$

This is the required diameter just to the left of gear G_i.

8. The required diameter just to the right of gear G_i is obtained by replacing T_i with T_{i+1} in the above computation.

9. The smallest permissible value of the diameter of the shaft is the largest of the values obtained for d_i.

PROGRAM OUTPUTS

```
Prob. 8.25
Omega = 600 rpm
Number of Gears: 2
Length of shaft = 400 mm
Tau = 60 MPa
For Gear 1
Power input = 80.00 kW
Radius of gear= 90 mm
Distance from A in mm = 120
For Gear 2
Power input = -80.00 kW
Radius of gear= 60 mm
Distance from A in mm = 280
AY= 11.141 kN, AZ= 6.586
BY= 4.775 kN, BZ= 14.854
Just to the left of Gear 1
MY= 763.94 Nm
MZ= -1336.91 Nm
T= 0.00 Nm
Diameter must be at least 52.75 mm
Just to the right of Gear 1
T=1273.24 Nm
Diameter must be at least 55.35 mm
Just to the left of Gear 2
MY=1782.51 Nm
MZ= 572.96 Nm
T=1273.24 Nm
Diameter must be at least 57.77 mm
Just to the right of Gear 2
T= 0.00 Nm
Diameter must be at least 54.17 mm
```

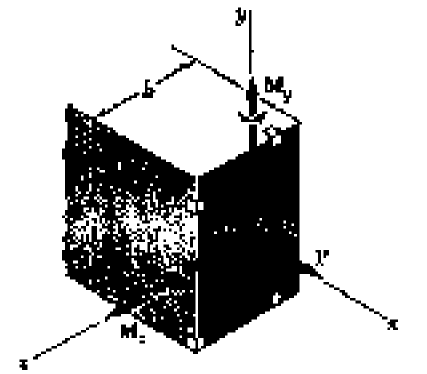
(CONTINUED)

PROBLEM 8.C4 CONTINUED

```
Prob. 8.29
Omega = 450 rpm
Number of Gears: 3
Length of shaft =   750 mm
Tau =   55 MPa
For Gear 1
Power input =   -8.00 kW
Radius of gear= 80 mm
Distance from A in mm = 150
For Gear 2
Power input =   20.00 kW
Radius of gear=100 mm
Distance from A in mm = 375
For Gear 3
Power input = -12.00 kW
Radius of gear= 60 mm
Distance from A in mm = 600
AY= -0.649 kN, AZ=    4.356
BY= -5.395 kN, BZ=    2.688
Just to the left of Gear 1
MY= 657.04 Nm
MZ= 127.32 Nm
T=    0.00 Nm
Diameter must be at least   39.59 mm
Just to the right of Gear 1
T=-169.77 Nm
Diameter must be at least   40.00 mm
Just to the left of Gear 2
MY=1007.98 Nm
MZ= 318.31 Nm
T=-169.77 Nm
Diameter must be at least   46.28 mm
Just to the right of Gear 2
T= 254.65 Nm
Diameter must be at least   46.52 mm
Just to the left of Gear 3
MY= 433.19 Nm
MZ= 509.30 Nm
T= 254.65 Nm
Diameter must be at least   40.33 mm
Just to the right of Gear 3
T=    0.00 Nm
Diameter must be at least   33.18 mm
```

```
Prob. 8.77
Omega = 600 rpm
Number of Gears: 3
Length of shaft =    24 in.
Tau =    8 ksi
For Gear 1
Power input =   60.00 hp
Radius of gear= 3.00 in.
Distance from A in inches =    4.0
FY=                 0
FZ =                2.100845
For Gear 2
Power input = -40.00 hp
Radius of gear= 4.00 in.
Distance from A in inches =   10.0
FY=                 1.050423
FZ =                0
For Gear 3
Power input = -20.00 hp
Radius of gear= 4.00 in.
Distance from A in inches =   18.0
FY=                 0
FZ =               -.5252133
AY=-0.6127 kips, AZ=-1.6194 kips
BY= 0.4377 kips, BZ= 0.0438 kips
Just to the left of Gear 1
MY= -6.478 kip.in.
MZ=  2.451 kip.in.
T=  0.000 kip.in.
Diameter must be at least   1.640 in.
Just to the right of Gear 1
T=  6.3025 kip.in.
Diameter must be at least   1.813 in.
Just to the left of Gear 2
MY= -3.889 kip.in.
MZ=  6.127 kip.in.
T=  6.303 kip.in.
Diameter must be at least   1.822 in.
Just to the right of Gear 2
T=  2.1008 kip.in.
Diameter must be at least   1.677 in.
Just to the left of Gear 3
MY=  0.263 kip.in.
MZ=  2.525 kip.in.
T=  2.101 kip.in.
Diameter must be at least   1.290 in.
Just to the right of Gear 3
T=  0.0000 kip.in.
Diameter must be at least   1.199 in.
```

PROBLEM 8.C5



8.C5 Write a computer program that can be used to calculate the normal and shearing stresses at points with given coordinates y and z located on the surface of a machine part having a rectangular cross section. The internal forces are known to be equivalent to the force-couple system shown. Write the program so that the loads and dimensions can be expressed in either SI or U.S. customary units. Use this program to solve (*a*) Prob. 8.50, (*b*) Prob. 8.53.

SOLUTION

Enter: b and h

Program: $A = bh \quad I_y = b^3h/12 \quad I_z = bh^3/12$

For point on surface, enter y and z

Note y and z must satisfy one of following:

$y^2 = h^2/4$ and $z^2 \le b^2/4$ (1)

or $z^2 = b^2/4$ and $y^2 \le h^2/4$ (2)

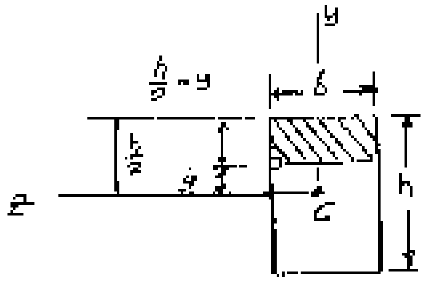
If either (1) or (2) are satisfied, compute

$$\sigma = \frac{F}{A} + \frac{M_y z}{I_y} - \frac{M_z y}{I_z}$$

If $z^2 = b^2/4$, then point is on vertical surface and

$$Q_z = b\left(\frac{h}{2} - y\right)\left(\frac{h}{2} + y\right)\frac{1}{2} = b\left(\frac{h^2}{8} - \frac{y^2}{2}\right)$$

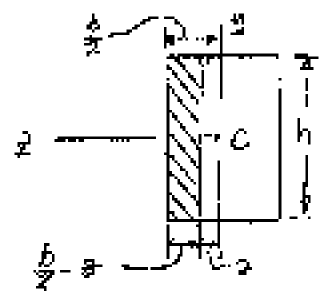
$$\tau = \frac{V_y Q_z}{I_z b}$$



If $y^2 = h^2/4$, the point is on horizontal surface, and

$$Q_y = h\left(\frac{b}{2} - z\right)\left(\frac{b}{2} + z\right)\frac{1}{2} = h\left(\frac{b^2}{8} - \frac{z^2}{2}\right)$$

$$\tau = \frac{V_z Q_y}{I_y h}$$



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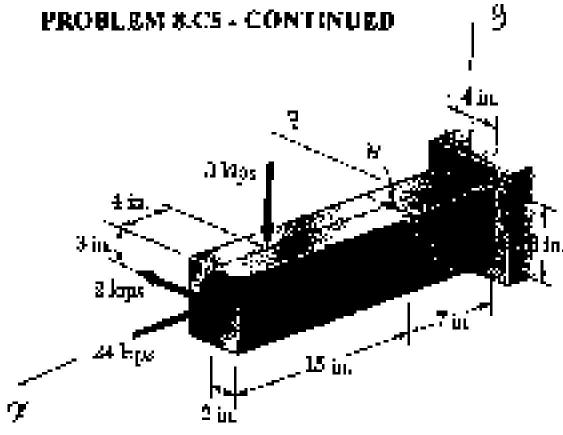
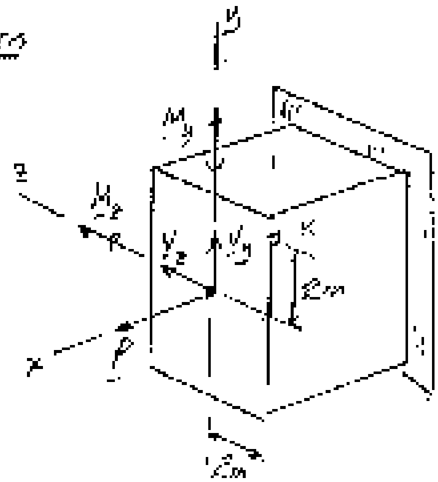
PROBLEM 8.C5 - CONTINUED



Problem 8.50

Point K



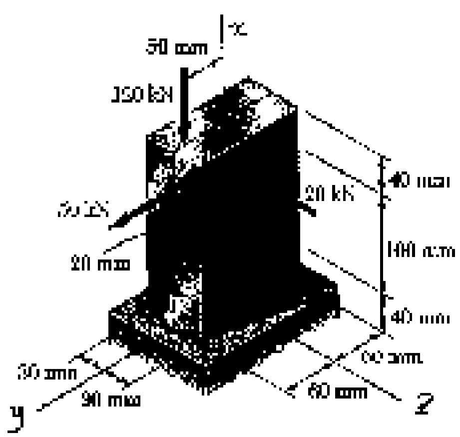
Force-Couple System

$P = 24$ kips $\qquad V_y = -3$ kips $\qquad V_z = 2$ kips

$M_y = -(2\text{ kips})(15\text{ in}) = -30$ kip·in $\qquad M_z = -(3\text{ kips})(15\text{ in} - 4\text{ in}) = -33$ kip·in

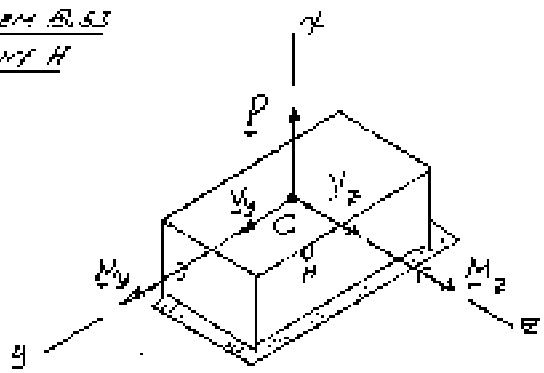
Point K $\qquad y = 2$ in $\qquad z = -2$ in

```
      Problem 8.50          Force-Couple at Centroid
                      P =    -24.000 kips
                     MY =    -30.000 kip·in.      MZ =    -33.000 kip·in.
                     VY =      3.000 kips         VZ =      2.000 kips
                      + + + + + + + + + + + + + + + + + + + + + + + + + + +
                   At point of coordinates:     y  =   2.000 in.    z =  -2.000 in.
                             sigma =   1.792 ksi
                               tau =   0.104 ksi
```



Problem 8.53

Point H



Force-Couple System $\qquad P = -120$ kN $\qquad V_y = 50$ kN $\qquad V_z = -20$ kN

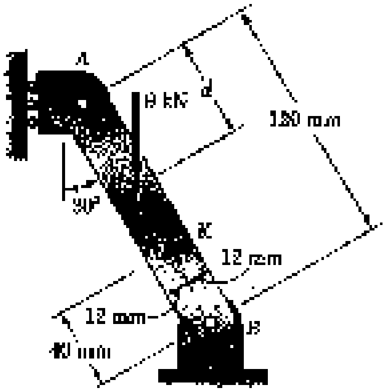
$M_y = (20\text{ kN})(0.1\text{ m}) = 2$ kN·m

$M_z = (120\text{ kN})(0.05\text{ m}) + (50\text{ kN})(0.10\text{ m}) = 11$ kN·m

Point H $\qquad y = 20$ mm $\qquad z - 30$ mm

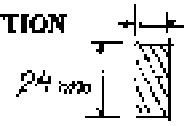
```
      Problem 8.53          Force-Couple at Centroid
                      P = -120000.00 N
                     MY =    2000.00 N·m      MZ =   11000.00 N·m
                     VY =   50000.00 N        VZ =  -20000.00 N
                      + + + + + + + + + + + + + + + + + + + + + + + + + + +
                   At point of coordinates:     y  =   20.00 mm    z =   30.00 mm
                             sigma = -14.352 MPa
                               tau =   9.259 MPa
```

PROBLEM 8.C6



8.C6 Member AB has a rectangular cross section of 10 × 24 mm. For the loading shown, write a computer program that can be used to determine the normal and shearing stresses at points H and K for values of d from 0 to 120 mm, using 15-mm increments. Use this program to solve Prob. 8.35.

SOLUTION



CROSS SECTION

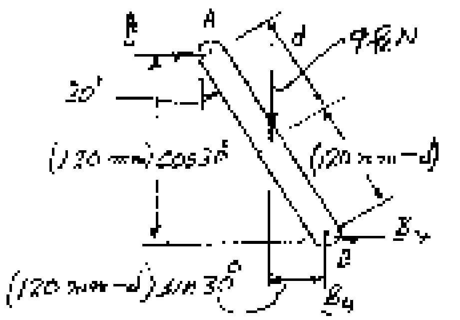
ENTER

$A = (0.010\text{ m})(0.024\text{ m}) = 240\times10^{-6}\text{ m}^2$

$I = (0.010\text{ m})(0.024\text{ m})^3/12 = 138.24\times10^{-9}\text{ m}^4$

$c = 0.5(0.024\text{ m}) = 12\times10^{-3}\text{ m}$

COMPUTE REACTION AT A.

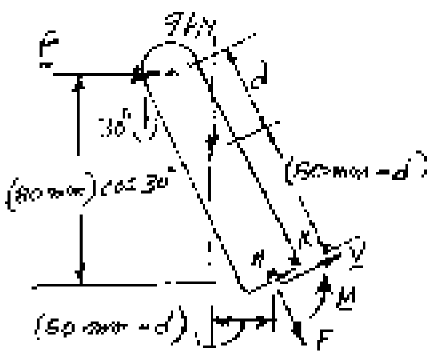


$+\circlearrowleft \Sigma M_B = 0:$

$(9\text{ kN})(120 - d)\sin 30° - A(120)\cos 30° = 0$

$A = (9\text{ kN})\dfrac{(120\text{ mm} - d)}{120\text{ mm}}\tan 30°$

FREE BODY FROM A TO SECTION CONTAINING POINTS H AND K.



DEFINE: IF $d < 80$ mm THEN STP = 1 ELSE STP = 0

PROGRAM FORCE-COUPLE SYSTEM

$F = -A\sin 30° - (9\text{ kN})\cos 30°(\text{STP})$

$V = -A\cos 30° + (9\text{ kN})\sin 30°(\text{STP})$

$M = A(80\text{ mm})\cos 30° - (9\text{ kN})(80\text{ mm} - d)\sin 30°(\text{STP})$

AT POINT H:

$\sigma_H = +F/A \qquad \tau_H = \frac{3}{2}V/A$

AT POINT K:

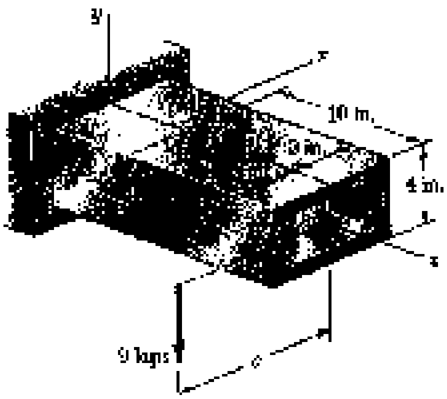
$\sigma_K = +F/A - Mc/I \qquad \tau_K = 0$

PROGRAM OUTPUT

Problem 8.35

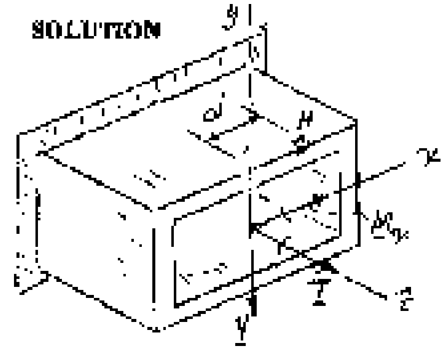
d mm	Stresses in MPa SigmaH	TauH	SigmaK	TauK
0.0	-43.30	0.00	-43.30	0.00
15.0	-41.95	3.52	-65.39	0.00
30.0	-40.59	7.03	-87.47	0.00
45.0	-39.24	10.55	-109.55	0.00
60.0	-37.89	14.06	-131.64	0.00
75.0	-36.54	17.58	-153.72	0.00
90.0	-2.71	-7.03	-96.46	0.00
105.0	-1.35	-3.52	-48.23	0.00
120.0	0.00	0.00	0.00	0.00

PROBLEM 8.C7



*8.C7 The structural tube shown has a uniform wall thickness of 0.3 in. A 9-kip force is applied to a bar (not shown) that is welded to the end of the tube. Write a computer program that can be used to determine, for any given value of c, the principal stresses, principal planes, and maximum shearing stress at point H for values of d from −3 in. to 3 in., using one-inch increments. Use this program to solve Prob. 8.72a.

SOLUTION

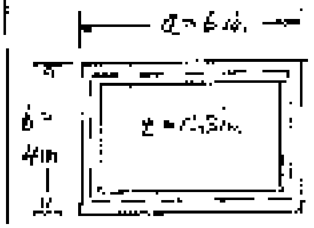


Force-Couple System at End:

$V = 9\text{ kips}\downarrow$

$M_z = (9\text{ kips})(10\text{ in.}) = 90\text{ kip}\cdot\text{in}$

$T = (9\text{ kips})\,c$

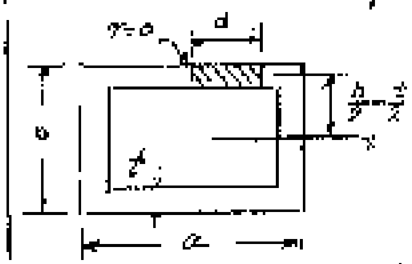


Area enclosed

$\mathcal{A} = (a-t)(b-t)$

$\tau_T = \dfrac{T}{2t\mathcal{A}} = \dfrac{9c}{2t\mathcal{A}}$

τ_T = shearing stress due to torsion

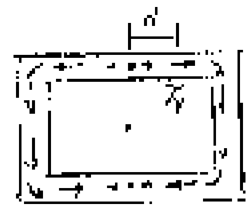


$Q = dt\left(\frac{b}{2} - \frac{t}{2}\right)$

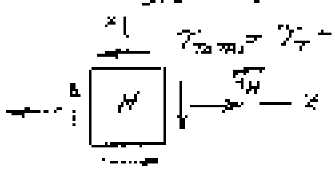
$I = ab^3/12 - (a-2t)(b-2t)^3/12$

$\tau_V = \dfrac{VQ}{It}$

τ_V = shearing stress due to V



$\tau_{total} = \tau_T - \tau_V$



Bending: $\sigma_H = \dfrac{M_z\left(\frac{b}{2}\right)}{I}$

Principal Stresses

$\sigma_{ave} = \frac{1}{2}\sigma_H\ ;\quad R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_{total}^2}$

$\sigma_{max} = \sigma_{ave} + R\ ;\quad \sigma_{min} = \sigma_{ave} - R\ ;\quad \theta_p = \frac{1}{2}\tan^{-1}\left(\frac{\tau_{total}}{\sigma_{ave}}\right)\ ;\quad \tau_{max} = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_{total}^2}$

```
Rectangular tube of uniform thickness t = 0.3 in.
Outside dimensions
  Horizontal width a = 6 in.
  Vertical depth   b = 4 in.
Vertical load     P = 9 kips; line of action at x = -c
Find normal and shearing stresses at
     Point H (x = d, y = b/2)
```

Problem 8.72   Program Output for Value of c = 2.85 in.

d in.	sigma ksi	tauV ksi	tauT ksi	tauTotal ksi	sigmaMax ksi	sigmaMin ksi	tauMax ksi	theta p degrees
-3.00	12.58	-3.49	-2.03	-5.52	14.65	-2.08	8.36	-18.49
-2.00	12.58	-2.33	-2.03	-4.35	13.94	-1.36	7.65	-16.00
-1.00	12.58	-1.16	-2.03	-3.19	13.34	-0.76	7.05	-12.78
0.00	12.58	0.00	-2.03	-2.03	12.89	-0.32	6.61	-8.73
1.00	12.58	1.16	-2.03	-0.86	12.63	-0.06	6.35	-3.89
2.00	12.58	2.33	-2.03	0.30	12.58	-0.01	6.30	1.36
3.00	12.58	3.49	-2.03	1.46	12.74	-0.17	6.46	6.46

CHAPTER 9