Solutions Manual for

Heat and Mass Transfer: Fundamentals & Applications
6th Edition
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Chapter 1
INTRODUCTION AND BASIC CONCEPTS

Thermodynamics and Heat Transfer

1-1C Thermodynamics deals with the amount of heat transfer as a system undergoes a process from one equilibrium state to another. Heat transfer, on the other hand, deals with the rate of heat transfer as well as the temperature distribution within the system at a specified time.

1-2C (a) The driving force for heat transfer is the temperature difference. (b) The driving force for electric current flow is the electric potential difference (voltage). (a) The driving force for fluid flow is the pressure difference.

1-3C The *rating* problems deal with the determination of the *heat transfer rate* for an existing system at a specified temperature difference. The *sizing* problems deal with the determination of the *size* of a system in order to transfer heat at a *specified rate* for a *specified temperature difference*.

1-4C The experimental approach (testing and taking measurements) has the advantage of dealing with the actual physical system, and getting a physical value within the limits of experimental error. However, this approach is expensive, time consuming, and often impractical. The analytical approach (analysis or calculations) has the advantage that it is fast and inexpensive, but the results obtained are subject to the accuracy of the assumptions and idealizations made in the analysis.

1-5C Modeling makes it possible to predict the course of an event before it actually occurs, or to study various aspects of an event mathematically without actually running expensive and time-consuming experiments. When preparing a mathematical model, all the variables that affect the phenomena are identified, reasonable assumptions and approximations are made, and the interdependence of these variables are studied. The relevant physical laws and principles are invoked, and the problem is formulated mathematically. Finally, the problem is solved using an appropriate approach, and the results are interpreted.

1-6C The right choice between a crude and complex model is usually the *simplest* model which yields *adequate* results.

Preparing very accurate but complex models is not necessarily a better choice since such models are not much use to an analyst if they are very difficult and time consuming to solve. At the minimum, the model should reflect the essential features of the physical problem it represents.

1-7C Warmer. Because energy is added to the room air in the form of electrical work.

1-8C Warmer. If we take the room that contains the refrigerator as our system, we will see that electrical work is supplied to this room to run the refrigerator, which is eventually dissipated to the room as waste heat.

1-9C The claim is false. The heater of a house supplies the energy that the house is losing, which is proportional to the temperature difference between the indoors and the outdoors. A turned off heater consumes no energy. The heat lost from a house to the outdoors during the warming up period is less than the heat lost from a house that is already at the temperature that the thermostat is set because of the larger cumulative temperature difference in the latter case. For best practice, the heater should be turned off when no one is at home during day (at subfreezing temperatures, the heater should be kept on at a low temperature to avoid freezing of water in pipes). Also, the thermostat should be lowered during bedtime to minimize the temperature difference between the indoors and the outdoors at night and thus the amount of heat that the heater needs to supply to the house.

1-10C No. The thermostat tells an airconditioner (or heater) at what interior temperature to stop. The air conditioner will cool the house at the same rate no matter what the thermostat setting is. So, it is best to set the thermostat at a comfortable temperature and then leave it alone. Setting the thermostat too low a home owner risks wasting energy and money (and comfort) by forgetting it at the set low temperature.

1-11C No. Since there is no temperature drop of water, the heater will never kick into make up for the heat loss. Therefore, it will not waste any energy during times of no use, and there is no need to use a timer. But if the family were on a time-of-use tariff, it would be possible to save money (but not energy) by turning on the heater when the rate was lowest at night and off during peak periods when rate is the highest.

1-12C For the constant pressure case. This is because the heat transfer to an ideal gas is $mc_p\Delta T$ at constant pressure and $mc_v\Delta T$ at constant volume, and c_p is always greater than c_v.

1-13C The rate of heat transfer per unit surface area is called heat flux $\dot{q}$. It is related to the rate of heat transfer by $\dot{Q} = \int_A \dot{q}dA$.

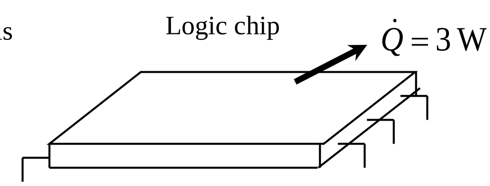
1-14C Energy can be transferred by heat and work to a close system. An energy transfer is heat transfer when its driving force is temperature difference.

1-15E A logic chip in a computer dissipates 3 W of power. The amount heat dissipated in 8 h and the heat flux on the surface of the chip are to be determined.

Assumptions Heat transfer from the surface is uniform.

Analysis (*a*) The amount of heat the chip dissipates during an 8-hour period is

$$Q = \dot{Q}\Delta t = (3\text{ W})(8\text{ h}) = 24\text{ Wh} = \mathbf{0.024\,kWh}$$



(*b*) The heat flux on the surface of the chip is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{3\text{ W}}{0.08\text{ in}^2} = \mathbf{37.5\ W/in^2}$$

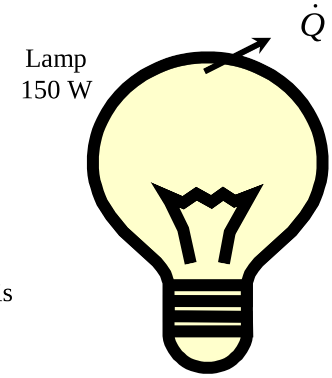
1-16 The filament of a 150 W incandescent lamp is 5 cm long and has a diameter of 0.5 mm. The heat flux on the surface of the filament, the heat flux on the surface of the glass bulb, and the annual electricity cost of the bulb are to be determined.

Assumptions Heat transfer from the surface of the filament and the bulb of the lamp is uniform.

Analysis (*a*) The heat transfer surface area and the heat flux on the surface of the filament are

$$A_s = \pi DL = \pi(0.05\,\text{cm})(5\,\text{cm}) = 0.785\,\text{cm}^2$$

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = \frac{150\ \text{W}}{0.785\,\text{cm}^2} = 191\ \text{W/cm}^2 = \mathbf{1.91\times10^6\ \ W/m^2}$$



(*b*) The heat flux on the surface of glass bulb is

$$A_s = \pi D^2 = \pi(8\,\text{cm})^2 = 201.1\,\text{cm}^2$$

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = \frac{150\ \text{W}}{201.1\ \text{cm}^2} = 0.75\ \text{W/cm}^2 = \mathbf{7500\ W/m^2}$$

(*c*) The amount and cost of electrical energy consumed during a one-year period is

$$\text{Electricity Consumption} = \dot{Q}\Delta t = (0.15\,\text{kW})(365\times 8\ \text{h/yr}) = 438\ \text{kWh/yr}$$

$$\text{Annual Cost} = (438\,\text{kWh/yr})(\$0.08/\text{kWh}) = \mathbf{\$35.04/yr}$$

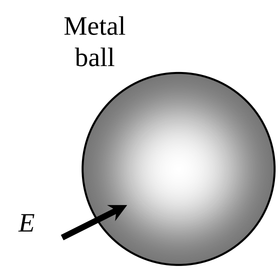
1-17 An aluminum ball is to be heated from 80°C to 200°C. The amount of heat that needs to be transferred to the aluminum ball is to be determined.

Assumptions The properties of the aluminum ball are constant.

Properties The average density and specific heat of aluminum are given to be ρ = 2700 kg/m^3 and c_p = 0.90 kJ/kg·°C.

Analysis The amount of energy added to the ball is simply the change in its internal energy, and is determined from

$$E_{\text{transfer}} = \Delta U = mc_p(T_2 - T_1)$$



where

$$m = \rho \boldsymbol{V} = \frac{\pi}{6}\rho D^3 = \frac{\pi}{6}(2700\ \text{kg/m}^3)(0.15\ \text{m})^3 = 4.77\ \text{kg}$$

Substituting,

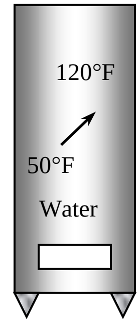
$$E_{\text{transfer}} = (4.77\ \text{kg})(0.90\,\text{kJ/kg}\cdot{}^\circ\text{C})(200-80)^\circ\text{C} = \mathbf{515\,kJ}$$

Therefore, 515 kJ of energy (heat or work such as electrical energy) needs to be transferred to the aluminum ball to heat it to 200°C.

1-18E A water heater is initially filled with water at 50°F. The amount of energy that needs to be transferred to the water to raise its temperature to 120°F is to be determined.

Assumptions **1** Water is an incompressible substance with constant specific. **2** No water flows in or out of the tank during heating.

Properties The density and specific heat of water at 85ºF from Table A-9E are: ρ = 62.17 lbm/ft^3 and c_p = 0.999 Btu/lbm·R.



Analysis The mass of water in the tank is

$$m = \rho V = (62.17 \text{ lbm/ft}^3)(60 \text{ gal})\left(\frac{1 \text{ ft}^3}{7.48 \text{ gal}}\right) = 498.7 \text{ lbm}$$

Then, the amount of heat that must be transferred to the water in the tank as it is heated from 50 to 120°F is determined to be

$$Q = mc_p(T_2 - T_1) = (498.7 \text{ lbm})(0.999 \text{ Btu/lbm}\cdot°\text{F})(120-50)°\text{F} = \mathbf{34{,}874 \text{ Btu}}$$

Discussion Referring to Table A-9E the density and specific heat of water at 50ºF are: ρ = 62.41 lbm/ft^3 and c_p = 1.000 Btu/lbm·R and at 120ºF are: ρ = 61.71 lbm/ft^3 and c_p = 0.999 Btu/lbm·R. We evaluated the water properties at an average temperature of 85ºF. However, we could have assumed constant properties and evaluated properties at the initial temperature of 50ºF or final temperature of 120ºF without loss of accuracy.

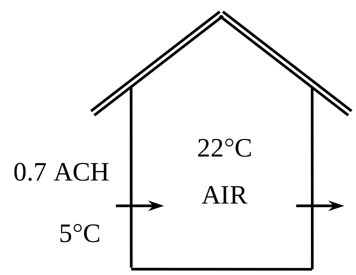
1-19 An electrically heated house maintained at 22°C experiences infiltration losses at a rate of 0.7 ACH. The amount of energy loss from the house due to infiltration per day and its cost are to be determined.

Assumptions **1** Air as an ideal gas with a constant specific heats at room temperature. **2** The volume occupied by the furniture and other belongings is negligible. **3** The house is maintained at a constant temperature and pressure at all times. **4** The infiltrating air exfiltrates at the indoors temperature of 22°C.

Properties The specific heat of air at room temperature is c_p = 1.007 kJ/kg·°C.

Analysis The volume of the air in the house is

$$V = (\text{floor space})(\text{height}) = (200 \text{ m}^2)(3 \text{ m}) = 600 \text{ m}^3$$



Noting that the infiltration rate is 0.7 ACH (air changes per hour) and thus the air in the house is completely replaced by the outdoor air 0.7×24 = 16.8 times per day, the mass flow rate of air through the house due to infiltration is

$$\dot{m}_{\text{air}} = \frac{P_o \dot{V}_{\text{air}}}{RT_o} = \frac{P_o(\text{ACH} \times V_{\text{house}})}{RT_o}$$

$$= \frac{(89.6 \text{ kPa})(16.8 \times 600 \text{ m}^3/\text{day})}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(5 + 273.15 \text{ K})} = 11{,}314 \text{ kg/day}$$

Noting that outdoor air enters at 5°C and leaves at 22°C, the energy loss of this house per day is

$$\dot{Q}_{\text{infilt}} = \dot{m}_{\text{air}} c_p (T_{\text{indoors}} - T_{\text{outdoors}})$$

$$= (11{,}314 \text{ kg/day})(1.007 \text{ kJ/kg.°C})(22-5)°\text{C} = 193{,}681 \text{ kJ/day} = \mathbf{53.8 \text{ kWh/day}}$$

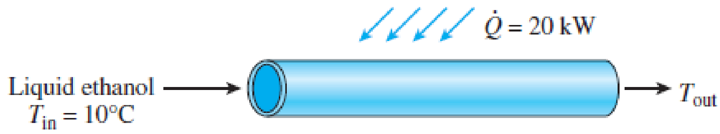
At a unit cost of \$0.082/kWh, the cost of this electrical energy lost by infiltration is

$$\text{Enegy Cost} = (\text{Energy used})(\text{Unit cost of energy}) = (53.8 \text{ kWh/day})(\$0.082/\text{kWh}) = \mathbf{\$4.41/day}$$

1-20 Liquid ethanol is being transported in a pipe where heat is added to the liquid. The volume flow rate that is necessary to keep the ethanol temperature below its flashpoint is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The specific heat and density of ethanol are constant.

Properties The specific heat and density of ethanol are given as 2.44 kJ/kg·K and 789 kg/m^3, respectively.



Analysis The rate of heat added to the ethanol being transported in the pipe is

$$\dot{Q} = \dot{m}c_p(T_{out} - T_{in})$$

or

$$\dot{Q} = \dot{V}\rho c_p(T_{out} - T_{in})$$

For the ethanol in the pipe to be below its flashpoint, it is necessary to keep T_{out} below 16.6°C. Thus, the volume flow rate should be

$$\dot{V} > \frac{\dot{Q}}{\rho c_p(T_{out} - T_{in})} = \frac{20\ \text{kJ/s}}{(789\ \text{kg/m}^3)(2.44\ \text{kJ/kg}\cdot\text{K})(16.6-10)\ \text{K}}$$

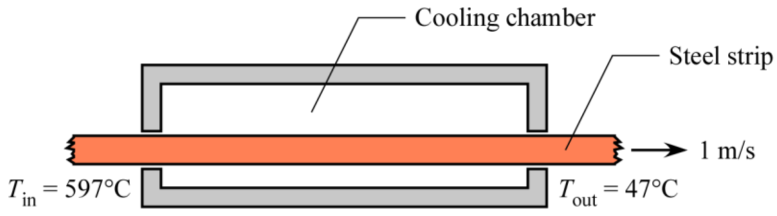
$$\dot{V} > \mathbf{0.00157\ m^3/s}$$

Discussion To maintain the ethanol in the pipe well below its flashpoint, it is more desirable to have a much higher flow rate than 0.00157 m^3/s.

1-21 A 2 mm thick by 3 cm wide AISI 1010 carbon steel strip is cooled in a chamber from 597 to 47°C to avoid instantaneous thermal burn upon contact with skin tissue. The amount of heat rate to be removed from the steel strip is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The stainless steel sheet has constant specific heat and density. **3** Changes in potential and kinetic energy are negligible.

Properties For AISI 1010 carbon steel, the specific heat of AISI 1010 steel at (597 + 47)°C / 2 = 322°C = 595 K is 682 J/kg·K (by interpolation from Table A-3), and the density is given as 7832 kg/m^3.



Analysis The mass of the steel strip being conveyed enters and exits the chamber at a rate of

$$\dot{m} = \rho V w t$$

The rate of heat being removed from the steel strip in the chamber is given as

$$\begin{aligned}\dot{Q}_{\text{removed}} &= \dot{m} c_p (T_{\text{in}} - T_{\text{out}}) \\ &= \rho V w t c_p (T_{\text{in}} - T_{\text{out}}) \\ &= (7832 \text{ kg/m}^3)(1 \text{ m/s})(0.030 \text{ m})(0.002 \text{ m})(682 \text{ J/kg}\cdot\text{K})(597-47) \text{ K} \\ &= \mathbf{176\ kW}\end{aligned}$$

Discussion By slowing down the conveyance speed of the steel strip would reduce the amount of heat rate needed to be removed from the steel strip in the cooling chamber. Since slowing the conveyance speed allows more time for the steel strip to cool.

1-22 Liquid water is to be heated in an electric teapot. The heating time is to be determined.

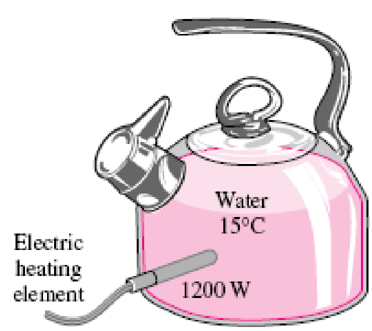
Assumptions **1** Heat loss from the teapot is negligible. **2** Constant properties can be used for both the teapot and the water.

Properties The average specific heats are given to be 0.7 kJ/kg·K for the teapot and 4.18 kJ/kg·K for water.

Analysis We take the teapot and the water in it as the system, which is a closed system (fixed mass). The energy balance in this case can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$E_{\text{in}} = \Delta U_{\text{system}} = \Delta U_{\text{water}} + \Delta U_{\text{teapot}}$$



Then the amount of energy needed to raise the temperature of water and the teapot from 15°C to 95°C is

$$\begin{aligned}E_{\text{in}} &= (mc\Delta T)_{\text{water}} + (mc\Delta T)_{\text{teapot}} \\ &= (1.2\text{ kg})(4.18\text{ kJ/kg}\cdot{}^\circ\text{C})(95-15)^\circ\text{C} + (0.5\text{ kg})(0.7\text{ kJ/kg}\cdot{}^\circ\text{C})(95-15)^\circ\text{C} \\ &= 429.3\text{ kJ}\end{aligned}$$

The 1200-W electric heating unit will supply energy at a rate of 1.2 kW or 1.2 kJ per second. Therefore, the time needed for this heater to supply 429.3 kJ of heat is determined from

$$\Delta t = \frac{\text{Total energy transferred}}{\text{Rate of energy transfer}} = \frac{E_{\text{in}}}{\dot{E}_{\text{transfer}}} = \frac{429.3\text{ kJ}}{1.2\text{ kJ/s}} = 358\text{ s} = \mathbf{6.0\ min}$$

Discussion In reality, it will take more than 6 minutes to accomplish this heating process since some heat loss is inevitable during heating. Also, the specific heat units kJ/kg · °C and kJ/kg · K are equivalent, and can be interchanged.

1-23 C&S A water heater uses 100 kW to heat 60 gallon (0.2271 m^3) of liquid water initially at 20°C. Determine the heating duration such that the water exiting the heater would be in compliance with the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015) service restrictions.

Assumptions **1** Heating of the heater material is negligible (i.e. 100 kW is for heating the water only). **2** Constant properties are used for the water. **3** No water flowing out of the heater during the heating.

Properties The average density and specific heat of water are given to be 970 kg/m^3 and 4.18 kJ/kg·K, respectively.

Analysis The mass of the water in the heater is

$$m = \rho V = (970\ \text{kg/m}^3)(0.2271\ \text{m}^3) = 220.29\ \text{kg}$$

The energy balance in this case can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$E_{\text{in}} = \Delta U_{\text{system}} = \Delta U_{\text{water}}$$

$$E_{\text{in}} = (mc\Delta T)_{\text{water}}$$

In terms of heat transfer rate

$$\dot{Q} = \frac{E_{\text{in}}}{\Delta t} = \frac{(mc\Delta T)_{\text{water}}}{\Delta t}$$

Solving for the heating duration from 20°C to 120°C,

$$\Delta t = \frac{(mc)_{\text{water}}(T_2 - T_1)}{\dot{Q}}$$

$$\Delta t = \frac{(220.29\ \text{kg})(4.18\ \text{kJ/kg}\cdot°\text{C})(100°\text{C})}{100\ \text{kJ/s}} = 920.8\ \text{s} = \mathbf{15.3\ min}$$

Discussion It takes about 15 minutes at 100 kW to heat 60 gallons of liquid water from 20°C to the ASME Boiler and Pressure Vessel Code service restrictions temperature of 120°C. If the heating duration is more than 15.3 minutes, then the final temperature of the water that would exit the heater would be higher than 120°C, which exceeds the code restrictions.

1-24 A boiler (10 kg) is used to heat 20 gallon (0.07571 m^3) of liquid water with 50 kW for 30 minutes. Determine whether this operating condition would be in compliance with the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015) service restrictions.

Assumptions **1** Heat loss from the boiler is negligible. **2** Constant properties are used for both the boiler and the water. **3** The raise in temperature for the boiler and the water is equal. **4** No water flowing out of the boiler during the heating.

Properties The average specific heats are given to be 0.48 kJ/kg·K for the boiler material and 4.18 kJ/kg·K for the water. The average density of the water is 850 kg/m^3.

Analysis We take the boiler and the water in it as a closed system. The energy balance in this case can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$E_{\text{in}} = \Delta U_{\text{system}} = \Delta U_{\text{water}} + \Delta U_{\text{boiler}}$$

$$E_{\text{in}} = (mc\Delta T)_{\text{system}} = (mc\Delta T)_{\text{water}} + (mc\Delta T)_{\text{boiler}}$$

$$E_{\text{in}} = [(mc)_{\text{water}} + (mc)_{\text{boiler}}]\Delta T$$

Solving for the raise in temperature

$$\Delta T = \frac{E_{\text{in}}}{[(\rho V c)_{\text{water}} + (mc)_{\text{boiler}}]}$$

$$\Delta T = \frac{(50\text{ kJ/s})(30 \times 60\text{ s})}{[(850\text{ kg/m}^3)(0.07571\text{ m}^3)(4.18\text{ kJ/kg}\cdot°\text{C}) + (10\text{ kg})(0.48\text{ kJ/kg}\cdot°\text{C})]} = 329°\text{C}$$

The final temperature is

$$T_2 = T_1 + \Delta T = 15°\text{C} + 329°\text{C} = \mathbf{344°C} > 120°\text{C}$$

Discussion The final temperature of the water after 30 minutes of heating at 50 kW is 224°C greater than the ASME Boiler and Pressure Vessel Code service restrictions of 120°C. Thus, the operating condition would not be in compliance with the code. A temperature control mechanism should be implemented to ensure that the water exiting the boiler stays below 120°C.

1-25 Water is heated in an insulated tube by an electric resistance heater. The mass flow rate of water through the heater is to be determined.

Assumptions **1** Water is an incompressible substance with a constant specific heat. **2** The kinetic and potential energy changes are negligible, $\Delta\text{ke} \cong \Delta\text{pe} \cong 0$. **3** Heat loss from the insulated tube is negligible.

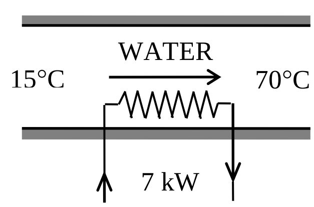
Properties The specific heat of water at room temperature is c_p = 4.18 kJ/kg·°C.

Analysis We take the tube as the system. This is a *control volume* since mass crosses the system boundary during the process. We *observe* that this is a steady-flow process since there is no change with time at any point and thus $\Delta m_{\text{CV}} = 0$ and $\Delta E_{\text{CV}} = 0$, there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and the tube is insulated. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0 \rightarrow \dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{\text{e,in}} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{W}_{\text{e,in}} = \dot{m}c_p(T_2 - T_1)$$



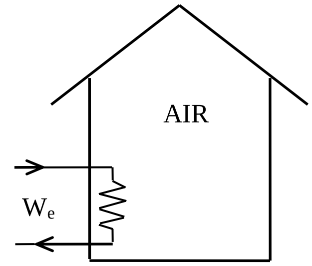
Thus,

$$\dot{m} = \frac{\dot{W}_{\text{e,in}}}{c_p(T_2 - T_1)} = \frac{7\text{ kJ/s}}{(4.18\text{ kJ/kg}\cdot°\text{C})(70-15)°\text{C}} = \mathbf{0.0304\ kg/s}$$

1-26 It is observed that the air temperature in a room heated by electric baseboard heaters remains constant even though the heater operates continuously when the heat losses from the room amount to 7000 kJ/h. The power rating of the heater is to be determined.

Assumptions **1** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of -141°C and 3.77 MPa. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** The temperature of the room remains constant during this process.

Analysis We take the room as the system. The energy balance in this case reduces to



$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$W_{e,in} - Q_{out} = \Delta U = 0$$

$$W_{e,in} = Q_{out}$$

since $\Delta U = mc_v\Delta T = 0$ for isothermal processes of ideal gases. Thus,

$$\dot{W}_{e,in} = \dot{Q}_{out} = 7000\text{kJ/h}\left(\frac{1\text{kW}}{3600\text{kJ/h}}\right) = \mathbf{1.94\ kW}$$

1-27 A room is heated by an electrical resistance heater placed in a short duct in the room in 15 min while the room is losing heat to the outside, and a 300-W fan circulates the air steadily through the heater duct. The power rating of the electric heater and the temperature rise of air in the duct are to be determined.

Assumptions **1** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of -141°C and 3.77 MPa. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** Constant specific heats at room temperature can be used for air. This assumption results in negligible error in heating and air-conditioning applications. **3** Heat loss from the duct is negligible. **4** The house is air-tight and thus no air is leaking in or out of the room.

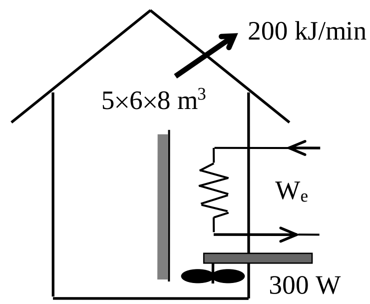
Properties The gas constant of air is R = 0.287 kPa.m^3/kg.K (Table A-1). Also, c_p = 1.007 kJ/kg·K for air at room temperature (Table A-15) and $c_v = c_p - R$ = 0.720 kJ/kg·K.

Analysis (*a*) We first take the air in the room as the system. This is a constant volume *closed system* since no mass crosses the system boundary. The energy balance for the room can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$W_{\text{e,in}} + W_{\text{fan, in}} - Q_{\text{out}} = \Delta U$$

$$(\dot{W}_{\text{e,in}} + \dot{W}_{\text{fan, in}} - \dot{Q}_{\text{out}})\Delta t = m(u_2 - u_1) \cong mc_v(T_2 - T_1)$$



The total mass of air in the room is

$$V = 5 \times 6 \times 8 \text{ m}^3 = 240 \text{ m}^3$$

$$m = \frac{P_1 V}{RT_1} = \frac{(98 \text{ kPa})(240 \text{ m}^3)}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(288 \text{ K})} = 284.6 \text{ kg}$$

Then the power rating of the electric heater is determined to be

$$\dot{W}_{\text{e,in}} = \dot{Q}_{\text{out}} - \dot{W}_{\text{fan, in}} + mc_v(T_2 - T_1)/\Delta t$$
$$= (200/60 \text{ kJ/s}) - (0.3 \text{ kJ/s}) + (284.6 \text{ kg})(0.720 \text{ kJ/kg}\cdot\text{°C})(25-15\text{°C})/(18\times 60 \text{ s}) = \mathbf{4.93\ kW}$$

(*b*) The temperature rise that the air experiences each time it passes through the heater is determined by applying the energy balance to the duct,

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{\text{e,in}} + \dot{W}_{\text{fan, in}} + \dot{m}h_1 = \dot{Q}_{\text{out}}^{\nearrow 0} + \dot{m}h_2 \quad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{W}_{\text{e,in}} + \dot{W}_{\text{fan, in}} = \dot{m}\Delta h = \dot{m}c_p\Delta T$$

Thus,

$$\Delta T = \frac{\dot{W}_{\text{e,in}} + \dot{W}_{\text{fan, in}}}{\dot{m}c_p} = \frac{(4.93 + 0.3)\text{kJ/s}}{(50/60 \text{ kg/s})(1.007 \text{ kJ/kg}\cdot\text{K})} = \mathbf{6.2°C}$$

1-28 Air is moved through the resistance heaters in a 1200-W hair dryer by a fan. The volume flow rate of air at the inlet and the velocity of the air at the exit are to be determined.

Assumptions **1** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of -141°C and 3.77 MPa. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** Constant specific heats at room temperature can be used for air. **4** The power consumed by the fan and the heat losses through the walls of the hair dryer are negligible.

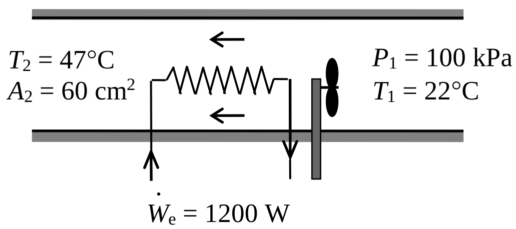
Properties The gas constant of air is R = 0.287 kPa.m^3/kg.K (Table A-1). Also, c_p = 1.007 kJ/kg·K for air at room temperature (Table A-15).

Analysis (a) We take the hair dryer as the system. This is a *control volume* since mass crosses the system boundary during the process. We observe that this is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$, and there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0 \rightarrow \dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{\text{e,in}} + \dot{W}_{\text{fan, in}}^{\nearrow 0} + \dot{m}h_1 = \dot{Q}_{out}^{\nearrow 0} + \dot{m}h_2 \quad (\text{since } \Delta ke \cong \Delta pe \cong 0)$$

$$\dot{W}_{\text{e,in}} = \dot{m}c_p(T_2 - T_1)$$



Thus,

$$\dot{m} = \frac{\dot{W}_{\text{e,in}}}{c_p(T_2 - T_1)} = \frac{1.2 \text{ kJ/s}}{(1.007 \text{ kJ/kg}\cdot{}^\circ\text{C})(47-22)^\circ\text{C}} = 0.04767 \text{ kg/s}$$

Then,

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(295 \text{ K})}{100 \text{ kPa}} = 0.8467 \text{ m}^3/\text{kg}$$

$$\dot{V}_1 = \dot{m}v_1 = (0.04767 \text{ kg/s})(0.8467\text{m}^3/\text{kg}) = \mathbf{0.0404 \; m^3/s}$$

(b) The exit velocity of air is determined from the conservation of mass equation,

$$v_2 = \frac{RT_2}{P_2} = \frac{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(320 \text{ K})}{100 \text{ kPa}} = 0.9184 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{1}{v_2}A_2V_2 \longrightarrow V_2 = \frac{\dot{m}v_2}{A_2} = \frac{(0.04767 \text{ kg/s})(0.9184 \text{ m}^3/\text{kg})}{60\times10^{-4} \text{ m}^2} = \mathbf{7.30 \; m/s}$$

1-29E Air gains heat as it flows through the duct of an air-conditioning system. The velocity of the air at the duct inlet and the temperature of the air at the exit are to be determined.

Assumptions **1** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values of -222°F and 548 psia. **2** The kinetic and potential energy changes are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** Constant specific heats at room temperature can be used for air. This assumption results in negligible error in heating and air-conditioning applications.

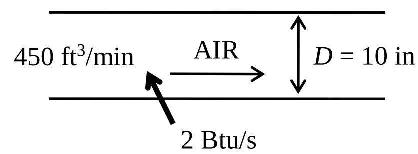
Properties The gas constant of air is R = 0.3704 psia·ft^3/lbm·R (Table A-1E). Also, c_p = 0.240 Btu/lbm·R for air at room temperature (Table A-15E).

Analysis We take the air-conditioning duct as the system. This is a *control volume* since mass crosses the system boundary during the process. We *observe* that this is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV} = 0$ and $\Delta E_{CV} = 0$, there is only one inlet and one exit and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and heat is lost from the system. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0 \rightarrow \dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{\text{in}} + \dot{m}h_1 = \dot{m}h_2 \quad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{Q}_{\text{in}} = \dot{m}c_p(T_2 - T_1)$$



(*a*) The inlet velocity of air through the duct is determined from

$$V_1 = \frac{\dot{V}_1}{A_1} = \frac{\dot{V}_1}{\pi r^2} = \frac{450\ \text{ft}^3/\text{min}}{\pi(5/12\ \text{ft})^2} = \mathbf{825\ ft/min}$$

(*b*) The mass flow rate of air becomes

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.3704\ \text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(510\ \text{R})}{15\ \text{psia}} = 12.6\ \text{ft}^3/\text{lbm}$$

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{450\ \text{ft}^3/\text{min}}{12.6\ \text{ft}^3/\text{lbm}} = 35.7\ \text{lbm/min} = 0.595\ \text{lbm/s}$$

Then the exit temperature of air is determined to be

$$T_2 = T_1 + \frac{\dot{Q}_{\text{in}}}{\dot{m}c_p} = 50°\text{F} + \frac{2\ \text{Btu/s}}{(0.595\ \text{lbm/s})(0.240\ \text{Btu/lbm}\cdot°\text{F})} = \mathbf{64.0°F}$$

1-30 Liquid water entering at 10°C and flowing at 10 g/s(0.01 kg/s) is heated in a circular tube by an electrical heater at 10 kW. Determine whether the water exit temperature would be below 79°C and comply with the ASME Code for Process Piping, and the minimum mass flow rate to keep the exit temperature below 79°C.

Assumptions **1** Water is an incompressible substance with constant properties. **2** Heat loss from the tube is negligible. **3** Steady operating conditions.

Properties The specific heat of water is given as 4.18 kJ/kg·K.

Analysis The tube is taken as a control volume. For steady state flow, the mass flow rate at the inlet is equal to the mass flow rate at the exit:

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

The energy balance for the control volume is

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}}$$

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{Q}_{\text{heater}} + \dot{m}h_1 = \dot{m}h_2$$

$$\dot{Q}_{\text{heater}} = \dot{m}c_p(T_2 - T_1)$$

Solving for the exit temperature,

$$T_2 = \frac{\dot{Q}_{\text{heater}}}{\dot{m}c_p} + T_1 = \frac{10 \text{ kJ/s}}{(0.01 \text{ kg/s})(4.18 \text{ kJ/kg}\cdot°\text{C})} + 10°\text{C} = \mathbf{249°C} > 79°\text{C}$$

Thus, the exit temperature is not in compliance with the ASME Code for Process Piping for PVDC lining.

The minimum mass flow rate needed to keep the water exit temperature from exceeding 79°C is

$$\dot{m} \geq \frac{\dot{Q}_{\text{heater}}}{(T_2 - T_1)c_p}$$

$$\geq \frac{10 \text{ kJ/s}}{(79 - 10)°\text{C}(4.18 \text{ kJ/kg}\cdot°\text{C})}$$

$$\geq \mathbf{0.0347 \text{ kg/s}}$$

The higher the value of mass flow rate, the lower the water exit temperature is achieved.

Discussion If the desire is to have higher exit temperature, then a different thermoplastic lining should be used. Polytetrafluoroethylene (PTFE) lining has a recommended maximum temperature of 260°C by the ASME Code for Process Piping (ASME B31.3-2014, A323).

Heat Transfer Mechanisms

1-31C The thermal conductivity of a material is the rate of heat transfer through a unit thickness of the material per unit area and per unit temperature difference. The thermal conductivity of a material is a measure of how fast heat will be conducted in that material.

1-32C Diamond is a better heat conductor.

1-33C The thermal conductivity of gases is proportional to the square root of absolute temperature. The thermal conductivity of most liquids, however, decreases with increasing temperature, with water being a notable exception.

1-34C Superinsulations are obtained by using layers of highly reflective sheets separated by glass fibers in an evacuated space. Radiation heat transfer between two surfaces is inversely proportional to the number of sheets used and thus heat loss by radiation will be very low by using this highly reflective sheets. At the same time, evacuating the space between the layers forms a vacuum under 0.000001 atm pressure which minimize conduction or convection through the air space between the layers.

1-35C Most ordinary insulations are obtained by mixing fibers, powders, or flakes of insulating materials with air. Heat transfer through such insulations is by conduction through the solid material, and conduction or convection through the air space as well as radiation. Such systems are characterized by apparent thermal conductivity instead of the ordinary thermal conductivity in order to incorporate these convection and radiation effects.

1-36C The mechanisms of heat transfer are conduction, convection and radiation. Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas which is in motion, and it involves combined effects of conduction and fluid motion. Radiation is energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules.

1-37C Conduction is expressed by Fourier's law of conduction as $\dot{Q}_{\text{cond}} = -kA\dfrac{dT}{dx}$ where dT/dx is the temperature gradient, k is the thermal conductivity, and A is the area which is normal to the direction of heat transfer.

Convection is expressed by Newton's law of cooling as $\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$ where h is the convection heat transfer coefficient, A_s is the surface area through which convection heat transfer takes place, T_s is the surface temperature and T_∞ is the temperature of the fluid sufficiently far from the surface.

Radiation is expressed by Stefan-Boltzman law as $\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4)$ where ε is the emissivity of surface, A_s is the surface area, T_s is the surface temperature, T_{surr} is the average surrounding surface temperature and $\sigma = 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4$ is the Stefan-Boltzman constant.

1-38C Convection involves fluid motion, conduction does not. In a solid we can have only conduction.

1-39C No. It is purely by radiation.

1-40C In forced convection the fluid is forced to move by external means such as a fan, pump, or the wind. The fluid motion in natural convection is due to buoyancy effects only.

1-41C In solids, conduction is due to the combination of the vibrations of the molecules in a lattice and the energy transport by free electrons. In gases and liquids, it is due to the collisions of the molecules during their random motion.

1-42C The parameters that effect the rate of heat conduction through a windowless wall are the geometry and surface area of wall, its thickness, the material of the wall, and the temperature difference across the wall.

1-43C In a typical house, heat loss through the wall with glass window will be larger since the glass is much thinner than a wall, and its thermal conductivity is higher than the average conductivity of a wall.

1-44C The house with the lower rate of heat transfer through the walls will be more energy efficient. Heat conduction is proportional to thermal conductivity (which is 0.72 W/m.°C for brick and 0.17 W/m.°C for wood, Table 1-1) and inversely proportional to thickness. The wood house is more energy efficient since the wood wall is twice as thick but it has about one-fourth the conductivity of brick wall.

1-45C The rate of heat transfer through both walls can be expressed as

$$\dot{Q}_{\text{wood}} = k_{\text{wood}} A \frac{T_1 - T_2}{L_{\text{wood}}} = (0.16 \text{ W/m}\cdot°\text{C}) A \frac{T_1 - T_2}{0.1 \text{ m}} = 1.6A(T_1 - T_2)$$

$$\dot{Q}_{\text{brick}} = k_{\text{brick}} A \frac{T_1 - T_2}{L_{\text{brick}}} = (0.72 \text{ W/m}\cdot°\text{C}) A \frac{T_1 - T_2}{0.25 \text{ m}} = 2.88A(T_1 - T_2)$$

where thermal conductivities are obtained from Table A-5. Therefore, heat transfer through the brick wall will be larger despite its higher thickness.

1-46C Emissivity is the ratio of the radiation emitted by a surface to the radiation emitted by a blackbody at the same temperature. Absorptivity is the fraction of radiation incident on a surface that is absorbed by the surface. The Kirchhoff's law of radiation states that the emissivity and the absorptivity of a surface are equal at the same temperature and wavelength.

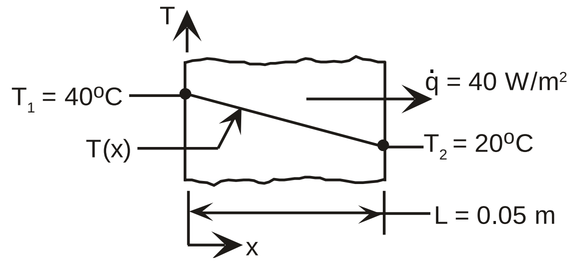
1-47C A blackbody is an idealized body which emits the maximum amount of radiation at a given temperature and which absorbs all the radiation incident on it. Real bodies emit and absorb less radiation than a blackbody at the same temperature.

1-48 The thermal conductivity of a wood slab subjected to a given heat flux of 40 W/m^2 with constant left and right surface temperatures of 40ºC and 20ºC is to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the wood slab remain constant at the specified values. **2** Heat transfer through the wood slab is one dimensional since the thickness of the slab is small relative to other dimensions. **3** Thermal conductivity of the wood slab is constant.

Analysis The thermal conductivity of the wood slab is determined directly from Fourier's relation to be

$$k = \dot{q}\frac{L}{T_1 - T_2} = \left(40\ \frac{\text{W}}{\text{m}^2}\right)\frac{0.05\ \text{m}}{(40-20)°\text{C}} = \mathbf{0.10\ W/m{\cdot}K}$$



Discussion Note that the ºC or K temperature units may be used interchangeably when evaluating a temperature difference.

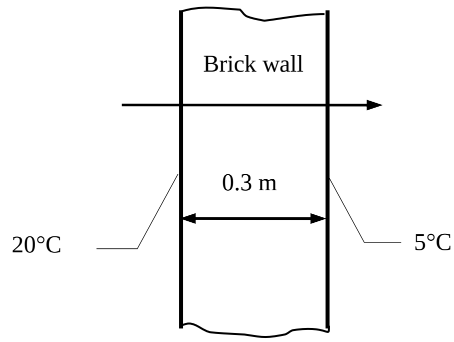
1-49 The inner and outer surfaces of a brick wall are maintained at specified temperatures. The rate of heat transfer through the wall is to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the wall remain constant at the specified values. **2** Thermal properties of the wall are constant.

Properties The thermal conductivity of the wall is given to be k = 0.69 W/m·°C.

Analysis Under steady conditions, the rate of heat transfer through the wall is

$$\dot{Q}_{\text{cond}} = kA\frac{\Delta T}{L} = (0.69\ \text{W/m}\cdot°\text{C})(4\times 7\ \text{m}^2)\frac{(20-5)°\text{C}}{0.3\ \text{m}} = \mathbf{966\ W}$$



1-50E The inner and outer glasses of a double pane window with a 0.5-in air space are at specified temperatures. The rate of heat transfer through the window is to be determined

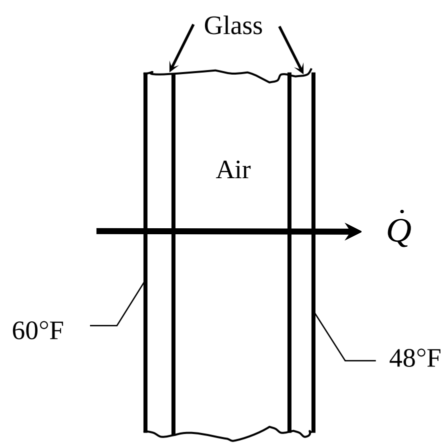
Assumptions **1** Steady operating conditions exist since the surface temperatures of the glass remain constant at the specified values. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the air are constant.

Properties The thermal conductivity of air at the average temperature of (60+48)/2 = 54°F is k = 0.01419 Btu/h·ft·°F (Table A-15E).

Analysis The area of the window and the rate of heat loss through it are

$$A = (4\ \text{ft})\times(4\ \text{ft}) = 16\ \text{ft}^2$$

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.01419\ \text{Btu/h.ft.°F})(16\ \text{ft}^2)\frac{(60-48)°\text{F}}{0.25/12\ \text{ft}} = \mathbf{131\ Btu/h}$$



1-51 The inner and outer surfaces of a window glass are maintained at specified temperatures. The amount of heat transfer through the glass in 5 h is to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the glass remain constant at the specified values. **2** Thermal properties of the glass are constant.

Properties The thermal conductivity of the glass is given to be k = 0.78 W/m·°C.

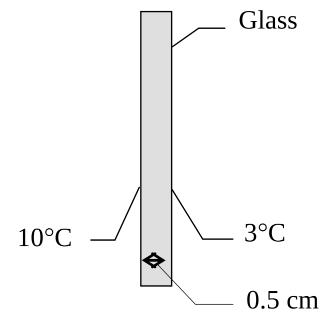
Analysis Under steady conditions, the rate of heat transfer through the glass by conduction is

$$\dot{Q}_{\text{cond}} = kA\frac{\Delta T}{L} = (0.78\text{ W/m}\cdot°\text{C})(2\times 2\text{ m}^2)\frac{(10-3)°\text{C}}{0.005\text{m}} = 4368\text{ W}$$

Then the amount of heat transfer over a period of 5 h becomes

$$Q = \dot{Q}_{\text{cond}}\Delta t = (4.368\text{ kJ/s})(5\times 3600\text{ s}) = \mathbf{78{,}620\ \ kJ}$$

If the thickness of the glass doubled to 1 cm, then the amount of heat transfer will go down by half to **39,310 kJ**.



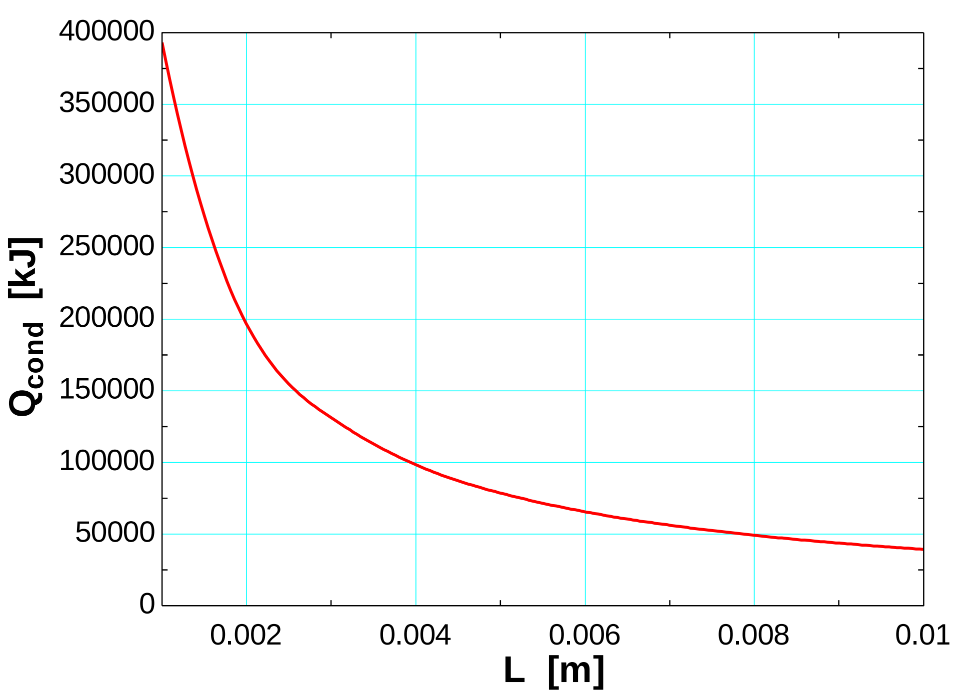
1-52 Prob. 1-51 is reconsidered. The amount of heat loss through the glass as a function of the window glass thickness is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.005 [m]
A=2*2 [m^2]
T_1=10 [C]
T_2=3 [C]
k=0.78 [W/m-C]
time=5*3600 [s]

"ANALYSIS"
Q_dot_cond=k*A*(T_1-T_2)/L
Q_cond=Q_dot_cond*time*Convert(J, kJ)
```

L [m]	Q_{cond} [kJ]
0.001	393120
0.002	196560
0.003	131040
0.004	98280
0.005	78624
0.006	65520
0.007	56160
0.008	49140
0.009	43680
0.01	39312



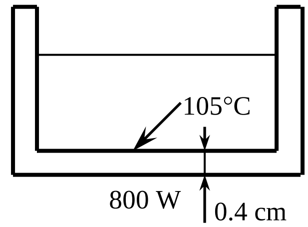
1-53 Heat is transferred steadily to boiling water in the pan through its bottom. The inner surface temperature of the bottom of the pan is given. The temperature of the outer surface is to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the pan remain constant at the specified values. **2** Thermal properties of the aluminum pan are constant.

Properties The thermal conductivity of the aluminum is given to be k = 237 W/m·°C.

Analysis The heat transfer area is

$$A = \pi r^2 = \pi (0.075 \text{ m})^2 = 0.0177 \text{ m}^2$$



Under steady conditions, the rate of heat transfer through the bottom of the pan by conduction is

$$\dot{Q} = kA\frac{\Delta T}{L} = kA\frac{T_2 - T_1}{L}$$

Substituting,

$$800 \text{ W} = (237 \text{ W/m}\cdot°\text{C})(0.0177 \text{ m}^2)\frac{T_2 - 105°\text{C}}{0.004 \text{ m}}$$

which gives

$$T_2 = \mathbf{105.76°C}$$

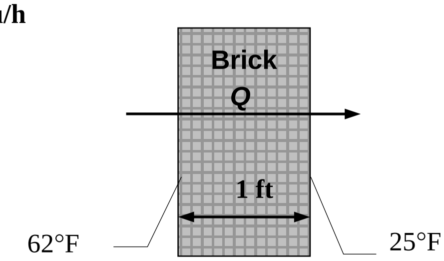
1-54E The inner and outer surface temperatures of the wall of an electrically heated home during a winter night are measured. The rate of heat loss through the wall that night and its cost are to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the wall remain constant at the specified values during the entire night. **2** Thermal properties of the wall are constant.

Properties The thermal conductivity of the brick wall is given to be k = 0.42 Btu/h·ft·°F.

Analysis (a) Noting that the heat transfer through the wall is by conduction and the surface area of the wall is $A = 20 \text{ ft} \times 10 \text{ ft} = 200 \text{ ft}^2$, the steady rate of heat transfer through the wall can be determined from

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.42 \text{ Btu/h.ft.°F})(200 \text{ ft}^2)\frac{(62-25)°\text{F}}{1 \text{ ft}} = \mathbf{3108 \text{ Btu/h}}$$



or 0.911 kW since 1 kW = 3412 Btu/h.

(b) The amount of heat lost during an 8 hour period and its cost are

$$Q = \dot{Q}\Delta t = (0.911 \text{ kW})(8 \text{ h}) = 7.288 \text{ kWh}$$

$$\begin{aligned}\text{Cost} &= (\text{Amount of energy})(\text{Unit cost of energy}) \\ &= (7.288 \text{ kWh})(\$0.07/\text{kWh}) \\ &= \mathbf{\$0.51}\end{aligned}$$

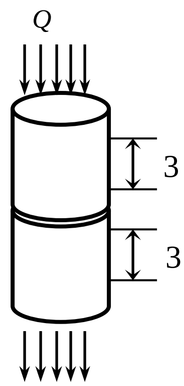
Therefore, the cost of the heat loss through the wall to the home owner that night is $0.51.

1-55 The thermal conductivity of a material is to be determined by ensuring one-dimensional heat conduction, and by measuring temperatures when steady operating conditions are reached.

Assumptions 1 Steady operating conditions exist since the temperature readings do not change with time. **2** Heat losses through the lateral surfaces of the apparatus are negligible since those surfaces are well-insulated, and thus the entire heat generated by the heater is conducted through the samples. **3** The apparatus possesses thermal symmetry.

Analysis The electrical power consumed by the heater and converted to heat is

$$\dot{W}_e = \mathbf{V}I = (110\ \text{V})(0.6\ \text{A}) = 66\ \text{W}$$



The rate of heat flow through each sample is

$$\dot{Q} = \frac{\dot{W}_e}{2} = \frac{66\ \text{W}}{2} = 33\ \text{W}$$

Then the thermal conductivity of the sample becomes

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.04\ \text{m})^2}{4} = 0.001257\ \text{m}^2$$

$$\dot{Q} = kA\frac{\Delta T}{L} \longrightarrow k = \frac{\dot{Q}L}{A\Delta T} = \frac{(33\ \text{W})(0.03\text{m})}{(0.001257\ \text{m}^2)(10°\text{C})} = \mathbf{78.8\ W/m.°C}$$

1-56 The thermal conductivity of a material is to be determined by ensuring one-dimensional heat conduction, and by measuring temperatures when steady operating conditions are reached.

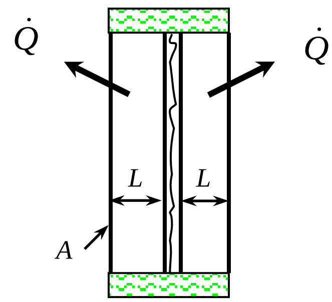
Assumptions 1 Steady operating conditions exist since the temperature readings do not change with time. **2** Heat losses through the lateral surfaces of the apparatus are negligible since those surfaces are well-insulated, and thus the entire heat generated by the heater is conducted through the samples. **3** The apparatus possesses thermal symmetry.

Analysis For each sample we have

$$\dot{Q} = 25/2 = 12.5\ \text{W}$$

$$A = (0.1\ \text{m})(0.1\ \text{m}) = 0.01\ \text{m}^2$$

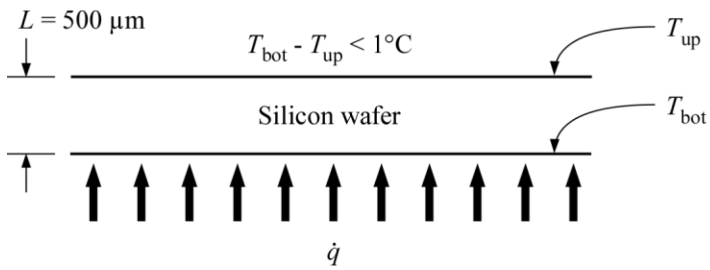
$$\Delta T = 82 - 74 = 8°\text{C}$$



Then the thermal conductivity of the material becomes

$$\dot{Q} = kA\frac{\Delta T}{L} \longrightarrow k = \frac{\dot{Q}L}{A\Delta T} = \frac{(12.5\ \text{W})(0.005\text{m})}{(0.01\ \text{m}^2)(8°\text{C})} = \mathbf{0.781\ W/m.°C}$$

1-57 To prevent a silicon wafer from warping, the temperature difference across its thickness cannot exceed 1°C. The maximum allowable heat flux on the bottom surface of the wafer is to be determined.



Assumptions **1** Heat conduction is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity is constant.

Properties The thermal conductivity of silicon at 27°C (300 K) is 148 W/m·K (Table A-3).

Analysis For steady heat transfer, the Fourier's law of heat conduction can be expressed as

$$\dot{q} = -k\frac{dT}{dx} = -k\frac{T_{up} - T_{bot}}{L}$$

Thus, the maximum allowable heat flux so that $T_{bot} - T_{up} < 1°C$ is

$$\dot{q} \le k\frac{T_{bot} - T_{up}}{L} = (148\ \text{W/m}\cdot\text{K})\frac{1\,\text{K}}{500\times10^{-6}\ \text{m}}$$

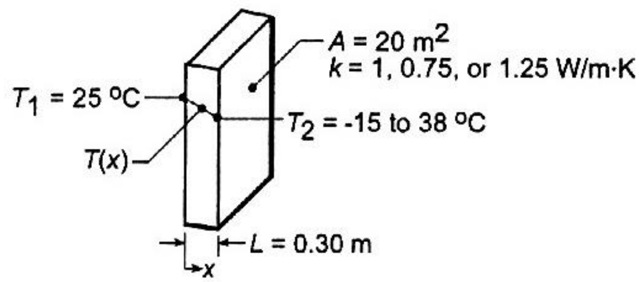
$$\dot{q} \le \mathbf{2.96\times10^{5}\ W/m^{2}}$$

Discussion With the upper surface of the wafer maintained at 27°C, if the bottom surface of the wafer is exposed to a flux greater than 2.96×10^5 W/m^2, the temperature gradient across the wafer thickness could be significant enough to cause warping.

1-58 Heat loss by conduction through a concrete wall as a function of ambient air temperatures ranging from -15 to 38°C is to be determined.

Assumptions **1** One-dimensional conduction. **2** Steady-state conditions exist. **3** Constant thermal conductivity. **4** Outside wall temperature is that of the ambient air.

Properties The thermal conductivity is given to be k = 0.75, 1 or 1.25 W/m·K.



Analysis From Fourier's law, it is evident that the gradient, $dT/dx = -\dot{q}/k$, is a constant, and hence the temperature distribution is linear, if $\dot{q}$ and k are each constant. The heat flux must be constant under one-dimensional, steady-state conditions; and k are each approximately constant if it depends only weakly on temperature. The heat flux and heat rate for the case when the outside wall temperature is $T_2 = -15°C$ and k = 1 W/m·K are:

$$\dot{q} = -k\frac{dT}{dx} = k\frac{T_1 - T_2}{L} = (1\,\text{W/m}\cdot\text{K})\frac{25°C - (-15°C)}{0.30\,\text{m}} = 133.3\,\text{W/m}^2 \tag{1}$$

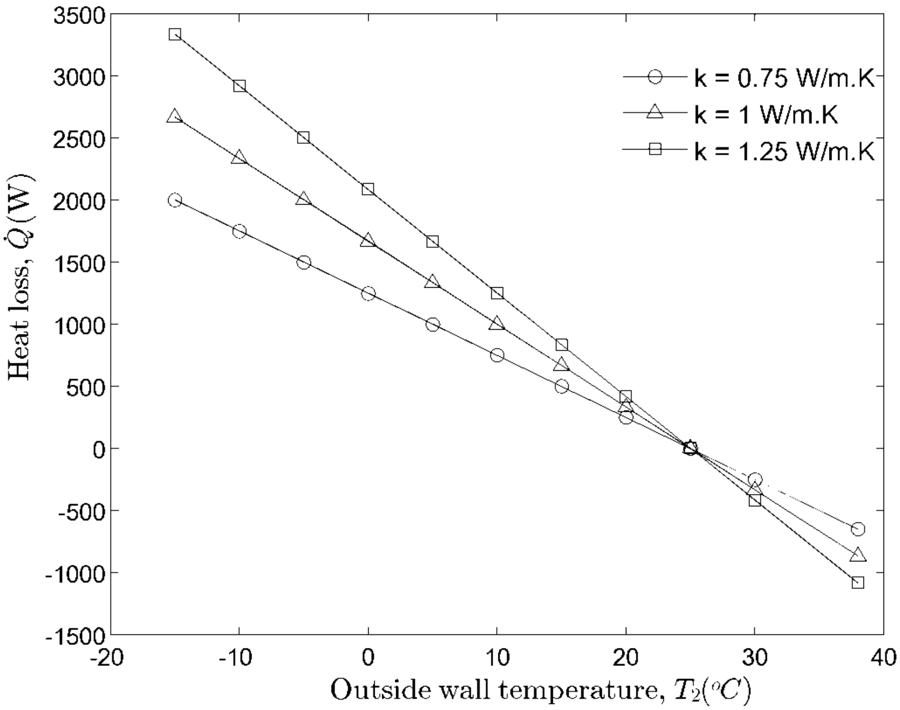
$$\dot{Q} = \dot{q}\cdot A = (133.3\,\text{W/m}^2)\cdot(20\,\text{m}^2) = \mathbf{2667\ W} \tag{2}$$

Combining Eqs. (1) and (2), the heat rate $\dot{Q}$ can be determined for the range of ambient temperature, $-15 \le T_2 \le 38$°C, with different wall thermal conductivities, k.

Discussion (1) Notice that from the graph, the heat loss curves are linear for all three thermal conductivities. This is true because under steady-state and constant k conditions, the temperature distribution in the wall is linear. (2) As the value of k increases, the slope of the heat loss curve becomes steeper. This shows that for insulating materials (very low k), the heat loss curve would be relatively flat. The magnitude of the heat loss also increases with increasing thermal conductivity. (3) At $T_2 = 25°C$, all the three heat loss curves intersect at zero; because $T_1 = T_2$ (when the inside and outside temperatures are the same), thus there is no heat conduction through the wall. This shows that heat conduction can only occur when there is temperature difference.

The results for the heat loss $\dot{Q}$ with different thermal conductivities k are tabulated and plotted as follows:

	$\dot{Q}$ [W]		
T_2 [°C]	k = 0.75 W/m·K	k = 1 W/m·K	k = 1.25 W/m·K
-15	2000	2667	3333
-10	1750	2333	2917
-5	1500	2000	2500
0	1250	1667	2083
5	1000	1333	1667
10	750	1000	1250
15	500	666.7	833.3
20	250	333.3	416.7
25	0	0	0
30	-250	-333.3	-416.7
38	-650	-866.7	-1083



k = 0.75 W/m.K
k = 1 W/m.K
k = 1.25 W/m.K
Heat loss, $\dot{Q}$(W)
Outside wall temperature, T_2(°C)
3500
3000
2500
2000
1500
1000
500
0
-500
-1000
-1500
-20
-10
0
10
20
30
40

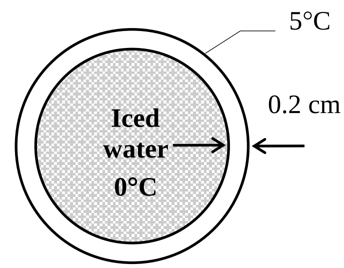
1-59 A hollow spherical iron container is filled with iced water at 0°C. The rate of heat gain by the iced water and the rate at which ice melts in the container are to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the wall remain constant at the specified values. **2** Heat transfer through the shell is one-dimensional. **3** Thermal properties of the iron shell are constant. **4** The inner surface of the shell is at the same temperature as the iced water, 0°C. **5** Treat the spherical shell as a plain wall and use the outer area.

Properties The thermal conductivity of iron is k = 80.2 W/m·°C (Table A-3). The heat of fusion of water is given to be 333.7 kJ/kg.

Analysis This spherical shell can be approximated as a plate of thickness 0.4 cm and area

$$A = \pi D^2 = \pi(0.2\text{ m})^2 = 0.126\text{ m}^2$$



Then the rate of heat transfer through the shell by conduction is

$$\dot{Q}_{\text{cond}} = kA\frac{\Delta T}{L} = (80.2\text{ W/m}\cdot°\text{C})(0.126\text{ m}^2)\frac{(5-0)°\text{C}}{0.002\text{ m}} = 25{,}263\text{ W} = \mathbf{25.3\,kW}$$

Considering that it takes 333.7 kJ of energy to melt 1 kg of ice at 0°C, the rate at which ice melts in the container can be determined from

$$\dot{m}_{\text{ice}} = \frac{\dot{Q}}{h_{if}} = \frac{25.263\text{ kJ/s}}{333.7\text{ kJ/kg}} = \mathbf{0.0757\ kg/s}$$

Discussion We should point out that this result is slightly in error for approximating a curved wall as a plain wall. The error in this case is very small because of the large diameter to thickness ratio. For better accuracy, we could use the inner surface area (D = 19.6 cm) or the mean surface area (D = 19.8 cm) in the calculations.

1-60 Prob. 1-59 is reconsidered. The rate at which ice melts as a function of the container thickness is to be plotted.

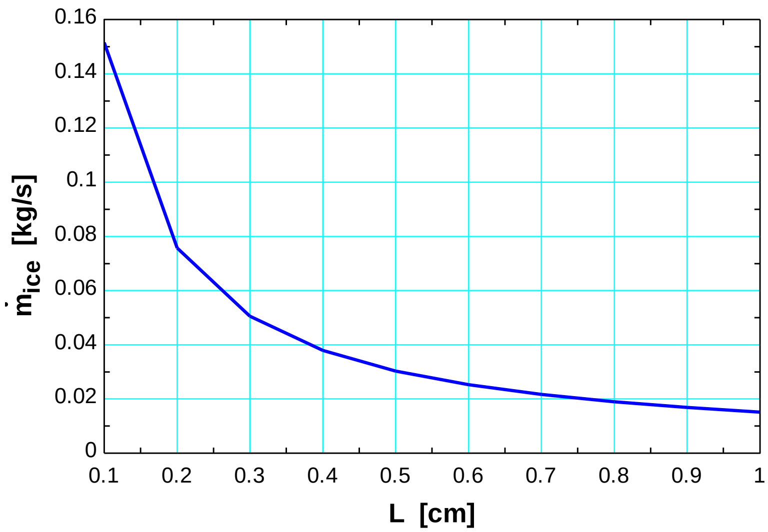
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.2 [m]
L=0.2 [cm]
T_1=0 [C]
T_2=5 [C]

"PROPERTIES"
h_if=333.7 [kJ/kg]
k=k_(Iron, 25)

"ANALYSIS"
A=pi*D^2
Q_dot_cond=k*A*(T_2-T_1)/(L*Convert(cm, m))
m_dot_ice=(Q_dot_cond*Convert(W, kW))/h_if
```

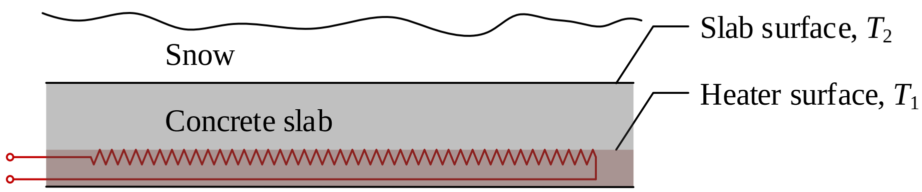
L [cm]	m_{ice} [kg/s]
0.1	0.1515
0.2	0.07574
0.3	0.0505
0.4	0.03787
0.5	0.0303
0.6	0.02525
0.7	0.02164
0.8	0.01894
0.9	0.01683
1	0.01515



1-61 C&S A 5 m × 5 m concrete slab with embedded heating cable melts snow at a rate of 0.1 kg/s. The power density (heat flux) for the embedded heater is to be determined whether it is in compliance with the NFPA 70 code. Also, the temperature difference between the heater surface and the slab surface is to be determined whether it exceeds 21°C, as recommended in the ASHRAE Handbook to minimize thermal stress.

Assumptions **1** Steady operating conditions. **2** Slab surface and heater surface temperatures are uniform. **3** Heat transfer through the concrete layer is one-dimensional. **3** Properties of the concrete are constant. **4** The heater heats the surface uniformly.

Properties The thermal conductivity of concrete is given as 1.4 W/m·K. The latent heat of fusion for water is 333.7 kJ/kg (Table A-2).



Analysis The heat rate required for melting snow at 0.1 kg/s is

$$\dot{Q} = \dot{m}_{\text{ice}} h_{if} = (0.1\,\text{kg/s})(333700\,\text{J/kg}) = 33370\,\text{W}$$

For a surface area of 5 m × 5 m, the power density (heat flux) is

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{33370\,\text{W}}{25\,\text{m}^2} = \mathbf{1335\ W/m^2} > 1300\,\text{W/m}^2$$

To determine the temperature difference between the heater surface(T_1) and the slab surface(T_2), we use the Fourier law of conduction:

$$\dot{q} = k\frac{T_1 - T_2}{L}$$

or

$$T_1 - T_2 = \frac{\dot{q}L}{k} = \frac{(1335\,\text{W/m}^2)(0.05\,\text{m})}{1.4\,\text{W/m}\cdot\text{K}} = \mathbf{47.7°C} > 21°\text{C}$$

Discussion The power density for the embedded heating cable in the concrete slab slightly exceeds the limit set by the National Electrical Code® (NFPA 70) of 1300 W/m^2. The temperature difference between the heater surface and the slab surface is about 27°C higher than the recommended value by the 2015 ASHRAE Handbook—HVAC Applications, Chapter 51.

1-62E Using the conversion factors between W and Btu/h, m and ft, and °C and °F, the convection coefficient in SI units is to be expressed in Btu/h·ft²·°F.

Analysis The conversion factors for W and m are straightforward, and are given in conversion tables to be

$$1\,\text{W} = 3.41214\,\text{Btu/h}$$

$$1\,\text{m} = 3.2808\,\text{ft}$$

The proper conversion factor between °C into °F in this case is

$$1°\text{C} = 1.8°\text{F}$$

since the °C in the unit W/m²·°C represents *per °C change in temperature*, and 1°C change in temperature corresponds to a change of 1.8°F. Substituting, we get

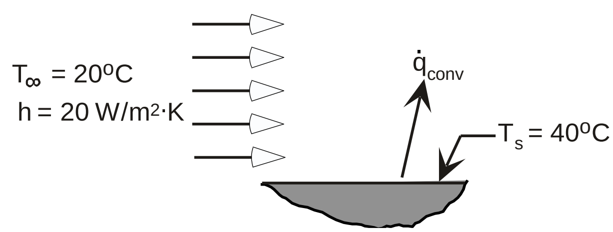
$$1\ \text{W/m}^2\cdot°\text{C} = \frac{3.41214\,\text{Btu/h}}{(3.2808\,\text{ft})^2(1.8°\text{F})} = 0.1761\,\text{Btu/h}\cdot\text{ft}^2\cdot°\text{F}$$

which is the desired conversion factor. Therefore, the given convection heat transfer coefficient in English units is

$$h = 14\ \text{W/m}^2\cdot°\text{C} = 14\times 0.1761\,\text{Btu/h}\cdot\text{ft}^2\cdot°\text{F} = \mathbf{2.47\,Btu/h\cdot ft^2\cdot°F}$$

1-63 The heat flux between air with a constant temperature and convection heat transfer coefficient blowing over a pond at a constant temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Convection heat transfer coefficient is uniform. **3** Heat transfer by radiation is negligible. **4** Air temperature and the surface temperature of the pond remain constant.



Analysis From Newton's law of cooling, the heat flux is given as

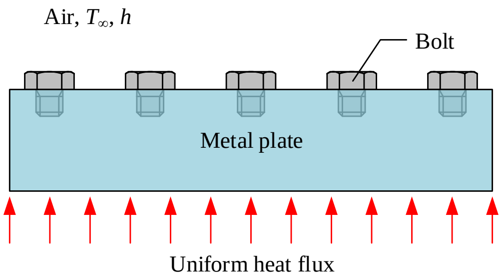
$$\dot{q}_{conv} = h\,(T_s - T_\infty)$$

$$\dot{q}_{conv} = 20\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}\ (40 - 20)°\text{C} = \mathbf{400\ W/m^2}$$

Discussion (1) Note the direction of heat flow is out of the surface since $T_s > T_\infty$; (2) Recognize why units of K in h and units of ºC in (T_s - T_∞) cancel.

1-64 C&S A series of ASME SA-193 carbon steel bolts are bolted to the upper surface of a metal plate. The upper surface is exposed to convection with the ambient air. The bottom surface is subjected to a uniform heat flux. Determine whether the use of the bolts complies with the ASME Boiler and Pressure Vessel Code, where 260°C is the maximum allowable use temperature.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through the metal plate. **3** Uniform heat flux on the bottom surface. **4** Uniform surface temperature at the upper plate surface. **5** The temperature of the bolts is equal to the upper surface temperature of the plate.



Analysis The uniform heat flux subjected on the bottom plate surface is equal to the heat flux transferred by convection on the upper surface.

$$\dot{q}_0 = \dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} = 5000 \text{ W/m}^2$$

From the Newton's law of cooling, we have

$$\dot{q}_{\text{conv}} = h(T_s - T_\infty)$$

Assuming the temperature of the bolts is equal to the upper surface temperature of the plate,

$$T_{\text{bolt}} = T_s = \frac{\dot{q}_{\text{conv}}}{h} + T_\infty = \frac{5000 \text{ W/m}^2}{10 \text{ W/m}^2\cdot\text{K}} + 30°\text{C} = \mathbf{530°C} > 260°\text{C}$$

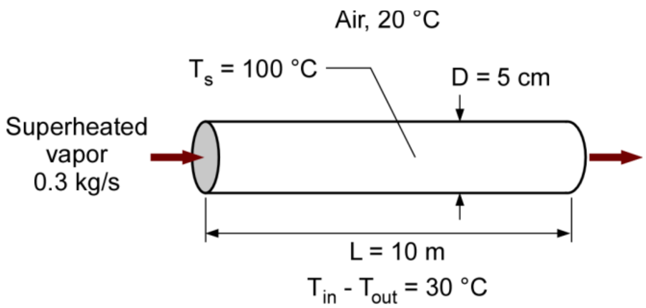
Discussion The temperature of the bolts exceeds the maximum allowable use temperature by 260°C. One way to keep the temperature of the bolts below 260°C is by increasing the convection heat transfer coefficient. Higher convection heat transfer coefficient can be achieved by having forced convection. To keep the upper surface temperature of the plate at 260°C or lower, the convection heat transfer coefficient should be higher than 21.7 W/m²·K.

$$h > \frac{\dot{q}_{\text{conv}}}{T_s - T_\infty} > \frac{5000 \text{ W/m}^2}{(260-30)\text{ K}} > 21.7 \text{ W/m}^2\cdot\text{K}$$

1-65 The convection heat transfer coefficient heat transfer between the surface of a pipe carrying superheated vapor and the surrounding is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by radiation is not considered. **3** Rate of heat loss from the vapor in the pipe is equal to the heat transfer rate by convection between pipe surface and the surrounding.

Properties The specific heat of vapor is given to be 2190 J/kg · °C.



Analysis The surface area of the pipe is

$$A_s = \pi DL = \pi(0.05\,\text{m})(10\,\text{m}) = 1.571\,\text{m}^2$$

The rate of heat loss from the vapor in the pipe can be determined from

$$\begin{aligned}\dot{Q}_{\text{loss}} &= \dot{m}c_p(T_{\text{in}} - T_{\text{out}})\\ &= (0.3\,\text{kg/s})(2190\,\text{J/kg}\cdot{}^\circ\text{C})(30)\,{}^\circ\text{C} = 19710\,\text{J/s}\\ &= 19710\,\text{W}\end{aligned}$$

With the rate of heat loss from the vapor in the pipe assumed equal to the heat transfer rate by convection, the heat transfer coefficient can be determined using the Newton's law of cooling:

$$\dot{Q}_{\text{loss}} = \dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

Rearranging, the heat transfer coefficient is determined to be

$$h = \frac{\dot{Q}_{\text{loss}}}{A_s(T_s - T_\infty)} = \frac{19710\,\text{W}}{(1.571\,\text{m}^2)(100-20)\,{}^\circ\text{C}} = \mathbf{157\ W/m^2\cdot{}^\circ C}$$

Discussion By insulating the pipe surface, heat loss from the vapor in the pipe can be reduced.

1-66 A boiler supplies hot water to a dishwasher through a pipe at 60 g/s. The pipe dimensions are given. The water exits the boiler at 95°C. The pipe section between the boiler and the dishwasher is exposed to convection. The water temperature entering the dishwasher is to be determined whether it meets the ANSI/NSF 3 standard.

Assumptions **1** Constant properties are used for the water. **2** Steady operating conditions. **3** Surface temperature of the pipe is uniform.

Properties The average specific heat of water is given to be 4.20 kJ/kg·K.

Analysis From energy balance, the rate of heat loss from the pipe is equal to the heat transfer rate by convection on the pipe surface:

$$\dot{Q}_{\text{pipe}} = \dot{Q}_{\text{conv}}$$

$$\dot{m} c_p (T_1 - T_2) = h A_s (T_s - T_\infty)$$

Solving for the water temperature entering the dishwasher T_2, we have

$$\begin{aligned} T_2 &= T_1 - \frac{h A_s}{\dot{m} c_p}(T_s - T_\infty) \\ &= T_1 - \frac{\pi D L h}{\dot{m} c_p}(T_s - T_\infty) \\ &= 95^\circ\text{C} - \frac{\pi (0.02\ \text{m})(20\ \text{m})(100\ \text{W/m}^2\cdot\text{K})}{(0.06\ \text{kg/s})(4200\ \text{J/kg}\cdot\text{K})}(50-20)^\circ\text{C} \\ &= \mathbf{80^\circ C} < 82^\circ\text{C} \end{aligned}$$

Discussion The hot water entering the dishwasher is 2°C lower than the temperature required by the ANSI/NSF 3 standard. To increase the water temperature entering the dishwasher, one or combination of the following steps can be taken: (a) add insulation on the pipe wall to reduce the heat loss from the pipe surface; (b) increase the water mass flow rate; (c) reduce the pipe distance between the boiler and the dishwasher; and (d) increase the water temperature coming out from the boiler.

1-67 Hot liquid flows in a pipe with PVDF lining on the inner surface. The pipe outer surface is subjected to uniform heat flux. The liquid mean temperature and convection heat transfer coefficient are given. Determine whether the surface temperature of the lining complies with the ASME Code for Process Piping.

Assumptions **1** Steady operating conditions. **2** Heat transfer is one-dimensional through the pipe wall. **3** Surface temperature is uniform. **4** Thermal properties are constant.

Analysis The surface energy balance on the PVDF lining is

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{Q}_o = \dot{Q}_{\text{conv}}$$

$$\dot{q}_0 A_{s,o} = h A_{s,i} (T_s - T_f)$$

The outer and inner surface areas of the pipe are

$$A_{s,o} = \pi D_o L \quad \text{and} \quad A_{s,i} = \pi D_i L$$

Solving for the lining surface temperature T_s,

$$\dot{q}_0 (\pi D_o L) = h(\pi D_i L)(T_s - T_f)$$

$$T_s = \frac{\dot{q}_0}{h}\frac{D_o}{D_i} + T_f = \frac{(1200\ \text{W/m}^2)(27\ \text{mm})}{(50\ \text{W/m}^2\cdot\text{K})(22\ \text{mm})} + 120^\circ\text{C} = \mathbf{149.5^\circ C} > 135^\circ\text{C}$$

Discussion The surface temperature of the lining exceeds the maximum temperature recommended by the ASME Process Piping code for PVDF lining. A different thermoplastic lining should be used. Polytetrafluoroethylene (PTFE) lining has a recommended maximum temperature of 260°C by the ASME Code for Process Piping (ASME B31.3-2014, A323), which would meet these conditions.

1-68 An electrical resistor with a uniform temperature of 90 °C is in a room at 20 °C. The heat transfer coefficient by convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation heat transfer is negligible. **3** No hot spot exists on the resistor.

Analysis The total heat transfer area of the resistor is

$$A_s = 2(\pi D^2/4) + \pi DL = 2\pi(0.025\,\text{m})^2/4 + \pi(0.025\,\text{m})(0.15\,\text{m}) = 0.01276\,\text{m}^2$$

The electrical energy converted to thermal energy is transferred by convection:

$$\dot{Q}_{\text{conv}} = IV = (5\,\text{A})(6\,\text{V}) = 30\,\text{W}$$

From Newton's law of cooling, the heat transfer by convection is given as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

Rearranging, the heat transfer coefficient is determined to be

$$h = \frac{\dot{Q}_{\text{conv}}}{A_s(T_s - T_\infty)} = \frac{30\,\text{W}}{(0.01276\,\text{m}^2)(90-20)\,°\text{C}} = \mathbf{33.6\,W/m^2\cdot°C}$$

Discussion By comparing the magnitude of the heat transfer coefficient determined here with the values presented in Table 1-5, one can conclude that it is likely that forced convection is taking place rather than free convection.

1-69 C&S An electrical cable is covered with polyethylene insulation and is subjected to convection with the ambient air. Determine whether the insulation surface temperature meets the ASTM D1351 standard for polyethylene insulation.

Assumptions**1** Steady operating conditions. **2** Radiation heat transfer is negligible. **3** No hot spot exists on the cable.

AnalysisThe electrical energy that is converted to thermal energy is determined using the Joule heating relation:

$$\dot{Q} = IV = (1\,\text{A})(30\,\text{V}) = 30\,\text{W}$$

The thermal energy for the joule heating is then transferred through the insulation layer by conduction, and then by convection at the outer surface of the insulation. From the Newton's law of cooling for convection, we have

$$\dot{Q} = hA_s(T_s - T_\infty)$$

Rearranging the equation and solving for the surface temperature,

$$T_s = \frac{\dot{Q}}{h\,A_s} + T_\infty = \frac{30\,\text{W}}{(5\,\text{W/m}^2\cdot\text{K})(0.1\,\text{m}^2)} + 20°\text{C} = \mathbf{80°C} > 75°\text{C}$$

Discussion With the surface temperature being 5°C higher than the specification of the ASTM D1351 standard for polyethylene insulation that means the temperature at the inner surface of the insulation being in contact with the cable would be higher than 80°C. To solve this problem, we will need to use a larger diameter (or thicker) cable. The electrical resistance decreases with increasing cable thickness, which would reduce joule heating. We can also use a different insulation material with a higher temperature rating. From the ASTM database, the crosslinked polyethylene insulation (ASTM D2655) is rated up to 90°C for normal operation.

1-70 An AISI 316 spherical container is used for storing chemical undergoing exothermic reaction that provide a uniform heat flux to its inner surface. The necessary convection heat transfer coefficient to keep the container's outer surface below 50°C is to be determined.

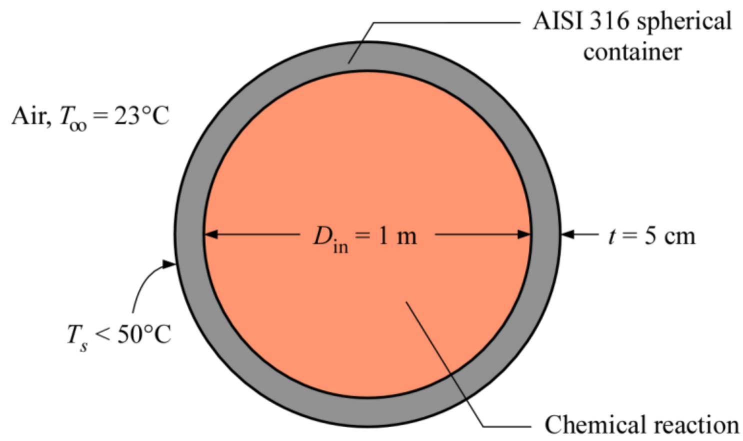
Assumptions **1** Steady operating conditions exist. **2** Negligible thermal storage for the container. **3** Temperature at the surface remained uniform.

Analysis The heat rate from the chemical reaction provided to the inner surface equal to heat rate removed from the outer surface by convection

$$\dot{Q}_{\text{in}} = \dot{Q}_{\text{out}}$$

$$\dot{q}_{\text{reaction}} A_{s,\text{in}} = hA_{s,\text{out}}(T_s - T_\infty)$$

$$\dot{q}_{\text{reaction}} (\pi D_{\text{in}}^2) = h(\pi D_{\text{out}}^2)(T_s - T_\infty)$$



The convection heat transfer coefficient can be determined as

$$h = \frac{\dot{q}_{\text{reaction}}}{T_s - T_\infty}\left(\frac{D_{\text{in}}}{D_{\text{out}}}\right)^2$$

$$= \frac{60000 \text{ W/m}^2}{(50-23)\text{ K}}\left(\frac{1\text{ m}}{1\text{ m} + 2\times 0.05\text{ m}}\right)^2$$

$$= 1840 \text{ W/m}^2\cdot\text{K}$$

To keep the container's outer surface temperature below 50°C, the convection heat transfer coefficient should be

$$h > \mathbf{1840\ W/m^2\cdot K}$$

Discussion From Table 1-5, the typical values for free convection heat transfer coefficient of gases are between 2–25 W/m²·K. Thus, the required h > 1840 W/m²·K is not feasible with free convection of air. To prevent thermal burn, the container's outer surface temperature should be covered with insulation.

1-71 A transistor mounted on a circuit board is cooled by air flowing over it. The transistor case temperature is not to exceed 70°C when the air temperature is 55°C. The amount of power this transistor can dissipate safely is to be determined.

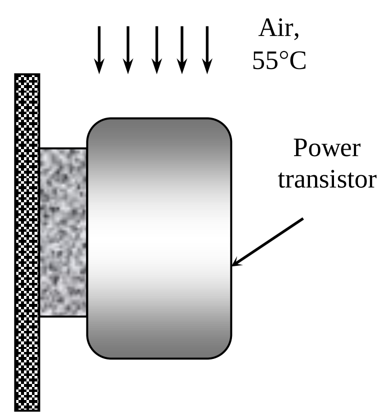
Assumptions **1** Steady operating conditions exist. **2** Heat transfer by radiation is disregarded. **3** The convection heat transfer coefficient is constant and uniform over the surface. **4** Heat transfer from the base of the transistor is negligible.

Analysis Disregarding the base area, the total heat transfer area of the transistor is

$$A_s = \pi DL + \pi D^2/4$$

$$= \pi(0.6\text{ cm})(0.4\text{cm}) + \pi(0.6\text{ cm})^2/4 = 1.037\text{ cm}^2$$

$$= 1.037\times 10^{-4}\text{ m}^2$$



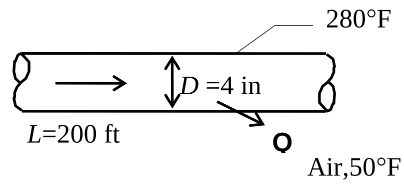
Then the rate of heat transfer from the power transistor at specified conditions is

$$\dot{Q} = hA_s(T_s - T_\infty) = (30\text{ W/m}^2\cdot°\text{C})(1.037\times10^{-4}\text{ m}^2)(70-55)°\text{C} = \mathbf{0.047\ W}$$

Therefore, the amount of power this transistor can dissipate safely is 0.047 W.

1-72E A 200-ft long section of a steam pipe passes through an open space at a specified temperature. The rate of heat loss from the steam pipe and the annual cost of this energy lost are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by radiation is disregarded. **3** The convection heat transfer coefficient is constant and uniform over the surface.



Analysis (*a*) The rate of heat loss from the steam pipe is

$$A_s = \pi DL = \pi(4/12\text{ ft})(200\text{ ft}) = 209.4\text{ ft}^2$$

$$\dot{Q}_{\text{pipe}} = hA_s(T_s - T_{\text{air}}) = (6\text{ Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(209.4\text{ ft}^2)(280-50)^\circ\text{F}$$
$$= \mathbf{289{,}000\ Btu/h}$$

(*b*) The amount of heat loss per year is

$$Q = \dot{Q}\Delta t = (289{,}000\text{ Btu/h})(365\times 24\text{ h/yr}) = 2.531\times 10^9\text{ Btu/yr}$$

The amount of gas consumption per year in the furnace that has an efficiency of 86% is

$$\text{Annual Energy Loss} = \frac{2.531\times 10^9\text{ Btu/yr}}{0.86}\left(\frac{1\text{ therm}}{100{,}000\text{ Btu}}\right) = 29{,}435\text{ therms/yr}$$

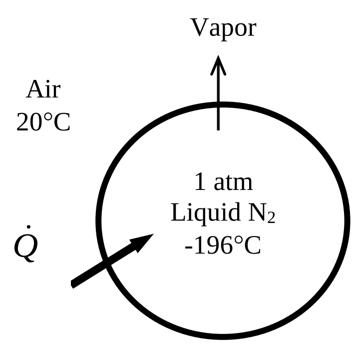
Then the annual cost of the energy lost becomes

$$\text{Energy cost} = (\text{Annual energy loss})(\text{Unit cost of energy})$$
$$= (29{,}435\text{ therms/yr})(\$1.10/\text{therm}) = \mathbf{\$32{,}380/yr}$$

1-73 A 4-m diameter spherical tank filled with liquid nitrogen at 1 atm and -196°C is exposed to convection with ambient air. The rate of evaporation of liquid nitrogen in the tank as a result of the heat transfer from the ambient air is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by radiation is disregarded. **3** The convection heat transfer coefficient is constant and uniform over the surface. **4** The temperature of the thin-shelled spherical tank is nearly equal to the temperature of the nitrogen inside.

Properties The heat of vaporization and density of liquid nitrogen at 1 atm are given to be 198 kJ/kg and 810 kg/m^3, respectively.



Analysis The rate of heat transfer to the nitrogen tank is

$$A_s = \pi D^2 = \pi(4\text{ m})^2 = 50.27\text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_{\text{air}}) = (25\text{ W/m}^2\cdot{}^\circ\text{C})(50.27\text{ m}^2)[20-(-196)]^\circ\text{C}$$
$$= 271{,}430\text{ W}$$

Then the rate of evaporation of liquid nitrogen in the tank is determined to be

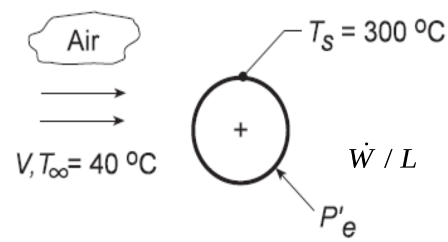
$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{271.430\text{ kJ/s}}{198\text{ kJ/kg}} = \mathbf{1.37\ kg/s}$$

1-74 Power required to maintain the surface temperature of a long, 25 mm diameter cylinder with an imbedded electrical heater for different air velocities.

Assumptions 1 Temperature is uniform over the cylinder surface. **2** Negligible radiation exchange between the cylinder surface and the surroundings. **3** Steady state conditions.

Analysis (*a*) From an overall energy balance on the cylinder, the power dissipated by the electrical heater is transferred by convection to the air stream. Using Newton's law of cooling on a per unit length basis,

$$\dot{W}/L = h A_s (T_s - T_\infty) = h (\pi D) (T_s - T_\infty)$$



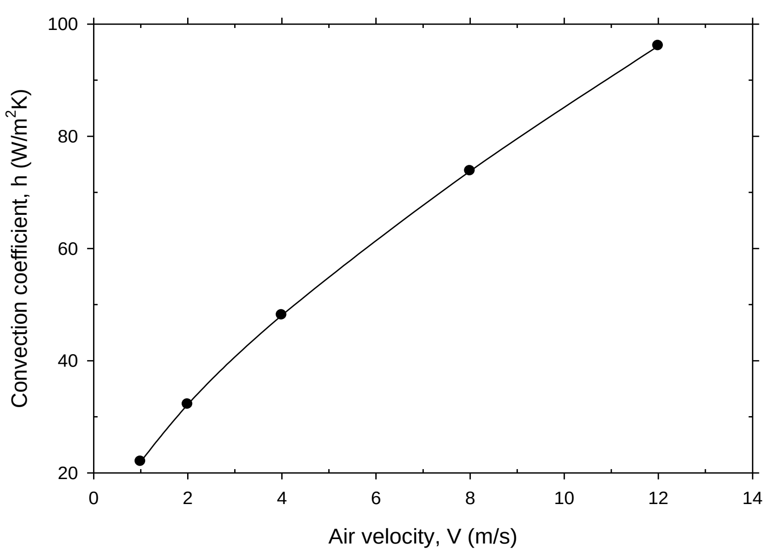
where $\dot{W}/L$ is the electrical power dissipated per unit length of the cylinder. For the V = 1 m/s condition, using the data from the table given in the problem statement, find

$$h = (\dot{W}/L) / (\pi D) (T_s - T_\infty)$$

$$h = 450 \text{ W/m} / (\pi \times 0.025 \text{ m}) (300 - 40) \text{ °C} = 22.0 \text{ W/m}^2\cdot\text{K}$$

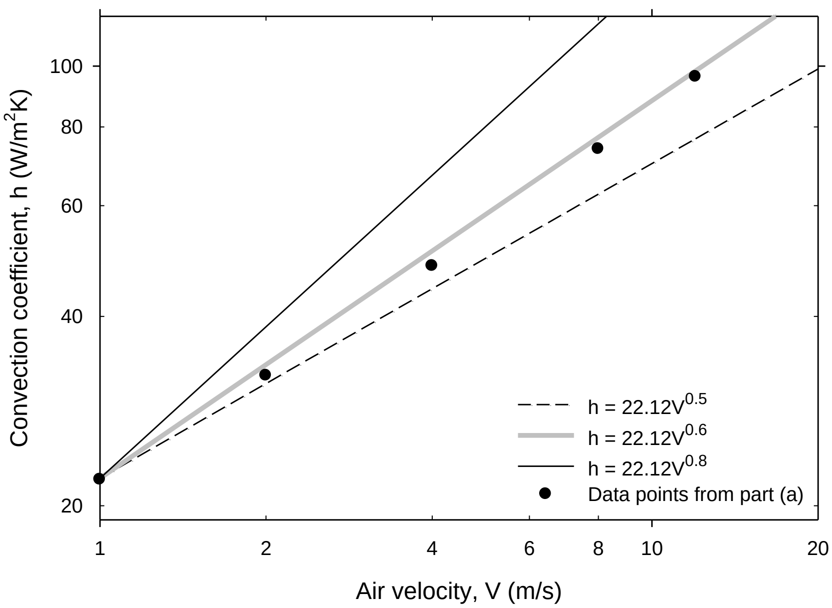
Repeating the calculations for the rest of the V values given, find the convection coefficients for the remaining conditions in the table. The results are tabulated and plotted below. Note that h is not linear with respect to the air velocity.

V (m/s)	$\dot{W}/L$ (W/m)	h (W/m^2·K)
1	450	22.0
2	658	32.2
4	983	48.1
8	1507	73.8
12	1963	96.1



Plot of convection coefficient (h) versus air velocity (V) (*b*) To determine the constants C and n, plot h vs. V on log-log coordinates. Choosing C = 22.12 W/m^2.K(s/m)^n, assuring a match at V = 1, we can readily find the exponent n from the slope of the h vs. V curve. From the trials with n = 0.8, 0.6 and 0.5, we recognize that n = 0.6 is a reasonable choice. Hence, the best values of the constants are: **C = 22.12** and **n = 0.6**. The details of these trials are given in the following table and plot.

V (m/s)	$\dot{W}/L$ (W/m)	h (W/m^2·K)	$h = 22.12V^n$ (W/m^2·K)		
			n = 0.5	n = 0.6	n = 0.8
1	450	22.0	22.12	22.12	22.12
2	658	32.2	31.28	33.53	38.51
4	983	48.1	44.24	50.82	67.06
8	1507	73.8	62.56	77.03	116.75
12	1963	96.1	76.63	98.24	161.48



Plots for $h = CV^n$ with C = 22.12 and n = 0.5, 0.6, and 0.8

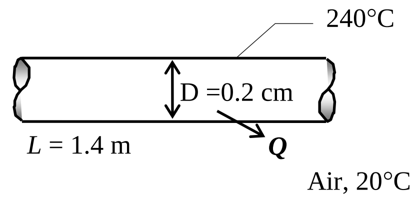
Discussion Radiation may not be negligible, depending on the surface emissivity.

1-75 The convection heat transfer coefficient for heat transfer from an electrically heated wire to air is to be determined by measuring temperatures when steady operating conditions are reached and the electric power consumed.

Assumptions 1 Steady operating conditions exist since the temperature readings do not change with time. **2** Radiation heat transfer is negligible.

Analysis In steady operation, the rate of heat loss from the wire equals the rate of heat generation in the wire as a result of resistance heating. That is,

$$\dot{Q} = \dot{E}_{\text{generated}} = \mathbf{V}I = (110\text{ V})(3\text{ A}) = 330\text{ W}$$



The surface area of the wire is

$$A_s = \pi DL = \pi(0.002\text{ m})(1.4\text{m}) = 0.00880\text{ m}^2$$

The Newton's law of cooling for convection heat transfer is expressed as

$$\dot{Q} = hA_s(T_s - T_\infty)$$

Disregarding any heat transfer by radiation, the convection heat transfer coefficient is determined to be

$$h = \frac{\dot{Q}}{A_s(T_1 - T_\infty)} = \frac{330\text{ W}}{(0.00880\text{ m}^2)(240-20)°\text{C}} = \mathbf{170.5\ W/m^2 \cdot °C}$$

Discussion If the temperature of the surrounding surfaces is equal to the air temperature in the room, the value obtained above actually represents the combined convection and radiation heat transfer coefficient.

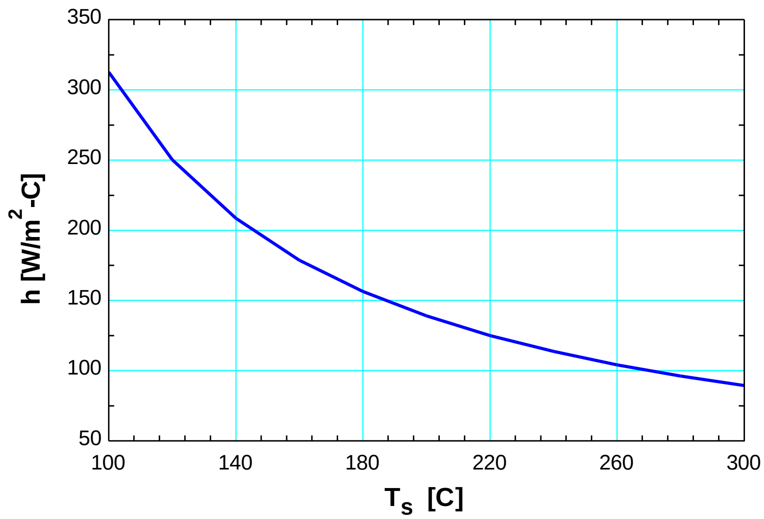
1-76 Prob. 1-75 is reconsidered. The convection heat transfer coefficient as a function of the wire surface temperature is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=2.1 [m]
D=0.002 [m]
T_infinity=20 [C]
T_s=180 [C]
V=110 [Volt]
I=3 [Ampere]

"ANALYSIS"
Q_dot=V*I
A=pi*D*L
Q_dot=h*A*(T_s-T_infinity)
```

T_s [C]	h [$W/m^2.C$]
100	312.6
120	250.1
140	208.4
160	178.6
180	156.3
200	138.9
220	125.1
240	113.7
260	104.2
280	96.19
300	89.32



1-77E Using the conversion factors between W and Btu/h, m and ft, and K and R, the Stefan-Boltzmann constant $\sigma = 5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4}$ is to be expressed in the English unit, $\mathrm{Btu/h\cdot ft^2\cdot R^4}$.

Analysis The conversion factors for W, m, and K are given in conversion tables to be

$$1\ \mathrm{W} = 3.41214\ \mathrm{Btu/h}$$

$$1\ \mathrm{m} = 3.2808\ \mathrm{ft}$$

$$1\ \mathrm{K} = 1.8\ \mathrm{R}$$

Substituting gives the Stefan-Boltzmann constant in the desired units,

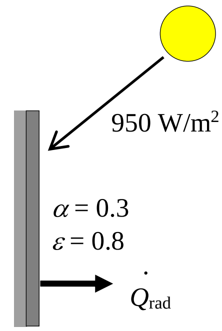
$$\sigma = 5.67\ \mathrm{W/m^2\cdot K^4} = 5.67\times\frac{3.41214\ \mathrm{Btu/h}}{(3.2808\ \mathrm{ft})^2(1.8\ \mathrm{R})^4} = \mathbf{0.171\ Btu/h\cdot ft^2\cdot R^4}$$

1-78 A spacecraft in space absorbs solar radiation while losing heat to deep space by thermal radiation. The surface temperature of the spacecraft is to be determined when steady conditions are reached.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the wall remain constant at the specified values. **2** Thermal properties of the wall are constant.

Properties The outer surface of a spacecraft has an emissivity of 0.8 and an absorptivity of 0.3.

Analysis When the heat loss from the outer surface of the spacecraft by radiation equals the solar radiation absorbed, the surface temperature can be determined from



$$\dot{Q}_{\text{solarabsorbed}} = \dot{Q}_{\text{rad}}$$

$$\alpha\dot{Q}_{\text{solar}} = \varepsilon\sigma A_s (T_s^4 - T_{\text{space}}^4)$$

$$0.3 \times A_s \times (950\ \text{W/m}^2) = 0.8 \times A_s \times (5.67\times10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)[T_s^4 - (0\ \text{K})^4]$$

Canceling the surface area A and solving for T_s gives

$$T_s = \mathbf{281.5\ K}$$

1-79 A person with a specified surface temperature is subjected to radiation heat transfer in a room at specified wall temperatures. The rate of radiation heat loss from the person is to be determined.

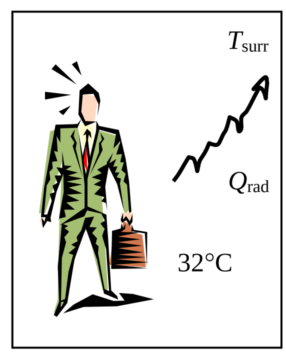
Assumptions **1** Steady operating conditions exist. **2** Heat transfer by convection is disregarded. **3** The emissivity of the person is constant and uniform over the exposed surface.

Properties The average emissivity of the person is given to be 0.5.

Analysis Noting that the person is completely enclosed by the surrounding surfaces, the net rates of radiation heat transfer from the body to the surrounding walls, ceiling, and the floor in both cases are

(*a*) T_{surr} = 300 K

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A_s (T_s^4 - T_{\text{surr}}^4)\\ &= (0.5)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1.7\,\text{m}^2)[(32+273)^4 - (300\,\text{K})^4]\text{K}^4\\ &= \mathbf{26.7\ W}\end{aligned}$$



(*b*) T_{surr} = 280 K

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A_s (T_s^4 - T_{\text{surr}}^4)\\ &= (0.5)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1.7\,\text{m}^2)[(32+273)^4 - (280\,\text{K})^4]\text{K}^4\\ &= \mathbf{121\ W}\end{aligned}$$

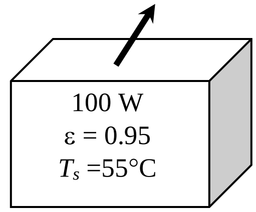
Discussion Note that the radiation heat transfer goes up by more than 4 times as the temperature of the surrounding surfaces drops from 300 K to 280 K.

1-80 A sealed electronic box dissipating a total of 100 W of power is placed in a vacuum chamber. If this box is to be cooled by radiation alone and the outer surface temperature of the box is not to exceed 55°C, the temperature the surrounding surfaces must be kept is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by convection is disregarded. **3** The emissivity of the box is constant and uniform over the exposed surface. **4** Heat transfer from the bottom surface of the box to the stand is negligible.

Properties The emissivity of the outer surface of the box is given to be 0.95.

Analysis Disregarding the base area, the total heat transfer area of the electronic box is



$$A_s = (0.4\,\text{m})(0.4\,\text{m}) + 4\times(0.2\,\text{m})(0.4\,\text{m}) = 0.48\,\text{m}^2$$

The radiation heat transfer from the box can be expressed as

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s (T_s^4 - T_{\text{surr}}^4)$$

$$100\text{ W} = (0.95)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(0.48\,\text{m}^2)\left[(55+273\,\text{K})^4 - T_{\text{surr}}^4\right]$$

which gives T_{surr} = **296.3 K = 23.3°C**. Therefore, the temperature of the surrounding surfaces must be less than 23.3°C.

1-81 One highly polished surface at 1070°C and one heavily oxidized surface are emitting the same amount of energy per unit area. The temperature of the heavily oxidized surface is to be determined.

Assumptions The emissivity of each surface is constant and uniform.

Properties The emissivity of the highly polished surface is $\varepsilon_1 = 0.1$, and the emissivity of heavily oxidized surface is $\varepsilon_2 = 0.78$.

Analysis The rate of energy emitted by radiation is

$$\dot{Q}_{\text{emit}} = \varepsilon\sigma A_s T_s^4$$

For both surfaces to emit the same amount energy per unit area

$$(\dot{Q}_{\text{emit}} / A_s)_1 = (\dot{Q}_{\text{emit}} / A_s)_2$$

or

$$\varepsilon_1 T_{s,1}^4 = \varepsilon_2 T_{s,2}^4$$

The temperature of the heavily oxidized surface is

$$T_{s,2} = \left(\frac{\varepsilon_1}{\varepsilon_2} T_{s,1}^4\right)^{1/4} = \left[\frac{0.1}{0.78}(1070+273)^4\right]^{1/4}\text{K} = \mathbf{803.6\,K}$$

Discussion If both surfaces are maintained at the same temperature, then the highly polished surface will emit less energy than the heavily oxidized surface.

1-82 A spherical probe in space absorbs solar radiation while losing heat to deep space by thermal radiation. The incident radiation rate on the probe surface is to be determined.

Assumptions **1** Steady operating conditions exist and surface temperature remains constant. **2** Heat generation is uniform.

Properties The outer surface the probe has an emissivity of 0.9 and an absorptivity of 0.1.

Analysis The rate of heat transfer at the surface of the probe can be expressed as

$$\dot{Q}_{\text{gen}} = \dot{Q}_{\text{rad}} - \dot{Q}_{\text{absorbed}}$$

$$\dot{e}_{\text{gen}} V = \varepsilon\sigma A_s (T_s^4 - T_{\text{space}}^4) - \alpha A_s \dot{q}_{\text{solar}}$$

$$\dot{e}_{\text{gen}} \left(\frac{4}{3}\pi r^3\right) = \varepsilon\sigma(4\pi r^2)(T_s^4 - T_{\text{space}}^4) - \alpha(4\pi r^2)\dot{q}_{\text{solar}}$$

Thus, incident radiation rate on the probe surface is

$$\dot{q}_{\text{solar}} = \frac{1}{\alpha}\left[\varepsilon\sigma(T_s^4 - T_{\text{space}}^4) - \frac{r}{3}\dot{e}_{\text{gen}}\right]$$

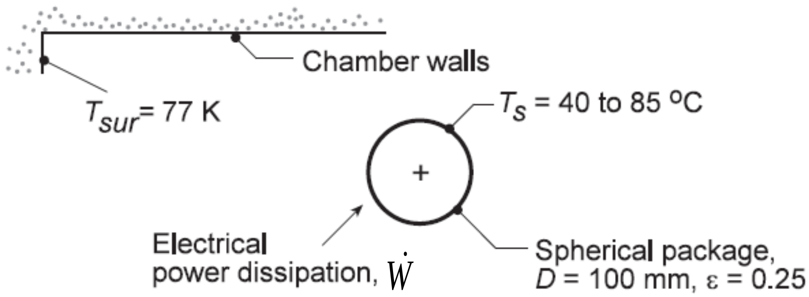
$$\dot{q}_{\text{solar}} = \frac{1}{0.1}\left[(0.9)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(-40+273)^4 - 0)]\text{K}^4 - \frac{(1\,\text{m})(100\ \text{W/m}^3)}{3}\right] = 1171\,\text{W/m}^2$$

$$\dot{Q}_{\text{solar}} = A_s\dot{q}_{\text{solar}} = (4\pi r^2)\dot{q}_{\text{solar}} = 4\pi(1\,\text{m})^2(1171\,\text{W/m}^2) = \mathbf{14{,}715\ W}$$

Discussion By adjusting the emissivity or absorptivity of the probe surface, the amount of incident radiation rate on the surface can be changed.

1-83 Spherical shaped instrumentation package with prescribed surface emissivity within a large laboratory room having walls at 77 K.

Assumptions 1 Uniform surface temperature. **2** Laboratory room walls are large compared to the spherical package. **3** Steady state conditions.



Analysis From an overall energy balance on the package, the internal power dissipation $\dot{W}$ will be transferred by radiation exchange between the package and the laboratory walls. The net rate of radiation between these two surfaces is given by

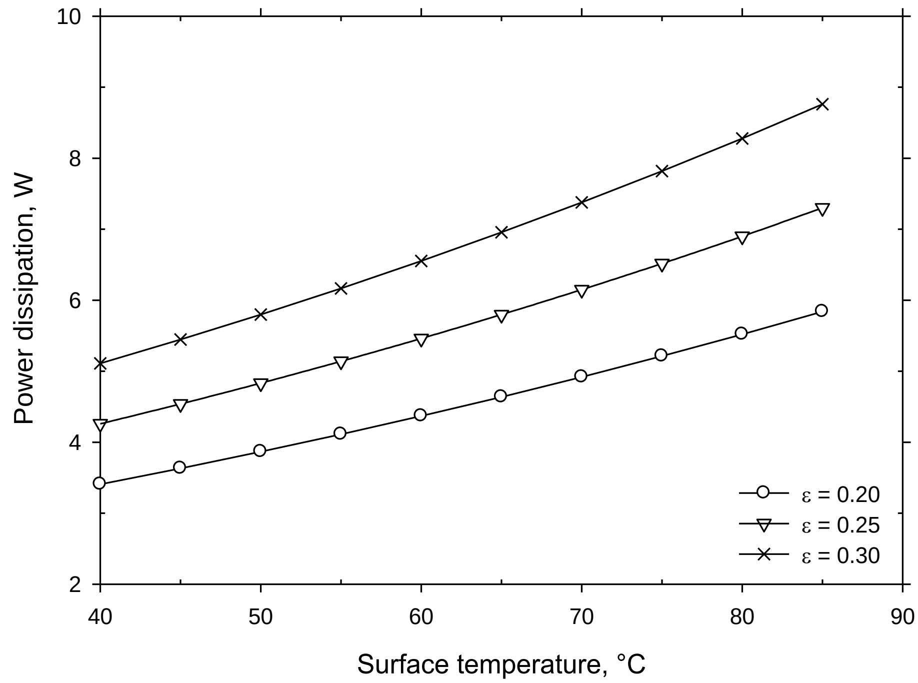
$$\dot{Q}_{rad} = \dot{W} = \varepsilon \sigma A_s (T_s^4 - T_{sur}^4)$$

For the condition when T_s = 40°C, ε = 0.25, with $A_s = \pi D^2$, the power dissipation will be

$$\dot{W} = 0.25\ (\pi \times 0.10^2\ \text{m}^2)\ (5.67 \times 10^{-8}\ \text{W/m}^2\cdot\ \text{K})\ [(40 + 273)^4 - 77^4]\ \text{K}^4 = 4.3\ \text{W}$$

Repeating this calculation for the range 40ºC ≤ T_s ≤ 85°C, we can obtain the power dissipation as a function of surface temperature for the ε = 0.25 condition. Similarly, the calculation can be repeated for with 0.2 or 0.3. The details of these trials are given in the following table and plot.

		$\dot{W}$ (W)	
T_s (°C)	ε = 0.20	ε = 0.25	ε = 0.30
40	3.41	4.26	5.11
45	3.63	4.54	5.45
50	3.87	4.83	5.80
55	4.11	5.14	6.17
60	4.37	5.46	6.55
65	4.64	5.80	6.96
70	4.92	6.15	7.38
75	5.21	6.52	7.82
80	5.52	6.90	8.28
85	5.84	7.30	8.76



Family of curves for power dissipation ($\dot{W}$) versus surface temperature (T_s) for different values of emissivity (ε)

Discussion:

(1) As expected, the internal power dissipation increases with increasing emissivity and surface temperature. Because the radiation rate equation is non-linear with respect to temperature, the power dissipation will likewise not be linear with surface temperature.

(2) At a constant surface temperature, the power dissipation is linear with respect to the emissivity. The trends of the $\dot{W}$ versus T_s curves remained similar at different values of emissivity. By increasing the emissivity, the $\dot{W}$ versus T_s curve is shifted higher.

(3) The emissivity of various materials is listed in Tables A-18 and A-19 of the text.

(4) The maximum power dissipation possible if the surface temperature is not to exceed 85 °C is $\dot{W}$ = 8.76, where $\varepsilon = 0.30$. Similarly, the minimum power dissipation possible for the surface temperature range 40ºC $\leq T_s \leq$ 85°C is $\dot{W}$ = 3.41 W, where T_s = 40 °C and $\varepsilon = 0.20$. Therefore, the possible range of power dissipation for 40ºC $\leq T_s \leq$ 85 °C and $0.2 \leq \varepsilon \leq 0.3$ is $3.41 \leq \dot{W} \leq 8.76$ W.

Simultaneous Heat Transfer Mechanisms

1-84C All three modes of heat transfer cannot occur simultaneously in a medium. A medium may involve two of them simultaneously.

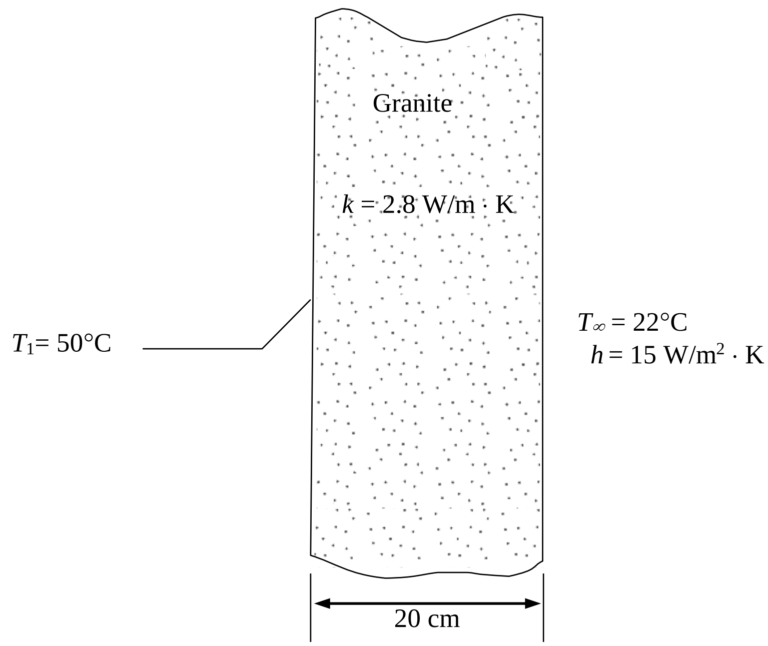
1-85C (*a*) Conduction and convection: No. (*b*) Conduction and radiation: Yes. Example: A hot surface on the ceiling. (*c*) Convection and radiation: Yes. Example: Heat transfer from the human body.

1-86C The human body loses heat by convection, radiation, and evaporation in both summer and winter. In summer, we can keep cool by dressing lightly, staying in cooler environments, turning a fan on, avoiding humid places and direct exposure to the sun. In winter, we can keep warm by dressing heavily, staying in a warmer environment, and avoiding drafts.

1-87C The fan increases the air motion around the body and thus the convection heat transfer coefficient, which increases the rate of heat transfer from the body by convection and evaporation. In rooms with high ceilings, ceiling fans are used in winter to force the warm air at the top downward to increase the air temperature at the body level. This is usually done by forcing the air up which hits the ceiling and moves downward in a gently manner to avoid drafts.

1-88 The right surface of a granite wall is subjected to convection heat transfer while the left surface is maintained as a constant temperature. The right wall surface temperature and the heat flux through the wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the granite wall is one dimensional. **3** Thermal conductivity of the granite wall is constant. **4** Radiation hest transfer is negligible.



Analysis The heat transfer through the wall by conduction is equal to heat transfer to the outer wall surface by convection:

$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}}$$

$$k\frac{T_1 - T_2}{L} = h(T_2 - T_\infty)$$

$$T_2 = \frac{(kT_1 / L) + hT_\infty}{(k / L) + h}$$

$$T_2 = \frac{(2.8\,\text{W/m}\cdot\text{K})(50°\text{C}) /(0.20\,\text{m}) + (15\,\text{W/m}^2\cdot\text{K})\,(22°\text{C})}{(2.8\,\text{W/m}\cdot\text{K}) /(0.20\,\text{m}) + 15\,\text{W/m}^2\cdot\text{K}}$$

$$T_2 = \mathbf{35.5\,°C}$$

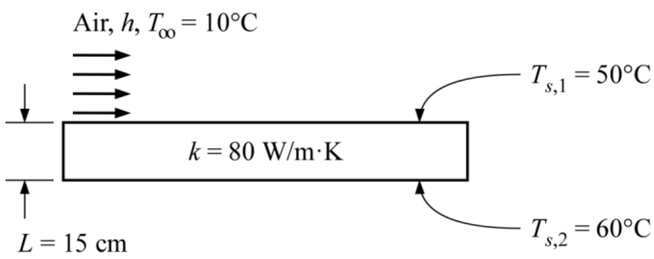
Now that T_2 is known, we can calculate the heat flux. Since the heat transfer through the wall by conduction is equal to heat transfer to the outer wall surface by convection, we may use either the Fourier's law of heat conduction or the Newton's law of cooling to find the heat flux. Using Fourier's law of heat conduction:

$$\dot{q}_{\text{cond}} = k\frac{T_1 - T_2}{L} = (2.8\text{ W/m}\cdot\text{K})\,\frac{(50-35.5)\,°\text{C}}{0.20\,\text{m}} = \mathbf{203\ W/m^2}$$

Discussion Alternatively using Newton's law of cooling to find the heat flux, we obtain the same result:

$$\dot{q}_{conv} = h(T_2 - T_\infty) = (15\,\text{W/m}^2\cdot\text{K})\,(35.5-22) = 203\,\text{W/m}^2$$

1-89 The upper surface of a solid plate is being cooled by air. The air convection heat transfer coefficient at the upper plate surface is to be determined.



Assumptions **1** Steady operating conditions exist. **2** Thermal conductivity of the plate is constant. **3** Heat conduction in solid is one-dimensional. **4** Temperatures at the surfaces remained constant.

Properties The thermal conductivity of the solid plate is given as k = 80 W/m·K.

Analysis Applying energy balance on the upper surface of the solid plate

$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}}$$

$$k\frac{T_{s,2}-T_{s,1}}{L} = h(T_{s,1}-T_\infty)$$

The convection heat transfer of the air is

$$h = \frac{k}{L}\frac{T_{s,2}-T_{s,1}}{T_{s,1}-T_\infty} = \frac{80\ \text{W/m}\cdot\text{K}}{0.15\ \text{m}}\left(\frac{60-50}{50-10}\right) = \mathbf{133\ W/m^2\cdot K}$$

Discussion A convection heat transfer coefficient of 133 W/m^2·K for forced convection of gas is reasonable when compared with the values listed in Table 1-5.

1-90 Air is blown over a hot horizontal plate which is maintained at a constant temperature. The surface also loses heat by radiation. The inside plate temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the steel plate is one dimensional. **3** Thermal conductivity of the steel plate is constant.

Analysis The heat transfer by conduction through the plate is equal to the sum of convection and radiation heat loses:

$$\dot{Q}_{\text{cond}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

where

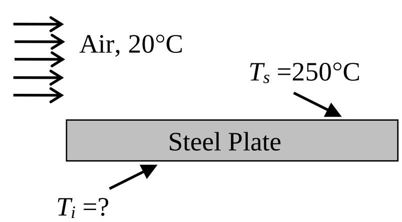
$$\dot{Q}_{conv} = hA(T_s - T_\infty) = (25\ \text{W/m}^2\cdot\text{K})(0.38\ \text{m}^2)(250-20)\text{K} = 2185\ \text{W}$$

$$\dot{Q}_{rad} = 300\ \text{W}$$

Then,

$$\dot{Q}_{cond} = kA\frac{T_i - T_s}{L} = 2185\ \text{W} + 300\ \text{W} = 2485\ \text{W}$$

Solving for the inside plate temperature



$$T_i = T_s + \frac{\dot{Q}_{cond}\,L}{kA} = 250°\text{C} + \frac{(2485\ \text{W})(0.02\ \text{m})}{(43\ \text{W/m}\cdot\text{K})(0.38\ \text{m}^2)} = \mathbf{253°C}$$

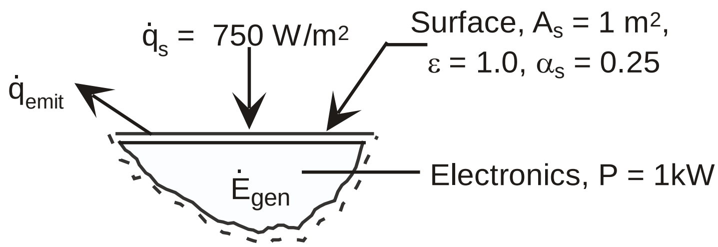
Discussion Heat loss by convection is much more dominant than heat loss by radiation. If we had not accounted for the heat loss by radiation in our calculation, the inside plate temperature would be 252.7ºC, which is only 0.3ºC less than the actual value. In this case we could have neglected the heat loss by radiation.

1-91 For an electronic package with given surface area, power dissipation, surface emissivity and absorptivity to solar radiation and the solar flux, the surface temperature with and without incident solar radiation is to be determined.

Assumptions **1** Steady operating conditions exist.

Analysis Apply conservation of energy (heat balance) to a control volume about the electronic package in rate form

$$\dot{Q}_{in} - \dot{Q}_{out} + \dot{E}_{gen} = \dot{E}_{stored} = 0$$



With the solar input, we have

$$\alpha_s A_s \dot{q}_s - A_s \dot{q}_{emit} + P = 0$$

where

$$\dot{q}_{emit} = \varepsilon\sigma T_s^4$$

Solving for the surface temperature T_s, we have

$$T_s = \left(\frac{\alpha_s A_s \dot{q}_s + P}{A_s \varepsilon\sigma}\right)^{1/4}$$

(a) Surface Temperature in the sun ($\dot{q}_s$ = 750 W/m^2)

$$T_s = \left[\frac{(0.25)(1\text{m}^2)(750\text{W/m}^2) + 1000\text{W}}{(1\text{m}^2)(1.0)(5.67\text{x}10^{-8}\text{W/m}^2\cdot\text{K}^4)}\right]^{1/4} = \mathbf{380.4\ K}$$

(b) Surface Temperature in the shade ($\dot{q}_s$ =0)

$$T_s = \left[\frac{1000\text{W}}{(1\text{m}^2)(1.0)(5.67\text{x}10^{-8}\text{W/m}^2\cdot\text{K}^4)}\right]^{1/4} = \mathbf{364.4\ K}$$

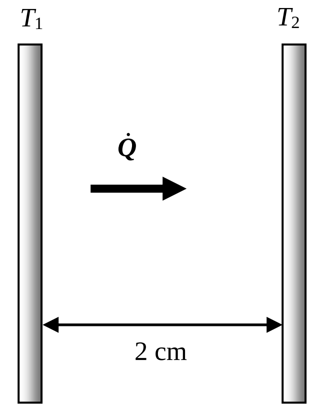
Discussion In orbit, the space station would be continuously cycling between shade and sunshine, and a steady-state condition would not exist.

1-92 Two large plates at specified temperatures are held parallel to each other. The rate of heat transfer between the plates is to be determined for the cases of still air, evacuation, regular insulation, and super insulation between the plates.

Assumptions **1** Steady operating conditions exist since the plate temperatures remain constant. **2** Heat transfer is one-dimensional since the plates are large. **3** The surfaces are black and thus $\varepsilon = 1$. **4** There are no convection currents in the air space between the plates.

Properties The thermal conductivities are $k = 0.00015$ W/m·°C for super insulation, $k = 0.01979$ W/m·°C at -50°C (Table A-15) for air, and $k = 0.036$ W/m·°C for fiberglass insulation (Table A-6).

Analysis (*a*) Disregarding any natural convection currents, the rates of conduction and radiation heat transfer



$$\dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.01979\ \text{W/m}^2\cdot{}^\circ\text{C})(1\,\text{m}^2)\frac{(290-150)\ \text{K}}{0.02\,\text{m}} = 139\ \text{W}$$

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A_s(T_1^4 - T_2^4)\\ &= 1(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1\text{m}^2)\left[(290\ \text{K})^4 - (150\ \text{K})^4\right] = 372\ \text{W}\end{aligned}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} + \dot{Q}_{\text{rad}} = 139 + 372 = \mathbf{511\ W}$$

(*b*) When the air space between the plates is evacuated, there will be radiation heat transfer only. Therefore,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{rad}} = \mathbf{372\ W}$$

(*c*) In this case there will be conduction heat transfer through the fiberglass insulation only,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.036\ \text{W/m}\cdot{}^\circ\text{C})(1\,\text{m}^2)\frac{(290-150)\ \text{K}}{0.02\,\text{m}} = \mathbf{252\ W}$$

(*d*) In the case of superinsulation, the rate of heat transfer will be

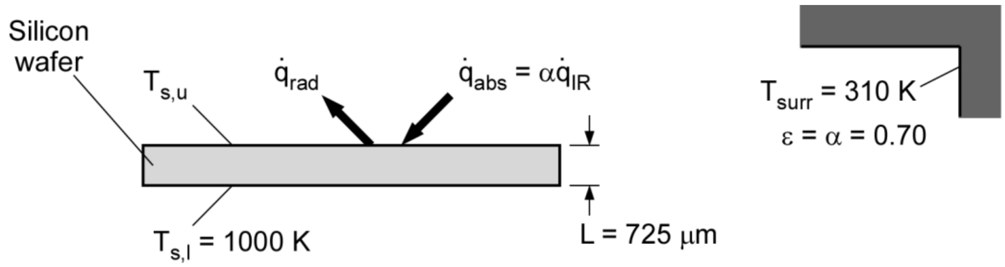
$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{cond}} = kA\frac{T_1 - T_2}{L} = (0.00015\ \text{W/m}\cdot{}^\circ\text{C})(1\,\text{m}^2)\frac{(290-150)\ \text{K}}{0.02\,\text{m}} = \mathbf{1.05\ W}$$

Discussion Note that superinsulators are very effective in reducing heat transfer between to surfaces.

1-93 The upper surface temperature of a silicon wafer undergoing heat treatment in a vacuum chamber by infrared heater is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation heat transfer between upper wafer surface and surroundings is between a small object and a large enclosure. **3** One-dimensional conduction in wafer. **4** The silicon wafer has constant properties. **5** No hot spot exists on the wafer.

Properties The thermal conductivity of silicon at 1000 K is 31.2 W/m · K (Table A-3).



Analysis The heat transfer through the thickness of the wafer by conduction is equal to net heat transfer at the upper wafer surface:

$$\dot{q}_{\text{cond}} = \dot{q}_{\text{abs}} - \dot{q}_{\text{rad}}$$

$$k\frac{T_{s,u} - T_{s,l}}{L} = \alpha\dot{q}_{\text{IR}} - \varepsilon\sigma(T_{s,u}^4 - T_{\text{surr}}^4)$$

$$(31.2\ \text{W/m}\cdot\text{K})\frac{(T_{s,u} - 1000)\ \text{K}}{(725\times10^{-6}\ \text{m})} = (0.70)(200000\ \text{W/m}^2) - (0.70)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(T_{s,u}^4 - 310^4)\ \text{K}^4$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
31.2*(T_su-1000)/725e-6=0.70*200000-0.70*5.67e-8*(T_su^4-310^4)
```

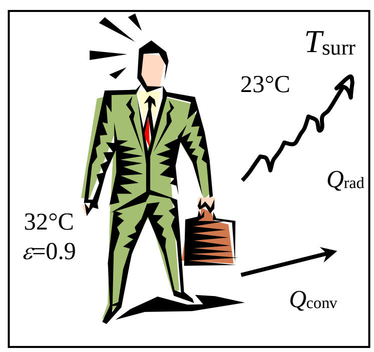
Solving by EES software, the upper surface temperature of silicon wafer is

$$T_{s,u} = \mathbf{1002\ K}$$

Discussion Excessive temperature difference across the wafer thickness will cause warping in the silicon wafer.

1-94 The total rate of heat transfer from a person by both convection and radiation to the surrounding air and surfaces at specified temperatures is to be determined.

Assumptions 1 Steady operating conditions exist. **2** The person is completely surrounded by the interior surfaces of the room. **3** The surrounding surfaces are at the same temperature as the air in the room. **4** Heat conduction to the floor through the feet is negligible. **5** The convection coefficient is constant and uniform over the entire surface of the person.



Properties The emissivity of a person is given to be ε = 0.9.

Analysis The person is completely enclosed by the surrounding surfaces, and he or she will lose heat to the surrounding air by convection and to the surrounding surfaces by radiation. The total rate of heat loss from the person is determined from

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4) = (0.90)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1.7\,\text{m}^2)[(32+273)^4 - (23+273)^4]\text{K}^4 = 84.8\ \text{W}$$

$$\dot{Q}_{\text{conv}} = hA_s\Delta T = (5\text{W/m}^2\cdot\text{K})(1.7\text{m}^2)(32-23)°\text{C} = 76.5\text{W}$$

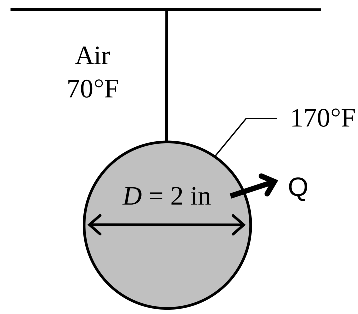
and

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 84.8 + 76.5 = \mathbf{161.3\ W}$$

Discussion Note that heat transfer from the person by evaporation, which is of comparable magnitude, is not considered in this problem.

1-95E A spherical ball whose surface is maintained at a temperature of 170°F is suspended in the middle of a room at 70°F. The total rate of heat transfer from the ball is to be determined.

Assumptions **1** Steady operating conditions exist since the ball surface and the surrounding air and surfaces remain at constant temperatures. **2** The thermal properties of the ball and the convection heat transfer coefficient are constant and uniform.



Properties The emissivity of the ball surface is given to be ε = 0.8.

Analysis The heat transfer surface area is

$$A_s = \pi D^2 = \pi(2/12\text{ ft})^2 = 0.08727\text{ ft}^2$$

Under steady conditions, the rates of convection and radiation heat transfer are

$$\dot{Q}_{\text{conv}} = hA_s\Delta T = (15\text{ Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(0.08727\text{ ft}^2)(170-70)^\circ\text{F} = 130.9\text{ Btu/h}$$

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A_s(T_s^4 - T_o^4)\\ &= 0.8(0.08727\text{ ft}^2)(0.1714\times10^{-8}\text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)[(170+460\text{ R})^4-(70+460\text{ R})^4]\\ &= 9.4\text{ Btu/h}\end{aligned}$$

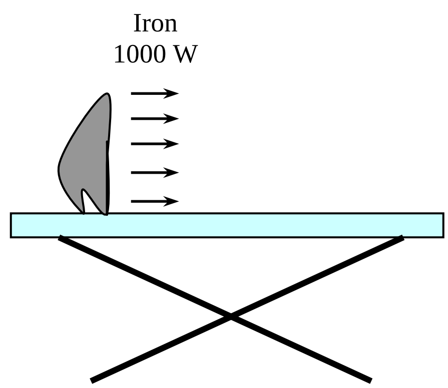
Therefore,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 130.9 + 9.4 = \mathbf{140.3\ Btu/h}$$

Discussion Note that heat loss by convection is several times that of heat loss by radiation. The radiation heat loss can further be reduced by coating the ball with a low-emissivity material.

1-96 A 1000-W iron is left on the iron board with its base exposed to the air at 20°C. The temperature of the base of the iron is to be determined in steady operation.

Assumptions **1** Steady operating conditions exist. **2** The thermal properties of the iron base and the convection heat transfer coefficient are constant and uniform. **3** The temperature of the surrounding surfaces is the same as the temperature of the surrounding air.



Properties The emissivity of the base surface is given to be ε = 0.6.

Analysis At steady conditions, the 1000 W energy supplied to the iron will be dissipated to the surroundings by convection and radiation heat transfer. Therefore,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 1000\text{ W}$$

where

$$\dot{Q}_{\text{conv}} = hA_s\Delta T = (35\text{ W/m}^2\cdot\text{K})(0.02\text{ m}^2)(T_s - 293\text{ K}) = 0.7(T_s - 293\text{ K})$$

and

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A_s(T_s^4 - T_o^4) = 0.6(0.02\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[T_s^4 - (293\text{ K})^4]\\ &= 0.06804\times10^{-8}[T_s^4 - (293\text{ K})^4]\end{aligned}$$

Substituting,

$$1000\text{ W} = 0.7(T_s - 293\text{ K}) + 0.06804\times10^{-8}[T_s^4 - (293\text{ K})^4]$$

Solving by trial and error gives

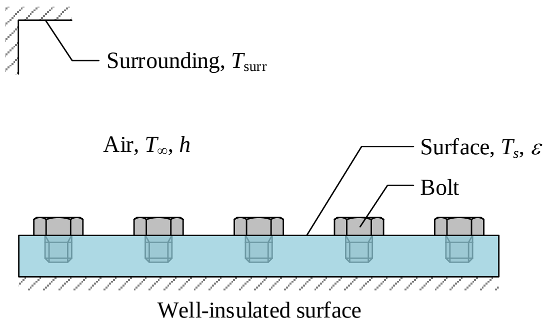
$$T_s = \mathbf{947\ K} = \mathbf{674^\circ C}$$

Discussion We note that the iron will dissipate all the energy it receives by convection and radiation when its surface temperature reaches 947 K.

1-97 C&S A series of ASTM B21 naval brass bolts are bolted on the upper surface of a plate. The upper surface is exposed to convection with air and radiation with the surrounding surface. Determine whether the use of the bolts complies with the ASME Code for Process Piping, where 149°C is the maximum use temperature for ASTM B21 bolts.

Assumptions **1** Heat transfer is steady. **2** Kirchhoff's law is applicable. **3** Surrounding surface is treated as blackbody. **4** Uniform surface temperature at the upper plate surface. **5** The temperature of the bolts is equal to the upper plate surface temperature.

Properties The emissivity of the plate and bolts is given as 0.3.



Analysis Since the bottom surface is well-insulated, heat transfer occurs at the upper surface only. From surface energy balance, we have

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{rad} = \dot{Q}_{conv}$$

$$\varepsilon \sigma A_s (T_{surr}^4 - T_s^4) = h A_s (T_s - T_\infty)$$

Assuming the temperature of the bolts is equal to the upper surface temperature of the plate,

$$(0.3)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(300+273)^4 - T_s^4]\,\text{K}^4 = (5\ \text{W/m}^2\cdot\text{K})[T_s - (50+273)]\,\text{K}$$

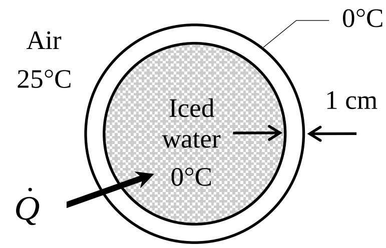
Solving for T_s yields

$$T_{bolt} = T_s = 491\,\text{K} = \mathbf{218°C} > 149°\text{C}$$

Discussion The temperature of the bolts exceeds the maximum use temperature by 149°C. The use of the ASTM B21 bolts under these conditions does not comply with the ASME Code for Process Piping (ASME B31.3-2014). Another type of bolts should be considered for use under these conditions. For example, the ASTM A193 B8M steel bolts have a maximum use temperature of 538°C.

1-98 A spherical tank located outdoors is used to store iced water at 0°C. The rate of heat transfer to the iced water in the tank and the amount of ice at 0°C that melts during a 24-h period are to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the wall remain constant at the specified values. **2** Thermal properties of the tank and the convection heat transfer coefficient is constant and uniform. **3** The average surrounding surface temperature for radiation exchange is 15°C. **4** The thermal resistance of the tank is negligible, and the entire steel tank is at 0°C.



Properties The heat of fusion of water at atmospheric pressure is $h_{if} = 333.7$ kJ/kg. The emissivity of the outer surface of the tank is 0.75.

Analysis (a) The outer surface area of the spherical tank is

$$A_s = \pi D^2 = \pi(3.02\text{ m})^2 = 28.65\text{ m}^2$$

Then the rates of heat transfer to the tank by convection and radiation become

$$\dot{Q}_{\text{conv}} = hA_s(T_\infty - T_s) = (30\text{ W/m}^2\cdot°\text{C})(28.65\text{ m}^2)(25-0)°\text{C} = 21{,}488\text{ W}$$

$$\dot{Q}_{\text{rad}} = \varepsilon A_s\sigma(T_{\text{surr}}^4 - T_s^4) = (0.75)(28.65\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(288\text{ K})^4 - (273\text{ K})^4] = 1614\text{ W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 21{,}488 + 1614 = 23{,}102\text{ W} = \mathbf{23.1 kW}$$

(b) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (23.102\text{ kJ/s})(24\times3600\text{ s}) = 1{,}996{,}000\text{ kJ}$$

Then the amount of ice that melts during this period becomes

$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{1{,}996{,}000\text{ kJ}}{333.7\text{ kJ/kg}} = \mathbf{5980\text{ kg}}$$

Discussion The amount of ice that melts can be reduced to a small fraction by insulating the tank.

1-99 A draw batch furnace front is subjected to uniform heat flux on the inside surface, while the outside surface is subjected to convection and radiation heat transfer. The outside surface temperature is to be determined.

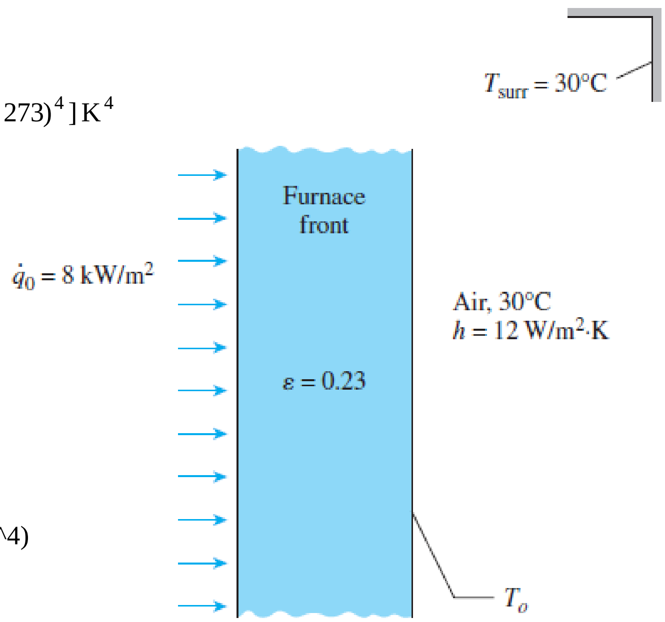
Assumptions **1** Heat conduction is steady. **2** One dimensional heat conduction across the furnace front thickness. **3** Uniform heat flux on inside surface.

Properties Emissivity and convective heat transfer coefficient are given to be 0.23 and 12 W/m²·K, respectively.

Analysis The uniform heat flux subjected on the inside surface is equal to the sum of heat fluxes transferred by convection and radiation on the outside surface

$$\dot{q}_0 = h(T_o - T_\infty) + \varepsilon\sigma(T_o^4 - T_{\text{surr}}^4)$$

$$8000\ \text{W/m}^2 = (12\ \text{W/m}^2\cdot\text{K})[T_o - (30+273)]\,\text{K}$$
$$+ (0.23)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[T_o^4 - (30+273)^4]\,\text{K}^4$$



Solving for the outer surface temperature yields

$$T_o = \mathbf{707\ K} = \mathbf{434°C}$$

Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
h=12 [W/m^2-K]
q_dot_0=8000 [W/m^2]
T_surr=303 [K]
epsilon=0.23
sigma=5.67e-8 [W/m^2-K^4]
q_dot_0=h*(T_o-T_surr)+epsilon*sigma#*(T_o^4-T_surr^4)
```

Discussion By insulating the furnace front, heat loss from the outer surface can be reduced.

1-100 A flat-plate solar absorber is exposed to an incident solar radiation. The efficiency of the solar absorber (the ratio of the usable heat collected by the absorber to the incident solar radiation on the absorber) is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Temperature at the surface remained constant.

Properties The absorber surface has an absorptivity of 0.93 and an emissivity of 0.9.

Analysis The rate of usable heat at the absorber plate can be expressed as

$$\dot{Q}_{\text{usable}} = \dot{Q}_{\text{absorbed}} - \dot{Q}_{\text{rad}} - \dot{Q}_{\text{conv}}$$

$$\dot{Q}_{\text{usable}} = \alpha A_s \dot{q}_{\text{solar}} - \varepsilon\sigma A_s (T_s^4 - T_{\text{surr}}^4) - hA_s (T_s - T_\infty)$$

Expressed in terms of heat flux, we have

$$\dot{q}_{\text{usable}} = \alpha\dot{q}_{\text{solar}} - \varepsilon\sigma(T_s^4 - T_{\text{surr}}^4) - h(T_s - T_\infty)$$

$$\dot{q}_{\text{usable}} = (0.93)(800\text{ W/m}^2) - (0.9)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(40+273)^4 - (-5+273)^4]\text{K}^4$$
$$- (7\text{ W/m}^2\cdot\text{K})(40-20)\text{ K}$$

$$\dot{q}_{\text{usable}} = 377.5\text{ W/m}^2$$

Thus, the efficiency of the solar absorber is

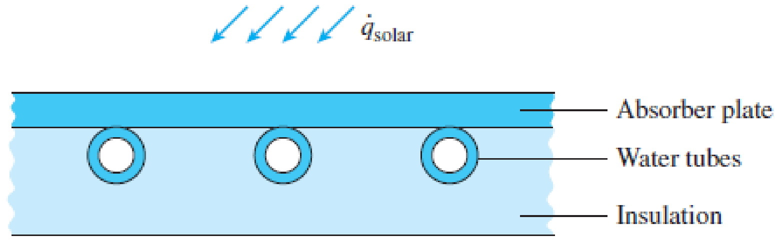
$$\eta = \frac{\dot{q}_{\text{usable}}}{\dot{q}_{\text{solar}}} = \frac{377.5\text{ W/m}^2}{800\text{ W/m}^2} = \mathbf{0.472}$$

Discussion The efficiency of the solar absorber is influenced by the surrounding and ambient temperatures, as well as the convective heat transfer coefficient. If the weather is particularly windy, thus causing higher level of heat loss via convection, then the efficiency of the solar absorber could be adversely affected.

1-101 A flat-plate solar collector is used to heat water. The temperature rise of the water heated by the net heat rate from the solar collector is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Specific heat of water is constant. **3** Temperature at the surface remained constant. **4** Conduction through the solar absorber is negligible. **5** Heat loss through the sides and back of the absorber is negligible.

Properties The absorber surface has an absorptivity of 0.9 and an emissivity of 0.9. The specific heat of water is given as 4.2 kJ/kg·K.



Analysis The net heat rate absorbed by the solar collector is

$$\dot{Q}_{net} = \dot{Q}_{absorbed} - \dot{Q}_{rad} - \dot{Q}_{conv}$$

$$\dot{Q}_{net} = A_s[\alpha\dot{q}_{solar} - \varepsilon\sigma(T_s^4 - T_{surr}^4) - h(T_s - T_\infty)]$$

$$\dot{Q}_{net} = (2\text{ m}^2)[(0.9)(500\text{ W/m}^2) - (0.9)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(35+273)^4 - (0+273)^4)]\text{K}^4$$
$$-(5\text{ W/m}^2\cdot\text{K})(35-25)\text{ K}]$$

$$\dot{Q}_{net} = 448.4\text{ W}$$

The temperature rise can be determined using

$$\dot{Q}_{net} = \dot{m}c_p(T_{out} - T_{in})$$

Thus,

$$T_{out} - T_{in} = \frac{\dot{Q}_{net}}{\dot{m}c_p} = \frac{448.4\text{ W}}{(0.005\text{ kg/s})(4200\text{ J/kg}\cdot\text{K})} = \mathbf{21.4°C}$$

Discussion The temperature rise of the water is influenced by the usable net heat rate absorbed by the solar collector and the water flow rate.

1-102 A draw batch furnace front is subjected to uniform heat flux on the inside surface, while the outside surface is subjected to convection and radiation heat transfer. The outside surface temperature is to be determined whether it is below 50°C or not.

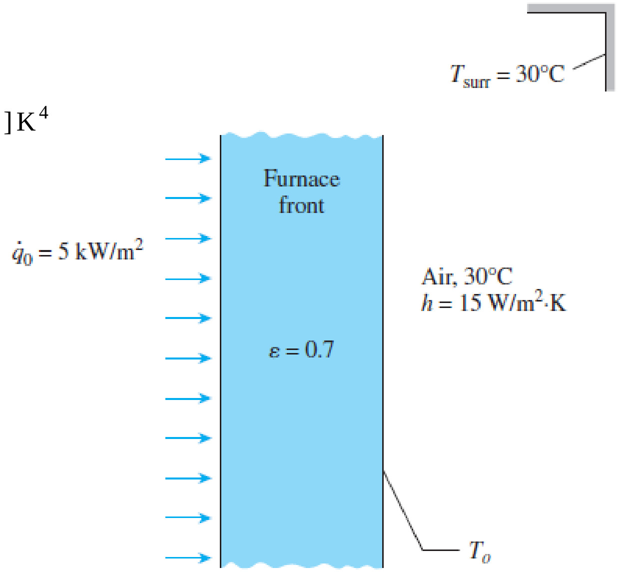
Assumptions **1** Heat conduction is steady. **2** One dimensional heat conduction across the furnace front thickness. **3** Uniform heat flux on inside surface.

Properties Emissivity and convective heat transfer coefficient are given to be 0.7 and 15 W/m²·K, respectively.

Analysis The uniform heat flux subjected on the inside surface is equal to the sum of heat fluxes transferred by convection and radiation on the outside surface

$$\dot{q}_0 = h(T_o - T_\infty) + \varepsilon\sigma(T_o^4 - T_{\rm surr}^4)$$

$$5000\ \text{W/m}^2 = (15\ \text{W/m}^2\cdot\text{K})[T_o - (30+273)]\,\text{K}$$
$$+ (0.7)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[T_o^4 - (30+273)^4]\,\text{K}^4$$



Solving for the outer surface temperature yields

$$T_o = \mathbf{497\ K} = \mathbf{224°C} > 50°\text{C}$$

Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
h=15 [W/m^2-K]
q_dot_0=5000 [W/m^2]
T_surr=303 [K]
epsilon=0.7
sigma=5.67e-8 [W/m^2-K^4]
q_dot_0=h*(T_o-T_surr)+epsilon*sigma#*(T_o^4-T_surr^4)
```

Discussion Yes, insulation is required on the furnace front surface to avoid thermal burn upon contact with skin.

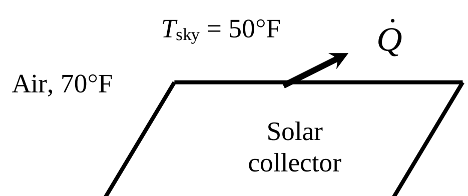
1-103E A flat plate solar collector is placed horizontally on the roof of a house. The rate of heat loss from the collector by convection and radiation during a calm day are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The emissivity and convection heat transfer coefficient are constant and uniform. **3** The exposed surface, ambient, and sky temperatures remain constant.

Properties The emissivity of the outer surface of the collector is given to be 0.9.

Analysis The exposed surface area of the collector is

$$A_s = (5\ \text{ft})(15\ \text{ft}) = 75\ \text{ft}^2$$



Noting that the exposed surface temperature of the collector is 100°F, the total rate of heat loss from the collector to the environment by convection and radiation becomes

$$\dot{Q}_{\rm conv} = hA_s(T_\infty - T_s) = (2.5\ \text{Btu/h.ft}^2\cdot°\text{F})(75\ \text{ft}^2)(100-70)°\text{F} = 5625\ \text{Btu/h}$$

$$\dot{Q}_{\rm rad} = \varepsilon A_s\sigma(T_{\rm surr}^4 - T_s^4) = (0.9)(75\ \text{ft}^2)(0.1714\times10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)[(100+460\ \text{R})^4 - (50+460\ \text{R})^4]$$
$$= 3551\ \text{Btu/h}$$

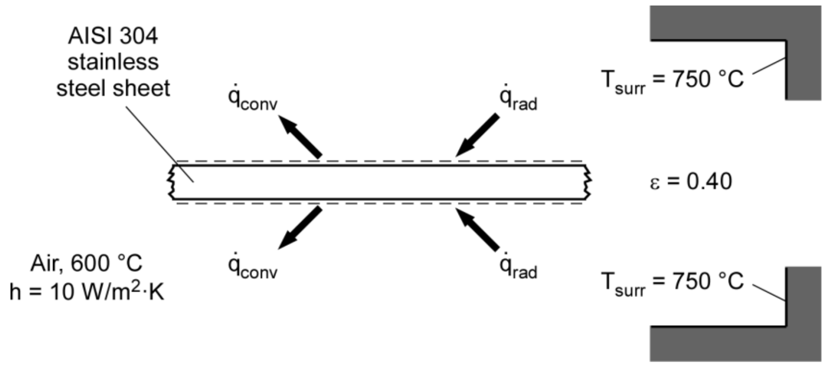
and

$$\dot{Q}_{\rm total} = \dot{Q}_{\rm conv} + \dot{Q}_{\rm rad} = 5625 + 3551 = \mathbf{9176\ Btu/h}$$

1-104 Temperature of the stainless steel sheet going through an annealing process inside an electrically heated oven is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Temperature of the stainless steel sheet is uniform. **3** Radiation heat transfer between stainless steel sheet and surrounding oven surfaces is between a small object and a large enclosure.

Properties The emissivity of the stainless steel sheet is given to be 0.40.



Analysis The amount of heat transfer by radiation between the sheet and the surrounding oven surfaces is balanced by the convection heat transfer between the sheet and the ambient air:

$$\dot{q}_{\text{rad}} - \dot{q}_{\text{conv}} = 0$$

$$\varepsilon\sigma(T_{\text{surr}}^4 - T_s^4) - h(T_s - T_\infty) = 0$$

$$(0.40)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(750+273)^4 - T_s^4]\,\text{K}^4 - (10\ \text{W/m}^2\cdot\text{K})[T_s - (600+273)]\,\text{K} = 0$$

Solving the above equation by EES software (Copy the following line and paste on a blank EES screen to verify solution):

```
0.40*5.67e-8*((750+273)^4-T_s^4)-10*(T_s-(600+273))=0
```

The temperature of the stainless steel sheet is

$$T_s = 1009\ \text{K} = \mathbf{736\,°C}$$

Discussion Note that the energy balance equation involving radiation heat transfer used for solving the stainless steel sheet temperature must be used with absolute temperature.

1-105 The roof of a house with a gas furnace consists of a 15-cm thick concrete that is losing heat to the outdoors by radiation and convection. The rate of heat transfer through the roof and the money lost through the roof that night during a 14 hour period are to be determined.

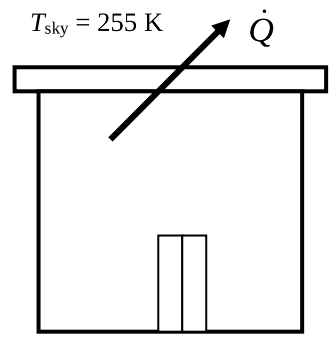
Assumptions **1** Steady operating conditions exist. **2** The emissivity and thermal conductivity of the roof are constant.

Properties The thermal conductivity of the concrete is given to be k = 2 W/m·°C. The emissivity of the outer surface of the roof is given to be 0.9.

Analysis In steady operation, heat transfer from the outer surface of the roof to the surroundings by convection and radiation must be equal to the heat transfer through the roof by conduction. That is,

$$\dot{Q} = \dot{Q}_{\text{roof, cond}} = \dot{Q}_{\text{roof to surroundings, conv+rad}}$$

The inner surface temperature of the roof is given to be $T_{s,in}$ = 15°C. Letting $T_{s,out}$ denote the outer surface temperatures of the roof, the energy balance above can be expressed as



$$\dot{Q} = kA\frac{T_{s,in} - T_{s,out}}{L} = h_o A(T_{s,out} - T_{surr}) + \varepsilon A\sigma(T_{s,out}{}^4 - T_{surr}{}^4)$$

$$\dot{Q} = (2\text{ W/m}\cdot{}^\circ\text{C})(300\text{ m}^2)\frac{15^\circ\text{C} - T_{s,out}}{0.15\text{ m}}$$

$$= (15\text{ W/m}^2.{}^\circ\text{C})(300\text{ m}^2)(T_{s,out} - 10)^\circ\text{C}$$

$$+ (0.9)(300\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(T_{s,out} + 273\text{ K})^4 - (255\text{ K})^4\right]$$

Solving the equations above using an equation solver (or by trial and error) gives

$$\dot{Q} = \mathbf{25{,}450\text{ W}} \text{ and } T_{s,\,out} = \mathbf{8.64^\circ C}$$

Then the amount of natural gas consumption during a 16-hour period is

$$E_{gas} = \frac{Q_{total}}{0.85} = \frac{\dot{Q}\Delta t}{0.85} = \frac{(25.450\text{ kJ/s})(14\times3600\text{ s})}{0.85}\left(\frac{1\text{ therm}}{105{,}500\text{ kJ}}\right) = 14.3\text{ therms}$$

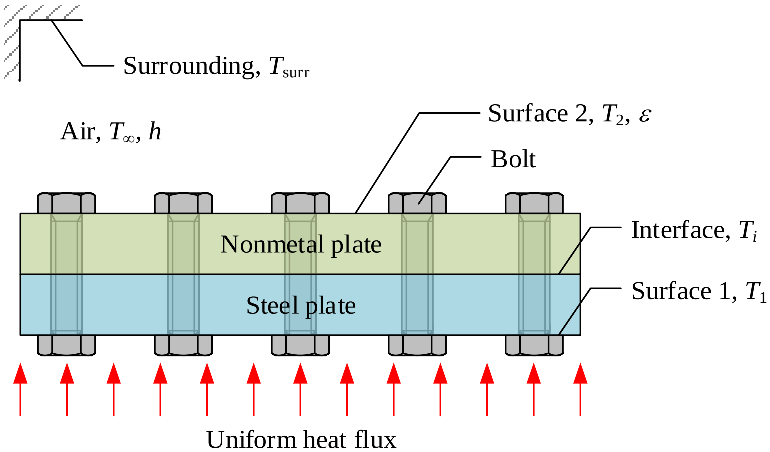
Finally, the money lost through the roof during that period is

$$\text{Money lost} = (14.3\text{ therms})(\$0.60 / \text{therm}) = \mathbf{\$8.58}$$

1-106 A nonmetal plate and an ASME SA-240 stainless steel plate are bolted together by ASTM B21 naval brass bolts. The bottom surface is subjected to uniform heat flux. The top surface is exposed to convection and radiation heat transfer. Determine whether the ASME Boiler and Pressure Vessel Code and the ASME Code for Process Piping are being complied.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through the plates. **3** Uniform heat flux on bottom surface. **4** Uniform surface temperatures. **5** No contact resistance at the interface.

Properties The thermal conductivities for the steel plate is given as k_1 = 15 W/m·K and for the nonmetal plate as k_2 = 0.05 W/m·K.



Analysis The uniform heat flux subjected on the bottom plate surface is transferred by conduction through the plates, and by convection and radiation on the top surface:

$$\dot{q}_0 = \dot{q}_{\text{cond,1}} = 200 \text{ W/m}^2$$

$$\dot{q}_0 = \dot{q}_{\text{cond,1}} = \dot{q}_{\text{cond,2}}$$

$$\dot{q}_0 = \dot{q}_{\text{cond,2}} = \dot{q}_{\text{conv}} + \dot{q}_{\text{rad}}$$

where

$$\dot{q}_{\text{cond,1}} = k_1 \frac{T_1 - T_i}{L}, \qquad \dot{q}_{\text{cond,2}} = k_2 \frac{T_i - T_2}{L}, \qquad \dot{q}_{\text{conv}} = h(T_2 - T_\infty), \qquad \dot{q}_{\text{rad}} = \varepsilon\sigma(T_2^4 - T_{\text{surr}}^4)$$

Combining the equations to solve for T_1, T_2, and T_i, we have

$$200 \text{ W/m}^2 = k_1 \frac{T_1 - T_i}{L} \qquad (1)$$

$$200 \text{ W/m}^2 = k_2 \frac{T_i - T_2}{L} \qquad (2)$$

$$200 \text{ W/m}^2 = h(T_2 - T_\infty) + \varepsilon\sigma(T_2^4 - T_{\text{surr}}^4) \qquad (3)$$

Solving for the top surface temperature T_2 using Eq. (3), yields

$$200 \text{ W/m}^2 = (5 \text{ W/m}^2\cdot\text{K})(T_2 - 20)\text{ K} + (0.3)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(T_2 + 273)^4 - (30 + 273)^4]\text{ K}^4$$

$$\rightarrow \qquad T_2 = 51.13°\text{C}$$

Solving for the interface temperature T_i using Eq. (2), yields

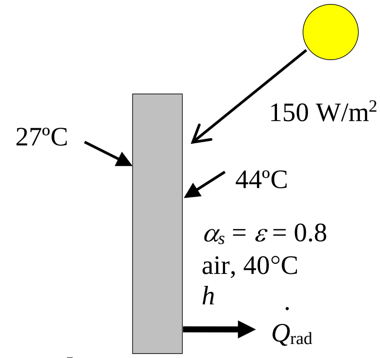
$$200 \text{ W/m}^2 = (0.05 \text{ W/m}\cdot\text{K})\frac{T_i - 51.13°\text{C}}{0.038 \text{ m}} \quad \rightarrow \quad T_i = 203.1°\text{C}$$

Solving for the bottom surface temperature T_1 using Eq. (1), yields

$$200 \text{ W/m}^2 = (15 \text{ W/m}\cdot\text{K})\frac{T_1 - 203.1°\text{C}}{0.038 \text{ m}} \quad \rightarrow \quad T_1 = \mathbf{203.6°C}$$

DiscussionThe ASME SA-240 stainless steel plate has an average temperature of 203.6°C, which complies with the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HF-300) which requires a temperature of less than 260°C. However, the ASTM B21 naval brass bolts are used in a steel plate that its temperature exceeds the maximum use temperature of 149°C required by the ASME Code for Process Piping (ASME B31.3-2014). Another type of bolts should be considered for use under these conditions. For example, the ASTM A193 B8M steel bolts have a maximum use temperature of 538°C (ASME B31.3-2014).

1-107 The outer surface of a wall is exposed to solar radiation. The effective thermal conductivity of the wall is to be determined.



Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is constant and uniform over the surface.

Properties Both the solar absorptivity and emissivity of the wall surface are given to be 0.8.

Analysis The heat transfer through the wall by conduction is equal to net heat transfer to the outer wall surface:

$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} + \dot{q}_{\text{rad}} + \dot{q}_{\text{solar}}$$

$$k\frac{T_2 - T_1}{L} = h(T_o - T_2) + \varepsilon\sigma(T_{\text{surr}}^4 - T_2^4) + \alpha_s q_{\text{solar}}$$

$$k\frac{(44-27)°\text{C}}{0.25\,\text{m}} = (8\text{ W/m}^2\cdot°\text{C})(40-44)°\text{C} + (0.8)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(40+273\,\text{K})^4 - (44+273\,\text{K})^4\right]$$

$$+(0.8)(150\text{ W/m}^2)$$

Solving for k gives

$$k = \mathbf{0.961\,W/m\cdot°C}$$

Solving Engineering Problems

1-108C (*a*) Despite the convenience and capability the engineering software packages offer, they are still just tools, and they will not replace the traditional engineering courses. They will simply cause a shift in emphasis in the course material from mathematics to physics. (*b*) They are of great value in engineering practice, however, as engineers today rely on software packages for solving large and complex problems in a short time, and perform optimization studies efficiently.

1-109 We are to determine a positive real root of the following equation using EES: $3.5x^3 - 10x^{0.5} - 3x = -4$.

Analysis Using EES software, copy the following lines and paste on a blank EES screen to verify the solution:

```
3.5*x^3-10*x^0.5-3*x = -4
```

Answer: ***x* = 1.554**

Discussion To obtain the solution in EES, click on the icon that looks like a calculator, or Calculate-Solve.

1-110 We are to solve a system of 2 equations and 2 unknowns using EES.

Analysis Using EES software, copy the following lines and paste on a blank EES screen to verify the solution:

```
x^3-y^2=10.5
3*x*y+y=4.6
```

Answers: ***x* = 2.215**, ***y* = 0.6018**

Discussion To obtain the solution in EES, click on the icon that looks like a calculator, or Calculate-Solve.

1-111 We are to solve a system of 3 equations with 3 unknowns using EES.

Analysis Using EES software, copy the following lines and paste on a blank EES screen to verify the solution:

```
2*x-y+z=5
3*x^2+2*y=z+2
x*y+2*z=8
```

Answers: ***x* = 1.141, *y* = 0.8159, *z* = 3.535.**

Discussion To obtain the solution in EES, click on the icon that looks like a calculator, or Calculate-Solve.

1-112 We are to solve a system of 3 equations with 3 unknowns using EES.

Analysis Using EES software, copy the following lines and paste on a blank EES screen to verify the solution:

```
x^2*y-z=1.5
x-3*y^0.5+x*z=-2
x+y-z=4.2
```

Answers: ***x* = 0.9149, *y* = 10.95, *z* = 7.665**

Discussion To obtain the solution in EES, click on the icon that looks like a calculator, or Calculate-Solve.

Special Topic: Thermal Comfort

1-113C The metabolism refers to the burning of foods such as carbohydrates, fat, and protein in order to perform the necessary bodily functions. The metabolic rate for an average man ranges from 108 W while reading, writing, typing, or listening to a lecture in a classroom in a seated position to 1250 W at age 20 (730 at age 70) during strenuous exercise. The corresponding rates for women are about 30 percent lower. Maximum metabolic rates of trained athletes can exceed 2000 W. We are interested in metabolic rate of the occupants of a building when we deal with heating and air conditioning because the metabolic rate represents the rate at which a body generates heat and dissipates it to the room. This body heat contributes to the heating in winter, but it adds to the cooling load of the building in summer.

1-114C The metabolic rate is proportional to the size of the body, and the metabolic rate of women, in general, is lower than that of men because of their smaller size. Clothing serves as insulation, and the thicker the clothing, the lower the environmental temperature that feels comfortable.

1-115C Asymmetric thermal radiation is caused by the *cold surfaces* of large windows, uninsulated walls, or cold products on one side, and the *warm surfaces* of gas or electric radiant heating panels on the walls or ceiling, solar heated masonry walls or ceilings on the other. Asymmetric radiation causes discomfort by exposing different sides of the body to surfaces at different temperatures and thus to different rates of heat loss or gain by radiation. A person whose left side is exposed to a cold window, for example, will feel like heat is being drained from that side of his or her body.

1-116C (*a*) Draft causes undesired local cooling of the human body by exposing parts of the body to high heat transfer coefficients. (*b*) Direct contact with *cold floor surfaces* causes localized discomfort in the feet by excessive heat loss by conduction, dropping the temperature of the bottom of the feet to uncomfortable levels.

1-117C Stratification is the formation of vertical still air layers in a room at difference temperatures, with highest temperatures occurring near the ceiling. It is likely to occur at places with high ceilings. It causes discomfort by exposing the head and the feet to different temperatures. This effect can be prevented or minimized by using destratification fans (ceiling fans running in reverse).

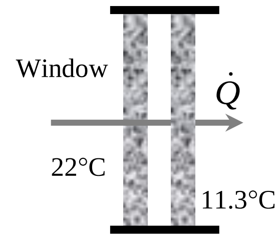
1-118C It is necessary to ventilate buildings to provide adequate fresh air and to get rid of excess carbon dioxide, contaminants, odors, and humidity. Ventilation increases the energy consumption for heating in winter by replacing the warm indoors air by the colder outdoors air. Ventilation also increases the energy consumption for cooling in summer by replacing the cold indoors air by the warm outdoors air. It is not a good idea to keep the bathroom fans on all the time since they will waste energy by expelling conditioned air (warm in winter and cool in summer) by the unconditioned outdoor air.

Review Problems

1-119 The windows of a house in Atlanta are of double door type with wood frames and metal spacers. The average rate of heat loss through the windows in winter is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses associated with the infiltration of air through the cracks/openings are not considered.

Analysis The rate of heat transfer through the window can be determined from



$$\dot{Q}_{\text{window, avg}} = U_{\text{overall}} A_{\text{window}} (T_{\text{i}} - T_{\text{o}})$$

where T_i and T_o are the indoor and outdoor air temperatures, respectively, U_{overall} is the *U*-factor (the overall heat transfer coefficient) of the window, and A_{window} is the window area. Substituting,

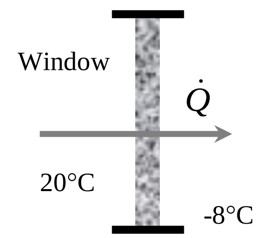
$$\dot{Q}_{\text{window, avg}} = (2.5\ \text{W/m}^2 \cdot {}^\circ\text{C})(20\ \text{m}^2)(22 - 11.3)^\circ\text{C} = \mathbf{535\ W}$$

Discussion This is the "average" rate of heat transfer through the window in winter in the absence of any infiltration.

1-120 The range of U-factors for windows are given. The range for the rate of heat loss through the window of a house is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses associated with the infiltration of air through the cracks/openings are not considered.

Analysis The rate of heat transfer through the window can be determined from



$$\dot{Q}_{\text{window}} = U_{\text{overall}} A_{\text{window}} (T_i - T_o)$$

where T_i and T_o are the indoor and outdoor air temperatures, respectively, U_{overall} is the U-factor (the overall heat transfer coefficient) of the window, and A_{window} is the window area. Substituting,

Maximum heat loss: $$\dot{Q}_{\text{window, max}} = (6.25\ \text{W/m}^2 \cdot {}^\circ\text{C})(1.2 \times 1.8\ \text{m}^2)[20 - (-8)]^\circ\text{C} = \mathbf{378\ W}$$

Minimum heat loss: $$\dot{Q}_{\text{window, min}} = (1.25\ \text{W/m}^2 \cdot {}^\circ\text{C})(1.2 \times 1.8\ \text{m}^2)[20 - (-8)]^\circ\text{C} = \mathbf{76\ W}$$

Discussion Note that the rate of heat loss through windows of identical size may differ by a factor of 5, depending on how the windows are constructed.

1-121 A room is to be heated by 1 ton of hot water contained in a tank placed in the room. The minimum initial temperature of the water is to be determined if it to meet the heating requirements of this room for a 24-h period.

Assumptions **1** Water is an incompressible substance with constant specific heats. **2** Air is an ideal gas with constant specific heats. **3** The energy stored in the container itself is negligible relative to the energy stored in water. **4** The room is maintained at 20°C at all times. **5** The hot water is to meet the heating requirements of this room for a 24-h period.

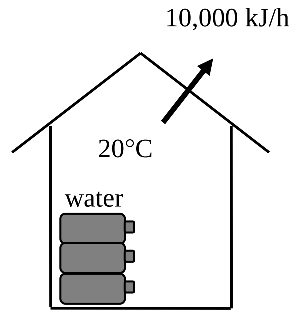
Properties The specific heat of water at room temperature is c = 4.18 kJ/kg·°C (Table A-9).

Analysis Heat loss from the room during a 24-h period is

$$Q_{\text{loss}} = (10{,}000 \text{ kJ/h})(24 \text{ h}) = 240{,}000 \text{ kJ}$$

Taking the contents of the room, including the water, as our system, the energy balance can be written as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc. energies}}} \rightarrow -Q_{\text{out}} = \Delta U = (\Delta U)_{\text{water}} + (\Delta U)_{\text{air}}^{\nearrow 0}$$



or

$$-Q_{\text{out}} = [mc(T_2 - T_1)]_{\text{water}}$$

Substituting,

$$-240{,}000 \text{ kJ} = (1000 \text{ kg})(4.18 \text{ kJ/kg·°C})(20 - T_1)$$

It gives

$$T_1 = \mathbf{77.4°C}$$

where T_1 is the temperature of the water when it is first brought into the room.

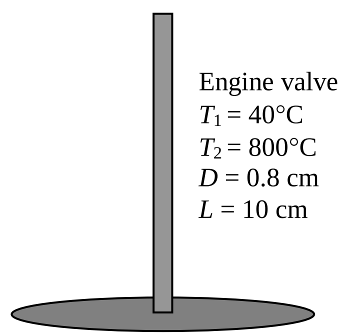
1-122 Engine valves are to be heated in a heat treatment section. The amount of heat transfer, the average rate of heat transfer, the average heat flux, and the number of valves that can be heat treated daily are to be determined.

Assumptions Constant properties given in the problem can be used.

Properties The average specific heat and density of valves are given to be c_p = 440 J/kg.°C and ρ = 7840 kg/m^3.

Analysis (*a*) The amount of heat transferred to the valve is simply the change in its internal energy, and is determined from

$$Q = \Delta U = mc_p(T_2 - T_1)$$
$$= (0.0788 \text{ kg})(0.440 \text{ kJ/kg·°C})(800 - 40)\text{°C} = \mathbf{26.35\ kJ}$$



(*b*) The average rate of heat transfer can be determined from

$$\dot{Q}_{\text{avg}} = \frac{Q}{\Delta t} = \frac{26.35 \text{ kJ}}{5 \times 60 \text{ s}} = 0.0878 \text{ kW} = \mathbf{87.8\ W}$$

(*c*) The average heat flux is determined from

$$\dot{q}_{\text{ave}} = \frac{\dot{Q}_{\text{avg}}}{A_s} = \frac{\dot{Q}_{\text{avg}}}{2\pi DL} = \frac{87.8 \text{ W}}{2\pi(0.008 \text{ m})(0.1 \text{m})} = \mathbf{1.75 \times 10^4\ \ W/m^2}$$

(*d*) The number of valves that can be heat treated daily is

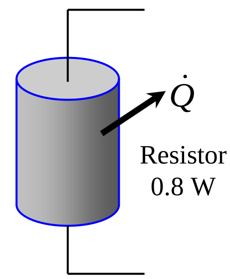
$$\text{Number of valves} = \frac{(10 \times 60 \text{ min})(25 \text{ valves})}{5 \text{ min}} = \mathbf{3000\ \ valves}$$

1-123 A cylindrical resistor on a circuit board dissipates 0.8 W of power. The amount of heat dissipated in 24 h, the heat flux, and the fraction of heat dissipated from the top and bottom surfaces are to be determined.

Assumptions Heat is transferred uniformly from all surfaces.

Analysis (*a*) The amount of heat this resistor dissipates during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.8\ \text{W})(24\ \text{h}) = \mathbf{19.2\ Wh = 69.1\ kJ} \quad \text{(since 1 Wh = 3600 Ws = 3.6 kJ)}$$



(*b*) The heat flux on the surface of the resistor is

$$A_s = 2\frac{\pi D^2}{4} + \pi DL = 2\frac{\pi(0.4\,\text{cm})^2}{4} + \pi(0.4\,\text{cm})(2\text{cm}) = 0.251 + 2.513 = 2.764\,\text{cm}^2$$

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = \frac{0.80\ \text{W}}{2.764\,\text{cm}^2} = \mathbf{0.289\ W/cm^2}$$

(*c*) Assuming the heat transfer coefficient to be uniform, heat transfer is proportional to the surface area. Then the fraction of heat dissipated from the top and bottom surfaces of the resistor becomes

$$\frac{Q_{\text{top-base}}}{Q_{\text{total}}} = \frac{A_{\text{top-base}}}{A_{\text{total}}} = \frac{0.251}{2.764} = \mathbf{0.091} \ \text{ or } \ (9.1\%)$$

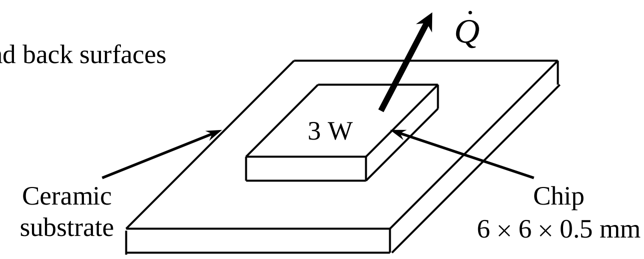
Discussion Heat transfer from the top and bottom surfaces is small relative to that transferred from the side surface.

1-124 The heat generated in the circuitry on the surface of a 3-W silicon chip is conducted to the ceramic substrate. The temperature difference across the chip in steady operation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Thermal properties of the chip are constant.

Properties The thermal conductivity of the silicon chip is given to be k = 130 W/m·°C.

Analysis The temperature difference between the front and back surfaces of the chip is



$$A = (0.006\,\text{m})(0.006\text{m}) = 0.000036\,\text{m}^2$$

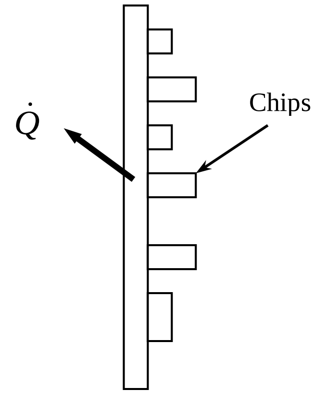
$$\dot{Q} = kA\frac{\Delta T}{L}$$

$$\Delta T = \frac{\dot{Q}L}{kA} = \frac{(3\ \text{W})(0.0005\,\text{m})}{(130\ \text{W/m}\cdot°\text{C})(0.000036\,\text{m}^2)} = \mathbf{0.32°C}$$

1-125 A circuit board houses 80 closely spaced logic chips on one side, each dissipating 0.06 W. All the heat generated in the chips is conducted across the circuit board. The temperature difference between the two sides of the circuit board is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Thermal properties of the board are constant. **3** All the heat generated in the chips is conducted across the circuit board.

Properties The effective thermal conductivity of the board is given to be k = 16 W/m·°C.



Analysis The total rate of heat dissipated by the chips is

$$\dot{Q} = 80 \times (0.06\text{ W}) = 4.8\text{ W}$$

Then the temperature difference between the front and back surfaces of the board is

$$A = (0.12\text{ m})(0.18\text{ m}) = 0.0216\ \text{m}^2$$

$$\dot{Q} = kA\frac{\Delta T}{L} \longrightarrow \Delta T = \frac{\dot{Q}L}{kA} = \frac{(4.8\text{ W})(0.003\text{ m})}{(16\text{ W/m}\cdot{}^\circ\text{C})(0.0216\text{ m}^2)} = \mathbf{0.042^\circ C}$$

Discussion Note that the circuit board is nearly isothermal.

1-126 An electric resistance heating element is immersed in water initially at 20°C. The time it will take for this heater to raise the water temperature to 80°C as well as the convection heat transfer coefficients at the beginning and at the end of the heating process are to be determined.

Assumptions **1** Steady operating conditions exist and thus the rate of heat loss from the wire equals the rate of heat generation in the wire as a result of resistance heating. **2** Thermal properties of water are constant. **3** Heat losses from the water in the tank are negligible.

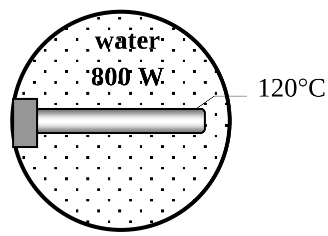
Properties The specific heat of water at room temperature is c = 4.18 kJ/kg·°C (Table A-9).

Analysis When steady operating conditions are reached, we have $\dot{Q} = \dot{E}_{\text{generated}} = 800\text{ W}$. This is also equal to the rate of heat gain by water. Noting that this is the only mechanism of energy transfer, the time it takes to raise the water temperature from 20°C to 80°C is determined to be

$$Q_{\text{in}} = mc(T_2 - T_1)$$

$$\dot{Q}_{\text{in}}\Delta t = mc(T_2 - T_1)$$

$$\Delta t = \frac{mc(T_2 - T_1)}{\dot{Q}_{\text{in}}} = \frac{(75\text{ kg})(4180\text{ J/kg}\cdot{}^\circ\text{C})(80-20)^\circ\text{C}}{800\text{ J/s}} = 23{,}510\text{ s} = \mathbf{6.53\,h}$$



The surface area of the wire is

$$A_s = \pi DL = \pi(0.005\text{ m})(0.4\text{m}) = 0.00628\text{ m}^2$$

The Newton's law of cooling for convection heat transfer is expressed as $\dot{Q} = hA_s(T_s - T_\infty)$. Disregarding any heat transfer by radiation and thus assuming all the heat loss from the wire to occur by convection, the convection heat transfer coefficients at the beginning and at the end of the process are determined to be

$$h_1 = \frac{\dot{Q}}{A_s(T_s - T_{\infty 1})} = \frac{800\text{ W}}{(0.00628\text{ m}^2)(120-20)^\circ\text{C}} = \mathbf{1274\ W/m^2\cdot{}^\circ C}$$

$$h_2 = \frac{\dot{Q}}{A_s(T_s - T_{\infty 2})} = \frac{800\text{ W}}{(0.00628\text{ m}^2)(120-80)^\circ\text{C}} = \mathbf{3185\ W/m^2\cdot{}^\circ C}$$

Discussion Note that a larger heat transfer coefficient is needed to dissipate heat through a smaller temperature difference for a specified heat transfer rate.

1-127 A standing man is subjected to high winds and thus high convection coefficients. The rate of heat loss from this man by convection in still air at 20°C, in windy air, and the wind chill temperature are to be determined.

Assumptions **1** A standing man can be modeled as a 30-cm diameter, 170-cm long vertical cylinder with both the top and bottom surfaces insulated. **2** The exposed surface temperature of the person and the convection heat transfer coefficient is constant and uniform. **3** Heat loss by radiation is negligible.

Analysis The heat transfer surface area of the person is

$$A_s = \pi DL = \pi(0.3\text{ m})(1.70\text{ m}) = 1.60\text{ m}^2$$

Windy weather

The rate of heat loss from this man by convection in still air is

$$Q_{\text{still air}} = hA_s\Delta T = (15\text{ W/m}^2\cdot°\text{C})(1.60\text{ m}^2)(34 - 20)°\text{C} = \mathbf{336\ W}$$

In windy air it would be

$$Q_{\text{windy air}} = hA_s\Delta T = (30\text{ W/m}^2\cdot°\text{C})(1.60\text{ m}^2)(34 - 20)°\text{C} = \mathbf{672\ W}$$

To lose heat at this rate in still air, the air temperature must be

$$672\text{ W} = (hA_s\Delta T)_{\text{still air}} = (15\text{ W/m}^2\cdot°\text{C})(1.60\text{ m}^2)(34 - T_{\text{effective}})°\text{C}$$

which gives

$$T_{\text{effective}} = \mathbf{6°C}$$

That is, the windy air at 20°C feels as cold as still air at 6°C as a result of the wind-chill effect.

1-128 The surface temperature of an engine block that generates 50 kW of power output is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Temperature inside the engine compartment is uniform. **3** Heat transfer by radiation is not considered.

Analysis With a net engine efficiency of 35%, which means 65% of the generated power output are heat loss by convection:

$$\dot{Q}_{\text{conv}} = \dot{W}_{\text{out}}(1-\eta) = (50\text{ kW})(1-0.35) = 32.5\text{ kW}$$

From Newton's law of cooling, the heat transfer by convection is given as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

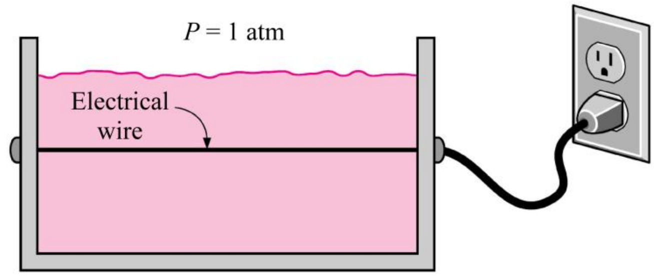
Rearranging, the engine block surface temperature is

$$T_s = \frac{\dot{Q}_{\text{conv}}}{hA_s} + T_\infty = \frac{32.5\times10^3\text{ W}}{(50\text{ W/m}^2\cdot°\text{C})(0.95\text{ m}^2)} + 157\,°\text{C} = \mathbf{841\,°C}$$

Discussion Due to the complex geometry of the engine block, hot spots are likely to occur with temperatures much higher than 841 °C.

1-129 Electric power required to maintain the surface temperature of an electrical wire submerged in boiling water at 115ºC.

Assumptions **1** Steady operating conditions exist. **2** Convection heat transfer coefficient is uniform. **3** Heat transfer by radiation is negligible. **4** Heat losses from the boiler are negligible.



Analysis From an overall energy balance on the electrical wire, the power dissipated by the wire is transferred by convection to the water. Using Newton's law of cooling,

$$\dot{Q}_{conv} = h\, A_s\, (T_s - T_\infty)$$

where the surface area of the wire is

$$A_s = \pi DL = \pi\,(1 \times 10^{-3})\,(15 \times 10^{-2}) = 4.712 \times 10^{-4}\ \text{m}^2$$

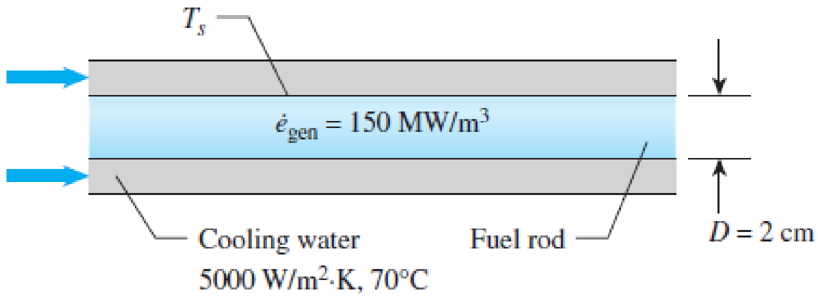
The heat transfer by convection with h = 51,250 W/m^2·K (average of the upper and lower values of the convection heat transfer coefficients given in Table 1-5) is

$$\dot{Q}_{conv} = h\, A_s\, (T_s - T_\infty) = (51{,}250\ \text{W/m}^2\cdot\text{K})\,(4.712 \times 10^{-4}\ \text{m}^2)\,(115 - 100)\ \text{K} = \mathbf{362.2\ W}$$

This is equal to the electric power that must be supplied to maintain the surface temperature of the wire at 115ºC.

Discussion If we had used the two extreme values of the convection heat transfer coefficient for boiling and condensation given in Table 1-5, the required electric power would range from 17.7 W (for h = 2500 W/m^2·K) to 706.8 W (for h = 100,000 W/m^2·K).

1-130 A cylindrical fuel rod is cooled by water flowing through its encased concentric tube. The surface temperature of the fuel rod is to be determined.



Assumptions **1** Steady operating conditions exist. **2** Heat generation in the fuel rod is uniform.

Analysis The total heat transfer area of the fuel rod is

$$A_s = \pi DL$$

The heat transfer rate from the fuel rod is equal to the rate of heat generation multiplied by the fuel rod volume

$$\dot{Q} = \dot{e}_{gen}(\pi D^2 L/4)$$

The rate of heat removal from the fuel rod by the cooling water is

$$\dot{Q} = hA_s(T_s - T_\infty)$$

Thus, $\dot{e}_{gen}(\pi D^2 L/4) = h\pi DL(T_s - T_\infty)$

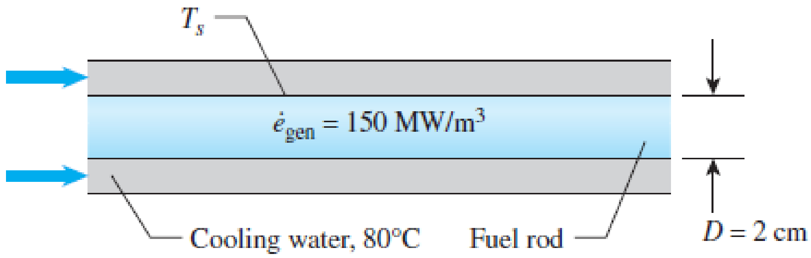
The surface temperature of the fuel rod is

$$T_s = T_\infty + \frac{D\dot{e}_{gen}}{4h}$$

$$T_s = 70°\text{C} + \frac{(0.02\text{ m})(150\times10^6\text{ W/m}^3)}{4(5000\text{ W/m}^2\cdot\text{K})} = \mathbf{220°C}$$

Discussion A convection heat transfer coefficient of 5000 W/m^2·K for forced convection of liquid is reasonable when compared with the values listed in Table 1-5.

1-131 A cylindrical fuel rod is cooled by water flowing through its encased concentric tube. The surface temperature of the fuel rod must be maintained below 300°C and the convection heat transfer coefficient is to be determined.



Assumptions **1** Steady operating conditions exist. **2** Heat generation in the fuel rod is uniform.

Analysis The total heat transfer area of the fuel rod is

$$A_s = \pi DL$$

The heat transfer rate from the fuel rod is equal to the rate of heat generation multiply the fuel rod volume

$$\dot{Q} = \dot{e}_{gen}(\pi D^2 L/4)$$

The heat transfer rate removed from the fuel rod by the cooling water is

$$\dot{Q} = hA_s(T_s - T_\infty)$$

Thus,

$$\dot{e}_{gen}(\pi D^2 L/4) = h\pi DL(T_s - T_\infty)$$

The convection heat transfer coefficient can be determined as

$$h = \frac{D\dot{e}_{gen}}{4(T_s - T_\infty)}$$

$$= \frac{(0.02\text{ m})(150\times 10^6\text{ W/m}^3)}{4(300-80)\text{ K}}$$

$$= 3410\text{ W/m}^2\cdot\text{K}$$

To maintained the fuel rod surface temperature below 300°C, the convection heat transfer coefficient should be

$$h > \mathbf{3410\ W/m^2\cdot K}$$

Discussion For the fuel rod surface temperature to be kept below 300°C, the cooling water convection heat transfer coefficient should be higher than 3410 W/m^2·K. Higher value of convection heat transfer coefficient can be achieved by increasing the flow rate of cooling water.

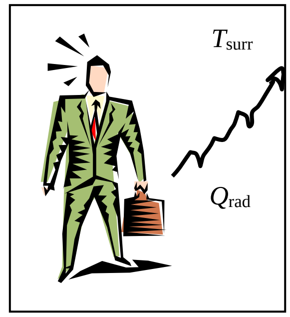
1-132 The rate of radiation heat transfer between a person and the surrounding surfaces at specified temperatures is to be determined in summer and in winter.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer by convection is not considered. **3** The person is completely surrounded by the interior surfaces of the room. **4** The surrounding surfaces are at a uniform temperature.

Properties The emissivity of a person is given to be ε = 0.95

Analysis Noting that the person is completely enclosed by the surrounding surfaces, the net rates of radiation heat transfer from the body to the surrounding walls, ceiling, and the floor in both cases are:

(*a*) Summer: T_{surr} = 23+273=296



$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon\sigma A_s (T_s^4 - T_{surr}^4) \\ &= (0.95)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1.6\,\text{m}^2)[(32+273)^4 - (296\,\text{K})^4]\text{K}^4 \\ &= \mathbf{84.2\ W}\end{aligned}$$

(*b*) Winter: T_{surr} = 12+273= 285 K

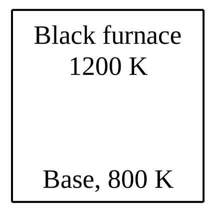
$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon\sigma A_s (T_s^4 - T_{surr}^4) \\ &= (0.95)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1.6\,\text{m}^2)[(32+273)^4 - (285\,\text{K})^4]\text{K}^4 \\ &= \mathbf{177.2\ W}\end{aligned}$$

Discussion Note that the radiation heat transfer from the person more than doubles in winter.

1-133 The base surface of a cubical furnace is surrounded by black surfaces at a specified temperature. The net rate of radiation heat transfer to the base surface from the top and side surfaces is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The top and side surfaces of the furnace closely approximate black surfaces. **3** The properties of the surfaces are constant.

Properties The emissivity of the base surface is ε = 0.7.



Analysis The base surface is completely surrounded by the top and side surfaces. Then using the radiation relation for a surface completely surrounded by another large (or black) surface, the net rate of radiation heat transfer from the top and side surfaces to the base is determined to be

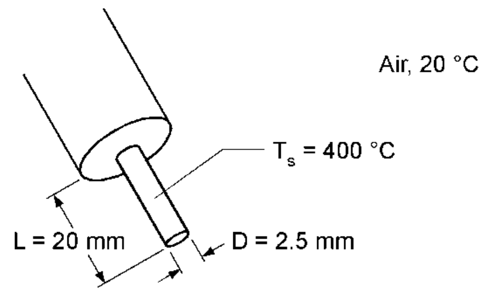
$$\begin{aligned}\dot{Q}_{rad,\,base} &= \varepsilon A\sigma(T_{base}^4 - T_{surr}^4) \\ &= (0.7)(3\times3\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)[(1200\,\text{K})^4 - (800\,\text{K})^4] \\ &= 594{,}400\ \text{W} = \mathbf{594\,kW}\end{aligned}$$

1-134 The power required to maintain the soldering iron tip at 400 °C is to be determined.

Assumptions **1** Steady operating conditions exist since the tip surface and the surrounding air temperatures remain constant. **2** The thermal properties of the tip and the convection heat transfer coefficient are constant and uniform. **3** The surrounding surfaces are at the same temperature as the air.

Properties The emissivity of the tip is given to be 0.80.

Analysis The total heat transfer area of the soldering iron tip is



$$\begin{aligned}A_s &= \pi D^2/4 + \pi DL \\ &= \pi(0.0025\text{ m})^2/4 + \pi(0.0025\text{ m})(0.02\text{ m}) \\ &= 1.62\times10^{-4}\text{ m}^2\end{aligned}$$

The rate of heat transfer by convection is

$$\begin{aligned}\dot{Q}_{\text{conv}} &= hA_s(T_{\text{tip}} - T_\infty) \\ &= (25\text{ W/m}^2\cdot{}^\circ\text{C})(1.62\times10^{-4}\text{ m}^2)(400-20)\,{}^\circ\text{C} \\ &= 1.54\text{ W}\end{aligned}$$

The rate of heat transfer by radiation is

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon\sigma A_s(T_{\text{tip}}^4 - T_{\text{surr}}^4) \\ &= (0.80)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(1.62\times10^{-4}\text{ m}^2)[(400+273)^4 - (20+273)^4]\text{ K}^4 \\ &= 1.45\text{ W}\end{aligned}$$

Thus, the power required is equal to the total rate of heat transfer from the tip by both convection and radiation:

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 1.54\text{ W} + 1.45\text{ W} = \mathbf{2.99\text{ W}}$$

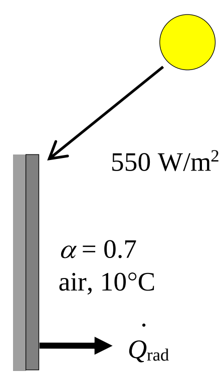
Discussion If the soldering iron tip is highly polished with an emissivity of 0.05, the power required to maintain the tip at 400 °C will reduce to 1.63 W, or by 45.5%.

1-135 The backside of the thin metal plate is insulated and the front side is exposed to solar radiation. The surface temperature of the plate is to be determined when it stabilizes.

Assumptions 1 Steady operating conditions exist. **2** Heat transfer through the insulated side of the plate is negligible. **3** The heat transfer coefficient is constant and uniform over the plate. **4** Radiation heat transfer is negligible.

Properties The solar absorptivity of the plate is given to be α = 0.7.

Analysis When the heat loss from the plate by convection equals the solar radiation absorbed, the surface temperature of the plate can be determined from



$$\begin{aligned}\dot{Q}_{\text{solarabsorbed}} &= \dot{Q}_{\text{conv}} \\ \alpha\dot{Q}_{\text{solar}} &= hA_s(T_s - T_o) \\ 0.7\times A\times550\text{ W/m}^2 &= (25\text{ W/m}^2\cdot{}^\circ\text{C})A_s(T_s - 10)\end{aligned}$$

Canceling the surface area A_s and solving for T_s gives

$$T_s = \mathbf{25.4^\circ C}$$

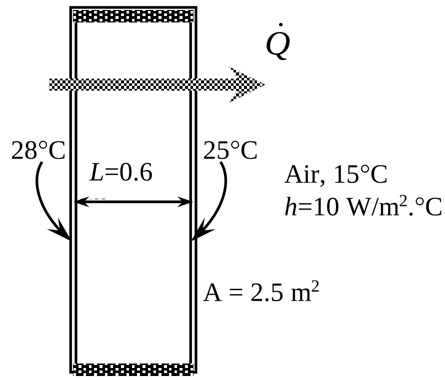
1-136 The glass cover of a flat plate solar collector with specified inner and outer surface temperatures is considered. The fraction of heat lost from the glass cover by radiation is to be determined.

Assumptions **1** Steady operating conditions exist since the surface temperatures of the glass remain constant at the specified values. **2** Thermal properties of the glass are constant.

Properties The thermal conductivity of the glass is given to be k = 0.7 W/m·°C.

Analysis Under steady conditions, the rate of heat transfer through the glass by conduction is

$$\dot{Q}_{\text{cond}} = kA\frac{\Delta T}{L} = (0.7\text{ W/m}\cdot°\text{C})(2.5\text{ m}^2)\frac{(28-25)°\text{C}}{0.006\text{ m}} = 875\text{ W}$$



The rate of heat transfer from the glass by convection is

$$\dot{Q}_{\text{conv}} = hA\Delta T = (10\text{ W/m}^2\cdot°\text{C})(2.5\text{ m}^2)(25-15)°\text{C} = 250\text{ W}$$

Under steady conditions, the heat transferred through the cover by conduction should be transferred from the outer surface by convection and radiation. That is,

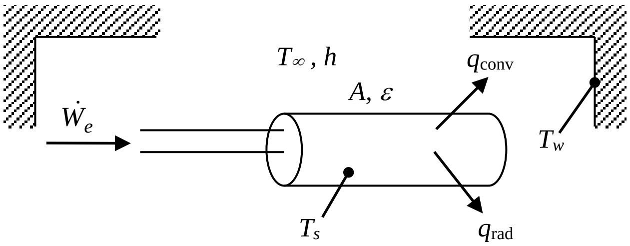
$$\dot{Q}_{\text{rad}} = \dot{Q}_{\text{cond}} - \dot{Q}_{\text{conv}} = 875 - 250 = 625\text{ W}$$

Then the fraction of heat transferred by radiation becomes

$$f = \frac{\dot{Q}_{\text{rad}}}{\dot{Q}_{\text{cond}}} = \frac{625}{875} = \mathbf{0.714} \quad \text{(or 71.4\%)}$$

1-137 An electric heater placed in a room consumes 500 W power when its surfaces are at 120°C. The surface temperature when the heater consumes 700 W is to be determined without and with the consideration of radiation.

Assumptions **1** Steady operating conditions exist. **2** The temperature is uniform over the surface.



Analysis (*a*) Neglecting radiation, the convection heat transfer coefficient is determined from

$$h = \frac{\dot{Q}}{A(T_s - T_\infty)} = \frac{500 \text{ W}}{(0.25 \text{ m}^2)(120-20)°\text{C}} = 20 \text{ W/m}^2\cdot°\text{C}$$

The surface temperature when the heater consumes 700 W is

$$T_s = T_\infty + \frac{\dot{Q}}{hA} = 20°\text{C} + \frac{700 \text{ W}}{(20 \text{ W/m}^2\cdot°\text{C})(0.25 \text{ m}^2)} = \mathbf{160°C}$$

(*b*) Considering radiation, the convection heat transfer coefficient is determined from

$$h = \frac{\dot{Q} - \varepsilon A\sigma(T_s^4 - T_{\text{surr}}^4)}{A(T_s - T_\infty)}$$

$$= \frac{500 \text{ W} - (0.75)(0.25 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)\left[(393 \text{ K})^4 - (283 \text{ K})^4\right]}{(0.25 \text{ m}^2)(120-20)°\text{C}}$$

$$= 12.58 \text{ W/m}^2\cdot°\text{C}$$

Then the surface temperature becomes

$$\dot{Q} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{\text{surr}}^4)$$

$$700 = (12.58)(0.25)(T_s - 293) + (0.75)(0.25)(5.67\times10^{-8})\left[T_s^4 - (283 \text{ K})^4\right]$$

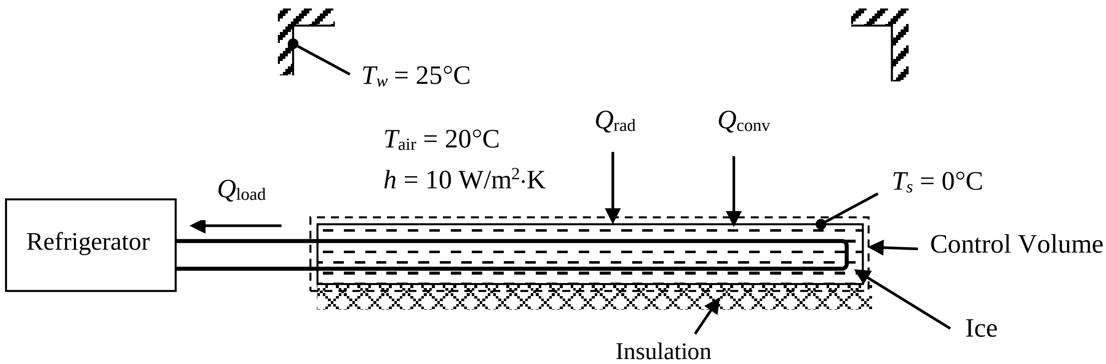
$$T_s = 425.9 \text{ K} = \mathbf{152.9°C}$$

Discussion Neglecting radiation changed T_s by more than 7°C, so assumption is not correct in this case.

1-138 An ice skating rink is located in a room is considered. The refrigeration load of the system and the time it takes to melt 3 mm of ice are to be determined.

Assumptions **1** Steady operating conditions exist in part (a). **2** The surface is insulated on the back side in part (b).

Properties The heat of fusion and the density of ice are given to be 333.7 kJ/kg and 920 kg/m^3, respectively.



Analysis (*a*) The refrigeration load is determined from

$$\dot{Q}_{\text{load}} = hA(T_{\text{air}} - T_s) + \varepsilon A\sigma(T_w^4 - T_s^4)$$
$$= (10)(40\times12)(20-0) + (0.95)(40\times12)(5.67\times10^{-8})\left[298^4 - 273^4\right] = \mathbf{156{,}300\ W}$$

(*b*) The time it takes to melt 3 mm of ice is determined from

$$t = \frac{LW\delta\rho h_{if}}{\dot{Q}_{\text{load}}} = \frac{(40\times12\ \text{m}^2)(0.003\text{m})(920\,\text{kg/m}^3)(333.7\times10^3\ \text{J/kg})}{156{,}300\ \text{J/s}} = 2831\,\text{s} = \mathbf{47.2\ min}$$

Fundamentals of Engineering (FE) Exam Problems

1-139 Which equation below is used to determine the heat flux for conduction?

(a) $-kA\dfrac{dT}{dx}$ (b) $-k\,\mathrm{grad}T$ (c) $h(T_2 - T_1)$ (d) $\varepsilon\sigma T^4$ (e) None of them

Answer (b) $-k\,\mathrm{grad}T$

1-140 Which equation below is used to determine the heat flux for convection?

(a) $-kA\dfrac{dT}{dx}$ (b) $-k\,\mathrm{grad}T$ (c) $h(T_2 - T_1)$ (d) $\varepsilon\sigma T^4$ (e) None of them

Answer (c) $h(T_2 - T_1)$

1-141 Which equation below is used to determine the heat flux emitted by thermal radiation from a surface?

(a) $-kA\dfrac{dT}{dx}$ (b) $-k\,\mathrm{grad}T$ (c) $h(T_2 - T_1)$ (d) $\varepsilon\sigma T^4$ (e) None of them

Answer (d) $\varepsilon\sigma T^4$

1-142 A 1-kW electric resistance heater in a room is turned on and kept on for 50 minutes. The amount of energy transferred to the room by the heater is

(a) 1 kJ (b) 50 kJ (c) 3000 kJ (d) 3600 kJ (e) 6000 kJ

Answer (c) 3000 kJ

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
We= 1 [kJ/s]
time=50*60 [s]
We_total=We*time [kJ]

"Wrong Solutions:"
W1_Etotal=We*time/60 "using minutes instead of s"
W2_Etotal=We "ignoring time"
```

1-143 A 2-kW electric resistance heater submerged in 30-kg water is turned on and kept on for 10 min. During the process, 500 kJ of heat is lost from the water. The temperature rise of water is

(a) 5.6°C (b) 9.6°C (c) 13.6°C (d) 23.3°C (e) 42.5°C

Answer (a) 5.6°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
C=4.18 [kJ/kg-K]
m=30 [kg]
Q_loss=500 [kJ]
time=10*60 [s]
W_e=2 [kJ/s]
"Applying energy balance E_in-E_out=dE_system gives"
time*W_e-Q_loss = dU_system
dU_system=m*C*DELTAT

“Some Wrong Solutions with Common Mistakes:”
time*W_e = m*C*W1_T "Ignoring heat loss"
time*W_e+Q_loss = m*C*W2_T  "Adding heat loss instead of subtracting"
time*W_e-Q_loss = m*1.0*W3_T "Using specific heat of air or not using specific heat"
```

1-144 Eggs with a mass of 0.15 kg per egg and a specific heat of 3.32 kJ/kg·°C are cooled from 32°C to 10°C at a rate of 300 eggs per minute. The rate of heat removal from the eggs is

(a) 11 kW (b) 80 kW (c) 25 kW (d) 657 kW (e) 55 kW

Answer (e) 55 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
C=3.32 [kJ/kg-K]
m_egg=0.15 [kg]
T1=32 [C]
T2=10 [C]
n=300 "eggs/min"
m=n*m_egg/60 "kg/s"
"Applying energy balance E_in-E_out=dE_system gives"
"-E_out = dU_system"
Qout=m*C*(T1-T2) "kJ/s"
“Some Wrong Solutions with Common Mistakes:”
W1_Qout = m*C*T1  "Using T1 only"
W2_Qout = m_egg*C*(T1-T2)  "Using one egg only"
W3_Qout = m*C*T2  "Using T2 only"
W4_Qout=m_egg*C*(T1-T2)*60   "Finding kJ/min"
```

1-145 A cold bottled drink (m = 2.5 kg, c_p = 4200 J/kg·°C) at 5°C is left on a table in a room. The average temperature of the drink is observed to rise to 15°C in 30 minutes. The average rate of heat transfer to the drink is

(a) 23 W (b) 29 W (c) 58 W (d) 88 W (e) 122 W

Answer: (c) 58 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
c=4200 [J/kg-K]
m=2.5 [kg]
T1=5 [C]
T2=15 [C]
time = 30*60 [s]
"Applying energy balance E_in-E_out=dE_system gives"
Q=m*c*(T2-T1)
Qave=Q/time

“Some Wrong Solutions with Common Mistakes:”
W1_Qave = m*c*T1/time  "Using T1 only"
W2_Qave = c*(T2-T1)/time  "Not using mass"
W3_Qave = m*c*T2/time  "Using T2 only"
```

1-146 Water enters a pipe at 20ºC at a rate of 0.25 kg/s and is heated to 60ºC. The rate of heat transfer to the water is

(a) 10 kW (b) 20.9 kW (c) 41.8 kW (d) 62.7 kW (e) 167.2 kW

Answer (c) 41.8 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_in=20 [C]
T_out=60 [C]
m_dot=0.25 [kg/s]
c_p=4.18 [kJ/kg-C]
Q_dot=m_dot*c_p*(T_out-T_in)

"Some Wrong Solutions with Common Mistakes"
W1_Q_dot=m_dot*(T_out-T_in) "Not using specific heat"
W2_Q_dot=c_p*(T_out-T_in) "Not using mass flow rate"
W3_Q_dot=m_dot*c_p*T_out "Using exit temperature instead of temperature change"
```

1-147 Air enters a 12-m-long, 7-cm-diameter pipe at 50ºC at a rate of 0.06 kg/s. The air is cooled at an average rate of 400 W per m^2 surface area of the pipe. The air temperature at the exit of the pipe is

(a) 4.3ºC (b) 17.5ºC (c) 32.5ºC (d) 43.4ºC (e) 45.8ºC

Answer (c) 32.5ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
L=12 [m]
D=0.07 [m]
T1=50 [C]
m_dot=0.06 [kg/s]
q=400 [W/m^2]
A=pi*D*L
Q_dot=q*A
c_p=1007 [J/kg-C] "Table A-15"
Q_dot=m_dot*c_p*(T1-T2)

"Some Wrong Solutions with Common Mistakes"
q=m_dot*c_p*(T1-W1_T2) "Using heat flux, q instead of rate of heat transfer, Q_dot"
Q_dot=m_dot*4180*(T1-W2_T2) "Using specific heat of water"
Q_dot=m_dot*c_p*W3_T2 "Using exit temperature instead of temperature change"
```

1-148 Heat is lost steadily through a 0.5-cm thick 2 m × 3 m window glass whose thermal conductivity is 0.7 W/m·°C. The inner and outer surface temperatures of the glass are measured to be 12°C to 9°C. The rate of heat loss by conduction through the glass is

(a) 420 W (b) 5040 W (c) 17,600 W (d) 1256 W (e) 2520 W

Answer (e) 2520 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
A=3*2 [m^2]
L=0.005 [m]
T1=12 [C]
T2=9 [C]
k=0.7 [W/m-C]
Q=k*A*(T1-T2)/L

“Some Wrong Solutions with Common Mistakes:”
W1_Q=k*(T1-T2)/L "Not using area"
W2_Q=k*2*A*(T1-T2)/L  "Using areas of both surfaces"
W3_Q=k*A*(T1+T2)/L  "Adding temperatures instead of subtracting"
W4_Q=k*A*L*(T1-T2) "Multiplying by thickness instead of dividing by it"
```

1-149 Steady heat conduction occurs through a 0.3-m thick 9 m by 3 m composite wall at a rate of 1.2 kW. If the inner and outer surface temperatures of the wall are 15°C and 7°C, the effective thermal conductivity of the wall is

(a) 0.61 W/m·°C (b) 0.83 W/m·°C (c) 1.7 W/m·°C (d) 2.2 W/m·°C (e) 5.1 W/m·°C

Answer (c) 1.7 W/m·°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
A=9*3 [m^2]
L=0.3 [m]
T1=15 [C]
T2=7 [C]
Q=1200 [W]
Q=k*A*(T1-T2)/L

"Wrong Solutions:"
Q=W1_k*(T1-T2)/L "Not using area"
Q=W2_k*2*A*(T1-T2)/L  "Using areas of both surfaces"
Q=W3_k*A*(T1+T2)/L  "Adding temperatures instead of subtracting"
Q=W4_k*A*L*(T1-T2) "Multiplying by thickness instead of dividing by it"
```

1-150 Heat is lost through a brick wall (k = 0.72 W/m·ºC), which is 4 m long, 3 m wide, and 25 cm thick at a rate of 500 W. If the inner surface of the wall is at 22ºC, the temperature at the midplane of the wall is

(a) 0ºC (b) 7.5ºC (c) 11.0ºC (d) 14.8ºC (e) 22ºC

Answer (d) 14.8ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k=0.72 [W/m-C]
Length=4 [m]
Width=3 [m]
L=0.25 [m]
Q_dot=500 [W]
T1=22 [C]
A=Length*Width
Q_dot=k*A*(T1-T_middle)/(0.5*L)

"Some Wrong Solutions with Common Mistakes"
Q_dot=k*A*(T1-W1_T_middle)/L "Using L instead of 0.5L"
W2_T_middle=T1/2 "Just taking the half of the given temperature"
```

1-151 A 10-cm high and 20-cm wide circuit board houses on its surface 100 closely spaced chips, each generating heat at a rate of 0.08 W and transferring it by convection and radiation to the surrounding medium at 40°C. Heat transfer from the back surface of the board is negligible. If the combined convection and radiation heat transfer coefficient on the surface of the board is 22 W/m^2·°C, the average surface temperature of the chips is

(a) 72.4°C (b) 66.5°C (c) 40.4°C (d) 58.2°C (e) 49.1°C

Answer (d) 58.2°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
A=0.1*0.2 [m^2]
Q= 100*0.08 [W]
Tair=40 [C]
h=22 [W/m^2-C]
Q= h*A*(Ts-Tair)

"Wrong Solutions:"
Q= h*(W1_Ts-Tair) "Not using area"
Q= h*2*A*(W2_Ts-Tair) "Using both sides of surfaces"
Q= h*A*(W3_Ts+Tair) "Adding temperatures instead of subtracting"
Q/100= h*A*(W4_Ts-Tair) "Considering 1 chip only"
```

1-152 A 40-cm-long, 0.4-cm-diameter electric resistance wire submerged in water is used to determine the convection heat transfer coefficient in water during boiling at 1 atm pressure. The surface temperature of the wire is measured to be 114°C when a wattmeter indicates the electric power consumption to be 7.6 kW. The heat transfer coefficient is

(a) 108 kW/m^2·°C (b) 13.3 kW/m^2·°C (c) 68.1 kW/m^2·°C (d) 0.76 kW/m^2·°C (e) 256 kW/m^2·°C

Answer (a) 108 kW/m^2·°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
L=0.4 [m]
D=0.004 [m]
A=pi*D*L [m^2]
We=7.6 [kW]
Ts=114 [C]
Tf=100 [C]  “Boiling temperature of water at 1 atm"
We= h*A*(Ts-Tf)

"Wrong Solutions:"
We= W1_h*(Ts-Tf) "Not using area"
We= W2_h*(L*pi*D^2/4)*(Ts-Tf) "Using volume instead of area"
We= W3_h*A*Ts "Using Ts instead of temp difference"
```

1-153 While driving down a highway early in the evening, the air flow over an automobile establishes an overall heat transfer coefficient of 25 W/m²·K. The passenger cabin of this automobile exposes 8 m² of surface to the moving ambient air. On a day when the ambient temperature is 33ºC, how much cooling must the air conditioning system supply to maintain a temperature of 20ºC in the passenger cabin?

(a) 0.65 MW (b) 1.4 MW (c) 2.6 MW (d) 3.5 MW (e) 0.94 MW

Answer (c) 2.6 MW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
h=25 [W/m^2-C]
A=8 [m^2]
T_1=33 [C]
T_2=20 [C]
Q=h*A*(T_2-T_1)
```

1-154 A room is heated by a 1.2 kW electric resistance heater whose wires have a diameter of 4 mm and a total length of 3.4 m. The air in the room is at 23ºC and the interior surfaces of the room are at 17ºC. The convection heat transfer coefficient on the surface of the wires is 8 W/m²·ºC. If the rates of heat transfer from the wires to the room by convection and by radiation are equal, the surface temperature of the wires is

(a) 3534ºC (b) 1778ºC (c) 1772ºC (d) 98ºC (e) 25ºC

Answer (b) 1778ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.004 [m]
L=3.4 [m]
W_dot_e=1200 [W]
T_infinity=23 [C]
T_surr=17 [C]
h=8 [W/m^2-C]
A=pi*D*L
Q_dot_conv=W_dot_e/2
Q_dot_conv=h*A*(T_s-T_infinity)

"Some Wrong Solutions with Common Mistakes"
Q_dot_conv=h*A*(W1_T_s-T_surr) "Using T_surr instead of T_infinity"
Q_dot_conv/1000=h*A*(W2_T_s-T_infinity) "Using kW unit for the rate of heat transfer"
Q_dot_conv=h*(W3_T_s-T_infinity) "Not using surface area of the wires"
W_dot_e=h*A*(W4_T_s-T_infinity) "Using total heat transfer"
```

1-155 Over 90 percent of the energy dissipated by an incandescent light bulb is in the form of heat, not light. What is the temperature of a vacuum-enclosed tungsten filament with an exposed surface area of 2.03 cm^2 in a 100 W incandescent light bulb? The emissivity of tungsten at the anticipated high temperatures is about 0.35. Note that the light bulb consumes 100 W of electrical energy, and dissipates all of it by radiation.

(a) 1870 K (b) 2230 K (c) 2640 K (d) 3120 K (e) 2980 K

Answer (b) 2230 K

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
e =0.35
Q=100 [W]
A=2.03E-4 [m^2]
Q=e*A*sigma#*T^4
```

1-156 On a still clear night, the sky appears to be a blackbody with an equivalent temperature of 250 K. What is the air temperature when a strawberry field cools to 0°C and freezes if the heat transfer coefficient between the plants and the air is 6 $W/m^2 \cdot {}^oC$ because of a light breeze and the plants have an emissivity of 0.9?

(a) 14ºC (b) 7ºC (c) 3ºC (d) 0ºC (e) –3°C

Answer (a) 14ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
e=0.9
h=6 [W/m^2-K]
T_1=273 [K]
T_2=250 [K]
h*(T-T_1)=e*sigma#*(T_1^4-T_2^4)
```

1-157 A 30-cm diameter black ball at 120°C is suspended in air, and is losing heat to the surrounding air at 25°C by convection with a heat transfer coefficient of 12 W/m^2·°C, and by radiation to the surrounding surfaces at 15°C. The total rate of heat transfer from the black ball is

(a) 322 W (b) 595 W (c) 234 W (d) 472 W (e) 2100 W

Answer: (b) 595 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
sigma=5.67E-8 [W/m^2-K^4]
eps=1
D=0.3 [m]
A=pi*D^2
h_conv=12 [W/m^2-C]
Ts=120 [C]
Tf=25 [C]
Tsurr=15 [C]
Q_conv=h_conv*A*(Ts-Tf)
Q_rad=eps*sigma*A*((Ts+273)^4-(Tsurr+273)^4)
Q_total=Q_conv+Q_rad

"Wrong Solutions:"
W1_Ql=Q_conv "Ignoring radiation"
W2_Q=Q_rad "ignoring convection"
W3_Q=Q_conv+eps*sigma*A*(Ts^4-Tsurr^4) "Using C in radiation calculations"
W4_Q=Q_total/A "not using area"
```

1-158 A 3-m^2 black surface at 140°C is losing heat to the surrounding air at 35°C by convection with a heat transfer coefficient of 16 W/m^2·°C, and by radiation to the surrounding surfaces at 15°C. The total rate of heat loss from the surface is

(a) 5105 W (b) 2940 W (c) 3779 W (d) 8819 W (e) 5040 W

Answer (d) 8819 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
sigma=5.67E-8 [W/m^2-K^4]
eps=1
A=3 [m^2]
h_conv=16 [W/m^2-C]
Ts=140 [C]
Tf=35 [C]
Tsurr=15 [C]
Q_conv=h_conv*A*(Ts-Tf)
Q_rad=eps*sigma*A*((Ts+273)^4-(Tsurr+273)^4)
Q_total=Q_conv+Q_rad

“Some Wrong Solutions with Common Mistakes:”
W1_Ql=Q_conv "Ignoring radiation"
W2_Q=Q_rad "ignoring convection"
W3_Q=Q_conv+eps*sigma*A*(Ts^4-Tsurr^4) "Using C in radiation calculations"
W4_Q=Q_total/A "not using area"
```

1-159 A person's head can be approximated as a 25-cm diameter sphere at 35°C with an emissivity of 0.95. Heat is lost from the head to the surrounding air at 25°C by convection with a heat transfer coefficient of 11 W/m^2·°C, and by radiation to the surrounding surfaces at 10°C. Disregarding the neck, determine the total rate of heat loss from the head.

(a) 22 W (b) 27 W (c) 49 W (d) 172 W (e) 249 W

Answer: (c) 49 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
sigma=5.67E-8 [W/m^2-K^4]
eps=0.95
D=0.25 [m]
A=pi*D^2
h_conv=11 [W/m^2-C]
Ts=35 [C]
Tf=25 [C]
Tsurr=10 [C]
Q_conv=h_conv*A*(Ts-Tf)
Q_rad=eps*sigma*A*((Ts+273)^4-(Tsurr+273)^4)
Q_total=Q_conv+Q_rad

"Wrong Solutions:"
W1_Ql=Q_conv "Ignoring radiation"
W2_Q=Q_rad "ignoring convection"
W3_Q=Q_conv+eps*sigma*A*(Ts^4-Tsurr^4) "Using C in radiation calculations"
W4_Q=Q_total/A "not using area"
```

1-160 A person standing in a room loses heat to the air in the room by convection and to the surrounding surfaces by radiation. Both the air in the room and the surrounding surfaces are at 20ºC. The exposed surfaces of the person is 1.5 m^2 and has an average temperature of 32ºC, and an emissivity of 0.90. If the rates of heat transfer from the person by convection and by radiation are equal, the combined heat transfer coefficient is

(a) 0.008 W/m^2·ºC (b) 3.0 W/m^2·ºC (c) 5.5 W/m^2·ºC (d) 8.3 W/m^2·ºC (e) 10.9 W/m^2·ºC

Answer (e) 10.9 W/m^2·ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_infinity=20 [C]
T_surr=20 [C]
T_s=32 [C]
A=1.5 [m^2]
epsilon=0.90
sigma=5.67E-8 [W/m^2-K^4]
Q_dot_rad=epsilon*A*sigma*((T_s+273)^4-(T_surr+273)^4)
Q_dot_total=2*Q_dot_rad
Q_dot_total=h_combined*A*(T_s-T_infinity)

"Some Wrong Solutions with Common Mistakes"
Q_dot_rad=W1_h_combined*A*(T_s-T_infinity) "Using radiation heat transfer instead of total heat transfer"
Q_dot_rad_1=epsilon*A*sigma*(T_s^4-T_surr^4) "Using C unit for temperature in radiation calculation"
2*Q_dot_rad_1=W2_h_combined*A*(T_s-T_infinity)
```

1-161 . . . 1-164 Design and Essay Problems

Solutions Manual for

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Chapter 2
HEAT CONDUCTION EQUATION

Introduction

2-1C The term *steady* implies *no change with time* at any point within the medium while *transient* implies *variation with time* or *time dependence*. Therefore, the temperature or heat flux remains unchanged with time during steady heat transfer through a medium at any location although both quantities may vary from one location to another. During transient heat transfer, the temperature and heat flux may vary with time as well as location. Heat transfer is one-dimensional if it occurs primarily in one direction. It is two-dimensional if heat tranfer in the third dimension is negligible.

2-2C Heat transfer is a *vector* quantity since it has direction as well as magnitude. Therefore, we must specify both direction and magnitude in order to describe heat transfer completely at a point. Temperature, on the other hand, is a scalar quantity.

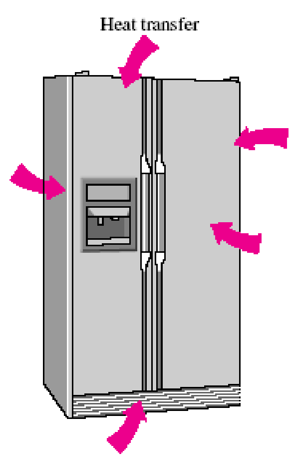
2-3C Yes, the heat flux vector at a point *P* on an isothermal surface of a medium has to be perpendicular to the surface at that point.

2-4C Isotropic materials have the same properties in all directions, and we do not need to be concerned about the variation of properties with direction for such materials. The properties of anisotropic materials such as the fibrous or composite materials, however, may change with direction.

2-5C In heat conduction analysis, the conversion of electrical, chemical, or nuclear energy into heat (or thermal) energy in solids is called heat generation.

2-6C The phrase "thermal energy generation" is equivalent to "heat generation," and they are used interchangeably. They imply the conversion of some other form of energy into thermal energy. The phrase "energy generation," however, is vague since the form of energy generated is not clear.

2-7C The heat transfer process from the kitchen air to the refrigerated space is transient in nature since the thermal conditions in the kitchen and the refrigerator, in general, change with time. However, we would analyze this problem as a steady heat transfer problem under the worst anticipated conditions such as the lowest thermostat setting for the refrigerated space, and the anticipated highest temperature in the kitchen (the so-called design conditions). If the compressor is large enough to keep the refrigerated space at the desired temperature setting under the presumed worst conditions, then it is large enough to do so under all conditions by cycling on and off. Heat transfer into the refrigerated space is three-dimensional in nature since heat will be entering through all six sides of the refrigerator. However, heat transfer through any wall or floor takes place in the direction normal to the surface, and thus it can be analyzed as being one-dimensional. Therefore, this problem can be simplified greatly by considering the heat transfer to be onedimensional at each of the four sides as well as the top and bottom sections, and then by adding the calculated values of heat transfer at each surface.



2-8C Heat transfer through the walls, door, and the top and bottom sections of an oven is transient in nature since the thermal conditions in the kitchen and the oven, in general, change with time. However, we would analyze this problem as a steady heat transfer problem under the worst anticipated conditions such as the highest temperature setting for the oven, and the anticipated lowest temperature in the kitchen (the so called "design" conditions). If the heating element of the oven is large enough to keep the oven at the desired temperature setting under the presumed worst conditions, then it is large enough to do so under all conditions by cycling on and off.

Heat transfer from the oven is three-dimensional in nature since heat will be entering through all six sides of the oven. However, heat transfer through any wall or floor takes place in the direction normal to the surface, and thus it can be analyzed as being one-dimensional. Therefore, this problem can be simplified greatly by considering the heat transfer as being one- dimensional at each of the four sides as well as the top and bottom sections, and then by adding the calculated values of heat transfers at each surface.

2-9C Heat transfer to a potato in an oven can be modeled as one-dimensional since temperature differences (and thus heat transfer) will exist in the radial direction only because of symmetry about the center point. This would be a transient heat transfer process since the temperature at any point within the potato will change with time during cooking. Also, we would use the spherical coordinate system to solve this problem since the entire outer surface of a spherical body can be described by a constant value of the radius in spherical coordinates. We would place the origin at the center of the potato.

2-10C Assuming the egg to be round, heat transfer to an egg in boiling water can be modeled as one-dimensional since temperature differences (and thus heat transfer) will primarily exist in the radial direction only because of symmetry about the center point. This would be a transient heat transfer process since the temperature at any point within the egg will change with time during cooking. Also, we would use the spherical coordinate system to solve this problem since the entire outer surface of a spherical body can be described by a constant value of the radius in spherical coordinates. We would place the origin at the center of the egg.

2-11C Heat transfer to a hot dog can be modeled as two-dimensional since temperature differences (and thus heat transfer) will exist in the radial and axial directions (but there will be symmetry about the center line and no heat transfer in the azimuthal direction. This would be a transient heat transfer process since the temperature at any point within the hot dog will change with time during cooking. Also, we would use the cylindrical coordinate system to solve this problem since a cylinder is best described in cylindrical coordinates. Also, we would place the origin somewhere on the center line, possibly at the center of the hot dog. Heat transfer in a very long hot dog could be considered to be one-dimensional in preliminary calculations.

2-12C Heat transfer to a roast beef in an oven would be transient since the temperature at any point within the roast will change with time during cooking. Also, by approximating the roast as a spherical object, this heat transfer process can be modeled as one-dimensional since temperature differences (and thus heat transfer) will primarily exist in the radial direction because of symmetry about the center point.

2-13C Heat loss from a hot water tank in a house to the surrounding medium can be considered to be a steady heat transfer problem. Also, it can be considered to be two-dimensional since temperature differences (and thus heat transfer) will exist in the radial and axial directions (but there will be symmetry about the center line and no heat transfer in the azimuthal direction.)

2-14C Heat transfer to a canned drink can be modeled as two-dimensional since temperature differences (and thus heat transfer) will exist in the radial and axial directions (but there will be symmetry about the center line and no heat transfer in the azimuthal direction. This would be a transient heat transfer process since the temperature at any point within the drink will change with time during heating. Also, we would use the cylindrical coordinate system to solve this problem since a cylinder is best described in cylindrical coordinates. Also, we would place the origin somewhere on the center line, possibly at the center of the bottom surface.

2-15 A certain thermopile used for heat flux meters is considered. The minimum heat flux this meter can detect is to be determined.

Assumptions **1** Steady operating conditions exist.

Properties The thermal conductivity of kapton is given to be 0.345 W/m·K.

Analysis The minimum heat flux can be determined from

$$\dot{q} = k\frac{\Delta T}{L} = (0.345 \text{ W/m}\cdot{}^\circ\text{C})\frac{0.1^\circ\text{C}}{0.002 \text{ m}} = \mathbf{17.3\ W/m^2}$$

2-16 Diameter, length, and mass of stainless steel rod is given. The rod is insulated on its exterior surface except for the ends. Temperature distribution in the rod is also given. The heat flux along the rod is to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional in the x-direction. **2** Thermal conductivity is constant.

Analysis The heat flux can be found from Fourier's law

$$\dot{q} = -k\,\frac{dT}{dx}$$

Table A-3 gives values for the thermal conductivity of stainless steels, however we are not told which type of stainless steel the rod is made of, and the thermal conductivity varies between them. We do know the mass of the rod, and can use this to calculate its density:

$$\rho = \frac{M}{Vol} = \frac{M}{\left(\frac{\pi D^2}{4}\right)L} = \frac{0.221 \text{ kg}}{[\pi(0.018 \text{ m})^2\ (0.11 \text{ m})]/4} = 7895 \text{ kg/m}^3$$

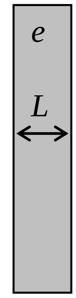
From Table A-3, with$\rho \approx$ 7900 kg/m^3,it appears that the material is AISI 304 stainless steel. The temperature of the rod from the given equation for temperature distribution varies from 310 K at x=0 to290 K at x = L = 110 mm. Evaluating the thermal conductivity at the average temperature of 300 K, from Table A-3, k = 14.9 W/m·K. Thus,

$$\dot{q} = -k\,\frac{dT}{dx} = -k\left(-\frac{20\text{K}}{L}\right) = -14.9 \text{ W/m·K}\left(-\frac{20\text{K}}{0.11 \text{ m}}\right) = \mathbf{2709\ W/m^2}$$

DiscussionIf the temperature of the rod varies significantly along its length, the thermal conductivity will vary along the rod as much or more than the variation in thermal conductivities between the different stainless steels.

2-17 The rate of heat generation per unit volume in a stainless steel plate is given. The heat flux on the surface of the plate is to be determined.

Assumptions Heat is generated uniformly in steel plate.



Analysis We consider a unit surface area of 1 m^2. The total rate of heat generation in this section of the plate is

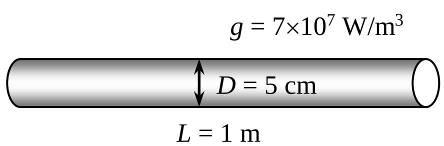
$$\dot{E}_{\text{gen}} = \dot{e}_{\text{gen}} V_{\text{plate}} = \dot{e}_{\text{gen}} (A \times L) = (5 \times 10^6 \text{ W/m}^3)(1\,\text{m}^2)(0.03\text{m}) = 1.5 \times 10^5 \text{ W}$$

Noting that this heat will be dissipated from both sides of the plate, the heat flux on either surface of the plate becomes

$$\dot{q} = \frac{\dot{E}_{\text{gen}}}{A_{\text{plate}}} = \frac{1.5 \times 10^5 \text{ W}}{2 \times 1\,\text{m}^2} = 75{,}000 \text{ W/m}^2 = \mathbf{75\,kW/m^2}$$

2-18 The rate of heat generation per unit volume in the uranium rods is given. The total rate of heat generation in each rod is to be determined.

Assumptions Heat is generated uniformly in the uranium rods.



Analysis The total rate of heat generation in the rod is determined by multiplying the rate of heat generation per unit volume by the volume of the rod

$$\dot{E}_{\text{gen}} = \dot{e}_{\text{gen}} V_{\text{rod}} = \dot{e}_{\text{gen}} (\pi D^2 / 4) L = (7 \times 10^7 \text{ W/m}^3)[\pi (0.05\,\text{m})^2 / 4](1\,\text{m}) = 1.374 \times 10^5 \text{ W} = \mathbf{137\,kW}$$

2-19 The variation of the absorption of solar energy in a solar pond with depth is given. A relation for the total rate of heat generation in a water layer at the top of the pond is to be determined.

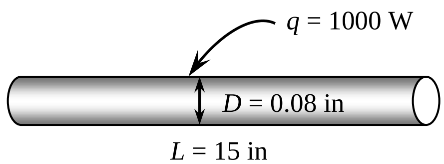
Assumptions Absorption of solar radiation by water is modeled as heat generation.

Analysis The total rate of heat generation in a water layer of surface area A and thickness L at the top of the pond is determined by integration to be

$$\dot{E}_{\text{gen}} = \int_V \dot{e}_{\text{gen}}\, dV = \int_{x=0}^{L} \dot{e}_0 e^{-bx} (A dx) = A \dot{e}_0 \left. \frac{e^{-bx}}{-b} \right|_0^L = \frac{\mathbf{A \dot{e}_0 (1 - e^{-bL})}}{\mathbf{b}}$$

2-20E The power consumed by the resistance wire of an iron is given. The heat generation and the heat flux are to be determined.

Assumptions Heat is generated uniformly in the resistance wire.



Analysis A 1000 W iron will convert electrical energy into heat in the wire at a rate of 1000 W. Therefore, the rate of heat generation in a resistance wire is simply equal to the power rating of a resistance heater. Then the rate of heat generation in the wire per unit volume is determined by dividing the total rate of heat generation by the volume of the wire to be

$$\dot{e}_{\text{gen}} = \frac{\dot{E}_{\text{gen}}}{V_{\text{wire}}} = \frac{\dot{E}_{\text{gen}}}{(\pi D^2/4)L} = \frac{1000\text{ W}}{[\pi(0.08/12\text{ ft})^2/4](15/12\text{ ft})}\left(\frac{3.412\text{ Btu/h}}{1\text{ W}}\right) = \mathbf{7.820\times10^7\ Btu/h\cdot ft^3}$$

Similarly, heat flux on the outer surface of the wire as a result of this heat generation is determined by dividing the total rate of heat generation by the surface area of the wire to be

$$\dot{q} = \frac{\dot{E}_{\text{gen}}}{A_{\text{wire}}} = \frac{\dot{E}_{\text{gen}}}{\pi DL} = \frac{1000\text{ W}}{\pi(0.08/12\text{ ft})(15/12\text{ ft})}\left(\frac{3.412\text{ Btu/h}}{1\text{ W}}\right) = \mathbf{1.303\times10^5\ Btu/h\cdot ft^2}$$

Discussion Note that heat generation is expressed per unit volume in Btu/h·ft^3 whereas heat flux is expressed per unit surface area in Btu/h·ft^2.

Heat Conduction Equation

2-21C The one-dimensional transient heat conduction equation for a plane wall with constant thermal conductivity and heat generation is $\frac{\partial^2 T}{\partial x^2}+\frac{\dot{e}_{\text{gen}}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$. Here T is the temperature, x is the space variable, $\dot{e}_{\text{gen}}$ is the heat generation per unit volume, k is the thermal conductivity, α is the thermal diffusivity, and t is the time.

2-22C The one-dimensional transient heat conduction equation for a long cylinder with constant thermal conductivity and heat generation is $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\dot{e}_{\text{gen}}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$. Here T is the temperature, r is the space variable, g is the heat generation per unit volume, k is the thermal conductivity, α is the thermal diffusivity, and t is the time.

2-23 We consider a thin element of thickness Δx in a large plane wall (see Fig. 2-12 in the text). The density of the wall is ρ, the specific heat is c, and the area of the wall normal to the direction of heat transfer is A. In the absence of any heat generation, an *energy balance* on this thin element of thickness Δx during a small time interval Δt can be expressed as

$$\dot{Q}_x-\dot{Q}_{x+\Delta x}=\frac{\Delta E_{\text{element}}}{\Delta t}$$

where

$$\Delta E_{\text{element}}=E_{t+\Delta t}-E_t=mc(T_{t+\Delta t}-T_t)=\rho cA\Delta x(T_{t+\Delta t}-T_t)$$

Substituting,

$$\dot{Q}_x-\dot{Q}_{x+\Delta x}=\rho cA\Delta x\frac{T_{t+\Delta t}-T_t}{\Delta t}$$

Dividing by $A\Delta x$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{x+\Delta x}-\dot{Q}_x}{\Delta x}=\rho c\frac{T_{t+\Delta t}-T_t}{\Delta t}$$

Taking the limit as $\Delta x\to 0$ and $\Delta t\to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial x}\left(kA\frac{\partial T}{\partial x}\right)=\rho c\frac{\partial T}{\partial t}$$

since from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta x\to 0}\frac{\dot{Q}_{x+\Delta x}-\dot{Q}_x}{\Delta x}=\frac{\partial Q}{\partial x}=\frac{\partial}{\partial x}\left(-kA\frac{\partial T}{\partial x}\right)$$

Noting that the area A of a plane wall is constant, the one-dimensional transient heat conduction equation in a plane wall with constant thermal conductivity k becomes

$$\frac{\partial^2 T}{\partial x^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where the property $\alpha=k/\rho c$ is the thermal diffusivity of the material.

2-24 We consider a thin cylindrical shell element of thickness Δr in a long cylinder (see Fig. 2-14 in the text). The density of the cylinder is ρ, the specific heat is c, and the length is L. The area of the cylinder normal to the direction of heat transfer at any location is $A = 2\pi rL$ where r is the value of the radius at that location. Note that the heat transfer area A depends on r in this case, and thus it varies with location. An *energy balance* on this thin cylindrical shell element of thickness Δr during a small time interval Δt can be expressed as

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{E}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

where

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho cA\Delta r(T_{t+\Delta t} - T_t)$$

$$\dot{E}_{\text{element}} = \dot{e}_{\text{gen}} V_{\text{element}} = \dot{e}_{\text{gen}} A\Delta r$$

Substituting,

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{e}_{\text{gen}} A\Delta r = \rho cA\Delta r \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

where $A = 2\pi rL$. Dividing the equation above by $A\Delta r$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} + \dot{e}_{\text{gen}} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the limit as $\Delta r \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial r}\left(kA\frac{\partial T}{\partial r}\right) + \dot{e}_{\text{gen}} = \rho c\frac{\partial T}{\partial t}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta r \to 0}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \frac{\partial Q}{\partial r} = \frac{\partial}{\partial r}\left(-kA\frac{\partial T}{\partial r}\right)$$

Noting that the heat transfer area in this case is $A = 2\pi rL$ and the thermal conductivity is constant, the one-dimensional transient heat conduction equation in a cylinder becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \dot{e}_{\text{gen}} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where $\alpha = k/\rho c$ is the thermal diffusivity of the material.

2-25 We consider a thin spherical shell element of thickness Δr in a sphere (see Fig. 2-16 in the text).. The density of the sphere is ρ, the specific heat is c, and the length is L. The area of the sphere normal to the direction of heat transfer at any location is $A = 4\pi r^2$ where r is the value of the radius at that location. Note that the heat transfer area A depends on r in this case, and thus it varies with location. When there is no heat generation, an *energy balance* on this thin spherical shell element of thickness Δr during a small time interval Δt can be expressed as

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

where

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c A \Delta r (T_{t+\Delta t} - T_t)$$

Substituting,

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} = \rho c A \Delta r \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

where $A = 4\pi r^2$. Dividing the equation above by $A\Delta r$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the limit as $\Delta r \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial r}\left(kA\frac{\partial T}{\partial r}\right) = \rho c \frac{\partial T}{\partial t}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta r \to 0} \frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \frac{\partial Q}{\partial r} = \frac{\partial}{\partial r}\left(-kA\frac{\partial T}{\partial r}\right)$$

Noting that the heat transfer area in this case is $A = 4\pi r^2$ and the thermal conductivity k is constant, the one-dimensional transient heat conduction equation in a sphere becomes

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where $\alpha = k/\rho c$ is the thermal diffusivity of the material.

2-26 For a medium in which the heat conduction equation is given in its simplest by $\dfrac{\partial^2 T}{\partial x^2} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}$:

(*a*) Heat transfer is transient, (*b*) it is one-dimensional, (*c*) there is no heat generation, and (*d*) the thermal conductivity is constant.

2-27 For a medium in which the heat conduction equation is given by $\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}$:

(*a*) Heat transfer is transient, (*b*) it is two-dimensional, (*c*) there is no heat generation, and (*d*) the thermal conductivity is constant.

2-28 For a medium in which the heat conduction equation is given in its simplest by $\frac{1}{r}\frac{d}{dr}\left(rk\frac{dT}{dr}\right)+\dot{e}_{\text{gen}}=0$:

(*a*) Heat transfer is steady, (*b*) it is one-dimensional, (*c*) there is heat generation, and (*d*) the thermal conductivity is variable.

2-29 For a medium in which the heat conduction equation is given by $\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{e}_{\text{gen}}=0$:

(*a*) Heat transfer is steady, (*b*) it is two-dimensional, (*c*) there is heat generation, and (*d*) the thermal conductivity is variable.

2-30 For a medium in which the heat conduction equation is given in its simplest by $r\frac{d^2T}{dr^2}+2\frac{dT}{dr}=0$:

(*a*) Heat transfer is steady, (*b*) it is one-dimensional, (*c*) there is no heat generation, and (*d*) the thermal conductivity is constant.

2-31 For a medium in which the heat conduction equation is given by $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

(*a*) Heat transfer is transient, (*b*) it is one-dimensional, (*c*) there is no heat generation, and (*d*) the thermal conductivity is constant.

2-32 For a medium in which the heat conduction equation is given by $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$

(*a*) Heat transfer is transient, (*b*) it is two-dimensional, (*c*) there is no heat generation, and (*d*) the thermal conductivity is constant.

2-33 We consider a small rectangular element of length Δx, width Δy, and height $\Delta z = 1$ (similar to the one in Fig. 2-20). The density of the body is ρ and the specific heat is *c*. Noting that heat conduction is two-dimensional and assuming no heat generation, an *energy balance* on this element during a small time interval Δt can be expressed as

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction at the}\\ \text{surfaces at } x \text{ and } y\end{pmatrix} - \begin{pmatrix}\text{Rate of heat conduction}\\ \text{at the surfaces at}\\ x+\Delta x \text{ and } y+\Delta y\end{pmatrix} = \begin{pmatrix}\text{Rate of change of}\\ \text{the energy content}\\ \text{of the element}\end{pmatrix}$$

or $$\dot{Q}_x + \dot{Q}_y - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

Noting that the volume of the element is $V_{\text{element}} = \Delta x \Delta y \Delta z = \Delta x \Delta y \times 1$, the change in the energy content of the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c \Delta x \Delta y (T_{t+\Delta t} - T_t)$$

Substituting, $$\dot{Q}_x + \dot{Q}_y - \dot{Q}_{x+\Delta x} - \dot{Q}_{y+\Delta y} = \rho c \Delta x \Delta y \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing by $\Delta x \Delta y$ gives

$$-\frac{1}{\Delta y}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} - \frac{1}{\Delta x}\frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the thermal conductivity *k* to be constant and noting that the heat transfer surface areas of the element for heat conduction in the *x* and *y* directions are $A_x = \Delta y \times 1$ and $A_y = \Delta x \times 1$, respectively, and taking the limit as $\Delta x, \Delta y$, and $\Delta t \to 0$ yields

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

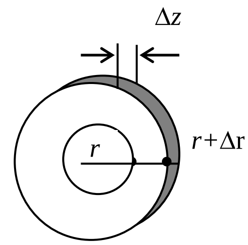
$$\lim_{\Delta x \to 0} \frac{1}{\Delta y \Delta z}\frac{\dot{Q}_{x+\Delta x} - \dot{Q}_x}{\Delta x} = \frac{1}{\Delta y \Delta z}\frac{\partial Q_x}{\partial x} = \frac{1}{\Delta y \Delta z}\frac{\partial}{\partial x}\left(-k \Delta y \Delta z \frac{\partial T}{\partial x}\right) = -\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = -k\frac{\partial^2 T}{\partial x^2}$$

$$\lim_{\Delta y \to 0} \frac{1}{\Delta x \Delta z}\frac{\dot{Q}_{y+\Delta y} - \dot{Q}_y}{\Delta y} = \frac{1}{\Delta x \Delta z}\frac{\partial Q_y}{\partial y} = \frac{1}{\Delta x \Delta z}\frac{\partial}{\partial y}\left(-k \Delta x \Delta z \frac{\partial T}{\partial y}\right) = -\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) = -k\frac{\partial^2 T}{\partial y^2}$$

Here the property $\alpha = k / \rho c$ is the thermal diffusivity of the material.

2-34 We consider a thin ring shaped volume element of width Δz and thickness Δr in a cylinder. The density of the cylinder is ρ and the specific heat is c. In general, an *energy balance* on this ring element during a small time interval Δt can be expressed as

$$(\dot{Q}_r - \dot{Q}_{r+\Delta r}) + (\dot{Q}_z - \dot{Q}_{z+\Delta z}) = \frac{\Delta E_{\text{element}}}{\Delta t}$$



But the change in the energy content of the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c(2\pi r\Delta r)\Delta z(T_{t+\Delta t} - T_t)$$

Substituting,

$$(\dot{Q}_r - \dot{Q}_{r+\Delta r}) + (\dot{Q}_z - \dot{Q}_{z+\Delta z}) = \rho c(2\pi r\Delta r)\Delta z \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing the equation above by $(2\pi r\Delta r)\Delta z$ gives

$$-\frac{1}{2\pi r\Delta z}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} - \frac{1}{2\pi r\Delta r}\frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Noting that the heat transfer surface areas of the element for heat conduction in the r and z directions are $A_r = 2\pi r\Delta z$ and $A_z = 2\pi r\Delta r$, respectively, and taking the limit as $\Delta r, \Delta z$ and $\Delta t \to 0$ yields

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) = \rho c\frac{\partial T}{\partial t}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta r\to 0}\frac{1}{2\pi r\Delta z}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \frac{1}{2\pi r\Delta z}\frac{\partial Q}{\partial r} = \frac{1}{2\pi r\Delta z}\frac{\partial}{\partial r}\left(-k(2\pi r\Delta z)\frac{\partial T}{\partial r}\right) = -\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)$$

$$\lim_{\Delta z\to 0}\frac{1}{2\pi r\Delta r}\frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} = \frac{1}{2\pi r\Delta r}\frac{\partial Q_z}{\partial z} = \frac{1}{2\pi r\Delta r}\frac{\partial}{\partial z}\left(-k(2\pi r\Delta r)\frac{\partial T}{\partial z}\right) = -\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)$$

For the case of constant thermal conductivity the equation above reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where $\alpha = k/\rho c$ is the thermal diffusivity of the material. For the case of steady heat conduction with no heat generation it reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} = 0$$

2-35 Consider a thin disk element of thickness Δz and diameter D in a long cylinder. The density of the cylinder is ρ, the specific heat is c, and the area of the cylinder normal to the direction of heat transfer is $A = \pi D^2/4$, which is constant. An *energy balance* on this thin element of thickness Δz during a small time interval Δt can be expressed as

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction at}\\ \text{the surface at } z\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction at the}\\ \text{surface at } z+\Delta z\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation inside}\\ \text{the element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change of}\\ \text{the energy content}\\ \text{of the element}\end{pmatrix}$$

or,

$$\dot{Q}_z - \dot{Q}_{z+\Delta z} + \dot{E}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t}$$

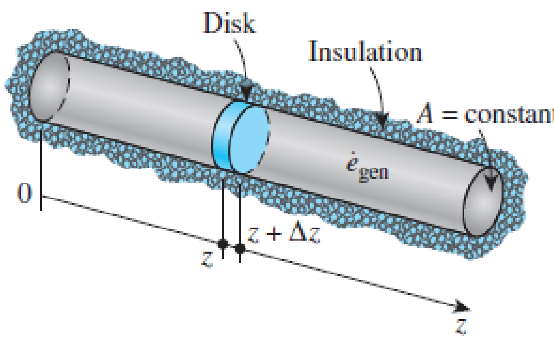
But the change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c A \Delta z (T_{t+\Delta t} - T_t)$$

and

$$\dot{E}_{\text{element}} = \dot{e}_{\text{gen}} V_{\text{element}} = \dot{e}_{\text{gen}} A \Delta z$$

Substituting,



$$\dot{Q}_z - \dot{Q}_{z+\Delta z} + \dot{e}_{\text{gen}} A \Delta z = \rho c A \Delta z \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Dividing by $A\Delta z$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} + \dot{e}_{\text{gen}} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t}$$

Taking the limit as $\Delta z \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial z}\left(kA\frac{\partial T}{\partial z}\right) + \dot{e}_{\text{gen}} = \rho c \frac{\partial T}{\partial t}$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta z \to 0} \frac{\dot{Q}_{z+\Delta z} - \dot{Q}_z}{\Delta z} = \frac{\partial Q}{\partial z} = \frac{\partial}{\partial z}\left(-kA\frac{\partial T}{\partial z}\right)$$

Noting that the area A and the thermal conductivity k are constant, the one-dimensional transient heat conduction equation in the axial direction in a long cylinder becomes

$$\frac{\partial^2 T}{\partial z^2} + \frac{\dot{e}_{\text{gen}}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where the property $\alpha = k/\rho c$ is the thermal diffusivity of the material.

Boundary and Initial Conditions; Formulation of Heat Conduction Problems

2-36C The mathematical expressions of the thermal conditions at the boundaries are called the **boundary conditions.** To describe a heat transfer problem completely, *two boundary conditions* must be given for *each direction* of the coordinate system along which heat transfer is significant. Therefore, we need to specify four boundary conditions for two-dimensional problems.

2-37C The mathematical expression for the temperature distribution of the medium initially is called the **initial condition**. We need only one initial condition for a heat conduction problem regardless of the dimension since the conduction equation is first order in time (it involves the first derivative of temperature with respect to time). Therefore, we need only 1 initial condition for a two-dimensional problem.

2-38C A heat transfer problem that is symmetric about a plane, line, or point is said to have thermal symmetry about that plane, line, or point. The thermal symmetry boundary condition is a mathematical expression of this thermal symmetry. It is equivalent to *insulation* or *zero heat flux* boundary condition, and is expressed at a point x_0 as $\partial T(x_0, t)/\partial x = 0$.

2-39C The boundary condition at a perfectly insulated surface (at $x = 0$, for example) can be expressed as

$-k\dfrac{\partial T(0,t)}{\partial x} = 0$ or $\dfrac{\partial T(0,t)}{\partial x} = 0$ which indicates zero heat flux.

2-40C Yes, the temperature profile in a medium must be perpendicular to an insulated surface since the slope $\partial T / \partial x = 0$ at that surface.

2-41C We try to avoid the radiation boundary condition in heat transfer analysis because it is a non-linear expression that causes mathematical difficulties while solving the problem; often making it impossible to obtain analytical solutions.

2-42 Heat conduction through the bottom section of an aluminum pan that is used to cook stew on top of an electric range is considered. Assuming variable thermal conductivity and one-dimensional heat transfer, the mathematical formulation (the differential equation and the boundary conditions) of this heat conduction problem is to be obtained for steady operation.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity is given to be variable. **3** There is no heat generation in the medium. **4** The top surface at $x = L$ is subjected to specified temperature and the bottom surface at $x = 0$ is subjected to uniform heat flux.

Analysis The heat flux at the bottom of the pan is

$$\dot{q}_s = \frac{\dot{Q}_s}{A_s} = \frac{\dot{E}_{\text{gen}}}{\pi D^2/4} = \frac{0.90 \times (900\text{ W})}{\pi (0.18\text{ m})^2/4} = 31{,}831\text{ W/m}^2$$

Then the differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

$$-k\frac{dT(0)}{dx} = \dot{q}_s = 31{,}831\text{ W/m}^2$$

$$T(L) = T_L = 108°\text{C}$$

2-43 Heat conduction through the bottom section of a steel pan that is used to boil water on top of an electric range is considered. Assuming constant thermal conductivity and one-dimensional heat transfer, the mathematical formulation (the differential equation and the boundary conditions) of this heat conduction problem is to be obtained for steady operation.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity is given to be constant. **3** There is no heat generation in the medium. **4** The top surface at $x = L$ is subjected to convection and the bottom surface at $x = 0$ is subjected to uniform heat flux.

Analysis The heat flux at the bottom of the pan is

$$\dot{q}_s = \frac{\dot{Q}_s}{A_s} = \frac{\dot{E}_{\text{gen}}}{\pi D^2/4} = \frac{0.85\times(1250\text{ W})}{\pi(0.20\text{ m})^2/4} = 33{,}820\text{ W/m}^2$$

Then the differential equation and the boundary conditions for this heat conduction problem can be expressed as

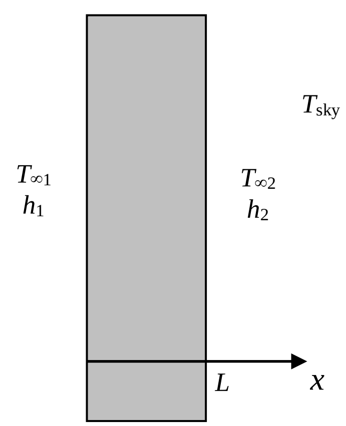
$$\frac{d^2T}{dx^2} = 0$$

$$-k\frac{dT(0)}{dx} = \dot{q}_s = 33{,}280\text{ W/m}^2$$

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty]$$

2-44 The outer surface of the East wall of a house exchanges heat with both convection and radiation., while the interior surface is subjected to convection only. Assuming the heat transfer through the wall to be steady and one-dimensional, the mathematical formulation (the differential equation and the boundary and initial conditions) of this heat conduction problem is to be obtained.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity is given to be constant. **3** There is no heat generation in the medium. **4** The outer surface at $x = L$ is subjected to convection and radiation while the inner surface at $x = 0$ is subjected to convection only.



Analysis Expressing all the temperatures in Kelvin, the differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

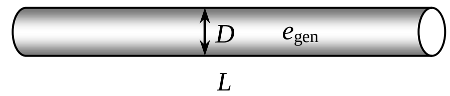
$$-k\frac{dT(0)}{dx} = h_1[T_{\infty 1} - T(0)]$$

$$-k\frac{dT(L)}{dx} = h_1[T(L) - T_{\infty 2}] + \varepsilon_2\sigma\left[T(L)^4 - T_{\text{sky}}^4\right]$$

2-45 Heat is generated in a long wire of radius r_o covered with a plastic insulation layer at a constant rate of $\dot{e}_{\text{gen}}$. The heat flux boundary condition at the interface (radius r_o) in terms of the heat generated is to be expressed. The total heat generated in the wire and the heat flux at the interface are

$$\dot{E}_{\text{gen}} = \dot{e}_{\text{gen}} V_{\text{wire}} = \dot{e}_{\text{gen}} (\pi r_o^2 L)$$

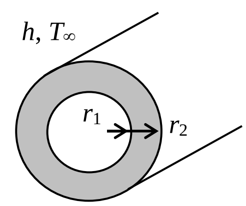
$$\dot{q}_s = \frac{\dot{Q}_s}{A} = \frac{\dot{E}_{\text{gen}}}{A} = \frac{\dot{e}_{\text{gen}} (\pi r_o^2 L)}{(2\pi r_o)L} = \frac{\dot{e}_{\text{gen}} r_o}{2}$$



Assuming steady one-dimensional conduction in the radial direction, the heat flux boundary condition can be expressed as

$$-k\frac{dT(r_o)}{dr} = \frac{\dot{e}_{\text{gen}} r_o}{2}$$

2-46 A long pipe of inner radius r_1, outer radius r_2, and thermal conductivity k is considered. The outer surface of the pipe is subjected to convection to a medium at T_∞ with a heat transfer coefficient of h. Assuming steady one-dimensional conduction in the radial direction, the convection boundary condition on the outer surface of the pipe can be expressed as

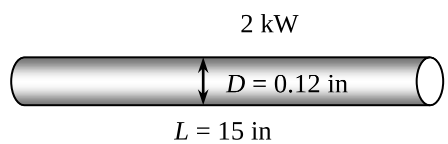


$$-k\frac{dT(r_2)}{dr} = h[T(r_2) - T_\infty]$$

2-47E A 2-kW resistance heater wire is used for space heating. Assuming constant thermal conductivity and one-dimensional heat transfer, the mathematical formulation (the differential equation and the boundary conditions) of this heat conduction problem is to be obtained for steady operation.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity is given to be constant. **3** Heat is generated uniformly in the wire.

Analysis The heat flux at the surface of the wire is



$$\dot{q}_s = \frac{\dot{Q}_s}{A_s} = \frac{\dot{E}_{\text{gen}}}{2\pi r_o L} = \frac{2000\text{ W}}{2\pi(0.06\text{ in})(15\text{ in})} = 353.7\text{ W/in}^2$$

Noting that there is thermal symmetry about the center line and there is uniform heat flux at the outer surface, the differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{e}_{\text{gen}}}{k} = 0$$

$$\frac{dT(0)}{dr} = 0$$

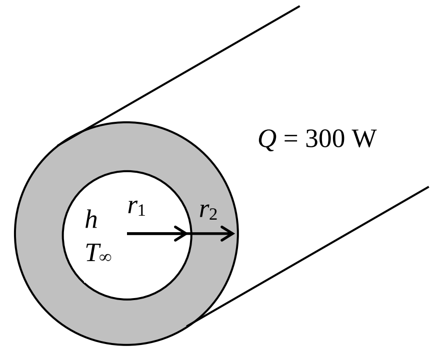
$$-k\frac{dT(r_o)}{dr} = \dot{q}_s = 353.7\text{ W/in}^2$$

2-48 Water flows through a pipe whose outer surface is wrapped with a thin electric heater that consumes 300 W per m length of the pipe. The exposed surface of the heater is heavily insulated so that the entire heat generated in the heater is transferred to the pipe. Heat is transferred from the inner surface of the pipe to the water by convection. Assuming constant thermal conductivity and one-dimensional heat transfer, the mathematical formulation (the differential equation and the boundary conditions) of the heat conduction in the pipe is to be obtained for steady operation.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity is given to be constant. **3** There is no heat generation in the medium. **4** The outer surface at $r = r_2$ is subjected to uniform heat flux and the inner surface at $r = r_1$ is subjected to convection.

Analysis The heat flux at the outer surface of the pipe is

$$\dot{q}_s = \frac{\dot{Q}_s}{A_s} = \frac{\dot{Q}_s}{2\pi r_2 L} = \frac{300\text{ W}}{2\pi(0.065\text{ cm})(1\text{m})} = 734.6\text{ W/m}^2$$



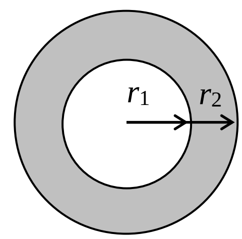
Noting that there is thermal symmetry about the center line and there is uniform heat flux at the outer surface, the differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$k\frac{dT(r_1)}{dr} = h[T(r_i) - T_\infty] = 85[T(r_i) - 70]$$

$$k\frac{dT(r_2)}{dr} = \dot{q}_s = 734.6\text{ W/m}^2$$

2-49 A spherical container of inner radius r_1, outer radius r_2, and thermal conductivity k is given. The boundary condition on the inner surface of the container for steady one-dimensional conduction is to be expressed for the following cases:



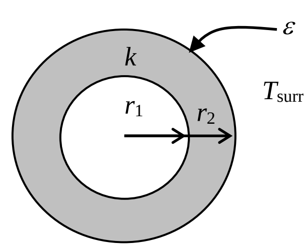
(*a*) Specified temperature of 50°C: $T(r_1) = 50°\text{C}$

(*b*) Specified heat flux of 30 W/m^2 towards the center: $k\dfrac{dT(r_1)}{dr} = 30\text{ W/m}^2$

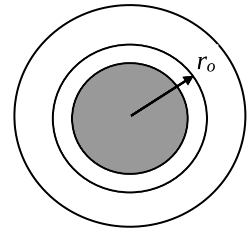
(*c*) Convection to a medium at T_∞ with a heat transfer coefficient of h: $k\dfrac{dT(r_1)}{dr} = h[T(r_1) - T_\infty]$

2-50 A spherical shell of inner radius r_1, outer radius r_2, and thermal conductivity k is considered. The outer surface of the shell is subjected to radiation to surrounding surfaces at T_{surr}. Assuming no convection and steady one-dimensional conduction in the radial direction, the radiation boundary condition on the outer surface of the shell can be expressed as

$$-k\frac{dT(r_2)}{dr} = \varepsilon\sigma\left[T(r_2)^4 - T_{\text{surr}}^4\right]$$



2-51 A spherical container consists of two spherical layers *A* and *B* that are at perfect contact. The radius of the interface is r_o. Assuming transient one-dimensional conduction in the radial direction, the boundary conditions at the interface can be expressed as



$$T_A(r_o,t) = T_B(r_o,t)$$

and $$-k_A \frac{\partial T_A(r_o,t)}{\partial r} = -k_B \frac{\partial T_B(r_o,t)}{\partial r}$$

2-52 A spherical metal ball that is heated in an oven to a temperature of T_i throughout is dropped into a large body of water at T_∞ where it is cooled by convection. Assuming constant thermal conductivity and transient one-dimensional heat transfer, the mathematical formulation (the differential equation and the boundary and initial conditions) of this heat conduction problem is to be obtained.

Assumptions **1** Heat transfer is given to be transient and one-dimensional. **2** Thermal conductivity is given to be constant. **3** There is no heat generation in the medium. **4** The outer surface at $r = r_0$ is subjected to convection.

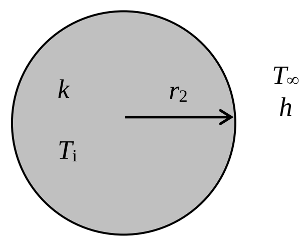
Analysis Noting that there is thermal symmetry about the midpoint and convection at the outer surface, the differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial T(0,t)}{\partial r} = 0$$

$$-k\frac{\partial T(r_o,t)}{\partial r} = h[T(r_o) - T_\infty]$$

$$T(r,0) = T_i$$



2-53 A spherical metal ball that is heated in an oven to a temperature of T_i throughout is allowed to cool in ambient air at T_∞ by convection and radiation. Assuming constant thermal conductivity and transient one-dimensional heat transfer, the mathematical formulation (the differential equation and the boundary and initial conditions) of this heat conduction problem is to be obtained.

Assumptions **1** Heat transfer is given to be transient and one-dimensional. **2** Thermal conductivity is given to be variable. **3** There is no heat generation in the medium. **4** The outer surface at $r = r_o$ is subjected to convection and radiation.

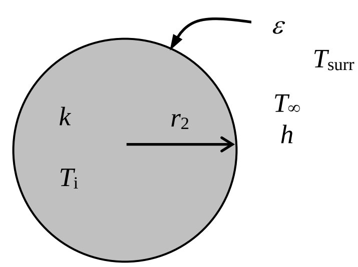
Analysis Noting that there is thermal symmetry about the midpoint and convection and radiation at the outer surface and expressing all temperatures in Rankine, the differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2 \frac{\partial T}{\partial r}\right) = \rho c\frac{\partial T}{\partial t}$$

$$\frac{\partial T(0,t)}{\partial r} = 0$$

$$-k\frac{\partial T(r_o,t)}{\partial r} = h[T(r_o) - T_\infty] + \varepsilon\sigma[T(r_o)^4 - T_{\text{surr}}^4]$$

$$T(r,0) = T_i$$



Solution of Steady One-Dimensional Heat Conduction Problems

2-54C Yes, the temperature in a plane wall with constant thermal conductivity and no heat generation will vary linearly during steady one-dimensional heat conduction even when the wall loses heat by radiation from its surfaces. This is because the steady heat conduction equation in a plane wall is $d^2T/dx^2 = 0$ whose solution is $T(x) = C_1x + C_2$ regardless of the boundary conditions. The solution function represents a straight line whose slope is C_1.

2-55C Yes, this claim is reasonable since in the absence of any heat generation the rate of heat transfer through a plain wall in steady operation must be constant. But the value of this constant must be zero since one side of the wall is perfectly insulated. Therefore, there can be no temperature difference between different parts of the wall; that is, the temperature in a plane wall must be uniform in steady operation.

2-56C Yes, this claim is reasonable since no heat is entering the cylinder and thus there can be no heat transfer from the cylinder in steady operation. This condition will be satisfied only when there are no temperature differences within the cylinder and the outer surface temperature of the cylinder is the equal to the temperature of the surrounding medium.

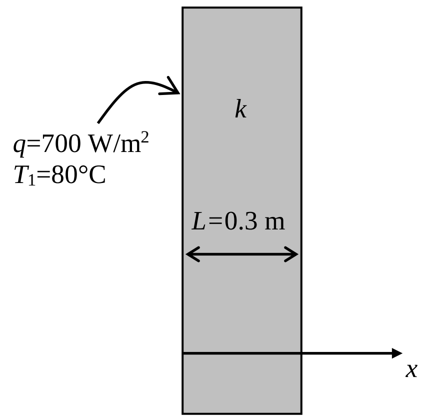
2-57C Yes, in the case of constant thermal conductivity and no heat generation, the temperature in a solid cylindrical rod whose ends are maintained at constant but different temperatures while the side surface is perfectly insulated will vary linearly during steady one-dimensional heat conduction. This is because the steady heat conduction equation in this case is $d^2T/dx^2 = 0$ whose solution is $T(x) = C_1x + C_2$ which represents a straight line whose slope is C_1.

2-58 A large plane wall is subjected to specified heat flux and temperature on the left surface and no conditions on the right surface. The mathematical formulation, the variation of temperature in the plate, and the right surface temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the wall is large relative to its thickness, and the thermal conditions on both sides of the wall are uniform. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall.

Properties The thermal conductivity is given to be k =2.5 W/m·°C.

Analysis (*a*) Taking the direction normal to the surface of the wall to be the *x* direction with x = 0 at the left surface, the mathematical formulation of this problem can be expressed as



$$\frac{d^2T}{dx^2} = 0$$

and $$-k\frac{dT(0)}{dx} = \dot{q}_0 = 700 \text{ W/m}^2$$

$$T(0) = T_1 = 80°\text{C}$$

(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

Heat flux at x = 0: $\quad -kC_1 = \dot{q}_0 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$

Temperature at x = 0: $\quad T(0) = C_1 \times 0 + C_2 = T_1 \rightarrow C_2 = T_1$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x) = -\frac{\dot{q}_0}{k}x + T_1 = -\frac{700 \text{ W/m}^2}{2.5 \text{ W/m}\cdot°\text{C}}x + 80°\text{C} = -280x + 80$$

(*c*) The temperature at $x = L$ (the right surface of the wall) is

$$T(L) = -280 \times (0.3 \text{ m}) + 80 = \mathbf{-4°C}$$

Note that the right surface temperature is lower as expected.

2-59 The base plate of a household iron is subjected to specified heat flux on the left surface and to specified temperature on the right surface. The mathematical formulation, the variation of temperature in the plate, and the inner surface temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the surface area of the base plate is large relative to its thickness, and the thermal conditions on both sides of the plate are uniform. **2** Thermal conductivity is constant. **3** There is no heat generation in the plate. **4** Heat loss through the upper part of the iron is negligible.

Properties The thermal conductivity is given to be k = 20 W/m·°C.

Analysis (*a*) Noting that the upper part of the iron is well insulated and thus the entire heat generated in the resistance wires is transferred to the base plate, the heat flux through the inner surface is determined to be

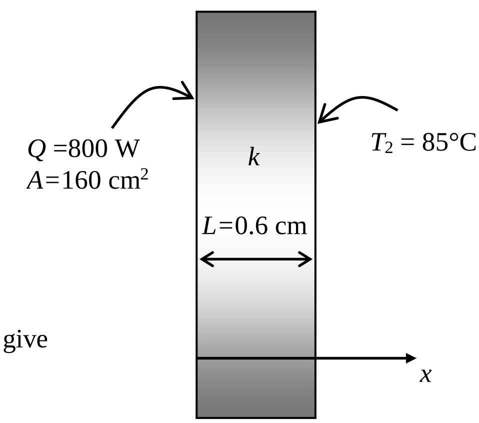
$$\dot{q}_0 = \frac{\dot{Q}_0}{A_{\text{base}}} = \frac{800\text{ W}}{160\times 10^{-4}\text{ m}^2} = 50{,}000\text{ W/m}^2$$

Taking the direction normal to the surface of the wall to be the x direction with x = 0 at the left surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and $$-k\frac{dT(0)}{dx} = \dot{q}_0 = 50{,}000\text{ W/m}^2$$

$$T(L) = T_2 = 85°\text{C}$$



(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

x = 0: $$-kC_1 = \dot{q}_0 \;\rightarrow\; C_1 = -\frac{\dot{q}_0}{k}$$

$x = L$: $$T(L) = C_1 L + C_2 = T_2 \;\rightarrow\; C_2 = T_2 - C_1 L \;\rightarrow\; C_2 = T_2 + \frac{\dot{q}_0 L}{k}$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x) = -\frac{\dot{q}_0}{k}x + T_2 + \frac{\dot{q}_0 L}{k} = \frac{\dot{q}_0(L-x)}{k} + T_2$$

$$= \frac{(50{,}000\text{ W/m}^2)(0.006 - x)\text{m}}{20\text{ W/m}\cdot°\text{C}} + 85°\text{C}$$

$$= 2500(0.006 - x) + 85$$

(*c*) The temperature at x = 0 (the inner surface of the plate) is

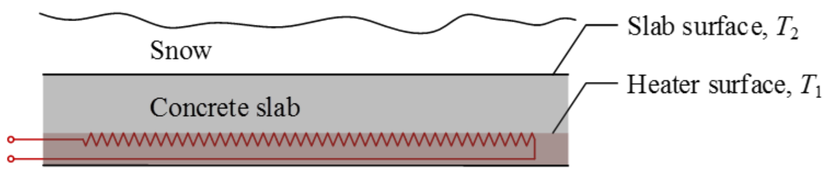
$$T(0) = 2500(0.006 - 0) + 85 = \mathbf{100°C}$$

Note that the inner surface temperature is higher than the exposed surface temperature, as expected.

2-60 A concrete slab with embedded heating cable to provide 1200 W/m^2 of heat to melt snow. Formulate the temperature profile in the concrete slab. Determine the slab thickness so that the temperature difference between the heater surface and the slab surface does not exceed 21°C, as recommended in ASHRAE Handbook to minimize thermal stress.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through the concrete slab. **3** The bottom surface at x = 0 is subjected to uniform heat flux from the heating cable. **4** The upper surface at x = L is at a constant temperature of 0°C from the snow melt. **5** There is no heat generation in the concrete slab. **6** Thermal properties are constant.

Properties The thermal conductivity of concrete is given as 1.4 W/m·K.



Analysis Taking the direction normal to the surface of the concrete slab to be the x direction with x = 0 at the bottom surface (the surface that is in contact with the heater surface), the differential equation for heat conduction can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions yields

$$x = 0: \qquad -k\frac{dT(0)}{dx} = \dot{q}_0 = -kC_1 \qquad \rightarrow \qquad C_1 = -\frac{\dot{q}_0}{k}$$

$$x = L: \qquad T(L) = T_2 = C_1 L + C_2 \qquad \rightarrow \qquad C_2 = T_L - C_1 L = T_2 + \frac{\dot{q}_0}{k}L$$

Substituting C_1 and C_2 into the general solution, the temperature profile in the concrete slab is determined to be

$$\boldsymbol{T(x) = \frac{\dot{q}_0}{k}(L - x) + T_2}$$

Note that at $x = L$, $T(L) = T_2 = 0$°C, where the snow melt occurs.

The concrete slab thickness such that the temperature difference between the heater surface (T_1) and the slab surface (T_2) does not exceed 21°C is

$$L = \frac{k}{\dot{q}_0}[T_1 - T_2] = \frac{1.4\ \text{W/m·K}}{1200\ \text{W/m}^2}[21\ \text{K}] = 0.0245\ \text{m} = \mathbf{24.5\ mm}$$

Discussion As the concrete slab thickness increases, the temperature difference between the heater surface and the slab surface will increase. So, 24.5 mm is the maximum thickness for the concrete slab to comply with the recommendation by the 2015 ASHRAE Handbook—HVAC Applications, Chapter 51, for $T_1 - T_2 \leq 21$°C.

2-61 A large plane wall is subjected to specified temperature on the left surface and convection on the right surface. The mathematical formulation, the variation of temperature, and the rate of heat transfer are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation.

Properties The thermal conductivity is given to be $k = 2.3$ W/m·°C.

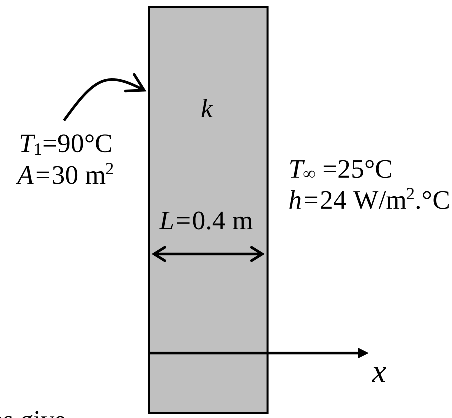
Analysis (*a*) Taking the direction normal to the surface of the wall to be the *x* direction with $x = 0$ at the left surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and

$$T(0) = T_1 = 90°\text{C}$$

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty]$$



(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$x = 0$: $\quad T(0) = C_1 \times 0 + C_2 \;\rightarrow\; C_2 = T_1$

$x = L$: $\quad -kC_1 = h[(C_1 L + C_2) - T_\infty] \;\rightarrow\; C_1 = -\dfrac{h(C_2 - T_\infty)}{k + hL} \;\rightarrow\; C_1 = -\dfrac{h(T_1 - T_\infty)}{k + hL}$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$\begin{aligned} T(x) &= -\frac{h(T_1 - T_\infty)}{k + hL}x + T_1 \\ &= -\frac{(24\ \text{W/m}^2\cdot°\text{C})(90-25)°\text{C}}{(2.3\ \text{W/m}\cdot°\text{C}) + (24\ \text{W/m}^2\cdot°\text{C})(0.4\ \text{m})}x + 90°\text{C} \\ &= 90 - 131.1x \end{aligned}$$

(*c*) The rate of heat conduction through the wall is

$$\begin{aligned} \dot{Q}_{\text{wall}} &= -kA\frac{dT}{dx} = -kAC_1 = kA\frac{h(T_1 - T_\infty)}{k + hL} \\ &= (2.3\ \text{W/m}\cdot°\text{C})(30\ \text{m}^2)\frac{(24\ \text{W/m}^2\cdot°\text{C})(90-25)°\text{C}}{(2.3\ \text{W/m}\cdot°\text{C}) + (24\ \text{W/m}^2\cdot°\text{C})(0.4\ \text{m})} \\ &= \mathbf{9045\ W} \end{aligned}$$

Note that under steady conditions the rate of heat conduction through a plain wall is constant.

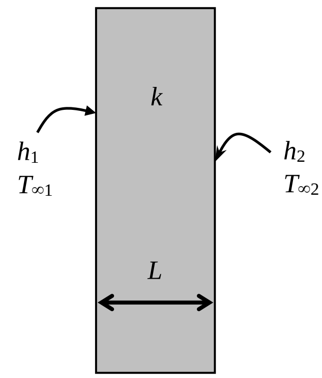
2-62 A large plane wall is subjected to convection on the inner and outer surfaces. The mathematical formulation, the variation of temperature, and the temperatures at the inner and outer surfaces to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation.

Properties The thermal conductivity is given to be $k = 0.77$ W/m·K.

Analysis (*a*) Taking the direction normal to the surface of the wall to be the x direction with $x = 0$ at the inner surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$



The boundary conditions for this problem are:

$$h_1[T_{\infty 1} - T(0)] = -k\frac{dT(0)}{dx}$$

$$-k\frac{dT(L)}{dx} = h_2[T(L) - T_{\infty 2}]$$

(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$x = 0$: $\quad h_1[T_{\infty 1} - (C_1 \times 0 + C_2)] = -kC_1$

$x = L$: $\quad -kC_1 = h_2[(C_1 L + C_2) - T_{\infty 2}]$

Substituting the given values, the above boundary condition equations can be written as

$$5(27 - C_2) = -0.77C_1$$

$$-0.77C_1 = (12)(0.2C_1 + C_2 - 8)$$

Solving these equations simultaneously give

$$C_1 = -45.45 \qquad C_2 = 20$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x) = 20 - 45.45x$$

(*c*) The temperatures at the inner and outer surfaces are

$$T(0) = 20 - 45.45 \times 0 = \mathbf{20^\circ C}$$

$$T(L) = 20 - 45.45 \times 0.2 = \mathbf{10.9^\circ C}$$

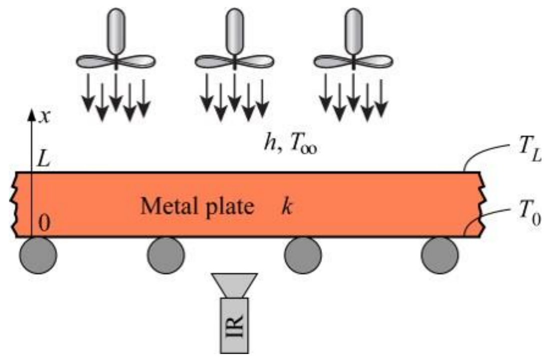
2-63 PtD In this example, the concepts of Prevention through Design (PtD) are applied in conjunction with the solution of steady one-dimensional heat conduction problem. The top surface of the plate is cooled by convection, and temperature at the bottom surface is measured by an IR thermometer. The variation of temperature in the metal plate and the convection heat transfer coefficient necessary to keep the top surface below 47°C are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the plate. **4** The bottom surface at $x = 0$ is at constant temperature while the top surface at $x = L$ is subjected to convection.

Properties The thermal conductivity of the metal plate is given to be $k = 13.5$ W/m·K.

Analysis Taking the direction normal to the surface of the wall to be the x direction with $x = 0$ at the lower surface, the mathematical formulation can be expressed as

$$\frac{d^2T}{dx^2} = 0$$



with boundary conditions

$$T(0) = T_0$$

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty]$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the first boundary condition yields

$$T(0) = C_1 \times 0 + C_2 \rightarrow C_2 = T_0$$

The application of the second boundary condition gives

$$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty] \rightarrow -kC_1 = h(C_1L + C_2 - T_\infty)$$

Solving for C_2 yields

$$C_1 = \frac{h(T_\infty - C_2)}{k + hL} = \frac{T_\infty - T_0}{(k/h) + L}$$

Now substituting C_1 and C_2 into the general solution and the variation of temperature is

$$\boldsymbol{T(x) = \frac{T_\infty - T_0}{(k/h) + L}x + T_0}$$

The minimum convection heat transfer coefficient necessary to maintain the top surface below 47°C can be determined from the variation of temperature:

$$T(L) = T_L = \frac{T_\infty - T_0}{(k/h) + L}L + T_0$$

Solving for h gives

$$h = \frac{k}{L}\frac{T_L - T_0}{T_\infty - T_L} = \left(\frac{13.5\ \text{W/m}\cdot\text{K}}{0.025\ \text{m}}\right)\frac{(47-60)°\text{C}}{(30-47)°\text{C}} = \mathbf{413\ W/m^2\cdot K}$$

Discussion To keep the top surface of the metal plate below 47°C, the convection heat transfer coefficient should be greater than 413 W/m^2·K. A convection heat transfer coefficient value of 413 W/m^2·K is very high for forced convection of gases. The typical values for forced convection of gases are 25–250 W/m^2·K (see Table 1-5 in Chapter 1). To protect workers from thermal burn, appropriate apparel should be worn when operating in an area where hot surfaces are present.

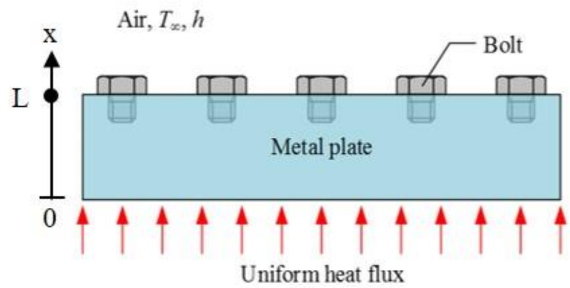
2-64 A series of ASME SA-193 carbon steel bolts of 1 cm thread length are bolted on the upper surface of a metal plate. The upper surface is exposed to convection with the ambient air. The bottom surface is subjected to a uniform heat flux. Formulate the temperature profile in the metal plate, and determine the location in the plate where the temperature begins to exceed 260°C. The compliance of the SA-193 bolts with the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HF-300) is to be determined.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through the metal plate. **3** The bottom surface at $x = 0$ is subjected to uniform heat flux while the upper surface at $x = L$ is at uniform temperature. **4** There is no heat generation in the plate. **5** Thermal properties are constant.

Properties The thermal conductivity of the metal plate is given as 15 W/m·K.

Analysis The uniform heat flux on the bottom plate surface ($x = 0$) is equal to the heat flux transferred by convection on the upper surface ($x = L$):

$$\dot{q}_0 = h[T(L) - T_\infty] \qquad \rightarrow \qquad T(L) = \frac{\dot{q}_0}{h} + T_\infty$$



Taking the direction normal to the surface of the plate to be the x direction with $x = 0$ at the bottom surface, the differential equation for heat conduction can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions yields

$$x = 0: \qquad -k\frac{dT(0)}{dx} = \dot{q}_0 = -kC_1 \qquad \rightarrow \qquad C_1 = -\frac{\dot{q}_0}{k}$$

$$x = L: \qquad T(L) = T_L = C_1 L + C_2 \qquad \rightarrow \qquad C_2 = -C_1 L + T_L = \frac{\dot{q}_0}{k}L + \frac{\dot{q}_0}{h} + T_\infty$$

Substituting C_1 and C_2 into the general solution, the temperature profile in the metal plate is determined to be

$$\boldsymbol{T(x) = -\frac{\dot{q}_0}{k}x + \frac{\dot{q}_0}{k}L + \frac{\dot{q}_0}{h} + T_\infty = \frac{\dot{q}_0}{k}(L - x) + \frac{\dot{q}_0}{h} + T_\infty}$$

Using the temperature profile of the metal plate, the location where the temperature begins to exceed 260°C is

$$T(x) = 260°\text{C}: \qquad x = L + \frac{k}{h} + \frac{k}{\dot{q}_0}[T_\infty - T(x)] = 0.05\text{ m} + \frac{15\text{ W/m·K}}{10\text{ W/m}^2\text{·K}} + \frac{15\text{ W/m·K}}{2250\text{ W/m}^2}(30 - 260)\text{K}$$
$$= 0.0167\text{ m} = \mathbf{1.67\text{ cm}}$$

Discussion Since the thread length of the SA-193 bolts is 1 cm; the bolt tips would only reach the location $x = 0.04$ cm in the metal plate. This is where the bolts are subjected to the highest temperature in the metal plate. At this location, the temperature in the plate can be determined from the temperature profile:

$$T(0.04) = \frac{2250\text{ W/m}^2}{15\text{ W/m·K}}(0.05 - 0.04)\text{m} + \frac{2250\text{ W/m}^2}{10\text{ W/m}^2\text{·K}} + 30°\text{C} = 256.5°\text{C}$$

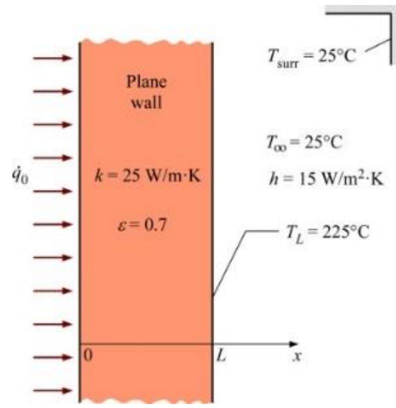
Therefore, the SA-193 bolts would comply with the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HF-300).

2-65 A plane wall is subjected to uniform heat flux on the left surface, while the right surface is subjected to convection and radiation heat transfer. The variation of temperature in the wall and the left surface temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Temperatures on both sides of the wall are uniform. **3** Thermal conductivity is constant. **4** There is no heat generation in the wall. **5** The surrounding temperature $T_\infty = T_{surr} = 25°C$.

Properties Emissivity and thermal conductivity are given to be 0.70 and 25 W/m·K, respectively.

Analysis(a) Taking the direction normal to the surface of the wall to be the x direction with $x = 0$ at the left surface, the mathematical formulation can be expressed as



$$\frac{d^2T}{dx^2} = 0$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

whereC_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$x = 0: \quad -k\frac{dT(0)}{dx} = \dot{q}_0 = -kC_1 \rightarrow C_1 = -\frac{\dot{q}_0}{k}$$

$$x = L: \quad T(L) = T_L = C_1 L + C_2 \rightarrow C_2 = -C_1 L + T_L = \frac{\dot{q}_0}{k}L + T_L$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x) = -\frac{\dot{q}_0}{k}x + \frac{\dot{q}_0}{k}L + T_L \rightarrow T(x) = \frac{\dot{q}_0}{k}(L - x) + T_L$$

(b) The uniform heat flux subjected on the left surface is equal to the sum of heat fluxes transferred by convection and radiation on the right surface:

$$\dot{q}_0 = h(T_L - T_\infty) + \varepsilon\sigma(T_L^4 - T_{surr}^4)$$

$$\dot{q}_0 = (15\ \text{W/m}^2\cdot\text{K})(225-25)\ \text{K} + (0.70)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(225+273)^4 - (25+273)^4]\ \text{K}^4$$

$$\dot{q}_0 = \mathbf{5130\ \ W/m^2}$$

(c) The temperature at $x = 0$ (the left surface of the wall) is

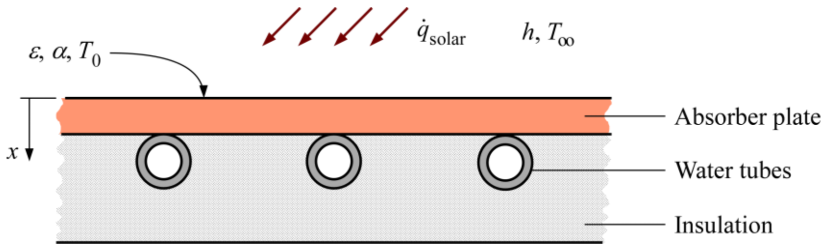
$$T(0) = \frac{\dot{q}_0}{k}(L-0) + T_L = \frac{5130\ \text{W/m}^2}{25\ \text{W/m}\cdot\text{K}}(0.50\ \text{m}) + 225°\text{C} = \mathbf{327.6\ °C}$$

Discussion As expected, the left surface temperature is higher than the right surface temperature. The absence of radiative boundary condition may lower the resistance to heat transfer at the right surface of the wall resulting in a temperature drop on the left wall surface by about 40°C.

2-66 A flat-plate solar collector is used to heat water. The top surface (x = 0) is subjected to convection, radiation, and incident solar radiation. The variation of temperature in the solar absorber and the net heat flux absorbed by the solar collector are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the plate. **4** The top surface at x = 0 is subjected to convection, radiation, and incident solar radiation.

Properties The absorber surface has an absorptivity of 0.9 and an emissivity of 0.9.



Analysis Taking the direction normal to the surface of the plate to be the x direction with $x = 0$ at the top surface, the mathematical formulation can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$x = 0: \qquad -k\frac{dT(0)}{dx} = \dot{q}_0 = -kC_1 \qquad \rightarrow \qquad C_1 = -\frac{\dot{q}_0}{k}$$

$$x = 0: \qquad T(0) = T_0 = C_2$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x) = -\frac{\dot{q}_0}{k}x + T_0$$

At the top surface (x = 0), the net heat flux absorbed by the solar collector is

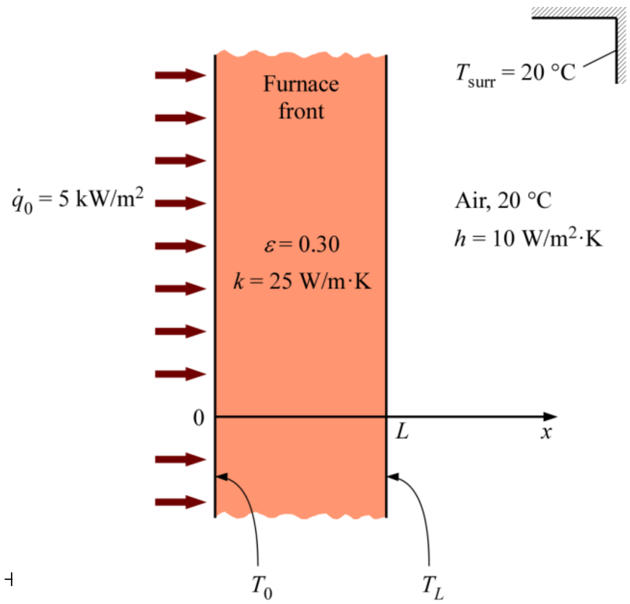
$$\dot{q}_0 = \alpha\dot{q}_{\text{solar}} - \varepsilon\sigma(T_0^4 - T_{\text{surr}}^4) - h(T_0 - T_\infty)$$

$$\dot{q}_0 = (0.9)(500\ \text{W/m}^2) - (0.9)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(35+273)^4 - (0+273)^4)]\,\text{K}^4 - (5\ \text{W/m}^2\cdot\text{K})(35-25)\,\text{K}$$

$$\dot{q}_0 = \mathbf{224\ W/m^2}$$

Discussion The absorber plate is generally very thin. Thus, the temperature difference between the top and bottom surface temperatures of the plate is miniscule. The net heat flux absorbed by the solar collector increases with the increase in the ambient and surrounding temperatures and thus the use of solar collectors is justified in hot climatic conditions.

2-67 A 20-mm thick draw batch furnace front is subjected to uniform heat flux on the inside surface, while the outside surface is subjected to convection and radiation heat transfer. The inside surface temperature of the furnace front is to be determined.



Assumptions **1** Heat conduction is steady. **2** One dimensional heat conduction across the furnace front thickness. **3** Thermal properties are constant. **4** Inside and outside surface temperatures are constant.

Properties Emissivity and thermal conductivity are given to be 0.30 and 25 W/m · K, respectively

Analysis The uniform heat flux subjected on the inside surface is equal to the sum of heat fluxes transferred by convection and radiation on the outside surface:

$$\dot{q}_0 = h(T_L - T_\infty) + \varepsilon\sigma(T_L^4 - T_{surr}^4)$$

$$5000 \text{ W/m}^2 = (10 \text{ W/m}^2\cdot\text{K})[T_L - (20+273)]\,\text{K}$$
$$+ (0.30)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[T_L^4 - (20+$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
5000=10*(T_L-(20+273))+0.30*5.67e-8*(T_L^4-(20+273)^4)
```

Solving by EES software, the outside surface temperature of the furnace front is

$$T_L = 594\,\text{K}$$

For steady heat conduction, the Fourier's law of heat conduction can be expressed as

$$\dot{q}_0 = -k\frac{dT}{dx}$$

Knowing that the heat flux and thermal conductivity are constant, integrating the differential equation once with respect to x yields

$$T(x) = -\frac{\dot{q}_0}{k}x + C_1$$

Applying the boundary condition gives

$$x = L: \quad T(L) = T_L = -\frac{\dot{q}_0}{k}L + C_1 \quad \rightarrow \quad C_1 = \frac{\dot{q}_0}{k}L + T_L$$

Substituting C_1 into the general solution, the variation of temperature in the furnace front is determined to be

$$T(x) = \frac{\dot{q}_0}{k}(L - x) + T_L$$

The inside surface temperature of the furnace front is

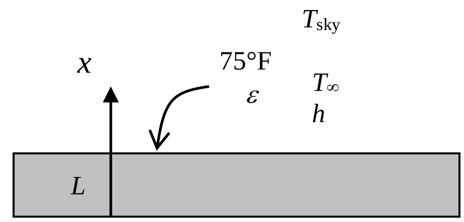
$$T(0) = T_0 = \frac{\dot{q}_0}{k}L + T_L = \frac{5000 \text{ W/m}^2}{25 \text{ W/m}\cdot\text{K}}(0.020\text{ m}) + 594\text{ K} = \mathbf{598\text{ K}}$$

Discussion By insulating the furnace front, heat loss from the outer surface can be reduced.

2-68E A large plate is subjected to convection, radiation, and specified temperature on the top surface and no conditions on the bottom surface. The mathematical formulation, the variation of temperature in the plate, and the bottom surface temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the plate is large relative to its thickness, and the thermal conditions on both sides of the plate are uniform. **2** Thermal conductivity is constant. **3** There is no heat generation in the plate.

Properties The thermal conductivity and emissivity are given to be k =7.2 Btu/h·ft·°F and ε = 0.7.



Analysis (*a*) Taking the direction normal to the surface of the plate to be the x direction with x = 0 at the bottom surface, and the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2}=0$$

and $$-k\frac{dT(L)}{dx}=h[T(L)-T_\infty]+\varepsilon\sigma[T(L)^4-T_{\text{sky}}^4]=h[T_2-T_\infty]+\varepsilon\sigma[(T_2+460)^4-T_{\text{sky}}^4]$$

$$T(L)=T_2=75°\text{F}$$

(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx}=C_1$$

$$T(x)=C_1x+C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

Convection at $x = L$: $$-kC_1=h[T_2-T_\infty]+\varepsilon\sigma[(T_2+460)^4-T_{\text{sky}}^4]$$
$$\rightarrow\ C_1=-\{h[T_2-T_\infty]+\varepsilon\sigma[(T_2+460)^4-T_{\text{sky}}^4]\}/k$$

Temperature at $x = L$: $T(L)=C_1\times L+C_2=T_2\rightarrow\ C_2=T_2-C_1L$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x)=C_1x+(T_2-C_1L)=T_2-(L-x)C_1=T_2+\frac{h[T_2-T_\infty]+\varepsilon\sigma[(T_2+460)^4-T_{\text{sky}}^4]}{k}(L-x)$$

$$=75°\text{F}+\frac{(12\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(75-90)°\text{F}+0.7(0.1714\times10^{-8}\text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)[(535\text{ R})^4-(480\text{ R})^4]}{7.2\text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}(4/12-x)\text{ ft}$$

$$=75-20.2(1/3-x)$$

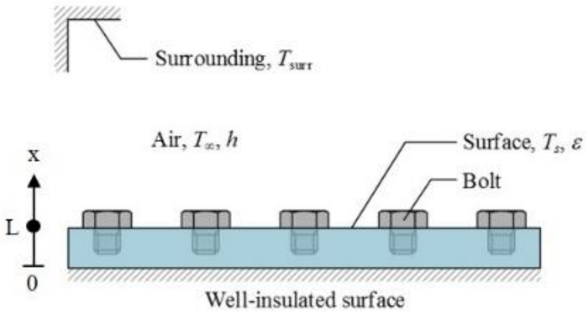
(*c*) The temperature at x = 0 (the bottom surface of the plate) is

$$T(0)=75-20.2\times(1/3-0)=\mathbf{68.3°F}$$

2-69 C&S A series of ASTM B21 naval brass bolts are bolted on the upper surface of a plate. The upper surface is exposed to convection with air and radiation with the surrounding surface. Formulate the temperature profile in the plate, and determine if the bolts comply with the ASME Code for Process Piping.

***Assumptions*1** Heat transfer is steady. **2** One dimensional heat conduction through the plate. **3** The bottom surface at $x = 0$ is well-insulated while the upper surface at $x = L$ is subjected to convection and radiation. **4** There is no heat generation in the plate. **5** Thermal properties are constant.

Properties The emissivity of the plate and bolts is given as 0.3.



Analysis Taking the direction normal to the surface of the plate to be the x direction with $x = 0$ at the bottom surface, the differential equation for heat conduction can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

whereC_1 and C_2 are arbitrary constants. Applying the well- insulated surface boundary condition at the bottom surface ($x = 0$) yields

$$x = 0: \qquad -k\frac{dT(0)}{dx} = -kC_1 = 0 \qquad \rightarrow \qquad C_1 = 0$$

Since $C_1 = 0$, the temperature profile in the plate is constant

$$T(x) = C_2$$

At the upper surface ($x = L$) we have

$$x = L: \qquad -k\frac{dT(L)}{dx} = -kC_1 = h[T(L) - T_\infty] + \varepsilon\sigma[T(L)^4 - T_{\text{surr}}^4] = 0$$

$$x = L: \qquad T(x) = T(L) = C_2$$

Solving for C_2 (note that absolute temperatures are used for the calculation),

$$h[C_2 - T_\infty] + \varepsilon\sigma[C_2^4 - T_{\text{surr}}^4] = 0$$

$$(2)\left(\frac{\text{W}}{\text{m}^2\cdot\text{K}}\right)(C_2 - 323)(\text{K}) + (0.3)(5.67\times10^{-8})\left(\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)(C_2^4 - 473^4)(\text{K}^4) = 0$$

$$T(x) = C_2 = 437\text{ K} = \mathbf{164}\text{°C}$$

Discussion The temperature in the plate exceeds the maximum use temperature by 15°C. The use of the ASTM B21 bolts under these conditions does not comply with the ASME Code for Process Piping (ASME B31.3-2014). One way to reduce the plate temperature is by increasing the convection heat transfer coefficient with forced convection. If the convection heat transfer coefficient were to increase to higher than 3.15 W/m^2·K, then the plate temperature would reduce to below 149°C:

$$h > \varepsilon\sigma\frac{T_{\text{surr}}^4 - T(x)^4}{T(x) - T_\infty}$$

$$> (0.3)(5.67\times10^{-8})\left(\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)\frac{473^4 - 422^4}{422 - 323}(\text{K}^3)$$

$$> 3.15\text{ W/m}^2\cdot\text{K}$$

2-70 Chilled water flows in a pipe that is well insulated from outside. The mathematical formulation and the variation of temperature in the pipe are to be determined for steady one-dimensional heat transfer.

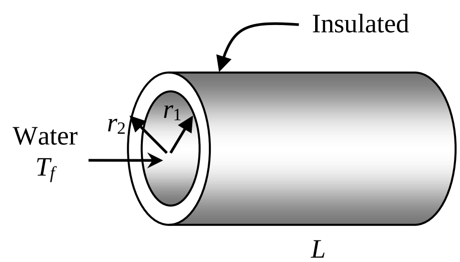
Assumptions **1** Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the center line. **2** Thermal conductivity is constant. **3** There is no heat generation in the pipe.

Analysis (*a*) Noting that heat transfer is one-dimensional in the radial *r* direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

and
$$-k\frac{dT(r_1)}{dr}=h[T_f-T(r_1)]$$

$$\frac{dT(r_2)}{dr}=0$$



(*b*) Integrating the differential equation once with respect to *r* gives

$$r\frac{dT}{dr}=C_1$$

Dividing both sides of the equation above by *r* to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr}=\frac{C_1}{r}$$

$$T(r)=C_1\ln r+C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_2$: $$\frac{C_1}{r_2}=0\rightarrow C_1=0$$

$r = r_1$: $$-k\frac{C_1}{r_1}=h[T_f-(C_1\ln r_1+C_2)]$$

$$0=h(T_f-C_2)\rightarrow C_2=T_f$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r)=T_f$$

This result is not surprising since steady operating conditions exist.

2-71E A steam pipe is subjected to convection on the inner surface and to specified temperature on the outer surface. The mathematical formulation, the variation of temperature in the pipe, and the rate of heat loss are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the center line. **2** Thermal conductivity is constant. **3** There is no heat generation in the pipe.

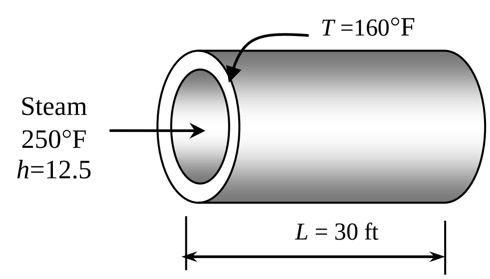
Properties The thermal conductivity is given to be k = 7.2 Btu/h·ft·°F.

Analysis (*a*) Noting that heat transfer is one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

and
$$-k\frac{dT(r_1)}{dr}=h[T_\infty - T(r_1)]$$

$$T(r_2)=T_2=160°\text{F}$$



(*b*) Integrating the differential equation once with respect to r gives

$$r\frac{dT}{dr}=C_1$$

Dividing both sides of the equation above by r to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr}=\frac{C_1}{r}$$

$$T(r)=C_1\ln r+C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_1$: $$-k\frac{C_1}{r_1}=h[T_\infty-(C_1\ln r_1+C_2)]$$

$r = r_2$: $$T(r_2)=C_1\ln r_2+C_2=T_2$$

Solving for C_1 and C_2 simultaneously gives

$$C_1=\frac{T_2-T_\infty}{\ln\dfrac{r_2}{r_1}+\dfrac{k}{hr_1}} \quad \text{and} \quad C_2=T_2-C_1\ln r_2=T_2-\frac{T_2-T_\infty}{\ln\dfrac{r_2}{r_1}+\dfrac{k}{hr_1}}\ln r_2$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r)=C_1\ln r+T_2-C_1\ln r_2=C_1(\ln r-\ln r_2)+T_2=\frac{T_2-T_\infty}{\ln\dfrac{r_2}{r_1}+\dfrac{k}{hr_1}}\ln\frac{r}{r_2}+T_2$$

$$=\frac{(160-250)°\text{F}}{\ln\dfrac{2.4}{2}+\dfrac{7.2\text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}{(12.5\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(2/12\text{ ft})}}\ln\frac{r}{2.4\text{ in}}+160°\text{F}=-24.74\ln\frac{r}{2.4\text{ in}}+160°\text{F}$$

(*c*) The rate of heat conduction through the pipe is

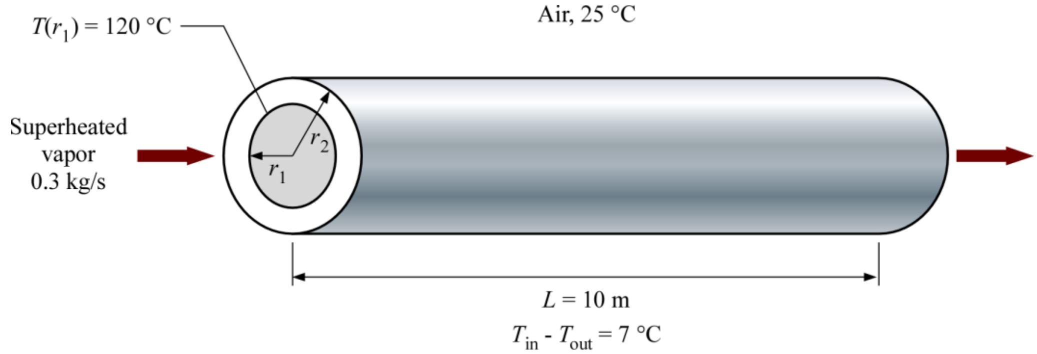
$$\dot{Q}=-kA\frac{dT}{dr}=-k(2\pi rL)\frac{C_1}{r}=-2\pi Lk\frac{T_2-T_\infty}{\ln\dfrac{r_2}{r_1}+\dfrac{k}{hr_1}}$$

$$=-2\pi(30\text{ ft})(7.2\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})\frac{(160-250)°\text{F}}{\ln\dfrac{2.4}{2}+\dfrac{7.2\text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}{(12.5\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(2/12\text{ ft})}}=\mathbf{33{,}600\text{ Btu/h}}$$

2-72 The convection heat transfer coefficient between the surface of a pipe carrying superheated vapor and the surrounding air is to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional and there is thermal symmetry about the centerline. **2** Thermal properties are constant. **3** There is no heat generation in the pipe. **4** Heat transfer by radiation is negligible.

Properties The constant pressure specific heat of vapor is given to be 2190 J/kg · °C and the pipe thermal conductivity is 17 W/m · °C.



Analysis The inner and outer radii of the pipe are

$$r_1 = 0.05\,\text{m}/2 = 0.025\,\text{m}$$

$$r_2 = 0.025\,\text{m} + 0.006\,\text{m} = 0.031\,\text{m}$$

The rate of heat loss from the vapor in the pipe can be determined from

$$\dot{Q}_{\text{loss}} = \dot{m}c_p(T_{\text{in}} - T_{\text{out}}) = (0.3\,\text{kg/s})(2190\,\text{J/kg}\cdot°\text{C})(7)\,°\text{C} = 4599\,\text{W}$$

For steady one-dimensional heat conduction in cylindrical coordinates, the heat conduction equation can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

and $\quad -k\dfrac{dT(r_1)}{dr} = \dfrac{\dot{Q}_{\text{loss}}}{A} = \dfrac{\dot{Q}_{\text{loss}}}{2\pi r_1 L}$ (heat flux at the inner pipe surface)

$$T(r_1) = 120\,°\text{C} \quad \text{(inner pipe surface temperature)}$$

Integrating the differential equation once with respect to r gives

$$\frac{dT}{dr} = \frac{C_1}{r}$$

Integrating with respect to r again gives

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions gives

$$r = r_1: \qquad \frac{dT(r_1)}{dr} = -\frac{1}{k}\frac{\dot{Q}_{\text{loss}}}{2\pi r_1 L} = \frac{C_1}{r_1} \quad \rightarrow \quad C_1 = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}$$

$$r = r_1: \qquad T(r_1) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_1 + C_2 \quad \rightarrow \quad C_2 = \frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_1 + T(r_1)$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r + \frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_1 + T(r_1)$$

$$= -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln(r/r_1) + T(r_1)$$

The outer pipe surface temperature is

$$T(r_2) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln(r_2/r_1) + T(r_1)$$

$$= -\frac{1}{2\pi}\frac{4599\text{ W}}{(17\text{ W/m}\cdot{}^\circ\text{C})(10\text{ m})}\ln\left(\frac{0.031}{0.025}\right) + 120\,^\circ\text{C}$$

$$= 119.1\,^\circ\text{C}$$

From Newton's law of cooling, the rate of heat loss at the outer pipe surface by convection is

$$\dot{Q}_{\text{loss}} = h(2\pi r_2 L)\left[T(r_2) - T_\infty\right]$$

Rearranging and the convection heat transfer coefficient is determined to be

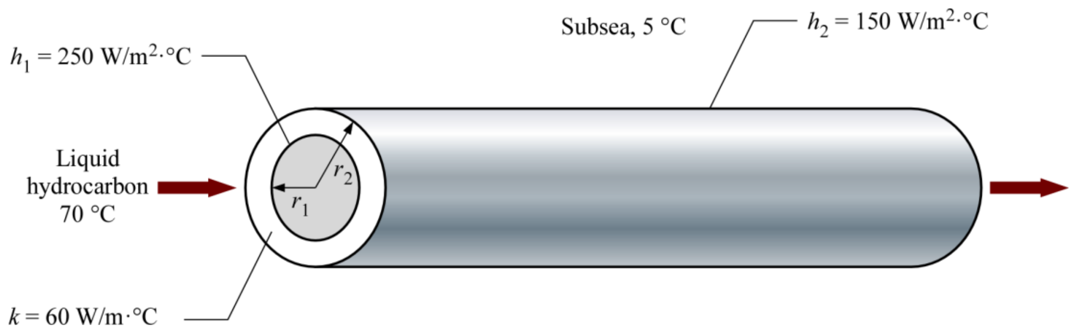
$$h = \frac{\dot{Q}_{\text{loss}}}{2\pi r_2 L[T(r_2) - T_\infty]} = \frac{4599\text{ W}}{2\pi(0.031\text{ m})(10\text{ m})(119.1 - 25)\,^\circ\text{C}} = \mathbf{25.1\ W/m^2\cdot{}^\circ C}$$

Discussion If the pipe wall is thicker, the temperature difference between the inner and outer pipe surfaces will be greater. If the pipe has very high thermal conductivity or the pipe wall thickness is very small, then the temperature difference between the inner and outer pipe surfaces may be negligible.

2-73 A subsea pipeline is transporting liquid hydrocarbon. The temperature variation in the pipeline wall, the inner surface temperature of the pipeline, the mathematical expression for the rate of heat loss from the liquid hydrocarbon, and the heat flux through the outer pipeline surface are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional and there is thermal symmetry about the centerline. **2** Thermal properties are constant. **3** There is no heat generation in the pipeline.

Properties The pipeline thermal conductivity is given to be 60 W/m · °C.



Analysis The inner and outer radii of the pipeline are

$$r_1 = 0.5\,\text{m}/2 = 0.25\,\text{m}$$

$$r_2 = 0.25\,\text{m} + 0.008\,\text{m} = 0.258\,\text{m}$$

(a) For steady one-dimensional heat conduction in cylindrical coordinates, the heat conduction equation can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

and $\quad -k\dfrac{dT(r_1)}{dr} = h_1[T_{\infty,1} - T(r_1)]$ (convection at the inner pipeline surface)

$$-k\frac{dT(r_2)}{dr} = h_2[T(r_2) - T_{\infty,2}] \quad \text{(convection at the outer pipeline surface)}$$

Integrating the differential equation once with respect to r gives

$$\frac{dT}{dr} = \frac{C_1}{r}$$

Integrating with respect to r again gives

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions gives

$$r = r_1: \qquad -k\frac{dT(r_1)}{dr} = -k\frac{C_1}{r_1} = h_1(T_{\infty,1} - C_1 \ln r_1 - C_2)$$

$$r = r_2: \qquad -k\frac{dT(r_2)}{dr} = -k\frac{C_1}{r_2} = h_2(C_1 \ln r_2 + C_2 - T_{\infty,2})$$

C_1 and C_2 can be expressed explicitly as

$$C_1 = -\frac{T_{\infty,1} - T_{\infty,2}}{k/(r_1 h_1) + \ln(r_2/r_1) + k/(r_2 h_2)}$$

$$C_2 = T_{\infty,1} - \frac{T_{\infty,1} - T_{\infty,2}}{k/(r_1 h_1) + \ln(r_2/r_1) + k/(r_2 h_2)}\left(\frac{k}{r_1 h_1} - \ln r_1\right)$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = -\frac{T_{\infty,1} - T_{\infty,2}}{k/(r_1 h_1) + \ln(r_2/r_1) + k/(r_2 h_2)}\left[\frac{k}{r_1 h_1} + \ln(r/r_1)\right] + T_{\infty,1}$$

(*b*) The inner surface temperature of the pipeline is

$$T(r_1) = -\frac{T_{\infty,1} - T_{\infty,2}}{k/(r_1 h_1) + \ln(r_2/r_1) + k/(r_2 h_2)}\left[\frac{k}{r_1 h_1} + \ln(r_1/r_1)\right] + T_{\infty,1}$$

$$= -\frac{(70-5)\,°\text{C}\left[\dfrac{60\ \text{W/m}\cdot°\text{C}}{(0.25\ \text{m})(250\ \text{W/m}^2\cdot°\text{C})}\right]}{\dfrac{60\ \text{W/m}\cdot°\text{C}}{(0.25\ \text{m})(250\ \text{W/m}^2\cdot°\text{C})} + \ln\left(\dfrac{0.258}{0.25}\right) + \dfrac{60\ \text{W/m}\cdot°\text{C}}{(0.258\ \text{m})(150\ \text{W/m}^2\cdot°\text{C})}} + 70\,°\text{C}$$

$$= \mathbf{45.5\,°C}$$

(*c*) The mathematical expression for the rate of heat loss through the pipeline can be determined from Fourier's law to be

$$\dot{Q}_{\text{loss}} = -kA\frac{dT}{dr}$$

$$= -k(2\pi r_2 L)\frac{dT(r_2)}{dr} = -2\pi L k C_1$$

$$= \frac{T_{\infty,1} - T_{\infty,2}}{\dfrac{1}{2\pi r_1 L h_1} + \dfrac{\ln(r_2/r_1)}{2\pi L k} + \dfrac{1}{2\pi r_2 L h_2}}$$

(*d*) Again from Fourier's law, the heat flux through the outer pipeline surface is

$$\dot{q}_2 = -k\frac{dT}{dr} = -k\frac{dT(r_2)}{dr} = -k\frac{C_1}{r_2}$$

$$= \frac{T_{\infty,1} - T_{\infty,2}}{k/(r_1 h_1) + \ln(r_2/r_1) + k/(r_2 h_2)}\frac{k}{r_2}$$

$$= \frac{(70-5)\,°\text{C}}{\dfrac{60\ \text{W/m}\cdot°\text{C}}{(0.25\ \text{m})(250\ \text{W/m}^2\cdot°\text{C})} + \ln\left(\dfrac{0.258}{0.25}\right) + \dfrac{60\ \text{W/m}\cdot°\text{C}}{(0.258\ \text{m})(150\ \text{W/m}^2\cdot°\text{C})}}\left(\frac{60\ \text{W/m}\cdot°\text{C}}{0.258\ \text{m}}\right)$$

$$= \mathbf{5947\ W/m^2}$$

Discussion Knowledge of the inner pipeline surface temperature can be used to control wax deposition blockages in the pipeline.

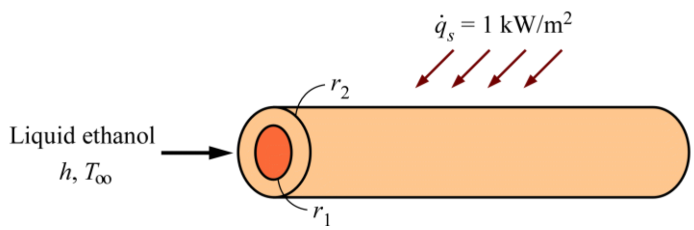
2-74 Liquid ethanol is being transported in a pipe where the outer surface is subjected to heat flux. Convection heat transfer occurs on the inner surface of the pipe. The variation of temperature in the pipe wall and the inner and outer surface temperatures are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall. **4** The inner surface at $r = r_1$ is subjected to convection while the outer surface at $r = r_2$ is subjected to uniform heat flux.

Properties Thermal conductivity is given to be 15 W/m·K.

Analysis For one-dimensional heat transfer in the radial r direction, the differential equation for heat conduction in cylindrical coordinate can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$



Integrating the differential equation twice with respect to r yields

$$r\frac{dT}{dr} = C_1 \quad \text{or} \quad \frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r = r_2: \quad -k\frac{dT(r_2)}{dr} = -\dot{q}_s = -k\frac{C_1}{r_2} \quad \rightarrow \quad C_1 = \dot{q}_s \frac{r_2}{k}$$

$$r = r_1: \quad -k\frac{dT(r_1)}{dr} = h[T_\infty - T(r_1)] \quad \rightarrow \quad k\frac{C_1}{r_1} = h(C_1 \ln r_1 + C_2 - T_\infty)$$

Solving for C_2 gives

$$C_2 = C_1\left(\frac{k}{h}\frac{1}{r_1} - \ln r_1\right) + T_\infty$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = C_1 \ln r + C_2 = C_1 \ln r + C_1\left(\frac{k}{h}\frac{1}{r_1} - \ln r_1\right) + T_\infty \quad \rightarrow \quad T(r) = \dot{q}_s \frac{r_2}{k}\left(\frac{k}{h}\frac{1}{r_1} + \ln\frac{r}{r_1}\right) + T_\infty$$

The temperature at $r = r_1 = 0.015$ m (the inner surface of the pipe) is

$$T(r_1) = \frac{\dot{q}_s}{h}\frac{r_2}{r_1} + T_\infty = \frac{1000\ \text{W/m}^2}{50\ \text{W/m}^2\cdot\text{K}}\left(\frac{0.018\ \text{m}}{0.015\ \text{m}}\right) + 10°\text{C} = \mathbf{34.0°C}$$

$$T(r_1) = \mathbf{34.0°C}$$

The temperature at $r = r_2 = 0.018$ m (the outer surface of the pipe) is

$$T(r_2) = \dot{q}_s \frac{r_2}{k}\left(\frac{k}{h}\frac{1}{r_1} + \ln\frac{r_2}{r_1}\right) + T_\infty = (1000\ \text{W/m}^2)\frac{0.018\ \text{m}}{15\ \text{W/m}\cdot\text{K}}\left[\left(\frac{15\ \text{W/m}\cdot\text{K}}{50\ \text{W/m}^2\cdot\text{K}}\right)\frac{1}{0.015\ \text{m}} + \ln\frac{0.018}{0.015}\right] + 10°\text{C}$$

$$T(r_2) = \mathbf{34.2°C}$$

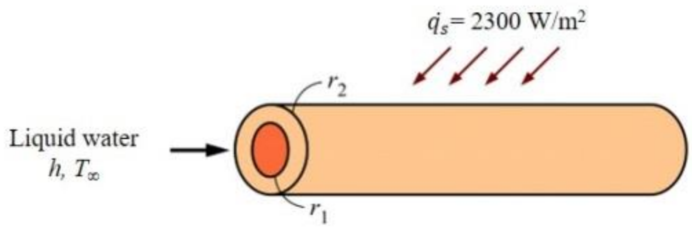
Both the inner and outer surfaces of the pipe are at higher temperatures than the flashpoint of ethanol (16.6°C).

Discussion The outer surface of the pipe should be wrapped with protective insulation to keep the heat input from heating the ethanol inside the pipe.

2-75 Liquid water flows in a tube with the inner surface lined with PVDC lining. The tube outer surface is subjected to a known uniform heat flux. The tube inner diameter, the tube wall thickness, the water temperature, and the convection heat transfer coefficient are known. Formulate the temperature profile in the tube wall, and determine if the PVDC lining is in compliance with the ASME Code for Process Piping.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the tube wall. **4** The inner surface at $r = r_1$ is subjected to convection while the outer surface at $r = r_2$ is subjected to uniform heat flux. **5** The PVDC lining is very thin and the temperature gradient in the lining is negligible.

Properties Thermal conductivity of the tube wall is given to be 15 W/m·K.



AnalysisFor one-dimensional heat transfer in the radial r direction, the differential equation for heat conduction in cylindrical coordinate can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

Integrating the differential equation twice with respect to r yields

$$r\frac{dT}{dr} = C_1 \text{or} \frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

whereC_1 and C_2 are arbitrary constants. Applying the boundary conditions yields

$$r = r_1: \quad -k\frac{dT(r_1)}{dr} = h[T_\infty - T(r_1)] \quad \rightarrow \quad k\frac{C_1}{r_1} = h(C_1 \ln r_1 + C_2 - T_\infty)$$

$$r = r_2: \quad -k\frac{dT(r_2)}{dr} = -k\frac{C_1}{r_2} = -\dot{q}_s \quad \rightarrow \quad C_1 = \dot{q}_s\frac{r_2}{k}$$

Solving for C_2 yields

$$C_2 = C_1\left(\frac{1}{r_1}\frac{k}{h} - \ln r_1\right) + T_\infty$$

Substituting C_1 and C_2 into the general solution, the temperature profile in the tube wall is determined to be

$$T(r) = C_1 \ln r + C_2 = C_1 \ln r + C_1\left(\frac{1}{r_1}\frac{k}{h} - \ln r_1\right) + T_\infty \quad \rightarrow \quad \boldsymbol{T(r) = \dot{q}_s\frac{r_2}{k}\left(\ln\frac{r}{r_1} + \frac{1}{r_1}\frac{k}{h}\right) + T_\infty}$$

At the tube inner surface ($r = r_1$ = D₁/2 = 0.012 m), which is lined with PVDC lining, the temperature is

$$T(r_1) = \dot{q}_s\frac{r_2}{k}\left(\ln\frac{r_1}{r_1} + \frac{1}{r_1}\frac{k}{h}\right) + T_\infty = \left(2300\ \frac{\text{W}}{\text{m}^2}\right)\frac{(0.017\text{ m})}{\left(15\frac{\text{W}}{\text{m}\cdot\text{K}}\right)}\left(\frac{1}{0.012\text{ m}} \times \frac{\left(15\frac{\text{W}}{\text{m}\cdot\text{K}}\right)}{50\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}}\right) + 20°\text{C}$$

$$= \mathbf{85.2}°\text{C} > 79°\text{C}$$

where and

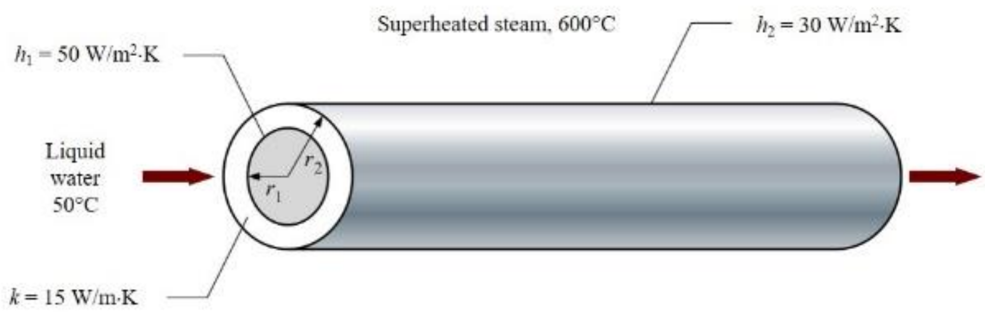
$r_2 = r_1$ + wall thickness = 0.012 m + 0.005 m = 0.017 m.

Discussion The tube inner surface temperature at $r = r_1$ =0.012 m is more than 6°C higher than the recommended maximum temperature of 79°C by the ASME Code for Process Piping (ASME B31.3-2014, A323) for PVDC. Therefore, having the tube inner surface lined with PVDC lining is not in compliance with the code.

2-76 C&S Liquid water flows in a tube with the inner surface lined with PTFE lining. The tube inner surface is subjected to convection with water, the tube outer surface is subjected to convection with superheated steam. Formulate the temperature profile in the tube wall, and determine if the PTFE lining is in compliance with the ASME Code for Process Piping.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the tube wall. **4** The inner surface at $r = r_1$ is subjected to convection with water. **5** The outer surface at $r = r_2$ is subjected to convection with superheated steam. **6** The PTFE lining is very thin and the temperature gradient in the lining is negligible.

Properties Thermal conductivity of the tube wall is given to be 15 W/m·K.



AnalysisFor one-dimensional heat transfer in the radial r direction, the differential equation for heat conduction in cylindrical coordinate can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

Integrating the differential equation twice with respect to r yields

$$r\frac{dT}{dr} = C_1 \text{or} \frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

whereC_1 and C_2 are arbitrary constants. Applying the boundary conditions yields

$$r = r_1: \quad -k\frac{dT(r_1)}{dr} = -k\frac{C_1}{r_1} = h_1[T_{\infty,1} - T(r_1)]$$

$$r = r_2: \quad -k\frac{dT(r_2)}{dr} = -k\frac{C_1}{r_2} = h_2[T(r_2) - T_{\infty,2}]$$

C_1 and C_2 can be expressed as

$$C_1 = -\frac{T_{\infty,1} - T_{\infty,2}}{\dfrac{k}{r_1 h_1} + \ln\dfrac{r_2}{r_1} + \dfrac{k}{r_2 h_2}}$$

$$C_2 = T_{\infty,1} - \frac{T_{\infty,1} - T_{\infty,2}}{\dfrac{k}{r_1 h_1} + \ln\dfrac{r_2}{r_1} + \dfrac{k}{r_2 h_2}}\left(\frac{k}{r_1 h_1} - \ln r_1\right)$$

Substituting C_1 and C_2 into the general solution, the temperature profile in the tube wall is determined to be

$$\boldsymbol{T(r) = -\frac{T_{\infty,1} - T_{\infty,2}}{\dfrac{k}{r_1 h_1} + \ln\dfrac{r_2}{r_1} + \dfrac{k}{r_2 h_2}}\left(\frac{k}{r_1 h_1} + \ln\frac{r}{r_1}\right) + T_{\infty,1}}$$

At the tube inner surface ($r = r_1 = 0.012$ m), which is lined with PTFE lining, the temperature is

$$T(r_1) = -\frac{T_{\infty,1} - T_{\infty,2}}{\dfrac{k}{r_1 h_1} + \ln\dfrac{r_2}{r_1} + \dfrac{k}{r_2 h_2}}\left(\frac{k}{r_1 h_1} + \ln\frac{r_1}{r_1}\right) + T_{\infty,1}$$

$$T(r_1) = -\frac{(50 - 600)°\text{C}}{\dfrac{15\,\dfrac{\text{W}}{\text{m·K}}}{(0.012\text{ m})\left(50\,\dfrac{\text{W}}{\text{m}^2\text{·K}}\right)} + \ln\dfrac{0.017\text{ m}}{0.012\text{ m}} + \dfrac{15\,\dfrac{\text{W}}{\text{m·K}}}{(0.017\text{ m})\left(30\,\dfrac{\text{W}}{\text{m}^2\text{·K}}\right)}}\left[\frac{15\,\dfrac{\text{W}}{\text{m·K}}}{(0.012\text{ m})\left(50\,\dfrac{\text{W}}{\text{m}^2\text{·K}}\right)}\right] + 50°\text{C}$$

$$= \mathbf{301}℃ > 260℃$$

whereand $r_2 = r_1$ + wall thickness = 0.012 m + 0.005 m = 0.017 m.

Discussion The tube inner surface temperature at $r = r_1$ =0.012 m is 41°C higher than the recommended maximum temperature of 260°C by the ASME Code for Process Piping (ASME B31.3-2014, A323) for PTFE. Therefore, having the tube inner surface lined with PTFE lining is not in compliance with the code.

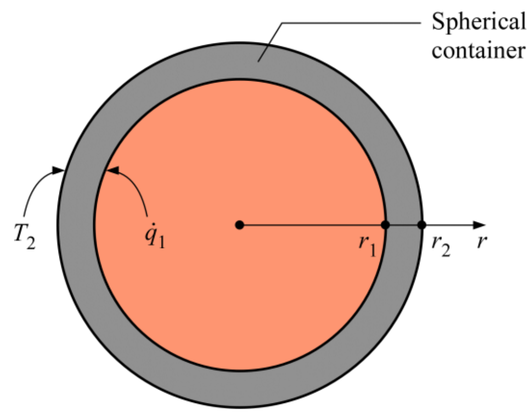
2-77 A spherical container is subjected to uniform heat flux on the inner surface, while the outer surface maintains a constant temperature. The variation of temperature in the container wall and the inner surface temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Temperatures on both surfaces are uniform. **3** Thermal conductivity is constant. **4** There is no heat generation in the wall. **5** The inner surface at $r = r_1$ is subjected to uniform heat flux while the outer surface at $r = r_2$ is at constant temperature T_2.

Properties Thermal conductivity is given to be k = 1.5 W/m·K.

Analysis For one-dimensional heat transfer in the radial direction, the differential equation for heat conduction in spherical coordinate can be expressed as

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$



Integrating the differential equation twice with respect to r yields

$$r^2\frac{dT}{dr} = C_1 \quad \text{or} \quad \frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T(r) = -\frac{C_1}{r} + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r = r_1: \quad -k\frac{dT(r_1)}{dr} = \dot{q}_1 = -k\frac{C_1}{r_1^2} \quad \rightarrow \quad C_1 = -\dot{q}_1\frac{r_1^2}{k}$$

$$r = r_2: \quad T(r_2) = T_2 = -\frac{C_1}{r_2} + C_2 \quad \rightarrow \quad C_2 = T_2 + \frac{C_1}{r_2} = T_2 - \frac{\dot{q}_1}{k}\frac{r_1^2}{r_2}$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = \frac{\dot{q}_1}{k}\frac{r_1^2}{r} + T_2 - \frac{\dot{q}_1}{k}\frac{r_1^2}{r_2} \quad \rightarrow \quad T(r) = \dot{q}_1\frac{r_1^2}{k}\left(\frac{1}{r} - \frac{1}{r_2}\right) + T_2$$

The temperature at $r = r_1 = 1$ m (the inner surface of the container) is

$$T(r_1) = T_1 = \dot{q}_1\frac{r_1^2}{k}\left(\frac{1}{r} - \frac{1}{r_2}\right) + T_2$$

$$T_1 = (7000\ \text{W/m}^2)\frac{(1\ \text{m})^2}{(1.5\ \text{W/m}\cdot\text{K})}\left(\frac{1}{1\ \text{m}} - \frac{1}{1.05\ \text{m}}\right) + 25°\text{C} = \mathbf{247°C}$$

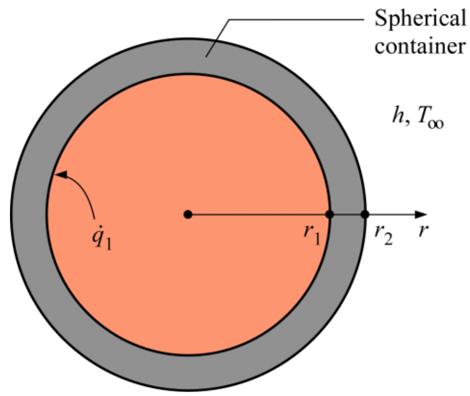
Discussion As expected the inner surface temperature is higher than the outer surface temperature.

2-78 A spherical shell is subjected to uniform heat flux on the inner surface, while the outer surface is subjected to convection heat transfer. The variation of temperature in the shell wall and the outer surface temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall. **4** The inner surface at $r = r_1$ is subjected to uniform heat flux while the outer surface at $r = r_2$ is subjected to convection.

Analysis For one-dimensional heat transfer in the radial r direction, the differential equation for heat conduction in spherical coordinate can be expressed as

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$



Integrating the differential equation twice with respect to r yields

$$r^2 \frac{dT}{dr} = C_1 \quad \text{or} \quad \frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T(r) = -\frac{C_1}{r} + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r = r_1: \quad -k\frac{dT(r_1)}{dr} = \dot{q}_1 = -k\frac{C_1}{r_1^2} \quad \rightarrow \quad C_1 = -\dot{q}_1 \frac{r_1^2}{k}$$

$$r = r_2: \quad -k\frac{dT(r_2)}{dr} = h[T(r_2) - T_\infty] \quad \rightarrow \quad -k\frac{C_1}{r_2^2} = h\left(-\frac{C_1}{r_2} + C_2 - T_\infty\right)$$

Solving for C_2 gives

$$C_2 = \dot{q}_1 \frac{r_1^2}{k}\left(\frac{k}{h}\frac{1}{r_2^2} - \frac{1}{r_2}\right) + T_\infty$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = -\frac{C_1}{r} + C_2 = \dot{q}_1 \frac{r_1^2}{k}\frac{1}{r} + \dot{q}_1 \frac{r_1^2}{k}\left(\frac{k}{h}\frac{1}{r_2^2} - \frac{1}{r_2}\right) + T_\infty \quad \rightarrow \quad T(r) = \dot{q}_1 \frac{r_1^2}{k}\left(\frac{1}{r} + \frac{k}{h}\frac{1}{r_2^2} - \frac{1}{r_2}\right) + T_\infty$$

The temperature at $r = r_2$ (the outer surface of the shell) can be expressed as

$$T(r_2) = \frac{\dot{q}_1}{h}\left(\frac{r_1}{r_2}\right)^2 + T_\infty$$

Discussion Increasing the convection heat transfer coefficient h would decrease the outer surface temperature $T(r_2)$.

2-79 A spherical container is subjected to specified temperature on the inner surface and convection on the outer surface. The mathematical formulation, the variation of temperature, and the rate of heat transfer are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since there is no change with time and there is thermal symmetry about the midpoint. **2** Thermal conductivity is constant. **3** There is no heat generation.

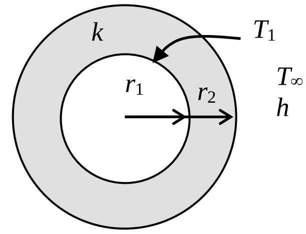
Properties The thermal conductivity is given to be $k = 30$ W/m·°C.

Analysis (*a*) Noting that heat transfer is one-dimensional in the radial *r* direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

and $$T(r_1) = T_1 = 0°\text{C}$$

$$-k\frac{dT(r_2)}{dr} = h[T(r_2) - T_\infty]$$



(*b*) Integrating the differential equation once with respect to *r* gives

$$r^2\frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by *r* to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T(r) = -\frac{C_1}{r} + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_1$: $$T(r_1) = -\frac{C_1}{r_1} + C_2 = T_1$$

$r = r_2$: $$-k\frac{C_1}{r_2^2} = h\left(-\frac{C_1}{r_2} + C_2 - T_\infty\right)$$

Solving for C_1 and C_2 simultaneously gives

$$C_1 = \frac{r_2(T_1 - T_\infty)}{1 - \dfrac{r_2}{r_1} - \dfrac{k}{hr_2}} \quad \text{and} \quad C_2 = T_1 + \frac{C_1}{r_1} = T_1 + \frac{T_1 - T_\infty}{1 - \dfrac{r_2}{r_1} - \dfrac{k}{hr_2}}\frac{r_2}{r_1}$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = -\frac{C_1}{r} + T_1 + \frac{C_1}{r_1} = C_1\left(\frac{1}{r_1} - \frac{1}{r}\right) + T_1 = \frac{T_1 - T_\infty}{1 - \dfrac{r_2}{r_1} - \dfrac{k}{hr_2}}\left(\frac{r_2}{r_1} - \frac{r_2}{r}\right) + T_1$$

$$= \frac{(0 - 25)°\text{C}}{1 - \dfrac{2.1}{2} - \dfrac{30\ \text{W/m}\cdot°\text{C}}{(18\ \text{W/m}^2\cdot°\text{C})(2.1\,\text{m})}}\left(\frac{2.1}{2} - \frac{2.1}{r}\right) + 0°\text{C} = 29.63(1.05 - 2.1/r)$$

(*c*) The rate of heat conduction through the wall is

$$\dot{Q} = -kA\frac{dT}{dr} = -k(4\pi r^2)\frac{C_1}{r^2} = -4\pi k C_1 = -4\pi k\frac{r_2(T_1 - T_\infty)}{1 - \dfrac{r_2}{r_1} - \dfrac{k}{hr_2}}$$

$$= -4\pi(30\ \text{W/m}\cdot°\text{C})\frac{(2.1\,\text{m})(0 - 25)°\text{C}}{1 - \dfrac{2.1}{2} - \dfrac{30\ \text{W/m}\cdot°\text{C}}{(18\ \text{W/m}^2\cdot°\text{C})(2.1\,\text{m})}} = \mathbf{23{,}460\ W}$$

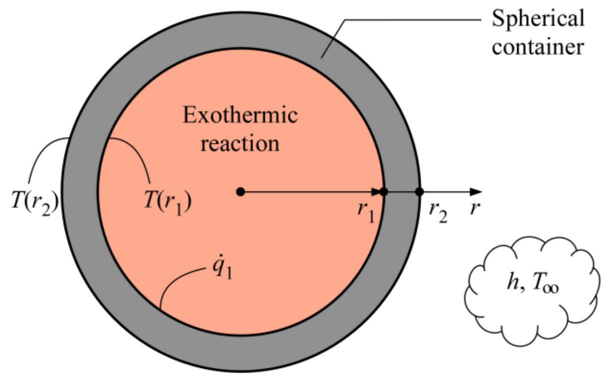
2-80 A spherical container is used for storing chemicals undergoing exothermic reaction that provides a uniform heat flux to its inner surface. The outer surface is subjected to convection heat transfer. The variation of temperature in the container wall and the inner and outer surface temperatures are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall. **4** The inner surface at $r = r_1$ is subjected to uniform heat flux while the outer surface at $r = r_2$ is subjected to convection.

Properties Thermal conductivity is given to be 15 W/m·K.

Analysis For one-dimensional heat transfer in the radial r direction, the differential equation for heat conduction in spherical coordinate can be expressed as

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0$$



Integrating the differential equation twice with respect to r yields

$$r^2\frac{dT}{dr}=C_1 \quad \text{or} \quad \frac{dT}{dr}=\frac{C_1}{r^2}$$

$$T(r)=-\frac{C_1}{r}+C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r=r_1: \quad -k\frac{dT(r_1)}{dr}=\dot{q}_1=-k\frac{C_1}{r_1^2} \quad \rightarrow \quad C_1=-\dot{q}_1\frac{r_1^2}{k}$$

$$r=r_2: \quad -k\frac{dT(r_2)}{dr}=h[T(r_2)-T_\infty] \quad \rightarrow \quad -k\frac{C_1}{r_2^2}=h\left(-\frac{C_1}{r_2}+C_2-T_\infty\right)$$

Solving for C_2 gives

$$C_2=\dot{q}_1\frac{r_1^2}{k}\left(\frac{k}{h}\frac{1}{r_2^2}-\frac{1}{r_2}\right)+T_\infty$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r)=-\frac{C_1}{r}+C_2=\dot{q}_1\frac{r_1^2}{k}\frac{1}{r}+\dot{q}_1\frac{r_1^2}{k}\left(\frac{k}{h}\frac{1}{r_2^2}-\frac{1}{r_2}\right)+T_\infty \quad \rightarrow \quad T(r)=\dot{q}_1\frac{r_1^2}{k}\left(\frac{k}{h}\frac{1}{r_2^2}+\frac{1}{r}-\frac{1}{r_2}\right)+T_\infty$$

The temperature at $r = r_1 = 0.5$ m (the inner surface of the container) is

$$T(r_1)=\dot{q}_1\frac{r_1^2}{k}\left(\frac{k}{h}\frac{1}{r_2^2}+\frac{1}{r_1}-\frac{1}{r_2}\right)+T_\infty$$

$$T(r_1)=(60000\ \text{W/m}^2)\frac{(0.5\ \text{m})^2}{(15\ \text{W/m}\cdot\text{K})}\left[\left(\frac{15\ \text{W/m}\cdot\text{K}}{1000\ \text{W/m}^2\cdot\text{K}}\right)\frac{1}{(0.55\ \text{m})^2}+\frac{1}{(0.5\ \text{m})}-\frac{1}{(0.55\ \text{m})}\right]+23°\text{C}$$

$$T(r_1)=\mathbf{254.4°C}$$

The temperature at $r = r_2 = 0.55$ m (the outer surface of the container) is

$$T(r_2)=\frac{\dot{q}_1}{h}\left(\frac{r_1}{r_2}\right)^2+T_\infty=\frac{60000\ \text{W/m}^2}{1000\ \text{W/m}^2\cdot\text{K}}\left(\frac{0.5\ \text{m}}{0.55\ \text{m}}\right)^2+23°\text{C}=\mathbf{72.6°C}$$

The outer surface temperature of the container is above the safe temperature of 50°C.

Discussion To prevent thermal burn, the container's outer surface should be covered with insulation.

2-81 A spherical container is subjected to uniform heat flux on the outer surface and specified temperature on the inner surface. The mathematical formulation, the variation of temperature in the pipe, and the outer surface temperature, and the maximum rate of hot water supply are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since there is no change with time and there is thermal symmetry about the mid point. **2** Thermal conductivity is constant. **3** There is no heat generation in the container.

Properties The thermal conductivity is given to be k = 1.5 W/m·°C. The specific heat of water at the average temperature of (100+20)/2 = 60°C is 4.185 kJ/kg·°C (Table A-9).

Analysis (*a*) Noting that the 90% of the 500 W generated by the strip heater is transferred to the container, the heat flux through the outer surface is determined to be

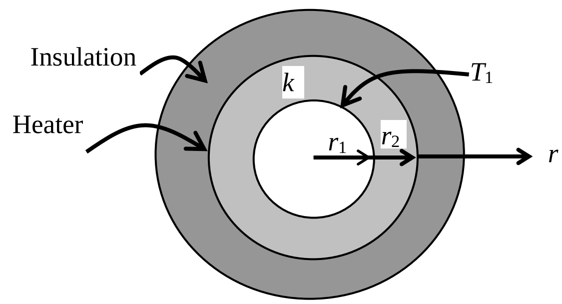
$$\dot{q}_s = \frac{\dot{Q}_s}{A_2} = \frac{\dot{Q}_s}{4\pi r_2^2} = \frac{0.90\times 500\ \text{W}}{4\pi(0.41\text{m})^2} = 213.0\ \text{W/m}^2$$

Noting that heat transfer is one-dimensional in the radial r direction and heat flux is in the negative r direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

and $$T(r_1) = T_1 = 100°\text{C}$$

$$k\frac{dT(r_2)}{dr} = \dot{q}_s$$



(*b*) Integrating the differential equation once with respect to r gives

$$r^2\frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by r^2 and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T(r) = -\frac{C_1}{r} + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_2$: $$k\frac{C_1}{r_2^2} = \dot{q}_s \rightarrow C_1 = \frac{\dot{q}_s r_2^2}{k}$$

$r = r_1$: $$T(r_1) = T_1 = -\frac{C_1}{r_1} + C_2 \rightarrow C_2 = T_1 + \frac{C_1}{r_1} = T_1 + \frac{\dot{q}_s r_2^2}{kr_1}$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = -\frac{C_1}{r} + C_2 = -\frac{C_1}{r} + T_1 + \frac{C_1}{r_1} = T_1 + \left(\frac{1}{r_1} - \frac{1}{r}\right)C_1 = T_1 + \left(\frac{1}{r_1} - \frac{1}{r}\right)\frac{\dot{q}_s r_2^2}{k}$$

$$= 100°\text{C} + \left(\frac{1}{0.40\ \text{m}} - \frac{1}{r}\right)\frac{(213\ \text{W/m}^2)(0.41\ \text{m})^2}{1.5\ \text{W/m}\cdot°\text{C}} = 100 + 23.87\left(2.5 - \frac{1}{r}\right)$$

(*c*) The outer surface temperature is determined by direct substitution to be

Outer surface ($r = r_2$): $$T(r_2) = 100 + 23.87\left(2.5 - \frac{1}{r_2}\right) = 100 + 23.87\left(2.5 - \frac{1}{0.41}\right) = \mathbf{101.5°C}$$

Noting that the maximum rate of heat supply to the water is $0.9\times 500\ \text{W} = 450\ \text{W}$, water can be heated from 20 to 100°C at a rate of

$$\dot{Q} = \dot{m}c_p\Delta T \rightarrow \dot{m} = \frac{\dot{Q}}{c_p\Delta T} = \frac{0.450\ \text{kJ/s}}{(4.185\ \text{kJ/kg}\cdot°\text{C})(100-20)°\text{C}} = 0.00134\ \text{kg/s} = \mathbf{4.84\ kg/h}$$

Heat Generation in a Solid

2-82C Heat generation in a solid is simply conversion of some form of energy into sensible heat energy. Some examples of heat generations are resistance heating in wires, exothermic chemical reactions in a solid, and nuclear reactions in nuclear fuel rods.

2-83C No. Heat generation in a solid is simply the conversion of some form of energy into sensible heat energy. For example resistance heating in wires is conversion of electrical energy to heat.

2-84C The cylinder will have a higher center temperature since the cylinder has less surface area to lose heat from per unit volume than the sphere.

2-85C The rate of heat generation inside an iron becomes equal to the rate of heat loss from the iron when steady operating conditions are reached and the temperature of the iron stabilizes.

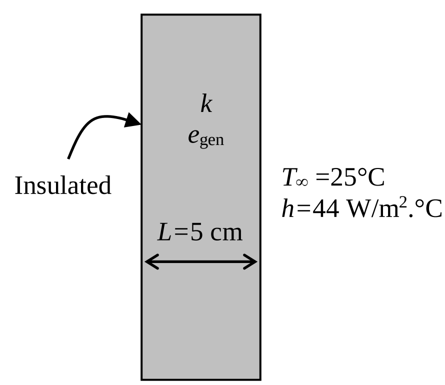
2-86C No, it is not possible since the highest temperature in the plate will occur at its center, and heat cannot flow "uphill."

2-87 Heat is generated uniformly in a large brass plate. One side of the plate is insulated while the other side is subjected to convection. The location and values of the highest and the lowest temperatures in the plate are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness, and there is thermal symmetry about the center plane **3** Thermal conductivity is constant. **4** Heat generation is uniform.

Properties The thermal conductivity is given to be k =111 W/m·°C.

Analysis This insulated plate whose thickness is L is equivalent to one-half of an uninsulated plate whose thickness is 2L since the midplane of the uninsulated plate can be treated as insulated surface. The highest temperature will occur at the insulated surface while the lowest temperature will occur at the surface which is exposed to the environment. Note that L in the following relations is the full thickness of the given plate since the insulated side represents the center surface of a plate whose thickness is doubled. The desired values are determined directly from



$$T_s = T_\infty + \frac{\dot{e}_{\text{gen}} L}{h} = 25°\text{C} + \frac{(2\times10^5 \text{ W/m}^3)(0.05 \text{ m})}{44 \text{ W/m}^2\cdot°\text{C}} = \mathbf{252.3°C}$$

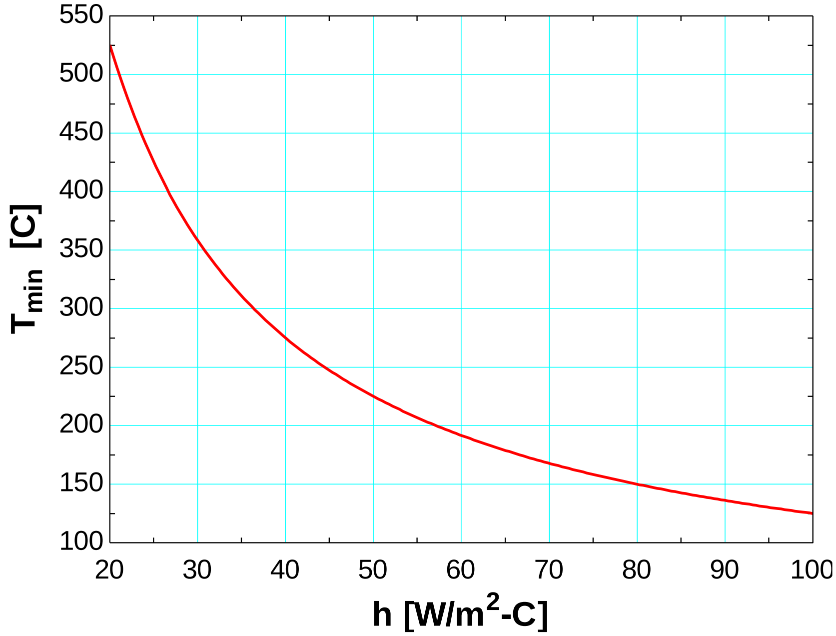
$$T_o = T_s + \frac{\dot{e}_{\text{gen}} L^2}{2k} = 252.3°\text{C} + \frac{(2\times10^5 \text{ W/m}^3)(0.05 \text{ m})^2}{2(111 \text{ W/m}\cdot°\text{C})} = \mathbf{254.6°C}$$

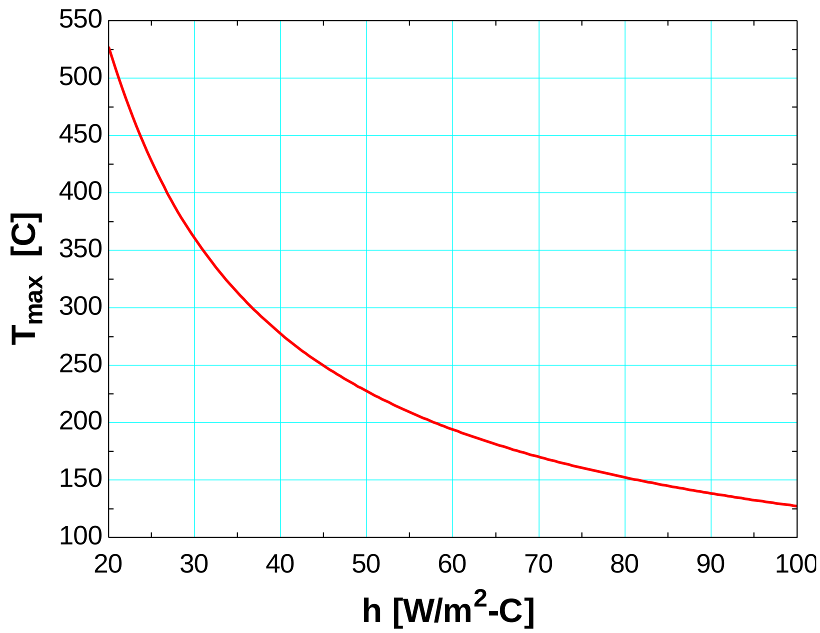
2-88 Prob. 2-87 is reconsidered. The effect of the heat transfer coefficient on the highest and lowest temperatures in the plate is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.05 [m]
k=111 [W/m-C]
g_dot=2E5 [W/m^3]
T_infinity=25 [C]
h=44 [W/m^2-C]
"ANALYSIS"
T_min=T_infinity+(g_dot*L)/h
T_max=T_min+(g_dot*L^2)/(2*k)
```

h $[W/m^2.C]$	T_{min} [C]	T_{max} [C]
20	525	527.3
25	425	427.3
30	358.3	360.6
35	310.7	313
40	275	277.3
45	247.2	249.5
50	225	227.3
55	206.8	209.1
60	191.7	193.9
65	178.8	181.1
70	167.9	170.1
75	158.3	160.6
80	150	152.3
85	142.6	144.9
90	136.1	138.4
95	130.3	132.5
100	125	127.3

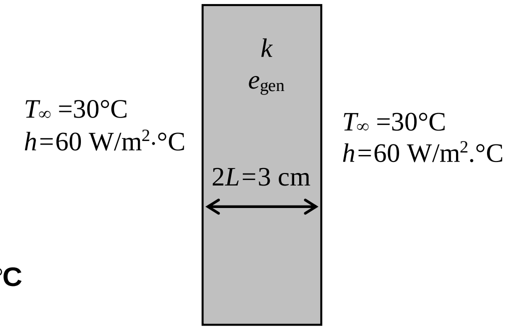




2-89 Both sides of a large stainless steel plate in which heat is generated uniformly are exposed to convection with the environment. The location and values of the highest and the lowest temperatures in the plate are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness, and there is thermal symmetry about the center plane **3** Thermal conductivity is constant. **4** Heat generation is uniform.

Properties The thermal conductivity is given to be k =15.1 W/m·°C.



Analysis The lowest temperature will occur at surfaces of plate while the highest temperature will occur at the midplane. Their values are determined directly from

$$T_s = T_\infty + \frac{\dot{e}_{\text{gen}} L}{h} = 30°\text{C} + \frac{(5\times 10^5 \text{ W/m}^3)(0.015 \text{ m})}{60 \text{ W/m}^2\cdot°\text{C}} = \mathbf{155°C}$$

$$T_o = T_s + \frac{\dot{e}_{\text{gen}} L^2}{2k} = 155°\text{C} + \frac{(5\times 10^5 \text{ W/m}^3)(0.015 \text{ m})^2}{2(15.1 \text{ W/m}\cdot°\text{C})} = \mathbf{158.7°C}$$

2-90 Heat is generated in a large plane wall whose one side is insulated while the other side is subjected to convection. The mathematical formulation, the variation of temperature in the wall, the relation for the surface temperature, and the relation for the maximum temperature rise in the plate are to be determined for steady one-dimensional heat transfer.

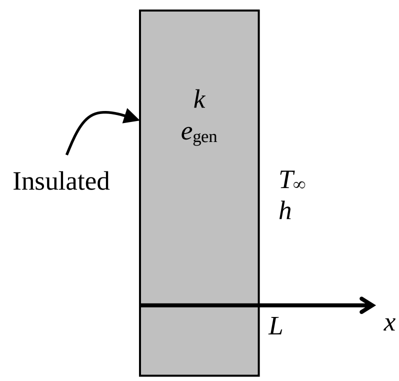
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since the wall is large relative to its thickness. **3** Thermal conductivity is constant. **4** Heat generation is uniform.

Analysis (*a*) Noting that heat transfer is steady and one-dimensional in *x* direction, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2}+\frac{\dot{e}_{\text{gen}}}{k}=0$$

and $\quad \dfrac{dT(0)}{dx}=0 \quad$ (insulated surface at x = 0)

$$-k\frac{dT(L)}{dx}=h[T(L)-T_\infty]$$



(b) Rearranging the differential equation and integrating,

$$\frac{d^2T}{dx^2}=-\frac{\dot{e}_{\text{gen}}}{k} \quad\rightarrow\quad \frac{dT}{dx}=-\frac{\dot{e}_{\text{gen}}}{k}x+C_1$$

Integrating one more time,

$$T(x)=\frac{-\dot{e}_{\text{gen}}x^2}{2k}+C_1x+C_2 \qquad (1)$$

Applying the boundary conditions:

B.C. at x = 0: $\quad \dfrac{dT(0)}{dx}=0\rightarrow\dfrac{-\dot{e}_{\text{gen}}}{k}(0)+C_1=0 \;\rightarrow\; C_1=0$

B. C. at $x = L$:

$$-k\left(\frac{-\dot{e}_{\text{gen}}}{k}L\right)=h\left(\frac{-\dot{e}_{\text{gen}}L^2}{2k}+C_2-T_\infty\right)$$

$$\dot{e}_{\text{gen}}L=\frac{-h\dot{e}_{\text{gen}}L^2}{2k}-hT_\infty+C_2\rightarrow C_2=\dot{e}_{\text{gen}}L+\frac{h\dot{e}_{\text{gen}}L^2}{2k}+hT_\infty$$

Dividing by h: $\quad C_2=\dfrac{\dot{e}_{\text{gen}}L}{h}+\dfrac{\dot{e}_{\text{gen}}L^2}{2k}+T_\infty$

Substituting the C_1 and C_2 relations into Eq. (1) and rearranging give

$$T(x)=\frac{-\dot{e}_{\text{gen}}x^2}{2k}+\frac{\dot{e}_{\text{gen}}L}{h}+\frac{\dot{e}_{\text{gen}}L^2}{2k}+T_\infty=\frac{\dot{e}_{\text{gen}}}{2k}(L^2-x^2)+\frac{\dot{e}_{\text{gen}}L}{h}+T_\infty$$

which is the desired solution for the temperature distribution in the wall as a function of *x*.

(*c*) The temperatures at two surfaces and the temperature difference between these surfaces are

$$T(0)=\frac{\dot{e}_{\text{gen}}L^2}{2k}+\frac{\dot{e}_{\text{gen}}L}{h}+T_\infty$$

$$T(L)=\frac{\dot{e}_{\text{gen}}L}{h}+T_\infty$$

$$\Delta T_{\max}=T(0)-T(L)=\frac{\dot{e}_{\text{gen}}L^2}{2k}$$

Discussion These relations are obtained without using differential equations in the text (see Eqs. 2-67 and 2-73).

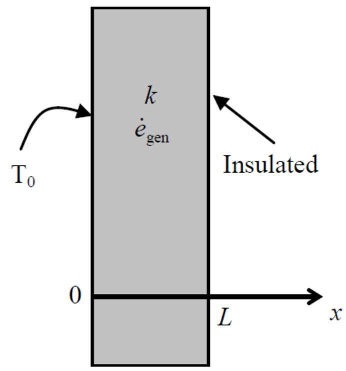
2-91E Heat is generated in a large plane wall whose one side is insulated while the other side is maintained at a specified temperature. The mathematical formulation, the variation of temperature in the wall, and the highest temperature in the wall are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat transfer is steady. **2** Heat transfer is one-dimensional, and there is thermal symmetry about the center plane. **3** Thermal conductivity is constant. **4** Heat generation varies with location in the x direction.

Properties The thermal conductivity is given to be k = 5 Btu/h·ft·°F.

Analysis (a) Noting that heat transfer is steady and one-dimensional in x direction, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2}+\frac{\dot{e}_{\text{gen}}(x)}{k}=0$$



where $\dot{e}_{gen}=ax^2$

$$\frac{d^2T}{dx^2}=-\frac{\dot{e}_{\text{gen}}(x)}{k}=-\frac{a}{k}x^2$$

The boundary conditions for this problem are:

$T(0)=T_0$ (specified surface temperature at x = 0)

$\frac{dT(L)}{dx}=0$ (insulated surface at x = L)

(b) Rearranging the differential equation and integrating,

$$\frac{d^2T}{dx^2}=-\frac{a}{k}x^2 \rightarrow \frac{dT}{dx}=-\frac{1}{3}\frac{a}{k}x^3+C_1$$

Integrating one more time,

$$T(x)=-\frac{1}{12}\frac{a}{k}x^4+C_1x+C_2 \qquad (1)$$

Applying the boundary conditions:

B.C. at x = 0: $T(0)=T_0=C_2$

B.C. at $x = L$: $\frac{dT(L)}{dx}=-\frac{1}{3}\frac{a}{k}L^3+C_1=0 \rightarrow C_1=\frac{aL^3}{3k}$

Substituting the C_1 and C_2 relations into Eq. (1) and rearranging gives

$$T(x)=-\frac{1}{12}\frac{a}{k}x^4+\frac{aL^3}{3k}x+T_0 \qquad (2)$$

(c) The highest (maximum) temperature occurs at the insulate surface (x = L) and is determined by substituting the given quantities into Eq. (2), the result is

$$T(L)=T_{\max}=-\frac{1}{12}\frac{a}{k}L^4+\frac{aL^3}{3k}L+T_0=\frac{aL^4}{4k}+T_0$$

$$=\frac{(1200\ \text{Btu/h}\cdot\text{ft}^5)(1\,\text{ft}^4)}{4(5\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})}+700{}^\circ\text{F}$$

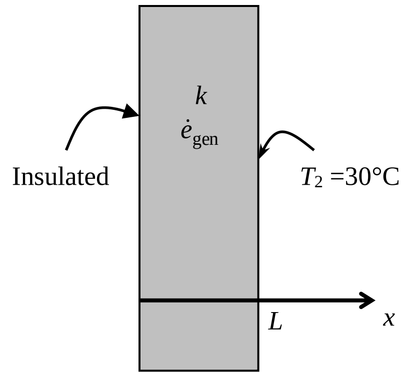
$$=\mathbf{760\ {}^\circ F}$$

2-92 Heat is generated in a large plane wall whose one side is insulated while the other side is maintained at a specified temperature. The mathematical formulation, the variation of temperature in the wall, and the temperature of the insulated surface are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since the wall is large relative to its thickness, and there is thermal symmetry about the center plane. **3** Thermal conductivity is constant. **4** Heat generation varies with location in the x direction.

Properties The thermal conductivity is given to be k = 30 W/m·°C.

Analysis (a) Noting that heat transfer is steady and one-dimensional in x direction, the mathematical formulation of this problem can be expressed as



$$\frac{d^2T}{dx^2}+\frac{\dot{e}_{\text{gen}}(x)}{k}=0$$

where $\dot{e}_{\text{gen}}=\dot{e}_0 e^{-0.5x/L}$ and $\dot{e}_0 = 8\times10^6\ \text{W/m}^3$

and $\dfrac{dT(0)}{dx}=0$ (insulated surface at x = 0)

$T(L)=T_2=30°\text{C}$ (specified surface temperature)

(b) Rearranging the differential equation and integrating,

$$\frac{d^2T}{dx^2}=-\frac{\dot{e}_0}{k}e^{-0.5x/L} \;\rightarrow\; \frac{dT}{dx}=-\frac{\dot{e}_0}{k}\frac{e^{-0.5x/L}}{-0.5/L}+C_1 \rightarrow \frac{dT}{dx}=\frac{2\dot{e}_0L}{k}e^{-0.5x/L}+C_1$$

Integrating one more time,

$$T(x)=\frac{2\dot{e}_0L}{k}\frac{e^{-0.5x/L}}{-0.5/L}+C_1x+C_2 \;\rightarrow\; T(x)=-\frac{4\dot{e}_0L^2}{k}e^{-0.5x/L}+C_1x+C_2 \qquad (1)$$

Applying the boundary conditions:

B.C. at x = 0: $$\frac{dT(0)}{dx}=\frac{2\dot{e}_0L}{k}e^{-0.5\times0/L}+C_1 \;\rightarrow\; 0=\frac{2\dot{e}_0L}{k}+C_1 \;\rightarrow\; C_1=-\frac{2\dot{e}_0L}{k}$$

B. C. at $x = L$: $$T(L)=T_2=-\frac{4\dot{e}_0L^2}{k}e^{-0.5L/L}+C_1L+C_2 \;\rightarrow\; C_2=T_2+\frac{4\dot{e}_0L^2}{k}e^{-0.5}+\frac{2\dot{e}_0L^2}{k}$$

Substituting the C_1 and C_2 relations into Eq. (1) and rearranging give

$$T(x)=T_2+\frac{\dot{e}_0L^2}{k}[4(e^{-0.5}-e^{-0.5x/L})+2(1-x/L)]$$

which is the desired solution for the temperature distribution in the wall as a function of x.

(c) The temperature at the insulate surface (x = 0) is determined by substituting the known quantities to be

$$\begin{aligned}T(0)&=T_2+\frac{\dot{e}_0L^2}{k}[4(e^{-0.5}-e^{0})+(2-0/L)]\\ &=30°\text{C}+\frac{(8\times10^6\ \text{W/m}^3)(0.05\ \text{m})^2}{(30\ \text{W/m}\cdot°\text{C})}[4(e^{-0.5}-1)+(2-0)]\\ &=\mathbf{314°C}\end{aligned}$$

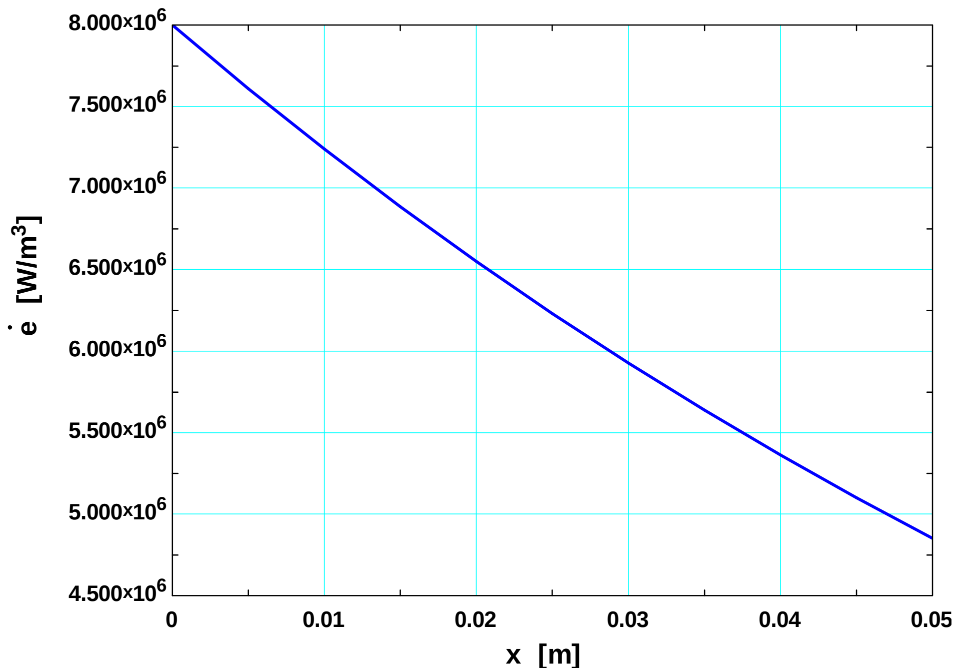
Therefore, there is a temperature difference of almost 300°C between the two sides of the plate.

2-93 Prob. 2-92 is reconsidered. The heat generation as a function of the distance is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

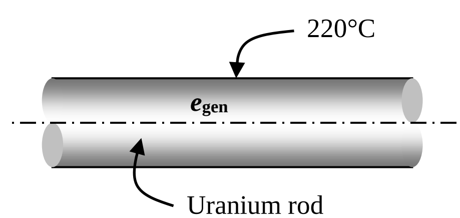
```
"GIVEN"
L=0.05 [m]
T_s=30 [C]
k=30 [W/m-C]
e_dot_0=8E6 [W/m^3]

"ANALYSIS"
e_dot=e_dot_0*exp((-0.5*x)/L) "Heat generation as a function of x"
```

x [m]	e [W/m^3]
0	8.000E+06
0.005	7.610E+06
0.01	7.239E+06
0.015	6.886E+06
0.02	6.550E+06
0.025	6.230E+06
0.03	5.927E+06
0.035	5.638E+06
0.04	5.363E+06
0.045	5.101E+06
0.05	4.852E+06



2-94 A nuclear fuel rod with a specified surface temperature is used as the fuel in a nuclear reactor. The center temperature of the rod is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the rod is uniform.



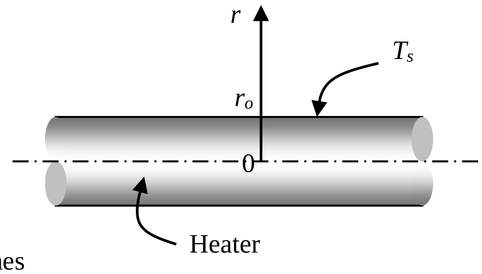
Properties The thermal conductivity is given to be k = 29.5 W/m·°C.

Analysis The center temperature of the rod is determined from

$$T_o = T_s + \frac{\dot{e}_{\text{gen}} r_o^2}{4k} = 220^\circ\text{C} + \frac{(4\times10^7\ \text{W/m}^3)(0.005\ \text{m})^2}{4(29.5\ \text{W/m.}^\circ\text{C})} = \mathbf{228^\circ C}$$

2-95E Heat is generated uniformly in a resistance heater wire. The temperature difference between the center and the surface of the wire is to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the heater is uniform.



Properties The thermal conductivity is given to be k = 5.8 Btu/h·ft·°F.

Analysis The resistance heater converts electric energy into heat at a rate of 3 kW. The rate of heat generation per unit length of the wire is

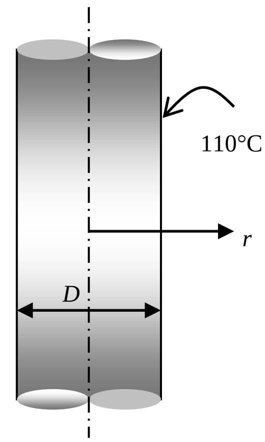
$$\dot{e}_{\text{gen}} = \frac{\dot{E}_{\text{gen}}}{V_{\text{wire}}} = \frac{\dot{E}_{gen}}{\pi r_o^2 L} = \frac{(3\times 3412.14\text{ Btu/h})}{\pi(0.04/12\text{ ft})^2(1\text{ ft})} = 2.933\times 10^8\ \text{Btu/h}\cdot\text{ft}^3$$

Then the temperature difference between the centerline and the surface becomes

$$\Delta T_{\max} = \frac{\dot{e}_{\text{gen}} r_o^2}{4k} = \frac{(2.933\times 10^8\ \text{Btu/h}\cdot\text{ft}^3)(0.04/12\text{ ft})^2}{4(5.8\text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})} = \mathbf{140.5^\circ F}$$

2-96 A 2-kW resistance heater wire with a specified surface temperature is used to boil water. The center temperature of the wire is to be determined.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the heater is uniform.



Properties The thermal conductivity is given to be k = 20 W/m·°C.

Analysis The resistance heater converts electric energy into heat at a rate of 2 kW. The rate of heat generation per unit volume of the wire is

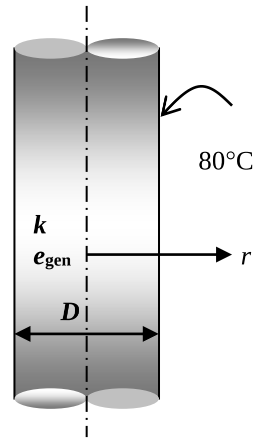
$$\dot{e}_{\text{gen}} = \frac{\dot{E}_{\text{gen}}}{V_{\text{wire}}} = \frac{\dot{E}_{\text{gen}}}{\pi r_o^2 L} = \frac{2000\text{ W}}{\pi(0.002\text{ m})^2(0.9\text{ m})} = 1.768\times 10^8\ \text{W/m}^3$$

The center temperature of the wire is then determined from Eq. 2-71 to be

$$T_o = T_s + \frac{\dot{e}_{\text{gen}} r_o^2}{4k} = 110^\circ\text{C} + \frac{(1.768\times 10^8\ \text{W/m}^3)(0.002\text{ m})^2}{4(20\text{ W/m.}^\circ\text{C})} = \mathbf{118.8^\circ C}$$

2-97 Heat is generated in a long solid cylinder with a specified surface temperature. The variation of temperature in the cylinder is given by

$$T(r) = \frac{\dot{e}_{\text{gen}} r_o^2}{k}\left[1-\left(\frac{r}{r_o}\right)^2\right] + T_s$$



(*a*) Heat conduction is steady since there is no time *t* variable involved.

(*b*) Heat conduction is a one-dimensional.

(*c*) Using Eq. (1), the heat flux on the surface of the cylinder at $r = r_o$ is determined from its definition to be

$$\dot{q}_s = -k\frac{dT(r_o)}{dr} = -k\left[\frac{\dot{e}_{\text{gen}} r_o^2}{k}\left(-\frac{2r}{r_o^2}\right)\right]_{r=r_0}$$

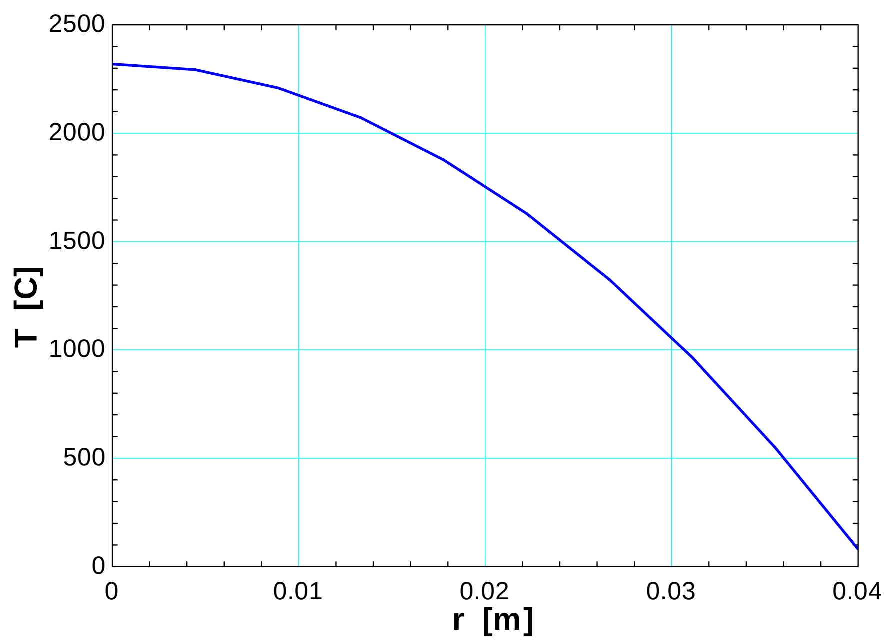
$$= -k\left[\frac{\dot{e}_{\text{gen}} r_o^2}{k}\left(-\frac{2r_o}{r_o^2}\right)\right] = 2\dot{e}_{\text{gen}} r_o = 2(35\ \text{W/cm}^3)(4\ \text{cm}) = \mathbf{280\ W/cm^2}$$

2-98  Prob. 2-97 is reconsidered. The temperature as a function of the radius is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
r_0=0.04 [m]
k=25 [W/m-C]
e_dot_gen=35E+6 [W/m^3]
T_s=80 [C]

"ANALYSIS"
T=(e_dot_gen*r_0^2)/k*(1-(r/r_0)^2)+T_s "Variation of temperature"
```

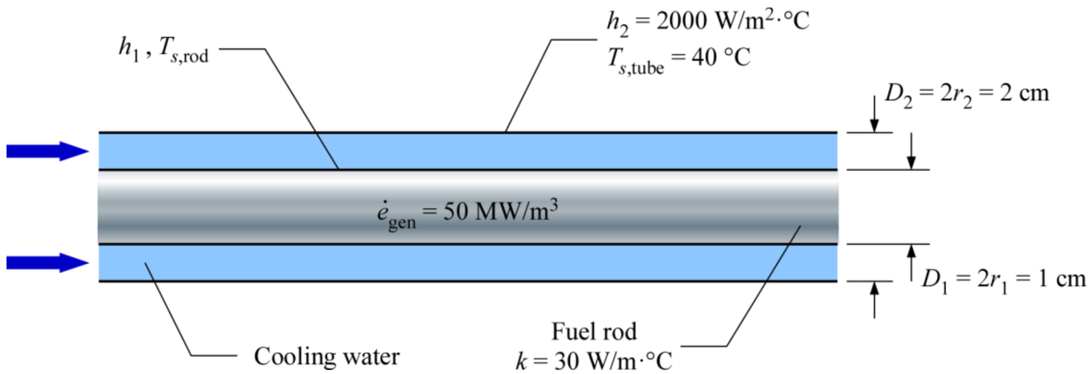
r [m]	**T [C]**
0	2320
0.004444	2292
0.008889	2209
0.01333	2071
0.01778	1878
0.02222	1629
0.02667	1324
0.03111	964.9
0.03556	550.1
0.04	80



2-99 A cylindrical nuclear fuel rod is cooled by water flowing through its encased concentric tube. The average temperature of the cooling water is to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat generation in the fuel rod is uniform.

Properties The thermal conductivity is given to be 30 W/m · °C.



Analysis The rate of heat transfer by convection at the fuel rod surface is equal to that of the concentric tube surface:

$$h_1 A_{s,1}(T_{s,\text{rod}} - T_\infty) = h_2 A_{s,2}(T_\infty - T_{s,\text{tube}})$$

$$h_1(2\pi\, r_1 L)(T_{s,\text{rod}} - T_\infty) = h_2(2\pi\, r_2 L)(T_\infty - T_{s,\text{tube}})$$

$$T_{s,\text{rod}} = \frac{h_2 r_2}{h_1 r_1}(T_\infty - T_{s,\text{tube}}) + T_\infty \qquad (a)$$

The average temperature of the cooling water can be determined by applying Eq. 2-68:

$$T_{s,\text{rod}} = T_\infty + \frac{\dot{e}_{\text{gen}} r_1}{2h_1} \qquad (b)$$

Substituting Eq. (a) into Eq. (b) and solving for the average temperature of the cooling water gives

$$\frac{h_2 r_2}{h_1 r_1}(T_\infty - T_{s,\text{tube}}) + T_\infty = T_\infty + \frac{\dot{e}_{\text{gen}} r_1}{2h_1}$$

$$T_\infty = \frac{r_1}{r_2}\frac{\dot{e}_{\text{gen}} r_1}{2h_2} + T_{s,\text{tube}}$$

$$= \frac{0.005\text{ m}}{0.010\text{ m}}\left[\frac{(50\times10^6\text{ W/m}^3)(0.005\text{ m})}{2(2000\text{ W/m}^2\cdot{}^\circ\text{C})}\right] + 40\,^\circ\text{C}$$

$$= \mathbf{71.3\,^\circ C}$$

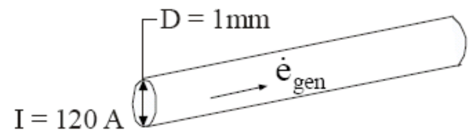
Discussion The given information is not sufficient for one to determine the fuel rod surface temperature. The convection heat transfer coefficient for the fuel rod surface (h_1) or the centerline temperature of the fuel rod (T_0) is needed to determine the fuel rod surface temperature.

2-100 The heat generation and the maximum temperature rise in a solid stainless steel wire.

Assumptions **1** Heat transfer is steady since there is no change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the heater is uniform.

Properties The thermal conductivity is given to be k = 14 W/m·K.

Analysis (a) The heat generation per unit volume of the wire is



$$\dot{e}_{gen} = \frac{\dot{E}_{gen,electric}}{V_{wire}} = \frac{I^2 R_e}{\pi r_o^2 L}$$

With electrical resistance defined as

$$R_e = \frac{\rho L}{A} \quad (\Omega)$$

where ρ = electrical resistivity (Ω·m), L = wire length (m), A = wire cross-sectional area $\pi D^2/4$ (m^2)

Combining equations for $\dot{e}_{gen}$ and R_e, we have

$$\dot{e}_{gen} = \frac{I^2 \rho}{A^2} = \frac{I^2 \rho}{(\pi D^2/4)^2} = \frac{16 I^2 \rho}{\pi^2 D^4}$$

$$\dot{e}_{gen} = \frac{16\,(120\text{A})^2\,(45\times10^{-8}\ \Omega\cdot\text{m})}{\pi^2\,(0.001\text{m})^4} = \mathbf{1.05\times10^{10}\ W/m^3}$$

(b) The maximum temperature rise in the solid stainless steel wire is obtained from

$$T_o - T_s = \Delta T_{max,cylinder} = \frac{\dot{e}_{gen}\, r_o^2}{4k} \quad (\text{W/m}^3)$$

$$\Delta T_{\max} = \frac{(1.05\times10^{10}\ \text{W/m}^3)(0.0005m)^2}{4\,(14\ \text{W/m}\cdot\text{K})} = \mathbf{47^\circ C}$$

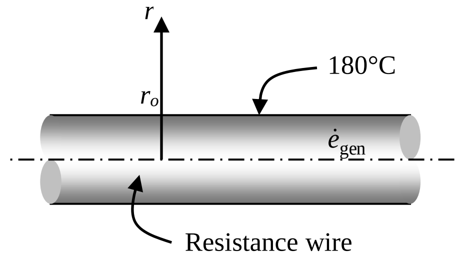
Discussion The maximum temperature rise in the wire can be reduced by increasing the convective heat transfer coefficient and thus reducing the surface temperature.

2-101 A long homogeneous resistance heater wire with specified surface temperature is used to heat the air. The temperature of the wire 3.5 mm from the center is to be determined in steady operation.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the wire is uniform.

Properties The thermal conductivity is given to be k = 8 W/m·°C.

Analysis Noting that heat transfer is steady and one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as



$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{e}_{\text{gen}}}{k}=0$$

and $T(r_o)=T_s=180°\text{C}$ (specified surface temperature)

$$\frac{dT(0)}{dr}=0 \quad \text{(thermal symmetry about the centerline)}$$

Multiplying both sides of the differential equation by r and rearranging gives

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{\dot{e}_{\text{gen}}}{k}r$$

Integrating with respect to r gives

$$r\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{k}\frac{r^2}{2}+C_1 \qquad (a)$$

It is convenient at this point to apply the boundary condition at the center since it is related to the first derivative of the temperature. It yields

B.C. at $r = 0$: $\quad 0\times\frac{dT(0)}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k}\times 0+C_1 \quad\rightarrow\quad C_1=0$

Dividing both sides of Eq. (a) by r to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k}r$$

and $\quad T(r)=-\frac{\dot{e}_{\text{gen}}}{4k}r^2+C_2 \qquad (b)$

Applying the other boundary condition at $r=r_o$,

B. C. at $r=r_o$: $\quad T_s=-\frac{\dot{e}_{\text{gen}}}{4k}r_o^2+C_2 \quad\rightarrow\quad C_2=T_s+\frac{\dot{e}_{\text{gen}}}{4k}r_o^2$

Substituting this C_2 relation into Eq. (b) and rearranging give

$$T(r)=T_s+\frac{\dot{e}_{\text{gen}}}{4k}(r_o^2-r^2)$$

which is the desired solution for the temperature distribution in the wire as a function of r. The temperature 3.5 mm from the center line (r = 0.0035 m) is determined by substituting the known quantities to be

$$T(0.0035\,\text{m})=T_s+\frac{\dot{e}_{\text{gen}}}{4k}(r_o^2-r^2)=180°\text{C}+\frac{5\times10^7\ \text{W/m}^3}{4\times(8\ \text{W/m}\cdot°\text{C})}[(0.005\,\text{m})^2-(0.0035\,\text{m})^2]=\mathbf{200°C}$$

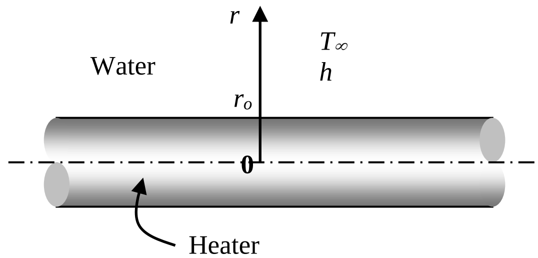
Thus the temperature at that location will be about 20°C above the temperature of the outer surface of the wire.

2-102 A long homogeneous resistance heater wire with specified convection conditions at the surface is used to boil water. The mathematical formulation, the variation of temperature in the wire, and the temperature at the centerline of the wire are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the wire is uniform.

Properties The thermal conductivity is given to be k = 15.2 W/m·K.

Analysis Noting that heat transfer is steady and one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as



$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{e}_{\text{gen}}}{k}=0$$

and $$-k\frac{dT(r_o)}{dr}=h[T(r_o)-T_\infty] \quad \text{(convection at the outer surface)}$$

$$\frac{dT(0)}{dr}=0 \quad \text{(thermal symmetry about the centerline)}$$

Multiplying both sides of the differential equation by r and rearranging gives

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{\dot{e}_{\text{gen}}}{k}r$$

Integrating with respect to r gives

$$r\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{k}\frac{r^2}{2}+C_1 \qquad (a)$$

It is convenient at this point to apply the second boundary condition since it is related to the first derivative of the temperature by replacing all occurrences of r and dT/dr in the equation above by zero. It yields

B.C. at $r = 0$: $$0\times\frac{dT(0)}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k}\times 0+C_1 \quad \rightarrow \quad C_1=0$$

Dividing both sides of Eq. (a) by r to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k}r$$

and $$T(r)=-\frac{\dot{e}_{\text{gen}}}{4k}r^2+C_2 \qquad (b)$$

Applying the second boundary condition at $r=r_o$,

B. C. at $r=r_o$: $$k\frac{\dot{e}_{\text{gen}}r_o}{2k}=h\left(-\frac{\dot{e}_{\text{gen}}}{4k}r_o^2+C_2-T_\infty\right) \quad \rightarrow \quad C_2=T_\infty+\frac{\dot{e}_{\text{gen}}r_o}{2h}+\frac{\dot{e}_{\text{gen}}}{4k}r_o^2$$

Substituting this C_2 relation into Eq. (b) and rearranging give

$$T(r)=T_\infty+\frac{\dot{e}_{\text{gen}}}{4k}(r_o^2-r^2)+\frac{\dot{e}_{\text{gen}}r_o}{2h}$$

which is the desired solution for the temperature distribution in the wire as a function of r. Then the temperature at the center line (r = 0) is determined by substituting the known quantities to be

$$T(0)=T_\infty+\frac{\dot{e}_{\text{gen}}}{4k}r_o^2+\frac{\dot{e}_{\text{gen}}r_o}{2h}$$
$$=100°\text{C}+\frac{(16.4\times10^6\ \text{W/m}^3)(0.006\ \text{m})^2}{4\times(15.2\ \text{W/m}\cdot\text{K})}+\frac{(16.4\times10^6\ \text{W/m}^3)(0.006\ \text{m})}{2\times(3200\ \text{W/m}^2\cdot\text{K})}=\mathbf{125°C}$$

Thus the centerline temperature will be 25°C above the temperature of the surface of the wire.

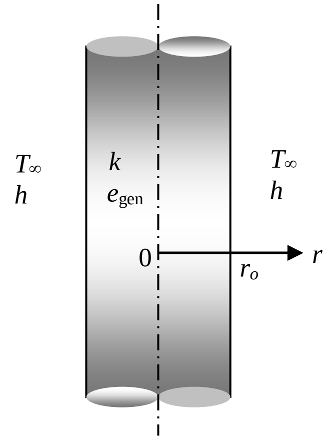
2-103 A long resistance heater wire is subjected to convection at its outer surface. The surface temperature of the wire is to be determined using the applicable relations directly and by solving the applicable differential equation.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no change in the axial direction. **3** Thermal conductivity is constant. **4** Heat generation in the wire is uniform.

Properties The thermal conductivity is given to be k = 15.1 W/m·°C.

Analysis (a) The heat generation per unit volume of the wire is

$$\dot{e}_{\text{gen}} = \frac{\dot{E}_{\text{gen}}}{V_{\text{wire}}} = \frac{\dot{E}_{\text{gen}}}{\pi r_o^{\,2} L} = \frac{2000\text{ W}}{\pi(0.001\,\text{m})^2(6\,\text{m})} = 1.061\times10^8\ \text{W/m}^3$$



The surface temperature of the wire is then (Eq. 2-68)

$$T_s = T_\infty + \frac{\dot{e}_{\text{gen}} r_o}{2h} = 20°\text{C} + \frac{(1.061\times10^8\ \text{W/m}^3)(0.001\,\text{m})}{2(175\ \text{W/m}^2\cdot°\text{C})} = \mathbf{323°C}$$

(b) The mathematical formulation of this problem can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{e}_{\text{gen}}}{k} = 0$$

and $\quad -k\dfrac{dT(r_o)}{dr} = h[T(r_o) - T_\infty]$ (convection at the outer surface)

$$\frac{dT(0)}{dr} = 0 \quad \text{(thermal symmetry about the centerline)}$$

Multiplying both sides of the differential equation by r and integrating gives

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{e}_{\text{gen}}}{k} r \quad \rightarrow r\frac{dT}{dr} = -\frac{\dot{e}_{\text{gen}}}{k}\frac{r^2}{2} + C_1 \qquad (a)$$

Applying the boundary condition at the center line,

B.C. at $r = 0$: $\quad 0\times\dfrac{dT(0)}{dr} = -\dfrac{\dot{e}_{\text{gen}}}{2k}\times 0 + C_1 \quad \rightarrow \quad C_1 = 0$

Dividing both sides of Eq. (a) by r to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr} = -\frac{\dot{e}_{\text{gen}}}{2k} r \qquad \rightarrow \qquad T(r) = -\frac{\dot{e}_{\text{gen}}}{4k} r^2 + C_2 \qquad (b)$$

Applying the boundary condition at $r = r_o$,

B. C. at $r = r_o$: $\quad -k\dfrac{\dot{e}_{\text{gen}} r_o}{2k} = h\left(-\dfrac{\dot{e}_{\text{gen}}}{4k} r_o^2 + C_2 - T_\infty\right) \quad \rightarrow \quad C_2 = T_\infty + \dfrac{\dot{e}_{\text{gen}} r_o}{2h} + \dfrac{\dot{e}_{\text{gen}}}{4k} r_o^2$

Substituting this C_2 relation into Eq. (b) and rearranging give

$$T(r) = T_\infty + \frac{\dot{e}_{\text{gen}}}{4k}(r_o^2 - r^2) + \frac{\dot{e}_{\text{gen}} r_o}{2h}$$

which is the temperature distribution in the wire as a function of r. Then the temperature of the wire at the surface ($r = r_o$) is determined by substituting the known quantities to be

$$T(r_0) = T_\infty + \frac{\dot{e}_{\text{gen}}}{4k}(r_o^2 - r_o^2) + \frac{\dot{e}_{\text{gen}} r_0}{2h} = T_\infty + \frac{\dot{e}_{\text{gen}} r_o}{2h} = 20°\text{C} + \frac{(1.061\times10^8\ \text{W/m}^3)(0.001\,\text{m})}{2(175\ \text{W/m}^2\cdot°\text{C})} = \mathbf{323°C}$$

Note that both approaches give the same result.

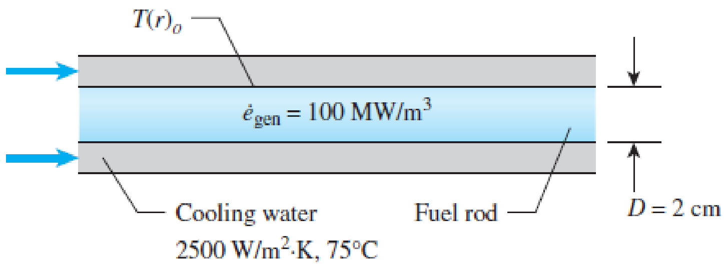
2-104 A cylindrical fuel rod is cooled by water flowing through its encased concentric tube while generating a uniform heat. The variation of temperature in the fuel rod and the center and surface temperatures are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat transfer is steady and one-dimensional with thermal symmetry about the center line. **2** Thermal conductivity is constant. **3** The rod surface at $r = r_o$ is subjected convection. **4** Heat generation in the rod is uniform.

Properties The thermal conductivity is given to be 30 W/m·K.

Analysis For one-dimensional heat transfer in the radial r direction, the differential equation for heat conduction in cylindrical coordinate with heat generation can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{e}_{\text{gen}}}{k}=0 \quad \text{or} \quad \frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{\dot{e}_{\text{gen}}}{k}r$$



Integrating the differential equation twice with respect to r yields

$$r\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k}r^2+C_1 \quad \text{or} \quad \frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k}r+\frac{C_1}{r}$$

$$T(r)=-\frac{\dot{e}_{\text{gen}}}{4k}r^2+C_1\ln r+C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r=0: \quad \frac{dT(0)}{dr}=0 \quad \rightarrow \quad C_1=0$$

$$r=r_o: \quad -k\frac{dT(r_o)}{dr}=h[T(r_o)-T_\infty] \quad \rightarrow \quad k\frac{\dot{e}_{\text{gen}}}{2k}r_o=h\left(-\frac{\dot{e}_{\text{gen}}}{4k}r_0^2+C_2-T_\infty\right)$$

Solving for C_2 gives

$$C_2=\frac{\dot{e}_{\text{gen}}}{2h}r_o+\frac{\dot{e}_{\text{gen}}}{4k}r_o^2+T_\infty$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r)=-\frac{\dot{e}_{\text{gen}}}{4k}r^2+C_2=-\frac{\dot{e}_{\text{gen}}}{4k}r^2+\frac{\dot{e}_{\text{gen}}}{2h}r_o+\frac{\dot{e}_{\text{gen}}}{4k}r_o^2+T_\infty \quad \rightarrow \quad T(r)=\frac{\dot{e}_{\text{gen}}}{4k}(r_o^2-r^2)+\frac{\dot{e}_{\text{gen}}}{2h}r_o+T_\infty$$

The temperature at $r = 0$ (the centerline of the rod) is

$$T(0)=\frac{\dot{e}_{\text{gen}}}{4k}r_o^2+\frac{\dot{e}_{\text{gen}}}{2h}r_o+T_\infty=\frac{100\times10^6\ \text{W/m}^3}{4(30\ \text{W/m}\cdot\text{K})}(0.01\,\text{m})^2+\frac{100\times10^6\ \text{W/m}^3}{2(2500\ \text{W/m}^2\cdot\text{K})}(0.01\,\text{m})+75°\text{C}$$

$$T(0)=\mathbf{358°C}$$

The temperature at $r = r_o = 0.01$ m (the surface of the rod) is

$$T(r_o)=\frac{\dot{e}_{\text{gen}}}{2h}r_o+T_\infty=\frac{100\times10^6\ \text{W/m}^3}{2(2500\ \text{W/m}^2\cdot\text{K})}(0.01\,\text{m})+75°\text{C}=\mathbf{275°C}$$

Fuel rod surface not cooled adequately.

Discussion The temperature of the fuel rod surface is 75°C higher than the temperature necessary to prevent the cooling water from reaching the CHF. To keep the temperature of the fuel rod surface below 200°C, the convection heat transfer coefficient of the cooling water should be kept above 4000 W/m^2·K. This can be done either by increasing the mass flow rate of the cooling water or by decreasing the inlet temperature of the cooling water. The topic of critical heat flux is covered in Chapter 10 (Boiling and Condensation).

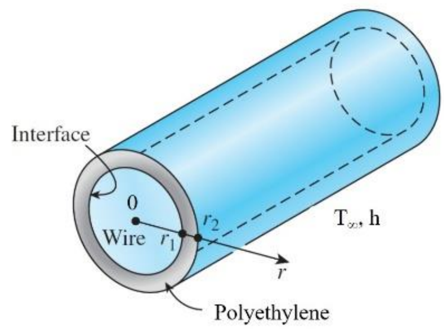
2-105 A long electrical resistance wire that is generating heat uniformly is covered with polyethylene insulation. Formulate the temperature profiles for the wire and the polyethylene insulation. Determine the temperature at the interface of the wire and the insulation, and the temperature at the center of the wire. Conclude whether the polyethylene insulation for the wire meets the ASTM D1351 standard.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivities are constant. **3** Heat generation in the wire is uniform. **4** There is no contact resistance at the interface of the wire and the insulation, $r = r_1$. **5** At the center of the wire, $r = 0$, is a symmetry boundary. **6** The outer surface of the insulation, $r = r_2$, is subjected to convection and radiation.

Properties The thermal conductivities of the wire and the polyethylene insulation are given to be $k_{\text{wire}} = 15$ W/m·K and $k_{\text{ins}} = 0.4$ W/m·K, respectively.

Analysis For one-dimensional heat transfer in the radial r direction with uniform heat generation, the differential equation for heat conduction in cylindrical coordinate for the wire can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT_{\text{wire}}}{dr}\right)+\frac{\dot{e}_{\text{gen}}}{k_{\text{wire}}}=0 \qquad \text{or} \quad \frac{d}{dr}\left(r\frac{dT_{\text{wire}}}{dr}\right)=-\frac{\dot{e}_{\text{gen}}}{k_{\text{wire}}}r$$



Integrating the differential equation twice with respect to r yields

$$r\frac{dT_{\text{wire}}}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k_{\text{wire}}}r^2+C_1$$

$$T_{\text{wire}}(r)=-\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}r^2+C_1\ln r+C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r=0: \qquad \frac{dT_{\text{wire}}(0)}{dr}=0 \qquad \rightarrow \qquad C_1=0$$

$$r=r_1: \qquad T_{\text{wire}}(r_1)=T_I=-\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}r_1^2+C_2 \qquad \rightarrow \qquad C_2=T_I+\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}r_1^2$$

Substituting C_1 and C_2 into the general solution, the temperature profile in the wire is determined to be

$$\boldsymbol{T_{\text{wire}}(r)=T_I+\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}\left(r_1^2-r^2\right)} \text{for} \qquad 0\le r\le r_1$$

The insulation layer does not involve any heat generation, the heat conduction equation in the insulation layer is

$$\frac{d}{dr}\left(r\frac{dT_{\text{ins}}}{dr}\right)=0$$

Integrating the differential equation twice with respect to r yields

$$r\frac{dT_{\text{ins}}}{dr}=C_3 \text{or} \frac{dT_{\text{ins}}}{dr}=\frac{C_3}{r}$$

$$T_{\text{ins}}(r)=C_3\ln r+C_4$$

where C_3 and C_4 are arbitrary constants. Applying the boundary conditions yields

$$r=r_1: \qquad -k_{\text{wire}}\frac{dT_{\text{wire}}(r_1)}{dr}=-k_{\text{ins}}\frac{dT_{\text{ins}}(r_1)}{dr} \qquad \rightarrow \qquad \frac{\dot{e}_{\text{gen}}}{2}r_1=-k_{\text{ins}}\frac{C_3}{r_1}$$

$$r=r_2: \qquad -k_{\text{ins}}\frac{dT_{\text{ins}}(r_2)}{dr}=-k_{\text{ins}}\frac{C_3}{r_2}=h_{\text{combined}}\left[T_{\text{ins}}(r_2)-T_{\text{surr}}\right]$$

Note that $h_{\text{combined}} = h_{\text{conv}} + h_{\text{rad}}$. The arbitrary constants C_3 and C_4 can be expressed as

$$C_3 = -\frac{\dot{e}_{\text{gen}}}{2}\frac{r_1^2}{k_{\text{ins}}}$$

$$C_4 = T_{\text{surr}} + \frac{\dot{e}_{\text{gen}} r_1^2}{2k_{\text{ins}}}\left(\frac{k_{\text{ins}}}{h_{\text{combined}}}\frac{1}{r_2} + \ln r_2\right)$$

Substituting C_3 and C_4 into the general solution, the temperature profile in the insulation layer is determined to be

$$\boldsymbol{T_{\text{ins}}(r) = \frac{\dot{e}_{\text{gen}} r_1^2}{2k_{\text{ins}}}\left(\frac{k_{\text{ins}}}{h_{\text{combined}}}\frac{1}{r_2} + \ln\frac{r_2}{r}\right) + T_{\text{surr}}} \text{ for } r_1 \le r \le r_2$$

At the interface of the wire and the insulation, $r = r_1$, we have

$$T_I = T_{\text{ins}}(r_1) = \frac{\dot{e}_{\text{gen}} r_1^2}{2k_{\text{ins}}}\left(\frac{k_{\text{ins}}}{h_{\text{combined}}}\frac{1}{r_2} + \ln\frac{r_2}{r_1}\right) + T_{\text{surr}}$$

$$T_I = \frac{\left(1.2\times 10^6\,\frac{\text{W}}{\text{m}^3}\right)(0.002\text{ m})^2}{2(0.4\text{ W/m}\cdot\text{K})}\left(\frac{0.4\text{ W/m}\cdot\text{K}}{7\,\frac{\text{W}}{\text{m}^2\cdot\text{K}}}\times\frac{1}{0.007\text{ m}} + \ln\frac{0.007\text{ m}}{0.002\text{ m}}\right) + 20°\text{C} = \mathbf{76.5}°\text{C} > 75°\text{C}$$

whereand $r_2 = r_1$ + wall thickness = 0.002 m + 0.005 m = 0.007 m.
The temperature at the center of the wire, $r = 0$, is

$$T_{\text{wire}}(0) = T_I + \frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}r_1^2 = 76.5°\text{C} + \frac{\left(1.2\times 10^6\,\frac{\text{W}}{\text{m}^3}\right)(0.002\text{ m})^2}{4(15\text{ W/m}\cdot\text{K})} = \mathbf{76.6}°\text{C}$$

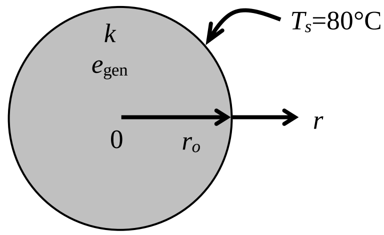
Discussion With the temperature at the interface of the wire and the insulation being 1.5°C higher than the specification of the ASTM D1351 standard for polyethylene insulation, the ASTM standard is not met. We can consider using a different insulation material with a higher temperature rating. From the ASTM database, the crosslinked polyethylene insulation (ASTM D2655) is rated up to 90°C for normal operation.

2-106 Heat is generated uniformly in a spherical radioactive material with specified surface temperature. The mathematical formulation, the variation of temperature in the sphere, and the center temperature are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat transfer is steady since there is no indication of any changes with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the mid point. **3** Thermal conductivity is constant. **4** Heat generation is uniform.

Properties The thermal conductivity is given to be k = 15 W/m·°C.

Analysis (*a*) Noting that heat transfer is steady and one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as



$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{\dot{e}_{\text{gen}}}{k}=0 \quad \text{with } \dot{e}_{\text{gen}}=\text{constant}$$

and $T(r_o)=T_s=80°\text{C}$ (specified surface temperature)

$$\frac{dT(0)}{dr}=0 \quad \text{(thermal symmetry about the mid point)}$$

(*b*) Multiplying both sides of the differential equation by r^2 and rearranging gives

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=-\frac{\dot{e}_{\text{gen}}}{k}r^2$$

Integrating with respect to r gives

$$r^2\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{k}\frac{r^3}{3}+C_1 \qquad (a)$$

Applying the boundary condition at the mid point,

B.C. at r = 0: $$0\times\frac{dT(0)}{dr}=-\frac{\dot{e}_{\text{gen}}}{3k}\times 0+C_1 \quad\rightarrow\quad C_1=0$$

Dividing both sides of Eq. (*a*) by r^2 to bring it to a readily integrable form and integrating,

$$\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{3k}r$$

and $$T(r)=-\frac{\dot{e}_{\text{gen}}}{6k}r^2+C_2 \qquad (b)$$

Applying the other boundary condition at $r=r_0$,

B. C. at $r=r_o$: $$T_s=-\frac{\dot{e}_{\text{gen}}}{6k}r_o^2+C_2 \quad\rightarrow\quad C_2=T_s+\frac{\dot{e}_{\text{gen}}}{6k}r_o^2$$

Substituting this C_2 relation into Eq. (*b*) and rearranging give

$$T(r)=T_s+\frac{\dot{e}_{\text{gen}}}{6k}(r_o^2-r^2)$$

which is the desired solution for the temperature distribution in the wire as a function of r.

(*c*) The temperature at the center of the sphere (r = 0) is determined by substituting the known quantities to be

$$T(0)=T_s+\frac{\dot{e}_{\text{gen}}}{6k}(r_o^2-0^2)=T_s+\frac{\dot{e}_{\text{gen}}r_o^2}{6k}=80°\text{C}+\frac{(4\times10^7\ \text{W/m}^3)(0.04\ \text{m})^2}{6\times(15\ \text{W/m}\cdot°\text{C})}=\mathbf{791°C}$$

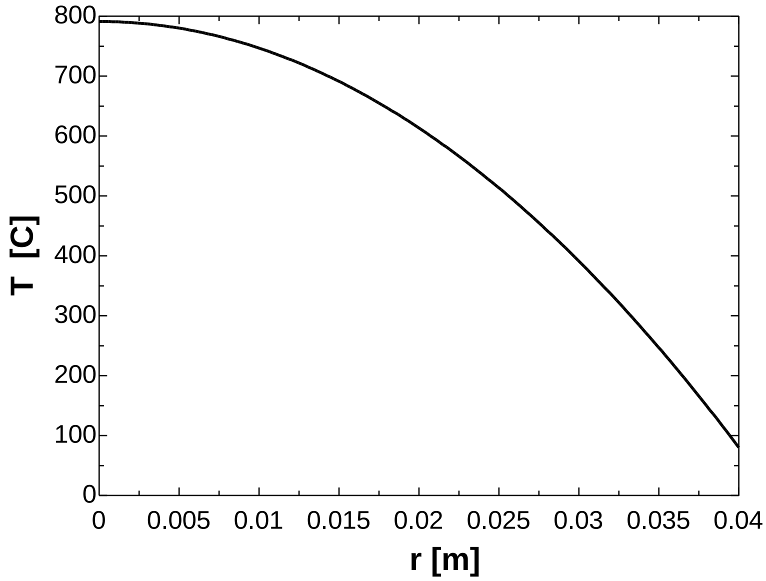
Thus the temperature at center will be about 711°C above the temperature of the outer surface of the sphere.

2-107 Prob. 2-106 is reconsidered. The temperature as a function of the radius is to be plotted. Also, the center temperature of the sphere as a function of the thermal conductivity is to be plotted.

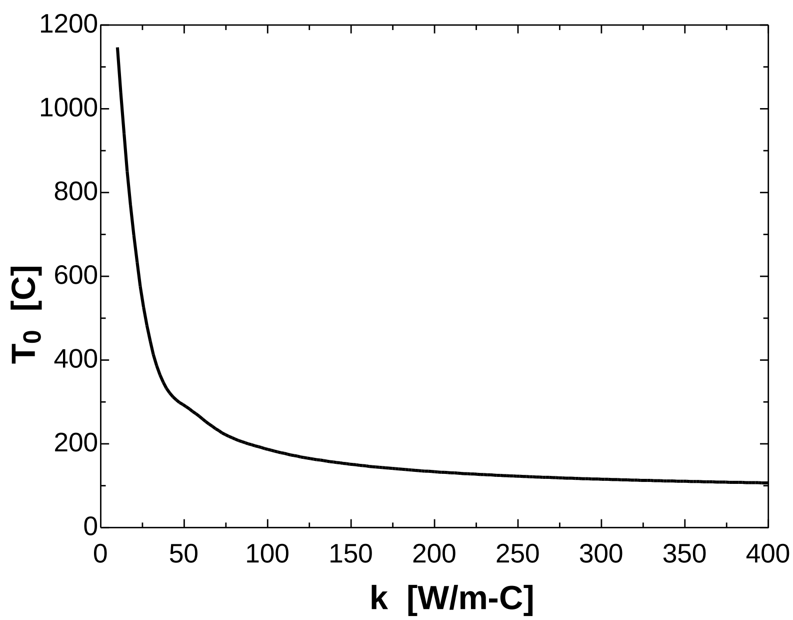
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
r_0=0.04 [m]
g_dot=4E7 [W/m^3]
T_s=80 [C]
k=15 [W/m-C]
"ANALYSIS"
T=T_s+g_dot/(6*k)*(r_0^2-r^2) "Temperature distribution as a function of r"
T_0=T_s+g_dot/(6*k)*r_0^2 "Temperature at the center (r=0)"
```

r [m]	T [C]
0	791.1
0.002105	789.1
0.004211	783.2
0.006316	773.4
0.008421	759.6
0.01053	741.9
0.01263	720.2
0.01474	694.6
0.01684	665
0.01895	631.6
0.02105	594.1
0.02316	552.8
0.02526	507.5
0.02737	458.2
0.02947	405
0.03158	347.9
0.03368	286.8
0.03579	221.8
0.03789	152.9
0.04	80



k [W/m.C]	T_0 [C]
10	1147
30.53	429.4
51.05	288.9
71.58	229
92.11	195.8
112.6	174.7
133.2	160.1
153.7	149.4
174.2	141.2
194.7	134.8
215.3	129.6
235.8	125.2
256.3	121.6
276.8	118.5
297.4	115.9
317.9	113.6
338.4	111.5
358.9	109.7
379.5	108.1
400	106.7



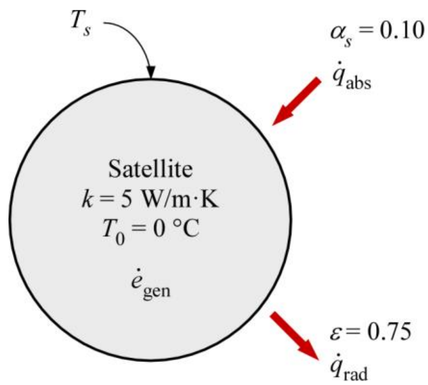
2-108 A spherical communication satellite orbiting in space absorbs solar radiation while losing heat to deep space by thermal radiation. The heat generation rate and the surface temperature of the satellite are to be determined.

Assumptions **1** Heat transfer is steady and one-dimensional. **2** Heat generation is uniform. **3** Thermal properties are constant.

Properties The properties of the satellite are given to be $\varepsilon = 0.75$, $\alpha = 0.10$, and $k = 5$ W/m · K.

Analysis For steady one-dimensional heat conduction in sphere, the differential equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)+\frac{\dot{e}_{\text{gen}}}{k}=0$$



and $\quad T(0) = T_0 = 273\,\text{K}$ (midpoint temperature of the satellite)

$$\frac{dT(0)}{dr}=0 \quad \text{(thermal symmetry about the midpoint)}$$

Multiply both sides of the differential equation by r^2 and rearranging gives

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=-\frac{\dot{e}_{\text{gen}}}{k}r^2$$

Integrating with respect to r gives

$$r^2\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{k}\frac{r^3}{3}+C_1 \qquad (a)$$

Applying the boundary condition at the midpoint (thermal symmetry about the midpoint),

$$r=0: \qquad 0\times\frac{dT(0)}{dr}=-\frac{\dot{e}_{\text{gen}}}{k}\times 0+C_1 \quad\rightarrow\quad C_1=0$$

Dividing both sides of Eq. (a) by r^2 and integrating,

$$\frac{dT}{dr}=-\frac{\dot{e}_{\text{gen}}}{3k}r$$

and $$T(r)=-\frac{\dot{e}_{\text{gen}}}{6k}r^2+C_2 \qquad (b)$$

Applying the boundary condition at the midpoint (midpoint temperature of the satellite),

$$r=0: \qquad T_0=-\frac{\dot{e}_{\text{gen}}}{6k}\times 0+C_2 \quad\rightarrow\quad C_2=T_0$$

Substituting C_2 into Eq. (b), the variation of temperature is determined to be

$$T(r)=-\frac{\dot{e}_{\text{gen}}}{6k}r^2+T_0$$

At the satellite surface ($r = r_o$), the temperature is

$$T_s=-\frac{\dot{e}_{\text{gen}}}{6k}r_o^2+T_0 \qquad (c)$$

Also, the rate of heat transfer at the surface of the satellite can be expressed as

$$\dot{e}_{\text{gen}}\left(\frac{4}{3}\pi r_o^3\right)=A_s\varepsilon\sigma(T_s^4-T_{\text{space}}^4)-A_s\alpha_s\dot{q}_{\text{solar}} \qquad \text{where} \qquad T_{\text{space}}=0$$

The surface temperature of the satellite can be explicitly expressed as

$$T_s=\left[\frac{1}{A_s\varepsilon\sigma}\left(\frac{4}{3}\pi r_o^3\dot{e}_{\text{gen}}+A_s\alpha_s\dot{q}_{\text{solar}}\right)\right]^{1/4}=\left(\frac{\dot{e}_{\text{gen}}r_o/3+\alpha_s\dot{q}_{\text{solar}}}{\varepsilon\sigma}\right)^{1/4} \qquad (d)$$

Substituting Eq. (c) into Eq. (d)

$$\left(\frac{\dot{e}_{\text{gen}}r_o/3+\alpha_s\dot{q}_{\text{solar}}}{\varepsilon\sigma}\right)^{1/4}=-\frac{\dot{e}_{\text{gen}}}{6k}r_o^2+T_0$$

$$\left[\frac{\dot{e}_{\text{gen}}(1.25\,\text{m})/3+(0.10)(1000\ \text{W/m}^2)}{(0.75)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}\right]^{1/4}=-\frac{\dot{e}_{\text{gen}}(1.25\,\text{m})^2}{6(5\ \text{W/m}\cdot\text{K})}+273\,\text{K}$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
((e_gen*1.25/3+0.10*1000)/(0.75*5.67e-8))^(1/4)=-e_gen*1.25^2/(6*5)+273
```

Solving by EES software, the heat generation rate is

$$\dot{e}_{\text{gen}} = \mathbf{233\ W/m^3}$$

Using Eq. (*c*), the surface temperature of the satellite is determined to be

$$T_s = -\frac{(233\ \text{W/m}^3)}{6(5\ \text{W/m}\cdot\text{K})}(1.25\ \text{m})^2 + 273\ \text{K} = \mathbf{261\ K}$$

Discussion The surface temperature of the satellite in space is well below freezing point of water.

Variable Thermal Conductivity, *k*(*T*)

2-109C The thermal conductivity of a medium, in general, varies with temperature.

2-110C Yes, when the thermal conductivity of a medium varies linearly with temperature, the average thermal conductivity is always equivalent to the conductivity value at the average temperature.

2-111C No, the temperature variation in a plain wall will not be linear when the thermal conductivity varies with temperature.

2-112C During steady one-dimensional heat conduction in a plane wall in which the thermal conductivity varies linearly, the error involved in heat transfer calculation by assuming constant thermal conductivity at the average temperature is (*a*) *none*.

2-113C During steady one-dimensional heat conduction in a plane wall, long cylinder, and sphere with constant thermal conductivity and no heat generation, the temperature in only the *plane wall* will vary linearly.

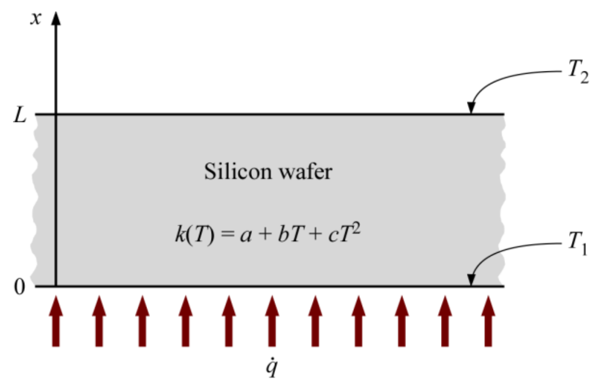
2-114 A silicon wafer with variable thermal conductivity is subjected to uniform heat flux at the lower surface. The maximum allowable heat flux such that the temperature difference across the wafer thickness does not exceed 2 °C is to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = (a + bT + cT^2)$ W/m · K.

Analysis For steady heat transfer, the Fourier's law of heat conduction can be expressed as

$$\dot{q} = -k(T)\frac{dT}{dx} = -(a + bT + cT^2)\frac{dT}{dx}$$



Separating variable and integrating from $x = 0$ where $T(0) = T_1$ to $x = L$ where $T(L) = T_2$, we obtain

$$\int_0^L \dot{q}dx = -\int_{T_1}^{T_2} (a + bT + cT^2)dT$$

Performing the integration gives

$$\dot{q}L = -\left[a(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2) + \frac{c}{3}(T_2^3 - T_1^3)\right]$$

The maximum allowable heat flux such that the temperature difference across the wafer thickness does not exceeding 2°C (where $T_1 = 602\,\text{K}$ and $T_2 = 600\,\text{K}$) is

$$\dot{q} = -\frac{\left[437(600 - 602) - \frac{1.29}{2}(600^2 - 602^2) + \frac{0.00111}{3}(600^3 - 602^3)\right]\text{W/m}}{(925 \times 10^{-6}\ \text{m})}$$

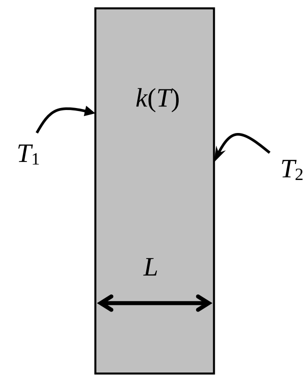
$$= \mathbf{1.35 \times 10^5\ W/m^2}$$

Discussion For heat flux less than 135 kW/m^2, the temperature difference across the silicon wafer thickness will be maintained below 2°C.

2-115 A plate with variable conductivity is subjected to specified temperatures on both sides. The rate of heat transfer through the plate is to be determined.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity varies linearly. **3** There is no heat generation.

Properties The thermal conductivity is given to be $k(T) = k_0(1+\beta T)$.



Analysis The average thermal conductivity of the medium in this case is simply the conductivity value at the average temperature since the thermal conductivity varies linearly with temperature, and is determined to be

$$\begin{aligned} k_{\text{ave}} &= k(T_{\text{avg}}) = k_0\left(1+\beta\frac{T_2+T_1}{2}\right) \\ &= (25\text{ W/m}\cdot\text{K})\left(1+(8.7\times10^{-4}\text{ K}^{-1})\frac{(500+350)\text{ K}}{2}\right) \\ &= 34.24\text{ W/m}\cdot\text{K} \end{aligned}$$

Then the rate of heat conduction through the plate becomes

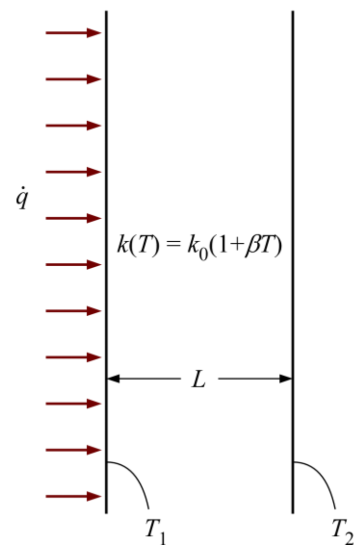
$$\dot{Q} = k_{\text{avg}} A\frac{T_1-T_2}{L} = (34.24\text{ W/m}\cdot\text{K})(1.5\text{ m}\times0.6\text{ m})\frac{(500-350)\text{K}}{0.15\text{ m}} = 30{,}820\text{ W} = \mathbf{30.8\,kW}$$

Discussion We would obtain the same result if we substituted the given $k(T)$ relation into the second part of Eq, 2-76, and performed the indicated integration.

2-116 On one side, a steel plate is subjected to a uniform heat flux and maintained at a constant temperature. On the other side, the temperature is maintained at a lower temperature. The plate thickness is to be determined.

Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = k_0(1+\beta T)$.



Analysis For steady heat transfer, the Fourier's law of heat conduction can be expressed as

$$\dot{q} = k_{\text{avg}}\frac{T_1-T_2}{L}$$

Solving for the plate thickness from the above equation

$$L = k_{\text{avg}}\frac{T_1-T_2}{\dot{q}}$$

The average thermal conductivity of the steel plate is

$$k_{\text{avg}} = k_0\left(1+\beta\frac{T_2+T_1}{2}\right) = (9.14\text{ W/m}\cdot\text{K})\left[1+(0.0023\text{K}^{-1})\frac{(600+800)\text{ K}}{2}\right] = 23.86\text{ W/m}\cdot\text{K}$$

Substituting into the equation for the plate thickness,

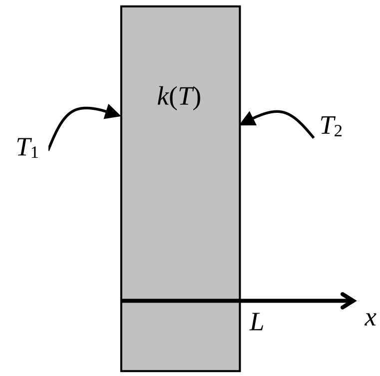
$$L = (23.86\text{ W/m}\cdot\text{K})\frac{(800-600)\text{ K}}{50000\text{ W/m}^2} = \mathbf{0.095\,m}$$

Discussion We would obtain the same result if we substituted the given $k(T)$ relation into the second part of Eq. 2-76, and performed the indicated integration.

2-117 A plate with variable conductivity is subjected to specified temperatures on both sides. The rate of heat transfer through the plate is to be determined.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity varies quadratically. **3** There is no heat generation.

Properties The thermal conductivity is given to be $k(T) = k_0(1+\beta T^2)$.



Analysis When the variation of thermal conductivity with temperature $k(T)$ is known, the average value of the thermal conductivity in the temperature range between T_1 and T_2 can be determined from

$$k_{avg} = \frac{\int_{T_1}^{T_2} k(T)dT}{T_2 - T_1} = \frac{\int_{T_1}^{T_2} k_0(1+\beta T^2)dT}{T_2 - T_1} = \frac{k_0\left(T + \frac{\beta}{3}T^3\right)_{T_1}^{T_2}}{T_2 - T_1}$$

$$= \frac{k_0\left[(T_2 - T_1) + \frac{\beta}{3}(T_2^3 - T_1^3)\right]}{T_2 - T_1}$$

$$= k_0\left[1 + \frac{\beta}{3}(T_2^2 + T_1T_2 + T_1^2)\right]$$

This relation is based on the requirement that the rate of heat transfer through a medium with constant average thermal conductivity k_{avg} equals the rate of heat transfer through the same medium with variable conductivity $k(T)$. Then the rate of heat conduction through the plate can be determined to be

$$\dot{Q} = k_{avg} A \frac{T_1 - T_2}{L} = k_0\left[1 + \frac{\beta}{3}\left(T_2^2 + T_1T_2 + T_1^2\right)\right] A \frac{T_1 - T_2}{L}$$

Discussion We would obtain the same result if we substituted the given $k(T)$ relation into the second part of Eq. 2-76, and performed the indicated integration.

2-118 The thermal conductivity of stainless steel has been characterized experimentally to vary with temperature. The average thermal conductivity over a given temperature range and the $k(T) = k_0(1 + \beta T)$ expression are to be determined.

Assumptions **1** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = 9.14 + 0.021T$ for $273 < T < 1500$ K.

Analysis The average thermal conductivity can be determined using

$$k_{\text{avg}} = \frac{\int_{T_1}^{T_2} k(T)\,dT}{T_2 - T_1} = \frac{\int_{300}^{1200} (9.14 + 0.021T)\,dT}{1200 - 300} = \frac{(9.14T + 0.0105T^2)\Big|_{300}^{1200}}{1200 - 300} = \mathbf{24.9\ W/m \cdot K}$$

To express $k(T) = 9.14 + 0.021T$ as $k(T) = k_0(1 + \beta T)$, we have

$$k(T) = k_0 + k_0 \beta T$$

and comparing with $k(T) = 9.14 + 0.021T$, we have

$$k_0 = 9.14\ \text{W/m}\cdot\text{K} \quad \text{and} \quad k_0\beta = 0.021\ \text{W/m}\cdot\text{K}^2$$

which gives

$$\beta = \frac{0.021\ \text{W/m}\cdot\text{K}^2}{k_0} = \frac{0.021\ \text{W/m}\cdot\text{K}^2}{9.14\ \text{W/m}\cdot\text{K}} = 0.0023\ \text{K}^{-1}$$

Thus,

$$k(T) = k_0(1 + \beta T) \quad \text{where} \quad k_0 = \mathbf{9.14\ W/m \cdot K} \quad \text{and} \quad \beta = \mathbf{0.0023\ K^{-1}}$$

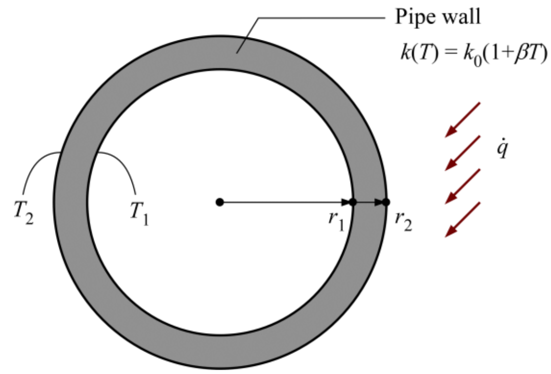
Discussion The average thermal conductivity can also be determined using the average temperature:

$$k_{\text{avg}} = k(T_{\text{avg}}) = 9.14 + 0.021\left(\frac{1200 + 300}{2}\right) = 24.9\ \text{W/m}\cdot\text{K}$$

2-119 A pipe outer surface is subjected to a uniform heat flux and has a known temperature. The metal pipe has a variable thermal conductivity. The inner surface temperature of the pipe is to be determined.

Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$.



Analysis For steady heat transfer, the heat conduction through a cylindrical layer can be expressed as

$$\dot{q} = \frac{\dot{Q}}{2\pi r_2 L} = \frac{2\pi L k_{avg}}{2\pi r_2 L}\frac{T_2 - T_1}{\ln(r_2/r_1)} = \frac{k_{avg}}{r_2}\frac{T_2 - T_1}{\ln(r_2/r_1)}$$

The inner and outer radii of the pipe are

$$r_1 = 0.1/2\,\text{m} = 0.05\,\text{m} \quad \text{and} \quad r_2 = (0.05 + 0.01)\,\text{m} = 0.06\,\text{m}$$

The average thermal conductivity is

$$k_{avg} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = (7.5\,\text{W/m}\cdot\text{K})\left[1 + (0.0012\,\text{K}^{-1})\frac{(773)\,\text{K} + T_1}{2}\right]$$
$$= [7.5 + 0.0045(773 + T_1)]\,\text{W/m}\cdot\text{K}$$

Thus,

$$5000\,\text{W/m}^2 = \frac{[7.5 + 0.0045(773 + T_1)]\,\text{W/m}\cdot\text{K}}{0.06\,\text{m}}\left[\frac{773 - T_1}{\ln(0.06/0.05)}\,\text{K}\right]$$

Solving for the inner pipe temperature T_1,

$$T_1 = 769.21\,\text{K} = \mathbf{496.2°C}$$

Discussion There is about 4°C drop in temperature across the pipe wall.

2-120 A pipe is used for transporting boiling water with a known inner surface temperature in surroundings of cooler ambient temperature and known convection heat transfer coefficient. The pipe wall has a variable thermal conductivity. The outer surface temperature of the pipe is to be determined to ensure that it is below 50°C.

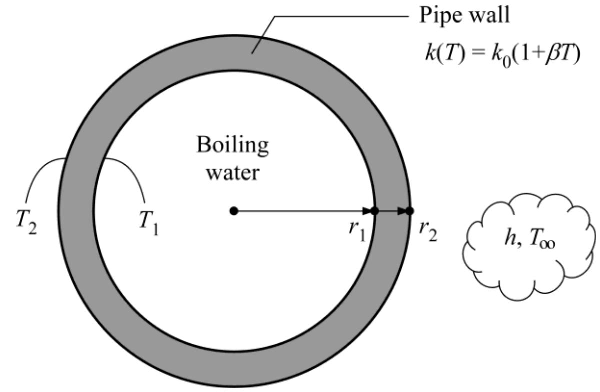
Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature. **4** Inner pipe surface temperature is constant at 100°C.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$.

Analysis The inner and outer radii of the pipe are

$$r_1 = 0.030/2\ \mathrm{m} = 0.015\ \mathrm{m}$$

$$r_2 = (0.015 + 0.003)\ \mathrm{m} = 0.018\ \mathrm{m}$$



The rate of heat transfer at the pipe's outer surface can be expressed as

$$\dot{Q}_{\text{cylinder}} = \dot{Q}_{\text{conv}}$$

$$2\pi k_{\text{avg}} L \frac{T_1 - T_2}{\ln(r_2/r_1)} = h(2\pi r_2 L)(T_2 - T_\infty)$$

$$\frac{k_{\text{avg}}}{r_2}\frac{T_1 - T_2}{\ln(r_2/r_1)} = h(T_2 - T_\infty) \qquad (1)$$

where

$h = 70\ \mathrm{W/m^2\,K}$, $T_1 = 373\ \mathrm{K}$, and $T_\infty = 283\ \mathrm{K}$

The average thermal conductivity is

$$k_{\text{avg}} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = (1.23\ \mathrm{W/m\cdot K})\left[1 + (0.002\ \mathrm{K^{-1}})\frac{T_2 + (373\ \mathrm{K})}{2}\right]$$

$$k_{\text{avg}} = [1.23 + 0.00123(T_2 + 373)]\ \mathrm{W/m\cdot K} \qquad (2)$$

Solving Eqs. (1) & (2) for the outer surface temperature yields

$$T_2 = \mathbf{364.3\ K = 91.3°C}$$

Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"GIVEN"
h=70 [W/(m^2*K)]  "convection heat transfer coefficient"
r_1=0.030/2 [m]  "inner radius"
r_2=r_1+0.003 [m]  "outer radius"
T_1=100+273 [K]  "inner surface temperature"
T_inf=10+273 [K]  "ambient temperature"
k_0=1.23 [W/(m*K)]
beta=0.002 [K^-1]
"SOLVING FOR OUTER SURFACE TEMPERATURE"
k_avg=k_0*(1+beta*(T_2+T_1)/2)
Q_dot_cylinder=2*pi*k_avg*(T_1-T_2)/ln(r_2/r_1)  "heat rate through the cylindrical layer"
Q_dot_conv=h*2*pi*r_2*(T_2-T_inf)  "heat rate by convection"
Q_dot_cylinder=Q_dot_conv
```

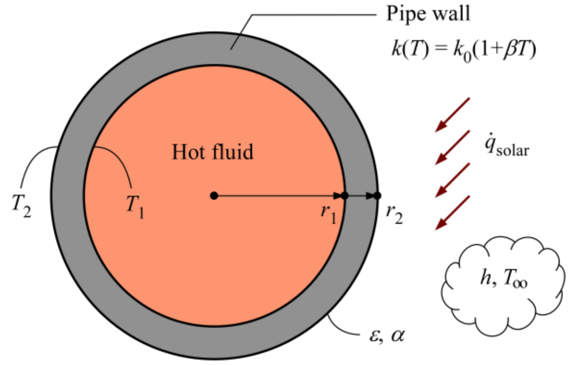
The outer surface temperature of the pipe is more than 40°C above the safe temperature of 50°C to prevent thermal burn on skin tissues.

Discussion It is necessary to wrap the pipe with insulation to prevent thermal burn.

2-121 A pipe is used for transporting hot fluid with a known inner surface temperature. The pipe wall has a variable thermal conductivity. The pipe's outer surface is subjected to radiation and convection heat transfer. The outer surface temperature of the pipe is to be determined.

Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$, $\alpha = \varepsilon = 0.9$ at the outer pipe surface.



Analysis The inner and outer radii of the pipe are

$$r_1 = 0.15/2\,\text{m} = 0.075\,\text{m}$$

$$r_2 = (0.075 + 0.005)\,\text{m} = 0.08\,\text{m}$$

The rate of heat transfer at the pipe's outer surface can be expressed as

$$\dot{Q}_{\text{cyl}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} - \dot{Q}_{\text{abs}}$$

$$2\pi k_{\text{avg}} L \frac{T_1 - T_2}{\ln(r_2/r_1)} = h(2\pi r_2 L)(T_2 - T_\infty) + \varepsilon\sigma(2\pi r_2 L)(T_2^4 - T_{\text{surr}}^4) - \alpha(2\pi r_2 L)\dot{q}_{\text{solar}}$$

$$\frac{k_{\text{avg}}}{r_2}\frac{T_1 - T_2}{\ln(r_2/r_1)} = h(T_2 - T_\infty) + \varepsilon\sigma(T_2^4 - T_{\text{surr}}^4) - \alpha\dot{q}_{\text{solar}} \quad (1)$$

where $h = 60\ \text{W/m}^2\ \text{K}$, $\dot{q}_{\text{solar}} = 100\ \text{W/m}^2$, $T_1 = 423\ \text{K}$, and $T_\infty = T_{\text{surr}} = 273\ \text{K}$

The average thermal conductivity is

$$k_{\text{avg}} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = (8.5\,\text{W/m}\cdot\text{K})\left[1 + (0.001\text{K}^{-1})\frac{T_2 + (423\ \text{K})}{2}\right]$$

$$k_{\text{avg}} = [8.5 + 0.00425(T_2 + 423)]\,\text{W/m}\cdot\text{K} \quad (2)$$

Solving Eqs. (1) & (2) for the outer surface temperature yields

$$T_2 = \mathbf{418.8\ K = 145.8°C}$$

Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

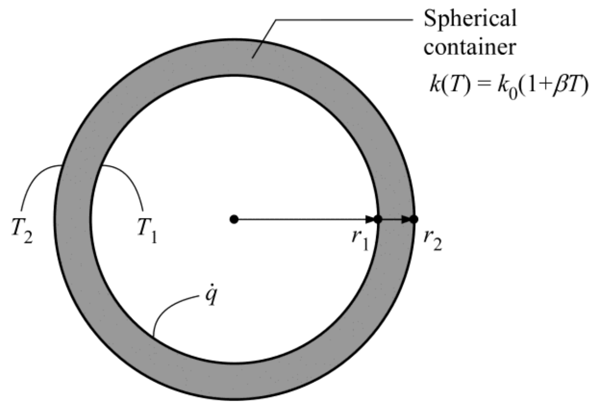
```
"GIVEN"
h=60 [W/(m^2*K)]  "outer surface h"
r_1=0.15/2 [m]  "inner radius"
r_2=r_1+0.005 [m]  "outer radius"
T_1=423 [K]  "inner surface T"
T_inf=273 [K]  "ambient T"
T_surr=273 [K]  "surrounding surface T"
alpha=0.9  "outer surface absorptivity"
epsilon=0.9  "outer surface emissivity"
q_dot_solar=100 [W/m^2]  "incident solar radiation"
k_0=8.5 [W/(m*K)]
beta=0.001 [K^-1]
"SOLVING FOR OUTER SURFACE TEMPERATURE"
k_avg=k_0*(1+beta*(T_2+T_1)/2)
q_dot_cyl=k_avg/r_2*(T_1-T_2)/ln(r_2/r_1)  "heat flux through the cylindrical layer"
q_dot_conv=h*(T_2-T_inf)  "heat flux by convection"
q_dot_rad=epsilon*sigma#*(T_2^4-T_surr^4)  "heat flux by radiation emission"
q_dot_abs=alpha*q_dot_solar  "heat flux by radiation absorption"
q_dot_cyl-q_dot_conv-q_dot_rad+q_dot_abs=0
```

Discussion Increasing h or decreasing k_{avg} would decrease the pipe's outer surface temperature.

2-122 A spherical container has its inner surface subjected to a uniform heat flux and its outer surface is at a known temperature. The container wall has a variable thermal conductivity. The temperature drop across the container wall thickness is to be determined.

Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$.



Analysis For steady heat transfer, the heat conduction through a spherical layer can be expressed as

$$\dot{q} = \frac{\dot{Q}}{4\pi r_1^2} = \frac{4\pi k_{\text{avg}} r_1 r_2}{4\pi r_1^2}\frac{T_1 - T_2}{r_2 - r_1} = k_{\text{avg}}\frac{r_2}{r_1}\frac{T_1 - T_2}{r_2 - r_1}$$

The inner and outer radii of the container are

$$r_1 = 1\,\text{m}$$

$$r_2 = 1\,\text{m} + 0.005\,\text{m} = 1.005\,\text{m}$$

The average thermal conductivity is

$$k_{\text{avg}} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = (1.33\,\text{W/m}\cdot\text{K})\left[1 + (0.0023\,\text{K}^{-1})\frac{(293)\,\text{K} + T_1}{2}\right]$$

$$= [1.33 + 0.00153(293 + T_1)]\,\text{W/m}\cdot\text{K}$$

Thus,

$$7000\,\text{W/m}^2 = [1.33 + 0.00153(293 + T_1)]\,\text{W/m}\cdot\text{K}\left(\frac{1.005\,\text{m}}{1\text{m}}\right)\left(\frac{T_1 - 293}{0.005}\frac{\text{K}}{\text{m}}\right)$$

Solving for the inner pipe temperature T_1,

$$T_1 = 308.5\,\text{K}$$

The temperature drop across the container wall is,

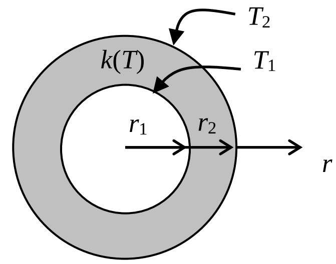
$$T_1 - T_2 = 308.5\,\text{K} - 293\,\text{K} = \mathbf{15.5°C}$$

Discussion The temperature drop across the container wall would decrease if a material with a higher k_{avg} value is used.

2-123 A spherical shell with variable conductivity is subjected to specified temperatures on both sides. The variation of temperature and the rate of heat transfer through the shell are to be determined.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity varies linearly. **3** There is no heat generation.

Properties The thermal conductivity is given to be $k(T) = k_0(1+\beta T)$.



Analysis (*a*) The rate of heat transfer through the shell is expressed as

$$\dot{Q}_{\text{sphere}} = 4\pi k_{\text{avg}} r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1}$$

where r_1 is the inner radius, r_2 is the outer radius, and

$$k_{\text{avg}} = k(T_{\text{avg}}) = k_0\left(1+\beta\frac{T_2+T_1}{2}\right)$$

is the average thermal conductivity.

(*b*) To determine the temperature distribution in the shell, we begin with the Fourier's law of heat conduction expressed as

$$\dot{Q} = -k(T)A\frac{dT}{dr}$$

where the rate of conduction heat transfer $\dot{Q}$ is constant and the heat conduction area $A = 4\pi r^2$ is variable. Separating the variables in the above equation and integrating from $r = r_1$ where $T(r_1) = T_1$ to any r where $T(r) = T$, we get

$$\dot{Q}\int_{r_1}^{r}\frac{dr}{r^2} = -4\pi\int_{T_1}^{T}k(T)dT$$

Substituting $k(T) = k_0(1+\beta T)$ and performing the integrations gives

$$\dot{Q}\left(\frac{1}{r_1}-\frac{1}{r}\right) = -4\pi k_0[(T-T_1)+\beta(T^2-T_1^2)/2]$$

Substituting the $\dot{Q}$ expression from part (*a*) and rearranging give

$$T^2 + \frac{2}{\beta}T + \frac{2k_{\text{avg}}}{\beta k_0}\frac{r_2(r-r_1)}{r(r_2-r_1)}(T_1-T_2) - T_1^2 - \frac{2}{\beta}T_1 = 0$$

which is a *quadratic* equation in the unknown temperature *T*. Using the quadratic formula, the temperature distribution $T(r)$ in the cylindrical shell is determined to be

$$T(r) = -\frac{1}{\beta} \pm \sqrt{\frac{1}{\beta^2} - \frac{2k_{\text{avg}}}{\beta k_0}\frac{r_2(r-r_1)}{r(r_2-r_1)}(T_1-T_2) + T_1^2 + \frac{2}{\beta}T_1}$$

Discussion The proper sign of the square root term (+ or -) is determined from the requirement that the temperature at any point within the medium must remain between T_1 and T_2.

2-124 A spherical vessel, filled with chemicals undergoing an exothermic reaction, has a known inner surface temperature. The wall of the vessel has a variable thermal conductivity. Convection heat transfer occurs on the outer surface of the vessel. The minimum wall thickness of the vessel is to be determined so that the outer surface temperature is 50°C or lower.

Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

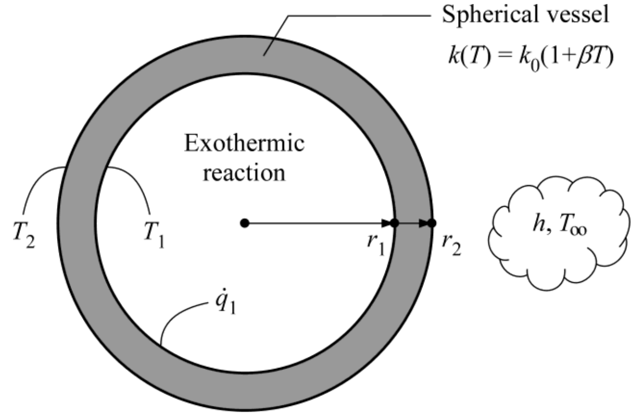
Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$.

Analysis The inner and outer radii of the vessel are

$$r_1 = 5/2\,\text{m} = 2.5\,\text{m}$$

and $\quad r_2 = (r_1 + t)$

where $\quad t$ = wall thickness



The rate of heat transfer at the vessel's outer surface can be expressed as

$$\dot{Q}_{\text{sph}} = \dot{Q}_{\text{conv}}$$

$$4\pi k_{\text{avg}} r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} = h(4\pi r_2^2)(T_2 - T_\infty)$$

$$k_{\text{avg}} \frac{r_1}{r_2} \frac{T_1 - T_2}{r_2 - r_1} = h(T_2 - T_\infty) \qquad (1)$$

where

h = 80 W/m^2 K, $\quad T_1$ = 393 K, $\quad T_2$ = 323 K, and $\quad T_\infty$ = 288 K

The average thermal conductivity is

$$k_{\text{avg}} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = (1.01\,\text{W/m}\cdot\text{K})\left[1 + (0.0018\,\text{K}^{-1})\frac{(323\,\text{K}) + (393\,\text{K})}{2}\right] = 1.6611\,\text{W/m}\cdot\text{K}$$

Solving Eq. (1) for r_2 yields

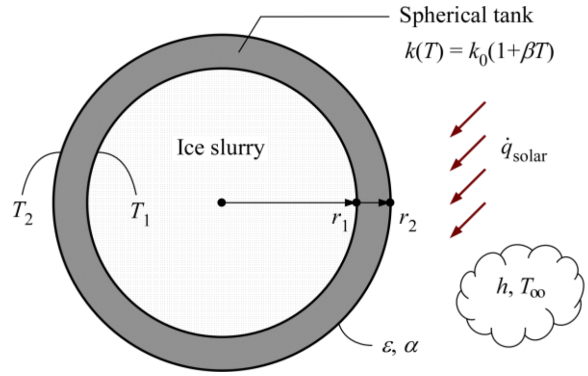
$$r_2 = 2.541\,\text{m}$$

Thus, the minimum wall thickness of the vessel should be

$$t = r_2 - r_1 = 0.041\,\text{m} = \mathbf{41\,mm}$$

Discussion To prevent the outer surface temperature of the vessel from causing thermal burn, the wall thickness should be at least 41 mm. As the wall thickness increases, it would decrease the outer surface temperature.

2-125 A spherical tank, filled with ice slurry, has a known inner surface temperature. The tank wall has a variable thermal conductivity. The tank's outer surface is subjected to radiation and convection heat transfer. The outer surface temperature of the tank is to be determined.



Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$, $\alpha = \varepsilon = 0.75$ at the outer tank surface.

Analysis The inner and outer radii of the tank are

$$r_1 = 9/2\,\text{m} = 4.5\,\text{m}$$

and $$r_2 = (4.5 + 0.02)\,\text{m} = 4.52\,\text{m}$$

The rate of heat transfer at the tank's outer surface can be expressed as

$$\dot{Q}_{\text{sph}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} + \dot{Q}_{\text{abs}}$$

$$4\pi k_{\text{avg}} r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} = h(4\pi r_2^2)(T_\infty - T_2) + \varepsilon\sigma(4\pi r_2^2)(T_{\text{surr}}^4 - T_2^4) + \alpha(4\pi r_2^2)\dot{q}_{\text{solar}}$$

$$k_{\text{avg}} \frac{r_1}{r_2}\frac{T_1 - T_2}{r_2 - r_1} = h(T_\infty - T_2) + \varepsilon\sigma(T_{\text{surr}}^4 - T_2^4) + \alpha\dot{q}_{\text{solar}} \qquad (1)$$

where

$$h = 70\ \text{W/m}^2\,\text{K},\quad \dot{q}_{\text{solar}} = 150\ \text{W/m}^2,\quad T_1 = 273\ \text{K},\quad \text{and}\quad T_\infty = T_{\text{surr}} = 308\ \text{K}$$

The average thermal conductivity is

$$k_{\text{avg}} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = (0.33\,\text{W/m}\cdot\text{K})\left[1 + (0.0025\,\text{K}^{-1})\frac{T_2 + (273.15\,\text{K})}{2}\right]$$

$$k_{\text{avg}} = [0.33 + 0.0004125(\text{T}_2 + 273)]\,\text{W/m}\cdot\text{K} \qquad (2)$$

Solving Eqs. (1) & (2) for the outer surface temperature yields

$$T_2 = \mathbf{299.5\,K} = \mathbf{26.5°C}$$

Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"GIVEN"
h=70 [W/(m^2*K)]  "outer surface h"
r_1=9/2 [m]  "inner radius"
r_2=r_1+0.020 [m]  "outer radius"
T_1=273 [K]  "inner surface T"
T_inf=308 [K]  "ambient T"
T_surr=308 [K]  "surrounding surface T"
alpha=0.75  "outer surface absorptivity"
epsilon=0.75  "outer surface emissivity"
q_dot_solar=150 [W/m^2]  "incident solar radiation"
k_0=0.33 [W/(m*K)]
beta=0.0025 [K^-1]
"SOLVING FOR OUTER SURFACE TEMPERATURE"
k_avg=k_0*(1+beta*(T_2+T_1)/2)
q_dot_sph=k_avg*r_1/r_2*(T_1-T_2)/(r_2-r_1)  "heat flux through the spherical layer"
q_dot_conv=h*(T_inf-T_2)  "heat flux by convection"
q_dot_rad=epsilon*sigma#*(T_surr^4-T_2^4)  "heat flux by radiation emission"
q_dot_abs=alpha*q_dot_solar  "heat flux by radiation absorption"
q_dot_sph+q_dot_conv+q_dot_rad+q_dot_abs=0
```

Discussion Increasing the tank wall thickness would increase the tanks' outer surface temperature.

Special Topic: Review of Differential equations

2-126C We utilize appropriate simplifying assumptions when deriving differential equations to obtain an equation that we can deal with and solve.

2-127C A **variable** is a quantity which may assume various values during a study. A variable whose value can be changed arbitrarily is called an **independent variable** (or argument). A variable whose value depends on the value of other variables and thus cannot be varied independently is called a **dependent variable** (or a function).

2-128C A differential equation may involve more than one dependent or independent variable. For example, the equation $\frac{\partial^2 T(x,t)}{\partial x^2} + \frac{\dot{e}_{\text{gen}}}{k} = \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t}$ has one dependent (T) and 2 independent variables (x and t). the equation $\frac{\partial^2 T(x,t)}{\partial x^2} + \frac{\partial W(x,t)}{\partial x} = \frac{1}{\alpha}\frac{\partial T(x,t)}{\partial t} + \frac{1}{\alpha}\frac{\partial W(x,t)}{\partial t}$ has 2 dependent (T and W) and 2 independent variables (x and t).

2-129C Geometrically, the **derivative** of a function $y(x)$ at a point represents the *slope* of the tangent line to the graph of the function at that point. The derivative of a function that depends on two or more independent variables with respect to one variable while holding the other variables constant is called the partial derivative. Ordinary and partial derivatives are equivalent for functions that depend on a single independent variable.

2-130C The order of a derivative represents the number of times a function is differentiated, whereas the degree of a derivative represents how many times a derivative is multiplied by itself. For example, y''' is the third order derivative of y, whereas $(y')^3$ is the third degree of the first derivative of y.

2-131C For a function $f(x, y)$, the partial derivative $\partial f / \partial x$ will be equal to the ordinary derivative df / dx when f does not depend on y or this dependence is negligible.

2-132C For a function $f(x)$, the derivative df / dx does not have to be a function of x. The derivative will be a constant when the f is a linear function of x.

2-133C Integration is the inverse of derivation. Derivation increases the order of a derivative by one, integration reduces it by one.

2-134C A differential equation involves derivatives, an algebraic equation does not.

2-135C A differential equation that involves only ordinary derivatives is called an ordinary differential equation, and a differential equation that involves partial derivatives is called a partial differential equation.

2-136C The order of a differential equation is the order of the highest order derivative in the equation.

2-137C A differential equation is said to be **linear** if the dependent variable and all of its derivatives are of the first degree, and their coefficients depend on the independent variable only. In other words, a differential equation is linear if it can be written in a form which does not involve (1) any powers of the dependent variable or its derivatives such as y^3 or $(y')^2$, (2) any products of the dependent variable or its derivatives such as yy' or $y'y'''$, and (3) any other nonlinear functions of the dependent variable such as $\sin y$ or e^y. Otherwise, it is **nonlinear**.

2-138C A linear homogeneous differential equation of order *n* is expressed in the most general form as

$$y^{(n)} + f_1(x)y^{(n-1)} + \cdots + f_{n-1}(x)y' + f_n(x)y = 0$$

Each term in a linear homogeneous equation contains the dependent variable or one of its derivatives after the equation is cleared of any common factors. The equation $y'' - 4x^2 y = 0$ is linear and homogeneous since each term is linear in y, and contains the dependent variable or one of its derivatives.

2-139C A differential equation is said to have **constant coefficients** if the coefficients of all the terms which involve the dependent variable or its derivatives are constants. If, after cleared of any common factors, any of the terms with the dependent variable or its derivatives involve the independent variable as a coefficient, that equation is said to have **variable coefficients** The equation $y'' - 4x^2 y = 0$ has variable coefficients whereas the equation $y'' - 4y = 0$ has constant coefficients.

2-140C A linear differential equation that involves a single term with the derivatives can be solved by direct integration.

2-141C The general solution of a 3rd order linear and homogeneous differential equation will involve 3 arbitrary constants.

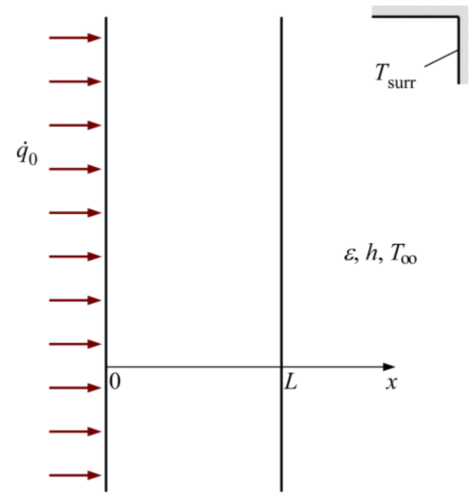
Review Problems

2-142 A plane wall is subjected to uniform heat flux on the left surface, while the right surface is subjected to convection and radiation heat transfer. The boundary conditions and the differential equation of this heat conduction problem are to be obtained.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall. **4** The left surface at $x = 0$ is subjected to uniform heat flux while the right surface at $x = L$ is subjected to convection and radiation. **5** The surrounding temperature is $T_\infty = T_{surr}$.

Analysis Taking the direction normal to the surface of the wall to be the x direction with $x = 0$ at the left surface, the differential equation for heat conduction can be expressed as

$$\frac{d^2T}{dx^2} = 0$$



The boundary conditions for the left and right surfaces are

$$x = 0: \qquad -k\frac{dT(0)}{dx} = \dot{q}_0$$

$$x = L: \qquad -k\frac{dT(L)}{dx} = h[T(L) - T_\infty] + \varepsilon\sigma[T(L)^4 - T_{surr}^4]$$

where

$$T_\infty = T_{surr}$$

Discussion Due to the radiation heat transfer equation, all temperatures are expressed in absolute temperatures, i.e. K or °R.

2-143 A long rectangular bar is initially at a uniform temperature of T_i. The surfaces of the bar at $x = 0$ and $y = 0$ are insulated while heat is lost from the other two surfaces by convection. The mathematical formulation of this heat conduction problem is to be expressed for transient two-dimensional heat transfer with no heat generation.

Assumptions **1** Heat transfer is transient and two-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation.

Analysis The differential equation and the boundary conditions for this heat conduction problem can be expressed as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

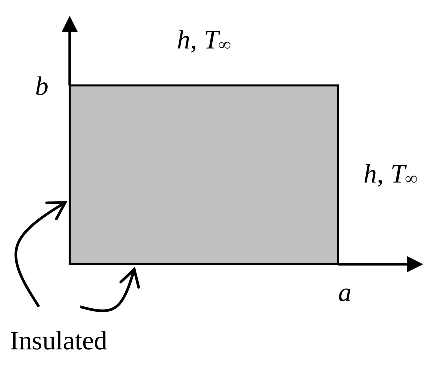
$$\frac{\partial T(x,0,t)}{\partial x} = 0$$

$$\frac{\partial T(0,y,t)}{\partial y} = 0$$

$$-k\frac{\partial T(a,y,t)}{\partial y} = h[T(a,y,t) - T_\infty]$$

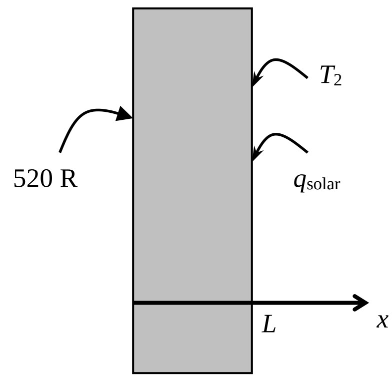
$$-k\frac{\partial T(x,b,t)}{\partial x} = h[T(x,b,t) - T_\infty]$$

$$T(x,y,0) = T_i$$



2-144E A large plane wall is subjected to a specified temperature on the left (inner) surface and solar radiation and heat loss by radiation to space on the right (outer) surface. The temperature of the right surface of the wall and the rate of heat transfer are to be determined when steady operating conditions are reached.

Assumptions **1** Steady operating conditions are reached. **2** Heat transfer is one-dimensional since the wall is large relative to its thickness, and the thermal conditions on both sides of the wall are uniform. **3** Thermal properties are constant. **4** There is no heat generation in the wall.



Properties The properties of the plate are given to be k = 1.2 Btu/h·ft·°F and ε = 0.80, and $\alpha_s = 0.60$.

Analysis In steady operation, heat conduction through the wall must be equal to net heat transfer from the outer surface. Therefore, taking the outer surface temperature of the plate to be T_2 (absolute, in *R*),

$$kA_s \frac{T_1 - T_2}{L} = \varepsilon\sigma A_s T_2^4 - \alpha_s A_s \dot{q}_{\text{solar}}$$

Canceling the area *A* and substituting the known quantities,

$$(1.2\ \text{Btu/h}\cdot\text{ft}\cdot°\text{F})\frac{(520\ \text{R}) - T_2}{0.8\ \text{ft}} = 0.8(0.1714\times10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)T_2^4 - 0.60(300\ \text{Btu/h}\cdot\text{ft}^2)$$

Solving for T_2 gives the outer surface temperature to be

T_2 = **553.9 R**

Then the rate of heat transfer through the wall becomes

$$\dot{q} = k\frac{T_1 - T_2}{L} = (1.2\ \text{Btu/h}\cdot\text{ft}\cdot°\text{F})\frac{(520 - 553.9)\ \text{R}}{0.8\ \text{ft}} = \mathbf{-50.9\ Btu/h\cdot ft^2} \quad \text{(per unit area)}$$

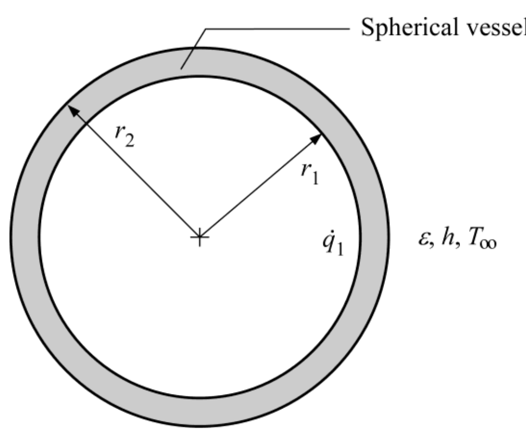
Discussion The negative sign indicates that the direction of heat transfer is from the outside to the inside. Therefore, the structure is gaining heat.

2-145 A spherical vessel is subjected to uniform heat flux on the inner surface, while the outer surface is subjected to convection and radiation heat transfer. The boundary conditions and the differential equation of this heat conduction problem are to be obtained.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation in the wall. **4** The inner surface at $r = r_1$ is subjected to uniform heat flux while the outer surface at $r = r_2$ is subjected to convection and radiation. **5** The surrounding temperature is $T_\infty = T_{\text{surr}}$.

Analysis For one-dimensional heat transfer in the radial *r* direction, the differential equation for heat conduction in spherical coordinate can be expressed as

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$



The boundary conditions for the inner and outer surfaces are

$$r = r_1: \quad -k\frac{dT(r_1)}{dr} = \dot{q}_1$$

$$r = r_2: \quad -k\frac{dT(r_2)}{dr} = h[T(r_2) - T_\infty] + \varepsilon\sigma[T(r_2)^4 - T_{\text{surr}}^4]$$

where $T_\infty = T_{\text{surr}}$

Discussion Due to the radiation heat transfer equation, all temperatures are expressed in absolute temperatures, i.e. K or °R.

2-146 Heat is generated at a constant rate in a short cylinder. Heat is lost from the cylindrical surface at $r = r_o$ by convection to the surrounding medium at temperature T_∞ with a heat transfer coefficient of h. The bottom surface of the cylinder at $r = 0$ is insulated, the top surface at $z = H$ is subjected to uniform heat flux $\dot{q}_h$, and the cylindrical surface at $r = r_o$ is subjected to convection. The mathematical formulation of this problem is to be expressed for steady two-dimensional heat transfer.

Assumptions **1** Heat transfer is given to be steady and two-dimensional. **2** Thermal conductivity is constant. **3** Heat is generated uniformly.

Analysis The differential equation and the boundary conditions for this heat conduction problem can be expressed as

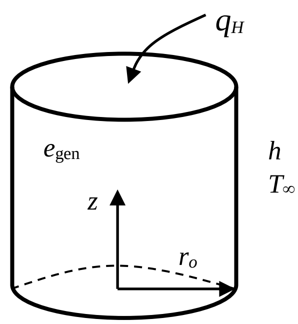
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{e}_{\text{gen}}}{k}=0$$

$$\frac{\partial T(r,0)}{\partial z}=0$$

$$k\frac{\partial T(r,H)}{\partial z}=\dot{q}_H$$

$$\frac{\partial T(0,z)}{\partial r}=0$$

$$-k\frac{\partial T(r_o,z)}{\partial r}=h[T(r_o,z)-T_\infty]$$

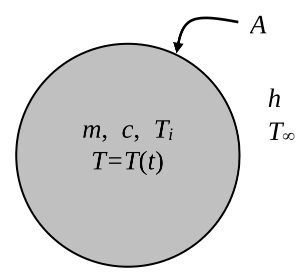


2-147 A small hot metal object is allowed to cool in an environment by convection. The differential equation that describes the variation of temperature of the ball with time is to be derived.

Assumptions **1** The temperature of the metal object changes uniformly with time during cooling so that $T = T(t)$. **2** The density, specific heat, and thermal conductivity of the body are constant. **3** There is no heat generation.

Analysis Consider a body of arbitrary shape of mass m, volume V, surface area A, density ρ, and specific heat c_p initially at a uniform temperature T_i. At time $t = 0$, the body is placed into a medium at temperature T_∞, and heat transfer takes place between the body and its environment with a heat transfer coefficient h.

During a differential time interval dt, the temperature of the body rises by a differential amount dT. Noting that the temperature changes with time only, an energy balance of the solid for the time interval dt can be expressed as



$$\begin{pmatrix}\text{Heat transfer from the body}\\ \text{during } dt\end{pmatrix}=\begin{pmatrix}\text{The decrease in the energy}\\ \text{of the body during } dt\end{pmatrix}$$

or $\quad hA_s(T-T_\infty)dt = mc_p(-dT)$

Noting that $m=\rho V$ and $dT = d(T-T_\infty)$ since $T_\infty =$ constant, the equation above can be rearranged as

$$\frac{d(T-T_\infty)}{T-T_\infty}=-\frac{hA_s}{\rho V c_p}dt$$

which is the desired differential equation.

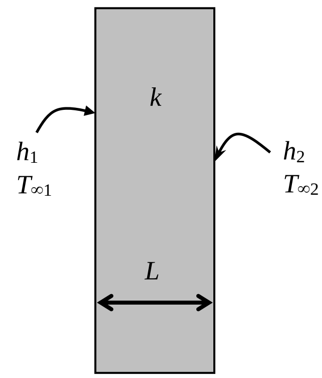
2-148 A large plane wall is subjected to convection on the inner and outer surfaces. The mathematical formulation, the variation of temperature, and the temperatures at the inner and outer surfaces to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation.

Properties The thermal conductivity is given to be k = 0.77 W/m·°C.

Analysis (*a*) Taking the direction normal to the surface of the wall to be the *x* direction with *x* = 0 at the inner surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$



and

$$h_1[T_{\infty 1} - T(0)] = -k\frac{dT(0)}{dx}$$

$$-k\frac{dT(L)}{dx} = h_2[T(L) - T_{\infty 2}]$$

(*b*) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

x = 0: $\quad h_1[T_{\infty 1} - (C_1 \times 0 + C_2)] = -kC_1$

x = *L*: $\quad -kC_1 = h_2[(C_1L + C_2) - T_{\infty 2}]$

Substituting the given values, these equations can be written as

$$8(22 - C_2) = -0.77C_1$$

$$-0.77C_1 = (12)(0.2C_1 + C_2 - 8)$$

Solving these equations simultaneously give

$$C_1 = -38.84 \qquad C_2 = 18.26$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(x) = 18.26 - 38.84x$$

(*c*) The temperatures at the inner and outer surfaces are

$$T(0) = 18.26 - 38.84 \times 0 = \mathbf{18.3°C}$$

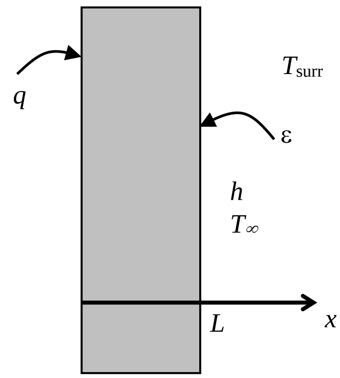
$$T(L) = 18.26 - 38.84 \times 0.2 = \mathbf{10.5°C}$$

2-149 The base plate of an iron is subjected to specified heat flux on the left surface and convection and radiation on the right surface. The mathematical formulation, and an expression for the outer surface temperature and its value are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation. **4** Heat loss through the upper part of the iron is negligible.

Properties The thermal conductivity and emissivity are given to be k = 18 W/m·°C and ε = 0.7.

Analysis (a) Noting that the upper part of the iron is well insulated and thus the entire heat generated in the resistance wires is transferred to the base plate, the heat flux through the inner surface is determined to be



$$\dot{q}_0 = \frac{\dot{Q}_0}{A_{\text{base}}} = \frac{1000\text{ W}}{150\times10^{-4}\text{ m}^2} = 66{,}667\text{ W/m}^2$$

Taking the direction normal to the surface of the wall to be the x direction with x = 0 at the left surface, the mathematical formulation of this problem can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

and $$-k\frac{dT(0)}{dx} = \dot{q}_0 = 66{,}667\text{ W/m}^2$$

$$-k\frac{dT(L)}{dx} = h[T(L)-T_\infty] + \varepsilon\sigma[T(L)^4 - T_{\text{surr}}^4] = h[T_2 - T_\infty] + \varepsilon\sigma[(T_2+273)^4 - T_{\text{surr}}^4]$$

(b) Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

x = 0: $$-kC_1 = \dot{q}_0 \;\rightarrow\; C_1 = -\frac{\dot{q}_0}{k}$$

$x = L$: $$-kC_1 = h[T_2 - T_\infty] + \varepsilon\sigma[(T_2+273)^4 - T_{\text{surr}}^4]$$

Eliminating the constant C_1 from the two relations above gives the following expression for the outer surface temperature T_2,

$$h(T_2 - T_\infty) + \varepsilon\sigma[(T_2+273)^4 - T_{\text{surr}}^4] = \dot{q}_0$$

(c) Substituting the known quantities into the implicit relation above gives

$$(30\text{ W/m}^2\cdot{}^\circ\text{C})(T_2 - 26) + 0.7(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(T_2+273)^4 - 295^4] = 66{,}667\text{ W/m}^2$$

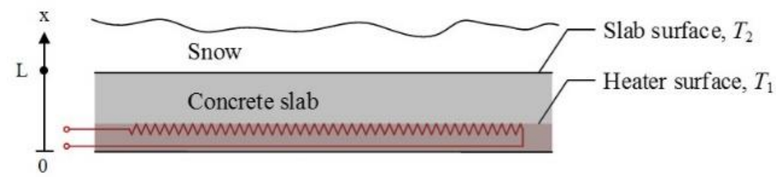
Using an equation solver (or a trial and error approach), the outer surface temperature is determined from the relation above to be

T_2 = **759°C**

2-150 A 30 m^2 concrete slab with embedded heating cable melts snow at a rate of 0.1 kg/s. Formulate the temperature profile in the concrete slab in terms of the snow melt rate. The power density for the embedded heater is to be determined whether it is in compliance with the NFPA 70 code.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through the concrete slab. **3** The bottom surface at $x = 0$ is subjected to uniform heat flux from the heating cable. **4** The upper surface at $x = L$ is at a constant temperature of 0°C from the snow melt. **5** There is no heat generation in the concrete slab. **6** Thermal properties are constant.

Properties The latent heat of fusion for water is 333.7 kJ/kg (Table A-2).



Analysis Taking the direction normal to the surface of the concrete slab to be the x direction with $x = 0$ at the bottom surface (the surface that is in contact with the heater surface), the differential equation for heat conduction can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1 x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions yields

$$x = 0: \qquad -k\frac{dT(0)}{dx} = \dot{q}_0 = -kC_1 \qquad \rightarrow \qquad C_1 = -\frac{\dot{q}_0}{k}$$

$$x = L: \qquad T(L) = T_L = C_1 L + C_2 \qquad \rightarrow \qquad C_2 = T_L - C_1 L = T_L + \frac{\dot{q}_0}{k}L$$

Substituting C_1 and C_2 into the general solution yields

$$T(x) = \frac{\dot{q}_0}{k}(L - x) + T_L$$

The heat rate required for melting snow can be determined from the latent heat of fusion for water,

$$\dot{Q} = \dot{m}_{\text{ice}} h_{if}$$

For a surface area of 30 m^2, the heat flux is determined using

$$\dot{q}_0 = \frac{\dot{Q}}{A_s} = \frac{\dot{m}_{\text{ice}} h_{if}}{A_s}$$

Therefore, the temperature profile in the concrete slab in terms of the snow melt rate is

$$\boldsymbol{T(x) = \frac{\dot{m}_{\text{ice}} h_{if}}{A_s k}(L - x) + T_L}$$

The power density (heat flux) for the heater to melt snow at 0.1 kg/s is

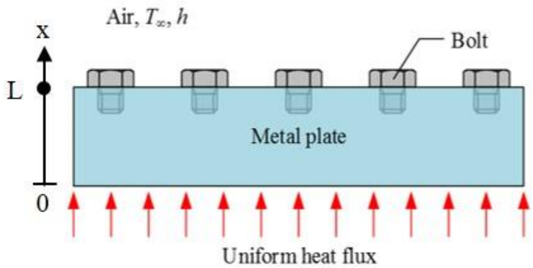
$$\dot{q}_0 = \frac{\dot{m}_{\text{ice}} h_{if}}{A_s} = \frac{(0.1\text{ kg/s})(333700\text{ J/kg})}{30\text{ m}^2} = \mathbf{1112\ W/m^2} < 1300\text{ W/m}^2$$

Discussion The power density for the embedded heating cable in the concrete slab is below the limit set by the National Electrical Code® (NFPA 70) of 1300 W/m^2. So, melting the snow at 0.1 kg/s is in compliance with the code.

2-151 C&S A series of ASME SA-193 carbon steel bolts are bolted on the upper surface of a metal plate. The upper surface is exposed to convection with the ambient air. The bottom surface is subjected to a uniform heat flux. Formulate the variation of temperature in the metal plate, and determine the temperatures at x = 0, 1.5, and 3.0 cm. The compliance of the SA-193 bolts with the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HF-300) is to be determined.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through the metal plate. **3** The bottom surface at x = 0 is subjected to uniform heat flux while the upper surface at $x = L$ is at uniform temperature. **4** There is no heat generation in the plate. **5** Thermal properties are constant.

Properties The thermal conductivity of the metal plate is given as 15 W/m·K.



Analysis Taking the direction normal to the surface of the plate to be the x direction with x = 0 at the bottom surface, the differential equation for heat conduction can be expressed as

$$\frac{d^2T}{dx^2} = 0$$

Integrating the differential equation twice with respect to x yields

$$\frac{dT}{dx} = C_1$$

$$T(x) = C_1x + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions yields

$$x = 0: \qquad -k\frac{dT(0)}{dx} = \dot{q}_0 = -kC_1 \qquad \rightarrow \qquad C_1 = -\frac{\dot{q}_0}{k}$$

$$x = L: \qquad T(L) = T_L = C_1L + C_2 \qquad \rightarrow \qquad C_2 = -C_1L + T_L = \frac{\dot{q}_0}{k}L + \frac{\dot{q}_0}{h} + T_\infty$$

Note that the uniform heat flux on the bottom plate surface (x = 0) is equal to the heat flux transferred by convection on the upper surface ($x = L$):

$$\dot{q}_0 = h[T(L) - T_\infty] \qquad \rightarrow \qquad T(L) = \frac{\dot{q}_0}{h} + T_\infty$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$\boldsymbol{T(x) = -\frac{\dot{q}_0}{k}x + \frac{\dot{q}_0}{k}L + \frac{\dot{q}_0}{h} + T_\infty = \frac{\dot{q}_0}{k}(L - x) + \frac{\dot{q}_0}{h} + T_\infty}$$

The temperatures in the metal plate at x = 0, 1.5, and 3.0 cm are

$$\text{At } x = 0: \qquad T(0) = \frac{5000\ \text{W/m}^2}{15\ \text{W/m·K}}(0.030\ \text{m} - 0) + \frac{5000\ \text{W/m}^2}{10\ \frac{\text{W}}{\text{m}^2\text{·K}}} + 30°\text{C} = \mathbf{540}°\text{C}$$

$$\text{At } x = 0.015\ \text{m}: \qquad T(0.015) = \frac{5000\ \text{W/m}^2}{15\ \text{W/m·K}}(0.030\ \text{m} - 0.015\ \text{m}) + \frac{5000\ \text{W/m}^2}{10\ \frac{\text{W}}{\text{m}^2\text{·K}}} + 30°\text{C} = \mathbf{535}°\text{C}$$

$$\text{At } x = 0.030\ \text{m}: \qquad T(0.030) = \frac{5000\ \text{W/m}^2}{15\ \text{W/m·K}}(0.030\ \text{m} - 0.030\ \text{m}) + \frac{5000\ \text{W/m}^2}{10\ \frac{\text{W}}{\text{m}^2\text{·K}}} + 30°\text{C} = \mathbf{530}°\text{C}$$

Discussion The entire metal plate is above 260°C. The minimum temperature in the metal plate is at the upper surface that is exposed to convection with the ambient air, 530°C at x = 3 cm. Therefore, the SA-193 bolts would not comply with the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HF-300).

2-152 A steam pipe is subjected to convection on both the inner and outer surfaces. The mathematical formulation of the problem and expressions for the variation of temperature in the pipe and on the outer surface temperature are to be obtained for steady one-dimensional heat transfer.

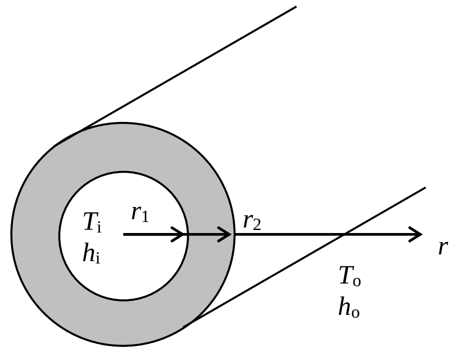
Assumptions **1** Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the center line. **2** Thermal conductivity is constant. **3** There is no heat generation in the pipe.

Analysis (*a*) Noting that heat transfer is steady and one-dimensional in the radial *r* direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

and
$$-k\frac{dT(r_1)}{dr}=h_i[T_i-T(r_1)]$$

$$-k\frac{dT(r_2)}{dr}=h_o[T(r_2)-T_o]$$



(*b*) Integrating the differential equation once with respect to r gives

$$r\frac{dT}{dr}=C_1$$

Dividing both sides of the equation above by r to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr}=\frac{C_1}{r}$$

$$T(r)=C_1\ln r+C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_1$: $$-k\frac{C_1}{r_1}=h_i[T_i-(C_1\ln r_1+C_2)]$$

$r = r_2$: $$-k\frac{C_1}{r_2}=h_o[(C_1\ln r_2+C_2)-T_o]$$

Solving for C_1 and C_2 simultaneously gives

$$C_1=\frac{T_0-T_i}{\ln\frac{r_2}{r_1}+\frac{k}{h_ir_1}+\frac{k}{h_or_2}} \quad \text{and} \quad C_2=T_i-C_1\left(\ln r_1-\frac{k}{h_ir_1}\right)=T_i-\frac{T_0-T_i}{\ln\frac{r_2}{r_1}+\frac{k}{h_ir_1}+\frac{k}{h_or_2}}\left(\ln r_1-\frac{k}{h_ir_1}\right)$$

Substituting C_1 and C_2 into the general solution and simplifying, we get the variation of temperature to be

$$T(r)=C_1\ln r+T_i-C_1(\ln r_1-\frac{k}{h_ir_1})=T_i+\frac{(T_0-T_i)\ln\frac{r}{r_1}+\frac{k}{h_ir_1}}{\ln\frac{r_2}{r_1}+\frac{k}{h_ir_1}+\frac{k}{h_or_2}}$$

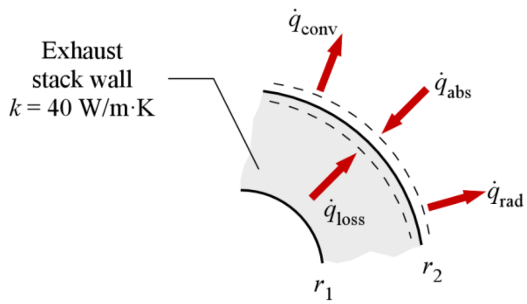
(*c*) The outer surface temperature is determined by simply replacing r in the relation above by r_2. We get

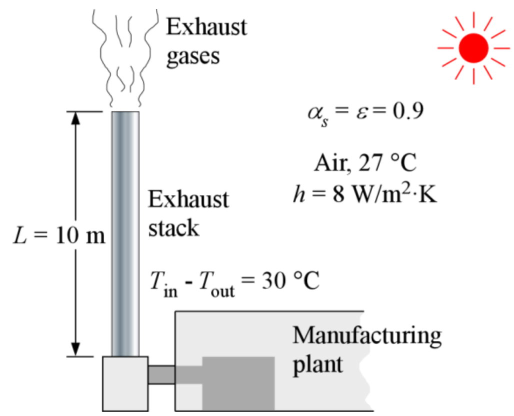
$$T(r_2)=T_i+\frac{(T_0-T_i)\ln\frac{r_2}{r_1}+\frac{k}{h_ir_1}}{\ln\frac{r_2}{r_1}+\frac{k}{h_ir_1}+\frac{k}{h_or_2}}$$

2-153 A 10-m tall exhaust stack discharging exhaust gases at a rate of 1.2 kg/s is subjected to solar radiation and convection at the outer surface. The variation of temperature in the exhaust stack and the inner surface temperature of the exhaust stack are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional and there is thermal symmetry about the centerline. **2** Thermal properties are constant. **3** There is no heat generation in the pipe.

Properties The constant pressure specific heat of exhaust gases is given to be 1600 J/kg · °C and the pipe thermal conductivity is 40 W/m · K. Both the emissivity and solar absorptivity of the exhaust stack outer surface are 0.9.





Analysis The outer and inner radii of the pipe are

$$r_2 = 1\,\text{m}/2 = 0.5\,\text{m}$$

$$r_1 = 0.5\,\text{m} - 0.1\,\text{m} = 0.4\,\text{m}$$

The outer surface area of the exhaust stack is

$$A_{s,2} = 2\pi r_2 L = 2\pi(0.5\,\text{m})(10\,\text{m}) = 31.42\,\text{m}^2$$

The rate of heat loss from the exhaust gases in the exhaust stack can be determined from

$$\dot{Q}_{\text{loss}} = \dot{m}c_p(T_{\text{in}} - T_{\text{out}}) = (1.2\,\text{kg/s})(1600\,\text{J/kg}\cdot{}^\circ\text{C})(30)\,{}^\circ\text{C} = 57600\,\text{W}$$

The heat loss on the outer surface of the exhaust stack by radiation and convection can be expressed as

$$\frac{\dot{Q}_{\text{loss}}}{A_{s,2}} = h[T(r_2) - T_\infty] + \varepsilon\sigma[T(r_2)^4 - T_{\text{surr}}^4] - \alpha_s\dot{q}_{\text{solar}}$$

$$\frac{57600\,\text{W}}{31.42\,\text{m}^2} = (8\,\text{W/m}^2\cdot\text{K})[T(r_2) - (27+273)]\,\text{K}$$
$$+ (0.9)(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4)[T(r_2)^4 - (27+273)^4]\,\text{K}^4 - (0.9)(150\,\text{W/m}^2)$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
57600/31.42=8*(T_r2-(27+273))+0.9*5.67e-8*(T_r2^4-(27+273)^4)-0.9*150
```

Solving by EES software, the outside surface temperature of the furnace front is

$$T(r_2) = 412.7\,\text{K}$$

(*a*) For steady one-dimensional heat conduction in cylindrical coordinates, the heat conduction equation can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

and $$-k\frac{dT(r_1)}{dr} = \frac{\dot{Q}_{\text{loss}}}{A_{s,1}} = \frac{\dot{Q}_{\text{loss}}}{2\pi r_1 L} \quad \text{(heat flux at the inner exhaust stack surface)}$$

$$T(r_2) = 412.7\,\text{K} \quad \text{(outer exhaust stack surface temperature)}$$

Integrating the differential equation once with respect to r gives

$$\frac{dT}{dr} = \frac{C_1}{r}$$

Integrating with respect to r again gives

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions gives

$$r = r_1: \qquad \frac{dT(r_1)}{dr} = -\frac{1}{k}\frac{\dot{Q}_{\text{loss}}}{2\pi r_1 L} = \frac{C_1}{r_1} \quad \rightarrow \quad C_1 = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}$$

$$r = r_2: \qquad T(r_2) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_2 + C_2 \quad \rightarrow \quad C_2 = \frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_2 + T(r_2)$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r + \frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln r_2 + T(r_2)$$

$$= -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln(r / r_2) + T(r_2)$$

(b) The inner surface temperature of the exhaust stack is

$$T(r_1) = -\frac{1}{2\pi}\frac{\dot{Q}_{\text{loss}}}{kL}\ln(r_1 / r_2) + T(r_2)$$

$$= -\frac{1}{2\pi}\frac{57600 \text{ W}}{(40 \text{ W/m}\cdot\text{K})(10 \text{ m})}\ln\left(\frac{0.4}{0.5}\right) + 412.7 \text{ K}$$

$$= 417.7 \text{ K} = \mathbf{418\ K}$$

Discussion There is a temperature drop of 5 °C from the inner to the outer surface of the exhaust stack.

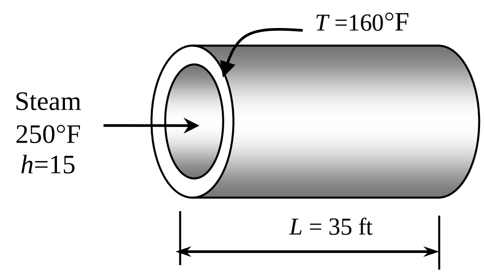
2-154E A steam pipe is subjected to convection on the inner surface and to specified temperature on the outer surface. The mathematical formulation, the variation of temperature in the pipe, and the rate of heat loss are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the center line. **2** Thermal conductivity is constant. **3** There is no heat generation in the pipe.

Properties The thermal conductivity is given to be $k = 8$ Btu/h·ft·°F.

Analysis (*a*) Noting that heat transfer is one-dimensional in the radial *r* direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$



The boundary conditions for this problem are:

$$-k\frac{dT(r_1)}{dr} = h[T_\infty - T(r_1)]$$

$$T(r_2) = T_2 = 160°\text{F}$$

(*b*) Integrating the differential equation once with respect to *r* gives

$$r\frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by *r* to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_1$: $\quad -k\dfrac{C_1}{r_1} = h[T_\infty - (C_1 \ln r_1 + C_2)]$

$r = r_2$: $\quad T(r_2) = C_1 \ln r_2 + C_2 = T_2$

Solving for C_1 and C_2 simultaneously gives

$$C_1 = \frac{T_2 - T_\infty}{ln\dfrac{r_2}{r_1} + \dfrac{k}{hr_1}} \quad \text{and} \quad C_2 = T_2 - C_1 ln\, r_2 = T_2 - \frac{T_2 - T_\infty}{ln\dfrac{r_2}{r_1} + \dfrac{k}{hr_1}} ln\, r_2$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = C_1 \ln r + T_2 - C_1 \ln r_2 = C_1(\ln r - \ln r_2) + T_2 = \frac{T_2 - T_\infty}{\ln\dfrac{r_2}{r_1} + \dfrac{k}{hr_1}} \ln\frac{r}{r_2} + T_2$$

$$= \frac{(160 - 250)°\text{F}}{\ln\dfrac{2.4}{2} + \dfrac{8\text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}{(15\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(2/12\text{ ft})}} \ln\frac{r}{2.4\text{ in}} + 160°\text{F} = -26.61\ln\frac{r}{2.4\text{ in}} + 160°\text{F}$$

(*c*) The rate of heat conduction through the pipe is

$$\dot{Q} = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{C_1}{r} = -2\pi Lk\frac{T_2 - T_\infty}{\ln\dfrac{r_2}{r_1} + \dfrac{k}{hr_1}}$$

$$= -2\pi(35\text{ ft})(8\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})\frac{(160 - 250)°\text{F}}{\ln\dfrac{2.4}{2} + \dfrac{8\text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}{(15\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(2/12\text{ ft})}} = \mathbf{46{,}813\ Btu/h}$$

2-155 A compressed air pipe is subjected to uniform heat flux on the outer surface and convection on the inner surface. The mathematical formulation, the variation of temperature in the pipe, and the surface temperatures are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the center line. **2** Thermal conductivity is constant. **3** There is no heat generation in the pipe.

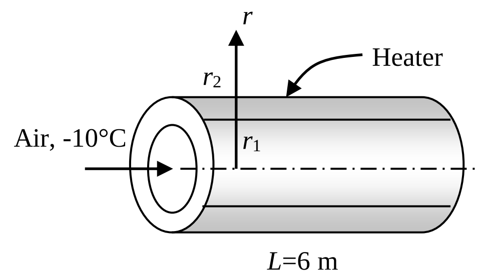
Properties The thermal conductivity is given to be k = 14 W/m·K.

Analysis (*a*) Noting that the 85% of the 300 W generated by the strip heater is transferred to the pipe, the heat flux through the outer surface is determined to be

$$\dot{q}_s = \frac{\dot{Q}_s}{A_2} = \frac{\dot{Q}_s}{2\pi r_2 L} = \frac{0.85\times 300\text{ W}}{2\pi(0.04\text{ m})(6\text{ m})} = 169.1\text{ W/m}^2$$

Noting that heat transfer is one-dimensional in the radial r direction and heat flux is in the negative r direction, the mathematical formulation of this problem can be expressed as

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$



The boundary conditions for this problem are:

$$-k\frac{dT(r_1)}{dr} = h[T_\infty - T(r_1)]$$

$$k\frac{dT(r_2)}{dr} = \dot{q}_s$$

(*b*) Integrating the differential equation once with respect to r gives

$$r\frac{dT}{dr} = C_1$$

Dividing both sides of the equation above by r to bring it to a readily integrable form and then integrating,

$$\frac{dT}{dr} = \frac{C_1}{r}$$

$$T(r) = C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_2$: $\quad k\frac{C_1}{r_2} = \dot{q}_s \;\rightarrow\; C_1 = \frac{\dot{q}_s r_2}{k}$

$r = r_1$: $\quad -k\frac{C_1}{r_1} = h[T_\infty - (C_1 \ln r_1 + C_2)] \;\rightarrow\; C_2 = T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)C_1 = T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)\frac{\dot{q}_s r_2}{k}$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = C_1 \ln r + T_\infty - \left(\ln r_1 - \frac{k}{hr_1}\right)C_1 = T_\infty + \left(\ln r - \ln r_1 + \frac{k}{hr_1}\right)C_1 = T_\infty + \left(\ln\frac{r}{r_1} + \frac{k}{hr_1}\right)\frac{\dot{q}_s r_2}{k}$$

$$= -10°\text{C} + \left(\ln\frac{\text{r}}{\text{r}_1} + \frac{14\text{ W/m}\cdot\text{K}}{(30\text{ W/m}^2\cdot\text{K})(0.037\text{ m})}\right)\frac{(169.1\text{ W/m}^2)(0.04\text{ m})}{14\text{ W/m}\cdot\text{K}} = -10 + 0.483\left(\ln\frac{r}{r_1} + 12.61\right)$$

(*c*) The inner and outer surface temperatures are determined by direct substitution to be

Inner surface ($r = r_1$): $\quad T(r_1) = -10 + 0.483\left(\ln\frac{r_1}{r_1} + 12.61\right) = -10 + 0.483(0 + 12.61) = \mathbf{-3.91°C}$

Outer surface ($r = r_2$): $\quad T(r_2) = -10 + 0.483\left(\ln\frac{r_2}{r_1} + 12.61\right) = -10 + 0.483\left(\ln\frac{0.04}{0.037} + 12.61\right) = \mathbf{-3.87°C}$

Discussion Note that the pipe is essentially isothermal at a temperature of about -3.9°C.

2-156 A hollow pipe is subjected to specified temperatures at the inner and outer surfaces. There is also heat generation in the pipe. The variation of temperature in the pipe and the center surface temperature of the pipe are to be determined for steady one-dimensional heat transfer.

Assumptions **1** Heat conduction is steady and one-dimensional since the pipe is long relative to its thickness, and there is thermal symmetry about the centerline. **2** Thermal conductivity is constant.

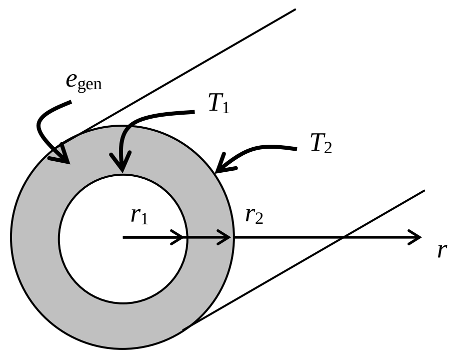
Properties The thermal conductivity is given to be $k = 14$ W/m·°C.

Analysis The rate of heat generation is determined from

$$\dot{e}_{\text{gen}} = \frac{\dot{W}}{V} = \frac{\dot{W}}{\pi(D_2^2 - D_1^2)L/4} = \frac{25{,}000 \text{ W}}{\pi\left[(0.4 \text{ m})^2 - (0.3 \text{ m})^2\right](17 \text{ m})/4} = 26{,}750 \text{ W/m}^3$$

Noting that heat transfer is one-dimensional in the radial r direction, the mathematical formulation of this problem can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{e}_{\text{gen}}}{k} = 0$$



and $\quad T(r_1) = T_1 = 60°\text{C}$

$$T(r_2) = T_2 = 80°\text{C}$$

Rearranging the differential equation

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = \frac{-\dot{e}_{\text{gen}}\, r}{k} = 0$$

and then integrating once with respect to r,

$$r\frac{dT}{dr} = \frac{-\dot{e}_{\text{gen}}\, r^2}{2k} + C_1$$

Rearranging the differential equation again

$$\frac{dT}{dr} = \frac{-\dot{e}_{\text{gen}}\, r}{2k} + \frac{C_1}{r}$$

and finally integrating again with respect to r, we obtain

$$T(r) = \frac{-\dot{e}_{\text{gen}}\, r^2}{4k} + C_1 \ln r + C_2$$

where C_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$r = r_1$: $\qquad T(r_1) = \dfrac{-\dot{e}_{\text{gen}}\, r_1^2}{4k} + C_1 \ln r_1 + C_2$

$r = r_2$: $\qquad T(r_2) = \dfrac{-\dot{e}_{\text{gen}}\, r_2^2}{4k} + C_1 \ln r_2 + C_2$

Substituting the given values, these equations can be written as

$$60 = \frac{-(26{,}750)(0.15)^2}{4(14)} + C_1 \ln(0.15) + C_2$$

$$80 = \frac{-(26{,}750)(0.20)^2}{4(14)} + C_1 \ln(0.20) + C_2$$

Solving for C_1 and C_2 simultaneously gives

$$C_1 = 98.58 \qquad C_2 = 257.8$$

Substituting C_1 and C_2 into the general solution, the variation of temperature is determined to be

$$T(r) = \frac{-26{,}750 r^2}{4(14)} + 98.58 \ln r + 257.8 = 257.8 - 477.7 r^2 + 98.58 \ln r$$

The temperature at the center surface of the pipe is determined by setting radius r to be 17.5 cm, which is the average of the inner radius and outer radius.

$$T(r) = 257.8 - 477.7(0.175)^2 + 98.58 \ln(0.175) = \mathbf{71.3°C}$$

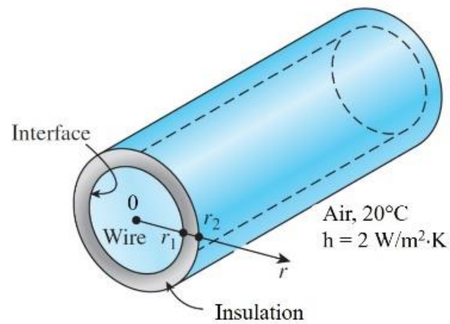
2-157 C&S A long electrical resistance wire that is generating heat uniformly is covered with polyethylene insulation. Formulate the temperature profiles for the wire and the polyethylene insulation. Determine the temperature at the interface of the wire and the insulation, and the temperature at the center of the wire. Conclude whether the polyethylene insulation for the wire meets the ASTM D1351 standard.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal conductivities are constant. **3** Heat generation in the wire is uniform. **4** There is no contact resistance at the interface of the wire and the insulation, $r = r_1$. **5** At the center of the wire, $r = 0$, is a symmetry boundary. **6** The outer surface of the insulation, $r = r_2$, is subjected to convection.

PropertiesThe thermal conductivities of the wire and the polyethylene insulation are given to be k_{wire} = 15 W/m·K and k_{ins} = 0.4 W/m·K, respectively.

AnalysisFor one-dimensional heat transfer in the radial r direction with uniform heat generation, the differential equation for heat conduction in cylindrical coordinate for the wire can be expressed as

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT_{\text{wire}}}{dr}\right)+\frac{\dot{e}_{\text{gen}}}{k_{\text{wire}}}=0 \qquad \text{or}\ \frac{d}{dr}\left(r\frac{dT_{\text{wire}}}{dr}\right)=-\frac{\dot{e}_{\text{gen}}}{k_{\text{wire}}}r$$



Integrating the differential equation twice with respect to r yields

$$r\frac{dT_{\text{wire}}}{dr}=-\frac{\dot{e}_{\text{gen}}}{2k_{\text{wire}}}r^2+C_1$$

$$T_{\text{wire}}(r)=-\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}r^2+C_1\ln r+C_2$$

whereC_1 and C_2 are arbitrary constants. Applying the boundary conditions give

$$r=0: \qquad \frac{dT_{\text{wire}}(0)}{dr}=0 \qquad \rightarrow \qquad C_1=0$$

$$r=r_1: \qquad T_{\text{wire}}(r_1)=T_I=-\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}r_1^2+C_2 \qquad \rightarrow \qquad C_2=T_I+\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}r_1^2$$

Substituting C_1 and C_2 into the general solution, the temperature profile in the wire is determined to be

$$\boldsymbol{T_{\text{wire}}(r)=T_I+\frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}}(r_1^2-r^2)}\text{for} \qquad 0\le r\le r_1$$

The insulation layer does not involve any heat generation, the heat conduction equation in the insulation layer is

$$\frac{d}{dr}\left(r\frac{dT_{\text{ins}}}{dr}\right)=0$$

Integrating the differential equation twice with respect to r yields

$$r\frac{dT_{\text{ins}}}{dr}=C_3\text{or}\frac{dT_{\text{ins}}}{dr}=\frac{C_3}{r}$$

$$T_{\text{ins}}(r)=C_3\ln r+C_4$$

whereC_3 and C_4 are arbitrary constants. Applying the boundary conditions yields

$$r=r_1: \qquad -k_{\text{wire}}\frac{dT_{\text{wire}}(r_1)}{dr}=-k_{\text{ins}}\frac{dT_{\text{ins}}(r_1)}{dr} \qquad \rightarrow \qquad \frac{\dot{e}_{\text{gen}}}{2}r_1=-k_{\text{ins}}\frac{C_3}{r_1}$$

$$r=r_2: \qquad -k_{\text{ins}}\frac{dT_{\text{ins}}(r_2)}{dr}=-k_{\text{ins}}\frac{C_3}{r_2}=h[T_{\text{ins}}(r_2)-T_\infty]$$

C_3 and C_4 can be expressed as

$$C_3 = -\frac{\dot{e}_{\text{gen}}}{2}\frac{r_1^2}{k_{\text{ins}}}$$

$$C_4 = T_\infty + \frac{\dot{e}_{\text{gen}} r_1^2}{2k_{\text{ins}}}\left(\frac{k_{\text{ins}}}{h}\frac{1}{r_2} + \ln r_2\right)$$

Substituting C_3 and C_4 into the general solution, the temperature profile in the insulation layer is determined to be

$$\boldsymbol{T_{\text{ins}}(r) = \frac{\dot{e}_{\text{gen}} r_1^2}{2k_{\text{ins}}}\left(\frac{k_{\text{ins}}}{h}\frac{1}{r_2} + \ln\frac{r_2}{r}\right) + T_\infty} \text{ for } r_1 \le r \le r_2$$

At the interface of the wire and the insulation, $r = r_1$, we have

$$T_I = T_{\text{ins}}(r_1) = \frac{\dot{e}_{\text{gen}} r_1^2}{2k_{\text{ins}}}\left(\frac{k_{\text{ins}}}{h}\frac{1}{r_2} + \ln\frac{r_2}{r_1}\right) + T_\infty$$

$$T_I = \frac{\left(5\times 10^5\,\frac{\text{W}}{\text{m}^3}\right)(0.0025\text{ m})^2}{2(0.4\text{ W/m·K})}\left(\frac{0.4\text{ W/m·K}}{2}\times\frac{1}{0.005\text{ m}} + \ln\frac{0.005\text{ m}}{0.0025\text{ m}}\right) + 20°\text{C} = \mathbf{178.96}°\text{C} > 75°\text{C}$$

whereand $r_2 = r_1$ + wall thickness = 0.0025 m + 0.0025 m = 0.005 m.

The temperature at the center of the wire, $r = 0$, is

$$T_{\text{wire}}(0) = T_I + \frac{\dot{e}_{\text{gen}}}{4k_{\text{wire}}} r_1^2 = 178.96 + \frac{\left(5\times 10^5\,\frac{\text{W}}{\text{m}^3}\right)(0.0025\text{ m})^2}{4(15\text{ W/m·K})} = \mathbf{179.01}°\text{C}$$

Discussion With the temperature at the interface of the wire and the insulation being about 104°C higher than the specification of the ASTM D1351 standard for polyethylene insulation, the ASTM standard is not met. If the convection heat transfer coefficient at the outer surface of the insulation is increase to 6 W/m^2·K or higher, the temperature at the interface of the wire and the insulation would be lower than 75°C.

2-158 In a quenching process, steel ball bearings at a given instant have a rate of temperature decrease of 50 K/s. The rate of heat loss is to be determined.

Assumptions **1** Heat conduction is one-dimensional. **2** There is no heat generation. **3** Thermal properties are constant.

Properties The properties of the steel ball bearings are given to be c = 500 J/kg · K, k = 60 W/m · K, and ρ = 7900 kg/m^3.

Analysis The thermal diffusivity on the steel ball bearing is

$$\alpha = \frac{k}{\rho c} = \frac{60\ \text{W/m}\cdot\text{K}}{(7900\ \text{kg/m}^3)(500\ \text{J/kg}\cdot\text{K})} = 15.19\times10^{-6}\ \text{m}^2\text{/s}$$

The given rate of temperature decrease can be expressed as

$$\frac{dT(r)}{dt} = -50\ \text{K/s}$$

For one-dimensional transient heat conduction in a sphere with no heat generation, the differential equation is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Substituting the thermal diffusivity and the rate of temperature decrease, the differential equation can be written as

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = \frac{-50\ \text{K/s}}{15.19\times10^{-6}\ \text{m}^2\text{/s}}$$

Multiply both sides of the differential equation by r^2 and rearranging gives

$$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = \frac{-50\ \text{K/s}}{15.19\times10^{-6}\ \text{m}^2\text{/s}}r^2$$

Integrating with respect to r gives

$$r^2\frac{dT}{dr} = \frac{-50\ \text{K/s}}{15.19\times10^{-6}\ \text{m}^2\text{/s}}\left(\frac{r^3}{3}\right) + C_1 \qquad (a)$$

Applying the boundary condition at the midpoint (thermal symmetry about the midpoint),

$$r = 0: \qquad 0\times\frac{dT(0)}{dr} = \frac{-50\ \text{K/s}}{15.19\times10^{-6}\ \text{m}^2\text{/s}}\left(\frac{0}{3}\right) + C_1 \quad\rightarrow\quad C_1 = 0$$

Dividing both sides of Eq. (a) by r^2 gives

$$\frac{dT}{dr} = \frac{-50\ \text{K/s}}{15.19\times10^{-6}\ \text{m}^2\text{/s}}\left(\frac{r}{3}\right)$$

The rate of heat loss through the steel ball bearing surface can be determined from Fourier's law to be

$$\begin{aligned}\dot{Q}_{\text{loss}} &= -kA\frac{dT}{dr}\\ &= -k(4\pi r_o^2)\frac{dT(r_o)}{dr} = k(4\pi r_o^2)\frac{50\ \text{K/s}}{15.19\times10^{-6}\ \text{m}^2\text{/s}}\left(\frac{r_o}{3}\right)\\ &= (60\ \text{W/m}\cdot\text{K})(4\pi)(0.125\ \text{m})^2\ \frac{50\ \text{K/s}}{15.19\times10^{-6}\ \text{m}^2\text{/s}}\left(\frac{0.125\ \text{m}}{3}\right)\\ &= \mathbf{1.62\ kW}\end{aligned}$$

Discussion The rate of heat loss through the steel ball bearing surface determined here is for the given instant when the rate of temperature decrease is 50 K/s.

2-159 A spherical reactor of 5-cm diameter operating at steady condition has its heat generation suddenly set to 9 MW/m^3. The time rate of temperature change in the reactor is to be determined.

Assumptions **1** Heat conduction is one-dimensional. **2** Heat generation is uniform. **3** Thermal properties are constant.

Properties The properties of the reactor are given to be c = 200 J/kg·°C, k = 40 W/m·°C, and ρ = 9000 kg/m^3.

Analysis The thermal diffusivity of the reactor is

$$\alpha = \frac{k}{\rho c} = \frac{40\ \text{W/m}\cdot{}^\circ\text{C}}{(9000\ \text{kg/m}^3)(200\ \text{J/kg}\cdot{}^\circ\text{C})} = 22.22\times10^{-6}\ \text{m}^2/\text{s}$$

For one-dimensional transient heat conduction in a sphere with heat generation, the differential equation is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{\dot{e}_{\text{gen}}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \text{or} \qquad \frac{\partial T}{\partial t}=\alpha\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{\dot{e}_{\text{gen}}}{k}\right]$$

At the instant when the heat generation of reactor is suddenly set to 90 MW/m^3 (t = 0), the temperature variation can be expressed by the given $T(r) = a - br^2$, hence

$$\frac{\partial T}{\partial t}=\alpha\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}(a-br^2)\right]+\frac{\dot{e}_{\text{gen}}}{k}\right\}=\alpha\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2(-2br)\right]+\frac{\dot{e}_{\text{gen}}}{k}\right\}$$

$$=\alpha\left[\frac{1}{r^2}(-6br^2)+\frac{\dot{e}_{\text{gen}}}{k}\right]=\alpha\left(-6b+\frac{\dot{e}_{\text{gen}}}{k}\right)$$

The time rate of temperature change in the reactor when the heat generation suddenly set to 9 MW/m^3 is determined to be

$$\frac{\partial T}{\partial t}=\alpha\left(-6b+\frac{\dot{e}_{\text{gen}}}{k}\right)=(22.22\times10^{-6}\ \text{m}^2/\text{s})\left[-6(5\times10^5\ {}^\circ\text{C/m}^2)+\frac{9\times10^6\ \text{W/m}^3}{40\ \text{W/m}\cdot{}^\circ\text{C}}\right]$$

$$= \mathbf{-61.7\,{}^\circ C/s}$$

Discussion Since the time rate of temperature change is a negative value, this indicates that the heat generation of reactor is suddenly decreased to 9 MW/m^3.

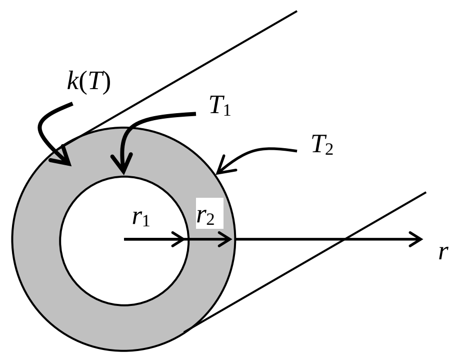
2-160 A cylindrical shell with variable conductivity is subjected to specified temperatures on both sides. The rate of heat transfer through the shell is to be determined.

Assumptions **1** Heat transfer is given to be steady and one-dimensional. **2** Thermal conductivity varies quadratically. **3** There is no heat generation.

Properties The thermal conductivity is given to be $k(T) = k_0(1+\beta T^2)$.

Analysis When the variation of thermal conductivity with temperature $k(T)$ is known, the average value of the thermal conductivity in the temperature range between T_1 and T_2 is determined from

$$k_{\text{avg}} = \frac{\int_{T_1}^{T_2} k(T)dT}{T_2 - T_1} = \frac{\int_{T_1}^{T_2} k_0(1+\beta T^2)dT}{T_2 - T_1} = \frac{k_0\left(T + \frac{\beta}{3}T^3\right)_{T_1}^{T_2}}{T_2 - T_1} = \frac{k_0\left[(T_2 - T_1) + \frac{\beta}{3}(T_2^3 - T_1^3)\right]}{T_2 - T_1} = k_0\left[1 + \frac{\beta}{3}(T_2^2 + T_1T_2 + T_1^2)\right]$$



This relation is based on the requirement that the rate of heat transfer through a medium with constant average thermal conductivity k_{avg} equals the rate of heat transfer through the same medium with variable conductivity $k(T)$. Then the rate of heat conduction through the cylindrical shell can be determined from Eq. 2-77 to be

$$\dot{Q}_{\text{cylinder}} = 2\pi k_{\text{avg}} L \frac{T_1 - T_2}{\ln(r_2 / r_1)} = 2\pi k_0\left[1 + \frac{\beta}{3}(T_2^2 + T_1T_2 + T_1^2)\right] L \frac{T_1 - T_2}{\ln(r_2 / r_1)}$$

Discussion We would obtain the same result if we substituted the given $k(T)$ relation into the second part of Eq. 2-77, and performed the indicated integration.

2-161 A pipe is used for transporting boiling water with a known inner surface temperature in a surrounding of cooler ambient temperature and known convection heat transfer coefficient. The pipe wall has a variable thermal conductivity. The outer surface temperature of the pipe is to be determined.

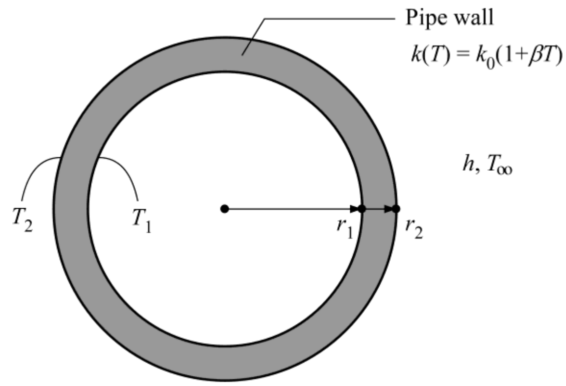
Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature. **4** Inner pipe surface temperature is constant at 100°C.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$.

Analysis The inner and outer radii of the pipe are

$$r_1 = 0.025/2\text{ m} = 0.0125\text{ m} \quad \text{and} \quad r_2 = (0.0125 + 0.003)\text{ m} = 0.0155\text{ m}$$

The rate of heat transfer at the pipe's outer surface can be expressed as



$$\dot{Q}_{\text{cylinder}} = \dot{Q}_{\text{conv}}$$

$$2\pi k_{\text{avg}} L \frac{T_1 - T_2}{\ln(r_2/r_1)} = h(2\pi r_2 L)(T_2 - T_\infty)$$

$$\frac{k_{\text{avg}}}{r_2} \frac{T_1 - T_2}{\ln(r_2/r_1)} = h(T_2 - T_\infty) \qquad (1)$$

where

$$h = 50\text{ W/m}^2\text{ K}, \quad T_1 = 373\text{ K}, \quad \text{and} \quad T_\infty = 293\text{ K}$$

The average thermal conductivity is

$$k_{\text{avg}} = k_0\left(1 + \beta \frac{T_2 + T_1}{2}\right) = (1.5\text{ W/m}\cdot\text{K})\left[1 + (0.003\text{K}^{-1})\frac{T_2 + (373\text{ K})}{2}\right]$$

$$k_{\text{avg}} = [1.5 + 0.00225(T_2 + 373)]\text{W/m}\cdot\text{K} \qquad (2)$$

Solving Eqs. (1) & (2) for the outer surface temperature yields

$$T_2 = \mathbf{369\text{ K}} = \mathbf{96°C}$$

Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"GIVEN"
h=50 [W/(m^2*K)]  "convection heat transfer coefficient"
r_1=0.025/2 [m]  "inner radius"
r_2=r_1+0.003 [m]  "outer radius"
T_1=373 [K]  "inner surface temperature"
T_inf=293 [K]  "ambient temperature"
k_0=1.5 [W/(m*K)]
beta=0.003 [K^-1]
"SOLVING FOR OUTER SURFACE TEMPERATURE"
k_avg=k_0*(1+beta*(T_2+T_1)/2)
Q_dot_cylinder=2*pi*k_avg*(T_1-T_2)/ln(r_2/r_1)  "heat rate through the cylindrical layer"
Q_dot_conv=h*2*pi*r_2*(T_2-T_inf)  "heat rate by convection"
Q_dot_cylinder=Q_dot_conv
```

Discussion Increasing h or decreasing k_{avg} would decrease the pipe's outer surface temperature.

2-162 A metal spherical tank, filled with chemicals undergoing an exothermic reaction, has a known inner surface temperature. The tank wall has a variable thermal conductivity. Convection heat transfer occurs on the outer tank surface. The heat flux on the inner surface of the tank is to be determined.

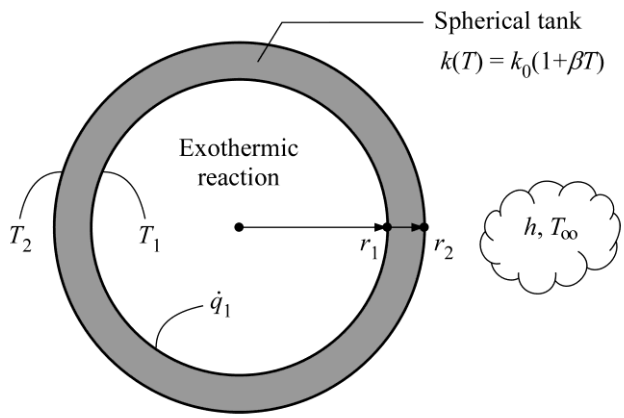
Assumptions **1** Heat transfer is steady and one-dimensional. **2** There is no heat generation. **3** Thermal conductivity varies with temperature.

Properties The thermal conductivity is given to be $k(T) = k_0(1 + \beta T)$.

Analysis The inner and outer radii of the tank are

$$r_1 = 5/2\,\text{m} = 2.5\,\text{m}$$

and $$r_2 = (2.5 + 0.01)\,\text{m} = 2.51\,\text{m}$$



The rate of heat transfer at the tank's outer surface can be expressed as

$$\dot{Q}_{\text{sph}} = \dot{Q}_{\text{conv}}$$

$$4\pi k_{\text{avg}} r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} = h(4\pi r_2^2)(T_2 - T_\infty)$$

$$k_{\text{avg}} \frac{r_1}{r_2} \frac{T_1 - T_2}{r_2 - r_1} = h(T_2 - T_\infty) \qquad (1)$$

where

$$h = 80\ \text{W/m}^2\,\text{K}, \quad T_1 = 393\ \text{K}, \quad \text{and} \quad T_\infty = 288\ \text{K}$$

The average thermal conductivity is

$$k_{\text{avg}} = k_0\left(1 + \beta\frac{T_2 + T_1}{2}\right) = (9.1\,\text{W/m}\cdot\text{K})\left[1 + (0.0018\,\text{K}^{-1})\frac{T_2 + (393\,\text{K})}{2}\right]$$

$$k_{\text{avg}} = [9.1 + 0.00819(T_2 + 393)]\,\text{W/m}\cdot\text{K} \qquad (2)$$

Solving Eqs. (1) & (2) for T_2 and k_{avg} yields

$$T_2 = 387.8\,\text{K} \quad \text{and} \quad k_{\text{avg}} = 15.5\,\text{W/m}\cdot\text{K}$$

Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"GIVEN"
h=80 [W/(m^2*K)]  "outer surface h"
r_1=5/2 [m]  "inner radius"
r_2=r_1+0.010 [m]  "outer radius"
T_1=120+273 [K]  "inner surface T"
T_inf=15+273 [K]  "ambient T"
k_0=9.1 [W/(m*K)]
beta=0.0018 [K^-1]
"SOLVING FOR OUTER SURFACE TEMPERATURE AND k_avg"
k_avg=k_0*(1+beta*(T_2+T_1)/2)
q_dot_sph=k_avg*r_1/r_2*(T_1-T_2)/(r_2-r_1)  "heat flux through the spherical layer"
q_dot_conv=h*(T_inf-T_2)  "heat flux by convection"
q_dot_sph+q_dot_conv=0
```

Thus, the heat flux on the inner surface of the tank is

$$\dot{q}_1 = \frac{\dot{Q}_{\text{sph}}}{4\pi r_1^2} = \frac{4\pi k_{\text{avg}} r_1 r_2}{4\pi r_1^2}\frac{T_1 - T_2}{r_2 - r_1} = k_{\text{avg}}\frac{r_2}{r_1}\frac{T_1 - T_2}{r_2 - r_1} = (15.5\,\text{W/m}\cdot\text{K})\left(\frac{2.51}{2.5}\right)\frac{(393 - 387.8)\,\text{K}}{0.01\,\text{m}}$$

$$\dot{q}_1 = \mathbf{8092.2\,W/m^2}$$

Discussion The inner-to-outer surface heat flux ratio can be related to r_1 and r_2: $\dot{q}_1/\dot{q}_2 = (r_2/r_1)^2$.

Fundamentals of Engineering (FE) Exam Problems

2-163 The heat conduction equation in a medium is given in its simplest form as $\frac{1}{r}\frac{d}{dr}\left(rk\frac{dT}{dr}\right)+\dot{e}_{\text{gen}}=0$ Select the wrong statement below.

(a) the medium is of cylindrical shape.
(b) the thermal conductivity of the medium is constant.
(c) heat transfer through the medium is steady.
(d) there is heat generation within the medium.
(e) heat conduction through the medium is one-dimensional.

Answer (b) thermal conductivity of the medium is constant

2-164 Consider a medium in which the heat conduction equation is given in its simplest form as

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

(*a*) Is heat transfer steady or transient?
(*b*) Is heat transfer one-, two-, or three-dimensional?
(*c*) Is there heat generation in the medium?
(*d*) Is the thermal conductivity of the medium constant or variable?
(*e*) Is the medium a plane wall, a cylinder, or a sphere?
(*f*) Is this differential equation for heat conduction linear or nonlinear?

Answers: (*a*) transient, (*b*) one-dimensional, (*c*) no, (*d*) constant, (*e*) sphere, (*f*) linear

2-165 Consider a large plane wall of thickness *L*, thermal conductivity *k*, and surface area *A*. The left surface of the wall is exposed to the ambient air at T_∞ with a heat transfer coefficient of *h* while the right surface is insulated. The variation of temperature in the wall for steady one-dimensional heat conduction with no heat generation is

(a) $T(x)=\frac{h(L-x)}{k}T_\infty$ (b) $T(x)=\frac{k}{h(x+0.5L)}T_\infty$ (c) $T(x)=\left(1-\frac{xh}{k}\right)T_\infty$ (d) $T(x)=(L-x)T_\infty$

(e) $T(x)=T_\infty$

Answer (e) $T(x)=T_\infty$

2-166 A solar heat flux $\dot{q}_s$ is incident on a sidewalk whose thermal conductivity is k, solar absorptivity is α_s and convective heat transfer coefficient is h. Taking the positive x direction to be towards the sky and disregarding radiation exchange with the surroundings surfaces, the correct boundary condition for this sidewalk surface is

(a) $-k\dfrac{dT}{dx}=\alpha_s\dot{q}_s$ (b) $-k\dfrac{dT}{dx}=h(T-T_\infty)$ (c) $-k\dfrac{dT}{dx}=h(T-T_\infty)-\alpha_s\dot{q}_s$

(d) $h(T-T_\infty)=\alpha_s\dot{q}_s$ (e) None of them

Answer (c) $-k\dfrac{dT}{dx}=h(T-T_\infty)-\alpha_s\dot{q}_s$

2-167 A plane wall of thickness L is subjected to convection at both surfaces with ambient temperature $T_{\infty 1}$ and heat transfer coefficient h_1 at inner surface, and corresponding $T_{\infty 2}$ and h_2 values at the outer surface. Taking the positive direction of x to be from the inner surface to the outer surface, the correct expression for the convection boundary condition is

(a) $k\dfrac{dT(0)}{dx}=h_1[T(0)-T_{\infty 1})]$ (b) $k\dfrac{dT(L)}{dx}=h_2[T(L)-T_{\infty 2})]$

(c) $-k\dfrac{dT(0)}{dx}=h_1[T_{\infty 1}-T_{\infty 2})]$ (d) $-k\dfrac{dT(L)}{dx}=h_2[T_{\infty 1}-T_{\infty 2})]$ (e) None of them

Answer (a) $k\dfrac{dT(0)}{dx}=h_1[T(0)-T_{\infty 1})]$

2-168 Consider steady one-dimensional heat conduction through a plane wall, a cylindrical shell, and a spherical shell of uniform thickness with constant thermophysical properties and no thermal energy generation. The geometry in which the variation of temperature in the direction of heat transfer be linear is

(a) plane wall (b) cylindrical shell (c) spherical shell (d) all of them (e) none of them

Answer (a) plane wall

2-169 The conduction equation boundary condition for an adiabatic surface with direction n being normal to the surface is

(a) $T = 0$ (b) $dT/dn = 0$ (c) $d^2T/dn^2 = 0$ (d) $d^3T/dn^3 = 0$ (e) $-kdT/dn = 1$

Answer (b) $dT/dn = 0$

2-170 The variation of temperature in a plane wall is determined to be $T(x)$=65x+25 where x is in m and T is in °C. If the temperature at one surface is 38ºC, the thickness of the wall is

(a) 2 m (b) 0.4 m (c) 0.2 m (d) 0.1 m (e) 0.05 m

Answer (c) 0.2 m

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
38=65*L+25
```

2-171 The variation of temperature in a plane wall is determined to be $T(x)$=110-48x where x is in m and T is in °C. If the thickness of the wall is 0.75 m, the temperature difference between the inner and outer surfaces of the wall is

(a) 110ºC (b) 74ºC (c) 55ºC (d) 36ºC (e) 18ºC

Answer (d) 36ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T1=110 [C]
L=0.75
T2=110-48*L
DELTAT=T1-T2
```

2-172 The temperatures at the inner and outer surfaces of a 15-cm-thick plane wall are measured to be 40ºC and 28ºC, respectively. The expression for steady, one-dimensional variation of temperature in the wall is

(a) $T(x) = 28x + 40$ (b) $T(x) = -40x + 28$ (c) $T(x) = 40x + 28$

(d) $T(x) = -80x + 40$ (e) $T(x) = 40x - 80$

Answer (d) $T(x) = -80x + 40$

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T1=40 [C]
T2=28 [C]
L=0.15 [m]
"T(x)=C1x+C2"
C2=T1
T2=C1*L+T1
```

2-173 The thermal conductivity of a solid depends upon the solid's temperature as $k = aT + b$ where a and b are constants. The temperature in a planar layer of this solid as it conducts heat is given by

(a) $aT + b = x + C_2$ (b) $aT + b = C_1x^2 + C_2$ (c) $aT^2 + bT = C_1x + C_2$
(d) $aT^2 + bT = C_1x^2 + C_2$ (e) None of them

Answer (c) $aT^2 + bT = C_1x + C_2$

2-174 Hot water flows through a PVC (k = 0.092 W/m·K) pipe whose inner diameter is 2 cm and outer diameter is 2.5 cm. The temperature of the interior surface of this pipe is 35ºC and the temperature of the exterior surface is 20ºC. The rate of heat transfer per unit of pipe length is

(a) 22.8 W/m (b) 38.9 W/m (c) 48.7 W/m (d) 63.6 W/m (e) 72.6 W/m

Answer (b) 38.9 W/m

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
do=2.5 [cm]
di=2.0 [cm]
k=0.092 [W/m-C]
T2=35 [C]
T1=20 [C]
Q=2*pi*k*(T2-T1)/LN(do/di)
```

2-175 Heat is generated in a long 0.3-cm-diameter cylindrical electric heater at a rate of 150 W/cm^3. The heat flux at the surface of the heater in steady operation is

(a) 42.7 W/cm^2 (b) 159 W/cm^2 (c) 150 W/cm^2 (d) 10.6 W/cm^2 (e) 11.3 W/cm^2

Answer (e) 11.3 W/cm^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"Consider a 1-cm long heater:"
L=1 [cm]
e=150 [W/cm^3]
D=0.3 [cm]
V=pi*(D^2/4)*L
A=pi*D*L "[cm^2]”
Egen=e*V "[W]"
Qflux=Egen/A "[W/cm^2]"

“Some Wrong Solutions with Common Mistakes:”
W1=Egen "Ignoring area effect and using the total"
W2=e/A "Threating g as total generation rate"
W3=e “ignoring volume and area effects”
```

2-176 Heat is generated uniformly in a 4-cm-diameter, 16-cm-long solid bar (k = 2.4 W/m·ºC). The temperatures at the center and at the surface of the bar are measured to be 210ºC and 45ºC, respectively. The rate of heat generation within the bar is

(a) 240 W (b) 796 W b) 1013 W (c) 79,620 W (d) 3.96×10^6 W

Answer (b) 796 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.04 [m]
L=0.16 [m]
k=2.4 [W/m-C]
T0=210 [C]
T_s=45 [C]
T0-T_s=(e*(D/2)^2)/(4*k)
V=pi*D^2/4*L
E_dot_gen=e*V

"Some Wrong Solutions with Common Mistakes"
W1_V=pi*D*L "Using surface area equation for volume"
W1_E_dot_gen=e*W14_1
T0=(W2_e*(D/2)^2)/(4*k) "Using center temperature instead of temperature difference"
W2_Q_dot_gen=W2_e*V
W3_Q_dot_gen=e "Using heat generation per unit volume instead of total heat generation as the result"
```

2-177 Heat is generated in a 8-cm-diameter spherical radioactive material whose thermal conductivity is 25 W/m.°C uniformly at a rate of 15 W/cm^3. If the surface temperature of the material is measured to be 120°C, the center temperature of the material during steady operation is

(a) 160°C (b) 280°C (c) 212°C (d) 360°C (e) 600°C

Answer (b) 280°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.08
Ts=120
k=25
e_gen=15E+6
T=Ts+g*(D/2)^2/(6*k)

“Some Wrong Solutions with Common Mistakes:”
W1_T= e_gen*(D/2)^2/(6*k) "Not using Ts"
W2_T= Ts+e_gen*(D/2)^2/(4*k) "Using the relation for cylinder"
W3_T= Ts+e_gen*(D/2)^2/(2*k) "Using the relation for slab"
```

2-178 Heat is generated in a 3-cm-diameter spherical radioactive material uniformly at a rate of 15 W/cm^3. Heat is dissipated to the surrounding medium at 25°C with a heat transfer coefficient of 120 W/m^2·°C. The surface temperature of the material in steady operation is

(a) 56°C (b) 84°C (c) 494°C (d) 650°C (e) 108°C

Answer (d) 650°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
h=120 [W/m^2-C]
e=15 [W/cm^3]
Tinf=25 [C]
D=3 [cm]
V=pi*D^3/6 "[cm^3]"
A=pi*D^2/10000 "[m^2]"
Egen=e*V "[W]"
Qgen=h*A*(Ts-Tinf)
```

2-179 .... 2-181 Design and Essay Problems

Solutions Manual for

Heat and Mass Transfer: Fundamentals & Applications

6th Edition

Yunus A. Çengel, Afshin J. Ghajar

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Chapter 3
STEADY HEAT CONDUCTION

Steady Heat Conduction in Plane Walls

3-1C The temperature distribution in a plane wall will be a straight line during steady and one dimensional heat transfer with constant wall thermal conductivity.

3-2C In steady heat conduction, the rate of heat transfer into the wall is equal to the rate of heat transfer out of it. Also, the temperature at any point in the wall remains constant. Therefore, the energy content of the wall does not change during steady heat conduction. However, the temperature along the wall and thus the energy content of the wall will change during transient conduction.

3-3C The thermal resistance of a medium represents the resistance of that medium against heat transfer.

3-4C Yes. The convection resistance can be defined as the inverse of the convection heat transfer coefficient per unit surface area since it is defined as $R_{conv} = 1/(hA)$.

3-5C Convection heat transfer through the wall is expressed as $\dot{Q} = hA_s(T_s - T_\infty)$. In steady heat transfer, heat transfer rate to the wall and from the wall are equal. Therefore at the outer surface which has convection heat transfer coefficient three times that of the inner surface will experience three times smaller temperature drop compared to the inner surface. Therefore, at the outer surface, the temperature will be closer to the surrounding air temperature.

3-6C The combined heat transfer coefficient represents the combined effects of radiation and convection heat transfers on a surface, and is defined as $h_{\text{combined}} = h_{\text{convection}} + h_{\text{radiation}}$. It offers the convenience of incorporating the effects of radiation in the convection heat transfer coefficient, and to ignore radiation in heat transfer calculations.

3-7C The convection and the radiation resistances at a surface are parallel since both the convection and radiation heat transfers occur simultaneously.

3-8C The temperature of each surface in this case can be determined from

$$\dot{Q} = (T_{\infty 1} - T_{s1})/R_{\infty 1-s1} \longrightarrow T_{s1} = T_{\infty 1} - (\dot{Q}R_{\infty 1-s1})$$
$$\dot{Q} = (T_{s2} - T_{\infty 2})/R_{s2-\infty 2} \longrightarrow T_{s2} = T_{\infty 2} + (\dot{Q}R_{s2-\infty 2})$$

where $R_{\infty - i}$ is the thermal resistance between the environment ∞ and surface i.

3-9C Yes, it is.

3-10C The blanket will introduce additional resistance to heat transfer and slow down the heat gain of the drink wrapped in a blanket. Therefore, the drink left on a table will warm up faster.

3-11C The new design introduces the thermal resistance of the copper layer in addition to the thermal resistance of the aluminum which has the same value for both designs. Therefore, the new design will be a poorer conductor of heat.

3-12C For a surface of *A* at which the convection and radiation heat transfer coefficients are h_{conv} and h_{rad}, the single equivalent heat transfer coefficient is $h_{eqv} = h_{conv} + h_{rad}$ when the medium and the surrounding surfaces are at the same temperature. Then the equivalent thermal resistance will be $R_{eqv} = 1/(h_{eqv}A)$.

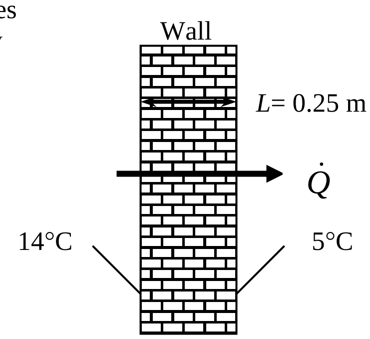
3-13C The thermal resistance network associated with a five-layer composite wall involves five single-layer resistances connected in series.

3-14C Once the rate of heat transfer $\dot{Q}$ is known, the temperature drop across any layer can be determined by multiplying heat transfer rate by the thermal resistance across that layer, $\Delta T_{layer} = \dot{Q}R_{layer}$

3-15C The window glass which consists of two 4 mm thick glass sheets pressed tightly against each other will probably have thermal contact resistance which serves as an additional thermal resistance to heat transfer through window, and thus the heat transfer rate will be smaller relative to the one which consists of a single 8 mm thick glass sheet.

3-16 The two surfaces of a wall are maintained at specified temperatures. The rate of heat loss through the wall is to be determined.

Assumptions **1** Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.



Properties The thermal conductivity is given to be k = 0.8 W/m·°C.

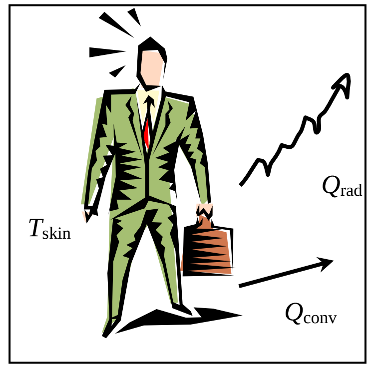
Analysis The surface area of the wall and the rate of heat loss through the wall are

$$A = (3\,\text{m}) \times (6\,\text{m}) = 18\,\text{m}^2$$

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.8\ \text{W/m}\cdot{}^\circ\text{C})(18\,\text{m}^2)\frac{(14-5)^\circ\text{C}}{0.25\,\text{m}} = \mathbf{518\ W}$$

3-17 A person is dissipating heat at a rate of 150 W by natural convection and radiation to the surrounding air and surfaces. For a given deep body temperature, the outer skin temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is constant and uniform over the entire exposed surface of the person. **3** The surrounding surfaces are at the same temperature as the indoor air temperature. **4** Heat generation within the 0.5-cm thick outer layer of the tissue is negligible.



Properties The thermal conductivity of the tissue near the skin is given to be $k = 0.3$ W/m·°C.

Analysis The skin temperature can be determined directly from

$$\dot{Q} = kA\frac{T_1 - T_{skin}}{L}$$

$$T_{skin} = T_1 - \frac{\dot{Q}L}{kA} = 37°\text{C} - \frac{(150\text{ W})(0.005\text{ m})}{(0.3\text{ W/m}\cdot°\text{C})(1.7\text{ m}^2)} = \mathbf{35.5°C}$$

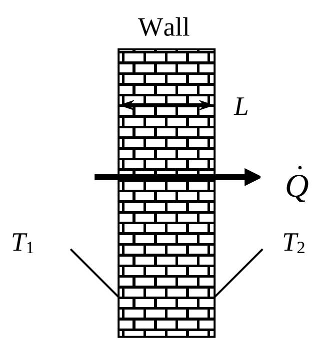
3-18E The inner and outer surfaces of the walls of an electrically heated house remain at specified temperatures during a winter day. The amount of heat lost from the house that day and its cost are to be determined.

Assumptions **1** Heat transfer through the walls is steady since the surface temperatures of the walls remain constant at the specified values during the time period considered. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity of the walls is constant.

Properties The thermal conductivity of the brick wall is given to be $k = 0.40$ Btu/h·ft·°F.

Analysis We consider heat loss through the walls only. The total heat transfer area is

$$A = 2(50\times 9 + 35\times 9) = 1530\text{ ft}^2$$



The rate of heat loss during the daytime is

$$\dot{Q}_{\text{day}} = kA\frac{T_1 - T_2}{L} = (0.40\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})(1530\text{ ft}^2)\frac{(55-45)°\text{F}}{1\text{ ft}} = 6120\text{ Btu/h}$$

The rate of heat loss during nighttime is

$$\dot{Q}_{\text{night}} = kA\frac{T_1 - T_2}{L}$$
$$= (0.40\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})(1530\text{ ft}^2)\frac{(55-35)°\text{C}}{1\text{ ft}} = 12{,}240\text{ Btu/h}$$

The amount of heat loss from the house that night will be

$$\dot{Q} = \frac{Q}{\Delta t} \longrightarrow Q = \dot{Q}\Delta t = 10\dot{Q}_{day} + 14\dot{Q}_{night} = (10\text{ h})(6120\text{ Btu/h}) + (14\text{ h})(12{,}240\text{ Btu/h})$$
$$= \mathbf{232{,}560\ Btu}$$

Then the cost of this heat loss for that day becomes

$$Cost = (232{,}560/3412\text{ kWh})(\$0.09/\text{kWh}) = \mathbf{\$6.13}$$

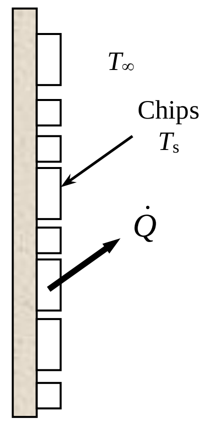
3-19 A circuit board houses 100 chips, each dissipating 0.06 W. The surface heat flux, the surface temperature of the chips, and the thermal resistance between the surface of the board and the cooling medium are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer from the back surface of the board is negligible. **2** Heat is transferred uniformly from the entire front surface.

Analysis (*a*) The heat flux on the surface of the circuit board is

$$A_s = (0.12\,\text{m})(0.18\text{m}) = 0.0216\,\text{m}^2$$

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{(100\times 0.06)\text{ W}}{0.0216\,\text{m}^2} = \mathbf{278\ W/m^2}$$



(*b*) The surface temperature of the chips is

$$\dot{Q} = hA_s(T_s - T_\infty)$$

$$T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 40°\text{C} + \frac{(100\times 0.06)\text{ W}}{(10\text{ W/m}^2\cdot°\text{C})(0.0216\,\text{m}^2)} = \mathbf{67.8°C}$$

(*c*) The thermal resistance is

$$R_{conv} = \frac{1}{hA_s} = \frac{1}{(10\text{ W/m}^2\cdot°\text{C})(0.0216\,\text{m}^2)} = \mathbf{4.63°C/W}$$

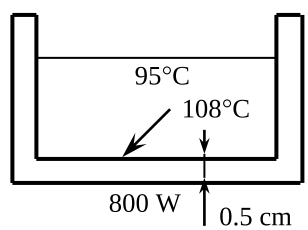
3-20 Heat is transferred steadily to the boiling water in an aluminum pan. The inner surface temperature of the bottom of the pan is given. The boiling heat transfer coefficient and the outer surface temperature of the bottom of the pan are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since the thickness of the bottom of the pan is small relative to its diameter. **3** The thermal conductivity of the pan is constant.

Properties The thermal conductivity of the aluminum pan is given to be k = 237 W/m·°C.

Analysis (*a*) The boiling heat transfer coefficient is

$$A_s = \frac{\pi D^2}{4} = \frac{\pi(0.25\,\text{m})^2}{4} = 0.0491\,\text{m}^2$$



$$\dot{Q} = hA_s(T_s - T_\infty)$$

$$h = \frac{\dot{Q}}{A_s(T_s - T_\infty)} = \frac{800\text{ W}}{(0.0491\,\text{m}^2)(108-95)°\text{C}} = \mathbf{1254\ W/m^2.°C}$$

(*b*) The outer surface temperature of the bottom of the pan is

$$\dot{Q} = kA\frac{T_{s,outer} - T_{s,inner}}{L}$$

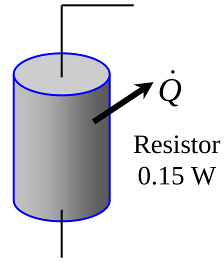
$$T_{s,outer} = T_{s,inner1} + \frac{\dot{Q}L}{kA} = 108°\text{C} + \frac{(800\text{ W})(0.005\,\text{m})}{(237\text{ W/m}\cdot°\text{C})(0.0491\text{m}^2)} = \mathbf{108.3°C}$$

3-21 A cylindrical resistor on a circuit board dissipates 0.15 W of power steadily in a specified environment. The amount of heat dissipated in 24 h, the surface heat flux, and the surface temperature of the resistor are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat is transferred uniformly from all surfaces of the resistor.

Analysis (*a*) The amount of heat this resistor dissipates during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.15\text{ W})(24\text{ h}) = \mathbf{3.6\text{ Wh}}$$



(*b*) The heat flux on the surface of the resistor is

$$A_s = 2\frac{\pi D^2}{4} + \pi DL = 2\frac{\pi(0.003\text{ m})^2}{4} + \pi(0.003\text{ m})(0.012\text{ m}) = 0.000127\text{ m}^2$$

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{0.15\text{ W}}{0.000127\text{ m}^2} = \mathbf{1179\text{ W/m}^2}$$

(*c*) The surface temperature of the resistor can be determined from

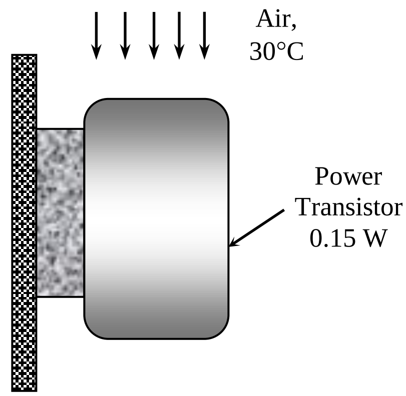
$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 35°\text{C} + \frac{0.15\text{ W}}{(9\text{ W/m}^2\cdot°\text{C})(0.000127\text{ m}^2)} = \mathbf{166°C}$$

3-22 A power transistor dissipates 0.15 W of power steadily in a specified environment. The amount of heat dissipated in 24 h, the surface heat flux, and the surface temperature of the resistor are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat is transferred uniformly from all surfaces of the transistor.

Analysis (*a*) The amount of heat this transistor dissipates during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.15\text{ W})(24\text{ h}) = 3.6\text{ Wh} = \mathbf{0.0036\text{ kWh}}$$



(*b*) The heat flux on the surface of the transistor is

$$A_s = 2\frac{\pi D^2}{4} + \pi DL$$
$$= 2\frac{\pi(0.005\text{ m})^2}{4} + \pi(0.005\text{ m})(0.004\text{ m}) = 0.0001021\text{ m}^2$$

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{0.15\text{ W}}{0.0001021\text{ m}^2} = \mathbf{1469\text{ W/m}^2}$$

(*c*) The surface temperature of the transistor can be determined from

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 30°\text{C} + \frac{0.15\text{ W}}{(18\text{ W/m}^2\cdot°\text{C})(0.0001021\text{ m}^2)} = \mathbf{111.6°C}$$

3-23 A double-pane window is considered. The rate of heat loss through the window and the temperature difference across the largest thermal resistance are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer coefficients are constant.

PropertiesThe thermal conductivities of glass and air are given to be 0.78 W/m·K and 0.025 W/m·K, respectively.

Analysis(*a*)The rate of heat transfer through the window is determined to be

$$\dot{Q} = \frac{A\Delta T}{\dfrac{1}{h_i} + \dfrac{L_g}{k_g} + \dfrac{L_a}{k_a} + \dfrac{L_g}{k_g} + \dfrac{1}{h_o}}$$

$$= \frac{(1\times 1.5\ \mathrm{m}^2)[20-(-20)]^\circ\mathrm{C}}{\dfrac{1}{40\ \mathrm{W/m^2\cdot {}^\circ C}} + \dfrac{0.004\ \mathrm{m}}{0.78\ \mathrm{W/m\cdot {}^\circ C}} + \dfrac{0.005\ \mathrm{m}}{0.025\ \mathrm{W/m\cdot {}^\circ C}} + \dfrac{0.004\ \mathrm{m}}{0.78\ \mathrm{W/m\cdot {}^\circ C}} + \dfrac{1}{20\ \mathrm{W/m^2\cdot {}^\circ C}}}$$

$$= \frac{(1\times 1.5\ \mathrm{m}^2)[20-(-20)]^\circ\mathrm{C}}{0.025 + 0.000513 + 0.2 + 0.000513 + 0.05} = \mathbf{210\ W}$$

(*b*) Noting that the largest resistance is through the air gap, the temperature difference across the air gap is determined from

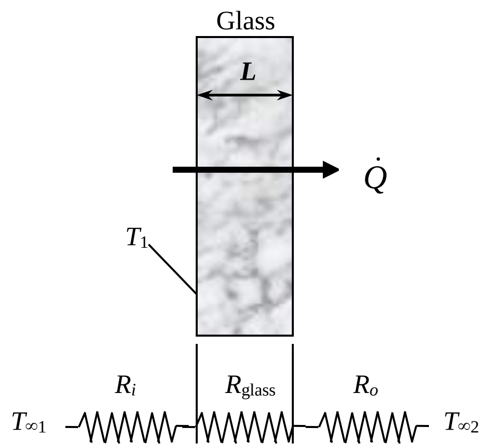
$$\Delta T_a = \dot{Q}R_a = \dot{Q}\frac{L_a}{k_a A} = (210\ \mathrm{W})\frac{0.005\ \mathrm{m}}{(0.025\ \mathrm{W/m\cdot {}^\circ C})(1\times 1.5\ \mathrm{m}^2)} = \mathbf{28^\circ C}$$

3-24 The two surfaces of a window are maintained at specified temperatures. The rate of heat loss through the window and the inner surface temperature are to be determined.

Assumptions **1** Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the glass is given to be k = 0.78 W/m·°C.

Analysis The area of the window and the individual resistances are



$$A = (1.5\ \mathrm{m})\times(2.4\ \mathrm{m}) = 3.6\ \mathrm{m}^2$$

$$R_\mathrm{i} = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\ \mathrm{W/m^2.{}^\circ C})(3.6\ \mathrm{m}^2)} = 0.02778\ \mathrm{{}^\circ C/W}$$

$$R_\mathrm{glass} = \frac{L}{k_1 A} = \frac{0.006\ \mathrm{m}}{(0.78\ \mathrm{W/m.{}^\circ C})(3.6\ \mathrm{m}^2)} = 0.00214\ \mathrm{{}^\circ C/W}$$

$$R_\mathrm{o} = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25\ \mathrm{W/m^2.{}^\circ C})(3.6\ \mathrm{m}^2)} = 0.01111\ \mathrm{{}^\circ C/W}$$

$$R_{total} = R_{conv,1} + R_{glass} + R_{conv,2}$$
$$= 0.02778 + 0.00214 + 0.01111 = 0.04103\ \mathrm{{}^\circ C/W}$$

The steady rate of heat transfer through window glass is then

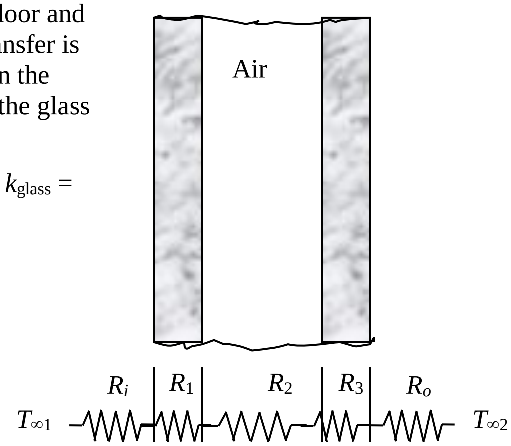
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[24-(-5)]^\circ\mathrm{C}}{0.04103\ \mathrm{{}^\circ C/W}} = \mathbf{707\ W}$$

The inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 24^\circ\mathrm{C} - (707\ \mathrm{W})(0.02778\ \mathrm{{}^\circ C/W}) = \mathbf{4.4^\circ C}$$

3-25 A double-pane window consists of two layers of glass separated by a stagnant air space. For specified indoors and outdoors temperatures, the rate of heat loss through the window and the inner surface temperature of the window are to be determined.

Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivities of the glass and air are constant. **4** Heat transfer by radiation is negligible.



Properties The thermal conductivity of the glass and air are given to be k_{glass} = 0.78 W/m·°C and k_{air} = 0.026 W/m·°C.

Analysis The area of the window and the individual resistances are

$$A = (1.5\,\text{m}) \times (2.4\,\text{m}) = 3.6\,\text{m}^2$$

$$R_{\text{i}} = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\text{ W/m}^2.°\text{C})(3.6\,\text{m}^2)} = 0.02778\,°\text{C/W}$$

$$R_1 = R_3 = R_{\text{glass}} = \frac{L_1}{k_1 A} = \frac{0.003\,\text{m}}{(0.78\text{ W/m.°C})(3.6\,\text{m}^2)} = 0.00107\,°\text{C/W}$$

$$R_2 = R_{air} = \frac{L_2}{k_2 A} = \frac{0.012\,\text{m}}{(0.026\text{ W/m.°C})(3.6\,\text{m}^2)} = 0.12821\,°\text{C/W}$$

$$R_{\text{o}} = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25\text{ W/m}^2.^\circ\text{C})(3.6\,\text{m}^2)} = 0.01111\ ^\circ\text{C/W}$$

$$\begin{aligned} R_{total} &= R_{conv,1} + 2R_1 + R_2 + R_{conv,2} = 0.02778 + 2(0.00107) + 0.12821 + 0.01111 \\ &= 0.16924\,°\text{C/W} \end{aligned}$$

The steady rate of heat transfer through window glass then becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[21 - (-5)]°\text{C}}{0.16924°\text{C/W}} = \mathbf{154\ W}$$

The inner surface temperature of the window glass can be determined from

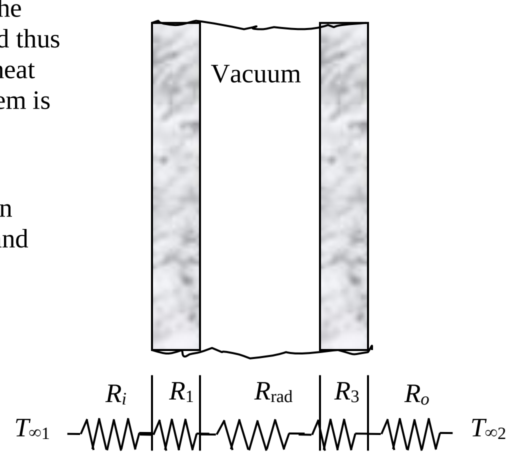
$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 21°\text{C} - (154\text{ W})(0.02778°\text{C/W}) = \mathbf{16.7°C}$$

3-26 A double-pane window consists of two layers of glass separated by an evacuated space. For specified indoors and outdoors temperatures, the rate of heat loss through the window and the inner surface temperature of the window are to be determined.

Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity of the glass is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the glass is given to be k_{glass} = 0.78 W/m·°C.

Analysis Heat cannot be conducted through an evacuated space since the thermal conductivity of vacuum is zero (no medium to conduct heat) and thus its thermal resistance is zero. Therefore, if radiation is disregarded, the heat transfer through the window will be zero. Then the answer of this problem is **zero** since the problem states to disregard radiation.



Discussion In reality, heat will be transferred between the glasses by radiation. We do not know the inner surface temperatures of windows. In order to determine radiation heat resistance we assume them to be 5°C and 15°C, respectively, and take the emissivity to be 1. Then individual resistances are

$$A = (1.5\,\text{m}) \times (2.4\,\text{m}) = 3.6\,\text{m}^2$$

$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2.°\text{C})(3.6\,\text{m}^2)} = 0.02778\ °\text{C/W}$$

$$R_1 = R_3 = R_{glass} = \frac{L_1}{k_1 A} = \frac{0.003\,\text{m}}{(0.78\ \text{W/m.°C})(3.6\,\text{m}^2)} = 0.00107\ °\text{C/W}$$

$$\begin{aligned} R_{rad} &= \frac{1}{\varepsilon\sigma A(T_s^{\,2} + T_{surr}^{\,2})(T_s + T_{surr})} \\ &= \frac{1}{1(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(3.6\,\text{m}^2)[288^2 + 278^2][288+278]K^3} \\ &= 0.05402\ °\text{C/W} \end{aligned}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(25\ \text{W/m}^2.^{o}\text{C})(3.6\,\text{m}^2)} = 0.01111\ ^{o}\text{C/W}$$

$$\begin{aligned} R_{total} &= R_{conv,1} + 2R_1 + R_{rad} + R_{conv,2} = 0.02778 + 2(0.00107) + 0.05402 + 0.01111 \\ &= 0.09505\ °\text{C/W} \end{aligned}$$

The steady rate of heat transfer through window glass then becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[21-(-5)]°\text{C}}{0.09505°\text{C/W}} = \mathbf{274\ W}$$

The inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{conv,1} = 21°\text{C} - (274\ \text{W})(0.02778°\text{C/W}) = \mathbf{13.4°C}$$

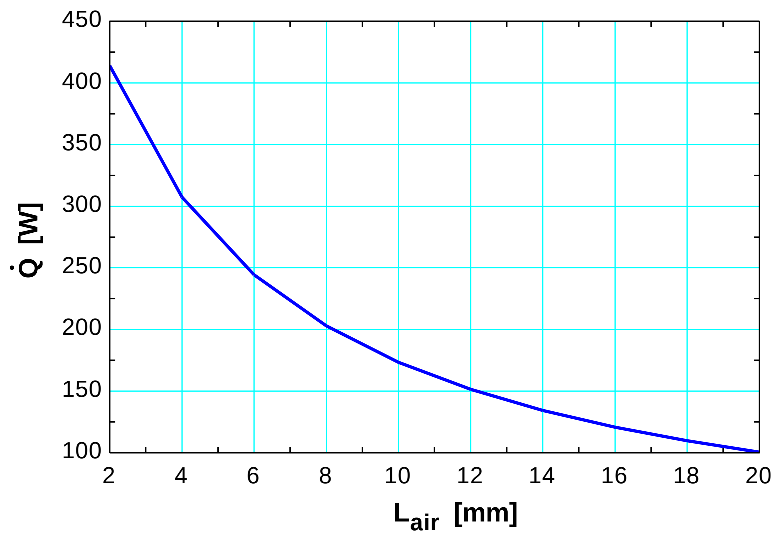
Similarly, the inner surface temperatures of the glasses are calculated to be 13.1 and -1.7°C (we had assumed them to be 15 and 5°C when determining the radiation resistance). We can improve the result obtained by reevaluating the radiation resistance and repeating the calculations.

3-27 Prob. 3-26 is reconsidered. The rate of heat transfer through the window as a function of the width of air space is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
A=1.5*2.4 [m^2]
L_glass=3 [mm]
k_glass=0.78 [W/m-C]
L_air=12 [mm]
T_infinity_1=21 [C]
T_infinity_2=-5 [C]
h_1=10 [W/m^2-C]
h_2=25 [W/m^2-C]

"PROPERTIES"
k_air=conductivity(Air,T=25)

"ANALYSIS"
R_conv_1=1/(h_1*A)
R_glass=(L_glass*Convert(mm, m))/(k_glass*A)
R_air=(L_air*Convert(mm, m))/(k_air*A)
R_conv_2=1/(h_2*A)
R_total=R_conv_1+2*R_glass+R_air+R_conv_2
Q_dot=(T_infinity_1-T_infinity_2)/R_total
```

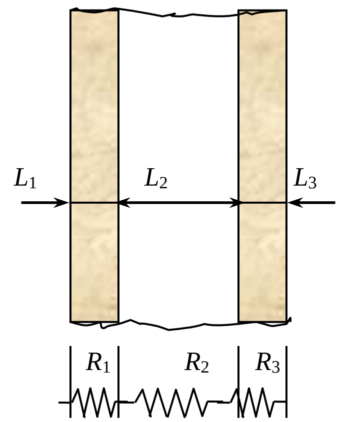
L_{air} [mm]	Q [W]
2	414
4	307.4
6	244.5
8	202.9
10	173.4
12	151.4
14	134.4
16	120.8
18	109.7
20	100.5



3-28E A wall is constructed of two layers of sheetrock with fiberglass insulation in between. The thermal resistance of the wall and its R-value of insulation are to be determined.

Assumptions **1** Heat transfer through the wall is one-dimensional. **2** Thermal conductivities are constant.

Properties The thermal conductivities are given to be $k_{\text{sheetrock}}$ = 0.10 Btu/h·ft·°F and $k_{\text{insulation}}$ = 0.020 Btu/h·ft·°F.



Analysis (*a*) The surface area of the wall is not given and thus we consider a unit surface area (A = 1 ft^2). Then the R-value of insulation of the wall becomes equivalent to its thermal resistance, which is determined from.

$$R_{sheetrock} = R_1 = R_3 = \frac{L_1}{k_1} = \frac{0.6/12\text{ ft}}{(0.10\text{ Btu/h.ft.°F})} = 0.500\text{ ft}^2.\text{°F.h/Btu}$$

$$R_{fiberglass} = R_2 = \frac{L_2}{k_2} = \frac{7/12\text{ ft}}{(0.020\text{ Btu/h.ft.°F})} = 29.17\text{ ft}^2.\text{°F.h/Btu}$$

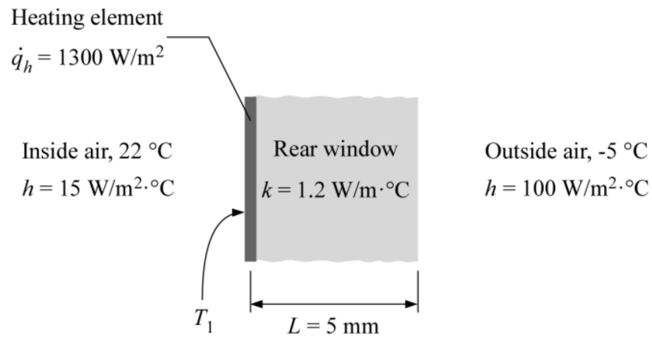
$$R_{total} = 2R_1 + R_2 = 2\times 0.500 + 29.17 = \mathbf{30.17\ ft^2.°F.h/Btu}$$

(*b*) Therefore, this is approximately a ***R*-30** wall in English units.

3-29 A very thin transparent heating element is attached to the inner surface of an automobile window for defogging purposes, the inside surface temperature of the window is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties are constant. **4** Heat transfer by radiation is negligible. **5** Thermal resistance of the thin heating element is negligible.

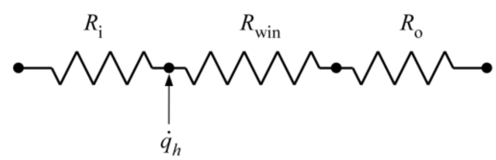
Properties Thermal conductivity of the window is given to be $k = 1.2$ W/m · °C.



Analysis The thermal resistances are

$$R_i = \frac{1}{h_i A} \quad R_o = \frac{1}{h_o A} \text{ and } R_{\text{win}} = \frac{L}{kA}$$

From energy balance and using the thermal resistance concept, the following equation is expressed:



$$\frac{T_{\infty,i} - T_1}{R_i} + \dot{q}_h A = \frac{T_1 - T_{\infty,o}}{R_{\text{win}} + R_o}$$

or

$$\frac{T_{\infty,i} - T_1}{1/(h_i A)} + \dot{q}_h A = \frac{T_1 - T_{\infty,o}}{L/(kA) + 1/(h_o A)}$$

$$\frac{T_{\infty,i} - T_1}{1/h_i} + \dot{q}_h = \frac{T_1 - T_{\infty,o}}{L/k + 1/h_o}$$

$$\frac{22\,°\text{C} - T_1}{1/15 \text{ W/m}^2\cdot°\text{C}} + 1300 \text{ W/m}^2 = \frac{T_1 - (-5\,°\text{C})}{(0.005\text{ m}/1.2 \text{ W/m}\cdot°\text{C}) + (1/100 \text{ W/m}^2\cdot°\text{C})}$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
(22-T_1)/(1/15)+1300=(T_1-(-5))/(0.005/1.2+1/100)
```

Solving by EES software, the inside surface temperature of the window is

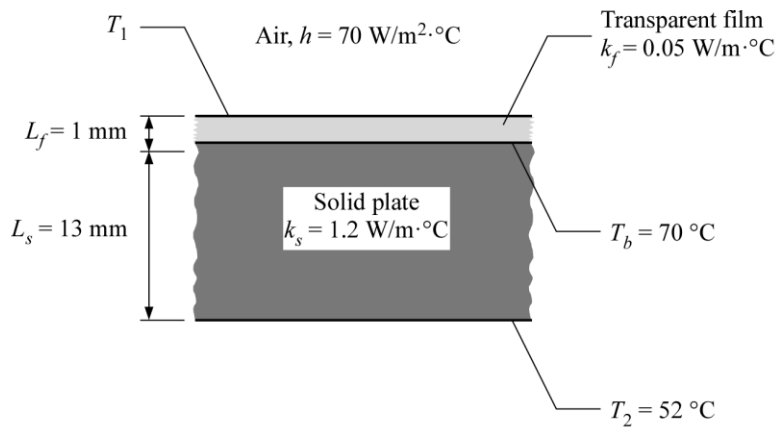
$$T_1 = \mathbf{14.9\,°C}$$

Discussion In actuality, the ambient temperature and the convective heat transfer coefficient outside the automobile vary with weather conditions and the automobile speed. To maintain the inner surface temperature of the window, it is necessary to vary the heat flux to the heating element according to the outside condition.

3-30 A process of bonding a transparent film onto a solid plate is taking place inside a heated chamber. The temperatures inside the heated chamber and on the transparent film surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal properties are constant. **4** Heat transfer by radiation is negligible. **5** Thermal contact resistance is negligible.

Properties The thermal conductivities of the transparent film and the solid plate are given to be 0.05 W/m · °C and 1.2 W/m · °C, respectively.

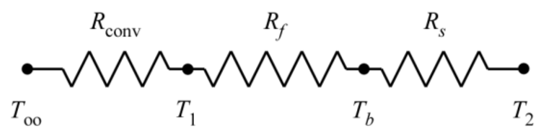


Analysis The thermal resistances are

$$R_{\text{conv}} = \frac{1}{hA}$$

$$R_f = \frac{L_f}{k_f A}$$

and $$R_s = \frac{L_s}{k_s A}$$



Using the thermal resistance concept, the following equation is expressed:

$$\frac{T_\infty - T_b}{R_{\text{conv}} + R_f} = \frac{T_b - T_2}{R_s}$$

Rearranging and solving for the temperature inside the chamber yields

$$T_\infty = \frac{T_b - T_2}{R_s}\left(R_{\text{conv}} + R_f\right) + T_b = \frac{T_b - T_2}{L_s / k_s}\left(\frac{1}{h} + \frac{L_f}{k_f}\right) + T_b$$

$$T_\infty = \frac{(70-52)\,°\text{C}}{0.013\,\text{m}/1.2\ \text{W/m}\cdot°\text{C}}\left(\frac{1}{70\ \text{W/m}^2\cdot°\text{C}} + \frac{0.001\,\text{m}}{0.05\ \text{W/m}\cdot°\text{C}}\right) + 70\,°\text{C} = \mathbf{127\,°C}$$

The surface temperature of the transparent film is

$$\frac{T_1 - T_b}{R_f} = \frac{T_b - T_2}{R_s}$$

$$T_1 = \frac{T_b - T_2}{R_s} R_f + T_b = \frac{T_b - T_2}{L_s / k_s}\left(\frac{L_f}{k_f}\right) + T_b$$

$$T_1 = \frac{(70-52)\,°\text{C}}{0.013\,\text{m}/1.2\ \text{W/m}\cdot°\text{C}}\left(\frac{0.001\,\text{m}}{0.05\ \text{W/m}\cdot°\text{C}}\right) + 70\,°\text{C} = \mathbf{103\,°C}$$

Discussion If a thicker transparent film were to be bonded on the solid plate, then the inside temperature of the heated chamber would have to be higher to maintain the temperature of the bond at 70 °C.

3-31 Warm air blowing over the inner surface of an automobile windshield is used for defrosting ice accumulated on the outer surface. The convection heat transfer coefficient for the warm air blowing over the inner surface of the windshield necessary to cause the accumulated ice to begin melting is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the windshield is one-dimensional. **3** Thermal properties are constant. **4** Heat transfer by radiation is negligible. **5** The automobile is operating at 1 atm.

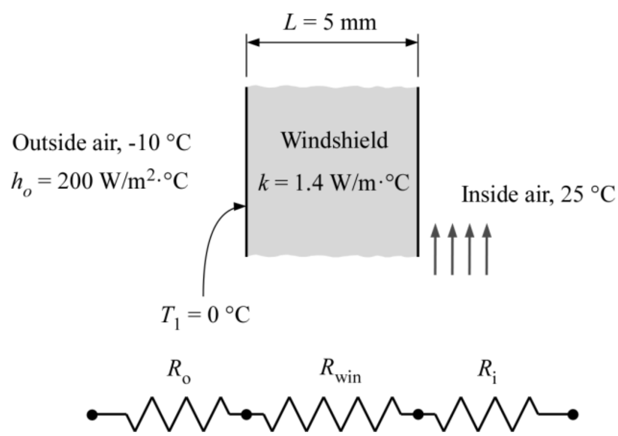
Properties Thermal conductivity of the windshield is given to be k = 1.4 W/m · °C.

Analysis The thermal resistances are

$$R_i = \frac{1}{h_i A}$$

$$R_o = \frac{1}{h_o A}$$

and $$R_{\text{win}} = \frac{L}{kA}$$



From energy balance and using the thermal resistance concept, the following equation is expressed:

$$\frac{T_{\infty,o} - T_1}{R_o} = \frac{T_1 - T_{\infty,i}}{R_{\text{win}} + R_i}$$

$$R_i = \frac{T_1 - T_{\infty,i}}{T_{\infty,o} - T_1} R_o - R_{\text{win}}$$

or $$\frac{1}{h_i} = \frac{T_1 - T_{\infty,i}}{T_{\infty,o} - T_1}\left(\frac{1}{h_o}\right) - \frac{L}{k}$$

For the ice to begin melting, the outer surface temperature of the windshield (T_1) should be at least 0 °C. The convection heat transfer coefficient for the warm air is

$$h_i = \left[\frac{T_1 - T_{\infty,i}}{T_{\infty,o} - T_1}\left(\frac{1}{h_o}\right) - \frac{L}{k}\right]^{-1}$$

$$= \left[\frac{(0-25)\,°\text{C}}{(-10-0)\,°\text{C}}\left(\frac{1}{200\ \text{W/m}^2\cdot°\text{C}}\right) - \frac{0.005\,\text{m}}{1.4\ \text{W/m}\cdot°\text{C}}\right]^{-1}$$

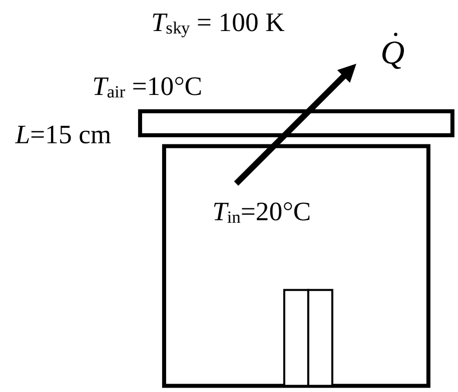
$$= \mathbf{112\ W/m^2\cdot°C}$$

Discussion In practical situations, the ambient temperature and the convective heat transfer coefficient outside the automobile vary with weather conditions and the automobile speed. Therefore the convection heat transfer coefficient of the warm air necessary to melt the ice should be varied as well. This is done by adjusting the warm air flow rate and temperature.

3-32 The roof of a house with a gas furnace consists of a concrete that is losing heat to the outdoors by radiation and convection. The rate of heat transfer through the roof and the money lost through the roof that night during a 14 hour period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The emissivity and thermal conductivity of the roof are constant.

Properties The thermal conductivity of the concrete is given to be k = 2 W/m·°C. The emissivity of both surfaces of the roof is given to be 0.9.



Analysis When the surrounding surface temperature is different than the ambient temperature, the thermal resistances network approach becomes cumbersome in problems that involve radiation. Therefore, we will use a different but intuitive approach.

In steady operation, heat transfer from the room to the roof (by convection and radiation) must be equal to the heat transfer from the roof to the surroundings (by convection and radiation), that must be equal to the heat transfer through the roof by conduction. That is,

$$\dot{Q} = \dot{Q}_{\text{room to roof, conv+rad}} = \dot{Q}_{\text{roof, cond}} = \dot{Q}_{\text{roof to surroundings, conv+rad}}$$

Taking the inner and outer surface temperatures of the roof to be $T_{s,in}$ and $T_{s,out}$, respectively, the quantities above can be expressed as

$$\dot{Q}_{\text{room to roof, conv+rad}} = h_i A(T_{room} - T_{s,in}) + \varepsilon A\sigma(T_{room}{}^4 - T_{s,in}{}^4) = (5\ \text{W/m}^2\cdot°\text{C})(300\ \text{m}^2)(20 - T_{s,in})°\text{C}$$
$$+ (0.9)(300\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(20+273\ \text{K})^4 - (T_{s,in}+273\ \text{K})^4\right]$$

$$\dot{Q}_{\text{roof, cond}} = kA\frac{T_{s,in} - T_{s,out}}{L} = (2\ \text{W/m}\cdot°\text{C})(300\ \text{m}^2)\frac{T_{s,in} - T_{s,out}}{0.15\ \text{m}}$$

$$\dot{Q}_{\text{roof to surr, conv+rad}} = h_o A(T_{s,out} - T_{surr}) + \varepsilon A\sigma(T_{s,out}{}^4 - T_{surr}{}^4) = (12\ \text{W/m}^2\cdot°\text{C})(300\ \text{m}^2)(T_{s,out} - 10)°\text{C}$$
$$+ (0.9)(300\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(T_{s,out}+273\ \text{K})^4 - (100\ \text{K})^4\right]$$

Solving the equations above simultaneously gives

$$\dot{Q} = \mathbf{37{,}440\ W},\ T_{s,in} = \mathbf{7.3°C},\ \text{and}\ T_{s,\text{out}} = -2.1°\text{C}$$

The total amount of natural gas consumption during a 14-hour period is

$$Q_{gas} = \frac{Q_{total}}{0.80} = \frac{\dot{Q}\Delta t}{0.80} = \frac{(37.440\ \text{kJ/s})(14\times3600\ \text{s})}{0.80}\left(\frac{1\ \text{therm}}{105{,}500\ \text{kJ}}\right) = 22.36\ \text{therms}$$

Finally, the money lost through the roof during that period is

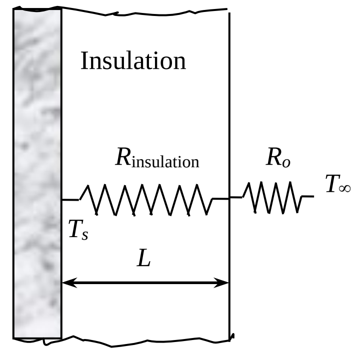
$$\text{Money lost} = (22.36\ \text{therms})(\$1.20\ /\ \text{therm}) = \mathbf{\$26.8}$$

3-33 An exposed hot surface of an industrial natural gas furnace is to be insulated to reduce the heat loss through that section of the wall by 90 percent. The thickness of the insulation that needs to be used is to be determined. Also, the length of time it will take for the insulation to pay for itself from the energy it saves will be determined.

Assumptions **1** Heat transfer through the wall is steady and one-dimensional. **2** Thermal conductivities are constant. **3** The furnace operates continuously. **4** The given heat transfer coefficient accounts for the radiation effects.

Properties The thermal conductivity of the glass wool insulation is given to be $k = 0.038$ W/m·°C.

Analysis The rate of heat transfer without insulation is



$$A = (2\text{ m})(1.5\text{m}) = 3\text{m}^2$$

$$\dot{Q} = hA(T_s - T_\infty) = (10\text{ W/m}^2\cdot°\text{C})(3\text{m}^2)(110-32)°\text{C} = 2340\text{ W}$$

In order to reduce heat loss by 90%, the new heat transfer rate and thermal resistance must be

$$\dot{Q} = 0.10 \times 2340\text{ W} = 234\text{ W}$$

$$\dot{Q} = \frac{\Delta T}{R_{total}} \longrightarrow R_{total} = \frac{\Delta T}{\dot{Q}} = \frac{(110-32)°\text{C}}{234\text{ W}} = 0.333\,°\text{C/W}$$

and in order to have this thermal resistance, the thickness of insulation must be

$$R_{total} = R_{\text{conv}} + R_{insulation} = \frac{1}{hA} + \frac{L}{kA}$$

$$= \frac{1}{(10\text{ W/m}^2\cdot°\text{C})(3\text{m}^2)} + \frac{L}{(0.038\text{ W/m.°C})(3\text{m}^2)} = 0.333\,°\text{C/W}$$

$$L = 0.034\text{ m} = \mathbf{3.4\,cm}$$

Noting that heat is saved at a rate of 0.9×2340 = 2106 W and the furnace operates continuously and thus 365×24 = 8760 h per year, and that the furnace efficiency is 78%, the amount of natural gas saved per year is

$$\text{Energy Saved} = \frac{\dot{Q}_{saved}\Delta t}{Efficiency} = \frac{(2.106\text{ kJ/s})(8760\text{h})}{0.78}\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{1\text{ therm}}{105{,}500\text{ kJ}}\right) = 807.1\text{ therms}$$

The money saved is

$$\text{Money saved} = (\text{Energy Saved})(\text{Cost of energy}) = (807.1\text{ therms})(\$1.10/\text{therm}) = \$887.8\text{ (per year)}$$

The insulation will pay for its cost of $250 in

$$\text{Payback period} = \frac{\text{Money spent}}{\text{Money saved}} = \frac{\$250}{\$887.8/\text{yr}} = \mathbf{0.282\,yr}$$

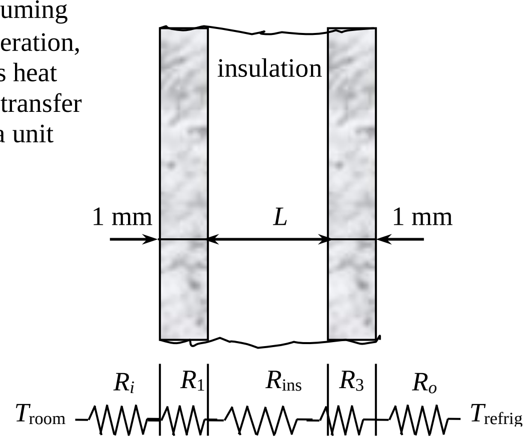
which is equal to 3.4 months.

3-34 The wall of a refrigerator is constructed of fiberglass insulation sandwiched between two layers of sheet metal. The minimum thickness of insulation that needs to be used in the wall in order to avoid condensation on the outer surfaces is to be determined.

Assumptions **1** Heat transfer through the refrigerator walls is steady since the temperatures of the food compartment and the kitchen air remain constant at the specified values. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation effects.

Properties The thermal conductivities are given to be k = 15.1 W/m·°C for sheet metal and 0.035 W/m·°C for fiberglass insulation.

Analysis The minimum thickness of insulation can be determined by assuming the outer surface temperature of the refrigerator to be 20°C. In steady operation, the rate of heat transfer through the refrigerator wall is constant, and thus heat transfer between the room and the refrigerated space is equal to the heat transfer between the room and the outer surface of the refrigerator. Considering a unit surface area,



$$\dot{Q} = h_o A(T_{room} - T_{s,out})$$
$$= (9\ \text{W/m}^2\cdot°\text{C})(1\ \text{m}^2)(24-20)°\text{C} = 36\ \text{W}$$

Using the thermal resistance network, heat transfer between the room and the refrigerated space can be expressed as

$$\dot{Q} = \frac{T_{room} - T_{refrig}}{R_{total}}$$

$$\dot{Q}/A = \frac{T_{room} - T_{refrig}}{\dfrac{1}{h_o} + 2\left(\dfrac{L}{k}\right)_{metal} + \left(\dfrac{L}{k}\right)_{insulation} + \dfrac{1}{h_i}}$$

Substituting,

$$36\ \text{W/m}^2 = \frac{(24-2)°\text{C}}{\dfrac{1}{9\ \text{W/m}^2\cdot°\text{C}} + \dfrac{2\times 0.001\,\text{m}}{15.1\,\text{W/m}^2\cdot°\text{C}} + \dfrac{L}{0.035\ \text{W/m}^2\cdot°\text{C}} + \dfrac{1}{4\ \text{W/m}^2\cdot°\text{C}}}$$

Solv ing for L, the minimum thickness of insulation is determined to be

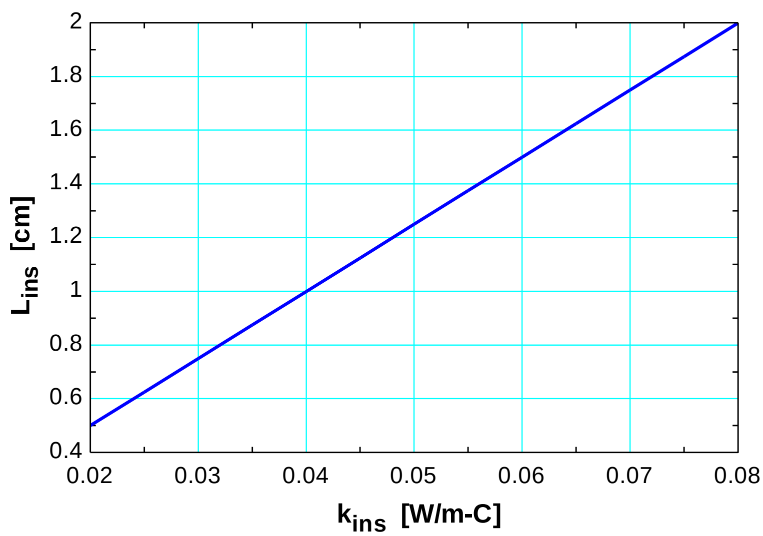
L = 0.00875 m = **0.875 cm**

3-35 Prob. 3-34 is reconsidered. The effects of the thermal conductivities of the insulation material and the sheet metal on the thickness of the insulation is to be investigated.

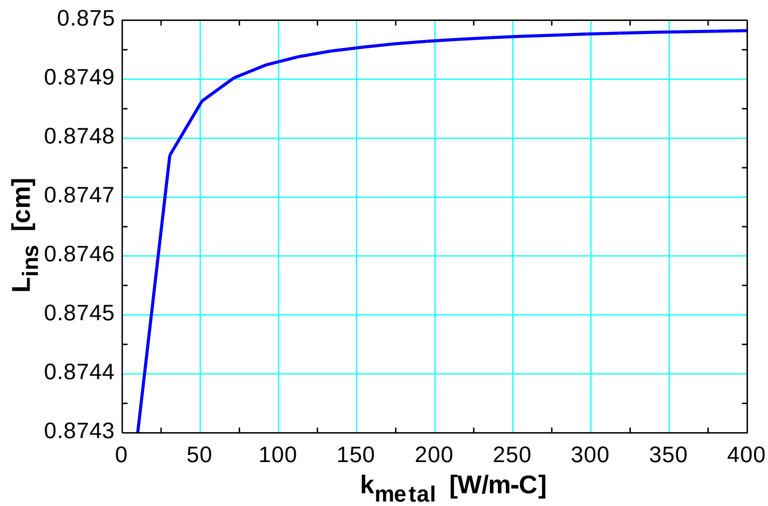
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
k_ins=0.035 [W/m-C]
L_metal=0.001 [m]
k_metal=15.1 [W/m-C]
T_refrig=2 [C]
T_kitchen=24 [C]
h_i=4 [W/m^2-C]
h_o=9 [W/m^2-C]
T_s_out=20 [C]

"ANALYSIS"
A=1 [m^2] “a unit surface area is considered"
Q_dot=h_o*A*(T_kitchen-T_s_out)
Q_dot=(T_kitchen-T_refrig)/R_total
R_total=R_conv_i+2*R_metal+R_ins+R_conv_o
R_conv_i=1/(h_i*A)
R_metal=L_metal/(k_metal*A)
R_ins=(L_ins*Convert(cm, m))/(k_ins*A) "L_ins is in cm"
R_conv_o=1/(h_o*A)
```

k_{ins} [W/m.C]	L_{ins} [cm]
0.02	0.4997
0.025	0.6247
0.03	0.7496
0.035	0.8745
0.04	0.9995
0.045	1.124
0.05	1.249
0.055	1.374
0.06	1.499
0.065	1.624
0.07	1.749
0.075	1.874
0.08	1.999



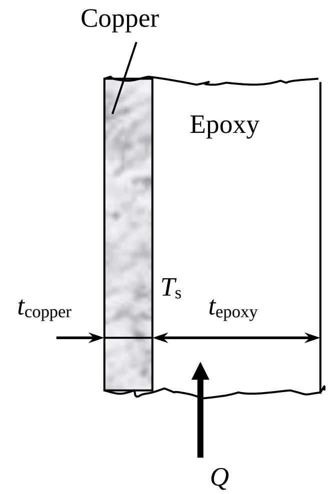
k_{metal} [W/m.C]	L_{ins} [cm]
10	0.8743
30.53	0.8748
51.05	0.8749
71.58	0.8749
92.11	0.8749
112.6	0.8749
133.2	0.8749
153.7	0.875
174.2	0.875
194.7	0.875
215.3	0.875
235.8	0.875
256.3	0.875
276.8	0.875
297.4	0.875
317.9	0.875
338.4	0.875
358.9	0.875
379.5	0.875
400	0.875



3-36 Heat is to be conducted along a circuit board with a copper layer on one side. The percentages of heat conduction along the copper and epoxy layers as well as the effective thermal conductivity of the board are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since heat transfer from the side surfaces is disregarded **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 386 W/m·°C for copper and 0.26 W/m·°C for epoxy layers.



Analysis We take the length in the direction of heat transfer to be L and the width of the board to be w. Then heat conduction along this two-layer board can be expressed as

$$\dot{Q} = \dot{Q}_{\text{copper}} + \dot{Q}_{\text{epoxy}} = \left(kA\frac{\Delta T}{L}\right)_{\text{copper}} + \left(kA\frac{\Delta T}{L}\right)_{\text{epoxy}}$$

$$= \left[(kt)_{\text{copper}} + (kt)_{\text{epoxy}}\right] w\frac{\Delta T}{L}$$

Heat conduction along an "equivalent" board of thickness $t = t_{\text{copper}} + t_{\text{epoxy}}$ and thermal conductivity k_{eff} can be expressed as

$$\dot{Q} = \left(kA\frac{\Delta T}{L}\right)_{\text{board}} = k_{\text{eff}}\ (t_{\text{copper}} + t_{\text{epoxy}})w\frac{\Delta T}{L}$$

Setting the two relations above equal to each other and solving for the effective conductivity gives

$$k_{eff}\,(t_{\text{copper}} + t_{\text{epoxy}}) = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} \longrightarrow k_{eff} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}}$$

Note that heat conduction is proportional to kt. Substituting, the fractions of heat conducted along the copper and epoxy layers as well as the effective thermal conductivity of the board are determined to be

$$(kt)_{copper} = (386\ \text{W/m}\cdot{}^\circ\text{C})(0.0001\,\text{m}) = 0.0386\ \text{W/}^\circ\text{C}$$

$$(kt)_{epoxy} = (0.26\ \text{W/m}\cdot{}^\circ\text{C})(0.0012\,\text{m}) = 0.000312\ \text{W/}^\circ\text{C}$$

$$(kt)_{total} = (kt)_{copper} + (kt)_{epoxy} = 0.0386 + 0.000312 = 0.038912\ \text{W/}^\circ\text{C}$$

$$f_{epoxy} = \frac{(kt)_{epoxy}}{(kt)_{total}} = \frac{0.000312}{0.038912} = 0.008 = \mathbf{0.8\%}$$

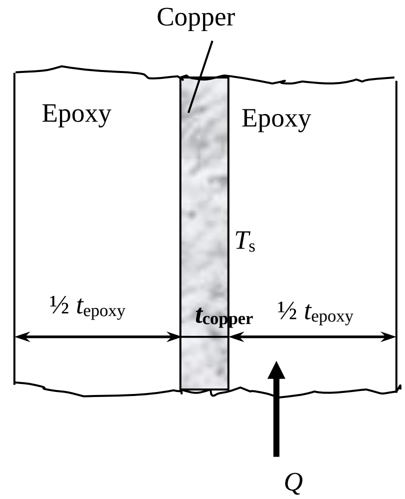
$$f_{copper} = \frac{(kt)_{copper}}{(kt)_{total}} = \frac{0.0386}{0.038912} = 0.992 = \mathbf{99.2\%}$$

and

$$k_{eff} = \frac{(386\times 0.0001 + 0.26\times 0.0012)\ \text{W/}^\circ\text{C}}{(0.0001 + 0.0012)\ \text{m}} = \mathbf{29.9\ W/m.^\circ C}$$

3-37E A thin copper plate is sandwiched between two layers of epoxy boards. The effective thermal conductivity of the board along its 9 in long side and the fraction of the heat conducted through copper along that side are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since heat transfer from the side surfaces are disregarded **3** Thermal conductivities are constant.



Properties The thermal conductivities are given to be k = 223 Btu/h·ft·°F for copper and 0.15 Btu/h·ft·°F for epoxy layers.

Analysis We take the length in the direction of heat transfer to be L and the width of the board to be w. Then heat conduction along this two-layer plate can be expressed as (we treat the two layers of epoxy as a single layer that is twice as thick)

$$\dot{Q} = \dot{Q}_{\text{copper}} + \dot{Q}_{\text{epoxy}}$$
$$= \left(kA\frac{\Delta T}{L}\right)_{\text{copper}} + \left(kA\frac{\Delta T}{L}\right)_{\text{epoxy}} = \left[(kt)_{\text{copper}} + (kt)_{\text{epoxy}}\right]w\frac{\Delta T}{L}$$

Heat conduction along an "equivalent" plate of thick ness $t = t_{\text{copper}} + t_{\text{epoxy}}$ and thermal conductivity k_{eff} can be expressed as

$$\dot{Q} = \left(kA\frac{\Delta T}{L}\right)_{\text{board}} = k_{\text{eff}}\ (t_{\text{copper}} + t_{\text{epoxy}})w\frac{\Delta T}{L}$$

Setting the two relations above equal to each other and solving for the effective conductivity gives

$$k_{eff}\,(t_{\text{copper}} + t_{\text{epoxy}}) = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} \longrightarrow k_{eff} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{t_{\text{copper}} + t_{\text{epoxy}}}$$

Note that heat conduction is proportional to kt. Substituting, the fraction of heat conducted along the copper layer and the effective thermal conductivity of the plate are determined to be

$$(kt)_{\text{copper}} = (223\text{ Btu/h.ft.°F})(0.05/12\text{ ft}) = 0.9292\text{ Btu/h.°F}$$
$$(kt)_{\text{epoxy}} = 2(0.15\text{ Btu/h.ft.°F})(0.15/12\text{ ft}) = 0.00375\text{ Btu/h.°F}$$
$$(kt)_{\text{total}} = (kt)_{\text{copper}} + (kt)_{\text{epoxy}} = 0.9292 + 0.00375 = 0.93292\text{ Btu/h.°F}$$

and

$$k_{eff} = \frac{(kt)_{\text{copper}} + (kt)_{\text{epoxy}}}{\text{t}_{\text{copper}} + t_{\text{epoxy}}}$$
$$= \frac{0.93292\text{ Btu/h.°F}}{[(0.05/12) + 2(0.15/12)]\text{ ft}} = \mathbf{32.0\ Btu/h.ft^2.°F}$$

$$f_{\text{copper}} = \frac{(kt)_{\text{copper}}}{(kt)_{\text{total}}} = \frac{0.9292}{0.93292} = 0.996 = \mathbf{99.6\%}$$

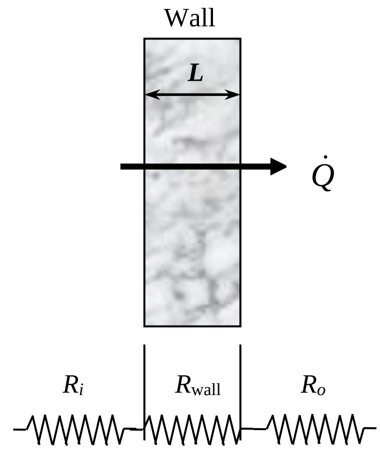
3-38 Two of the walls of a house have no windows while the other two walls have single- or double-pane windows. The average rate of heat transfer through each wall, and the amount of money this household will save per heating season by converting the single pane windows to double pane windows are to be determined.

Assumptions **1** Heat transfer through the window is steady since the indoor and outdoor temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivities of the glass and air are constant. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be k = 0.026 W/m·°C for air, and 0.78 W/m·°C for glass.

Analysis The rate of heat transfer through each wall can be determined by applying thermal resistance network. The convection resistances at the inner and outer surfaces are common in all cases.

Walls without windows:



$$R_i = \frac{1}{h_i A} = \frac{1}{(7\ \text{W/m}^2\cdot{}^\circ\text{C})(10\times 4\ \text{m}^2)} = 0.003571\,{}^\circ\text{C/W}$$

$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\,\text{m}^2\cdot{}^\circ\text{C/W}}{(10\times 4\,\text{m}^2)} = 0.05775\,{}^\circ\text{C/W}$$

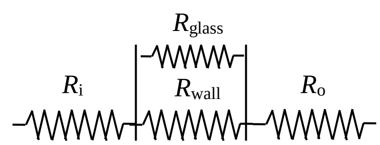
$$R_o = \frac{1}{h_o A} = \frac{1}{(18\ \text{W/m}^2\cdot{}^\circ\text{C})(10\times 4\ \text{m}^2)} = 0.001389{}^\circ\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{wall}} + R_o = 0.003571 + 0.05775 + 0.001389 = 0.06271\,{}^\circ\text{C/W}$$

Then

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(24-8){}^\circ\text{C}}{0.06271{}^\circ\text{C/W}} = \mathbf{255.1\ W}$$

Wall with single pane windows:



$$R_i = \frac{1}{h_i A} = \frac{1}{(7\ \text{W/m}^2\cdot{}^\circ\text{C})(20\times 4\ \text{m}^2)} = 0.001786\,{}^\circ\text{C/W}$$

$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R-value}{A} = \frac{2.31\,\text{m}^2\cdot{}^\circ\text{C/W}}{(20\times 4) - 5(1.2\times 1.8)\ \text{m}^2} = 0.033382\,{}^\circ\text{C/W}$$

$$R_{\text{glass}} = \frac{L_{\text{glass}}}{kA} = \frac{0.005\ \text{m}}{(0.78\ \text{W/m}^2\cdot{}^\circ\text{C})(1.2\times 1.8)\text{m}^2} = 0.002968\,{}^\circ\text{C/W}$$

$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_{\text{wall}}} + 5\frac{1}{R_{\text{glass}}} = \frac{1}{0.033382} + 5\frac{1}{0.002968} \rightarrow R_{eqv} = 0.000583\ {}^\circ\text{C/W}$$

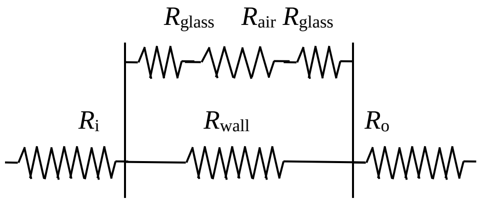
$$R_o = \frac{1}{h_o A} = \frac{1}{(18\ \text{W/m}^2\cdot{}^\circ\text{C})(20\times 4\ \text{m}^2)} = 0.000694\,{}^\circ\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{eqv}} + R_o = 0.001786 + 0.000583 + 0.000694 = 0.003063\,{}^\circ\text{C/W}$$

Then

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(24-8){}^\circ\text{C}}{0.003063{}^\circ\text{C/W}} = \mathbf{5224\ W}$$

4th wall with double pane windows:



$$R_{\text{wall}} = \frac{L_{\text{wall}}}{kA} = \frac{R - value}{A} = \frac{2.31\,\text{m}^2 \cdot °\text{C/W}}{(20\times 4) - 5(1.2\times 1.8)\text{m}^2} = 0.033382\,°\text{C/W}$$

$$R_{\text{glass}} = \frac{L_{\text{glass}}}{kA} = \frac{0.005\,\text{m}}{(0.78\ \text{W/m}^2 \cdot °\text{C})(1.2\times 1.8)\text{m}^2} = 0.002968\,°\text{C/W}$$

$$R_{\text{air}} = \frac{L_{\text{air}}}{kA} = \frac{0.015\,\text{m}}{(0.026\ \text{W/m}^2 \cdot °\text{C})(1.2\times 1.8)\text{m}^2} = 0.267094\,°\text{C/W}$$

$$R_{\text{window}} = 2R_{\text{glass}} + R_{\text{air}} = 2\times 0.002968 + 0.267094 = 0.27303\,°\text{C/W}$$

$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_{\text{wall}}} + 5\frac{1}{R_{\text{window}}} = \frac{1}{0.033382} + 5\frac{1}{0.27303} \longrightarrow R_{\text{eqv}} = 0.020717\,°\text{C/W}$$

$$R_{\text{total}} = R_i + R_{\text{eqv}} + R_o = 0.001786 + 0.020717 + 0.000694 = 0.023197\,°\text{C/W}$$

Then

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(24-8)°\text{C}}{0.023197°\text{C/W}} = \mathbf{690\ W}$$

The rate of heat transfer which will be saved if the single pane windows are converted to double pane windows is

$$\dot{Q}_{\text{save}} = \dot{Q}_{\substack{\text{single}\\\text{pane}}} - \dot{Q}_{\substack{\text{double}\\\text{pane}}} = 5224 - 690 = 4534\ \text{W}$$

The amount of energy and money saved during a 7-month long heating season by switching from single pane to double pane windows become

$$Q_{save} = \dot{Q}_{save}\Delta t = (4.534\,\text{kW})(7\times 30\times 24\,\text{h}) = 22{,}851\,\text{kWh}$$

Money savings = (Energy saved)(Unit cost of energy) = (22,851 kWh)($0.08/kWh) = **$1828**

3-39E Two of the walls of a house have no windows while the other two walls have 4 windows each. The ratio of heat transfer through the walls with and without windows is to be determined.

Assumptions **1** Heat transfer through the walls and the windows is steady and one-dimensional. **2** Thermal conductivity of each wall is constant. **3** Any direct radiation gain or loss through the windows is negligible. **4** Heat transfer coefficients are constant and uniform over the entire surface.

Properties The thermal conductivity of the glass is given to be k_{glass} = 0.45 Btu/h·ft·°F. The R-value of the wall is given to be 19 h·ft^2·°F/Btu.

Analysis The thermal resistances through the wall without windows are

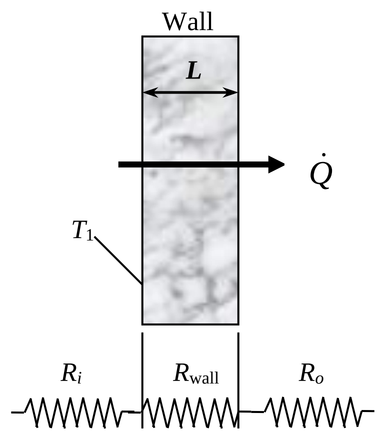
$$A = (12\text{ ft})(40\text{ ft}) = 480\text{ ft}^2$$

$$R_i = \frac{1}{h_i A} = \frac{1}{(2\text{ Btu/h.ft}^2\cdot°\text{F})(480\text{ ft}^2)} = 0.0010417\text{ h}\cdot°\text{F/Btu}$$

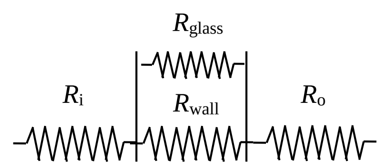
$$R_{\text{wall}} = \frac{L}{kA} = \frac{19\text{ h}\cdot\text{ft}^2\,°\text{F/Btu}}{480\text{ ft}^2} = 0.03958\text{ h}\cdot°\text{F/Btu}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(4\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(480\text{ ft}^2)} = 0.00052\text{ h}\cdot°\text{F/Btu}$$

$$R_{total,1} = R_i + R_{wall} + R_o$$
$$= 0.0010417 + 0.03958 + 0.00052 = 0.0411417\text{ h}\cdot°\text{F/Btu}$$



The thermal resistances through the wall with windows are



$$A_{windows} = 4(3\times 5) = 60\text{ ft}^2$$

$$A_{wall} = A_{total} - A_{windows} = 480 - 60 = 420\text{ ft}^2$$

$$R_2 = R_{glass} = \frac{L}{kA} = \frac{0.25/12\text{ ft}}{(0.45\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})(60\text{ ft}^2)} = 0.0007716\text{ h}\cdot°\text{F/Btu}$$

$$R_4 = R_{wall} = \frac{L}{kA} = \frac{19\text{ h}\cdot\text{ft}^2\cdot°\text{F/Btu}}{(420\text{ ft}^2)} = 0.04524\text{ h}\cdot°\text{F/Btu}$$

$$\frac{1}{R_{eqv}} = \frac{1}{R_{glass}} + \frac{1}{R_{wall}} = \frac{1}{0.0007716} + \frac{1}{0.04524} \longrightarrow R_{eqv} = 0.00076\cdot\text{h°F/Btu}$$

$$R_{total,2} = R_i + R_{eqv} + R_o = 0.001047 + 0.00076 + 0.00052 = 0.002327\text{ h}\cdot°\text{F/Btu}$$

Then the ratio of the heat transfer through the walls with and without windows becomes

$$\frac{\dot{Q}_{total,2}}{\dot{Q}_{total,1}} = \frac{\Delta T/R_{total,2}}{\Delta T/R_{total,1}} = \frac{R_{total,1}}{R_{total,2}} = \frac{0.0411417}{0.002327} = \mathbf{17.7}$$

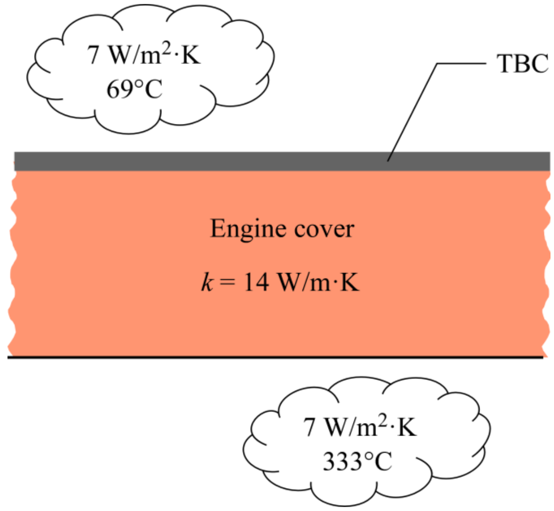
Discussion In case of heat transfer through the walls with widows, additional heat transfer may occur due to air infiltration through the cracks around the window edges. Heat transfer calculations for building heating and cooling applications account for this additional heat transfer through the 'crack' method.

3-40 An engine cover is subjected to convection heat transfer on the inner surface and the outer surface. The thickness of a thermal barrier coating (TBC) layer applied on the engine cover outer surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal conductivitiesare constant.**4** Thermal contact resistance at interface is negligible.

PropertiesThe thermal conductivities of the stainless steel and the TBC are given to bek_1= 14 W/m·K andk_2 = 1.1 W/m·K, respectively.

AnalysisThe thermal resistances of different layers are



$$R_{\text{conv},1} = \frac{1}{h_1 A} \quad \text{(inside surface convection resistance)}$$

$$R_1 = \frac{L_1}{k_1 A} \quad \text{(stainless steel layer resistance)}$$

$$R_2 = \frac{L_2}{k_2 A} \quad \text{(TBC layer resistance)}$$

$$R_{\text{conv},2} = \frac{1}{h_2 A} \quad \text{(outside surface convection resistance)}$$

Then,

$$\begin{aligned} AR_{\text{total}} &= A(R_{\text{conv},1} + R_1 + R_2 + R_{\text{conv},2}) \\ &= \frac{1}{h_1} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{1}{h_2} \\ &= \frac{1}{7\ \text{W/m}^2\cdot\text{K}} + \frac{0.01\ \text{m}}{14\ \text{W/m}\cdot\text{K}} + \frac{0.004\ \text{m}}{1.1\ \text{W/m}\cdot\text{K}} + \frac{1}{7\ \text{W/m}^2\cdot\text{K}} \\ &= 0.2901\ \text{m}^2\cdot\text{K/W} \end{aligned}$$

and

$$AR_{\text{conv},2} = \frac{1}{h_2} = \frac{1}{7\ \text{W/m}^2\cdot\text{K}} = 0.1429\ \text{m}^2\cdot\text{K/W}$$

The heat flux through the layers is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_{\infty,1} - T_{\infty,2}}{AR_{\text{total}}} = \frac{T_2 - T_{\infty,2}}{AR_{\text{conv},2}} \rightarrow T_2 = \frac{R_{\text{conv},2}}{R_{\text{total}}}(T_{\infty,1} - T_{\infty,2}) + T_{\infty,2}$$

$$T_2 = \frac{0.1429}{0.2901}(333 - 69)°\text{C} + 69°\text{C} = \mathbf{199°C}$$

Yes, a TBC layer with a thickness of 4 mm will keep the engine cover surface below 200°C.

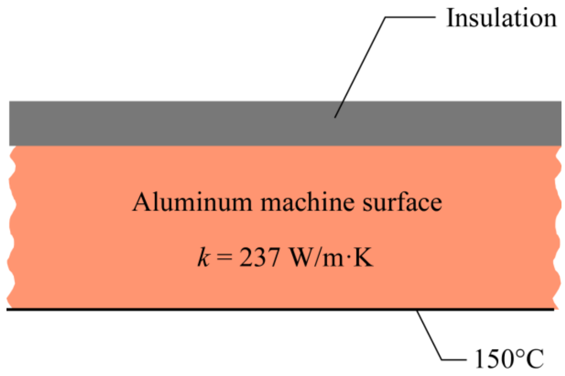
DiscussionSince the calculated cover surface temperature of 199°C is very close to the required temperature of 200°C to prevent fire hazard, it is recommended to increase the TBC layer thickness and use a TBC layer of lower thermal conductivity. Doubling the TBC layer (8mm) would reduce the cover surface temperature to about 197°C, a reduction of 2°C.

3-41 PtD To prevent hot spots on a machine surface from causing thermal burns, the thickness of an insulation to cover the machine surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal conductivitiesare constant.**4** Thermal contact resistance at interface is negligible.

PropertiesThe thermal conductivities of the aluminum and the insulation are given to bek_1= 237 W/m·K andk_2 = 0.06 W/m·K, respectively. The thermal contact conductance at the interface is given as 3000 W/m^2·K.

AnalysisThe thermal resistances of different layers are



$$R_1 = \frac{L_1}{k_1 A} \quad \text{(aluminum layer resistance)}$$

$$R_{\text{interface}} = \frac{1}{h_c A}$$

$$R_2 = \frac{L_2}{k_2 A} \quad \text{(insulation layer resistance)}$$

Then,

$$AR_{\text{total}} = A(R_1 + R_{\text{interface}} + R_2) = \frac{L_1}{k_1} + \frac{1}{h_c} + \frac{L_2}{k_2}$$

The heat flux through the layers is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_1 - T_2}{AR_{\text{total}}} = \frac{T_1 - T_2}{\dfrac{L_1}{k_1} + \dfrac{1}{h_c} + \dfrac{L_2}{k_2}} \rightarrow L_2 = k_2\left[\frac{T_1 - T_2}{\dot{q}} - \left(\frac{L_1}{k_1} + \frac{1}{h_c}\right)\right]$$

$$L_2 = (0.06 \text{ W/m}\cdot\text{K})\left[\frac{(150-45)\text{ K}}{300 \text{ W/m}^2} - \left(\frac{0.005\text{ m}}{237 \text{ W/m}\cdot\text{K}} + \frac{1}{3000 \text{ W/m}^2\cdot\text{K}}\right)\right]$$

$$= 0.021\text{ m}$$

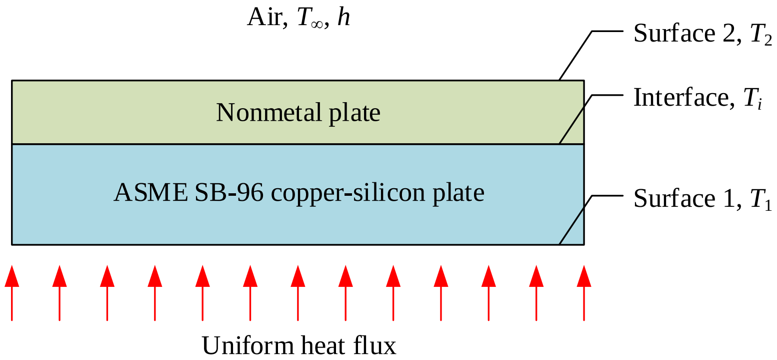
$$= \mathbf{21\ mm}$$

DiscussionBy covering the surface of the machine with 21mm of insulation, the surface temperature can be kept below 45°C.

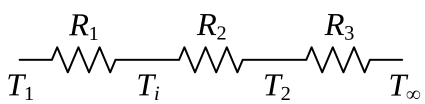
3-42 A nonmetal plate and an ASME SB-96 copper-silicon plate are attached together. The bottom surface is subjected to uniform heat flux. The top surface is exposed to convection heat transfer. Determine the temperatures T_i and T_1, and whether the use of the ASME SB-96 plate complies with the ASME Boiler and Pressure Vessel Code.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through plates. **3** Uniform heat flux on bottom surface. **4** Uniform surface temperatures. **5** No contact resistance at the interface. **6** Thermal properties are constant.

Properties The thermal conductivity for the ASME SB-96 copper-silicon plate is given as k_1 = 36 W/m·K, and for the nonmetal plate as k_2 = 0.05 W/m·K.



Analysis The thermal resistances encountered by the heat flow are



$$R_1 = \frac{L_1}{k_1 A}$$

$$R_2 = \frac{L_2}{k_2 A}$$

$$R_3 = \frac{1}{hA}$$

The heat flux through each thermal resistance is

$$\dot{q} = \frac{T_1 - T_i}{AR_1} = \frac{T_i - T_2}{AR_2} = \frac{T_2 - T_\infty}{AR_3}$$

The temperature at the upper surface of the nonmetal plate T_2 can be determined with

$$\dot{q} = \frac{T_2 - T_\infty}{AR_3} = \frac{T_2 - T_\infty}{1/h} \quad \rightarrow \quad T_2 = \frac{\dot{q}}{h} + T_\infty = \frac{200\ \text{W/m}^2}{10\ \text{W/m}^2{\cdot}\text{K}} + 15°\text{C} = 35°\text{C}$$

The temperature at the interface T_i is

$$\dot{q} = \frac{T_i - T_2}{AR_2} = \frac{T_i - T_2}{L_2/k_2} \quad \rightarrow \quad T_i = \dot{q}\frac{L_2}{k_2} + T_2 = \frac{(200\ \text{W/m}^2)(0.020\ \text{m})}{0.05\ \text{W/m}{\cdot}\text{K}} + 35°\text{C} = \mathbf{115}°\text{C}$$

The temperature at the bottom surface of the copper-silicon plate T_1 is

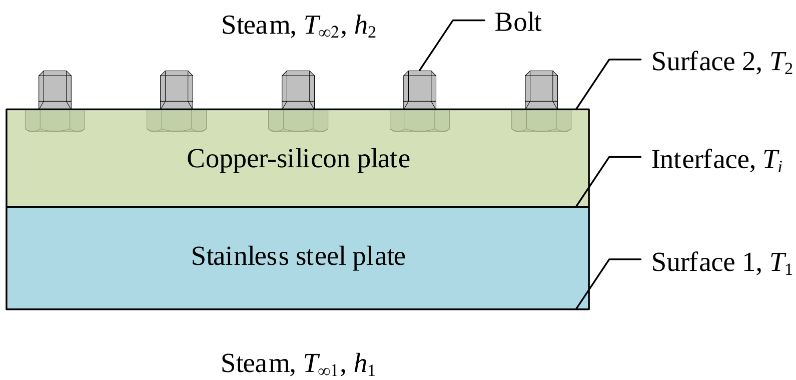
$$\dot{q} = \frac{T_1 - T_i}{AR_1} = \frac{T_1 - T_i}{L_1/k_1} \quad \rightarrow \quad T_1 = \dot{q}\frac{L_1}{k_1} + T_i = \frac{(200\ \text{W/m}^2)(0.035\ \text{m})}{36\ \text{W/m}{\cdot}\text{K}} + 115°\text{C} = \mathbf{115.2}°\text{C}$$

Discussion The ASME SB-96 copper-silicon plate has an average temperature of 115.2°C. Therefore, it does not comply with the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HF-300). The code limits the use of equipment constructed with the ASME SB-96 plate to operate at below 93°C.

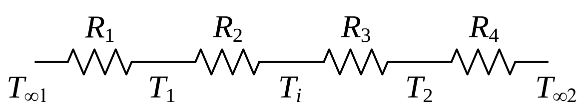
3-43 A series of ASTM B21 naval brass bolts are attached on the upper surface of a copper-silicon (ASME SB-96) plate. The copper-silicon plate is attached on a stainless steel (ASTM A240 904L) plate. The upper and bottom surfaces are exposed to convection heat transfer with steam. Determine the temperatures T_1, T_2, and T_i, and whether the use of the ASME SB-96 plate, ASTM A240 904L plate, and ASTM B21 bolts complies with the ASME Boiler and Pressure Vessel Code and the ASME Code for Process Piping.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through plates. **3** Uniform surface temperatures. **4** No contact resistance at the interface. **5** Thermal properties are constant.

Properties The thermal conductivity for the ASTM A240 904L stainless steel plate is k_1 = 13 W/m·K, and for the ASME SB-96 copper-silicon plate is k_2 = 36 W/m·K.



Analysis The thermal resistances encountered by the heat flow are



$$R_1 = \frac{1}{h_1 A} \text{ or } \quad AR_1 = \frac{1}{h_1} = \frac{1}{500 \text{ W/m}^2\cdot\text{K}} = 0.002 \text{ m}^2\cdot\text{K/W}$$

$$R_2 = \frac{L_1}{k_1 A} \text{ or } \quad AR_2 = \frac{L_1}{k_1} = \frac{0.025 \text{ m}}{13 \text{ W/m}\cdot\text{K}} = 0.0019231 \text{ m}^2\cdot\text{K/W}$$

$$R_3 = \frac{L_2}{k_2 A} \text{ or } \quad AR_3 = \frac{L_2}{k_2} = \frac{0.025 \text{ m}}{36 \text{ W/m}\cdot\text{K}} = 0.0006944 \text{ m}^2\cdot\text{K/W}$$

$$R_4 = \frac{1}{h_2 A} \text{ or } \quad AR_4 = \frac{1}{h_2} = \frac{1}{300 \text{ W/m}^2\cdot\text{K}} = 0.0033333 \text{ m}^2\cdot\text{K/W}$$

where

$$AR_{\text{tot}} = AR_1 + AR_2 + AR_3 + AR_4 = 0.0079508 \text{ m}^2\cdot\text{K/W}$$

The heat flux through the thermal circuit is

$$\dot{q} = \frac{T_{\infty 1} - T_{\infty 2}}{AR_{\text{tot}}} = \frac{(260 - 80)\text{K}}{0.0079508 \text{ m}^2\cdot\text{K/W}} = 22639 \text{ W/m}^2$$

The temperatures T_1, T_2, and T_i can be calculated using the heat flux and the individual thermal resistance:

$$\dot{q} = \frac{T_{\infty 1} - T_1}{AR_1} \quad \rightarrow \quad T_1 = T_{\infty 1} - \dot{q}AR_1 = 260°\text{C} - (22639 \text{ W/m}^2)(0.002 \text{ m}^2\cdot\text{K/W}) = \mathbf{214.7}°\text{C}$$

$$\dot{q} = \frac{T_2 - T_{\infty 2}}{AR_4} \quad \rightarrow \quad T_2 = \dot{q}AR_4 + T_{\infty 2} = (22639 \text{ W/m}^2)(0.0033333 \text{ m}^2\cdot\text{K/W}) + 80°\text{C} = \mathbf{155.5}°\text{C}$$

$$\dot{q} = \frac{T_1 - T_i}{AR_2} \qquad \rightarrow \qquad T_i = T_1 - \dot{q}AR_2 = 214.7°\text{C} - (22639\ \text{W/m}^2)(0.0019231\ \text{m}^2\text{·K/W}) = \mathbf{171.2}°\text{C}$$

Discussion The ASME SB-96 copper-silicon plate is operating in the temperature range of 155°C to 171°C, which is above the temperature (93°C) specified by the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HF-300). The ASTM B21 naval brass bolts are attached to the plate's upper surface, which is at T_2 = 155.5°C. Therefore, the ASTM B21 bolts do not comply with the ASME Code for Process Piping (ASME B31.3-2014), which has a maximum use temperature of 149°C. Although the bottom surface of the ASTM A240 904L stainless steel plate is exposed to a steam temperature of 260°C, which is the maximum use temperature for the plate as stipulated by the ASME Code for Process Piping (ASME B31.3-2014), the plate's bottom surface temperature is 214.7°C. Therefore the ASTM A240 904L plate complies with the code.

Thermal Contact Resistance

3-44C The resistance that an interface offers to heat transfer per unit interface area is called thermal contact resistance, R_c. The inverse of thermal contact resistance is called the thermal contact conductance.

3-45C The thermal contact resistance will be greater for rough surfaces because an interface with rough surfaces will contain more air gaps whose thermal conductivity is low.

3-46C Thermal contact resistance can be minimized by (1) applying a thermally conducting liquid on the surfaces before they are pressed against each other, (2) by replacing the air at the interface by a better conducting gas such as helium or hydrogen, (3) by increasing the interface pressure, and (4) by inserting a soft metallic foil such as tin, silver, copper, nickel, or aluminum between the two surfaces.

3-47C An interface acts like a very thin layer of insulation, and thus the thermal contact resistance has significance only for highly conducting materials like metals. Therefore, the thermal contact resistance can be ignored for two layers of insulation pressed against each other.

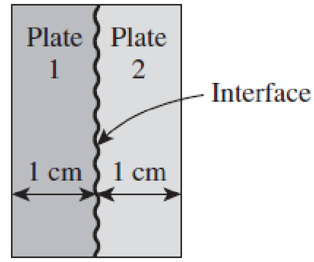
3-48C An interface acts like a very thin layer of insulation, and thus the thermal contact resistance is significant for highly conducting materials like metals. Therefore, the thermal contact resistance must be considered for two layers of metals pressed against each other.

3-49C Heat transfer through the voids at an interface is by conduction and radiation. Evacuating the interface eliminates heat transfer by conduction, and thus increases the thermal contact resistance.

3-50 The thickness of aluminum plate whose thermal resistance is equal to the thermal contact resistance is to be determined.

Properties The thermal conductivity of aluminum is k = 237 W/m·K (Table A-3).

Analysis Noting that thermal contact resistance is the inverse of thermal contact conductance, the thermal contact resistance is determined to be



$$R_c = \frac{1}{h_c} = \frac{1}{11{,}000 \text{ W/m}^2.\text{K}} = 0.909 \times 10^{-4} \text{ m}^2 \cdot \text{K/W}$$

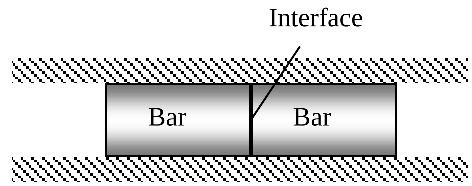
For a unit surface area, the thermal resistance of a flat plate is defined as $R = \frac{L}{k}$ where L is the thickness of the plate and k is the thermal conductivity. Setting $R = R_c$, the equivalent thickness is determined from the relation above to be

$$L = kR = kR_c = (237 \text{ W/m} \cdot \text{K})(0.909 \times 10^{-4} \text{ m}^2 \cdot \text{K/W}) = 0.0215 \text{ m} = \mathbf{2.15 \text{ cm}}$$

Therefore, the interface between the two plates offers as much resistance to heat transfer as a 2.14 cm thick copper. Note that the thermal contact resistance in this case is greater than the sum of the thermal resistances of both plates.

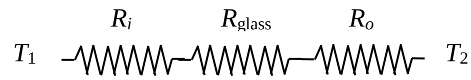
Discussion Note that the interface between the two plates offers as much resistance to heat transfer as a 2.15-cm-thick aluminum plate. It is interesting that the thermal contact resistance in this case is greater than the sum of the thermal resistances of both plates.

3-51 Two cylindrical aluminum bars with ground surfaces are pressed against each other in an insulation sleeve. For specified top and bottom surface temperatures, the rate of heat transfer along the cylinders and the temperature drop at the interface are to be determined.



Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional in the axial direction since the lateral surfaces of both cylinders are well-insulated. **3** Thermal conductivities are constant.

Properties The thermal conductivity of aluminum bars is given to be k = 176 W/m·°C. The contact conductance at the interface of aluminum-aluminum plates for the case of ground surfaces and of 20 atm ≈ 2 MPa pressure is h_c= 11,400 W/m^2·°C (Table 3-2).



Analysis (a) The thermal resistance network in this case consists of two conduction resistance and the contact resistance, and they are determined to be

$$R_{\text{contact}} = \frac{1}{h_c A_c} = \frac{1}{(11{,}400\ \text{W/m}^2 \cdot °\text{C})[\pi(0.05\,\text{m})^2/4]} = 0.0447\ °\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.15\,\text{m}}{(176\ \text{W/m}\cdot°\text{C})[\pi(0.05\,\text{m})^2/4]} = 0.4341\,°\text{C/W}$$

Then the rate of heat transfer is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_{\text{contact}} + 2R_{\text{bar}}} = \frac{(150-20)°\text{C}}{(0.0447 + 2\times 0.4341)\ °\text{C/W}} = \mathbf{142.4\ W}$$

Therefore, the rate of heat transfer through the bars is 142.4 W.

(b) The temperature drop at the interface is determined to be

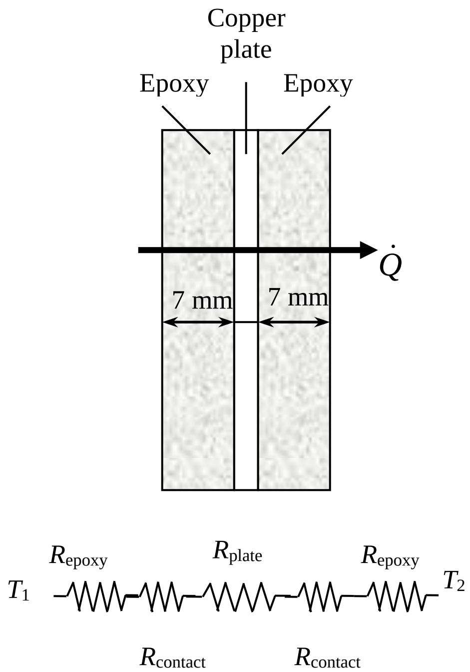
$$\Delta T_{\text{interface}} = \dot{Q}R_{\text{contact}} = (142.4\ \text{W})(0.0447°\text{C/W}) = \mathbf{6.4°C}$$

3-52 A thin copper plate is sandwiched between two epoxy boards. The error involved in the total thermal resistance of the plate if the thermal contact conductances are ignored is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional since the plate is large. **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 401 W/m·°C for copper plates and k = 0.26 W/m·°C for epoxy boards. The contact conductance at the interface of copper-epoxy layers is given to be h_c = 6000 W/m²·°C.

Analysis The thermal resistances of different layers for unit surface area of 1 m² are



$$R_{\text{contact}} = \frac{1}{h_c A_c} = \frac{1}{(6000\ \text{W/m}^2\cdot°\text{C})(1\,\text{m}^2)} = 0.00017\ °\text{C/W}$$

$$R_{\text{plate}} = \frac{L}{kA} = \frac{0.001\,\text{m}}{(401\,\text{W/m}\cdot°\text{C})(1\,\text{m}^2)} = 2.5\times10^{-6}\ °\text{C/W}$$

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.007\ \text{m}}{(0.26\,\text{W/m}\cdot°\text{C})(1\,\text{m}^2)} = 0.02692\ °\text{C/W}$$

The total thermal resistance is

$$\begin{aligned} R_{\text{total}} &= 2R_{\text{contact}} + R_{\text{plate}} + 2R_{\text{epoxy}} \\ &= 2\times0.00017 + 2.5\times10^{-6} + 2\times0.02692 = 0.05418\ °\text{C/W} \end{aligned}$$

Then the percent error involved in the total thermal resistance of the plate if the thermal contact resistances are ignored is determined to be

$$\%\text{Error} = \frac{2R_{\text{contact}}}{R_{\text{total}}}\times100 = \frac{2\times0.00017}{0.05418}\times100 = \mathbf{0.63\%}$$

which is negligible.

3-53 Two identical aluminum plates are pressed against each other, where the interface is filled with glycerin. The thermal contact conductance of the glycerin is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal conductivity is constant.

PropertiesThe thermal conductivity of the aluminum plates is given to be k = 237 W/m·K.

AnalysisThe thermal resistances of different layers are

$$R_{\text{interface}} = \frac{1}{h_c A} \text{ and } R_{\text{plate}} = \frac{L}{kA}$$

The total thermal resistance is

$$R_{\text{total}} = R_{\text{interface}} + 2R_{\text{plate}} = \frac{1}{h_c A} + \frac{2L}{kA}$$

The rate of heat transfer through the layers is

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{\dfrac{1}{h_c A} + \dfrac{2L}{kA}} \text{ or } \dot{q} = \frac{\dot{Q}}{A} = \frac{\Delta T}{1/h_c + 2L/k}$$

Thus, the thermal contact conductance of the glycerin is

$$h_c = \left(\frac{\Delta T}{\dot{q}} - \frac{2L}{k} \right)^{-1} = \left[\frac{(50-30)\text{ K}}{7800\text{ W/m}^2} - \frac{2(0.30\text{ m})}{237\text{ W/m}\cdot\text{K}} \right]^{-1} = \mathbf{30{,}810\ W/m^2\cdot K}$$

Discussion By comparing the calculated value of h_c = 30,810 W/m^2·K for glycerinwith the value listed in Table 3-1for glycerin (37,700 W/m^2·K), the calculated value is about 18% lower. The discrepancy between the calculated h_c and the value listed in Table 3-1 may be due to the surface roughness of the aluminum plates that causes imperfect contact between plate surface and glycerin.

3-54 A two-layer wall is made of stainless steel and aluminum plates pressed together. The stainless steel surface is subjected to uniform heat flux, while the aluminum surface is subjected to convection heat transfer. The surface temperature of the stainless steel plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant.

Properties The thermal conductivities of the stainless steel and aluminum plates are given to be k_1 = 14 W/m·K and k_2 = 237 W/m·K, respectively. The thermal contact conductance of the stainless steel-aluminum interface with a surface roughness of about 25μm at an average pressure of 10 MPa is h_c = 2900 W/m^2·K (Table 3-2).

Analysis The thermal resistances of different layers are

$$R_1 = \frac{L_1}{k_1 A} \quad \text{(stainless steel plate resistance)}$$

$$R_{\text{interface}} = \frac{1}{h_c A}$$

$$R_2 = \frac{L_2}{k_2 A} \quad \text{(aluminum plate resistance)}$$

$$R_{\text{conv}} = \frac{1}{h_{\text{conv}} A}$$

The total thermal resistance is

$$R_{\text{total}} = R_1 + R_{\text{interface}} + R_2 + R_{\text{conv}}$$

The heat flux through the layers is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_1 - T_\infty}{AR_{\text{total}}} = \frac{T_1 - T_\infty}{\dfrac{L_1}{k_1} + \dfrac{1}{h_c} + \dfrac{L_2}{k_2} + \dfrac{1}{h_{\text{conv}}}}$$

The surface temperature of the stainless steel plate is

$$\begin{aligned} T_1 &= \dot{q}\left(\frac{L_1}{k_1} + \frac{1}{h_c} + \frac{L_2}{k_2} + \frac{1}{h_{\text{conv}}}\right) + T_\infty \\ &= (800\ \text{W/m}^2)\left[\frac{0.005\ \text{m}}{14\ \text{W/m}\cdot\text{K}} + \frac{1}{2900\ \text{W/m}^2\cdot\text{K}} + \frac{0.015\ \text{m}}{237\ \text{W/m}\cdot\text{K}} + \frac{1}{12\ \text{W/m}^2\cdot\text{K}}\right] + 20°\text{C} \\ &= \mathbf{87.3°C} \end{aligned}$$

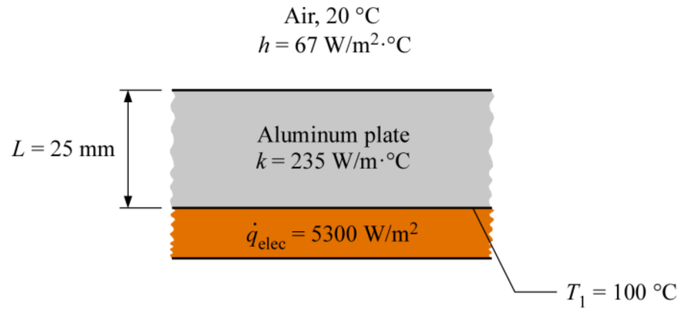
Discussion Among the thermal resistances considered in this problem, the convection resistance has the largest value.

3-55 The thermal contact conductance for an aluminum plate attached on a copper plate, that is heated electrically, is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal properties are constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the aluminum plate is given to be 235 W/m · °C.

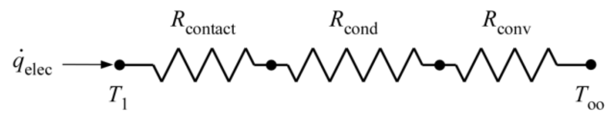
Analysis The thermal resistances are



$$R_{\text{cond}} = \frac{L}{kA}$$

and $$R_{\text{conv}} = \frac{1}{hA}$$

From energy balance and using the thermal resistance concept, the following equation is expressed:



$$\dot{q}_{\text{elec}} A = \frac{T_1 - T_\infty}{R_c / A + R_{\text{cond}} + R_{\text{conv}}}$$

or $$\dot{q}_{\text{elec}} A = \frac{T_1 - T_\infty}{R_c / A + L/(kA) + 1/(hA)}$$

Rearranging the equation and solving for the contact resistance yields

$$R_c = \frac{T_1 - T_\infty}{\dot{q}_{\text{elec}}} - \frac{L}{k} - \frac{1}{h}$$
$$= \frac{(100 - 20)\ °\text{C}}{5300\ \text{W/m}^2} - \frac{0.025\ \text{m}}{235\ \text{W/m} \cdot °\text{C}} - \frac{1}{67\ \text{W/m}^2 \cdot °\text{C}} = 6.258 \times 10^{-5}\ \text{m}^2 \cdot °\text{C/W}$$

The thermal contact conductance is

$$h_c = 1/R_c = \mathbf{16000\ W/m^2 \cdot °C}$$

Discussion By comparing the value of the thermal contact conductance with the values listed in Table 3-2, the surface conditions of the plates appear to be milled.

3-56 An aluminum plate and a stainless steel plate are pressed against each other. The impact of the plate surface roughness on the temperature drop at the interface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional.

Properties From Table 3-2, the thermal contact conductance of the stainless steel-aluminum interface at an average pressure of 20 MPa is $h_{c,\text{rough}}$ = 3600 W/m^2·K (for roughness = 20 μm) and $h_{c,\text{smooth}}$ = 20,800 W/m^2·K (for roughness = 2 μm).

Analysis The heat rate through the interface is

$$\dot{Q} = \frac{\Delta T_{\text{interface}}}{R_{\text{interface}}} = \frac{\Delta T_{\text{interface}}}{\dfrac{1}{h_c A}} \text{ or } \Delta T_{\text{interface}} = \dot{Q} R_{\text{interface}}$$

Thus,

$$\frac{(\Delta T_{\text{interface}})_{\text{rough}}}{(\Delta T_{\text{interface}})_{\text{smooth}}} = \frac{\dot{Q}(R_{\text{interface}})_{\text{rough}}}{\dot{Q}(R_{\text{interface}})_{\text{smooth}}} = \frac{\left(\dfrac{1}{h_{c,\text{rough}} A}\right)}{\left(\dfrac{1}{h_{c,\text{smooth}} A}\right)} = \frac{h_{c,\text{smooth}}}{h_{c,\text{rough}}}$$

$$\frac{(\Delta T_{\text{interface}})_{\text{rough}}}{(\Delta T_{\text{interface}})_{\text{smooth}}} = \frac{20{,}800 \text{ W/m}^2\cdot\text{K}}{3600 \text{ W/m}^2\cdot\text{K}} = 5.78$$

If the surface roughness of the plates is increased by tenfold, then the temperature drop at the interface would increase by about a factor of six

Discussion Thus, thermal contact resistance between two plates can be minimized by reducing the plate surface roughness or by increasing the contact pressure.

3-57 A thin electronic component is cooled by dissipating heat through a heat sink attached on its top surface. There is contact resistance at the interface of the electronic component and the heat sink, and the temperature of the electronic component is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional.**3**The electronic component maintains a constant temperature.

PropertiesThe thermal contact conductance at the electronic component/heat sink interface is given as h_c = 2000 W/m^2·K, the combined convection and radiation thermal resistance of the heat sink is given as 0.3 K/W.

AnalysisThe thermal resistances of different layers are

$$R_{\text{interface}} = \frac{1}{h_c A} = \frac{1}{(2000\ \text{W/m}^2\cdot\text{K})(950\ \text{cm}^2)(1/100\ \text{m/cm})^2} = 0.005263\ \text{K/W}$$

$$R_{\text{heat sink}} = 0.3\ \text{K/W}$$

The total thermal resistance is

$$R_{\text{total}} = R_{\text{interface}} + R_{\text{heat sink}} = 0.3 + 0.005263 = 0.30526\ \text{K/W}$$

The rate of heat transfer through the layers is

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{\text{total}}}$$

The temperature of the electronic component is

$$T_s = \dot{Q}R_{\text{total}} + T_\infty = (45\ \text{W})(0.30526\ \text{K/W}) + 30°\text{C} = \mathbf{43.7°C}$$

The contact resistance at the interface is about 1.7% (0.005236/0.30526) of the total thermal resistance, thus it is negligible in this case and does not play a significant role in the heat dissipation.

3-58 An engine cover made with two layers of metal (stainless steel and aluminum) pressed together. Both the inside and outside surface is subjected to convection heat transfer.The heat flux through the engine cover is to be determined

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant.

PropertiesThe thermal conductivities of the stainless steel and aluminum plates are given to be k_1= 14 W/m·K and k_2 = 237 W/m·K, respectively. The thermal contact conductance of the stainless steel-aluminum interface with a surface roughness of about 23 μm and at an average pressure of 20 MPa is h_c = 3600 W/m^2·K (Table 3-2).

AnalysisThe thermal resistances of different layers are

$$R_{\text{conv},1} = \frac{1}{h_1 A} \quad \text{(inside surface convection resistance)}$$

$$R_1 = \frac{L_1}{k_1 A} \quad \text{(stainless steel layer resistance)}$$

$$R_{\text{interface}} = \frac{1}{h_c A}$$

$$R_2 = \frac{L_2}{k_2 A} \quad \text{(aluminum layer resistance)}$$

$$R_{\text{conv},2} = \frac{1}{h_2 A} \quad \text{(outside surface convection resistance)}$$

The total thermal resistance is

$$R_{\text{total}} = R_{\text{conv},1} + R_1 + R_{\text{interface}} + R_2 + R_{\text{conv},2}$$

The heat flux through the layers is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_{\infty,1} - T_{\infty,2}}{AR_{\text{total}}}$$

$$= \frac{T_{\infty,1} - T_{\infty,2}}{\frac{1}{h_1} + \frac{L_1}{k_1} + \frac{1}{h_c} + \frac{L_2}{k_2} + \frac{1}{h_2}}$$

$$= \frac{(150-40)\text{ K}}{\frac{1}{10 \text{ W/m}^2\cdot\text{K}} + \frac{0.010\text{ m}}{14 \text{ W/m}\cdot\text{K}} + \frac{1}{3600 \text{ W/m}^2\cdot\text{K}} + \frac{0.005\text{ m}}{237 \text{ W/m}\cdot\text{K}} + \frac{1}{25 \text{ W/m}^2\cdot\text{K}}} = \mathbf{780\ W/m^2}$$

DiscussionThe contact resistance at the interface is about 0.2% of the total thermal resistance.

3-59 An Inconel® plate covered with a layer of thermal barrier coating (TBC). The plate is exposed to hot combustion gases with known convection heat transfer coefficient. The temperature of the surface exposed to the hot gases is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant.

Properties The thermal conductivities of the Inconel® and the thermal barrier coating are given to be k_1= 25 W/m·K and k_2 = 1.5 W/m·K, respectively. The thermal contact conductance at the interface is given as h_c = 3500 W/m²·K.

Analysis The thermal resistances of different layers are

$$R_1 = \frac{L_1}{k_1 A} \quad \text{(Inconel layer resistance)}$$

$$R_{\text{interface}} = \frac{1}{h_c A}$$

$$R_2 = \frac{L_2}{k_2 A} \quad \text{(TBC layer resistance)}$$

$$R_{\text{conv}} = \frac{1}{hA}$$

Then,

$$\begin{aligned} AR_{\text{total}} &= A(R_1 + R_{\text{interface}} + R_2 + R_{\text{conv}}) \\ &= \frac{L_1}{k_1} + \frac{1}{h_c} + \frac{L_2}{k_2} + \frac{1}{h} \\ &= \frac{0.012/2 \text{ m}}{25 \text{ W/m}\cdot\text{K}} + \frac{1}{3500 \text{ W/m}^2\cdot\text{K}} + \frac{300\times10^{-6} \text{ m}}{1.5 \text{ W/m}\cdot\text{K}} + \frac{1}{750 \text{ W/m}^2\cdot\text{K}} \\ &= 0.0020590 \text{ m}^2\cdot\text{K/W} \end{aligned}$$

and

$$AR_{\text{conv}} = \frac{1}{h} = \frac{1}{750 \text{ W/m}^2\cdot\text{K}} = 0.001333 \text{ m}^2\cdot\text{K/W}$$

The heat flux through the layers is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_\infty - T_2}{AR_{\text{conv}}} = \frac{T_\infty - T_1}{AR_{\text{total}}}$$

Thus,

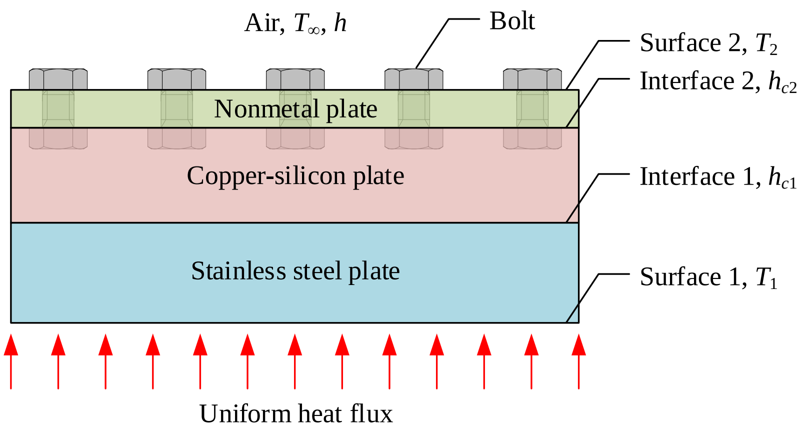
$$\begin{aligned} T_1 &= T_\infty - \frac{AR_{\text{total}}}{AR_{\text{conv}}}(T_\infty - T_2) \\ &= 1500°\text{C} - \frac{0.002059 \text{ m}^2\cdot\text{K/W}}{0.001333 \text{ m}^2\cdot\text{K/W}}(1500 - 1200)°\text{C} \\ &= \mathbf{1036°C} \end{aligned}$$

Discussion If the contact resistance is neglected in the analysis, the mid-plane temperature would be 1100°C.

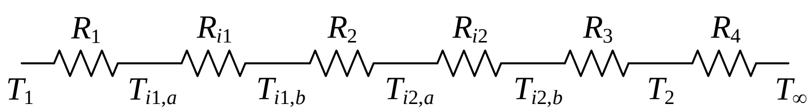
3-60 A wall is made of a composite stainless steel, copper-silicon, and nonmetal plates. A series of ASTM B21 naval brass bolts are bolted to a nonmetal plate. The upper surface is exposed to convection with air, while the bottom surface is subjected to a uniform heat flux. Thermal contact resistances exist at the plate interfaces. The total thermal resistance for the wall for an area of 1 m^2 is to be calculated. Determine whether the ASTM B21 bolts are in compliance with the ASME Code for Process Piping.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through plates. **3** Uniform surface temperatures. **4**. Thermal properties are constant.

Properties The thermal conductivity for the stainless steel plate is k_1 = 13 W/m·K, for the copper-silicon plate is k_2 = 36 W/m·K, and for the nonmetal plate is k_3 = 0.1 W/m·K. The thermal contact conductance between the stainless steel and copper-silicon plates is h_{c1} = 20000 W/m^2·K, and between the copper-silicon and nonmetal plates is h_{c2} = 10000 W/m^2·K.



Analysis The thermal resistances encountered by the heat flow are (R_{i1} and R_{i2} are contact resistances for interfaces 1 and 2, respectively)



$$R_1 = \frac{L_1}{k_1 A} \text{ or } \quad AR_1 = \frac{L_1}{k_1} = \frac{0.03 \text{ m}}{13 \text{ W/m·K}} = 0.002308 \text{ m}^2\text{·K/W}$$

$$R_{i1} = \frac{1}{h_{c1} A} \text{ or } \quad AR_{i1} = \frac{1}{h_{c1}} = \frac{1}{20000 \text{ W/m}^2\text{·K}} = 0.00005 \text{ m}^2\text{·K/W}$$

$$R_2 = \frac{L_2}{k_2 A} \text{ or } \quad AR_2 = \frac{L_2}{k_2} = \frac{0.03 \text{ m}}{36 \text{ W/m·K}} = 0.0008333 \text{ m}^2\text{·K/W}$$

$$R_{i2} = \frac{1}{h_{c2} A} \text{ or } \quad AR_{i2} = \frac{1}{h_{c2}} = \frac{1}{10000 \text{ W/m}^2\text{·K}} = 0.0001 \text{ m}^2\text{·K/W}$$

$$R_3 = \frac{L_3}{k_3 A} \text{ or } \quad AR_3 = \frac{L_3}{k_3} = \frac{0.015 \text{ m}}{0.1 \text{ W/m·K}} = 0.15 \text{ m}^2\text{·K/W}$$

$$R_4 = \frac{1}{hA} \text{ or } \quad AR_4 = \frac{1}{h} = \frac{1}{20 \text{ W/m}^2\text{·K}} = 0.05 \text{ m}^2\text{·K/W}$$

where

$$AR_{\text{tot}} = AR_1 + AR_{i1} + AR_2 + AR_{i2} + AR_3 + AR_4 = 0.20329 \text{ m}^2\text{·K/W}$$

Thus, the total thermal resistance for an area of 1 m^2 is $R_{\text{tot}} = \mathbf{0.20329\ K/W}$.

The bolts are in the nonmetal plate, so to determine if they are in compliance with the code, we will calculate the temperatures $T_{i2,b}$ and T_2. The heat flux through the thermal resistance R_4 is

$$\dot{q} = \frac{T_2 - T_\infty}{AR_4} \quad \rightarrow \quad T_2 = T_\infty + \dot{q}AR_4 = 20\text{°C} + (2000 \text{ W/m}^2)(0.05 \text{ m}^2\text{·K/W}) = \mathbf{120}\text{°C}$$

and

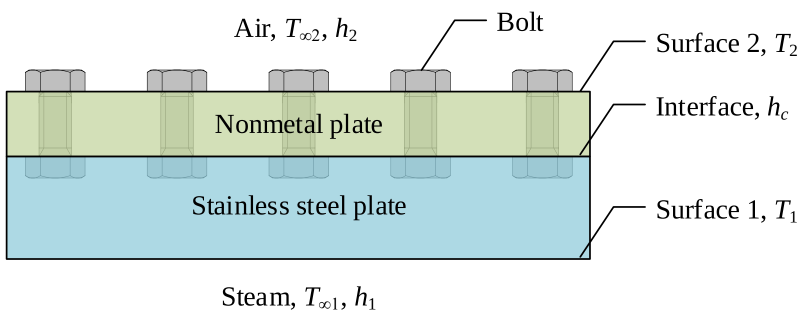
$$\dot{q} = \frac{T_{i2,b} - T_2}{AR_3} \quad \rightarrow \quad T_{i2,b} = T_2 + \dot{q}AR_3 = 120\text{°C} + (2000 \text{ W/m}^2)(0.15 \text{ m}^2\text{·K/W}) = \mathbf{420}\text{°C}$$

DiscussionThe ASTM B21 bolts are mainly in the nonmetal plate, and the bolts have a maximum use temperature of 149°C (ASME B31.3-2014). The temperatures in the nonmetal plate are between T_2 = 120°C and $T_{i2,b}$ = 420°C, which gives an average temperature of 270°C. Thus, the ASTM B21 bolts would be operating in an average temperature that is above the maximum use temperature, which would make the bolts not in compliance with the code.

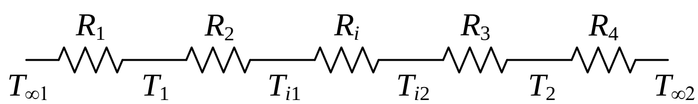
3-61 A nonmetal plate is bolted on an ASTM A240 904L stainless steel plate by ASTM B211 6061 aluminum alloy bolts. The upper surface is exposed to convection with air, and the bottom surface is exposed to convection with hot steam. Thermal contact resistance exists in the interface between the plates. Determine whether the use of the ASTM A240 904L plate and ASTM B211 6061 bolts complies with the ASME Code for Process Piping.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through plates. **3** Uniform surface temperatures. **4** Thermal properties are constant.

Properties The thermal conductivity for the ASTM A240 904L stainless steel plate is k_1 = 13 W/m·K and for the nonmetal plate is k_2 = 3 W/m·K. The thermal contact conductance between the stainless steel and nonmetal plates is h_c = 10000 W/m²·K.



Analysis The thermal resistances encountered by the heat flow are (R_i is the thermal contact resistance at the interface)



$$R_1 = \frac{1}{h_1 A} \text{ or } \quad AR_1 = \frac{1}{h_1} = \frac{1}{500\ \text{W/m}^2\cdot\text{K}} = 0.002\ \text{m}^2\cdot\text{K/W}$$

$$R_2 = \frac{L_1}{k_1 A} \text{ or } \quad AR_2 = \frac{L_1}{k_1} = \frac{0.03\ \text{m}}{13\ \text{W/m}\cdot\text{K}} = 0.002308\ \text{m}^2\cdot\text{K/W}$$

$$R_i = \frac{1}{h_c A} \text{ or } \quad AR_i = \frac{1}{h_c} = \frac{1}{10000\ \text{W/m}^2\cdot\text{K}} = 0.0001\ \text{m}^2\cdot\text{K/W}$$

$$R_3 = \frac{L_2}{k_2 A} \text{ or } \quad AR_3 = \frac{L_2}{k_2} = \frac{0.01\ \text{m}}{3\ \text{W/m}\cdot\text{K}} = 0.003333\ \text{m}^2\cdot\text{K/W}$$

$$R_4 = \frac{1}{h_2 A} \text{ or } \quad AR_4 = \frac{1}{h_2} = \frac{1}{50\ \text{W/m}^2\cdot\text{K}} = 0.02\ \text{m}^2\cdot\text{K/W}$$

where

$$AR_{\text{tot}} = AR_1 + AR_2 + AR_i + AR_3 + AR_4 = 0.027741\ \text{m}^2\cdot\text{K/W}$$

The heat flux through the thermal circuit is

$$\dot{q} = \frac{T_{\infty 1} - T_{\infty 2}}{AR_{\text{tot}}} = \frac{(260 - 20)\text{K}}{0.027741\ \text{m}^2\cdot\text{K/W}} = 8651.5\ \text{W/m}^2$$

The temperatures T_1, T_{i1}, T_{i2}, and T_2 can be calculated using the heat flux and the individual thermal resistance:

$$\dot{q} = \frac{T_{\infty 1} - T_1}{AR_1} \quad \rightarrow \quad T_1 = T_{\infty 1} - \dot{q} AR_1 = 260°\text{C} - (8651.5\ \text{W/m}^2)(0.002\ \text{m}^2\cdot\text{K/W}) = \mathbf{242.7}°\text{C}$$

$$\dot{q} = \frac{T_1 - T_{i1}}{AR_2} \qquad \rightarrow \qquad T_{i1} = T_1 - \dot{q}AR_2 = 242.7°\text{C} - (8651.5 \text{ W/m}^2)(0.002308 \text{ m}^2\text{·K/W}) = \mathbf{222.7}°\text{C}$$

Thus, the range of temperature experienced by stainless steel plate is between T_{i1} and T_1, or 222.7 $\leq T \leq$ 242.7°C.

$$\dot{q} = \frac{T_2 - T_{\infty 2}}{AR_4} \qquad \rightarrow \qquad T_2 = \dot{q}AR_4 + T_{\infty 2} = (8651.5 \text{ W/m}^2)(0.02 \text{ m}^2\text{·K/W}) + 20°\text{C} = \mathbf{193.0}°\text{C}$$

$$\dot{q} = \frac{T_{i2} - T_2}{AR_3} \qquad \rightarrow \qquad T_{i2} = \dot{q}AR_3 + T_2 = (8651.5 \text{ W/m}^2)(0.003333 \text{ m}^2\text{·K/W}) + 193.0°\text{C} = \mathbf{221.8}°\text{C}$$

Thus, the range of temperature experienced by nonmetal plate is between T_2 and T_{i2}, or 193°C$\leq T \leq$ 221.8°C.

Discussion Since the operating temperature for the ASTM A240 904L plate is between 222.7°C and 242.7°C, therefore it operates below the maximum use temperature of 260°C and in compliance with the ASME Code for Process Piping. The ASTM B211 6061 bolts are mainly in the nonmetal plate, and the bolts have a maximum use temperature of 204°C (ASME B31.3-2014). The temperatures in the nonmetal plate are between 193°C and 221.8°C. Thus, part of the bolts is operating in temperatures above the maximum use temperature, and would not be in compliance with the ASME code.

Generalized Thermal Resistance Networks

3-62C Two approaches used in development of the thermal resistance network in the x-direction for multi-dimensional problems are (1) to assume any plane wall normal to the x-axis to be isothermal and (2) to assume any plane parallel to the x-axis to be adiabatic.

3-63C The thermal resistance network approach will give adequate results for multi-dimensional heat transfer problems if heat transfer occurs predominantly in one direction.

3-64C Parallel resistances indicate simultaneous heat transfer (such as convection and radiation on a surface). Series resistances indicate sequential heat transfer (such as two homogeneous layers of a wall).

3-65 A wall is to be constructed of 10-cm thick wood studs or with pairs of 5-cm thick wood studs nailed to each other. The rate of heat transfer through the solid stud and through a stud pair nailed to each other, as well as the effective conductivity of the nailed stud pair are to be determined.

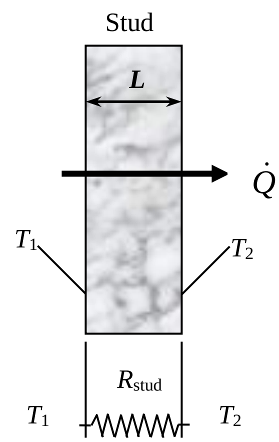
Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer can be approximated as being one-dimensional since it is predominantly in the *x* direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance between the two layers is negligible. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be k = 0.11 W/m·°C for wood studs and k = 50 W/m·°C for manganese steel nails.

Analysis (*a*) The heat transfer area of the stud is A = (0.1 m)(2.5 m) = 0.25 m^2. The thermal resistance and heat transfer rate through the solid stud are

$$R_{stud} = \frac{L}{kA} = \frac{0.1\,\text{m}}{(0.11\,\text{W/m}\cdot°\text{C})(0.25\,\text{m}^2)} = 3.636\,°\text{C/W}$$

$$\dot{Q} = \frac{\Delta T}{R_{stud}} = \frac{8°\text{C}}{3.636\,°\text{C/W}} = \mathbf{2.2\ W}$$



(*b*) The thermal resistances of stud pair and nails are in parallel

$$A_{nails} = 50\frac{\pi D^2}{4} = 50\left[\frac{\pi(0.004\text{ m})^2}{4}\right] = 0.000628\text{ m}^2$$

$$R_{nails} = \frac{L}{kA} = \frac{0.1\text{ m}}{(50\text{ W/m}\cdot°\text{C})(0.000628\text{ m}^2)} = 3.18\ °\text{C/W}$$

$$R_{stud} = \frac{L}{kA} = \frac{0.1\text{ m}}{(0.11\text{ W/m}\cdot°\text{C})(0.25 - 0.000628\text{ m}^2)} = 3.65\ °\text{C/W}$$

$$\frac{1}{R_{total}} = \frac{1}{R_{stud}} + \frac{1}{R_{nails}} = \frac{1}{3.65} + \frac{1}{3.18} \longrightarrow R_{total} = 1.70\ °\text{C/W}$$

$$\dot{Q} = \frac{\Delta T}{R_{total}} = \frac{8°\text{C}}{1.70\ °\text{C/W}} = \mathbf{4.7\ W}$$

(*c*) The effective conductivity of the nailed stud pair can be determined from

$$\dot{Q} = k_{eff}A\frac{\Delta T}{L} \longrightarrow k_{eff} = \frac{\dot{Q}L}{\Delta TA} = \frac{(4.7\text{ W})(0.1\text{ m})}{(8°\text{C})(0.25\text{ m}^2)} = \mathbf{0.235\ W/m.°C}$$

3-66E The thermal resistance of an epoxy glass laminate across its thickness is to be reduced by planting cylindrical copper fillings throughout. The thermal resistance of the epoxy board for heat conduction across its thickness as a result of this modification is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the plate is one-dimensional. **3** Thermal conductivities are constant.

Properties The thermal conductivities are given to be k = 0.10 Btu/h·ft·°F for epoxy glass laminate and k = 223 Btu/h·ft·°F for copper fillings.

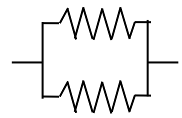
Analysis The thermal resistances of copper fillings and the epoxy board are in parallel. The number of copper fillings in the board and the area they comprise are

$$A_{total} = (10/12\text{ ft})(12/12\text{ ft}) = 0.8333\text{ ft}^2$$

$$n_{copper} = \frac{0.8333\text{ ft}^2}{(0.06/12\text{ ft})(0.06/12\text{ ft})} = 33{,}333 \text{ (number of copper fillings)}$$

$$A_{copper} = n\frac{\pi D^2}{4} = 33{,}333\frac{\pi(0.02/12\text{ ft})^2}{4} = 0.07272\text{ ft}^2$$

$$A_{epoxy} = A_{total} - A_{copper} = 0.8333 - 0.07272 = 0.7606\text{ ft}^2$$



The thermal resistances are evaluated to be

$$R_{copper} = \frac{L}{kA} = \frac{0.05/12\text{ ft}}{(223\text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})(0.07272\text{ ft}^2)} = 0.000257\text{ h}\cdot{}^\circ\text{F/Btu}$$

$$R_{epoxy} = \frac{L}{kA} = \frac{0.05/12\text{ ft}}{(0.10\text{ Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})(0.7606\text{ ft}^2)} = 0.0548\text{ h}\cdot{}^\circ\text{F/Btu}$$

Then the thermal resistance of the entire epoxy board becomes

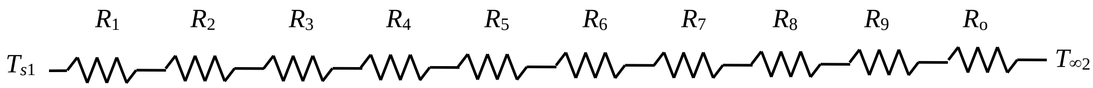
$$\frac{1}{R_{board}} = \frac{1}{R_{copper}} + \frac{1}{R_{epoxy}} = \frac{1}{0.000257} + \frac{1}{0.0548} \longrightarrow R_{board} = \mathbf{0.000256\ h\cdot{}^\circ F/Btu}$$

3-67 A coat is made of 5 layers of 0.15 mm thick synthetic fabric separated by 1.5 mm thick air space. The rate of heat loss through the jacket is to be determined, and the result is to be compared to the heat loss through a jackets without the air space. Also, the equivalent thickness of a wool coat is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the jacket is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer.

Properties The thermal conductivities are given to be k = 0.13 W/m·°C for synthetic fabric, k = 0.026 W/m·°C for air, and k = 0.035 W/m·°C for wool fabric.

Analysis The thermal resistance network and the individual thermal resistances are



$$R_{fabric} = R_1 = R_3 = R_5 = R_7 = R_9 = \frac{L}{kA} = \frac{0.00015\text{ m}}{(0.13\text{ W/m}\cdot°\text{C})(1.25\text{ m}^2)} = 0.0009\text{ °C/W}$$

$$R_{air} = R_2 = R_4 = R_6 = R_8 = \frac{L}{kA} = \frac{0.0015\text{ m}}{(0.026\text{ W/m}\cdot°\text{C})(1.25\text{ m}^2)} = 0.0462\text{ °C/W}$$

$$R_o = \frac{1}{hA} = \frac{1}{(25\text{ W/m}^2\cdot°\text{C})(1.25\text{ m}^2)} = 0.0320\text{ °C/W}$$

$$R_{total} = 5R_{fabric} + 4R_{air} + R_o = 5\times 0.0009 + 4\times 0.0462 + 0.0320 = 0.2214\text{ °C/W}$$

and

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{(25-0)°\text{C}}{0.2214\text{ °C/W}} = \mathbf{113\ W}$$

If the jacket is made of a single layer of 0.75 mm thick synthetic fabric, the rate of heat transfer would be

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{T_{s1} - T_{\infty 2}}{5\times R_{fabric} + R_o} = \frac{(25-0)°\text{C}}{(5\times 0.0009 + 0.0320)\text{ °C/W}} = \mathbf{685\ W}$$

The thickness of a wool fabric that has the same thermal resistance is determined from

$$R_{\text{total}} = R_{\substack{\text{wool}\\ \text{fabric}}} + R_o = \frac{L}{kA} + \frac{1}{hA}$$

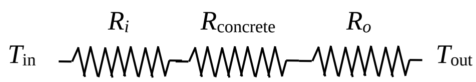
$$0.2214\text{ °C/W} = \frac{L}{(0.035\text{ W/m}\cdot°\text{C})(1.25\text{ m}^2)} + 0.0320 \longrightarrow L = 0.00829\text{ m} = \mathbf{8.29\ mm}$$

3-68 A kiln is made of 20 cm thick concrete walls and ceiling. The two ends of the kiln are made of thin sheet metal covered with 2-cm thick styrofoam. For specified indoor and outdoor temperatures, the rate of heat transfer from the kiln is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the walls and ceiling is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer. **5** Heat loss through the floor is negligible. **6** Thermal resistance of sheet metal is negligible.

Properties The thermal conductivities are given to be k = 0.9 W/m·°C for concrete and k = 0.033 W/m·°C for styrofoam insulation.

Analysis In this problem there is a question of which surface area to use. We will use the outer surface area for outer convection resistance, the inner surface area for inner convection resistance, and the average area for the conductionresistance. Or we could use the inner or the outer surface areas in the calculation of all thermal resistances with little loss in accuracy. For top and the two side surfaces:



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3000\ \text{W/m}^2\cdot{}^\circ\text{C})[(40\ \text{m})(13-1.2)\ \text{m}]} = 0.0071\times10^{-4}\ {}^\circ\text{C/W}$$

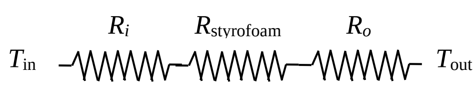
$$R_{concrete} = \frac{L}{kA_{ave}} = \frac{0.2\ \text{m}}{(0.9\ \text{W/m}\cdot{}^\circ\text{C})[(40\ \text{m})(13-0.6)\ \text{m}]} = 4.480\times10^{-4}\ {}^\circ\text{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(25\ \text{W/m}^2\cdot{}^\circ\text{C})[(40\ \text{m})(13\text{m})]} = 0.769\times10^{-4}\ {}^\circ\text{C/W}$$

$$R_{total} = R_i + R_{concrete} + R_o = (0.0071+4.480+0.769)\times10^{-4} = 5.256\times10^{-4}\ {}^\circ\text{C/W}$$

and $$\dot{Q}_{top+sides} = \frac{T_{in}-T_{out}}{R_{total}} = \frac{[40-(-4)]^\circ\text{C}}{5.256\times10^{-4}\ {}^\circ\text{C/W}} = 83{,}700\ \text{W}$$

Heat loss through the end surface of the kiln with styrofoam:



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3000\ \text{W/m}^2\cdot{}^\circ\text{C})[(4-0.4)(5-0.4)\ \text{m}^2]} = 0.201\times10^{-4}\ {}^\circ\text{C/W}$$

$$R_{styrofoam} = \frac{L}{kA_{ave}} = \frac{0.02\ \text{m}}{(0.033\ \text{W/m}\cdot{}^\circ\text{C})[(4-0.2)(5-0.2)\ \text{m}^2]} = 0.0332\ {}^\circ\text{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(25\ \text{W/m}^2\cdot{}^\circ\text{C})[4\times5\ \text{m}^2]} = 0.0020\ {}^\circ\text{C/W}$$

$$R_{total} = R_i + R_{styrpfoam} + R_o = 0.201\times10^{-4} + 0.0332 + 0.0020 = 0.0352\ {}^\circ\text{C/W}$$

and $$\dot{Q}_{end\ surface} = \frac{T_{in}-T_{out}}{R_{total}} = \frac{[40-(-4)]^\circ\text{C}}{0.0352\ {}^\circ\text{C/W}} = 1250\ \text{W}$$

Then the total rate of heat transfer from the kiln becomes

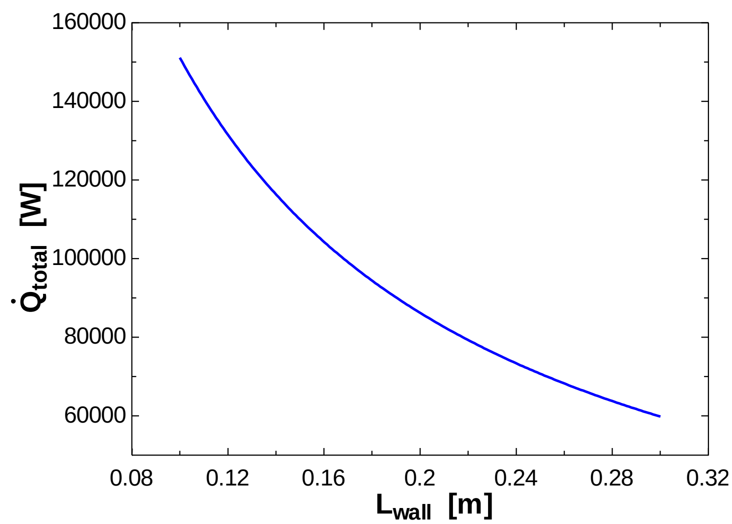
$$\dot{Q}_{total} = \dot{Q}_{top+sides} + 2\dot{Q}_{side} = 83{,}700 + 2\times1250 = \mathbf{86{,}200\ W}$$

3-69 Prob. 3-68 is reconsidered. The effects of the thickness of the wall and the convection heat transfer coefficient on the outer surface of the rate of heat loss from the kiln are to be investigated.

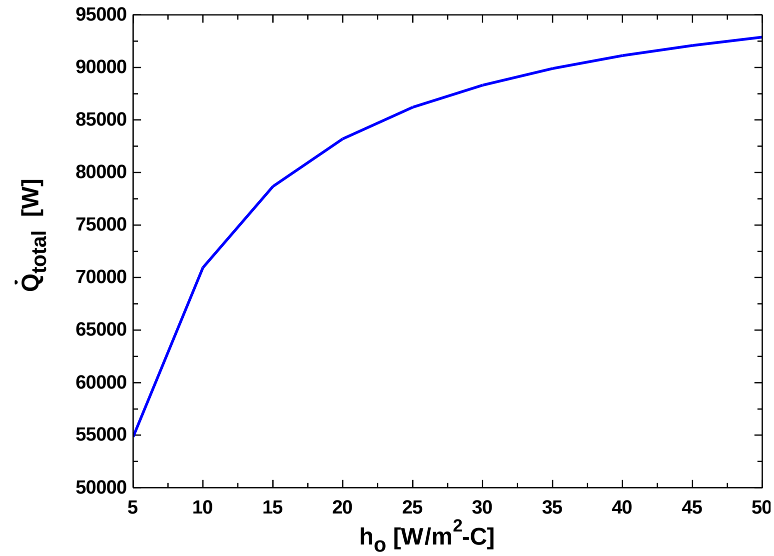
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
width=5 [m]
height=4 [m]
length=40 [m]
L_wall=0.2 [m]
k_concrete=0.9 [W/m-C]
T_in=40 [C]
T_out=-4 [C]
L_sheet=0.003 [m]
L_styrofoam=0.02 [m]
k_styrofoam=0.033 [W/m-C]
h_i=3000 [W/m^2-C]
h_o=25 [W/m^2-C]

"ANALYSIS"
R_conv_i=1/(h_i*A_1)
A_1=(2*height+width-6*L_wall)*length
R_concrete=L_wall/(k_concrete*A_2)
A_2=(2*height+width-3*L_wall)*length
R_conv_o=1/(h_o*A_3)
A_3=(2*height+width)*length
R_total_top_sides=R_conv_i+R_concrete+R_conv_o
Q_dot_top_sides=(T_in-T_out)/R_total_top_sides "Heat loss from top and the two side surfaces"

R_conv_i_end=1/(h_i*A_4)
A_4=(height-2*L_wall)*(width-2*L_wall)
R_styrofoam=L_styrofoam/(k_styrofoam*A_5)
A_5=(height-L_wall)*(width-L_wall)
R_conv_o_end=1/(h_o*A_6)
A_6=height*width
R_total_end=R_conv_i_end+R_styrofoam+R_conv_o_end
Q_dot_end=(T_in-T_out)/R_total_end "Heat loss from one end surface"
Q_dot_total=Q_dot_top_sides+2*Q_dot_end
```

L_{wall} [m]	Q_{total} [W]
0.1	151098
0.12	131499
0.14	116335
0.16	104251
0.18	94395
0.2	86201
0.22	79281
0.24	73359
0.26	68233
0.28	63751
0.3	59800



h_o [W/m^2.C]	Q_{total} [W]
5	54834
10	70939
15	78670
20	83212
25	86201
30	88318
35	89895
40	91116
45	92089
50	92882

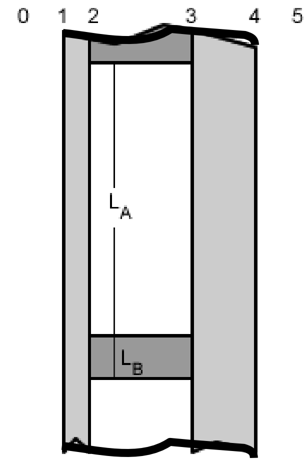


3-70 A typical section of a building wall is considered. The average heat flux through the wall is to be determined.

Assumptions **1** Steady operating conditions exist.

Properties The thermal conductivities are given to be k_{23b} = 50 W/m·K, k_{23a} = 0.03 W/m·K, k_{12} = 0.5 W/m·K, k_{34} = 1.0 W/m·K.

Analysis We consider 1 m^2 of wall area. The thermal resistances are

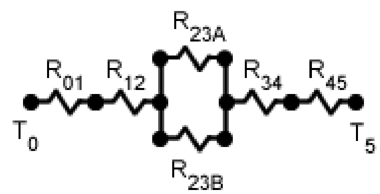


$$R_{12} = \frac{t_{12}}{k_{12}} = \frac{0.01\,\text{m}}{(0.5\text{ W/m}\cdot{}^\circ\text{C})} = 0.02\,\text{m}^2\cdot{}^\circ\text{C/W}$$

$$R_{23a} = t_{23}\frac{L_a}{k_{23a}(L_a+L_b)} = (0.08\,\text{m})\frac{0.6\,\text{m}}{(0.03\text{ W/m}\cdot{}^\circ\text{C})(0.6+0.005)} = 2.645\,\text{m}^2\cdot{}^\circ\text{C/W}$$

$$R_{23b} = t_{23}\frac{L_b}{k_{23b}(L_a+L_b)} = (0.08\,\text{m})\frac{0.005\,\text{m}}{(50\text{ W/m}\cdot{}^\circ\text{C})(0.6+0.005)} = 1.32\times10^{-5}\ \text{m}^2\cdot{}^\circ\text{C/W}$$

$$R_{34} = \frac{t_{34}}{k_{34}} = \frac{0.1\,\text{m}}{(1.0\text{ W/m}\cdot{}^\circ\text{C})} = 0.1\ \text{m}^2\cdot{}^\circ\text{C/W}$$



The total thermal resistance and the rate of heat transfer are

$$R_{\text{total}} = R_{12} + \left(\frac{R_{23a}R_{23b}}{R_{23a}+R_{23b}}\right) + R_{34} = 0.02 + 2.645\left(\frac{1.32\times10^{-5}}{2.645+1.32\times10^{-5}}\right) + 0.1 = 0.120\,\text{m}^2\cdot{}^\circ\text{C/W}$$

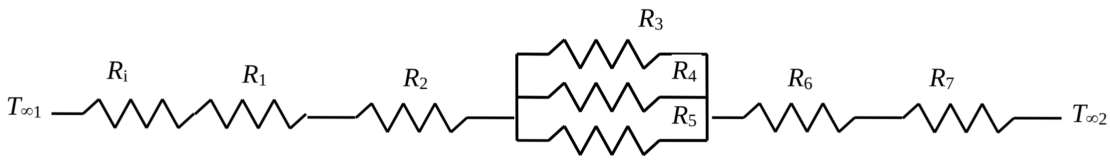
$$\dot{q} = \frac{T_4 - T_1}{R_{\text{total}}} = \frac{(35-20)^\circ\text{C}}{0.120\,\text{m}^2\cdot\text{C/W}} = \mathbf{125\ W/m^2}$$

3-71 A wall consists of horizontal bricks separated by plaster layers. There are also plaster layers on each side of the wall, and a rigid foam on the inner side of the wall. The rate of heat transfer through the wall is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is disregarded.

Properties The thermal conductivities are given to be k = 0.72 W/m·°C for bricks, k = 0.22 W/m·°C for plaster layers, and k = 0.026 W/m·°C for the rigid foam.

Analysis We consider 1 m deep and 0.28 m high portion of wall which is representative of the entire wall. The thermal resistance network and individual resistances are



$$R_i = R_{conv,1} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot °\text{C})(0.28 \times 1 \text{ m}^2)} = 0.357 \text{ °C/W}$$

$$R_1 = R_{foam} = \frac{L}{kA} = \frac{0.02 \text{ m}}{(0.026 \text{ W/m} \cdot °\text{C})(0.28 \times 1 \text{ m}^2)} = 2.747 \text{ °C/W}$$

$$R_2 = R_6 = R_{\substack{plaster\\side}} = \frac{L}{kA} = \frac{0.02 \text{ m}}{(0.22 \text{ W/m} \cdot °\text{C})(0.28 \times 1 \text{ m}^2)} = 0.325 \text{ °C/W}$$

$$R_3 = R_5 = R_{\substack{plaster\\center}} = \frac{L}{h_o A} = \frac{0.15 \text{ m}}{(0.22 \text{ W/m} \cdot °\text{C})(0.015 \times 1 \text{ m}^2)} = 45.45 \text{°C/W}$$

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{0.15 \text{ m}}{(0.72 \text{ W/m} \cdot °\text{C})(0.25 \times 1 \text{ m}^2)} = 0.833 \text{ °C/W}$$

$$R_o = R_{conv,2} = \frac{1}{h_2 A} = \frac{1}{(20 \text{ W/m} \cdot °\text{C})(0.28 \times 1 \text{ m}^2)} = 0.179 \text{ °C/W}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{45.45} + \frac{1}{0.833} + \frac{1}{45.45} \longrightarrow R_{mid} = 0.804 \text{ °C/W}$$

$$R_{total} = R_i + R_1 + 2R_2 + R_{mid} + R_o = 0.357 + 2.747 + 2(0.325) + 0.804 + 0.179 = 4.737 \text{ °C/W}$$

The steady rate of heat transfer through the wall per 0.28 m^2 is

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{[(22 - (-4)]°\text{C}}{4.737 \text{°C/W}} = 5.49 \text{ W}$$

Then steady rate of heat transfer through the entire wall becomes

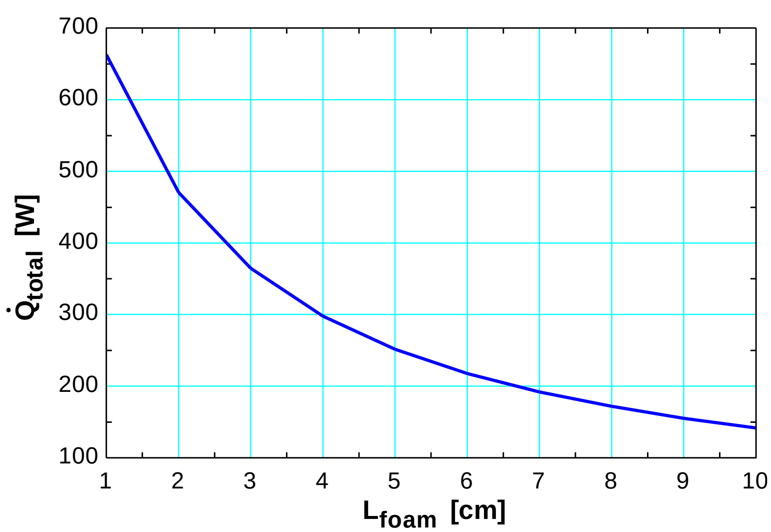
$$\dot{Q}_{total} = (5.49 \text{ W}) \frac{(4 \times 6)\text{m}^2}{0.28 \text{ m}^2} = \mathbf{470\ W}$$

3-72 Prob. 3-71 is reconsidered. The rate of heat transfer through the wall as a function of the thickness of the rigid foam is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
A=4*6 [m^2]
L_brick=0.15 [m]
L_plaster_center=0.15 [m]
L_plaster_side=0.02 [m]
L_foam=2 [cm]
k_brick=0.72 [W/m-C]
k_plaster=0.22 [W/m-C]
k_foam=0.026 [W/m-C]
T_infinity_1=22 [C]
T_infinity_2=-4 [C]
h_1=10 [W/m^2-C]
h_2=20 [W/m^2-C]

A_1=0.28*1 [m^2]
A_2=0.25*1 [m^2]
A_3=0.015*1 [m^2]

"ANALYSIS"
R_conv_1=1/(h_1*A_1)
R_foam=(L_foam*Convert(cm, m))/(k_foam*A_1) "L_foam is in cm"
R_plaster_side=L_plaster_side/(k_plaster*A_1)
R_plaster_center=L_plaster_center/(k_plaster*A_3)
R_brick=L_brick/(k_brick*A_2)
R_conv_2=1/(h_2*A_1)
1/R_mid=2*1/R_plaster_center+1/R_brick
R_total=R_conv_1+R_foam+2*R_plaster_side+R_mid+R_conv_2
Q_dot=(T_infinity_1-T_infinity_2)/R_total
Q_dot_total=Q_dot*A/A_1
```

L_{foam} [cm]	Q_{total} [W]
1	662.8
2	470.5
3	364.8
4	297.8
5	251.6
6	217.8
7	192
8	171.7
9	155.3
10	141.7

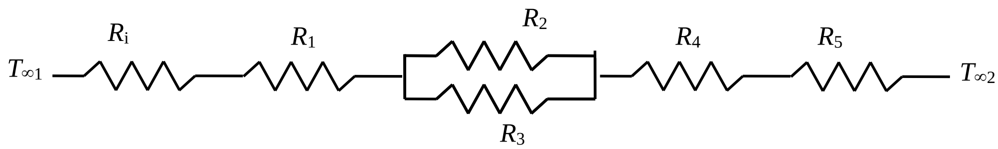


3-73 A wall is constructed of two layers of sheetrock spaced by 5 cm×16 cm wood studs. The space between the studs is filled with fiberglass insulation. The thermal resistance of the wall and the rate of heat transfer through the wall are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer.

Properties The thermal conductivities are given to be k = 0.17 W/m·°C for sheetrock, k = 0.11 W/m·°C for wood studs, and k = 0.034 W/m·°C for fiberglass insulation.

Analysis (a) The representative surface area is $A = 1\times 0.65 = 0.65\,\text{m}^2$. The thermal resistance network and the individual thermal resistances are



$$R_i = \frac{1}{h_i A} = \frac{1}{(8.3\ \text{W/m}^2\cdot{}^\circ\text{C})(0.65\,\text{m}^2)} = 0.185\,{}^\circ\text{C/W}$$

$$R_1 = R_4 = R_{sheetrock} = \frac{L}{kA} = \frac{0.01\,\text{m}}{(0.17\ \text{W/m}\cdot{}^\circ\text{C})(0.65\,\text{m}^2)} = 0.090\,{}^\circ\text{C/W}$$

$$R_2 = R_{stud} = \frac{L}{kA} = \frac{0.16\,\text{m}}{(0.11\,\text{W/m}\cdot{}^\circ\text{C})(0.05\,\text{m}^2)} = 29.091\,{}^\circ\text{C/W}$$

$$R_3 = R_{fiberglass} = \frac{L}{kA} = \frac{0.16\,\text{m}}{(0.034\ \text{W/m}\cdot{}^\circ\text{C})(0.60\,\text{m}^2)} = 7.843\,{}^\circ\text{C/W}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(34\ \text{W/m}^2\cdot{}^\circ\text{C})(0.65\,\text{m}^2)} = 0.045\,{}^\circ\text{C/W}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{29.091} + \frac{1}{7.843} \longrightarrow R_{mid} = 6.178\,{}^\circ\text{C/W}$$

$$\begin{aligned} R_{total} &= R_i + R_1 + R_{mid} + R_4 + R_o \\ &= 0.185 + 0.090 + 6.178 + 0.090 + 0.045 = \mathbf{6.588\,{}^\circ C/W}\ (\text{for a } 1\,\text{m} \times 0.65\,\text{m section}) \end{aligned}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[20-(-9)]^\circ\text{C}}{6.588\,{}^\circ\text{C/W}} = 4.40\ \text{W}$$

(b) Then steady rate of heat transfer through entire wall becomes

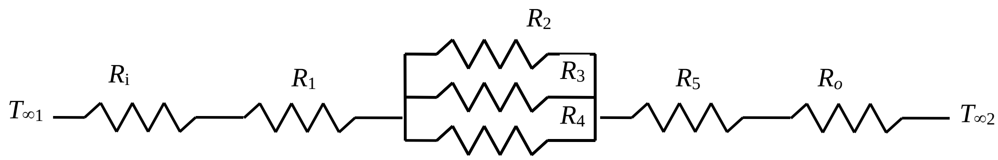
$$\dot{Q}_{total} = (4.40\ \text{W})\frac{(12\,\text{m})(5\,\text{m})}{0.65\,\text{m}^2} = \mathbf{406\ W}$$

3-74E A wall is to be constructed using solid bricks or identical size bricks with 9 square air holes. There is a 0.5 in thick sheetrock layer between two adjacent bricks on all four sides, and on both sides of the wall. The rates of heat transfer through the wall constructed of solid bricks and of bricks with air holes are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer coefficients account for the radiation heat transfer.

Properties The thermal conductivities are given to be k = 0.40 Btu/h·ft·°F for bricks, k = 0.015 Btu/h·ft·°F for air, and k = 0.10 Btu/h·ft·°F for sheetrock.

Analysis (a) The representative surface area is $A = (7.5/12)(7.5/12) = 0.3906\text{ ft}^2$. The thermal resistance network and the individual thermal resistances if the wall is constructed of solid bricks are



$$R_i = \frac{1}{h_i A} = \frac{1}{(1.5\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(0.3906\text{ ft}^2)} = 1.7068\text{ h°F/Btu}$$

$$R_1 = R_5 = R_{plaster} = \frac{L}{kA} = \frac{0.5/12\text{ ft}}{(0.10\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})(0.3906\text{ ft}^2)} = 1.0667\text{ h}\cdot°\text{F/Btu}$$

$$R_2 = R_{plaster} = \frac{L}{kA} = \frac{9/12\text{ ft}}{(0.10\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})[(7.5/12)\times(0.5/12)]\text{ft}^2} = 288\text{ h}\cdot°\text{F/Btu}$$

$$R_3 = R_{plaster} = \frac{L}{kA} = \frac{9/12\text{ ft}}{(0.10\text{ Btu/h}\cdot\text{ft}\cdot^{o}\text{ F})[(7/12)\times(0.5/12)]\text{ft}^2} = 308.57\text{ h}\cdot°\text{F/Btu}$$

$$R_4 = R_{brick} = \frac{L}{kA} = \frac{9/12\text{ ft}}{(0.40\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})[(7/12)\times(7/12)]\text{ft}^2} = 5.51\text{ h}\cdot°\text{F/Btu}$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(6\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(0.3906\text{ ft}^2)} = 0.4267\text{ h}\cdot°\text{F/Btu}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{288} + \frac{1}{308.57} + \frac{1}{5.51} \longrightarrow R_{mid} = 5.3135\text{ h}\cdot°\text{F/Btu}$$

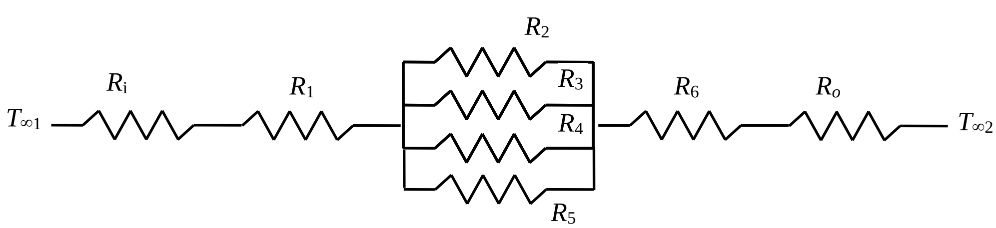
$$R_{total} = R_i + R_1 + R_{mid} + R_5 + R_o = 1.7068 + 1.0667 + 5.3135 + 1.0667 + 0.4267 = 9.5804\text{ h}\cdot°\text{F/Btu}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(80-35)°\text{F}}{9.5804\text{ h}\cdot°\text{F/Btu}} = 4.6971\text{ Btu/h}$$

Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{total} = (4.6971\text{ Btu/h})\frac{(30\text{ ft})(10\text{ ft})}{0.3906\text{ m}^2} = \mathbf{3610\text{ Btu/h}}$$

(b) The thermal resistance network and the individual thermal resistances if the wall is constructed of bricks with air holes are



$$A_{airholes} = 9(1.5/12)\times(1.5/12) = 0.1406\text{ ft}^2$$

$$A_{bricks} = (7/12\text{ ft})^2 - 0.1406 = 0.1997\text{ ft}^2$$

$$R_4 = R_{airholes} = \frac{L}{kA} = \frac{9/12\ \text{ft}}{(0.015\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})(0.1406\ \text{ft}^2)} = 355.62\ \text{h}\cdot{}^\circ\text{F/Btu}$$

$$R_5 = R_{brick} = \frac{L}{kA} = \frac{9/12\ \text{ft}}{(0.40\ \text{Btu/h}\cdot\text{ft}\cdot{}^\circ\text{F})(0.1997\ \text{ft}^2)} = 9.389\ \text{h}\cdot{}^\circ\text{F/Btu}$$

$$\frac{1}{R_{mid}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{288} + \frac{1}{308.57} + \frac{1}{355.62} + \frac{1}{9.389} \longrightarrow R_{mid} = 8.618\ \text{h}\cdot{}^\circ\text{F/Btu}$$

$$R_{total} = R_i + R_1 + R_{mid} + R_6 + R_o = 1.7068 + 1.0667 + 8.618 + 1.0667 + 0.4267 = 12.885\ \text{h}\cdot{}^\circ\text{F/Btu}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(80-35)^\circ\text{F}}{12.885\ \text{h}\cdot{}^\circ\text{F/Btu}} = 3.492\ \text{Btu/h}$$

Then steady rate of heat transfer through entire wall becomes

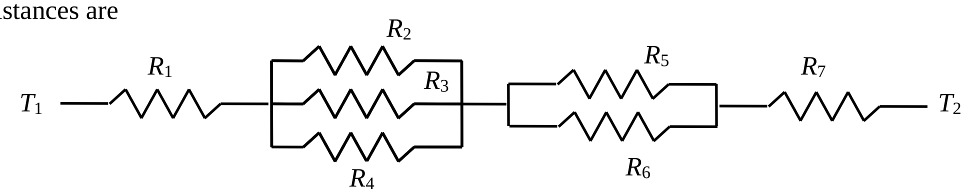
$$\dot{Q}_{total} = (3.492\ \text{Btu/h})\,\frac{(30\ \text{ft})(10\ \text{ft})}{0.3906\ \text{ft}^2} = \mathbf{2680\ Btu/h}$$

3-75 A composite wall consists of several horizontal and vertical layers. The left and right surfaces of the wall are maintained at uniform temperatures. The rate of heat transfer through the wall, the interface temperatures, and the temperature drop across the section F are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall is one-dimensional. **3** Thermal conductivities are constant. **4** Thermal contact resistances at the interfacesare disregarded.

Properties The thermal conductivities are given to be $k_A = k_F = 2$, $k_B = 8$, $k_C = 20$, $k_D = 15$, $k_E = 35$ W/m·°C.

Analysis (*a*) The representative surface area is $A = 0.12 \times 1 = 0.12\text{ m}^2$. The thermal resistance network and the individual thermal resistances are



$$R_1 = R_A = \left(\frac{L}{kA}\right)_A = \frac{0.01\text{ m}}{(2\text{ W/m}\cdot{}^\circ\text{C})(0.12\text{ m}^2)} = 0.04\ {}^\circ\text{C/W}$$

$$R_2 = R_4 = R_C = \left(\frac{L}{kA}\right)_C = \frac{0.05\text{ m}}{(20\text{ W/m}\cdot{}^\circ\text{C})(0.04\text{ m}^2)} = 0.06\ {}^\circ\text{C/W}$$

$$R_3 = R_B = \left(\frac{L}{kA}\right)_B = \frac{0.05\text{ m}}{(8\text{ W/m}\cdot{}^\circ\text{C})(0.04\text{ m}^2)} = 0.16\ {}^\circ\text{C/W}$$

$$R_5 = R_D = \left(\frac{L}{kA}\right)_D = \frac{0.1\text{ m}}{(15\text{ W/m}\cdot{}^\circ\text{C})(0.06\text{ m}^2)} = 0.11\ {}^\circ\text{C/W}$$

$$R_6 = R_E = \left(\frac{L}{kA}\right)_E = \frac{0.1\text{ m}}{(35\text{ W/m}\cdot{}^\circ\text{C})(0.06\text{ m}^2)} = 0.05\ {}^\circ\text{C/W}$$

$$R_7 = R_F = \left(\frac{L}{kA}\right)_F = \frac{0.06\text{ m}}{(2\text{ W/m}\cdot{}^\circ\text{C})(0.12\text{ m}^2)} = 0.25\ {}^\circ\text{C/W}$$

$$\frac{1}{R_{mid,1}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{0.06} + \frac{1}{0.16} + \frac{1}{0.06} \longrightarrow R_{mid,1} = 0.025\ {}^\circ\text{C/W}$$

$$\frac{1}{R_{mid,2}} = \frac{1}{R_5} + \frac{1}{R_6} = \frac{1}{0.11} + \frac{1}{0.05} \longrightarrow R_{mid,2} = 0.034\ {}^\circ\text{C/W}$$

$$R_{total} = R_1 + R_{mid,1} + R_{mid,2} + R_7 = 0.04 + 0.025 + 0.034 + 0.25 = 0.349\ {}^\circ\text{C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(300 - 100)^\circ\text{C}}{0.349\ {}^\circ\text{C/W}} = 572\text{ W}\ \ (\text{for a } 0.12\text{ m} \times 1\text{ m section})$$

Then steady rate of heat transfer through entire wall becomes

$$\dot{Q}_{total} = (572\text{ W})\frac{(5\text{ m})(8\text{ m})}{0.12\text{ m}^2} = \mathbf{1.91 \times 10^5\ W}$$

(*b*) The total thermal resistance between left surface and the point where the sections B, D, and E meet is

$$R_{total} = R_1 + R_{mid,1} = 0.04 + 0.025 = 0.065\ {}^\circ\text{C/W}$$

Then the temperature at the point where the sections B, D, and E meet becomes

$$\dot{Q} = \frac{T_1 - T}{R_{total}} \longrightarrow T = T_1 - \dot{Q}R_{total} = 300^\circ\text{C} - (572\text{ W})(0.065^\circ\text{C/W}) = \mathbf{263^\circ C}$$

(*c*) The temperature drop across the section F can be determined from

$$\dot{Q} = \frac{\Delta T}{R_F} \rightarrow \Delta T = \dot{Q}R_F = (572\text{ W})(0.25^\circ\text{C/W}) = \mathbf{143^\circ C}$$

3-76 In an experiment, the convection heat transfer coefficients of (a) air and (b) water flowing over the metal foil are to be determined.

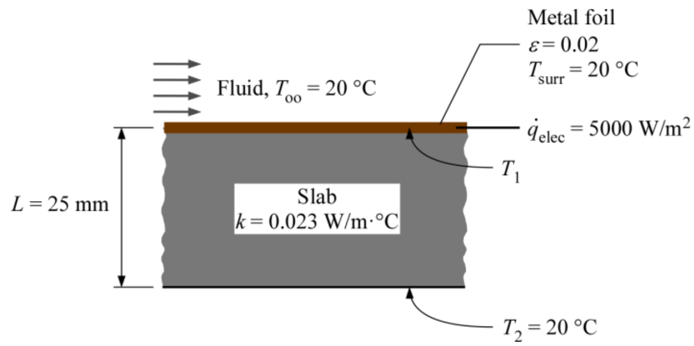
Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** Thermal properties are constant. **4** Thermal resistance of the thin metal foil is negligible.

Properties Thermal conductivity of the slab is given to be $k = 0.023$ W/m · K and the emissivity of the metal foil is 0.02.

Analysis The thermal resistances are

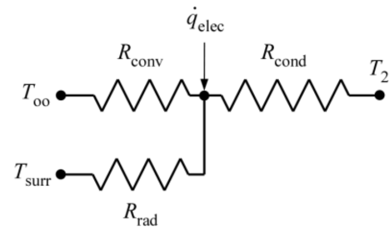
$$R_{\text{cond}} = \frac{L}{kA} \quad R_{\text{conv}} = \frac{1}{hA}$$

and $$R_{\text{rad}} = \frac{1}{h_{\text{rad}} A}$$



From energy balance and using the thermal resistance concept, the following equation is expressed:

$$\frac{T_\infty - T_1}{R_{\text{conv}}} + \frac{T_{\text{surr}} - T_1}{R_{\text{rad}}} + \dot{q}_{\text{elec}} A = \frac{T_1 - T_2}{R_{\text{cond}}}$$



or $$\frac{1}{R_{\text{conv}}} = \left(\frac{T_1 - T_2}{R_{\text{cond}}} - \frac{T_{\text{surr}} - T_1}{R_{\text{rad}}} - \dot{q}_{\text{elec}} A \right) \frac{1}{T_\infty - T_1}$$

$$h = \left(\frac{T_1 - T_2}{L/k} - \frac{T_{\text{surr}} - T_1}{1/h_{\text{rad}}} - \dot{q}_{\text{elec}} \right) \frac{1}{T_\infty - T_1}$$

(a) For air flowing over the metal foil, the radiation heat transfer coefficient is

$$\begin{aligned} h_{\text{rad}} &= \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}}) \\ &= (0.02)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(423^2 + 293^2)\text{ K}^2(423 + 293)\text{ K} \\ &= 0.215 \text{ W/m}^2 \cdot \text{K} \end{aligned}$$

The convection heat transfer coefficient for air flowing over the metal foil is

$$\begin{aligned} h &= \left[\frac{(150-20)\text{ K}}{0.025\text{ m}/0.023\text{ W/m}\cdot\text{K}} - \frac{(20-150)\text{ K}}{1/0.215\text{ W/m}^2\cdot K} - 5000\text{ W/m}^2 \right] \frac{1}{(20-150)\text{ K}} \\ &= \mathbf{37.3\ W/m^2 \cdot K} \end{aligned}$$

(b) For water flowing over the metal foil, the radiation heat transfer coefficient is

$$\begin{aligned} h_{\text{rad}} &= \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}}) \\ &= (0.02)(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(303^2 + 293^2)\text{ K}^2(303 + 293)\text{ K} \\ &= 0.1201 \text{ W/m}^2 \cdot \text{K} \end{aligned}$$

The convection heat transfer coefficient for water flowing over the metal foil is

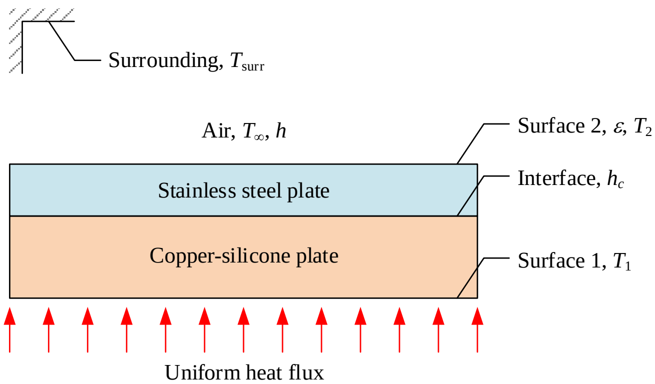
$$\begin{aligned} h &= \left[\frac{(30-20)\text{ K}}{0.025\text{ m}/0.023\text{ W/m}\cdot\text{K}} - \frac{(20-30)\text{ K}}{1/0.1201\text{ W/m}^2\cdot K} - 5000\text{ W/m}^2 \right] \frac{1}{(20-30)\text{ K}} \\ &= \mathbf{499\ W/m^2 \cdot K} \end{aligned}$$

Discussion If heat transfer by conduction through the slab and radiation on the metal foil surface is neglected, the convection heat transfer coefficient for the case with air flow would deviate by 3.2% from the result in part (a), while the convection heat transfer coefficient for the case with water flow would deviate by 0.2% from the result in part (b).

3-77 A stainless steel plate is attached on a copper-silicon plate. The upper surface is exposed to convection with air and thermal radiation with the surroundings. The bottom surface is subjected to a uniform heat flux. Thermal contact resistance exists in the plate interface. The total thermal resistance, between T_1 and T_∞, for an area of 1 m^2 is to be calculated. Determine whether the use of the ASME SB-96 plate complies with the ASME Boiler and Pressure Vessel Code.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through plates. **3** Uniform surface temperatures. **4** Thermal properties are constant.

Properties The thermal conductivity for the copper-silicon plate is k_1 = 36 W/m·K and for the stainless steel plate is k_2 = 13 W/m. The thermal contact conductance between the stainless steel and nonmetal plates is h_c = 5000 W/m^2·K. The emissivity of the stainless steel plate surface is 0.3.



Analysis The uniform heat flux subjected on the bottom surface (surface 1) is transferred by conduction through the plates, and by convection and radiation on the top surface (surface 2):

$$\dot{q} = \dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} + \dot{q}_{\text{rad}} \qquad \rightarrow \qquad \dot{q} = h(T_2 - T_\infty) + \varepsilon\sigma(T_2^4 - T_{\text{surr}}^4)$$

Solving for T_2,

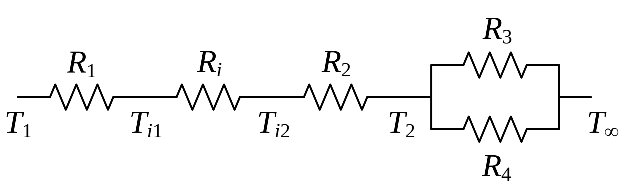
$$750\ \text{W/m}^2 = 5\ \text{W/m}^2\cdot\text{K}\,(T_2 - 20)\text{K} + (0.3)(5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4)[(T_2+273)^4 - (20+273)^4]\text{K}^4 \ \rightarrow\ T_2 = 116.7°\text{C}$$

Having determined the top surface temperature T_2, we then can find the radiation heat transfer coefficient:

$$h_{\text{rad}} = \varepsilon\sigma(T_2^2 + T_{\text{surr}}^2)(T_2 + T_{\text{surr}})$$

$$h_{\text{rad}} = (0.3)\left(5.67\times10^{-8}\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)[(116.7+273)^2 + (20+273)^2](\text{K}^2)[(116.7+273)+(20+273)](\text{K})$$
$$= 2.761\ \text{W/m}^2\cdot\text{K}$$

The thermal resistances for the thermal network between T_1 and T_∞ (R_i is the thermal contact resistance at the interface) are



$$R_1 = \frac{L_1}{k_1A} \text{ or } \qquad AR_1 = \frac{L_1}{k_1} = \frac{0.03\ \text{m}}{36\ \text{W/m}\cdot\text{K}} = 0.0008333\ \text{m}^2\cdot\text{K/W}$$

$$R_i = \frac{1}{h_cA} \text{ or } \qquad AR_i = \frac{1}{h_c} = \frac{1}{5000\ \text{W/m}^2\cdot\text{K}} = 0.0002\ \text{m}^2\cdot\text{K/W}$$

$$R_2 = \frac{L_2}{k_2A} \text{ or } \qquad AR_2 = \frac{L_2}{k_2} = \frac{0.01\ \text{m}}{13\ \text{W/m}\cdot\text{K}} = 0.0007692\ \text{m}^2\cdot\text{K/W}$$

$$R_3 = \frac{1}{hA} \text{ or } \qquad AR_3 = \frac{1}{h} = \frac{1}{5\ \text{W/m}^2{\cdot}\text{K}} = 0.2\ \text{m}^2{\cdot}\text{K/W}$$

$$R_4 = \frac{1}{h_{\text{rad}}A} \text{ or } \qquad AR_4 = \frac{1}{h_{\text{rad}}} = \frac{1}{2.761\ \text{W/m}^2{\cdot}\text{K}} = 0.36219\ \text{m}^2{\cdot}\text{K/W}$$

The two parallel resistances R_3 and R_4 can be combined as an equivalent resistance R_{equiv}:

$$\frac{1}{AR_{\text{equiv}}} = \frac{1}{AR_3} + \frac{1}{AR_4} = \frac{1}{0.2\ \text{m}^2{\cdot}\text{K/W}} + \frac{1}{0.36219\ \text{m}^2{\cdot}\text{K/W}} = 7.761\ \text{W/m}^2{\cdot}\text{K} \text{or} \quad AR_{\text{equiv}} = 0.12885\ \text{m}^2{\cdot}\text{K/W}$$

The total resistance for the thermal network is

$$AR_{\text{tot}} = AR_1 + AR_i + AR_2 + AR_{\text{equiv}} = 0.13065\ \text{m}^2{\cdot}\text{K/W}$$

Thus, the total thermal resistance for an area of 1 m^2 is $R_{\text{tot}} = \mathbf{0.13065\ K/W}$.

To determine the temperatures that the copper-silicon plate experiences, we need to find T_1 and T_{i1}, which can be calculated using the heat flux and the individual thermal resistance:

$$\dot{q} = \frac{T_{i2} - T_2}{AR_2} \quad \rightarrow \quad T_{i2} = T_2 + \dot{q}AR_2 = 116.7\text{°C} + (750\ \text{W/m}^2)(0.0007692\ \text{m}^2{\cdot}\text{K/W}) = 117.3\text{°C}$$

$$\dot{q} = \frac{T_{i1} - T_{i2}}{AR_i} \quad \rightarrow \quad T_{i1} = T_{i2} + \dot{q}AR_i = 117.3\text{°C} + (750\ \text{W/m}^2)(0.0002\ \text{m}^2{\cdot}\text{K/W}) = \mathbf{117.5}\text{°C}$$

$$\dot{q} = \frac{T_1 - T_{i1}}{AR_1} \quad \rightarrow \quad T_1 = T_{i1} + \dot{q}AR_1 = 117.5\text{°C} + (750\ \text{W/m}^2)(0.0008333\ \text{m}^2{\cdot}\text{K/W}) = \mathbf{118.1}\text{°C}$$

Discussion The average temperature that the ASME SB-96 copper-silicon plate would operate is 117.8°C, which is above the temperature (93°C) specified by the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HF-300). Thus, the copper-silicon plate is not in compliance with the ASME code.

Heat Conduction in Cylinders and Spheres

3-78C When the diameter of cylinder is very small compared to its length, it can be treated as an infinitely long cylinder. Cylindrical rods can also be treated as being infinitely long when dealing with heat transfer at locations far from the top or bottom surfaces. However, it is not proper to use this model when finding temperatures near the bottom and the top of the cylinder.

3-79C No. In steady-operation the temperature of a solid cylinder or sphere does not change in radial direction (unless there is heat generation).

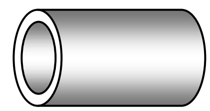
3-80C Heat transfer in this short cylinder is one-dimensional since there will be no heat transfer in the axial and tangential directions.

3-81 A steam pipe covered with 3-cm thick glass wool insulation is subjected to convection on its surfaces. The rate of heat transfer per unit length and the temperature drops across the pipe and the insulation are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

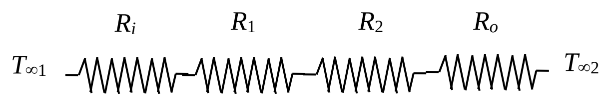
Properties The thermal conductivities are given to be k = 15 W/m·°C for steel and k = 0.038 W/m·°C for glass wool insulation

Analysis The inner and the outer surface areas of the insulated pipe per unit length are



$$A_i = \pi D_i L = \pi(0.05\text{ m})(1\text{ m}) = 0.157\text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.055 + 0.06\text{ m})(1\text{ m}) = 0.361\text{ m}^2$$

The individual thermal resistances are



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(80\text{ W/m}^2\cdot°\text{C})(0.157\text{ m}^2)} = 0.08\ °\text{C/W}$$
$$R_1 = R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(15\text{ W/m}\cdot°\text{C})(1\text{ m})} = 0.00101\ °\text{C/W}$$
$$R_2 = R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.038\text{ W/m}\cdot°\text{C})(1\text{ m})} = 3.089\ °\text{C/W}$$
$$R_o = \frac{1}{h_o A_o} = \frac{1}{(22\text{ W/m}^2\cdot°\text{C})(0.361\text{ m}^2)} = 0.1259\ °\text{C/W}$$
$$R_{total} = R_i + R_1 + R_2 + R_o = 0.08 + 0.00101 + 3.089 + 0.1259 = 3.296\ °\text{C/W}$$

Then the steady rate of heat loss from the steam per m. pipe length becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(280-5)°\text{C}}{3.296\ °\text{C/W}} = \mathbf{83.4\ W}$$

The temperature drops across the pipe and the insulation are

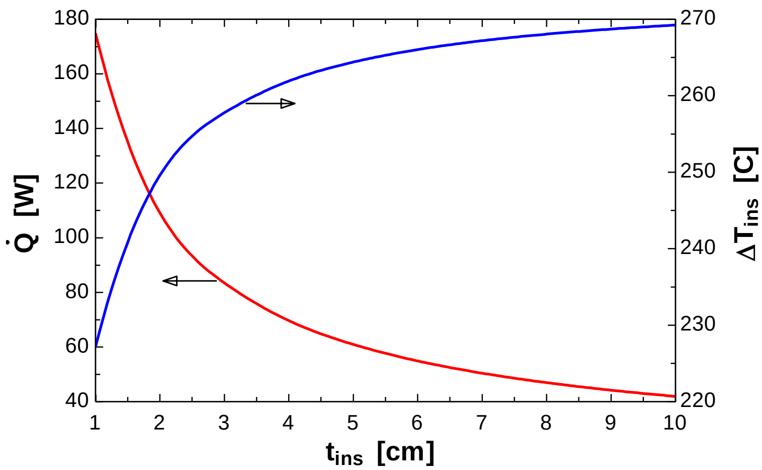
$$\Delta T_{pipe} = \dot{Q}R_{pipe} = (83.4\text{ W})(0.00101\ °\text{C/W}) = \mathbf{0.084°C}$$
$$\Delta T_{insulation} = \dot{Q}R_{insulation} = (83.4\text{ W})(3.089°\text{C/W}) = \mathbf{257.6°C}$$

3-82 Prob. 3-81 is reconsidered. The effect of the thickness of the insulation on the rate of heat loss from the steam and the temperature drop across the insulation layer are to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_infinity_1=280 [C]
T_infinity_2=5 [C]
k_steel=15 [W/m-C]
D_i=0.05 [m]
D_o=0.055 [m]
r_1=D_i/2
r_2=D_o/2
t_ins=3 [cm]
k_ins=0.038 [W/m-C]
h_o=22 [W/m^2-C]
h_i=80 [W/m^2-C]
L=1 [m]

"ANALYSIS"
A_i=pi*D_i*L
A_o=pi*(D_o+2*t_ins*Convert(cm, m))*L
R_conv_i=1/(h_i*A_i)
R_pipe=ln(r_2/r_1)/(2*pi*k_steel*L)
R_ins=ln(r_3/r_2)/(2*pi*k_ins*L)
r_3=r_2+t_ins*Convert(cm, m) "t_ins is in cm"
R_conv_o=1/(h_o*A_o)
R_total=R_conv_i+R_pipe+R_ins+R_conv_o
Q_dot=(T_infinity_1-T_infinity_2)/R_total
DELTAT_pipe=Q_dot*R_pipe
DELTAT_ins=Q_dot*R_ins
```

T_{ins} [cm]	Q [W]	ΔT_{ins} [C]
1	174.9	227.2
2	109	249.6
3	83.44	257.8
4	69.64	261.9
5	60.93	264.4
6	54.88	266
7	50.41	267.2
8	46.95	268.1
9	44.18	268.7
10	41.91	269.2

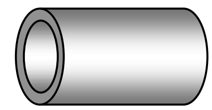


3-83 A 50-m long section of a steam pipe passes through an open space at 15°C. The rate of heat loss from the steam pipe, the annual cost of this heat loss, and the thickness of fiberglass insulation needed to save 90 percent of the heat lost are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivity is constant. **4** The thermal contact resistance at the interface is negligible. **5** The pipe temperature remains constant at about 150°C with or without insulation. **6** The combined heat transfer coefficient on the outer surface remains constant even after the pipe is insulated.

Properties The thermal conductivity of fiberglass insulation is given to be k = 0.035 W/m·°C.

Analysis (a) The rate of heat loss from the steam pipe is



$$A_o = \pi DL = \pi(0.1\,\text{m})(50\,\text{m}) = 15.71\,\text{m}^2$$

$$\dot{Q}_{bare} = h_o A(T_s - T_{air}) = (20\ \text{W/m}^2\cdot°\text{C})(15.71\,\text{m}^2)(150-15)°\text{C} = \mathbf{42{,}412\ W}$$

(b) The amount of heat loss per year is

$$Q = \dot{Q}\Delta t = (42.412\,\text{kJ/s})(365\times 24\times 3600\,\text{s/yr}) = 1.337\times 10^9\ \text{kJ/yr}$$

The amount of gas consumption from the natural gas furnace that has an efficiency of 75% is

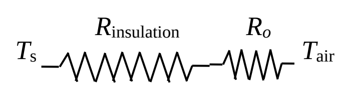
$$Q_{gas} = \frac{1.337\times 10^9\ \text{kJ/yr}}{0.75}\left(\frac{1\,\text{therm}}{105{,}500\,\text{kJ}}\right) = 16{,}903\ \text{therms/yr}$$

The annual cost of this energy lost is

$$\begin{aligned}\text{Energy cost} &= (\text{Energy used})(\text{Unit cost of energy})\\ &= (16{,}903\,\text{therms/yr})(\$0.52/\text{therm}) = \mathbf{\$8790/yr}\end{aligned}$$

(c) In order to save 90% of the heat loss and thus to reduce it to 0.1×42,412 = 4241 W, the thickness of insulation needed is determined from

$$\dot{Q}_{insulated} = \frac{T_s - T_{air}}{R_o + R_{insulation}} = \frac{T_s - T_{air}}{\dfrac{1}{h_o A_o} + \dfrac{\ln(r_2/r_1)}{2\pi kL}}$$



Substituting and solving for r_2, we get

$$4241\ \text{W} = \frac{(150-15)°\text{C}}{\dfrac{1}{(20\ \text{W/m}^2\cdot°\text{C})[(2\pi r_2(50\,\text{m})]} + \dfrac{\ln(r_2/0.05)}{2\pi(0.035\ \text{W/m}\cdot°\text{C})(50\,\text{m})}} \longrightarrow r_2 = 0.0692\ \text{m}$$

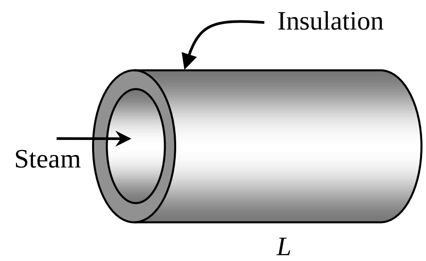
Then the thickness of insulation becomes

$$t_{insulation} = r_2 - r_1 = 6.92 - 5 = \mathbf{1.92\,cm}$$

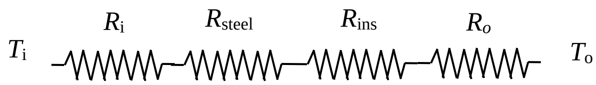
3-84 Steam flows in a steel pipe, which is insulated by gypsum plaster. The rate of heat transfer from the steam and the temperature on the outside surface of the insulation are be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties (*a*) The thermal conductivities of steel and gypsum plaster are given to be 50 and 0.5 W/m·°C, respectively.



Analysis The thermal resistances are



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(800\ \text{W/m}^2\cdot°\text{C})\pi(0.06\,\text{m})(20\,\text{m})} = 0.0003316°\text{C/W}$$

$$R_{\text{steel}} = \frac{\ln(D_2/D_1)}{2\pi k_{\text{steel}} L} = \frac{\ln(8/6)}{2\pi(50\ \text{W/m}\cdot°\text{C})(20\,\text{m})} = 0.0000458°\text{C/W}$$

$$R_{\text{ins}} = \frac{\ln(D_3/D_2)}{2\pi k_{\text{ins}} L} = \frac{\ln(16/8)}{2\pi(0.5\ \text{W/m}\cdot°\text{C})(20\,\text{m})} = 0.011032°\text{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(200\ \text{W/m}^2\cdot°\text{C})\pi(0.16\,\text{m})(20\,\text{m})} = 0.0004974°\text{C/W}$$

The total thermal resistance and the rate of heat transfer are

$$R_{\text{total}} = R_i + R_{\text{steel}} + R_{\text{ins}} + R_o = 0.0003316 + 0.0000458 + 0.011032 + 0.0004974 = 0.011907°\text{C/W}$$

$$\dot{Q} = \frac{T_i - T_o}{R_{\text{total}}} = \frac{(200-10)°\text{C}}{0.011907\ \text{m}^2\cdot\text{C/W}} = \mathbf{15{,}957\ W}$$

(*b*) The temperature at the outer surface of the insulation is determined from

$$\dot{Q} = \frac{T_s - T_o}{R_o} \longrightarrow 15{,}957\ \text{W} = \frac{(T_s - 10)°\text{C}}{0.0004974\ \text{m}^2\cdot°\text{C/W}} \longrightarrow T_s = \mathbf{17.9°C}$$

3-85 Hot liquid flows in a pipe with FEP lining on the inner surface. The pipe outer surface is subjected to uniform heat flux. The liquid mean temperature and convection heat transfer coefficient are given. Thermal contact resistance exists in the interface between the FEP lining and the steel surface. Determine the temperatures at the FEP lining T_1, and at the pipe outer surface T_2. Does the FEP lining comply with the recommendation of the ASME code?

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction. **3** Uniform surface temperatures. **4** Thermal properties are constant.

Properties The thermal conductivity for the pipe wall is k = 15 W/m·K.

Analysis The heat transfer rate through the pipe is

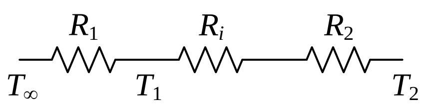
$$\dot{Q} = \dot{q}A_2 = \dot{q}\pi D_2 L = (1200\ \mathrm{W/m^2})\pi(0.027\ \mathrm{m})(1\ \mathrm{m}) = 101.79\ \mathrm{W}$$

The inner and outer surface areas of the pipe are

$$A_1 = \pi D_1 L = \pi(0.022\ \mathrm{m})(1\ \mathrm{m}) = 0.06912\ \mathrm{m^2}$$

$$A_2 = \pi D_2 L = \pi(0.027\ \mathrm{m})(1\ \mathrm{m}) = 0.08482\ \mathrm{m^2}$$

The thermal circuit network between the mean temperature of the liquid T_∞ and the outer pipe surface temperature T_2 is



$$R_1 = \frac{1}{hA_1} = \frac{1}{(50\ \mathrm{W/m^2{\cdot}K})(0.06912\ \mathrm{m^2})} = 0.2894\ \mathrm{K/W}$$

$$R_i = \frac{1}{h_c A_1} = \frac{1}{(1500\ \mathrm{W/m^2{\cdot}K})(0.06912\ \mathrm{m^2})} = 0.009645\ \mathrm{K/W}$$

$$R_2 = \frac{\ln(D_2/D_1)}{2\pi kL} = \frac{\ln(27\ \mathrm{mm}/22\ \mathrm{mm})}{2\pi(15\ \mathrm{W/m\cdot K})(1\ \mathrm{m})} = 0.002173\ \mathrm{K/W}$$

The total thermal resistance between T_∞ and T_2 is

$$R_{\mathrm{tot}} = R_1 + R_i + R_2 = 0.2894\ \mathrm{K/W} + 0.009645\ \mathrm{K/W} + 0.002173\ \mathrm{K/W} = \mathbf{0.30122\ K/W}$$

The temperature at the pipe outer surface T_2 is (note that the heat is assumed flowing from T_2 to T_∞)

$$\dot{Q} = \frac{T_2 - T_\infty}{R_{\mathrm{tot}}} \quad \rightarrow \quad T_2 = T_\infty + \dot{Q}R_{\mathrm{tot}} = 180°\mathrm{C} + (101.79\ \mathrm{W})(0.30122\ \mathrm{K/W}) = \mathbf{210.7}°\mathrm{C}$$

The temperature at the lining T_1 is

$$\dot{Q} = \frac{T_1 - T_\infty}{R_1} \quad \rightarrow \quad T_1 = T_\infty + \dot{Q}R_1 = 180°\mathrm{C} + (101.79\ \mathrm{W})(0.2894\ \mathrm{K/W}) = \mathbf{209.5}°\mathrm{C}$$

Discussion The temperature at the lining (T_1 = 209.5°C) exceeds the maximum temperature (204°C) recommended by the ASME Process Piping code for FEP lining. A different thermoplastic lining should be used. Polytetrafluoroethylene (PTFE) lining has a recommended maximum temperature of 260°C by the ASME Code for Process Piping (ASME B31.3-2014, A323), which would comply with the code.

3-86 Hot water flows in an ABS pipe. The pipe outer surface is subjected convection with air. Determine the maximum water temperature, such that the ABS pipe is operating at the recommended temperature or lower by the ASME Code for Process Piping.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction. **3** Uniform surface temperatures. **4** Thermal properties are constant.

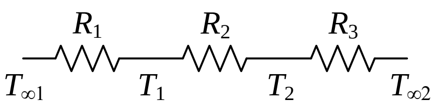
Properties The thermal conductivity for the pipe wall is k = 0.1 W/m·K.

Analysis The inner and outer surface areas of the pipe are

$$A_1 = \pi D_1 L = \pi(0.022 \text{ m})(1 \text{ m}) = 0.06912 \text{ m}^2$$

$$A_2 = \pi D_2 L = \pi(0.027 \text{ m})(1 \text{ m}) = 0.08482 \text{ m}^2$$

The thermal circuit network between the water temperature $T_{\infty 1}$ and the air temperature $T_{\infty 2}$ is



$$R_1 = \frac{1}{h_1 A_1} = \frac{1}{(50 \text{ W/m}^2\cdot\text{K})(0.06912 \text{ m}^2)} = 0.2894 \text{ K/W}$$

$$R_2 = \frac{\ln(D_2/D_1)}{2\pi k L} = \frac{\ln(27 \text{ mm}/22 \text{ mm})}{2\pi(0.1 \text{ W/m}\cdot\text{K})(1 \text{ m})} = 0.3259 \text{ K/W}$$

$$R_3 = \frac{1}{h_2 A_2} = \frac{1}{(10 \text{ W/m}^2\cdot\text{K})(0.08482 \text{ m}^2)} = 1.1790 \text{ K/W}$$

The total thermal resistance between $T_{\infty 1}$ and $T_{\infty 2}$ is

$$R_{\text{tot}} = R_1 + R_2 + R_3 = 0.2894 \text{ K/W} + 0.3259 \text{ K/W} + 1.1790 \text{ K/W} = 1.7943 \text{ K/W}$$

The maximum water temperature is limited by the temperature at the inner pipe surface at T_1 = 80°C. Thus, the heat transfer rate through the thermal circuit is

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_2 + R_3} = \frac{(80 - 20)\text{K}}{0.3259 \text{ K/W} + 1.1790 \text{ K/W}} = 39.87 \text{ W}$$

The maximum water temperature $T_{\infty 1}$ is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{tot}}} \qquad \rightarrow \qquad T_{\infty 1} = T_{\infty 2} + \dot{Q} R_{\text{tot}} = 20°\text{C} + (39.87 \text{ W})(1.7943 \text{ K/W}) = \mathbf{91.5}°\text{C}$$

The temperature at the outer pipe surface T_2 is

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_3} \qquad \rightarrow \qquad T_2 = T_{\infty 2} + \dot{Q} R_3 = 20°\text{C} + (39.87 \text{ W})(1.1790 \text{ K/W}) = \mathbf{67.0}°\text{C}$$

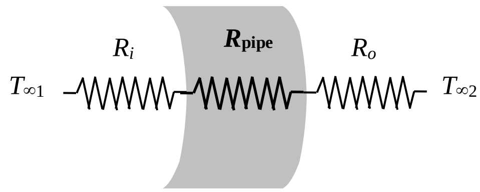
Discussion The water at $T_{\infty 1}$ = 91.5°C would keep the ABS pipe in compliance with the ASME Code for Process Piping, where the inner pipe surface is at 80°C and the outer pipe surface is at 67°C. If the water temperature goes above 91.5°C, the inner pipe surface temperature would exceed the recommended value of 80°C.

3-87E Steam exiting the turbine of a steam power plant at 100°F is to be condensed in a large condenser by cooling water flowing through copper tubes. For specified heat transfer coefficients, the length of the tube required to condense steam at a rate of 250 lbm/h is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal properties are constant. **4** Heat transfer coefficients are constant and uniform over the surfaces.

Properties The thermal conductivity of copper tube is given to be k = 223 Btu/h·ft·°F. The heat of vaporization of water at 100°F is given to be 1037 Btu/lbm.

Analysis The individual resistances are



$$A_i = \pi D_i L = \pi(0.4/12\text{ ft})(1\text{ ft}) = 0.105\text{ ft}^2$$

$$A_o = \pi D_o L = \pi(0.6/12\text{ ft})(1\text{ ft}) = 0.157\text{ ft}^2$$

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(35\text{ Btu/h.ft}^2.°\text{F})(0.105\text{ ft}^2)} = 0.27211\text{ h°F/Btu}$$

$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3/2)}{2\pi(223\text{ Btu/h.ft.°F})(1\text{ ft})} = 0.00029\text{ h°F/Btu}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(2400\text{ Btu/h.ft}^2.°\text{F})(0.157\text{ ft}^2)} = 0.00265\text{ h°F/Btu}$$

$$R_{total} = R_i + R_{pipe} + R_o = 0.27211 + 0.00029 + 0.00265 = 0.27505\text{ h°F/Btu}$$

The heat transfer rate per ft length of the tube is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(100-70)°\text{F}}{0.27505\,°\text{F/Btu}} = 109.1\text{ Btu/h}$$

The total rate of heat transfer required to condense steam at a rate of 250 lbm/h and the length of the tube required is determined to be

$$\dot{Q}_{total} = \dot{m}h_{fg} = (250\text{ lbm/h})(1037\text{ Btu/lbm}) = 259{,}250\text{ Btu/h}$$

$$\text{Tube length} = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{259{,}250}{109.1} = 2376\text{ ft} \cong \mathbf{2380\text{ ft}}$$

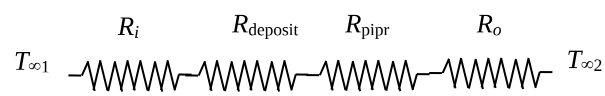
3-88E Steam exiting the turbine of a steam power plant at 100°F is to be condensed in a large condenser by cooling water flowing through copper tubes. For specified heat transfer coefficients and 0.01-in thick scale build up on the inner surface, the length of the tube required to condense steam at a rate of 250 lbm/h is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** Heat transfer coefficients are constant and uniform over the surfaces.

Properties The thermal conductivities are given to be k = 223 Btu/h·ft·°F for copper tube and be k = 0.5 Btu/h·ft·°F for the mineral deposit. The heat of vaporization of water at 100°F is given to be 1037 Btu/lbm.

Analysis When a 0.01-in thick layer of deposit forms on the inner surface of the pipe, the inner diameter of the pipe will reduce from 0.4 in to 0.38 in. The individual thermal resistances are

$$A_i = \pi D_i L = \pi(0.38/12\text{ ft})(1\text{ ft}) = 0.09948\text{ ft}^2$$
$$A_o = \pi D_o L = \pi(0.6/12\text{ ft})(1\text{ ft}) = 0.1571\text{ ft}^2$$



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(35\text{ Btu/h.ft}^2.°\text{F})(0.09948\text{ ft}^2)} = 0.28720\text{ h°F/Btu}$$
$$R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3/2)}{2\pi(223\text{ Btu/h.ft.°F})(1\text{ ft})} = 0.000289\text{ h°F/Btu}$$
$$R_{\text{deposit}} = \frac{\ln(r_1/r_{\text{dep}})}{2\pi k_2 L} = \frac{\ln(0.2/0.19)}{2\pi(0.5\text{ Btu/h.ft.°F})(1\text{ ft})} = 0.01633\text{ h.°F/Btu}$$
$$R_o = \frac{1}{h_o A_o} = \frac{1}{(2400\text{ Btu/h.ft}^2.°\text{F})(0.1571\text{ ft}^2)} = 0.00265\text{ h°F/Btu}$$
$$R_{\text{total}} = R_i + R_{\text{pipe}} + R_{\text{deposit}} + R_o = 0.28720 + 0.000289 + 0.01633 + 0.00265 = 0.306469\text{ h°F/Btu}$$

The heat transfer rate per ft length of the tube is

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{\text{total}}} = \frac{(100-70)°\text{F}}{0.306469\text{ °F/Btu}} = 97.89\text{ Btu/h}$$

The total rate of heat transfer required to condense steam at a rate of 250 lbm/h and the length of the tube required can be determined to be

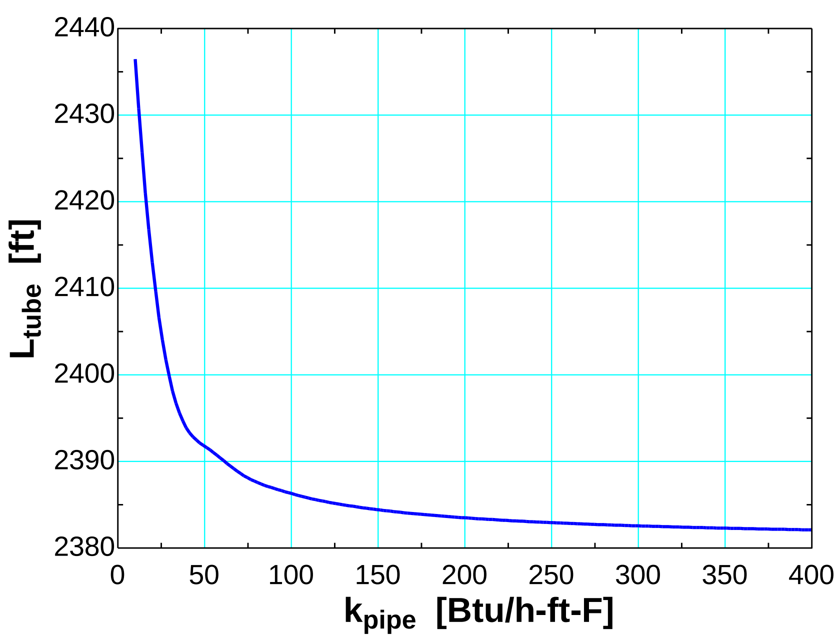
$$\dot{Q}_{\text{total}} = \dot{m}h_{fg} = (120\text{ lbm/h})(1037\text{ Btu/lbm}) = 124{,}440\text{ Btu/h}$$
$$\text{Tube length} = \frac{\dot{Q}_{\text{total}}}{\dot{Q}} = \frac{124{,}440}{97.38} = \mathbf{2648\text{ ft}}$$

3-89E  Prob. 3-87E is reconsidered. The effects of the thermal conductivity of the pipe material and the outer diameter of the pipe on the length of the tube required are to be investigated.

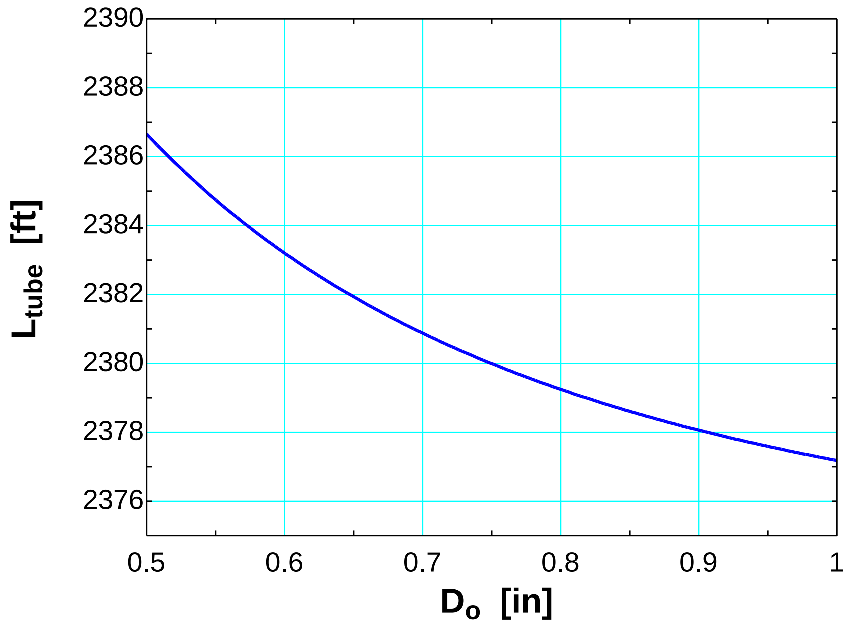
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_infinity_1=100 [F]
T_infinity_2=70 [F]
k_pipe=223 [Btu/h-ft-F]
D_i=0.4 [in]
D_o=0.6 [in]
r_1=D_i/2
r_2=D_o/2
h_fg=1037 [Btu/lbm]
h_o=2400 [Btu/h-ft^2-F]
h_i=35 [Btu/h-ft^2-F]
m_dot=250 [lbm/h]

"ANALYSIS"
L=1 [ft] "for 1 ft length of the tube"
A_i=pi*(D_i*Convert(in, ft))*L
A_o=pi*(D_o*Convert(in, ft))*L
R_conv_i=1/(h_i*A_i)
R_pipe=ln(r_2/r_1)/(2*pi*k_pipe*L)
R_conv_o=1/(h_o*A_o)
R_total=R_conv_i+R_pipe+R_conv_o
Q_dot=(T_infinity_1-T_infinity_2)/R_total
Q_dot_total=m_dot*h_fg
L_tube=Q_dot_total/Q_dot
```

k_{pipe} [Btu/h.ft.F]	L_{tube} [ft]
10	2436
30.53	2399
51.05	2392
71.58	2388
92.11	2387
112.6	2386
133.2	2385
153.7	2384
174.2	2384
194.7	2384
215.3	2383
235.8	2383
256.3	2383
276.8	2383
297.4	2383
317.9	2382
338.4	2382
358.9	2382
379.5	2382
400	2382



D_o [in]	L_{tube} [ft]
0.5	2387
0.525	2386
0.55	2385
0.575	2384
0.6	2383
0.625	2383
0.65	2382
0.675	2381
0.7	2381
0.725	2380
0.75	2380
0.775	2380
0.8	2379
0.825	2379
0.85	2379
0.875	2378
0.9	2378
0.925	2378
0.95	2378
0.975	2377
1	2377



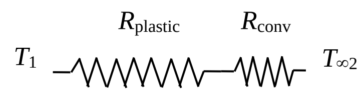
3-90 An electric wire is tightly wrapped with a 1-mm thick plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible. **5** Heat transfer coefficient accounts for the radiation effects, if any.

Properties The thermal conductivity of plastic cover is given to be k = 0.15 W/m·°C.

Analysis In steady operation, the rate of heat transfer from the wire is equal to the heat generated within the wire,

$$\dot{Q} = \dot{W}_e = \mathbf{V}I = (8\text{ V})(13\text{ A}) = 104\text{ W}$$



The total thermal resistance is

$$R_{\text{conv}} = \frac{1}{h_o A_o} = \frac{1}{(24\text{ W/m}^2.°\text{C})[\pi(0.0042\text{ m})(14\text{ m})]} = 0.2256\text{ °C/W}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(2.1/1.1)}{2\pi(0.15\text{ W/m.°C})(14\text{ m})} = 0.0490\text{ °C/W}$$

$$R_{\text{total}} = R_{\text{conv}} + R_{\text{plastic}} = 0.2256 + 0.0490 = 0.2746\text{ °C/W}$$

Then the interface temperature becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{\text{total}} = 30°\text{C} + (104\text{ W})(0.2746\text{ °C/W}) = \mathbf{58.6°C}$$

The critical radius of plastic insulation is

$$r_{cr} = \frac{k}{h} = \frac{0.15\text{ W/m.°C}}{24\text{ W/m}^2.°\text{C}} = 0.00625\text{ m} = 6.25\text{ mm}$$

Doubling the thickness of the plastic cover will increase the outer radius of the wire to 3 mm, which is less than the critical radius of insulation. Therefore, doubling the thickness of plastic cover will increase the rate of heat loss and decrease the interface temperature.

3-91 An electric hot water tank is made of two concentric cylindrical metal sheets with foam insulation in between. The fraction of the hot water cost that is due to the heat loss from the tank and the payback period of the do-it-yourself insulation kit are to be determined.

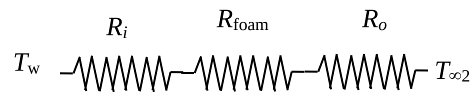
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal resistances of the water tank and the outer thin sheet metal shell are negligible. **5** Heat loss from the top and bottom surfaces is negligible.

Properties The thermal conductivities are given to be k = 0.03 W/m·°C for foam insulation and k = 0.035 W/m·°C for fiber glass insulation

Analysis We consider only the side surfaces of the water heater for simplicity, and disregard the top and bottom surfaces (it will make difference of about 10 percent). The individual thermal resistances are

$$A_i = \pi D_i L = \pi(0.40\text{ m})(1.5\text{ m}) = 1.885\text{ m}^2$$

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(50\text{ W/m}^2.°\text{C})(1.885\text{ m}^2)} = 0.0106\text{ °C/W}$$



$$A_o = \pi D_o L = \pi(0.46\text{ m})(1.5\text{ m}) = 2.168\text{ m}^2$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(12\text{ W/m}^2.°\text{C})(2.168\text{ m}^2)} = 0.0384\text{ °C/W}$$

$$R_{foam} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(23/20)}{2\pi(0.03\text{ W/m}^2\cdot°\text{C})(1.5\text{ m})} = 0.4943\text{ °C/W}$$

$$R_{total} = R_i + R_o + R_{foam} = 0.0106 + 0.0384 + 0.4943 = 0.5433\text{ °C/W}$$

The rate of heat loss from the hot water tank is

$$\dot{Q} = \frac{T_w - T_{\infty 2}}{R_{total}} = \frac{(60-27)°\text{C}}{0.5433\text{ °C/W}} = 60.74\text{ W}$$

The amount and cost of heat loss per year are

$$Q = \dot{Q}\Delta t = (0.06074\text{ kW})(365\times 24\text{ h/yr}) = 532.1\text{ kWh/yr}$$

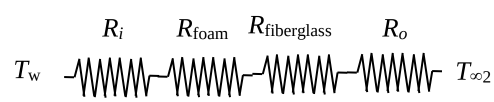
$$\text{Cost of Energy} = (\text{Amount of energy})(\text{Unit cost}) = (532.1\text{ kWh})(\$0.08/\text{kWh}) = \$42.57$$

$$f = \frac{\$42.57}{\$280} = 0.152 = \mathbf{15.2\%}$$

If 3 cm thick fiber glass insulation is used to wrap the entire tank, the individual resistances becomes

$$A_o = \pi D_o L = \pi(0.52\text{ m})(1.5\text{ m}) = 2.450\text{ m}^2$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(12\text{ W/m}^2.^\circ\text{C})(2.450\text{ m}^2)} = 0.0340°\text{C/W}$$



$$R_{foam} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(23/20)}{2\pi(0.03\text{ W/m}^2\cdot°\text{C})(1.5\text{ m})} = 0.4943\text{ °C/W}$$

$$R_{fiberglass} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(26/23)}{2\pi(0.035\text{ W/m}^2\cdot°\text{C})(1.5\text{ m})} = 0.3717\text{ °C/W}$$

$$R_{total} = R_i + R_o + R_{foam} + R_{fiberglass} = 0.0106 + 0.0340 + 0.4943 + 0.3717 = 0.9106\text{ °C/W}$$

The rate of heat loss from the hot water heater in this case is

$$\dot{Q} = \frac{T_w - T_{\infty 2}}{R_{total}} = \frac{(60-27)°\text{C}}{0.9106\text{ °C/W}} = 36.24\text{ W}$$

The energy saving is

saving = 60.74 - 36.24 = 24.5 W

The time necessary for this additional insulation to pay for its cost of $30 is then determined to be

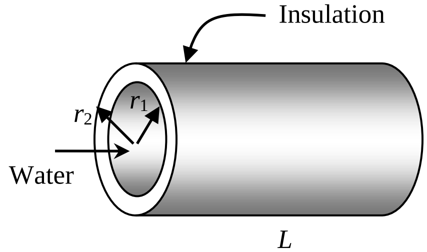
$$\text{Cost} = (0.0245\text{ kW})(\text{Time period})(\$0.08/\text{kWh}) = \$30$$

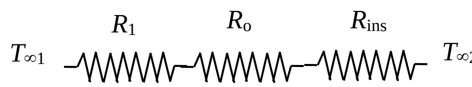
$$\longrightarrow \text{Time period} = 15{,}306\text{ hours} = 638\text{ days} \approx \mathbf{21\ months}$$

3-92 Chilled water is flowing inside a pipe. The thickness of the insulation needed to reduce the temperature rise of water to one-fourth of the original value is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity is given to be k = 0.05 W/m·°C for insulation.





Analysis The rate of heat transfer without the insulation is

$$\dot{Q}_{\text{old}} = \dot{m}c_p\Delta T = (0.98\,\text{kg/s})(4180\,\text{J/kg}\cdot{}^\circ\text{C})(8-7)^\circ\text{C} = 4096\text{ W}$$

The total resistance in this case is

$$\dot{Q}_{\text{old}} = \frac{T_\infty - T_w}{R_{\text{total}}}$$

$$4096\text{ W} = \frac{(30-7.5)^\circ\text{C}}{R_{\text{total}}} \longrightarrow R_{\text{total}} = 0.005493^\circ\text{C/W}$$

The convection resistance on the outer surface is

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(9\text{ W/m}^2\cdot{}^\circ\text{C})\pi(0.04\,\text{m})(200\,\text{m})} = 0.004421\,^\circ\text{C/W}$$

The rest of thermal resistances are due to convection resistance on the inner surface and the resistance of the pipe and it is determined from

$$R_1 = R_{\text{total}} - R_{\text{o}} = 0.005493 - 0.004421 = 0.001072\,^\circ\text{C/W}$$

The rate of heat transfer with the insulation is

$$\dot{Q}_{\text{new}} = \dot{m}c_p\Delta T = (0.98\,\text{kg/s})(4180\,\text{J/kg}\cdot{}^\circ\text{C})(0.25^\circ\text{C}) = 1024\text{ W}$$

The total thermal resistance with the insulation is

$$\dot{Q}_{\text{new}} = \frac{T_\infty - T_w}{R_{\text{total,new}}} \longrightarrow 1024\text{ W} = \frac{[30-(7+7.25)/2)]^\circ\text{C}}{R_{\text{total,new}}} \longrightarrow R_{\text{total,new}} = 0.02234^\circ\text{C/W}$$

It is expressed by

$$R_{\text{total,new}} = R_1 + R_{\text{o,new}} + R_{\text{ins}} = R_1 + \frac{1}{h_o A_o} + \frac{\ln(D_2/D_1)}{2\pi k_{\text{ins}} L}$$

$$0.02234^\circ\text{C/W} = 0.001072 + \frac{1}{(9\text{ W/m}^2\cdot{}^\circ\text{C})\pi D_2(200\,\text{m})} + \frac{\ln(D_2/0.04)}{2\pi(0.05\text{ W/m}\cdot{}^\circ\text{C})(200\,\text{m})}$$

Solving this equation by trial-error or by using an equation solver such as EES, we obtain

$$D_2 = 0.1406\,\text{m}$$

The following line in EES is used:

```
0.02234=0.001072+1/(9*pi*D2*200)+ln(D2/0.04)/(2*pi*0.05*200)
```

Then the required thickness of the insulation becomes

$$t_{\text{ins}} = (D_2 - D_1)/2 = (0.1406 - 0.04)/2 = 0.0503\,\text{m} = \mathbf{5.03\,cm}$$

3-93E A steam pipe covered with 2-in thick fiberglass insulation is subjected to convection on its surfaces. The rate of heat loss from the steam per unit length and the error involved in neglecting the thermal resistance of the steel pipe in calculations are to be determined.

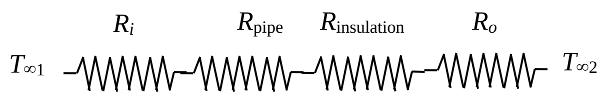
Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be k = 8.7 Btu/h·ft·°F for steel and k = 0.020 Btu/h·ft·°F for fiberglass insulation.

Analysis The inner and outer surface areas of the insulated pipe are

$$A_i = \pi D_i L = \pi(3.5/12\text{ ft})(1\text{ ft}) = 0.916\text{ ft}^2$$

$$A_o = \pi D_o L = \pi(8/12\text{ ft})(1\text{ ft}) = 2.094\text{ ft}^2$$



The individual resistances are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(30\text{ Btu/h.ft}^2.°\text{F})(0.916\text{ ft}^2)} = 0.036\text{ h}\cdot°\text{F/Btu}$$

$$R_1 = R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2/1.75)}{2\pi(8.7\text{ Btu/h.ft.}°\text{F})(1\text{ ft})} = 0.002\text{ h}\cdot°\text{F/Btu}$$

$$R_2 = R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(4/2)}{2\pi(0.020\text{ Btu/h.ft.}°\text{F})(1\text{ ft})} = 5.516\text{ h}\cdot°\text{F/Btu}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(5\text{ Btu/h.ft}^2.^\text{o}\text{ F})(2.094\text{ ft}^2)} = 0.096\text{ h}\cdot°\text{F/Btu}$$

$$R_{total} = R_i + R_1 + R_2 + R_o = 0.036 + 0.002 + 5.516 + 0.096 = 5.65\text{ h}\cdot°\text{F/Btu}$$

Then the steady rate of heat loss from the steam per ft. pipe length becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(450-55)°\text{F}}{5.65\text{ h}\cdot°\text{F/Btu}} = \mathbf{69.91 Btu/h}$$

If the thermal resistance of the steel pipe is neglected, the new value of total thermal resistance will be

$$R_{total} = R_i + R_2 + R_o = 0.036 + 5.516 + 0.096 = 5.648\text{ h°F/Btu}$$

Then the percentage error involved in calculations becomes

$$error\,\% = \frac{(5.65 - 5.648)\text{h°F/Btu}}{5.65\text{ h°F/Btu}} \times 100 = \mathbf{0.035\%}$$

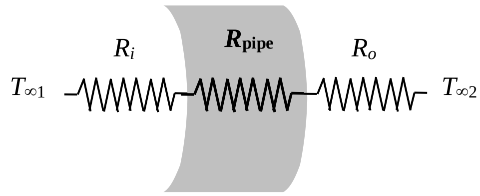
which is insignificant.

3-94 Hot water is flowing through a 15-m section of a cast iron pipe. The pipe is exposed to cold air and surfaces in the basement. The rate of heat loss from the hot water and the average velocity of the water in the pipe as it passes through the basement are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the center line and no variation in the axial direction. **3** Thermal properties are constant.

Properties The thermal conductivity and emissivity of cast iron are given to be k = 52 W/m·°C and ε = 0.7.

Analysis The individual resistances are



$$A_i = \pi D_i L = \pi(0.04\text{ m})(15\text{ m}) = 1.885\text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.046\text{ m})(15\text{ m}) = 2.168\text{ m}^2$$

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(120\text{ W/m}^2.°\text{C})(1.885\text{ m}^2)} = 0.00442\text{ °C/W}$$
$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.3/2)}{2\pi(52\text{ W/m.°C})(15\text{ m})} = 0.00003\text{ °C/W}$$

The outer surface temperature of the pipe will be somewhat below the water temperature. Assuming the outer surface temperature of the pipe to be 80°C (we will check this assumption later), the radiation heat transfer coefficient is determined to be

$$\begin{aligned} h_{rad} &= \varepsilon\sigma(T_2^{\,2} + T_{surr}^{\,2})(T_2 + T_{surr}) \\ &= (0.7)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(353\text{ K})^2 + (283\text{ K})^2](353+283) = 5.167\text{ W/m}^2.\text{K} \end{aligned}$$

Since the surrounding medium and surfaces are at the same temperature, the radiation and convection heat transfer coefficients can be added and the result can be taken as the combined heat transfer coefficient. Then,

$$h_{combined} = h_{rad} + h_{conv,2} = 5.167 + 15 = 20.17\text{ W/m}^2.°\text{C}$$
$$R_o = \frac{1}{h_{combined} A_o} = \frac{1}{(20.17\text{ W/m}^2.°\text{C})(2.168\text{ m}^2)} = 0.02287\text{ °C/W}$$
$$R_{total} = R_i + R_{pipe} + R_o = 0.00442 + 0.00003 + 0.02287 = 0.02732\text{ °C/W}$$

The rate of heat loss from the hot water pipe then becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(90-10)°\text{C}}{0.02732\text{ °C/W}} = \mathbf{2928\ W}$$

For a temperature drop of 3°C, the mass flow rate of water and the average velocity of water must be

$$\dot{Q} = \dot{m}c_p\Delta T \longrightarrow \dot{m} = \frac{\dot{Q}}{c_p\Delta T} = \frac{2928\text{ J/s}}{(4180\text{ J/kg.°C})(3\text{ °C})} = 0.2335\text{ kg/s}$$
$$\dot{m} = \rho V A_c \longrightarrow V = \frac{\dot{m}}{\rho A_c} = \frac{0.2335\text{ kg/s}}{(1000\text{ kg/m}^3)\dfrac{\pi(0.04\text{ m})^2}{4}} = \mathbf{0.186\ m/s}$$

Discussion The outer surface temperature of the pipe is

$$\dot{Q} = \frac{T_{\infty 1} - T_s}{R_i + R_{pipe}} \rightarrow 2\,928\text{ W} = \frac{(90 - T_s)°\text{C}}{(0.00442 + 0.00003)°\text{C/W}} \rightarrow T_s = 77.0°\text{C}$$

which is close to the value assumed for the surface temperature in the evaluation of the radiation resistance. Therefore, there is no need to repeat the calculations.

3-95 Liquid flows in a pipe with PVDC lining on the inner surface. The pipe outer surface is subjected to convection and radiation heat transfer. The pipe outer surface temperature is known. Determine the temperature at the PVDC lining T_1, and the temperature of the liquid $T_{\infty 1}$. Does the PVDC lining comply with the recommendation of the ASME code?

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction. **3** Uniform surface temperatures. **4** Thermal properties are constant. **5** Negligible contact resistance.

Properties The thermal conductivity for the pipe wall is k = 12 W/m·K. The emissivity of the outer pipe surface is 0.3.

Analysis The inner and outer surface areas of the pipe are

$$A_1 = \pi D_1 L = \pi(0.022 \text{ m})(1 \text{ m}) = 0.06912 \text{ m}^2$$

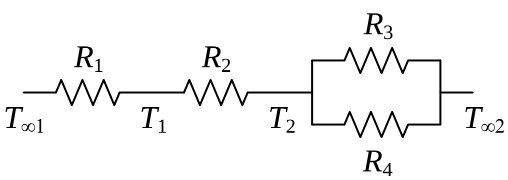
$$A_2 = \pi D_2 L = \pi(0.032 \text{ m})(1 \text{ m}) = 0.10053 \text{ m}^2$$

The radiation heat transfer coefficient is

$$h_{\text{rad}} = \varepsilon\sigma(T_2^2 + T_{\text{surr}}^2)(T_2 + T_{\text{surr}})$$

$$h_{\text{rad}} = (0.3)\left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}\right)[(85+273)^2 + (100+273)^2](\text{K}^2)[(85+273)+(100+273)](\text{K})$$
$$= 3.3236 \text{ W/m}^2 \cdot \text{K}$$

The thermal circuit network between $T_{\infty 1}$ and $T_{\infty 2}$ is



$$R_1 = \frac{1}{h_1 A_1} = \frac{1}{(50 \text{ W/m}^2 \cdot \text{K})(0.06912 \text{ m}^2)} = 0.2894 \text{ K/W}$$

$$R_2 = \frac{\ln(D_2/D_1)}{2\pi k L} = \frac{\ln(32 \text{ mm}/22 \text{ mm})}{2\pi(12 \text{ W/m} \cdot \text{K})(1 \text{ m})} = 0.00497 \text{ K/W}$$

$$R_3 = \frac{1}{h_2 A_2} = \frac{1}{(5 \text{ W/m}^2 \cdot \text{K})(0.10053 \text{ m}^2)} = 1.9895 \text{ K/W}$$

$$R_4 = \frac{1}{h_{\text{rad}} A_2} = \frac{1}{(3.3236 \text{ W/m}^2 \cdot \text{K})(0.10053 \text{ m}^2)} = 2.9929 \text{ K/W}$$

The two parallel resistances R_3 and R_4 can be combined as an equivalent resistance R_{equiv}:

$$\frac{1}{R_{\text{equiv}}} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{1.9895 \text{ K/W}} + \frac{1}{2.9929 \text{ K/W}} = 0.83676 \text{ W/K} \text{or} R_{\text{equiv}} = 1.1951 \text{ K/W}$$

The total resistance for the thermal network is

$$R_{\text{tot}} = R_1 + R_2 + R_{\text{equiv}} = 1.4895 \text{ K/W}$$

The heat transfer rate through R_{tot} can be determined using the given T_2 and $T_{\infty 2}$:

$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_{\text{equiv}}} = \frac{(85-100)\text{K}}{1.1951 \text{ K/W}} = -12.551 \text{ W}$$

(Note the negative sign indicates the heat flows from $T_{\infty 2}$ to $T_{\infty 1}$.)

The temperature of the liquid $T_{\infty 1}$ in the pipe is

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{tot}}} \qquad \rightarrow \qquad T_{\infty 1} = T_{\infty 2} + \dot{Q} R_{\text{tot}} = 100°\text{C} + (-12.551 \text{ W})(1.4895 \text{ K/W}) = \mathbf{81.3}°\text{C}$$

The temperature at the lining T_1 is

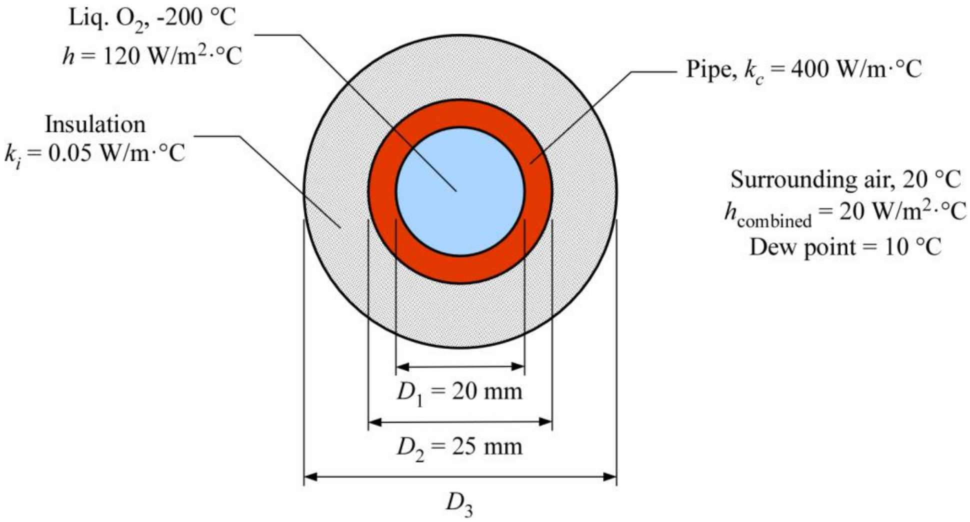
$$\dot{Q} = \frac{T_1 - T_2}{R_2} \qquad \rightarrow \qquad T_1 = T_2 + \dot{Q} R_2 = 85°\text{C} + (-12.551 \text{ W})(0.00497 \text{ K/W}) = \mathbf{84.9}°\text{C}$$

DiscussionThe temperature at the lining (T_1 = 84.9°C) exceeds the maximum temperature (79°C) recommended by the ASME Process Piping code for PVDC lining. A different thermoplastic lining should be used. Polyvinylidene fluoride (PVDF) lining has a recommended maximum temperature of 135°C by the ASME Code for Process Piping (ASME B31.3-2014, A323), which would comply with the code.

3-96 To avoid condensation on the outer surface, the necessary thickness of the insulation around a copper pipe that carries liquid oxygen is to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Thermal contact resistance is negligible.

Properties The thermal conductivities of the copper pipe and the insulation are given to be 400 W/m · °C and 0.05 W/m · °C, respectively.



Analysis From energy balance and using the thermal resistance concept, the following equation is expressed:

$$\frac{T_{\infty,o}-T_{\infty,i}}{R_{\text{combined}}+R_{\text{cond,i}}+R_{\text{cond,c}}+R_{\text{conv}}}=\frac{T_{\infty,o}-T_s}{R_{\text{combined}}}$$

$$\frac{T_{\infty,o}-T_{\infty,i}}{\dfrac{1}{h_{\text{combined}}A}+\dfrac{\ln(D_3/D_2)}{2\pi k_i L}+\dfrac{\ln(D_2/D_1)}{2\pi k_c L}+\dfrac{1}{hA}}=\frac{T_{\infty,o}-T_s}{\dfrac{1}{h_{\text{combined}}A}}$$

$$\frac{T_{\infty,o}-T_{\infty,i}}{\dfrac{1}{h_{\text{combined}}\pi D_3 L}+\dfrac{\ln(D_3/D_2)}{2\pi k_i L}+\dfrac{\ln(D_2/D_1)}{2\pi k_c L}+\dfrac{1}{h\pi D_1 L}}=\frac{T_{\infty,o}-T_s}{\dfrac{1}{h_{\text{combined}}\pi D_3 L}}$$

Rearranging yields

$$\frac{T_{\infty,o}-T_{\infty,i}}{T_{\infty,o}-T_s}=1+h_{\text{combined}}D_3\left[\frac{\ln(D_3/D_2)}{2k_i}+\frac{\ln(D_2/D_1)}{2k_c}+\frac{1}{hD_1}\right]$$

$$\frac{(20+200)\ {}^\circ\text{C}}{(20-10)\ {}^\circ\text{C}}=1+(20\ \text{W/m}^2\cdot{}^\circ\text{C})D_3\left[\frac{\ln(D_3/0.025\ \text{m})}{2(0.05\ \text{W/m}\cdot{}^\circ\text{C})}\right.$$
$$\left.+\frac{\ln(0.025\ \text{m}/0.020\ \text{m})}{2(400\ \text{W/m}\cdot{}^\circ\text{C})}+\frac{1}{(120\ \text{W/m}^2\cdot{}^\circ\text{C})(0.020\ \text{m})}\right]$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
(20+200)/(20-10)=1+20*D_3*(ln(D_3/25e-3)/(2*0.05)+ln(25/20)/(2*400)+1/(120*20e-3))
```

Solving by EES software, the outer diameter of the insulation is

$$D_3=0.0839\ \text{m}$$

The thickness of the insulation necessary to avoid condensation on the outer surface is

$$t>\frac{D_3-D_2}{2}=\frac{0.0839\ \text{m}-0.025\ \text{m}}{2}=\mathbf{0.0295\ m}$$

Discussion If the insulation thickness is less than 29.5 mm, the outer surface temperature would decrease to the dew point at 10 °C where condensation would occur.

3-97 Liquid H_2 flows in a pipe, which is insulated. The insulation thickness on the pipe that is necessary to keep the liquid H_2 temperature below −300°C is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities of the pipe and the insulation are given to be k_{pipe} = 23 W/m·K and k_{ins} = 0.6 W/m·K, respectively.

Analysis The thermal resistances of different layers are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{h_i \pi D_1 L} \quad \text{(liq. H}_2\text{ convection resistance)}$$

$$R_{\text{pipe}} = \frac{\ln(D_2/D_1)}{2\pi k_{\text{pipe}} L} \quad \text{(pipe layer resistance)}$$

$$R_{\text{ins}} = \frac{\ln(D_3/D_2)}{2\pi k_{\text{ins}} L} \quad \text{(insulation layer resistance)}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o \pi D_3 L} \quad \text{(ambient air convection resistance)}$$

The total thermal resistance and the rate of heat transfer are

$$R_{\text{total}} = R_i + R_{\text{pipe}} + R_{\text{ins}} + R_o \text{ and } \dot{Q} = \frac{T_o - T_i}{R_{\text{total}}} = \frac{T_o - T_3}{R_o}$$

and the insulation thickness is

$$t = \frac{D_3 - D_2}{2}$$

Solving for the insulation thickness yields $t = 0.051\,\text{m} = \mathbf{5.1\,cm}$

Solved by EES Software. Copy-and-paste the following lines on a blank EES screen to verify the solutions.

```
"GIVEN"
h_i=200 [W/m^2-K]   "liq. H2 convection heat transfer coefficient"
h_o=50 [W/m^2-K]   "ambient air convection heat transfer coefficient"
k_pipe=23 [W/m-K]   "pipe thermal conductivity"
k_ins=0.6 [W/m-K]   "insulation thermal conductivity"
L=20 [m]   "pipe length"
D_1=0.03 [m] "inner pipe diameter"
D_2=0.04 [m]   "outer pipe diameter"
T_3=5 [C]   "outer insulation surface temperature"
T_i=-300 [C]   "liq. NH3 temperature"
T_o=40 [C]   "ambient air temperature"
"THERMAL RESISTANCES"
R_i=1/(h_i*pi*D_1*L)   "liq. H2 convection resistance"
R_pipe=ln(D_2/D_1)/(2*pi*k_pipe*L)   "pipe layer resistance"
R_ins=ln(D_3/D_2)/(2*pi*k_ins*L)   "insulation layer resistance"
R_o=1/(h_o*pi*D_3*L)   "ambient air convection resistance"
R_total=R_i+R_pipe+R_ins+R_o
"SOLVING FOR THE INSULATION THICKNESS"
Q_dot=(T_o-T_i)/(R_total)
Q_dot=(T_o-T_3)/(R_o)
t=(D_3-D_2)/2
```

Discussion To keep the liquid H_2 below −300°C, the pipe insulation thickness must be at least 5.1 cm thick.

3-98 Liquid NH_3 flows in a pipe, which is insulated. The insulation thickness on the pipe that is necessary to keep the liquid NH_3 temperature below −35°C is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities of the pipe and the insulation are given to be k_{pipe} = 25 W/m·K and k_{ins} = 0.75 W/m·K, respectively.

Analysis The thermal resistances of different layers are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{h_i \pi D_1 L} \quad \text{(liq. NH}_3\text{ convection resistance)}$$

$$R_{\text{pipe}} = \frac{\ln(D_2/D_1)}{2\pi k_{\text{pipe}} L} \quad \text{(pipe layer resistance)}$$

$$R_{\text{ins}} = \frac{\ln(D_3/D_2)}{2\pi k_{\text{ins}} L} \quad \text{(insulation layer resistance)}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o \pi D_3 L} \quad \text{(ambient air convection resistance)}$$

The total thermal resistance and the rate of heat transfer are

$$R_{\text{total}} = R_i + R_{\text{pipe}} + R_{\text{ins}} + R_o \text{ and } \dot{Q} = \frac{T_o - T_i}{R_{\text{total}}} = \frac{T_o - T_3}{R_o}$$

and the insulation thickness is

$$t = \frac{D_3 - D_2}{2}$$

Solving for the insulation thickness yields $t = 0.063\text{ m} = \mathbf{6.3\,cm}$

Solved by EES Software. Copy-and-paste the following lines on a blank EES screen to verify the solutions.

```
"GIVEN"
h_i=100 [W/m^2-K]   "liq. NH3 convection heat transfer coefficient"
h_o=20 [W/m^2-K]   "ambient air convection heat transfer coefficient"
k_pipe=25 [W/m-K]   "pipe thermal conductivity"
k_ins=0.75 [W/m-K]   "insulation thermal conductivity"
L=10 [m]   "pipe length"
D_1=0.025 [m]   "inner pipe diameter"
D_2=0.04 [m]   "outer pipe diameter"
T_3=10 [C]   "outer insulation surface temperature"
T_i=-35 [C]   "liq. NH3 temperature"
T_o=20 [C]   "ambient air temperature"
"THERMAL RESISTANCES"
R_i=1/(h_i*pi*D_1*L)   "liq. NH3 convection resistance"
R_pipe=ln(D_2/D_1)/(2*pi*k_pipe*L)   "pipe layer resistance"
R_ins=ln(D_3/D_2)/(2*pi*k_ins*L)   "insulation layer resistance"
R_o=1/(h_o*pi*D_3*L)   "ambient air convection resistance"
R_total=R_i+R_pipe+R_ins+R_o
"SOLVING FOR THE INSULATION THICKNESS"
Q_dot=(T_o-T_i)/(R_total)
Q_dot=(T_o-T_3)/(R_o)
t=(D_3-D_2)/2
```

Discussion To keep the liquid NH_3 below −35°C, the pipe insulation thickness must be at least 6.3 cm thick.

3-99 A mixture of chemicals undergoing exothermic reaction is being transported in a pipe. The insulation thermal conductivity on the pipe that is necessary to keep the outer surface at 45°C or lower is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities of the pipe is given to be k_{pipe} = 14 W/m·K.

Analysis The thermal resistances of different layers are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{h_i \pi D_1 L} \quad \text{(mixture convection resistance)}$$

$$R_{\text{pipe}} = \frac{\ln(D_2 / D_1)}{2\pi k_{\text{pipe}} L} \quad \text{(pipe layer resistance)}$$

$$R_{\text{ins}} = \frac{\ln(D_3 / D_2)}{2\pi k_{\text{ins}} L} \quad \text{(insulation layer resistance)}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o \pi D_3 L} \quad \text{(ambient air convection resistance)}$$

where, $D_3 = 2t + D_2$

the rate of heat transfer is

$$\dot{Q} = \frac{T_i - T_3}{R_i + R_{\text{pipe}} + R_{\text{ins}}} = \frac{T_3 - T_O}{R_o}$$

Solving for the insulation thermal conductivity yields $k_{\text{ins}} = \mathbf{0.321\ W/m \cdot K}$

Solved by EES Software. Copy-and-paste the following lines on a blank EES screen to verify the solutions.

```
"GIVEN"
h_i=150 [W/m^2-K]   "mix.convection heat transfer coefficient"
h_o=25 [W/m^2-K]   "ambient air convection heat transfer coefficient"
k_pipe=14 [W/m-K]   "pipe thermal conductivity"
L=10 [m]   "pipe length"
D_1=0.025 [m]   "inner pipe diameter"
D_2=0.03 [m]   "outer pipe diameter"
D_3=0.08 [m]
T_3=45 [C]   "outer insulation surface temperature"
T_i=135 [C]   "avg. mix.temperature"
T_o=20 [C]   "ambient air temperature"
"THERMAL RESISTANCES"
R_i=1/(h_i*pi*D_1*L)   "mix.convection resistance"
R_pipe=ln(D_2/D_1)/(2*pi*k_pipe*L)   "pipe layer resistance"
R_ins=ln(D_3/D_2)/(2*pi*k_ins*L)   "insulation layer resistance"
R_o=1/(h_o*pi*D_3*L)   "ambient air convection resistance"
"SOLVING FOR THE INSULATION THICKNESS"
Q_dot=(T_i-T_3)/(R_i+R_pipe+R_ins)
Q_dot=(T_3-T_o)/(R_o)
```

Discussion To keep the outer surface below 45°C, the thermal conductivity of the pipe insulation must be 0.321 W/m·K or lower.

3-100 Ice slurry is being transported in an insulated pipe. The insulation thickness on the pipe that is necessary to prevent condensation on the outer surface is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities of the pipe and the insulation are given to be k_{pipe} = 15 W/m·K and k_{ins} = 0.95 W/m·K, respectively.

Analysis The thermal resistances of different layers are

$$R_{\text{pipe}} = \frac{\ln(D_2/D_1)}{2\pi k_{\text{pipe}} L} \quad \text{(pipe layer resistance)}$$

$$R_{\text{ins}} = \frac{\ln(D_3/D_2)}{2\pi k_{\text{ins}} L} \quad \text{(insulation layer resistance)}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{h_o \pi D_3 L} \quad \text{(ambient air convection resistance)}$$

The total thermal resistance and the rate of heat transfer are

$$R_{\text{total}} = R_{\text{pipe}} + R_{\text{ins}} + R_o \text{ and } \dot{Q} = \frac{T_o - T_1}{R_{\text{total}}} = \frac{T_o - T_3}{R_o}$$

and the insulation thickness is

$$t = \frac{D_3 - D_2}{2}$$

Solving for the insulation thickness yields $t = 0.0496\,\text{m} = \mathbf{5.0\,cm}$

Solved by EES Software. Copy-and-paste the following lines on a blank EES screen to verify the solutions.

```
"GIVEN"
h_o=10 [W/m^2-K]  "ambient air convection heat transfer coefficient"
k_pipe=15 [W/m-K]  "pipe thermal conductivity"
k_ins=0.95 [W/m-K]  "insulation thermal conductivity"
L=5 [m]  "pipe length"
D_1=0.025 [m]  "inner pipe diameter"
D_2=0.03 [m]  "outer pipe diameter"
T_1=0 [C]  "inner pipe surface temperature"
T_3=10 [C]  "outer insulation surface temperature"
T_o=20 [C]  "ambient air temperature"
"THERMAL RESISTANCES"
"R_i=1/(h_i*PI*D_1*L)""liq. NH3 convection resistance"
R_pipe=ln(D_2/D_1)/(2*pi*k_pipe*L)  "pipe layer resistance"
R_ins=ln(D_3/D_2)/(2*pi*k_ins*L)  "insulation layer resistance"
R_o=1/(h_o*pi*D_3*L)  "ambient air convection resistance"
R_total=R_pipe+R_ins+R_o
"SOLVING FOR THE INSULATION THICKNESS"
Q_dot=(T_o-T_1)/(R_total)
Q_dot=(T_o-T_3)/(R_o)
t=(D_3-D_2)/2
```

Discussion To keep the outer surface temperature from dropping below the dew point at 10°C, the insulation thickness needs to be 5 cm or more.

3-101 A spherical container filled with iced water is subjected to convection and radiation heat transfer at its outer surface. The rate of heat transfer and the amount of ice that melts per day are to be determined.

Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal conductivity is constant.

Properties The thermal conductivity of steel is given to be k = 15 W/m·°C. The heat of fusion of water at 1 atm is $h_{if} = 333.7\text{ kJ/kg}$. The outer surface of the tank is black and thus its emissivity is ε = 1.

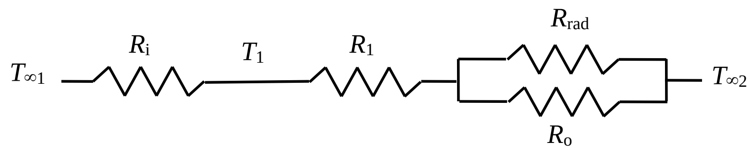
Analysis (a) The inner and the outer surface areas of sphere are

$$A_i = \pi D_i^{\,2} = \pi(8\text{ m})^2 = 201.06\text{ m}^2$$
$$A_o = \pi D_o^{\,2} = \pi(8.03\text{ m})^2 = 202.57\text{ m}^2$$

We assume the outer surface temperature T_2 to be 5°C after comparing convection heat transfer coefficients at the inner and the outer surfaces of the tank. With this assumption, the radiation heat transfer coefficient can be determined from

$$\begin{aligned} h_{rad} &= \varepsilon\sigma(T_2^{\,2} + T_{surr}^{\,2})(T_2 + T_{surr}) \\ &= 1(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(273+5\text{ K})^2 + (273+25\text{ K})^2](273+25\text{ K})(273+5\text{ K})] \\ &= 5.424\text{ W/m}^2.\text{K} \end{aligned}$$

The individual thermal resistances are



$$R_{conv,i} = \frac{1}{h_i A} = \frac{1}{(80\text{ W/m}^2\cdot°\text{C})(201.06\text{ m}^2)} = 0.000062\text{ °C/W}$$

$$R_1 = R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(4.015-4.0)\text{ m}}{4\pi(15\text{ W/m}\cdot°\text{C})(4.015\text{ m})(4.0\text{ m})} = 0.000005\text{ °C/W}$$

$$R_{conv,o} = \frac{1}{h_o A} = \frac{1}{(10\text{ W/m}^2\cdot°\text{C})(202.57\text{ m}^2)} = 0.000494\text{ °C/W}$$

$$R_{rad} = \frac{1}{h_{rad} A} = \frac{1}{(5.424\text{ W/m}^2\cdot°\text{C})(202.57\text{ m}^2)} = 0.000910\text{ °C/W}$$

$$\frac{1}{R_{eqv}} = \frac{1}{R_{conv,o}} + \frac{1}{R_{rad}} = \frac{1}{0.000494} + \frac{1}{0.000910} \longrightarrow R_{eqv} = 0.000320\text{ °C/W}$$

$$R_{total} = R_{conv,i} + R_1 + R_{eqv} = 0.000062 + 0.000005 + 0.000320 = 0.000387\text{ °C/W}$$

Then the steady rate of heat transfer to the iced water becomes

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{(25-0)°\text{C}}{0.000387\text{ °C/W}} = \mathbf{64{,}600\text{ W}}$$

(b) The total amount of heat transfer during a 24-hour period and the amount of ice that will melt during this period are

$$Q = \dot{Q}\Delta t = (64.600\text{ kJ/s})(24\times3600\text{ s}) = 5.581\times10^6\text{ kJ}$$

$$m_{ice} = \frac{Q}{h_{if}} = \frac{5.581\times10^6\text{ kJ}}{333.7\text{ kJ/kg}} = \mathbf{16{,}730\,kg}$$

Check: The outer surface temperature of the tank is

$$\dot{Q} = h_{conv+rad} A_o (T_{\infty1} - T_s)$$
$$\rightarrow T_s = T_{\infty1} - \frac{\dot{Q}}{h_{conv+rad} A_o} = 25°\text{C} - \frac{64{,}600\text{ W}}{(10+5.424\text{ W/m}^2\cdot°\text{C})(202.57\text{ m}^2)} = 4.3°\text{C}$$

which is very close to the assumed temperature of 5°C for the outer surface temperature used in the evaluation of the radiation heat transfer coefficient. Therefore, there is no need to repeat the calculations.

3-102 A spherical tank filled with liquid nitrogen at 1 atm and -196°C is exposed to convection and radiation with the surrounding air and surfaces. The rate of evaporation of liquid nitrogen in the tank as a result of the heat gain from the surroundings for the cases of no insulation, 5-cm thick fiberglass insulation, and 2-cm thick superinsulation are to be determined.

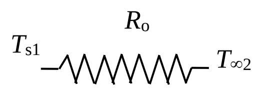
Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** The combined heat transfer coefficient is constant and uniform over the entire surface. **4** The temperature of the thin-shelled spherical tank is said to be nearly equal to the temperature of the nitrogen inside, and thus thermal resistance of the tank and the internal convection resistance are negligible.

Properties The heat of vaporization and density of liquid nitrogen at 1 atm are given to be 198 kJ/kg and 810 kg/m^3, respectively. The thermal conductivities are given to be k = 0.035 W/m·°C for fiberglass insulation and k = 0.00005 W/m·°C for super insulation.

Analysis (*a*)The heat transfer rate and the rate of evaporation of the liquid without insulation are

$$A = \pi D^2 = \pi(3\,\text{m})^2 = 28.27\,\text{m}^2$$

$$R_o = \frac{1}{h_o A} = \frac{1}{(35\ \text{W/m}^2.°\text{C})(28.27\,\text{m}^2)} = 0.00101\,°\text{C/W}$$

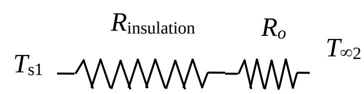


$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_o} = \frac{[15-(-196)]°\text{C}}{0.00101°\text{C/W}} = 208{,}910\ \text{W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{208.910\,\text{kJ/s}}{198\,\text{kJ/kg}} = \mathbf{1.055\,kg/s}$$

(*b*)The heat transfer rate and the rate of evaporation of the liquid with a 5-cm thick layer of fiberglass insulation are

$$A = \pi D^2 = \pi(3.1\,\text{m})^2 = 30.19\,\text{m}^2$$



$$R_o = \frac{1}{h_o A} = \frac{1}{(35\ \text{W/m}^2.°\text{C})(30.19\,\text{m}^2)} = 0.000946\,°\text{C/W}$$

$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.55-1.5)\ \text{m}}{4\pi(0.035\ \text{W/m.°C})(1.55\,\text{m})(1.5\,\text{m})} = 0.0489\,°\text{C/W}$$

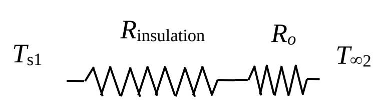
$$R_{total} = R_o + R_{insulation} = 0.000946 + 0.0489 = 0.0498\,°\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-196)]°\text{C}}{0.0498\,°\text{C/W}} = 4233\ \text{W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{4.233\,\text{kJ/s}}{198\,\text{kJ/kg}} = \mathbf{0.0214\,kg/s}$$

(*c*)The heat transfer rate and the rate of evaporation of the liquid with 2-cm thick layer of superinsulation is

$$A = \pi D^2 = \pi(3.04\,\text{m})^2 = 29.03\,\text{m}^2$$



$$R_o = \frac{1}{h_o A} = \frac{1}{(35\ \text{W/m}^2.°\text{C})(29.03\,\text{m}^2)} = 0.000984\,°\text{C/W}$$

$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52-1.5)\ \text{m}}{4\pi(0.00005\ \text{W/m.°C})(1.52\,\text{m})(1.5\,\text{m})} = 13.96\,°\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.000984 + 13.96 = 13.96\,°\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-196)]°\text{C}}{13.96\,°\text{C/W}} = 15.11\,\text{W}$$

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{0.01511\,\text{kJ/s}}{198\,\text{kJ/kg}} = \mathbf{0.000076\,kg/s}$$

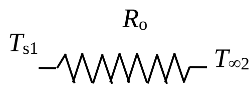
3-103 A spherical tank filled with liquid oxygen at 1 atm and -183°C is exposed to convection and radiation with the surrounding air and surfaces. The rate of evaporation of liquid oxygen in the tank as a result of the heat gain from the surroundings for the cases of no insulation, 5-cm thick fiberglass insulation, and 2-cm thick superinsulation are to be determined.

Assumptions **1** Heat transfer is steady since the specified thermal conditions at the boundaries do not change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** The combined heat transfer coefficient is constant and uniform over the entire surface. **4** The temperature of the thin-shelled spherical tank is said to be nearly equal to the temperature of the oxygen inside, and thus thermal resistance of the tank and the internal convection resistance are negligible.

Properties The heat of vaporization and density of liquid oxygen at 1 atm are given to be 213 kJ/kg and 1140 kg/m^3, respectively. The thermal conductivities are given to be k = 0.035 W/m·°C for fiberglass insulation and k = 0.00005 W/m·°C for super insulation.

Analysis (*a*)The heat transfer rate and the rate of evaporation of the liquid without insulation are

$$A = \pi D^2 = \pi(3\,\text{m})^2 = 28.27\,\text{m}^2$$



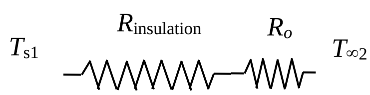
$$R_o = \frac{1}{h_o A} = \frac{1}{(35\ \text{W/m}^2.°\text{C})(28.27\,\text{m}^2)} = 0.00101\,°\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_o} = \frac{[15-(-183)]°\text{C}}{0.00101°\text{C/W}} = 196{,}040\ \text{W}$$

$$\dot{Q} = \dot{m} h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{196.040\,\text{kJ/s}}{213\,\text{kJ/kg}} = \mathbf{0.920\,kg/s}$$

(*b*)The heat transfer rate and the rate of evaporation of the liquid with a 5-cm thick layer of fiberglass insulation are

$$A = \pi D^2 = \pi(3.1\,\text{m})^2 = 30.19\,\text{m}^2$$



$$R_o = \frac{1}{h_o A} = \frac{1}{(35\ \text{W/m}^2.°\text{C})(30.19\,\text{m}^2)} = 0.000946\,°\text{C/W}$$

$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.55-1.5)\ \text{m}}{4\pi(0.035\ \text{W/m.°C})(1.55\,\text{m})(1.5\,\text{m})} = 0.0489\,°\text{C/W}$$

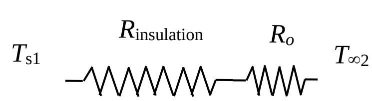
$$R_{total} = R_o + R_{insulation} = 0.000946 + 0.0489 = 0.0498\,°\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-183)]°\text{C}}{0.0498\,°\text{C/W}} = 3976\ \text{W}$$

$$\dot{Q} = \dot{m} h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{3.976\,\text{kJ/s}}{213\,\text{kJ/kg}} = \mathbf{0.0187\,kg/s}$$

(*c*)The heat transfer rate and the rate of evaporation of the liquid with a 2-cm superinsulation is

$$A = \pi D^2 = \pi(3.04\,\text{m})^2 = 29.03\,\text{m}^2$$



$$R_o = \frac{1}{h_o A} = \frac{1}{(35\ \text{W/m}^2.°\text{C})(29.03\,\text{m}^2)} = 0.000984\,°\text{C/W}$$

$$R_{insulation} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(1.52-1.5)\ \text{m}}{4\pi(0.00005\ \text{W/m.°C})(1.52\,\text{m})(1.5\,\text{m})} = 13.96\,°\text{C/W}$$

$$R_{total} = R_o + R_{insulation} = 0.000984 + 13.96 = 13.96\,°\text{C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{total}} = \frac{[15-(-183)]°\text{C}}{13.96\,°\text{C/W}} = 14.18\ \text{W}$$

$$\dot{Q} = \dot{m} h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{0.01418\,\text{kJ/s}}{213\,\text{kJ/kg}} = \mathbf{0.000067\,kg/s}$$

Critical Radius of Insulation

3-104C In a cylindrical pipe or a spherical shell, the additional insulation increases the conduction resistance of insulation, but decreases the convection resistance of the surface because of the increase in the outer surface area. Due to these opposite effects, a critical radius of insulation is defined as the outer radius that provides maximum rate of heat transfer. For a cylindrical layer, it is defined as $r_{cr} = k / h$ where k is the thermal conductivity of insulation and h is the external convection heat transfer coefficient.

3-105C For a cylindrical pipe, the critical radius of insulation is defined as $r_{cr} = k / h$. On windy days, the external convection heat transfer coefficient is greater compared to calm days. Therefore critical radius of insulation will be greater on calm days.

3-106C Yes, the measurements can be right. If the radius of insulation is less than critical radius of insulation of the pipe, the rate of heat loss will increase.

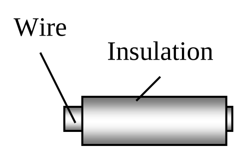
3-107C No.

3-108C It will decrease.

3-109E An electrical wire is covered with 0.02-in thick plastic insulation. It is to be determined if the plastic insulation on the wire will increase or decrease heat transfer from the wire.

Assumptions **1** Heat transfer from the wire is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity of plastic cover is given to be k = 0.075 Btu/h·ft·°F.



Analysis The critical radius of plastic insulation is

$$r_{cr} = \frac{k}{h} = \frac{0.075 \text{ Btu/h.ft.°F}}{2.5 \text{ Btu/h.ft}^2\text{.°F}} = 0.03 \text{ ft} = 0.36 \text{ in} > r_2 (= 0.0615 \text{ in})$$

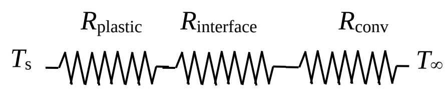
Since the outer radius of the wire with insulation is smaller than critical radius of insulation, plastic insulation will **increase** heat transfer from the wire.

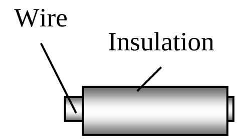
3-110E An electrical wire is covered with 0.02-in thick plastic insulation. By considering the effect of thermal contact resistance, it is to be determined if the plastic insulation on the wire will increase or decrease heat transfer from the wire.

Assumptions **1** Heat transfer from the wire is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties are constant

Properties The thermal conductivity of plastic cover is given to be k = 0.075 Btu/h·ft·°F.

Analysis Without insulation, the total thermal resistance is (per ft length of the wire)





$$R_{tot} = R_{conv} = \frac{1}{h_o A_o} = \frac{1}{(2.5\text{ Btu/h.ft}^2.°\text{F})[\pi(0.083/12\text{ ft})(1\text{ft})]} = 18.4\text{ h.°F/Btu}$$

With insulation, the total thermal resistance is

$$R_{conv} = \frac{1}{h_o A_o} = \frac{1}{(2.5\text{ Btu/h.ft}^2.°\text{F})[\pi(0.123/12\text{ ft})(1\text{ft})]} = 12.42\text{ h.°F/Btu}$$

$$R_{plastic} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(0.123/0.083)}{2\pi(0.075\text{ Btu/h.ft.°F})(1\text{ ft})} = 0.835\text{ h.°F/Btu}$$

$$R_{interface} = \frac{h_c}{A_c} = \frac{0.001\text{ h.ft}^2.°\text{F/Btu}}{[\pi(0.083/12\text{ ft})(1\text{ft})]} = 0.046\text{ h.°F/Btu}$$

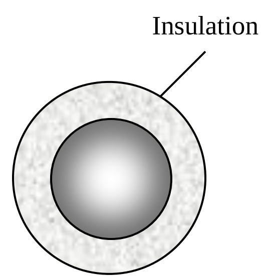
$$R_{total} = R_{conv} + R_{plastic} + R_{interface} = 12.42 + 0.835 + 0.046 = 13.30\text{ h.°F/Btu}$$

Since the total thermal resistance decreases after insulation, plastic insulation **will increase** heat transfer from the wire. The thermal contact resistance appears to have negligible effect in this case.

3-111 A spherical ball is covered with 1-mm thick plastic insulation. It is to be determined if the plastic insulation on the ball will increase or decrease heat transfer from it.

Assumptions **1** Heat transfer from the ball is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the midpoint. **3** Thermal properties are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivity of plastic cover is given to be k = 0.13 W/m·°C.



Analysis The critical radius of plastic insulation for the spherical ball is

$$r_{cr} = \frac{2k}{h} = \frac{2(0.13\text{ W/m.°C})}{20\text{ W/m}^2.°\text{C}} = 0.013\text{ m} = 13\text{ mm} > r_2 (= 3\text{ mm})$$

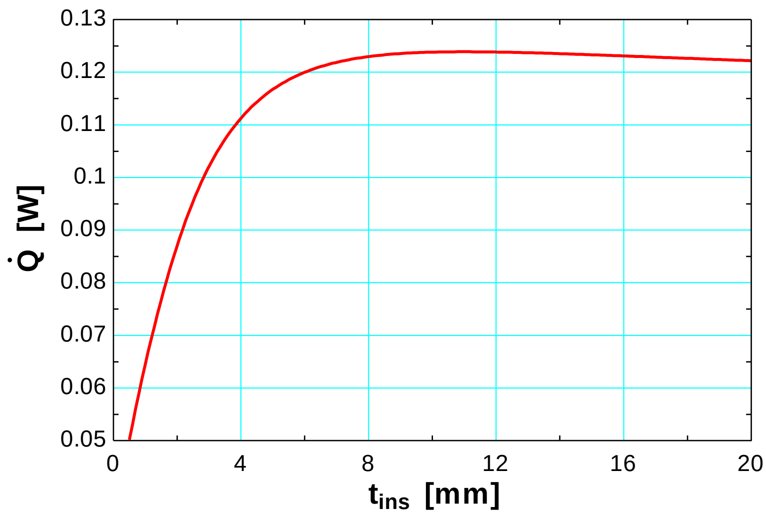
Since the outer radius of the ball with insulation is smaller than critical radius of insulation, plastic insulation will **increase** heat transfer from the wire.

3-112 Prob. 3-111 is reconsidered. The rate of heat transfer from the ball as a function of the plastic insulation thickness is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D_1=0.004 [m]
t_ins=1 [mm]
k_ins=0.13 [W/m-C]
T_ball=50 [C]
T_infinity=15 [C]
h_o=20 [W/m^2-C]

"ANALYSIS"
D_2=D_1+2*t_ins*Convert(mm, m)
A_o=pi*D_2^2
R_conv_o=1/(h_o*A_o)
R_ins=(r_2-r_1)/(4*pi*r_1*r_2*k_ins)
r_1=D_1/2
r_2=D_2/2
R_total=R_conv_o+R_ins
Q_dot=(T_ball-T_infinity)/R_total
```

t_{ins} [mm]	Q [W]
0.5	0.05016
1.526	0.07736
2.553	0.09626
3.579	0.108
4.605	0.1149
5.632	0.119
6.658	0.1213
7.684	0.1227
8.711	0.1234
9.737	0.1238
10.76	0.1239
11.79	0.1238
12.82	0.1237
13.84	0.1236
14.87	0.1233
15.89	0.1231
16.92	0.1229
17.95	0.1226
18.97	0.1224
20	0.1222



Heat Transfer from Finned Surfaces

3-113C Fins should be attached to the outside since the heat transfer coefficient inside the tube will be higher due to forced convection. Fins should be added to both sides of the tubes when the convection coefficients at the inner and outer surfaces are comparable in magnitude.

3-114C Increasing the rate of heat transfer from a surface by increasing the heat transfer surface area.

3-115C The fin efficiency is defined as the ratio of actual heat transfer rate from the fin to the ideal heat transfer rate from the fin if the entire fin were at base temperature, and its value is between 0 and 1. Fin effectiveness is defined as the ratio of heat transfer rate from a finned surface to the heat transfer rate from the same surface if there were no fins, and its value is expected to be greater than 1.

3-116C Heat transfer rate will decrease since a fin effectiveness smaller than 1 indicates that the fin acts as insulation.

3-117C Fins enhance heat transfer from a surface by increasing heat transfer surface area for convection heat transfer. However, adding too many fins on a surface can suffocate the fluid and retard convection, and thus it may cause the overall heat transfer coefficient and heat transfer to decrease.

3-118C Effectiveness of a single fin is the ratio of the heat transfer rate from the entire exposed surface of the fin to the heat transfer rate from the fin base area. The overall effectiveness of a finned surface is defined as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins.

3-119C Fins should be attached on the air side since the convection heat transfer coefficient is lower on the air side than it is on the water side.

3-120C Welding or tight fitting introduces thermal contact resistance at the interface, and thus retards heat transfer. Therefore, the fins formed by casting or extrusion will provide greater enhancement in heat transfer.

3-121C If the fin is too long, the temperature of the fin tip will approach the surrounding temperature and we can neglect heat transfer from the fin tip. Also, if the surface area of the fin tip is very small compared to the total surface area of the fin, heat transfer from the tip can again be neglected.

3-122C Increasing the length of a fin decreases its efficiency but increases its effectiveness.

3-123C Increasing the diameter of a fin increases its efficiency but decreases its effectiveness.

3-124C The thicker fin has higher efficiency; the thinner one has higher effectiveness.

3-125C The fin with the lower heat transfer coefficient has the higher efficiency and the higher effectiveness.

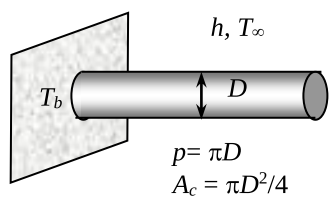
3-126 A relation is to be obtained for the fin efficiency for a fin of constant cross-sectional area A_c, perimeter *p*, length *L*, and thermal conductivity *k* exposed to convection to a medium at T_∞ with a heat transfer coefficient *h*. The relation is to be simplified for circular fin of diameter *D* and for a rectangular fin of thickness *t*.

Assumptions **1**T fins are sufficiently long so that the temperature of the fin at the tip is nearly T_∞. **2** Heat transfer from the fin tips is negligible.

Analysis Taking the temperature of the fin at the base to be T_b and using the heat transfer relation for a long fin, fin efficiency for long fins can be expressed as

$$\eta_{\text{fin}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if the entire fin were at base temperature}}$$

$$= \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)}{hA_{fin}(T_b - T_\infty)} = \frac{\sqrt{hpkA_c}}{hpL} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}}$$



This relation can be simplified for a circular fin of diameter *D* and rectangular fin of thickness t and width w to be

$$\eta_{\text{fin, circular}} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}} = \frac{1}{L}\sqrt{\frac{k(\pi D^2/4)}{(\pi D)h}} = \frac{1}{2L}\sqrt{\frac{kD}{h}}$$

$$\eta_{\text{fin, rectangular}} = \frac{1}{L}\sqrt{\frac{kA_c}{ph}} = \frac{1}{L}\sqrt{\frac{k(wt)}{2(w+t)h}} \cong \frac{1}{L}\sqrt{\frac{k(wt)}{2wh}} = \frac{1}{L}\sqrt{\frac{kt}{2h}}$$

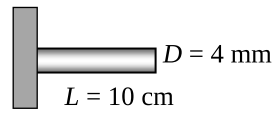
3-127 A fin is attached to a surface. The percent error in the rate of heat transfer from the fin when the infinitely long fin assumption is used instead of the adiabatic fin tip assumption is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** The heat transfer coefficient is constant and uniform over the entire fin surface. **4** The thermal properties of the fins are constant. **5** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the aluminum fin is given to be k = 237 W/m·°C.

Analysis The expressions for the heat transfer from a fin under infinitely long fin and adiabatic fin tip assumptions are

$$\dot{Q}_{\text{long fin}} = \sqrt{hpkA_c}\,(T_b - T_\infty)$$
$$\dot{Q}_{\text{ins. tip}} = \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh(mL)$$



The percent error in using long fin assumption can be expressed as

$$\%\,\text{Error} = \frac{\dot{Q}_{\text{long fin}} - \dot{Q}_{\text{ins. tip}}}{\dot{Q}_{\text{ins. tip}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty) - \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh(mL)}{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh(mL)} = \frac{1}{\tanh(mL)} - 1$$

where

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{(12\text{ W/m}^2.°\text{C})\pi(0.004\text{ m})}{(237\text{ W/m.°C})\pi(0.004\text{ m})^2/4}} = 7.116\text{ m}^{-1}$$

Substituting,

$$\%\,\text{Error} = \frac{1}{\tanh(mL)} - 1 = \frac{1}{\tanh\left[(7.116\text{ m}^{-1})(0.10\text{ m})\right]} - 1 = 0.635 = \mathbf{63.5\%}$$

This result shows that using infinitely long fin assumption may yield results grossly in error.

3-128 A very long fin is attached to a flat surface. The fin temperature at a certain distance from the base and the rate of heat loss from the entire fin are to be determined.

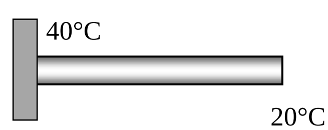
Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** The heat transfer coefficient is constant and uniform over the entire fin surface. **4** The thermal properties of the fins are constant. **5** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the fin is given to be k = 200 W/m·°C.

Analysis The fin temperature at a distance of 5 cm from the base is determined from

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{(20\text{ W/m}^2.°\text{C})(2\times0.05 + 2\times0.001)\text{m}}{(200\text{ W/m.°C})(0.05\times0.001)\text{m}^2}} = 14.3\text{ m}^{-1}$$

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx} \longrightarrow \frac{T - 20}{40 - 20} = e^{-(14.3)(0.05)} \longrightarrow T = \mathbf{29.8°C}$$



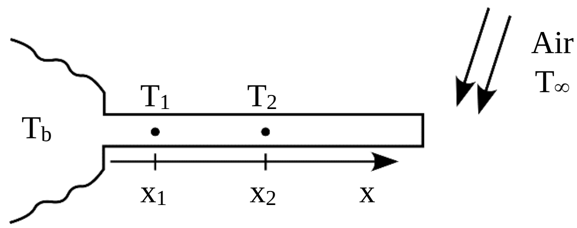
The rate of heat loss from this very long fin is

$$\begin{aligned}\dot{Q}_{\text{long fin}} &= \sqrt{hpkA_c}\,(T_b - T_\infty)\\ &= \sqrt{(20)(2\times0.05 + 2\times0.001)(200(0.05\times0.001)}\,(40 - 20)\\ &= \mathbf{2.9\ W}\end{aligned}$$

3-129 A very long rod has one end maintained at T_b and the other end isexposed to air at 400°C. Thermocouples imbedded in the rod at locations 25 and 120 mm from the base surface register temperatures of 325°C and 375°C, respectively. (a) Calculate the rod base temperature (°C) and (b) Determine the rod length (mm) for the case where the ratio of the heat transfer from a finite length fin to the heat transfer from a very long fin under the same conditions is 99%.

Assumptions **1** Steady-state conditions. **2** Rod is infinitely long with uniform cross-sectional area. **3** Uniform convection coefficient along the rod.

Analysis (a) For an infinitely long fin, use Eq. 3-60



$$\frac{\theta\ x}{\theta_b} = \frac{T\ \ x - T_\infty}{T_b - T_\infty} = e^{-mx}$$

Substitute values for T_1 and T_2 at their respective distances, x_1 and x_2, into Eq. 3-60 and evaluation "m"

$$\frac{\theta(x_1)}{\theta(x_2)} = \frac{T(x_1) - T_\infty}{T(x_2) - T_\infty} = \frac{\theta_b e^{-mx_1}}{\theta_b e^{-mx_2}} = e^{-m(x_1 - x_2)}$$

$$\frac{(325-400)°C}{(375-400)°C} = e^{-m(0.025-0.120)} \qquad \Rightarrow \qquad m = 11.56\ \ 1/\text{meter}$$

Using the value for "m" with Eq. 3-60 at location x_1 or x_2, determine T_b

$$\theta(x_1) = T(x_1) - T_\infty = (T_b - T_\infty)e^{-mx_1}$$

$$(325-400)°C = (T_b - 400)e^{-11.56(0.025)} \Rightarrow \quad T_b = \mathbf{300°C}$$

(b) From Eq. 3-83

$$\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{long fin}}} = \frac{\sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL}{\sqrt{hpkA_c}\,(T_b - T_\infty)} = \tanh mL = 0.99$$

$$\therefore\ mL = 2.65$$

or

$$L = \frac{2.65}{m} = \frac{2.65}{11.56\ \ 1/\text{meter}} = 0.229\ \text{meter} \approx \mathbf{230\,mm}$$

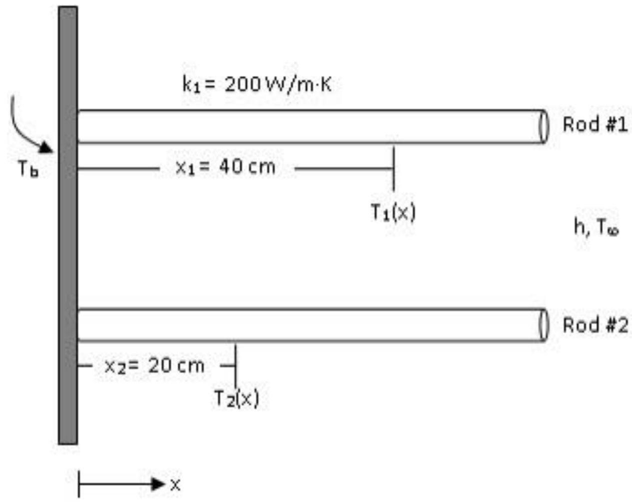
Discussion The infinitely long fin assumption is justified as long as the fin length is ≥ 230 mm. See Table 3-5 for the variation of heat transfer from a fin relative to that from an infinitely long fin.

3-130 Positions of equal temperature on two long rods of the same diameter and length,but different thermal conductivity, which are exposed to the same base temperature and ambient condition.

Assumptions **1** Steady-state conditions. **2** Rods are infinitely long fins of uniform cross-sectional area. **3** Uniform convection coefficient along the rods. **4** Constant properties.

Properties The thermal conductivity for the aluminum rod is k_1 = 200 W/m·K.

Analysis



$$T_1(x)\big|_{x_1=40\text{cm}} = T_2(x)\big|_{x_2=20\text{cm}}$$

For very long fins (case 1), use Eq. 3-60

$$\frac{T(x)-T_\infty}{T_b-T_\infty} = \frac{\theta(x)}{\theta_b} = e^{-mx}$$

where $\quad m = (hP/kA_c)^{1/2}$

$$\theta_1(x_1) = \theta_b e^{-m_1 x_1}$$

$$\theta_2(x_2) = \theta_b e^{-m_2 x_2}$$

We have $\theta_1(0.4\text{m}) = \theta_2(0.2\text{m})$

Then

$$\theta_b e^{-m_1 x_1} = \theta_b e^{-m_2 x_2}$$

$$-m_1 x_1 = -m_2 x_2$$

$$m_1^2 x_1^2 = m_2^2 x_2^2$$

$$\left(\frac{hP}{k_1 A_c}\right)x_1^2 = \left(\frac{hP}{k_2 A_c}\right)x_2^2$$

Solving for

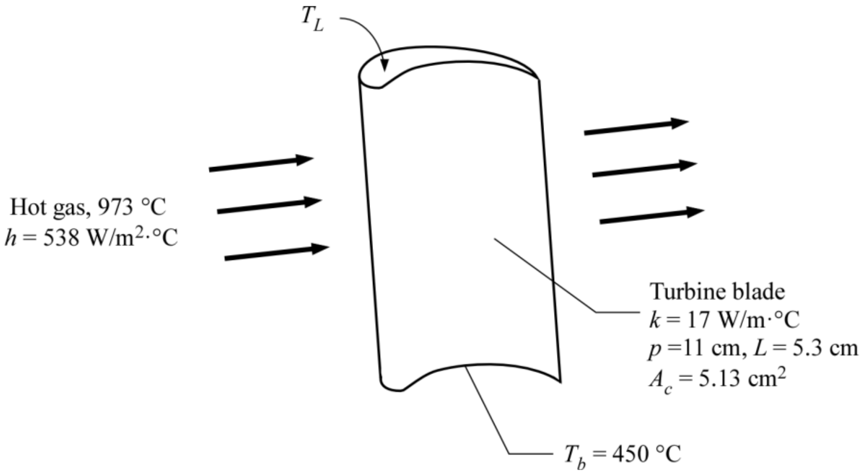
$$k_2 = \frac{x_2^2}{x_1^2}k_1 = \left(\frac{0.2\text{m}}{0.4\text{m}}\right)^2 (200\,\text{W/m}\cdot\text{K}) = \mathbf{50\ W/m\cdot K}$$

Discussion This approach has been used as a method for determining the thermal conductivity. It has the attractive feature of not requiring power or temperature measurements, assuming of course, a reference material of known thermal conductivity is available.

3-131 A turbine blade is exposed to hot gas from the combustion chamber. The heat transfer rate to the turbine blade and the temperature at the tip are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** The cross-sectional area of the turbine blade is uniform.

Properties The thermal conductivity of the turbine blade is given as 17 W/m · °C.



Analysis The turbine blade can be treated as a uniform cross section fin with adiabatic tip. The heat transfer rate to the turbine blade can be expressed as

$$\dot{Q}_{\text{blade}} = \sqrt{hpkA_c}\,(T_\infty - T_b)\tanh mL$$

where

$$mL = \left(\frac{hp}{kA_c}\right)^{0.5} L = \left[\frac{(538\ \text{W/m}^2\cdot{}^\circ\text{C})(0.11\,\text{m})}{(17\ \text{W/m}\cdot{}^\circ\text{C})(5.13\times10^{-4}\ \text{m}^2)}\right]^{0.5}(0.053\,\text{m}) = 4.366$$

$$\sqrt{hpkA_c} = \sqrt{(538\ \text{W/m}^2\cdot{}^\circ\text{C})(0.11\,\text{m})(17\ \text{W/m}\cdot{}^\circ\text{C})(5.13\times10^{-4}\ \text{m}^2)} = 0.7184\ \text{W/}^\circ\text{C}$$

The heat transfer rate to the turbine blade is

$$\dot{Q}_{\text{blade}} = (0.7184\ \text{W/}^\circ\text{C})(973-450)\,{}^\circ\text{C}(\tanh 4.366) = \mathbf{376\ W}$$

For adiabatic tip, the temperature distribution is expressed as

$$\frac{T(x)-T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}$$

The temperature at the tip of the turbine blade is

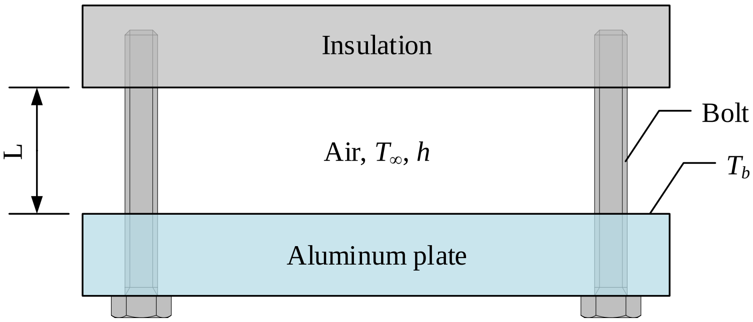
$$T_L = \frac{T_b - T_\infty}{\cosh mL} + T_\infty = \frac{(450-973)\,{}^\circ\text{C}}{\cosh 4.366} + 973\,{}^\circ\text{C} = \mathbf{960\,{}^\circ C}$$

Discussion The tolerance of the turbine blade to high temperature can be increased by applying Zirconia based thermal barrier coatings (TBCs) on the blade surface.

3-132 C&S An ASTM B209 5154 aluminum alloy plate is connected to an insulation plate by long metal bolts. Portion of the bolts are exposed to convection with ambient air. The temperature of the bolt at mid-length is known. The surface temperature at the base T_b and the heat transfer rate from each bolt are to be determined. Is the use of the ASTM B209 5154 plate in compliance with the ASME Code for Process Piping, which has a maximum use temperature of 65°C?

Assumptions **1** Heat transfer is steady. **2** The portion of the bolt exposed to convection behaves as finned surface. **3** The temperature T_b is uniform for the aluminum plate surface and bolts. **4** Thermal properties are constant. **5** The tip of the bolt is adiabatic.

Properties The thermal conductivity for the bolts is 15 W/m·K.



Analysis The bolts can be treated as pin fins of rectangular profile. Since the bolts are attached to an insulation plate, the tip of the bolts can be treated as adiabatic. The equation for an adiabatic fin tip is

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh m(L-x)}{\cosh mL}$$

where

$$x=\frac{L}{2}=\frac{0.05\text{ m}}{2}=0.025\text{ m}$$

and $$m=\sqrt{\frac{4h}{kD}}=\sqrt{\frac{4(20\text{W/m}^2\cdot\text{K})}{(15\text{W/m}\cdot\text{K})(0.0048\text{m})}}=33.333\text{ m}^{-1}$$

The temperature T_b is

$$T_b=\frac{\cosh mL}{\cosh m(L-x)}[T(x)-T_\infty]+T_\infty=\frac{\cosh[(33.333\text{m}^{-1})(0.05\text{m})]}{\cosh[(33.333\text{m}^{-1})(0.05-0.025)(\text{m})]}[50-20]℃+20℃=\mathbf{80.13}℃$$

The rate of heat loss from each bolt to convection is

$$\dot{Q}=\sqrt{hpkA_c}(T_b-T_\infty)\tanh mL=\sqrt{hk\pi^2D^3/4}(T_b-T_\infty)\tanh mL$$

$$\dot{Q}=\sqrt{(20\text{W/m}^2\cdot\text{K})(15\text{W/m}\cdot\text{K})\pi^2(0.0048\text{m})^3/4}(80.13-20)(\text{K})\tanh[(33.333\text{m}^{-1})(0.05\text{m})]=\mathbf{0.5066\text{ W}}$$

Discussion The temperature T_b is assumed uniform for the bolts and the upper surface of the aluminum plate. With the temperature T_b found to be 80.1°C, the aluminum plate surface is operating at higher than the maximum use temperature of 65°C. Therefore, the ASTM B209 5154 plate is not in compliance with the ASME Code for Process Piping.

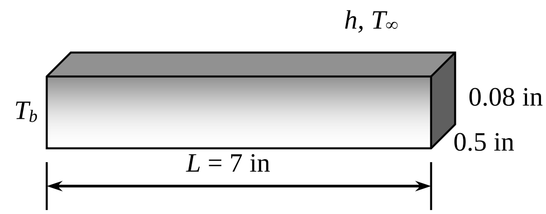
3-133E The handle of a stainless steel spoon partially immersed in boiling water extends 7 in. in the air from the free surface of the water. The temperature difference across the exposed surface of the spoon handle is to be determined.

Assumptions **1** The temperature of the submerged portion of the spoon is equal to the water temperature. **2** The temperature in the spoon varies in the axial direction only (along the spoon), $T(x)$. **3** The heat transfer from the tip of the spoon is negligible. **4** The heat transfer coefficient is constant and uniform over the entire spoon surface. **5** The thermal properties of the spoon are constant. **6** The heat transfer coefficient accounts for the effect of radiation from the spoon.

Properties The thermal conductivity of the spoon is given to be $k = 8.7$ Btu/h·ft·°F.

Analysis Noting that the cross-sectional area of the spoon is constant and measuring x from the free surface of water, the variation of temperature along the spoon can be expressed as

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh m(L-x)}{\cosh mL}$$



where

$$p = 2(0.5/12\text{ ft}+0.08/12\text{ ft}) = 0.0967\text{ ft}$$

$$A_c = (0.5/12\text{ ft})(0.08/12\text{ ft}) = 0.000278\text{ ft}^2$$

$$m=\sqrt{\frac{hp}{kA_c}}=\sqrt{\frac{(3\text{ Btu/h.ft}^2.°\text{F})(0.0967\text{ ft})}{(8.7\text{ Btu/h.ft.°F})(0.000278\text{ ft}^2)}}=10.95\text{ ft}^{-1}$$

Noting that $x = L = 7/12 = 0.583$ ft at the tip and substituting, the tip temperature of the spoon is determined to be

$$\begin{aligned} T(L) &= T_\infty+(T_b-T_\infty)\frac{\cosh m(L-L)}{\cosh mL} \\ &= 75°\text{F}+(200-75)\frac{\cosh 0}{\cosh(10.95\times 0.583)} \\ &= 75°\text{F}+(200-75)\frac{1}{296} \\ &= 75.4°\text{F} \end{aligned}$$

Therefore, the temperature difference across the exposed section of the spoon handle is

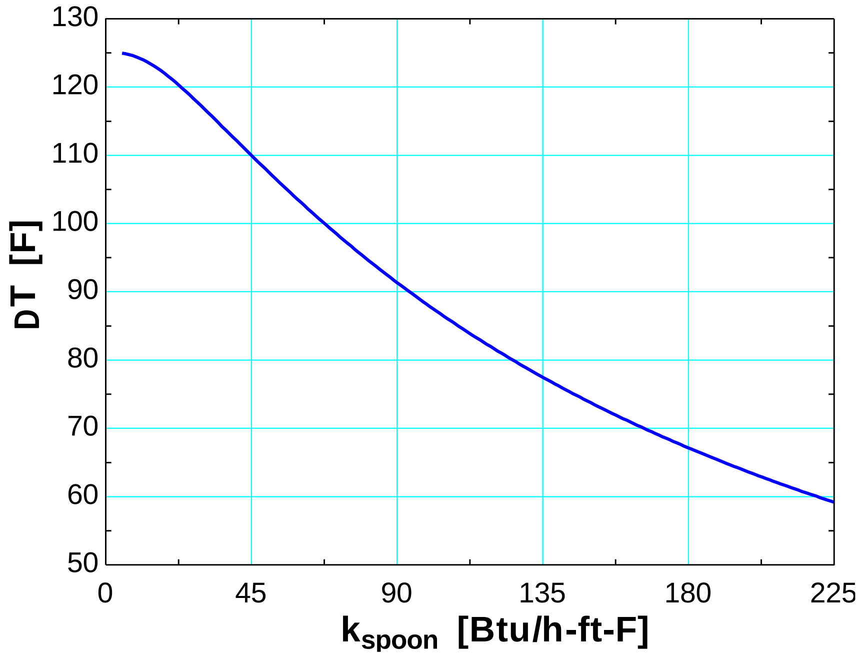
$$\Delta T = T_b - T_{\text{tip}} = (200-75.4)°\text{F} = \mathbf{124.6°F}$$

3-134E  Prob. 3-133E is reconsidered. The effects of the thermal conductivity of the spoon material and the length of its extension in the air on the temperature difference across the exposed surface of the spoon handle are to be investigated.

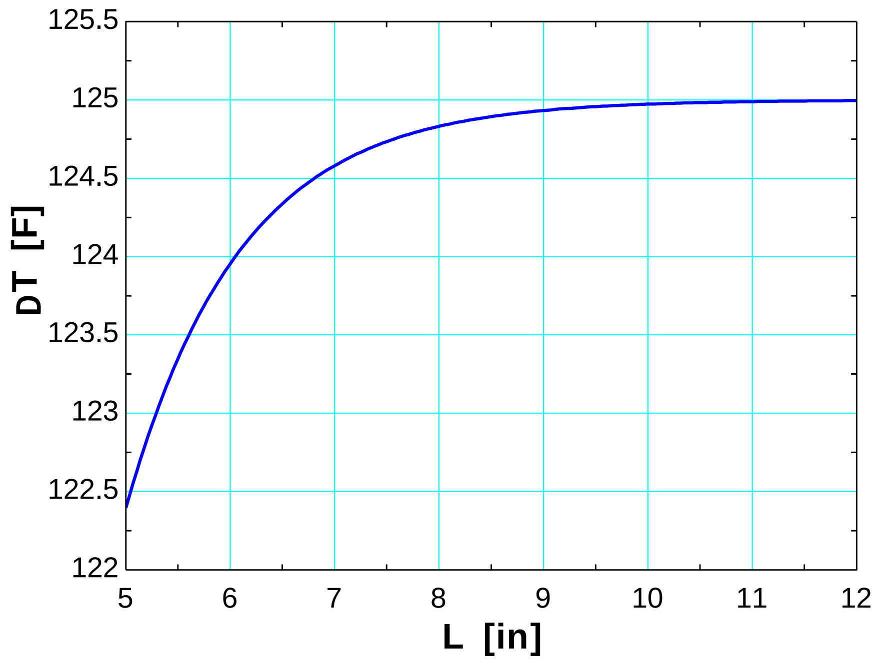
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
k_spoon=8.7 [Btu/h-ft-F]
T_w=200 [F]
T_infinity=75 [F]
A_c=(0.08/12*0.5/12) [ft^2]
L=7 [in]
h=3 [Btu/h-ft^2-F]

"ANALYSIS"
p=2*(0.08/12+0.5/12)
a=sqrt((h*p)/(k_spoon*A_c))
(T_tip-T_infinity)/(T_w-T_infinity)=cosh(a*(L-x)*Convert(in, ft))/cosh(a*L*Convert(in, ft))
x=L "for tip temperature"
DELTAT=T_w-T_tip
```

k_{spoon} [Btu/h.ft.F]	ΔT [F]
5	124.9
16.58	122.6
28.16	117.8
39.74	112.5
51.32	107.1
62.89	102
74.47	97.21
86.05	92.78
97.63	88.69
109.2	84.91
120.8	81.42
132.4	78.19
143.9	75.19
155.5	72.41
167.1	69.82
178.7	67.4
190.3	65.14
201.8	63.02
213.4	61.04
225	59.17



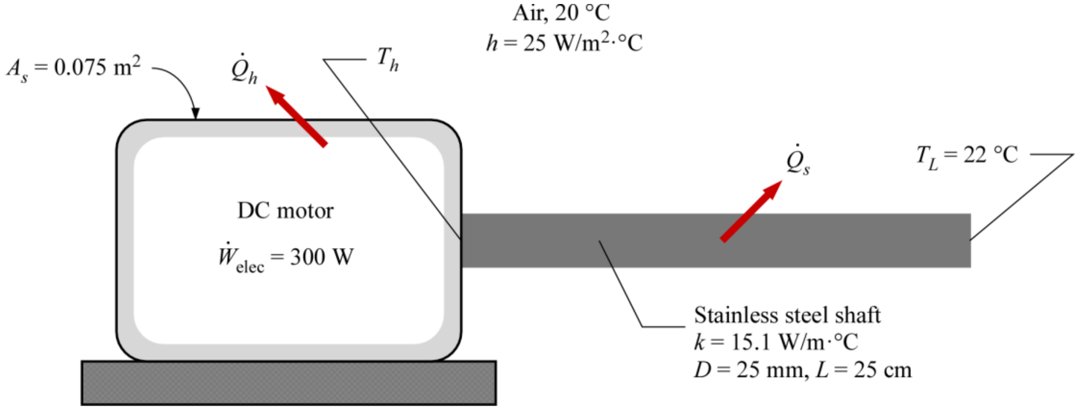
L [in]	ΔT [F]
5	122.4
5.5	123.4
6	124
6.5	124.3
7	124.6
7.5	124.7
8	124.8
8.5	124.9
9	124.9
9.5	125
10	125
10.5	125
11	125
11.5	125
12	125



3-135 A DC motor draws electrical power and delivers mechanical power to rotate a stainless steel shaft. The surface temperature of the motor housing is to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** The surface temperature of the motor housing is uniform. **5** The base temperature of the shaft is equal to the surface temperature of the motor housing.

Properties The thermal conductivity of the stainless steel shaft is given as 15.1 W/m · °C.



Analysis From energy balance, the following equation is expressed:

$$\dot{W}_{\text{elec}} = \dot{W}_{\text{mech}} + \dot{Q}_h + \dot{Q}_s \text{ or } \dot{W}_{\text{elec}} = 0.55\dot{W}_{\text{elec}} + \dot{Q}_h + \dot{Q}_s$$

The heat transfer rate from the motor housing surface is

$$\dot{Q}_h = hA_s(T_h - T_\infty)$$

The motor shaft can be treated as a circular fin with a specified fin tip temperature. The heat transfer rate from the motor shaft can be written as

$$\dot{Q}_s = \sqrt{hpkA_c}(T_h - T_\infty)\frac{\cosh mL - (T_L - T_\infty)/(T_h - T_\infty)}{\sinh mL}$$

$$= \sqrt{hkD^3\frac{\pi^2}{4}}(T_h - T_\infty)\frac{\cosh mL - (T_L - T_\infty)/(T_h - T_\infty)}{\sinh mL}$$

where

$$mL = \left(\frac{hp}{kA_c}\right)^{0.5} L = \left(\frac{4h}{kD}\right)^{0.5} L = \left[\frac{4(25\text{ W/m}^2\cdot°\text{C})}{(15.1\text{ W/m}\cdot°\text{C})(0.025\text{ m})}\right]^{0.5}(0.25\text{ m}) = 4.069$$

$$\sqrt{hk\frac{\pi^2}{4}D^3} = \sqrt{(25\text{ W/m}^2\cdot°\text{C})(15.1\text{ W/m}\cdot°\text{C})(0.025\text{ m})^3\frac{\pi^2}{4}} = 0.1206\text{ W/°C}$$

Substituting the listed terms into the energy balance equation we get

$$0.45\dot{W}_{\text{elec}} = hA_s(T_h - T_\infty) + \sqrt{hk\frac{\pi^2}{4}D^3}(T_h - T_\infty)\frac{\cosh mL - (T_L - T_\infty)/(T_h - T_\infty)}{\sinh mL}$$

Rearranging the equation, the surface temperature of the motor housing is

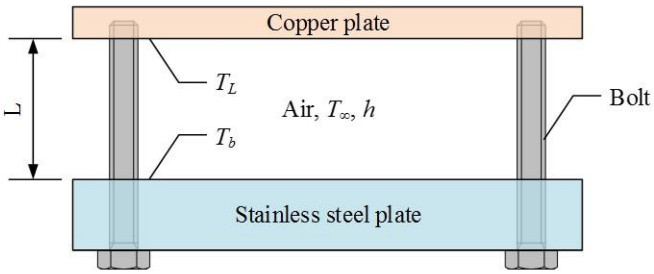
$$T_h = T_\infty + \frac{0.45\dot{W}_{\text{elec}} + \sqrt{hk\frac{\pi^2}{4}D^3}\,\frac{(T_L - T_\infty)}{\sinh mL}}{hA_s + \sqrt{hk\frac{\pi^2}{4}D^3}\left(\frac{\cosh mL}{\sinh mL}\right)} = 20\text{ °C} + \frac{0.45(300\text{ W}) + (0.1206\text{ W/°C})\,\frac{(22-20)\text{ °C}}{\sinh 4.069}}{(25\text{ W/m}^2\cdot°\text{C})(0.075\text{ m}^2) + (0.1206\text{ W/°C})\left(\frac{\cosh 4.069}{\sinh 4.069}\right)} = \mathbf{87.7\text{ °C}}$$

Discussion If the surface of the motor housing has a high emissivity, heat transfer by radiation from the motor housing would decrease the surface temperature.

3-136 C&S A stainless steel plate is connected to a copper plate by long ASTM B98 copper-silicon bolts. Portion of the bolts are exposed to convection with ambient air. The temperature T_L at the tip of the bolts is known, as well as the heat transfer rate from each bolt. The surface temperature at the base T_b is to be determined. Is the use of the ASTM B98 bolts in compliance with the ASME Code for Process Piping, which has a maximum use temperature of 149°C?

Assumptions**1** Heat transfer is steady. **2** The portion of the bolt exposed to convection behaves as finned surface. **3** The temperature T_b is uniform for the stainless steel plate surface and bolts. **4** Thermal properties are constant. **5** The tip of the bolt has specified temperature.

Properties The thermal conductivity for the bolts is 36 W/m·K.



Analysis The bolts can be treated as pin fins of rectangular profile. Since the bolts are attached to the copper plate with a uniform temperature, the tip of the bolts has a specified T_L = 70°C. Thus, the heat transfer rate equation for a fin with specified tip temperature is

$$\dot{Q} = \sqrt{hpkA_c}(T_b - T_\infty)\frac{\cosh mL - [(T_L - T_\infty)/(T_b - T_\infty)]}{\sinh mL}$$

$$\dot{Q} = \sqrt{hk\pi^2 D^3/4}(T_b - T_\infty)\frac{\cosh mL - [(T_L - T_\infty)/(T_b - T_\infty)]}{\sinh mL}$$

Where

$$m = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(5\text{W/m}^2\cdot\text{K})}{(36\text{W/m}\cdot\text{K})(0.0095\text{m})}} = 7.6472\ \text{m}^{-1}$$

The temperature T_b can be solved implicitly by an equation solver or trail-and-error or:

$$5\text{W} = \sqrt{(5\text{W/m}^2\cdot\text{K})(36\text{W/m}\cdot\text{K})\pi^2(0.0095\text{m})^3/4}(T_b - 20)(\text{K})\frac{\cosh[(7.6472)(0.05)] - [(70-20)/(T_b - 20)]}{\sinh[(7.6472)(0.05)]}$$

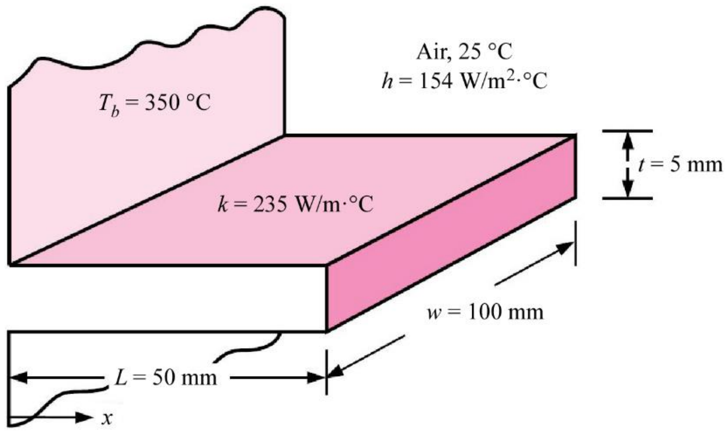
which gives $T_b = \mathbf{160}$°C.

Discussion With the temperature T_b = 160°C, this indicates that part of the bolt is operating at temperature above the maximum use temperature of 149°C (ASME B31.3-2014). Therefore, the ASTM B98 bolts are not suitable for this condition, since they do not comply with the ASME Code for Process Piping.

3-137 Using Table 3-3 and Figure 3-43, the efficiency, heat transfer rate, and effectiveness of a straight rectangular fin are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fin is given as 235 W/m · °C.



Analysis (a) From Table 3-3, for straight rectangular fins, we have

$$m = \sqrt{\frac{2h}{kt}} = \sqrt{\frac{2(154\ \text{W/m}^2\cdot°\text{C})}{(235\ \text{W/m}\cdot°\text{C})(0.005\ \text{m})}} = 16.19\ \text{m}^{-1}$$

$$L_c = L + t/2 = (0.05\ \text{m}) + (0.005\ \text{m})/2 = 0.0525\ \text{m}$$

$$A_{\text{fin}} = 2wL_c = 2(0.1\ \text{m})(0.0525\ \text{m}) = 0.0105\ \text{m}^2$$

The fin efficiency is

$$\eta_{\text{fin}} = \frac{\tanh mL_c}{mL_c} = \frac{\tanh\left[(16.19\ \text{m}^{-1})(0.0525\ \text{m})\right]}{(16.19\ \text{m}^{-1})(0.0525\ \text{m})} = \mathbf{0.813}$$

The heat transfer rate for a single fin is

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) = (0.813)(154\ \text{W/m}^2\cdot°\text{C})(0.0105\ \text{m}^2)(350-25)\ °\text{C} = \mathbf{427\ W}$$

The fin effectiveness is

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\dot{Q}_{\text{fin}}}{h(tw)(T_b - T_\infty)} = \frac{427\ \text{W}}{(154\ \text{W/m}^2\cdot°\text{C})(0.005\ \text{m})(0.1\ \text{m})(350-25)\ °\text{C}} = \mathbf{17.1}$$

(b) To use Figure 3-43, we need

$$L_c = 0.0525\ \text{m and } A_p = L_c t$$

Hence,

$$L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} = (0.0525\ \text{m})^{3/2}\left[\frac{154\ \text{W/m}^2\cdot°\text{C}}{(235\ \text{W/m}\cdot°\text{C})(0.0525\ \text{m})(0.005\ \text{m})}\right]^{1/2} \approx 0.60$$

Using Figure 3-43, the fin efficiency is

$$\eta_f \approx \mathbf{0.81}$$

The heat transfer rate for a single fin is

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) = (0.81)(154\ \text{W/m}^2\cdot°\text{C})(0.0105\ \text{m}^2)(350-25)\ °\text{C} = \mathbf{426\ W}$$

The fin effectiveness is

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\dot{Q}_{\text{fin}}}{h(tw)(T_b - T_\infty)} = \frac{426\ \text{W}}{(154\ \text{W/m}^2\cdot°\text{C})(0.005\ \text{m})(0.1\ \text{m})(350-25)\ °\text{C}} = \mathbf{17.0}$$

Discussion The results determined using Table 3-3 and Figure 3-43 are very comparable. However, it should be noted that results determined using Table 3-3 are more accurate.

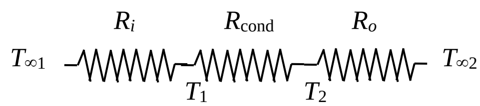
3-138 Two cast iron steam pipes are connected to each other through two 1-cm thick flanges exposed to cold ambient air. The average outer surface temperature of the pipe, the fin efficiency, the rate of heat transfer from the flanges, and the equivalent pipe length of the flange for heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the flanges (fins) varies in one direction only (normal to the pipe). **3** The heat transfer coefficient is constant and uniform over the entire fin surface. **4** The thermal properties of the fins are constant. **5** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the cast iron is given to be k = 52 W/m·°C.

Analysis (*a*) We treat the flanges as fins. The individual thermal resistances are

$$A_i = \pi D_i L = \pi(0.092\text{ m})(8\text{ m}) = 2.312\text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.1\text{ m})(8\text{ m}) = 2.513\text{ m}^2$$



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(180\text{ W/m}^2.°\text{C})(2.312\text{ m}^2)} = 0.00240\text{ °C/W}$$
$$R_{cond} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(5/4.6)}{2\pi(52\text{ W/m.°C})(8\text{ m})} = 0.00003\text{ °C/W}$$
$$R_o = \frac{1}{h_o A_o} = \frac{1}{(25\text{ W/m}^2.°\text{C})(2.513\text{ m}^2)} = 0.01592\text{ °C/W}$$

$$R_{total} = R_i + R_{cond} + R_o = 0.00240 + 0.00003 + 0.01592 = 0.01835\text{ °C/W}$$

The rate of heat transfer and average outer surface temperature of the pipe are

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(200-12)°\text{C}}{0.01835\text{ °C}} = 10{,}245\text{ W}$$
$$\dot{Q} = \frac{T_2 - T_{\infty 2}}{R_o} \longrightarrow T_2 = T_{\infty 2} + \dot{Q}R_o = 12\text{ °C} + (10{,}245\text{ W})(0.01592\text{ °C/W}) = \mathbf{175.1°C}$$

(*b*) The fin efficiency can be determined from (Fig. 3-44)

$$\left.\begin{aligned} &\frac{r_2 + \frac{t}{2}}{r_1} = \frac{0.09 + \frac{0.02}{2}}{0.05} = 2.0 \\ &\xi = L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} = \left(L + \frac{t}{2}\right)\sqrt{\frac{h}{kt}} = \left(0.04\text{ m} + \frac{0.02}{2}\text{ m}\right)\sqrt{\frac{25\text{ W/m}^2\text{°C}}{(52\text{ W/m°C})(0.02\text{ m})}} = 0.245 \end{aligned}\right\}\eta_{fin} = \mathbf{0.94}$$

$$A_{fin} = 2\pi(r_2^2 - r_1^2) + 2\pi r_2 t = 2\pi[(0.09\text{ m})^2 - (0.05\text{ m})^2] + 2\pi(0.09\text{ m})(0.02\text{m}) = 0.0465\text{ m}^2$$

The heat transfer rate from the flanges is

$$\begin{aligned}\dot{Q}_{finned} &= \eta_{fin}\dot{Q}_{fin,max} = \eta_{fin} h A_{fin}(T_b - T_\infty) \\ &= 0.94(25\text{ W/m}^2.°\text{C})(0.0465\text{ m}^2)(175.1-12)°\text{C} = \mathbf{178\ W}\end{aligned}$$

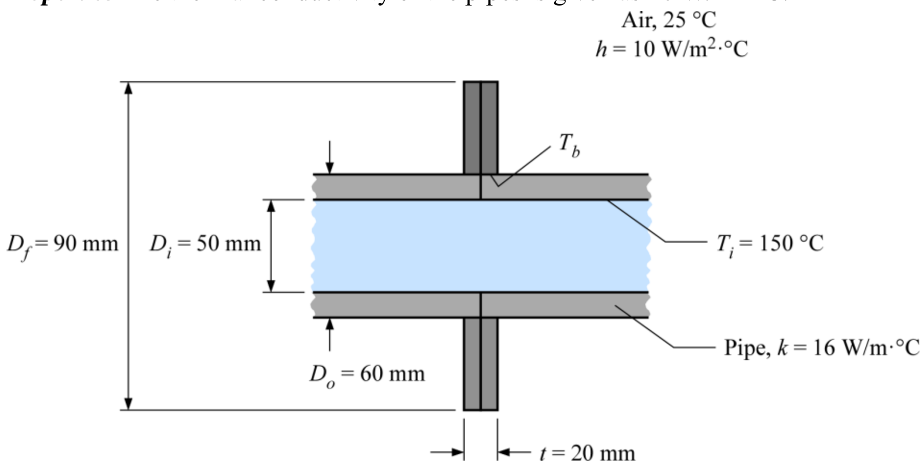
(*c*) An 8-m long section of the steam pipe is losing heat at a rate of 10,245 W or 10,245/8 = 1280 W per m length. Then for heat transfer purposes the flange section is equivalent to

$$\text{Equivalent length} = \frac{178\text{ W}}{1280\text{ W/m}} = 0.139\text{ m} = \mathbf{13.9\ cm}$$

3-139 Pipes used for transporting superheated vapor are connected together by flanges. The temperature at the base of the flange and the rate of heat loss through the flange are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** The flanges profile is similar to circular fins of rectangular profile.

Properties The thermal conductivity of the pipes is given as 16 W/m · °C.



Analysis The heat transfer rate through the pipe wall is equal to the heat transfer rate through the flanges:

$$\dot{Q}_{\text{pipe}} = \dot{Q}_f \text{ or } 2tk\pi \frac{T_i - T_b}{\ln(D_o/D_i)} = \eta_f hA_f (T_b - T_\infty)$$

Rearranging the equation yields

$$T_b = \frac{\eta_f hA_f T_\infty + \dfrac{2tk\pi}{\ln(D_o/D_i)} T_i}{\eta_f hA_f + \dfrac{2tk\pi}{\ln(D_o/D_i)}}$$

From Table 3-3, for circular fins of rectangular profile we have

$$r_{2c} = r_2 + t/2 = \frac{0.09\text{ m}}{2} + \frac{0.02\text{ m}}{2} = 0.055\text{ m}$$

$$A_f = 2\pi(r_{2c}^2 - r_1^2) = 2\pi[(0.055\text{ m})^2 - (0.06/2\text{ m})^2] = 0.01335\text{ m}^2$$

$$L_c = L + t/2 = \frac{0.09\text{ m} - 0.06\text{ m}}{2} + \frac{0.02\text{ m}}{2} = 0.025\text{ m}$$

$$A_p = L_c t = (0.025\text{ m})(0.02\text{ m}) = 0.0005\text{ m}^2$$

Hence,

$$\xi = L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} = (0.025\text{ m})^{3/2}\left[\frac{10\text{ W/m}^2\cdot°\text{C}}{(16\text{ W/m}\cdot°\text{C})(0.0005\text{ m}^2)}\right]^{1/2} = 0.1398$$

$$r_{2c}/r_1 = \frac{0.055\text{ m}}{0.030\text{ m}} = 1.83$$

Using Figure 3-44, the fin efficiency is $\eta_f \approx 0.97$. The temperature at the base of the flange is

$$T_b = \frac{(0.97)(10\text{ W/m}^2\cdot°\text{C})(0.01335\text{ m}^2)(25\,°\text{C}) + \dfrac{2(0.02\text{ m})(16\text{ W/m}\cdot°\text{C})\pi}{\ln(60/50)}(150\,°\text{C})}{(0.97)(10\text{ W/m}^2\cdot°\text{C})(0.01335\text{ m}^2) + \dfrac{2(0.02\text{ m})(16\text{ W/m}\cdot°\text{C})\pi}{\ln(60/50)}} = \mathbf{148.5\,°C}$$

The rate of heat loss through the flange is

$$\dot{Q}_f = \eta_f hA_f (T_b - T_\infty) = (0.97)(10\text{ W/m}^2\cdot°\text{C})(0.01335\text{ m}^2)(148.5 - 25)\,°\text{C} = \mathbf{16.0\text{ W}}$$

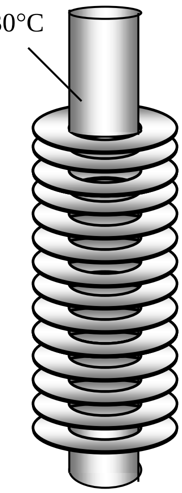
Discussion The flanges act as extended surfaces, which enhanced heat transfer from the pipes.

3-140 Circular aluminum fins are to be attached to the tubes of a heating system. The increase in heat transfer from the tubes per unit length as a result of adding fins is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is constant and uniform over the entire fin surfaces. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fins is given to be k = 186 W/m·°C.

Analysis In case of no fins, heat transfer from the tube per meter of its length is



$$A_{\text{no fin}} = \pi D_1 L = \pi(0.05\,\text{m})(1\,\text{m}) = 0.1571\,\text{m}^2$$
$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty) = (40\ \text{W/m}^2.°\text{C})(0.1571\,\text{m}^2)(130-25)°\text{C} = 660\ \text{W}$$

The efficiency of these circular fins is, from the efficiency curve, Fig. 3-43

$$\left.\begin{aligned} L &= (D_2 - D_1)/2 = (0.06-0.05)/2 = 0.005\,\text{m} \\ \frac{r_2 + (t/2)}{r_1} &= \frac{0.03+(0.001/2)}{0.025} = 1.22 \\ L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} &= \left(L+\frac{t}{2}\right)\sqrt{\frac{h}{kt}} \\ &= \left(0.005+\frac{0.001}{2}\right)\sqrt{\frac{40\ \text{W/m}^2\,^\circ\text{C}}{(186\ \text{W/m}^\circ\text{C})(0.001\text{m})}} = 0.08 \end{aligned}\right\} \eta_{\text{fin}} = 0.97$$

Heat transfer from a single fin is

$$A_{\text{fin}} = 2\pi(r_2^{\,2} - r_1^{\,2}) + 2\pi r_2 t = 2\pi(0.03^2 - 0.025^2) + 2\pi(0.03)(0.001) = 0.001916\,\text{m}^2$$
$$\begin{aligned} \dot{Q}_{\text{fin}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) \\ &= 0.97(40\ \text{W/m}^2.°\text{C})(0.001916\,\text{m}^2)(130-25)°\text{C} \\ &= 7.81\,\text{W} \end{aligned}$$

Heat transfer from a single unfinned portion of the tube is

$$A_{\text{unfin}} = \pi D_1 s = \pi(0.05\,\text{m})(0.003\text{m}) = 0.0004712\,\text{m}^2$$
$$\dot{Q}_{\text{unfin}} = hA_{\text{unfin}}(T_b - T_\infty) = (40\ \text{W/m}^2.°\text{C})(0.0004712\,\text{m}^2)(130-25)°\text{C} = 1.98\ \text{W}$$

There are 250 fins and thus 250 interfin spacings per meter length of the tube. The total heat transfer from the finned tube is then determined from

$$\dot{Q}_{\text{total,fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 250(7.81+1.98) = 2448\ \text{W}$$

Therefore the increase in heat transfer from the tube per meter of its length as a result of the addition of the fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total,fin}} - \dot{Q}_{\text{no fin}} = 2448 - 660 = \mathbf{1788\ W}$$

Discussion The The overall effectiveness of the finned tube is 2448/660 = 3.7. That is, the rate of heat transfer from the steam tube increases by a factor of 10 as a result of adding fins. This explains the widespread use of finned surfaces.

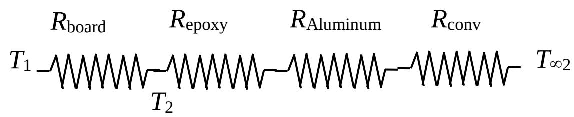
3-141 A circuit board houses 80 logic chips on one side, dissipating 0.04 W each through the back side of the board to the surrounding medium. The temperatures on the two sides of the circuit board are to be determined for the cases of no fins and 864 aluminum pin fins on the back surface.

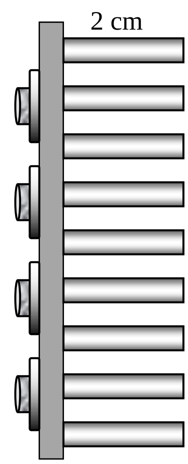
Assumptions **1** Steady operating conditions exist. **2** The temperature in the board and along the fins varies in one direction only (normal to the board). **3** All the heat generated in the chips is conducted across the circuit board, and is dissipated from the back side of the board. **4** Heat transfer from the fin tips is negligible. **5** The heat transfer coefficient is constant and uniform over the entire fin surface. **6** The thermal properties of the fins are constant. **7** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivities are given to be k = 30 W/m·°C for the circuit board, k = 237 W/m·°C for the aluminum plate and fins, and k = 1.8 W/m·°C for the epoxy adhesive.

Analysis (a) The total rate of heat transfer dissipated by the chips is

$$\dot{Q} = 80 \times (0.04\text{ W}) = 3.2\text{ W}$$





The individual resistances are

$$A = (0.12\text{ m})(0.18\text{ m}) = 0.0216\text{ m}^2$$

$$R_{\text{board}} = \frac{L}{kA} = \frac{0.004\text{ m}}{(30\text{ W/m.°C})(0.0216\text{ m}^2)} = 0.00617\text{ °C/W}$$

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(52\text{ W/m}^2\text{.°C})(0.0216\text{ m}^2)} = 0.89031\text{ °C/W}$$

$$R_{\text{total}} = R_{\text{board}} + R_{\text{conv}} = 0.00617 + 0.89031 = 0.8965\text{ °C/W}$$

The temperatures on the two sides of the circuit board are

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_{\infty 2} + \dot{Q}R_{\text{total}} = 40\text{°C} + (3.2\text{ W})(0.8965\text{ °C/W}) = 42.87\text{°C} \cong \mathbf{42.9°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 42.87\text{°C} - (3.2\text{ W})(0.00617\text{ °C/W}) = 42.85\text{°C} \cong \mathbf{42.9°C}$$

Therefore, the board is nearly isothermal.

(b) Noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{h\pi D}{k\pi D^2/4}} = \sqrt{\frac{4h}{kD}} = \sqrt{\frac{4(52\text{ W/m}^2\text{.°C})}{(237\text{ W/m.°C})(0.0025\text{ m})}} = 18.74\text{ m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh mL}{mL} = \frac{\tanh(18.74\text{ m}^{-1} \times 0.02\text{ m})}{18.74\text{ m}^{-1} \times 0.02\text{ m}} = 0.956$$

The fins can be assumed to be at base temperature provided that the fin area is modified by multiplying it by 0.956. Then the various thermal resistances are

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.0002\text{ m}}{(1.8\text{ W/m.°C})(0.0216\text{ m}^2)} = 0.00514\text{ °C/W}$$

$$R_{\text{Al}} = \frac{L}{kA} = \frac{0.002\text{ m}}{(237\text{ W/m.°C})(0.0216\text{ m}^2)} = 0.00039\text{ °C/W}$$

$$A_{\text{finned}} = \eta_{\text{fin}}\, n\pi DL = 0.956 \times 864\pi(0.0025\text{ m})(0.02\text{ m}) = 0.1297\text{ m}^2$$

$$A_{\text{unfinned}} = 0.0216 - 864\frac{\pi D^2}{4} = 0.0216 - 864 \times \frac{\pi(0.0025)^2}{4} = 0.0174\text{ m}^2$$

$$A_{\text{total,with fins}} = A_{\text{finned}} + A_{\text{unfinned}} = 0.1297 + 0.0174 = 0.1471\text{ m}^2$$

$$R_{\text{conv}} = \frac{1}{hA_{\text{total,with fins}}} = \frac{1}{(52\text{ W/m}^2\text{.°C})(0.1471\text{ m}^2)} = 0.1307\text{ °C/W}$$

$$R_{\text{total}} = R_{\text{board}} + R_{\text{epoxy}} + R_{\text{aluminum}} + R_{\text{conv}} = 0.00617 + 0.00514 + 0.00039 + 0.1307 = 0.1424\text{ °C/W}$$

Then the temperatures on the two sides of the circuit board becomes

$$\dot{Q} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} \longrightarrow T_1 = T_{\infty 2} + \dot{Q}R_{\text{total}} = 40\text{°C} + (3.2\text{ W})(0.1424\text{ °C/W}) = 40.46\text{°C} \cong \mathbf{40.5°C}$$

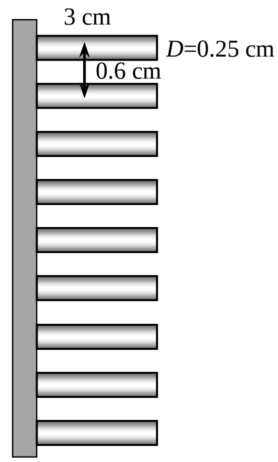
$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 40.46\text{°C} - (3.2\text{ W})(0.00617\text{ °C/W}) = 40.44 \cong \mathbf{40.4°C}$$

3-142 A hot plate is to be cooled by attaching aluminum pin fins on one side. The rate of heat transfer from the 1 m by 1 m section of the plate and the effectiveness of the fins are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** Heat transfer from the fin tips is negligible. **4** The heat transfer coefficient is constant and uniform over the entire fin surface. **5** The thermal properties of the fins are constant. **6** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the aluminum plate and fins is given to be k = 237 W/m·°C.

Analysis Noting that the cross-sectional areas of the fins are constant, the efficiency of the circular fins can be determined to be



$$m=\sqrt{\frac{hp}{kA_c}}=\sqrt{\frac{h\pi D}{k\pi D^2/4}}=\sqrt{\frac{4h}{kD}}=\sqrt{\frac{4(35\text{ W/m}^2.°\text{C})}{(237\text{ W/m.°C})(0.0025\text{ m})}}=15.37\text{ m}^{-1}$$

$$\eta_{\text{fin}}=\frac{\tanh mL}{mL}=\frac{\tanh(15.37\text{ m}^{-1}\times 0.03\text{ m})}{15.37\text{ m}^{-1}\times 0.03\text{ m}}=0.935$$

The number of fins, finned and unfinned surface areas, and heat transfer rates from those areas are

$$n=\frac{1\text{ m}^2}{(0.006\text{ m})(0.006\text{ m})}=27{,}777$$

$$\begin{aligned}
A_{\text{fin}}&=27777\left[\pi DL+\frac{\pi D^2}{4}\right]=27777\left[\pi(0.0025)(0.03)+\frac{\pi(0.0025)^2}{4}\right]\\
&=6.68\text{ m}^2\\
A_{\text{unfinned}}&=1-27777\left(\frac{\pi D^2}{4}\right)=1-27777\left[\frac{\pi(0.0025)^2}{4}\right]=0.86\text{ m}^2\\
\dot{Q}_{\text{finned}}&=\eta_{\text{fin}}\dot{Q}_{\text{fin, max}}=\eta_{\text{fin}}hA_{\text{fin}}(T_b-T_\infty)\\
&=0.935(35\text{ W/m}^2.°\text{C})(6.68\text{ m}^2)(100-30)°\text{C}\\
&=15{,}300\text{ W}\\
\dot{Q}_{\text{unfinned}}&=hA_{\text{unfinned}}(T_b-T_\infty)=(35\text{ W/m}^2.°\text{C})(0.86\text{ m}^2)(100-30)°\text{C}\\
&=2107\text{ W}
\end{aligned}$$

Then the total heat transfer from the finned plate becomes

$$\dot{Q}_{\text{total,fin}}=\dot{Q}_{\text{finned}}+\dot{Q}_{\text{unfinned}}=15{,}300+2107=1.74\times10^4\text{ W}=\mathbf{17.4\,kW}$$

The rate of heat transfer if there were no fin attached to the plate would be

$$\begin{aligned}
A_{\text{no fin}}&=(1\text{ m})(1\text{ m})=1\text{ m}^2\\
\dot{Q}_{\text{no fin}}&=hA_{\text{no fin}}(T_b-T_\infty)=(35\text{ W/m}^2.°\text{C})(1\text{ m}^2)(100-30)°\text{C}=2450\text{ W}
\end{aligned}$$

Then the fin effectiveness becomes

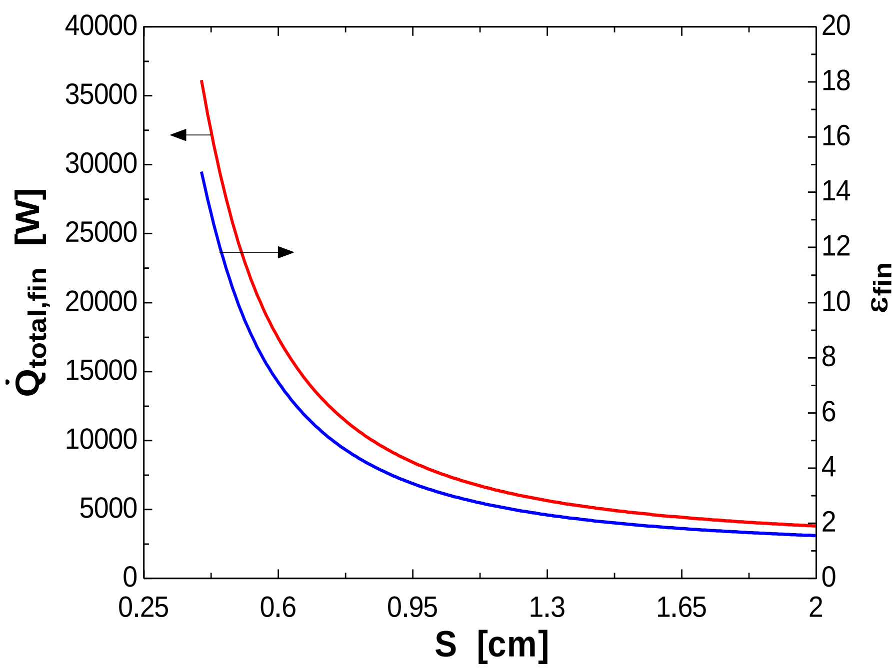
$$\varepsilon_{\text{fin}}=\frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}}=\frac{17{,}400}{2450}=\mathbf{7.10}$$

3-143 Prob. 3-142 is reconsidered. The effect of the center-to center distance of the fins on the rate of heat transfer from the surface and the overall effectiveness of the fins is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_b=100 [C]
L=0.03 [m]
D=0.0025 [m]
k=237 [W/m-C]
S=0.6 [cm]
T_infinity=30 [C]
h=35 [W/m^2-C]
A_surface=1*1 [m^2]

"ANALYSIS"
p=pi*D
A_c=pi*D^2/4
a=sqrt((h*p)/(k*A_c))
eta_fin=tanh(a*L)/(a*L)
n=A_surface/(S^2*Convert(cm^2, m^2)) "number of fins"
A_fin=n*(pi*D*L+pi*D^2/4)
A_unfinned=A_surface-n*(pi*D^2/4)
Q_dot_finned=eta_fin*h*A_fin*(T_b-T_infinity)
Q_dot_unfinned=h*A_unfinned*(T_b-T_infinity)
Q_dot_total_fin=Q_dot_finned+Q_dot_unfinned
Q_dot_nofin=h*A_surface*(T_b-T_infinity)
epsilon_fin=Q_dot_total_fin/Q_dot_nofin
```

S [cm]	$Q_{total\ fin}$ [W]	ε_{fin}
0.4	36123	14.74
0.5	24001	9.796
0.6	17416	7.108
0.7	13445	5.488
0.8	10868	4.436
0.9	9101	3.715
1	7838	3.199
1.1	6903	2.817
1.2	6191	2.527
1.3	5638	2.301
1.4	5199	2.122
1.5	4845	1.977
1.6	4555	1.859
1.7	4314	1.761
1.8	4113	1.679
1.9	3942	1.609
2	3797	1.55

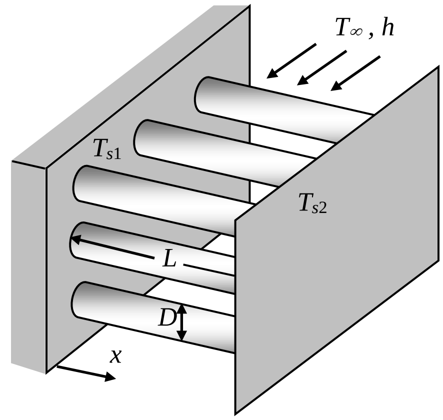


3-144 Circular fins made of copper are considered. The function $\theta(x) = T(x) - T_\infty$ along a fin is to be expressed and the temperature at the middle is to be determined. Also, the rate of heat transfer from each fin, the fin effectiveness, and the total rate of heat transfer from the wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** The heat transfer coefficient is constant and uniform over the entire finned and unfinned wall surfaces. **4** The thermal properties of the fins are constant. **5** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the copper fin is given to be k = 380 W/m·°C.

Analysis (*a*)



For $x = L/2$:

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{(100)\pi(0.001)}{(380)\pi(0.001)^2/4}} = 32.44\ \text{m}^{-1}$$

Noting that $T_b - T_\infty = T_{s1}$ and $T_L - T_\infty = 0$,

$$\frac{T(x)-T_\infty}{T_b - T_\infty} = \frac{\left(\dfrac{T_L - T_\infty}{T_b - T_\infty}\right)\sinh(mx) + \sinh m(L-x)}{\sinh mL} = \frac{\sinh[m(L-x)]}{\sinh mL}$$

$$\frac{T(L/2)-0}{132-0} = \frac{\sinh[m(L-x)]}{\sinh mL}$$

$$T(L/2) = 132\frac{\sinh[32.44(0.030-0.015)]}{\sinh(32.44\times 0.030)} = \mathbf{58.9°C}$$

(*b*) The rate of heat transfer from a single fin is

$$\begin{aligned}\dot{Q}_{\text{one fin}} &= (T_b - T_\infty)\sqrt{hpkA_c}\,\frac{\cosh(mL) - \left(\dfrac{T_L - T_\infty}{T_b - T_\infty}\right)}{\sinh(mL)} \\ &= (132-0)\sqrt{(100)\pi(0.001)(380)\pi(0.001)^2/4}\,\frac{\cosh(32.44\times 0.030) - 0}{\sinh(32.44\times 0.030)} \\ &= \mathbf{1.704\ W}\end{aligned}$$

The effectiveness of the fin is

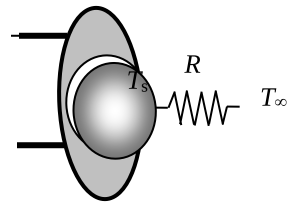
$$\varepsilon = \frac{\dot{Q}}{hA_c(T_b - T_\infty)} = \frac{1.704}{(100)0.25\pi(0.001)^2(132-0)} = \mathbf{164.4}$$

Since ε >> 2, the fins are well justified.

(*c*) The total rate of heat transfer is

$$\begin{aligned}\dot{Q}_{\text{total}} &= \dot{Q}_{\text{fins}} + \dot{Q}_{\text{base}} \\ &= n_{\text{fin}}\dot{Q}_{\text{one fin}} + (A_{\text{wall}} - n_{\text{fin}}A_c)h(T_b - T_\infty) \\ &= (625)(1.704) + [0.1\times 0.1 - 625\times 0.25\pi(0.001)^2](100)(132) \\ &= \mathbf{1191\ W}\end{aligned}$$

3-145 A commercially available heat sink is to be selected to keep the case temperature of a transistor below 90°C in an environment at 20°C.



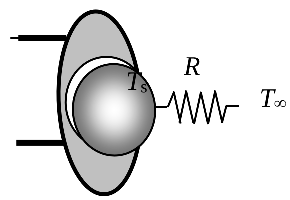
Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 90°C. **3** The contact resistance between the transistor and the heat sink is negligible.

Analysis The thermal resistance between the transistor attached to the sink and the ambient air is determined to be

$$\dot{Q} = \frac{\Delta T}{R_{\text{case}-\text{ambient}}} \longrightarrow R_{\text{case}-\text{ambient}} = \frac{T_{\text{transistor}} - T_\infty}{\dot{Q}} = \frac{(90-20)°\text{C}}{40\text{ W}} = \mathbf{1.75\ °C/W}$$

The thermal resistance of the heat sink must be below 1.75°C/W.Table 3-6 reveals that HS6071 in vertical position, HS5030 and HS6115 in both horizontal and vertical position can be selected.

3-146 A commercially available heat sink is to be selected to keep the case temperature of a transistor below 90°C in an environment at 30°C.



Assumptions **1** Steady operating conditions exist. **2** The transistor case is isothermal at 90°C. **3** The contact resistance between the transistor and the heat sink is negligible.

Analysis The thermal resistance between the transistor attached to the sink and the ambient air is determined to be

$$\dot{\text{Q}} = \frac{\Delta\text{T}}{\text{R}_{\text{case}-\text{ambient}}} \longrightarrow \text{R}_{\text{case}-\text{ambient}} = \frac{\text{T}_{\text{transistor}} - \text{T}_\infty}{\dot{\text{Q}}} = \frac{(90-30)°\text{C}}{60\text{ W}} = \mathbf{1.0\ °C/W}$$

The thermal resistance of the heat sink must be below 1.0°C/W.Table 3-6 reveals that HS5030, whose thermal resistance is 0.9°C/W in the vertical position, is the only heat sink that will meet this requirement.

Bioheat Transfer Equation

3-147 The different human body types with three different thicknesses of skin/fat layes are subjected to the varying ambient temperature. The rate of metabolic heat generation is to be determined so as to maintain the skin temperarue constant at 34ºC.

Assumptions **1** Muscle and skin/fat layer considered as a 1-D plain wall. **2** Steady state conditions. **3** Blood properties, thermal conductivities, arterial temperature, core body temperature, and perfusion rate are all constant. **4** The surrounding temperature is the same as that of the ambient temperature. **5** Solar radiation is negligible.

Propertie: Constant thermophysical properties for skin and blood. Ambient temperature is varied between -20 and 20ºC. L_{sf} = 0.075, 0.005 and 0.003 m for three different body types.

Analysis Solve the bioheat transfer differential equation along with the appropriate boundary conditions to develop an expression for the interface temperature (T_i) between the muscle and the outer skin/fat layer. The bioheat transfer differential equation is given by Eq. 3-88.

$$\frac{d^2\theta}{dx^2} - B^2\theta = 0$$

where $B^2 = \dot{p}\rho_b c_b / k$ has units of (1/m) and $\theta = T - T_a - \dot{e}_m / \dot{p}\rho_b c_b$. The boundary conditions for the problem in terms of temperature excess θ are:

$$\theta(0) = T_c - T_a - \dot{e}_m / \dot{p}\rho_b c_b = \theta_c \quad \text{and} \quad \theta(L_m) = T_i - T_a - \dot{e}_m / \dot{p}\rho_b c_b = \theta_i$$

The solution to Eq. 3-88 with the two specified temperature boundary conditions θ_c and θ_i, is given by Eq. 3-67 developed for fins (case 3 – specified temperature). For our case Eq. 3-67 becomes

$$\frac{\theta}{\theta_c} = \frac{(\theta_i/\theta_c)\sinh Bx + \sinh B(L_m - x)}{\sinh BL_m}$$

Using the Fourier's law of heat conduction, the rate of heat transfer that leaves the muscle at $x = L_m$ and enters the skin/fat layer is

$$\dot{Q}_{specified\ temp.} = -k_m A \left.\frac{dT}{dx}\right|_{x=L_m} = -k_m A \left.\frac{d\theta}{dx}\right|_{x=L_m} = -k_m AB\theta_c \frac{(\theta_i/\theta_c)\cosh BL_m - 1}{\sinh BL_m}$$

The rate at which heat is transferred through the skin/fat layer and into the environment is obtained by using the thermal resistance network concept. The total rate of heat transfer through the skin/fat layer and into the environment (the rate of heat loss from the body) is,

$$\dot{Q}_b = \frac{T_i - T_\infty}{R_{total}} = \frac{T_i - T_s}{R_{sf}} = (T_s - T_\infty)\frac{R_{conv} + R_{rad}}{R_{conv} \times R_{rad}}$$

where the total resistance is

$$R_{total} = R_{sf} + R_{conv-rad} = R_{sf} + \frac{R_{conv}\,R_{rad}}{R_{conv} + R_{rad}}$$

and the individual resistances are

$$R_{sf} = \frac{L_{sf}}{k_{sf}\,A}$$

$$R_{conv} = \frac{1}{h_{conv}\,A} = \frac{1}{2(W/m^2\cdot K)\times 1.8m^2} = 0.2778\ \text{K/W}$$

$$R_{rad} = \frac{1}{h_{rad}\,A} = \frac{1}{5.9(W/m^2\cdot K)\times 1.8m^2} = 0.0941\ \text{K/W}$$

From the above heat balance, for the known values of skin temperature and the environment temperature, we find interface temperature as,

$$T_i = T_s + \dot{Q}_b R_{sf}$$

Equating the rate of heat transfer that leaves the muscle at $x = L_m$ and enters the skin/fat layer with the rate at which heat is transferred through the skin/fat layer and into the environment yields

$$-k_m AB\theta_c \frac{(\theta_i/\theta_c)\cosh BL_m - 1}{\sinh BL_m} = \frac{T_i - T_\infty}{R_{total}}$$

The rearrangement of above equation by replacing θ_i and θ_c with appropriate equations results in the equation to determine the metabolic heat generation rate,

$$\dot{e}_m = \left(\frac{(T_i - T_\infty)\sinh(BL_m)}{k_m ABR_{tot}} + (T_i - T_a)\cosh(BL_m) \right) \times \frac{\dot{p}\rho_b c_b}{\cosh(BL_m) - 1}$$

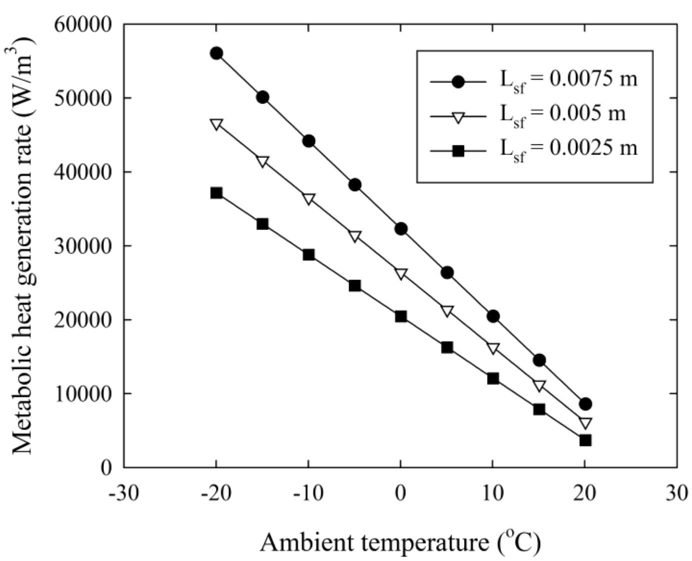
where, $B = \sqrt{\dfrac{\dot{p}\rho_b c_b}{k_m}} = 60\ \text{m}^{-1}$, $\sinh(BL_m) = 2.94$, $\cosh(BL_m) = 3.107$, $k_m = 0.5\ \text{W/m}\cdot\text{K}$ and $A = 1.8\ \text{m}^2$.

The muscle and skin/fat interface temperature is a variable that will change with the change in the ambient temperature and the thickness of skin/fat layer. For all other known values, the metabolic heat generation rate ($\dot{e}_m$) is calculated as shown in table below.

Metabolic heat generation rate as a function of ambient air temperature and skin/fat layer thickness

	Metabolic heat generation rate $\dot{e}_m$ (*W/m*^{3})		
T_∞ (°C)	L_{sf} = 0.0075 m	L_{sf} = 0.005 m	L_{sf} = 0.0025 m
-20	56077	46642	37207
-15	50148	41586	33024
-10	44218	36530	28842
-5	38289	31474	24660
0	32359	26418	20477
5	26429	21362	16295
10	20500	16306	12113
15	14570	11250	7931
20	8641	6194	3748

Variation of the metabolic heat generation rate with change in the ambient temperature for three different skin/fat layer thicknesses is shown graphically in the figure below.



Discussion It is clear that the metabolic heat generation rate required to keep a skin temperature increases with drop in the ambient temperature. The effect of increase in the skin/fat layer thickness on the metabolic heat generation rate is also evident. This also implies that, in oder to maintain the interface and core body temperature within a comfortable zone, a human being with lesser skin/fat layer thickness will have to have higher metabolism.

3-148 The metabolic heat generation rate within human body is to be determined so as to maintain the skin temperature at 34ºC.

***Assumptions*1** Muscle and skin/fat layer considered as a 1-D plain wall. **2** Steady state conditions. **3** Blood properties, thermal conductivities, arterial temperature, core body temperature, and perfusion rate are all constant. **4** The surrounding temperature is the same as that of the ambient temperature. **5** Solar radiation is negligible. **6. S**ame heat transfer everywhere throughout the different parts of body.

Properties The convective heat transfer coefficients at T_∞ = 15ºC for air and water are 2 W/m²·K and 20 W/m²·K, respectively. All other properties remain same as that of the example problem 3-14 in the text.

Analysis For this problem, the properties of human body parameters remain the same as that considered in example problem 3-14 in the text. However, the ambient and surrounding temperature is lowered to 15ºC and has different convective heat transfer coefficient when the human body is exposed to air and water.

The thermal resistance due to convection is

For air: $$R_{conv} = \frac{1}{h_{conv}A} = \frac{1}{2\ \mathrm{W/m^2\cdot K} \times 1.8\ \mathrm{m^2}} = 0.277\ \mathrm{K/W}$$

For water: $$R_{conv} = \frac{1}{h_{conv}A} = \frac{1}{20\ \mathrm{W/m^2\cdot K} \times 1.8\ \mathrm{m^2}} = 0.0277\ \mathrm{K/W}$$

The thermal resistance due to radiation remains unchanged and is same as that in the example problem 3-14 in the text,

$$R_{rad} = \frac{1}{h_{rad}A} = 0.09416\ \mathrm{K/W}$$

Since we know the environment and skin temperatures, we find the rate of heat loss from the skin surface to the environment as a combined effect of convection and radiation.

$$\dot{Q}_b = (T_s - T_\infty)\frac{R_{conv} + R_{rad}}{R_{conv} \times R_{rad}} = 270.2\ \mathrm{W}$$

From of heat transfer calculated above we can then find the skin and muscle/fat layer interface temperature using heat balance.

$$\dot{Q}_b = k_{sf} A \frac{T_i - T_s}{L_{sf}} \quad \text{and}\ T_i = T_s + \frac{\dot{Q}_b L_{sf}}{k_{sf} A} = 34°C + \frac{270.2\,W \times 0.003\,m}{0.3\,W/m\cdot K \times 1.8\,m^2} = 35.5°C$$

We also know that, the muscle and skin/fat layer interface temperature T_i can be calculated using

$$T_i = \frac{T_\infty \sinh BL_m + k_m ABR_{total}\left[\theta_c + \left(T_a + \dfrac{\dot{e}_m}{\dot{p}\rho_b c_b}\right)\cosh BL_m\right]}{\sinh BL_m + k_m ABR_{total} \cosh BL_m}$$

Rearrangement of this equation to find unknown metabolic heat generation rate yields

$$\dot{e}_m = \dot{p}\rho_b c_b \left(\frac{(T_i - T_\infty)\sinh(BL_m)}{k_m ABR_{tot} \cosh(BL_m)} + (T_i - T_a)\cosh(BL_m)\right) \times \frac{1}{\cosh(BL_m) + 1}$$

For the given conditions,

$$B = \sqrt{\frac{\dot{p}\rho_b c_b}{k_m}} = \sqrt{\frac{0.0005\,(\mathrm{s^{-1}}) \times 1000\,(\mathrm{kg/m^3}) \times 3600\,(\mathrm{J/kg\cdot K})}{0.5\,(\mathrm{W/m\cdot K})}} = 60\ \mathrm{m^{-1}}$$

$$\sinh(BL_m) = 2.94,\ \cosh(BL_m) = 3.107$$

Putting these values in the equation for metabolic heat generation rate gives,

$$\dot{e}_m = 4401\ \mathrm{W/m^3}\ \text{(air environment)}$$

Similarly using the convective heat transfer coefficient of 20 W/m²·K for water, we find,

$$\dot{Q}_b = (T_s - T_\infty)\frac{R_{conv} + R_{rad}}{R_{conv} \times R_{rad}} = (34 - 15)°C \times \left(\frac{0.0277 + 0.09416}{0.0277 \times 0.09416}\right) = 887.7\,W$$

and $$T_i = T_s + \frac{\dot{Q}_b\, L_{sf}}{k_{sf}\, A} = 34°C + \frac{887.7\,W \times 0.003\,m}{0.3\,W/m \cdot K \times 1.8\,m^2} = 38.9°C$$

Using the equation for metabolic heat generation rate as above we get,

$\dot{e}_m = 23937\,W/m^3$ (water environment).

Discussion The human body is adaptable to adjust with the surrounding thermal environment. For instance, if the environmental conditions are cold, human body will adjust itself through the involuntarily motion of shivering. In this case, for lower ambient temperature, metabolic heat generation achieved through shivering is about 6 times higher than that given in example problem 3-14 in the text. In case of the water as surrounding fluid with about 10 times higher convective heat transfer coefficient, the metabolic heat generation rate required to maintain the skin temperature at 34 ºC is about 35 times higher compared to that given in the example problem. This indicates that, for such a case only shivering motion may not be sufficient to raise the body temperature and the human body may have to perform certain physical activity to increase the metabolic heat generation rate.

3-149 The metabolic heat generation rate in a person's body increases from 700 to 7000 W/m^3 due to rigorous exercise over a period of time. The perspiration rate in lit/s is to be determined to maintain the skin temperature at 34 ^{o}C.

Assumptions **1** Muscle and skin/fat layer considered as a 1-D plain wall. **2** Steady state conditions. **3** Blood properties, thermal conductivities, arterial temperature, core body temperature, and perfusion rate are all constant. **4** The surrounding temperature is the same as that of the ambient temperature. **5** Solar radiation is negligible. **6.** **S**ame heat transfer everywhere throughout the different parts of body.

Properties Constant thermophysical properties for skin and blood. The perfusion rate stays constant at 0.0005 s^{-1}.

Analysis Since the convective heat transfer coefficient, ambient and surrounding temperatures, skin and blood thermophysical properties and the perfusion rate are same as that in the example problem 3-14 in the text, the values of B, R_{sf}, R_{conv}, R_{rad} and R_{total} remain unchanged. Thus, B = 60 m^{-1}, $\rho_b = 1000\,\text{kg/m}^3$, $c_b = 3600\,\text{J/kg}\cdot\text{K}$, $L_m = 0.03\,\text{m}$, R_{sf} = 0.00555 K/W, R_{conv} = 0.2778 K/W, R_{rad} = 0.0941 K/W and R_{total} = 0.07588 K/W.

With the increase in metabolic rate from 700 to 7000 W/m^3 and for the ambient temperature of 30^{o}C, the skin and muscle/fat layer interface temperature is calculated using,

$$T_i = \frac{T_\infty \sinh BL_m + k_m ABR_{total}\left[\theta_c + \left(T_a + \dfrac{\dot{e}_m}{\dot{p}\rho_b c_b}\right)\cosh BL_m\right]}{\sinh BL_m + k_m ABR_{total}\cosh BL_m}$$

where

$$\sinh(BL_m) = 2.94,\ \cosh(BL_m) = 3.107 \text{ and } \theta_c = T_c - T_a - \frac{\dot{e}_m}{\dot{p}\rho_b c_b} = -3.88°\text{C}$$

$$T_i = \frac{30°\text{C}\times 2.94 + 0.5\,\text{W/m}\cdot\text{K}\times 1.8\,\text{m}^2\times 60\,\text{m}^{-1}\times 0.0758\,\text{K/W}\,[-3.88°\text{C} + (37°\text{C}+3.88°\text{C})\times 3.107]}{2.94 + (0.5\,\text{W/m}\cdot\text{K}\times 1.8\,\text{m}^2\times 60\,\text{m}^{-1}\times 0.0758\,\text{K/W}\times 3.107)}$$

$$\therefore T_i = 37.4°\text{C}$$

From the value of interface temperature, the total heat loss from the body to the environment is calculated as,

$$\dot{Q}_b = \frac{T_i - T_\infty}{R_{total}} = \frac{37.4°\text{C} - 30°\text{C}}{0.07588\,\text{K/W}} = 97.5\,\text{W}$$

Now, the skin temperature is calculated using Fourier's law of heat conduction to the skin/fat layer i.e.,

$$\dot{Q}_b = k_{sf} A \frac{T_i - T_s}{L_{sf}}$$

Thus the skin temperature is calculated as,

$$T_{skin} = T_i - \frac{\dot{Q}_b L_{sf}}{k_{sf} A} = 36.9°\text{C}$$

The skin temperature is 2.9^{o}C higher than the desired temperature. If the skin temperature is to be maintained at 34 ^{o}C, then the excess heat generated within the body due to increased metabolic heat generation rate has to be removed through perspiration. The heat removal rate from the body when the skin temperature is 36.9^{o}C is 97.5 W. However in order to maintain the skin temperature at 34 ^{o}C, the amount of heat to be removed is

$$\dot{Q}_b = \frac{T_i - 34}{R_{sf}} = 612.6\,\text{W}$$

Thus the excess amount of heat to be removed from body is 515.1 W. If this heat has to be removed through perspiration then,

$$\dot{Q}_p = \dot{m}_p h_{fg}$$

Given that the perspiration properties are same as that of water evaluated at a skin surface temperature of 35.5^{o}C, from Table A-9, we have $\rho_p = 994\,\text{kg/m}^3$ and $h_{fg} = 2417\,\text{kJ/kg}$.

Thus the volume of perspiration in lit/s is determined as,

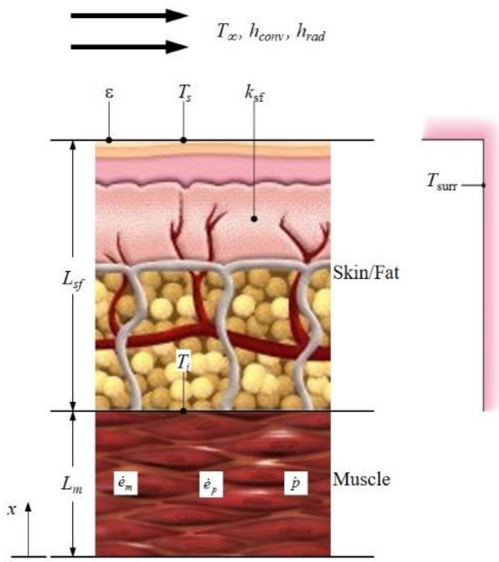
$$\dot{V}_p = \frac{\dot{m}_p}{\rho_p} = \frac{\dot{Q}_p}{\rho_p h_{fg}} = \frac{515.1\,\text{J/s}}{2417\times 10^3\,\text{J/kg}\times 994\,\text{kg/m}^3} = 2.144\times 10^{-7}\,\text{m}^3\text{/s} = 2.144\times 10^{-4}\,\text{L/s}$$

Discussion The volumetric perspiration rate of $2.144\times 10^{-4}\,\text{L/s} = 0.772\,\text{L/h}$ is moderate perspiration rate expected during rigorous exercise. If the perspiration rate exceeds about 2 L/h, human body may get dehydrated quickly resulting in sudden collapse. It was assumed in this problem that the perfusion rate stays constant. However, in reality, the body will automatically increase the perfusion rate that will increase the temperature at the muscle and skin/fat layer interface. Thus the body heat is rejected to the environment in form of convection and radiation and hence may result into less perspiration.

3-150 For a forearm we have been given the dimensions and thermal conductivities of a muscle layer and a skin/fat layer, metabolic heat generation and perfusion rate within the muscle layer, arterial temperature, blood density, specific heat, and ambient conditions. The mathematical formulation, the temperature at the outer surface of the muscle, and the maximum temperature in the forearm are to be determined.

Assumptions **1** Muscle and skin/fat layer considered as a 1-D cylinder. **2** Steady state conditions. **3** Blood properties, thermal conductivities, arterial temperature, core body temperature, metabolic heat generation rate, and perfusion rate are all constant. **4** Radiation exchange between the skin surface and the surroundings is between a small surface and a large enclosure at the air temperature. **5** Solar radiation is negligible.

Properties Muscle thermal conductivity k_m = 0.5 W/m·K, skin/fat layer thermal conductivity k_{sf} = 0.3 W/m·K, blood density ρ_b = 1000 kg/m^3 and blood specific heat c_b = 3600 J/kg·K.



Analysis (a) The bioheat transfer differential equation in cylindrical coordinates with constant properties is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{e}_m+\dot{e}_P}{k}=0$$

where $\dot{e}_m$ and $\dot{e}_p$ are the *metabolic* and *perfusion* heat source terms (W/m^3). The expression proposed by Pennes for the exchange of thermal energy between flowing blood and the surrounding tissue (perfusion) is as

$$\dot{e}_p = \dot{p}\,\rho_b c_b\,(T_a - T)$$

Substituting the Pennes' perfusion heat source term expression, into the differential equation in cylindrical coordinates, results in

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{e}_m+\dot{p}\,\rho_b\,c_b\,(T_a-T)}{k}=0$$

With the following boundary conditions:

$$\left.\frac{dT}{dr}\right|_{r=0}=0 \qquad \text{and} \qquad T(r_m) = T_i$$

Assuming constant $\dot{e}_m, \dot{p}, \rho_b, c_b$ and T_a, the above differential equation reduces to

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right)-B^2\,\theta=0 \quad \text{or} \quad \frac{d^2T}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}-B^2\,\theta=0$$

where $B^2 = \dot{p}\,\rho_b\,c_b\,/\,k$ has units of (1/m) and $\theta = T - T_a - \dot{e}_m\,/\,\dot{p}\,\rho_b c_b$. The boundary conditions for the problem in terms of temperature excess θ are:

@ r = 0 $\left.\dfrac{d\theta}{dr}\right|_{r=0} = 0$

@ r = r_m $\theta(r_m) = T_i - T_a - \dot{e}_m \ / \ \dot{p}\ \rho_b\, c_b = \theta_i$

The differential equation in terms of temperature excess θ is a *modified Bessel equation* of order zero, and its general solution is of the form

$\theta(r) = C_1 I_0\,(Br) + C_2 K_0\,(Br)$

where I_0 and K_0 are modified, zero-order Bessel functions of the first and second kinds, respectively.

Applying the boundary condition at r = 0: $\left.\dfrac{d\theta}{dr}\right|_{r=0} = C_1 B I_1\,(0) + C_2 B K_1(0) = 0$

Since $K_1(0)$ is infinite, we must have $C_2 = 0$.

Applying the boundary condition at r = r_m: $\theta(r_m) = T_i - T_a - \dot{e}_m \ / \ \dot{p}\ \rho_b\, c_b = \theta_i = C_1 I_0\,(Br_m)$

Solving for C_1 we get $C_1 = \theta_i / I_0\,(Br_m)$

The complete solution for $\theta\,(r)$ *is*

$$\theta = \frac{\theta_i\, I_0(Br)}{I_0(Br_m)} = \left(T_i - T_a - \frac{\dot{e}_m}{\dot{p}\,\rho_b\, c_b}\right)\frac{I_0(Br)}{I_0(Br_m)} \qquad (1)$$

(b) In order to find T_i, use the above equation (note that T_i that appears in θ_i in Eq. (1) above is unknown). Follow the general procedure used in the example problem on the application of the bioheat transfer equation. Use Eq. (1) to calculate the rate at which heat leaves the muscle and enters the skin/fat layer at $r = r_m$ and equate it with the rate at which heat is transferred through the skin/fat layer and into the environment.

Using the Fourier's law of heat conduction, the rate of heat transfer that leaves the muscle at $r = r_m$ and enters the skin/fat layer is

$$\dot{Q}_{specified\ temp.} = -k_m A_r \left.\frac{dT}{dr}\right|_{r=r_m} = -k_m A_r \left.\frac{d\theta}{dr}\right|_{r=r_m} = -k_m\,(2\pi\, r_m)B\,\theta_i\,\frac{I_1(Br_m)}{I_0(Br_m)} \qquad (2)$$

The rate at which heat is transferred through the skin/fat layer and into the environment is obtained by using the thermal resistance network concept. In this case the thermal resistance is a combined series-parallel arrangement. Heat is transferred through the skin/fat layer by conduction in series and is in parallel with heat transfer by convection and radiation. The total rate of heat transfer through the skin/fat layer and into the environment (the rate of heat loss from the forearm) is

$$\dot{Q}_b = \frac{T_i - T_\infty}{R_{total}} \qquad (3)$$

where the total resistance is $R_{total} = R_{sf} + R_{conv-rad} = R_{sf} + \dfrac{R_{conv}\, R_{rad}}{R_{conv} + R_{rad}}$

and the individual resistances assuming unit length for the cylinder are

$$R_{sf} = \frac{ln\left(\dfrac{r_m + t_{sf}}{r_m}\right)}{2\pi\, k_{sf}}\ ,\ R_{conv} = \frac{1}{2\pi(r_m + t_{sf})h_{conv}} \ \text{ and } \ R_{rad} = \frac{1}{2\pi(r_m + t_{sf})h_{rad}}$$

Equating the rate of heat transfer that leaves the muscle at $r = r_m$ and enters the skin/fat layer, Eq. (2), with the rate at which heat is transferred through the skin/fat layer and into the environment, Eq. (3), yields

$$-k_m\,(2\pi\, r_m)B\,\theta_i\,\frac{I_1(Br_m)}{I_0(Br_m)} = \frac{T_i - T_\infty}{R_{total}}$$

The above equation can be solved for T_i, the final expression is

$$T_i = \frac{T_\infty I_0(Br_m) + k_m(2\pi r_m)BR_{total}\left(T_a + \dfrac{\dot{e}_m}{\dot{p}\rho_b c_b}\right)I_1(Br_m)}{I_0(Br_m) + k_m(2\pi r_m)BR_{total}\;I_1(Br_m)} \tag{4}$$

(c) Using the data given in the problem statement and the expression for the interface temperature (T_i) between the muscle and the outer skin/fat layer, Eq. (4), the interface temperature between the muscle and the outer skin/fat layer is

T_i= **34.2ºC**

The maximum temperature in the forearm (T_{max}) occurs at the center of the forearm (r = 0). Thus from Eq. (1), with $I_0\,(Br) = I_0\,(0) = 1$, we have

$$\theta_{max} = \left(T_{max} - T_a - \frac{\dot{e}_m}{\dot{p}\,\rho_b\,c_b}\right) = \frac{\theta_i\,I_0(0)}{I_0(Br_m)} = \left(T_i - T_a - \frac{\dot{e}_m}{\dot{p}\,\rho_b\,c_b}\right)\frac{1}{I_0(Br_m)}$$

or

$$T_{max} = T_a + \frac{\dot{e}_m}{\dot{p}\,\rho_b\,c_b} + \left(T_i - T_a - \frac{\dot{e}_m}{\dot{p}\,\rho_b\,c_b}\right)\frac{1}{I_0(Br_m)} \tag{5}$$

Using the data given in the problem statement and the expression for the maximum temperature (T_{max}) in the forearm, Eq. (5), we get

T_{max} = **36.7ºC**

Discussion The core body temperature is 37ºC. The maximum temperature is very close to the core body temperature which appears to be very reasonable.

Heat Transfer in Common Configurations

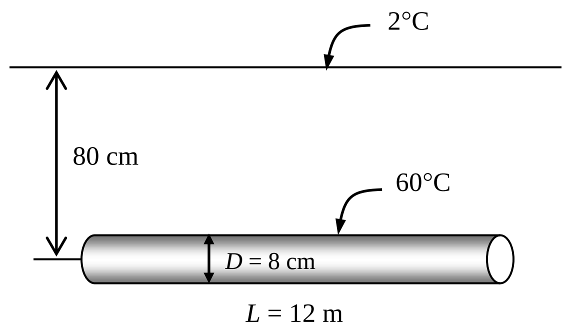
3-151C Under steady conditions, the rate of heat transfer between two surfaces is expressed as $\dot{Q} = Sk(T_1 - T_2)$ where S is the conduction shape factor. It is related to the thermal resistance by $S=1/(kR)$.

3-152C It provides an easy way of calculating the steady rate of heat transfer between two isothermal surfaces in common configurations.

3-153 The hot water pipe of a district heating system is buried in the soil. The rate of heat loss from the pipe is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the soil is constant.

Properties The thermal conductivity of the soil is given to be k = 0.9 W/m·°C.



Analysis Since z >1.5D, the shape factor for this configuration is given in Table 3-7 to be

$$S = \frac{2\pi L}{\ln(4z/D)} = \frac{2\pi(12\text{ m})}{\ln[4(0.8\text{ m})/(0.08\text{ m})]} = 20.44\text{ m}$$

Then the steady rate of heat transfer from the pipe becomes

$$\dot{Q} = Sk(T_1 - T_2) = (20.44\text{ m})(0.9\text{ W/m.}^\circ\text{C})(60-2)^\circ\text{C} = \mathbf{1067\ W}$$

3-154 A thin-walled cylindrical container, filled with chemicals undergoing exothermic reaction, is buried in fresh snow. The reaction provides a uniform heat generation. The snow surface is maintained at a specified temperature. The container surface temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the ground is constant. **4** Isothermal container surface.

Properties The thermal conductivity of fresh snow is k = 0.60 W/m·K (Table A-8).

Analysis The shape factor for this configuration is given in Table 3-7 (Case 1) to be

$$S = \frac{2\pi L}{\ln(4z/D)} = \frac{2\pi(1.5\text{ m})}{\ln[4(0.30\text{ m})/(0.10\text{ m})]} = 3.793\text{ m}$$

The heat transfer rate from the cylindrical container is

$$\dot{Q} = \dot{e}_{\text{gen}}\boldsymbol{V} = kS(T_1 - T_2)$$

Thus, the surface temperature of the container is

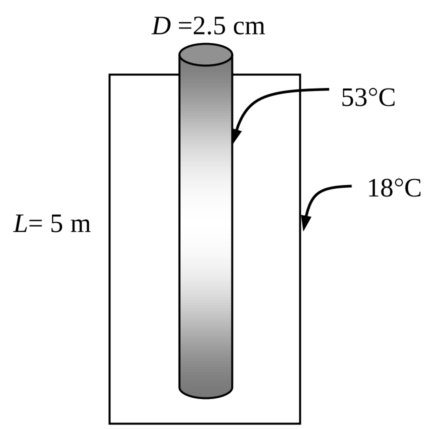
$$T_1 = \frac{\dot{e}_{\text{gen}}\boldsymbol{V}}{kS} + T_2 = \frac{\dot{e}_{\text{gen}}\pi D^2 L}{4kS} + T_2 = \frac{(900\text{ W/m}^3)\pi(0.10\text{ m})^2(1.5\text{ m})}{4(0.60\text{ W/m}\cdot\text{K})(3.793\text{ m})} + (-5°\text{C}) = \mathbf{-0.34°C}$$

Discussion The surface temperature of the container is below the freezing point of water, therefore the snow around the container will not melt.

3-155 Hot water flows through a 5-m long section of a thin walled hot water pipe that passes through the center of a 14-cm thick wall filled with fiberglass insulation. The rate of heat transfer from the pipe to the air in the rooms and the temperature drop of the hot water as it flows through the pipe are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the fiberglass insulation is constant. **4** The pipe is at the same temperature as the hot water.

Properties The thermal conductivity of fiberglass insulation is given to be k = 0.035 W/m·°C.



Analysis (*a*) The shape factor for this configuration is given in Table 3-7 to be

$$S = \frac{2\pi L}{\ln\left(\frac{8z}{\pi D}\right)} = \frac{2\pi(5\text{ m})}{\ln\left[\frac{8(0.07\text{ m})}{\pi(0.025\text{ m})}\right]} = 16\text{ m}$$

Then the steady rate of heat transfer from the pipe becomes

$$\dot{Q} = Sk(T_1 - T_2) = (16\text{ m})(0.035\text{ W/m.°C})(53 - 18)°\text{C} = \mathbf{19.6\ W}$$

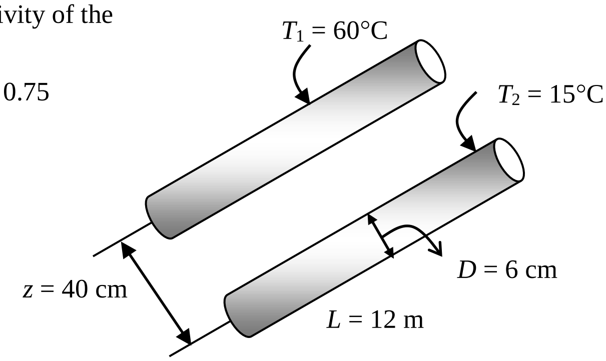
(*b*) Using the water properties at the room temperature, the temperature drop of the hot water as it flows through this 5-m section of the wall becomes

$$\dot{Q} = \dot{m}c_p\Delta T$$

$$\Delta T = \frac{\dot{Q}}{\dot{m}c_p} = \frac{\dot{Q}}{\rho\dot{\boldsymbol{V}}c_p} = \frac{\dot{Q}}{\rho VA_c c_p} = \frac{19.6\text{ J/s}}{(1000\text{ kg/m}^3)(0.4\text{ m/s})\left[\frac{\pi(0.025\text{ m})^2}{4}\right](4180\text{ J/kg.°C})} = \mathbf{0.024°C}$$

3-156 Hot and cold water pipes run parallel to each other in a thick concrete layer. The rate of heat transfer between the pipes is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the concrete is constant.



Properties The thermal conductivity of concrete is given to be k = 0.75 W/m·°C.

Analysis The shape factor for this configuration is given in Table 3-7 to be

$$S = \frac{2\pi L}{\cosh^{-1}\left(\dfrac{4z^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$$

$$= \frac{2\pi(12\text{ m})}{\cosh^{-1}\left(\dfrac{4(0.4\text{ m})^2 - (0.06\text{ m})^2 - (0.06\text{ m})^2}{2(0.06\text{ m})(0.06\text{ m})}\right)} = 14.59\text{ m}$$

Then the steady rate of heat transfer between the pipes becomes

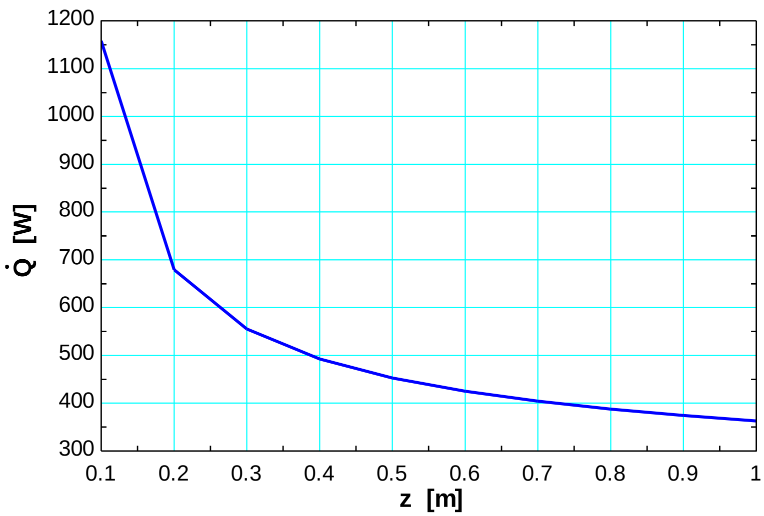
$$\dot{Q} = Sk(T_1 - T_2) = (14.59\text{ m})(0.75\text{ W/m.°C})(60-15)\text{°C} = \mathbf{492\ W}$$

3-157 Prob. 3-156 is reconsidered. The rate of heat transfer between the pipes as a function of the distance between the centerlines of the pipes is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=12 [m]
D_1=0.06 [m]
D_2=D_1
z=0.40 [m]
T_1=60 [C]
T_2=15 [C]
k=0.75 [W/m-C]

"ANALYSIS"
S=(2*pi*L)/(arccosh((4*z^2-D_1^2-D_2^2)/(2*D_1*D_2)))
Q_dot=S*k*(T_1-T_2)
```

z [m]	Q [W]
0.1	1158
0.2	679
0.3	555
0.4	492.3
0.5	452.8
0.6	425.1
0.7	404.2
0.8	387.7
0.9	374.2
1	362.9

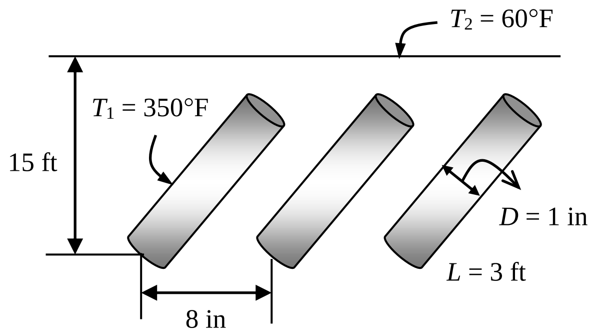


3-158E A row of used uranium fuel rods are buried in the ground parallel to each other. The rate of heat transfer from the fuel rods to the atmosphere through the soil is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the soil is constant.

Properties The thermal conductivity of the soil is given to be k = 0.6 Btu/h·ft·°F.

Analysis The shape factor for this configuration is given in Table 3-7 to be



$$S_{\text{total}} = 4\times\frac{2\pi L}{\ln\left(\dfrac{2w}{\pi D}\sinh\dfrac{2\pi z}{w}\right)}$$

$$= 4\times\frac{2\pi(3\text{ ft})}{\ln\left(\dfrac{2(8/12\text{ ft})}{\pi(1/12\text{ ft})}\sinh\dfrac{2\pi(15\text{ ft})}{(8/12\text{ ft})}\right)} = 0.5298\text{ ft}$$

Then the steady rate of heat transfer from the fuel rods becomes

$$\dot{Q} = S_{\text{total}}k(T_1 - T_2) = (0.5298\text{ ft})(0.6\text{ Btu/h.ft.°F})(350-60)\text{°F} = \mathbf{92.2\ Btu/h}$$

3-159 Hot water is flowing through a pipe that extends 2 m in the ambient air and continues in the ground before it enters the next building. The surface of the ground is covered with snow at 0°C. The total rate of heat loss from the hot water and the temperature drop of the hot water in the pipe are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the ground is constant. **4** The pipe is at the same temperature as the hot water.

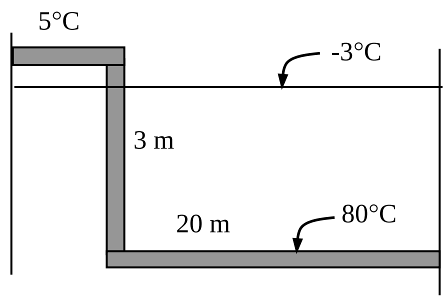
Properties The thermal conductivity of the ground is given to be k = 1.5 W/m·°C.

Analysis (a) We assume that the surface temperature of the tube is equal to the temperature of the water. Then the heat loss from the part of the tube that is on the ground is

$$A_s = \pi DL = \pi(0.05\text{ m})(2\text{ m}) = 0.3142\text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$

$$= (22\text{ W/m}^2.\text{°C})(0.3142\text{ m}^2)(80-5)\text{°C} = 518\text{ W}$$



Considering the shape factor, the heat loss for vertical part of the tube can be determined from

$$S = \frac{2\pi L}{\ln\left(\frac{4L}{D}\right)} = \frac{2\pi(3\text{ m})}{\ln\left[\frac{4(3\text{ m})}{(0.05\text{ m})}\right]} = 3.44\text{ m}$$

$$\dot{Q} = Sk(T_1 - T_2) = (3.44\text{ m})(1.5\text{ W/m.°C})[80-(-3)]\text{°C} = 428\text{ W}$$

The shape factor, and the rate of heat loss on the horizontal part that is in the ground are

$$S = \frac{2\pi L}{\ln\left(\frac{4z}{D}\right)} = \frac{2\pi(20\text{ m})}{\ln\left[\frac{4(3\text{ m})}{(0.05\text{ m})}\right]} = 22.9\text{ m}$$

$$\dot{Q} = Sk(T_1 - T_2) = (22.9\text{ m})(1.5\text{ W/m.°C})[80-(-3)]\text{°C} = 2851\text{ W}$$

and the total rate of heat loss from the hot water becomes

$$\dot{Q}_{\text{total}} = 518 + 428 + 2851 = \mathbf{3797\ W}$$

(b) Using the water properties at the room temperature, the temperature drop of the hot water as it flows through this 25-m section of the wall becomes

$$\dot{Q} = \dot{m}c_p\Delta T$$

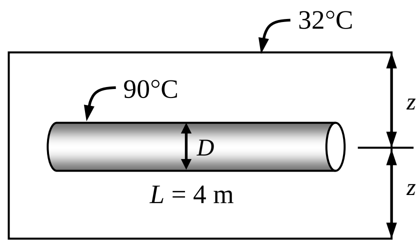
$$\Delta T = \frac{\dot{Q}}{\dot{m}c_p} = \frac{\dot{Q}}{(\rho\dot{\boldsymbol{V}})c_p} = \frac{\dot{Q}}{(\rho VA_c)c_p} = \frac{3797\text{ J/s}}{(1000\text{ kg/m}^3)(1.5\text{ m/s})\left[\frac{\pi(0.05\text{ m})^2}{4}\right](4180\text{ J/kg.°C})} = \mathbf{0.31°C}$$

3-160 Hot water passes through a row of 8 parallel pipes placed vertically in the middle of a concrete wall whose surfaces are exposed to a medium at 32°C with a heat transfer coefficient of 8 W/m^2.°C. The rate of heat loss from the hot water, and the surface temperature of the wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of concrete is constant.

Properties The thermal conductivity of concrete is given to be k = 0.75 W/m·°C.

Analysis The shape factor for this configuration is given in Table 3-7 to be



$$S = \frac{2\pi L}{\ln\left(\dfrac{8z}{\pi D}\right)} = \frac{2\pi(4\text{ m})}{\ln\left(\dfrac{8(0.075\text{ m})}{\pi(0.03\text{ m})}\right)} = 13.58\text{ m}$$

Then rate of heat loss from the hot water in 8 parallel pipes becomes

$$\dot{Q} = 8Sk(T_1 - T_2) = 8(13.58\text{ m})(0.75\text{ W/m.°C})(90-32)\text{°C} = \mathbf{4726\ W}$$

The surface temperature of the wall can be determined from

$$A_s = 2(4\text{ m})(8\text{ m}) = 64\text{ m}^2 \quad \text{(from both sides)}$$

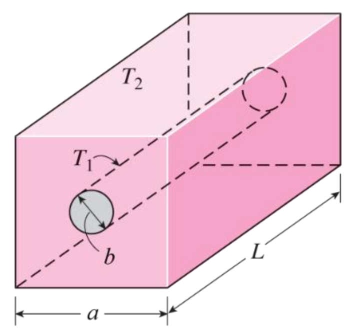
$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 32\text{°C} + \frac{4726\text{ W}}{(12\text{ W/m}^2\text{.°C})(64\text{ m}^2)} = \mathbf{38.2°C}$$

3-161 Two flow passages of the same length but of different cross-sectional shapes. Each flow passage is centered in a square solid bar of the same length. The configuration that has the higher rate of heat transfer through the square solid bar is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivitiesare constant. **4** Isothermal surfaces.

Analysis The shape factor of a circular flow passage centered in a square solid bar is given in Table 3-7 (Case 6) to be

$$S_{\text{cir}} = \frac{2\pi L}{\ln(1.08w/D)} = \frac{2\pi L}{\ln(1.08a/b)}$$



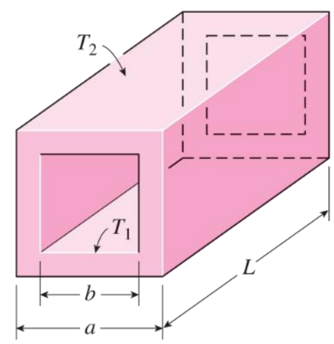
The shape factor of a square flow passage centered in a square solid bar is given in Table 3-7 (Case 10), for $a/b > 1.4$, to be

$$S_{\text{sq}} = \frac{2\pi L}{0.785\ln(a/b)} \quad (\text{for}\, a/b < 1.4)$$

and

$$S_{\text{sq}} = \frac{2\pi L}{0.93\ln(0.948a/b)} \quad (\text{for}\, a/b > 1.4)$$

The rate of heat transfer for both configurations can be expressed as



$$\frac{\dot{Q}_{\text{sq}}}{\dot{Q}_{\text{cir}}} = \frac{kS_{\text{sq}}(T_1 - T_2)}{kS_{\text{cir}}(T_1 - T_2)} = \frac{S_{\text{sq}}}{S_{\text{cir}}}$$

(*a*) For $a = 1.2b$ ($a/b < 1.4$),

$$\frac{\dot{Q}_{\text{sq}}}{\dot{Q}_{\text{cir}}} = \frac{S_{\text{sq}}}{S_{\text{cir}}} = \frac{\ln(1.08a/b)}{0.785\ln(a/b)} = \frac{\ln(1.08\times 1.2)}{0.785\ln(1.2)} = 1.812$$

(*b*) For $a = 2b$ ($a/b > 1.4$),

$$\frac{\dot{Q}_{\text{sq}}}{\dot{Q}_{\text{cir}}} = \frac{S_{\text{sq}}}{S_{\text{cir}}} = \frac{\ln(1.08a/b)}{0.93\ln(0.948a/b)} = \frac{\ln(1.08\times 2)}{0.93\ln(0.948\times 2)} = 1.294$$

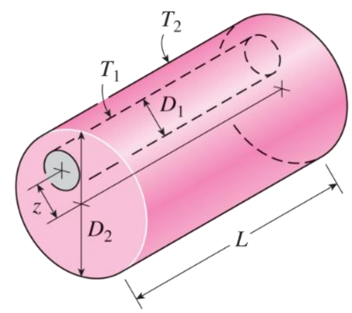
DiscussionFor both cases, thesquare flow passage has higher rate of heat transfer through the square solid bar than the circular flow passage. For $a/b < 1.4$, thesquare flow passage has 81.2% higher heat transfer rate than the circular flow passage. For $a/b > 1.4$, thesquare flow passage has 29.4% higher heat transfer rate than the circular flow passage.

3-162 A tube transporting steam is inserted eccentrically in a cylindrically insulation. The rate of heat transfer per unit length is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivityis constant. **4** Isothermal surfaces.

Properties The thermal conductivity of the insulation is given as$k$ = 0.73 W/m·K.

Analysis The shape factor for this configuration is given in Table 3-7 (Case 7) to be



$$\frac{S}{L}=\frac{2\pi}{\cosh^{-1}\left(\dfrac{D_1^2+D_2^2-4z^2}{2D_1D_2}\right)}=\frac{2\pi}{\cosh^{-1}\left[\dfrac{(0.02\text{ m})^2+(0.04\text{ m})^2-4(0.005\text{ m})^2}{2(0.02\text{ m})(0.04\text{ m})}\right]}=11.26$$

The heat transfer rate per unit lengththrough the insulation is

$$\frac{\dot{Q}}{L}=k\frac{S}{L}(T_1-T_2)=(0.73\text{ W/m}\cdot\text{K})(11.26)(100-30)(\text{K})=\mathbf{575\ W/m}$$

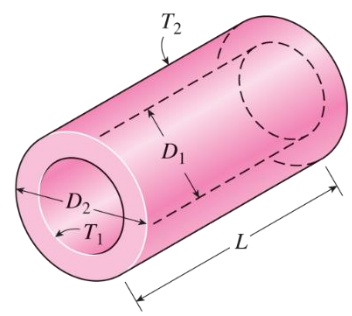
Discussion To reduce heat loss from the steam, the tube should be centered in the insulation.

3-163 Two circular tubes, one is properly centered in a cylindrical insulation material but the other is not. The configuration that has the higher rate of heat transfer through the insulation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivityis constant. **4** Isothermal surfaces.

Analysis The shape factor for a tube centered in a cylindrical insulation is given in Table 3-7 (Case 9) to be

$$S_{\text{cen}} = \frac{2\pi L}{\ln(D_2/D_1)}$$

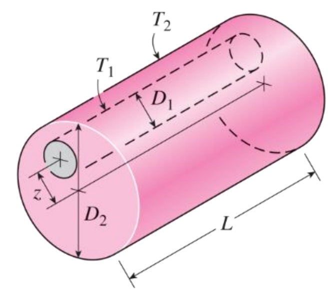


The shape factor for a tube that is eccentric in a cylindrical insulation is given in Table 3-7 (Case 7) to be

$$S_{\text{eccen}} = \frac{2\pi L}{\cosh^{-1}\left(\dfrac{D_1^2 + D_2^2 - 4z^2}{2D_1D_2}\right)}$$

The rate of heat transfer for both configurations can be expressed as

$$\begin{aligned}\frac{\dot{Q}_{\text{eccen}}}{\dot{Q}_{\text{cen}}} &= \frac{kS_{\text{eccen}}(T_1-T_2)}{kS_{\text{cen}}(T_1-T_2)} = \frac{S_{\text{eccen}}}{S_{\text{cen}}} \\ &= \frac{\ln(D_2/D_1)}{\cosh^{-1}\left(\dfrac{D_1^2 + D_2^2 - 4z^2}{2D_1D_2}\right)} \\ &= \frac{\ln(0.04\text{ m}/0.02\text{ m})}{\cosh^{-1}\left[\dfrac{(0.02\text{ m})^2 + (0.04\text{ m})^2 - 4(0.005\text{ m})^2}{2(0.02\text{ m})(0.04\text{ m})}\right]} \\ &= 1.149\end{aligned}$$



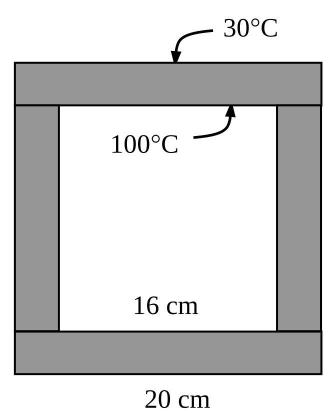
The tube that is eccentric in the cylindrical insulation has a 14.9% higher rate of heat transfer through the insulation than the one that is properly centered.

Discussion Thus, to limit heat loss the tube should be centered in the insulation.

3-164 The inner and outer surfaces of a long thick-walled concrete duct are maintained at specified temperatures. The rate of heat transfer through the walls of the duct is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the concrete is constant.

Properties The thermal conductivity of concrete is given to be $k = 0.75$ W/m·°C.



Analysis The shape factor for this configuration is given in Table 3-7 to be

$$\frac{a}{b} = \frac{20}{16} = 1.25 < 1.41 \longrightarrow S = \frac{2\pi L}{0.785\ln\left(\dfrac{a}{b}\right)} = \frac{2\pi(25\text{ m})}{0.785\ln 1.25} = 896.7\text{ m}$$

Then the steady rate of heat transfer through the walls of the duct becomes

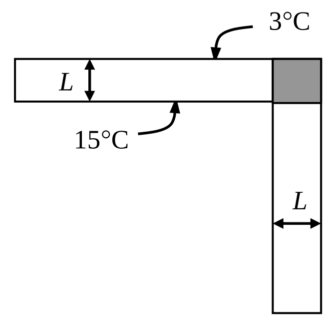
$$\dot{Q} = Sk(T_1 - T_2) = (896.7\text{ m})(0.75\text{ W/m.°C})(100-30)\text{°C} = 4.71\times10^4\ \text{W} = \mathbf{47.1kW}$$

3-165 The walls and the roof of the house are made of 20-cm thick concrete, and the inner and outer surfaces of the house are maintained at specified temperatures. The rate of heat loss from the house through its walls and the roof is to be determined, and the error involved in ignoring the edge and corner effects is to be assessed.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer at the edges and corners is two-or three-dimensional. **3** Thermal conductivity of the concrete is constant. **4** The edge effects of adjoining surfaces on heat transfer are to be considered.

Properties The thermal conductivity of the concrete is given to be k = 0.75 W/m·°C.

Analysis The rate of heat transfer excluding the edges and corners is first determined to be



$$A_{\text{total}} = (12-0.4)(12-0.4)+4(12-0.4)(6-0.2) = 403.7\ \text{m}^2$$

$$\dot{Q} = \frac{kA_{\text{total}}}{L}(T_1 - T_2) = \frac{(0.75\ \text{W/m.°C})(403.7\ \text{m}^2)}{0.2\ \text{m}}(15-3)\text{°C} = 18{,}167\ \text{W}$$

The heat transfer rate through the edges can be determined using the shape factor relations in Table 3-7,

$$\begin{aligned} S_{\text{corners+edges}} &= 4\times\text{corners} + 4\times\text{edges} = 4\times 0.15L + 4\times 0.54w \\ &= 4\times 0.15(0.2\ \text{m}) + 4\times 0.54(12\ \text{m}) = 26.04\ \text{m} \end{aligned}$$

$$\dot{Q}_{\text{corners+edges}} = S_{\text{corners+edges}}k(T_1 - T_2) = (26.04\ \text{m})(0.75\ \text{W/m.°C})(15-3)\text{°C} = 234\ \text{W}$$

and $\dot{Q}_{\text{total}} = 18{,}167 + 234 = 1.840\times 10^4\ \text{W} = \mathbf{18.4\ kW}$

Ignoring the edge effects of adjoining surfaces, the rate of heat transfer is determined from

$$A_{\text{total}} = (12)(12)+4(12)(6) = 432\ \text{m}^2$$

$$\dot{Q} = \frac{kA_{\text{total}}}{L}(T_1 - T_2) = \frac{(0.75\ \text{W/m.°C})(432\ \text{m}^2)}{0.2\ \text{m}}(15-3)\text{°C} = 1.94\times 10^4 = 19.4\ \text{kW}$$

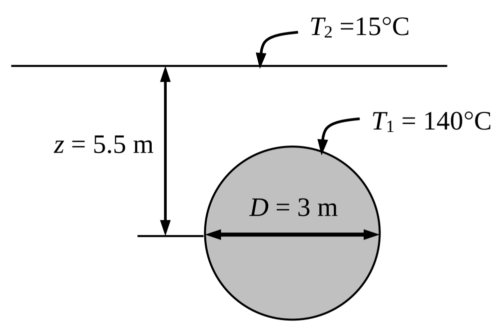
The percentage error involved in ignoring the effects of the edges then becomes

$$\%\text{error} = \frac{19.4-18.4}{18.4}\times 100 = \mathbf{5.4\%}$$

3-166 A spherical tank containing some radioactive material is buried in the ground. The tank and the ground surface are maintained at specified temperatures. The rate of heat transfer from the tank is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the ground is constant.

Properties The thermal conductivity of the ground is given to be k = 1.4 W/m·°C.



Analysis The shape factor for this configuration is given in Table 3-7 to be

$$S = \frac{2\pi D}{1-0.25\dfrac{D}{z}} = \frac{2\pi(3\ \text{m})}{1-0.25\dfrac{3\ \text{m}}{5.5\ \text{m}}} = 21.83\ \text{m}$$

Then the steady rate of heat transfer from the tank becomes

$$\dot{Q} = Sk(T_1 - T_2) = (21.83\ \text{m})(1.4\ \text{W/m.°C})(140-15)\text{°C} = \mathbf{3820\ W}$$

3-167 Radioactive material is stored in a spherical vessel that is buried underground. The ground surface temperature directly above the vessel is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the ground is constant.**4** Isothermal surfaces.

Properties The thermal conductivity of the ground is given to be k = 2.0 W/m·K.

Analysis The shape factor for this configuration is given in Table 3-7 (Case 15) to be

$$S = \frac{2\pi D}{1 - 0.25\dfrac{D}{z}} = \frac{2\pi(3.5\,\text{m})}{1 - 0.25\dfrac{3.5\,\text{m}}{10\,\text{m}}} = 24.1\,\text{m}$$

The heat transfer rate from the spherical container is

$$\dot{Q} = \dot{e}_{\text{gen}}\boldsymbol{V} = kS(T_1 - T_2)$$

Thus, the surface temperature of the ground is

$$T_2 = T_1 - \frac{\dot{e}_{\text{gen}}\boldsymbol{V}}{kS} = T_1 - \frac{\dot{e}_{\text{gen}}}{kS}\frac{\pi}{6}D^3$$

$$= 480°\text{C} - \frac{(1000\,\text{W/m}^3)(3.5\,\text{m})^3\pi}{(2.0\,\text{W/m}\cdot\text{K})(24.1\,\text{m})6} = \mathbf{14.2°C}$$

Since the ground surface directly above the vessel is at a temperature above freezing, snow will not cover that area but will be melted away.

Special Topic: Heat Transfer through the Walls and Roofs

3-168C The*R*-value of a wall is the thermal resistance of the wall per unit surface area. It is the same as the unit thermal resistance of the wall. It is the inverse of the *U*-factor of the wall, $R = 1/U$.

3-169C The effective emissivity for a plane-parallel air space is the "equivalent" emissivity of one surface for use in the relation $\dot{Q}_{\rm rad} = \varepsilon_{\rm effective}\, \sigma A_s (T_2^4 - T_1^4)$ that results in the same rate of radiation heat transfer between the two surfaces across the air space. It is determined from

$$\frac{1}{\varepsilon_{\rm effective}} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1$$

where ε_1 and ε_2 are the emissivities of the surfaces of the air space. When the effective emissivity is known, the radiation heat transfer through the air space is determined from the $\dot{Q}_{\rm rad}$ relation above.

3-170C The unit thermal resistances (*R*-value) of both 40-mm and 90-mm vertical air spaces are given to be the same, which implies that more than doubling the thickness of air space in a wall has no effect on heat transfer through the wall. This is not surprising since the convection currents that set in in the thicker air space offset any additional resistance due to a thicker air space.

3-171C Radiant barriers are highly reflective materials that minimize the radiation heat transfer between surfaces. Highly reflective materials such as aluminum foil or aluminum coated paper are suitable for use as radiant barriers. Yes, it is worthwhile to use radiant barriers in the attics of homes by covering at least one side of the attic (the roof or the ceiling side) since they reduce radiation heat transfer between the ceiling and the roof considerably.

3-172C The roof of a house whose attic space is ventilated effectively so that the air temperature in the attic is the same as the ambient air temperature at all times will still have an effect on heat transfer through the ceiling since the roof in this case will act as a radiation shield, and reduce heat transfer by radiation.

3-173 The *R*-value and the *U*-factor of a wood frame wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

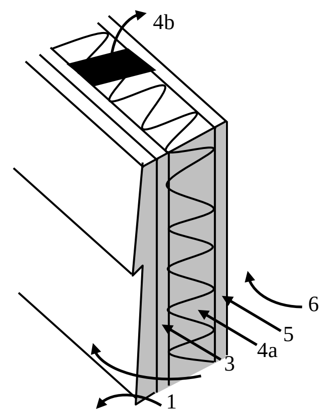
Properties The R-values of different materials are given in Table 3-8.

AnalysisThe schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the insulation and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factorsfor the insulation and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \text{ where } U_{\text{overall}} = (Uf_{\text{area}})_{\text{insulation}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.80 for insulation section and 0.20 for stud section since the headers that constitute a small part of the wall are to be treated as studs. Using the available *R*-values from Table 3-8 andcalculating others, the total *R*-values for each section is determined in the table below.

Construction	*R* -value, m^2.°C/W	
	Between studs	At studs
1. Outside surface, 12 km/h wind	0.044	0.044
2. Wood bevel lapped siding	0.14	0.14
3. Fiberboard sheathing, 13 mm	0.23	0.23
4a. Mineral fiber insulation, 140 mm	3.696	--
4b. Wood stud, 38 mm by 140 mm	--	0.98
5. Gypsum wallboard, 13 mm	0.079	0.079
6. Inside surface, still air	0.12	0.12



Total unit thermal resistance of each section, *R* (in m^2.°C/W)	4.309	1.593
The *U*-factor of each section, $U = 1/R$, in W/m^2.°C	0.232	0.628
Area fraction of each section, f_{area}	0.80	0.20
Overall *U*-factor, $U = \Sigma f_{\text{area,i}} U_{\text{i}} = 0.80\times0.232+0.20\times0.628$	**0.311 W/m^2.°C**	
Overall unit thermal resistance, $R = 1/U$	**3.213 m^2.°C/W**	

Therefore, the *R*-value and *U*-factor of the wall are R = 3.213 m^2.°C/W and U = 0.311 W/m^2.°C.

3-174 The change in the *R*-value of a wood frame wall due to replacing fiberwood sheathing in the wall by rigid foam sheathing is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

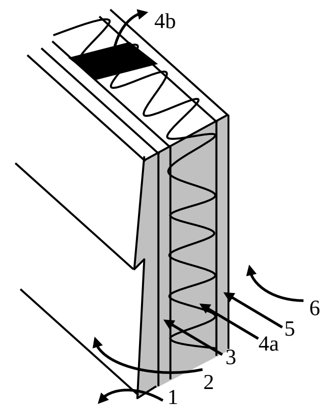
Properties The *R*-values of different materials are given in Table 3-8.

AnalysisThe schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the insulation and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factorsfor the insulation and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \text{ where } U_{\text{overall}} = (Uf_{\text{area}})_{\text{insulation}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.80 for insulation section and 0.20 for stud section since the headers that constitute a small part of the wall are to be treated as studs. Using the available *R*-values from Table 3-6 andcalculating others, the total *R*-values for each section of the existing wall is determined in the table below.

Construction	*R* -value, m^2.°C/W	
	Between studs	At studs
1. Outside surface, 12 km/h wind	0.044	0.044
2. Wood bevel lapped siding	0.14	0.14
3. Rigid foam, 25 mm	0.98	0.98
4a. Mineral fiber insulation, 140 mm	3.696	--
4b. Wood stud, 38 mm by 140 mm	--	0.98
5. Gypsum wallboard, 13 mm	0.079	0.079
6. Inside surface, still air	0.12	0.12



Total unit thermal resistance of each section, *R* (in m^2.°C/W)	5.059	2.343
The *U*-factor of each section, $U = 1/R$, in W/m^2.°C	0.198	0.426
Area fraction of each section, f_{area}	0.80	0.20
Overall *U*-factor, $U = \Sigma f_{\text{area,i}} U_{\text{i}}$ = 0.80×0.232+0.20×0.628	0.2436 W/m^2.°C	
Overall unit thermal resistance, $R = 1/U$	4.105 m^2.°C/W	

The *R*-value of the existing wall is R = 3.213 m^2.°C/W. Then the change in the *R*-value becomes

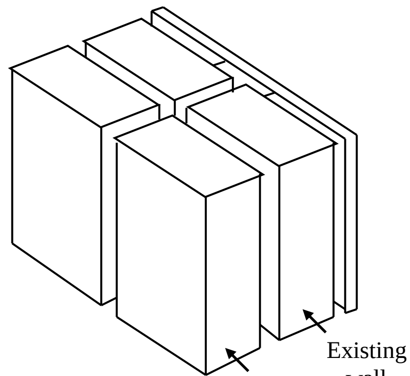
$$\%\,\text{Change} = \frac{\Delta R - \text{value}}{R - \text{value, old}} = \frac{4.105 - 3.213}{4.105} = 0.217 \quad (\text{or } \mathbf{21.7\%})$$

3-175 The U-value of a wall is given. A layer of face brick is added to the outside of a wall, leaving a 20-mm air space between the wall and the bricks. The new U-value of the wall and the rate of heat transfer through the wall is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The U-value of a wall is given to be U = 2.25 W/m^2.°C. The R - values of 100-mm face brick and a 20-mm air space between the wall and the bricks various layers are 0.075 and 0.170 m^2.°C/W, respectively.

Analysis The R-value of the existing wall for the winter conditions is



$$R_{\text{existing wall}} = 1/U_{\text{existing wall}} = 1/2.25 = 0.444\ \text{m}^2\cdot°\text{C/W}$$

Noting that the added thermal resistances are in series, the overall R-value of the wall becomes

$$\begin{aligned}R_{\text{modified wall}} &= R_{\text{existing wall}} + R_{\text{brick}} + R_{\text{air layer}}\\ &= 0.44 + 0.075 + 0.170 = 0.689\ \text{m}^2\cdot°\text{C/W}\end{aligned}$$

Then the U-value of the wall after modification becomes

$$R_{\text{modified wall}} = 1/U_{\text{modified wall}} = 1/0.689 = \mathbf{1.45\ m^2\cdot °C/W}$$

The rate of heat transfer through the modified wall is

$$\dot{Q}_{\text{wall}} = (UA)_{\text{wall}}(T_i - T_o) = (1.45\ \text{W/m}^2\cdot°\text{C})(3\times 7\ \text{m}^2)[22-(-25)°\text{C}] = \mathbf{1431\ W}$$

3-176 The winter R-value and the U-factor of a flat ceiling with an air space are to be determined for the cases of air space with reflective and nonreflective surfaces.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the ceiling is one-dimensional. **3** Thermal properties of the ceiling and the heat transfer coefficients are constant.

Properties The R-values are given in Table 3-8 for different materials, and in Table 3-11 for air layers.

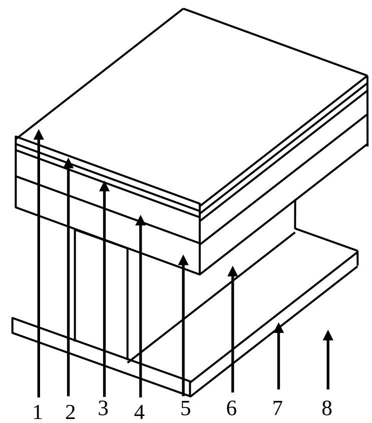
AnalysisThe schematic of the ceiling as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the U-factorsfor the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \text{ where } U_{\text{overall}} = (Uf_{\text{area}})_{\text{air space}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.82 for air space and 0.18 for stud section since the headers which constitute a small part of the wall are to be treated as studs.

(a) Nonreflective surfaces, $\varepsilon_1 = \varepsilon_2 = 0.9$ and thus $\varepsilon_{\text{effective}} = \dfrac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \dfrac{1}{1/0.9 + 1/0.9 - 1} = 0.82$

Construction	R -value, m^2.°C/W	
	Between studs	At studs
1. Still air above ceiling	0.12	0.044
2. Linoleum (R = 0.009 m^2.°C/W)	0.009	0.14
3. Felt (R = 0.011 m^2.°C/W)	0.011	0.23
4. Plywood, 13 mm	0.11	
5. Wood subfloor (R = 0.166 m^2.°C/W)	0.166	
6a. Air space, 90 mm, nonreflective	0.16	---
6b. Wood stud, 38 mm by 90 mm	---	0.63
7. Gypsum wallboard, 13 mm	0.079	0.079
8. Still air below ceiling	0.12	0.12



Total unit thermal resistance of each section, R (in m^2.°C/W)	0.775	1.243
The U-factor of each section, $U = 1/R$, in W/m^2.°C	1.290	0.805
Area fraction of each section, f_{area}	0.82	0.18
Overall U-factor, $U = \Sigma f_{\text{area,i}} U_{\text{i}}$ = 0.82×1.290+0.18×0.805	**1.203 W/m^2.°C**	
Overall unit thermal resistance, $R = 1/U$	**0.831 m^2.°C/W**	

(b) One-reflective surface, $\varepsilon_1 = 0.05$ and $\varepsilon_2 = 0.9 \rightarrow \varepsilon_{\text{effective}} = \dfrac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \dfrac{1}{1/0.05 + 1/0.9 - 1} = 0.05$

In this case we replace item 6a from 0.16 to 0.47 m^2.°C/W. It gives R = 1.085 m^2.°C/W and U = 0.922 W/ m^2.°C for the air space. Then,

Overall U-factor, $U = \Sigma f_{\text{area,i}} U_{\text{i}}$ = 0.82×1.085+0.18×0.805	**1.035 W/m^2.°C**
Overall unit thermal resistance, $R = 1/U$	**0.967 m^2.°C/W**

(c) Two-reflective surface, $\varepsilon_1 = \varepsilon_2 = 0.05 \rightarrow \varepsilon_{\text{effective}} = \dfrac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \dfrac{1}{1/0.05 + 1/0.05 - 1} = 0.03$

In this case we replace item 6a from 0.16 to 0.49 m^2.°C/W. It gives R = 1.105 m^2.°C/W and U = 0.905 W/ m^2.°C for the air space. Then,

Overall U-factor, $U = \Sigma f_{\text{area,i}} U_{\text{i}}$ = 0.82×1.105+0.18×0.805	**1.051 W/m^2.°C**
Overall unit thermal resistance, $R = 1/U$	**0.951 m^2.°C/W**

3-177 The winter R-value and the U-factor of a masonry cavity wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

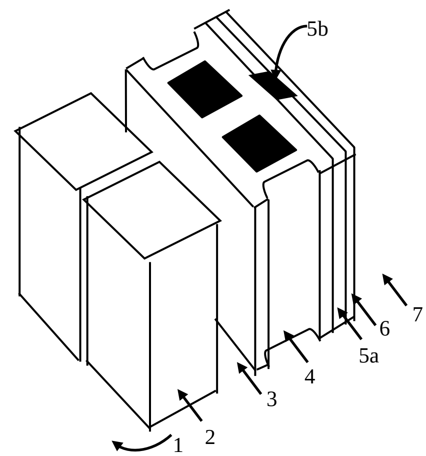
Properties The R-values of different materials are given in Table 3-8.

AnalysisThe schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the U-factorsfor the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \text{where } U_{\text{overall}} = (Uf_{\text{area}})_{\text{air space}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.84 for air space and 0.16 for the ferrings and similar structures. Using the available R-values from Tables 3-8 and 3-11 andcalculating others, the total R-values for each section of the existing wall is determined in the table below.

Construction	R -value, m^2.°C/W	
	Between furring	At furring
1. Outside surface, 24 km/h	0.030	0.030
2. Face brick, 100 mm	0.12	0.12
3. Air space, 90-mm, nonreflective	0.16	0.16
4. Concrete block, lightweight, 100-mm	0.27	0.27
5a. Air space, 20 mm, nonreflective	0.17	---
5b. Vertical ferring, 20 mm thick	---	0.94
6. Gypsum wallboard, 13	0.079	0.079
7. Inside surface, still air	0.12	0.12



Total unit thermal resistance of each section, R	0.949	1.719
The U-factor of each section, $U = 1/R$, in W/m^2.°C	1.054	0.582
Area fraction of each section, f_{area}	0.84	0.16
Overall U-factor, $U = \Sigma f_{\text{area,i}} U_{\text{i}} = 0.84\times1.054+0.16\times0.582$	**0.978 W/m^2.°C**	
Overall unit thermal resistance, $R = 1/U$	**1.02 m^2.°C/W**	

Therefore, the overall unit thermal resistance of the wall is R = 1.02 m^2.°C/W and the overall U-factor is U = 0.978 W/m^2.°C. These values account for the effects of the vertical ferring.

3-178 The winter *R*-value and the *U*-factor of a masonry cavity wall with a reflective surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 3-8. The *R*-values of air spaces are given in Table 3-11.

AnalysisThe schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factorsfor the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

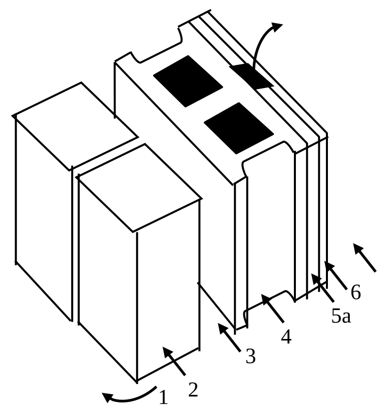
$$R_{\text{overall}} = 1/U_{\text{overall}} \text{ where } U_{\text{overall}} = (Uf_{\text{area}})_{\text{air space}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.84 for air space and 0.16 for the ferrings and similar structures. For an air space with one-reflective surface, we have $\varepsilon_1 = 0.05$ and $\varepsilon_2 = 0.9$, and thus

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \frac{1}{1/0.05 + 1/0.9 - 1} = 0.05$$

Using the available *R*-values from Tables 3-8 and 3-11 andcalculating others, the total *R*-values for each section of the existing wall is determined in the table below.

Construction	*R* -value, m^2.°C/W	
	Between furring	At furring
1. Outside surface, 24 km/h	0.030	0.030
2. Face brick, 100 mm	0.12	0.12
3. Air space, 90-mm, reflective with ε = 0.05	0.45	0.45
4. Concrete block, lightweight, 100-mm	0.27	0.27
5a. Air space, 20 mm, reflective with ε =0.05	0.49	---
5b. Vertical ferring, 20 mm thick	---	0.94
6. Gypsum wallboard, 13	0.079	0.079
7. Inside surface, still air	0.12	0.12



Total unit thermal resistance of each section, *R*	1.559	2.009
The *U*-factor of each section, *U* = 1/*R*, in W/m^2.°C	0.641	0.498
Area fraction of each section, f_{area}	0.84	0.16
Overall *U*-factor, $U = \Sigma f_{\text{area,i}} U_i = 0.84\times0.641+0.16\times0.498$	**0.618 W/m^2.°C**	
Overall unit thermal resistance, *R* = 1/*U*	**1.62 m^2.°C/W**	

Therefore, the overall unit thermal resistance of the wall is *R* = 1.62 m^2.°C/W and the overall U-factor is *U* = 0.618 W/m^2.°C. These values account for the effects of the vertical ferring.

DiscussionThe change in the *U*-value as a result of adding reflective surfaces is

$$\text{Change} = \frac{\Delta U - \text{value}}{U - \text{value, nonreflective}} = \frac{0.978 - 0.618}{0.978} = 0.368$$

Therefore, the rate of heat transfer through the wall will decrease by 36.8% as a result of adding a reflective surface.

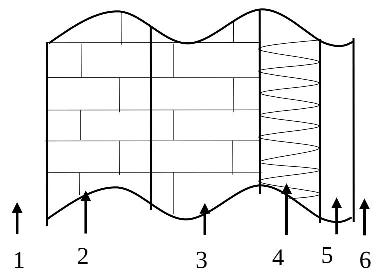
3-179 The winter *R*-value and the *U*-factor of a masonry wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 3-8.

AnalysisUsing the available *R*-values from Tables 3-8, the total *R*-value of the wall is determined in the table below.

Construction	*R*-value, $m^2.°C/W$
1. Outside surface, 24 km/h	0.030
2. Face brick, 100 mm	0.075
3. Common brick, 100 mm	0.12
4. Urethane foam insulation, 25-mm	0.98
5. Gypsum wallboard, 13 mm	0.079
6. Inside surface, still air	0.12



Total unit thermal resistance of each section, *R*	**1.404 m²·°C/W**
The *U*-factor of each section, *U* = 1/*R*	**0.712 W/m²·°C**

Therefore, the overall unit thermal resistance of the wall is R = 1.404 $m^2.°C/W$ and the overall U-factor is U = 0.712 $W/m^2.°C$.

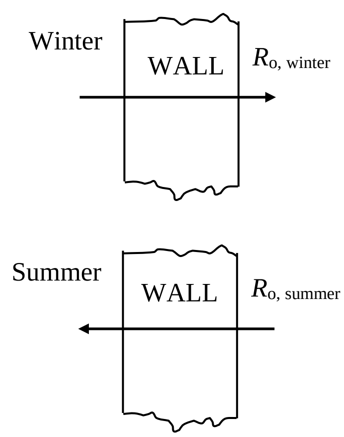
3-180 The *U*-value of a wall under winter design conditions is given. The *U*-value of the wall under summer design conditions is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant except the one at the outer surface.

Properties The R-values at the outer surface of a wall for summer (12 km/h winds) and winter (24 km/h winds) conditions are given in Table 3-8 to be $R_{\text{o, summer}}$ = 0.044 $m^2.°C/W$ and $R_{\text{o, winter}}$ =0.030 $m^2.°C/W$.

Analysis The *R*-value of the existing wall is

$$R_{\text{winter}} = 1/U_{\text{winter}} = 1/1.40 = 0.714\ \text{m}^2\cdot°\text{C/W}$$



Noting that the added and removed thermal resistances are in series, the overall *R*-value of the wall under summer conditions becomes

$$\begin{aligned} R_{\text{summer}} &= R_{\text{winter}} - R_{\text{o,winter}} + R_{\text{o,summer}} \\ &= 0.714 - 0.030 + 0.044 \\ &= 0.728\ \text{m}^2\cdot°\text{C/W} \end{aligned}$$

Then the summer *U*-value of the wall becomes

$$R_{\text{summer}} = 1/U_{\text{summer}} = 1/0.728 = \mathbf{1.37\ m^2\cdot°C/W}$$

3-181E The *R*-value and the *U*-factor of a masonry cavity wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

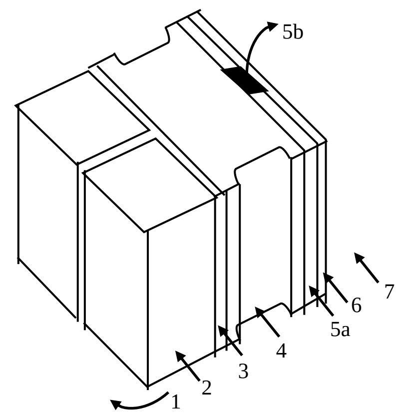
Properties The R-values of different materials are given in Table 3-8.

AnalysisThe schematic of the wall as well as the different elements used in its construction are shown below. Heat transfer through the air space and through the studs will meet different resistances, and thus we need to analyze the thermal resistance for each path separately. Once the unit thermal resistances and the *U*-factorsfor the air space and stud sections are available, the overall average thermal resistance for the entire wall can be determined from

$$R_{\text{overall}} = 1/U_{\text{overall}} \text{ where } U_{\text{overall}} = (Uf_{\text{area}})_{\text{air space}} + (Uf_{\text{area}})_{\text{stud}}$$

and the value of the area fraction f_{area} is 0.80 for air space and 0.20 for the ferrings and similar structures. Using the available *R*-values from Table 3-8 andcalculating others, the total *R*-values for each section of the existing wall is determined in the table below.

Construction	*R* -value, h.ft^2.°F/Btu	
	Between furring	At furring
1. Outside surface, 15 mph wind	0.17	0.17
2. Face brick, 4 in	0.43	0.43
3. Cement mortar, 0.5 in	0.10	0.10
4. Concrete block, 4-in	1.51	1.51
5a. Air space, 3/4-in, nonreflective	2.91	--
5b. Nominal 1 × 3 vertical furring	--	0.94
6. Gypsum wallboard, 0.5 in	0.45	0.45
7. Inside surface, still air	0.68	0.68



Total unit thermal resistance of each section, *R*	6.25	4.28
The *U*-factor of each section, $U = 1/R$, in Btu/h.ft^2.°F	0.160	0.234
Area fraction of each section, f_{area}	0.80	0.20
Overall *U*-factor, $U = \Sigma f_{\text{area,i}} U_i = 0.80\times0.160+0.20\times0.234$	**0.175 Btu/h.ft^2.°F**	
Overall unit thermal resistance, $R = 1/U$	**5.72 h.ft^2.°F/Btu**	

Therefore, the overall unit thermal resistance of the wall is $R = 5.72$ h.ft^2.°F/Btu and the overall U-factor is $U = 0.175$ Btu/h.ft^2.°F. These values account for the effects of the vertical ferring.

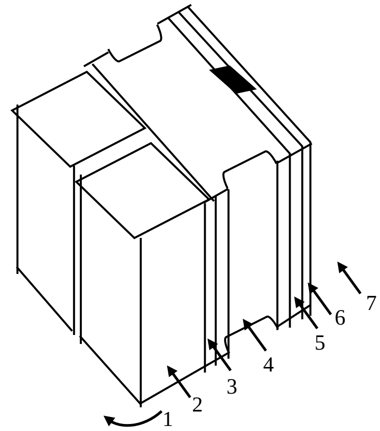
3-182 The summer and winter *R*-values of a masonry wall are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant. **4** The air cavity does not have any reflecting surfaces.

Properties The R-values of different materials are given in Table 3-8.

AnalysisUsing the available *R*-values from Tables 3-8, the total *R*-value of the wall is determined in the table below.

	R -value, m^2.°C/W	
Construction	Summer	Winter
1a. Outside surface, 24 km/h (winter)	---	0.030
1b. Outside surface, 12 km/h (summer)	0.044	---
2. Face brick, 100 mm	0.075	0.075
3. Cement mortar, 13 mm	0.018	0.018
4. Concrete block, lightweight, 100 mm	0.27	0.27
5. Air space, nonreflecting, 40-mm	0.16	0.16
5. Plaster board, 20 mm	0.122	0.122
6. Inside surface, still air	0.12	0.12



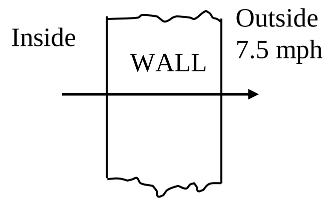
Total unit thermal resistance of each section (the *R*-value) ,m^2.°C/W	**0.809**	**0.795**

Therefore, the overall unit thermal resistance of the wall is R = 0.809 m^2.°C/W in summer and R = 0.795 m^2.°C/W in winter.

3-183E The *U*-value of a wall for 7.5 mph winds outside are given. The *U*-value of the wall for the case of 15 mph winds outside is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant except the one at the outer surface.

Properties The R-values at the outer surface of a wall for summer (7.5 mph winds) and winter (15 mph winds) conditions are given in Table 3-8 to be



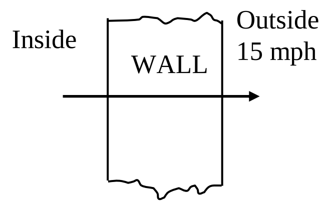
$$R_{\text{o, 7.5 mph}} = R_{\text{o, summer}} = 0.25 \text{ h.ft}^2.°\text{F/Btu}$$

and $$R_{\text{o, 15 mph}} = R_{\text{o, winter}} = 0.17 \text{ h.ft}^2.°\text{F/Btu}$$

Analysis The *R*-value of the wall at 7.5 mph winds (summer) is

$$R_{\text{wall, 7.5mph}} = 1/U_{\text{wall, 7.5mph}} = 1/0.075 = 13.33 \text{ h.ft}^2 \cdot °\text{F/Btu}$$

Noting that the added and removed thermal resistances are in series, the overall *R*-value of the wall at 15 mph (winter) conditions is obtained by replacing the summer value of outer convection resistance by the winter value,



$$R_{\text{wall,15 mph}} = R_{\text{wall,7.5 mph}} - R_{\text{o,7.5 mph}} + R_{\text{o,15 mph}}$$
$$= 13.33 - 0.25 + 0.17 = 13.25 \text{ h.ft}^2 \cdot °\text{F/Btu}$$

Then the U-value of the wall at 15 mph winds becomes

$$R_{\text{wall,15 mph}} = 1/U_{\text{wal,15 mph}} = 1/13.25 = \mathbf{0.0755\ Btu/h.ft^2 \cdot °F}$$

Discussion Note that the effect of doubling the wind velocity on the *U*-value of the wall is less than 1 percent since

$$\text{Change} = \frac{\Delta U - \text{value}}{U - \text{value}} = \frac{0.0755 - 0.075}{0.075} = 0.0067 \quad (\text{or } 0.67\%)$$

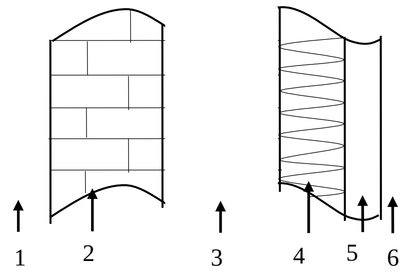
3-184 Two homes are identical, except that their walls are constructed differently. The house that is more energy efficient is to be determined.

Assumptions **1** The homes are identical, except that their walls are constructed differently. **2** Heat transfer through the wall is one-dimensional. **3** Thermal properties of the wall and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Table 3-8.

AnalysisUsing the available *R*-values from Tables 3-8, the total *R*-value of the masonry wall is determined in the table below.

Construction	*R* -value, m^2.°C/W
1. Outside surface, 24 km/h (winter)	0.030
2. Concrete block, light weight, 200 mm	2×0.27=0.54
3. Air space, nonreflecting, 20 mm	0.17
5. Plasterboard, 20 mm	0.12
6. Inside surface, still air	0.12



Total unit thermal resistance (the *R*-value)	**0.98 m^2.°C/W**

which is less than 2.4 m^2.°C/W. Therefore, the standard *R*-2.4 m^2.°C/W wall is better insulated and thus it is more energy efficient.

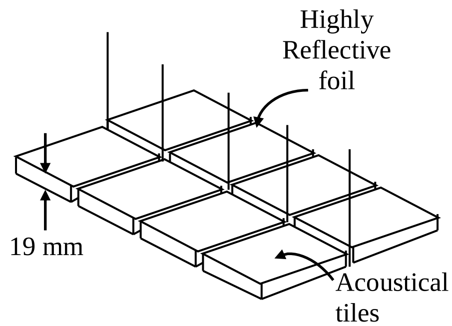
3-185 A ceiling consists of a layer of reflective acoustical tiles. The *R*-value of the ceiling is to be determined for winter conditions.

Assumptions **1** Heat transfer through the ceiling is one-dimensional. **3** Thermal properties of the ceiling and the heat transfer coefficients are constant.

Properties The R-values of different materials are given in Tables 3-8 and 3-9.

AnalysisUsing the available *R*-values, the total *R*-value of the ceiling is determined in the table below.

Construction	*R* -value, m^2.°C/W
1. Still air, reflective horizontal surface facing up	R = 1/h = 1/4.32 = 0.23
2. Acoustic tile, 19 mm	0.32
3. Still air, horizontal surface, facing down	R = 1/h = 1/9.26 = 0.11



Total unit thermal resistance (the *R*-value)	**0.66 m^2.°C/W**

Therefore, the R-value of the hanging ceiling is 0.66 m^2.°C/W.

Review Problems

3-186 Two persons are wearing different clothes made of different materials with different surface areas. The fractions of heat lost from each person's body by perspiration are to be determined.

Assumptions **1** Heat transfer is steady. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is accounted for in the heat transfer coefficient. **5** The human body is assumed to be cylindrical in shape for heat transfer purposes.

Properties The thermal conductivities of the leather and synthetic fabric are given to be k = 0.159 W/m·°C and k = 0.13 W/m·°C, respectively.

Analysis The surface area of each body is first determined from

$$A_1 = \pi DL/2 = \pi(0.25\,\text{m})(1.7\,\text{m})/2 = 0.6675\,\text{m}^2$$

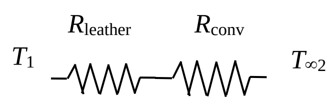
$$A_2 = 2A_1 = 2\times 0.6675 = 1.335\,\text{m}^2$$

The sensible heat lost from the first person's body is

$$R_{\text{leather}} = \frac{L}{kA} = \frac{0.001\,\text{m}}{(0.159\ \text{W/m.°C})(0.6675\,\text{m}^2)} = 0.00942\ \text{°C/W}$$

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(15\ \text{W/m}^2.\text{°C})(0.6675\,\text{m}^2)} = 0.09988\ \text{°C/W}$$

$$R_{\text{total}} = R_{\text{leather}} + R_{\text{conv}} = 0.00942 + 0.09988 = 0.10930\ \text{°C/W}$$



The total sensible heat transfer is the sum of heat transferred through the clothes and the skin

$$\dot{Q}_{\text{clothes}} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} = \frac{(32-30)\text{°C}}{0.10930\text{°C/W}} = 18.3\ \text{W}$$

$$\dot{Q}_{\text{skin}} = \frac{T_1 - T_{\infty 2}}{R_{\text{conv}}} = \frac{(32-30)\text{°C}}{0.09988\text{°C/W}} = 20.0\ \text{W}$$

$$\dot{Q}_{\text{sensible}} = \dot{Q}_{\text{clothes}} + \dot{Q}_{\text{skin}} = 18.3 + 20 = 38.3\ \text{W}$$

Then the fraction of heat lost by respiration becomes

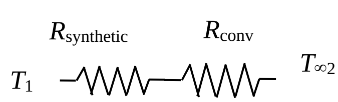
$$f \doteq \frac{\dot{Q}_{\text{respiration}}}{\dot{Q}_{\text{total}}} = \frac{\dot{Q}_{\text{total}} - \dot{Q}_{\text{sensible}}}{\dot{Q}_{\text{total}}} = \frac{60-38.3}{60} = \mathbf{0.362}$$

Repeating similar calculations for the second person's body

$$R_{\text{synthetic}} = \frac{L}{kA} = \frac{0.001\,\text{m}}{(0.13\ \text{W/m.°C})(1.335\,\text{m}^2)} = 0.00576\ \text{°C/W}$$

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(15\ \text{W/m}^2.\text{°C})(1.335\,\text{m}^2)} = 0.04994\ \text{°C/W}$$

$$R_{\text{total}} = R_{\text{leather}} + R_{\text{conv}} = 0.00576 + 0.04994 = 0.05570\ \text{°C/W}$$



$$\dot{Q}_{\text{sensible}} = \frac{T_1 - T_{\infty 2}}{R_{\text{total}}} = \frac{(32-30)\text{°C}}{0.05570\text{°C/W}} = 35.9\ \text{W}$$

$$f \doteq \frac{\dot{Q}_{\text{respiration}}}{\dot{Q}_{\text{total}}} = \frac{\dot{Q}_{\text{total}} - \dot{Q}_{\text{sensible}}}{\dot{Q}_{\text{total}}} = \frac{60-35.9}{60} = \mathbf{0.402}$$

3-187 Cold conditioned air is flowing inside a duct of square cross-section. The maximum length of the duct for a specified temperature increase in the duct is to be determined.

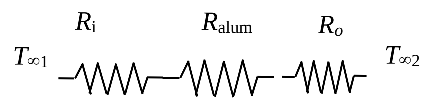
Assumptions **1** Heat transfer is steady. **2** Heat transfer is one-dimensional. **3** Thermal conductivities are constant. **4** Steady one-dimensional heat conduction relations can be used due to small thickness of the duct wall. **5** When calculating the conduction thermal resistance of aluminum, the average of inner and outer surface areas will be used.

Properties The thermal conductivity of aluminum is given to be 237 W/m·°C. The specific heat of air at the given temperature is c_p = 1006 J/kg·°C (Table A-15).

Analysis The inner and the outer surface areas of the duct per unit length and the individual thermal resistances are

$$A_1 = 4a_1 L = 4(0.22\text{ m})(1\text{ m}) = 0.88\text{ m}^2$$

$$A_2 = 4a_2 L = 4(0.25\text{ m})(1\text{ m}) = 1.0\text{ m}^2$$



$$R_\text{i} = \frac{1}{h_1 A} = \frac{1}{(75\text{ W/m}^2.°\text{C})(0.88\text{ m}^2)} = 0.01515°\text{C/W}$$

$$R_\text{alum} = \frac{L}{kA} = \frac{0.015\text{ m}}{(237\text{ W/m.°C})\left[(0.88+1)/2\right]\text{m}^2} = 0.00007\text{ °C/W}$$

$$R_\text{o} = \frac{1}{h_2 A} = \frac{1}{(13\text{ W/m}^2.°\text{C})(1.0\text{ m}^2)} = 0.07692°\text{C/W}$$

$$R_\text{total} = R_\text{i} + R_\text{alum} + R_\text{o} = 0.01515 + 0.00007 + 0.07692 = 0.09214\text{ °C/W}$$

The rate of heat loss from the air inside the duct is

$$\dot{Q} = \frac{T_{\infty 2} - T_{\infty 1}}{R_\text{total}} = \frac{(33-12)°\text{C}}{0.09214°\text{C/W}} = 228\text{ W}$$

For a temperature rise of 1°C, the air inside the duct should gain heat at a rate of

$$\dot{Q}_\text{total} = \dot{m}c_p\Delta T = (0.8\text{ kg/s})(1006\text{ J/kg.°C})(1°\text{C}) = 805\text{ W}$$

Then the maximum length of the duct becomes

$$L = \frac{\dot{Q}_\text{total}}{\dot{Q}} = \frac{805\text{ W}}{228\text{ W}} = \mathbf{3.53\text{ m}}$$

3-188 Hot water is flowing through a 15-m section of a cast iron pipe. The pipe is exposed to cold air and surfaces in the basement, and it experiences a 3°C-temperature drop. The combined convection and radiation heat transfer coefficient at the outer surface of the pipe is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any significant change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no significant variation in the axial direction. **3** Thermal properties are constant.

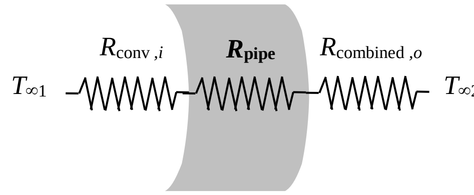
Properties The thermal conductivity of cast iron is given to be k = 52 W/m·°C.

Analysis Using water properties at room temperature, the mass flow rate of water and rate of heat transfer from the water are determined to be

$$\dot{m} = \rho \dot{\mathbf{V}}_c = \rho V A_c = (1000\ \text{kg/m}^3)(1.5\ \text{m/s})\left[\pi(0.03)^2/4\right]\text{m}^2 = 1.06\ \text{kg/s}$$

$$\dot{Q} = \dot{m} c_p \Delta T = (1.06\ \text{kg/s})(4180\ \text{J/kg.°C})(70-67)\text{°C} = 13{,}296\ \text{W}$$

The thermal resistances for convection in the pipe and the pipe itself are



$$R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k L} = \frac{\ln(1.75/1.5)}{2\pi(52\ \text{W/m.°C})(15\ \text{m})} = 0.000031\ \text{°C/W}$$

$$R_{\text{conv,i}} = \frac{1}{h_i A_i} = \frac{1}{(400\ \text{W/m}^2.\text{°C})[\pi(0.03)(15)]\text{m}^2} = 0.001768\ \text{°C/W}$$

Using arithmetic mean temperature (70+67)/2 = 68.5°C for water, the heat transfer can be expressed as

$$\dot{Q} = \frac{T_{\infty,1,ave} - T_{\infty 2}}{R_{\text{total}}} = \frac{T_{\infty,1,ave} - T_{\infty 2}}{R_{\text{conv,i}} + R_{\text{pipe}} + R_{\text{combined,o}}} = \frac{T_{\infty,1,ave} - T_{\infty 2}}{R_{\text{conv,i}} + R_{\text{pipe}} + \dfrac{1}{h_{\text{combined}} A_o}}$$

Substituting,

$$13{,}296\ \text{W} = \frac{(68.5-15)\text{°C}}{(0.000031\ \text{°C/W}) + (0.001768\ \text{°C/W}) + \dfrac{1}{h_{\text{combined}}[\pi(0.035)(15)]\text{m}^2}}$$

Solving for the combined heat transfer coefficient gives

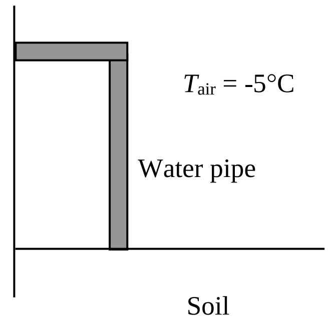
$$h_{\text{combined}} = \mathbf{272.5\ W/m^2.°C}$$

3-189 The plumbing system of a house involves some section of a plastic pipe exposed to the ambient air. The pipe is initially filled with stationary water at 0°C. It is to be determined if the water in the pipe will completely freeze during a cold night.

Assumptions **1** Heat transfer is transient, but can be treated as steady since the water temperature remains constant during freezing. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal properties of water are constant. **4** The water in the pipe is stationary, and its initial temperature is 0°C. **5** The convection resistance inside the pipe is negligible so that the inner surface temperature of the pipe is 0°C.

Properties The thermal conductivity of the pipe is given to be k = 0.16 W/m·°C. The density and latent heat of fusion of water at 0°C are ρ = 1000 kg/m^3 and h_{if} = 333.7 kJ/kg (Table A-9).

Analysis We assume the inner surface of the pipe to be at 0°C at all times. The thermal resistances involved and the rate of heat transfer are



$$R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(1.2/1)}{2\pi(0.16\text{ W/m.°C})(0.5\text{ m})} = 0.3627\text{ °C/W}$$

$$R_{\text{conv,o}} = \frac{1}{h_o A} = \frac{1}{(40\text{ W/m}^2\text{.°C})[\pi(0.024\text{ m})(0.5\text{ m})]} = 0.6631\text{ °C/W}$$

$$R_{\text{total}} = R_{\text{pipe}} + R_{\text{conv,o}} = 0.3627 + 0.6631 = 1.0258\text{ °C/W}$$

$$\dot{Q} = \frac{T_{s1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[0-(-5)]\text{°C}}{1.0258\text{ °C/W}} = 4.874\text{ W}$$

The total amount of heat lost by the water during a 14-h period that night is

$$Q = \dot{Q}\Delta t = (4.874\text{ J/s})(14\times 3600\text{ s}) = 245.7\text{ kJ}$$

The amount of heat required to freeze the water in the pipe completely is

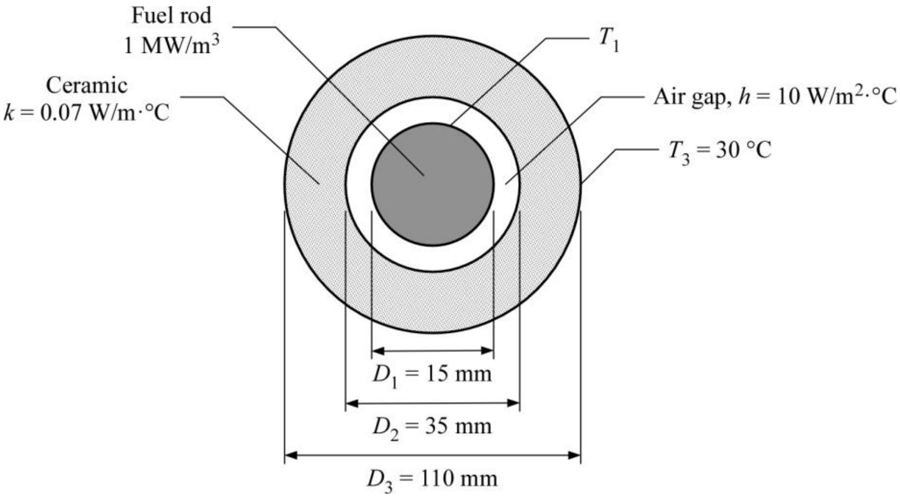
$$m = \rho \boldsymbol{V} = \rho\pi r^2 L = (1000\text{ kg/m}^3)\pi(0.01\text{ m})^2(0.5\text{ m}) = 0.157\text{ kg}$$
$$Q = mh_{fg} = (0.157\text{ kg})(333.7\text{ kJ/kg}) = 52.4\text{ kJ}$$

The water in the pipe will **freeze completely** that night since the amount heat loss is greater than the amount it takes to freeze the water completely $(245.7 > 52.4)$.

3-190 A nuclear fuel rod is encased in a concentric hollow ceramic cylinder, which created an air gap between the rod and the hollow cylinder. The surface temperature of the fuel rod is to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat generation in the fuel rod is uniform. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of ceramic is given to be 0.07 W/m · °C.



Analysis The combined thermal resistance between the nuclear fuel rod surface and the outer surface of the ceramic cylinder is

$$R_{\text{combined}} = R_{\text{conv, rod}} + R_{\text{conv, cyl}} + R_{\text{cond, cyl}}$$
$$= \frac{1}{\pi D_1 L h} + \frac{1}{\pi D_2 L h} + \frac{\ln(D_3 / D_2)}{2\pi L k}$$

or

$$R_{\text{combined}} L = \frac{1}{\pi D_1 h} + \frac{1}{\pi D_2 h} + \frac{\ln(D_3 / D_2)}{2\pi k}$$
$$= \frac{1}{\pi(0.015\text{ m})(10\text{ W/m}^2\cdot{}^\circ\text{C})} + \frac{1}{\pi(0.035\text{ m})(10\text{ W/m}^2\cdot{}^\circ\text{C})} + \frac{\ln(0.110/0.035)}{2\pi(0.07\text{ W/m}\cdot{}^\circ\text{C})}$$
$$= 5.635\text{ m}\cdot{}^\circ\text{C/W}$$

The heat generated by the fuel rod is dissipated through the air gap and the ceramic cylinder, and can be expressed as

$$\dot{Q}_{\text{gen}} = \frac{T_1 - T_3}{R_{\text{combined}}} \text{ or } \frac{\dot{Q}_{\text{gen}}}{L} = \frac{T_1 - T_3}{R_{\text{combined}} L}$$

The surface temperature of the fuel rod is

$$T_1 = \left(\frac{\dot{Q}_{\text{gen}}}{L}\right) R_{\text{combined}} L + T_3$$

$$T_1 = (1\times10^6\text{ W/m}^3)\frac{\pi}{4}(0.015\text{ m})^2(5.635\text{ m}\cdot{}^\circ\text{C/W}) + 30\,^\circ\text{C} = \mathbf{1026\,^\circ C}$$

Discussion The air gap between the fuel rod and the hollow ceramic cylinder contributed about 54% to the combined thermal resistance between the nuclear fuel rod surface and the outer surface of the ceramic cylinder.

3-191 Steam is flowing inside a steel pipe. The thickness of the insulation needed to reduce the heat loss by 95 percent and the thickness of the insulation needed to reduce outer surface temperature to 40°C are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

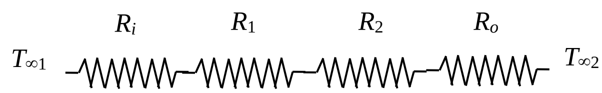
Properties The thermal conductivities are given to be k = 61 W/m·°C for steel and k = 0.038 W/m·°C for insulation.

Analysis (a) Considering a unit length of the pipe, the inner and the outer surface areas of the pipe and the insulation are

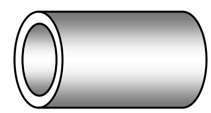
$$A_1 = \pi D_i L = \pi(0.10\text{ m})(1\text{ m}) = 0.3142\text{ m}^2$$

$$A_2 = \pi D_o L = \pi(0.12\text{ m})(1\text{ m}) = 0.3770\text{ m}^2$$

$$A_3 = \pi D_3 L = \pi D_3(1\text{ m}) = 3.1416 D_3\text{ m}^2$$



The individual thermal resistances are



$$R_i = \frac{1}{h_i A_i} = \frac{1}{(120\text{ W/m}^2.°\text{C})(0.3142\text{ m}^2)} = 0.02652\text{ °C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(6/5)}{2\pi(61\text{ W/m.°C})(1\text{ m})} = 0.00048\text{ °C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(D_3/0.12)}{2\pi(0.038\text{ W/m.°C})(1\text{ m})} = \frac{\ln(D_3/0.12)}{0.23876}\text{ °C/W}$$

$$R_{\text{o,steel}} = \frac{1}{h_o A_o} = \frac{1}{(14\text{ W/m}^2.°\text{C})(0.3770\text{ m}^2)} = 0.18947\text{ °C/W}$$

$$R_{\text{o,insulation}} = \frac{1}{h_o A_o} = \frac{1}{(14\text{ W/m}^2.°\text{C})(3.1416 D_3\text{ m}^2)} = \frac{0.02274}{D_3}\text{ °C/W}$$

$$R_{\text{total, no insulation}} = R_i + R_1 + R_{\text{o,steel}} = 0.02652 + 0.00048 + 0.18947 = 0.2165\text{ °C/W}$$

$$R_{\text{total,insulation}} = R_i + R_1 + R_2 + R_{\text{o,insulation}} = 0.02652 + 0.00048 + \frac{\ln(D_3/0.12)}{0.23876} + \frac{0.02274}{D_3}$$

$$= 0.0270 + \frac{\ln(D_3/0.12)}{0.23876} + \frac{0.02274}{D_3}\text{ °C/W}$$

Then the steady rate of heat loss from the steam per meter pipe length for the case of no insulation becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{(260-20)°\text{C}}{0.2165\text{ °C/W}} = 1109\text{ W}$$

The thickness of the insulation needed in order to save 95 percent of this heat loss can be determined from

$$\dot{Q}_{\text{insulation}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total,insulation}}} \longrightarrow (0.05\times 1109)\text{ W} = \frac{(260-20)°\text{C}}{\left(0.0270 + \dfrac{\ln(D_3/0.12)}{0.23876} + \dfrac{0.02274}{D_3}\right)°\text{C/W}}$$

whose solution is

$$D_3 = 0.3296\text{ m} \longrightarrow \text{thickness} = \frac{D_3 - D_2}{2} = \frac{32.96 - 12}{2} = \mathbf{10.5\text{ cm}}$$

(b) The thickness of the insulation needed that would maintain the outer surface of the insulation at a maximum temperature of 40°C can be determined from

$$\dot{Q}_{\text{insulation}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total,insulation}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{o, insulation}}} \longrightarrow \frac{(260-20)°\text{C}}{\left(0.0270 + \dfrac{\ln(D_3/0.12)}{0.23876} + \dfrac{0.02274}{D_3}\right)°\text{C/W}} = \frac{(40-20)°\text{C}}{\dfrac{0.02274}{D_3}\text{ °C/W}}$$

whose solution is

$$D_3 = 0.1696\text{ m} \longrightarrow \text{thickness} = \frac{D_3 - D_2}{2} = \frac{16.96 - 12}{2} = \mathbf{2.48\text{ cm}}$$

3-192 A spherical vessel is used to store a fluid. The thermal resistances, the rate of heat transfer, and the temperature difference across the insulation layer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional.

PropertiesThe thermal conductivity of the insulation is given to be 0.20 W/m·K.

Analysis(*a*) The thermal resistances are

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(40 \text{ W/m}^2\cdot\text{K})\pi(3\text{ m})^2} = \mathbf{8.84\times10^{-4}\ K/W}$$

$$R_{ins} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{(1.55-1.5)\text{m}}{4\pi(1.5\text{ m})(1.55\text{m})(0.2\text{ W/m}\cdot\text{K})} = \mathbf{8.56\times10^{-3}\ K/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(10 \text{ W/m}^2\cdot\text{K})\pi(3.10\text{ m})^2} = \mathbf{3.31\times10^{-3}\ K/W}$$

(*b*) The rate of heat transfer is

$$\dot{Q} = \frac{\Delta T}{R_i + R_{ins} + R_o} = \frac{(22-0)\text{ K}}{(8.84\times10^{-4} + 8.56\times10^{-3} + 3.31\times10^{-3})\text{ K/W}} = \mathbf{1725\ W}$$

(*c*) The temperature difference across the insulation layer is

$$\dot{Q} = \frac{\Delta T_{ins}}{R_{ins}} \longrightarrow 1725\text{ W} = \frac{\Delta T_{ins}}{8.56\times10^{-3}\text{ K/W}} \longrightarrow \Delta T_{ins} = \mathbf{14.8\ K}$$

3-193 One wall of a refrigerated warehouse is made of three layers. The rates of heat transfer across the warehouse without and with the metal bolts, and the percent change in the rate of heat transfer across the wall due to metal bolts are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer coefficients are constant.

Properties The thermal conductivities are given to be k_{Al} = 200 W/m·K, $k_{fiberglass}$ = 0.038 W/m·K, k_{gypsum} = 0.48 W/m·K, and k_{bolts} = 43 W/m·K.

Analysis (*a*) The rate of heat transfer through the warehouse is

$$U_1 = \frac{1}{\frac{1}{h_i} + \frac{L_{Al}}{k_{Al}} + \frac{L_{fg}}{k_{fg}} + \frac{L_{gy}}{k_{gy}} + \frac{1}{h_o}}$$

$$= \frac{1}{\frac{1}{40\ \text{W/m}^2\cdot°\text{C}} + \frac{0.01\ \text{m}}{200\ \text{W/m}\cdot°\text{C}} + \frac{0.08\ \text{m}}{0.038\ \text{W/m}\cdot°\text{C}} + \frac{0.03\ \text{m}}{0.48\ \text{W/m}\cdot°\text{C}} + \frac{1}{40\ \text{W/m}^2\cdot°\text{C}}} = 0.451\ \text{W/m}^2\cdot°\text{C}$$

$$\dot{Q}_1 = U_1 A(T_o - T_i) = (0.451\ \text{W/m}^2\cdot°\text{C})(5\times 10\ \text{m}^2)\left[20-(-10)°\text{C}\right] = \mathbf{676\ W}$$

(*b*) The rate of heat transfer with the consideration of metal bolts is

$$\dot{Q}_1 = U_1 A_1 (T_o - T_i) = (0.451)\left[10\times 5 - 400\times 0.25\pi(0.02)^2\right]\left[20-(-10)\right] = 674.8\ \text{W}$$

$$U_2 = \frac{1}{\frac{1}{h_i} + \frac{L_{bolts}}{k_{bolts}} + \frac{1}{h_o}} = \frac{1}{\frac{1}{40\ \text{W/m}^2\cdot°\text{C}} + \frac{0.12\ \text{m}}{43\ \text{W/m}\cdot°\text{C}} + \frac{1}{40\ \text{W/m}^2\cdot°\text{C}}} = 18.94\ \text{W/m}^2\cdot°\text{C}$$

$$\dot{Q}_2 = U_2 A_2 (T_o - T_i) = (18.94\ \text{W/m}^2\cdot°\text{C})[400\times 0.25\pi(0.02)^2\ \text{m}^2]\left[20-(-10)°\text{C}\right] = 71.4\ \text{W}$$

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = 674.8 + 71.4 = \mathbf{746\ W}$$

(*c*) The percent change in the rate of heat transfer across the wall due to metal bolts is

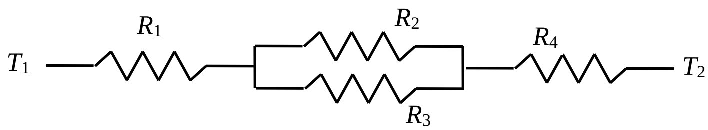
$$\%\ \text{change} = \frac{746-676}{676} = 0.103 = \mathbf{10.3\%}$$

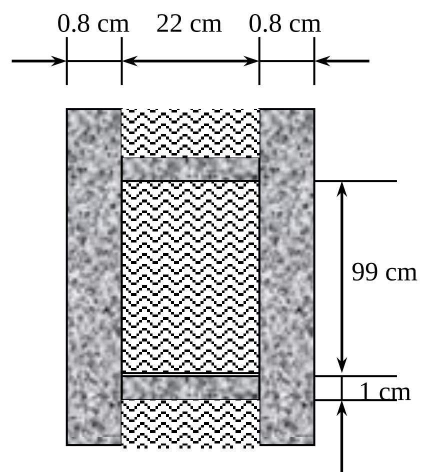
3-194 A wall is constructed of two large steel plates separated by 1-cm thick steel bars placed 99 cm apart. The remaining space between the steel plates is filled with fiberglass insulation. The rate of heat transfer through the wall is to be determined, and it is to be assessed if the steel bars between the plates can be ignored in heat transfer analysis since they occupy only 1 percent of the heat transfer surface area.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer through the wall can be approximated to be one-dimensional. **3** Thermal conductivities are constant. **4** The surfaces of the wall are maintained at constant temperatures.

Properties The thermal conductivities are given to be k = 15 W/m·°C for steel plates and k = 0.035 W/m·°C for fiberglass insulation.

Analysis We consider 1 m high and 1 m wide portion of the wall which is representative of entire wall.
Thermal resistance network and individual resistances are





$$R_1 = R_4 = R_{\text{steel}} = \frac{L}{kA} = \frac{0.008\text{ m}}{(15\text{ W/m.°C})(1\text{ m}^2)} = 0.00053\text{ °C/W}$$

$$R_2 = R_{\text{steel}} = \frac{L}{kA} = \frac{0.22\text{ m}}{(15\text{ W/m.°C})(0.01\text{ m}^2)} = 1.4667\text{ °C/W}$$

$$R_3 = R_{\text{insulation}} = \frac{L}{kA} = \frac{0.22\text{ m}}{(0.035\text{ W/m.°C})(0.99\text{ m}^2)} = 6.3492\text{ °C/W}$$

$$\frac{1}{R_{\text{eqv}}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.4667} + \frac{1}{6.3492} \longrightarrow R_{eqv} = 1.1915\text{ °C/W}$$

$$R_{\text{total}} = R_1 + R_{\text{eqv}} + R_4 = 0.00053 + 1.1915 + 0.00053 = 1.1926\text{ °C/W}$$

The rate of heat transfer per m^2 surface area of the wall is

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{22\text{ °C}}{1.1926\text{ °C/W}} = 18.45\text{ W}$$

The total rate of heat transfer through the entire wall is then determined to be

$$\dot{Q}_{\text{total}} = (4\times 6)\dot{Q} = 24(18.45\text{ W}) = \mathbf{442.7\ W}$$

If the steel bars were ignored since they constitute only 1% of the wall section, the R_{equiv} would simply be equal to the thermal resistance of the insulation, and the heat transfer rate in this case would be

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{\Delta T}{R_1 + R_{\text{insulation}} + R_4} = \frac{22\text{ °C}}{(0.00053 + 6.3492 + 0.00053)\text{°C/W}} = 3.46\text{ W}$$

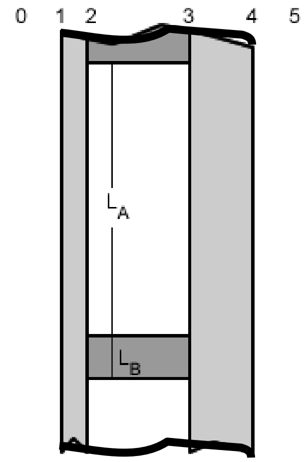
which is mush less than 18.45 W obtained earlier. Therefore, (18.45-3.46)/18.45 = 81.2% of the heat transfer occurs through the steel bars across the wall despite the negligible space that they occupy, and obviously their effect cannot be neglected. The connecting bars are serving as "thermal bridges."

3-195 A typical section of a building wall is considered. The temperature on the interior brick surface is to be determined.

Assumptions 1 Steady operating conditions exist.

Properties The thermal conductivities are given to be k_{23b} = 50 W/m·K, k_{23a} = 0.03 W/m·K, k_{12} = 0.5 W/m·K, k_{34} = 1.0 W/m·K.

Analysis We consider 1 m^2 of wall area. The thermal resistances are

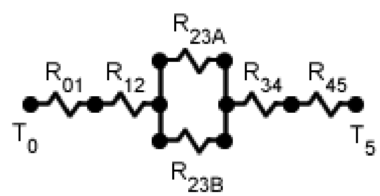


$$R_{12} = \frac{t_{12}}{k_{12}} = \frac{0.01\text{ m}}{(0.5\text{ W/m}\cdot{}^\circ\text{C})} = 0.02\text{ m}^2\cdot{}^\circ\text{C/W}$$

$$R_{23a} = t_{23}\frac{L_a}{k_{23a}(L_a+L_b)}$$
$$= (0.08\text{ m})\frac{0.6\text{ m}}{(0.03\text{ W/m}\cdot{}^\circ\text{C})(0.6+0.005)} = 2.645\text{ m}^2\cdot{}^\circ\text{C/W}$$

$$R_{23b} = t_{23}\frac{L_b}{k_{23b}(L_a+L_b)}$$
$$= (0.08\text{ m})\frac{0.005\text{ m}}{(50\text{ W/m}\cdot{}^\circ\text{C})(0.6+0.005)} = 1.32\times10^{-5}\text{ m}^2\cdot{}^\circ\text{C/W}$$

$$R_{34} = \frac{t_{34}}{k_{34}} = \frac{0.1\text{ m}}{(1.0\text{ W/m}\cdot{}^\circ\text{C})} = 0.1\text{ m}^2\cdot{}^\circ\text{C/W}$$



The total thermal resistance and the rate of heat transfer are

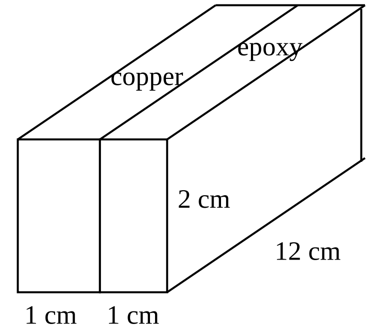
$$R_{\text{total}} = R_{12} + \left(\frac{R_{23a}R_{23b}}{R_{23a}+R_{23b}}\right) + R_{34}$$
$$= 0.02 + 2.645\left(\frac{(2.645)(1.32\times10^{-5})}{2.645+1.32\times10^{-5}}\right) + 0.1 = 0.120\text{ m}^2\cdot{}^\circ\text{C/W}$$

$$\dot{q} = \frac{T_4 - T_1}{R_{\text{total}}} = \frac{(35-20)^\circ\text{C}}{0.120\text{ m}^2\cdot\text{C/W}} = 125\text{ W/m}^2$$

The temperature on the interior brick surface is

$$\dot{q} = \frac{T_4 - T_3}{R_{34}} \longrightarrow 125\text{ W/m}^2 = \frac{(35-T_3)^\circ\text{C}}{0.1\text{ m}^2\cdot\text{C/W}} \longrightarrow T_3 = \mathbf{22.5^\circ C}$$

3-196 A square cross-section bar consists of a copper layer and an epoxy layer. The rates of heat transfer in different directions are to be determined.



Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional.

Properties The thermal conductivities of copper and epoxy are given to be 380 and 0.4 W/m·K, respectively.

Analysis (*a*) Noting that the resistances in this case are in parallel, the heat transfer from front to back is

$$R = \left[\left(\frac{kA}{L}\right)_{Cu} + \left(\frac{kA}{L}\right)_{Ep}\right]^{-1}$$

$$= \left[\left(\frac{(380\ \text{W/m}\cdot\text{K})(0.02\times 0.01)\ \text{m}^2}{0.12\ \text{m}}\right) + \left(\frac{(0.4\ \text{W/m}\cdot\text{K})(0.02\times 0.01)\ \text{m}^2}{0.12\ \text{m}}\right)\right]^{-1}$$

$$= 1.577\ \text{K/W}$$

$$\dot{Q} = \frac{\Delta T}{R} = \frac{50\ \text{K}}{1.577\ \text{K/W}} = \mathbf{31.7\ W}$$

(*b*) Noting that the resistances in this case are in series, the heat transfer from left to right is

$$R = R_{Cu} + R_{Ep} = \left(\frac{L}{kA}\right)_{Cu} + \left(\frac{L}{kA}\right)_{Ep}$$

$$= \left(\frac{0.01\ \text{m}}{(380\ \text{W/m}\cdot\text{K})(0.02\times 0.12)\ \text{m}^2}\right) + \left(\frac{0.01\ \text{m}}{(0.4\ \text{W/m}\cdot\text{K})(0.02\times 0.12)\ \text{m}^2}\right) = 10.43\ \text{K/W}$$

$$\dot{Q} = \frac{\Delta T}{R} = \frac{50\ \text{K}}{10.43\ \text{K/W}} = \mathbf{4.8\ W}$$

(*c*) Noting that the resistances in this case are in parallel, the heat transfer from top to bottom is

$$R = \left[\left(\frac{kA}{L}\right)_{Cu} + \left(\frac{kA}{L}\right)_{Ep}\right]^{-1}$$

$$= \left[\left(\frac{(380\ \text{W/m}\cdot\text{K})(0.01\times 0.12)\ \text{m}^2}{0.02\ \text{m}}\right) + \left(\frac{(0.4\ \text{W/m}\cdot\text{K})(0.01\times 0.12)\ \text{m}^2}{0.02\ \text{m}}\right)\right]^{-1} = 0.04381\ \text{K/W}$$

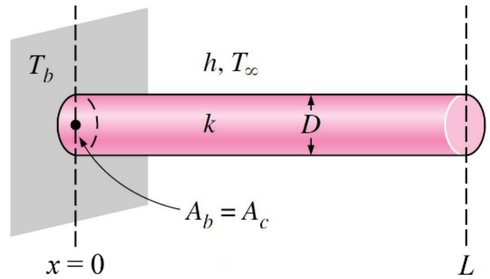
$$\dot{Q} = \frac{\Delta T}{R} = \frac{50\ \text{K}}{0.04381\ \text{K/W}} = \mathbf{1141\ W}$$

3-197 The heat transfer rates are to be determined and the temperature variations are to be plotted for infinitely long fin, adiabatic fin tip, fin tip with temperature of 250 °C, and convection from the fin tip.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fin is given as 240 W/m · °C.

Analysis For a circular fin with uniform cross section, the perimeter and cross section area are



$$p = \pi D = \pi(0.01\,\text{m}) = 0.03142\,\text{m}$$

and $$A_c = \frac{\pi D^2}{4} = \frac{\pi(0.01\,\text{m})^2}{4} = 7.854\times10^{-5}\ \text{m}^2$$

Also, we have

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{(250\ \text{W/m}^2\cdot{}^\circ\text{C})(0.03142\ \text{m})}{(240\ \text{W/m}\cdot{}^\circ\text{C})(7.854\times10^{-5}\ \text{m}^2)}} = 20.41\,\text{m}^{-1}$$

$$\sqrt{hpkA_c} = \sqrt{(250\ \text{W/m}^2\cdot{}^\circ\text{C})(0.03142\ \text{m})(240\ \text{W/m}\cdot{}^\circ\text{C})(7.854\times10^{-5}\ \text{m}^2)} = 0.3848\ \text{W/}^\circ\text{C}$$

(a) For an infinitely long fin, the heat transfer rate can be calculated as

$$\dot{Q}_{\text{long fin}} = \sqrt{hpkA_c}\,(T_b - T_\infty) = (0.3848\ \text{W/}^\circ\text{C})(350-25)\ {}^\circ\text{C} = \mathbf{125\ W}$$

The temperature variation along the fin is given as

$$\frac{T(x)-T_\infty}{T_b - T_\infty} = e^{-mx}$$

(b) For an adiabatic fin tip, the heat transfer rate can be calculated as

$$\begin{aligned}\dot{Q}_{\text{adiabatic tip}} &= \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL\\ &= (0.3848\ \text{W/}^\circ\text{C})(350\,^\circ\text{C} - 25\,^\circ\text{C})\tanh\left[(20.41\,\text{m}^{-1})(0.050\ \text{m})\right]\\ &= \mathbf{96.3\ W}\end{aligned}$$

The temperature variation along the fin is given as

$$\frac{T(x)-T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}$$

(c) For fin with tip temperature of 250 °C, the heat transfer rate can be calculated as

$$\begin{aligned}\dot{Q}_{\text{specified temp}} &= \sqrt{hpkA_c}\,(T_b - T_\infty)\frac{\cosh mL - (T_L - T_\infty)/(T_b - T_\infty)}{\sinh mL}\\ &= (0.3848\ \text{W/}^\circ\text{C})(350\,^\circ\text{C} - 25\,^\circ\text{C})(0.7250)\\ &= \mathbf{90.7\ W}\end{aligned}$$

The temperature variation along the fin is as

$$\frac{T(x)-T_\infty}{T_b - T_\infty} = \frac{(T_L - T_\infty)/(T_b - T_\infty)\sinh mx + \sinh m(L-x)}{\sinh mL}$$

(d) For fin with convection from the tip, the heat transfer rate can be calculated as

$$\begin{aligned}\dot{Q}_{\text{conv tip}} &= \sqrt{hpkA_c}\,(T_b - T_\infty)\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}\\ &= (0.3848\ \text{W/}^\circ\text{C})(350\,^\circ\text{C} - 25\,^\circ\text{C})(0.7901)\\ &= \mathbf{98.8\ W}\end{aligned}$$

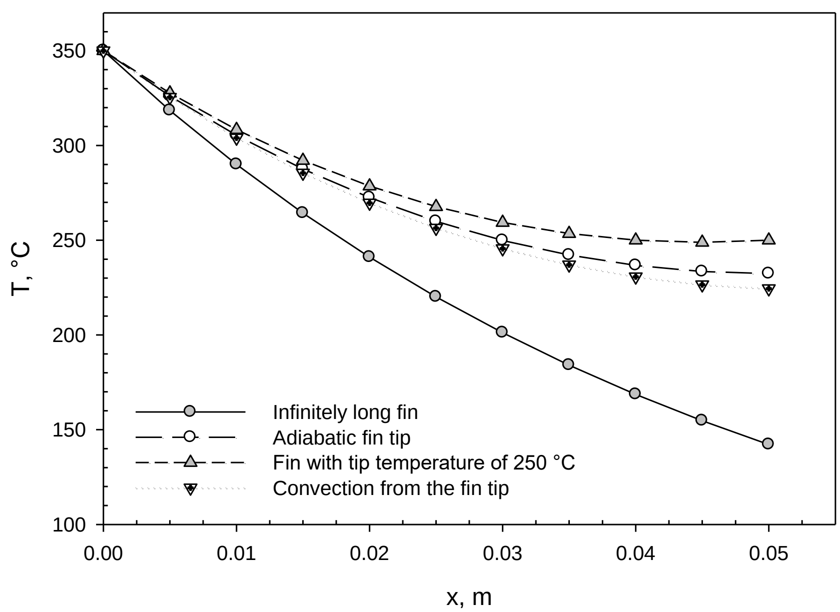
The temperature variation along the fin is given as

$$\frac{T(x)-T_\infty}{T_b-T_\infty}=\frac{\cosh m(L-x)+(h/mk)\sinh m(L-x)}{\cosh mL+(h/mk)\sinh mL}$$

The values for the temperature variations for parts (a) to (d) are tabulated in the following table:

L, m	T(x), °C			
	Part (a)	Part (b)	Part (c)	Part (d)
0	350	350	350	350
0.005	318	326	328	325
0.010	290	305	308	304
0.015	264	288	292	285
0.020	241	272	279	270
0.025	220	260	268	256
0.030	201	250	259	246
0.035	184	242	253	237
0.040	169	237	250	231
0.045	155	233	249	227
0.050	142	232	250	224

The temperature variations for parts (a) to (d) are plotted in the following figure:



Discussion The differences in the temperature variations show that applying the proper boundary condition is very important in order to perform the analysis correctly.

3-198 Ten rectangular aluminum fins are placed on the outside surface of an electronic device. The rate of heat loss from the electronic device to the surrounding air and the fin effectiveness are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature along the fins varies in one direction only (normal to the plate). **3** The heat transfer coefficient is constant and uniform over the entire fin surface. **4** The thermal properties of the fins are constant. **5** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivity of the aluminum fin is given to be k = 203 W/m·K.

Analysis The fin efficiency is to be determined using Fig. 3-43 in the text.

$$\xi = L_c^{3/2}\sqrt{h/(kA_p)} = (L+t/2)^{3/2}\sqrt{h/(kL_c t)} = (0.020+0.004/2)^{3/2}\sqrt{\frac{80}{(203)(0.022)(0.004)}} = 0.218 \longrightarrow \eta_{\text{fin}} = 0.97$$

The rate of heat loss can be determined as follows

$$A_{\text{fin}} = 2\times 10(0.020\times 0.100 + 0.004\times 0.020) = 0.0416\ \text{m}^2$$

$$A_{\text{base}} = 10(0.100\times 0.004) = 0.004\ \text{m}^2$$

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\dot{Q}_{\text{fin}}}{hA_{\text{fin}}(T_b - T_\infty)} \longrightarrow 0.97 = \frac{\dot{Q}_{\text{fin}}}{(80)(0.0416)(72-20)} \longrightarrow \dot{Q}_{\text{fin}} = 167.9\ \text{W}$$

$$\dot{Q}_{\text{base}} = hA_{\text{base}}(T_b - T_\infty) = (80)(0.004)(72-20) = 16.64\ \text{W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{fin}} + \dot{Q}_{\text{base}} = 167.9 + 16.64 = \mathbf{184.54\ W}$$

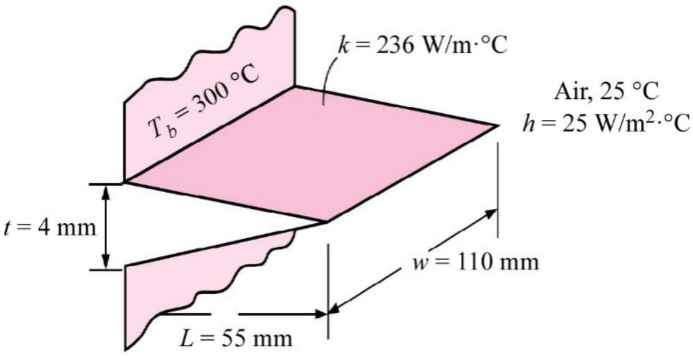
The fin effectiveness is

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_{\text{base, no fin}}(T_b - T_\infty)} = \frac{184.54}{(80)(0.080\times 0.100)(72-20)} = \mathbf{5.55}$$

3-199 Using Table 3-3, the efficiency, heat transfer rate, and effectiveness of a straight triangular fin are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fin is given as 236 W/m · °C.



Analysis From Table 3-3, for straight triangular fins, we have

$$mL = \sqrt{\frac{2h}{kt}}L = \sqrt{\frac{2(25\ \text{W/m}^2\cdot°\text{C})}{(236\ \text{W/m}\cdot°\text{C})(0.004\ \text{m})}}(0.055\ \text{m}) = 0.4$$

$$A_{\text{fin}} = 2w\sqrt{L^2+(t/2)^2} = 2(0.110\ \text{m})\sqrt{(0.055\ \text{m})^2+(0.004\ \text{m}/2)^2} = 0.01211\ \text{m}^2$$

$$\eta_{\text{fin}} = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$

From Table 3-4, the modified Bessel functions are

$$e^{-2mL}I_0(2mL) = e^{-0.8}I_0(0.8) = 0.5241 \text{ or } I_0(0.8) = 1.166$$

$$e^{-2mL}I_1(2mL) = e^{-0.8}I_1(0.8) = 0.1945 \text{ or } I_1(0.8) = 0.4329$$

Hence, the fin efficiency is

$$\eta_{\text{fin}} = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)} = \frac{1}{0.4}\left(\frac{0.4329}{1.166}\right) = \mathbf{0.928}$$

The heat transfer rate for a single fin is

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) = (0.928)(25\ \text{W/m}^2\cdot°\text{C})(0.01211\ \text{m}^2)(300-25)\ °\text{C} = \mathbf{77.3\ W}$$

The fin effectiveness is

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{hA_b(T_b - T_\infty)} = \frac{\dot{Q}_{\text{fin}}}{h(tw)(T_b - T_\infty)} = \frac{77.3\ \text{W}}{(25\ \text{W/m}^2\cdot°\text{C})(0.004\ \text{m})(0.11\ \text{m})(300-25)\ °\text{C}} = \mathbf{25.5}$$

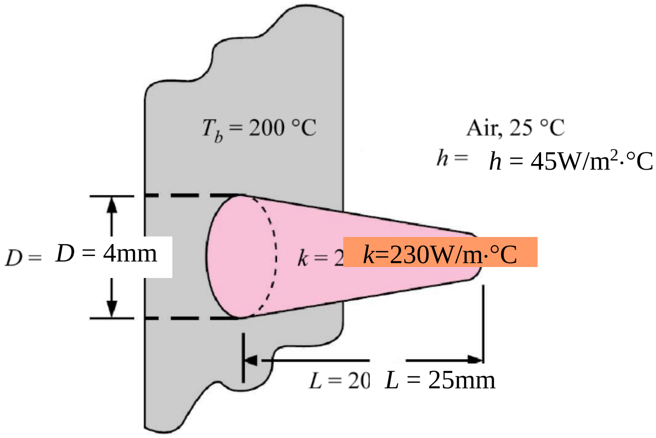
Discussion The fin efficiency can also be determined using the EES with the following line:

```
eta_fin=1/0.4*Bessel_I1(0.8)/Bessel_I0(0.8)
```

3-200 Aluminum pin fins of parabolic profile with blunt tips are attached to a plane surface. The heat transfer rate from a single fin and the increase in the heat transfer as a result of attaching fins are to be determined.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fin is given as 230 W/m · °C.



Analysis From Table 3-3, for pin fins of parabolic profile (blunt tip), we have

$$mL = \sqrt{\frac{4h}{kD}}L = \sqrt{\frac{4(45\text{ W/m}^2\cdot°\text{C})}{(230\text{ W/m}\cdot°\text{C})(0.004\text{ m})}}(0.025\text{ m}) = 0.3497$$

$$A_{\text{fin}} = \frac{\pi D^4}{96L^2}\left\{\left[16\left(\frac{L}{D}\right)^2+1\right]^{3/2}-1\right\} = \frac{\pi(0.004\text{ m})^4}{96(0.025\text{ m})^2}\left\{\left[16\left(\frac{0.025\text{ m}}{0.004\text{ m}}\right)^2+1\right]^{3/2}-1\right\}$$

$$= 2.099\times10^{-4}\text{ m}^2$$

$$\eta_{\text{fin}} = \frac{3}{2mL}\frac{I_1(4mL/3)}{I_0(4mL/3)} = \frac{3}{2(0.3497)}\frac{I_1[4(0.3497)/3]}{I_0[4(0.3497)/3]}$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
eta_fin=3/(2*0.3497)*Bessel_I1(4*0.3497/3)/Bessel_I0(4*0.3497/3)
```

Solving by EES software, the fin efficiency is

$$\eta_{\text{fin}} = 0.9738$$

The heat transfer rate for a single fin is

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty) = (0.9738)(45\text{ W/m}^2\cdot°\text{C})(2.099\times10^{-4}\text{ m}^2)(200-25)\,°\text{C} = \mathbf{1.610\ W}$$

Heat transfer from 100 fins is

$$\dot{Q}_{\text{fin, total}} = (100)(1.610\text{ W}) = 161\text{ W}$$

The surface area of the unfinned portion is

$$A_{\text{unfin}} = (1\times1)\text{ m}^2 - 100(\pi D^2/4) = 1 - 100\pi(0.004\text{ m})^2/4 = 0.9987\text{ m}^2$$

The heat transfer from the unfinned portion is

$$\dot{Q}_{\text{unfin}} = hA_{\text{unfin}}(T_b - T_\infty) = (0.9987\text{ m}^2)(45\text{ W/m}^2\cdot°\text{C})(200-25)\,°\text{C} = 7865\text{ W}$$

The total heat transfer from the surface is

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{fin, total}} + \dot{Q}_{\text{unfin}} = 161 + 7865 = 8026\text{ W}$$

If there was no fin at the surface,

$$\dot{Q}_{\text{nofin}} = hA_{\text{unfin}}(T_b - T_\infty) = (1\text{ m}^2)(45\text{ W/m}^2\cdot°\text{C})(200-25)\,°\text{C} = 7875\text{ W}$$

The increase in heat transfer as a result of attaching fins is then

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total}} - \dot{Q}_{\text{nofin}} = 8026 - 7875 = \mathbf{151\,W}$$

Discussion The values for the Bessel functions may also be approximated using Table 3-4.

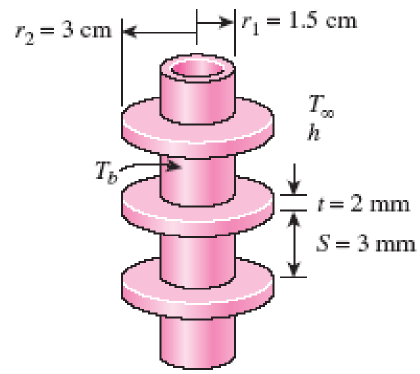
3-201 Circular aluminum alloy fins are to be attached to the tubes of a heating system. The increase in heat transfer from the tubes per unit length as a result of adding fins is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat transfer coefficient is constant and uniform over the entire fin surfaces. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fins is given to be k = 180 W/m·°C.

Analysis In case of no fins, heat transfer from the tube per meter of its length is

$$A_{\text{no fin}} = \pi D_1 L = \pi(0.03\text{ m})(1\text{ m}) = 0.0942\text{ m}^2$$
$$\dot{Q}_{\text{no fin}} = hA_{\text{no fin}}(T_b - T_\infty) = (60\text{ W/m}^2.°\text{C})(0.0942\text{ m}^2)(120 - 25)°\text{C} = 537\text{ W}$$



The efficiency of these circular fins is, from the efficiency curve, Fig. 3-44

$$\left.\begin{aligned}
L &= (D_2 - D_1)/2 = (0.06 - 0.03)/2 = 0.015\text{ m}\\
\frac{r_2 + (t/2)}{r_1} &= \frac{0.03 + (0.002/2)}{0.015} = 2.07\\
L_c^{3/2}\left(\frac{h}{kA_p}\right)^{1/2} &= \left(L + \frac{t}{2}\right)\sqrt{\frac{h}{kt}}\\
&= \left(0.015 + \frac{0.002}{2}\right)\sqrt{\frac{60\text{ W/m}^2\,{}^\circ\text{C}}{(180\text{ W/m}^\circ\text{C})(0.002\text{ m})}} = 0.207
\end{aligned}\right\}\eta_{\text{fin}} = 0.96$$

Heat transfer from a single fin is

$$A_{\text{fin}} = 2\pi(r_2{}^2 - r_1{}^2) + 2\pi r_2 t = 2\pi(0.03^2 - 0.015^2) + 2\pi(0.03)(0.002) = 0.004624\text{ m}^2$$
$$\begin{aligned}
\dot{Q}_{\text{fin}} &= \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_b - T_\infty)\\
&= 0.96(60\text{ W/m}^2.°\text{C})(0.004624\text{ m}^2)(120 - 25)°\text{C}\\
&= 25.3\text{ W}
\end{aligned}$$

Heat transfer from a single unfinned portion of the tube is

$$A_{\text{unfin}} = \pi D_1 s = \pi(0.03\text{ m})(0.003\text{m}) = 0.000283\text{ m}^2$$
$$\dot{Q}_{\text{unfin}} = hA_{\text{unfin}}(T_b - T_\infty) = (60\text{ W/m}^2.°\text{C})(0.000283\text{ m}^2)(120 - 25)°\text{C} = 1.6\text{ W}$$

There are 200 fins and thus 200 interfin spacings per meter length of the tube. The total heat transfer from the finned tube is then determined from

$$\dot{Q}_{\text{total,fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 200(25.3 + 1.6) = 5380\text{ W}$$

Therefore, the increase in heat transfer from the tube per meter of its length as a result of the addition of the fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total,fin}} - \dot{Q}_{\text{no fin}} = 5380 - 537 = \mathbf{4843\text{ W}}$$

Discussion The The overall effectiveness of the finned tube is 5380/537 = 10. That is, the rate of heat transfer from the steam tube increases by a factor of 10 as a result of adding fins. This explains the widespread use of finned surfaces.

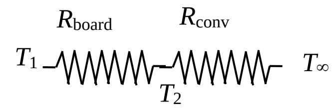
3-202 A circuit board houses electronic components on one side, dissipating a total of 15 W through the backside of the board to the surrounding medium. The temperatures on the two sides of the circuit board are to be determined for the cases of no fins and 20 aluminum fins of rectangular profile on the backside.

Assumptions **1** Steady operating conditions exist. **2** The temperature in the board and along the fins varies in one direction only (normal to the board). **3** All the heat generated in the chips is conducted across the circuit board, and is dissipated from the backside of the board. **4** Heat transfer from the fin tips is negligible. **5** The heat transfer coefficient is constant and uniform over the entire fin surface. **6** The thermal properties of the fins are constant. **7** The heat transfer coefficient accounts for the effect of radiation from the fins.

Properties The thermal conductivities are given to be k = 12 W/m·°C for the circuit board, k = 237 W/m·°C for the aluminum plate and fins, and k = 1.8 W/m·°C for the epoxy adhesive.

Analysis (a) The thermal resistance of the board and the convection resistance on the backside of the board are

$$R_{\text{board}} = \frac{L}{kA} = \frac{0.002\text{ m}}{(12\text{ W/m.°C})(0.1\text{ m})(0.15\text{ m})} = 0.011\text{ °C/W}$$



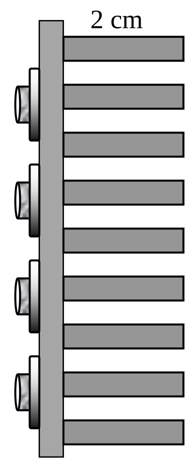
$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{(45\text{ W/m.°C})(0.1\text{ m})(0.15\text{ m})} = 1.481\text{ °C/W}$$

$$R_{\text{total}} = R_{\text{board}} + R_{\text{conv}} = 0.011 + 1.481 = 1.492\text{ °C/W}$$

Then surface temperatures on the two sides of the circuit board becomes

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{\text{total}} = 37\text{°C} + (15\text{ W})(1.492\text{°C/W}) = \mathbf{59.4°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 59.4\text{°C} - (15\text{ W})(0.011\text{°C/W}) = \mathbf{59.2°C}$$



(b) Noting that the cross-sectional areas of the fins are constant, the efficiency of these rectangular fins is determined to be

$$m = \sqrt{\frac{hp}{kA_c}} \cong \sqrt{\frac{h(2w)}{k(tw)}} = \sqrt{\frac{2h}{kt}} = \sqrt{\frac{2(45\text{ W/m}^2\text{.°C})}{(237\text{ W/m.°C})(0.002\text{ m})}} = 13.78\text{ m}^{-1}$$

$$\eta_{\text{fin}} = \frac{\tanh mL}{mL} = \frac{\tanh(13.78\text{ m}^{-1} \times 0.02\text{ m})}{13.78\text{ m}^{-1} \times 0.02\text{ m}} = 0.975$$

The finned and unfinned surface areas are

$$A_{\text{finned}} = (20)2w\left(L + \frac{t}{2}\right) = (20)2(0.15)\left(0.02 + \frac{0.002}{2}\right) = 0.126\text{ m}^2$$

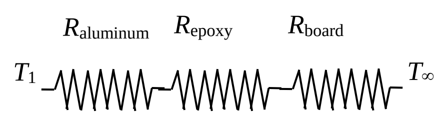
$$A_{\text{unfinned}} = (0.1)(0.15) - 20(0.002)(0.15) = 0.0090\text{ m}^2$$

Then,

$$\dot{Q}_{\text{finned}} = \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}} hA_{\text{fin}}(T_{\text{base}} - T_\infty)$$

$$\dot{Q}_{\text{unfinned}} = hA_{\text{unfinned}}(T_{\text{base}} - T_\infty)$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{unfinned}} + \dot{Q}_{\text{finned}} = h(T_{\text{base}} - T_\infty)(\eta_{\text{fin}} A_{\text{fin}} + A_{\text{unfinned}})$$



Substituting, the base temperature of the finned surfaces is determined to be

$$T_{\text{base}} = T_\infty + \frac{\dot{Q}_{\text{total}}}{h(\eta_{\text{fin}} A_{\text{fin}} + A_{\text{unfinned}})} = 37\text{°C} + \frac{15\text{ W}}{(45\text{ W/m}^2\text{.°C})[(0.975)(0.126\text{ m}^2) + (0.0090\text{ m}^2)]} = \mathbf{39.5°C}$$

Then the temperatures on both sides of the board are determined using the thermal resistance network to be

$$R_{\text{aluminum}} = \frac{L}{kA} = \frac{0.001\text{ m}}{(237\text{ W/m.°C})(0.1\text{ m})(0.15\text{ m})} = 0.00028\text{ °C/W}$$

$$R_{\text{epoxy}} = \frac{L}{kA} = \frac{0.0003\text{ m}}{(1.8\text{ W/m.°C})(0.1\text{ m})(0.15\text{ m})} = 0.01111\text{ °C/W}$$

$$\dot{Q} = \frac{T_1 - T_{\text{base}}}{R_{\text{aluminum}} + R_{\text{epoxy}} + R_{\text{board}}} = \frac{(T_1 - 39.5)\text{°C}}{(0.00028 + 0.01111 + 0.011)\text{ °C/W}} \longrightarrow T_1 = 39.5\text{°C} + (15\text{ W})(0.02239\text{ °C/W}) = \mathbf{39.8°C}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{board}}} \longrightarrow T_2 = T_1 - \dot{Q}R_{\text{board}} = 39.8\text{°C} - (15\text{ W})(0.011\text{°C/W}) = \mathbf{39.6°C}$$

3-203 PtD An electronic device is cooled by dissipating heat through a heat sink attached on its top surface. There is contact resistance at the interface of the electronic component and the heat sink. The surface temperature of the electronic device is to be determined whether it is below 85°C or not.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is one-dimensional. **3** The electronic device maintains a constant surface temperature.

Properties The thermal contact conductance for aluminum plates with air at the interface, a roughness of about 10 μm and an average interface pressure of 1 atm is h_c = 3640 W/m^2·K (Table 3-1), the combined thermal resistance of an HS 5030 heat sink, attached horizontally, is 1.2 K/W (Table 3-6).

Analysis The thermal resistances of different layers are

$$R_{\text{interface}} = \frac{1}{h_c A} = \frac{1}{(3640\ \text{W/m}^2\cdot\text{K})(0.100\ \text{m})(0.040\ \text{m})} = 0.06868\ \text{K/W}$$

$$R_{\text{heat sink}} = 1.2\ \text{K/W}$$

The total thermal resistance is

$$R_{\text{total}} = R_{\text{interface}} + R_{\text{heat sink}} = 1.26868\ \text{K/W}$$

The rate of heat transfer through the layers is

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{R_{\text{total}}}$$

The surface temperature of the electronic device is

$$T_s = \dot{Q}R_{\text{total}} + T_\infty = (45\ \text{W})(1.26868\ \text{K/W}) + 30°\text{C} = \mathbf{87.1°C} > 85°\text{C}$$

Since the surface temperature of the electronic device is above 85°C, there is a risk of overheating. To reduce the surface temperature, the total thermal resistance needs to be reduced to promote more heat dissipation through the heat sink. One way to solve this problem is by reducing the contact resistance at the interface. This can be achieved by filling the interface with a fluid having higher thermal contact conductance than air.

$$\dot{Q} = \frac{\Delta T}{R_{\text{total}}} = \frac{T_s - T_\infty}{\dfrac{1}{h_c A} + R_{\text{heat sink}}} \rightarrow h_c = \left[A\left(\frac{T_s - T_\infty}{\dot{Q}} - R_{\text{heat sink}}\right)\right]^{-1}$$

$$h_c = \left[(0.100\ \text{m})(0.040\ \text{m})\left(\frac{85-30}{45} - 1.2\right)\left(\frac{\text{K}}{\text{W}}\right)\right]^{-1} = 11{,}250\ \text{W/m}^2\cdot\text{K}$$

Thus, the surface temperature of the electronic device can be reduced to below 85°C by filling the interface with a fluid having a thermal contact conductance value higher than 11,250 W/m^2·K. From Table 3-1 hydrogen, silicone oil and glycerin all have thermal contact conductance greater than 11,250 W/m^2·K.

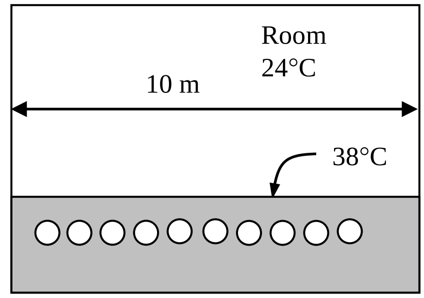
Discussion In practice, the interfaces between electronic devices and heat sinks are filled with thermally conductive epoxy adhesives to reduce thermal contact resistance.

3-204 Steam passes through a row of 10 parallel pipes placed horizontally in a concrete floor exposed to room air at 24°C with a heat transfer coefficient of 12 W/m^2.°C. If the surface temperature of the concrete floor is not to exceed 38°C, the minimum burial depth of the steam pipes below the floor surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the concrete is constant.

Properties The thermal conductivity of concrete is given to be k = 0.75 W/m·°C.

Analysis In steady operation, the rate of heat loss from the steam through the concrete floor by conduction must be equal to the rate of heat transfer from the concrete floor to the room by combined convection and radiation, which is determined to be



$$\dot{Q} = hA_s(T_s - T_\infty)$$
$$= (12\ \text{W/m}^2.°\text{C})[(10\ \text{m})(5\ \text{m})](38-24)°\text{C} = 8400\ \text{W}$$

Then the depth the steam pipes should be buried can be determined with the aid of shape factor for this configuration from Table 3-7 to be

$$\dot{Q} = nSk(T_1 - T_2) \longrightarrow S = \frac{\dot{Q}}{nk(T_1 - T_2)} = \frac{8400\ \text{W}}{10(0.75\ \text{W/m.°C})(145-38)°\text{C}} = 10.47\ \text{m}\ \ \text{(per pipe)}$$

$$w = \frac{a}{n} = \frac{10\ \text{m}}{10} = 1\ \text{m}\ \text{(center-to-center distance of pipes)}$$

$$S = \frac{2\pi L}{\ln\left(\dfrac{2w}{\pi D}\sinh\dfrac{2\pi z}{w}\right)}$$

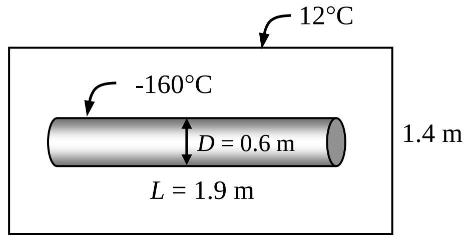
$$10.47\ \text{m} = \frac{2\pi(5\ \text{m})}{\ln\left[\dfrac{2(1\ \text{m})}{\pi(0.06\ \text{m})}\sinh\dfrac{2\pi z}{(1\ \text{m})}\right]} \longrightarrow z = 0.222\ \text{m} = \mathbf{22.2\ cm}$$

3-205 A cylindrical tank containing liquefied natural gas (LNG) is placed at the center of a square solid bar. The rate of heat transfer to the tank and the LNG temperature at the end of a one-month period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the bar is constant. **4** The tank surface is at the same temperature as the LNG.

Properties The thermal conductivity of the bar is given to be k = 0.0002 W/m·°C. The density and the specific heat of LNG are given to be 425 kg/m^3 and 3.475 kJ/kg·°C, respectively,

Analysis The shape factor for this configuration is given in Table 3-7 to be



$$S = \frac{2\pi L}{\ln\left(\frac{1.08w}{D}\right)} = \frac{2\pi(1.9\text{ m})}{\ln\left(1.08\frac{1.4\text{ m}}{0.6\text{ m}}\right)} = 12.92\text{ m}$$

Then the steady rate of heat transfer to the tank becomes

$$\dot{Q} = Sk(T_1 - T_2) = (12.92\text{ m})(0.0002\text{ W/m.°C})[12-(-160)]°\text{C} = \mathbf{0.4444\ W}$$

The mass of LNG is

$$m = \rho \boldsymbol{V} = \rho\pi\frac{D^3}{6} = (425\text{ kg/m}^3)\pi\frac{(0.6\text{ m})^3}{6} = 48.07\text{ kg}$$

The amount heat transfer to the tank for a one-month period is

$$Q = \dot{Q}\Delta t = (0.4444\text{ W})(30\times 24\times 3600\text{ s}) = 1.152\times 10^6\text{ J}$$

Then the temperature of LNG at the end of the month becomes

$$Q = mc_p(T_1 - T_2)$$

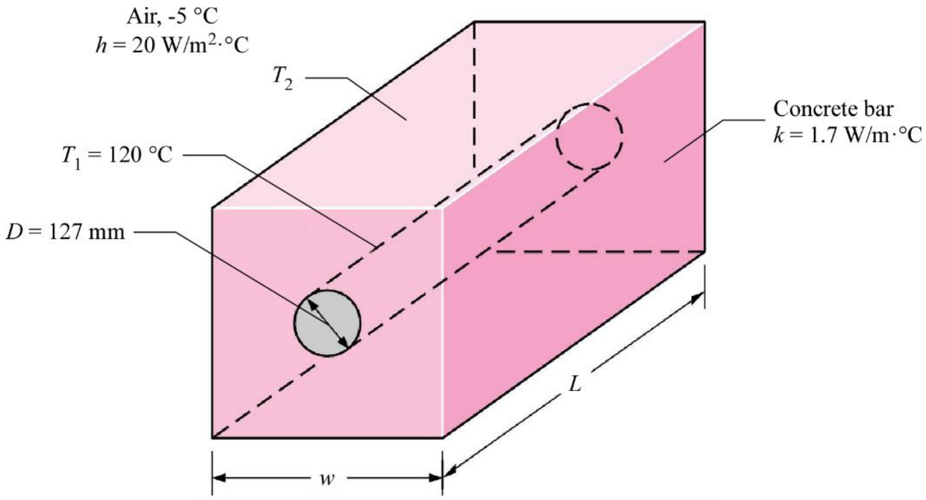
$$1.152\times 10^6\text{ J} = (48.07\text{ kg})(3475\text{ J/kg.°C})[(-160) - T_2]°\text{C}$$

$$T_2 = \mathbf{-153.1°C}$$

3-206 A tube carrying hot steam is centered at a square cross-section concrete bar. The width of the square concrete bar and the rate of heat loss in (W/m) are to be determined for the temperature difference between the outer surface of the square concrete bar and the ambient air to be maintained at 5 °C.

Assumptions **1** Heat conduction is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** Heat conduction through the tube wall is negligible. **5** Thermal contact resistance between the tube and the concrete bar is negligible.

Properties The thermal conductivity of the concrete is given as 1.7 W/m · °C.



Analysis Using Table 3-7 (Case 6), the shape factor is given to be

$$S = \frac{2\pi L}{\ln(1.08w/D)}$$

From energy balance, we have

$$kS(T_1 - T_2) = hA_s(T_2 - T_\infty)$$

or $$\frac{2\pi kL}{\ln(1.08w/D)}(T_1 - T_2) = 4hwL(T_2 - T_\infty)$$

Rearrange to get

$$w\ln\left(\frac{1.08w}{D}\right) = \frac{T_1 - T_2}{T_2 - T_\infty}\left(\frac{\pi k}{2h}\right)$$

$$w\ln\left(\frac{1.08w}{0.127\text{ m}}\right) = \frac{(120-0)\,°\text{C}}{5\,°\text{C}}\left[\frac{\pi(1.7\text{ W/m}\cdot°\text{C})}{2(20\text{ W/m}^2\cdot°\text{C})}\right]$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
w*ln(1.08*w/0.127)=120/5*(3.1416*1.7)/(2*20)
```

Solving by EES software, the width of the square concrete bar is

$$w = \mathbf{1.324\,m}$$

The heat loss to the ambient air is

$$\dot{Q}/L = 4hw(T_2 - T_\infty) = 4(20\text{ W/m}^2\cdot°\text{C})(1.324\text{ m})(5\,°\text{C}) = \mathbf{530\ W/m}$$

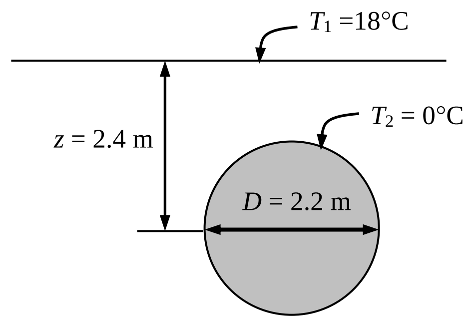
Discussion If the width of the concrete bar were less than 1.324 m, then the temperature difference between the outer surface of the concrete bar and the ambient air would be greater than 5 °C. This would mean more heat loss to the ambient air.

3-207 A spherical tank containing iced water is buried underground. The rate of heat transfer to the tank is to be determined for the insulated and uninsulated ground surface cases.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the soil is constant. **4** The tank surface is assumed to be at the same temperature as the iced water because of negligible resistance through the steel.

Properties The thermal conductivity of the soil is given to be k = 0.55 W/m·°C.

Analysis The shape factor for this configuration is given in Table 3-7 to be



$$S = \frac{2\pi D}{1 - 0.25\dfrac{D}{z}} = \frac{2\pi(2.2\text{ m})}{1 - 0.25\dfrac{2.2\text{ m}}{2.4\text{ m}}} = 17.93\text{ m}$$

Then the steady rate of heat transfer from the tank becomes

$$\dot{Q} = Sk(T_1 - T_2) = (17.93\text{ m})(0.55\text{ W/m.°C})(18 - 0)\text{°C} = \mathbf{178\ W}$$

If the ground surface is insulated,

$$S = \frac{2\pi D}{1 + 0.25\dfrac{D}{z}} = \frac{2\pi(2.2\text{ m})}{1 + 0.25\dfrac{2.2\text{ m}}{2.4\text{ m}}} = 11.25\text{ m}$$

$$\dot{Q} = Sk(T_1 - T_2) = (11.25\text{ m})(0.55\text{ W/m.°C})(18 - 0)\text{°C} = \mathbf{111\ W}$$

3-208 A thin-walled spherical tank, filled with chemicals undergoing exothermic reaction, is buried in the ground. The reaction provides uniform heat flux on the tank inner surface and the ground surface is maintained at a specified temperature. The tank surface temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer is two-dimensional (no change in the axial direction). **3** Thermal conductivity of the ground is constant.**4** Isothermal tank surface.

Properties The thermal conductivity of the ground is given to be k = 1.3 W/m·K.

Analysis The shape factor for this configuration is given in Table 3-7 (Case 15) to be

$$S = \frac{2\pi D}{1 - 0.25\dfrac{D}{z}} = \frac{2\pi(1.5\text{ m})}{1 - 0.25\dfrac{1.5\text{ m}}{3\text{ m}}} = 10.77\text{ m}$$

The heat transfer rate from the spherical tank is

$$\dot{Q}_{\text{sph}} = \dot{q}A_s = kS(T_1 - T_2)$$

Thus, the surface temperature of the tank is

$$T_1 = \frac{\dot{q}A_s}{kS} + T_2 = \frac{\dot{q}\pi D^2}{kS} + T_2 = \frac{(1000\text{ W/m}^2)\pi(1.5\text{ m})^2}{(1.3\text{ W/m}\cdot\text{K})(10.77\text{ m})} + 10\text{°C} = \mathbf{515°C}$$

DiscussionThe deeper the tank is buried in the ground, the higher its surface temperature will be. This is because the ground depth acts as a thermal resistance, limiting heat transfer between the tank surface and the ground surface.

Fundamentals of Engineering (FE) Exam Problems

3-209 Heat is lost at a rate of 275 W per m^2 area of a 15-cm-thick wall with a thermal conductivity of k=1.1 W/m·ºC. The temperature drop across the wall is

(a) 37.5ºC (b) 27.5ºC (c) 16.0ºC (d) 8.0ºC (e) 4.0ºC

Answer (a) 37.5ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
L=0.15 [m]
k=1.1 [W/m-C]
q=275 [W/m^2]
q=k*DELTAT/L
```

3-210 Consider a wall that consists of two layers, A and B, with the following values: k_A = 1.2 W/m·ºC, L_A = 8 cm, k_B = 0.2 W/m·ºC, L_B = 5 cm. If the temperature drop across the wall is 18ºC, the rate of heat transfer through the wall per unit area of the wall is

(a) 56.8 W/m^2 (b) 72.1 W/m^2 (c) 114 W/m^2 (d) 201 W/m^2 (e) 270 W/m^2

Answer (a) 56.8 W/m^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k_A=1.2 [W/m-C]
L_A=0.08 [m]
k_B=0.2 [W/m-C]
L_B=0.05 [m]
DELTAT=18 [C]
R_total=L_A/k_A+L_B/k_B
q_dot=DELTAT/R_total

"Some Wrong Solutions with Common Mistakes"
W1_q_dot=DELTAT/(L_A/k_A) "Considering layer A only"
W2_q_dot=DELTAT/(L_B/k_B) "Considering layer B only"
```

3-211 Heat is generated steadily in a 3-cm-diameter spherical ball. The ball is exposed to ambient air at 26ºC with a heat transfer coefficient of 7.5 W/m^2·ºC. The ball is to be covered with a material of thermal conductivity 0.15 W/m·ºC. The thickness of the covering material that will maximize heat generation within the ball while maintaining ball surface temperature constant is

(a) 0.5 cm (b) 1.0 cm (c) 1.5 cm (d) 2.0 cm (e) 2.5 cm

Answer (e) 2.5 cm

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.03 [m]
r=D/2
T_infinity=26 [C]
h=7.5 [W/m^2-C]
k=0.15 [W/m-C]
r_cr=(2*k)/h r_cr=(2*k)/h "critical radius of insulation for a sphere"
thickness=r_cr-r
"Some Wrong Solutions with Common Mistakes"
W_r_cr=k/h
W1_thickness=W_r_cr-r "Using the equation for cylinder"
```

3-212 Consider a 1.5-m-high and 2-m-wide triple pane window. The thickness of each glass layer (k = 0.80 W/m.°C) is 0.5 cm, and the thickness of each air space (k = 0.025 W/m.°C) is 1.2 cm. If the inner and outer surface temperatures of the window are 10°C and 0°C, respectively, the rate of heat loss through the window is

(a) 3.4 W (b) 10.2 W (c) 30.7 W (d) 61.7 W (e) 86.8 W

Answer: (c) 30.7 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"Using the thermal resistances per unit area, Q can be expressed as Q=A*DeltaT/R_total"
Lglass=0.005 {m}
kglass=0.80 {W/mC}
Rglass=Lglass/kglass
Lair=0.012 {m}
kair=0.025 {W/mC}
Rair=Lair/kair
A=1.5*2
T1=10
T2=0
Q=A*(T1-T2)/(3*Rglass+2*Rair)

"Some Wrong Solutions with Common Mistakes:"
W1_Q=(T1-T2)/(3*Rglass+2*Rair) "Not using area"
W2_Q=A*(T1-Tinf)*(3*Rglass+2*Rair) "Multiplying resistance instead of dividing"
W3_Q=A*(T1-T2)/(Rglass+Rair) "Using one layer only"
W4_Q=(T1-T2)/(3*Rglass+2*Rair)/A "Dividing by area instead of multiplying"
```

3-213 Consider two metal plates pressed against each other. Other things being equal, which of the measures below will cause the thermal contact resistance to increase?

(a) Cleaning the surfaces to make them shinier

(b) Pressing the plates against each other with a greater force

(c) Filling the gab with a conducting fluid

(d) Using softer metals

(e) Coating the contact surfaces with a thin layer of soft metal such as tin

Answer (a) Cleaning the surfaces to make them shinier

3-214 A 10-m-long, 8-cm-outer-radius cylindrical steam pipe is covered with 3-cm thick cylindrical insulation with a thermal conductivity of 0.05 W/m.°C. If the rate of heat loss from the pipe is 1000 W, the temperature drop across the insulation is

(a) 58°C (b) 101°C (c) 143°C (d) 282°C (e) 600°C

Answer (b) 101°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
R1=0.08
S=0.03
R2=0.11
L=10
K=0.05
Q=1000
R=ln(r2/r1)/(2*pi*L*k)
dT=Q*R

“Some Wrong Solutions with Common Mistakes:”
W1_T=Q/k "Wrong relation"
RR1=ln(s/r1)/(2*pi*L*k)
W2_T=Q*RR1"Wrong radius"
RR2=s/k
W3_T=Q*RR2"Wrong radius"
```

3-215 A 5-m diameter spherical tank is filled with liquid oxygen (ρ = 1141 kg/m^3, c_p = 1.71 kJ/kg·°C) at -184ºC. It is observed that the temperature of oxygen increases to -183ºC in a 144-hour period. The average rate of heat transfer to the tank is

(a) 124 W (b) 185 W (c) 246 W (d) 348 W (e) 421 W

Answer (c) 246 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=5 [m]
rho=1141 [kg/m^3]
c_p=1710 [J/kg-C]
T1=-184 [C]
T2=-183 [C]
time=144*3600 [s]
V=pi*D^3/6
m=rho*V
Q=m*c_p*(T2-T1)
Q_dot=Q/time

"Some Wrong Solutions with Common Mistakes"
W1_Q_dot=Q "Using amount of heat transfer as the answer"
Q1=m*(T2-T1)
W2_Q_dot=Q1/time "Not using specific heat in the equation"
```

3-216 A 2.5-m-high, 4-m-wide, and 20-cm-thick wall of a house has a thermal resistance of 0.025ºC/W. The thermal conductivity of the wall is

(a) 0.8 W/m·ºC (b) 1.2 W/m·ºC (c) 3.4 W/m·ºC (d) 5.2 W/m·ºC (e) 8.0 W/m·ºC

Answer (a) 0.8 W/m·ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Height=2.5 [m]
Width=4 [m]
L=0.20 [m]
R_wall=0.025 [C/W]
A=Height*Width
R_wall=L/(k*A)

"Some Wrong Solutions with Common Mistakes"
R_wall=L/W1_k "Not using area in the equation"
```

3-217 Consider two walls, *A* and *B*, with the same surface areas and the same temperature drops across their thicknesses. The ratio of thermal conductivities is $k_A/k_B = 4$ and the ratio of the wall thicknesses is $L_A/L_B = 2$. The ratio of heat transfer rates through the walls $\dot{Q}_A / \dot{Q}_B$ is

(a) 0.5 (b) 1 (c) 2 (d) 4 (e) 8

Answer (c) 2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k_A\k_B=4
L_A\L_B=2
Q_dot_A\Q_dot_B=k_A\k_B*(1/L_A\L_B) "From Fourier's Law of Heat Conduction"
```

3-218 A hot plane surface at 100°C is exposed to air at 25°C with a combined heat transfer coefficient of 20 W/m^2·°C. The heat loss from the surface is to be reduced by half by covering it with sufficient insulation with a thermal conductivity of 0.10 W/m·°C. Assuming the heat transfer coefficient to remain constant, the required thickness of insulation is

(a) 0.1 cm (b) 0.5 cm (c) 1.0 cm (d) 2.0 cm (e) 5 cm

Answer (b) 0.5 cm

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Ts=100
Tinf=25
h=20
k=0.1
Rconv=1/h
Rins=L/k
Rtotal=Rconv+Rins
Q1=h*(Ts-Tinf)
Q2=(Ts-Tinf)/(Rconv+Rins)
Q2=Q1/2
```

3-219 A room at 20°C air temperature is loosing heat to the outdoor air at 0°C at a rate of 1000 W through a 2.5-m-high and 4-m-long wall. Now the wall is insulated with 2-cm-thick insulation with a conductivity of 0.02 W/m.°C. Determine the rate of heat loss from the room through this wall after insulation. Assume the heat transfer coefficients on the inner and outer surface of the wall, the room air temperature, and the outdoor air temperature to remain unchanged. Also, disregard radiation.

(a) 20 W (b) 561 W (c) 388 W (d) 167 W (e) 200 W

Answer (d) 167 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Tin=20
Tout=0
Q=1000
A=2.5*4
L=0.02
k=0.02
Rins=L/(k*A)
Q=(Tin-Tout)/R
Qnew=(Tin-Tout)/(R+Rins)
"Some Wrong Solutions with Common Mistakes:"
W1_Q=(Tin-Tout)/Rins "Disregarding original resistance"
W2_Q=(Tin-Tout)*(R+L/k) "Disregarding area"
W3_Q=(Tin-Tout)*(R+Rins)"Multiplying by resistances"
```

3-220 A 1-cm-diameter, 30-cm long fin made of aluminum (k = 237 W/m·ºC) is attached to a surface at 80ºC. The surface is exposed to ambient air at 22ºC with a heat transfer coefficient of 18 W/m^2·ºC. If the fin can be assumed to be very long, the rate of heat transfer from the fin is

(a) 2.0 W (b) 3.2 W (c) 4.4 W (d) 5.5 W (e) 6.0 W

Answer (e) 6.0 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.01 [m]
L=0.30 [m]
k=237 [W/m-C]
T_b=80 [C]
T_infinity=22 [C]
h=18 [W/m^2-C]
p=pi*D
A_c=pi*D^2/4
Q_dot=sqrt(h*p*k*A_c)*(T_b-T_infinity)

"Some Wrong Solutions with Common Mistakes"
a=sqrt((h*p)/(k*A_c))
W1_Q_dot=sqrt(h*p*k*A_c)*(T_b-T_infinity)*tanh(a*L) "Using the relation for insulated fin tip"
```

3-221 A 1-cm-diameter, 30-cm-long fin made of aluminum (k = 237 W/m·ºC) is attached to a surface at 80ºC. The surface is exposed to ambient air at 22ºC with a heat transfer coefficient of 11 W/m^2·ºC. If the fin can be assumed to be very long, its efficiency is

(a) 0.60 (b) 0.67 (c) 0.72 (d) 0.77 (e) 0.88

Answer (d) 0.77

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.01 [m]
L=0.30 [m]
k=237 [W/m-C]
T_b=80 [C]
T_infinity=22 [C]
h=11 [W/m^2-C]
p=pi*D
A_c=pi*D^2/4
a=sqrt((h*p)/(k*A_c))
eta_fin=1/(a*L)
"Some Wrong Solutions with Common Mistakes"
W1_eta_fin=tanh(a*L)/(a*L) "Using the relation for insulated fin tip"
```

3-222 A hot surface at 80°C in air at 20°C is to be cooled by attaching 10-cm-long and 1-cm-diameter cylindrical fins. The combined heat transfer coefficient is 30 W/m^2·°C, and heat transfer from the fin tip is negligible. If the fin efficiency is 0.75, the rate of heat loss from 100 fins is

(a) 325 W (b) 707 W (c) 566 W (d) 424 W (e) 754 W

Answer (d) 424 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
N=100
Ts=80
Tinf=20
L=0.1
D=0.01
h=30
Eff=0.75
A=N*pi*D*L
Q=Eff*h*A*(Ts-Tinf)
“Some Wrong Solutions with Common Mistakes:”
W1_Q= h*A*(Ts-Tinf)"Using Qmax"
W2_Q= h*A*(Ts-Tinf)/Eff"Dividing by fin efficiency"
W3_Q= Eff*h*A*(Ts+Tinf) "Adding temperatures"
W4_Q= Eff*h*(pi*D^2/4)*L*N*(Ts-Tinf) "Wrong area"
```

3-223 A cylindrical pin fin of diameter 0.6 cm and length of 3 cm with negligible heat loss from the tip has an efficiency of 0.7. The effectiveness of this fin is

(a) 0.3 (b) 0.7 (c) 2 (d) 8 (e) 14

Answer (e) 14

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
"The relation between fin efficiency and fin effectiveness is effect = (A_fin/A_base)*Efficiency"
D=0.6 {cm}
L=3 {cm}
Effici=0.7
Effect=(pi*D*L/(pi*D^2/4))*Effici
"Some Wrong Solutions with Common Mistakes:"
W1_Effect= Effici "Taking it equal to efficiency"
W2_Effect= (D/L)*Effici "Using wrong ratio"
W3_Effect= 1-Effici "Using wrong relation"
W4_Effect= (pi*D*L/(pi*D))*Effici "Using area over perimeter"
```

3-224 A 3-cm-long, 2 mm×2 mm rectangular cross-section aluminum fin (k = 237 W/m·ºC) is attached to a surface. If the fin efficiency is 65 percent, the effectiveness of this single fin is

(a) 39 (b) 30 (c) 24 (d) 18 (e) 7

Answer (a) 39

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
L=0.03 [m]
s=0.002 [m]
k=237 [W/m-C]
eta_fin=0.65
A_fin=4*s*L
A_b=s*s
epsilon_fin=A_fin/A_b*eta_fin
```

3-225 Two finned surfaces with long fins are identical, except that the convection heat transfer coefficient for the first finned surface is twice that of the second one. What statement below is accurate for the efficiency and effectiveness of the first finned surface relative to the second one?

(a) higher efficiency and higher effectiveness

(b) higher efficiency but lower effectiveness

(c) lower efficiency but higher effectiveness

(d) lower efficiency and lower effectiveness

(e) equal efficiency and equal effectiveness

Answer (d) lower efficiency and lower effectiveness

Solution The efficiency of long fin is given by $\eta = \sqrt{kA_c / hp} / L$, which is inversely proportional to convection coefficient *h*. Therefore, efficiency of first finned surface with higher *h* will be lower. This is also the case for effectiveness since effectiveness is proportional to efficiency, $\varepsilon = \eta(A_{fin} / A_{base})$.

3-226 A 20-cm-diameter hot sphere at 120°C is buried in the ground with a thermal conductivity of 1.2 W/m·°C. The distance between the center of the sphere and the ground surface is 0.8 m, and the ground surface temperature is 15°C. The rate of heat loss from the sphere is

(a) 169 W (b) 20 W (c) 217 W (d) 312 W (e) 1.8 W

Answer (a) 169 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.2
T1=120
T2=15
K=1.2
Z=0.8
S=2*pi*D/(1-0.25*D/z)
Q=S*k*(T1-T2)
“Some Wrong Solutions with Common Mistakes:”
A=pi*D^2
W1_Q=2*pi*z/ln(4*z/D) "Using the relation for cylinder"
W2_Q=k*A*(T1-T2)/z "Using wrong relation"
W3_Q= S*k*(T1+T2) "Adding temperatures"
W4_Q= S*k*A*(T1-T2)"Multiplying vy area also"
```

3-227 A 25-cm-diameter, 2.4-m-long vertical cylinder containing ice at 0ºC is buried right under the ground. The cylinder is thin-shelled and is made of a high thermal conductivity material. The surface temperature and the thermal conductivity of the ground are 18ºC and 0.85 W/m·ºC, respectively. The rate of heat transfer to the cylinder is

(a) 37.2 W (b) 63.2 W (c) 158 W (d) 480 W (e) 1210 W

Answer (b) 63.2 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.25 [m]
L=2.4 [m]
T_ice=0 [C]
T_ground=18 [C]
k=0.85 [W/m-C]
S=(2*pi*L)/ln((4*L)/D)
Q_dot=S*k*(T_ground-T_ice)
```

3-228 Hot water (c = 4.179 kJ/kg·K) flows through a 80 m long PVC (k = 0.092 W/m·K) pipe whose inner diameter is 2 cm and outer diameter is 2.5 cm at a rate of 1 kg/s, entering at 40°C. If the entire interior surface of this pipe is maintained at 35ºC and the entire exterior surface at 20ºC, the outlet temperature of water is

(a) 35ºC (b) 36ºC (c) 37ºC (d) 38ºC (e) 39°C

Answer (e) 39ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
do=2.5 [cm]
di=2.0 [cm]
k=0.092 [W/m-C]
T2=35 [C]
T1=20 [C]
Q=2*pi*k*l*(T2-T1)/LN(do/di)
Tin=40 [C]
c=4179 [J/kg-K]
m=1 [kg/s]
l=80 [m]
Q=m*c*(Tin-Tout)
```

3-229 The walls of a food storage facility are made of a 2-cm-thick layer of wood (k = 0.1 W/m·K) in contact with a 5-cm-thick layer of polyurethane foam (k = 0.03 W/m·K). If the temperature of the surface of the wood is -10ºC and the temperature of the surface of the polyurethane foam is 20ºC, the temperature of the surface where the two layers are in contact is

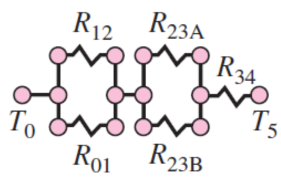
(a) -7ºC (b) -2ºC (c) 3ºC (d) 8ºC (e) 11°C

Answer (a) -7ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
kw=0.1 [W/m-C]
tkw=0.02 [m]
Tw=-10 [C]
kf=0.03 [W/m-C]
tkf=0.05 [W/m-C]
Tf=20 [C]
T=((kw*Tw/tkw)+(kf*Tf/tkf))/((kw/tkw)+(kf/tkf))
```

3-230 The equivalent thermal resistance for the thermal circuit shown here is



(a) $R_{12}R_{01} + R_{23A}R_{23B} + R_{34}$

(b) $R_{12}R_{01} + \left(\dfrac{R_{23A}R_{23B}}{R_{23A}+R_{23B}}\right) + R_{34}$

(c) $\left(\dfrac{R_{12}R_{01}}{R_{12}+R_{01}}\right) + \left(\dfrac{R_{23A}R_{23B}}{R_{23A}+R_{23B}}\right) + \dfrac{1}{R_{34}}$

(d) $\left(\dfrac{R_{12}R_{01}}{R_{12}+R_{01}}\right) + \left(\dfrac{R_{23A}R_{23B}}{R_{23A}+R_{23B}}\right) + R_{34}$

(e) None of them

Answer (d) $\left(\dfrac{R_{12}R_{01}}{R_{12}+R_{01}}\right) + \left(\dfrac{R_{23A}R_{23B}}{R_{23A}+R_{23B}}\right) + R_{34}$

3-231 The 700 m^2 ceiling of a building has a thermal resistance of 0.52 m^2·K/W. The rate at which heat is lost through this ceiling on a cold winter day when the ambient temperature is -10ºC and the interior is at 20ºC is

(a) 23.1kW (b) 40.4kW (c) 55.6kW (d) 68.1kW (e) 88.6kW

Answer (b) 40.4kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
R=0.52 [m^2-C/W]
A=700 [m^2]
T_1=20 [C]
T_2=-10 [C]
Q=A*(T_2-T_1)/R
```

3-232 A 1 m-inner diameter liquid oxygen storage tank at a hospital keeps the liquid oxygen at 90 K. This tank consists of a 0.5-cm thick aluminum (k = 170 W/m·K) shell whose exterior is covered with a 10-cm-thick layer of insulation (k = 0.02 W/m·K). The insulation is exposed to the ambient air at 20ºC and the heat transfer coefficient on the exterior side of the insulation is 5 W/m^2·K. The rate at which the liquid oxygen gains heat is

(a) 141 W (b) 176 W (c) 181 W (d) 201 W (e) 221 W

Answer (b) 176 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
R1=0.5 [m]
R2=0.55 [m]
R3=0.65 [m]
k1=170 [W/m-K]
k2=0.02 [W/m-K]
h=5[W/m^2-K]
T2=293 [K]
T1=90 [K]
R12=(R2-R1)/(4*pi*k1*R1*R2)
R23=(R3-R2)/(4*pi*k2*R2*R3)
R45=1/(h*4*pi*R3^2)
Re=R12+R23+R45
Q=(T2-T1)/Re
```

3-233 A 1-m-inner diameter liquid oxygen storage tank at a hospital keeps the liquid oxygen at 90 K. This tank consists of a 0.5-cm-thick aluminum (k = 170 W/m·K) shell whose exterior is covered with a 10-cm-thick layer of insulation (k = 0.02 W/m·K). The insulation is exposed to the ambient air at 20ºC and the heat transfer coefficient on the exterior side of the insulation is 5 W/m^2·K. The temperature of the exterior surface of the insulation is

(a) 13ºC (b) 9ºC (c) 2ºC (d) -3ºC (e) -12°C

Answer (a) 13ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
R1=0.5 [m]
R2=0.505 [m]
R3=0.605 [m]
k1=170 [W/m-K]
k2=0.02 [W/m-K]
h=5[W/m^2-K]
T2=293 [K]
T1=90 [K]
R12=(R2-R1)/(4*pi*k1*R1*R2)
R23=(R3-R2)/(4*pi*k2*R2*R3)
R45=1/(h*4*pi*R3^2)
Re=R12+R23+R45
Q=(T2-T1)/Re
Q=(T2-T3)/R45
```

3-234 The fin efficiency is defined as the ratio of the actual heat transfer from the fin to

(a) The heat transfer from the same fin with an adiabatic tip

(b) The heat transfer from an equivalent fin which is infinitely long

(c) The heat transfer from the same fin if the temperature along the entire length of the fin is the same as the base temperature

(d) The heat transfer through the base area of the same fin

(e) None of the above

Answer: (c) The heat transfer from the same fin if the temperature along the entire length of the fin is the same as the base temperature

3-235 Computer memory chips are mounted on a finned metallic mount to protect them from overheating. A 512 MB memory chip dissipates 5 W of heat to air at 25ºC. If the temperature of this chip is not exceed 60ºC, the overall heat transfer coefficient – area product of the finned metal mount must be at least

(a) 0.14 W/ºC (b) 0.20 W/ºC (c) 0.32 W/ºC (d) 0.48 W/ºC (e) 0.76 W/ºC

Answer (a) 0.14 W/ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_1=60 [C]
T_2=25 [C]
Q=5 [W]
Q=UA*(T_1-T_2)
```

3-236 In the United States, building insulation is specified by the R-value (thermal resistance in h·ft^2·ºF/Btu units). A home owner decides to save on the cost of heating the home by adding additional insulation in the attic. If the total R-value is increased from 15 to 25, the home owner can expect the heat loss through the ceiling to be reduced by

(a) 25% (b) 40% (c) 50% (d) 60% (e) 75%

Answer (b) 40%

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
R_1=15
R_2=25
DeltaT=1 "Any value can be chosen"
Q1=DeltaT/R_1
Q2=DeltaT/R_2
Reduction%=100*(Q1-Q2)/Q1
```

3-237 A triangular shaped fin on a motorcycle engine is 0.5-cm thick at its base and 3-cm long (normal distance between the base and the tip of the triangle), and is made of aluminum (k = 150 W/m·K). This fin is exposed to air with a convective heat transfer coefficient of 30 W/m^2·K acting on its surfaces. The efficiency of the fin is 75 percent. If the fin base temperature is 130ºC and the air temperature is 25ºC, the heat transfer from this fin per unit width is

(a) 32 W/m (b) 57 W/m (c) 102 W/m (d) 124 W/m (e) 142 W/m

Answer (e) 142 W/m

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
h=30 [W/m-K]
b=0.005 [m]
l=0.03 [m]
eff=0.75
Ta=25 [C]
Tb=130 [C]
A=2*(l^2+(b/2)^2)^0.5
Qideal=h*A*(Tb-Ta)
Q=eff*Qideal
```

3-238 A plane brick wall (k = 0.7 W/m·K) is 10 cm thick. The thermal resistance of this wall per unit of wall area is

(a) 0.143 m^2·K/W (b) 0.250 m^2·K/W (c) 0.327 m^2·K/W (d) 0.448 m^2·K/W (e) 0.524 m^2·K/W

Answer (a) 0.143 m^2·K/W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k=0.7 [W/m-K]
t=0.1 [m]
R=t/k
```

3-239 ··· 3-245 Design and Essay Problems

Solutions Manual for

Heat and Mass Transfer: Fundamentals & Applications
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Chapter 4
TRANSIENT HEAT CONDUCTION

Lumped System Analysis

4-1C Biot number represents the ratio of conduction resistance within the body to convection resistance at the surface of the body. The Biot number is more likely to be larger for poorly conducting solids since such bodies have larger resistances against heat conduction.

4-2C In heat transfer analysis, some bodies are observed to behave like a "lump" whose entire body temperature remains essentially uniform at all times during a heat transfer process. The temperature of such bodies can be taken to be a function of time only. Heat transfer analysis which utilizes this idealization is known as the lumped system analysis. It is applicable when the Biot number (the ratio of conduction resistance within the body to convection resistance at the surface of the body) is less than or equal to 0.1.

4-3C The lumped system analysis is more likely to be applicable in air than in water since the convection heat transfer coefficient and thus the Biot number is much smaller in air.

4-4C The lumped system analysis is more likely to be applicable for a golden apple than for an actual apple since the thermal conductivity is much larger and thus the Biot number is much smaller for gold.

4-5C The lumped system analysis is more likely to be applicable to slender bodies than the well-rounded bodies since the characteristic length (ratio of volume to surface area) and thus the Biot number is much smaller for slender bodies.

4-6C The lumped system analysis is more likely to be applicable for the body cooled naturally since the Biot number is proportional to the convection heat transfer coefficient, which is proportional to the air velocity. Therefore, the Biot number is more likely to be less than 0.1 for the case of natural convection.

4-7C The lumped system analysis is more likely to be applicable for the body allowed to cool in the air since the Biot number is proportional to the convection heat transfer coefficient, which is larger in water than it is in air because of the larger thermal conductivity of water. Therefore, the Biot number is more likely to be less than 0.1 for the case of the solid cooled in the air

4-8C The temperature drop of the potato during the second minute will be less than 4°C since the temperature of a body approaches the temperature of the surrounding medium asymptotically, and thus it changes rapidly at the beginning, but slowly later on.

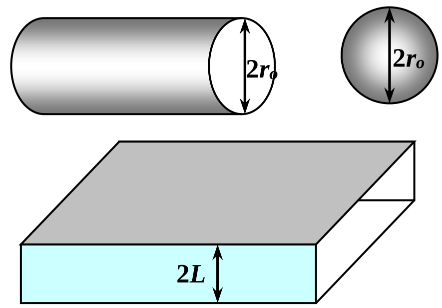
4-9C The temperature rise of the potato during the second minute will be less than 5°C since the temperature of a body approaches the temperature of the surrounding medium asymptotically, and thus it changes rapidly at the beginning, but slowly later on.

4-10C The heat transfer is proportional to the surface area. Two half pieces of the roast have a much larger surface area than the single piece and thus a higher rate of heat transfer. As a result, the two half pieces will cook much faster than the single large piece.

4-11C The cylinder will cool faster than the sphere since heat transfer rate is proportional to the surface area, and the sphere has the smallest area for a given volume.

4-12 Relations are to be obtained for the characteristic lengths of a large plane wall of thickness 2*L*, a very long cylinder of radius r_o and a sphere of radius r_o.

Analysis Relations for the characteristic lengths of a large plane wall of thickness 2*L*, a very long cylinder of radius r_o and a sphere of radius r_o are



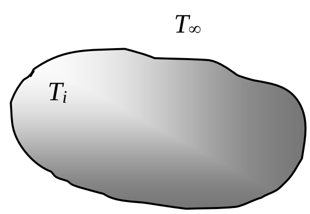
$$L_{c,wall} = \frac{V}{A_{\text{surface}}} = \frac{2LA}{2A} = L$$

$$L_{c,cylinder} = \frac{V}{A_{\text{surface}}} = \frac{\pi r_o^2 h}{2\pi r_o h} = \frac{r_o}{2}$$

$$L_{c,sphere} = \frac{V}{A_{\text{surface}}} = \frac{4\pi r_o^3/3}{4\pi r_o{}^2} = \frac{r_o}{3}$$

4-13 A relation for the time period for a lumped system to reach the average temperature $(T_i + T_\infty)/2$ is to be obtained.

Analysis The relation for time period for a lumped system to reach the average temperature $(T_i + T_\infty)/2$ can be determined as



$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{\frac{T_i + T_\infty}{2} - T_\infty}{T_i - T_\infty} = e^{-bt}$$

$$\frac{T_i - T_\infty}{2(T_i - T_\infty)} = e^{-bt} \longrightarrow \frac{1}{2} = e^{-bt}$$

$$-bt = -\ln 2 \longrightarrow t = \mathbf{\frac{\ln 2}{b}} = \mathbf{\frac{0.693}{b}}$$

4-14 The time required to cool a brick from 1100°C to a temperature difference of 5°C from the ambient air temperature is to be determined.

Assumptions **1** Thermal properties are constant. **2** Convection heat transfer coefficient is uniform. **3** Heat transfer by radiation is negligible.

Properties The properties of the brick are given as ρ = 1920 kg/m^3, c_p = 790 J/kg · K, and k = 0.90 W/m · K.

Analysis For a brick, the characteristic length and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{(0.203\times 0.102\times 0.057)\ \text{m}^3}{[2(0.203\times 0.102)+2(0.102\times 0.057)+2(0.203\times 0.057)]\text{m}^2} = 0.01549\ \text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(5\ \text{W/m}^2\cdot\text{K})(0.01549\ \text{m})}{0.90\ \text{W/m}\cdot\text{K}} = 0.0861 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable. Then the time required to cool a brick from 1100°C to a temperature difference of 5°C from the ambient air temperature is

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{5\ \text{W/m}^2\cdot\text{K}}{(1920\ \text{kg/m}^3)(790\ \text{J/kg}\cdot\text{K})(0.01549\ \text{m})} = 2.128\times 10^{-4}\ \text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt}$$

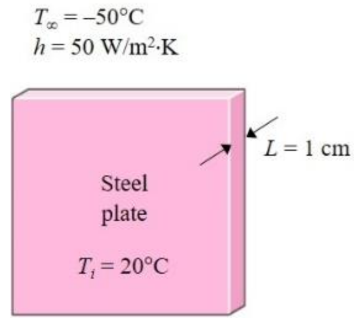
or

$$t = -\frac{1}{b}\ln\left[\frac{T(t)-T_\infty}{T_i - T_\infty}\right] = -\frac{1}{2.128\times 10^{-4}\ \text{s}^{-1}}\ln\left[\frac{5}{1100-30}\right] = 2.522\times 10^4\ \text{s} = \mathbf{7\ hours}$$

Discussion In practice, it takes days to cool bricks coming out of kilns, since they are being burned and cooled in bulk.

4-15 An ASTM A203 B steel plate is exposed to cryogenic fluid at $T_\infty = -50°\text{C}$ and $h = 50\ \text{W/m}^2\cdot\text{K}$. The plate has an initial temperature of 20°C. Determine the duration that the plate can be in contact with the cold fluid before it reaches the minimum suitable temperature of −30°C (ASME B31.3-2014, Table A-1M).

Assumptions **1** Heat transfer at the plate edges is assumed negligible. **2** Thermal properties are constant. **3** Radiation effects are negligible. **4** The convection heat transfer coefficient is constant.



Properties The thermal properties given are $c_p = 470$ J/kg·K, $k =$ 52 W/m·K, and $\rho = 7900\ \text{kg/m}^3$.

Analysis The characteristic length of the plate is

$$L_c = \frac{\text{V}}{A_s} = \frac{H \times W \times L}{2(H \times W)} = \frac{L}{2} = \frac{0.010\ \text{m}}{2} = 0.005\ \text{m}$$

The Biot number becomes

$$\text{Bi} = \frac{hL_c}{k} = \frac{(50\ \text{W/m}^2\cdot\text{K})(0.005\ \text{m})}{52\ \text{W/m}\cdot\text{K}} = 0.004808 < 0.1$$

Therefore, the lumped system analysis is applicable. The value of exponent b inEq. 4-4

$$b = \frac{h}{\rho c_p L_c} = \frac{50\ \text{W/m}^2\cdot\text{K}}{(7900\ \text{kg/m}^3)(470\ \text{J/kg}\cdot\text{K})(0.005\ \text{m})} = 0.002693\ \text{s}^{-1}$$

The time duration for each plate to reach the minimum suitable temperature of −30°C is

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

$$t = -\frac{1}{b}\ln\left[\frac{T(t) - T_\infty}{T_i - T_\infty}\right] = -\frac{1}{0.002693\ \text{s}^{-1}}\ln\left(\frac{-30 + 50}{20 + 50}\right) = \mathbf{465.2\ s}$$

Discussion The steel plate is estimated to reach the minimum suitable temperature of −30°C (ASME B31.3-2014, Table A-1M) in 7 minutes and 45 seconds. Exposure of the plate to the cryogenic fluid for duration longer than 465 s would not comply with the ASME Code for Process Piping

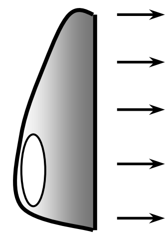
4-16 An iron whose base plate is made of an aluminum alloy is turned on. The time for the plate temperature to reach 140°C and whether it is realistic to assume the plate temperature to be uniform at all times are to be determined.

Assumptions **1** 85 percent of the heat generated in the resistance wires is transferred to the plate. **2** The thermal properties of the plate are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The density, specific heat, and thermal diffusivity of the aluminum alloy plate are given to be ρ = 2770 kg/m^3, c_p = 875 kJ/kg.°C, and α = 7.3×10^{-5} m^2/s. The thermal conductivity of the plate can be determined from $k = \alpha\rho c_p$ = 177 W/m.°C (or it can be read from Table A-3).

Analysis The mass of the iron's base plate is

$$m = \rho V = \rho LA = (2770\ \text{kg/m}^3)(0.005\ \text{m})(0.03\ \text{m}^2) = 0.4155\ \text{kg}$$



Noting that only 85 percent of the heat generated is transferred to the plate, the rate of heat transfer to the iron's base plate is

$$\dot{Q}_{\text{in}} = 0.85 \times 800\ \text{W} = 680\ \text{W}$$

The temperature of the plate, and thus the rate of heat transfer from the plate, changes during the process. Using the average plate temperature, the average rate of heat loss from the plate is determined from

$$\dot{Q}_{\text{loss}} = hA(T_{\text{plate, ave}} - T_\infty) = (12\ \text{W/m}^2.°\text{C})(0.03\ \text{m}^2)\left(\frac{140+22}{2} - 22\right)°\text{C} = 21.2\ \text{W}$$

Energy balance on the plate can be expressed as

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{plate}} \rightarrow \dot{Q}_{\text{in}}\Delta t - \dot{Q}_{\text{out}}\Delta t = \Delta E_{\text{plate}} = mc_p \Delta T_{\text{plate}}$$

Solving for Δt and substituting,

$$\Delta t = \frac{mc_p \Delta T_{\text{plate}}}{\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}} = \frac{(0.4155\ \text{kg})(875\ \text{J/kg.°C})(140-22)°\text{C}}{(680-21.2)\ \text{J/s}} = \mathbf{65.1\,s}$$

which is the time required for the plate temperature to reach 140 °C . To determine whether it is realistic to assume the plate temperature to be uniform at all times, we need to calculate the Biot number,

$$L_c = \frac{V}{A_s} = \frac{LA}{A} = L = 0.005\ \text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\ \text{W/m}^2.°\text{C})(0.005\ \text{m})}{(177.0\ \text{W/m.°C})} = 0.00034 < 0.1$$

It is realistic to assume uniform temperature for the plate since Bi < 0.1.

Discussion This problem can also be solved by obtaining the differential equation from an energy balance on the plate for a differential time interval, and solving the differential equation. It gives

$$T(t) = T_\infty + \frac{\dot{Q}_{\text{in}}}{hA}\left(1 - \exp(-\frac{hA}{mc_p}t)\right)$$

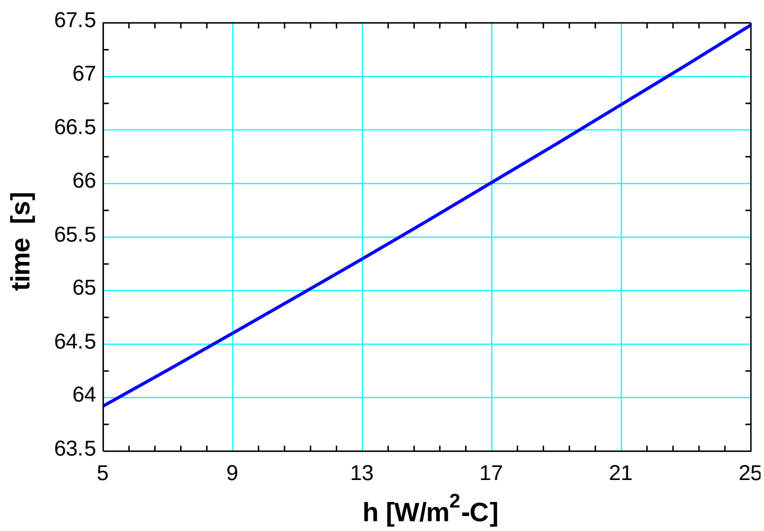
Substituting the known quantities and solving for t again gives 65.1 s.

4-17 Prob. 4-16 is reconsidered. The effects of the heat transfer coefficient and the final plate temperature on the time it will take for the plate to reach this temperature are to be investigated.

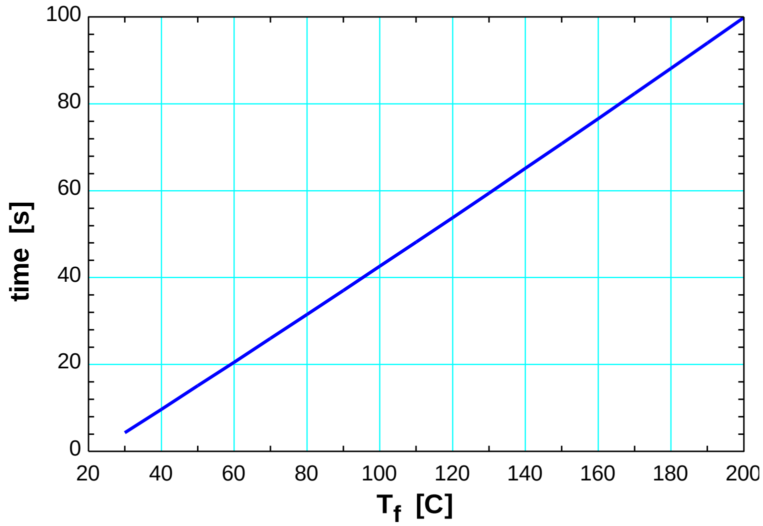
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
E_dot=800 [W]
L=0.005 [m]
A=0.03 [m^2]
T_infinity=22 [C]
T_i=T_infinity
h=12 [W/m^2-C]
f_heat=0.85
T_f=140 [C]

"PROPERTIES"
rho=2770 [kg/m^3]
c_p=875 [J/kg-C]
alpha=7.3E-5 [m^2/s]

"ANALYSIS"
V=L*A
m=rho*V
Q_dot_in=f_heat*E_dot
Q_dot_out=h*A*(T_ave-T_infinity)
T_ave=1/2*(T_i+T_f)
(Q_dot_in-Q_dot_out)*time=m*c_p*(T_f-T_i) "energy balance on the plate"
```

h [W/m^2.C]	time [s]
5	63.92
7	64.26
9	64.6
11	64.95
13	65.3
15	65.65
17	66.01
19	66.37
21	66.74
23	67.11
25	67.48



T_f [C]	time [s]
30	4.286
40	9.67
50	15.08
60	20.52
70	25.99
80	31.49
90	37.02
100	42.58
110	48.17
120	53.79
130	59.44
140	65.12
150	70.84
160	76.58
170	82.35
180	88.16
190	94
200	99.87



4-18 Metal plates are heated in an oven. The temperature of the plates exiting the oven is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface of all the metal plates. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the metal plates are given as k = 180 W/m·K, ρ = 2800 kg/m^3, and c_p = 880 J/kg·K.

Analysis The characteristic length and the Biot number of the metal plate are

$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{10\text{ mm}}{2} = 5\text{ mm}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{(200\text{ W/m}^2\cdot\text{K})(5\times10^{-3}\text{ m})}{(180\text{ W/m}\cdot\text{K})} = 0.00556 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable, then

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{200\text{ W/m}^2\cdot\text{K}}{(2800\text{ kg/m}^3)(880\text{ J/kg}\cdot\text{K})(5\times10^{-3}\text{ m})} = 0.01623\text{ s}^{-1}$$

Thus, the temperature of the plates as they exit the oven is (at t = 120 s)

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \quad\rightarrow\quad T(t) = T_\infty + (T_i - T_\infty)e^{-bt}$$

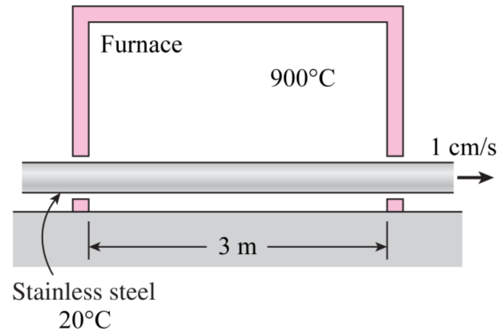
$$T(t) = 800°\text{C} + (20-800)(°\text{C})\exp[-(0.01623\text{ s}^{-1})(120\text{ s})] = \mathbf{689°C}$$

Discussion As the metal plates exit the oven, they have reached about 88% of the initial temperature difference.

4-19 Stainless steel strip is heat treated as it moves through a furnace. The temperature of the strip exiting the furnace is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of stainless steel are given as k = 21 W/m·K, ρ = 8000 kg/m^3, and c_p = 570 J/kg·K.



Analysis The characteristic length and the Biot number of the stainless steel strip

$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{5\text{ mm}}{2} = 2.5\text{ mm}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{(80\text{ W/m}^2\cdot\text{K})(2.5\times10^{-3}\text{ m})}{(21\text{ W/m}\cdot\text{K})} = 0.00952 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable, then

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{80\text{ W/m}^2\cdot\text{K}}{(8000\text{ kg/m}^3)(570\text{ J/kg}\cdot\text{K})(2.5\times10^{-3}\text{ m})} = 0.007018\text{ s}^{-1}$$

The time for the stainless steel strip being heated can be determined from the furnace length and the speed of the moving strip:

$$t = \frac{3\text{ m}}{0.01\text{ m/s}} = 300\text{ s}$$

Thus, the temperature of the strip as it exits the oven is (at t = 300 s)

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \quad \rightarrow \quad T(t) = T_\infty + (T_i - T_\infty)e^{-bt}$$

$$T(t) = 900°\text{C} + (20-900)(°\text{C})\exp[-(0.007018\text{ s}^{-1})(300\text{ s})] = \mathbf{793°C}$$

Discussion As the stainless steel strip exits the furnace, it has reached about 90% of the initial temperature difference.

4-20 C&S ASTM A479 904L bars are exposed to hot liquid at T_∞ = 300°C and h = 96 W/m²·K. The bars have an initial temperature of 20°C. If the bars are submerged in the hot liquid for 5 minutes, would they be in compliance with the ASME code? How long will it take for the bars to reach the maximum use temperature?

Assumptions **1** The bars have square cross section. **2** Thermal properties are constant. **3** Radiation effects are negligible. **4** The convection heat transfer coefficient is constant. **5** Heat transfer at the two end surfaces of the bar is negligible.

Properties The thermal properties given are c_p = 500 J/kg·K, k = 12 W/m·K, and ρ = 7900 kg/m³.

Analysis The characteristic length of the bar is

$$L_c = \frac{V}{A_s} = \frac{a^2 L}{4aL} = \frac{a}{4} = \frac{0.012\text{ m}}{4} = 0.003\text{ m}$$

The Biot number becomes

$$\text{Bi} = \frac{hL_c}{k} = \frac{(96\text{ W/m}^2\cdot\text{K})(0.003\text{ m})}{12\text{ W/m}\cdot\text{K}} = 0.024 < 0.1$$

Therefore, the lumped system analysis is applicable. The value of exponent b inEq. 4-4

$$b = \frac{h}{\rho c_p L_c} = \frac{96\text{ W/m}^2\cdot\text{K}}{(7900\text{ kg/m}^3)(500\text{ J/kg}\cdot\text{K})(0.003\text{ m})} = 0.008101\text{ s}^{-1}$$

The temperature of the bar after being submerged in the hot liquid for 300 s is

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \quad\rightarrow\quad T(t) = (T_i - T_\infty)e^{-bt} + T_\infty = (20-300)e^{-(0.008101\text{ s}^{-1})(300\text{ s})} + 300 = \mathbf{275.4}℃$$

The time duration for each bar to reach the maximum use temperature of 260°C is

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt}$$

$$t = -\frac{1}{b}\ln\left[\frac{T(t)-T_\infty}{T_i-T_\infty}\right] = -\frac{1}{0.008101\text{ s}^{-1}}\ln\left(\frac{260-300}{20-300}\right) = \mathbf{240.2\text{ s}}$$

Discussion If the stainless steel bars are submerged in the hot liquid for 5 minutes, then they would not comply with the ASME Code for Process Piping. At that duration, the temperature of the bars would exceed the maximum use temperature by more than 15°C. In order to keep the bars from exceeding the maximum use temperature of 260°C, the bars cannot be submerged in the hot liquid for more than 4 minutes.

4-21 Stainless steel plates are heat treated as they move through a furnace. The effect of the plate velocity on the plate temperature at the furnace exit is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

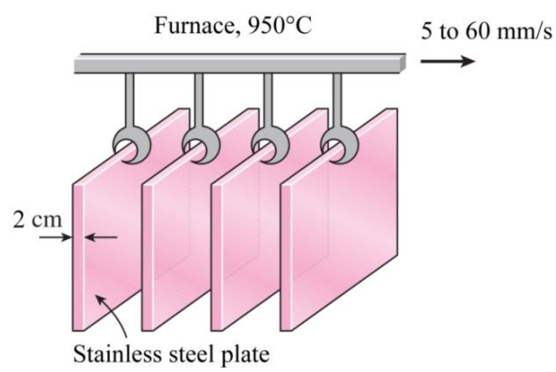
Properties The properties of stainless steel are given as k = 21 W/m·K, ρ = 8000 kg/m^3, and c_p = 570 J/kg·K.

Analysis The characteristic length and the Biot number of the plate are

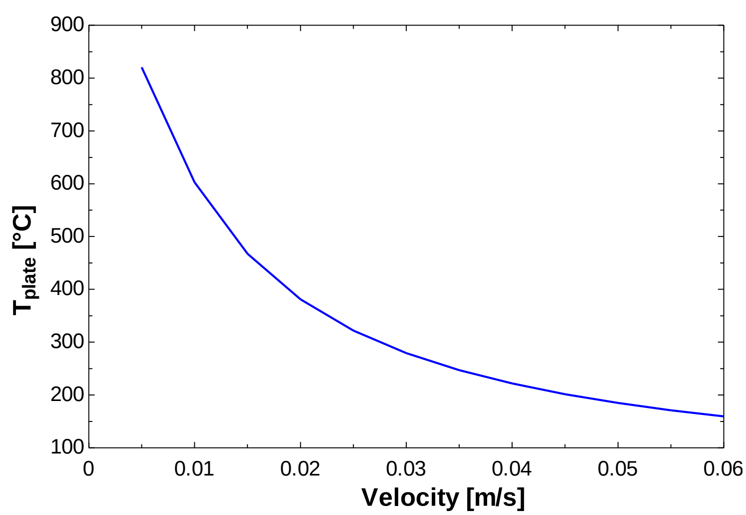
$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{2\,\text{cm}}{2} = 1\,\text{cm} \quad \text{and} \quad \text{Bi} = \frac{hL_c}{k} = \frac{(150\ \text{W/m}^2\cdot\text{K})(1\times10^{-2}\ \text{m})}{(21\ \text{W/m}\cdot\text{K})} = 0.07143 < 0.1$$

Thus, lumped system analysis is applicable. The problem is solved using EES, and the solution is given below.

```
"GIVEN"
h=150 [W/m^2-K]
L_c=2e-2/2 [m]
L=3 [m]  "length of furnace"
T_infinity=950 [C]
T_i=18 [C]

"PROPERTIES"
c_p=570 [J/kg-K]
k=21 [W/m-K]
rho=8000 [kg/m^3]

"ANALYSIS"
t=L/Velocity
Bi=h*L_c/k
b=h/(rho*c_p*L_c)
(T_plate-T_infinity)/(T_i-T_infinity)=exp(-b*t)
```



Velocity [m/s]	T_{plate} [°C]
0.005	820.5
0.010	602.6
0.015	467.3
0.020	381.0
0.025	322.0
0.030	279.3
0.035	247.0
0.040	221.8
0.045	201.5
0.050	184.9
0.055	171.1
0.060	159.3



Discussion As the plate velocity increases, the duration of the plates being heated in the furnace decreases. Thus, the plate temperature at the furnace exit decreases with increasing plate velocity.

4-22 Stainless steel strip exiting an oven is allowed to cool within a distance of 5 m. The maximum speed of the strip, such that it is cooled to 45°C within 5 m, is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of stainless steel are given as k = 21 W/m·K, ρ = 8000 kg/m^3, and c_p = 570 J/kg·K.

Analysis We take the thickness of the strip as 2L = 6 mm, and the top (or bottom) area to be A and neglect the edge areas. Then, the total surface area is the sum of top and bottom areas, which is 2A. Then, the characteristic length and the Biot number of the stainless steel strip are

$$L_c = \frac{\boldsymbol{V}}{A_s} = \frac{2LA}{2A} = L = \frac{6\text{ mm}}{2} = 3\text{ mm}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{(120\text{ W/m}^2\cdot\text{K})(3\times 10^{-3}\text{ m})}{(21\text{ W/m}\cdot\text{K})} = 0.01714 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable, then

$$b = \frac{hA_s}{\rho c_p \boldsymbol{V}} = \frac{h}{\rho c_p L_c} = \frac{120\text{ W/m}^2\cdot\text{K}}{(8000\text{ kg/m}^3)(570\text{ J/kg}\cdot\text{K})(3\times 10^{-3}\text{ m})} = 0.008772\text{ s}^{-1}$$

The time for the stainless steel strip to cool can be determined from the cooling distance of x = 5 m and the speed of the moving strip V:

$$\text{t} = \frac{\text{x}}{\text{V}}$$

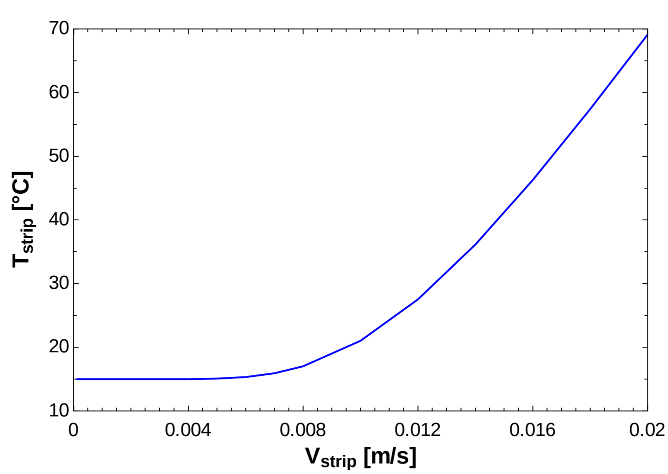
At the end of the cooling distance, the temperature of the strip should be 45°C, thus

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \quad \rightarrow \quad \ln\left[\frac{\text{T(t)}-\text{T}_\infty}{\text{T}_\text{i}-\text{T}_\infty}\right] = -\text{bt} = -\text{b}\frac{\text{x}}{\text{V}}$$

$$V = -bx\left\{\ln\left[\frac{T(t)-T_\infty}{T_i - T_\infty}\right]\right\}^{-1} = -(0.008772\text{ s}^{-1})(5\text{ m})\left[\ln\left(\frac{45-15}{500-15}\right)\right]^{-1} = \mathbf{0.0157\text{ m/s}}$$

Discussion To prevent thermal burn by cooling the strip from 500°C to 45°C within the distance of 5 m, the maximum speed of the moving strip should be 0.0157 m/s. The effect of the moving strip speed on the temperature at the end of the cooling distance is shown in the figure. As long as the strip is moving slower than 0.0157 m/s, the temperature of the strip at the end of the cooling distance would stay below 45°C.

In many industrial applications, air jets/blowers are used in the buffer zone for cooling of the heat treated metal parts. This may speed up the process without reducing the speed of stainless steel strip.



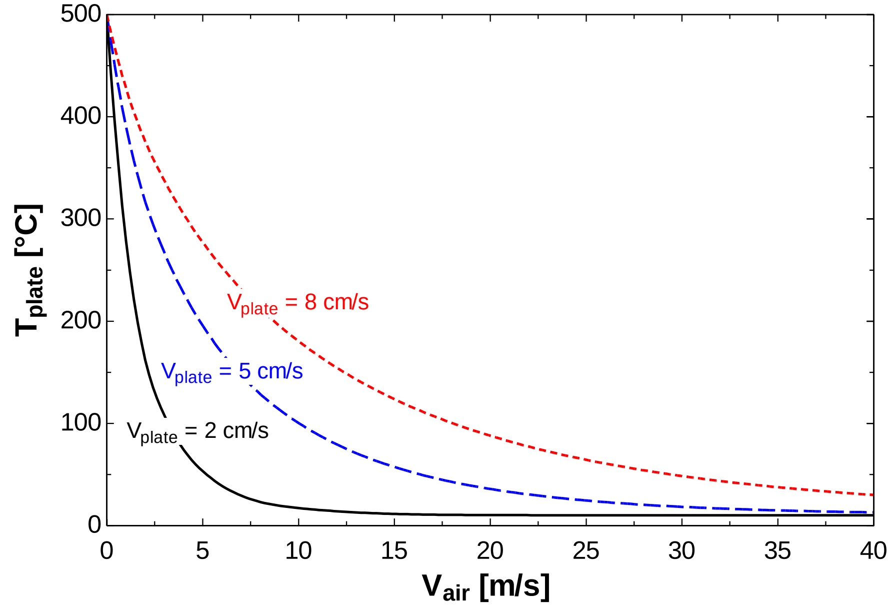
4-23 Metal plates exiting an oven are being cooled by air in a cooling chamber. The temperatures of the plates exiting the cooling chamber at different speed as a function of the air velocity are to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the metal plates are given as k = 180 W/m·K, ρ = 2800 kg/m^3, and c_p = 880 J/kg·K.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L_c=0.02/2 [m]
L_chamber=10 [m]
T_i=500 [C]
T_inf=10 [C]
V_plate1=0.02 [m/s]
V_plate2=0.05 [m/s]
V_plate3=0.08 [m/s]

"PROPERTIES"
c_p=880 [J/kg-K]
k=180 [W/m-K]
rho=2800 [kg/m^3]

"ANALYSIS"
t_1=L_chamber/V_plate1
t_2=L_chamber/V_plate2
t_3=L_chamber/V_plate3
h=33*V_air^0.8 [W/m^2-K]
Bi=h*L_c/k
b=h/(rho*c_p*L_c)
((T_plate1-T_inf)/(T_i-T_inf))=exp(-b*t_1)
((T_plate2-T_inf)/(T_i-T_inf))=exp(-b*t_2)
((T_plate3-T_inf)/(T_i-T_inf))=exp(-b*t_3)
```



V_{air} [m/s]	T_{plate} [°C] V_{plate} = 0.02 m/s	0.05 m/s	0.08 m/s
0	500	500	500
2	163	317	376
3	108	267	337
4	74.4	228	305
5	53.3	196	277
10	17.2	100	180
15	11.4	57.3	124
20	10.3	35.8	87.9
25	10.1	24.5	64.4
30	10.0	18.4	48.5
35	10.0	14.9	37.6
40	10.0	12.9	29.9

Since this analysis was carried out under the assumption that it is a lumped system, and for this assumption to be applicable, the condition Bi < 0.1 needs to be satisfied. In this problem, the highest Bi occurs at V_{air} = 40 m/s, which gives h = 631 W/m^2·K.

$$\text{Bi} = \frac{hL_c}{k} = \frac{(631\ \text{W/m}^2\cdot\text{K})(0.01\ \text{m})}{180\ \text{W/m}\cdot\text{K}} = 0.0351 < 0.1$$

Discussion The air velocities required to cool the plates to 50°C before exiting the cooling chamber are 5.2, 16.4, and 29.4 m/s for the plates moving at the speed of 0.02, 0.05, and 0.08 m/s, respectively.

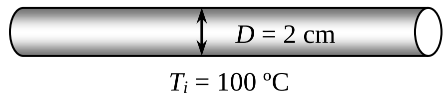
4-24 A long copper rod is cooled to a specified temperature. The cooling time is to be determined.

Assumptions **1** The thermal properties of the geometry are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The properties of copper are k = 401 W/m·ºC, ρ = 8933 kg/m^3, and c_p = 0.385 kJ/kg·ºC (Table A-3).

Analysis For cylinder, the characteristic length and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{(\pi D^2/4)L}{\pi DL} = \frac{D}{4} = \frac{0.02\text{ m}}{4} = 0.005\text{ m}$$



$$Bi = \frac{hL_c}{k} = \frac{(200\text{ W/m}^2.^\circ\text{C})(0.005\text{ m})}{(401\text{ W/m}.^\circ\text{C})} = 0.0025 < 0.1$$

Since $Bi < 0.1$, the lumped system analysis is applicable. Then the cooling time is determined from

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{200\text{ W/m}^2.^\circ\text{C}}{(8933\text{ kg/m}^3)(385\text{ J/kg}.^\circ\text{C})(0.005\text{ m})} = 0.01163\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25-20}{100-20} = e^{-(0.01163\text{ s}^{-1})t} \longrightarrow t = 238\text{ s} = \mathbf{4.0\text{ min}}$$

4-25 The ambient temperature in the oven necessary to heat the steel rods from 20°C to 450°C within 10 minutes is to be determined.

Assumptions **1** Thermal properties are constant. **2** Convection heat transfer coefficient is uniform. **3** Heat transfer by radiation is negligible.

Properties The properties of the steel rods are given as ρ = 7832 kg/m^3, c_p = 434 J/kg · K, and k = 63.9 W/m · K.

Analysis For a cylindrical rod, the characteristic length and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{(\pi D^2/4)L}{\pi DL} = \frac{D}{4} = \frac{0.025\text{ m}}{4} = 0.00625\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(20\text{ W/m}^2\cdot\text{K})(0.00625\text{ m})}{63.9\text{ W/m}\cdot\text{K}} = 0.00196 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable. Then the ambient temperature in the oven is

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{20\text{ W/m}^2\cdot\text{K}}{(7832\text{ kg/m}^3)(434\text{ J/kg}\cdot\text{K})(0.00625\text{ m})} = 9.414\times10^{-4}\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt}$$

or

$$T_\infty = \frac{T_i e^{-bt} - T(t)}{e^{-bt} - 1} = \frac{(20\,^\circ\text{C})e^{-(9.414\times10^{-4})(600)} - 450\,^\circ\text{C}}{e^{-(9.414\times10^{-4})(600)} - 1} = \mathbf{1016\,^\circ C}$$

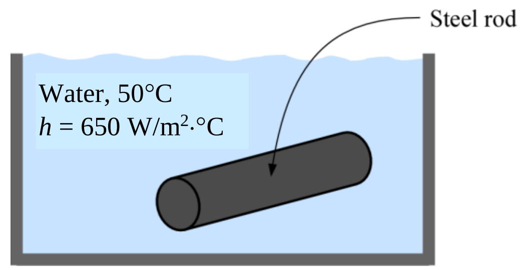
Discussion By increasing the ambient temperature in the oven, the time required to heat the steel rods to the desired temperature would be reduced.

4-26 Steel rods are quenched in a hardening process. The average temperature of rods when they are taken out of oven is to be determined.

Assumptions **1** Thermal properties are constant. **2** Convection heat transfer coefficient is uniform. **3** Heat transfer by radiation is negligible.

Properties The properties of the steel rod are given as ρ = 7832 kg/m^3, c_p = 434 J/kg · K, and k = 63.9 W/m·K.

Analysis (a) For a cylindrical rod, the characteristic length and the Biot number are



$$L_c = \frac{V}{A_s} = \frac{(\pi D^2/4)L}{\pi DL} = \frac{D}{4} = \frac{0.040\text{ m}}{4} = 0.01\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(650\text{ W/m}^2\cdot\text{K})(0.01\text{ m})}{63.9\text{ W/m}\cdot\text{K}} = 0.102 \approx 0.1$$

Since Biot number is close to 0.1, we can use the lumped system analysis. Then,

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{650\text{ W/m}^2\cdot\text{K}}{(7832\text{ kg/m}^3)(434\text{ J/kg}\cdot\text{K})(0.01\text{ m})} = 0.01912\text{ s}^{-1}$$

The average temperature of rods when they are taken out of the water bath is determined from

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T(t)-50}{850-50} = e^{-(0.01912\text{ s}^{-1})(40\text{ s})} \longrightarrow T(t) = \mathbf{422.3°C}$$

Discussion For the temperature of the water bath to remain constant, it is assumed that the heat capacity of the water is much larger than that of the steel rod.

4-27 ASTM B98 bolts are exposed to hot steam at T_∞ = 200°C and h = 50 W/m^2·K. The bolts have an initial temperature of 20°C. The duration that the bolts can be in the hot steam before they reach the maximum use temperature (ASME B31.3-2014, Table A-2M) is to be determined.

Assumptions **1** The bolts are cylindrical. **2** Thermal properties are constant. **3** Radiation effects are negligible. **4** The convection heat transfer coefficient is constant. **5** Heat transfer at the two end surfaces of the cylinder is assumed negligible.

Properties The thermal properties given are c_p = 377 J/kg·K, k = 36 W/m·K, and ρ = 8550 kg/m^3.

Analysis The characteristic length of the bolt is

$$L_c = \frac{\mathrm{V}}{A_s} = \frac{\pi D^2 L/4}{\pi D L} = \frac{D}{4} = \frac{0.003\ \mathrm{m}}{4} = 0.00075\ \mathrm{m}$$

The Biot number becomes

$$\mathrm{Bi} = \frac{hL_c}{k} = \frac{(50\ \mathrm{W/m^2\cdot K})(0.00075\ \mathrm{m})}{36\ \mathrm{W/m\cdot K}} = 0.001042 < 0.1$$

Therefore, the lumped system analysis is applicable. The value of exponent b inEq. 4-4

$$b = \frac{h}{\rho c_p L_c} = \frac{50\ \mathrm{W/m^2\cdot K}}{(8550\ \mathrm{kg/m^3})(377\ \mathrm{J/kg\cdot K})(0.00075\ \mathrm{m})} = 0.02068\ \mathrm{s^{-1}}$$

The time duration for each bolt to reach the maximum use temperature of 149°C is

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

$$t = -\frac{1}{b}\ln\left[\frac{T(t) - T_\infty}{T_i - T_\infty}\right] = -\frac{1}{0.02068\ \mathrm{s^{-1}}}\ln\left(\frac{149-200}{20-200}\right) = \mathbf{60.98\ s}$$

Discussion The bolts are estimated to reach the maximum use temperature of 149°C (ASME B31.3-2014, Table A-2M) in 61 seconds. If the bolts are exposed to the hot steam for more than 61 s, then that would violate the ASME Code for Process Piping.

4-28 C&S ASTM A437 B4B stainless steel bolts are exposed to cryogenic fluid at T_∞ = −40°C and h = 40 W/m^2·K. The bolts have an initial temperature of 10°C. If the bolts are submerged in the cold fluid for 12 minutes, would they be in compliance with the ASME code? How long will it take for the bolts to reach the minimum suitable temperature?

Assumptions **1** The bolts are cylindrical. **2** Thermal properties are constant. **3** Radiation effects are negligible. **4** The convection heat transfer coefficient is constant. **5** Heat transfer at the two end surfaces of the cylinder is assumed negligible.

Properties The thermal properties given are c_p = 460 J/kg·K, k = 23.9 W/m·K, and ρ = 7800 kg/m^3.

Analysis The characteristic length of the bolt is

$$L_c = \frac{\mathrm{V}}{A_s} = \frac{\pi D^2 L/4}{\pi D L} = \frac{D}{4} = \frac{0.025\ \mathrm{m}}{4} = 0.00625\ \mathrm{m}$$

The Biot number becomes

$$\mathrm{Bi} = \frac{hL_c}{k} = \frac{(40\ \mathrm{W/m^2\cdot K})(0.00625\ \mathrm{m})}{23.9\ \mathrm{W/m\cdot K}} = 0.01046 < 0.1$$

Therefore, the lumped system analysis is applicable. The value of exponent b inEq. 4-4

$$b = \frac{h}{\rho c_p L_c} = \frac{40\ \mathrm{W/m^2\cdot K}}{(7800\ \mathrm{kg/m^3})(460\ \mathrm{J/kg\cdot K})(0.00625\ \mathrm{m})} = 0.001784\ \mathrm{s^{-1}}$$

The temperature of the bolt after being submerged in the cryogenic fluid for 720 s is

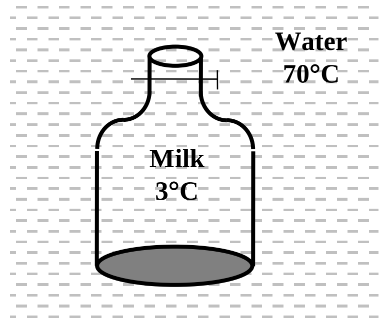
$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \qquad \rightarrow \qquad T(t) = (T_i - T_\infty)e^{-bt} + T_\infty = (10+40)e^{-(0.001784\ \mathrm{s^{-1}})(720\ \mathrm{s})} - 40 = \mathbf{-26.2}°\mathrm{C}$$

The time duration for each bolt to reach the minimum suitable temperature of −30°C is

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$
$$t = -\frac{1}{b}\ln\left[\frac{T(t) - T_\infty}{T_i - T_\infty}\right] = -\frac{1}{0.001784\ \mathrm{s^{-1}}}\ln\left(\frac{-30+40}{10+40}\right) = \mathbf{902.2\ s}$$

Discussion Submerging the ASTM A437 B4B stainless steel bolts in the cryogenic fluid for 12 minutes would cool the plate temperature to about −26°C. This is still in compliance with the ASME Code for Process Piping. The stainless steel bolts are estimated to reach the minimum suitable temperature of −30°C when they are submerged in the cold fluid for 15 minutes. Exposure of the bolts to the cryogenic fluid longer than 15 minutes would not comply with the ASME Code for Process Piping.

4-29 Milk in a thin-walled glass container is to be warmed up by placing it into a large pan filled with hot water. The warming time of the milk is to be determined.



Assumptions **1** The glass container is cylindrical in shape with a radius of r_0 = 3 cm. **2** The thermal properties of the milk are taken to be the same as those of water. **3** Thermal properties of the milk are constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the milk is stirred constantly, so that its temperature remains uniform at all times.

Properties The thermal conductivity, density, and specific heat of the milk at 20°C are k = 0.598 W/m.°C, ρ = 998 kg/m^3, and c_p = 4.182 kJ/kg.°C (Table A-9).

Analysis The characteristic length and Biot number for the glass of milk are

$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi (0.03\,\text{m})^2 (0.07\,\text{m})}{2\pi(0.03\,\text{m})(0.07\,\text{m}) + 2\pi(0.03\,\text{m})^2} = 0.01050\,\text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(120\text{ W/m}^2.°\text{C})(0.0105\,\text{m})}{(0.598\text{ W/m.°C})} = 2.107 > 0.1$$

For the reason explained above we can use the lumped system analysis to determine how long it will take for the milk to warm up to 38°C:

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{120\text{ W/m}^2.°\text{C}}{(998\,\text{kg/m}^3)(4182\text{ J/kg.°C})(0.0105\,\text{m})} = 0.002738\,\text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{38-70}{3-70} = e^{-(0.002738\,\text{s}^{-1})t} \longrightarrow t = \mathbf{270\,s} = \mathbf{4.50\,min}$$

Therefore, it will take 4.5 minutes to warm the milk from 3 to 38°C.

4-30 A body is found while still warm. The time of death is to be estimated.

Assumptions **1** The body can be modeled as a 30-cm-diameter, 1.70-m-long cylinder. **2** The thermal properties of the body and the heat transfer coefficient are constant. **3** The radiation effects are negligible. **4** The person was healthy(!) when he or she died with a body temperature of 37°C.

Properties The average human body is 72 percent water by mass, and thus we can assume the body to have the properties of water at the average temperature of (37 + 25)/2 = 31°C; k = 0.617 W/m · °C, ρ = 996 kg/m^3, and c_p = 4178 J/kg·°C (Table A–9).

Analysis The characteristic length and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(0.15\text{ m})^2(1.7\text{ m})}{2\pi(0.15\text{ m})(1.7\text{ m}) + 2\pi(0.15\text{ m})(0.15\text{ m})^2} = 0.0689\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(8\text{ W/m}^2.°\text{C})(0.0689\text{ m})}{0.617\text{ W/m.°C}} = 0.89 > 0.1$$

Therefore, lumped system analysis is *not* applicable. However, we can still use it to get a “rough” estimate of the time of death. Then,

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{8\text{ W/m}^2.°\text{C}}{(996\text{ kg/m}^3)(4178\text{ J/kg.°C})(0.0689\text{ m})} = 2.79\times 10^{-5}\text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25-20}{37-20} = \exp[(-2.79\times 10^{-5}\text{ s}^{-1})t] \longrightarrow t = 43{,}860\text{ s} = \mathbf{12.2\,h}$$

Therefore, as a rough estimate, the person died about 12 h before the body was found, and thus the time of death is 5 AM.

Discussion This example demonstrates how to obtain “ball park” values using a simple analysis. A similar analysis is used in practice by incorporating constants to account for deviation from lumped system analysis.

4-31 The temperature of a gas stream is to be measured by a thermocouple. The time it takes to register 99 percent of the initial ΔT is to be determined.

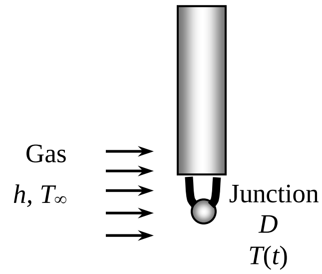
Assumptions **1** The junction is spherical in shape with a diameter of D = 0.0012 m. **2** The thermal properties of the junction are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** Radiation effects are negligible. **5** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the junction are given to be $k = 35\ \text{W/m.°C}$, $\rho = 8500\ \text{kg/m}^3$, and $c_p = 320\ \text{J/kg.°C}$.

Analysis The characteristic length of the junction and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.0012\ \text{m}}{6} = 0.0002\ \text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(110\ \text{W/m}^2.\text{°C})(0.0002\ \text{m})}{35\ \text{W/m.°C}} = 0.000629 < 0.1$$



Since $Bi < 0.1$, the lumped system analysis is applicable. Then the time period for the thermocouple to read 99% of the initial temperature difference is determined from

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = 0.01$$

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{110\ \text{W/m}^2.\text{°C}}{(8500\ \text{kg/m}^3)(320\ \text{J/kg.°C})(0.0002\ \text{m})} = 0.2022\ \text{s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow 0.01 = e^{-(0.2022\ \text{s}^{-1})t} \longrightarrow t = \mathbf{22.8\ s}$$

4-32 The temperature of a gas stream is to be measured by a thermocouple. The time it takes to register 99 percent of the initial ΔT should be within 5 s, and the junction diameter is to be determined.

Assumptions **1** The junction is spherical in shape. **2** The thermal properties of the junction are constant. **3** The heat transfer coefficient is constant and uniform over the junction surface. **4** Radiation effects are negligible. **5** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the junction are given to be k = 35 W/m·K, ρ = 8500 kg/m^3, and c_p = 320 J/kg·K.

Analysis The characteristic length of the thermocouple junction is

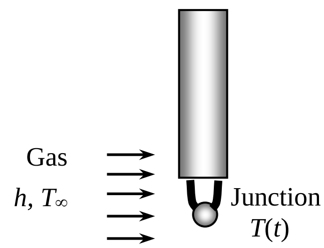
$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6}$$

The time period for the thermocouple to read 99% of the initial temperature difference is determined from

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{6h}{\rho c_p D} = \frac{6(250\ \text{W/m}^2\cdot\text{K})}{(8500\ \text{kg/m}^3)(320\ \text{J/kg}\cdot\text{K})}\frac{1}{D}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} = 0.01 \quad \rightarrow \quad 0.01 = \exp\left(-\frac{6h}{\rho c_p D}t\right)$$

$$0.01 = \exp\left[-\frac{6(250\ \text{W/m}^2\cdot\text{K})(5\ \text{s})}{(8500\ \text{kg/m}^3)(320\ \text{J/kg}\cdot\text{K})}\frac{1}{D}\right] = \exp\left(-\frac{0.002757\ \text{m}}{D}\right)$$



Solving for the junction diameter yields

$$D = 5.99\times10^{-4}\ \text{m} = \mathbf{0.599\,mm}$$

Since this analysis was carried out under the assumption that it is a lumped system, and for this assumption to be applicable, the condition Bi < 0.1 needs to be satisfied:

$$\text{Bi} = \frac{hL_c}{k} = \frac{hD}{6k} = \frac{(250\ \text{W/m}^2\cdot\text{K})(0.000599\ \text{m})}{6(35\ \text{W/m}\cdot\text{K})} = 0.000713 < 0.1$$

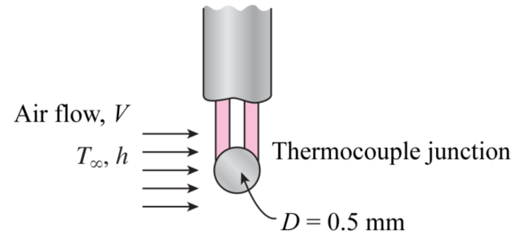
Thus, the lumped system analysis is applicable.

Discussion For the thermocouple to register 99% of the initial temperature difference within 5 s, the junction diameter should be less than 0.6 mm. The smaller the junction size, the faster the thermocouple would respond.

4-33 The temperature of an air flow is to be measured by a thermocouple. The time it takes to register 99 percent of the initial ΔT should be within 5 s, and the air flow velocity (given as a function of h) is to be determined.

Assumptions **1** The junction is spherical in shape. **2** The thermal properties of the junction are constant. **3** The heat transfer coefficient is uniform over the entire surface. **4** Radiation effects are negligible. **5** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the junction are given to be k = 35 W/m·K, ρ = 8500 kg/m^3, and c_p = 320 J/kg·K.



Analysis The characteristic length of the thermocouple junction is

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6}$$

The time period for the thermocouple to read 99% of the initial temperature difference is determined from

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{6h}{\rho c_p D} = \frac{6h}{(8500\ \text{kg/m}^3)(320\ \text{J/kg}\cdot\text{K})(0.0005\ \text{m})}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} = 0.01 \quad \rightarrow \quad 0.01 = \exp\left(-\frac{6h}{\rho c_p D}t\right)$$

$$0.01 = \exp\left[-\frac{6h}{(8500\ \text{kg/m}^3)(320\ \text{J/kg}\cdot\text{K})(0.0005\ \text{m})}(5\ \text{s})\right]$$

Solving for the convection heat transfer coefficient yields

$$h = 208.77\ \text{W/m}^2\cdot\text{K}$$

The lowest air flow velocity that the thermocouple can be used to register 99% of the initial temperature difference in 5 s is determined from

$$h = 2.2\left(\frac{V}{D}\right)^{0.5}$$

where $\quad D = 0.0005\ \text{m} \quad$ and $\quad h = 208.77\ \text{W/m}^2\cdot\text{K}$

Thus, $\quad V = \mathbf{4.50\ m/s}$

Since this analysis was carried out under the assumption that it is a lumped system, and for this assumption to be applicable, the condition Bi < 0.1 needs to be satisfied

$$\text{Bi} = \frac{hL_c}{k} = \frac{hD}{6k} = \frac{(208.77\ \text{W/m}^2\cdot\text{K})(0.0005\ \text{m})}{6(35\ \text{W/m}\cdot\text{K})} = 0.000497 < 0.1$$

Thus, the lumped system analysis is applicable.

Discussion The lower the convection heat transfer coefficient is, the longer the time it takes for the thermocouple to register 99% of the initial temperature difference.

4-34 Coal particles suspended in hot air flow. The time it takes for the particles to reach 2/3 of the initial temperature difference is to be determined.

Assumptions **1** The thermal properties of coal particles are constant. **2** The heat transfer coefficient is uniform over the entire particle surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of coal are k = 0.26 W/m·K, ρ = 1350 kg/m^3, and c_p = 1260 J/kg·K (from Table A-8).

Analysis The characteristic length and the Biot number of the coal particle are

$$L_c = \frac{V}{A_s} = \frac{0.5\,\text{mm}^3}{3.1\,\text{mm}^2} = 0.1613\,\text{mm}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{(100\ \text{W/m}^2\cdot\text{K})(0.1613\times10^{-3}\ \text{m})}{(0.26\ \text{W/m}\cdot\text{K})} = 0.062 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable. Then the time period for a coal particle to heat up to 2/3 of the initial temperature difference is determined from

$$b = \frac{hA_s}{\rho c_p V} = \frac{(100\ \text{W/m}^2\cdot\text{K})(3.1\times10^{-6}\ \text{m}^2)}{(1350\ \text{kg/m}^3)(1260\ \text{J/kg}\cdot\text{K})(0.5\times10^{-9}\ \text{m}^3)} = 0.3645\ \text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} = \frac{1}{3} \quad\rightarrow\quad \exp[-(0.3645\ \text{s}^{-1})\,t] = \frac{1}{3}$$

Thus,

$$t = \mathbf{3.01\,s}$$

Discussion If the coal particles are treated as spheres, then the diameter of the coal particles would be about $D = 6L_c$ = 0.97 mm and the time required to heat the coal particles to two thirds of the initial temperature difference is 2.88s.

4-35 Coal particles suspended in hot air are flowing through a heated tube at V = 2 m/s. The temperature of the particles exiting the heated tube is to be determined.

Assumptions **1** The thermal properties of coal particles are constant. **2** The heat transfer coefficient is uniform over the entire particle surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of coal are k = 0.26 W/m·K, ρ = 1350 kg/m^3, and c_p = 1260 J/kg·K (from Table A-8).

Analysis The characteristic length and the Biot number of the coal particle are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{300\ \mu\text{m}}{6} = 50\ \mu\text{m}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{(250\ \text{W/m}^2\cdot\text{K})(50\times10^{-6}\ \text{m})}{(0.26\ \text{W/m}\cdot\text{K})} = 0.0481 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable, then

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{250\ \text{W/m}^2\cdot\text{K}}{(1350\ \text{kg/m}^3)(1260\ \text{J/kg}\cdot\text{K})(50\times10^{-6}\ \text{m})} = 2.939\ \text{s}^{-1}$$

The time of the coal particles being heated in the tube can be determined from the tube length and the particle velocity:

$$t = \frac{3\ \text{m}}{2\ \text{m/s}} = 1.5\ \text{s}$$

Thus, the temperature of the coal particles exiting the heated tube is (at t = 1.5 s)

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \quad\rightarrow\quad T(t) = T_\infty + (T_i - T_\infty)e^{-bt}$$

$$T(t) = 900°\text{C} + (20-900)(°\text{C})\exp[-(2.939\ \text{s}^{-1})(1.5\ \text{s})] = \mathbf{889°C}$$

Discussion As the coal particles exit the heated tube, they have reached about 99% of the initial temperature difference.

4-36 Alumina particles are injected into a plasma jet. The time it would take for the particles to reach their melting point is to be determined.

Assumptions **1** The thermal properties of alumina are constant. **2** The heat transfer coefficient is uniform over the entire particle surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of alumina are given as k = 30 W/m·K, ρ = 3970 kg/m^3, and c_p = 800 J/kg·K.

Analysis The characteristic length and the Biot number of the alumina particle are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{60\ \mu\text{m}}{6} = 10\ \mu\text{m}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{(10{,}000\ \text{W/m}^2\cdot\text{K})(10\times10^{-6}\ \text{m})}{(30\ \text{W/m}\cdot\text{K})} = 0.00333 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable, then

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{10{,}000\ \text{W/m}^2\cdot\text{K}}{(3970\ \text{kg/m}^3)(800\ \text{J/kg}\cdot\text{K})(10\times10^{-6}\ \text{m})} = 314.86\ \text{s}^{-1}$$

Thus, the time for the alumina particles to reach their melting point is

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \quad \rightarrow \quad t = -\frac{1}{b}\ln\left[\frac{T(t)-T_\infty}{T_i - T_\infty}\right]$$

$$t = -\frac{1}{314.86\ \text{s}^{-1}}\ln\left(\frac{2300-15{,}000}{20-15{,}000}\right) = \mathbf{5.24\times10^{-4}\ s}$$

Discussion It takes about half a millisecond for the particles to be heated to their melting point.

4-37 The satellite shell temperature after 5 minutes of reentry is to be determined

Assumptions **1** Thermal properties are constant. **2** Convection heat transfer coefficient is uniform. **3** Heat transfer is uniform over the outer surface of the shell. **4** Heat transfer is limited to the shell only. **5** Heat transfer by radiation is negligible.

Properties The properties of stainless steel are given as ρ = 8238 kg/m^3, c_p = 468 J/kg · K, and k = 13.4 W/m · K.

Analysis For a spherical shell, the characteristic length and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{\pi/6\left[D^3-(D-2L)^3\right]}{\pi D^2} = \frac{4^3-[4-2(0.01)]^3}{6(4)^2}\ \text{m} = 0.00995\ \text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(130\ \text{W/m}^2\cdot\text{K})(0.00995\ \text{m})}{13.4\ \text{W/m}\cdot\text{K}} = 0.0965 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable. Then the shell temperature after 5 minutes of reentry is

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{130\ \text{W/m}^2\cdot\text{K}}{(8283\ \text{kg/m}^3)(468\ \text{J/kg}\cdot\text{K})(0.00995\ \text{m})} = 0.003389\ \text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt}$$

or

$$T(t) = (T_i - T_\infty)e^{-bt} + T_\infty$$

$$T(5\ \text{min}) = (10\ ^\circ\text{C} - 1250\ ^\circ\text{C})e^{-(0.003389)(300)} + 1250\ ^\circ\text{C} = \mathbf{801\ ^\circ C}$$

Discussion The analysis to this problem has been simplified by assuming the shell temperature to be uniform during the reentry.

4-38 A number of carbon steel balls are to be annealed by heating them first and then allowing them to cool slowly in ambient air at a specified rate. The time of annealing and the total rate of heat transfer from the balls to the ambient air are to be determined.

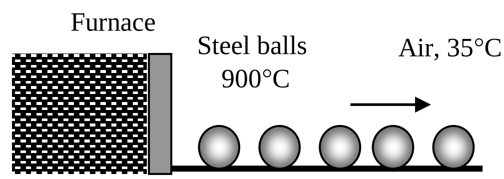
Assumptions **1** The balls are spherical in shape with a radius of r_o = 4 mm. **2** The thermal properties of the balls are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the balls are given to be k = 54 W/m.°C, ρ = 7833 kg/m^3, and c_p = 0.465 kJ/kg.°C.

Analysis The characteristic length of the balls and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.008\text{ m}}{6} = 0.0013\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(75\text{ W/m}^2.°\text{C})(0.0013\text{ m})}{(54\text{ W/m}.°\text{C})} = 0.0018 < 0.1$$



Therefore, the lumped system analysis is applicable. Then the time for the annealing process is determined to be

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{75\text{ W/m}^2.°\text{C}}{(7833\text{ kg/m}^3)(465\text{ J/kg}.°\text{C})(0.0013\text{ m})} = 0.01584\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{100-35}{900-35} = e^{-(0.01584\text{ s}^{-1})t} \longrightarrow t = \mathbf{163\text{ s} = 2.7\text{ min}}$$

The amount of heat transfer from a single ball is

$$m = \rho V = \rho\frac{\pi D^3}{6} = (7833\text{ kg/m}^3)\frac{\pi(0.008\text{ m})^3}{6} = 0.0021\text{ kg}$$

$$Q = mc_p[T_f - T_i] = (0.0021\text{ kg})(465\text{ J/kg}.°\text{C})(900-100)°\text{C} = 781\text{ J} = 0.781\text{ kJ (per ball)}$$

Then the total rate of heat transfer from the balls to the ambient air becomes

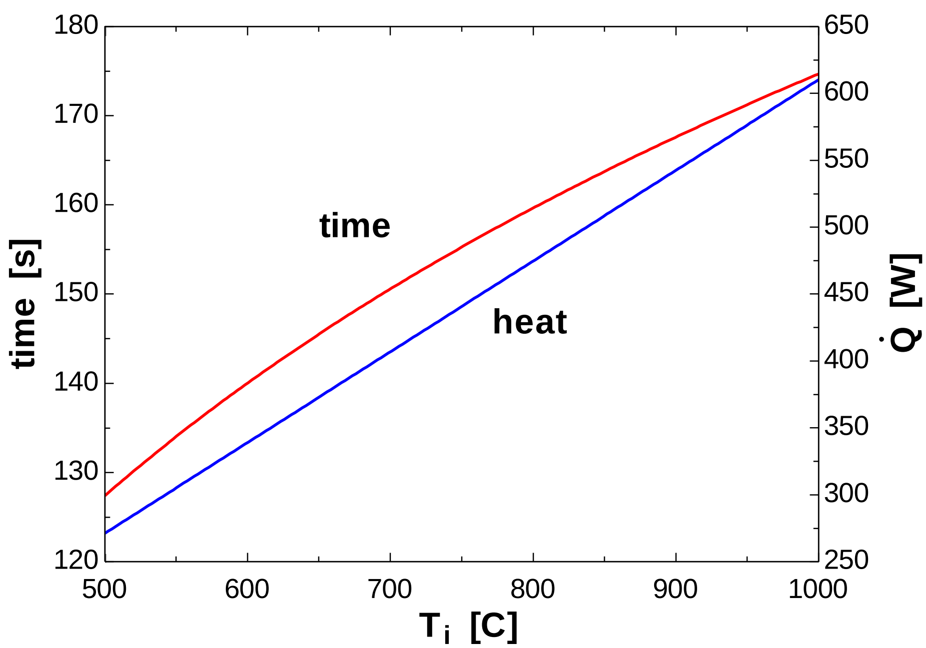
$$\dot{Q} = \dot{n}_{\text{ball}}Q = (2500\text{ balls/h})\times(0.781\text{ kJ/ball}) = 1{,}953\text{ kJ/h} = \mathbf{543\text{ W}}$$

4-39 Prob. 4-38 is reconsidered. The effect of the initial temperature of the balls on the annealing time and the total rate of heat transfer is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.008 [m]
T_i=900 [C]
T_f=100 [C]
T_infinity=35 [C]
h=75 [W/m^2-C]
n_dot_ball=2500 [1/h]

"PROPERTIES"
rho=7833 [kg/m^3]
k=54 [W/m-C]
c_p=465 [J/kg-C]
alpha=1.474E-6 [m^2/s]

"ANALYSIS"
A=pi*D^2
V=pi*D^3/6
L_c=V/A
Bi=(h*L_c)/k "if Bi < 0.1, the lumped sytem analysis is applicable"
b=(h*A)/(rho*c_p*V)
(T_f-T_infinity)/(T_i-T_infinity)=exp(-b*time)
m=rho*V
Q=m*c_p*(T_i-T_f)
Q_dot=n_dot_ball*Q*Convert(J/h, W)
```

T_i [C]	time [s]	Q [W]
500	127.4	271.2
550	134	305.1
600	140	339
650	145.5	372.9
700	150.6	406.9
750	155.3	440.8
800	159.6	474.7
850	163.7	508.6
900	167.6	542.5
950	171.2	576.4
1000	174.7	610.3

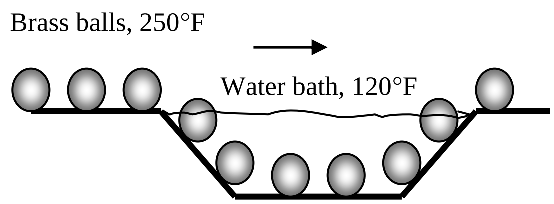


4-40E A number of brass balls are to be quenched in a water bath at a specified rate. The temperature of the balls after quenching and the rate at which heat needs to be removed from the water in order to keep its temperature constant are to be determined.

Assumptions **1** The balls are spherical in shape with a radius of r_o = 1 in. **2** The thermal properties of the balls are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the brass balls are given to be k = 64.1 Btu/h.ft.°F, ρ = 532 lbm/ft^3, and c_p = 0.092 Btu/lbm.°F.

Analysis (*a*) The characteristic length and the Biot number for the brass balls are



$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{2/12 \text{ ft}}{6} = 0.02778 \text{ ft}$$

$$Bi = \frac{hL_c}{k} = \frac{(42 \text{ Btu/h.ft}^2.°\text{F})(0.02778 \text{ ft})}{(64.1 \text{ Btu/h.ft.°F})} = 0.01820 < 0.1$$

The lumped system analysis is applicable since Bi < 0.1. Then the temperature of the balls after quenching becomes

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{42 \text{ Btu/h.ft}^2.°\text{F}}{(532 \text{ lbm/ft}^3)(0.092 \text{ Btu/lbm.°F})(0.02778 \text{ ft})} = 30.9 \text{ h}^{-1} = 0.00858 \text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T(t)-120}{250-120} = e^{-(0.00858 \text{ s}^{-1})(120 \text{ s})} \longrightarrow T(t) = \mathbf{166\ °F}$$

(*b*) The total amount of heat transfer from a ball during a 2-minute period is

$$m = \rho V = \rho \frac{\pi D^3}{6} = (532 \text{ lbm/ft}^3)\frac{\pi (2/12 \text{ ft})^3}{6} = 1.290 \text{ lbm}$$

$$Q = mc_p[T_i - T(t)] = (1.29 \text{ lbm})(0.092 \text{ Btu/lbm.°F})(250-166)°\text{F} = 9.97 \text{ Btu}$$

Then the rate of heat transfer from the balls to the water becomes

$$\dot{Q}_{total} = \dot{n}_{\text{ball}} Q_{\text{ball}} = (120 \text{ balls/min}) \times (9.97 \text{ Btu}) = \mathbf{1196\ Btu/min}$$

Therefore, heat must be removed from the water at a rate of 1196 Btu/min in order to keep its temperature constant at 120°F.

4-41 The heating times of a sphere, a cube, and a rectangular prism with similar dimensions are to be determined.

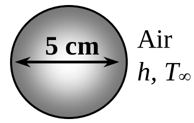
Assumptions **1** The thermal properties of the geometries are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The properties of silver are given to be k = 429 W/m·ºC, ρ = 10,500 kg/m^3, and c_p = 0.235 kJ/kg·ºC.

Analysis For sphere, the characteristic length and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.05\text{ m}}{6} = 0.008333\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\text{ W/m}^2.°\text{C})(0.008333\text{ m})}{(429\text{ W/m.°C})} = 0.00023 < 0.1$$



Since $Bi < 0.1$, the lumped system analysis is applicable. Then the time period for the sphere temperature to reach to 25ºC is determined from

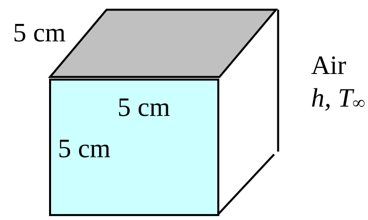
$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{12\text{ W/m}^2.°\text{C}}{(10{,}500\text{ kg/m}^3)(235\text{ J/kg.°C})(0.008333\text{ m})} = 0.0005836\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25-33}{0-33} = e^{-(0.0005836\text{ s}^{-1})t} \longrightarrow t = 2428\text{ s} = \mathbf{40.5\text{ min}}$$

Cube:

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{L^3}{6L^2} = \frac{L}{6} = \frac{0.05\text{ m}}{6} = 0.008333\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\text{ W/m}^2.°\text{C})(0.008333\text{ m})}{(429\text{ W/m.°C})} = 0.00023 < 0.1$$



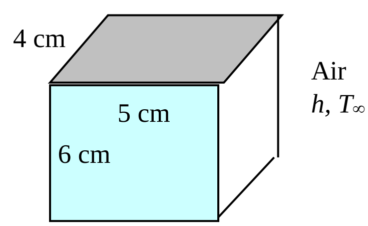
$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{12\text{ W/m}^2.°\text{C}}{(10{,}500\text{ kg/m}^3)(235\text{ J/kg.°C})(0.008333\text{ m})} = 0.0005836\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25-33}{0-33} = e^{-(0.0005836\text{ s}^{-1})t} \longrightarrow t = 2428\text{ s} = \mathbf{40.5\text{ min}}$$

Rectangular prism:

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{(0.04\text{ m})(0.05\text{ m})(0.06\text{ m})}{2(0.04\text{ m})(0.05\text{ m}) + 2(0.04\text{ m})(0.06\text{ m}) + 2(0.05\text{ m})(0.06\text{ m})} = 0.008108\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\text{ W/m}^2.°\text{C})(0.008108\text{ m})}{(429\text{ W/m.°C})} = 0.00023 < 0.1$$



$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c}$$

$$= \frac{12\text{ W/m}^2.°\text{C}}{(10{,}500\text{ kg/m}^3)(235\text{ J/kg.°C})(0.008108\text{ m})} = 0.0005998\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25-33}{0-33} = e^{-(0.0005998\text{ s}^{-1})t} \longrightarrow t = 2363\text{ s} = \mathbf{39.4\text{ min}}$$

The heating times are same for the sphere and cube while it is smaller in rectangular prism.

4-42 An electronic device is on for 5 minutes, and off for several hours. The temperature of the device at the end of the 5-min operating period is to be determined for the cases of operation with and without a heat sink.

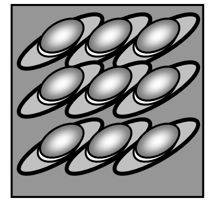
Assumptions **1** The device and the heat sink are isothermal. **2** The thermal properties of the device and of the sink are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The specific heat of the device is given to be c_p = 850 J/kg.°C. The specific heat of the aluminum sink is 903 J/kg.°C (Table A-3), but can be taken to be 850 J/kg.°C for simplicity in analysis.

Analysis

(*a*) Approximate solution

Electronic device, 18 W



This problem can be solved approximately by using an average temperature for the device when evaluating the heat loss. An energy balance on the device can be expressed as

$$E_{\text{in}} - E_{\text{out}} + E_{\text{generation}} = \Delta E_{\text{device}} \longrightarrow -\dot{Q}_{\text{out}}\Delta t + \dot{E}_{\text{generation}}\Delta t = mc_p \Delta T_{\text{device}}$$

or,
$$\dot{E}_{\text{generation}}\Delta t - hA_s\left(\frac{T+T_\infty}{2} - T_\infty\right)\Delta t = mc_p(T - T_\infty)$$

Substituting the given values,

$$(18\text{ J/s})(5\times 60\text{ s}) - (12\text{ W/m}^2.°\text{C})(0.0004\text{ m}^2)\left(\frac{T-25}{2}\right)°\text{C}(5\times 60\text{ s}) = (0.02\text{ kg})(850\text{ J/kg.°C})(T-25)°\text{C}$$

which gives T = **329.7°C**

If the device were attached to an aluminum heat sink, the temperature of the device would be

$$(18\text{ J/s})(5\times 60\text{ s}) - (12\text{ W/m}^2.°\text{C})(0.0084\text{ m}^2)\left(\frac{T-25}{2}\right)°\text{C}(5\times 60\text{ s})$$
$$= (0.20 + 0.02)\text{kg}\times(850\text{ J/kg.°C})(T-25)°\text{C}$$

which gives T = **51.7°C**

Note that the temperature of the electronic device drops considerably as a result of attaching it to a heat sink.

(*b*) Exact solution

This problem can be solved exactly by obtaining the differential equation from an energy balance on the device for a differential time interval *dt*. We will get

$$\frac{d(T-T_\infty)}{dt} + \frac{hA_s}{mc_p}(T-T_\infty) = \frac{\dot{E}_{\text{generation}}}{mc_p}$$

It can be solved to give

$$T(t) = T_\infty + \frac{\dot{E}_{\text{generation}}}{hA_s}\left(1 - \exp\left(-\frac{hA_s}{mc_p}t\right)\right)$$

Substituting the known quantities and solving for *t* gives **329.6°C** for the first case and **51.7°C** for the second case, which are practically identical to the results obtained from the approximate analysis.

Transient Heat Conduction in Large Plane Walls, Long Cylinders, and Spheres with Spatial Effects

4-43C Yes. Although rapid boiling will not change the boiling temperature, it will increase the heat transfer coefficient because of the higher level of agitation of bubbles. As a result, the cooking time will be shortened.

4-44C A cylinder whose diameter is small relative to its length can be treated as an infinitely long cylinder. When the diameter and length of the cylinder are comparable, it is not proper to treat the cylinder as being infinitely long. It is also not proper to use this model when finding the temperatures near the bottom or top surfaces of a cylinder since heat transfer at those locations can be two-dimensional.

4-45C The Fourier number is a measure of heat conducted through a body relative to the heat stored. Thus a large value of Fourier number indicates faster propagation of heat through body. Since Fourier number is proportional to time, doubling the time will also double the Fourier number.

4-46C Yes. A plane wall whose one side is insulated is equivalent to a plane wall that is twice as thick and is exposed to convection from both sides. The midplane in the latter case will behave like an insulated surface because of thermal symmetry.

4-47C This case can be handled by setting the heat transfer coefficient *h* to infinity ∞ since the temperature of the surrounding medium in this case becomes equivalent to the surface temperature.

4-48C When the Biot number is less than 0.1, the temperature of the sphere will be nearly uniform at all times. Therefore, it is more convenient to use the lumped system analysis in this case.

4-49C The maximum possible amount of heat transfer will occur when the temperature of the body reaches the temperature of the medium, and can be determined from $Q_{\max} = mc_p(T_\infty - T_i)$.

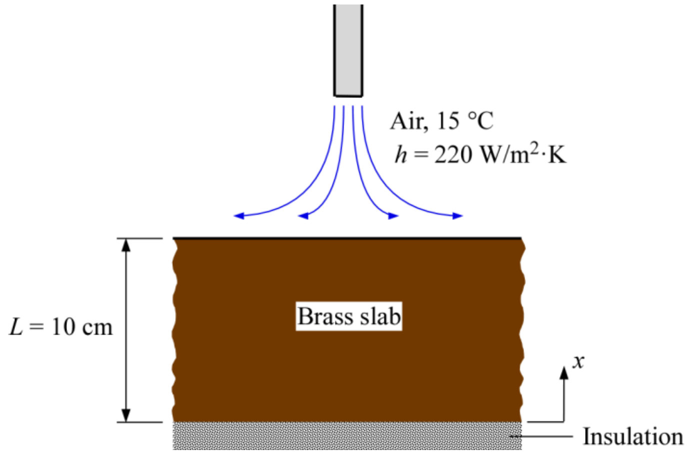
4-50 The temperature at the center plane of a brass plate after 3 minutes of cooling by impinging air jet is to be determined.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of the brass plate are given as $\rho = 8530$ kg/m³, $c_p = 380$ J/kg · K, $k = 110$ W/m · K, and $\alpha = 33.9 \times 10^{-6}$ m²/s.

Analysis This geometry can be considered to be a large plane wall with a thickness of $2L = 20$ cm subjected to convection at both sides. The surface with insulation becomes the center surface of the wall. Then the Biot number for this process is

$$Bi = \frac{hL}{k} = \frac{(220 \text{ W/m}^2 \cdot \text{K})(0.10 \text{ m})}{110 \text{ W/m} \cdot \text{K}} = 0.2$$



From Table 4-2, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 0.4328 \quad \text{and} \quad A_1 = 1.0311$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(33.9 \times 10^{-6} \text{ m}^2\text{/s})(3 \times 60 \text{ s})}{(0.10 \text{ m})^2} = 0.6102 > 0.2$$

Therefore, the one-term approximate solution is applicable.

The temperature at the center plane of the plate ($x/L = 0.5$) after 3 minutes of cooling is

$$\theta_{\text{wall}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x / L)$$

$$T(x,t) = (T_i - T_\infty) A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x / L) + T_\infty$$

$$T(0.05 \text{ m}, 180 \text{ s}) = (650\,°\text{C} - 15\,°\text{C})(1.0311) e^{-(0.4328)^2 (0.6102)} \cos[(0.4328)(0.5)] + 15\,°\text{C}$$
$$= \mathbf{585\,°C}$$

Discussion The insulated bottom surface of the brass plate is treated as a thermally symmetric boundary.

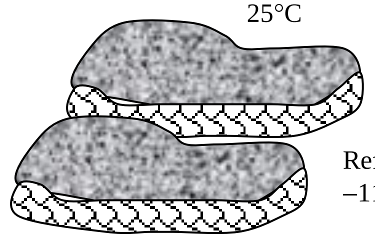
4-51 Steaks are cooled by passing them through a refrigeration room. The time of cooling is to be determined.

Assumptions **1** Heat conduction in the steaks is one-dimensional since the steaks are large relative to their thickness and there is thermal symmetry about the center plane. **2** The thermal properties of the steaks are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of steaks are given to be $k = 0.45$ W/m.°C and $\alpha = 0.91\times10^{-7}$ m^2/s

Analysis The Biot number is

$$Bi = \frac{hL}{k} = \frac{(9\ \text{W/m}^2.°\text{C})(0.01\,\text{m})}{(0.45\ \text{W/m}.°\text{C})} = 0.200$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 0.4328 \quad \text{and} \quad A_1 = 1.0311$$

The Fourier number is

$$\frac{T(L,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}\cos(\lambda_1 L/L)$$

$$\frac{2-(-11)}{25-(-11)} = (1.0311)e^{-(0.4328)^2\tau}\cos(0.4328) \longrightarrow \tau = 5.085 > 0.2$$

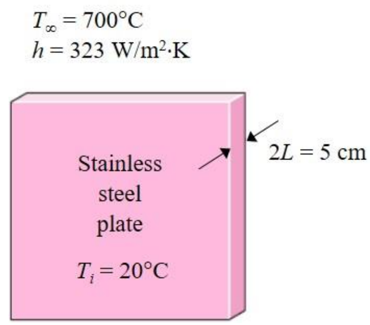
Therefore, the one-term approximate solution is applicable. Then the length of time for the steaks to be kept in the refrigerator is determined to be

$$t = \frac{\tau L^2}{\alpha} = \frac{(5.085)(0.01\,\text{m})^2}{0.91\times10^{-7}\ \text{m}^2/\text{s}} = 5590\,\text{s} = \mathbf{93.1\,min}$$

4-52 C&S ASTM A240 410S stainless steel plate is exposed to hot fluid at T_∞ = 700°C and h = 323 W/m^2·K. The plate has an initial temperature of 20°C. How long can the plate expose to the hot fluid before reaching its maximum use temperature?

Assumptions **1** Heat conduction is transient and one dimensional. **2** Thermal properties are constant. **3** Radiation effects are negligible. **4** The convection heat transfer coefficient is constant. **5** The Fourier number is τ> 0.2 so that the one term-term approximate solutions areapplicable.

Properties The thermal properties given are c_p = 460 J/kg·K, k = 26.9 W/m·K, and ρ = 7730 kg/m^3.



Analysis The plate is considered as a large plane wall with a thickness of 2L = 5 cm subjected to convection on both sides. The Biot number for this process is

$$\mathrm{Bi} = \frac{hL}{k} = \frac{(323\ \mathrm{W/m^2 \cdot K})(0.025\ \mathrm{m})}{26.9\ \mathrm{W/m \cdot K}} = 0.3002 > 0.1$$

The coefficients λ_1 and A_1 for a plane wall corresponding to this Bi are determined from Table 4-2 to be

$$\lambda_1 = 0.5218 \qquad \text{and} A_1 = 1.0450$$

The surface (x = L) would first reach the maximum use temperature, so from Eq. 4-23 we have

$$\frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L) \qquad \rightarrow \qquad \frac{T(L,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1)$$

Solving for the Fourier number (dimensionless time), we have

$$\tau = -\frac{1}{\lambda_1^2} \ln\left[\frac{1}{A_1 \cos(\lambda_1)} \frac{T(L,t) - T_\infty}{T_i - T_\infty}\right] = -\frac{1}{(0.5218)^2} \ln\left[\frac{1}{1.0450 \cos(0.5218)}\left(\frac{649 - 700}{20 - 700}\right)\right] = 9.151\ > 0.2$$

The time for the surface of the plate to reach the maximum use temperature is

$$t = L^2 \frac{\tau}{\alpha} = (0.025\ \mathrm{m})^2 \frac{9.151}{7.565 \times 10^{-6} \mathrm{m^2/s}} = \mathbf{756\ s}$$

where the thermal diffusivity of the plate is

$$\alpha = \frac{k}{\rho c_p} = \frac{26.9\ \mathrm{W/m \cdot K}}{(7730\ \mathrm{kg/m^3})(460\ \mathrm{J/kg \cdot K})} = 7.565 \times 10^{-6} \mathrm{m^2/s}$$

Discussion The steel plate surface is estimated to reach the maximum use temperature of 649°C (ASME B31.3-2014, Table A-1M) in 12 minutes and 36 seconds. Exposure of the plate to the hot fluid for duration longer than 756 s would not comply with the ASME Code for Process Piping.

4-53 The temperature at the center plane of an aluminum plate with $T_s \approx T_\infty$, after 15 seconds of heating, is to be determined.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal diffusivity erties of the aluminum plate is given as $\alpha = 97.1 \times 10^{-6}$ m^2/s.

Analysis For $T_s \approx T_\infty$, it implies that $h \to \infty$. Thus, the Biot number is

$$Bi = \frac{hL}{k} \to \infty$$

From Table 4-2 with $Bi \to \infty$, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 1.5708 \quad \text{and} \quad A_1 = 1.2732$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(97.1 \times 10^{-6}\ \text{m}^2/\text{s})(15\ \text{s})}{(0.05\ \text{m})^2} = 0.5826$$

The temperature at the center plane after 15 seconds of heating is

$$\theta_{0,\ \text{wall}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$T_0 = (T_i - T_\infty) A_1 e^{-\lambda_1^2 \tau} + T_\infty$$

$$T_0 = (25\,°\text{C} - 500\,°\text{C})(1.2732) e^{-(1.5708)^2 (0.5826)} + 500\,°\text{C} = \mathbf{356\,°C}$$

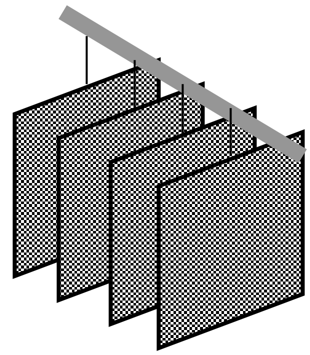
Discussion Since $\tau > 0.2$, the one-term approximate solution is applicable for this problem.

4-54 Large brass plates are heated in an oven. The surface temperature of the plates leaving the oven is to be determined.

Assumptions **1** Heat conduction in the plate is one-dimensional since the plate is large relative to its thickness and there is thermal symmetry about the center plane. **2** The thermal properties of the plate are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of brass at room temperature are given to be $k = 110$ W/m·°C, $\alpha = 33.9\times10^{-6}$ m²/s

Analysis The Biot number for this process is



$$Bi = \frac{hL}{k} = \frac{(80 \text{ W/m}^2.°\text{C})(0.015 \text{ m})}{(110 \text{ W/m.°C})} = 0.0109$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 0.1035 \quad \text{and} \quad A_1 = 1.0018$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(33.9\times10^{-6} \text{ m}^2/\text{s})(10 \text{ min}\times 60 \text{ s/min})}{(0.015 \text{ m})^2} = 90.4 > 0.2$$

Therefore, the one-term approximate solution is applicable. Then the temperature at the surface of the plates becomes

$$\theta(L,t)_{wall} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L) = (1.0018)e^{-(0.1035)^2(90.4)}\cos(0.1035) = 0.378$$

$$\frac{T(L,t)-700}{25-700} = 0.378 \longrightarrow T(L,t) = \mathbf{445\ °C}$$

Discussion This problem can be solved easily using the lumped system analysis since Bi < 0.1, and thus the lumped system analysis is applicable. It gives

$$\alpha = \frac{k}{\rho c_p} \rightarrow \rho c_p = \frac{k}{\alpha} = \frac{110 \text{ W/m}\cdot°\text{C}}{33.9\times10^{-6} \text{ m}^2/s} = 3.245\times10^6 \text{ W}\cdot\text{s/m}^3\cdot°\text{C}$$

$$b = \frac{hA}{\rho V c_p} = \frac{hA}{\rho(LA)c_p} = \frac{h}{\rho L c_p} = \frac{h}{L(k/\alpha)} = \frac{80 \text{ W/m}^2\cdot°\text{C}}{(0.015 \text{ m})(3.245\times10^6 \text{ W}\cdot\text{s/m}^3\cdot°\text{C})} = 0.001644 \text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \quad \rightarrow \quad T(t) = T_\infty + (T_i - T_\infty)e^{-bt} = 700°\text{C} + (25-700°\text{C})e^{-(0.001644\text{ s}^{-1})(600\text{ s})} = \mathbf{448\ °C}$$

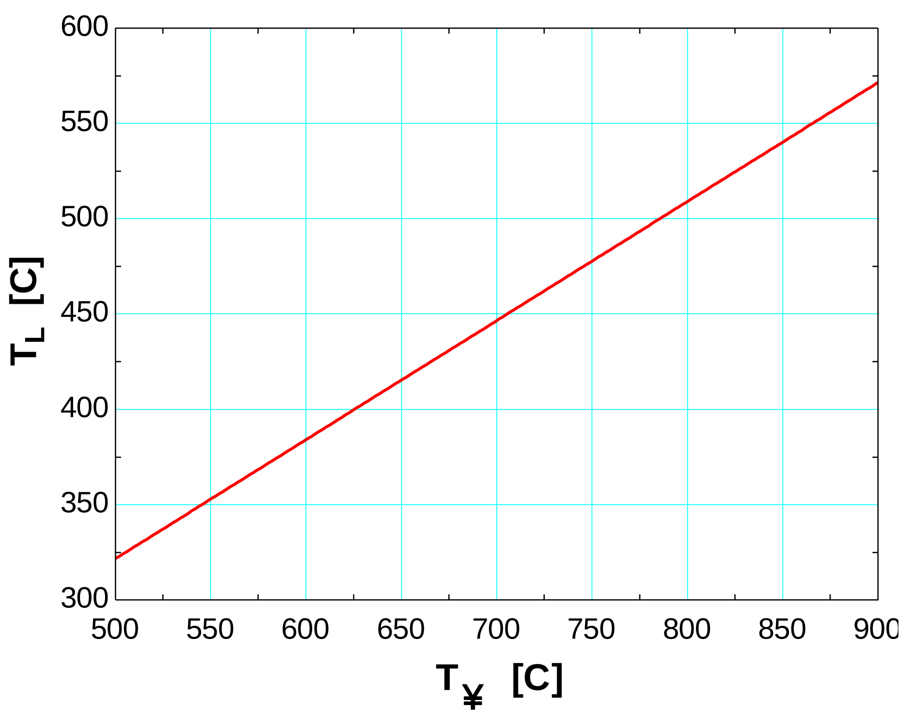
which is almost identical to the result obtained above.

4-55  Prob. 4-54 is reconsidered. The effects of the temperature of the oven and the heating time on the final surface temperature of the plates are to be investigated.

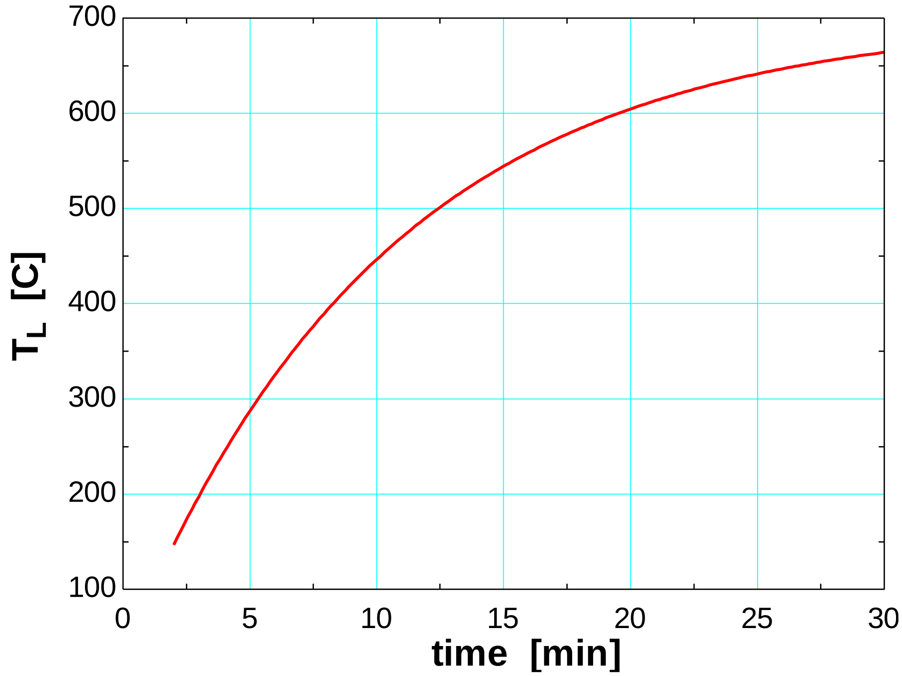
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=(0.03/2) [m]
T_i=25 [C]
T_infinity=700 [C]
time=10 [min]
h=80 [W/m^2-C]

"PROPERTIES"
k=110 [W/m-C]
alpha=33.9E-6 [m^2/s]

"ANALYSIS"
Bi=(h*L)/k
"From Table 4-2, corresponding to this Bi number, we read"
lambda_1=0.1039
A_1=1.0018
tau=(alpha*time*Convert(min, s))/L^2
(T_L-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)*Cos(lambda_1*L/L)
```

T_∞ [C]	T_L [C]
500	321.6
525	337.2
550	352.9
575	368.5
600	384.1
625	399.7
650	415.3
675	430.9
700	446.5
725	462.1
750	477.8
775	493.4
800	509
825	524.6
850	540.2
875	555.8
900	571.4



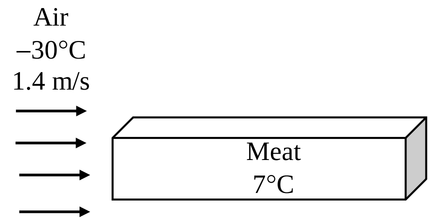
time [min]	T_L [C]
2	146.7
4	244.8
6	325.5
8	391.9
10	446.5
12	491.5
14	528.5
16	558.9
18	583.9
20	604.5
22	621.4
24	635.4
26	646.8
28	656.2
30	664



4-56 The center temperature of meat slabs is to be lowered to -18°C during cooling. The cooling time and the surface temperature of the slabs at the end of the cooling process are to be determined.

Assumptions **1** The meat slabs can be approximated as very large plane walls of half-thickness L = 11.5 cm. **2** Heat conduction in the meat slabs is one-dimensional because of the symmetry about the centerplane. **3** The thermal properties of the meat slabs are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified). **6** The phase change effects are not considered, and thus the actual cooling time will be much longer than the value determined.

Properties The thermal conductivity and thermal diffusivity of meat slabs are given to be k = 0.47 W/m·°C and $\alpha = 0.13\times10^{-6}$ m^2/s. These properties will be used for both fresh and frozen meat.



Analysis First we find the Biot number:

$$\text{Bi} = \frac{\text{hL}}{\text{k}} = \frac{(20\ \text{W/m}^2.°\text{C})(0.115\ \text{m})}{0.47\ \text{W/m}.°\text{C}} = 4.89$$

From Table 4-2 we read, for a plane wall, λ_1 = 1.308 and A_1=1.239. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \to \frac{-18-(-30)}{7-(-30)} = 1.239 e^{-(1.308)^2\tau} \to \tau = 0.783$$

which is greater than 0.2 and thus the one-term solution is applicable. Then the cooling time becomes

$$\tau = \frac{\alpha t}{L^2} \to t = \frac{\tau L^2}{\alpha} = \frac{(0.783)(0.115\ \text{m})^2}{0.13\times10^{-6}\ \text{m}^2/\text{s}} = 79{,}650\ \text{s} = \mathbf{22.1\ h}$$

which is greater than 0.2. Therefore, the one-term approximate solution is applicable.

The lowest temperature during cooling will occur on the surface (x/L = 1), and is determined to be

$$\frac{T(x)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 x/L) \to \frac{T(L)-T_\infty}{T_i - T_\infty} = \theta_0 \cos(\lambda_1 L/L) = \frac{T_o - T_\infty}{T_i - T_\infty}\cos(\lambda_1)$$

Substituting,

$$\frac{T(L)-(-30)}{7-(-30)} = \left(\frac{-18-(-30)}{7-(-30)}\right)\cos(\lambda_1) = 0.3243\times0.2598 = 0.08425 \longrightarrow T(L) = \mathbf{-26.9°C}$$

which is close the temperature of the refrigerated air.

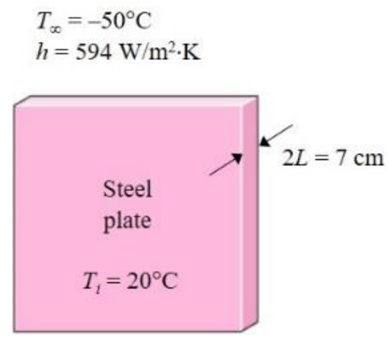
4-57 ASTM A203 B steel plate is exposed to cryogenic fluid at T_∞ = −50°C and h = 594 W/m^2·K. The plate has an initial temperature of 20°C. Would the plate comply with the ASME Code for Process Piping if it is exposed to the cold fluid for 6 minutes?

Assumptions **1** Heat conduction is transient and one dimensional. **2** Thermal properties are constant. **3** Radiation effects are negligible. **4** The convection heat transfer coefficient is constant. **5** The Fourier number is $\tau > 0.2$ so that the one term-term approximate solutions are applicable.

Properties The thermal properties given are c_p = 470 J/kg·K, k = 52 W/m·K, and ρ = 7900 kg/m^3.

Analysis The plate is considered as a large plane wall with a thickness of $2L$ = 7 cm subjected to convection on both sides. The Biot number for this process is

$$\text{Bi} = \frac{hL}{k} = \frac{(594\ \text{W/m}^2\cdot\text{K})(0.035\ \text{m})}{52\ \text{W/m}\cdot\text{K}} = 0.3998 > 0.1$$



The coefficients λ_1 and A_1 for a plane wall corresponding to this Bi are determined from Table 4-2 to be

$$\lambda_1 = 0.5932 \qquad \text{and} A_1 = 1.0580$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.4005\times10^{-5}\text{m}^2/\text{s})(360\ \text{s})}{(0.035\ \text{m})^2} = 4.1158\ > 0.2$$

where the thermal diffusivity of the plate is

$$\alpha = \frac{k}{\rho c_p} = \frac{52\ \text{W/m}\cdot\text{K}}{(7900\ \text{kg/m}^3)(470\ \text{J/kg}\cdot\text{K})} = 1.4005\times10^{-5}\text{m}^2/\text{s}$$

The surface (x = L) would first reach the minimum suitable temperature, so from Eq. 4-23 we have

$$\frac{T(x,t)-T_\infty}{T_i-T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 x/L) \qquad \rightarrow \qquad \frac{T(L,t)-T_\infty}{T_i-T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1)$$
$$T(L,t) = (T_i - T_\infty)A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1) + T_\infty$$
$$T(L,t) = (20+50)(1.0580)e^{-(0.5932)^2(4.1158)}\cos(0.5932) - 50 = \mathbf{-35.6}℃ < -30℃$$

Discussion Exposing the ASTM A203 B steel plate to the cryogenic fluid for 6 minutes would cool the surface temperature of the plate to about −36°C. This means that the plate would not be in compliance with the ASME code that limits the minimum suitable temperature of ASTM A203 B steel plate to −30°C (ASME B31.3-2014, Table A-1M).

4-58 The heat transfer from the Pyroceram plate during the cooling process of 286 seconds is to be determined using the one-term approximate solutions.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of the Pyroceram plate are given as ρ = 2600 kg/m^3, c_p = 808 J/kg · K, k = 3.98 W/m · K, and α = 1.89 × 10^{-6} m^2/s.

Analysis The maximum amount heat transfer from the Pyroceram plate is

$$Q_{\max} = mc_p(T_i - T_\infty) = (10\text{ kg})(808\text{ J/kg}\cdot\text{K})(500-25)\text{ K} = 3.838\times10^6\text{ J}$$

The Biot number for this process is

$$Bi = \frac{hL}{k} = \frac{(13.3\text{ W/m}^2\cdot\text{K})(0.003\text{ m})}{3.98\text{ W/m}\cdot\text{K}} = 0.01$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.89\times10^{-6}\text{ m}^2\text{/s})(286\text{ s})}{(0.003\text{ m})^2} = 60.06$$

which is greater than 0.2. Therefore, the one-term approximate solution is applicable.

(a) From Table 4-2 with Bi = 0.01, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 0.0998 \quad \text{and} \quad A_1 = 1.0017$$

For plane wall, we have

$$\theta_{0,\text{ wall}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = 1.0017e^{-(0.0998)^2 60.06} = 0.5507$$

The heat transfer from the Pyroceram plate during the cooling process of 286 seconds is

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{wall}} = 1 - \theta_{0,\text{ wall}}\frac{\sin\lambda_1}{\lambda_1} = 1-(0.5507)\frac{\sin(0.0998)}{0.0998} = 0.4502$$

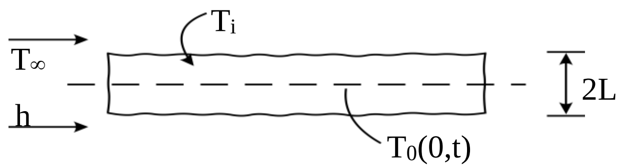
$$Q = 0.4502Q_{\max} = \mathbf{1.73\times10^6\ J}$$

4-59 The time required and the amount of energy removed to thaw a frozen steak to a specified temperature is to be determined.

Assumptions **1** Heat conduction is transient and one dimensional. **2** Thermal properties are constant. **3** Radiation effects are negligible. **4** The convection heat transfer coefficient is constant. **5** Neglect the heat of fusion associated with the melting phase change **6** The Fourier number is $\tau > 0.2$ so that the one term-term approximate solutions are applicable.

Properties The thermal properties given are c_p = 4472 J/kg·K, k = 0.625 W/m·K, and ρ = 1000 kg/m^3.

Analysis (a) The steak is considered as a large plane wall with a thickness of $2L$ = 50mm subjected to convection on both sides. The Biot number for this process is



$$\text{Bi} = \frac{hL}{k} = \frac{(10\,\text{W/m}^2\cdot\text{K})(0.025\,\text{m})}{0.625\,\text{W/m}\cdot\text{K}} = 0.40$$

The coefficients λ_1 and A_1 for a plane wall corresponding to this Bi are determined from Table 4-2 to be

$$\lambda_1 = 0.5932 \qquad \text{and} A_1 = 1.0580$$

Solving for the Fourier number (dimensionless time) from Eq. 4-26, we have

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$\tau = -\frac{1}{\lambda_1^2}\ln\left[\frac{1}{A_1}\frac{T_0 - T_\infty}{T_i - T_\infty}\right] = -\frac{1}{(0.5932)^2}\ln\left[\frac{1}{1.0580}\left(\frac{4-22}{-8-22}\right)\right] = 1.61 \ > 0.2$$

Solving for the time it takes for the center temperature of the steak to reach 4°C

$$t = L^2\frac{\tau}{\alpha} = (0.025\text{ m})^2\frac{1.61}{1.3976\times 10^{-7}\text{m}^2/\text{s}} = 7{,}200\text{ s} = \mathbf{2\ hr}$$

where the thermal diffusivity of the plate is

$$\alpha = \frac{k}{\rho c_p} = \frac{0.625\text{ W/m}\cdot\text{K}}{(1000\text{ kg/m}^3)(4472\text{ J/kg}\cdot\text{K})} = 1.3976\times 10^{-7}\text{m}^2/\text{s}$$

(b) To calculate the amount of energy per unit area that has been removed from the steak during this period of thawing, use Eq. 4-33,

$$\left(\frac{Q}{Q_{max}}\right)_{wall} = 1 - \theta_{0,\text{wall}}\frac{\sin\lambda_1}{\lambda_1} = 1 - (0.60)\frac{\sin 0.5932}{0.5932} = 0.4346$$

where $\theta_0 = \dfrac{T_0 - T_\infty}{T_i - T_\infty} = 0.60$ and $\lambda_1 = 0.5932$ (from Table 4-2 with Bi = 0.40)

From Eq. 4-30 $\quad Q_{max} = mc_p(T_\infty - T_i) = \rho V c_p(T_\infty - T_i)$

$$Q_{max}/A = \dot{q}_{max} = (1000\,\text{kg/m}^3)(50\times 10^{-3}\,\text{m})[22-(-8)]°\text{C} = 6.326\times 10^6\,\text{J/m}^2$$

$$\dot{q} = 0.4346\dot{q}_{max} = 0.4346(6.326\times 10^6\,\text{J/m}^2) = \mathbf{2.75\times 10^6\ J/m^2}$$

(c) Since Bi > 0.1, lumped system analysis is not appropriate or internal thermal resistance **cannot** be neglected.

4-60 The temperature at the center of a Pyroceram rod after 3 minutes of cooling is to be determined using analytical one-term approximation method.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of Pyroceram rod are given as $\rho = 2600$ kg/m^3, $c_p = 808$ J/kg · K, $k = 3.98$ W/m · K, and $\alpha = 1.89 \times 10^{-6}$ m^2/s

Analysis The Biot number for this process is

$$Bi = \frac{hr_o}{k} = \frac{(80\ \text{W/m}^2 \cdot \text{K})(0.005\ \text{m})}{3.98\ \text{W/m} \cdot \text{K}} = 0.10$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.89 \times 10^{-6}\ \text{m}^2/\text{s})(3 \times 60\ \text{s})}{(0.005\ \text{m})^2} = 13.61$$

which is greater than 0.2. Therefore, the one-term approximate solution can be used.

From Table 4-2 with $Bi = 0.10$, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 0.4417 \quad \text{and} \quad A_1 = 1.0246$$

The temperature at the center of the rod after 3 minutes is

$$\theta_{0,\,\text{cyl}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$T_0 = (T_i - T_\infty) A_1 e^{-\lambda_1^2 \tau} + T_\infty = (1000\,°\text{C} - 25\,°\text{C})(1.0246) e^{-(0.4417)^2 (13.61)} + 25\,°\text{C} = \mathbf{95.2\,°C}$$

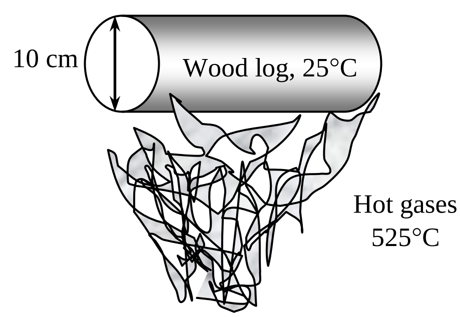
4-61 A long cylindrical wood log is exposed to hot gases in a fireplace. The time for the ignition of the wood is to be determined.

Assumptions **1** Heat conduction in the wood is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the wood are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of wood are given to be $k = 0.17$ W/m.°C, $\alpha = 1.28 \times 10^{-7}$ m^2/s

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(13.6\ \text{W/m}^2.°\text{C})(0.05\ \text{m})}{(0.17\ \text{W/m}.°\text{C})} = 4.00$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 1.9081 \quad \text{and} \quad A_1 = 1.4698$$

Once the constant J_0 is determined from Table 4-3 corresponding to the constant λ_1 =1.9081, the Fourier number is determined to be

$$\frac{T(r_o, t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r_o / r_o)$$

$$\frac{375 - 525}{25 - 525} = (1.4698) e^{-(1.9081)^2 \tau} (0.2771) \longrightarrow \tau = 0.203$$

which is above the value of 0.2. We can use the one-term approximate solution. Then the length of time before the log ignites is

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.203)(0.05\ \text{m})^2}{1.28 \times 10^{-7}\ \text{m}^2/\text{s}} = 3965\ \text{s} = \mathbf{66.1\ min}$$

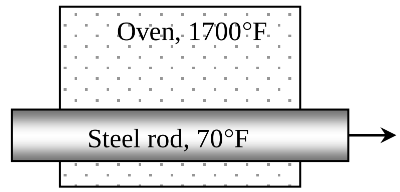
4-62E Long cylindrical steel rods are heat-treated in an oven. Their centerline temperature when they leave the oven is to be determined.

Assumptions **1** Heat conduction in the rods is one-dimensional since the rods are long and they have thermal symmetry about the center line. **2** The thermal properties of the rod are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of AISI stainless steel rods are given to be k = 7.74 Btu/h.ft.°F, α = 0.135 ft²/h.

Analysis The time the steel rods stays in the oven can be determined from

$$t = \frac{\text{length}}{\text{velocity}} = \frac{21\text{ ft}}{7\text{ ft/min}} = 3\text{ min} = 180\text{ s}$$



The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(20\text{ Btu/h.ft}^2.°\text{F})(2/12\text{ ft})}{(7.74\text{ Btu/h.ft.°F})} = 0.4307$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 0.8790 \quad \text{and} \quad A_1 = 1.0996$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.135\text{ ft}^2/\text{h})(3/60\text{ h})}{(2/12\text{ ft})^2} = 0.243$$

which is greater than 0.2. Therefore, the one-term approximate solution is applicable.

Then the temperature at the center of the rods becomes

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.0996)e^{-(0.8790)^2(0.243)} = 0.911$$

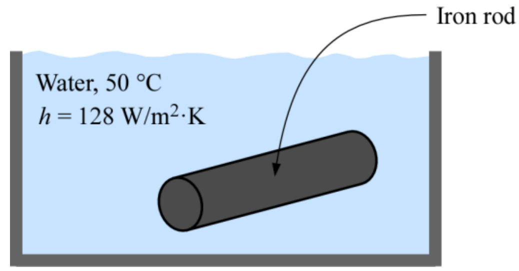
$$\frac{T_0 - 1700}{70 - 1700} = 0.911 \longrightarrow T_o = \mathbf{215°F}$$

4-63 The time required for a long iron rod surface temperature to cool to 200°C in a water bath is to be determined.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of iron rod are given as ρ = 7870 kg/m^3, c_p = 447 J/kg · K, k = 80.2 W/m · K, and α = 23.1 × 10^{-6} m^2/s.

Analysis The Biot number for this process is



$$Bi = \frac{hr_o}{k} = \frac{(128\ \text{W/m}^2\cdot\text{K})(0.0125\ \text{m})}{80.2\ \text{W/m}\cdot\text{K}} = 0.02$$

From Table 4-2, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 0.1995 \quad \text{and} \quad A_1 = 1.0050$$

For the temperature at the rod surface ($r = r_o$) to be 200°C, we have

$$\theta_{\text{cyl}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r / r_o)$$

From Table 4-3, we have $J_0(0.1995) \approx 0.9900$. Hence

$$\frac{200-50}{700-50} = (1.0050)e^{-(0.1995)^2 \tau}(0.9900) \quad \rightarrow \quad \tau = 36.72$$

The time required for the iron rod surface to cool to 200°C is

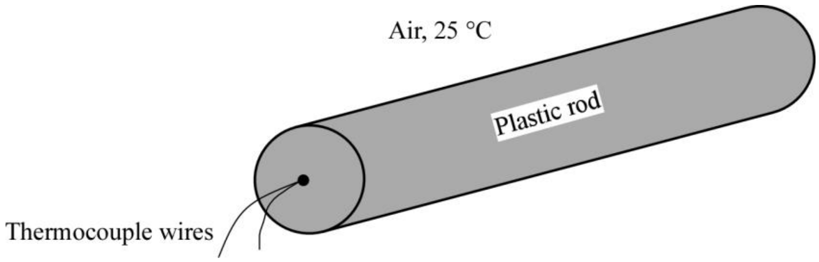
$$\tau = \frac{\alpha t}{r_o^2} = 36.72 \quad \rightarrow \quad t = \frac{36.72 r_o^2}{\alpha} = \frac{36.72(0.0125\ \text{m})^2}{23.1\times10^{-6}\ \text{m}^2/\text{s}} = \mathbf{248\ s}$$

Discussion Since τ > 0.2, the one-term approximate solution is applicable for this problem.

4-64 The convection heat transfer coefficient for a plastic rod being cooled is to be determined.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of the plastic rod are given as ρ = 1190 kg/m^3, c_p = 1465 J/kg · K, and k = 0.19 W/m · K.



Analysis The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{kt}{\rho c_p r_o^2} = \frac{(0.19 \text{ W/m}\cdot\text{K})(1388\text{ s})}{(1190\text{ kg/m}^3)(1465\text{ J/kg}\cdot\text{K})(0.01\text{ m})^2} = 1.513$$

which is greater than 0.2. Therefore, the one-term approximate solution can be used.

After 1388 s of cooling, the temperature at the center of the rod is 30°C. So, we have

$$\theta_{0,\text{cyl}} = A_1 e^{-\lambda_1^2 \tau} = \frac{T_0 - T_\infty}{T_i - T_\infty} = \frac{30-25}{70-25} = 0.111$$

To determine the convection heat transfer coefficient, we need to find the corresponding Biot number by trial-and-error:

Trial 1: Let Bi = 0.8 and from Table 4-2 we have

$$\lambda_1 = 1.1490 \quad \text{and} \quad A_1 = 1.1724$$

$$A_1 e^{-\lambda_1^2 \tau} = (1.1724)e^{-(1.1490)^2(1.513)} = 0.159 > 0.111 \quad \text{(does no match)}$$

Trial 2: Let Bi = 2.0 and from Table 4-2 we have

$$\lambda_1 = 1.5995 \quad \text{and} \quad A_1 = 1.3384$$

$$A_1 e^{-\lambda_1^2 \tau} = (1.3384)e^{-(1.5995)^2(1.513)} = 0.0279 < 0.111 \quad \text{(does no match)}$$

Trial 3: Let Bi = 1.0 and from Table 4-2 we have

$$\lambda_1 = 1.2558 \quad \text{and} \quad A_1 = 1.2071$$

$$A_1 e^{-\lambda_1^2 \tau} = (1.2071)e^{-(1.2558)^2(1.513)} = 0.111 = 0.111 \quad \text{(match)}$$

Therefore the Biot number for this process is

$$Bi = \frac{hr_o}{k} = 1.0 \quad \rightarrow \quad h = \frac{k}{r_o} = \frac{0.19 \text{ W/m}\cdot\text{K}}{0.01\text{ m}} = \mathbf{19\ W/m^2\cdot K}$$

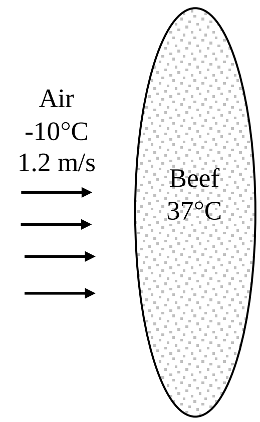
Discussion Speeding up the cooling process can be achieved by increasing the convection heat transfer coefficient.

4-65 The center temperature of a beef carcass is to be lowered to 4°C during cooling. The cooling time and if any part of the carcass will suffer freezing injury during this cooling process are to be determined.

Assumptions **1** The beef carcass can be approximated as a cylinder with insulated top and base surfaces having a radius of r_o = 12 cm and a height of H = 1.4 m. **2** Heat conduction in the carcass is one-dimensional in the radial direction because of the symmetry about the centerline. **3** The thermal properties of the carcass are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of carcass are given to be k = 0.47 W/m·°C and $\alpha = 0.13\times10^{-6}$ m^2/s.

Analysis First we find the Biot number:



$$\text{Bi} = \frac{hr_o}{k} = \frac{(22\ \text{W/m}^2.°\text{C})(0.12\ \text{m})}{0.47\ \text{W/m.°C}} = 5.62$$

From Table 4-2 we read, for a cylinder, λ_1 = 2.027 and A_1 = 1.517. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \rightarrow \frac{4-(-10)}{37-(-10)} = 1.517 e^{-(2.027)^2\tau} \rightarrow \tau = 0.396$$

which is greater than 0.2 and thus the one-term solution is applicable. Then the cooling time becomes

$$\tau = \frac{\alpha t}{r_o^2} \rightarrow t = \frac{\tau r_o^2}{\alpha} = \frac{(0.396)(0.12\ \text{m})^2}{0.13\times10^{-6}\ \text{m}^2/\text{s}} = 43{,}865\ \text{s} = \mathbf{12.2\ h}$$

The lowest temperature during cooling will occur on the surface (r/r_o = 1), and is determined to be

$$\frac{T(r)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r/r_o) \rightarrow \frac{T(r_o)-T_\infty}{T_i - T_\infty} = \theta_0 J_0(\lambda_1 r/r_o) = \frac{T_o - T_\infty}{T_i - T_\infty} J_0(\lambda_1 r_o/r_o)$$

Substituting,

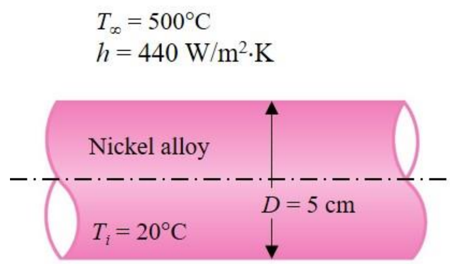
$$\frac{T(r_0)-(-10)}{37-(-10)} = \left(\frac{4-(-10)}{37-(-10)}\right) J_0(\lambda_1) = 0.2979\times0.2084 = 0.0621 \longrightarrow T(r_o) = -7.1°\text{C}$$

which is below the freezing temperature of -1.7 °C. Therefore, the outer part of the beef carcass will freeze during this cooling process.

4-66 ASTM B335 rod is submerged in hot fluid at T_∞ = 500°C and h = 440 W/m²·K. The rod has an initial temperature of 20°C. How long can the rod submerge in the hot fluid before reaching its maximum use temperature?

Assumptions**1** Heat conduction is transient and one dimensional. **2** Thermal properties are constant. **3** Radiation effects are negligible. **4** The convection heat transfer coefficient is constant. **5** The Fourier number is τ> 0.2 so that the one term-term approximate solutions are applicable.

Properties The thermal properties given are c_p = 380 J/kg·K, k = 11 W/m·K, and ρ = 9300 kg/m³.



Analysis The rod is considered as a long cylinder with a radius of r_o = 25 mm subjected to convection at the surface. The Biot number for this process is

$$\text{Bi} = \frac{hr_o}{k} = \frac{(440\ \text{W/m}^2\cdot\text{K})(0.025\ \text{m})}{11\ \text{W/m}\cdot\text{K}} = 1.0 > 0.1$$

The coefficients λ_1 and A_1 for a plane wall corresponding to this Bi are determined from Table 4-2 to be

$$\lambda_1 = 1.2558 \qquad \text{and} A_1 = 1.2071$$

The surface ($r = r_o$) would first reach the maximum use temperature, so from Eq. 4-24 we have

$$\frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o) \qquad \rightarrow \qquad \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1)$$

From Table 4-3, the zeroth-order Bessel function of the first kind is

$$J_0(\lambda_1) = J_0(1.2588) = 0.6429$$

Solving for the Fourier number (dimensionless time), we have

$$\tau = -\frac{1}{\lambda_1^2}\ln\left[\frac{1}{A_1 J_0(\lambda_1)}\frac{T(r_o,t) - T_\infty}{T_i - T_\infty}\right] = -\frac{1}{(1.2558)^2}\ln\left[\frac{1}{(1.2071)(0.6429)}\left(\frac{427-500}{20-500}\right)\right] = 1.0334\ > 0.2$$

The time for the surface of the rod to reach the maximum use temperature is

$$t = r_o^2\frac{\tau}{\alpha} = (0.025\ \text{m})^2\frac{1.0334}{3.1126\times 10^{-6}\text{m}^2/\text{s}} = \mathbf{207.5\ s}$$

where the thermal diffusivity of the rod is

$$\alpha = \frac{k}{\rho c_p} = \frac{11\ \text{W/m}\cdot\text{K}}{(9300\ \ \text{kg/m}^3)(380\ \text{J/kg}\cdot\text{K})} = 3.1126\times 10^{-6}\text{m}^2/\text{s}$$

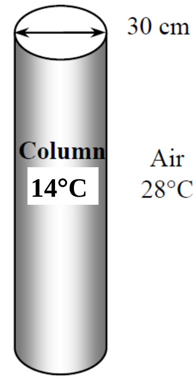
Discussion The nickel alloy rod surface is estimated to reach the maximum use temperature of 427°C (ASME B31.3-2014, Table A-1M) in about 3 minutes and 28 seconds. Exposure of the rod to the hot fluid for duration longer than 208 s would not comply with the ASME Code for Process Piping.

4-67 A cold cylindrical concrete column is exposed to warm ambient air during the day. The time it will take for the surface temperature to rise to a specified value, the amounts of heat transfer for specified values of center and surface temperatures are to be determined using the approximate analytical solutions.

Assumptions **1** Heat conduction in the column is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the column are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions is applicable (this assumption will be verified).

Properties The properties of concrete are given to be $k = 0.79$ W/m.K, $\alpha = 5.94\times10^{-7}$ m^2/s, $\rho = 1600$ kg/m^3 and $c_p = 0.84$ kJ/kg.K

Analysis (*a*) The Biot number is



$$Bi = \frac{hr_o}{k} = \frac{(14\ \text{W/m}^2\cdot\text{K})(0.15\ \text{m})}{(0.79\ \text{W/m}\cdot\text{K})} = 2.658$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 1.7240 \quad \text{and} \quad A_1 = 1.3915$$

Once the constant J_0=0.3841 is determined from Table 4-3 corresponding to the constant λ_1, the Fourier number is determined to be

$$\frac{T(r_o,t)-T_\infty}{T_i-T_\infty} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r_o/r_o) \longrightarrow \frac{27-28}{14-28} = (1.3915)e^{-(1.7240)^2\tau}(0.3841) \longrightarrow \tau$$

which is greater than 0.2. Therefore, the one-term approximate solution can be used. Then the time it will take for the column surface temperature to rise to 27°C becomes

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.6771)(0.15\ \text{m})^2}{5.94\times10^{-7}\ \text{m}^2/\text{s}} = 25{,}650\ \text{s} = \mathbf{7.1\,hours}$$

(*b*) The heat transfer to the column will stop when the center temperature of column reaches to the ambient temperature, which is 28°C. That is, we are asked to determine the maximum heat transfer between the ambient air and the column.

$$m = \rho \boldsymbol{V} = \rho\pi r_o^2 L = (1600\ \text{kg/m}^3)[\pi(0.15\ \text{m})^2(4\ \text{m})] = 452.4\ \text{kg}$$

$$Q_{\max} = mc_p[T_\infty - T_i] = (452.4\ \text{kg})(0.84\ \text{kJ/kg}.K)(28-14)°\text{C} = \mathbf{5320\ kJ}$$

(*c*) To determine the amount of heat transfer until the surface temperature reaches to 27°C, we first determine

$$\frac{T(0,t)-T_\infty}{T_i-T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.3915)e^{-(1.7240)^2(0.6771)} = 0.1860$$

Once the constant J_1 = 0.5787 is determined from Table 4-3 corresponding to the constant λ_1, the amount of heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 2\left(\frac{T_0-T_\infty}{T_i-T_\infty}\right)\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2\times0.1860\times\frac{0.5787}{1.7240} = 0.875$$

$$Q = 0875Q_{\max}$$

$$Q = 0.875(5320\ \text{kJ}) = \mathbf{4660\ kJ}$$

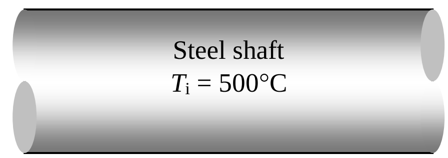
4-68 A long cylindrical shaft at 400°C is allowed to cool slowly. The center temperature and the heat transfer per unit length of the cylinder are to be determined.

Assumptions **1** Heat conduction in the shaft is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the shaft are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of stainless steel 304 at room temperature are given to be k = 14.9 W/m.°C, ρ = 7900 kg/m^3, c_p = 477 J/kg.°C, $\alpha = 3.95\times10^{-6}$ m^2/s

Analysis First the Biot number is calculated to be

$$Bi = \frac{hr_o}{k} = \frac{(60\ \text{W/m}^2.°\text{C})(0.175\ \text{m})}{(14.9\ \text{W/m.°C})} = 0.705$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 1.0904 \quad \text{and} \quad A_1 = 1.1548$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.95\times10^{-6}\ \text{m}^2/\text{s})(20\times 60\ \text{s})}{(0.175\ \text{m})^2} = 0.1548$$

which is very close to the value of 0.2. Therefore, the one-term approximate solution can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the temperature at the center of the shaft becomes

$$\theta_{0,\text{cyl}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.1548)e^{-(1.0904)^2(0.1548)} = 0.9607$$

$$\frac{T_0 - 150}{500 - 150} = 0.9607 \longrightarrow T_0 = \mathbf{486.2°C}$$

The maximum heat can be transferred from the cylinder per meter of its length is

$$m = \rho \boldsymbol{V} = \rho \pi r_o^2 L = (7900\ \text{kg/m}^3)[\pi(0.175\ \text{m})^2(1\ \text{m})] = 760.1\ \text{kg}$$
$$Q_{max} = mc_p[T_\infty - T_i] = (760.1\ \text{kg})(0.477\ \text{kJ/kg.°C})(500-150)°\text{C} = 126{,}894\ \text{kJ}$$

Once the constant J_1 = 0.4679 is determined from Table 4-3 corresponding to the constant λ_1 =1.0904, the actual heat transfer becomes

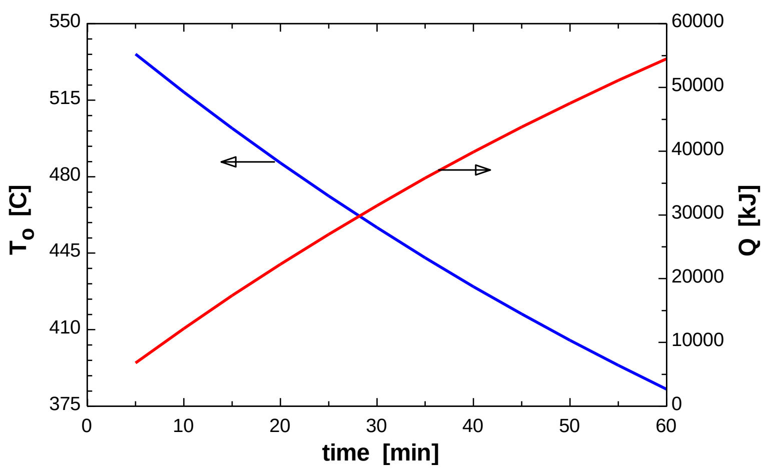
$$\left(\frac{Q}{Q_{max}}\right)_{cyl} = 1 - 2\left(\frac{T_o - T_\infty}{T_i - T_\infty}\right)\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2\left(\frac{486.2-150}{500-150}\right)\frac{0.4679}{1.0904} = 0.1756$$
$$Q = 0.1754(126{,}894\ \text{kJ}) = \mathbf{22{,}270\ kJ}$$

4-69 Prob. 4-68 is reconsidered. The effect of the cooling time on the final center temperature of the shaft and the amount of heat transfer is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
r_o=(0.35/2) [m]
T_i=500 [C]
T_infinity=150 [C]
h=60 [W/m^2-C]
time=20 [min]

 "PROPERTIES"
k=14.9 [W/m-C]
rho=7900 [kg/m^3]
c_p=477 [J/kg-C]
alpha=3.95E-6 [m^2/s]

"ANALYSIS"
Bi=(h*r_o)/k
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1=1.0904
A_1=1.1548
J_1=0.4679 "From Table 4-3, corresponding to lambda_1"
tau=(alpha*time*Convert(min, s))/r_o^2
(T_o-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)
L=1 "[m], 1 m length of the cylinder is considered"
V=pi*r_o^2*L
m=rho*V
Q_max=m*c_p*(T_i-T_infinity)*Convert(J, kJ)
Q/Q_max=1-2*(T_o-T_infinity)/(T_i-T_infinity)*J_1/lambda_1
```

time [min]	T_o [C]	Q [kJ]
5	536	6788
10	518.7	12188
15	502.1	17346
20	486.2	22272
25	471.1	26976
30	456.7	31468
35	442.9	35759
40	429.7	39857
45	417.1	43770
50	405.1	47508
55	393.7	51077
60	382.7	54486



4-70 The amount of heat transfer to a steel rod being drawn through an oven is to be determined using analytical one-term approximate method.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of the steel rod are given as ρ = 7832 kg/m^3, c_p = 434 J/kg · K, k = 63.9 W/m · K, and α = 18.8 × 10^{-6} m^2/s.

Analysis The maximum amount of heat transfer to a steel rod is

$$\begin{aligned} Q_{\max} &= \rho V c_p(T_\infty - T_i) = \rho \pi L r_o^2 c_p (T_\infty - T_i) \\ &= (7832\ \text{kg/m}^3)\pi(2\ \text{m})(0.03\ \text{m})^2(434\ \text{J/kg}\cdot\text{K})(800-30)\ \text{K} = 1.48\times10^7\ \text{J} \end{aligned}$$

The Biot number for this process is

$$Bi = \frac{hr_o}{k} = \frac{(128\ \text{W/m}^2\cdot\text{K})(0.030\ \text{m})}{63.9\ \text{W/m}\cdot\text{K}} = 0.06$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(18.8\times10^{-6}\ \text{m}^2/\text{s})(133\ \text{s})}{(0.03\ \text{m})^2} = 2.778$$

which is greater than 0.2. Therefore, the one-term approximate solution can be used.

From Table 4-2 with Bi = 0.06, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 0.3438 \quad \text{and} \quad A_1 = 1.0148$$

For cylindrical rod, we have

$$\theta_{0,\,\text{cyl}} = A_1 e^{-\lambda_1^2 \tau} = (1.0148)e^{-(0.3438)^2\,2.778} = 0.7308$$

The heat transfer to a steel rod after 133 s is

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 2\theta_{0,\,\text{cyl}}\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.7308)\frac{0.1694}{0.3438} = 0.2798$$

where from Table 4-3, $J_1(0.3438)$ = 0.1694. Thus

$$Q = 0.2798 Q_{\max} = \mathbf{4.14\times10^6\ J}$$

Discussion The value of the Bessel function $J_1(\lambda_1)$ for part (a) can also be calculated using the EES with the following line:

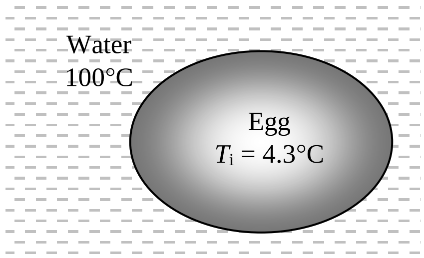
```
J_1=Bessel_J1(0.3438)
```

4-71 An egg is dropped into boiling water. The cooking time of the egg is to be determined.

Assumptions **1** The egg is spherical in shape with a radius of r_0 = 2.75 cm. **2** Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the egg are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is τ > 0.2 so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs can be approximated by those of water at room temperature to be k = 0.607 W/m.°C, $\alpha = k/\rho c_p = 0.146\times10^{-6}$ m²/s (Table A-9).

Analysis The Biot number is



$$Bi = \frac{hr_o}{k} = \frac{(800\text{ W/m}^2.°\text{C})(0.0275\text{ m})}{(0.607\text{ W/m.°C})} = 36.2$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 3.0533 \quad \text{and} \quad A_1 = 1.9925$$

Then the Fourier number and the time period become

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{71-100}{4.3-100} = (1.9925)e^{-(3.0533)^2\tau} \longrightarrow \tau = 0.202$$

which is greater than value of 0.2. Therefore, the one-term approximate solution can be used. Then the length of time for the egg to be kept in boiling water is determined to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.202)(0.0275\text{ m})^2}{0.146\times10^{-6}\text{ m}^2/\text{s}} = 1{,}046.3\text{ s} = \mathbf{17.4\,min}$$

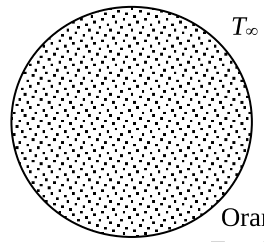
4-72 An orange is exposed to very cold ambient air. It is to be determined whether the orange will freeze in 4 h in subfreezing temperatures.

Assumptions **1** The orange is spherical in shape with a diameter of 8 cm. **2** Heat conduction in the orange is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the orange are constant, and are those of water. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of the orange are approximated by those of water at the average temperature of about 5°C, k = 0.571 W/m.°C and $\alpha = k/\rho c_p = 0.571/(999.9\times 4205) = 0.136\times 10^{-6}\ \text{m}^2/\text{s}$ (Table A-9).

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(15\ \text{W/m}^2.°\text{C})(0.04\ \text{m})}{(0.571\ \text{W/m.°C})} = 1.051 \approx 1.0$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 1.5708 \quad \text{and} \quad A_1 = 1.2732$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.136\times 10^{-6}\ \text{m}^2/\text{s})(4\,\text{h}\times 3600\ \text{s/h})}{(0.04\ \text{m})^2} = 1.224 > 0.2$$

Therefore, the one-term approximate solution is applicable. Then the temperature at the surface of the oranges becomes

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.2732)e^{-(1.5708)^2(1.224)}\frac{\sin(1.5708\ \text{rad})}{1.5708} = 0.0396$$

$$\frac{T(r_o,t)-(-6)}{15-(-6)} = 0.0396 \longrightarrow T(r_o,t) = \mathbf{-5.2\,°C}$$

which is less than 0°C. Therefore, the oranges will freeze.

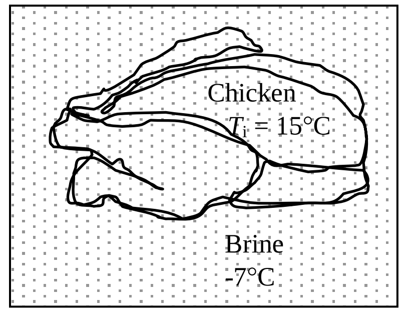
4-73 Chickens are to be chilled by holding them in agitated brine for 2.75 h. The center and surface temperatures of the chickens are to be determined, and if any part of the chickens will freeze during this cooling process is to be assessed.

Assumptions **1** The chickens are spherical in shape. **2** Heat conduction in the chickens is one-dimensional in the radial direction because of symmetry about the midpoint. **3** The thermal properties of the chickens are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified). **6** The phase change effects are not considered, and thus the actual the temperatures will be much higher than the values determined since a considerable part of the cooling process will occur during phase change (freezing of chicken).

Properties The thermal conductivity, thermal diffusivity, and density of chickens are given to be k = 0.45 W/m·°C, α = 0.13×10^{-6} m²/s, and ρ = 950 kg/ m³. These properties will be used for both fresh and frozen chicken.

Analysis We first find the volume and equivalent radius of the chickens:

$$V = m/\rho = 1700\text{g}/(0.95\text{g/cm}^3) = 1789\text{cm}^3$$



$$r_o = \left(\frac{3}{4\pi}V\right)^{1/3} = \left(\frac{3}{4\pi}1789\text{ cm}^3\right)^{1/3} = 7.53\text{ cm} = 0.0753\text{ m}$$

Then the Biot and Fourier numbers become

$$\text{Bi} = \frac{hr_o}{k} = \frac{(440\text{ W/m}^2.°\text{C})(0.0753\text{ m})}{0.45\text{ W/m.°C}} = 73.6$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.13\times10^{-6}\text{ m}^2\text{/s})(2.75\times3600\text{ s})}{(0.0753\text{m})^2} = 0.2270$$

Note that $\tau = 0.2270 > 0.2$, and thus the one-term solution is applicable. From Table 4-2 we read, for a sphere, λ_1 = 3.094 and A_1 = 1.998. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \rightarrow \frac{T_0 - (-7)}{15 - (-7)} = 1.998e^{-(3.094)^2(0.2270)} = 0.2274 \longrightarrow T_0 = \mathbf{-2.0°C}$$

The lowest temperature during cooling will occur on the surface (r/r_o = 1), and is determined to be

$$\frac{T(r) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o} \rightarrow \frac{T(r_o) - T_\infty}{T_i - T_\infty} = \theta_0 \frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = \frac{T_o - T_\infty}{T_i - T_\infty}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o}$$

Substituting,

$$\frac{T(r_o) - (-7)}{15 - (-7)} = 0.2274\frac{\sin(3.094\text{ rad})}{3.094} \rightarrow T(r_o) = \mathbf{-6.9°C}$$

Most parts of chicken will freeze during this process since the freezing point of chicken is -2.8°C.

4-74 The time it takes for the surface of a falling hailstone to reach melting point is to be determined.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of ice at 253 K are ρ = 922 kg/m^3, c_p = 1945 J/kg · K, and k = 2.03 W/m · K (from Table A-8).

Analysis The Biot number for this process is

$$Bi = \frac{hr_o}{k} = \frac{(163\ \text{W/m}^2\cdot\text{K})(0.010\ \text{m})}{2.03\ \text{W/m}\cdot\text{K}} = 0.80$$

From Table 4-2, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 1.4320 \quad \text{and} \quad A_1 = 1.2236$$

For a sphere, we have

$$\theta_{\text{sph}} = \frac{T(r,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}$$

For the hailstone surface ($r = r_o$) to reach melting point (0°C), the Fourier number is

$$\frac{0-15}{-20-15} = (1.2236)e^{-(1.4320)^2\tau}\frac{\sin(1.4320)}{1.4320} \quad\rightarrow\quad \tau = 0.3318$$

which is greater than 0.2. Therefore one-term solution is applicable.

The time required for hailstone surface to reach melting point is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{kt}{\rho c_p r_o^2} = 0.3318$$

$$t = \frac{0.3318 r_o^2 \rho c_p}{k} = \frac{0.3318(0.01\ \text{m})^2(922\ \text{kg/m}^3)(1945\ \text{J/kg}\cdot\text{K})}{2.03\ \text{W/m}\cdot\text{K}} = \mathbf{29.3\ s}$$

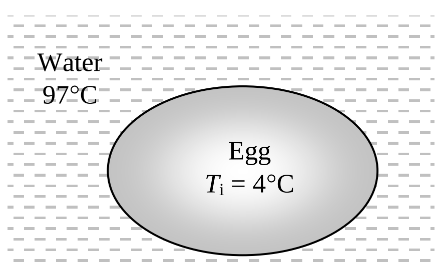
Discussion Depending on the altitude in which the hailstone is formed, its surface may not even reach melting point before hitting the ground.

4-75 An egg is dropped into boiling water. The cooking time of the egg is to be determined.

Assumptions **1** The egg is spherical in shape with a radius of r_o = 2.75 cm. **2** Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the egg are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs are given to be k = 0.6 W/m.°C and α = 0.14×10^{-6} m^2/s.

Analysis The Biot number for this process is



$$Bi = \frac{hr_o}{k} = \frac{(1400\ \text{W/m}^2.°\text{C})(0.0275\ \text{m})}{(0.6\ \text{W/m}.°\text{C})} = 64.2$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 3.0877 \quad \text{and} \quad A_1 = 1.9969$$

Then the Fourier number becomes

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{70-97}{4-97} = (1.9969)e^{-(3.0877)^2\tau} \longrightarrow \tau = 0.2023 > 0.2$$

Therefore, the one-term approximate solution is applicable. Then the time required for the temperature of the center of the egg to reach 70°C is determined to be

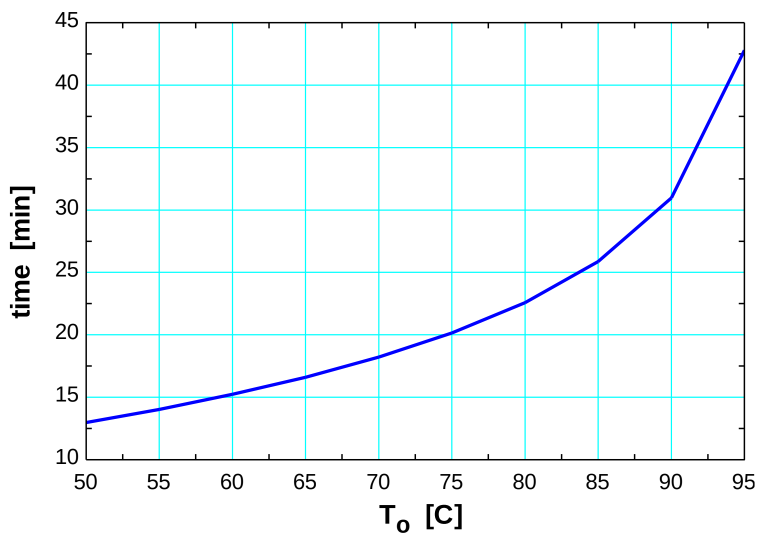
$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.2023)(0.0275\ \text{m})^2}{0.14\times10^{-6}\ \text{m}^2/\text{s}} = 1093\ \text{s} = \mathbf{18.2\ min}$$

4-76 Prob. 4-75 is reconsidered. The effect of the final center temperature of the egg on the time it will take for the center to reach this temperature is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.055 [m]
T_i=4 [C]
T_o=70 [C]
T_infinity=97 [C]
h=1400 [W/m^2-C]

"PROPERTIES"
k=0.6 [W/m-C]
alpha=0.14E-6 [m^2/s]

"ANALYSIS"
Bi=(h*r_o)/k
r_o=D/2
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1=1.9969
A_1=3.0863
(T_o-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)
time=(tau*r_o^2)/alpha*Convert(s, min)
```

T_o [C]	time [min]
50	12.98
55	14.04
60	15.23
65	16.61
70	18.21
75	20.14
80	22.58
85	25.87
90	30.96
95	42.79



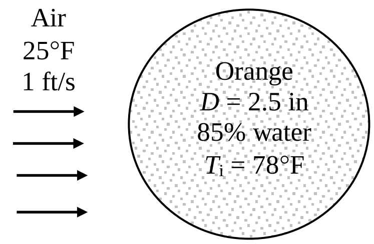
4-77E The center temperature of oranges is to be lowered to 40°F during cooling. The cooling time and if any part of the oranges will freeze during this cooling process are to be determined.

Assumptions **1** The oranges are spherical in shape with a radius of r_o =1.25 in = 0.1042 ft. **2** Heat conduction in the orange is one-dimensional in the radial direction because of the symmetry about the midpoint. **3** The thermal properties of the orange are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of oranges are given to be k = 0.26 Btu/h·ft·°F and $\alpha = 1.4\times10^{-6}$ ft^2/s.

Analysis First we find the Biot number:

$$\text{Bi} = \frac{hr_o}{k} = \frac{(4.6\ \text{Btu/h.ft}^2.°\text{F})(1.25/12\ \text{ft})}{0.26\ \text{Btu/h.ft.°F}} = 1.843$$



From Table 4-2 we read, for a sphere, λ_1 = 1.9569 and A_1 = 1.447. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \rightarrow \frac{40-25}{78-25} = 1.447 e^{-(1.9569)^2 \tau} \rightarrow \tau = 0.426$$

which is greater than 0.2 and thus the one-term solution is applicable. Then the cooling time becomes

$$\tau = \frac{\alpha t}{r_o^2} \rightarrow t = \frac{\tau r_o^2}{\alpha} = \frac{(0.426)(1.25/12\ \text{ft})^2}{1.4\times10^{-6}\ \text{ft}^2/\text{s}} = 3302\ \text{s} = \mathbf{55.0\ min}$$

The lowest temperature during cooling will occur on the surface (r/r_0 = 1), and is determined to be

$$\frac{T(r)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o} \rightarrow \frac{T(r_o)-T_\infty}{T_i - T_\infty} = \theta_0 \frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = \frac{T_o - T_\infty}{T_i - T_\infty} \frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o}$$

Substituting,

$$\frac{T(r_o)-25}{78-25} = \left(\frac{40-25}{78-25}\right) \frac{\sin(1.9569\ \text{rad})}{1.9569} \longrightarrow T(r_o) = 32.1°\text{F}$$

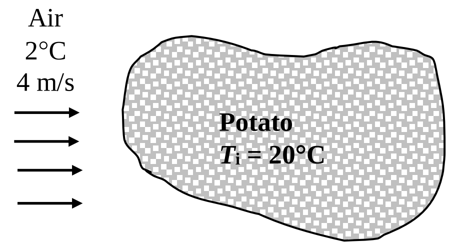
which is above the freezing temperature of 31°F for oranges. Therefore, no part of the oranges will freeze during this cooling process.

4-78 The center temperature of potatoes is to be lowered to 6°C during cooling. The cooling time and if any part of the potatoes will suffer chilling injury during this cooling process are to be determined.

Assumptions **1** The potatoes are spherical in shape with a radius of r_0 = 3 cm. **2** Heat conduction in the potato is one-dimensional in the radial direction because of the symmetry about the midpoint. **3** The thermal properties of the potato are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of potatoes are given to be k = 0.50 W/m·°C and $\alpha = 0.13\times10^{-6}$ m^2/s.

Analysis First we find the Biot number:



$$\text{Bi} = \frac{hr_o}{k} = \frac{(19\ \text{W/m}^2.°\text{C})(0.03\ \text{m})}{0.5\ \text{W/m.°C}} = 1.14$$

From Table 4-2 we read, for a sphere, $\lambda_1 = 1.635$ and $A_1 = 1.302$. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \rightarrow \frac{6-2}{20-2} = 1.302 e^{-(1.635)^2 \tau} \rightarrow \tau = 0.661$$

which is greater than 0.2 and thus the one-term solution is applicable. Then the cooling time becomes

$$\tau = \frac{\alpha t}{r_o^2} \longrightarrow t = \frac{\tau r_o^2}{\alpha} = \frac{(0.661)(0.03\ \text{m})^2}{0.13\times10^{-6}\ \text{m}^2/\text{s}} = 4579\ \text{s} = \mathbf{1.27\ h}$$

The lowest temperature during cooling will occur on the surface (r/r_0 = 1), and is determined to be

$$\frac{T(r) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r / r_o)}{\lambda_1 r / r_o} \rightarrow \frac{T(r_o) - T_\infty}{T_i - T_\infty} = \theta_0 \frac{\sin(\lambda_1 r_o / r_o)}{\lambda_1 r_o / r_o} = \frac{T_o - T_\infty}{T_i - T_\infty} \frac{\sin(\lambda_1 r_o / r_o)}{\lambda_1 r_o / r_o}$$

Substituting, $$\frac{T(r_o) - 2}{20-2} = \left(\frac{6-2}{20-2}\right)\frac{\sin(1.635\ \text{rad})}{1.635} \longrightarrow T(r_o) = 4.44°\text{C}$$

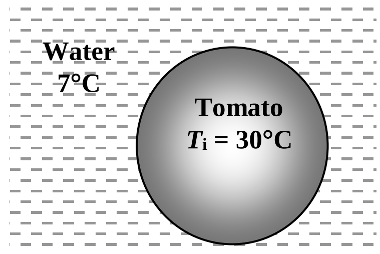
which is above the temperature range of 3 to 4 °C for chilling injury for potatoes. Therefore, **no part** of the potatoes will experience chilling injury during this cooling process.

4-79 Tomatoes are placed into cold water to cool them. The heat transfer coefficient and the amount of heat transfer are to be determined.

Assumptions **1** The tomatoes are spherical in shape. **2** Heat conduction in the tomatoes is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the tomatoes are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of the tomatoes are given to be $k = 0.59$ W/m.°C, $\alpha = 0.141\times10^{-6}$ m^2/s, $\rho = 999$ kg/m^3 and $c_p = 3.99$ kJ/kg.°C.

Analysis The Fourier number is



$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.141\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}{(0.04\ \text{m})^2} = 0.635$$

which is greater than 0.2. Therefore one-term solution is applicable. The ratio of the dimensionless temperatures at the surface and center of the tomatoes are

$$\frac{\theta_{\text{s,sph}}}{\theta_{\text{0,sph}}} = \frac{\dfrac{T_s - T_\infty}{T_i - T_\infty}}{\dfrac{T_0 - T_\infty}{T_i - T_\infty}} = \frac{T_s - T_\infty}{T_0 - T_\infty} = \frac{A_1 e^{-\lambda_1^2\tau}\,\dfrac{\sin(\lambda_1)}{\lambda_1}}{A_1 e^{-\lambda_1^2\tau}} = \frac{\sin(\lambda_1)}{\lambda_1}$$

Substituting,

$$\frac{7.1-7}{10-7} = \frac{\sin(\lambda_1)}{\lambda_1} \longrightarrow \lambda_1 = 3.0401$$

From Table 4-2, the corresponding Biot number and the heat transfer coefficient are

$$\text{Bi} = 31.1$$

$$\text{Bi} = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.59\ \text{W/m.°C})(31.1)}{(0.04\ \text{m})} = \mathbf{459\ W/m^2.°C}$$

The maximum amount of heat transfer is

$$m = 8\rho V = 8\rho\pi D^3/6 = 8(999\ \text{kg/m}^3)[\pi(0.08\ \text{m})^3/6] = 2.143\ \text{kg}$$

$$Q_{\max} = mc_p[T_i - T_\infty] = (2.143\ \text{kg})(3.99\ \text{kJ/kg.°C})(30-7)\text{°C} = 196.6\ \text{kJ}$$

Then the actual amount of heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 3\left(\frac{T_0 - T_\infty}{T_i - T_\infty}\right)\frac{\sin\lambda_1 - \lambda_1\cos\lambda_1}{\lambda_1^3} = 1 - 3\left(\frac{10-7}{30-7}\right)\frac{\sin(3.0401) - (3.0401)\cos(3.0401)}{(3.0401)^3} = 0.9565$$

$$Q = 0.9565 Q_{\max}$$

$$Q = 0.9565(196.6\ \text{kJ}) = \mathbf{188\ kJ}$$

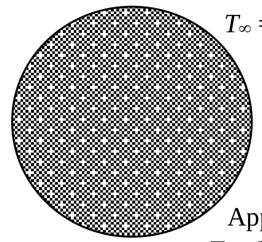
4-80 A person puts apples into the freezer to cool them quickly. The center and surface temperatures of the apples, and the amount of heat transfer from each apple in 1 h are to be determined.

Assumptions **1** The apples are spherical in shape with a diameter of 9 cm. **2** Heat conduction in the apples is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the apples are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of the apples are given to be k = 0.418 W/m.°C, ρ = 840 kg/m^3, c_p = 3.81 kJ/kg.°C, and α = 1.3×10^{-7} m^2/s.

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(8 \text{ W/m}^2.°\text{C})(0.045 \text{ m})}{(0.418 \text{ W/m.°C})} = 0.861$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 1.476 \quad \text{and} \quad A_1 = 1.2390$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.3\times 10^{-7} \text{ m}^2\text{/s})(1\text{h}\times 3600 \text{ s/h})}{(0.045 \text{ m})^2} = 0.231 > 0.2$$

which is greater than 0.2. Therefore one-term solution is applicable.

Then the temperature at the center of the apples becomes

$$\theta_{0,\text{sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2390)e^{-(1.476)^2(0.231)} = 0.749$$

$$\frac{T_0 - (-15)}{25-(-15)} = 0.749 \longrightarrow T_0 = \mathbf{15.0°C}$$

The temperature at the surface of the apples is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r_o / r_o)}{\lambda_1 r_o / r_o} = (1.239)e^{-(1.476)^2(0.231)} \frac{\sin(1.476 \text{ rad})}{1.476} = 0.505$$

$$\frac{T(r_o,t) - (-15)}{25-(-15)} = 0.505 \longrightarrow T(r_o,t) = \mathbf{5.2°C}$$

The maximum possible heat transfer is

$$m = \rho \boldsymbol{V} = \rho \frac{4}{3}\pi r_o^3 = (840 \text{ kg/m}^3)\left[\frac{4}{3}\pi(0.045 \text{ m})^{3.}\right] = 0.3206 \text{ kg}$$

$$Q_{\max} = mc_p(T_i - T_\infty) = (0.3206 \text{ kg})(3.81 \text{ kJ/kg.°C})[25-(-15)]°\text{C} = 48.9 \text{ kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1) - \lambda_1 \cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.749)\frac{\sin(1.476 \text{ rad}) - (1.476)\cos(1.476 \text{ rad})}{(1.476)^3} = 0.402$$

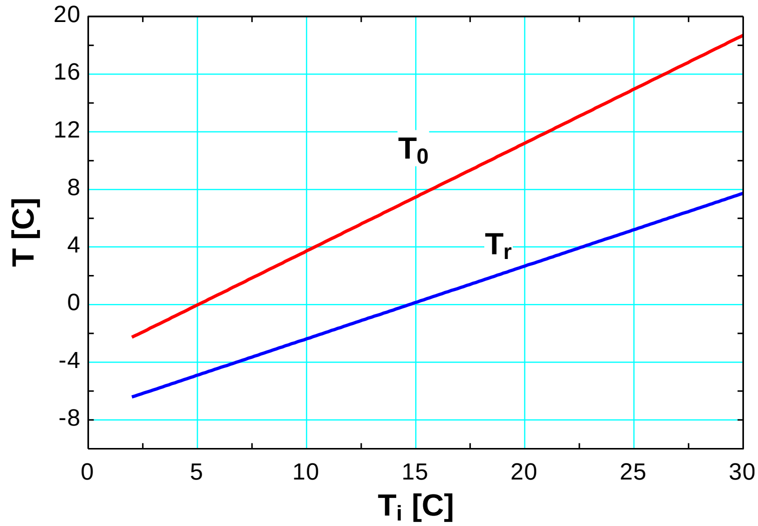
$$Q = 0.402 Q_{\max} = (0.402)(48.9 \text{ kJ}) = \mathbf{19.6\,kJ}$$

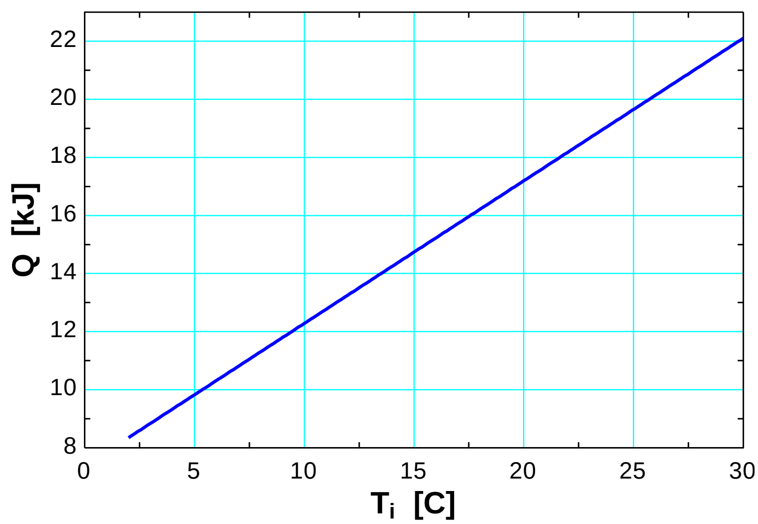
4-81 Prob. 4-80 is reconsidered. The effect of the initial temperature of the apples on the final center and surface temperatures and the amount of heat transfer is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_infinity=-15 [C]
T_i=25 [C]
h=8 [W/m^2-C]
r_o=(0.09/2) [m]
time=1*3600 [s]

"PROPERTIES"
k=0.418 [W/m-C]
rho=840 [kg/m^3]
c_p=3.81 [kJ/kg-C]
alpha=1.3E-7 [m^2/s]

"ANALYSIS"
Bi=(h*r_o)/k
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1=1.476
A_1=1.2390
tau=(alpha*time)/r_o^2
(T_o-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)
(T_r-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)*Sin(lambda_1*r_o/r_o)/(lambda_1*r_o/r_o)
V=4/3*pi*r_o^3
m=rho*V
Q_max=m*c_p*(T_i-T_infinity)
Q/Q_max=1-3*(T_o-T_infinity)/(T_i-T_infinity)*(Sin(lambda_1)-lambda_1*Cos(lambda_1))/lambda_1^3
```

T_i [C]	T_o [C]	T_r [C]	Q [kJ]
2	-2.269	-6.414	8.35
4	-0.7715	-5.403	9.333
6	0.7263	-4.393	10.31
8	2.224	-3.383	11.3
10	3.722	-2.373	12.28
12	5.22	-1.363	13.26
14	6.717	-0.3525	14.24
16	8.215	0.6577	15.23
18	9.713	1.668	16.21
20	11.21	2.678	17.19
22	12.71	3.688	18.17
24	14.21	4.698	19.16
26	15.7	5.709	20.14
28	17.2	6.719	21.12
30	18.7	7.729	22.1

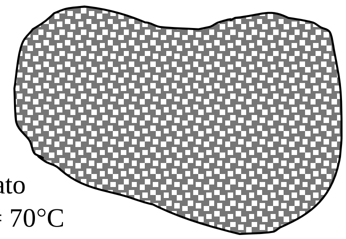

T [C]
20
16
12
8
4
0
-4
-8
0
5
10
15
20
25
30
T_0
T_r
T_i [C]


Q [kJ]
22
20
18
16
14
12
10
8
0
5
10
15
20
25
30
T_i [C]

4-82 A hot baked potato is taken out of the oven and wrapped so that no heat is lost from it. The time the potato is baked in the oven and the final equilibrium temperature of the potato after it is wrapped are to be determined.

Assumptions **1** The potato is spherical in shape with a diameter of 9 cm. **2** Heat conduction in the potato is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the potato are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of the potato are given to be $k = 0.6$ W/m.°C, $\rho = 1100$ kg/m^3, $c_p = 3.9$ kJ/kg.°C, and $\alpha = 1.4\times10^{-7}$ m^2/s.



Analysis (*a*) The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(40\ \text{W/m}^2.°\text{C})(0.045\ \text{m})}{(0.6\ \text{W/m.°C})} = 3$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 2.2889 \quad \text{and} \quad A_1 = 1.6227$$

Then the Fourier number and the time period become

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70-170}{25-170} = 0.69 = (1.6227)e^{-(2.2889)^2 \tau} \longrightarrow \tau = 0.163$$

which is not greater than 0.2 but it is close. We may use one-term approximation knowing that the result may be somewhat in error. Then the baking time of the potatoes is determined to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.163)(0.045\ \text{m})^2}{1.4\times10^{-7}\ \text{m}^2/\text{s}} = 2358\ \text{s} = \mathbf{39.3\ min}$$

(*b*) The maximum amount of heat transfer is

$$m = \rho \boldsymbol{V} = \rho \frac{4}{3}\pi r_o^3 = (1100\ \text{kg/m}^3)\left[\frac{4}{3}\pi(0.045\ \text{m})^{3.}\right] = 0.420\ \text{kg}$$

$$Q_{\max} = mc_p(T_\infty - T_i) = (0.420\ \text{kg})(3.900\ \text{kJ/kg.°C})(170-25)°\text{C} = 237\ \text{kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1) - \lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.69)\frac{\sin(2.2889) - (2.2889)\cos(2.2889)}{(2.2889)^3} = 0.610$$

$$Q = 0.610 Q_{\max} = (0.610)(237\ \text{kJ}) = 145\ \text{kJ}$$

The final equilibrium temperature of the potato after it is wrapped is

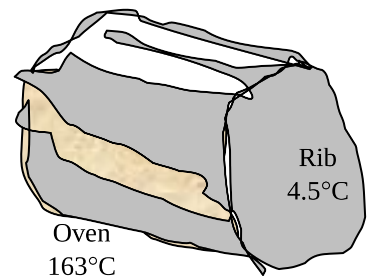
$$Q = mc_p(T_{eqv} - T_i) \longrightarrow T_{eqv} = T_i + \frac{Q}{mc_p} = 25°\text{C} + \frac{145\ \text{kJ}}{(0.420\ \text{kg})(3.9\ \text{kJ/kg.°C})} = \mathbf{114°C}$$

4-83 A rib is roasted in an oven. The heat transfer coefficient at the surface of the rib, the temperature of the outer surface of the rib and the amount of heat transfer when it is rare done are to be determined. The time it will take to roast this rib to medium level is also to be determined.

Assumptions **1** The rib is a homogeneous spherical object. **2** Heat conduction in the rib is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the rib are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of the rib are given to be k = 0.45 W/m.°C, ρ = 1200 kg/m^3, c_p = 4.1 kJ/kg.°C, and $\alpha = 0.91\times10^{-7}$ m^2/s.

Analysis (*a*) The radius of the roast is determined to be



$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{3.2\text{ kg}}{1200\text{ kg/m}^3} = 0.002667\text{ m}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.002667\text{ m}^3)}{4\pi}} = 0.08603\text{ m}$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.91\times10^{-7}\text{ m}^2/\text{s})(2\times3600 + 45\times60)\text{s}}{(0.08603\text{ m})^2} = 0.1217$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the one-term solution can be written in the form

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{60-163}{4.5-163} = 0.65 = A_1 e^{-\lambda_1^2(0.1217)}$$

It is determined from Table 4-2 by trial and error that this equation is satisfied when Bi = 30, which corresponds to $\lambda_1 = 3.0372$ and $A_1 = 1.9898$. Then the heat transfer coefficient can be determined from

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.45\text{ W/m.°C})(30)}{(0.08603\text{ m})} = \mathbf{156.9\text{ W/m}^2.°C}$$

This value seems to be larger than expected for problems of this kind. This is probably due to the Fourier number being less than 0.2.

(*b*) The temperature at the surface of the rib is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.9898)e^{-(3.0372)^2(0.1217)}\frac{\sin(3.0372\text{ rad})}{3.0372}$$

$$\frac{T(r_o,t)-163}{4.5-163} = 0.0222 \longrightarrow T(r_o,t) = \mathbf{159.5\,°C}$$

(*c*) The maximum possible heat transfer is

$$Q_{\max} = mc_p(T_\infty - T_i) = (3.2\text{ kg})(4.1\text{ kJ/kg.°C})(163-4.5)°\text{C} = 2080\text{ kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1)-\lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1-3(0.65)\frac{\sin(3.0372)-(3.0372)\cos(3.0372)}{(3.0372)^3} = 0.783$$

$$Q = 0.783Q_{\max} = (0.783)(2080\text{ kJ}) = \mathbf{1629\text{ kJ}}$$

(*d*) The cooking time for medium-done rib is determined to be

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{71-163}{4.5-163} = (1.9898)e^{-(3.0372)^2\tau} \longrightarrow \tau = 0.1336$$

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.1336)(0.08603\text{ m})^2}{(0.91\times10^{-7}\text{ m}^2/\text{s})} = 10{,}866\text{ s} = 181\text{ min} \cong \mathbf{3\text{ hr}}$$

This result is close to the listed value of 3 hours and 20 minutes. The difference between the two results is due to the Fourier number being less than 0.2 and thus the error in the one-term approximation.

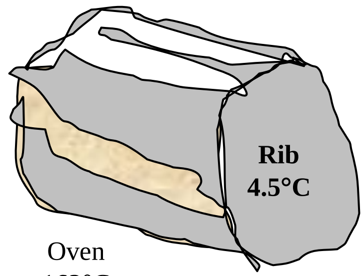
Discussion The temperature of the outer parts of the rib is greater than that of the inner parts of the rib after it is taken out of the oven. Therefore, there will be a heat transfer from outer parts of the rib to the inner parts as a result of this temperature difference. The recommendation is logical.

4-84 A rib is roasted in an oven. The heat transfer coefficient at the surface of the rib, the temperature of the outer surface of the rib and the amount of heat transfer when it is well-done are to be determined. The time it will take to roast this rib to medium level is also to be determined.

Assumptions **1** The rib is a homogeneous spherical object. **2** Heat conduction in the rib is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the rib are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of the rib are given to be $k = 0.45$ W/m.°C, $\rho = 1200$ kg/m^3, $c_p = 4.1$ kJ/kg.°C, and $\alpha = 0.91\times10^{-7}$ m^2/s

Analysis (*a*) The radius of the rib is determined to be



$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{3.2\text{ kg}}{1200\text{ kg/m}^3} = 0.00267\text{ m}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.00267\text{ m}^3)}{4\pi}} = 0.08603\text{ m}$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.91\times10^{-7}\text{ m}^2/\text{s})(4\times3600+15\times60)\text{s}}{(0.08603\text{ m})^2} = 0.1881$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the one-term solution formulation can be written in the form

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{77-163}{4.5-163} = 0.543 = A_1 e^{-\lambda_1^2(0.1881)}$$

It is determined from Table 4-2 by trial and error that this equation is satisfied when $Bi = 4.3$, which corresponds to $\lambda_1 = 2.4900$ and $A_1 = 1.7402$. Then the heat transfer coefficient can be determined from.

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.45\text{ W/m.°C})(4.3)}{(0.08603\text{ m})} = \mathbf{22.5\ W/m^2.°C}$$

(*b*) The temperature at the surface of the rib is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.7402)e^{-(2.49)^2(0.1881)}\frac{\sin(2.49)}{2.49}$$

$$\frac{T(r_o,t)-163}{4.5-163} = 0.132 \longrightarrow T(r_o,t) = \mathbf{142.1°C}$$

(*c*) The maximum possible heat transfer is

$$Q_{\max} = mc_p(T_\infty - T_i) = (3.2\text{ kg})(4.1\text{ kJ/kg.°C})(163-4.5)\text{°C} = 2080\text{ kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1)-\lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1-3(0.543)\frac{\sin(2.49)-(2.49)\cos(2.49)}{(2.49)^3} = 0.727$$

$$Q = 0.727 Q_{\max} = (0.727)(2080\text{ kJ}) = \mathbf{1512\ kJ}$$

(*d*) The cooking time for medium-done rib is determined to be

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{71-163}{4.5-163} = (1.7402)e^{-(2.49)^2\tau} \longrightarrow \tau = 0.177$$

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.177)(0.08603\text{ m})^2}{(0.91\times10^{-7}\text{ m}^2/\text{s})} = 14{,}400\text{ s} = 240\text{ min} = \mathbf{4\ hr}$$

This result is close to the listed value of 4 hours and 15 minutes. The difference between the two results is probably due to the Fourier number being less than 0.2 and thus the error in the one-term approximation.

Discussion The temperature of the outer parts of the rib is greater than that of the inner parts of the rib after it is taken out of the oven. Therefore, there will be a heat transfer from outer parts of the rib to the inner parts as a result of this temperature difference. The recommendation is logical.

Transient Heat Conduction in Semi-Infinite Solids

4-85C A thick plane wall can be treated as a semi-infinite medium if all we are interested in is the variation of temperature in a region near one of the surfaces for a time period during which the temperature in the mid section of the wall does not experience any change.

4-86C A semi-infinite medium is an idealized body which has a single exposed plane surface and extends to infinity in all directions. The earth and thick walls can be considered to be semi-infinite media.

4-87C The total amount of heat transfer from a semi-infinite solid up to a specified time t_0 can be determined by integration from

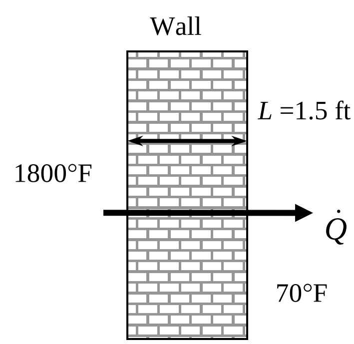
$$Q = \int_0^{t_o} Ah[T(0,t) - T_\infty]dt$$

where the surface temperature $T(0, t)$ is obtained from Eq. 4-47 by substituting $x = 0$.

4-88E The walls of a furnace made of concrete are exposed to hot gases at the inner surfaces. The time it will take for the temperature of the outer surface of the furnace to change is to be determined.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at inner surfaces only and the convection heat transfer coefficient inside is given to be very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature of 1800°F. **2** The thermal properties of the concrete wall are constant.

Properties The thermal properties of the concrete are given to be $k = 0.64$ Btu/h.ft.°F and $\alpha = 0.023$ ft^2/h.



Analysis The one-dimensional transient temperature distribution in the wall for that time period can be determined from

$$\frac{T(x,t) - T_i}{T_s - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

But,

$$\frac{T(x,t) - T_i}{T_s - T_i} = \frac{70.1 - 70}{1800 - 70} = 0.00006 \;\rightarrow\; 0.00006 = erfc(2.85) \quad \text{(Table 4-4)}$$

Therefore,

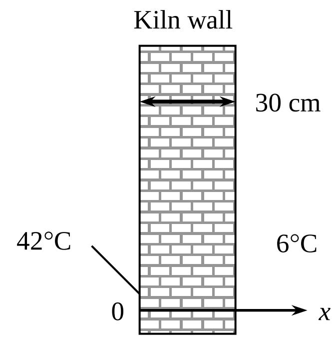
$$\frac{x}{2\sqrt{\alpha t}} = 2.85 \longrightarrow t = \frac{x^2}{4 \times (2.85)^2 \alpha} = \frac{(1.5\text{ ft})^2}{4 \times (2.85)^2 (0.023\text{ ft}^2/\text{h})} = \mathbf{3.0\,h}$$

4-89 A curing kiln is heated by injecting steam into it and raising its inner surface temperature to a specified value. It is to be determined whether the temperature at the outer surfaces of the kiln changes during the curing period.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at inner surfaces only and the convection heat transfer coefficient inside is very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature of 45°C. **2** The thermal properties of the concrete wall are constant.

Properties The thermal diffusivity of the concrete wall is given to be $\alpha = 0.23\times10^{-5}$ m^2/s.

Analysis We determine the temperature at a depth of x = 0.3 m in 2.5 h using the analytical solution,



$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Substituting,

$$\frac{T(x,t)-6}{42-6} = erfc\left(\frac{0.3\,\text{m}}{2\sqrt{(0.23\times10^{-5}\text{ m}^2/\text{s})(2.5\,\text{h}\times3600\,\text{s/h})}}\right)$$

$$= erfc(1.043) = 0.1402$$

$$T(x,t) = \mathbf{11.0\,°C}$$

which is greater than the initial temperature of 6°C. Therefore, heat will propagate through the 0.3 m thick wall in 2.5 h, and thus it may be desirable to insulate the outer surface of the wall to save energy.

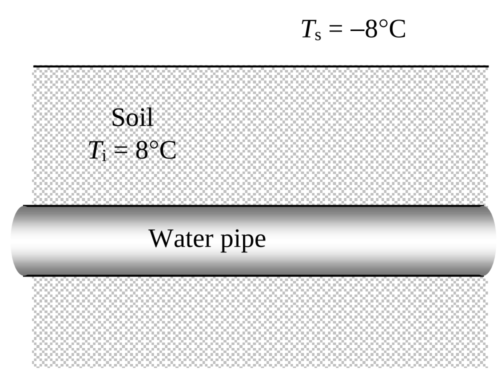
4-90 The water pipes are buried in the ground to prevent freezing. The minimum burial depth at a particular location is to be determined.

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the soil are constant.

Properties The thermal properties of the soil are given to be k = 0.35 W/m.°C and $\alpha = 0.15\times10^{-6}$ m^2/s.

Analysis The length of time the snow pack stays on the ground is

$$t = (60\,\text{days})(24\,\text{hr/days})(3600\,\text{s/hr}) = 5.184\times10^6\text{ s}$$



The surface is kept at -8°C at all times. The depth at which freezing at 0°C occurs can be determined from the analytical solution,

$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{\sqrt{\alpha t}}\right)$$

$$\frac{0-8}{-8-8} = erfc\left(\frac{x}{2\sqrt{(0.15\times10^{-6}\text{ m}^2/\text{s})(5.184\times10^6\text{ s})}}\right)$$

$$0.5 = erfc\left(\frac{x}{1.7636}\right)$$

Then from Table 4-4 we get

$$\frac{x}{1.7636} = 0.4796 \longrightarrow x = \mathbf{0.846\,m}$$

Discussion The solution could also be determined using the chart, but it would be subject to reading error.

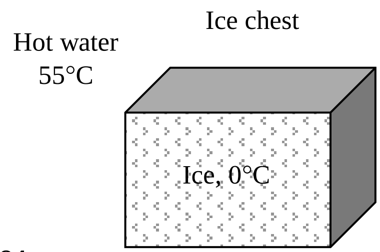
4-91 The outer surfaces of a large cast iron container filled with ice are exposed to hot water. The time before the ice starts melting and the rate of heat transfer to the ice are to be determined.

Assumptions **1** The temperature in the container walls is affected by the thermal conditions at outer surfaces only and the convection heat transfer coefficient outside is given to be very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the wall are constant.

Properties The thermal properties of the cast iron are given to be k = 52 W/m.°C and $\alpha = 1.70\times10^{-5}$ m^2/s.

Analysis The one-dimensional transient temperature distribution in the wall for that time period can be determined from

$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$



But,

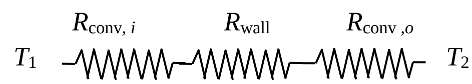
$$\frac{T(x,t)-T_i}{T_s-T_i} = \frac{0.1-0}{55-0} = 0.00182 \rightarrow \quad 0.00182 = erfc(2.206) \quad \text{(Table 4-4)}$$

Therefore,

$$\frac{x}{2\sqrt{\alpha t}} = 2.206 \longrightarrow t = \frac{x^2}{4\times(2.206)^2\alpha} = \frac{(0.04\text{ m})^2}{4(2.206)^2(1.7\times10^{-5}\text{ m}^2/\text{s})} = \mathbf{4.84\,s}$$

The rate of heat transfer to the ice when steady operation conditions are reached can be determined by applying the thermal resistance network concept as

$$R_{conv,i} = \frac{1}{h_i A} = \frac{1}{(250\text{ W/m}^2.°\text{C})(1.2\times 2\text{ m}^2)} = 0.00167°\text{C/W}$$



$$R_{wall} = \frac{L}{kA} = \frac{0.04\text{ m}}{(52\text{ W/m.°C})(1.2\times 2\text{ m}^2)} = 0.00032°\text{C/W}$$

$$R_{conv,o} = \frac{1}{h_o A} = \frac{1}{(\infty)(1.2\times 2\text{ m}^2)} \cong 0°\text{C/W}$$

$$R_{total} = R_{conv,i} + R_{wall} + R_{conv,o} = 0.00167 + 0.00032 + 0 = 0.00199°\text{C/W}$$

$$\dot{Q} = \frac{T_2 - T_1}{R_{total}} = \frac{(55-0)°\text{C}}{0.00199^\circ\text{ C/W}} = \mathbf{27{,}600\ W}$$

4-92 With the highway surface temperature maintained at 25°C, the temperature at the depth of 3 cm from surface and the heat flux transferred after 60 minutes are to be determined.

Assumptions **1** The highway is treated as semi-infinite solid. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The properties of asphalt are ρ = 2115 kg/m^3, c_p = 920 J/kg · K, and k = 0.062 W/m · K (from Table A-8).

Analysis The thermal diffusivity for asphalt is

$$\alpha = \frac{k}{\rho c_p} = \frac{0.062\ \text{W/m}\cdot\text{K}}{(2115\ \text{kg/m}^3)(920\ \text{J/kg}\cdot\text{K})} = 3.186\times10^{-8}\ \text{m}^2/\text{s}$$

For semi-infinite solid with specified surface temperature, we have

$$\frac{T(x,t)-T_i}{T_s - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

where

$$\frac{x}{2\sqrt{\alpha t}} = \frac{0.03\ \text{m}}{2\sqrt{(3.186\times10^{-8}\ \text{m}^2/\text{s})(60\times60\ \text{s})}} = 1.40$$

From Table 4-4, erfc(1.40) = 0.04772. Hence the temperature at the depth of 3 cm from the highway surface after 60 minutes is

$$T(0.03\ \text{m}, 3600\ \text{s}) = (T_s - T_i)\text{erfc}(1.40) + T_i = (25\,°\text{C} - 55\,°\text{C})(0.04772) + 55\,°\text{C} = \mathbf{53.6\,°C}$$

The heat flux transferred from the highway after 60 minutes is

$$\dot{q}_s(t) = \frac{k(T_i - T_s)}{\sqrt{\pi\alpha t}} \quad\rightarrow\quad \dot{q}_s(3600\ \text{s}) = \frac{(0.062\ \text{W/m}\cdot\text{K})(55-25)\ \text{K}}{\sqrt{\pi(3.186\times10^{-8}\ \text{m}^2/\text{s})(60\times60\ \text{s})}} = \mathbf{98\ W/m^2}$$

Discussion Having very low thermal diffusivity, asphalt diffuses heat so slowly that even after 60 minutes of the surface maintained at 25°C, the temperature at the depth of 3 cm only drops by less than 2°C.

4-93 An aluminum block is subjected to heat flux. The surface temperature of the block is to be determined.

Assumptions **1** All heat flux is absorbed by the block. **2** Heat loss from the block is disregarded (and thus the result obtained is the maximum temperature). **3** The block is sufficiently thick to be treated as a semi-infinite solid, and the properties of the block are constant.

Properties Thermal conductivity and diffusivity of aluminum at room temperature are k = 237 kg/m^3 and α = 97.1×10^{-6} m^2/s.

Analysis This is a transient conduction problem in a semi-infinite medium subjected to constant surface heat flux, and the surface temperature can be determined to be

$$T_s = T_i + \frac{\dot{q}_s}{k}\sqrt{\frac{4\alpha t}{\pi}} = 20°\text{C} + \frac{4000\ \text{W/m}^2}{237\ \text{W/m}\cdot°\text{C}}\sqrt{\frac{4(9.71\times10^{-5}\ \text{m}^2/\text{s})(30\times60\ \text{s})}{\pi}} = 28.0°\text{C}$$

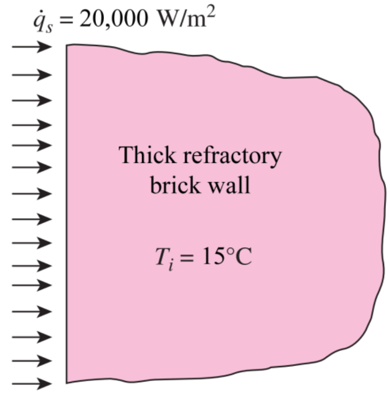
Then the temperature rise of the surface becomes

$$\Delta T_s = 28 - 20 = \mathbf{8.0°C}$$

4-94 A thick refractory brick wall is subjected to uniform heat flux. The temperature at the depth of 10 cm from the wall surface after an hour is to be determined.

Assumptions **1** The wall is thick and can be treated as a semi-infinite medium with a specified surface heat flux. **2** The thermal properties of the wall are constant.

Properties The properties of the brick wall are given as k = 1.0 W/m·K and $\alpha = 5.08 \times 10^{-7}$ m^2/s.



Analysis This is a transient conduction problem in a semi-infinite medium subjected to constant surface heat flux, and the wall temperature at x = 0.1 m and t = 3600 s can be determined from

$$T(x,t) - T_i = \frac{\dot{q}_s}{k}\left[\sqrt{\frac{4\alpha t}{\pi}}\exp\left(-\frac{x^2}{4\alpha t}\right) - x\,\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right]$$

where

$$\frac{x}{2\sqrt{\alpha t}} = \frac{0.1\,\text{m}}{2\sqrt{(5.08\times 10^{-7}\ \text{m}^2/\text{s})(3600\,\text{s})}} = 1.169$$

From Table 4-4, erfc(1.169) = 0.09832. Hence, the temperature at the depth of 10 cm from the wall surface after an hour is

$$T(x,t) = \frac{20{,}000\ \text{W/m}^2}{1.0\ \text{W/m}\cdot\text{K}}\left[\sqrt{\frac{4(5.08\times 10^{-7}\ \text{m}^2/\text{s})(3600\,\text{s})}{\pi}}\exp\left[-\frac{(0.1\,\text{m})^2}{4(5.08\times 10^{-7}\ \text{m}^2/\text{s})(3600\,\text{s})}\right]\right.$$

$$\left. - (0.1\,\text{m})\text{erfc}\left(\frac{0.1\,\text{m}}{2\sqrt{(5.08\times 10^{-7}\ \text{m}^2/\text{s})(3600\,\text{s})}}\right)\right] + 15°\text{C}$$

$$T(x,t) = \mathbf{64.5°C} \quad \text{where} \quad x = 0.1\ \text{m} \quad \text{and} \quad t = 3600\ \text{s}$$

Discussion At the wall surface after one hour, the temperature is 980°C

$$T(x,t) = \frac{20{,}000\ \text{W/m}^2}{1.0\ \text{W/m}\cdot\text{K}}\sqrt{\frac{4(5.08\times 10^{-7}\ \text{m}^2/\text{s})(3600\,\text{s})}{\pi}} + 15°\text{C} = 980°\text{C}$$

4-95  A thick refractory brick wall is subjected to uniform heat flux. The temperatures at the surface and at the depths of 1 cm and 5 cm from the wall surface as a function of heating time are to be determined.

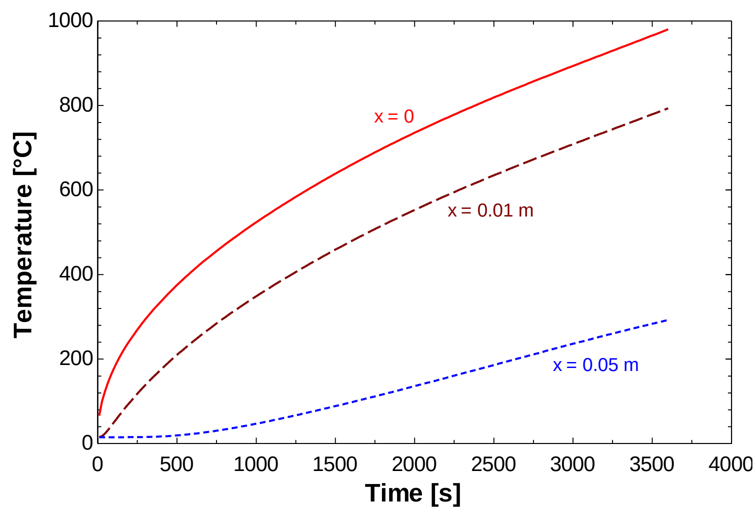
Assumptions **1** The wall is thick and can be treated as a semi-infinite medium with a specified surface heat flux. **2** The thermal properties of the wall are constant.

Properties The properties of the brick wall are given as k = 1.0 W/m·K and α = 5.08 × 10^{-7} m^2/s.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
q_dot_s=20000 [W/m^2]
T_i=15 [C]
x1=0.01 [m]
x2=0.05 [m]

"PROPERTIES"
k=1.0 [W/m-K]
alpha=5.08e-7 [m^2/s]

"ANALYSIS"
T_0-T_i=q_dot_s/k*((4*alpha*t/pi#)^0.5)
T_1cm-T_i=q_dot_s/k*((4*alpha*t/pi#)^0.5*exp(-x1^2/(4*alpha*t))-x1*erfc(x1/(2*(alpha*t)^0.5)))
T_5cm-T_i=q_dot_s/k*((4*alpha*t/pi#)^0.5*exp(-x2^2/(4*alpha*t))-x2*erfc(x2/(2*(alpha*t)^0.5)))
```

		T (x, t) [°C]	
Time [s]	x = 0	x = 0.01 m	x = 0.05 m
10	65.9	15.0	15.0
15	77.3	15.3	15.0
30	103.1	18.1	15.0
50	128.7	25.4	15.0
75	154.3	36.9	15.0
100	175.8	49.1	15.0
200	242.5	96.3	15.0
400	336.7	175.5	16.7
800	469.9	297.7	33.3
1200	572.2	394.9	62.7
2400	803.0	619.1	175.7
3600	980.1	793.3	292.3



Discussion The temperature at the wall depth of 5 cm remained at the initial temperature and did not increase until after 200 s have elapsed.

4-96  Thick stainless steel and copper slabs are subjected to uniform heat flux. The temperatures of both slabs at the depth of 8 cm from the surface as a function of time are to be determined.

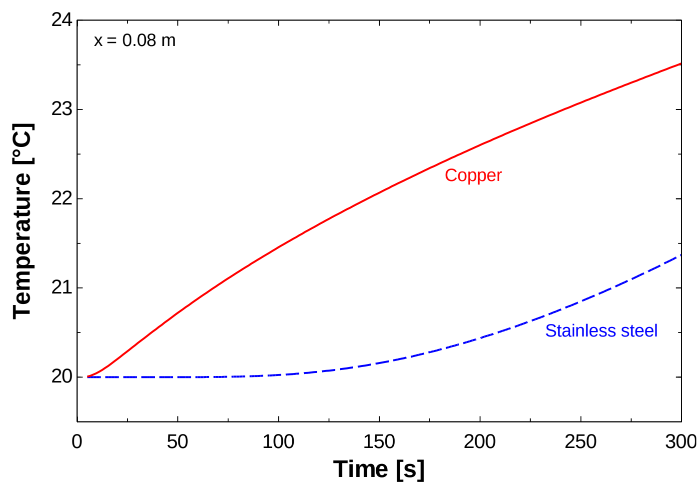
Assumptions **1** The slabs are treated as a semi-infinite medium with a specified surface heat flux. **2** The thermal properties of the slabs are constant.

Properties The properties of stainless steel are given as k = 14.9 W/m·K and $\alpha = 3.95\times 10^{-6}$ m^2/s; the properties of copper are given as k = 401 W/m·K and $\alpha = 117 \times 10^{-6}$ m^2/s.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
q_dot_s=10000 [W/m^2]
T_i=20 [C]
x=0.08 [m]

"PROPERTIES"
"stainless steel"
k_ss=14.9 [W/m-K]
alpha_ss=3.95e-6 [m^2/s]
"copper"
k_cu=401 [W/m-K]
alpha_cu=117e-6 [m^2/s]

"ANALYSIS"
T_ss-T_i=q_dot_s/k_ss*((4*alpha_ss*t/pi#)^0.5*exp(-x^2/(4*alpha_ss*t))-x*erfc(x/(2*(alpha_ss*t)^0.5)))
T_cu-T_i=q_dot_s/k_cu*((4*alpha_cu*t/pi#)^0.5*exp(-x^2/(4*alpha_cu*t))-x*erfc(x/(2*(alpha_cu*t)^0.5)))
```

	$T(x, t)$ [°C]	
Time [s]	SS	Cu
5	20.0	20.0
10	20.0	20.1
20	20.0	20.2
30	20.0	20.4
40	20.0	20.6
50	20.0	20.7
60	20.0	20.9
80	20.0	21.2
100	20.0	21.5
150	20.2	22.1
200	20.4	22.6
300	21.4	23.5



Discussion The copper slab, having a much higher thermal diffusivity value, diffuses the heat energy through the medium faster than the stainless steel slab. Due to the low thermal diffusivity of stainless steel, the tempearure of the slab stays virtually constant for the first 100 seconds.

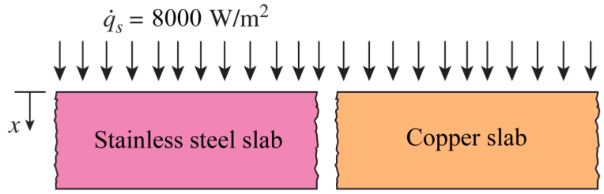
4-97 Thick stainless steel and copper slabs are subjected to uniform heat flux. The temperatures of both slabs at the depth of 1 cm from the surface after 60 s are to be determined.

Assumptions **1** The slabs are treated as a semi-infinite medium with a specified surface heat flux. **2** The thermal properties of the slabs are constant.

Properties The properties of stainless steel are given as k = 14.9 W/m·K and α = 3.95× 10^{-6} m^2/s; the properties of copper are given as k = 401 W/m·K and α = 117 × 10^{-6} m^2/s.

Analysis This is a transient conduction problem in a semi-infinite medium subjected to constant surface heat flux, and the temperatures at x = 0.01 m and t = 60 s can be determined from

$$T(x,t)-T_i=\frac{\dot{q}_s}{k}\left[\sqrt{\frac{4\alpha t}{\pi}}\exp\left(-\frac{x^2}{4\alpha t}\right)-x\,\mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right]$$



For stainless steel,

$$\frac{x}{2\sqrt{\alpha t}}=\frac{0.01\,\text{m}}{2\sqrt{(3.95\times10^{-6}\ \text{m}^2/\text{s})(60\,\text{s})}}=0.3248$$

From Table 4-4, erfc(0.3248) = 0.646. The temperature at the depth of 1 cm from the surface after 60 s is

$$T(x,t)=\frac{8000\ \text{W/m}^2}{14.9\ \text{W/m}\cdot\text{K}}\left[\sqrt{\frac{4(3.95\times10^{-6}\ \text{m}^2/\text{s})(60\,\text{s})}{\pi}}\exp\left[-\frac{(0.01\,\text{m})^2}{4(3.95\times10^{-6}\ \text{m}^2/\text{s})(60\,\text{s})}\right]\right.$$
$$\left.-(0.01\,\text{m})\mathrm{erfc}\left(\frac{0.01\,\text{m}}{2\sqrt{(3.95\times10^{-6}\ \text{m}^2/\text{s})(60\,\text{s})}}\right)\right]+20°\text{C}$$

$$T(x,t)=\mathbf{24.9°C}\quad\text{(stainless steel slab)}\quad\text{where}\quad x=0.01\text{ m}\quad\text{and}\quad t=60\text{ s}$$

For copper,

$$\frac{x}{2\sqrt{\alpha t}}=\frac{0.01\,\text{m}}{2\sqrt{(117\times10^{-6}\ \text{m}^2/\text{s})(60\,\text{s})}}=0.05968$$

From Table 4-4, erfc(0.05968) = 0.9327. The temperature at the depth of 1 cm from the surface after 60 s is

$$T(x,t)=\frac{8000\ \text{W/m}^2}{401\ \text{W/m}\cdot\text{K}}\left[\sqrt{\frac{4(117\times10^{-6}\ \text{m}^2/\text{s})(60\,\text{s})}{\pi}}\exp\left[-\frac{(0.01\,\text{m})^2}{4(117\times10^{-6}\ \text{m}^2/\text{s})(60\,\text{s})}\right]\right.$$
$$\left.-(0.01\,\text{m})\mathrm{erfc}\left(\frac{0.01\,\text{m}}{2\sqrt{(117\times10^{-6}\ \text{m}^2/\text{s})(60\,\text{s})}}\right)\right]+20°\text{C}$$

$$T(x,t)=\mathbf{21.7°C}\quad\text{(copper slab)}\quad\text{where}\quad x=0.01\text{ m}\quad\text{and}\quad t=60\text{ s}$$

Discussion Aside from using Table 4-4, the complementary error function can be solved using the EES software with the following lines:

```
EF_ss=erfc(0.3248)
EF_cu=erfc(0.05968)
```

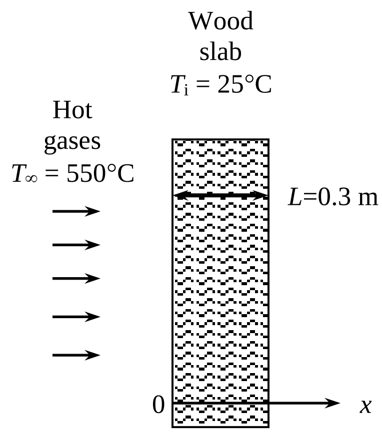
4-98 A thick wood slab is exposed to hot gases for a period of 5 minutes. It is to be determined whether the wood will ignite.

Assumptions **1** The wood slab is treated as a semi-infinite medium subjected to convection at the exposed surface. **2** The thermal properties of the wood slab are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The thermal properties of the wood are k = 0.17 W/m.°C and α = 1.28×10^{-7} m^2/s.

Analysis The one-dimensional transient temperature distribution in the wood can be determined from

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$



where

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(35\text{ W/m}^2.°\text{C})\sqrt{(1.28\times10^{-7}\text{ m}^2\text{ /s})(5\times 60\text{ s})}}{0.17\text{ W/m.°C}} = 1.276$$

$$\frac{h^2\alpha t}{k^2} = \left(\frac{h\sqrt{\alpha t}}{k}\right)^2 = 1.276^2 = 1.628$$

Noting that x = 0 at the surface and using Table 4-4 for *erfc* values,

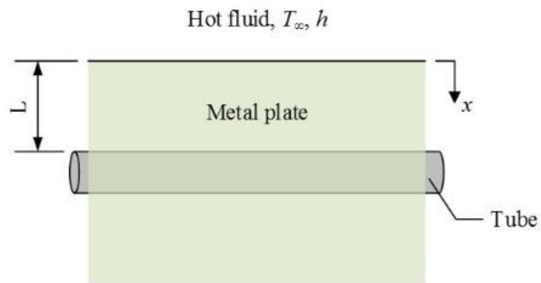
$$\frac{T(x,t)-25}{550-25} = erfc(0) - \exp(0+1.628)erfc(0+1.276)$$
$$= 1-(5.0937)(0.0712)$$
$$= 0.637$$

Solving for $T(x, t)$ gives

$$T(x,t) = \mathbf{360°C}$$

which is less than the ignition temperature of 450°C. Therefore, the wood will not ignite.

4-99 C&S ASTM F441 CPVC tube is embedded in a metal plate, while the plate's upper surface is subjected to convection with hot fluid at T_∞ = 300°C and h = 200 W/m^2·K. The plate has an initial temperature of 20°C. Would the tube comply with the ASME Code for Process Piping if the plate's upper surface is exposed to the hot fluid for 10 minutes?



***Assumptions*1** The plate is thick and is considered as a semi-infinite solid. **2**. Heat conduction is transient in the semi-infinite solid. **3** Thermal properties are constant. **4** Radiation effects are negligible. **5** The convection heat transfer coefficient is constant.

Properties The thermal properties of the metal plate are c_p = 460 J/kg·K, k = 26.9 W/m·K, and ρ = 7730 kg/m^3.

Analysis This a transient conduction problem in a semi-infinite medium subjected to convection at the surface. So from Eq. 4-47 we have

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = \mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)$$

Where $x = L = 0.02$ mand $t = 600$ s. First we determine the various quantities in parentheses in Eq. 4-47:

$$\alpha = \frac{k}{\rho c_p} = \frac{26.9\ \mathrm{W/m\cdot K}}{(7730\ \mathrm{kg/m^3})(460\ \mathrm{J/kg\cdot K})} = 7.5651\times 10^{-6}\mathrm{m^2/s}$$

$$\frac{x}{2\sqrt{\alpha t}} = \frac{0.02\ \mathrm{m}}{2\sqrt{(7.5651\times 10^{-6}\mathrm{m^2/s})(600\ \mathrm{s})}} = 0.14843$$

$$\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k} = 0.14843 + \frac{(200\ \mathrm{W/m^2\cdot K})\sqrt{(7.5651\times 10^{-6}\mathrm{m^2/s})(600\ \mathrm{s})}}{26.9\ \mathrm{W/m\cdot K}} = 0.64934$$

$$\frac{hx}{k} = \frac{(200\ \mathrm{W/m^2\cdot K})(0.02\ \mathrm{m})}{26.9\ \mathrm{W/m\cdot K}} = 0.148699$$

$$\frac{h^2\alpha t}{k^2} = \frac{(200\ \mathrm{W/m^2\cdot K})^2(7.5651\times 10^{-6}\mathrm{m^2/s})(600\ \mathrm{s})}{(26.9\ \mathrm{W/m\cdot K})^2} = 0.25091$$

$$\mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) = \mathrm{erfc}(0.14843) = 0.83374$$

$$\mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right) = \mathrm{erfc}(0.64934) = 0.35846$$

Thus,

$$\frac{T(L,t)-T_i}{T_\infty - T_i} = 0.83374 - \exp(0.148699+0.25091)\,(0.35846) = 0.29919$$

And

$$T(L,t) = (300-20)0.29919 + 20 = \mathbf{103.8}℃ > 93.3℃$$

Discussion After 10 minutes of subjecting the upper surface to convection with the hot fluid, the plate temperature at a depth of x = 2 cm from the upper surface is 104°C. This means that the tube surface is at a temperature that exceeds the maximum use temperature specified by the ASME Code for Process Piping. Therefore, the tube would not be in compliance.

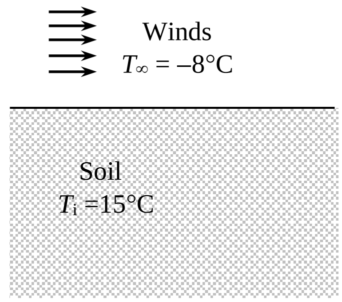
4-100 An area is subjected to cold air for a 10-h period. The soil temperatures at distances 0, 10, 20, and 50 cm from the earth's surface are to be determined.

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the soil are constant.

Properties The thermal properties of the soil are given to be k = 0.9 W/m.°C and $\alpha = 1.6\times10^{-5}$ m²/s.

Analysis The one-dimensional transient temperature distribution in the ground can be determined from

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$



where

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(40\ \text{W/m}^2.°\text{C})\sqrt{(1.6\times10^{-5}\ \text{m}^2/\text{s})(10\times3600\ \text{s})}}{0.9\ \text{W/m.°C}} = 33.7$$

$$\frac{h^2\alpha t}{k^2} = \left(\frac{h\sqrt{\alpha t}}{k}\right)^2 = 33.7^2 = 1138$$

Then we conclude that the last term in the temperature distribution relation above must be zero regardless of x despite the exponential term tending to infinity since (1) $erfc(\eta)\to 0$ for $\eta > 4$ (see Table 4-4) and (2) the term has to remain less than 1 to have physically meaningful solutions. That is,

$$\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right] = \exp\left(\frac{hx}{k}+1138\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+33.7\right)\right] \cong 0$$

Therefore, the temperature distribution relation simplifies to

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) \to T(x,t) = T_i + (T_\infty - T_i)erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Then the temperatures at 0, 10, 20, and 50 cm depth from the ground surface become

x = 0:

$$T(0{,}10\ \text{h}) = T_i + (T_\infty - T_i)erfc\left(\frac{0}{2\sqrt{\alpha t}}\right) = T_i + (T_\infty - T_i)erfc(0) = T_i + (T_\infty - T_i)\times 1 = T_\infty = \mathbf{-8°C}$$

x = 0.1m:

$$T(0.1\ \text{m},10\ \text{h}) = 15 + (-8-15)erfc\left(\frac{0.1\ \text{m}}{2\sqrt{(1.6\times10^{-5}\ \text{m}^2/\text{s})(10\ \text{h}\times3600\ \text{s/h})}}\right)$$
$$= 15 - 23erfc(0.066) = 15 - 23\times0.9257 = \mathbf{-6.3°C}$$

x = 0.2 m:

$$T(0.2\ \text{m},10\ \text{h}) = 15 + (-8-15)erfc\left(\frac{0.2\ \text{m}}{2\sqrt{(1.6\times10^{-5}\ \text{m}^2/\text{s})(10\ \text{h}\times3600\ \text{s/h})}}\right)$$
$$= 15 - 23erfc(0.132) = 15 - 23\times0.8519 = \mathbf{-4.6°C}$$

x = 0.5 m:

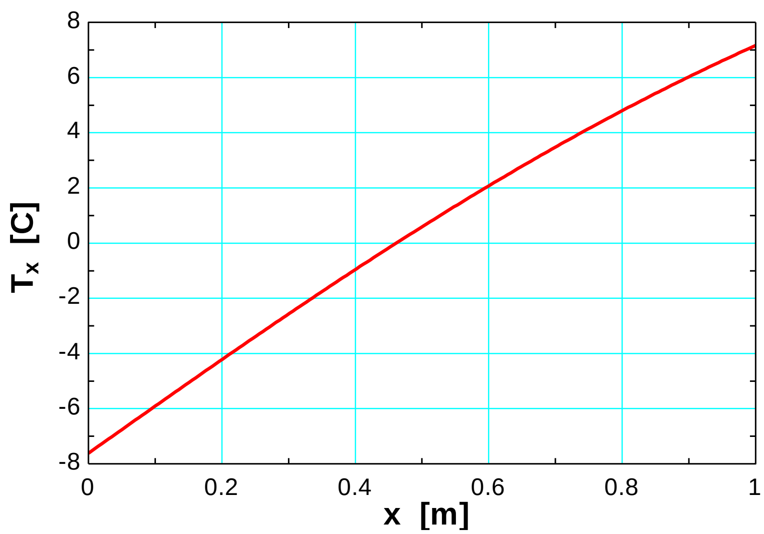
$$T(0.5\ m,10\ h) = 15 + (-8-15)erfc\left(\frac{0.5\ m}{2\sqrt{(1.6\times10^{-5}\ m^2/s)(10\ h\times3600\ s/h)}}\right)$$
$$= 15 - 23erfc(0.329) = 15 - 23\times0.6418 = \mathbf{0.2°C}$$

4-101  Prob. 4-100 is reconsidered. The soil temperature as a function of the distance from the earth's surface is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_i=15 [C]
T_infinity=-8 [C]
h=40 [W/m^2-C]
time=10*3600 [s]
x=0.1 [m]

"PROPERTIES"
k=0.9 [W/m-C]
alpha=1.6E-5 [m^2/s]

"ANALYSIS"
(T_x-T_i)/(T_infinity-T_i)=erfc(x/(2*sqrt(alpha*time)))-
exp((h*x)/k+(h^2*alpha*time)/k^2)*erfc(x/(2*sqrt(alpha*time))+(h*sqrt(alpha*time)/k))
```

x [m]	T_x [C]
0	-7.615
0.05	-6.762
0.1	-5.911
0.15	-5.064
0.2	-4.224
0.25	-3.391
0.3	-2.569
0.35	-1.758
0.4	-0.96
0.45	-0.1764
0.5	0.5912
0.55	1.342
0.6	2.074
0.65	2.786
0.7	3.478
0.75	4.149
0.8	4.797
0.85	5.423
0.9	6.026
0.95	6.605
1	7.16

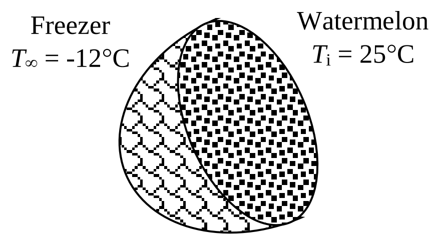


4-102 A spherical watermelon that is cut into two equal parts is put into a freezer. The time it will take for the center of the exposed cut surface to cool from 25°C to 3°C is to be determined.

Assumptions **1** The temperature of the exposed surfaces of the watermelon is affected by the convection heat transfer at those surfaces only. Therefore, the watermelon can be considered to be a semi-infinite medium **2** The thermal properties of the watermelon are constant.

Properties The thermal properties of the water is closely approximated by those of water at room temperature, k = 0.607 W/m.K and $\alpha = 0.146\times10^{-6}$ m^2/s (Table A-9).

Analysis We use the transient chart in Fig. 4-31 in this case for convenience (instead of the analytic solution),



$$\left.\begin{aligned} 1-\frac{T(x,t)-T_\infty}{T_i-T_\infty} &= 1-\frac{3-(-12)}{25-(-12)} = 0.595 \\ \eta &= \frac{x}{2\sqrt{\alpha t}} = 0 \end{aligned}\right\} \frac{h\sqrt{\alpha t}}{k} = 1$$

Therefore,

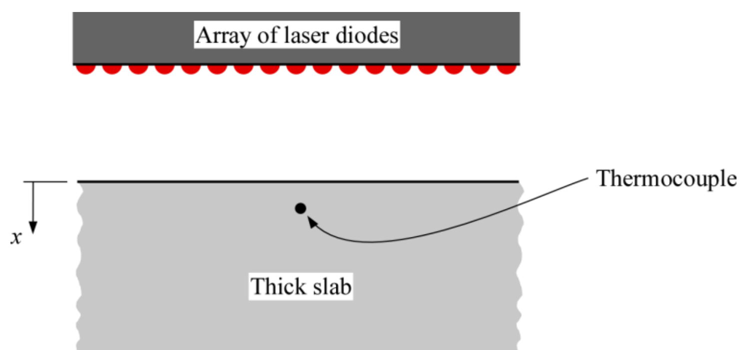
$$t = \frac{(1)^2 k^2}{h^2\alpha} = \frac{(0.607\text{ W/m.}K)^2}{(22\text{ W/m}^2.K)^2(0.146\times10^{-6}\text{ m}^2/\text{s})} = 5214\text{ s} = \mathbf{86.9\,min}$$

4-103 A slab surface has been exposed to laser pulse, (a) the amount of energy per unit surface area directed on the slab surface and (b) the thermocouple reading (at x = 25 mm) after 60 s has elapsed are to be determined.

Assumptions **1** The slab is treated as semi-infinite solid. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The properties of the slab are given to be k = 63.9 W/m · K and $\alpha = 18.8\times10^{-6}$ m^2/s.

Analysis (a) For semi-infinite solid with energy pulse at surface, we have



$$T(x,t)-T_i = \frac{e_s}{k\sqrt{\pi\, t/\alpha}}\exp\left(-\frac{x^2}{4\alpha\, t}\right)$$

$$e_s = k\sqrt{\frac{\pi\, t}{\alpha}}\exp\left(\frac{x^2}{4\alpha\, t}\right)\left[T(x,t)-T_i\right]$$

$$= (63.9\text{ W/m}\cdot\text{K})\sqrt{\frac{\pi(30\text{ s})}{18.8\times10^{-6}\text{ m}^2/\text{s}}}\exp\left(\frac{(0.025\text{ m})^2}{4(18.8\times10^{-6}\text{ m}^2/\text{s})(30\text{ s})}\right)(130-20)\text{ K}$$

$$= \mathbf{2.076\times10^7\ J/m^2}$$

(b) After 60 s has elapsed, the thermocouple reading is

$$T(x,t) = \frac{2.076\times10^7\text{ J/m}^2}{(63.9\text{ W/m}\cdot°\text{C})\sqrt{\dfrac{\pi(60\text{ s})}{18.8\times10^{-6}\text{ m}^2/\text{s}}}}\exp\left(-\frac{(0.025\text{ m})^2}{4(18.8\times10^{-6}\text{ m}^2/\text{s})(60\text{ s})}\right)+20\,°\text{C}$$

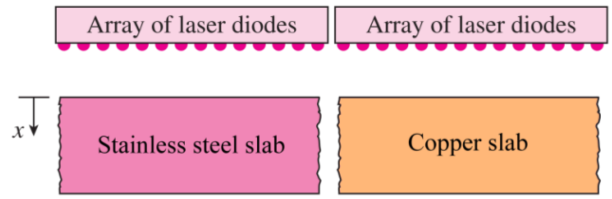
$$T(0.025\text{ m}, 60\text{ s}) = \mathbf{109\,°C}$$

Discussion High-power laser diodes can be used in many industrial applications, such as welding, heat treatment, and cladding.

4-104 Thick stainless steel and copper slabs are subjected to an energy pulse. The temperatures of both slabs at the depth of 5 cm from the surface, after 60 s of receiving the energy pulse, are to be determined.

Assumptions **1** The slabs are treated as semi-infinite solids. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** Entire energy from the pulse enters the slabs.

Properties The properties of stainless steel are given as k = 14.9 W/m·K and α = 3.95× 10^{-6} m^2/s; the properties of copper are given as k = 401 W/m·K and α = 117 × 10^{-6} m^2/s.



Analysis This is a transient conduction problem in a semi-infinite medium subjected to an energy pulse at the surface, and the temperatures at x = 0.05 m and t = 60 s can be determined from

$$T(x,t)-T_i=\frac{e_s}{k\sqrt{\pi t/\alpha}}\exp\left(-\frac{x^2}{4\alpha\ t}\right)$$

For the stainless steel slab,

$$T(x,t)=\frac{5\times10^7\ \text{J/m}^2}{(14.9\ \text{W/m}\cdot\text{K})\sqrt{\dfrac{\pi(60\ \text{s})}{3.95\times10^{-6}\ \text{m}^2/\text{s}}}}\exp\left(-\frac{(0.05\ \text{m})^2}{4(3.95\times10^{-6}\ \text{m}^2/\text{s})(60\ \text{s})}\right)+20°\text{C}=\mathbf{54.8°C}$$

For the copper slab,

$$T(x,t)=\frac{5\times10^7\ \text{J/m}^2}{(401\ \text{W/m}\cdot\text{K})\sqrt{\dfrac{\pi(60\ \text{s})}{117\times10^{-6}\ \text{m}^2/\text{s}}}}\exp\left(-\frac{(0.05\ \text{m})^2}{4(117\times10^{-6}\ \text{m}^2/\text{s})(60\ \text{s})}\right)+20°\text{C}=\mathbf{109.9°C}$$

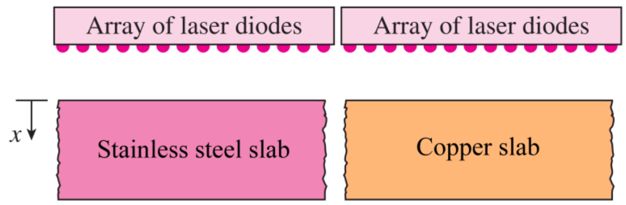
Discussion High-power laser diodes are used in many industrial applications, such as welding, heat treatment, and cladding.

4-105 Thick stainless steel and copper slabs are subjected to an energy pulse. The temperatures of both slabs at the depth of 5 cm from the surface as a function of time are to be determined.

Assumptions **1** The slabs are treated as semi-infinite solids. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** Entire energy from the pulse enters the slabs.

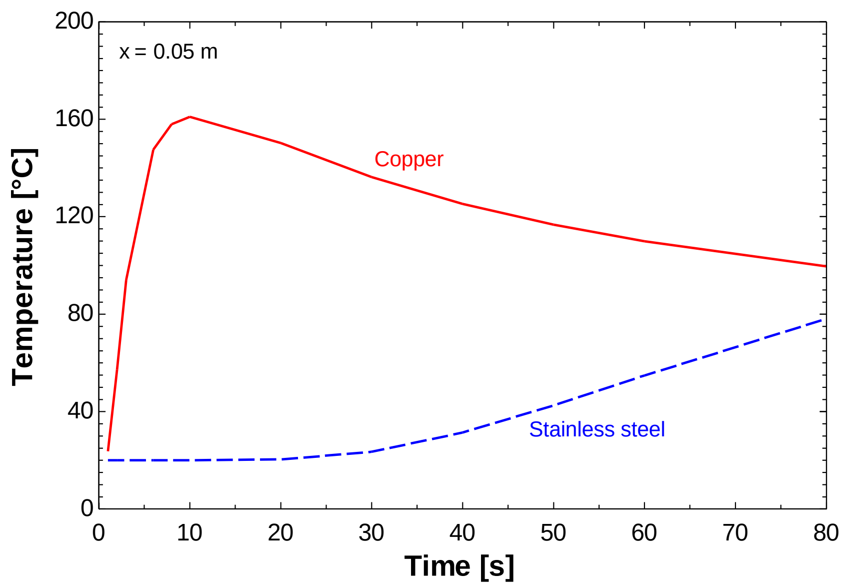
Properties The properties of stainless steel are given as k = 14.9 W/m·K and $\alpha = 3.95\times 10^{-6}$ m^2/s; the properties of copper are given as k = 401 W/m·K and $\alpha = 117 \times 10^{-6}$ m^2/s.

Analysis The problem is solved using EES, and the solution is given below.




```
"GIVEN"
e_s=5e7 [J/m^2]
T_i=20 [C]
x=0.05 [m]

"PROPERTIES"
"stainless steel"
k_ss=14.9 [W/m-K]
alpha_ss=3.95e-6 [m^2/s]
"copper"
k_cu=401 [W/m-K]
alpha_cu=117e-6 [m^2/s]

"ANALYSIS"
T_ss-T_i=e_s/(k_ss*(pi#*t/alpha_ss)^0.5)*exp(-x^2/(4*alpha_ss*t))
T_cu-T_i=e_s/(k_cu*(pi#*t/alpha_cu)^0.5)*exp(-x^2/(4*alpha_cu*t))
```

	$T(x, t)$ [°C]	
Time [s]	SS	Cu
1	20.0	23.6
2	20.0	57.2
3	20.0	94.0
6	20.0	147.5
8	20.0	158.0
10	20.0	161.0
20	20.3	150.3
30	23.5	136.3
40	31.4	125.3
50	42.5	116.7
60	54.8	109.9
80	78.2	99.6



Discussion The copper slab, having a much higher thermal diffusivity value, diffuses the heat energy from the pulse much quicker than the stainless steel slab. As shown in the table and figure, the copper temperature increases sharply for the first 10 seconds from an initial temperature of 20°C to a temperature of 161°C, while the temperature of stainless steel stays constant at its initial temperature of 20°C .

4-106 The contact surface temperatures when a bare footed person steps on aluminum and wood blocks are to be determined.

Assumptions **1** Both bodies can be treated as the semi-infinite solids. **2** Heat loss from the solids is disregarded. **3** The properties of the solids are constant.

Properties The $\sqrt{k\rho c_p}$ value is 24 kJ/m²·°C for aluminum, 0.38 kJ/m²·°C for wood, and 1.1 kJ/m²·°C for the human flesh.

Analysis The surface temperature is determined from Eq. 4-49 to be

$$T_s = \frac{\sqrt{(k\rho c_p)_{\text{human}}}T_{\text{human}} + \sqrt{(k\rho c_p)_{\text{Al}}}T_{\text{Al}}}{\sqrt{(k\rho c_p)_{\text{human}}} + \sqrt{(k\rho c_p)_{\text{Al}}}} = \frac{(1.1\,\text{kJ/m}^2\cdot°\text{C})(32°\text{C}) + (24\,\text{kJ/m}^2\cdot°\text{C})(20°\text{C})}{(1.1\,\text{kJ/m}^2\cdot°\text{C}) + (24\,\text{kJ/m}^2\cdot°\text{C})} = \mathbf{20.5°C}$$

In the case of wood block, we obtain

$$\begin{aligned} T_s &= \frac{\sqrt{(k\rho c_p)_{\text{human}}}T_{\text{human}} + \sqrt{(k\rho c_p)_{\text{wood}}}T_{\text{wood}}}{\sqrt{(k\rho c_p)_{\text{human}}} + \sqrt{(k\rho c_p)_{\text{wood}}}} \\ &= \frac{(1.1\,\text{kJ/m}^2\cdot°\text{C})(32°\text{C}) + (0.38\,\text{kJ/m}^2\cdot°\text{C})(20°\text{C})}{(1.1\,\text{kJ/m}^2\cdot°\text{C}) + (0.38\,\text{kJ/m}^2\cdot°\text{C})} \\ &= \mathbf{28.9°C} \end{aligned}$$

Transient Heat Conduction in Multidimensional Systems

4-107C The product solution enables us to determine the dimensionless temperature of two- or three-dimensional heat transfer problems as the product of dimensionless temperatures of one-dimensional heat transfer problems. The dimensionless temperature for a two-dimensional problem is determined by determining the dimensionless temperatures in both directions, and taking their product.

4-108C The dimensionless temperature for a three-dimensional heat transfer is determined by determining the dimensionless temperatures of one-dimensional geometries whose intersection is the three dimensional geometry, and taking their product.

4-109C This short cylinder is physically formed by the intersection of a long cylinder and a plane wall. The dimensionless temperatures at the center of plane wall and at the center of the cylinder are determined first. Their product yields the dimensionless temperature at the center of the short cylinder.

4-110C The heat transfer in this short cylinder is one-dimensional since there is no heat transfer in the axial direction. The temperature will vary in the radial direction only.

4-111 A cubic block and a cylindrical block are exposed to hot gases on all of their surfaces. The center temperatures of each geometry in 10, 20, and 60 min are to be determined.

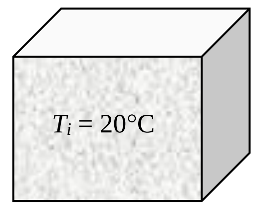
Assumptions **1** Heat conduction in the cubic block is three-dimensional, and thus the temperature varies in all *x*-, *y*, and *z*-directions. **2** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial *x*- and radial *r*- directions. **3** The thermal properties of the granite are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The thermal properties of the granite are given to be $k = 2.5$ W/m.°C and $\alpha = 1.15\times10^{-6}$ m²/s.

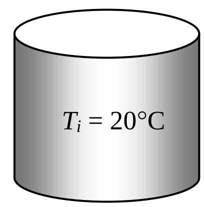
Analysis

Cubic block: This cubic block can physically be formed by the intersection of three infinite plane walls of thickness $2L = 5$ cm.

5 cm × 5 cm × 5 cm



Hot gases
500°C



<u>After 10 minutes</u>: The Biot number, the corresponding constants, and the Fourier number are

$$Bi = \frac{hL}{k} = \frac{(40\text{ W/m}^2.°\text{C})(0.025\text{ m})}{(2.5\text{ W/m.°C})} = 0.400 \longrightarrow \lambda_1 = 0.5932 \text{ and } A_1 = 1.0580$$

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\text{ m}^2/\text{s})(10\text{ min}\times 60\text{ s/min})}{(0.025\text{ m})^2} = 1.104 > 0.2$$

To determine the center temperature, the product solution can be written as

$$\theta(0,0,0,t)_{\text{block}} = \left[\theta(0,t)_{\text{wall}}\right]^3$$

$$\frac{T(0,0,0,t)-T_\infty}{T_i - T_\infty} = \left(A_1 e^{-\lambda_1^2\tau}\right)^3$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(1.104)}\right\}^3 = 0.369$$

$$T(0,0,0,t) = \mathbf{323°C}$$

<u>After 20 minutes</u>

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\text{ m}^2/\text{s})(20\text{ min}\times 60\text{ s/min})}{(0.025\text{ m})^2} = 2.208 > 0.2$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(2.208)}\right\}^3 = 0.115 \longrightarrow T(0,0,0,t) = \mathbf{445°C}$$

<u>After 60 minutes</u>

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\text{ m}^2/\text{s})(60\text{ min}\times 60\text{ s/min})}{(0.025\text{ m})^2} = 6.624 > 0.2$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(6.624)}\right\}^3 = 0.00109 \longrightarrow T(0,0,0,t) = \mathbf{500°C}$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

Cylinder: This cylindrical block can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = 2.5$ cm and a plane wall of thickness $2L = 5$ cm.

<u>After 10 minutes</u>: The Biot number and the corresponding constants for the long cylinder are

$$Bi = \frac{hr_o}{k} = \frac{(40\text{ W/m}^2.°\text{C})(0.025\text{ m})}{(2.5\text{ W/m.°C})} = 0.400 \longrightarrow \lambda_1 = 0.8516 \text{ and } A_1 = 1.0931$$

To determine the center temperature, the product solution can be written as

$$\theta(0,0,t)_{block} = \left[\theta(0,t)_{wall}\right]\left[\theta(0,t)_{cyl}\right]$$

$$\frac{T(0,0,t)-T_\infty}{T_i - T_\infty} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{wall}\left(A_1 e^{-\lambda_1^2\tau}\right)_{cyl}$$

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(1.104)}\right\}\left\{(1.0931)e^{-(0.8516)^2(1.104)}\right\} = 0.352 \longrightarrow T(0,0,t) = \mathbf{331°C}$$

<u>After 20 minutes</u>

$$\frac{T(0,0,t)-500}{20-500}=\left\{(1.0580)e^{-(0.5932)^2(2.208)}\right\}\left\{(1.0931)e^{-(0.8516)^2(2.208)}\right\}=0.107\longrightarrow T(0,0,t)=\mathbf{449°C}$$

<u>After 60 minutes</u>

$$\frac{T(0,0,t)-500}{20-500}=\left\{(1.0580)e^{-(0.5932)^2(6.624)}\right\}\left\{(1.0931)e^{-(0.8516)^2(6.624)}\right\}=0.00092\longrightarrow T(0,0,t)=\mathbf{500°C}$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

4-112 A cubic block and a cylindrical block are exposed to hot gases on all of their surfaces. The center temperatures of each geometry in 10, 20, and 60 min are to be determined.

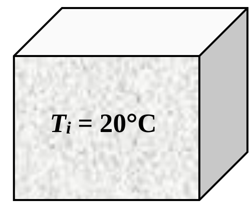
Assumptions **1** Heat conduction in the cubic block is three-dimensional, and thus the temperature varies in all *x*-, *y*, and *z*-directions. **2** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial *x*- and radial *r*- directions. **3** The thermal properties of the granite are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The thermal properties of the granite are $k = 2.5$ W/m.°C and $\alpha = 1.15\times10^{-6}$ m²/s.

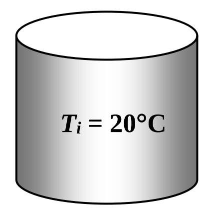
Analysis

Cubic block: This cubic block can physically be formed by the intersection of three infinite plane wall of thickness $2L = 5$ cm. Two infinite plane walls are exposed to the hot gases with a heat transfer coefficient of $h = 40\ \text{W/m}^2.°\text{C}$ and one with $h = 80\ \text{W/m}^2.°\text{C}$.

5 cm × 5 cm × 5 cm



Hot gases
500°C



<u>After 10 minutes</u>: The Biot number and the corresponding constants for $h = 40\ \text{W/m}^2.°\text{C}$ are

$$Bi = \frac{hL}{k} = \frac{(40\ \text{W/m}^2.°\text{C})(0.025\ \text{m})}{(2.5\ \text{W/m.°C})} = 0.400 \longrightarrow \lambda_1 = 0.5932 \quad \text{and} \quad A_1 = 1.0580$$

The Biot number and the corresponding constants for $h = 80\ \text{W/m}^2.°\text{C}$ are

$$Bi = \frac{hL}{k} = \frac{(80\ \text{W/m}^2.°\text{C})(0.025\ \text{m})}{(2.5\ \text{W/m.°C})} = 0.800$$

$$\longrightarrow \lambda_1 = 0.7910 \quad \text{and} \quad A_1 = 1.1016$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(10\,\text{min}\times 60\ \text{s/min})}{(0.025\ \text{m})^2} = 1.104 > 0.2$$

To determine the center temperature, the product solution method can be written as

$$\theta(0,0,0,t)_{\text{block}} = \left[\theta(0,t)_{\text{wall}}\right]^2 \left[\theta(0,t)_{\text{wall}}\right]$$

$$\frac{T(0,0,0,t) - T_\infty}{T_i - T_\infty} = \left(A_1 e^{-\lambda_1^2 \tau}\right)^2 \left(A_1 e^{-\lambda_1^2 \tau}\right)$$

$$\frac{T(0,0,0,t) - 500}{20 - 500} = \left\{(1.0580)e^{-(0.5932)^2(1.104)}\right\}^2 \left\{(1.1016)e^{-(0.7910)^2(1.104)}\right\} = 0.284$$

$$T(0,0,0,t) = \mathbf{364°C}$$

<u>After 20 minutes</u>

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(20\,\text{min}\times 60\ \text{s/min})}{(0.025\ \text{m})^2} = 2.208 > 0.2$$

$$\frac{T(0,0,0,t) - 500}{20 - 500} = \left\{(1.0580)e^{-(0.5932)^2(2.208)}\right\}^2 \left\{(1.1016)e^{-(0.7910)^2(2.208)}\right\} = 0.0654$$

$$\longrightarrow T(0,0,0,t) = \mathbf{469°C}$$

<u>After 60 minutes</u>

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(60\,\text{min}\times 60\ \text{s/min})}{(0.025\ \text{m})^2} = 6.624 > 0.2$$

$$\frac{T(0,0,0,t) - 500}{20 - 500} = \left\{(1.0580)e^{-(0.5932)^2(6.624)}\right\}^2 \left\{(1.1016)e^{-(0.7910)^2(6.624)}\right\} = 0.000186$$

$$\longrightarrow T(0,0,0,t) = \mathbf{500°C}$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

Cylinder: This cylindrical block can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = 2.5$ cm exposed to the hot gases with a heat transfer coefficient of $h = 40\ \text{W/m}^2.°\text{C}$ and a plane wall of thickness $2L = 5$ cm exposed to the hot gases with $h = 80\ \text{W/m}^2.°\text{C}$.

After 10 minutes: The Biot number and the corresponding constants for the long cylinder are

$$Bi = \frac{hr_o}{k} = \frac{(40\ \text{W/m}^2.°\text{C})(0.025\ \text{m})}{(2.5\ \text{W/m.°C})} = 0.400 \longrightarrow \lambda_1 = 0.8516 \quad \text{and} \quad A_1 = 1.0931$$

To determine the center temperature, the product solution method can be written as

$$\theta(0,0,t)_{\text{block}} = [\theta(0,t)_{\text{wall}}][\theta(0,t)_{cyl}]$$

$$\frac{T(0,0,t)-T_\infty}{T_i - T_\infty} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}}\left(A_1 e^{-\lambda_1^2\tau}\right)_{cyl}$$

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.1016)e^{-(0.7910)^2(1.104)}\right\}\left\{(1.0931)e^{-(0.8516)^2(1.104)}\right\} = 0.271$$

$$T(0,0,t) = \mathbf{370°C}$$

After 20 minutes

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.1016)e^{-(0.7910)^2(2.208)}\right\}\left\{(1.0931)e^{-(0.8516)^2(2.208)}\right\} = 0.06094 \longrightarrow T(0,0,t) = \mathbf{471°C}$$

After 60 minutes

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.1016)e^{-(0.7910)^2(6.624)}\right\}\left\{(1.0931)e^{-(0.8516)^2(6.624)}\right\} = 0.0001568 \longrightarrow T(0,0,t) = \mathbf{500°C}$$

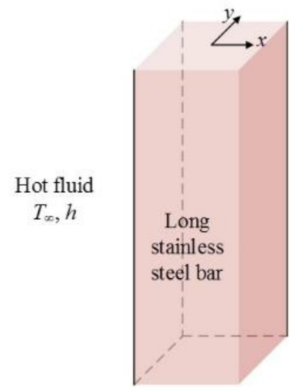
Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

4-113 ASTM A479 904L bar is submerged is hot liquid at T_∞ = 300°C and h = 288 W/m^2·K. The bar has an initial temperature of 20°C. If the bar is submerged in the hot liquid for 7 minutes, would it be in compliance with the ASME code? How long will it take for the bar to reach the maximum use temperature?

***Assumptions*1** The bars have square cross section with infinite length. **2** Thermal properties are constant. **3** Radiation effects are negligible. **4** The convection heat transfer coefficient is constant. **5** The Fourier number is $\tau > 0.2$ so that the one term-term approximate solutions are applicable.

Properties The thermal properties given are c_p = 500 J/kg·K, k = 12 W/m·K, and ρ = 7900 kg/m^3.

Analysis The bar is treated as an infinite square bar and can be formed by the intersection of two plane walls with the same thickness 2L = 5 cm. The Biot number for this process is



$$\mathrm{Bi} = \frac{hL}{k} = \frac{(288\ \mathrm{W/m^2\cdot K})(0.025\ \mathrm{m})}{12\ \mathrm{W/m\cdot K}} = 0.6 > 0.1$$

The coefficients λ_1 and A_1 for a plane wall corresponding to this Bi are determined from Table 4-2 to be

$$\lambda_1 = 0.7051 \qquad \text{and} A_1 = 1.0814$$

The transient temperature distribution for the square bar is

$$\frac{T(x,y,t) - T_\infty}{T_i - T_\infty} = \theta_{\text{wall}}(x,t)\theta_{\text{wall}}(y,t) = \left[A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 x/L)\right]\left[A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 y/L)\right]$$

The edge of the bar ($x = y = L$ = 0.05 m) would first reach the maximum use temperature, so we have

$$\frac{T(L,L,t) - T_\infty}{T_i - T_\infty} = A_1^2 e^{-2\lambda_1^2\tau}\cos^2(\lambda_1) \qquad \rightarrow \qquad T(L,L,t) = (T_i - T_\infty)A_1^2 e^{-2\lambda_1^2\tau}\cos^2(\lambda_1) + T_\infty$$

So,

$$T(L,L,t) = (20 - 300)(1.0814)^2 e^{-2(0.7051)^2(2.0415)}\cos^2(0.7051) + 300 = \mathbf{275.1}℃ > 260℃$$

where

$$\alpha = \frac{k}{\rho c_p} = \frac{12\ \mathrm{W/m\cdot K}}{(7900\ \mathrm{kg/m^3})(500\ \mathrm{J/kg\cdot K})} = 3.0380 \times 10^{-6}\mathrm{m^2/s}$$

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.0380 \times 10^{-6}\mathrm{m^2/s})(420\ \mathrm{s})}{(0.025\ \mathrm{m})^2} = 2.0415 \ > 0.2$$

To find the duration for the edge of the bar to reach the maximum use temperature, we need to find the Fourier number (dimensionless time) when $T(L, L, t)$ = 260°C:

$$\tau = -\frac{1}{2\lambda_1^2}\ln\left[\frac{1}{A_1^2\cos^2(\lambda_1)}\frac{T(L,L,t) - T_\infty}{T_i - T_\infty}\right] = -\frac{1}{2(0.7051)^2}\ln\left[\frac{1}{(1.0814)^2\cos^2(0.7051)}\left(\frac{260 - 300}{20 - 300}\right)\right] = 1.5665$$

The time for the edge of the bar to reach the maximum use temperature is

$$t = L^2\frac{\tau}{\alpha} = (0.025\ \mathrm{m})^2\frac{1.5665}{3.0380 \times 10^{-6}\mathrm{m^2/s}} = \mathbf{322.3\ s}$$

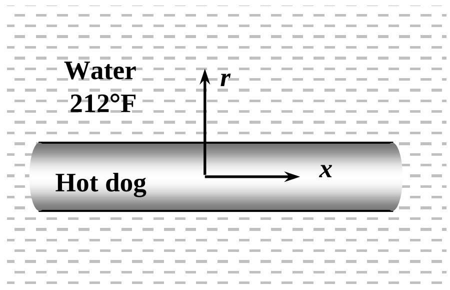
Discussion If the stainless steel bar is submerged in the hot liquid for 7 minutes, then it would not comply with the ASME Code for Process Piping. At that duration, the temperature at the bar edges would exceed the maximum use temperature by more than 15°C. In order to keep the bar from exceeding the maximum use temperature of 260°C, the bar cannot be submerged in the hot liquid for more than 322 seconds.

4-114E A hot dog is dropped into boiling water. The center temperature of the hot dog is do be determined by treating hot dog as a finite cylinder and also as an infinitely long cylinder.

Assumptions **1** When treating hot dog as a finite cylinder, heat conduction in the hot dog is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. When treating hot dog as an infinitely long cylinder, heat conduction is one-dimensional in the radial *r*- direction. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The thermal properties of the hot dog are given to be $k = 0.44$ Btu/h.ft.°F, $\rho = 61.2$ lbm/ft^3 $c_p = 0.93$ Btu/lbm.°F, and $\alpha = 0.0077$ ft^2/h.

Analysis (a) This hot dog can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = (0.4/12)$ ft and a plane wall of thickness $2L = (5/12)$ ft. The distance x is measured from the midplane.



After 5 minutes

First the Biot number is calculated for the plane wall to be

$$Bi = \frac{hL}{k} = \frac{(120\ \text{Btu/h.ft}^2.°\text{F})(2.5/12\ \text{ft})}{(0.44\ \text{Btu/h.ft.°F})} = 56.8$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 1.5421 \quad \text{and} \quad A_1 = 1.2728$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(5/60\ \text{h})}{(2.5/12\ \text{ft})^2} = 0.015 < 0.2 \quad \text{(Be cautious!)}$$

Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.015)} = 1.228$$

We repeat the same calculations for the long cylinder,

$$Bi = \frac{hr_o}{k} = \frac{(120\ \text{Btu/h.ft}^2.°\text{F})(0.4/12\ \text{ft})}{(0.44\ \text{Btu/h.ft.°F})} = 9.1$$

$$\lambda_1 = 2.1589 \quad \text{and} \quad A_1 = 1.5618$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(5/60\ \text{h})}{(0.4/12\ \text{ft})^2} = 0.578 > 0.2$$

$$\theta_{o,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(0.578)} = 0.106$$

Then the center temperature of the short cylinder becomes

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1.228 \times 0.106 = 0.130$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.130 \longrightarrow T(0,0,t) = \mathbf{190°F}$$

After 10 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(10/60\ \text{h})}{(2.5/12\ \text{ft})^2} = 0.03 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.03)} = 1.185$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(10/60\ \text{h})}{(0.4/12\ \text{ft})^2} = 1.156 > 0.2$$

$$\theta_{o,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(1.156)} = 0.0071$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\\text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1.185 \times 0.0071 = 0.0084$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.0084 \longrightarrow T(0,0,t) = \mathbf{211°F}$$

<u>After 15 minutes</u>

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077\text{ ft}^2/\text{h})(15/60\text{ h})}{(2.5/12\text{ ft})^2} = 0.045 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.045)} = 1.143$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077\text{ ft}^2/\text{h})(15/60\text{ h})}{(0.4/12\text{ ft})^2} = 1.734 > 0.2$$

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(1.734)} = 0.00048$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\\text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1.143 \times 0.00048 = 0.00055$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.00055 \longrightarrow T(0,0,t) = \mathbf{212\ °F}$$

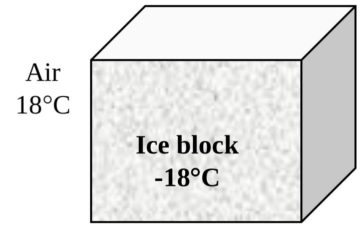
(*b*) Treating the hot dog as an infinitely long cylinder will not change the results obtained in the part (a) since dimensionless temperatures for the plane wall is 1 for all cases.

4-115 A rectangular ice block is placed on a table. The time the ice block starts melting is to be determined.

Assumptions **1** Heat conduction in the ice block is two-dimensional, and thus the temperature varies in both *x*- and *y*-directions. **2** The thermal properties of the ice block are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The thermal properties of the ice are given to be k = 2.22 W/m.°C and $\alpha = 0.124\times10^{-7}$ m^2/s.

Analysis This rectangular ice block can be treated as a short rectangular block that can physically be formed by the intersection of two infinite plane wall of thickness $2L$ = 4 cm and an infinite plane wall of thickness $2L$ = 12 cm. We measure x from the bottom surface of the block since this surface represents the adiabatic center surface of the plane wall of thickness $2L$ = 12 cm. Since the melting starts at the corner of the top surface, we need to determine the time required to melt ice block which will happen when the temperature drops below 0°C at this location. The Biot numbers and the corresponding constants are first determined to be



$$Bi_{\text{wall},1} = \frac{hL_1}{k} = \frac{(12\text{ W/m}^2.°\text{C})(0.02\text{ m})}{(2.22\text{ W/m.°C})} = 0.1081 \longrightarrow \lambda_1 = 0.3208 \text{ and } A_1 = 1.0173$$

$$Bi_{\text{wall},3} = \frac{hL_3}{k} = \frac{(12\text{ W/m}^2.°\text{C})(0.06\text{ m})}{(2.22\text{ W/m.°C})} = 0.3243 \longrightarrow \lambda_1 = 0.5392 \text{ and } A_1 = 1.0482$$

The ice will start melting at the corners because of the maximum exposed surface area there. Noting that $\tau = \alpha t / L^2$ and assuming that $\tau > 0.2$ in all dimensions so that the one-term approximate solution for transient heat conduction is applicable, the product solution method can be written for this problem as

$$\theta(L_1, L_2, L_3, t)_{block} = \theta(L_1, t)_{\text{wall},1}{}^2\,\theta(L_3, t)_{\text{wall},2}$$

$$\frac{0-18}{-18-18} = \left[A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L_1 / L_1)\right]^2\left[A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L_3 / L_3)\right]$$

$$0.500 = \left\{(1.0173)\exp\left[-(0.3208)^2\frac{(0.124\times10^{-7})t}{(0.02)^2}\right]\cos(0.3208)\right\}^2$$

$$\times\left\{(1.0482)\exp\left[-(0.5392)^2\frac{(0.124\times10^{-7})t}{(0.06)^2}\right]\cos(0.5392)\right\}$$

$$\longrightarrow t = 70{,}020\text{ s} = 1167\text{ min} = \mathbf{19.5\ hours}$$

Therefore, the ice will start melting in about 20 hours.

Discussion Note that

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.124\times10^{-7}\text{ m}^2\text{/s})(70{,}020\text{s})}{(0.06\text{ m})^2} = 0.241 > 0.2$$

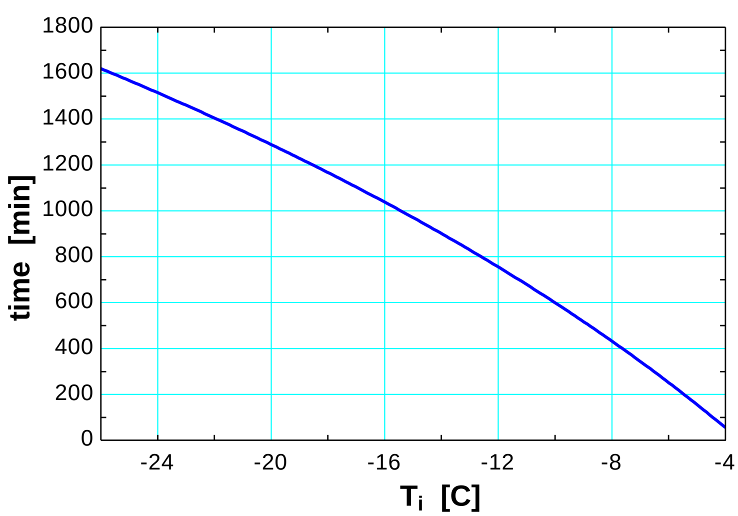
and thus the assumption of $\tau > 0.2$ for the applicability of the one-term approximate solution is verified.

4-116 Prob. 4-115 is reconsidered. The effect of the initial temperature of the ice block on the time period before the ice block starts melting is to be investigated.

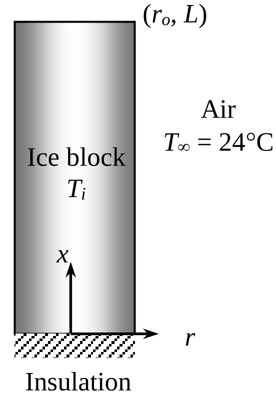
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
2*L_1=0.04 [m]
L_2=L_1
2*L_3=0.12 [m]
T_i=-18 [C]
T_infinity=18 [C]
h=12 [W/m^2-C]
T_L1_L2_L3=0 [C]

"PROPERTIES"
k=2.22 [W/m-C]
alpha=0.124E-7 [m^2/s]

"ANALYSIS"
"This block can physically be formed by the intersection of two infinite plane wall of thickness 2L=4 cm and an infinite plane wall of thickness 2L=10 cm"
"For the two plane walls"
Bi_w1=(h*L_1)/k
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1_w1=0.3208 "w stands for wall"
A_1_w1=1.0173
time*Convert(min, s)=tau_w1*L_1^2/alpha
"For the third plane wall"
Bi_w3=(h*L_3)/k
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1_w3=0.5392
A_1_w3=1.0482
time*Convert(min, s)=tau_w3*L_3^2/alpha
theta_L_w1=A_1_w1*exp(-lambda_1_w1^2*tau_w1)*Cos(lambda_1_w1*L_1/L_1) "theta_L_w1=(T_L_w1-T_infinity)/(T_i-T_infinity)"
theta_L_w3=A_1_w3*exp(-lambda_1_w3^2*tau_w3)*Cos(lambda_1_w3*L_3/L_3) "theta_L_w3=(T_L_w3-T_infinity)/(T_i-T_infinity)"
(T_L1_L2_L3-T_infinity)/(T_i-T_infinity)=theta_L_w1^2*theta_L_w3 "corner temperature"
```

T_i [C]	time [min]
-26	1620
-24	1515
-22	1405
-20	1289
-18	1167
-16	1038
-14	900.8
-12	755.1
-10	599.4
-8	432
-6	251.3
-4	54.89



4-117 A cylindrical ice block is placed on a table. The initial temperature of the ice block to avoid melting for 2 h is to be determined.



Assumptions **1** Heat conduction in the ice block is two-dimensional, and thus the temperature varies in both *x*- and *r*- directions. **2** Heat transfer from the base of the ice block to the table is negligible. **3** The thermal properties of the ice block are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The thermal properties of the ice are given to be $k = 2.22$ W/m.°C and $\alpha = 0.124\times10^{-7}$ m²/s.

Analysis This cylindrical ice block can be treated as a short cylinder that can physically be formed by the intersection of a long cylinder of diameter $D = 2$ cm and an infinite plane wall of thickness $2L = 4$ cm. We measure *x* from the bottom surface of the block since this surface represents the adiabatic center surface of the plane wall of thickness $2L = 4$ cm. The melting starts at the outer surfaces of the top surface when the temperature drops below 0°C at this location. The Biot numbers, the corresponding constants, and the Fourier numbers are

$$Bi_{\text{wall}} = \frac{hL}{k} = \frac{(13\text{ W/m}^2.°\text{C})(0.02\text{ m})}{(2.22\text{ W/m.°C})} = 0.1171 \longrightarrow \lambda_1 = 0.3319 \quad \text{and} \quad A_1 = 1.0187$$

$$Bi_{\text{cyl}} = \frac{hr_o}{k} = \frac{(13\text{ W/m}^2.°\text{C})(0.01\text{ m})}{(2.22\text{ W/m.°C})} = 0.05856 \longrightarrow \lambda_1 = 0.3393 \quad \text{and} \quad A_1 = 1.0144$$

$$\tau_{\text{wall}} = \frac{\alpha t}{L^2} = \frac{(0.124\times10^{-7}\text{ m}^2/\text{s})(3\text{h}\times3600\text{ s/h})}{(0.02\text{ m})^2} = 0.3348 > 0.2$$

$$\tau_{\text{cyl}} = \frac{\alpha t}{r_o^2} = \frac{(0.124\times10^{-7}\text{ m}^2/\text{s})(3\text{h}\times3600\text{ s/h})}{(0.01\text{ m})^2} = 1.3392 > 0.2$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable. The product solution for this problem can be written as

$$\theta(L, r_o, t)_{\text{block}} = \theta(L,t)_{\text{wall}}\,\theta(r_o,t)_{\text{cyl}}$$

$$\frac{0-24}{T_i-24} = \left[A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L)\right]\left[A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r_o/r_o)\right]$$

$$\frac{0-24}{T_i-24} = \left[(1.0187)e^{-(0.3319)^2(0.3348)}\cos(0.3319)\right]\left[(1.0146)e^{-(0.3393)^2(1.3392)}(0.9708)\right]$$

which gives

$$T_i = \mathbf{-6.6°C}$$

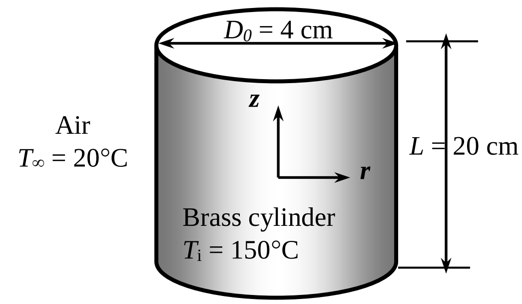
Therefore, the ice will not start melting for at least 3 hours if its initial temperature is -6.6°C or below.

4-118 A short cylinder is allowed to cool in atmospheric air. The temperatures at the centers of the cylinder and the top surface as well as the total heat transfer from the cylinder for 15 min of cooling are to be determined.

Assumptions **1** Heat conduction in the short cylinder is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. **2** The thermal properties of the cylinder are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The thermal properties of brass are given to be $\rho = 8530\ \text{kg/m}^3$, $c_p = 0.389\ \text{kJ/kg}\cdot°\text{C}$, $k = 110\ \text{W/m}\cdot°\text{C}$, and $\alpha = 3.39\times10^{-5}\ \text{m}^2/\text{s}$.

Analysis This short cylinder can physically be formed by the intersection of a long cylinder of radius $D/2$ = 2 cm and a plane wall of thickness $2L$ = 20 cm. We measure *x* from the midplane.



(*a*) The Biot number is calculated for the plane wall to be

$$Bi = \frac{hL}{k} = \frac{(40\ \text{W/m}^2.°\text{C})(0.10\ \text{m})}{(110\ \text{W/m}.°\text{C})} = 0.03636$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 0.1882 \quad \text{and} \quad A_1 = 1.0060$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.39\times10^{-5}\ \text{m}^2/\text{s})(15\,\text{min}\times 60\ \text{s/min})}{(0.10\ \text{m})^2} = 3.051 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0060)e^{-(0.1882)^2(3.051)} = 0.9030$$

We repeat the same calculations for the long cylinder,

$$Bi = \frac{hr_o}{k} = \frac{(40\ \text{W/m}^2.°\text{C})(0.02\ \text{m})}{(110\ \text{W/m}.°\text{C})} = 0.00727$$

Approximating Biot number as 0.01 for use in Table 4-2,

$$\lambda_1 = 0.1412 \quad \text{and} \quad A_1 = 1.0025$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(3.39\times10^{-5}\ \text{m}^2/\text{s})(15\times 60\ \text{s})}{(0.02\ \text{m})^2} = 76.275 > 0.2$$

$$\theta_{o,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0025)e^{-(0.1412)^2(76.275)} = 0.2191$$

Then the center temperature of the short cylinder becomes

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{short\\cylinder}} = \theta_{o,wall}\times\theta_{o,cyl} = 0.9030\times0.2191 = 0.1978$$

$$\frac{T(0,0,t) - 20}{150 - 20} = 0.1978 \longrightarrow T(0,0,t) = \mathbf{45.7°C}$$

(*b*) The center of the top surface of the cylinder is still at the center of the long cylinder (r = 0), but at the outer surface of the plane wall ($x = L$). Therefore, we first need to determine the dimensionless temperature at the surface of the wall.

$$\theta(L,t)_{wall} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L) = (1.0060)e^{-(0.1882)^2(3.051)}\cos(0.1882) = 0.8871$$

Then the center temperature of the top surface of the cylinder becomes

$$\left[\frac{T(L,0,t)-T_\infty}{T_i-T_\infty}\right]_{\substack{short\\cylinder}} = \theta(L,t)_{wall}\times\theta_{o,cyl} = 0.8871\times 0.2191 = 0.1944$$

$$\frac{T(L,0,t)-20}{150-20} = 0.1944 \longrightarrow T(L,0,t) = \mathbf{45.3°C}$$

(c) We first need to determine the maximum heat can be transferred from the cylinder

$$m = \rho \boldsymbol{V} = \rho\pi r_o^2 L = (8530\text{ kg/m}^3)\left[\pi(0.02\text{ m})^2(0.20\text{ m})\right] = 2.144\text{ kg}$$

$$Q_{\max} = mc_p(T_i - T_\infty) = (2.144\text{ kg})(0.389\text{ kJ/kg.°C})(150-20)\text{°C} = 108.4\text{ kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left(\frac{Q}{Q_{\max}}\right)_{wall} = 1-\theta_{o,wall}\frac{\sin(\lambda_1)}{\lambda_1} = 1-(0.9030)\frac{\sin(0.1882)}{0.1882} = 0.1023$$

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1-2\theta_{o,cyl}\frac{J_1(\lambda_1)}{\lambda_1} = 1-2(0.2191)\frac{0.07034}{0.1412} = 0.7817$$

since $J_1(0.1677) = 0.08348$ from Table 4-3. The heat transfer ratio for the short cylinder is

$$\left(\frac{Q}{Q_{\max}}\right)_{\substack{short\\cylinder}} = \left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}} + \left(\frac{Q}{Q_{\max}}\right)_{\substack{long\\cylinder}}\left[1-\left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}}\right] = 0.1023+(0.7817)(1-0.1023) = 0.8040$$

Then the total heat transfer from the short cylinder during the first 15 minutes of cooling becomes

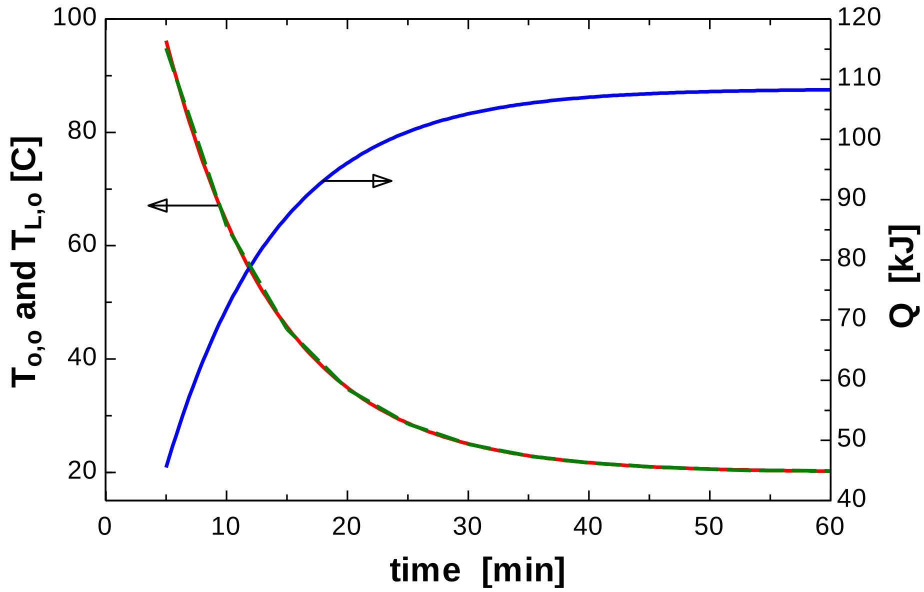
$$Q = 0.8040Q_{\max} = (0.8040)(108.4\text{ kJ}) = \mathbf{87.2\,kJ}$$

4-119 Prob. 4-118 is reconsidered. The effect of the cooling time on the center temperature of the cylinder, the center temperature of the top surface of the cylinder, and the total heat transfer is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.04 [m]
r_o=D/2
height=0.20 [m]
L=height/2
T_i=150 [C]
T_infinity=20 [C]
h=40 [W/m^2-C]
time=15 [min]

"PROPERTIES"
k=110 [W/m-C]
rho=8530 [kg/m^3]
c_p=0.389 [kJ/kg-C]
alpha=3.39E-5 [m^2/s]

"ANALYSIS"
"(a)"
"This short cylinder can physically be formed by the intersection of a long cylinder of radius r_o and a plane wall
of thickness 2L"
"For plane wall"
Bi_w=(h*L)/k
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1_w=0.1882 "w stands for wall"
A_1_w=1.0060
tau_w=(alpha*time*Convert(min, s))/L^2
theta_o_w=A_1_w*exp(-lambda_1_w^2*tau_w) "theta_o_w=(T_o_w-T_infinity)/(T_i-T_infinity)"
"For long cylinder"
Bi_c=(h*r_o)/k "c stands for cylinder"
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1_c=0.1412
A_1_c=1.0025
tau_c=(alpha*time*Convert(min, s))/r_o^2
theta_o_c=A_1_c*exp(-lambda_1_c^2*tau_c) "theta_o_c=(T_o_c-T_infinity)/(T_i-T_infinity)"
(T_o_o-T_infinity)/(T_i-T_infinity)=theta_o_w*theta_o_c "center temperature of short cylinder"
"(b)"
theta_L_w=A_1_w*exp(-lambda_1_w^2*tau_w)*Cos(lambda_1_w*L/L) "theta_L_w=(T_L_w-T_infinity)/(T_i-
T_infinity)"
(T_L_o-T_infinity)/(T_i-T_infinity)=theta_L_w*theta_o_c "center temperature of the top surface"
"(c)"
V=pi*r_o^2*(2*L)
m=rho*V
Q_max=m*c_p*(T_i-T_infinity)
Q_w=1-theta_o_w*Sin(lambda_1_w)/lambda_1_w "Q_w=(Q/Q_max)_w"
Q_c=1-2*theta_o_c*J_1/lambda_1_c "Q_c=(Q/Q_max)_c"
J_1=0.07034 "From Table 4-3, at lambda_1_c"
Q/Q_max=Q_w+Q_c*(1-Q_w) "total heat transfer"
```

time [min]	$T_{o,o}$ [C]	$T_{L,o}$ [C]	Q [kJ]
5	96.18	94.83	45.49
10	64.26	63.48	71.85
15	45.72	45.26	87.17
20	34.94	34.68	96.07
25	28.68	28.53	101.2
30	25.05	24.96	104.2
35	22.93	22.88	106
40	21.7	21.67	107
45	20.99	20.97	107.6
50	20.58	20.56	107.9
55	20.33	20.33	108.1
60	20.19	20.19	108.3



4-120 A semi-infinite aluminum cylinder is cooled by water. The temperature at the center of the cylinder 5 cm from the end surface is to be determined.

Assumptions **1** Heat conduction in the semi-infinite cylinder is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. **2** The thermal properties of the cylinder are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The thermal properties of aluminum are given to be $k = 237$ W/m.°C and $\alpha = 9.71\times10^{-5}$ m²/s.

Analysis This semi-infinite cylinder can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = 7.5$ cm and a semi-infinite medium. The dimensionless temperature 5 cm from the surface of a semi-infinite medium is first determined from

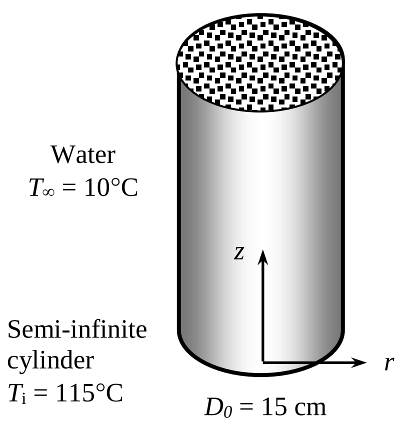
$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$

$$= erfc\left(\frac{0.05}{2\sqrt{(9.71\times10^{-5})(8\times60)}}\right) - \exp\left(\frac{(140)(0.05)}{237}+\frac{(140)^2(9.71\times10^{-5})(8\times60)}{(237)^2}\right)$$

$$\times\left[erfc\left(\frac{0.05}{2\sqrt{(9.71\times10^{-5})(8\times60)}}+\frac{(140)\sqrt{(9.71\times10^{-5})(8\times60)}}{237}\right)\right]$$

$$= erfc(0.1158) - \exp(0.0458)erfc(0.2433) = 0.8699 - (1.0468)(0.7308) = 0.1049$$

$$\theta_{semi-\inf} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = 1 - 0.1049 = 0.8951$$



The Biot number is calculated for the long cylinder to be

$$Bi = \frac{hr_o}{k} = \frac{(140\text{ W/m}^2.°\text{C})(0.075\text{ m})}{237\text{ W/m.°C}} = 0.0443$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$\lambda_1 = 0.2948$ and $A_1 = 1.0110$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(9.71\times10^{-5}\text{ m}^2/\text{s})(8\times60\text{ s})}{(0.075\text{ m})^2} = 8.286 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{o,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0110)e^{-(0.2948)^2(8.286)} = 0.4921$$

The center temperature of the semi-infinite cylinder then becomes

$$\left[\frac{T(x,0,t)-T_\infty}{T_i - T_\infty}\right]_{\substack{\text{semi-infinite}\\ \text{cylinder}}} = \theta_{\text{semi-inf}}(x,t)\times\theta_{o,cyl} = 0.8951\times0.4921 = 0.4405$$

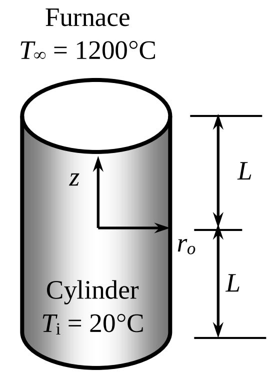
$$\left[\frac{T(x,0,t)-10}{115-10}\right]_{\substack{\text{semi-infinite}\\ \text{cylinder}}} = 0.4405 \longrightarrow T(x,0,t) = \mathbf{56.3°C}$$

4-121 A cylindrical aluminum block is heated in a furnace. The length of time the block should be kept in the furnace and the amount of heat transfer to the block are to be determined.

Assumptions **1** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial *x*- and radial *r*- directions. **2** The thermal properties of the aluminum are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (it will be verified).

Properties The thermal properties of the aluminum block are given to be k = 236 W/m.°C, ρ = 2702 kg/m^3, c_p = 0.896 kJ/kg.°C, and $\alpha = 9.75\times10^{-5}$ m^2/s.

Analysis This cylindrical aluminum block can physically be formed by the intersection of an infinite plane wall of thickness 2L = 30 cm, and a long cylinder of radius $r_o = D/2$ = 7.5 cm. The Biot numbers and the corresponding constants are first determined to be



$$Bi = \frac{hL}{k} = \frac{(80\text{ W/m}^2.°\text{C})(0.15\text{ m})}{(236\text{ W/m.°C})} = 0.0508 \longrightarrow \lambda_1 = 0.2224 \quad\text{and}\quad A_1 = 1.0083$$

$$Bi = \frac{hr_0}{k} = \frac{(80\text{ W/m}^2.°\text{C})(0.075\text{ m})}{236\text{ W/m.°C}} = 0.0254 \longrightarrow \lambda_1 = 0.2217 \quad\text{and}\quad A_1 = 1.0063$$

Noting that $\tau = \alpha t / L^2$ and assuming $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable, the product solution for this problem can be written as

$$\theta(0,0,t)_{\text{block}} = \theta(0,t)_{\text{wall}}\theta(0,t)_{\text{cyl}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}}\left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{cyl}}$$

$$\frac{300-1200}{20-1200} = \left\{(1.0083)\exp\left[-(0.2224)^2\frac{(9.75\times10^{-5})t}{(0.15)^2}\right]\right\}\times\left\{(1.0063)\exp\left[-(0.2217)^2\frac{(9.75\times10^{-5})t}{(0.075)^2}\right]\right\}$$
$$= 0.7627$$

Solving for the time t gives

t = 268 s = **4.46 min**

We note that

$$\tau_{\text{wall}} = \frac{\alpha t}{L^2} = \frac{(9.75\times10^{-5}\text{ m}^2/\text{s})(268\text{s})}{(0.15\text{ m})^2} = 1.16 > 0.2$$

$$\tau_{\text{cyl}} = \frac{\alpha t}{r_o^2} = \frac{(9.75\times10^{-5}\text{ m}^2/\text{s})(268\text{s})}{(0.075\text{ m})^2} = 4.64 > 0.2$$

and thus the assumption of $\tau > 0.2$ for the applicability of the one-term approximate solution is verified. The dimensionless temperatures at the center are

$$\theta(0,t)_{\text{wall}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}} = (1.0083)\exp\left[-(0.2224)^2(1.16)\right] = 0.9521$$

$$\theta(0,t)_{\text{cyl}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{cyl}} = (1.0063)\exp\left[-(0.2217)^2(4.64)\right] = 0.8011$$

The maximum amount of heat transfer is

$$m = \rho \boldsymbol{V} = \rho\pi r_o^2(2L) = (2702\text{ kg/m}^3)\left[\pi(0.075\text{ m})^2(0.3\text{ m})\right] = 14.32\text{ kg}$$
$$Q_{\max} = mc_p(T_i - T_\infty) = (14.32\text{ kg})(0.896\text{ kJ/kg.°C})(20-1200)°\text{C} = 15{,}140\text{ kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left(\frac{Q}{Q_{\max}}\right)_{wall} = 1-\theta_{o,wall}\frac{\sin(\lambda_1)}{\lambda_1} = 1-(0.9521)\frac{\sin(0.2224)}{0.2224} = 0.05575$$

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1-2\theta_{o,cyl}\frac{J_1(\lambda_1)}{\lambda_1} = 1-2(0.8011)\frac{0.1101}{0.2217} = 0.2043$$

The heat transfer ratio for the short cylinder is

$$\left(\frac{Q}{Q_{\max}}\right)_{\substack{short\\cylinder}} = \left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}} + \left(\frac{Q}{Q_{\max}}\right)_{\substack{long\\cylinder}} \left[1-\left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}}\right]$$

$$= 0.05575 + (0.2043)(1-0.05575) = 0.2487$$

Then the total heat transfer from the short cylinder as it is cooled from 300°C at the center to 20°C becomes

$$Q = 0.2487 Q_{\max} = (0.2487)(15{,}140 \text{ kJ}) = \mathbf{3765 \text{ kJ}}$$

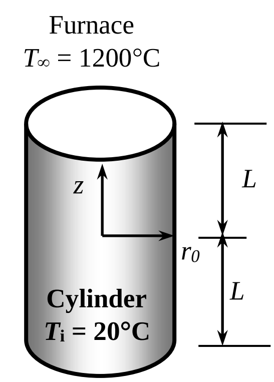
which is identical to the heat transfer to the cylinder as the cylinder at 20°C is heated to 300°C at the center.

4-122 A cylindrical aluminum block is heated in a furnace. The length of time the block should be kept in the furnace and the amount of heat transferred to the block are to be determined.

Assumptions **1** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial *x*- and radial *r*- directions. **2** Heat transfer from the bottom surface of the block is negligible. **3** The thermal properties of the aluminum are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the aluminum block are given to be k = 236 W/m.°C, ρ = 2702 kg/m^3, c_p = 0.896 kJ/kg.°C, and $\alpha = 9.75\times10^{-5}$ m^2/s.

Analysis This cylindrical aluminum block can physically be formed by the intersection of an infinite plane wall of thickness $2L$ = 60 cm and a long cylinder of radius $r_o = D/2$ = 7.5 cm. Note that the height of the short cylinder represents the half thickness of the infinite plane wall where the bottom surface of the short cylinder is adiabatic. The Biot numbers and corresponding constants are first determined to be



$$Bi = \frac{hL}{k} = \frac{(80\ \text{W/m}^2.°\text{C})(0.3\ \text{m})}{(236\ \text{W/m.°C})} = 0.102 \rightarrow \lambda_1 = 0.3135 \quad \text{and} \quad A_1 = 1.0164$$

$$Bi = \frac{hr_o}{k} = \frac{(80\ \text{W/m}^2.°\text{C})(0.075\ \text{m})}{(236\ \text{W/m.°C})} = 0.0254 \rightarrow \lambda_1 = 0.2217 \text{ and } A_1 = 1.0063$$

Noting that $\tau = \alpha t / L^2$ and assuming $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable, the product solution for this problem can be written as

$$\theta(0,0,t)_{\text{block}} = \theta(0,t)_{\text{wall}}\,\theta(0,t)_{\text{cyl}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}}\left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{cyl}}$$

$$\frac{300-1200}{20-1200} = \left\{(1.0164)\exp\left[-(0.3135)^2\frac{(9.75\times10^{-5})t}{(0.3)^2}\right]\right\}\left\{(1.0063)\exp\left[-(0.2217)^2\frac{(9.75\times10^{-5})t}{(0.075)^2}\right]\right\}$$
$$= 0.7627$$

Solving for the time t gives

t = 306 s = **5.1 min**

We note that

$$\tau_{\text{wall}} = \frac{\alpha t}{L^2} = \frac{(9.75\times10^{-5}\ \text{m}^2/\text{s})(306\text{s})}{(0.3\ \text{m})^2} = 0.3317 > 0.2$$

$$\tau_{\text{cyl}} = \frac{\alpha t}{r_o^2} = \frac{(9.75\times10^{-5}\ \text{m}^2/\text{s})(306\text{s})}{(0.075\ \text{m})^2} = 5.307 > 0.2$$

and thus the assumption of $\tau > 0.2$ for the applicability of the one-term approximate solution is verified. The dimensionless temperatures at the center are

$$\theta(0,t)_{\text{wall}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}} = (1.0164)\exp\left[-(0.3135)^2(0.3317)\right] = 0.9838$$

$$\theta(0,t)_{\text{cyl}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{cyl}} = (1.0063)\exp\left[-(0.2217)^2(5.307)\right] = 0.7753$$

The maximum amount of heat transfer is

$$m = \rho V = \rho\pi r_o^2(2L) = (2702\ \text{kg/m}^3)\left[\pi(0.075\ \text{m})^2(0.3\ \text{m})\right] = 14.32\ \text{kg}$$
$$Q_{\max} = mc_p(T_i - T_\infty) = (14.32\ \text{kg})(0.896\ \text{kJ/kg.°C})(20-1200)°\text{C} = 15{,}140\ \text{kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left(\frac{Q}{Q_{\max}}\right)_{wall} = 1 - \theta_{o,wall}\frac{\sin(\lambda_1)}{\lambda_1} = 1-(0.9838)\frac{\sin(0.3135)}{0.3135} = 0.03223$$

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\theta_{o,cyl}\,\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.7753)\frac{0.1101}{0.2217} = 0.2300$$

The heat transfer ratio for the short cylinder is

$$\left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{short}\\\text{cylinder}}} = \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{plane}\\\text{wall}}} + \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{long}\\\text{cylinder}}}\left[1 - \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{plane}\\\text{wall}}}\right]$$

$$= 0.03223 + (0.2300)(1 - 0.03223) = 0.2548$$

Then the total heat transfer from the short cylinder as it is cooled from 300°C at the center to 20°C becomes

$$Q = 0.2507 Q_{\max} = (0.2548)(15{,}140\ \text{kJ}) = \mathbf{3860\ kJ}$$

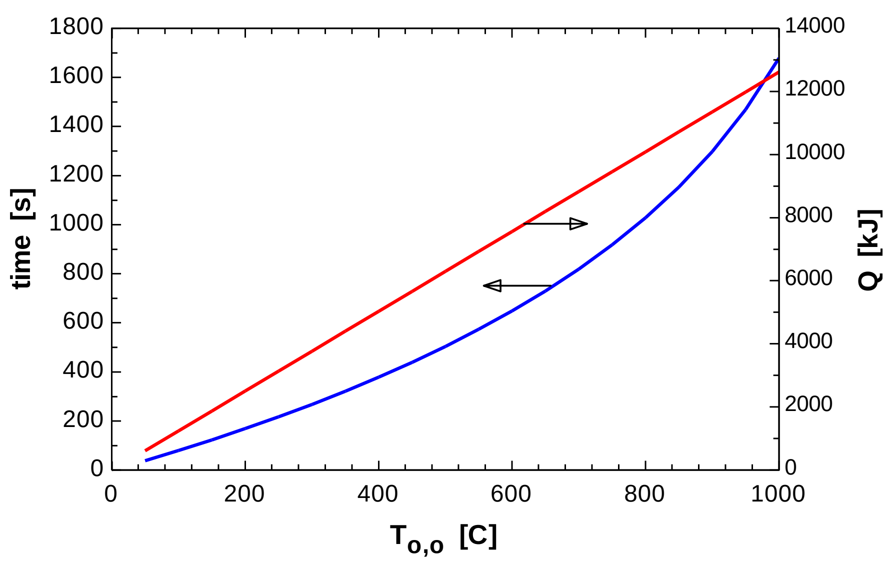
which is identical to the heat transfer to the cylinder as the cylinder at 20°C is heated to 300°C at the center.

4-123 Prob. 4-121 is reconsidered. The effect of the final center temperature of the block on the heating time and the amount of heat transfer is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
2*L=0.30 [m]
2*r_o=0.15 [m]
T_i=20 [C]
T_infinity=1200 [C]
T_o_o=300 [C]
h=80 [W/m^2-C]

"PROPERTIES"
k=236 [W/m-C]
rho=2702 [kg/m^3]
c_p=0.896 [kJ/kg-C]
alpha=9.75E-5 [m^2/s]

"ANALYSIS"
"This short cylinder can physically be formed by the intersection of a long cylinder of radius r_o and a plane wall of thickness 2L"
"For plane wall"
Bi_w=(h*L)/k
"From Table 4-1 corresponding to this Bi number, we read"
lambda_1_w=0.2224 "w stands for wall"
A_1_w=1.0083
tau_w=(alpha*time)/L^2
theta_o_w=A_1_w*exp(-lambda_1_w^2*tau_w) "theta_o_w=(T_o_w-T_infinity)/(T_i-T_infinity)"
"For long cylinder"
Bi_c=(h*r_o)/k "c stands for cylinder"
"From Table 4-2 corresponding to this Bi number, we read"
lambda_1_c=0.2217
A_1_c=1.0063
tau_c=(alpha*time)/r_o^2
theta_o_c=A_1_c*exp(-lambda_1_c^2*tau_c) "theta_o_c=(T_o_c-T_infinity)/(T_i-T_infinity)"
(T_o_o-T_infinity)/(T_i-T_infinity)=theta_o_w*theta_o_c "center temperature of cylinder"
V=pi*r_o^2*(2*L)
m=rho*V
Q_max=m*c_p*(T_infinity-T_i)
Q_w=1-theta_o_w*Sin(lambda_1_w)/lambda_1_w "Q_w=(Q/Q_max)_w"
Q_c=1-2*theta_o_c*J_1/lambda_1_c "Q_c=(Q/Q_max)_c"
J_1=0.1101 "From Table 4-3, at lambda_1_c"
Q/Q_max=Q_w+Q_c*(1-Q_w) "total heat transfer"
```

$T_{o,o}$ [C]	time [s]	Q [kJ]
50	37.79	605.5
100	79.48	1238
150	123.1	1870
200	168.9	2502
250	217	3134
300	267.7	3766
350	321.3	4398
400	378.1	5031
450	438.7	5663
500	503.4	6295
550	572.9	6927
600	647.9	7559
650	729.5	8191
700	818.9	8823
750	917.7	9456
800	1028	10088
850	1153	10720
900	1298	11352
950	1469	11984
1000	1678	12616



Special Topic: Refrigeration and Freezing of Foods

4-124C The common kinds of microorganisms are bacteria, yeasts, molds, and viruses. The undesirable changes caused by microorganisms are off-flavors and colors, slime production, changes in the texture and appearances, and the spoilage of foods.

4-125C Microorganisms are the prime cause for the spoilage of foods. Refrigeration prevents or delays the spoilage of foods by reducing the rate of growth of microorganisms. Freezing extends the storage life of foods for months by preventing the growths of microorganisms.

4-126C The environmental factors that affect of the growth rate of microorganisms are the temperature, the relative humidity, the oxygen level of the environment, and air motion.

4-127C Cooking kills the microorganisms in foods, and thus prevents spoilage of foods. It is important to raise the internal temperature of a roast in an oven above 70ºC since most microorganisms, including some that cause diseases, may survive temperatures below 70ºC.

4-128C The contamination of foods with microorganisms can be prevented or minimized by (1) preventing contamination by following strict sanitation practices such as washing hands and using fine filters in ventilation systems, (2) inhibiting growth by altering the environmental conditions, and (3) destroying the organisms by heat treatment or chemicals.

The growth of microorganisms in foods can be retarded by keeping the temperature below 4ºC and relative humidity below 60 percent. Microorganisms can be destroyed by heat treatment, chemicals, ultraviolet light, and solar radiation.

4-129C (a) High air motion retards the growth of microorganisms in foods by keeping the food surfaces dry, and creating an undesirable environment for the microorganisms. (b) Low relative humidity (dry) environments also retard the growth of microorganisms by depriving them of water that they need to grow. Moist air supplies the microorganisms with the water they need, and thus encourages their growth. Relative humidities below 60 percent prevent the growth rate of most microorganisms on food surfaces.

4-130C Cooling the carcass with refrigerated air is at -10ºC would certainly reduce the cooling time, but this proposal should be rejected since it will cause the outer parts of the carcasses to freeze, which is undesirable. Also, the refrigeration unit will consume more power to reduce the temperature to -10ºC, and thus it will have a lower efficiency.

4-131C The freezing time could be decreased by (a) lowering the temperature of the refrigerated air, (b) increasing the velocity of air, (c) increasing the capacity of the refrigeration system, and (d) decreasing the size of the meat boxes.

4-132C The rate of freezing can affect color, tenderness, and drip. Rapid freezing increases tenderness and reduces the tissue damage and the amount of drip after thawing.

4-133C This claim is reasonable since the lower the storage temperature, the longer the storage life of beef. This is because some water remains unfrozen even at subfreezing temperatures, and the lower the temperature, the smaller the unfrozen water content of the beef.

4-134C A refrigerated shipping dock is a refrigerated space where the orders are assembled and shipped out. Such docks save valuable storage space from being used for shipping purpose, and provide a more acceptable working environment for the employees. The refrigerated shipping docks are usually maintained at 1.5ºC, and therefore the air that flows into the freezer during shipping is already cooled to about 1.5ºC. This reduces the refrigeration load of the cold storage rooms.

4-135C (*a*) The heat transfer coefficient during immersion cooling is much higher, and thus the cooling time during immersion chilling is much lower than that during forced air chilling. (*b*) The cool air chilling can cause a moisture loss of 1 to 2 percent while water immersion chilling can actually cause moisture absorption of 4 to 15 percent. (*c*) The chilled water circulated during immersion cooling encourages microbial growth, and thus immersion chilling is associated with more microbial growth. The problem can be minimized by adding chloride to the water.

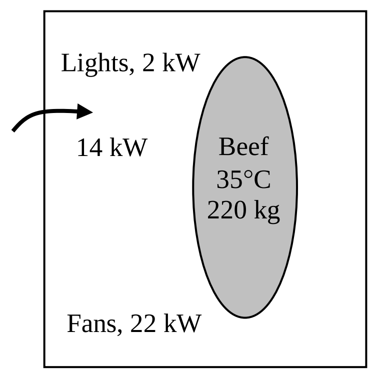
4-136C The proper storage temperature of frozen poultry is about -18ºC or below. The primary freezing methods of poultry are the air blast tunnel freezing, cold plates, immersion freezing, and cryogenic cooling.

4-137C The factors, which affect the quality of frozen, fish are the condition of the fish before freezing, the freezing method, and the temperature and humidity during storage and transportation, and the length of storage time.

4-138 The chilling room of a meat plant with a capacity of 350 beef carcasses is considered. The cooling load and the air flow rate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Specific heats of beef carcass and air are constant.

Properties The density and specific heat of air at 0°C are given to be 1.28 kg/m^3 and 1.0 kJ/kg·°C. The specific heat of beef carcass is given to be 3.14 kJ/kg·°C.



Analysis (*a*) The amount of beef mass that needs to be cooled per unit time is

$$\begin{aligned}\dot{m}_{beef} &= \text{(Total beef mass cooled)/(cooling time)}\\ &= (350\times 220\text{ kg/carcass})/(12\text{ h}\times 3600\text{ s}) = 1.782\text{ kg/s}\end{aligned}$$

The product refrigeration load can be viewed as the energy that needs to be removed from the beef carcass as it is cooled from 35 to 16ºC at a rate of 2.27 kg/s, and is determined to be

$$\begin{aligned}\dot{Q}_{beef} &= (\dot{m}c_p\Delta T)_{beef}\\ &= (1.782\text{ kg/s})(3.14\text{ kJ/kg.°C})(35-16)^\circ\text{C} = 106\text{ kW}\end{aligned}$$

Then the total refrigeration load of the chilling room becomes

$$\dot{Q}_{\text{total,chillingroom}} = \dot{Q}_{beef} + \dot{Q}_{fan} + \dot{Q}_{lights} + \dot{Q}_{heat\ gain} = 106+22+2+14 = \mathbf{144\ kW}$$

(*b*) Heat is transferred to air at the rate determined above, and the temperature of air rises from -2.2ºC to 0.5ºC as a result. Therefore, the mass flow rate of air is

$$\dot{m}_{air} = \frac{\dot{Q}_{air}}{(c_p\Delta T)_{air}} = \frac{144\text{ kW}}{(1.0\text{ kJ/kg.°C})[0.5-(-2.2)\text{°C}]} = 53.3\text{ kg/s}$$

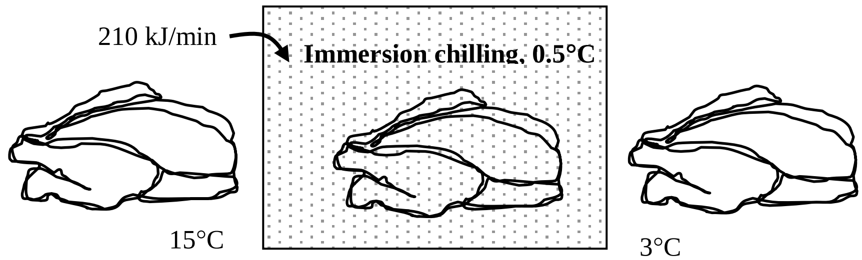
Then the volume flow rate of air becomes

$$\dot{V}_{air} = \frac{\dot{m}_{air}}{\rho_{air}} = \frac{53.3\text{ kg/s}}{1.28\text{ kg/m}^3} = \mathbf{41.7\ m^3/s}$$

4-139 Chickens are to be cooled by chilled water in an immersion chiller. The rate of heat removal from the chicken and the mass flow rate of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal properties of chickens are constant.

Properties The specific heat of chicken are given to be 3.54 kJ/kg.°C. The specific heat of water is 4.18 kJ/kg.°C (Table A-9).



Analysis (*a*) Chickens are dropped into the chiller at a rate of 500 per hour. Therefore, chickens can be considered to flow steadily through the chiller at a mass flow rate of

$$\dot{m}_{\text{chicken}} = (500\,\text{chicken/h})(2.2\,\text{kg/chicken}) = 1100\,\text{kg/h} = 0.3056\text{kg/s}$$

Then the rate of heat removal from the chickens as they are cooled from 15°C to 3ºC at this rate becomes

$$\dot{Q}_{\text{chicken}} = (\dot{m}c_p\Delta T)_{\text{chicken}} = (0.3056\text{ kg/s})(3.54\,\text{kJ/kg.°C})(15-3)\text{° C} = \mathbf{13.0\,kW}$$

(*b*) The chiller gains heat from the surroundings as a rate of 210 kJ/min = 3.5 kJ/s. Then the total rate of heat gain by the water is

$$\dot{Q}_{\text{water}} = \dot{Q}_{\text{chicken}} + \dot{Q}_{\text{heat gain}} = 13.0 + 3.5 = 16.5\text{ kW}$$

Noting that the temperature rise of water is not to exceed 2ºC as it flows through the chiller, the mass flow rate of water must be at least

$$\dot{m}_{\text{water}} = \frac{\dot{Q}_{\text{water}}}{(c_p\Delta T)_{\text{water}}} = \frac{16.5\text{kW}}{(4.18\,\text{kJ/kg.°C})(2\text{° C})} = \mathbf{1.97\,kg/s}$$

If the mass flow rate of water is less than this value, then the temperature rise of water will have to be more than 2°C.

4-140E Chickens are to be frozen by refrigerated air. The cooling time of the chicken is to be determined for the cases of cooling air being at –40°F and -80°F.

Assumptions **1** Steady operating conditions exist. **2** The thermal properties of chickens are constant.

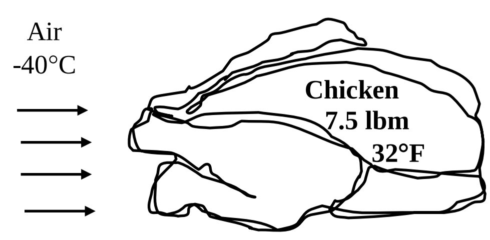
Analysis The time required to reduce the inner surface temperature of the chickens from 32ºF to 25ºF with refrigerated air at -40ºF is determined from Fig. 4-51 to be

$t \cong$ **2.3 hours**

If the air temperature were -80ºF, the freezing time would be

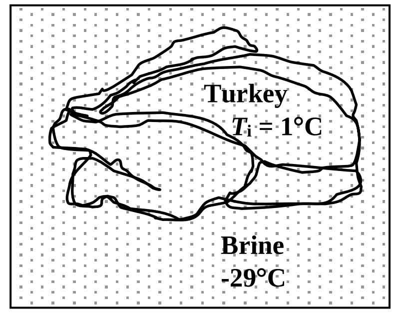
$t \cong$ **1.4 hours**

Therefore, the time required to cool the chickens to 25°F is reduced considerably when the refrigerated air temperature is decreased.



4-141 Turkeys are to be frozen by submerging them into brine at -29°C. The time it will take to reduce the temperature of turkey breast at a depth of 3.8 cm to -18°C and the amount of heat transfer per turkey are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal properties of turkeys are constant.



Properties It is given that the specific heats of turkey are 2.98 and 1.65 kJ/kg.°C above and below the freezing point of -2.8°C, respectively, and the latent heat of fusion of turkey is 214 kJ/kg.

Analysis The time required to freeze the turkeys from 1°C to -18ºC with brine at -29ºC can be determined directly from Fig. 4-52 to be

$t \cong 180$ min. $\cong$ **3 hours**

(*a*) Assuming the entire water content of turkey is frozen, the amount of heat that needs to be removed from the turkey as it is cooled from 1°C to -18°C is

Cooling to -2.8ºC:

$$Q_{\text{cooling,fresh}} = (mc_p\Delta T)_{\text{fresh}} = (7\text{ kg})(2.98\text{ kJ/kg}\cdot{}^\circ\text{C})[1-(-2.8)^\circ\text{C}] = 79.3\text{ kJ}$$

Freezing at -2.8ºC:

$$Q_{\text{freezing}} = mh_{\text{latent}} = (7\text{ kg})(214\text{ kJ/kg}) = 1498\text{ kJ}$$

Cooling -18ºC:

$$Q_{\text{cooling,frozen}} = (mc_{\text{p}}\Delta T)_{\text{frozen}} = (7\text{ kg})(1.65\text{ kJ/kg.}^\circ\text{C})[-2.8-(-18)]^\circ\text{C} = 175.6\text{ kJ}$$

Therefore, the total amount of heat removal per turkey is

$$Q_{\text{total}} = Q_{\text{cooling,fresh}} + Q_{\text{freezing}} + Q_{\text{cooling,frozen}} = 79.3 + 1498 + 175.6 \cong \mathbf{1753\text{ kJ}}$$

(*b*) Assuming only 90 percent of the water content of turkey is frozen, the amount of heat that needs to be removed from the turkey as it is cooled from 1°C to -18°C is

Cooling to -2.8ºC:

$$Q_{\text{cooling,fresh}} = (mc_{\text{p}}\Delta T)_{\text{fresh}} = (7\text{ kg})(2.98\text{ kJ/kg}\cdot{}^\circ\text{C})[1-(-2.98)^\circ\text{C}] = 79.3\text{ kJ}$$

Freezing at -2.8ºC:

$$Q_{\text{freezing}} = mh_{\text{latent}} = (7\times 0.9\text{ kg})(214\text{ kJ/kg}) = 1348\text{ kJ}$$

Cooling -18ºC:

$$Q_{\text{cooling,frozen}} = (mc_p\Delta T)_{\text{frozen}} = (7\times 0.9\text{ kg})(1.65\text{kJ/kg.}^\circ\text{C})[-2.8-(-18)]^\circ\text{C} = 158\text{ kJ}$$

$$Q_{\text{cooling,unfrozen}} = (mc_p\Delta T)_{\text{fresh}} = (7\times 0.1\text{ kg})(2.98\text{ kJ/kg.}^\circ\text{C})[-2.8-(-18)^\circ\text{C}] = 31.7\text{ kJ}$$

Therefore, the total amount of heat removal per turkey is

$$Q_{\text{total}} = Q_{\text{cooling,fresh}} + Q_{\text{freezing}} + Q_{\text{cooling,frozen \&unfrozen}} = 79.3 + 1348 + 158 + 31.7 = \mathbf{1617\text{ kJ}}$$

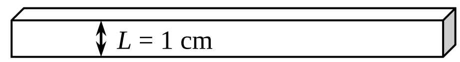
Review Problems

4-142 Large steel plates are quenched in an oil reservoir. The quench time is to be determined.

Assumptions **1** The thermal properties of the plates are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The properties of steel plates are given to be k = 45 W/m·K, ρ = 7800 kg/m^3, and c_p = 470 J/kg·K.

Analysis For sphere, the characteristic length and the Biot number are



$$L_c = \frac{V}{A_{\text{surface}}} = \frac{L}{2} = \frac{0.01\,\text{m}}{2} = 0.005\,\text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(400\ \text{W/m}^2.°\text{C})(0.005\,\text{m})}{45\ \text{W/m.°C}} = 0.044 < 0.1$$

Since $Bi < 0.1$, the lumped system analysis is applicable. Then the cooling time is determined from

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{400\ \text{W/m}^2.°\text{C}}{(7800\,\text{kg/m}^3)(470\ \text{J/kg.°C})(0.005\,\text{m})} = 0.02182\,\text{s}^{-1}$$

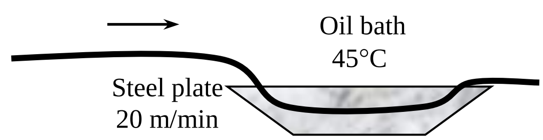
$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{100-30}{600-30} = e^{-(0.02182\,\text{s}^{-1})t} \longrightarrow t = 96\,\text{s} = \mathbf{1.6\,min}$$

4-143 A long roll of large 1-Mn manganese steel plate is to be quenched in an oil bath at a specified rate. The temperature of the sheet metal after quenching and the rate at which heat needs to be removed from the oil in order to keep its temperature constant are to be determined.

Assumptions **1** The thermal properties of the steel plate are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface. **3** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be checked).

Properties The properties of the steel plate are k = 60.5 W/m.°C, ρ = 7854 kg/m^3, and c_p = 434 J/kg.°C (Table A-3).

Analysis The characteristic length of the steel plate and the Biot number are



$$L_c = \frac{V}{A_s} = L = 0.0025\,\text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(860\ \text{W/m}^2.°\text{C})(0.0025\,\text{m})}{60.5\ \text{W/m.°C}} = 0.036 < 0.1$$

Since $\text{Bi} < 0.1$, the lumped system analysis is applicable. Therefore,

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{860\ \text{W/m}^2.°\text{C}}{(7854\,\text{kg/m}^3)(434\ \text{J/kg.°C})(0.0025\,\text{m})} = 0.10092\,\text{s}^{-1}$$

$$\text{time} = \frac{\text{length}}{\text{velocity}} = \frac{9\,\text{m}}{20\,\text{m/min}} = 0.45\,\text{min} = 27\,\text{s}$$

Then the temperature of the sheet metal when it leaves the oil bath is determined to be

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T(t) - 45}{820 - 45} = e^{-(0.10092\,\text{s}^{-1})(27\,\text{s})} \longrightarrow T(t) = \mathbf{95.8°C}$$

The mass flow rate of the sheet metal through the oil bath is

$$\dot{m} = \rho \dot{V} = \rho wtV = (7854\,\text{kg/m}^3)(2\,\text{m})(0.005\,\text{m})(20\,\text{m/min}) = 1571\,\text{kg/min}$$

Then the rate of heat transfer from the sheet metal to the oil bath and thus the rate at which heat needs to be removed from the oil in order to keep its temperature constant at 45°C becomes

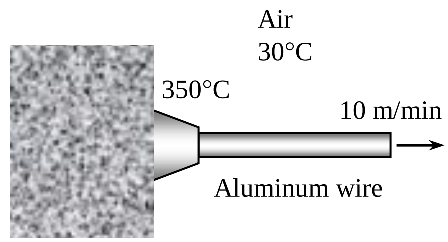
$$\dot{Q} = \dot{m}c_p[T_i - T(t)] = (1571\,\text{kg/min})(0.434\,\text{kJ/kg.°C})(820 - 95.8)°\text{C} = 493{,}770\,\text{kJ/min} = \mathbf{8230\,kW}$$

4-144 Long aluminum wires are extruded and exposed to atmospheric air. The time it will take for the wire to cool, the distance the wire travels, and the rate of heat transfer from the wire are to be determined.

Assumptions **1** Heat conduction in the wires is one-dimensional in the radial direction. **2** The thermal properties of the aluminum are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of aluminum are given to be k = 236 W/m.°C, ρ = 2702 kg/m^3, c_p = 0.896 kJ/kg.°C, and α = 9.75×10^{-5} m^2/s.

Analysis (*a*) The characteristic length of the wire and the Biot number are



$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L} = \frac{r_o}{2} = \frac{0.0015\text{ m}}{2} = 0.00075\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(35\text{ W/m}^2.°\text{C})(0.00075\text{ m})}{236\text{ W/m.°C}} = 0.00011 < 0.1$$

Since $\text{Bi} < 0.1$, the lumped system analysis is applicable. Then,

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{35\text{ W/m}^2.°\text{C}}{(2702\text{ kg/m}^3)(896\text{ J/kg.°C})(0.00075\text{ m})} = 0.0193\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{50-30}{350-30} = e^{-(0.0193\text{ s}^{-1})t} \longrightarrow t = \mathbf{144\text{ s}}$$

(*b*) The wire travels a distance of

$$\text{velocity} = \frac{\text{length}}{\text{time}} \to \text{length} = (10/60\text{ m/s})(144\text{s}) = \mathbf{24\text{ m}}$$

This distance can be reduced by cooling the wire in a water or oil bath.

(*c*) The mass flow rate of the extruded wire through the air is

$$\dot{m} = \rho \dot{V} = \rho(\pi r_o^2)V = (2702\text{ kg/m}^3)\pi(0.0015\text{ m})^2(10\text{ m/min}) = 0.191\text{ kg/min}$$

Then the rate of heat transfer from the wire to the air becomes

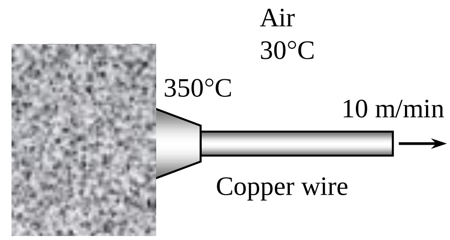
$$\dot{Q} = \dot{m}c_p[T(t)-T_\infty] = (0.191\text{ kg/min})(0.896\text{ kJ/kg.°C})(350-50)\text{°C} = 51.3\text{ kJ/min} = \mathbf{856\text{ W}}$$

4-145 Long copper wires are extruded and exposed to atmospheric air. The time it will take for the wire to cool, the distance the wire travels, and the rate of heat transfer from the wire are to be determined.

Assumptions **1** Heat conduction in the wires is one-dimensional in the radial direction. **2** The thermal properties of the copper are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of copper are given to be k = 386 W/m.°C, ρ = 8950 kg/m^3, c_p = 0.383 kJ/kg.°C, and $\alpha = 1.13\times10^{-4}$ m^2/s.

Analysis (a) The characteristic length of the wire and the Biot number are



$$L_c = \frac{V}{A_s} = \frac{\pi r_o^{\,2} L}{2\pi r_o L} = \frac{r_o}{2} = \frac{0.0015\text{ m}}{2} = 0.00075\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(35\text{ W/m}^2.\text{°C})(0.00075\text{ m})}{386\text{ W/m.°C}} = 0.000068 < 0.1$$

Since $\text{Bi} < 0.1$ the lumped system analysis is applicable. Then,

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{35\text{ W/m}^2.\text{°C}}{(8950\text{ kg/m}^3)(383\text{ J/kg.°C})(0.00075\text{ m})} = 0.0136\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{50-30}{350-30} = e^{-(0.0136\text{ s}^{-1})t} \longrightarrow t = \mathbf{204\text{ s}}$$

(b) The wire travels a distance of

$$\text{velocity} = \frac{\text{length}}{\text{time}} \longrightarrow \text{length} = \left(\frac{10\text{ m/min}}{60\text{ s/min}}\right)(204\text{ s}) = \mathbf{34\text{ m}}$$

This distance can be reduced by cooling the wire in a water or oil bath.

(c) The mass flow rate of the extruded wire through the air is

$$\dot{m} = \rho\dot{V} = \rho(\pi r_o^2)V = (8950\text{ kg/m}^3)\pi(0.0015\text{ m})^2(10\text{ m/min}) = 0.633\text{ kg/min}$$

Then the rate of heat transfer from the wire to the air becomes

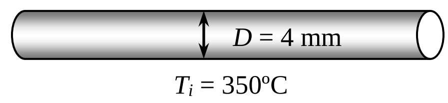
$$\dot{Q} = \dot{m}c_p[T(t)-T_\infty] = (0.633\text{ kg/min})(0.383\text{ kJ/kg.°C})(350-50)\text{°C} = 72.7\text{ kJ/min} = \mathbf{1212\text{ W}}$$

4-146 Aluminum wires leaving the extruder at a specified rate are cooled in air. The necessary length of the wire is to be determined.

Assumptions **1** The thermal properties of the geometry are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The properties of aluminum are k = 237 W/m·°C, ρ = 2702 kg/m^3, and c_p = 0.903 kJ/kg·°C (Table A-3).

Analysis For a long cylinder, the characteristic length and the Biot number are



$$L_c = \frac{V}{A_{\text{surface}}} = \frac{(\pi D^2/4)L}{\pi DL} = \frac{D}{4} = \frac{0.004\text{ m}}{4} = 0.001\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(50\text{ W/m}^2.°\text{C})(0.001\text{ m})}{237\text{ W/m.°C}} = 0.000211 < 0.1$$

Since $Bi < 0.1$, the lumped system analysis is applicable. Then the cooling time is determined from

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{50\text{ W/m}^2.°\text{C}}{(2702\text{ kg/m}^3)(903\text{ J/kg.°C})(0.001\text{m})} = 0.02049\text{ s}^{-1}$$

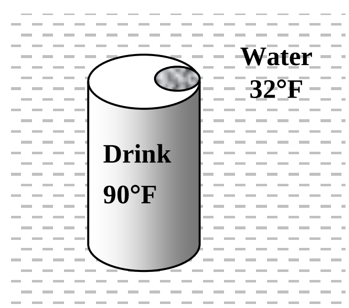
$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{50-25}{350-25} = e^{-(0.02049\text{ s}^{-1})t} \longrightarrow t = 125.2\text{ s}$$

Then the necessary length of the wire in the cooling section is determined to be

$$\text{Length} = \frac{t}{V} = \frac{(125.2/60)\text{ min}}{10\text{ m/min}} = \mathbf{0.209\,m}$$

4-147E A person shakes a can of drink in an iced water to cool it. The cooling time of the drink is to be determined.

Assumptions **1** The can containing the drink is cylindrical in shape with a radius of r_o = 1.25 in. **2** The thermal properties of the drink are taken to be the same as those of water. **3** Thermal properties of the drink are constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the drink is stirred constantly, so that its temperature remains uniform at all times.



Properties The density and specific heat of water at room temperature (90°F) are ρ = 62.12 lbm/ft^3, c_p = 0.999 Btu/lbm.°F, k = 0.358 Btu/h.ft.°F (Table A-9E).

Analysis The characteristic length and Biot number for the can of drink are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(1.25/12\text{ ft})^2(5/12\text{ ft})}{2\pi(1.25/12\text{ ft})(5/12\text{ ft}) + 2\pi(1.25/12\text{ ft})^2} = 0.04167\text{ ft}$$

$$Bi = \frac{hL_c}{k} = \frac{(30\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(0.04167\text{ ft})}{0.358\text{ Btu/h}\cdot\text{ft}\cdot°\text{F}} = 3.49 > 0.1$$

For the reason explained above we can use the lumped system analysis to determine how long it will take for the canned drink to cool to 40°F

$$b = \frac{hA}{\rho C_p V} = \frac{h}{\rho C_p L_c} = \frac{30\text{ Btu/h.ft}^2.°\text{F}}{(62.22\text{ lbm/ft}^3)(0.999\text{ Btu/lbm.°F})(0.04167\text{ ft})} = 11.599\text{ h}^{-1} = 0.00322\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{40-32}{90-32} = e^{-(0.00322\text{ s}^{-1})t} \longrightarrow t = \mathbf{615\text{ s}}$$

Therefore, it will take 10 minutes and 15 seconds to cool the canned drink to 40°F.

4-148 The average temperatures of aluminum and stainless steel rods, after 5 minutes elapsed time, are to be determined.

Assumptions **1** Thermal properties are constant. **2** Convection heat transfer coefficient is uniform. **3** Heat transfer by radiation is negligible.

Properties The properties of the aluminum rod are given as ρ = 2702 kg/m^3, c_p = 903 J/kg · K, and k = 237 W/m · K; the properties of the stainless steel rod are given as ρ = 8238 kg/m^3, c_p = 468 J/kg · K, and k = 13.4 W/m · K.

Analysis The characteristic length of both rod A and rod B is

$$L_c = \frac{V}{A_s} = \frac{(\pi D^2/4)L}{\pi DL} = \frac{D}{4} = \frac{0.025\,\text{m}}{4} = 0.00625\,\text{m}$$

For rod A (aluminum), the Biot number is

$$Bi_{\text{rod A}} = \frac{hL_c}{k_{\text{rod A}}} = \frac{(20\ \text{W/m}^2\cdot\text{K})(0.00625\,\text{m})}{237\ \text{W/m}\cdot\text{K}} = 5.274\times10^{-4} < 0.1$$

Since, $Bi_{\text{rod A}} < 0.1$, the lumped system analysis is applicable. Then the average temperature of rod A after 5 minutes elapsed time is

$$b_{\text{rod A}} = \frac{h}{\rho_{\text{rod A}}c_{p,\text{rod A}}L_c} = \frac{20\ \text{W/m}^2\cdot\text{K}}{(2702\,\text{kg/m}^3)(903\,\text{J/kg}\cdot\text{K})(0.00625\,\text{m})} = 0.001311\,\text{s}^{-1}$$

$$T(t) = (T_i - T_\infty)e^{-bt} + T_\infty$$

$$T(5\,\text{min}) = (15\,°\text{C} - 1000\,°\text{C})e^{-(0.001311)(300)} + 1000\,°\text{C} = \mathbf{335\,°C}\ \text{(rod A)}$$

For rod B (stainless steel), the Biot number is

$$Bi_{\text{rod B}} = \frac{hL_c}{k_{\text{rod B}}} = \frac{(20\ \text{W/m}^2\cdot\text{K})(0.00625\,\text{m})}{13.4\ \text{W/m}\cdot\text{K}} = 0.009328 < 0.1$$

Since $Bi_{\text{rod B}} < 0.1$, the lumped system analysis is applicable. Then the average temperature of rod B after 5 minutes elapsed time is

$$b_{\text{rod B}} = \frac{h}{\rho_{\text{rod B}}c_{p,\text{rod B}}L_c} == \frac{20\ \text{W/m}^2\cdot\text{K}}{(8238\,\text{kg/m}^3)(468\,\text{J/kg}\cdot\text{K})(0.00625\,\text{m})} = 8.3\times10^{-4}\ \text{s}^{-1}$$

$$T(t) = (T_i - T_\infty)e^{-bt} + T_\infty$$

$$T(5\,\text{min}) = (15\,°\text{C} - 1000\,°\text{C})e^{-(8.3\times10^{-4})(300)} + 1000\,°\text{C} = \mathbf{232\,°C}\ \text{(rod B)}$$

Discussion The results indicate that it is quicker to heat the aluminum rod to a desired temperature than the stainless steel rod, because $b_{\text{rod A}} > b_{\text{rod B}}$.

4-149 Ball bearings leaving the oven at a uniform temperature of 900°C are exposed to air for a while before they are dropped into the water for quenching. The time they can stand in the air before their temperature falls below 850°C is to be determined.

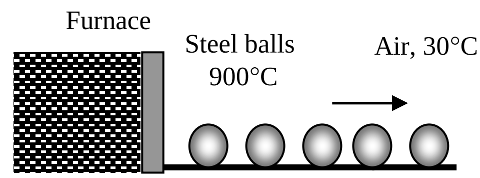
Assumptions **1** The bearings are spherical in shape with a radius of r_o = 0.6 cm. **2** The thermal properties of the bearings are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the bearings are given to be k = 15.1 W/m.°C, ρ = 8085 kg/m^3, and c_p = 0.480 kJ/kg.°C.

Analysis The characteristic length of the steel ball bearings and Biot number are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.012\text{ m}}{6} = 0.002\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(125\text{ W/m}^2.°\text{C})(0.002\text{ m})}{(15.1\text{ W/m.°C})} = 0.0166 < 0.1$$



Therefore, the lumped system analysis is applicable. Then the allowable time is determined to be

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{125\text{ W/m}^2.°\text{C}}{(8085\text{ kg/m}^3)(480\text{ J/kg.°C})(0.002\text{ m})} = 0.01610\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{850-30}{900-30} = e^{-(0.0161\text{s}^{-1})t} \longrightarrow t = \mathbf{3.68\ s}$$

The result indicates that the ball bearing can stay in the air about 4 s before being dropped into the water.

4-150 The time that a stainless steel plate should be heated in the furnace to at least 600°C is to be determined using analytical one-term approximation method.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible. **5** The Fourier number is τ> 0.2 so that the one term-term approximate solutions are applicable.

Properties The properties of stainless steel are given as ρ = 8238 kg/m^3, c_p = 468 J/kg · K, k = 13.4 W/m · K, and α = 3.48 × 10^{-6} m^2/s.

Analysis The Biot number for this process is

$$Bi = \frac{hL}{k} = \frac{(215\text{ W/m}^2\cdot\text{K})(0.025\text{ m})}{13.4\text{ W/m}\cdot\text{K}} = 0.4$$

The coefficients λ_1 and A_1 for a plane wall corresponding to this Bi are determined from Table 4-2 to be

$$\lambda_1 = 0.5932 \quad \text{and} \quad A_1 = 1.0580$$

For plane wall with the temperature at the center plane being 600°C, we have

$$\theta_{0,\text{ wall}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \quad \rightarrow \quad \frac{600-1000}{230-1000} = 1.0580 e^{-(0.5932)^2\tau} \quad \rightarrow \quad \tau = 2.021$$

Hence, the time that the plate should be heated in the furnace is

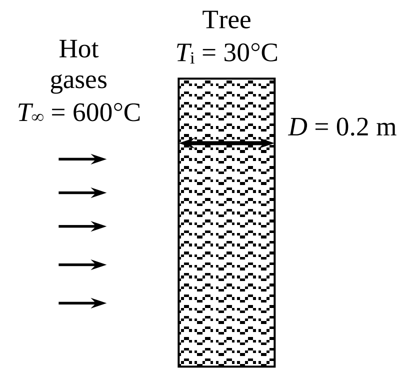
$$\tau = \frac{\alpha t}{L^2} = 2.021 > 0.2 \quad \rightarrow \quad t = \frac{2.021L^2}{\alpha} = \frac{2.021(0.025\text{ m})^2}{3.48\times10^{-6}\text{ m}^2/\text{s}} = \mathbf{363\ s}$$

4-151 The trunks of some dry oak trees are exposed to hot gases. The time for the ignition of the trunks is to be determined.

Assumptions **1** Heat conduction in the trunks is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the trunks are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of the trunks are given to be k = 0.17 W/m.°C and $\alpha = 1.28\times10^{-7}$ m²/s.

Analysis We treat the trunks of the trees as an infinite cylinder since heat transfer is primarily in the radial direction. Then the Biot number becomes



$$Bi = \frac{hr_o}{k} = \frac{(65\ \text{W/m}^2.°\text{C})(0.1\,\text{m})}{(0.17\ \text{W/m.°C})} = 38.24$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 4-2,

$$\lambda_1 = 2.3420 \quad \text{and} \quad A_1 = 1.5989$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.28\times10^{-7}\ \text{m}^2/\text{s})(4\,\text{h}\times3600\,\text{s/h})}{(0.1\,\text{m})^2} = 0.184$$

which is greater than 0.2. Therefore, assuming the one-term approximate solution for transient heat conduction to be applicable, the temperature at the surface of the trees in 5 h becomes

$$\theta(r_o,t)_{cyl} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r/r_o)$$

$$\frac{T(r_o,t)-600}{30-600} = (1.5989)e^{-(2.3420)^2(0.184)}(0.0332) = 0.01935 \longrightarrow T(r_o,t) = \mathbf{589\ °C} \ > \ 410°\text{C}$$

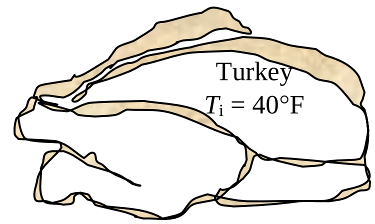
Therefore, the trees will ignite. (Note: J_0 is read from Table 4-3).

4-152E A stuffed turkey is cooked in an oven. The average heat transfer coefficient at the surface of the turkey, the temperature of the skin of the turkey in the oven and the total amount of heat transferred to the turkey in the oven are to be determined.

Assumptions **1** The turkey is a homogeneous spherical object. **2** Heat conduction in the turkey is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the turkey are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of the turkey are given to be $k = 0.26$ Btu/h.ft.°F, $\rho = 75$ lbm/ft^3, $c_p = 0.98$ Btu/lbm.°F, and $\alpha = 0.0035$ ft^2/h.

Analysis (*a*) Assuming the turkey to be spherical in shape, its radius is determined to be



$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{14\text{ lbm}}{75\text{ lbm/ft}^3} = 0.1867\text{ ft}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.1867\text{ ft}^3)}{4\pi}} = 0.3545\text{ ft}$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(3.5\times10^{-3}\text{ ft}^2/\text{h})(5\text{ h})}{(0.3545\text{ ft})^2} = 0.1392$$

which is close to 0.2 but a little below it. Therefore, assuming the one-term approximate solution for transient heat conduction to be applicable, the one-term solution formulation at one-third the radius from the center of the turkey can be expressed as

$$\theta(x,t)_{sph} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\,\frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}$$

$$\frac{185-325}{40-325} = 0.491 = A_1 e^{-\lambda_1^2(0.14)}\,\frac{\sin(0.333\lambda_1)}{0.333\lambda_1}$$

By trial and error, it is determined from Table 4-2 that the equation above is satisfied when $Bi = 20$ corresponding to $\lambda_1 = 2.9857$ and $A_1 = 1.9781$. Then the heat transfer coefficient can be determined from

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.26\text{ Btu/h.ft.°F})(20)}{(0.3545\text{ ft})} = \mathbf{14.7\text{ Btu/h.ft}^2.°F}$$

(*b*) The temperature at the surface of the turkey is

$$\frac{T(r_o,t) - 325}{40-325} = A_1 e^{-\lambda_1^2\tau}\,\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.9781)e^{-(2.9857)^2(0.14)}\,\frac{\sin(2.9857)}{2.9857} = 0.02953$$

$$\longrightarrow T(r_o,t) = \mathbf{317\text{ °F}}$$

(*c*) The maximum possible heat transfer is

$$Q_{\max} = mc_p(T_\infty - T_i) = (14\text{ lbm})(0.98\text{ Btu/lbm.°F})(325-40)\text{°F} = 3910\text{ Btu}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\,\frac{\sin(\lambda_1) - \lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.491)\,\frac{\sin(2.9857) - (2.9857)\cos(2.9857)}{(2.9857)^3} = 0.828$$

$$Q = 0.828Q_{\max} = (0.828)(3910\text{ Btu}) = \mathbf{3240\text{ Btu}}$$

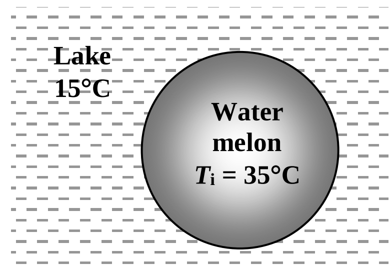
Discussion The temperature of the outer parts of the turkey will be greater than that of the inner parts when the turkey is taken out of the oven. Then heat will continue to be transferred from the outer parts of the turkey to the inner as a result of temperature difference. Therefore, after 5 minutes, the thermometer reading will probably be more than 185°F.

4-153 A watermelon is placed into a lake to cool it. The heat transfer coefficient at the surface of the watermelon and the temperature of the outer surface of the watermelon are to be determined.

Assumptions **1** The watermelon is a homogeneous spherical object. **2** Heat conduction in the watermelon is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the watermelon are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of the watermelon are given to be $k = 0.618$ W/m.°C, $\alpha = 0.15\times10^{-6}$ m^2/s, $\rho = 995$ kg/m^3 and $c_p = 4.18$ kJ/kg.°C.

Analysis The Fourier number is



$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.15\times10^{-6}\ \text{m}^2/\text{s})\left[(4\times60+40\ \text{min})\times60\ \text{s/min}\right]}{(0.10\ \text{m})^2} = 0.252$$

which is greater than 0.2. Then the one-term solution can be written in the form

$$\theta_{0,\text{sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{20-15}{35-15} = 0.25 = A_1 e^{-\lambda_1^2(0.252)}$$

It is determined from Table 4-2 by trial and error that this equation is satisfied when $Bi = 10$, which corresponds to $\lambda_1 = 2.8363$ and $A_1 = 1.9249$. Then the heat transfer coefficient can be determined from

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.618\ \text{W/m.°C})(10)}{(0.10\ \text{m})} = \mathbf{61.8\ W/m^2.°C}$$

The temperature at the surface of the watermelon is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.9249)e^{-(2.8363)^2(0.252)}\frac{\sin(2.8363\ \text{rad})}{2.8363}$$

$$\frac{T(r_o,t)-15}{35-15} = 0.0269 \longrightarrow T(r_o,t) = \mathbf{15.5\ °C}$$

4-154 The temperature at the center of a spherical glass bead after 3 minutes of cooling is to be determined using analytical one-term approximation method.

Assumptions **1** Heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible.

Properties The properties of glass are given to be $\rho = 2800$ kg/m^3, $c_p = 750$ J/kg · K, and $k = 0.7$ W/m · K.

Analysis The Biot number for this process is

$$Bi = \frac{hr_o}{k} = \frac{(28\ \text{W/m}^2\cdot\text{K})(0.005\ \text{m})}{0.7\ \text{W/m}\cdot\text{K}} = 0.2$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{kt}{\rho c_p r_o^2} = \frac{(0.7\ \text{W/m}\cdot\text{K})(3\times60\ \text{s})}{(2800\ \text{kg/m}^3)(750\ \text{J/kg}\cdot\text{K})(0.005\ \text{m})^2} = 2.4$$

From Table 4-2 with $Bi = 0.2$, the corresponding constants λ_1 and A_1 are

$$\lambda_1 = 0.7593 \quad \text{and} \quad A_1 = 1.0592$$

For a sphere, we have

$$\theta_{0,\text{sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}$$

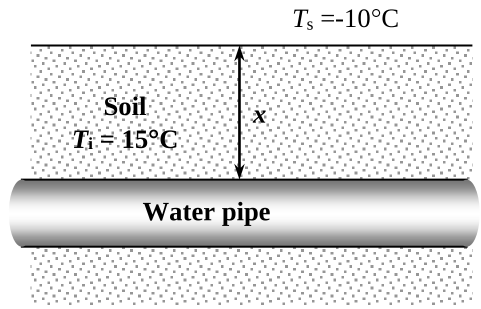
The temperature at the center of the glass bead is

$$T_0 = (T_i - T_\infty)A_1 e^{-\lambda_1^2\tau} + T_\infty = (400\ °\text{C} - 30\ °\text{C})(1.0592)e^{-(0.7593)^2(2.4)} + 30\ °\text{C} = \mathbf{128\ °C}$$

4-155 The water pipes are buried in the ground to prevent freezing. The minimum burial depth at a particular location is to be determined.

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature of -10°C. **2** The thermal properties of the soil are constant.

Properties The thermal properties of the soil are given to be $k = 0.7$ W/m.°C and $\alpha = 1.4\times10^{-5}$ m²/s.



Analysis The depth at which the temperature drops to 0°C in 75 days is determined using the analytical solution,

$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Substituting and using Table 4-4, we obtain

$$\frac{0-15}{-10-15} = erfc\left(\frac{x}{2\sqrt{(1.4\times10^{-5}\ \text{m}^2/\text{s})(75\ \text{day}\times24\ \text{h/day}\times3600\ \text{s/h})}}\right)$$

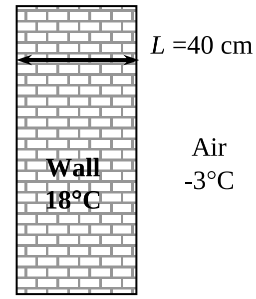
$$\longrightarrow x = \mathbf{7.05\ m}$$

Therefore, the pipes must be buried at a depth of at least 7.05 m.

4-156 A thick wall is exposed to cold outside air. The wall temperatures at distances 15, 30, and 40 cm from the outer surface at the end of 2-hour cooling period are to be determined.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at outer surfaces only. Therefore, the wall can be considered to be a semi-infinite medium **2** The thermal properties of the wall are constant.

Properties The thermal properties of the brick are given to be $k = 0.72$ W/m.°C and $\alpha = 1.6\times10^{-7}$ m²/s.



Analysis For a 15 cm distance from the outer surface, from Fig. 4-27 we have

$$\left.\begin{aligned}\frac{h\sqrt{\alpha t}}{k} &= \frac{(20\ \text{W/m}^2.°\text{C})\sqrt{(1.6\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}}{0.72\ \text{W/m.°C}} = 2.98 \\ \eta = \frac{x}{2\sqrt{\alpha t}} &= \frac{0.15\ \text{m}}{2\sqrt{(1.6\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}} = 0.70\end{aligned}\right\} 1-\frac{T-T_\infty}{T_i-T_\infty} = 0.25$$

$$1-\frac{T-(-3)}{18-(-3)} = 0.25 \longrightarrow T = \mathbf{12.8°C}$$

For a 30 cm distance from the outer surface, from Fig. 4-27 we have

$$\left.\begin{aligned}\frac{h\sqrt{\alpha t}}{k} &= \frac{(20\ \text{W/m}^2.°\text{C})\sqrt{(1.6\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}}{0.72\ \text{W/m.°C}} = 2.98 \\ \eta = \frac{x}{2\sqrt{\alpha t}} &= \frac{0.3\ \text{m}}{2\sqrt{(1.6\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}} = 1.40\end{aligned}\right\} 1-\frac{T-T_\infty}{T_i-T_\infty} = 0.038$$

$$1-\frac{T-(-3)}{18-(-3)} = 0.038 \longrightarrow T = \mathbf{17.2°C}$$

For a 40 cm distance from the outer surface, that is for the inner surface, from Fig. 4-27 we have

$$\left.\begin{aligned}\frac{h\sqrt{\alpha t}}{k} &= \frac{(20\ \text{W/m}^2.°\text{C})\sqrt{(1.6\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}}{0.72\ \text{W/m.°C}} = 2.98 \\ \eta = \frac{x}{2\sqrt{\alpha t}} &= \frac{0.4\ \text{m}}{2\sqrt{(1.6\times10^{-6}\ \text{m}^2/\text{s})(2\times3600\ \text{s})}} = 1.87\end{aligned}\right\} 1-\frac{T-T_\infty}{T_i-T_\infty} = 0$$

$$1-\frac{T-(-3)}{18-(-3)} = 0 \longrightarrow T = \mathbf{18.0°C}$$

Discussion This last result shows that the semi-infinite medium assumption is a valid one.

4-157 The surface temperature and heat flux with lava flow on the ground are to be determined.

Assumptions **1** The ground is treated as semi-infinite solid. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is constant. **4** Heat transfer by radiation is negligible.

Properties The properties of the ground (dry soil) are ρ = 1500 kg/m^3, c_p = 1900 J/kg · K, and k = 1.0 W/m · K (from Table A-8).

Analysis (a) For semi-infinite solid with convection on the surface, the temperature of the ground surface (x = 0) can be determined with

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = \operatorname{erfc}(0) - \exp\left(\frac{h^2\alpha t}{k^2}\right)\operatorname{erfc}\left(\frac{h\sqrt{\alpha t}}{k}\right)$$

where

$$\alpha = \frac{k}{\rho c_p} = \frac{1.0\ \text{W/m}\cdot\text{K}}{(1500\ \text{kg/m}^3)(1900\ \text{J/kg}\cdot\text{K})} = 3.509\times10^{-7}\ \text{m}^2/\text{s}$$

$$\frac{h^2\alpha t}{k^2} = \frac{(3500\ \text{W/m}^2\cdot\text{K})^2(3.509\times10^{-7}\ \text{m}^2/\text{s})(2\ \text{s})}{(1.0\ \text{W/m}\cdot\text{K})^2} = 8.597$$

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(3500\ \text{W/m}^2\cdot\text{K})\sqrt{(3.509\times10^{-7}\ \text{m}^2/\text{s})(2\ \text{s})}}{1.0\ \text{W/m}\cdot\text{K}} = 2.932$$

Hence

$$T(0, 2\ \text{s}) = (1200\ °\text{C} - 15\ °\text{C})\left[1 - \exp(8.597)\operatorname{erfc}(2.932)\right] + 15\ °\text{C}$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
T=(1200-15)*(1-exp(8.597)*erfc(2.932))+15
```

Solving by EES software, the temperature of ground surface after 2 s of lava flowing on it is

$$T(0, 2\ \text{s}) = \mathbf{983\ °C}$$

(b) The heat flux from the lava flow to the ground surface at t = 2 s is

$$\dot{q}_s(2\ \text{s}) = h\left[T_\infty - T(0, 2\ \text{s})\right] = (3500\ \text{W/m}^2\cdot\text{K})(1200-983)\ \text{K} = \mathbf{7.595\times10^5\ W/m^2}$$

Discussion The surface temperature of the ground can also be determined using Figure 4-27:

At $\eta = 0$ and $h\sqrt{\alpha t}/k \approx 2.9$, Figure 4-27 gives

$$\frac{T(x,t)-T_i}{T_\infty - T_i} \approx 0.81 \quad \rightarrow \quad T(0, 2\ \text{s}) = \mathbf{975\ °C}$$

The result determined using Figure 4-27 is about 0.8% lower than the result obtained for part (a).

4-158 The temperature at the edge of a steel block after 10 minutes of cooling is to be determined.

Assumptions **1** Two-dimensional heat conduction in x and y directions. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is constant. **4** Heat transfer by radiation is negligible.

Properties The properties of steel are (ρ = 7832 kg/m^3, c_p = 434 J/kg · K, k = 63.9 W/m · K, and α = 18.8 × 10^{-6} m^2/s.

Analysis For a quarter-infinite medium, at the edge of the steel block ($x = y = 0$), we have

$$\theta(0,0,t) = \theta_{\text{semi-inf}}(0,t)\theta_{\text{semi-inf}}(0,t) = [\theta_{\text{semi-inf}}(0,t)]^2$$

where

$$1-\theta_{\text{semi-inf}}(0,t) = \frac{T(0,t)-T_i}{T_\infty - T_i} = \text{erfc}(0) - \exp\left(\frac{h^2\alpha t}{k^2}\right)\text{erfc}\left(\frac{h\sqrt{\alpha t}}{k}\right)$$

At t = 10 minutes, we have

$$\frac{h^2\alpha t}{k^2} = \frac{(25\ \text{W/m}^2\cdot\text{K})^2(18.8\times10^{-6}\ \text{m}^2/\text{s})(10\times60\ \text{s})}{(63.9\ \text{W/m}\cdot\text{K})^2} = 1.727\times10^{-3}$$

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(25\ \text{W/m}^2\cdot\text{K})\sqrt{(18.8\times10^{-6}\ \text{m}^2/\text{s})(10\times60\ \text{s})}}{63.9\ \text{W/m}\cdot\text{K}} = 0.04155$$

Hence

$$1-\theta_{\text{semi-inf}}(0, 600\ \text{s}) = \text{erfc}(0) - \exp(1.727\times10^{-3})\text{erfc}(0.04155)$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
1-theta=erfc(0)-exp(1.727e-3)*erfc(0.04155)
```

Solving by EES software, we get

$$\theta_{\text{semi-inf}}(0, 600\ \text{s}) = 0.9548$$

The temperature at the edge of the steel block after 10 minutes of cooling is

$$\theta(0,0,600\ \text{s}) = \frac{T(0,0,600\ \text{s})-T_i}{T_\infty - T_i} = [\theta_{\text{semi-inf}}(0,600\ \text{s})]^2 = 0.9548^2$$

$$T(0,0,600\ \text{s}) = (T_\infty - T_i)0.9548^2 + T_i = (450\,°\text{C} - 25\,°\text{C})0.9548^2 + 25\,°\text{C} = \mathbf{412\,°C}$$

Discussion The temperature at the steel block edge can also be determined using Figure 4-27:

At $\eta = 0$ and $h\sqrt{\alpha t}/k \approx 0.04$, Figure 4-27 gives

$$1-\frac{T(x,t)-T_\infty}{T_i - T_\infty} \approx 0.04 \quad \rightarrow \quad \theta_{\text{semi-inf}}(0,600\ \text{s}) = 0.96$$

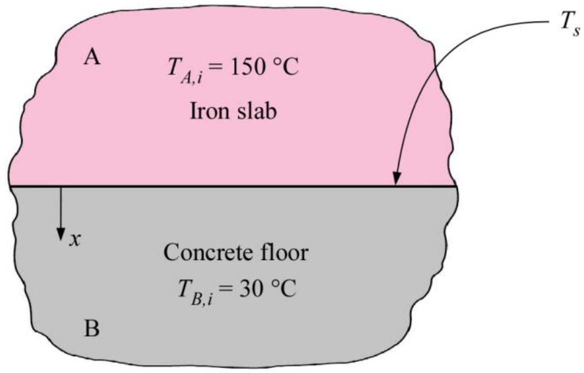
$$T(0,0,600\ \text{s}) = (450\,°\text{C} - 25\,°\text{C})0.96^2 + 25\,°\text{C} = \mathbf{417\,°C}$$

The result determined using Figure 4-27 is about 1.2% higher than the result obtained using the EES software.

4-159 A heated large iron slab is placed on a concrete floor; (a) the surface temperature and (b) the temperature of the concrete floor at the depth of 25 mm are to be determined.

Assumptions **1** The iron slab and concrete floor are treated as semi-infinite solids. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** Contact resistance is negligible.

Properties The properties of iron slab are given to be ρ = 7870 kg/m^3, c_p = 447 J/kg · K, and k = 80.2 W/m · K; the properties of concrete floor are given to be ρ = 1600 kg/m^3, c_p = 840 J/kg · K, and k = 0.79 W/m · K.



Analysis (a) For contact of two semi-infinite solids, the surface temperature is

$$T_s = \frac{\sqrt{(k\rho c_p)_A}T_{A,i} + \sqrt{(k\rho c_p)_B}T_{B,i}}{\sqrt{(k\rho c_p)_A} + \sqrt{(k\rho c_p)_B}} = \frac{(16797)(150\,°\text{C}) + (1030)(30\,°\text{C})}{16797 + 1030} = \mathbf{143\,°C}$$

where

$$\sqrt{(k\rho c_p)_A} = \sqrt{(80.2\text{ W/m}\cdot\text{K})(7870\text{ kg/m}^3)(447\text{ J/kg}\cdot\text{K})} = 16797$$

$$\sqrt{(k\rho c_p)_B} = \sqrt{(0.79\text{ W/m}\cdot\text{K})(1600\text{ kg/m}^3)(840\text{ J/kg}\cdot\text{K})} = 1030$$

(b) For semi-infinite solid with specified surface temperature, we have

$$\frac{T(x,t) - T_{B,i}}{T_s - T_{B,i}} = \text{erfc}\left[\frac{x}{2\sqrt{kt/(\rho c_p)}}\right]$$

At t = 15 minutes and x = 25 mm with T_s = 143°C,

$$\frac{T(x,t) - 30\,°\text{C}}{143\,°\text{C} - 30\,°\text{C}} = \text{erfc}\left(\frac{0.025\text{ m}}{2\sqrt{\dfrac{(0.79\text{ W/m}\cdot\text{K})(15\times 60\text{ s})}{(1600\text{ kg/m}^3)(840\text{ J/kg}\cdot\text{K})}}}\right)$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
(T-30)/(143-30)=erfc(0.025/(2*sqrt(0.79*15*60/(1600*840))))
```

Solving by EES software, the temperature of the concrete floor at x = 25 mm and t = 15 minutes is

$$T(0.025\text{ m}, 900\text{ s}) = \mathbf{80\,°C}$$

Discussion Depending on surface condition of the concrete floor, contact resistance may be significant and cannot be neglected.

4-160 A hot dog is to be cooked by dropping it into boiling water. The time of cooking is to be determined.

Assumptions **1** Heat conduction in the hot dog is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. **2** The thermal properties of the hot dog are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The thermal properties of the hot dog are given to be $k = 0.76$ W/m.°C, $\rho = 980$ kg/m^3, $c_p = 3.9$ kJ/kg.°C, and $\alpha = 2\times10^{-7}$ m^2/s.

Analysis This hot dog can physically be formed by the intersection of an infinite plane wall of thickness $2L = 12$ cm, and a long cylinder of radius $r_o = D/2 = 1$ cm. The Biot numbers and corresponding constants are first determined to be

$$Bi = \frac{hL}{k} = \frac{(600\text{ W/m}^2.°\text{C})(0.06\text{ m})}{(0.76\text{ W/m.°C})} = 47.37 \longrightarrow \lambda_1 = 1.5380 \text{ and } A_1 = 1.2726$$

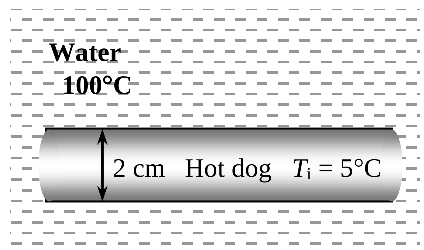
$$Bi = \frac{hr_o}{k} = \frac{(600\text{ W/m}^2.°\text{C})(0.01\text{ m})}{(0.76\text{ W/m.°C})} = 7.895 \longrightarrow \lambda_1 = 2.1249 \text{ and } A_1 = 1.5514$$

Noting that $\tau = \alpha t / L^2$ and assuming $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable, the product solution for this problem can be written as

$$\theta(0,0,t)_{block} = \theta(0,t)_{wall}\,\theta(0,t)_{cyl} = \left(A_1 e^{-\lambda_1^2\tau}\right)\left(A_1 e^{-\lambda_1^2\tau}\right)$$

$$\frac{80-100}{5-100} = \left\{(1.2726)\exp\left[-(1.5380)^2\frac{(2\times10^{-7})t}{(0.06)^2}\right]\right\}$$

$$\times\left\{(1.5514)\exp\left[-(2.1249)^2\frac{(2\times10^{-7})t}{(0.01)^2}\right]\right\} = 0.2105$$



which gives

$$t = \mathbf{244\ s = 4.1\ min}$$

Therefore, it will take about 4.1 min for the hot dog to cook. Note that

$$\tau_{cyl} = \frac{\alpha t}{r_o^2} = \frac{(2\times10^{-7}\text{ m}^2\text{/s})(244\text{s})}{(0.01\text{ m})^2} = 0.49 > 0.2$$

and thus the assumption $\tau > 0.2$ for the applicability of the one-term approximate solution is verified.

Discussion This problem could also be solved by treating the hot dog as an infinite cylinder since heat transfer through the end surfaces will have little effect on the mid section temperature because of the large distance.

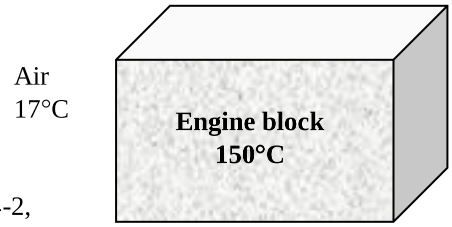
4-161 The engine block of a car is allowed to cool in atmospheric air. The temperatures at the center of the top surface and at the corner after a specified period of cooling are to be determined.

Assumptions **1** Heat conduction in the block is three-dimensional, and thus the temperature varies in all three directions. **2** The thermal properties of the block are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The thermal properties of cast iron are given to be k = 52 W/m.°C and $\alpha = 1.7\times10^{-5}$ m²/s.

Analysis This rectangular block can physically be formed by the intersection of two infinite plane walls of thickness 2L = 40 cm (call planes A and B) and an infinite plane wall of thickness 2L = 80 cm (call plane C). We measure x from the center of the block.

(*a*) The Biot number is calculated for each of the plane wall to be



$$Bi_A = Bi_B = \frac{hL}{k} = \frac{(6\text{ W/m}^2.\text{°C})(0.2\text{ m})}{(52\text{ W/m.°C})} = 0.0231$$

$$Bi_C = \frac{hL}{k} = \frac{(6\text{ W/m}^2.\text{°C})(0.4\text{ m})}{(52\text{ W/m.°C})} = 0.0462$$

The constants λ_1 and A_1 corresponding to these Biot numbers are, from Table 4-2,

$$\lambda_{1(A,B)} = 0.150 \quad\text{and}\quad A_{1(A,B)} = 1.0038$$

$$\lambda_{1(C)} = 0.212 \quad\text{and}\quad A_{1(C)} = 1.0076$$

The Fourier numbers are

$$\tau_{A,B} = \frac{\alpha t}{L^2} = \frac{(1.70\times10^{-5}\text{ m}^2\text{/s})(45\text{ min}\times 60\text{ s/min})}{(0.2\text{ m})^2} = 1.1475 > 0.2$$

$$\tau_C = \frac{\alpha t}{L^2} = \frac{(1.70\times10^{-5}\text{ m}^2\text{/s})(45\text{ min}\times 60\text{ s/min})}{(0.4\text{ m})^2} = 0.2869 > 0.2$$

The center of the top surface of the block (whose sides are 80 cm and 40 cm) is at the center of the plane wall with 2L = 80 cm, at the center of the plane wall with 2L = 40 cm, and at the surface of the plane wall with 2L = 40 cm. The dimensionless temperatures are

$$\theta_{o,\text{wall (A)}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0038)e^{-(0.150)^2(1.1475)} = 0.9782$$

$$\theta(L,t)_{\text{wall (B)}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L) = (1.0038)e^{-(0.150)^2(1.1475)}\cos(0.150) = 0.9672$$

$$\theta_{o,\text{wall (C)}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0076)e^{-(0.212)^2(0.2869)} = 0.9947$$

Then the center temperature of the top surface of the block becomes

$$\left[\frac{T(L,0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\text{block}} = \theta(L,t)_{\text{wall (B)}} \times \theta_{o,\text{wall (A)}} \times \theta_{o,\text{wall (C)}} = 0.9672\times0.9782\times0.9947 = 0.9411$$

$$\frac{T(L,0,0,t) - 17}{150-17} = 0.9411 \longrightarrow T(L,0,0,t) = \mathbf{142.2°C}$$

(*b*) The corner of the block is at the surface of each plane wall. The dimensionless temperature for the surface of the plane walls with 2L = 40 cm is determined in part (*a*). The dimensionless temperature for the surface of the plane wall with 2L = 80 cm is determined from

$$\theta(L,t)_{\text{wall (C)}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L) = (1.0076)e^{-(0.212)^2(0.2869)}\cos(0.212) = 0.9724$$

Then the corner temperature of the block becomes

$$\left[\frac{T(L,L,L,t) - T_\infty}{T_i - T_\infty}\right]_{\text{block}} = \theta(L,t)_{\text{wall,C}} \times \theta(L,t)_{\text{wall,B}} \times \theta(L,t)_{\text{wall,A}} = 0.9724\times0.9672\times0.9672 = 0.9097$$

$$\frac{T(L,L,L,t) - 17}{150-17} = 0.9097 \longrightarrow T(L,L,L,t) = \mathbf{138.0°C}$$

4-162 A man is found dead in a room. The time passed since his death is to be estimated.

Assumptions **1** Heat conduction in the body is two-dimensional, and thus the temperature varies in both radial r- and x-directions. **2** The thermal properties of the body are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The human body is modeled as a cylinder. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

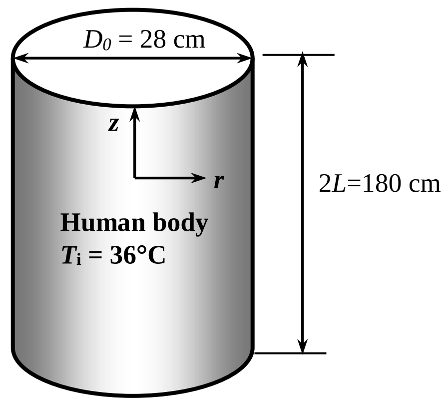
Properties The thermal properties of body are given to be $k = 0.62$ W/m.°C and $\alpha = 0.15\times10^{-6}$ m^2/s.

Analysis A short cylinder can be formed by the intersection of a long cylinder of radius $D/2 = 14$ cm and a plane wall of thickness $2L = 180$ cm. We measure x from the midplane. The temperature of the body is specified at a point that is at the center of the plane wall but at the surface of the cylinder. The Biot numbers and the corresponding constants are first determined to be

$$Bi_{\text{wall}} = \frac{hL}{k} = \frac{(9\ \text{W/m}^2.°\text{C})(0.90\ \text{m})}{(0.62\ \text{W/m.°C})} = 13.06 \longrightarrow \lambda_1 = 1.4495 \quad \text{and} \quad A_1 = 1.2644$$

$$Bi_{\text{cyl}} = \frac{hr_o}{k} = \frac{(9\ \text{W/m}^2.°\text{C})(0.14\ \text{m})}{(0.62\ \text{W/m.°C})} = 2.03 \longrightarrow \lambda_1 = 1.6052 \quad \text{and} \quad A_1 = 1.3408$$

Noting that $\tau = \alpha t / L^2$ for the plane wall and $\tau = \alpha t / r_o^2$ for cylinder and $J_0(1.6052)=0.4524$ from Table 4-3, and assuming that $\tau > 0.2$ in all dimensions so that the one-term approximate solution for transient heat conduction is applicable, the product solution method can be written for this problem as



$$\theta(0, r_0, t)_{block} = \theta(0,t)_{\text{wall}}\,\theta(r_0,t)_{\text{cyl}}$$

$$\frac{23-12}{36-12} = (A_1 e^{-\lambda_1^2\tau})\left[A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r/r_0)\right]$$

$$0.4583 = \left\{(1.2644)\exp\left[-(1.4495)^2\frac{(0.15\times10^{-6})t}{(0.90)^2}\right]\right\}$$

$$\times\left\{(1.3408)\exp\left[-(1.6052)^2\frac{(0.15\times10^{-6})t}{(0.14)^2}\right](0.4524)\right\}$$

$$\longrightarrow t = 25{,}600\ \text{s} = \mathbf{7.11\ hours}$$

Fundamentals of Engineering (FE) Exam Problems

4-163 The Biot number can be thought of as the ratio of

(a) the conduction thermal resistance to the convective thermal resistance
(b) the convective thermal resistance to the conduction thermal resistance
(c) the thermal energy storage capacity to the conduction thermal resistance
(d) the thermal energy storage capacity to the convection thermal resistance
(e) None of the above

Answer (a) the conduction thermal resistance to the convective thermal resistance

4-164 Lumped system analysis of transient heat conduction situations is valid when the Biot number is

(a) very small (b) approximately one (c) very large
(d) any real number (e) cannot say unless the Fourier number is also known.

Answer (a) very small

4-165 Polyvinylchloride automotive body panels (k = 0.092 W/m·K, c_p = 1.05 kJ/kg·K, ρ = 1714 kg/m^3), 1-mm thick, emerge from an injection molder at 120°C. They need to be cooled to 40°C by exposing both sides of the panels to 20°C air before they can be handled. If the convective heat transfer coefficient is 15 W/m^2·K and radiation is not considered, the time that the panels must be exposed to air before they can be handled is

(a) 0.8 min (b) 1.6 min (c) 2.4 min (d) 3.1 min (e) 5.6 min

Answer (b) 1.6 min

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=40 [C]
Ti=120 [C]
Ta=20 [C]
r=1714 [kg/m^3]
k=0.092 [W/m-K]
c=1050 [J/kg-K]
h=15 [W/m^2-K]
L=0.001 [m]
Lc=L/2
b=h/(r*c*Lc)
(T-Ta)/(Ti-Ta)=exp(-b*time)
```

4-166 A steel casting cools to 90 percent of the original temperature difference in 30 min in still air. The time it takes to cool this same casting to 90 percent of the original temperature difference in a moving air stream whose convective heat transfer coefficient is 5 times that of still air is

(a) 3 min (b) 6 min (c) 9 min (d) 12 min (e) 15 min

Answer (b) 6 min

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
t1=30 [min]
per=0.9
a=ln(per)/t1
t2=ln(per)/(5*a)
```

4-167 An 18-cm-long, 16-cm-wide, and 12-cm-high hot iron block (ρ = 7870 kg/m^3, c_p = 447 J/kg·ºC) initially at 20ºC is placed in an oven for heat treatment. The heat transfer coefficient on the surface of the block is 100 W/m^2·ºC. If it is required that the temperature of the block rises to 750ºC in a 25-min period, the oven must be maintained at

(a) 750ºC (b) 830ºC (c) 875ºC (d) 910ºC (e) 1000ºC

Answer (d) 910ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Length=0.18 [m]
Width=0.16 [m]
Height=0.12 [m]
rho=7870 [kg/m^3]
c_p=447 [J/kg-C]
T_i=20 [C]
T_f=750 [C]
h=100 [W/m^2-C]
t=25*60 [s]
A_s=2*Length*Width+2*Length*Height+2*Width*Height
V=Length*Width*Height
b=(h*A_s)/(rho*c_p*V)
(T_f-T_infinity)/(T_i-T_infinity)=exp(-b*t)
```

4-168 A 10-cm-inner diameter, 30-cm long can filled with water initially at 25ºC is put into a household refrigerator at 3ºC. The heat transfer coefficient on the surface of the can is 14 $W/m^2 \cdot ºC$. Assuming that the temperature of the water remains uniform during the cooling process, the time it takes for the water temperature to drop to 5ºC is

(a) 0.55 h (b) 1.17 h (c) 2.09 h (d) 3.60 h (e) 4.97 h

Answer (e) 4.97 h

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.10 [m]
L=0.30 [m]
T_i=25 [C]
T_infinity=3 [C]
T_f=5 [C]
h=14 [W/m^2-C]
A_s=pi*D*L
V=pi*D^2/4*L
rho=1000 [kg/m^3]
c_p=4180 [J/kg-C]
b=(h*A_s)/(rho*c_p*V)
(T_f-T_infinity)/(T_i-T_infinity)=exp(-b*t)
t_hour=t*Convert(s, h)
```

7-169 A 6-cm-diameter 13-cm-high canned drink (ρ = 977 kg/m^3, k = 0.607 W/m·°C, c_p = 4180 J/kg·°C) initially at 25°C is to be cooled to 5°C by dropping it into iced water at 0°C. Total surface area and volume of the drink are A_s = 301.6 cm^2 and V = 367.6 cm^3. If the heat transfer coefficient is 120 W/m^2·°C, determine how long it will take for the drink to cool to 5°C. Assume the can is agitated in water and thus the temperature of the drink changes uniformly with time.

(a) 1.5 min (b) 8.7 min (c) 11.1 min (d) 26.6 min (e) 6.7 min

Answer (c) 11.1 min

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.06 [m]
L=0.13 [m]
Cp=4180 [J/kg-K]
rho= 977 [kg/m^3]
k=0.607 [W/m-K]
V=pi*L*D^2/4
A=2*pi*D^2/4+pi*D*L
m=rho*V
h=120 [W/m^2-C]
Ti=25 [C]
Tinf=0 [C]
T=5 [C]
b=h*A/(rho*V*Cp)

"Lumped system analysis is applicable. Applying the lumped system analysis equation:"
(T-Tinf)/(Ti-Tinf)=exp(-b*time)
t_min=time/60

"Some Wrong Solutions with Common Mistakes:"
(T-0)/(Ti-0)=exp(-b*W1_time); W1_t=W1_time/60 "Tinf is ignored"
(T-Tinf)/(Ti-Tinf)=exp(-b*W2_time); W2_t=W2_time/60 "Sign error"
(T-Ti)/(Tinf-Ti)=exp(-b*W3_time); W3_t=W3_time/60 "Switching Ti and Tinf"
(T-Tinf)/(Ti-Tinf)=exp(-b*W4_time) "Using seconds instead of minutes"
```

4-170 Copper balls (ρ = 8933 kg/m^3, k = 401 W/m·°C, c_p = 385 J/kg·°C, $\alpha = 1.166\times10^{-4}$ m^2/s) initially at 180°C are allowed to cool in air at 30°C for a period of 2 minutes. If the balls have a diameter of 2 cm and the heat transfer coefficient is 80 W/m^2·°C, the center temperature of the balls at the end of cooling is

(a) 78°C (b) 95°C (c) 118°C (d) 134°C (e) 151°C

Answer (b) 95°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.02 [m]
Cp=385 [J/kg-K]
rho= 8933 [kg/m^3]
k=401 [W/m-K]
V=pi*D^3/6
A=pi*D^2
m=rho*V
h=80 [W/m^2-C]
Ti=180 [C]
Tinf=30 [C]
b=h*A/(rho*V*Cp)
time=2*60 [s]
Bi=h*(V/A)/k

"Lumped system analysis is applicable. Applying the lumped system analysis equation:"
(T-Tinf)/(Ti-Tinf)=exp(-b*time)

“Some Wrong Solutions with Common Mistakes:”
(W1_T-0)/(Ti-0)=exp(-b*time) “Tinf is ignored”
(-W2_T+Tinf)/(Ti-Tinf)=exp(-b*time) “Sign error”
(W3_T-Ti)/(Tinf-Ti)=exp(-b*time) “Switching Ti and Tinf”
(W4_T-Tinf)/(Ti-Tinf)=exp(-b*time/60) “Using minutes instead of seconds”
```

4-171 Carbon steel balls (ρ = 7830 kg/m^3, k = 64 W/m·°C, c_p = 434 J/kg·°C) initially at 200°C are quenched in an oil bath at 20°C for a period of 3 minutes. If the balls have a diameter of 5 cm and the convection heat transfer coefficient is 450 W/m^2·°C, the center temperature of the balls after quenching will be (Hint: Check the Biot number).

(a) 30.3°C (b) 46.1°C (c) 55.4°C (d) 68.9°C (e) 79.4°C

Answer (a) 30.3°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.05 [m]
Cp=434 [J/kg-K]
rho= 7830 [kg/m^3]
k=64 [W/m-K]
V=pi*D^3/6
A=pi*D^2
m=rho*V
h=450 [W/m^2-C]
Ti=200 [C]
Tinf=20 [C]
b=h*A/(rho*V*Cp)
time=3*60 [s]
Bi=h*(V/A)/k

"Applying the lumped system analysis equation:"
(T-Tinf)/(Ti-Tinf)=exp(-b*time)

“Some Wrong Solutions with Common Mistakes:”
(W1_T-0)/(Ti-0)=exp(-b*time) “Tinf is ignored”
(-W2_T+Tinf)/(Ti-Tinf)=exp(-b*time) “Sign error”
(W3_T-Ti)/(Tinf-Ti)=exp(-b*time) “Switching Ti and Tinf”
(W4_T-Tinf)/(Ti-Tinf)=exp(-b*time/60) “Using minutes instead of seconds”
```

4-172 In a production facility, large plates made of stainless steel (k = 15 W/m·ºC, $\alpha = 3.91\times10^{-6}$ m^2/s) of 40 cm thickness are taken out of an oven at a uniform temperature of 750ºC. The plates are placed in a water bath that is kept at a constant temperature of 20ºC with a heat transfer coefficient of 600 W/m^2·ºC. The time it takes for the surface temperature of the plates to drop to 120ºC is

(a) 0.6 h (b) 0.8 h (c) 1.4 h (d) 2.6 h (e) 3.2 h

Answer (e) 3.2 h

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k=15 [W/m-C]
alpha=3.91E-6 [m^2/s]
2*L=0.4 [m]
T_i=750 [C]
T_infinity=20 [C]
h=600 [W/m^2-C]
T_s=120 [C]

Bi=(h*L)/k "The coefficients lambda_1 and A_1 corresponding to the calculated Bi number of 8 are obtained from Table 4-2 of the text as"
lambda_1=1.3978
A_1=1.2570
tau=(alpha*t)/L^2
(T_s-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)*cos(lambda_1)

"Some Wrong Solutions with Common Mistakes"
tau_1=(alpha*W1_t)/L^2
(T_s-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau_1) "Using the relation for center temperature"
```

4-173 A long 18-cm-diameter bar made of hardwood (k = 0.159 W/m·ºC, $\alpha = 1.75\times10^{-7}$ m^2/s) is exposed to air at 30ºC with a heat transfer coefficient of 8.83 W/m^2·ºC. If the center temperature of the bar is measured to be 15ºC after a period of 3-hours, the initial temperature of the bar is

(a) 11.9ºC (b) 4.9ºC (c) 1.7ºC(d) 0ºC (e) -9.2ºC

Answer (b) 4.9ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.18 [m]
k=0.159 [W/m-C]
alpha=1.75E-7 [m^2/s]
T_infinity=30 [C]
h=8.83 [W/m^2-C]
T_0=15 [C]
t=3*3600 [s]
r_0=D/2
Bi=(h*r_0)/k "The coefficients lambda_1 and A_1 corresponding to the calculated Bi = 5 are obtained from Table 4-2 of the text as"
lambda_1=1.9898
A_1=1.5029
tau=(alpha*t)/r_0^2
(T_0-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)

"Some Wrong Solutions with Common Mistakes"
lambda_1a=1.3138
A_1a=1.2403
(T_0-T_infinity)/(W1_T_i-T_infinity)=A_1a*exp(-lambda_1a^2*tau) "Using coefficients for plane wall in Table 4-2"
 lambda_1b=2.5704
A_1b=1.7870
(T_0-T_infinity)/(W2_T_i-T_infinity)=A_1b*exp(-lambda_1b^2*tau) "Using coefficients for sphere in Table 4-2"
```

4-174 Consider a 7.6-cm-long and 3-cm-diameter cylindrical lamb meat chunk (ρ = 1030 kg/m^3, c_p = 3.49 kJ/kg·ºC, k = 0.456 W/m·ºC, $\alpha = 1.3\times10^{-7}$ m^2/s). Such a meat chunk initially at 2ºC is dropped into boiling water at 95ºC with a heat transfer coefficient of 1200 W/m^2·ºC. The amount of heat transfer during the first 8 minutes of cooking is

(a) 71 kJ (b) 227 kJ (c) 238 kJ (d) 269 kJ (e) 307 kJ

Answer (c) 269 kJ

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
2*L=0.076 [m]
D=0.03 [m]
n=15
rho=1030 [kg/m^3]
c_p=3490 [J/kg-C]
k=0.456 [W/m-C]
alpha=1.3E-7 [m^2/s]
T_i=2 [C]
T_infinity=95 [C]
h=1200 [W/m^2-C]
t=8*60 [s]
Bi_wall=(h*L)/k
lambda_1_wall=1.5552 "for Bi_wall = 100 from Table 4-2"
A_1_wall=1.2731
tau_wall=(alpha*t)/L^2
theta_wall=A_1_wall*exp(-lambda_1_wall^2*tau_wall)
Q\Q_max_wall=1-theta_wall*sin(lambda_1_wall)/lambda_1_wall
r_0=D/2
Bi_cyl=(h*r_0)/k
lambda_1_cyl=2.3455 "for Bi_cyl = 40 from Table 4-2"
A_1_cyl=1.5993
tau_cyl=(alpha*t)/r_0^2
theta_cyl=A_1_cyl*exp(-lambda_1_cyl^2*tau_cyl)
J_1=0.5309 "For xi = lambda_a_cyl = 2.3455 from Table 4-2"
Q\Q_max_cyl=1-2*theta_cyl*J_1/lambda_1_cyl
V=pi*D^2/4*(2*L)
Q_max=n*rho*V*c_p*(T_infinity-T_i)
Q\Q_max=Q\Q_max_wall+Q\Q_max_cyl*(1-Q\Q_max_wall)
Q=Q_max*Q\Q_max

"Some Wrong Solutions with Common Mistakes"
W1_Q=Q_max "Using Q_max as the result"
W2_Q=Q_max*Q\Q_max_wall "Considering large plane wall only"
W3_Q=Q_max*Q\Q_max_cyl "Considering long cylinder only"
```

4-175 Consider a 7.6-cm-long and 3-cm-diameter cylindrical lamb meat chunk (ρ = 1030 kg/m^3, c_p = 3.49 kJ/kg·ºC, k = 0.456 W/m·ºC, $\alpha = 1.3\times10^{-7}$ m^2/s). Such a meat chunk initially at 2ºC is dropped into boiling water at 95ºC with a heat transfer coefficient of 1200 W/m^2·ºC. The time it takes for the center temperature of the meat chunk to rise to 75ºC is

(a) 136 min (b) 21.2 min (c) 13.6 min (d) 11.0 min (e) 8.5 min

Answer (d) 11.0 min

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
2*L=0.076 [m]
D=0.03 [m]
rho=1030 [kg/m^3]
c_p=3490 [J/kg-C]
k=0.456 [W/m-C]
alpha=1.3E-7 [m^2/s]
T_i=2 [C]
T_infinity=95 [C]
h=1200 [W/m^2-C]
T_0=75 [C]
Bi_wall=(h*L)/k
lambda_1_wall=1.5552 "for Bi_wall = 100 from Table 4-2"
A_1_wall=1.2731
r_0=D/2
Bi_cyl=(h*r_0)/k
lambda_1_cyl=2.3455 "for Bi_cyl = 40 from Table 4-2"
A_1_cyl=1.5993
tau_wall=(alpha*t)/L^2
theta_wall=A_1_wall*exp(-lambda_1_wall^2*tau_wall)
tau_cyl=(alpha*t)/r_0^2
theta_cyl=A_1_cyl*exp(-lambda_1_cyl^2*tau_cyl)
theta=theta_wall*theta_cyl
theta=(T_0-T_infinity)/(T_i-T_infinity)

"Some Wrong Solutions with Common Mistakes"
tau_wall_w=(alpha*W1_t)/L^2
theta_wall_w=A_1_wall*exp(-lambda_1_wall^2*tau_wall_w)
theta_wall_w=(T_0-T_infinity)/(T_i-T_infinity) "Considering only large plane wall solution"
tau_cyl_w=(alpha*W2_t)/r_0^2
theta_cyl_w=A_1_wall*exp(-lambda_1_wall^2*tau_cyl_w)
theta_cyl_w=(T_0-T_infinity)/(T_i-T_infinity) "Considering only long cylinder solution"
```

4-176 A potato that may be approximated as a 5.7-cm-diameter solid sphere with the properties ρ = 910 kg/m^3, c_p = 4.25 kJ/kg·ºC, k = 0.68 W/m·ºC, and α = 1.76×10^{-7} m^2/s. Twelve such potatoes initially at 25ºC are to be cooked by placing them in an oven maintained at 250ºC with a heat transfer coefficient of 95 W/m^2·ºC. The amount of heat transfer to the potatoes by the time the center temperature reaches 90ºC is

(a) 1012 kJ (b) 1366 kJ (c) 1788 kJ (d) 2046 kJ (e) 3270 kJ

Answer (b) 1366 kJ

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.057 [m]
rho=910 [kg/m^3]
c_p=4250 [J/kg-C]
k=0.68 [W/m-C]
alpha=1.76E-7 [m^2/s]
n=12
T_i=25 [C]
T_infinity=250 [C]
h=95 [W/m^2-C]
T_0=90 [C]
r_0=D/2
Bi=(h*r_0)/k "The coefficients lambda_1 and A_1 corresponding to the calculated Bi = 4 are obtained from Table 4-2 of the text as"
lambda_1=2.4556
A_1=1.7202
Theta_0=(T_0-T_infinity)/(T_i-T_infinity)
V=pi*D^3/6
Q_max=n*rho*V*c_p*(T_infinity-T_i)
Q=Q_max*(1-3*Theta_0*(sin(lambda_1)-lambda_1*cos(lambda_1))/lambda_1^3)

"Some Wrong Solutions with Common Mistakes"
W1_Q=Q_max "Using Q_max as the result"
W2_Q=Q_max*(1-Theta_0*(sin(lambda_1))/lambda_1) "Using the relation for plane wall"
W3_Q_max=rho*V*c_p*(T_infinity-T_i)
W3_Q=W3_Q_max*(1-3*Theta_0*(sin(lambda_1)-lambda_1*cos(lambda_1))/lambda_1^3) "Not multiplying with the number of potatoes"
```

4-177 A small chicken (k = 0.45 W/m·K, $\alpha = 0.15\times10^{-6}$ m^2/s) can be approximated as an 11.25-cm-diameter solid sphere. The chicken is initially at a uniform temperature of 8ºC and is to be cooked in an oven maintained at 220ºC with a heat transfer coefficient of 80 W/m^2·K. With this idealization, the temperature at the center of the chicken after a 90-min period is

(a) 25ºC (b) 61ºC (c) 89ºC (d) 122ºC (e) 168ºC

Answer (e) 168ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k=0.45 [W/m-C]
alpha=0.15E-6 [m^2/s]
D=0.1125 [m]
T_i=8 [C]
T_infinity=220 [C]
h=80 [W/m^2-C]
t=90*60 [s]
r_0=D/2
Bi=(h*r_0)/k "The coefficients lambda_1 and A_1 corresponding to the calculated Bi number of 10 are obtained from Table 4-2 of the text as"
lambda_1=2.8363
A_1=1.9249
tau=(alpha*t)/r_0^2
(T_0-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)

"Some Wrong Solutions with Common Mistakes"
lambda_1a=1.4289
A_1a=1.2620
(W1_T_0-T_infinity)/(T_i-T_infinity)=A_1a*exp(-lambda_1a^2*tau) "Using coefficients for plane wall in Table 4-2"
lambda_1b=2.1795
A_1b=1.5677
(W2_T_0-T_infinity)/(T_i-T_infinity)=A_1b*exp(-lambda_1b^2*tau) "Using coefficients for cylinder in Table 4-2"
```

4-178 A potato may be approximated as a 5.7-cm-diameter solid sphere with the properties ρ = 910 kg/m^3, c_p = 4.25 kJ/kg·K, k = 0.68 W/m·K, and α = 1.76×10^{-7} m^2/s. Twelve such potatoes initially at 25ºC are to be cooked by placing them in an oven maintained at 250ºC with a heat transfer coefficient of 95 W/m^2·K. The amount of heat transfer to the potatoes during a 30-min period is

(a) 77 kJ (b) 483 kJ (c) 927 kJ (d) 970 kJ (e) 1012 kJ

Answer (c) 927 kJ

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.057 [m]
rho=910 [kg/m^3]
c_p=4250 [J/kg-C]
k=0.68 [W/m-C]
alpha=1.76E-7 [m^2/s]
n=12
T_i=25 [C]
T_infinity=250 [C]
h=95 [W/m^2-C]
t=30*60 [s]
r_0=D/2
Bi=(h*r_0)/k "The coefficients lambda_1 and A_1 corresponding to the calculated Bi = 4 are obtained from Table 4-2 of the text as"
lambda_1=2.4556
A_1=1.7202
tau=(alpha*t)/r_0^2
Theta_0=A_1*exp(-lambda_1^2*tau)
V=pi*D^3/6
Q_max=n*rho*V*c_p*(T_infinity-T_i)
Q=Q_max*(1-3*Theta_0*(sin(lambda_1)-lambda_1*cos(lambda_1))/lambda_1^3)

"Some Wrong Solutions with Common Mistakes"
W1_Q=Q_max "Using Q_max as the result"
W2_Q=Q_max*(1-Theta_0*(sin(lambda_1))/lambda_1) "Using the relation for plane wall"
W2_Q_max=rho*V*c_p*(T_infinity-T_i)
W3_Q=W2_Q_max*(1-3*Theta_0*(sin(lambda_1)-lambda_1*cos(lambda_1))/lambda_1^3) "Not multiplying with the number of potatoes"
```

4-179 When water, as in a pond or lake, is heated by warm air above it, it remains stable, does not move, and forms a warm layer of water on top of a cold layer. Consider a deep lake (k = 0.6 W/m·K, c_p = 4.179 kJ/kg·K) that is initially at a uniform temperature of 2ºC and has its surface temperature suddenly increased to 20ºC by a spring weather front. The temperature of the water 1 m below the surface 400 hours after this change is

(a) 2.1ºC (b) 4.2ºC (c) 6.3ºC (d) 8.4ºC (e) 10.2ºC

Answer (b) 4.2ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k=0.6 [W/m-C]
c=4179 [J/kg-C]
rho=1000 [kg/m^3]
T_i=2 [C]
T_s=20 [C]
x=1 [m]
time=400*3600 [s]
alpha=k/(rho*c)
xi=x/(2*sqrt(alpha*time))
(T-T_i)/(T_s-T_i)=erfc(xi)
```

4-180 A large chunk of tissue at 35°C with a thermal diffusivity of 1×10^{-7} m^2/s is dropped into iced water. The water is well-stirred so that the surface temperature of the tissue drops to 0°C at time zero and remains at 0°C at all times. The temperature of the tissue after 4 minutes at a depth of 1 cm is

(a) 5°C (b) 30°C (c) 25°C (d) 20°C (e) 10°C

Answer (b) 30°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
X=0.01 [m]
Alpha=1E-7 [m^2/s]
Ti=35 [C]
Ts=0 [C]
time=4*60 [s]
a=0.5*x/sqrt(alpha*time)
b=erfc(a)
(T-Ti)/(Ts-Ti)=b
```

4-181 The 35-cm-thick roof of a large room made of concrete (k = 0.79 W/m·ºC, $\alpha = 5.88\times10^{-7}$ m^2/s) is initially at a uniform temperature of 15ºC. After a heavy snow storm, the outer surface of the roof remains covered with snow at -5ºC. The roof temperature at 12 cm distance from the outer surface after a period of 2 hours is

(a) 13ºC (b) 11ºC (c) 7ºC (d) 3ºC (e) -5ºC

Answer (b) 11ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Thickness=0.35 [m]
k=0.79 [W/m-C]
alpha=5.88E-7 [m^2/s]
T_i=15 [C]
T_s=-5 [C]
x=0.12 [m]
time=2*3600 [s]
xi=x/(2*sqrt(alpha*time))
(T-T_i)/(T_s-T_i)=erfc(xi)
```

4-182 ··· 4-185 Design and Essay Problems

Solutions Manual for

Heat and Mass Transfer: Fundamentals & Applications
6th Edition
Yunus A. Çengel, Afshin J. Ghajar
McGraw-Hill Education, 2020

Chapter 5
NUMERICAL METHODS IN HEAT CONDUCTION

Why Numerical Methods?

5-1C Analytical solutions provide insight to the problems, and allows us to observe the degree of dependence of solutions on certain parameters. They also enable us to obtain quick solution, and to verify numerical codes. Therefore, analytical solutions are not likely to disappear from engineering curricula.

5-2C Analytical solution methods are limited to *highly simplified problems* in *simple geometries*. The geometry must be such that its entire surface can be described mathematically in a coordinate system by setting the variables equal to constants. Also, heat transfer problems can not be solved analytically if the *thermal conditions* are not sufficiently simple. For example, the consideration of the variation of thermal conductivity with temperature, the variation of the heat transfer coefficient over the surface, or the radiation heat transfer on the surfaces can make it impossible to obtain an analytical solution. Therefore, analytical solutions are limited to problems that are simple or can be simplified with reasonable approximations.

5-3C In practice, we are most likely to use a software package to solve heat transfer problems even when analytical solutions are available since we can do parametric studies very easily and present the results graphically by the press of a button. Besides, once a person is used to solving problems numerically, it is very difficult to go back to solving differential equations by hand.

5-4C The *energy balance method* is based on *subdividing* the medium into a sufficient number of volume elements, and then applying an *energy balance* on each element. The formal *finite difference method* is based on replacing derivatives by their finite difference approximations. For a specified nodal network, these two methods will result in the same set of equations.

5-5C The *analytical solutions* are based on (1) driving the governing differential equation by performing an energy balance on a differential volume element, (2) expressing the boundary conditions in the proper mathematical form, and (3) solving the differential equation and applying the boundary conditions to determine the integration constants. The *numerical solution* methods are based on replacing the *differential equations* by *algebraic equations*. In the case of the popular *finite difference* method, this is done by replacing the *derivatives* by *differences*. The analytical methods are simple and they provide solution functions applicable to the entire medium, but they are limited to simple problems in simple geometries. The numerical methods are usually more involved and the solutions are obtained at a number of points, but they are applicable to any geometry subjected to any kind of thermal conditions.

5-6C The experiments will most likely prove engineer B right since an approximate solution of a more realistic model is more accurate than the exact solution of a crude model of an actual problem.

Finite Difference Formulation of Differential Equations

5-7C A point at which the finite difference formulation of a problem is obtained is called a *node*, and all the nodes for a problem constitute the *nodal network*. The region about a node whose properties are represented by the property values at the nodal point is called the *volume element*. The distance between two consecutive nodes is called the *nodal spacing*, and a differential equation whose derivatives are replaced by differences is called a *difference equation*.

5-8 The finite difference formulation of steady two-dimensional heat conduction in a medium with heat generation and constant thermal conductivity is given by

$$\frac{T_{m-1,n} - 2T_{m,n} + T_{m+1,n}}{\Delta x^2} + \frac{T_{m,n-1} - 2T_{m,n} + T_{m,n+1}}{\Delta y^2} + \frac{\dot{e}_{m,n}}{k} = 0$$

in rectangular coordinates. This relation can be modified for the three-dimensional case by simply adding another index j to the temperature in the z direction, and another difference term for the z direction as

$$\frac{T_{m-1,n,j} - 2T_{m,n,j} + T_{m+1,n,j}}{\Delta x^2} + \frac{T_{m,n-1,j} - 2T_{m,n,j} + T_{m,n+1,j}}{\Delta y^2} + \frac{T_{m,n,j-1} - 2T_{m,n,j} + T_{m,n,j+1}}{\Delta z^2} + \frac{\dot{e}_{m,n,j}}{k} = 0$$

5-9 Finite difference formulation for an interior node, boundary node subject to convection and constant heat flux in case of variable thermal conductivity is to be determined.

Analysis The one dimensional steady state heat conduction equation with variable thermal conductivity is expressed as

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)+\dot{e}=0$$

Using Eq. (5-6), the first derivative of the temperature at the midpoints surrounding the node with variable thermal conductivity can be expressed for x as

$$k(T)\frac{dT}{dx}\bigg|_{m-\frac{1}{2}}=k_o\left[1+\beta\frac{(T_{m-1}+T_m)}{2}\right]\frac{(T_{m-1}-T_m)}{\Delta x}\ \text{ and }\ k(T)\frac{dT}{dx}\bigg|_{m+\frac{1}{2}}=k_o\left[1+\beta\frac{(T_{m+1}+T_m)}{2}\right]\frac{(T_{m+1}-T_m)}{\Delta x}$$

Using the definition of second derivative as the derivative of the first derivative we get

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)+\dot{e}=\frac{k_o\left[1+\beta\frac{(T_{m+1}+T_m)}{2}\right]\frac{(T_{m+1}-T_m)}{\Delta x}-k_o\left[1+\beta\frac{(T_{m-1}+T_m)}{2}\right]\frac{(T_{m-1}-T_m)}{\Delta x}}{\Delta x}+\dot{e}_m=0$$

Simplifying above equation yields,

$$(T_{m+1}-T_m)\left[1+\frac{\beta}{2}(T_{m+1}+T_m)\right]+(T_{m-1}-T_m)\left[1+\frac{\beta}{2}(T_{m-1}+T_m)\right]+\dot{e}_m\frac{\Delta x^2}{k_o}=0$$

$$\therefore(T_{m-1}-2T_m+T_{m+1})+\frac{\beta}{2}\left[T_{m-1}^2-2T_m^2+T_{m+1}^2\right]+\dot{e}_m\frac{\Delta x^2}{k_o}=0$$

For left boundary node exposed to constant heat flux apply energy balance to the half volume around the boundary node with all the heat transfer entering the volume element.

Replacing k by k(T) in Eq. 5-22 we get,

$$k_o\left[1+\beta\frac{(T_{m+1}+T_m)}{2}\right]\frac{(T_{m+1}-T_m)}{\Delta x}+\dot{q}+\dot{e}_m\frac{\Delta x}{2}=0\quad\therefore(T_{m+1}-T_m)+\frac{\beta}{2}\left(T_{m+1}^2-T_m^2\right)+\frac{\dot{q}\Delta x}{k_o}+\frac{\dot{e}_m\Delta x^2}{2k_o}=0$$

For right boundary node exposed to convection environment, apply energy balance to the half volume around the boundary node with all the heat transfer entering the volume element.

$$k_o\left[1+\beta\frac{(T_{m-1}+T_m)}{2}\right]\frac{(T_{m-1}-T_m)}{\Delta x}+h(T_\infty-T_m)+\frac{\dot{e}_m\Delta x}{2}=0$$

$$\therefore(T_{m-1}-T_m)+\frac{\beta}{2}\left(T_{m-1}^2-T_m^2\right)+\frac{h\Delta x}{k_o}(T_\infty-T_m)+\frac{\dot{e}_m\Delta x^2}{2k_o}=0$$

5-10 For a three dimensional steady state heat transfer without internal heat generation finite difference formulations are to be determined.

Analysis The three dimensional heat conduction equation for steady state conditions with variable thermal conductivity is expressed as

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)=0$$

Using Eq. (5-6), the first derivative of the temperature at the midpoints surrounding the node can be expressed for x, y and z directions as

x-direction: $$k\frac{dT}{dx}\bigg|_{m-\frac{1}{2},n,j} \cong k_{m-\frac{1}{2},n,j}\frac{T_{m,n,j}-T_{m-1,n,j}}{\Delta x} \quad and \quad k\frac{dT}{dx}\bigg|_{m+\frac{1}{2},n,j} \cong k_{m+\frac{1}{2},n,j}\frac{T_{m+1,n,j}-T_{m,n,j}}{\Delta x}$$

y-direction: $$k\frac{dT}{dy}\bigg|_{n-\frac{1}{2},m,j} \cong k_{n-\frac{1}{2},m,j}\frac{T_{n,m,j}-T_{n-1,m,j}}{\Delta y} \quad and \quad k\frac{dT}{dy}\bigg|_{n+\frac{1}{2},m,j} \cong k_{n+\frac{1}{2},m,j}\frac{T_{n+1,m,j}-T_{n,m,j}}{\Delta y}$$

z-direction: $$k\frac{dT}{dz}\bigg|_{j-\frac{1}{2},m,n} \cong k_{j-\frac{1}{2},m,n}\frac{T_{j,m,n}-T_{j-1,m,n}}{\Delta z} \quad and \quad k\frac{dT}{dz}\bigg|_{j+\frac{1}{2},m,n} \cong k_{j+\frac{1}{2},m,n}\frac{T_{j+1,m,n}-T_{j,m,n}}{\Delta z}$$

Using the definition of second derivative as the derivative of the first derivative we get

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)=\frac{k\frac{dT}{dx}\Big|_{m+\frac{1}{2},n,j}-k\frac{dT}{dx}\Big|_{m-\frac{1}{2},n,j}}{\Delta x}=k_{m+\frac{1}{2},n,j}\frac{T_{m+1,n,j}-T_{m,n,j}}{\Delta x^2}-k_{m-\frac{1}{2},n,j}\frac{T_{m,n,j}-T_{m-1,n,j}}{\Delta x^2}$$

Similar for y and z directions we can show that

$$\frac{d}{dy}\left(k\frac{dT}{dy}\right)=k_{n+\frac{1}{2},m,j}\frac{T_{n+1,m,j}-T_{n,m,j}}{\Delta y^2}-k_{n-\frac{1}{2},m,j}\frac{T_{n,m,j}-T_{n-1,m,j}}{\Delta y^2}$$

$$\frac{d}{dz}\left(k\frac{dT}{dz}\right)=k_{j+\frac{1}{2},m,n}\frac{T_{j+1,m,n}-T_{j,m,n}}{\Delta z^2}-k_{j-\frac{1}{2},m,n}\frac{T_{j,m,n}-T_{j-1,m,n}}{\Delta z^2}$$

Putting these equations of the second derivatives in the three dimensional heat conduction equation with variable thermal conductivity we get the required finite difference formulation.

$$k_{m+\frac{1}{2},n,j}\frac{T_{m+1,n,j}-T_{m,n,j}}{\Delta x^2}-k_{m-\frac{1}{2},n,j}\frac{T_{m,n,j}-T_{m-1,n,j}}{\Delta x^2}+k_{n+\frac{1}{2},m,j}\frac{T_{n+1,m,j}-T_{n,m,j}}{\Delta y^2}-k_{n-\frac{1}{2},m,j}\frac{T_{n,m,j}-T_{n-1,m,j}}{\Delta y^2}+$$
$$k_{j+\frac{1}{2},m,n}\frac{T_{j+1,m,n}-T_{j,m,n}}{\Delta z^2}-k_{j-\frac{1}{2},m,n}\frac{T_{j,m,n}-T_{j-1,m,n}}{\Delta z^2}=0$$

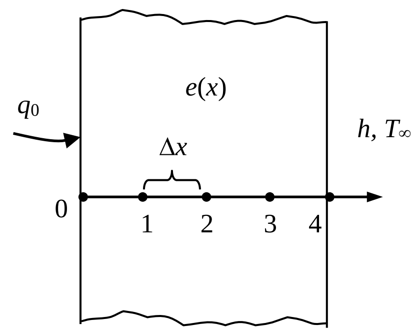
5-11 A plane wall with variable heat generation and constant thermal conductivity is subjected to uniform heat flux $\dot{q}_0$ at the left (node 0) and convection at the right boundary (node 4). Using the finite difference form of the 1st derivative, the finite difference formulation of the boundary nodes is to be determined.

Assumptions **1** Heat transfer through the wall is steady since there is no indication of change with time. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness. **3** Thermal conductivity is constant and there is nonuniform heat generation in the medium. **4** Radiation heat transfer is negligible.

Analysis The boundary conditions at the left and right boundaries can be expressed analytically as

at $x = 0$: $$-k\frac{dT(0)}{dx} = q_0$$

at $x = L$: $$-k\frac{dT(L)}{dx} = h[T(L) - T_\infty]$$



Replacing derivatives by differences using values at the closest nodes, the finite difference form of the 1st derivative of temperature at the boundaries (nodes 0 and 4) can be expressed as

$$\left.\frac{dT}{dx}\right|_{\text{left, m}=0} \cong \frac{T_1 - T_0}{\Delta x} \quad \text{and} \quad \left.\frac{dT}{dx}\right|_{\text{right, m}=4} \cong \frac{T_4 - T_3}{\Delta x}$$

Substituting, the finite difference formulation of the boundary nodes become

at $x = 0$: $$-k\frac{T_1 - T_0}{\Delta x} = q_0$$

at $x = L$: $$-k\frac{T_4 - T_3}{\Delta x} = h[T_4 - T_\infty]$$

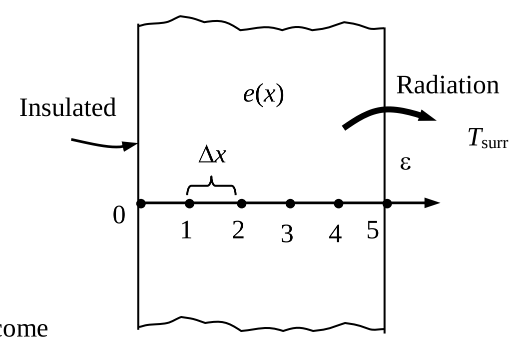
5-12 A plane wall with variable heat generation and constant thermal conductivity is subjected to insulation at the left (node 0) and radiation at the right boundary (node 5). Using the finite difference form of the 1st derivative, the finite difference formulation of the boundary nodes is to be determined.

Assumptions **1** Heat transfer through the wall is steady since there is no indication of change with time. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness. **3** Thermal conductivity is constant and there is nonuniform heat generation in the medium. **4** Convection heat transfer is negligible.

Analysis The boundary conditions at the left and right boundaries can be expressed analytically as

At $x = 0$: $$-k\frac{dT(0)}{dx} = 0 \quad \text{or} \quad \frac{dT(0)}{dx} = 0$$

At $x = L$: $$-k\frac{dT(L)}{dx} = \varepsilon\sigma[T^4(L) - T_{surr}^4]$$



Replacing derivatives by differences using values at the closest nodes, the finite difference form of the 1st derivative of temperature at the boundaries (nodes 0 and 5) can be expressed as

$$\left.\frac{dT}{dx}\right|_{\text{left, m}=0} \cong \frac{T_1 - T_0}{\Delta x} \quad \text{and} \quad \left.\frac{dT}{dx}\right|_{\text{right, m}=5} \cong \frac{T_5 - T_4}{\Delta x}$$

Substituting, the finite difference formulation of the boundary nodes become

At $x = 0$: $$-k\frac{T_1 - T_0}{\Delta x} = 0 \quad \text{or} \quad T_1 = T_0$$

At $x = L$: $$-k\frac{T_5 - T_4}{\Delta x} = \varepsilon\sigma[T_5^4 - T_{surr}^4]$$

One-Dimensional Steady Heat Conduction

5-13C The finite difference form of a heat conduction problem by the *energy balance method* is obtained by *subdividing* the medium into a sufficient number of volume elements, and then applying an *energy balance* on each element. This is done by first *selecting* the nodal points (or nodes) at which the temperatures are to be determined, and then *forming elements* (or control volumes) over the nodes by drawing lines through the midpoints between the nodes. The properties *at the node* such as the temperature and the rate of heat generation represent the *average* properties of the element. The temperature is assumed to vary *linearly* between the nodes, especially when expressing heat conduction between the elements using Fourier's law.

5-14C The basic steps involved in the iterative Gauss-Seidel method are: (1) Writing the equations explicitly for each unknown (the unknown on the left-hand side and all other terms on the right-hand side of the equation), (2) making a reasonable initial guess for each unknown, (3) calculating new values for each unknown, *always using the most recent values*, and (4) repeating the process until desired convergence is achieved.

5-15C In a medium in which the finite difference formulation of a general interior node is given in its simplest form as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0$$

(*a*) heat transfer in this medium is **steady**, (*b*) it is **one-dimensional**, (*c*) there **is** heat generation, (*d*) the nodal spacing is **constant**, and (*e*) the thermal conductivity is **constant**.

5-16C In the finite difference formulation of a problem, an insulated boundary is best handled by replacing the insulation by a mirror, and treating the node on the boundary as an *interior* node. Also, a thermal symmetry line and an insulated boundary are treated the same way in the finite difference formulation.

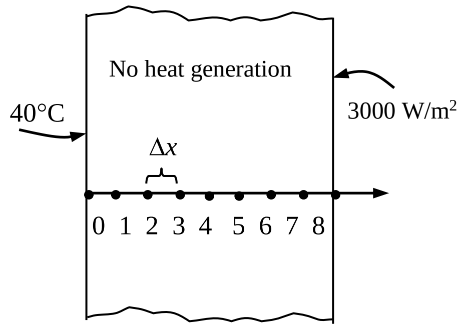
5-17C A node on an insulated boundary can be treated as an interior node in the finite difference formulation of a plane wall by replacing the insulation on the boundary by a *mirror*, and considering the reflection of the medium as its extension. This way the node next to the boundary node appears on both sides of the boundary node because of symmetry, converting it into an interior node.

5-18C In the energy balance formulation of the finite difference method, it is recommended that all heat transfer at the boundaries of the volume element be assumed to be *into* the volume element even for steady heat conduction. This is a valid recommendation even though it seems to violate the conservation of energy principle since the assumed direction of heat conduction at the surfaces of the volume elements has no effect on the formulation, and some heat conduction terms turn out to be negative.

5-19 A plane wall with no heat generation is subjected to specified temperature at the left (node 0) and heat flux at the right boundary (node 8). The finite difference formulation of the boundary nodes and the finite difference formulation for the rate of heat transfer at the left boundary are to be determined.

Assumptions **1** Heat transfer through the wall is given to be steady, and the thermal conductivity to be constant. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness. **3** There is no heat generation in the medium.

Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the finite difference formulations become



Left boundary node: $T_0 = 40$

Right boundary node: $kA\dfrac{T_7 - T_8}{\Delta x} + \dot{q}_0 A = 0$ or $k\dfrac{T_7 - T_8}{\Delta x} + 3000 = 0$

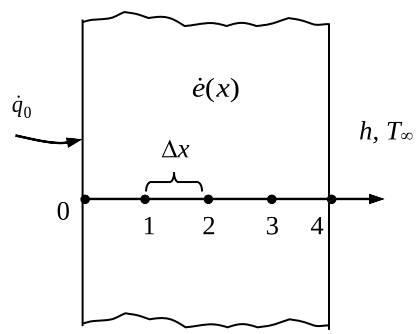
Heat transfer at left surface:

$$\dot{Q}_{\text{left surface}} + kA\frac{T_1 - T_0}{\Delta x} = 0$$

5-20 A plane wall with variable heat generation and constant thermal conductivity is subjected to uniform heat flux $\dot{q}_0$ at the left (node 0) and convection at the right boundary (node 4). The finite difference formulation of the boundary nodes is to be determined.

Assumptions **1** Heat transfer through the wall is given to be steady, and the thermal conductivity to be constant. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness. **3** Radiation heat transfer is negligible.

Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the finite difference formulations become



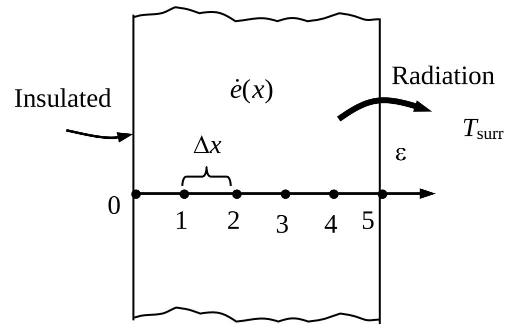
Left boundary node: $\dot{q}_0 A + kA\dfrac{T_1 - T_0}{\Delta x} + \dot{e}_0 (A\Delta x/2) = 0$

Right boundary node: $kA\dfrac{T_3 - T_4}{\Delta x} + hA(T_\infty - T_4) + \dot{e}_4 (A\Delta x/2) = 0$

5-21 A plane wall with variable heat generation and constant thermal conductivity is subjected to insulation at the left (node 0) and radiation at the right boundary (node 5). The finite difference formulation of the boundary nodes is to be determined.

Assumptions **1** Heat transfer through the wall is given to be steady and one-dimensional, and the thermal conductivity to be constant. **2** Convection heat transfer is negligible.

Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the finite difference formulations become



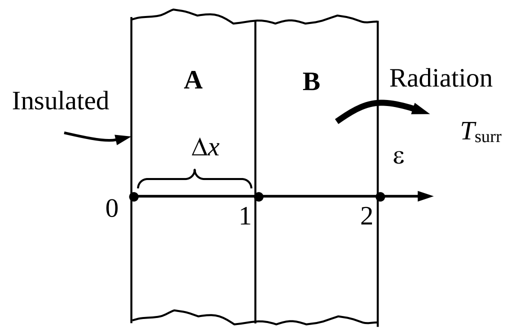
Left boundary node: $kA\dfrac{T_1 - T_0}{\Delta x} + \dot{e}_0(A\Delta x/2) = 0$

Right boundary node: $\varepsilon\sigma A(T_{surr}^4 - T_5^4) + kA\dfrac{T_4 - T_5}{\Delta x} + \dot{e}_5(A\Delta x/2) = 0$

5-22 A composite plane wall consists of two layers A and B in perfect contact at the interface where node 1 is. The wall is insulated at the left (node 0) and subjected to radiation at the right boundary (node 2). The complete finite difference formulation of this problem is to be obtained.

Assumptions **1** Heat transfer through the wall is given to be steady and one-dimensional, and the thermal conductivity to be constant. **2** Convection heat transfer is negligible. **3** There is no heat generation.

Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the finite difference formulations become



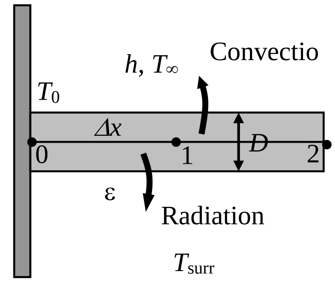
Node 0 (at left boundary): $k_A A\dfrac{T_1 - T_0}{\Delta x} = 0 \;\rightarrow\; T_1 = T_0$

Node 1 (at the interface): $k_A A\dfrac{T_0 - T_1}{\Delta x} + k_B A\dfrac{T_2 - T_1}{\Delta x} = 0$

Node 2 (at right boundary): $\varepsilon\sigma A(T_{surr}^4 - T_2^4) + k_B A\dfrac{T_1 - T_2}{\Delta x} = 0$

5-23 A pin fin with negligible heat transfer from its tip is considered. The complete finite difference formulation for the determination of nodal temperatures is to be obtained.

Assumptions **1** Heat transfer through the pin fin is given to be steady and one-dimensional, and the thermal conductivity to be constant. **2** Convection heat transfer coefficient is constant and uniform. **3** Heat loss from the fin tip is given to be negligible.



Analysis The nodal network consists of 3 nodes, and the base temperature T_0 at node 0 is specified. Therefore, there are two unknowns T_1 and T_2, and we need two equations to determine them. Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the finite difference formulations become

Node 1 (at midpoint):

$$kA\frac{T_0-T_1}{\Delta x}+kA\frac{T_2-T_1}{\Delta x}+h(p\Delta x)(T_\infty-T_1)+\varepsilon\sigma(p\Delta x)\left[(T_{\rm surr}+273)^4-(T_1+273)^4\right]=0$$

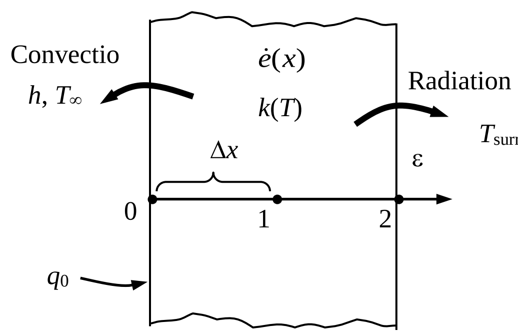
Node 2 (at fin tip):

$$kA\frac{T_1-T_2}{\Delta x}+h(p\Delta x/2)(T_\infty-T_2)+\varepsilon\sigma(p\Delta x/2)\left[(T_{\rm surr}+273)^4-(T_2+273)^4\right]=0$$

where $A=\pi D^2/4$ is the cross-sectional area and $p=\pi D$ is the perimeter of the fin.

5-24 A plane wall with variable heat generation and variable thermal conductivity is subjected to specified heat flux $\dot{q}_0$ and convection at the left boundary (node 0) and radiation at the right boundary (node 5). The complete finite difference formulation of this problem is to be obtained.

Assumptions **1** Heat transfer through the wall is given to be steady and one-dimensional, and the thermal conductivity and heat generation to be variable. **2** Convection heat transfer at the right surface is negligible.



Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the finite difference formulations become

Node 0 (at left boundary):

$$\dot{q}_0A+hA(T_\infty-T_0)+k_0A\frac{T_1-T_0}{\Delta x}+\dot{e}_0(A\Delta x/2)=0$$

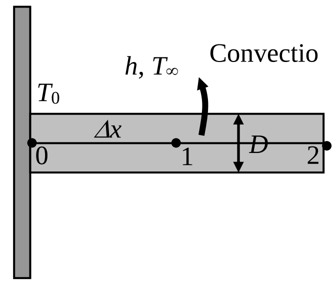
Node 1 (at the mid plane): $$k_1A\frac{T_0-T_1}{\Delta x}+k_1A\frac{T_2-T_1}{\Delta x}+\dot{e}_1(A\Delta x)=0$$

Node 2 (at right boundary): $$\varepsilon\sigma A(T_{\rm surr}^4-T_2^4)+k_2A\frac{T_1-T_2}{\Delta x}+\dot{e}_2(A\Delta x/2)=0$$

5-25 A pin fin with negligible heat transfer from its tip is considered. The complete finite difference formulation for the determination of nodal temperatures is to be obtained.

Assumptions **1** Heat transfer through the pin fin is given to be steady and one-dimensional, and the thermal conductivity to be constant. **2** Convection heat transfer coefficient is constant and uniform. **3** Radiation heat transfer is negligible. **4** Heat loss from the fin tip is given to be negligible.

Analysis The nodal network consists of 3 nodes, and the base temperature T_0 at node 0 is specified. Therefore, there are two unknowns T_1 and T_2, and we need two equations to determine them. Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the finite difference formulations become



Node 1 (at midpoint): $$kA\frac{T_0 - T_1}{\Delta x} + kA\frac{T_2 - T_1}{\Delta x} + hp\Delta x(T_\infty - T_1) = 0$$

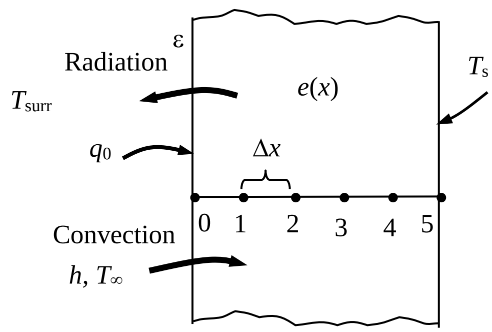
Node 2 (at fin tip): $$kA\frac{T_1 - T_2}{\Delta x} + h(p\Delta x/2)(T_\infty - T_2) = 0$$

where $A = \pi D^2/4$ is the cross-sectional area and $p = \pi D$ is the perimeter of the fin.

5-26 A plane wall with variable heat generation and constant thermal conductivity is subjected to combined convection, radiation, and heat flux at the left (node 0) and specified temperature at the right boundary (node 5). The finite difference formulation of the left boundary node (node 0) and the finite difference formulation for the rate of heat transfer at the right boundary (node 5) are to be determined.

Assumptions **1** Heat transfer through the wall is given to be steady and one-dimensional. **2** The thermal conductivity is given to be constant.

Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the finite difference formulations become



Left boundary node (all temperatures are in K):

$$\varepsilon\sigma A(T_{surr}^4 - T_0^4) + hA(T_\infty - T_0) + kA\frac{T_1 - T_0}{\Delta x} + \dot{q}_0 A + \dot{e}_0(A\Delta x/2) = 0$$

Heat transfer at right surface:

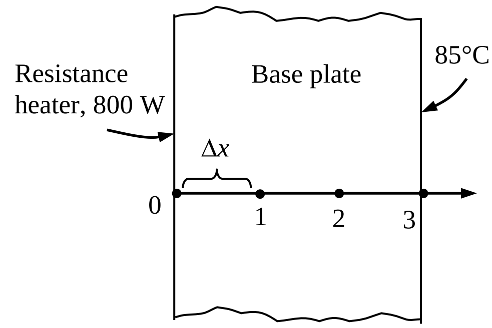
$$\dot{Q}_{\text{right surface}} + kA\frac{T_4 - T_5}{\Delta x} + \dot{e}_5(A\Delta x/2) = 0$$

5-27 A plate is subjected to specified heat flux on one side and specified temperature on the other. The finite difference formulation of this problem is to be obtained, and the unknown surface temperature under steady conditions is to be determined.

Assumptions **1** Heat transfer through the base plate is given to be steady. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness. **3** There is no heat generation in the plate. **4** Radiation heat transfer is negligible. **5** The entire heat generated by the resistance heaters is transferred through the plate.

Properties The thermal conductivity is given to be k = 20 W/m·°C.

Analysis The nodal spacing is given to be Δx=0.2 cm. Then the number of nodes M becomes



$$M = \frac{L}{\Delta x} + 1 = \frac{0.6\,\text{cm}}{0.2\,\text{cm}} + 1 = 4$$

The right surface temperature is given to be T_3 =85°C. This problem involves 3 unknown nodal temperatures, and thus we need to have 3 equations to determine them uniquely. Nodes 1 and 2 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0 \rightarrow T_{m-1} - 2T_m + T_{m+1} = 0 \quad (\text{since } \dot{e} = 0), \quad \text{for } m = 1 \text{ and } 2$$

The finite difference equation for node 0 on the left surface subjected to uniform heat flux is obtained by applying an energy balance on the half volume element about node 0 and taking the direction of all heat transfers to be towards the node under consideration:

$$\text{Node 0 (left surface- heat flux)}: \qquad \dot{q}_0 + k\frac{T_1 - T_0}{\Delta x} = 0$$

$$\text{Node 1 (interior)}: \qquad T_0 - 2T_1 + T_2 = 0$$

$$\text{Node 2 (interior)}: \qquad T_1 - 2T_2 + T_3 = 0$$

where

$$\Delta x = 0.2\,\text{cm},\ k = 20\ \text{W/m}\cdot°\text{C},\ T_3 = 85°\text{C, and } \dot{q}_0 = \dot{Q}_0 / A = (800\text{W})/(0.0160\,\text{m}^2) = 50{,}000\ \text{W/m}^2.$$

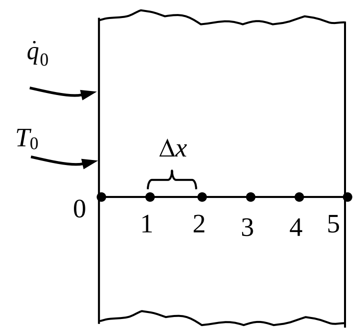
The system of 3 equations with 3 unknown temperatures constitute the finite difference formulation of the problem.

(*b*) The nodal temperatures under steady conditions are determined by solving the 3 equations above simultaneously with an equation solver to be

T_0 = **100°C**, T_1 =95°C, and T_2 =90°C

Discussion This problem can be solved analytically by solving the differential equation as described in Chap. 2, and the analytical (exact) solution can be used to check the accuracy of the numerical solution above.

5-28 A plane wall is subjected to specified heat flux and specified temperature on one side, and no conditions on the other. The finite difference formulation of this problem is to be obtained, and the temperature of the other side under steady conditions is to be determined.



Assumptions **1** Heat transfer through the plate is given to be steady and one-dimensional. **2** There is no heat generation in the plate.

Properties The thermal conductivity is given to be k = 1.8 W/m·°C.

Analysis The nodal spacing is given to be Δx=0.06 m.

Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{0.3\,\text{m}}{0.06\,\text{m}} + 1 = 6$$

Nodes 1, 2, 3, and 4 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0 \rightarrow T_{m+1} - 2T_m + T_{m-1} = 0 \quad (\text{since}\,\dot{e} = 0), \quad \text{for} \;\; m = 1, 2, 3, \text{and } 4$$

The finite difference equation for node 0 on the left surface is obtained by applying an energy balance on the half volume element about node 0 and taking the direction of all heat transfers to be towards the node under consideration,

$$\dot{q}_0 + k\frac{T_1 - T_0}{\Delta x} = 0 \quad \longrightarrow \quad 350\ \text{W/m}^2 + (1.8\ \text{W/m}\cdot°\text{C})\frac{T_1 - 60°\text{C}}{0.06\,\text{m}} = 0 \quad \longrightarrow \quad T_1 = 48.3°\text{C}$$

Other nodal temperatures are determined from the general interior node relation as follows:

$$m = 1: \qquad T_2 = 2T_1 - T_0 = 2\times 48.3 - 60 = 36.6°\text{C}$$
$$m = 2: \qquad T_3 = 2T_2 - T_1 = 2\times 36.6 - 48.3 = 24.9°\text{C}$$
$$m = 3: \qquad T_4 = 2T_3 - T_2 = 2\times 24.9 - 36.6 = 13.2°\text{C}$$
$$m = 4: \qquad T_5 = 2T_4 - T_3 = 2\times 13.2 - 24.9 = \mathbf{1.5°C}$$

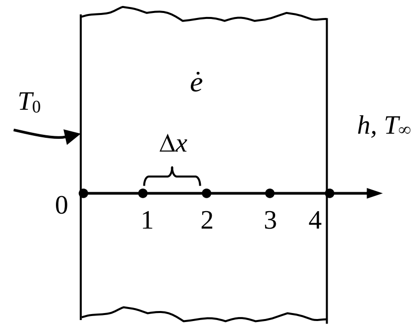
Therefore, the temperature of the other surface will be 1.5°C

Discussion This problem can be solved analytically by solving the differential equation as described in Chap. 2, and the analytical (exact) solution can be used to check the accuracy of the numerical solution above.

5-29 A plane wall is subjected to specified temperature on one side and convection on the other. The finite difference formulation of this problem is to be obtained, and the nodal temperatures under steady conditions as well as the rate of heat transfer through the wall are to be determined.

Assumptions **1** Heat transfer through the wall is given to be steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation. **4** Radiation heat transfer is negligible.

Properties The thermal conductivity is given to be k = 2.3 W/m·°C.



Analysis The nodal spacing is given to be Δx=0.1 m. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{0.4\,\text{m}}{0.1\,\text{m}} + 1 = 5$$

The left surface temperature is given to be T_0 = 95°C. This problem involves 4 unknown nodal temperatures, and thus we need to have 4 equations to determine them uniquely. Nodes 1, 2, and 3 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0 \rightarrow T_{m-1} - 2T_m + T_{m+1} = 0 \quad (\text{since}\, \dot{e} = 0), \quad \text{for } m = 0, 1, 2, \text{and } 3$$

The finite difference equation for node 4 on the right surface subjected to convection is obtained by applying an energy balance on the half volume element about node 4 and taking the direction of all heat transfers to be towards the node under consideration:

$$\text{Node 1 (interior)}: \quad T_0 - 2T_1 + T_2 = 0$$

$$\text{Node 2 (interior)}: \quad T_1 - 2T_2 + T_3 = 0$$

$$\text{Node 3 (interior)}: \quad T_2 - 2T_3 + T_4 = 0$$

$$\text{Node 4 (right surface- convection)}: \quad h(T_\infty - T_4) + k\frac{T_3 - T_4}{\Delta x} = 0$$

where

$$\Delta x = 0.1\,\text{m},\; k = 2.3\,\text{W/m}\cdot{}^\circ\text{C},\; h = 18\,\text{W/m}^2\cdot{}^\circ\text{C},\, T_0 = 95^\circ\text{C and } T_\infty = 15^\circ\text{C}.$$

The system of 4 equations with 4 unknown temperatures constitute the finite difference formulation of the problem.

(b) The nodal temperatures under steady conditions are determined by solving the 4 equations above simultaneously with an equation solver to be

T_1 = **79.8°C**, T_2 = **64.7°C**, T_3 = **49.5°C**, and T_4 = **34.4°C**

(c) The rate of heat transfer through the wall is simply convection heat transfer at the right surface,

$$\dot{Q}_{\text{wall}} = \dot{Q}_{\text{conv}} = hA(T_4 - T_\infty) = (18\,\text{W/m}^2.{}^\circ\text{C})(20\,\text{m}^2)(34.37\text{-}15)^\circ\text{C} = \mathbf{6970\ W}$$

Discussion This problem can be solved analytically by solving the differential equation as described in Chap. 2, and the analytical (exact) solution can be used to check the accuracy of the numerical solution above.

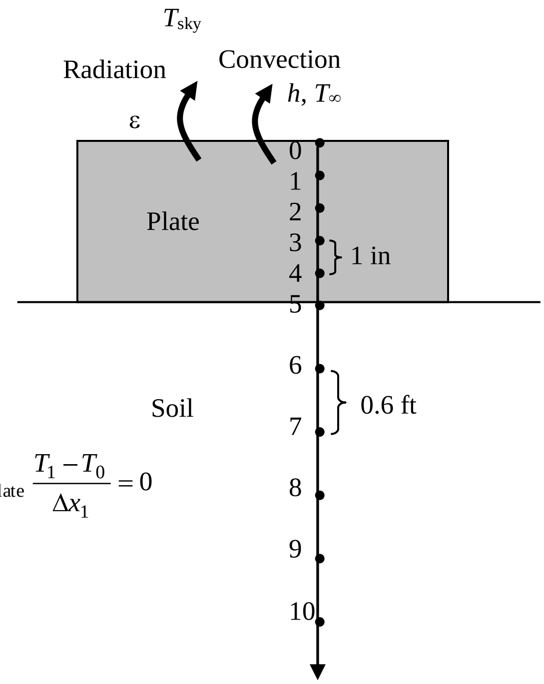
5-30E A large plate lying on the ground is subjected to convection and radiation. Finite difference formulation is to be obtained, and the top and bottom surface temperatures under steady conditions are to be determined.

Assumptions **1** Heat transfer through the plate is given to be steady and one-dimensional. **2** There is no heat generation in the plate and the soil. **3** Thermal contact resistance at plate-soil interface is negligible.

Properties The thermal conductivity of the plate and the soil are given to be k_{plate} = 7.2 Btu/h·ft·°F and k_{soil} = 0.49 Btu/h·ft·°F.

Analysis The nodal spacing is given to be Δx_1=1 in. in the plate, and be Δx_2=0.6 ft in the soil. Then the number of nodes becomes

$$M = \left(\frac{L}{\Delta x}\right)_{\text{plate}} + \left(\frac{L}{\Delta x}\right)_{\text{soil}} + 1 = \frac{5\text{ in}}{1\text{ in}} + \frac{3\text{ ft}}{0.6\text{ ft}} + 1 = 11$$



The temperature at node 10 (bottom of thee soil) is given to be T_{10} =50°F. Nodes 1, 2, 3, and 4 in the plate and 6, 7, 8, and 9 in the soil are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0 \rightarrow T_{m-1} - 2T_m + T_{m+1} = 0 \quad (\text{since } \dot{e} = 0)$$

The finite difference equation for node 0 on the left surface and node 5 at the interface are obtained by applying an energy balance on their respective volume elements and taking the direction of all heat transfers to be towards the node under consideration:

$$\text{Node 0 (top surface):} \quad h(T_\infty - T_0) + \varepsilon\sigma[T_{sky}^4 - (T_0 + 460)^4] + k_{\text{plate}} \frac{T_1 - T_0}{\Delta x_1} = 0$$

$$\text{Node 1 (interior):} \quad T_0 - 2T_1 + T_2 = 0$$

$$\text{Node 2 (interior):} \quad T_1 - 2T_2 + T_3 = 0$$

$$\text{Node 3 (interior):} \quad T_2 - 2T_3 + T_4 = 0$$

$$\text{Node 4 (interior):} \quad T_3 - 2T_4 + T_5 = 0$$

$$\text{Node 5 (interface):} \quad k_{\text{plate}} \frac{T_4 - T_5}{\Delta x_1} + k_{\text{soil}} \frac{T_6 - T_5}{\Delta x_2} = 0$$

$$\text{Node 6 (interior):} \quad T_5 - 2T_6 + T_7 = 0$$

$$\text{Node 7 (interior):} \quad T_6 - 2T_7 + T_8 = 0$$

$$\text{Node 8 (interior):} \quad T_7 - 2T_8 + T_9 = 0$$

$$\text{Node 9 (interior):} \quad T_8 - 2T_9 + T_{10} = 0$$

where

Δx_1=1/12 ft, Δx_2=0.6 ft, k_{plate} = 7.2 Btu/h·ft·°F, k_{soil} = 0.49 Btu/h·ft·°F,

h = 3.5 Btu/h·ft²·°F, T_{sky} =510 R, ε = 0.6, T_∞ = 80°F, and T_{10} =50°F.

This system of 10 equations with 10 unknowns constitute the finite difference formulation of the problem.

(*b*) The temperatures are determined by solving equations above to be

T_0 = **74.71°F**, T_1 =74.67°F, T_2 =74.62°F, T_3 =74.58°F, T_4 =74.53°F,

T_5 = **74.48°F**, T_6 =69.6°F, T_7 =64.7°F, T_8 =59.8°F, T_9 =54.9°F

Discussion Note that the plate is essentially isothermal at about 74.6°F. Also, the temperature in each layer varies linearly and thus we could solve this problem by considering 3 nodes only (one at the interface and two at the boundaries).

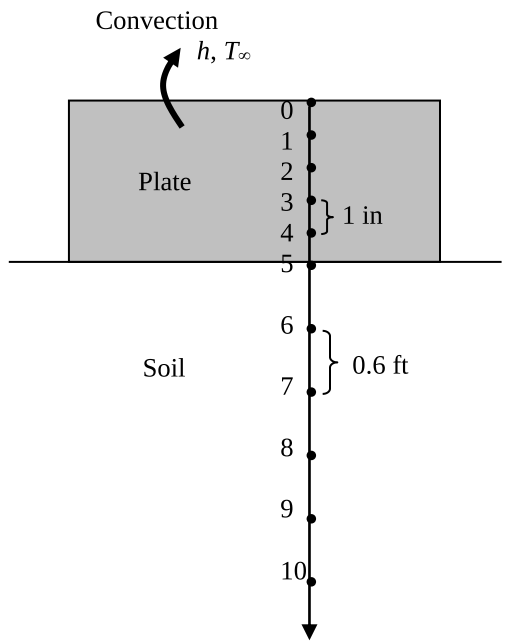
5-31E A large plate lying on the ground is subjected to convection from its exposed surface. The finite difference formulation of this problem is to be obtained, and the top and bottom surface temperatures under steady conditions are to be determined.

Assumptions **1** Heat transfer through the plate is given to be steady and one-dimensional. **2** There is no heat generation in the plate and the soil. **3** The thermal contact resistance at the plate-soil interface is negligible. **4** Radiation heat transfer is negligible.

Properties The thermal conductivity of the plate and the soil are given to be k_{plate} = 7.2 Btu/h·ft·°F and k_{soil} = 0.49 Btu/h·ft·°F.

Analysis The nodal spacing is given to be Δx_1=1 in. in the plate, and be Δx_2=0.6 ft in the soil. Then the number of nodes becomes

$$M = \left(\frac{L}{\Delta x}\right)_{plate} + \left(\frac{L}{\Delta x}\right)_{soil} + 1 = \frac{5\text{ in}}{1\text{ in}} + \frac{3\text{ ft}}{0.6\text{ ft}} + 1 = 11$$



The temperature at node 10 (bottom of thee soil) is given to be T_{10} =50°F. Nodes 1, 2, 3, and 4 in the plate and 6, 7, 8, and 9 in the soil are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0 \rightarrow T_{m-1} - 2T_m + T_{m+1} = 0 \quad (\text{since } \dot{e} = 0)$$

The finite difference equation for node 0 on the left surface and node 5 at the interface are obtained by applying an energy balance on their respective volume elements and taking the direction of all heat transfers to be towards the node under consideration:

$$\text{Node 0 (top surface):} \quad h(T_\infty - T_0) + k_{plate}\frac{T_1 - T_0}{\Delta x_1} = 0$$

$$\text{Node 1 (interior):} \quad T_0 - 2T_1 + T_2 = 0$$

$$\text{Node 2 (interior):} \quad T_1 - 2T_2 + T_3 = 0$$

$$\text{Node 3 (interior):} \quad T_2 - 2T_3 + T_4 = 0$$

$$\text{Node 4 (interior):} \quad T_3 - 2T_4 + T_5 = 0$$

$$\text{Node 5 (interface):} \quad k_{plate}\frac{T_4 - T_5}{\Delta x_1} + k_{soil}\frac{T_6 - T_5}{\Delta x_2} = 0$$

$$\text{Node 6 (interior):} \quad T_5 - 2T_6 + T_7 = 0$$

$$\text{Node 7 (interior):} \quad T_6 - 2T_7 + T_8 = 0$$

$$\text{Node 8 (interior):} \quad T_7 - 2T_8 + T_9 = 0$$

$$\text{Node 9 (interior):} \quad T_8 - 2T_9 + T_{10} = 0$$

where

Δx_1=1/12 ft, Δx_2=0.6 ft, k_{plate} = 7.2 Btu/h·ft·°F, k_{soil} = 0.49 Btu/h·ft·°F,

h = 3.5 Btu/h·ft²·°F, $T_\infty = 80°\text{F}$, and T_{10} =50°F.

This system of 10 equations with 10 unknowns constitute the finite difference formulation of the problem.

(*b*) The temperatures are determined by solving equations above to be

T_0 = **78.67°F**, T_1 =78.62°F, T_2 =78.57°F, T_3 =78.51°F, T_4 =78.46°F,

T_5 = **78.41°F**, T_6 =72.7°F, T_7 =67.0°F, T_8 =61.4°F, T_9 =55.7°F

Discussion Note that the plate is essentially isothermal at about 78.6°F. Also, the temperature in each layer varies linearly and thus we could solve this problem by considering 3 nodes only (one at the interface and two at the boundaries).

5-32 A steel plate with no internal heat generation is subjected to a uniform heat flux on its top surface while the bottom surface is cooled convectively by a fluid at 10ºC and having h = 150 W/m^2·K. Using finite difference formulation, the temperature at the midpoint of the plate is to be determined.

Assumptions **1** Steady state 1-D heat transfer in lateral direction. **2** Constant thermal conductivity of the steel plate.

Properties Thermal conductivity of the steel plate is given as 35 W/m·K.

Analysis To discretize the plate of thickness 0.1 m into four equal parts, each part must be of length 0.025 m i.e.,

$$\Delta x = 0.025 \text{ m}$$

And hence the number of nodes is

$$M = 1 + \frac{L}{\Delta x} = 1 + \frac{0.1}{0.025} = 5$$

This problem involves 5 unknown nodal temperatures and hence we need 5 equations to determine these temperatures. The steel plate thickness is discretized such that the node 0 is on the bottom of the plate exposed to convective environment while the node 4 is on the top surface exposed to the uniform heat flux. Nodes 1, 2 and 3 are the internal nodes and their temperature can be expressed using general form of the finite difference formulation.

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0 \quad \text{for } m = 1, 2, \text{ and } 3$$

The finite difference formulation at the top and bottom surfaces can be obtained by applying energy balance on the half volume element around these nodes and considering all heat transfers towards these nodes. The finite difference equations for different nodes without internal heat generation are as follows.

Node 0 (Bottom node): $hA(T_\infty - T_0) + kA\left(\frac{T_1 - T_0}{\Delta x}\right) = 0$

Node 1 (Interior node): $\frac{T_0 - 2T_1 + T_2}{\Delta x^2} = 0$

Node 2 (Interior node): $\frac{T_1 - 2T_2 + T_3}{\Delta x^2} = 0$

Node 3 (Interior node): $\frac{T_2 - 2T_3 + T_4}{\Delta x^2} = 0$

Node 4 (Top node): $\dot{q}_0 A + kA\left(\frac{T_3 - T_4}{\Delta x}\right) = 0$

where

$$\Delta x = 0.025 \text{ m}, h = 150 \text{ W/m}^2\cdot\text{K}, k = 35 \text{ W/m}\cdot\text{K}, \dot{q}_0 = 5500 \text{ W/m}^2 \text{ and } T_\infty = 10^\circ\text{C}$$

Solving these five equations for five unknowns using EES or any other software gives

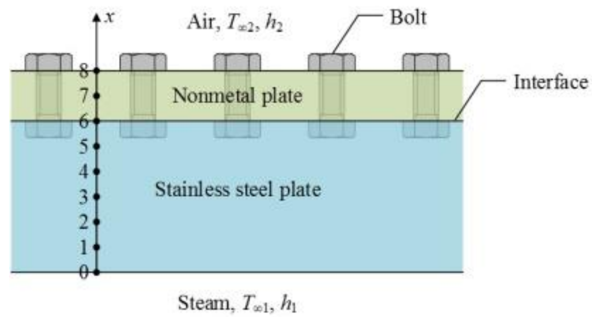
T_0 = **46.67ºC**, T_1 = **50.6ºC**, T_2 = **54.52ºC**, T_3 = **58.45ºC**, T_4 = **62.38ºC**.

5-33 A nonmetal plate is bolted on an ASTM A240 904L stainless steel plate by ASTM B211 6061 aluminum alloy bolts. The upper surface is exposed to convection with air, and the bottom surface is exposed to convection with hot steam. Determine the nodal temperatures and whether the use of the ASTM A240 904L plate and ASTM B211 6061 bolts complies with the ASME Code for Process Piping.

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through plates. **3** Uniform surface temperatures. **4** Thermal properties are constant. **5** Thermal radiation is negligible. **6** Thermal contact resistance is negligible.

Properties The thermal conductivity for the ASTM A240 904L stainless steel plate is k_1 = 13 W/m·K and for the nonmetal plate is k_2 = 3 W/m·K.

Analysis The nodal spacing is given as Δx = 5 mm. So, the number of nodes is



$$M = \frac{L}{\Delta x} + 1 = \frac{(30 + 10)\text{mm}}{5\text{ mm}} + 1 = 9$$

The nodes are numbered from m = 0 to 8. The finite difference formulations for the nodes are

$$m = 0: \quad h_1(T_{\infty 1} - T_0) + \frac{k_1}{\Delta x}(T_1 - T_0) = 0 \quad \text{or} \quad h_1 T_{\infty 1} - \left(h_1 + \frac{k_1}{\Delta x}\right)T_0 + \frac{k_1}{\Delta x}T_1 = 0$$

$$m = 1 \text{ to } 5: \quad \frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x} = 0 \quad \text{or} \quad T_{m-1} - 2T_m + T_{m+1} = 0$$

$$m = 6: \quad k_1(T_5 - T_6) + k_2(T_7 - T_6) = 0 \quad \text{or} \quad k_1 T_5 - (k_1 + k_2)T_6 + k_2 T_7 = 0$$

$$m = 7: \quad \frac{T_6 - 2T_7 + T_8}{\Delta x} = 0 \quad \text{or} \quad T_6 - 2T_7 + T_8 = 0$$

$$m = 8: \quad h_2(T_{\infty 2} - T_8) + \frac{k_2}{\Delta x}(T_7 - T_8) = 0 \quad \text{or} \quad h_2 T_{\infty 2} + \frac{k_2}{\Delta x}T_7 - \left(h_2 + \frac{k_2}{\Delta x}\right)T_8 = 0$$

Note that nodes 0 and 8 are convection boundaries; nodes 1–5 and 7 are interior nodes; node 6 is an interface boundary.

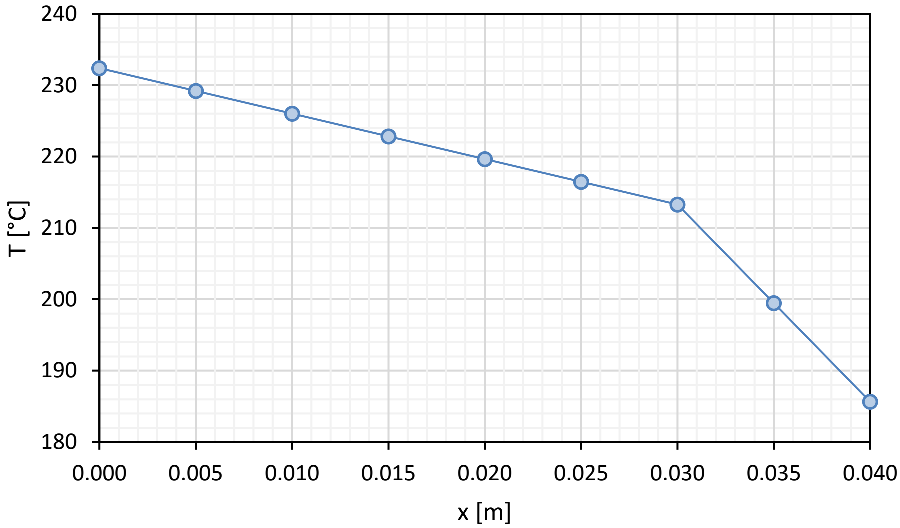
Solving for the nodal temperatures, T_0 to T_8, yields

$$T_0 = \mathbf{232.4}°\text{C}, \ T_1 = \mathbf{229.2}°\text{C}, \ T_2 = \mathbf{226.0}°\text{C}, \ T_3 = \mathbf{222.8}°\text{C}, \ T_4 = \mathbf{219.6}°\text{C},$$
$$T_5 = \mathbf{216.5}°\text{C}, \ T_6 = \mathbf{213.3}°\text{C}, \ T_7 = \mathbf{199.5}°\text{C}, \ T_8 = \mathbf{185.7}°\text{C}$$

where,

$$h_1 = 300 \text{ W/m}^2\text{·K}, \ h_2 = 50 \text{ W/m}^2\text{·K}$$
$$k_1 = 13 \text{ W/m·K}, \ k_2 = 3 \text{ W/m·K}$$
$$T_{\infty 1} = 260°\text{C}, \ T_{\infty 2} = 20°\text{C}$$
$$\frac{k_1}{\Delta x} = 2600 \text{ W/m}^2\text{·K}, \quad \frac{k_2}{\Delta x} = 600 \text{ W/m}^2\text{·K}$$

The temperature distribution in the plates as a function of $x$is plotted in the following figure:

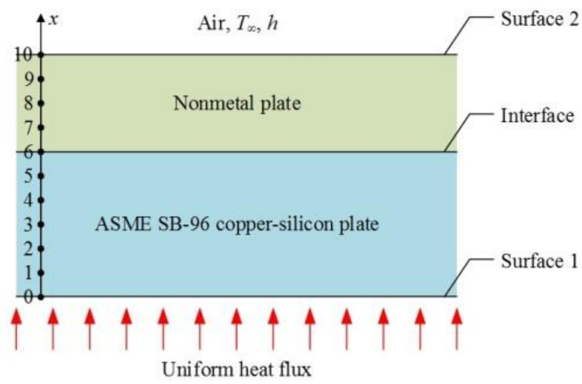


Thus, the range of temperature experienced by nonmetal plate is between T_6 and T_8, or $185.7 \le T \le 213.3$°C.

Discussion Since the operating temperature for the ASTM A240 904L plate is between T_0 = 232.4°C and T_6 = 213.3°C, therefore it operates below the maximum use temperature of 260°C and is in compliance with the ASME Code for Process Piping. The ASTM B211 6061 bolts are mainly in the nonmetal plate, and the bolts have a maximum use temperature of 204°C (ASME B31.3-2014). The temperatures in the nonmetal plate are between T_6 = 213.3°C and T_8 = 185.7°C. Thus, part of the bolts is operating in temperatures above the maximum use temperature, and would not be in compliance with the ASME code.

5-34 C&S A nonmetal plate and an ASME SB-96 copper-silicon plate are attached together. The bottom surface is subjected to uniform heat flux. The top surface is exposed to convection heat transfer. Determine the nodal temperatures, and whether the use of the ASME SB-96 plate complies with the ASME Boiler and Pressure Vessel Code. Also, what is the lowest value of the convection heat transfer coefficient for the air on the upper surface so that the ASME SB-96 plate is below 93°C?

Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through plates. **3** Uniform heat flux on bottom surface. **4** Uniform surface temperature. **5** No contact resistance at the interface. **6** Thermal properties are constant. **7** Thermal radiation is negligible.



Properties The thermal conductivity for the ASME SB-96 copper-silicon plate is given as k_1 = 36 W/m·K, and for the nonmetal plate as k_2 = 0.05 W/m·K.

Analysis The nodal spacing is given as Δx = 5 mm. So, the number of nodes is

$$M = \frac{L}{\Delta x} + 1 = \frac{(30 + 20)\text{mm}}{5 \text{ mm}} + 1 = 11$$

The nodes are numbered from m = 0 to 10. The finite difference formulations for the nodes are

$$m = 0: \qquad \dot{q}_0 + \frac{k_1}{\Delta x}(T_1 - T_0) = 0 \qquad \text{or} \quad \dot{q}_0 - \frac{k_1}{\Delta x}T_0 + \frac{k_1}{\Delta x}T_1 = 0$$

$$m = 1 \text{ to } 5: \qquad \frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x} = 0 \qquad \text{or} \quad T_{m-1} - 2T_m + T_{m+1} = 0$$

$$m = 6: \qquad k_1(T_5 - T_6) + k_2(T_7 - T_6) = 0 \qquad \text{or} \quad k_1T_5 - (k_1 + k_2)T_6 + k_2T_7 = 0$$

$$m = 7 \text{ to } 9: \qquad \frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x} = 0 \qquad \text{or} \quad T_{m-1} - 2T_m + T_{m+1} = 0$$

$$m = 10: \qquad h(T_\infty - T_{10}) + \frac{k_2}{\Delta x}(T_9 - T_{10}) = 0 \qquad \text{or} \qquad hT_\infty + \frac{k_2}{\Delta x}T_9 - \left(h + \frac{k_2}{\Delta x}\right)T_{10} = 0$$

Note that node 0 is a specified heat flux boundary; nodes 1–5 and 7–9 are interior nodes; node 6 is an interface boundary; node 10 is a convection boundary.

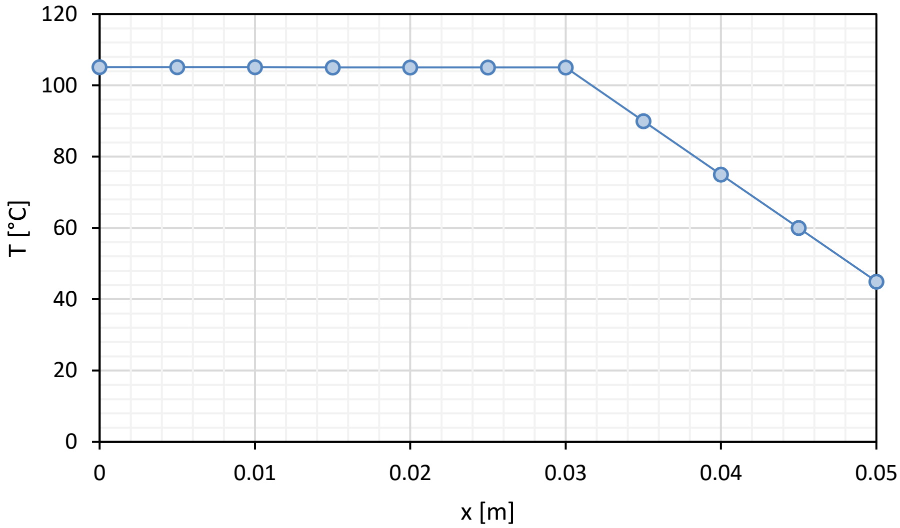
Solving for the nodal temperatures, T_0 to T_{10}, yields

$$T_0 = \mathbf{105.1}℃,\ T_1 = \mathbf{105.1}℃,\ T_2 = \mathbf{105.1}℃,\ T_3 = \mathbf{105.1}℃,\ T_4 = \mathbf{105}℃,$$
$$T_5 = \mathbf{105}℃,\ T_6 = \mathbf{105}℃,\ T_7 = \mathbf{90}℃,\ T_8 = \mathbf{75}℃,\ T_9 = \mathbf{60}℃,\ T_{10} = \mathbf{45}℃$$

where,

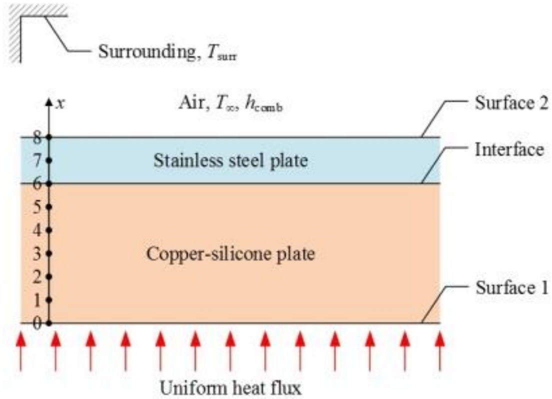
$$h = 5 \text{ W/m}^2\text{·K},\ k_1 = 36 \text{ W/m·K},\ k_2 = 0.05 \text{ W/m·K},\ T_\infty = 15℃,\ \dot{q}_0 = 150 \text{ W/m}^2$$
$$\frac{k_1}{\Delta x} = 7200 \text{ W/m}^2\text{·K}, \qquad \frac{k_2}{\Delta x} = 10 \text{ W/m}^2\text{·K}$$

The temperature distribution in the plates as a function of $x$is plotted in the following figure:



Discussion The ASME SB-96 copper-silicon plate is practically at a uniform temperature of 105°C. Therefore, it does not comply with the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HF-300). The code limits the use of equipment constructed with the ASME SB-96 plate to operate at below 93°C. To keep the ASME SB-96 plate below 93°C, the convection heat transfer coefficient for the air on the upper surface should be h> 8.5 W/m^2·K.

5-35 A stainless steel plate is attached on a copper-silicon plate. The upper surface is exposed to convection with air and thermal radiation with the surroundings. The bottom surface is subjected to a uniform heat flux. Determine the nodal temperatures, and whether the use of the ASME SB-96 plate complies with the ASME Boiler and Pressure Vessel Code. What is the highest heat flux that the bottom surface can be subjected to such that the ASME SB-96 plate is still operating at below 93°C?



Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through plates. **3** Uniform heat flux on bottom surface. **4** Uniform surface temperature. **5** No contact resistance at the interface. **6** Thermal properties are constant.

Properties The thermal conductivity for the copper-silicon plate is k_1 = 36 W/m·K and for the stainless steel plate is k_2 = 13 W/m.

Analysis The nodal spacing is given as Δx = 5 mm. So, the number of nodes is

$$M = \frac{L}{\Delta x} + 1 = \frac{(30+10)\text{mm}}{5\text{ mm}} + 1 = 9$$

The nodes are numbered from m = 0 to 8. The finite difference formulations for the nodes are

$$m = 0: \qquad \dot{q}_0 + \frac{k_1}{\Delta x}(T_1 - T_0) = 0 \qquad \text{or} \quad \dot{q}_0 - \frac{k_1}{\Delta x}T_0 + \frac{k_1}{\Delta x}T_1 = 0$$

$$m = 1 \text{ to } 5: \qquad \frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x} = 0 \qquad \text{or} \quad T_{m-1} - 2T_m + T_{m+1} = 0$$

$$m = 6: \qquad k_1(T_5 - T_6) + k_2(T_7 - T_6) = 0 \qquad \text{or} \quad k_1T_5 - (k_1 + k_2)T_6 + k_2T_7 = 0$$

$$m = 7: \qquad \frac{T_6 - 2T_7 + T_8}{\Delta x} = 0 \qquad \text{or} \quad T_6 - 2T_7 + T_8 = 0$$

$$m = 8: \qquad h_{\text{comb}}(T_\infty - T_8) + \frac{k_2}{\Delta x}(T_7 - T_8) = 0 \qquad \text{or} \quad h_{\text{comb}}T_\infty + \frac{k_2}{\Delta x}T_7 - \left(h + \frac{k_2}{\Delta x}\right)T_8 = 0$$

Note that node 0 is a specified heat flux boundary; nodes 1–5 and 7 are interior nodes; node 6 is an interface boundary; node 8 is a combined convection and radiation boundary. Solving for the nodal temperatures, T_0 to T_8, yields

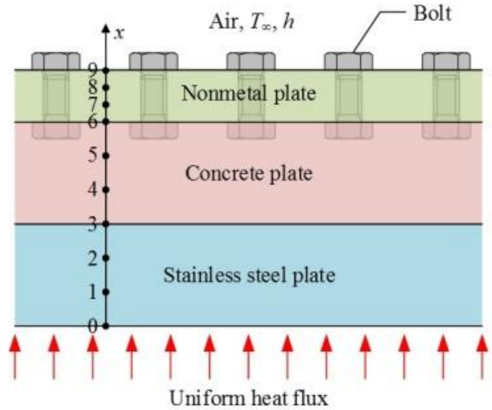
$$T_0 = \mathbf{117.9}°\text{C},\ T_1 = \mathbf{117.7}°\text{C},\ T_2 = \mathbf{117.6}°\text{C},\ T_3 = \mathbf{117.5}°\text{C},\ T_4 = \mathbf{117.4}°\text{C},$$
$$T_5 = \mathbf{117.3}°\text{C},\ T_6 = \mathbf{117.2}°\text{C},\ T_7 = \mathbf{116.9}°\text{C},\ T_8 = \mathbf{116.6}°\text{C}$$

where

$$h_{\text{comb}} = 7.76 \text{ W/m}^2\cdot\text{K},\ k_1 = 36 \text{ W/m}\cdot\text{K},\ k_2 = 13 \text{ W/m}\cdot\text{K},\ T_\infty = 20°\text{C},\ \dot{q}_0 = 750 \text{ W/m}^2$$
$$\frac{k_1}{\Delta x} = 7200 \text{ W/m}^2\cdot\text{K}, \qquad \frac{k_2}{\Delta x} = 2600 \text{ W/m}^2\cdot\text{K}$$

Discussion The average temperature that the ASME SB-96 copper-silicon plate would operate is 117.5°C, which is above the temperature (93°C) specified by the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HF-300). Thus, the copper-silicon plate is not in compliance with the ASME code. To keep the ASME SB-96 plate below 93°C, the heat flux subjected on the bottom surface should be $\dot{q}_0$ < 559 W/m^2.

5-36 A wall is made of a composite stainless steel, concrete, and nonmetal plates. A series of ASTM B21 naval brass bolts are bolted to the nonmetal plate. The upper surface is exposed to convection with air, while the bottom surface is subjected to a uniform heat flux. Determine the nodal temperatures, and whether the ASTM B21 bolts are in compliance with the ASME Code for Process Piping.



Assumptions **1** Heat transfer is steady. **2** One dimensional heat conduction through plates. **3** Uniform heat flux on bottom surface. **4** Uniform surface temperature. **5** No contact resistance at the interface. **6** Thermal properties are constant. **7** Thermal radiation is neglected.

Properties The thermal conductivity for the stainless steel plate is k_1 = 13 W/m·K, for the concrete plate is k_2 = 1.1 W/m·K, and for the nonmetal plate is k_3 = 0.1 W/m·K.

Analysis The nodal spacing is given as Δx_1 = 10 mm for the stainless steel and concrete plates, and Δx_2 = 5 mm for the nonmetal plate. So, the number of nodes is

$$M = \frac{L_1}{\Delta x_1} + \frac{L_2}{\Delta x_2} + 1 = \frac{(30+30)\text{mm}}{10\text{ mm}} + \frac{15}{5\text{ mm}} + 1 = 10$$

The nodes are numbered from m = 0 to 9. The finite difference formulations for the nodes are

$$m = 0: \qquad \dot{q}_0 + \frac{k_1}{\Delta x_1}(T_1 - T_0) = 0 \qquad \text{or} \qquad \dot{q}_0 - \frac{k_1}{\Delta x_1}T_0 + \frac{k_1}{\Delta x_1}T_1 = 0$$

$$m = 1 \text{ to } 2: \qquad \frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x_1} = 0 \qquad \text{or} \qquad T_{m-1} - 2T_m + T_{m+1} = 0$$

$$m = 3: \qquad k_1(T_2 - T_3) + k_2(T_4 - T_3) = 0 \qquad \text{or} \qquad k_1T_2 - (k_1 + k_2)T_3 + k_2T_4 = 0$$

$$m = 4 \text{ to } 5: \qquad \frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x_1} = 0 \qquad \text{or} \qquad T_{m-1} - 2T_m + T_{m+1} = 0$$

$$m = 6: \qquad \frac{k_2}{\Delta x_1}(T_5 - T_6) + \frac{k_3}{\Delta x_2}(T_7 - T_6) = 0 \qquad \text{or} \qquad \frac{k_2}{\Delta x_1}T_5 - \left(\frac{k_2}{\Delta x_1} + \frac{k_3}{\Delta x_2}\right)T_6 + \frac{k_3}{\Delta x_2}T_7 = 0$$

$$m = 7 \text{ to } 8: \qquad \frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x_2} = 0 \qquad \text{or} \qquad T_{m-1} - 2T_m + T_{m+1} = 0$$

$$m = 9: \qquad h(T_\infty - T_9) + \frac{k_3}{\Delta x_2}(T_8 - T_9) = 0 \qquad \text{or} \qquad hT_\infty + \frac{k_3}{\Delta x_2}T_8 - \left(h + \frac{k_3}{\Delta x_2}\right)T_9 = 0$$

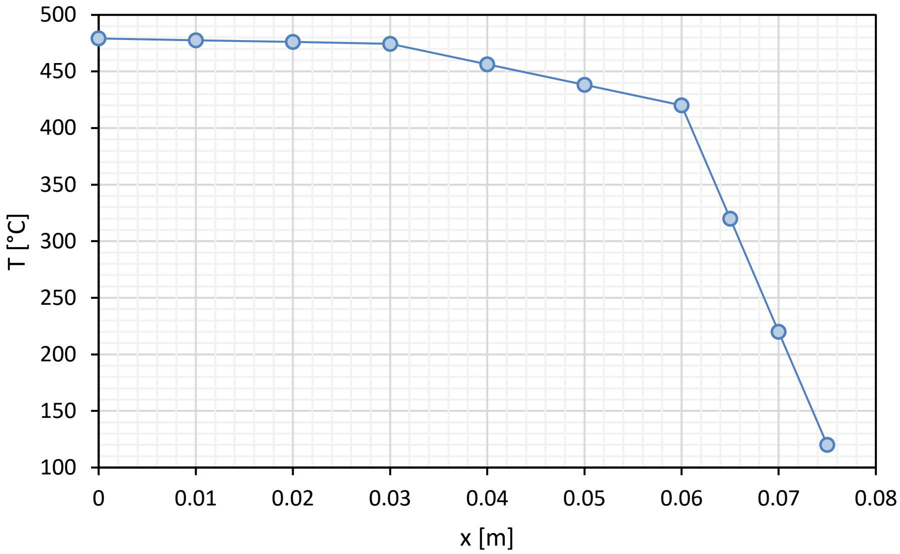
Note that node 0 is a specified heat flux boundary; nodes 1–2, 4–5 and 7–8 are interior nodes; nodes 3 and 6 are interface boundaries; node 9 is a convection boundary. Also, for node 6, the nodal spacing transitions from Δx_1 = 10 mm to Δx_2 = 5 mm.Solving for the nodal temperatures, T_0 to T_9, yields

$$T_0 = \mathbf{479.2}°\text{C},\ T_1 = \mathbf{477.6}°\text{C},\ T_2 = \mathbf{476.1}°\text{C},\ T_3 = \mathbf{474.5}°\text{C},\ T_4 = \mathbf{456.4}°\text{C},$$
$$T_5 = \mathbf{438.2}°\text{C},\ T_6 = \mathbf{420}°\text{C},\ T_7 = \mathbf{320}°\text{C},\ T_8 = \mathbf{220}°\text{C},\ T_9 = \mathbf{120}°\text{C}$$

where

$$h = 20\text{ W/m}^2\cdot\text{K},\ k_1 = 13\text{ W/m}\cdot\text{K},\ k_2 = 1.1\text{ W/m}\cdot\text{K},\ k_3 = 0.1\text{ W/m}\cdot\text{K},$$
$$T_\infty = 20°\text{C},\ \dot{q}_0 = 2000\text{ W/m}^2,\ \Delta x_1 = 10\text{ mm},\ \Delta x_2 = 5\text{ mm}$$
$$\frac{k_1}{\Delta x_1} = 1300\text{ W/m}^2\cdot\text{K}, \qquad \frac{k_2}{\Delta x_1} = 110\text{ W/m}^2\cdot\text{K}, \qquad \frac{k_3}{\Delta x_2} = 20\text{ W/m}^2\cdot\text{K}$$

The temperature distribution in the plates as a function of $x$is plotted in the following figure:



DiscussionThe ASTM B21 bolts are mainly in the nonmetal plate, and the bolts have a maximum use temperature of 149°C (ASME B31.3-2014). The temperatures in the nonmetal plate are between T_9 = 120°C and T_6 = 420°C, which gives an average temperature of 270°C. Thus, the ASTM B21 bolts would be operating in an average temperature that is above the maximum use temperature (149°C), which would make the bolts not in compliance with the code.

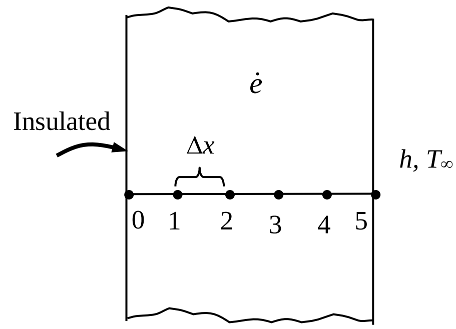
5-37 A uranium plate is subjected to insulation on one side and convection on the other. The finite difference formulation of this problem is to be obtained, and the nodal temperatures under steady conditions are to be determined.

Assumptions **1** Heat transfer through the wall is steady since there is no indication of change with time. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness. **3** Thermal conductivity is constant. **4** Radiation heat transfer is negligible.

Properties The thermal conductivity is given to be k = 34 W/m·°C.

Analysis The number of nodes is specified to be M = 6. Then the nodal spacing Δx becomes

$$\Delta x = \frac{L}{M-1} = \frac{0.05\,\text{m}}{6\text{-}1} = 0.01\,\text{m}$$



This problem involves 6 unknown nodal temperatures, and thus we need to have 6 equations to determine them uniquely. Node 0 is on insulated boundary, and thus we can treat it as an interior note by using the mirror image concept. Nodes 1, 2, 3, and 4 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0\,,\quad \text{for } m = 0, 1, 2, 3, \text{and } 4$$

Finally, the finite difference equation for node 5 on the right surface subjected to convection is obtained by applying an energy balance on the half volume element about node 5 and taking the direction of all heat transfers to be towards the node under consideration:

$$\text{Node 0 (Left surface- insulated)}: \quad \frac{T_1 - 2T_0 + T_1}{\Delta x^2} + \frac{\dot{e}}{k} = 0$$

$$\text{Node 1 (interior)}: \quad \frac{T_0 - 2T_1 + T_2}{\Delta x^2} + \frac{\dot{e}}{k} = 0$$

$$\text{Node 2 (interior)}: \quad \frac{T_1 - 2T_2 + T_3}{\Delta x^2} + \frac{\dot{e}}{k} = 0$$

$$\text{Node 3 (interior)}: \quad \frac{T_2 - 2T_3 + T_4}{\Delta x^2} + \frac{\dot{e}}{k} = 0$$

$$\text{Node 4 (interior)}: \quad \frac{T_3 - 2T_4 + T_5}{\Delta x^2} + \frac{\dot{e}}{k} = 0$$

$$\text{Node 5 (right surface- convection)}: \quad h(T_\infty - T_5) + k\frac{T_4 - T_5}{\Delta x} + \dot{e}(\Delta x/2) = 0$$

where

$$\Delta x = 0.01\,\text{m}, \dot{e} = 6\times 10^5\ \text{W/m}^3,\ k = 34\ \text{W/m}\cdot{}^\circ\text{C},\ h = 60\ \text{W/m}^2\cdot{}^\circ\text{C},\ \text{and } T_\infty = 30^\circ\text{C}.$$

This system of 6 equations with six unknown temperatures constitute the finite difference formulation of the problem.

(*b*) The 6 nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver to be

T_0 =**552.1°C**, T_1 =**551.2°C**, T_2 =**548.5°C**, T_3 =**544.1°C**, T_4 =**537.9°C**, and T_5 =**530.0°C**

Discussion This problem can be solved analytically by solving the differential equation as described in Chap. 2, and the analytical (exact) solution can be used to check the accuracy of the numerical solution above.

5-38 Prob. 5-37 is reconsidered. The nodal temperatures under steady conditions are to be determined.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
e_gen=6e5 [W/m^3] "heat generation"
dx=0.01 [m] "mesh size"
h=60 [W/m^2-K] "convection coefficient"
k=34 [W/m-K] "thermal conductivity"
T_inf=30 [C] "ambient temperature"

"ANALYSIS"
"Using the finite difference method, the nodal temperatures can be determined"
(T_1-T_0)/dx^2+e_gen/(2*k)=0 "for node 0"
(T_0-2*T_1+T_2)/dx^2+e_gen/k=0 "for node 1"
(T_1-2*T_2+T_3)/dx^2+e_gen/k=0 "for node 2"
(T_2-2*T_3+T_4)/dx^2+e_gen/k=0 "for node 3"
(T_3-2*T_4+T_5)/dx^2+e_gen/k=0 "for node 4"
h*(T_inf-T_5)+k*(T_4-T_5)/dx+e_gen*dx/2=0 "for node 5"
```

The nodal temperatures are determined to be

T_0 = **552°C**, T_1 = **551°C**, T_2 = **549°C**, T_3 = **544°C**, T_4 = **538°C**, and T_5 = **530°C**

5-39 A composite wall made up of two different materials is subjected to constant temperature and radiation boundary condition. The finite difference formulations and the temperature distribution across the wall thickness is to be determined

Assumptions **1** 1-D steady state heat conduction. **2** No internal heat generation in material B. **3** Constant thermal conductivities. **4** Perfect contact at the material A and B interface.

Properties Thermal conductivity of material A is k = 45 W/m·K and that of material B is k = 28 W/m·K.

Analysis Using a nodal spacing of 2.5 cm, the composite wall of thickness 20 cm (10 cm thick material A and B each) can be discretized into 8 equal parts (4 parts of material A and B each).

And hence the number of nodes is

$$M = 1 + \frac{L}{\Delta x} = 1 + \frac{0.2}{0.025} = 9$$

This problem involves 9 unknown nodal temperatures and hence we need 9 equations to determine these temperatures. The composite wall thickness is discretized such that the node 1 is on the left side boundary of the wall exposed to a constant heat flux while node 9 is placed such that it is on the right boundary of the composite wall. Node 5 is at the interface of material A and B.

The finite difference equations at all nodes are as follows

Node 1: (Left boundary node) $$\dot{q} + k_A \frac{(T_2 - T_1)}{\Delta x} + \dot{e}_m \frac{\Delta x}{2} = 0$$

Node 2: (Internal node) $$k_A \frac{(T_1 - T_2)}{\Delta x} + k_A \frac{(T_3 - T_2)}{\Delta x} + \dot{e}_m \Delta x = 0$$

Node 3: (Internal node) $$k_A \frac{(T_2 - T_3)}{\Delta x} + k_A \frac{(T_4 - T_3)}{\Delta x} + \dot{e}_m \Delta x = 0$$

Node 4: (Internal node) $$k_A \frac{(T_3 - T_4)}{\Delta x} + k_A \frac{(T_5 - T_4)}{\Delta x} + \dot{e}_m \Delta x = 0$$

Node 5: (Interface node) $$k_A \frac{(T_4 - T_5)}{\Delta x} + \dot{e}_m \frac{\Delta x}{2} + k_B \frac{(T_6 - T_5)}{\Delta x} = 0$$

Node 6: (Internal node) $$k_B \frac{(T_5 - T_6)}{\Delta x} + k_B \frac{(T_7 - T_6)}{\Delta x} = 0$$

Node 7: (Internal node) $$k_B \frac{(T_6 - T_7)}{\Delta x} + k_B \frac{(T_8 - T_7)}{\Delta x} = 0$$

Node 8: (Internal node) $$k_B \frac{(T_7 - T_{87})}{\Delta x} + k_B \frac{(T_9 - T_8)}{\Delta x} = 0$$

Node 9: (Right boundary node) $$h(T_\infty - T_9) + \varepsilon\sigma\left(T_{surr}^4 - T_9^4\right) + k_B \frac{(T_8 - T_9)}{\Delta x} = 0$$

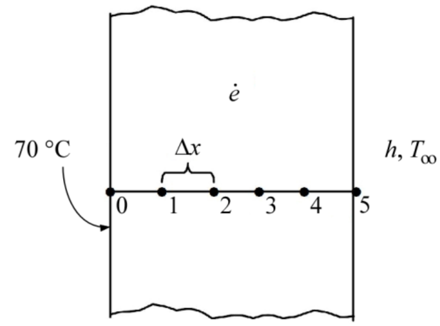
T_1 = **216.9ºC**, T_2 = **213.9ºC**, T_3 = **209.9ºC**, T_4 = **205ºC**, T_5 = **199.1ºC**,
T_6 = **188.8ºC**, T_7 = **178.6ºC**, T_8 = **168.3ºC**, T_9 = **158ºC**.

5-40 A stainless steel plane wall experiencing a uniform heat generation is subjected to constant temperature on one side and convection on the other. The finite difference equations and the nodal temperatures are to be determined.

Assumptions **1** Heat transfer through the wall is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity is given as 15.1 W/m·K.

Analysis (*a*) The nodal spacing is given to be Δx = 2 cm. Then the number of nodes *M* becomes



$$M = \frac{L}{\Delta x} + 1 = \frac{1\,\text{m}}{0.2\,\text{m}} + 1 = 6$$

The left surface temperature is given to be T_0 = 70°C. There are 5 unknown nodal temperatures, thus we need to have 5 equations to determine them uniquely. Nodes 1, 2, 3, and 4 are interior nodes, and we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0 \quad \rightarrow \quad T_{m-1} - 2T_m + T_{m+1} + \frac{\dot{e}_m}{k}\Delta x^2 = 0$$

The finite difference equation for node 5 on the right surface subjected to convection is obtained by applying an energy balance on the half volume element about that node:

$$k\frac{T_4 - T_5}{\Delta x} + \dot{e}_5\frac{\Delta x}{2} + h(T_\infty - T_5) = 0 \quad \rightarrow \quad T_4 - \left(1 + \frac{h}{k}\Delta x\right)T_5 + \frac{\Delta x^2}{2k}\dot{e}_5 + \frac{h}{k}\Delta x T_\infty = 0$$

Then

$$m = 1: \quad T_0 - 2T_1 + T_2 + (\dot{e}_1/k)\Delta x^2 = 0$$

$$m = 2: \quad T_1 - 2T_2 + T_3 + (\dot{e}_2/k)\Delta x^2 = 0$$

$$m = 3: \quad T_2 - 2T_3 + T_4 + (\dot{e}_3/k)\Delta x^2 = 0$$

$$m = 4: \quad T_3 - 2T_4 + T_5 + (\dot{e}_4/k)\Delta x^2 = 0$$

$$m = 5: \quad T_4 - (1 + h\Delta x/k)T_5 + (\Delta x^2\dot{e}_5)/(2k) + (h\Delta x/k)T_\infty = 0$$

(*b*) The nodal temperatures under steady conditions are determined by solving the 5 equations above simultaneously with an equation solver. Copy the following lines and paste on a blank EES screen to solve the above equations:

```
e_gen=1000
h=250
k=15.1
Dx=0.2
T_inf=0
T_0=70
T_0-2*T_1+T_2+(e_gen/k)*Dx^2=0
T_1-2*T_2+T_3+(e_gen/k)*Dx^2=0
T_2-2*T_3+T_4+(e_gen/k)*Dx^2=0
T_3-2*T_4+T_5+(e_gen/k)*Dx^2=0
T_4-(1+h*Dx/k)*T_5+(Dx^2*e_gen)/(2*k)+(h*Dx/k)*T_inf=0
```

Solving by EES software, we get

$$T_1 = \mathbf{62.5\,°C}, \quad T_2 = \mathbf{52.3\,°C}, \quad T_3 = \mathbf{39.5\,°C}, \quad T_4 = \mathbf{24.0\,°C}, \quad T_5 = \mathbf{5.87\,°C}$$

Discussion For a very large value of convection heat transfer coefficient (e.g. 20000 W/m^2·K), the right surface temperature would become approximately the same as the ambient fluid temperature ($T_5 \approx T_\infty$).

5-41 For a 0.1 m thick stainless steel plate exposed to a constant heat flux and convection environment, temperature distribution is to be determined for a case of variable thermal conductivity.

Assumptions **1** One-dimensional steady state heat conduction. **2** No internal heat generation.

Properties The thermal conductivity of the stainless steel plate is given as $k(T) = k_o(1+\beta T)$ where k_o = 48 W/m·K and β = 9.21x10^{-4} ^{o}C^{-1}

Analysis The one-dimensional first derivative of the finite difference formulation for an interior node 'm' in between node 'm-1' and 'm+2' is expressed as

$$k(T)\frac{dT}{dx}\bigg|_{m-\frac{1}{2}} = k_o\left[1+\beta\frac{(T_{m-1}+T_m)}{2}\right]\frac{(T_{m-1}-T_m)}{\Delta x} \quad \text{and} \quad k(T)\frac{dT}{dx}\bigg|_{m+\frac{1}{2}} = k_o\left[1+\beta\frac{(T_{m+1}+T_m)}{2}\right]\frac{(T_{m+1}-T_m)}{\Delta x}$$

Now accounting for the internal heat generation and combining these two equations to get the finite difference equation for the interior node 'm' gives

$$(T_{m+1}-T_m)\left[1+\frac{\beta}{2}(T_{m+1}+T_m)\right]+(T_{m-1}-T_m)\left[1+\frac{\beta}{2}(T_{m-1}+T_m)\right]+\dot{e}_m\frac{\Delta x^2}{k_o}=0$$

$$\therefore (T_{m-1}-2T_m+T_{m+1})+\frac{\beta}{2}\left[T_{m-1}^2-2T_m^2+T_{m+1}^2\right]+\dot{e}_m\frac{\Delta x^2}{k_o}=0$$

For the left boundary node exposed to constant heat flux we replace thermal conductivity '*k*' in Eq. (5-22) by $k(T) = k_o(1+\beta T)$ resulting into following equation

$$\therefore (T_{m+1}-T_m)+\frac{\beta}{2}\left(T_{m+1}^2-T_m^2\right)+\frac{\dot{q}\Delta x}{k_o}+\frac{\dot{e}_m\Delta x^2}{2k_o}=0$$

For the right boundary node exposed to convection environment we replace thermal conductivity '*k*' in Eq. (5-24) by $k(T) = k_o(1+\beta T)$ resulting into following equation

$$\therefore (T_{m-1}-T_m)+\frac{\beta}{2}\left(T_{m-1}^2-T_m^2\right)+\frac{h\Delta x}{k_o}(T_\infty-T_m)+\frac{\dot{e}_m\Delta x^2}{2k_o}=0$$

The finite difference equations for the interior and boundary nodes based on the above equations are expressed as follows

Node 0: (Left boundary node) $\quad (T_1-T_0)+\frac{\beta}{2}\left(T_1^2-T_0^2\right)+\frac{\dot{q}\Delta x}{k_o}+\frac{\dot{e}_m\Delta x^2}{2k_o}=0$

Node 1: (Internal node) $\quad (T_0-2T_1+T_2)+\frac{\beta}{2}\left(T_0^2-2T_1^2+T_2^2\right)+\frac{\dot{e}_m\Delta x^2}{k_o}=0$

Node 2: (Internal node) $\quad (T_1-2T_2+T_3)+\frac{\beta}{2}\left(T_1^2-2T_2^2+T_3^2\right)+\frac{\dot{e}_m\Delta x^2}{k_o}=0$

Node 3: (Internal node) $\quad (T_2-2T_3+T_4)+\frac{\beta}{2}\left(T_2^2-2T_3^2+T_4^2\right)+\frac{\dot{e}_m\Delta x^2}{k_o}=0$

Node 4: (Internal node) $\quad (T_3-2T_4+T_5)+\frac{\beta}{2}\left(T_3^2-2T_4^2+T_5^2\right)+\frac{\dot{e}_m\Delta x^2}{k_o}=0$

Node 5: (Right boundary node) $\quad (T_4-T_5)+\frac{\beta}{2}\left(T_4^2-T_5^2\right)+\frac{h\Delta x}{k_o}(T_\infty-T_5)+\frac{\dot{e}_m\Delta x^2}{2k_o}=0$

The temperature distribution in the stainless steel plate is found by solving these 5 equations simultaneously in EES or any other software.

```
"Given data"
k_o = 48 [W/mC] " Thermal conductivity"
beta = 9.21e-4 [C^-1] "Temperature coefficient of thermal conductivity"
e_dot = 8e5 [W/m^3] " Internal heat generation per unit volume"
q_dot = 2000 [W/m^2] "Heat flux at left boundary"
DELTAx = 0.02 [m] " Mesh size"
h = 400 [W/m^2C] "Convective heat transfer coefficient at right boundary"
T_infi = 0 [C] " Conective environment temperature"

"Finite difference equations"
"Node 0"(T[1]-T[0])+beta/2*(T[1]^2-T[0]^2)+q_dot*DELTAx/k_o+e_dot*DELTAx^2/(2*k_o) = 0
"Node 1"(T[0]-2*T[1]+T[2])+beta/2*(T[0]^2-2*T[1]^2+T[2]^2)+e_dot*DELTAx^2/k_o = 0
"Node 2"(T[1]-2*T[2]+T[3])+beta/2*(T[1]^2-2*T[2]^2+T[3]^2)+e_dot*DELTAx^2/k_o = 0
"Node 3"(T[2]-2*T[3]+T[4])+beta/2*(T[2]^2-2*T[3]^2+T[4]^2)+e_dot*DELTAx^2/k_o = 0
"Node 4"(T[3]-2*T[4]+T[5])+beta/2*(T[3]^2-2*T[4]^2+T[5]^2)+e_dot*DELTAx^2/k_o = 0
"Node 5"(T[4]-T[5])+beta/2*(T[4]^2-T[5]^2)+e_dot*DELTAx^2/(2*k_o)+h*DELTAx/k_o*(T_infi-T[5]) = 0
```

T_0 = **276.6 ºC**, T_1 = **273.3 ºC**, T_2 = **264.6 ºC**, T_3 = **250.5 ºC**, T_4 = **230.7 ºC**, T_5 = **205 ºC.**

Discussion Thermal conductivity as a function of temperature must be accounted for during finite difference formulations for high temperature applications such as metallurgical processes and thermal power generation.

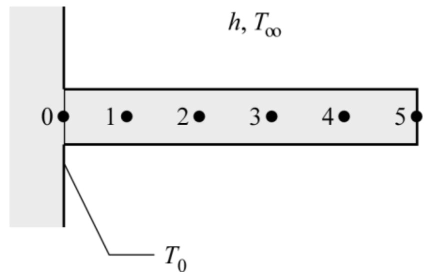
5-42 Straight rectangular fins are attached to a plane wall. For a single fin, (*a*) the finite difference equations, (*b*) the nodal temperatures, and (*c*) heat transfer rate are to be determined. The heat transfer rate is also to be compared with analytical solution.

Assumptions **1** Heat transfer along the fin is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity is given as 235 W/m·K.

Analysis (*a*) The nodal spacing is given to be Δx = 10 cm. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{50\text{ mm}}{10\text{ mm}} + 1 = 6$$



The base temperature at node 0 is given to be T_0 = 350°C. There are 5 unknown nodal temperatures, thus we need to have 5 equations to determine them uniquely. Nodes 1, 2, 3, and 4 are interior nodes, and we can use the general finite difference relation expressed as

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0$$

$$T_{m-1} - \left(2 + \frac{hp\Delta x^2}{kA}\right)T_m + T_{m+1} + \frac{hp\Delta x^2}{kA}T_\infty = 0$$

where

$$\frac{hp\Delta x^2}{kA} = \frac{h(2t+2w)\Delta x^2}{k(wt)} = \frac{(154\text{ W/m}^2\cdot\text{K})2(0.005\text{ m} + 0.1\text{ m})(0.01\text{ m})^2}{(235\text{ W/m}\cdot\text{K})(0.005\text{ m})(0.1\text{ m})} = 0.0275$$

The finite difference equation for node 5 at the fin tip (convection boundary) is obtained by applying an energy balance on the half volume element about that node:

$$kA\frac{T_4 - T_5}{\Delta x} + h\left(\frac{p\Delta x}{2} + A\right)(T_\infty - T_5) = 0$$

$$T_4 - \left[1 + \frac{h\Delta x}{kA}\left(\frac{p\Delta x}{2} + A\right)\right]T_5 + \frac{h\Delta x}{kA}\left(\frac{p\Delta x}{2} + A\right)T_\infty = 0$$

where

$$\frac{h\Delta x}{kA}\left(\frac{p\Delta x}{2} + A\right) = \frac{h\Delta x}{k}\left[\frac{(t+w)\Delta x}{wt} + 1\right] = 0.0203$$

Then,

$$m = 1:\quad T_0 - 2.0275T_1 + T_2 + 0.0275T_\infty = 0$$

$$m = 2:\quad T_1 - 2.0275T_2 + T_3 + 0.0275T_\infty = 0$$

$$m = 3:\quad T_2 - 2.0275T_3 + T_4 + 0.0275T_\infty = 0$$

$$m = 4:\quad T_3 - 2.0275T_4 + T_5 + 0.0275T_\infty = 0$$

$$m = 5:\quad T_4 - 1.0203T_5 + 0.0203T_\infty = 0$$

(*b*) The nodal temperatures under steady conditions are determined by solving the 5 equations above simultaneously with an equation solver. Copy the following lines and paste on a blank EES screen to solve the above equations:

```
T_0=350
T_0-2.0275*T_1+T_2+0.0275*25=0
T_1-2.0275*T_2+T_3+0.0275*25=0
T_2-2.0275*T_3+T_4+0.0275*25=0
T_3-2.0275*T_4+T_5+0.0275*25=0
T_4-1.0203*T_5+0.0203*25=0
```

Solving by EES software, we get

$$T_1 = \mathbf{316.6°C},\ T_2 = \mathbf{291.2°C},\ T_3 = \mathbf{273.2°C},\ T_4 = \mathbf{261.9°C},\ T_5 = \mathbf{257.2°C}$$

(c) The rate of heat transfer from a single fin is simply the sum of the heat transfer from the nodal elements,

$$\begin{aligned}\dot{Q}_{\text{fin, num}} &= \sum_{m=0}^{5} \dot{Q}_{\text{element}, m} = \sum_{m=0}^{5} hA_{\text{surface},\ m}(T_m - T_\infty) \\ &= hp\frac{\Delta x}{2}(T_0 - T_\infty) + hp\Delta x(T_1 + T_2 + T_3 + T_4 - 4T_\infty) + h\left(p\frac{\Delta x}{2} + A\right)(T_5 - T_\infty) \\ &= \mathbf{445\ W}\end{aligned}$$

For straight rectangular fins, the analytical solution from Chapter 3 for the heat transfer rate is,

$$\dot{Q}_{\text{fin, exact}} = \eta_{\text{fin}}\, hA_{\text{fin}}\,(T_b - T_\infty) = (0.813)(154\ \text{W/m}^2\cdot\text{K})(0.0105\ \text{m}^2)(350 - 25)\ ^\circ\text{C} = \mathbf{427\ W}$$

where

$$m = \sqrt{\frac{2h}{kt}} = 16.19\ \text{m}^{-1}$$

$$L_c = L + t/2 = 0.0525\ \text{m}$$

$$A_{\text{fin}} = 2wL_c = 0.0105\ \text{m}^2$$

$$\eta_{\text{fin}} = \frac{\tanh mL_c}{mL_c} = 0.813$$

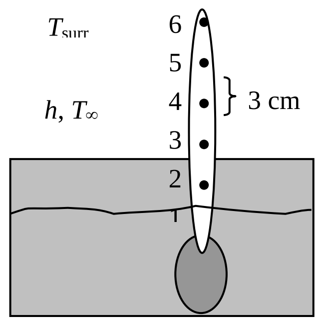
Discussion The comparison between the analytical and numerical solutions is within ±4.3% agreement. One way to increase the accuracy of the numerical solution is by reducing the nodal spacing, thereby increasing the number of nodes.

5-43 The handle of a stainless steel spoon partially immersed in boiling water loses heat by convection and radiation. The finite difference formulation of the problem is to be obtained, and the tip temperature of the spoon as well as the rate of heat transfer from the exposed surfaces are to be determined.

Assumptions **1** Heat transfer through the handle of the spoon is given to be steady and one-dimensional. **2** Thermal conductivity and emissivity are constant. **3** Convection heat transfer coefficient is constant and uniform.

Properties The thermal conductivity and emissivity are given to be k = 15.1 W/m·°C and ε = 0.6.

Analysis The nodal spacing is given to be Δx=3 cm. Then the number of nodes M becomes



$$M = \frac{L}{\Delta x} + 1 = \frac{18\,\text{cm}}{3\,\text{cm}} + 1 = 7$$

The base temperature at node 0 is given to be T_0 = 100°C. This problem involves 6 unknown nodal temperatures, and thus we need to have 6 equations to determine them uniquely. Nodes 1, 2, 3, 4, and 5 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) + \varepsilon\sigma(p\Delta x)[T_{\text{surr}}^4 - (T_m + 273)^4] = 0$$

or $T_{m-1} - 2T_m + T_{m+1} + h(p\Delta x^2 / kA)(T_\infty - T_m) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_m + 273)^4] = 0$, m = 1,2,3,4,5

The finite difference equation for node 6 at the fin tip is obtained by applying an energy balance on the half volume element about node 6. Then,

m= 1: $T_0 - 2T_1 + T_2 + h(p\Delta x^2 / kA)(T_\infty - T_1) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_1 + 273)^4] = 0$

m= 2: $T_1 - 2T_2 + T_3 + h(p\Delta x^2 / kA)(T_\infty - T_2) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_2 + 273)^4] = 0$

m= 3: $T_2 - 2T_3 + T_4 + h(p\Delta x^2 / kA)(T_\infty - T_3) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_3 + 273)^4] = 0$

m= 4: $T_3 - 2T_4 + T_5 + h(p\Delta x^2 / kA)(T_\infty - T_4) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_4 + 273)^4] = 0$

m= 5: $T_4 - 2T_5 + T_6 + h(p\Delta x^2 / kA)(T_\infty - T_5) + \varepsilon\sigma(p\Delta x^2 / kA)[T_{\text{surr}}^4 - (T_5 + 273)^4] = 0$

Node 6: $kA\dfrac{T_5 - T_6}{\Delta x} + h(p\Delta x / 2 + A)(T_\infty - T_6) + \varepsilon\sigma(p\Delta x / 2 + A)[T_{\text{surr}}^4 - (T_6 + 273)^4] = 0$

where

$$\Delta x = 0.03\,\text{m},\ k = 15.1\,\text{W/m}\cdot°\text{C},\ \varepsilon = 0.6,\ T_\infty = 32°\text{C}, T_0 = 100°\text{C},\ T_{surr} = 295\,\text{K},\ h = 13\,\text{W/m}^2\cdot°\text{C}$$

and $A = (1\,\text{cm})(0.2\text{cm}) = 0.2\,\text{cm}^2 = 0.2\times10^{-4}\ \text{m}^2$ and $p = 2(1 + 0.2\,\text{cm}) = 2.4\,\text{cm} = 0.024\,\text{m}$

The system of 6 equations with 6 unknowns constitute the finite difference formulation of the problem.

(b) The nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver to be

T_1 = 54.1°C, T_2 = 38.3°C, T_3 = 32.8°C, T_4 = 30.9°C, T_5 = 30.2°C, and T_6 =**30.1°C**,

(c) The total rate of heat transfer from the spoon handle is simply the sum of the heat transfer from each nodal element, and is determined from

$$\dot{Q}_{\text{fin}} = \sum_{m=0}^{6} \dot{Q}_{\text{element},\,m} = \sum_{m=0}^{6} hA_{\text{surface},\,m}(T_m - T_\infty) + \sum_{m=0}^{6} \varepsilon\sigma A_{\text{surface},\,m}[(T_m + 273)^4 - T_{\text{surr}}^4] = \mathbf{0.92\ \ W}$$

where $A_{\text{surface, m}}$ =$p\Delta x$/2for node 0, $A_{\text{surface, m}}$ =$p\Delta x$/2+Afor node 6, and $A_{\text{surface, m}}$ =$p\Delta x$ for other nodes.

5-44 A circular fin of uniform cross section is attached to a wall. The finite difference equations for all nodes are to be obtained, the nodal temperatures along the fin and the heat transfer rate are to be determined and compared with analytical solutions.

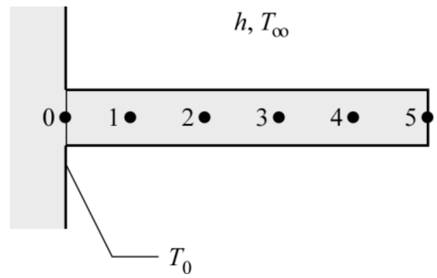
Assumptions **1** Heat transfer along the fin is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fin is given as 240 W/m·K.

Analysis (*a*) The nodal spacing is given to be Δx = 10 mm. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{50 \text{ mm}}{10 \text{ mm}} + 1 = 6$$

The base temperature at node 0 is given to be T_0 = 350°C. There are 5 unknown nodal temperatures, thus we need to have 5 equations to determine them uniquely. Nodes 1, 2, 3, and 4 are interior nodes, and we can use the general finite difference relation expressed as



$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0$$

$$T_{m-1} - 2T_m + T_{m+1} + \frac{hp\Delta x^2}{kA}(T_\infty - T_m) = 0$$

where

$$\frac{hp\Delta x^2}{kA} = \frac{4h\Delta x^2}{kD} = \frac{4(250 \text{ W/m}^2 \cdot \text{K})(0.01 \text{ m})^2}{(240 \text{ W/m} \cdot \text{K})(0.01 \text{ m})} = 0.04167$$

The finite difference equation for node 5 at the fin tip (convection boundary) is obtained by applying an energy balance on the half volume element about that node:

$$kA\frac{T_4 - T_5}{\Delta x} + h\left(\frac{p\Delta x}{2} + A\right)(T_\infty - T_5) = 0 \quad \rightarrow \quad T_4 - T_5 + \frac{h\Delta x}{kA}\left(\frac{p\Delta x}{2} + A\right)(T_\infty - T_5) = 0$$

where

$$\frac{h\Delta x}{kA}\left(\frac{p\Delta x}{2} + A\right) = \frac{h\Delta x}{k}\left(\frac{2\Delta x}{D} + 1\right) = 0.03125$$

Then,

$$m = 1: \quad T_0 - 2T_1 + T_2 + 0.04167(T_\infty - T_1) = 0$$

$$m = 2: \quad T_1 - 2T_2 + T_3 + 0.04167(T_\infty - T_2) = 0$$

$$m = 3: \quad T_2 - 2T_3 + T_4 + 0.04167(T_\infty - T_3) = 0$$

$$m = 4: \quad T_3 - 2T_4 + T_5 + 0.04167(T_\infty - T_4) = 0$$

$$m = 5: \quad T_4 - T_5 + 0.03125(T_\infty - T_5) = 0$$

(*b*) The nodal temperatures under steady conditions are determined by solving the 5 equations above simultaneously with an equation solver. Copy the following lines and paste on a blank EES screen to solve the above equations:

```
T_0=350
T_0-2*T_1+T_2+0.04167*(25-T_1)=0
T_1-2*T_2+T_3+0.04167*(25-T_2)=0
T_2-2*T_3+T_4+0.04167*(25-T_3)=0
T_3-2*T_4+T_5+0.04167*(25-T_4)=0
T_4-T_5+0.03125*(25-T_5)=0
```

Solving by EES software, we get

$$T_1 = \mathbf{304.1°C}, \quad T_2 = \mathbf{269.9°C}, \quad T_3 = \mathbf{245.9°C}, \quad T_4 = \mathbf{231.0°C}, \quad T_5 = \mathbf{224.8°C}$$

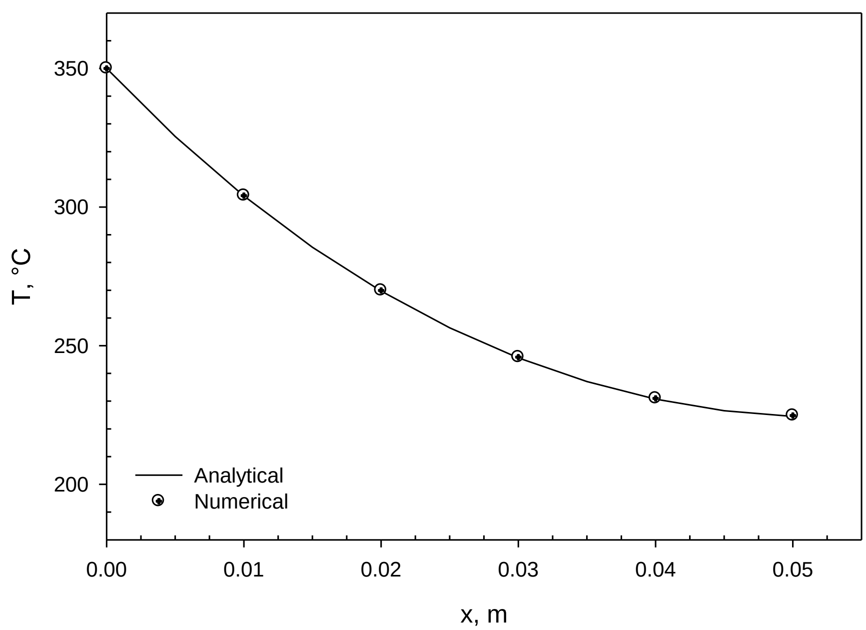
From Chapter 3, the analytical solution for the temperature variation along the fin (for convection from fin tip) is given as

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x) + (h/mk)\sinh m(L - x)}{\cosh mL + (h/mk)\sinh mL}$$

The nodal temperatures for analytical and numerical solutions are tabulated in the following table:

x, m	$T(x)$,°C	
	Analytical	Numerical
0	350.0	350.0
0.01	304.0	304.1
0.02	269.7	269.9
0.03	245.6	245.9
0.04	230.7	231.0
0.05	224.5	224.8

The comparison of the analytical and numerical solutions is shown in the following figure:



(c) The rate of heat transfer from a single fin is simply the sum of the heat transfer from the nodal elements,

$$\dot{Q}_{\text{fin}} = \sum_{m=0}^{5} \dot{Q}_{\text{element},\,m} = \sum_{m=0}^{5} hA_{\text{surface},\ m}(T_m - T_\infty)$$

$$= hp\frac{\Delta x}{2}(T_0 - T_\infty) + hp\Delta x(T_1 + T_2 + T_3 + T_4 - 4T_\infty) + h\left(p\frac{\Delta x}{2} + A\right)(T_5 - T_\infty)$$

$$= \mathbf{99.2\ W}$$

From Chapter 3, the analytical solution for the heat transfer rate of fin with convection from the tip is,

$$\dot{Q}_{\text{conv tip}} = \sqrt{hpkA_c}\,(T_b - T_\infty)\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

$$= (0.3848\ \text{W/°C})(350\,°\text{C} - 25\,°\text{C})(0.7901)$$

$$= \mathbf{98.8\ W}$$

where

$$m = \sqrt{\frac{hp}{kA_c}} = 20.41\ \text{m}^{-1}, \qquad p = \pi D = 0.03142\ \text{m}, \qquad A_c = \frac{\pi D^2}{4} = 7.854\times 10^{-5}\ \text{m}^2$$

Discussion For part (b), the comparison between the analytical and numerical solutions is excellent, with agreement within ±0.15%. For part (c), the comparison between the analytical and numerical solutions is within ±0.5%.

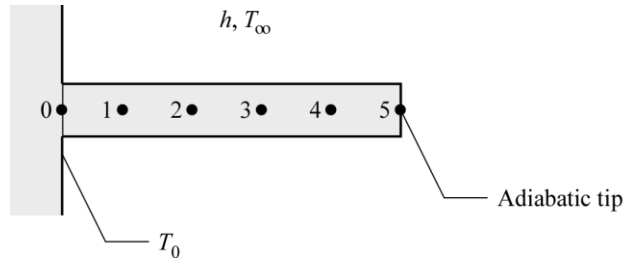
5-45 A circular aluminum fin of uniform cross section with adiabatic tip is attached to a wall. The finite difference equations for all nodes are to be obtained and solved using Gauss-Seidel iterative method, and the nodal temperatures along the fin are to be determined and compared with analytical solution.

Assumptions **1** Heat transfer along the fin is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fin is given as 237 W/m·K.

Analysis (*a*) The nodal spacing is given to be Δx = 10 mm. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{5\,\text{cm}}{1\,\text{cm}} + 1 = 6$$



The base temperature at node 0 is given to be T_0 = 300°C. There are 5 unknown nodal temperatures, thus we need to have 5 equations to determine them uniquely. Nodes 1, 2, 3, and 4 are interior nodes, and we can use the general finite difference relation expressed in explicit form as

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0$$

$$T_m = \left(2 + \frac{hp\Delta x^2}{kA}\right)^{-1}\left(T_{m-1} + T_{m+1} + \frac{hp\Delta x^2}{kA}T_\infty\right)$$

The finite difference equation for node 5 at the fin tip (adiabatic) is obtained by applying an energy balance on the half volume element about that node:

$$2kA\frac{T_4 - T_5}{\Delta x} + h(p\Delta x)(T_\infty - T_5) = 0 \quad \rightarrow \quad T_5 = \left(2 + \frac{hp\Delta x^2}{kA}\right)^{-1}\left(2T_4 + \frac{hp\Delta x^2}{kA}T_\infty\right)$$

Then,

m = 1: $T_1 = 0.4938T_0 + 0.4938T_2 + 0.1875$

m = 2: $T_2 = 0.4938T_1 + 0.4938T_3 + 0.1875$

m = 3: $T_3 = 0.4938T_2 + 0.4938T_4 + 0.1875$

m = 4: $T_4 = 0.4938T_3 + 0.4938T_5 + 0.1875$

m = 5: $T_5 = 0.9876T_4 + 0.1875$

(*b*) By letting the initial guesses as $T_1 = T_2 = T_3 = T_4 = T_5 = 250\,°\text{C}$, the results obtained from successive iterations are listed in the following table:

Iteration	Nodal temperature,°C				
	T_1	T_2	T_3	T_4	T_5
1	271.8	257.8	251.0	247.6	244.7
2	275.6	260.2	250.9	244.9	242.1
3	276.8	260.8	249.9	243.1	240.3
4	277.1	260.4	248.8	241.7	238.9
5	276.9	259.8	247.8	240.5	237.7
6	276.6	259.2	246.9	239.5	236.7
7	276.3	258.6	246.1	238.6	235.9
8	276.0	258.0	245.4	237.9	235.1
...	...	...	...	...	...
52	273.7	253.9	240.1	232.0	229.3

Hence, the converged nodal temperatures are

$T_1 = \mathbf{273.7\,°C}$, $T_2 = \mathbf{253.9\,°C}$, $T_3 = \mathbf{240.1\,°C}$, $T_4 = \mathbf{232.0\,°C}$, $T_5 = \mathbf{229.3\,°C}$

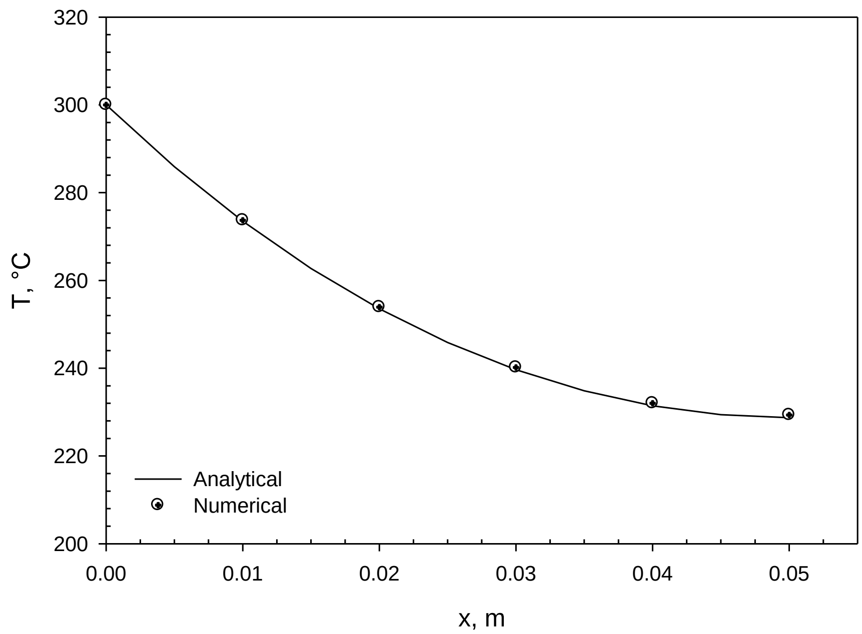
From Chapter 3, the analytical solution for the temperature variation along the fin (for adiabatic tip) is given as

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL}$$

The nodal temperatures for analytical and numerical solutions are tabulated in the following table:

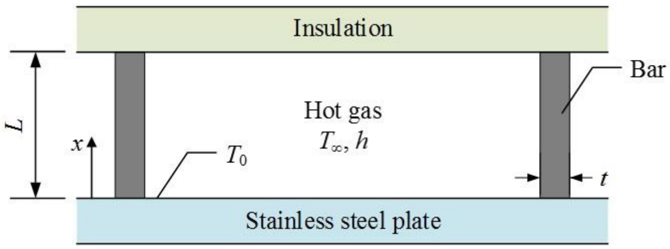
x, m	*T*(*x*),°C	
	Analytical	Numerical
0	300.0	300.0
0.01	273.5	273.7
0.02	253.5	253.9
0.03	239.6	240.1
0.04	231.4	232.0
0.05	228.7	229.3

The comparison of the analytical and numerical solutions is shown in the following figure:



Discussion The comparison between the analytical and numerical solutions is excellent, with agreement within ±0.3%.

5-46 A stainless steel plate is connected to an insulation plate by square ASTM A479 904L stainless steel bars. The bars are exposed to convection with hot gas. The temperature, T_0 at $x = 0$, is known. Determine the nodal temperatures and compare them with the analytical solution. Would any part of the ASTM A479 904L bars be above the maximum use temperature of 260°C.



Assumptions **1** Heat transfer is steady and one dimensional. **2** The part of the bar exposed to convection behaves as finned surface. **3** Thermal properties are constant. **4** Thermal radiation is neglected.

Properties The thermal conductivity for the bars is 12 W/m·K.

Analysis The nodal spacing is given as Δx = 5 mm. So, the number of nodes is

$$M = \frac{L}{\Delta x} + 1 = \frac{5\text{ cm}}{0.5\text{ cm}} + 1 = 11$$

The nodes are numbered from m = 0 to 10, where T_0 = 100°C is given. The finite difference formulations for nodes 1–9 are

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0$$

$$T_{m-1} - 2T_m + T_{m+1} + \frac{hp\Delta x^2}{kA_c}(T_\infty - T_m) = 0$$

or

$$m = 1 \text{ to } 9: \quad T_{m-1} - \left(2 + \frac{hp\Delta x^2}{kA_c}\right)T_m + T_{m+1} + \left(\frac{hp\Delta x^2}{kA_c}\right)T_\infty = 0$$

For node 10 (insulated fin tip),

$$m = 10: \quad 2T_{m-1} - \left(2 + \frac{hp\Delta x^2}{kA_c}\right)T_m + \left(\frac{hp\Delta x^2}{kA_c}\right)T_\infty = 0$$

where

$$\frac{hp\Delta x^2}{kA_c} = \frac{4h\Delta x^2}{kt} = \frac{4(25\text{ W/m}^2\cdot\text{K})(0.005\text{ m})^2}{(12\text{ W/m}\cdot\text{K})(0.01\text{ m})} = 0.02083$$

where $A_c = t^2$ and $p = 4t$

Solving for the nodal temperatures, T_1 to T_{10}, yields

$$T_1 = \mathbf{123.8}℃,\ T_2 = \mathbf{143.9}℃,\ T_3 = \mathbf{160.8}℃,\ T_4 = \mathbf{174.8}℃,\ T_5 = \mathbf{186.1}℃,$$
$$T_6 = \mathbf{195.1}℃,\ T_7 = \mathbf{201.9}℃,\ T_8 = \mathbf{206.7}℃,\ T_9 = \mathbf{209.5}℃,\ T_{10} = \mathbf{210.4}℃$$

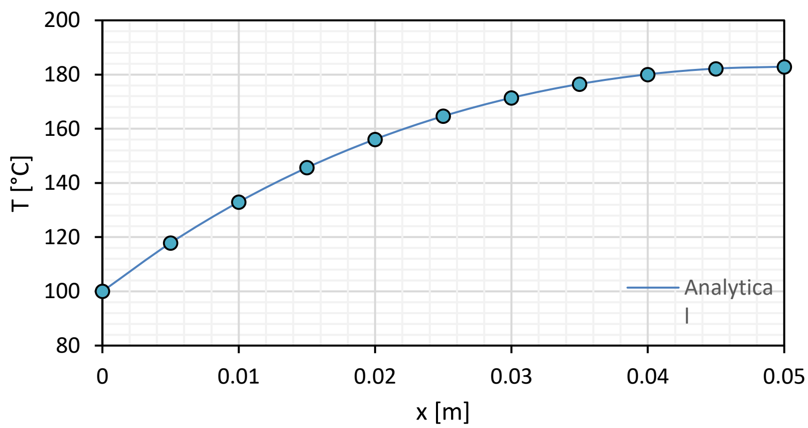
The analytical solution for the temperature variation along a fin with adiabatic tip is given in Chapter 3:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}, \qquad m^2 = \frac{hp}{kA_c}, \qquad T_b = T_0$$

The analytical and numerical results are tabulated in the following table:

x [m]	Analytical T [°C]	Numerical T [°C]
0	100.0	100.0
0.005	123.8	123.8
0.010	144.0	143.9
0.015	160.9	160.8
0.020	174.8	174.8
0.025	186.2	186.1
0.030	195.2	195.1
0.035	202.0	201.9
0.040	206.8	206.7
0.045	209.6	209.5
0.050	210.5	210.4

The temperature distribution along the bar, as a function of x, is plotted in the following figure:



DiscussionThe comparison between the analytical solution and the numerical results is excellent, with agreement within 0.05%. The entire bar is at a temperature below the maximum use temperature of 260°C, with the highest temperature reaching about 210°C. Even though the hot gas has a temperature of 300°C, the convection thermal resistance between the bar surface and the hot gas keeps the bars from reaching the fluid temperature.

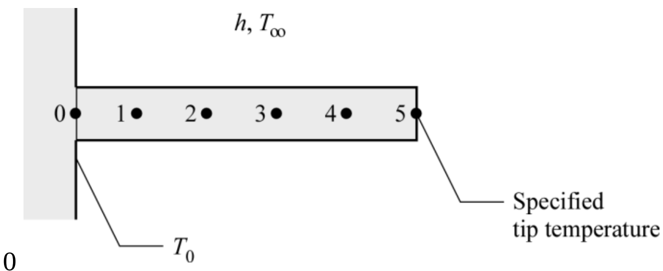
5-47 A circular fin of uniform cross section is attached to a wall with the fin tip temperature specified as 250°C. The finite difference equations for all nodes are to be obtained and the nodal temperatures along the fin are to be determined and compared with analytical solution.

Assumptions **1** Heat transfer along the fin is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fin is given as 240 W/m·K.

Analysis (*a*) The nodal spacing is given to be Δx = 10 mm. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x}+1 = \frac{50\text{ mm}}{10\text{ mm}}+1 = 6$$



The base temperature at node 0 is given to be T_0 =350°C and the tip temperature at node 5 is given as T_5 = 200°C. There are 4 unknown nodal temperatures, thus we need to have 4 equations to determine them uniquely. Nodes 1, 2, 3, and 4 are interior nodes, and we can use the general finite difference relation expressed as

$$kA\frac{T_{m-1}-T_m}{\Delta x} + kA\frac{T_{m+1}-T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0$$

$$T_{m-1} - 2T_m + T_{m+1} + \frac{hp\Delta x^2}{kA}(T_\infty - T_m) = 0$$

where

$$\frac{hp\Delta x^2}{kA} = \frac{4h\Delta x^2}{kD} = \frac{4(250\text{ W/m}^2\cdot\text{K})(0.01\text{ m})^2}{(240\text{ W/m}\cdot\text{K})(0.01\text{ m})} = 0.04167$$

Then,

m = 1: $\quad T_0 - 2T_1 + T_2 + 0.04167(T_\infty - T_1) = 0$

m = 2: $\quad T_1 - 2T_2 + T_3 + 0.04167(T_\infty - T_2) = 0$

m = 3: $\quad T_2 - 2T_3 + T_4 + 0.04167(T_\infty - T_3) = 0$

m = 4: $\quad T_3 - 2T_4 + T_5 + 0.04167(T_\infty - T_4) = 0$

(*b*) The nodal temperatures under steady conditions are determined by solving the 4 equations above simultaneously with an equation solver. Copy the following lines and paste on a blank EES screen to solve the above equations:

```
T_0=350
T_5=200
T_0-2*T_1+T_2+0.04167*(25-T_1)=0
T_1-2*T_2+T_3+0.04167*(25-T_2)=0
T_2-2*T_3+T_4+0.04167*(25-T_3)=0
T_3-2*T_4+T_5+0.04167*(25-T_4)=0
```

Solving by EES software, we get

$$T_1 = \mathbf{299.9\,°C},\ \ T_2 = \mathbf{261.3\,°C},\ \ T_3 = \mathbf{232.5\,°C},\ \ T_4 = \mathbf{212.3\,°C}$$

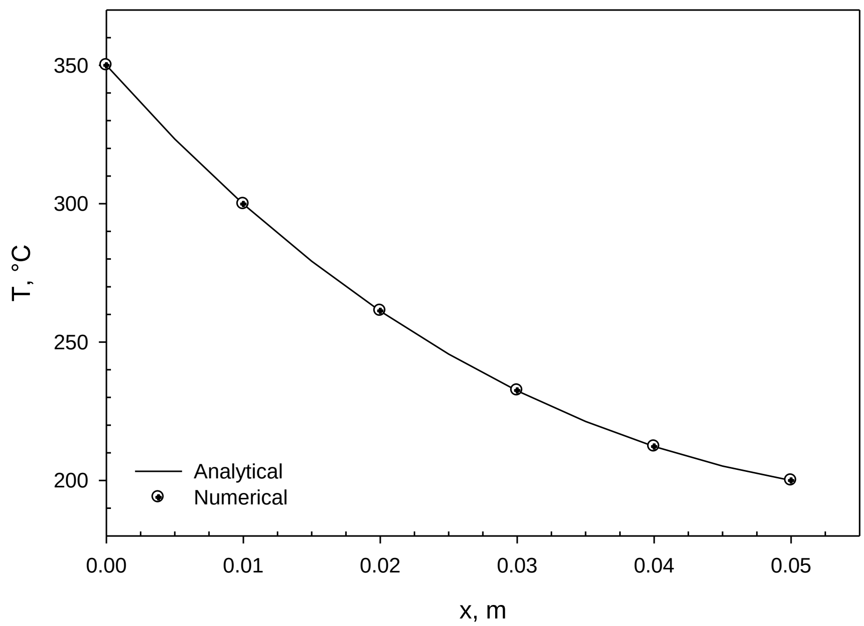
From Chapter 3, the analytical solution for the temperature variation along the fin (for specified tip temperature) is given as

$$\frac{T(x)-T_\infty}{T_b - T_\infty} = \frac{(T_L - T_\infty)/(T_b - T_\infty)\sinh mx + \sinh m(L-x)}{\sinh mL}$$

The nodal temperatures for analytical and numerical solutions are tabulated in the following table:

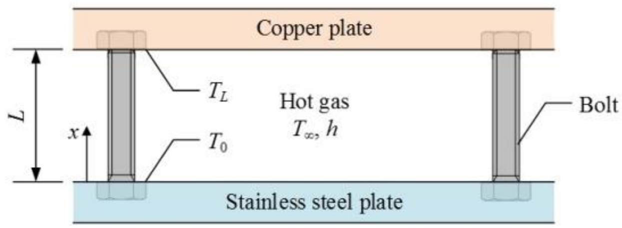
x, m	$T(x)$,°C	
	Analytical	Numerical
0	350.0	350.0
0.01	299.8	299.9
0.02	261.2	261.3
0.03	232.4	232.5
0.04	212.3	212.3
0.05	200.0	200.0

The comparison of the analytical and numerical solutions is shown in the following figure:



Discussion The comparison between the analytical and numerical solutions is excellent, with agreement within ±0.05%.

5-48 C&S A stainless steel plate is connected to a copper plate by long ASTM B98 copper-silicon bolts. Portion of the bolts are exposed to convection with hot gas. The temperatures, T_0 at x = 0 and T_L at x = L, are known. Determine the nodal temperatures and compare them with the analytical solution. Would any part of the ASTM B98 bolts be above the maximum use temperature of 149°C?



Assumptions **1** Heat transfer is steady and one dimensional. **2** The part of the bolt exposed to convection behaves as finned surface. **3** Thermal properties are constant. **4** Thermal radiation is neglected.

Properties The thermal conductivity for the bolts is 36 W/m·K.

Analysis The nodal spacing is given as Δx = 5 mm. So, the number of nodes is

$$M = \frac{L}{\Delta x} + 1 = \frac{5\text{ cm}}{0.5\text{ cm}} + 1 = 11$$

The nodes are numbered from m = 0 to 10, where T_0 = 100°C and T_{10} = 80°C are given. The finite difference formulations for nodes 1–9 are

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0$$

$$T_{m-1} - 2T_m + T_{m+1} + \frac{hp\Delta x^2}{kA_c}(T_\infty - T_m) = 0$$

or

$$m = 1\text{ to }9: \quad T_{m-1} - \left(2 + \frac{hp\Delta x^2}{kA_c}\right)T_m + T_{m+1} + \left(\frac{hp\Delta x^2}{kA_c}\right)T_\infty = 0$$

where

$$\frac{hp\Delta x^2}{kA_c} = \frac{4h\Delta x^2}{kD} = \frac{4(50\text{ W/m}^2\text{·K})(0.005\text{ m})^2}{(36\text{ W/m·K})(0.0095\text{ m})} = 0.01462$$

where $A_c = \pi D^2/4$ and $p = \pi D$

Solving for the nodal temperatures, T_1 to T_9, yields

$$T_1 = \mathbf{121.7}°\text{C},\ T_2 = 1\mathbf{37.8}°\text{C},\ T_3 = \mathbf{148.6}°\text{C},\ T_4 = \mathbf{154.3}°\text{C},$$
$$T_5 = \mathbf{154.9}°\text{C},\ T_6 = \mathbf{150.5}°\text{C},\ T_7 = \mathbf{141}°\text{C},\ T_8 = \mathbf{126.2}°\text{C},\ T_9 = \mathbf{106}°\text{C}$$

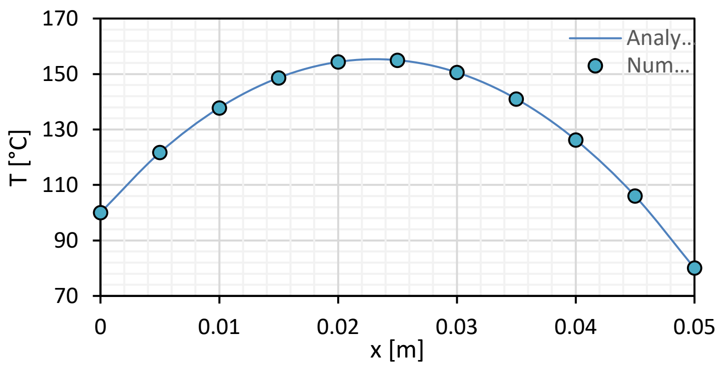
The analytical solution for the temperature variation along a fin with specified tip temperature is given in Chapter 3:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{[(T_L - T_\infty)/(T_b - T_\infty)]\sinh mx + \sinh m(L - x)}{\sinh mL}, \qquad m^2 = \frac{hp}{kA_c}, \qquad T_b = T_0$$

The analytical and numerical results are tabulated in the following table:

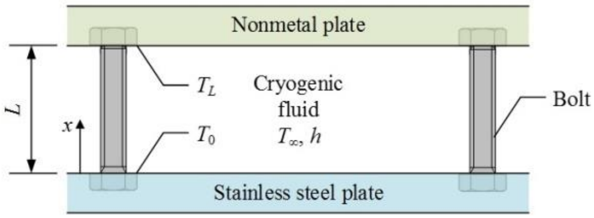
	Analytical	Numerical
x [m]	*T* [°C]	*T* [°C]
0	100.0	100.0
0.005	121.7	121.7
0.010	137.8	137.8
0.015	148.7	148.6
0.020	154.4	154.3
0.025	155.0	154.9
0.030	150.6	150.5
0.035	141.1	141.0
0.040	126.3	126.2
0.045	106.0	106.0
0.050	80.0	80.0

The temperature distribution along the bolt, as a function of *x*, is plotted in the following figure:



Discussion The comparison between the analytical solution and the numerical results is excellent, with agreement within 0.05%. Part of the bolts, 0.015 <*x*< 0.032 m, experiences temperature above the maximum use temperature (149°C). This makes the bolts unsuitable for this application to meet the ASME Code for Process Piping (ASME B31.3-2014, Table A-2M).

5-49 C&S A nonmetal plate is connected to a stainless steel plate by long ASTM A437 B4B stainless steel bolts. Portion of the bolts are exposed to convection with cryogenic fluid. The temperatures, T_0 at $x = 0$ and T_L at $x = L$, are known. Determine the nodal temperatures and compare them with the analytical solution. Would any part of the ASTM A437 B4B bolts be above the minimum suitable temperature of −30°C?



Assumptions **1** Heat transfer is steady and one dimensional. **2** The part of the bolt exposed to convection behaves as finned surface. **3** Thermal properties are constant. **4** Thermal radiation is neglected.

Properties The thermal conductivity for the bolts is 23.9 W/m·K.

Analysis The nodal spacing is given as Δx = 5 mm. So, the number of nodes is

$$M = \frac{L}{\Delta x} + 1 = \frac{5\text{ cm}}{0.5\text{ cm}} + 1 = 11$$

The nodes are numbered from m = 0 to 10, where $T_0 = T_{10}$ = 0°C is given. The finite difference formulations for nodes 1–9 are

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0$$

$$T_{m-1} - 2T_m + T_{m+1} + \frac{hp\Delta x^2}{kA_c}(T_\infty - T_m) = 0$$

or

$$m = 1\text{ to }9:\quad T_{m-1} - \left(2 + \frac{hp\Delta x^2}{kA_c}\right)T_m + T_{m+1} + \left(\frac{hp\Delta x^2}{kA_c}\right)T_\infty = 0$$

where

$$\frac{hp\Delta x^2}{kA_c} = \frac{4h\Delta x^2}{kD} = \frac{4(100\text{ W/m}^2\cdot\text{K})(0.005\text{ m})^2}{(23.9\text{ W/m}\cdot\text{K})(0.0095\text{ m})} = 0.044043$$

where $A_c = \pi D^2/4$ and $p = \pi D$

Solving for the nodal temperatures, T_1 to T_9, yields

$$T_1 = T_9 = \mathbf{-7.1}\text{°C},\ T_2 = T_8 = \mathbf{-12.4}\text{°C},$$
$$T_3 = T_7 = \mathbf{-16.0}\text{°C},\ T_4 = T_6 = \mathbf{-18.1}\text{°C},\ T_5 = \mathbf{-18.8}\text{°C}$$

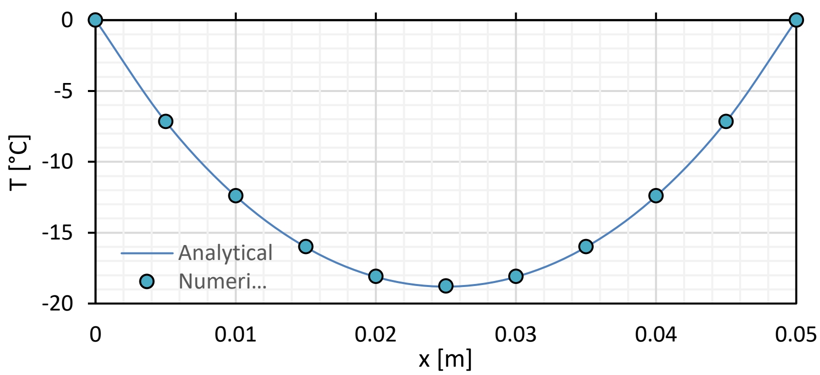
The analytical solution for the temperature variation along a fin with specified tip temperature is given in Chapter 3:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{[(T_L - T_\infty)/(T_b - T_\infty)]\sinh mx + \sinh m(L-x)}{\sinh mL},\qquad m^2 = \frac{hp}{kA_c},\qquad T_b = T_0$$

The analytical and numerical results are tabulated in the following table:

x [m]	Analytical T [°C]	Numerical T [°C]
0	0.0	0.0
0.005	-7.2	-7.1
0.010	-12.4	-12.4
0.015	-16.0	-16.0
0.020	-18.1	-18.1
0.025	-18.8	-18.8
0.030	-18.1	-18.1
0.035	-16.0	-16.0
0.040	-12.4	-12.4
0.045	-7.2	-7.1
0.050	0.0	0.0

The temperature distribution along the bolt, as a function of x, is plotted in the following figure:



Discussion The comparison between the analytical solution and the numerical results is excellent, with agreement within 0.3%. The entire bolt is at a temperature above the minimum suitable temperature of −30°C, with the lowest temperature reaching −18.8°C at$x$ = 0.025 m. Even though the cryogenic fluid has a temperature of −50°C, the convection thermal resistance between the bolt surface and the cryogenic fluid keeps the bolts from reaching the fluid temperature.

Note that the numerical solution can be solved by applying symmetry at node 5, at x = 0.025 m.

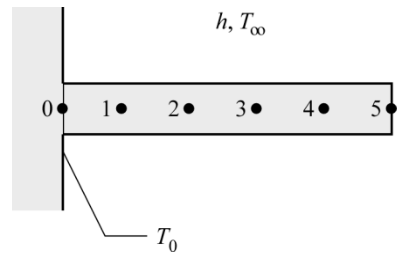
5-50 A DC motor delivers mechanical power to a rotating stainless steel shaft. With a uniform nodal spacing of 5 cm along shaft, the finite difference equations and the nodal temperatures are to be determined.

Assumptions **1** Heat transfer along the shaft is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity of the shaft is given as 15.1 W/m·K.

Analysis (*a*) The nodal spacing is given to be Δx = 5 cm. Then the number of nodes *M* becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{25\text{ cm}}{5\text{ cm}} + 1 = 6$$



The base temperature at node 0 is given to be T_0 = 90°C. There are 5 unknown nodal temperatures, thus we need to have 5 equations to determine them uniquely. Nodes 1, 2, 3, and 4 are interior nodes, and we can use the general finite difference relation expressed as

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0$$

$$T_{m-1} - \left(2 + \frac{hp\Delta x^2}{kA}\right)T_m + T_{m+1} + \frac{hp\Delta x^2}{kA}T_\infty = 0$$

where $\dfrac{hp\Delta x^2}{kA} = \dfrac{4h\Delta x^2}{kD} = \dfrac{4(25\text{ W/m}^2\cdot\text{K})(0.05\text{ m})^2}{(15.5\text{ W/m}\cdot\text{K})(0.025\text{ m})} = 0.6452$

The finite difference equation for node 5 at the fin tip (convection boundary) is obtained by applying an energy balance on the half volume element about that node:

$$kA\frac{T_4 - T_5}{\Delta x} + h\left(\frac{p\Delta x}{2} + A\right)(T_\infty - T_5) = 0$$

$$T_4 - \left[1 + \frac{h\Delta x}{kA}\left(\frac{p\Delta x}{2} + A\right)\right]T_5 + \frac{h\Delta x}{kA}\left(\frac{p\Delta x}{2} + A\right)T_\infty = 0$$

where $\dfrac{h\Delta x}{kA}\left(\dfrac{p\Delta x}{2} + A\right) = \dfrac{h\Delta x}{k}\left(\dfrac{2\Delta x}{D} + 1\right) = 0.4032$

Then,

m = 1: $T_0 - 2.6452T_1 + T_2 + 0.6452T_\infty = 0$

m = 2: $T_1 - 2.6452T_2 + T_3 + 0.6452T_\infty = 0$

m = 3: $T_2 - 2.6452T_3 + T_4 + 0.6452T_\infty = 0$

m = 4: $T_3 - 2.6452T_4 + T_5 + 0.6452T_\infty = 0$

m = 5: $T_4 - 1.4032T_5 + 0.4032T_\infty = 0$

The nodal temperatures under steady conditions are determined by solving the 5 equations above simultaneously with an equation solver. Copy the following lines and paste on a blank EES screen to solve the above equations:

```
T_0=90
T_0-2.6452*T_1+T_2+0.6452*20=0
T_1-2.6452*T_2+T_3+0.6452*20=0
T_2-2.6452*T_3+T_4+0.6452*20=0
T_3-2.6452*T_4+T_5+0.6452*20=0
T_4-1.4032*T_5+0.4032*20=0
```

Solving by EES software, we get

$T_1 = \mathbf{52.03\,°C}$, $T_2 = \mathbf{34.72\,°C}$, $T_3 = \mathbf{26.92\,°C}$, $T_4 = \mathbf{23.58\,°C}$, $T_5 = \mathbf{22.55\,°C}$

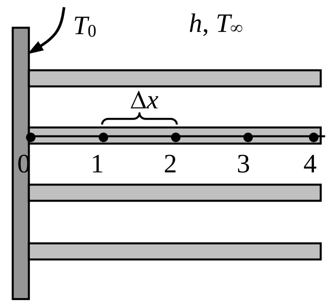
Discussion The nodal temperatures along the motor shaft can be compared with the analytical solution from Chapter 3 for fin with convection fin tip boundary condition:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$$

5-51 One side of a hot vertical plate is to be cooled by attaching aluminum fins of rectangular profile. The finite difference formulation of the problem for all nodes is to be obtained, and the nodal temperatures, the rate of heat transfer from a single fin and from the entire surface of the plate are to be determined.

Assumptions **1** Heat transfer along the fin is given to be steady and one-dimensional. **2** The thermal conductivity is constant. **3** Combined convection and radiation heat transfer coefficient is constant and uniform.

Properties The thermal conductivity is given to be k = 237 W/m·°C.



Analysis (a) The nodal spacing is given to be Δx=0.5 cm. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{2\text{ cm}}{0.5\text{ cm}} + 1 = 5$$

The base temperature at node 0 is given to be T_0 = 80°C. This problem involves 4 unknown nodal temperatures, and thus we need to have 4 equations to determine them uniquely. Nodes 1, 2, and 3 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0 \rightarrow T_{m-1} - 2T_m + T_{m+1} + h(p\Delta x^2 / kA)(T_\infty - T_m) = 0$$

The finite difference equation for node 4 at the fin tip is obtained by applying an energy balance on the half volume element about that node. Then,

$$m=1:\ T_0 - 2T_1 + T_2 + h(p\Delta x^2 / kA)(T_\infty - T_1) = 0$$

$$m=2:\ T_1 - 2T_2 + T_3 + h(p\Delta x^2 / kA)(T_\infty - T_2) = 0$$

$$m=3:\ T_2 - 2T_3 + T_4 + h(p\Delta x^2 / kA)(T_\infty - T_3) = 0$$

$$\text{Node 4:}\quad kA\frac{T_3 - T_4}{\Delta x} + h(p\Delta x/2 + A)(T_\infty - T_4) = 0$$

where $\Delta x = 0.005\text{ m},\ k = 237\text{ W/m}\cdot°\text{C},\ T_\infty = 35°\text{C}, T_0 = 80°\text{C},\ h = 30\text{ W/m}^2\cdot°\text{C}$

and $A = (3\text{ m})(0.003\text{m}) = 0.009\text{ m}^2$ and $p = 2(3 + 0.003\text{ m}) = 6.006\text{ m}$.

This system of 4 equations with 4 unknowns constitute the finite difference formulation of the problem.

(b) The nodal temperatures under steady conditions are determined by solving the 4 equations above simultaneously with an equation solver to be

T_1 = **79.64°C**, T_2 = **79.38°C**, T_3 = **79.21°C**, T_4 = **79.14°C**

(c) The rate of heat transfer from a single fin is simply the sum of the heat transfer from each nodal element,

$$\begin{aligned}\dot{Q}_{\text{fin}} &= \sum_{m=0}^{4} \dot{Q}_{\text{element},\,m} = \sum_{m=0}^{4} hA_{\text{surface},\,m}(T_m - T_\infty) \\ &= hp(\Delta x/2)(T_0 - T_\infty) + hp\Delta x(T_1 + T_2 + T_3 - 3T_\infty) + h(p\Delta x/2 + A)(T_4 - T_\infty) = \mathbf{172\ W}\end{aligned}$$

(d) The number of fins on the surface is

$$\text{No. of fins} = \frac{\text{Plate height}}{\text{Fin thickness} + \text{fin spacing}} = \frac{2\text{ m}}{(0.003 + 0.004)\text{ m}} = 286\text{ fins}$$

Then the rate of heat tranfer from the fins, the unfinned portion, and the entire finned surface become

$$\dot{Q}_{\text{fin, total}} = (\text{No. of fins})\dot{Q}_{\text{fin}} = 286(172\text{ W}) = 49{,}192\text{ W}$$

$$\dot{Q}_{\text{unfinned}} = hA_{\text{unfinned}}(T_0 - T_\infty) = (30\text{ W/m}^2\cdot°\text{C})(286 \times 3\text{ m} \times 0.004\text{ m})(80 - 35)°\text{C} = 4633\text{ W}$$

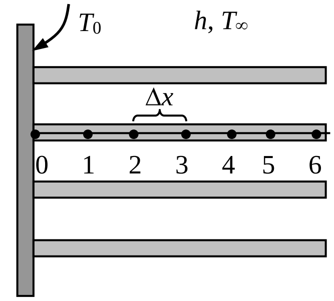
$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{fin, total}} + \dot{Q}_{\text{unfinned}} = 49{,}192 + 4633 = 53{,}825\text{ W} \cong \mathbf{53.8\,kW}$$

5-52 One side of a hot vertical plate is to be cooled by attaching aluminum pin fins. The finite difference formulation of the problem for all nodes is to be obtained, and the nodal temperatures, the rate of heat transfer from a single fin and from the entire surface of the plate are to be determined.

Assumptions **1** Heat transfer along the fin is given to be steady and one-dimensional. **2** The thermal conductivity is constant. **3** Combined convection and radiation heat transfer coefficient is constant and uniform.

Properties The thermal conductivity is given to be k = 237 W/m·°C.

Analysis (*a*) The nodal spacing is given to be Δx=0.5 cm. Then the number of nodes M becomes



$$M = \frac{L}{\Delta x} + 1 = \frac{3\,\text{cm}}{0.5\,\text{cm}} + 1 = 7$$

The base temperature at node 0 is given to be T_0 = 100°C. This problem involves 6 unknown nodal temperatures, and thus we need to have 6 equations to determine them uniquely. Nodes 1, 2, 3, 4, and 5 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$kA\frac{T_{m-1}-T_m}{\Delta x} + kA\frac{T_{m+1}-T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0 \;\rightarrow\; T_{m-1} - 2T_m + T_{m+1} + h(p\Delta x^2 / kA)(T_\infty - T_m) = 0$$

The finite difference equation for node 6 at the fin tip is obtained by applying an energy balance on the half volume element about that node. Then,

$$m\text{= 1: } T_0 - 2T_1 + T_2 + h(p\Delta x^2 / kA)(T_\infty - T_1) = 0$$

$$m\text{= 2: } T_1 - 2T_2 + T_3 + h(p\Delta x^2 / kA)(T_\infty - T_2) = 0$$

$$m\text{= 3: } T_2 - 2T_3 + T_4 + h(p\Delta x^2 / kA)(T_\infty - T_3) = 0$$

$$m\text{= 4: } T_3 - 2T_4 + T_5 + h(p\Delta x^2 / kA)(T_\infty - T_4) = 0$$

$$m\text{= 5: } T_4 - 2T_5 + T_6 + h(p\Delta x^2 / kA)(T_\infty - T_5) = 0$$

$$\text{Node 6: } kA\frac{T_5 - T_6}{\Delta x} + h(p\Delta x/2 + A)(T_\infty - T_6) = 0$$

where $\Delta x = 0.005\,\text{m},\ k = 237\text{ W/m}\cdot°\text{C},\ T_\infty = 30°\text{C}, T_0 = 100°\text{C},\ h = 35\text{ W/m}^2\cdot°\text{C}$

and
$$A = \pi D^2/4 = \pi(0.25\,\text{cm})^2/4 = 0.0491\,\text{cm}^2 = 0.0491\times10^{-4}\ \text{m}^2$$
$$p = \pi D = \pi(0.0025\,\text{m}) = 0.00785\,\text{m}$$

(*b*) The nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver to be

T_1 = **97.9°C**, T_2 = **96.1°C**, T_3 = **94.7°C**, T_4 = **93.8°C**, T_5 = **93.1°C**, T_6 = **92.9°C**

(*c*) The rate of heat transfer from a single fin is simply the sum of the heat transfer from the nodal elements,

$$\dot{Q}_{\text{fin}} = \sum_{m=0}^{6}\dot{Q}_{\text{element},\,m} = \sum_{m=0}^{6} hA_{\text{surface},\,m}(T_m - T_\infty)$$
$$= hp\Delta x/2(T_0 - T_\infty) + hp\Delta x(T_1 + T_2 + T_3 + T_4 + T_5 - 5T_\infty) + h(p\Delta x/2 + A)(T_6 - T_\infty) = \mathbf{0.5496\ \ W}$$

(*d*) The number of fins on the surface is

$$\text{No. of fins} = \frac{1\,\text{m}^2}{(0.006\,\text{m})(0.006\,\text{m})} = 27{,}778\,\text{fins}$$

Then the rate of heat tranfer from the fins, the unfinned portion, and the entire finned surface become

$$\dot{Q}_{\text{fin, total}} = (\text{No. of fins})\dot{Q}_{\text{fin}} = 27{,}778(0.5496\text{ W}) = 15{,}267\text{ W}$$

$$\dot{Q}`_{\text{unfinned}} = hA_{\text{unfinned}}(T_0 - T_\infty) = (35\text{ W/m}^2\cdot°\text{C})(1\text{-}27{,}778\times0.0491\times10^{-4}\ \text{m}^2)(100\text{-}30)°\text{C} = 2116\text{ W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{fin, total}} + \dot{Q}_{\text{unfinned}} = 15{,}267 + 2116 = \mathbf{17{,}383\ W} \cong \mathbf{17.4\ \ kW}$$

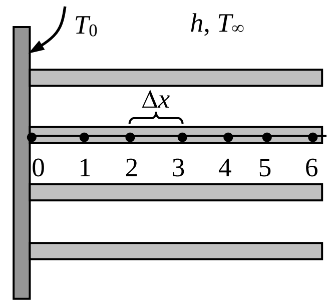
5-53 One side of a hot vertical plate is to be cooled by attaching copper pin fins. The finite difference formulation of the problem for all nodes is to be obtained, and the nodal temperatures, the rate of heat transfer from a single fin and from the entire surface of the plate are to be determined.

Assumptions **1** Heat transfer along the fin is given to be steady and one-dimensional. **2** The thermal conductivity is constant. **3** Combined convection and radiation heat transfer coefficient is constant and uniform.

Properties The thermal conductivity is given to be k = 386 W/m·°C.

Analysis (*a*) The nodal spacing is given to be Δx=0.5 cm. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{3\,\text{cm}}{0.5\,\text{cm}} + 1 = 7$$



The base temperature at node 0 is given to be T_0 = 100°C. This problem involves 6 unknown nodal temperatures, and thus we need to have 6 equations to determine them uniquely. Nodes 1, 2, 3, 4, and 5 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$kA\frac{T_{m-1} - T_m}{\Delta x} + kA\frac{T_{m+1} - T_m}{\Delta x} + h(p\Delta x)(T_\infty - T_m) = 0 \;\rightarrow\; T_{m-1} - 2T_m + T_{m+1} + h(p\Delta x^2 / kA)(T_\infty - T_m) = 0$$

The finite difference equation for node 6 at the fin tip is obtained by applying an energy balance on the half volume element about that node. Then,

$$m=1:\; T_0 - 2T_1 + T_2 + h(p\Delta x^2 / kA)(T_\infty - T_1) = 0$$

$$m=2:\; T_1 - 2T_2 + T_3 + h(p\Delta x^2 / kA)(T_\infty - T_2) = 0$$

$$m=3:\; T_2 - 2T_3 + T_4 + h(p\Delta x^2 / kA)(T_\infty - T_3) = 0$$

$$m=4:\; T_3 - 2T_4 + T_5 + h(p\Delta x^2 / kA)(T_\infty - T_4) = 0$$

$$m=5:\; T_4 - 2T_5 + T_6 + h(p\Delta x^2 / kA)(T_\infty - T_5) = 0$$

$$\text{Node 6:}\; kA\frac{T_5 - T_6}{\Delta x} + h(p\Delta x/2 + A)(T_\infty - T_6) = 0$$

where $\Delta x = 0.005\,\text{m},\; k = 386\ \text{W/m}\cdot{}^\circ\text{C},\; T_\infty = 30^\circ\text{C}, T_0 = 100^\circ\text{C},\; h = 35\ \text{W/m}^2\cdot{}^\circ\text{C}$

and
$$A = \pi D^2/4 = \pi(0.25\,\text{cm})^2/4 = 0.0491\,\text{cm}^2 = 0.0491\times 10^{-4}\ \text{m}^2$$
$$p = \pi D = \pi(0.0025\,\text{m}) = 0.00785\,\text{m}$$

(*b*) The nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver to be

T_1 = **98.6°C**, T_2 = **97.5°C**, T_3 = **96.7°C**, T_4 = **96.0°C**, T_5 = **95.7°C**, T_6 = **95.5°C**

(*c*) The rate of heat transfer from a single fin is simply the sum of the heat transfer from the nodal elements,

$$\dot{Q}_{\text{fin}} = \sum_{m=0}^{6} \dot{Q}_{\text{element},\,m} = \sum_{m=0}^{6} hA_{\text{surface},\,m}(T_m - T_\infty)$$
$$= hp\Delta x/2(T_0 - T_\infty) + hp\Delta x(T_1 + T_2 + T_3 + T_4 + T_5 - 5T_\infty) + h(p\Delta x/2 + A)(T_6 - T_\infty) = \mathbf{0.5641\ W}$$

(*d*) The number of fins on the surface is

$$\text{No. of fins} = \frac{1\,\text{m}^2}{(0.006\,\text{m})(0.006\,\text{m})} = 27{,}778\,\text{fins}$$

Then the rate of heat tranfer from the fins, the unfinned portion, and the entire finned surface become

$$\dot{Q}_{\text{fin, total}} = (\text{No. of fins})\dot{Q}_{\text{fin}} = 27{,}778(0.5641\ \text{W}) = 15{,}670\ \text{W}$$

$$\dot{Q}_{\text{unfinned}} = hA_{\text{unfinned}}(T_0 - T_\infty) = (35\ \text{W/m}^2\cdot{}^\circ\text{C})(1 - 27{,}778\times 0.0491\times 10^{-4}\ \text{m}^2)(100-30)^\circ\text{C} = 2116\ \text{W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{fin, total}} + \dot{Q}_{\text{unfinned}} = 15{,}670 + 2116 = \mathbf{17{,}786\ W} \cong \mathbf{17.8\,kW}$$

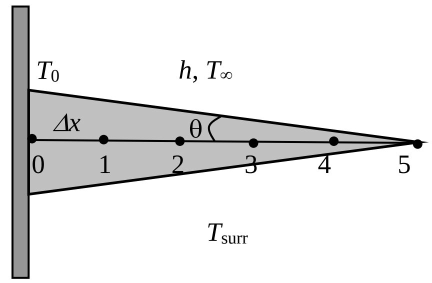
5-54 A long triangular fin attached to a surface is considered. The nodal temperatures, the rate of heat transfer, and the fin efficiency are to be determined numerically using 6 equally spaced nodes.

Assumptions **1** Heat transfer along the fin is given to be steady, and the temperature along the fin to vary in the x direction only so that $T = T(x)$. **2** Thermal conductivity is constant.

Properties The thermal conductivity is given to be k = 180 W/m·°C. The emissivity of the fin surface is 0.9.

Analysis The fin length is given to be L = 5 cm, and the number of nodes is specified to be M = 6. Therefore, the nodal spacing Δx is

$$\Delta x = \frac{L}{M-1} = \frac{0.05\ \text{m}}{6-1} = 0.01\ \text{m}$$



The temperature at node 0 is given to be T_0 = 180°C, and the temperatures at the remaining 5 nodes are to be determined. Therefore, we need to have 5 equations to determine them uniquely. Nodes 1, 2, 3, and 4 are interior nodes, and the finite difference formulation for a *general interior node m* is obtained by applying an energy balance on the volume element of this node. Noting that heat transfer is steady and there is no heat generation in the fin and assuming heat transfer to be into the medium from all sides, the energy balance can be expressed as

$$\sum_{\text{all sides}} \dot{Q} = 0 \rightarrow kA_{\text{left}} \frac{T_{m-1}-T_m}{\Delta x} + kA_{\text{right}} \frac{T_{m+1}-T_m}{\Delta x} + hA_{\text{conv}}(T_\infty - T_m) + \varepsilon\sigma A_{\text{surface}}[T_{\text{surr}}^4 - (T_m+273)^4\} = 0$$

Note that heat transfer areas are different for each node in this case, and using geometrical relations, they can be expressed as

$$A_{\text{left}} = (\text{Height} \times \text{width})_{@\,m-1/2} = 2w\left[L-(m-1/2)\Delta x\right]\tan\theta$$

$$A_{\text{right}} = (\text{Height} \times \text{width})_{@\,m+1/2} = 2w\left[L-(m+1/2)\Delta x\right]\tan\theta$$

$$A_{\text{surface}} = 2\times\text{Length}\times\text{width} = 2w(\Delta x/\cos\theta)$$

Substituting,

$$2kw[L-(m-0.5)\Delta x]\tan\theta\frac{T_{m-1}-T_m}{\Delta x} + 2kw[L-(m+0.5)\Delta x]\tan\theta\frac{T_{m+1}-T_m}{\Delta x}$$
$$+2w(\Delta x/\cos\theta)\{h(T_\infty - T_m) + \varepsilon\sigma[T_{\text{surr}}^4 - (T_m+273)^4]\} = 0$$

Dividing each term by $2kwL\tan\theta/\Delta x$ gives

$$\left[1-(m-1/2)\frac{\Delta x}{L}\right](T_{m-1}-T_m) + \left[1-(m+1/2)\frac{\Delta x}{L}\right](T_{m+1}-T_m) + \frac{h(\Delta x)^2}{kL\sin\theta}(T_\infty - T_m) + \frac{\varepsilon\sigma(\Delta x)^2}{kL\sin\theta}[T_{\text{surr}}^4 - (T_m+273)^4] = 0$$

Substituting,

m = 1: $$\left[1-0.5\frac{\Delta x}{L}\right](T_0-T_1) + \left[1-1.5\frac{\Delta x}{L}\right](T_2-T_1) + \frac{h(\Delta x)^2}{kL\sin\theta}(T_\infty - T_1) + \frac{\varepsilon\sigma(\Delta x)^2}{kL\sin\theta}[T_{\text{surr}}^4 - (T_1+273)^4] = 0$$

m = 2: $$\left[1-1.5\frac{\Delta x}{L}\right](T_1-T_2) + \left[1-2.5\frac{\Delta x}{L}\right](T_3-T_2) + \frac{h(\Delta x)^2}{kL\sin\theta}(T_\infty - T_2) + \frac{\varepsilon\sigma(\Delta x)^2}{kL\sin\theta}[T_{\text{surr}}^4 - (T_2+273)^4] = 0$$

m = 3: $$\left[1-2.5\frac{\Delta x}{L}\right](T_2-T_3) + \left[1-3.5\frac{\Delta x}{L}\right](T_4-T_3) + \frac{h(\Delta x)^2}{kL\sin\theta}(T_\infty - T_3) + \frac{\varepsilon\sigma(\Delta x)^2}{kL\sin\theta}[T_{\text{surr}}^4 - (T_3+273)^4] = 0$$

m = 4: $$\left[1-3.5\frac{\Delta x}{L}\right](T_3-T_4) + \left[1-4.5\frac{\Delta x}{L}\right](T_5-T_4) + \frac{h(\Delta x)^2}{kL\sin\theta}(T_\infty - T_4) + \frac{\varepsilon\sigma(\Delta x)^2}{kL\sin\theta}[T_{\text{surr}}^4 - (T_4+273)^4] = 0$$

An energy balance on the 5$^{\text{th}}$ node gives the 5$^{\text{th}}$ equation,

m = 5: $$2k\frac{\Delta x}{2}\tan\theta\frac{T_4-T_5}{\Delta x} + 2h\frac{\Delta x/2}{\cos\theta}(T_\infty - T_5) + 2\varepsilon\sigma\frac{\Delta x/2}{\cos\theta}[T_{\text{surr}}^4 - (T_5+273)^4] = 0$$

Solving the 5 equations above simultaneously for the 5 unknown nodal temperatures gives

T_1 = **177.0°C**, T_2 = **174.1°C**, T_3 = **171.2°C**, T_4 = **168.4°C**, and T_5 = **165.5°C**

(*b*) The total rate of heat transfer from the fin is simply the sum of the heat transfer from each volume element to the ambient, and for w = 1 m it is determined from

$$\dot{Q}_{\text{fin}} = \sum_{m=0}^{5} \dot{Q}_{\text{element},\, m} = \sum_{m=0}^{5} hA_{\text{surface},\ m}(T_m - T_\infty) + \sum_{m=0}^{5} \varepsilon\sigma A_{\text{surface},\ m}[(T_m + 273)^4 - T_{\text{surr}}^4]$$

Noting that the heat transfer surface area is $w\Delta x / \cos\theta$ for the boundary nodes 0 and 5, and twice as large for the interior nodes 1, 2, 3, and 4, we have

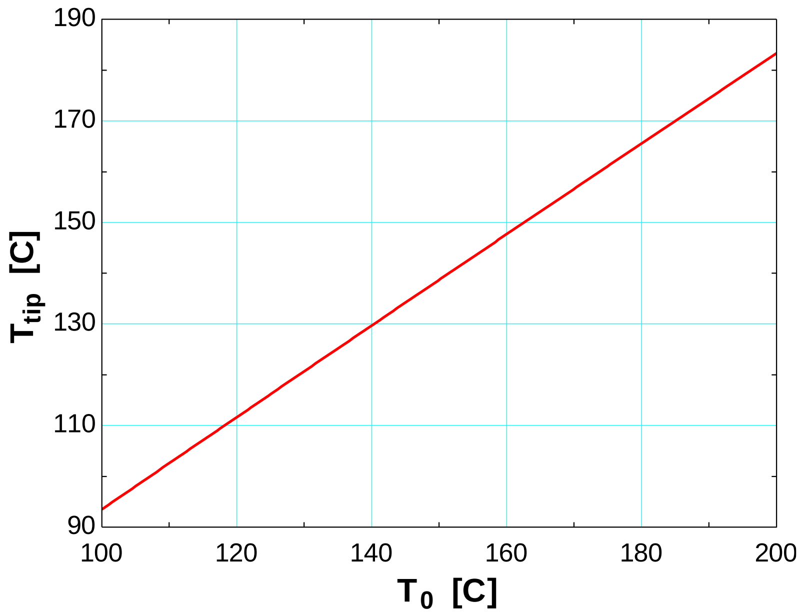
$$\begin{aligned}\dot{Q}_{\text{fin}} &= h\frac{w\Delta x}{\cos\theta}\left[(T_0 - T_\infty) + 2(T_1 - T_\infty) + 2(T_2 - T_\infty) + 2(T_3 - T_\infty) + 2(T_4 - T_\infty) + (T_5 - T_\infty)\right] \\ &+ \varepsilon\sigma\frac{w\Delta x}{\cos\theta}\{[(T_0 + 273)^4 - T_{\text{surr}}^4] + 2[(T_1 + 273)^4 - T_{\text{surr}}^4] + 2[(T_2 + 273)^4 - T_{\text{surr}}^4] + 2[(T_3 + 273)^4 - T_{\text{surr}}^4] \\ &+ 2[(T_4 + 273)^4 - T_{\text{surr}}^4] + [(T_5 + 273)^4 - T_{\text{surr}}^4]\} \\ &= \mathbf{537\ W}\end{aligned}$$

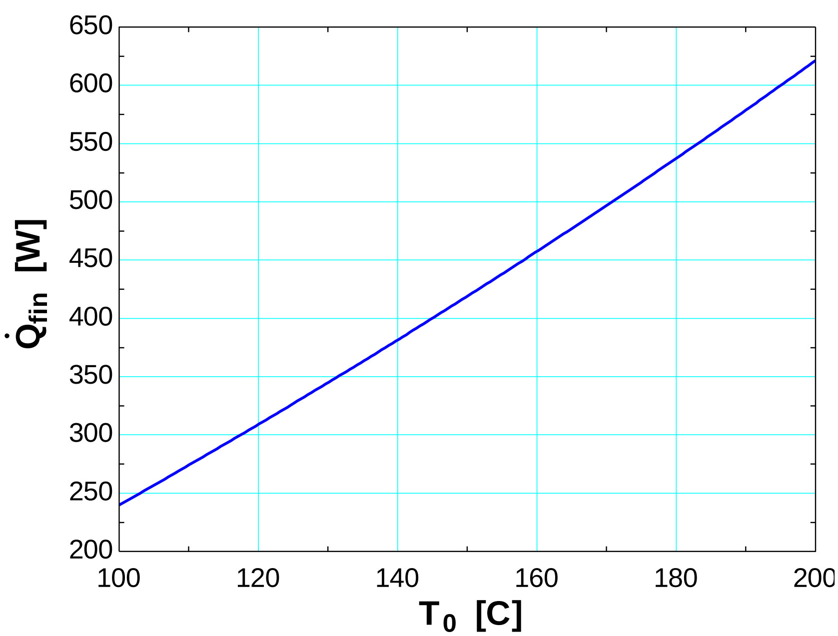
5-55 Prob. 5-54 is reconsidered. The effect of the fin base temperature on the fin tip temperature and the rate of heat transfer from the fin is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
k=180 [W/m-C]
L=0.05 [m]
b=0.01 [m]
w=1 [m]
T_0=180 [C]
T_infinity=25 [C]
h=25 [W/m^2-C]
T_surr=290 [K]
M=6
epsilon=0.9
tan(theta)=(0.5*b)/L
sigma=5.67E-8 [W/m^2-K^4] “Stefan-Boltzmann constant"

"ANALYSIS"
"(a)"
DELTAx=L/(M-1)
"Using the finite difference method, the five equations for the temperatures at 5 nodes are determined to be"
(1-0.5*DELTAx/L)*(T_0-T_1)+(1-1.5*DELTAx/L)*(T_2-T_1)+(h*DELTAx^2)/(k*L*sin(theta))*(T_infinity-T_1)+(epsilon*sigma*DELTAX^2)/(k*L*sin(theta))*(T_surr^4-(T_1+273)^4)=0 "for mode 1"

(1-1.5*DELTAx/L)*(T_1-T_2)+(1-2.5*DELTAx/L)*(T_3-T_2)+(h*DELTAx^2)/(k*L*sin(theta))*(T_infinity-T_2)+(epsilon*sigma*DELTAX^2)/(k*L*sin(theta))*(T_surr^4-(T_2+273)^4)=0 "for mode 2"

(1-2.5*DELTAx/L)*(T_2-T_3)+(1-3.5*DELTAx/L)*(T_4-T_3)+(h*DELTAx^2)/(k*L*sin(theta))*(T_infinity-T_3)+(epsilon*sigma*DELTAX^2)/(k*L*sin(theta))*(T_surr^4-(T_3+273)^4)=0 "for mode 3"

(1-3.5*DELTAx/L)*(T_3-T_4)+(1-4.5*DELTAx/L)*(T_5-T_4)+(h*DELTAx^2)/(k*L*sin(theta))*(T_infinity-T_4)+(epsilon*sigma*DELTAX^2)/(k*L*sin(theta))*(T_surr^4-(T_4+273)^4)=0 "for mode 4"

2*k*DELTAx/2*tan(theta)*(T_4-T_5)/DELTAx+2*h*(0.5*DELTAx)/cos(theta)*(T_infinity-T_5)+2*epsilon*sigma*(0.5*DELTAx)/cos(theta)*(T_surr^4-(T_5+273)^4)=0 "for mode 5"

T_tip=T_5

"(b)"
Q_dot_fin=C+D "where"

C=h*(w*DELTAx)/cos(theta)*((T_0-T_infinity)+2*(T_1-T_infinity)+2*(T_2-T_infinity)+2*(T_3-T_infinity)+2*(T_4-T_infinity)+(T_5-T_infinity))

D=epsilon*sigma*(w*DELTAx)/cos(theta)*(((T_0+273)^4-T_surr^4)+2*((T_1+273)^4-T_surr^4)+2*((T_2+273)^4-T_surr^4)+2*((T_3+273)^4-T_surr^4)+2*((T_4+273)^4-T_surr^4)+((T_5+273)^4-T_surr^4))
```

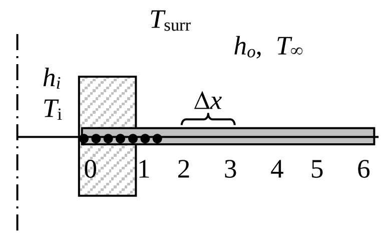
T_0 [C]	T_{tip} [C]	$\dot{Q}_{fin}$ [W]
100	93.51	239.8
105	98.05	256.8
110	102.6	274
115	107.1	291.4
120	111.6	309
125	116.2	326.8
130	120.7	344.8
135	125.2	363.1
140	129.7	381.5
145	134.2	400.1
150	138.7	419
155	143.2	438.1
160	147.7	457.5
165	152.1	477.1
170	156.6	496.9
175	161.1	517
180	165.5	537.3
185	170	557.9
190	174.4	578.7
195	178.9	599.9
200	183.3	621.2





5-56 Two cast iron steam pipes are connected to each other through two 1-cm thick flanges, and heat is lost from the flanges by convection and radiation. The finite difference formulation of the problem for all nodes is to be obtained, and the temperature of the tip of the flange as well as the rate of heat transfer from the exposed surfaces of the flange are to be determined.

Assumptions **1** Heat transfer through the flange is stated to be steady and one-dimensional. **2** The thermal conductivity and emissivity are constants. **3** Convection heat transfer coefficient is constant and uniform.



Properties The thermal conductivity and emissivity are given to be k = 52 W/m·°C and ε = 0.8.

Analysis(*a*) The distance between nodes 0 and 1 is the thickness of the pipe, Δx_1=0.4 cm=0.004 m. The nodal spacing along the flange is given to be Δx_2=1 cm = 0.01 m. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 2 = \frac{5\,\text{cm}}{1\,\text{cm}} + 2 = 7$$

This problem involves 7 unknown nodal temperatures, and thus we need to have 7 equations to determine them uniquely. Noting that the total thickness of the flange is t = 0.02 m, the heat conduction area at any location along the flange is $A_{\text{cond}} = 2\pi rt$ where the values of radii at the nodes and between the nodes (the mid points) are

r_0=0.046 m, r_1=0.05 m, r_2=0.06 m, r_3=0.07 m, r_4=0.08 m, r_5=0.09 m, r_6=0.10 m

r_{01}=0.048 m, r_{12}=0.055 m, r_{23}=0.065 m, r_{34}=0.075 m, r_{45}=0.085 m, r_{56}=0.095 m

Then the finite difference equations for each node are obtained from the energy balance to be as follows:

Node 0: $$h_i(2\pi tr_0)(T_i - T_0) + k(2\pi tr_{01})\frac{T_1 - T_0}{\Delta x_1} = 0$$

Node 1: $$k(2\pi tr_{01})\frac{T_0 - T_1}{\Delta x_1} + k(2\pi tr_{12})\frac{T_2 - T_1}{\Delta x_2} + 2[2\pi t(r_1 + r_{12})/2)](\Delta x_2/2)\{h(T_\infty - T_1) + \varepsilon\sigma[T_{\text{surr}}^4 - (T_1 + 273)^4]\} = 0$$

Node 2: $$k(2\pi tr_{12})\frac{T_1 - T_2}{\Delta x_2} + k(2\pi tr_{23})\frac{T_3 - T_2}{\Delta x_2} + 2(2\pi r_2 \Delta x_2)\{h(T_\infty - T_2) + \varepsilon\sigma[T_{\text{surr}}^4 - (T_2 + 273)^4]\} = 0$$

Node 3: $$k(2\pi tr_{23})\frac{T_2 - T_3}{\Delta x_2} + k(2\pi tr_{34})\frac{T_4 - T_3}{\Delta x_2} + 2(2\pi r_3 \Delta x_2)\{h(T_\infty - T_3) + \varepsilon\sigma[T_{\text{surr}}^4 - (T_3 + 273)^4]\} = 0$$

Node 4: $$k(2\pi tr_{34})\frac{T_3 - T_4}{\Delta x_2} + k(2\pi tr_{45})\frac{T_5 - T_4}{\Delta x_2} + 2(2\pi r_4 \Delta x_2)\{h(T_\infty - T_4) + \varepsilon\sigma[T_{\text{surr}}^4 - (T_4 + 273)^4]\} = 0$$

Node 5: $$k(2\pi tr_{45})\frac{T_4 - T_5}{\Delta x_2} + k(2\pi tr_{56})\frac{T_6 - T_5}{\Delta x_2} + 2(2\pi r_5 \Delta x_2)\{h(T_\infty - T_5) + \varepsilon\sigma[T_{\text{surr}}^4 - (T_5 + 273)^4]\} = 0$$

Node 6: $$k(2\pi tr_{56})\frac{T_5 - T_6}{\Delta x_2} + 2[2\pi t(\Delta x_2/2)(r_{56} + r_6)/2 + 2\pi r_6 t]\{h(T_\infty - T_6) + \varepsilon\sigma[T_{\text{surr}}^4 - (T_6 + 273)^4]\} = 0$$

where $\Delta x_1 = 0.004\,\text{m},\ \Delta x_2 = 0.01\,\text{m},\ k = 52\ \text{W/m}\cdot{}^\circ\text{C},\ \varepsilon = 0.8,\ T_\infty = 12^\circ\text{C},\ T_{in} = 250^\circ\text{C},\ T_{surr} = 290\,\text{K}$

and $h = 25\ \text{W/m}^2\cdot{}^\circ\text{C},\ h_i = 180\ \text{W/m}^2\cdot{}^\circ\text{C},\ \sigma = 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4$.

The system of 7 equations with 7 unknowns constitutes the finite difference formulation of the problem.

(*b*) The nodal temperatures under steady conditions are determined by solving the 7 equations above simultaneously with an equation solver to be

T_0 =148.4°C, T_1 =147.0°C, T_2 = 144.1°C, T_3 =141.6°C, T_4 =139.5°C, T_5 =137.7°C, and T_6 = **136.0°C**

(*c*) Knowing the inner surface temperature, the rate of heat transfer from the flange under steady conditions is simply the rate of heat transfer from the steam to the pipe at flange section

$$\dot{Q}_{\text{fin}} = \sum_{m=1}^{6}\dot{Q}_{\text{element},\,m} = \sum_{m=1}^{6} hA_{\text{surface},\,m}(T_m - T_\infty) + \sum_{m=1}^{6}\varepsilon\sigma A_{\text{surface},\,m}[(T_m + 273)^4 - T_{\text{surr}}^4] = \mathbf{105.7\ \ W}$$

where $A_{\text{surface, m}}$ are as given above for different nodes.

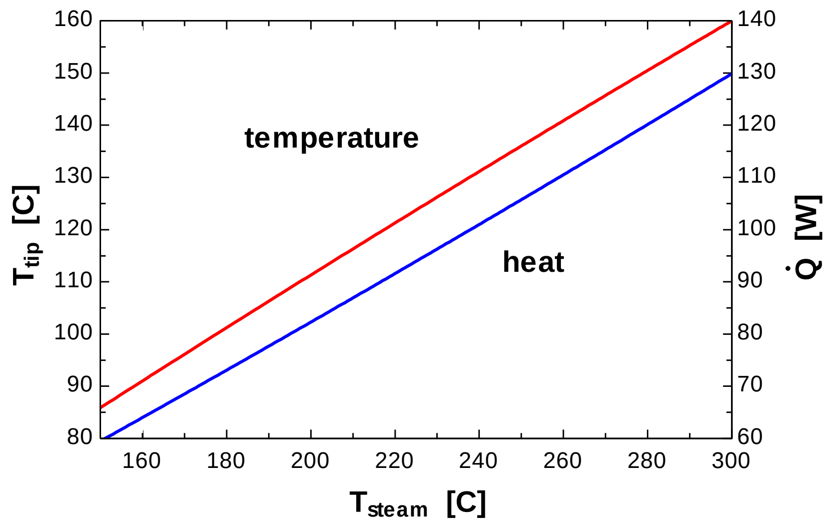
5-57 Prob. 5-56 is reconsidered. The effects of the steam temperature and the outer heat transfer coefficient on the flange tip temperature and the rate of heat transfer from the exposed surfaces of the flange are to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

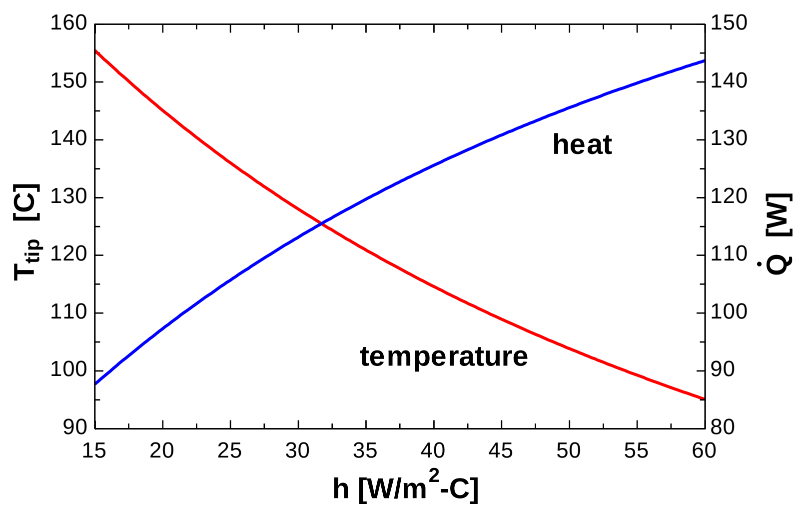
```
"GIVEN"
t_pipe=0.004 [m]
k=52 [W/m-C]
epsilon=0.8
D_o_pipe=0.10 [m]
t_flange=0.01 [m]
D_o_flange=0.20 [m]
T_steam=250 [C]
h_i=180 [W/m^2-C]
T_infinity=12 [C]
h=25 [W/m^2-C]
T_surr=290 [K]
DELTAx=0.01 [m]
sigma=5.67E-8 [W/m^2-K^4]"Stefan-Boltzmann constant"

"ANALYSIS"
"(b)"
DELTAx_1=t_pipe "the distance between nodes 0 and 1"
DELTAx_2=t_flange "nodal spacing along the flange"
L=(D_o_flange-D_o_pipe)/2
M=L/DELTAx_2+2 "Number of nodes"
t=2*t_flange "total thixkness of the flange"
"The values of radii at the nodes and between the nodes /-(the midpoints) are"
r_0=0.046 "[m]"
r_1=0.05 "[m]"
r_2=0.06 "[m]"
r_3=0.07 "[m]"
r_4=0.08 "[m]"
r_5=0.09 "[m]"
r_6=0.10 "[m]"
r_01=0.048 "[m]"
r_12=0.055 "[m]"
r_23=0.065 "[m]"
r_34=0.075 "[m]"
r_45=0.085 "[m]"
r_56=0.095 "[m]"
"Using the finite difference method, the five equations for the unknown temperatures at 7 nodes are determined to be"
h_i*(2*pi*t*r_0)*(T_steam-T_0)+k*(2*pi*t*r_01)*(T_1-T_0)/DELTAx_1=0 "node 0"
k*(2*pi*t*r_01)*(T_0-T_1)/DELTAx_1+k*(2*pi*t*r_12)*(T_2-T_1)/DELTAx_2+2*2*pi*t*(r_1+r_12)/2*(DELTAx_2/2)*(h*(T_infinity-T_1)+epsilon*sigma*(T_surr^4-(T_1+273)^4))=0 "node 1"
k*(2*pi*t*r_12)*(T_1-T_2)/DELTAx_2+k*(2*pi*t*r_23)*(T_3-T_2)/DELTAx_2+2*2*pi*t*r_2*DELTAx_2*(h*(T_infinity-T_2)+epsilon*sigma*(T_surr^4-(T_2+273)^4))=0 "node 2"
k*(2*pi*t*r_23)*(T_2-T_3)/DELTAx_2+k*(2*pi*t*r_34)*(T_4-T_3)/DELTAx_2+2*2*pi*t*r_3*DELTAx_2*(h*(T_infinity-T_3)+epsilon*sigma*(T_surr^4-(T_3+273)^4))=0 "node 3"
k*(2*pi*t*r_34)*(T_3-T_4)/DELTAx_2+k*(2*pi*t*r_45)*(T_5-T_4)/DELTAx_2+2*2*pi*t*r_4*DELTAx_2*(h*(T_infinity-T_4)+epsilon*sigma*(T_surr^4-(T_4+273)^4))=0 "node 4"
k*(2*pi*t*r_45)*(T_4-T_5)/DELTAx_2+k*(2*pi*t*r_56)*(T_6-T_5)/DELTAx_2+2*2*pi*t*r_5*DELTAx_2*(h*(T_infinity-T_5)+epsilon*sigma*(T_surr^4-(T_5+273)^4))=0 "node 5"
k*(2*pi*t*r_56)*(T_5-T_6)/DELTAx_2+2*(2*pi*t*(r_56+r_6)/2*(DELTAx_2/2)+2*pi*t*r_6)*(h*(T_infinity-T_6)+epsilon*sigma*(T_surr^4-(T_6+273)^4))=0 "node 6"
T_tip=T_6
```

```
"(c)"
Q_dot=Q_dot_1+Q_dot_2+Q_dot_3+Q_dot_4+Q_dot_5+Q_dot_6 "where"
Q_dot_1=h*2*2*pi*t*(r_1+r_12)/2*DELTAx_2/2*(T_1-
T_infinity)+epsilon*sigma*2*2*pi*t*(r_1+r_12)/2*DELTAx_2/2*((T_1+273)^4-T_surr^4)
Q_dot_2=h*2*2*pi*t*r_2*DELTAx_2*(T_2-T_infinity)+epsilon*sigma*2*2*pi*t*r_2*DELTAx_2*((T_2+273)^4-
T_surr^4)
Q_dot_3=h*2*2*pi*t*r_3*DELTAx_2*(T_3-T_infinity)+epsilon*sigma*2*2*pi*t*r_3*DELTAx_2*((T_3+273)^4-
T_surr^4)
Q_dot_4=h*2*2*pi*t*r_4*DELTAx_2*(T_4-T_infinity)+epsilon*sigma*2*2*pi*t*r_4*DELTAx_2*((T_4+273)^4-
T_surr^4)
Q_dot_5=h*2*2*pi*t*r_5*DELTAx_2*(T_5-T_infinity)+epsilon*sigma*2*2*pi*t*r_5*DELTAx_2*((T_5+273)^4-
T_surr^4)
Q_dot_6=h*2*(2*pi*t*(r_56+r_6)/2*(DELTAx_2/2)+2*pi*t*r_6)*(T_6-
T_infinity)+epsilon*sigma*2*(2*pi*t*(r_56+r_6)/2*(DELTAx_2/2)+2*pi*t*r_6)*((T_6+273)^4-T_surr^4)
```

T_{steam} [C]	T_{tip} [C]	$\dot{Q}$ [W]
150	85.87	59.48
160	91.01	63.99
170	96.12	68.52
180	101.2	73.08
190	106.3	77.66
200	111.3	82.27
210	116.3	86.91
220	121.3	91.57
230	126.2	96.26
240	131.1	101
250	136	105.7
260	140.9	110.5
270	145.7	115.3
280	150.5	120.1
290	155.2	125
300	160	129.9



h [W/m^2.C]	T_{tip} [C]	$\dot{Q}$ [W]
15	155.4	87.7
20	145.1	97.31
25	136	105.7
30	128	113.2
35	120.9	119.7
40	114.6	125.6
45	108.9	130.8
50	103.9	135.6
55	99.26	139.8
60	95.08	143.7



5-58 Using EES, the solutions of the systems of algebraic equations are determined to be as follows:

```
"(a)"
3*x_1-x_2+3*x_3=0
-x_1+2*x_2+x_3=3
2*x_1-x_2-x_3=2
```

Solution**:** $x_1 = 2$, $x_2 = 3$, $x_3 = -1$

```
"(b)"
4*x_1-2*x_2^2+0.5*x_3=-2
x_1^3-x_2+-x_3=11.964
x_1+x_2+x_3=3
```

Solution**:** $x_1 = 2.33$, $x_2 = 2.29$, $x_3 = -1.62$

5-59 Using EES, the solutions of the systems of algebraic equations are determined to be as follows:

```
"(a)"
3*x_1+2*x_2-x_3+x_4=6
x_1+2*x_2-x_4=-3
-2*x_1+x_2+3*x_3+x_4=2
3*x_2+x_3-4*x_4=-6
```

Solution**:** $x_1 = 13$, $x_2 = -9$, $x_3 = 13$, $x_4 = -2$

```
"(b)"
3*x_1+x_2^2+2*x_3=8
-x_1^2+3*x_2+2*x_3=-6.293
2*x_1-x_2^4+4*x_3=-12
```

Solution**:** $x_1 = 2.825$, $x_2 = 1.791$, $x_3 = -1.841$

5-60 Using EES, the solutions of the systems of algebraic equations are determined to be as follows:

```
"(a)"
4*x_1-x_2+2*x_3+x_4=-6
x_1+3*x_2-x_3+4*x_4=-1
-x_1+2*x_2+5*x_4=5
2*x_2-4*x_3-3*x_4=-5
```

***Solution*:** $x_1 = -2$, $x_2 = -1$, $x_3 = 0$, $x_4 = 1$

```
"(b)"
2*x_1+x_2^4-2*x_3+x_4=1
x_1^2+4*x_2+2*x_3^2-2*x_4=-3
-x_1+x_2^4+5*x_3=10
3*x_1-x_3^2+8*x_4=15
```

***Solution*:** $x_1 = 0.263$, $x_2 = -1.15$, $x_3 = 1.70$, $x_4 = 2.14$

Two-Dimensional Steady Heat Conduction

5-61C A region that cannot be filled with simple volume elements such as strips for a plane wall, and rectangular elements for two-dimensional conduction is said to have *irregular boundaries*. A practical way of dealing with such geometries in the finite difference method is to replace the elements bordering the irregular geometry by a series of simple volume elements.

5-62C For a medium in which the finite difference formulation of a general interior node is given in its simplest form as $T_{\text{node}} = (T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}})/4$:

(*a*) Heat transfer is steady, (*b*) heat transfer is two-dimensional, (*c*) there is no heat generation in the medium, (*d*) the nodal spacing is constant, and (*e*) the thermal conductivity of the medium is constant.

5-63C For a medium in which the finite difference formulation of a general interior node is given in its simplest form as

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{e}_{\text{node}} l^2}{k} = 0:$$

(*a*) Heat transfer is steady, (*b*) heat transfer is two-dimensional, (*c*) there is heat generation in the medium, (*d*) the nodal spacing is constant, and (*e*) the thermal conductivity of the medium is constant.

5-64 Starting with an energy balance on a volume element, the steady two-dimensional finite difference equation for a general interior node in rectangular coordinates for $T(x, y)$ for the case of variable thermal conductivity and uniform heat generation is to be obtained.

Analysis We consider a *volume element* of size $\Delta x \times \Delta y \times 1$ centered about a general interior node (m, n) in a region in which heat is generated at a constant rate of $\dot{e}$ and the thermal conductivity k is variable (see Fig. 5-24 in the text). Assuming the direction of heat conduction to be *towards* the node under consideration at all surfaces, the energy balance on the volume element can be expressed as

$$\dot{Q}_{\text{cond, left}} + \dot{Q}_{\text{cond, top}} + \dot{Q}_{\text{cond, right}} + \dot{Q}_{\text{cond, bottom}} + \dot{G}_{\text{element}} = \frac{\Delta E_{\text{element}}}{\Delta t} = 0$$

for the *steady* case. Again assuming the temperatures between the adjacent nodes to vary linearly and noting that the heat transfer area is $\Delta y \times 1$ in the x direction and $\Delta x \times 1$ in the y direction, the energy balance relation above becomes

$$k_{m,n}(\Delta y \times 1)\frac{T_{m-1,n} - T_{m,n}}{\Delta x} + k_{m,n}(\Delta x \times 1)\frac{T_{m,n+1} - T_{m,n}}{\Delta y} + k_{m,n}(\Delta y \times 1)\frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$
$$+ k_{m,n}(\Delta x \times 1)\frac{T_{m,n-1} - T_{m,n}}{\Delta y} + \dot{e}_0(\Delta x \times \Delta y \times 1) = 0$$

Dividing each term by $\Delta x \times \Delta y \times 1$ and simplifying gives

$$\frac{T_{m-1,n} - 2T_{m,n} + T_{m+1,n}}{\Delta x^2} + \frac{T_{m,n-1} - 2T_{m,n} + T_{m,n+1}}{\Delta y^2} + \frac{\dot{e}_0}{k_{m,n}} = 0$$

For a square mesh with $\Delta x = \Delta y = l$, and the relation above simplifies to

$$T_{m-1,n} + T_{m+1,n} + T_{m,n-1} + T_{m,n+1} - 4T_{m,n} + \frac{\dot{e}_0 l^2}{k_{m,n}} = 0$$

It can also be expressed in the following easy-to-remember form

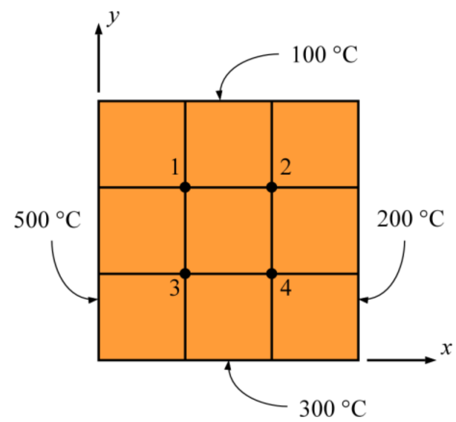
$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{e}_0 l^2}{k_{\text{node}}} = 0$$

5-65 A square cross section is undergoing a steady two-dimensional heat transfer. The finite difference equations and the nodal temperatures are to be determined.

Assumptions **1** Steady heat conduction is two-dimensional. **2** Thermal properties are constant. **3** There is no heat generation in the body.

Analysis (*a*) There are 4 unknown nodal temperatures, thus we need to have 4 equations to determine them uniquely. For nodes 1 to 4, we can use the general finite difference relation expressed as

$$\frac{T_{m-1,n} - 2T_{m,n} + T_{m+1,n}}{\Delta x^2} + \frac{T_{m,n-1} - 2T_{m,n} + T_{m,n+1}}{\Delta y^2} = 0$$



Since $\Delta x = \Delta y$, we have

$$T_{m,n} = 0.25(T_{m,n+1} + T_{m+1,n} + T_{m,n-1} + T_{m-1,n})$$

or $$T_{\text{node}} = 0.25(T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} + T_{\text{left}})$$

Then

Node 1: $T_1 = 0.25(100 + T_2 + T_3 + 500)$

Node 2: $T_2 = 0.25(100 + 200 + T_4 + T_1)$

Node 3: $T_3 = 0.25(T_1 + T_4 + 300 + 500)$

Node 4: $T_4 = 0.25(T_2 + 200 + 300 + T_3)$

(*b*) By letting the initial guesses as T_1 = 300°C, T_2 = 150°C, T_3 = 400°C, and T_4 = 250°C the results obtained from successive iterations are listed in the following table:

Iteration	Nodal temperature,°C			
	T_1	T_2	T_3	T_4
1	287.5	209.4	334.4	260.9
2	285.9	211.7	336.7	262.1
3	287.1	212.3	337.3	262.4
4	287.4	212.5	337.5	262.5
5	287.5	212.5	337.5	262.5
6	287.5	212.5	337.5	262.5

Hence, the converged nodal temperatures are

T_1 = **287.5°C**, T_2 = **212.5°C**, T_3 = **337.5°C**, T_4 = **262.5°C**

Discussion The finite difference equations can also be calculated using the EES. Copy the following lines and paste on a blank EES screen to solve the above equations:

```
T_1=0.25*(100+T_2+T_3+500)
T_2=0.25*(100+200+T_4+T_1)
T_3=0.25*(T_1+T_4+300+500)
T_4=0.25*(T_2+200+300+T_3)
```

Solving by EES software, we get the same results:

T_1 = **287.5°C**, T_2 = **212.5°C**, T_3 = **337.5°C**, T_4 = **262.5°C**

5-66 A long solid body is subjected to steady two-dimensional heat transfer. The unknown nodal temperatures are to be determined.

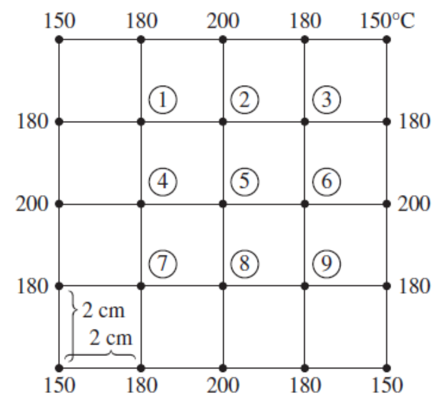
Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** There is no heat generation in the body.

Properties The thermal conductivity is given to be k = 45 W/m·°C.

Analysis The nodal spacing is given to be Δx = Δx = l = 0.02 m, and the general finite difference form of an interior node for steady two-dimensional heat conduction for the case of no heat generation is expressed as

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} = 0 \;\rightarrow\; T_{\text{node}} = (T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}})/4$$

There is symmetry about the horizontal, vertical, and diagonal lines passing through the midpoint, and thus we need to consider only 1/8th of the region. Then,



$$T_1 = T_3 = T_7 = T_9$$

$$T_2 = T_4 = T_6 = T_8$$

Therefore, there are there are only 3 unknown nodal temperatures, T_1, T_2, and T_5, and thus we need only 3 equations to determine them uniquely. Also, we can replace the symmetry lines by insulation and utilize the mirror-image concept when writing the finite difference equations for the interior nodes.

Node 1 (interior) : $T_1 = (180 + 180 + 2T_2)/4$

Node 2 (interior) : $T_2 = (200 + T_5 + 2T_1)/4$

Node 5 (interior) : $T_5 = 4T_2/4 = T_2$

Solving the equations above simultaneously gives

$$T_1 = T_3 = T_7 = T_9 = \mathbf{185°C}$$

$$T_2 = T_4 = T_5 = T_6 = T_8 = \mathbf{190°C}$$

Discussion Note that taking advantage of symmetry simplified the problem greatly.

5-67 A long solid body is subjected to steady two-dimensional heat transfer. The unknown nodal temperatures are to be determined.

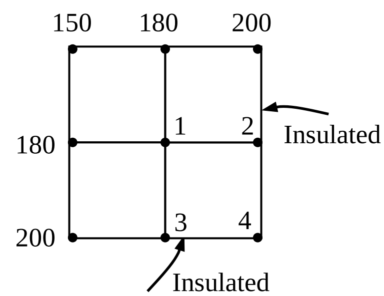
Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** There is no heat generation in the body.

Properties The thermal conductivity is given to be k = 20 W/m·°C.

Analysis The nodal spacing is given to be Δx = Δx = l = 0.01 m, and the general finite difference form of an interior node for steady two-dimensional heat conduction for the case of no heat generation is expressed as

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} = 0 \;\rightarrow\; T_{\text{node}} = (T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}})/4$$

(a) There is symmetry about the insulated surfaces as well as about the diagonal line. Therefore $T_3 = T_2$, and $T_1, T_2,$ and T_4 are the only 3 unknown nodal temperatures. Thus we need only 3 equations to determine them uniquely. Also, we can replace the symmetry lines by insulation and utilize the mirror-image concept when writing the finite difference equations for the interior nodes.



$$\text{Node 1 (interior)}: \quad T_1 = (180 + 180 + T_2 + T_3)/4$$

$$\text{Node 2 (interior)}: \quad T_2 = (200 + T_4 + 2T_1)/4$$

$$\text{Node 4 (interior)}: \quad T_4 = (2T_2 + 2T_3)/4$$

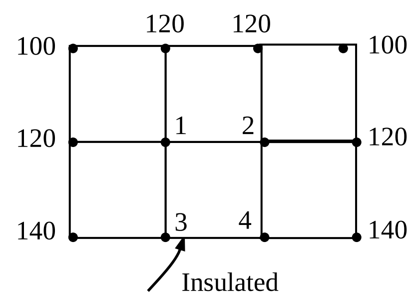
Also, $T_3 = T_2$

Solving the equations above simultaneously gives

$$T_2 = T_3 = T_4 = \mathbf{190°C}$$

$$T_1 = \mathbf{185°C}$$

(b) There is symmetry about the insulated surface as well as the diagonal line. Replacing the symmetry lines by insulation, and utilizing the mirror-image concept, the finite difference equations for the interior nodes can be written as



$$\text{Node 1 (interior)}: \quad T_1 = (120 + 120 + T_2 + T_3)/4$$

$$\text{Node 2 (interior)}: \quad T_2 = (120 + 120 + T_4 + T_1)/4$$

$$\text{Node 3 (interior)}: \quad T_3 = (140 + 2T1 + T_4)/4 = T_2$$

$$\text{Node 4 (interior)}: \quad T_4 = (2T_2 + 140 + 2T_3)/4$$

Solving the equations above simultaneously gives

$$T_1 = T_2 = \mathbf{122.9°C}$$

$$T_3 = T_4 = \mathbf{128.6°C}$$

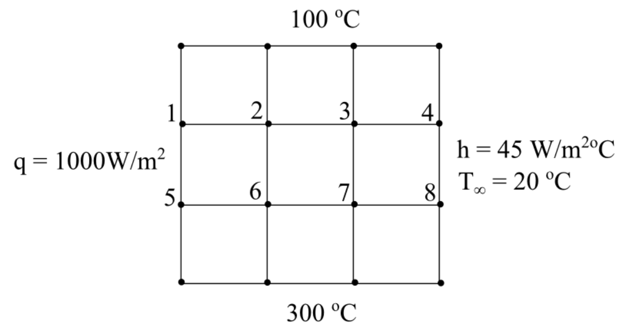
Discussion Note that taking advantage of symmetry simplified the problem greatly.

5-68 A square cross section geometry is subjected to four different boundary conditions. The temperature distribution within the geometry is to be determined using Gauss-Seidel iteration method.

Assumptions **1** Steady heat conduction is two-dimensional without internal heat generation. **2** Thermal conductivity is constant.

Properties Thermal conductivity is given as k=20 W/m·K.

Analysis: (a) There are 4 internal nodes (node 2, 2, 6 and 7) and 4 boundary nodes (node 1, 4, 5 and 8). Thus we need to have 8 equations for 8 unknown temperatures. For internal nodes we can use the general form of the finite difference equation expressed as



$$\frac{T_{m-1,n}-2T_{m,n}+T_{m+1,n}}{\Delta x^2}+\frac{T_{m,n-1}-2T_{m,n}+T_{m,n+1}}{\Delta y^2}=0$$

For $\Delta x=\Delta y$, above equation simplifies to

$$T_{m,n}=0.25(T_{m,n+1}+T_{m+1,n}+T_{m,n-1}+T_{m-1,n})$$

The finite difference equation of the boundary nodes can be found by energy balance at the boundary node control volume assuming all heat transfer is to the node volume.

Node 1: (Left boundary node)

$$\dot{q}\Delta y+k\Delta y\frac{(T_2-T_1)}{\Delta x}+k\frac{\Delta x}{2}\frac{(100-T_1)}{\Delta y}+k\frac{\Delta x}{2}\frac{(T_5-T_1)}{\Delta y}=0\ \Rightarrow\ T_1=25.25+0.5T_1+0.25T_5$$

Node 2: (Internal node)

$$T_2=0.25(T_1+100+T_3+T_6)$$

Node 3: (Internal node)

$$T_3=0.25(T_2+100+T_4+T_7)$$

Node 4: (Right boundary node)

$$k\frac{\Delta x}{2}\frac{(100-T_4)}{\Delta y}+k\frac{\Delta x}{2}\frac{(T_8-T_4)}{\Delta y}+k\Delta y\frac{(T_3-T_4)}{\Delta x}+h\Delta y(T_\infty-T_4)=0\ \Rightarrow\ T_4=0.4944(50.45+0.5T_8+T_3)$$

Node 5: (Left boundary node)

$$\dot{q}\Delta y+k\Delta y\frac{(T_6-T_5)}{\Delta x}+k\frac{\Delta x}{2}\frac{(T_1-T_5)}{\Delta y}+k\frac{\Delta x}{2}\frac{(300-T_1)}{\Delta y}=0\ \Rightarrow\ T_5=0.5(0.5T_1+150.5+T_6)$$

Node 6: (Internal node)

$$T_6=0.25(T_5+T_2+T_7+300)$$

Node 7: (Internal node

$$T_7=0.25(T_6+T_3+T_8+300)$$

Node 8: (Right boundary node)

$$k\frac{\Delta x}{2}\frac{(300-T_8)}{\Delta y}+k\frac{\Delta x}{2}\frac{(T_4-T_8)}{\Delta y}+k\Delta y\frac{(T_7-T_8)}{\Delta x}+h\Delta y(T_\infty-T_8)=0\ \Rightarrow\ T_8=0.4944(150.45+0.5T_4+T_7)$$

(b) By letting the initial guess as 200ºC at each node, the temperature distribution obtained using Gauss-Seidel iteration method is as follows

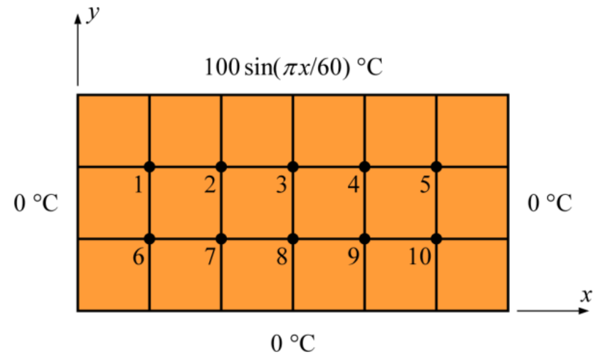
	Nodal temperature, ºC							
Iteration	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
1	175.3	168.8	167.2	157.0	219.1	222.0	222.3	223.1
2	167.6	164.2	160.9	159.6	228.1	228.7	228.2	226.6
3	166.1	163.9	162.9	161.5	231.1	230.8	230.1	228.1
4	166.1	165.0	164.1	162.5	232.2	231.8	231.0	228.8
5	166.3	165.6	164.8	162.9	232.7	232.3	231.5	229.1
6	166.6	165.9	165.1	163.2	233.1	232.6	231.7	229.3
7	166.8	166.1	165.3	163.3	233.3	232.8	231.8	229.4
8	167.0	166.2	165.3	163.4	233.4	232.9	231.9	229.4
9	167.1	166.3	165.4	163.4	233.5	232.9	231.9	229.5
10	167.2	166.4	165.4	163.5	233.5	233.0	232.0	229.5
11	167.2	166.4	165.5	163.5	233.5	233.0	232.0	229.5

5-69 A rectangular cross section is undergoing a steady two-dimensional heat transfer. The finite difference equations and the nodal temperatures are to be determined.

Assumptions **1** Steady heat conduction is two-dimensional. **2** Thermal properties are constant. **3** There is no heat generation in the body.

Analysis (*a*) There are 10 unknown nodal temperatures, thus we need to have 10 equations to determine them uniquely. For nodes 1 to 10, we can use the general finite difference relation expressed as

$$\frac{T_{m-1,n} - 2T_{m,n} + T_{m+1,n}}{\Delta x^2} + \frac{T_{m,n-1} - 2T_{m,n} + T_{m,n+1}}{\Delta y^2} = 0$$



Since $\Delta x = \Delta y$, we have

$$T_{m,n} = 0.25(T_{m,n+1} + T_{m-1,n} + T_{m,n-1} + T_{m+1,n})$$

or $$T_{\text{node}} = 0.25(T_{\text{top}} + T_{\text{left}} + T_{\text{bottom}} + T_{\text{right}})$$

Then

Node 1: $T_1 = 0.25[100\sin(\pi/6) + 0 + T_6 + T_2]$

Node 2: $T_2 = 0.25[100\sin(2\pi/6) + T_1 + T_7 + T_3]$

Node 3: $T_3 = 0.25[100\sin(3\pi/6) + T_2 + T_8 + T_4]$

Node 4: $T_4 = 0.25[100\sin(4\pi/6) + T_3 + T_9 + T_5]$

Node 5: $T_5 = 0.25[100\sin(5\pi/6) + T_4 + T_{10} + 0]$

Node 6: $T_6 = 0.25[T_1 + 0 + 0 + T_7]$

Node 7: $T_7 = 0.25[T_2 + T_6 + 0 + T_8]$

Node 8: $T_8 = 0.25[T_3 + T_7 + 0 + T_9]$

Node 9: $T_9 = 0.25[T_4 + T_8 + 0 + T_{10}]$

Node 10: $T_{10} = 0.25[T_5 + T_9 + 0 + 0]$

(*b*) The nodal temperatures under steady conditions are determined by solving the 10 equations above simultaneously with an equation solver. Copy the following lines and paste on a blank EES screen to solve the above equations:

```
T_1=0.25*(100*sin(pi/6)+0+T_6+T_2)
T_2=0.25*(100*sin(2*pi/6)+T_1+T_7+T_3)
T_3=0.25*(100*sin(3*pi/6)+T_2+T_8+T_4)
T_4=0.25*(100*sin(4*pi/6)+T_3+T_9+T_5)
T_5=0.25*(100*sin(5*pi/6)+T_4+T_10+0)
T_6=0.25*(T_1+0+0+T_7)
T_7=0.25*(T_2+T_6+0+T_8)
T_8=0.25*(T_3+T_7+0+T_9)
T_9=0.25*(T_4+T_8+0+T_10)
T_10=0.25*(T_5+T_9+0+0)
```

Solving by EES software, we get

T_1 = **27.4°C**, T_2 = **47.4°C**, T_3 = **54.7°C**, T_4 = **47.4°C**, T_5 = **27.4°C**

T_6 = **12.1°C**, T_7 = **20.9°C**, T_8 = **24.1°C**, T_9 = **20.9°C**, T_{10} = **12.1°C**

Discussion The numerical solution can be verified using the following analytical solution:

$$T(x,y) = \frac{100\sinh(\pi\ y/60)\sin(\pi\ x/60)}{\sinh(30\pi/60)}$$

For example, at x = 30 cm and y = 20 cm, we have

$$T(30\text{ cm}, 20\text{ cm}) = \frac{100\sinh(20\pi/60)\sin(30\pi/60)}{\sinh(30\pi/60)} = 54.3\,^\circ\text{C}$$

When compared with the numerical solution, T_3 = 54.7°C, the difference is within 0.8%.

5-70 A rectangular metal block is subjected to specified boundary conditions. The finite difference equations and the nodal temperatures are to be determined.

Assumptions **1**Two-dimensional steady state heat conduction. **2** No internal heat generation. **3** Thermal conductivity is constant.

Properties Thermal conductivity of the metal block is given as k = 35 W/m·K.

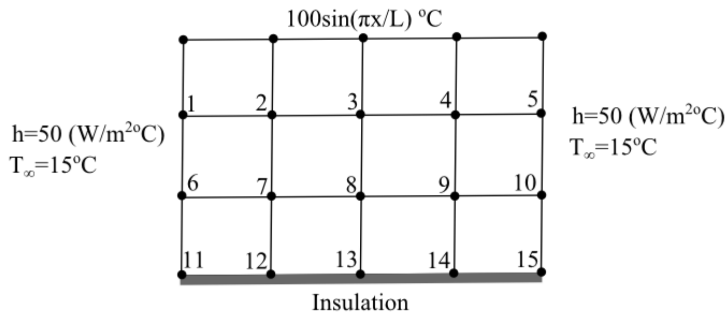
Analysis (a) There are 15 unknown temperatures while the temperatures on the top surface of the block are to be determined from the given sinusoidal temperature distribution. Given that $\Delta x = \Delta y = 25 cm$, the number of nodes in x and y directions are 5 and 4, respectively. For internal nodes i.e., 2, 3, 4, 7, 8 and 9 we can use the general form of the finite difference equations without heat generation expressed as

$$\frac{T_{m-1,n} - 2T_{m,n} + T_{m+1,n}}{\Delta x^2} + \frac{T_{m,n-1} - 2T_{m,n} + T_{m,n+1}}{\Delta y^2} = 0$$

Since $\Delta x = \Delta y$, we have

$$T_{m,n} = 0.25(T_{m,n+1} + T_{m-1,n} + T_{m,n-1} + T_{m+1,n})$$

For the boundary (external nodes) the finite difference formulation is obtained using energy balance and considering all heat transfer towards these nodes.



Thus the finite difference equations at each node are expressed as follows

Node 1: $h\Delta y(T_\infty - T_1) + k\Delta y\frac{(T_2 - T_1)}{\Delta x} + k\frac{\Delta x}{2}\frac{(T_6 - T_1)}{\Delta y} + k\frac{\Delta x}{2}\frac{(100\sin(0\pi) - T_1)}{\Delta y} = 0$

Node 2: $T_1 + T_7 + T_3 + 100\sin(\pi/4) - 4T_2 = 0$

Node 3: $T_2 + T_8 + T_4 + 100\sin(\pi/2) - 4T_3 = 0$

Node 4: $T_3 + T_9 + T_5 + 100\sin(3\pi/4) - 4T_4 = 0$

Node 5: $h\Delta y(T_\infty - T_5) + k\Delta y\frac{(T_4 - T_5)}{\Delta x} + k\frac{\Delta x}{2}\frac{(100\sin(\pi) - T_5)}{\Delta y} + k\frac{\Delta x}{2}\frac{(T_{10} - T_5)}{\Delta y} = 0$

Node 6: $h\Delta y(T_\infty - T_6) + k\Delta y\frac{(T_7 - T_6)}{\Delta x} + k\frac{\Delta x}{2}\frac{(T_1 - T_6)}{\Delta y} + k\frac{\Delta x}{2}\frac{(T_{11} - T_6)}{\Delta y} = 0$

Node 7: $T_6 + T_{12} + T_8 + T_2 - 4T_7 = 0$

Node 8: $T_7 + T_{13} + T_9 + T_3 - 4T_8 = 0$

Node 9: $T_8 + T_{14} + T_{10} + T_4 - 4T_9 = 0$

Node 10: $h\Delta y(T_\infty - T_{10}) + k\Delta y\frac{(T_9 - T_{10})}{\Delta x} + k\frac{\Delta x}{2}\frac{(T_5 - T_{10})}{\Delta y} + k\frac{\Delta x}{2}\frac{(T_{15} - T_{10})}{\Delta y} = 0$

Node 11: $h\frac{\Delta y}{2}(T_\infty - T_{11}) + \dot{q}\frac{\Delta x}{2} + k\frac{\Delta y}{2}\frac{(T_{12} - T_{11})}{\Delta x} + k\frac{\Delta x}{2}\frac{(T_6 - T_{11})}{\Delta y} = 0$

Node 12: $k\frac{\Delta y}{2}\frac{(T_{11} - T_{12})}{\Delta x} + k\frac{\Delta y}{2}\frac{(T_{13} - T_{12})}{\Delta x} + k\Delta x\frac{(T_7 - T_{12})}{\Delta y} + \dot{q}\Delta x = 0$

Node 13: $k\frac{\Delta y}{2}\frac{(T_{12} - T_{13})}{\Delta x} + k\frac{\Delta y}{2}\frac{(T_{14} - T_{13})}{\Delta x} + k\Delta x\frac{(T_8 - T_{13})}{\Delta y} + \dot{q}\Delta x = 0$

Node 14: $k\frac{\Delta y}{2}\frac{(T_{13} - T_{14})}{\Delta x} + k\frac{\Delta y}{2}\frac{(T_{15} - T_{14})}{\Delta x} + k\Delta x\frac{(T_9 - T_{14})}{\Delta y} + \dot{q}\Delta x = 0$

Node 15: $h\frac{\Delta y}{2}(T_\infty - T_{15}) + \dot{q}\frac{\Delta x}{2} + k\frac{\Delta y}{2}\frac{(T_{14} - T_{15})}{\Delta x} + k\frac{\Delta x}{2}\frac{(T_{10} - T_{15})}{\Delta y} = 0$

(b) The nodal temperatures at these different nodes can be determined by solving the equations simultaneously in equation solver. Solving these equations simultaneously in EES or any other software gives

T_1 = **32.91ºC**, T_2 = **53.77ºC,** T_3 = **64.77ºC,** T_4 = **53.77ºC,** T_5 = **32.91ºC,**
T_6 = **36.87ºC,** T_7 = **46.69ºC,** T_8 = **51.55ºC,** T_9 = **46.69ºC,** T_{10} = **36.87ºC,**
T_{11} = **36.82ºC,** T_{12} = **44.56ºC,** T_{13} = **48.06ºC,** T_{14} = **44.56ºC,** T_{15} = **36.82ºC.**

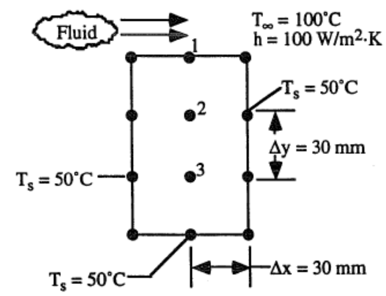
Discussion Thermal symmetry is observed about the centerline due to sinusoidal temperature distribution on the top surface and similar convective environments on both sides of the metal block. Thermal symmetry may not exist in case of the change in environment temperature or heat transfer coefficient on either side.

5-71 A rectangular cross section is subjected to convection on the top surface and constant temperature boundary condition on the other three surfaces. The temperatures at nodes 1, 2, and 3 are to be determined using Gauss-Seidel iteration method.

Assumptions **1** Steady heat conduction is two-dimensional without internal heat generation. **2** Thermal conductivity is constant.

Properties Thermal conductivity is given as k = 1 W/m·K.

Analysis: Node 1: Follow the development of the equation for node 7 of example problem 5-3 in the text with $\Delta x = \Delta y$



$$h\Delta x(T_\infty - T_1) + k\,\frac{\Delta x}{2}\,\frac{50-T_1}{\Delta x} + k\Delta x\,\frac{T_2-T_1}{\Delta x} + k\,\frac{\Delta x}{2}\,\frac{50-T_1}{\Delta x} = 0$$

$$\frac{h\Delta x}{k}(T_\infty - T_1) + 50 - T_1 + T_2 - T_1 = 0$$

where

$$\frac{h\Delta x}{k} = \frac{\left(100\frac{W}{m^2}\cdot K\right)(0.03\ m)}{1\ \mathrm{W/m\cdot K}} = 3 \quad \text{and } T_\infty = 100\,℃$$

The eq. for Node 1 reduces to

$$3(100 - T_1) + 50 - 2T_1 + T_2 = 0 \;\rightarrow\; T_1 = 0.2\,T_2 + 70$$

Node 2: This is an interior node, use Eq. 5-35

$$50 + T_1 + 50 + T_3 - 4T_2 = 0 \;\rightarrow T_2 = 0.25T_1 + 0.25T_3 + 25$$

Node 3: This is an interior node, use Eq. 5-35

$$50 + T_2 + 50 + 50 - 4T_3 = 0 \;\rightarrow T_3 = 0.25T_2 + 37.5$$

Nodal temperature, ºC				
Iteration	T_1	T_2	T_3	
Initial Guess	0	0	0	
1	70.00	42.50	48.13	
2	78.50	56.66	51.67	
3	81.33	58.25	52.06	
4	**81.65**	**58.43**	**52.11**	Final answer ε < 0.35 for all temps.

5-72 A long solid body is subjected to steady two-dimensional heat transfer. The unknown nodal temperatures and the rate of heat loss from the top surface are to be determined.

Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** Heat is generated uniformly in the body.

Properties The thermal conductivity is given to be k = 150 W/m·°C.

Analysis (*a*) The nodal spacing is given to be $\Delta x=\Delta x=l=0.1$ m, and the general finite difference form of an interior node equation for steady two-dimensional heat conduction for the case of constant heat generation is expressed as

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{e}_{\text{node}} l^2}{k} = 0$$

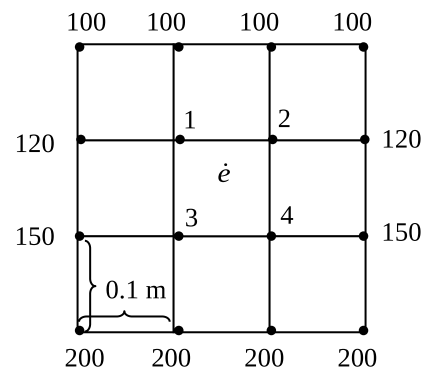
There is symmetry about a vertical line passing through the middle of the region, and thus we need to consider only half of the region. Then,

$$T_1 = T_2 \text{ and } T_3 = T_4$$

Therefore, there are there are only 2 unknown nodal temperatures, T_1 and T_3, and thus we need only 2 equations to determine them uniquely. Also, we can replace the symmetry lines by insulation and utilize the mirror-image concept when writing the finite difference equations for the interior nodes.

Node 1 (interior): $\quad 100 + 120 + T_2 + T_3 - 4T_1 + \dfrac{\dot{e} l^2}{k} = 0$

Node 3 (interior): $\quad 150 + 200 + T_1 + T_4 - 4T_3 + \dfrac{\dot{e} l^2}{k} = 0$



Noting that $T_1 = T_2$ and $T_3 = T_4$ and substituting,

$$220 + T_3 - 3T_1 + \frac{(3\times10^7 \text{ W/m}^3)(0.1\text{m})^2}{150 \text{ W/m}\cdot°\text{C}} = 0$$

$$350 + T_1 - 3T_3 + \frac{(3\times10^7 \text{ W/m}^3)(0.1\text{m})^2}{150 \text{ W/m}\cdot°\text{C}} = 0$$

The solution of the above system is

$$T_1 = T_2 = \mathbf{1126°C}$$
$$T_3 = T_4 = \mathbf{1159°C}$$

(*b*) The total rate of heat transfer from the top surface $\dot{Q}_{\text{top}}$ can be determined from an energy balance on a volume element at the top surface whose height is $l/2$, length 0.3 m, and depth 1 m:

$$\dot{Q}_{\text{top}} + \dot{e}_0(0.3\times1\times l/2) + \left(2k\frac{l\times1}{2}\frac{120-100}{l} + 2kl\times1\frac{T_1-100}{l}\right) = 0$$

$$\dot{Q}_{\text{top}} = -(3\times10^7 \text{ W/m}^3)(0.3\times0.1/2)\text{m}^3 - 2(150 \text{ W/m}\cdot°\text{C})\left(\frac{1\,\text{m}}{2}(120-100)°\text{C} + (1\,\text{m})(1126\text{-}100)°\text{C}\right)$$

$$= \mathbf{760{,}900\ W} \qquad \text{(per } m \text{ depth)}$$

5-73 Prob. 5-72 is reconsidered. The unknown nodal temperatures and the rate of heat loss from the top surface are to be determined.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
e_gen=3e7 [W/m^3] "heat generation"
k=150 [W/m-K] "thermal conductivity"
L=0.10 [m] "mesh size"

"ANALYSIS"
"(a) Using the finite difference method, the nodal temperatures can be determined"
100+120+T_2+T_3-4*T_1+e_gen*L^2/k=0 "for node 1"
T_2=T_1 "for node 2"
150+200+T_1+T_4-4*T_3+e_gen*L^2/k=0 "for node 3"
T_4=T_3 "for node 4"

"(b) The rate of heat loss from the top surface is calculated using"
Q_dot=e_gen*(0.3*1*L/2)+(2*k*L/2*(120-100)/L+2*k*L*(T_1-100)/L)
```

(*a*) The nodal temperatures are determined to be

$$T_1 = T_2 = \mathbf{1126°C} \quad \text{and} \quad T_3 = T_4 = \mathbf{1159°C}$$

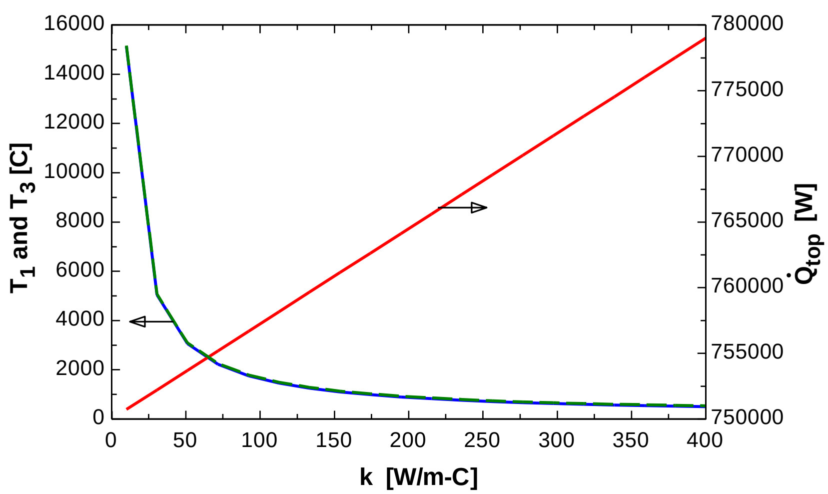
(*b*) The rate of heat loss from the top surface is $\dot{Q} = \mathbf{760900\ W}$.

5-74 Prob. 5-72 is reconsidered. The effects of the thermal conductivity and the heat generation rate on the temperatures at nodes 1 and 3, and the rate of heat loss from the top surface are to be investigated.

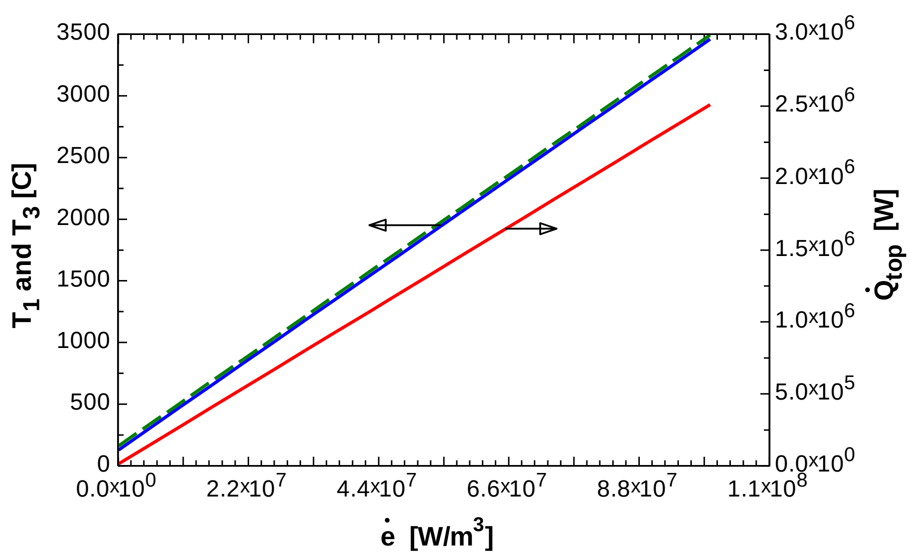
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
k=150 [W/m-C]
e_dot=3E7 [W/m^3]
DELTAx=0.10 [m]
DELTAy=0.10 [m]
d=1 [m] "depth"
"Temperatures at the selected nodes are also given in the figure"

"ANALYSIS"
"(a)"
l=DELTAx
T_1=T_2 "due to symmetry"
T_3=T_4 "due to symmetry"
"Using the finite difference method, the two equations for the two unknown temperatures are determined to be"
100+120+T_2+T_3-4*T_1+(e_dot*l^2)/k=0
150+200+T_1+T_4-4*T_3+(e_dot*l^2)/k=0
"(b)"
"The rate of heat loss from the top surface can be determined from an energy balance on a volume element
whose height is l/2, length 3*l, and depth d=1 m"
-Q_dot_top+e_dot*(3*l*d*l/2)+2*(k*(l*d)/2*(120-100)/l+k*l*d*(T_1-100)/l)=0
```

k [W/m.C]	T_1 [C]	T_3 [C]	$\dot{Q}_{top}$ [W]
10	15126	15159	750725
30.53	5040	5073	752213
51.05	3064	3097	753701
71.58	2222	2254	755189
92.11	1755	1787	756678
112.6	1458	1491	758166
133.2	1253	1285	759654
153.7	1102	1135	761142
174.2	987.3	1020	762630
194.7	896.5	929	764118
215.3	823.1	855.6	765607
235.8	762.4	794.9	767095
256.3	711.5	744	768583
276.8	668.1	700.6	770071
297.4	630.7	663.2	771559
317.9	598.1	630.6	773047
338.4	569.5	602	774536
358.9	544.1	576.6	776024
379.5	521.5	554	777512
400	501.2	533.7	779000



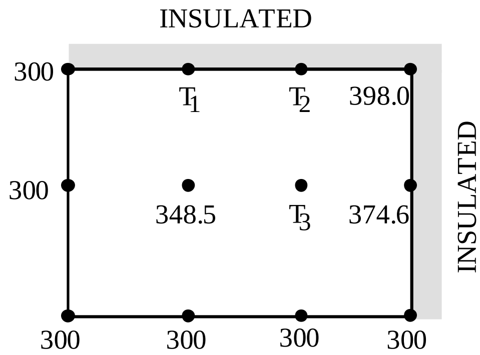
$\dot{e}$ [W/m^3]	T_1 [C]	T_3 [C]	$\dot{Q}_{top}$ [W]
100000	129.6	162.1	13375
5.358E+06	304.8	337.3	144822
1.061E+07	480.1	512.6	276270
1.587E+07	655.4	687.9	407717
2.113E+07	830.6	863.1	539164
2.639E+07	1006	1038	670612
3.165E+07	1181	1214	802059
3.691E+07	1356	1389	933507
4.216E+07	1532	1564	1.065E+06
4.742E+07	1707	1739	1.196E+06
5.268E+07	1882	1915	1.328E+06
5.794E+07	2057	2090	1.459E+06
6.319E+07	2233	2265	1.591E+06
6.845E+07	2408	2441	1.722E+06
7.371E+07	2583	2616	1.854E+06
7.897E+07	2759	2791	1.985E+06
8.423E+07	2934	2966	2.117E+06
8.948E+07	3109	3142	2.248E+06
9.474E+07	3284	3317	2.379E+06
1.000E+08	3460	3492	2.511E+06



5-75 A two-dimensional system is subjected to steady heat transfer. The unknown nodal temperatures are to be determined.

Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** Heat is generated uniformly in the body. **3** Radiation heat transfer is negligible.

Properties The thermal conductivity is given to be k = 20 W/m·K.



Analysis The nodal spacing is given as $\Delta x = \Delta y$= 5 mm, and the general finite difference equation form of an interior node for steady two-dimensional heat conduction is expressed as

$$T_{m-1,n} + T_{m+1,n} + T_{m,n+1} + T_{m,n-1} - 4T_{m,n} + \frac{\dot{e}_{m,n}\,\Delta x^2}{k} = 0$$

where $$\frac{\dot{e}_{m,n}\,\Delta x^2}{k} = \frac{(5\times 10^7\ \mathrm{W/m^3})(0.005\ \mathrm{m})^2}{20\ \mathrm{W/m{\cdot}K}} = 62.5\ \mathrm{K}$$

Apply the general finite difference equation form of an interior node to the upper right corner node to find T_2 and recognize that the insulated boundary can be treated as an interior node by replacing the insulation with a mirror. In this case, $T_{m-1,n} = T_{m+1,n}$ and $T_{m,n-1} = T_{m,n+1}$.

$$T_2 + T_2 + 374.6 + 374.6 - 4(398.0) + 62.5 = 0$$

$$T_2 = \frac{1}{2}(-2\times 374.6 + 4\times 398.0 - 62.5) = \mathbf{390.2\ K}$$

Apply the general finite difference equation form of an interior node to node 3 to find T_3

$$348.5 + 374.6 + T_2 + 300 - 4T_3 + 62.5 = 0$$

$$T_3 = \frac{1}{4}(348.5 + 374.6 + 390.2 + 300 + 62.5) \approx \mathbf{369\,K}$$

Apply the general finite difference equation form of an interior node to node 1 to find T_1 with $T_{m,n-1} = T_{m,n+1}$

$$300 + T_2 + 348.5 + 348.5 - 4T_1 + 62.5 = 0$$

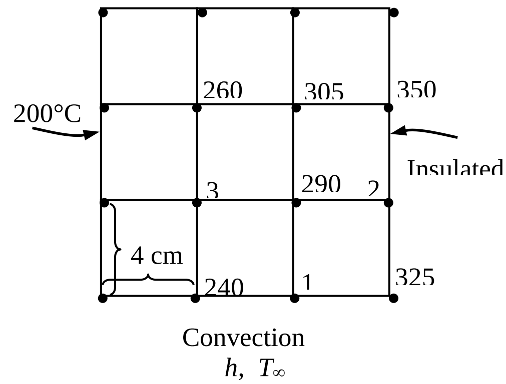
$$T_1 = \frac{1}{4}(300 + 390.2 + 2\times 348.5 + 62.5) = \mathbf{362.5K}$$

5-76 A long solid body is subjected to steady two-dimensional heat transfer. The unknown nodal temperatures and the rate of heat loss from the bottom surface through a 1-m long section are to be determined.

Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** Heat is generated uniformly in the body. **3** Radiation heat transfer is negligible.

Properties The thermal conductivity is given to be k = 45 W/m·°C.

Analysis The nodal spacing is given to be $\Delta x = \Delta x = l = 0.04$ m, and the general finite difference form of an interior node for steady two-dimensional heat conduction is expressed as



$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{e}_{\text{node}} l^2}{k} = 0$$

where

$$\frac{\dot{e}_{\text{node}} l^2}{k} = \frac{\dot{e}_0 l^2}{k} = \frac{(4\times10^6 \text{ W/m}^3)(0.04\text{ m})^2}{45 \text{ W/m}\cdot\text{°C}} = 142.2\text{°C}$$

The finite difference equations for boundary nodes are obtained by applying an energy balance on the volume elements and taking the direction of all heat transfers to be towards the node under consideration:

$$\text{Node 1 (convection)}: \quad k\frac{l}{2}\frac{240-T_1}{l} + kl\frac{290-T_1}{l} + k\frac{l}{2}\frac{325-T_1}{l} + hl(T_\infty - T_1) + \frac{\dot{e}_0 l^2}{2k} = 0$$

$$\text{Node 2 (interior)}: \quad 350+290+325+290-4T_2 + \frac{\dot{e}_0 l^2}{k} = 0$$

$$\text{Node 3 (interior)}: \quad 260+290+240+200-4T_3 + \frac{\dot{e}_0 l^2}{k} = 0$$

where

$$k = 45 \text{ W/m.°C}, \quad h = 50 \text{ W/m}^2\text{.°C}, \quad \dot{e} = 4\times10^6 \text{ W/m}^3, \quad T_\infty = 20\text{°C}$$

Substituting,

T_1 **= 281.2°C**, T_2 **= 349.3°C**, T_3 **= 283.1°C**,

(*b*) The rate of heat loss from the bottom surface through a 1-m long section is

$$\begin{aligned}
\dot{Q} &= \sum_m \dot{Q}_{\text{element},\,m} = \sum_m hA_{\text{surface},\,m}(T_m - T_\infty) \\
&= h(l/2)(200-T_\infty) + hl(240-T_\infty) + hl(T_1 - T_\infty) + h(l/2)(325-T_\infty) \\
&= (50 \text{ W/m}^2\cdot\text{°C})(0.04\text{ m}\times 1\text{ m})[(200-20)/2 + (240-20) + (281.2-20) + (325-20)/2]\text{°C} \\
&= \mathbf{1447\ \ W}
\end{aligned}$$

5-77 Prob. 5-76 is reconsidered. The unknown nodal temperatures and the rate of heat loss from the bottom surface through a 1-m long section are to be determined.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
e_gen=4e6 [W/m^3] "heat generation"
h=50 [W/m^2-K] "convection coefficient"
k=45 [W/m-K] "thermal conductivity"
L=0.04 [m] "mesh size"
T_inf=20 [C] "ambient temperature"

"ANALYSIS"
"(a) Using the finite difference method, the 3 equations for the 3 nodal temperatures can be determined"
k*L/2*(240-T_1)/L+k*L*(290-T_1)/L+k*L/2*(325-T_1)/L+h*L*(T_inf-T_1)+e_gen*L^2/(2*k)=0 "for node 1"
350+290+325+290-4*T_2+e_gen*L^2/k=0 "for node 2"
260+290+240+200-4*T_3+e_gen*L^2/k=0 "for node 3"

"(b) The rate of heat loss from the bottom surface is calculated by summing the heat loss from each node"
Q_dot=h*L/2*(200-T_inf)+h*L*(240-T_inf)+h*L*(T_1-T_inf)+h*L/2*(325-T_inf)
```

(*a*) The nodal temperatures are determined to be

$$T_1 = \mathbf{281°C}, \quad T_2 = \mathbf{349°C}, \quad \text{and} \quad T_3 = \mathbf{283°C}$$

(*b*) The rate of heat loss from the bottom surface is $\dot{Q} = \mathbf{1447\ W}$.

5-78 A rectangular block is subjected to uniform heat flux at the top, and iced water at 0°C at the sides. The steady finite difference formulation of the problem is to be obtained, and the unknown nodal temperatures as well as the rate of heat transfer to the iced water are to be determined.

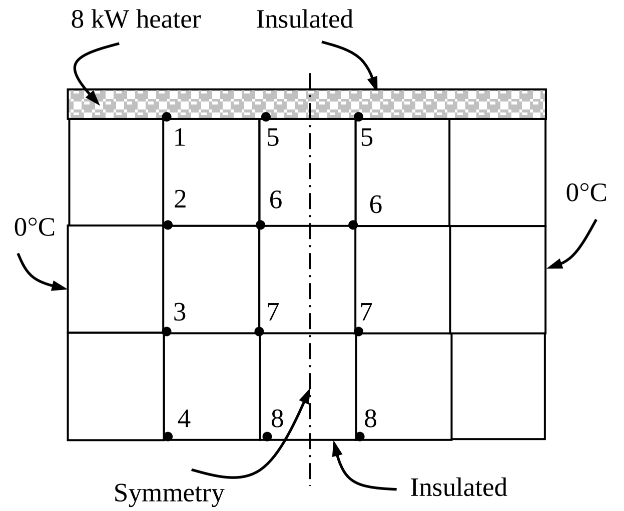
Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** There is no heat generation within the block. **3** The heat transfer coefficient is very high so that the temperatures on both sides of the block can be taken to be 0°C. **4** Heat transfer through the bottom surface is negligible.

Properties The thermal conductivity is given to be k = 23 W/m·°C.

Analysis The nodal spacing is given to be $\Delta x = \Delta x = l = 0.1$ m, and the general finite difference form of an interior node equation for steady 2-D heat conduction is expressed as

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{e}_{\text{node}} l^2}{k} = 0$$

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} = 0$$



There is symmetry about a vertical line passing through the middle of the region, and we need to consider only half of the region. Note that all side surfaces are at T_0 = 0°C, and there are 8 nodes with unknown temperatures. Replacing the symmetry lines by insulation and utilizing the mirror-image concept, the finite difference equations are obtained to be as follows:

Node 1 (heat flux): $\dot{q}_0 l + k\frac{l}{2}\frac{T_0 - T_1}{l} + k\frac{l}{2}\frac{T_5 - T_1}{l} + kl\frac{T_2 - T_1}{l} = 0$

Node 2 (interior): $T_0 + T_1 + T_3 + T_6 - 4T_2 = 0$

Node 3 (interior): $T_0 + T_2 + T_4 + T_7 - 4T_3 = 0$

Node 4 (insulation): $T_0 + 2T_3 + T_8 - 4T_4 = 0$

Node 5 (heat flux): $\dot{q}_0 l + k\frac{l}{2}\frac{T_1 - T_5}{l} + kl\frac{T_6 - T_5}{l} + 0 = 0$

Node 6 (interior): $T_2 + T_5 + T_6 + T_7 - 4T_6 = 0$

Node 7 (interior): $T_3 + T_6 + T_7 + T_8 - 4T_7 = 0$

Node 8 (insulation): $T_4 + 2T_7 + T_8 - 4T_8 = 0$

where

l = 0.1 m, k = 23 W/m·°C, T_0 =0°C, and $\dot{q}_0 = \dot{Q}_0 / A = (8000\text{ W})/(5\times 0.5\text{ m}^2) = 3200\text{ W/m}^2$

This system of 8 equations with 8 unknowns constitutes the finite difference formulation of the problem.

(*b*) The 8 nodal temperatures under steady conditions are determined by solving the 8 equations above simultaneously with an equation solver to be

T_1 = **18.2°C**, T_2 = **9.9°C**, T_3 = **6.2°C**, T_4 = **5.2°C**, T_5 = **25.4°C**, T_6 = **15.0°C**, T_7 = **9.9°C**, T_8 = **8.3°C**

(*c*) The rate of heat transfer from the block to the iced water is 8 kW since all the heat supplied to the block from the top must be equal to the heat transferred from the block. Therefore, $\dot{Q} = \mathbf{8\,kW}$.

Discussion The rate of heat transfer can also be determined by calculating the heat loss from the side surfaces using the heat conduction relation.

5-79 A square cross section with uniform heat generation is undergoing a steady two-dimensional heat transfer. The finite difference equations and the nodal temperatures are to be determined.

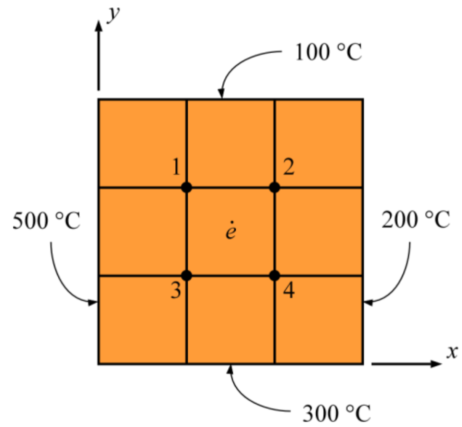
Assumptions **1** Steady heat conduction is two-dimensional. **2** Thermal properties are constant. **3** The heat generation in the body is uniform.

Properties The conductivity is given to be $k = 25$ W/m·K .

Analysis (*a*) There are 4 unknown nodal temperatures, thus we need to have 4 equations to determine them uniquely. For nodes 1 to 4, we can use the general finite difference relation expressed as

$$\frac{T_{m-1,n} - 2T_{m,n} + T_{m+1,n}}{\Delta x^2} + \frac{T_{m,n-1} - 2T_{m,n} + T_{m,n+1}}{\Delta y^2} + \frac{\dot{e}_{m,n}}{k} = 0$$

$$T_{m,n} = 0.25(T_{m-1,n} + T_{m+1,n} + T_{m,n-1} + T_{m,n+1} + \dot{e}_{m,n}\Delta x^2 / k)$$



Since $\Delta x = \Delta y$, we have

$$T_{m,n} = 0.25(T_{m,n+1} + T_{m+1,n} + T_{m,n-1} + T_{m-1,n} + \dot{e}_{m,n}\Delta x^2 / k)$$

or $$T_{\text{node}} = 0.25(T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} + T_{\text{left}} + \dot{e}_{\text{node}}\Delta x^2 / k)$$

Then

Node 1: $T_1 = 0.25(100 + T_2 + T_3 + 500 + \dot{e}_{\text{node}}\Delta x^2 / k)$

Node 2: $T_2 = 0.25(100 + 200 + T_4 + T_1 + \dot{e}_{\text{node}}\Delta x^2 / k)$

Node 3: $T_3 = 0.25(T_1 + T_4 + 300 + 500 + \dot{e}_{\text{node}}\Delta x^2 / k)$

Node 4: $T_4 = 0.25(T_2 + 200 + 300 + T_3 + \dot{e}_{\text{node}}\Delta x^2 / k)$

where $\dot{e}_{\text{node}}\Delta x^2 / k = 20\,°\text{C}$.

(*b*) By letting the initial guesses as T_1 = 300°C, T_2 = 150°C, T_3 = 400°C, and T_4 = 250°C the results obtained from successive iterations are listed in the following table:

Iteration	Nodal temperature,°C			
	T_1	T_2	T_3	T_4
1	292.5	215.6	340.6	269.1
2	294.1	220.8	345.8	271.6
3	296.6	222.1	347.1	272.3
4	297.3	222.4	347.4	272.4
5	297.4	222.5	347.5	272.5
6	297.5	222.5	347.5	272.5
7	297.5	222.5	347.5	272.5

Hence, the converged nodal temperatures are

T_1 = **297.5°C**, T_2 = **222.5°C**, T_3 = **347.5°C**, T_4 = **272.5°C**

Discussion The finite difference equations can also be calculated using the EES. Copy the following lines and paste on a blank EES screen to solve the above equations:

```
T_1=0.25*(100+T_2+T_3+500+20)
T_2=0.25*(100+200+T_4+T_1+20)
T_3=0.25*(T_1+T_4+300+500+20)
T_4=0.25*(T_2+200+300+T_3+20)
```

Solving by EES software, we get the same results:

T_1 = **297.5°C**, T_2 = **222.5°C**, T_3 = **347.5°C**, T_4 = **272.5°C**

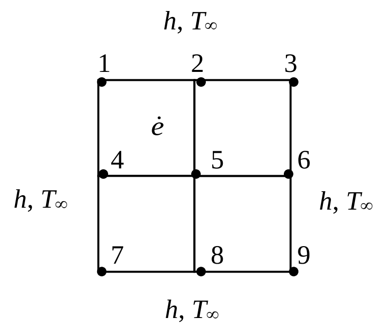
5-80E A long solid bar is subjected to steady two-dimensional heat transfer. The unknown nodal temperatures and the rate of heat loss from the bar through a 1-ft long section are to be determined.

Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** Heat is generated uniformly in the body. **3** The heat transfer coefficient also includes the radiation effects.

Properties The thermal conductivity is given to be k = 16 Btu/h.ft·°C.

Analysis The nodal spacing is given to be $\Delta x=\Delta y=l=0.25$ ft, and the general finite difference form of an interior node for steady two-dimensional heat conduction is expressed as

$$T_{\text{left}}+T_{\text{top}}+T_{\text{right}}+T_{\text{bottom}}-4T_{\text{node}}+\frac{\dot{e}_{\text{node}}l^2}{k}=0$$



(a) There is symmetry about the vertical, horizontal, and diagonal lines passing through the center. Therefore, $T_1=T_3=T_7=T_9$ and $T_2=T_4=T_6=T_8$, and T_1, T_2, and T_5 are the only 3 unknown nodal temperatures, and thus we need only 3 equations to determine them uniquely. Also, we can replace the symmetry lines by insulation and utilize the mirror-image concept for the interior nodes.

The finite difference equations for boundary nodes are obtained by applying an energy balance on the volume elements and taking the direction of all heat transfers to be towards the node under consideration:

$$\text{Node 1 (convection)}: \quad 2k\frac{l}{2}\frac{T_2-T_1}{l}+2h\frac{l}{2}(T_\infty-T_1)+\frac{\dot{e}_0 l^2}{4}=0$$

$$\text{Node 2 (convection)}: \quad 2k\frac{l}{2}\frac{T_1-T_2}{l}+kl\frac{T_5-T_2}{l}+hl(T_\infty-T_2)+\frac{\dot{e}_0 l^2}{2}=0$$

$$\text{Node 5 (interior)}: \quad 4T_2-4T_5+\frac{\dot{e}_0 l^2}{k}=0$$

where $\dot{e}_0=0.19\times10^5$ Btu/h·ft^3, l = 0.25 ft, k = 16 Btu/h.ft·°F, h =7.9 Btu/h.ft^2·°F, and T_∞ =70°F. The 3 nodal temperatures under steady conditions are determined by solving the 3 equations above simultaneously with an equation solver to be

$T_1=T_3=T_7=T_9$ **= 361.89°F,**

$T_2=T_4=T_6=T_8$ **= 379.37°F,** T_5 = **397.93°F**

(b) The rate of heat loss from the bar through a 1-ft long section is determined from an energy balance on one-eight section of the bar, and multiplying the result by 8:

$$\dot{Q}=8\times\dot{Q}_{\text{one-eight section, conv}}=8\times\left[h\frac{l}{2}(T_1-T_\infty)+h\frac{l}{2}(T_2-T_\infty)\right](1\text{ ft})=8\times h\frac{l}{2}\left[T_1+T_2-2T_\infty\right](1\text{ ft})$$

$$=8(7.9\text{ Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(0.25/2\text{ ft})(1\text{ft})[361.89+379.37-2\times70]^\circ\text{F}$$

$$=\mathbf{4750\ Btu/h} \quad \text{(per ft flength)}$$

Discussion Under steady conditions, the rate of heat loss from the bar is equal to the rate of heat generati on within the bar per unit length, and is determined to be

$$\dot{Q}=\dot{E}_{\text{gen}}=\dot{e}_0 V=(0.19\times10^5\text{ Btu/h.ft}^3)(0.5\text{ ft}\times0.5\text{ ft}\times1\text{ ft})=4750\text{ Btu/h} \quad \text{(per ft length)}$$

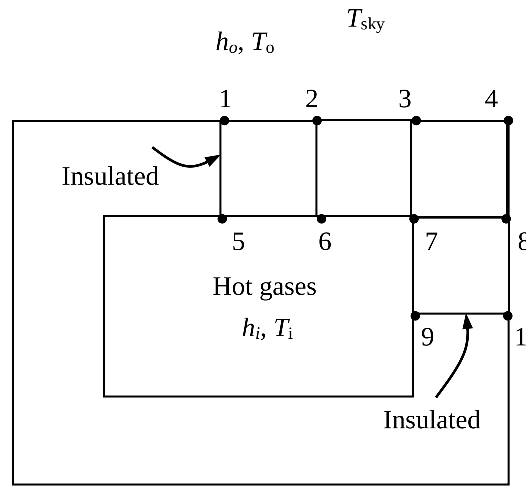
which confirms the results obtained by the finite difference method.

5-81 Heat transfer through a square chimney is considered. The nodal temperatures and the rate of heat loss per unit length are to be determined with the finite difference method.

Assumptions **1** Heat transfer is given to be steady and two-dimensional since the height of the chimney is large relative to its cross-section, and thus heat conduction through the chimney in the axial direction is negligible. It is tempting to simplify the problem further by considering heat transfer in each wall to be one dimensional which would be the case if the walls were thin and thus the corner effects were negligible. This assumption cannot be justified in this case since the walls are very thick and the corner sections constitute a considerable portion of the chimney structure. **2** There is no heat generation in the chimney. **3** Thermal conductivity is constant.

Properties The thermal conductivity and emissivity are given to be k = 1.4 W/m·°C and ε = 0.9.

Analysis (*a*) The most striking aspect of this problem is the apparent symmetry about the horizontal and vertical lines passing through the midpoint of the chimney. Therefore, we need to consider only one-fourth of the geometry in the solution whose nodal network consists of 10 equally spaced nodes. No heat can cross a symmetry line, and thus symmetry lines can be treated as insulated surfaces and thus "mirrors" in the finite-difference formulation. Considering a unit depth and using the energy balance approach for the boundary nodes (again assuming all heat transfer to be into the volume element for convenience), the finite difference formulation is obtained to be



$$\text{Node 1:}\ \ h_o \frac{l}{2}(T_0 - T_1) + k\frac{l}{2}\frac{T_2 - T_1}{l} + k\frac{l}{2}\frac{T_5 - T_1}{l} + \varepsilon\sigma\frac{l}{2}[T_{sky}^4 - (T_1 + 273)^4] = 0$$

$$\text{Node 2:}\ \ h_o l(T_0 - T_2) + k\frac{l}{2}\frac{T_1 - T_2}{l} + k\frac{l}{2}\frac{T_3 - T_2}{l} + kl\frac{T_6 - T_2}{l} + \varepsilon\sigma l[T_{sky}^4 - (T_2 + 273)^4] = 0$$

$$\text{Node 3:}\ \ h_o l(T_0 - T_3) + k\frac{l}{2}\frac{T_2 - T_3}{l} + k\frac{l}{2}\frac{T_4 - T_3}{l} + kl\frac{T_7 - T_3}{l} + \varepsilon\sigma l[T_{sky}^4 - (T_3 + 273)^4] = 0$$

$$\text{Node 4:}\ \ h_o l(T_0 - T_4) + k\frac{l}{2}\frac{T_3 - T_4}{l} + k\frac{l}{2}\frac{T_8 - T_4}{l} + \varepsilon\sigma l[T_{sky}^4 - (T_4 + 273)^4] = 0$$

$$\text{Node 5:}\ \ h_i \frac{l}{2}(T_i - T_5) + k\frac{l}{2}\frac{T_6 - T_5}{l} + k\frac{l}{2}\frac{T_1 - T_5}{l} = 0$$

$$\text{Node 6:}\ \ h_i l(T_i - T_6) + k\frac{l}{2}\frac{T_5 - T_6}{l} + k\frac{l}{2}\frac{T_7 - T_6}{l} + kl\frac{T_2 - T_6}{l} = 0$$

$$\text{Node 7:}\ \ h_i l(T_i - T_7) + k\frac{l}{2}\frac{T_6 - T_7}{l} + k\frac{l}{2}\frac{T_9 - T_7}{l} + kl\frac{T_3 - T_7}{l} + kl\frac{T_8 - T_7}{l} = 0$$

$$\text{Node 8:}\ \ h_0 l(T_0 - T_8) + k\frac{l}{2}\frac{T_4 - T_8}{l} + k\frac{l}{2}\frac{T_{10} - T_8}{l} + kl\frac{T_7 - T_8}{l} + \varepsilon\sigma l[T_{sky}^4 - (T_8 + 273)^4] = 0$$

$$\text{Node 9:}\ \ h_i \frac{l}{2}(T_i - T_9) + k\frac{l}{2}\frac{T_7 - T_9}{l} + k\frac{l}{2}\frac{T_{10} - T_9}{l} = 0$$

$$\text{Node 10:}\ \ h_o \frac{l}{2}(T_0 - T_{10}) + k\frac{l}{2}\frac{T_8 - T_{10}}{l} + k\frac{l}{2}\frac{T_9 - T_{10}}{l} + \varepsilon\sigma\frac{l}{2}[T_{sky}^4 - (T_{10} + 273)^4] = 0$$

where l = 0.1 m, k = 1.4 W/m·°C, h_i = 75 W/m²·°C, T_i =280°C, h_o = 18 W/m²·°C, T_0 =15°C, T_{surr} =250 K, ε = 0.9, and σ = 5.67×10^{-8} W/m².K^4. This system of 10 equations with 10 unknowns constitutes the finite difference formulation of the problem.

(*b*) The 10 nodal temperatures under steady conditions are determined by solving the 10 equations above simultaneously with an equation solver to be

T_1 = 94.5°C, T_2 = 92.9°C, T_3 = 81.8°C, T_4 = 35.8°C, T_5 = 250.5°C,

T_6 = 249.0°C, T_7 = 228.7°C, T_8 = 81.5°C, T_9 = 247.3°C, T_{10} = 90.8°C

(*c*) The rate of heat loss through a 1-m long section of the chimney is determined from

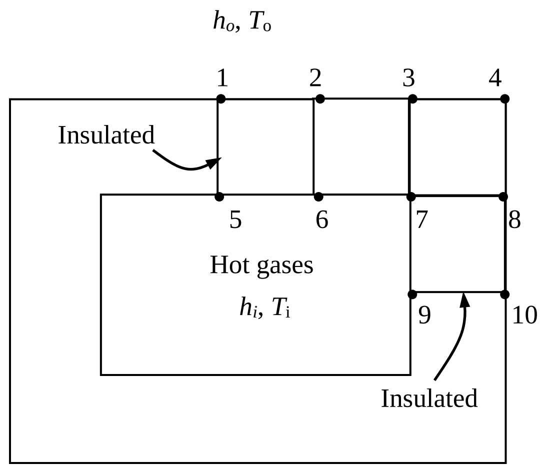
$$\begin{aligned}\dot{Q} &= 4\sum \dot{Q}_{\text{one-fourth of chimney}} = 4\sum \dot{Q}_{\text{element, inner surface}} = 4\sum_m h_i A_{\text{surface, }m}(T_i - T_m)\\ &= 4[h_i(l/2)(T_i - T_5) + h_i l(T_i - T_6) + h_i l(T_i - T_7) + h_i(l/2)(T_i - T_9)]\\ &= 4(75\ \text{W/m}^2\cdot{}^\circ\text{C})(0.1\,\text{m}\times 1\,\text{m})[(280 - 250.5)/2 + (280 - 249.0) + (280 - 228.7) + (280 - 247.3)/2]^\circ\text{C}\\ &= \mathbf{3402\ W}\end{aligned}$$

Discussion The rate of heat transfer can also be determined by calculating the heat loss from the outer surface by convection and radiation.

5-82 Heat transfer through a square chimney is considered. The nodal temperatures and the rate of heat loss per unit length are to be determined with the finite difference method.

Assumptions **1** Heat transfer is given to be steady and two-dimensional since the height of the chimney is large relative to its cross-section, and thus heat conduction through the chimney in the axial direction is negligible. It is tempting to simplify the problem further by considering heat transfer in each wall to be one dimensional which would be the case if the walls were thin and thus the corner effects were negligible. This assumption cannot be justified in this case since the walls are very thick and the corner sections constitute a considerable portion of the chimney structure. **2** There is no heat generation in the chimney. **3** Thermal conductivity is constant. **4** Radiation heat transfer is negligible.

Properties The thermal conductivity of chimney is given to be k = 1.4 W/m·°C.



Analysis (a) The most striking aspect of this problem is the apparent symmetry about the horizontal and vertical lines passing through the midpoint of the chimney. Therefore, we need to consider only one-fourth of the geometry in the solution whose nodal network consists of 10 equally spaced nodes. No heat can cross a symmetry line, and thus symmetry lines can be treated as insulated surfaces and thus "mirrors" in the finite-difference formulation. Considering a unit depth and using the energy balance approach for the boundary nodes (again assuming all heat transfer to be into the volume element for convenience), the finite difference formulation is obtained to be

Node 1: $$h_o \frac{l}{2}(T_0 - T_1) + k\frac{l}{2}\frac{T_2 - T_1}{l} + k\frac{l}{2}\frac{T_5 - T_1}{l} = 0$$

Node 2: $$h_o l(T_0 - T_2) + k\frac{l}{2}\frac{T_1 - T_2}{l} + k\frac{l}{2}\frac{T_3 - T_2}{l} + kl\frac{T_6 - T_2}{l} = 0$$

Node 3: $$h_o l(T_0 - T_3) + k\frac{l}{2}\frac{T_2 - T_3}{l} + k\frac{l}{2}\frac{T_4 - T_3}{l} + kl\frac{T_7 - T_3}{l} = 0$$

Node 4: $$h_o l(T_0 - T_4) + k\frac{l}{2}\frac{T_3 - T_4}{l} + k\frac{l}{2}\frac{T_8 - T_4}{l} = 0$$

Node 5: $$h_i \frac{l}{2}(T_i - T_5) + k\frac{l}{2}\frac{T_6 - T_5}{l} + k\frac{l}{2}\frac{T_1 - T_5}{l} = 0$$

Node 6: $$h_i l(T_i - T_6) + k\frac{l}{2}\frac{T_5 - T_6}{l} + k\frac{l}{2}\frac{T_7 - T_6}{l} + kl\frac{T_2 - T_6}{l} = 0$$

Node 7: $$h_i l(T_i - T_7) + k\frac{l}{2}\frac{T_6 - T_7}{l} + k\frac{l}{2}\frac{T_9 - T_7}{l} + kl\frac{T_3 - T_7}{l} + kl\frac{T_8 - T_7}{l} = 0$$

Node 8: $$h_o l(T_0 - T_8) + k\frac{l}{2}\frac{T_4 - T_8}{l} + k\frac{l}{2}\frac{T_{10} - T_8}{l} + kl\frac{T_7 - T_8}{l} = 0$$

Node 9: $$h_i \frac{l}{2}(T_i - T_9) + k\frac{l}{2}\frac{T_7 - T_9}{l} + k\frac{l}{2}\frac{T_{10} - T_9}{l} = 0$$

Node 10: $$h_o \frac{l}{2}(T_0 - T_{10}) + k\frac{l}{2}\frac{T_8 - T_{10}}{l} + k\frac{l}{2}\frac{T_9 - T_{10}}{l} = 0$$

where l = 0.1 m, k = 1.4 W/m·°C, h_i = 75 W/m^2·°C, T_i =280°C, h_o = 18 W/m^2·°C, T_0 =15°C, and $\sigma = 5.67\times10^{-8}$ W/m^2.K^4. This system of 10 equations with 10 unknowns constitutes the finite difference formulation of the problem.

(b) The 10 nodal temperatures under steady conditions are determined by solving the 10 equations above simultaneously with an equation solver to be

T_1 = **118.6°C**, T_2 = **116.4°C**, T_3 = **103.0°C**, T_4 = **53.3°C**, T_5 = **254.3°C**,
T_6 = **252.8°C**, T_7 = **234.1°C**, T_8 = **102.4°C**, T_9 = **251.0°C**, T_{10} = **113.4°C**

(c) The rate of heat loss through a 1-m long section of the chimney is determined from

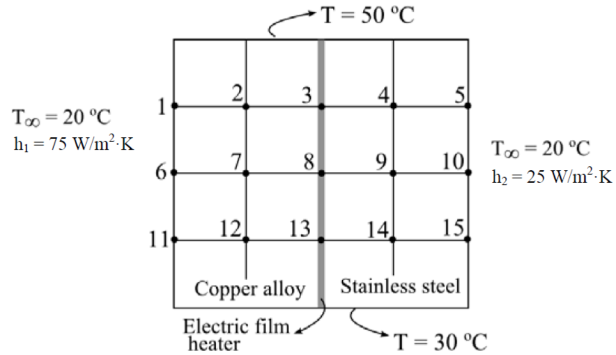
$$\begin{aligned}\dot{Q} &= 4\sum \dot{Q}_{\text{one-fourth of chimney}} = 4\sum \dot{Q}_{\text{element, inner surface}} = 4\sum_m h_i A_{\text{surface, } m}(T_i - T_m)\\ &= 4[h_i(l/2)(T_i - T_5) + h_i l(T_i - T_6) + h_i l(T_i - T_7) + h_i(l/2)(T_i - T_9)]\\ &= 4(75\ \text{W/m}^2\cdot{}^\circ\text{C})(0.1\,\text{m}\times 1\,\text{m})[(280-254.3)/2 + (280-252.8) + (280-234.1) + (280-251.0)/2]^\circ\text{C}\\ &= \mathbf{3014\ W}\end{aligned}$$

Discussion The rate of heat transfer can also be determined by calculating the heat loss from the outer surface by convection.

5-83 A thin film heater is sandwiched between copper alloy and stainless steel blocks. By developing finite difference formulation at different nodes, the location of maximum temperature is to be determined.

Assumptions **1** Heat transfer is steady state and two-dimensional without heat generation. **2** Thermal conductivities of both copper alloy and stainless steel stay constant.

Properties: The thermal conductivity of the copper alloy and stainless steel is given to be 120 W/m·K and 15 W/m·K, respectively.



Analysis: (a) The finite difference formulation at each node is calculated using energy balance at each node such that all heat transfer is to the node control volume. Nodes 1, 5, 6, 10, 11 and 15 are the boundary nodes subjected to convection boundary condition while nodes 2, 4, 7, 9, 12 and 14 are the internal nodes. The nodes 3, 8 and 13 are the internal node but are subjected to a uniform heat flux by a thin film heater.

Node 1: $$h_1\Delta y(T_\infty - T_1) + k_1\frac{\Delta x}{2}\frac{(50-T_1)}{\Delta y} + k_1\Delta y\frac{(T_2-T_1)}{\Delta x} + k_1\frac{\Delta x}{2}\frac{(T_6-T_1)}{\Delta y} = 0$$

Node 2: $$k_1\Delta y\frac{(T_1-T_2)}{\Delta x} + k_1\Delta x\frac{(50-T_2)}{\Delta y} + k_1\Delta x\frac{(T_7-T_2)}{\Delta y} + k_1\Delta y\frac{(T_3-T_2)}{\Delta x} = 0$$

Node 3: $$k_1\Delta y\frac{(T_2-T_3)}{\Delta x} + (k_1+k_2)\frac{\Delta x}{2}\frac{(50-T_3)}{\Delta y} + (k_1+k_2)\frac{\Delta x}{2}\frac{(T_8-T_3)}{\Delta y} + k_2\Delta y\frac{(T_4-T_3)}{\Delta x} + \dot{q}\Delta y = 0$$

Node 4: $$k_2\Delta y\frac{(T_3-T_4)}{\Delta x} + k_2\Delta x\frac{(50-T_4)}{\Delta y} + k_2\Delta x\frac{(T_9-T_4)}{\Delta y} + k_2\Delta y\frac{(T_5-T_4)}{\Delta x} = 0$$

Node 5: $$h_2\Delta y(T_\infty - T_5) + k_2\frac{\Delta x}{2}\frac{(50-T_5)}{\Delta y} + k_2\Delta y\frac{(T_4-T_5)}{\Delta x} + k_2\frac{\Delta x}{2}\frac{(T_{10}-T_5)}{\Delta y} = 0$$

Node 6: $$h_1\Delta y(T_\infty - T_6) + k_1\frac{\Delta x}{2}\frac{(T_1-T_6)}{\Delta y} + k_1\Delta y\frac{(T_7-T_6)}{\Delta x} + k_1\frac{\Delta x}{2}\frac{(T_{11}-T_6)}{\Delta y} = 0$$

Node 7: $$k_1\Delta y\frac{(T_6-T_7)}{\Delta x} + k_1\Delta x\frac{(T_2-T_7)}{\Delta y} + k_1\Delta x\frac{(T_{12}-T_7)}{\Delta y} + k_1\Delta y\frac{(T_8-T_7)}{\Delta x} = 0$$

Node 8: $$k_1\Delta y\frac{(T_7-T_8)}{\Delta x} + (k_1+k_2)\frac{\Delta x}{2}\frac{(T_3-T_8)}{\Delta y} + (k_1+k_2)\frac{\Delta x}{2}\frac{(T_{13}-T_8)}{\Delta y} + k_2\Delta y\frac{(T_9-T_8)}{\Delta x} + \dot{q}\Delta y = 0$$

Node 9: $$k_2\Delta y\frac{(T_8-T_9)}{\Delta x} + k_2\Delta x\frac{(T_4-T_9)}{\Delta y} + k_2\Delta x\frac{(T_{14}-T_9)}{\Delta y} + k_2\Delta y\frac{(T_{10}-T_9)}{\Delta x} = 0$$

Node 10: $$h_2\Delta y(T_\infty - T_{10}) + k_2\frac{\Delta x}{2}\frac{(T_5-T_{10})}{\Delta y} + k_2\Delta y\frac{(T_9-T_{10})}{\Delta x} + k_2\frac{\Delta x}{2}\frac{(T_{15}-T_{10})}{\Delta y} = 0$$

Node 11: $$h_1\Delta y(T_\infty - T_{11}) + k_1\frac{\Delta x}{2}\frac{(T_6-T_{11})}{\Delta y} + k_1\Delta y\frac{(T_{12}-T_{11})}{\Delta x} + k_1\frac{\Delta x}{2}\frac{(30-T_{11})}{\Delta y} = 0$$

Node 12: $$k_1\Delta y\frac{(T_{11}-T_{12})}{\Delta x} + k_1\Delta x\frac{(T_7-T_{12})}{\Delta y} + k_1\Delta x\frac{(30-T_{12})}{\Delta y} + k_1\Delta y\frac{(T_{13}-T_{12})}{\Delta x} = 0$$

Node 13: $$k_1\Delta y\frac{(T_{12}-T_{13})}{\Delta x} + (k_1+k_2)\frac{\Delta x}{2}\frac{(T_8-T_{13})}{\Delta y} + (k_1+k_2)\frac{\Delta x}{2}\frac{(30-T_{13})}{\Delta y} + k_2\Delta y\frac{(T_{14}-T_{13})}{\Delta x} + \dot{q}\Delta y = 0$$

Node 14: $$k_2\Delta y\frac{(T_{13}-T_{14})}{\Delta x} + k_2\Delta x\frac{(T_9-T_{14})}{\Delta y} + k_2\Delta x\frac{(30-T_{14})}{\Delta y} + k_2\Delta y\frac{(T_{15}-T_{14})}{\Delta x} = 0$$

Node 15: $$h_2\Delta y(T_\infty - T_{15}) + k_2\frac{\Delta x}{2}\frac{(T_{10}-T_{15})}{\Delta y} + k_2\Delta y\frac{(T_{14}-T_{15})}{\Delta x} + k_2\frac{\Delta x}{2}\frac{(30-T_{15})}{\Delta y} = 0$$

For a 500 W heater, the heat flux on each slab of 100 mm × 100 mm cross section is 50000 W/m^2.

(b) The temperature at each node is determined by solving the 15 equations for 15 unknown nodal temperatures using EES or any other software. The temperature at different nodes are as follows

T_1 = **49.07ºC**, T_2 = **50.71ºC**, T_3 = **55.97ºC**, T_4 = **50.46ºC**, T_5 = **48.38ºC,**

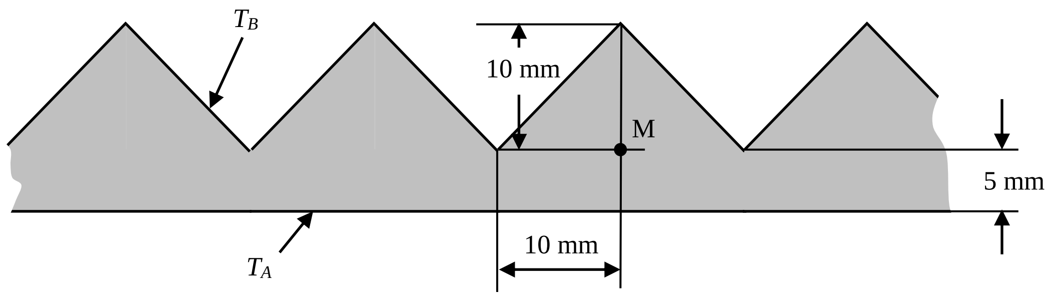
T_6 = **45.76ºC**, T_7 = **47.8ºC**, T_8 = **54ºC**, T_9 = **47.49ºC**, T_{10} = **44.98ºC,**

T_{11} = **39.16ºC**, T_{12} = **40.74ºC**, T_{13} = **45.99ºC**, T_{14} = **40.52ºC**, T_{15} = **38.62ºC.**

The maximum temperature occurs at node 8 (Center node) where the temperature is **54ºC**.

5-84 Two dimensional ridges are machined on the cold side of a heat exchanger. The smallest section of the wall is to be identified. A two-dimensional grid is to be constructed and the unknown temperatures in the grid are to be determined.

Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation.



Analysis (*a*) From symmetry, the smallest domain is between the top and the base of one ridge.

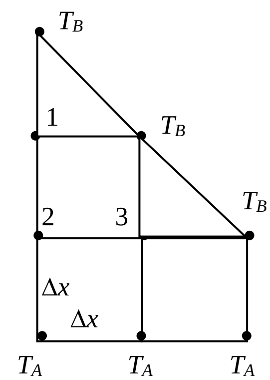
(*b*) The unknown temperatures at nodes 1, 2, and 3 are to be determined from finite difference formulations

Node 1:

$$k\frac{T_B-T_1}{\Delta x}\Delta x+k\frac{T_2-T_1}{\Delta x}\frac{\Delta x}{2}+k\frac{T_B-T_1}{\Delta x}\frac{\Delta x}{2}=0$$

$$2T_B-2T_1+T_2-T_1+T_B-T_1=0$$

$$4T_1-T_2=3T_B=3\times 10=30$$



Node 2:

$$k\frac{T_1-T_2}{\Delta x}\frac{\Delta x}{2}+k\frac{T_3-T_2}{\Delta x}\Delta x+k\frac{T_A-T_2}{\Delta x}\frac{\Delta x}{2}=0$$

$$T_1-T_2+2T_3-2T_2+T_A-T_2=0$$

$$-T_1+4T_2-2T_3=T_A=90$$

Node 3:

$$4T_3=T_2+T_A+T_B+T_B$$

$$-T_2+4T_3=2T_B+T_A=2\times 10+90=110$$

The matrix equation is

$$\begin{bmatrix}4 & -1 & 0\\ -1 & 4 & -2\\ 0 & -1 & 4\end{bmatrix}\begin{bmatrix}T_1\\ T_2\\ T_3\end{bmatrix}=\begin{bmatrix}30\\ 90\\ 110\end{bmatrix}$$

(*c*) The temperature T_2 is 46.9ºC. Then the temperatures T_1 and T_3 are determined from equations 1 and 3.

$$4T_1-T_2=30$$

$$4T_1-46.9=30\longrightarrow T_1=\mathbf{19.2^\circ C}$$

$$-T_2+4T_3=110$$

$$-46.9+4T_3=110\longrightarrow T_3=\mathbf{39.2^\circ C}$$

5-85 Two long solid bodies are subjected to steady two-dimensional heat transfer. The unknown nodal temperatures are to be determined.

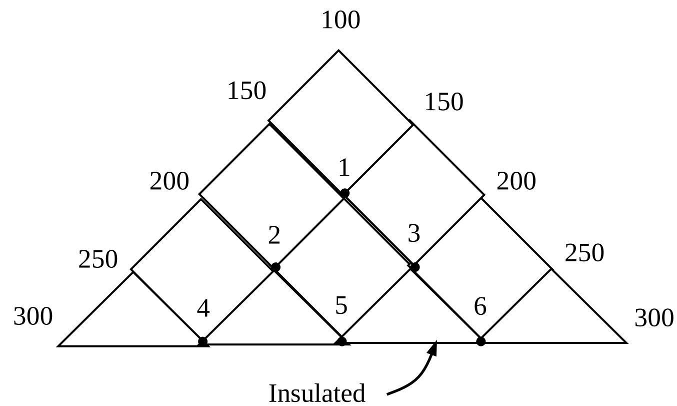
Assumptions **1** Heat transfer through the bodies are given to be steady and two-dimensional. **2** There is no heat generation in the body.

Properties The thermal conductivity is given to be k = 20 W/m·°C.

Analysis The nodal spacing is given to be Δx=Δx=l=0.01m, and the general finite difference form of an interior node for steady two-dimensional heat conduction for the case of no heat generation is expressed as

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{e}_{\text{node}} l^2}{k} = 0 \longrightarrow T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} = 0$$

(*a*) There is symmetry about a vertical line passing through the nodes 1 and 3. Therefore, $T_3 = T_2$, $T_6 = T_4$, and T_1, T_2, T_4, and T_5 are the only 4 unknown nodal temperatures, and thus we need only 4 equations to determine them uniquely. Also, we can replace the symmetry lines by insulation and utilize the mirror-image concept when writing the finite difference equations for the interior nodes.



Node 1 (interior) : $150 + 150 + 2T_2 - 4T_1 = 0$

Node 2 (interior) : $200 + T_1 + T_5 + T_4 - 4T_2 = 0$

Node 4 (interior) : $250 + 250 + T_2 + T_2 - 4T_4 = 0$

Node 5 (interior) : $4T_2 - 4T_5 = 0$

Solving the 4 equations above simultaneously gives

T_1 = **175°C**

$T_2 = T_3$ = **200°C**

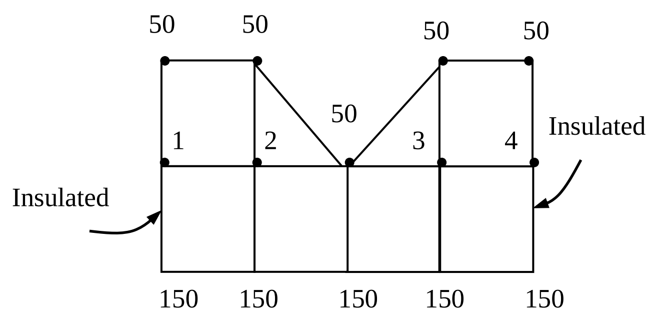
$T_4 = T_6$ = **225°C**

T_5 = **200°C**

(*b*) There is symmetry about a vertical line passing through the middle. Therefore, $T_3 = T_2$ and $T_4 = T_1$. Replacing the symmetry lines by insulation and utilizing the mirror-image concept, the finite difference equations for the interior nodes 1 and 2 are determined to be

Node 1 (interior) : $50 + 150 + 2T_2 - 4T_1 = 0$

Node 2 (interior) : $50 + 50 + 150 + T_1 - 4T_2 = 0$



Solving the 2 equations above simultaneously gives

$T_1 = T_4$ = **92.9°C,** $T_2 = T_3$ = **85.7°C**

Discussion Note that taking advantage of symmetry simplified the problem greatly.

5-86 The exposed surface of a long concrete damn of triangular cross-section is subjected to solar heat flux and convection and radiation heat transfer. The vertical section of the damn is subjected to convection with water. The temperatures at the top, middle, and bottom of the exposed surface of the damn are to be determined.

Assumptions **1** Heat transfer through the damn is given to be steady and two-dimensional. **2** There is no heat generation within the damn. **3** Heat transfer through the base is negligible. **4** Thermal properties and heat transfer coefficients are constant.

Properties The thermal conductivity and solar absorptivity are given to be k = 0.6 W/m·°C and α_s = 0.7.

Analysis The nodal spacing is given to be $\Delta x=\Delta x=l$=1 m, and all nodes are boundary nodes. Node 5 on the insulated boundary can be treated as an interior node for which $T_{\text{left}}+T_{\text{top}}+T_{\text{right}}+T_{\text{bottom}}-4T_{\text{node}}=0$. Using the energy balance approach and taking the direction of all heat transfer to be towards the node, the finite difference equations for the nodes are obtained to be as follows:

Node 1: $$h_i\frac{l}{2}(T_i-T_1)+k\frac{l}{2}\frac{T_2-T_1}{l}+\frac{l/2}{\sin 45}\left[\alpha_s\dot{q}_s+h_0(T_0-T_1)\right]=0$$

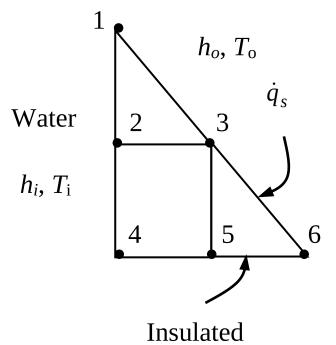
Node 2: $$h_i l(T_i-T_2)+k\frac{l}{2}\frac{T_1-T_2}{l}+k\frac{l}{2}\frac{T_4-T_2}{l}+kl\frac{T_3-T_2}{l}=0$$

Node 3: $$kl\frac{T_2-T_3}{l}+kl\frac{T_5-T_3}{l}+\frac{l}{\sin 45}\left[\alpha_s\dot{q}_s+h_0(T_0-T_3)\right]=0$$

Node 4: $$h_i\frac{l}{2}(T_i-T_4)+k\frac{l}{2}\frac{T_2-T_4}{l}+k\frac{l}{2}\frac{T_5-T_4}{l}=0$$

Node 5: $$T_4+2T_3+T_6-4T_5=0$$

Node 6: $$k\frac{l}{2}\frac{T_5-T_6}{l}+\frac{l/2}{\sin 45}\left[\alpha_s\dot{q}_s+h_0(T_0-T_6)\right]=0$$



where

l = 1 m, k = 0.6 W/m·°C, h_i =150 W/m^2·°C, T_i =15°C, h_o = 30 W/m^2·°C, T_0 =25°C, α_s = 0.7, and $\dot{q}_s$ =800 W/m^2.

The system of 6 equations with 6 unknowns constitutes the finite difference formulation of the problem. The 6 nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver to be

$T_1 = T_{top}$ =**21.3**°C, T_2 =15.1°C, $T_3 = T_{middle}$ =**43.2**°C

T_4 =15.1°C, T_5 =36.3°C, $T_6 = T_{bottom}$ =**43.6**°C

Discussion Note that the highest temperature occurs at a location furthest away from the water, as expected.

EES SOLUTION

```
hi*l/2*(Ti-T1)+k*l/2*(T2-T1)/l+(l/2)/sin(45)*(alpha_s*qs+h0*(T0-T1))=0
hi*l*(Ti-T2)+k*l/2*(T1-T2)/l+k*l/2*(T4-T2)/l+k*l*(T3-T2)/l=0
k*l*(T2-T3)/l+k*l*(T5-T3)/l+l/sin(45)*(alpha_s*qs+h0*(T0-T3))=0
hi*l/2*(Ti-T4)+k*l/2*(T2-T4)/l+k*l/2*(T5-T4)/l=0
T4+2*T3+T6-4*T5=0
k*l/2*(T5-T6)/l+(l/2)/sin(45)*(alpha_s*qs+h0*(T0-T6))=0
l=1
k=0.6
hi=150
h0=30
Ti=15
T0=25
alpha_s=0.7
qs=800
```

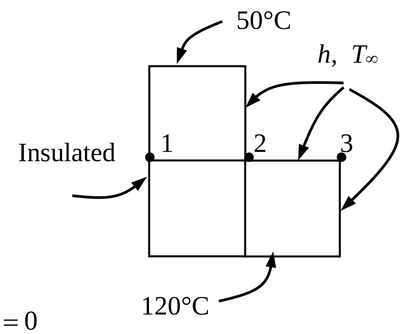
Solution

alpha_s=0.7 h0=30 hi=150 k=0.6 l=1 qs=800 T0=25 T1=21.3 T2=15.12 T3=43.17
T4=15.08 T5=36.25 T6=43.56 Ti=15

5-87 The top and bottom surfaces of an L-shaped long solid bar are maintained at specified temperatures while the left surface is insulated and the remaining 3 surfaces are subjected to convection. The finite difference formulation of the problem is to be obtained, and the unknown nodal temperatures are to be determined.

Assumptions **1** Heat transfer through the bar is given to be steady and two-dimensional. **2** There is no heat generation within the bar. **3** Thermal properties and heat transfer coefficients are constant. **4** Radiation heat transfer is negligible.

Properties The thermal conductivity is given to be k = 5 W/m·°C.



Analysis (*a*) The nodal spacing is given to be $\Delta x=\Delta x=l$=0.1 m, and all nodes are boundary nodes. Node 1 on the insulated boundary can be treated as an interior node for which $T_{\text{left}}+T_{\text{top}}+T_{\text{right}}+T_{\text{bottom}}-4T_{\text{node}}=0$. Using the energy balance approach and taking the direction of all heat transfer to be towards the node, the finite difference equations for the nodes are obtained to be as follows:

Node 1: $\quad 50+120+2T_2-4T_1=0$

Node 2: $\quad hl(T_\infty-T_2)+k\dfrac{l}{2}\dfrac{50-T_2}{l}+k\dfrac{l}{2}\dfrac{T_3-T_2}{l}+kl\dfrac{T_1-T_2}{l}+kl\dfrac{120-T_2}{l}=0$

Node 3: $\quad hl(T_\infty-T_3)+k\dfrac{l}{2}\dfrac{T_2-T_3}{l}+k\dfrac{l}{2}\dfrac{120-T_3}{l}=0$

where

l = 0.1 m, k = 5 W/m·°C, h = 40 W/m^2·°C, and T_∞ =25°C.

This system of 3 equations with 3 unknowns constitute the finite difference formulation of the problem.

(*b*) The 3 nodal temperatures under steady conditions are determined by solving the 3 equations above simultaneously with an equation solver to be

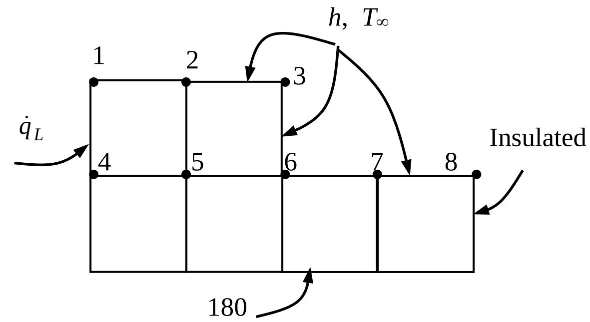
T_1 = **78.8°C,** T_2 = **72.7°C,** T_3 = **64.6°C**

5-88 Heat conduction through a long L-shaped solid bar with specified boundary conditions is considered. The unknown nodal temperatures are to be determined with the finite difference method.

Assumptions **1** Heat transfer through the body is given to be steady and two-dimensional. **2** Thermal conductivity is constant. **3** Heat generation is uniform.

PropertiesThe thermal conductivity is given to be k = 45 W/m·°C.

Analysis (a) The nodal spacing is given to be Δx=Δx=l=0.015 m, and the general finite difference form of an interior node for steady two-dimensional heat conduction for the case of constant heat generation is expressed as



$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{e}_{\text{node}} l^2}{k} = 0$$

We observe that all nodes are boundary nodes except node 5 that is an interior node. Therefore, we will have to rely on energy balances to obtain the finite difference equations. Using energy balances, the finite difference equations for each of the 8 nodes are obtained as follows:

Node 1: $$\dot{q}_L \frac{l}{2} + h\frac{l}{2}(T_\infty - T_1) + k\frac{l}{2}\frac{T_2 - T_1}{l} + k\frac{l}{2}\frac{T_4 - T_1}{l} + \dot{e}_0\frac{l^2}{4} = 0$$

Node 2: $$hl(T_\infty - T_2) + k\frac{l}{2}\frac{T_1 - T_2}{l} + k\frac{l}{2}\frac{T_3 - T_2}{l} + kl\frac{T_5 - T_2}{l} + \dot{e}_0\frac{l^2}{2} = 0$$

Node 3: $$hl(T_\infty - T_3) + k\frac{l}{2}\frac{T_2 - T_3}{l} + k\frac{l}{2}\frac{T_6 - T_3}{l} + \dot{e}_0\frac{l^2}{4} = 0$$

Node 4: $$\dot{q}_L l + k\frac{l}{2}\frac{T_1 - T_4}{l} + k\frac{l}{2}\frac{180 - T_4}{l} + kl\frac{T_5 - T_4}{l} + \dot{e}_0\frac{l^2}{2} = 0$$

Node 5: $$T_4 + T_2 + T_6 + 180 - 4T_5 + \frac{\dot{e}_0 l^2}{k} = 0$$

Node 6: $$hl(T_\infty - T_6) + k\frac{l}{2}\frac{T_3 - T_6}{l} + kl\frac{T_5 - T_6}{l} + kl\frac{180 - T_6}{l} + k\frac{l}{2}\frac{T_7 - T_6}{l} + \dot{e}_0\frac{3l^2}{4} = 0$$

Node 7: $$hl(T_\infty - T_7) + k\frac{l}{2}\frac{T_6 - T_7}{l} + k\frac{l}{2}\frac{T_8 - T_7}{l} + kl\frac{180 - T_7}{l} + \dot{e}_0\frac{l^2}{2} = 0$$

Node 8: $$h\frac{l}{2}(T_\infty - T_8) + k\frac{l}{2}\frac{T_7 - T_8}{l} + k\frac{l}{2}\frac{180 - T_8}{l} + \dot{e}_0\frac{l^2}{4} = 0$$

where

$\dot{e}_0 = 5\times 10^6$ W/m^3, $\dot{q}_L = 8000$ W/m^2, l = 0.015 m, k = 45 W/m·°C, h = 55 W/m^2·°C, and T_∞ =30°C.

This system of 8 equations with 8 unknowns is the finite difference formulation of the problem.

(b) The 8 nodal temperatures under steady conditions are determined by solving the 8 equations above simultaneously with an equation solver to be

T_1 = **221.1°C**, T_2 = **217.9°C**, T_3 = **213.3°C**, T_4 = **212.7°C**, T_5 = **209.6°C**, T_6 = **202.8°C**,

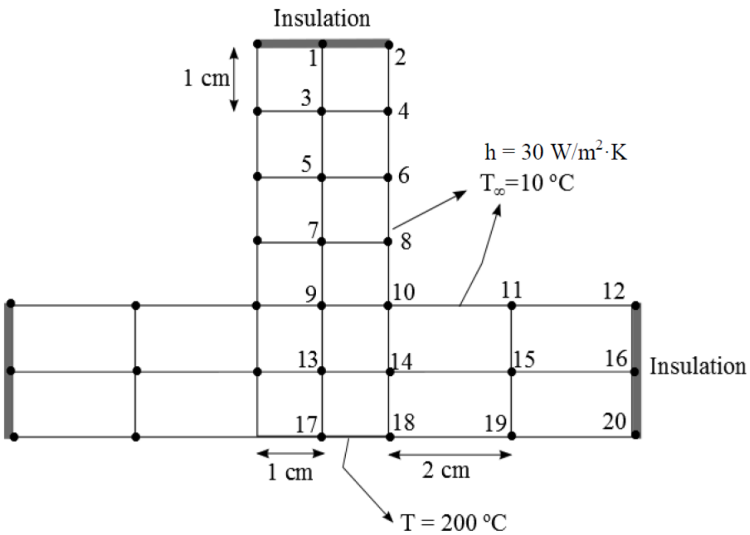
T_7 = **193.3°C**, T_8 = **191.4°C**

Discussion The accuracy of the solution can be improved by using more nodal points.

5-89 T shaped bar with known thermal properties is subjected to convection environment. Taking the advantage of symmetry develop finite difference formulation and determine the nodal temperatures.

Assumptions **1** Steady state two-dimensional heat conduction. **2** Constant thermal conductivities. 3 No internal heat generation.

Properties: The thermal conductivity of the T shaped bar is given as 28 W/m·K.



Analysis The finite difference formulation at all boundary nodes is done by doing energy balance at each node assuming all heat transfer entering the node.

Node 1: $$k\frac{\Delta y}{2}\frac{2(T_2-T_1)}{\Delta x}+2k\Delta x\frac{(T_3-T_1)}{\Delta y}=0$$

Node 2: $$h\frac{\Delta y}{2}(T_\infty-T_2)+k\frac{\Delta y}{2}\frac{(T_1-T_2)}{\Delta x}+k\frac{\Delta x}{2}\frac{(T_4-T_2)}{\Delta y}=0$$

Node 3: $$2T_4+T_1+T_5-4T_3=0$$

Node 4: $$h\Delta y(T_\infty-T_4)+k\Delta y\frac{(T_3-T_4)}{\Delta x}+k\frac{\Delta x}{2}\frac{(T_2-T_4)}{\Delta y}+k\frac{\Delta x}{2}\frac{(T_6-T_4)}{\Delta y}=0$$

Node 5: $$2T_6+T_3+T_7-4T_5=0$$

Node 6: $$h\Delta y(T_\infty-T_6)+k\Delta y\frac{(T_5-T_6)}{\Delta x}+k\frac{\Delta x}{2}\frac{(T_4-T_6)}{\Delta y}+k\frac{\Delta x}{2}\frac{(T_8-T_6)}{\Delta y}=0$$

Node 7: $$2T_8+T_5+T_9-4T_7=0$$

Node 8: $$h\Delta y(T_\infty-T_8)+k\Delta y\frac{(T_7-T_8)}{\Delta x}+k\frac{\Delta x}{2}\frac{(T_6-T_8)}{\Delta y}+k\frac{\Delta x}{2}\frac{(T_{10}-T_8)}{\Delta y}=0$$

Node 9: $$2T_{10}+T_7+T_{13}-4T_9=0$$

Node 10: $$h\left(\frac{2\Delta x}{2}+\frac{\Delta y}{2}\right)(T_\infty-T_{10})+k\Delta y\frac{(T_9-T_{10})}{\Delta x}+k(1.5\Delta x)\frac{(T_{14}-T_{10})}{\Delta y}+k\frac{\Delta y}{2}\frac{(T_{11}-T_{10})}{2\Delta x}+k(1.5\Delta x)\frac{(T_8-T_{10})}{\Delta y}=0$$

Node 11: $$h(2\Delta x)(T_\infty-T_{11})+k\frac{\Delta y}{2}\frac{(T_{10}-T_{11})}{2\Delta x}+k\frac{\Delta y}{2}\frac{(T_{12}-T_{11})}{2\Delta x}+k(2\Delta x)\frac{(T_{15}-T_{11})}{\Delta y}=0$$

Node 12: $$h\Delta x(T_\infty-T_{12})+k\frac{\Delta y}{2}\frac{(T_{11}-T_{12})}{2\Delta x}+k\frac{2\Delta x}{2}\frac{(T_{16}-T_{12})}{\Delta y}=0$$

Node 13: $$2T_{14}+T_9+T_{17}-4T_{13}=0$$

Node 14: $$k\Delta y\frac{(T_{13}-T_{14})}{\Delta x}+k\Delta y\frac{(T_{15}-T_{14})}{2\Delta x}+k(1.5\Delta x)\frac{(T_{10}-T_{14})}{\Delta y}+k(1.5\Delta x)\frac{(T_{18}-T_{14})}{\Delta y}=0$$

This equation for node 14 reduces to

$$T_{13} + 0.5T_{15} + 1.5T_{10} + 1.5T_{18} - 4.5T_{14} = 0$$

Node 15:

$$\frac{(T_{14} + T_{16} - 2T_{15})}{(2\Delta x)^2} + \frac{(T_{11} + T_{19} - 2T_{15})}{(\Delta y)^2} = 0$$

This equation for node 15 reduces to

$$T_{14} + T_{16} + 4T_{11} + 4T_{19} - 10T_{15} = 0$$

Node 16:

$$k\Delta y \frac{(T_{15} - T_{16})}{2\Delta x} + k\frac{2\Delta x}{2}\frac{(T_{12} - T_{16})}{\Delta y} + k\frac{2\Delta x}{2}\frac{(T_{20} - T_{16})}{\Delta y} = 0$$

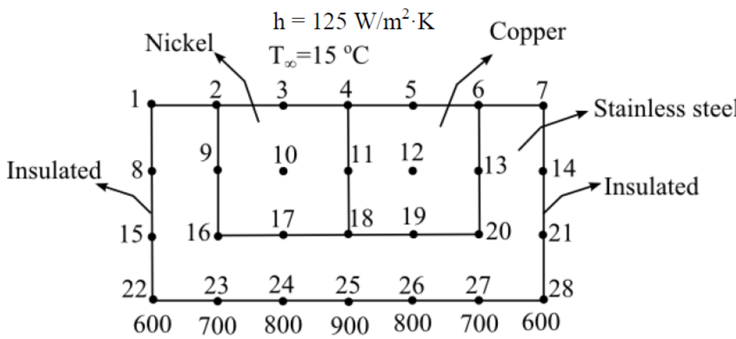
Solving these 16 equations simultaneously for 16 unknown nodal temperatures using EES or any other software gives

T_1 = **174.8ºC**, T_2 = **173.7ºC**, T_3 = **175.3ºC**, T_4 = **174.4ºC**, T_5 = **177.7ºC**,
T_6 = **176.8ºC**, T_7 = **181.8ºC**, T_8 = **181ºC**, T_9 = **187.5ºC**, T_{10} = **187.2ºC**,
T_{11} = **194.1ºC**, T_{12} = **195.2ºC**, T_{13} = **193.9ºC**, T_{14} = **194ºC**, T_{15} = **196.8ºC**,
T_{16} = **197.4ºC**, T_{17} = **200ºC**, T_{18} = **200ºC**, T_{19} = **200ºC**, T_{20} = **200ºC**.

5-90 Nickel and Copper are embedded into stainless steel material during a sintering process. For the given boundary conditions, the finite difference equations and the temperatures at different nodes are to be determined.

Assumptions **1** Two-dimensional steady state heat conduction with no heat generation. **2** Thermal conductivities for each material are constant. **3** Perfect contact at the interface.

Properties: The thermal conductivities of Nickel, Copper, and stainless steel are given as k = 90.7 W/m·K, k = 401 W/m·K, and k = 15.1 W/m·K, respectively.



Analysis The top surface is exposed to the convection while the two sides are insulated. The bottom surface is subjected to a non-uniform temperature. Nodes 9, 10, 11, 12, 13, 16, 17, 18, 19, 20 are the internal nodes. However due to different thermal conductivity of the three materials in use, Eq. (5-35) cannot be used for all internal nodes except node 10 and node 12. The nodes 8, 14, 15 and 21 at the two insulated sides can also be treated as the interior node using the mirror image concept at adiabatic boundary.

For all other nodes, the finite difference formulation can be obtained by doing an energy balance at the node volume and assuming all heat transfers entering the node volume. At node 18, the effect of thermal conductivity of stainless steel, nickel and copper on the heat transfer and hence the nodal temperature must be considered.

For the case of two-dimensional steady state heat conduction without internal heat generation, the finite difference formulations for different nodes are expressed as follows

Node 1: $$h\frac{\Delta x}{2}(T_\infty - T_1) + k_A\frac{\Delta y}{2}\frac{(T_2 - T_1)}{\Delta x} + k_A\frac{\Delta x}{2}\frac{(T_8 - T_1)}{\Delta y} = 0$$

Node 2: $$h\Delta x(T_\infty - T_2) + k_A\frac{\Delta y}{2}\frac{(T_1 - T_2)}{\Delta x} + k_B\frac{\Delta y}{2}\frac{(T_3 - T_2)}{\Delta x} + (k_A + k_B)\frac{\Delta x}{2}\frac{(T_9 - T_2)}{\Delta y} = 0$$

Node 3: $$h\Delta x(T_\infty - T_3) + k_B\frac{\Delta y}{2}\frac{(T_2 - T_3)}{\Delta x} + k_B\frac{\Delta y}{2}\frac{(T_4 - T_3)}{\Delta x} + k_B\Delta x\frac{(T_{10} - T_3)}{\Delta y} = 0$$

Node 4: $$h\Delta x(T_\infty - T_4) + k_B\frac{\Delta y}{2}\frac{(T_3 - T_4)}{\Delta x} + k_C\frac{\Delta y}{2}\frac{(T_5 - T_4)}{\Delta x} + (k_B + k_C)\frac{\Delta x}{2}\frac{(T_{11} - T_4)}{\Delta y} = 0$$

Node 5: $$h\Delta x(T_\infty - T_5) + k_C\frac{\Delta y}{2}\frac{(T_4 - T_5)}{\Delta x} + k_C\frac{\Delta y}{2}\frac{(T_6 - T_5)}{\Delta x} + k_C\Delta x\frac{(T_{12} - T_5)}{\Delta y} = 0$$

Node 6: $$h\Delta x(T_\infty - T_6) + k_A\frac{\Delta y}{2}\frac{(T_7 - T_6)}{\Delta x} + k_C\frac{\Delta y}{2}\frac{(T_5 - T_6)}{\Delta x} + (k_A + k_C)\frac{\Delta x}{2}\frac{(T_{13} - T_6)}{\Delta y} = 0$$

Node 7: $$h\frac{\Delta x}{2}(T_\infty - T_7) + k_A\frac{\Delta y}{2}\frac{(T_6 - T_7)}{\Delta x} + k_A\frac{\Delta x}{2}\frac{(T_{14} - T_7)}{\Delta y} = 0$$

Node 8: $$T_1 + T_{15} + 2T_9 - 4T_8 = 0$$

Node 9: $$k_A\Delta y\frac{(T_8 - T_9)}{\Delta x} + k_B\Delta y\frac{(T_{10} - T_9)}{\Delta x} + (k_A + k_B)\frac{\Delta x}{2}\frac{(T_2 - T_9)}{\Delta y} + (k_A + k_B)\frac{\Delta x}{2}\frac{(T_{16} - T_9)}{\Delta y} = 0$$

Node 10: $$T_3 + T_{17} + T_9 + T_{11} - 4T_{10} = 0$$

Node 11: $$k_B\Delta y\frac{(T_{10} - T_{11})}{\Delta x} + k_C\Delta y\frac{(T_{12} - T_{11})}{\Delta x} + (k_B + k_C)\frac{\Delta x}{2}\frac{(T_4 - T_{11})}{\Delta y} + (k_B + k_C)\frac{\Delta x}{2}\frac{(T_{18} - T_{11})}{\Delta y} = 0$$

Node 12: $$T_{11} + T_5 + T_{19} + T_{13} - 4T_{12} = 0$$

Node 13: $k_A \Delta y \dfrac{(T_{14} - T_{13})}{\Delta x} + k_C \Delta y \dfrac{(T_{12} - T_{13})}{\Delta x} + (k_A + k_C)\dfrac{\Delta x}{2}\dfrac{(T_6 - T_{13})}{\Delta y} + (k_A + k_C)\dfrac{\Delta x}{2}\dfrac{(T_{20} - T_{13})}{\Delta y} = 0$

Node 14: $T_7 + T_{21} + 2T_{13} - 4T_{14} = 0$

Node 15: $T_8 + T_{22} + 2T_{16} - 4T_{15} = 0$

Node 16: $k_A \Delta y \dfrac{(T_{15} - T_{16})}{\Delta x} + k_A \Delta x \dfrac{(T_{23} - T_{16})}{\Delta y} + (k_A + k_B)\dfrac{\Delta x}{2}\dfrac{(T_9 - T_{16})}{\Delta y} + (k_A + k_B)\dfrac{\Delta y}{2}\dfrac{(T_{17} - T_{16})}{\Delta x} = 0$

Node 17: $k_B \Delta x \dfrac{(T_{10} - T_{17})}{\Delta y} + k_A \Delta x \dfrac{(T_{24} - T_{17})}{\Delta y} + (k_A + k_B)\dfrac{\Delta y}{2}\dfrac{(T_{16} - T_{17})}{\Delta x} + (k_A + k_B)\dfrac{\Delta y}{2}\dfrac{(T_{18} - T_{17})}{\Delta x} = 0$

Node 18: $(k_A + k_B)\dfrac{\Delta y}{2}\dfrac{(T_{17} - T_{18})}{\Delta x} + (k_A + k_C)\dfrac{\Delta y}{2}\dfrac{(T_{19} - T_{18})}{\Delta x} + (k_B + k_C)\dfrac{\Delta x}{2}\dfrac{(T_{11} - T_{18})}{\Delta y} + k_A \Delta x \dfrac{(T_{25} - T_{18})}{\Delta y} = 0$

Node 19: $k_C \Delta x \dfrac{(T_{12} - T_{19})}{\Delta y} + k_A \Delta x \dfrac{(T_{26} - T_{19})}{\Delta y} + (k_A + k_C)\dfrac{\Delta y}{2}\dfrac{(T_{18} - T_{19})}{\Delta x} + (k_A + k_C)\dfrac{\Delta y}{2}\dfrac{(T_{20} - T_{19})}{\Delta x} = 0$

Node 20: $k_A \Delta y \dfrac{(T_{21} - T_{20})}{\Delta x} + k_A \Delta x \dfrac{(T_{27} - T_{20})}{\Delta y} + (k_A + k_C)\dfrac{\Delta x}{2}\dfrac{(T_{13} - T_{20})}{\Delta y} + (k_A + k_C)\dfrac{\Delta y}{2}\dfrac{(T_{19} - T_{20})}{\Delta x} = 0$

Node 21: $T_{14} + T_{28} + 2T_{20} - 4T_{21} = 0$

Solving these 21 equations simultaneously for 21 unknown nodal temperatures using EES or any other software we get,

T_1 = **605.4ºC**, T_2 = **641.6ºC**, T_3 = **652.9ºC**, T_4 = **664.7ºC**, T_5 = **665.7ºC**, T_6 = **663.5ºC**, T_7 = **620ºC**,
T_8 = **642.4ºC**, T_9 = **659.3ºC**, T_{10} = **665.9ºC**, T_{11} = **671ºC**, T_{12} = **670.3ºC**, T_{13} = **668.8ºC**, T_{14} = **651.7ºC**,
T_{15} = **645.8ºC**, T_{16} = **670.5ºC**, T_{17} = **680.2ºC**, T_{18} = **680.5ºC**, T_{19} = **675.7ºC**, T_{20} = **672.4ºC**, T_{21} = **649.1ºC**.

5-91E The top and bottom surfaces of a V-grooved long solid bar are maintained at specified temperatures while the left and right surfaces are insulated. The temperature at the middle of the insulated surface is to be determined.

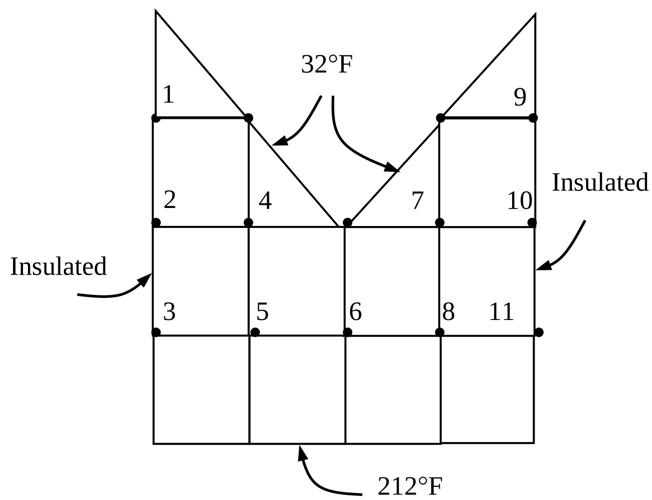
Assumptions 1 Heat transfer through the bar is given to be steady and two-dimensional. **2** There is no heat generation within the bar. **3** Thermal conductivity is constant.

Analysis The nodal spacing is given to be $\Delta x=\Delta y=l=1$ ft, and the general finite difference form of an interior node for steady two-dimensional heat conduction with no heat generation is expressed as

$$T_{\text{left}}+T_{\text{top}}+T_{\text{right}}+T_{\text{bottom}}-4T_{\text{node}}+\frac{\dot{e}_{\text{node}}l^2}{k}=0 \;\rightarrow\; T_{\text{left}}+T_{\text{top}}+T_{\text{right}}+T_{\text{bottom}}-4T_{\text{node}}=0$$

There is symmetry about the vertical plane passing through the center. Therefore, $T_1 = T_9$, $T_2 = T_{10}$, $T_3 = T_{11}$, $T_4 = T_7$, and $T_5 = T_8$. Therefore, there are only 6 unknown nodal temperatures, and thus we need only 6 equations to determine them uniquely. Also, we can replace the symmetry lines by insulation and utilize the mirror-image concept when writing the finite difference equations for the interior nodes.

The finite difference equations for boundary nodes are obtained by applying an energy balance on the volume elements and taking the direction of all heat transfers to be towards the node under consideration:



Node 1: $k\frac{l}{2}\frac{32-T_1}{l}+kl\frac{32-T_1}{l}+k\frac{l}{2}\frac{T_2-T_1}{l}=0$

(Note that k and l cancel out)

Node 2: $T_1+2T_4+T_3-4T_2=0$

Node 3: $T_2+212+2T_5-4T_3=0$

Node 4: $2\times 32+T_2+T_5-4T_4=0$

Node 5: $T_3+212+T_4+T_6-4T_5=0$

Node 6: $32+212+2T_5-4T_6=0$

The 6 nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver to be

T_1 = 44.7°F, T_2 =82.8°F, T_3 =143.4°F, T_4 = 71.6°F, T_5 =139.4°F, T_6 =130.7°F

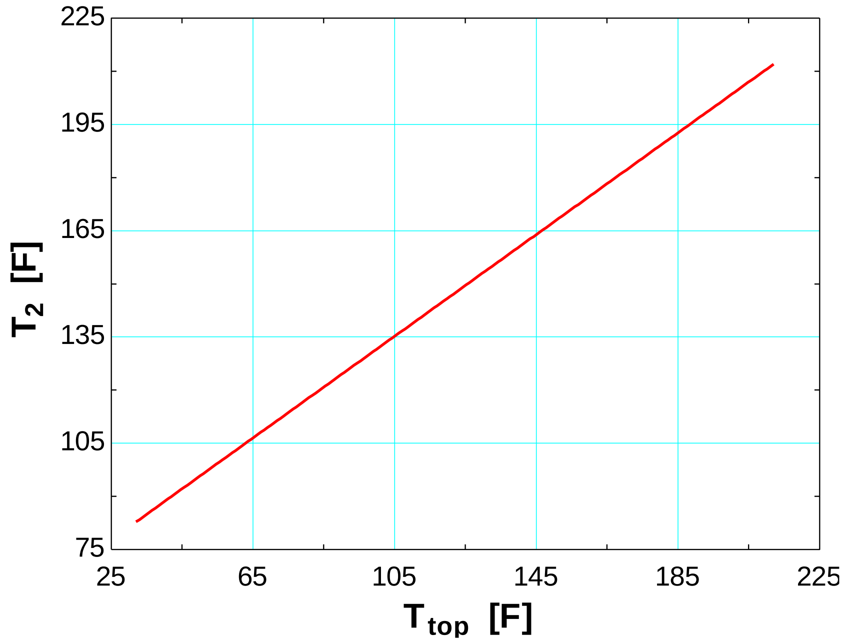
Therefore, the temperature at the middle of the insulated surface will be T_2 = **82.8°F.**

5-92E Prob. 5-91E is reconsidered. The effects of the temperatures at the top and bottom surfaces on the temperature in the middle of the insulated surface are to be investigated.

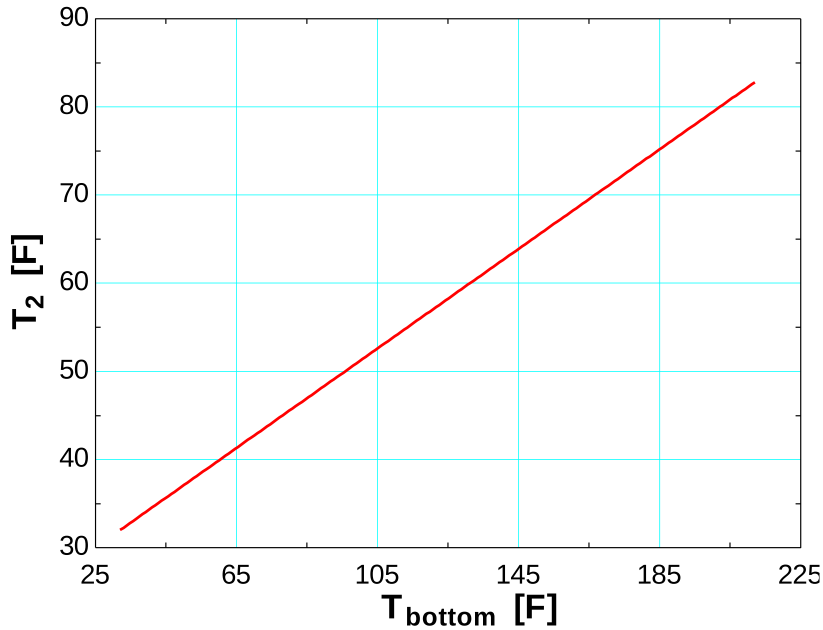
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_top=32 [F]
T_bottom=212 [F]
DELTAx=1 [ft]
DELTAy=1 [ft]

"ANALYSIS"
l=DELTAx
T_1=T_9 "due to symmetry"
T_2=T_10 "due to symmetry"
T_3=T_11 "due to symmetry"
T_4=T_7 "due to symmetry"
T_5=T_8 "due to symmetry"
"Using the finite difference method, the six equations for the six unknown temperatures are determined to be"
"k*l/2*(T_top-T_1)/l+k*l*(T_top-T_1)/l+k*l/2*(T_2-T_1)/l=0 simplifies to for Node 1"
1/2*(T_top-T_1)+(T_top-T_1)+1/2*(T_2-T_1)=0 "Node 1"
T_1+2*T_4+T_3-4*T_2=0 "Node 2"
T_2+T_bottom+2*T_5-4*T_3=0 "Node 3"
2*T_top+T_2+T_5-4*T_4=0 "Node 4"
T_3+T_bottom+T_4+T_6-4*T_5=0 "Node 5"
T_top+T_bottom+2*T_5-4*T_6=0 "Node 6"
```

T_{top} [F]	T_2 [F]
32	82.81
41.47	89.61
50.95	96.41
60.42	103.2
69.89	110
79.37	116.8
88.84	123.6
98.32	130.4
107.8	137.2
117.3	144
126.7	150.8
136.2	157.6
145.7	164.4
155.2	171.2
164.6	178
174.1	184.8
183.6	191.6
193.1	198.4
202.5	205.2
212	212



T_{bottom} [F]	T_2 [F]
32	32
41.47	34.67
50.95	37.35
60.42	40.02
69.89	42.7
79.37	45.37
88.84	48.04
98.32	50.72
107.8	53.39
117.3	56.07
126.7	58.74
136.2	61.41
145.7	64.09
155.2	66.76
164.6	69.44
174.1	72.11
183.6	74.78
193.1	77.46
202.5	80.13
212	82.81



Transient Heat Conduction

5-93C The formulation of a transient heat conduction problem differs from that of a steady heat conduction problem in that the transient problem involves an *additional term* that represents the *change in the energy content* of the medium with time. This additional term $\rho A\Delta x c_p (T_m^{i+1} - T_m^i)/\Delta t$ represent the change in the internal energy content during Δt in the transient finite difference formulation.

5-94C The two basic methods of solution of transient problems based on finite differencing are the *explicit* and the *implicit methods*. The heat transfer terms are expressed at time step *i* in the explicit method, and at the future time step *i* + 1 in the implicit method as

Explicit method: $$\sum_{\text{All sides}} \dot{Q}^i + \dot{E}^{\text{i}}_{\text{gen, element}} = \rho V_{\text{element}} c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Implicit method: $$\sum_{\text{All sides}} \dot{Q}^{i+1} + \dot{E}^{\text{i+1}}_{\text{gen, element}} = \rho V_{\text{element}} c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

5-95C The explicit finite difference formulation of a general interior node for transient heat conduction in a plane wall is given by $T_{m-1}^i - 2T_m^i + T_{m+1}^i + \frac{\dot{e}_m^i \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau}$. The finite difference formulation for the steady case is obtained by simply setting $T_m^{i+1} = T_m^i$ and disregarding the time index *i*. It yields

$$T_{m-1} - 2T_m + T_{m+1} + \frac{\dot{e}_m \Delta x^2}{k} = 0$$

5-96C For transient one-dimensional heat conduction in a plane wall with both sides of the wall at specified temperatures, the stability criteria for the explicit method can be expressed in its simplest form as

$$\tau = \frac{\alpha \Delta t}{(\Delta x)^2} \le \frac{1}{2}$$

5-97C For transient one-dimensional heat conduction in a plane wall with specified heat flux on both sides, the stability criteria for the explicit method can be expressed in its simplest form as

$$\tau = \frac{\alpha \Delta t}{(\Delta x)^2} \le \frac{1}{2}$$

which is identical to the one for the interior nodes. This is because the heat flux boundary conditions have no effect on the stability criteria.

5-98C The explicit finite difference formulation of a general interior node for transient two-dimensional heat conduction is given by $T_{\text{node}}^{i+1} = \tau(T_{\text{left}}^{\text{i}} + T_{\text{top}}^{\text{i}} + T_{\text{right}}^{\text{i}} + T_{\text{bottom}}^{\text{i}}) + (1-4\tau)T_{\text{node}}^{\text{i}} + \tau \frac{\dot{e}_{\text{node}}^{\text{i}} l^2}{k}$. The finite difference formulation for the steady case is obtained by simply setting $T_m^{i+1} = T_m^i$ and disregarding the time index *i*. It yields

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{e}_{\text{node}} l^2}{k} = 0$$

5-99C There is a limitation on the size of the time step Δt in the solution of transient heat conduction problems using the explicit method, but there is no such limitation in the implicit method.

5-100C The general stability criteria for the explicit method of solution of transient heat conduction problems is expressed as follows: *The coefficients of all* T_m^i *in the* T_m^{i+1} *expressions* (*called the primary coefficient*) *in the simplified expressions must be greater than or equal to zero for all nodes m.*

5-101C For transient two-dimensional heat conduction in a rectangular region with insulation or specified temperature boundary conditions, the stability criteria for the explicit method can be expressed in its simplest form as

$$\tau = \frac{\alpha \Delta t}{(\Delta x)^2} \le \frac{1}{4}$$

which is identical to the one for the interior nodes. This is because the insulation or specified temperature boundary conditions have no effect on the stability criteria.

5-102C The implicit method is unconditionally stable and thus any value of time step Δt can be used in the solution of transient heat conduction problems since there is no danger of unstability. However, using a very large value of Δt is equivalent to replacing the time derivative by a very large difference, and thus the solution will not be accurate. Therefore, we should still use the smallest time step practical to minimize the numerical error.

5-103 Starting with an energy balance on a volume element, the two-dimensional transient *explicit* finite difference equation for a general interior node in rectangular coordinates for $T(x,y,t)$ for the case of constant thermal conductivity and no heat generation is to be obtained.

Analysis (See Figure 5-49 in the text). We consider a rectangular region in which heat conduction is significant in the x and y directions, and consider a unit depth of $\Delta z = 1$ in the *z* direction. There is no heat generation in the medium, and the thermal conductivity *k* of the medium is constant. Now we divide the *x*-*y* plane of the region into a *rectangular mesh* of nodal points which are spaced Δx and Δy apart in the *x* and *y* directions, respectively, and consider a general interior node (*m*, *n*) whose coordinates are $x = m\Delta x$ and $y = n\Delta y$. Noting that the volume element centered about the general interior node (*m*, *n*) involves heat conduction from four sides (right, left, top, and bottom) and expressing them at previous time step *i*, the transient explicit finite difference formulation for a general interior node can be expressed as

$$k(\Delta y \times 1)\frac{T_{m-1,n}^i - T_{m,n}^i}{\Delta x} + k(\Delta x \times 1)\frac{T_{m,n+1}^i - T_{m,n}^i}{\Delta y} + k(\Delta y \times 1)\frac{T_{m+1,n}^i - T_{m,n}^i}{\Delta x} + k(\Delta x \times 1)\frac{T_{m,n-1}^i - T_{m,n}^i}{\Delta y}$$
$$= \rho(\Delta x \times \Delta y \times 1)c_p \frac{T_{m,n}^{i+1} - T_{m,n}^i}{\Delta t}$$

Taking a square mesh ($\Delta x = \Delta y = l$) and dividing each term by *k* gives, after simplifying,

$$T_{m-1,n}^i + T_{m+1,n}^i + T_{m,n+1}^i + T_{m,n-1}^i - 4T_{m,n}^i = \frac{T_{m,n}^{i+1} - T_{m,n}^i}{\tau}$$

where $\alpha = k/\rho c_p$ is the thermal diffusivity of the material and $\tau = \alpha \Delta t / l^2$ is the dimensionless mesh Fourier number. It can also be expressed in terms of the temperatures at the neighboring nodes in the following easy-to-remember form:

$$T_{\text{left}}^i + T_{\text{top}}^i + T_{\text{right}}^i + T_{\text{bottom}}^i - 4T_{\text{node}}^i = \frac{T_{\text{node}}^{i+1} - T_{\text{node}}^i}{\tau}$$

Discussion We note that setting $T_{\text{node}}^{i+1} = T_{\text{node}}^i$ gives the steady finite difference formulation.

5-104 Starting with an energy balance on a volume element, the two-dimensional transient *implicit* finite difference equation for a general interior node in rectangular coordinates for $T(x, y, t)$ for the case of constant thermal conductivity and no heat generation is to be obtained.

Analysis (See Figure 5-49 in the text). We consider a rectangular region in which heat conduction is significant in the x and y directions, and consider a unit depth of $\Delta z = 1$ in the z direction. There is no heat generation in the medium, and the thermal conductivity k of the medium is constant. Now we divide the x-y plane of the region into a *rectangular mesh* of nodal points which are spaced Δx and Δy apart in the x and y directions, respectively, and consider a general interior node (m, n) whose coordinates are $x = m\Delta x$ and $y = n\Delta y$. Noting that the volume element centered about the general interior node (m, n) involves heat conduction from four sides (right, left, top, and bottom) and expressing them at previous time step i, the transient *implicit* finite difference formulation for a general interior node can be expressed as

$$k(\Delta y \times 1)\frac{T_{m-1,n}^{i+1} - T_{m,n}^{i+1}}{\Delta x} + k(\Delta x \times 1)\frac{T_{m,n+1}^{i+1} - T_{m,n}^{i+1}}{\Delta y} + k(\Delta y \times 1)\frac{T_{m+1,n}^{i+1} - T_{m,n}^{i}}{\Delta x} + k(\Delta x \times 1)\frac{T_{m,n-1}^{i+1} - T_{m,n}^{i+1}}{\Delta y}$$

$$= \rho(\Delta x \times \Delta y \times 1)c_p \frac{T_{m,n}^{i+1} - T_{m,n}^{i}}{\Delta t}$$

Taking a square mesh ($\Delta x = \Delta y = l$) and dividing each term by k gives, after simplifying,

$$T_{m-1,n}^{i+1} + T_{m+1,n}^{i+1} + T_{m,n+1}^{i+1} + T_{m,n-1}^{i+1} - 4T_{m,n}^{i+1} = \frac{T_m^{i+1} - T_m^{i}}{\tau}$$

where $\alpha = k/\rho c_p$ is the thermal diffusivity of the material and $\tau = \alpha \Delta t / l^2$ is the dimensionless mesh Fourier number. It can also be expressed in terms of the temperatures at the neighboring nodes in the following easy-to-remember form:

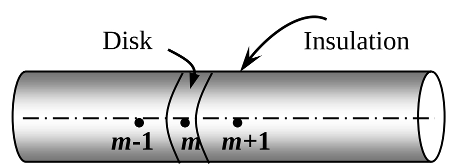
$$T_{\text{left}}^{i+1} + T_{\text{top}}^{i+1} + T_{\text{right}}^{i+1} + T_{\text{bottom}}^{i+1} - 4T_{\text{node}}^{i+1} = \frac{T_{\text{node}}^{i+1} - T_{\text{node}}^{i}}{\tau}$$

Discussion We note that setting $T_{\text{node}}^{i+1} = T_{\text{node}}^{i}$ gives the steady finite difference formulation.

5-105 Starting with an energy balance on a disk volume element, the one-dimensional transient explicit finite difference equation for a general interior node for $T(z, t)$ in a cylinder whose side surface is insulated for the case of constant thermal conductivity with uniform heat generation is to be obtained.

Analysis We consider transient one-dimensional heat conduction in the axial z direction in an insulated cylindrical rod of constant cross-sectional area A with constant heat generation $\dot{g}_0$ and constant conductivity k with a mesh size of Δz in the z direction. Noting that the volume element of a general interior node m involves heat conduction from two sides and the volume of the element is $\boldsymbol{V}_{\text{element}} = A\Delta z$, the transient explicit finite difference formulation for an interior node can be expressed as

$$kA\frac{T_{m-1}^{i} - T_m^{i}}{\Delta x} + kA\frac{T_{m+1}^{i} - T_m^{i}}{\Delta x} + \dot{e}_0 A\Delta x = \rho A \Delta x c_p \frac{T_m^{i+1} - T_m^{i}}{\Delta t}$$



Canceling the surface area A and multiplying by $\Delta x/k$, it simplifies to

$$T_{m-1}^{i} - 2T_m^{i} + T_{m+1}^{i} + \frac{\dot{e}_0 \Delta x^2}{k} = \frac{(\Delta x)^2}{\alpha \Delta t}(T_m^{i+1} - T_m^{i})$$

where $\alpha = k/\rho c_p$ is the *thermal diffusivity* of the wall material.

Using the definition of the dimensionless *mesh Fourier number* $\tau = \dfrac{\alpha \Delta t}{\Delta x^2}$, the last equation reduces to

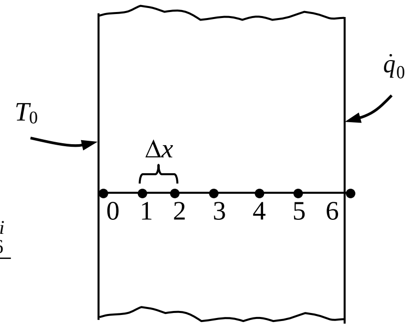
$$T_{m-1}^{i} - 2T_m^{i} + T_{m+1}^{i} + \frac{\dot{e}_0 \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^{i}}{\tau}$$

Discussion We note that setting $T_{\text{m}}^{i+1} = T_{\text{m}}^{i}$ gives the steady finite difference formulation.

5-106 A plane wall with no heat generation is subjected to specified temperature at the left (node 0) and heat flux at the right boundary (node 6). The explicit transient finite difference formulation of the boundary nodes and the finite difference formulation for the total amount of heat transfer at the left boundary during the first 3 time steps are to be determined.

Assumptions **1** Heat transfer through the wall is given to be transient, and the thermal conductivity to be constant. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness. **3** There is no heat generation in the medium.

Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the *explicit* finite difference formulations become



Left boundary node: $T_0^i = T_0 = 80°\text{C}$

Right boundary node: $k\dfrac{T_5^i - T_6^i}{\Delta x} + \dot{q}_0 = \rho\dfrac{\Delta x}{2}c_p\dfrac{T_6^{i+1} - T_6^i}{\Delta t}$

Heat transfer at left surface: $\dot{Q}^i_{\text{left surface}} + kA\dfrac{T_1^i - T_0}{\Delta x} = \rho A\dfrac{\Delta x}{2}c_p\dfrac{T_6^{i+1} - T_6^i}{\Delta t}$

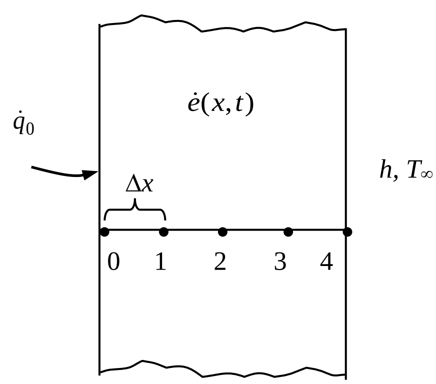
Noting that $Q = \dot{Q}\Delta t = \sum_i \dot{Q}^i\Delta t$, the total amount of heat transfer becomes

$$Q_{\text{left surface}} = \sum_{i=1}^{3}\dot{Q}^i_{\text{left surface}}\Delta t = \sum_{i=1}^{3}\left(kA\frac{T_0 - T_1^i}{\Delta x} + \rho A\frac{\Delta x}{2}c_p\frac{T_0^{i+1} - T_0^i}{\Delta t}\right)\Delta t$$

5-107 A plane wall with variable heat generation and constant thermal conductivity is subjected to uniform heat flux $\dot{q}_0$ at the left (node 0) and convection at the right boundary (node 4). The explicit transient finite difference formulation of the boundary nodes is to be determined.

Assumptions **1** Heat transfer through the wall is given to be transient, and the thermal conductivity to be constant. **2** Heat transfer is one-dimensional since the wall is large relative to its thickness. **3** Radiation heat transfer is negligible.

Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the *explicit* finite difference formulations become



Left boundary node:

$$kA\frac{T_1^i - T_0^i}{\Delta x} + \dot{q}_0 A + \dot{e}_0^i(A\Delta x/2) = \rho A\frac{\Delta x}{2}c_p\frac{T_0^{i+1} - T_0^i}{\Delta t}$$

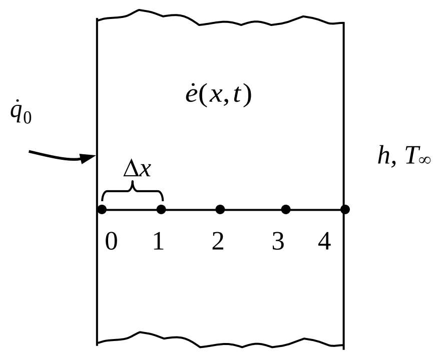
Right boundary node:

$$kA\frac{T_3^i - T_4^i}{\Delta x} + hA(T_\infty^i - T_4^i) + \dot{e}_4^i(A\Delta x/2) = \rho A\frac{\Delta x}{2}c_p\frac{T_4^{i+1} - T_4^i}{\Delta t}$$

5-108 A plane wall with variable heat generation and constant thermal conductivity is subjected to uniform heat flux $\dot{q}_0$ at the left (node 0) and convection at the right boundary (node 4). The implicit transient finite difference formulation of the boundary nodes is to be determined.

Assumptions **1** Heat transfer through the wall is given to be transient, and the thermal conductivity to be constant. **2** Heat transfer is one-dimensional since the wall is large relative to its thickness. **3** Radiation heat transfer is negligible.

Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the *implicit* finite difference formulations become



Left boundary node:

$$kA\frac{T_1^{i+1}-T_0^{i+1}}{\Delta x}+\dot{q}_0 A+\dot{e}_0^{i+1}(A\Delta x/2)=\rho A\frac{\Delta x}{2}c_p\,\frac{T_0^{i+1}-T_0^i}{\Delta t}$$

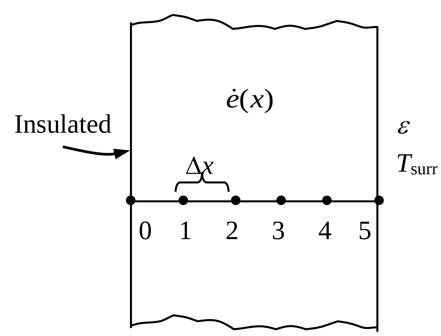
Right boundary node:

$$kA\frac{T_3^{i+1}-T_4^{i+1}}{\Delta x}+hA(T_\infty^{i+1}-T_4^{i+1})+\dot{e}_4^{i+1}(A\Delta x/2)=\rho A\frac{\Delta x}{2}c_p\,\frac{T_4^{i+1}-T_4^i}{\Delta t}$$

5-109 A plane wall with variable heat generation and constant thermal conductivity is subjected to insulation at the left (node 0) and radiation at the right boundary (node 5). The explicit transient finite difference formulation of the boundary nodes is to be determined.

Assumptions **1** Heat transfer through the wall is given to be transient and one-dimensional, and the thermal conductivity to be constant. **2** Convection heat transfer is negligible.

Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the *explicit* transient finite difference formulations become



Left boundary node:

$$kA\frac{T_1^i-T_0^i}{\Delta x}+\dot{e}_0^i A\frac{\Delta x}{2}=\rho A\frac{\Delta x}{2}c_p\,\frac{T_0^{i+1}-T_0^i}{\Delta t}$$

Right boundary node:

$$\varepsilon\sigma A[(T_{surr}^i)^4-(T_5^i)^4]+kA\frac{T_4^i-T_5^i}{\Delta x}+\dot{e}_5^i A\frac{\Delta x}{2}=\rho A\frac{\Delta x}{2}c_p\,\frac{T_5^{i+1}-T_5^i}{\Delta t}$$

5-110 A plane wall with variable heat generation and constant thermal conductivity is subjected to combined convection, radiation, and heat flux at the left (node 0) and specified temperature at the right boundary (node 4). The explicit finite difference formulation of the left boundary and the finite difference formulation for the total amount of heat transfer at the right boundary are to be determined.

Assumptions **1** Heat transfer through the wall is given to be transient and one-dimensional, and the thermal conductivity to be constant.

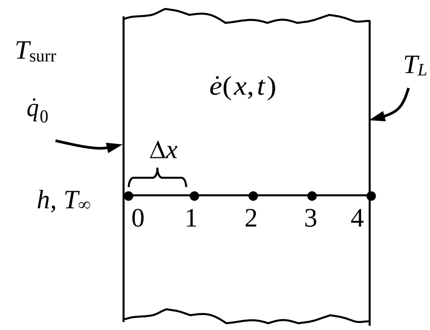
Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the *explicit* transient finite difference formulations become

Left boundary node:

$$\dot{q}_0 A + \varepsilon\sigma A[(T_{\text{surr}}^i)^4 - (T_0^i)^4] + hA(T_\infty^i - T_0^i) + kA\frac{T_1^i - T_0^i}{\Delta x} + \dot{e}_0^i A\frac{\Delta x}{2} = \rho A\frac{\Delta x}{2}c_p\frac{T_0^{i+1} - T_0^i}{\Delta t}$$

Heat transfer at right surface:

$$\dot{Q}_{\text{right surface}}^i + kA\frac{T_3^i - T_4^i}{\Delta x} + \dot{e}_4^i A\frac{\Delta x}{2} = \rho A\frac{\Delta x}{2}c_p\frac{T_4^{i+1} - T_4^i}{\Delta t}$$



Noting that

$$Q = \dot{Q}\Delta t = \sum_i \dot{Q}^i \Delta t$$

the total amount of heat transfer becomes

$$Q_{\text{right surface}} = \sum_{i=1}^{20} \dot{Q}_{\text{right surface}}^i \Delta t$$

$$= \sum_{i=1}^{20}\left(kA\frac{T_4^i - T_3^i}{\Delta x} - \dot{e}_4^i A\frac{\Delta x}{2} + \rho A\frac{\Delta x}{2}c_p\frac{T_4^{i+1} - T_4^i}{\Delta t}\right)\Delta t$$

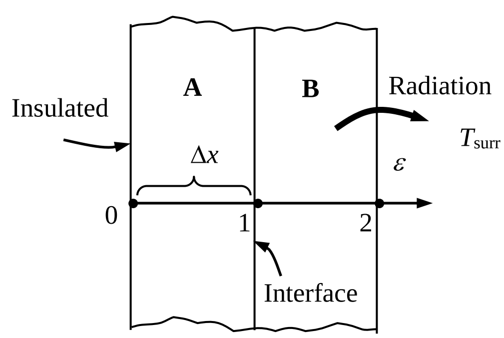
5-111 A composite plane wall consists of two layers A and B in perfect contact at the interface where node 1 is at the interface. The wall is insulated at the left (node 0) and subjected to radiation at the right boundary (node 2). The complete transient explicit finite difference formulation of this problem is to be obtained.

Assumptions **1** Heat transfer through the wall is given to be transient and one-dimensional, and the thermal conductivity to be constant. **2** Convection heat transfer is negligible. **3** There is no heat generation.

Analysis Using the energy balance approach with a unit area A = 1 and taking the direction of all heat transfers to be towards the node under consideration, the finite difference formulations become

Node 0 (at left boundary):

$$k_A\frac{T_1^i - T_0^i}{\Delta x} = \rho_A\frac{\Delta x}{2}c_{p,A}\frac{T_0^{i+1} - T_0^i}{\Delta t}$$



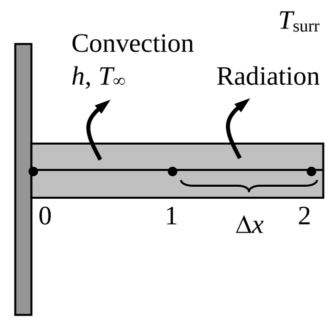
Node 1 (at interface):

$$k_A\frac{T_0^i - T_1^i}{\Delta x} + k_B\frac{T_2^i - T_1^i}{\Delta x} = \left(\rho_A\frac{\Delta x}{2}c_{p,A} + \rho_B\frac{\Delta x}{2}c_B\right)\frac{T_1^{i+1} - T_1^i}{\Delta t}$$

Node 2 (at right boundary):

$$\varepsilon\sigma[T_{\text{surr}}^4 - (T_2^i)^4] + k_B\frac{T_1^i - T_2^i}{\Delta x} = \rho_B\frac{\Delta x}{2}c_{p,B}\frac{T_2^{i+1} - T_2^i}{\Delta t}$$

5-112 A pin fin with negligible heat transfer from its tip is considered. The complete explicit finite difference formulation for the determination of nodal temperatures is to be obtained.



Assumptions **1** Heat transfer through the pin fin is given to be steady and one-dimensional, and the thermal conductivity to be constant. **2** Convection heat transfer coefficient is constant and uniform. **3** Heat loss from the fin tip is given to be negligible.

Analysis The nodal network consists of 3 nodes, and the base temperature T_0 at node 0 is specified. Therefore, there are two unknowns T_1 and T_2, and we need two equations to determine them. Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the explicit transient finite difference formulations become

Node 1 (at midpoint):

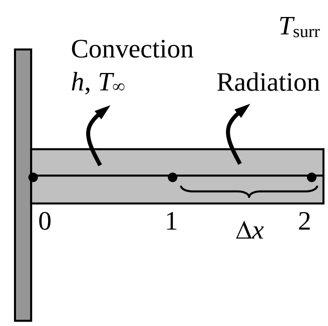
$$\varepsilon\sigma p\Delta x[T_{surr}^4-(T_1^i)^4]+hp\Delta x(T_\infty-T_1^i)+kA\frac{T_2^i-T_1^i}{\Delta x}+kA\frac{T_0^i-T_1^i}{\Delta x}=\rho A\Delta xc_p\,\frac{T_1^{i+1}-T_1^i}{\Delta t}$$

Node 2 (at fin tip):

$$\varepsilon\sigma\left(p\frac{\Delta x}{2}\right)[T_{surr}^4-(T_2^i)^4]+h\left(p\frac{\Delta x}{2}\right)(T_\infty-T_2^i)+kA\frac{T_1^i-T_2^i}{\Delta x}=\rho A\frac{\Delta x}{2}c_p\,\frac{T_2^{i+1}-T_2^i}{\Delta t}$$

where $A=\pi D^2/4$ is the cross-sectional area and $p=\pi D$ is the perimeter of the fin.

5-113 A pin fin with negligible heat transfer from its tip is considered. The complete implicit finite difference formulation for the determination of nodal temperatures is to be obtained.



Assumptions **1** Heat transfer through the pin fin is given to be steady and one-dimensional, and the thermal conductivity to be constant. **2** Convection heat transfer coefficient is constant and uniform. **3** Heat loss from the fin tip is given to be negligible.

Analysis The nodal network consists of 3 nodes, and the base temperature T_0 at node 0 is specified. Therefore, there are two unknowns T_1 and T_2, and we need two equations to determine them. Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the implicit transient finite difference formulations become

Node 1: $$\varepsilon\sigma p\Delta x[T_{surr}^4-(T_1^{i+1})^4]+hp\Delta x(T_\infty-T_1^{i+1})+kA\frac{T_2^{i+1}-T_1^{i+1}}{\Delta x}+kA\frac{T_0^{i+1}-T_1^{i+1}}{\Delta x}=\rho A\Delta xc_p\,\frac{T_1^{i+1}-T_1^i}{\Delta t}$$

Node 2: $$\varepsilon\sigma\left(p\frac{\Delta x}{2}\right)[T_{surr}^4-(T_2^{i+1})^4]+h\left(p\frac{\Delta x}{2}\right)(T_\infty-T_2^{i+1})+kA\frac{T_1^{i+1}-T_2^{i+1}}{\Delta x}=\rho A\frac{\Delta x}{2}c_p\,\frac{T_2^{i+1}-T_2^i}{\Delta t}$$

where $A=\pi D^2/4$ is the cross-sectional area and $p=\pi D$ is the perimeter of the fin.

5-114 A hot brass plate is having its upper surface cooled by impinging jet while its lower surface is insulated. The implicit finite difference equations and the nodal temperatures of the brass plate after 10 seconds of cooling are to be determined.

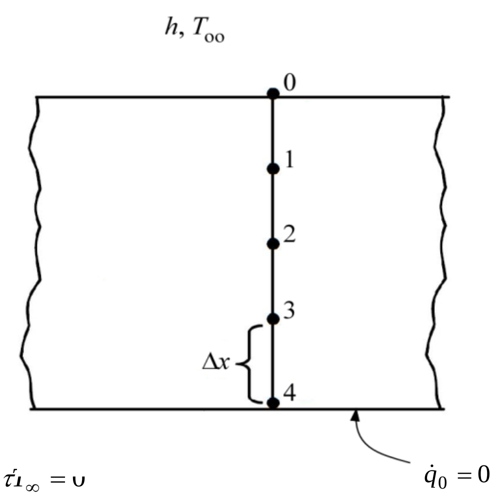
Assumptions **1** Transient heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible. **5** There is no heat generation.

Properties The properties of the brass plate are given as ρ = 8530 kg/m^3, c_p = 380 J/kg·K, k = 110 W/m·K, and α = 33.9 × 10^{-6} m^2/s.

Analysis The nodal spacing is given to be Δx = 2.5 cm. Then the number of nodes becomes $M = L/\Delta x + 1 = 10/2.5 + 1 = 5$. This problem involves 5 unknown nodal temperatures, and thus we need to have 5 equations. The finite difference equation for node 0 on the top surface subjected to convection is obtained by applying an energy balance on the half volume element about node 0 and taking the direction of all heat transfers to be towards the node under consideration:

$$h(T_\infty - T_0^{i+1}) + k\frac{T_1^{i+1} - T_0^{i+1}}{\Delta x} = \rho\frac{\Delta x}{2}c_p\frac{T_0^{i+1} - T_0^i}{\Delta t}$$

or
$$-\left(1 + 2\tau + 2\frac{h\Delta x}{k}\tau\right)T_0^{i+1} + 2\tau T_1^{i+1} + T_0^i + 2\frac{h\Delta x}{k}\tau T_\infty = 0$$



Node 4 is on insulated boundary, and thus we can treat it as an interior node by using the mirror image concept. Nodes 1, 2, and 3 are interior nodes, and thus for them we can use the general implicit finite difference relation expressed as

$$T_{m-1}^{i+1} - 2T_m^{i+1} + T_{m+1}^{i+1} = \frac{T_m^{i+1} - T_m^i}{\tau}$$

or
$$\tau T_{m-1}^{i+1} - (1 + 2\tau)T_m^{i+1} + \tau T_{m+1}^{i+1} + T_m^i = 0$$

Thus, the implicit finite difference equations are

Node 0: $$-\left(1 + 2\tau + 2\frac{h\Delta x}{k}\tau\right)T_0^{i+1} + 2\tau T_1^{i+1} + T_0^i + 2\frac{h\Delta x}{k}\tau T_\infty = 0$$

Node 1: $$\tau T_0^{i+1} - (1 + 2\tau)T_1^{i+1} + \tau T_2^{i+1} + T_1^i = 0$$

Node 2: $$\tau T_1^{i+1} - (1 + 2\tau)T_2^{i+1} + \tau T_3^{i+1} + T_2^i = 0$$

Node 3: $$\tau T_2^{i+1} - (1 + 2\tau)T_3^{i+1} + \tau T_4^{i+1} + T_3^i = 0$$

Node 4: $$\tau T_3^{i+1} - (1 + 2\tau)T_4^{i+1} + \tau T_3^{i+1} + T_4^i = 0$$

where Δx = 2.5 cm, k = 110 W/m·K, h = 220 W/m^2·K, T_∞ = 15°C, α = 33.9 × 10^{-6} m^2/s, and $h\Delta x/k$ = 0.05. For time step of Δt = 10 s. Then the mesh Fourier number becomes

$$\tau = \frac{\alpha\Delta t}{\Delta x^2} = \frac{(33.9\times10^{-6}\text{ m}^2/\text{s})(10\text{ s})}{(0.025\text{ m})^2} = 0.5424 \qquad \text{(for } \Delta t = 10\text{ s)}$$

(*b*) The nodal temperatures of the brass plate after 10 seconds of cooling can be determined by solving the 5 equations above simultaneously with an equation solver. Copy the following lines and paste on a blank EES screen to solve the above equations:

```
-(1+2*0.5424+2*0.05*0.5424)*T_0+2*0.5424*T_1+650+2*0.05*0.5424*15=0
0.5424*T_0-(1+2*0.5424)*T_1+0.5424*T_2+650=0
0.5424*T_1-(1+2*0.5424)*T_2+0.5424*T_3+650=0
0.5424*T_2-(1+2*0.5424)*T_3+0.5424*T_4+650=0
0.5424*T_3-(1+2*0.5424)*T_4+0.5424*T_3+650=0
```

Solving by EES software, we get the same results:

T_0 = **631.2°C**, T_1 = **644.7°C**, T_2 = **648.5°C**, T_3 = **649.6°C**, T_4 = **649.8°C**

Discussion Unlike the explicit method, the implicit method does not require any stability criterion, and the solution will converge with large values of time step. However, the large time step tends to give less accurate the results.

5-115 A uranium plate initially at a uniform temperature is subjected to insulation on one side and convection on the other. The transient finite difference formulation of this problem is to be obtained, and the nodal temperatures after 5 min and under steady conditions are to be determined.

Assumptions **1** Heat transfer is one-dimensional since the plate is large relative to its thickness. **2** Thermal conductivity is constant. **3** Radiation heat transfer is negligible.

Properties The conductivity and diffusivity are given to be k = 28 W/m·°C and $\alpha = 12.5\times10^{-6}$ m^2/s .

Analysis The nodal spacing is given to be Δx = 0.015 m. Then the number of nodes becomes $M = L/\Delta x + 1$ = 0.09/0.015+1 = 7. This problem involves 7 unknown nodal temperatures, and thus we need to have 7 equations. Node 0 is on insulated boundary, and thus we can treat it as an interior note by using the mirror image concept. Nodes 1, 2, and 3 are interior nodes, and thus for them we can use the general explicit finite difference relation expressed as

$$T_{m-1}^i - 2T_m^i + T_{m+1}^i + \frac{\dot{e}_m^i \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau} \quad \rightarrow \quad T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1-2\tau)T_m^i + \tau\frac{\dot{e}_m^i \Delta x^2}{k}$$

The finite difference equation for node 4 on the right surface subjected to convection is obtained by applying an energy balance on the half volume element about node 4 and taking the direction of all heat transfers to be towards the node under consideration:

Node 0 (insulated) : $T_0^{i+1} = \tau(T_1^i + T_1^i) + (1-2\tau)T_0^i + \tau\frac{\dot{e}_0 \Delta x^2}{k}$

Node 1 (interior) : $T_1^{i+1} = \tau(T_0^i + T_2^i) + (1-2\tau)T_1^i + \tau\frac{\dot{e}_0 \Delta x^2}{k}$

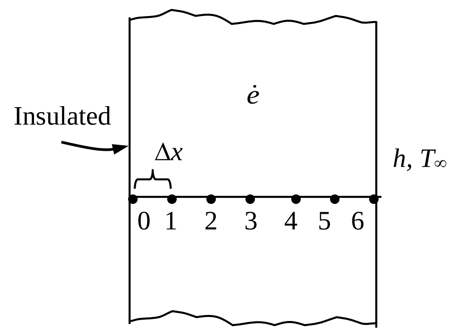
Node 2 (interior) : $T_2^{i+1} = \tau(T_1^i + T_3^i) + (1-2\tau)T_2^i + \tau\frac{\dot{e}_0 \Delta x^2}{k}$

Node 3 (interior) : $T_3^{i+1} = \tau(T_2^i + T_4^i) + (1-2\tau)T_3^i + \tau\frac{\dot{e}_0 \Delta x^2}{k}$

Node 4 (interior) : $T_4^{i+1} = \tau(T_3^i + T_5^i) + (1-2\tau)T_4^i + \tau\frac{\dot{e}_0 \Delta x^2}{k}$

Node 5 (interior) : $T_5^{i+1} = \tau(T_4^i + T_6^i) + (1-2\tau)T_5^i + \tau\frac{\dot{e}_0 \Delta x^2}{k}$

Node 6 (convection) : $h(T_\infty - T_6^i) + k\frac{T_5^i - T_6^i}{\Delta x} + \dot{e}_0\frac{\Delta x}{2} = \rho\frac{\Delta x}{2}c_p\frac{T_6^{i+1} - T_6^i}{\Delta t}$



or $$T_6^{i+1} = \left(1 - 2\tau - 2\tau\frac{h\Delta x}{k}\right)T_6^i + 2\tau T_5^i + 2\tau\frac{h\Delta x}{k}T_\infty + \tau\frac{\dot{e}_0(\Delta x)^2}{k}$$

where

$$\Delta x = 0.015\text{ m}, \dot{e}_0 = 10^6\text{ W/m}^3,\; k = 28\text{ W/m}\cdot°\text{C},\; h = 35\text{ W/m}^2\cdot°\text{C},\; T_\infty = 20°\text{C}\text{, and } \alpha = 12.5\times10^{-6}\text{ m}^2\text{/s.}$$

The upper limit of the time step Δt is determined from the stability criteria that requires all primary coefficients to be greater than or equal to zero. The coefficient of T_4^i is smaller in this case, and thus the stability criteria for this problem can be expressed as

$$1 - 2\tau - 2\tau\frac{h\Delta x}{k} \ge 0 \quad \rightarrow \quad \tau \le \frac{1}{2(1 + h\Delta x/k)} \quad \rightarrow \quad \Delta t \le \frac{\Delta x^2}{2\alpha(1 + h\Delta x/k)}$$

since $\tau = \alpha\Delta t/\Delta x^2$. Substituting the given quantities, the maximum allowable the time step becomes

$$\Delta t \le \frac{(0.015\text{ m})^2}{2(12.5\times10^{-6}\text{ m}^2\text{/s})[1 + (35\text{ W/m}^2.°\text{C})(0.015\text{ m})/(28\text{ W/m.°C})]} = 8.8\text{ s}$$

Therefore, any time step less than 8.8 s can be used to solve this problem. For convenience, let us choose the time step to be Δt = 7.5 s. Then the mesh Fourier number becomes

$$\tau = \frac{\alpha\Delta t}{\Delta x^2} = \frac{(12.5\times10^{-6}\text{ m}^2\text{/s})(7.5\text{s})}{(0.015\text{ m})^2} = 0.4167$$

Substituting this value of τ and other given quantities, the nodal temperatures after 5×60/7.5 = 40 time steps (5 min) are determined to be

After 5 min:

T_0 = **229.9°C**, T_1 = **229.7°C**, T_2 = **229.0°C**, T_3 = **227.7°C**, T_4 = **225.8°C**, T_5 = **223.3°C**, and T_6 = **220.0 °C**

(*b*) The time needed for transient operation to be established is determined by increasing the number of time steps until the nodal temperatures no longer change. Using EES, we increased time steps for 16.8 hours (60500 seconds). The temperatures seem to remain constant at about this time. Then, the nodal temperatures under steady conditions are

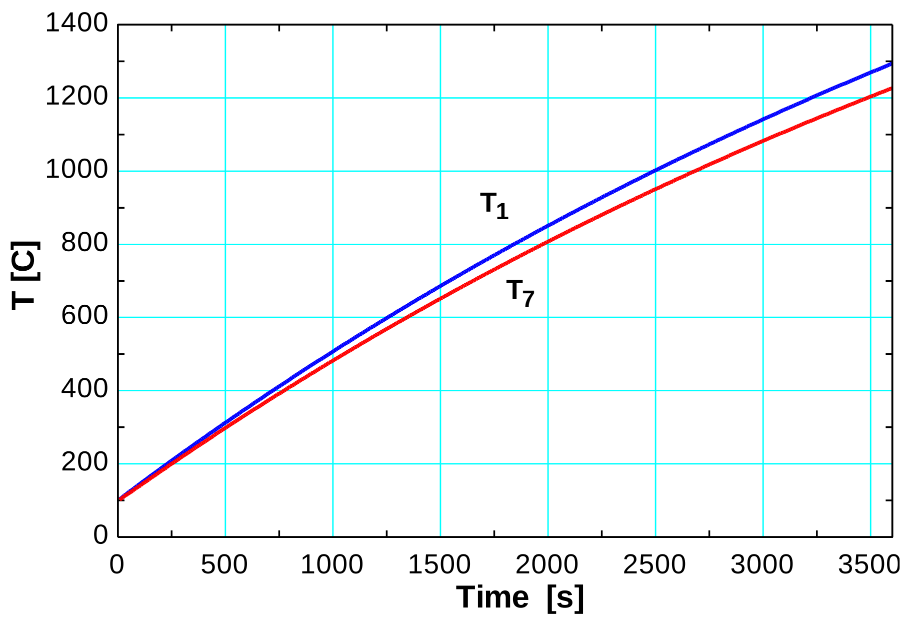
T_0 = **2736°C**, T_1 = **2732°C**, T_2 = **2720°C**, T_3 = **2700°C**, T_4 = **2672°C**, T_5 = **2636°C**, and T_6 = **2591 °C**

5-116 Prob. 5-115 is reconsidered. The effect of the cooling time on the temperatures of the left and right sides of the plate is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.09 [m]
k=28 [W/m-C]
alpha=12.5E-6 [m^2/s]
T_i=100 [C]
g_dot=1E6 [W/m^3]
T_infinity=20 [C]
h=35 [W/m^2-C]
DELTAx=0.015 [m]
"time=300 [s]"

"ANALYSIS"
M=L/DELTAx+1 "Number of nodes"
"DELTAt=7.5 [s]"
tau=(alpha*DELTAt)/DELTAx^2

"The technique is to store the temperatures in the parametric table and recover them (as old temperatures) using
the variable ROW.  The first row contains the initial values so Solve Table must begin at row 2. Use the
DUPLICATE statement to reduce the number of equations that need to be typed.  Column 1 contains the time,
column 2 the value of T[1], column 3, the value of T[2], etc., and column 9 the Row."
Time=TableValue(Row-1,#Time)+DELTAt
Duplicate i=1,7
    T_old[i]=TableValue(Row-1,#T[i])
end

"Using the explicit finite difference approach, the six equations for the six unknown temperatures are determined
to be"
T[1]=tau*(T_old[2]+T_old[2])+(1-2*tau)*T_old[1]+tau*(g_dot*DELTAx^2)/k "Node 0, insulated"
T[2]=tau*(T_old[1]+T_old[3])+(1-2*tau)*T_old[2]+tau*(g_dot*DELTAx^2)/k "Node 1"
T[3]=tau*(T_old[2]+T_old[4])+(1-2*tau)*T_old[3]+tau*(g_dot*DELTAx^2)/k "Node 2"
T[4]=tau*(T_old[3]+T_old[5])+(1-2*tau)*T_old[4]+tau*(g_dot*DELTAx^2)/k "Node 3"
T[5]=tau*(T_old[4]+T_old[6])+(1-2*tau)*T_old[5]+tau*(g_dot*DELTAx^2)/k "Node 4"
T[6]=tau*(T_old[5]+T_old[7])+(1-2*tau)*T_old[6]+tau*(g_dot*DELTAx^2)/k "Node 5"
T[7]=(1-2*tau-
2*tau*(h*DELTAx)/k)*T_old[7]+2*tau*T_old[6]+2*tau*(h*DELTAx)/k*T_infinity+tau*(g_dot*DELTAx^2)/k "Node 6,
convection"
```

Time [s]	T_1 [C]	T_2 [C]	T_3 [C]	T_4 [C]	T_5 [C]	T_6 [C]	T_7 [C]	Row
0	100	100	100	100	100	100	100	1
7.5	103.3	103.3	103.3	103.3	103.3	103.3	103.3	2
15	106.7	106.7	106.7	106.7	106.7	106.2	106.7	3
22.5	110	110	110	110	109.8	109.3	109.8	4
30	113.4	113.4	113.4	113.3	113.1	112.3	113.1	5
37.5	116.7	116.7	116.7	116.6	116.2	115.5	116.2	6
45	120.1	120.1	120	119.8	119.4	118.5	119.4	7
52.5	123.4	123.4	123.3	123.1	122.6	121.7	122.6	8
60	126.8	126.7	126.6	126.3	125.7	124.8	125.7	9
67.5	130.1	130	129.9	129.5	128.9	127.9	128.9	10
…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…
3525	1276	1274	1268	1259	1246	1230	1209	471
3533	1277	1276	1270	1261	1248	1232	1211	472
3540	1279	1277	1272	1263	1250	1233	1213	473
3548	1281	1279	1274	1265	1252	1235	1215	474
3555	1283	1281	1276	1266	1254	1237	1216	475
3563	1285	1283	1277	1268	1255	1239	1218	476
3570	1287	1285	1279	1270	1257	1240	1220	477
3578	1288	1287	1281	1272	1259	1242	1222	478
3585	1290	1288	1283	1274	1261	1244	1223	479
3593	1292	1290	1285	1275	1262	1246	1225	480
3600	1294	1292	1286	1277	1264	1247	1227	481



5-117E A plain window glass initially at a uniform temperature is subjected to convection on both sides. The transient finite difference formulation of this problem is to be obtained, and it is to be determined how long it will take for the fog on the windows to clear up (i.e., for the inner surface temperature of the window glass to reach 54°F).

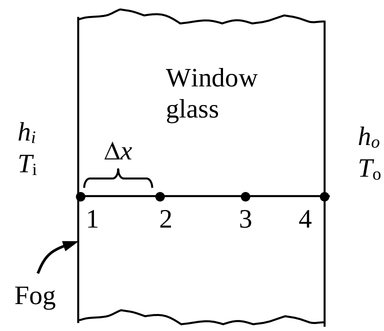
Assumptions **1** Heat transfer is one-dimensional since the window is large relative to its thickness. **2** Thermal conductivity is constant. **3** Radiation heat transfer is negligible.

Properties The conductivity and diffusivity are given to be k = 0.48 Btu/h.ft·°F and $\alpha = 4.2\times10^{-6}\ \text{ft}^2/\text{s}$.

Analysis The nodal spacing is given to be Δx = 0.125 in. Then the number of nodes becomes $M = L/\Delta x + 1$ = 0.375/0.125+1 = 4. This problem involves 4 unknown nodal temperatures, and thus we need to have 4 equations. Nodes 2 and 3 are interior nodes, and thus for them we can use the general explicit finite difference relation expressed as

$$T_{m-1}^i - 2T_m^i + T_{m+1}^i + \frac{\dot{e}_m^i \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau} \quad \to \quad T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1-2\tau)T_m^i$$

since there is no heat generation. The finite difference equation for nodes 1 and 4 on the surfaces subjected to convection is obtained by applying an energy balance on the half volume element about the node, and taking the direction of all heat transfers to be towards the node under consideration:



$$\text{Node 1 (convection)}: \ h_i(T_i - T_1^i) + k\frac{T_2^i - T_1^i}{\Delta x} = \rho\frac{\Delta x}{2}c_p\frac{T_1^{i+1} - T_1^i}{\Delta t}$$

or

$$T_1^{i+1} = \left(1 - 2\tau - 2\tau\frac{h_i \Delta x}{k}\right)T_1^i + 2\tau T_2^i + 2\tau\frac{h_i \Delta x}{k}T_i$$

$$\text{Node 2 (interior)}: \quad T_2^{i+1} = \tau(T_1^i + T_3^i) + (1-2\tau)T_2^i$$

$$\text{Node 3 (interior)}: \quad T_3^{i+1} = \tau(T_2^i + T_4^i) + (1-2\tau)T_3^i$$

$$\text{Node 4 (convection)}: \ h_o(T_o - T_4^i) + k\frac{T_3^i - T_4^i}{\Delta x} = \rho\frac{\Delta x}{2}c\frac{T_4^{i+1} - T_4^i}{\Delta t}$$

or

$$T_4^{i+1} = \left(1 - 2\tau - 2\tau\frac{h_o \Delta x}{k}\right)T_4^i + 2\tau T_3^i + 2\tau\frac{h_o \Delta x}{k}T_o$$

where Δx = 0.125/12 ft , k = 0.48 Btu/h.ft·°F, h_i = 1.2 Btu/h.ft²·°F, T_i =35+2*(t/60)°F (t in seconds), h_o = 2.6 Btu/h.ft²·°F, and T_o =35°F. The upper limit of the time step Δt is determined from the stability criteria that requires all primary coefficients to be greater than or equal to zero. The coefficient of T_4^i is smaller in this case, and thus the stability criteria for this problem can be expressed as

$$1 - 2\tau - 2\tau\frac{h\Delta x}{k} \geq 0 \quad \to \quad \tau \leq \frac{1}{2(1 + h\Delta x/k)} \quad \to \quad \Delta t \leq \frac{\Delta x^2}{2\alpha(1 + h\Delta x/k)}$$

since $\tau = \alpha\Delta t/\Delta x^2$. Substituting the given quantities, the maximum allowable time step becomes

$$\Delta t \leq \frac{(0.125/12\ \text{ft})^2}{2(4.2\times10^{-6}\ \text{ft}^2/\text{s})[1 + (2.6\ \text{Btu/h.ft}^2.°\text{F})(0.125/12\ \text{m})/(0.48\ \text{Btu/h.ft.°F})]} = 12.2\ \text{s}$$

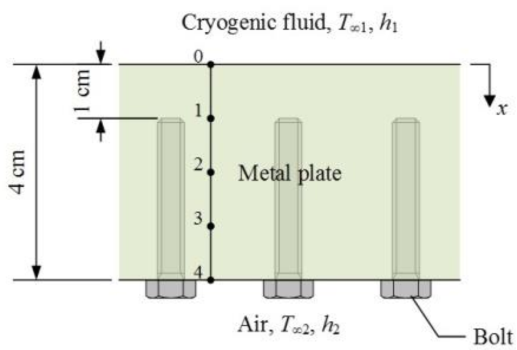
Therefore, any time step less than 12.2 s can be used to solve this problem. For convenience, let us choose the time step to be Δt = 10 s. Then the mesh Fourier number becomes

$$\tau = \frac{\alpha\Delta t}{\Delta x^2} = \frac{(4.2\times10^{-6}\ \text{ft}^2/\text{s})(10\text{s})}{(0.125/12\ \text{ft})^2} = 0.3871$$

Substituting this value of τ and other given quantities, the time needed for the inner surface temperature of the window glass to reach 54°F to avoid fogging is determined to be *never*. This is because steady conditions are reached in about 156 min, and the inner surface temperature at that time is determined to be 48.0°F. Therefore, the window will be fogged at all times.

5-118 C&S ASTM A437 B4B stainless steel bolts are fastened in a metal plate from the bottom surface, while the plate's upper surface is subjected to convection with cryogenic fluid at $T_{\infty 1} = -70°C$ and $h_1 = 300$ W/m²·K. The plate's bottom surface is subjected to convection with air at $T_{\infty 2} = 10°C$ and $h_2 = 10$ W/m²·K. The plate has an initial temperature of 10°C. Would the bolts comply with the ASME Code for Process Piping if the plate's upper surface is exposed to the cold fluid for 9 minutes?

Assumptions **1** Transient heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** There is no heat generation. **5** The convection heat transfer coefficients are constant.



Properties The thermal properties of the metal plate are $c_p = 500$ J/kg·K, $k = 16.3$ W/m·K, and $\rho = 8000$ kg/m³. (The analysis is on the plate not the bolts.)

Analysis The nodal spacing is given to be $\Delta x = 1$ cm. So, the number of nodes is

$$M = \frac{L}{\Delta x} + 1 = \frac{4\text{ cm}}{1\text{ cm}} + 1 = 5$$

The nodes are numbered from $m = 0$ to 4. Node 0 is a convection boundary with cryogenic fluid,

$$h_1\left(T_{\infty 1} - T_0^i\right) + k\frac{T_1^i - T_0^i}{\Delta x} = \rho\frac{\Delta x}{2}c_p\frac{T_0^{i+1} - T_0^i}{\Delta t}$$

or

$$m = 0: \qquad T_0^{i+1} = \left(1 - 2\tau - 2\tau\frac{h_1\Delta x}{k}\right)T_0^i + \tau\left(2T_1^i + 2\frac{h_1\Delta x}{k}T_{\infty 1}\right)$$

Nodes 1–3 are interior nodes,

$$m = 1 \text{ to } 3: \qquad T_m^{i+1} = (1 - 2\tau)T_m^i + \tau\left(T_{m-1}^i + T_{m+1}^i\right)$$

Node 4 is a convection boundary with air,

$$h_2\left(T_{\infty 2} - T_4^i\right) + k\frac{T_3^i - T_4^i}{\Delta x} = \rho\frac{\Delta x}{2}c_p\frac{T_4^{i+1} - T_4^i}{\Delta t}$$

or

$$m = 4: \qquad T_4^{i+1} = \left(1 - 2\tau - 2\tau\frac{h_2\Delta x}{k}\right)T_4^i + \tau\left(2T_3^i + 2\frac{h_2\Delta x}{k}T_{\infty 2}\right)$$

The upper limit of the time step Δt is determined from the stability criteria that requires all primary coefficients to be greater than or equal to zero. The coefficient T_0^i is smaller in this case, and thus the stability criterion for this problem can be expressed as

$$1 - 2\tau - 2\tau\frac{h_1\Delta x}{k} \geq 0 \ \rightarrow \tau \leq \frac{1}{2(1 + h_1\Delta x/k)} \text{ or } \quad \Delta t \leq \frac{\Delta x^2}{2\alpha(1 + h_1\Delta x/k)} = 10.36\text{ s} \quad \Longrightarrow \quad \Delta t = 10\text{ s}$$

where,

$$c_p = 500\text{ J/kg·K}, \ k = 16.3\text{ W/m·K}, \ \rho = 8000\text{ kg/m}^3, \ \Delta x = 1\text{ cm}$$

$$h_1 = 300\text{ W/m}^2\text{·K}, \ h_2 = 10\text{ W/m}^2\text{·K}, \ T_{\infty 1} = -70°\text{C}, \ T_{\infty 2} = 10°\text{C}$$

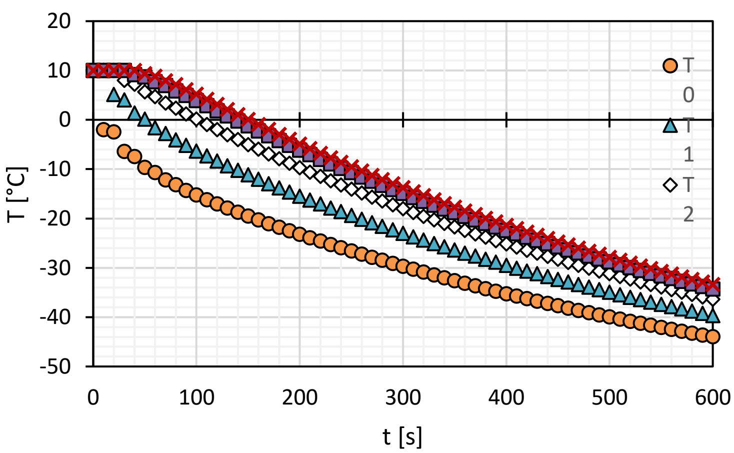
$$\alpha = \frac{k}{\rho c_p} = \frac{16.3}{(8000)(500)} = 4.075 \times 10^{-6} \text{m}^2/\text{s}$$

$$\tau = \frac{\alpha \Delta t}{\Delta x^2} = \frac{(4.075 \times 10^{-6})(10)}{(0.01)^2} = 0.4075$$

The numerical results can be solved iteratively with the initial nodal temperatures, T_i = 10°C, and time step, Δt = 10 s. The following table tabulates the iterated nodal temperatures at every minute:

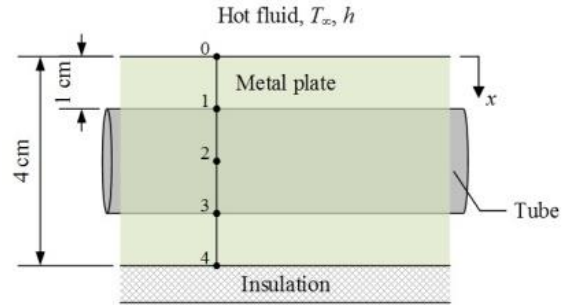
x [m] =	0	0.01	0.02	0.03	0.04
t [s]	T_0 [°C]	T_1 [°C]	T_2 [°C]	T_3 [°C]	T_4 [°C]
0	10.0	10.0	10.0	10.0	10.0
60	-10.7	-1.6	4.7	7.7	8.8
120	-17.0	-8.3	-2.0	1.8	3.1
180	-21.8	-13.8	-7.9	-4.2	-2.9
240	-25.9	-18.6	-13.2	-9.8	-8.6
300	-29.7	-23.0	-18.0	-14.9	-13.8
360	-33.1	-27.0	-22.4	-19.6	-18.5
420	-36.2	-30.6	-26.4	-23.8	-22.7
480	-39.1	-33.9	-30.1	-27.6	-26.6
540	**-41.7**	**-36.9**	**-33.4**	**-31.1**	**-30.2**
600	-44.0	-39.7	-36.4	-34.3	-33.4

The nodal temperatures, as a function of t, are plotted in the following figure:



Discussion After 9 minutes of subjecting the upper surface to convection with the cryogenic fluid, the entire plate temperature is below −30°C. This means that the bolts are at temperatures that are below the minimum suitable temperature specified by the ASME Code for Process Piping. Therefore, the bolts would not be in compliance.

5-119 C&S ASTM F441 CPVC tube is embedded in a metal plate, while the plate's upper surface is subjected to convection with hot fluid at T_∞ = 300°C and h = 200 W/m²·K. The plate's bottom surface is well insulated. The plate has an initial temperature of 20°C. Would the tube comply with the ASME Code for Process Piping if the plate's upper surface is exposed to the hot fluid for 5 minutes?



Assumptions **1** Transient heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible. **4** There is no heat generation. **5** The convection heat transfer coefficients are constant.

Properties The thermal properties of the metal plate are c_p = 460 J/kg·K, k = 26.9 W/m·K, and ρ = 7730 kg/m³.

Analysis The nodal spacing is given to be Δx = 1 cm. So, the number of nodes is

$$M = \frac{L}{\Delta x} + 1 = \frac{4\text{ cm}}{1\text{ cm}} + 1 = 5$$

The nodes are numbered from m = 0 to 4. Node 0 is a convection boundary with hot fluid,

$$h(T_\infty - T_0^i) + k\frac{T_1^i - T_0^i}{\Delta x} = \rho\frac{\Delta x}{2}c_p\frac{T_0^{i+1} - T_0^i}{\Delta t}$$

or

$$m = 0: \qquad T_0^{i+1} = \left(1 - 2\tau - 2\tau\frac{h\Delta x}{k}\right)T_0^i + \tau\left(2T_1^i + 2\frac{h\Delta x}{k}T_\infty\right)$$

Nodes 1–3 are interior nodes,

$$m = 1\text{ to }3: \qquad T_m^{i+1} = (1 - 2\tau)T_m^i + \tau\left(T_{m-1}^i + T_{m+1}^i\right)$$

Node 4 is an insulation boundary,

$$m = 4: \qquad T_4^{i+1} = (1 - 2\tau)T_4^i + \tau\left(2T_3^i\right)$$

The upper limit of the time step Δt is determined from the stability criteria that requires all primary coefficients to be greater than or equal to zero. The coefficient T_0^i is smaller in this case, and thus the stability criterion for this problem can be expressed as

$$1 - 2\tau - 2\tau\frac{h\Delta x}{k} \geq 0 \;\rightarrow \tau \leq \frac{1}{2(1 + h\Delta x/k)} \text{ or } \quad \Delta t \leq \frac{\Delta x^2}{2\alpha(1 + h\Delta x/k)} = 6.15\text{ s} \quad \Longrightarrow \quad \Delta t = 5\text{ s}$$

where,

$$c_p = 460\text{ J/kg·K},\; k = 26.9\text{ W/m·K},\; \rho = 7730\text{ kg/m}^3,\; \Delta x = 1\text{ cm}$$

$$h = 200\text{ W/m}^2\text{·K},\; T_\infty = 300°\text{C}$$

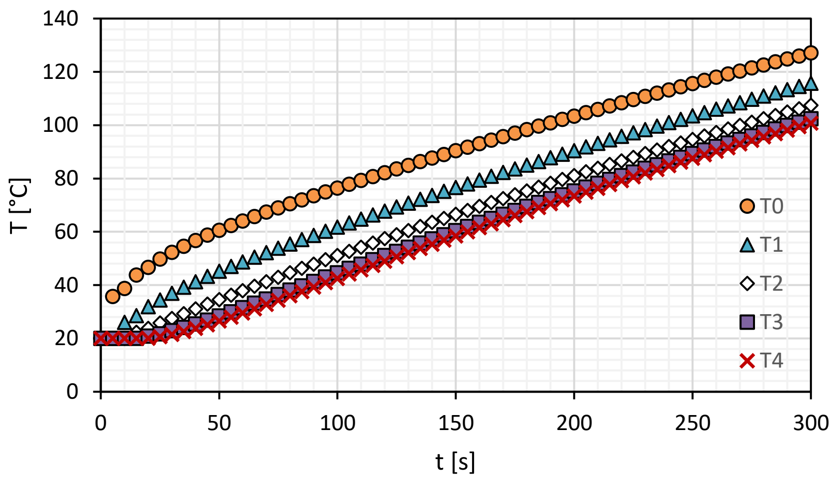
$$\alpha = \frac{k}{\rho c_p} = \frac{26.9}{(7730)(460)} = 7.5651 \times 10^{-6}\text{m}^2\text{/s}$$

$$\tau = \frac{\alpha\Delta t}{\Delta x^2} = \frac{(7.5651 \times 10^{-6})(5)}{(0.01)^2} = 0.37826$$

The numerical results can be solved iteratively with the initial nodal temperatures, T_i = 20°C, and time step, Δt = 5 s. The following table tabulates the iterated nodal temperatures at every minute:

x [m] =	0	0.01	0.02	0.03	0.04
t [s]	T_0 [°C]	T_1 [°C]	T_2 [°C]	T_3 [°C]	T_4 [°C]
0	20.0	20.0	20.0	20.0	20.0
60	64.1	48.7	37.9	31.6	29.5
120	82.1	67.8	57.4	51.0	48.9
180	98.3	85.0	75.3	69.5	67.5
240	113.2	100.9	92.0	86.5	84.7
300	**127.1**	**115.7**	**107.4**	**102.4**	**100.7**

The nodal temperatures, as a function of t, are plotted in the following figure:



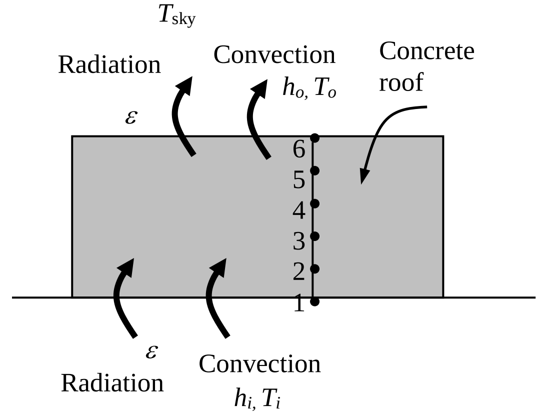
DiscussionAfter 5 minutes of subjecting the upper surface to convection with the hot fluid, the entire plate temperature is above 93.3°C. This means that the tube surface is at a temperature that exceeds the maximum use temperature specified by the ASME Code for Process Piping. Therefore, the tube would not be in compliance.

5-120 The roof of a house initially at a uniform temperature is subjected to convection and radiation on both sides. The temperatures of the inner and outer surfaces of the roof at 6 am in the morning as well as the average rate of heat transfer through the roof during that night are to be determined.

Assumptions **1** Heat transfer is one-dimensional. **2** Thermal properties, heat transfer coefficients, and the indoor and outdoor temperatures are constant. **3** Radiation heat transfer is significant.

Properties The conductivity and diffusivity are given to be k = 1.4 W/m.°C and $\alpha = 0.69\times10^{-6}\ \text{m}^2/\text{s}$. The emissivity of both surfaces of the concrete roof is 0.9.

Analysis The nodal spacing is given to be Δx = 0.03 m. Then the number of nodes becomes $M = L/\Delta x + 1$ = 0.15/0.03+1 = 6. This problem involves 6 unknown nodal temperatures, and thus we need to have 6 equations. Nodes 2, 3, 4, and 5 are interior nodes, and thus for them we can use the general explicit finite difference relation expressed as



$$T_{m-1}^i - 2T_m^i + T_{m+1}^i + \frac{\dot{e}_m^i \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau}$$

$$\rightarrow\ T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1-2\tau)T_m^i + \tau\frac{\dot{e}_m^i \Delta x^2}{k}$$

The finite difference equations for nodes 1 and 6 subjected to convection and radiation are obtained from an energy balance by taking the direction of all heat transfers to be towards the node under consideration:

$$\text{Node 1 (convection)}:\quad h_i(T_i - T_1^i) + k\frac{T_2^i - T_1^i}{\Delta x} + \varepsilon\sigma\left[T_{wall}^4 - (T_1^i + 273)^4\right] = \rho\frac{\Delta x}{2}c_p\frac{T_1^{i+1} - T_1^i}{\Delta t}$$

$$\text{Node 2 (interior)}:\quad T_2^{i+1} = \tau(T_1^i + T_3^i) + (1-2\tau)T_2^i$$

$$\text{Node 3 (interior)}:\quad T_3^{i+1} = \tau(T_2^i + T_4^i) + (1-2\tau)T_3^i$$

$$\text{Node 4 (interior)}:\quad T_4^{i+1} = \tau(T_3^i + T_5^i) + (1-2\tau)T_4^i$$

$$\text{Node 5 (interior)}:\quad T_5^{i+1} = \tau(T_4^i + T_6^i) + (1-2\tau)T_5^i$$

$$\text{Node 6 (convection)}:\quad h_o(T_0 - T_6^i) + k\frac{T_5^i - T_6^i}{\Delta x} + \varepsilon\sigma\left[T_{sky}^4 - (T_6^i + 273)^4\right] = \rho\frac{\Delta x}{2}c_p\frac{T_6^{i+1} - T_6^i}{\Delta t}$$

where k = 1.4 W/m.°C, $\alpha = k/\rho c_p = 0.69\times10^{-6}\ \text{m}^2/\text{s}$, T_i = 20°C, T_{wall} = 293 K, T_o = 6°C, T_{sky} =260 K, h_i = 5 W/m^2.°C, h_o = 12 W/m^2.°C, Δx = 0.03 m, and Δt = 5 min. Also, the mesh Fourier number is

$$\tau = \frac{\alpha\Delta t}{\Delta x^2} = \frac{(0.69\times10^{-6}\ \text{m}^2/\text{s})(300\,\text{s})}{(0.03\,\text{m})^2} = 0.230$$

Substituting this value of τ and other given quantities, the inner and outer surface temperatures of the roof after 12×(60/5) = 144 time steps (12 h) are determined to be T_1 = **10.3°C** and T_6 = **-0.97°C.**

(*b*) The average temperature of the inner surface of the roof can be taken to be

$$T_{1,avg} = \frac{T_{1@6\,\text{PM}} + T_{1@6\,\text{AM}}}{2} = \frac{18+10.3}{2} = 14.15°\text{C}$$

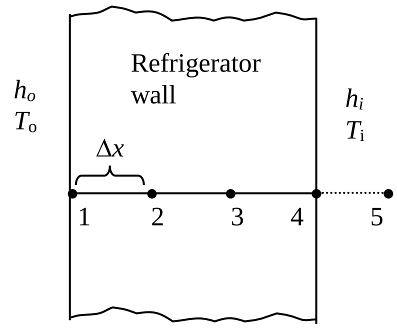
Then the average rate of heat loss through the roof that night becomes

$$\begin{aligned}\dot{Q}_{avg} &= h_i A_s(T_i - T_{1,ave}) + \varepsilon\sigma A_s\left[T_{wall}^4 - (T_1^i + 273)^4\right]\\ &= (5\ \text{W/m}^2\cdot°\text{C})(18\times32\ \text{m}^2)(20-14.15)°\text{C}\\ &\quad + 0.9(18\times32\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(293\,\text{K})^4 - (14.15+273\,\text{K})^4]\\ &= \mathbf{33{,}640\ W}\end{aligned}$$

5-121 A refrigerator whose walls are constructed of 3-cm thick urethane insulation malfunctions, and stops running for 6 h. The temperature inside the refrigerator at the end of this 6 h period is to be determined.

Assumptions **1** Heat transfer is one-dimensional since the walls are large relative to their thickness. **2** Thermal properties, heat transfer coefficients, and the outdoor temperature are constant. **3** Radiation heat transfer is negligible. **4** The temperature of the contents of the refrigerator, including the air inside, rises uniformly during this period. **5** The local atmospheric pressure is 1 atm. **6** The space occupied by food and the corner effects are negligible. **7** Heat transfer through the bottom surface of the refrigerator is negligible.

Properties The conductivity and diffusivity are given to be k = 0.026 W/m·K and $\alpha = 0.36\times10^{-6}$ m^2/s. The average specific heat of food items is given to be 3.6 kJ/kg·K. The specific heat and density of air at 1 atm and 3°C are c_p = 1.006 kJ/kg·K and ρ = 1.28 kg/m^3 (Table A-15).



Analysis The nodal spacing is given to be Δx = 0.01 m. Then the number of nodes becomes $M = L/\Delta x + 1$ = 0.03/0.01+1 = 4. This problem involves 4 unknown nodal temperatures, and thus we need to have 4 equations. Nodes 2 and 3 are interior nodes, and thus for them we can use the general explicit finite difference relation expressed as

$$T_{m-1}^i - 2T_m^i + T_{m+1}^i + \frac{\dot{e}_m^i \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau} \rightarrow T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1-2\tau)T_m^i + \tau\frac{\dot{e}_m^i \Delta x^2}{k}$$

The finite difference equations for nodes 1 and 4 subjected to convection and radiation are obtained from an energy balance by taking the direction of all heat transfers to be towards the node under consideration:

$$\text{Node 1 (convection)}: \quad h_o(T_0 - T_1^i) + k\frac{T_2^i - T_1^i}{\Delta x} = \rho\frac{\Delta x}{2}c_p\frac{T_1^{i+1} - T_1^i}{\Delta t}$$

$$\text{Node 2 (interior)}: \quad T_2^{i+1} = \tau(T_1^i + T_3^i) + (1-2\tau)T_2^i$$

$$\text{Node 3 (interior)}: \quad T_3^{i+1} = \tau(T_2^i + T_4^i) + (1-2\tau)T_3^i$$

$$\text{Node 4 (convection)}: \quad h_i(T_5^i - T_4^i) + k\frac{T_3^i - T_4^i}{\Delta x} = \rho\frac{\Delta x}{2}c_p\frac{T_4^{i+1} - T_4^i}{\Delta t}$$

where $T_5 = T_i$ = 3°C (initially), T_o = 25°C, h_i = 6 W/m^2·K, h_o = 9 W/m^2·K, Δx = 0.01 m, and Δt = 1 min. Also, the mesh Fourier number is

$$\tau = \frac{\alpha\Delta t}{\Delta x^2} = \frac{(0.36\times10^{-6}\text{ m}^2/\text{s})(60\text{ s})}{(0.01\text{ m})^2} = 0.216$$

The volume of the refrigerator cavity and the mass of air inside are

$$V = (1.80-0.03)(0.8-0.03)(0.7-0.03) = 0.913\text{ m}^3$$

$$m_{air} = \rho V = (1.28\text{ kg/m}^3)(0.913\text{m}^3) = 1.17\text{ kg}$$

Energy balance for the air space of the refrigerator can be expressed as

$$\text{Node 5 (refrig. air)}: \quad h_i A_i(T_4^i - T_5^i) = (mc_p\Delta T)_{air} + (mc_p\Delta T)_{food}$$

or

$$h_i A_i(T_4^i - T_5^i) = \left[(mc_p)_{air} + (mc_p)_{food}\right]\frac{T_5^{i+1} - T_5^i}{\Delta t}$$

where $$A_i = 2(1.77\times0.77) + 2(1.77\times0.67) + (0.77\times0.67) = 5.6135\text{ m}^2$$

Substituting, temperatures of the refrigerated space after 6×60 = 360 time steps (6 h) is determined to be

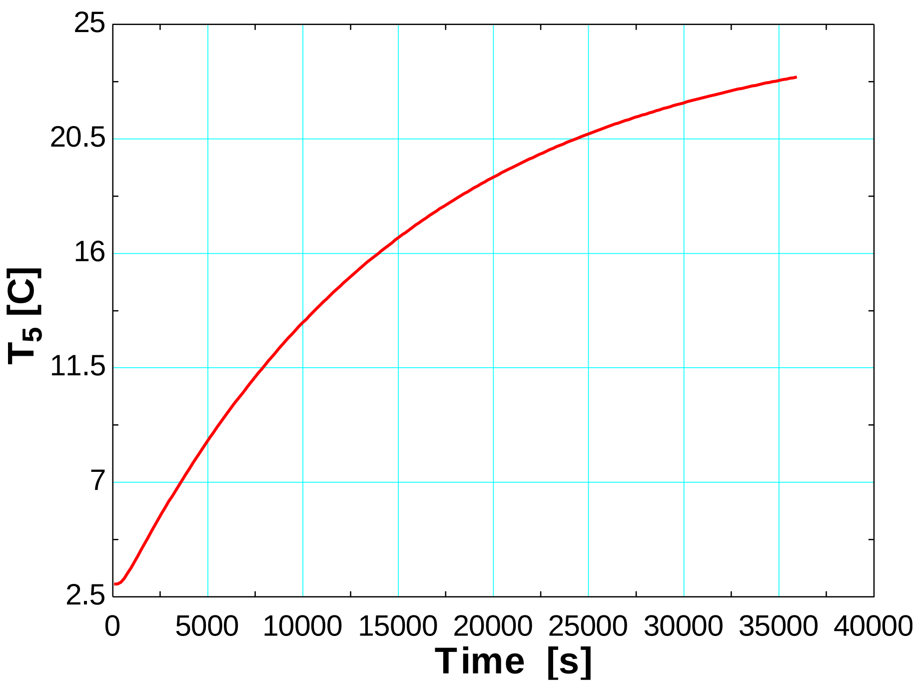
$T_{in} = T_5 =$ **19.6°C**

5-122 Prob. 5-121 is reconsidered. The temperature inside the refrigerator as a function of heating time is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
t_ins=0.03 [m]
k=0.026 [W/m-C]
alpha=0.36E-6 [m^2/s]
T_i=3 [C]
h_i=6 [W/m^2-C]
h_o=9 [W/m^2-C]
T_infinity=25 [C]
m_food=15 [kg]
C_food=3600 [J/kg-C]
DELTAx=0.01 [m]
DELTAt=60 [s]
time=6*3600 [s]

"PROPERTIES"
rho_air=density(air, T=T_i, P=101.3)
C_air=CP(air, T=T_i)*Convert(kJ/kg-C, J/kg-C)

"ANALYSIS"
M=t_ins/DELTAx+1 "Number of nodes"
tau=(alpha*DELTAt)/DELTAx^2
RhoC=k/alpha "RhoC=rho*C"

"The technique is to store the temperatures in the parametric table and recover them (as old temperatures)
using the variable ROW.  The first row contains the initial values so Solve Table must begin at row 2.
Use the DUPLICATE statement to reduce the number of equations that need to be typed.  Column 1
contains the time, column 2 the value of T[1], column 3, the value of T[2], etc., and column 7 the Row."
Time=TableValue('Table 1',Row-1,#Time)+DELTAt
Duplicate i=1,5
    T_old[i]=TableValue('Table 1',Row-1,#T[i])
end

"Using the explicit finite difference approach, the six equations for the six unknown temperatures are
determined to be"
h_o*(T_infinity-T_old[1])+k*(T_old[2]-T_old[1])/DELTAx=RhoC*DELTAx/2*(T[1]-T_old[1])/DELTAt "Node 1,
convection"
T[2]=tau*(T_old[1]+T_old[3])+(1-2*tau)*T_old[2] "Node 2"
T[3]=tau*(T_old[2]+T_old[4])+(1-2*tau)*T_old[3] "Node 3"
h_i*(T_old[5]-T_old[4])+k*(T_old[3]-T_old[4])/DELTAx=RhoC*DELTAx/2*(T[4]-T_old[4])/DELTAt "Node 4,
convection"
h_i*A_i*(T_old[4]-T_old[5])=m_air*C_air*(T[5]-T_old[5])/DELTAt+m_food*C_food*(T[5]-T_old[5])/DELTAt
"Node 5, refrig. air"

A_i=2*(1.8-0.03)*(0.8-0.03)+2*(1.8-0.03)*(0.7-0.03)+(0.8-0.03)*(0.7-0.03)
m_air=rho_air*V_air
V_air=(1.8-0.03)*(0.8-0.03)*(0.7-0.03)
```

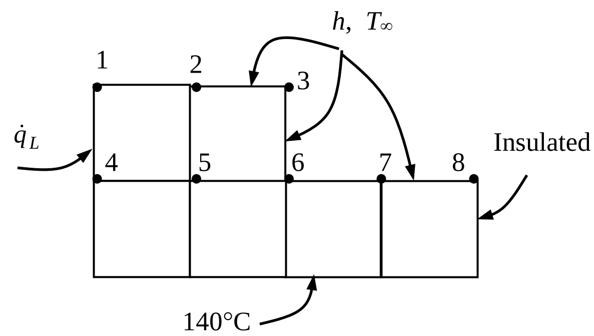
Time [s]	T_1 [C]	T_2 [C]	T_3 [C]	T_4 [C]	T_5 [C]	Row
0	3	3	3	3	3	1
60	35.9	3	3	3	3	2
120	5.389	10.11	3	3	3	3
180	36.75	7.552	4.535	3	3	4
240	6.563	13.21	4.855	3.663	3	5
300	37	9.968	6.402	3.517	3.024	6
360	7.374	15.04	6.549	4.272	3.042	7
420	37.04	11.55	7.891	4.03	3.087	8
480	8.021	16.27	7.847	4.758	3.122	9
540	36.97	12.67	8.998	4.461	3.182	10
…	…	…	…	…	…	…
…	…	…	…	…	…	…
35460	24.85	24.23	23.65	23.09	22.86	592
35520	24.81	24.24	23.65	23.1	22.87	593
35580	24.85	24.23	23.66	23.11	22.88	594
35640	24.81	24.24	23.67	23.12	22.88	595
35700	24.85	24.24	23.67	23.12	22.89	596
35760	24.81	24.25	23.68	23.13	22.9	597
35820	24.85	24.25	23.68	23.14	22.91	598
35880	24.81	24.26	23.69	23.15	22.92	599
35940	24.85	24.25	23.69	23.15	22.93	600
36000	24.82	24.26	23.7	23.16	22.94	601



5-123 Heat conduction through a long L-shaped solid bar with specified boundary conditions is considered. The temperature at the top corner (node #3) of the body after 2, 5, and 30 min is to be determined with the transient explicit finite difference method.

Assumptions **1** Heat transfer through the body is given to be transient and two-dimensional. **2** Thermal conductivity is constant. **3** Heat generation is uniform.

Properties The conductivity and diffusivity are given to be k = 15 W/m·°C and $\alpha = 3.2\times10^{-6}$ m^2/s .



Analysis The nodal spacing is given to be Δx=Δx=l=0.015 m. The explicit finite difference equations are determined on the basis of the energy balance for the transient case expressed as

$$\sum_{\text{All sides}} \dot{Q}^i + \dot{E}^{\text{i}}_{\text{element}} = \rho \mathcal{V}_{\text{element}} c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

The quantities $h, T_\infty, \dot{e}$, and $\dot{q}_R$ do not change with time, and thus we do not need to use the superscript i for them. Also, the energy balance expressions can be simplified using the definitions of thermal diffusivity $\alpha = k/\rho c_p$ and the dimensionless mesh Fourier number $\tau = \alpha\Delta t / l^2$ where $\Delta x = \Delta y = l$. We note that all nodes are boundary nodes except node 5 that is an interior node. Therefore, we will have to rely on energy balances to obtain the finite difference equations. Using energy balances, the finite difference equations for each of the 8 nodes are obtained as follows:

Node 1: $$\dot{q}_L \frac{l}{2} + h\frac{l}{2}(T_\infty - T_1^i) + k\frac{l}{2}\frac{T_2^i - T_1^i}{l} + k\frac{l}{2}\frac{T_4^i - T_1^i}{l} + \dot{e}_0\frac{l^2}{4} = \rho\frac{l^2}{4}c\frac{T_1^{i+1} - T_1^i}{\Delta t}$$

Node 2: $$hl(T_\infty - T_2^i) + k\frac{l}{2}\frac{T_1^i - T_2^i}{l} + k\frac{l}{2}\frac{T_3^i - T_2^i}{l} + kl\frac{T_5^i - T_2^i}{l} + \dot{e}_0\frac{l^2}{2} = \rho\frac{l^2}{2}c_p\frac{T_2^{i+1} - T_2^i}{\Delta t}$$

Node 3: $$hl(T_\infty - T_3^i) + k\frac{l}{2}\frac{T_2^i - T_3^i}{l} + k\frac{l}{2}\frac{T_6^i - T_3^i}{l} + \dot{e}_0\frac{l^2}{4} = \rho\frac{l^2}{4}c\frac{T_3^{i+1} - T_3^i}{\Delta t}$$

(It can be rearranged as $$T_3^{i+1} = \left(1 - 4\tau - 4\tau\frac{hl}{k}\right)T_3^i + 2\tau\left(T_4^i + T_6^i + 2\frac{hl}{k}T_\infty + \frac{\dot{e}_0 l^2}{2k}\right))$$

Node 4: $$\dot{q}_L l + k\frac{l}{2}\frac{T_1^i - T_4^i}{l} + k\frac{l}{2}\frac{140 - T_4^i}{l} + kl\frac{T_5^i - T_4^i}{l} + \dot{e}_0\frac{l^2}{2} = \rho\frac{l^2}{2}c\frac{T_4^{i+1} - T_4^i}{\Delta t}$$

Node 5 (interior): $$T_5^{i+1} = (1 - 4\tau)T_5^i + \tau\left(T_2^i + T_4^i + T_6^i + 140 + \frac{\dot{e}_0 l^2}{k}\right)$$

Node 6: $$hl(T_\infty - T_6^i) + k\frac{l}{2}\frac{T_3^i - T_6^i}{l} + kl\frac{T_5^i - T_6^i}{l} + kl\frac{140 - T_6^i}{l} + k\frac{l}{2}\frac{T_7^i - T_6^i}{l} + \dot{e}_0\frac{3l^2}{4} = \rho\frac{3l^2}{4}c\frac{T_6^{i+1} - T_6^i}{\Delta t}$$

Node 7: $$hl(T_\infty - T_7^i) + k\frac{l}{2}\frac{T_6^i - T_7^i}{l} + k\frac{l}{2}\frac{T_8^i - T_7^i}{l} + kl\frac{140 - T_7^i}{l} + \dot{e}_0\frac{l^2}{2} = \rho\frac{l^2}{2}c\frac{T_7^{i+1} - T_7^i}{\Delta t}$$

Node 8: $$h\frac{l}{2}(T_\infty - T_8^i) + k\frac{l}{2}\frac{T_7^i - T_8^i}{l} + k\frac{l}{2}\frac{140 - T_8^i}{l} + \dot{e}_0\frac{l^2}{4} = \rho\frac{l^2}{4}c\frac{T_8^{i+1} - T_8^i}{\Delta t}$$

where

$\dot{e}_0 = 2\times10^7$ W/m^3, $\dot{q}_L = 8000$ W/m^2, l = 0.015 m, k =15 W/m·°C, h = 80 W/m^2·°C, and T_∞ =25°C.

The upper limit of the time step Δt is determined from the stability criteria that requires the coefficient of T_m^i in the T_m^{i+1} expression (the primary coefficient) be greater than or equal to zero for all nodes. The smallest primary coefficient in the 8 equations above is the coefficient of T_3^i in the T_3^{i+1} expression since it is exposed to most convection per unit volume (this can be verified), and thus the stability criteria for this problem can be expressed as

$$1 - 4\tau - 4\tau\frac{hl}{k} \geq 0 \quad\rightarrow\quad \tau \leq \frac{1}{4(1 + hl/k)} \quad\rightarrow\quad \Delta t \leq \frac{l^2}{4\alpha(1 + hl/k)}$$

since $\tau = \alpha\Delta t / l^2$. Substituting the given quantities, the maximum allowable value of the time step is determined to be

$$\Delta t \le \frac{(0.015\,\text{m})^2}{4(3.2\times10^{-6}\ \text{m}^2/\text{s})[1+(80\ \text{W/m}^2.°\text{C})(0.015\,\text{m})/(15\ \text{W/m.°C})]} = 16.3\,\text{s}$$

Therefore, any time step less than 16.3 s can be used to solve this problem. For convenience, we choose the time step to be Δt = 15 s. Then the mesh Fourier number becomes

$$\tau = \frac{\alpha\Delta t}{l^2} = \frac{(3.2\times10^{-6}\ \text{m}^2/\text{s})(15\text{s})}{(0.015\,\text{m})^2} = 0.2133 \qquad (\text{for}\ \ \Delta t = 15\ \text{s})$$

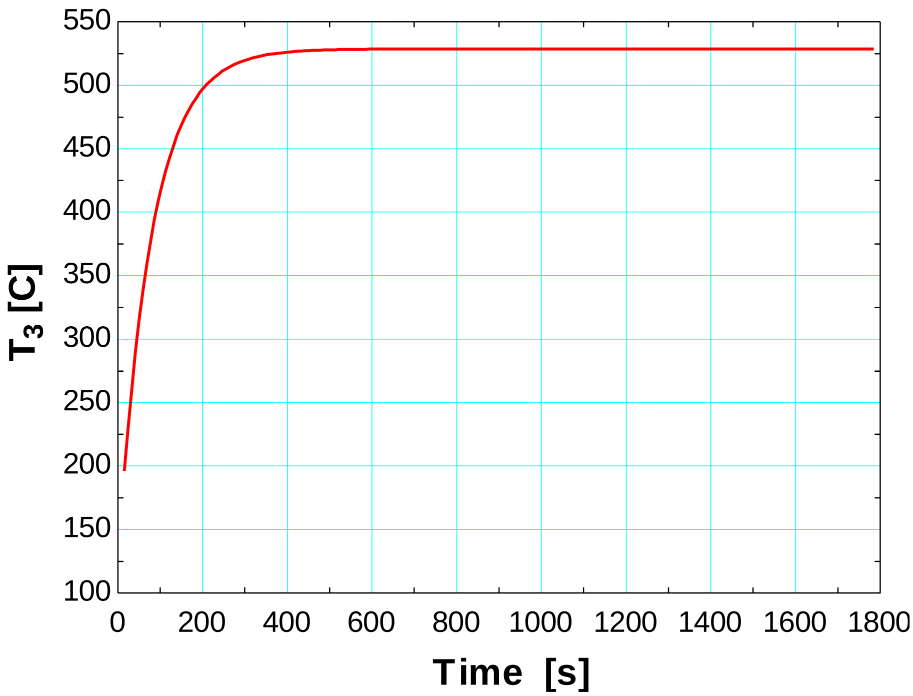
Using the specified initial condition as the solution at time t = 0 (for i = 0), sweeping through the 9 equations above will give the solution at intervals of 15 s. Using a computer, the solution at the upper corner node (node 3) is determined to be **441, 520**, and **529°C** at 2, 5, and 30 min, respectively. It can be shown that the steady state solution at node 3 is 531°C.

5-124 Prob. 5-123 is reconsidered. The temperature at the top corner as a function of heating time is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_i=140 [C]
k=15 [W/m-C]
alpha=3.2E-6 [m^2/s]
e_dot=2E7 [W/m^3]
T_bottom=140 [C]
T_infinity=25 [C]
h=80 [W/m^2-C]
q_dot_L=8000 [W/m^2]
DELTAx=0.015 [m]
DELTAy=0.015 [m]
time=120 [s]

"ANALYSIS"
l=DELTAx
DELTAt=15 [s]
tau=(alpha*DELTAt)/l^2
RhoC=k/alpha "RhoC=rho*C"

"The technique is to store the temperatures in the parametric table and recover them (as old temperatures)
using the variable ROW.  The first row contains the initial values so Solve Table must begin at row 2.
Use the DUPLICATE statement to reduce the number of equations that need to be typed.  Column 1
contains the time, column 2 the value of T[1], column 3, the value of T[2], etc., and column 10 the Row."
Time=TableValue('Table 1',Row-1,#Time)+DELTAt
Duplicate i=1,8
    T_old[i]=TableValue('Table 1',Row-1,#T[i])
end

"Using the explicit finite difference approach, the eight equations for the eight unknown temperatures are
determined to be"
q_dot_L*l/2+h*l/2*(T_infinity-T_old[1])+k*l/2*(T_old[2]-T_old[1])/l+k*l/2*(T_old[4]-
T_old[1])/l+e_dot*l^2/4=RhoC*l^2/4*(T[1]-T_old[1])/DELTAt "Node 1"
h*l*(T_infinity-T_old[2])+k*l/2*(T_old[1]-T_old[2])/l+k*l/2*(T_old[3]-T_old[2])/l+k*l*(T_old[5]-
T_old[2])/l+e_dot*l^2/2=RhoC*l^2/2*(T[2]-T_old[2])/DELTAt "Node 2"
h*l*(T_infinity-T_old[3])+k*l/2*(T_old[2]-T_old[3])/l+k*l/2*(T_old[6]-T_old[3])/l+e_dot*l^2/4=RhoC*l^2/4*(T[3]-
T_old[3])/DELTAt "Node 3"
q_dot_L*l+k*l/2*(T_old[1]-T_old[4])/l+k*l/2*(T_bottom-T_old[4])/l+k*l*(T_old[5]-
T_old[4])/l+e_dot*l^2/2=RhoC*l^2/2*(T[4]-T_old[4])/DELTAt "Node 4"
T[5]=(1-4*tau)*T_old[5]+tau*(T_old[2]+T_old[4]+T_old[6]+T_bottom+e_dot*l^2/k) "Node 5"
h*l*(T_infinity-T_old[6])+k*l/2*(T_old[3]-T_old[6])/l+k*l*(T_old[5]-T_old[6])/l+k*l*(T_bottom-
T_old[6])/l+k*l/2*(T_old[7]-T_old[6])/l+e_dot*3/4*l^2=RhoC*3/4*l^2*(T[6]-T_old[6])/DELTAt "Node 6"
h*l*(T_infinity-T_old[7])+k*l/2*(T_old[6]-T_old[7])/l+k*l/2*(T_old[8]-T_old[7])/l+k*l*(T_bottom-
T_old[7])/l+e_dot*l^2/2=RhoC*l^2/2*(T[7]-T_old[7])/DELTAt "Node 7"
h*l/2*(T_infinity-T_old[8])+k*l/2*(T_old[7]-T_old[8])/l+k*l/2*(T_bottom-
T_old[8])/l+e_dot*l^2/4=RhoC*l^2/4*(T[8]-T_old[8])/DELTAt "Node 8"
```

Time [s]	T_1 [C]	T_2 [C]	T_3 [C]	T_4 [C]	T_5 [C]	T_6 [C]	T_7 [C]	T_8 [C]	Row
0	140	140	140	140	140	140	140	140	1
15	203.5	200.1	196.1	207.4	204	201.4	200.1	200.1	2
30	265	259.7	252.4	258.2	253.7	243.7	232.7	232.5	3
45	319	312.7	300.3	299.9	293.5	275.7	252.4	250.1	4
60	365.5	357.4	340.3	334.6	326.4	300.7	265.2	260.4	5
75	404.6	394.9	373.2	363.6	353.5	320.6	274.1	267	6
90	437.4	426.1	400.3	387.8	375.9	336.7	280.8	271.6	7
105	464.7	451.9	422.5	407.9	394.5	349.9	286	275	8
120	487.4	473.3	440.9	424.5	409.8	360.7	290.1	277.5	9
135	506.2	491	456.1	438.4	422.5	369.6	293.4	279.6	10
…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…
1650	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	111
1665	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	112
1680	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	113
1695	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	114
1710	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	115
1725	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	116
1740	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	117
1755	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	118
1770	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	119
1785	596.3	575.7	528.5	504.6	483.1	411.9	308.8	288.9	120

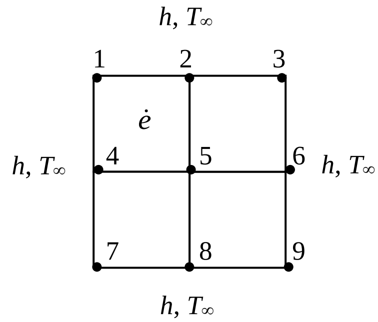


5-125 A long solid bar is subjected to transient two-dimensional heat transfer. The centerline temperature of the bar after 20 min and after steady conditions are established are to be determined.

Assumptions **1** Heat transfer through the body is given to be transient and two-dimensional. **2** Heat is generated uniformly in the body. **3** The heat transfer coefficient also includes the radiation effects.

Properties The conductivity and diffusivity are given to be k = 28 W/m·°C and $\alpha = 12\times10^{-6}\ \text{m}^2/\text{s}$.

Analysis The nodal spacing is given to be Δx=Δx=l=0.1 m. The explicit finite difference equations are determined on the basis of the energy balance for the transient case expressed as



$$\sum_{\text{All sides}} \dot{Q}^i + \dot{E}^{\mathrm{i}}_{\text{element}} = \rho V_{\text{element}} c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

The quantities h, T_∞, and $\dot{e}_0$ do not change with time, and thus we do not need to use the superscript i for them. The general explicit finite difference form of an interior node for transient two-dimensional heat conduction is expressed as

$$T_{\text{node}}^{i+1} = \tau(T_{\text{left}}^{\mathrm{i}} + T_{\text{top}}^{\mathrm{i}} + T_{\text{right}}^{\mathrm{i}} + T_{\text{bottom}}^{\mathrm{i}}) + (1-4\tau)T_{\text{node}}^{\mathrm{i}} + \tau \frac{\dot{e}_{\text{node}}^{\mathrm{i}} l^2}{k}$$

There is symmetry about the vertical, horizontal, and diagonal lines passing through the center. Therefore, $T_1 = T_3 = T_7 = T_9$ and $T_2 = T_4 = T_6 = T_8$, and T_1, T_2, and T_5 are the only 3 unknown nodal temperatures, and thus we need only 3 equations to determine them uniquely. Also, we can replace the symmetry lines by insulation and utilize the mirror-image concept when writing the finite difference equations for the interior nodes. The finite difference equations for boundary nodes are obtained by applying an energy balance on the volume elements and taking the direction of all heat transfers to be towards the node under consideration:

Node 1: $$hl(T_\infty - T_1^i) + k\frac{l}{2}\frac{T_2^i - T_1^i}{l} + k\frac{l}{2}\frac{T_4^i - T_1^i}{l} + \dot{e}_0\frac{l^2}{4} = \rho\frac{l^2}{4}c\frac{T_1^{i+1} - T_1^i}{\Delta t}$$

Node 2: $$h\frac{l}{2}(T_\infty - T_2^i) + k\frac{l}{2}\frac{T_1^i - T_2^i}{l} + k\frac{l}{2}\frac{T_5^i - T_2^i}{l} + \dot{e}_0\frac{l^2}{4} = \rho\frac{l^2}{4}c\frac{T_2^{i+1} - T_2^i}{\Delta t}$$

Node 5 (interior): $$T_5^{i+1} = (1-4\tau)T_5^i + \tau\left(4T_2^i + \frac{\dot{e}_0 l^2}{k}\right)$$

where $\dot{e}_0 = 8\times10^5\ \text{W/m}^3$, l = 0.1 m, and k = 28 W/m·°C, h = 45 W/m²·°C, and T_∞ = 30°C.

The upper limit of the time step Δt is determined from the stability criteria that requires the coefficient of T_m^i in the T_m^{i+1} expression (the primary coefficient) be greater than or equal to zero for all nodes. The smallest primary coefficient in the 3 equations above is the coefficient of T_1^i in the T_1^{i+1} expression since it is exposed to most convection per unit volume (this can be verified), and thus the stability criteria for this problem can be expressed as

$$1 - 4\tau - 4\tau\frac{hl}{k} \ge 0 \quad \rightarrow \quad \tau \le \frac{1}{4(1+hl/k)} \quad \rightarrow \quad \Delta t \le \frac{l^2}{4\alpha(1+hl/k)}$$

since $\tau = \alpha\Delta t / l^2$. Substituting the given quantities, the maximum allowable value of the time step is determined to be

$$\Delta t \le \frac{(0.1\,\text{m})^2}{4(12\times10^{-6}\ \text{m}^2/\text{s})[1 + (45\ \text{W/m}^2.°\text{C})(0.1\,\text{m})/(28\ \text{W/m.°C})]} = 179\,\text{s}$$

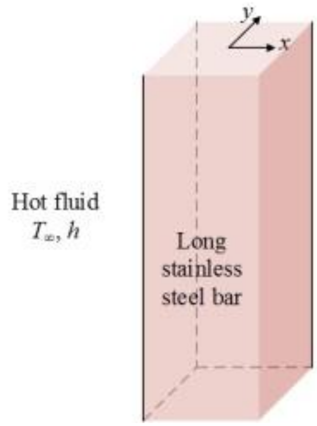
Therefore, any time step less than 179 s can be used to solve this problem. For convenience, we choose the time step to be Δt = 60 s. Then the mesh Fourier number becomes

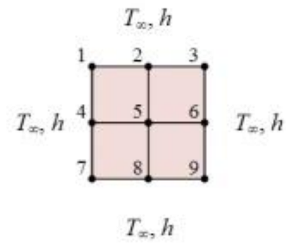
$$\tau = \frac{\alpha\Delta t}{l^2} = \frac{(12\times10^{-6}\ \text{m}^2/\text{s})(60\,\text{s})}{(0.1\,\text{m})^2} = 0.072 \qquad (\text{for } \Delta t = 60\text{ s})$$

Using the specified initial condition as the solution at time t = 0 (for i = 0), sweeping through the 3 equations above will give the solution at intervals of 1 min. Using a computer, the solution at the center node (node 5) is determined to be 227.5°C, 312.0°C, **387.6°C**, 455.1°C, 515.5°C, 617.7°C, 699.3°C, and 764.5°C at 10, 15, 20, 25, 30, 40, 50, and 60 min, respectively. Continuing in this manner, it is observed that steady conditions are reached in the medium after about 6 hours for which the temperature at the center node is **1023°C**.

5-126 C&S ASTM A479 904L bar is submerged is hot fluid at T_∞ = 300°C and h = 288 W/m^2·K. The bar has an initial temperature of 20°C. If the bar is submerged in the hot liquid for 7 minutes, would it be in compliance with the ASME code? How long will it take for the bar to reach the maximum use temperature?

Assumptions **1** The bars have square cross section with infinite length. **2** Thermal properties are constant. **3** Radiation effects are negligible. **4** The convection heat transfer coefficient is constant. **5** Transient heat conduction is two-dimensional.





Properties The thermal properties given are c_p = 500 J/kg·K, k = 12 W/m·K, and ρ = 7900 kg/m^3.

Analysis There is symmetry about the vertical, horizontal, and diagonal lines passing through the center. The symmetry lines are treated as insulation boundary. Therefore, $T_1 = T_3 = T_7 = T_9$ and $T_2 = T_4 = T_6 = T_8$. The three unknown nodal temperatures are T_1, T_2, and T_5. Thus, we need 3 equations to determine them uniquely.

Node 1 is a corner node with convection boundary on both sides,

$$m = 1: \qquad T_1^{i+1} = \left(1 - 4\tau - 4\tau\frac{h\Delta x}{k}\right)T_1^i + 2\tau\left(2T_2^i + 2\frac{h\Delta x}{k}T_\infty\right)$$

Node 2 is a convection boundary,

$$m = 2: \qquad T_2^{i+1} = \left(1 - 4\tau - 2\tau\frac{h\Delta x}{k}\right)T_2^i + \tau\left(2T_1^i + 2T_5^i + 2\frac{h\Delta x}{k}T_\infty\right)$$

Node 5 is an interior node,

$$m = 5: \qquad T_5^{i+1} = (1 - 4\tau)T_5^i + \tau\left(4T_2^i\right)$$

The upper limit of the time step Δt is determined from the stability criteria that requires all primary coefficients to be greater than or equal to zero. The coefficient T_1^i is smaller in this case, and thus the stability criterion for this problem can be expressed as

$$1 - 4\tau - 4\tau\frac{h\Delta x}{k} \geq 0 \;\rightarrow \tau \leq \frac{1}{4(1 + h\Delta x/k)} \text{ or } \;\; \Delta t \leq \frac{\Delta x^2}{4\alpha(1 + h\Delta x/k)} = 32.15 \text{ s} \quad \Longrightarrow \quad \Delta t = 10 \text{ s}$$

where,

$$c_p = 500 \text{ J/kg·K}, \;\; k = 12 \text{ W/m·K}, \;\; \rho = 7900 \text{ kg/m}^3, \;\; \Delta x = 2.5 \text{ cm}$$

$$h = 288 \text{ W/m}^2\text{·K}, \;\; T_\infty = 300°\text{C}$$

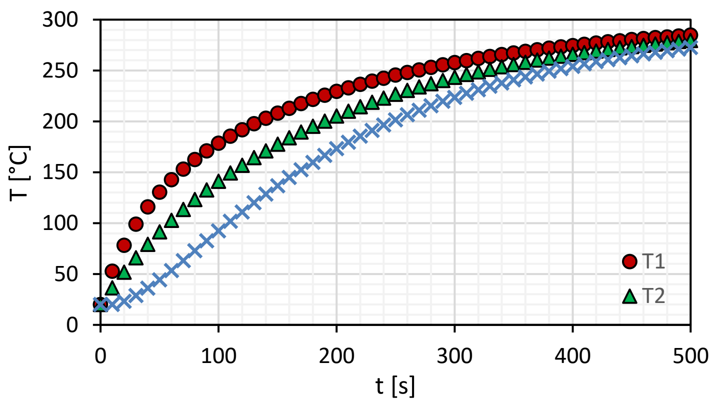
$$\alpha = \frac{k}{\rho c_p} = \frac{12}{(7900)(500)} = 3.038 \times 10^{-6} \text{m}^2\text{/s}$$

$$\tau = \frac{\alpha \Delta t}{\Delta x^2} = \frac{(3.038 \times 10^{-6})(10)}{(0.025)^2} = 0.04861$$

The numerical results can be solved iteratively with the initial nodal temperatures, T_i = 20°C, and time step, Δt = 10 s. The following table tabulates the iterated nodal temperatures at every minute:

t [s]	T_1 [°C]	T_2 [°C]	T_5 [°C]
0	20.0	20.0	20.0
60	142.6	102.9	53.5
120	191.8	157.0	111.1
180	221.7	195.1	159.6
240	242.6	222.9	196.4
300	257.8	243.2	223.7
360	268.9	258.2	243.8
420	**277.1**	**269.2**	**258.7**
480	283.1	277.4	269.6

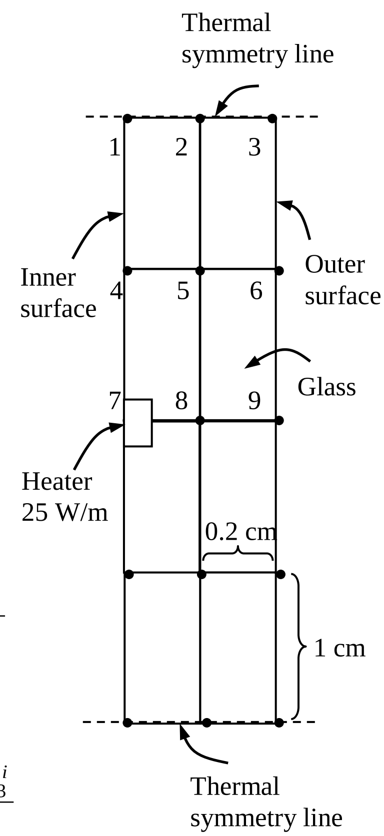
The nodal temperatures, as a function of t, are plotted in the following figure:



Discussion If the stainless steel bar is submerged in the hot liquid for 7 minutes, then it would not comply with the ASME Code for Process Piping. At that duration, the surface temperatures of the bar (T_1 and T_2) would exceed the maximum use temperature of 260°C. At t = 7 minutes, the temperature at the center of the bar (T_5) is 1.3°C from the maximum use temperature. To keep the bar from exceeding the maximum use temperature of 260°C, the bar cannot be submerged in the hot fluid for more than 310 seconds. At t = 310 s, T_1 = 259.9°C, T_2 = 246.1°C, and T_5 = 227.5°C.

5-127 The formation of fog on the glass surfaces of a car is to be prevented by attaching electric resistance heaters to the inner surfaces. The temperature distribution throughout the glass 15 min after the strip heaters are turned on and also when steady conditions are reached are to be determined using the explicit method.

Assumptions **1** Heat transfer through the glass is given to be transient and two-dimensional. **2** Thermal conductivity is constant. **3** There is heat generation only at the inner surface, which will be treated as prescribed heat flux.



Properties The conductivity and diffusivity are given to be k = 0.84 W/m·°C and $\alpha = 0.39\times10^{-6}$ m^2/s .

Analysis The nodal spacing is given to be Δx = 0.2 cm and Δy = 1 cm. The explicit finite difference equations are determined on the basis of the energy balance for the transient case expressed as

$$\sum_{\text{All sides}} \dot{Q}^i + \dot{E}^{\,i}_{\text{gen,element}} = \rho V_{\text{element}} c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

We consider only 9 nodes because of symmetry. Note that we do not have a square mesh in this case, and thus we will have to rely on energy balances to obtain the finite difference equations. Using energy balances, the finite difference equations for each of the 9 nodes are obtained as follows:

Node 1: $$h_i \frac{\Delta y}{2}(T_i - T_1^i) + k\frac{\Delta x}{2}\frac{T_4^i - T_1^i}{\Delta y} + k\frac{\Delta y}{2}\frac{T_2^i - T_1^i}{\Delta x} = \rho c_p \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_1^{i+1} - T_1^i}{\Delta t}$$

Node 2: $$k\frac{\Delta y}{2}\frac{T_1^i - T_2^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_3^i - T_2^i}{\Delta x} + k\Delta x\frac{T_5^i - T_2^i}{\Delta y} = \rho c_p \Delta x\frac{\Delta y}{2}\frac{T_2^{i+1} - T_2^i}{\Delta t}$$

Node 3: $$h_o \frac{\Delta y}{2}(T_o - T_3^i) + k\frac{\Delta x}{2}\frac{T_6^i - T_3^i}{\Delta y} + k\frac{\Delta y}{2}\frac{T_2^i - T_3^i}{\Delta x} = \rho c_p \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_3^{i+1} - T_3^i}{\Delta t}$$

Node 4: $$h_i \Delta y(T_i - T_4^i) + k\frac{\Delta x}{2}\frac{T_1^i - T_4^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_7^i - T_4^i}{\Delta y} + k\Delta y\frac{T_5^i - T_4^i}{\Delta x} = \rho c_p \Delta y\frac{\Delta x}{2}\frac{T_4^{i+1} - T_4^i}{\Delta t}$$

Node 5: $$k\Delta y\frac{T_4^i - T_5^i}{\Delta x} + k\Delta y\frac{T_6^i - T_5^i}{\Delta x} + k\Delta x\frac{T_8^i - T_5^i}{\Delta y} + k\Delta x\frac{T_2^i - T_5^i}{\Delta y} = \rho c_p \Delta x\Delta y\frac{T_5^{i+1} - T_5^i}{\Delta t}$$

Node 6: $$h_o \Delta y(T_i - T_6^i) + k\frac{\Delta x}{2}\frac{T_3^i - T_6^i}{\Delta y} + k\frac{\Delta x}{2}\frac{T_9^i - T_6^i}{\Delta y} + k\Delta y\frac{T_5^i - T_6^i}{\Delta x} = \rho c_p \Delta y\frac{\Delta x}{2}\frac{T_6^{i+1} - T_6^i}{\Delta t}$$

Node 7: $$12.5\text{ W} + h_i \frac{\Delta y}{2}(T_i - T_7^i) + k\frac{\Delta x}{2}\frac{T_4^i - T_7^i}{\Delta y} + k\frac{\Delta y}{2}\frac{T_8^i - T_7^i}{\Delta x} = \rho c_p \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_7^{i+1} - T_7^i}{\Delta t}$$

Node 8: $$k\frac{\Delta y}{2}\frac{T_7^i - T_8^i}{\Delta x} + k\frac{\Delta y}{2}\frac{T_9^i - T_8^i}{\Delta x} + k\Delta x\frac{T_5^i - T_8^i}{\Delta y} = \rho c_p \Delta x\frac{\Delta y}{2}\frac{T_8^{i+1} - T_8^i}{\Delta t}$$

Node 9: $$h_o \frac{\Delta y}{2}(T_o - T_9^i) + k\frac{\Delta x}{2}\frac{T_6^i - T_9^i}{\Delta y} + k\frac{\Delta y}{2}\frac{T_8^i - T_9^i}{\Delta x} = \rho c_p \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_9^{i+1} - T_9^i}{\Delta t}$$

where

k = 0.84 W/m.°C, $\alpha = k/\rho c = 0.39\times10^{-6}$ m^2/s , $T_i = T_o$ = -3°C h_i = 6 W/m^2.°C,

h_o = 20 W/m^2.°C, Δx = 0.002 m, and Δy = 0.01 m.

The upper limit of the time step Δt is determined from the stability criteria that requires the coefficient of T_m^i in the T_m^{i+1} expression (the primary coefficient) be greater than or equal to zero for all nodes. The smallest primary coefficient in the 9 equations above is the coefficient of T_6^i in the T_6^{i+1} expression since it is exposed to most convection per unit volume (this can be verified). The equation for node 6 can be rearranged as

$$T_6^{i+1} = \left[1 - 2\alpha\Delta t\left(\frac{h_o}{k\Delta x} + \frac{1}{\Delta y^2} + \frac{1}{\Delta x^2}\right)\right]T_6^i + 2\alpha\Delta t\left(\frac{h_o}{k\Delta x}T_0 + \frac{T_3^i + T_9^i}{\Delta y^2} + \frac{T_5^i}{\Delta x^2}\right)$$

Therefore, the stability criteria for this problem can be expressed as

$$1 - 2\alpha\Delta t\left(\frac{h_o}{k\Delta x} + \frac{1}{\Delta y^2} + \frac{1}{\Delta x^2}\right) \ge 0 \;\rightarrow\; \Delta \mathrm{t} \le \frac{1}{2\alpha\left(\dfrac{h_o}{k\Delta x} + \dfrac{1}{\Delta y^2} + \dfrac{1}{\Delta x^2}\right)}$$

Substituting the given quantities, the maximum allowable value of the time step is determined to be

or,

$$\Delta \mathrm{t} \le \frac{1}{2\times(0.39\times10^6\ \mathrm{m^2}/s)\left(\dfrac{20\ \mathrm{W/m^2\cdot{}^\circ C}}{(0.84\ \mathrm{W/m\cdot{}^\circ C})(0.002\ \mathrm{m})} + \dfrac{1}{(0.002\ \mathrm{m})^2} + \dfrac{1}{(0.01\,\mathrm{m})^2}\right)} = 4.7\ \mathrm{s}$$

Therefore, any time step less than 4.8 s can be used to solve this problem. For convenience, we choose the time step to be Δt = 4 s. Then the temperature distribution throughout the glass 15 min after the strip heaters are turned on and when steady conditions are reached are determined to be

15 min: T_1 = 10.2°C, T_2 = 10.2°C, T_3 = 9.8°C, T_4 = 16.0°C, T_5 = 15.9°C,
T_6 = 15.3°C, T_7 = 41.1°C, T_8 = 36.8°C, T_9 = 34.3°C

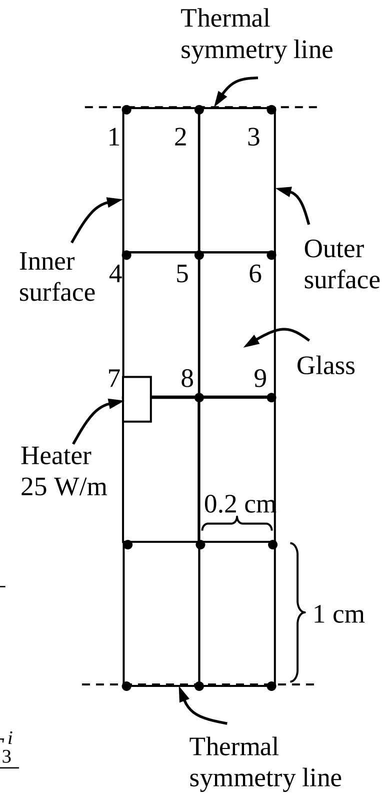
Steady-state: T_1 = 11.8°C, T_2 = 11.7°C, T_3 = 11.3°C, T_4 = 17.6°C, T_5 = 17.5°C,
T_6 = 16.8°C, T_7 = 42.7°C, T_8 = 38.4°C, T_9 = 35.8°C

Discussion Steady operating conditions are reached in about 20 min.

5-128 The formation of fog on the glass surfaces of a car is to be prevented by attaching electric resistance heaters to the inner surfaces. The temperature distribution throughout the glass 15 min after the strip heaters are turned on and also when steady conditions are reached are to be determined using the implicit method with a time step of Δt = 1 min.

Assumptions **1** Heat transfer through the glass is given to be transient and two-dimensional. **2** Thermal conductivity is constant. **3** There is heat generation only at the inner surface, which will be treated as prescribed heat flux.

Properties The conductivity and diffusivity are given to be k = 0.84 W/m·°C and $\alpha = 0.39\times10^{-6}$ m^2/s .



Analysis The nodal spacing is given to be Δx = 0.2 cm and Δy = 1 cm. The implicit finite difference equations are determined on the basis of the energy balance for the transient case expressed as

$$\sum_{\text{All sides}} \dot{Q}^{i+1} + \dot{E}^{\,i+1}_{\text{gen,element}} = \rho V_{\text{element}} c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

We consider only 9 nodes because of symmetry. Note that we do not have a square mesh in this case, and thus we will have to rely on energy balances to obtain the finite difference equations. Using energy balances, the finite difference equations for each of the 9 nodes are obtained as follows:

Node 1: $h_i \frac{\Delta y}{2}(T_i - T_1^{i+1}) + k\frac{\Delta x}{2}\frac{T_4^{i+1} - T_1^{i+1}}{\Delta y} + k\frac{\Delta y}{2}\frac{T_2^{i+1} - T_1^{i+1}}{\Delta x} = \rho c_p \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_1^{i+1} - T_1^i}{\Delta t}$

Node 2: $k\frac{\Delta y}{2}\frac{T_1^{i+1} - T_2^{i+1}}{\Delta x} + k\frac{\Delta y}{2}\frac{T_3^{i+1} - T_2^{i+1}}{\Delta x} + k\Delta x\frac{T_5^{i+1} - T_2^{i+1}}{\Delta y} = \rho c_p \Delta x\frac{\Delta y}{2}\frac{T_2^{i+1} - T_2^i}{\Delta t}$

Node 3: $h_o \frac{\Delta y}{2}(T_o - T_3^{i+1}) + k\frac{\Delta x}{2}\frac{T_6^{i+1} - T_3^{i+1}}{\Delta y} + k\frac{\Delta y}{2}\frac{T_2^{i+1} - T_3^{i+1}}{\Delta x} = \rho c_p \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_3^{i+1} - T_3^i}{\Delta t}$

Node 4: $h_i \Delta y(T_i - T_4^{i+1}) + k\frac{\Delta x}{2}\frac{T_1^{i+1} - T_4^{i+1}}{\Delta y} + k\frac{\Delta x}{2}\frac{T_7^{i+1} - T_4^{i+1}}{\Delta y} + k\Delta y\frac{T_5^{i+1} - T_4^{i+1}}{\Delta x} = \rho c_p \Delta y\frac{\Delta x}{2}\frac{T_4^{i+1} - T_4^i}{\Delta t}$

Node 5: $k\Delta y\frac{T_4^{i+1} - T_5^{i+1}}{\Delta x} + k\Delta y\frac{T_6^{i+1} - T_5^{i+1}}{\Delta x} + k\Delta x\frac{T_8^{i+1} - T_5^{i+1}}{\Delta y} + k\Delta x\frac{T_2^{i+1} - T_5^{i+1}}{\Delta y} = \rho c_p \Delta x\Delta y\frac{T_5^{i+1} - T_5^i}{\Delta t}$

Node 6: $h_o \Delta y(T_o - T_6^{i+1}) + k\frac{\Delta x}{2}\frac{T_3^{i+1} - T_6^{i+1}}{\Delta y} + k\frac{\Delta x}{2}\frac{T_9^{i+1} - T_6^{i+1}}{\Delta y} + k\Delta y\frac{T_5^{i+1} - T_6^{i+1}}{\Delta x} = \rho c_p \Delta y\frac{\Delta x}{2}\frac{T_6^{i+1} - T_6^i}{\Delta t}$

Node 7: $12.5\text{ W} + h_i \frac{\Delta y}{2}(T_i - T_7^{i+1}) + k\frac{\Delta x}{2}\frac{T_4^{i+1} - T_7^{i+1}}{\Delta y} + k\frac{\Delta y}{2}\frac{T_8^{i+1} - T_7^{i+1}}{\Delta x} = \rho c_p \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_7^{i+1} - T_7^i}{\Delta t}$

Node 8: $k\frac{\Delta y}{2}\frac{T_7^{i+1} - T_8^{i+1}}{\Delta x} + k\frac{\Delta y}{2}\frac{T_9^{i+1} - T_8^{i+1}}{\Delta x} + k\Delta x\frac{T_5^{i+1} - T_8^{i+1}}{\Delta y} = \rho c_p \Delta x\frac{\Delta y}{2}\frac{T_8^{i+1} - T_8^i}{\Delta t}$

Node 9: $h_o \frac{\Delta y}{2}(T_o - T_9^{i+1}) + k\frac{\Delta x}{2}\frac{T_6^{i+1} - T_9^{i+1}}{\Delta y} + k\frac{\Delta y}{2}\frac{T_8^{i+1} - T_9^{i+1}}{\Delta x} = \rho c_p \frac{\Delta x}{2}\frac{\Delta y}{2}\frac{T_9^{i+1} - T_9^i}{\Delta t}$

where k = 0.84 W/m.°C, $\alpha = k/\rho c_p = 0.39\times10^{-6}$ m^2/s , $T_i = T_o$ = -3°C h_i = 6 W/m^2.°C, h_o = 20 W/m^2.°C, Δx = 0.002 m, and Δy = 0.01 m. Taking time step to be Δt = 1 min, the temperature distribution throughout the glass 15 min after the strip heaters are turned on and when steady conditions are reached are determined to be

15 min: T_1 = 10.2°C, T_2 = 10.2°C, T_3 = 9.8°C, T_4 = 16.0°C, T_5 = 15.9°C, T_6 = 15.3°C, T_7 = 41.1°C, T_8 = 36.8°C, T_9 = 34.3°C

Steady-state: T_1 = 11.8°C, T_2 = 11.7°C, T_3 = 11.3°C, T_4 = 17.6°C, T_5 = 17.5°C, T_6 = 16.8°C, T_7 = 42.7°C, T_8 = 38.4°C, T_9 = 35.8°C

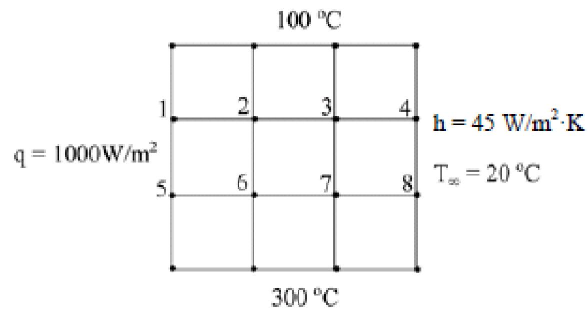
Discussion Steady operating conditions are reached in about 20 min.

5-129 Square cross section geometry is subjected to four different boundary conditions. The transient temperature distribution within the geometry is to be determined after 15 seconds using the explicit finite difference formulation.

Assumptions **1** Two-dimensional transient heat conduction without internal heat generation. **2** Thermal properties are constant.

Properties Thermal conductivity is given as k=20 W/m·K and the thermal diffusivity value is $\alpha = 6.694 \times 10^{-6}$ m^2/s.

Analysis: (a) There are 4 internal nodes (node 2, 3, 6 and 7) and 4 boundary nodes (node 1, 4, 5 and 8). Thus we need to have 8 equations for 8 unknown temperatures. Finite difference equations for internal nodes can be expressed directly using Eq. (5-60) while for the boundary nodes energy balance is carried out around the node volume.



The explicit finite difference formulations for different nodes are as follows.

Node 1: (Left boundary node)

$$\dot{q}\Delta y + k\frac{\Delta x}{2}\frac{(T_5^i - T_1^i)}{\Delta y} + k\frac{\Delta x}{2}\frac{(100 - T_1^i)}{\Delta y} + k\Delta y\frac{(T_2^i - T_1^i)}{\Delta x} = \rho c_p \Delta y\frac{\Delta x}{2}\frac{(T_1^{i+1} - T_1^i)}{\Delta t}$$

$$\Rightarrow T_1^{i+1} = (1-4\tau)T_1^i + \tau\left(100 + T_5^i + 2T_2^i + \frac{2\dot{q}\Delta y}{k}\right)$$

Node 2: (Interior node) $T_2^{i+1} = (1-4\tau)T_2^i + \tau(100 + T_6^i + T_1^i + T_3^i)$

Node 3: (Interior node) $T_3^{i+1} = (1-4\tau)T_3^i + \tau(100 + T_2^i + T_4^i + T_7^i)$

Node 4: (Right boundary node)

$$k\frac{\Delta x}{2}\frac{(100 - T_4^i)}{\Delta y} + k\frac{\Delta x}{2}\frac{(T_8^i - T_4^i)}{\Delta y} + k\Delta y\frac{(T_3^i - T_4^i)}{\Delta x} + h\Delta y(T_\infty - T_4^i) = \rho c_p \Delta y\frac{\Delta x}{2}\frac{(T_4^{i+1} - T_4^i)}{\Delta t}$$

$$\Rightarrow T_4^{i+1} = \left(1 - 4\tau - \frac{2\tau\Delta y h}{k}\right)T_4^i + \tau\left(100 + T_8^i + 2T_3^i + \frac{2h\Delta y}{k}T_\infty\right)$$

Node 5: (Left boundary node)

$$\dot{q}\Delta y + k\frac{\Delta x}{2}\frac{(T_1^i - T_5^i)}{\Delta y} + k\frac{\Delta x}{2}\frac{(300 - T_5^i)}{\Delta y} + k\Delta y\frac{(T_{62}^i - T_1^i)}{\Delta x} = \rho c_p \Delta y\frac{\Delta x}{2}\frac{(T_5^{i+1} - T_5^i)}{\Delta t}$$

$$\Rightarrow T_5^{i+1} = (1-4\tau)T_5^i + \tau\left(300 + T_1^i + 2T_6^i + \frac{2\dot{q}\Delta y}{k}\right)$$

Node 6: (Interior node) $T_6^{i+1} = (1-4\tau)T_6^i + \tau(300 + T_5^i + T_2^i + T_7^i)$

Node 7: (Interior node) $T_7^{i+1} = (1-4\tau)T_7^i + \tau(300 + T_6^i + T_3^i + T_8^i)$

Node 8: (Right boundary node)

$$k\frac{\Delta x}{2}\frac{(300 - T_8^i)}{\Delta y} + k\frac{\Delta x}{2}\frac{(T_4^i - T_8^i)}{\Delta y} + k\Delta y\frac{(T_7^i - T_8^i)}{\Delta x} + h\Delta y(T_\infty - T_8^i) = \rho c_p \Delta y\frac{\Delta x}{2}\frac{(T_8^{i+1} - T_8^i)}{\Delta t}$$

$$\Rightarrow T_8^{i+1} = \left(1 - 4\tau - \frac{2\tau\Delta y h}{k}\right)T_8^i + \tau\left(300 + T_4^i + 2T_7^i + \frac{2h\Delta y}{k}T_\infty\right)$$

where $\Delta x = \Delta y = 0.01m$, $\mathrm{h} = 45(\mathrm{W/m^2 \cdot K})$, $\mathrm{k} = 20(\mathrm{W/m \cdot K})$ and $T_\infty = 20^o C$

Next we need to determine the upper limit of the time step from the stability criterion which requires the primary coefficient of T_m^i in the expression of T_m^{i+1} to be greater than or equal to zero at all nodes. The smallest primary coefficient is for node at right boundary exposed to convective environment. Thus using the stability criterion we get

$$\tau \le \frac{1}{2(2 + h\Delta y/k)} \Rightarrow \Delta t \le \frac{\Delta y^2}{2\alpha(2 + h\Delta y/k)}$$

Thus the maximum allowable time step is

$$\Delta t \le \frac{0.01^2 (\mathrm{m}^2)}{2 \times 6.694 \times 10^{-6} (\mathrm{m}^2/\mathrm{s}) \left(2 + 45 (\mathrm{W/m^2 \cdot K}) \times 0.01 (\mathrm{m}) / 20 (\mathrm{W/m \cdot K})\right)} \le 3.69 (\mathrm{s})$$

For convenience let's select a time step of $\Delta t = 3s$ .This gives a mesh Fourier number of

$$\tau = \frac{\alpha \Delta t}{\Delta y^2} = 0.20082$$

(b) The temperature distribution is obtained by running the following EES code.

```
"Given data"
k = 20 [W/mC] " Thermal conductivity"
h = 45 [W/m^2C] " Convective heat transfer coefficient"
T_infi = 20 [C] " Convective environment temperature"
q_dot = 1000 [W/m^2] " Heat flux at left boundary"
DELTAt = 3 [s] "Time step"
DELTAy = 0.01 [m] " Mesh size"
tau = 0.20082 "Mesh Fourier number"
"The technique is to store the temperatures in the parametric table and recover them (as old temperatures) using the variable ROW.  The first row contains the initial values so Solve Table must begin at row 2. Use the DUPLICATE statement to reduce the number of equations that need to be typed.  Column 1 contains the time, column 2 the value of T[1], column 3, the value of T[2], etc., and column 9 the Row.To start the Solve Table at 2 go to 'Calculate' and select 'Solve table' (or hit F3) and make the 'First Run Number' as 2. The initial temperatures and the initial time '0' can be set manually in the parametric table"

Row = TableRun#
Time = TableValue('Table 1',Row-1,#Time)+DELTAt
        Duplicate i=1,8
                T_old[i] = TableValue('Table 1',Row-1,#T[i])
         End
"Finite difference Equation"
"Node 1" T[1]=(1-4*tau)*T_old[1]+tau*(100+T_old[5]+2*T_old[2]+2*q_dot*DELTAy/k)
"Node 2" T[2] = (1-4*tau)*T_old[2]+tau*(100+T_old[6]+T_old[1]+T_old[3])
"Node 3" T[3] = (1-4*tau)*T_old[3]+tau*(100+T_old[2]+T_old[4]+T_old[7])
"Node4"T[4] = (1-4*tau-2*tau*h*DELTAy/k)*T_old[4]+tau*(100+T_old[8]+2*T_old[3]+2*h*DELTAy*T_infi/k)
"Node 5" T[5] = (1-4*tau)*T_old[5]+tau*(300+T_old[1]+2*T_old[6]+2*q_dot*DELTAy/k)
"Node 6" T[6] = (1-4*tau)*T_old[6]+tau*(300+T_old[5]+T_old[2]+T_old[7])
"Node 7" T[7] = (1-4*tau)*T_old[7]+tau*(300+T_old[6]+T_old[3]+T_old[8])
"Node8"T[8] = (1-4*tau-tau*2*h*DELTAy/k)*T_old[8]+tau*(300+T_old[4]+2*T_old[7]+2*h*DELTAy*T_infi/k)
```

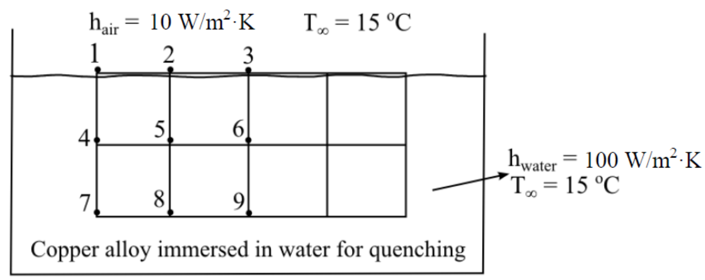
Temperature distribution at different time steps.

	Nodal temperature, ºC							
Time (s)	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
0	200.0	200.0	200.0	200.0	200.0	200.0	200.0	200.0
3	180.0	179.8	179.8	178.2	220.1	219.9	219.9	218.3
6	172.0	171.7	171.4	169.6	228.1	227.8	227.5	225.4
9	168.8	168.4	167.9	166.1	231.2	230.9	230.3	228.0
12	167.4	167.1	166.4	164.5	232.4	232.0	231.2	228.9
15	**166.9**	**166.5**	**165.7**	**163.8**	**232.8**	**232.4**	**231.5**	**229.1**

5-130 A copper alloy with known thermal properties and initially heated to 800°C is subjected to quenching in water with top surface exposed to air environment. The temperature distribution in the copper alloy after 10 min is to be determined using explicit finite difference formulation.

Assumptions **1** Two-dimensional transient heat transfer without internal heat generation. 2 Constant thermal properties 3 The top surface of the copper alloy is always in contact with the air environment.

Properties Thermal conductivity is given as k = 120 W/m·K and the thermal diffusivity value is $\alpha = 3.91 \times 10^{-6}$ m²/s.



Analysis The nodes 1, 2 and 3 are exposed to air environment while nodes 4, 7, 8 and 9 are in contact with the water environment. Since there is geometrical symmetry we need to find the temperatures for only the nodes shown in figure. Node 5 can be treated as an internal node and for all other nodes, the temperature is found using energy balance at the node volume. The explicit finite difference formulation at different nodes is as follows

Node 1: $$\left(h_1\frac{\Delta x}{2}+h_2\frac{\Delta y}{2}\right)\left(T_\infty-T_1^i\right)+k\frac{\Delta x}{2}\frac{\left(T_4^i-T_1^i\right)}{\Delta y}+k\frac{\Delta y}{2}\frac{\left(T_2^i-T_1^i\right)}{\Delta x}=\rho\frac{\Delta x}{2}\frac{\Delta y}{2}c_p\frac{\left(T_1^{i+1}-T_1^i\right)}{\Delta t}$$

$$\Rightarrow T_1^{i+1}=\left(1-4\tau-2\tau\frac{(h_1+h_2)l}{k}\right)T_1^i+2\tau\left(T_4^i+T_2^i+\frac{l}{k}(h_1+h_2)T_\infty\right)$$

Node 2: $$h_1\Delta x\left(T_\infty-T_2^i\right)+k\frac{\Delta y}{2}\frac{\left(T_1^i-T_2^i\right)}{\Delta x}+k\frac{\Delta y}{2}\frac{\left(T_3^i-T_2^i\right)}{\Delta x}+k\Delta x\frac{\left(T_5^i-T_2^i\right)}{\Delta y}=\rho\Delta x\frac{\Delta y}{2}c_p\frac{\left(T_2^{i+1}-T_2^i\right)}{\Delta t}$$

$$\Rightarrow T_2^{i+1}=\left(1-4\tau-2\tau\frac{h_1 l}{k}\right)T_2^i+\tau\left(T_1^i+T_3^i+2T_5^i+2\frac{h_1 l}{k}T_\infty\right)$$

Node 3: $$h_1\Delta x\left(T_\infty-T_3^i\right)+2k\frac{\Delta y}{2}\frac{\left(T_2^i-T_3^i\right)}{\Delta x}+k\Delta x\frac{\left(T_6^i-T_3^i\right)}{\Delta y}=\rho\Delta x\frac{\Delta y}{2}c_p\frac{\left(T_3^{i+1}-T_3^i\right)}{\Delta t}$$

$$\Rightarrow T_3^{i+1}=\left(1-4\tau-2\tau\frac{h_1 l}{k}\right)T_3^i+\tau\left(2T_2^i+2T_6^i+2\frac{h_1 l}{k}T_\infty\right)$$

Node 4: $$h_2\Delta y\left(T_\infty-T_4^i\right)+k\frac{\Delta x}{2}\frac{\left(T_1^i-T_4^i\right)}{\Delta y}+k\frac{\Delta x}{2}\frac{\left(T_7^i-T_4^i\right)}{\Delta y}+k\Delta y\frac{\left(T_5^i-T_4^i\right)}{\Delta x}=\rho\Delta y\frac{\Delta x}{2}c_p\frac{\left(T_4^{i+1}-T_4^i\right)}{\Delta t}$$

$$\Rightarrow T_4^{i+1}=\left(1-4\tau-2\tau\frac{h_2 l}{k}\right)T_4^i+\tau\left(T_1^i+T_7^i+2T_5^i+2\frac{h_2 l}{k}T_\infty\right)$$

Node 5: $$T_4^i+T_2^i+T_6^i+T_8^i-4T_5^i=\rho\Delta x\Delta y c_p\frac{\left(T_5^{i+1}-T_5^i\right)}{\Delta t}$$

$$\Rightarrow T_5^{i+1}=(1-4\tau)T_5^i+\tau\left(T_2^i+T_8^i+T_6^i+T_4^i\right)$$

Node 6: $$k\Delta x\frac{\left(T_9^i-T_6^i\right)}{\Delta y}+2k\Delta y\frac{\left(T_5^i-T_6^i\right)}{\Delta x}+k\Delta x\frac{\left(T_3^i-T_6^i\right)}{\Delta y}=\rho\Delta x\Delta y c_p\frac{\left(T_6^{i+1}-T_6^i\right)}{\Delta t}$$

$$\Rightarrow T_6^{i+1}=(1-4\tau)T_6^i+\tau\left(2T_5^i+T_3^i+T_9^i\right)$$

Node 7: $$h_2\left(\frac{\Delta y}{2}+\frac{\Delta x}{2}\right)\left(T_\infty-T_7^i\right)+k\frac{\Delta x}{2}\frac{\left(T_4^i-T_7^i\right)}{\Delta y}+k\frac{\Delta y}{2}\frac{\left(T_8^i-T_7^i\right)}{\Delta y}=\rho\frac{\Delta y}{2}\frac{\Delta x}{2}c_p\frac{\left(T_7^{i+1}-T_7^i\right)}{\Delta t}$$

$$\Rightarrow T_7^{i+1}=\left(1-4\tau-4\tau\frac{h_2 l}{k}\right)T_7^i+2\tau\left(T_4^i+T_8^i+2\frac{h_2 l}{k}T_\infty\right)$$

Node 8: $$h_2\Delta x\left(T_\infty-T_8^i\right)+k\frac{\Delta y}{2}\frac{\left(T_7^i-T_8^i\right)}{\Delta x}+k\frac{\Delta y}{2}\frac{\left(T_9^i-T_8^i\right)}{\Delta x}+k\Delta x\frac{\left(T_5^i-T_8^i\right)}{\Delta y}=\rho\frac{\Delta y}{2}\Delta x c_p\frac{\left(T_8^{i+1}-T_8^i\right)}{\Delta t}$$

$$\Rightarrow T_8^{i+1} = \left(1 - 4\tau - 2\tau\frac{h_2 l}{k}\right)T_8^i + \tau\left(2T_5^i + T_7^i + T_9^i + 2\frac{h_2 l}{k}T_\infty\right)$$

Node 9: $h_2\Delta x(T_\infty - T_9^i) + 2k\frac{\Delta y}{2}\frac{(T_8^i - T_9^i)}{\Delta x} + k\Delta x\frac{(T_6^i - T_9^i)}{\Delta y} = \rho\frac{\Delta y}{2}\Delta x c_p\frac{(T_9^{i+1} - T_9^i)}{\Delta t}$

$$\Rightarrow T_9^{i+1} = \left(1 - 4\tau - 2\tau\frac{h_2 l}{k}\right)T_9^i + \tau\left(2T_6^i + 2T_8^i + 2\frac{h_2 l}{k}T_\infty\right)$$

The unknown nodal temperatures are found by running the following EES program.

```
"Given"
l = 0.1 [m] "mesh size"
alpha = 3.91e-6 "Thermal diffusivity [m^2/s]"
DELTAt = 10 [s] "Time step"
k = 120 [W/moC] "Thermal conductivity"
h_1 = 10 [W/m^2oC] “Convective heat transfer coefficient of air"
h_2 = 100 [W/m^2oC] “Convective heat transfer coefficient of water"
T_infi = 15 [C] " Ambient temperature"

tau = (alpha*DELTAt)/l^2 "Mesh Fourier number"

"The technique is to store the temperatures in the parametric table and recover them (as old temperatures) using the variable ROW.  The first row contains the initial values so Solve Table must begin at row 2. Use the DUPLICATE statement to reduce the number of equations that need to be typed.  Column 1 contains the time, column 2 the value of T[1], column 3, the value of T[2], etc., and column 9 the Row.

To start the Solve Table at 2 go to 'Calculate' and select 'Solve table' (or hit F3) and make the 'First Run Number' as 2. The initial temperatures and the initial time ‘0’ can be set manually in the parametric table"

Row = TableRun#
Time = TableValue('Table 1',Row-1,#Time)+DELTAt
        Duplicate i=1,9
                  T_old[i] = TableValue('Table 1',Row-1,#T[i])
         end

"Finite difference formulation"
"Node 1" T[1] = (1-4*tau-2*tau*(h_1+h_2)*l/k)*T_old[1]+2*tau*(T_old[4]+T_old[2]+(h_1+h_2)*l*T_infi/k)
"Node 2" T[2] = (1-4*tau-2*tau*h_1*l/k)*T_old[2]+tau*(T_old[1]+T_old[3]+2*T_old[5]+2*h_1*l*T_infi/k)
"Node 3" T[3] = (1-4*tau-2*tau*h_1*l/k)*T_old[3]+tau*(2*T_old[2]+2*T_old[6]+2*h_1*l*T_infi/k)
"Node 4" T[4] = (1-4*tau-2*tau*h_2*l/k)*T_old[4] + tau*(T_old[1]+T_old[7]+2*T_old[5]+2*h_2*l*T_infi/k)
"Node 5" T[5] = (1-4*tau)*T_old[5]+tau*(T_old[2]+T_old[4]+T_old[6]+T_old[8])
"Node 6" T[6] = (1-4*tau)*T_old[6]+tau*(2*T_old[5]+T_old[3]+T_old[9])
"Node 7" T[7] = (1-4*tau-4*tau*h_2*l/k)*T_old[7]+2*tau*(T_old[4]+T_old[8]+2*h_2*l*T_infi/k)
"Node 8" T[8] = (1-4*tau-2*tau*h_2*l/k)*T_old[8]+tau*(T_old[7]+T_old[9]+2*T_old[5]+2*h_2*l*T_infi/k)
"Node 9" T[9] = (1-4*tau-2*tau*h_2*l/k)*T_old[9]+tau*(2*T_old[8]+2*T_old[6]+2*h_2*l*T_infi/k)
```

Temperature distribution in the copper alloy block after 10 min is as follows,

T_1 = **772.7ºC**, T_2 = **794.5ºC**, T_3 = **796.7ºC**, T_4 = **772.7ºC**, T_5 = **794.5ºC**,

T_6 = **796.7ºC**, T_7 = **751.7ºC**, T_8 = **772.9ºC**, T_9 = **775.1ºC**.

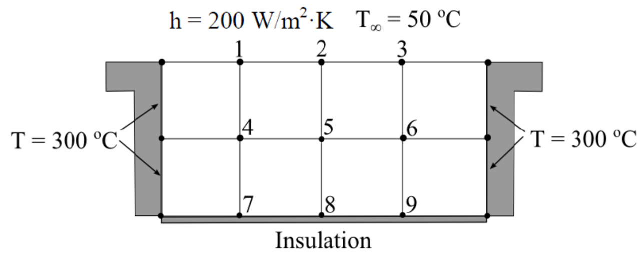
Discussion If the mass and/or heat capacity of the fluid used for quenching is not large enough then the heat released by the metal parts is absorbed by the fluid thus raising its temperature considerably. In such a case constant temperature of the convective environment may not be a good approximation as it will alter the rate of heat transfer by convection. Note that thermal diffusivity of the material dictates the temperature distribution and the quenching time to attain specified temperature. Use of flowing water (forced convective cooling) instead of stagnant water bath may further drop the temperature of copper alloy significantly.

5-131 A ceramic strip at a specified initial temperature is exposed to convective environment at top and constant temperature boundary conditions at its two sides. Using implicit finite difference formulation, the temperature distribution in the ceramic strip is to be determined after 12 seconds.

Assumptions **1** Two-dimensional transient heat conduction without heat generation. **2** Thermal properties of the ceramic strip stays constant.

Properties Thermal properties of ceramic strip are given as: k = 3 W/m·K, ρ = 1600 kg/m^3 and c_p = 800 J/kg·K.

Analysis The finite difference equation at each node is developed by doing an energy balance assuming all heat transfer is to the control volume of the node under consideration. Nodes 1, 2 and 3 are the boundary nodes exposed to the convection environment. Nodes 7, 8 and 9 are the boundary nodes at the insulated surface. The implicit finite difference formulation for all the nodes is as follows



Node 1: $$h\Delta x\left(T_\infty - T_1^{i+1}\right) + k\frac{\Delta y}{2}\frac{\left(300 - T_1^{i+1}\right)}{\Delta x} + k\frac{\Delta y}{2}\frac{\left(T_2^{i+1} - T_1^{i+1}\right)}{\Delta x} + k\Delta x\frac{\left(T_4^{i+1} - T_1^{i+1}\right)}{\Delta y} = \rho c_p \Delta x\frac{\Delta y}{2}\frac{\left(T_1^{i+1} - T_1^{i}\right)}{\Delta t}$$

Node 2: $$h\Delta x\left(T_\infty - T_2^{i+1}\right) + 2k\frac{\Delta y}{2}\frac{\left(T_1^{i+1} - T_2^{i+1}\right)}{\Delta x} + k\Delta x\frac{\left(T_5^{i+1} - T_2^{i+1}\right)}{\Delta y} = \rho c_p \Delta x\frac{\Delta y}{2}\frac{\left(T_2^{i+1} - T_2^{i}\right)}{\Delta t}$$

Node 4: $$k\Delta y\frac{\left(300 - T_4^{i+1}\right)}{\Delta x} + k\Delta y\frac{\left(T_5^{i+1} - T_4^{i+1}\right)}{\Delta x} + k\Delta x\frac{\left(T_1^{i+1} - T_4^{i+1}\right)}{\Delta y} + k\Delta x\frac{\left(T_7^{i+1} - T_4^{i+1}\right)}{\Delta y} = \rho c_p \Delta x\Delta y\frac{\left(T_4^{i+1} - T_4^{i}\right)}{\Delta t}$$

Node 5: $$k\Delta x\frac{\left(T_2^{i+1} - T_5^{i+1}\right)}{\Delta y} + k\Delta x\frac{\left(T_8^{i+1} - T_5^{i+1}\right)}{\Delta y} + 2k\Delta y\frac{\left(T_4^{i+1} - T_5^{i+1}\right)}{\Delta x} = \rho c_p \Delta x\Delta y\frac{\left(T_5^{i+1} - T_5^{i}\right)}{\Delta t}$$

Node 7: $$k\frac{\Delta y}{2}\frac{\left(300 - T_7^{i+1}\right)}{\Delta x} + k\frac{\Delta y}{2}\frac{\left(T_8^{i+1} - T_7^{i+1}\right)}{\Delta x} + 2k\Delta x\frac{\left(T_4^{i+1} - T_7^{i+1}\right)}{\Delta y} = \rho c_p \Delta x\Delta y\frac{\left(T_7^{i+1} - T_7^{i}\right)}{\Delta t}$$

Node 8: $$2k\frac{\Delta y}{2}\frac{\left(T_7^{i+1} - T_8^{i+1}\right)}{\Delta x} + 2k\Delta x\frac{\left(T_5^{i+1} - T_8^{i+1}\right)}{\Delta y} = \rho c_p \Delta x\Delta y\frac{\left(T_8^{i+1} - T_8^{i}\right)}{\Delta t}$$

The unknown temperatures at theses nodes are determined by running the following EES code.

```
"Given data"
k = 3 [W/mC] "Thermal conductivity of ceramic strip"
h = 200 [W/m^2C] "Convective heat transfer coefficient"
T_infi = 50 [C] " Ambient temperature"
DELTAx = 0.01 [m] "Mesh size in x direction"
DELTAy = 0.01 [m] "Mesh size in y direction"
DELTAt = 2 [s] "Time step"
rho = 1600 [kg/m^3] "Density of ceramic strip"
c = 800 [J/kgK] " Specific heat capacity of ceramic strip"
alpha = k/(rho*c)  "Thermal diffusivity of ceramic strip [m^2/s]"
tau = DELTAx^2/(alpha*DELTAt)"Mesh Fourier number"

"The technique is to store the temperatures in the parametric table and recover them (as old temperatures) using the variable ROW.  The first row contains the initial values so Solve Table must begin at row 2. Use the DUPLICATE statement to reduce the number of equations that need to be typed.  Column 1 contains the time, column 2 the value of T[1], column 3, the value of T[2], etc., and column 9 the Row.

To start the Solve Table at 2 go to 'Calculate' and select 'Solve table' (or hit F3) and make the 'First Run Number' as 2. The initial temperatures and the initial time ‘0’ can be set manually in the parametric table"

Row = TableRun#
Time = TableValue('Table 1',Row-1,#Time)+DELTAt
Duplicate i = 1,9
```

```
T_old[i] = TableValue('Table 1', Row-1,#T[i])
End
 "Implicit finite difference formulation"
"Node 1"
h*DELTAx*(T_infi-T[1])+k*DELTAy/2*(300-T[1])/DELTAx+k*DELTAy/2*(T[2]-T[1])/DELTAx+k*DELTAx*(T[4]-T[1])/DELTAy = rho*c*DELTAx*DELTAy/(2*DELTAt)*(T[1]-T_old[1])
"Node 2"
h*DELTAx*(T_infi-T[2])+k*DELTAy/2*(T[1]-T[2])/DELTAx+k*DELTAy/2*(T[3]-T[2])/DELTAx+k*DELTAx*(T[5]-T[2])/DELTAy = rho*c*DELTAx*DELTAy/(2*DELTAt)*(T[2]-T_old[2])
"Node 4"
k*DELTAy*(300-T[4])/DELTAx+k*DELTAx*(T[1]-T[4])/DELTAy+k*DELTAy*(T[5]-T[4])/DELTAx+k*DELTAx*(T[7]-T[4])/DELTAy = rho*c*DELTAx*DELTAy/DELTAt*(T[4]-T_old[4])
"Node 5"
k*DELTAy*(T[4]-T[5])/DELTAx+k*DELTAy*(T[6]-T[5])/DELTAx+k*DELTAx*(T[2]-T[5])/DELTAy+k*DELTAx*(T[8]-T[5])/DELTAy = rho*c*DELTAx*DELTAy/DELTAt*(T[5]-T_old[5])
"Node 7"
k*DELTAy/2*(300-T[7])/DELTAx+k*DELTAy/2*(T[8]-T[7])/DELTAx+2*k*DELTAx*(T[4]-T[7])/DELTAy = rho*c*DELTAx*DELTAy/(1*DELTAt)*(T[7]-T_old[7])
"Node 8"
k*DELTAy/2*(T[7]-T[8])/DELTAx+k*DELTAy/2*(T[9]-T[8])/DELTAx+2*k*DELTAx*(T[5]-T[8])/DELTAy = rho*c*DELTAx*DELTAy/(1*DELTAt)*(T[8]-T_old[8])

"Due to symmetry"
T[3] = T[1]
T[6] = T[4]
T[9] = T[7]
```

Temperature distribution in the ceramic strip after 12 seconds is as follows

T_1 = **246.6ºC,** T_2 = **241.3ºC,** T_3 = **246.6ºC,** T_4 = **292.8ºC,** T_5 = **291.6ºC,** T_6 = **292.8ºC,** T_7 = **298.4ºC,** T_8 = **298.1ºC,** T_9 = **298.4ºC.**

Due to symmetry we have,

$T_1 = T_3$, $T_4 = T_6$ and $T_7 = T_9$

Discussion Usually the ceramic strips in electronic components are embedded into high thermal conductivity material. In such cases the temperature drop at their interface must be accounted.

Special Topic: Controlling the Numerical Error

5-132C The results obtained using a numerical method differ from the exact results obtained analytically because the results obtained by a numerical method are approximate. The difference between a numerical solution and the exact solution (the error) is primarily due to two sources: The *discretization error* (also called the *truncation* or *formulation* error) which is caused by the approximations used in the formulation of the numerical method, and the *round-off error* which is caused by the computers' representing a number by using a limited number of significant digits and continuously rounding (or chopping) off the digits it cannot retain.

5-133C The *discretization error* (also called the *truncation* or *formulation* error) is due to replacing the derivatives by differences in each step, or replacing the actual temperature distribution between two adjacent nodes by a straight line segment. The difference between the two solutions at each time step is called the *local discretization error*. The total discretization error at any step is called the *global* or *accumulated discretization error*. The local and global discretization errors are identical for the first time step.

5-134C Yes, the global (accumulated) discretization error be less than the local error during a step. The global discretization error usually increases with increasing number of steps, but the opposite may occur when the solution function changes direction frequently, giving rise to local discretization errors of opposite signs which tend to cancel each other.

5-135C The Taylor series expansion of the temperature at a specified nodal point *m* about time t_i is

$$T(x_m, t_i + \Delta t) = T(x_m, t_i) + \Delta t \frac{\partial T(x_m, t_i)}{\partial t} + \frac{1}{2}\Delta t^2 \frac{\partial^2 T(x_m, t_i)}{\partial t^2} + \cdots$$

The finite difference formulation of the time derivative at the same nodal point is expressed as

$$\frac{\partial T(x_m, t_i)}{\partial t} \cong \frac{T(x_m, t_i + \Delta t) - T(x_m, t_i)}{\Delta t} = \frac{T_m^{i+1} - T_m^i}{\Delta t} \quad \text{or} \quad T(x_m, t_i + \Delta t) \cong T(x_m, t_i) + \Delta t \frac{\partial T(x_m, t_i)}{\partial t}$$

which resembles the Taylor series expansion terminated after the first two terms.

5-136C The Taylor series expansion of the temperature at a specified nodal point *m* about time t_i is

$$T(x_m, t_i + \Delta t) = T(x_m, t_i) + \Delta t \frac{\partial T(x_m, t_i)}{\partial t} + \frac{1}{2}\Delta t^2 \frac{\partial^2 T(x_m, t_i)}{\partial t^2} + \cdots$$

The finite difference formulation of the time derivative at the same nodal point is expressed as

$$\frac{\partial T(x_m, t_i)}{\partial t} \cong \frac{T(x_m, t_i + \Delta t) - T(x_m, t_i)}{\Delta t} = \frac{T_m^{i+1} - T_m^i}{\Delta t} \quad \text{or} \quad T(x_m, t_i + \Delta t) \cong T(x_m, t_i) + \Delta t \frac{\partial T(x_m, t_i)}{\partial t}$$

which resembles the Taylor series expansion terminated after the first two terms. Therefore, the 3rd and following terms in the Taylor series expansion represent the error involved in the finite difference approximation. For a sufficiently small time step, these terms decay rapidly as the order of derivative increases, and their contributions become smaller and smaller. The first term neglected in the Taylor series expansion is proportional to $(\Delta t)^2$, and thus the local discretization error is also proportional to $(\Delta t)^2$.

The global discretization error is proportional to the step size to Δt itself since, at the worst case, the accumulated discretization error after I time steps during a time period t_0 is $I\Delta t^2 = (t_0 / \Delta t)\Delta t^2 = t_0 \Delta t$ which is proportional to Δt.

5-137C The *round-off error* is caused by retaining a limited number of digits during calculations. It depends on the number of calculations, the method of rounding off, the type of the computer, and even the sequence of calculations. Calculations that involve the alternate addition of small and large numbers are most susceptible to round-off error.

5-138C As the step size is decreased, the discretization error decreases but the round-off error increases.

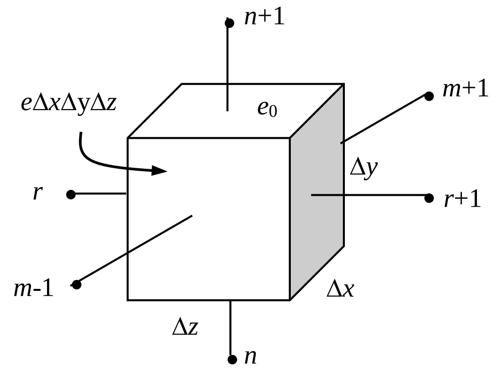
5-139C The round-off error can be reduced by avoiding extremely small mess sizes (smaller than necessary to keep the discretization error in check) and sequencing the terms in the program such that the addition of small and large numbers is avoided.

5-140C A practical way of checking if the round-off error has been significant in calculations is to repeat the calculations using double precision holding the mesh size and the size of the time step constant. If the changes are not significant, we conclude that the round-off error is not a problem.

5-141C A practical way of checking if the discretization error has been significant in calculations is to start the calculations with a reasonable mesh size Δx (and time step size Δt for transient problems), based on experience, and then to repeat the calculations using a mesh size of $\Delta x/2$. If the results obtained by halving the mesh size do not differ significantly from the results obtained with the full mesh size, we conclude that the discretization error is at an acceptable level.

Review Problems

5-142 Starting with an energy balance on a volume element, the steady three-dimensional finite difference equation for a general interior node in rectangular coordinates for $T(x, y, z)$ for the case of constant thermal conductivity and uniform heat generation is to be obtained.



Analysis We consider a *volume element* of size $\Delta x \times \Delta y \times \Delta z$ centered about a general interior node (m, n, r) in a region in which heat is generated at a constant rate of $\dot{e}_0$ and the thermal conductivity k is variable. Assuming the direction of heat conduction to be *towards* the node under consideration at all surfaces, the energy balance on the volume element can be expressed as

$$\dot{Q}_{\text{cond, left}} + \dot{Q}_{\text{cond, top}} + \dot{Q}_{\text{cond, right}} + \dot{Q}_{\text{cond, bottom}} + \dot{Q}_{\text{cond, front}} + \dot{Q}_{\text{cond, back}} + \dot{E}_{\text{gen,element}} = \frac{\Delta E_{\text{element}}}{\Delta t} = 0$$

for the *steady* case. Again assuming the temperatures between the adjacent nodes to vary linearly, the energy balance relation above becomes

$$\begin{aligned} & k(\Delta y \times \Delta z)\frac{T_{m-1,n,r} - T_{m,n,r}}{\Delta x} + k(\Delta x \times \Delta z)\frac{T_{m,n+1,r} - T_{m,n,r}}{\Delta y} \\ & + k(\Delta y \times \Delta z)\frac{T_{m+1,n,r} - T_{m,n,r}}{\Delta x} + k(\Delta x \times \Delta z)\frac{T_{m,n-1,r} - T_{m,n,r}}{\Delta y} \\ & + k(\Delta x \times \Delta y)\frac{T_{m,n,r-1} - T_{m,n,r}}{\Delta z} + k(\Delta x \times \Delta y)\frac{T_{m,n,r+1} - T_{m,n,r}}{\Delta z} + \dot{e}_0(\Delta x \times \Delta y \times \Delta z) = 0 \end{aligned}$$

Dividing each term by $k\,\Delta x \times \Delta y \times \Delta z$ and simplifying gives

$$\frac{T_{m-1,n,r} - 2T_{m,n,r} + T_{m+1,n,r}}{\Delta x^2} + \frac{T_{m,n-1,r} - 2T_{m,n,r} + T_{m,n+1,r}}{\Delta y^2} + \frac{T_{m,n,r-1} - 2T_{m,n,r} + T_{m,n,r+1}}{\Delta z^2} + \frac{\dot{e}_0}{k} = 0$$

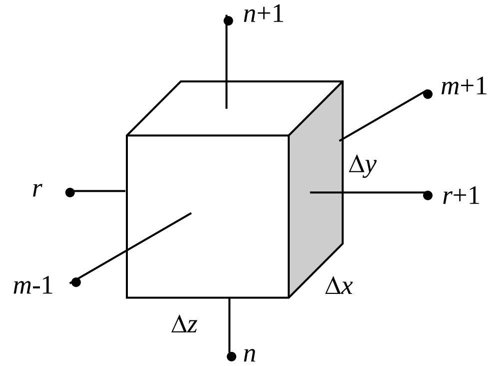
For a cubic mesh with $\Delta x = \Delta y = \Delta z = l$, and the relation above simplifies to

$$T_{m-1,n,r} + T_{m+1,n,r} + T_{m,n-1,r} + T_{m,n-1,r} + T_{m,n,r-1} + T_{m,n,r+1} - 6T_{m,n,r} + \frac{\dot{e}_0 l^2}{k} = 0$$

It can also be expressed in the following easy-to-remember form:

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} + T_{\text{front}} + T_{\text{back}} - 6T_{\text{node}} + \frac{\dot{e}_0 l^2}{k} = 0$$

5-143 Starting with an energy balance on a volume element, the three-dimensional transient *explicit* finite difference equation for a general interior node in rectangular coordinates for $T(x, y, z, t)$ for the case of constant thermal conductivity k and no heat generation is to be obtained.



Analysis We consider a rectangular region in which heat conduction is significant in the x and y directions. There is no heat generation in the medium, and the thermal conductivity k of the medium is constant. Now we divide the x-y-z region into a *mesh* of nodal points which are spaced Δx, Δy, and Δz apart in the x, y, and z directions, respectively, and consider a general interior node (m, n, r) whose coordinates are $x = m\Delta x$, $y = n\Delta y$, are $z = r\Delta z$. Noting that the volume element centered about the general interior node (m, n, r) involves heat conduction from six sides (right, left, front, rear, top, and bottom) and expressing them at previous time step i, the transient explicit finite difference formulation for a general interior node can be expressed as

$$k(\Delta y \times \Delta z)\frac{T^i_{m-1,n,r} - T^i_{m,n,r}}{\Delta x} + k(\Delta x \times \Delta z)\frac{T^i_{m,n+1,r} - T^i_{m,n,r}}{\Delta y} + k(\Delta y \times \Delta z)\frac{T^i_{m+1,n,r} - T^i_{m,n,r}}{\Delta x}$$

$$+ k(\Delta x \times \Delta z)\frac{T^i_{m,n-1,r} - T^i_{m,n,r}}{\Delta y} + k(\Delta x \times \Delta y)\frac{T^i_{m,n,r-1} - T^i_{m,n,r}}{\Delta z} + k(\Delta x \times \Delta y)\frac{T^i_{m,n,r+1} - T^i_{m,n,r}}{\Delta z}$$

$$= \rho(\Delta x \times \Delta y \times 1)c\frac{T^{i+1}_{m,n} - T^i_{m,n}}{\Delta t}$$

Taking a cubic mesh ($\Delta x = \Delta y = \Delta z = l$) and dividing each term by k gives, after simplifying,

$$T^i_{m-1,n,r} + T^i_{m+1,n,r} + T^i_{m,n+1,r} + T^i_{m,n-1,r} + T^i_{m,n,r-1} + T^i_{m,n,r+1} - 6T^i_{m,n,r} = \frac{T^{i+1}_{m,n,r} - T^i_{m,n,r}}{\tau}$$

where $\alpha = k/\rho c$ is the thermal diffusivity of the material and $\tau = \alpha\Delta t/l^2$ is the dimensionless mesh Fourier number. It can also be expressed in terms of the temperatures at the neighboring nodes in the following easy-to-remember form:

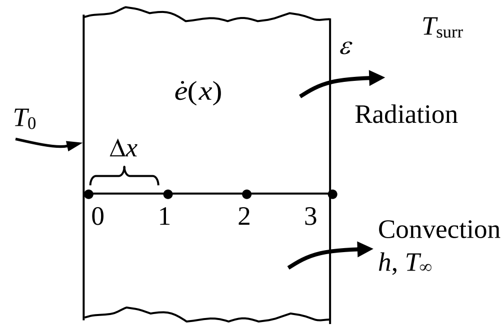
$$T^i_{\text{left}} + T^i_{\text{top}} + T^i_{\text{right}} + T^i_{\text{bottom}} + T^i_{\text{front}} + T^i_{\text{back}} - 6T^i_{\text{node}} = \frac{T^{i+1}_{\text{node}} - T^i_{\text{node}}}{\tau}$$

Discussion We note that setting $T^{i+1}_{\text{node}} = T^i_{\text{node}}$ gives the steady finite difference formulation.

5-144 A plane wall with variable heat generation and constant thermal conductivity is subjected to combined convection and radiation at the right (node 3) and specified temperature at the left boundary (node 0). The finite difference formulation of the right boundary node (node 3) and the finite difference formulation for the rate of heat transfer at the left boundary (node 0) are to be determined.

Assumptions **1** Heat transfer through the wall is given to be steady and one-dimensional. **2** The thermal conductivity is given to be constant.

Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the finite difference formulations become



Right boundary node (all temperatures are in K):

$$\varepsilon\sigma A(T_{surr}^4 - T_3^4) + hA(T_\infty - T_3) + kA\frac{T_2 - T_3}{\Delta x} + \dot{e}_3(A\Delta x/2) = 0$$

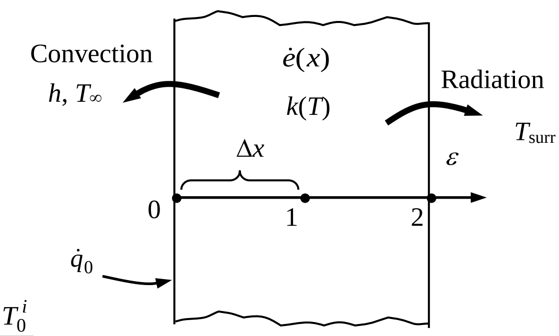
Heat transfer at left surface:

$$\dot{Q}_{\text{left surface}} + kA\frac{T_1 - T_0}{\Delta x} + \dot{e}_0(A\Delta x/2) = 0$$

5-145 A plane wall with variable heat generation and variable thermal conductivity is subjected to uniform heat flux $\dot{q}_0$ and convection at the left (node 0) and radiation at the right boundary (node 2). The explicit transient finite difference formulation of the problem using the energy balance approach method is to be determined.

Assumptions **1** Heat transfer through the wall is given to be transient, and the thermal conductivity and heat generation to be variables. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness. **3** Radiation from the left surface, and convection from the right surface are negligible.

Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the *explicit* finite difference formulations become



Left boundary node (node 0):

$$k_0^i A\frac{T_1^i - T_0^i}{\Delta x} + \dot{q}_0 A + hA(T_\infty - T_0^i) + \dot{e}_0^i(A\Delta x/2) = \rho A\frac{\Delta x}{2}c_p\frac{T_0^{i+1} - T_0^i}{\Delta t}$$

Interior node (node 1):

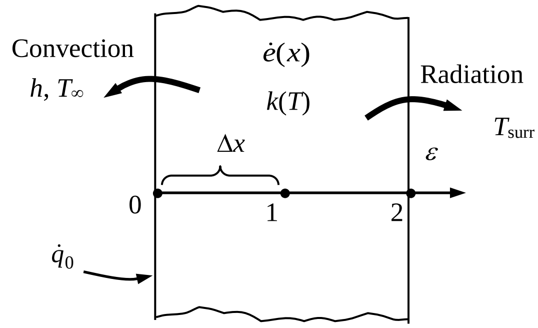
$$k_1^i A\frac{T_0^i - T_1^i}{\Delta x} + k_1^i A\frac{T_2^i - T_1^i}{\Delta x} + \dot{e}_1^i(A\Delta x) = \rho A\Delta x c_p\frac{T_1^{i+1} - T_1^i}{\Delta t}$$

Right boundary node (node 2):

$$k_2^i A\frac{T_1^i - T_2^i}{\Delta x} + \varepsilon\sigma A[(T_{surr}^i + 273)^4 - (T_2^i + 273)^4] + \dot{e}_2^i(A\Delta x/2) = \rho A\frac{\Delta x}{2}c_p\frac{T_2^{i+1} - T_2^i}{\Delta t}$$

5-146 A plane wall with variable heat generation and variable thermal conductivity is subjected to uniform heat flux $\dot{q}_0$ and convection at the left (node 0) and radiation at the right boundary (node 2). The implicit transient finite difference formulation of the problem using the energy balance approach method is to be determined.

Assumptions **1** Heat transfer through the wall is given to be transient, and the thermal conductivity and heat generation to be variables. **2** Heat transfer is one-dimensional since the plate is large relative to its thickness. **3** Radiation from the left surface, and convection from the right surface are negligible.



Analysis Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the *implicit* finite difference formulations become

Left boundary node (node 0):

$$k_0^{i+1}A\frac{T_1^{i+1}-T_0^{i+1}}{\Delta x}+\dot{q}_0A+hA(T_\infty-T_0^{i+1})+\dot{e}_0^{i+1}(A\Delta x/2)=\rho A\frac{\Delta x}{2}c_p\frac{T_0^{i+1}-T_0^i}{\Delta t}$$

Interior node (node 1):

$$k_1^{i+1}A\frac{T_0^{i+1}-T_1^{i+1}}{\Delta x}+k_1^{i+1}A\frac{T_2^{i+1}-T_1^{i+1}}{\Delta x}+\dot{e}_1^{i+1}(A\Delta x)=\rho A\Delta x c_p\frac{T_1^{i+1}-T_1^i}{\Delta t}$$

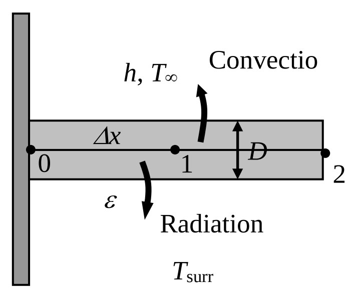
Right boundary node (node 2):

$$k_2^{i+1}A\frac{T_1^{i+1}-T_2^{i+1}}{\Delta x}+\varepsilon\sigma A[(T_{surr}^{i+1}+273)^4-(T_2^{i+1}+273)^4]+\dot{e}_2^{i+1}(A\Delta x/2)=\rho A\frac{\Delta x}{2}c_p\frac{T_2^{i+1}-T_2^i}{\Delta t}$$

5-147 A pin fin with convection and radiation heat transfer from its tip is considered. The complete finite difference formulation for the determination of nodal temperatures is to be obtained.

Assumptions **1** Heat transfer through the pin fin is given to be steady and one-dimensional, and the thermal conductivity to be constant. **2** Convection heat transfer coefficient and emissivity are constant and uniform.

Assumptions **1** Heat transfer through the wall is given to be steady and one-dimensional, and the thermal conductivity and heat generation to be variable. **2** Convection heat transfer at the right surface is negligible.



Analysis The nodal network consists of 3 nodes, and the base temperature T_0 at node 0 is specified. Therefore, there are two unknowns T_1 and T_2, and we need two equations to determine them. Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the finite difference formulations become

Node 1 (at midpoint):

$$kA\frac{T_0-T_1}{\Delta x}+kA\frac{T_2-T_1}{\Delta x}+h(p\Delta x)(T_\infty-T_1)+\varepsilon\sigma(p\Delta x)[T_{surr}^4-(T_1+273)^4]=0$$

Node 2 (at fin tip):

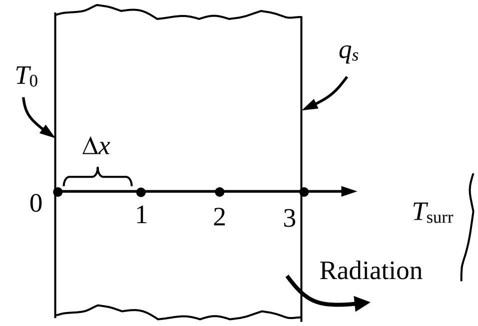
$$kA\frac{T_1-T_2}{\Delta x}+h(p\Delta x/2+A)(T_\infty-T_2)+\varepsilon\sigma(p\Delta x/2+A)[T_{surr}^4-(T_2+273)^4]=0$$

where $A=\pi D^2/4$ is the cross-sectional area and $p=\pi D$ is the perimeter of the fin.

5-148E A plane wall in space is subjected to specified temperature on one side and radiation and heat flux on the other. The finite difference formulation of this problem is to be obtained, and the nodal temperatures under steady conditions are to be determined.

Assumptions **1** Heat transfer through the wall is given to be steady and one-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation. **4** There is no convection in space.

Properties The properties of the wall are given to be k=1.2 Btu/h·ft·°F, ε = 0.80, and α_s = 0.6.



Analysis The nodal spacing is given to be Δx = 0.1 ft. Then the number of nodes becomes $M = L/\Delta x + 1$ = 0.3/0.1+1 = 4. The left surface temperature is given to be T_0 = 520 R = 60°F. This problem involves 3 unknown nodal temperatures, and thus we need to have 3 equations to determine them uniquely. Nodes 1 and 2 are interior nodes, and thus for them we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0 \rightarrow T_{m-1} - 2T_m + T_{m+1} = 0 \quad (\text{since } \dot{e} = 0), \quad \text{for } m = 1 \text{ and } 2$$

The finite difference equation for node 3 on the right surface subjected to convection and solar heat flux is obtained by applying an energy balance on the half volume element about node 3 and taking the direction of all heat transfers to be towards the node under consideration:

$$\text{Node 1 (interior)}: \qquad T_0 - 2T_1 + T_2 = 0$$

$$\text{Node 2 (interior)}: \qquad T_1 - 2T_2 + T_3 = 0$$

$$\text{Node 3 (right surface)}: \qquad \alpha_s \dot{q}_s + \varepsilon\sigma[T_{\text{space}}^4 - (T_3 + 460)^4] + k\frac{T_2 - T_3}{\Delta x} = 0$$

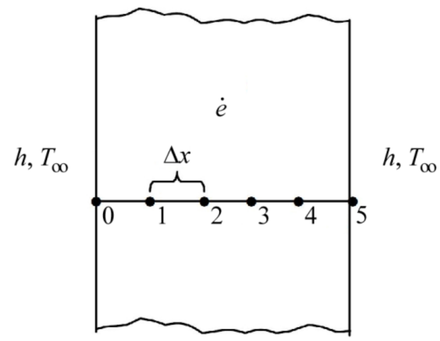
where k = 1.2 Btu/h.ft.°F, ε = 0.80, α_s = 0.60, $\dot{q}_s = 350$ Btu/h.ft^2, T_{space} = 0 R, and σ = 0.1714×10^{-8} Btu/h.ft^2.R^4 The system of 3 equations with 3 unknown temperatures constitute the finite difference formulation of the problem.

(*b*) The nodal temperatures under steady conditions are determined by solving the 3 equations above simultaneously with an equation solver to be

T_1 = **67.6°F = 527.6 R**, T_2 = **75.2°F = 535.2 R**, and T_3 = **82.8°F = 542.8 R**

5-149 A nuclear fuel element, modeled as a plane wall, generates 3×10^7 W/m^3 of heat uniformly with both side surfaces cooled by liquid. The finite difference equations and the nodal temperatures are to be determined, and the surface temperatures of both sides of the fuel element are to be compared with analytical solution.

Assumptions **1** Heat transfer through the nuclear fuel element is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.



Properties The thermal conductivity is given as 57 W/m·K.

Analysis (*a*) The nodal spacing is given as Δx = 8 mm. Then the number of nodes M becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{40\text{ mm}}{8\text{ mm}} + 1 = 6$$

There are 6 unknown nodal temperatures, thus we need to have 6 equations to determine them uniquely. The finite difference equation for node 0 on the left surface subjected to convection is obtained by applying an energy balance on the half volume element about that node:

$$h(T_\infty - T_0) + k\frac{T_1 - T_0}{\Delta x} + \dot{e}_0 \frac{\Delta x}{2} = 0 \quad \rightarrow \quad T_1 - \left(1 + \frac{h}{k}\Delta x\right)T_0 + \dot{e}_0 \frac{\Delta x^2}{2k} + \frac{h}{k}\Delta x T_\infty = 0$$

Nodes 1, 2, 3, and 4 are interior nodes, and we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0 \quad \rightarrow \quad T_{m-1} - 2T_m + T_{m+1} + \frac{\dot{e}_m}{k}\Delta x^2 = 0$$

The finite difference equation for node 5 on the right surface subjected to convection is obtained by applying an energy balance on the half volume element about that node:

$$k\frac{T_4 - T_5}{\Delta x} + \dot{e}_5 \frac{\Delta x}{2} + h(T_\infty - T_5) = 0 \quad \rightarrow \quad T_4 - \left(1 + \frac{h}{k}\Delta x\right)T_5 + \frac{\Delta x^2}{2k}\dot{e}_5 + \frac{h}{k}\Delta x T_\infty = 0$$

Then

m = 0: $\quad T_1 - (1 + h\Delta x / k)T_0 + (\Delta x^2 \dot{e}_0)/(2k) + (h\Delta x / k)T_\infty = 0$

m = 1: $\quad T_0 - 2T_1 + T_2 + (\dot{e}_1 / k)\Delta x^2 = 0$

m = 2: $\quad T_1 - 2T_2 + T_3 + (\dot{e}_2 / k)\Delta x^2 = 0$

m = 3: $\quad T_2 - 2T_3 + T_4 + (\dot{e}_3 / k)\Delta x^2 = 0$

m = 4: $\quad T_3 - 2T_4 + T_5 + (\dot{e}_4 / k)\Delta x^2 = 0$

m = 5: $\quad T_4 - (1 + h\Delta x / k)T_5 + (\Delta x^2 \dot{e}_5)/(2k) + (h\Delta x / k)T_\infty = 0$

(*b*) The nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver. Copy the following lines and paste on a blank EES screen to solve the above equations:

```
e_gen=3E7
h=8000
k=57
Dx=8E-3
T_inf=80
T_1-(1+h*Dx/k)*T_0+(Dx^2*e_gen)/(2*k)+(h*Dx/k)*T_inf=0
T_0-2*T_1+T_2+(e_gen/k)*Dx^2=0
T_1-2*T_2+T_3+(e_gen/k)*Dx^2=0
T_2-2*T_3+T_4+(e_gen/k)*Dx^2=0
T_3-2*T_4+T_5+(e_gen/k)*Dx^2=0
T_4-(1+h*Dx/k)*T_5+(Dx^2*e_gen)/(2*k)+(h*Dx/k)*T_inf=0
```

Solving by EES software, we get

$T_0 = \mathbf{155\,°C}$, $\quad T_1 = \mathbf{222\,°C}$, $\quad T_2 = \mathbf{256\,°C}$

$T_3 = \mathbf{256\,°C}$, $\quad T_4 = \mathbf{222\,°C}$, $\quad T_5 = \mathbf{155\,°C}$

(*c*) Using the analytical solution from Chapter 2, for a plane wall of thickness 2*L* with heat generation, the surface temperature exposed to convection can be determined using

$$T_{s,\,\text{plane wall}} = T_\infty + \frac{\dot{e}_{\text{gen}} L}{h} = 80\,°\text{C} + \frac{(3\times10^7\ \text{W/m}^3)(0.02\ \text{m})}{8000\ \text{W/m}^2\cdot\text{K}} = \mathbf{155\,°C} \quad \text{(for both sides)}$$

The analytical solution matches exactly with the results obtained using numerical method for both sides of the surface temperatures, $T_0 = T_5 = \mathbf{155\,°C}$.

Discussion Since both side of the fuel element are exposed to the same liquid temperature and convection heat transfer coefficient, it is possible to solve half of the plane wall by treating the centerline of the plane wall as symmetry line and get the same results.

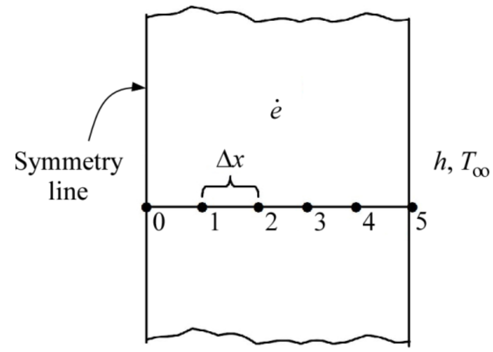
5-150 A fuel element, modeled as a plane wall, generates 5×10^7 W/m^3 of heat uniformly with both side surfaces cooled by liquid. The finite difference equations and the nodal temperatures are to be determined by making use of the symmetry line of the plane wall.

Assumptions **1** Heat transfer through the fuel element is steady and one-dimensional. **2** Thermal properties are constant. **3** Heat transfer by radiation is negligible.

Properties The thermal conductivity is given as 67 W/m·K.

Analysis (*a*) The nodal spacing is given to be Δx = 4 mm. Then the number of nodes *M* becomes

$$M = \frac{L}{\Delta x} + 1 = \frac{20\text{ mm}}{4\text{ mm}} + 1 = 6$$



There are 6 unknown nodal temperatures, thus we need to have 6 equations to determine them uniquely. The finite difference equation for node 0 on the symmetry line is obtained by applying an energy balance on the half volume element about that node (the symmetry boundary is similar to the insulated boundary):

$$k\frac{T_1 - T_0}{\Delta x} + \dot{e}_0 \frac{\Delta x}{2} = 0 \quad \rightarrow \quad T_1 - T_0 + \dot{e}_0 \frac{\Delta x^2}{2k} = 0$$

Nodes 1, 2, 3, and 4 are interior nodes, and we can use the general finite difference relation expressed as

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{\Delta x^2} + \frac{\dot{e}_m}{k} = 0 \quad \rightarrow \quad T_{m-1} - 2T_m + T_{m+1} + \frac{\dot{e}_m}{k}\Delta x^2 = 0$$

The finite difference equation for node 5 on the right surface subjected to convection is obtained by applying an energy balance on the half volume element about that node:

$$k\frac{T_4 - T_5}{\Delta x} + \dot{e}_5 \frac{\Delta x}{2} + h(T_\infty - T_5) = 0 \quad \rightarrow \quad T_4 - \left(1 + \frac{h}{k}\Delta x\right)T_5 + \frac{\Delta x^2}{2k}\dot{e}_5 + \frac{h}{k}\Delta x T_\infty = 0$$

Then

$m = 0$: $\quad T_1 - T_0 + (\dot{e}_0 \Delta x^2)/(2k) = 0$

$m = 1$: $\quad T_0 - 2T_1 + T_2 + (\dot{e}_1 / k)\Delta x^2 = 0$

$m = 2$: $\quad T_1 - 2T_2 + T_3 + (\dot{e}_2 / k)\Delta x^2 = 0$

$m = 3$: $\quad T_2 - 2T_3 + T_4 + (\dot{e}_3 / k)\Delta x^2 = 0$

$m = 4$: $\quad T_3 - 2T_4 + T_5 + (\dot{e}_4 / k)\Delta x^2 = 0$

$m = 5$: $\quad T_4 - (1 + h\Delta x / k)T_5 + (\Delta x^2 \dot{e}_5)/(2k) + (h\Delta x / k)T_\infty = 0$

(*b*) The nodal temperatures under steady conditions are determined by solving the 6 equations above simultaneously with an equation solver. Copy the following lines and paste on a blank EES screen to solve the above equations:

```
e_gen=5E7
h=5000
k=67
Dx=4E-3
T_inf=90
T_1-T_0+(Dx^2*e_gen)/(2*k)=0
T_0-2*T_1+T_2+(e_gen/k)*Dx^2=0
T_1-2*T_2+T_3+(e_gen/k)*Dx^2=0
T_2-2*T_3+T_4+(e_gen/k)*Dx^2=0
T_3-2*T_4+T_5+(e_gen/k)*Dx^2=0
T_4-(1+h*Dx/k)*T_5+(Dx^2*e_gen)/(2*k)+(h*Dx/k)*T_inf=0
```

Solving by EES software, we get

$$T_0 = \mathbf{439\,°C}, \quad T_1 = \mathbf{433\,°C}, \quad T_2 = \mathbf{415\,°C}$$

$$T_3 = \mathbf{386\,°C}, \quad T_4 = \mathbf{344\,°C}, \quad T_5 = \mathbf{290\,°C}$$

Discussion Using the analytical solution from Chapter 2, for a plane wall of thickness 2*L* with heat generation, the surface temperature exposed to convection can be determined using

$$T_{s,\text{ plane wall}} = T_\infty + \frac{\dot{e}_{\text{gen}} L}{h} = 90\text{ °C} + \frac{(5 \times 10^7 \text{ W/m}^3)(0.02\text{ m})}{5000\text{ W/m}^2 \cdot \text{K}} = 290\text{ °C}$$

The analytical solution matches exactly with the results obtained using numerical method for both sides of the surface temperatures, $T_5 = 290$ °C.

5-151 A square cross section with uniform heat generation is undergoing a steady two-dimensional heat transfer. The top and right surfaces are subjected to convection while the left and bottom surfaces maintain a constant temperature. The finite difference equations and the nodal temperatures are to be determined.

Assumptions **1** Steady heat conduction is two-dimensional. **2** Thermal properties are constant. **3** The heat generation in the body is uniform.

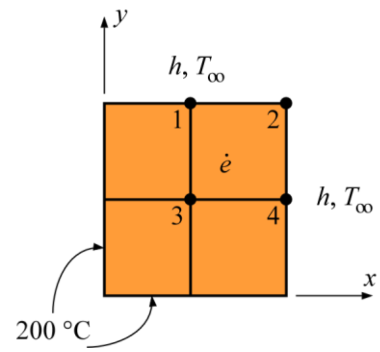
Properties The conductivity is given to be k = 25 W/m·K.

Analysis (a) There is symmetry about the diagonal line passing through the center. Therefore, $T_1 = T_4$, and the unknown temperatures are T_1, T_2, and T_3. Thus, we need to have 3 equations to determine them uniquely.

$$\text{Node 1:}\quad h\Delta x(T_\infty - T_1) + k\frac{\Delta y}{2}\frac{T_2 - T_1}{\Delta x} + k\Delta x\frac{T_3 - T_1}{\Delta y} + k\frac{\Delta y}{2}\frac{200 - T_1}{\Delta x} + \dot{e}_1\Delta x\frac{\Delta y}{2} = 0$$

$$\text{Node 2:}\quad h\left(\frac{\Delta x}{2} + \frac{\Delta y}{2}\right)(T_\infty - T_2) + k\frac{\Delta x}{2}\frac{T_4 - T_2}{\Delta y} + k\frac{\Delta y}{2}\frac{T_1 - T_2}{\Delta x} + \dot{e}_2\frac{\Delta x}{2}\frac{\Delta y}{2} = 0$$

$$\text{Node 3:}\quad \frac{200 - 2T_3 + T_4}{\Delta x^2} + \frac{200 - 2T_3 + T_1}{\Delta y^2} + \frac{\dot{e}_{m,n}}{k} = 0$$



or

$$\text{Node 1:}\quad T_1 = \frac{1}{4 + 2h\Delta x/k}\left(200 + T_2 + 2T_3 + 2\frac{h\Delta x}{k}T_\infty + \frac{\dot{e}_1\Delta x^2}{k}\right)$$

$$\text{Node 2:}\quad T_2 = \frac{1}{2 + 2h\Delta x/k}\left(2T_1 + 2\frac{h\Delta x}{k}T_\infty + \frac{\dot{e}_2\Delta x^2}{2k}\right)$$

$$\text{Node 3:}\quad T_3 = 0.25(400 + 2T_1 + \dot{e}_3\Delta x^2/k)$$

Then

$$\text{Node 1:}\quad T_1 = (200 + T_2 + 2T_3 + 0.2T_\infty + 12)/4.2$$

$$\text{Node 2:}\quad T_2 = (2T_1 + 0.2T_\infty + 6)/2.2$$

$$\text{Node 3:}\quad T_3 = 0.25(400 + 2T_1 + 12)$$

where $\dot{e}_{\text{node}}\Delta x^2/k = 12\,°\text{C}$ and $2h\Delta x/k = 0.2$

(b) By letting the initial guesses as $T_1 = T_2 = T_3$ = 200°C, the results obtained from successive iterations are listed in the following table:

Iteration	Nodal temperature,°C		
	T_1	T_2	T_3
1	198.1	191.9	202.0
2	197.1	191.0	201.6
3	196.7	190.6	201.4
4	196.5	190.5	201.3
5	196.4	190.4	201.2
6	196.4	190.3	201.2
7	196.4	190.3	201.2

Hence, the converged nodal temperatures are

$T_1 = T_4 =$ **196.4°C**, $T_2 =$ **190.3°C**, $T_3 =$ **201.2°C**

Discussion The finite difference equations can also be calculated using the EES. Copy the following lines and paste on a blank EES screen to solve the above equations:

```
T_1=(200+T_2+2*T_3+0.2*100+12)/4.2
T_2=(2*T_1+0.2*100+6)/2.2
T_3=0.25*(400+2*T_1+12)
```

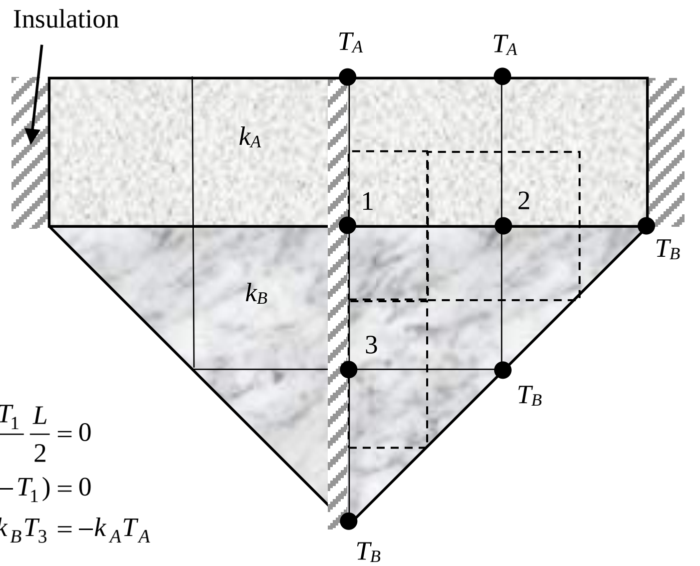
Solving by EES software, we get the same results:

$T_1 = T_4 =$ **196.4°C**, $T_2 =$ **190.3°C**, $T_3 =$ **201.2°C**

5-152 A two-dimensional bar shown in the figure is considered. The simplest form of the matrix equation is to be written and the grid notes with energy balance equations are to be identified on the figure.

__Assumptions__ **1** Heat transfer through the body is given to be steady and two-dimensional. **2** Thermal conductivity is constant. **3** There is no heat generation.

__Analysis__ From symmetry, we have only three unknown temperatures at nodes 1, 2, and 3. The finite difference formulations are



Node 1:

$$k_A \frac{T_A - T_1}{L}\frac{L}{2} + k_A \frac{T_2 - T_1}{L}\frac{L}{2} + k_B \frac{T_2 - T_1}{L}\frac{L}{2} + k_B \frac{T_3 - T_1}{L}\frac{L}{2} = 0$$

$$k_A(T_A - T_1) + k_A(T_2 - T_1) + k_B(T_2 - T_1) + k_B(T_3 - T_1) = 0$$

$$-2(k_A + k_B)T_1 + (k_A + k_B)T_2 + k_B T_3 = -k_A T_A$$

Node 2:

$$k_A \frac{T_A - T_2}{L} L + k_A \frac{T_1 - T_2}{L}\frac{L}{2} + k_B \frac{T_1 - T_2}{L}\frac{L}{2} + k_B \frac{T_B - T_2}{L} L + k_B \frac{T_B - T_2}{L}\frac{L}{2} + k_A \frac{T_B - T_2}{L}\frac{L}{2} = 0$$

$$2k_A(T_A - T_2) + k_A(T_1 - T_2) + k_B(T_1 - T_2) + 2k_B(T_B - T_2) + k_B(T_B - T_2) + k_A(T_B - T_2) = 0$$

$$(k_A + k_B)T_1 - 4(k_A + k_B)T_2 = -2k_A T_A - (k_A + 3k_B)T_B$$

Node 3:

$$k_B \frac{T_1 - T_3}{L}\frac{L}{2} + k_B \frac{T_B - T_3}{L} L + k_B \frac{T_B - T_3}{L}\frac{L}{2} = 0$$

$$T_1 - T_3 + 2(T_B - T_3) + T_B - T_3 = 0$$

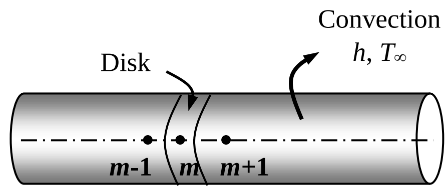
$$T_1 + -4T_3 = -3T_B$$

The matrix equation is

$$\begin{bmatrix} -2(k_A + k_B) & k_A + k_B & k_B \\ k_A + k_B & -4(k_A + k_B) & 0 \\ 1 & 0 & -4 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} k_A T_A \\ -2k_A T_A - (k_A + 3k_B)T_B \\ -3T_B \end{bmatrix}$$

__Discussion__ Note that the results do not depend on L (size of the system). If you don't use the symmetry and get a 4×4 linear system, two of the equations must be equivalent.

5-153 Starting with an energy balance on a disk volume element, the one-dimensional transient implicit finite difference equation for a general interior node for $T(z,t)$ in a cylinder whose side surface is subjected to convection with a convection coefficient of *h* and an ambient temperature of T_∞ for the case of constant thermal conductivity with uniform heat generation is to be obtained.



Analysis We consider transient one-dimensional heat conduction in the axial *z* direction in a cylindrical rod of constant cross-sectional area *A* with constant heat generation $\dot{e}_0$ and constant conductivity *k* with a mesh size of Δz in the *z* direction. Noting that the volume element of a general interior node *m* involves heat conduction from two sides, convection from its lateral surface, and the volume of the element is $V_{\text{element}} = A\Delta z$, the transient implicit finite difference formulation for an interior node can be expressed as

$$hp\Delta z(T_\infty - T_m^{i+1}) + kA\frac{T_{m-1}^{i+1} - T_m^{i+1}}{\Delta z} + kA\frac{T_{m+1}^{i+1} - T_m^{i+1}}{\Delta z} + \dot{e}_0 A\Delta z = \rho A\Delta z c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

where $A = \pi D^2/4$ is the cross-sectional area. Multiplying both sides of equation by $\Delta z/(kA)$,

$$\frac{hp\Delta z^2}{kA}(T_\infty - T_m^{i+1}) + (T_{m-1}^{i+1} - T_m^{i+1}) + (T_{m+1}^{i+1} - T_m^{i+1}) + \frac{\dot{e}_0 \Delta z^2}{k} = \frac{\rho \Delta z^2 c_p}{k\Delta t}(T_m^{i+1} - T_m^i)$$

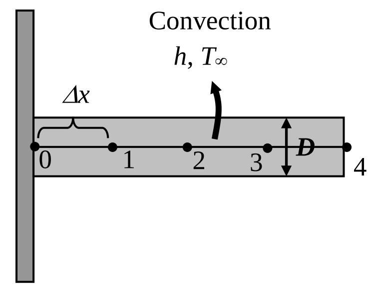
Using the definitions of *thermal diffusivity* $\alpha = k/\rho c_p$ and the dimensionless *mesh Fourier number* $\tau = \dfrac{\alpha\Delta t}{\Delta z^2}$ the equation reduces to

$$\frac{hp\Delta z^2}{kA}(T_\infty - T_m^{i+1}) + (T_{m-1}^{i+1} + T_{m+1}^{i+1} - 2T_m^{i+1}) + \frac{\dot{e}_0 \Delta z^2}{k} = \frac{(T_m^{i+1} - T_m^i)}{\tau}$$

Discussion We note that setting $T_{\text{m}}^{i+1} = T_{\text{m}}^i$ gives the steady finite difference formulation.

5-154 A hot surface is to be cooled by aluminum pin fins. The nodal temperatures after 10 min are to be determined using the explicit finite difference method. Also to be determined is the time it takes for steady conditions to be reached.

Assumptions **1** Heat transfer through the pin fin is given to be one-dimensional. **2** The thermal properties of the fin are constant. **3** Convection heat transfer coefficient is constant and uniform. **4** Radiation heat transfer is negligible. **5** Heat loss from the fin tip is considered.



Analysis The nodal network of this problem consists of 5 nodes, and the base temperature T_0 at node 0 is specified. Therefore, there are 4 unknown nodal temperatures, and we need 4 equations to determine them. Using the energy balance approach and taking the direction of all heat transfers to be towards the node under consideration, the explicit transient finite difference formulations become

Node 1 (interior): $$hp\Delta x(T_\infty - T_1^i) + kA\frac{T_2^i - T_1^i}{\Delta x} + kA\frac{T_0 - T_1^i}{\Delta x} = \rho A\Delta x c_p \frac{T_1^{i+1} - T_1^i}{\Delta t}$$

Node 2 (interior): $$hp\Delta x(T_\infty - T_2^i) + kA\frac{T_3^i - T_2^i}{\Delta x} + kA\frac{T_1^i - T_2^i}{\Delta x} = \rho A\Delta x c_p \frac{T_2^{i+1} - T_2^i}{\Delta t}$$

Node 3 (interior): $$hp\Delta x(T_\infty - T_3^i) + kA\frac{T_4^i - T_3^i}{\Delta x} + kA\frac{T_2^i - T_3^i}{\Delta x} = \rho A\Delta x c_p \frac{T_3^{i+1} - T_3^i}{\Delta t}$$

Node 4 (fin tip): $$h(p\Delta x/2 + A)(T_\infty - T_4^i) + kA\frac{T_3^i - T_4^i}{\Delta x} = \rho A(\Delta x/2) c_p \frac{T_4^{i+1} - T_4^i}{\Delta t}$$

where $A = \pi D^2/4$ is the cross-sectional area and $p = \pi D$ is the perimeter of the fin. Also, $D = 0.008$ m, $k = 237$ W/m.°C, $\alpha = k/\rho c_p = 97.1\times10^{-6}\ \text{m}^2/\text{s}$, $\Delta x = 0.02$ m, $T_\infty = 15$°C, $T_0 = T_i = 120$°C, $h_o = 35$ W/m^2.°C, and $\Delta t = 1$ s. Also, the mesh Fourier number is

$$\tau = \frac{\alpha\Delta t}{\Delta x^2} = \frac{(97.1\times10^{-6}\ \text{m}^2/\text{s})(1\text{s})}{(0.02\ \text{m})^2} = 0.24275$$

Substituting these values, the nodal temperatures along the fin after 10×60 = 600 time steps (4 h) are determined to be

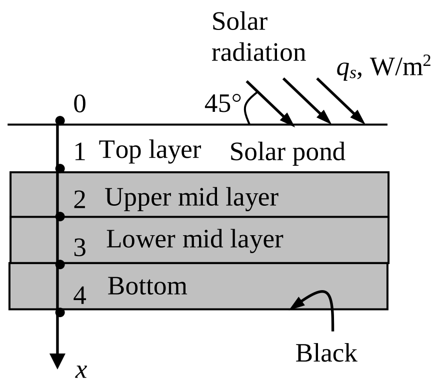
T_0 = **120°C**, T_1 = **110.6°C**, T_2 = **103.9°C**, T_3 = **100.0°C**, and T_4 = **98.5°C.**

Printing the temperatures after each time step and examining them, we observe that the nodal temperatures stop changing after about 3.8 min. Thus we conclude that steady conditions are reached after **3.8 min.**

5-155 A large pond is initially at a uniform temperature. Solar energy is incident on the pond surface at for 4 h The temperature distribution in the pond under the most favorable conditions is to be determined.

Assumptions **1** Heat transfer is one-dimensional since the pond is large relative to its depth. **2** Thermal properties, heat transfer coefficients, and the indoor temperatures are constant. **3** Radiation heat transfer is significant. **4** There are no convection currents in the water. **5** The given time step Δt = 15 min is less than the critical time step so that the stability criteria is satisfied. **6** All heat losses from the pond are negligible. **7** Heat generation due to absorption of radiation is uniform in each layer.

Properties The conductivity and diffusivity are given to be k = 0.61 W/m.°C and $\alpha = 0.15\times10^{-6}$ m^2/s. The volumetric absorption coefficients of water are as given in the problem.



Analysis The nodal spacing is given to be Δx = 0.25 m. Then the number of nodes becomes $M = L/\Delta x + 1$ = 1/0.25+1 = 4. This problem involves 5 unknown nodal temperatures, and thus we need to have 5 equations. Nodes 2, 3, and 4 are interior nodes, and thus for them we can use the general explicit finite difference relation expressed as

$$T_{m-1}^i - 2T_m^i + T_{m+1}^i + \frac{\dot{e}_m^i \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau} \;\rightarrow\; T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1-2\tau)T_m^i + \tau\frac{\dot{e}_m^i \Delta x^2}{k}$$

Node 0 can also be treated as an interior node by using the mirror image concept. The finite difference equation for node 4 subjected to heat flux is obtained from an energy balance by taking the direction of all heat transfers to be towards the node:

$$\text{Node 0 (insulation):}\quad T_0^{i+1} = \tau(T_1^i + T_1^i) + (1-2\tau)T_0^i + \tau\dot{e}_0(\Delta x)^2/k$$

$$\text{Node 0 (insulation):}\quad T_1^{i+1} = \tau(T_0^i + T_2^i) + (1-2\tau)T_1^i + \tau\dot{e}_1(\Delta x)^2/k$$

$$\text{Node 2 (interior):}\quad T_2^{i+1} = \tau(T_1^i + T_3^i) + (1-2\tau)T_2^i + \tau\dot{e}_2(\Delta x)^2/k$$

$$\text{Node 3 (interior):}\quad T_3^{i+1} = \tau(T_2^i + T_4^i) + (1-2\tau)T_3^i + \tau\dot{e}_3(\Delta x)^2/k$$

$$\text{Node 6 (convection):}\quad \dot{q}_b + k\frac{T_3^i - T_4^i}{\Delta x} + \tau\dot{e}_4(\Delta x)^2/k = \rho\frac{\Delta x}{2}c_p\frac{T_4^{i+1} - T_4^i}{\Delta t}$$

where k = 0.61 W/m.°C, $\alpha = k/\rho c_p = 0.15\times10^{-6}$ m^2/s, Δx = 0.25 m, and Δt = 15 min = 900 s. Also, the mesh Fourier number is

$$\tau = \frac{\alpha\Delta t}{\Delta x^2} = \frac{(0.15\times10^{-6}\text{ m}^2/\text{s})(900\text{s})}{(0.25\text{ m})^2} = 0.002160$$

The values of heat generation rates at the nodal points are determined as follows:

$$\dot{e}_0 = \frac{\dot{E}_0}{\text{Volume}} = \frac{0.473\times500\text{ W}}{(1\text{m}^2)(0.25\text{m})} = 946\text{ W/m}^3$$

$$\dot{e}_1 = \frac{\dot{E}_1}{\text{Volume}} = \frac{[(0.473+0.061)/2]\times500\text{ W}}{(1\text{m}^2)(0.25\text{m})} = 534\text{ W/m}^3$$

$$\dot{e}_4 = \frac{\dot{E}_4}{\text{Volume}} = \frac{0.024\times500\text{ W}}{(1\text{m}^2)(0.25\text{m})} = 48\text{ W/m}^3$$

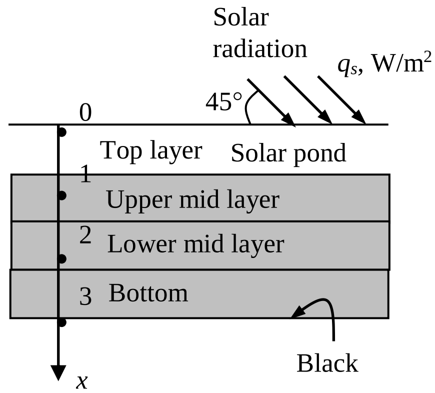
Also, the heat flux at the bottom surface is $\dot{q}_b = 0.379\times500\text{ W/m}^2 = 189.5\text{ W/m}^2$. Substituting these values, the nodal temperatures in the pond after 4×(60/15) = 16 time steps (4 h) are determined to be

T_0 = **18.3°C**, T_1 = **16.9°C**, T_2 = **15.4°C**, T_3 = **15.3°C**, and T_4 = **20.2°C**

5-156 A large 1-m deep pond is initially at a uniform temperature of 15°C throughout. Solar energy is incident on the pond surface at 45° at an average rate of 500 W/m^2 for a period of 4 h The temperature distribution in the pond under the most favorable conditions is to be determined.

Assumptions **1** Heat transfer is one-dimensional since the pond is large relative to its depth. **2** Thermal properties, heat transfer coefficients, and the indoor temperatures are constant. **3** Radiation heat transfer is significant. **4** There are no convection currents in the water. **5** The given time step Δt = 15 min is less than the critical time step so that the stability criteria is satisfied. **6** All heat losses from the pond are negligible. **7** Heat generation due to absorption of radiation is uniform in each layer.

Properties The conductivity and diffusivity are given to be k = 0.61 W/m.°C and $\alpha = 0.15\times10^{-6}\ \text{m}^2/\text{s}$. The volumetric absorption coefficients of water are as given in the problem.



Analysis The nodal spacing is given to be Δx = 0.25 m. Then the number of nodes becomes $M = L/\Delta x + 1$ = 1/0.25+1 = 4. This problem involves 5 unknown nodal temperatures, and thus we need to have 5 equations. Nodes 2, 3, and 4 are interior nodes, and thus for them we can use the general explicit finite difference relation expressed as

$$T_{m-1}^i - 2T_m^i + T_{m+1}^i + \frac{\dot{e}_m^i \Delta x^2}{k} = \frac{T_m^{i+1} - T_m^i}{\tau}$$

$$\rightarrow\quad T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1-2\tau)T_m^i + \tau\frac{\dot{e}_m^i \Delta x^2}{k}$$

Node 0 can also be treated as an interior node by using the mirror image concept. The finite difference equation for node 4 subjected to heat flux is obtained from an energy balance by taking the direction of all heat transfers to be towards the node:

Node 0 (insulation): $T_0^{i+1} = \tau(T_1^i + T_1^i) + (1-2\tau)T_0^i + \tau\dot{e}_0(\Delta x)^2/k$

Node 0 (insulation): $T_1^{i+1} = \tau(T_0^i + T_2^i) + (1-2\tau)T_1^i + \tau\dot{e}_1(\Delta x)^2/k$

Node 2 (interior): $T_2^{i+1} = \tau(T_1^i + T_3^i) + (1-2\tau)T_2^i + \tau\dot{e}_2(\Delta x)^2/k$

Node 3 (interior): $T_3^{i+1} = \tau(T_2^i + T_4^i) + (1-2\tau)T_3^i + \tau\dot{e}_3(\Delta x)^2/k$

Node 6 (convection): $\dot{q}_b + k\dfrac{T_3^i - T_4^i}{\Delta x} + \tau\dot{e}_4(\Delta x)^2/k = \rho\dfrac{\Delta x}{2}c_p\dfrac{T_4^{i+1} - T_4^i}{\Delta t}$

where k = 0.61 W/m.°C, $\alpha = k/\rho c_p = 0.15\times10^{-6}\ \text{m}^2/\text{s}$, Δx = 0.25 m, and Δt = 15 min = 900 s. Also, the mesh Fourier number is

$$\tau = \frac{\alpha\Delta t}{\Delta x^2} = \frac{(0.15\times10^{-6}\ \text{m}^2/\text{s})(900\text{s})}{(0.25\ \text{m})^2} = 0.002160$$

The absorption of solar radiation is given to be $\dot{e}(x) = \dot{q}_s(0.859 - 3.415x + 6.704x^2 - 6.339x^3 + 2.278x^4)$

where $\dot{q}_s$ is the solar flux incident on the surface of the pond in W/m^2, and x is the distance form the free surface of the pond, in m. Then the values of heat generation rates at the nodal points are determined to be

Node 0 (x = 0): $\dot{e}_0 = 500(0.859 - 3.415\times0 + 6.704\times0^2 - 6.339\times0^3 + 2.278\times0^4) = 429.5\ \text{W/m}^3$

Node 1(x=0.25): $\dot{e}_1 = 500(0.859 - 3.415\times0.25 + 6.704\times0.25^2 - 6.339\times0.25^3 + 2.278\times0.25^4) = 167.1\ \text{W/m}^3$

Node 2 (x=0.50): $\dot{e}_2 = 500(0.859 - 3.415\times0.5 + 6.704\times0.5^2 - 6.339\times0.5^3 + 2.278\times0.5^4) = 88.8\ \text{W/m}^3$

Node3 (x=0.75): $\dot{e}_3 = 500(0.859 - 3.415\times0.75 + 6.704\times0.75^2 - 6.339\times0.75^3 + 2.278\times0.75^4) = 57.6\ \text{W/m}^3$

Node 4 (x = 1.00): $\dot{e}_4 = 500(0.859 - 3.415\times1 + 6.704\times1^2 - 6.339\times1^3 + 2.278\times1^4) = 43.5\ \text{W/m}^3$

Also, the heat flux at the bottom surface is $\dot{q}_b = 0.379\times500\ \text{W/m}^2 = 189.5\ \text{W/m}^2$. Substituting these values, the nodal temperatures in the pond after 4×(60/15) = 16 time steps (4 h) are determined to be

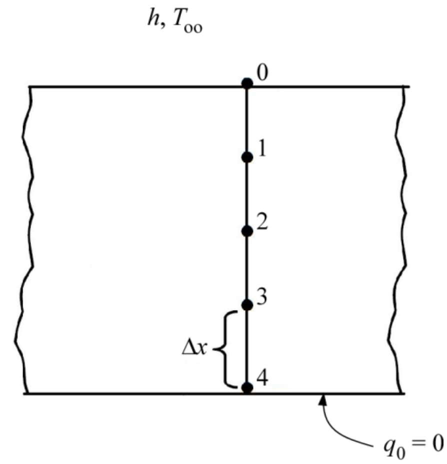
T_0 = **16.5°C**, T_1 = **15.6°C**, T_2 = **15.3°C**, T_3 = **15.3°C**, and T_4 = **20.2°C**

5-157 A hot brass plate is having its upper surface cooled by impinging jet while its lower surface is insulated. The explicit finite difference equations, the maximum allowable value of the time step, and the temperature at the center plane of the brass plate after 1 minute of cooling are to be determined.

Assumptions **1** Transient heat conduction is one-dimensional. **2** Thermal properties are constant. **3** Convection heat transfer coefficient is uniform. **4** Heat transfer by radiation is negligible. **5** There is no heat generation.

Properties The properties of the brass plate are given as ρ = 8530 kg/m^3, c_p = 380 J/kg·K, k = 110 W/m·K, and α = 33.9 × 10^{-6} m^2/s.

Analysis The nodal spacing is given to be Δx = 2.5 cm. Then the number of nodes becomes $M = L/\Delta x + 1$ = 10/2.5 + 1 = 5. This problem involves 5 unknown nodal temperatures, and thus we need to have 5 equations. The finite difference equation for node 0 on the top surface subjected to convection is obtained by applying an energy balance on the half volume element about node 0 and taking the direction of all heat transfers to be towards the node under consideration:



$$h(T_\infty - T_0^i) + k\frac{T_1^i - T_0^i}{\Delta x} = \rho\frac{\Delta x}{2}c_p\frac{T_0^{i+1} - T_0^i}{\Delta t}$$

or

$$T_0^{i+1} = \left(1 - 2\tau - 2\tau\frac{h\Delta x}{k}\right)T_0^i + \tau\left(2T_1^i + 2\frac{h\Delta x}{k}T_\infty\right)$$

Node 4 is on insulated boundary, and thus we can treat it as an interior node by using the mirror image concept. Nodes 1, 2, and 3 are interior nodes, and thus for them we can use the general explicit finite difference relation expressed as

$$T_{m-1}^i - 2T_m^i + T_{m+1}^i = \frac{T_m^{i+1} - T_m^i}{\tau}$$

or $$T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1 - 2\tau)T_m^i$$

Thus, the explicit finite difference equations are

Node 0: $T_0^{i+1} = \left(1 - 2\tau - 2\tau\frac{h\Delta x}{k}\right)T_0^i + \tau\left(2T_1^i + 2\frac{h\Delta x}{k}T_\infty\right)$

Node 1: $T_1^{i+1} = \tau(T_0^i + T_2^i) + (1 - 2\tau)T_1^i$

Node 2: $T_2^{i+1} = \tau(T_1^i + T_3^i) + (1 - 2\tau)T_2^i$

Node 3: $T_3^{i+1} = \tau(T_2^i + T_4^i) + (1 - 2\tau)T_3^i$

Node 4: $T_4^{i+1} = \tau(T_3^i + T_3^i) + (1 - 2\tau)T_4^i$

where

Δx = 2.5 cm, k = 110 W/m·K, h = 220 W/m^2·K, T_∞ = 15°C, α = 33.9 × 10^{-6} m^2/s, and $h\Delta x/k$ = 0.05.

(*b*) The upper limit of the time step Δt is determined from the stability criterion that requires all primary coefficients to be greater than or equal to zero. The coefficient of T_0^i is smaller in this case, and thus the stability criterion for this problem can be expressed as

$$1 - 2\tau - 2\tau\frac{h\Delta x}{k} \geq 0 \quad \rightarrow \quad \tau \leq \frac{1}{2(1 + h\Delta x/k)} \quad \rightarrow \quad \Delta t \leq \frac{\Delta x^2}{2\alpha(1 + h\Delta x/k)}$$

Since $\tau = \alpha\Delta t/\Delta x^2$. Substituting the given quantities, the maximum allowable value of the time step is determined to be

$$\Delta t \leq \frac{(0.025\,\text{m})^2}{2(33.9\times10^{-6}\ \text{m}^2/\text{s})[1 + (220\ \text{W/m}^2\cdot\text{K})(0.025\,\text{m})/(110\ \text{W/m}\cdot\text{K})]} = 8.779\,\text{s}$$

Therefore, any time step less than 8.779 s can be used to solve this problem. For convenience, let us choose the time step to be Δt = 6 s. Then the mesh Fourier number becomes

$$\tau = \frac{\alpha \Delta t}{\Delta x^2} = \frac{(33.9\times 10^{-6}\ \mathrm{m^2/s})(6\ \mathrm{s})}{(0.025\ \mathrm{m})^2} = 0.32544 \qquad (\text{for } \Delta t = 6\ \mathrm{s})$$

(*c*) With the initial nodal temperatures of 650°C, the results obtained from successive iterations are listed in the following table:

Time step, *i*	Time, s	Nodal temperature,°C				
		T_0^i	T_1^i	T_2^i	T_3^i	T_4^i
0	0	650	650	650	650	650
1	6	629.3	650	650	650	650
2	12	622.8	643.3	650	650	650
3	18	616.3	638.8	647.8	650	650
4	24	611.4	634.4	645.6	649.3	650
5	30	607.0	630.6	643.2	648.3	649.5
6	36	603.1	627.0	640.7	647.0	648.7
7	42	599.5	623.7	638.3	645.5	647.6
8	48	596.2	620.6	635.9	643.9	646.3
9	54	593.2	617.6	633.5	642.1	644.7
10	60	590.3	614.8	**631.1**	640.1	643.0

The temperature at the center plane of the brass plate after 1 minute of cooling is

$$T_2^{10} = T(0.05\ \mathrm{m}, 60\ \mathrm{s}) = \mathbf{631.1°C}$$

(*d*) From Chapter 4, the approximate analytical solution is given as

$$\theta_{\mathrm{wall}} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}\cos(\lambda_1 x/L)$$

where

$$Bi = \frac{hL}{k} = \frac{(220\ \mathrm{W/m^2\cdot K})(0.10\ \mathrm{m})}{110\ \mathrm{W/m\cdot K}} = 0.2$$

$$\tau = \frac{\alpha t}{L^2} = \frac{(33.9\times 10^{-6}\ \mathrm{m^2/s})(60\ \mathrm{s})}{(0.10\ \mathrm{m})^2} = 0.2034 > 0.2$$

$$\lambda_1 = 0.4328 \quad \text{and} \quad A_1 = 1.0311 \ \ (\text{from Table 4-2})$$

Hence,

$$T(x,t) = (T_i - T_\infty)A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 x/L) + T_\infty$$

$$\mathrm{T}(0.05\ \mathrm{m}, 60\ \mathrm{s}) = (650\ \mathrm{°C} - 15\ \mathrm{°C})(1.0311)\mathrm{e}^{-(0.4328)^2(0.2034)}\cos[(0.4328)(0.5)] + 15\ \mathrm{°C}$$
$$= 630.6\ \mathrm{°C}$$

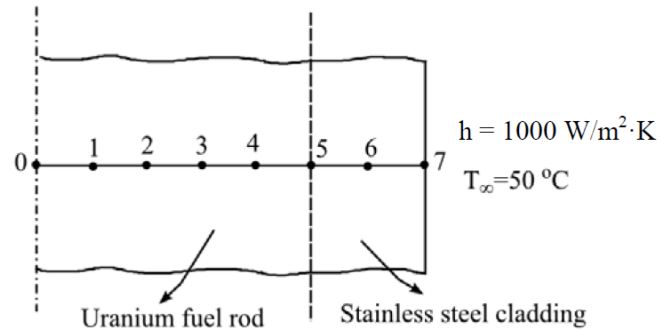
Discussion The comparison between the approximate analytical and numerical solutions is within ±0.08% agreement.

5-158 A uranium nuclear fuel rod experiencing a uniform heat generation and enclosed in a stainless steel cladding is cooled by pressurized water at specified conditions. The temperature distribution in the nuclear rod and the cladding material is to be determined at different time spans.

Assumptions **1** One-dimensional transient heat transfer with constant thermal conductivity. **2** Perfect contact between the fuel rod and cladding material. **3** No internal heat generation in the cladding material.

Properties For uranium nuclear fuel element we are given: k = 35 W/m·K, ρ = 19070 kg/m^3 and c_p = 116 J/kg·K. For stainless steel we are given: k = 15 W/m·K, ρ = 8055 kg/m^3 and c_p = 480 J/kg·K.

Analysis For the given geometry, nodes 0 and 7 are the boundary nodes while node 5 is the interface node. Use the finite difference formulation for the internal node as given by Eq. (5-48) for nodes 1, 2, 3, 4 and 6. Due to thermal symmetry about the centerline at node 0 it can be also treated as an internal node.



Node 0: $2T_1^{i+1} - 2T_0^{i+1} + \dfrac{\dot{e}_0 \Delta x^2}{k_1} = \dfrac{T_0^{i+1} - T_0^i}{\tau_1}$

Node 1: $T_0^{i+1} - 2T_1^{i+1} + T_2^{i+1} + \dfrac{\dot{e}_1 \Delta x^2}{k_1} = \dfrac{T_1^{i+1} - T_1^i}{\tau_1}$

Node 2: $T_1^{i+1} - 2T_2^{i+1} + T_3^{i+1} + \dfrac{\dot{e}_2 \Delta x^2}{k_1} = \dfrac{T_2^{i+1} - T_2^i}{\tau_1}$

Node 3: $T_2^{i+1} - 2T_3^{i+1} + T_4^{i+1} + \dfrac{\dot{e}_3 \Delta x^2}{k_1} = \dfrac{T_3^{i+1} - T_3^i}{\tau_1}$

Node 4: $T_3^{i+1} - 2T_4^{i+1} + T_5^{i+1} + \dfrac{\dot{e}_4 \Delta x^2}{k_1} = \dfrac{T_4^{i+1} - T_4^i}{\tau_1}$

Node 5: (Interface node) $k_1 \dfrac{\left(T_4^{i+1} - T_5^{i+1}\right)}{\Delta x} + k_2 \dfrac{\left(T_6^{i+1} - T_5^{i+1}\right)}{\Delta x} + \dot{e}_5 \dfrac{\Delta x}{2} = \dfrac{\Delta x}{2\Delta t}\left(\rho_1 c_{p,1} + \rho_2 c_{p,2}\right)\left(T_5^{i+1} - T_5^i\right)$

Node 6: $T_5^{i+1} - 2T_6^{i+1} + T_7^{i+1} = \dfrac{T_6^{i+1} - T_6^i}{\tau_2}$

Node 7: (Right boundary node) $2\left(T_6^{i+1} - T_7^{i+1}\right) + 2\dfrac{h\Delta x}{k_2}\left(T_\infty - T_7^{i+1}\right) = \dfrac{T_7^{i+1} - T_7^i}{\tau_2}$

The EES code used to solve the implicit finite difference equations is as follows

```
"Given data"
k_1 = 35 [W/mK] " Thermal conductivity of nuclear rod"
k_2 = 15 [W/mK] " Thermal conductivity of caldding material"
h = 1000 [W/m^2K] "Convective heat transfer coefficient"
T_infi = 50 [C]  " Temperature of cooling water"
DELTAx = 0.02 [m] "mesh size"
DELTAt = 60 [s]  " time step"
rho_1 = 19070 [kg/m^3] "Density of nuclear rod"
rho_2 = 8055 [kg/m^3] "Density of cladding material"
c_1 = 116 [J/kgK] "Specific heat of uranium rod"
c_2 = 480 [J/kgK] "Specific heat of cladding material"
e_gen = 4e5 [W/m^3] " Volumetric heat generation"

alpha_1 = k_1/(rho_1*c_1) " Thermal diffusivity of uranium rod"
alpha_2 = k_2/(rho_2*c_2) " Thermal diffusivity of cladding material"

tau_1 = (alpha_1*DELTAt)/DELTAx^2  " Mesh Fourier number"
tau_2 = (alpha_2*DELTAt)/DELTAx^2

"The technique is to store the temperatures in the parametric table and recover them (as old temperatures) using the variable ROW.  The first row contains the initial values so Solve Table must begin at row 2. Use the DUPLICATE statement to reduce the number of equations that need to be typed.  Column 1 contains the time, column 2 the value of T[1], column 3, the value of T[2], etc., and column 9 the Row.
```

To start the Solve Table at 2 go to 'Calculate' and select 'Solve table' (or hit F3) and make the 'First Run Number' as 2. The initial temperatures and the initial time ‘0’ can be set manually in the parametric table"

```
Row = TableRun#
Time = TableValue('Table 1', Row-1, #Time)+DELTAt/60

        Duplicate i = 0,7
                T_old[i] = TableValue('Table 1', Row-1, #T[i])
        End
" Finite difference equations"

"Node 0" 2*T[1]-2*T[0]+e_gen*DELTAx^2/k_1 = (T[0] - T_old[0])/tau_1
"Node 1" T[0]-2*T[1]+T[2]+e_gen*DELTAx^2/k_1 = (T[1] - T_old[1])/tau_1
"Node 2" T[1]-2*T[2]+T[3]+e_gen*DELTAx^2/k_1 = (T[2] - T_old[2])/tau_1
"Node 3" T[2]-2*T[3]+T[4]+e_gen*DELTAx^2/k_1 = (T[3] - T_old[3])/tau_1
"Node 4" T[3]-2*T[4]+T[5]+e_gen*DELTAx^2/k_1 = (T[4] - T_old[4])/tau_1
"Node 5" k_1*(T[4]-T[5])/DELTAx + k_2*(T[6]-T[5])/DELTAx+e_gen*(DELTAx/2) =
1/2*(((rho_1*c_1*DELTAx)/DELTAt)+(((rho_2*c_2*DELTAx)/DELTAt)))*(T[5]-T_old[5])
"Node 6" T[5]-2*T[6]+T[7] = (T[6] - T_old[6])/tau_2
"Node 7" 2*(T[6]-T[7])+2*DELTAx*h/k_2*(T_infi-T[7]) = (T[7]-T_old[7])/tau_2
```

Temperature distribution in the fuel rod and cladding at different times

Node temperature (ºC)	Time = 10 min	Time = 20 min	Time = 30 min
	Node temperature (ºC)		
T_0	468.3	389.8	339.1
T_1	464.1	386.1	335.9
T_2	451.4	375.2	326.5
T_3	430.3	357.1	310.9
T_4	400.8	331.9	289.1
T_5	362.9	299.8	261.3
T_6	257.2	213	187
T_7	140	120.4	109

Discussion In most of the practical cases, during the cooling of nuclear reactor rods, the water used for convective cooling undergoes a phase change process that enhances the rate of heat removal from the nuclear rods. Determination of the exact heat transfer rates to the cooling water and the estimation of pressure drop due to two- phase flow of water (as the water undergoes phase change) are quite challenging.

5-159 Starting with an energy balance on a volume element, the two-dimensional transient *explicit* finite difference equation for a general interior node in rectangular coordinates for $T(x, y, t)$ for the case of constant thermal conductivity k and uniform heat generation $\dot{e}_0$ is to be obtained.

Analysis (See Figure 5-24 in the text)**.** We consider a rectangular region in which heat conduction is significant in the x and y directions, and consider a unit depth of $\Delta z = 1$ in the z direction. There is uniform heat generation in the medium, and the thermal conductivity k of the medium is constant. Now we divide the x-y plane of the region into a *rectangular mesh* of nodal points which are spaced Δx and Δy apart in the x and y directions, respectively, and consider a general interior node (m, n) whose coordinates are $x = m\Delta x$ and $y = n\Delta y$. Noting that the volume element centered about the general interior node (m, n) involves heat conduction from four sides (right, left, top, and bottom) and expressing them at previous time step i, the transient explicit finite difference formulation for a general interior node can be expressed as

$$k(\Delta y\times 1)\frac{T^i_{m-1,n}-T^i_{m,n}}{\Delta x}+k(\Delta x\times 1)\frac{T^i_{m,n+1}-T^i_{m,n}}{\Delta y}+k(\Delta y\times 1)\frac{T^i_{m+1,n}-T^i_{m,n}}{\Delta x}+k(\Delta x\times 1)\frac{T^i_{m,n-1}-T^i_{m,n}}{\Delta y}$$

$$+\dot{e}_0(\Delta x\times\Delta y\times 1)=\rho(\Delta x\times\Delta y\times 1)c_p\frac{T^{i+1}_{m,n}-T^i_{m,n}}{\Delta t}$$

Taking a square mesh ($\Delta x = \Delta y = l$) and dividing each term by k gives, after simplifying,

$$T^i_{m-1,n}+T^i_{m+1,n}+T^i_{m,n+1}+T^i_{m,n-1}-4T^i_{m,n}+\frac{\dot{e}_0 l^2}{k}=\frac{T^{i+1}_{m,n}-T^i_{m,n}}{\tau}$$

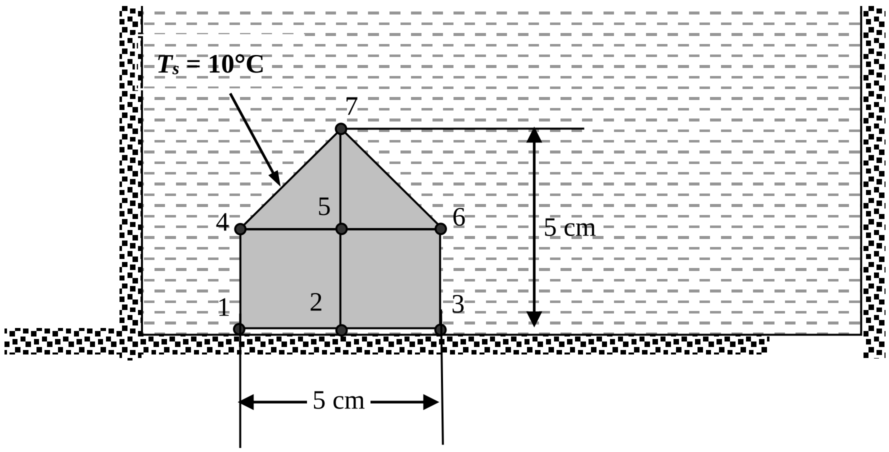
where $\alpha = k/\rho c_p$ is the thermal diffusivity of the material and $\tau = \alpha\Delta t/l^2$ is the dimensionless mesh Fourier number. It can also be expressed in terms of the temperatures at the neighboring nodes in the following easy-to-remember form:

$$T^i_{\text{left}}+T^i_{\text{top}}+T^i_{\text{right}}+T^i_{\text{bottom}}-4T^i_{\text{node}}+\frac{\dot{e}_0 l^2}{k}=\frac{T^{i+1}_{\text{node}}-T^i_{\text{node}}}{\tau}$$

Discussion We note that setting $T^{i+1}_{\text{node}} = T^i_{\text{node}}$ gives the steady finite difference formulation.

5-160 A two-dimensional long steel bar shown in the figure is considered. The finite difference equations for the unknown temperatures in the grid using the explicit method is to be written and dimensionless parameters are to be identified. Also, the range of time steps for stability condition and the temperature field at certain times are to be determined.

Assumptions **1** Heat transfer through the body is transient and two-dimensional. **2** All surafces of the bar except the bottom surface are maintained at a constant temperature. **3** Thermal conductivity is constant. **4** There is no heat generation.



Analysis (*a*) the finite difference equations for the unknown temperatures in the grid using the explicit method are

Node	T (10 s)	T (20 s)
1	10	10
2	443.3	234.4
3	10	10
4	10	10
5	315	168.6
6	10	10
7	10	10

Node 5:

$$k\frac{T_7^i - T_5^i}{\Delta x}\Delta x + k\frac{T_6^i - T_5^i}{\Delta x}\Delta x + k\frac{T_2^i - T_5^i}{\Delta x}\Delta x + k\frac{T_4^i - T_5^i}{\Delta x}\Delta x = \rho c_p \Delta x^2 \frac{T_5^{i+1} - T_5^i}{\Delta t}$$

$$T_7^i + T_6^i + T_2^i + T_4^i - 4T_5^i = \frac{\rho c \Delta x^2}{k\Delta t}(T_5^{i+1} - T_5^i) \qquad (1)$$

$$T_5^{i+1} = T_5^i(1 - 4\text{Fo}) + \text{Fo}T_2^i + \text{Fo}\times 30$$

where $\text{Fo} = \dfrac{k\Delta t}{\rho c_p \Delta x^2}$

Node 2:

$$k\frac{T_1^i - T_2^i}{\Delta x}\frac{\Delta x}{2} + k\frac{T_5^i - T_2^i}{\Delta x}\Delta x + k\frac{T_3^i - T_2^i}{\Delta x}\frac{\Delta x}{2} = \rho c_p \frac{\Delta x^2}{2}\frac{T_2^{i+1} - T_2^i}{\Delta t}$$

$$T_1^i + T_5^i + T_3^i - 4T_2^i = \frac{-\rho c_p \Delta x^2}{k\Delta t}(T_2^{i+1} - T_2^i) \qquad (2)$$

$$T_2^{i+1} = T_2^i(1 - 4\text{Fo}) + 2\text{Fo}T_5^i + \text{Fo}\times 20$$

(*b*) For both steps, stability condition is

$$1-4\text{Fo} \geq 0 \longrightarrow \text{Fo} \leq \frac{1}{4} \longrightarrow \frac{k\Delta t}{\rho c_p \Delta x^2} \leq \frac{1}{4}$$

$$\Delta t \leq \frac{\rho c_p \Delta x^2}{4k} = \frac{(8000)(430)(0.025)^2}{4(40)} = \mathbf{13.44\ s}$$

(*c*) For Δt = 10 s,

$$\text{Fo} = \frac{k\Delta t}{\rho c_p \Delta x^2} = \frac{(40)(10)}{(8000)(430)(0.025)^2} = 0.186$$

Then, Eq. (1) and (2) become

$$T_5^{i+1} = 0.256T_5^i + 0.186T_2^i + 5.58$$
$$T_2^{i+1} = 0.256T_2^i + 0.372T_5^i + 3.72$$

Substituting at Δt = 10 s,

$$T_5^1 = 0.256(700) + 0.186(700) + 5.58 = \mathbf{315°C}$$
$$T_2^1 = 0.256(700) + 0.372(700) + 3.72 = \mathbf{443.3°C}$$

Substituting at Δt = 20 s,

$$T_5^2 = 0.256(315) + 0.186(443) + 5.58 = \mathbf{168.6°C}$$
$$T_2^1 = 0.256(443.3) + 0.372(315) + 3.72 = \mathbf{234.4°C}$$

Fundamentals of Engineering (FE) Exam Problems

5-161 The unsteady forward-difference heat conduction for a constant area, A, pin fin with perimeter, p, exposed to air whose temperature is T_0 with a convection heat transfer coefficient of h is

$$T_m^{*+1} = \frac{k}{\rho c_p \Delta x^2}\left[T_{m-1}^* + T_{m+1}^* + \frac{hp\Delta x^2}{A}T_0\right] - \left[1 - \frac{2k}{\rho c_p \Delta x^2} - \frac{hp}{\rho c_p A}\right]T_m^*$$

In order for this equation to produce a stable solution, the quantity $\frac{2k}{\rho c_p \Delta x^2} + \frac{hp}{\rho c_p A}$ must be

(a) Negative (b) zero (c) Positive (d) Greater than 1 (e) Less than 1

Answer (d) Greater than 1

5-162 Air at T_0 acts on top surface of the rectangular solid shown in Fig. P5-162 with a convection heat transfer coefficient of h. The correct steady-state finite-difference heat conduction equation for node 3 of this solid is

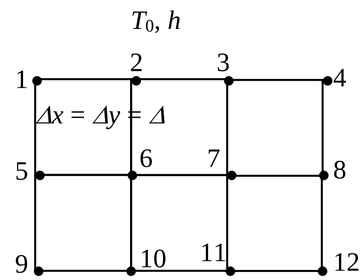
(a) $T_3 = [(k/2\Delta)(T_2 + T_4 + T_7) + hT_0] / [(k/\Delta) + h]$

(b) $T_3 = [(k/2\Delta)(T_2 + T_4 + 2T_7) + hT_0] / [(2k/\Delta) + h]$

(c) $T_3 = [(k/\Delta)(T_2 + T_4) + hT_0] / [(2k/\Delta) + h]$

(d) $T_3 = [(k/\Delta)(T_2 + T_4 + T_7) + hT_0] / [(k/\Delta) + h]$

(e) $T_3 = [(k/\Delta)(2T_2 + 2T_4 + T_7) + hT_0] / [(k/\Delta) + h]$



Answer (b)

5-163 What is the correct unsteady forward-difference heat conduction equation of node 6 of the rectangular solid shown in Fig. P5-163 if its temperature at the previous time (Δt) is T_6^*?

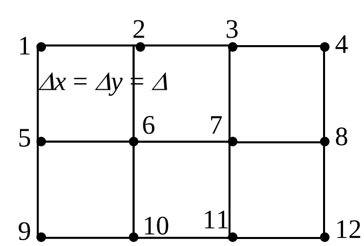
$(a)\ T_6^{i+1} = \left[k\Delta t/(\rho c_p \Delta^2)\right](T_5^* + T_2^* + T_7^* + T_{10}^*) + \left[1 - 4k\Delta t/(\rho c_p \Delta^2)\right]T_6^*$

$(b)\ T_6^{i+1} = \left[k\Delta t/(\rho c_p \Delta^2)\right](T_5^* + T_2^* + T_7^* + T_{10}^*) + \left[1 - k\Delta t/(\rho c_p \Delta^2)\right]T_6^*$

$(c)\ T_6^{i+1} = \left[k\Delta t/(\rho c_p \Delta^2)\right](T_5^* + T_2^* + T_7^* + T_{10}^*) + \left[2k\Delta t/(\rho c_p \Delta^2)\right]T_6^*$

$(d)\ T_6^{i+1} = \left[2k\Delta t/(\rho c_p \Delta^2)\right](T_5^* + T_2^* + T_7^* + T_{10}^*) + \left[1 - 2k\Delta t/(\rho c_p \Delta^2)\right]T_6^*$

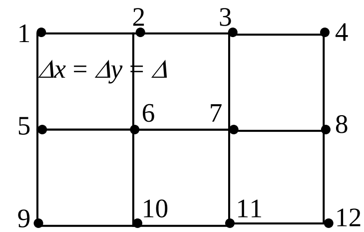
$(e)\ T_6^{i+1} = \left[2k\Delta t/(\rho c_p \Delta^2)\right](T_5^* + T_2^* + T_7^* + T_{10}^*) + \left[1 - 4k\Delta t/(\rho c_p \Delta^2)\right]T_6^*$



Answer (a)

5-164 What is the correct steady-state finite-difference heat conduction equation of node 6 of the rectangular solid shown in Fig. P5-164?

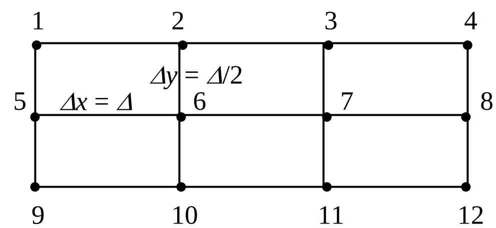
(a) $T_6 = (T_1 + T_3 + T_9 + T_{11}) / 2$
(b) $T_6 = (T_5 + T_7 + T_2 + T_{10}) / 2$
(c) $T_6 = (T_1 + T_3 + T_9 + T_{11}) / 4$
(d) $T_6 = (T_2 + T_5 + T_7 + T_{10}) / 4$
(e) $T_6 = (T_1 + T_2 + T_9 + T_{10}) / 4$



Answer (d) $T_6 = (T_2 + T_5 + T_7 + T_{10}) / 4$

5-165 The height of the cells for a finite-difference solution of the temperature in the rectangular solid shown in Fig. P5-165 is one-half the cell width to improve the accuracy of the solution. The correct steady-state finite-difference heat conduction equation for cell 6 is

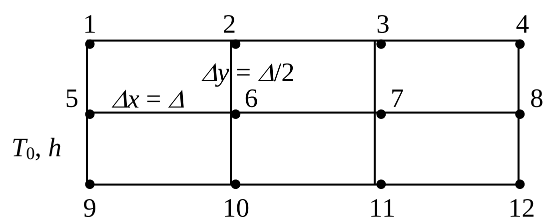
(a) $T_6 = 0.1(T_5 + T_7) + 0.4(T_2 + T_{10})$
(b) $T_6 = 0.25(T_5 + T_7) + 0.25(T_2 + T_{10})$
(c) $T_6 = 0.5(T_5 + T_7) + 0.5(T_2 + T_{10})$
(d) $T_6 = 0.4(T_5 + T_7) + 0.1(T_2 + T_{10})$
(e) $T_6 = 0.5(T_5 + T_7) + 0.5(T_2 + T_{10})$



Answer (a) $T_6 = 0.1(T_5 + T_7) + 0.4(T_2 + T_{10})$

5-166 The height of the cells for a finite-difference solution of the temperature in the rectangular solid shown in Fig. P5-166 is one-half the cell width to improve the accuracy of the solution. If the left surface is exposed to air at T_0 with a heat transfer coefficient of h, the correct finite-difference heat conduction energy balance for node 5 is

(a) $2T_1 + 2T_9 + T_6 - T_5 + h\Delta/k\,(T_0 - T_5) = 0$
(b) $2T_1 + 2T_9 + T_6 - 2T_5 + h\Delta/k\,(T_0 - T_5) = 0$
(c) $2T_1 + 2T_9 + T_6 - 3T_5 + h\Delta/k\,(T_0 - T_5) = 0$
(d) $2T_1 + 2T_9 + T_6 - 4T_5 + h\Delta/k\,(T_0 - T_5) = 0$
(e) $2T_1 + 2T_9 + T_6 - 5T_5 + h\Delta/k\,(T_0 - T_5) = 0$



Answer (e) $2T_1 + 2T_9 + T_6 - 5T_5 + h\Delta/k\,(T_0 - T_5) = 0$

5-167 ….. 5-170 Design and Essay Problems

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Chapter 6
FUNDAMENTALS OF CONVECTION

Mechanism and Types of Convection

6-1C A fluid flow during which the density of the fluid remains nearly constant is called *incompressible flow*. A fluid whose density is practically independent of pressure (such as a liquid) is called an incompressible fluid. The flow of compressible fluid (such as air) is not necessarily compressible since the density of a compressible fluid may still remain constant during flow.

6-2C In forced convection, the fluid is forced to flow over a surface or in a tube by external means such as a pump or a fan. In natural convection, any fluid motion is caused by natural means such as the buoyancy effect that manifests itself as the rise of the warmer fluid and the fall of the cooler fluid. The convection caused by winds is natural convection for the earth, but it is forced convection for bodies subjected to the winds since for the body it makes no difference whether the air motion is caused by a fan or by the winds.

6-3C If the fluid is forced to flow over a surface, it is called external forced convection. If it is forced to flow in a tube, it is called internal forced convection. A heat transfer system can involve both internal and external convection simultaneously. Example: A pipe transporting a fluid in a windy area.

6-4C The convection heat transfer coefficient will usually be higher in forced convection since heat transfer coefficient depends on the fluid velocity, and forced convection involves higher fluid velocities.

6-5C The potato will normally cool faster by blowing warm air to it despite the smaller temperature difference in this case since the fluid motion caused by blowing enhances the heat transfer coefficient considerably.

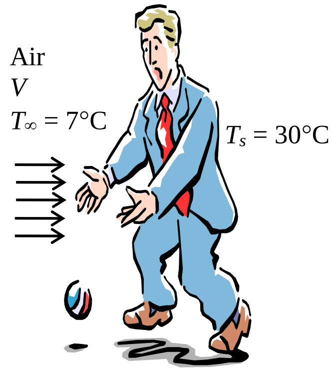
6-6C Nusselt number is the dimensionless convection heat transfer coefficient, and it represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer. It is defined as $Nu = \frac{hL_c}{k}$ where L_c is the characteristic length of the surface and k is the thermal conductivity of the fluid.

6-7C Heat transfer through a fluid is conduction in the absence of bulk fluid motion, and convection in the presence of it. The rate of heat transfer is higher in convection because of fluid motion. The value of the convection heat transfer coefficient depends on the fluid motion as well as the fluid properties. Thermal conductivity is a fluid property, and its value does not depend on the flow.

6-8 The rate of heat loss from an average man walking in still air is to be determined at different walking velocities.

Assumptions **1** Steady operating conditions exist. **2** Convection heat transfer coefficient is constant over the entire surface.

Analysis The convection heat transfer coefficients and the rate of heat losses at different walking velocities are



(*a*) $h = 8.6V^{0.53} = 8.6(0.5\text{ m/s})^{0.53} = 5.956\text{ W/m}^2.°\text{C}$

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.956\text{ W/m}^2.°\text{C})(1.8\text{ m}^2)(30-7)°\text{C} = \mathbf{246.6\ W}$$

(*b*) $h = 8.6V^{0.53} = 8.6(1.0\text{ m/s})^{0.53} = 8.60\text{ W/m}^2.°\text{C}$

$$\dot{Q} = hA_s(T_s - T_\infty) = (8.60\text{ W/m}^2.°\text{C})(1.8\text{ m}^2)(30-7)°\text{C} = \mathbf{356.0\ W}$$

(*c*) $h = 8.6V^{0.53} = 8.6(1.5\text{ m/s})^{0.53} = 10.66\text{ W/m}^2.°\text{C}$

$$\dot{Q} = hA_s(T_s - T_\infty) = (10.66\text{ W/m}^2.°\text{C})(1.8\text{ m}^2)(30-7)°\text{C} = \mathbf{441.3\ W}$$

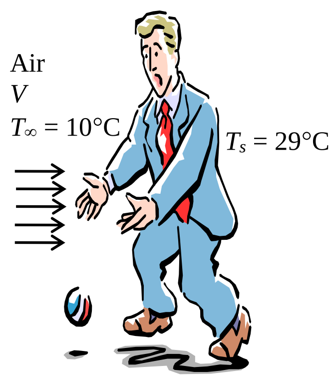
(*d*) $h = 8.6V^{0.53} = 8.6(2.0\text{ m/s})^{0.53} = 12.42\text{ W/m}^2.°\text{C}$

$$\dot{Q} = hA_s(T_s - T_\infty) = (12.42\text{ W/m}^2.°\text{C})(1.8\text{ m}^2)(30-7)°\text{C} = \mathbf{514.2\ W}$$

6-9 The rate of heat loss from an average man in windy air is to be determined at different wind velocities.

Assumptions **1** Steady operating conditions exist. **2** Convection heat transfer coefficient is constant over the entire surface.

Analysis The convection heat transfer coefficients and the rate of heat losses at different wind velocities are



(*a*) $h = 14.8V^{0.69} = 14.8(0.5\text{ m/s})^{0.69} = 9.174\text{ W/m}^2.°\text{C}$

$$\dot{Q} = hA_s(T_s - T_\infty) = (9.174\text{ W/m}^2.°\text{C})(1.7\text{ m}^2)(29-10)°\text{C} = \mathbf{296.3\ W}$$

(*b*) $h = 14.8V^{0.69} = 14.8(1.0\text{ m/s})^{0.69} = 14.8\text{ W/m}^2.°\text{C}$

$$\dot{Q} = hA_s(T_s - T_\infty) = (14.8\text{ W/m}^2.°\text{C})(1.7\text{ m}^2)(29-10)°\text{C} = \mathbf{478.0\ W}$$

(*c*) $h = 14.8V^{0.69} = 14.8(1.5\text{ m/s})^{0.69} = 19.58\text{ W/m}^2.°\text{C}$

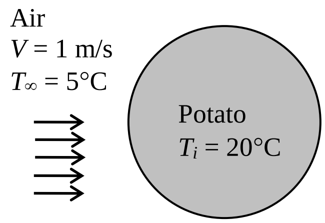
$$\dot{Q} = hA_s(T_s - T_\infty) = (19.58\text{ W/m}^2.°\text{C})(1.7\text{ m}^2)(29-10)°\text{C} = \mathbf{632.4\ W}$$

6-10 Heat transfer coefficients at different air velocities are given during air cooling of potatoes. The initial rate of heat transfer from a potato and the temperature gradient at the potato surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Potato is spherical in shape. **3** Convection heat transfer coefficient is constant over the entire surface.

Properties The thermal conductivity of the potato at the film temperature of $T_f = (T_s + T_\infty)/2 = (20°\text{C} + 5°\text{C})/2 = 12.5°\text{C}$ is $k_{\text{fluid}} = 0.02458$ W/m·K (from Table A-15).

Analysis The initial rate of heat transfer from a potato is



$$A_s = \pi D^2 = \pi(0.08\,\text{m})^2 = 0.02011\,\text{m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (19.1\,\text{W/m}^2.°\text{C})(0.02011\text{m}^2)(20-5)°\text{C} = \mathbf{5.8\ W}$$

where the heat transfer coefficient is obtained from the table at 1 m/s velocity. The initial value of the temperature gradient at the potato surface is

$$\dot{q}_{\text{conv}} = \dot{q}_{\text{cond}} = -k_{\text{fluid}}\left(\frac{\partial T}{\partial r}\right)_{r=R} = h(T_s - T_\infty)$$

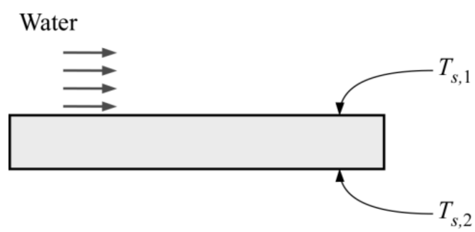
$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = -\frac{h(T_s - T_\infty)}{k} = -\frac{(19.1\,\text{W/m}^2.°\text{C})(20-5)°\text{C}}{0.02458\ \text{W/m.°C}} = \mathbf{-11{,}666°C/m}$$

6-11 The upper surface of a solid plate is being cooled by water. The water convection heat transfer coefficient and the water temperature gradient at the upper plate surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Heat conduction in solid is one-dimensional. **4** No-slip condition at the plate surface.

Properties The thermal conductivity of the solid plate is given as $k = 237$ W/m·K. The thermal conductivity of water at the film temperature of $T_f = (T_{s,1} + T_\infty)/2 = (60°\text{C} + 20°\text{C})/2 = 40°\text{C}$ is $k_{\text{fluid}} = 0.631$ W/m·K (from Table A-9).

Analysis Applying energy balance on the upper surface of the solid plate ($x = 0$), we have



$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} \quad \rightarrow \quad k\frac{T_{s,2} - T_{s,1}}{L} = h(T_{s,1} - T_\infty)$$

The convection heat transfer coefficient for the water is

$$h = \frac{k}{L}\left(\frac{T_{s,2} - T_{s,1}}{T_{s,1} - T_\infty}\right) = \frac{237\ \text{W/m}\cdot\text{K}}{0.50\ \text{m}}\left(\frac{120-60}{60-20}\right) = \mathbf{711\ W/m^2\cdot K}$$

The temperature gradient at the upper plate surface ($x = 0$) for the water is

$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = -\frac{h}{k_{\text{fluid}}}(T_{s,1} - T_\infty) = -\frac{711\,\text{W/m}^2\cdot\text{K}}{0.631\,\text{W/m}\cdot\text{K}}(60-20)\,\text{K} = \mathbf{-45{,}071\,K/m}$$

Discussion The film temperature is used to evaluate the thermal conductivity of water (k_{fluid}). This is to account for the effect of temperature on the thermal conductivity.

6-12 Airflow over a plate surface has a given temperature profile. The heat flux on the plate surface and the convection heat transfer coefficient of the airflow are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** No-slip condition at the plate surface. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivity and thermal diffusivity of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (220°\text{C} + 20°\text{C})/2 = 120°\text{C}$ are $k_{\text{fluid}} = 0.03235$ W/m·K and $\alpha_{\text{fluid}} = 3.565 \times 10^{-5}$ m^2/s (from Table A-15).

Analysis For no-slip condition, heat flux from the solid surface to the fluid layer adjacent to the surface is

$$\dot{q} = \dot{q}_{\text{cond}} = -k_{\text{fluid}} \left.\frac{\partial T}{\partial y}\right|_{y=0} = -(0.03235 \text{ W/m}\cdot\text{K})(-4.488\times 10^5 \text{ °C/m}) = \mathbf{1.452\times 10^4 \ W/m^2}$$

where the temperature gradient at the plate surface is

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = (T_\infty - T_s)\left(\frac{V}{\alpha_{\text{fluid}}}\right)\exp\left(-\frac{V}{\alpha_{\text{fluid}}}y\right)\Bigg|_{y=0}$$

$$= (T_\infty - T_s)\left(\frac{V}{\alpha_{\text{fluid}}}\right) = (20°\text{C} - 220°\text{C})\left(\frac{0.08 \text{ m/s}}{3.565\times 10^{-5} \text{ m}^2/\text{s}}\right)$$

$$= -4.488\times 10^5 \text{ °C/m}$$

The convection heat transfer coefficient of the airflow is

$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty} = \frac{-(0.03235 \text{ W/m}\cdot\text{K})(-4.488\times 10^5 \text{ °C/m})}{(220°\text{C} - 20°\text{C})} = \mathbf{72.6 \ W/m^2\cdot K}$$

Discussion The positive heat flux means that the plate is being cooled by the airflow that passes over the surface of the plate.

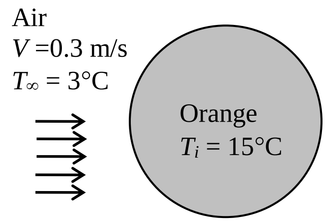
6-13 The expression for the heat transfer coefficient for air cooling of some fruits is given. The initial rate of heat transfer from an orange, the temperature gradient at the orange surface, and the value of the Nusselt number are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Orange is spherical in shape. **3** Convection heat transfer coefficient is constant over the entire surface. **4** Properties of water is used for orange.

Properties The thermal conductivity of the orange is given to be k = 0.70 W/m.°C. The thermal conductivity and the kinematic viscosity of air at the film temperature of $(T_s + T_\infty)/2 = (15+3)/2 = 9$°C are (Table A-15)

$$k = 0.02431\ \text{W/m.°C}, \qquad \nu = 1.417\times10^{-5}\ \text{m}^2/\text{s}$$

Analysis (*a*) The Reynolds number, the heat transfer coefficient, and the initial rate of heat transfer from an orange are



$$A_s = \pi D^2 = \pi(0.07\ \text{m})^2 = 0.01539\ \text{m}^2$$

$$\text{Re} = \frac{VD}{\nu} = \frac{(0.3\ \text{m/s})(0.07\ \text{m})}{1.417\times10^{-5}\ \text{m}^2/\text{s}} = 1482$$

$$h = \frac{5.05 k_{air}\,\text{Re}^{1/3}}{D} = \frac{5.05(0.02431\ \text{W/m.°C})(1482)^{1/3}}{0.07\ \text{m}} = 20.0\ \text{W/m}^2.\text{°C}$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (20.0\ \text{W/m}^2.\text{°C})(0.01539\ \text{m}^2)(15-3)\text{°C} = \mathbf{3.69\ W}$$

(*b*) The temperature gradient at the orange surface is determined from

$$\dot{q}_{conv} = \dot{q}_{cond} = -k\left(\frac{\partial T}{\partial r}\right)_{r=R} = h(T_s - T_\infty)$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = -\frac{h(T_s - T_\infty)}{k} = -\frac{(20.0\ \text{W/m}^2.\text{°C})(15-3)\text{°C}}{0.70\ \text{W/m.°C}} = \mathbf{-343\ °C/m}$$

(*c*) The Nusselt number is

$$\text{Nu} = \frac{hD}{k} = \frac{(20.0\ \text{W/m}^2.\text{°C})(0.07\ \text{m})}{0.02431\ \text{W/m.°C}} = \mathbf{57.6}$$

6-14 Heat transfer coefficient as a function of air velocity is given during air cooling of steel balls. The initial values of the heat flux and the temperature gradient in the steel ball at the surface are to be determined.

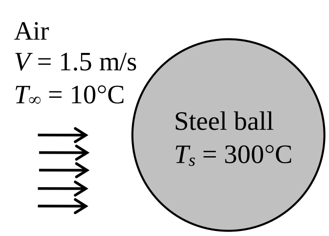
Assumptions **1** Steady operating conditions exist. **2** The thermal conductivity is constant. **3** Convection heat transfer coefficient is constant over the entire surface.

Properties The thermal conductivity of the steel ball is given to be k = 15 W/m·K.

Analysis The initial value of the heat flux in the steel ball at the surface is

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = h(T_s - T_\infty) = (22.28\ \text{W/m}^2\cdot\text{K})(300-10)\ \text{K} = \mathbf{6462\ W/m^2}$$

where



$$h = 17.9V^{0.54} = 17.9(1.5)^{0.54} = 22.28\ \text{W/m}^2\cdot\text{K}$$

The initial value of the temperature gradient in the steel ball at the surface is

$$\dot{q}_s = \dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} \quad\to\quad -k\left(\frac{\partial T}{\partial r}\right)_{r=R} = h(T_s - T_\infty)$$

$$\left.\frac{\partial T}{\partial r}\right|_{r=R} = -\frac{h(T_s - T_\infty)}{k} = -\frac{\dot{q}_s}{k} = -\frac{6462\ \text{W/m}^2}{15\ \text{W/m}\cdot\text{K}} = -\ \mathbf{431\ K/m}$$

Discussion The higher the temperature gradient in the steel ball, the higher the value of the heat flux would be.

6-15 An ASTM B152 copper plate is cooled by air at 20°C. The average convection heat transfer coefficient for the plate is to be evaluated from the correlation for local heat transfer coefficient. Determine whether the plate meets the ASME Code for Process Piping.

Assumptions **1** Steady operating conditions exist. **2** Uniform surface temperature. **3** Edge effects of plate are negligible.

Analysis The average convection heat transfer coefficient for the entire plate is determined by integrating h_x for $0 \le x \le L$:

$$h = \frac{1}{L}\int_0^L h_x\,dx = \frac{1}{L}\int_0^L 1.36x^{-0.5}\,dx = \frac{1.36}{L}\left(\frac{L^{0.5}}{0.5}\right) = 2.72L^{-0.5} = 2.72\ \text{W/m}^2\cdot\text{K}$$

where L = 1 m

From the Newton's law of cooling, the convection heat transfer rate is

$$\dot{Q} = hA(T_s - T_\infty)$$

Solving for the surface temperature T_s yields,

$$T_s = \frac{\dot{Q}}{hA} + T_\infty = \frac{700\ \text{W}}{(2.72\ \text{W/m}^2\cdot\text{K})(1\text{m}^2)} + 20\ ℃ = \mathbf{277}℃ > 260℃$$

Discussion The average convection heat transfer coefficient of 2.72 W/m^2·K is not sufficient to cool the plate surface to below the maximum use temperature of 260°C for the ASTM B152 copper plate. Thus, the use of the plate is not in compliance with the ASME Code for Process Piping.

6-16 An ASTM A240 410S stainless steel plate is exposed to cold gas, at −50°C, flowing in parallel over its surface. The correlation for the average convection heat transfer coefficient is known. The maximum velocity that the gas can achieve without cooling the plate below the suitable temperature set by the ASME Code for Process Piping is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Uniform surface temperature. **3** Edge effects of plate are negligible.

Analysis From the Newton's law of cooling, the convection heat transfer rate is

$$\dot{Q} = hA(T_s - T_\infty)$$

With $h = 6.5V^{0.8}$, we have

$$\dot{Q} \ge 6.5V^{0.8}A(T_s - T_\infty)$$

Rearranging to solve for the gas velocity V,

$$V \le \left[\frac{\dot{Q}}{6.5A(T_s - T_\infty)}\right]^{1/0.8} = \left[\frac{840}{6.5(1)(-30+50)}\right]^{1/0.8} = \mathbf{10.3\ m/s}$$

Discussion The maximum velocity that the gas can achieve without cooling the plate below the suitable temperature of −30°C is 10.3 m/s. If the gas velocity goes above 10.3 m/s, the convection heat transfer coefficient would increase. This renders the heat rate of 840 W supplied to the plate insufficient to keep the plate surface above −30°C.

6-17 C&S An ASTM B98 copper-silicon bolt connects two metal plates. Hot gas flows across the bolt. The minimum heat removal rate required to keep the bolt surface from going above the maximum use temperature set by the ASME Code for Process Piping is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Uniform surface temperature. **3** Wall effects from the plates are negligible.

Analysis From the Newton's law of cooling, the convection heat transfer rate is

$$\dot{Q} = hA(T_\infty - T_s)$$

With $h = 24.6V^{0.62}$, we have

$$\dot{Q} \geq 24.6V^{0.62}(\pi DL)(T_\infty - T_s) = [24.6(10.4)^{0.62}](\text{W/m}^2\cdot\text{K})\pi(0.0095\text{ m})(0.10\text{ m})(200 - 149)°\text{C} = \mathbf{16\ W}$$

Discussion To keep the surface temperature of the bolt from going above the maximum use temperature of 149°C, the minimum heat removal rate required is 16 W. If the heat removal rate is below 16 W, the surface temperature of the bolt will be higher than 149°C. Increasing the heat removal rate will reduce the surface temperature:
$T_s = T_\infty - \dot{Q}/(hA)$.

6-18 C&S An ASTM A479 904L stainless steel bar connects two metal plates. Hot gas flows across the square bar. The maximum velocity that the gas can achieve without heating the stainless steel bar above the maximum use temperature set by the ASME Code for Process Piping is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Uniform surface temperature. **3** Wall effects from the plates are negligible.

Analysis From the Newton's law of cooling, the convection heat transfer rate is

$$\dot{Q} = hA(T_\infty - T_s)$$

With $h = 13.6V^{0.675}$, we have

$$\dot{Q} \geq 13.6V^{0.675}(4wL)(T_\infty - T_s)$$

$$V \leq \left[\frac{\dot{Q}}{13.6(4wL)(T_\infty - T_s)}\right]^{1/0.675} = \left[\frac{100}{13.6(4 \times 0.02 \times 0.10)(400 - 260)}\right]^{1/0.675} = \mathbf{16.2\ m/s}$$

Discussion The maximum velocity that the gas can achieve without heating the stainless steel bar above the maximum use temperature of 260°C is 16.2 m/s. If the gas velocity goes above 16.2 m/s, the convection heat transfer coefficient would increase. This renders the heat removal rate of 100 W by the cooling mechanism insufficient to keep the surface temperature of the bar below 260°C.

6-19 Heat transfer coefficient as a function of air velocity is given during air cooling of chromium steel balls. The effect of air velocity on the temperature gradient in the chromium steel ball at the surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal conductivity is constant. **3** Convection heat transfer coefficient is constant over the entire surface.

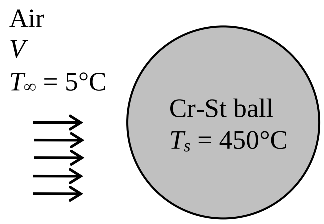
Properties The thermal conductivity of the steel ball is given to be k = 40 W/m·K.

Analysis The equation for the temperature gradient in the chromium steel ball at the surface is

$$\dot{q}_s = \dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} \quad \rightarrow \quad -k\left(\frac{\partial T}{\partial r}\right)_{r=R} = h(T_s - T_\infty)$$

or

$$\left(\frac{\partial T}{\partial r}\right)_{r=R} = -\frac{h(T_s - T_\infty)}{k} \qquad \text{where} \qquad h = 18.05V^{0.56}$$



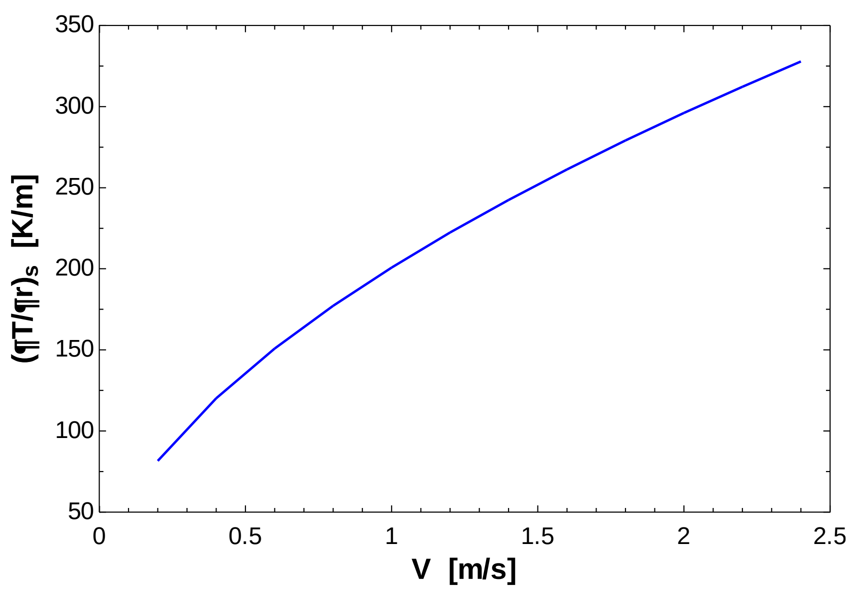
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
T_s=450 [C]
T_inf=5 [C]

"PROPERTIES"
k=40 [W/m-K]

"ANALYSIS"
h=18.05*V^0.56
dTdr_s=h*(T_s-T_inf)/k   "Temperature gradient at the surface"
```

V [m/s]	$(\partial T/\partial r)_{r=R}$ [K/m]
0.2	81.54
0.4	120.2
0.6	150.8
0.8	177.2
1.0	200.8
1.2	222.4
1.4	242.4
1.6	261.3
1.8	279.1
2.0	296.0
2.2	312.3
2.4	327.9



Discussion As the air velocity increases the surface temperature gradient in the chromium steel ball increases as well. Thus, the rate of heat removal from the chromium steel ball increases with increasing air velocity.

6-20 A metal plate surface is being cooled by convection. The ratio of the temperature gradient in the fluid to the temperature gradient in the plate at the surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal conductivities of the plate and the fluid are constant. **3** Convection heat transfer coefficient is constant over the entire surface. **4** No-slip condition at the plate surface.

Properties The thermal conductivities of the plate and the fluid are constant.

Analysis The temperature gradient in the fluid at the plate surface is

$$h = \frac{-k_{\text{fluid}}\,(\partial T/\partial y)_{\text{fluid},\,y=0}}{T_s - T_\infty} \quad\rightarrow\quad \left(\frac{\partial T}{\partial y}\right)_{\text{fluid},\,y=0} = -\frac{h}{k_{\text{fluid}}}(T_s - T_\infty)$$

The temperature gradient in the plate at the surface is

$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} \quad\rightarrow\quad -k_{\text{plate}}\left(\frac{\partial T}{\partial y}\right)_{\text{plate},\,y=0} = h(T_s - T_\infty)$$

or

$$\left(\frac{\partial T}{\partial y}\right)_{\text{plate},\,y=0} = -\frac{h}{k_{\text{plate}}}(T_s - T_\infty)$$

Thus, the ratio of the temperature gradient in the fluid to the temperature gradient in the plate at the surface is

$$\frac{(\partial T/\partial y)_{\text{fluid},\,y=0}}{(\partial T/\partial y)_{\text{plate},\,y=0}} = \frac{-\dfrac{h}{k_{\text{fluid}}}(T_s - T_\infty)}{-\dfrac{h}{k_{\text{plate}}}(T_s - T_\infty)} = \frac{k_{\text{plate}}}{k_{\text{fluid}}}$$

Discussion The temperature gradient in the fluid is larger than the temperature gradient in the plate, because the thermal conductivity of solid is generally larger than the thermal conductivity of fluid ($k_{\text{plate}} > k_{\text{fluid}}$).

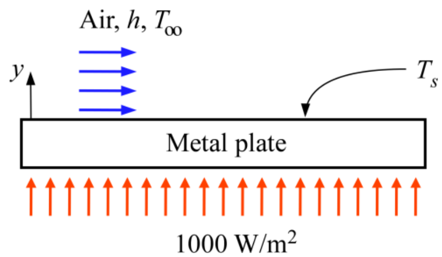
6-21 The upper surface of a metal plate is being cooled by air while the bottom surface is subjected to uniform heat flux. The temperature gradient in the air at the upper plate surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal conductivity of the fluid is constant. **3** Convection heat transfer coefficient is constant over the entire surface. **4** Heat conduction in solid is one-dimensional. **5** No-slip condition at the plate surface.

Properties The thermal conductivity of air is given as k_{fluid} = 0.259 W/m·K

Analysis The heat flux through the plate is

$$\dot{q}_s = \dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} \quad\rightarrow\quad \dot{q}_s = h(T_s - T_\infty)$$



The temperature gradient in the air at the upper plate surface is

$$h = \frac{-k_{\text{fluid}}\,(\partial T/\partial y)_{y=0}}{T_s - T_\infty}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = -\frac{h(T_s - T_\infty)}{k_{\text{fluid}}} = -\frac{\dot{q}_s}{k_{\text{fluid}}} = -\frac{1000\ \text{W/m}^2}{0.259\ \text{W/m}\cdot\text{K}} = \mathbf{-3861\ K/m}$$

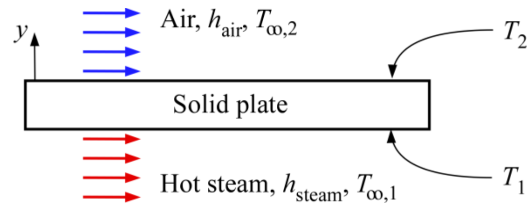
Discussion The temperature gradient in the air at the plate surface increases with decreasing k_{fluid}.

6-22 The top surface of a metal plate is being cooled by air while the bottom surface is subjected to a hot steam. The temperature gradient in the air and the temperature gradient in the plate at the top surface of the plate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal conductivity of fluid and the plate are constant. **3** Convection heat transfer coefficient is constant over the entire surface. **4** Heat conduction in solid is one-dimensional. **5** No-slip condition at the plate surface.

Properties The thermal conductivity of air is given as k_{air} = 0.243 W/m·K and the thermal conductivity of the plate is given as k_{plate} = 237 W/m·K.

Analysis The heat transfer rate through the plate is



$$\dot{Q}_{conv,1} = \dot{Q}_{conv,2} \quad \rightarrow \quad h_{steam}A(T_{\infty 1} - T_1) = h_{air}A(T_2 - T_{\infty 2})$$

The temperature gradient in the air at the top surface of the plate is

$$h_{air} = \frac{-k_{air}(\partial T/\partial y)_{air,\,y=0}}{T_2 - T_{\infty,2}}$$

$$\left(\frac{\partial T}{\partial y}\right)_{air,\,y=0} = -\frac{h_{air}(T_2 - T_{\infty 2})}{k_{air}} = -\frac{h_{steam}(T_{\infty 1} - T_1)}{k_{air}}$$

Thus,

$$\left(\frac{\partial T}{\partial y}\right)_{air,\,y=0} = -\frac{(30\ \text{W/m}^2\cdot\text{K})(100-80)\ \text{K}}{0.243\ \text{W/m}\cdot\text{K}} = \mathbf{-2469\ K/m}$$

The temperature gradient in the plate at the top surface is

$$-k_{plate}\left(\frac{\partial T}{\partial y}\right)_{plate,\,y=0} = h_{air}(T_2 - T_{\infty 2}) = h_{steam}(T_{\infty 1} - T_1) \quad \rightarrow \quad \left(\frac{\partial T}{\partial y}\right)_{plate,\,y=0} = -\frac{h_{steam}(T_{\infty 1} - T_1)}{k_{plate}}$$

Thus,

$$\left(\frac{\partial T}{\partial y}\right)_{plate,\,y=0} = -\frac{(30\ \text{W/m}^2\cdot\text{K})(100-80)\ \text{K}}{237\ \text{W/m}\cdot\text{K}} = \mathbf{-2.53\ K/m}$$

Discussion The temperature gradient in the air is larger than the temperature gradient in the plate, because the thermal conductivity of the plate is larger than the thermal conductivity of air ($k_{plate} > k_{air}$).

6-23 Heat transfer coefficient as a function of air velocity is given during air cooling of a flat plate. The effect of air velocity on the air temperature and plate temperature gradients at the plate surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal conductivities or air and plate are constant. **3** Convection heat transfer coefficient is constant over the entire surface. **4** No-slip condition at the plate surface.

Properties The thermal conductivity of the plate is given as k_{plate} = 1.4 W/m·K and the thermal conductivity of air is given as k_{air} = 0.0266 W/m·K.

Analysis The air temperature gradient at the plate surface is

$$h = \frac{-k_{\text{air}}(\partial T/\partial y)_{\text{air},y=0}}{T_s - T_\infty} \quad \rightarrow \quad \left(\frac{\partial T}{\partial y}\right)_{\text{air},y=0} = -\frac{h}{k_{\text{air}}}(T_s - T_\infty) \quad \text{where} \quad h = 27V^{0.85}$$

The temperature gradient in the plate at the surface is

$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} \quad \rightarrow \quad -k_{\text{plate}}\left(\frac{\partial T}{\partial y}\right)_{\text{plate},y=0} = h(T_s - T_\infty) \quad \text{or} \quad \left(\frac{\partial T}{\partial y}\right)_{\text{plate},y=0} = -\frac{h}{k_{\text{plate}}}(T_s - T_\infty)$$

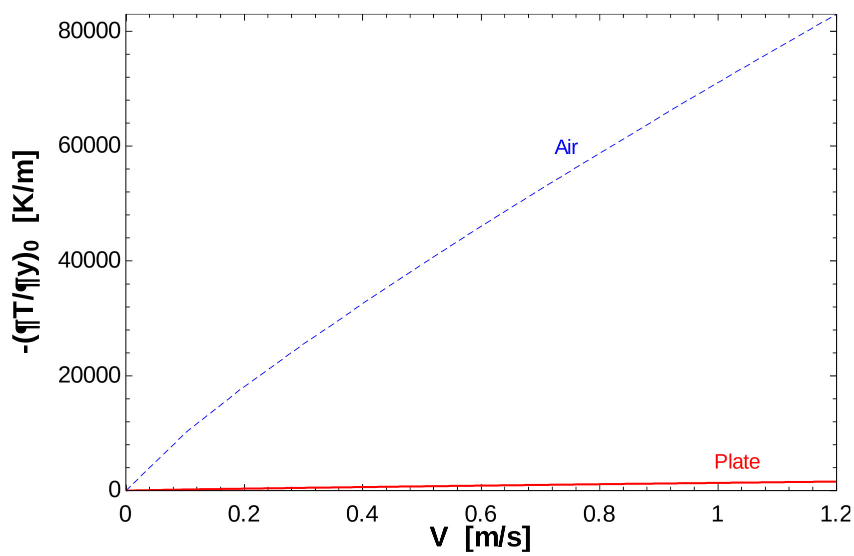
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
T_s=75 [C]
T_inf=5 [C]

"PROPERTIES"
k_plate=1.4 [W/m-K]
k_air=0.0266 [W/m-K]

"ANALYSIS"
h=27*V^0.85
dTdy_air=h*(T_s-T_inf)/k_air   "Air temperature gradient at the surface"
dTdy_plate=h*(T_s-T_inf)/k_plate   "Plate temperature gradient at the surface"
```

V [m/s]	$-(\partial T/\partial y)_{0,\text{air}}$ [K/m]	$-(\partial T/\partial y)_{0,\text{plate}}$ [K/m]
0	0	0
0.1	10036	190.7
0.2	18091	343.7
0.3	25535	485.2
0.4	32609	619.6
0.5	39419	749.0
0.6	46027	874.5
0.7	52470	996.9
0.8	58777	1117
0.9	64966	1234
1.0	71053	1350
1.1	77048	1464
1.2	82963	1576



Discussion The air temperature gradient is larger than the temperature gradient for the plate, because the thermal conductivity of the plate is larger than the thermal conductivity of air ($k_{\text{plate}} > k_{\text{air}}$).

6-24 A metal plate is being cooled by air, the plate temperature gradient at the surface after 2 minutes of cooling is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the metal plate are given as k = 180 W/m·K, ρ = 2800 kg/m^3, and c_p = 880 J/kg·K.

Analysis The characteristic length and the Biot number of the metal plate are

$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{10\text{ mm}}{2} = 5\text{ mm} \qquad \text{(Note: the plate thickness is } 2L = 10\text{ mm)}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{(30\text{ W/m}^2\cdot\text{K})(5\times10^{-3}\text{ m})}{(180\text{ W/m}\cdot\text{K})} = 0.0008333 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable, then

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{30\text{ W/m}^2\cdot\text{K}}{(2800\text{ kg/m}^3)(880\text{ J/kg}\cdot\text{K})(5\times10^{-3}\text{ m})} = 0.002435\text{ s}^{-1}$$

The temperature of the plate after 2 minutes (t = 120 s) of cooling is

$$\frac{T_s(t)-T_\infty}{T_{s,i}-T_\infty} = e^{-bt} \quad \rightarrow \quad T_s(t) = T_\infty + (T_{s,i}-T_\infty)e^{-bt}$$

$$T_s(t) = 5°\text{C} + (300-5)(°\text{C})\exp[-(0.002435\text{ s}^{-1})(120\text{ s})] = 225.3°\text{C}$$

The plate temperature gradient at the surface at t = 120 s is

$$-k\left.\frac{\partial T(t)}{\partial y}\right|_{y=0} = h[T_s(t)-T_\infty] \quad \rightarrow \quad \left.\frac{\partial T(t)}{\partial y}\right|_{y=0} = -\frac{h[T_s(t)-T_\infty]}{k}$$

Thus,

$$\left.\frac{\partial T(t)}{\partial y}\right|_{y=0} = -\frac{(30\text{ W/m}^2\cdot\text{K})(225.3-5)\text{ K}}{180\text{ W/m}\cdot\text{K}} = \mathbf{-36.7\text{ K/m}}$$

Discussion With respect to the y-direction on the plate, the negative temperature gradient at the surface indicates that heat is being removed from the plate.

6-25  Metal plates are being cooled by air. The effect of cooling time on the plates' temperature gradient at the surface is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the metal plate are given as k = 180 W/m·K, ρ = 2800 kg/m^3, and c_p = 880 J/kg·K.

Analysis The characteristic length and the Biot number of the metal plates are (note: plate thickness is 2L = 10 mm),

$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{10\text{ mm}}{2} = 5\text{ mm}, \qquad \text{Bi} = \frac{hL_c}{k} = \frac{(30\text{ W/m}^2\cdot\text{K})(5\times10^{-3}\text{ m})}{(180\text{ W/m}\cdot\text{K})} = 0.0008333 < 0.1$$

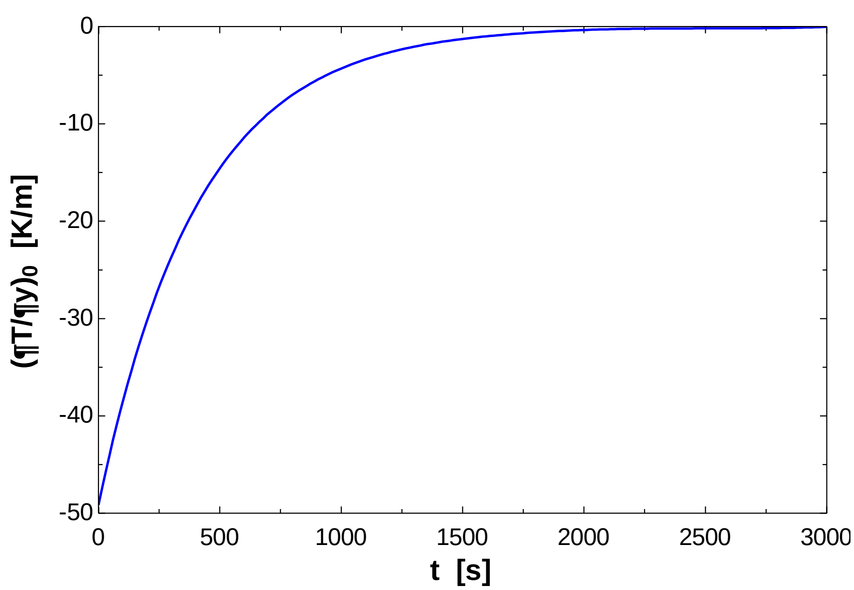
Thus, lumped system analysis is applicable. The problem is solved using EES, and the solution is given below:

```
"GIVEN"
h=30 [W/m^2-K]
L_c=10e-3/2 [m]
T_infinity=5 [C]
T_i=300 [C]

"PROPERTIES"
k=180 [W/m-K]
c_p=880 [J/kg-K]
rho=2800 [kg/m^3]

"ANALYSIS"
Bi=h*L_c/k
b=h/(rho*c_p*L_c)
(T_s-T_infinity)/(T_i-T_infinity)=exp(-b*t)
dTdy_s=-h*(T_s-T_infinity)/k   "Plate temperature gradient at the surface"
```

t [s]	$(\partial T/\partial y)_{0,\text{plate}}$ [K/m]
0	−49.17
100	−38.54
200	−30.21
300	−23.68
400	−18.56
500	−14.55
600	−11.41
700	−8.941
800	−7.009
900	−5.494
1000	−4.307
1200	−2.646
1600	−0.9991
2200	−0.2318
3000	−0.03304



Discussion As the plates cool to the air temperature, the temperature gradient at the surface approaches zero.

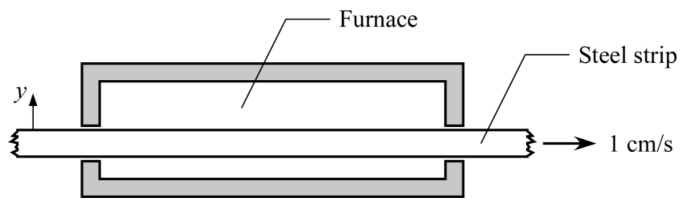
6-26 A stainless steel strip is heat treated as it moves through a furnace. The surface temperature gradient of the strip at mid-length of the furnace is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of stainless steel are given as k = 21 W/m·K, ρ = 8000 kg/m^3, and c_p = 570 J/kg·K.

Analysis The characteristic length and the Biot number of the stainless steel strip

$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{5\,\text{mm}}{2} = 2.5\,\text{mm}$$



(Note: the strip thickness is 2L = 5 mm)

$$\text{Bi} = \frac{hL_c}{k} = \frac{(80\ \text{W/m}^2\cdot\text{K})(2.5\times10^{-3}\ \text{m})}{(21\,\text{W/m}\cdot\text{K})} = 0.00952 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable, then

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{80\ \text{W/m}^2\cdot\text{K}}{(8000\,\text{kg/m}^3)(570\ \text{J/kg}\cdot\text{K})(2.5\times10^{-3}\ \text{m})} = 0.007018\,\text{s}^{-1}$$

The time of the stainless steel strip being heated can be determined from the furnace mid-length and the speed of the moving strip:

$$t = \frac{3\,\text{m}/2}{0.01\,\text{m/s}} = 150\,\text{s}$$

Thus, the temperature of the strip at mid-length of the furnace is (at t = 150 s)

$$\frac{T_s(t) - T_\infty}{T_{s,i} - T_\infty} = e^{-bt} \quad \rightarrow \quad T_s(t) = T_\infty + (T_{s,i} - T_\infty)e^{-bt}$$

$$T_s(t) = 900°\text{C} + (20 - 900)(°\text{C})\exp[-(0.007018\,\text{s}^{-1})(150\,\text{s})] = 592.9°\text{C}$$

The strip temperature gradient at the surface at t = 150 s is

$$-k\left.\frac{\partial T(t)}{\partial y}\right|_{y=0} = h[T_s(t) - T_\infty] \quad \rightarrow \quad \left.\frac{\partial T(t)}{\partial y}\right|_{y=0} = -\frac{h[T_s(t) - T_\infty]}{k}$$

Thus,

$$\left.\frac{\partial T(t)}{\partial y}\right|_{y=0} = -\frac{(80\ \text{W/m}^2\cdot\text{K})(592.9 - 900)\,\text{K}}{21\,\text{W/m}\cdot\text{K}} = \mathbf{1170\,K/m}$$

Discussion With respect to the y-direction on the strip, the positive temperature gradient at the surface indicates that heat is being added to the strip.

6-27 A steel strip is heat treated as it moves through a furnace. The surface temperature gradient of the strip as a function of the furnace location is to be determined.

Assumptions **1** The thermal properties are constant. **2** The heat transfer coefficient is uniform over the entire surface. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of stainless steel are given as k = 21 W/m·K, ρ = 8000 kg/m^3, and c_p = 570 J/kg·K.

Analysis The characteristic length and the Biot number of the steel strip are (note: plate thickness is $2L$ = 5 mm),

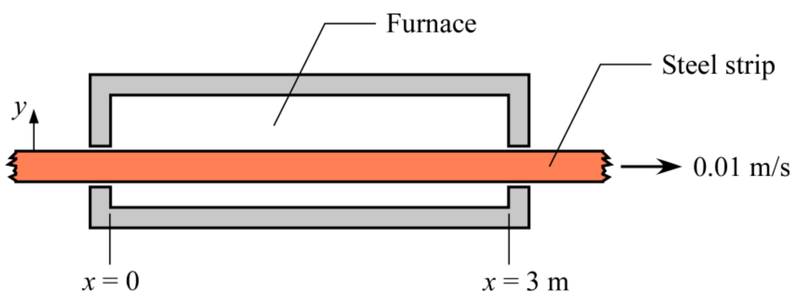
$$L_c = \frac{V}{A_s} = \frac{2LA}{2A} = L = \frac{5\,\text{mm}}{2} = 2.5\,\text{mm}, \qquad \text{Bi} = \frac{hL_c}{k} = \frac{(80\ \text{W/m}^2\cdot\text{K})(2.5\times10^{-3}\ \text{m})}{(21\,\text{W/m}\cdot\text{K})} = 0.00952 < 0.1$$

Thus, lumped system analysis is applicable. The problem is solved using EES, and the solution is given below:

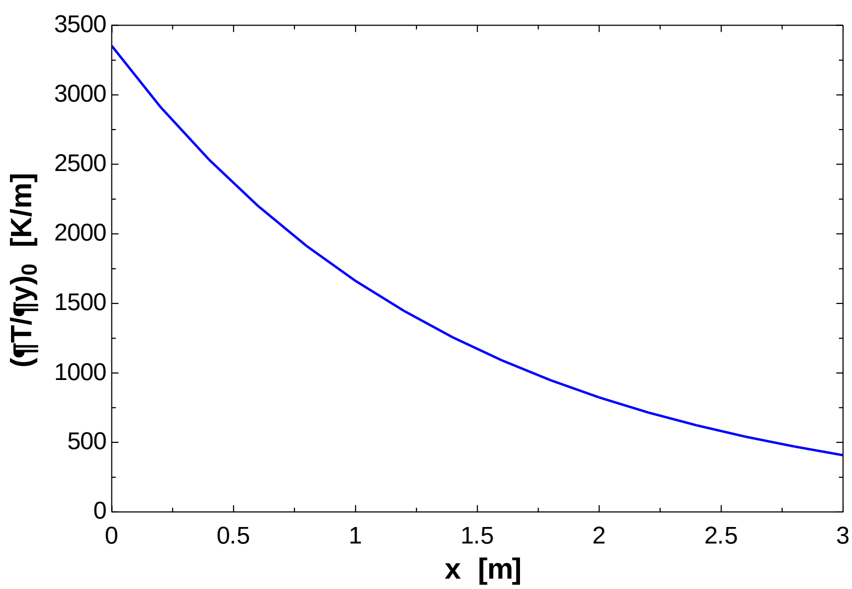
```
"GIVEN"
h=80 [W/m^2-K]
L_c=5e-3/2 [m]
T_infinity=900 [C]
T_i=20 [C]
V=0.01 [m/s]

"PROPERTIES"
k=21 [W/m-K]
c_p=570 [J/kg-K]
rho=8000 [kg/m^3]

"ANALYSIS"
t=x/V   "Cooling time"
Bi=h*L_c/k
b=h/(rho*c_p*L_c)
(T_s-T_infinity)/(T_i-T_infinity)=exp(-b*t)
dTdy_s=-h*(T_s-T_infinity)/k   "Surface temperature gradient of the strip"
```



x [m]	$(\partial T/\partial y)_{0,\text{strip}}$ [K/m]
0	3352
0.2	2913
0.4	2532
0.6	2200
0.8	1912
1.0	1662
1.2	1444
1.4	1255
1.6	1091
1.8	947.9
2.0	823.8
2.2	715.9
2.4	622.2
2.6	540.7
2.8	469.9
3.0	408.4



Discussion As the strip heats to the hot air in the furnace, the surface temperature gradient of the strip approaches zero. With respect to the y-direction on the strip, the positive temperature gradient at the surface indicates that heat is being added to the strip.

Boundary Layers and Flow Regimes

6-28C A fluid in direct contact with a solid surface sticks to the surface and there is no slip. This is known as the *no-slip condition*, and it is due to the viscosity of the fluid.

6-29C The fluids whose shear stress is proportional to the velocity gradient are called *Newtonian fluids*. Most common fluids such as water, air, gasoline, and oil are Newtonian fluids.

6-30C Viscosity is a measure of the “stickiness” or “resistance to deformation” of a fluid. It is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other. Viscosity is caused by the cohesive forces between the molecules in liquids, and by the molecular collisions in gases. Liquids have higher dynamic viscosities than gases.

6-31C The ball reaches the bottom of the container first in water due to lower viscosity of water compared to oil.

6-32C (*a*) The dynamic viscosity of liquids decreases with temperature. (*b*) The dynamic viscosity of gases increases with temperature.

6-33C The fluid viscosity is responsible for the development of the velocity boundary layer. For the idealized inviscid fluids (fluids with zero viscosity), there will be no velocity boundary layer.

6-34C The Prandtl number $\mathrm{Pr} = \nu / \alpha$ is a measure of the relative magnitudes of the diffusivity of momentum (and thus the development of the velocity boundary layer) and the diffusivity of heat (and thus the development of the thermal boundary layer). The Pr is a fluid property, and thus its value is independent of the type of flow and flow geometry. The Pr changes with temperature, but not pressure.

6-35C A thermal boundary layer will not develop in flow over a surface if both the fluid and the surface are at the same temperature since there will be no heat transfer in that case.

6-36C Reynolds number is the ratio of the inertial forces to viscous forces, and it serves as a criterion for determining the flow regime. For flow over a plate of length L it is defined as $\mathrm{Re} = VL/\nu$ where V is flow velocity and ν is the kinematic viscosity of the fluid.

6-37C A fluid motion is laminar when it involves smooth streamlines and highly ordered motion of molecules, and turbulent when it involves velocity fluctuations and highly disordered motion. The heat transfer coefficient is higher in turbulent flow.

6-38C The friction coefficient represents the resistance to fluid flow over a flat plate. It is proportional to the drag force acting on the plate. The drag coefficient for a flat surface is equivalent to the mean friction coefficient.

6-39C In turbulent flow, it is the *turbulent eddies* due to enhanced mixing that cause the friction factor to be larger.

6-40C Turbulent viscosity μ_t is caused by turbulent eddies, and it accounts for momentum transport by turbulent eddies. It is expressed as $\tau_t = -\rho\overline{u'v'} = \mu_t \dfrac{\partial \bar{u}}{\partial y}$ where $\bar{u}$ is the mean value of velocity in the flow direction and u' and u' are the fluctuating components of velocity.

6-41C Turbulent thermal conductivity k_t is caused by turbulent eddies, and it accounts for thermal energy transport by turbulent eddies. It is expressed as $\dot{q}_t = \rho c_p \overline{v'T'} = -k_t \dfrac{\partial \bar{T}}{\partial y}$ where T' is the eddy temperature relative to the mean value, and $\dot{q}_t = \rho c_p v'T'$ the rate of thermal energy transport by turbulent eddies.

6-42 Using the given velocity profile, the wall shear stresses for air and liquid water, are to be determined.

Assumptions **1** The fluid is Newtonian. **2** Properties are constant.

Properties The dynamic viscosities for air and liquid water at 20°C are 1.825 × 10^{-5} kg/m·s (Table A-15) and 1.002 × 10^{-3} kg/m·s (Table A-9), respectively.

Analysis The shear stress at the wall surface is

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = 100\mu\left[1 + 4y - 1.5y^2\right]_{y=0} = 100\mu$$

(*a*) For air

$$\tau_{s,\,\text{air}} = 100\mu_{\text{air}} = 100(1.825\times10^{-5})\ \text{N/m}^2 = \mathbf{1.825\times10^{-3}\ N/m^2}$$

(*b*) For water

$$\tau_{s,\,\text{H2O}} = 100\mu_{\text{H2O}} = 100(1.002\times10^{-3})\ \text{N/m}^2 = \mathbf{0.1002\ N/m^2}$$

Discussion For the same velocity profile, the wall shear stress ratio for liquid water and air is simply the ratio of the dynamic viscosity for both fluids:

$$\frac{\tau_{s,\,\text{H2O}}}{\tau_{s,\,\text{air}}} = \frac{\mu_{\text{H2O}}\left.(\partial u/\partial y)\right|_{y=0}}{\mu_{\text{air}}\left.(\partial u/\partial y)\right|_{y=0}} = \frac{\mu_{\text{H2O}}}{\mu_{\text{air}}} = 54.9$$

Hence the wall shear stress of liquid water flow over the surface is approximately fifty five times larger than that of air flow. Since liquid water is about fifty five times more viscous than air.

6-43 Using the given velocity and temperature profiles, the expressions for friction coefficient and convection heat transfer coefficient are to be determined.

Assumptions **1** The fluid is Newtonian. **2** Properties are constant. **3** No-slip condition at the plate surface.

Analysis The shear stress at the wall surface is

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = C_1\mu\left[1+2y-3y^2\right]_{y=0} = C_1\mu$$

The friction coefficient is

$$C_f = \frac{2\tau_s}{\rho V^2} = 2C_1\frac{\mu}{\rho V^2} = 2C_1\frac{\nu}{V^2}$$

The heat transfer convection coefficient is

$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty} = \frac{-k_{\text{fluid}}\left[2C_2 e^{-2C_2 y}\right]_{y=0}}{T_s - T_\infty} = \frac{-2C_2 k_{\text{fluid}}}{C_2 - 1 - T_\infty} \quad \text{where} \quad T_s = T(0) = C_2 - 1$$

Discussion Obtaining the expressions for friction and convection heat transfer coefficients is simple if the velocity and temperature profiles are known. However, determining the velocity and temperature profiles is generally not a simple matter in practice.

6-44 For air flowing over a flat plate, the wall shear stress and the air velocity gradient on the plate surface at mid-length of the plate are to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant. **3** Edge effects are negligible.

Properties The properties of air at 20°C are μ = 1.825 × 10^{-5} kg/m·s, ν = 1.516 × 10^{-5} m^2/s, and ρ = 1.204 kg/m^3 (Table A-15).

Analysis At mid-length of the plate (x = 0.5 m), the friction coefficient is

$$C_f = 0.664\left(\frac{Vx}{\nu}\right)^{-0.5} = 0.664\left[\frac{(7\text{ m/s})(0.5\text{ m})}{1.516\times10^{-5}\text{ m}^2/\text{s}}\right]^{-0.5} = 0.001382$$

The wall shear stress on the plate can be determined using

$$\tau_s = C_f\frac{\rho V^2}{2} = (0.001382)\frac{(1.204\text{ kg/m}^3)(7\text{ m/s})^2}{2} = \mathbf{0.04076\ N/m^2}$$

The velocity gradient at the plate surface is

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\tau_w}{\mu} = \frac{0.04076\text{ N/m}^2}{1.825\times10^{-5}\text{ kg/m}\cdot\text{s}} = \mathbf{2233.4\ s^{-1}}$$

Discussion For Newtonian fluids, such as air, the shear stress is proportional to the velocity gradient at the wall surface.

6-45 For air flowing over a flat plate, the effect of air velocity on the wall shear stress at x = 0.5 m and 1 m is to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant. **3** Edge effects are negligible.

Properties The properties of air at 20°C are $\nu = 1.516 \times 10^{-5}$ m²/s and ρ = 1.204 kg/m³ (Table A-15).

Analysis The friction coefficient and the wall shear stress can be determined with

$$C_f = 0.664\left(\frac{Vx}{\nu}\right)^{-0.5} \quad \text{and} \quad \tau_s = C_f \frac{\rho V^2}{2}$$

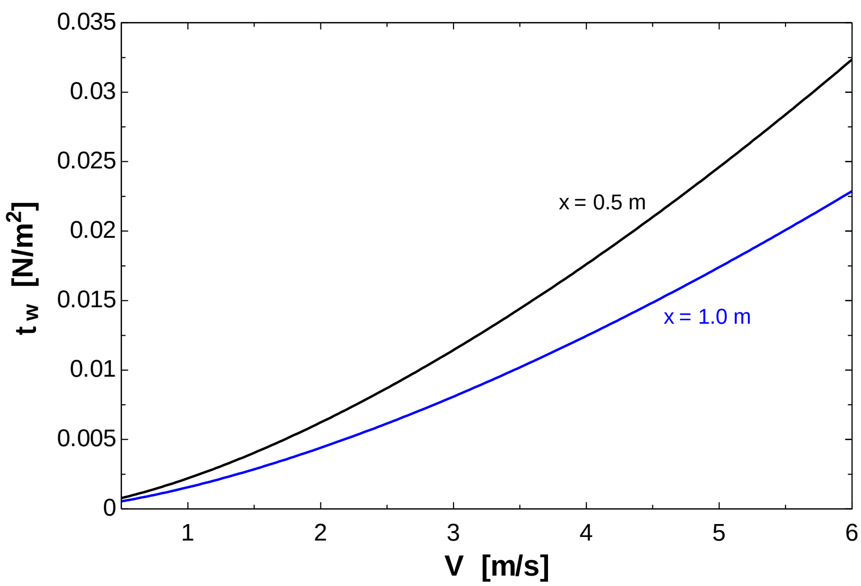
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
x_1=0.5 [m]
x_2=1.0 [m]

"PROPERTIES"
nu=1.516e-5 [m^2/s]
rho=1.204 [kg/m^3]

"ANALYSIS"
C_f1=0.664*(V*x_1/nu)^(-0.5)
C_f2=0.664*(V*x_2/nu)^(-0.5)
tau_w1=C_f1*rho*V^2/2   "Wall shear stress at x = 0.5 m"
tau_w2=C_f2*rho*V^2/2   "Wall shear stress at x = 1.0 m"
```

V [m/s]	τ_w (x = 0.5 m) [N/m²]	τ_w (x = 1 m) [N/m²]
0.5	0.0007782	0.0005503
1.0	0.002201	0.001556
1.5	0.004044	0.002859
2.0	0.006225	0.004402
2.5	0.008700	0.006152
3.0	0.01144	0.008087
3.5	0.01441	0.01019
4.0	0.01761	0.01245
4.5	0.02101	0.01486
5.0	0.02461	0.01740
5.5	0.02839	0.02008
6.0	0.03235	0.02287



Discussion As the air velocity increases, the wall shear stress also increases.

6-46 For air flowing over a flat plate at 5 m/s, the effect of plate location on the wall shear stress is to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant. **3** Edge effects are negligible.

Properties The properties of air at 20°C are $\nu = 1.516 \times 10^{-5}$ m^2/s and $\rho = 1.204$ kg/m^3 (Table A-15).

Analysis The friction coefficient and the wall shear stress can be determined with

$$C_f = 0.664\left(\frac{Vx}{\nu}\right)^{-0.5} \quad \text{and} \quad \tau_s = C_f \frac{\rho V^2}{2}$$

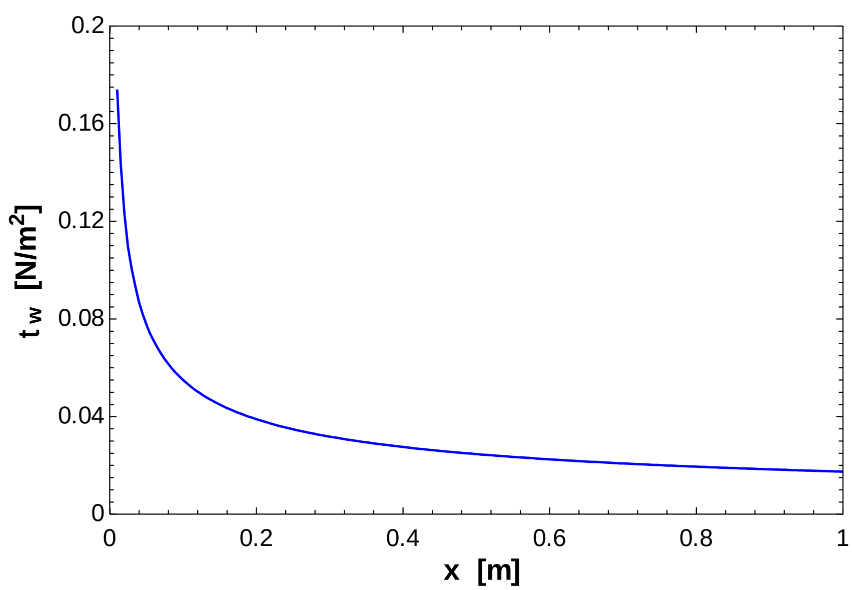
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
V=5 [m/s]

"PROPERTIES"
nu=1.516e-5 [m^2/s]
rho=1.204 [kg/m^3]

"ANALYSIS"
C_f=0.664*(V*x/nu)^(-0.5)
tau_w=C_f*rho*V^2/2
```

x [m]	τ_w [N/m^2]
0.01	0.1740
0.02	0.1230
0.03	0.1005
0.04	0.0870
0.06	0.07104
0.08	0.06152
0.10	0.05503
0.15	0.04493
0.20	0.03891
0.30	0.03177
0.40	0.02751
0.50	0.02461
0.60	0.02246
0.80	0.01945
1.00	0.01740



Discussion The wall shear stress decreases with increasing x. This is because as the air flows along the plate, the velocity gradient decreases with increasing velocity boundary layer thickness.

6-47 A flat plate is positioned inside a wind tunnel. The minimum length of the plate necessary for the Reynolds number to reach 1×10^5 is to be determined. The type of flow regime at 0.2 m from the leading edge is to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant.

Properties The kinematic viscosity for air at 20°C is $\nu = 1.516 \times 10^{-5}$ m²/s (Table A-15).

Analysis The Reynolds number is given as

$$\mathrm{Re} = \frac{VL_c}{\nu}$$

For the Reynolds number to reach 2×10^7, we need the minimum length of

$$L_c = \frac{\nu\,\mathrm{Re}}{V} = \frac{(1.516\times10^{-5}\ \mathrm{m^2/s})(2\times10^7)}{60\ \mathrm{m/s}} = \mathbf{5.053\,m}$$

At $L_c = 0.2$ m, the flow regime is

$$\mathrm{Re} = \frac{VL_c}{\nu} = \frac{(60\ \mathrm{m/s})(0.2\ \mathrm{m})}{1.516\times10^{-5}\ \mathrm{m^2/s}} = 7.92\times10^5 > 5\times10^5 \quad \rightarrow \quad \textbf{Flow is turbulent}$$

Discussion The distance from the leading edge necessary for the flow to reach turbulent regime is

$$x_{\mathrm{cr}} = \frac{\nu\,\mathrm{Re}_{\mathrm{cr}}}{V} = \frac{(1.516\times10^{-5}\ \mathrm{m^2/s})(5\times10^5)}{60\ \mathrm{m/s}} = 0.1263\ \mathrm{m}$$

6-48 For air flowing over a flat plate, the plate length to achieve a Reynolds number of 1×10^8 at the end of the plate and the distance from the leading edge of the plate at which transition would occur are to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant. **3** Edge effects are negligible.

Properties The kinematic viscosity of air at 25°C is $\nu = 1.562 \times 10^{-5}$ m²/s (Table A-15).

Analysis (a) The Reynolds number is given as

$$\mathrm{Re} = \frac{VL_c}{\nu}$$

For the Reynolds number to reach 1×10^8, we need a plate length of

$$L_c = \frac{\nu\,\mathrm{Re}}{V} = \frac{(1.562\times10^{-5}\ \mathrm{m^2/s})(1\times10^8)}{40\ \mathrm{m/s}} = \mathbf{39.1\,m}$$

(b) The distance from the leading edge for the transition to take place with a critical Reynolds number of 5×10^5 is

$$x_{\mathrm{cr}} = \frac{\nu\,\mathrm{Re}_{\mathrm{cr}}}{V} = \frac{(1.562\times10^{-5}\ \mathrm{m^2/s})(5\times10^5)}{40\ \mathrm{m/s}} = \mathbf{0.195\ m}$$

Discussion The expressions for the Reynolds number and the critical Reynolds number when combined would provide the following expression.

$$x_{cr} = \frac{\mathrm{Re}_{cr}}{\mathrm{Re}}\,L_c = \frac{5\times10^5}{1\times10^8}\,(39.1\,\mathrm{m}) = \mathbf{0.195\ m}$$

The above expression the knowledge of Re and $\mathrm{Re}_{\mathrm{cr}}$ can be used to quickly establish the location of the transition along the plate.

6-49 Fluid is flowing over a flat plate. The distance from the leading edge at which the transition from laminar to turbulent flow occurs for different fluids is to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant.

Properties The kinematic viscosities for the different fluids are listed in the following table:

Fluid	*Table*	*Kinematic viscosity*, m^2/s
Air (1 atm, 20°C)	A-15	1.516×10^{-5}
Liq. water (20°C)	A-9	1.004×10^{-6}
Methanol (20°C)	A-13	7.429×10^{-7}
Engine oil (20°C)	A-13	9.429×10^{-4}
Mercury (25°C)	A-14	1.133×10^{-7}

Analysis The distance from the leading edge at which the transition from laminar to turbulent flow occurs is calculated using

$$x_{\mathrm{cr}} = \frac{\nu \,\mathrm{Re}_{\mathrm{cr}}}{V}$$

Substituting the appropriate kinematic viscosities, we have

Fluid	x_c, m
Air (1 atm, 20°C)	1.52
Liq. water (20°C)	0.100
Methanol (20°C)	0.0743
Engine oil (20°C)	94.3
Mercury (25°C)	0.0113

Discussion The distance required by the flow to reach turbulent regime increases with increasing value of kinematic viscosity. Engine oil has the highest kinematic viscosity and requires the longest length and mercury with the smallest value of kinematic viscosity requires the shortest distance.

6-50E Fluid is flowing over a flat plate. The distance from the leading edge at which the transition from laminar to turbulent flow occurs for different fluids is to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant.

Properties The kinematic viscosities for the different fluids at 50°F are listed in the following table:

Fluid	*Table*	*Kinematic viscosity*, ft^2/s
Air (1 atm)	A-15E	1.535×10^{-4}
Liq. water	A-9E	1.407×10^{-5}
Isobutane	A-13E	3.368×10^{-6}
Engine oil	A-13E	2.169×10^{-2}
Mercury	A-14E	1.289×10^{-6}

Analysis The distance from the leading edge at which the transition from laminar to turbulent flow occurs is calculated using

$$x_{cr} = \frac{\nu \, \mathrm{Re}_{cr}}{V}$$

Substituting the appropriate kinematic viscosities, we have

Fluid	x_c, ft
Air (1 atm)	76.8
Liq. water	7.04
Isobutane	1.68
Engine oil	10845
Mercury	0.645

Discussion The distance required by the flow to reach turbulent regime increases with increasing value of kinematic viscosity. Mercury due to its low kinematic viscosity value, can achieve turbulent flow at a relatively short distance from the leading edge.

Convection Equations and Similarity Solutions

6-51C For steady, laminar, two-dimensional, incompressible flow with constant properties and a Prandtl number of unity and a given geometry, yes, it is correct to say that both the average friction and heat transfer coefficients depend on the Reynolds number only since $C_f = f_4(\mathrm{Re}_L)$ and $\mathrm{Nu} = g_3(\mathrm{Re}_L, \mathrm{Pr})$ from non-dimensionalized momentum and energy equations.

6-52C The continuity equation for steady two-dimensional flow is expressed as $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$. When multiplied by density, the first and the second terms represent net mass fluxes in the *x* and *y* directions, respectively.

6-53C *Steady* simply means no change with time at a specified location (and thus $\partial u/\partial t = 0$), but the value of a quantity may change from one location to another (and thus $\partial u/\partial x$ and $\partial u/\partial y$ may be different from zero). Even in steady flow and thus constant mass flow rate, a fluid may accelerate. In the case of a water nozzle, for example, the velocity of water remains constant at a specified point, but it changes from inlet to the exit (water accelerates along the nozzle).

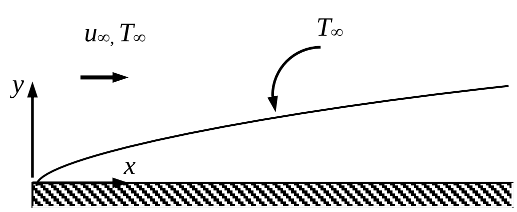
6-54C In a boundary layer during steady two-dimensional flow, the velocity component in the flow direction is much larger than that in the normal direction, and thus $u >> v$, and $\partial v/\partial x$ and $\partial v/\partial y$ are negligible. Also, *u* varies greatly with *y* in the normal direction from zero at the wall surface to nearly the free-stream value across the relatively thin boundary layer, while the variation of *u* with *x* along the flow is typically small. Therefore, $\partial u/\partial y >> \partial u/\partial x$. Similarly, if the fluid and the wall are at different temperatures and the fluid is heated or cooled during flow, heat conduction will occur primarily in the direction normal to the surface, and thus $\partial T/\partial y >> \partial T/\partial x$. That is, the velocity and temperature gradients normal to the surface are much greater than those along the surface. These simplifications are known as the **boundary layer approximations**.

6-55C For flows with low velocity and for fluids with low viscosity the viscous dissipation term in the energy equation is likely to be negligible.

6-56C For steady two-dimensional flow over an isothermal flat plate in the *x*-direction, the boundary conditions for the velocity components *u* and *v*, and the temperature *T* at the plate surface and at the edge of the boundary layer are expressed as follows:

At $y = 0$: $u(x, 0) = 0$, $v(x, 0) = 0$, $T(x, 0) = T_s$

As $y \to \infty$: $u(x, \infty) = u_\infty$, $T(x, \infty) = T_\infty$



6-57C An independent variable that makes it possible to transforming a set of partial differential equations into a single ordinary differential equation is called a **similarity variable**. A similarity solution is likely to exist for a set of partial differential equations if there is a function that remains unchanged (such as the non-dimensional velocity profile on a flat plate).

6-58C During steady, laminar, two-dimensional flow over an isothermal plate, the thickness of the velocity boundary layer (*a*) increases with distance from the leading edge, (*b*) decreases with free-stream velocity, and (*c*) and increases with kinematic viscosity

6-59C During steady, laminar, two-dimensional flow over an isothermal plate, the wall shear stress decreases with distance from the leading edge

6-60C A major advantage of nondimensionalizing the convection equations is the significant reduction in the number of parameters [the original problem involves 6 parameters (L, V , T_∞, T_s, ν, α), but the nondimensionalized problem involves just 2 parameters (Re_L and Pr)]. Nondimensionalization also results in similarity parameters (such as Reynolds and Prandtl numbers) that enable us to group the results of a large number of experiments and to report them conveniently in terms of such parameters.

6-61C A curved surface can be treated as a flat surface if there is no flow separation and the curvature effects are negligible.

6-62 A shaft rotating in a bearing is considered. The power required to rotate the shaft is to be determined for different fluids in the gap.

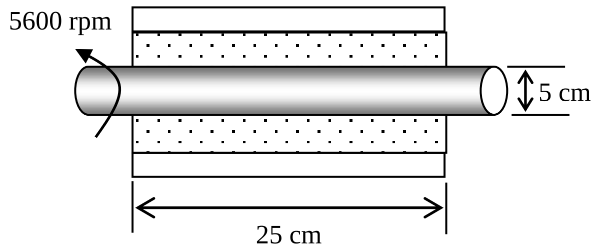
Assumptions **1** Steady operating conditions exist. **2** The fluid has constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of air, water, and oil at 40°C are (Tables A-15, A-9, A-13)

Air: $\mu = 1.918\times10^{-5}$ N·s/m²

Water: $\mu = 0.653\times10^{-3}$ N·s/m²

Oil: $\mu = 0.2177$ N·s/m²



Analysis A shaft rotating in a bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. Therefore, we solve this problem considering such a flow with the plates separated by a L=0.5 mm thick fluid film similar to the problem given in Example 6-1. By simplifying and solving the continuity, momentum, and energy equations it is found in Example 6-1 that

$$\dot{W}_{\text{mech}} = \dot{Q}_0 = -\dot{Q}_L = -kA\left.\frac{dT}{dy}\right|_{y=0} = -kA\frac{\mu V^2}{2kL}(1-0) = -A\frac{\mu V^2}{2L} = -A\frac{\mu V^2}{2L}$$

First, the velocity and the surface area are

$$V = \pi D\dot{N} = \pi(0.05\,\text{m})(5600\,\text{rev/min})\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 14.66\,\text{m/s}$$

$$A = \pi DL_{\text{bearing}} = \pi(0.05\,\text{m})(0.25\,\text{m}) = 0.03927\,\text{m}^2$$

(*a*) Air:

$$\dot{W}_{\text{mech}} = -A\frac{\mu V^2}{2L} = -(0.03927\,\text{m}^2)\frac{(1.918\times10^{-5}\ \text{N}\cdot\text{s/m}^2)(14.66\,\text{m/s})^2}{2(0.0005\,\text{m})}\left(\frac{1\,\text{W}}{1\,\text{N}\cdot\text{m/s}}\right) = \mathbf{-0.162\,W}$$

(*b*) Water:

$$\dot{W}_{\text{mech}} = \dot{Q}_0 = -A\frac{\mu V^2}{2L} = -(0.03927\,\text{m}^2)\frac{(0.653\times10^{-3}\ \text{N}\cdot\text{s/m}^2)(14.66\,\text{m/s})^2}{2(0.0005\,\text{m})}\left(\frac{1\,\text{W}}{1\,\text{N}\cdot\text{m/s}}\right) = \mathbf{-5.51\,W}$$

(*c*) Oil:

$$\dot{W}_{\text{mech}} = \dot{Q}_0 = -A\frac{\mu V^2}{2L} = -(0.03927\,\text{m}^2)\frac{(0.2177\ \text{N}\cdot\text{s/m}^2)(14.66\,\text{m/s})^2}{2(0.0005\,\text{m})}\left(\frac{1\,\text{W}}{1\,\text{N}\cdot\text{m/s}}\right) = \mathbf{-1837\,W}$$

6-63 Parallel flow of oil between two plates is considered. The velocity and temperature distributions, the maximum temperature, and the heat flux are to be determined.

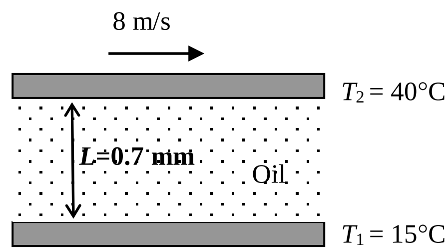
Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible. **4** The plates are large so that there is no variation in *z* direction.

Properties The properties of oil at the average temperature of (40+15)/2 = 27.5°C are (Table A-13):

k = 0.145 W/m·K and μ = 0.605 kg/m·s = 0.605 N·s/m^2

Analysis (*a*) We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus v = 0. Then the continuity equation reduces to

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u=u(y)$$



Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, v = 0, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation (Eq. 6-28) reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2 u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1 y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0)$ = 0 and $u(L) = V$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}V$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and v = 0. Then the energy equation with dissipation (Eqs. 6-36 and 6-37) reduce to

Energy: $$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2 T}{dy^2}=-\mu\left(\frac{V}{L}\right)^2$$

since $\partial u/\partial y = V/L$. Dividing both sides by *k* and integrating twice give

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}V\right)^2+C_3 y+C_4$$

Applying the boundary conditions $T(0) = T_1$ and $T(L) = T_2$ gives the temperature distribution to be

$$T(y)=\frac{T_2-T_1}{L}y+T_1+\frac{\mu V^2}{2k}\left(\frac{y}{L}-\frac{y^2}{L^2}\right)$$

(*b*) The temperature gradient is determined by differentiating $T(y)$ with respect to *y*,

$$\frac{dT}{dy}=\frac{T_2-T_1}{L}+\frac{\mu V^2}{2kL}\left(1-2\frac{y}{L}\right)$$

The location of maximum temperature is determined by setting dT/dy = 0 and solving for *y*,

$$\frac{dT}{dy}=\frac{T_2-T_1}{L}+\frac{\mu V^2}{2kL}\left(1-2\frac{y}{L}\right)=0 \longrightarrow y=L\left(k\frac{T_2-T_1}{\mu V^2}+\frac{1}{2}\right)$$

The maximum temperature is the value of temperature at this *y*, whose numeric value is

$$y=L\left(k\frac{T_2-T_1}{\mu V^2}+\frac{1}{2}\right)=(0.0007\text{ m})\left[(0.145\text{ W/m.°C})\frac{(40-15)\text{°C}}{(0.605\text{ N.s/m}^2)(8\text{ m/s})^2}+\frac{1}{2}\right]$$
$$=0.0004155\text{ m}=\mathbf{0.4155\ mm}$$

Then

$$T_{\max} = T(0.0004155) = \frac{T_2 - T_1}{L} y + T_1 + \frac{\mu V^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

$$= \frac{(40-15)°\text{C}}{0.0007\text{ m}}(0.0004155\text{ m}) + 15°\text{C} + \frac{(0.605\text{ N}\cdot\text{s/m}^2)(8\text{ m/s})^2}{2(0.145\text{ W/m}\cdot°\text{C})}\left(\frac{0.0004155\text{ m}}{0.0007\text{ m}} - \frac{(0.0004155\text{ m})^2}{(0.0007\text{ m})^2}\right)$$

$$= \mathbf{62.0°C}$$

(c) Heat flux at the plates is determined from the definition of heat flux,

$$\dot{q}_0 = -k\left.\frac{dT}{dy}\right|_{y=0} = -k\frac{T_2 - T_1}{L} - k\frac{\mu V^2}{2kL}(1-0) = -k\frac{T_2 - T_1}{L} - \frac{\mu V^2}{2L}$$

$$= -(0.145\text{ W/m.°C})\frac{(40-15)°\text{C}}{0.0007\text{ m}} - \frac{(0.605\text{ N}\cdot\text{s/m}^2)(8\text{ m/s})^2}{2(0.0007\text{ m})}\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = \mathbf{-3.28\times 10^4\ W/m^2}$$

$$\dot{q}_L = -k\left.\frac{dT}{dy}\right|_{y=L} = -k\frac{T_2 - T_1}{L} - k\frac{\mu V^2}{2kL}(1-2) = -k\frac{T_2 - T_1}{L} + \frac{\mu V^2}{2L}$$

$$= -(0.145\text{ W/m.°C})\frac{(40-15)°\text{C}}{0.0007\text{ m}} + \frac{(0.605\text{ N}\cdot\text{s/m}^2)(8\text{ m/s})^2}{2(0.0007\text{ m})}\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = \mathbf{2.25\times 10^4\ W/m^2}$$

Discussion A temperature rise of about 35°C confirms our suspicion that viscous dissipation is very significant. Calculations are done using oil properties at 27.5°C, but the oil temperature turned out to be much higher. Therefore, knowing the strong dependence of viscosity on temperature, calculations should be repeated using properties at the average temperature of about 45°C to improve accuracy.

6-64 Parallel flow of oil between two plates is considered. The velocity and temperature distributions, the maximum temperature, and the heat flux are to be determined.

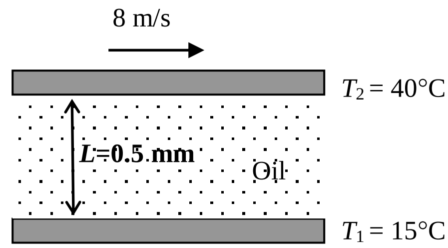
Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible. **4** The plates are large so that there is no variation in *z* direction.

Properties The properties of oil at the average temperature of (40+15)/2 = 27.5°C are (Table A-13):

$k = 0.145$ W/m·K and $\mu = 0.605$ kg/m·s = 0.605 N·s/m^2

Analysis (*a*) We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation (Eq. 6-21) reduces to

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u = u(y)$$



Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y) = C_1 y + C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = V$, and applying them gives the velocity distribution to be

$$u(y) = \frac{y}{L}V$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with dissipation reduces to

Energy: $$0 = k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2T}{dy^2}=-\mu\left(\frac{V}{L}\right)^2$$

since $\partial u/\partial y = V/L$. Dividing both sides by *k* and integrating twice give

$$T(y) = -\frac{\mu}{2k}\left(\frac{y}{L}V\right)^2 + C_3 y + C_4$$

Applying the boundary conditions $T(0) = T_1$ and $T(L) = T_2$ gives the temperature distribution to be

$$T(y) = \frac{T_2 - T_1}{L}y + T_1 + \frac{\mu V^2}{2k}\left(\frac{y}{L}-\frac{y^2}{L^2}\right)$$

(*b*) The temperature gradient is determined by differentiating $T(y)$ with respect to *y*,

$$\frac{dT}{dy} = \frac{T_2 - T_1}{L} + \frac{\mu V^2}{2kL}\left(1 - 2\frac{y}{L}\right)$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for *y*,

$$\frac{dT}{dy} = \frac{T_2 - T_1}{L} + \frac{\mu V^2}{2kL}\left(1 - 2\frac{y}{L}\right) = 0 \longrightarrow y = L\left(k\frac{T_2 - T_1}{\mu V^2}+\frac{1}{2}\right)$$

The maximum temperature is the value of temperature at this *y*, whose numeric value is

$$y = L\left(k\frac{T_2 - T_1}{\mu V^2}+\frac{1}{2}\right) = (0.0005\text{ m})\left[(0.145\text{ W/m.°C})\frac{(40-15)\text{°C}}{(0.605\text{ N.s/m}^2)(8\text{ m/s})^2}+\frac{1}{2}\right]$$

$$= 0.0002968\text{ m} = \mathbf{0.2968\,mm}$$

Then

$$T_{\max} = T(0.0002166) = \frac{T_2 - T_1}{L} y + T_1 + \frac{\mu V^2}{2k}\left(\frac{y}{L} - \frac{y^2}{L^2}\right)$$

$$= \frac{(40-15)°\text{C}}{0.0005\text{ m}}(0.0002968\text{ m}) + 15°\text{C} + \frac{(0.605\text{ N}\cdot\text{s/m}^2)(8\text{ m/s})^2}{2(0.145\text{ W/m}\cdot°\text{C})}\left(\frac{0.0002968\text{ m}}{0.0005\text{ m}} - \frac{(0.0002968\text{ m})^2}{(0.0005\text{ m})^2}\right)$$

$$= \mathbf{62.0°C}$$

(*c*) Heat flux at the plates is determined from the definition of heat flux,

$$\dot{q}_0 = -k\left.\frac{dT}{dy}\right|_{y=0} = -k\frac{T_2 - T_1}{L} - k\frac{\mu V^2}{2kL}(1-0) = -k\frac{T_2 - T_1}{L} - \frac{\mu V^2}{2L}$$

$$= -(0.145\text{ W/m.°C})\frac{(40-15)°\text{C}}{0.0005\text{ m}} - \frac{(0.605\text{ N}\cdot\text{s/m}^2)(8\text{ m/s})^2}{2(0.0005\text{ m})}\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = \mathbf{-4.60\times10^4\ W/m^2}$$

$$\dot{q}_L = -k\left.\frac{dT}{dy}\right|_{y=L} = -k\frac{T_2 - T_1}{L} - k\frac{\mu V^2}{2kL}(1-2) = -k\frac{T_2 - T_1}{L} + \frac{\mu V^2}{2L}$$

$$= -(0.145\text{ W/m.°C})\frac{(40-15)°\text{C}}{0.0005\text{ m}} + \frac{(0.58\text{ N}\cdot\text{s/m}^2)(8\text{ m/s})^2}{2(0.0005\text{ m})}\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = \mathbf{2.99\times10^4\ W/m^2}$$

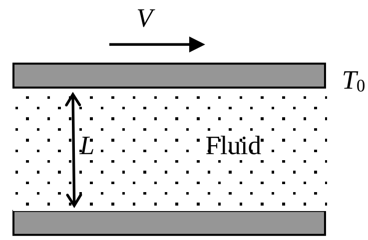
Discussion A temperature rise of about 35°C confirms our suspicion that viscous dissipation is very significant. Calculations are done using oil properties at 27.5°C, but the oil temperature turned out to be much higher. Therefore, knowing the strong dependence of viscosity on temperature, calculations should be repeated using properties at the average temperature of about 45°C to improve accuracy.

6-65 The flow of fluid between two large parallel plates is considered. The relations for the maximum temperature of fluid, the location where it occurs, and heat flux at the upper plate are to be obtained.

Assumptions **1** Steady operating conditions exist. **2** The fluid has constant properties. **3** Body forces such as gravity are negligible.

Analysis We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation reduces to

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u = u(y)$$



Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P / \partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2 u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1 y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = V$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}V$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with dissipation (Eqs. 6-36 and 6-37) reduce to

Energy: $$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2 T}{dy^2}=-\mu\left(\frac{V}{L}\right)^2$$

since $\partial u / \partial y = V / L$. Dividing both sides by *k* and integrating twice give

$$\frac{dT}{dy}=-\frac{\mu}{k}\left(\frac{V}{L}\right)^2 y+C_3$$

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}V\right)^2+C_3 y+C_4$$

Applying the two boundary conditions give

B.C. 1: $y=0$ $$-k\left.\frac{dT}{dy}\right|_{y=0}=0 \longrightarrow C_3=0$$

B.C. 2: $y=L$ $$T(L)=T_0 \longrightarrow C_4=T_0+\frac{\mu V^2}{2k}$$

Substituting the constants give the temperature distribution to be

$$T(y)=T_0+\frac{\mu V^2}{2k}\left(1-\frac{y^2}{L^2}\right)$$

The temperature gradient is determined by differentiating $T(y)$ with respect to *y*,

$$\frac{dT}{dy}=\frac{-\mu V^2}{kL^2}y$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for *y*,

$$\frac{dT}{dy}=\frac{-\mu V^2}{kL^2}y=0 \longrightarrow y=0$$

Therefore, maximum temperature will occur at the lower plate surface, and it s value is

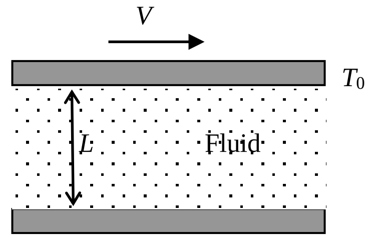
$$T_{\max} = T(0) = T_0 + \frac{\mu V^2}{2k}$$

.The heat flux at the upper plate is

$$\dot{q}_L = -k \frac{dT}{dy}\bigg|_{y=L} = k\,\frac{\mu V^2}{kL^2}\,L = \frac{\mu V^2}{L}$$

6-66 The flow of fluid between two large parallel plates is considered. Using the results of Problem 6-45, a relation for the volumetric heat generation rate is to be obtained using the conduction problem, and the result is to be verified.

Assumptions **1** Steady operating conditions exist. **2** The fluid has constant properties. **3** Body forces such as gravity are negligible.



Analysis The energy equation in Prob. 6-55 was determined to be

$$k\frac{d^2T}{dy^2} = -\mu\left(\frac{V}{L}\right)^2 \qquad (1)$$

The steady one-dimensional heat conduction equation with constant heat generation is

$$\frac{d^2T}{dy^2} + \frac{\dot{e}_{\text{gen}}}{k} = 0 \qquad (2)$$

Comparing the two equation above, the volumetric heat generation rate is determined to be

$$\dot{e}_{\text{gen}} = \mu\left(\frac{V}{L}\right)^2$$

Integrating Eq. (2) twice gives

$$\frac{dT}{dy} = -\frac{\dot{e}_{\text{gen}}}{k}y + C_3$$

$$T(y) = -\frac{\dot{e}_{\text{gen}}}{2k}y^2 + C_3y + C_4$$

Applying the two boundary conditions give

B.C. 1: $y=0$ $\qquad -k\left.\frac{dT}{dy}\right|_{y=0} = 0 \longrightarrow C_3 = 0$

B.C. 2: $y=L$ $\qquad T(L) = T_0 \longrightarrow C_4 = T_0 + \frac{\dot{e}_{\text{gen}}}{2k}L^2$

Substituting, the temperature distribution becomes

$$T(y) = T_0 + \frac{\dot{e}_{\text{gen}}L^2}{2k}\left(1 - \frac{y^2}{L^2}\right)$$

Maximum temperature occurs at $y = 0$, and it value is

$$T_{\max} = T(0) = T_0 + \frac{\dot{e}_{\text{gen}}L^2}{2k}$$

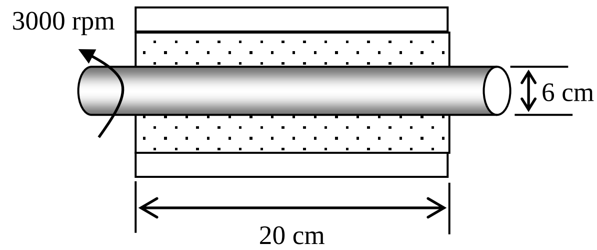
which is equivalent to the result $T_{\max} = T(0) = T_0 + \dfrac{\mu V^2}{2k}$ obtained in Prob. 6-55.

6-67 The oil in a journal bearing is considered. The velocity and temperature distributions, the maximum temperature, the rate of heat transfer, and the mechanical power wasted in oil are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of oil at 50°C are given to be

$k = 0.17$ W/m·K and $\mu = 0.05$ N·s/m^2



Analysis (*a*) Oil flow in journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation reduces to

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u = u(y)$$

Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the shaft/lower plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1 y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Taking $x = 0$ at the surface of the shaft/lower plate, the boundary conditions are $u(0) = V$ and $u(L) = 0$, and applying them gives the velocity distribution to be

$$u(y)=\frac{-V}{L}y+V$$

The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with viscous dissipation reduce to

Energy: $$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2T}{dy^2}=-\mu\left(\frac{-V}{L}\right)^2$$

since $\partial u/\partial y = -V/L$. Dividing both sides by *k* and integrating twice give

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}(-V)\right)^2+C_3 y+C_4$$

Applying the boundary conditions $T(0) = T_0$ and $T(L) = T_0$ gives the temperature distribution to be

$$T(y)=T_0+\frac{\mu V^2}{2k}\left(\frac{y}{L}-\frac{y^2}{L^2}\right)$$

The temperature gradient is determined by differentiating $T(y)$ with respect to *y*,

$$\frac{dT}{dy}=\frac{\mu V^2}{2kL}\left(1-2\frac{y}{L}\right)$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for *y*,

$$\frac{dT}{dy}=\frac{\mu V^2}{2kL}\left(1-2\frac{y}{L}\right)=0 \longrightarrow y=\frac{L}{2}$$

Therefore, maximum temperature will occur at mid plane in the oil. The velocity and the surface area are

$$V=\pi D\dot{N}=\pi(0.06\text{ m})(3000\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right)=9.425\text{ m/s}$$

$$A=\pi DL_{bearing}=\pi(0.06\text{ m})(0.20\text{ m})=0.0377\text{ m}^2$$

The maximum temperature is

$$T_{\max} = T(L/2) = T_0 + \frac{\mu V^2}{2k}\left(\frac{L/2}{L} - \frac{(L/2)^2}{L^2}\right)$$

$$= T_0 + \frac{\mu V^2}{8k} = 50°\text{C} + \frac{(0.05\ \text{N}\cdot\text{s/m}^2)(9.425\ \text{m/s})^2}{8(0.17\ \text{W/m}\cdot°\text{C})}\left(\frac{1\ \text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = \mathbf{53.3°C}$$

(*b*) The rates of heat transfer are

$$\dot{Q}_0 = -kA\frac{dT}{dy}\bigg|_{y=0} = -kA\frac{\mu V^2}{2kL}(1-0) = -A\frac{\mu V^2}{2L}$$

$$= -(0.0377\ \text{m}^2)\frac{(0.05\ \text{N}\cdot\text{s/m}^2)(9.425\ \text{m/s})^2}{2(0.0002\ \text{m})}\left(\frac{1\ \text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = \mathbf{-419\ W}$$

$$\dot{Q}_L = -kA\frac{dT}{dy}\bigg|_{y=L} = -kA\frac{\mu V^2}{2kL}(1-2) = A\frac{\mu V^2}{2L} = -\dot{Q}_0 = \mathbf{419\ W}$$

(*c*) Therefore, rates of heat transfer at the two plates are equal in magnitude but opposite in sign. The mechanical power wasted is equal to the rate of heat transfer.

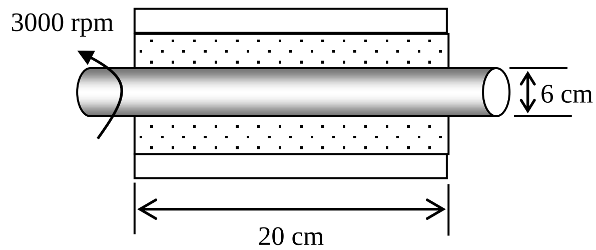
$$\dot{W}_{\text{mech}} = \dot{Q} = 2\times 419 = \mathbf{838\ W}$$

6-68 The oil in a journal bearing is considered. The velocity and temperature distributions, the maximum temperature, the rate of heat transfer, and the mechanical power wasted in oil are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of oil at 50°C are given to be

$k = 0.17$ W/m·K and $\mu = 0.05$ N·s/m^2



Analysis (*a*) Oil flow in journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus $v = 0$. Then the continuity equation reduces to

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u = u(y)$$

Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, $v = 0$, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the shaft/lower plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1 y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Taking $x = 0$ at the surface of the shaft/lower plate, the boundary conditions are $u(0) = V$ and $u(L) = 0$, and applying them gives the velocity distribution to be

$$u(y)=\frac{-V}{L}y+V$$

Frictional heating due to viscous dissipation in this case is significant because of the high viscosity of oil and the large plate velocity. The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and $v = 0$. Then the energy equation with dissipation reduce to

Energy: $$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2T}{dy^2}=-\mu\left(\frac{-V}{L}\right)^2$$

since $\partial u/\partial y = V/L$. Dividing both sides by *k* and integrating twice give

$$\frac{dT}{dy}=-\frac{\mu}{k}\left(\frac{(-V)}{L}\right)^2 y+C_3$$

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}(-V)\right)^2+C_3 y+C_4$$

Applying the two boundary conditions give

B.C. 1: y=0 $$-k\left.\frac{dT}{dy}\right|_{y=0}=0 \longrightarrow C_3=0$$

B.C. 2: y=L $$T(L)=T_1 \longrightarrow C_4=T_1+\frac{\mu V^2}{2k}$$

Substituting the constants give the temperature distribution to be

$$T(y)=T_1+\frac{\mu V^2}{2k}\left(1-\frac{y^2}{L^2}\right)$$

The temperature gradient is determined by differentiating $T(y)$ with respect to *y*,

$$\frac{dT}{dy} = -\frac{\mu V^2 y}{kL^2}$$

The location of maximum temperature is determined by setting $dT/dy = 0$ and solving for y,

$$\frac{dT}{dy} = -\frac{\mu V^2 y}{kL^2} = 0 \longrightarrow y = 0$$

This result is also known from the first boundary condition. Therefore, maximum temperature will occur at the shaft surface, for $y = 0$. The velocity and the surface area are

$$V = \pi D\dot{N} = \pi(0.06\text{ m})(3000\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right) = 9.425\text{ m/s}$$

$$A = \pi D L_{bearing} = \pi(0.06\text{ m})(0.20\text{ m}) = 0.0377\text{ m}^2$$

The maximum temperature is

$$T_{\max} = T(0) = T_1 + \frac{\mu V^2}{2k} = 50°\text{C} + \frac{(0.05\text{ N}\cdot\text{s/m}^2)(9.425\text{ m/s})^2}{2(0.17\text{ W/m}\cdot°\text{C})}\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = \mathbf{63.1°C}$$

(b) The rate of heat transfer to the bearing is

$$\dot{Q}_L = -kA\left.\frac{dT}{dy}\right|_{y=L} = kA\frac{\mu V^2 L}{kL^2} = A\frac{\mu V^2}{L} = (0.0377\text{ m}^2)\frac{(0.05\text{ N}\cdot\text{s/m}^2)(9.425\text{ m/s})^2}{0.0002\text{ m}}\left(\frac{1\text{ W}}{1\text{ N}\cdot\text{m/s}}\right) = \mathbf{837\ W}$$

(c) The rate of heat transfer to the shaft is zero. The mechanical power wasted is equal to the rate of heat transfer,

$$\dot{W}_{\text{mech}} = \dot{Q} = \mathbf{837\ W}$$

6-69 Prob. 6-67 is reconsidered. The effect of shaft velocity on the mechanical power wasted by viscous dissipation is to be investigated.

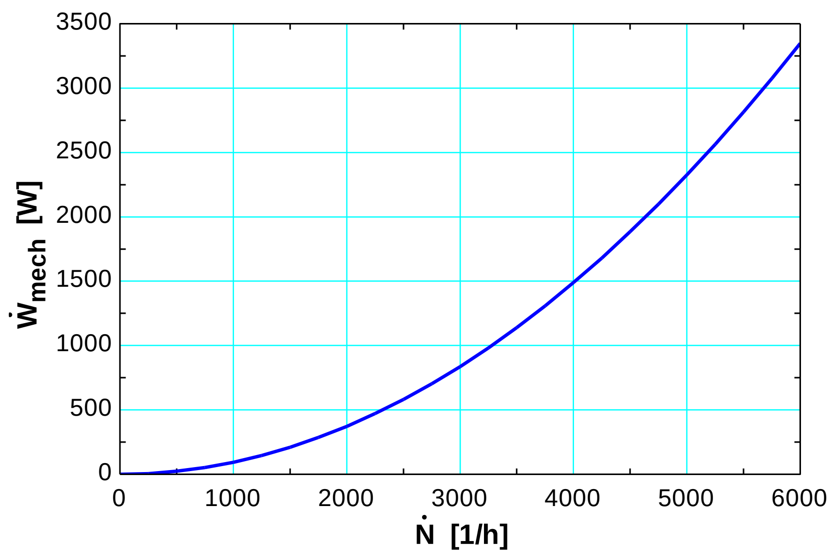
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.06 [m]
N_dot=3000 [1/h]
L_bearing=0.20 [m]
L=0.0002 [m]
T_0=50 [C]

"PROPERTIES"
k=0.17 [W/m-K]
mu=0.05 [N-s/m^2]

"ANALYSIS"
Vel=pi*D*N_dot*Convert(1/min, 1/s)
A=pi*D*L_bearing
T_max=T_0+(mu*Vel^2)/(8*k)
Q_dot=A*(mu*Vel^2)/(2*L)
W_dot_mech=2*Q_dot
```

$\dot{N}$ [rpm]	$\dot{W}_{mech}$ [W]
0	0
250	5.814
500	23.25
750	52.32
1000	93.02
1250	145.3
1500	209.3
1750	284.9
2000	372.1
2250	470.9
2500	581.4
2750	703.5
3000	837.2
3250	982.5
3500	1139
3750	1308
4000	1488
4250	1680
4500	1884
4750	2099
5000	2325
5250	2564
5500	2814
5750	3075
6000	3349

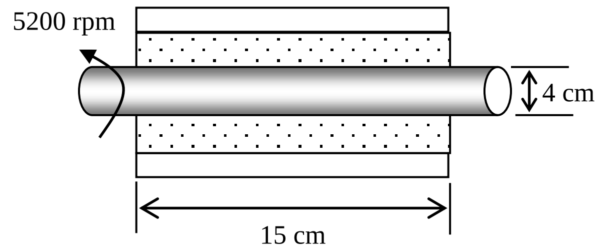


6-70 The oil in a journal bearing is considered. The bearing is cooled externally by a liquid. The surface temperature of the shaft, the rate of heat transfer to the coolant, and the mechanical power wasted are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of oil are given to be k = 0.14 W/m·K and μ = 0.03 N·s/m^2. The thermal conductivity of bearing is given to be k = 70 W/m·K.

Analysis (*a*) Oil flow in a journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus v = 0. Then the continuity equation reduces to



Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u=u(y)$$

Therefore, the *x*-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, v = 0, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the *x*-momentum equation reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1 y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0)$ = 0 and $u(L) = V$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}V$$

where

$$V=\pi D\dot{N}=\pi(0.04\,\text{m})(5200\,\text{rev/min})\left(\frac{1\,\text{min}}{60\,\text{s}}\right)=10.89\,\text{m/s}$$

The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on *y* only, $T = T(y)$. Also, $u = u(y)$ and v = 0. Then the energy equation with viscous dissipation reduces to

Energy: $$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2T}{dy^2}=-\mu\left(\frac{V}{L}\right)^2$$

since $\partial u/\partial y = V/L$. Dividing both sides by *k* and integrating twice give

$$\frac{dT}{dy}=-\frac{\mu}{k}\left(\frac{V}{L}\right)^2 y+C_3$$

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}V\right)^2+C_3 y+C_4$$

Applying the two boundary conditions give

B.C. 1: y=0 $$-k\left.\frac{dT}{dy}\right|_{y=0}=0 \longrightarrow C_3=0$$

B.C. 2: y=L $$T(L)=T_0 \longrightarrow C_4=T_0+\frac{\mu V^2}{2k}$$

Substituting the constants give the temperature distribution to be

$$T(y)=T_0+\frac{\mu V^2}{2k}\left(1-\frac{y^2}{L^2}\right)$$

The temperature gradient is determined by differentiating $T(y)$ with respect to *y*,

$$\frac{dT}{dy} = \frac{-\mu V^2}{kL^2} y$$

.The heat flux at the upper surface is

$$\dot{q}_L = -k\frac{dT}{dy}\bigg|_{y=L} = k\frac{\mu V^2}{kL^2} L = \frac{\mu V^2}{L}$$

Noting that heat transfer along the shaft is negligible, all the heat generated in the oil is transferred to the shaft, and the rate of heat transfer is

$$\dot{Q} = A_s \dot{q}_L = (\pi DW)\frac{\mu V^2}{L} = \pi(0.04\,\text{m})(0.15\text{m})\frac{(0.03\,\text{N}\cdot\text{s/m}^2)(10.89\,\text{m/s})^2}{0.0006\,\text{m}} = \mathbf{111.8\,W}$$

(b) This is equivalent to the rate of heat transfer through the cylindrical sleeve by conduction, which is expressed as

$$\dot{Q} = k\frac{2\pi W(T_0 - T_s)}{\ln(D_0 / D)} \quad \rightarrow (70\ \text{W/m}\cdot{}^\circ\text{C})\frac{2\pi(0.15\,\text{m})(T_0 \text{ - } 40^\circ\text{C})}{\ln(8/4)} = 111.8\ \text{W}$$

which gives the surface temperature of the shaft to be

T_o = **41.2°C**

(c) The mechanical power wasted by the viscous dissipation in oil is equivalent to the rate of heat generation,

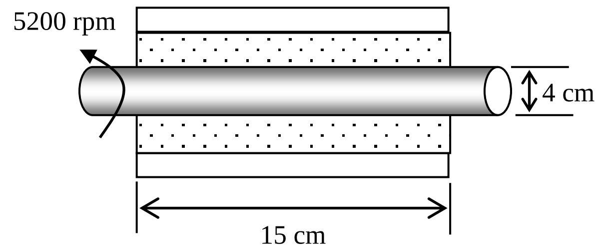
$$\dot{W}_{lost} = \dot{Q} = \mathbf{111.8\,W}$$

6-71 The oil in a journal bearing is considered. The bearing is cooled externally by a liquid. The surface temperature of the shaft, the rate of heat transfer to the coolant, and the mechanical power wasted are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Oil is an incompressible substance with constant properties. **3** Body forces such as gravity are negligible.

Properties The properties of oil are given to be k = 0.14 W/m·K and μ = 0.03 N·s/m^2. The thermal conductivity of bearing is given to be k = 70 W/m·K.

Analysis (*a*) Oil flow in a journal bearing can be approximated as parallel flow between two large plates with one plate moving and the other stationary. We take the *x-axis* to be the flow direction, and *y* to be the normal direction. This is parallel flow between two plates, and thus v = 0. Then the continuity equation reduces to



Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \longrightarrow \frac{\partial u}{\partial x}=0 \longrightarrow u=u(y)$$

Therefore, the x-component of velocity does not change in the flow direction (i.e., the velocity profile remains unchanged). Noting that $u = u(y)$, v = 0, and $\partial P/\partial x = 0$ (flow is maintained by the motion of the upper plate rather than the pressure gradient), the x-momentum equation reduces to

x-momentum: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\partial P}{\partial x} \longrightarrow \frac{d^2u}{dy^2}=0$$

This is a second-order ordinary differential equation, and integrating it twice gives

$$u(y)=C_1y+C_2$$

The fluid velocities at the plate surfaces must be equal to the velocities of the plates because of the no-slip condition. Therefore, the boundary conditions are $u(0) = 0$ and $u(L) = V$, and applying them gives the velocity distribution to be

$$u(y)=\frac{y}{L}V$$

where

$$V=\pi D\dot{N}=\pi(0.04\text{ m})(5200\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right)=10.89\text{ m/s}$$

The plates are isothermal and there is no change in the flow direction, and thus the temperature depends on y only, $T = T(y)$. Also, $u = u(y)$ and v = 0. Then the energy equation with viscous dissipation reduces to

Energy: $$0=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \longrightarrow k\frac{d^2T}{dy^2}=-\mu\left(\frac{V}{L}\right)^2$$

since $\partial u/\partial y = V/L$. Dividing both sides by k and integrating twice give

$$\frac{dT}{dy}=-\frac{\mu}{k}\left(\frac{V}{L}\right)^2 y+C_3$$

$$T(y)=-\frac{\mu}{2k}\left(\frac{y}{L}V\right)^2+C_3y+C_4$$

Applying the two boundary conditions give

B.C. 1: y=0 $$-k\left.\frac{dT}{dy}\right|_{y=0}=0\longrightarrow C_3=0$$

B.C. 2: y=L $$T(L)=T_0\longrightarrow C_4=T_0+\frac{\mu V^2}{2k}$$

Substituting the constants give the temperature distribution to be

$$T(y)=T_0+\frac{\mu V^2}{2k}\left(1-\frac{y^2}{L^2}\right)$$

The temperature gradient is determined by differentiating $T(y)$ with respect to y,

$$\frac{dT}{dy} = \frac{-\mu V^2}{kL^2} y$$

.The heat flux at the upper surface is

$$\dot{q}_L = -k\frac{dT}{dy}\bigg|_{y=L} = k\frac{\mu V^2}{kL^2}L = \frac{\mu V^2}{L}$$

Noting that heat transfer along the shaft is negligible, all the heat generated in the oil is transferred to the shaft, and the rate of heat transfer is

$$\dot{Q} = A_s\dot{q}_L = (\pi DW)\frac{\mu V^2}{L} = \pi(0.04\,\text{m})(0.15\text{m})\frac{(0.03\,\text{N}\cdot\text{s/m}^2)(10.89\,\text{m/s})^2}{0.001\,\text{m}} = \mathbf{67.1W}$$

(b) This is equivalent to the rate of heat transfer through the cylindrical sleeve by conduction, which is expressed as

$$\dot{Q} = k\frac{2\pi W(T_0 - T_s)}{\ln(D_0/D)} \quad \rightarrow (70\text{ W/m}\cdot{}^\circ\text{C})\frac{2\pi(0.15\,\text{m})(T_0 - 40^\circ\text{C})}{\ln(8/4)} = 67.1\,\text{W}$$

which gives the surface temperature of the shaft to be

T_o = **40.7°C**

(c) The mechanical power wasted by the viscous dissipation in oil is equivalent to the rate of heat generation,

$$\dot{W}_{lost} = \dot{Q} = \mathbf{67.1W}$$

6-72E Glycerin is flowing over a flat plate. The velocity and thermal boundary layer thicknesses are to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant.

Properties The properties of glycerin at 50°F are ν = 0.03594 ft^2/s and Pr = 34561 (Table A-13E).

Analysis The Reynolds number at x = 0.5 ft is

$$\text{Re}_x = \frac{Vx}{\nu} = \frac{(6\text{ ft/s})(0.5\text{ ft})}{0.03594\text{ ft}^2\text{/s}} = 83.47 < 5\times10^5 \qquad \text{(flow is laminar at } x = 0.5\text{ ft)}$$

The velocity boundary thickness for laminar flow over a flat plate is

$$\delta = \frac{4.91x}{\sqrt{\text{Re}_x}} = \frac{4.91(0.5\text{ ft})}{\sqrt{83.47}} = \mathbf{0.2687\,ft}$$

The thermal boundary thickness over a flat plate is

$$\delta_t = \frac{\delta}{\text{Pr}^{1/3}} = \frac{0.2687\text{ ft}}{34561^{1/3}} = \mathbf{0.008249\,ft}$$

Discussion The ratio of the velocity boundary thickness to the thermal boundary layer thickness can be expressed as

$$\frac{\delta}{\delta_t} = \text{Pr}^{1/3} = 32.57$$

Since glycerin has a large Prandtl number, this implies that the velocity boundary thickness is larger than the thermal boundary layer thickness, and this case by an order of about 33 times.

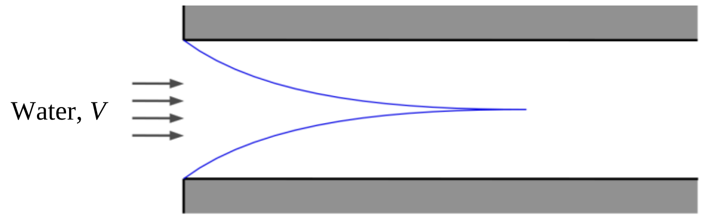
6-73 Water is flowing between two parallel flat plates. The distances from the entrance at which the velocity and thermal boundary layers meet are to be determined.

Assumptions **1** Isothermal condition exists between the flat plates and fluid flow. **2** Properties are constant.

Properties The properties of water at 20°C are ρ = 998.0 kg/m^3, μ = 1.002 × 10^{-3} kg/m·s and Pr = 7.01 (Table A-9).

Analysis The kinematic viscosity for water at 20°C is

$$\nu = \frac{\mu}{\rho} = \frac{1.002\times10^{-3}\ \text{kg/m}\cdot\text{s}}{998.0\ \text{kg/m}^3} = 1.004\times10^{-6}\ \text{m}^2/\text{s}$$



Both the velocity and thermal boundary layers meet at the centerline between the two parallel plates when

$$\delta = \delta_t = \frac{1\,\text{cm}}{2} = 0.005\ \text{m}$$

Assuming the flow is laminar, the velocity and thermal boundary layer thicknesses are

$$\delta = \frac{4.91}{\sqrt{V/(\nu x)}} \qquad \text{and} \qquad \delta_t = \frac{4.91}{\text{Pr}^{1/3}\sqrt{V/(\nu x_t)}}$$

The distance from the entrance at which the velocity boundary layers meet is

$$x = \frac{\delta^2 V}{(4.91)^2 \nu} = \frac{(0.005\ \text{m})^2 (0.5\ \text{m/s})}{(4.91)^2 (1.004\times10^{-6}\ \text{m}^2/\text{s})} = \mathbf{0.516\ m}$$

The distance from the entrance at which the thermal boundary layers meet is

$$x_t = \frac{\delta_t^2\, \text{Pr}^{2/3} V}{(4.91)^2 \nu} = \frac{(0.005\ \text{m})^2 (7.01)^{2/3} (0.5\ \text{m/s})}{(4.91)^2 (1.004\times10^{-6}\ \text{m}^2/\text{s})} = \mathbf{1.89\ m}$$

Discussion The analysis for this problem assumed that the flow is laminar. To check whether the flow is indeed laminar, the Reynolds number at x = 0.516 m is calculated:

$$\text{Re}_x = \frac{Vx}{\nu} = \frac{(0.5\ \text{m/s})(0.516\ \text{m})}{1.004\times10^{-6}\ \text{m}^2/\text{s}} = 2.57\times10^5 < 5\times10^5 \qquad \text{(flow is laminar at } x = 0.516\text{ m)}$$

Therefore, the laminar flow assumption for this analysis is valid.

6-74E The δ / δ_t ratios for different fluids in laminar boundary layer flow over a flat plate are to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant.

Properties The Prandtl numbers for the different fluids at 50°F are listed in the following table:

Fluid	*Table*	Pr
Air (1 atm)	A-15E	0.7336
Liq. water	A-9E	9.44
Isobutane	A-13E	4.114
Engine oil	A-13E	22963
Mercury	A-14E	0.02737

Analysis The velocity and thermal boundary layers for laminar flow can be related using

$$\delta_t = \frac{\delta}{\Pr^{1/3}} \quad \rightarrow \quad \frac{\delta}{\delta_t} = \Pr^{1/3}$$

Hence the δ / δ_t ratio for each fluid is

Fluid	δ / δ_t
Air (1 atm)	0.902
Liq. water	2.11
Isobutane	1.60
Engine oil	28.4
Mercury	0.301

Discussion For Pr > 1, the velocity boundary layer is thicker than the thermal boundary layer ($\delta > \delta_t$). For Pr < 1, the velocity boundary layer is thinner than the thermal boundary layer ($\delta < \delta_t$).

6-75 For laminar boundary layer flow over a flat plate with air (100ºC and 1 atm), δ_t is 15% larger than δ. Determine the ratio of δ/δ_t for engine oil (unused) at the same conditions.

Assumptions **1** Laminar flow.

Properties For air at 100ºC and 1 atm, Pr = 0.7111 (Table A-15). For engine oil (unused) at 100ºC, Pr = 279.1 (Table A-13)

Analysis The Prandtl number strongly influences growth of the velocity δ, and thermal δ_t, boundary layers. For laminar flow, the approximate relationship is given by

$$\delta / \delta_t \approx \Pr^n$$

Where n is a positive coefficient. Substituting the values for air

$$\delta / 1.15\delta = (0.7111)^n \ \rightarrow n = 0.4099$$

For engine oil (unused) it follows that

$$\delta / \delta_t = (279.1)^{0.4099} = \mathbf{10.06}$$

Discussion Large value of Prandtl number for engine oil causes $\delta >> \delta_t$.

6-76 The hydrodynamic boundary layer and the thermal boundary layer both as a function of x are to be plotted for the flow of air over a plate.

Analysis The problem is solved using Excel, and the solution is given below.

Assumptions

1. The flow is steady and incompressible
2. The critical Reynolds number is 500,000
3. Air is an ideal gas
4. The plate is smooth
5. Edge effects are negligible and the upper surface of the plate is considered

Input Properties

The average film temperature is 40°C (Property data from Table A-15)

ρ = 1.127 kg/m^3

c_p = 1007 J/kg·°C

μ = 0.00001918 kg/m·s

ν = 1.702×10^{-5} m^2/s

k = 0.02662 W/m·°C

Pr = 0.7255

Input Parameters

W = 0.3 m

$T_{f,\text{avg}}$ = 40°C

V = 3 m/s

T_{fluid} = 10°C

T_s = 70°C

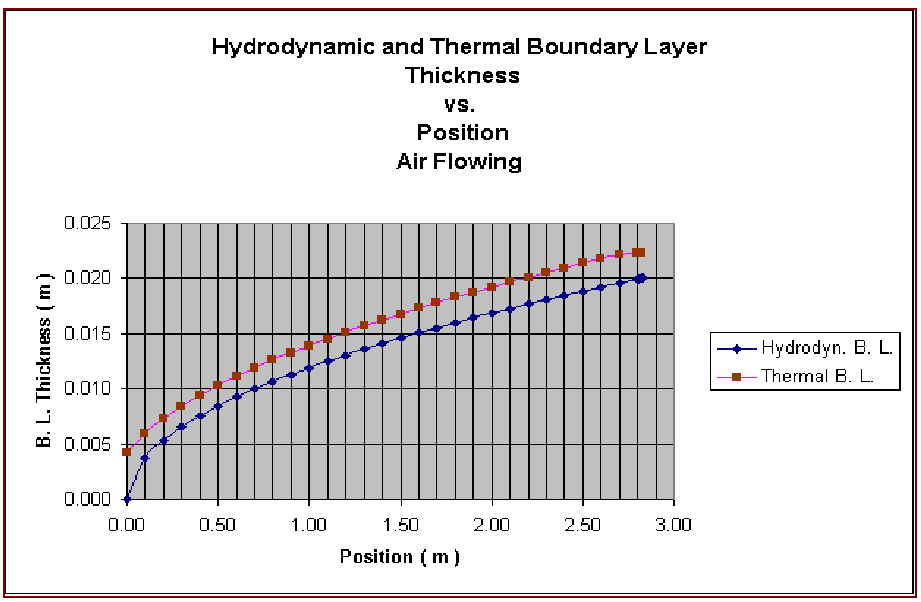
Analysis

The critical length: $\text{Re} = \dfrac{Vx_{cr}}{\nu} \longrightarrow x_{cr} = \dfrac{\text{Re}\,\nu}{V} = \dfrac{(500{,}000)(1.702\times10^{-5}\ \text{m}^2/\text{s})}{3\,\text{m/s}} = 2.84\,\text{m}$

Hydrodynamic boundary layer thickness: $\delta = \dfrac{4.91x}{\sqrt{\text{Re}_x}}$

Thermal boundary layer thickness: $\delta_t = \dfrac{4.91x}{\text{Pr}^{1/3}\sqrt{\text{Re}_x}}$

x (m)	Re_x	δ	δ_t
0.00	0	0	0
0.10	17628	0.0038	0.0042
0.20	35255	0.0053	0.0059
0.30	52883	0.0065	0.0073
0.40	70511	0.0075	0.0084
0.50	88139	0.0084	0.0094
0.60	105766	0.0092	0.0103
0.70	123394	0.0100	0.0111
0.80	141022	0.0107	0.0119
0.90	158650	0.0113	0.0126
1.00	176277	0.0119	0.0133
1.10	193905	0.0125	0.0139
1.20	211533	0.0130	0.0145
1.30	229161	0.0136	0.0151
1.40	246788	0.0141	0.0157
1.50	264416	0.0146	0.0162
1.60	282044	0.0151	0.0168
1.70	299672	0.0155	0.0173
1.80	317299	0.0160	0.0178
1.90	334927	0.0164	0.0183
2.00	352555	0.0168	0.0187
2.10	370182	0.0173	0.0192
2.20	387810	0.0177	0.0197
2.30	405438	0.0181	0.0201
2.40	423066	0.0184	0.0205
2.50	440693	0.0188	0.0210
2.60	458321	0.0192	0.0214
2.70	475949	0.0196	0.0218
2.80	493577	0.0199	0.0222
2.81	495339	0.0200	0.0222
2.82	497102	0.0200	0.0223
2.83	498865	0.0200	0.0223



6-77 The hydrodynamic boundary layer and the thermal boundary layer both as a function of x are to be plotted for the flow of liquid water over a plate.

Analysis The problem is solved using Excel, and the solution is given below.

Assumptions

1. The flow is steady and incompressible
2. The critical Reynolds number is 500,000
3. Air is an ideal gas
4. The plate is smooth
5. Edge effects are negligible and the upper surface of the plate is considered

Input Properties

The average film temperature is 40°C (Property data from Table A-9)

ρ = 992.1 kg/m^3

c_p = 4179 J/kg·°C

μ = 0.000653 kg/m·s

k = 0.631 W/m·°C

Pr = 4.32

Input Parameters

W = 0.3 m

$T_{f,\text{avg}}$ = 40°C

V = 3 m/s

T_{fluid} = 15°C

T_s = 65°C

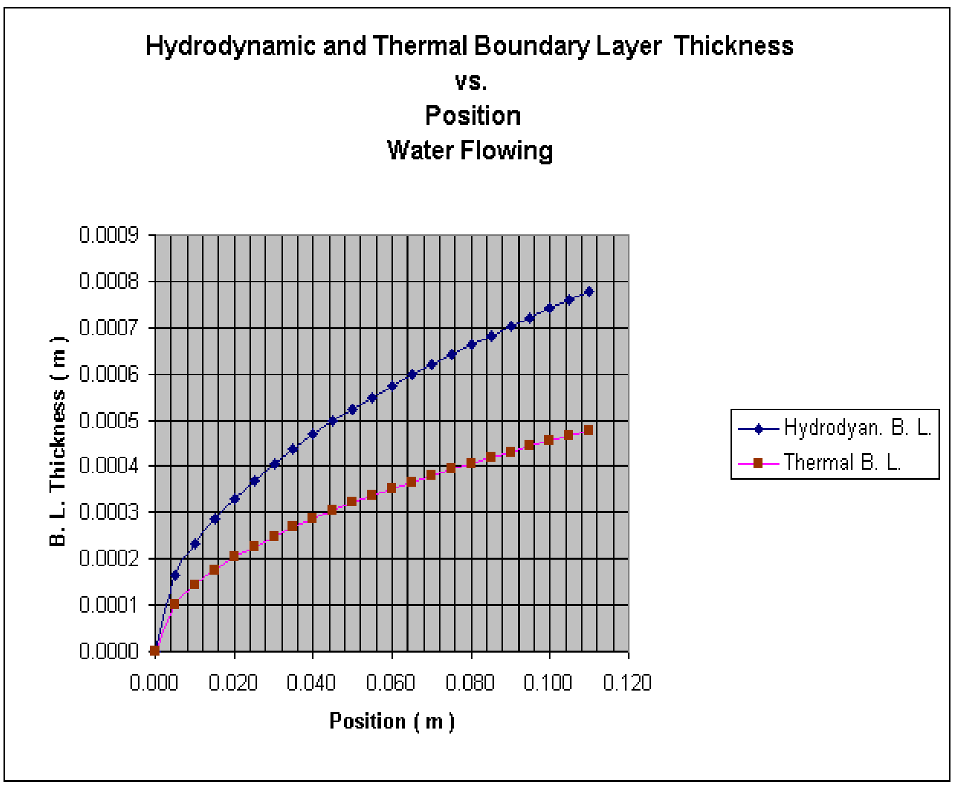
Analysis

The critical length: $\text{Re} = \dfrac{Vx_{cr}}{\nu} \longrightarrow x_{cr} = \dfrac{\text{Re}\,\nu}{V} = \dfrac{\text{Re}\,\mu}{V\rho} = \dfrac{(500{,}000)(0.000653\,\text{kg/m}\cdot\text{s})}{(3\,\text{m/s})(992.1\text{kg/m}^3)} = 0.11\,\text{m}$

Hydrodynamic boundary layer thickness: $\delta = \dfrac{4.91x}{\sqrt{\text{Re}_x}}$

Thermal boundary layer thickness: $\delta_t = \dfrac{4.91x}{\text{Pr}^{1/3}\sqrt{\text{Re}_x}}$

x (m)	Re_x	δ	δ_t
0.000	0.000	0	0
0.005	22789	0.0002	0.0001
0.010	45579	0.0002	0.0001
0.015	68368	0.0003	0.0002
0.020	91158	0.0003	0.0002
0.025	113947	0.0004	0.0002
0.030	136737	0.0004	0.0002
0.035	159526	0.0004	0.0003
0.040	182315	0.0005	0.0003
0.045	205105	0.0005	0.0003
0.050	227894	0.0005	0.0003
0.055	250684	0.0005	0.0003
0.060	273473	0.0006	0.0004
0.065	296263	0.0006	0.0004
0.070	319052	0.0006	0.0004
0.075	341842	0.0006	0.0004
0.080	364631	0.0007	0.0004
0.085	387420	0.0007	0.0004
0.090	410210	0.0007	0.0004
0.095	432999	0.0007	0.0004
0.100	455789	0.0007	0.0005
0.105	478578	0.0008	0.0005
0.110	501368	0.0008	0.0005



6-78 For saturated liquid water flowing over a flat plate, the effect of plate location on the velocity and thermal boundary layer thicknesses is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of saturated liquid water at 5°C are $\mu = 1.519 \times 10^{-3}$ kg/m·s, $\rho = 999.9$ kg/m^3, and Pr = 11.2 (Table A-9).

Analysis The Reynolds number at x = 0.5m is

$$\text{Re}_x = \frac{\rho V x}{\mu} = \frac{(999.9\,kg/m^3)(1\,m/s)(0.5\,\text{ft})}{1.519\times10^{-3}\,kg/m\cdot s} = 3.29\times10^5 < 5\times10^5 \quad \text{(flow is laminar at } x = 0.5\text{m)}$$

The velocity and thermal boundary thicknesses for laminar flow over a flat plate are

$$\delta = \frac{4.91x}{\sqrt{\text{Re}_x}} \quad \text{and} \quad \delta_t = \frac{\delta}{\text{Pr}^{1/3}} \quad \text{where} \quad \text{Re}_x = \frac{\rho V x}{\mu}$$

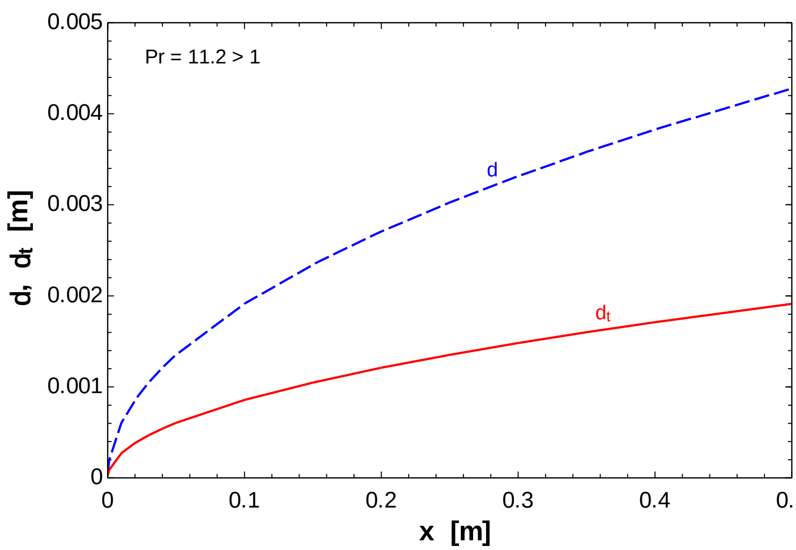
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
V=1 [m/s]

"PROPERTIES"
mu=1.519e-3 [kg/m-s]
rho=999.9 [kg/m^3]
Pr=11.2

"ANALYSIS"
Re_x=rho*V*x/mu
delta=4.91*x/Re_x^0.5
delta_t=delta/Pr^(1/3)
```

x [m]	δ [m]	δ_t [m]
0.0001	0.00006052	0.00002705
0.001	0.0001914	0.00008553
0.01	0.0006052	0.0002705
0.02	0.0008558	0.0003825
0.03	0.001048	0.0004685
0.04	0.001210	0.0005410
0.05	0.001353	0.0006048
0.10	0.001914	0.0008553
0.15	0.002344	0.001048
0.20	0.002706	0.001210
0.25	0.003026	0.001352
0.30	0.003315	0.001482
0.35	0.003580	0.001600
0.40	0.003827	0.001711
0.50	0.004279	0.001913



Discussion The ratio of the velocity boundary layer thickness to the thermal boundary layer thickness can be expressed as $\delta/\delta_t = \text{Pr}^{1/3}$. Since Pr > 1 for saturated liquid water, the velocity boundary layer develops quicker than the thermal boundary layer along the plate.

6-79 For mercury flowing over a flat plate, the effect of plate location on the velocity and thermal boundary layer thicknesses is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of mercury at 0°C are $\nu = 1.241 \times 10^{-7}$ m²/s, and Pr = 0.0289 (Table A-14).

Analysis The Reynolds number at x = 0.5 m is

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(0.1\,\mathrm{m/s})(0.5\,\mathrm{m})}{1.241\times10^{-7}\,\mathrm{m^2/s}} = 4.03\times10^5 < 5\times10^5 \qquad \text{(flow is laminar at } x = 0.5\text{m)}$$

The velocity and thermal boundary thicknesses for laminar flow over a flat plate are

$$\delta = \frac{4.91x}{\sqrt{\mathrm{Re}_x}} \quad \text{and} \quad \delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} \quad \text{where} \quad \mathrm{Re}_x = \frac{Vx}{\nu}$$

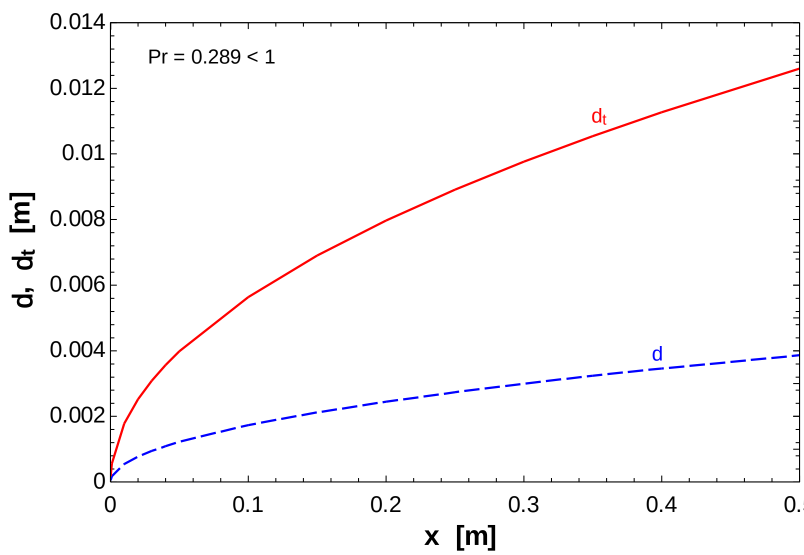
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
V=0.1 [m/s]

"PROPERTIES"
nu=1.241e-7 [m^2/s]
Pr=0.0289

"ANALYSIS"
Re_x=V*x/nu
delta=4.91*x/Re_x^0.5
delta_t=delta/Pr^(1/3)
```

x [m]	δ [m]	δ_t [m]
0.0001	0.0000547	0.0001782
0.001	0.0001730	0.0005636
0.01	0.0005470	0.001782
0.02	0.0007735	0.002521
0.03	0.0009474	0.003087
0.04	0.001094	0.003565
0.05	0.001223	0.003986
0.10	0.001730	0.005636
0.15	0.002118	0.006903
0.20	0.002446	0.007971
0.25	0.002735	0.008912
0.30	0.002996	0.009763
0.35	0.003236	0.01054
0.40	0.003459	0.01127
0.50	0.003868	0.01260



Discussion The ratio of the velocity boundary layer thickness to the thermal boundary layer thickness can be expressed as $\delta/\delta_t = \mathrm{Pr}^{1/3}$. Since Pr < 1 for mercury, the velocity boundary layer develops slower than the thermal boundary layer along the plate.

6-80 For water vapor flowing over a flat plate, the effect of plate location on the velocity and thermal boundary layer thicknesses is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of water vapor at 0°C and 1 atm are $\nu = 1.114 \times 10^{-5}$ m²/s, and Pr = 1.0033 (Table A-16).

Analysis The Reynolds number at x = 0.5 m is

$$\text{Re}_x = \frac{Vx}{\nu} = \frac{(10\,\text{m/s})(0.5\,\text{m})}{1.114\times10^{-5}\,\text{m}^2/\text{s}} = 4.49\times10^5 < 5\times10^5 \quad \text{(flow is laminar at } x = 0.5\text{m)}$$

The velocity and thermal boundary thicknesses for laminar flow over a flat plate are

$$\delta = \frac{4.91x}{\sqrt{\text{Re}_x}} \quad \text{and} \quad \delta_t = \frac{\delta}{\text{Pr}^{1/3}} \quad \text{where} \quad \text{Re}_x = \frac{Vx}{\nu}$$

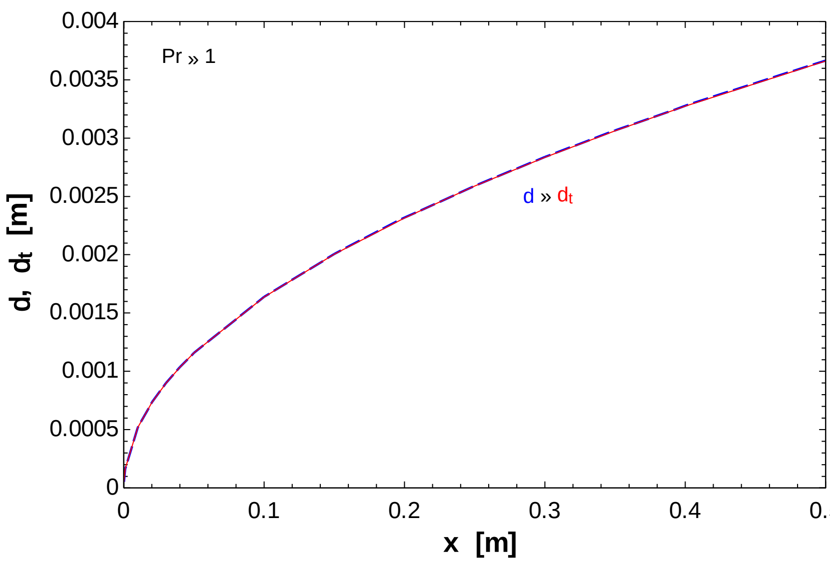
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
V=10 [m/s]

"PROPERTIES"
nu=1.114e-5 [m^2/s]
Pr=1.0033

"ANALYSIS"
Re_x=V*x/nu
delta=4.91*x/Re_x^0.5
delta_t=delta/Pr^(1/3)
```

x [m]	δ [m]	δ_t [m]
0.0001	0.00005182	0.00005177
0.001	0.0001639	0.0001637
0.01	0.0005182	0.0005177
0.02	0.0007329	0.0007321
0.03	0.0008976	0.0008966
0.04	0.001036	0.001035
0.05	0.001159	0.001158
0.10	0.001639	0.001637
0.15	0.002007	0.002005
0.20	0.002318	0.002315
0.25	0.002591	0.002588
0.30	0.002838	0.002835
0.35	0.003066	0.003063
0.40	0.003278	0.003274
0.50	0.003664	0.003660



Discussion The ratio of the velocity boundary layer thickness to the thermal boundary layer thickness can be expressed as $\delta/\delta_t = \text{Pr}^{1/3}$. Since Pr ≈ 1 for water vapor, the velocity and thermal boundary layer thicknesses are approximately the same along the plate.

6-81 A laminar ideal gas flows over a flat plate. Using the given $\mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$ expression, the formulation for local convection heat transfer coefficient, $h_x = C[V/(xT)]^m$, is to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Gas behaves as ideal gas. **3** Flow is laminar.

Analysis Using the definitions for Nusselt, Prandtl, and Reynolds numbers, we have

$$\mathrm{Nu}_x = \frac{h_x x}{k}, \qquad \mathrm{Pr} = \frac{c_p \mu}{k}, \qquad \text{and} \qquad \mathrm{Re}_x = \frac{\rho V x}{\mu}$$

Hence

$$\mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \quad \rightarrow \quad \frac{h_x x}{k} = 0.332\left(\frac{\rho V x}{\mu}\right)^{1/2}\left(\frac{c_p \mu}{k}\right)^{1/3}$$

For ideal gas, the density is $\rho = P/(RT)$, thus

$$\frac{h_x x}{k} = 0.332\left(\frac{PVx}{RT\mu}\right)^{1/2}\left(\frac{c_p \mu}{k}\right)^{1/3}$$

Simplifying we get

$$h_x = 0.332\,\frac{c_p^{1/3} k^{2/3}}{\mu^{1/6}}\left(\frac{P}{R}\right)^{1/2}\left(\frac{V}{xT}\right)^{1/2}$$

Or $\qquad h_x = C[V/(xT)]^m$

where

$$C = 0.332\,\frac{c_p^{1/3} k^{2/3}}{\mu^{1/6}}\left(\frac{P}{R}\right)^{1/2} \qquad \text{and} \qquad m = \frac{1}{2}$$

Discussion The temperature in the $h_x = C[V/(xT)]^m$ expression is an absolute temperature, since it was derived from the ideal gas law.

6-82 For air flowing over a flat plate, the convection heat transfer coefficients and the Nusselt numbers at x = 0.5 m and 0.75 m are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Edge effects are negligible.

Properties The properties of air at 40°C are k = 0.02662 W/m·K, $\nu = 1.702 \times 10^{-5}$ m^2/s, and Pr = 0.7255 (Table A-15).

Analysis The Reynolds number at x = 0.75 m is

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(3\,\mathrm{m/s})(0.75\,\mathrm{m})}{1.702\times 10^{-5}\ m^2/s} = 1.32\times 10^5 < 5\times 10^5 \qquad \text{(flow is laminar at } x = 0.75\text{m)}$$

For laminar flow over a flat plate, the local convection heat transfer coefficients can be determined using

$$h_x = 0.332\,\mathrm{Pr}^{1/3}\,k\left(\frac{V}{\nu\,x}\right)^{0.5}$$

At x = 0.5 m

$$h_x = 0.332(0.7255)^{1/3}(0.02662\ \mathrm{W/m\cdot K})\left[\frac{3\,\mathrm{m/s}}{(1.702\times 10^{-5}\ \mathrm{m^2/s})(0.5\,\mathrm{m})}\right]^{0.5} = \mathbf{4.715\ W/m^2\cdot K}$$

At x = 0.75 m

$$h_x = 0.332(0.7255)^{1/3}(0.02662\ \mathrm{W/m\cdot K})\left[\frac{3\,\mathrm{m/s}}{(1.702\times 10^{-5}\ \mathrm{m^2/s})(0.75\,\mathrm{m})}\right]^{0.5} = \mathbf{3.850\ W/m^2\cdot K}$$

The Nusselt numbers are, at x = 0.5 m,

$$\mathrm{Nu}_x = \frac{h_x x}{k} = \frac{(4.715\ \mathrm{W/m^2\cdot K})(0.5\,\mathrm{m})}{0.02662\ \mathrm{W/m\cdot K}} = \mathbf{88.6}$$

At x = 0.75 m

$$\mathrm{Nu}_x = \frac{h_x x}{k} = \frac{(3.850\ \mathrm{W/m^2\cdot K})(0.75\,\mathrm{m})}{0.02662\ \mathrm{W/m\cdot K}} = \mathbf{108.5}$$

Discussion The average convection heat transfer coefficient of the plate can be determined by integrating h_x over the plate length $0 \le x \le 1$ m.

6-83 Cold gas flows in parallel over the surface of an ASTM A240 410S stainless steel plate. The total heat flux on the plate surface necessary to maintain the surface temperature at the minimum suitable temperature set by the ASME Code for Process Piping is to be determined.

Assumptions**1** Steady operating conditions exist. **2** Uniform surface temperature. **3** Edge effects of plate are negligible.

Properties The gas properties are given as c_p = 1.002 kJ/kg·K, k = 0.02057 W/m·K, μ = 1.527×10^{-5} kg/m·s, and ρ = 1.514 kg/m^3.

Analysis The Reynolds number at x = 1 m is

$$\mathrm{Re}_x = \frac{\rho V x}{\mu} = \frac{(1.514\ \mathrm{kg/m^3})(5\ \mathrm{m/s})(1\ \mathrm{m})}{1.527\times10^{-5}\mathrm{kg/m\cdot s}} = 4.96\ \times 10^5 < \ 5\ \times 10^5 (\text{flow is laminar at } x = 1\text{ m})$$

The Prandtl number for the gas is

$$\mathrm{Pr} = \frac{\mu c_p}{k} = \frac{(1.527\times10^{-5}\mathrm{kg/m\cdot s})(1002\ \mathrm{J/kg\cdot K})}{0.02057\ \mathrm{W/m\cdot K}} = 0.7438$$

The local heat flux on the plate surface is given from the solutions of convection equations for a flat plate:

$$\dot{q}_s(x) = 0.332\ \mathrm{Pr}^{1/3}\ k\left(\frac{V}{\nu x}\right)^{1/2}(T_s - T_\infty)$$

Note that the kinematic viscosity is $\nu = \mu / \rho$, and $T_\infty = -50$°C and $T_s = -30$°C.

The total heat flux for the entire plate is determined by integrating $\dot{q}_s(x)$ for $0 \le x \le L$, where L =1 m:

$$\dot{q}_s = \frac{1}{L}\int_0^L 0.332\ \mathrm{Pr}^{1/3}\ k\left(\frac{\rho V}{\mu x}\right)^{1/2}(T_s - T_\infty)\,dx = \frac{0.664\ \mathrm{Pr}^{1/3}\ k}{L}\left(\frac{\rho V L}{\mu}\right)^{1/2}(T_s - T_\infty)$$

So,

$$\dot{q}_s = \frac{0.664(0.7438)^{1/3}(0.02057\ \mathrm{W/m\cdot K})}{1\ \mathrm{m}}[4.96\ \times 10^5]^{1/2}(-30+50)\mathrm{K} = \mathbf{174.3\ W/m^2}$$

Discussion To keep the surface temperature of the ASTM A240 410S plate from cooling below the minimum suitable temperature of −30°C, the minimum total heat flux needed is 174.3 W/m^2. If the total heat flux supplied to the plate is less than 174.3 W/m^2, then the surface temperature will decrease below −30°C.

6-84 An ASME SB-96 copper-silicon plate is heated by hot air at 200°C flowing in parallel over its surface. The ASME Boiler and Pressure Vessel Code limits the maximum operating temperature of the plate at 93°C. The variation of the local heat flux on the plate surface for $0 < x \leq 1$ m is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Uniform surface temperature. **3** Edge effects of plate are negligible.

Properties The air properties are given as c_p = 1.016 kJ/kg·K, k = 0.03419 W/m·K, μ = 2.371×10^{-5} kg/m·s, and ρ = 0.8412 kg/m^3.

Analysis The Reynolds number at x = 1 m is

$$\mathrm{Re}_x = \frac{\rho V x}{\mu} = \frac{(0.8412\ \mathrm{kg/m^3})(7.5\ \mathrm{m/s})(1\ \mathrm{m})}{2.371\times10^{-5}\,\mathrm{kg/m\cdot s}} = 2.66\times10^5 < 5\times10^5 \quad \text{(flow is laminar at } x = 1 \text{ m)}$$

The Prandtl number for the air is

$$\mathrm{Pr} = \frac{\mu c_p}{k} = \frac{(2.371\times10^{-5}\,\mathrm{kg/m\cdot s})(1016\ \mathrm{J/kg\cdot K})}{0.03419\ \mathrm{W/m\cdot K}} = 0.7046$$

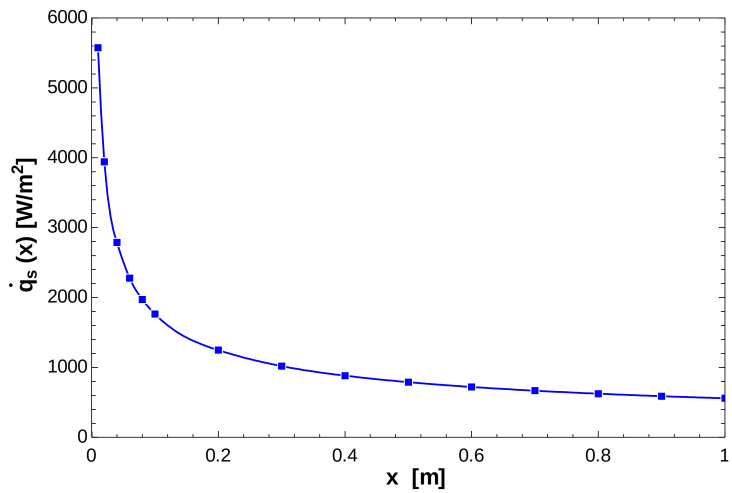
The local heat flux on the plate surface can be determined using the solutions of convection equations for a flat plate:

$$\dot{q}_s(x) = 0.332\,\mathrm{Pr}^{1/3}\,k\left(\frac{V}{\nu x}\right)^{1/2}(T_\infty - T_s)$$

Note that the kinematic viscosity is $\nu = \mu/\rho$, and T_∞ = 200°C and T_s = 93°C.

Using the $\dot{q}_s(x)$ function, the results for the variation of the local heat flux on the plate surface for $0 < x \leq 1$ m are calculated. They are tabulated and plotted as follows:

x [m]	$\dot{q}_s(x)$ [W/m^2]	x [m]	$\dot{q}_s(x)$ [W/m^2]
0.01	5574	0.4	881.4
0.02	3942	0.5	788.3
0.04	2787	0.6	719.7
0.06	2276	0.7	666.3
0.08	1971	0.8	623.2
0.1	1763	0.9	587.6
0.2	1246	1.0	557.4
0.3	1018		



Discussion The local heat flux at the leading edge (x = 0) is the highest and it decreases along the plate. The local heat flux is minimum at the trailing edge (x = 1 m) of the plate. When implementing a cooling mechanism for the plate, the emphasis should be on the first 1/3 of the plate. For the last 2/3 of the plate, the local heat flux is below 1000 W/m^2.

6-85 C&S An ASTM B152 copper plate is heated by hot air at 400°C flowing in parallel over its surface. The ASME Code for Process Piping set the maximum use temperature for the plate at 260°C. The variation of the local heat flux with the thermal boundary layer thickness on the plate surface for $0 < x \leq 1$ m is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Uniform surface temperature. **3** Edge effects of plate are negligible.

Properties The air properties are given as c_p = 1.052 kJ/kg·K, k = 0.04601 W/m·K, μ = 3.035×10^{-5} kg/m·s, and ρ = 0.5852 kg/m^3.

Analysis The Reynolds number at x = 1 m is

$$\mathrm{Re}_x = \frac{\rho V x}{\mu} = \frac{(0.5852\ \mathrm{kg/m^3})(6\ \mathrm{m/s})(1\ \mathrm{m})}{3.035\times10^{-5}\mathrm{kg/m\cdot s}} = 1.157\ \times 10^5 < \ 5\ \times 10^5 \quad \text{(flow is laminar at } x = 1\text{ m)}$$

The Prandtl number for the air is

$$\mathrm{Pr} = \frac{\mu c_p}{k} = \frac{(3.035\times10^{-5})(1052)}{0.04601} = 0.6939$$

The local heat flux on the plate surface can be determined using the solutions of convection equations for a flat plate:

$$\dot{q}_s(x) = 0.332\ \mathrm{Pr}^{1/3}\ k\left(\frac{V}{\nu x}\right)^{1/2}(T_\infty - T_s) = 0.332\ \mathrm{Pr}^{1/3}\ k\left(\frac{\rho V}{\mu x}\right)^{1/2}(T_\infty - T_s)$$

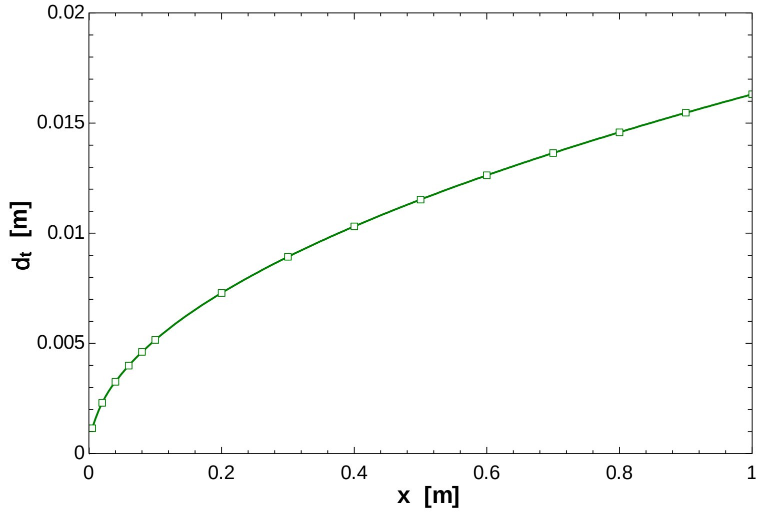
Note that the kinematic viscosity is $\nu = \mu / \rho$, and T_∞ = 400°C and T_s = 260°C.

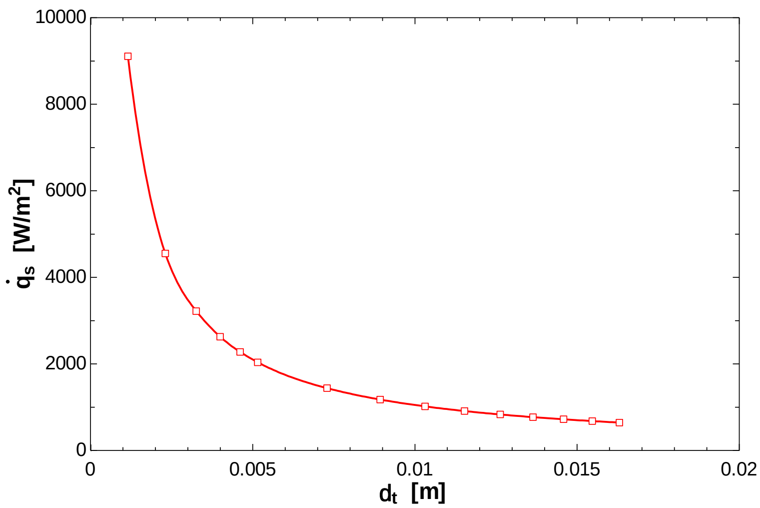
The thermal boundary layer thickness, from the solutions of convection equations for a flat plate, is

$$\delta_t(x) = \frac{\delta}{\mathrm{Pr}^{1/3}} = \frac{4.91x}{\mathrm{Pr}^{1/3}\sqrt{\mathrm{Re}_x}} = \frac{4.91x}{\mathrm{Pr}^{1/3}(\rho V x/\mu)^{1/2}}$$

Using the $\delta_t(x)$ and $\dot{q}_s(x)$ functions, the results for the variation of the local heat flux with the thermal boundary layer thickness on the plate surface for $0 < x \leq 1$ m are calculated. They are tabulated and plotted as follows:

x [m]	$\delta_t(x)$ [m]	$\dot{q}_s(x)$ [W/m^2]	x [m]	$\delta_t(x)$ [m]	$\dot{q}_s(x)$ [W/m^2]
0.005	0.001153	9107	0.4	0.01031	1018
0.02	0.002306	4554	0.5	0.01153	910.7
0.04	0.003261	3220	0.6	0.01263	831.4
0.06	0.003994	2629	0.7	0.01364	769.7
0.08	0.004612	2277	0.8	0.01458	720.0
0.1	0.005156	2036	0.9	0.01547	678.8
0.2	0.007292	1440	1	0.01631	644.0
0.3	0.008931	1176			





Discussion The thermal boundary layer thickness grow starting from the plate's leading edge (x = 0) until it reaches its maximum thickness at the plate's trailing edge (x = 1 m). From the $\dot{q}_s$ versus δ_t plot, the local heat flux on the plate surface decreases with increasing thermal boundary layer thickness. The thermal boundary layer thickness acts as a thermal resistance to convection. Thus, with the growth of the thermal boundary layer thickness, the local heat flux on the plate surface decreases.

6-86 For air flowing over a flat plate, the effect of the location along the plate (x) on the heat transfer coefficient and the surface temperature gradient of the plate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of air at the film temperature 40°C and 1 atm are k = 0.02662 W/m·K, $\nu = 1.702 \times 10^{-5}$ m²/s, and Pr = 0.7255 (Table A-15). The thermal conductivity of the plate is k_{plate} = 15 W/m·K.

Analysis The Reynolds number at x = 0.50 m is

$$\text{Re}_x = \frac{Vx}{\nu} = \frac{(3\,\text{m/s})(0.50\,\text{m})}{1.702 \times 10^{-5}\,\text{m}^2/\text{s}} = 8.813 \times 10^4 < 5 \times 10^5 \qquad \text{(flow is laminar at } x = 0.50\text{m)}$$

For laminar flow over a flat plate, the local convection heat transfer coefficients can be determined using

$$h_x = 0.332\,\text{Pr}^{1/3}\,k\left(\frac{V}{\nu x}\right)^{0.5}$$

The surface temperature gradient of the plate can be determined using

$$\left(\frac{\partial T}{\partial y}\right)_{\text{plate},y=0} = -\frac{h_x}{k_{\text{plate}}}(T_s - T_\infty)$$

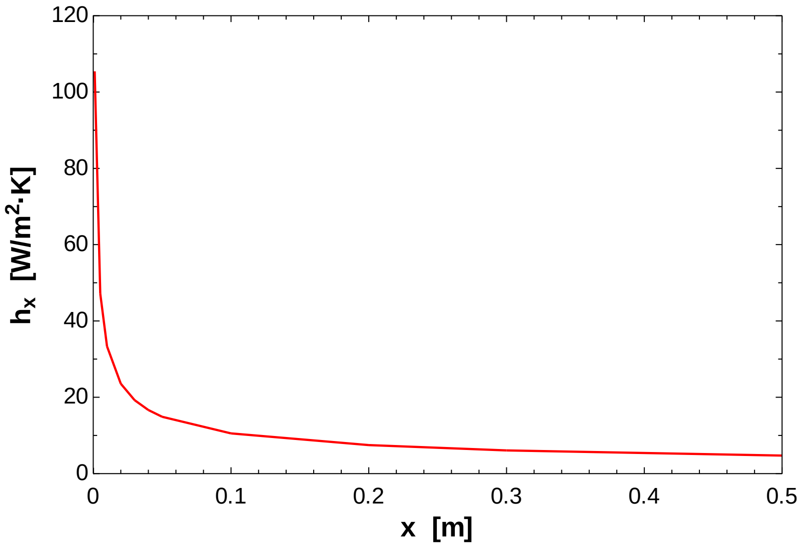
The problem is solved using EES, and the solution is given below:

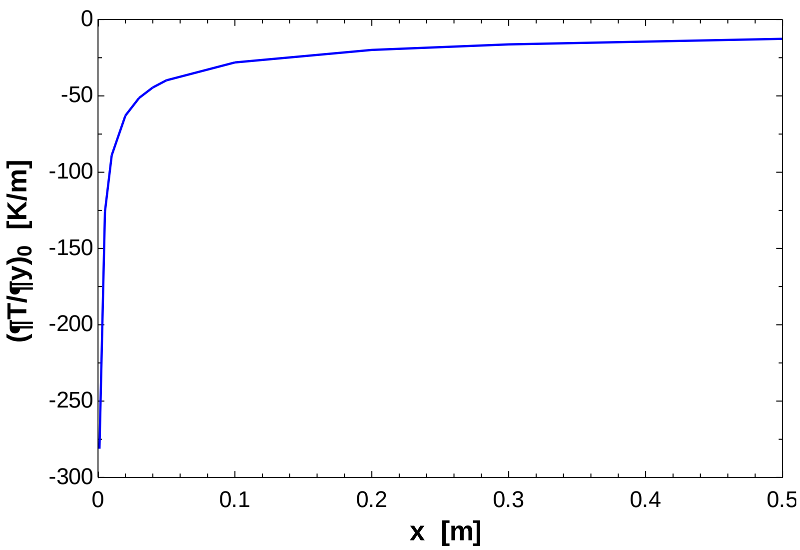
```
"GIVEN"
V=3 [m/s]
T_infinity=20 [C]
T_s=60 [C]

"PROPERTIES"
"For Air"
nu=1.702e-5 [m^2/s]
k=0.02662 [W/m-K]
Pr=0.7255
"For Plate"
k_plate=15 [W/m-K]

"ANALYSIS"
h_x=0.332*Pr^(1/3)*k*(V/(nu*x))^0.5
dTdy_0=-h_x/k_plate*(T_s-T_infinity)   "Surface temperature gradient of the plate"
```

x [m]	h_x [W/m²·K]	$(\partial T/\partial r)_{0,\text{plate}}$ [K/m]
0.001	105.4	−281.2
0.005	47.15	−125.7
0.01	33.34	−88.91
0.02	23.58	−62.87
0.03	19.25	−51.33
0.04	16.67	−44.45
0.05	14.91	−39.76
0.1	10.54	−28.12
0.2	7.455	−19.88
0.3	6.087	−16.23
0.5	4.715	−12.57





Discussion As the magnitude of the surface temperature gradient decreases along *x*, so does the value of the convection heat transfer coefficient.

6-87 The ratio of the average convection heat transfer coefficient (h) to the local convection heat transfer coefficient (h_x) is to be determined from a given relationship for h_x.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Analysis From the given relationship for $h_x = Cx^{-0.5}$

At $x = L$, the local convection heat transfer coefficient is $h_{x=L} = CL^{-0.5}$. The average convection heat transfer coefficient over the entire plate length is

$$h = \frac{1}{L}\int_0^L h_x dx = \frac{C}{L}\int_0^L x^{-0.5} dx = \frac{2C}{L} L^{0.5} = 2CL^{-0.5}$$

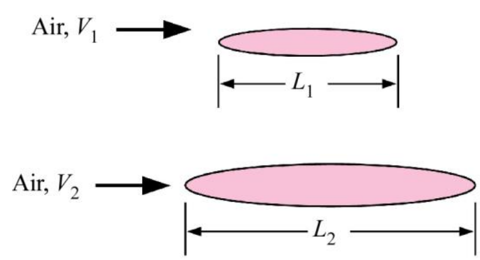
Taking the ratio of h to h_x at $x = L$, we get

$$\frac{h}{h_{x=L}} = \frac{2CL^{-0.5}}{CL^{-0.5}} = \mathbf{2}$$

Discussion For laminar flow and constant properties, it should be noted that ratio of the average Nusselt number over the entire plate length to the local Nusselt number at the end of the plate is also $\mathrm{Nu}/\mathrm{Nu}_{x=L} = 2$.

6-88E Two airfoils with different characteristic lengths are placed in airflow of different free stream velocities at 1 atm and 60°F. The heat flux from the second airfoil is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Both airfoils are geometrically similar.



Analysis The relation for Nusselt, Prandtl, and Reynolds numbers is given as

$$\mathrm{Nu} = g(\mathrm{Re}, \mathrm{Pr})$$

where

$$\mathrm{Nu} = \frac{hL}{k}, \quad \mathrm{Pr} = \frac{c_p \mu}{k}, \quad \text{and} \quad \mathrm{Re} = \frac{VL}{\nu}$$

Then

$$\textit{Airfoil 1}: \quad \mathrm{Re}_1 = \frac{VL_1}{\nu} = \frac{(150\ \mathrm{ft/s})(0.2\ \mathrm{ft})}{\nu} = \frac{30\ \mathrm{ft^2/s}}{\nu} \quad \text{and} \quad \mathrm{Pr}_1 = \frac{c_p \mu}{k}$$

$$\textit{Airfoil 2}: \quad \mathrm{Re}_2 = \frac{VL_2}{\nu} = \frac{(75\ \mathrm{ft/s})(0.4\ \mathrm{ft})}{\nu} = \frac{30\ \mathrm{ft^2/s}}{\nu} \quad \text{and} \quad \mathrm{Pr}_2 = \frac{c_p \mu}{k}$$

Since the fluid properties are constant, we have $\mathrm{Re}_1 = \mathrm{Re}_2$ and $\mathrm{Pr}_1 = \mathrm{Pr}_2$, which implies

$$\mathrm{Nu}_1 = g(\mathrm{Re}_1, \mathrm{Pr}_1) = \mathrm{Nu}_2 = g(\mathrm{Re}_2, \mathrm{Pr}_2) \quad \rightarrow \quad \mathrm{Nu}_1 = \mathrm{Nu}_2$$

Hence

$$\frac{h_1 L_1}{k} = \frac{h_2 L_2}{k} \quad \rightarrow \quad h_2 = h_1 \frac{L_1}{L_2} = h_1 \frac{0.2}{0.4} = 0.5 h_1$$

Therefore the average convection heat transfer coefficient for airfoil 2 is

$$h_2 = 0.5 h_1 = 0.5(21\ \mathrm{Btu/h \cdot ft^2 \cdot °F}) = 10.5\ \mathrm{Btu/h \cdot ft^2 \cdot °F}$$

The heat flux from the airfoil 2 is

$$\dot{q}_2 = h_2 (T_s - T_\infty) = (10.5\ \mathrm{Btu/h \cdot ft^2 \cdot °F})(180 - 60)\mathrm{°F} = \mathbf{1260\ Btu/h \cdot ft^2}$$

Discussion The relation for the Nusselt numbers $\mathrm{Nu}_1 = \mathrm{Nu}_2$ is valid due to $\mathrm{Re}_1 = \mathrm{Re}_2$ and $\mathrm{Pr}_1 = \mathrm{Pr}_2$.

6-89 Two irregularly shaped objects with different characteristic lengths and the same surface temperatures are placed in atmospheric airflow of different air velocities at 250 K. The average heat flux from the first object is known. The average convection heat transfer coefficient from the second object is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Both objects are geometrically similar.

Analysis The relation for Nusselt, Prandtl, and Reynolds numbers is given as

$$\mathrm{Nu} = g(\mathrm{Re}, \mathrm{Pr})$$

where $\mathrm{Nu} = \dfrac{hL}{k}$, $\mathrm{Pr} = \dfrac{c_p \mu}{k}$, and $\mathrm{Re} = \dfrac{VL}{\nu}$

Then

Object 1: $\mathrm{Re}_1 = \dfrac{V_1 L_1}{\nu} = \dfrac{(20\ \mathrm{m/s})(0.5\ \mathrm{m})}{\nu} = \dfrac{10\ \mathrm{m^2/s}}{\nu}$ and $\mathrm{Pr}_1 = \dfrac{c_p \mu}{k}$

Object 2: $\mathrm{Re}_2 = \dfrac{V_2 L_2}{\nu} = \dfrac{(4\ \mathrm{m/s})(2.5\ \mathrm{m})}{\nu} = \dfrac{10\ \mathrm{m^2/s}}{\nu}$ and $\mathrm{Pr}_2 = \dfrac{c_p \mu}{k}$

Since the fluid properties are constant, we have $\mathrm{Re}_1 = \mathrm{Re}_2$ and $\mathrm{Pr}_1 = \mathrm{Pr}_2$, which implies

$$\mathrm{Nu}_1 = g(\mathrm{Re}_1, \mathrm{Pr}_1) = \mathrm{Nu}_2 = g(\mathrm{Re}_2, \mathrm{Pr}_2) \quad \rightarrow \quad \mathrm{Nu}_1 = \mathrm{Nu}_2$$

Hence

$$\frac{h_1 L_1}{k} = \frac{h_2 L_2}{k} \quad \rightarrow h_2 = h_1 \frac{L_1}{L_2} = h_1 \frac{0.5\ \mathrm{m}}{2.5\ \mathrm{m}} = 0.2\, h_1$$

The average heat transfer coefficient for object 1 is

$$\dot{q}_1 = h_1 \left(T_{s,1} - T_{\infty,1}\right) \rightarrow \ h_1 = \frac{\dot{q}_1}{\left(T_{s,1} - T_{\infty,1}\right)} = \frac{8{,}000 \frac{\mathrm{W}}{\mathrm{m^2}}}{(350 - 250)\mathrm{K}} = 80\, \frac{\mathrm{W}}{\mathrm{m^2}} \cdot \mathrm{K}$$

Hence, it follows that for object 2, the average heat transfer coefficient is

$$h_2 = 0.2 \left(80 \frac{\mathrm{W}}{\mathrm{m^2}} \cdot \mathrm{K}\right) = \mathbf{16\ \frac{W}{m^2} \cdot K}$$

Discussion The relation for the Nusselt numbers $\mathrm{Nu}_1 = \mathrm{Nu}_2$ is valid due to $\mathrm{Re}_1 = \mathrm{Re}_2$ and $\mathrm{Pr}_1 = \mathrm{Pr}_2$. Slight variation of ν from object 1 to object 2 would cause Re_2 to differ from Re_1. However, for the given conditions, the variable property effect is small. If Re_2 were not equal to Re_1, it would be necessary to know the specific form of g (Re, Pr) before h_2 can be determined.

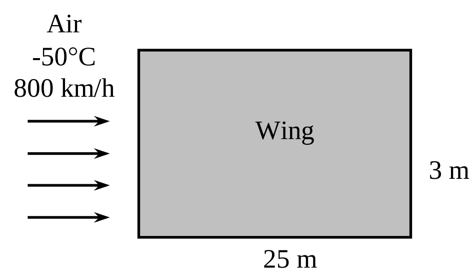
Momentum and Heat Transfer Analogies

6-90C Reynolds analogy is expressed as $C_{f,x}\dfrac{\mathrm{Re}_L}{2}=\mathrm{Nu}_x$. It allows us to calculate the heat transfer coefficient from a knowledge of friction coefficient. It is limited to flow of fluids with a Prandtl number of near unity (such as gases), and negligible pressure gradient in the flow direction (such as flow over a flat plate).

6-91C Modified Reynolds analogy is expressed as $C_{f,x}\dfrac{\mathrm{Re}_L}{2}=\mathrm{Nu}_x\,\mathrm{Pr}^{-1/3}$ or $\dfrac{C_{f,x}}{2}=\dfrac{h_x}{\rho c_p V}\mathrm{Pr}^{2/3}\equiv j_H$. It allows us to calculate the heat transfer coefficient from a knowledge of friction coefficient. It is valid for a Prandtl number range of 0.6 < Pr < 60. This relation is developed using relations for laminar flow over a flat plate, but it is also applicable approximately for turbulent flow over a surface, even in the presence of pressure gradients.

6-92 An airplane cruising is considered. The average heat transfer coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.



Properties The properties of air at -50°C and 1 atm are (Table A-15)

c_p =0.999 kJ/kg·K   Pr = 0.7440

The density of air at -50°C and 26.5 kPa is

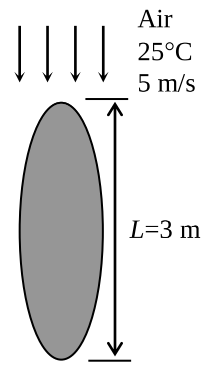
$$\rho=\frac{P}{RT}=\frac{26.5\,\text{kPa}}{(0.287\,\text{kJ/kg.K})(-50+273)\text{K}}=0.4141\,\text{kg/m}^3$$

Analysis The average heat transfer coefficient can be determined from the modified Reynolds analogy to be

$$h=\frac{C_f}{2}\frac{\rho V c_p}{\mathrm{Pr}^{2/3}}=\frac{0.0016}{2}\frac{(0.4141\,\text{kg/m}^3)(800/3.6\,\text{m/s})(999\,\text{J/kg}\cdot{}^\circ\text{C})}{(0.7440)^{2/3}}=\mathbf{89.6\ W/m^2\cdot C}$$

6-93 A metallic airfoil is subjected to air flow. The average friction coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.



Properties The properties of air at 25°C and 1 atm are (Table A-15)

ρ = 1.184 kg/m^3, c_p =1.007 kJ/kg·K, Pr = 0.7296

Analysis First, we determine the rate of heat transfer from

$$\dot{Q} = \frac{mc_{\text{p,airfoil}}(T_2 - T_1)}{\Delta t} = \frac{(50\ \text{kg})(500\,\text{J/kg}\cdot{}^\circ\text{C})(160-150)^\circ\text{C}}{(2\times 60\,\text{s})} = 2083\,\text{W}$$

Then the average heat transfer coefficient is

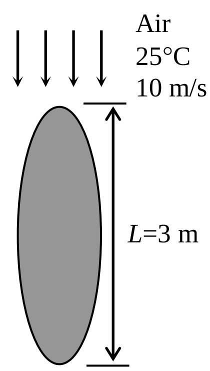
$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_s(T_s - T_\infty)} = \frac{2083\,\text{W}}{(12\,\text{m}^2)(155-25)^\circ\text{C}} = 1.335\,\text{W/m}^2\cdot{}^\circ\text{C}$$

where the surface temperature of airfoil is taken as its average temperature, which is (150+160)/2=155°C. The average friction coefficient of the airfoil is determined from the modified Reynolds analogy to be

$$C_f = \frac{2h\text{Pr}^{2/3}}{\rho V c_p} = \frac{2(1.335\,\text{W/m}^2\cdot{}^\circ\text{C})(0.7296)^{2/3}}{(1.184\,\text{kg/m}^3)(5\,\text{m/s})(1007\,\text{J/kg}\cdot{}^\circ\text{C})} = \mathbf{0.000363}$$

6-94 A metallic airfoil is subjected to air flow. The average friction coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.



Properties The properties of air at 25°C and 1 atm are (Table A-15)

ρ = 1.184 kg/m^3, c_p =1.007 kJ/kg·K, Pr = 0.7296

Analysis First, we determine the rate of heat transfer from

$$\dot{Q} = \frac{mc_{\text{p,airfoil}}(T_2 - T_1)}{\Delta t} = \frac{(50\ \text{kg})(500\,\text{J/kg}\cdot{}^\circ\text{C})(160-150)^\circ\text{C}}{(2\times 60\,\text{s})} = 2083\,\text{W}$$

Then the average heat transfer coefficient is

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_s(T_s - T_\infty)} = \frac{2083\,\text{W}}{(12\,\text{m}^2)(155-25)^\circ\text{C}} = 1.335\,\text{W/m}^2\cdot{}^\circ\text{C}$$

where the surface temperature of airfoil is taken as its average temperature, which is (150+160)/2=155°C. The average friction coefficient of the airfoil is determined from the modified Reynolds analogy to be

$$C_f = \frac{2h\text{Pr}^{2/3}}{\rho V c_p} = \frac{2(1.335\,\text{W/m}^2\cdot{}^\circ\text{C})(0.7296)^{2/3}}{(1.184\,\text{kg/m}^3)(10\,\text{m/s})(1007\,\text{J/kg}\cdot{}^\circ\text{C})} = \mathbf{0.000181}$$

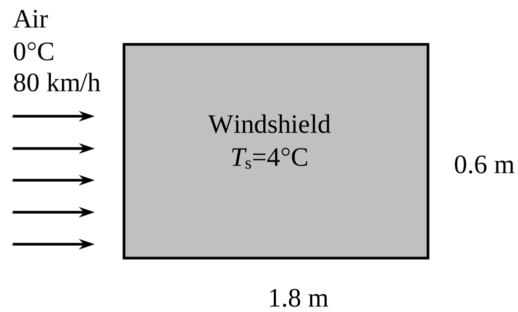
6-95 The windshield of a car is subjected to parallel winds. The drag force the wind exerts on the windshield is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.

Properties The properties of air at 0°C and 1 atm are (Table A-15)

ρ = 1.292 kg/m^3, c_p =1.006 kJ/kg·K, Pr = 0.7362

Analysis The average heat transfer coefficient is



$$\dot{Q} = hA_s(T_s - T_\infty)$$

$$h = \frac{\dot{Q}}{A_s(T_s - T_\infty)} = \frac{70 \text{ W}}{(0.6 \times 1.8 \text{ m}^2)(4-0)°\text{C}} = 16.20 \text{ W/m}^2 \cdot °\text{C}$$

The average friction coefficient is determined from the modified Reynolds analogy to be

$$C_f = \frac{2h\text{Pr}^{2/3}}{\rho V c_p} = \frac{2(16.20 \text{ W/m}^2 \cdot °\text{C})(0.7362)^{2/3}}{(1.292 \text{ kg/m}^3)(80/3.6 \text{ m/s})(1006 \text{J/kg} \cdot °\text{C})} = 0.0009146$$ The

drag force is determined from

$$F_f = C_f A_s \frac{\rho V^2}{2} = (0.0009146)(0.6 \times 1.8 \text{ m}^2)\frac{(1.292 \text{ kg/m}^3)(80/3.6 \text{ m/s})^2}{2}\left(\frac{1 \text{ N}}{1 \text{ kg.m/s}^2}\right) = \mathbf{0.315 \text{ N}}$$

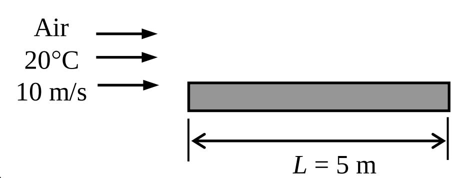
6-96 A flat plate is subjected to air flow, and the drag force acting on it is measured. The average convection heat transfer coefficient and the rate of heat transfer on the upper surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.

Properties The properties of air at 50°C and 1 atm are (Table A-15)

ρ = 1.092 kg/m^3, c_p =1.007 kJ/kg-K, Pr = 0.7228

Analysis The flow is along the 5-m side of the plate, and thus the characteristic length is L = 5 m. The surface area of the upper surface is



$$A_s = WL = (5 \text{ m})(5 \text{ m}) = 25 \text{ m}^2$$

For flat plates, the drag force is equivalent to friction force. The average friction coefficient C_f can be determined from

$$F_f = C_f A_s \frac{\rho V^2}{2} \longrightarrow C_f = \frac{F_f}{\rho A_s V^2/2} = \frac{2.4 \text{ N}}{(1.092 \text{ kg/m}^3)(25 \text{ m}^2)(10 \text{ m/s})^2/2}\left(\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}}\right) = 0.001758$$

Then the average heat transfer coefficient can be determined from the modified Reynolds analogy to be

$$h = \frac{C_f}{2}\frac{\rho V c_p}{\text{Pr}^{2/3}} = \frac{0.001758}{2}\frac{(1.092 \text{ kg/m}^3)(10 \text{ m/s})(1007 \text{J/kg} \cdot °\text{C})}{(0.7228)^{2/3}} = \mathbf{12.0 \text{ W/m}^2 \cdot \text{C}}$$

Then the rate of heat transfer becomes

$$\dot{Q} = hA_s(T_s - T_\infty) = (12.0 \text{ W/m}^2 \cdot °\text{C})(25 \text{ m}^2)(80-20)°\text{C} = \mathbf{18{,}000 \text{ W}}$$

6-97 Air is flowing in parallel to a stationary thin flat plate over the top and bottom surfaces. The rate of heat transfer from the plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The edge effects are negligible.

Properties The properties of air (1 atm) at the film temperature of $T_f = (T_s + T_\infty)/2 = 20°C$ are given in Table A-15: $\rho = 1.204$ kg/m^3, $c_p = 1007$ J/kg·K, $\nu = 1.516 \times 10^{-5}$ m^2/s, and Pr = 0.7309.

Analysis The flow is over the top and bottom surfaces of the plate, hence the total surface area is

$$A_s = 2(1\,\text{m})(1\,\text{m}) = 2\,\text{m}^2$$

For flat plate, the friction force can be determined using

$$F_f = C_f A_s \frac{\rho V^2}{2} \quad \rightarrow \quad C_f = \frac{2F_f}{\rho A_s V^2}$$

Using the Chilton-Colburn analogy, the convection heat transfer coefficient is determined to be:

$$\frac{C_f}{2} = \frac{h}{\rho c_p V}\text{Pr}^{2/3} \quad \rightarrow \quad h = \frac{C_f}{2}\rho c_p V\,\text{Pr}^{-2/3}$$

$$h = \frac{F_f}{A_s V} c_p\,\text{Pr}^{-2/3} = \frac{(0.1\,\text{N})}{(2\,\text{m}^2)(2\,\text{m/s})}(1007\,\text{J/kg}\cdot\text{K})(0.7309)^{-2/3} = 31.03\,\text{W/m}^2\cdot\text{K}$$

The rate of heat transfer from the flat plate is

$$\dot{Q} = hA_s(T_s - T_\infty) = (31.03\,\text{W/m}^2\cdot\text{K})(2\,\text{m}^2)(35-5)\,\text{K} = \mathbf{1862\,W}$$

Discussion The friction force asserted on the flat plate is due to the shear stress on the plate surfaces.

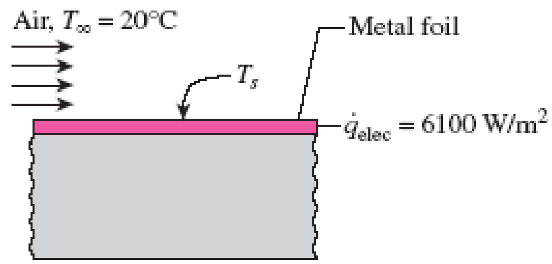
6-98 Air is flowing in parallel to a stationary thin flat plate over the top surface. The rate of heat transfer from the plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of air (1 atm) at the 100°C are given in Table A-15: $\rho = 0.9458$ kg/m^3, $c_p = 1009$ J/kg·K, $\nu = 2.306 \times 10^{-5}$ m^2/s, and Pr = 0.7111.

Analysis The flow is over the top surface of the metal foil, hence the surface area is

$$A_s = (0.2\text{ m})(0.5\text{ m}) = 0.1\text{ m}^2$$



For flat plate, the friction force can be determined using

$$F_f = C_f A_s \frac{\rho V^2}{2} \quad \rightarrow \quad C_f = \frac{2F_f}{\rho A_s V^2}$$

Using the Chilton-Colburn analogy, the convection heat transfer coefficient is determined to be:

$$\frac{C_f}{2} = \frac{h}{\rho c_p V}\text{Pr}^{2/3} \quad \rightarrow \quad h = \frac{C_f}{2}\rho c_p V\,\text{Pr}^{-2/3}$$

$$h = \frac{F_f}{A_s V}c_p\,\text{Pr}^{-2/3} = \frac{(0.3\text{ N})}{(0.1\text{ m}^2)(100\text{ m/s})}(1009\text{ J/kg}\cdot\text{K})(0.7111)^{-2/3} = 38\text{ W/m}^2\cdot\text{K}$$

The surface temperature of the metal foil is

$$\dot{q} = h(T_s - T_\infty) \quad \rightarrow \quad T_s = \frac{\dot{q}}{h} + T_\infty = \frac{6100\text{ W/m}^2}{38\text{ W/m}^2\cdot\text{K}} + 20°\text{C} = \mathbf{181°C}$$

Discussion The temperature, at 100°C, used for evaluating the fluid properties turned out to be appropriate, since the film temperature is

$$T_f = \frac{T_s + T_\infty}{2} = 101°\text{C} \approx 100°\text{C}$$

6-99 C&S An ASME SB-96 copper-silicon plate is heated by hot air at 200°C flowing in parallel over its surface. The minimum heat removal rate required to keep the plate surface from going above 93°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Uniform surface temperature. **3** Edge effects of plate are negligible.

Properties The air properties are given as c_p = 1.016 kJ/kg·K, k = 0.03419 W/m·K, μ = 2.371×10^{-5} kg/m·s, and ρ = 0.8412 kg/m^3.

Analysis The Prandtl number for the air is

$$\mathrm{Pr} = \frac{\mu c_p}{k} = \frac{(2.371 \times 10^{-5}\,\mathrm{kg/m \cdot s})(1016\ \mathrm{J/kg \cdot K})}{0.03419\ \mathrm{W/m \cdot K}} = 0.7046$$

The Reynolds number for the plate is

$$\mathrm{Re}_L = \frac{\rho VL}{\mu} = \frac{(0.8412\ \ \mathrm{kg/m^3})(6.5\ \mathrm{m/s})(1\ \mathrm{m})}{2.371 \times 10^{-5}\mathrm{kg/m \cdot s}} = 230612$$

Using the modified Reynolds analogy, the Nusselt number for the plate surface is

$$\mathrm{Nu} = \frac{C_f}{2}\mathrm{Re}_L\mathrm{Pr}^{1/3} = \frac{0.0023}{2}(230612)(0.7046)^{1/3} = 236$$

The convection heat transfer coefficient becomes

$$h = \mathrm{Nu}\frac{k}{L} = (236)\frac{0.03419\ \mathrm{W/m \cdot K}}{1\ \mathrm{m}} = 8.0688 \mathrm{W/m^2 \cdot K}$$

Thus, the heat transfer rate for the plate surface is

$$\dot{Q} = hA_s(T_\infty - T_s) = (8.0688\ \mathrm{W/m^2 \cdot K})(1\ \mathrm{m^2})(200 - 93)\ \mathrm{K} = \mathbf{863.4\ W}$$

Discussion To keep the surface temperature of the plate from going above 93°C, the minimum heat removal rate required is 863.4 W. If the heat removal rate is below 863.4 W, the surface temperature of the plate will be higher than the maximum operating temperature set by the ASME Boiler and Pressure Vessel Code. Increasing the heat removal rate will reduce the surface temperature.

6-100 Air at 1 atm is flowing over a flat plate. The friction coefficient and wall shear stress at a location 2 m from the leading edge are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of air (1 atm) at 20°C are given in Table A-15: $\rho = 1.204$ kg/m^3, $c_p = 1007$ J/kg·K, $\nu = 1.516 \times 10^{-5}$ m^2/s, and Pr = 0.7309.

Analysis At the location x = 2 m from the leading edge, the Reynolds number is

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(70\ \mathrm{m/s})(2\ \mathrm{m})}{1.516\times10^{-5}\ \mathrm{m^2/s}} = 9.235\times10^6$$

Applying the modified Reynolds analogy,

$$\frac{C_{f,x}}{2} = \mathrm{St}_x\,\mathrm{Pr}^{2/3} = \frac{\mathrm{Nu}_x}{\mathrm{Re}_x\,\mathrm{Pr}}\mathrm{Pr}^{2/3}$$

Substituting the given correlation for Nusselt number, we get

$$\frac{C_{f,x}}{2} = \frac{0.03\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{\mathrm{Re}_x\,\mathrm{Pr}}\mathrm{Pr}^{2/3} = 0.03\,\mathrm{Re}_x^{-0.2} \quad \text{or} \quad C_{f,x} = 0.06\,\mathrm{Re}_x^{-0.2}$$

The friction coefficient at x = 2 m is

$$C_{f,x} = 0.06\,\mathrm{Re}_x^{-0.2} = 0.06(9.235\times10^6)^{-0.2} = \mathbf{0.002427}$$

The wall shear stress is

$$\tau_s = C_{f,x}\frac{\rho V^2}{2} = (0.002427)\frac{(1.204\ \mathrm{kg/m^3})(70\ \mathrm{m/s})^2}{2} = \mathbf{7.16\ N/m^2}$$

Discussion At x = 2 m from the leading edge, the flow is turbulent. Since the Reynolds number at that location is greater than 5×10^5.

6-101 Metal plates are subject to parallel air flow cooling. The average convection heat transfer coefficient for the plates are to be determined from a given average friction coefficient over each plate.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of air at 20°C and 1 atm are $\nu = 1.516 \times 10^{-5}$ m^2/s, $\rho = 1.204$ kg/m^3, c_p = 1007J/kg·K, and Pr = 0.7309 (Table A-15).

Analysis The Reynolds number for the plate is

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(5\ \mathrm{m/s})(1\ \mathrm{m})}{1.516\times10^{-5}\ \mathrm{m^2/s}} = 3.298\times10^5 < 5\times10^5 \quad \text{(flow is laminar at } x = 1\text{m)}$$

The average friction coefficient over the plate is

$$C_f = 1.33\,\mathrm{Re}_L^{-0.5} = 1.33(3.298\times10^5)^{-0.5} = 0.002316$$

The average convection heat transfer coefficient can be determined from the modified Reynolds analogy to be

$$h = \frac{C_f}{2}\frac{\rho V c_p}{\mathrm{Pr}^{2/3}} = \frac{0.002316}{2}\frac{(1.204\ \mathrm{kg/m^3})(5\ \mathrm{m/s})(1007\ \mathrm{J/kg\cdot K})}{(0.7309)^{2/3}} = \mathbf{8.65\ W/m^2\cdot K}$$

Discussion The given $C_f = 1.33(\mathrm{Re}_L)^{-0.5}$ equation is valid up to V = 7.58 m/s to satisfy the $\mathrm{Re}_L < 5 \times 10^5$ condition.

6-102 A flat plate is subjected to air flow parallel to its surface. The effect of air velocity on the average convection heat transfer coefficient for the plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of air at 20°C and 1 atm are $\nu = 1.516 \times 10^{-5}$ m^2/s, $\rho = 1.204$ kg/m^3, $c_p = 1007$ J/kg·K, and Pr = 0.7309 (Table A-15).

Analysis The air velocity for $\text{Re}_L < 5 \times 10^5$ is

$$\text{Re}_L = \frac{VL}{\nu} \quad \rightarrow \quad V = \frac{\nu \, \text{Re}_L}{L} = \frac{(1.516\times10^{-5}\ \text{m}^2/\text{s})(5\times10^5)}{1\,\text{m}} = 7.58\ \text{m/s} \quad \rightarrow \quad V < 7.58\ \text{m/s}$$

For $V < 7.58$ m/s, we have $\text{Re}_L < 5 \times 10^5$; and for $V > 7.58$ m/s, we have $\text{Re}_L > 5 \times 10^5$.

Thus,

$$C_f = 1.33\,\text{Re}_L^{-1/2} \quad \text{for} \quad V < 7.58\ \text{m/s} \quad \text{(laminar flow)}$$

$$C_f = 0.074\,\text{Re}_L^{-1/5} \quad \text{for} \quad 7.58 \le V \le 20\ \text{m/s} \quad \text{(turbulent flow)}$$

The average convection heat transfer coefficient can be determined from the modified Reynolds analogy

$$h = \frac{C_f}{2}\frac{\rho V c_p}{\text{Pr}^{2/3}}$$

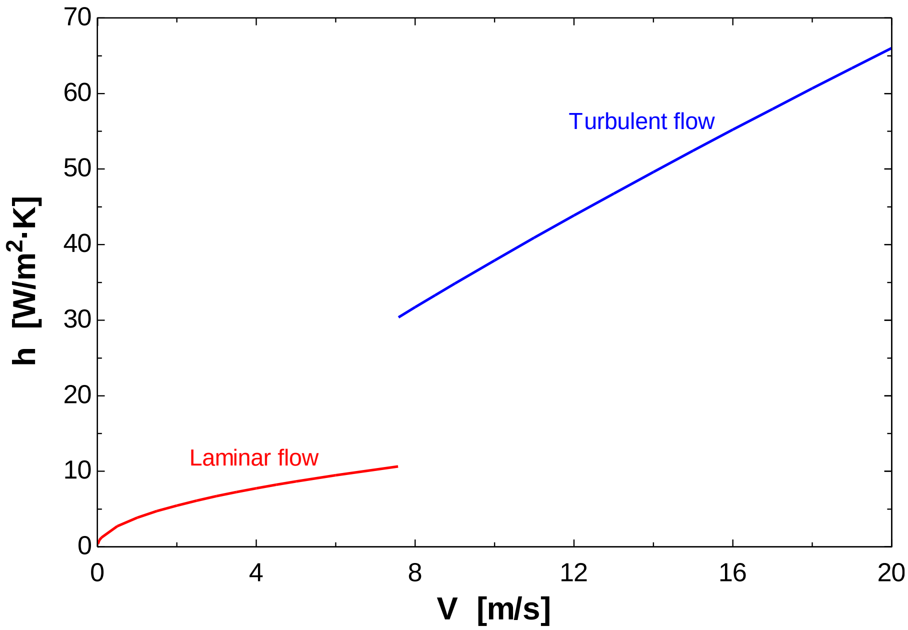
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
L=1 [m]

"PROPERTIES"
nu=1.516e-5 [m^2/s]
rho=1.204 [kg/m^3]
c_p=1007 [J/kg-K]
Pr=0.7309

"ANALYSIS"
C_f_lam=1.33*(V_lam*L/nu)^(-1/2)
C_f_turb=0.074*(V_turb*L/nu)^(-1/5)
h_lam=(C_f_lam/2)*(rho*V_lam*c_p)/(Pr^(2/3))
h_turb=(C_f_turb/2)*(rho*V_turb*c_p)/(Pr^(2/3))
```

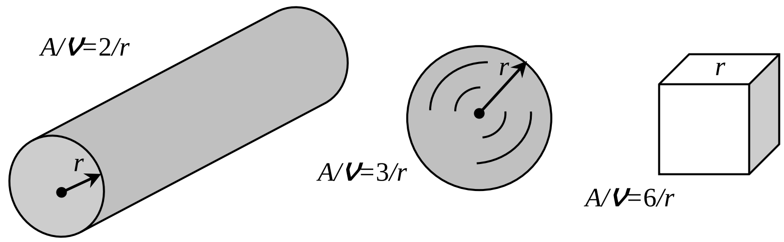
V [m/s]	h [W/m^2·K]	V [m/s]	h [W/m^2·K]
0.01	0.3869	7.58	30.37
0.05	0.8651	8.0	31.71
0.1	1.223	8.5	33.29
0.5	2.736	9.0	34.85
1.0	3.869	10	37.91
1.5	4.738	11	40.91
2.0	5.471	12	43.86
2.5	6.117	13	46.76
3.0	6.701	14	49.62
3.5	7.238	15	52.44
4.0	7.738	16	55.21
4.5	8.207	17	57.96
5.0	8.651	18	60.67
6.0	9.477	19	63.35
7.57	10.64	20	66.01



Discussion The convection heat transfer coefficient in turbulent flow is significantly higher than that of laminar flow.

Special Topic: Microscale Heat Transfer

6-103 It is to be shown that the rate of heat transfer is inversely proportional to the size of an object.



Analysis Consider a cylinder of radius *r* and length *l*. The surface area of this cylinder is $A = 2\pi r(l + r)$ and its volume is $A = \pi r^2 l$. Therefore, the area per unit volume is $\frac{2(l+r)}{rl}$ which, for a long tube *l* <<*r*, becomes $\frac{A}{V} = \frac{2}{r}$. Similarly, it can be shown that the surface area to volume ratio is $\frac{3}{r}$ for a sphere of radius *r*, and $\frac{6}{r}$ for a cube of side *r*.

Note that as *r* becomes smaller, the surface to volume ratio increases. Specifically, this means that while the surface area is about the same order of that of the volume of macroscale (meter, centimeter scale) objects, but the surface becomes million or more times the volume as the size of the object goes to micrometer scale or below. Since, convective heat transfer is proportional to $A(T - T_\infty)$, heat flow increases as *A* increases.

6-104 For specified wall and fluid temperatures, the heat flux at the wall of a microchannel is to be determined.

Assumptions Steady operating conditions exist.

Properties The properties for both cases are given.

***Analysis*:** (*a*) The gas and wall temperatures are T_g =100°C = 373 K, T_w = 50°C = 323 K. Then,

$$T_g - T_w = \frac{2-\sigma_T}{\sigma_T}\left(\frac{2\gamma}{\gamma+1}\right)\left(\frac{\lambda}{\mathrm{Pr}}\right)\left(\frac{\partial T}{\partial y}\right)_w = \left(\frac{2-1}{1}\right)\left(\frac{2\times 1.667}{2.667}\right)(0.5)\left(\frac{\partial T}{\partial y}\right)_w$$

$$\left(\frac{\partial T}{\partial y}\right)_w = \frac{T_g - T_w}{0.625} = \frac{373-323}{0.625} = 80\ \mathrm{K/m}$$

Therefore, the wall heat flux is

$$-k\left(\frac{\partial T}{\partial y}\right)_w = (0.15\ \mathrm{W/m\cdot K})(80\ \mathrm{K/m}) = \mathbf{12\ W/m^2}$$

(*b*) Repeating the same calculations for a different set of properties,

$$T_g - T_w = \frac{2-\sigma_T}{\sigma_T}\left(\frac{2\gamma}{\gamma+1}\right)\left(\frac{\lambda}{\mathrm{Pr}}\right)\left(\frac{\partial T}{\partial y}\right)_w = \left(\frac{2-0.8}{0.8}\right)\left(\frac{2\times 2}{2+1}\right)(5)\left(\frac{\partial T}{\partial y}\right)_w$$

$$\left(\frac{\partial T}{\partial y}\right)_w = \frac{T_g - T_w}{10} = \frac{373-323}{10} = 5\ \mathrm{K/m}$$

$$-k\left(\frac{\partial T}{\partial y}\right)_w = (0.1\ \mathrm{W/m\cdot K})(5\ \mathrm{K/m}) = \mathbf{0.5\ W/m^2}$$

6-105 For a specified temperature gradient, the Nusselt numbers associated with ambient air and nitrogen gas are to be determined.

Assumptions Steady operating conditions exist.

***Analysis*:** At the outer surface of the microchannel (assuming it to be infinitesimally thin), the heat transferred through the channel fluid (gas-wall interface) outward should balance the heat convected outside, and

$$h(T_a - T_w) = -k\left(\frac{\partial T}{\partial y}\right)_w$$

Therefore, the Nusselt number for cooling is

$$\text{Nu} = \frac{h}{k}L = \frac{-(\partial T/dy)_w}{(T_a - T_w)}L$$

The channel is 1.2 μm thick, i.e., $L = 1.2 \times 10^{-6}$ m.

(*a*) For an ambient air temperature of 30°C,

$$\frac{h}{k} = \frac{-(\partial T/dy)_w}{(T_a - T_w)} = \frac{65\text{ K/m}}{(50-30)\text{K}} = 3.25\text{ m}^{-1}$$

Thus,

$$\text{Nu} = hL/k = (3.25\text{ m}^{-1})(1.2\times10^{-6}\text{ m}) = \mathbf{3.90 \times 10^{-6}}$$

(*b*) For a nitrogen gas temperature of -100°C,

$$\frac{h}{k} = \frac{-(\partial T/dy)_w}{(T_a - T_w)} = \frac{65\text{ K/m}}{[50-(-100)]\text{K}} = 0.433\text{ m}^{-1}$$

Thus,

$$\text{Nu} = hL/k = (0.433\text{ m}^{-1})(1.2\times10^{-6}\text{ m}) = \mathbf{5.20 \times 10^{-7}}$$

Review problems

6-106E Prantl number is to be determined for a given set of properties.

Assumptions None.

Properties The given properties are: c_p = 0.5 Btu/lbm·R, k = 2 Btu/h·ft·R, μ = 0.3 lbm/ft·s.

Analysis The Prandtl number is

$$\Pr = \frac{c_p\,\mu}{k} = \frac{(0.5\,\text{Btu/lbm}\cdot\text{R})(0.3\,\text{lbm/ft}\cdot\text{s})(3600\,\text{s/h})}{2\,\text{Btu/h}\cdot\text{ft}\cdot\text{R}} = \mathbf{270}$$

Discussion Typically Prandtl number of liquids and gases is listed along with other properties in the property tables in the Appendix. However, if it is not listed, it can be obtained from the other properties. An alternative to the above equation is Pr = ν/α where $\nu = \mu/\rho$ and $\alpha = k/\rho c_p$.

6-107 Determine whether the flow is laminar or turbulent over a flat plate for different fluids at a given temperature.

Assumptions **1** Transition from laminar to turbulent flow over the flat plate occurs at a Reynolds number of 5×10^5

Properties The properties of the fluids are evaluated at 50°C. For Air: $\nu = 1.798 \times 10^{-5}$ m²/s (Table A-15). For CO_2: $\nu = 9.714 \times 10^{-6}$ m²/s (Table A-16). For Water: ρ = 988.1 kg/m³, $\mu = 0.547 \times 10^{-3}$ kg/m·s $\rightarrow$ $\nu = \mu/\rho = 5.536 \times 10^{-7}$ m²/s (Table A-9). For Engine oil (unused): $\nu = 1.671 \times 10^{-4}$ m²/s (Table A-13).

Analysis The Reynolds number for the plate is

$$\mathrm{Re}_L = \frac{VL}{\nu}$$

For Air: $$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(5\,\text{m/s})(0.15\,\text{m})}{1.798\times10^{-5}\,\text{m}^2\text{/s}} = \mathbf{4.17\times10^4} < 5\times10^5 \quad (\textbf{Laminar Flow})$$

For CO_2: $$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(5\,\text{m/s})(0.15\,\text{m})}{9.714\times10^{-6}\,\text{m}^2\text{/s}} = \mathbf{7.72\times10^4} < 5\times10^5 \quad (\textbf{Laminar Flow})$$

For Water: $$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(5\,\text{m/s})(0.15\,\text{m})}{5.536\times10^{-7}\,\text{m}^2\text{/s}} = \mathbf{1.35\times10^6} < 5\times10^5 \quad (\textbf{Turbulent Flow})$$

For Engine oil (unused): $$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(5\,\text{m/s})(0.15\,\text{m})}{1.671\times10^{-4}\,\text{m}^2\text{/s}} = \mathbf{4.49\times10^3} < 5\times10^5 \quad (\textbf{Laminar Flow})$$

Discussion The value of the kinematic viscosity dictates whether the flow will be laminar or turbulent for the four fluids. The smaller the value, the higher the Reynolds number.

6-108E Fluid is flowing over a flat plate. The characteristic length (L_c) at which the Reynolds number is 1×10^5 is to be determined.

Assumptions **1** Isothermal condition exists between the flat plate and fluid flow. **2** Properties are constant.

Analysis Using the definition for Reynolds number, we have

$$\mathrm{Re} = \frac{VL_c}{\nu} \quad \text{and} \quad \mathrm{Re}_{\mathrm{cr}} = \frac{Vx_{\mathrm{cr}}}{\nu}$$

Taking the ratio yields

$$\frac{\mathrm{Re}}{\mathrm{Re}_{\mathrm{cr}}} = \frac{VL_c/\nu}{Vx_{\mathrm{cr}}/\nu} \quad \rightarrow \quad \frac{\mathrm{Re}}{\mathrm{Re}_{\mathrm{cr}}} = \frac{L_c}{x_{\mathrm{cr}}}$$

The characteristic length (L_c) at which Re = 1×10^5 is

$$L_c = x_{\mathrm{cr}} \frac{\mathrm{Re}}{\mathrm{Re}_{\mathrm{cr}}} = (7\text{ ft})\frac{1\times10^5}{5\times10^5} = \mathbf{1.4\,ft}$$

Discussion In some flow conditions, the value of $\mathrm{Re}_{\mathrm{cr}}$ may change substantially, depending on the free stream turbulence level.

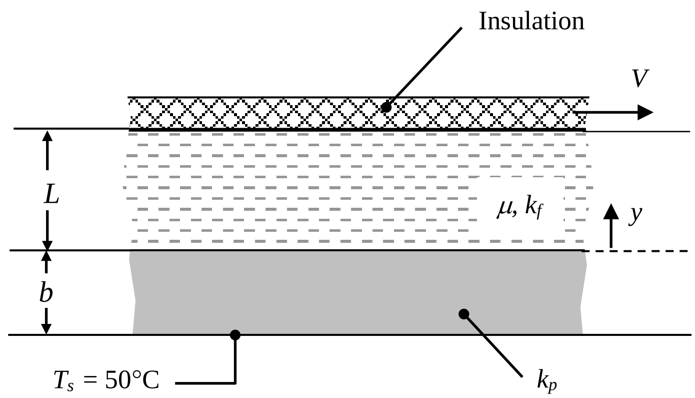
6-109 The Coutte flow of a fluid between two parallel plates is considered. The temperature distribution is to be sketched and determined, and the maximum temperature of the fluid, as well as the temperature of the fluid at the contact surfaces with the lower and upper plates are to be determined.

Assumptions Steady operating conditions exist.

Properties The viscosity and thermal conductivity of the fluid are given to be $\mu = 0.8$ N·s/m^2 and $k_f = 0.145$ W/m·K. The thermal conductivity of lower plate is given to be $k_p = 1.5$ W/m·K.

***Analysis*:** (*a*) The sketch of temperature distribution is given in the figure. We observe from this figure that there are different slopes at the interface ($y = 0$) because of different conductivities ($k_p > k_f$). The slope is zero at the upper plate ($y = L$) because of adiabatic condition.

(*b*) The general solution of the relevant differential equation is obtained as follows:



$$u = \frac{y}{L}V \longrightarrow \frac{du}{dy} = \frac{V}{L}$$

$$\frac{d^2T}{dy^2} = \frac{-\mu}{k_f}\frac{V^2}{L^2} \longrightarrow \frac{dT}{dy} = \frac{-\mu}{k_f}\frac{V^2}{L^2}y + C_1$$

$$T = \frac{-\mu}{2k_f}\frac{V^2}{L^2}y^2 + C_1 y + C_2$$

Applying the boundary conditions:

$y = 0$

$$\dot{q}_f = \dot{q}_p \longrightarrow -k_f \left.\frac{dT}{dy}\right|_0 = \frac{T(0) - T_s}{b/k_p}$$

$$k_f C_1 = \frac{k_p}{b}(C_2 - T_s) \qquad (1)$$

$y = L$, adiabatic $\quad \left.\frac{dT}{dy}\right|_L = 0 \longrightarrow C_1 = \frac{\mu}{k_f}\frac{V^2}{L}$

From Eq. (1), $\quad C_2 = b\frac{k_f}{k_p}C_1 + T_s = b\frac{\mu}{k_p}\frac{V^2}{L} + T_s$

Substituting the coefficients, the temperature distribution becomes

$$T(y) = \frac{-\mu}{2k_f}\frac{V^2}{L^2}y^2 + \frac{\mu}{k_f}\frac{V^2}{L}y + \frac{\mu}{k_p}\frac{V^2}{L}b + T_s$$

(c) Then the temperatures at the contact surfaces are determined to be

$$T(0) = 0 + 0 + \frac{0.8}{1.5}\frac{7^2}{0.005}0.003 + 50 = \mathbf{65.7°C}$$

$$T(y) = \frac{-0.8}{2(0.145)}\frac{7^2}{0.005^2}0.005^2 + \frac{0.8}{0.145}\frac{7^2}{0.005}0.005 + \frac{0.8}{1.5}\frac{7^2}{0.005}0.003 + 50 = \mathbf{201°C}$$

The maximum temperature is $T_{\max} = T(L) = 201$°C because of the adiabatic condition at $y = L$.

6-110 The hydrodynamic boundary layer and the thermal boundary layer both as a function of *x* are to be plotted for the flow of engine oil over a plate.

Analysis The problem is solved using Excel, and the solution is given below.

Assumptions

1. The flow is steady and incompressible
2. The critical Reynolds number is 500,000
3. Air is an ideal gas
4. The plate is smooth
5. Edge effects are negligible and the upper surface of the plate is considered

Input Properties

The average film temperature is 40°C (Property data from Table A-13)

ρ = 876 kg/m^3

c_p = 1964 J/kg·°C

μ = 0.2177 kg/m·s

k = 0.1444 W/m·°C

Pr = 2962

Input Parameters

W = 0.3 m

$T_{f,\text{avg}}$ = 40°C

V = 3 m/s

T_{fluid} = 15°C

T_s = 65°C

Analysis

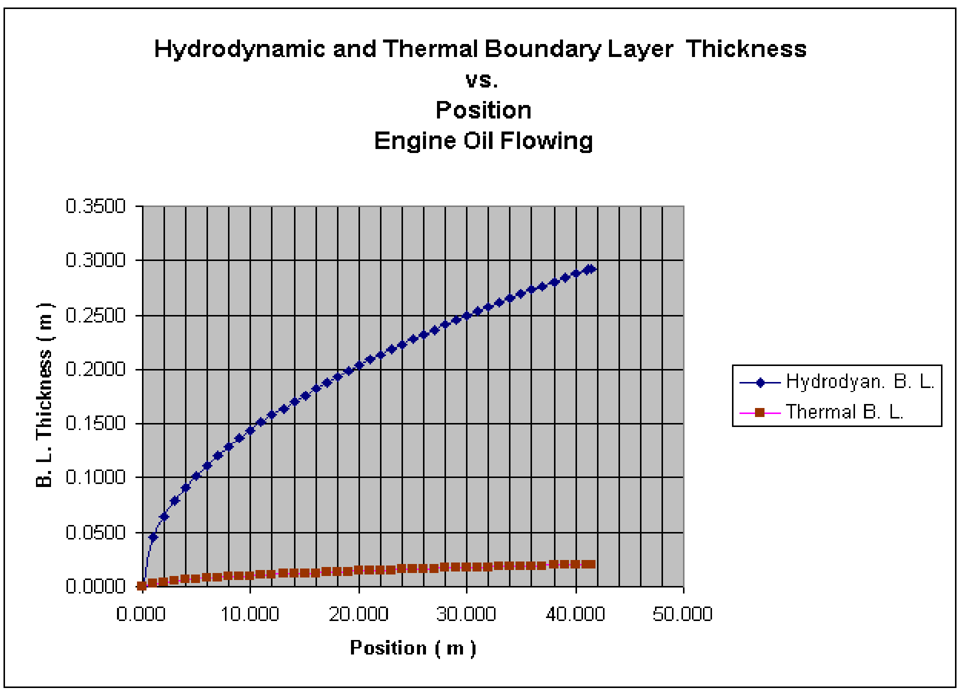
The critical length: $$\text{Re} = \frac{Vx_{cr}}{\nu} \longrightarrow x_{cr} = \frac{\text{Re}\,\nu}{V} = \frac{\text{Re}\,\mu}{V\rho} = \frac{(500{,}000)(0.2177\text{ kg/m}\cdot\text{s})}{(3\text{ m/s})(876\text{ kg/m}^3)} = 41.42\text{ m}$$

Hydrodynamic boundary layer thickness: $$\delta = \frac{4.91x}{\sqrt{\text{Re}_x}}$$

Thermal boundary layer thickness: $$\delta_t = \frac{4.91x}{\text{Pr}^{1/3}\sqrt{\text{Re}_x}}$$

x (m)	Re_x	δ	δ_t
0.000	0.000	0	0
1.000	12072	0.0455	0.0032
2.000	24143	0.0644	0.0045
3.000	36215	0.0788	0.0055
4.000	48287	0.0910	0.0063
5.000	60358	0.1018	0.0071
6.000	72430	0.1115	0.0078
7.000	84502	0.1204	0.0084
8.000	96573	0.1287	0.0090
9.000	108645	0.1365	0.0095
10.000	120717	0.1439	0.0100
11.000	132788	0.1509	0.0105
12.000	144860	0.1576	0.0110
13.000	156932	0.1641	0.0114

14.000	169003	0.1703	0.0119
15.000	181075	0.1763	0.0123
16.000	193147	0.1820	0.0127
17.000	205218	0.1876	0.0131
18.000	217290	0.1931	0.0134
19.000	229362	0.1984	0.0138
20.000	241433	0.2035	0.0142
21.000	253505	0.2085	0.0145
22.000	265576	0.2135	0.0149
23.000	277648	0.2182	0.0152
24.000	289720	0.2229	0.0155
25.000	301791	0.2275	0.0158
26.000	313863	0.2320	0.0162
27.000	325935	0.2365	0.0165
28.000	338006	0.2408	0.0168
29.000	350078	0.2451	0.0171
30.000	362150	0.2493	0.0174
31.000	374221	0.2534	0.0176
32.000	386293	0.2574	0.0179
33.000	398365	0.2614	0.0182
34.000	410436	0.2654	0.0185
35.000	422508	0.2692	0.0187
36.000	434580	0.2730	0.0190
37.000	446651	0.2768	0.0193
38.000	458723	0.2805	0.0195
39.000	470795	0.2842	0.0198
40.000	482866	0.2878	0.0200
41.000	494938	0.2914	0.0203
41.210	497473	0.2921	0.0203
41.420	500008	0.2929	0.0204



6-111 Object 1 and object 2 with same shape and geometry, but different characteristic lengths, are placed in airflow of different free stream velocities at 1 atm and 20°C. The average convection heat transfer coefficient for object 2 is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Analysis The relation for Nusselt, Prandtl, and Reynolds numbers is given as

$$\mathrm{Nu} = g(\mathrm{Re}, \mathrm{Pr}) \quad \text{where} \quad \mathrm{Nu} = \frac{hL}{k}, \quad \mathrm{Pr} = \frac{c_p \mu}{k}, \quad \text{and} \quad \mathrm{Re} = \frac{VL}{\nu}$$

Then

Object 1: $$\mathrm{Re}_1 = \frac{VL_1}{\nu} = \frac{(50\ \mathrm{m/s})(0.5\ \mathrm{m})}{\nu} = \frac{25\ \mathrm{m^2/s}}{\nu} \quad \text{and} \quad \mathrm{Pr}_1 = \frac{c_p \mu}{k}$$

Object 2: $$\mathrm{Re}_2 = \frac{VL_2}{\nu} = \frac{(5\ \mathrm{m/s})(5\ \mathrm{m})}{\nu} = \frac{25\ \mathrm{m^2/s}}{\nu} \quad \text{and} \quad \mathrm{Pr}_2 = \frac{c_p \mu}{k}$$

Since the fluid properties are constant, we have $\mathrm{Re}_1 = \mathrm{Re}_2$ and $\mathrm{Pr}_1 = \mathrm{Pr}_2$, which implies

$$\mathrm{Nu}_1 = g(\mathrm{Re}_1, \mathrm{Pr}_1) = \mathrm{Nu}_2 = g(\mathrm{Re}_2, \mathrm{Pr}_2) \quad \rightarrow \quad \mathrm{Nu}_1 = \mathrm{Nu}_2$$

Hence

$$\frac{h_1 L_1}{k} = \frac{h_2 L_2}{k} \quad \rightarrow \quad h_2 = h_1 \frac{L_1}{L_2} = h_1 \frac{0.5}{5} = 0.1 h_1$$

The average convection heat transfer coefficient for object 1 is

$$\dot{q}_1 = h_1 (T_s - T_\infty) \quad \rightarrow \quad h_1 = \frac{\dot{q}_1}{T_s - T_\infty} = \frac{12000\ \mathrm{W/m^2}}{(120 - 20)\ \mathrm{K}} = 120\ \mathrm{W/m^2 \cdot K}$$

Therefore the average convection heat transfer coefficient for object 2 is

$$h_2 = 0.1 h_1 = 0.1(120\ \mathrm{W/m^2 \cdot K}) = \mathbf{12\ W/m^2 \cdot K}$$

Discussion The conditions where $\mathrm{Re}_1 = \mathrm{Re}_2$ and $\mathrm{Pr}_1 = \mathrm{Pr}_2$ allowed one to easily relate the Nusselt numbers as $\mathrm{Nu}_1 = \mathrm{Nu}_2$. If $\mathrm{Re}_1 \neq \mathrm{Re}_2$, then the specific expression for $g(\mathrm{Re}, \mathrm{Pr})$ is needed to relate Nu_1 and Nu_2.

6-112 A rectangular bar is placed in a free stream flow. Using the given expression for Nusselt number, the heat transfer coefficients, for different characteristic lengths and free stream velocities, are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Analysis From the given expression for Nusselt number

$$\mathrm{Nu} = C\,\mathrm{Re}^m\,\mathrm{Pr}^n \quad \rightarrow \quad \frac{hL}{k} = C\left(\frac{VL}{\nu}\right)^m \left(\frac{\nu}{\alpha}\right)^n$$

From the given information, we have

Case 1: $h_1 = 100\ \mathrm{W/m^2\cdot K}$ when $V_1 = 25\ \mathrm{m/s}$ and $L_1 = 0.5\ \mathrm{m}$

Case 2: $h_2 = 50\ \mathrm{W/m^2\cdot K}$ when $V_2 = 5\ \mathrm{m/s}$ and $L_2 = 0.5\ \mathrm{m}$

Hence

$$\frac{\mathrm{Nu}_1}{\mathrm{Nu}_2} = \frac{C\,\mathrm{Re}_1^m\,\mathrm{Pr}_1^n}{C\,\mathrm{Re}_2^m\,\mathrm{Pr}_2^n} = \frac{\mathrm{Re}_1^m}{\mathrm{Re}_2^m} \quad \rightarrow \quad \frac{h_1 L_1}{h_2 L_2} = \left(\frac{V_1 L_1}{V_2 L_2}\right)^m$$

where $\mathrm{Pr}_1 = \mathrm{Pr}_2$ is due to constant properties. Then

$$\frac{h_1 L_1}{h_2 L_2} = \left(\frac{V_1 L_1}{V_2 L_2}\right)^m \quad \rightarrow \quad \frac{(100\ \mathrm{W/m^2\cdot K})(0.5\ \mathrm{m})}{(50\ \mathrm{W/m^2\cdot K})(0.5\ \mathrm{m})} = \left[\frac{(25\ \mathrm{m/s})(0.5\ \mathrm{m})}{(5\ \mathrm{m/s})(0.5\ \mathrm{m})}\right]^m \quad \rightarrow \quad 2 = 5^m$$

Solving for the constant m yields $m = 0.4307$.

(*a*) For $L = 1$ m and $V = 5$ m/s, the convection heat transfer coefficient is

$$h = h_2 \frac{L_2}{L}\left(\frac{VL}{V_2 L_2}\right)^{0.4307} = (50\ \mathrm{W/m^2\cdot K})\frac{(0.5\ \mathrm{m})}{(1\ \mathrm{m})}\left[\frac{(5\ \mathrm{m/s})(1\ \mathrm{m})}{(5\ \mathrm{m/s})(0.5\ \mathrm{m})}\right]^{0.4307} = \mathbf{33.7\ W/m^2\cdot K}$$

(*b*) For $L = 2$ m and $V = 50$ m/s, the convection heat transfer coefficient is

$$h = h_2 \frac{L_2}{L}\left(\frac{VL}{V_2 L_2}\right)^{0.4307} = (50\ \mathrm{W/m^2\cdot K})\frac{(0.5\ \mathrm{m})}{(2\ \mathrm{m})}\left[\frac{(50\ \mathrm{m/s})(2\ \mathrm{m})}{(5\ \mathrm{m/s})(0.5\ \mathrm{m})}\right]^{0.4307} = \mathbf{61.2\ W/m^2\cdot K}$$

Discussion The Nusselt number relation, $\mathrm{Nu} = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$, is in general a reasonably accurate representation for convection heat transfer coefficient. However, more complex relations for Nusselt number are used for better accuracy.

6-113 Electrical heaters are embedded inside the wing to prevent formation of ice. The heat flux necessary to keep the wing surface above 0°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of air (1 atm) at −10°C are given in Table A-15: $\nu = 1.252 \times 10^{-5}$ m²/s, $k = 0.02288$ W/m·K, and Pr = 0.7387.

Analysis With a characteristic length of 2.5 m, the Reynolds number is

$$\text{Re} = \frac{VL}{\nu} = \frac{(200\ \text{m/s})(2.5\ \text{m})}{1.252\times10^{-5}\ \text{m}^2/\text{s}} = 3.994\times10^7$$

Applying the modified Reynolds analogy,

$$\frac{C_f\ \text{Re}}{2} = \text{Nu}\,\text{Pr}^{-1/3} \quad\rightarrow\quad \text{Nu} = \frac{C_f}{2}\text{Re}\,\text{Pr}^{1/3} \quad \text{or} \quad h = \frac{C_f}{2}\frac{k}{L}\text{Re}\,\text{Pr}^{1/3}$$

$$h = \frac{0.001}{2}\frac{(0.02288\ \text{W/m}\cdot\text{K})}{(2.5\ \text{m})}(3.994\times10^7)(0.7387)^{1/3} = 165.2\ \text{W/m}^2\cdot\text{K}$$

The heat flux necessary to keep the wing surface above 0°C is

$$\dot{q} \ge h(T_s - T_\infty) = (165.2\ \text{W/m}^2\cdot\text{K})[0-(-20)]\ \text{K} = 3304\ \text{W/m}^2 \quad\rightarrow\quad \dot{q} \ge \mathbf{3304\ W/m^2}$$

Discussion The modified Reynolds analogy is applicable approximately for turbulent flow over a surface, even when pressure gradient is present.

6-114 Forced convection of air is used for cooling the surface of a circuit board. The temperature difference between the circuit board surface temperature and the airstream temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

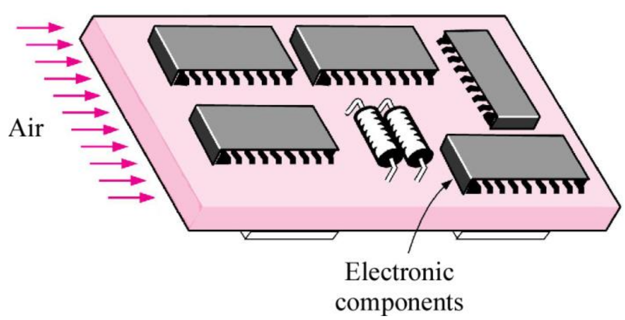
Properties The properties of air (1 atm) at 40°C are given in Table A-15: $\rho = 1.127$ kg/m³, $c_p = 1007$ J/kg·K, $\nu = 1.702 \times 10^{-5}$ m²/s, and Pr = 0.7255.

Analysis Applying the modified Reynolds analogy

$$\frac{C_f}{2} = \text{St}\,\text{Pr}^{2/3}$$

with $\quad \text{St} = \dfrac{h}{\rho c_p V}$

and $\quad \tau_s = C_f \dfrac{\rho V^2}{2}$



Substituting yields

$$\frac{\tau_s}{\rho V^2} = \frac{h}{\rho c_p V}\text{Pr}^{2/3} \quad\rightarrow\quad h = \frac{\tau_s}{V}c_p\,\text{Pr}^{-2/3}$$

The convection heat transfer coefficient is

$$h = \frac{\tau_s}{V}c_p\,\text{Pr}^{-2/3} = \frac{(0.075\ \text{N/m}^2)}{(3\ \text{m/s})}(1007\ \text{J/kg}\cdot\text{K})(0.7255)^{-2/3} = 31.18\ \text{W/m}^2\cdot\text{K}$$

Therefore the temperature difference between the circuit board surface temperature and the airstream temperature is

$$T_s - T_\infty = \frac{\dot{q}}{h} = \frac{1000\ \text{W/m}^2}{31.18\ \text{W/m}^2\cdot\text{K}} = \mathbf{32.1°C}$$

Discussion To reduce the temperature difference between the circuit board surface temperature and the airstream temperature, the value of convection heat transfer coefficient needs to be increased. This can be achieved by increasing the airstream velocity.

6-115 A flat flat is subjected to air flow, and the drag force acting on it is measured. The electrical power needed to maintain the prescribed heater surface temperature of 80°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The edge effects are negligible.

Properties The properties of air at 50°C and 1 atm are (Table A-15)

ρ = 1.092 kg/m^3, c_p=1.007 kJ/kg-K, Pr = 0.7228

Analysis For flat plates, the drag force is equivalent to friction force. The average friction coefficient C_f can be determined from

$$F_f = C_f A_s \frac{\rho V^2}{2} \longrightarrow C_f = \frac{F_f}{\rho A_s V^2/2} = \frac{0.2\text{ N}}{(1.092\text{ kg/m}^3)(0.30\text{ m}^2)(10\text{ m/s})^2/2}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = 0.01221$$

Then the average heat transfer coefficient can be determined from the modified Reynolds analogy to be

$$h = \frac{C_f}{2}\frac{\rho V c_p}{\Pr^{2/3}} = \frac{0.01221}{2}\frac{(1.092\text{ kg/m}^3)(10\text{ m/s})(1007\text{ J/kg}\cdot°\text{C})}{(0.7228)^{2/3}} = 83.35\text{ W/m}^2\cdot\text{C}$$

Then the rate of heat transfer becomes

$$\dot{Q} = hA_s(T_s - T_\infty) = (83.35\text{W/m}^2\cdot\text{K})(0.30\text{m}^2)(80-20)\text{ K} = \mathbf{1500W}$$

Fundamentals of Engineering (FE) Exam Problems

6-116 The transition from laminar flow to turbulent flow in a forced convection situation is determined by which one of the following dimensionless numbers?

(a) Grasshof (b) Nusselt (c) Reynolds (d) Stanton (e) Mach

Answer (c) Reynolds

6-117 The ______ number is a significant dimensionless parameter for forced convection and the _____ number is a significant dimensionless parameter for natural convection.

(a) Reynolds, Grashof (b) Reynolds, Mach (c) Reynolds, Eckert

(d) Reynolds, Schmidt (e) Grashof, Sherwood

Answer (a) Reynolds, Grashof

6-118 In any forced or natural convection situation, the velocity of the flowing fluid is zero where the fluid wets any stationary surface. The magnitude of heat flux where the fluid wets a stationary surface is given by

(a) $k(T_{\text{fluid}} - T_{\text{wall}})$ (b) $k\left.\dfrac{dT}{dy}\right|_{\text{wall}}$ (c) $k\left.\dfrac{d^2T}{dy^2}\right|_{\text{wall}}$ (d) $h\left.\dfrac{dT}{dy}\right|_{\text{wall}}$ (e) None of them

Answer (b) $k\left.\dfrac{dT}{dy}\right|_{\text{wall}}$

6-119 The coefficient of friction C_f for a fluid flowing across a surface in terms of the surface shear stress, τ_s, is given by

(a) $2\rho V^2/\tau_s$ (b) $2\tau_s/\rho V^2$ (c) $2\tau_s/\rho V^2\Delta T$ (d) $4\tau_s/\rho V^2$ (e) None of them

Answer (b) $2\tau_s/\rho V^2$

6-120 Most correlations for the convection heat transfer coefficient use the dimensionless Nusselt number, which is defined as

(a) h/k (b) k/h (c) hL_c/k (d) kL_c/h (e) $k/\rho c_p$

Answer (c) hL_c/k

6-121 For the same initial conditions, one can expect the laminar thermal and momentum boundary layers on a flat plate to have the same thickness when the Prandtl number of the flowing fluid is

(a) Close to zero (b) Small (c) Approximately one

(d) Large (e) Very large

Answer (c) Approximately one

6-122 One can expect the heat transfer coefficient for turbulent flow to be ____ for laminar flow

(a) less than (b) same as (c) greater than

Answer (c) greater than

6-123 An electrical water (k = 0.61 W/m·K) heater uses natural convection to transfer heat from a 1-cm diameter by 0.65-m long, 110 V electrical resistance heater to the water. During operation, the surface temperature of this heater is 120ºC while the temperature of the water is 35ºC, and the Nusselt number (based on the diameter) is 6. Considering only the side surface of the heater (and thus $A = \pi DL$), the current passing through the electrical heating element is

(a) 3.2 A (b) 3.7 A (c) 4.6 A (d) 5.8 A (e) 6.6 A

Answer (d) 5.8 A

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k=0.61 [W/m-K]
d=0.01 [m]
L=0.65 [m]
Nus=6
DT=85 [K]
DV=110 [Volt]
h=Nus*k/d
Q=h*pi*d*L*DT
I=Q/DV
```

6-124 In turbulent flow, one can estimate the Nusselt number using the analogy between heat and momentum transfer (Colburn analogy). This analogy relates the Nusselt number to the coefficient of friction, C_f, as

(a) $Nu = 0.5\ C_f Re\ Pr^{1/3}$ (b) $Nu = 0.5\ C_f Re\ Pr^{2/3}$ (c) $Nu = C_f Re\ Pr^{1/3}$
(d) $Nu = C_f Re\ Pr^{2/3}$ (e) $Nu = C_f Re^{1/2}\ Pr^{1/3}$

Answer (a) $Nu = 0.5\ C_f Re\ Pr^{1/3}$

6-125, 6-126 Design and Essay Problems

Solutions Manual for

Heat and Mass Transfer: Fundamentals & Applications

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Chapter 7
EXTERNAL FORCED CONVECTION

Drag Force and Heat Transfer in External Flow

7-1C The velocity of the fluid relative to the immersed solid body sufficiently far away from a body is called the *free-stream velocity*, V_∞. The *upstream*(or *approach*) *velocityV*is the velocity of the approaching fluid far ahead of the body. These two velocities are equal if the flow is uniform and the body is small relative to the scale of the free-stream flow.

7-2C The force a flowing fluid exerts on a body in the flow direction is called *drag*. Drag is caused by friction between the fluid and the solid surface, and the pressure difference between the front and back of the body. We try to minimize drag in order to reduce fuel consumption in vehicles, improve safety and durability of structures subjected to high winds, and to reduce noise and vibration.

7-3C The force a flowing fluid exerts on a body in the normal direction to flow that tend to move the body in that direction is called *lift*. It is caused by the components of the pressure and wall shear forces in the normal direction to flow. The wall shear also contributes to lift (unless the body is very slim), but its contribution is usually small.

7-4C When the drag force F_D, the upstream velocity$V$, and the fluid density ρ are measured during flow over a body, the drag coefficient can be determined from

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

where A is ordinarily the *frontal area* (the area projected on a plane normal to the direction of flow) of the body.

7-5C The *frontal area* of a body is the area seen by a person when looking from upstream. The frontal area is appropriate to use in drag and lift calculations for blunt bodies such as cars, cylinders, and spheres.

7-6C The part of drag that is due directly to wall shear stress τ_w is called the *skin friction drag*$F_{D,\text{ friction}}$ since it is caused by frictional effects, and the part that is due directly to pressure P and depends strongly on the shape of the body is called the *pressure drag*$F_{D,\text{ pressure}}$. For slender bodies such as airfoils, the friction drag is usually more significant.

7-7C A body is said to be *streamlined* if a conscious effort is made to align its shape with the anticipated streamlines in the flow. Otherwise, a body tends to block the flow, and is said to be *blunt*. A tennis ball is a blunt body (unless the velocity is very low and we have "creeping flow").

7-8C As a result of streamlining, (*a*) friction drag increases, (*b*) pressure drag decreases, and (*c*) total drag decreases at high Reynolds numbers (the general case), but increases at very low Reynolds numbers since the friction drag dominates at low Reynolds numbers.

7-9C The friction drag coefficient is independent of surface roughness in *laminar flow*, but is a strong function of surface roughness in *turbulent flow* due to surface roughness elements protruding further into the highly viscous laminar sublayer.

7-10C At sufficiently high velocities, the fluid stream detaches itself from the surface of the body. This is called *separation*. It is caused by a fluid flowing over a curved surface at a high velocity (or technically, by adverse pressure gradient). Separation increases the drag coefficient drastically.

Flow over Flat Plates

7-11C The friction coefficient represents the resistance to fluid flow over a flat plate. It is proportional to the drag force acting on the plate. The drag coefficient for a flat surface is equivalent to the mean friction coefficient.

7-12C The friction and the heat transfer coefficients change with position in laminar flow over a flat plate.

7-13C The average friction and heat transfer coefficients in flow over a flat plate are determined by integrating the local friction and heat transfer coefficients over the entire plate, and then dividing them by the length of the plate.

7-14 Air is flowing over a long flat plate with a specified velocity. The distance from the leading edge of the plate where the flow becomes turbulent, and the thickness of the boundary layer at that location are to be determined.

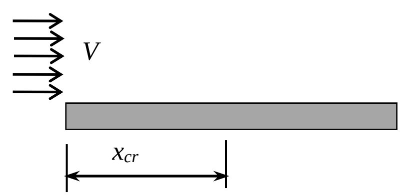
Assumptions **1**The flow is steady and incompressible. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The surface of the plate is smooth.

Properties The density and kinematic viscosity of air at 1 atm and 25°C are $\rho = 1.184$ kg/m^3 and $\nu = 1.562\times10^{-5}$ m^2/s (Table A-15).

AnalysisThe distance from the leading edge of the plate where the flow becomes turbulent is the distance $x_{cr}$where the Reynolds number becomes equal to the critical Reynolds number,

$$\mathrm{Re}_{cr} = \frac{Vx_{cr}}{\nu} \rightarrow$$

$$x_{cr} = \frac{\nu\,\mathrm{Re}_{cr}}{V} = \frac{(1.562\times10^{-5}\ \mathrm{m^2/s})(5\times10^5)}{8\ \mathrm{m/s}} = \mathbf{0.976\ m}$$



The thickness of the boundary layer at that location is obtained by substituting this value of x into the laminar boundary layer thickness relation,

$$\delta_x = \frac{5x}{\mathrm{Re}_x^{1/2}} \rightarrow \delta_{cr} = \frac{5x_{cr}}{\mathrm{Re}_{cr}^{1/2}} = \frac{5(0.976\ \mathrm{m})}{(5\times10^5)^{1/2}} = 0.006903\ \mathrm{m} = \mathbf{0.69\ cm}$$

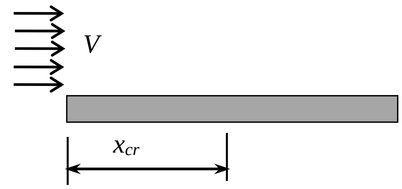
Discussion When the flow becomes turbulent, the boundary layer thickness starts to increase, and the value of its thickness can be determined from the boundary layer thickness relation for turbulent flow.

7-15 Water is flowing over a long flat plate with a specified velocity. The distance from the leading edge of the plate where the flow becomes turbulent, and the thickness of the boundary layer at that location are to be determined.

Assumptions **1**The flow is steady and incompressible. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **3**The surface of the plate is smooth.

Properties The density and dynamic viscosity of water at 1 atm and 25°C are $\rho = 997$ kg/m^3 and $\mu = 0.891\times10^{-3}$ kg/m·s (Table A-9).

AnalysisThe distance from the leading edge of the plate where the flow becomes turbulent is the distance $x_{cr}$where the Reynolds number becomes equal to the critical Reynolds number,



$$\mathrm{Re}_{cr} = \frac{\rho V x_{cr}}{\mu} \rightarrow$$

$$x_{cr} = \frac{\mu \,\mathrm{Re}_{cr}}{\rho V} = \frac{(0.891\times10^{-3}\text{ kg/m}\cdot\text{s})(5\times10^5)}{(997\text{ kg/m}^3)(8\text{ m/s})} = 0.056\text{ m} = \mathbf{5.6\,cm}$$

The thickness of the boundary layer at that location is obtained by substituting this value of x into the laminar boundary layer thickness relation,

$$\delta_{cr} = \frac{5x}{\mathrm{Re}_x^{1/2}} \rightarrow \delta_{cr} = \frac{5x_{cr}}{\mathrm{Re}_{cr}^{1/2}} = \frac{5(0.056\text{ m})}{(5\times10^5)^{1/2}} = 0.00040\text{ m} = \mathbf{0.4\,mm}$$

Therefore, the flow becomes turbulent after about 5 cm from the leading edge of the plate, and the thickness of the boundary layer at that location is 0.4 mm.

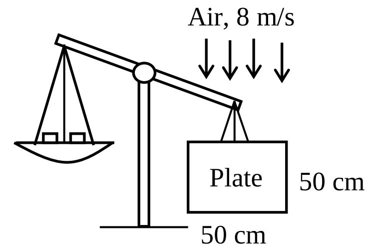
Discussion When the flow becomes turbulent, the boundary layer thickness starts to increase, and the value of its thickness can be determined from the boundary layer thickness relation for turbulent flow.

7-16 The weight of a thin flat plate exposed to air flow on both sides is balanced by a counterweight. The mass of the counterweight that needs to be added in order to balance the plate is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The surfaces of the plate are smooth.

Properties The density and kinematic viscosity of air at 1 atm and 25°C are $\rho = 1.184$ kg/m^3 and $\nu = 1.562\times10^{-5}$ m^2/s (Table A-15).

Analysis The Reynolds number is



$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(8\,\mathrm{m/s})(0.5\mathrm{m})}{1.562\times10^{-5}\ \mathrm{m^2/s}} = 2.561\times10^5$$

which is less than the critical Reynolds number of 5×10^5 . Therefore the flow is laminar. The average friction coefficient, drag force and the corresponding mass are

$$C_f = \frac{1.33}{\mathrm{Re}_L^{\,0.5}} = \frac{1.33}{(2.561\times10^5)^{0.5}} = 0.002628$$

$$\begin{aligned} F_D &= C_f A_s \frac{\rho V^2}{2} \\ &= (0.002628)[(2\times0.5\times0.5)\,\mathrm{m^2}]\frac{(1.184\,\mathrm{kg/m^3})(8\,\mathrm{m/s})^2}{2} = 0.04978\,\mathrm{kg\cdot m/s^2} \\ &= 0.04978\,\mathrm{N} \end{aligned}$$

The mass whose weight is 0.04978 N is

$$m = \frac{F_D}{g} = \frac{0.04978\,\mathrm{kg.m/s^2}}{9.81\,\mathrm{m/s^2}} = 0.00507\,\mathrm{kg} = \mathbf{5.07\,g}$$

Therefore, the mass of the counterweight must be 5 g to counteract the drag force acting on the plate.

Discussion Note that the apparatus described in this problem provides a convenient mechanism to measure drag force and thus drag coefficient.

7-17 Air flows over a plate. Various quantities are to be determined at x = 0.3 m.

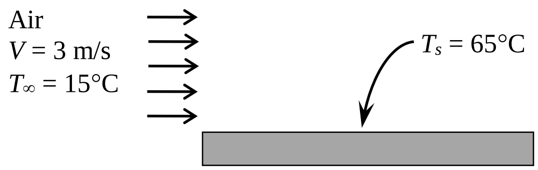
Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The plate is smooth. **5** Edge effects are negligible and the upper surface of the plate is considered.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (65+15)/2 = 40°\mathrm{C}$ are (Table A-15)

$$\rho = 1.127\ \mathrm{kg/m^3},\quad c_p = 1007\ \mathrm{J/kg\cdot K}\ \ k = 0.02662\ \mathrm{W/m\cdot K},\quad \mu = 1.918\times10^{-5}\ \mathrm{kg/m\cdot s},\ \mathrm{Pr} = 0.7255$$

AnalysisThe critical length of the plate is first determined to be

$$x_{cr} = \frac{\mathrm{Re}_{cr}\ \mu}{V\rho} = \frac{(5\times10^5)(1.918\times10^{-5}\ \mathrm{kg/m\cdot s})}{(3\ \mathrm{m/s})(1.127\mathrm{kg/m^3})} = 2.84\ \mathrm{m}$$



Thus flow at x = 0.3m is in the laminar region.

The calculations at x = 0.3 m are

$$\mathrm{Re}_x = \frac{Vx\rho}{\mu} = \frac{(3\ \mathrm{m/s})(0.3\mathrm{m})(1.127\mathrm{kg/m^3})}{1.918\times10^{-5}\ \mathrm{kg/m\cdot s}} = 52{,}883$$

(a) Hydrodynamic boundary layer thickness, Eq. 7-12a:

$$\delta = \frac{4.91x}{\sqrt{\mathrm{Re}_x}} = \frac{4.91(0.3\ \mathrm{m})}{\sqrt{52{,}883}} = \mathbf{0.00641m}$$

(b) Local friction coefficient, Eq. 7-12b:

$$C_{f,x} = 0.664\,\mathrm{Re}_x^{-1/2} = 0.664(52{,}883)^{-1/2} = \mathbf{0.0029}$$

(c) Average friction coefficient, Eq. 7-14:

$$C_f = \frac{1.33}{\mathrm{Re}_x^{1/2}} = \frac{1.33}{52{,}883^{1/2}} = \mathbf{0.0058}$$

(d) Total drag force due to friction, Eq. 7-1:

$$F_f = C_f A_s \frac{\rho V^2}{2} = (0.0058)(0.3\times0.3\ \mathrm{m^2})\frac{(1.127\ \mathrm{kg/m^3})(3\ \mathrm{m/s})^2}{2} = \mathbf{0.0026\,N}$$

(e) Local convection heat transfer coefficient, Eq. 7-19:

$$\mathrm{Nu_x} = 0.332\,\mathrm{Re_x^{1/2}}\,\mathrm{Pr^{1/3}} = 0.332(52{,}883)^{1/2}(0.7255)^{1/3} = 68.6$$

$$h_x = \frac{k}{x}\mathrm{Nu_x} = \frac{0.02662\ \mathrm{W/m}\cdot K}{0.3\ \mathrm{m}}(68.6) = \mathbf{6.09\ W/m^2\cdot K}$$

(f) Average convection heat transfer coefficient, Eq. 7-21:

$$\mathrm{Nu} = 0.664\,\mathrm{Re^{1/2}}\,\mathrm{Pr^{1/3}} = 0.664(52{,}883)^{1/2}(0.7255)^{1/3} = 2\,\mathrm{Nu_x} = 137.2$$

$$h = \frac{k}{x}\mathrm{Nu_x} = \frac{0.02662\ \mathrm{W/m}\cdot K}{0.3\ \mathrm{m}}(137.2) = 2\,h_x = \mathbf{12.2\ W/m^2\cdot K}$$

(g) Rate of convective heat transfer, Eq. 7-9:

$$\dot{Q} = hA_s(T_s - T_\infty) = \left(1.22\,\frac{\mathrm{W}}{\mathrm{m^2}}\cdot\mathrm{K}\right)(0.3\ \times 0.3\ \mathrm{m^2})(65-15)°\mathrm{C} = \mathbf{54.9\ W}$$

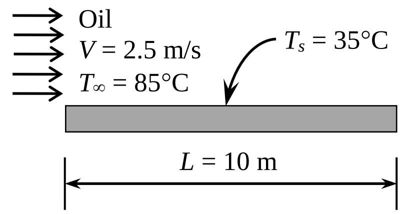
7-18 Hot engine oil flows over a flat plate. The total drag force and the rate of heat transfer per unit width of the plate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible.

Properties The properties of engine oil at the film temperature of $(T_s + T_\infty)/2 = (85+35)/2 = 60$°C are (Table A-13)

$\rho = 863.9$ kg/m^3 $\quad\quad$ $\nu = 8.565 \times 10^{-5}$ m^2/s

$k = 0.1404$ W/m·K $\quad\quad$ Pr = 1080



Analysis Noting that L = 10 m, the Reynolds number at the end of the plate is

$$\text{Re} = \frac{VL}{\nu} = \frac{(2.5\text{ m/s})\ (10\text{ m})}{8.565\times10{-5}\ \text{ m}^2/\text{s}} = 2.919\times10^5$$

which is less than the critical Reynolds number. Thus we have laminar flow over the entire plate. The average friction coefficient and the drag force per unit width are determined from

$$C_f = 1.33\, Re_L^{-0.5} = 1.33\ (2.919\times10^5)^{-0.5} = 0.00246$$

$$F_D = C_f A_s \frac{\rho V^2}{2} = (0.00246)(10\times1\text{ m}^2)\frac{(863.9\text{ kg}/m^3)(2.5\text{ m/s})^2}{2} = \mathbf{66.5\ N}$$

Similarly, the average Nusselt number and the heat transfer coefficient are determined using the laminar flow relations for a flat plate,

$$Nu = \frac{h\,L}{k} = 0.664\ \text{Re}_L^{0.5}\text{Pr}^{1/3} = 0.664\ (2.919\times10^5)^{0.5}(1080)^{1/3} = 3681$$

$$h = \frac{k}{L}\ Nu = \frac{0.1404\text{ W/m·K}}{10\text{ m}}(3681) = 51.68\text{ W/m}^2\text{·K}$$

The rate of heat transfer is then determined from Newton's law of cooling to be

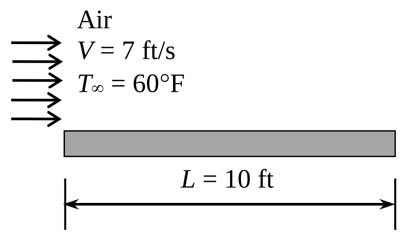
$$\dot{Q} = h\,A_s(T_\infty - T_s) = (51.68\text{ W/m}^2\text{·K})(10\times1\text{ m}^2)(85-35)\text{K} = 25{,}840\text{ W} = \mathbf{25.84\ kW}$$

7-19E Air flows over a flat plate. The local friction and heat transfer coefficients at intervals of 1 ft are to be determined and plotted against the distance from the leading edge.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties.

Properties The properties of air at 1 atm and 60°F are (Table A-15E)

$$k = 0.01433 \text{ Btu/h.ft.°F}$$
$$\nu = 0.1588\times10^{-3} \text{ ft}^2\text{/s}$$
$$\Pr = 0.7321$$



Analysis For the first 1 ft interval, the Reynolds number is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{(7 \text{ ft/s})(1\text{ft})}{0.1588\times10^{-3} \text{ ft}^2\text{/s}} = 4.408\times10^4$$

which is less than the critical value of 5×10^5. Therefore, the flow is laminar. The local Nusselt number is

$$Nu_x = \frac{hx}{k} = 0.332\,\text{Re}_x^{0.5}\,\text{Pr}^{1/3} = 0.332(4.408\times10^4)^{0.5}(0.7321)^{1/3} = 62.82$$

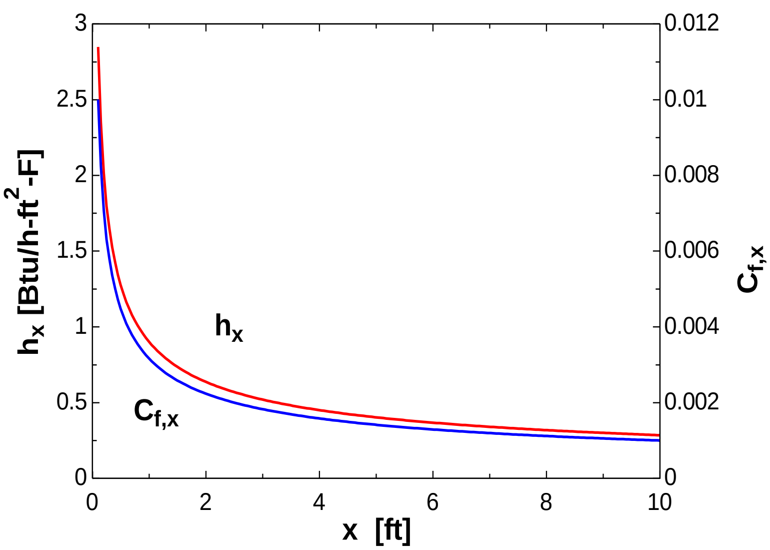
The local heat transfer and friction coefficients are

$$h_x = \frac{k}{x}Nu = \frac{0.01433 \text{ Btu/h.ft.°F}}{1 \text{ ft}}(62.82) = 0.9002 \text{ Btu/h.ft}^2\text{.°F}$$

$$C_{f,x} = \frac{0.664}{\text{Re}^{0.5}} = \frac{0.664}{(4.408\times10^4)^{0.5}} = 0.00316$$

We repeat calculations for all 1-ft intervals. The results are

x [ft]	h_x [Btu/h.ft^2.F]	$C_{f,x}$
1	0.9005	0.003162
2	0.6367	0.002236
3	0.5199	0.001826
4	0.4502	0.001581
5	0.4027	0.001414
6	0.3676	0.001291
7	0.3404	0.001195
8	0.3184	0.001118
9	0.3002	0.001054
10	0.2848	0.001

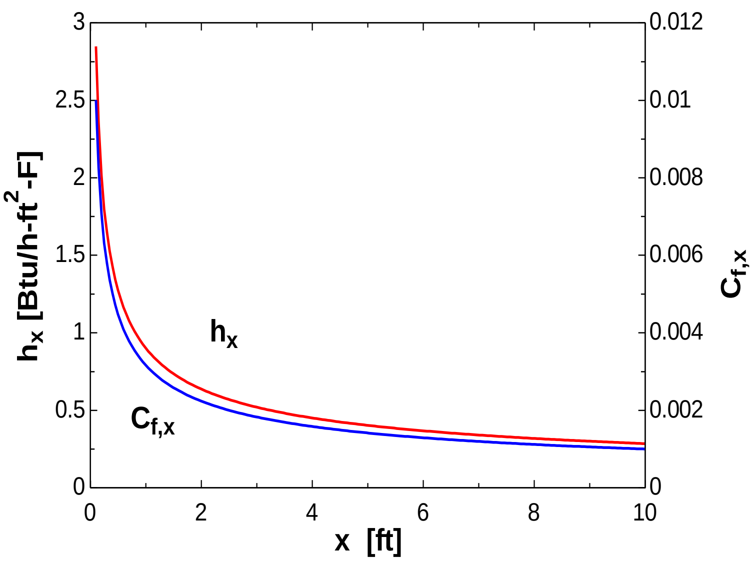


7-20E Prob. 7-19E is reconsidered. The local friction and heat transfer coefficients along the plate are to be plotted against the distance from the leading edge.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_air=60 [F]
x=10 [ft]
Vel=7 [ft/s]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_air)
Pr=Prandtl(Fluid$, T=T_air)
rho=Density(Fluid$, T=T_air, P=14.7)
mu=Viscosity(Fluid$, T=T_air)*Convert(lbm/ft-h, lbm/ft-s)
nu=mu/rho

"ANALYSIS"
Re_x=(Vel*x)/nu
"Reynolds number is calculated to be smaller than the critical Re number. The flow is laminar."
Nusselt_x=0.332*Re_x^0.5*Pr^(1/3)
h_x=k/x*Nusselt_x
C_f_x=0.664/Re_x^0.5
```

x [ft]	h_x [Btu/h.ft^2.F]	$C_{f,x}$
0.1	2.848	0.01
0.2	2.014	0.007071
0.3	1.644	0.005774
0.4	1.424	0.005
0.5	1.273	0.004472
0.6	1.163	0.004083
0.7	1.076	0.00378
0.8	1.007	0.003536
0.9	0.9492	0.003333
1	0.9005	0.003162
…	…	…
…	…	…
9.1	0.2985	0.001048
9.2	0.2969	0.001043
9.3	0.2953	0.001037
9.4	0.2937	0.001031
9.5	0.2922	0.001026
9.6	0.2906	0.001021
9.7	0.2891	0.001015
9.8	0.2877	0.00101
9.9	0.2862	0.001005
10	0.2848	0.001



7-21 Laminar flow of a fluid over a flat plate is considered. The change in the drag force and the rate of heat transfer are to be determined when the free-stream velocity of the fluid is doubled.

Analysis For the laminar flow of a fluid over a flat plate maintained at a constant temperature the drag force is given by

$$F_{D1} = C_f A_s \frac{\rho V^2}{2} \quad \text{where} \quad C_f = \frac{1.33}{\text{Re}^{0.5}}$$

Therefore

$$F_{D1} = \frac{1.33}{\text{Re}^{0.5}} A_s \frac{\rho V^2}{2}$$

Substituting Reynolds number relation, we get

$$F_{D1} = \frac{1.33}{\left(\frac{VL}{\nu}\right)^{0.5}} A_s \frac{\rho V^2}{2} = 0.664 V^{3/2} A_s \frac{\nu^{0.5}}{L^{0.5}}$$

When the free-stream velocity of the fluid is doubled, the new value of the drag force on the plate becomes

$$F_{D2} = \frac{1.33}{\left(\frac{(2V)L}{\nu}\right)^{0.5}} A_s \frac{\rho (2V)^2}{2} = 0.664 (2V)^{3/2} A_s \frac{\nu^{0.5}}{L^{0.5}}$$

The ratio of drag forces corresponding to V and $2V$ is

$$\frac{F_{D2}}{F_{D1}} = \frac{(2V)^{3/2}}{V^{3/2}} = \mathbf{2^{3/2}}$$

We repeat similar calculations for heat transfer rate ratio corresponding to V and $2V$

$$\dot{Q}_1 = hA_s(T_s - T_\infty) = \left(\frac{k}{L} Nu\right) A_s (T_s - T_\infty) = \left(\frac{k}{L}\right)\left(0.664\,\text{Re}^{0.5}\ \text{Pr}^{1/3}\right) A_s (T_s - T_\infty)$$

$$= \frac{k}{L} 0.664 \left(\frac{VL}{\nu}\right)^{0.5} \text{Pr}^{1/3} A_s (T_s - T_\infty)$$

$$= 0.664 V^{0.5} \frac{k}{L^{0.5} \nu^{0.5}} \text{Pr}^{1/3} A_s (T_s - T_\infty)$$

When the free-stream velocity of the fluid is doubled, the new value of the heat transfer rate between the fluid and the plate becomes

$$\dot{Q}_2 = 0.664 (2V)^{0.5} \frac{k}{L^{0.5} \nu^{0.5}} \text{Pr}^{1/3} A_s (T_s - T_\infty)$$

Then the ratio is

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{(2V)^{0.5}}{V^{0.5}} = 2^{0.5} = \mathbf{\sqrt{2}}$$

7-22 The ratio of the average convection heat transfer coefficient (h) to the local convection heat transfer coefficient (h_x) is to be determined from a given correlation.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Analysis From the given correlation in the form of local Nusselt number, the local convection heat transfer coefficient is

$$\mathrm{Nu}_x = 0.035\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \quad \rightarrow \quad h_x = \mathrm{Nu}_x \frac{k}{x} = 0.035\frac{k}{x}\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$$

or $$h_x = 0.035k\left(\frac{V}{\nu}\right)^{0.8}\mathrm{Pr}^{1/3}\,x^{-0.2} = Cx^{-0.2} \quad \text{where} \quad C = 0.035k\left(\frac{V}{\nu}\right)^{0.8}\mathrm{Pr}^{1/3}$$

At $x = L$, the local convection heat transfer coefficient is $h_{x=L} = CL^{-0.2}$. The average convection heat transfer coefficient over the entire plate length is

$$h = \frac{1}{L}\int_0^L h_x\,dx = \frac{C}{L}\int_0^L x^{-0.2}dx = 1.25\frac{C}{L}L^{0.8} = 1.25CL^{-0.2}$$

Taking the ratio of h to h_x at $x = L$, we get

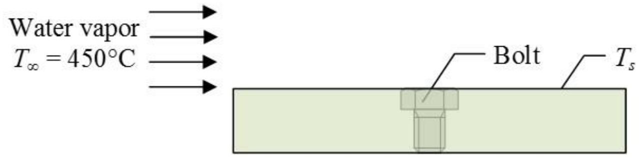
$$\frac{h}{h_{x=L}} = \frac{1.25CL^{-0.2}}{CL^{-0.2}} = \mathbf{1.25}$$

Discussion For constant properties, it should be noted that $\mathrm{Nu}/\mathrm{Nu}_{x=L} = 1.25$.

7-23 An ASTM B98 copper-silicon bolt is embedded at mid-length of a 1-m long plate. The maximum use temperature for the bolt is 149°C. Hot water vapor flows in parallel over the plate's upper surface. The local heat flux at the location where the bolt is embedded is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Uniform surface temperature. **3** Edge effects of plate are negligible. **4** The critical Reynolds number is $\mathrm{Re}_{cr} = 5 \times 10^5$.

Properties The properties of water vapor at the film temperature of $T_f = (T_s + T_\infty)/2 = (149 + 450)°\mathrm{C}/2 \approx 300°\mathrm{C}$ are (Table A–16): Pr = 0.9401, k = 0.04345 W/m·K, and $\nu = 5.340 \times 10^{-5}$ m²/s



Analysis The Reynolds number at the end of the plate is

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(10\ \mathrm{m/s})(1\ \mathrm{m})}{5.340 \times 10^{-5} \mathrm{m^2/s}} = 1.8727 \times 10^5 < 5 \times 10^5$$

Thus, we have laminar flow over the entire plate. The local heat transfer convection coefficient at mid-length (x = 0.5 m) is determined using the laminar flow relation for a flat plate,

$$h_x = \left(\frac{k}{x}\right) 0.332\ \mathrm{Re}_x^{0.5} \mathrm{Pr}^{1/3} = \left(\frac{0.04345\ \mathrm{W/m \cdot K}}{0.5\ \mathrm{m}}\right)(0.332)(93633)^{0.5}(0.9401)^{1/3} = 8.65\ \mathrm{W/m^2{\cdot}K}$$

Where

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(10\ \mathrm{m/s})(0.5\ \mathrm{m})}{5.340 \times 10^{-5} \mathrm{m^2/s}} = 93633$$

From the Newton's law of cooling, the local heat flux from the water vapor at the location where the bolt is embedded (x = 0.5 m) is

$$\dot{q}_x = h_x(T_\infty - T_s) = (8.65\ \mathrm{W/m^2{\cdot}K})(450 - 149)K = \mathbf{2604\ W/m^2}$$

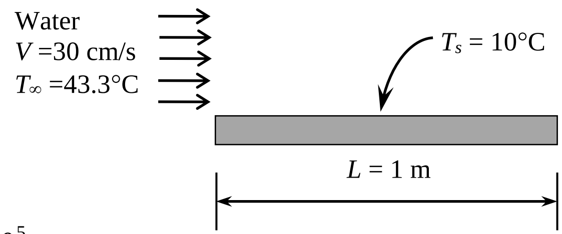
Discussion The local heat flux from the hot water vapor to the bolt is 2604 W/m². This indicates that to keep the bolt from heating to above the maximum use temperature of 149°C (ASME Code for Process Piping), the cooling mechanism needs to be able to remove 2604 W/m² of heat at the minimum.

7-24 Water flows over a large plate. The rate of heat transfer per unit width of the plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible.

Properties The properties of water at the film temperature of $(T_s + T_\infty)/2 = (10+43.3)/2 = 27°C$ are (Table A-9)

$$\rho = 996.6 \text{ kg/m}^3$$
$$k = 0.610 \text{ W/m.°C}$$
$$\mu = 0.854\times10^{-3} \text{ kg//m}\cdot\text{s}$$
$$\text{Pr} = 5.85$$



Analysis(*a*)The Reynolds number is

$$\text{Re}_L = \frac{VL\rho}{\mu} = \frac{(0.3\text{ m/s})(1.0\text{m})(996.6\text{kg/m}^3)}{0.854\times10^{-3}\text{ m}^2\text{/s}} = 3.501\times10^5$$

which is smaller than the critical Reynolds number. Thus we have laminar flow for the entire plate. The Nusselt number and the heat transfer coefficient are

$$\text{Nu} = 0.664\,\text{Re}_L^{1/2}\,\text{Pr}^{1/3} = 0.664(3.501\times10^5)^{1/2}(5.85)^{1/3} = 707.9$$

$$h = \frac{k}{L}\text{Nu} = \frac{0.610 \text{ W/m.°C}}{1.0 \text{ m}}(707.9) = 431.8 \text{ W/m}^2\text{.°C}$$

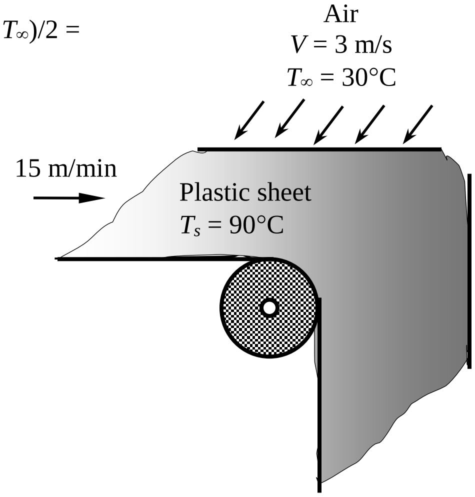
Then the rate of heat transfer per unit width of the plate is determined to be

$$\dot{Q} = hA_s(T_s - T_\infty) = (431.8 \text{ W/m}^2\text{.°C})(1\text{m})(1\text{m})](43.3\text{–}10)\text{°C} = \mathbf{14{,}400\ W}$$

7-25 Air flows on both sides of a continuous sheet of plastic. The rate of heat transfer from the plastic sheet is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (90+30)/2 = 60°C$ are (Table A-15)



$$\rho = 1.059 \text{ kg/m}^3$$
$$k = 0.02808 \text{ W/m.°C}$$
$$\nu = 1.896\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7202$$

Analysis The width of the cooling section is first determined from

$$W = V\Delta t = [(15/60) \text{ m/s}](2 \text{ s}) = 0.5 \text{ m}$$

The Reynolds number is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{(3 \text{ m/s})(1.2 \text{m})}{1.896\times10^{-5} \text{ m}^2\text{/s}} = 1.899\times10^5$$

which is less than the critical Reynolds number. Thus the flow is laminar. Using the proper relation in laminar flow for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = 0.664 \text{Re}_L^{0.5} \Pr^{1/3} = 0.664(1.899\times10^5)^{0.5}(0.7202)^{1/3} = 259.3$$
$$h = \frac{k}{L} Nu = \frac{0.02808 \text{ W/m.°C}}{1.2 \text{ m}}(259.3) = 6.07 \text{ W/m}^2\text{.°C}$$

$$A_s = 2LW = 2(1.2 \text{ m})(0.5 \text{m}) = 1.2 \text{ m}^2$$
$$\dot{Q}_{conv} = hA_s(T_\infty - T_s) = (6.07 \text{ W/m}^2\text{.°C})(1.2 \text{ m}^2)(90-30)\text{°C} = \mathbf{437 \text{ W}}$$

7-26 Hot carbon dioxide exhaust gas is being cooled by flat plates, (*a*) the local convection heat transfer coefficient at 1 m from the leading edge, (*b*) the average convection heat transfer coefficient over the entire plate, and (*c*) the total heat flux transfer to the plate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperature is uniform throughout the plate. **3** Thermal properties are constant. **4** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **5** Heat transfer by radiation is negligible.

Properties The properties of CO_2 at T_f = (220°C + 80°C)/2 = 150°C are k = 0.02652 W/m·K, $\nu = 1.627 \times 10^{-5}$ m²/s, Pr = 0.7445 (from Table A-16).

Analysis (*a*) The Reynolds number at x = 1 m is

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(3\ \mathrm{m/s})(1\ \mathrm{m})}{1.627\times10^{-5}\ \mathrm{m^2/s}} = 1.844\times10^5$$

Since $\mathrm{Re}_x < 5 \times 10^5$, the flow is laminar. Using the proper relation for Nusselt number, the local heat transfer coefficient at 1 m from the leading edge of the flat plate is

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} = 0.332(1.844\times10^5)^{0.5}(0.7445)^{1/3} = 129.2$$

$$h_x = 129.2\frac{k}{x} = 129.2\frac{0.02652\ \mathrm{W/m\cdot K}}{1\ \mathrm{m}} = \mathbf{3.426\ W/m^2\cdot K}$$

(*b*) The Reynolds number at L = 1.5 m is

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(3\ \mathrm{m/s})(1.5\ \mathrm{m})}{1.627\times10^{-5}\ \mathrm{m^2/s}} = 2.766\times10^5$$

Since $\mathrm{Re}_x < 5 \times 10^5$, the flow is laminar. Using the proper relation for Nusselt number, the average heat transfer coefficient of the entire flat plate is

$$\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} = 0.664(2.766\times10^5)^{0.5}(0.7445)^{1/3} = 316.5$$

$$h = 316.5\frac{k}{L} = 316.5\frac{0.02652\ \mathrm{W/m\cdot K}}{1.5\ \mathrm{m}} = \mathbf{5.596\ W/m^2\cdot K}$$

(*c*) The total heat flux transfer to the flat plate on the upper and lower surfaces is

$$\dot{q}_{\mathrm{conv}} = 2h(T_\infty - T_s) = 2(5.596\ \mathrm{W/m^2\cdot K})(220-80)\ \mathrm{K} = \mathbf{1567\ W/m^2}$$

Discussion The average convection heat transfer coefficient calculated in part (*b*) is relatively low, which indicates that the role of natural convection may be important.

7-27 An ASTM B152 copper plate is cooled by air at 20°C. The average convection heat transfer rate from the plate needed to keep the surface temperature from going above 260°C is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Uniform surface temperature. **3** Edge effects of plate are negligible. **4** The critical Reynolds number is $\mathrm{Re_{cr}} = 5 \times 10^5$.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (260 + 20)°\mathrm{C}/2 = 140°\mathrm{C}$ are (Table A-15):Pr = 0.7041, k = 0.03374 W/m·K, and$\nu = 2.745 \times 10^{-5}$ m^2/s

Analysis The Reynolds number at the end of the plate is

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(0.5\ \mathrm{m/s})(1\ m)}{2.745\times10^{-5}\mathrm{m^2/s}} = 18215 < 5 \times 10^5$$

Thus, we have laminar flow over the entire plate. The average heat transfer convection coefficient is determined using the laminar flow relation for a flat plate,

$$h = \left(\frac{k}{L}\right) 0.664\ \mathrm{Re}_L^{0.5}\mathrm{Pr}^{1/3} = \left(\frac{0.03374\ \mathrm{W/m\cdot K}}{1\ \mathrm{m}}\right)(0.664)(18215)^{0.5}(0.7041)^{1/3} = 2.69\ \mathrm{W/m^2{\cdot}K}$$

From the Newton's law of cooling, the average convection heat transfer rate is

$$\dot{Q} = hA(T_s - T_\infty) = (2.69\ \mathrm{W/m^2{\cdot}K})(1\ \mathrm{m} \times 1\ \mathrm{m})(260 - 20)\mathrm{K} = \mathbf{646\ W}$$

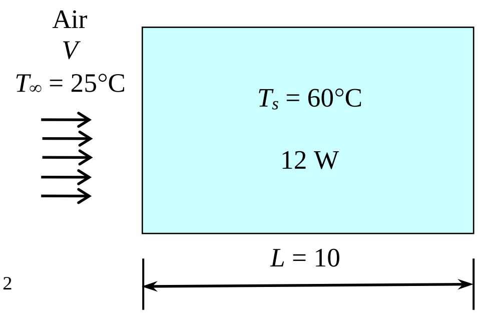
Discussion The average convection heat transfer rate from the plate surface (i.e. heat removal rate by convection) required to keep the plate surface from going above the maximum use temperature of 260°C is 646 W.

7-28 A fan blows air parallel to the passages between the fins of a heat sink attached to a transformer. The minimum free-stream velocity that the fan should provide to avoid overheating is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** The fins and the base plate are nearly isothermal (fin efficiency is equal to 1) **5** Air is an ideal gas with constant properties. **6** The local atmospheric pressure is 1 atm.**7** The flow is laminar over the entire finned surface of the transformer.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (60+25)/2 = 42.5°\text{C}$ are (Table A-15)

$$k = 0.02681\ \text{W/m.°C}$$
$$\nu = 1.726\times10^{-5}\ \text{m}^2\text{/s}$$
$$\Pr = 0.7248$$



Analysis The total heat transfer surface area for this finned surface is

$$A_{s,\text{finned}} = (2\times7)(0.1\,\text{m})(0.005\text{m}) = 0.007\,\text{m}^2$$
$$A_{s,\text{unfinned}} = (0.1\,\text{m})(0.062\text{m}) - 7\times(0.002\,\text{m})(0.1\text{m}) = 0.0048\,\text{m}^2$$
$$A_{s,\text{total}} = A_{s,\text{finned}} + A_{s,\text{unfinned}} = 0.007\,\text{m}^2 + 0.0048\,\text{m}^2 = 0.0118\,\text{m}^2$$

The convection heat transfer coefficient can be determined from Newton's law of cooling relation for a finned surface.

$$\dot{Q} = \eta h A_s (T_\infty - T_s) \longrightarrow h = \frac{\dot{Q}}{\eta A_s (T_\infty - T_s)} = \frac{12\ \text{W}}{(1)(0.0118\text{m}^2)(60-25)°\text{C}} = 29.06\ \text{W/m}^2.°\text{C}$$

Starting from heat transfer coefficient, Nusselt number, Reynolds number and finally free-stream velocity will be determined. We assume the flow is laminar over the entire finned surface of the transformer.

$$Nu = \frac{hL}{k} = \frac{(29.06\ \text{W/m}^2.°\text{C})(0.1\text{m})}{0.02681\ \text{W/m.°C}} = 108.4$$

$$Nu = 0.664\,\mathrm{Re}_L^{0.5}\,\Pr^{1/3} \longrightarrow \mathrm{Re}_L = \frac{Nu^2}{0.664^2\,\Pr^{2/3}} = \frac{(108.4)^2}{(0.664)^2(0.7248)^{2/3}} = 3.302\times10^4$$

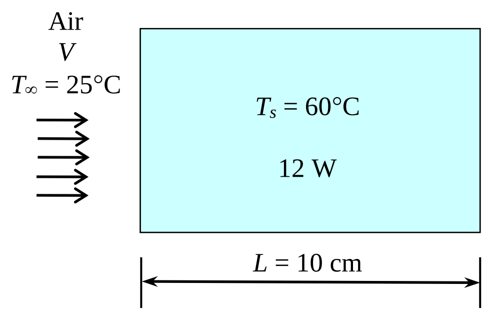
$$\mathrm{Re}_L = \frac{VL}{\nu} \longrightarrow V = \frac{\mathrm{Re}_L\,\nu}{L} = \frac{(3.302\times10^4)(1.726\times10^{-5}\ \text{m}^2\text{/s})}{0.1\,\text{m}} = \mathbf{5.70\ m/s}$$

7-29 A fan blows air parallel to the passages between the fins of a heat sink attached to a transformer. The minimum free-stream velocity that the fan should provide to avoid overheating is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** The fins and the base plate are nearly isothermal (fin efficiency is equal to 1) **4** Air is an ideal gas with constant properties. **5** The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $(T_s + T_\infty)/2 = (60+25)/2 = 42.5°C$ are (Table A-15)

$$k = 0.02681 \text{ W/m.°C}$$
$$\nu = 1.726\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7248$$



Analysis We first need to determine radiation heat transfer rate. Note that we will use the base area and we assume the temperature of the surrounding surfaces are at the same temperature with the air ($T_{surr} = 25°C$)

$$\dot{Q}_{rad} = \varepsilon A_s \sigma (T_s{}^4 - T_{surr}{}^4)$$
$$= (0.90)[(0.1\text{ m})(0.062\text{ m})](5.67\times10^{-8}\text{ W/m}^2\text{.°C})[(60+273\text{ K})^4 - (25+273\text{ K})^4]$$
$$= 1.4\text{ W}$$

The heat transfer rate by convection will be 1.4 W less than total rate of heat transfer from the transformer. Therefore

$$\dot{Q}_{conv} = \dot{Q}_{total} - \dot{Q}_{rad} = 12 - 1.4 = 10.6\text{ W}$$

The total heat transfer surface area for this finned surface is

$$A_{s,\text{finned}} = (2\times7)(0.1\text{ m})(0.005\text{m}) = 0.007\text{ m}^2$$
$$A_{s,\text{unfinned}} = (0.1\text{ m})(0.062\text{m}) - 7\times(0.002\text{ m})(0.1\text{m}) = 0.0048\text{ m}^2$$
$$A_{s,\text{total}} = A_{s,\text{finned}} + A_{s,\text{unfinned}} = 0.007\text{ m}^2 + 0.0048\text{ m}^2 = 0.0118\text{ m}^2$$

The convection heat transfer coefficient can be determined from Newton's law of cooling relation for a finned surface.

$$\dot{Q}_{conv} = \eta h A_s (T_\infty - T_s) \longrightarrow h = \frac{\dot{Q}_{conv}}{\eta A_s (T_\infty - T_s)} = \frac{10.6\text{ W}}{(1)(0.0118\text{m}^2)(60-25)°\text{C}} = 25.67\text{ W/m}^2\text{.°C}$$

Starting from heat transfer coefficient, Nusselt number, Reynolds number and finally free-stream velocity will be determined. We assume the flow is laminar over the entire finned surface of the transformer.

$$Nu = \frac{hL}{k} = \frac{(25.67\text{ W/m}^2\text{.°C})(0.1\text{m})}{0.02681\text{ W/m.°C}} = 95.73$$

$$Nu = 0.664\,\text{Re}_L{}^{0.5}\,\Pr^{1/3} \longrightarrow \text{Re}_L = \frac{Nu^2}{0.664^2\,\Pr^{2/3}} = \frac{(95.73)^2}{(0.664)^2(0.7248)^{2/3}} = 2.576\times10^4$$

$$\text{Re}_L = \frac{VL}{\nu} \longrightarrow V = \frac{\text{Re}_L\,\nu}{L} = \frac{(2.576\times10^4)(1.726\times10^{-5}\text{ m}^2\text{/s})}{0.1\text{ m}} = \mathbf{4.45\text{ m/s}}$$

7-30 Hot engine oil is flowing in parallel over a flat plate, the local convection heat transfer coefficient at 0.2 m from the leading edge and the average convection heat transfer coefficient over the entire plate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperature is uniform throughout the plate. **3** Thermal properties are constant. **4** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$.

Properties The properties of engine oil at T_f = (150°C + 50°C)/2 = 100°C are k = 0.1367 W/m·K, $\nu = 2.046\times10^{-5}$ m²/s, Pr = 279.1 (from Table A-13).

Analysis (*a*) The Reynolds number at x = 0.2 m is

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(2\ \mathrm{m/s})(0.2\ \mathrm{m})}{2.046\times10^{-5}\ \mathrm{m^2/s}} = 1.955\times10^4$$

The Reynolds number at L = 0.5 m is

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(2\ \mathrm{m/s})(0.5\ \mathrm{m})}{2.046\times10^{-5}\ \mathrm{m^2/s}} = 4.888\times10^4$$

Since $\mathrm{Re}_L < 5\times10^5$ at the trailing edge, the flow is laminar over the entire plate. Using the proper relation for Nusselt number, the local convection heat transfer coefficient at x = 0.2 m from the leading edge is

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \quad \rightarrow \quad h_x = \frac{k}{x}0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$$

$$h_x = \frac{(0.1367\ \mathrm{W/m\cdot K})}{(0.2\ \mathrm{m})}0.332(1.955\times10^4)^{0.5}(279.1)^{1/3} = \mathbf{207.3\ W/m^2\cdot K}$$

The average convection heat transfer coefficient over the entire plate is

$$\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} \quad \rightarrow \quad h = \frac{k}{L}0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3}$$

$$h = \frac{(0.1367\ \mathrm{W/m\cdot K})}{(0.5\ \mathrm{m})}0.664(4.888\times10^4)^{0.5}(279.1)^{1/3} = 262.3\ \mathrm{W/m^2\cdot K}$$

(*b*) Using the Churchill and Ozoe (1973) relation for Nusselt number, the local convection heat transfer coefficient at x = 0.2 m from the leading edge is

$$\mathrm{Nu}_x = \frac{h_x x}{k} = \frac{0.3387\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}}{[1+(0.0468/\mathrm{Pr})^{2/3}]^{1/4}} \quad \rightarrow \quad h_x = \frac{k}{x}\frac{0.3387\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}}{[1+(0.0468/\mathrm{Pr})^{2/3}]^{1/4}}$$

$$h_x = \frac{(0.1367\ \mathrm{W/m\cdot K})}{(0.2\ \mathrm{m})}\frac{0.3387(279.1)^{1/3}(1.955\times10^4)^{1/2}}{[1+(0.0468/279.1)^{2/3}]^{1/4}} = \mathbf{211.4\ W/m^2\cdot K}$$

The average convection heat transfer coefficient over the entire plate is

$$h = \frac{1}{L}\int_0^L h_x\,dx = \frac{C}{L}\int_0^L x^{-1/2}dx = 2\frac{C}{L}L^{1/2} \quad \text{where} \quad C = k\frac{0.3387\,\mathrm{Pr}^{1/3}(V/\nu)^{1/2}}{[1+(0.0468/\mathrm{Pr})^{2/3}]^{1/4}}$$

or

$$h = 2\frac{k}{L}\frac{0.3387\,\mathrm{Pr}^{1/3}(VL/\nu)^{1/2}}{[1+(0.0468/\mathrm{Pr})^{2/3}]^{1/4}} = 2\frac{k}{L}\frac{0.3387\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_L^{1/2}}{[1+(0.0468/\mathrm{Pr})^{2/3}]^{1/4}}$$

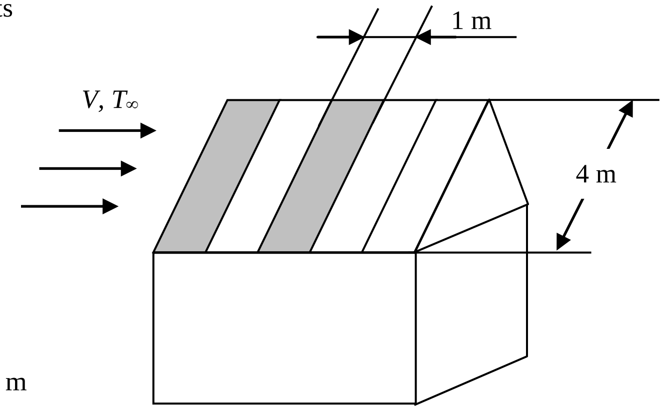
Hence

$$h = \frac{2(0.1367\ \mathrm{W/m\cdot K})}{(0.5\ \mathrm{m})}\frac{0.3387(279.1)^{1/3}(4.888\times10^4)^{1/2}}{[1+(0.0468/279.1)^{2/3}]^{1/4}} = \mathbf{267.4\ W/m^2\cdot K}$$

Discussion Since the fluid properties are constant, it should be noted that $\mathrm{Nu} = 2\mathrm{Nu}_x$. The comparison of the results from parts (*a*) and (*b*) show that the Churchill and Ozoe (1973) relation calculated both local and average heat transfer coefficients by about 2% larger.

7-31 Ambient air flows over parallel plates of a solar collector that is maintained at a specified temperature. The rates of convection heat transfer from the first and third plate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Atmospheric pressure is taken 1 atm.



Properties The properties of air at the film temperature of (15+10)/2=12.5°C are (Table A-15)

$$k = 0.02458 \text{ W/m.°C}$$
$$\nu = 1.448\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7330$$

Analysis (*a*) The critical length of the plate is first determined to be

$$x_{cr} = \frac{\mathrm{Re}_{cr}\,\nu}{V} = \frac{(5\times10^5)(1.448\times10^{-5}\text{ m}^2\text{/s})}{2\text{ m/s}} = 3.62\text{ m}$$

Therefore, all three plates are under laminar flow. The Reynolds number for the first plate is

$$\mathrm{Re}_1 = \frac{VL_1}{\nu} = \frac{(2\text{ m/s})(1\text{m})}{1.448\times10^{-5}\text{ m}^2\text{/s}} = 1.381\times10^5$$

Using the relation for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$\mathrm{Nu}_1 = 0.664\,\mathrm{Re}_1^{1/2}\,\mathrm{Pr}^{1/3} = 0.664(1.381\times10^5)^{1/2}(0.7330)^{1/3} = 222.5$$
$$h_1 = \frac{k}{L_1}Nu = \frac{0.02458\text{ W/m.°C}}{1\text{ m}}(222.5) = 5.469\text{ W/m}^2\text{.°C}$$

$$A = wL = (4\text{ m})(1\text{m}) = 4\text{ m}^2$$
$$\dot{Q} = hA(T_s - T_\infty) = (5.469\text{ W/m}^2\text{.°C})(4\text{ m}^2)(15-10)\text{°C} = \mathbf{109\ W}$$

(*b*) Repeating the calculations for the second and third plates,

$$\mathrm{Re}_2 = \frac{VL_2}{\nu} = \frac{(2\text{ m/s})(2\text{m})}{1.448\times10^{-5}\text{ m}^2\text{/s}} = 2.762\times10^5$$

$$\mathrm{Nu}_2 = 0.664\,\mathrm{Re}_2^{1/2}\,\mathrm{Pr}^{1/3} = 0.664(2.762\times10^5)^{1/2}(0.7330)^{1/3} = 314.7$$
$$h_2 = \frac{k}{L_2}Nu = \frac{0.02458\text{ W/m.°C}}{2\text{ m}}(314.7) = 3.867\text{ W/m}^2\text{.°C}$$

$$\mathrm{Re}_3 = \frac{VL_3}{\nu} = \frac{(2\text{ m/s})(3\text{m})}{1.448\times10^{-5}\text{ m}^2\text{/s}} = 4.144\times10^5$$

$$\mathrm{Nu}_3 = 0.664\,\mathrm{Re}_3^{1/2}\,\mathrm{Pr}^{1/3} = 0.664(4.144\times10^5)^{1/2}(0.7330)^{1/3} = 385.4$$
$$h_3 = \frac{k}{L_3}Nu = \frac{0.02458\text{ W/m.°C}}{3\text{ m}}(385.4) = 3.158\text{ W/m}^2\text{.°C}$$

Then

$$h_{2-3} = \frac{h_3L_3 - h_2L_2}{L_3 - L_2} = \frac{3.158\times3 - 3.867\times2}{3-2} = 1.739\text{ W/m}^2\text{.°C}$$

The rate of heat loss from the third plate is

$$\dot{Q} = hA(T_s - T_\infty) = (1.739\text{ W/m}^2\text{.°C})(4\text{ m}^2)(15-10)\text{°C} = \mathbf{34.8\ W}$$

Alternative solution for part (b)

(*b*) The average heat transfer coefficient for the combined first and second plates is determined as

$$h_2 = 3.867\text{ W/m}^2\text{.°C}$$

Consequently, the rate of heat loss from the combined first and second plates is

$$\dot{Q}_{1-2} = hA(T_s - T_\infty) = (3.867\ \text{W/m}^2.°\text{C})(4 \times 2\ \text{m}^2)(15-10)°\text{C} = 154.7\ \text{W}$$

The average heat transfer coefficient for the combined first, second, and third plates is

$$h_3 = 3.158\ \text{W/m}^2.°\text{C}$$

Consequently, the rate of heat loss from the combined first, second, and third plates is

$$\dot{Q}_{1-3} = hA(T_s - T_\infty) = (3.158\ \text{W/m}^2.°\text{C})(4 \times 3\ \text{m}^2)(15-10)°\text{C} = 189.5\ \text{W}$$

Then the rate of heat loss from the third plate is the difference between these values

$$\dot{Q}_3 = \dot{Q}_{1-3} - \dot{Q}_{1-2} = 189.5 - 154.7 = \mathbf{34.8\ W}$$

The result is the same as before, as expected.

7-32  Hydrogen gas flows in parallel over the upper and lower surfaces of a flat plate. The local convection heat transfer coefficient and the localtotal convection heat flux along the plate are to be evaluated.

Assumptions **1** Steady operating conditions exist. **2** Surface temperature is uniform over the entire plate. **3**Local atmospheric pressure is 1 atm. **4** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **5** Heat transfer by radiation is negligible. **7** Flow is laminar (this assumption will be verified).

Analysis For laminar flow, the relation for local Nusselt number along a flat plate is

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$$

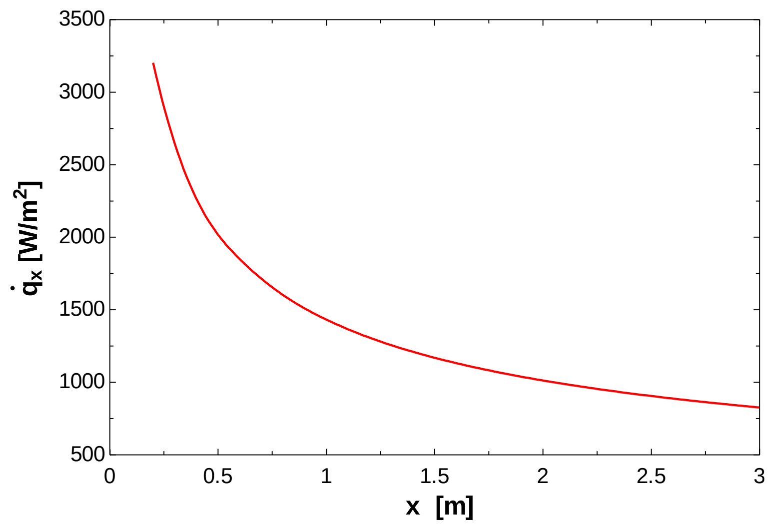
The total local convection heat flux at the plate upper and lower surfaces is

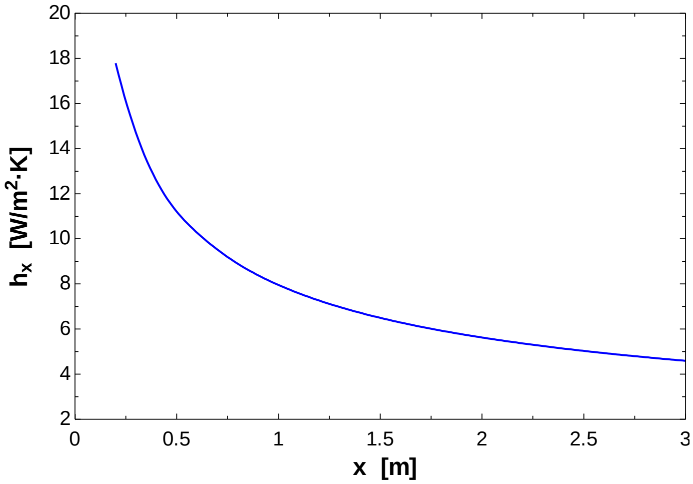
$$\dot{q}_x = 2h_x(T_\infty - T_s)$$

The problem is solved using EES, and the solution is given below:

```
"GIVEN"
T_infinity=120 [C]
T_s=30 [C]
V=2.5 [m/s]

"PROPERTIES"
T_film=1/2*(T_s+T_infinity)
Fluid$='hydrogen'
k=Conductivity(Fluid$, T=T_film, P=101.3)
Pr=Prandtl(Fluid$, T=T_film, P=101.3)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film, P=101.3)
nu=mu/rho

"ANALYSIS"
Re_x=V*x/nu
Nusselt_x=0.332*Re_x^0.5*Pr^(1/3)
h_x=Nusselt_x*k/x
q_dot_x=2*h_x*(T_infinity-T_s)
```

x [m]	Re_x	h_x [W/m²·K]	$\dot{q}_x$ [W/m²]
0.2	3512	17.79	3202
0.4	7023	12.58	2264
0.6	10535	10.27	1849
0.8	14047	8.895	1601
1.0	17558	7.956	1432
1.2	21070	7.263	1307
1.4	24582	6.724	1210
1.6	28093	6.290	1132
1.8	31605	5.930	1067
2.0	35117	5.626	1013
2.2	38628	5.364	965.5
2.4	42140	5.136	924.4
2.6	45652	4.934	888.1
2.8	49163	4.755	855.8
3.0	52675	4.593	826.8





Discussion As shown in the table above, for $0.2 \leq x \leq 3$ m, the local Reynolds number varies from 3512 to 52,675 which is less than the $Re_{cr} = 5\times10^5$. Thus, the flow is laminar. As shown in the figure, the local convection heat transfer coefficient decreases along the plate. This causes the total convection heat flux along the plate to decrease also.

7-33 CO_2 and H_2 as ideal gases flow in parallel over a flat plate. The local Reynolds number, local Nusselt number, and local convection heat transfer coefficient along the plate are to be evaluated.

Assumptions **1** Steady operating conditions exist. **2**Surface temperature is uniform over the entire plate. **3** Local atmospheric pressure is 1 atm. **4** The critical Reynolds number is $\mathrm{Re}_{cr} = 5 \times 10^5$. **5** Flow is laminar (this assumption will be verified).

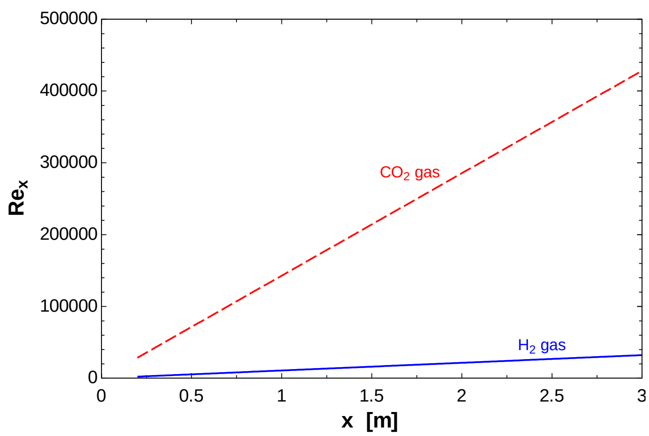
Analysis For laminar flow, the relation for local Nusselt number along a flat plate is

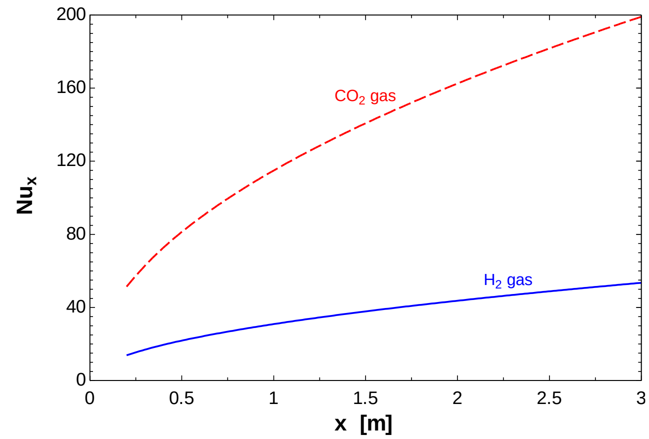
$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$$

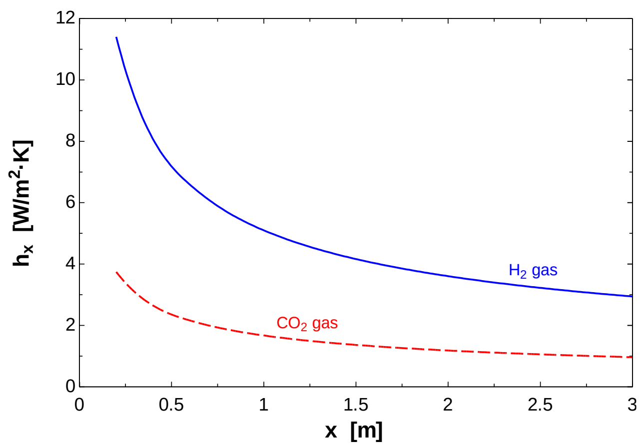
The problem is solved using EES, and the solution is given below:

```
"GIVEN"
V=1 [m/s]
T_s=20 [C]
T_infinity=-20 [C]

"PROPERTIES"
"CO2 gas"
T_film=1/2*(T_s+T_infinity)
k_CO2=Conductivity(CO2, T=T_film)
Pr_CO2=Prandtl(CO2, T=T_film)
rho_CO2=Density(CO2, T=T_film, P=101.3)
mu_CO2=Viscosity(CO2, T=T_film)
nu_CO2=mu_CO2/rho_CO2
"H2 gas"
k_H2=Conductivity(H2, T=T_film)
Pr_H2=Prandtl(H2, T=T_film)
rho_H2=Density(H2, T=T_film, P=101.3)
mu_H2=Viscosity(H2, T=T_film)
nu_H2=mu_H2/rho_H2

"ANALYSIS"
"CO2 gas"
Re_x_CO2=V*x/nu_CO2
Nusselt_x_CO2=0.332*(Re_x_CO2)^0.5*(Pr_CO2)^(1/3)
h_x_CO2=Nusselt_x_CO2*k_CO2/x
"H2 gas"
Re_x_H2=V*x/nu_H2
Nusselt_x_H2=0.332*(Re_x_H2)^0.5*(Pr_H2)^(1/3)
h_x_H2=Nusselt_x_H2*k_H2/x
```

x [m]	$Re_{x,CO2}$	$Nu_{x,CO2}$	$h_{x,CO2}$ [W/m²·K]	$Re_{x,H2}$	$Nu_{x,H2}$	$h_{x,H2}$ [W/m²·K]
	CO_2 gas			H_2 gas		
0.2	28552	51.4	3.741	2143	13.8	11.40
0.4	57104	72.69	2.645	4287	19.52	8.062
0.6	85656	89.03	2.160	6430	23.91	6.583
0.8	114207	102.8	1.871	8574	27.61	5.701
1.0	142759	114.9	1.673	10717	30.87	5.099
1.2	171311	125.9	1.527	12861	33.81	4.655
1.4	199863	136.0	1.414	15004	36.52	4.309
1.6	228415	145.4	1.323	17147	39.05	4.031
1.8	256967	154.2	1.247	19291	41.41	3.800
2.0	285518	162.5	1.183	21434	43.65	3.605
2.2	314070	170.5	1.128	23578	45.78	3.438
2.4	342622	178.1	1.080	25721	47.82	3.291
2.6	371174	185.3	1.038	27865	49.77	3.162
2.8	399726	192.3	0.9999	30008	51.65	3.047
3.0	428278	199.1	0.9660	32151	53.46	2.944







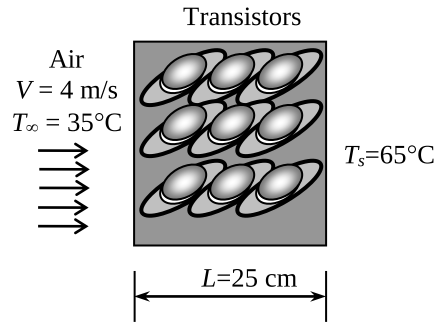
Discussion As shown in the table above, for $0.2 \le x \le 3$ m, the local Reynolds number is less than the $Re_{cr} = 5\times10^5$.Thus, the flow is laminar for both gases. As shown in the figure, the local Nusselt number of the CO_2 gas is higher than that of the H_2 gas. This is because CO_2 gas has higher local Reynolds number (due to its lower kinematic viscosity) than H_2 gas. However, the H_2 gas has higher local convection heat transfer coefficient, due to its higher thermal conductivity.

7-34 Air is blown over an aluminum plate mounted on an array of power transistors. The number of transistors that can be placed on this plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **3** Radiation effects are negligible **4** Heat transfer from the back side of the plate is negligible. **5** Air is an ideal gas with constant properties. **6** The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $(T_s + T_\infty)/2 = (65+35)/2 = 50°\mathrm{C}$ are (Table A-15)

$$k = 0.02735 \text{ W/m.°C}$$
$$\nu = 1.798\times10^{-5} \text{ m}^2/\text{s}$$
$$\mathrm{Pr} = 0.7228$$



Analysis The Reynolds number is

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(4 \text{ m/s})(0.25\text{m})}{1.798\times10^{-5} \text{ m}^2/\text{s}} = 55{,}617$$

which is less than the critical Reynolds number. Thus the flow is laminar. Using the proper relation in laminar flow for Nusselt number, heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{\,0.5}\,\mathrm{Pr}^{1/3} = 0.664(55{,}617)^{0.5}(0.7228)^{1/3} = 140.5$$

$$h = \frac{k}{L}Nu = \frac{0.02735 \text{ W/m.°C}}{0.25 \text{ m}}(140.5) = 15.37 \text{ W/m}^2.°\text{C}$$

$$A_s = wL = (0.25\text{ m})(0.25\text{m}) = 0.0625\text{ m}^2$$

$$\dot{Q}_{conv} = hA_s(T_\infty - T_s) = (15.37 \text{ W/m}^2.°\text{C})(0.0625\text{ m}^2)(65-35)°\text{C} = 28.83\text{ W}$$

Considering that each transistor dissipates 5 W of power, the number of transistors that can be placed on this plate becomes

$$n = \frac{28.8 \text{ W}}{5 \text{ W}} = 5.8 \longrightarrow \mathbf{5}$$

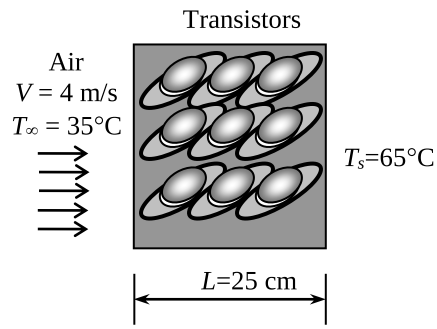
This result is conservative since the transistors will cause the flow to be turbulent, and the rate of heat transfer to be higher.

7-35 Air is blown over an aluminum plate mounted on an array of power transistors. The number of transistors that can be placed on this plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible **4** Heat transfer from the backside of the plate is negligible. **5** Air is an ideal gas with constant properties. **6** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (65+35)/2 = 50°C$ are (Table A-15)

$$k = 0.02735 \text{ W/m.°C}$$
$$\nu = 1.798\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7228$$



Note that the atmospheric pressure will only affect the kinematic viscosity. The atmospheric pressure in atm is

$$P = (83.4\,\text{kPa})\frac{1\,\text{atm}}{101.325\,\text{kPa}} = 0.823\,\text{atm}$$

The kinematic viscosity at this atmospheric pressure will be

$$\nu = (1.798\times10^{-5} \text{ m}^2\text{/s}) / 0.823 = 2.184\times10^{-5} \text{ m}^2\text{/s}$$

Analysis The Reynolds number is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{(4\,\text{m/s})(0.25\text{m})}{2.184\times10^{-5} \text{ m}^2\text{/s}} = 4.579\times10^4$$

which is less than the critical Reynolds number. Thus the flow is laminar. Using the proper relation in laminar flow for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = 0.664\,\text{Re}_L^{\,0.5}\,\Pr^{1/3} = 0.664(4.579\times10^4)^{0.5}(0.7228)^{1/3} = 127.5$$

$$h = \frac{k}{L}Nu = \frac{0.02735 \text{ W/m.°C}}{0.25\,\text{m}}(127.5) = 13.95 \text{ W/m}^2\text{.°C}$$

$$A_s = wL = (0.25\,\text{m})(0.25\text{m}) = 0.0625\,\text{m}^2$$

$$\dot{Q}_{\text{conv}} = hA_s(T_\infty - T_s) = (13.95 \text{ W/m}^2\text{.°C})(0.0625\,\text{m}^2)(65-35)\text{°C} = 26.2 \text{ W}$$

Considering that each transistor dissipates 5 W of power, the number of transistors that can be placed on this plate becomes

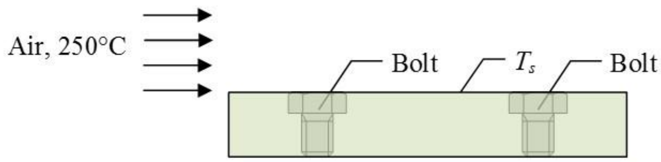
$$n = \frac{26.2 \text{ W}}{5 \text{ W}} = 5.2 \longrightarrow \mathbf{5}$$

This result is conservative since the transistors will cause the flow to be turbulent, and the rate of heat transfer to be higher.

7-36 C&S Two ASTM B98 copper-silicon bolts are embedded at 0.25 m and 1.75 m from the plate's leading edge. The maximum use temperature for the bolt is 149°C. The air flows in parallel over the plate's upper surface. The local heat fluxes at the locations where the bolts are embedded are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Uniform surface temperature. **3** Edge effects of plate are negligible. **4** The critical Reynolds number is $\text{Re}_{cr} = 5 \times 10^5$.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (149 + 250)°\text{C}/2 \approx 200°\text{C}$ are (Table A–15): Pr = 0.6974, k = 0.03779 W/m·K, and $\nu = 3.455 \times 10^{-5}$ m²/s



Analysis The location where the critical Reynolds number occurs is

$$x_{cr} = \text{Re}_{\text{cr}}\frac{\nu}{V} = 5 \times 10^5 \left(\frac{3.455 \times 10^{-5}\text{m}^2/\text{s}}{20\ \text{m/s}}\right) = 0.8638\ \text{m}$$

At the location of the first bolt (x_1 = 0.25 m), the flow is laminar. The local heat transfer convection coefficient is determined using the laminar flow relation for a flat plate,

$$h_{x_1} = \left(\frac{k}{x_1}\right) 0.332\ \text{Re}_{x_1}^{0.5}\text{Pr}^{1/3} = \left(\frac{0.03779\ \text{W/m}\cdot\text{K}}{0.25\ \text{m}}\right)(0.332)(1.4472 \times 10^5)^{0.5}(0.6974)^{1/3} = 16.93\ \text{W/m}^2\cdot\text{K}$$

where

$$\text{Re}_{x_1} = \frac{Vx_1}{\nu} = \frac{(20\ \text{m/s})(0.25\ \text{m})}{3.455{\times}10^{-5}\text{m}^2/\text{s}} = 1.4472 \times 10^5 < 5 \times 10^5$$

From the Newton's law of cooling, the local heat flux from the air at the location of the first bolt (x_1 = 0.25 m) is

$$\dot{q}_{x_1} = h_{x_1}(T_\infty - T_s) = (16.93\ \text{W/m}^2\cdot\text{K})(250 - 149)\text{K} = \mathbf{1710\ W/m^2}$$

At the location of the second bolt (x_2 = 1.75 m), the flow is turbulent. The local heat transfer convection coefficient is determined using the turbulent flow relation for a flat plate,

$$h_{x_2} = \left(\frac{k}{x_2}\right) 0.0296\ \text{Re}_{x_2}^{0.8}\text{Pr}^{1/3} = \left(\frac{0.03779\ \text{W/m}\cdot\text{K}}{1.75\ \text{m}}\right)(0.0296)(1.0130 \times 10^6)^{0.8}(0.6974)^{1/3} = 36.14\ \text{W/m}^2\cdot\text{K}$$

where

$$\text{Re}_{x_2} = \frac{Vx_2}{\nu} = \frac{(20\ \text{m/s})(1.75\ \text{m})}{3.455{\times}10^{-5}\text{m}^2/\text{s}} = 1.0130 \times 10^6 > 5 \times 10^5$$

From the Newton's law of cooling, the local heat flux from the air at the location of the second bolt (x_2 = 1.75 m) is

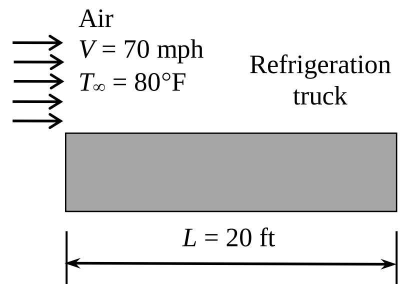
$$\dot{q}_{x_2} = h_{x_2}(T_\infty - T_s) = (36.14\ \text{W/m}^2\cdot\text{K})(250 - 149)\text{K} = \mathbf{3650\ W/m^2}$$

Discussion The local heat fluxes from the air to the first and second bolts are 1710 W/m² and 3650 W/m², respectively. This indicates that to keep the bolts from heating to above the maximum use temperature of 149°C (ASME Code for Process Piping), the cooling mechanism needs to be able to remove at least 1710 W/m² of heat from the first bolt and 3650 W/m² from the second bolt.

7-37E A refrigeration truck is traveling at 70 mph. The average temperature of the outer surface of the refrigeration compartment of the truck is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties. **5** The local atmospheric pressure is 1 atm.

Properties Assuming the film temperature to be approximately 80°F, the properties of air at this temperature and 1 atm are (Table A-15E)



$$k = 0.01481\ \text{Btu/h.ft.°F}$$
$$\nu = 1.697\times10^{-4}\ \text{ft}^2/\text{s}$$
$$\text{Pr} = 0.7290$$

Analysis The Reynolds number is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{[70\times5280/3600)\ \text{ft/s}](20\,\text{ft})}{1.697\times10^{-4}\ \text{ft}^2/\text{s}} = 1.210\times10^7$$

We assume the air flow over the entire outer surface to be turbulent. Therefore using the proper relation in turbulent flow for Nusselt number, the average heat transfer coefficient is determined to be

$$Nu = \frac{hL}{k} = 0.037\,\text{Re}_L^{\,0.8}\,\text{Pr}^{1/3} = 0.037(1.210\times10^7)^{0.8}(0.7290)^{1/3} = 1.544\times10^4$$

$$h = \frac{k}{L}Nu = \frac{0.01481\ \text{Btu/h.ft.°F}}{20\ \text{ft}}(1.544\times10^4) = 11.43\ \text{Btu/h.ft}^2.\text{°F}$$

Since the refrigeration system is operated at half the capacity, we will take half of the heat removal rate

$$\dot{Q} = \frac{(600\times60)\ \text{Btu/h}}{2} = 18{,}000\ \text{Btu/h}$$

The total heat transfer surface area and the average surface temperature of the refrigeration compartment of the truck are determined from

$$A = 2\big[(20\,\text{ft})(9\,\text{ft}) + (20\,\text{ft})(7\,\text{ft}) + (9\,\text{ft})(7\,\text{ft})\big] = 766\ \text{ft}^2$$

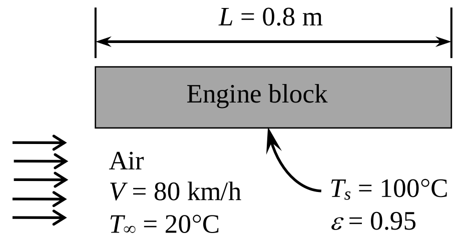
$$\dot{Q} = hA_s(T_\infty - T_s) \longrightarrow T_s = T_\infty - \frac{\dot{Q}}{hA_s} = 80\text{°F} - \frac{18{,}000\ \text{Btu/h}}{(11.43\ \text{Btu/h.ft}^2.\text{°F})(766\,\text{ft}^2)} = \mathbf{77.9°F}$$

7-38 A car travels at a velocity of 80 km/h. The rate of heat transfer from the bottom surface of the hot automotive engine block is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas with constant properties. **4** The flow is turbulent over the entire surface because of the constant agitation of the engine block.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (100+20)/2 = 60°C$ are (Table A-15)

$$k = 0.02808 \text{ W/m.°C}$$
$$\nu = 1.896\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7202$$



Analysis Air flows parallel to the 0.4 m side. The Reynolds number in this case is

$$\text{Re}_L = \frac{V_\infty L}{\nu} = \frac{[(80\times1000/3600)\text{ m/s}](0.8\text{m})}{1.896\times10^{-5}\text{ m}^2\text{/s}} = 9.376\times10^5$$

which is greater than the critical Reynolds number and thus the flow is laminar + turbulent. But the flow is assumed to be turbulent over the entire surface because of the constant agitation of the engine block. Using the proper relations, the Nusselt number, the heat transfer coefficient, and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = 0.037\,\text{Re}_L^{\,0.8}\,\Pr^{1/3} = 0.037(9.376\times10^5)^{0.8}(0.7202)^{1/3} = 1988$$

$$h = \frac{k}{L}Nu = \frac{0.02808\text{ W/m.°C}}{0.8\text{ m}}(1988) = 69.78\text{ W/m}^2\text{.°C}$$

$$A_s = wL = (0.8\text{ m})(0.4\text{m}) = 0.32\text{ m}^2$$

$$\dot{Q}_{conv} = hA_s(T_\infty - T_s) = (69.78\text{ W/m}^2\text{.°C})(0.32\text{ m}^2)(100-20)°\text{C} = \mathbf{1786\ W}$$

The radiation heat transfer from the same surface is

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})\\ &= (0.95)(0.32\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(100+273\text{ K})^4 - (25+273\text{ K})^4]\\ &= \mathbf{198\ W}\end{aligned}$$

Then the total rate of heat transfer from that surface becomes

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = (1786+198)\text{W} = \mathbf{1984\ W}$$

7-39 Air flows in parallel over a flat plate. The distance from the plate's leading edge where the critical Reynolds number is reached is to be determined. The local convection heat transfer coefficient along the plate is to be evaluated.

Assumptions **1** Steady operating conditions exist. **2**Surface temperature is uniform over the entire plate. **3** Local atmospheric pressure is 1 atm. **4** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$.

Properties The kinematic viscosity of air at $T_f = (120°\text{C} + 20°\text{C})/2 = 70°\text{C}$ is $\nu = 1.995\times 10^{-5}\ \text{m}^2/\text{s}$ (Table A-15).

Analysis The distance from the plate's leading edge when $\mathrm{Re}_{cr} = 5\times 10^5$ is

$$\mathrm{Re}_{cr} = \frac{Vx_{cr}}{\nu} \rightarrow x_{cr} = \frac{\mathrm{Re}_{cr}\,\nu}{V} = \frac{(5\times10^5)(1.995\times10^{-5}\ \text{m}^2/\text{s})}{7\ \text{m/s}} = \mathbf{1.425\,m}$$

For laminar flow ($x<x_{cr}$), the relation for local Nusselt number along a flat plate is

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$$

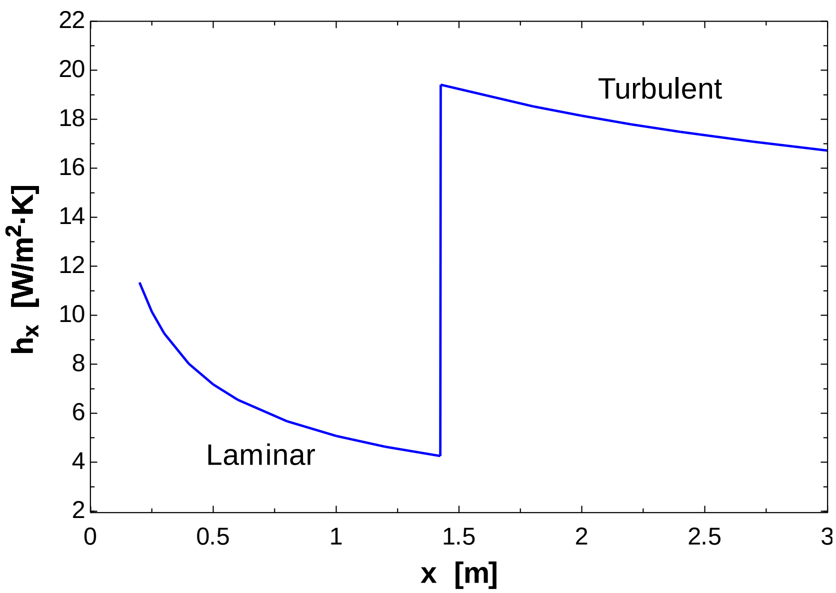
For turbulent flow ($x>x_{cr}$), the relation for local Nusselt number along a flat plate is

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$$

The problem is solved using EES, and the solution is given below:

```
"GIVEN"
T_infinity=120 [C]
T_s=20 [C]
V=7 [m/s]

"PROPERTIES"
Fluid$='air'
T_film=1/2*(T_s+T_infinity)
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho

"ANALYSIS"
Re_x=V*x/nu
Nusselt_lam=0.332*Re_x^0.5*Pr^(1/3)     "Laminar flow Nusselt_x"
Nusselt_turb=0.0296*Re_x^0.8*Pr^(1/3)     "Turbulent flow Nusselt_x"
Nusselt_x=if(Re_x,5e5,Nusselt_lam,Nusselt_turb,Nusselt_turb)
h_x=Nusselt_x*k/x
```

x [m]	Re_x	h_x [W/m^2·K]
0.20	70161	11.34
0.25	87701	10.14
0.30	105241	9.260
0.40	140321	8.019
0.50	175402	7.172
0.60	210482	6.548
0.80	280643	5.670
1.0	350803	5.072
1.2	420964	4.630
1.424	499544	4.250
1.425	500000	19.41
1.8	631446	18.53
2.0	701607	18.14
2.2	771768	17.80
2.4	841928	17.49
2.7	947169	17.08
3.0	1.052E+06	16.73



Discussion When the flow becomes turbulent at $x = x_{cr}$, there is a jump in the local convection heat transfer coefficient. The higher level of mixing in the turbulent region promotes more convection heat transfer than that in the laminar region.

7-40 To prevent local hot spots on a machine surface from causing thermal burns, the thickness of an insulation to cover the machine surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** One-dimensional heat conduction through the plate. **3** Thermal conductivities of plate and insulationare constant. **4** Uniform surface temperature. **5** Local atmospheric pressure is 1 atm. **6** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$.

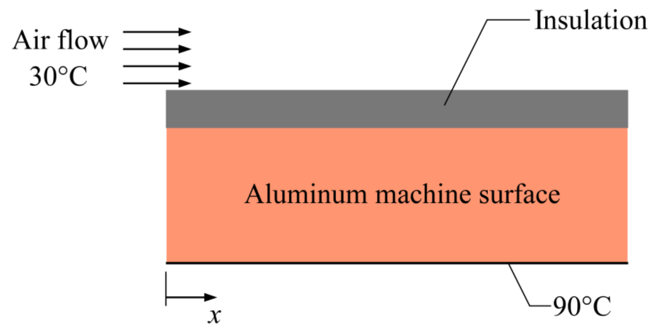
PropertiesThe thermal conductivities of the aluminum and the insulation are given to bek_{al}= 237 W/m·K andk_{ins} = 0.06 W/m·K, respectively. The thermal contact conductance at the interface is given as h_c = 3000 W/m²·K.

AnalysisFrom Chapter 3, the thermal resistances of different layers are

$$R_{al} = \frac{L_{al}}{k_{al}A} \quad \text{(aluminum layer resistance)}$$

$$R_{interface} = \frac{1}{h_c A} \quad \text{(contact resistance)}$$

$$R_{ins} = \frac{L_{ins}}{k_{ins}A} \quad \text{(insulation layer resistance)}$$



The heat balance at the outer surface is

$$\dot{q}_{cond} = \dot{q}_{conv} \rightarrow \frac{T_{s,i} - T_{s,o}}{\dfrac{L_{al}}{k_{al}} + \dfrac{1}{h_c} + \dfrac{L_{ins}}{k_{ins}}} = \frac{T_{s,o} - T_\infty}{\dfrac{1}{h_x}}$$

For laminar flow ($x<x_{cr}$), the relation for local Nusselt number along a flat plate is

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$$

For turbulent flow ($x>x_{cr}$), the relation for local Nusselt number along a flat plate is

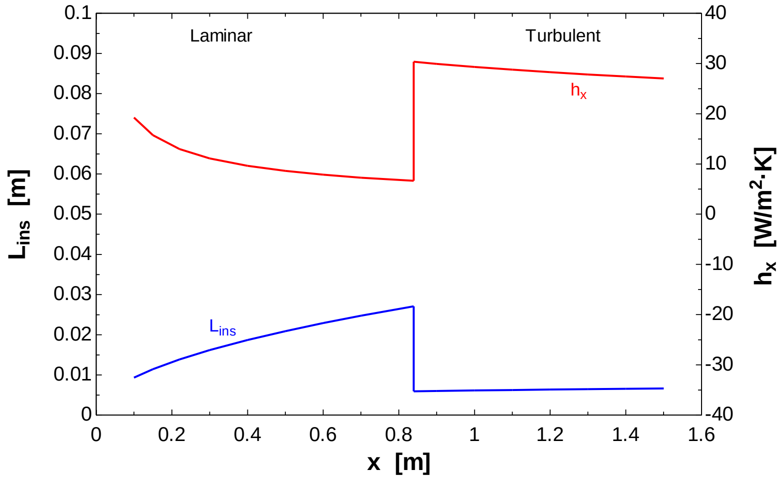
$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$$

The problem is solved using EES, and the solution is given below:

```
"GIVEN"
h_c=3000 [W/m^2-K]
L_al=0.005 [m]
T_infinity=30 [C]
T_s_i=90 [C]     "Inner surface T"
T_s_o=45 [C]     "Outer surface T"
V=10 [m/s]

"PROPERTIES"
"Air"
Fluid$='air'
T_film=1/2*(T_s_o+T_infinity)
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
"Aluminum layer"
k_al=237 [W/m-K]
"Insulation layer"
k_ins=0.06 [W/m-K]

"ANALYSIS"
```

```
Re_x=V*x/nu
Nusselt_lam=0.332*Re_x^0.5*Pr^(1/3)      "Laminar flow Nusselt_x"
Nusselt_turb=0.0296*Re_x^0.8*Pr^(1/3)     "Turbulent flow Nusselt_x"
Nusselt_x=if(Re_x,5e5,Nusselt_lam,Nusselt_turb,Nusselt_turb)
h_x=Nusselt_x*k/x
"Heat balance at the outer surface"
q_dot_cond=(T_s_i-T_s_o)/(L_al/k_al+1/h_c+L_ins/k_ins)
q_dot_conv=(T_s_o-T_infinity)/(1/h_x)
q_dot_cond=q_dot_conv
```

x [m]	Re_x	h_x [W/m^2·K]	L_{ins} [m]
0.10	59585	19.25	0.009331
0.15	89378	15.71	0.01143
0.22	131087	12.98	0.01385
0.30	178756	11.11	0.01618
0.40	238341	9.623	0.01868
0.50	297926	8.607	0.02089
0.60	357511	7.857	0.02289
0.70	417096	7.275	0.02472
0.8391	499979	6.644	0.02707
0.8392	500039	30.36	0.005908
0.90	536267	29.94	0.005991
1.0	595852	29.31	0.006120
1.1	655437	28.76	0.006238
1.2	715022	28.26	0.006348
1.3	774607	27.81	0.006450
1.4	834192	27.4	0.006547
1.5	893778	27.03	0.006638



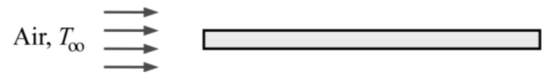
Discussion To ensure the entire outer surface of the machine is below 45°C, without local hot spots, the insulation should be at least 3 cm thick. The location that requires the thickest insulation is where the local convection heat transfer coefficient is the lowest, which is near the end of the laminar region.

7-41 Air is flowing in parallel over a stationary thin flat plate: (*a*) the average friction coefficient, (*b*) the average convection heat transfer coefficient, and (*c*) the average convection heat transfer coefficient using the modified Reynolds analogy are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The edge effects are negligible. **4** The critical Reynolds number is $Re_{cr} = 5\times10^5$.

Properties The properties of air (1 atm) at the T_f = (20°C + 180°C)/2 = 100°C are given in Table A-15: k = 0.03095 W/m·K, $\nu = 2.306 \times 10^{-5}$ m²/s, and Pr = 0.7111.

Analysis (*a*) The Reynolds at the trailing edge of the plate is



$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(50\text{ m/s})(0.5\text{ m})}{2.306\times10^{-5}\text{ m}^2/\text{s}} = 1.084\times10^6$$

Since $5 \times 10^5 < \mathrm{Re}_L < 10^7$ at the trailing edge, the flow is a combined laminar and turbulent flow. The friction coefficient is therefore

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} = \mathbf{0.00299}$$

(*b*) Using the proper relation for Nusselt number for combined laminar and turbulent flow, the average convection heat transfer coefficient is

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\,\mathrm{Pr}^{1/3} \quad \rightarrow \quad h = \frac{k}{L}(0.037\,\mathrm{Re}_L^{0.8} - 871)\,\mathrm{Pr}^{1/3}$$

$$h = \frac{(0.03095\text{ W/m}\cdot\text{K})}{(0.5\text{ m})}[0.037(1.084\times10^6)^{0.8} - 871](0.7111)^{1/3} = \mathbf{89.46\ W/m^2 \cdot K}$$

(*c*) Using the modified Reynolds analogy from Chapter 6, the average convection heat transfer coefficient is

$$\mathrm{Nu} = C_f\,\frac{\mathrm{Re}_L}{2}\,\mathrm{Pr}^{1/3} \quad \rightarrow \quad h = \frac{k}{L}C_f\,\frac{\mathrm{Re}_L}{2}\,\mathrm{Pr}^{1/3}$$

$$h = \frac{(0.03095\text{ W/m}\cdot\text{K})}{(0.5\text{ m})}(0.00299)\frac{1.084\times10^6}{2}(0.7111)^{1/3} = \mathbf{89.54\ W/m^2 \cdot K}$$

Discussion There is practically no difference in the results between parts (*b*) and (*c*). The two results differ by less than 0.1%.

7-42 C&S An ASTM A240 410S stainless steel plate is exposed to cold gas, at −70°C, flowing in parallel over its surface. The average heat transfer rate required to keep the plate surface from getting below −30°C is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Uniform surface temperature. **3** Edge effects of plate are negligible. **4** The critical Reynolds number is $\text{Re}_{cr} = 5 \times 10^5$.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (-70 - 30)°\text{C}/2 = -50°\text{C}$ are (Table A–15): Pr = 0.7440, k = 0.01979 W/m·K, and $\nu = 9.319 \times 10^{-6}$ m²/s

Analysis The Reynolds number at the end of the plate is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{(5 \text{ m/s})(2 \text{ m})}{9.319 \times 10^{-6} \text{m}^2/\text{s}} = 1.0731 \times 10^6 > 5 \times 10^5$$

Thus, we have turbulent flow at the end of the plate and combined laminar and turbulent flow over the entire plate. The average convection heat transfer coefficient is determined using the combined laminar and turbulent flow relation for a flat plate,

$$h = \left(\frac{k}{L}\right)(0.037 \text{ Re}_L^{0.8} - 871)\text{Pr}^{1/3} = \left(\frac{0.01979 \frac{\text{W}}{\text{m}} \cdot \text{K}}{2 \text{ m}}\right)[0.037(1.0731 \times 10^6)^{0.8} - 871](0.7440)^{\frac{1}{3}}$$
$$= 14.34 \text{ W/m}^2\cdot\text{K}$$

From the Newton's law of cooling, the average convection heat transfer rate is

$$\dot{Q} = hA(T_s - T_\infty) = (14.34 \text{ W/m}^2\cdot\text{K})(2 \text{ m} \times 2 \text{ m})(-30 + 70)\text{K} = \mathbf{2294\ W}$$

Discussion The average heat transfer rate (i.e. heat added to the plate) required to keep the plate surface from going colder than the minimum suitable temperature of −30°C is 2294 W. If the heat rate is less than 2294 W, that would risk having the plate surface operating at a temperature lower than the value set by the ASME Code for Process Piping.

7-43 A 5-m long strip of sheet metal is being transported on a conveyor, while the coating on the upper surface is being cured by infrared lamps. The surface temperature of the sheet metal is to be determined.

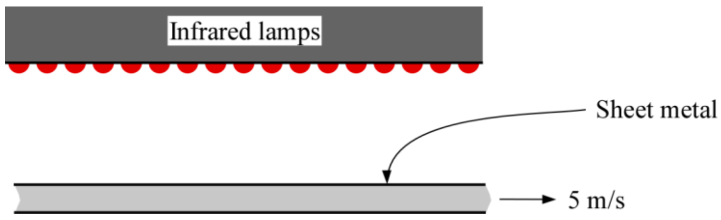
Assumptions **1** Steady operating conditions exist. **2** Heat conduction through the sheet metal is negligible. **3** Thermal properties are constant. **4** The surrounding ambient air is at 1 atm. **5** The critical Reynolds number is $Re_{cr} = 5\times10^5$.

Properties The properties of air at 80°C are (Table A-15)

$k = 0.02953$ W/m·K

$\nu = 2.097 \times 10^{-5}$ m²/s

Pr = 0.7154



Analysis The Reynolds number for L = 5 m is

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(5\ \mathrm{m/s})(5\ \mathrm{m})}{2.097\times10^{-5}\ \mathrm{m^2/s}} = 1.192\times10^6$$

Since $5 \times 10^5 < \mathrm{Re}_L < 10^7$, the flow is a combined laminar and turbulent flow. Using the proper relation for Nusselt number, the average heat transfer coefficient on the sheet metal is

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\,\mathrm{Pr}^{1/3} = [0.037(1.192\times10^6)^{0.8} - 871](0.7154)^{1/3} = 1624$$

$$h = 1624\frac{k}{L} = 1624\frac{0.02953\ \mathrm{W/m\cdot K}}{5\ \mathrm{m}} = 9.591\ \mathrm{W/m^2\cdot K}$$

From energy balance, we have

$$\dot{Q}_{\mathrm{absorbed}} - \dot{Q}_{\mathrm{rad}} - \dot{Q}_{\mathrm{conv}} = 0 \quad \rightarrow \quad A\dot{q}_{\mathrm{absorbed}} - A\dot{q}_{\mathrm{rad}} - 2A\dot{q}_{\mathrm{conv}} = 0$$

or $$\alpha\dot{q}_{\mathrm{incident}} - \varepsilon\sigma(T_s^4 - T_{\mathrm{surr}}^4) - 2h(T_s - T_\infty) = 0$$

Copy the following lines and paste on a blank EES screen to solve the above equation:

```
h=9.591
T_inf=25+273
T_surr=25+273
q_incindent=5000
alpha=0.6
epsilon=0.7
sigma=5.670e-8
alpha*q_incindent-epsilon*sigma*(T_s^4-T_surr^4)-2*h*(T_s-T_inf)=0
```

Solving by EES software, the surface temperature of the sheet metal is

$$T_s = 411\ \mathrm{K} = \mathbf{138°C}$$

Discussion Note that absolute temperatures must be used in calculations involving the radiation heat transfer equation. The assumed temperature of 80°C for evaluating the air properties turned out to be a good estimation, since T_f = (138°C + 25°C)/2 = 82°C.

7-44 Prob. 7-43 is reconsidered. The effect of the sheet metal velocity on its surface temperature is to be evaluated.

Assumptions **1** Steady operating conditions exist. **2** Heat conduction through the sheet metal is negligible. **3** The surrounding ambient air is at 1 atm. **4** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **5** Flow is combined laminar and turbulent (this assumption will be verified).

Analysis For combined laminar and turbulent flow, the relation for Nusselt number is

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\,\mathrm{Pr}^{1/3}$$

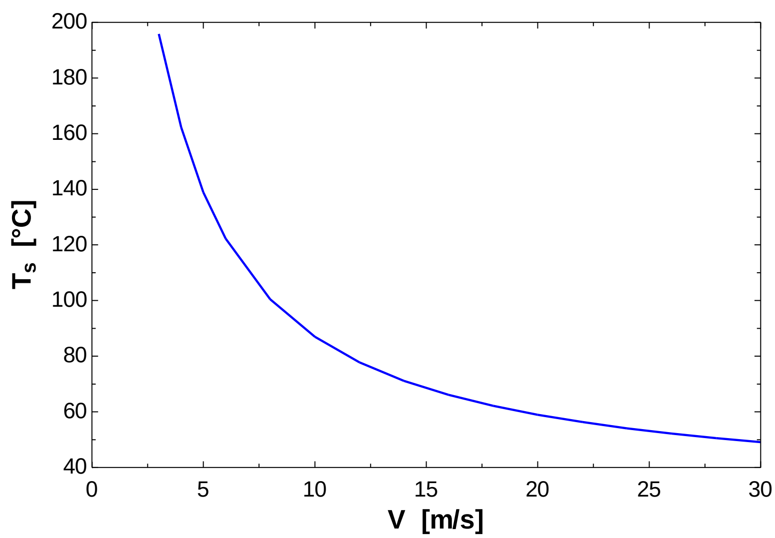
The surface temperature of the sheet metal is determined by applying energy balance on the sheet metal:

$$\alpha \dot{q}_{\text{incident}} - \varepsilon\sigma(T_s^4 - T_{\text{surr}}^4) - 2h(T_s - T_\infty) = 0$$

The problem is solved using EES, and the solution is given below:

```
"GIVEN"
L=5 [m]
T_infinity=25 [C]
T_surr=25 [C]
q_incindent=5000 [W/m^2]
alpha=0.6
epsilon=0.7

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
T_film=1/2*(T_s+T_infinity)

"ANALYSIS"
Re=V*L/nu
Nusselt_L=(0.037*Re^0.8-871)*Pr^(1/3)
h=Nusselt_L*k/L
alpha*q_incindent-epsilon*sigma#*((T_s+273)^4-(T_surr+273)^4)-2*h*((T_s+273)-(T_infinity+273))=0
```

V [m/s]	Re_L	T_s [°C]
3	620478	195.8
4	893402	162.3
5	1.181E+06	138.9
6	1.476E+06	122.2
8	2.081E+06	100.4
10	2.694E+06	86.94
12	3.312E+06	77.79
14	3.934E+06	71.16
16	4.558E+06	66.13
18	5.184E+06	62.17
20	5.811E+06	58.96
22	6.440E+06	56.32
24	7.069E+06	54.09
26	7.699E+06	52.18
28	8.329E+06	50.54
30	8.960E+06	49.10

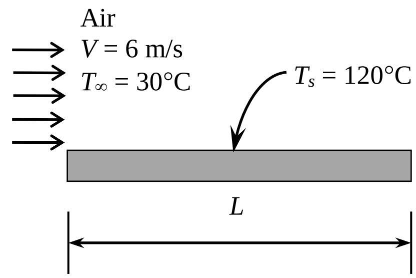


Discussion As the velocity increases, the effect of convective cooling on the sheet metal increases also. Thus, the surface temperature decreases with increasing velocity. For the sheet metal to maintain a surface temperature above 100°C, the velocity should not go below 8 m/s.As shown in the table above, between 3 and 30 m/s, the Reynolds number is $5 \times 10^5 < \mathrm{Re}_L < 10^7$.Thus, the flow is combined laminar and turbulent.

7-45 The top surface of a hot block is to be cooled by forced air. The rate of heat transfer is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties.

Properties The atmospheric pressure in atm is



$$P = (83.4\text{ kPa})\frac{1\text{ atm}}{101.325\text{ kPa}} = 0.823\text{ atm}$$

For an ideal gas, the thermal conductivity and the Prandtl number are independent of pressure, but the kinematic viscosity is inversely proportional to the pressure. With these considerations, the properties of air at 0.823 atm and at the film temperature of (120+30)/2=75°C are (Table A-15)

$$k = 0.02917\text{ W/m.°C}$$
$$\nu = \nu_{@1atm} / P_{atm} = (2.046\times10^{-5}\text{ m}^2\text{/s}) / 0.823 = 2.486\times10^{-5}\text{ m}^2\text{/s}$$
$$\Pr = 0.7166$$

Analysis (*a*) If the air flows parallel to the 8 m side, the Reynolds number in this case becomes

$$\text{Re}_L = \frac{VL}{\nu} = \frac{(6\text{ m/s})(8\text{m})}{2.486\times10^{-5}\text{ m}^2\text{/s}} = 1.931\times10^6$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = (0.037\,\text{Re}_L^{0.8} - 871)\Pr^{1/3} = [0.037(1.931\times10^6)^{0.8} - 871](0.7166)^{1/3} = 2757$$
$$h = \frac{k}{L}Nu = \frac{0.02917\text{ W/m.°C}}{8\text{ m}}(2757) = 10.05\text{ W/m}^2\text{.°C}$$

$$A_s = wL = (2.5\text{ m})(8\text{ m}) = 20\text{ m}^2$$
$$\dot{Q} = hA_s(T_\infty - T_s) = (10.05\text{ W/m}^2\text{.°C})(20\text{ m}^2)(120-30)\text{°C} = 18{,}100\text{ W} = \mathbf{18.10\,kW}$$

(*b*) If the air flows parallel to the 2.5 m side, the Reynolds number is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{(6\text{ m/s})(2.5\text{m})}{2.486\times10^{-5}\text{ m}^2\text{/s}} = 6.034\times10^5$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = (0.037\,\text{Re}_L^{0.8} - 871)\Pr^{1/3} = [0.037(6.034\times10^5)^{0.8} - 871](0.7166)^{1/3} = 615.1$$
$$h = \frac{k}{L}Nu = \frac{0.02917\text{ W/m.°C}}{2.5\text{ m}}(615.1) = 7.177\text{ W/m}^2\text{.°C}$$

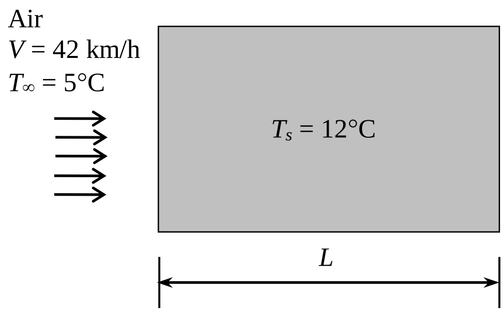
$$\dot{Q} = hA_s(T_\infty - T_s) = (7.177\text{ W/m}^2\text{.°C})(20\text{ m}^2)(120-30)\text{°C} = 12{,}920\text{ W} = \mathbf{12.92\,kW}$$

7-46 Wind is blowing parallel to the wall of a house. The rate of heat loss from that wall is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (12+5)/2 = 8.5$°C are (Table A-15)

$$k = 0.02428 \text{ W/m}\cdot°\text{C}$$
$$\nu = 1.413\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7340$$



Analysis *Air flows parallel to the 10 m side*:

The Reynolds number in this case is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{[(42\times1000/3600)\text{m/s}](10\text{m})}{1.413\times10^{-5} \text{ m}^2/\text{s}} = 8.257\times10^6$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, heat transfer coefficient and then heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = (0.037\,\text{Re}_L^{0.8} - 871)\,\text{Pr}^{1/3} = [0.037(8.257\times10^6)^{0.8} - 871](0.7340)^{1/3} = 1.061\times10^4$$

$$h = \frac{k}{L}Nu = \frac{0.02428 \text{ W/m.°C}}{10 \text{ m}}(1.061\times10^4) = 25.77 \text{ W/m}^2.°\text{C}$$

$$A_s = wL = (6\text{ m})(10\text{ m}) = 60\text{ m}^2$$
$$\dot{Q} = hA_s(T_\infty - T_s) = (25.77 \text{ W/m}^2.°\text{C})(60\text{ m}^2)(12-5)°\text{C} = 10{,}820 \text{ W} = \mathbf{10.8\,kW}$$

If the wind velocity is doubled:

$$\text{Re}_L = \frac{VL}{\nu} = \frac{[(84\times1000/3600)\text{m/s}](10\text{m})}{1.413\times10^{-5} \text{ m}^2/\text{s}} = 1.651\times10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = (0.037\,\text{Re}_L^{0.8} - 871)\,\text{Pr}^{1/3} = [0.037(1.651\times10^7)^{0.8} - 871](0.7340)^{1/3} = 1.906\times10^4$$

$$h = \frac{k}{L}Nu = \frac{0.02428 \text{ W/m.°C}}{10 \text{ m}}(1.906\times10^4) = 46.28 \text{ W/m}^2.°\text{C}$$

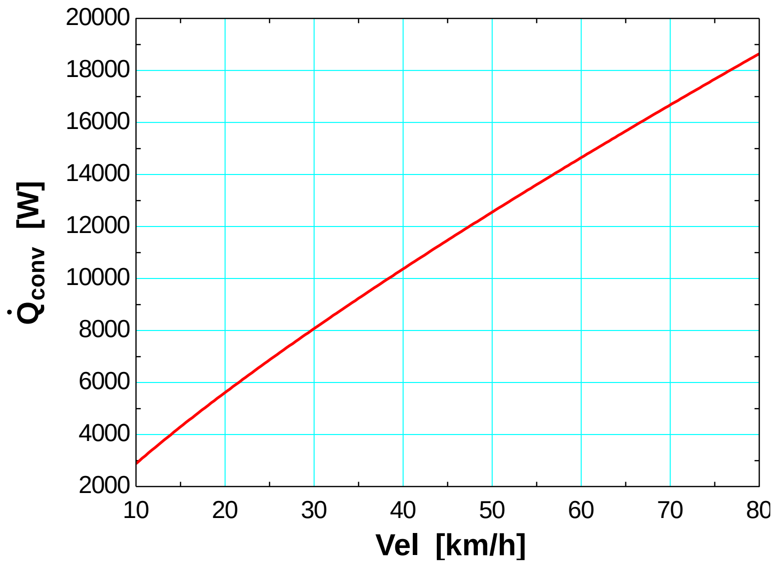
$$\dot{Q} = hA_s(T_\infty - T_s) = (46.28 \text{ W/m}^2.°\text{C})(60\text{ m}^2)(12-5)°\text{C} = 19{,}440 \text{ W} = \mathbf{19.4\,kW}$$

7-47 Prob. 7-46 is reconsidered. The effects of wind velocity and outside air temperature on the rate of heat loss from the wall by convection are to be investigated.

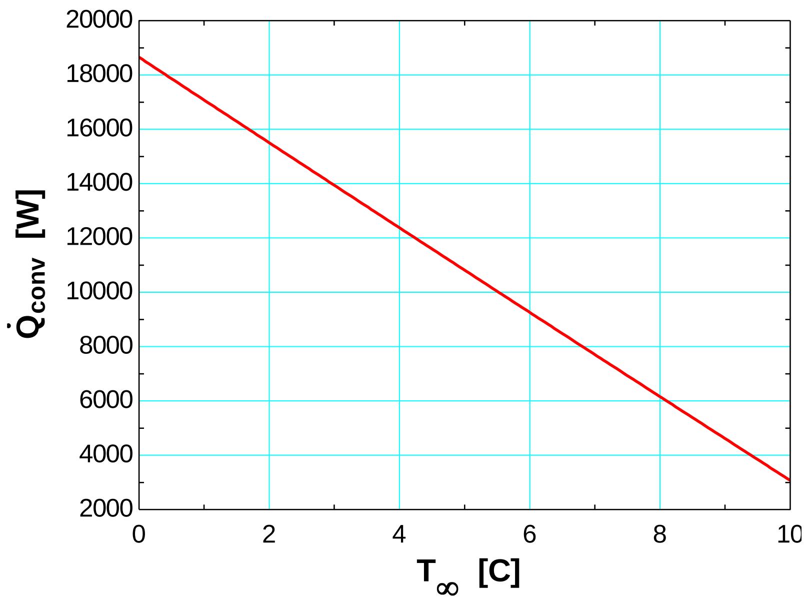
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
Vel=42 [km/h]
height=6 [m]
L=10 [m]
T_infinity=5 [C]
T_s=12 [C]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
T_film=1/2*(T_s+T_infinity)

"ANALYSIS"
Re=(Vel*Convert(km/h, m/s)*L)/nu
"We use combined laminar and turbulent flow relation for Nusselt number"
Nusselt=(0.037*Re^0.8-871)*Pr^(1/3)
h=k/L*Nusselt
A=height*L
Q_dot_conv=h*A*(T_s-T_infinity)
```

Vel [km/h]	Q_{conv} [W]
10	2884
15	4296
20	5614
25	6868
30	8072
35	9237
40	10368
45	11472
50	12551
55	13609
60	14648
65	15670
70	16676
75	17667
80	18646



T_∞ [C]	Q_{conv} [W]
0	18649
0.5	17861
1	17074
1.5	16288
2	15503
2.5	14719
3	13936
3.5	13154
4	12373
4.5	11592
5	10813
5.5	10035
6	9257
6.5	8481
7	7705
7.5	6930
8	6157
8.5	5384
9	4612
9.5	3841
10	3071



7-48E Warm air blowing over the inner surface of an automobile windshield is used for defrosting ice accumulated on the outer surface. The convection heat transfer coefficient for the warm air blowing over the inner surface of the windshield, necessary to cause the accumulated ice to begin melting, is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the windshield is one-dimensional. **3** Thermal properties are constant. **4** Heat transfer by radiation is negligible. **5** The outside air pressure is 1 atm. **6** The critical Reynolds number is $Re_{cr} = 5\times10^5$.

Properties The properties of air at the film temperature of T_f = (8°F + 32°F)/2 = 20°F are k = 0.01336 Btu/h·ft·R, $\nu = 1.379 \times 10^{-4}$ ft²/s, Pr = 0.7378 (from Table A-15E).

Analysis On the outer surface of the windshield, the Reynolds number at L = 20 in. is

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(50\times1.46667\ \mathrm{ft/s})(20/12\ \mathrm{ft})}{1.379\times10^{-4}\ \mathrm{ft^2/s}} = 8.863\times10^5$$

Since $5 \times 10^5 < \mathrm{Re}_L < 10^7$, the flow is a combined laminar and turbulent flow. Using the proper relation for Nusselt number, the average heat transfer coefficient on the outer surface of the windshield is

$$\mathrm{Nu}_o = \frac{h_o L}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\,\mathrm{Pr}^{1/3} = [0.037(8.863\times10^5)^{0.8} - 871](0.7378)^{1/3} = 1128$$

$$h_o = 1128\frac{k}{L} = 1128\frac{0.01336\ \mathrm{Btu/h\cdot ft\cdot R}}{20/12\ \mathrm{ft}} = 9.042\ \mathrm{Btu/h\cdot ft^2\cdot R}$$

From energy balance, the heat transfer through the windshield thickness can be written as

$$\frac{T_{\infty,o} - T_{s,o}}{1/h_o} = \frac{T_{s,o} - T_{\infty,i}}{t/k_w + 1/h_i}$$

For the ice to begin melting, the outer surface temperature of the windshield ($T_{s,o}$) should be at least 32°F. The convection heat transfer coefficient for the warm air blowing over the inner surface of the windshield is

$$h_i = \left(\frac{1}{h_o}\frac{T_{s,o} - T_{\infty,i}}{T_{\infty,o} - T_{s,o}} - \frac{t}{k_w}\right)^{-1}$$

$$= \left[\frac{(32-77)°\mathrm{F}}{(8-32)°\mathrm{F}}\left(\frac{1}{9.042\ \mathrm{Btu/h\cdot ft^2\cdot R}}\right) - \frac{0.2/12\ \mathrm{ft}}{0.8\ \mathrm{Btu/h\cdot ft\cdot R}}\right]^{-1}$$

$$= \mathbf{5.36\ Btu/h\cdot ft^2\cdot R}$$

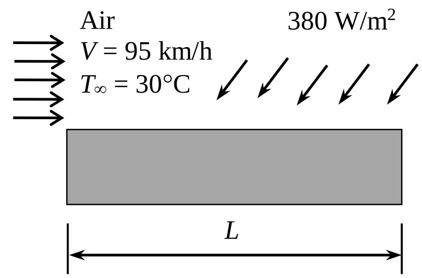
Discussion To keep the ice from accumulating for the given conditions, the convection heat transfer coefficient for the warm air blowing over the inner surface of the windshield needs to be at least 5.36 Btu/h·ft²·R or higher.

7-49 The top surface of the passenger car of a train in motion is absorbing solar radiation. The equilibrium temperature of the top surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation heat exchange with the surroundings is negligible. **4** Air is an ideal gas with constant properties.

Properties The properties of air at 30°C are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$
$$\nu = 1.608\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.7282$$



Analysis The rate of convection heat transfer from the top surface of the car to the air must be equal to the solar radiation absorbed by the same surface in order to reach steady operation conditions. The Reynolds number is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{[95\times1000/3600)\text{ m/s}](8\text{m})}{1.608\times10^{-5}\text{ m}^2/\text{s}} = 1.313\times10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = (0.037\,\text{Re}_L^{\,0.8} - 871)\Pr^{1/3} = [0.037(1.313\times10^7)^{0.8} - 871](0.7282)^{1/3} = 1.569\times10^4$$

$$h = \frac{k}{L}Nu = \frac{0.02588\text{ W/m.°C}}{8\text{ m}}(1.569\times10^4) = 50.77\text{ W/m}^2.\text{°C}$$

The equilibrium temperature of the top surface is then determined by taking convection and radiation heat fluxes to be equal to each other

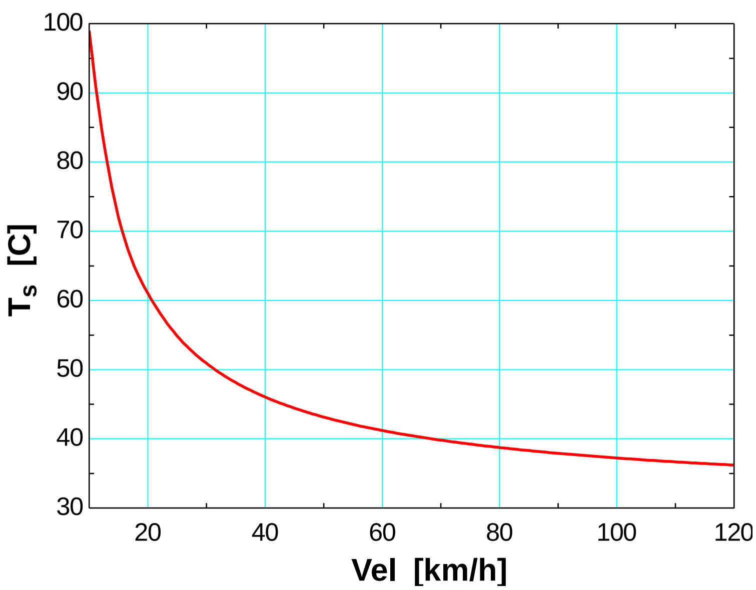
$$\dot{q}_{rad} = \dot{q}_{conv} = h(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{q}_{conv}}{h} = 30\text{°C} + \frac{380\text{ W/m}^2}{50.77\text{ W/m}^2.\text{°C}} = \mathbf{37.5°C}$$

7-50  Prob. 7-49 is reconsidered. The effects of the train velocity and the rate of absorption of solar radiation on the equilibrium temperature of the top surface of the car are to be investigated.

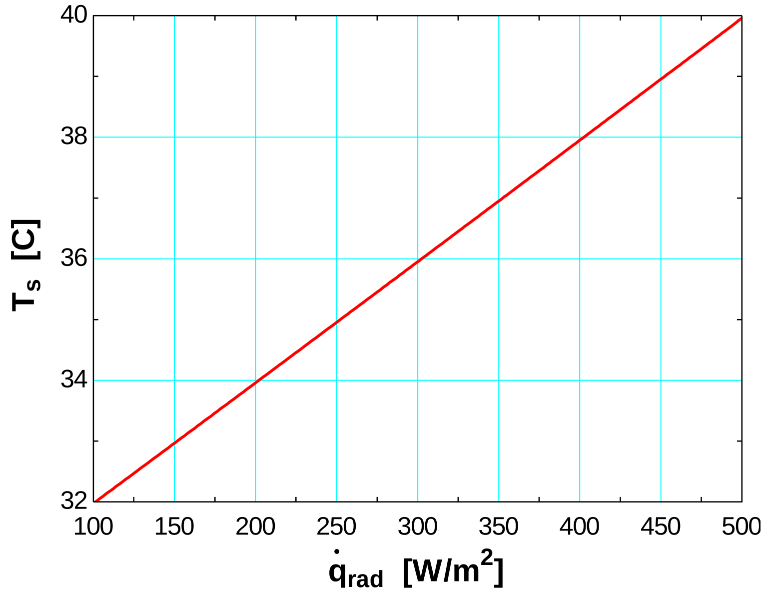
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
Vel=95 [km/h]
w=2.8 [m]
L=8 [m]
q_dot_rad=380 [W/m^2]
T_infinity=30 [C]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
T_film=1/2*(T_s+T_infinity)

"ANALYSIS"
Re=(Vel*Convert(km/h, m/s)*L)/nu
"Reynolds number is greater than the critical Reynolds number. We use combined laminar and turbulent flow relation for Nusselt number"
Nusselt=(0.037*Re^0.8-871)*Pr^(1/3)
h=k/L*Nusselt
q_dot_conv=h*(T_s-T_infinity)
q_dot_conv=q_dot_rad
```

Vel [km/h]	T_s [C]
10	99
15	72.06
20	61.02
25	54.87
30	50.91
35	48.12
40	46.04
45	44.43
50	43.14
55	42.08
60	41.2
65	40.45
70	39.8
75	39.23
80	38.74
85	38.3
90	37.9
95	37.55
100	37.23
105	36.94
110	36.67
115	36.43
120	36.2



Q_{rad} [W/m^2]	T_s [C]
100	31.98
125	32.47
150	32.97
175	33.46
200	33.96
225	34.46
250	34.95
275	35.45
300	35.95
325	36.45
350	36.95
375	37.45
400	37.95
425	38.45
450	38.95
475	39.45
500	39.96

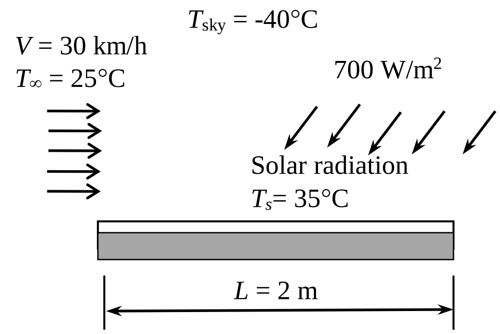


7-51 Solar radiation is incident on the glass cover of a solar collector. The total rate of heat loss from the collector, the collector efficiency, and the temperature rise of water as it flows through the collector are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Heat exchange on the back surface of the absorber plate is negligible. **4** Air is an ideal gas with constant properties. **5** The local atmospheric pressure is 1 atm.

Properties The properties of air at the film temperature of $(35+25)/2 = 30°C$ are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$
$$\nu = 1.608\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7282$$



Analysis (*a*) Assuming wind flows across 2 m surface, the Reynolds number is determined from

$$\text{Re}_L = \frac{VL}{\nu} = \frac{(30\times1000/3600 \text{ m/s})(2\text{ m})}{1.608\times10^{-5} \text{ m}^2\text{/s}} = 1.036\times10^6$$

which is greater than the critical Reynolds number. Using the Nusselt number relation for combined laminar and turbulent flow, the average heat transfer coefficient is determined to be

$$Nu = \frac{hL}{k} = (0.037\,\text{Re}^{0.8} - 871)\Pr^{1/3} = [0.037(1.036\times10^6)^{0.8} - 871](0.7282)^{1/3} = 1378$$

$$h = \frac{k}{L}Nu = \frac{0.02588 \text{ W/m.°C}}{2\text{ m}}(1378) = 17.83 \text{ W/m}^2\text{.°C}$$

Then the rate of heat loss from the collector by convection is

$$\dot{Q}_{conv} = hA_s(T_\infty - T_s) = (17.83 \text{ W/m}^2\text{.°C})(2\times1.2\text{ m}^2)(35-25)\text{°C} = 427.9 \text{ W}$$

The rate of heat loss from the collector by radiation is

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4}) \\ &= (0.90)(2\times1.2\text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.°C})\left[(35+273\text{ K})^4 - (-40+273\text{ K})^4\right] \\ &= 741.2 \text{ W}\end{aligned}$$

and

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 427.9 + 741.2 = \mathbf{1169\ W}$$

(*b*) The net rate of heat transferred to the water is

$$\begin{aligned}\dot{Q}_{net} &= \dot{Q}_{in} - \dot{Q}_{out} = \alpha AI - \dot{Q}_{out} \\ &= (0.88)(2\times1.2\text{ m}^2)(700 \text{ W/m}^2) - 1169 \text{ W} \\ &= 1478 - 1169 = 309 \text{ W}\end{aligned}$$

$$\eta_{collector} = \frac{\dot{Q}_{net}}{\dot{Q}_{in}} = \frac{309\text{ W}}{1478\text{ W}} = \mathbf{0.209}$$

(*c*) The temperature rise of water as it flows through the collector is

$$\dot{Q}_{net} = \dot{m}c_p\Delta T \longrightarrow \Delta T = \frac{\dot{Q}_{net}}{\dot{m}c_p} = \frac{309.4\text{ W}}{(1/60\text{ kg/s})(4180\text{ J/kg.°C})} = \mathbf{4.44°C}$$

7-52 PtD An engine cover with a layer of insulation is subjected to convection heat transfer on the inner and outer surfaces. To prevent fire hazard by keeping the engine outer surface temperature below 180°C, it is suggested to increase the insulation thickness by threefold. The effectiveness of this method is to be evaluated.

Assumptions **1** The thermal properties of the plate and insulation are constant. **2** One-dimensional heat conduction through the plate. **3** Uniform plate surface temperature. **4** Thermal contact resistance at interface is negligible. **5** Radiation effects are negligible. **6** Local atmospheric pressure is 1 atm. **7** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$.

Properties The thermal conductivities of the stainless steel and the insulation are given to be k_{ss} = 14 W/m·K and k_{ins} = 0.5 W/m·K, respectively. The properties of air are evaluated at T_f = 120°C: k = 0.03235 W/m·K, $\nu = 2.522 \times 10^{-5}$ m²/s, and Pr = 0.7073 (from Table A-15).

Analysis With increasing the insulation thickness by threefold, the Reynolds number for the 2-m long plateis

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(7\text{ m/s})(2\text{m})}{2.522\times10^{-5}\text{ m}^2/\text{s}} = 555115 > 5\times10^5$$

With the Reynolds number between $5 \times 10^5 < \mathrm{Re}_L < 10^7$,the proper equation is the combined laminar and turbulent relation for the Nusselt number:

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\,\mathrm{Pr}^{1/3} = [0.037(555115)^{0.8} - 871](0.7073)^{1/3} = 522.86$$

The convection heat transfer coefficient on the engine outer surface is

$$h = \mathrm{Nu}\frac{k}{L} = 522.86\left(\frac{0.03235\text{ W/m}\cdot\text{K}}{2\text{ m}}\right) = 8.457\text{ W/m}^2\cdot\text{K}$$

From Chapter 3, the thermal resistances of different layers are

$$R_{\text{conv},i} = \frac{1}{h_i A} \quad \text{(inside surface convection resistance)}, \quad R_{\text{ss}} = \frac{L_{\text{ss}}}{k_{\text{ss}}A} \quad \text{(stainless steel layer resistance)},$$

$$R_{\text{ins}} = \frac{L_{\text{ins}}}{k_{\text{ins}}A} \quad \text{(insulation layer resistance)}, \quad R_{\text{conv},o} = \frac{1}{h_o A} \quad \text{(outside surface convection resistance)}$$

Then,

$$AR_{\text{total}} = A(R_{\text{conv},i} + R_{\text{ss}} + R_{\text{ins}} + R_{\text{conv},o}) = \frac{1}{h_i} + \frac{L_{\text{ss}}}{k_{\text{ss}}} + \frac{L_{\text{ins}}}{k_{\text{ins}}} + \frac{1}{h_o}$$

$$= \frac{1}{7\text{ W/m}^2\cdot\text{K}} + \frac{0.01\text{ m}}{14\text{ W/m}\cdot\text{K}} + \frac{3(0.005)\text{ m}}{0.5\text{ W/m}\cdot\text{K}} + \frac{1}{8.457\text{ W/m}^2\cdot\text{K}} = 0.29182\text{ m}^2\cdot\text{K/W}$$

and

$$AR_{\text{conv},o} = \frac{1}{h_o} = \frac{1}{8.457\text{ W/m}^2\cdot\text{K}} = 0.11825\text{ m}^2\cdot\text{K/W}$$

The heat flux through the layers is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_{\infty,i} - T_{\infty,o}}{AR_{\text{total}}} = \frac{T_{s,o} - T_{\infty,o}}{AR_{\text{conv},o}} \rightarrow T_{s,o} = \frac{R_{\text{conv},o}}{R_{\text{total}}}(T_{\infty,i} - T_{\infty,o}) + T_{\infty,o}$$

$$T_{s,o} = \frac{0.11825}{0.29182}(350 - 60)°\text{C} + 60°\text{C} = \mathbf{177.5°C}$$

Yes, the suggested method is a viable method.

Discussion Increasing the insulation thickness by threefold would reduce $T_{s,o}$ to 177.5°C. However, from Example 7-2 results, increasing the cooling air velocity by only 10% reduced $T_{s,o}$ to 173.5°C. Thus, increasing the cooling air velocity is the more effective approach.

7-53 PtD An engine cover with a layer of thermal barrier coating (TBC) is subjected to convection heat transfer on its inner and outer surfaces. The engine outer surface temperature is to be determined whether it is above 180°C or not.

Assumptions **1** The thermal conductivities of the plate and TBC are constant. **2** One-dimensional heat conduction through the plate. **3** Uniform plate surface temperature. **4** Thermal contact resistance at interface is negligible. **5** Radiation effects are negligible. **6** Local atmospheric pressure is 1 atm. **7** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$.

Properties The thermal conductivities of the stainless steel and the TBC are given to be k_{ss}= 14 W/m·K and k_{TBC} = 1.1 W/m·K, respectively. The properties of air are evaluated at T_f = 120°C:

k = 0.03235 W/m·K, $\nu = 2.522\times 10^{-5}$ m²/s, and Pr = 0.7073 (Table A-15).

Analysis The Reynolds number for the 2-m long plate can be determined with

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(7.1\ \mathrm{m/s})(2\mathrm{m})}{2.522\times10^{-5}\ \mathrm{m^2/s}} = 563045 > 5\times10^5$$

With the Reynolds number between $5\times10^5 < \mathrm{Re}_L < 10^7$, the proper equation is the combined laminar and turbulent relation for the Nusselt number:

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\,\mathrm{Pr}^{1/3} = [0.037(563045)^{0.8} - 871](0.7073)^{1/3} = 537.68$$

The convection heat transfer coefficient on the engine outer surface is

$$h = \mathrm{Nu}\frac{k}{L} = 537.68\left(\frac{0.03235\ \mathrm{W/m\cdot K}}{2\ \mathrm{m}}\right) = 8.697\ \mathrm{W/m^2\cdot K}$$

From Chapter 3, the thermal resistances of different layers are

$$R_{\mathrm{conv},i} = \frac{1}{h_i A}\quad \text{(inside surface convection resistance)},\quad R_{ss} = \frac{L_{ss}}{k_{ss}A}\quad \text{(stainless steel layer resistance)},$$

$$R_{\mathrm{TBC}} = \frac{L_{\mathrm{TBC}}}{k_{\mathrm{TBC}}A}\quad \text{(TBC layer resistance)},\quad R_{\mathrm{conv},o} = \frac{1}{h_o A}\quad \text{(outside surface convection resistance)}$$

Then,

$$AR_{\mathrm{total}} = A(R_{\mathrm{conv},i} + R_{ss} + R_{\mathrm{TBC}} + R_{\mathrm{conv},o}) = \frac{1}{h_i} + \frac{L_{ss}}{k_{ss}} + \frac{L_{\mathrm{TBC}}}{k_{\mathrm{TBC}}} + \frac{1}{h_o}$$
$$= \frac{1}{7\ \mathrm{W/m^2\cdot K}} + \frac{0.01\ \mathrm{m}}{14\ \mathrm{W/m\cdot K}} + \frac{0.004\ \mathrm{m}}{1.1\ \mathrm{W/m\cdot K}} + \frac{1}{8.697\ \mathrm{W/m^2\cdot K}} = 0.26219\ \mathrm{m^2\cdot K/W}$$

and

$$AR_{\mathrm{conv},o} = \frac{1}{h_o} = \frac{1}{8.697\ \mathrm{W/m^2\cdot K}} = 0.11498\ \mathrm{m^2\cdot K/W}$$

The heat flux through the layers is

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_{\infty,i} - T_{\infty,o}}{AR_{\mathrm{total}}} = \frac{T_{s,o} - T_{\infty,o}}{AR_{\mathrm{conv},o}} \rightarrow T_{s,o} = \frac{R_{\mathrm{conv},o}}{R_{\mathrm{total}}}(T_{\infty,i} - T_{\infty,o}) + T_{\infty,o}$$

$$T_{s,o} = \frac{0.11498}{0.26219}(333 - 60)°\mathrm{C} + 60°\mathrm{C} = \mathbf{179.7°C}$$

Discussion The outer surface temperature does not exceed 180°C, thus the TBC layer with a thickness of 4 mm in conjunction with 7.1 m/s air cooling is a sufficient thickness. Note that the assumed 120°C is an appropriate film temperature $T_f = (T_\infty + T_{s,o})/2$ for evaluating the air properties.

7-54 Metal plates exiting an oven are being cooled by air in a cooling chamber. The air velocity that is required to cool the plates so that they exit the cooling chamber at 45°C is to be determined.

Assumptions **1** The thermal properties of the plates are constant. **2** Uniform plate surface temperature. **3** Radiation effects are negligible. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified). **5** Local atmospheric pressure is 1 atm. **6** Flow is combined laminar and turbulent(this assumption will be verified). **7** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$.

Properties The properties of the metal plates are given as k_{plate} = 180 W/m·K, ρ_{plate} = 2800 kg/m^3, and $c_{p,\text{plate}}$ = 880 J/kg·K. The properties of air are evaluated at $T_f = (T_\infty + T_{s,\text{ave}})/2$ = 55°C, where $T_{s,\text{ave}}$= (155°C + 45°C)/2 = 100°C:

$$k = 0.02772 \text{ W/m·K}, \nu = 1.847\times 10^{-5} \text{ m}^2\text{/s, and Pr} = 0.7210 \quad \text{(Table A-15).}$$

Analysis The Reynolds number and the Nusselt number for combined laminar and turbulent flow over a flat plate can be determined with

$$\mathrm{Re}_L = \frac{VL}{\nu} \quad \text{and} \quad \mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\,\mathrm{Pr}^{1/3}$$

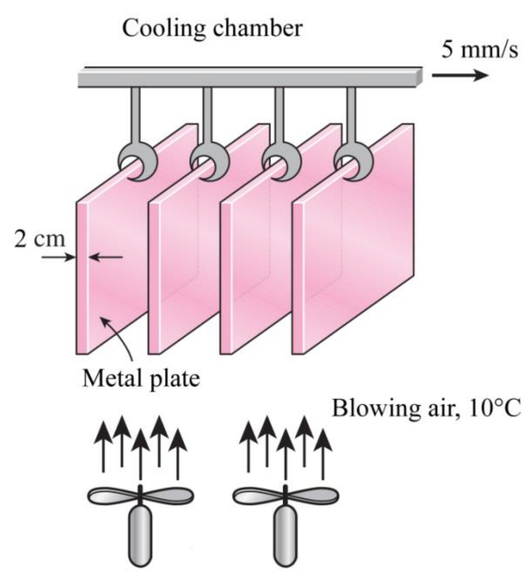
From lumped system analysis (see Chapter 4)

$$L_c = \frac{\boldsymbol{V}}{A_s} = \frac{\text{Plate thickness}}{2} = \frac{20\text{ mm}}{2} = 10\text{ mm}, \qquad b = \frac{hA_s}{\rho c_p \boldsymbol{V}} = \frac{h}{\rho c_p L_c}, \; \frac{T_{s,\text{f}} - T_\infty}{T_{s,\text{i}} - T_\infty} = \frac{45°\text{C} - 10°\text{C}}{155°\text{C} - 10°\text{C}} = e^{-bt}$$

The duration of cooling can be determined from the cooling chamber length and the speed of the plates,

$$t = \frac{10\text{ m}}{0.005\text{ m/s}} = 2000\text{ s}$$

The problem is solved using EES, and the solution is given below:




```
"GIVEN"
L=1 [m]     "Plate length"
T_s_i=155 [C]     "Initial surface T"
T_s_f=45 [C]     "Final surface T"
T_infinity=10 [C]
thickness=20e-3 [m]     "Plate thickness"
V_plate=0.005 [m/s]     "Plate speed"
Distance_cool=10 [m]     "Cooling distance"
t=Distance_cool/V_plate     "Cooling time"

"PROPERTIES"
"Air"
Fluid$='air'
T_s_ave=1/2*(T_s_i+T_s_f)
T_film=1/2*(T_s_ave+T_infinity)
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
"Plate"
c_p_plate=880 [J/kg-K]
k_plate=180 [W/m-K]
rho_plate=2800 [kg/m^3]

"ANALYSIS"
Re_L=V*L/nu
Nusselt_L=(0.037*Re_L^0.8-871)*Pr^(1/3)
h=Nusselt_L*k/L
"Lumped system analysis"
L_c=thickness/2
Bi=h*L_c/k_plate
```

```
b=h/(rho_plate*c_p_plate*L_c)
ln((T_s_f-T_infinity)/(T_s_i-T_infinity))=-b*t
```

Thus, the final results are

$\mathrm{Re}_L = 611691$, $\mathrm{Nu} = 631.8$, $h = 17.51\ \mathrm{W/m^2 \cdot K}$, $\mathrm{Bi} = 0.0009729$, $b = 0.0007107\ \mathrm{s^{-1}}$,

$V = \mathbf{11.29\ m/s}$

Discussion For the plates to exit the cooling chamber at 45°C, the blowers should produce an air velocity of 11.3 m/s. Since Bi < 0.1, the lumped system analysis is valid. With the Reynolds number between $5 \times 10^5 < \mathrm{Re}_L < 10^7$,the use of the combined laminar and turbulent relation for the Nusselt number is appropriate.

7-55 Mercury flows over a flat plate that is maintained at a specified temperature. The rate of heat transfer from the entire plate is to be determined.

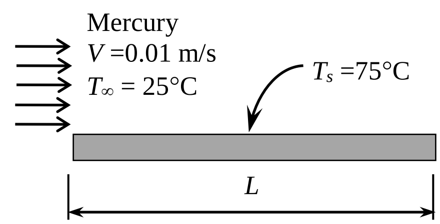
Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Atmospheric pressure is taken 1 atm.

Properties The properties of mercury at the film temperature of (75+25)/2=50°C are (Table A-14)

$$k = 8.83632 \text{ W/m.°C}$$

$$\nu = 1.056\times10^{-7} \text{ m}^2\text{/s}$$

$$\Pr = 0.0223$$



Analysis The local Nusselt number relation for liquid metals is given by Eq. 7-25 to be

$$Nu_x = \frac{h_x x}{k} = 0.565(\text{Re}_x \Pr)^{1/2}$$

The average heat transfer coefficient for the entire surface can be determined from

$$h = \frac{1}{L}\int_0^L h_x dx$$

Substituting the local Nusselt number relation into the above equation and performing the integration we obtain

$$Nu = 1.13(\text{Re}_L \Pr)^{1/2}$$

The Reynolds number is

$$Re_L = \frac{VL}{\nu} = \frac{(0.01\text{ m/s})(3\text{m})}{1.056\times10^{-7}\text{ m}^2\text{/s}} = 2.841\times10^5 \quad (\text{Re}_x < 5\times10^5 \text{ – Laminar Flow})$$

Using the relation for Nusselt number, the average heat transfer coefficient and the heat transfer rate are determined to be

$$Nu = \frac{hL}{k} = 1.13(\text{Re}_L \Pr)^{1/2} = 1.13[(2.8413\times10^5)(0.0223)]^{1/2} = 89.94$$

$$h = \frac{k}{L}Nu = \frac{8.83632 \text{ W/m.°C}}{3\text{ m}}(89.94) = 264.9 \text{ W/m}^2\text{.°C}$$

$$A = wL = (2\text{ m})(3\text{ m}) = 6\text{ m}^2$$

$$\dot{Q} = hA(T_s - T_\infty) = (264.9\text{W/m}^2\text{.°C})(6\text{ m}^2)(75-25)\text{°C} = 79470 \text{ W} = \mathbf{79.47\ kW}$$

7-56 Liquid mercury is flowing in parallel over a flat plate, (*a*) the local convection heat transfer coefficient at 5 cm from the leading edge and (*b*) the average convection heat transfer coefficient over the entire plate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperature is uniform throughout the plate. **3** Thermal properties are constant. **4** The critical Reynolds number is $Re_{cr} = 5\times10^5$.

Properties The properties of liquid mercury at T_f = (250°C + 50°C)/2 = 150°C are k = 10.07780 W/m·K, $\nu = 8.514 \times 10^{-8}$ m²/s, Pr = 0.0152 (from Table A-14).

Analysis (*a*) The Reynolds number at x = 0.05 m is

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(0.3\ \mathrm{m/s})(0.05\ \mathrm{m})}{8.514\times10^{-8}\ \mathrm{m^2/s}} = 1.762\times10^5$$

Since Pr < 0.60, the Churchill and Ozoe (1973) relation for Nusselt number is used. The local convection heat transfer coefficient at 0.05 m from the leading edge of the flat plate is

$$\mathrm{Nu}_x = \frac{h_x x}{k} = \frac{0.3387\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}}{[1+(0.0468/\mathrm{Pr})^{2/3}]^{1/4}} \quad \rightarrow \quad h_x = \frac{k}{x}\frac{0.3387\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}}{[1+(0.0468/\mathrm{Pr})^{2/3}]^{1/4}}$$

$$h_x = \frac{(10.0778\ \mathrm{W/m\cdot K})}{(0.05\ \mathrm{m})}\frac{0.3387(0.0152)^{1/3}(1.762\times10^5)^{1/2}}{[1+(0.0468/0.0152)^{2/3}]^{1/4}} = \mathbf{5343\ W/m^2\cdot K}$$

(*b*) The Reynolds number at L = 0.1 m is

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(0.3\ \mathrm{m/s})(0.1\ \mathrm{m})}{8.514\times10^{-8}\ \mathrm{m^2/s}} = 3.524\times10^5$$

The average convection heat transfer coefficient over the entire plate is

$$h = \frac{1}{L}\int_0^L h_x\,dx = \frac{C}{L}\int_0^L x^{-1/2}dx = 2\frac{C}{L}L^{1/2} \quad \text{where} \quad C = k\frac{0.3387\,\mathrm{Pr}^{1/3}(V/\nu)^{1/2}}{[1+(0.0468/\mathrm{Pr})^{2/3}]^{1/4}}$$

or

$$h = 2\frac{k}{L}\frac{0.3387\,\mathrm{Pr}^{1/3}(VL/\nu)^{1/2}}{[1+(0.0468/\mathrm{Pr})^{2/3}]^{1/4}} = 2\frac{k}{L}\frac{0.3387\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_L^{1/2}}{[1+(0.0468/\mathrm{Pr})^{2/3}]^{1/4}}$$

Hence

$$h = \frac{2(10.0778\ \mathrm{W/m\cdot K})}{(0.1\ \mathrm{m})}\frac{0.3387(0.0152)^{1/3}(3.524\times10^5)^{1/2}}{[1+(0.0468/0.0152)^{2/3}]^{1/4}} = \mathbf{7555\ W/m^2\cdot K}$$

Discussion Since the fluid properties are constant, it should be noted that $\mathrm{Nu} = 2\mathrm{Nu}_x$.

7-57 Water vapor flows in parallel over a 2-m long flat plate where there is an unheated starting length of 0.5 m, (*a*) the local convection heat transfer coefficient at x = 2 m, (*b*) the average convection heat transfer coefficient for the heated section, and (*c*) the rate of heat transfer per unit width for the heated section are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperature is uniform throughout the heated section. **3** Thermal properties are constant. **4** The critical Reynolds number is $\text{Re}_{cr} = 5\times10^5$.

Properties The properties of water vapor at T_f = (250°C + 50°C)/2 = 150°C are k = 0.02861 W/m·K, $\nu = 2.806\times 10^{-5}$ m²/s, Pr = 0.9712 (from Table A-16).

Analysis (*a*) The Reynolds number at x = 2 m is:

$$\text{Re}_{\text{x=L}} = \frac{VL}{\nu} = \frac{(5\text{ m/s})(2\text{ m})}{2.806\times10^{-5}\text{ m}^2\text{/s}} = 3.564\times10^5$$

Since $\text{Re}_{x=L} < 5\times10^5$ at the trailing edge, the flow over the entire heated section is laminar. Using the proper relation for Nusselt number, the local heat transfer coefficient at the trailing edge (x = 2 m) can be determined:

$$\text{Nu}_x = \frac{Nu_{x\,(\text{for }\xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} \;\rightarrow\; \text{h}_x = \frac{k}{x}\frac{0.332\text{Re}_x^{0.5}\text{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}}$$

$$\text{h}_{x=L} = \frac{0.02861\text{ W/m}\cdot\text{K}}{2\text{ m}}\frac{0.332(3.564\times10^5)^{0.5}(0.9712)^{1/3}}{[1-(0.5\text{ m/2 m})^{3/4}]^{1/3}} = \mathbf{3.25\ W/m^2\cdot K}$$

(*b*) The average convection heat transfer coefficient over the heated section is

$$h = \text{Nu}\frac{k}{L} = 391.4\left(\frac{0.02861\text{W/m}\cdot\text{K}}{2\text{ m}}\right) = \mathbf{5.6\ W/m^2\cdot K}$$

where $\quad \text{Nu} = \text{Nu}_{(\text{for }\xi=0)}\frac{L}{L-\xi}[1-(\xi/L)^{3/4}]^{2/3} = (392.6)\frac{2\text{ m}}{2\text{ m}-0.5\text{ m}}\left[1-\left(\frac{0.5\text{ m}}{2\text{ m}}\right)^{\frac{3}{4}}\right]^{\frac{2}{3}} = 391.4$

and $\quad \text{Nu}_{(\text{for }\xi=0)} = 0.664\,\text{Re}_L^{0.5}\text{Pr}^{1/3} = 0.664\,(3.564\times10^5)^{0.5}\,(0.9712)^{1/3} = 392.6$

(*c*) The rate of heat transfer per unit width (w) for the heated section is

$$\dot{Q} = hA(T_\infty - T_s) = h\,(L-\xi)w(T_\infty - T_s)$$

$$\dot{Q}/w = (5.6\text{ W/m}^2\cdot\text{K})(2\text{ m}-0.5\text{ m})(250-50)\text{K} = \mathbf{1680\ W/m}$$

Discussion For plate with unheated starting length, the thermal boundary layer does not begin to grow until the heated section, while the velocity boundary layer begins at the leading edge.

7-58 A silicon chip is mounted flush in a substrate that provides an unheated starting length. The surface temperature at the trailing edge of the chip is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Thermal properties are constant. **3** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **4** Only the upper surface of the chip is conditioned for heat transfer. **5** Heat transfer by radiation is negligible.

Properties The properties of air at 50°C are k = 0.02735 W/m·K, $\nu = 1.798 \times 10^{-5}$ m²/s, Pr = 0.7228 (from Table A-15).

Analysis The Reynolds number at the trailing edge (x = 0.030 m) is

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(25\ \mathrm{m/s})(0.030\ \mathrm{m})}{1.798\times10^{-5}\ \mathrm{m^2/s}} = 4.171\times10^4$$

Since $\mathrm{Re}_x < 5 \times 10^5$ at the trailing edge, the flow over the entire heated section is laminar. Using the proper relation for Nusselt number, the local heat transfer coefficient at the trailing edge (x = 0.030 m) can be determined:

$$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} \quad \rightarrow \quad h_x = \frac{k}{x}\frac{0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}}$$

$$h_x = \frac{(0.02735\ \mathrm{W/m\cdot K})}{(0.030\ \mathrm{m})}\frac{0.453(4.171\times10^4)^{0.5}(0.7228)^{1/3}}{[1-(15/30)^{3/4}]^{1/3}} = 102.3\ \mathrm{W/m^2\cdot K}$$

Then the surface temperature at the trailing edge of the chip is

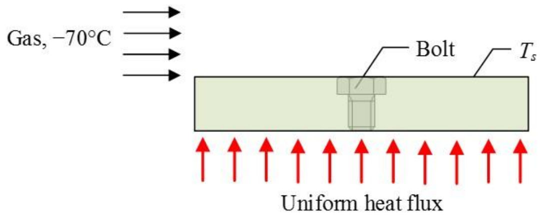
$$\dot{Q}/A = h(T_s - T_\infty) \quad \rightarrow \quad T_s = \frac{\dot{Q}/A}{h} + T_\infty = \frac{(1.4\ \mathrm{W})/(0.015\ \mathrm{m})^2}{102.3\ \mathrm{W/m^2\cdot K}} + 20°\mathrm{C} = \mathbf{80.8°C}$$

Discussion The assumed temperature of 50°C for evaluating the air properties turned out to be a good estimation, since T_f = (80.8°C + 20°C)/2 = 50.4°C.

7-59 C&S An ASTM A437 B4B stainless steel bolt is embedded at mid-length of a 1-m long plate. The minimum suitable temperature for the bolt is −30°C. Cold gas flows in parallel over the plate's upper surface. The plate is subjected to a uniform heat flux of 250 W/m^2. The surface temperature of the plate at the location where the bolt is embedded is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Uniform heat flux subjected to plate. **3** Edge effects of plate are negligible. **4** The critical Reynolds number is $\mathrm{Re}_{cr} = 5 \times 10^5$.

Properties The properties of the gas are given as Pr = 0.7440, k = 0.01979 W/m·K, and $\nu = 9.319 \times 10^{-6}$ m^2/s



Analysis The Reynolds number at the end of the plate is

$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(4\ \mathrm{m/s})(1\ \mathrm{m})}{9.319 \times 10^{-6}\mathrm{m^2/s}} = 4.2923 \times 10^5 < 5 \times 10^5$$

Thus, we have laminar flow over the entire plate. The local heat transfer convection coefficient at mid-length (x = 0.5 m) is determined using the laminar flow relation for a flat plate subjected to uniform heat flux,

$$h_x = \left(\frac{k}{x}\right) 0.453\ \mathrm{Re}_x^{0.5}\mathrm{Pr}^{1/3} = \left(\frac{0.01979\ \mathrm{W/m\cdot K}}{0.5\ \mathrm{m}}\right)(0.453)(2.1462 \times 10^5)^{0.5}(0.7440)^{1/3} = 7.53\ \mathrm{W/m^2{\cdot}K}$$

where

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(4\ \mathrm{m/s})(0.5\ \mathrm{m})}{9.319 \times 10^{-6}\mathrm{m^2/s}} = 2.1462 \times 10^5$$

From the Newton's law of cooling, the surface temperature at the location where the bolt is embedded (x = 0.5 m) is

$$\dot{q}_x = h_x(T_s - T_\infty) \quad \Rightarrow \quad T_s = \frac{\dot{q}_x}{h_x} + T_\infty = \frac{250\ \mathrm{W/m^2}}{7.53\ \mathrm{W/m^2{\cdot}K}} - 70°\mathrm{C} = \mathbf{-36.8}°\mathrm{C} < -30°\mathrm{C}$$

Discussion The heat flux subjected to the plate is not sufficient to keep the bolt from getting below the minimum suitable temperature of −30°C. To keep the bolt above −30°C, the heat flux to the plate needs to be above 301 W/m^2.

7-60 Air flow in parallel over the upper surface of a flat plate while the lower surface is subjected to uniform heat flux. The surface temperature at x = 1.5 m is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Local atmospheric pressure is 1 atm. **3** The critical Reynolds number is $Re_{cr} = 5\times10^5$.

Properties The properties of air are obtained from Table A-15.

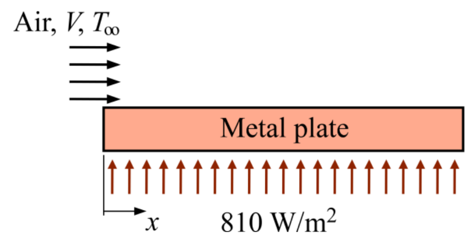
Analysis Since the surface temperature T_s is not known, it is necessary to guess a value for T_s so that the properties of air can be evaluated using the film temperature T_f. To begin, we guess T_s = 45°C, so

$$T_f = (T_s + T_\infty)/2 = (45+15)°\text{C}/2 = 30°\text{C}$$

From Table A-15, the properties of air are k = 0.02588 W/m·K, $\nu = 1.608 \times 10^{-5}$ m²/s, and Pr = 0.7282

$$\text{Re}_x = \frac{Vx}{\nu} = \frac{(2.5\text{ m/s})(1.5\text{m})}{1.608\times10^{-5}\text{ m}^2/\text{s}} = 233209 \text{ (flow is laminar at } x = 1.5\text{ m)}$$

For uniform heat flux on a flat plate with laminar flow, we have



$$\text{Nu}_x = \frac{h_x x}{k} = 0.453\text{Re}_x^{0.5}\text{Pr}^{1/3} = 0.453(233209)^{0.5}(0.7282)^{1/3} = 196.8$$

$$h_x = \text{Nu}_x\frac{k}{x} = 196.8\left(\frac{0.02588\text{ W/m}\cdot\text{K}}{1.5\text{ m}}\right) = 3.395\text{ W/m}^2\cdot\text{K}$$

Thus,

$$T_s = T_\infty + \frac{\dot{q}_s}{h_x} = 15°\text{C} + \frac{810\text{ W/m}^2}{3.395\text{ W/m}^2\cdot\text{K}} = 254°\text{C}$$

Next, repeat the above calculations iteratively with T_s = 254°C until the value of T_s converges. The results of the iterations are

Iter	T_s [°C]	T_f [°C]	k [W/m²·K]	ν [m²/s]	Pr	Re_x	Nu_x	h_x [W/m²·K]
1	45	30	0.02588	1.608×10^{-5}	0.7282	233209	197	3.395
2	254	135	0.03336	2.683×10^{-5}	0.7062	139768	151	3.354
3	257	136	0.03347	2.700×10^{-5}	0.7060	138894	150	3.354
4	**256.5**	135.8	0.03345	2.697×10^{-5}	0.7060	139039	150	3.354

Discussion After 4 iterations, with initial guess of T_s = 45°C, the final value of the flat plate surface temperature is $\boldsymbol{T_s}$ **= 257°C**.

7-61 Air flows in parallel over the upper surface of a flat plate while the lower surface is subjected to uniform heat flux.The local convection heat transfer coefficient, local surface temperature, and local film temperature along the plate are to be evaluated.

Assumptions **1** Steady operating conditions exist. **2** Local atmospheric pressure is 1 atm. **3** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$. **4** Flow is laminar (this assumption will be verified).

Analysis For laminar flow, the relation for local Nusselt number along a flat plate subjected to uniform heat flux is

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$$

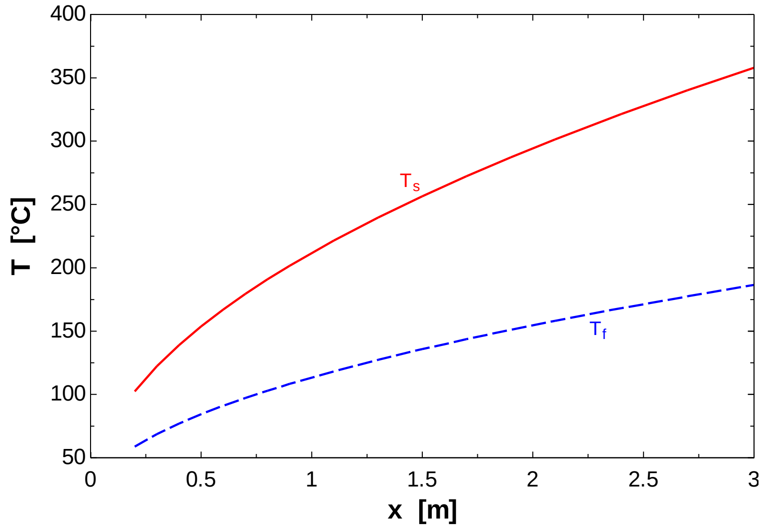
The local surface temperature and the local film temperature can be determined using

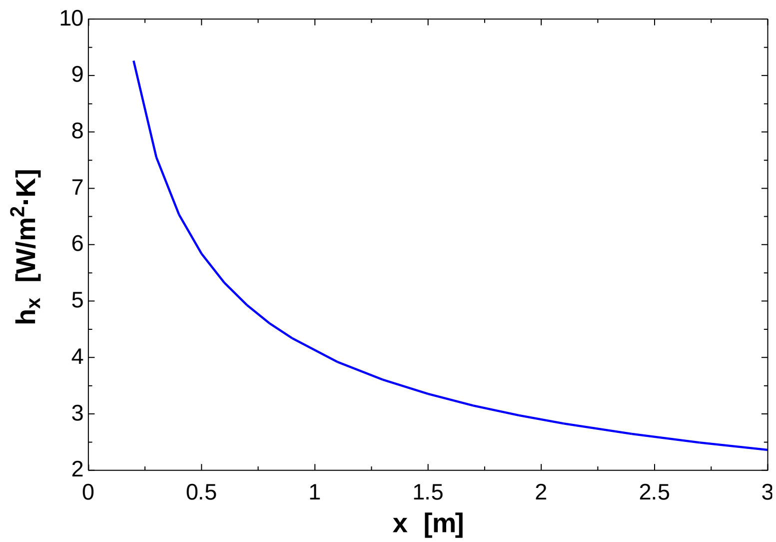
$$T_s = T_\infty + \dot{q}_s / h_x \qquad \text{and} \qquad T_f = (T_s + T_\infty)/2$$

The problem is solved using EES, and the solution is given below:

```
"GIVEN"
T_infinity=15 [C]
V=2.5 [m/s]
q_dot_s=810 [W/m^2]

"PROPERTIES"
Fluid$='air'
T_film=1/2*(T_s+T_infinity)
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho

"ANALYSIS"
Re_x=V*x/nu
Nusselt_x=0.453*Re_x^0.5*Pr^(1/3)      "Laminar flow Nusselt_x"
T_s=T_infinity+q_dot_s/h_x
h_x=Nusselt_x*k/x
```

x [m]	Re_x	T_s [°C]	T_f [°C]	h_x [W/m²·K]
0.2	26550	102.5	58.74	9.260
0.3	37847	122.3	68.63	7.552
0.4	48397	139.0	76.99	6.534
0.5	58358	153.7	84.36	5.839
0.6	67831	167.1	91.04	5.326
0.7	76889	179.4	97.18	4.928
0.8	85585	190.8	102.9	4.607
0.9	93962	201.6	108.3	4.341
1.1	109884	221.5	118.2	3.923
1.3	124860	239.7	127.3	3.605
1.5	139035	256.5	135.8	3.354
1.7	152521	272.3	143.6	3.148
1.9	165401	287.2	151.1	2.976
2.1	177745	301.3	158.2	2.829
2.4	195373	321.4	168.2	2.644
2.7	212068	340.2	177.6	2.491
3.0	227947	358	186.5	2.361



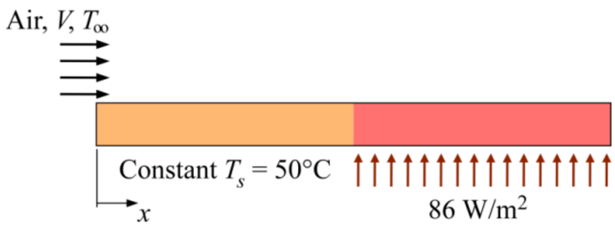


Discussion From the table above, for $0.2 \leq x \leq 3$ m, the local Reynolds numbersare less than the critical Reynolds number of 5×10^5. Thus, the flow is laminar over the entire plate.

7-62 Air flows in parallel over a flat plate where the first-half length has a constant surface temperature and the second-half length is subjected to uniform heat flux.The local convection heat transfer coefficients at x = 1 and 3 m are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Local atmospheric pressure is 1 atm. **3** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **4** The boundary layer over the second portion of the plate with uniform heat flux has not been affected by the first half of the plate with constant surface temperature.

Properties The properties of air at T_f =30°C are k = 0.02588 W/m·K, $\nu = 1.608 \times 10^{-5}$ m²/s, and Pr = 0.7282 (Table A-15).



Analysis The Reynolds numbers at x = 1 m and 3 m are

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(2\,\mathrm{m/s})(1\mathrm{m})}{1.608\times10^{-5}\ \mathrm{m^2/s}} = 124378 \text{ (flow is laminar at } x = 1 \text{ m)}$$

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(2\,\mathrm{m/s})(3\mathrm{m})}{1.608\times10^{-5}\ \mathrm{m^2/s}} = 373134 \text{ (flow is laminar at } x = 3 \text{ m)}$$

At x = 1 m (constant T_s), the relation for local Nusselt number is

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} = 0.332(124378)^{0.5}(0.7282)^{1/3} = 105.34$$

Thus,

$$h_x = \mathrm{Nu}_x \frac{k}{x} = 105.34\left(\frac{0.02588\ \mathrm{W/m\cdot K}}{1\,\mathrm{m}}\right) = \mathbf{2.73\ W/m^2\cdot K} \qquad \text{(for } x = 1 \text{ m)}$$

At x = 3 m (constant heat flux), the relation for local Nusselt number is (as a first approximation the constanrt heat flux equation can be used since we are assuming the boundary layer over the second portion of the plate with uniform heat flux has not been affected by the first half of the plate with constant surface temperature, this assumption is correct only if the entire plate was at uniform heat flux)

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} = 0.453(373134)^{0.5}(0.7282)^{1/3} = 248.95$$

Thus,

$$h_x = \mathrm{Nu}_x \frac{k}{x} = 248.95\left(\frac{0.02588\ \mathrm{W/m\cdot K}}{3\,\mathrm{m}}\right) = \mathbf{2.15\ W/m^2\cdot K} \qquad \text{(for } x = 3 \text{ m)}$$

Discussion The surface temperature at x = 3 m is

$$T_s = T_\infty + \frac{\dot{q}_s}{h_x} = 10°\mathrm{C} + \frac{86\ \mathrm{W/m^2}}{2.15\ \mathrm{W/m^2\cdot K}} = 50°\mathrm{C}$$

Thus, T_f = (50°C + 10°C)/2 = 30°C is applicable at x = 3 m.

7-63 Air flows in parallel over a flat plate where the first-half length has a constant surface temperature and the second-half length is subjected to uniform heat flux. The local convection heat transfer coefficient, local surface temperature, and local film temperature along the plate are to be evaluated.

Assumptions **1** Steady operating conditions exist. **2** Local atmospheric pressure is 1 atm. **3** The critical Reynolds number is $\text{Re}_{cr} = 5\times10^5$. **4** Flow is laminar (this assumption will be verified).

Analysis For constant T_s, the laminar flow relation for local Nusselt number along a flat plate is

$$\text{Nu}_x = \frac{h_x x}{k} = 0.332\,\text{Re}_x^{0.5}\,\text{Pr}^{1/3}$$

For uniform heat flux, the laminar flow relation for local Nusselt number along a flat plate is

$$\text{Nu}_x = \frac{h_x x}{k} = 0.453\,\text{Re}_x^{0.5}\,\text{Pr}^{1/3}$$

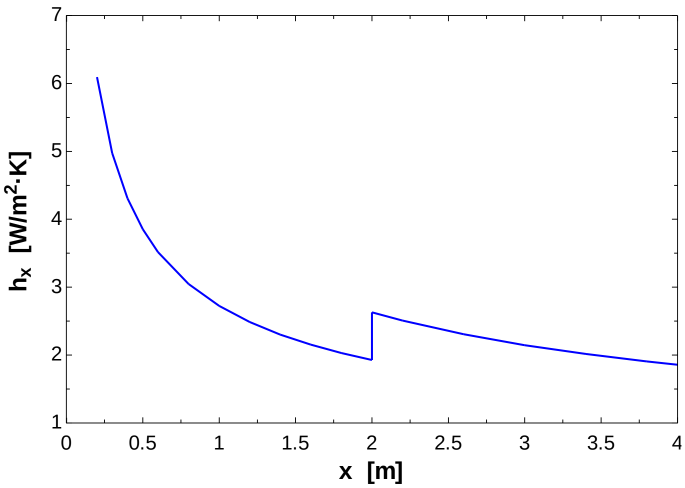
The local surface temperature and the local film temperature for uniform heat flux can be determined using

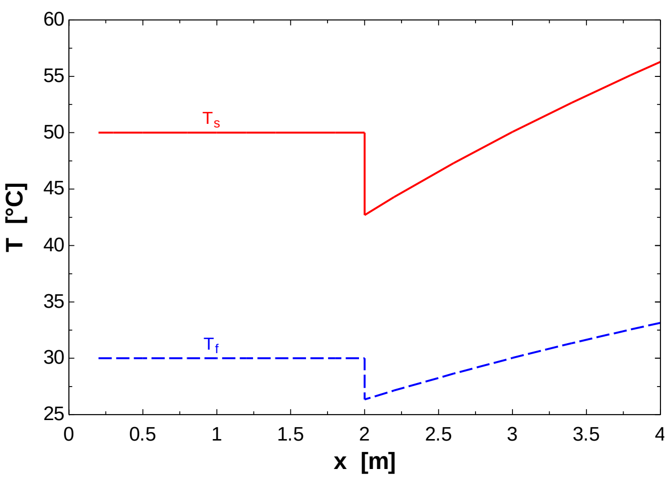
$$T_s = T_\infty + \dot{q}_s / h_x \quad \text{and} \quad T_f = (T_s + T_\infty)/2$$

The problem is solved using EES, and the solution is given below:

```
"GIVEN"
T_infinity=10 [C]
V=2 [m/s]
q_dot_s=86 [W/m^2]

"PROPERTIES"
Fluid$='air'
T_film=1/2*(T_s+T_infinity)
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho

"ANALYSIS"
Re_x=V*x/nu
Nusselt_T_s=0.332*Re_x^0.5*Pr^(1/3)    "Laminar flow Nusselt_x, constant T_s"
Nusselt_q_dot=0.453*Re_x^0.5*Pr^(1/3)    "Laminar flow Nusselt_x, constant q_dot"
Nusselt_x=if(x,2,Nusselt_T_s,Nusselt_q_dot,Nusselt_q_dot)
T_s_x1=50 [C]    "T_s for first-half plate length"
T_s_x2=T_infinity+q_dot_s/h_x    "T_s for second-half plate length"
T_s=if(x,2,T_s_x1,T_s_x2,T_s_x2)
h_x=Nusselt_x*k/x
```

x [m]	Re_x	T_s [°C]	T_f [°C]	h_x [W/m^2·K]
0.2	24875	50	30	6.092
0.3	37313	50	30	4.974
0.4	49751	50	30	4.308
0.5	62188	50	30	3.853
0.6	74626	50	30	3.517
0.8	99501	50	30	3.046
1.0	124377	50	30	2.725
1.2	149252	50	30	2.487
1.4	174128	50	30	2.303
1.6	199003	50	30	2.154
1.8	223878	50	30	2.031
2.0	248754	50	30	1.927
2.0	254093	42.70	26.35	2.630
2.2	278198	44.30	27.15	2.507
2.6	325924	47.29	28.65	2.306
3.0	373058	50.07	30.03	2.146
3.4	419648	52.66	31.33	2.016
3.8	465736	55.11	32.56	1.906
4.0	488603	56.29	33.14	1.858



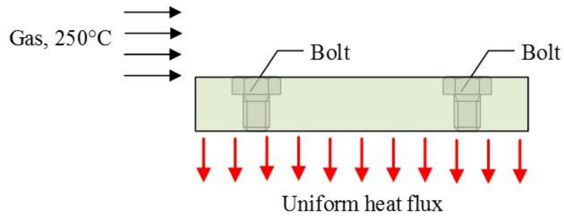


Discussion From the above table, for $0.2 \leq x \leq 4$ m, the local Reynolds numbersare less than the critical Reynolds number of 5×10^5. Thus, the flow is laminar over the entire plate.

7-64 Two ASTM B98 copper-silicon bolts are embedded at 0.5 m and 1.5 m from the plate's leading edge. The maximum use temperature for the bolt is 149°C. The hot gas flows in parallel over the plate's upper surface. Heat is being removed from the plate uniformly at 2000 W/m^2. The surface temperatures of the plate at the locations where the bolts are embedded are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Uniform heat flux subjected to the plate. **3** Edge effects of plate are negligible. **4** The critical Reynolds number is $\text{Re}_{cr} = 5 \times 10^5$.

Properties The properties of the gas are given as Pr = 0.6974, k = 0.03779 W/m·K, and$\nu = 3.455 \times 10^{-5}$ m^2/s



Analysis The location where the critical Reynolds number occurs is

$$x_{cr} = \text{Re}_{\text{cr}}\frac{\nu}{V} = 5 \times 10^5\left(\frac{3.455 \times 10^{-5}\text{m}^2/\text{s}}{17\ \text{m/s}}\right) = 1.016\ \text{m}$$

At the location of the first bolt (x_1 = 0.5 m), the flow is laminar. The local heat transfer convection coefficient is determined using the laminar flow relation for a flat plate subjected to uniform heat flux,

$$h_{x_1} = \left(\frac{k}{x_1}\right)0.453\ \text{Re}_{x_1}^{0.5}\text{Pr}^{1/3} = \left(\frac{0.03779\ \text{W/m}\cdot\text{K}}{0.5\ \text{m}}\right)(0.453)(2.4602 \times 10^5)^{0.5}(0.6974)^{1/3} = 15.06\ \text{W/m}^2\cdot\text{K}$$

where

$$\text{Re}_{x_1} = \frac{Vx_1}{\nu} = \frac{(17\ \text{m/s})(0.5\ \text{m})}{3.455 \times 10^{-5}\text{m}^2/\text{s}} = 2.4602 \times 10^5 < 5 \times 10^5$$

From the Newton's law of cooling, the surface temperature at the location where the first bolt is embedded (x_1 = 0.5 m) is

$$\dot{q}_x = h_{x_1}(T_\infty - T_{s,1}) \quad \Rightarrow \quad T_{s,1} = -\frac{\dot{q}_x}{h_{x_1}} + T_\infty = -\frac{2000\ \text{W/m}^2}{15.06\ \text{W/m}^2\cdot\text{K}} + 250°\text{C} = \mathbf{117.2}°\text{C} < 149°\text{C}$$

At the location of the second bolt (x_2 = 1.5 m), the flow is turbulent. The local heat transfer convection coefficient is determined using the turbulent flow relation for a flat plate subjected to uniform heat flux,

$$h_{x_2} = \left(\frac{k}{x_2}\right)0.0308\ \text{Re}_{x_2}^{0.8}\text{Pr}^{1/3} = \left(\frac{0.03779\ \text{W/m}\cdot\text{K}}{1.5\ \text{m}}\right)(0.0308)(7.3806 \times 10^5)^{0.8}(0.6974)^{1/3} = 34.05\ \text{W/m}^2\cdot\text{K}$$

where

$$\text{Re}_{x_2} = \frac{Vx_2}{\nu} = \frac{(17\ \text{m/s})(1.5\ \text{m})}{3.455 \times 10^{-5}\text{m}^2/\text{s}} = 7.3806 \times 10^5 > 5 \times 10^5$$

From the Newton's law of cooling, the surface temperature at the location where the second bolt is embedded (x_2 = 1.5 m) is

$$\dot{q}_x = h_{x_2}(T_\infty - T_{s,2}) \quad \Rightarrow \quad T_{s,2} = -\frac{\dot{q}_x}{h_{x_2}} + T_\infty = -\frac{2000\ \text{W/m}^2}{34.05\ \text{W/m}^2\cdot\text{K}} + 250°\text{C} = \mathbf{191.3}°\text{C} > 149°\text{C}$$

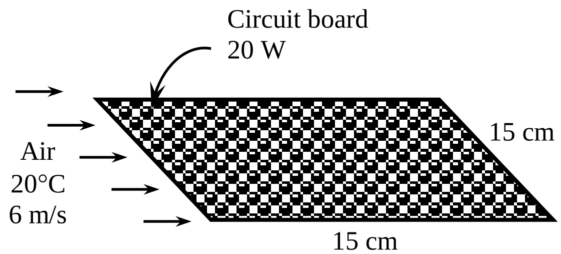
Discussion The heat being removed from the plate uniformly at 2000 W/m^2 is sufficient to keep the first bolt at x_1 = 0.5 m below the maximum use temperature of 149°C. However, it is insufficient to keep the second bolt at x_2 = 1.5 m below 149°C. To keep both bolts from heating above the maximum use temperature, the cooling device needs to remove heat from the plate at 3440 W/m^2 or higher.

7-65 A circuit board is cooled by air. The surface temperatures of the electronic components at the leading edge and the end of the board are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Any heat transfer from the back surface of the board is disregarded. **5** Air is an ideal gas with constant properties.

Properties We assume a film temperature of 35°C based on the problem statement, the properties of air are evaluated at this temperature to be (Table A-15)

$$k = 0.0265 \text{ W/m.°C}$$
$$\nu = 1.655\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7268$$



Analysis (*a*) The convection heat transfer coefficient at the leading edge approaches infinity, and thus the surface temperature there must approach the air temperature, which is 20°C.

(*b*) The Reynolds number is

$$\text{Re}_x = \frac{Vx}{\nu} = \frac{(6 \text{ m/s})(0.15\text{m})}{1.655\times10^{-5} \text{ m}^2\text{/s}} = 5.438\times10^4$$

which is less than the critical Reynolds number but we assume the flow to be turbulent since the electronic components are expected to act as turbulators. Using the Nusselt number uniform heat flux, the local heat transfer coefficient at the end of the board is determined to be

$$Nu_x = \frac{h_x x}{k} = 0.0308 \,\text{Re}_x^{0.8} \Pr^{1/3} = 0.0308(5.438\times10^4)^{0.8}(0.7268)^{1/3} = 170.1$$
$$h_x = \frac{k_x}{x} Nu_x = \frac{0.02625 \text{ W/m.°C}}{0.15 \text{ m}}(170.1) = 29.77 \text{ W/m}^2\text{.°C}$$

Then the surface temperature at the end of the board becomes

$$\dot{q} = h_x(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{q}}{h_x} = 20\text{°C} + \frac{(20 \text{ W})/(0.15\text{m})^2}{29.77 \text{ W/m}^2\text{.°C}} = \mathbf{49.9°C}$$

Now, the film temperature can be determined to be $T_f = (T_s + T_\infty)/2 = (49.9 + 20)/2 = 35$°C. This is the same as the assumed film temperature verifying the validity of this assumption.

Discussion The heat flux can also be determined approximately using the relation for isothermal surfaces,

$$Nu_x = \frac{h_x x}{k} = 0.0296 \,\text{Re}_x^{0.8} \Pr^{1/3} = 0.0296(5.438\times10^4)^{0.8}(0.7268)^{1/3} = 163.5$$
$$h_x = \frac{k_x}{x} Nu_x = \frac{0.02625 \text{ W/m.°C}}{0.15 \text{ m}}(163.5) = 28.61 \text{ W/m}^2\text{.°C}$$

Then the surface temperature at the end of the board becomes

$$\dot{q} = h_x(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{q}}{h_x} = 20\text{°C} + \frac{(20 \text{ W})/(0.15\text{m})^2}{28.61 \text{ W/m}^2\text{.°C}} = \mathbf{51.1°C}$$

Note that the two results are close to each other.

Flow across Cylinders and Spheres

7-66C Friction drag is due to the shear stress at the surface whereas the pressure drag is due to the pressure differential between the front and back sides of the body when a wake is formed in the rear.

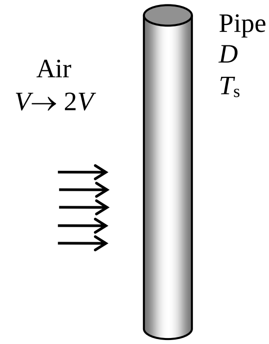
7-67C Turbulence moves the fluid separation point further back on the rear of the body, reducing the size of the wake, and thus the magnitude of the pressure drag (which is the dominant mode of drag). As a result, the drag coefficient suddenly drops. In general, turbulence increases the drag coefficient for flat surfaces, but the drag coefficient usually remains constant at high Reynolds numbers when the flow is turbulent.

7-68C Flow separation in flow over a cylinder is delayed in turbulent flow because of the extra mixing due to random fluctuations and the transverse motion.

7-69C For the laminar flow, the heat transfer coefficient will be the highest at the stagnation point which corresponds to $\theta \approx 0°$. In turbulent flow, on the other hand, it will be highest when θ is between $90°$ and $120°$.

7-70 The flow of a fluid across an isothermal cylinder is considered. The change in the drag force and the rate of heat transfer when the free-stream velocity of the fluid is doubled is to be determined.

Analysis The drag force on a cylinder is given by



$$F_{D1} = C_D A_N \frac{\rho V^2}{2}$$

When the free-stream velocity of the fluid is doubled, the drag force becomes

$$F_{D2} = C_D A_N \frac{\rho (2V)^2}{2}$$

Taking the ratio of them yields

$$\frac{F_{D2}}{F_{D1}} = \frac{(2V)^2}{V^2} = \mathbf{4}$$

The rate of heat transfer between the fluid and the cylinder is given by Newton's law of cooling. We assume the Nusselt number is proportional to the nth power of the Reynolds number with 0.33 $<n<$ 0.805. Then,

$$\dot{Q}_1 = hA_s(T_s - T_\infty) = \left(\frac{k}{D} Nu\right) A_s(T_s - T_\infty) = \frac{k}{D}(\mathrm{Re})^n A_s(T_s - T_\infty)$$

$$= \frac{k}{D}\left(\frac{VD}{\nu}\right)^n A_s(T_s - T_\infty)$$

$$= V^n \frac{k}{D}\left(\frac{D}{\nu}\right)^n A_s(T_s - T_\infty)$$

When the free-stream velocity of the fluid is doubled, the heat transfer rate becomes

$$\dot{Q}_2 = (2V)^n \frac{k}{D}\left(\frac{D}{\nu}\right)^n A(T_s - T_\infty)$$

Taking the ratio of them yields

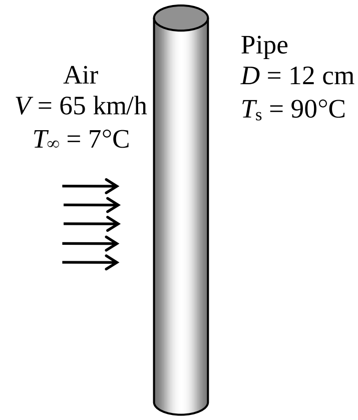
$$\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{(2V)^n}{V^n} = \mathbf{2^n}$$

7-71 A steam pipe is exposed to windy air. The rate of heat loss from the steam is to be determined.√

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (90+7)/2 = 48.5°\text{C}$ are (Table A-15)

$$k = 0.02724 \text{ W/m.°C}$$
$$\nu = 1.784\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7232$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{[(65 \text{ km/h})(1000 \text{ m/km})/(3600 \text{ s/h})](0.12 \text{ m})}{1.784\times10^{-5} \text{ m}^2\text{/s}} = 1.214\times10^5$$

The Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(1.214\times10^5)^{0.5}(0.7232)^{1/3}}{\left[1+(0.4/0.7232)^{2/3}\right]^{1/4}}\left[1+\left(\frac{1.214\times10^5}{282{,}000}\right)^{5/8}\right]^{4/5} = 247.5$$

The heat transfer coefficient and the heat transfer rate become

$$h = \frac{k}{D}Nu = \frac{0.02724 \text{ W/m.°C}}{0.12 \text{ m}}(247.5) = 56.18 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.12 \text{ m})(1 \text{m}) = 0.3770 \text{ m}^2$$

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = (56.18 \text{ W/m}^2\text{.°C})(0.3770 \text{ m}^2)(90-7)°\text{C} = \mathbf{1758\ W} \text{ (per m length)}$$

7-72 An ASTM B98 copper-silicon bolt connects two metal plates. Hot air flows across the cylindrical bolt. A cooling mechanism removes heat at a rate of 30 W from the bolt. The surface temperature of the bolt is to be determined to see if it exceeds the maximum use temperature of 149°C.

Assumptions **1** The flow is steady and incompressible. **2** Uniform surface temperature. **3** Wall effects from the plates on the bolt are negligible.

Properties Using the maximum use temperature for the bolt as the surface temperature, the properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (149 + 250)$°C/2 ≈ 200°C are (Table A–15): Pr = 0.6974, k = 0.03779 W/m·K, and $\nu = 3.455 \times 10^{-5}$ m^2/s

Analysis The Reynolds number is

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(17\ \mathrm{m/s})(0.0095\ \mathrm{m})}{3.455 \times 10^{-5}\mathrm{m^2/s}} = 4674$$

Using the Churchill and Bernstein correlation for cross flow over a cylinder,

$$h = \left(\frac{k}{D}\right)\left\{0.3 + \frac{0.62\ \mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282000}\right)^{5/8}\right]^{4/5}\right\}$$

$$h = \left(\frac{0.03779\ \mathrm{W/m\cdot K}}{0.0095\ \mathrm{m}}\right)\left\{0.3 + \frac{0.62\ (4674)^{1/2}(0.6974)^{1/3}}{[1 + (0.4/0.6974)^{2/3}]^{1/4}}\left[1 + \left(\frac{4674}{282000}\right)^{5/8}\right]^{4/5}\right\} = 140.4\ \mathrm{W/m^2\cdot K}$$

From the Newton's law of cooling, the surface temperature of the bolt is

$$\dot{Q} = hA(T_\infty - T_s) \quad \Rightarrow \quad T_s = -\frac{\dot{Q}}{h(\pi DL)} + T_\infty = -\frac{30\ \mathrm{W}}{(140.4\ \mathrm{W/m^2\cdot K})\pi(0.0095\ \mathrm{m})(0.10\ \mathrm{m})} + 250℃ = \mathbf{178.4}℃ > 149℃$$

Discussion The heat being removed from the bolt at 30 W is insufficient to keep the bolt below the maximum use temperature of 149°C. To keep the bolt from heating above the maximum use temperature, the cooling mechanism needs to remove heat from the bolt at a rate higher than 42 W.

7-73 A heated long cylindrical rod is placed in a cross flow of air. The rod surface has an emissivity of 0.95 and its surface temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The surface temperature is constant. **4** Heat flux dissipated from the rod is uniform.

Properties The properties of air (1 atm) at 70°C are given in Table A-15: k = 0.02881 W/m·K, $\nu = 1.995 \times 10^{-5}$ m^2/s, and Pr = 0.7177.

Analysis The Reynolds number for the air flowing across the rod is

$$\mathrm{Re}_D = \frac{VD}{\nu} = \frac{(10\ \mathrm{m/s})(0.005\ \mathrm{m})}{1.995\times10^{-5}\ \mathrm{m^2/s}} = 2506$$

Using the Churchill and Bernstein relation for Nusselt number, the convection heat transfer coefficient is

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1+(0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282000}\right)^{5/8}\right]^{4/5}$$

$$h = \frac{0.02881\ \mathrm{W/m\cdot K}}{0.005\ \mathrm{m}}\left\{0.3 + \frac{0.62(2506)^{1/2}(0.7177)^{1/3}}{[1+(0.4/0.7177)^{2/3}]^{1/4}}\left[1+\left(\frac{2506}{282000}\right)^{5/8}\right]^{4/5}\right\}$$

$$= 148.3\ \mathrm{W/m^2\cdot K}$$

From energy balance, we obtain

$$16000\ \mathrm{W/m^2} = \dot{q}_{\mathrm{conv}} + \dot{q}_{\mathrm{rad}} \quad \rightarrow \quad 16000\ \mathrm{W/m^2} = h(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_{\mathrm{surr}}^4)$$

Copy the following line and paste on a blank EES screen to solve the above equation:

```
h=148.3
epsilon=0.95
sigma=5.670e-8
T_inf=20+273
T_surr=20+273
16000=h*(T_s-T_inf)+epsilon*sigma*(T_s^4-T_surr^4)
```

Solving by EES software, the surface temperature of the rod is $T_s = 395\ \mathrm{K} = \mathbf{122°C}$

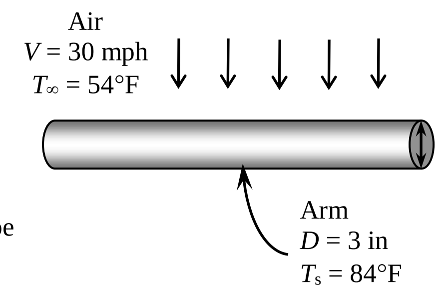
Discussion Note that absolute temperatures must be used in calculations involving the radiation heat transfer equation.

7-74E A person extends his uncovered arms into the windy air outside. The rate of heat loss from the arm is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. 4 The arm is treated as a 2-ft-long and 3-in-diameter cylinder with insulated ends. **5** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (84+54)/2 = 69°F$ are (Table A-15E)

$$k = 0.01455 \text{ Btu/h.ft.°F}$$
$$\nu = 0.1638\times10^{-3} \text{ ft}^2/\text{s}$$
$$\text{Pr} = 0.7308$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{[(30\times5280/3600)\text{ ft/s}](3/12)\text{ ft}}{0.1638\times10^{-3}\text{ ft}^2/\text{s}} = 6.716\times10^4$$

The Nusselt number corresponding to this Reynolds number is determined to be

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+\left(\frac{0.4}{\text{Pr}}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(6.716\times10^4)^{0.5}(0.7308)^{1/3}}{\left[1+\left(\frac{0.4}{0.7308}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{6.716\times10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 167.7$$

Then the heat transfer coefficient and the heat transfer rate from the arm becomes

$$h = \frac{k}{D}Nu = \frac{0.01455\text{ Btu/h.ft.°F}}{(3/12)\text{ ft}}(167.7) = 9.760\text{ Btu/h.ft}^2\text{.°F}$$

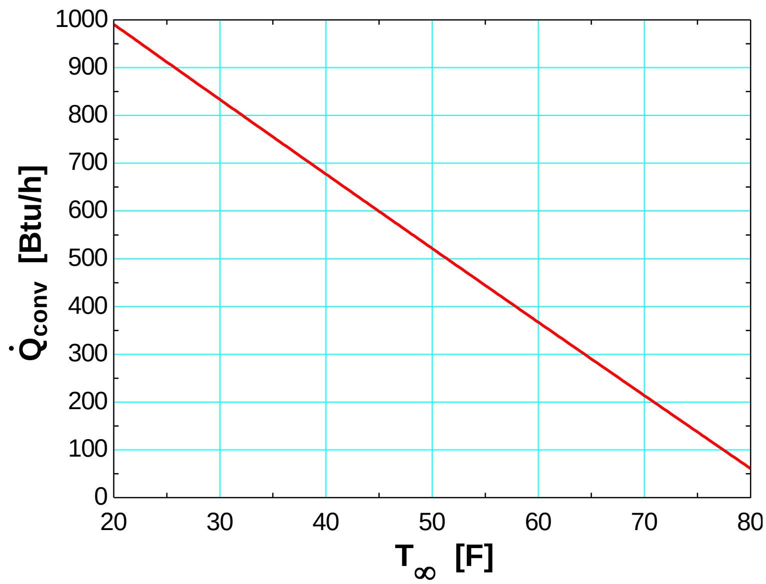
$$A_s = \pi DL = \pi(3/12\text{ ft})(2\text{ ft}) = 1.571\text{ft}^2$$
$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = (9.760\text{ Btu/h.ft}^2\text{.°F})(1.571\text{ft}^2)(84-54)\text{°F} = \mathbf{460\ Btu/h}$$

7-75E Prob. 7-74E is reconsidered. The effects of air temperature and wind velocity on the rate of heat loss from the arm are to be investigated.

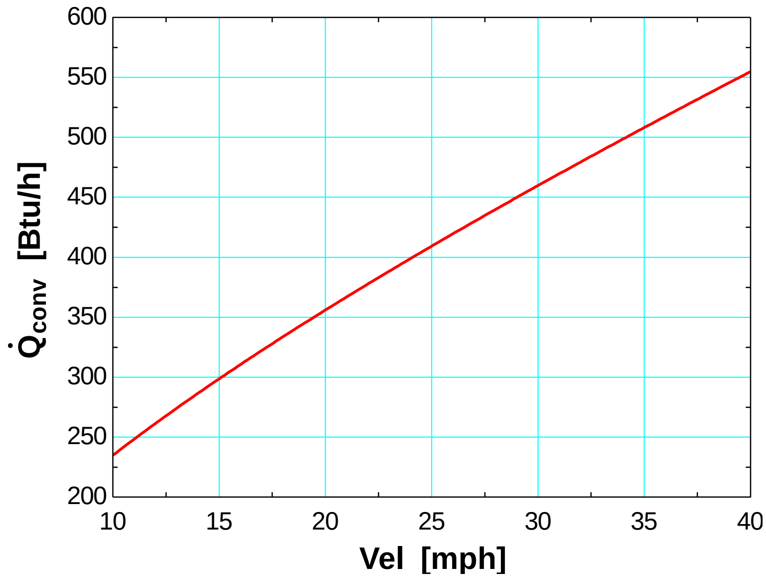
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_infinity=54 [F]
Vel=30 [mph]
T_s=84 [F]
L=2 [ft]
D=(3/12) [ft]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=14.7)
mu=Viscosity(Fluid$, T=T_film)*Convert(lbm/ft-h, lbm/ft-s)
nu=mu/rho
T_film=1/2*(T_s+T_infinity)

"ANALYSIS"
Re=(Vel*Convert(mph, ft/s)*D)/nu
Nusselt=0.3+(0.62*Re^0.5*Pr^(1/3))/(1+(0.4/Pr)^(2/3))^0.25*(1+(Re/282000)^(5/8))^(4/5)
h=k/D*Nusselt
A=pi*D*L
Q_dot_conv=h*A*(T_s-T_infinity)
```

T_∞ [F]	Q_{conv} [Btu/h]
20	990.6
25	911.9
30	833.3
35	755.1
40	677
45	599.2
50	521.6
55	444.3
60	367.2
65	290.3
70	213.6
75	137.1
80	60.86



Vel [mph]	Q_{conv} [Btu/h]
10	234.8
12	261.3
14	286.5
16	310.5
18	333.6
20	356
22	377.7
24	398.9
26	419.6
28	439.8
30	459.7
32	479.3
34	498.6
36	517.6
38	536.3
40	554.8

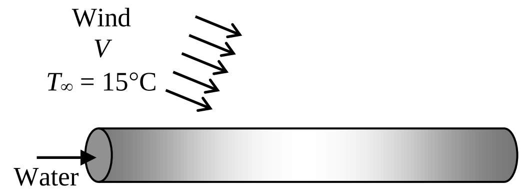


7-76 The wind is blowing across a geothermal water pipe. The average wind velocity is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties The specific heat of water at the average temperature of 75ºC is 4193 J/kg.ºC. The properties of air at the film temperature of (75+15)/2=45ºC are (Table A-15)

$$k = 0.02699 \text{ W/m.°C}$$
$$\nu = 1.75\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7241$$



Analysis The rate of heat transfer from the pipe is the energy change of the water from inlet to exit of the pipe, and it can be determined from

$$\dot{Q} = \dot{m}c_p\Delta T = (8.5\text{ kg/s})(4193\text{J/kg.°C})(80-70)\text{°C} = 356{,}400\text{ W}$$

The surface area and the heat transfer coefficient are

$$A = \pi DL = \pi(0.15\text{ m})(400\text{ m}) = 188.5\text{ m}^2$$

$$\dot{Q} = hA(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A(T_s - T_\infty)} = \frac{356{,}400\text{ W}}{(188.5\text{ m}^2)(75-15)\text{°C}} = 31.51\text{ W/m}^2\text{.°C}$$

The Nusselt number is

$$Nu = \frac{hD}{k} = \frac{(31.51\text{ W/m}^2\text{.°C})(0.15\text{ m})}{0.02699\text{ W/m.°C}} = 175.1$$

The Reynolds number may be obtained from the Nusselt number relation by trial-error or using an equation solver such as EES:

$$Nu = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\Pr^{1/3}}{\left[1+(0.4/\Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$175.1 = 0.3 + \frac{0.62\,\text{Re}^{0.5}(0.7241)^{1/3}}{\left[1+(0.4/0.7241)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5} \longrightarrow \text{Re} = 71{,}900$$

The average wind velocity can be determined from Reynolds number relation

$$\text{Re} = \frac{VD}{\nu} \longrightarrow 71{,}900 = \frac{V(0.15\text{ m})}{1.75\times10^{-5}\text{ m}^2\text{/s}} \longrightarrow V = 8.39\text{ m/s} = \mathbf{30.2\ km/h}$$

7-77 C&S An ASTM A437 B4B stainless steel bolt connects two metal plates. Cold gas flows across the cylindrical bolt. A heating mechanism heats the bolt at a rate of 15 W. The maximum velocity that the gas can achieve without cooling the bolt below −30°C is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Uniform surface temperature. **3** Wall effects from the plates on the bolt are negligible.

Properties The properties of the gas are given as Pr = 0.74, k = 0.020 W/m·K, and $\nu = 9.3 \times 10^{-6}$ m^2/s

Analysis The Reynolds number is given as

$$\mathrm{Re} = \frac{VD}{\nu}$$

Using the Churchill and Bernstein correlation,

$$h = \left(\frac{k}{D}\right)\left\{0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}}{[1+(0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282000}\right)^{5/8}\right]^{4/5}\right\}$$

we have

$$h = \left(\frac{k}{D}\right)\left\{0.3 + \frac{0.62\,(VD/\nu)^{1/2}\mathrm{Pr}^{1/3}}{[1+(0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1+\left(\frac{VD/\nu}{282000}\right)^{5/8}\right]^{4/5}\right\}$$

With the Newton's law of cooling, we have

$$\dot{Q} = h(\pi DL)(T_s - T_\infty)$$

Hence,

$$\dot{Q} = \left(\frac{k}{D}\right)\left\{0.3 + \frac{0.62\,(VD/\nu)^{1/2}\mathrm{Pr}^{1/3}}{[1+(0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1+\left(\frac{VD/\nu}{282000}\right)^{5/8}\right]^{4/5}\right\}(\pi DL)(T_s - T_\infty)$$

where

D = 0.0095 m, L = 0.10 m, $\dot{Q}$ = 15 W, T_s = −30°C, T_∞ = −70°C,
Pr = 0.74, k = 0.020 W/m·K, and $\nu = 9.3 \times 10^{-6}$ m^2/s.

The unknown is the velocity V, which can be solved implicitly by trial-and-error. Solving for V yields

$$V = \mathbf{11.52\ m/s}$$

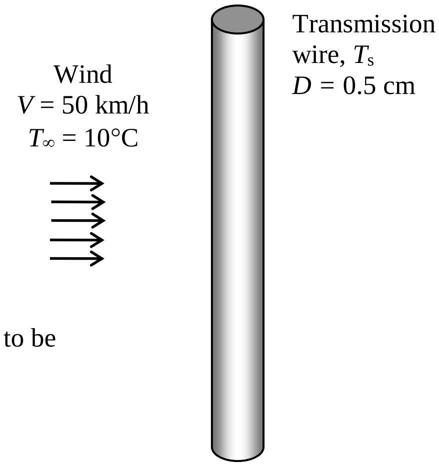
Discussion The maximum velocity that the gas can achieve without cooling the stainless steel bolt below the minimum suitable temperature of −30°C is 11.52 m/s. If the cold gas velocity goes above 11.52 m/s, the convection heat transfer coefficient would increase. This renders the heating rate of 15 W insufficient to keep the surface temperature of the bolt above the minimum suitable temperature.

7-78 The wind is blowing across the wire of a transmission line. The surface temperature of the wire is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties We assume the film temperature to be 10°C. The properties of air at this temperature are (Table A-15)

$$\rho = 1.246\ \text{kg/m}^3$$
$$k = 0.02439\ \text{W/m.°C}$$
$$\nu = 1.426\times10^{-5}\ \text{m}^2\text{/s}$$
$$\text{Pr} = 0.7336$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{[(50\times1000/3600)\ \text{m/s}](0.005\ \text{m})}{1.426\times10^{-5}\ \text{m}^2\text{/s}} = 4870$$

The Nusselt number corresponding to this Reynolds number is determined to be

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(4870)^{0.5}(0.7336)^{1/3}}{\left[1+(0.4/0.7336)^{2/3}\right]^{1/4}}\left[1+\left(\frac{4870}{282{,}000}\right)^{5/8}\right]^{4/5} = 36.80$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02439\ \text{W/m.°C}}{0.005\ \text{m}}(36.80) = 179.5\ \text{W/m}^2\text{.°C}$$

The rate of heat generated in the electrical transmission lines per meter length is

$$\dot{W} = \dot{Q} = I^2R = (50\ \text{A})^2(0.002\ \text{Ohm}) = 5.0\ \text{W}$$

The entire heat generated in electrical transmission line has to be transferred to the ambient air. The surface temperature of the wire then becomes

$$A_s = \pi DL = \pi(0.005\ \text{m})(1\ \text{m}) = 0.01571\ \text{m}^2$$

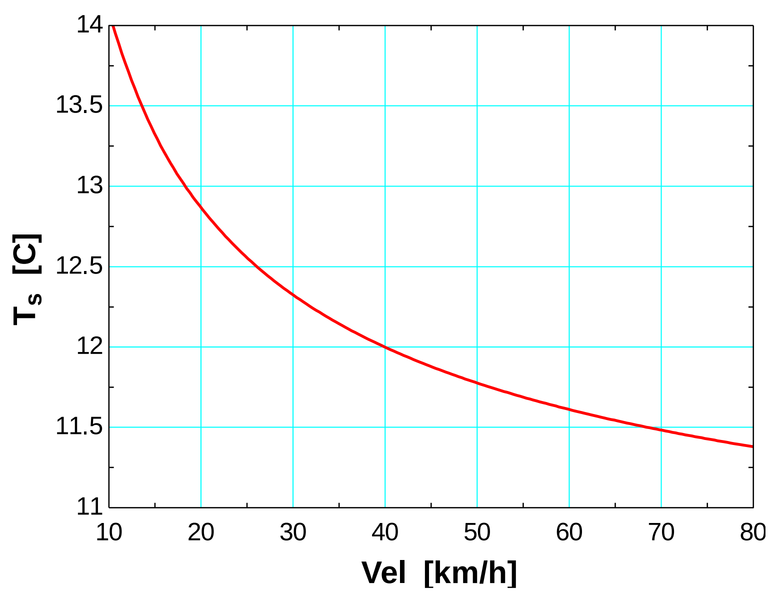
$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 10°\text{C} + \frac{5\ \text{W}}{(179.5\ \text{W/m}^2\text{.°C})(0.01571\ \text{m}^2)} = \mathbf{11.8°C}$$

7-79 Prob. 7-78 is reconsidered. The effect of the wind velocity on the surface temperature of the wire is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.005 [m]
L=1 [m] "unit length is considered"
I=50 [Ampere]
R=0.002 [Ohm]
T_infinity=10 [C]
Vel=50 [km/h]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
T_film=1/2*(T_s+T_infinity)

"ANALYSIS"
Re=(Vel*Convert(km/h, m/s)*D)/nu
Nusselt=0.3+(0.62*Re^0.5*Pr^(1/3))/(1+(0.4/Pr)^(2/3))^0.25*(1+(Re/282000)^(5/8))^(4/5)
h=k/D*Nusselt
W_dot=I^2*R
Q_dot=W_dot
A=pi*D*L
Q_dot=h*A*(T_s-T_infinity)
```

Vel [km/h]	T_s [C]
10	14.08
15	13.32
20	12.87
25	12.56
30	12.32
35	12.14
40	12
45	11.88
50	11.78
55	11.69
60	11.61
65	11.54
70	11.48
75	11.43
80	11.38

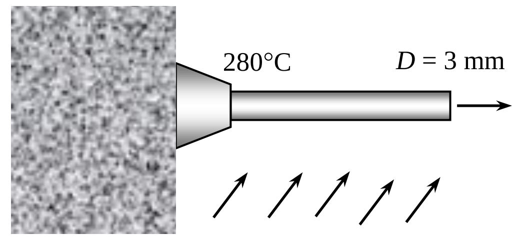


7-80 A long aluminum wire is cooled by cross air flowing over it. The rate of heat transfer from the wire per meter length when it is first exposed to the air is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (280+20)/2 = 150°C$ are (Table A-15)

$$k = 0.03443 \text{ W/m.°C}$$
$$\nu = 2.860\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.7028$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(6 \text{ m/s})(0.003\text{m})}{2.860\times10^{-5} \text{ m}^2/\text{s}} = 629.4$$

The Nusselt number corresponding to this Reynolds number is determined to be

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\Pr^{1/3}}{\left[1+(0.4/\Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$
$$= 0.3 + \frac{0.62(629.8)^{0.5}(0.7028)^{1/3}}{\left[1+(0.4/0.7028)^{2/3}\right]^{1/4}}\left[1+\left(\frac{629.8}{282{,}000}\right)^{5/8}\right]^{4/5}$$
$$= 12.65$$

Then the heat transfer coefficient and the heat transfer rate from the wire per meter length become

$$h = \frac{k}{D}Nu = \frac{0.03443 \text{ W/m.°C}}{0.003 \text{ m}}(12.65) = 145.2 \text{ W/m}^2\text{.°C}$$

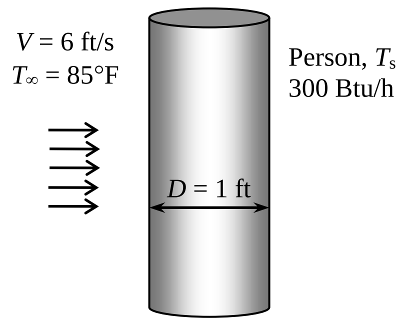
$$A_s = \pi DL = \pi(0.003 \text{ m})(1\text{m}) = 0.009425 \text{ m}^2$$
$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = (145.2 \text{ W/m}^2\text{.°C})(0.009425 \text{ m}^2)(280-20)°\text{C} = \mathbf{356\ W}$$

7-81E A fan is blowing air over the entire body of a person. The average temperature of the outer surface of the person is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The average human body can be treated as a 1-ft-diameter cylinder with an exposed surface area of 18 ft^2. **5** The local atmospheric pressure is 1 atm.

Properties We evaluate the air properties at 100°F based on the problem statement. The properties of air at this temperature are (Table A-15E)

$$k = 0.01529 \text{ Btu/h.ft.°F}$$
$$\nu = 1.809\times10^{-4} \text{ ft}^2\text{/s}$$
$$\Pr = 0.7260$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(6 \text{ ft/s})(1 \text{ ft})}{1.809\times10^{-4} \text{ ft}^2\text{/s}} = 3.317\times10^4$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\Pr^{1/3}}{\left[1+(0.4/\Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(3.317\times10^4)^{0.5}(0.7260)^{1/3}}{\left[1+(0.4/0.7260)^{2/3}\right]^{1/4}}\left[1+\left(\frac{3.317\times10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 107.8$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.01529 \text{ Btu/h.ft.°F}}{1 \text{ ft}}(107.8) = 1.649 \text{ Btu/h.ft}^2\text{.°F}$$

Then the average temperature of the outer surface of the person becomes

$$\dot{Q} = hA_s(T_s - T_\infty) \rightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 85°\text{F} + \frac{300 \text{ Btu/h}}{(1.649 \text{ Btu/h.ft}^2\text{.°F})(18 \text{ ft}^2)} = \mathbf{95.1°F}$$

If the air velocity were doubled, the Reynolds number would be

$$\text{Re} = \frac{VD}{\nu} = \frac{(12 \text{ ft/s})(1 \text{ ft})}{1.809\times10^{-4} \text{ ft}^2\text{/s}} = 6.633\times10^4$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\Pr^{1/3}}{\left[1+(0.4/\Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(6.633\times10^4)^{0.5}(0.7260)^{1/3}}{\left[1+(0.4/0.7260)^{2/3}\right]^{1/4}}\left[1+\left(\frac{6.633\times10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 165.9$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.01529 \text{ Btu/h.ft.°F}}{1 \text{ ft}}(165.9) = 2.537 \text{ Btu/h.ft}^2\text{.°F}$$

Then the average temperature of the outer surface of the person becomes

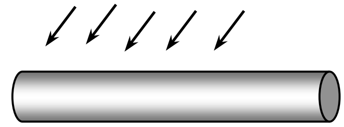
$$\dot{Q} = hA_s(T_s - T_\infty) \rightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 85°\text{F} + \frac{300 \text{ Btu/h}}{(2.537 \text{ Btu/h.ft}^2\text{.°F})(18 \text{ ft}^2)} = \mathbf{91.6°F}$$

7-82E An electrical resistance wire is cooled by a fan. The surface temperature of the wire is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties As an initial guess, we assume the film temperature to be 200°F. The properties of air at this temperature are (Table A-15E)

$$k = 0.01761\ \text{Btu/h.ft.°F}$$
$$\nu = 2.406\times10^{-4}\ \text{ft}^2/\text{s}$$
$$\text{Pr} = 0.7124$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(20\ \text{ft/s})(0.1/12\ \text{ft})}{2.406\times10^{-4}\ \text{ft}^2/\text{s}} = 692.7$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$
$$= 0.3 + \frac{0.62(692.7)^{0.5}(0.7124)^{1/3}}{\left[1+(0.4/0.7124)^{2/3}\right]^{1/4}}\left[1+\left(\frac{692.7}{282{,}000}\right)^{5/8}\right]^{4/5} = 13.34$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.01761\ \text{Btu/h.ft.°F}}{(0.1/12\ \text{ft})}(13.34) = 28.19\ \text{Btu/h.ft}^2.\text{°F}$$

Then the average temperature of the outer surface of the wire becomes

$$A_s = \pi DL = \pi(0.1/12\ \text{ft})(12\ \text{ft}) = 0.3142\ \text{ft}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA} = 85\text{°F} + \frac{(1500\times3.41214)\ \text{Btu/h}}{(28.19\ \text{Btu/h.ft}^2.\text{°F})(0.3142\ \text{ft}^2)} = \mathbf{662.9°F}$$

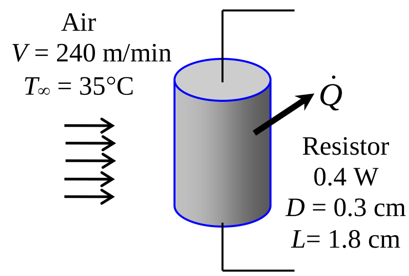
Discussion Repeating the calculations at the new film temperature of (85+662.9)/2=374°F gives T_s=668.3°F.

7-83 A cylindrical electronic component mounted on a circuit board is cooled by air flowing across it. The surface temperature of the component is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties We assume the film temperature to be 50°C based on the problem statement. The properties of air at 1 atm and at this temperature are (Table A-15)

$$k = 0.02735 \text{ W/m.°C}$$
$$\nu = 1.798\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7228$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(240/60 \text{ m/s})(0.003\text{m})}{1.798\times10^{-5} \text{ m}^2\text{/s}} = 667.4$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(667.4)^{0.5}(0.7228)^{1/3}}{\left[1+(0.4/0.7228)^{2/3}\right]^{1/4}}\left[1+\left(\frac{667.4}{282{,}000}\right)^{5/8}\right]^{4/5} = 13.17$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02735 \text{ W/m.°C}}{0.003 \text{ m}}(13.17) = 120.0 \text{ W/m}^2\text{.°C}$$

Then the surface temperature of the component becomes

$$A_s = \pi DL = \pi(0.003 \text{ m})(0.018 \text{ m}) = 0.0001696 \text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA} = 35\text{ °C} + \frac{0.4 \text{ W}}{(120.0 \text{ W/m}^2\text{.°C})(0.0001696 \text{ m}^2)} = \mathbf{54.6°C}$$

The film temperature is (54.6+35)/2=44.8°C, which is sufficiently close to the assumed value of 50°C. Therefore, there is no need to repeat calculations.

7-84 A cylindrical hot water tank is exposed to windy air. The temperature of the tank after a 45-min cooling period is to be estimated.

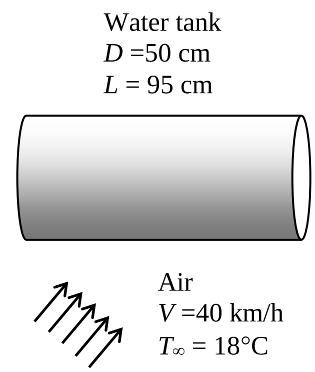
Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The surface of the tank is at the same temperature as the water temperature. **5** The heat transfer coefficient on the top and bottom surfaces is the same as that on the side surfaces.

Properties The properties of water at 80°C are (Table A-9)

$$\rho = 971.8\ \text{kg/m}^3$$
$$c_p = 4197\ \text{J/kg.°C}$$

The properties of air at 1 atm and at the film temperature of 50°C (based on the problem statement) are (Table A-15)

$$k = 0.02735\ \text{W/m.°C}$$
$$\nu = 1.798\times10^{-5}\ \text{m}^2/\text{s}$$
$$\Pr = 0.7228$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{\left(\dfrac{40\times1000}{3600}\ \text{m/s}\right)(0.50\,\text{m})}{1.798\times10^{-5}\ \text{m}^2/\text{s}} = 3.090\times10^5$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$\text{Nu} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\Pr^{1/3}}{\left[1+(0.4/\Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(3.090\times10^5)^{0.5}(0.7228)^{1/3}}{\left[1+(0.4/0.7228)^{2/3}\right]^{1/4}}\left[1+\left(\frac{3.090\times10^5}{282{,}000}\right)^{5/8}\right]^{4/5} = 484.8$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02735\ \text{W/m.°C}}{0.50\ \text{m}}(484.8) = 26.52\ \text{W/m}^2.\text{°C}$$

The surface area of the tank is

$$A_s = \pi DL + 2\pi\frac{D^2}{4} = \pi(0.5)(0.95) + 2\pi(0.5)^2/4 = 1.885\,\text{m}^2$$

The rate of heat transfer is determined from

$$\dot{Q} = hA_s(T_s - T_\infty) = (26.52\ \text{W/m}^2.\text{°C})(1.885\,\text{m}^2)\left(\frac{80+T_2}{2} - 18\right)\text{°C} \qquad \text{(Eq. 1)}$$

where T_2 is the final temperature of water so that $(80+T_2)/2$ gives the average temperature of water during the cooling process. The mass of water in the tank is

$$m = \rho \mathbf{V} = \rho\pi\frac{D^2}{4}L = (971.8\,\text{kg/m}^3)\pi(0.50\,\text{m})^2(0.95\,\text{m})/4 = 181.3\,\text{kg}$$

The amount of heat transfer from the water is determined from

$$Q = mc_p(T_2 - T_1) = (181.3\,\text{kg})(4197\,\text{J/kg.°C})(80 - T_2)\text{°C}$$

Then average rate of heat transfer is

$$\dot{Q} = \frac{Q}{\Delta t} = \frac{(181.3\,\text{kg})(4197\,\text{J/kg.°C})(80-T_2)\text{°C}}{45\times60\,\text{s}} \qquad \text{(Eq. 2)}$$

Setting Eq. 1 to be equal to Eq. 2 we obtain the final temperature of water

$$\dot{Q} = (26.52\ \text{W/m}^2.\text{°C})(1.885\,\text{m}^2)\left(\frac{80+T_2}{2} - 18\right)\text{°C} = \frac{(181.3\,\text{kg})(4197\,\text{J/kg.°C})(80-T_2)\text{°C}}{45\times60\,\text{s}}$$

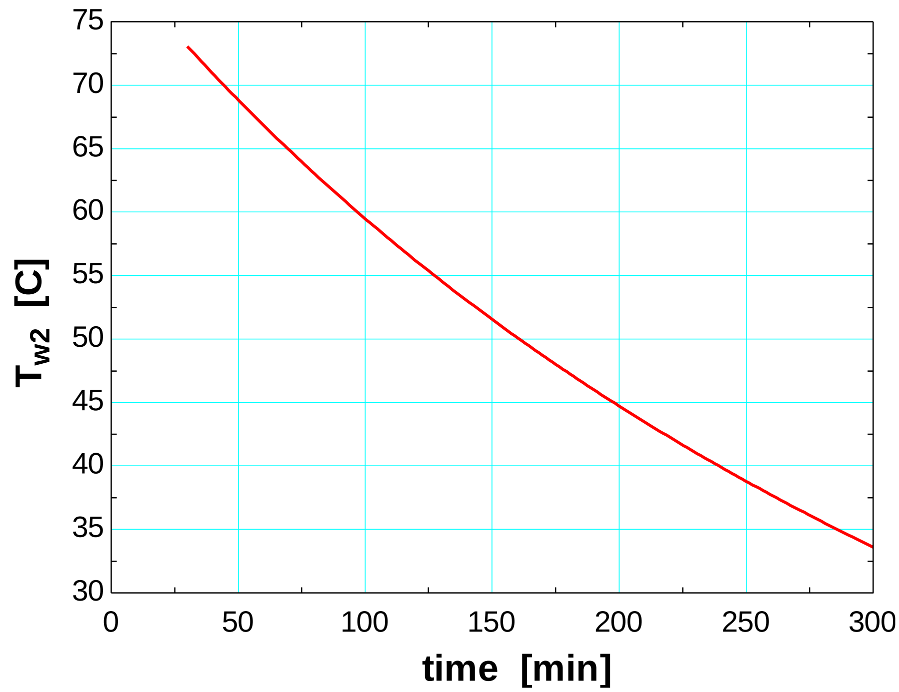
$$T_2 = \mathbf{69.9°C}$$

7-85 Prob. 7-84 is reconsidered. The temperature of the tank as a function of the cooling time is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.50 [m]
L=0.95 [m]
T_w1=80 [C]
T_infinity=18 [C]
Vel=40 [km/h]
time=45 [min]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
T_film=1/2*(T_w_ave+T_infinity)
rho_w=Density(water, T=T_w_ave, P=101.3)
c_p_w=CP(Water, T=T_w_ave, P=101.3)*Convert(kJ/kg-C, J/kg-C)
T_w_ave=1/2*(T_w1+T_w2)

"ANALYSIS"
Re=(Vel*Convert(km/h, m/s)*D)/nu
Nusselt=0.3+(0.62*Re^0.5*Pr^(1/3))/(1+(0.4/Pr)^(2/3))^0.25*(1+(Re/282000)^(5/8))^(4/5)
h=k/D*Nusselt
A=pi*D*L+2*pi*D^2/4
Q_dot=h*A*(T_w_ave-T_infinity)
m_w=rho_w*V_w
V_w=pi*D^2/4*L
Q=m_w*c_p_w*(T_w1-T_w2)
Q_dot=Q/(time*Convert(min, s))
```

time [min]	T_{w2} [C]
30	73.06
45	69.86
60	66.83
75	63.96
90	61.23
105	58.63
120	56.16
135	53.8
150	51.54
165	49.39
180	47.33
195	45.36
210	43.47
225	41.65
240	39.91
255	38.24
270	36.63
285	35.09
300	33.6

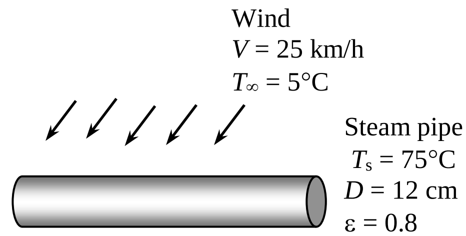


7-86 A steam pipe is exposed to a light winds in the atmosphere. The amount of heat loss from the steam during a certain period and the money the facility will save a year as a result of insulating the steam pipe are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The plant operates every day of the year for 10 h a day. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (75+5)/2 = 40°C$ are (Table A-15)

$$k = 0.02662 \text{ W/m.°C}$$
$$\nu = 1.702\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7255$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{[(25\times1000/3600)\text{ m/s}](0.12\text{ m})}{1.702\times10^{-5}\text{ m}^2\text{/s}} = 4.896\times10^4$$

The Nusselt number corresponding to this Reynolds number is determined to be

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\Pr^{1/3}}{\left[1+(0.4/\Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(4.896\times10^4)^{0.5}(0.7255)^{1/3}}{\left[1+(0.4/0.7255)^{2/3}\right]^{1/4}}\left[1+\left(\frac{4.896\times10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 136.9$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02662\text{ W/m.°C}}{0.12\text{ m}}(136.9) = 30.37\text{ W/m}^2\text{.°C}$$

The rate of heat loss by convection is

$$A_s = \pi DL = \pi(0.12\text{ m})(12\text{ m}) = 4.524\text{ m}^2$$

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = (30.37\text{ W/m}^2\text{.°C})(4.524\text{ m}^2)(75-5)\text{°C} = 9617\text{ W}$$

The rate of heat loss by radiation is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4}) = (0.8)(4.524\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)\left[(75+273\text{ K})^4 - (0+273\text{ K})^4\right] = 1870\text{ W}$$

The total rate of heat loss then becomes

$$\dot{Q}_{\text{total}} = \dot{Q}_{conv} + \dot{Q}_{rad} = 9617 + 1870 = 11{,}487\text{ W}$$

The amount of heat loss from the steam during a 10-hour work day is

$$Q = \dot{Q}_{total}\Delta t = (11.487\text{ kJ/s})(10\text{ h/day}\times3600\text{ s/h}) = \mathbf{4.135\times10^5\ kJ/day}$$

The total amount of heat loss from the steam per year is

$$Q_{total} = \dot{Q}_{day}(\text{no. of days}) = (4.135\times10^5\text{ kJ/day})(365\text{ days/yr}) = 1.509\times10^8\text{ kJ/yr}$$

Noting that the steam generator has an efficiency of 80%, the amount of gas used is

$$Q_{gas} = \frac{Q_{total}}{0.80} = \frac{1.509\times10^8\text{ kJ/yr}}{0.80}\left(\frac{1\text{ therm}}{105{,}500\text{ kJ}}\right) = 1788\text{ therms/yr}$$

Insulation reduces this amount by 90%. The amount of energy and money saved becomes

$$\text{Energy saved} = (0.90)Q_{gas} = (0.90)(1788\text{ therms/yr}) = 1609\text{ therms/yr}$$

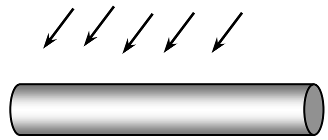
$$\text{Money saved} = (\text{Energy saved})(\text{Unit cost of energy}) = (1609\text{ therms/yr})(\$1.05/\text{therm}) = \mathbf{\$1690}$$

7-87 A steam pipe is exposed to light winds in the atmosphere. The amount of heat loss from the steam during a certain period and the money the facility will save a year as a result of insulating the steam pipes are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The plant operates every day of the year for 10 h. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (75+5)/2 = 40°C$ are (Table A-15)

$$k = 0.02662 \text{ W/m.°C}$$
$$\nu = 1.702\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7255$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{[(10\times1000/3600)\text{ m/s}](0.10\text{ m})}{1.702\times10^{-5}\text{ m}^2/\text{s}} = 1.632\times10^4$$

The Nusselt number corresponding to this Reynolds number is determined to be

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(1.632\times10^4)^{0.5}(0.7255)^{1/3}}{\left[1+(0.4/0.7255)^{2/3}\right]^{1/4}}\left[1+\left(\frac{1.632\times10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 71.19$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02662\text{ W/m.°C}}{0.10\text{ m}}(71.19) = 18.95\text{ W/m}^2.\text{°C}$$

The rate of heat loss by convection is

$$A_s = \pi DL = \pi(0.10\text{ m})(12\text{ m}) = 3.770\text{ m}^2$$

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = (18.95\text{ W/m}^2.\text{°C})(3.770\text{ m}^2)(75-5)\text{°C} = 5001\text{ W}$$

For an average surrounding temperature of 0°C, the rate of heat loss by radiation and the total rate of heat loss are

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4}) = (0.8)(3.770\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)\left[(75+273\text{ K})^4 - (0+273\text{ K})^4\right] = 1558\text{ W}$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{conv} + \dot{Q}_{rad} = 5001 + 1558 = 6559\text{ W}$$

If the average surrounding temperature is -20°C, the rate of heat loss by radiation and the total rate of heat loss become

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4}) = (0.8)(3.770\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)\left[(75+273\text{ K})^4 - (-20+273\text{ K})^4\right] = 1807\text{ W}$$

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 5001 + 1807 = 6808\text{ W}$$

which is 6808 – 6559 = 249 W more than the value for a surrounding temperature of 0°C. This corresponds to

$$\%\text{ change} = \frac{\dot{Q}_{\text{difference}}}{\dot{Q}_{\text{total,0°C}}}\times100 = \frac{249\text{ W}}{6559\text{ W}}\times100 = \mathbf{3.80\%}\quad\text{(increase)}$$

If the average surrounding temperature is 25°C, the rate of heat loss by radiation and the total rate of heat loss become

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4}) = (0.8)(3.770\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)\left[(75+273\text{ K})^4 - (25+273\text{ K})^4\right] = 1159\text{ W}$$

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 5001 + 1159 = 6160\text{ W}$$

which is 6559 – 6160 = 399 W less than the value for a surrounding temperature of 0°C. This corresponds to

$$\%\text{ change} = \frac{\dot{Q}_{\text{difference}}}{\dot{Q}_{\text{total,0°C}}}\times100 = \frac{399\text{ W}}{6559\text{ W}}\times100 = \mathbf{6.08\%}\quad\text{(decrease)}$$

Therefore, the effect of the temperature variations of the surrounding surfaces on the total heat transfer is less than about 6%.

7-88 A cylindrical bottle containing cold water is exposed to windy air. The average wind velocity is to be estimated.

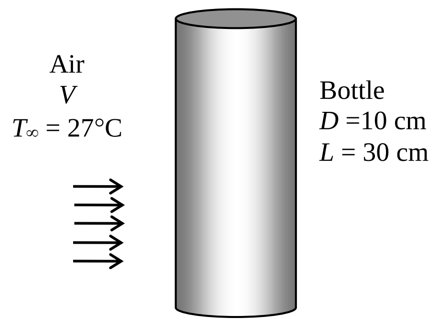
Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** Heat transfer at the top and bottom surfaces is negligible.

Properties The properties of water at the average temperature of $(T_1 + T_2)/2=(3+11)/2=7°C$ are (Table A-9)

$$\rho = 999.8\ \text{kg/m}^3$$
$$c_p = 4200\ \text{J/kg.°C}$$

The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (7+27)/2 = 17°C$ are (Table A-15)

$$k = 0.02491\ \text{W/m.°C}$$
$$\nu = 1.488\times10^{-5}\ \text{m}^2\text{/s}$$
$$\text{Pr} = 0.7317$$



Analysis The mass of water in the bottle is

$$m = \rho \boldsymbol{V} = \rho\pi\frac{D^2}{4}L = (999.8\ \text{kg/m}^3)\pi(0.10\ \text{m})^2(0.30\ \text{m})/4 = 2.356\ \text{kg}$$

Then the amount of heat transfer to the water is

$$Q = mc_p(T_2 - T_1) = (2.356\ \text{kg})(4200\ \text{J/kg.°C})(11\text{-}3)\text{°C} = 79{,}162\ \text{J}$$

The average rate of heat transfer is

$$\dot{Q} = \frac{Q}{\Delta t} = \frac{79{,}162\ \text{J}}{45\times60\ \text{s}} = 29.32\ \text{W}$$

The heat transfer coefficient is

$$A_s = \pi DL = \pi(0.10\ \text{m})(0.30\ \text{m}) = 0.09425\ \text{m}^2$$
$$\dot{Q}_{\text{conv}} = hA_s(T_\infty - T_s) \longrightarrow 29.32\ \text{W} = h(0.09425\ \text{m}^2)(27-7)\text{°C} \longrightarrow h = 15.55\ \text{W/m}^2.\text{°C}$$

The Nusselt number is

$$Nu = \frac{hD}{k} = \frac{(15.55\ \text{W/m}^2.\text{°C})(0.10\ \text{m})}{0.02491\ \text{W/m.°C}} = 62.42$$

Reynolds number can be obtained from the Nusselt number relation for a flow over the cylinder

$$Nu = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$
$$62.42 = 0.3 + \frac{0.62\,\text{Re}^{0.5}(0.7317)^{1/3}}{\left[1+(0.4/0.7317)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5} \longrightarrow \text{Re} = 12{,}856$$

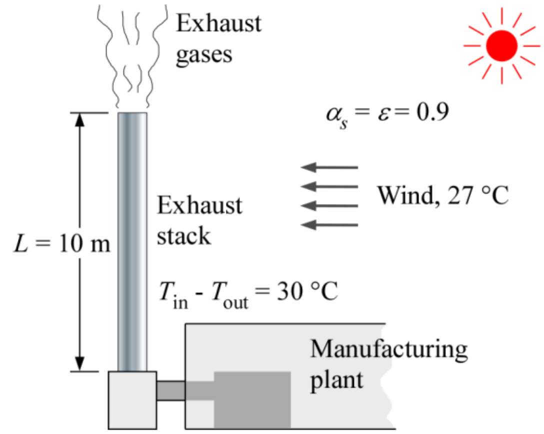
Then using the Reynolds number relation we determine the wind velocity

$$\text{Re} = \frac{VD}{\nu} \longrightarrow 12{,}856 = \frac{V(0.10\ \text{m})}{1.488\times10^{-5}\ \text{m}^2\text{/s}} \longrightarrow V = \mathbf{1.91\ m/s}$$

7-89 A 10-m tall exhaust stack discharging exhaust gases at a rate of 1.2 kg/s is subjected to solar radiation and convection at the outer surface. The outer surface temperature of the exhaust stack is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The surface temperature is constant.

Properties The properties of air at 80°C are k = 0.02953 W/m·K, $\nu = 2.097 \times 10^{-5}$ m²/s, Pr = 0.7154 (from Table A-15).



Analysis The Reynolds number for the air flowing across the exhaust stack is

$$\mathrm{Re}_D = \frac{VD}{\nu} = \frac{(10\text{ m/s})(1\text{ m})}{2.097\times10^{-5}\text{ m}^2/\text{s}} = 4.769\times10^5$$

Using the Churchill and Bernstein relation for Nusselt number, the convection heat transfer coefficient is

$$\mathrm{Nu}_{cyl} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1+(0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282000}\right)^{5/8}\right]^{4/5}$$

$$h = \frac{0.02953\text{ W/m}\cdot\text{K}}{1\text{ m}}\left\{0.3 + \frac{0.62(476900)^{1/2}(0.7154)^{1/3}}{[1+(0.4/0.7154)^{2/3}]^{1/4}}\left[1+\left(\frac{476900}{282000}\right)^{5/8}\right]^{4/5}\right\}$$

$$= 19.95\text{ W/m}^2\cdot\text{K}$$

The outer surface area of the exhaust stack is

$$A_s = \pi DL = \pi(1\text{ m})(10\text{ m}) = 31.42\text{ m}^2$$

The rate of heat loss from the exhaust gases in the exhaust stack can be determined from

$$\dot{Q}_{\text{loss}} = \dot{m}c_p(T_{\text{in}} - T_{\text{out}}) = (1.2\text{ kg/s})(1600\text{ J/kg}\cdot°\text{C})(30)\,°\text{C} = 57600\text{ W}$$

The heat loss on the outer surface of the exhaust stack by radiation and convection can be expressed as

$$\frac{\dot{Q}_{\text{loss}}}{A_s} = h[T_s - T_\infty] + \varepsilon\sigma[T_s^4 - T_{\text{surr}}^4] - \alpha_s\dot{q}_{\text{solar}}$$

Copy the following lines and paste on a blank EES screen to solve the above equation:

```
A_s=31.42
h=19.95
q_incindent=1400
Q_loss=57600
T_inf=27+273
T_surr=27+273
alpha=0.9
epsilon=0.9
sigma=5.670e-8
Q_loss/A_s=h*(T_s-T_inf)+epsilon*sigma*(T_s^4-T_surr^4)-alpha*q_incindent
```

Solving by EES software, the surface temperature of exhaust stack is

$$T_s = 406\text{ K} = \mathbf{133°C}$$

Discussion Since the value of the (force) convection heat transfer coefficient is relatively small, this indicates that natural convection may play an important role.

7-90 Liquid NH_3 flows in a pipe, which is insulated. The insulation thickness on the pipe that is necessary to keep the liquid NH_3 temperature below −35°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with local atmospheric pressureat 1 atm. **4** One-dimensional heat conduction through walls. **5** The thermal conductivities are constant. **6** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities of the pipe and the insulation are given to be k_{pipe}= 25 W/m·K and k_{ins} = 0.75 W/m·K, respectively.

The properties of air at 1 atm and $T_f = (T_s + T_\infty)/2 = (10 + 20)/2 = 15$°C are k = 0.02476 W/m·K, $\nu = 1.470\times 10^{-5}$ m²/s, and Pr = 0.7323 (Table A-15).

Analysis The convection heat transfer coefficient on the outer surface can be determined using the Nusselt number relation for flow across a cylinder. The Reynolds number and Nusselt number can be determined using

$$\mathrm{Re} = \frac{VD_o}{\nu} \quad \text{and} \quad \mathrm{Nu} = \frac{h_{\mathrm{air}} D_o}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1+(0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

From Chapter 3, the thermal resistances of different layers are

$$R_{\mathrm{conv},i} = \frac{1}{h_{\mathrm{NH3}} A_i} = \frac{1}{h_{\mathrm{NH3}} \pi D_i L} \quad \text{(liq. NH}_3\text{ convection resistance)}$$

$$R_{\mathrm{pipe}} = \frac{\ln(D_{\mathrm{interface}}/D_i)}{2\pi k_{\mathrm{pipe}} L} \quad \text{(pipe layer resistance)}$$

$$R_{\mathrm{ins}} = \frac{\ln(D_o/D_{\mathrm{interface}})}{2\pi k_{\mathrm{ins}} L} \quad \text{(insulation layer resistance)}$$

$$R_{\mathrm{conv},o} = \frac{1}{h_{\mathrm{air}} A_o} = \frac{1}{h_{\mathrm{air}} \pi D_o L} \quad \text{(air flow across cylinder convection resistance)}$$

The total thermal resistance and the rate of heat transfer are

$$R_{\mathrm{total}} = R_{\mathrm{conv},i} + R_{\mathrm{pipe}} + R_{\mathrm{ins}} + R_{\mathrm{conv}} \quad \text{and} \quad \dot{Q} = \frac{T_\infty - T_{\mathrm{NH3}}}{R_{\mathrm{total}}} = \frac{T_\infty - T_{s,o}}{R_{\mathrm{conv},o}}$$

and the insulation thickness is

$$t_{\mathrm{ins}} = \frac{D_o - D_{\mathrm{interface}}}{2}$$

Solving for the insulation thickness yields $t_{\mathrm{ins}} = 0.0426\,\mathrm{m} = \mathbf{4.26\,cm}$

Solved by EES Software. Copy-and-paste the following lines on a blank EES screen to verify the solutions.

```
"GIVEN"
h_NH3=100 [W/m^2-K]   "liq. NH3 convection heat transfer coefficient"
L=10 [m]   "pipe length"
D_i=0.025 [m]   "inner pipe diameter"
D_interface=0.04 [m]   "outer pipe diameter"
T_s_o=10 [C]   "outer insulation surface temperature"
T_s_i=-35 [C]   "liq. NH3 temperature"
T_infinity=20 [C]   "ambient air temperature"
V=7 [m/s]

"PROPERTIES"
"Air"
Fluid$='air'
T_film=1/2*(T_s_o+T_infinity)
k=Conductivity(Fluid$, T=T_film)
```

```
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
"Walls - pipe and insulation"
k_pipe=25 [W/m-K]   "pipe thermal conductivity"
k_ins=0.75 [W/m-K]   "insulation thermal conductivity"

"ANALYSIS"
Re=V*D_o/nu
Nusselt=0.3+(0.62*Re^0.5*Pr^(1/3))/(1+(0.4/Pr)^(2/3))^0.25*(1+(Re/282000)^(5/8))^(4/5)
h_air=Nusselt*k/D_o
"Thermal resistances for diffenrent layers"
R_conv_i=1/(h_NH3*pi#*D_i*L)   "liq. NH3 convection resistance"
R_pipe=ln(D_interface/D_i)/(2*pi#*k_pipe*L)   "pipe layer resistance"
R_ins=ln(D_o/D_interface)/(2*pi#*k_ins*L)   "insulation layer resistance"
R_conv_o=1/(h_air*pi#*D_o*L)   "ambient air convection resistance"
R_total=R_conv_i+R_pipe+R_ins+R_conv_o
"Solving for the insulation thickness"
Q_dot=(T_infinity-T_s_i)/(R_total)
Q_dot=(T_infinity-T_s_o)/(R_conv_o)
t_ins=(D_o-D_interface)/2
```

Discussion To keep the liquid NH_3 below −35°C, the pipe insulation thickness must be at least 4.26 cm thick.

7-91 Ice slurry is being transported in an insulated pipe. The insulation thickness on the pipe that is necessary to prevent condensation on the outer surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with local atmospheric pressureat 1 atm. **4** One-dimensional heat conduction through walls. **5** The thermal conductivities are constant. **6** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities of the pipe and the insulation are given to be k_{pipe}= 15 W/m·K and k_{ins} = 0.95 W/m·K, respectively.

The properties of air at 1 atm and $T_f = (T_{s,o} + T_\infty)/2 = (10 + 20)/2 = 15°\text{C}$ are k = 0.02476 W/m·K, $\nu = 1.470\times 10^{-5}$ m²/s, and Pr = 0.7323 (Table A-15).

Analysis The convection heat transfer coefficient on the outer surface can be determined using the Nusselt number relation for flow across a cylinder. The Reynolds number and Nusselt number can be determined with

$$\text{Re} = \frac{VD_o}{\nu} \quad \text{and} \quad \text{Nu} = \frac{hD_o}{k} = 0.3 + \frac{0.62\,\text{Re}^{1/2}\,\text{Pr}^{1/3}}{[1+(0.4/\text{Pr})^{2/3}]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

From Chapter 3, the thermal resistances of different layers are

$$R_{\text{pipe}} = \frac{\ln(D_{\text{interface}}/D_i)}{2\pi k_{\text{pipe}} L} \quad \text{(pipe layer resistance)}$$

$$R_{\text{ins}} = \frac{\ln(D_o/D_{\text{interface}})}{2\pi k_{\text{ins}} L} \quad \text{(insulation layer resistance)}$$

$$R_{\text{conv}} = \frac{1}{hA_{s,o}} = \frac{1}{h\pi D_o L} \quad \text{(air flow across cylinder convection resistance)}$$

The total thermal resistance and the rate of heat transfer are

$$R_{\text{total}} = R_{\text{pipe}} + R_{\text{ins}} + R_{\text{conv}} \quad \text{and} \quad \dot{Q} = \frac{T_\infty - T_{s,i}}{R_{\text{total}}} = \frac{T_\infty - T_{s,o}}{R_{\text{conv}}}$$

and the insulation thickness is

$$t_{\text{ins}} = \frac{D_o - D_{\text{interface}}}{2}$$

Solving for the insulation thickness yields $t_{\text{ins}} = 0.0343\text{ m} = \mathbf{3.43\text{ cm}}$

Solved by EES Software. Copy-and-paste the following lines on a blank EES screen to verify the solutions.

```
"GIVEN"
L=5 [m]   "pipe length"
D_i=0.025 [m]   "inner pipe diameter"
D_interface=0.03 [m]   "outer pipe diameter"
T_s_i=0 [C]   "inner pipe surface temperature"
T_s_o=10 [C]   "outer insulation surface temperature"
T_infinity=20 [C]   "ambient air temperature"
V=2 [m/s]

"PROPERTIES"
"Air"
Fluid$='air'
T_film=1/2*(T_s_o+T_infinity)
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
```

```
"Walls - pipe and insulation"
k_pipe=15 [W/m-K]   "pipe thermal conductivity"
k_ins=0.95 [W/m-K]   "insulation thermal conductivity"

"ANALYSIS"
Re=V*D_o/nu
Nusselt=0.3+(0.62*Re^0.5*Pr^(1/3))/(1+(0.4/Pr)^(2/3))^0.25*(1+(Re/282000)^(5/8))^(4/5)
h=Nusselt*k/D_o
"THERMAL RESISTANCES"
R_pipe=ln(D_interface/D_i)/(2*pi#*k_pipe*L)   "pipe layer resistance"
R_ins=ln(D_o/D_interface)/(2*pi#*k_ins*L)   "insulation layer resistance"
R_conv=1/(h*pi#*D_o*L)   "ambient air convection resistance"
R_total=R_pipe+R_ins+R_conv
"SOLVING FOR THE INSULATION THICKNESS"
Q_dot=(T_infinity-T_s_i)/(R_total)
Q_dot=(T_infinity-T_s_o)/(R_conv)
t_ins=(D_o-D_interface)/2
```

Discussion To prevent condensation on the outer surface of the pipe, the insulation should be at least 3.43 cm thick.

7-92 Air is flowing over a 5-cm diameter sphere, (*a*) the average drag coefficient on the sphere and (*b*) the heat transfer rate from the sphere are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The surface temperature is constant.

Properties The properties of air (1 atm) at the free stream temperature T_∞ = 20°C (Table A-15): ρ = 1.204 kg/m^3, k = 0.02514 W/m·K, μ = 1.825 × 10^{-5} kg/m·s, and Pr = 0.7309; at the surface temperature T_s = 80°C: μ_s = 2.096 × 10^{-5} kg/m·s; at the film temperature T_f = (80°C + 20°C)/2 = 50°C: ρ = 1.092 kg/m^3 and ν = 1.798 × 10^{-5} m^2/s.

Analysis (*a*) The Reynolds number for air properties evaluated from the film temperature is

$$\mathrm{Re}_D = \frac{VD}{\nu} = \frac{(3.5\ \mathrm{m/s})(0.05\ \mathrm{m})}{1.798\times10^{-5}\ \mathrm{kg/m\cdot s}} = 9733$$

From Fig. 7-17, the average drag coefficient is $C_D \approx \mathbf{0.4}$.

(*b*) The Reynolds number for air properties evaluated from the free stream temperature is

$$\mathrm{Re_D} = \frac{\rho \mathrm{VD}}{\mu} = \frac{(1.204\ kg/m^3)(3.5\ m/s)(0.05\ m)}{1.825\times10^{-5}\ kg/m\cdot s} = 1.155\times10^4$$

Using the Whitaker relation for Nusselt number, the convection heat transfer coefficient is

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\,\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$\mathrm{Nu}_{\mathrm{sph}} = \left\{2 + [0.4(1.155\times10^4)^{1/2} + 0.06(1.155\times10^4)^{2/3}](0.7309)^{0.4}\left(\frac{1.825}{2.096}\right)^{1/4}\right\} = 64.76$$

Hence

$$h = 64.76\left(\frac{0.02514\ \mathrm{W/m\cdot K}}{0.05\ \mathrm{m}}\right) = 32.56\ \mathrm{W/m^2\cdot K}$$

The heat transfer rate from the sphere is

$$\dot{Q} = hA(T_s - T_\infty) = h\pi D^2(T_s - T_\infty) = (32.56\ \mathrm{W/m^2\cdot K})\pi(0.05\ \mathrm{m})^2(80-20)\ \mathrm{K} = \mathbf{15.34\ W}$$

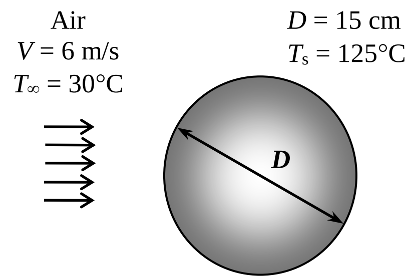
Discussion If the difference in the free stream temperature and the surface temperature is small, then the assumption that μ_∞ / $\mu_s \approx 1$ is appropriate.

7-93 A hot stainless steel ball is cooled by forced air. The average convection heat transfer coefficient and the cooling time are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The outer surface temperature of the ball is uniform at all times.

Properties The average surface temperature is (125+75)/2 = 100°C, and the properties of air at 1 atm pressure and the free stream temperature of 30°C are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$
$$\nu = 1.608\times10^{-5} \text{ m}^2/\text{s}$$
$$\mu_\infty = 1.872\times10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@\,100\,°\text{C}} = 2.181\times10^{-5} \text{ kg/m.s}$$
$$\text{Pr} = 0.7282$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(6\text{ m/s})(0.15\text{m})}{1.608\times10^{-5}\text{ m}^2/\text{s}} = 5.597\times10^4$$

The Nusselt number corresponding to this Reynolds number is determined to be

$$Nu = \frac{hD}{k} = 2 + \left[0.4\text{Re}^{0.5} + 0.06\text{Re}^{2/3}\right]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$
$$= 2 + \left[0.4(5.597\times10^4)^{0.5} + 0.06(5.597\times10^4)^{2/3}\right](0.7282)^{0.4}\left(\frac{1.872\times10^{-5}}{2.181\times10^{-5}}\right)^{1/4} = 156.7$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02588 \text{ W/m.°C}}{0.15\text{ m}}(156.7) = \mathbf{27.04\ W/m^2.°C}$$

The average rate of heat transfer can be determined from Newton's law of cooling by using average surface temperature of the ball

$$A_s = \pi D^2 = \pi(0.15\text{ m})^2 = 0.07069\text{ m}^2$$
$$\dot{Q}_{avg} = hA_s(T_s - T_\infty) = (27.04\text{ W/m}^2.°\text{C})(0.07069\text{ m}^2)(100-30)°\text{C} = 133.8\text{ W}$$

Assuming the ball temperature to be nearly uniform, the total heat transferred from the ball during the cooling from 350°C to 250°C can be determined from

$$Q_{\text{total}} = mc_p(T_1 - T_2)$$

where

$$m = \rho \boldsymbol{V} = \rho\frac{\pi D^3}{6} = (8055\text{ kg/m}^3)\frac{\pi(0.15\text{m})^3}{6} = 14.23\text{ kg}$$

Therefore,

$$Q_{\text{total}} = mc_p(T_1 - T_2) = (14.23\text{ kg})(480\text{ J/kg.°C})(125-75)°\text{C} = 341{,}520\text{ J}$$

Then the time of cooling becomes

$$\Delta t = \frac{Q}{\dot{Q}_{avg}} = \frac{341{,}5210\text{ J}}{133.8\text{ J/s}} = 2552\text{ s} = \mathbf{42.5\ min}$$

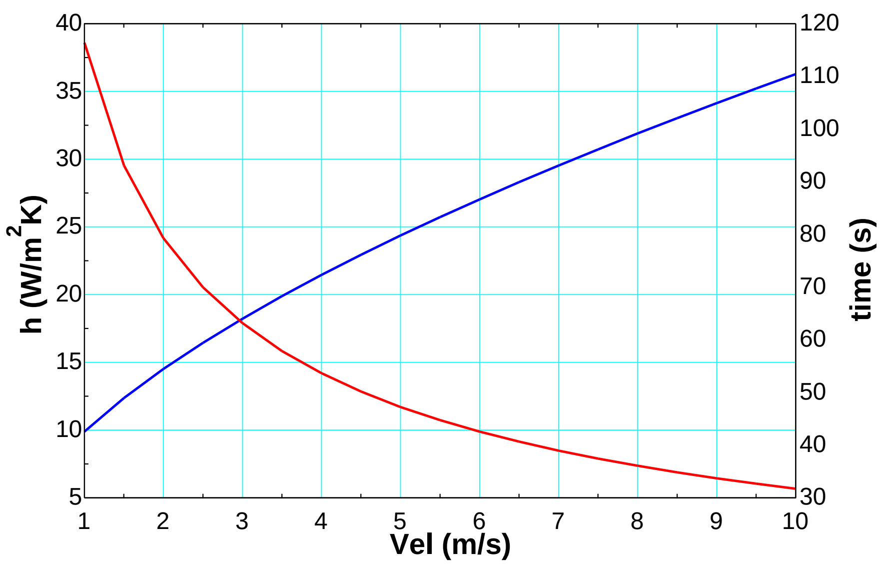
Discussion The Nusselt number relation for flow over a sphere (Eq. 7-36) is valid for $1.0 \le \mu_\infty/\mu_s \le 3.2$. In this problem, $\mu_\infty/\mu_s = 0.86$, which is less than 1.0. However, it is sufficiently close to 1, which is appropriate for an engineering heat transfer analysis. Note that the effect of viscosity ratio on the value of Nusselt number is minor.

7-94  Prob. 7-93 is reconsidered. The effect of air velocity on the average convection heat transfer coefficient and the cooling time is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.15 [m]
T_1=125 [C]
T_2=75 [C]
T_infinity=30 [C]
P=101.3 [kPa]
Vel=6 [m/s]
rho_ball=8055 [kg/m^3]
c_p_ball=480 [J/kg-C]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_infinity)
Pr=Prandtl(Fluid$, T=T_infinity)
rho=Density(Fluid$, T=T_infinity, P=P)
mu_infinity=Viscosity(Fluid$, T=T_infinity)
nu=mu_infinity/rho
mu_s=Viscosity(Fluid$, T=T_s_ave)
T_s_ave=1/2*(T_1+T_2)

"ANALYSIS"
Re=(Vel*D)/nu
Nusselt=2+(0.4*Re^0.5+0.06*Re^(2/3))*Pr^0.4*(mu_infinity/mu_s)^0.25
h=k/D*Nusselt
A=pi*D^2
Q_dot_ave=h*A*(T_s_ave-T_infinity)
Q_total=m_ball*c_p_ball*(T_1-T_2)
m_ball=rho_ball*V_ball
V_ball=(pi*D^3)/6
time=Q_total/Q_dot_ave*Convert(s, min)
```

Vel [m/s]	h [W/m^2.C]	time [min]
1	9.886	116.4
1.5	12.36	93.08
2	14.51	79.3
2.5	16.45	69.97
3	18.22	63.14
3.5	19.88	57.87
4	21.45	53.65
4.5	22.94	50.17
5	24.36	47.25
5.5	25.72	44.74
6	27.03	42.57
6.5	28.3	40.66
7	29.53	38.97
7.5	30.73	37.45
8	31.89	36.09
8.5	33.02	34.85
9	34.13	33.72
9.5	35.21	32.68
10	36.27	31.73

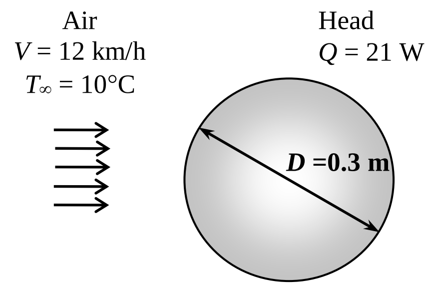


7-95 The average surface temperature of the head of a person when it is not covered and is subjected to winds is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** One-quarter of the heat the person generates is lost from the head. **5** The head can be approximated as a 30-cm-diameter sphere. **6** The local atmospheric pressure is 1 atm.

Properties We assume the surface temperature to be 15°C for viscosity based on the problem statement. The properties of air at 1 atm pressure and the free stream temperature of 10°C are (Table A-15)

$$k = 0.02439 \text{ W/m.°C}$$
$$\nu = 1.426\times10^{-5} \text{ m}^2\text{/s}$$
$$\mu_\infty = 1.778\times10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@15°C} = 1.802\times10^{-5} \text{ kg/m.s}$$
$$\text{Pr} = 0.7336$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{[(12\times1000/3600)\text{ m/s}](0.3\text{ m})}{1.426\times10^{-5}\text{ m}^2\text{/s}} = 70{,}126$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 2 + \left[0.4\text{Re}^{0.5} + 0.06\text{Re}^{2/3}\right]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$
$$= 2 + \left[0.4(70{,}126)^{0.5} + 0.06(70{,}126)^{2/3}\right](0.7336)^{0.4}\left(\frac{1.778\times10^{-5}}{1.802\times10^{-5}}\right)^{1/4} = 185.1$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02439\text{ W/m.°C}}{0.3\text{ m}}(185.1) = 15.05\text{ W/m}^2\text{.°C}$$

Then the surface temperature of the head is determined to be

$$A_s = \pi D^2 = \pi(0.3\text{ m})^2 = 0.2827\text{ m}^2$$
$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 10\text{ °C} + \frac{(84/4)\text{ W}}{(15.05\text{ W/m}^2\text{.°C})(0.2827\text{ m}^2)} = \mathbf{14.9\text{ °C}}$$

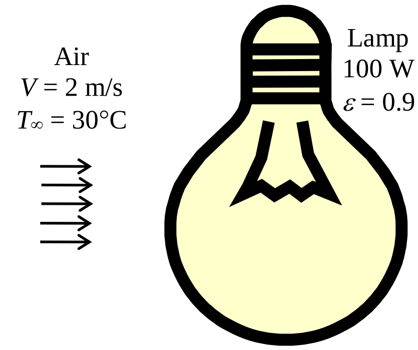
Discussion This calculated surface temperature is close to the assumed temperature of 15°C making this a good assumption. Also, the Nusselt number relation for flow over a sphere (Eq. 7-36) is valid for $1.0 \le \mu_\infty/\mu_s \le 3.2$. In this problem, $\mu_\infty/\mu_s = 0.99$, which is very close to 1.0.

7-96 A light bulb is cooled by a fan. The equilibrium temperature of the glass bulb is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The light bulb is in spherical shape. **4** The local atmospheric pressure is 1 atm.

Properties We assume the surface temperature to be 100°C for viscosity based on the problem statement. The properties of air at 1 atm pressure and the free stream temperature of 30°C are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$
$$\nu = 1.608\times10^{-5} \text{ m}^2\text{/s}$$
$$\mu_\infty = 1.872\times10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@100°C} = 2.181\times10^{-5} \text{ kg/m.s}$$
$$\Pr = 0.7282$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(2\text{ m/s})(0.1\text{m})}{1.608\times10^{-5}\text{ m}^2\text{/s}} = 1.244\times10^4$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 2 + \left[0.4\,\text{Re}^{0.5} + 0.06\,\text{Re}^{2/3}\right]\Pr^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$
$$= 2 + \left[0.4(1.244\times10^4)^{0.5} + 0.06(1.244\times10^4)^{2/3}\right](0.7282)^{0.4}\left(\frac{1.872\times10^{-5}}{2.181\times10^{-5}}\right)^{1/4}$$
$$= 67.14$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02588\text{ W/m.°C}}{0.1\text{ m}}(67.14) = 17.37\text{ W/m}^2\text{.°C}$$

Noting that 90 % of electrical energy is converted to heat,

$$\dot{Q} = (0.90)(100\text{ W}) = 90\text{ W}$$

The bulb loses heat by both convection and radiation. The equilibrium temperature of the glass bulb can be determined by iteration or by an equation solver:

$$A_s = \pi D^2 = \pi(0.1\text{ m})^2 = 0.0314\text{ m}^2$$

$$\dot{Q}_{\text{total}} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$
$$90\text{ W} = (17.37\text{ W/m}^2\text{.°C})(0.0314\text{ m}^2)\left[T_s - (30+273)\text{K}\right]$$
$$+ (0.9)(0.0314\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)\left[T_s^4 - (30+273\text{ K})^4\right]$$

$$T_s = 409.9\text{ K} = \mathbf{136.9°C}$$

Discussion This surface temperature is not close to the assumed surface temperature of 100°C. For better accuracy, we can repeat the calculations using a new viscosity value at 136.9°C: $\mu_{s@136.9°C} = 2.332\times10^{-5}$ kg/m.s (Table A-15). It gives T_s = 412.6 K = 139.6°C. The difference between the two results is 2.7°C.

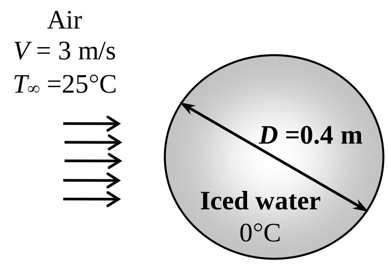
Also, the Nusselt number relation for flow over a sphere (Eq. 7-36) is valid for $1.0 \le \mu_\infty/\mu_s \le 3.2$. In this problem, $\mu_\infty/\mu_s = 0.86$, which is less than 1.0. However, it is sufficiently close to 1, which is appropriate for an engineering heat transfer analysis. Note that the effect of viscosity ratio on the value of Nusselt number is minor.

7-97 Air flows over a spherical tank containing iced water. The rate of heat transfer to the tank and the rate at which ice melts are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm pressure and the free stream temperature of 25°C are (Table A-15)

$$k = 0.02551\ \text{W/m.°C}$$
$$\nu = 1.562\times10^{-5}\ \text{m}^2/\text{s}$$
$$\mu_\infty = 1.849\times10^{-5}\ \text{kg/m.s}$$
$$\mu_{s,\,@\,0°\text{C}} = 1.729\times10^{-5}\ \text{kg/m.s}$$
$$\text{Pr} = 0.7296$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(3\ \text{m/s})(0.4\ \text{m})}{1.562\times10^{-5}\ \text{m}^2/\text{s}} = 76{,}825$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 2 + \left[0.4\text{Re}^{0.5} + 0.06\text{Re}^{2/3}\right]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$
$$= 2 + \left[0.4(76{,}825)^{0.5} + 0.06(76{,}825)^{2/3}\right](0.7296)^{0.4}\left(\frac{1.849\times10^{-5}}{1.729\times10^{-5}}\right)^{1/4} = 198.6$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02551\ \text{W/m.°C}}{0.4\ \text{m}}(198.6) = 12.67\ \text{W/m}^2.\text{°C}$$

Then the rate of heat transfer is determined to be

$$A_s = \pi D^2 = \pi(0.4\ \text{m})^2 = 0.5027\ \text{m}^2$$
$$\dot{Q} = hA_s(T_s - T_\infty) = (12.67\ \text{W/m}^2.\text{°C})(0.5027\ \text{m}^2)(25-0)\text{°C} = \mathbf{159.2\ W}$$

The rate at which ice melts is

$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{0.1592\ \text{kW}}{333.7\ \text{kJ/kg}} = 4.771\times10^{-4}\ \text{kg/s} = \mathbf{1.72\ kg/h}$$

7-98 The temperature of a hot air stream is to be measured by a spherical thermocouple junction. The time it takes to register 99%of the initial ΔT should be within 5 s, and the junction diameter is to be determined.

Assumptions **1** The junction is spherical in shape. **2** The thermal properties of the junction are constant.**3**Air behaves as ideal gas at 1 atm. **4** Radiation effects are negligible. **5** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the junction are given to be k_{TC} = 35 W/m·K, ρ_{TC} = 8500 kg/m^3, and $c_{p,TC}$ = 320 J/kg·K. The properties of air at T_∞ = 140°C are k = 0.03374 W/m·K, $\nu = 2.745\times 10^{-5}$ m^2/s,$\mu_\infty = 2.345\times 10^{-5}$kg/m·s, and Pr = 0.7054 (Table A-15). The dynamic viscosity of air at the surface T_{ave} = (20°C + 140°C)/2 = 80°C is $\mu_s = 2.096\times 10^{-5}$kg/m·s.

Analysis The Reynolds number and the Nusselt number for flow across a sphere are

$$\mathrm{Re} = \frac{VD}{\nu} \quad \text{and} \quad \mathrm{Nu} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{0.5} + 0.06\,\mathrm{Re}^{2/3}]\,\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

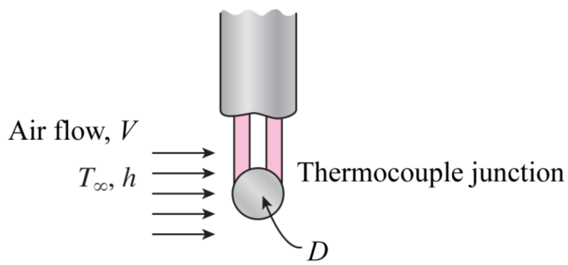
Using lumped system analysis on the thermocouple junction (see Chapter 4),

$$L_c = \frac{\mathcal{V}}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} \quad \text{and} \quad \mathrm{Bi} = \frac{hL_c}{k_{\mathrm{TC}}} = \frac{hD}{6k_{\mathrm{TC}}}$$

The time period for the thermocouple to read 99% of the initial temperature difference is determined from

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} = 0.01 \quad \text{where} \quad b = \frac{hA_s}{\rho_{\mathrm{TC}}\, c_{p,\mathrm{TC}}\, \mathcal{V}} = \frac{h}{\rho_{\mathrm{TC}}\, c_{p,\mathrm{TC}}\, L_c} = \frac{6h}{\rho_{\mathrm{TC}}\, c_{p,\mathrm{TC}}\, D}$$

The problem is solved using EES, and the solution is given below:




```
"GIVEN"
T_infinity=140 [C]
T_s_i=20 [C]
V=3 [m/s]
t=5 [s]
"PROPERTIES"
"Air"
Fluid$='air'
T_s_ave=1/2*(T_s_i+T_infinity)
k=Conductivity(Fluid$, T=T_infinity)
Pr=Prandtl(Fluid$, T=T_infinity)
rho=Density(Fluid$, T=T_infinity, P=101.3)
mu_infinity=Viscosity(Fluid$, T=T_infinity)
mu_s=Viscosity(Fluid$, T=T_s_ave)
nu=mu_infinity/rho
"Thermocouple junction"
c_p_TC=320 [J/kg-K]
k_TC=35 [W/m-K]
rho_TC=8500 [kg/m^3]
"ANALYSIS"
"Flow across a sphere"
Re=V*D/nu
Nusselt=2+(0.4*Re^0.5+0.06*Re^(2/3))*Pr^0.4*(mu_infinity/mu_s)^0.25
h=Nusselt*k/D
"Lumped system analysis"
L_c=D/6
Bi=h*L_c/k_TC
b=h/(rho_TC*c_p_TC*L_c)
ln(0.01)=-b*t
```

Thus, the final results are

$$\mathrm{Re} = 76.75,\ \mathrm{Nu} = 6.104,\quad h = 293.2\ \mathrm{W/m^2\cdot K},\quad L_c = 0.0001171\,\mathrm{m},\quad \mathrm{Bi} = 0.0009807,$$

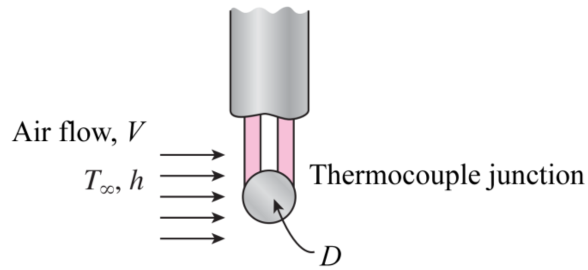
$$b = 0.921\,\mathrm{s}^{-1},\quad D = \mathbf{0.0007023\,m}$$

Discussion For the thermocouple to register 99% of the initial temperature difference within 5 s, the junction diameter should be 0.7 mm or less. The smaller the junction size, the faster the thermocouple would response. Since Bi < 0.1, the lumped system analysis is valid.

7-99  The temperature of a hot air stream is to be measured by a spherical thermocouple junction. The time it takes to register 99% of the initial ΔT should be within 5 s. The effect of the air velocity on the thermocouple junction diameter that would satisfy the required response time of 5 s is to be evaluated.

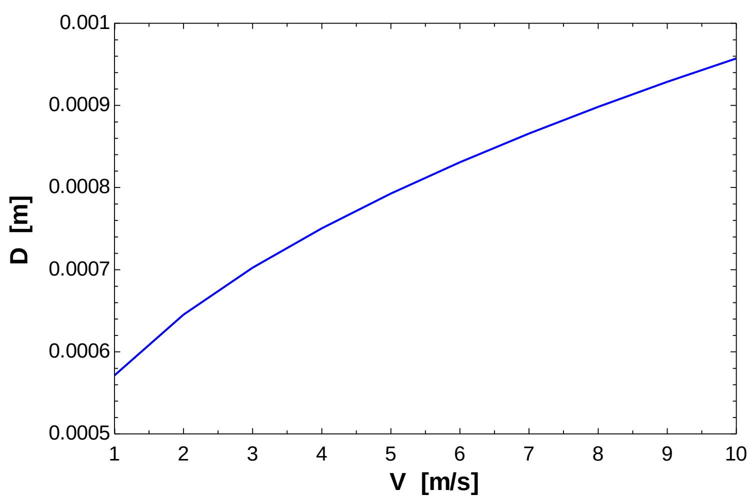
Assumptions **1** The junction is spherical in shape. **2** The thermal properties of the junction are constant. **3** Air behaves as ideal gas at 1 atm. **4** Radiation effects are negligible. **5** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Analysis The problem is solved using EES, and the solution is given below:




```
"GIVEN"
T_infinity=140 [C]
T_s_i=20 [C]
t=5 [s]

"PROPERTIES"
"Air"
Fluid$='air'
T_s_ave=1/2*(T_s_i+T_infinity)
k=Conductivity(Fluid$, T=T_infinity)
Pr=Prandtl(Fluid$, T=T_infinity)
rho=Density(Fluid$, T=T_infinity, P=101.3)
mu_infinity=Viscosity(Fluid$, T=T_infinity)
mu_s=Viscosity(Fluid$, T=T_s_ave)
nu=mu_infinity/rho
"Thermocouple junction"
c_p_TC=320 [J/kg-K]
k_TC=35 [W/m-K]
rho_TC=8500 [kg/m^3]

"ANALYSIS"
"Flow across a sphere"
Re=V*D/nu
Nusselt=2+(0.4*Re^0.5+0.06*Re^(2/3))*Pr^0.4*(mu_infinity/mu_s)^0.25
h=Nusselt*k/D
"Lumped system analysis"
L_c=D/6
Bi=h*L_c/k_TC
b=h/(rho_TC*c_p_TC*L_c)
ln(0.01)=-b*t
```

V [m/s]	D [m]	Bi	Re
1	0.0005712	0.0006488	20.81
2	0.0006453	0.0008278	47.01
3	0.0007023	0.0009807	76.75
4	0.0007504	0.0011200	109.3
5	0.0007927	0.0012490	144.4
6	0.0008309	0.0013730	181.6
7	0.0008658	0.0014910	220.8
8	0.0008983	0.0016040	261.8
9	0.0009287	0.0017150	304.5
10	0.0009573	0.0018220	348.7



Discussion As the air velocity increases, thereby increasing the h, the thermocouple junction diameter can be increased while still satisfying the required response time of 5 s. The smaller the junction size, the faster the thermocouple would response.

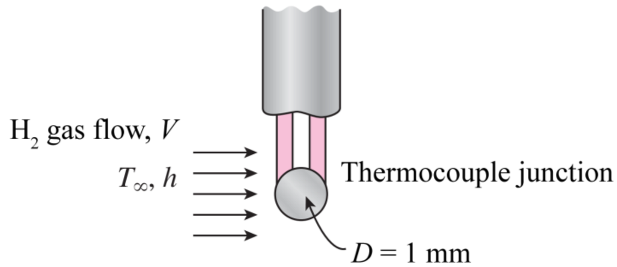
Since Bi < 0.1, the lumped system analysis is valid.

7-100 The temperature of H_2 gas stream is to be measured by a spherical thermocouple junction. The time it takes to register 99%of the initial $\Delta T$and the convection heat transfer coefficient as functions of the free stream velocity are to be evaluated.

Assumptions **1** The junction is spherical in shape. **2** The thermal properties of the junction are constant. **3** H_2 gas behaves as ideal gas at 1 atm. **4** Radiation effects are negligible. **5** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

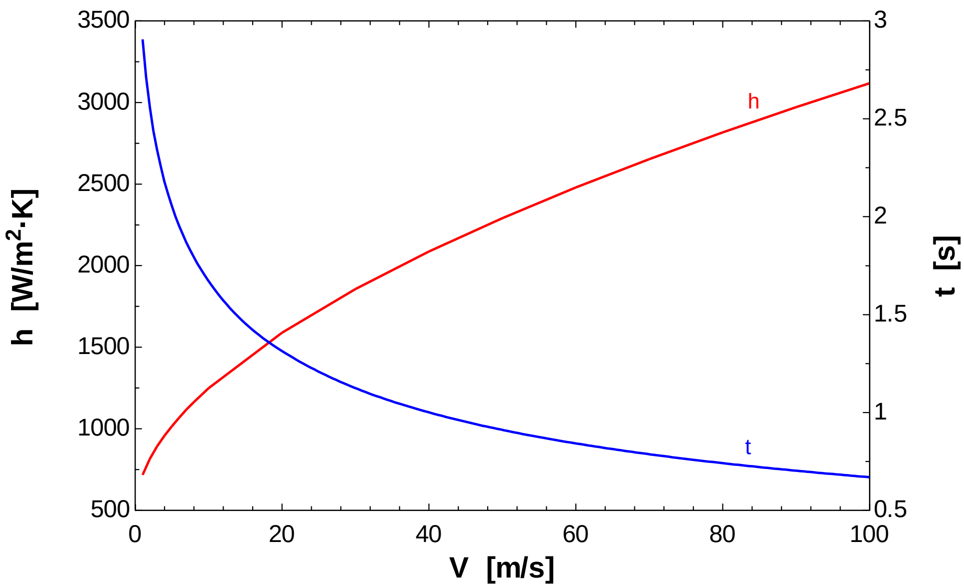
Properties The properties of the junction are given to be k_{TC} = 35 W/m·K, ρ_{TC} = 8500 kg/m^3, and $c_{p,TC}$ = 320 J/kg·K. The properties of H_2 gas are evaluated at T_∞ = 200°C, and the dynamic viscosity μ_s of H_2 gas at the sphere surface is evaluated at T_{ave} = (10°C + 200°C)/2 = 105°C

Analysis The problem is solved using EES, and the solution is given below:




```
"GIVEN"
T_infinity=200 [C]
T_s_i=10 [C]
D=0.001 [m]

"PROPERTIES"
"H2 gas"
Fluid$='H2'
T_s_ave=1/2*(T_s_i+T_infinity)
k=Conductivity(Fluid$, T=T_infinity)
Pr=Prandtl(Fluid$, T=T_infinity)
rho=Density(Fluid$, T=T_infinity, P=101.3)
mu_infinity=Viscosity(Fluid$, T=T_infinity)
mu_s=Viscosity(Fluid$, T=T_s_ave)
nu=mu_infinity/rho
"Thermocouple junction"
c_p_TC=320 [J/kg-K]
k_TC=35 [W/m-K]
rho_TC=8500 [kg/m^3]

"ANALYSIS"
"Flow across a sphere"
Re=V*D/nu
Nusselt=2+(0.4*Re^0.5+0.06*Re^(2/3))*Pr^0.4*(mu_infinity/mu_s)^0.25
h=Nusselt*k/D
"Lumped system analysis"
L_c=D/6
Bi=h*L_c/k_TC
b=h/(rho_TC*c_p_TC*L_c)
ln(0.01)=-b*t
```

V [m/s]	t [s]	h [W/m^2·K]	Re	Bi
1	2.906	718.4	4.227	0.003421
2	2.558	816.2	8.453	0.003886
3	2.339	892.6	12.68	0.004251
4	2.179	958.0	16.91	0.004562
5	2.054	1016	21.13	0.004839
6	1.953	1069	25.36	0.005091
7	1.867	1118	29.59	0.005325
8	1.793	1164	33.81	0.005544
9	1.729	1208	38.04	0.005751
10	1.671	1249	42.27	0.005948
20	1.314	1589	84.53	0.007566
30	1.124	1857	126.8	0.008841
40	1.001	2086	169.1	0.009935
50	0.911	2292	211.3	0.010910
60	0.8421	2479	253.6	0.011800
70	0.7869	2653	295.9	0.012630
80	0.7413	2816	338.1	0.013410
90	0.7027	2971	380.4	0.014150
100	0.6696	3118	422.7	0.014850



Discussion The convection heat transfer coefficient increases with increasing H_2 gas velocity. As the convection heat transfer coefficient increases, the time for the thermocouple to register 99% of the initial temperature difference decreases. To decrease the response time at low velocity, a smaller thermocouple junction should be used.

For this analysis, Bi < 0.1, therefore the lumped system analysis is valid.

7-101 A glass spherical tank that is filled with chemicals undergoing exothermic reaction, has a known inner surface temperature. The outer surface of the tank is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with local atmospheric pressureat 1 atm. **4** One-dimensional heat conduction through tank wall. **5** The thermal conductivity of the tank wallis constant.

Properties The thermal conductivity of the tank wall is given to be k_{tank} = 1.1 W/m·K.

The properties of air at T_∞ = 15°C are k = 0.02476 W/m·K,ν = 1.470× 10^{-5} m^2/s,μ_∞ = 1.802× 10^{-5}kg/m·s, and Pr = 0.7323 (Table A-15). The dynamic viscosity of air at the surface $T_{s,o}$ is to be solved using EES.

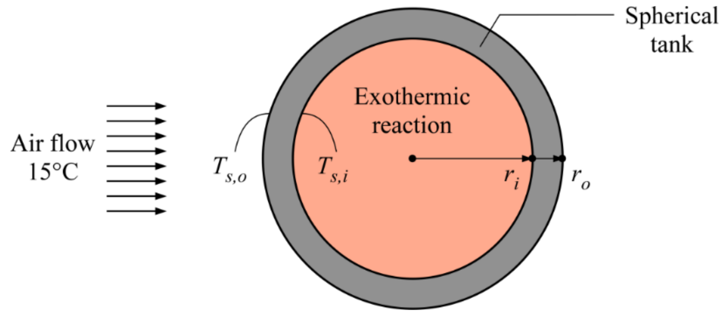
Analysis The convection heat transfer coefficient on the outer surface can be determined using the Nusselt number relation for flow across a sphere. The Reynolds number and the Nusselt number for flow across a sphere are

$$\text{Re} = \frac{VD_o}{\nu} \quad \text{and} \quad \text{Nu} = \frac{hD_o}{k} = 2 + [0.4\,\text{Re}^{0.5} + 0.06\,\text{Re}^{2/3}]\,\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

The inner and outer radii of the tank are

$$r_i = 0.5\,\text{m} \quad \text{and} \quad r_o = (0.5 + 0.01)\,\text{m} = 0.51\,\text{m}$$

From Chapter 2 the rate of heat transfer at the tank's outer surface can be expressed as



$$\dot{Q}_{\text{sph}} = \dot{Q}_{\text{conv}}$$

$$4\pi k_{\text{tank}}\, r_i\, r_o \frac{T_{s,i} - T_{s,o}}{r_o - r_i} = h(4\pi r_o^2)(T_{s,o} - T_\infty)$$

$$k_{\text{tank}} \frac{r_i}{r_o} \frac{T_{s,i} - T_{s,o}}{r_o - r_i} = h(T_{s,o} - T_\infty)$$

where

h = 70 W/m^2 K, $T_{s,i}$ = 80°C, and T_∞ = 15°C.

The problem is solved using EES, and the solution is given below:

```
"GIVEN"
"h=70 [W/(m^2*K)]""convection heat transfer coefficient"
r_i=0.5 [m]   "inner radius"
r_o=r_i+0.010 [m]   "outer radius"
T_s_i=80 [C]   "inner surface temperature"
T_infinity=15 [C]   "ambient temperature"
V=5 [m/s]

"PROPERTIES"
"Air"
Fluid$='air'
k=Conductivity(Fluid$, T=T_infinity)
Pr=Prandtl(Fluid$, T=T_infinity)
rho=Density(Fluid$, T=T_infinity, P=101.3)
mu_infinity=Viscosity(Fluid$, T=T_infinity)
mu_s=Viscosity(Fluid$, T=T_s_o)
nu=mu_infinity/rho
k_tank=1.1 [W/(m*K)]   "Thermal conductivity of tank wall"

"ANALYSIS"
"Flow across a sphere"
D_o=2*r_o
Re=V*D_o/nu
Nusselt=2+(0.4*Re^0.5+0.06*Re^(2/3))*Pr^0.4*(mu_infinity/mu_s)^0.25
```

```
h=Nusselt*k/D_o
"SOLVING FOR OUTER SURFACE TEMPERATURE AND k_avg"
q_dot_sph=k_tank*r_i/r_o*(T_s_i-T_s_o)/(r_o-r_i)   "heat flux through the spherical layer"
q_dot_conv=h*(T_infinity-T_s_o)   "heat flux by convection"
q_dot_sph+q_dot_conv=0
```

Thus,

$T_{s,o}$ = **76.96°C**

Discussion The tank's outer surface temperature is about 27°C higher than the safe temperature of 50°C. Preventive measures, such as insulating the tank's outer surface, should be taken to reduce risks of thermal burn hazards.

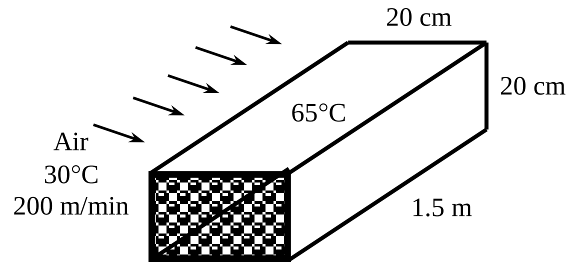
This problem can also be solved by hand calculation with an initial guess of $T_{s,o}$, which is used for evaluating μ_s. Then, the solution is solved iteratively until $T_{s,o}$ converges.

7-102 The components of an electronic system located in a horizontal duct is cooled by air flowing over the duct. The total power rating of the electronic device is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (65+30)/2 = 47.5°C$ are (Table A-15)

$$k = 0.02717 \text{ W/m.°C}$$
$$\nu = 1.774\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7235$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{[(200/60)\text{ m/s}](0.2\text{ m})}{1.774\times10^{-5}\text{ m}^2\text{/s}} = 3.758\times10^4$$

Using the relation for a square duct from Table 7-1, the Nusselt number is determined to be

$$Nu = \frac{hD}{k} = 0.094\,\text{Re}^{0.675}\,\text{Pr}^{1/3} = 0.094(3.758\times10^4)^{0.675}(0.7235)^{1/3} = 103.4$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02717\text{ W/m.°C}}{0.2\text{ m}}(103.4) = 14.05\text{ W/m}^2\text{.°C}$$

Then the rate of heat transfer from the duct becomes

$$A_s = (4\times0.2\text{ m})(1.5\text{ m}) = 1.2\text{ m}^2$$

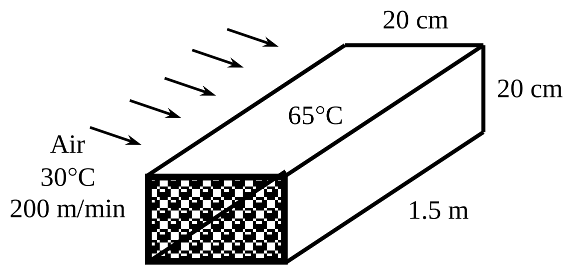
$$\dot{Q} = hA_s(T_s - T_\infty) = (14.05\text{ W/m}^2\text{.°C})(1.2\text{ m}^2)(65-30)\text{°C} = \mathbf{590\ W}$$

7-103 The components of an electronic system located in a horizontal duct is cooled by air flowing over the duct. The total power rating of the electronic device is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties.

Properties The properties of air at 1 atm and the film temperature of (T_s + T_∞)/2 = (65+30)/2 = 47.5°C are (Table A-15)

$$k = 0.02717 \text{ W/m.°C}$$
$$\nu = 1.774\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7235$$



For a location at 3000 m altitude where the atmospheric pressure is 70.12 kPa, only kinematic viscosity of air will be affected. Thus,

$$\nu_{@61.66\,\text{kPa}} = \left(\frac{101.325}{70.12}\right)(1.774\times10^{-5}) = 2.563\times10^{-5} \text{ m}^2\text{/s}$$

Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{[(200/60)\text{ m/s}](0.2\text{ m})}{2.563\times10^{-5}\text{ m}^2\text{/s}} = 2.601\times10^4$$

Using the relation for a square duct from Table 7-1, the Nusselt number is determined to be

$$Nu = \frac{hD}{k} = 0.094\,\text{Re}^{0.675}\,\text{Pr}^{1/3} = 0.094(2.601\times10^4)^{0.675}(0.7235)^{1/3} = 80.63$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02717\text{ W/m.°C}}{0.2\text{ m}}(80.63) = 10.95\text{ W/m}^2\text{.°C}$$

Then the rate of heat transfer from the duct becomes

$$A_s = (4\times0.2\text{ m})(1.5\text{ m}) = 1.2\text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (10.95\text{ W/m}^2\text{.°C})(1.2\text{ m}^2)(65-30)\text{°C} = \mathbf{460\ W}$$

7-104 C&S An ASTM A479 904L stainless steel bar connects two metal plates. Hot air flows across the square bar. A cooling mechanism removes heat at a rate of 50 W from the bar. The surface temperature of the bar is to be determined to see if it exceeds the maximum use temperature of 260°C.

Assumptions **1** The flow is steady and incompressible. **2** Uniform surface temperature. **3** Wall effects from the plates on the bar are negligible.

Properties Using the maximum use temperature for the bar as the surface temperature, the properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (260 + 340)$°C/2 = 300°C are (Table A–15): Pr = 0.6935, k = 0.04418 W/m·K, and $\nu = 4.765 \times 10^{-5}$ m^2/s

Analysis The Reynolds number is

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(25 \text{ m/s})(0.010 \text{ m})}{4.765 \times 10^{-5} \text{m}^2/\text{s}} = 5247$$

Using the empirical correlation for square bar (Table 7-1), the convection heat transfer coefficient is

$$h = \left(\frac{k}{D}\right) 0.094 \,\mathrm{Re}^{0.675}\mathrm{Pr}^{1/3} = \left(\frac{0.04418 \text{ W/m}\cdot\text{K}}{0.010 \text{ m}}\right) 0.094\,(5247)^{0.675}(0.6935)^{1/3} = 119.2 \text{ W/m}^2\cdot\text{K}$$

From the Newton's law of cooling, the surface temperature of the bar is

$$\dot{Q} = hA(T_\infty - T_s) \quad \Longrightarrow \quad T_s = -\frac{\dot{Q}}{h(4DL)} + T_\infty = -\frac{50 \text{ W}}{(119.2 \text{ W/m}^2\cdot\text{K})(4 \times 0.010 \text{ m} \times 0.10 \text{ m})} + 340 = \mathbf{235.1}℃ < 260℃$$

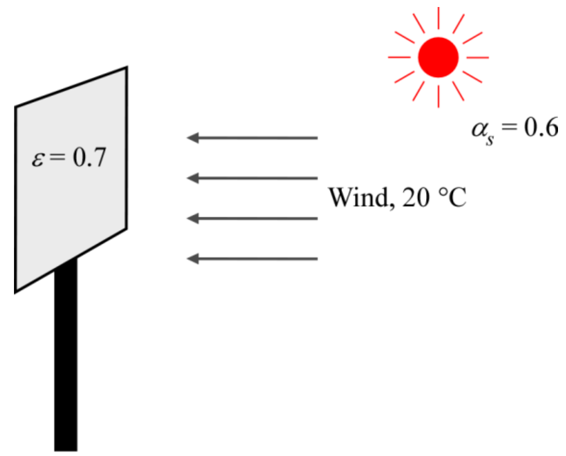
Discussion The heat being removed from the bar at 50 W is sufficient to keep the bar below the maximum use temperature of 260°C. To maintain the surface temperature of the bar at 260°C, the cooling mechanism needs to remove heat at a rate of 41.4 W.

7-105 A street sign surface is subjected to radiation and cross flow wind, the surface temperature of the street sign is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The surface temperature is constant. **4** The street sign is treated a vertical plate in cross flow.

Properties The properties of air (1 atm) at 30°C are given in Table A-15: k = 0.02588 W/m·K, $\nu = 1.608 \times 10^{-5}$ m²/s, and Pr = 0.7282.

Analysis The Reynolds number is



$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(1\,\mathrm{m/s})(0.2\,\mathrm{m})}{1.608 \times 10^{-5}\,\mathrm{m^2/s}} = 1.244 \times 10^4$$

From Table 7-1, the relation for Nusselt number is

$$\mathrm{Nu} = \frac{hD}{k} = 0.257\,\mathrm{Re}^{0.731}\,\mathrm{Pr}^{1/3}$$

$$h = \frac{0.02588\ \mathrm{W/m \cdot K}}{0.2\ \mathrm{m}}(0.257)(12440)^{0.731}(0.7282)^{1/3} = 29.46\ \mathrm{W/m^2 \cdot K}$$

From energy balance, we obtain

$$\alpha_s \dot{q}_{\mathrm{solar}} = h[T_s - T_\infty] + \varepsilon\sigma[T_s^4 - T_{\mathrm{surr}}^4]$$

Copy the following lines and paste on a blank EES screen to solve the above equation:

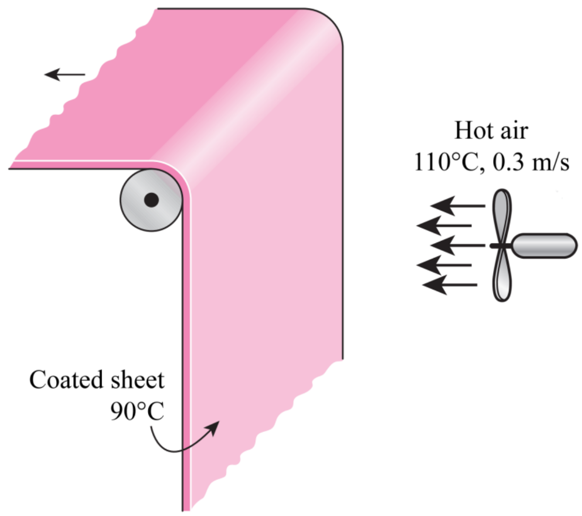
```
h=29.46
q_incindent=1100
T_inf=20+273
T_surr=20+273
alpha=0.6
epsilon=0.7
sigma=5.670e-8
alpha*q_incindent=h*(T_s-T_inf)+epsilon*sigma*(T_s^4-T_surr^4)
```

Solving by EES software, the surface temperature of the street sign is

$$T_s = 312.5\,\mathrm{K} = \mathbf{39.5°C}$$

Discussion Note that absolute temperatures must be used in calculations involving the radiation heat transfer equation.

7-106 A coated sheet is being dried with hot air in cross flow. The convection heat transfer coefficient and the heat flux added to the sheet surface are to be determined.



Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The surface temperature is constant. **4** The coated sheet is treated as a vertical plate in cross flow.

Properties The properties of air at T_f = (110°C + 90°C)/2 = 100°C are k = 0.03095 W/m·K, ν = 2.306 × 10^{-5} m^2/s, and Pr = 0.7111 (Table A-15).

Analysis The Reynolds number is

$$\text{Re} = \frac{\text{VD}}{\nu} = \frac{(0.3\text{m/s})(1\text{m})}{2.306\times10^{-5}\ \text{m}^2/\text{s}} = 13010$$

From Table 7-1, the relation for Nusselt number is

$$\text{Nu} = \frac{\text{hD}}{\text{k}} = 0.257\,\text{Re}^{0.731}\,\text{Pr}^{1/3}$$

$$h = \left(\frac{0.03095\ \text{W/m}\cdot\text{K}}{1\,\text{m}}\right)0.257(13010)^{0.731}(0.7111)^{1/3} = \mathbf{7.224\ W/m^2\cdot K}$$

The heat flux added to the coated sheet surface is

$$\dot{q}_s = h(T_\infty - T_s) = (7.224\ \text{W/m}^2\cdot\text{K})(110-90)\ \text{K} = \mathbf{144.5\ W/m^2}$$

Discussion The relation for the Nusselt number is valid for 6300 ≤ Re ≤ 23,600. Thus, the applicability of the relation is limited to low air velocity (less than 0.55 m/s) for the 1-m long plate.

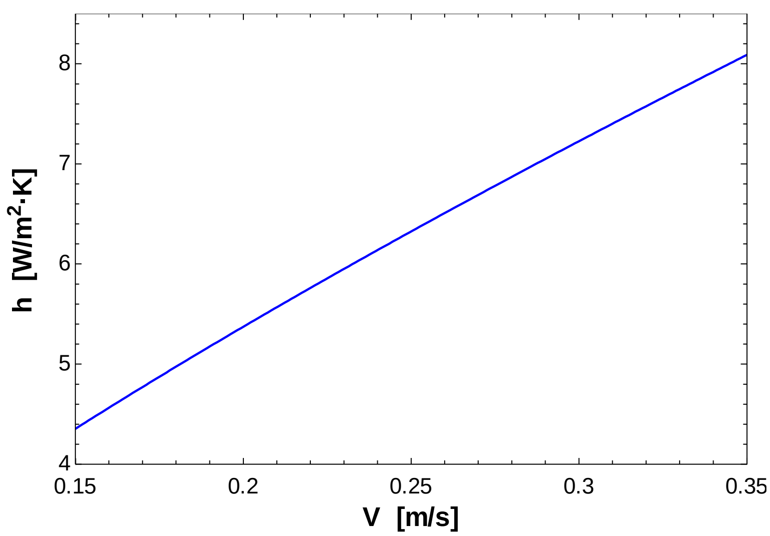
7-107 A coated sheet is being dried with hot air in cross flow. The effect of air velocity on the convection heat transfer coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The surface temperature is constant. **4** The coated sheet is treated a vertical plate in cross flow.

Analysis The problem is solved using EES, and the solution is given below:

```
"GIVEN"
T_infinity=110 [C]
T_s=90 [C]
L=1 [m]

"PROPERTIES"
Fluid$='air'
T_film=1/2*(T_s+T_infinity)
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho

"ANALYSIS"
Re=V*L/nu
Nusselt=0.257*Re^0.731*Pr^(1/3)      "Nusselt number for vertical plate in cross flow"
h=Nusselt*k/L
q_dot=h*(T_infinity-T_s)
```

V [m/s]	h [W/m^2·K]	$\dot{q}$ [W/m^2]
0.15	4.354	87.08
0.175	4.873	97.46
0.20	5.373	107.5
0.225	5.856	117.1
0.25	6.325	126.5
0.275	6.781	135.6
0.30	7.227	144.5
0.325	7.662	153.2
0.35	8.089	161.8



Discussion The low values of convection heat transfer coefficient indicate that natural convection might be a significant factor. The Reynolds number range corresponding to 0.15 ≤V≤ 0.35 m/s is 6500 < Re < 15200. Thus, the heat transfer correlation is still valid in this range.

Flow across Tube Banks

7-108C The level of turbulence, and thus the heat transfer coefficient, increases with row number because of the combined effects of upstream rows in turbulence caused and the wakes formed. But there is no significant change in turbulence level after the first few rows, and thus the heat transfer coefficient remains constant. There is no change in transverse direction.

7-109C In tube banks, the flow characteristics are dominated by the *maximum velocity* V_{max} that occurs within the tube bank rather than the approach velocity V. Therefore, the Reynolds number is defined on the basis of maximum velocity.

7-110 Air is heated by hot tubes in a tube bank. The average heat transfer coefficient is to be determined.

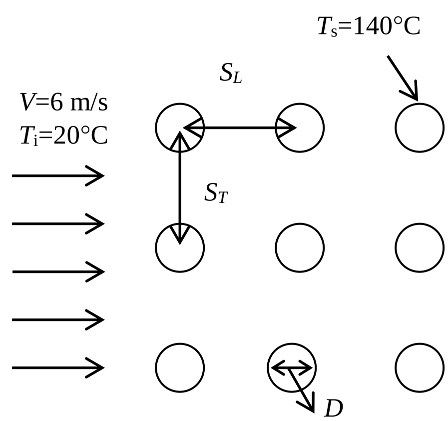
Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the tubes is constant.

Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of 70°C and 1 atm based on the problem statement (Table A-15):

k = 0.02881 W/m-K $\qquad \rho$ = 1.028 kg/m^3

c_p=1.007 kJ/kg-K $\qquad$ Pr = 0.7177

$\mu = 2.052\times10^{-5}$ kg/m-s $\qquad \mathrm{Pr}_s = \mathrm{Pr}_{@\,T_s\,=\,140°C} = 0.7041$



Analysis It is given that D = 0.02 m, S_L= S_T= 0.06 m, and V= 6 m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become

$$V_{\max} = \frac{S_T}{S_T - D}V = \frac{0.06}{0.06-0.02}(6\text{ m/s}) = 9\text{ m/s}$$

$$\mathrm{Re}_D = \frac{\rho V_{\max} D}{\mu} = \frac{(1.028\text{ kg/m}^3)(9\text{ m/s})(0.02\text{m})}{2.052\times10^{-5}\text{ kg/m}\cdot\text{s}} = 9018$$

The average Nusselt number is determined using the proper relation from Table 7-2 to be

$$\begin{aligned}\mathrm{Nu}_D &= 0.27\,\mathrm{Re}_D^{0.63}\,\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}\\ &= 0.27(9018)^{0.63}(0.7177)^{0.36}(0.7177/0.7041)^{0.25}\\ &= 74.70\end{aligned}$$

Since N_L> 16, the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\mathrm{Nu}_{D,N_L} = \mathrm{Nu}_D = 74.70$$

$$h = \frac{Nu_{D,N_L}k}{D} = \frac{74.70(0.02881\text{ W/m}\cdot°\text{C})}{0.02\text{ m}} = \mathbf{107.6\ W/m^2\cdot°C}$$

7-111 Water is heated by a bundle of resistance heater rods. The number of tube rows is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the rods is constant.

Properties The properties of water at the mean temperature of (15°C +65°C)/2=40°C are (Table A-9):

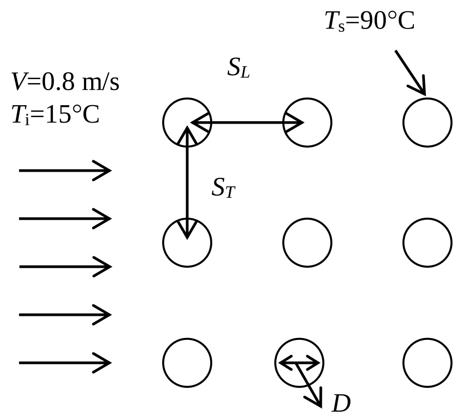
$k = 0.631$ W/m·K $\rho = 992.1$ kg/m^3

c_p=4.179 kJ/kg·K Pr = 4.32

$\mu = 0.653\times10^{-3}$ kg/m·s $Pr_s = Pr_{@\,Ts = 90°C} = 1.96$

Also, the density of water at the inlet temperature of 15°C (for use in the mass flow rate calculation at the inlet) is ρ_i =999.1 kg/m^3.

Analysis It is given that D = 0.01 m, S_L= 0.04 m and S_T= 0.03 m, and V= 0.8 m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become



$$V_{\max} = \frac{S_T}{S_T - D}V = \frac{0.03}{0.03-0.01}(0.8\text{ m/s}) = 1.20\text{ m/s}$$

$$\text{Re}_D = \frac{\rho V_{\max} D}{\mu} = \frac{(992.1\text{ kg/m}^3)(1.20\text{ m/s})(0.01\text{m})}{0.653\times10^{-3}\text{ kg/m}\cdot\text{s}} = 18{,}232$$

The average Nusselt number is determined using the proper relation from Table 7-2 to be

$$\begin{aligned}\text{Nu}_D &= 0.27\,\text{Re}_D^{0.63}\,\text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}\\ &= 0.27(18{,}232)^{0.63}(4.32)^{0.36}(4.32/1.96)^{0.25}\\ &= 269.3\end{aligned}$$

Assuming that N_L> 16, the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\text{Nu}_{D,N_L} = \text{Nu}_D = 269.3$$

$$h = \frac{Nu_{D,N_L}k}{D} = \frac{269.3(0.631\text{ W/m}\cdot°\text{C})}{0.01\text{m}} = 16{,}994\text{ W/m}^2\cdot°\text{C}$$

Consider one-row of tubes in the transpose direction (normal to flow), and thus take N_T =1. Then the heat transfer surface area becomes

$$A_s = N_{tube}\pi DL = (1\times N_L)\pi(0.01\text{m})(4\text{m}) = 0.1257N_L$$

Then the log mean temperature difference, and the expression for the rate of heat transfer become

$$\Delta T_{lm} = \frac{(T_s - T_i)-(T_s - T_e)}{\ln[(T_s - T_i)/(T_s - T_e)]} = \frac{(90-15)-(90-65)}{\ln[(90-15)/(90-65)]} = 45.51°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{lm} = (16{,}994\text{ W/m}^2\cdot°\text{C})(0.1257N_L)(45.51°\text{C}) = 97{,}220N_L$$

The mass flow rate of water through a cross-section corresponding to N_T =1 and the rate of heat transfer are

$$\dot{m} = \rho A_c V = (999.1\text{kg/m}^3)(4\times0.03\text{m}^2)(0.8\text{ m/s}) = 95.91\text{kg/s}$$

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (95.91\text{kg/s})(4179\text{J/kg.C})(65-15)°\text{C} = 2.004\times10^7\text{ W}$$

Substituting this result into the heat transfer expression above we find the number of tube rows

$$\dot{Q} = hA_s\Delta T_{lm} \rightarrow 2.004\times10^7\text{ W} = 97{,}220N_L \rightarrow N_L = \mathbf{206}$$

7-112 Combustion air is heated by condensing steam in a tube bank. The rate of heat transfer to air, the pressure drop of air, and the rate of condensation of steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the tubes is equal to the temperature of steam.

Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of 35°C based on the problem statement (will be checked later) and 1 atm (Table A-15):

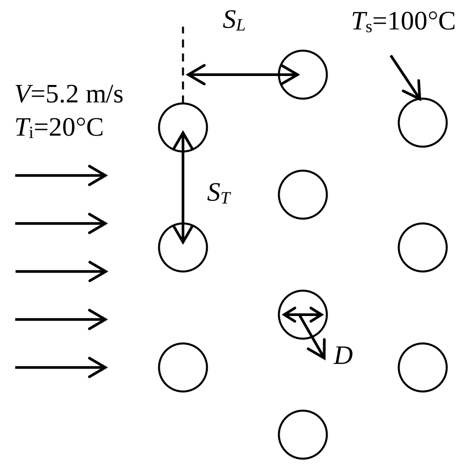
k = 0.02625 W/m·K $\quad$ ρ = 1.145 kg/m^3

c_p=1.007 kJ/kg·K $\quad$ Pr = 0.7268

μ = 1.895×10^{-5} kg/m·s $\quad$ $\Pr_s = \Pr_{@\,T_s = 100°C} = 0.7111$

Also, the density of air at the inlet temperature of 20°C (for use in the mass flow rate calculation at the inlet) is ρ_i = 1.204 kg/m^3. The enthalpy of vaporization of water at 100°C is h_{fg}= 2257 kJ/kg-K (Table A-9).

Analysis (*a*) It is given that D = 0.016 m, S_L= S_T= 0.04 m, and V= 5.2 m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become



$$V_{\max} = \frac{S_T}{S_T - D}V = \frac{0.04}{0.04-0.016}(5.2\text{ m/s}) = 8.667\text{ m/s}$$

since $S_D > (S_T + D)/2$

$$\text{Re}_D = \frac{\rho V_{\max} D}{\mu} = \frac{(1.145\text{ kg/m}^3)(8.667\text{ m/s})(0.016\text{m})}{1.895\times10^{-5}\text{ kg/m}\cdot\text{s}} = 8379$$

The average Nusselt number is determined using the proper relation from Table 7-2 to be

$$\text{Nu}_D = 0.35(S_T/S_L)^{0.2}\text{Re}_D^{0.6}\Pr^{0.36}(\Pr/\Pr_s)^{0.25}$$
$$= 0.35(0.04/0.04)^{0.2}(8379)^{0.6}(0.7268)^{0.36}(0.7268/0.7111)^{0.25} = 70.87$$

Since N_L =20, which is greater than 16, the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\text{Nu}_{D,N_L} = \text{Nu}_D = 70.87$$

$$h = \frac{Nu_{D,N_L}k}{D} = \frac{70.87(0.02625\text{ W/m}\cdot°\text{C})}{0.016\text{ m}} = 116.3\text{ W/m}^2\cdot°\text{C}$$

The total number of tubes is $N = N_L \times N_T$ = 20×10 = 200. For the given tube length (L = 3 m), the heat transfer surface area and the mass flow rate of air (evaluated at the inlet) are

$$A_s = N\pi DL = 200\pi(0.016\text{ m})(3\text{m}) = 30.16\text{ m}^2$$

$$\dot{m} = \dot{m}_i = \rho_i V(N_T S_T L) = (1.204\text{ kg/m}^3)(5.2\text{ m/s})(10)(0.04\text{ m})(3\text{m}) = 7.513\text{ kg/s}$$

Then the fluid exit temperature, the log mean temperature difference, and the rate of heat transfer become

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}c_p}\right) = 100 - (100-20)\exp\left(-\frac{(30.16\text{ m}^2)(116.3\text{ W/m}^2\cdot°\text{C})}{(7.513\text{ kg/s})(1007\text{ J/kg}\cdot°\text{C})}\right) = 49.68°\text{C}$$

$$\Delta T_{lm} = \frac{(T_s - T_i)-(T_s - T_e)}{\ln[(T_s - T_i)/(T_s - T_e)]} = \frac{(100-20)-(100-49.68)}{\ln[(100-20)/(100-49.68)]} = 64.01°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{lm} = (116.3\text{ W/m}^2\cdot°\text{C})(30.16\text{ m}^2)(64.02°\text{C}) = \mathbf{224{,}557\ W}$$

(*b*) For this staggered tube bank, the friction coefficient corresponding to Re$_D$ = 8379 and S_T/D = 4/1.6 = 2.5is, from Fig. 7-27b, f = 0.33. Also, χ = 1 for the square arrangements. Then the pressure drop across the tube bank becomes

$$\Delta P = N_L f\chi\frac{\rho V_{\max}^2}{2} = 20(0.33)(1)\frac{(1.145\text{ kg/m}^3)(8.667\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{284\ \ Pa}$$

(*c*) The rate of condensation of steam is

$$\dot{Q} = \dot{m}_{\text{cond}}h_{fg@100°\text{C}} \longrightarrow \dot{m}_{\text{cond}} = \frac{\dot{Q}}{h_{fg@100°\text{C}}} = \frac{224.6\text{ kW}}{2257\text{ kJ/kg}\cdot°\text{C}} = 0.0995\text{ kg/s} = \mathbf{5.97\,kg/min}$$

Discussion The arithmetic mean fluid temperature is (T_i+ T_e)/2 = (20 + 49.7)/2 = 34.9°C, which is very close to the assumed value of 35°C. Therefore, there is no need to repeat calculations.

7-113 Combustion air is heated by condensing steam in a tube bank. The rate of heat transfer to air, the pressure drop of air, and the rate of condensation of steam are to be determined.

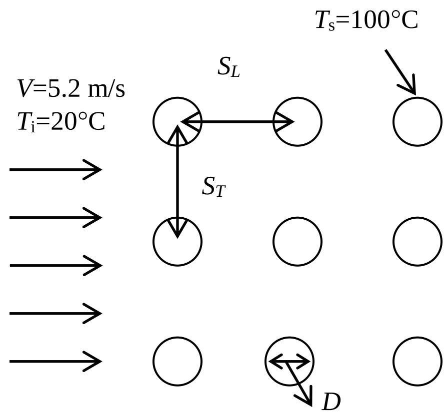
Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the tubes is equal to the temperature of steam.

Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of 35°C based on the problem statement (will be checked later) and 1 atm (Table A-15):

k = 0.02625 W/m·K　　　ρ = 1.145 kg/m^3
c_p=1.007 kJ/kg·K　　　Pr = 0.7268
$\mu = 1.895\times10^{-5}$ kg/m·s　　　$\Pr_s = \Pr_{@\,T_s = 100°C} = 0.7111$

Also, the density of air at the inlet temperature of 20°C (for use in the mass flow rate calculation at the inlet) is ρ_i = 1.204 kg/m^3. The enthalpy of vaporization of water at 100°C is h_{fg}= 2257 kJ/kg-K (Table A-9).

Analysis (*a*) It is given that D = 0.016 m, $S_L = S_T$= 0.06 m, and V= 5.2 m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become



$$V_{\max} = \frac{S_T}{S_T - D}V = \frac{0.06}{0.06-0.016}(5.2\text{ m/s}) = 7.091\text{ m/s}$$

$$\text{Re}_D = \frac{\rho V_{\max} D}{\mu} = \frac{(1.145\text{ kg/m}^3)(7.091\text{ m/s})(0.016\text{m})}{1.895\times10^{-5}\text{ kg/m}\cdot\text{s}} = 6855$$

The average Nusselt number is determined using the proper relation from Table 7-2 to be

$$\text{Nu}_D = 0.27\,\text{Re}_D^{0.63}\,\text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}$$
$$= 0.27(6855)^{0.63}(0.7268)^{0.36}(0.7268/0.7111)^{0.25} = 63.17$$

Since N_L =20, which is greater than 16, the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\text{Nu}_{D,N_L} = \text{Nu}_D = 63.17$$

$$h = \frac{Nu_{D,N_L}k}{D} = \frac{63.17(0.02625\text{ W/m}\cdot°\text{C})}{0.016\text{ m}} = 103.6\text{ W/m}^2\cdot°\text{C}$$

The total number of tubes is $N = N_L \times N_T$ = 20×10 = 200. For the given tube length (L = 3 m), the heat transfer surface area and the mass flow rate of air (evaluated at the inlet) are

$$A_s = N\pi DL = 200\pi(0.016\text{ m})(3\text{m}) = 30.16\text{ m}^2$$

$$\dot{m} = \dot{m}_i = \rho_i V(N_T S_T L) = (1.204\text{ kg/m}^3)(5.2\text{ m/s})(10)(0.06\text{ m})(3\text{m}) = 11.27\text{ kg/s}$$

Then the fluid exit temperature, the log mean temperature difference, and the rate of heat transfer become

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}c_p}\right) = 100 - (100-20)\exp\left(-\frac{(30.16\text{ m}^2)(103.6\text{ W/m}^2\cdot°\text{C})}{(11.27\text{ kg/s})(1007\text{ J/kg}\cdot°\text{C})}\right) = 39.25°\text{C}$$

$$\Delta T_{lm} = \frac{(T_s - T_i)-(T_s - T_e)}{\ln[(T_s - T_i)/(T_s - T_e)]} = \frac{(100-20)-(100-39.25)}{\ln[(100-20)/(100-39.25)]} = 69.93°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{lm} = (103.6\text{ W/m}^2\cdot°\text{C})(30.16\text{m}^2)(69.93°\text{C}) = 218{,}502\text{ W} = \mathbf{218.5\,kW}$$

(*b*) For this in-line arrangement tube bank, the friction coefficient corresponding to Re$_D$ = 6855 and S_L/D = 6/1.6 = 3.75is, from Fig. 7-27a, f = 0.12. Note that an accurate reading of friction factor does not seem to be possible in this case. Also, χ = 1 for the square arrangements. Then the pressure drop across the tube bank becomes

$$\Delta P = N_L f\chi\frac{\rho V_{\max}^2}{2} = 20(0.12)(1)\frac{(1.145\text{ kg/m}^3)(7.091\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{69.1\,Pa}$$

(*c*) The rate of condensation of steam is

$$\dot{Q} = \dot{m}_{\text{cond}}h_{fg@100°\text{C}} \longrightarrow \dot{m}_{\text{cond}} = \frac{\dot{Q}}{h_{fg@100°\text{C}}} = \frac{218.5\text{ kW}}{2257\text{ kJ/kg}\cdot°\text{C}} = 0.0968\text{ kg/s} = \mathbf{5.81\,kg/min}$$

Discussion The arithmetic mean fluid temperature is $(T_i + T_e)/2$ = (20 + 43.4)/2 = 31.7°C, which is fairly close to the assumed value of 35°C. Therefore, there is no need to repeat calculations.

7-114 Water is preheated by exhaust gases in a tube bank. The rate of heat transfer, the pressure drop of exhaust gases, and the temperature rise of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** For exhaust gases, air properties are used.

Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of 250°C based on the problem statement (will be checked later) and 1 atm (Table A-15):

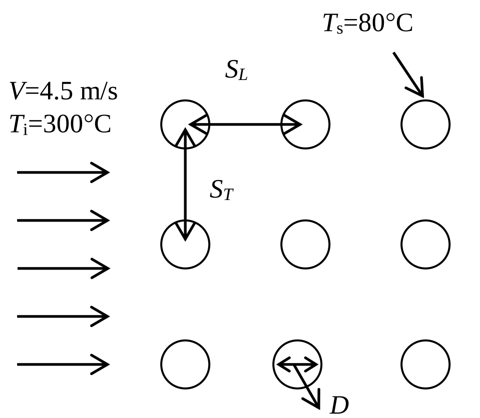
k = 0.04104 W/m·K ρ = 0.6746 kg/m^3

c_p=1.033 kJ/kg·K Pr = 0.6946

μ = 2.76×10^{-5} kg/m·s Pr$_s$ = Pr$_{@\ Ts\ =\ 80°C}$ = 0.7154

The density of air at the inlet temperature of 300°C (for use in the mass flow rate calculation at the inlet) is ρ_i = 0.6158 kg/m^3. The specific heat of water at 80°C is 4.197 kJ/kg.°C (Table A-9).

Analysis (*a*) It is given that D = 0.021 m, S_L= S_T= 0.08 m, and V= 4.5 m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become



$$V_{\max} = \frac{S_T}{S_T - D}V = \frac{0.08}{0.08 - 0.021}(4.5\,\text{m/s}) = 6.102\,\text{m/s}$$

$$\text{Re}_D = \frac{\rho V_{\max} D}{\mu} = \frac{(0.6746\,\text{kg/m}^3)(6.102\,\text{m/s})(0.021\text{m})}{2.76\times10^{-5}\,\text{kg/m}\cdot\text{s}} = 3132$$

The average Nusselt number is determined using the proper relation from Table 7-2 to be

$$\text{Nu}_D = 0.27\,\text{Re}_D^{0.63}\,\text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}$$
$$= 0.27(3132)^{0.63}(0.6946)^{0.36}(0.6946/0.7154)^{0.25} = 37.46$$

Since N_L =16, the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\text{Nu}_{D,N_L} = \text{Nu}_D = 37.46$$

$$h = \frac{Nu_{D,N_L}k}{D} = \frac{37.46(0.04104\,\text{W/m}\cdot°\text{C})}{0.021\,\text{m}} = 73.2\,\text{W/m}^2\cdot°\text{C}$$

The total number of tubes is $N = N_L \times N_T$ = 16×8 = 128. For a unit tube length (L = 1 m), the heat transfer surface area and the mass flow rate of air (evaluated at the inlet) are

$$A_s = N\pi DL = 128\pi(0.021\,\text{m})(1\text{m}) = 8.445\ \text{m}^2$$

$$\dot{m} = \dot{m}_i = \rho_i V(N_T S_T L) = (0.6158\,\text{kg/m}^3)(4.5\,\text{m/s})(8)(0.08\,\text{m})(1\text{m}) = 1.774\,\text{kg/s}$$

Then the fluid exit temperature, the log mean temperature difference, and the rate of heat transfer become

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}c_p}\right) = 80 - (80 - 300)\exp\left(-\frac{(8.445\,\text{m}^2)(73.2\,\text{W/m}^2\cdot°\text{C})}{(1.774\,\text{kg/s})(1033\text{J/kg}\cdot°\text{C})}\right) = 237.0°\text{C}$$

$$\Delta T_{lm} = \frac{(T_s - T_i) - (T_s - T_e)}{\ln[(T_s - T_i)/(T_s - T_e)]} = \frac{(80 - 300) - (80 - 237)}{\ln[(80 - 300)/(80 - 237)]} = 186.7°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{lm} = (73.2\,\text{W/m}^2\cdot°\text{C})(8.445\,\text{m}^2)(186.7°\text{C}) = \mathbf{115{,}430\ W}$$

(*b*) For this in-line arrangement tube bank, the friction coefficient corresponding to Re$_D$ = 3132 and S_L/D = 8/2.1 = 3.81is, from Fig. 7-27a, f = 0.18. Also, χ = 1 for the square arrangements. Then the pressure drop across the tube bank becomes

$$\Delta P = N_L f\chi\frac{\rho V_{\max}^2}{2} = 16(0.18)(1)\frac{(0.6746\,\text{kg/m}^3)(6.102\,\text{m/s})^2}{2}\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{36.2\ \ Pa}$$

(*c*)The temperature rise of water is

$$\dot{Q} = \dot{m}_{\text{water}}c_{p,water}\Delta T_{water} \longrightarrow \Delta T_{water} = \frac{\dot{Q}}{\dot{m}_{\text{water}}c_{p,water}} = \frac{115.43\,\text{kW}}{(6\,\text{kg/s})(4.197\,\text{kJ/kg}\cdot°\text{C})} = \mathbf{4.6°C}$$

Discussion The arithmetic mean fluid temperature is $(T_i + T_e)/2$ = (300 + 237)/2 = 269°C, which is sufficiently close to the assumed value of 250°C. Therefore, there is no need to repeat calculations.

7-115 Air is cooled by an evaporating refrigerator. The refrigeration capacity and the pressure drop across the tube bank are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the tubes is equal to the temperature of refrigerant.

Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of -5°C (will be checked later) and 1 atm (Table A-15):

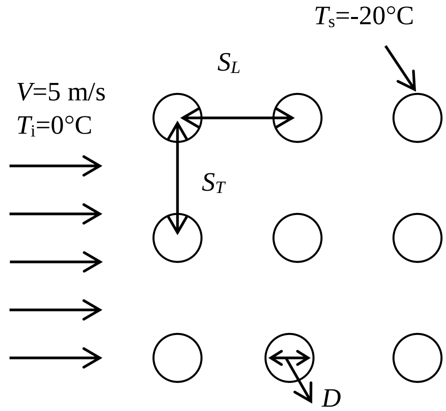
k = 0.02326 W/m·K ρ = 1.317 kg/m^3

c_p =1.006 kJ/kg·K Pr = 0.7375

μ = 1.705×10^{-5} kg/m·s $\mathrm{Pr}_s = \mathrm{Pr}_{@\,T_s = -20°C}$ = 0.7408

Also, the density of air at the inlet temperature of 0°C (for use in the mass flow rate calculation at the inlet) is ρ_i = 1.292 kg/m^3.

Analysis It is given that D = 0.008 m, $S_L = S_T$ = 0.015 m, and V = 5 m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become



$$V_{\max} = \frac{S_T}{S_T - D}V = \frac{0.015}{0.015 - 0.008}(5\,\mathrm{m/s}) = 10.71\,\mathrm{m/s}$$

$$\mathrm{Re}_D = \frac{\rho V_{\max} D}{\mu} = \frac{(1.317\,\mathrm{kg/m^3})(10.71\,\mathrm{m/s})(0.008\mathrm{m})}{1.705\times10^{-5}\,\mathrm{kg/m\cdot s}} = 6621$$

The average Nusselt number is determined using the proper relation from Table 7-2 to be

$$\begin{aligned}\mathrm{Nu}_D &= 0.27\,\mathrm{Re}_D^{0.63}\,\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}\\ &= 0.27(6621)^{0.63}(0.7375)^{0.36}(0.7375/0.7408)^{0.25} = 61.72\end{aligned}$$

Since N_L > 16. the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\mathrm{Nu}_{D,N_L} = F\mathrm{Nu}_D = 61.72$$

$$h = \frac{Nu_{D,N_L}k}{D} = \frac{61.72(0.02326\,\mathrm{W/m\cdot°C})}{0.008\,\mathrm{m}} = 179.5\,\mathrm{W/m^2\cdot°C}$$

The total number of tubes is $N = N_L \times N_T$ = 25×15 = 375. The heat transfer surface area and the mass flow rate of air (evaluated at the inlet) are

$$A_s = N\pi DL = 375\pi(0.008\,\mathrm{m})(0.8\mathrm{m}) = 7.540\,\mathrm{m^2}$$

$$\dot m = \dot m_i = \rho_i V(N_T S_T L) = (1.292\,\mathrm{kg/m^3})(5\,\mathrm{m/s})(15)(0.015\,\mathrm{m})(0.8\mathrm{m}) = 1.163\,\mathrm{kg/s}$$

Then the fluid exit temperature, the log mean temperature difference, and the rate of heat transfer (refrigeration capacity) become

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot m c_p}\right) = -20-(-20-0)\exp\left(-\frac{(7.540\,\mathrm{m^2})(179.5\,\mathrm{W/m^2\cdot°C})}{(1.163\,\mathrm{kg/s})(1006\,\mathrm{J/kg\cdot°C})}\right) = -13.71°\mathrm{C}$$

$$\Delta T_{lm} = \frac{(T_s - T_i)-(T_s - T_e)}{\ln[(T_s - T_i)/(T_s - T_e)]} = \frac{(-20-0)-[-20-(-13.71)]}{\ln[(-20-0)/(-20+13.71)]} = 11.85°\mathrm{C}$$

$$\dot Q = hA_s\Delta T_{lm} = (179.5\,\mathrm{W/m^2\cdot°C})(7.540\,\mathrm{m^2})(11.85°\mathrm{C}) = \mathbf{16{,}040\ W}$$

For this square in-line tube bank, the friction coefficient corresponding to Re_D = 6621 and S_L/D = 1.5/0.8 = 1.875 is, from Fig. 7-27a, f = 0.28. Also, χ = 1 for the square arrangements. Then the pressure drop across the tube bank becomes

$$\Delta P = N_L f\chi\frac{\rho V_{\max}^2}{2} = 25(0.28)(1)\frac{(1.317\,\mathrm{kg/m^3})(10.71\,\mathrm{m/s})^2}{2}\left(\frac{1\,\mathrm{N}}{1\,\mathrm{kg\cdot m/s^2}}\right) = \mathbf{529\ Pa}$$

Discussion The arithmetic mean fluid temperature is $(T_i + T_e)/2$ = (0 -13.7)/2 = -6.9°C, which is fairly close to the assumed value of -5°C. Therefore, there is no need to repeat calculations.

7-116 Air is cooled by an evaporating refrigerator. The refrigeration capacity and the pressure drop across the tube bank are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the tubes is equal to the temperature of refrigerant.

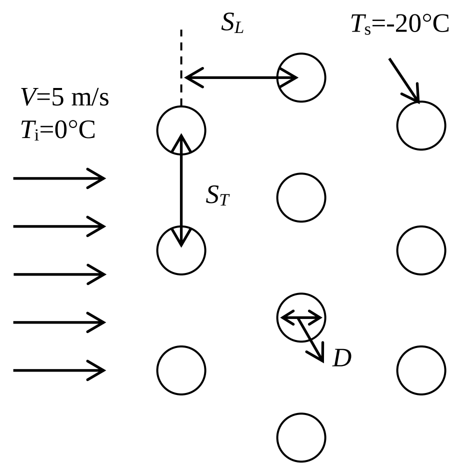
Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of -5°C (will be checked later) and 1 atm (Table A-15):

k = 0.02326 W/m·K $\qquad$ ρ = 1.316 kg/m^3

c_p =1.006 kJ/kg·K $\qquad$ Pr = 0.7375

μ = 1.705×10^{-5} kg/m·s $\qquad$ $Pr_s = Pr_{@\,T_s = -20°C}$ = 0.7408

Also, the density of air at the inlet temperature of 0°C (for use in the mass flow rate calculation at the inlet) is ρ_i = 1.292 kg/m^3.



Analysis It is given that D = 0.008 m, $S_L = S_T$ = 0.015 m, and V = 5 m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become

$$V_{max} = \frac{S_T}{S_T - D}V = \frac{0.015}{0.015 - 0.008}(5\text{ m/s}) = 10.71\text{ m/s}$$

$$\text{Re}_D = \frac{\rho V_{max} D}{\mu} = \frac{(1.317\text{ kg/m}^3)(10.71\text{ m/s})(0.008\text{m})}{1.705\times 10^{-5}\text{ kg/m}\cdot\text{s}} = 6621$$

The average Nusselt number is determined using the proper relation from Table 7-2 to be

$$\begin{aligned}\text{Nu}_D &= 0.35(S_T/S_L)^{0.2}\text{Re}_D^{0.6}\text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}\\ &= 0.35(0.015/0.015)^{0.2}(6621)^{0.6}(0.7375)^{0.36}(0.7375/0.7408)^{0.25} = 61.45\end{aligned}$$

Since N_L > 16. the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\text{Nu}_{D,N_L} = F\text{Nu}_D = 61.45$$

$$h = \frac{Nu_{D,N_L}k}{D} = \frac{61.45(0.02326\text{ W/m}\cdot°\text{C})}{0.008\text{ m}} = 178.7\text{ W/m}^2\cdot°\text{C}$$

The total number of tubes is $N = N_L \times N_T$ = 25×15 = 375. The heat transfer surface area and the mass flow rate of air (evaluated at the inlet) are

$$A_s = N\pi DL = 375\pi(0.008\text{ m})(0.8\text{m}) = 7.540\text{ m}^2$$

$$\dot{m} = \dot{m}_i = \rho_i V(N_T S_T L) = (1.292\text{ kg/m}^3)(5\text{ m/s})(15)(0.015\text{ m})(0.8\text{m}) = 1.163\text{ kg/s}$$

Then the fluid exit temperature, the log mean temperature difference, and the rate of heat transfer (refrigeration capacity) become

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}c_p}\right) = -20 - (-20-0)\exp\left(-\frac{(7.540\text{ m}^2)(178.7\text{ W/m}^2\cdot°\text{C})}{(1.163\text{kg/s})(1006\text{J/kg}\cdot°\text{C})}\right) = -13.68°\text{C}$$

$$\Delta T_{lm} = \frac{(T_s - T_i) - (T_s - T_e)}{\ln[(T_s - T_i)/(T_s - T_e)]} = \frac{(-20-0) - [-20-(-13.68)]}{\ln[(-20-0)/(-20+13.68)]} = 11.87°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{lm} = (178.7\text{ W/m}^2\cdot°\text{C})(7.540\text{ m}^2)(11.87°\text{C}) = \mathbf{16{,}000\ W}$$

For this staggered arrangement tube bank, the friction coefficient corresponding to Re_D = 6621 and S_L/D = 1.5/0.8 = 1.875 is, from Fig. 7-27b, f = 0.42. Also, χ = 1 for the square arrangements. Then the pressure drop across the tube bank becomes

$$\Delta P = N_L f\chi\frac{\rho V_{max}^2}{2} = 25(0.42)(1)\frac{(1.317\text{ kg/m}^3)(10.71\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{793\ Pa}$$

Discussion The arithmetic mean fluid temperature is $(T_i + T_e)/2$ = (0 -13.7)/2 = -6.9°C, which is fairly close to the assumed value of -5°C. Therefore, there is no need to repeat calculations.

7-117 Combustion air is preheated by hot water in a tube bank. The rate of heat transfer to air and the pressure drop of air are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the tubes is equal to the temperature of hot water.

Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of 20°C (will be checked later) and 1 atm (Table A-15):

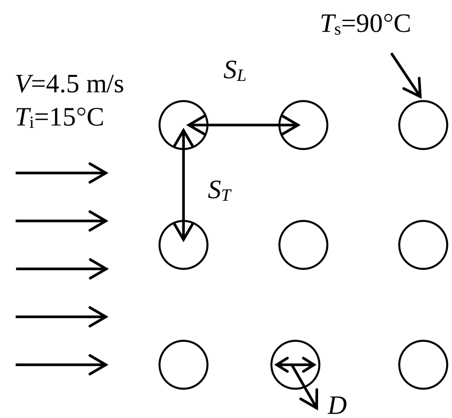
k = 0.02514 W/m·K ρ = 1.204 kg/m^3

c_p =1.007 kJ/kg·K   Pr = 0.7309

μ = 1.825×10^{-5} kg/m·s $Pr_s = Pr_{@\,Ts = 90°C}$ = 0.7132

Also, the density of air at the inlet temperature of 15°C (for use in the mass flow rate calculation at the inlet) is ρ_i = 1.225 kg/m^3.

Analysis It is given that D = 0.022 m, $S_L = S_T$ = 0.05 m, and V = 4.5 m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become



$$V_{\max} = \frac{S_T}{S_T - D}V = \frac{0.05}{0.05 - 0.022}(4.5\text{ m/s}) = 8.036\text{ m/s}$$

$$\text{Re}_D = \frac{\rho V_{\max} D}{\mu} = \frac{(1.204\text{ kg/m}^3)(8.036\text{ m/s})(0.022\text{m})}{1.825\times 10^{-5}\text{ kg/m}\cdot\text{s}} = 11{,}663$$

The average Nusselt number is determined using the proper relation from Table 7-2 to be

$$\begin{aligned}\text{Nu}_D &= 0.27\,\text{Re}_D^{0.63}\,\text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}\\ &= 0.27(11{,}663)^{0.63}(0.7309)^{0.36}(0.7309/0.7132)^{0.25} = 88.53\end{aligned}$$

This Nusselt number is applicable to tube banks with N_L > 16. In our case the number of rows is N_L = 8, and the corresponding correction factor from Table 7-3 is F = 0.967. Then the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\text{Nu}_{D,N_L} = F\text{Nu}_D = (0.967)(88.53) = 85.61$$

$$h = \frac{Nu_{D,N_L} k}{D} = \frac{85.61(0.02514\text{ W/m}\cdot°\text{C})}{0.022\text{ m}} = 97.83\text{ W/m}^2\cdot°\text{C}$$

The total number of tubes is $N = N_L \times N_T$ = 8×8 = 64. For a unit tube length (L = 1 m), the heat transfer surface area and the mass flow rate of air (evaluated at the inlet) are

$$A_s = N\pi DL = 64\pi(0.022\text{ m})(1\text{m}) = 4.423\text{ m}^2$$

$$\dot m = \dot m_i = \rho_i V(N_T S_T L) = (1.225\text{ kg/m}^3)(4.5\text{ m/s})(8)(0.05\text{ m})(1\text{m}) = 2.205\text{ kg/s}$$

Then the fluid exit temperature, the log mean temperature difference, and the rate of heat transfer become

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot m c_p}\right) = 90 - (90-15)\exp\left(-\frac{(4.423\text{ m}^2)(97.83\text{ W/m}^2\cdot°\text{C})}{(2.205\text{ kg/s})(1007\text{ J/kg}\cdot°\text{C})}\right) = 28.28°\text{C}$$

$$\Delta T_{lm} = \frac{(T_s - T_i) - (T_s - T_e)}{\ln[(T_s - T_i)/(T_s - T_e)]} = \frac{(90-15)-(90-28.28)}{\ln[(90-15)/(90-28.28)]} = 68.14°\text{C}$$

$$\dot Q = hA_s\Delta T_{lm} = (97.83\text{ W/m}^2\cdot°\text{C})(4.423\text{m}^2)(68.14°\text{C}) = \mathbf{29{,}490\ W}$$

For this square in-line tube bank, the friction coefficient corresponding to Re_D = 11,663 and S_L/D = 5/2.2 = 2.27 is, from Fig. 7-27a, f = 0.25. Also, χ = 1 for the square arrangements. Then the pressure drop across the tube bank becomes

$$\Delta P = N_L f\chi\frac{\rho V_{\max}^2}{2} = 8(0.25)(1)\frac{(1.204\text{ kg/m}^3)(8.036\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{77.8\ Pa}$$

Discussion The arithmetic mean fluid temperature is $(T_i + T_e)/2$ = (15 + 28.3)/2 = 21.7°C, which is fairly close to the assumed value of 20°C. Therefore, there is no need to repeat calculations.

7-118 Combustion air is preheated by hot water in a tube bank. The rate of heat transfer to air and the pressure drop of air are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the tubes is equal to the temperature of hot water.

Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of 20°C (will be checked later) and 1 atm (Table A-15):

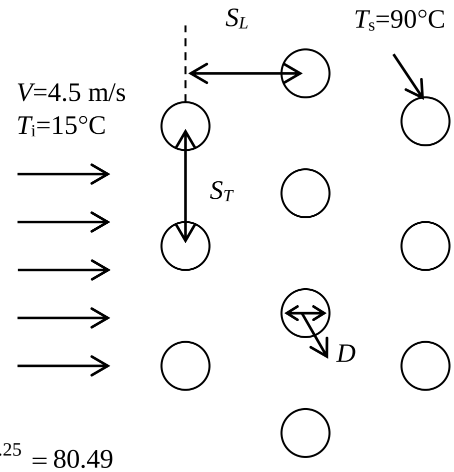
$k = 0.02514$ W/m·K $\qquad \rho = 1.204$ kg/m^3

$c_p = 1.007$ kJ/kg·K $\qquad \Pr = 0.7309$

$\mu = 1.825\times10^{-5}$ kg/m·s $\qquad \Pr_s = \Pr_{@\,T_s = 90°C} = 0.7132$

Also, the density of air at the inlet temperature of 15°C (for use in the mass flow rate calculation at the inlet) is $\rho_i = 1.225$ kg/m^3.

Analysis It is given that $D = 0.022$ m, $S_L = S_T = 0.06$ m, and $V = 4.5$ m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become



$$V_{\max} = \frac{S_T}{S_T - D}V = \frac{0.06}{0.06 - 0.022}(4.5\text{ m/s}) = 7.105\text{ m/s}$$

since $S_D > (S_T + D)/2$

$$\text{Re}_D = \frac{\rho V_{\max} D}{\mu} = \frac{(1.204\text{ kg/m}^3)(7.105\text{ m/s})(0.022\text{m})}{1.825\times10^{-5}\text{ kg/m}\cdot\text{s}} = 10{,}313$$

The average Nusselt number is determined using the proper relation from Table 7-2 to be

$$\begin{aligned}\text{Nu}_D &= 0.35(S_T/S_L)^{0.2}\text{Re}_D^{0.6}\Pr^{0.36}(\Pr/\Pr_s)^{0.25}\\ &= 0.35(0.06/0.06)^{0.2}(10{,}313)^{0.6}(0.7309)^{0.36}(0.7309/0.7132)^{0.25} = 80.49\end{aligned}$$

This Nusselt number is applicable to tube banks with $N_L > 16$. In our case the number of rows is $N_L = 8$, and the corresponding correction factor from Table 7-3 is $F = 0.967$. Then the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\text{Nu}_{D,N_L} = F\text{Nu}_D = (0.967)(80.49) = 77.84$$

$$h = \frac{Nu_{D,N_L}k}{D} = \frac{77.84(0.02514\text{ W/m}\cdot°\text{C})}{0.022\text{ m}} = 88.94\text{ W/m}^2\cdot°\text{C}$$

The total number of tubes is $N = N_L \times N_T = 8\times8 = 64$. For a unit tube length ($L = 1$ m), the heat transfer surface area and the mass flow rate of air (evaluated at the inlet) are

$$A_s = N\pi DL = 64\pi(0.022\text{ m})(1\text{m}) = 4.423\text{ m}^2$$

$$\dot{m} = \dot{m}_i = \rho_i V(N_T S_T L) = (1.225\text{ kg/m}^3)(4.5\text{ m/s})(8)(0.06\text{ m})(1\text{m}) = 2.646\text{ kg/s}$$

Then the fluid exit temperature, the log mean temperature difference, and the rate of heat transfer become

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}c_p}\right) = 90 - (90-15)\exp\left(-\frac{(4.423\text{ m}^2)(88.94\text{ W/m}^2\cdot°\text{C})}{(2.646\text{ kg/s})(1007\text{ J/kg}\cdot°\text{C})}\right) = 25.29°\text{C}$$

$$\Delta T_{lm} = \frac{(T_s - T_i) - (T_s - T_e)}{\ln[(T_s - T_i)/(T_s - T_e)]} = \frac{(90-15) - (90-25.29)}{\ln[(90-15)/(90-25.29)]} = 69.73°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{lm} = (88.94\text{ W/m}^2\cdot°\text{C})(4.423\text{m}^2)(69.73°\text{C}) = \mathbf{27{,}430\text{ W}}$$

For this staggered tube bank, the friction coefficient corresponding to $\text{Re}_D = 10{,}313$ and $S_T/D = 6/2.2 = 2.73$ is, from Fig. 7-27b, $f = 0.27$. Also, $\chi = 1$ for the square arrangements. Then the pressure drop across the tube bank becomes

$$\Delta P = N_L f\chi\frac{\rho V_{\max}^2}{2} = 8(0.27)(1)\frac{(1.204\text{ kg/m}^3)(7.105\text{ m/s})^2}{2}\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{65.6\text{ Pa}}$$

Discussion The arithmetic mean fluid temperature is $(T_i + T_e)/2 = (15 + 25.3)/2 = 20.2$°C, which is fairly close to the assumed value of 20°C. Therefore, there is no need to repeat calculations.

Review Problems

7-119 Air flows over a plate. Various quantities are to be determined at $x = x_{cr}$.

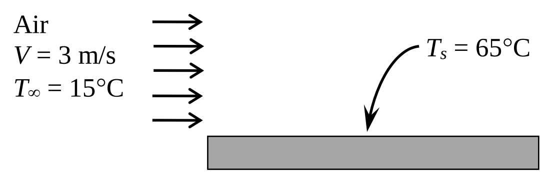
Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Air is an ideal gas. **4** The plate is smooth. **5** Edge effects are negligible and the upper surface of the plate is considered.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (65+15)/2 = 40°C$ are (Table A-15)

$$\rho = 1.127\ \text{kg/m}^3,\quad c_p = 1007\ \text{J/kg}\cdot\text{K}\quad k = 0.02662\ \text{W/m}\cdot\text{K},\quad \mu = 1.918\times10^{-5}\ \text{kg/m}\cdot\text{s},\ \Pr = 0.7255$$

Analysis The critical length of the plate is

$$x_{cr} = \frac{Re_{cr}\,\mu}{V\rho} = \frac{(5\times10^5)(1.918\times10^{-5}\ \text{kg/m}\cdot\text{s})}{(3\ \text{m/s})(1.127\text{kg/m}^3)} = 2.8364\ \text{m}$$



The calculations at x = 2.84 m are

(*a*) Hydrodynamic boundary layer thickness, Eq. 7-12a:

$$\delta = \frac{4.91x}{\sqrt{Re_x}} = \frac{4.91(2.84\ \text{m})}{\sqrt{5\times10^5}} = \mathbf{0.0197m}$$

(*b*) Local friction coefficient, Eq. 7-12b:

$$C_{f,x} = 0.664\,Re_x^{-1/2} = 0.664(5\times10^5)^{-1/2} = \mathbf{0.00094}$$

(*c*) Average friction coefficient, Eq. 7-14:

$$C_f = \frac{1.33}{Re_x^{1/2}} = \frac{1.33}{(5\times10^5)^{1/2}} = \mathbf{0.00188}$$

(*d*) Total drag force due to friction, Eq. 7-1:

$$F_f = C_f A_s \frac{\rho V^2}{2} = (0.00188)(0.3\times0.3\ \text{m}^2)\frac{(1.127\ \text{kg/m}^3)(3\ \text{m/s})^2}{2} = \mathbf{0.00086\,N}$$

(*e*) Local convection heat transfer coefficient, Eq. 7-19:

$$Nu_x = 0.332\,Re_x^{1/2}\Pr^{1/3} = 0.332(5\times10^5)^{1/2}(0.7255)^{1/3} = 210.9$$

$$h_x = \frac{k}{x}Nu_x = \frac{0.02662\ \text{W/m}\cdot K}{2.8364\ \text{m}}(210.9) = \mathbf{1.98\ W/m^2\cdot K}$$

(*f*) Average convection heat transfer coefficient, Eq. 7-21:

$$Nu = 0.664\,Re^{1/2}\Pr^{1/3} = 0.664(5\times10^5)^{1/2}(0.7255)^{1/3} = 2\,Nu_x = 421.8$$

$$h = \frac{k}{x}Nu_x = \frac{0.02662\ \text{W/m}\cdot K}{0.3\ \text{m}}(421.8) = 2\,h_x = \mathbf{3.96\ W/m^2\cdot K}$$

(*g*) Rate of convective heat transfer, Eq. 7-9:

$$\dot{Q} = hA_s(T_s - T_\infty) = (3.96\ \text{W/m}^2\cdot K)(0.3\times0.3\ \text{m}^2)(65-15)°\text{C} = \mathbf{17.8\,W}$$

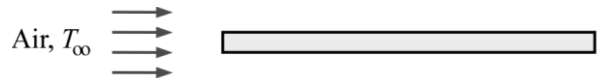
7-120 Air is flowing in parallel to a stationary thin flat plate over the top and bottom surfaces: (*a*) the average friction coefficient, (*b*) the average convection heat transfer coefficient, (*c*) the average convection heat transfer coefficient using the modified Reynolds analogy are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The edge effects are negligible. **4** The critical Reynolds number is $\mathrm{Re}_{cr} = 5\times10^5$.

Properties The properties of air (1 atm) at the film temperature of $T_f = (T_s + T_\infty)/2 = 20°\text{C}$ are given in Table A-15:

$$k = 0.02514 \text{ W/m·K},\ \nu = 1.516 \times 10^{-5} \text{ m}^2\text{/s, and Pr} = 0.7309.$$

Analysis (*a*) The Reynolds at the trailing edge of the plate is



$$\mathrm{Re}_L = \frac{VL}{\nu} = \frac{(2\text{ m/s})(1\text{ m})}{1.516\times10^{-5}\text{ m}^2\text{/s}} = 1.319\times10^5$$

Since $\mathrm{Re}_x < 5 \times 10^5$ at the trailing edge, the flow over the plate is laminar. The average friction coefficient over the plate is

$$C_f = \frac{1.33}{\mathrm{Re}_L^{1/2}} = \frac{1.33}{(1.319\times10^5)^{1/2}} = \mathbf{0.00366}$$

(*b*) Using the proper relation for Nusselt number for laminar flow, the average convection heat transfer coefficient is

$$\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} \quad \rightarrow \quad h = \frac{k}{L}0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3}$$

$$h = \frac{(0.02514\text{ W/m}\cdot\text{K})}{(1\text{ m})}0.664(1.319\times10^5)^{0.5}(0.7309)^{1/3} = \mathbf{5.461\ W/m^2\cdot K}$$

(*c*) Using the modified Reynolds analogy from Chapter 6, the average convection heat transfer coefficient is

$$\mathrm{Nu} = C_f\frac{\mathrm{Re}_L}{2}\mathrm{Pr}^{1/3} \quad \rightarrow \quad h = \frac{k}{L}C_f\frac{\mathrm{Re}_L}{2}\mathrm{Pr}^{1/3}$$

$$h = \frac{(0.02514\text{ W/m}\cdot\text{K})}{(1\text{ m})}(0.00366)\frac{1.319\times10^5}{2}(0.7309)^{1/3} = \mathbf{5.466\ W/m^2\cdot K}$$

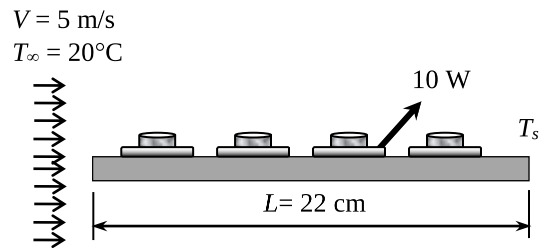
Discussion The comparison of the results obtained for parts (*b*) and (*c*) shows that the discrepancy between the two values is less than 0.1%. This demonstrates that the modified Reynolds analogy is, at times, a very useful method.

7-121 The heat generated by four transistors mounted on a thin vertical plate is dissipated by air blown over the plate on both surfaces. The temperature of the aluminum plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** The entire plate is nearly isothermal. **5** The exposed surface area of the transistor is taken to be equal to its base area. **6** Air is an ideal gas with constant properties. **7** The pressure of air is 1 atm.

Properties Assuming a film temperature of 40°C, the properties of air are evaluated to be (Table A-15)

$$k = 0.02662 \text{ W/m.°C}$$
$$\nu = 1.702\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7255$$



Analysis The Reynolds number in this case is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{(5 \text{ m/s})(0.22\text{m})}{1.702\times10^{-5} \text{ m}^2\text{/s}} = 6.463\times10^4$$

which is smaller than the critical Reynolds number. Thus we have laminar flow. Using the proper relation for Nusselt number, heat transfer coefficient is determined to be

$$Nu = \frac{hL}{k} = 0.664\,\text{Re}_L^{0.5}\,\text{Pr}^{1/3} = 0.664(6.463\times10^4)^{0.5}(0.7255)^{1/3} = 151.7$$

$$h = \frac{k}{L}Nu = \frac{0.02662 \text{ W/m.°C}}{0.22 \text{ m}}(151.7) = 18.35 \text{ W/m}^2\text{.°C}$$

The temperature of aluminum plate then becomes

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 20°\text{C} + \frac{(4\times10) \text{ W}}{(18.35 \text{ W/m}^2\text{.°C})[2(0.22 \text{ m})^2]} = \mathbf{42.5°C}$$

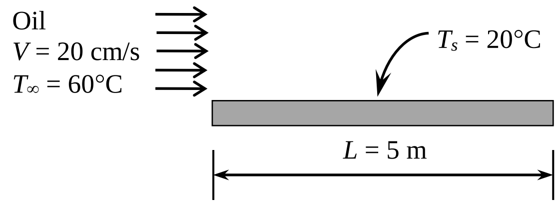
Discussion In reality, the heat transfer coefficient will be higher since the transistors will cause turbulence in the air. Also, the film temperature is (20 + 42.5)/2 = 31.3°C, which is not close to the assumed value of 40°C. Therefore, calculations should be repeated with a lower film temperature.

7-122 Oil flows over a flat plate that is maintained at a specified temperature. The rate of heat transfer is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible.

Properties The properties of oil are given to be ρ = 880 kg/m^3, μ = 0.005 kg/m.s, k = 0.15 W/m·K, and c_p = 2.0 kJ/kg·K.

Analysis The Prandtl and Reynolds numbers are



$$\Pr = \frac{\mu c_p}{k} = \frac{(0.005\text{ kg/m}\cdot\text{s})(2000\text{ J/kg}\cdot°\text{C})}{0.15\text{ W/m}\cdot°\text{C}} = 66.7$$

$$\text{Re}_L = \frac{VL\rho}{\mu} = \frac{(0.2\text{ m/s})(5\text{m})(880\text{ kg/m}^3)}{5\times10^{-3}\text{ kg/m}\cdot\text{s}} = 176{,}000$$

which is smaller than the critical Reynolds number. Thus we have laminar flow for the entire plate. The Nusselt number and the heat transfer coefficient are

$$\text{Nu} = 0.664\,\text{Re}_L^{1/2}\,\Pr^{1/3} = 0.664(176{,}000)^{1/2}(66.7)^{1/3} = 1130$$

$$h = \frac{k}{L}\text{Nu} = \frac{0.15\text{ W/m.°C}}{5\text{ m}}(1130) = 33.9\text{ W/m}^2\text{.°C}$$

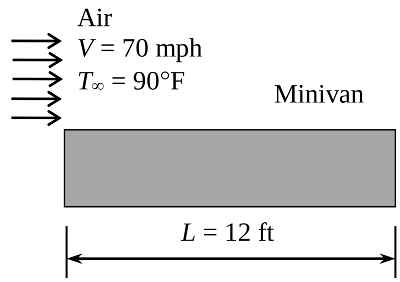
Then the rate of heat transfer is determined to be

$$\dot{Q} = hA_s(T_s - T_\infty) = (33.9\text{ W/m}^2\text{.°C})(5\times1\text{ m}^2)(60-20)°\text{C} = \mathbf{6780\ W}$$

7-123E A minivan is traveling at 70 mph. The rate of heat transfer to the van is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air flow is turbulent because of the intense vibrations involved. **5** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties Assuming a film temperature of T_f = 80°F, the properties of air are evaluated to be (Table A-15E)



$$k = 0.01481\ \text{Btu/h.ft.°F}$$
$$\nu = 1.697\times10^{-4}\ \text{ft}^2\text{/s}$$
$$\text{Pr} = 0.7290$$

Analysis Air flows along 12 ft long side. The Reynolds number in this case is

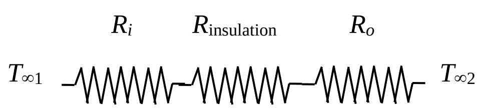
$$\text{Re}_L = \frac{VL}{\nu} = \frac{[(70\times5280/3600)\ \text{ft/s}](12\ \text{ft})}{1.697\times10^{-4}\ \text{ft}^2\text{/s}} = 7.260\times10^6$$

which is greater than the critical Reynolds number. The air flow is assumed to be entirely turbulent because of the intense vibrations involved. Then the Nusselt number and the heat transfer coefficient are determined to be

$$Nu = \frac{h_o L}{k} = 0.037\,\text{Re}_L^{0.8}\,\text{Pr}^{1/3} = 0.037(7.260\times10^6)^{0.8}(0.7290)^{1/3} = 10{,}261$$

$$h_o = \frac{k}{L}Nu = \frac{0.01481\ \text{Btu/h.ft.°F}}{12\ \text{ft}}(10{,}261) = 12.66\ \text{Btu/h.ft}^2\text{.°F}$$

The thermal resistances are



$$A_s = 2\left[(3.2\ \text{ft})(6\ \text{ft}) + (3.2\ \text{ft})(12\ \text{ft}) + (6\ \text{ft})(12\ \text{ft})\right] = 259.2\ \text{ft}^2$$

$$R_i = \frac{1}{h_i A_s} = \frac{1}{(1.2\ \text{Btu/h.ft}^2\text{.°F})(259.2\ \text{ft}^2)} = 0.00322\ \text{h.°F/Btu}$$

$$R_{insulation} = \frac{(R-3)_{value}}{A_s} = \frac{3\ \text{h.ft}^2\text{.°F/Btu}}{(259.2\ \text{ft}^2)} = 0.01157\ \text{h.°F/Btu}$$

$$R_o = \frac{1}{h_o A_s} = \frac{1}{(12.66\ \text{Btu/h.ft}^2\text{.°F})(259.2\ \text{ft}^2)} = 0.00030\ \text{h.°F/Btu}$$

Then the total thermal resistance and the heat transfer rate into the minivan are determined to be

$$R_{total} = R_i + R_{insulation} + R_o = 0.00322 + 0.01157 + 0.00030 = 0.01509\ \text{h.°F/Btu}$$

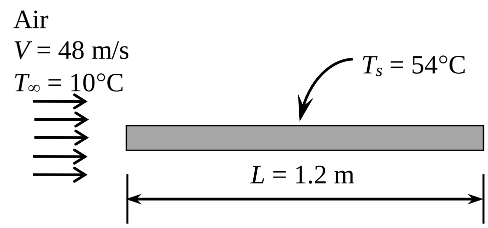
$$\dot{Q} = \frac{T_{\infty2} - T_{\infty1}}{R_{total}} = \frac{(90-70)\text{°F}}{0.01509\ \text{h.°F/Btu}} = \mathbf{1325\ Btu/h}$$

7-124 Air flows over the top and bottom surfaces of a thin, square plate. The total heat transfer rate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible.

Properties The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (54+10)/2 = 32°C$ are (Table A-15)

$$\rho = 1.156 \text{ kg/m}^3 \qquad \nu = 1.627\times10^{-5} \text{ m}^2\text{/s}$$
$$c_p = 1007 \text{ J/kg}\cdot\text{K} \qquad \text{Pr} = 0.7276$$
$$k = 0.02603 \text{ W/m}\cdot\text{K}$$



Analysis The Reynolds number is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{(48 \text{ m/s})(1.2\text{m})}{1.627\times10^{-5} \text{ m}^2\text{/s}} = 3.540\times10^6$$

which is greater than the critical Reynolds number. Since the surface of the plate on the top and the bottom is very rough, it can be assumed that the flow over the entire plate is turbulent.

We use modified Reynolds analogy to determine the heat transfer coefficient and the rate of heat transfer

$$\tau_s = \frac{F}{A} = \frac{1.5 \text{ N}}{2(1.2 \text{ m})^2} = 0.5208 \text{ N/m}^2$$

$$C_f = \frac{\tau_s}{0.5\rho V^2} = \frac{0.5208 \text{ N/m}^2}{0.5(1.156 \text{ kg/m}^3)(48 \text{ m/s})^2} = 3.911\times10^{-4}$$

$$\frac{C_f}{2} = \text{St Pr}^{2/3} = \frac{\text{Nu}_L}{\text{Re}_L \text{ Pr}}\text{Pr}^{2/3} = \frac{\text{Nu}_L}{\text{Re}_L \text{ Pr}^{1/3}}$$

$$\text{Nu} = \text{Re}_L \text{ Pr}^{1/3}\frac{C_f}{2} = (3.540\times10^6)(0.7276)^{1/3}\frac{(3.911\times10^{-4})}{2} = 622.6$$

$$h = \frac{k}{L}\text{Nu} = \frac{0.02603 \text{ W/m}\cdot K}{1.2 \text{ m}}(622.6) = 13.51 \text{ W/m}^2\cdot K$$

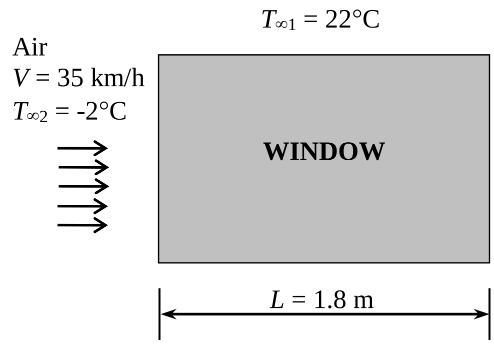
$$\dot{Q} = hA_s(T_s - T_\infty) = (13.51 \text{ W/m}^2\cdot K)[2\times(1.2 \text{ m})^2](54-10)°\text{C} = \mathbf{1711\ W}$$

7-125 Wind is blowing parallel to the walls of a house with windows. The rate of heat loss through the window is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties Assuming a film temperature of 5°C based on the problem statement, the properties of air at 1 atm and this temperature are evaluated to be (Table A-15)

$$k = 0.02401 \text{ W/m.°C}$$
$$\nu = 1.382\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7350$$



Analysis Air flows along 1.8 m side. The Reynolds number in this case is

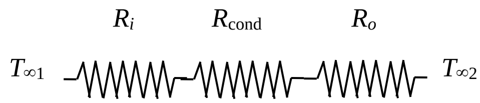
$$\text{Re}_L = \frac{VL}{\nu} = \frac{[(35\times1000/3600)\text{ m/s}](1.8\text{ m})}{1.382\times10^{-5}\text{ m}^2\text{/s}} = 1.266\times10^6$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. Using the proper relation for Nusselt number, heat transfer coefficient is determined to be

$$Nu = \frac{hL}{k} = (0.037\,\text{Re}_L^{0.8} - 871)\,\text{Pr}^{1/3} = [0.037(1.266\times10^6)^{0.8} - 871](0.7350)^{1/3} = 1759$$

$$h = \frac{k}{L}Nu = \frac{0.02401\text{ W/m.°C}}{1.8\text{ m}}(1759) = 23.46\text{ W/m}^2\text{.°C}$$

The thermal resistances are



$$A_s = 3(1.8\text{ m})(1.5\text{m}) = 8.1\text{m}^2$$

$$R_{conv,i} = \frac{1}{h_i A_s} = \frac{1}{(8\text{ W/m}^2\text{.°C})(8.1\text{m}^2)} = 0.0154\text{ °C/W}$$

$$R_{cond} = \frac{L}{kA_s} = \frac{0.005\text{ m}}{(0.78\text{ W/m.°C})(8.1\text{m}^2)} = 0.0008\text{ °C/W}$$

$$R_{conv,o} = \frac{1}{h_o A_s} = \frac{1}{(23.46\text{ W/m}^2\text{.°C})(8.1\text{m}^2)} = 0.0053\text{ °C/W}$$

Then the total thermal resistance and the heat transfer rate through the 3 windows become

$$R_{total} = R_{conv,i} + R_{cond} + R_{conv,o} = 0.0154 + 0.0008 + 0.0053 = 0.0215\text{ °C/W}$$

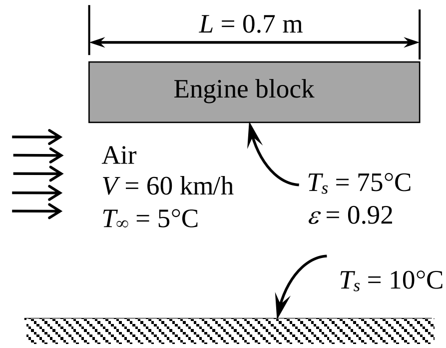
$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{R_{total}} = \frac{[22-(-2)]\text{°C}}{0.0215\text{ °C/W}} = \mathbf{1116\ W}$$

7-126 A car travels at a velocity of 60 km/h. The rate of heat transfer from the bottom surface of the hot automotive engine block is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $\text{Re}_{cr} = 5\times10^5$. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm. **5** The flow is turbulent over the entire surface because of the constant agitation of the engine block. **6** The bottom surface of the engine is a flat surface.

Properties The properties of air at 1 atm and the film temperature of $(T_s + T_\infty)/2 = (75+5)/2 = 40°\text{C}$ are (Table A-15)

$$k = 0.02662 \text{ W/m.°C}$$
$$\nu = 1.702\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7255$$



Analysis The Reynolds number is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{[(60\times1000/3600)\text{ m/s}](0.7\text{ m})}{1.702\times10^{-5}\text{ m}^2\text{/s}} = 6.855\times10^5$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow. But we will assume turbulent flow because of the constant agitation of the engine block.

$$Nu = \frac{hL}{k} = 0.037\,\text{Re}_L^{0.8}\,\text{Pr}^{1/3} = 0.037(6.855\times10^5)^{0.8}(0.7255)^{1/3} = 1551$$

$$h = \frac{k}{L}Nu = \frac{0.02662\text{ W/m.°C}}{0.7\text{ m}}(1551) = 58.97\text{ W/m}^2\text{.°C}$$

$$\dot{Q}_{conv} = hA_s(T_\infty - T_s) = (58.97\text{ W/m}^2\text{.°C})[(0.6\text{m})(0.7\text{m})](75-5)°\text{C} = 1734\text{ W}$$

The heat loss by radiation is then determined from Stefan-Boltzman law to be

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s\sigma(T_s^4 - T_{surr}^4)\\ &= (0.92)(0.6\text{ m})(0.7\text{ m})(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)\left[(75+273\text{ K})^4 - (10+273\text{ K})^4\right]\\ &= 181\text{ W}\end{aligned}$$

Then the total rate of heat loss from the bottom surface of the engine block becomes

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 1734 + 181 = \mathbf{1915\ W}$$

The gunk will introduce an additional resistance to heat dissipation from the engine. The total heat transfer rate in this case can be calculated from

$$\dot{Q} = \frac{T_\infty - T_s}{\dfrac{1}{hA_s} + \dfrac{L}{kA_s}} = \frac{(75\text{-}5)°\text{C}}{\dfrac{1}{(58.97\text{ W/m}^2\text{.°C})[(0.6\text{m})(0.7\text{m})]} + \dfrac{(0.002\text{ m})}{(3\text{ W/m.°C})(0.6\text{ m}\times0.7\text{ m})}} = 1668\text{ W}$$

The decrease in the heat transfer rate is

1734 – 1668 = **66 W**

Percent decrease = 66/1915 = 0.034 =**3.4%**

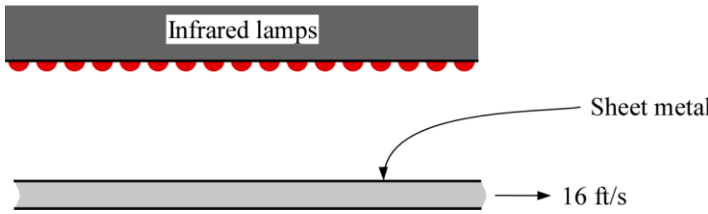
7-127E A 15-ft long strip of sheet metal is being transported on a conveyor, while the coating on the upper surface is being cured by infrared lamps. The surface temperature of the sheet metal is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat conduction through the sheet metal is negligible. **3** Thermal properties are constant. **4** The surrounding ambient air is at 1 atm. **5** The critical Reynolds number is $\text{Re}_{cr} = 5\times10^5$.

Properties The properties of air at 180°F are $k = 0.01715$ Btu/h·ft·R, $\nu = 2.281 \times 10^{-4}$ ft²/s, Pr = 0.7148 (from Table A-15E).

Analysis The Reynolds number for L = 15 ft is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{(16\text{ ft/s})(15\text{ ft})}{2.281\times10^{-4}\text{ ft}^2/\text{s}} = 1.052\times10^6$$



Since $5 \times 10^5 < \text{Re}_L < 10^7$, the flow is a combined laminar and turbulent flow. Using the proper relation for Nusselt number, the average heat transfer coefficient on the sheet metal is

$$\text{Nu} = \frac{hL}{k} = (0.037\,\text{Re}_L^{0.8} - 871)\,\text{Pr}^{1/3} = [0.037(1.052\times10^6)^{0.8} - 871](0.7148)^{1/3} = 1395$$

$$h = 1395\frac{k}{L} = 1395\frac{0.01715\text{ Btu/h}\cdot\text{ft}\cdot\text{R}}{15\text{ ft}} = 1.595\text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}$$

From energy balance, we have

$$\dot{Q}_{\text{absorbed}} - \dot{Q}_{\text{rad}} - \dot{Q}_{\text{conv}} = 0 \quad \rightarrow \quad A\dot{q}_{\text{absorbed}} - A\dot{q}_{\text{rad}} - 2A\dot{q}_{\text{conv}} = 0$$

or $$\alpha\dot{q}_{\text{incident}} - \varepsilon\sigma(T_s^4 - T_{\text{surr}}^4) - 2h(T_s - T_\infty) = 0$$

Copy the following lines and paste on a blank EES screen to solve the above equation:

```
h=1.595
T_inf=77+460
T_surr=77+460
q_incindent=1500
alpha=0.6
epsilon=0.7
sigma=0.1714e-8
alpha*q_incindent-epsilon*sigma*(T_s^4-T_surr^4)-2*h*(T_s-T_inf)=0
```

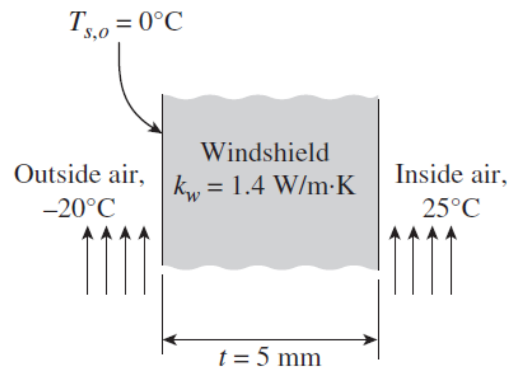
Solving by EES software, the surface temperature of the sheet metal is

$$T_s = 739\text{ R} = \mathbf{279°F}$$

Discussion Since the value of the (force) convection heat transfer coefficient is relatively small, this indicates that natural convection may play an important role.

7-128 Warm air blowing over the inner surface of a windshield is used for defrosting. The required inner convection heat transfer coefficient to cause the accumulated ice to begin melting is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the windshield is one-dimensional. **3** Thermal conductivity is constant. **4** Heat transfer by radiation is negligible. **5** The outside air pressure is 1 atm. **6** The critical Reynolds number is $Re_{cr} = 5\times10^5$.



Properties The properties of air at the film temperature of $T_f = (-20°C + 0°C)/2 = -10°C$ are $k = 0.02288$ W/m·K, $\nu = 1.252\times10^{-5}$ m²/s, and Pr = 0.7387 (from Table A-15).

Analysis On the outer surface of the windshield, the Reynolds number at $L = 0.5$ m is

$$\text{Re}_L = \frac{VL}{\nu} = \frac{(80/3.60\ \text{m/s})(0.5\ \text{m})}{1.252\times10^{-5}\ \text{m}^2/\text{s}} = 8.875\times10^5$$

Since $5\times10^5 < \text{Re}_L < 10^7$, the flow is a combined laminar and turbulent flow. Using the proper relation for Nusselt number, the average heat transfer coefficient on the outer surface of the windshield is

$$\text{Nu}_o = \frac{h_o L}{k} = (0.037\,\text{Re}_L^{0.8} - 871)\,\text{Pr}^{1/3} = [0.037(8.875\times10^5)^{0.8} - 871](0.7387)^{1/3} = 1131$$

$$h_o = Nu\frac{k}{L} = 1131\frac{0.02288\ \text{W/m}\cdot\text{K}}{0.5\ \text{m}} = 51.75\ \text{W/m}^2\cdot\text{K}$$

From energy balance, the heat transfer through the windshield thickness can be written as

$$\frac{T_{\infty,o} - T_{s,o}}{1/h_o} = \frac{T_{s,o} - T_{\infty,i}}{t/k_w + 1/h_i}$$

For the ice to begin melting, the outer surface temperature of the windshield ($T_{s,o}$) should be at least 0°C. Then the convection heat transfer coefficient for the warm air blowing over the inner surface of the windshield must be

$$h_i = \left(\frac{1}{h_o}\frac{T_{s,o} - T_{\infty,i}}{T_{\infty,o} - T_{s,o}} - \frac{t}{k_w}\right)^{-1}$$

$$= \left[\frac{(0-25)°\text{C}}{(-20-0)°\text{C}}\left(\frac{1}{51.75\ \text{W/m}^2\cdot K}\right) - \frac{0.005\ \text{m}}{1.4\ \text{W/m}\cdot K}\right]^{-1}$$

$$= \mathbf{48.6\ W/m^2\cdot K}$$

Discussion In practical situations, the ambient temperature and the convective heat transfer coefficient outside the automobile vary with weather conditions and the automobile speed. Therefore the convection heat transfer coefficient of the warm air necessary to melt the ice should be varied as well. This is done by adjusting the warm air flow rate and temperature.

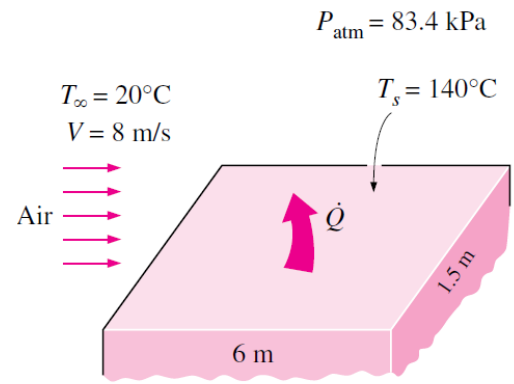
7-129 The top surface of a hot block is to be cooled by forced air. The rate of heat transfer is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** The critical Reynolds number is $Re_{cr} = 5\times10^5$. **3** Radiation effects are negligible. **4** Air is an ideal gas.

Properties The properties k, μ, c_p, and Pr of ideal gases are independent of pressure, while the properties ν and α are inversely proportional to density and thus pressure. The properties of air at the film temperature of $T_f = (T_s + T_\infty)/2 = (140 + 20)/2 = 80°C$ and 1 atm pressure are (Table A–15):

$k = 0.02953$ W/m · K, Pr = 0.7154, $\nu_{@\,1\,atm} = 2.097\times10^{-5}$ m^2/s

The atmospheric pressure in Denver is P = (83.4 kPa)/(101.325 kPa/atm) = 0.823 atm. Then the kinematic viscosity of air in Denver becomes: $\nu = \nu_{@\,1\,atm}/P = (2.097\times10^{-5}\ \text{m}^2/\text{s})/0.823 = 2.548\times10^{-5}\ \text{m}^2/\text{s}$



Analysis (*a*) When air flow is parallel to the long side, we have L= 6 m, andthe Reynolds number at the end of the plate becomes

$$Re_L = \frac{VL}{\nu} = \frac{(8\ \text{m/s})(6\ \text{m})}{2.548\times10^{-5}\ \text{m}^2/\text{s}} = 1.884\times10^6$$

which is greater than the critical Reynolds number. Thus, we have combinedlaminar and turbulent flow, and the average Nusselt number for the entire plateis determined to be

$$\text{Nu} = \frac{hL}{k} = (0.037\,Re_L^{0.8} - 871)\,Pr^{1/3} = [0.037(1.884\times10^6)^{0.8} - 871](0.7154)^{1/3} = 2687$$

$$h = Nu\frac{k}{L} = (2687)\frac{0.02953\ \text{W/m}\cdot\text{K}}{6\ \text{m}} = 13.2\ \text{W/m}^2\cdot\text{K}$$

$$A_s = wL = (1.5\ \text{m})(6\ \text{m}) = 9\ \text{m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (13.2\ \text{W/m}^2\cdot K)(9\,\text{m}^2)(140-20)°\text{C} = \mathbf{1.43\times10^4\ W}$$

Note that if we disregarded the laminar region and assumed turbulent flow overthe entire plate, we would get Nu = 3466 from Eq. 7–22, which is 29 percenthigher than the value calculated above.

(*b*) When air flow is along the short side, we have L = 1.5 m, and the Reynoldsnumber at the end of the plate becomes

$$Re_L = \frac{VL}{\nu} = \frac{(8\ \text{m/s})(1.5\ \text{m})}{2.548\times10^{-5}\ \text{m}^2/\text{s}} = 4.71\times10^5$$

which is less than the critical Reynolds number. Thus we have laminar flowover the entire plate, and the average Nusselt number is

$$\text{Nu} = \frac{hL}{k} = 0.664\,Re_x^{0.5}\,Pr^{1/3} = 0.664(4.71\times10^5)^{0.5}(0.7154)^{1/3} = 408$$

$$h = Nu\frac{k}{L} = (408)\frac{0.02953\ \text{W/m}\cdot\text{K}}{1.5\ \text{m}} = 8.03\ \text{W/m}^2\cdot\text{K}$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (8.03\ \text{W/m}^2\cdot K)(9\,\text{m}^2)(140-20)°\text{C} = \mathbf{8670\ W}$$

which is considerably less than the heat transfer rate determined in case (*a*).

Discussion Note that the *direction* of fluid flow can have a significant effecton convection heat transfer to or from a surface. In this case, wecan increase the heat transfer rate by 65 percent by simply blowing the airalong the long side of the rectangular plate instead of the short side.

7-130 A silicon chip is mounted flush in a substrate that provides an unheated starting length. The maximum allowable power dissipation is to be determined such that the surface temperature of the chip cannot exceed 75°C.

Assumptions **1** Steady operating conditions exist. **2** Thermal properties are constant. **3** The flow is turbulent. **4** Only the upper surface of the chip is conditioned for heat transfer. **5** Heat transfer by radiation is negligible. **6** Heat dissipated from the chip is uniform.

Properties The properties of air at T_f = (75°C + 25°C)/2 = 50°C are k = 0.02735 W/m·K, $\nu = 1.798 \times 10^{-5}$ m^2/s, Pr = 0.7228 (from Table A-15).

Analysis For uniform heat flux on the chip surface, the maximum surface temperature occurs at the trailing edge, where the convection heat transfer coefficient is at minimum. The Reynolds number at the trailing edge (x = 0.040 m) is

$$\mathrm{Re}_x = \frac{Vx}{\nu} = \frac{(25\ \mathrm{m/s})(0.040\ \mathrm{m})}{1.798\times10^{-5}\ \mathrm{m^2/s}} = 5.562\times10^4$$

Since the flow is turbulent, use the turbulent flow relation for Nusselt number, the local heat transfer coefficient at the trailing edge (x = 0.040 mm) can be determined from:

$$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for }\xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} \quad \rightarrow \quad h_x = \frac{k}{x}\frac{0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}}$$

$$h_x = \frac{(0.02735\ \mathrm{W/m\cdot K})}{(0.040\ \mathrm{m})}\frac{0.0308(5.562\times10^4)^{0.8}(0.7228)^{1/3}}{[1-(20/40)^{9/10}]^{1/9}} = 128.7\ \mathrm{W/m^2\cdot K}$$

Hence, the maximum allowable power dissipation on the chip surface is

$$\dot{Q}_{\max} = hA(T_s - T_\infty) = (128.7\ \mathrm{W/m^2\cdot K})(0.020\ \mathrm{m})^2(75-25)\ \mathrm{K} = \mathbf{2.57\ W}$$

Discussion Turbulator is a device that trips the velocity boundary layer to turbulence. The turbulator caused airflow over the chip to be turbulent. Hence the Nusselt number relation for turbulent flow is used, even though Re_x is less than the generally accepted value of critical Reynolds number ($\mathrm{Re}_{cr} = 5 \times 10^5$).

7-131 Airstream flows in parallel over a 3-m long flat plate where there is an unheated starting length of 1 m, (*a*) the local convection heat transfer coefficient at x = 3 m and (*b*) the average convection heat transfer coefficient for the heated section are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperature is uniform throughout the heated section. **3** Thermal properties are constant. **4** The critical Reynolds number is $\text{Re}_{cr} = 5\times10^5$.

Properties The properties of air at T_f = (80°C + 20°C)/2 = 50°C are k = 0.02735 W/m·K, $\nu = 1.798 \times 10^{-5}$ m²/s, Pr = 0.7228 (from Table A-15).

Analysis (*a*) The Reynolds number at x = 1 m is

$$\text{Re}_x = \frac{Vx}{\nu} = \frac{(15\text{ m/s})(1\text{ m})}{1.798\times10^{-5}\text{ m}^2\text{/s}} = 8.343\times10^5$$

Since $\text{Re}_x > 5 \times 10^5$ at the start of heating, the flow over the entire heated section is turbulent. Using the proper relation for Nusselt number, the local heat transfer coefficient at the trailing edge (x = 3 m) can be determined:

$$\text{Re}_{x=L} = \frac{VL}{\nu} = \frac{(15\text{ m/s})(3\text{ m})}{1.798\times10^{-5}\text{ m}^2\text{/s}} = 2.503\times10^6$$

$$\text{Nu}_x = \frac{0.0296\,\text{Re}_x^{0.8}\,\text{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}} \quad \rightarrow \quad h_x = \frac{k}{x}\frac{0.0296\,\text{Re}_x^{0.8}\,\text{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}}$$

$$h_{x=L} = \frac{(0.02735\text{ W/m}\cdot\text{K})}{(3\text{ m})}\frac{0.0296(2.503\times10^6)^{0.8}(0.7228)^{1/3}}{[1-(1/3)^{9/10}]^{1/9}} = \mathbf{33.52\ W/m^2\cdot K}$$

(*b*) The average convection heat transfer coefficient over the heated section is

$$h = \text{Nu}\frac{k}{L} = 4{,}329.6\left(\frac{0.02735\text{W/m}\cdot\text{K}}{3\text{ m}}\right) = \mathbf{39.47\ W/m^2\cdot K}$$

where $\text{Nu} = \text{Nu}_{(\text{for }\xi=0)}\frac{L}{L-\xi}[1-(\xi/L)^{9/10}]^{8/9} = (4{,}364.9)\frac{3\text{ m}}{3\text{ m}-1\text{ m}}\left[1-\left(\frac{1\text{ m}}{3\text{ m}}\right)^{9/10}\right]^{8/9} = 4{,}329.6$

and $\text{Nu}_{(\text{for }\xi=0)} = 0.037\,\text{Re}_L^{0.8}\text{Pr}^{1/3} = 0.037\,(2.503\times10^6)^{0.8}\,(0.7228)^{1/3} = 4{,}364.9$

Discussion The ratio of the average to the local convection heat transfer coefficient is

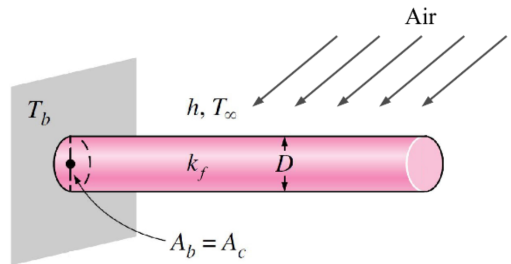
$$\frac{h}{h_{x=L}} = \frac{39.47\text{ W/m}^2\cdot\text{K}}{33.52\text{ W/m}^2\cdot\text{K}} = 1.18$$

7-132 Air is flowing across a cylindrical pin fin that is attached to the hot surface. The maximum possible rate of heat transfer from the pin fin is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Radiation effects are negligible. **4** Flow over pin fin can be treated as flow across a cylinder. **5** The film temperature is assumed to be 70°C.

Properties The properties of air (1 atm) at 70°C are given in Table A-15: k = 0.02881 W/m·K, $\nu = 1.995 \times 10^{-5}$ m²/s, and Pr = 0.7177.

Analysis The Reynolds number for the air flowing across the pin fin is



$$\mathrm{Re}_D = \frac{VD}{\nu} = \frac{(10\ \mathrm{m/s})(0.005\ \mathrm{m})}{1.995\times10^{-5}\ \mathrm{m^2/s}} = 2506$$

Using the Churchill and Bernstein relation for Nusselt number, the convection heat transfer coefficient is

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1+(0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282000}\right)^{5/8}\right]^{4/5}$$

$$h = \frac{0.02881\ \mathrm{W/m\cdot K}}{0.005\ \mathrm{m}}\left\{0.3 + \frac{0.62(2506)^{1/2}(0.7177)^{1/3}}{[1+(0.4/0.7177)^{2/3}]^{1/4}}\left[1+\left(\frac{2506}{282000}\right)^{5/8}\right]^{4/5}\right\}$$

$$= 148.3\ \mathrm{W/m^2\cdot K}$$

Maximum rate of heat transfer from pin fin occurs when fin is infinitely long. Therefore from Chapter 3, the maximum possible heat transfer rate is

$$\dot{Q}_{\mathrm{long\,fin}} = \sqrt{hpk_f A_c}\,(T_b - T_\infty)$$

where

$$p = \pi D = 0.01571\ \mathrm{m}, \quad A_c = \pi D^2/4 = 1.963\times10^{-5}\ \mathrm{m^2}$$

Hence

$$\dot{Q}_{\mathrm{long\,fin}} = \sqrt{(148.3\ \mathrm{W/m^2\cdot K})(0.01571\ \mathrm{m})(237\ \mathrm{W/m\cdot K})(1.963\times10^{-5}\ \mathrm{m^2})}\,(120-20)\ \mathrm{K}$$
$$= \mathbf{10.4\ W}$$

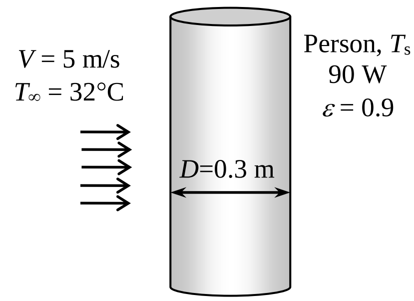
Discussion For infinitely long fin, the fin tip temperature is equal to the air temperature. Hence, evaluating the air properties at 70°C is reasonable, since it is the average of the air and fin base temperatures.

7-133 A fan is blowing air over the entire body of a person. The average temperature of the outer surface of the person is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The pressure of air is 1 atm. **4** The average human body can be treated as a 30-cm-diameter cylinder with an exposed surface area of 1.7 m^2.

Properties We assume the film temperature to be 35°C based on the problem statement. The properties of air at 1 atm and this temperature are (Table A-15)

$$k = 0.02625 \text{ W/m.°C}$$
$$\nu = 1.655\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7268$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(5\text{ m/s})(0.3\text{m})}{1.655\times10^{-5}\text{ m}^2\text{/s}} = 9.063\times10^4$$

The proper relation for Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\Pr^{1/3}}{\left[1+(0.4/\Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(9.063\times10^4)^{0.5}(0.7268)^{1/3}}{\left[1+(0.4/0.7268)^{2/3}\right]^{1/4}}\left[1+\left(\frac{9.063\times10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 203.6$$

Then

$$h = \frac{k}{D}Nu = \frac{0.02655\text{ W/m.°C}}{0.3\text{ m}}(203.6) = 18.02\text{ W/m}^2\text{.°C}$$

Considering that there is heat generation in that person's body at a rate of 90 W and body gains heat by radiation from the surrounding surfaces, an energy balance can be written as

$$\dot{Q}_{generated} + \dot{Q}_{radiation} = \dot{Q}_{convection}$$

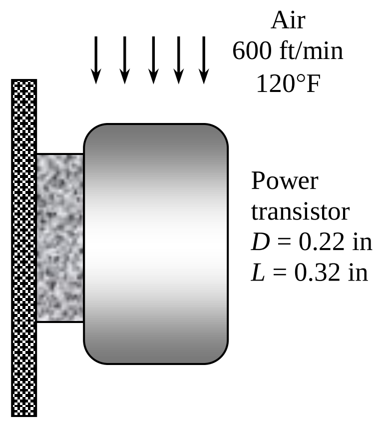
Substituting values with proper units and then application of trial & error method or the use of an equation solver yields the average temperature of the outer surface of the person.

$$90\text{ W} + \varepsilon A_s \sigma (T_{surr}^4 - T_s^4) = hA_s(T_s - T_\infty)$$
$$90 + (0.9)(1.7)(5.67\times10^{-8})[(40+273)^4 - T_s^{\,4}] = (18.02)(1.7)[T_s - (32+273)]$$
$$\longrightarrow T_s = \mathbf{309.2\ K} = \mathbf{36.2°C}$$

7-134E A cylindrical transistor mounted on a circuit board is cooled by air flowing over it. The maximum power rating of the transistor is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $T_f = (180+120)/2 = 150°\text{F}$ are (Table A-15E)



$$k = 0.01646 \text{ Btu/h.ft.°F}$$
$$\nu = 2.099\times10^{-4} \text{ ft}^2/\text{s}$$
$$\text{Pr} = 0.7188$$

Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(600/60 \text{ ft/s})(0.22/12 \text{ ft})}{2.099\times10^{-4} \text{ ft}^2/\text{s}} = 873.4$$

The Nusselt number corresponding to this Reynolds number is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(873.4)^{0.5}(0.7188)^{1/3}}{\left[1+(0.4/0.7188)^{2/3}\right]^{1/4}}\left[1+\left(\frac{873.4}{282{,}000}\right)^{5/8}\right]^{4/5} = 15.04$$

and $$h = \frac{k}{D}Nu = \frac{0.01646 \text{ Btu/h.ft.°F}}{(0.22/12 \text{ ft})}(15.04) = 13.50 \text{ Btu/h.ft}^2.°\text{F}$$

Then the amount of power this transistor can dissipate safely becomes

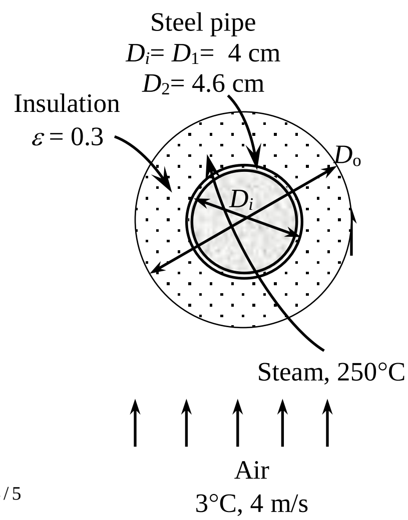
$$\dot{Q} = hA_s(T_s - T_\infty) = h(\pi DL)(T_s - T_\infty)$$
$$= (13.50 \text{ Btu/h.ft}^2.°\text{F})\left[\pi(0.22/12 \text{ ft})(0.32/12 \text{ ft})\right](180-120)°\text{F}$$
$$= \mathbf{1.244\ Btu/h = 0.365\ W} \quad (1 \text{ W} = 3.412 \text{ Btu/h})$$

7-135 Steam is flowing in a stainless steel pipe while air is flowing across the pipe. The rate of heat loss from the steam per unit length of the pipe is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The pressure of air is 1 atm.

Properties Assuming a film temperature of 10°C based on the problem statement, the properties of air are (Table A-15)

$$k = 0.02439 \text{ W/m.°C}$$
$$\nu = 1.426\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7336$$



Analysis The outer diameter of insulated pipe is

$$D_o = 4.6+2\times3.5 = 11.6 \text{ cm} = 0.116 \text{ m.}$$

The Reynolds number is

$$\text{Re} = \frac{VD_o}{\nu} = \frac{(4\text{ m/s})(0.116\text{m})}{1.426\times10^{-5}\text{ m}^2\text{/s}} = 3.254\times10^4$$

The Nusselt number for flow across a cylinder is determined from

$$Nu = \frac{hD_o}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\Pr^{1/3}}{\left[1+(0.4/\Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(3.254\times10^4)^{0.5}(0.7336)^{1/3}}{\left[1+(0.4/0.7336)^{2/3}\right]^{1/4}}\left[1+\left(\frac{3.254\times10^4}{282{,}000}\right)^{5/8}\right]^{4/5} = 107.0$$

and $\quad h_o = \dfrac{k}{D_o}Nu = \dfrac{0.02439 \text{ W/m}\cdot\text{°C}}{0.116\text{ m}}(107.0) = 22.50 \text{ W/m}^2\cdot\text{°C}$

Area of the outer surface of the pipe per m length of the pipe is

$$A_o = \pi D_o L = \pi(0.116\text{ m})(1\text{ m}) = 0.3644\text{ m}^2$$

In steady operation, heat transfer from the steam through the pipe and the insulation to the outer surface (by first convection and then conduction) must be equal to the heat transfer from the outer surface to the surroundings (by simultaneous convection and radiation). That is,

$$\dot{Q} = \dot{Q}_{\text{pipe and insulation}} = \dot{Q}_{\text{surface to surroundings}}$$

Using the thermal resistance network, heat transfer from the steam to the outer surface is expressed as

$$R_{conv,i} = \frac{1}{h_i A_i} = \frac{1}{(80\text{ W/m}^2\text{.°C})[\pi(0.04\text{ m})(1\text{ m})]} = 0.0995\text{ °C/W}$$

$$R_{pipe} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(2.3/2)}{2\pi(15\text{ W/m.°C})(1\text{ m})} = 0.0015\text{ °C/W}$$

$$R_{insulation} = \frac{\ln(r_3/r_2)}{2\pi kL} = \frac{\ln(5.8/2.3)}{2\pi(0.038\text{ W/m.°C})(1\text{ m})} = 3.874\text{ °C/W}$$

and $\quad \dot{Q}_{\text{pipe and ins}} = \dfrac{T_{\infty1}-T_s}{R_{conv,i}+R_{pipe}+R_{insulation}} = \dfrac{(250-T_s)\text{°C}}{(0.0995+0.0015+3.874)\text{ °C/W}}$

Heat transfer from the outer surface can be expressed as

$$\dot{Q}_{\text{surface to surr, conv+rad}} = h_o A_o(T_s - T_{surr}) + \varepsilon A_o\sigma(T_s^{\,4} - T_{surr}^{\,4}) = (22.50\text{ W/m}^2\text{.°C})(0.3644\text{ m}^2)(T_s - 3)\text{°C}$$
$$+(0.3)(0.3644\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)\left[(T_s+273\text{ K})^4 - (3+273\text{ K})^4\right]$$

Solving the two equations above simultaneously, the surface temperature and the heat transfer rate per m length of the pipe are determined to be

$$T_s = 9.9\text{°C} \quad \text{and} \quad \dot{Q} = \mathbf{60.4\ W} \quad \text{(per m length)}$$

7-136 A cylindrical rod is placed in a cross flow of air, (*a*) the average drag coefficient, (*b*) the convection heat transfer coefficient using the Churchill and Bernstein relation, and (*c*) the convection heat transfer coefficient using Table 7-1 are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The surface temperature is constant.

Properties The properties of air (1 atm) at T_f = (120°C + 20°C)/2 =70°C are given in Table A-15: k = 0.02881 W/m·K, ν = 1.995×10^{-5} m²/s, and Pr = 0.7177.

Analysis (*a*) The Reynolds number for the air flowing across the rod is

$$\mathrm{Re}_D = \frac{VD}{\nu} = \frac{(10\ \mathrm{m/s})(0.005\ \mathrm{m})}{1.995\times10^{-5}\ \mathrm{m^2/s}} = 2506$$

From Fig. 7-17, the average drag coefficient is $C_D \approx \mathbf{0.85}$.

(*b*) Using the Churchill and Bernstein relation for Nusselt number, the convection heat transfer coefficient is

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1+(0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282000}\right)^{5/8}\right]^{4/5}$$

$$h = \frac{0.02881\ \mathrm{W/m\cdot K}}{0.005\ \mathrm{m}}\left\{0.3 + \frac{0.62(2506)^{1/2}(0.7177)^{1/3}}{[1+(0.4/0.7177)^{2/3}]^{1/4}}\left[1+\left(\frac{2506}{282000}\right)^{5/8}\right]^{4/5}\right\}$$

$$= \mathbf{148.3\ W/m^2\cdot K}$$

(*c*) Using Table 7-1, the relation for Nusselt number with Re = 2506 is

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.683\,\mathrm{Re}^{0.466}\,\mathrm{Pr}^{1/3}$$

Hence the convection heat transfer coefficient is

$$h = \frac{0.02881\ \mathrm{W/m\cdot K}}{0.005\ \mathrm{m}}\,0.683(2506)^{0.466}(0.7177)^{1/3} = \mathbf{135.2\ W/m^2\cdot K}$$

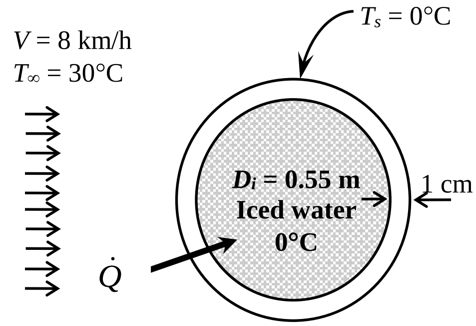
Discussion The Churchill and Bernstein relation is more accurate, and should be preferred whenever possible. The result from (*c*) is approximately 9% lower than the result from (*b*).

7-137 A spherical tank used to store iced water is subjected to winds. The rate of heat transfer to the iced water and the amount of ice that melts during a 24-h period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Thermal resistance of the tank is negligible. **3** Radiation effects are negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties The properties of air at 1 atm pressure and the free stream temperature of 30°C are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$
$$\nu = 1.608\times10^{-5} \text{ m}^2\text{/s}$$
$$\mu_\infty = 1.872\times10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@\,0°\text{C}} = 1.729\times10^{-5} \text{ kg/m.s}$$
$$\text{Pr} = 0.7282$$



Analysis (*a*) The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{[(8\times1000/3600)\text{ m/s}](0.57\text{ m})}{1.608\times10^{-5}\text{ m}^2\text{/s}} = 78{,}773$$

The Nusselt number corresponding to this Reynolds number is determined from

$$Nu = \frac{hD}{k} = 2 + \left[0.4\text{Re}^{0.5} + 0.06\text{Re}^{2/3}\right]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$
$$= 2 + \left[0.4(78{,}773)^{0.5} + 0.06(78{,}773)^{2/3}\right](0.7282)^{0.4}\left(\frac{1.872\times10^{-5}}{1.729\times10^{-5}}\right)^{1/4}$$
$$= 201.9$$

and

$$h = \frac{k}{D}Nu = \frac{0.02588\text{ W/m.°C}}{0.57\text{ m}}(201.9) = 9.167\text{ W/m}^2\text{.°C}$$

The rate of heat transfer to the iced water is

$$\dot{Q} = hA_s(T_s - T_\infty) = h(\pi D^2)(T_s - T_\infty) = (9.167\text{ W/m}^2\text{.°C})[\pi(0.57\text{ m})^2](30-0)\text{°C} = \mathbf{280.7\ W}$$

(*b*) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.2807\text{ kJ/s})(24\times3600\text{ s}) = 24{,}253\text{ kJ}$$

Then the amount of ice that melts during this period becomes

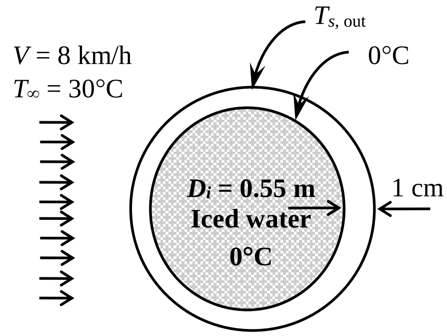
$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{24{,}253\text{ kJ}}{333.7\text{ kJ/kg}} = \mathbf{72.7\ kg}$$

7-138 A spherical tank used to store iced water is subjected to winds. The rate of heat transfer to the iced water and the amount of ice that melts during a 24-h period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The pressure of air is 1 atm.

Properties The properties of air at 1 atm pressure and the free stream temperature of 30°C are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$
$$\nu = 1.608\times10^{-5} \text{ m}^2\text{/s}$$
$$\mu_\infty = 1.872\times10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@0°C} = 1.729\times10^{-5} \text{ kg/m.s}$$
$$\text{Pr} = 0.7282$$



Analysis (*a*) The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{[(8\times1000/3600)\text{ m/s}](0.57\text{ m})}{1.608\times10^{-5}\text{ m}^2\text{/s}} = 78{,}773$$

The Nusselt number corresponding to this Reynolds number is determined from

$$Nu = \frac{hD}{k} = 2 + \left[0.4\text{Re}^{0.5} + 0.06\text{Re}^{2/3}\right]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$
$$= 2 + \left[0.4(78{,}773)^{0.5} + 0.06(78{,}773)^{2/3}\right](0.7282)^{0.4}\left(\frac{1.872\times10^{-5}}{1.729\times10^{-5}}\right)^{1/4}$$
$$= 201.9$$

and $$h = \frac{k}{D}Nu = \frac{0.02588\text{ W/m.°C}}{0.57\text{ m}}(201.9) = 9.167\text{ W/m}^2\text{.°C}$$

In steady operation, heat transfer through the tank by conduction is equal to the heat transfer from the outer surface of the tank by convection and radiation. Therefore,

$$\dot{Q} = \dot{Q}_{\text{through tank}} = \dot{Q}_{\text{from tank, conv+rad}}$$
$$\dot{Q} = \frac{T_{s,out} - T_{s,in}}{R_{sphere}} = h_o A_o (T_{surr} - T_{s,out}) + \varepsilon A_o \sigma (T_{surr}^4 - T_{s,out}^4)$$

where $$R_{sphere} = \frac{r_2 - r_1}{4\pi k r_1 r_2} = \frac{(0.285 - 0.275)\text{ m}}{4\pi(15\text{ W/m.°C})(0.285\text{ m})(0.275\text{ m})} = 6.769\times10^{-4}\text{ °C/W}$$

$$A_o = \pi D^2 = \pi(0.57\text{ m})^2 = 1.021\text{ m}^2$$

Substituting,

$$\dot{Q} = \frac{T_{s,out} - 0°\text{C}}{6.769\times10^{-4}\text{ °C/W}} = (9.167\text{ W/m}^2\text{.°C})(1.021\text{ m}^2)(30 - T_{s,out})°\text{C}$$
$$+ (0.75)(1.021\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(25 + 273\text{ K})^4 - (T_{s,out} + 273\text{ K})^4]$$

whose solution is $T_s = 0.26°\text{C}$ and $\dot{Q} = 378.7\text{ W} \cong$ **379 W**

(*b*) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.3787\text{ kJ/s})(24\times3600\text{ s}) = 32{,}720\text{ kJ}$$

Then the amount of ice that melts during this period becomes

$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{32{,}720\text{ kJ}}{333.7\text{ kJ/kg}} = \mathbf{98.1\text{ kg}}$$

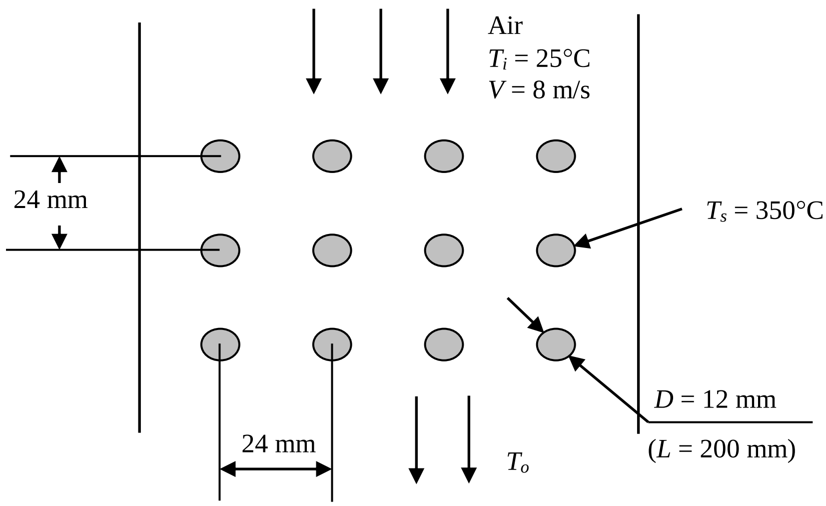
7-139 Air is heated by an array of electrical heating elements. The rate of heat transfer to air and the exit temperature of air are to be determined.

Assumptions **1** Steady operating conditions exist.

Properties The exit temperature of air, and thus the mean temperature, is not known. We evaluate the air properties at the assumed mean temperature of 35°C (will be checked later) and 1 atm (Table A-15):

k = 0.02625 W/m-K ρ = 1.145 kg/m^3
c_p =1.007 kJ/kg-K   Pr = 0.7268
μ = 1.895×10^{-5} kg/m-s $Pr_s = Pr_{@\,Ts}$ = 0.6937

Also, the density of air at the inlet temperature of 25°C (for use in the mass flow rate calculation at the inlet) is ρ_i = 1.184 kg/m^3.



Analysis It is given that D = 0.012 m, $S_L = S_T$ = 0.024 m, and V = 8 m/s. Then the maximum velocity and the Reynolds number based on the maximum velocity become

$$V_{\max} = \frac{S_T}{S_T - D}V = \frac{24}{24-12}(8\text{ m/s}) = 16\text{ m/s}$$

$$\text{Re}_D = \frac{\rho V_{\max} D}{\mu} = \frac{(1.145\text{ kg/m}^3)(16\text{ m/s})(0.012\text{m})}{1.895\times10^{-5}\text{ kg/m}\cdot\text{s}} = 11{,}600$$

The average Nusselt number is determined using the proper relation from Table 7-2 to be

$$\text{Nu}_D = 0.27\,\text{Re}_D^{0.63}\,\text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25}$$
$$= 0.27(11{,}600)^{0.63}(0.7268)^{0.36}(0.7268/0.6937)^{0.25} = 88.55$$

This Nusselt number is applicable to tube banks with N_L > 16. In our case the number of rows is N_L = 3, and the corresponding correction factor from Table 7-3 is F = 0.86. Then the average Nusselt number and heat transfer coefficient for all the tubes in the tube bank become

$$\text{Nu}_{D,N_L} = F\text{Nu}_D = (0.86)(88.55) = 76.15$$

$$h = \frac{Nu_{D,N_L}k}{D} = \frac{76.15(0.02625\text{ W/m}\cdot{}^\circ\text{C})}{0.012\text{ m}} = 166.6\text{ W/m}^2\cdot{}^\circ\text{C}$$

The total number of tubes is $N = N_L \times N_T$ = 3×4 = 12. The heat transfer surface area and the mass flow rate of air (evaluated at the inlet) are

$$A_s = N\pi DL = 12\pi(0.012\text{ m})(0.200\text{m}) = 0.09048\text{ m}^2$$

$$\dot m = \dot m_i = \rho_i V(N_T S_T L) = (1.184\text{ kg/m}^3)(8\text{ m/s})(4)(0.024\text{ m})(0.200\text{m}) = 0.1819\text{ kg/s}$$

Then the fluid exit temperature, the log mean temperature difference, and the rate of heat transfer become

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot m c_p}\right) = 350-(350-25)\exp\left(-\frac{(0.09048\text{ m}^2)(166.6\text{ W/m}^2\cdot{}^\circ\text{C})}{(0.1819\text{ kg/s})(1007\text{ J/kg}\cdot{}^\circ\text{C})}\right) = 50.67^\circ\text{C}$$

$$\Delta T_{lm} = \frac{(T_s - T_i)-(T_s - T_e)}{\ln[(T_s - T_i)/(T_s - T_e)]} = \frac{(350-25)-(350-50.67)}{\ln[(350-25)/(350-50.67)]} = 312.0^\circ\text{C}$$

$$\dot Q = hA_s\Delta T_{lm} = (166.6\text{ W/m}^2\cdot{}^\circ\text{C})(0.09048\text{m}^2)(312.0^\circ\text{C}) = \mathbf{4703\ W}$$

Fundamentals of Engineering (FE) Exam Problems

7-140 For laminar flow of a fluid along a flat plate, one would expect the largest local convection heat transfer coefficient for the same Reynolds and Prandtl numbers when

(a) The same temperature is maintained on the surface

(b) The same heat flux is maintained on the surface

(c) The plate has an unheated section

(d) The plate surface is polished

(e) None of the above

Answer (b)

7-141 Air at 20ºC flows over a 4-m long and 3-m wide surface of a plate whose temperature is 80ºC with a velocity of 7 m/s. The length of the surface for which the flow remains laminar is

(a) 0.9 m (b) 1.3 m (c) 1.8 m (d) 2.2 m (e) 3.7 m

(For air, use k = 0.02735 W/m·°C, Pr = 0.7228, $\nu = 1.798\times10^{-5}$ m^2/s.)

Answer (b) 1.3 m

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_infinity=20 [C]
T_s=80 [C]
L=4 [m]
W=3 [m]
V=7 [m/s]
"Properties of air at the film temperature of (80+20)/2=50C are (Table A-15)"
k=0.02735 [W/m-C]
nu=1.798E-5 [m^2/s]
Pr=0.7228
Re_cr=5E5
x_cr=(Re_cr*nu)/V
```

7-142 Air at 20ºC flows over a 4-m long and 3-m wide surface of a plate whose temperature is 80ºC with a velocity of 5 m/s. The rate of heat transfer from the laminar flow region of the surface is

(a) 950 W (b) 1037 W (c) 2074 W (d) 2640 W (e) 3075 W

(For air, use k=0.02735 W/m·°C, Pr = 0.7228, ν =1.798×10^{-5} m^2/s.)

Answer (c) 2074 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_infinity=20 [C]
T_s=80 [C]
L=4 [m]
W=3 [m]
V=5 [m/s]
"Properties of air at the film temperature of (80+20)/2=50C are (Table A-15)"
k=0.02735 [W/m-C]
nu=1.798E-5 [m^2/s]
Pr=0.7228
Re_cr=5E5
x_cr=(Re_cr*nu)/V
Nus=0.664*Re_cr^0.5*Pr^(1/3)
h=k/x_cr*Nus
A_laminar=x_cr*W
Q_dot=h*A_laminar*(T_s-T_infinity)

"Some Wrong Solutions with Common Mistakes"
W_Nus=0.332*Re_cr^0.5*Pr^(1/3) "Using local Nusselt number relation"
W_h=k/x_cr*W_Nus
W_Q_dot=W_h*A_laminar*(T_s-T_infinity)
```

7-143 Engine oil at 105ºC flows over the surface of a flat plate whose temperature is 15ºC with a velocity of 1.5 m/s. The local drag force per unit surface area 0.8 m from the leading edge of the plate is

(a) 21.8 N/m^2 (b) 14.3 N/m^2 (c) 10.9 N/m^2 (d) 8.5 N/m^2 (e) 5.5 N/m^2

(For oil, use $\nu = 8.565\times10^{-5}$ m^2/s, $\rho = 864$ kg/m^3.)

Answer (e) 5.5 N/m^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_infinity=105 [C]
T_s=15 [C]
V=1.5 [m/s]
x=0.8 [m]
"Properties of oil at the film temperature of (105+15)/2=60C are (Table A-13)"
rho=864 [kg/m^3]
nu=8.565E-5 [m^2/s]
Re_x=(V*x)/nu "The calculated Re number is smaller than the critical number, and therefore we have laminar flow"
C_f_x=0.664/Re_x^(1/2)
F_D=C_f_x*(rho*V^2)/2

"Some Wrong Solutions with Common Mistakes"
W1_C_f_x=0.0592/Re_x^(1/5) "Using local turbulent flow relation"
W1_F_D=W1_C_f_x*(rho*V^2)/2
W2_C_f_x=1.328/Re_x^(1/2) "Using average laminar flow relation"
W2_F_D=W2_C_f_x*(rho*V^2)/2
```

7-144 Air (k = 0.028 W/m·K, Pr = 0.7) at 50ºC flows along a 1 m long flat plate whose temperature is maintained at 20ºC with a velocity such that the Reynolds number at the end of the plate is 10,000. The heat transfer per unit width between the plate and air is

(a) 20 W/m (b) 30 W/m (c) 40 W/m (d) 50 W/m (e) 60 W/m

Answer (d) 50 W/m

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Re= 10000
Pr=0.7
l=1 [m]
k=0.028 [W/m-K]
Ta=50 [C]
Tp=20 [C]
h=0.664*k*Re^0.5*Pr^0.333/l
Q=h*l*(Ta-Tp)
```

7-145 Air at 15ºC flows over a flat plate subjected to a uniform heat flux of 240 W/m^2 with a velocity of 3.5 m/s. The surface temperature of the plate 6 m from the leading edge is

(a) 40.5ºC (b) 41.5ºC (c) 58.2 ºC (d) 95.4ºC (e) 134ºC

(For air, use k=0.02551 W/m·°C, Pr = 0.7296, ν =1.562×10^{-5} m^2/s.)

Answer (a) 40.5ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_infinity=15 [C]
q_dot=240 [W/m^2]
V=3.5 [m/s]
x=6 [m]
"Properties of air at 25 C are (Table A-15)"
k=0.02551 [W/m-C]
nu=1.562E-5 [m^2/s]
Pr=0.7296
Re_x=(V*x)/nu "The calculated Re number is greater than critical number, and therefore we have turbulent flow at the specified location"
Nus=0.0308*Re_x^0.8*Pr^(1/3)
h=k/x*Nus
q_dot=h*(T_s-T_infinity)

"Some Wrong Solutions with Common Mistakes"
W1_Nus=0.453*Re_x^0.5*Pr^(1/3) "Using laminar flow Nusselt number relation for q_dot = constant"
W1_h=k/x*W1_Nus
q_dot=W1_h*(W1_T_s-T_infinity)
W2_Nus=0.0296*Re_x^0.8*Pr^(1/3) "Using turbulent flow Nusselt number relation for T_s = constant"
W2_h=k/x*W2_Nus
q_dot=W2_h*(W2_T_s-T_infinity)
```

7-146 Air at 20ºC flows over a 4-m long and 3-m wide surface of a plate whose temperature is 80ºC with a velocity of 5 m/s. The rate of heat transfer from the surface is

(a) 7383 W (b) 8985 W (c) 11,231 W (d) 14,672 W (e) 20,402 W

(For air, use k=0.02735 W/m·°C, Pr = 0.7228, ν =1.798×10^{-5} m^2/s.)

Answer (a) 7383 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_infinity=20 [C]
T_s=80 [C]
L=4 [m]
W=3 [m]
V=5 [m/s]
"Properties of air at the film temperature of (80+20)/2=50C are (Table A-15)"
k=0.02735 [W/m-C]
nu=1.798E-5 [m^2/s]
Pr=0.7228
Re=(V*L)/nu "The calculated Re number is greater than critical number, and therefore we have combined laminar-turbulent flow"
Nus=(0.037*Re^0.8-871)*Pr^(1/3)
h=k/L*Nus
A_s=L*W
Q_dot=h*A_s*(T_s-T_infinity)

"Some Wrong Solutions with Common Mistakes"
W1_Nus=0.037*Re^0.8*Pr^(1/3) "Using turbulent flow relation"
W1_h=k/L*W1_Nus
W1_Q_dot=W1_h*A_s*(T_s-T_infinity)
```

7-147 Water at 75ºC flows over a 2-m-long, 2-m-wide surface of a plate whose temperature is 5ºC with a velocity of 1.5 m/s. The total drag force acting on the plate is

(a) 2.8 N (b) 12.3 N (c) 13.7 N (d) 15.4 N (e) 20.0 N

(For air, use $\nu = 0.658\times10^{-6}$ m^2/s, $\rho = 992$ kg/m^3.)

Answer (c) 13.7 N

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_infinity=75 [C]
T_s=5 [C]
L=2 [m]
W=2 [m]
V=1.5 [m/s]
"Properties of water at the film temperature of (75+5)/2=40C are (Table A-9)"
nu=0.658E-6 [m^2/s]
rho=992 [kg/m^3]
Re=(V*L)/nu "The calculated Re number is greater than critical number, and therefore we have combined laminar-turbulent flow"
C_f=0.074/Re^(1/5)-1742/Re
A_s=L*W
F_D=C_f*A_s*(rho*V^2)/2

"Some Wrong Solutions with Common Mistakes"
W1_C_f=0.074/Re^(1/5) "Using turbulent flow relation"
W1_F_D=W1_C_f*A_s*(rho*V^2)/2
W2_C_f=1.328/Re^(1/2) "Using laminar flow relation"
W2_F_D=W2_C_f*A_s*(rho*V^2)/2
W3_C_f=0.0592/Re^(1/5) "Using local turbulent flow relation"
W3_F_D=W3_C_f*A_s*(rho*V^2)/2
```

7-148 Air at 25ºC flows over a 5-cm-diameter, 1.7-m-long smooth pipe with a velocity of 4 m/s. A refrigerant at -15ºC flows inside the pipe and the surface temperature of the pipe is essentially the same as the refrigerant temperature inside. The drag force exerted on the pipe by the air is

(a) 0.4 N (b) 1.1 N (c) 8.5 N (d) 13 N (e) 18 N

(For air, use $\nu = 1.382\times10^{-5}$ m^2/s, ρ = 1.269 kg/m^3.)

Answer (b) 1.1 N

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_infinity=25 [C]
T_s=-15 [C]
D=0.05 [m]
L=1.7 [m]
V=4 [m/s]
"Properties of air at the film temperature of (25-15)/2=5 C are (Table A-15)"
rho=1.269 [kg/m^3]
nu=1.382E-5 [m^2/s]
Re=(V*D)/nu "The drag coefficient corresponding to the calculated Re = 14,472 is (Fig. 7-17)"
C_D=1.3
A=L*D
F_D=C_D*A*rho*V^2/2
```

7-149 Air at 25ºC flows over a 4-cm-diameter, 1.7-m-long pipe with a velocity of 4 m/s. A refrigerant at –15ºC flows inside the pipe and the surface temperature of the pipe is essentially the same as the refrigerant temperature inside. Air properties at the average temperature are k=0.0240 W/m·°C, Pr = 0.735, $\nu = 1.382\times10^{-5}$ m^2/s. The rate of heat transfer to the pipe is

(a) 126 W (b) 245 W (c) 302 W (d) 415 W (e) 556 W

Answer (c) 302 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_infinity=25 [C]
T_s=-15 [C]
D=0.04 [m]
L=1.7 [m]
V=4 [m/s]
"Properties of air at the film temperature of (25-15)/2=5 C are (Table A-15)"
k=0.0240 [W/m-C]
nu=1.382E-5 [m^2/s]
Pr=0.735
Re=(V*D)/nu
Nus=0.3+(0.62*Re^(1/2)*Pr^(1/3))/(1+(0.4/Pr)^(2/3))^(1/4)*(1+(Re/282000)^(5/8))^(4/5)
h=k/D*Nus
A_s=pi*D*L
Q_dot=h*A_s*(T_infinity-T_s)
```

7-150 Kitchen water at 10ºC flows over a 10-cm-diameter pipe with a velocity of 1.1 m/s. Geothermal water enters the pipe at 90ºC at a rate of 1.25 kg/s. For calculation purposes, the surface temperature of the pipe may be assumed to be 70ºC. If the geothermal water is to leave the pipe at 50ºC, the required length of the pipe is

(a) 1.1 m (b) 1.8 m (c) 2.9 m (d) 4.3 m (e) 7.6 m

(For both water streams, use k = 0.631 W/m·°C, Pr = 4.32, ν =0.658×10^{-6} m^2/s, c_p= 4179 J/kg·°C.)

Answer (c) 2.9 m

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_infinity=10 [C]
D=0.10 [m]
V=1.1 [m/s]
T_s=70 [C]
T_geo_in=90 [C]
T_geo_out=50 [C]
m_dot_geo=1.25 [kg/s]
"Properties of water at the film temperature of (10+70)/2=40 C are (Table A-9)"
k=0.631 [W/m-C]
Pr=4.32
c_p=4179 [J/kg-C]
nu=0.658E-6 [m^2/s]
Re=(V*D)/nu
Nus=0.3+(0.62*Re^(1/2)*Pr^(1/3))/(1+(0.4/Pr)^(2/3))^(1/4)*(1+(Re/282000)^(5/8))^(4/5)
h=k/D*Nus
q=h*(T_s-T_infinity)
Q_dot=m_dot_geo*c_p*( T_geo_in-T_geo_out)
A_s=Q_dot/q
L=A_s/(pi*D)
```

7-151 Wind at 30ºC flows over a 0.5-m-diameter spherical tank containing iced water at 0ºC with a velocity of 25 km/h. If the tank is thin-shelled with a high thermal conductivity material, the rate at which ice melts is

(a) 4.78 kg/h (b) 6.15 kg/h (c) 7.45 kg/h (d) 11.8 kg/h (e) 16.0 kg/h

(Take h_{if}= 333.7 kJ/kg and use the following for air: k=0.02588 W/m·°C, Pr = 0.7282, ν =1.608×10^{-5} m^2/s, μ_∞ =1.872×10^{-5} kg/m·s, μ_s = 1.729×10^{-5} kg/m·s)

Answer (a) 4.78 kg/h

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.5 [m]
T_infinity=30 [C]
T_s=0 [C]
V=25 [km/h]*Convert(km/h, m/s)
"Properties of air at the free-stream temperature of 30 C are (Table A-15)"
k=0.02588 [W/m-C]
nu=1.608E-5 [m^2/s]
Pr=0.7282
mu_infinity=1.872E-5 [kg/m-s]
mu_s=1.729E-5 [kg/m-s] "at the surface temperature of 0 C"
Re=(V*D)/nu
Nus=2+(0.4*Re^(1/2)+0.06*Re^(2/3))*Pr^0.4*(mu_infinity/mu_s)^(1/4)
h=k/D*Nus
A_s=pi*D^2
Q_dot=h*A_s*(T_infinity-T_s)*Convert(W, kW)
h_if=333.7 [kJ/kg] "Heat of fusion of water at 0 C"
m_dot_cond=Q_dot/h_if*Convert(kg/s, kg/h)
```

7-152 Ambient air at 20ºC flows over a 30-cm-diameter hot spherical object with a velocity of 4.2 m/s. If the average surface temperature of the object is 200ºC, the average convection heat transfer coefficient during this process is

(a) 8.6 W/m^2·ºC (b) 15.7 W/m^2·ºC (c) 18.6 W/m^2·ºC (d) 21.0 W/m^2·ºC (e) 32.4 W/m^2·ºC

(For air, use k=0.02514 W/m·°C, Pr = 0.7309, ν =1.516×10^{-5} m^2/s, μ_∞ =1.825×10^{-5} kg/m·s, μ_s = 2.577×10^{-5} kg/m·s.)

Answer (b) 15.7 W/m^2·ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.3 [m]
T_infinity=20 [C]
T_s=200 [C]
V=4.2 [m/s]
"Properties of air at the free-stream temperature of 20 C are (Table A-15)"
k=0.02514 [W/m-C]
nu=1.516E-5 [m^2/s]
Pr=0.7309
mu_infinity=1.825E-5 [kg/m-s]
mu_s=2.577E-5 [kg/m-s] "at the surface temperature of 200 C"
Re=(V*D)/nu
Nus=2+(0.4*Re^(1/2)+0.06*Re^(2/3))*Pr^0.4*(mu_infinity/mu_s)^(1/4)
h=k/D*Nus
```

7-153 Jakob suggests the following correlation be used for square tubes in a liquid cross-flow situation: $\mathrm{Nu} = 0.102\mathrm{Re}^{0.675}\,\mathrm{Pr}^{1/3}$. Water ($k$ = 0.61 W/m·K, Pr = 6) at 50ºC flows across a 1 cm square tube with a Reynolds number of 10,000 and surface temperature of 75ºC. If the tube is 3 m long, the rate of heat transfer between the tube and water is

(a) 9.8 kW (b) 12.4 kW (c) 17.0 kW (d) 19.6 kW (e) 24.0 kW

Answer (c) 17.0 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k=0.61 [W/m-K]
Pr = 6
L=0.01 [m]
Lg=3 [m]
DT=25 [K]
Re=10000
Nus=0.102*Re^0.675*Pr^0.333
h=Nus*k/L
Q=4*L*Lg*h*DT
```

7-154 Air (Pr = 0.7, k = 0.026 W/m·K) at 200ºC flows across 3-cm-diameter tubes whose surface temperature is 50ºC with a Reynolds number of 8000. The Churchill and Bernstein convective heat transfer correlation for the average Nusselt number in this situation is $\mathrm{Nu} = 0.3 + \frac{0.62\mathrm{Re}^{0.5}\,\mathrm{Pr}^{0.33}}{\left[1+(0.4/\mathrm{Pr})^{0.67}\right]^{0.25}}$. The average heat flux in this case is

(a) 1.3 kW/m^2 (b) 2.4 kW/m^2 (c) 4.1 kW/m^2 (d) 5.7 kW/m^2 (e) 8.2 kW/m^2

Answer (d) 5.7 kW/m^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Pr=0.7
k=0.026 [W/m-K]
Re=8000
dT=150 [K]
D=0.03 [m]
Nusselt=0.3+0.62*Re^0.5*Pr^0.33/(1+(0.4/Pr)^0.67)^0.25
Q=k*Nusselt*dT/D
```

7-155 Jakob suggests the following correlation be used for square tubes in a liquid cross-flow situation: $\mathrm{Nu} = 0.102\mathrm{Re}^{0.675}\,\mathrm{Pr}^{1/3}$. Water ($k$ = 0.61 W/m·K, Pr = 6) flows across a 1 cm square tube with a Reynolds number of 10,000. The convection heat transfer coefficient is

(a) 5.7 kW/m^2·K (b) 8.3 kW/m^2·K (c) 11.2 kW/m^2·K (d) 15.6 kW/m^2·K (e) 18.1 kW/m^2·K

Answer (a) 5.7 kW/m^2·K

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
k=0.61 [W/m-K]
Pr = 6
L=0.01 [m]
Re=10000
Nus=0.102*Re^0.675*Pr^0.333
h=Nus*k/L
```

7-156 …. 7-158 Design and Essay Problems

Solutions Manual for

Heat and Mass Transfer: Fundamentals & Applications
6th Edition
Yunus A. Çengel, Afshin J. Ghajar
McGraw-Hill Education, 2020

Chapter 8
INTERNAL FORCED CONVECTION

General Flow Analysis

8-1C Engine oil requires a larger pump because of its much larger viscosity.

8-2C In fluid flow, it is convenient to work with an average or mean velocity V_{avg} and an average or mean temperature T_m which remain constant in incompressible flow when the cross-sectional area of the tube is constant. The V_{avg} and T_m represent the velocity and temperature, respectively, at a cross section if all the particles were at the same velocity and temperature.

8-3C The generally accepted value of the Reynolds number above which the flow in a smooth pipe is turbulent is 4000.

8-4C For flow through non-circular tubes, the Reynolds number as well as the Nusselt number and the friction factor are based on the hydraulic diameter D_h defined as $D_h = \dfrac{4A_c}{p}$ where A_c is the cross-sectional area of the tube and p is its perimeter. The hydraulic diameter is defined such that it reduces to ordinary diameter D for circular tubes since

$$D_h = \frac{4A_c}{p} = \frac{4\pi D^2/4}{\pi D} = D.$$

8-5C The fluid viscosity is responsible for the development of the velocity boundary layer. For the idealized inviscid fluids (fluids with zero viscosity), there will be no velocity boundary layer.

8-6C In the fully developed region of flow in a circular tube, the velocity profile will not change in the flow direction but the temperature profile may.

8-7C The friction factor is highest at the tube inlet where the thickness of the boundary layer is zero, and decreases gradually to the fully developed value. The same is true for turbulent flow.

8-8C The friction factor f remains constant along the flow direction in the fully developed region in both laminar and turbulent flow.

8-9C In turbulent flow, the tubes with rough surfaces have much higher friction factors than the tubes with smooth surfaces. In the case of laminar flow, the effect of surface roughness on the friction factor is negligible.

8-10C The region from the tube inlet to the point at which the boundary layer merges at the centerline is called the *hydrodynamic entry region*, and the length of this region is called *hydrodynamic entry length*. The entry length is much longer in laminar flow than it is in turbulent flow. But at very low Reynolds numbers, L_h is very small (L_h = 1.2D at Re = 20).

8-11C The hydrodynamic and thermal entry lengths are given as $L_h = 0.05\,\text{Re}\,D$ and $L_t = 0.05\,\text{Re}\,\text{Pr}\,D$ for laminar flow, and $L_h \approx L_t \approx 10D$ in turbulent flow. Noting that $\text{Pr} >> 1$ for oils, the thermal entry length is larger than the hydrodynamic entry length in laminar flow. In turbulent, the hydrodynamic and thermal entry lengths are independent of Re or Pr numbers, and are comparable in magnitude.

8-12C The hydrodynamic and thermal entry lengths are given as $L_h = 0.05\,\text{Re}\,D$ and $L_t = 0.05\,\text{Re}\,\text{Pr}\,D$ for laminar flow, and $L_h \approx L_t \approx 10D$ in turbulent flow. Noting that $\text{Pr} << 1$ for liquid metals, the thermal entry length is smaller than the hydrodynamic entry length in laminar flow. In turbulent, the hydrodynamic and thermal entry lengths are independent of Re or Pr numbers, and are comparable in magnitude.

8-13C The region of flow over which the thermal boundary layer develops and reaches the tube center is called the thermal entry region, and the length of this region is called the thermal entry length. The region in which the flow is both hydrodynamically (the velocity profile is fully developed and remains unchanged) and thermally (the dimensionless temperature profile remains unchanged) developed is called the fully developed region.

8-14C The heat flux will be higher near the inlet because the heat transfer coefficient is highest at the tube inlet where the thickness of thermal boundary layer is zero, and decreases gradually to the fully developed value.

8-15C The heat flux will be higher near the inlet because the heat transfer coefficient is highest at the tube inlet where the thickness of thermal boundary layer is zero, and decreases gradually to the fully developed value.

8-16C The logarithmic mean temperature difference ΔT_{ln} is an exact representation of the average temperature difference between the fluid and the surface for the entire tube. It truly reflects the exponential decay of the local temperature difference. The error in using the arithmetic mean temperature increases to undesirable levels when ΔT_e differs from ΔT_i by great amounts. Therefore we should always use the logarithmic mean temperature.

8-17C When the surface temperature of tube is constant, the appropriate temperature difference for use in the Newton's law of cooling is logarithmic mean temperature difference that can be expressed as

$$\Delta T_{\text{lm}} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e / \Delta T_i)}$$

8-18C The number of transfer units NTU is a measure of the heat transfer area and effectiveness of a heat transfer system. A small value of NTU (NTU < 5) indicates more opportunities for heat transfer whereas a large NTU value (NTU >5) indicates that heat transfer will not increase no matter how much we extend the length of the tube.

8-19 The average velocity and mean temperature are to be determined from the given velocity and temperature profiles.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Analysis The average velocity in a tube with a radius of $R = D/2$ is

$$V_{\text{avg}} = \frac{2}{R^2}\int_0^R u(r)r\,dr$$

$$V_{\text{avg}} = \frac{2}{R^2}\int_0^R 0.05r[1-(r/R)^2]\,dr = \frac{2}{R^2}\left[\frac{-0.0125(r^2-R^2)^2}{R^2}\right]_0^R = \mathbf{0.025\,m/s}$$

The mean temperature in a tube with a radius of $R = D/2$ is

$$T_m = \frac{2}{V_{\text{avg}}R^2}\int_0^R T(r)u(r)r\,dr$$

$$T_m = \frac{2(0.05)}{V_{\text{avg}}R^2}\int_0^R r[400+80(r/R)^2-30(r/R)^3][1-(r/R)^2]\,dr$$

$$= \frac{4}{R^2}\int_0^R [400r+80r(r/R)^2-30r(r/R)^3]-[400r(r/R)^2+80r(r/R)^4-30r(r/R)^5]\,dr$$

$$= \frac{4}{R^2}\left(105R^2\right) = \mathbf{420\,K}$$

Discussion Note that the average velocity is half of the maximum velocity (velocity at the center of the tube) for the given profile. This suggests that the given velocity profile has a profile of a fully developed laminar flow or it is parabolic.

8-20 The mass flow rate and the surface heat flux are to be determined from the given velocity and temperature profiles.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The density of 1 atm air at 20°C is ρ = 1.204 kg/m^3 (Table A-15).

Analysis The average velocity in a tube with a radius of $R = D/2$ is

$$V_{\text{avg}} = \frac{2}{R^2}\int_0^R u(r)r\,dr$$

$$V_{\text{avg}} = \frac{2}{R^2}\int_0^R 0.2[1-(r/R)^2]r\,dr = \frac{2}{R^2}\left[\frac{-0.05(r^2-R^2)^2}{R^2}\right]_0^R = 0.1\text{ m/s}$$

The mean temperature in a tube with a radius of $R = D/2$ is

$$T_m = \frac{2}{V_{\text{avg}}R^2}\int_0^R T(r)u(r)r\,dr$$

$$\begin{aligned}T_m &= \frac{2(0.2)}{V_{\text{avg}}R^2}\int_0^R [250+200(r/R)^3][1-(r/R)^2]r\,dr\\ &= \frac{4}{R^2}\int_0^R [250r+200r(r/R)^3]-[250r(r/R)^2+200r(r/R)^5]\,dr\\ &= \frac{4}{R^2}(73.93R^2)\\ &= 295.7\text{ K}\end{aligned}$$

The mass flow rate is

$$\dot{m} = \rho V_{\text{avg}}A_c = \rho V_{\text{avg}}\pi D^2/4 = (1.204\text{ kg/m}^3)(0.1\text{ m/s})\pi(0.08\text{ m})^2/4 = \mathbf{6.05\times10^{-4}\ kg/s}$$

The surface heat flux is determined using

$$\dot{q}_s = h(T_s - T_m)$$

where $T_s = T(r_o) = 250 + 200(R/R)^3 = 450\text{ K}$

$$\dot{q}_s = (100\text{ W/m}^2\cdot\text{K})(450-295.7)\text{ K} = \mathbf{15.4\ kW/m^2}$$

Discussion Since $T_s > T_m$, this indicates that the air is being heated, thus a positive value for the surface heat flux.

8-21 Liquid water entering at 5°C and flowing at 0.007 kg/s is heated in a circular tube with a constant surface temperature. The heat rate transferred to the water is to be determined. The inner surface temperature of the tube is to be determined whether it exceeds the maximum temperature of 79°C recommended by the ASME Code for Process Piping.

Assumptions **1** The flow is steady and incompressible. **2** Uniform tube surface temperature. **3** Inner surface of the tube is smooth. **4** The convection heat transfer coefficient is constant. **5** The PVDC lining is very thin.

Properties The specific heat of water at the bulk mean temperature of $T_b = (T_i + T_e)/2$ = (5 + 15)°C/2 = 10°C is (Table A–9) c_p = 4194 J/kg·K.

Analysis The heat transfer rate to the water is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.007\ \text{kg/s})(4194\ \text{J/kg}\cdot\text{K})(15 - 5)\text{K} = 293.6\ \text{W}$$

The inner surface temperature of the tube is

$$T_e = T_s - (T_s - T_i)\exp\left(-hA_s/\dot{m}c_p\right)$$

where

$$A_s = \pi DL$$

Solving for T_s, yields

$$15 = T_s - (T_s - 5)\exp[-(20\ \text{W/m}^2\cdot\text{K})\pi(0.025\ \text{m})(3\ \text{m})/(0.007\ \text{kg/s})(4194\ \text{J/kg}\cdot\text{K})]$$

$$T_s = \mathbf{72.4}℃ < 79℃$$

Discussion The inner surface temperature of the tube is below the maximum temperature of 79°C for PVDC lining (ASME Code for Process Piping, B31.3-2014, Table A323.4.3). Thus, the use of PVDC as the tube lining is suitable for this operation.

8-22 Liquid water entering at 5°C and flowing at 0.0035 kg/s is heated in a circular tube at a rate of 1000 W. The water temperature at the tube exit is to be determined if it exceeds 120°C. The inner surface temperature of the tube is to be determined if it exceeds 79°C.

Assumptions **1** The flow is steady and incompressible. **2** Uniform tube surface temperature. **3** Inner surface of the tube is smooth. **4** The convection heat transfer coefficient is constant. **5** The PVDC lining is very thin.

Properties The specific heat of water at 40°C is (Table A–9) c_p = 4179 J/kg·K.

Analysis (*a*) The heat transfer rate to the water is

$$\dot{Q} = \dot{m}c_p(T_e - T_i)$$

which can be used to solve for the exit temperature of water:

$$T_e = \frac{\dot{Q}}{\dot{m}c_p} + T_i = \frac{1000\ \text{W}}{(0.0035\ \text{kg/s})(4179\ \text{J/kg}\cdot\text{K})} + 5℃ = \mathbf{73.4}℃ < 120℃$$

(*b*) To solve for the inner surface temperature of the tube, we use

$$T_e = T_s - (T_s - T_i)\exp\left(-hA_s/\dot{m}c_p\right)$$

where

$$\text{NTU} = hA_s/\dot{m}c_p$$

Solving for T_s, yields

$$73.4℃ = T_s - (T_s - 5)\exp(-2)$$

$$T_s = \mathbf{84.1}℃ > 79℃$$

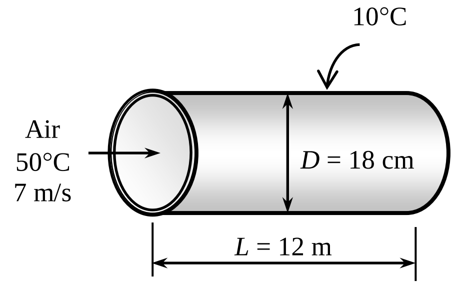
Discussion The water exiting the tube is at a temperature below 120°C, which complies with the ASME Boiler and Pressure Vessel Code. However, the inner surface temperature of the tube is above the maximum temperature of 79°C for PVDC lining, which is noncompliant with the ASME Code for Process Piping.

A different thermoplastic lining should be considered. For example, polytetrafluoroethylene (PTFE) lining has a recommended maximum temperature of 260°C by the ASME Code for Process Piping (ASME B31.3-2014, A323, Table A323.4.3).

With the exit temperature at 73.4°C and the inlet temperature at 5°C, the bulk mean fluid temperature becomes 39.2°C. Thus, 40°C is an appropriate temperature to evaluate the specific heat of water. If desired, the calculations can be repeated by using 39.2°C as the temperature to evaluate the specific heat of water.

8-23 Air flows inside a duct and it is cooled by water outside. The exit temperature of air and the rate of heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the duct is constant. **3** The thermal resistance of the duct is negligible.



Properties The properties of air at the anticipated average temperature of 30°C are (Table A-15)

$$\rho = 1.164\ \text{kg/m}^3$$
$$c_p = 1007\ \text{J/kg.°C}$$

Analysis The mass flow rate of water is

$$\dot{m} = \rho A_c V_{\text{avg}} = \rho\left(\frac{\pi D^2}{4}\right)V_{\text{avg}} = (1.164\ \text{kg/m}^3)\frac{\pi(0.18\ \text{m})^2}{4}(7\ \text{m/s}) = 0.2073\ \text{kg/s}$$

$$A_s = \pi DL = \pi(0.18\ \text{m})(12\ \text{m}) = 6.786\ \text{m}^2$$

The exit temperature of air is determined from

$$T_e = T_s - (T_s - T_i)e^{-hA_s/(\dot{m}c_p)} = 10 - (10-50)e^{-\frac{(65)(6.786)}{(0.2073)(1007)}} = \mathbf{14.84\,°C}$$

The logarithmic mean temperature difference and the rate of heat transfer are

$$\Delta T_{\text{lm}} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{14.84 - 50}{\ln\left(\frac{10 - 14.84}{10 - 50}\right)} = 16.65°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\text{lm}} = (65\ \text{W/m}^2.°\text{C})(6.786\ \text{m}^2)(16.65°\text{C}) = 7343\ \text{W} = \mathbf{7.34\,kW}$$

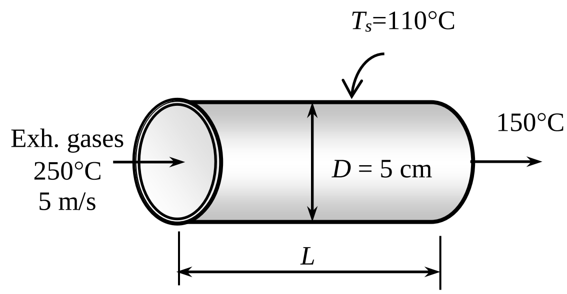
8-24 Combustion gases passing through a tube are used to vaporize waste water. The tube length and the rate of evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is constant. **3** The thermal resistance of the pipe is negligible. **4** Air properties are to be used for exhaust gases.

Properties The properties of air at the average temperature of (250+150)/2=200°C are (Table A-15)

$$c_p = 1023 \text{ J/kg.°C}$$
$$R = 0.287 \text{ kJ/kg.K}$$

Also, the heat of vaporization of water at 1 atm or 100°C is $h_{fg} = 2257$ kJ/kg (Table A-9).



Analysis The density of air at the inlet and the mass flow rate of exhaust gases are

$$\rho = \frac{P}{RT} = \frac{115 \text{ kPa}}{(0.287 \text{ kJ/kg.K})(250+273 \text{ K})} = 0.7662 \text{ kg/m}^3$$

$$\dot{m} = \rho A_c V_{\text{avg}} = \rho\left(\frac{\pi D^2}{4}\right)V_{\text{avg}} = (0.7662 \text{ kg/m}^3)\frac{\pi(0.05 \text{ m})^2}{4}(5 \text{ m/s}) = 0.007522 \text{ kg/s}$$

The rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_i - T_e) = (0.007522 \text{ kg/s})(1023 \text{ J/kg.°C})(250-150\text{°C}) = 769.5 \text{ W}$$

The logarithmic mean temperature difference and the surface area are

$$\Delta T_{\text{lm}} = \frac{T_e - T_i}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} = \frac{150-250}{\ln\left(\dfrac{110-150}{110-250}\right)} = 79.82\text{°C}$$

$$\dot{Q} = hA_s\Delta T_{\text{lm}} \longrightarrow A_s = \frac{\dot{Q}}{h\Delta T_{\text{lm}}} = \frac{769.5 \text{ W}}{(120 \text{ W/m}^2.\text{°C})(79.82\text{°C})} = 0.08034 \text{ m}^2$$

Then the tube length becomes

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{0.08034 \text{ m}^2}{\pi(0.05 \text{ m})} = 0.5115 \text{ m} = \mathbf{51.2 \text{ cm}}$$

The rate of evaporation of water is determined from

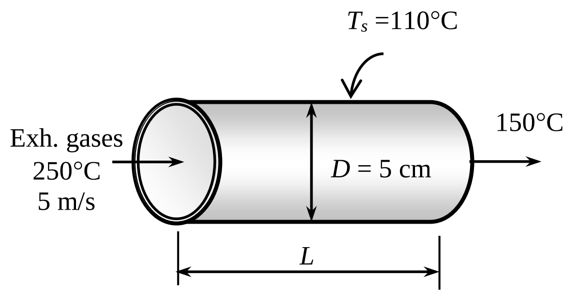
$$\dot{Q} = \dot{m}_{evap}h_{fg} \longrightarrow \dot{m}_{evap} = \frac{\dot{Q}}{h_{fg}} = \frac{0.7695 \text{ kW}}{2257 \text{ kJ/kg}} = 0.0003409 \text{ kg/s} = \mathbf{1.23 \text{ kg/h}}$$

8-25 Combustion gases passing through a tube are used to vaporize waste water. The tube length and the rate of evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is constant. **3** The thermal resistance of the pipe is negligible. **4** Air properties are to be used for exhaust gases.

Properties The properties of air at the average temperature of (250+150)/2=200°C are (Table A-15)

$$c_p = 1023 \text{ J/kg.°C}$$
$$R = 0.287 \text{ kJ/kg.K}$$



Also, the heat of vaporization of water at 1 atm or 100°C is $h_{fg} = 2257$ kJ/kg (Table A-9).

Analysis The density of air at the inlet and the mass flow rate of exhaust gases are

$$\rho = \frac{P}{RT} = \frac{115 \text{ kPa}}{(0.287 \text{ kJ/kg.K})(250+273 \text{ K})} = 0.7662 \text{ kg/m}^3$$

$$\dot{m} = \rho A_c V_{\text{avg}} = \rho\left(\frac{\pi D^2}{4}\right)V_{\text{avg}} = (0.7662 \text{ kg/m}^3)\frac{\pi(0.05 \text{ m})^2}{4}(5 \text{ m/s}) = 0.007522 \text{ kg/s}$$

The rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_i - T_e) = (0.007522 \text{ kg/s})(1023 \text{ J/kg.°C})(250-150°\text{C}) = 769.5 \text{ W}$$

The logarithmic mean temperature difference and the surface area are

$$\Delta T_{\text{lm}} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{150-250}{\ln\left(\frac{110-150}{110-250}\right)} = 79.82°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\text{lm}} \longrightarrow A_s = \frac{\dot{Q}}{h\Delta T_{\text{lm}}} = \frac{769.5 \text{ W}}{(40 \text{ W/m}^2.°\text{C})(79.82°\text{C})} = 0.2410 \text{ m}^2$$

Then the tube length becomes

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{0.2410 \text{ m}^2}{\pi(0.05 \text{ m})} = 1.534 \text{ m} = \mathbf{153\ cm}$$

The rate of evaporation of water is determined from

$$\dot{Q} = \dot{m}_{evap}h_{fg} \longrightarrow \dot{m}_{evap} = \frac{\dot{Q}}{h_{fg}} = \frac{0.7695 \text{ kW}}{2257 \text{ kJ/kg}} = 0.0003409 \text{ kg/s} = \mathbf{1.23\ kg/h}$$

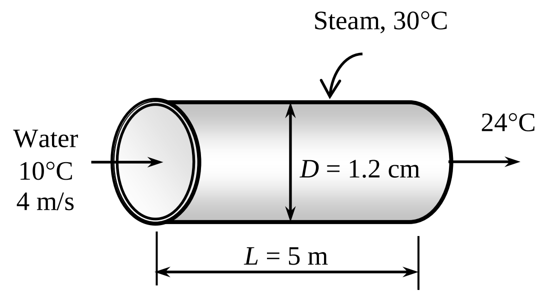
8-26 Steam is condensed by cooling water flowing inside copper tubes. The average heat transfer coefficient and the number of tubes needed are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is constant. **3** The thermal resistance of the pipe is negligible.

Properties The properties of water at the average temperature of (10+24)/2=17°C are (Table A-9)

$$\rho = 998.7 \text{ kg/m}^3$$
$$c_p = 4183.8 \text{ J/kg.°C}$$

Also, the heat of vaporization of water at 30°C is $h_{fg} = 2431 \text{ kJ/kg}$.



Analysis The mass flow rate of water and the surface area are

$$\dot{m} = \rho A_c V_{\text{avg}} = \rho\left(\frac{\pi D^2}{4}\right)V_{\text{avg}} = (998.7 \text{ kg/m}^3)\frac{\pi(0.012 \text{ m})^2}{4}(4 \text{ m/s}) = 0.4518 \text{ kg/s}$$

The rate of heat transfer for one tube is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.4518 \text{ kg/s})(4183.8 \text{ J/kg.°C})(24-10\text{°C}) = 26{,}460 \text{ W}$$

The logarithmic mean temperature difference and the surface area are

$$\Delta T_{\text{lm}} = \frac{T_e - T_i}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} = \frac{24-10}{\ln\left(\dfrac{30-24}{30-10}\right)} = 11.63\text{°C}$$

$$A_s = \pi DL = \pi(0.012 \text{ m})(5 \text{ m}) = 0.1885 \text{ m}^2$$

The average heat transfer coefficient is determined from

$$\dot{Q} = hA_s\Delta T_{\text{lm}} \longrightarrow h = \frac{\dot{Q}}{A_s\Delta T_{\text{lm}}} = \frac{26{,}460 \text{ W}}{(0.1885 \text{ m}^2)(11.63\text{°C})}\left(\frac{1 \text{ kW}}{1000 \text{ W}}\right) = \mathbf{12.1\ kW/m^2.°C}$$

The total rate of heat transfer is determined from

$$\dot{Q}_{total} = \dot{m}_{cond}h_{fg} = (0.15 \text{ kg/s})(2431 \text{ kJ/kg}) = 364.65 \text{ kW}$$

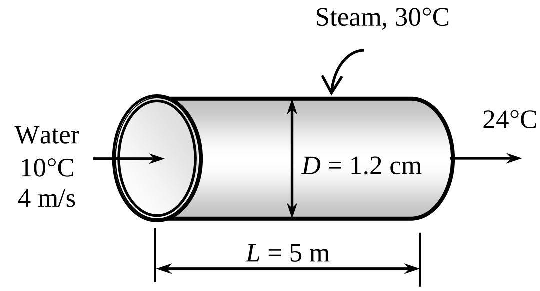
Then the number of tubes becomes

$$N_{tube} = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{364{,}650 \text{ W}}{26{,}460 \text{ W}} = \mathbf{13.8}$$

8-27 Steam is condensed by cooling water flowing inside copper tubes. The average heat transfer coefficient and the number of tubes needed are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is constant. **3** The thermal resistance of the pipe is negligible.

Properties The properties of water at the average temperature of (10+24)/2=17°C are (Table A-9)



$$\rho = 998.7 \text{ kg/m}^3$$
$$c_p = 4183.8 \text{ J/kg.°C}$$

Also, the heat of vaporization of water at 30°C is $h_{fg} = 2431 \text{ kJ/kg}$.

Analysis The mass flow rate of water is

$$\dot{m} = \rho A_c V_{\text{avg}} = \rho\left(\frac{\pi D^2}{4}\right)V_{\text{avg}} = (998.7 \text{ kg/m}^3)\frac{\pi(0.012\text{ m})^2}{4}(4\text{ m/s}) = 0.4518 \text{ kg/s}$$

The rate of heat transfer for one tube is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.4518\text{ kg/s})(4183.8\text{ J/kg.°C})(24-10\text{°C}) = 26{,}460\text{ W}$$

The logarithmic mean temperature difference and the surface area are

$$\Delta T_{\text{lm}} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{24-10}{\ln\left(\frac{30-24}{30-10}\right)} = 11.63\text{°C}$$

$$A_s = \pi DL = \pi(0.012\text{ m})(5\text{ m}) = 0.1885\text{ m}^2$$

The average heat transfer coefficient is determined from

$$\dot{Q} = hA_s\Delta T_{\text{lm}} \longrightarrow h = \frac{\dot{Q}}{A_s\Delta T_{\text{lm}}} = \frac{26{,}460\text{ W}}{(0.1885\text{ m}^2)(11.63\text{°C})}\left(\frac{1\text{ kW}}{1000\text{ W}}\right) = \mathbf{12.1\,kW/m^2.°C}$$

The total rate of heat transfer is determined from

$$\dot{Q}_{total} = \dot{m}_{cond}h_{fg} = (0.60\text{ kg/s})(2431\text{ kJ/kg}) = 1458.6\text{ kW}$$

Then the number of tubes becomes

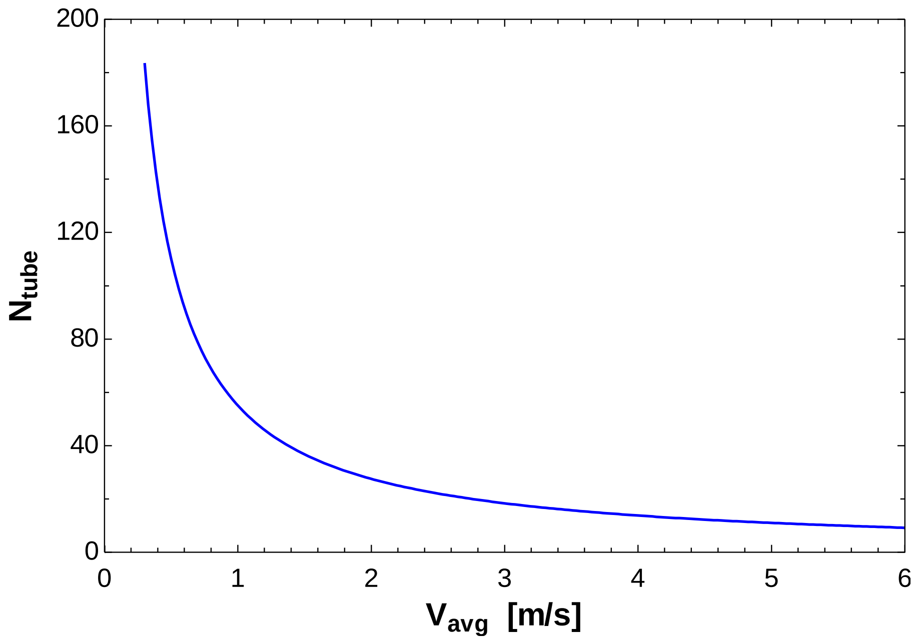
$$N_{tube} = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{1{,}458{,}600\text{ W}}{26{,}460\text{ W}} = \mathbf{55.1}$$

8-28  Prob. 8-26 is reconsidered. The effect of the cooling water average velocity on the number of tubes needed to achieve the indicated heat transfer rate in the condenser is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_i=10 [C]
T_e=24 [C]
T_s=30 [C]
L=5 [m]
D=1.2e-2 [m]
m_dot_cond=0.15

"PROPERTIES"
Fluid$='water'
C_p=CP(Fluid$, T=T_ave, x=0)*Convert(kJ/kg-C, J/kg-C)
rho=Density(Fluid$, T=T_ave, x=0)
h_f=enthalpy(Fluid$, T=T_sat, x=0)
h_g=enthalpy(Fluid$, T=T_sat, x=1)
h_fg=h_g-h_f
T_ave=(T_i+T_e)/2     "T_ave = 1/2*(T_i+T_e)"
T_sat=T_s

"ANALYSIS"
A_c=pi#*D^2/4     "Cross-section area"
A_s=pi#*D*L     "Surface area"
m_dot=rho*A_c*V_avg
DELTAT_lm=(T_i-T_e)/ln((T_s-T_e)/(T_s-T_i))
Q_dot=m_dot*C_p*(T_e-T_i)
Q_dot=h*A_s*DELTAT_lm
Q_dot_total=m_dot_cond*h_fg*1e3
N_tube=Q_dot_total/Q_dot
```

V_{avg} [m/s]	N_{tubes}
0.3	183.6
0.4	137.7
0.5	110.2
0.6	91.81
0.7	78.70
0.8	68.86
0.9	61.21
1.0	55.09
1.2	45.91
1.4	39.35
1.6	34.43
1.8	30.60
2.0	27.54
2.5	22.03
3.0	18.36
3.5	15.74
4.0	13.77
5.0	11.02
6.0	9.181



Discussion At lower velocities ($0.3 \le V_{avg} \le 3$ m/s), the number of tubes needed decreases sharply with increasing V_{avg}. At higher velocities ($3 < V_{avg} \le 6$ m/s), the number of tubes needed does not change significantly with increasing V_{avg}.

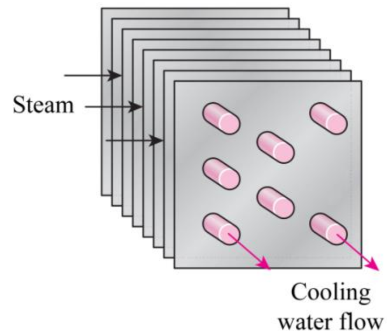
8-29 Steam is condensed by cooling water flowing inside copper tubes. The average heat transfer coefficient and the cooling water mean velocity needed to achieve the indicated heat transfer rate in the condenser are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is constant. **3** The thermal resistance of the tubes is negligible.

Properties The properties of liquid water at the bulk mean fluid temperature of $T_b = (T_i + T_e)/2 = (60°C + 5°C)/2 = 32.5°C$ are (Table A-9):

$$c_p = 4178 \text{ J/kg·K} \quad \text{and} \quad \rho = 994.8 \text{ kg/m}^3$$

Also, the heat of vaporization of water at 68°C is h_{fg} = 2338 kJ/kg



Analysis The total rate of heat transfer from the condensation is

$$\dot{Q}_{\text{total}} = \dot{m}_{\text{cond}} h_{fg} = (0.6\text{ kg/s})(2338\text{ kJ/kg}) = 1402.8\text{ kW}$$

The rate of heat transfer for one tube is

$$\dot{Q} = \frac{\dot{Q}_{\text{total}}}{N_{\text{tube}}} = \frac{1402.8\text{ kW}}{7} = 200.4\text{ kW}$$

Thus, the average heat transfer coefficient can be determined from

$$\dot{Q} = hA_s\Delta T_{\text{lm}} \quad \rightarrow \quad h = \frac{\dot{Q}}{A_s\Delta T_{\text{lm}}} = \frac{200.4\text{ kW}}{(0.3927\text{ m}^2)(26.65\text{ K})} = \mathbf{19.15}$$

where

$$\Delta T_{\text{lm}} = \frac{T_i - T_e}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} = \frac{(5-60)\text{ K}}{\ln\left(\dfrac{68-60}{68-5}\right)} = 26.65\text{ K} \quad \text{and} \quad A_s = \pi DL = \pi(0.025\text{ m})(5\text{ m}) = 0.3927\text{ m}^2$$

Also, the rate of heat transfer for one tube is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) \quad \rightarrow \quad \dot{m} = \frac{\dot{Q}}{c_p(T_e - T_i)} = \frac{200400\text{ kW}}{(4178\text{ J/kg}\cdot\text{K})(60-5)\text{ K}} = 0.8721\text{ kg/s}$$

Thus, water mean velocity is

$$V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{4\dot{m}}{\rho\pi D^2} = \frac{4(0.8721\text{ kg/s})}{(994.8\text{ kg/m}^3)\pi(0.025\text{ m})^2} = \mathbf{1.79\ m/s}$$

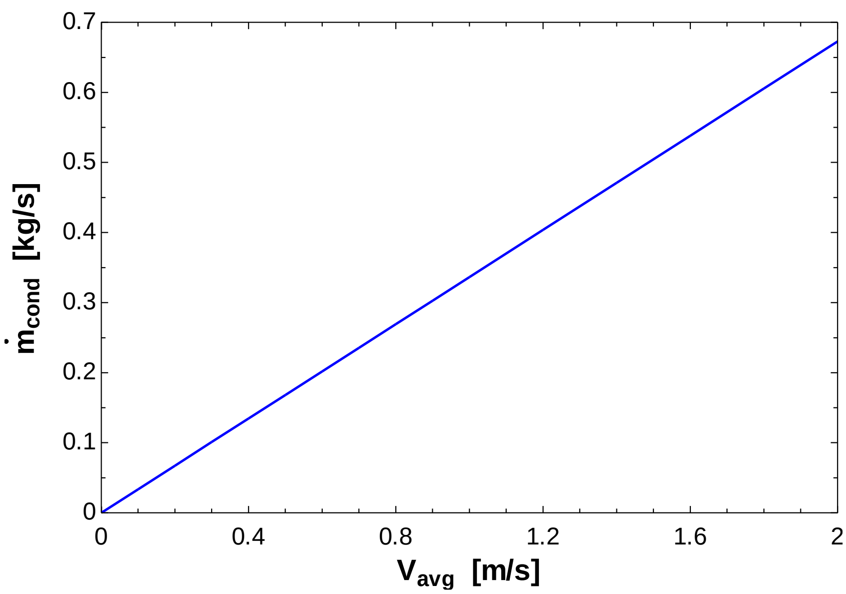
Discussion The water mean velocity needed to achieve the indicated heat transfer rate in the condenser can be reduced by increasing the number of tubes.

8-30 Prob. 8-29 is reconsidered. Steam is condensed by cooling water flowing inside copper tubes. The effect of the cooling water mean velocity on the steam condensation rate is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_i=5 [C]
T_e=60 [C]
T_s=68 [C]
L=5 [m]
D=2.5e-2 [m]
N_tube=7

"PROPERTIES"
Fluid$='water'
C_p=CP(Fluid$, T=T_b, x=0)*Convert(kJ/kg-C, J/kg-C)
rho=Density(Fluid$, T=T_b, x=0)
h_f=enthalpy(Fluid$, T=T_sat, x=0)
h_g=enthalpy(Fluid$, T=T_sat, x=1)
h_fg=h_g-h_f
T_b=(T_i+T_e)/2     "T_b = 1/2*(T_i+T_e)"
T_sat=T_s

"ANALYSIS"
A_c=pi#*D^2/4     "Cross-section area"
A_s=pi#*D*L     "Surface area"
m_dot=rho*A_c*V_avg
DELTAT_lm=(T_i-T_e)/ln((T_s-T_e)/(T_s-T_i))
Q_dot=m_dot*C_p*(T_e-T_i)
Q_dot=h*A_s*DELTAT_lm
Q_dot_total=m_dot_cond*h_fg*1e3
N_tube=Q_dot_total/Q_dot
```

V_{avg} [m/s]	$\dot{m}_{cond}$ [kg/s]
0.001	0.0003364
0.1	0.03364
0.2	0.06728
0.3	0.1009
0.4	0.1346
0.5	0.1682
0.6	0.2018
0.7	0.2355
0.8	0.2691
0.9	0.3027
1.0	0.3364
1.2	0.4037
1.4	0.4709
1.6	0.5382
1.8	0.6055
2.0	0.6728



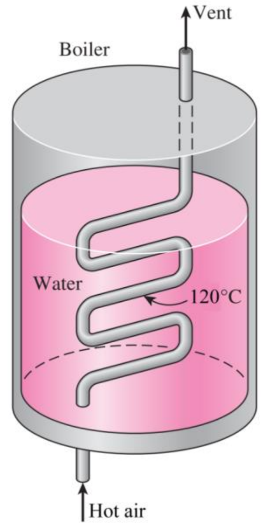
Discussion The rate of steam condensation increases linearly with increasing mean velocity of the cooling water.

8-31 Hot air at 1 atm passing through a tube is used to boil water. The average heat transfer coefficient and the rate of evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is constant. **3** The thermal resistance of the tube is negligible.

Properties The properties of air at the bulk mean fluid temperature of $T_b = (T_i + T_e)/2 = (300°C + 150°C)/2 = 225°C$ are (Table A-15):

$$c_p = 1029 \text{ J/kg·K} \quad \text{and} \quad \rho = 0.7085 \text{ kg/m}^3$$



Also, the heat of vaporization of water at 120°C is $h_{fg} = 2203$ kJ/kg (Table A-9).

Analysis The density of air at the inlet and the mass flow rate of exhaust gases are

$$\dot{m} = \rho A_c V_{avg} = \rho\left(\frac{\pi D^2}{4}\right)V_{avg} = (0.7085\,\text{kg/m}^3)\frac{\pi(0.05\,\text{m})^2}{4}(7\,\text{m/s}) = 0.009738\,\text{kg/s}$$

The rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_i - T_e) = (0.009738\,\text{kg/s})(1029\,\text{J/kg}\cdot\text{K})(300-150)\text{K} = 1503\,\text{W}$$

Thus, the average heat transfer coefficient can be determined from

$$\dot{Q} = hA_s\Delta T_{lm} \quad \rightarrow \quad h = \frac{\dot{Q}}{A_s\Delta T_{lm}} = \frac{1503\,\text{W}}{(0.7854\,\text{m}^2)(83.72\,\text{K})} = \mathbf{22.86\ W/m^2\cdot K}$$

where

$$\Delta T_{lm} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{(150-300)\text{K}}{\ln\left(\frac{120-150}{120-300}\right)} = 83.72\,\text{K}$$

and

$$A_s = \pi DL = \pi(0.05\,\text{m})(5\,\text{m}) = 0.7854\,\text{m}^2$$

Also, the rate of heat transfer from the condensation is

$$\dot{Q} = \dot{m}_{cond}h_{fg}$$

Thus, the rate of water evaporation is

$$\dot{m}_{cond} = \frac{\dot{Q}}{h_{fg}} = \frac{(1503\,\text{J/s})(3600\,\text{s/h})}{2203\times10^3\ \text{J/kg}} = \mathbf{2.457\ kg/h}$$

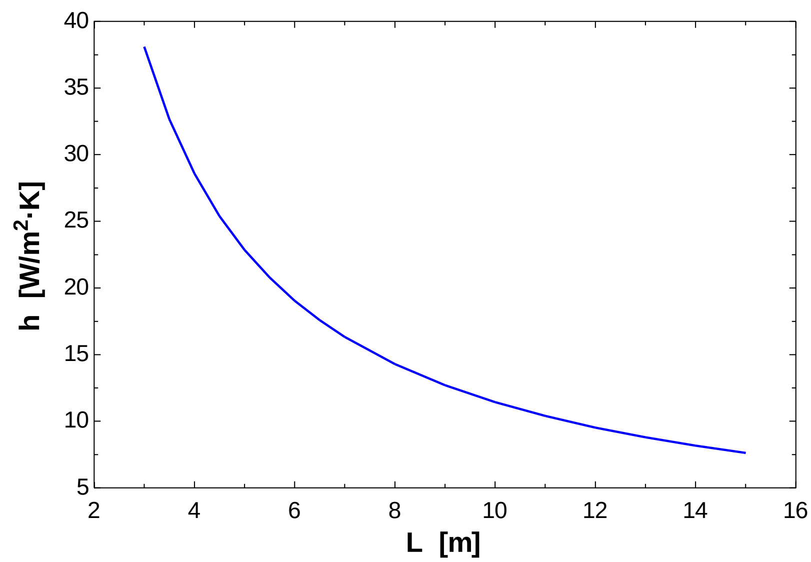
Discussion The rate of water evaporation can be increased by increasing the heat transfer rate.

8-32 Prob. 8-31 is reconsidered. Hot air at 1 atm passing through a tube is used to boil water. The effect of the tube length on the average convection heat transfer coefficient is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_i=300 [C]
T_e=150 [C]
T_s=120 [C]
D=5e-2 [m]
V_avg=7 [m/s]

"PROPERTIES"
T_b=(T_i+T_e)/2     "T_b = 1/2*(T_i+T_e)"
C_p=CP(air, T=T_b)*Convert(kJ/kg-C, J/kg-C)
rho=Density(air, T=T_b, P=101.3)
T_sat=T_s
h_f=enthalpy(water, T=T_sat, x=0)
h_g=enthalpy(water, T=T_sat, x=1)
h_fg=h_g-h_f

"ANALYSIS"
A_c=pi#*D^2/4     "Cross-section area"
A_s=pi#*D*L     "Surface area"
m_dot=rho*A_c*V_avg
DELTAT_lm=(T_e-T_i)/ln((T_s-T_e)/(T_s-T_i))
Q_dot=m_dot*C_p*(T_i-T_e)
Q_dot=h*A_s*DELTAT_lm
Q_dot=m_dot_cond*h_fg*1e3
```

L [m]	h [W/m^2·K]
3.0	38.1
3.5	32.65
4.0	28.57
4.5	25.40
5.0	22.86
5.5	20.78
6.0	19.05
6.5	17.58
7.0	16.33
8.0	14.29
9.0	12.70
10	11.43
11	10.39
12	9.524
13	8.792
14	8.164
15	7.619



Discussion As the tube length increases, so does its surface area. Thus, the average convection heat transfer coefficient decreases with increasing tube length.

8-33 Liquid water entering at 5°C and flowing at 0.0035 kg/s is heated in a circular tube with a constant surface heat flux. The inner surface temperature of the tube is to be determined whether it exceeds the maximum temperature of 79°C recommended by the ASME Code for Process Piping. The axial location where the inner surface temperature reaches 79°C is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Uniform surface heat flux. **3** Fully developed velocity and temperature profiles. **4** Inner surface of the tube is smooth. **5** The convection heat transfer coefficient is constant. **6** The PVDC lining is very thin.

Properties The specific heat of water at 15°C is (Table A–9) c_p = 4185 J/kg·K.

Analysis For constant surface heat flux, the tube inner surface temperature is maximum at the exit. So, with

$$\dot{Q} = \dot{m}c_p(T_e - T_i)$$

the exit temperature of water is

$$T_e = \frac{\dot{Q}}{\dot{m}c_p} + T_i = \frac{300\text{ W}}{(0.0035\text{ kg/s})(4185\text{ J/kg}\cdot\text{K})} + 5℃ = 25.48℃$$

The tube inner surface temperature at the tube exit ($x = L$) is

$$T_{s,L} = T_e + \frac{\dot{q}_s}{h} = 25.48℃ + \frac{1273.2\text{ W/m}^2}{20\text{ W/m}^2\cdot\text{K}} = \mathbf{89.14}℃ > 79℃$$

where the constant surface heat flux is

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = \frac{\dot{Q}}{\pi DL} = \frac{300\text{ W}}{\pi(0.025\text{ m})(3\text{ m})} = 1273.2\text{ W/m}^2$$

At the tube exit, the inner surface temperature exceeds the maximum temperature for PVDC lining.

To determine the axial location where the tube's inner surface temperature reaches $T_{s,x}$ = 79°C, we first determine the mean fluid temperature at that axial location:

$$T_{m,x} = T_{s,x} - \frac{\dot{q}_s}{h} = 79℃ - \frac{1273.2\text{ W/m}^2}{20\text{ W/m}^2\cdot\text{K}} = 15.34℃$$

To find the axial location, we use

$$T_{m,x} = T_i + \frac{\dot{q}_s\pi D}{\dot{m}c_p}x$$

$$x = \frac{\dot{m}c_p}{\dot{q}_s\pi D}\left(T_{m,x} - T_i\right) = \frac{(0.0035\text{ kg/s})(4185\text{ J/kg}\cdot\text{K})}{(1273.2\text{ W/m}^2)\pi(0.025\text{ m})}(15.34 - 5)\text{K} = \mathbf{1.51\ m}$$

Discussion The inner surface temperature at the tube exit exceeds the maximum temperature of 79°C for PVDC lining (ASME Code for Process Piping, B31.3-2014, Table A323.4.3).

The tube's inner surface temperature reaches 79°C at the axial location of 1.51 m. The axial location is measured from the tube inlet along the tube length. So, from x = 1.51 m to the tube exit at x = 3 m, the inner surface temperature is at 79°C and higher. Thus, the use of PVDC as the tube lining is not suitable for this operation.

With the exit temperature at 25.5°C and the inlet temperature at 5°C, the bulk mean fluid temperature becomes 15.3°C. Thus, 15°C is an appropriate temperature to evaluate the specific heat of water.

Laminar and Turbulent Flow in Tubes

8-34C Yes, the volume flow rate in a circular pipe with laminar flow can be determined by measuring the velocity at the centerline in the fully developed region, multiplying it by the cross-sectional area, and dividing the result by 2 since $\dot{V} = V_{avg} A_c = (V_{max}/2) A_c$.

8-35C No, the average velocity in a circular pipe in fully developed laminar flow **cannot** be determined by simply measuring the velocity at $R/2$ (midway between the wall surface and the centerline). The mean velocity is $V_{max}/2$, but the velocity at $R/2$ is

$$V(R/2) = V_{max}\left(1 - \frac{r^2}{R^2}\right)_{r=R/2} = \frac{3V_{max}}{4}$$

8-36C The friction factor for flow in a tube is proportional to the pressure drop. Since the pressure drop along the flow is directly related to the power requirements of the pump to maintain flow, the friction factor is also proportional to the power requirements. The applicable relations are

$$\Delta P = f\,\frac{L}{D}\frac{\rho V^2}{2} \quad \text{and} \quad \dot{W}_{pump} = \frac{\dot{m}\Delta P}{\rho}$$

8-37C Yes, the shear stress at the surface of a tube during fully developed turbulent flow is maximum since the shear stress is proportional to the velocity gradient, which is maximum at the tube surface.

8-38C In fully developed flow in a circular pipe with negligible entrance effects, if the length of the pipe is doubled, the pressure drop will also *double* (the pressure drop is proportional to length).

8-39C In fully developed laminar flow in a circular pipe, the pressure drop is given by

$$\Delta P = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

The mean velocity can be expressed in terms of the flow rate as $V_{avg} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4}$. Substituting,

$$\Delta P = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2} = \frac{32\mu L}{D^2}\frac{\dot{V}}{\pi D^2/4} = \frac{128\mu L \dot{V}}{\pi D^4}$$

Therefore, at constant flow rate and pipe length, the pressure drop is inversely proportional to the 4th power of diameter, and thus reducing the pipe diameter by half will increase the pressure drop **by a factor of 16** .

8-40C In fully developed laminar flow in a circular pipe, the pressure drop is given by

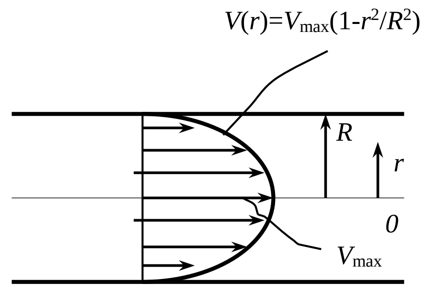
$$\Delta P = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

When the flow rate and thus mean velocity are held constant, the pressure drop becomes proportional to viscosity. Therefore, pressure drop will be **reduced by half** when the viscosity is reduced by half.

8-41 In fully developed laminar flow in a circular pipe, the velocity at $r = R/2$ is measured. The velocity at the center of the pipe ($r = 0$) is to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is given by



$$V(r) = V_{max}\left(1 - \frac{r^2}{R^2}\right)$$

where V_{max} is the maximum velocity which occurs at pipe center, $r = 0$. At $r = R/2$,

$$V(R/2) = V_{max}\left(1 - \frac{(R/2)^2}{R^2}\right) = V_{max}\left(1 - \frac{1}{4}\right) = \frac{3V_{max}}{4}$$

Solving for V_{max} and substituting,

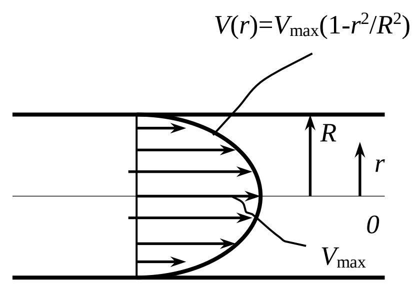
$$V_{max} = \frac{4V(R/2)}{3} = \frac{4(6\text{ m/s})}{3} = \mathbf{8\text{ m/s}}$$

which is the velocity at the pipe center.

8-42 The velocity profile in fully developed laminar flow in a circular pipe is given. The mean and maximum velocities and the volume flow rate are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is given by



$$V(r) = V_{max}\left(1 - \frac{r^2}{R^2}\right)$$

The velocity profile in this case is given by

$$V(r) = 4(1 - r^2/R^2)$$

Comparing the two relations above gives the maximum velocity to be V_{max} = 4 m/s. Then the mean velocity and volume flow rate become

$$V_{avg} = \frac{V_{max}}{2} = \frac{4\text{ m/s}}{2} = \mathbf{2\text{ m/s}}$$

$$\dot{V} = V_{avg}A_c = V_{avg}(\pi R^2) = (2\text{ m/s})[\pi(0.10\text{ m})^2] = \mathbf{0.0628\text{ m}^3\text{/s}}$$

8-43  In fully developed laminar flow inside a circular pipe, the velocities at r/R = 0.5 are measured. For each measured velocity, (*a*) the maximum velocity is to be determined, and (*b*) the velocity profile is to be plotted.

Assumptions The flow is steady, laminar, and fully developed.

Analysis (*a*) The velocity profile in fully developed laminar flow in a circular pipe is given by

$$u(r/R) = V_{max}[1-(r/R)^2]$$

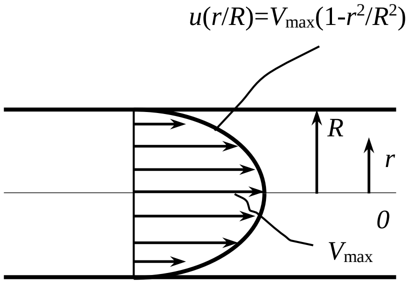
where V_{max} is the maximum velocity which occurs at pipe center, r/R = 0. At r/R = 0.5,

$$u(r/R) = V_{max}[1-0.5^2] = 0.75V_{max} \quad \rightarrow \quad V_{max} = \frac{u(r/R=0.5)}{0.75}$$

Thus, $V_{max,1} = \dfrac{3\text{ m/s}}{0.75} = \mathbf{4\ m/s}$ for midway velocity of 3 m/s

$$V_{max,2} = \frac{6\text{ m/s}}{0.75} = \mathbf{8\ m/s} \quad \text{for midway velocity of 6 m/s}$$

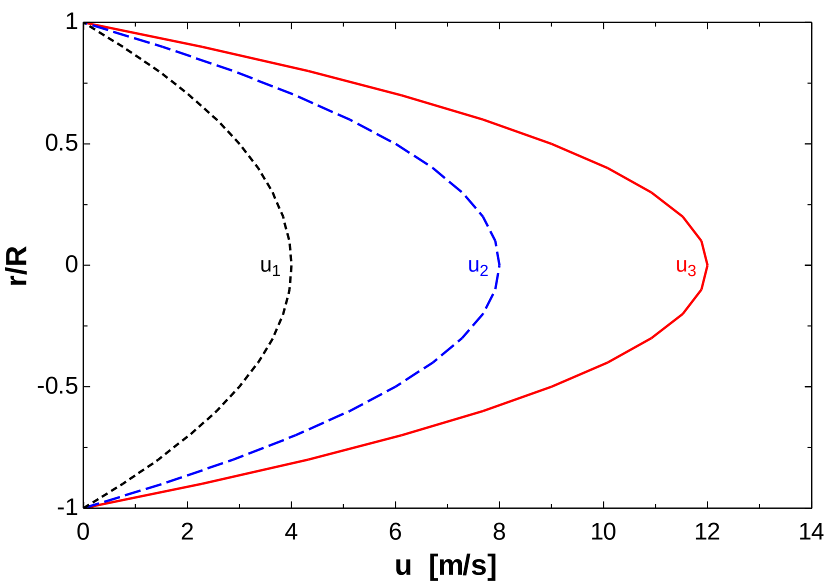
$$V_{max,3} = \frac{9\text{ m/s}}{0.75} = \mathbf{12\ m/s} \quad \text{for midway velocity of 9 m/s}$$



(*b*) The problem is solved using EES, and the solution is given below

```
"GIVEN"
V_midway_1=3 [m/s]    "u(r = R/2) = 3 m/s"
V_midway_2=6 [m/s]    "u(r = R/2) = 6 m/s"
V_midway_3=9 [m/s]    "u(r = R/2) = 9 m/s"
"ANALYSIS"
u_1=4/3*V_midway_1*(1-r_ratio^2)
u_2=4/3*V_midway_2*(1-r_ratio^2)
u_3=4/3*V_midway_3*(1-r_ratio^2)
```

r/R	u_1 [m/s]	u_2 [m/s]	u_3 [m/s]
-1	0	0	0
-0.9	0.76	1.52	2.28
-0.8	1.44	2.88	4.32
-0.7	2.04	4.08	6.12
-0.6	2.56	5.12	7.68
-0.5	3.00	6.00	9.00
-0.4	3.36	6.72	10.08
-0.3	3.64	7.28	10.92
-0.2	3.84	7.68	11.52
-0.1	3.96	7.92	11.88
0	4.00	8.00	12.00
0.1	3.96	7.92	11.88
0.2	3.84	7.68	11.52
0.3	3.64	7.28	10.92
0.4	3.36	6.72	10.08
0.5	3.00	6.00	9.00
0.6	2.56	5.12	7.68
0.7	2.04	4.08	6.12
0.8	1.44	2.88	4.32
0.9	0.76	1.52	2.28
1	0	0	0



Discussion At the pipe wall (r/R = −1 and 1), the velocity is zero because of no-slip condition.

8-44 Water flowing in fully developed conditions through a tube, (*a*) the maximum velocity of the flow in the tube and (*b*) the pressure gradient for the flow are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Flow is isothermal.

Properties The properties of water at 15°C are ρ = 999.1 kg/m^3 and μ = 1.138 × 10^{-3} kg/m·s (from Table A-9).

Analysis The Reynolds number of the flow is

$$\text{Re} = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.02\ \text{kg/s})}{\pi(0.03\ \text{m})(1.138\times10^{-3}\ \text{kg/m}\cdot\text{s})} = 745.9$$

Since Re < 2300, the flow is laminar. The average velocity is

$$V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{4\dot{m}}{\rho \pi D^2} = \frac{4(0.02\ \text{kg/s})}{(999.1\ \text{kg/m}^3)\pi(0.03\ \text{m})^2} = 0.02832\ \text{m/s}$$

(*a*) For fully developed laminar flow, the maximum velocity occurs at the centerline and is determined as

$$u_{\max} = 2V_{\text{avg}} = \mathbf{0.0566\ m/s}$$

(*b*) The pressure gradient can be obtained from the average velocity,

$$V_{\text{avg}} = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right) \quad \rightarrow \quad \left(\frac{dP}{dx}\right) = -V_{\text{avg}}\frac{8\mu}{R^2} = -V_{\text{avg}}\frac{32\mu}{D^2}$$

$$\left(\frac{dP}{dx}\right) = -(0.02832\ \text{m/s})\frac{32(1.138\times10^{-3}\ \text{kg/m}\cdot\text{s})}{(0.03\ \text{m})^2} = \mathbf{-1.15\ Pa/m}$$

Discussion The negative sign for the pressure gradient indicates that there is pressure loss along the tube. Another indication is that fluid flows from high pressure point to low pressure point.

8-45 Water flowing through a tube, the Darcy friction factor and pressure loss associated with the tube for (*a*) mass flow rate of 0.02 kg/s and (*b*) mass flow rate of 0.3 kg/s are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Flow is isothermal.

Properties The properties of water at 15°C are ρ = 999.1 kg/m^3 and μ = 1.138 × 10^{-3} kg/m·s (from Table A-9).

Analysis (*a*) The Reynolds number and the hydrodynamic entry length for the 0.02 kg/s flow are

$$\text{Re} = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.02\ \text{kg/s})}{\pi(0.05\ \text{m})(1.138\times10^{-3}\ \text{kg/m}\cdot\text{s})} = 447.5 < 2300 \quad \text{(laminar flow)}$$

$$L_{h,\,\text{lam}} \approx 0.05\,\text{Re}\,D = 0.05(447.5)(0.05\ \text{m}) = 1.12\ \text{m} < 200\ \text{m} \quad \text{(fully developed flow)}$$

The average velocity is

$$V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{4\dot{m}}{\rho \pi D^2} = \frac{4(0.02\ \text{kg/s})}{(999.1\ \text{kg/m}^3)\pi(0.05\ \text{m})^2} = 0.0102\ \text{m/s}$$

For laminar fully developed flow, the Darcy friction factor and pressure loss are

$$f = \frac{64}{\text{Re}} = \frac{64}{447.5} = \mathbf{0.143}$$

$$\Delta P_L = f\,\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.143\frac{(200\ \text{m})}{(0.05\ \text{m})}\frac{(999.1\ \text{kg/m}^3)(0.0102\ \text{m/s})^2}{2} = \mathbf{29.7\ Pa}$$

(*b*) The Reynolds number and the hydrodynamic entry length for the 0.5 kg/s flow are

$$\text{Re} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.5\ \text{kg/s})}{\pi(0.05\ \text{m})(1.138\times10^{-3}\ \text{kg/m}\cdot\text{s})} = 1.119\times10^4 > \mathbf{10{,}000} \quad \text{(turbulent flow)}$$

$$L_{h,\,\text{turb}} \approx 10D = 10(0.05\ \text{m}) = 0.5\ \text{m} < 200\ \text{m} \quad \text{(fully developed flow)}$$

The average velocity is

$$V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{4\dot{m}}{\rho \pi D^2} = \frac{4(0.5\ \text{kg/s})}{(999.1\ \text{kg/m}^3)\pi(0.05\ \text{m})^2} = 0.2549\ \text{m/s}$$

For turbulent flow, the Darcy friction factor and pressure loss are

$$f = (0.790\ln \text{Re} - 1.64)^{-2} = \mathbf{0.0305}$$

$$\Delta P_L = f\,\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.0305\frac{(200\ \text{m})}{(0.05\ \text{m})}\frac{(999.1\ \text{kg/m}^3)(0.2549\ \text{m/s})^2}{2} = \mathbf{3960\ Pa}$$

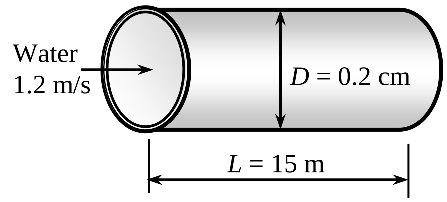
Discussion Even though the laminar flow friction factor is higher than the turbulent flow friction factor, the pressure loss in turbulent flow is larger due to larger average velocity of the flow.

8-46 The average flow velocity in a pipe is given. The pressure drop and the pumping power are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of water are given to be ρ = 999.7 kg/m^3 and μ = 1.307×10^{-3} kg/m·s, respectively.

Analysis (*a*) First we need to determine the flow regime. The Reynolds number of the flow is



$$\text{Re} = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{(999.7\ \text{kg/m}^3)(1.2\ \text{m/s})(2\times10^{-3}\ \text{m})}{1.307\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 1836$$

which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the pressure drop become

$$f = \frac{64}{\text{Re}} = \frac{64}{1836} = 0.0349$$

$$\Delta P = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.0349\frac{15\ \text{m}}{0.002\ \text{m}}\frac{(999.7\ \text{kg/m}^3)(1.2\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) = \mathbf{188\ kPa}$$

(*b*) The volume flow rate and the pumping power requirements are

$$\dot{V} = V_{\text{avg}} A_c = V_{\text{avg}}(\pi D^2/4) = (1.2\ \text{m/s})[\pi(0.002\ \text{m})^2/4] = 3.77\times10^{-6}\ \text{m}^3/s$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (3.77\times10^{-6}\ \text{m}^3/s)(188\ \text{kPa})\left(\frac{1000\ \text{W}}{1\ \text{kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{0.71\ W}$$

Therefore, power input in the amount of 0.71 W is needed to overcome the frictional losses in the flow due to viscosity.

Discussion Note that for turbulent flow, the entry length is $L_{h,\,\text{turb}} \approx 10D = 2\ \text{cm}$. Therefore, the assumption for fully developed flow is valid for this 15-m long pipe.

8-47 For a given mass flow rate, the hydrodynamic and thermal entry lengths for water, engine oil, and liquid mercury flowing through a tube are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Flow is isothermal.

Properties The properties of water, engine oil, and liquid mercury at 100°C are listed in the following table:

Liquid	**μ (kg/m·s)**	**Pr**
Water (Table A-9)	0.282×10^{-3}	1.75
Engine oil (Table A-13)	17.18×10^{-3}	279.1
Liq. mercury (Table A-14)	1.245×10^{-3}	0.0180

Analysis The hydrodynamic and thermal entry lengths can be calculated using the following equations:

$$L_{h,\,\mathrm{lam}} \approx 0.05\,\mathrm{Re}\,D\,, \quad L_{t,\,\mathrm{lam}} \approx 0.05\,\mathrm{Re}\,\mathrm{Pr}\,D\,, \quad \text{where} \quad \mathrm{Re} = \frac{4\dot{m}}{\pi D\mu}$$

Hence, the calculated Reynolds numbers, hydrodynamic and thermal entry lengths are

Liquid	**Pr**	**Re**	**$L_{h,\,\mathrm{lam}}$ (m)**	**$L_{t,\,\mathrm{lam}}$ (m)**
Water	1.75	1806	2.26	3.95
Engine oil	279.1	29.64	0.0371	10.3
Liq. mercury	0.018	409.1	0.511	0.00920

Discussion Note that for Pr > 1, $L_{t,\,\mathrm{lam}} > L_{h,\,\mathrm{lam}}$, and for Pr < 1, $L_{t,\,\mathrm{lam}} < L_{h,\,\mathrm{lam}}$.

8-48E For a given mass flow rate, the average velocity, hydrodynamic and thermal entry lengths for water, engine oil, and liquid mercury flowing through a standard 2-in Schedule 40 pipe are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Flow is isothermal.

Properties The properties of water, engine oil, and liquid mercury at 100°F are listed in the following table:

Liquid	**ρ (lbm/ft^3)**	**μ (lbm/ft·s)**	**Pr**
Water (Table A-9E)	62.00	4.578×10^{-4}	4.54
Engine oil (Table A-13E)	54.77	163.0×10^{-3}	3275
Liq. mercury (Table A-14E)	842.9	9.919×10^{-4}	0.02363

Analysis The average velocity, hydrodynamic and thermal entry lengths can be calculated using the following equations:

$$V_{\text{avg}} = \frac{\dot{m}}{\rho A_c}, \quad L_{h,\text{lam}} \approx 0.05\,\text{Re}\,D, \quad L_{t,\text{lam}} \approx 0.05\,\text{Re}\,\text{Pr}\,D, \quad \text{where} \quad \text{Re} = \frac{4\dot{m}}{\pi D \mu}$$

From Table 8-2, the actual inside diameter for a standard 2-in Schedule 40 pipe is 2.067 in. Hence, the calculated average velocities, hydrodynamic and thermal entry lengths are

Liquid	**Pr**	**Re**	**V_{avg} (ft/s)**	**$L_{h,\text{lam}}$ (ft)**	**$L_{t,\text{lam}}$ (ft)**
Water	4.54	1615	0.0692	13.9	63.1
Engine oil	3275	4.535	0.0784	0.0391	128
Liq. mercury	0.02363	745.2	0.00509	6.42	0.152

Discussion As viscosity increases, the hydrodynamic entry length decreases. As Prandtl number increases, the thermal entry length increases also.

8-49 An engineer is to design an experimental apparatus that consists of a 25-mm diameter smooth tube; (*a*) the minimum tube length and (*b*) the required pumping power to overcome the pressure loss in the tube at largest allowable flow rate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Flow is isothermal.

Properties The properties of water, engine oil, and liquid mercury at 100°C are listed in the following table:

Liquid	ρ (kg/m^3)	μ (kg/m·s)	Pr
Water (Table A-9)	957.9	0.282×10^{-3}	1.75
Engine oil (Table A-13)	840.0	17.18×10^{-3}	279.1
Liq. mercury (Table A-14)	13351	1.245×10^{-3}	0.0180

Analysis The upper limit of the Reynolds number for laminar flow in tubes is Re ≈ 2300. The hydrodynamic and thermal entry lengths can be calculated using the following equations:

$$L_{h,\,\mathrm{lam}} \approx 0.05\,\mathrm{Re}\,D\,, \quad L_{t,\,\mathrm{lam}} \approx 0.05\,\mathrm{Re}\,\mathrm{Pr}\,D\,, \quad \text{where} \quad \mathrm{Re} = \frac{\rho V_{\mathrm{avg}} D}{\mu}$$

(*a*) Hence, the calculated average velocities, hydrodynamic and thermal entry lengths for Re = 2300 and D = 0.025 m are

Liquid	Pr	V_{avg} (m/s)	$L_{h,\,lam}$ (m)	$L_{t,\,lam}$ (m)
Water	1.75	0.02708	2.88	5.03
Engine oil	279.1	1.882	2.88	802
Liq. mercury	0.018	0.008579	2.88	0.0518

In order for the experimental apparatus to be equipped for the entire Reynolds number range for water, engine oil, and liquid mercury to flow in hydrodynamically and thermally fully developed laminar flow conditions, the tube length should be **802 m** or longer.

(*b*) For fully developed laminar flow at Re = 2300, engine oil requires the longest tube length and has the highest average velocity among the three fluids. The required pumping power to overcome the pressure loss should be determined using the properties and flow parameters of engine oil. The Darcy friction factor is calculated using the upper limit of the Reynolds number for fully developed laminar flow:

$$f = \frac{64}{\mathrm{Re}} = \frac{64}{2300} = 0.02783$$

The pressure loss is

$$\Delta P_L = f\,\frac{L}{D}\,\frac{\rho V_{\mathrm{avg}}^2}{2} = 0.02783\,\frac{(802.4\ \mathrm{m})}{(0.025\ \mathrm{m})}\,\frac{(840.0\ \mathrm{kg/m^3})(1.882\ \mathrm{m/s})^2}{2} = 1.329\times10^6\ \mathrm{Pa}$$

Hence, the required pumping power to overcome the pressure loss in the tube at largest allowable flow rate is

$$\dot{W}_{\mathrm{pump},L} = \dot{V}\Delta P_L = V_{\mathrm{avg}} A_c \Delta P_L = V_{\mathrm{avg}}\,\frac{\pi D^2}{4}\,\Delta P_L$$

$$\dot{W}_{\mathrm{pump},L} = V_{\mathrm{avg}}\,\frac{\pi D^2}{4}\,\Delta P_L = (1.882\ \mathrm{m/s})\,\frac{\pi(0.025\ \mathrm{m})^2}{4}(1.329\times10^6\ \mathrm{Pa}) = \mathbf{1230\ W}$$

Discussion Note that engine oil with very large Prandtl number has resulted in a very large thermal entry length. Therefore a different fluid with lower Pr should be considered as an alternative for this experiment, so that the tube in this experimental apparatus can be made much shorter than the 802m required for the engine oil.

8-50 A tube with constant surface heat flux, the convection heat transfer coefficients at the tube outlet are to be determined for water, engine oil, and liquid mercury.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of water, engine oil, and liquid mercury at $T_b = (T_i + T_e)/2 = 100°C$ are listed in the following table:

Liquid	c_p, J/kg·K	k, W/m·K	μ, kg/m·s	Pr
Water (Table A-9)	4217	0.679	0.282×10^{-3}	1.75
Engine oil (Table A-13)	2220	0.1367	17.18×10^{-3}	279.1
Liq. mercury (Table A-14)	137.1	9.46706	1.245×10^{-3}	0.0180

Analysis The hydrodynamic and thermal entry lengths can be calculated using the following equations:

$$L_{h,\,\mathrm{lam}} \approx 0.05\,\mathrm{Re}\,D\,, \quad L_{t,\,\mathrm{lam}} \approx 0.05\,\mathrm{Re}\,\mathrm{Pr}\,D\,, \quad \text{where} \quad \mathrm{Re} = \frac{4\dot{m}}{\pi D\mu}$$

Hence, the calculated Reynolds numbers, hydrodynamic and thermal entry lengths are

Liquid	Pr	Re	$L_{h,\,\mathrm{lam}}$, m	$L_{t,\,\mathrm{lam}}$, m
Water	1.75	1806	2.258	3.951
Engine oil	279.1	29.64	0.03706	10.34
Liq. mercury	0.018	409.1	0.5113	0.009204

Since the Reynolds numbers are less than 2300, and the hydrodynamic and thermal entry lengths are less than 15 m, therefore the flow is laminar and fully developed at the tube outlet. Hence, the Nusselt number for constant surface heat flux is Nu = 4.36. The convection heat transfer coefficients at the tube outlet are

Liquid	$h = (k/D)\mathrm{Nu}$, W/m²·K
Water	118
Engine oil	23.8
Liq. mercury	1650

Discussion Liquid mercury has high h due to its large k value.

8-51 Liquid water entering at 40°C and flowing at 0.0036 kg/s is heated in a circular copper tube with a constant surface heat flux. The surface temperature of the tube is to be determined whether it exceeds the maximum use temperature of 204°C set by the ASME Code for Process Piping. The axial location where the surface temperature reaches 204°C is to be determined.

Assumptions **1** The flow is steady and incompressible. **2** Uniform surface heat flux. **3** Inner surface of the tube is smooth.

Properties The properties of water at 100°C are (Table A–9) c_p = 4217 J/kg·K, k = 0.679 W/m·K, $\mu = 0.282 \times 10^{-3}$ kg/m·s, and Pr = 1.75

Analysis The Reynolds number for the flow in circular tube is

$$\mathrm{Re} = \frac{4\dot{m}}{\pi D\mu} = \frac{4(0.0036\ \mathrm{kg/s})}{\pi(0.025\ \mathrm{m})(0.000282\ \mathrm{kg/m\cdot s})} = 650.16$$

So, the flow is laminar. The hydrodynamic and thermal entry lengths are

$$L_h \approx 0.05\,\mathrm{Re}\,D = 0.05(650.16)(0.025\ \mathrm{m}) = 0.813\ \mathrm{m}$$

$$L_t \approx L_h\mathrm{Pr} = (0.813\ \mathrm{m})(1.75) = 1.423\ \mathrm{m}$$

At the axial location of x> 1.423 m, the flow is hydrodynamically and thermally developed. For fully-developed laminar flow, the Nusselt number for constant surface heat flux is

$$\mathrm{Nu} = \frac{hD}{k} = 4.36 \qquad \Longrightarrow \qquad h = \mathrm{Nu}\frac{k}{D} = (4.36)\frac{0.679\ \mathrm{W/m\cdot K}}{0.025\ \mathrm{m}} = 118.42\ \mathrm{W/m^2\cdot K}$$

For constant surface heat flux, the tube surface temperature is maximum at the exit. So, with

$$\dot{Q} = \dot{m}c_p(T_e - T_i)$$

The exit temperature of water is

$$T_e = \frac{\dot{Q}}{\dot{m}c_p} + T_i = \frac{1800\ \mathrm{W}}{(0.0036\ \mathrm{kg/s})(4217\ \mathrm{J/kg\cdot K})} + 40℃ = 158.57℃$$

The tube surface temperature at the exit ($x = L$) is

$$T_{s,L} = T_e + \frac{\dot{q}_s}{h} = 158.57℃ + \frac{7639.4\ \mathrm{W/m^2}}{118.42\ \mathrm{W/m^2\cdot K}} = \mathbf{223.08}℃ > 204℃$$

where the constant surface heat flux is

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = \frac{\dot{Q}}{\pi DL} = \frac{1800\ \mathrm{W}}{\pi(0.025\ \mathrm{m})(3\ \mathrm{m})} = 7639.4\ \mathrm{W/m^2}$$

At the tube exit, the surface temperature exceeds the maximum temperature for the copper tube.

To determine the axial location where the tube's surface temperature reaches $T_{s,x}$ = 204°C, we first determine the mean fluid temperature at that axial location:

$$T_{m,x} = T_{s,x} - \frac{\dot{q}_s}{h} = 204℃ - \frac{7639.4\ \mathrm{W/m^2}}{118.42\ \mathrm{W/m^2\cdot K}} = 139.49℃$$

To find the axial location, we use

$$T_{m,x} = T_i + \frac{\dot{q}_s\pi D}{\dot{m}c_p}x$$

$$x = \frac{\dot{m}c_p}{\dot{q}_s\pi D}\left(T_{m,x} - T_i\right) = \frac{(0.0036\ \mathrm{kg/s})(4217\ \mathrm{J/kg\cdot K})}{(7639.4\ \mathrm{W/m^2})\pi(0.025\ \mathrm{m})}(139.49 - 40)\mathrm{K} = \mathbf{2.52\ m}$$

Discussion The surface temperature at the tube exit ($x = L$) exceeds the maximum temperature of 204°C for ASTM B75 copper tube (ASME Code for Process Piping, ASME B31.3-2014, Table A-1M).

The tube's surface temperature reaches 204°C at the axial location of 2.52 m. The axial location is measured from the tube inlet along the tube length. So, from x = 2.52 m to the tube exit at x = 3 m, the surface temperature is at 204°C and higher. Thus, the use of ASTM B75 copper tube is not suitable for this operation.

With the exit temperature at 159°C and the inlet temperature at 40°C, the bulk mean fluid temperature becomes 99.5°C. Thus, 100°C is an appropriate temperature to evaluate the properties of water.

8-52 The tube surface temperatures necessary to heat water, engine oil, and liquid mercury to the desired outlet temperature of 150°C are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Constant tube surface temperature.

Properties The properties of water, engine oil, and liquid mercury at $T_b = (T_i + T_e)/2 = 100$°C are listed in the following table:

Liquid	c_p, J/kg·K	k, W/m·K	μ, kg/m·s	Pr
Water (Table A-9)	4217	0.679	0.282×10^{-3}	1.75
Engine oil (Table A-13)	2220	0.1367	17.18×10^{-3}	279.1
Liq. mercury (Table A-14)	137.1	9.46706	1.245×10^{-3}	0.0180

Analysis The hydrodynamic and thermal entry lengths can be calculated using the following equations:

$$L_{h,\,\mathrm{lam}} \approx 0.05\,\mathrm{Re}\,D\,, \qquad L_{t,\,\mathrm{lam}} \approx 0.05\,\mathrm{Re}\,\mathrm{Pr}\,D\,, \qquad \text{where} \qquad \mathrm{Re} = \frac{4\dot{m}}{\pi D \mu}$$

Hence, the calculated Reynolds numbers, hydrodynamic and thermal entry lengths are

Liquid	Pr	Re	$L_{h,\,\mathrm{lam}}$, m	$L_{t,\,\mathrm{lam}}$, m
Water	1.75	1806	2.258	3.951
Engine oil	279.1	29.64	0.03706	10.34
Liq. mercury	0.018	409.1	0.5113	0.009204

Since the Reynolds numbers are less than 2300, and the hydrodynamic and thermal entry lengths are less than 15 m, therefore the flow is laminar and fully developed at the tube outlet. Hence, the Nusselt number for constant surface temperature is Nu = 3.66. The convection heat transfer coefficients and the tube surface temperatures can be determined using

$$h = \frac{k}{D}\mathrm{Nu}$$

and
$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) \quad \rightarrow \quad T_s = \frac{T_e - T_i\exp[-hA_s/(\dot{m}c_p)]}{1 - \exp[-hA_s/(\dot{m}c_p)]}$$

The calculated convection heat transfer coefficients and tube surface temperatures are

Liquid	h, W/m²·K	T_s, °C
Water	99.4	157
Engine oil	20.0	203
Liq. mercury	1390	150

Discussion Liquid metals such as mercury, due to their high thermal conductivities, are particularly applicable to cases where large amount of energy must be removed from a relatively small space.

8-53 Liquid water entering at 15°C is heated in a circular tube with a constant surface temperature. The laminar flow pressure loss in the tube is 5 Pa. The water temperature at the tube exit is to be determined if it exceeds 120°C. The inner surface temperature of the tube is to be determined if it exceeds 135°C.

Assumptions **1** The flow is steady and incompressible. **2** Uniform surface temperature. **3** Inner surface of the tube is smooth. **4** The flow is laminar.

Properties The properties of water at 80°C are (Table A–9) c_p = 4197 J/kg·K, k = 0.670 W/m·K, $\mu = 0.355 \times 10^{-3}$ kg/m·s, ρ = 971.8 kg/m^3, and Pr = 2.22

Analysis The following equations relate the pressure loss for internal flow with the Reynolds number and the friction factor for laminar flow in a tube:

$$\mathrm{Re} = \frac{\rho V_{\mathrm{avg}} D}{\mu}$$

$$f = \frac{64}{\mathrm{Re}}$$

$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{\mathrm{avg}}^2}{2} = 64\frac{L}{D^3}\frac{\mu^2 \mathrm{Re}}{2\rho}$$

Solving for the Reynolds number Re,

$$\Delta P_L = 64\frac{L}{D^3}\frac{\mu^2\mathrm{Re}}{2\rho} \quad\Longrightarrow\quad \mathrm{Re} = \frac{1}{32}\frac{\rho}{\mu^2}\frac{D^3}{L}\Delta P_L = \frac{1}{32}\frac{971.8\ \mathrm{kg/m^3}}{(0.000355\ \mathrm{kg/m\cdot s})^2}\frac{(0.0125\ \mathrm{m})^3}{3}(5\ \mathrm{Pa}) = 784.42$$

So, the flow is laminar and agrees with the assumption. The hydrodynamic and thermal entry lengths are

$$L_h \approx 0.05\ \mathrm{Re}\ D = 0.05(784.42)(0.0125\ \mathrm{m}) = 0.49\ \mathrm{m}$$

$$L_t \approx L_h \mathrm{Pr} = 2.22(0.49\ \mathrm{m}) = 1.09\ \mathrm{m}$$

At the tube exit (L = 3 m), the flow is hydrodynamically and thermally developed. For fully-developed laminar flow, the Nusselt number for constant surface temperature is

$$\mathrm{Nu} = \frac{hD}{k} = 3.66 \quad\Longrightarrow\quad h = \mathrm{Nu}\frac{k}{D} = (3.66)\frac{0.670\ \mathrm{W/m\cdot K}}{0.0125\ \mathrm{m}} = 196.18\ \mathrm{W/m^2\cdot K}$$

The average velocity and the mass flow rate are

$$V_{\mathrm{avg}} = \frac{\mu\,\mathrm{Re}}{\rho\, D} = \frac{(0.000355\ \mathrm{kg/m\cdot s})(784.42)}{(971.8\ \mathrm{kg/m^3})(0.0125\ \mathrm{m})} = 0.02292\ \mathrm{m/s}$$

$$\dot{m} = \rho V_{\mathrm{avg}} A = \rho V_{\mathrm{avg}}\frac{\pi D^2}{4} = (971.8\ \mathrm{kg/m^3})(0.02292\ \mathrm{m/s})\frac{\pi(0.0125\ \mathrm{m})^2}{4} = 0.002733\ \mathrm{kg/s}$$

(*a*) The heat transfer rate to the water is

$$\dot{Q} = \dot{m}c_p(T_e - T_i)$$

which can be used to solve for the exit temperature of water:

$$T_e = \frac{\dot{Q}}{\dot{m}c_p} + T_i = \frac{1500\ \mathrm{W}}{(0.002733\ \mathrm{kg/s})(4197\ \mathrm{J/kg\cdot K})} + 15°\mathrm{C} = \mathbf{145.77}°\mathrm{C} > 120°\mathrm{C}$$

(*b*) To solve for the surface temperature of the tube, we use

$$T_e = T_s - (T_s - T_i)\exp\left(-hA_s/\dot{m}c_p\right)$$

where

$$A_s = \pi D L$$

Solving for T_s, yields

$$145.77°\mathrm{C} = T_s - (T_s - 15)\exp[-(196.18\ \mathrm{W/m^2\cdot K})\pi(0.0125\ \mathrm{m})(3\ \mathrm{m})/(0.002733\ \mathrm{kg/s})(4197\ \mathrm{J/kg\cdot K})]$$

$$T_s = \mathbf{165.89}°\mathrm{C} > 135°\mathrm{C}$$

Discussion The water exiting the tube is at a temperature above 120°C, which is noncompliant with the ASME Boiler and Pressure Vessel Code. The inner surface temperature of the tube is above the maximum temperature of 135°C for PVDF lining, which is noncompliant with the ASME Code for Process Piping. Thus, both ASME codes are violated.

With the exit temperature at 146°C and the inlet temperature at 15°C, the bulk mean fluid temperature becomes 80.5°C. Thus, 80°C is an appropriate temperature to evaluate the properties of water.

8-54E Liquid isobutane is flowing through a standard 3/4-in Schedule 40 cast iron pipe, (*a*) the pressure loss and (*b*) the pumping power required to overcome the pressure loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Flow is isothermal. **4** Flow is fully developed.

Properties The properties of isobutane at 50°F are ρ = 35.52 lbm/ft^3 and μ = 1.196 × 10^{-4} lbm/ft·s (from Table A-13E).

Analysis From Table 8-2, a standard 3/4-in Schedule 40 pipe has an actual inside diameter of

$$D = 0.824\text{ in.}$$

The Reynolds number for the flow is

$$\text{Re} = \frac{4\dot{m}}{\pi D\mu} = \frac{4(0.4\text{ lbm/s})}{\pi(0.824/12\text{ ft})(1.196\times10^{-4}\text{ lbm/ft}\cdot\text{s})} = 6.201\times10^4 > 10{,}000 \text{ (turbulent flow)}$$

The average velocity is

$$V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{4\dot{m}}{\rho\pi D^2} = \frac{4(0.4\text{ lbm/s})}{(35.52\text{ lbm/ft}^3)\pi(0.824/12\text{ ft})^2} = 3.041\text{ ft/s}$$

From Table 8-3, the equivalent roughness of cast iron is ε = 0.00085 ft. Since accuracy is an important issue, the Darcy friction factor is calculated using the Colebrook equation rather than the Haaland equation:

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

Copy the following lines and paste on a blank EES screen to solve the above equation:

```
D=0.824/12
epsilon=0.00085
Re=6.201e4
1/f_sqrt=-2.0*log10((epsilon/D)/3.7+2.51/(Re*f_sqrt))
```

Solving by EES software, the Darcy friction factor is

$$\sqrt{f} = 0.204 \quad\rightarrow\quad f = 0.0416$$

(*a*) The pressure loss in the pipe is

$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.0416\frac{(30\text{ ft})}{(0.824/12\text{ ft})}\frac{(35.52\text{ lbm/ft}^3)(3.041\text{ ft/s})^2}{2(32.2\text{ lbm}\cdot\text{ft/lbf}\cdot\text{s}^2)} = \mathbf{92.7\ lbf/ft^2}$$

(*b*) The pumping power required to overcome the pressure loss is

$$\dot{W}_{\text{pump},L} = \dot{V}\Delta P_L = \frac{\dot{m}}{\rho}\Delta P_L = \frac{(0.4\text{ lbm/s})}{(35.52\text{ lbm/ft}^3)}92.7\text{ lbf/ft}^2 = \mathbf{1.04\ lbf\cdot ft/s}$$

Discussion Note that for turbulent flow, the entry length is $L_{h,\text{ turb}} \approx 10D = 0.687\text{ ft}$. Therefore, the assumption for fully developed flow is valid for this 35-ft long pipe.

8-55 Water is flowing through a standard 1-in Schedule 40 cast iron pipe, (*a*) the pressure loss and (*b*) the pumping power required to overcome the pressure loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Flow is isothermal. **4** Flow is fully developed.

Properties The properties of water at 15°C are ρ = 999.1 kg/m^3 and μ = 1.138 × 10^{-3} kg/m·s (from Table A-9).

Analysis From Table 8-2, a standard 1-in Schedule 40 pipe has an actual inside diameter of

$$D = 1.049 \text{ in.} = 0.02664 \text{ m}$$

The Reynolds number for the flow is

$$\text{Re} = \frac{4\dot{m}}{\pi D\mu} = \frac{4(0.5 \text{ kg/s})}{\pi(0.02664 \text{ m})(1.138\times10^{-3} \text{ kg/m}\cdot\text{s})} = 2.10\times10^4 > 10{,}000 \qquad \text{(turbulent flow)}$$

The average velocity is

$$V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{4\dot{m}}{\rho\pi D^2} = \frac{4(0.5 \text{ kg/s})}{(999.1 \text{ kg/m}^3)\pi(0.02664 \text{ m})^2} = 0.8978 \text{ m/s}$$

From Table 8-3, the equivalent roughness of cast iron is ε = 0.26 mm. Since accuracy is an important issue, the Darcy friction factor is calculated using the Colebrook equation rather than the Haaland equation:

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

Copy the following lines and paste on a blank EES screen to solve the above equation:

```
D=0.02664
epsilon=0.26e-3
Re=2.10e4
1/f_sqrt=-2.0*log10((epsilon/D)/3.7+2.51/(Re*f_sqrt))
```

Solving by EES software, the Darcy friction factor is

$$\sqrt{f} = 0.2008 \quad \rightarrow \quad f = 0.0403$$

(*a*) The pressure loss in the pipe is

$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.0403\frac{(200 \text{ m})}{(0.02664 \text{ m})}\frac{(999.1 \text{ kg/m}^3)(0.8978 \text{ m/s})^2}{2} = \mathbf{122 \text{ kPa}}$$

(*b*) The pumping power required to overcome the pressure loss is

$$\dot{W}_{\text{pump},L} = \dot{V}\Delta P_L = \frac{\dot{m}}{\rho}\Delta P_L = \frac{(0.5 \text{ kg/s})}{(999.1 \text{ kg/m}^3)}121.8 \text{ kPa} = \mathbf{61 \text{ W}}$$

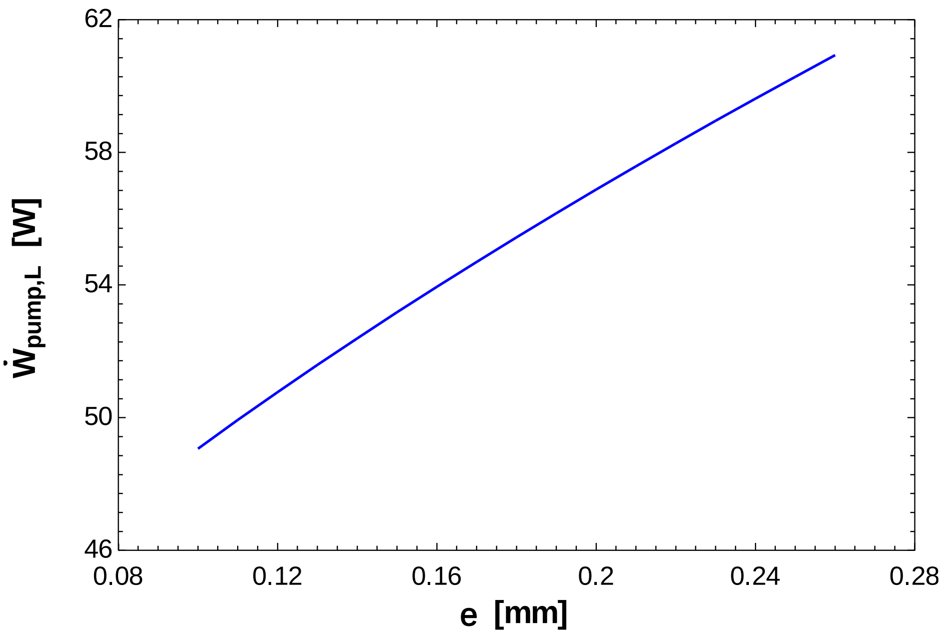
Discussion Note that for turbulent flow, the entry length is $L_{h,\text{ turb}} \approx 10D = 0.2664 \text{ m}$. Therefore, the assumption for fully developed flow is valid for this 200-m long pipe.

8-56 Prob. 8-55 is reconsidered. The effect of the pipe roughness on the pumping power is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_b=15 [C]
D=1.049*0.0254 [m]
L=200 [m]
m_dot=0.5 [kg/s]

"PROPERTIES"
rho=Density(water, T=T_b, P=101.3)
mu=Viscosity(water, T=T_b, P=101.3)

"ANALYSIS"
A_c=pi#*D^2/4     "Cross-section area"
Re=4*m_dot/(pi#*D*mu)
V_avg=m_dot/(rho*A_c)
1/f^0.5=-2.0*log10((epsilon*1e-3/D)/3.7+2.51/(Re*f^0.5))
DELTAP_L=f*L/D*rho*V_avg^2/2
W_dot_pump_L=m_dot*DELTAP_L/rho
```

ε [mm]	$\dot{W}_{\text{pump},L}$ [W]
0.1	49.07
0.11	49.93
0.12	50.77
0.13	51.59
0.14	52.39
0.15	53.18
0.16	53.95
0.17	54.7
0.18	55.44
0.19	56.17
0.20	56.88
0.21	57.58
0.22	58.27
0.23	58.95
0.24	59.62
0.25	60.28
0.26	60.93



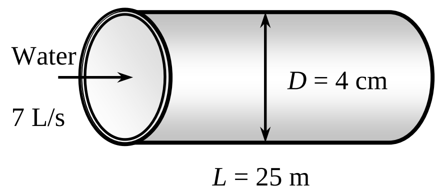
Discussion A 100% increase in the pipe roughness from 0.13 to 0.26 mm would increase the pumping power required to overcome the pressure loss by 18.1%.

8-57 The flow rate through a specified water pipe is given. The pressure drop and the pumping power requirements are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of water are given to be ρ = 999.1 kg/m^3 and μ = 1.138×10^{-3} kg/m·s, respectively. The roughness of stainless steel is 0.002 mm (Table 8-3).

Analysis First, we calculate the mean velocity and the Reynolds number to determine the flow regime:



$$V_{avg} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.007\ \text{m}^3/s}{\pi(0.04\ \text{m})^2/4} = 5.570\ \text{m}/s$$

$$\text{Re} = \frac{\rho V_{avg} D}{\mu} = \frac{(999.1\ \text{kg/m}^3)(5.570\ \text{m/s})(0.04\text{m})}{1.138\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 1.956\times10^5$$

which is greater than 10,000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon / D = \frac{2\times10^{-6}\ \text{m}}{0.04\ \text{m}} = 5\times10^{-5}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{5\times10^{-5}}{3.7} + \frac{2.51}{1.956\times10^5\sqrt{f}}\right)$$

It gives f = 0.0161. Then the pressure drop and the required power input become

$$\Delta P = f\frac{L}{D}\frac{\rho V_{avg}^2}{2} = 0.0161\frac{25\ \text{m}}{0.04\ \text{m}}\frac{(999.1\ \text{kg/m}^3)(5.570\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) = \mathbf{156.0\ kPa}$$

$$\dot{W}_{pump,u} = \dot{V}\Delta P = (0.007\ \text{m}^3/s)(156.0\ \text{kPa})\left(\frac{1\ \text{kW}}{1\ \text{kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{1.09\ kW}$$

Therefore, useful power input in the amount of 1.09 kW is needed to overcome the frictional losses in the pipe.

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give f = 0.01589, which is sufficiently close to 0.0161. Also, the friction factor corresponding to ε = 0 in this case is 0.01557, which indicates that stainless steel pipes can be assumed to be smooth with an error of about 3%. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

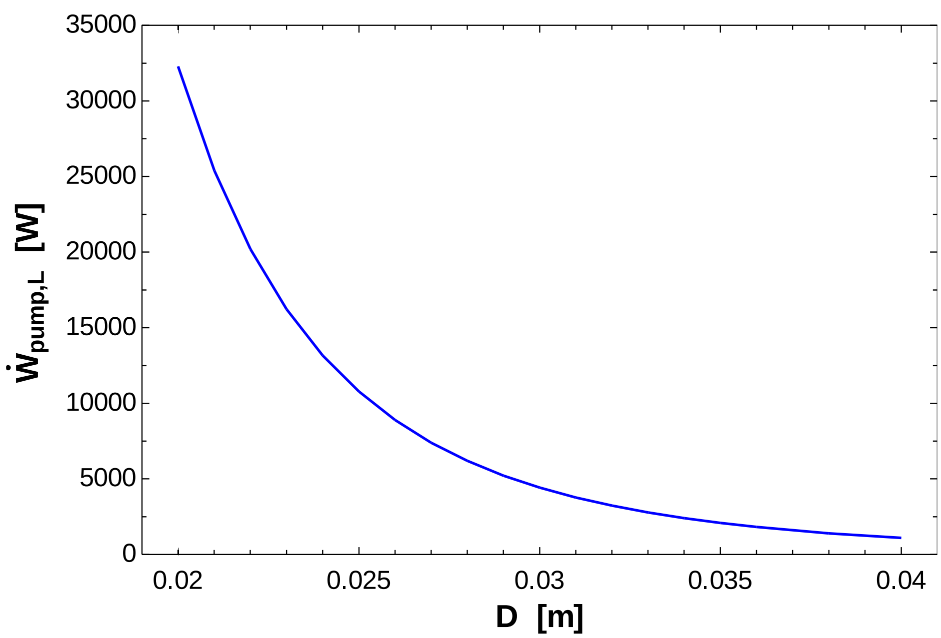
Note that for turbulent flow, the entry length is $L_{h,\ turb} \approx 10D = 10(0.04\ \text{m}) = 0.4\ \text{m}$. Therefore, the assumption for fully developed flow is valid for this 25-m long pipe.

8-58 Prob. 8-57 is reconsidered. The effect of the pipe diameter on the pumping power is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_b=15 [C]
L=25 [m]
Vol_dot=0.007 [m^3/s]
epsilon=0.002 [mm]

"PROPERTIES"
rho=Density(water, T=T_b, P=101.3)
mu=Viscosity(water, T=T_b, P=101.3)

"ANALYSIS"
A_c=pi#*D^2/4     "Cross-section area"
V_avg=Vol_dot/A_c
Re=rho*V_avg*D/mu
1/f^0.5=-2.0*log10((epsilon*1e-3/D)/3.7+2.51/(Re*f^0.5))
DELTAP_L=f*L/D*rho*V_avg^2/2
W_dot_pump_L=Vol_dot*DELTAP_L
```

D [m]	$\dot{W}_{pump,\,L}$ [W]
0.020	32292
0.021	25385
0.022	20188
0.023	16225
0.024	13165
0.025	10777
0.026	8893
0.027	7394
0.028	6190
0.029	5215
0.030	4420
0.031	3767
0.032	3227
0.033	2779
0.034	2403
0.035	2087
0.036	1820
0.038	1400
0.040	1092



Discussion A decrease in the pipe diameter by one-half, from 4 to 2 cm, would cause almost a 30 times increase in the pumping power requirement.

8-59 A fluid with mean inlet temperature T_i is flowing through a tube, of diameter D and length L, at a mass flow rate $\dot{m}$; an expression for the mean temperature of the fluid $T_m(x)$ is to be determined from the given surface heat flux, $\dot{q}_s(x) = a + b\sin(x\pi / L)$.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Heat conduction in the x-direction is negligible. **4** Work done by viscous forces is negligible.

Analysis Applying the energy balance to a differential control volume in a tube gives

$$\dot{m}c_p dT_m = \dot{q}_s(x) p dx \quad \rightarrow \quad dT_m = \dot{q}_s(x) \frac{p}{\dot{m}c_p} dx$$

Integrating from the inlet (x = 0) to x yields

$$\int_{T_i}^{T_m(x)} dT_m = \frac{p}{\dot{m}c_p} \int_0^x \dot{q}_s(x)\, dx$$

$$T_m(x) - T_i = \frac{p}{\dot{m}c_p} \int_0^x a + b\sin(x\pi / L)\, dx$$

$$T_m(x) - T_i = \frac{p}{\dot{m}c_p} \left[ax - \frac{bL}{\pi} \cos\left(\frac{x\pi}{L}\right)\right]_0^x$$

$$T_m(x) - T_i = \frac{p}{\dot{m}c_p} \left[ax + \frac{bL}{\pi} - \frac{bL}{\pi} \cos\left(\frac{x\pi}{L}\right)\right]$$

Noting that the perimeter of the tube is $p = \pi D$, hence the expression for the mean temperature of the fluid as a function the x-coordinate is

$$T_m(x) = T_i + \frac{\pi D}{\dot{m}c_p} \left[ax + \frac{bL}{\pi} - \frac{bL}{\pi} \cos\left(\frac{x\pi}{L}\right)\right]$$

Discussion The mean fluid temperature difference of the tube inlet (x = 0) and outlet (x = L) is

$$T_e - T_i = T_m(L) - T_i = \frac{\pi D}{\dot{m}c_p} \left[aL + \frac{bL}{\pi} - \frac{bL}{\pi} \cos\left(\frac{L\pi}{L}\right)\right] = \frac{\pi D}{\dot{m}c_p} \left[aL + 2\frac{bL}{\pi}\right]$$

8-60 A fluid is flowing in fully developed laminar conditions in a tube with diameter D and length L at a mass flow rate $\dot{m}$; an expression for the difference in mean temperature at the tube inlet and outlet is to be determined from the given surface heat flux, $\dot{q}_s(x) = a\exp(-bx/2)$.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Heat conduction in the x-direction is negligible. **4** Work done by viscous forces is negligible.

Analysis Applying the energy balance to a differential control volume in a tube gives

$$\dot{m}c_p dT_m = \dot{q}_s(x)p\,dx \quad \rightarrow \quad dT_m = \dot{q}_s(x)\frac{p}{\dot{m}c_p}dx$$

Integrating from the inlet (x = 0) to outlet (x = L) yields

$$\int_{T_i}^{T_o} dT_m = \frac{p}{\dot{m}c_p}\int_0^L \dot{q}_s(x)\,dx$$

$$T_e - T_i = \frac{p}{\dot{m}c_p}\int_0^L a\exp(-bx/2)\,dx$$

$$T_e - T_i = \frac{p}{\dot{m}c_p}\left[-\frac{2a}{b}\exp(-bx/2)\right]_0^L$$

$$T_e - T_i = \frac{pa}{\dot{m}c_p}\left[\frac{2}{b} - \frac{2}{b}\exp(-bL/2)\right]$$

Noting that the perimeter of the tube is $p = \pi D$, hence the expression for the difference in mean temperature at the tube inlet and outlet is

$$T_e - T_i = \frac{\pi D a}{\dot{m}c_p}\left[\frac{2}{b} - \frac{2}{b}\exp(-bL/2)\right]$$

Discussion Note that if c_p is to be determined, it should be evaluated at the bulk mean temperature $T_b = (T_i + T_e)$.

8-61 Water enters a 25-mm diameter and 23-m long circular tube, (*a*) an expression for the mean temperature $T_m(x)$, (*b*) the outlet mean temperature, and (*c*) the value of a uniform heat flux on the tube surface that would result in the same outlet mean temperature calculated in part (*b*) are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Heat conduction in the *x*-direction is negligible. **4** Work done by viscous forces is negligible.

Properties The constant pressure specific heat of water at 35°C is c_p = 4178 J/kg·K (Table A-9).

Analysis (*a*) Applying the energy balance to a differential control volume in a tube gives

$$\dot{m}c_p dT_m = \dot{q}_s(x)p dx \quad \rightarrow \quad dT_m = \dot{q}_s(x)\frac{p}{\dot{m}c_p}dx$$

Integrating from the inlet ($x = 0$) to x yields

$$\int_{T_i}^{T_m(x)} dT_m = \frac{p}{\dot{m}c_p}\int_0^x \dot{q}_s(x)\,dx$$

$$T_m(x) - T_i = \frac{p}{\dot{m}c_p}\int_0^x ax\,dx$$

$$T_m(x) - T_i = \frac{ap}{\dot{m}c_p}\left(\frac{x^2}{2}\right)$$

Noting that the perimeter of the tube is $p = \pi D$, hence the expression for the mean temperature $T_m(x)$ is

$$T_m(x) = T_i + \frac{a\pi D}{2\dot{m}c_p}x^2$$

(*b*) The outlet mean temperature is at $x = L$, hence

$$T_e = T_m(L) = T_i + \frac{a\pi D}{2\dot{m}c_p}L^2 = 25°\text{C} + \frac{(400\ \text{W/m}^3)\pi(0.025\ \text{m})}{2(0.1\ \text{kg/s})(4178\ \text{J/kg}\cdot\text{K})}(23\ \text{m})^2 = \mathbf{44.9°C}$$

(*c*) For uniform heat flux on the tube surface, the overall energy balance on the tube can be expressed as

$$\dot{q}_s = \frac{\dot{m}c_p}{A_s}(T_\text{e} - T_\text{i}) = \frac{\dot{m}c_p}{\pi DL}(T_\text{e} - T_\text{i})$$

Using the outlet mean temperature from part (*b*), the value of a uniform heat flux on the tube surface that would result in the same outlet mean temperature is

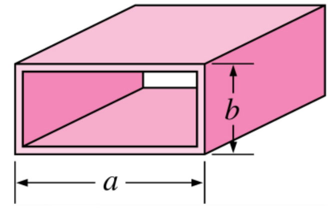
$$\dot{q}_s = \frac{(0.1\ \text{kg/s})(4178\ \text{J/kg}\cdot\text{K})}{\pi(0.025\ \text{m})(23\ \text{m})}(44.9 - 25)\ \text{K} = \mathbf{4600\ W/m^2}$$

Discussion Using 35°C as the temperature to evaluate the constant pressure specific heat of water turned out to be appropriate, since the bulk mean temperature is $T_b = (T_i + T_e) = 35°\text{C}$.

8-62 The rectangular tube surface temperature necessary to heat water to the desired outlet temperature of 80°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Constant tube surface temperature.

Properties The properties of water at $T_b = (T_i + T_e)/2 = 50°C$:
$c_p = 4181$ J/kg·K, $k = 0.644$ W/m·K, $\mu = 0.547 \times 10^{-3}$ kg/m·s,
and Pr = 3.55 (Table A-15).



Analysis The hydraulic diameter is

$$D_h = \frac{4A_c}{p} = \frac{4ab}{2(a+b)} = 0.03333\,\text{m}$$

The Reynolds number, hydrodynamic and thermal entry lengths are

$$\text{Re} = \frac{\rho V_{\text{avg}} D_h}{\mu} = \frac{\dot{m}(4A_c/p)}{A_c\mu} = \frac{4\dot{m}}{p\mu} = \frac{4\dot{m}}{2(a+b)\mu} = \frac{4(0.01\,\text{kg/s})}{2(0.075\,\text{m})(0.547\times10^{-3}\,\text{kg/m}\cdot\text{s})}$$
$$= 488 < 2300$$

$$L_{h,\,\text{lam}} \approx 0.05\,\text{Re}\,D_h = 0.813\,\text{m} < 10\,\text{m} \quad \text{and} \quad L_{t,\,\text{lam}} \approx 0.05\,\text{Re}\,\text{Pr}\,D = 2.89\,\text{m} < 10\,\text{m}$$

Hence, the flow is laminar and fully developed. From Table 8-1 with $a/b = 2$ for constant surface temperature, we have

$$\text{Nu} = 3.39 \quad \rightarrow \quad h = \frac{k}{D_h}\text{Nu} = 3.39\left(\frac{0.644\,\text{W/m}\cdot\text{K}}{0.03333\,\text{m}}\right) = 65.5\,\text{W/m}^2\cdot\text{K}$$

The tube surface temperature can be determined using

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) \quad \rightarrow \quad T_s = \frac{T_e - T_i\exp[-hA_s/(\dot{m}c_p)]}{1-\exp[-hA_s/(\dot{m}c_p)]}$$

$$T_s = \frac{80°\text{C} - (20°\text{C})\exp(-2.35)}{1-\exp(-2.35)} = \mathbf{86.3°C}$$

where

$$\frac{hA_s}{\dot{m}c_p} = \frac{(65.5\,\text{W/m}^2\cdot\text{K})2(10\,\text{m})(0.025\,\text{m} + 0.050\,\text{m})}{(0.01\,\text{kg/s})(4181\,\text{J/kg}\cdot\text{K})} = 2.35$$

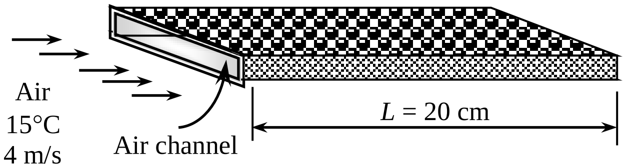
Discussion The Nu = 3.39 for rectangular tube with $a/b = 2$ for laminar fully developed flow with constant surface temperature is slightly lower than its circular tube counterpart, Nu = 3.66.

8-63 A circuit board is cooled by passing cool air through a channel drilled into the board. The maximum total power of the electronic components is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat flux at the top surface of the channel is uniform, and heat transfer through other surfaces is negligible. **3** The inner surfaces of the channel are smooth. **4** Air is an ideal gas with constant properties. **5** The pressure of air in the channel is 1 atm. **5** Flow is fully developed in the channel.

Properties The properties of air at 1 atm and estimated average temperature of 25°C based on the problem statement are (Table A-15)

$$\rho = 1.184 \text{ kg/m}^3$$
$$k = 0.02551 \text{ W/m.°C}$$
$$\nu = 1.562 \times 10^{-5} \text{ m}^2\text{/s}$$
$$c_p = 1007 \text{ J/kg.°C}$$
$$\text{Pr} = 0.7296$$



Analysis The cross-sectional and heat transfer surface areas are

$$A_c = (0.002 \text{ m})(0.14 \text{ m}) = 0.00028 \text{ m}^2$$
$$A_s = (0.14 \text{ m})(0.2 \text{ m}) = 0.028 \text{ m}^2$$

To determine heat transfer coefficient, we first need to find the Reynolds number,

$$D_h = \frac{4A_c}{P} = \frac{4(0.00028 \text{ m}^2)}{2(0.002 \text{ m} + 0.14 \text{ m})} = 0.003944 \text{ m}$$

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(4 \text{ m/s})(0.003944 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2\text{/s}} = 1010$$

which is less than 2300. Therefore, the flow is laminar and the thermal entry length is

$$L_t = 0.05 \text{ Re Pr } D_h = 0.05(1010)(0.7296)(0.003944 \text{ m}) = 0.1453 \text{ m} < 0.20 \text{ m}$$

Therefore, we have developing flow through most of the channel. However, we take the conservative approach and assume fully developed flow, and from Table 8-1 we read Nu = 8.24. Then the heat transfer coefficient becomes

$$h = \frac{k}{D_h} Nu = \frac{0.02551 \text{ W/m.°C}}{0.003944 \text{ m}}(8.24) = 53.30 \text{ W/m}^2\text{.°C}$$

Also,

$$\dot{m} = \rho V A_c = (1.184 \text{ kg/m}^3)(4 \text{ m/s})(0.00028 \text{ m}^2) = 0.001326 \text{ kg/s}$$

Heat flux at the exit can be written as $\dot{q} = h(T_s - T_e)$ where $T_s = 50°\text{C}$ at the exit. Then the heat transfer rate can be expressed as $\dot{Q} = \dot{q}A_s = hA_s(T_s - T_e)$, and the exit temperature of the air can be determined from

$$hA_s(T_s - T_e) = \dot{m}c_p(T_e - T_i)$$
$$(53.30 \text{ W/m}^2\text{.°C})(0.028 \text{ m}^2)(50°\text{C} - T_e) = (0.001326 \text{ kg/s})(1007 \text{ J/kg.°C})(T_e - 15°\text{C})$$
$$T_e = 33.5°\text{C}$$

Then the maximum total power of the electronic components that can safely be mounted on this circuit board becomes

$$\dot{Q}_{\max} = \dot{m}c_p(T_e - T_i) = (0.001326 \text{ kg/s})(1007 \text{ J/kg.°C})(33.5 - 15°\text{C}) = \mathbf{24.7\ W}$$

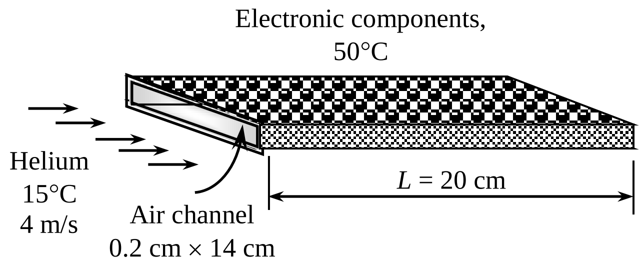
Discussion The bulk mean temperature of air is (15 + 33.5)/2 = 24.3°C. This is very close to the assumed temperature of 25°C. Therefore, there is no need to repeat calculations.

8-64 A circuit board is cooled by passing cool helium gas through a channel drilled into the board. The maximum total power of the electronic components is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat flux at the top surface of the channel is uniform, and heat transfer through other surfaces is negligible. **3** The inner surfaces of the channel are smooth. **4** Helium is an ideal gas. **5** The pressure of helium in the channel is 1 atm. **6** Flow is fully developed in the channel.

Properties Use the following properties for helium:

$$\rho = 0.1636\ \text{kg/m}^3$$
$$k = 0.1502\ \text{W/m.°C}$$
$$\nu = 1.214\times10^{-4}\ \text{m}^2/\text{s}$$
$$c_p = 5193\ \text{J/kg.°C}$$
$$\text{Pr} = 0.6867$$



Analysis The cross-sectional and heat transfer surface areas are

$$A_c = (0.002\ \text{m})(0.14\ \text{m}) = 0.00028\ \text{m}^2$$
$$A_s = (0.14\ \text{m})(0.2\ \text{m}) = 0.028\ \text{m}^2$$

To determine heat transfer coefficient, we need to first find the Reynolds number

$$D_h = \frac{4A_c}{p} = \frac{4(0.00028\ \text{m}^2)}{2(0.002\ \text{m}+0.14\ \text{m})} = 0.003944\ \text{m}$$

$$\text{Re} = \frac{V_{\text{avg}}D_h}{\nu} = \frac{(4\ \text{m/s})(0.003944\ \text{m})}{1.214\times10^{-4}\ \text{m}^2/\text{s}} = 130.0$$

which is less than 2300. Therefore, the flow is laminar and the thermal entry length is

$$L_t = 0.05\,\text{Re}\,\text{Pr}\,D_h = 0.05(130.0)(0.6867)(0.003944\ \text{m}) = 0.0176\ \text{m} << 0.20\ \text{m}$$

Therefore, the flow is fully developed flow, and from Table 8-1 we read Nu = 8.24. Then the heat transfer coefficient becomes

$$h = \frac{k}{D_h}Nu = \frac{0.1502\ \text{W/m.°C}}{0.003944\ \text{m}}(8.24) = 313.8\ \text{W/m}^2.\text{°C}$$

Also,

$$\dot{m} = \rho V A_c = (0.1636\ \text{kg/m}^3)(4\ \text{m/s})(0.00028\ \text{m}^2) = 0.0001832\ \text{kg/s}$$

Heat flux at the exit can be written as $\dot{q} = h(T_s - T_e)$ where $T_s = 50°\text{C}$ at the exit. Then the heat transfer rate can be expressed as $\dot{Q} = \dot{q}A_s = hA_s(T_s - T_e)$, and the exit temperature of the air can be determined from

$$\dot{m}c_p(T_e - T_i) = hA_s(T_s - T_e)$$
$$(0.0001832\ \text{kg/s})(5193\ \text{J/kg.°C})(T_e - 15°\text{C}) = (313.8\ \text{W/m}^2.\text{°C})(0.028\ \text{m}^2)(50°\text{C} - T_e)$$
$$T_e = 46.58°\text{C}$$

Then the maximum total power of the electronic components that can safely be mounted on this circuit board becomes

$$\dot{Q}_{\max} = \dot{m}c_p(T_e - T_i) = (0.0001832\ \text{kg/s})(5193\ \text{J/kg.°C})(46.58 - 15°\text{C}) = \mathbf{30.0\ W}$$

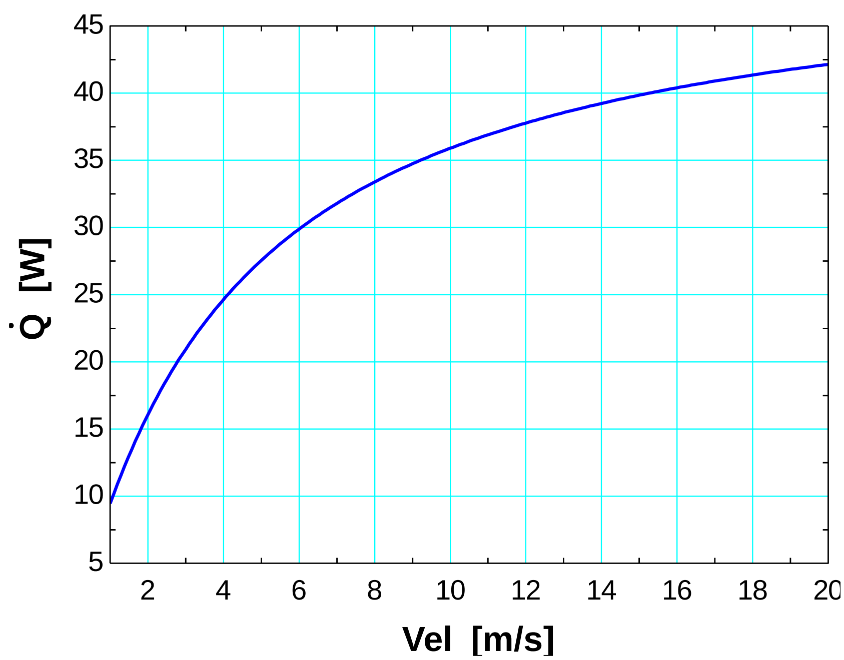
Discussion The maximum total power of the electronic components that can safely be mounted on this circuit board with helium as the working fluid is about 22% higher in comparison to air. This is maily due to the high thermal conductivity of helim compared to air, which leads to a much higher (six time more) heat transfer coefficient.

8-65 Prob. 8-63 is reconsidered. The effects of air velocity at the inlet of the channel and the maximum surface temperature on the maximum total power dissipation of electronic components are to be investigated.

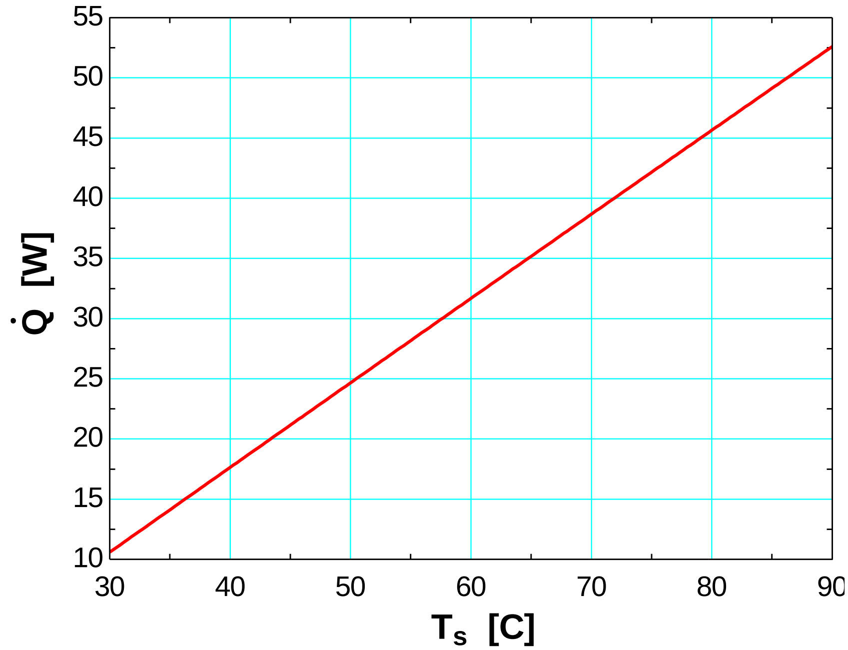
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.20 [m]
width=0.14 [m]
height=0.002 [m]
T_i=15 [C]
Vel=4 [m/s]
T_s=50 [C]

"PROPERTIES"
Fluid$='air'
c_p=CP(Fluid$, T=T_ave)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(Fluid$, T=T_ave)
Pr=Prandtl(Fluid$, T=T_ave)
rho=Density(Fluid$, T=T_ave, P=101.3)
mu=Viscosity(Fluid$, T=T_ave)
nu=mu/rho
T_ave=1/2*(T_i+T_e)

"ANALYSIS"
A_c=width*height
A=width*L
p=2*(width+height)
D_h=(4*A_c)/p
Re=(Vel*D_h)/nu "The flow is laminar"
L_t=0.05*Re*Pr*D_h
"Taking conservative approach and assuming fully developed laminar flow, from Table 8-1 we read"
Nusselt=8.24
h=k/D_h*Nusselt
m_dot=rho*Vel*A_c
Q_dot=h*A*(T_s-T_e)
Q_dot=m_dot*c_p*(T_e-T_i)
```

Vel [m/s]	$\dot{Q}$ [W]
1	9.438
2	16.07
3	20.94
4	24.64
5	27.54
6	29.88
7	31.79
8	33.39
9	34.74
10	35.9
11	36.9
12	37.78
13	38.55
14	39.23
15	39.85
16	40.4
17	40.9
18	41.35
19	41.76
20	42.14



T_s [C]	$\dot{Q}$ [W]
30	10.58
35	14.1
40	17.62
45	21.13
50	24.64
55	28.15
60	31.65
65	35.14
70	38.64
75	42.13
80	45.61
85	49.09
90	52.56



8-66 A computer is cooled by a fan blowing air through its case. The flow rate of the air, the fraction of the temperature rise of air that is due to heat generated by the fan, and the highest allowable inlet air temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** Heat flux is uniformly distributed. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 25°C. The properties of air at 1 atm and this temperature are (Table A-15)

$$\rho = 1.184 \text{ kg/m}^3$$
$$k = 0.02551 \text{ W/m.°C}$$
$$\nu = 1.562\times10^{-5} \text{ m}^2/\text{s}$$
$$c_p = 1007 \text{ J/kg.°C}$$
$$\Pr = 0.7296$$

Analysis (*a*) Noting that the electric energy consumed by the fan is converted to thermal energy, the mass flow rate of air is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) \rightarrow \dot{m} = \frac{\dot{Q} + \dot{W}_{\text{elect, fan}}}{c_p(T_e - T_i)} = \frac{(8\times12+10)\text{ W}}{(1007 \text{ J/kg.°C})(10\text{°C})} = \mathbf{0.01053\,kg/s}$$

(*b*) The fraction of temperature rise of air that is due to the heat generated by the fan and its motor is

$$\dot{Q} = \dot{m}c_p\Delta T \rightarrow \Delta T = \frac{\dot{Q}}{\dot{m}c_p} = \frac{10 \text{ W}}{(0.01053\text{ kg/s})(1007\text{ J/kg.°C})} = 0.943\text{°C}$$

$$f = \frac{0.943\text{°C}}{10\text{°C}} = 0.0943 = \mathbf{9.43\%}$$

(*c*) The mean velocity of air is

$$\dot{m} = \rho A_c V_{\text{avg}} \rightarrow V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{(0.01053/8)\text{ kg/s}}{(1.184\text{ kg/m}^3)\left[(0.003\text{ m})(0.12\text{ m})\right]} = 3.088\text{ m/s}$$

and $$D_h = \frac{4A_c}{P} = \frac{4(0.003\text{ m})(0.12\text{m})}{2(0.003\text{ m} + 0.12\text{ m})} = 0.00585\text{ m}$$

Therefore,

$$\text{Re} = \frac{V_{\text{avg}}D_h}{\nu} = \frac{(3.088\text{ m/s})(0.00585\text{ m})}{1.562\times10^{-5}\text{ m}^2/\text{s}} = 1157$$

which is less than 2300. Therefore, the flow is laminar. Assuming fully developed flow, the Nusselt number is determined from Table 8-4 corresponding to a/b = 12/0.3 = 40 to be Nu = 8.24. Then,

$$h = \frac{k}{D_h}Nu = \frac{0.02551\text{ W/m.°C}}{0.00585\text{ m}}(8.24) = 35.9\text{ W/m}^2\text{.°C}$$

The highest component surface temperature will occur at the exit of the duct. Assuming uniform heat flux, the air temperature at the exit is determined from

$$\dot{q} = h(T_{s,\max} - T_e) \rightarrow T_e = T_{s,\max} - \frac{\dot{q}}{h} = 70\text{°C} - \frac{[(8\times12+10)\text{ W}]/[8\times2(0.12\times0.15+0.003\times0.15)\text{ m}^2]}{35.9\text{ W/m}^2\text{.°C}} = 60.0\text{°C}$$

The highest allowable inlet temperature then becomes

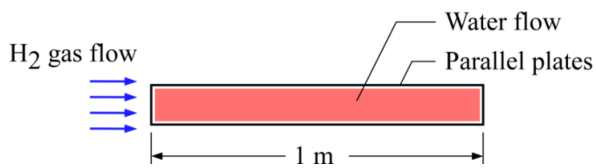
$$T_e - T_i = 10\text{°C} \rightarrow T_i = T_e - 10\text{°C} = 60.0\text{°C} - 10\text{°C} = \mathbf{50.0°C}$$

Discussion Although the Reynolds number is less than 2300, the flow in this case will most likely be turbulent because of the electronic components that that protrude into flow. Therefore, the heat transfer coefficient determined above is probably conservative.

8-67 Water is flowing between two parallel 1-m wide plates with 12.5-mm spacing. Hydrogen gas flows width-wise in parallel over the upper and lower surfaces of the two plates. The outlet mean temperature of the water, the surface temperature of the plates, and the total rate of heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Isothermal parallel plates. **4** The thermal resistance of the plates is negligible (thin plates). **5** The bulk mean fluid temperature of the water is 30°C (this will be validated). **6** The film temperature of the H_2 gas is 100°C (this will be validated).

Properties The properties of liquid water at 30°C are c_p = 4178 J/kg·K, k = 0.615 W/m·K, $\mu = 0.798 \times 10^{-3}$ kg/m·s, and Pr = 5.42 (Table A-9). The properties of H_2 gas at 100°C are k_{H2} = 0.2095 W/m·K, $\nu_{H2} = 1.582 \times 10^{-4}$ m²/s, and Pr_{H2} = 0.7196 (Table A-16)



Analysis The Reynolds number, hydrodynamic and thermal entry lengths can be determined to be

$$p = 2(1 + 0.0125)\text{ m} = 2.025\text{ m}$$

$$A_c = (1\text{ m})(0.0125\text{ m}) = 0.0125\text{ m}^2$$

$$D_h = 4A_c / p = 0.02469\text{ m}$$

$$\text{Re} = \frac{4\dot{m}}{p\mu} = \frac{4(0.58\text{ kg/s})}{(2.025\text{ m})(0.798\times10^{-3}\text{ kg/m}\cdot\text{s})} = 1436 < 2300 \qquad \text{(laminar flow)}$$

$$L_{h,\text{lam}} \approx 0.05\,\text{Re}\,D_h = 1.773\text{ m} < 10\text{ m}$$

and $$L_{t,\text{lam}} \approx 0.05\,\text{Re}\,\text{Pr}\,D = 9.608\text{ m} < 10\text{ m}$$

Therefore the flow is laminar and fully-developed. The appropriate equation to determine the Nusselt number is from Table 8-1 (a/b →∞ for parallel plates):

$$\text{Nu} = 7.54 \quad \rightarrow \quad h = \frac{k}{D_h}\text{Nu} = 187.81\text{ W/m}^2\cdot\text{K}$$

From Chap. 7, the convection heat transfer coefficient for H_2 gas parallel flow over the plates can be determined as follows:

$$\text{Re}_{\text{H2}} = \frac{V_\infty \text{width}}{\nu_{\text{H2}}} = \frac{(5\text{ m/s})(1\text{m})}{1.582\times10^{-4}\text{ m}^2\text{/s}} = 31606 < 5\times10^5 \qquad \text{(flow is laminar)}$$

$$\text{Nu}_{\text{H2}} = \frac{h_{\text{H2}}\text{width}}{k_{\text{H2}}} = 0.664\,\text{Re}_{\text{H2}}^{0.5}\,\text{Pr}_{\text{H2}}^{1/3} = 0.664(31606)^{0.5}(0.7196)^{1/3} = 105.83$$

$$h_{\text{H2}} = \frac{k_{\text{H2}}}{\text{width}}\text{Nu}_{\text{H2}} = 22.171\text{ W/m}^2\cdot\text{K}$$

The total rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) \qquad (1)$$

and $$\dot{Q} = 2(\text{width}\times L)h_{\text{H2}}(T_\infty - T_s) \qquad (2)$$

Also, the outlet mean temperature is

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) \qquad (3)$$

where $A_s = (2.025\text{ m})(10\text{ m}) = 20.25\text{ m}^2$

Solving for equations (1) to (3) simultaneously to obtain the final results:

$$T_e = \mathbf{40.1°C}, \quad T_s = \mathbf{45.3°C}, \quad \text{and} \quad \dot{Q} = \mathbf{48.6\ kW}$$

Discussion The bulk mean fluid temperature is $T_b = (T_i + T_e)/2 = 30.1°C$, thus 30°C is an appropriate temperature for evaluating the properties of glycerin. The film temperature of the H_2 gas is $T_f = (T_\infty + T_s)/2 = 100.28°C$, thus 100°C is an appropriate temperature for evaluating the properties of H_2 gas.

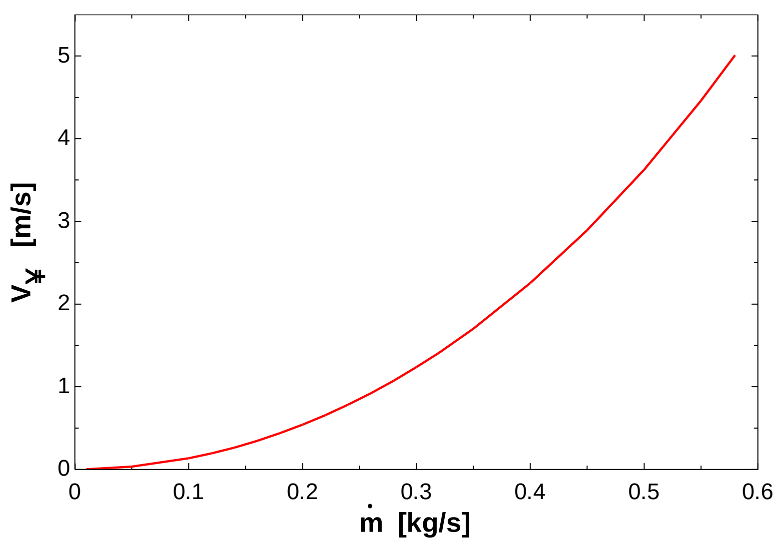
Equations (1) to (3) can be solved using the EES software with the following lines:

```
"GIVEN"
c_p=4178 [J/kg-K]
h=187.81 [W/m^2-K]
h_H2=22.171 [W/m^2-K]
A_s=20.25 [m^2]
m_dot=0.58 [kg/s]
T_i=20 [C]
T_infinity=155 [C]
V_infinity=5 [m/s]
L=10 [m]
width=1 [m]

"ANALYSIS"
Q_dot=2*(width*L)*h_H2*(T_infinity-T_s)
Q_dot=m_dot*c_p*(T_e-T_i)
T_e=T_s-(T_s-T_i)*exp(-(h*A_s)/(m_dot*c_p))
```

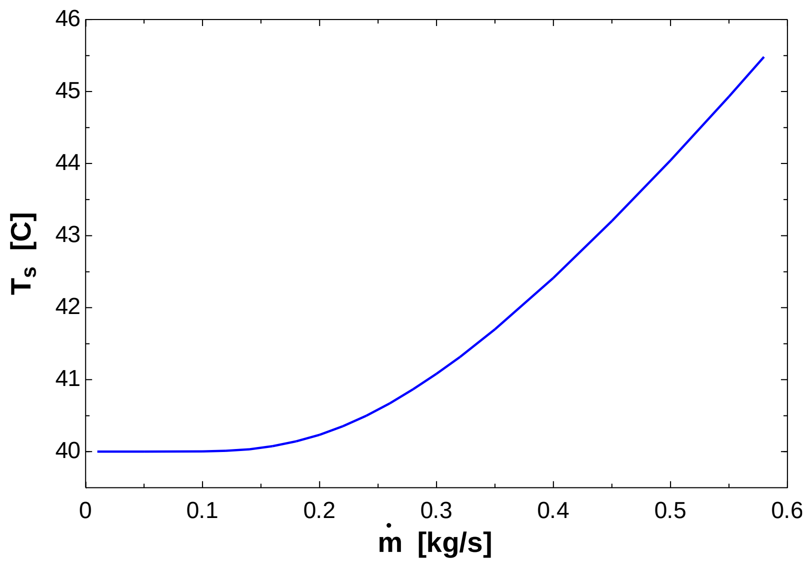
8-68 Reconsider Prob. 8-67. Water is flowing between two parallel 1-m wide plates with 12.5-mm spacing. Hydrogen gas flows width-wise in parallel over the upper and lower surfaces of the two plates. The effect of water mass flow rate on the free-stream velocity of the H_2 gas and the surface temperature of the parallel plates, and the effect of the free-stream velocity of the H_2 gas on the total heat transfer rate are to be evaluated.

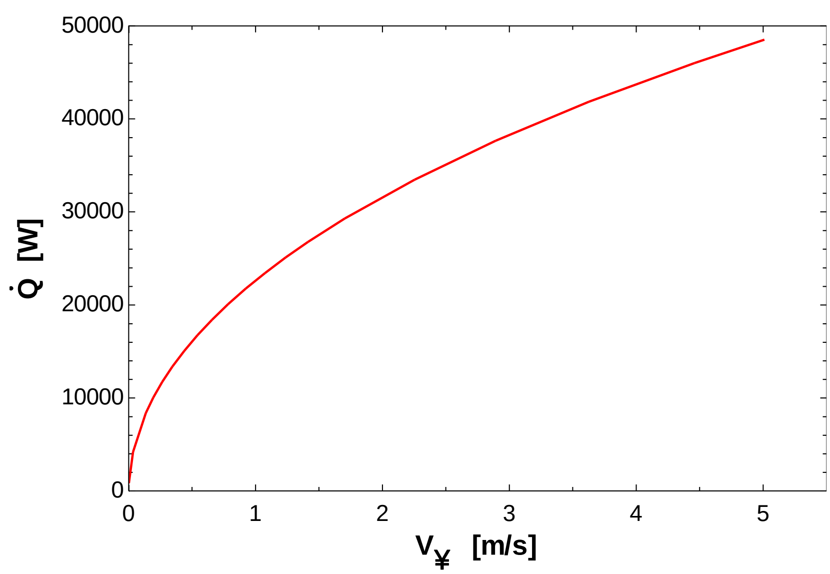
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=10 [m]
spacing=12.5e-3 [m]
width=1 [m]
T_i=20 [C]
T_e=40 [C]
T_infinity=155 [C]

"PROPERTIES"
"Water at 30°C"
T_b=(T_i+T_e)/2     "T_b = 1/2*(T_i+T_e)"
c_p=cP(water, T=T_b, x=0)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(water, T=T_b, x=0)
rho=Density(water, T=T_b, x=0)
Pr=Prandtl(water, T=T_b, x=0)
mu=Viscosity(water, T=T_b, x=0)

"H2 gas"
T_film=1/2*(T_s+T_infinity)
Fluid$='H2'
k_H2=Conductivity(Fluid$, T=T_film)
Pr_H2=Prandtl(Fluid$, T=T_film)
rho_H2=Density(Fluid$, T=T_film, P=101.3)
mu_H2=Viscosity(Fluid$, T=T_film)
nu_H2=mu_H2/rho_H2

"ANALYSIS"
A_c=width*spacing     "Cross-section area"
p=2*(width+spacing)     "Perimeter"
D_h=(4*A_c)/p     "Hydraulic diameter"
A_s=p*L     "Surface area"
"Flow between plates"
Re=4*m_dot/(mu*p)
L_t=0.05*Re*Pr*D_h
L_h=0.05*Re*D_h
Nusselt=7.54
h=k/D_h*Nusselt
T_e=T_s-(T_s-T_i)*exp(-(h*A_s)/(m_dot*c_p))
Q_dot=m_dot*c_p*(T_e-T_i)
"Flow over plates"
Re_H2=V_infinity*width/nu_H2
Nusselt_H2=0.664*Re_H2^0.5*Pr_H2^(1/3)
h_H2=Nusselt_H2*k_H2/width
Q_dot=2*h_H2*width*L*(T_infinity-T_s)
```



$\dot{m}$ [kg/s]	V_∞ [m/s]	T_s [°C]	$\dot{Q}$ [W]
0.01	0.001348	40	836.7
0.05	0.0337	40	4183
0.10	0.1348	40	8367
0.12	0.1942	40.01	10040
0.14	0.2644	40.03	11713
0.16	0.3456	40.08	13387
0.18	0.4379	40.14	15060
0.20	0.5415	40.23	16733
0.22	0.6566	40.35	18407
0.24	0.7834	40.50	20080
0.26	0.9222	40.67	21753
0.28	1.073	40.87	23427
0.30	1.237	41.08	25100
0.32	1.413	41.32	26773
0.35	1.702	41.70	29283
0.40	2.252	42.41	33467
0.45	2.891	43.20	37650
0.50	3.625	44.04	41833
0.55	4.459	44.93	46017
0.58	5.009	45.48	48527





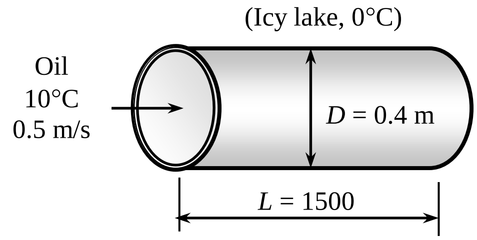
Discussion As the mass flow rate of water increases, the free-stream velocity of the H_2 gas, the surface temperature of the parallel plates, and the total heat transfer rate increase as well in order to keep T_e = 40°C. For $\dot{m} \leq 0.58$ kg/s, the water flow between the parallel plates is laminar and fully-developed. The flow of H_2 gas over the plates is also laminar with $Re_{H2} < 5 \times 10^5$.

8-69 Oil flows through a pipeline that passes through icy waters of a lake. The exit temperature of the oil and the rate of heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is very nearly 0°C. **3** The thermal resistance of the pipe is negligible. **4** The inner surfaces of the pipeline are smooth. **5** The flow is hydrodynamically developed when the pipeline reaches the lake.

Properties The properties of oil at 10°C are (Table A-13)

$$\rho = 893.6 \text{ kg/m}^3, \qquad k = 0.1460 \text{ W/m.°C}$$
$$\mu = 2.326 \text{ kg/m.s}, \qquad \nu = 2.592\times10^{-3} \text{ m}^2\text{/s}$$
$$c_p = 1839 \text{ J/kg.°C}, \qquad \text{Pr} = 28{,}750$$



Analysis (a) The Reynolds number in this case is

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(0.5 \text{ m/s})(0.4 \text{ m})}{2.592\times10^{-3} \text{ m}^2\text{/s}} = 77.16$$

which is less than 2300. Therefore, the flow is laminar, and the thermal entry length is roughly

$$L_t = 0.05 \text{ Re Pr } D = 0.05(77.16)(28{,}750)(0.4 \text{ m}) = 44{,}367 \text{ m}$$

which is much longer than the total length of the pipe. Therefore, we assume thermally developing flow, and determine the Nusselt number from

$$Nu = \frac{hD}{k} = 3.66 + \frac{0.065(D/L)\text{ Re Pr}}{1+0.04\left[(D/L)\text{ Re Pr}\right]^{2/3}} = 3.66 + \frac{0.065\left(\frac{0.4 \text{ m}}{1500 \text{ m}}\right)(77.16)(28{,}750)}{1+0.04\left[\left(\frac{0.4 \text{ m}}{1500 \text{ m}}\right)(77.16)(28{,}750)\right]^{2/3}} = 13.73$$

and $$h = \frac{k}{D} Nu = \frac{0.1460 \text{ W/m.°C}}{0.4 \text{ m}}(13.73) = 5.011 \text{ W/m}^2\text{.°C}$$

Next we determine the exit temperature of oil

$$A_s = \pi DL = \pi(0.4 \text{ m})(1500 \text{ m}) = 1885 \text{ m}^2$$

$$\dot{m} = \rho\dot{V} = \rho A_c V_{\text{avg}} = \rho\left(\frac{\pi D^2}{4}\right)V_{\text{avg}} = (893.6 \text{ kg/m}^3)\frac{\pi(0.4 \text{ m})^2}{4}(0.5 \text{ m/s}) = 56.15 \text{ kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA_s/(\dot{m}c_p)} = 0-(0-10)e^{-\frac{(5.011)(1885)}{(56.15)(1839)}} = \mathbf{9.13\,°C}$$

(*b*) The logarithmic mean temperature difference and the rate of heat loss from the oil are

$$\Delta T_{\text{lm}} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{9.13-10}{\ln\left(\frac{0-9.13}{0-10}\right)} = 9.56°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\text{lm}} = (5.011 \text{ W/m}^2\text{.°C})(1885 \text{ m}^2)(9.56°\text{C}) = 90{,}300 \text{ W} = \mathbf{90.3\,kW}$$

The friction factor is

$$f = \frac{64}{\text{Re}} = \frac{64}{77.16} = 0.8294$$

Then the pressure drop in the pipe and the required pumping power become

$$\Delta P = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.8294\frac{1500 \text{ m}}{0.4 \text{ m}}\frac{(893.6 \text{ kg/m}^3)(0.5 \text{ m/s})^2}{2}\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1 \text{ kN/m}^2}\right) = 347.4 \text{ kPa}$$

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = A_c V_{avg}\Delta P = \frac{\pi(0.4 \text{ m})^2}{4}(0.5 \text{ m/s})(347.4 \text{ kPa})\left(\frac{1 \text{ kW}}{1 \text{ kPa}\cdot\text{m}^3\text{/s}}\right) = \mathbf{21.8\,kW}$$

Discussion The power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be much more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

8-70 Glycerin is being heated by flowing through a 25-mm diameter and 10-m long tube. The outlet mean temperature and the total rate of heat transfer for the tube are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Constant tube surface temperature.

Properties The properties of glycerin at 30°C are c_p = 2447 J/kg·K, k = 0.2860 W/m·K, μ = 0.6582 kg/m·s, and Pr = 5631 (Table A-13).

Analysis The Reynolds number, hydrodynamic and thermal entry lengths are

$$\mathrm{Re} = \frac{4\dot{m}}{\pi D\mu} = \frac{4(0.5\ \mathrm{kg/s})}{\pi(0.025\ \mathrm{m})(0.6582\ \mathrm{kg/m\cdot s})} = 38.7 < 2300 \qquad \text{(laminar flow)}$$

$$L_{h,\,\mathrm{lam}} \approx 0.05\,\mathrm{Re}\,D = 0.0484\ \mathrm{m} < 10\ \mathrm{m} \qquad \text{and} \qquad L_{t,\,\mathrm{lam}} \approx 0.05\,\mathrm{Re}\,\mathrm{Pr}\,D = 273\ \mathrm{m} > 10\ \mathrm{m}$$

Therefore the flow is laminar and hydrodynamically developed but still thermally developing. The appropriate equation to determine the Nusselt number is from (Edwards et al., 1979)

$$\begin{aligned}\mathrm{Nu} &= 3.66 + \frac{0.065(D/L)\,\mathrm{Re}\,\mathrm{Pr}}{1+0.04[(D/L)\,\mathrm{Re}\,\mathrm{Pr}]^{2/3}} \\ &= 3.66 + \frac{0.065(0.025/10)(38.7)(5631)}{1+0.04[(0.025/10)(38.7)(5631)]^{2/3}} \\ &= 13.31\end{aligned}$$

$$h = \frac{k}{D}\mathrm{Nu} = 152\ \mathrm{W/m^2\cdot K}$$

The outlet mean temperature is

$$\begin{aligned}T_e &= T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) \\ &= 140^\circ\mathrm{C} - (140^\circ\mathrm{C} - 25^\circ\mathrm{C})\exp\left[-\frac{(152\ \mathrm{W/m^2\cdot K})\pi(0.025\ \mathrm{m})(10\ \mathrm{m})}{(0.5\ \mathrm{kg/s})(2447\ \mathrm{J/kg\cdot K})}\right] = \mathbf{35.7^\circ C}\end{aligned}$$

The total rate of heat transfer for the tube is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.5\ \mathrm{kg/s})(2447\ \mathrm{J/kg\cdot K})(35.7-25)\ \mathrm{K} = \mathbf{13.1\,kW}$$

Discussion The total rate of heat transfer for the tube can also be calculated using $\dot{Q} = hA_s\Delta T_{\mathrm{ln}}$.

8-71 Liquid glycerin is flowing through a 25-mm diameter and 10-m long tube, the constant surface temperature of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Constant tube surface temperature.

Properties The properties of glycerin at $T_b = (T_i + T_e)/2 = 30°\text{C}$ are $c_p = 2447$ J/kg·K, $k = 0.2860$ W/m·K, $\mu = 0.6582$ kg/m·s, and Pr = 5631 (Table A-13).

Analysis The Reynolds number, hydrodynamic and thermal entry lengths are

$$\text{Re} = \frac{4\dot{m}}{\pi D\mu} = \frac{4(0.5\text{ kg/s})}{\pi(0.025\text{ m})(0.6582\text{ kg/m}\cdot\text{s})} = 38.7 < 2300 \qquad \text{(laminar flow)}$$

$$L_{h,\text{lam}} \approx 0.05\,\text{Re}\,D = 0.0484\text{ m} < 10\text{ m} \qquad \text{and} \qquad L_{t,\text{lam}} \approx 0.05\,\text{Re}\,\text{Pr}\,D = 273\text{ m} > 10\text{ m}$$

Therefore the flow is laminar and hydrodynamically developed but still thermally developing. The appropriate equation to determine the Nusselt number is from (Edwards et al., 1979)

$$\begin{aligned}\text{Nu} &= 3.66 + \frac{0.065(D/L)\,\text{Re}\,\text{Pr}}{1+0.04[(D/L)\,\text{Re}\,\text{Pr}]^{2/3}} \\ &= 3.66 + \frac{0.065(0.025/10)(38.7)(5631)}{1+0.04[(0.025/10)(38.7)(5631)]^{2/3}} \\ &= 13.31\end{aligned}$$

$$h = \frac{k}{D}\text{Nu} = 152\text{ W/m}^2\cdot\text{K}$$

The surface temperature of the tube is

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) \quad \rightarrow \quad T_s = \frac{T_e - T_i\exp[-hA_s/(\dot{m}c_p)]}{1-\exp[-hA_s/(\dot{m}c_p)]}$$

$$T_s = \frac{40°\text{C} - (20°\text{C})\exp(-0.09757)}{1-\exp(-0.09757)} = \mathbf{235°C}$$

where

$$\frac{hA_s}{\dot{m}c_p} = \frac{(152\text{ W/m}^2\cdot\text{K})\pi(0.025\text{ m})(10\text{ m})}{(0.5\text{ kg/s})(2447\text{ J/kg}\cdot\text{K})} = 0.09757$$

Discussion For laminar hydrodynamically and thermally fully developed flow in constant surface temperature tube, the Nu is 3.66. This problem shows that the development of the temperature profile in the entry region contributed to the increase in the value of Nu.

8-72 Liquid water entering at 60°C is heated in a circular tube with a constant surface temperature. The mass flow rate of the flow is 11 g/s. The surface temperature of the tube is to be determined and whether a HNBR o-ring attached to the tube surface is suitable. The maximum temperature permitted for the o-ring is 150°C.

Assumptions **1** The flow is steady and incompressible. **2** Uniform surface temperature. **3** Inner surface of the tube is smooth.

Properties The properties of water at 100°C are (Table A–9) c_p = 4217 J/kg·K, k = 0.679 W/m·K, μ = 0.282 × 10^{-3} kg/m·s, and Pr = 1.75

Analysis The Reynolds number for the flow in circular tube is

$$\text{Re} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.011\ \text{kg/s})}{\pi(0.025\ \text{m})(0.000282\ \text{kg/m}\cdot\text{s})} = 1987$$

So, the flow is laminar. The hydrodynamic and thermal entry lengths are

$$L_h \approx 0.05\ \text{Re}\ D = 0.05(1987)(0.025\ \text{m}) = 2.48\ \text{m}$$
$$L_t \approx L_h \text{Pr} = (2.48\ \text{m})(1.75) = 4.34\ \text{m}$$

At the tube exit (x = 3 m), the flow is hydrodynamically developed and thermally developing. For thermal entrance region, the Nusselt number for constant surface temperature is

$$\text{Nu} = \frac{hD}{k} = 3.66 + \frac{0.065(D/L)\text{RePr}}{1 + 0.04[(D/L)\text{RePr}]^{2/3}} = 3.66 + \frac{0.065(0.025\ \text{m}/3\ \text{m})(1987)(1.75)}{1 + 0.04[(0.025\ \text{m}/3\ \text{m})(1987)(1.75)]^{2/3}} = 5.0275$$

$$h = \text{Nu}\frac{k}{D} = (5.0275)\frac{0.679\ \text{W/m}\cdot\text{K}}{0.025\ \text{m}} = 136.55\ \text{W/m}^2\cdot\text{K}$$

The heat transfer rate to the water is

$$\dot{Q} = \dot{m}c_p(T_e - T_i)$$

which can be used to solve for the exit temperature of water:

$$T_e = \frac{\dot{Q}}{\dot{m}c_p} + T_i = \frac{3800\ \text{W}}{(0.011\ \text{kg/s})(4217\ \text{J/kg}\cdot\text{K})} + 60℃ = 141.92℃$$

To solve for the surface temperature of the tube, we use

$$T_e = T_s - (T_s - T_i)\exp\left(-hA_s/\dot{m}c_p\right) = T_s - (T_s - T_i)\exp\left(-h\pi DL/\dot{m}c_p\right)$$

Solving for T_s, yields

$$141.92℃ = T_s - (T_s - 60)\exp[-(136.55\ \text{W/m}^2\cdot\text{K})\pi(0.025\ \text{m})(3\ m)/(0.011\ \text{kg/s})(4217\ \text{J/kg}\cdot\text{K})]$$

$$T_s = \mathbf{223.77}℃ > 150℃$$

Discussion The surface temperature of the tube exceeds the maximum temperature (150°C) permitted for the HNBR o-ring (ASME Boiler and Pressure Vessel Code, BPVC.IV-2015, HG-360). Therefore, the HNBR o-ring is not suitable for this operation.

With the exit temperature at 142°C and the inlet temperature at 60°C, the bulk mean fluid temperature becomes 101°C. Thus, 100°C is a reasonable temperature to evaluate the properties of water.

8-73 Air at 20°C (1 atm) enters into a 5-mm diameter and 10-cm long circular tube, the convection heat transfer coefficient and the outlet mean temperature are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Constant tube surface temperature.

Properties The properties of air at 50°C: c_p = 1007 J/kg·K, k = 0.02735 W/m·K, ρ = 1.092 kg/m^3, μ = 1.963 × 10^{-5} kg/m·s, ν = 1.798 × 10^{-5} m^2/s, and Pr = 0.7228; at T_s = 160°C: μ_s = 2.420 × 10^{-5} kg/m·s (Table A-15).

Analysis The Reynolds number, hydrodynamic and thermal entry lengths are

$$\text{Re} = \frac{V_{\text{avg}} D}{\nu} = \frac{(5\text{ m/s})(0.005\text{ m})}{(1.798\times10^{-5}\text{ m}^2\text{/s})} = 1390 < 2300 \qquad \text{(laminar flow)}$$

$$L_{h,\text{lam}} \approx 0.05\,\text{Re}\,D = 34.8\text{ cm} > 10\text{ cm} \quad \text{and} \quad L_{t,\text{lam}} \approx 0.05\,\text{Re}\,\text{Pr}\,D = 25.1\text{ cm} > 10\text{ cm}$$

Therefore the flow is laminar, hydrodynamically and thermally developing. The appropriate equation to determine the Nusselt number is from (Sieder and Tate, 1936)

$$\text{Nu} = 1.86\left(\frac{\text{Re}\,\text{Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 1.86\left[\frac{(1390)(0.7228)(0.005\text{ m})}{0.1\text{ m}}\right]^{1/3}\left(\frac{1.963}{2.420}\right)^{0.14} = 6.665$$

$$h = \frac{k}{D}\text{Nu} = \mathbf{36.5\ W/m^2 \cdot K}$$

The outlet mean temperature is

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right)$$

$$= 160°\text{C} - (160°\text{C} - 20°\text{C})\exp\left[-\frac{(36.5\text{ W/m}^2\cdot\text{K})\pi(0.005\text{ m})(0.1\text{ m})}{(1.072\times10^{-4}\text{ kg/s})(1007\text{ J/kg}\cdot\text{K})}\right]$$

$$= \mathbf{77.7°C}$$

where

$$\dot{m} = \rho V_{\text{avg}} \pi D^2/4 = 1.072\times10^{-4}\text{ kg/s}$$

Discussion Note that the bulk mean temperature is $T_b = (T_i + T_e)/2$ = 48.9°C, thus evaluating the air properties at 50°C is reasonable.

8-74 C&S Liquid methane flows at 1.7 mm/s between two isothermal parallel ASTM A240 410S stainless steel plates. The inlet and exit temperatures are −150°C and −50°C, respectively. The plates are heated at 700 W/m². The surface temperature of the parallel plates is to be determined and whether the ASTM A240 410S plates are suitable. The minimum temperature for the plates is −30°C.

Assumptions **1** The flow is steady and incompressible. **2** The plates are isothermal.

Properties The properties of liquid methane at the bulk mean temperature of $T_b = (T_i + T_e)/2 = (-150 - 50)$°C/2 = −100°C are (Table A–13) $k = 0.0967$ W/m·K, $\mu = 3.577 \times 10^{-5}$ kg/m·s, $\rho = 301.0$ kg/m³, and Pr = 2.063.

Analysis The Reynolds number for the flow between the parallel plates is

$$\mathrm{Re} = \frac{\rho V_{\mathrm{avg}} D_h}{\mu} = \frac{\rho V_{\mathrm{avg}}(2b)}{\mu} = \frac{(301\ \mathrm{kg/m^3})(0.0017\ \mathrm{m/s})(2 \times 0.025\ \mathrm{m})}{3.577 \times 10^{-5}\ \dfrac{\mathrm{kg}}{\mathrm{m}} \cdot \mathrm{s}} = 715$$

So, the flow is laminar. The hydrodynamic and thermal entry lengths are

$$L_h \approx 0.05\ \mathrm{Re}\ D_h = 0.05(715)(2 \times 0.025\ \mathrm{m}) = 1.79\ \mathrm{m}$$

$$L_t \approx L_h \mathrm{Pr} = (1.79\ \mathrm{m})(2.063) = 3.69\ \mathrm{m}$$

At the exit of parallel plates ($x = 2$ m), the flow is hydrodynamically developed and thermally developing. For thermal entrance region, the Nusselt number for isothermal parallel plates is

$$\mathrm{Nu} = \frac{hD_h}{k} = 7.54 + \frac{0.03(D_h/L)\mathrm{RePr}}{1 + 0.016[(D_h/L)\mathrm{RePr}]^{2/3}} = 7.54 + \frac{0.03(0.05\ \mathrm{m}/2\ \mathrm{m})(715)(2.063)}{1 + 0.016[(0.05\ \mathrm{m}/2\ \mathrm{m})(715)(2.063)]^{2/3}} = 8.48$$

$$h = \mathrm{Nu}\frac{k}{D_h} = (8.48)\frac{0.0967\ \mathrm{W/m \cdot K}}{0.05\ \mathrm{m}} = 16.4\ \mathrm{W/m^2 \cdot K}$$

The heat transfer per unit area to the liquid methane is

$$\dot{q} = h\Delta T_{lm} = \frac{h(T_i - T_e)}{\ln\left[\dfrac{T_s - T_e}{T_s - T_i}\right]}$$

Solving for the surface temperature,

$$\dot{q} = \frac{h(T_i - T_e)}{\ln\left[\dfrac{T_s - T_e}{T_s - T_i}\right]} \qquad \Longrightarrow \qquad 700 = \frac{(16.4\ \mathrm{W/m^2 \cdot K})(-150 + 50)\mathrm{K}}{\ln\left[\dfrac{T_s + 50}{T_s + 150}\right]}$$

Thus,

$$T_s = \mathbf{-39.4}℃ < -30℃$$

Discussion The surface temperature (T_s = −39.4°C) of the parallel plates is below the minimum temperature (−30°C) suitable for ASTM A240 410S stainless steel plate. Therefore, the ASTM A240 410S stainless steel plates are not suitable for this operation as per the ASME Code for Process Piping.

8-75 Glycerin is being heated by flowing between two parallel 1-m wide plates with 12.5-mm spacing. The outlet mean temperature and the total rate of heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Isothermal parallel plates. **4** Bulk mean fluid temperature is 30°C (this will be validated).

Properties The properties of glycerin at 30°C are c_p = 2447 J/kg·K, k = 0.2860 W/m·K, μ = 0.6582 kg/m·s, and Pr = 5631 (Table A-13).

Analysis The Reynolds number, hydrodynamic and thermal entry lengths can be determined to be

$$p = 2(1 + 0.0125)\text{ m} = 2.025\text{ m}$$

$$A_c = (1\text{ m})(0.0125\text{ m}) = 0.0125\text{ m}^2$$

$$D_h = 4A_c / p = 0.02469\text{ m}$$

$$\text{Re} = \frac{4\dot{m}}{p\mu} = \frac{4(0.7\text{ kg/s})}{(2.025\text{ m})(0.6582\text{ kg/m}\cdot\text{s})} = 2.101 < 2300 \qquad \text{(laminar flow)}$$

$$L_{h,\text{lam}} \approx 0.05\,\text{Re}\,D_h = 0.002594\text{ m} < 10\text{ m}$$

$$L_{t,\text{lam}} \approx 0.05\,\text{Re}\,\text{Pr}\,D = 14.6\text{ m} > 10\text{ m}$$

Therefore the flow is laminar and hydrodynamically developed but still thermally developing. The appropriate equation to determine the Nusselt number is from (Edwards et al., 1979)

$$\begin{aligned}\text{Nu} &= 7.54 + \frac{0.03(D_h / L)\,\text{Re}\,\text{Pr}}{1 + 0.016[(D_h / L)\,\text{Re}\,\text{Pr}]^{2/3}} \\ &= 7.54 + \frac{0.03(0.02469/10)(2.101)(5631)}{1 + 0.016[(0.02469/10)(2.101)(5631)]^{2/3}} \\ &= 8.301\end{aligned}$$

$$h = \frac{k}{D_h}\text{Nu} = 96.156\text{ W/m}^2\cdot\text{K}$$

The outlet mean temperature is

$$\begin{aligned}T_e &= T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) \\ &= 40°\text{C} - (40°\text{C} - 25°\text{C})\exp\left[-\frac{(96.156\text{ W/m}^2\cdot\text{K})(2.025\text{ m})(10\text{ m})}{(0.7\text{ kg/s})(2447\text{ J/kg}\cdot\text{K})}\right] \\ &= \mathbf{35.19°C}\end{aligned}$$

The total rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.7\text{ kg/s})(2447\text{ J/kg}\cdot\text{K})(35.19 - 25)\text{ K} = \mathbf{17.45\text{ kW}}$$

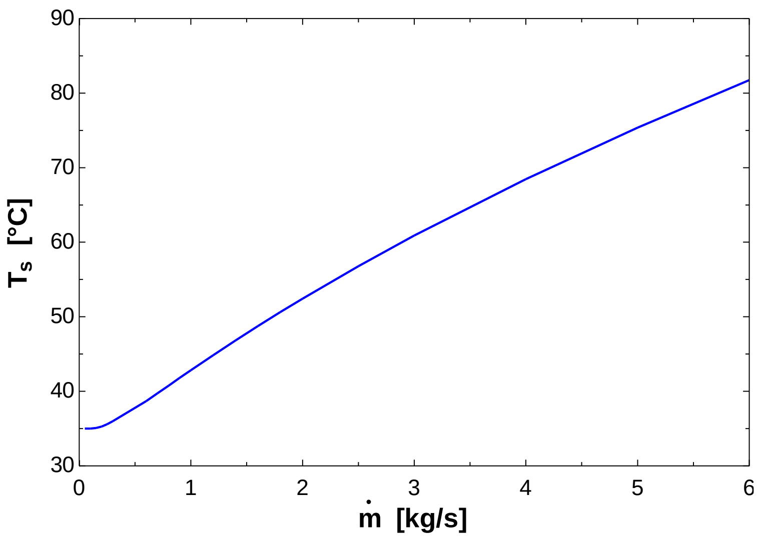
Discussion The bulk mean fluid temperature is T_b = (T_i + T_e)/2 = (25 + 35.19)/2 = 30.1°C, thus 30°C is an appropriate temperature for evaluating the properties of glycerin.

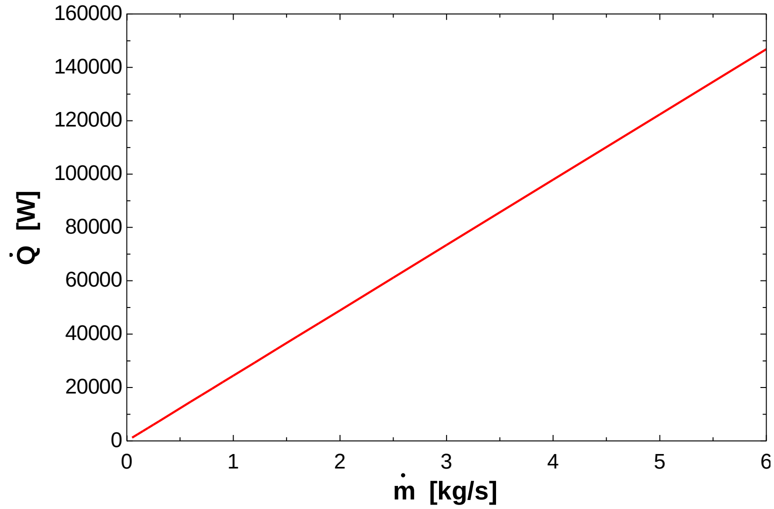
8-76 Reconsider Prob. 8-75. Glycerin is being heated by flowing between two parallel 1-m wide plates with 12.5-mm spacing. The effect of glycerin mass flow rate on the surface temperature of the parallel plates and the total rate of heat transfer necessary to keep T_e = 35°C are to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=10 [m]
spacing=12.5e-3 [m]
width=1 [m]
T_i=25 [C]
T_e=35 [C]

"PROPERTIES"
c_p=2447 [J/kg-K]
k=0.2860 [W/m-K]
Pr=5631
rho=1258 [kg/m^3]
mu=0.6582 [kg/m-s]
T_b=1/2*(T_i+T_e)

"ANALYSIS"
A_c=width*spacing     "Cross-section area"
p=2*(width+spacing)     "Perimeter"
D_h=(4*A_c)/p     "Hydraulic diameter"
A_s=p*L     "Surface area"
Re=4*m_dot/(mu*p)
L_t=0.05*Re*Pr*D_h     "Thermal entry length"
L_h=0.05*Re*D_h     "Hydrodynamic entry length"
Nusselt_fd=7.54     "Fully-developed Nu"
Nusselt_ent=7.54+0.03*(D_h/L)*Re*Pr/(1+0.016*((D_h/L)*Re*Pr)^(2/3))     "Entry region Nu"
Nusselt=if(L,L_t,Nusselt_ent,Nusselt_fd,Nusselt_fd)     "If L > L_t, then fully-developed"
h=k/D_h*Nusselt
T_e=T_s-(T_s-T_i)*exp(-(h*A_s)/(m_dot*c_p))
Q_dot=m_dot*c_p*(T_e-T_i)
```

$\dot{m}$ [kg/s]	h [W/m^2·K]	T_e [°C]	$\dot{Q}$ [W]
0.05	87.34	35	1224
0.1	87.34	35.01	2447
0.15	87.34	35.08	3671
0.2	87.34	35.28	4894
0.25	87.34	35.59	6118
0.3	87.34	35.99	7341
0.6	94.99	38.69	14682
0.7	96.15	39.73	17129
0.8	97.29	40.76	19576
0.9	98.40	41.80	22023
1.0	99.49	42.82	24470
1.2	101.6	44.85	29364
1.4	103.7	46.82	34258
1.6	105.7	48.75	39152
1.8	107.6	50.62	44046
2.0	109.5	52.44	48940
2.5	114.1	56.79	61175
3.0	118.4	60.89	73410
4.0	126.4	68.46	97880
5.0	133.7	75.37	122350
6.0	140.5	81.76	146820





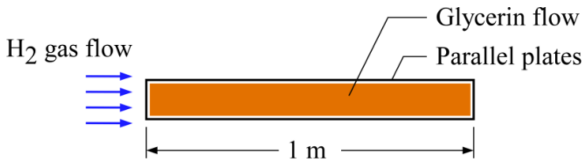
Discussion For $\dot{m}$ > 0.48 kg/s, the thermal entry length $L_{t,\text{lam}} > L$, thus the flow becomes thermally developing. For all the evaluated $\dot{m}$, the flow is hydrodynamically developed and laminar.

As $\dot{m}$ decreases, the outlet mean temperature of glycerin approaches the surface temperature of the parallel plates T_s = 40°C. The total heat transfer rate increases with increasing mass flow rate.

8-77 Glycerin is being heated by flowing between two parallel 1-m wide plates with 12.5-mm spacing. Hydrogen gas flows width-wise in parallel over the upper and lower surfaces of the two plates. The outlet mean temperature of the glycerin, the surface temperature of the plates, and the total rate of heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Isothermal parallel plates. **4** The thermal resistance of the plates is negligible (thin plates). **5** The bulk mean fluid temperature of the glycerin is 30°C (this will be validated). **6** The film temperature of the H_2 gas is 100°C (this will be validated).

Properties The properties of glycerin at 30°C are c_p = 2447 J/kg·K, k = 0.2860 W/m·K, μ = 0.6582 kg/m·s, and Pr = 5631 (Table A-13). The properties of H_2 gas at 100°C are k_{H2} = 0.2095 W/m·K, $\nu_{H2} = 1.582 \times 10^{-4}$ m²/s, and Pr_{H2} = 0.7196 (Table A-16)



Analysis The Reynolds number, hydrodynamic and thermal entry lengths can be determined to be

$$p = 2(1+0.0125)\text{ m} = 2.025\text{ m}, \quad A_c = (1\text{ m})(0.0125\text{ m}) = 0.0125\text{ m}^2, \quad D_h = 4A_c / p = 0.02469\text{ m}$$

$$\text{Re} = \frac{4\dot{m}}{p\mu} = \frac{4(0.7\text{ kg/s})}{(2.025\text{ m})(0.6582\text{ kg/m}\cdot\text{s})} = 2.101 < 2300 \qquad \text{(laminar flow)}$$

$$L_{h,\text{lam}} \approx 0.05\,\text{Re}\,D_h = 0.002594\text{ m} < 10\text{ m}$$

and $$L_{t,\text{lam}} \approx 0.05\,\text{Re}\,\text{Pr}\,D = 14.6\text{ m} > 10\text{ m}$$

Therefore the flow is laminar and hydrodynamically developed but still thermally developing. The appropriate equation to determine the Nusselt number is from Edwards et al. (1979):

$$\text{Nu} = 7.54 + \frac{0.03(D_h / L)\,\text{Re}\,\text{Pr}}{1+0.016[(D_h / L)\,\text{Re}\,\text{Pr}]^{2/3}} = 7.54 + \frac{0.03(0.02469/10)(2.101)(5631)}{1+0.016[(0.02469/10)(2.101)(5631)]^{2/3}} = 8.301$$

$$h = \frac{k}{D_h}\text{Nu} = 96.156\text{ W/m}^2\cdot\text{K}$$

From Chap. 7, the convection heat transfer coefficient for H_2 gas parallel flow over the plates can be determined as follows:

$$\text{Re}_{\text{H2}} = \frac{V_\infty\,\text{width}}{\nu_{\text{H2}}} = \frac{(3\text{ m/s})(1\text{m})}{1.582\times10^{-4}\text{ m}^2\text{/s}} = 18963 < 5\times10^5 \qquad \text{(flow is laminar)}$$

$$\text{Nu}_{\text{H2}} = \frac{h_{\text{H2}}\,\text{width}}{k_{\text{H2}}} = 0.664\,\text{Re}_{\text{H2}}^{0.5}\,\text{Pr}_{\text{H2}}^{1/3} = 0.664(18963)^{0.5}(0.7196)^{1/3} = 81.94$$

$$h_{\text{H2}} = \frac{k_{\text{H2}}}{\text{width}}\text{Nu}_{\text{H2}} = 17.166\text{ W/m}^2\cdot\text{K}$$

The total rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) \qquad (1)$$

and $$\dot{Q} = 2(\text{width}\times L)h_{\text{H2}}(T_\infty - T_s) \qquad (2)$$

Also, the outlet mean temperature is

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) \qquad (3)$$

where $$A_s = (2.025\text{ m})(10\text{ m}) = 20.25\text{ m}^2$$

Solving for equations (1) to (3) simultaneously to obtain the final results:

$$T_e = \mathbf{40.1°C}, \quad T_s = \mathbf{49.6°C}, \quad \text{and} \quad \dot{Q} = \mathbf{34.5\ kW}$$

Discussion The bulk mean fluid temperature is $T_b = (T_i + T_e)/2 = (20 + 40.1)/2 = 30.1°C$, thus 30°C is an appropriate temperature for evaluating the properties of glycerin. The film temperature of the H_2 gas is $T_f = (T_\infty + T_s)/2 = (150 + 49.6)/2 = 99.8°C$, thus 100°C is an appropriate temperature for evaluating the properties of H_2 gas.

Equations (1) to (3) can be solved using the EES software with the following lines:

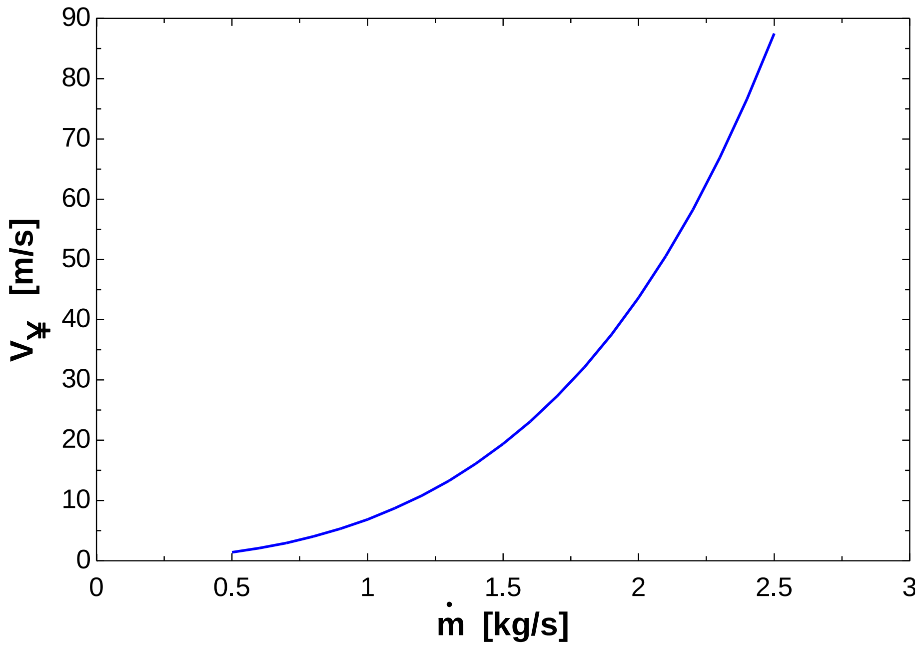
```
"GIVEN"
c_p=2447 [J/kg-K]
h=96.156 [W/m^2-K]
h_H2=17.166 [W/m^2-K]
A_s=20.25 [m^2]
m_dot=0.7 [kg/s]
T_i=20 [C]
T_infinity=150 [C]
V_infinity=3 [m/s]
L=10 [m]
width=1 [m]

"ANALYSIS"
Q_dot=2*(width*L)*h_H2*(T_infinity-T_s)
Q_dot=m_dot*c_p*(T_e-T_i)
T_e=T_s-(T_s-T_i)*exp(-(h*A_s)/(m_dot*c_p))
```

8-78 Reconsider Prob. 8-77. Glycerin is being heated by flowing between two parallel 1-m wide plates with 12.5-mm spacing. Hydrogen gas flows width-wise in parallel over the upper and lower surfaces of the two plates. The effect of glycerin mass flow rate on the free-stream velocity of the H_2 gas needed to keep T_e = 40°C is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=10 [m]
spacing=12.5e-3 [m]
width=1 [m]
T_i=20 [C]
T_e=40 [C]
T_infinity=150 [C]

"PROPERTIES"
"Glycerin at 30°C"
c_p=2447 [J/kg-K]
k=0.2860 [W/m-K]
Pr=5631
rho=1258 [kg/m^3]
mu=0.6582 [kg/m-s]
T_b=1/2*(T_i+T_e)
"H2 gas"
T_film=1/2*(T_s+T_infinity)
Fluid$='H2'
k_H2=Conductivity(Fluid$, T=T_film)
Pr_H2=Prandtl(Fluid$, T=T_film)
rho_H2=Density(Fluid$, T=T_film, P=101.3)
mu_H2=Viscosity(Fluid$, T=T_film)
nu_H2=mu_H2/rho_H2

"ANALYSIS"
A_c=width*spacing    "Cross-section area"
p=2*(width+spacing)    "Perimeter"
D_h=(4*A_c)/p    "Hydraulic diameter"
A_s=p*L    "Surface area"
"Flow between plates"
Re=4*m_dot/(mu*p)
L_t=0.05*Re*Pr*D_h
L_h=0.05*Re*D_h
Nusselt=7.54+0.03*(D_h/L)*Re*Pr/(1+0.016*((D_h/L)*Re*Pr)^(2/3))
h=k/D_h*Nusselt
T_e=T_s-(T_s-T_i)*exp(-(h*A_s)/(m_dot*c_p))
Q_dot=m_dot*c_p*(T_e-T_i)
"Flow over plates"
Re_H2=V_infinity*width/nu_H2
Nusselt_H2=0.664*Re_H2^0.5*Pr_H2^(1/3)
h_H2=Nusselt_H2*k_H2/width
Q_dot=2*h_H2*width*L*(T_infinity-T_s)
```

$\dot{m}$ [kg/s]	V_∞ [m/s]
0.5	1.393
0.6	2.087
0.7	2.961
0.8	4.034
0.9	5.331
1.0	6.876
1.1	8.696
1.2	10.82
1.3	13.29
1.4	16.13
1.5	19.39
1.6	23.11
1.7	27.33
1.8	32.13
1.9	37.54
2.0	43.65
2.1	50.53
2.2	58.26
2.3	66.92
2.4	76.64



Discussion As the mass flow rate of the glycerin increases, the free-stream velocity of the H_2 gas needs to increase as well in order to keep T_e = 40°C

8-79E Air is to be heated in a constant surface temperature tube. The exit temperature of the air and the total rate of heat transfer from the tube wall is to be determined.

Assumptions **1** The flow issteady. **2** The flow is fully developed. **3** The tube is isothermal and smooth.

Properties The properties of air at 80°F are (Table A-15E): ρ = 0.07350 lbm/ft^3, k = 0.01481 Btu/h·ft·R, $\nu = 1.697 \times 10^{-4}$ ft^2/s, c_p = 0.2404 Btu/lbm·R, and Pr=0.7290.

Analysis First the cross sectional area, surface area, and average velocity will be determined.

$$A_c = \frac{\pi(D)^2}{4} = \frac{\pi(0.167\ \text{ft})^2}{4} = 0.022\ \text{ft}^2$$

$$A_s = \pi DL = \pi(0.167\ \text{ft})(10\ \text{ft}) = 5.25\ \text{ft}^2$$

$$V_{avg} = \frac{\dot{m}}{\rho A_c} = \frac{18.2\ \text{lbm/h}}{(0.07350\ \text{lbm/ ft}^3)(0.022\ \text{ft}^2)(3600\ \frac{\text{s}}{\text{h}})} = 3.13\ \text{ft/s}$$

The Reynolds number for the flow is

$$\text{Re} = \frac{V_{avg}D}{\nu} = \frac{\left(3.13\ \frac{\text{ft}}{\text{s}}\right)(0.167\ \text{ft})}{1.697 \times 10^{-4}\ \frac{\text{ft}^2}{\text{s}}} = 3080$$

which is between 2300 and 4000. Therefore, the flow is in the transitional region. With the Reynolds number known, it is possible to obtain the friction factor and the Nusselt number. The friction factor for the transitional flow with Re = 3080 is found from

$$f = 3.03 \times 10^{-12}\text{Re}^3 - 3.67 \times 10^{-8}\text{Re}^2 + 1.46 \times 10^{-4}\text{Re} - 0.151 = 0.039$$

and the Nusslet number with f = 0.039, Re = 3080, and Pr = 0.7290 is found from

$$\text{Nu} = \frac{\frac{f}{8}(\text{Re}-1000)\text{Pr}}{1+12.7\sqrt{\frac{f}{8}}(\text{Pr}^{2/3}-1)} = 8.89$$

and from the Nusselt number, the convective heat transfer coefficient can be found from

$$\text{h} = \frac{\text{Nu k}}{\text{D}} = \frac{8.89\ (0.01481\ \text{Btu/h·ft·R})}{0.167\ \text{ft}} = 0.788\ \frac{\text{Btu}}{\text{h·ft}^2\text{·R}}$$

Next, we determine the exit temperature of air for the constant wall temperature case from

$$T_e = T_s - (T_s - T_i)\exp\left(-hA_s/\dot{m}c_p\right)$$

$$\text{T}_\text{e} = 120°\text{F} - (120°\text{F} - 60°\text{F})\exp\left[-\frac{(0.788\ \frac{\text{Btu}}{\text{h·ft}^2\text{·R}})(5.25\ \text{ft}^2)}{\left(18.2\ \frac{\text{lbm}}{\text{h}}\right)\left(0.2404\ \frac{\text{Btu}}{\text{lbm·R}}\right)}\right] = \mathbf{96.7}°\text{F}$$

$$\dot{Q}_{total} = \dot{m}c(T_e - T_i) = (18.2\ \frac{\text{lbm}}{\text{h}})(0.2404\ \frac{\text{Btu}}{\text{lbm·R}})(96.7 - 60)°\text{F} = \mathbf{160.6 Btu/h}$$

Discussion Note that the convective coefficient is quite low. This reflects the slow speed of the air within the channel. Also note that the air temperature increases quite a lot within the tube. This is due to the very low heat capacity (both mass flow and specific heat) of the air.

The bulk mean temperature is $T_b = (T_i + T_e)/2$ = (60 + 96.7)/2 = 78.4°F. This is very close to the assumed temperature of 80°F for air properties. Therefore, there is no need to repeat calculations.

8-80 Water is uniformly heated in a circular pipe. The heat flux to raise the water temperature by 1℃ at the exit of the pipe and the pipe surface temperature at the exit is to be determined.

Assumptions **1** The flow issteady and incompressible. **2** The flow is fully developed. **3** The tube is uniformly heated and smooth.

Properties The properties of liquid water at20°C are (Table A-9): ρ = 998.0 kg/m^3, k = 0.598 W/m·K, μ= 1.002 × 10^{-3} kg/m·s, c_p = 4182 J/kg·K, and Pr=7.01

Analysis First the cross sectional area, surface area, and mass flow rate will be determined.

$$A_c = \frac{\pi D^2}{4} = \frac{\pi(0.02\text{ m})^2}{4} = 0.000314\text{ m}^2$$

$$A_s = \pi DL = \pi(0.02\text{ m})(6\text{ m}) = 0.377\text{ m}^2$$

$$\dot{m} = \rho A_c V_{avg} = (998.0\ \frac{\text{kg}}{\text{m}^3})(0.000314\text{ m}^2)(0.15\ \frac{\text{m}}{\text{s}}) = 0.047\text{ kg/s}$$

The rate of heat transfer required to cause a 1°C rise can be determined from

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.047\ \frac{\text{kg}}{\text{s}})(4182\ \frac{\text{J}}{\text{kg·K}})(1℃) = 196.6\text{ W}$$

Since the heating is uniform, the flux can be found by dividing the total rate of heat transfer by the surface area so that

$$\dot{q}_s = \frac{\dot{Q}}{A_s} = \frac{196.6\text{ W}}{0.377\text{ m}^2} = \mathbf{521.5\ \frac{W}{m^2}}$$

Next, The Reynolds number for the flow will be found.

$$\text{Re} = \frac{\rho V_{avg} D}{\mu} = \frac{(998.0\ \frac{\text{kg}}{\text{m}^3})(0.15\ \frac{\text{m}}{\text{s}})(0.02\text{ m})}{1.002 \times 10^{-3}\text{kg/m·s}} = 2988$$

which is between 2300 and 4000. Therefore, the flow is in the transitional region. With the Reynolds number known, it is possible to obtain the friction factor and the Nusselt number. The friction factor for the transitional flow with Re = 2988 is found from

$$f = 3.03 \times 10^{-12}\text{Re}^3 - 3.67 \times 10^{-8}\text{Re}^2 + 1.46 \times 10^{-4}\text{Re} - 0.151 = 0.038$$

and the Nusslet number with f = 0.038, Re = 2988, and Pr = 7.01 is found from

$$\text{Nu} = \frac{\frac{f}{8}(\text{Re} - 1000)\text{Pr}}{1 + 12.7\sqrt{\frac{f}{8}}(\text{Pr}^{2/3} - 1)} = 19.9$$

and from the Nusselt number, the convective heat transfer coefficient can be found.

$$h = \frac{Nu\ k}{D} = \frac{(19.9)(0.598\ \frac{\text{W}}{\text{m·K}})}{0.02\text{ m}} = 595\ \frac{\text{W}}{\text{m}^2\text{·K}}$$

and the surface temperature of the pipe at the exit becomes

$$T_s = T_e + \frac{\dot{q}_s}{h}$$

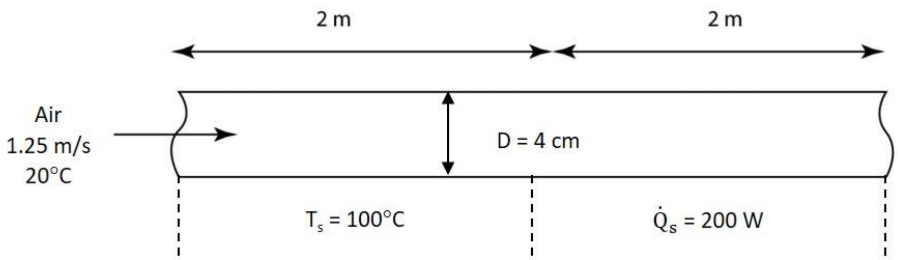
$$T_s = 21℃ + \frac{521.5\ \frac{\text{W}}{\text{m}^2}}{595\ \frac{\text{W}}{\text{m}^2\text{·K}}} = \mathbf{21.9}℃$$

Discussion Notice that the convective coefficient is very high even though the speed of the fluid is slow. This is because water is an excellent fluid for convective heat transfer. As a consequence of the large convection coefficient, the wall temperature is very close to the water temperature. Also, notice how small the temperature increase is in the water. This is primarily due to the very high thermal inertia of the water (mass flowrate and specific heat).

8-81 Air is being heated by constant wall temperature over the first 2 m of the pipe and then the heating takes place by constant heat flux. The air temperature at the 2 m length, the air temperature at the exit, the total heat transfer to the air, and the wall temperature at the exit of the tube are to be determined.

Assumptions **1** The flow issteady. **2** The flow is fully developed. **3** The air in first half of the pipe is being heat at constant wall temperature. **4** The air in second half of the pipe is being heated by constant heat flux. **5** The pipe wall is smooth.

Properties The properties of air at80°C are (Table A-15): ρ = 0.9994 kg/m^3, k = 0.02953 W/m·K, ν= 2.097 × 10^{-5} m^2/s, c_p = 1008 J/kg·K, and Pr=0.7154



Analysis (a) First the cross sectional and surface areas will be determined.

$$A_c = \frac{\pi D^2}{4} = \frac{\pi(0.04\text{ m})^2}{4} = 0.00126\text{ m}^2$$

$$A_s = \pi DL = \pi(0.04\text{ m})(2\text{ m}) = 0.251\text{ m}^2\text{ (for half of entire pipe)}$$

Next, the mass flow rate and Reynolds number will be determined

$$\dot{m} = \rho A_c V_{avg} = (0.9994\ \frac{\text{kg}}{\text{m}^3})(0.00126\text{ m}^2)(1.25\ \frac{\text{m}}{\text{s}}) = 0.00157\text{ kg/s}$$

$$\text{Re} = \frac{V_{avg}D}{\nu} = \frac{(1.25\ \frac{\text{m}}{\text{s}})(0.04\text{ m})}{2.097\times10^{-5}\ \frac{\text{m}^2}{\text{s}}} = 2384$$

which is between 2300 and 4000. Therefore, the flow is in the transitional region. With the Reynolds number known, it is possible to obtain the friction factor and the Nusselt number. The friction factor for the transitional flow with Re = 2384 is found from

$$f = 3.03\times10^{-12}\text{Re}^3 - 3.67\times10^{-8}\text{Re}^2 + 1.46\times10^{-4}\text{Re} - 0.151 = 0.0295$$

and the Nusslet number with f = 0.0295, Re = 2384, and Pr = 0.7154 is found from

$$\text{Nu} = \frac{\frac{f}{8}(\text{Re}-1000)\text{Pr}}{1+12.7\sqrt{\frac{f}{8}}(\text{Pr}^{2/3}-1)} = 4.32$$

and from the Nusselt number, the convective heat transfer coefficient can be found.

$$h = \frac{Nu\,k}{D} = \frac{(4.32)(0.02953\ \frac{\text{W}}{\text{m·K}})}{0.04\text{ m}} = 3.19\ \frac{\text{W}}{\text{m}^2\text{·K}}$$

Next, we determine the air temperature at the 2 m length for the constant wall temperature case from

$$T\ (2\text{ m}) = T_s - (T_s - T_i)\exp\left(-hA_s/\dot{m}c_p\right)$$

$$T(2\text{ m}) = 100℃ - (100℃ - 20℃)\exp\left[-\frac{\left(3.19\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}\right)(0.251\text{ m}^2)}{\left(0.00157\frac{\text{kg}}{\text{s}}\right)\left(1008\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)}\right] = \mathbf{51.8}℃$$

(*b*) The air temperature at the exit (4 m location) can be determined from

$$\dot{Q}_s = \dot{m}c_p[T(4\text{ m}) - T(2\text{ m})]$$

$$T(4\text{ m}) = T(2\text{ m}) + \frac{\dot{Q}_s}{\dot{m}c_p} = 51.8℃ + \frac{200\text{ W}}{\left(0.00157\ \frac{\text{kg}}{\text{s}}\right)\left(1008\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)} = \mathbf{178.2}℃$$

(*c*) The total rate of heat transfer over the entire pipe can be determined from

$$\dot{Q}_{total} = \dot{m}c_p(T(4\text{ m}) - T_i) = (0.00157\ \frac{kg}{s})(1008\ \frac{\text{J}}{\text{kg}\cdot\text{K}})(178.2 - 20)℃ = \mathbf{250.4\ W}$$

(*d*) The temperature of the wall at the exit of the pipe (4 m location) can be determined from

$$T_s = T(4\text{ m}) + \frac{\dot{q}_s}{h}$$

$$T_s = 178.2℃ + \frac{796.8\text{ W/m}^2}{3.19\ \frac{\text{W}}{\text{m}^2\cdot\text{K}}h} = \mathbf{428}℃$$

where

$$\dot{q}_s = \dot{Q}_s/A_s = (200\text{ W})/\ (0.251\text{ m}^2) = 796.8\text{ W/m}^2$$

Discussion Splitting a problem like this is common when the boundary conditions change. These two parts of the pipe can be treated separately and the outflow conditions from the first half become the inlet conditions for the second half.

8-82E Air is to be heated in a constant surface temperature duct. The temperature of the air as it leaves the duct, the rate of heat transfer from the duct to the air, and the heat flux at the duct exit are to be determined.

Assumptions **1** The flow issteady. **2** The flow is fully developed. **3** The duct is isothermal and smooth. **4** Treat the duct as a parallel plate channel.

Properties The properties of air at 75°F are (Table A-15E): ρ = 0.0742 lbm/ft^3, k = 0.01469 Btu/h·ft·R, ν = 1.67 × 10^{-4} ft^2/s, c_p = 0.2404 Btu/lbm·R, and Pr=0.73.

Analysis (*a*)First the cross sectional area, surface area, and mass flow rate are found from

$$A_c = \text{width} \times \text{height} = (1\text{ ft})(0.083\text{ ft}) = 0.083\text{ ft}^2$$

$$A_s = 2(\text{width} + \text{height})\times \text{length} = 2(1\text{ ft} + 0.083\text{ ft})(40\text{ ft}) = 86.7\text{ ft}^2$$

$$\dot{m} = \rho A_c V_{avg} = \left(0.0742\frac{\text{lbm}}{\text{ft}^3}\right)(0.083\text{ ft}^2)\left(3\ \frac{\text{ft}}{\text{s}}\right) = 0.0185\frac{\text{lbm}}{\text{s}} = 66.6\ \frac{\text{lbm}}{\text{h}}$$

The hydraulic diameter is found from

$$D_h = \frac{4A_c}{p} = \frac{4\ (0.083\text{ ft}^2)}{2(1 + 0.083)\text{ft}} = 0.153\text{ ft}$$

Next, The Reynolds number for the flow will be determined from

$$\text{Re} = \frac{V_{avg}D_h}{\nu} = \frac{(3\ \frac{\text{ft}}{\text{s}})(0.153\text{ ft})}{1.67\times 10^{-4}\frac{\text{ft}^2}{\text{s}}} = 2749$$

which is between 2300 and 4000. Therefore, the flow is in the transitional region. With the Reynolds number known, it is possible to obtain the friction factor and the Nusselt number. The friction factor for the transitional flow in a smooth parallel plate channel with Re = 2749 is found from

$$f = -6.38\times 10^{-13}\text{Re}^3 + 1.17\times 10^{-8}\text{Re}^2 - 6.69\times 10^{-5}\text{Re} + 0.147 = 0.038$$

and the Nusslet number with f = 0.038, Re = 2749, and Pr = 0.73 is found from

$$\text{Nu} = \frac{\frac{f}{8}(\text{Re} - 1000)\text{Pr}}{1 + 12.7\sqrt{\frac{f}{8}}(\text{Pr}^{2/3} - 1)} = 7.3$$

and from the Nusselt number, the convective heat transfer coefficient can be found from

$$\text{h} = \frac{\text{Nu k}}{D_h} = \frac{7.3\ (0.01469\text{ Btu/h·ft·R})}{0.153\text{ ft}} = 0.70\ \frac{\text{Btu}}{\text{h·ft}^2\text{·R}}$$

Next, we determine the exit temperature of air for the constant wall temperature case from

$$T_e = T_s - (T_s - T_i)\exp\left(-hA_s/\dot{m}c_p\right)$$

$$\text{T}_\text{e} = 100°\text{F} - (100°\text{F} - 50°\text{F})\exp\left[-\frac{(0.70\ \frac{\text{Btu}}{\text{h·ft}^2\text{·R}})(86.7\text{ ft}^2)}{\left(66.6\ \frac{\text{lbm}}{\text{h}}\right)\left(0.2404\ \frac{\text{Btu}}{\text{lbm·R}}\right)}\right] = \mathbf{98.9}°\text{F}$$

(*b*) The rate of heat transfer from the duct to the air is determined from

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (66.6\ \frac{\text{lbm}}{\text{h}})(0.2404\ \frac{\text{Btu}}{\text{lbm·R}})(98.9 - 50)°\text{F} = \mathbf{782.9\ Btu/h}$$

(*c*) The heat flux at the duct exit is determined from

$$\dot{q} = h(T_s - T_e) = \left(0.70\ \frac{\text{Btu}}{\text{h·ft}^2\text{·R}}\right)(100 - 98.9)°\text{F} = \mathbf{0.77}\ \frac{\mathbf{Btu}}{\mathbf{h·ft^2}}$$

Discussion For the slow moving gases we obtain low values for the convection coefficient. We also see that the air temperature changes significantly in the duct because of the thermal inertia of the fluid.

The bulk mean temperature is $T_b = (T_i + T_e)/2$ = (50 + 98.9)/2 = 74.5°F. This is very close to the assumed temperature of 75°F for air properties. Therefore, there is no need to repeat calculations.

8-83 The convection heat transfer coefficients for the flow of air and water are to be determined under similar conditions.

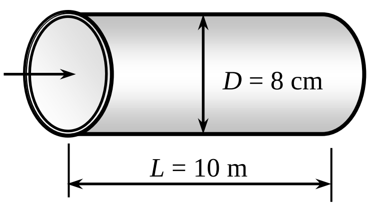
Assumptions **1** Steady flow conditions exist. **2** The surface heat flux is uniform. **3** The inner surfaces of the tube are smooth.

Properties The properties of air at 25°C are (Table A-15)

$$k = 0.02551 \text{ W/m.°C}$$
$$\nu = 1.562\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7296$$

The properties of water at 25°C are (Table A-9)

$$\rho = 997 \text{ kg/m}^3$$
$$k = 0.607 \text{ W/m.°C}$$
$$\nu = \mu/\rho = 0.891\times10^{-3}/997 = 8.937\times10^{-7} \text{ m}^2\text{/s}$$
$$\text{Pr} = 6.14$$



Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(5\text{ m/s})(0.08\text{m})}{1.562\times10^{-5} \text{ m}^2\text{/s}} = 25{,}608$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.08\text{ m}) = 0.8\text{ m}$$

which is much shorter than the total length of the tube. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD}{k} = 0.023\text{Re}^{0.8}\text{Pr}^{0.4} = 0.023(25{,}608)^{0.8}(0.7296)^{0.4} = 68.18$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02551\text{ W/m.°C}}{0.08\text{ m}}(68.18) = \mathbf{21.7\ W/m^2.°C}$$

Repeating calculations for water:

$$\text{Re} = \frac{VD}{\nu} = \frac{(5\text{ m/s})(0.08\text{m})}{8.937\times10^{-7} \text{ m}^2\text{/s}} = 447{,}577$$

$$Nu = \frac{hD}{k} = 0.023\text{Re}^{0.8}\text{Pr}^{0.4} = 0.023(447{,}577)^{0.8}(6.14)^{0.4} = 1576$$

$$h = \frac{k}{D}Nu = \frac{0.607\text{ W/m.°C}}{0.08\text{ m}}(1576) = \mathbf{11{,}960\ W/m^2.°C}$$

Discussion The heat transfer coefficient for water is about 550 times that of air.

8-84 Liquid water entering at 65°C is heated in a circular tube with a constant surface temperature. The mass flow rate of the flow is 0.12 kg/s. The inner surface temperature of the tube is to be determined and whether it exceeds the maximum temperature of 79°C recommended by the ASME Code for Process Piping for PVDC lining.

Assumptions **1** The flow is steady and incompressible. **2** Uniform surface temperature. **3** Inner surface of the tube is smooth.

Properties The properties of water at 70°C are (Table A–9) c_p = 4190 J/kg·K, k = 0.663 W/m·K, $\mu = 0.404 \times 10^{-3}$ kg/m·s, and Pr = 2.55.

Analysis The Reynolds number for the flow in circular tube is

$$\mathrm{Re} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.12\ \mathrm{kg/s})}{\pi(0.025\ \mathrm{m})(0.000404\ \mathrm{kg/m\cdot s})} = 15128$$

So, the flow is turbulent and the entry length is this case are roughly

$$L_h \approx L_t \approx 10\,D = 10(0.025\ \mathrm{m}) = 0.25\ \mathrm{m}$$

which is much shorter than the total length of the tube of 3 m. The friction factor for a tube with smooth surface is

$$f = (0.790 \ln \mathrm{Re} - 1.64)^{-2} = (0.790 \ln(15128) - 1.64)^{-2} = 0.02812$$

For turbulent flow, the Nusselt number is determined using the Gnielinski correlation:

$$\mathrm{Nu} = \frac{hD}{k} = \frac{(f/8)(\mathrm{Re} - 1000)\mathrm{Pr}}{1 + 12.7(f/8)^{0.5}(\mathrm{Pr}^{2/3} - 1)} = \frac{(0.02812/8)(15128 - 1000)(2.55)}{1 + 12.7(0.02812/8)^{0.5}(2.55^{2/3} - 1)} = 76.63$$

$$h = \mathrm{Nu}\frac{k}{D} = (76.63)\frac{0.663\ \mathrm{W/m\cdot K}}{0.025\ \mathrm{m}} = 2032.2\ \mathrm{W/m^2\cdot K}$$

The heat transfer rate to the water is

$$\dot{Q} = \dot{m}c_p(T_e - T_i)$$

which can be used to solve for the exit temperature of water:

$$T_e = \frac{\dot{Q}}{\dot{m}c_p} + T_i = \frac{5500\ \mathrm{W}}{(0.12\ \mathrm{kg/s})(4190\ \mathrm{J/kg\cdot K})} + 65℃ = 75.94℃$$

To solve for the surface temperature of the tube, we use

$$T_e = T_s - (T_s - T_i)\exp\left(-hA_s/\dot{m}c_p\right) = T_s - (T_s - T_i)\exp\left(-h\pi DL/\dot{m}c_p\right)$$

Solving for T_s, yields

$$75.94 = T_s - (T_s - 65)\exp[-(2032.2\ \mathrm{W/m^2\cdot K})\pi(0.025\ \mathrm{m})(3\ \mathrm{m})/(0.12\ \mathrm{kg/s})(4190\ \ \mathrm{J/kg\cdot K})]$$

$$T_s = \mathbf{82.8}℃ > 79℃$$

Discussion The surface temperature of the tube exceeds the maximum temperature (79°C) recommended for the PVCD lining (ASME Code for Process Piping, ASME B31.3-2014, Table A323.4.3). Therefore, the PVCD lining is not suitable to be used as the tube inner surface lining.

With the exit temperature at 76°C and the inlet temperature at 65°C, the bulk mean fluid temperature becomes 70.5°C. Thus, 70°C is a reasonable temperature for evaluating the properties of water.

8-85 Saturated liquid propane entering at −50°C is heated in a circular tube with a constant surface temperature. The mass flow rate of the flow is 42 g/s. The inner tube surface has a relative roughness of 0.05. The surface temperature of the tube is to be determined and whether the tube is compliant with the ASME Code for Process Piping.

Assumptions **1** The flow is steady and incompressible. **2** Uniform surface temperature.

Properties The properties of saturated liquid propane at −40°C are (Table A–12) c_p = 2258 J/kg·K, k = 0.1281 W/m·K, μ = 1.926×10^{-4} kg/m·s, and Pr = 3.395.

Analysis The Reynolds number for the flow in circular tube is

$$\text{Re} = \frac{4\dot{m}}{\pi D\mu} = \frac{4(0.042\ \text{kg/s})}{\pi(0.025\ \text{m})(1.926 \times 10^{-4}\text{kg/m}\cdot\text{s})} = 11106$$

So, the flow is turbulent. and the entry length is this case are roughly

$$L_h \approx L_t \approx 10\,D = 10(0.025\ \text{m}) = 0.25\ \text{m}$$

which is much shorter than the total length of the tube of 3 m. The friction factor corresponding to the relative roughness of ε/D = 0.05 can be determined using the Colebrook equation:

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad \Rightarrow \quad \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.05}{3.7} + \frac{2.51}{11106\sqrt{f}}\right)$$

Using an equation solver or an iterative method, the friction factor is determined as

$$f = 0.07358$$

For turbulent flow, the Nusselt number is determined using the Gnielinski correlation:

$$\text{Nu} = \frac{hD}{k} = \frac{(f/8)(\text{Re}-1000)\text{Pr}}{1+12.7(f/8)^{0.5}(\text{Pr}^{2/3}-1)} = \frac{(0.07358/8)(11106-1000)(3.395)}{1+12.7(0.07358/8)^{0.5}(3.395^{2/3}-1)} = 124.57$$

$$h = \text{Nu}\frac{k}{D} = (124.6)\frac{0.1281\ \text{W/m}\cdot\text{K}}{0.025\ \text{m}} = 638.3\ \text{W/m}^2\cdot\text{K}$$

The heat transfer rate to the liquid propane is

$$\dot{Q} = \dot{m}c_p(T_e - T_i)$$

which can be used to solve for the exit temperature of the liquid propane:

$$T_e = \frac{\dot{Q}}{\dot{m}c_p} + T_i = \frac{1300\ W}{(0.042\ \text{kg/s})(2258\ \text{J/kg}\cdot\text{K})} - 50℃ = -36.29℃$$

To solve for the surface temperature of the tube, we use

$$T_e = T_s - (T_s - T_i)\exp\left(-hA_s/\dot{m}c_p\right) = T_s - (T_s - T_i)\exp\left(-h\pi DL/\dot{m}c_p\right)$$

Solving for T_s, yields

$$-36.29℃ = T_s - (T_s + 50)\exp[-(638.3\ \text{W/m}^2\cdot\text{K})\pi(0.025\ \text{m})(3\ \text{m})/(0.042\ \text{kg/s})(2258\ \text{J/kg}\cdot\text{K})]$$

$$T_s = \mathbf{-32.8}℃ < -30℃$$

Discussion The surface temperature (T_s = −32.8°C) of the tube is below the minimum temperature (−30°C) suitable for ASTM A268 TP443 stainless steel plate. Therefore, the stainless steel tube is not suitable for this operation as per the ASME Code for Process Piping.

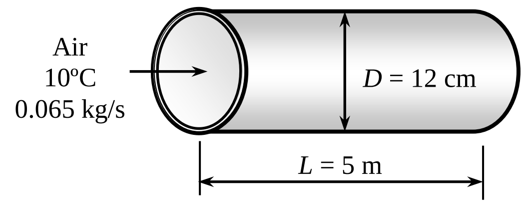
With the exit temperature at −36°C and the inlet temperature at −50°C, the bulk mean fluid temperature becomes −43°C. Thus, −40°C is a reasonable temperature for evaluating the properties of saturated liquid propane. If desired, the calculations can be repeated by using −43°C as the temperature to evaluate the saturated liquid propane properties.

8-86 Air flows in a pipe whose inner surface is not smooth. The rate of heat transfer is to be determined using two different Nusselt number relations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The pressure of air is 1 atm.

Properties Assuming a bulk mean fluid temperature of 20°C based on the problem statement, the properties of air are (Table A-15)

$$\rho = 1.204 \text{ kg/m}^3$$
$$k = 0.02514 \text{ W/m.°C}$$
$$\nu = 1.516\times10^{-5} \text{ m}^2\text{/s}$$
$$c_p = 1007 \text{ J/kg.°C}$$
$$\text{Pr} = 0.7309$$



Analysis The mean velocity of air and the Reynolds number are

$$V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{0.065 \text{ kg/s}}{(1.204 \text{ kg/m}^3)\pi(0.12 \text{ m})^2/4} = 4.773 \text{ m/s}$$

$$\text{Re} = \frac{V_{\text{avg}} D}{\nu} = \frac{(4.773 \text{ m/s})(0.12 \text{ m})}{1.516\times10^{-5} \text{ m}^2\text{/s}} = 37{,}785$$

which is greater than 10,0000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.12 \text{ m}) = 1.2 \text{ m}$$

which is much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire duct. The friction factor may be determined from Colebrook equation using EES to be

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \longrightarrow \frac{1}{\sqrt{f}} = -2\log\left(\frac{0.00022/0.12}{3.7} + \frac{2.51}{37{,}785\sqrt{f}}\right) \longrightarrow f = 0.02695$$

The Nusselt number from Eq. 8-66 is

$$Nu = 0.125 f \text{ Re Pr}^{1/3} = 0.125(0.02695)(37{,}785)(0.7309)^{1/3} = 114.7$$

Heat transfer coefficient is

$$h = \frac{k}{D} Nu = \frac{0.02514 \text{ W/m.°C}}{0.12 \text{ m}}(114.7) = 24.02 \text{ W/m}^2\text{.°C}$$

Next we determine the exit temperature of air

$$A = \pi DL = \pi(0.12 \text{ m})(5 \text{ m}) = 1.885 \text{ m}^2$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 50 - (50-10)e^{-\frac{(24.02)(1.885)}{(0.065)(1007)}} = 30.0°\text{C}$$

This result verifies our assumption of bulk mean fluid temperature that we used for property evaluation. Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.065 \text{ kg/s})(1007 \text{ J/kg.°C})(30.0-10)°\text{C} = \mathbf{1307\ W}$$

Repeating the calculations using the Nusselt number from Eq. 8-71:

$$Nu = \frac{(f/8)(\text{Re}-1000)\text{ Pr}}{1+12.7(f/8)^{0.5}(\text{Pr}^{2/3}-1)} = \frac{(0.02695/8)(37{,}785-1000)(0.7309)}{1+12.7(0.02695/8)^{0.5}(0.7309^{2/3}-1)} = 105.2$$

$$h = \frac{k}{D} Nu = \frac{0.02514 \text{ W/m.°C}}{0.12 \text{ m}}(105.2) = 22.04 \text{ W/m}^2\text{.°C}$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 50 - (50-10)e^{-\frac{(22.04)(1.885)}{(0.065)(1007)}} = 28.8°\text{C}$$

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.065 \text{ kg/s})(1007 \text{ J/kg.°C})(28.8-10)°\text{C} = \mathbf{1230\ W}$$

The result by Eq. 8-66 is about 6 percent greater than that by Eq. 8-71.

Discussion The average temperature of air is 20°C in the first part (same as the assumed value) and 19.4°C in the second part (very close to the assumed value). Therefore, there is no need to repeat calculations.

8-87 C&S Liquid water entering at 10°C is heated in a circular tube with a constant surface temperature. The pumping power to overcome the turbulent flow pressure loss is 100 W. The surface temperature of the tube is to be determined and whether an EPDM o-ring attached to the tube surface is suitable. The maximum temperature permitted for the o-ring is 150°C.

Assumptions **1** The flow is steady and incompressible. **2** Uniform surface temperature. **3** The flow is turbulent. **4** Inner surface of the tube is smooth. **5** The convection heat transfer coefficient is constant. **6** Thin-walled tube where thermal resistant by conduction is negligible.

Properties The properties of water at 10°C are (Table A–9) c_p = 4194 J/kg·K, $\mu = 1.307 \times 10^{-3}$ kg/m·s, and ρ = 999.7 kg/m^3.

Analysis The following equations relate the pressure loss and the pumping power for internal flow with the Reynolds number and the friction factor for turbulent flow in a smooth tube:

$$\mathrm{Re} = \frac{\rho V_{\mathrm{avg}} D}{\mu}$$

$$f = (0.790 \ln \mathrm{Re} - 1.64)^{-2}$$

$$\Delta P_L = f \frac{L}{D} \frac{\rho V_{\mathrm{avg}}^2}{2} = (0.790 \ln \mathrm{Re} - 1.64)^{-2} \frac{L}{D^3} \frac{\mu^2 \mathrm{Re}^2}{2\rho}$$

$$\dot{W}_{\mathrm{pump},L} = V_{\mathrm{avg}}\, A\, \Delta P_L = (0.790 \ln \mathrm{Re} - 1.64)^{-2} \frac{\pi}{4} \frac{L}{D^2} \frac{\mu^3 \mathrm{Re}^3}{2\rho^2}$$

Solving for the Reynolds number Re by trail-and-error or a numerical solver,

$$100 = (0.790 \ln \mathrm{Re} - 1.64)^{-2} \frac{\pi}{4} \frac{3}{(0.0125\ \mathrm{m})^2} \frac{(0.001307\ \mathrm{kg/m \cdot s})^3 \mathrm{Re}^3}{2(999.7\ \mathrm{kg/m^3})^2} \qquad \Longrightarrow \qquad \mathrm{Re} = 67140$$

(Note: Another solution that exists for the above equation is Re = 7.972; but at this Reynolds number the flow is not turbulent).

The average velocity and the mass flow rate are

$$V_{\mathrm{avg}} = \frac{\mu\, \mathrm{Re}}{\rho\, D} = \frac{(0.001307\ \mathrm{kg/m \cdot s})(67140)}{(999.7\ \mathrm{kg/m^3})(0.0125\ \mathrm{m})} = 7.0223\ \mathrm{m/s}$$

$$\dot{m} = \rho V_{\mathrm{avg}} A = \rho V_{\mathrm{avg}} \frac{\pi D^2}{4} = (999.7\ \mathrm{kg/m^3})(7.0223\ \mathrm{m/s}) \frac{\pi (0.0125\ \mathrm{m})^2}{4} = 0.86151\ \mathrm{kg/s}$$

The heat transfer rate to the water is

$$\dot{Q} = \dot{m} c_p (T_e - T_i)$$

which can be used to solve for the exit temperature of water:

$$T_e = \frac{\dot{Q}}{\dot{m} c_p} + T_i = \frac{3600\ \mathrm{W}}{(0.86151\ \mathrm{kg/s})(4194\ \mathrm{J/kg \cdot K})} + 10℃ = 11℃$$

To solve for the surface temperature of the tube, we use

$$T_e = T_s - (T_s - T_i) \exp\left(-hA_s/\dot{m} c_p\right)$$

where

$$A_s = \pi D L$$

Solving for T_s, yields

$$11℃ = T_s - (T_s - 10) \exp[-(120\ \mathrm{W/m^2 \cdot K}) \pi (0.0125\ \mathrm{m})(3\ \mathrm{m})/(0.86151\ \mathrm{kg/s})(4194\ \mathrm{J/kg \cdot K})]$$

$$T_s = \mathbf{266}℃ > 150℃$$

Discussion The surface temperature of the tube exceeds the maximum temperature (150°C) permitted for the EPDM o-ring (ASME Boiler and Pressure Vessel Code, BPVC.IV-2015, HG-360). Therefore, the EPDM o-ring is not suitable for this operation.

Note that even though the water at the tube exit is at 11°C, the tube surface temperature is significantly higher at 266°C. This is due to the low value of the convection heat transfer coefficient. By increasing the convection heat transfer coefficient to 220 W/m^2·K or higher, the tube surface temperature would reduce to below 150°C.

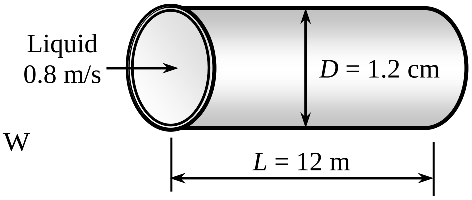
With the exit temperature at 11°C and the inlet temperature at 10°C, the bulk mean fluid temperature becomes 10.5°C. Thus, 10°C is an appropriate temperature to evaluate the properties of water.

8-88 A liquid is heated as it flows in a pipe that is wrapped by electric resistance heaters. The required surface heat flux, the surface temperature at the exit, and the pressure loss through the pipe and the minimum power required to overcome the resistance to flow are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface heat flux is uniform. **3** The inner surfaces of the tube are smooth. **4** Heat transfer to the surroundings is negligible.

Properties The properties of the fluid are given to be ρ = 1000 kg/m^3, c_p = 4000 J/kg·K, μ = 2x10^{-3} kg/s·m, k = 0.48 W/m·K, and Pr = 10

Analysis (*a*) The mass flow rate of the liquid is



$$\dot{m} = \rho AV = (1000\ \text{kg/m}^3)\left(\pi(0.012\ \text{m})^2/4\right)(0.8\ \text{m/s}) = 0.09048\ \text{kg/s}$$

The rate of heat transfer and the heat flux are

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.09048\ \text{kg/s})(4000\ \text{J/kg.°C})(75-25)\text{°C} = 18{,}100\ \text{W}$$

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{18{,}100\ \text{W}}{\pi(0.012\ \text{m})(12\ \text{m})} = \mathbf{40{,}000\ W/m^2}$$

(*b*) The Reynolds number is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(1000\ \text{kg/m}^3)(0.8\ \text{m/s})(0.012\ \text{m})}{0.002\ \text{kg/m}\cdot\text{s}} = 4800$$

which is greater than 2300 and smaller than 10,000. Therefore, we have transitional flow. However, we use turbulent flow relation. The entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.012\ \text{m}) = 0.12\ \text{m}$$

which is much shorter than the total length of the tube. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(4800)^{0.8}(10)^{0.4} = 50.90$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.48\ \text{W/m.°C}}{0.012\ \text{m}}(50.90) = 2036\ \text{W/m}^2\text{.°C}$$

The surface temperature at the exit is

$$\dot{q} = h(T_s - T_e) \longrightarrow 40{,}000\ \text{W} = (2036\ \text{W/m}^2\cdot\text{°C})(T_s - 75)\text{°C} \longrightarrow T_s = \mathbf{94.6°C}$$

(*c*) From Moody chart (or Colebrook equation) :

Re = 4800, ε = 0.045 mm, ε/D = 0.045/12 = 0.00375 $\rightarrow f$ = 0.04175

Then the pressure drop and the minimum power required to overcome this pressure drop are determined to be

$$\Delta P = f\frac{\rho V^2}{2D}L = (0.04175)\frac{(1000\ \text{kg/m}^3)(0.8\ \text{m/s})^2}{2(0.012\ \text{m})}(12\ \text{m}) = \mathbf{13{,}360\ Pa}$$

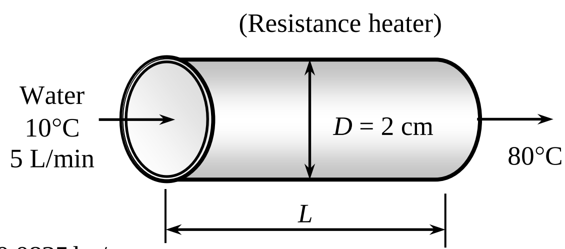
$$\dot{W} = \dot{V}\Delta P = \left(\pi(0.012\ \text{m})^2/4\right)(0.8\ \text{m/s})(13{,}360\ \text{Pa}) = \mathbf{1.21\ W}$$

8-89 Water is to be heated in a tube equipped with an electric resistance heater on its surface. The power rating of the heater and the inner surface temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface heat flux is uniform. **3** The inner surfaces of the tube are smooth.

Properties The properties of water at the average temperature of (80+10) / 2 = 45°C are (Table A-9)

$$\rho = 990.1\ \text{kg/m}^3$$
$$k = 0.637\ \text{W/m.°C}$$
$$\nu = \mu/\rho = 0.602\times10^{-6}\ \text{m}^2\text{/s}$$
$$c_p = 4180\ \text{J/kg.°C}$$
$$\text{Pr} = 3.91$$



Analysis The power rating of the resistance heater is

$$\dot{m} = \rho\dot{V} = (990.1\ \text{kg/m}^3)(0.005\ \text{m}^3\text{/min}) = 4.951\ \text{kg/min} = 0.0825\ \text{kg/s}$$

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.0825\ \text{kg/s})(4180\ \text{J/kg.°C})(80-10)\text{°C} = \mathbf{24{,}140\ W}$$

The velocity of water and the Reynolds number are

$$V_{\text{avg}} = \frac{\dot{V}}{A_c} = \frac{(5\times10^{-3}/60)\ \text{m}^3\text{/s}}{\pi(0.02\ \text{m})^2/4} = 0.2653\ \text{m/s}$$

$$\text{Re} = \frac{V_{\text{avg}}D_h}{\nu} = \frac{(0.2653\ \text{m/s})(0.02\ \text{m})}{0.602\times10^{-6}\ \text{m}^2\text{/s}} = 8813$$

which is less than 10,000 but much greater than 2300. We assume the flow to be turbulent. The entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.02\ \text{m}) = 0.20\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(8813)^{0.8}(3.91)^{0.4} = 56.85$$

Heat transfer coefficient is

$$h = \frac{k}{D_h}Nu = \frac{0.637\ \text{W/m.°C}}{0.02\ \text{m}}(56.85) = 1811\ \text{W/m}^2\text{.°C}$$

Then the inner surface temperature of the pipe at the exit becomes

$$\dot{Q} = hA_s(T_{s,e} - T_e)$$
$$24{,}140\ \text{W} = (1811\ \text{W/m}^2\text{.°C})[\pi(0.02\ \text{m})(13\ \text{m})](T_s - 80)\text{°C}$$
$$T_{s,e} = \mathbf{96.3°C}$$

8-90 The Nusselt numbers for various Reynolds numbers are to be determined using the Colburn, Petukhov, and Gnielinski equations.

Analysis The Colburn, Petukhov, and Gnielinski equations are

Colburn equation:

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}$$

Petukhov equation:

$$\mathrm{Nu} = \frac{(f/8)\,\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7(f/8)^{0.5}(\mathrm{Pr}^{2/3} - 1)} \qquad \text{where} \qquad f = (0.790\ln \mathrm{Re} - 1.64)^{-2}$$

Gnielinski equation:

$$\mathrm{Nu} = \frac{(f/8)(\mathrm{Re} - 1000)\,\mathrm{Pr}}{1 + 12.7(f/8)^{0.5}(\mathrm{Pr}^{2/3} - 1)} \qquad \text{where} \qquad f = (0.790\ln \mathrm{Re} - 1.64)^{-2}$$

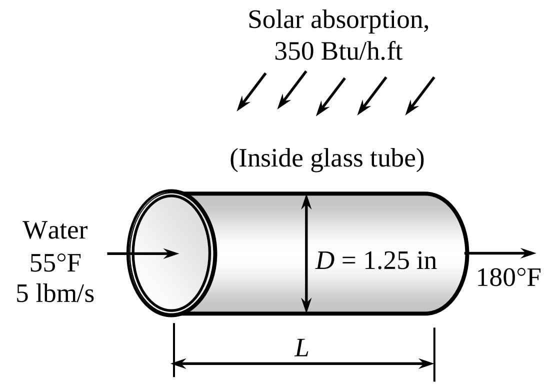
The calculated Nusselt numbers are listed in the following table:

Re	Nu		
	Colburn	Petukhov	Gnielinski
3500	30.1	37.3	27.2
10^4	69.7	86.4	79.5
5×10^5	1590	2360	2420

Discussion The Gnielinski equation is preferred in calculations since it has higher accuracy. In the transition region (Re = 3500), the Colburn equation performed better than the Petukhov equation. When compared with the Gnielinski equation at Re = 3500, the Colburn equation over-predicted the Nu by about 11%, while the Petukhov equation over-predicted the Nu by about 37%. When compared with the Gnielinski equation at Re = 10^4, the Colburn equation under-predicted the Nu by about 12%, while the Petukhov equation over-predicted the Nu by about 9%. When compared with the Gnielinski equation at Re = 5×10^5, the Colburn equation under-predicted the Nu by about 34%, while the Petukhov equation under-predicted the Nu by about 3%. The Petukhov equation compared better than the Colburn equation in the turbulent region (Re > 10^4).

8-91E Water is heated in a parabolic solar collector. The required length of parabolic collector and the surface temperature of the collector tube are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal resistance of the tube is negligible. **3** The inner surfaces of the tube are smooth.



Properties The properties of water at the average temperature of (55+180)/2 = 117.5°F are (Table A-9E)

$$\rho = 61.75 \text{ lbm/ft}^3$$
$$k = 0.370 \text{ Btu/ft.°F}$$
$$\nu = \mu/\rho = 3.84\times10^{-4}/61.75 = 0.6219\times10^{-5} \text{ ft}^2\text{/s}$$
$$c_p = 0.999 \text{Btu/lbm.°F}$$
$$\text{Pr} = 3.75$$

Analysis The total rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (5 \text{ lbm/s})(0.999 \text{ Btu/lbm.°F})(180-55)\text{°F} = 624.4 \text{ Btu/s} = 2.248\times10^6 \text{ Btu/h}$$

The length of the tube required is

$$L = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{2.248\times10^6 \text{ Btu/h}}{350 \text{ Btu/h.ft}} = \mathbf{6422 \text{ ft}}$$

The velocity of water and the Reynolds number are

$$V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{5 \text{ lbm/s}}{(61.75 \text{ lbm/m}^3)\pi\dfrac{(1.25/12 \text{ ft})^2}{4}} = 9.501 \text{ ft/s}$$

$$\text{Re} = \frac{V_{\text{avg}}D_h}{\nu} = \frac{(9.501 \text{ m/s})(1.25/12 \text{ ft})}{0.6219\times10^{-5} \text{ ft}^2\text{/s}} = 1.591\times10^5$$

which is greater than 10,000. Therefore, we can assume fully developed turbulent flow in the entire tube, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(1.591\times10^5)^{0.8}(3.75)^{0.4} = 565.9$$

The heat transfer coefficient is

$$h = \frac{k}{D_h}Nu = \frac{0.370 \text{ Btu/h.ft.°F}}{1.25/12 \text{ ft}}(565.9) = 2010 \text{ Btu/h.ft}^2\text{.°F}$$

The heat flux on the tube is

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{2.248\times10^6 \text{ Btu/h}}{\pi(1.25/12 \text{ ft})(6422 \text{ ft})} = 1070 \text{ Btu/h.ft}^2$$

Then the surface temperature of the tube at the exit becomes

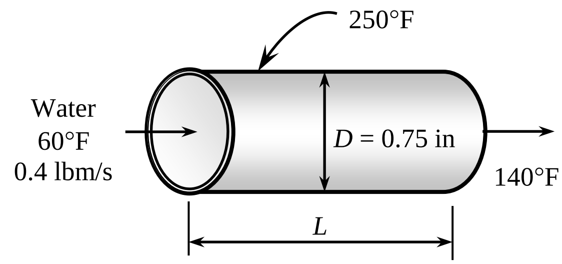
$$\dot{q} = h(T_s - T_e) \longrightarrow T_s = T_e + \frac{\dot{q}}{h} = 180\text{°F} + \frac{1070 \text{ Btu/h.ft}^2}{2010 \text{ Btu/h.ft}^2\text{.°F}} = \mathbf{180.5°F}$$

8-92E Water is heated by passing it through thin-walled copper tubes. The length of the copper tube that needs to be used is to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the tube are smooth. **3** The thermal resistance of the tube is negligible. **4** The temperature at the tube surface is constant.

Properties The properties of water at the bulk mean fluid temperature of $T_{b,avg} = (60+140)/2 = 100°\text{F}$ are (Table A-9E)

$$\rho = 62.0 \text{ lbm/ft}^3$$
$$k = 0.363 \text{ Btu/h.ft.°F}$$
$$\nu = \mu/\rho = 0.738\times10^{-5} \text{ ft}^2\text{/s}$$
$$c_p = 0.999 \text{ Btu/lbm.°F}$$
$$\text{Pr} = 4.54$$



Analysis (*a*) The mass flow rate and the Reynolds number are

$$\dot{m} = \rho A_c V_{\text{avg}} \rightarrow V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{0.4 \text{ lbm/s}}{(62 \text{ lbm/ft}^3)[\pi(0.75/12 \text{ ft})^2/4]} = 2.10 \text{ ft/s}$$

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(2.10 \text{ ft/s})(0.75/12 \text{ ft})}{0.738\times10^{-5} \text{ ft}^2\text{/s}} = 17{,}810$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.75 \text{ in}) = 7.5 \text{ in}$$

which is probably shorter than the total length of the pipe we will determine. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(17{,}810)^{0.8}(4.54)^{0.4} = 105.9$$

and $$h = \frac{k}{D_h} Nu = \frac{0.363 \text{ Btu/h.ft.°F}}{(0.75/12) \text{ ft}}(105.9) = 615 \text{ Btu/h.ft}^2\text{.°F}$$

The logarithmic mean temperature difference and then the rate of heat transfer per ft length of the tube are

$$\Delta T_{\text{lm}} = \frac{T_e - T_i}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} = \frac{140-60}{\ln\left(\dfrac{250-140}{250-60}\right)} = 146.4°\text{F}$$

$$\dot{Q} = hA_s\Delta T_{\text{lm}} = (615 \text{ Btu/h.ft}^2\text{.°F})[\pi(0.75/12 \text{ ft})(1 \text{ ft})](146.4°\text{F}) = 17{,}675 \text{ Btu/h}$$

The rate of heat transfer needed to raise the temperature of water from 60°F to 140°F is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.4\times3600 \text{ lbm/h})(0.999 \text{ Btu/lbm.°F})(140-60)°\text{F} = 115{,}085 \text{ Btu/h}$$

Then the length of the copper tube that needs to be used becomes

$$Length = \frac{115{,}085 \text{ Btu/h}}{17{,}675 \text{ Btu/h}} = \mathbf{6.51 \text{ ft}}$$

(*b*) The friction factor, the pressure drop, and then the pumping power required to overcome this pressure drop can be determined for the case of fully developed turbulent flow to be

$$f = 0.184\,\text{Re}^{-0.2} = 0.184(17{,}810)^{-0.2} = 0.02598$$

$$\Delta P = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.02598\frac{(6.51 \text{ ft})}{(0.75/12 \text{ ft})}\frac{(62 \text{ lbm/ft}^3)(2.10 \text{ ft/s})^2}{2}\left(\frac{1 \text{ lbf}}{32.174 \text{ lbm}\cdot\text{ft/s}^2}\right) = 11.50 \text{ lbf/ft}^2$$

$$\dot{W}_{pump} = \frac{\dot{m}\Delta P}{\rho} = \frac{(0.4 \text{ lbm/s})(11.50 \text{ lbf/ft}^2)}{62 \text{ lbm/ft}^3}\left(\frac{1 \text{ hp}}{550 \text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{0.00013 \text{ hp}}$$

8-93 Air (1 atm) enters into a 5-cm diameter circular tube at 20°C with an average velocity of 5 m/s. The length of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Constant tube surface temperature.

Properties The properties of air at $T_b = (T_i + T_e)/2 = 50°C$: $c_p = 1007$ J/kg·K, $k = 0.02735$ W/m·K, $\rho = 1.092$ kg/m^3, $\mu = 1.963 \times 10^{-5}$ kg/m·s, $\nu = 1.798 \times 10^{-5}$ m^2/s, and Pr = 0.7228 (Table A-15).

Analysis The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D}{\nu} = \frac{(5\ \text{m/s})(0.05\ \text{m})}{(1.798 \times 10^{-5}\ \text{m}^2/\text{s})} = 13904 > 10{,}000 \qquad \text{(turbulent flow)}$$

Since the flow is turbulent, we can use the Dittus-Boelter equation to calculate the Nusselt number:

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(13904)^{0.8}(0.7228)^{0.4} = 41.67 \qquad \rightarrow \qquad h = 22.8\ \text{W/m}^2 \cdot \text{K}$$

The length of the tube can be determined using

$$\ln\frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m}c_p} = -\frac{h\pi DL}{\dot{m}c_p} \qquad \text{where} \qquad \dot{m} = \rho V_{\text{avg}} \pi D^2 / 4 = 0.01072\ \text{kg/s}$$

Hence, length of the tube is

$$L = -\frac{\dot{m}c_p}{\pi Dh}\ln\frac{T_s - T_e}{T_s - T_i} = -\frac{(0.01072\ \text{kg/s})(1007\ \text{J/kg}\cdot\text{K})}{\pi(0.05\ \text{m})(22.8\ \text{W/m}^2\cdot\text{K})}\ln\frac{160-80}{160-20} = \mathbf{1.69\ m}$$

Discussion Since $L/D = 33.8 > 10$, the turbulent flow is fully developed.

8-94 PtD Hot water flows in a pipe with known inlet and outlet temperatures. The outer surface temperature of the pipe is to be determined whether it is safe from thermal burn hazards.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Convection effects on the outer pipe surface are negligible. **4** One-dimensional heat conduction through pipe wall. **5** The thermal conductivity of pipe wall is constant. **6** The outer pipe surface temperature is constant. **7** The inner surfaces of the tube are smooth.

Properties The properties of water at the bulk mean temperature of $T_b = (T_i + T_e)/2 = (100 + 60)/2 = 80°C$ are (Table A-9): $c_p = 4197$ J/kg·K, $k = 0.670$ W/m·K, $\mu = 0.355 \times 10^{-3}$ kg/m·s, and Pr = 2.22. The thermal conductivity of the pipe is given to be $k_{\text{pipe}} = 15$ W/m·K.

Analysis The Reynolds number of the water flow in the pipe is

$$\text{Re} = \frac{4\dot{m}}{\pi D_i \mu} = \frac{4(0.15\text{ kg/s})}{\pi(0.025\text{ m})(0.355\times10^{-3}\text{ kg/m}\cdot\text{s})} = 21{,}520 > 10{,}000$$

Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$\text{L}_\text{h} \approx \text{L}_\text{t} \approx 10\text{D} = 10(0.025\text{ m}) = 0.25\text{ m} >> 10\text{ m} \quad \text{(assume fully-developed turbulent flow)}$$

The Nusselt number can be determined from the Gnielinski correlation:

$$\text{Nu} = \frac{(f/8)(\text{Re}-1000)\,\text{Pr}}{1+12.7(f/8)^{0.5}(\text{Pr}^{2/3}-1)} = 98.13 \quad \rightarrow \quad h = \frac{k}{D_i}\text{Nu} = 2603\text{ W/m}^2\cdot\text{K}$$

where $f = (0.790\ln\text{Re} - 1.64)^{-2} = 0.02567$

The inner pipe surface temperature is

$$T_e = T_{s,i} - (T_{s,i} - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) \quad \rightarrow \quad T_{s,i} = 58.38°\text{C}$$

where $A_s = \pi(0.025\text{ m})(10\text{ m}) = 0.7854\text{ m}^2$

From Chapter 3, the thermal resistance for the pipe wall is

$$R_{\text{pipe}} = \frac{\ln(D_o/D_i)}{2\pi k_{\text{pipe}}L} = 0.0001934\text{ K/W} \qquad \text{(pipe wall resistance)}$$

The rate of heat transfer through the pipe wall is

$$\dot{Q} = \frac{T_{s,i} - T_{s,o}}{R_{\text{pipe}}} = \dot{m}c_p(T_i - T_e)$$

Thus, the outer pipe surface temperature is

$$T_{s,o} = T_{s,i} - R_{\text{pipe}}\dot{m}c_p(T_i - T_e) = \mathbf{53.5°C}$$

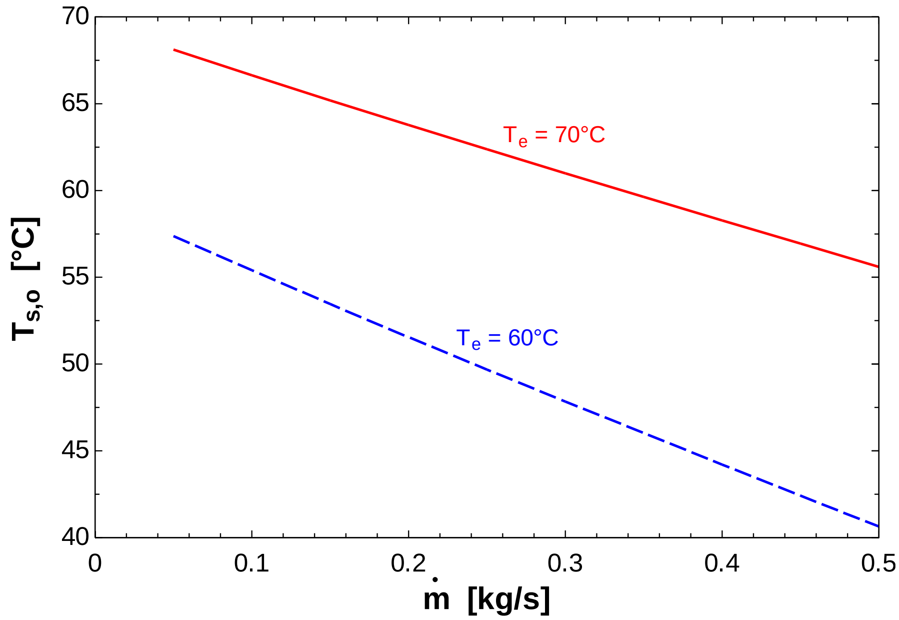
Discussion The pipe's outer surface temperature is 8.5°C higher than the safe temperature of 45°C. Thus, the risk of thermal burn upon accidental contact with skin tissue for individuals working in the vicinity of the pipe is present. Preventive measures, such as insulating the pipe's outer surface, should be taken to reduce the risk of thermal burn.

8-95 Reconsider Prob. 8-94. Hot water flows in a pipe with known inlet and outlet temperatures. The effect of the hot water mass flow rate on the outer surface temperature of the pipe is to be investigated for two different water outlet temperatures. The condition of the hot water mass flow rate to prevent thermal burn from the pipe's outer surface is to be determined.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=10 [m]
D_i=0.025 [m]
D_o=0.03 [m]
T_i=100 [C]
T_e=60 [C]     "Adjust T_e value for 60°C and 70°C"

"PROPERTIES"
"Water at T_b"
T_b=(T_i+T_e)/2     "T_b = 1/2*(T_i+T_e)"
c_p=cP(water, T=T_b, x=0)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(water, T=T_b, x=0)
rho=Density(water, T=T_b, x=0)
Pr=Prandtl(water, T=T_b, x=0)
mu=Viscosity(water, T=T_b, x=0)
"Pipe wall"
k_pipe=15 [W/m-K]   "pipe thermal conductivity"

"ANALYSIS"
A_c=pi#*D_i^2/4     "Cross-section area"
A_s=pi#*D_i*L     "Surface area"
"Flow inside tube"
Re=4*m_dot/(mu*pi#*D_i)
f=(0.790*ln(Re)-1.64)^(-2)     "Petukhov correlation"
Nusselt=((f/8)*(Re-1000)*Pr)/(1+12.7*(f/8)^0.5*(Pr^(2/3)-1))     "Gnielinski correlation"
h=k/D_i*Nusselt
Q_dot=m_dot*c_p*(T_i-T_e)
T_e=T_s_i-(T_s_i-T_i)*exp(-(h*A_s)/(m_dot*c_p))
"Pipe thermal resistance"
R_pipe=ln(D_o/D_i)/(2*pi#*k_pipe*L)   "pipe wall resistance"
Q_dot=(T_s_i-T_s_o)/(R_pipe)
```

$\dot{m}$ [kg/s]	(*a*) $T_{s,o}$ [°C]	(*b*) T_s [°C]
0.05	57.37	68.12
0.10	55.41	66.64
0.15	53.46	65.19
0.20	51.56	63.77
0.25	49.69	62.37
0.30	47.84	60.99
0.35	46.02	59.63
0.40	44.22	58.28
0.45	42.42	56.94
0.50	40.64	55.60



Discussion (*a*) When the hot water exits the pipe at 60°C, the pipe's outer surface temperature can be reduced to below 45°C by setting the mass flow rate for $\dot{m} > 0.38$ kg/s.

(*b*) When the hot water exits the pipe at 70°C, the pipe's outer surface temperature is above 45°C for $0.05 \le \dot{m} \le 0.5$ kg/s. Thus, preventive measures, such as insulating the pipe's outer surface, should be taken to reduce the risk of thermal burn.

8-96 A metal pipe is used for transporting hot saturated water vapor in an engine room. The needed insulation layer thickness to keep the outer surface temperature below 180°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Convection effects on the outer pipe surface are negligible. **4** One-dimensional heat conduction through pipe wall. **5** The thermal properties of pipe wall and insulation are constant. **6** Thermal resistance at the interface is negligible. **7** The surface temperatures are uniform. **8** The inner surfaces of the tube are smooth.

Properties The properties of sat. water vapor at $T_b = (T_i + T_e)/2 = (325 + 290)/2 = 307.5$°C are $c_p = 6554$ J/kg·K, $k = 0.06746$ W/m·K, $\mu = 2.006 \times 10^{-5}$ kg/m·s, and Pr = 1.949 (EES or Table A-9). The thermal conductivities of the pipe and the insulation are given to be $k_{\text{pipe}} = 15$ W/m·K and $k_{\text{ins}} = 0.95$ W/m·K, respectively.

Analysis The Reynolds number of the sat. water vapor flow in the pipe is

$$\text{Re} = \frac{4\dot{m}}{\pi D_i \mu} = 38{,}083 > 10{,}000 \qquad \text{(turbulent flow)}$$

Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.05\text{ m}) = 0.5\text{ m} \qquad \text{(fully-developed)}$$

The Nusselt number can be determined from the Gnielinski correlation:

$$\text{Nu} = \frac{(f/8)(\text{Re}-1000)\,\text{Pr}}{1+12.7(f/8)^{0.5}(\text{Pr}^{2/3}-1)} = 146.6 \quad \rightarrow \quad h = \frac{k}{D_i}\text{Nu} = 197.8\text{ W/m}^2\cdot\text{K}$$

where

$$f = (0.790\ln\text{Re} - 1.64)^{-2} = 0.02233$$

The inner pipe surface temperature is

$$T_e = T_{s,i} - (T_{s,i} - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) \quad \rightarrow \quad T_{s,i} = 280.9°\text{C}$$

where $A_s = \pi(0.05\text{ m})(10\text{ m}) = 1.571\text{ m}^2$

From Chapter 3, the thermal resistances for the pipe wall and the insulation are

$$R_{\text{pipe}} = \frac{\ln(D_{\text{interface}}/D_i)}{2\pi k_{\text{pipe}}L} \qquad \text{(pipe wall resistance)}$$

$$R_{\text{ins}} = \frac{\ln(D_o/D_{\text{interface}})}{2\pi k_{\text{ins}}L} \qquad \text{(insulation layer resistance)}$$

The total thermal resistance and the rate of heat transfer are

$$R_{\text{total}} = R_{\text{pipe}} + R_{\text{ins}} \quad \text{and} \quad \dot{Q} = \frac{T_{s,i} - T_{s,o}}{R_{\text{total}}} = \dot{m}c_p(T_i - T_e)$$

and the insulation thickness is

$$t_{\text{ins}} = \frac{D_o - D_{\text{interface}}}{2}$$

Solving for the insulation thickness yields $t_{\text{ins}} = 0.0412\text{ m} = \mathbf{4.12\text{ cm}}$

Solved by EES Software. Copy-and-paste the following lines on a blank EES screen to verify the solutions.

```
"GIVEN"
L=10 [m]
D_i=0.05 [m]
D_interface=0.06 [m]
T_s_o=180 [C]
T_i=325 [C]
T_e=290 [C]
m_dot=0.03 [kg/s]

"PROPERTIES"
"Sat. water vapor"
```

```
T_b=(T_i+T_e)/2     "T_b = 1/2*(T_i+T_e)"
c_p=cP(water, T=T_b, x=1)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(water, T=T_b, x=1)
rho=Density(water, T=T_b, x=1)
Pr=Prandtl(water, T=T_b, x=1)
mu=Viscosity(water, T=T_b, x=1)
"Pipe & insulation"
k_pipe=15 [W/m-K]   "pipe thermal conductivity"
k_ins=0.95 [W/m-K]   "insulation thermal conductivity"

"ANALYSIS"
A_c=pi#*D_i^2/4     "Cross-section area"
A_s=pi#*D_i*L     "Surface area"
"Flow inside tube"
Re=4*m_dot/(mu*pi#*D_i)
f=(0.790*ln(Re)-1.64)^(-2)     "Petukhov correlation"
Nusselt=((f/8)*(Re-1000)*Pr)/(1+12.7*(f/8)^0.5*(Pr^(2/3)-1))     "Gnielinski correlation"
h=k/D_i*Nusselt
Q_dot=m_dot*c_p*(T_i-T_e)
T_e=T_s_i-(T_s_i-T_i)*exp(-(h*A_s)/(m_dot*c_p))
"Pipe & insulation thermal resistances"
R_pipe=ln(D_interface/D_i)/(2*pi#*k_pipe*L)   "pipe wall resistance"
R_ins=ln(D_o/D_interface)/(2*pi#*k_ins*L)   "insulation resistance"
R_total=R_pipe+R_ins
"Solving for the insulation thickness"
Q_dot=(T_s_i-T_s_o)/(R_total)
t_ins=(D_o-D_interface)/2
```

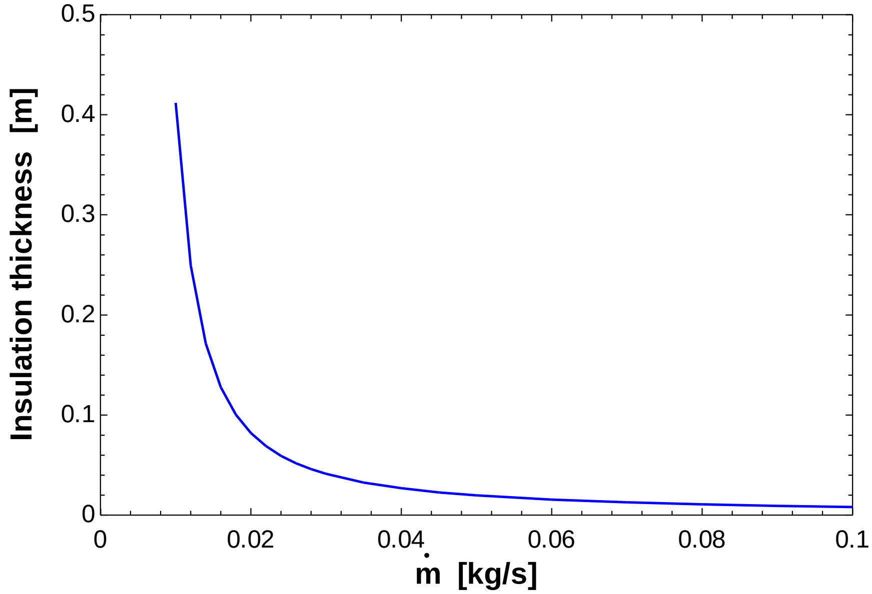
Discussion The Dittus-Boelter correlation can be used for this problem in place of the Gnielinski correlation.

8-97 Reconsider Prob. 8-96. A metal pipe is used for transporting hot saturated water vapor in an engine room. The effect of the satuarated water vapor mass flow rate on the needed insulation layer thickness to keep the outer surface temperature below 180°C is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=10 [m]
D_i=0.05 [m]
D_interface=0.06 [m]
T_s_o=180 [C]
T_i=325 [C]
T_e=290 [C]

"PROPERTIES"
"Sat. water vapor"
T_b=(T_i+T_e)/2     "T_b = 1/2*(T_i+T_e)"
c_p=cP(water, T=T_b, x=1)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(water, T=T_b, x=1)
rho=Density(water, T=T_b, x=1)
Pr=Prandtl(water, T=T_b, x=1)
mu=Viscosity(water, T=T_b, x=1)
"Pipe & insulation"
k_pipe=15 [W/m-K]   "pipe thermal conductivity"
k_ins=0.95 [W/m-K]   "insulation thermal conductivity"

"ANALYSIS"
A_c=pi#*D_i^2/4     "Cross-section area"
A_s=pi#*D_i*L     "Surface area"
"Flow inside tube"
Re=4*m_dot/(mu*pi#*D_i)
f=(0.790*ln(Re)-1.64)^(-2)     "Petukhov correlation"
Nusselt=((f/8)*(Re-1000)*Pr)/(1+12.7*(f/8)^0.5*(Pr^(2/3)-1))     "Gnielinski correlation"
h=k/D_i*Nusselt
Q_dot=m_dot*c_p*(T_i-T_e)
T_e=T_s_i-(T_s_i-T_i)*exp(-(h*A_s)/(m_dot*c_p))
"Pipe & insulation thermal resistances"
R_pipe=ln(D_interface/D_i)/(2*pi#*k_pipe*L)   "pipe wall resistance"
R_ins=ln(D_o/D_interface)/(2*pi#*k_ins*L)   "insulation resistance"
R_total=R_pipe+R_ins
"Solving for the insulation thickness"
Q_dot=(T_s_i-T_s_o)/(R_total)
t_ins=(D_o-D_interface)/2
```

$\dot{m}$ [kg/s]	t_{ins} [m]
0.01	0.4119
0.012	0.2493
0.014	0.1714
0.016	0.1277
0.018	0.1004
0.020	0.0820
0.022	0.06894
0.024	0.05924
0.026	0.05179
0.028	0.04591
0.030	0.04116
0.035	0.03256
0.040	0.02682
0.045	0.02274
0.050	0.01969
0.060	0.01546
0.070	0.01267
0.080	0.01071
0.090	0.009245
0.10	0.008118



Discussion The insulation layer thickness that is suitable for the flow rate range from 0.03 to 0.1 kg/s is 0.0412 m. When the flow rate decreases, the needed insulation layer thickness increases.

8-98 Liquid NH_3 flows in a pipe, which is insulated. The insulation thickness on the pipe that is necessary to keep the liquid NH_3 exit temperature at −20°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Radiation effects are negligible. **3** Convection effects on the outer pipe surface are negligible. **4** One-dimensional heat conduction through pipe wall. **5** The thermal properties of pipe wall and insulation are constant. **6** Thermal resistance at the interface is negligible. **7** The surface temperatures are uniform. **8** The inner surfaces of the tube are smooth.

Properties The properties of liquid NH_3 at $T_b = (T_i + T_e)/2 = [(-30 + (-20)]/2 = -25$°C are $c_p = 4489$ J/kg·K, $k = 0.5968$ W/m·K, $\mu = 2.492 \times 10^{-4}$ kg/m·s, and Pr = 1.875 (EES or Table A-11). The thermal conductivities of the pipe and the insulation are given to be $k_{pipe} = 15$ W/m·K and $k_{ins} = 0.95$ W/m·K, respectively.

Analysis The Reynolds number of the sat. water vapor flow in the pipe is

$$\text{Re} = \frac{4\dot{m}}{\pi D_i \mu} = 15{,}328 > 10{,}000 \qquad \text{(turbulent flow)}$$

Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.05\text{ m}) = 0.5\text{ m} \qquad \text{(fully-developed)}$$

The Nusselt number can be determined from the Gnielinski correlation:

$$\text{Nu} = \frac{(f/8)(\text{Re}-1000)\,\text{Pr}}{1+12.7(f/8)^{0.5}(\text{Pr}^{2/3}-1)} = 67.64 \quad \rightarrow \quad h = \frac{k}{D_i}\text{Nu} = 807.4\text{ W/m}^2\cdot\text{K}$$

where

$$f = (0.790\ln\text{Re} - 1.64)^{-2} = 0.02802$$

The inner pipe surface temperature is

$$T_e = T_{s,i} - (T_{s,i} - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) \quad \rightarrow \quad T_{s,i} = -18.21\text{°C}$$

where $\quad A_s = \pi(0.05\text{ m})(10\text{ m}) = 1.571\text{ m}^2$

From Chapter 3, the thermal resistances for the pipe wall and the insulation are

$$R_{pipe} = \frac{\ln(D_{interface}/D_i)}{2\pi k_{pipe} L} \qquad \text{(pipe wall resistance)}$$

$$R_{ins} = \frac{\ln(D_o/D_{interface})}{2\pi k_{ins} L} \qquad \text{(insulation layer resistance)}$$

The total thermal resistance and the rate of heat transfer are

$$R_{total} = R_{pipe} + R_{ins} \qquad \text{and} \qquad \dot{Q} = \frac{T_{s,i} - T_{s,o}}{R_{total}} = \dot{m}c_p(T_i - T_e)$$

and the insulation thickness is

$$t_{ins} = \frac{D_o - D_{interface}}{2}$$

Solving for the insulation thickness yields $t_{ins} = 0.0116\text{ m} = \mathbf{1.16\text{ cm}}$

Solved by EES Software. Copy-and-paste the following lines on a blank EES screen to verify the solutions.

```
"GIVEN"
L=10 [m]
D_i=0.05 [m]
D_interface=0.06 [m]
T_s_o=20 [C]
T_i=-30 [C]
T_e=-20 [C]
m_dot=0.15 [kg/s]

"PROPERTIES"
"Liquid NH3"
```

```
T_b=(T_i+T_e)/2     "T_b = 1/2*(T_i+T_e)"
c_p=4489 [J/kg-K]
k=0.5968 [W/m-K]
rho=671.5 [kg/m^2]
Pr=1.875
mu=2.492e-4 [kg/m-s]
"Pipe & insulation"
k_pipe=15 [W/m-K]   "pipe thermal conductivity"
k_ins=0.95 [W/m-K]   "insulation thermal conductivity"

"ANALYSIS"
A_c=pi#*D_i^2/4     "Cross-section area"
A_s=pi#*D_i*L     "Surface area"
"Flow inside tube"
Re=4*m_dot/(mu*pi#*D_i)
f=(0.790*ln(Re)-1.64)^(-2)     "Petukhov correlation"
Nusselt=((f/8)*(Re-1000)*Pr)/(1+12.7*(f/8)^0.5*(Pr^(2/3)-1))     "Gnielinski correlation"
h=k/D_i*Nusselt
Q_dot=m_dot*c_p*(T_i-T_e)
T_e=T_s_i-(T_s_i-T_i)*exp(-(h*A_s)/(m_dot*c_p))
"Pipe & insulation thermal resistances"
R_pipe=ln(D_interface/D_i)/(2*pi#*k_pipe*L)   "pipe wall resistance"
R_ins=ln(D_o/D_interface)/(2*pi#*k_ins*L)   "insulation resistance"
R_total=R_pipe+R_ins
"Solving for the insulation thickness"
Q_dot=(T_s_i-T_s_o)/(R_total)
t_ins=(D_o-D_interface)/2
```

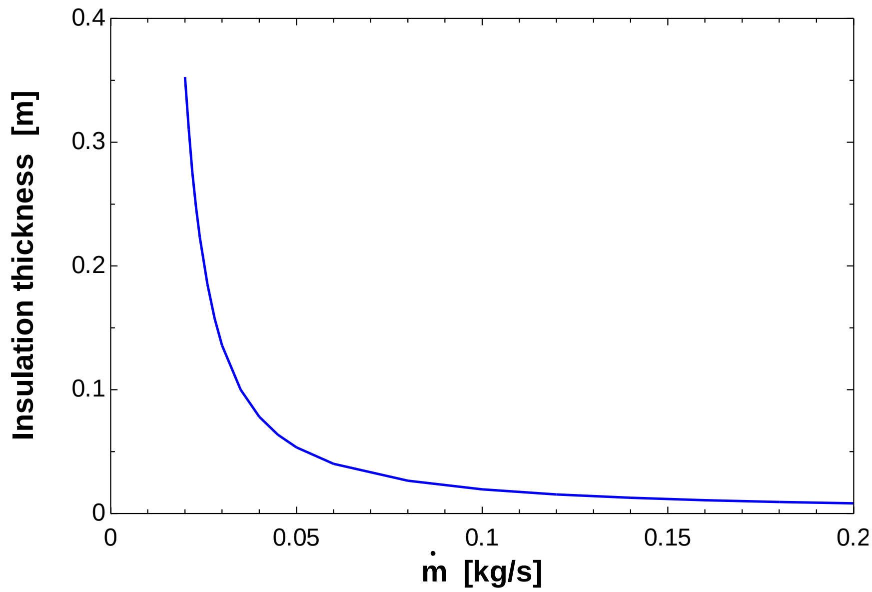
Discussion The Dittus-Boelter correlation can be used for this problem in place of the Gnielinski correlation.

8-99 Reconsider Prob. 8-98. Liquid NH_3 flows in a pipe, which is insulated. The effect of the NH_3 mass flow rate on the needed insulation layer thickness to keep the liquid NH_3 exit temperature at −20°C is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=10 [m]
D_i=0.05 [m]
D_interface=0.06 [m]
T_s_o=20 [C]
T_i=-30 [C]
T_e=-20 [C]

"PROPERTIES"
"Liquid NH3"
T_b=(T_i+T_e)/2     "T_b = 1/2*(T_i+T_e)"
c_p=cP(ammonia, T=T_b, x=0)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(ammonia, T=T_b, x=0)
rho=Density(ammonia, T=T_b, x=0)
Pr=Prandtl(ammonia, T=T_b, x=0)
mu=Viscosity(ammonia, T=T_b, x=0)
"Pipe & insulation"
k_pipe=15 [W/m-K]   "pipe thermal conductivity"
k_ins=0.95 [W/m-K]   "insulation thermal conductivity"

"ANALYSIS"
A_c=pi#*D_i^2/4     "Cross-section area"
A_s=pi#*D_i*L     "Surface area"
"Flow inside tube"
Re=4*m_dot/(mu*pi#*D_i)
f=(0.790*ln(Re)-1.64)^(-2)     "Petukhov correlation"
Nusselt=((f/8)*(Re-1000)*Pr)/(1+12.7*(f/8)^0.5*(Pr^(2/3)-1))     "Gnielinski correlation"
h=k/D_i*Nusselt
Q_dot=m_dot*c_p*(T_i-T_e)
T_e=T_s_i-(T_s_i-T_i)*exp(-(h*A_s)/(m_dot*c_p))
"Pipe & insulation thermal resistances"
R_pipe=ln(D_interface/D_i)/(2*pi#*k_pipe*L)   "pipe wall resistance"
R_ins=ln(D_o/D_interface)/(2*pi#*k_ins*L)   "insulation resistance"
R_total=R_pipe+R_ins
"Solving for the insulation thickness"
Q_dot=(T_s_i-T_s_o)/(R_total)
t_ins=(D_o-D_interface)/2
```

$\dot{m}$ [kg/s]	t_{ins} [m]
0.02	0.3528
0.021	0.3106
0.022	0.2760
0.023	0.2473
0.024	0.2233
0.026	0.1855
0.028	0.1574
0.030	0.1360
0.035	0.09998
0.040	0.07811
0.045	0.06363
0.050	0.05344
0.060	0.04017
0.080	0.02649
0.10	0.01960
0.12	0.01548
0.14	0.01275
0.16	0.01081
0.18	0.009365
0.20	0.008248



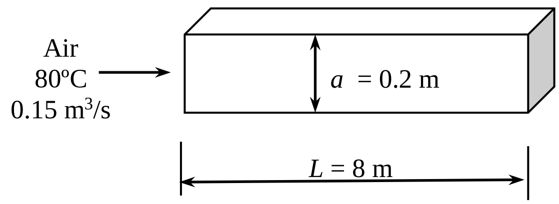
Discussion The insulation layer thickness that is suitable for the flow rate range from 0.04 to 0.2 kg/s is 0.0781 m. When the flow rate decreases, the needed insulation layer thickness increases.

8-100 Air flows in a square cross section pipe. The rate of heat loss and the pressure difference between the inlet and outlet sections of the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The pressure of air is 1 atm.

Properties Taking a bulk mean fluid temperature of 80°C based on the problem statement (this assumes that the air does not loose much heat to the attic), the properties of air are (Table A-15)

$$\rho = 0.9994\ \text{kg/m}^3$$
$$k = 0.02953\ \text{W/m.°C}$$
$$\nu = 2.097\times10^{-5}\ \text{m}^2\text{/s}$$
$$c_p = 1008\ \text{J/kg.°C}$$
$$\text{Pr} = 0.7154$$



Analysis The mean velocity of air, the hydraulic diameter, and the Reynolds number are

$$V = \frac{\dot{V}}{A} = \frac{0.15\ \text{m}^3\text{/s}}{(0.2\ \text{m})^2} = 3.75\ \text{m/s}$$

$$D_h = \frac{4A}{P} = \frac{4a^2}{4a} = a = 0.2\ \text{m}$$

$$\text{Re} = \frac{VD_h}{\nu} = \frac{(3.75\ \text{m/s})(0.2\ \text{m})}{2.097\times10^{-5}} = 35{,}765$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D_h = 10(0.2\ \text{m}) = 2\ \text{m}$$

which is much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(35{,}765)^{0.8}(0.7154)^{0.3} = 91.4$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02953\ \text{W/m.°C}}{0.2\ \text{m}}(91.4) = 13.5\ \text{W/m}^2\text{.°C}$$

Next we determine the exit temperature of air

$$A = 4aL = 4(0.2\ \text{m})(8\ \text{m}) = 6.4\ \text{m}^2$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 60 - (60-80)e^{-\frac{(13.5)(6.4)}{(0.9994)(0.15)(1008)}} = 71.3\text{°C}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.9994\ \text{kg/m}^3)(0.15\ \text{m}^3\text{/s})(1008\ \text{J/kg.°C})(80-71.3)\text{°C} = \mathbf{1315\ W}$$

From Moody chart:

Re = 35,765 and ε/D = 0.001 $\rightarrow f$ = 0.026

Then the pressure drop is determined to be

$$\Delta P = f\frac{\rho V^2}{2D}L = (0.026)\frac{(0.9994\ \text{kg/m}^3)(3.75\ \text{m/s})^2}{2(0.2\ \text{m})}(8\ \text{m}) = \mathbf{7.3\ Pa}$$

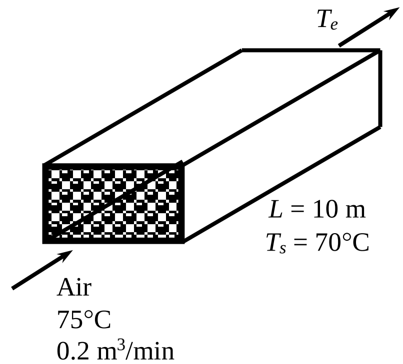
Discussion The average temperature of air is (80 + 71.3)/2 = 75.7°C, which is sufficiently close to the assumed value of 80°C. Therefore, there is no need to repeat calculations.

8-101 Flow of hot air through uninsulated square ducts of a heating system in the attic is considered. The exit temperature and the rate of heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 75°C since the mean temperature of air at the inlet will not drop significantly because the surfaces are at 70°C. The properties of air at 1 atm and this temperature are (Table A-15)

$$\rho = 1.014 \text{ kg/m}^3$$
$$k = 0.02917 \text{ W/m.°C}$$
$$\nu = 2.046 \times 10^{-5} \text{ m}^2\text{/s}$$
$$c_p = 1007.5 \text{ J/kg.°C}$$
$$\text{Pr} = 0.7166$$



Analysis The characteristic length that is the hydraulic diameter, the mean velocity of air, and the Reynolds number are

$$D_h = \frac{4A_c}{P} = \frac{4a^2}{4a} = a = 0.15 \text{ m}$$

$$V_{\text{avg}} = \frac{\dot{V}}{A_c} = \frac{0.20 \text{ m}^3\text{/s}}{(0.15 \text{ m})^2} = 8.889 \text{ m/s}$$

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(8.889 \text{ m/s})(0.15\text{m})}{2.046 \times 10^{-5} \text{ m}^2\text{/s}} = 65{,}168$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D_h = 10(0.15 \text{ m}) = 1.5 \text{ m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(65{,}168)^{0.8}(0.7166)^{0.3} = 147.8$$

Heat transfer coefficient is

$$h = \frac{k}{D_h} Nu = \frac{0.02917 \text{ W/m.°C}}{0.15 \text{ m}}(147.8) = 28.73 \text{ W/m}^2\text{.°C}$$

Next we determine the exit temperature of air,

$$A_s = 4aL = 4(0.15 \text{ m})(10 \text{ m}) = 6 \text{ m}^2$$
$$\dot{m} = \rho\dot{V} = (1.014 \text{ kg/m}^3)(0.20 \text{m}^3\text{/s}) = 0.2028 \text{ kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 70 - (70 - 75)e^{-\frac{(28.73)(6)}{(0.2028)(1007.5)}} = \mathbf{72.15°C}$$

Then the logarithmic mean temperature difference and the rate of heat loss from the air becomes

$$\Delta T_{\text{lm}} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{72.15 - 75}{\ln\left(\frac{70 - 72.15}{70 - 75}\right)} = 3.377°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\text{lm}} = (28.73 \text{ W/m}^2\text{.°C})(6 \text{ m}^2)(3.377°\text{C}) = \mathbf{582\ W}$$

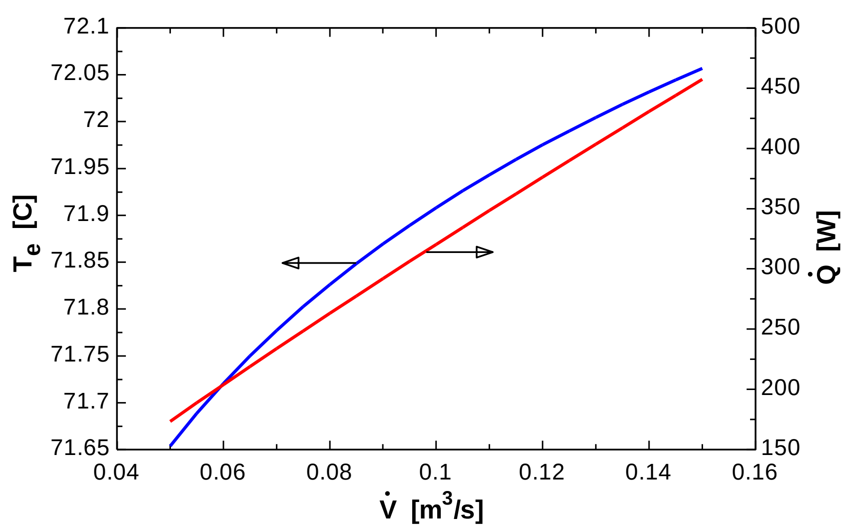
Note that the temperature of air drops by about 3°C as it flows in the duct as a result of heat loss.

8-102 Prob. 8-101 is reconsidered. The effect of the volume flow rate of air on the exit temperature of air and the rate of heat loss is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_i=75 [C]
L=10 [m]
side=0.15 [m]
V_dot=0.20 [m^3/s]
T_s=70 [C]

"PROPERTIES"
Fluid$='air'
C_p=CP(Fluid$, T=T_ave)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(Fluid$, T=T_ave)
Pr=Prandtl(Fluid$, T=T_ave)
rho=Density(Fluid$, T=T_ave, P=101.3)
mu=Viscosity(Fluid$, T=T_ave)
nu=mu/rho
T_ave=70 "T_ave = 1/2*(T_i+T_e)"

"ANALYSIS"
D_h=(4*A_c)/p
A_c=side^2
p=4*side
Vel=V_dot/A_c
Re=(Vel*D_h)/nu "The flow is turbulent"
L_t=10*D_h "The entry length is much shorter than the total length of the duct."
Nusselt=0.023*Re^0.8*Pr^0.3
h=k/D_h*Nusselt
A=4*side*L
m_dot=rho*V_dot
T_e=T_s-(T_s-T_i)*exp((-h*A)/(m_dot*C_p))
DELTAT_ln=(T_e-T_i)/ln((T_s-T_e)/(T_s-T_i))
Q_dot=h*A*DELTAT_ln
```

$\dot{V}$ [m^3/s]	T_e [C]	$\dot{Q}$ [W]
0.05	71.65	173.4
0.055	71.69	188.7
0.06	71.72	203.9
0.065	71.75	218.9
0.07	71.78	233.8
0.075	71.8	248.5
0.08	71.83	263.1
0.085	71.85	277.6
0.09	71.87	291.9
0.095	71.89	306.2
0.1	71.91	320.4
0.105	71.93	334.4
0.11	71.94	348.4
0.115	71.96	362.3
0.12	71.98	376.1
0.125	71.99	389.8
0.13	72	403.5
0.135	72.02	417.1
0.14	72.03	430.6
0.145	72.04	444
0.15	72.06	457.4



8-103 Hot air enters a sheet metal duct located in a basement. The exit temperature of hot air and the rate of heat loss are to be determined.

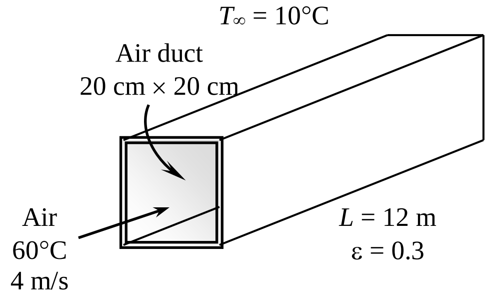
Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties We evaluate the air properties at 1 atm and the estimated bulk mean temperature of 50°C based on the problem statement (Table A-15),

$$\rho = 1.092 \text{ kg/m}^3; \qquad k = 0.02735 \text{ W/m.°C}$$

$$\nu = 1.798\times10^{-5} \text{ m}^2\text{/s}; \quad c_p = 1007 \text{ J/kg.°C}$$

$$\Pr = 0.7228$$



Analysis The surface area and the Reynolds number are

$$A_s = 4aL = 4\times(0.2\text{ m})(12\text{ m}) = 9.6\text{ m}^2$$

$$D_h = \frac{4A_c}{p} = \frac{4a^2}{4a} = a = 0.2\text{ m}$$

$$\text{Re} = \frac{V_{\text{avg}}D_h}{\nu} = \frac{(4\text{ m/s})(0.20\text{m})}{1.798\times10^{-5}\text{ m}^2\text{/s}} = 44{,}494$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D_h = 10(0.2\text{ m}) = 2.0\text{ m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow for the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(44{,}494)^{0.8}(0.7228)^{0.3} = 109.2$$

and $$h = \frac{k}{D_h}Nu = \frac{0.02735\text{ W/m.°C}}{0.2\text{ m}}(109.2) = 14.93\text{ W/m}^2\text{.°C}$$

The mass flow rate of air is

$$\dot{m} = \rho A_c V = (1.092\text{ kg/m}^3)(0.2\times0.2\text{ m}^2)(4\text{ m/s}) = 0.1747\text{ kg/s}$$

In steady operation, heat transfer from hot air to the duct must be equal to the heat transfer from the duct to the surrounding (by convection and radiation), which must be equal to the energy loss of the hot air in the duct. That is,

$$\dot{Q} = \dot{Q}_{\text{conv,in}} = \dot{Q}_{\text{conv+rad,out}} = \Delta\dot{E}_{\text{hot air}}$$

Assuming the duct to be at an average temperature of T_s , the quantities above can be expressed as

$\dot{Q}_{\text{conv,in}}$:
$$\dot{Q} = h_i A_s \Delta T_{\text{lm}} = h_i A_s \frac{T_e - T_i}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} \rightarrow \dot{Q} = (14.93\text{ W/m}^2\text{.°C})(9.6\text{ m}^2)\frac{T_e - 60}{\ln\left(\dfrac{T_s - T_e}{T_s - 60}\right)}$$

$\dot{Q}_{\text{conv+rad,out}}$:
$$\dot{Q} = h_o A_s (T_s - T_o) + \varepsilon A_s \sigma\left(T_s^4 - T_o^4\right) \rightarrow \dot{Q} = (10\text{ W/m}^2\text{.°C})(9.6\text{ m}^2)(T_s - 10)\text{°C}$$
$$+0.3(9.6\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)\left[(T_s + 273)^4 - (10+273)^4\right]\text{K}^4$$

$\Delta\dot{E}_{\text{hot air}}$:
$$\dot{Q} = \dot{m}c_p(T_e - T_i) \rightarrow \dot{Q} = (0.1747\text{ kg/s})(1007\text{ J/kg.°C})(60 - T_e)\text{°C}$$

This is a system of three equations with three unknowns whose solution is

$$\dot{Q} = \mathbf{2622\ W},\ T_e = \mathbf{45.1°C},\ \text{and } T_s = 33.3\text{°C}$$

Therefore, the hot air will lose heat at a rate of 2622 W and exit the duct at 45.1°C.

Discussion The bulk mean temperature of air is (60 + 45.1)/2 = 52.6°C. This is very close to the assumed temperature of 50°C. Therefore, there is no need to repeat calculations.

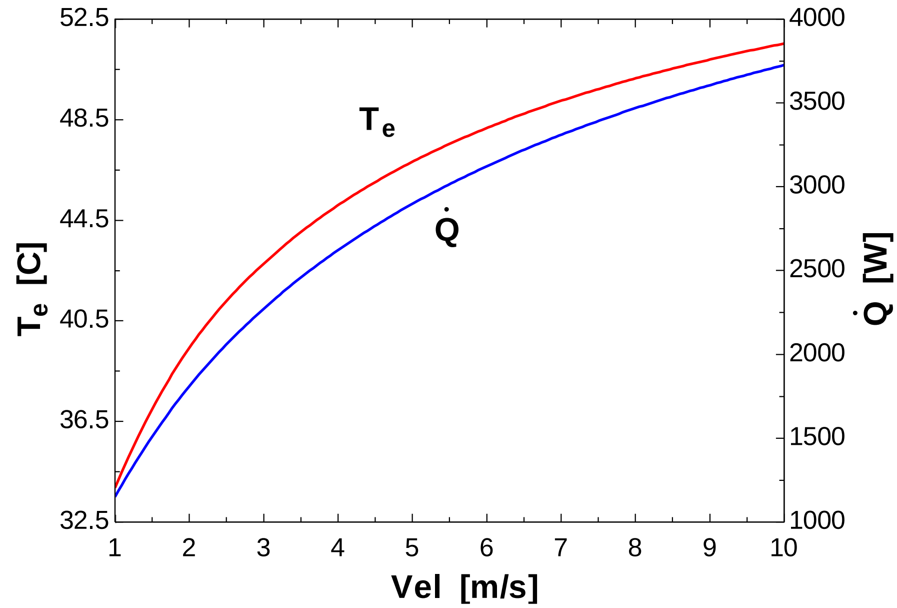
8-104 Prob. 8-103 is reconsidered. The effects of air velocity and the surface emissivity on the exit temperature of air and the rate of heat loss are to be investigated.

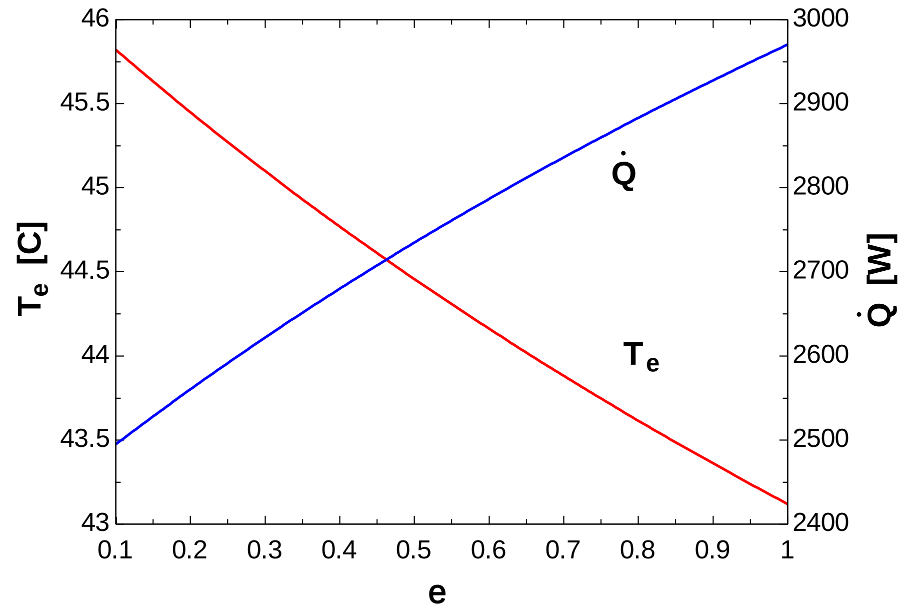
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_i=60 [C]
L=12 [m]
side=0.20 [m]
Vel=4 [m/s]
epsilon=0.3
T_o=10 [C]
h_o=10 [W/m^2-C]
T_surr=10 [C]

"PROPERTIES"
Fluid$='air'
c_p=CP(Fluid$, T=T_ave)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(Fluid$, T=T_ave)
Pr=Prandtl(Fluid$, T=T_ave)
rho=Density(Fluid$, T=T_ave, P=101.3)
mu=Viscosity(Fluid$, T=T_ave)
nu=mu/rho
T_ave=T_i-10 "assumed average bulk mean temperature"

"ANALYSIS"
A=4*side*L
A_c=side^2
p=4*side
D_h=(4*A_c)/p
Re=(Vel*D_h)/nu "The flow is turbulent"
L_t=10*D_h "The entry length is much shorter than the total length of the duct."
Nusselt=0.023*Re^0.8*Pr^0.3
h_i=k/D_h*Nusselt
m_dot=rho*Vel*A_c
Q_dot=Q_dot_conv_in
Q_dot_conv_in=Q_dot_conv_out+Q_dot_rad_out
Q_dot_conv_in=h_i*A*DELTAT_ln
DELTAT_ln=(T_e-T_i)/ln((T_s-T_e)/(T_s-T_i))
Q_dot_conv_out=h_o*A*(T_s-T_o)
Q_dot_rad_out=epsilon*A*sigma*((T_s+273)^4-(T_surr+273)^4)
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"
Q_dot=m_dot*c_p*(T_i-T_e)
```

Vel [m/s]	T_e [C]	$\dot{Q}$ [W]
1	33.85	1150
2	39.43	1810
3	42.78	2273
4	45.1	2622
5	46.83	2898
6	48.17	3122
7	49.25	3310
8	50.14	3469
9	50.89	3606
10	51.53	3726

ε	T_e [C]	$\dot{Q}$ [W]
0.1	45.82	2495
0.2	45.45	2560
0.3	45.1	2622
0.4	44.77	2680
0.5	44.46	2735
0.6	44.16	2787
0.7	43.88	2836
0.8	43.61	2883
0.9	43.36	2928
1	43.12	2970

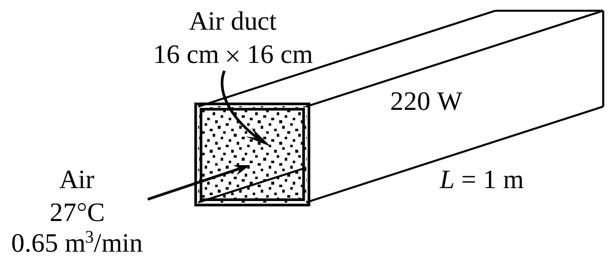




8-105 The components of an electronic system located in a rectangular horizontal duct are cooled by forced air. The exit temperature of the air and the highest component surface temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm. **6** Fully developed turbulent flow in the channel.

Properties We assume the bulk mean temperature for air to be 35°C since the mean temperature of air at the inlet will rise somewhat as a result of heat gain through the duct whose surface is exposed to a constant heat flux. The properties of air at 1 atm and this temperature are (Table A-15)



$$\rho = 1.145 \text{ kg/m}^3$$
$$k = 0.02625 \text{ W/m.°C}$$
$$\nu = 1.655\times10^{-5} \text{ m}^2\text{/s}$$
$$c_p = 1007 \text{ J/kg.°C}$$
$$\text{Pr} = 0.7268$$

Analysis (*a*) The mass flow rate of air and the exit temperature are determined from

$$\dot{m} = \rho\dot{V} = (1.145 \text{ kg/m}^3)(0.65 \text{m}^3\text{/min}) = 0.7443 \text{ kg/min} = 0.0124 \text{ kg/s}$$

$$\dot{Q} = \dot{m}c_p(T_e - T_i) \rightarrow T_e = T_i + \frac{\dot{Q}}{\dot{m}c_p} = 27°\text{C} + \frac{(0.85)(220\text{ W})}{(0.0124 \text{ kg/s})(1007 \text{ J/kg.°C})} = \mathbf{42.0°C}$$

(*b*) The mean fluid velocity and hydraulic diameter are

$$V_m = \frac{\dot{V}}{A_c} = \frac{0.65 \text{ m/min}}{(0.16 \text{ m})(0.16 \text{ m})} = 25.4 \text{ m/min} = 0.4232 \text{ m/s}$$

$$D_h = \frac{4A_c}{p} = \frac{4(0.16 \text{ m})(0.16 \text{ m})}{4(0.16 \text{ m})} = 0.16 \text{ m}$$

Then

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(0.4232 \text{ m/s})(0.16 \text{m})}{1.655\times10^{-5} \text{ m}^2\text{/s}} = 4091$$

which is not greater than 10,000 but the components will cause turbulence and thus we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} = 0.023(4091)^{0.8}(0.7268)^{0.4} = 15.69$$

and

$$h = \frac{k}{D_h} Nu = \frac{0.02625 \text{ W/m.°C}}{0.16 \text{ m}}(15.69) = 2.574 \text{ W/m}^2\text{.°C}$$

The highest component surface temperature will occur at the exit of the duct. Assuming uniform surface heat flux, its value is determined from

$$\dot{Q}/A_s = h(T_{s,highest} - T_e)$$

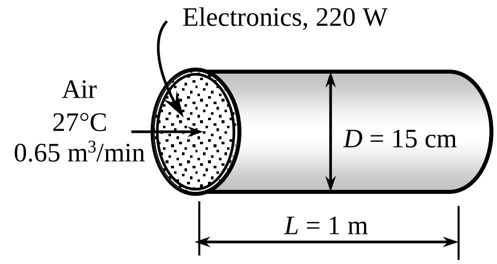
$$T_{s,highest} = T_e + \frac{\dot{Q}/A_s}{h} = 42.0°\text{C} + \frac{(0.85)(220 \text{ W})/[4(0.16 \text{ m})(1\text{m})]}{2.574 \text{ W/m}^2\text{.°C}} = \mathbf{155.5°C}$$

8-106 The components of an electronic system located in a circular horizontal duct are cooled by forced air. The exit temperature of the air and the highest component surface temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 35°C since the mean temperature of air at the inlet will rise somewhat as a result of heat gain through the duct whose surface is exposed to a constant heat flux. The properties of air at 1 atm and this temperature are (Table A-15)

$$\rho = 1.145 \text{ kg/m}^3$$
$$k = 0.02625 \text{ W/m.°C}$$
$$\nu = 1.655\times10^{-5} \text{ m}^2\text{/s}$$
$$c_p = 1007 \text{ J/kg.°C}$$
$$\Pr = 0.7268$$



Analysis (a) The mass flow rate of air and the exit temperature are determined from

$$\dot{m} = \rho\dot{V} = (1.145 \text{ kg/m}^3)(0.65 \text{ m}^3\text{/min}) = 0.7443 \text{ kg/min} = 0.0124 \text{ kg/s}$$

$$\dot{Q} = \dot{m}c_p(T_e - T_i) \rightarrow T_e = T_i + \frac{\dot{Q}}{\dot{m}c_p} = 27\,°\text{C} + \frac{(0.85)(220 \text{ W})}{(0.0124 \text{ kg/s})(1007 \text{ J/kg.°C})} = \mathbf{42.0\,°C}$$

(b) The mean fluid velocity is

$$V_{\text{avg}} = \frac{\dot{V}}{A_c} = \frac{0.65 \text{ m/min}}{\pi(0.15 \text{ m})^2/4} = 36.8 \text{ m/min} = 0.613 \text{ m/s}$$

Then,

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(0.613 \text{ m/s})(0.15 \text{m})}{1.655\times10^{-5} \text{ m}^2\text{/s}} = 5556$$

which is not greater than 10,000 but the components will cause turbulence and thus we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\Pr^{0.4} = 0.023(5556)^{0.8}(0.7268)^{0.4} = 20.05$$

and

$$h = \frac{k}{D_h}Nu = \frac{0.02625 \text{ W/m.°C}}{0.15 \text{ m}}(20.05) = 3.51 \text{ W/m}^2\text{.°C}$$

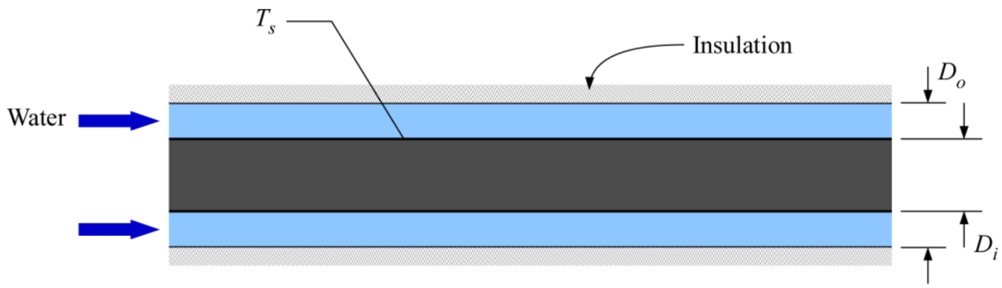
The highest component surface temperature will occur at the exit of the duct. Assuming uniform heat flux, its value is determined from

$$\dot{q} = h(T_{s,highest} - T_e) \rightarrow T_{s,highest} = T_e + \frac{\dot{q}}{h} = 42.0°\text{C} + \frac{(0.85)(220 \text{ W})/[\pi(0.15 \text{ m})(1 \text{m})]}{3.51 \text{ W/m}^2\text{.°C}} = \mathbf{155.1°C}$$

8-107 Water flows through a concentric annulus tube with constant inner surface temperature and insulated outer surface, the length of the annulus tube is to be determined.

Assumptions **1** Steady operating conditions. **2** Constant properties. **3** Constant inner tube surface temperature. **4** Insulated outer tube surface. **5** Fully developed flow.

Properties The properties of water at $T_b = (T_i + T_e)/2 = 50°\text{C}$: $c_p = 4181$ J/kg·K, $k = 0.644$ W/m·K, $\mu = 0.547 \times 10^{-3}$ kg/m·s, and Pr = 3.55 (Table A-15).



Analysis The Reynolds number is

$$\text{Re} = \frac{\rho V_{\text{avg}}(D_o - D_i)}{\mu} = \frac{\dot{m}(D_o - D_i)}{(\pi/4)(D_o^2 - D_i^2)\mu} = \frac{4\dot{m}}{\pi(D_o + D_i)\mu}$$

$$= \frac{4(0.05\ \text{kg/s})}{\pi(0.025\ \text{m} + 0.1\ \text{m})(0.547 \times 10^{-3}\ \text{kg/m}\cdot\text{s})}$$

$$= 931$$

Since Re < 2300, the flow through the annulus is laminar. Assuming fully developed flow, the Nusselt number for the inner tube surface is (from Table 8-4)

$$\text{Nu}_i = \frac{h_i D_h}{k} = 7.37 \qquad \text{for} \qquad D_i / D_o = 0.25$$

Hence, the convection heat transfer coefficient is

$$h_i = 7.37\left(\frac{0.644\ \text{W/m}\cdot\text{K}}{0.075\ \text{m}}\right) = 63.28\ \text{W/m}^2\cdot\text{K}$$

The length of the concentric annulus tube is

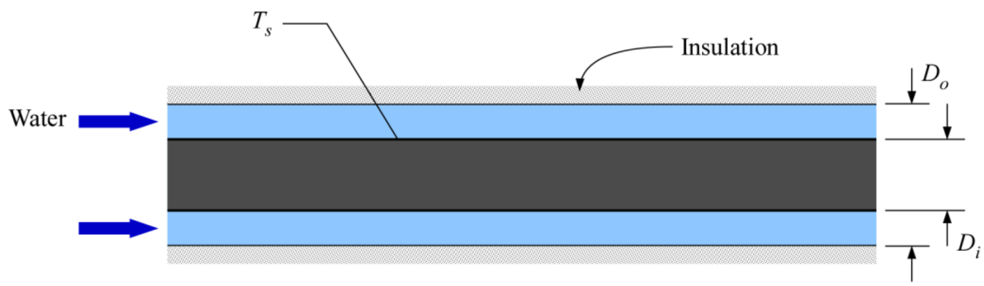
$$L = -\frac{\dot{m}c_p}{\pi D_i h_i}\ln\frac{T_s - T_e}{T_s - T_i} = -\frac{(0.05\ \text{kg/s})(4181\ \text{J/kg}\cdot\text{K})}{\pi(0.025\ \text{m})(63.28\ \text{W/m}^2\cdot\text{K})}\ln\frac{120 - 80}{120 - 20} = \mathbf{38.5\ m}$$

Discussion Similar to regular tubes, the total rate of heat transfer in the annulus tube can be determined using $\dot{Q} = \dot{m}c_p(T_e - T_i)$.

8-108 Water flows through a concentric annulus tube with constant inner surface temperature and insulated outer surface, the length of the annulus tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Constant inner tube surface temperature. **4** Insulated outer tube surface. **5** Fully developed flow.

Properties The properties of water at $T_b = (T_i + T_e)/2 = 50°C$: $c_p = 4181$ J/kg·K, $k = 0.644$ W/m·K, $\mu = 0.547 \times 10^{-3}$ kg/m·s, and Pr = 3.55 (Table A-15).



Analysis The Reynolds number is

$$\text{Re} = \frac{\rho V_{\text{avg}}(D_o - D_i)}{\mu} = \frac{\dot{m}(D_o - D_i)}{(\pi/4)(D_o^2 - D_i^2)\mu} = \frac{4\dot{m}}{\pi(D_o + D_i)\mu}$$
$$= \frac{4(0.7\text{ kg/s})}{\pi(0.1\text{ m} + 0.01\text{ m})(0.547\times10^{-3}\text{ kg/m}\cdot\text{s})}$$
$$= 14{,}812$$

Since Re > 10000, the flow through the annulus is turbulent. Using the Dittus-Boelter equation, we have

$$\text{Nu} = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(14{,}812)^{0.8}(3.55)^{0.4} = 82.86$$

Hence, the convection heat transfer coefficient is

$$h = 82.86\left(\frac{0.644\text{ W/m}\cdot\text{K}}{(0.1-0.01)\text{ m}}\right) = 592.9\text{ W/m}^2\cdot\text{K}$$

The length of the concentric annulus tube is

$$L = -\frac{\dot{m}c_p}{\pi D_h h_i}\ln\frac{T_s - T_e}{T_s - T_i} = -\frac{(0.7\text{ kg/s})(4181\text{ J/kg}\cdot\text{K})}{\pi[(0.1-0.01)\text{ m}](592.9\text{ W/m}^2\cdot\text{K})}\ln\frac{120-80}{120-20} = \mathbf{16.0\text{ m}}$$

Discussion For fully developed turbulent flow, $h_i \approx h_o$, hence the tube annulus can be treated as a noncircular duct with $D_h = D_o - D_i$.

Since the flow is turbulent, following the recommendation of Petukhov and Roizen, the accuracy of the Nusselt number calculated from the Dittus-Boelter equation can be improved by using the correction factor (F_i) given by Eq. 8-79 for the case of adiabatic outer wall. For D_i/D_o= 0.10, F_i = 1.243, Nu = 1.243 (82.86) = 103, h = 737 W/m^2·K, and $\boldsymbol{L}$ **= 12.9 m**.

Special Topic: Transitional Flow

8-109 A liquid mixture flowing in a tube with a bell-mouth inlet is subjected to uniform wall heat flux. The friction coefficient is to be determined.

Assumptions Steady operating conditions exist.

Properties The properties of the ethylene glycol-distilled water mixture are given to be Pr = 14.85, $\nu = 1.93\times10^{-6}$ m^2/s and $\mu_b/\mu_s = 1.07$.

Analysis**:** For the calculation of the non-isothermal fully developed friction coefficient, it is necessary to determine the flow regime before making any decision regarding which friction coefficient relation to use. The Reynolds number at the specified location is

$$\text{Re} = \frac{(\dot{V}/A_c)D}{\nu} = \frac{[(1.43\times10^{-4}\ \text{m}^3/\text{s})/(1.961\times10^{-4}\ \text{m}^2)](0.0158\ \text{m})}{1.93\times10^{-6}\ \text{m}^2/\text{s}} = 5973$$

since $\quad A_c = \pi D^2/4 = \pi(0.0158\ \text{m})^2/4 = 1.961\times10^{-4}\ \text{m}^2$

From Table 8-6, we see that for a bell-mouth inlet and a heat flux of 3 kW/m^2 the flow is in the transition region. Therefore, Eq. 8-83 applies. Reading the constants A, B, C and m_1, m_2, m_3, and m_4 from Table 8-5, the friction coefficient is determined to be

$$C_{f,\text{trans}} = \left[1+\left(\frac{\text{Re}}{A}\right)^B\right]^C \left(\frac{\mu_b}{\mu_s}\right)^m$$

$$= \left[1+\left(\frac{5973}{5340}\right)^{-0.099}\right]^{-6.32} (1.07)^{-2.58-0.42\times16600^{-0.41}\times14.85^{2.46}}$$

$$= \mathbf{0.0073}$$

8-110 A liquid mixture flowing in a tube with a bell-mouth inlet is subjected to uniform wall heat flux. The friction coefficient is to be determined.

Assumptions Steady operating conditions exist.

Properties The properties of the ethylene glycol-distilled water mixture are given to be Pr = 14.85, $\nu = 1.93\times10^{-6}$ m^2/s and $\mu_b/\mu_s = 1.07$.

Analysis**:** For the calculation of the non-isothermal fully developed friction coefficient, it is necessary to determine the flow regime before making any decision regarding which friction coefficient relation to use. If the volume flow rate is increased by 50%, the Reynolds number becomes

$$\text{Re} = \frac{(\dot{V}/A_c)D}{\nu} = \frac{[(1.5\times1.43\times10^{-4}\ \text{m}^3/\text{s})/(1.961\times10^{-4}\ \text{m}^2)](0.0158\ \text{m})}{1.93\times10^{-6}\ \text{m}^2/\text{s}} = 8960$$

since $\quad A_c = \pi D^2/4 = \pi(0.0158\ \text{m})^2/4 = 1.961\times10^{-4}\ \text{m}^2$

From Table 8-6 for a bell-mouth inlet and a heat flux of 3 kW/m^2, the flow is in the turbulent region. To calculate the fully developed friction coefficient for this case, Eq. 8-82 for turbulent flow with m = - 0.25 is used.

$$C_{f,\text{turb}} = \left(\frac{0.0791}{\text{Re}^{0.25}}\right)\left(\frac{\mu_b}{\mu_s}\right)^m = \left(\frac{0.0791}{8960^{0.25}}\right)(1.07)^{-0.25} = \mathbf{0.0080}$$

8-111E A liquid mixture flowing in a tube with a square-edged inlet is subjected to uniform wall heat flux. The friction coefficient is to be determined.

Assumptions Steady operating conditions exist.

Properties The properties of the ethylene glycol-distilled water mixture are given to be Pr = 13.8, ν = 18.4×10^{-6} ft^2/s and μ_b/μ_s = 1.12.

***Analysis*:** For the calculation of the non-isothermal fully developed friction coefficient, it is necessary to determine the flow regime before making any decision regarding which friction coefficient relation to use. The Reynolds number at the specified location is

$$\mathrm{Re} = \frac{(\dot{V}/A_c)\,D}{\nu} = \frac{[(2.16\ \mathrm{gal/min})/(2.11\times10^{-3}\ \mathrm{ft}^2)](0.622/12\ \mathrm{ft})}{18.4\times10^{-6}\ \mathrm{ft}^2/\mathrm{s}}\left(\frac{1\ \mathrm{ft}^3/\mathrm{s}}{448.8\ \mathrm{gal/min}}\right) = 6425$$

since

$$A_c = \pi D^2/4 = \pi(0.622/12\ \mathrm{ft})^2/4 = 2.110\times10^{-3}\ \mathrm{ft}^2$$

From Table 8-6, the transition Reynolds number range for this case (square-edged inlet and a heat flux of 8 kW/m^2) is 3860 < Re < 5200, which means that the flow in this case is turbulent and Eq. 8-82 is the appropriate equation to use. It gives

$$C_{f,\mathrm{turb}} = \left(\frac{0.0791}{\mathrm{Re}^{0.25}}\right)\left(\frac{\mu_b}{\mu_s}\right)^m = \left(\frac{0.0791}{6425^{0.25}}\right)(1.12)^{-0.25} = \mathbf{0.00859}$$

Repeating the calculations when the volume flow rate is increased by 50%, we obtain

$$\mathrm{Re} = \frac{(\dot{V}/A_c)\,D}{\nu} = \frac{[1.5(2.16\ \mathrm{gal/min})/(2.11\times10^{-3}\ \mathrm{ft}^2)](0.622/12\ \mathrm{ft})}{18.4\times10^{-6}\ \mathrm{ft}^2/\mathrm{s}}\left(\frac{1\ \mathrm{ft}^3/\mathrm{s}}{448.8\ \mathrm{gal/min}}\right) = 9639$$

$$C_{f,\mathrm{turb}} = \left(\frac{0.0791}{\mathrm{Re}^{0.25}}\right)\left(\frac{\mu_b}{\mu_s}\right)^m = \left(\frac{0.0791}{9639^{0.25}}\right)(1.12)^{-0.25} = \mathbf{0.00776}$$

8-112 A liquid mixture flowing in a tube is subjected to uniform wall heat flux. The apparent friction factor for two different inlets of re-entrant and square-edged is to be determined.

Assumptions Steady operating conditions exist.

Properties The properties of the ethylene glycol-distilled water mixture are given to be Pr = 20.9, $\nu = 2.33 \times 10^{-6}$ m²/s, and $\mu_b/\mu_s = 1.25$.

Analysis**:** For the calculation of the non-isothermal apparent (developing) friction factor, it is necessary to determine the flow regime before making any decision regarding which friction factor relation to use. The Reynolds number at the specified location is

$$\mathrm{Re} = \frac{(\dot{V}/A_c)D}{\nu} = \frac{[(7.8\times10^{-5}\ \mathrm{m^3/s})/(1.744\times10^{-4}\ \mathrm{m^2})](0.0149\ \mathrm{m})}{2.33\times10^{-6}\ \mathrm{m^2/s}} = 2860$$

since

$$A_c = \pi D^2/4 = \pi(0.0149\ \mathrm{m})^2/4 = 1.744\times10^{-4}\ \mathrm{m^2}$$

From Table 8–7, we see that for both inlets the flow is in the transition region. Therefore, Eq. 8-86 applies. The value of m and the constants a, b, and c are found in Table 8-7.

$$f_{app,trans} = \left\{\left(\frac{64}{Re}\right)\left[(1 + (0.0049Re^{0.75})^{a})^{1/a} + b\right]\left[1 + \left(\frac{\mathrm{c}}{\mathrm{x/D}}\right)\right]\right\}\left(\frac{\mu_b}{\mu_s}\right)^m$$

(a) For re-entrant inlet: From Table 8-7

$m = -1.8+0.46\ Gr^{-0.13}\ Pr^{0.41} = -1.8+0.46\ (24{,}000)^{-0.13}\ (20.9)^{0.41} = -1.3689$; a = 0.52, b = -3.47, c = 4.8

$$f_{app,trans} = \left\{\left(\frac{64}{2860}\right)\left[(1 + (0.0049\times2860^{0.75})^{0.52})^{1/0.52} - 3.47\right]\left[1 + \left(\frac{4.8}{20}\right)\right]\right\}(1.25)^{-1.3776}$$

$f_{app,trsns}$ = **0.03936**

(b) For square-edged inlet: From Table 8-7

$m = -1.13+0.48\ Gr^{-0.15}\ Pr^{0.55} = -1.13+0.48\ (24{,}000)^{-0.15}\ (20.9)^{0.55} = -\ 0.56728$; a = 0.50, b = -4.0, c = 3.0

$$f_{app,trans} = \left\{\left(\frac{64}{2860}\right)\left[(1 + (0.0049\times2860^{0.75})^{0.5})^{1/0.5} - 4\right]\left[1 + \left(\frac{3}{20}\right)\right]\right\}(1.25)^{-0.56728}$$

$f_{app,trsns}$ = **0.03820**

Discussion If the flow was considered to be isothermal, in the above calculations the viscosity ratio should be set to unity or m = 0. The apparent friction factor for the re-entrant inlet would be $f_{app,trsns}$ = **0.05342** (about 36% increase) and for the square-edged inlet would be $f_{app,trsns}$ = **0.04336** (about 14% increase). Heating causes a decrease in the friction factor.

8-113 A liquid mixture flowing in a tube is subjected to uniform wall heat flux. The Nusselt number at a specified location is to be determined for two different tube inlet configurations.

Assumptions Steady operating conditions exist.

Properties The properties of the ethylene glycol-distilled water mixture are given to be Pr = 33.46, $\nu = 3.45\times10^{-6}$ m^2/s and $\mu_b/\mu_s = 2.0$.

Analysis For a tube with a known diameter and volume flow rate, the type of flow regime is determined before making any decision regarding which Nusselt number correlation to use. The Reynolds number at the specified location is

$$\text{Re} = \frac{(\dot{V}/A_c)\,D}{\nu} = \frac{[(2.05\times10^{-4}\ \text{m}^3/\text{s})/(1.961\times10^{-4}\ \text{m}^2)](0.0158\,\text{m})}{3.45\times10^{-6}\ \text{m}^2/\text{s}} = 4790$$

since

$$A_c = \pi D^2/4 = \pi(0.0158\,\text{m})^2/4 = 1.961\times10^{-4}\ \text{m}^2.$$

Therefore, the flow regime is in the transition region for all three inlet configurations (thus use the information given in Table 8-9 with x/D = 10) and therefore Eq. 8-87 should be used with the constants *a, b, c* found in Table 8-8. However, Nu_{lam} and Nu_{turb} are the inputs to Eq. 8-87 and they need to be evaluated first from Eqs. 8-88 and 8-89, respectively. It should be mentioned that the correlations for Nu_{lam} and Nu_{turb} have no inlet dependency.

From Eq. 8-88:

$$\begin{aligned}\text{Nu}_{\text{lam}} &= 1.24\left[\left(\frac{\text{Re Pr}\,D}{x}\right) + 0.025\left(\text{GrPr}\right)^{0.75}\right]^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} \\ &= 1.24\left[\left(\frac{(4790)(33.46)}{10}\right) + 0.025[(60{,}000)(33.46)]^{0.75}\right]^{1/3}(2.0)^{0.14} \\ &= 35.4\end{aligned}$$

From Eq. 8-89:

$$\begin{aligned}\text{Nu}_{\text{turb}} &= 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.385}\left(\frac{x}{D}\right)^{-0.0054}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} \\ &= 0.023(4790)^{0.8}(33.46)^{0.385}(10)^{-0.0054}(2.0)^{0.14} \\ &= 85.1\end{aligned}$$

Then the transition Nusselt number can be determined from Eq. 8-87,

$$\text{Nu}_{\text{trans}} = \text{Nu}_{\text{lam}} + \left\{\exp[(a-\text{Re})/b] + \text{Nu}_{\text{turb}}^{c}\right\}^{c}$$

Case 1: For bell-mouth inlet:

$$\text{Nu}_{\text{trans}} = 35.4 + \left\{\exp[(6628-4790)/237] + 85.1^{-0.980}\right\}^{-0.980} = \mathbf{35.4}$$

Case 2: For re-entrant inlet:

$$\text{Nu}_{\text{trans}} = 35.4 + \left\{\exp[(1766-4790)/276] + 85.1^{-0.955}\right\}^{-0.955} = \mathbf{92.9}$$

Discussion Comparing the two results, it can be seen that under the same conditions, the Nusselt number for the re-entrant inlet is much higher than that for the bell-mouth inlet. To verify this trend, refer to Fig. 8-38.

8-114 A liquid mixture flowing in a tube is subjected to uniform wall heat flux. The Nusselt number at a specified location is to be determined for two different tube inlet configurations.

Assumptions Steady operating conditions exist.

Properties The properties of the ethylene glycol-distilled water mixture are given to be Pr = 33.46, $\nu = 3.45\times10^{-6}$ m^2/s and $\mu_b/\mu_s = 2.0$.

Analysis For a tube with a known diameter and volume flow rate, the type of flow regime is determined before making any decision regarding which Nusselt number correlation to use. The Reynolds number at the specified location is

$$\mathrm{Re} = \frac{(\dot{V}/A_c)\,D}{\nu} = \frac{[(2.05\times10^{-4}\ \mathrm{m^3/s})/(1.961\times10^{-4}\ \mathrm{m^2})](0.0158\,\mathrm{m})}{3.45\times10^{-6}\ \mathrm{m^2/s}} = 4790$$

since

$$A_c = \pi D^2/4 = \pi(0.0158\,\mathrm{m})^2/4 = 1.961\times10^{-4}\ \mathrm{m^2}.$$

Therefore, the flow regime is in the transition region for all three inlet configurations (thus use the information given in Table 8-9 with *x/D* = 90) and therefore Eq. 8-87 should be used with the constants *a, b, c* found in Table 8-8. However, $\mathrm{Nu_{lam}}$ and $\mathrm{Nu_{turb}}$ are the inputs to Eq. 8-87 and they need to be evaluated first from Eqs. 8-88 and 8-89, respectively. It should be mentioned that the correlations for $\mathrm{Nu_{lam}}$ and $\mathrm{Nu_{turb}}$ have no inlet dependency.

From Eq. 8-88:

$$\begin{aligned}\mathrm{Nu_{lam}} &= 1.24\left[\left(\frac{\mathrm{Re\,Pr}\,D}{x}\right)+0.025(\mathrm{GrPr})^{0.75}\right]^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}\\ &= 1.24\left[\left(\frac{(4790)(33.46)}{90}\right)+0.025[(60{,}000)(33.46)]^{0.75}\right]^{1/3}(2.0)^{0.14}\\ &= 20.0\end{aligned}$$

From Eq. 8-89:

$$\begin{aligned}\mathrm{Nu_{turb}} &= 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.385}\left(\frac{x}{D}\right)^{-0.0054}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}\\ &= 0.023(4790)^{0.8}(33.46)^{0.385}(90)^{-0.0054}(2.0)^{0.14}\\ &= 84.1\end{aligned}$$

Then the transition Nusselt number can be determined from Eq. 8-87,

$$\mathrm{Nu_{trans}} = \mathrm{Nu_{lam}} + \left\{\exp[(a-\mathrm{Re})/b] + \mathrm{Nu_{turb}^{\,c}}\right\}^{c}$$

Case 1: For bell-mouth inlet:

$$\mathrm{Nu_{trans}} = 20.0 + \left\{\exp[(6628-4790)/237] + 84.1^{-0.980}\right\}^{-0.980} = \mathbf{20.0}$$

Case 2: For re-entrant inlet:

$$\mathrm{Nu_{trans}} = 20.0 + \left\{\exp[(1766-4790)/276] + 84.1^{-0.955}\right\}^{-0.955} = \mathbf{76.9}$$

Discussion Comparing the two results, it can be seen that under the same conditions, the Nusselt number for the re-entrant inlet is much higher than that for the bell-mouth inlet. To verify this trend, refer to Fig. 8-38.

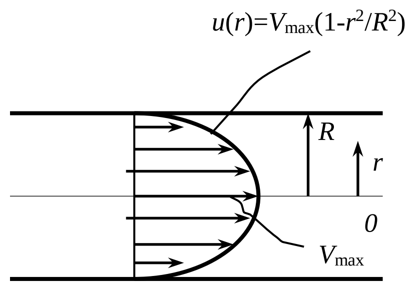
Review Problems

8-115 The velocity profile in fully developed laminar flow in a circular pipe is given. The radius of the pipe, the mean velocity, and the maximum velocity are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is

$$u(r) = V_{max}\left(1 - \frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by

$$u(r) = 4(1 - 100r^2)$$

Comparing the two relations above gives the pipe radius, the maximum velocity, and the mean velocity to be

$$R^2 = \frac{1}{100} \quad \rightarrow \quad R = \mathbf{0.10\,m}$$

$$V_{max} = \mathbf{4\ m/s}$$

$$V_{avg} = \frac{V_{max}}{2} = \frac{4\,\text{m/s}}{2} = \mathbf{2\,m/s}$$

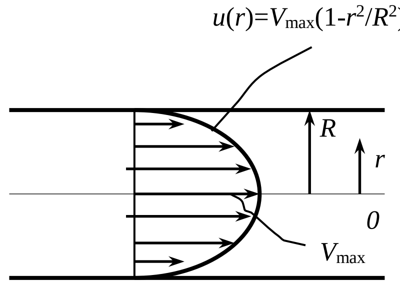
8-116E The velocity profile in fully developed laminar flow in a circular pipe is given. The volume flow rate, the pressure drop, and the useful pumping power required to overcome this pressure drop are to be determined.

Assumptions **1** The flow is steady, laminar, and fully developed. **2** The pipe is horizontal.

Properties The density and dynamic viscosity of water at 40°F are $\rho = 62.42$ lbm/ft^3 and $\mu = 1.308\times10^{-3}$ lbm/ft·s, respectively (Table A-9E).

Analysis The velocity profile in fully developed laminar flow in a circular pipe is

$$u(r) = V_{\max}\left(1-\frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by

$$u(r) = 0.8(1-625r^2)$$

Comparing the two relations above gives the pipe radius, the maximum velocity, and the mean velocity to be

$$R^2 = \frac{1}{625} \quad \rightarrow \quad R = 0.04\text{ ft}$$

V_{max} = 0.8 ft/s

$$V_{avg} = \frac{V_{\max}}{2} = \frac{0.8\text{ ft/s}}{2} = 0.4\text{ ft/s}$$

Then the volume flow rate and the pressure drop become

$$\dot{V} = V_{avg}A_c = V_{avg}(\pi R^2) = (0.4\text{ ft/s})[\pi(0.04\text{ ft})^2] = \mathbf{0.00201\ ft^3/s}$$

$$\dot{V}_{\text{horiz}} = \frac{\Delta P \pi D^4}{128\mu L} \quad \rightarrow \quad 0.00201\text{ ft}^3\text{/s} = \frac{(\Delta P)\pi(0.08\text{ ft})^4}{128(1.308\times10^{-3}\text{ lbm/ft}\cdot\text{s})(140\text{ ft})}\left(\frac{32.2\text{ lbm}\cdot\text{ft/s}^2}{1\text{ lbf}}\right)$$

It gives

$$\Delta P = 11.37\text{ lbf/ft}^2 = \mathbf{0.0790\ psi}$$

Then the useful pumping power requirement becomes

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (0.00201\text{ ft}^3\text{/s})(11.37\text{ lbf/ft}^2)\left(\frac{1\text{ W}}{0.737\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{0.031\ W}$$

Checking The flow was assumed to be laminar. To verify this assumption, we determine the Reynolds number:

$$\text{Re} = \frac{\rho V_{avg} D}{\mu} = \frac{(62.42\text{ lbm/ft}^3)(0.4\text{ ft/s})(0.08\text{ft})}{1.308\times10^{-3}\text{ lbm/ft}\cdot\text{s}} = 1527$$

which is less than 2300. Therefore, the flow is laminar.

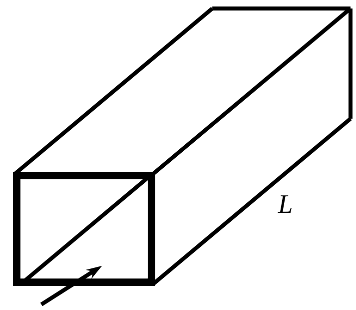
Discussion Note that the pressure drop across the water pipe and the required power input to maintain flow is negligible. This is due to the very low flow velocity. Such water flows are the exception in practice rather than the rule.

8-117 Laminar flow of a fluid through an isothermal square channel is considered. The change in the pressure drop and the rate of heat transfer are to be determined when the mean velocity is doubled.

Assumptions **1** The flow is fully developed. **2** The effect of the change in $\Delta T_{\rm lm}$ on the rate of heat transfer is not considered.

Analysis The pressure drop of the fluid for laminar flow is expressed as

$$\Delta P_1 = f\,\frac{L}{D}\frac{\rho V_{\rm avg}^2}{2} = \frac{64}{\rm Re}\frac{L}{D}\frac{\rho V_{\rm avg}^2}{2} = \frac{64\nu}{V_{\rm avg}D}\frac{L}{D}\frac{\rho V_{\rm avg}^2}{2} = 32V_{\rm avg}\frac{\nu L\rho}{D^2}$$



When the free-stream velocity of the fluid is doubled, the pressure drop becomes

$$\Delta P_2 = f\,\frac{L}{D}\frac{\rho (2V_{\rm avg})^2}{2} = \frac{64}{\rm Re}\frac{L}{D}\frac{\rho 4V_{\rm avg}^2}{2} = \frac{64\nu}{2V_{\rm avg}D}\frac{L}{D}\frac{\rho 4V_{\rm avg}^2}{2} = 64V_{\rm avg}\frac{\nu L\rho}{D^2}$$

Their ratio is

$$\frac{\Delta P_2}{\Delta P_1} = \frac{64}{32} = \mathbf{2}$$

The rate of heat transfer between the fluid and the walls of the channel is expressed as

$$\dot{Q}_1 = hA_s\Delta T_{lm} = \frac{k}{D}NuA_s\Delta T_{lm} = \frac{k}{D}2.98A_s\Delta T_{lm}$$

When the effect of the change in $\Delta T_{\rm lm}$ on the rate of heat transfer is disregarded, the rate of heat transfer remains the same. Therefore,

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \mathbf{1}$$

Therefore, doubling the velocity will double the pressure drop but it will not affect the heat transfer rate.

8-118 Repeat Prob. 8-117 for turbulent flow. Turbulent flow of a fluid through an isothermal square channel is considered. The change in the pressure drop and the rate of heat transfer are to be determined when the free-stream velocity is doubled.

Assumptions **1** The flow is fully developed. **2** The effect of the change in ΔT_{lm} on the rate of heat transfer is not considered.

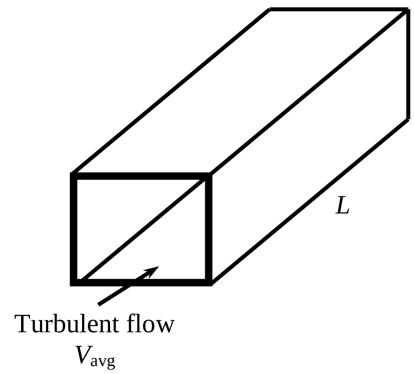
Analysis The pressure drop of the fluid for turbulent flow is expressed as

$$\Delta P_1 = f\,\frac{L}{D}\frac{\rho V_{avg}^2}{2} = 0.184\,\mathrm{Re}^{-0.2}\,\frac{L}{D}\frac{\rho V_{avg}^2}{2} = 0.184\,\frac{V_{avg}^{-0.2}D^{-0.2}}{\nu^{-0.2}}\,\frac{L}{D}\frac{\rho V_{avg}^2}{2} = 0.092 V_{avg}^{1.8}\left(\frac{D}{\nu}\right)^{-0.2}\frac{L\rho}{D}$$

When the free-stream velocity of the fluid is doubled, the pressure drop becomes

$$\Delta P_2 = f\,\frac{L}{D}\frac{\rho(2V_{avg})^2}{2} = 0.184\,\mathrm{Re}^{-0.2}\,\frac{L}{D}\frac{\rho 4V_{avg}^2}{2} = 0.184\,\frac{(\mathbf{2}V_{avg})^{-0.2}D^{-0.2}}{\nu^{-0.2}}\,\frac{L}{D}\frac{\rho 4V_{avg}^2}{2}$$

$$= 0.368(2)^{-0.2}V_{avg}^{1.8}\left(\frac{D}{\nu}\right)^{-0.2}\frac{L\rho}{D}$$



Their ratio is

$$\frac{\Delta P_2}{\Delta P_1} = \frac{0.368(2)^{-0.2}V_{avg}^{1.8}}{0.092V_{avg}^{1.8}} = 4(2)^{-0.2} = \mathbf{3.48}$$

The rate of heat transfer between the fluid and the walls of the channel is expressed as

$$\dot{Q}_1 = hA\Delta T_{lm} = \frac{k}{D}NuA\Delta T_{lm} = \frac{k}{D}0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}\,A\Delta T_{lm}$$

$$= 0.023V_{avg}^{0.8}\left(\frac{D}{\nu}\right)^{0.8}\frac{k}{D}\mathrm{Pr}^{1/3}\,A\Delta T_{lm}$$

When the free-stream velocity of the fluid is doubled, the heat transfer rate becomes

$$\dot{Q}_2 = 0.023(2V_{avg})^{0.8}\left(\frac{D}{\nu}\right)^{0.8}\frac{k}{D}\mathrm{Pr}^{1/3}\,A\Delta T_{lm}$$

Their ratio is

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{(2V_{avg})^{0.8}}{V_{avg}^{0.8}} = 2^{0.8} = \mathbf{1.74}$$

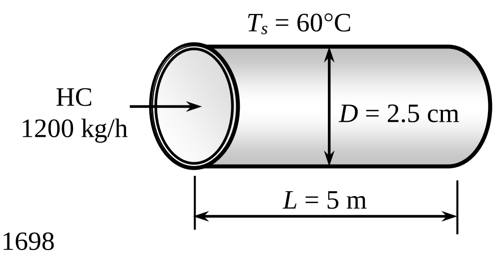
Therefore, doubling the velocity will increase the pressure drop 3.8 times but it will increase the heat transfer rate by only 74%.

8-119 A liquid hydrocarbon is heated as it flows in a tube that is maintained at a specified temperature. The flow rate and the exit temperature of the liquid are given. The exit temperature is to be determined for a different flow rate.

Assumptions **1** Steady flow conditions exist. **2** The surface temperature is constant and uniform. **3** The inner surfaces of the tube are smooth. **4** Heat transfer to the surroundings is negligible.

Properties The properties of the liquid are given to c_p = 2.0 kJ/kg·K, μ = 10 mPa·s = 0.01 kg/m·s, and ρ = 900 kg/m^3.

Analysis We first determine the heat transfer coefficient from an energy balance:



$$\dot{m}c_p(T_e - T_i) = hA_s(T_s - T_{b,\text{avg}})$$

$$(1200/3600)(2000)(30-20) = h(\pi \times 0.025 \times 5)\left(60 - \frac{20+30}{2}\right)$$

$$h = 485 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

The Reynolds number for the first case is

$$\text{Re}_1 = \frac{\rho D}{\mu}V = \frac{\rho D}{\mu}\frac{\dot{m}}{\rho\pi D^2/4} = \frac{4\dot{m}}{\pi D\mu} = \frac{4(1200/3600\text{ kg/s})}{\pi(0.025\text{ m})(0.01\text{kg/m}\cdot\text{s})} = 1698$$

which is smaller than 2300 and therefore the flow is laminar. The Reynolds number in the second case is

$$\text{Re}_2 = \frac{4\dot{m}}{\pi D\mu} = \frac{4(400/3600\text{ kg/s})}{\pi(0.025\text{ m})(0.01\text{kg/m}\cdot\text{s})} = 566$$

Using the relationship between the Nusselt and Reynolds numbers for the laminar flow,

$$\frac{\text{Nu}_2}{\text{Nu}_1} = \frac{\text{Re}_2^{1/3}}{\text{Re}_1^{1/3}} \longrightarrow \frac{h_2}{h_1} = \frac{\dot{m}_2^{1/3}}{\dot{m}_1^{1/3}} \longrightarrow h_2 = \left(\frac{400}{1200}\right)^{1/3}(485) = 336.3 \text{ W/m}^2 \cdot \text{C}$$

Using this new heat transfer coefficient value, the exit temperature for the second case is determined to be

$$\dot{m}c_p(T_e - T_i) = hA_s(T_s - T_{b,\text{avg}})$$

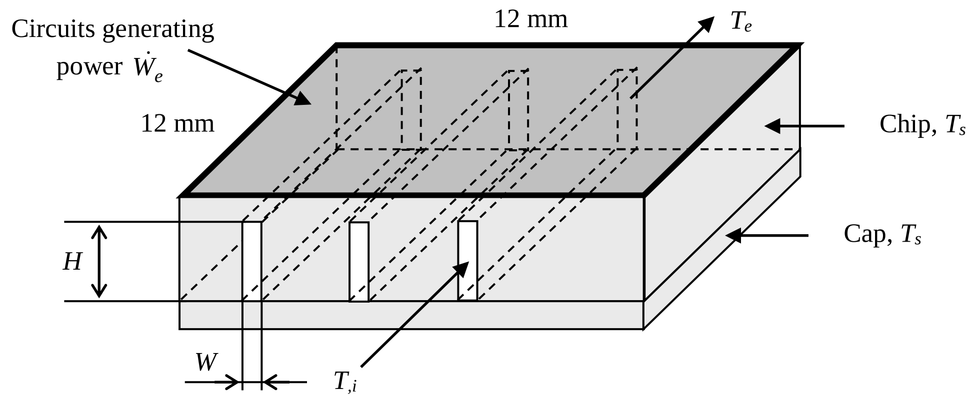
$$(400/3600)(2000)(T_e - 20) = (336.3)(\pi \times 0.025 \times 5)\left(60 - \frac{20+T_e}{2}\right)$$

$$T_e = \mathbf{38.3°C}$$

8-120 A silicon chip is cooled by passing water through microchannels etched in the back of the chip. The outlet temperature of water and the chip power dissipation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The flow of water is fully developed. **3** All the heat generated by the circuits on the top surface of the chip is transferred to the water.

Properties The properties of water at an anticipated average temperature of 25°C (298 K) based on the problem statement are k = 0.607 W/m·K, $\mu = 0.891 \times 10^{-3}$ kg/m·s, and c_p = 4180 J/kg·°C (Table A-9).



Analysis (a) The mass flow rate for one channel, the hydraulic diameter, and the Reynolds number are

$$\dot{m} = \frac{\dot{m}_{\text{total}}}{n_{\text{channel}}} = \frac{0.005\text{ kg/s}}{60} = 8.333 \times 10^{-5}\text{ kg/s}$$

$$D_h = \frac{4A}{p} = \frac{4(H \times W)}{2(H + W)} = \frac{4(50 \times 200)}{2(50 + 200)} = 80\ \mu\text{m} = 8 \times 10^{-5}\text{ m}$$

$$\text{Re} = \frac{\rho V D_h}{\mu} = \frac{\rho \dot{m} V D_h}{\rho A_c \mu} = \frac{\dot{m} D_h}{A_c \mu} = \frac{(8.333 \times 10^{-5}\text{ kg/s})(8 \times 10^{-5}\text{ m})}{(50 \times 200 \times 10^{-12}\text{ m}^2)(0.891 \times 10^{-3}\text{ kg/m}\cdot\text{s})} = 748.2$$

which is smaller than 2300. Therefore, the flow is laminar. We take fully developed laminar flow in the entire duct. The Nusselt number in this case is

$$Nu = 3.66$$

Heat transfer coefficient is

$$h = \frac{k}{D} Nu = \frac{0.607\text{ W/m.°C}}{8 \times 10^{-5}\text{ m}}(3.66) = 27{,}770\text{ W/m}^2.\text{°C}$$

Next we determine the exit temperature of water

$$A = 2WL + 2HL = 2(0.05 \times 12) + 2(0.2 \times 12) = 6\text{ mm}^2 = 6 \times 10^{-6}\text{ m}^2$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 350 - (350 - 290)\exp\left[-\frac{(27{,}770)(6 \times 10^{-6})}{(8.333 \times 10^{-5})(4180)}\right] = \mathbf{312.8\ K}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m} c_p (T_e - T_i) = (8.333 \times 10^{-5}\text{ kg/s})(4180\text{ J/kg.°C})(312.8 - 290)\text{°C} = 7.94\text{ W}$$

(b) Noting that there are 60 such channels, the chip power dissipation becomes

$$\dot{W}_e = N_{\text{channel}} \dot{Q}_{\text{one channel}} = 60(7.94\text{ W}) = \mathbf{477\ W}$$

Discussion The average temperature of water is (290+312.8)/2 = 301.4 K = 28.4°C, which is very close to the assumed temperature of 25°C. There is no need to repeat the calculations.

8-121 A fluid flows through a tube subjected to uniform heat flux, (*a*) the convection heat transfer coefficient, (*b*) the value of $T_s - T_m$, and (*c*) the value of $T_e - T_i$ are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant.

Properties The properties of the fluid are given: $\rho = 1000$ kg/m^3, $\mu = 1.4 \times 10^{-3}$ kg/m·s, $c_p = 4.2$ kJ/kg·K, and $k = 0.58$ W/m·K.

Analysis (*a*) The Reynolds number, hydrodynamic and thermal entry lengths are

$$\text{Re} = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{(1000\ \text{kg/m}^3)(0.3\ \text{m/s})(0.01\ \text{m})}{1.4\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 2143 < 2300 \qquad \text{(laminar flow)}$$

$$L_{h,\text{lam}} \approx 0.05\,\text{Re}\,D_h = 1.07\ \text{m} < 14\ \text{m} \qquad \text{and} \qquad L_{t,\text{lam}} \approx 0.05\,\text{Re}\,\text{Pr}\,D = 10.9\ \text{m} < 14\ \text{m}$$

where

$$\text{Pr} = \frac{c_p \mu}{k} = \frac{(1.4\times10^{-3}\ \text{kg/m}\cdot\text{s})(4.2\times10^3\ \text{J/kg}\cdot\text{K})}{0.58\ \text{W/m}\cdot\text{K}} = 10.14$$

Therefore flow is laminar and fully developed, and the Nusselt number and the convection heat transfer coefficient for constant surface heat flux is

$$Nu = hD/k = 4.36 \quad \rightarrow \quad h = 4.36\frac{k}{D} = 4.36\frac{0.58\ \text{W/m}\cdot\text{K}}{0.01\ \text{m}} = \mathbf{253\ W/m^2K}$$

(*b*) The value of $T_s - T_m$ can be determined using

$$T_s - T_m = \frac{11}{48}\frac{\dot{q}_s D}{k} = \frac{11}{48}\frac{(1500\ \text{W/m}^2)(0.01\ \text{m})}{0.58\ \text{W/m}\cdot\text{K}} = \mathbf{5.93^\circ C}$$

(*c*) The value of $T_e - T_i$ can be determined using

$$\dot{Q} = \dot{m}c_p(T_e - T_i) \quad \rightarrow \quad T_e - T_i = \frac{\dot{Q}}{\dot{m}c_p} = \frac{4\dot{q}_s(\pi DL)}{\rho V_{\text{avg}}\pi D^2 c_p} = \frac{4\dot{q}L}{\rho V_{\text{avg}} D c_p}$$

$$T_e - T_i = \frac{4(1500\ \text{W/m}^2)(14\ \text{m})}{(1000\ \text{kg/m}^3)(0.3\ \text{m/s})(0.01\ \text{m})(4.2\times10^3\ \text{J/kg}\cdot\text{K})} = \mathbf{6.67^\circ C}$$

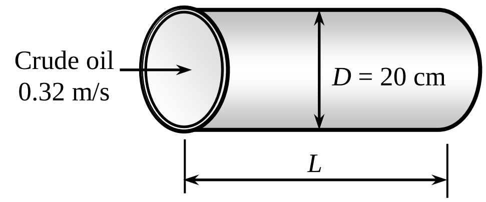
Discussion The value of $T_s - T_m$ can also be calculated using $\dot{q} = h(T_s - T_m)$.

8-122 Crude oil is cooled as it flows in a pipe. The rate of heat transfer and the pipe length are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface temperature is constant and uniform. **3** The inner surfaces of the tubes are smooth. **4** Heat transfer to the surroundings is negligible.

Properties The properties of crude oil are given in the table.

Analysis The mass flow rate of air is



$$\begin{aligned}\dot{m} &= \rho AV \\ &= (890\ \text{kg/m}^3)\left[\pi(0.20\ \text{m})^2/4\right](0.32\ \text{m/s}) \\ &= 8.947\ \text{kg/s}\end{aligned}$$

Finding the specific heat of oil at 21°C by interpolation, the rate of heat transfer is determined to be

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (8.947\ \text{kg/s})(1895\ \text{J/kg.°C})(22-20)\text{°C} = \mathbf{33{,}910\ W}$$

The Prandtl and Reynolds number are

$$\Pr = \frac{\mu c_p}{k} = \frac{(0.022\ \text{kg/m}\cdot\text{s})(1895\ \text{J/kg}\cdot\text{K})}{0.145\ \text{W/m}\cdot\text{K}} = 287.5$$

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(890\ \text{kg/m}^3)(0.32\ \text{m/s})(0.2\text{m})}{0.022\ \text{kg/m}\cdot\text{s}} = 2589$$

which is a little greater than 2300 but much smaller than 10,000. Therefore, we assume laminar flow. Assuming fully developed flow in the entire tube, the Nusselt number is determined from

$$Nu = \frac{hD}{k} = 3.66 + \frac{0.065(D/L)\,\text{Re}\,\Pr}{1+0.04[(D/L)\,\text{Re}\,\Pr]^{2/3}} = 3.66 + \frac{0.065(0.2/L)(2589)(287.5)}{1+0.04[(0.2/L)(2589)(287.5)]^{2/3}}$$

The heat transfer coefficient is expressed as

$$h = \frac{k}{D}\text{Nu} = \left(\frac{0.145}{0.2}\right)\left(3.66 + \frac{0.065(0.2/L)(2589)(287.5)}{1+0.04[(0.2/L)(2589)(287.5)]^{2/3}}\right) = 0.725\left[3.66 + \frac{9676/L}{1+112.3L^{-2/3}}\right]$$

From Newton's law of cooling

$$\dot{Q} = hA(T_s - T_{b,\text{avg}})$$

$$33{,}910 = h\pi(0.2)L\left(\frac{22+20}{2} - 2\right)$$

$$hL = 2840$$

Setting both h equations to each other

$$hL = 0.725L\left[3.66 + \frac{9676/L}{1+112.3L^{-2/3}}\right] = 2840$$

By trial error or using an equation solver such as EES, we obtain

L = **307 m**

8-123 Air (1 atm) enters into a 5-mm diameter circular tube, the convection heat transfer coefficient for (*a*) a 10-cm long tube and (*b*) a 50-cm long are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Constant tube surface temperature.

Properties The properties of air at 50°C: c_p = 1007 J/kg·K, k = 0.02735 W/m·K, ρ = 1.092 kg/m^3, $\mu = 1.963 \times 10^{-5}$ kg/m·s, $\nu = 1.798 \times 10^{-5}$ m^2/s, and Pr = 0.7228; at T_s = 160°C: $\mu_s = 2.420 \times 10^{-5}$ kg/m·s (Table A-15).

Analysis The Reynolds number, hydrodynamic and thermal entry lengths are

$$\text{Re} = \frac{V_{\text{avg}} D}{\nu} = \frac{(5\ \text{m/s})(0.005\ \text{m})}{(1.798\times 10^{-5}\ \text{m}^2/\text{s})} = 1390 < 2300 \qquad \text{(laminar flow)}$$

$$L_{h,\text{lam}} \approx 0.05\,\text{Re}\,D = 34.8\ \text{cm} \qquad \text{and} \qquad L_{t,\text{lam}} \approx 0.05\,\text{Re}\,\text{Pr}\,D = 25.1\ \text{cm}$$

(*a*) For the 10-cm long tube, the flow is laminar, hydrodynamically and thermally developing. The appropriate equation to determine the Nusselt number is from (Sieder and Tate, 1936)

$$\text{Nu} = 1.86\left(\frac{\text{Re}\,\text{Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 1.86\left[\frac{(1390)(0.7228)(0.005\ \text{m})}{0.1\ \text{m}}\right]^{1/3}\left(\frac{1.963}{2.420}\right)^{0.14} = 6.665$$

$$h = \frac{k}{D}\text{Nu} = \mathbf{36.5\ W/m^2 \cdot K}$$

(*b*) For the 50-cm long tube, the flow is laminar and fully developed. The Nusselt number for constant surface temperature is

$$\text{Nu} = 3.66 \quad \rightarrow \quad h = \frac{k}{D}\text{Nu} = \mathbf{20.0\ W/m^2 \cdot K}$$

Discussion The convection heat transfer coefficient in the hydrodynamic and thermal entrance regions is larger than that in the fully developed flow.

8-124 Air is flowing through a thin smooth copper tube that is submerged in the nearby lake; the necessary copper tube length for the air to exit with an outlet mean temperature of 20°C is to be determined.

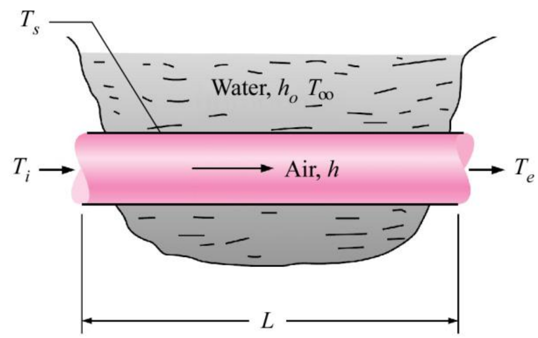
Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Constant tube surface temperature. **4** Conduction through the copper tube wall is negligible.

Properties The properties of air at $T_b = (T_i + T_e)/2 = 25°C$: $c_p = 1007$ J/kg·K, $k = 0.02551$ W/m·K, $\rho = 1.184$ kg/m^3, $\nu = 1.562 \times 10^{-5}$ m^2/s, and Pr = 0.7296 (Table A-15).

Analysis The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D}{\nu} = \frac{(2.5\ \text{m/s})(0.1\ \text{m})}{(1.562 \times 10^{-5}\ \text{m}^2/\text{s})} = 16005 > 10{,}000 \qquad \text{(turbulent flow)}$$

Since the flow inside the copper tube is turbulent, we can use the Dittus-Boelter equation to calculate the Nusselt number:



$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(16005)^{0.8}(0.7296)^{0.3} = 48.31$$

$$\rightarrow \quad h = 12.32\ \text{W/m}^2 \cdot \text{K}$$

The length of the copper tube can be determined using

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{h\pi DL}{\dot{m}c_p}\right)$$

where the surface temperature of the tube can be determined by applying energy balance on the tube surface:

$$\dot{m}c_p(T_i - T_e) = h_o(\pi DL)(T_s - T_\infty)$$

where $\quad \dot{m} = \rho V_{\text{avg}} \pi D^2 / 4 = 0.02325\ \text{kg/s}$

Copy the following lines and paste on a blank EES screen to solve the above equation:

```
c_p=1007
D=0.1
h=12.32
h_o=1000
T_i=30
T_e=20
T_inf=15
V_avg=2.5
mdot=0.02325
mdot*c_p*(T_i-T_e)=h_o*pi#*D*L*(T_s-T_inf)
T_e=T_s-(T_s-T_i)*exp(-h*pi#*D*L/(mdot*c_p))
```

Solving by EES software, the necessary copper tube length is

L = **6.74 m**

Discussion It is reasonable to neglect heat conduction through the copper tube wall, since copper has high thermal conductivity and the tube wall is thin.

8-125 Liquid mercury flowing through a tube, the tube length is to be determined using (*a*) the appropriate Nusselt number relation for liquid metals and (*b*) the Dittus-Boelter equation. The results of (*a*) and (*b*) are to be compared.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Constant tube surface temperature. **4** Fully developed flow.

Properties The properties of liquid mercury at $T_b = (T_i + T_e)/2 = 150°C$: $c_p = 136.1$ J/kg·K, $k = 10.0778$ W/m·K, $\mu = 1.126\times10^{-3}$ kg/m·s, and Pr = 0.0152; at $T_s = 250°C$: $Pr_s = 0.0119$ (Table A-14).

Analysis The Reynolds number is

$$\text{Re} = \frac{4\dot{m}}{\pi D\mu} = \frac{4(0.6\text{ kg/s})}{\pi(0.05\text{ m})(1.126\times10^{-3}\text{ kg/m}\cdot\text{s})} = 13570 > 10{,}000 \qquad \text{(turbulent flow)}$$

(*a*) The flow is turbulent and the appropriate Nusselt number relation for liquid metals is

$$\text{Nu} = 4.8 + 0.0156\,\text{Re}^{0.85}\,\text{Pr}_s^{0.93} = 4.8 + 0.0156(13570)^{0.85}(0.0119)^{0.93} = 5.624$$

$$h = (k/D)\text{Nu} = 1134\text{ W/m}^2\cdot\text{K}$$

Hence, length of the tube is

$$L = -\frac{\dot{m}c_p}{\pi Dh}\ln\frac{T_s - T_e}{T_s - T_i} = -\frac{(0.6\text{ kg/s})(136.1\text{ J/kg}\cdot\text{K})}{\pi(0.05\text{ m})(1134\text{ W/m}^2\cdot\text{K})}\ln\frac{250-200}{250-100} = \mathbf{0.504\,m}$$

(*b*) Using the Dittus-Boelter equation, we have

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(13570)^{0.8}(0.0152)^{0.4} = 8.72 \quad \rightarrow \quad h = 1758\text{ W/m}^2\cdot\text{K}$$

Hence, length of the tube is

$$L = -\frac{\dot{m}c_p}{\pi Dh}\ln\frac{T_s - T_e}{T_s - T_i} = -\frac{(0.6\text{ kg/s})(136.1\text{ J/kg}\cdot\text{K})}{\pi(0.05\text{ m})(1758\text{ W/m}^2\cdot\text{K})}\ln\frac{250-200}{250-100} = \mathbf{0.325\,m}$$

(*c*) Comparing the results calculated for (*a*) and (*b*) showed that when using the Dittus-Boelter equation, the tube length is underestimated by approximately 36%.

Discussion When using the relations for calculating Nusselt number, it is necessary to consider their applicability and their associated limits and conditions.

8-126 The convection heat transfer coefficients for air (*a*) flowing through and (*b*) flowing across a thin-walled tube are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Constant tube surface temperature. **4** Fully developed flow.

Properties The properties of air at 50°C: k = 0.02735 W/m·K, $\nu = 1.798 \times 10^{-5}$ m^2/s, and Pr = 0.7228 (Table A-15).

Analysis The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D}{\nu} = \frac{(25\ \text{m/s})(0.05\ \text{m})}{(1.798 \times 10^{-5}\ \text{m}^2/\text{s})} = 69522$$

(*a*) The flow inside the tube is turbulent (Re > 10,000), and using the Dittus-Boelter equation, we have

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(69522)^{0.8}(0.7228)^{0.4} = 151$$

Hence the convection heat transfer coefficient is

$$h = \frac{k}{D}\text{Nu} = \left(\frac{0.02735\ \text{W/m}\cdot\text{K}}{0.05\ \text{m}}\right)151 = \mathbf{82.6\ W/m^2 \cdot K} \qquad \text{(internal forced convection)}$$

(*b*) Using the Zukauskas (1972) equation from Table 7-1, we have

$$\text{Nu} = 0.027\,\text{Re}^{0.805}\,\text{Pr}^{1/3} = 0.027(69522)^{0.805}(0.7228)^{1/3} = 192$$

Hence the convection heat transfer coefficient is

$$h = \frac{k}{D}\text{Nu} = \left(\frac{0.02735\ \text{W/m}\cdot\text{K}}{0.05\ \text{m}}\right)192 = \mathbf{105\ W/m^2 \cdot K} \qquad \text{(external forced convection)}$$

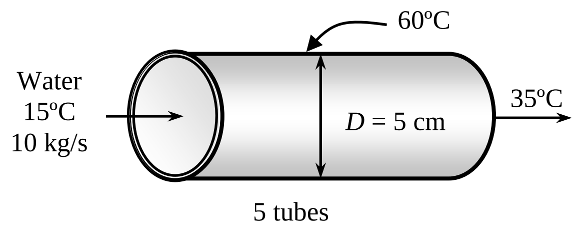
Discussion The convection heat transfer coefficient for the case when air is flowing across the tube outer surface is approximately 27% larger than the case when air is flowing inside the tube.

8-127 Water is heated by passing it through five identical tubes that are maintained at a specified temperature. The rate of heat transfer and the length of the tubes necessary are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface temperature is constant and uniform. **3** The inner surfaces of the tubes are smooth. **4** Heat transfer to the surroundings is negligible.

Properties The properties of water at the bulk mean fluid temperature of (15+35)/2=25ºC are (Table A-9)

$$\rho = 997 \text{ kg/m}^3$$
$$k = 0.607 \text{ W/m.°C}$$
$$\mu = 0.891\times10^{-3} \text{ m}^2\text{/s}$$
$$c_p = 4180 \text{ J/kg.°C}$$
$$\text{Pr} = 6.14$$



Analysis (*a*) The rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (10 \text{ kg/s})(4180 \text{ J/kg.°C})(35-15)\text{°C} = \mathbf{836{,}000\ W}$$

(*b*) The water velocity is

$$V = \frac{\dot{m}}{\rho A_c} = \frac{(10/5) \text{ kg/s}}{(997 \text{ kg/m}^3)\pi(0.05 \text{ m})^2 / 4} = 1.02 \text{ m/s}$$

The Reynolds number is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(997 \text{ kg/m}^3)(1.02 \text{ m/s})(0.05\text{m})}{0.891\times10^{-3} \text{ kg/m}\cdot\text{s}} = 57{,}067$$

which is greater than 10,000. Therefore, we have turbulent flow. Assuming fully developed flow in the entire tube, the Nusselt number is determined from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(57{,}067)^{0.8}(6.14)^{0.4} = 303.5$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.607 \text{ W/m.°C}}{0.05 \text{ m}}(303.5) = 3684 \text{ W/m}^2\text{.°C}$$

Considering that there are 5 tubes, the logarithmic mean temperature difference, the surface area and the lenghth of the tubes are determined as follows:

$$\Delta T_{\text{lm}} = \frac{T_i - T_e}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} = \frac{15-35}{\ln\left(\dfrac{60-35}{60-15}\right)} = 34.03\text{°C}$$

$$\dot{Q} = hA_s\Delta T_{\text{lm}} \longrightarrow 836{,}000 \text{ W} = (3684 \text{ W/m}^2\text{.°C})A_s(34.03\text{°C}) \longrightarrow A_s = 6.668 \text{ m}^2$$

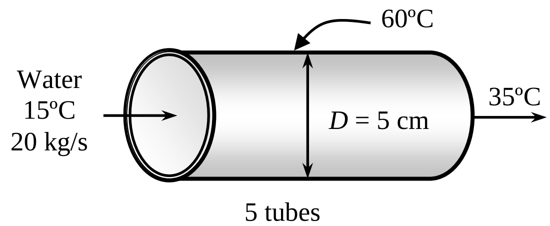
$$A_s = 5\pi DL \longrightarrow L = \frac{A_s}{5\pi D} = \frac{6.668 \text{ m}^2}{5\pi(0.05 \text{ m})} = \mathbf{8.49\ m}$$

8-128 Water is heated by passing it through five identical tubes that are maintained at a specified temperature. The rate of heat transfer and the length of the tubes necessary are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface temperature is constant and uniform. **3** The inner surfaces of the tubes are smooth. **4** Heat transfer to the surroundings is negligible.

Properties The properties of water at the bulk mean fluid temperature of (15+35)/2=25ºC are (Table A-9)

$$\rho = 997 \text{ kg/m}^3$$
$$k = 0.607 \text{ W/m.°C}$$
$$\mu = 0.891\times10^{-3} \text{ m}^2\text{/s}$$
$$c_p = 4180 \text{ J/kg.°C}$$
$$\Pr = 6.14$$



Analysis (*a*) The rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (20 \text{ kg/s})(4180 \text{ J/kg.°C})(35-15)\text{°C} = \mathbf{1{,}672{,}000\ W}$$

(*b*) The water velocity is

$$V = \frac{\dot{m}}{\rho A_c} = \frac{(20/5) \text{ kg/s}}{(997 \text{ kg/m}^3)\left[\pi(0.05 \text{ m})^2/4\right]} = 2.04 \text{ m/s}$$

The Reynolds number is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(997 \text{ kg/m}^3)(2.04 \text{ m/s})(0.05\text{m})}{0.891\times10^{-3} \text{ kg/m}\cdot\text{s}} = 114{,}320$$

which is greater than 10,000. Therefore, we have turbulent flow. Assuming fully developed flow in the entire tube, the Nusselt number is determined from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\Pr^{0.4} = 0.023(114{,}320)^{0.8}(6.14)^{0.4} = 529.0$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.607 \text{ W/m.°C}}{0.05 \text{ m}}(529.0) = 6423 \text{ W/m}^2\text{.°C}$$

Considering that there are 5 tubes, the logarithmic mean temperature difference, the surface area and the lenghth of the tubes are determined as follows:

$$\Delta T_{\text{lm}} = \frac{T_i - T_e}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{15-35}{\ln\left(\frac{60-35}{60-15}\right)} = 34.03\text{°C}$$

$$\dot{Q} = hA_s\Delta T_{\text{lm}} \longrightarrow 1{,}672{,}000 \text{ W} = (6423 \text{ W/m}^2\text{.°C})A_s(34.03\text{°C}) \longrightarrow A_s = 7.650 \text{ m}^2$$

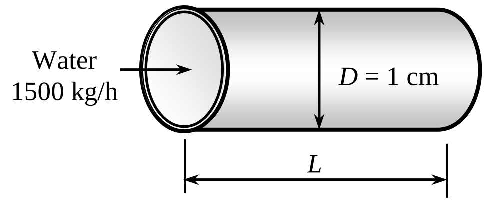
$$A_s = 5\pi DL \longrightarrow L = \frac{A_s}{5\pi D} = \frac{7.650 \text{ m}^2}{5\pi(0.05 \text{ m})} = \mathbf{9.74\ m}$$

8-129 Water is heated as it flows in a smooth tube that is maintained at a specified temperature. The necessary tube length and the water outlet temperature if the tube length is doubled are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface temperature is constant and uniform. **3** The inner surfaces of the tube are smooth. **4** Heat transfer to the surroundings is negligible.

Properties The properties of water at the bulk mean fluid temperature of (10+40)/2=25ºC are (Table A-9)

$$\rho = 997 \text{ kg/m}^3$$
$$k = 0.607 \text{ W/m.°C}$$
$$\mu = 0.891\times10^{-3} \text{ m}^2\text{/s}$$
$$c_p = 4180 \text{ J/kg.°C}$$
$$\text{Pr} = 6.14$$



Analysis (a) The rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (1500/3600 \text{ kg/s})(4180 \text{ J/kg.°C})(40-10)\text{°C} = 52{,}250 \text{ W}$$

The water velocity is

$$V = \frac{\dot{m}}{\rho A_c} = \frac{(1500/3600) \text{ kg/s}}{(997 \text{ kg/m}^3)\left[\pi(0.01\text{m})^2/4\right]} = 5.321 \text{ m/s}$$

The Reynolds number is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(997 \text{ kg/m}^3)(5.321\text{m/s})(0.01\text{m})}{0.891\times10^{-3} \text{ kg/m}\cdot\text{s}} = 59{,}540$$

which is greater than 10,000. Therefore, we have turbulent flow. Assuming fully developed flow in the entire tube, the Nusselt number is determined from

$$Nu = \frac{hD_h}{k} = 0.023\text{Re}^{0.8}\text{Pr}^{0.4} = 0.023(59{,}540)^{0.8}(6.14)^{0.4} = 313.9$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.607 \text{ W/m.°C}}{0.01 \text{ m}}(313.9) = 19{,}054 \text{ W/m}^2\text{.°C}$$

The logarithmic mean temperature difference, the surface area and the lenghth of the tube are determined as follows:

$$\Delta T_{\text{lm}} = \frac{T_i - T_e}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} = \frac{10-40}{\ln\left(\dfrac{49-40}{49-10}\right)} = 20.46\text{°F}$$

$$\dot{Q} = hA_s\Delta T_{\text{lm}} \longrightarrow 52{,}250 \text{ W} = (19{,}054 \text{ W/m}^2\text{.°C})A_s(20.46\text{°C}) \longrightarrow A_s = 0.1340 \text{ m}^2$$

$$A_s = \pi DL \longrightarrow 0.1340 \text{ m}^2 = \pi(0.01 \text{ m})L \longrightarrow L = \mathbf{4.27\ m}$$

(b) If the tube length is doubled, the surface area doubles, and the outlet water temperature may be obtained from an energy balance to be

$$\dot{m}c_p(T_e - T_i) = hA_s\Delta T_{\text{lm}} = hA_s\frac{T_i - T_e}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)}$$

$$(1500/3600)(4180)(T_e - 10) = (19{,}054)(2\times0.1340)\left(\frac{10 - T_e}{\ln\left(\dfrac{49 - T_e}{49-10}\right)}\right)$$

By trial-error or using an equation solver such as EES, we obtain

T_e = **46.9°C**

8-130E The exhaust gases of an automotive engine enter a steel exhaust pipe. The velocity of exhaust gases at the inlet and the temperature of exhaust gases at the exit are to be determined.

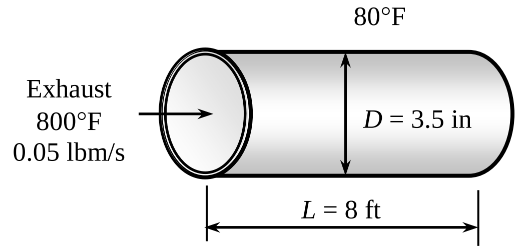
Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the pipe are smooth. **3** The thermal resistance of the pipe is negligible. **4** Exhaust gases have the properties of air, which is an ideal gas with constant properties.

Properties We take the bulk mean temperature for exhaust gases to be 700°F since the mean temperature of gases at the inlet will drop somewhat as a result of heat loss through the exhaust pipe whose surface is at a lower temperature. The properties of air at this temperature and 1 atm pressure are (Table A-15E)

$$\rho = 0.03421\ \text{lbm/ft}^3 \qquad c_p = 0.2535\ \text{Btu/lbm.°F}$$

$$k = 0.0280\ \text{Btu/h.ft.°F} \qquad \text{Pr} = 0.694$$

$$\nu = 6.225\times10^{-4}\ \text{ft}^2\text{/s}$$



Noting that 1 atm = 14.7 psia, the pressure in atm is P = (15.5 psia)/(14.7 psia) = 1.054 atm. Then,

$$\rho = (0.03421\ \text{lbm/ft}^3)(1.054) = 0.03606\ \text{lbm/ft}^3$$

$$\nu = (6.225\times10^{-4}\ \text{ft}^2\text{/s})/(1.054) = 5.906\times10^{-4}\ \text{ft}^2\text{/s}$$

Analysis (a) The velocity of exhaust gases at the inlet of the exhaust pipe is

$$\dot{m} = \rho V_{\text{avg}} A_c \longrightarrow V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{0.05\ \text{lbm/s}}{(0.03606\ \text{lbm/ft}^3)\left(\pi(3.5/12\ \text{ft})^2/4\right)} = \mathbf{20.75\ ft/s}$$

(b) The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(20.75\ \text{ft/s})(3.5/12\ \text{ft})}{5.906\times10^{-4}\ \text{ft}^2\text{/s}} = 10{,}249$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(3.5/12\ \text{ft}) = 2.917\ \text{ft}$$

which are shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(10{,}249)^{0.8}(0.6940)^{0.3} = 33.32$$

and $$h_i = h = \frac{k}{D_h} Nu = \frac{0.0280\ \text{Btu/h.ft.°F}}{(3.5/12)\ \text{ft}}(33.32) = 3.198\ \text{Btu/h.ft}^2\text{.°F}$$

$$A_s = \pi DL = \pi(3.5/12\ \text{ft})(8\ \text{ft}) = 7.33\ \text{ft}^2$$

In steady operation, heat transfer from exhaust gases to the duct must be equal to the heat transfer from the duct to the surroundings, which must be equal to the energy loss of the exhaust gases in the pipe. That is,

$$\dot{Q} = \dot{Q}_{\text{internal}} = \dot{Q}_{\text{external}} = \Delta\dot{E}_{\text{exhaust gases}}$$

Assuming the duct to be at an average temperature of T_s, the quantities above can be expressed as

$\dot{Q}_{\text{internal}}$: $$\dot{Q} = h_i A_s \Delta T_{lm} = h_i A_s \frac{T_e - T_i}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} \rightarrow \dot{Q} = (3.198\ \text{Btu/h.ft}^2\text{.°F})(7.33\ \text{ft}^2)\frac{T_e - 800\text{°F}}{\ln\left(\dfrac{T_s - T_e}{T_s - 800}\right)}$$

$\dot{Q}_{\text{external}}$: $$\dot{Q} = h_o A_s (T_s - T_o) \rightarrow \dot{Q} = (3\ \text{Btu/h.ft}^2\text{.°F})(7.33\ \text{ft}^2)(T_s - 80)\text{°F}$$

$\Delta\dot{E}_{\text{exhaust gases}}$: $$\dot{Q} = \dot{m}c_p(T_e - T_i) \rightarrow \dot{Q} = (0.05\times3600\ \text{lbm/h})(0.2535\ \text{Btu/lbm.°F})(800 - T_e)\text{°F}$$

This is a system of three equations with three unknowns whose solution is

$$\dot{Q} = 7234\ \text{Btu/h},\ T_e = \mathbf{641.5°F},\ \text{and}\ T_s = 408.9\text{°F}$$

8-131E Air is flowing through a smooth thin-walled copper tube that is submerged in water; the necessary copper tube length for the air to exit with an outlet mean temperature of 68°F is to be determined.

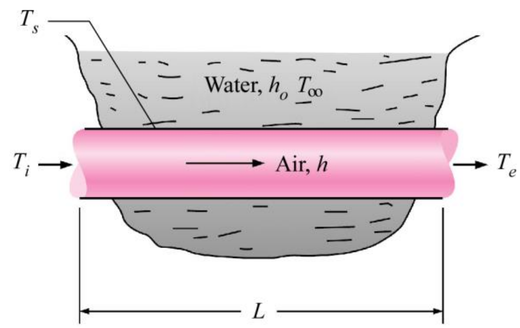
Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Constant tube surface temperature. **4** Conduction through the copper tube wall is negligible.

Properties The properties of air at $T_b = (T_i + T_e)/2 = 80$°F: $c_p = 0.2404$ Btu/lbm·R, $k = 0.01409$ Btu/h·ft·R, $\rho = 0.07783$ lbm/ft^3, $\nu = 1.535 \times 10^{-4}$ ft^2/s, and Pr = 0.7336 (Table A-15E).

Analysis The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D}{\nu} = \frac{(8\text{ ft/s})(4/12\text{ ft})}{(1.535\times10^{-4}\text{ ft}^2\text{/s})} = 17370 > 10{,}000 \qquad \text{(turbulent flow)}$$

Since the flow inside the copper tube is turbulent, we can use the Dittus-Boelter equation to calculate the Nusselt number:



$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(17370)^{0.8}(0.7336)^{0.3} = 51.67$$

$$h = 2.184\text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}$$

The length of the copper tube can be determined using

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{h\pi DL}{\dot{m}c_p}\right)$$

where the surface temperature of the tube can be determined by applying energy balance on the tube surface:

$$\dot{m}c_p(T_i - T_e) = h_o(\pi DL)(T_s - T_\infty)$$

where $\quad \dot{m} = \rho V_{\text{avg}}\pi D^2/4 = 195.6\text{ lbm/h}$

Copy the following lines and paste on a blank EES screen to solve the above equation:

```
c_p=0.2404
D=4/12
h=2.184
h_o=176
T_i=90
T_e=70
T_inf=60
V_avg=8
mdot=195.6
mdot*c_p*(T_i-T_e)=h_o*pi#*D*L*(T_s-T_inf)
T_e=T_s-(T_s-T_i)*exp(-h*pi#*D*L/(mdot*c_p))
```

Solving by EES software, the necessary copper tube length is

L = **22.9 ft**

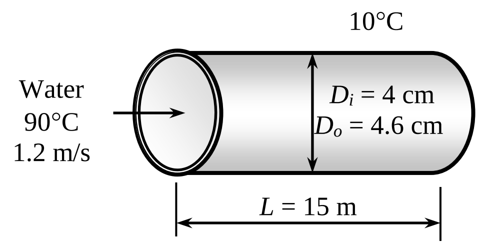
Discussion It is reasonable to neglect heat conduction through the copper tube wall, since copper has high thermal conductivity and the tube wall is thin.

8-132 Hot water enters a cast iron pipe whose outer surface is exposed to cold air with a specified heat transfer coefficient. The rate of heat loss from the water and the exit temperature of the water are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the pipe are smooth.

Properties We assume the water temperature not to drop significantly since the pipe is not very long. We will check this assumption later. The properties of water at 90°C are (Table A-9)

$$\rho = 965.3\ \text{kg/m}^3; \qquad k = 0.675\ \text{W/m.°C}$$
$$\nu = \mu/\rho = 0.326\times10^{-6}\ \text{m}^2/\text{s}; \quad c_p = 4206\ \text{J/kg.°C}$$
$$\text{Pr} = 1.96$$



Analysis (a) The mass flow rate of water is

$$\dot{m} = \rho A_c V_{\text{avg}} = (965.3\ \text{kg/m}^3)\frac{\pi(0.04\ \text{m})^2}{4}(1.2\ \text{m/s}) = 1.456\ \text{kg/s}$$

The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(1.2\ \text{m/s})(0.04\text{m})}{0.326\times10^{-6}\ \text{m}^2/\text{s}} = 147{,}240$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.04\ \text{m}) = 0.4\ \text{m}$$

which are much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire pipe. The friction factor corresponding to Re = 147,240 and ε/D = (0.026 cm)/(4 cm) = 0.0065 is determined from the Moody chart to be f = 0.032. Then the Nusselt number becomes

$$Nu = \frac{hD_h}{k} = 0.125 f\ \text{Re}\ \text{Pr}^{1/3} = 0.125\times0.032\times147{,}240\times1.96^{1/3} = 737.1$$

and
$$h_i = h = \frac{k}{D_h} Nu = \frac{0.675\ \text{W/m.°C}}{0.04\ \text{m}}(737.1) = 12{,}440\ \text{W/m}^2.\text{°C}$$

which is much greater than the convection heat transfer coefficient of 12 W/m^2.°C. Therefore, the convection thermal resistance inside the pipe is negligible, and thus the inner surface temperature of the pipe can be taken to be equal to the water temperature. Also, we expect the pipe to be nearly isothermal since it is made of thin metal (we check this later). Then the rate of heat loss from the pipe will be the sum of the convection and radiation from the outer surface at a temperature of 90°C, and is determined to be

$$A_o = \pi D_0 L = \pi(0.046\ \text{m})(15\text{m}) = 2.168\ \text{m}^2$$
$$\dot{Q}_{conv} = h_o A_o (T_s - T_{surr}) = (12\ \text{W/m}^2.\text{°C})(2.168\ \text{m}^2)(90-10)\text{°C} = 2081\ \text{W}$$
$$\dot{Q}_{rad} = \varepsilon A_0 \sigma (T_s{}^4 - T_{surr}{}^4)$$
$$= (0.7)(2.168\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)\left[(90+273\ \text{K})^4 - (10+273\ \text{K})^4\right] = 942\ \text{W}$$
$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 2081 + 942 = \mathbf{3023\ W}$$

(b) The temperature at which water leaves the basement is

$$\dot{Q} = \dot{m}c_p(T_i - T_e) \longrightarrow T_e = T_i - \frac{\dot{Q}}{\dot{m}c_p} = 90\text{°C} - \frac{3023\ \text{W}}{(1.456\ \text{kg/s})(4206\ \text{J/kg.°C})} = \mathbf{89.5°C}$$

The result justifies our assumption that the temperature drop of water is negligible. Also, the thermal resistance of the pipe and temperature drop across it are

$$R_{pipe} = \frac{\ln(D_2/D_1)}{2\pi kL} = \frac{\ln(4.6/4)}{4\pi(52\ \text{W/m.°C})(15\ \text{m})} = 2.85\times10^{-5}\ \text{°C/W}$$
$$\Delta T_{pipe} = \dot{Q}_{total} R_{pipe} = (3023\ \text{W})(2.85\times10^{-5}\ \text{°C/W}) = 0.09\text{°C}$$

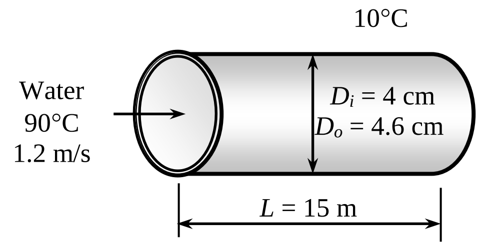
which justifies our assumption that the temperature drop across the pipe is negligible.

8-133 Hot water enters a copper pipe whose outer surface is exposed to cold air with a specified heat transfer coefficient. The rate of heat loss from the water and the exit temperature of the water are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the pipe are smooth.

Properties We assume the water temperature not to drop significantly since the pipe is not very long. We will check this assumption later. The properties of water at 90°C are (Table A-15)

$$\rho = 965.3\ \text{kg/m}^3; \qquad k = 0.675\ \text{W/m.°C}$$
$$\nu = \mu/\rho = 0.326\times10^{-6}\ \text{m}^2\text{/s}; \quad c_p = 4206\ \text{J/kg.°C}$$
$$\Pr = 1.96$$



Analysis (a) The mass flow rate of water is

$$\dot{m} = \rho A_c V_{\text{avg}} = (965.3\ \text{kg/m}^3)\frac{\pi(0.04\ \text{m})^2}{4}(1.2\ \text{m/s}) = 1.456\ \text{kg/s}$$

The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(1.2\ \text{m/s})(0.04\text{m})}{0.326\times10^{-6}\ \text{m}^2\text{/s}} = 147{,}240$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.04\ \text{m}) = 0.4\ \text{m}$$

which are much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire pipe. The friction factor corresponding to Re = 147,240 and ε/D = (0.026 cm)/(4 cm) = 0.0065 is determined from the Moody chart to be f = 0.032. Then the Nusselt number becomes

$$Nu = \frac{hD_h}{k} = 0.125 f\ \text{Re}\Pr^{1/3} = 0.125\times0.032\times147{,}240\times1.96^{1/3} = 737.1$$

and $$h_i = h = \frac{k}{D_h}Nu = \frac{0.675\ \text{W/m.°C}}{0.04\ \text{m}}(737.1) = 12{,}440\ \text{W/m}^2\text{.°C}$$

which is much greater than the convection heat transfer coefficient of 12 W/m^2.°C. Therefore, the convection thermal resistance inside the pipe is negligible, and thus the inner surface temperature of the pipe can be taken to be equal to the water temperature. Also, we expect the pipe to be nearly isothermal since it is made of thin metal (we check this later). Then the rate of heat loss from the pipe will be the sum of the convection and radiation from the outer surface at a temperature of 90°C, and is determined to be

$$A_o = \pi D_0 L = \pi(0.046\ \text{m})(15\text{m}) = 2.168\ \text{m}^2$$
$$\dot{Q}_{conv} = h_o A_o (T_s - T_{surr}) = (12\ \text{W/m}^2\text{.°C})(2.168\ \text{m}^2)(90-10)\text{°C} = 2081\ \text{W}$$
$$\dot{Q}_{rad} = \varepsilon A_0 \sigma (T_s^{\ 4} - T_{surr}^{\ 4})$$
$$= (0.7)(2.168\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\text{.K}^4)\left[(90+273\ \text{K})^4 - (10+273\ \text{K})^4\right] = 942\ \text{W}$$
$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 2081 + 942 = \mathbf{3023\ W}$$

(b) The temperature at which water leaves the basement is

$$\dot{Q} = \dot{m}c_p(T_i - T_e) \longrightarrow T_e = T_i - \frac{\dot{Q}}{\dot{m}c_p} = 90\text{°C} - \frac{3023\ \text{W}}{(1.456\ \text{kg/s})(4206\ \text{J/kg.°C})} = \mathbf{89.5°C}$$

The result justifies our assumption that the temperature drop of water is negligible. Also, the thermal resistance of the pipe and temperature drop across it are

$$R_{pipe} = \frac{\ln(D_2/D_1)}{4\pi kL} = \frac{\ln(4.6/4)}{2\pi(386\ \text{W/m.°C})(15\ \text{m})} = 3.84\times10^{-6}\ \text{°C/W}$$
$$\Delta T_{pipe} = \dot{Q}_{total} R_{pipe} = (3023\ \text{W})(3.84\times10^{-6}\ \text{°C/W}) = 0.012\text{°C}$$

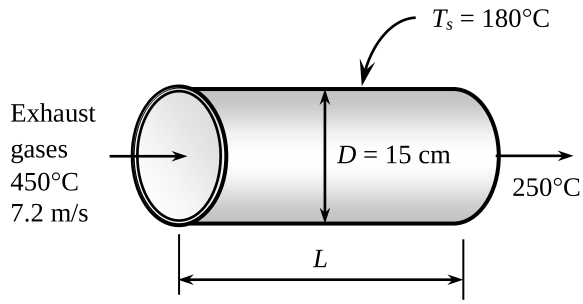
which justifies our assumption that the temperature drop across the pipe is negligible.

8-134 Hot exhaust gases flow through a pipe. For a specified exit temperature, the pipe length is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surface of the pipe is smooth. **3** For hot gases, air properties are used. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties The properties of air at 1 atm and the bulk mean temperature of (450+250)/2 = 350°C are (Table A-15)

$$\rho = 0.5664\ \text{kg/m}^3$$
$$k = 0.04721\ \text{W/m.°C}$$
$$\nu = 5.475\times10^{-5}\ \text{m}^2\text{/s}$$
$$c_p = 1056\ \text{J/kg.°C}$$
$$\text{Pr} = 0.6937$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}}D}{\nu} = \frac{(7.2\ \text{m/s})(0.15\text{m})}{5.475\times10^{-5}\ \text{m}^2\text{/s}} = 19{,}726$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.15\ \text{m}) = 1.5\ \text{m}$$

which is probably much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(19{,}726)^{0.8}(0.6937)^{0.3} = 56.25$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.04721\ \text{W/m.°C}}{0.15\ \text{m}}(56.25) = 17.70\ \text{W/m}^2\text{.°C}$$

The logarithmic mean temperature difference is

$$\Delta T_{\text{lm}} = \frac{T_e - T_i}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} = \frac{250 - 450}{\ln\left(\dfrac{180 - 250}{180 - 450}\right)} = 148.2\text{°C}$$

The rate of heat loss from the exhaust gases can be expressed as

$$\dot{Q} = hA_s\Delta T_{\text{lm}} = (17.70\ \text{W/m}^2\text{.°C})\left[\pi(0.15\ \text{m})L\right](148.2\text{°C}) = 1236L$$

where L is the length of the pipe. The rate of heat loss can also be determined from

$$\dot{m} = \rho V_{\text{avg}}A_c = (0.5664\ \text{kg/m}^3)(7.2\,\text{m/s})\left[\pi(0.15\text{m})^2/4\right] = 0.07207\ \text{kg/s}$$

$$\dot{Q} = \dot{m}c_p\Delta T = (0.07207\ \text{kg/s})(1056\,\text{J/kg.°C})(450 - 250)\text{°C} = 15{,}220\ \text{W}$$

Setting this equal to rate of heat transfer expression above, the pipe length is determined to be

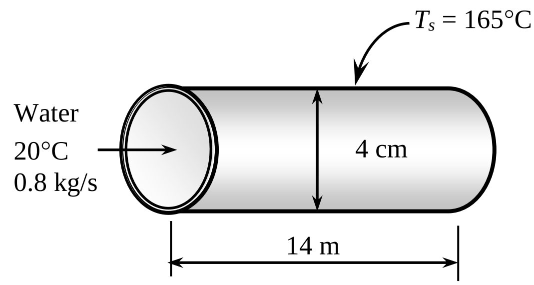
$$\dot{Q} = 1236L = 15{,}220\ \text{W} \longrightarrow L = \mathbf{12.3\ m}$$

8-135 Water is heated in a heat exchanger by the condensing geothermal steam. The exit temperature of water and the rate of condensation of geothermal steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the tube are smooth. **3** The surface temperature of the pipe is 165°C, which is the temperature at which the geothermal steam is condensing.

Properties The properties of water at the anticipated mean temperature of 85°C are (Table A-9)

$$\rho = 968.1\ \text{kg/m}^3$$
$$k = 0.673\ \text{W/m.°C}$$
$$c_p = 4201\ \text{J/kg.°C}$$
$$\text{Pr} = 2.08$$
$$\nu = \frac{\mu}{\rho} = \frac{0.333\times10^{-3}\ \text{kg/m.s}}{968.1\ \text{kg/m}^3} = 3.44\times10^{-7}\ \text{m}^2/\text{s}$$
$$h_{fg@165°\text{C}} = 2066.5\ \text{kJ/kg}$$



Analysis The velocity of water and the Reynolds number are

$$\dot{m} = \rho A V_{\text{avg}} \longrightarrow 0.8\ \text{kg/s} = (968.1\ \text{kg/m}^3)\pi\frac{(0.04\ \text{m})^2}{4}V_{\text{avg}} \longrightarrow V_{\text{avg}} = 0.6576\ \text{m/s}$$

$$\text{Re} = \frac{V_{\text{avg}} D}{\nu} = \frac{(0.6576\ \text{m/s})(0.04\text{m})}{3.44\times10^{-7}\ \text{m}^2/\text{s}} = 76{,}465$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.04\ \text{m}) = 0.4\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(76{,}465)^{0.8}(2.08)^{0.4} = 248.7$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.673\ \text{W/m.°C}}{0.04\ \text{m}}(248.7) = 4184\ \text{W/m}^2.\text{°C}$$

Next we determine the exit temperature of air,

$$A_s = \pi DL = \pi(0.04\ \text{m})(14\ \text{m}) = 1.759\ \text{m}^2$$

$$T_e = T_s - (T_s - T_i)e^{-hA_s/(\dot{m}c_p)} = 165 - (165-20)e^{-\frac{(4184)(1.759)}{(0.8)(4201)}} = \mathbf{148.8°C}$$

The logarithmic mean temperature difference is

$$\Delta T_{\text{lm}} = \frac{T_e - T_i}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} = \frac{148.8 - 20}{\ln\left(\dfrac{165 - 148.8}{165 - 20}\right)} = 58.77°\text{C}$$

The rate of heat transfer can be expressed as

$$\dot{Q} = hA_s\Delta T_{\text{lm}} = (4185\ \text{W/m}^2.\text{°C})(1.759\ \text{m}^2)(58.77°\text{C}) = 432{,}500\ \text{W}$$

The rate of condensation of steam is determined from

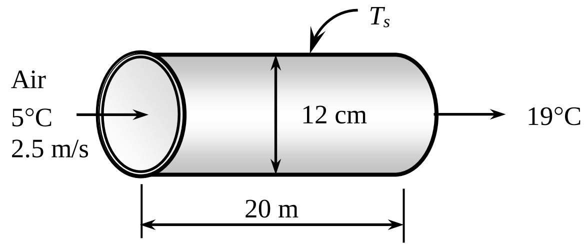
$$\dot{Q} = \dot{m}h_{fg} \longrightarrow 432.5\ \text{kW} = \dot{m}(2066.5\ \text{kJ/kg}) \longrightarrow \dot{m} = \mathbf{0.209\ kg/s}$$

8-136 Cold-air flows through an isothermal pipe. The pipe temperature is to be estimated.

Assumptions **1** Steady operating conditions exist. **2** The inner surface of the duct is smooth. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties The properties of air at 1 atm and the bulk mean temperature of (5+19)/2=12°C are (Table A-15)

$$\rho = 1.238\ \text{kg/m}^3$$
$$k = 0.02454\ \text{W/m.°C}$$
$$\nu = 1.444\times10^{-5}\ \text{m}^2/\text{s}$$
$$c_p = 1007\ \text{J/kg.°C}$$
$$\Pr = 0.7331$$



Analysis The rate of heat transfer to the air is

$$\dot{m} = \rho A_c V_{\text{avg}} = (1.238\ \text{kg/m}^3)\pi\frac{(0.12\ \text{m})^2}{4}(2.5\ \text{m/s}) = 0.0350\ \text{m/s}$$

$$\dot{Q} = \dot{m}c_p\Delta T = (0.0350\ \text{kg/s})(1007\ \text{J/kg.°C})(19-5)\text{°C} = 493.1\ \text{W}$$

Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}}D}{\nu} = \frac{(2.5\ \text{m/s})(0.12\ \text{m})}{1.444\times10^{-5}\ \text{m}^2/\text{s}} = 20{,}775$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.12\ \text{m}) = 1.2\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(20{,}775)^{0.8}(0.7331)^{0.4} = 57.79$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02454\ \text{W/m.°C}}{0.12\ \text{m}}(57.79) = 11.82\ \text{W/m}^2.\text{°C}$$

The logarithmic mean temperature difference is determined from

$$\dot{Q} = hA_s\Delta T_{\text{lm}} \longrightarrow 493.1\ \text{W} = (11.82\ \text{W/m}^2.\text{°C})[\pi(0.12\ \text{m})(20\ \text{m})]\Delta T_{\text{ln}} \longrightarrow \Delta T_{\text{ln}} = 5.533\text{°C}$$

Then the pipe temperature is determined from the definition of the logarithmic mean temperature difference

$$\Delta T_{\text{lm}} = \frac{T_e - T_i}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} \longrightarrow 5.533\text{°C} = \frac{19-5}{\ln\left(\dfrac{T_s - 19}{T_s - 5}\right)} \longrightarrow T_s = \mathbf{3.8°C}$$

8-137 Crude oil is heated as it flows in the tube-side of a multi-tube heat exchanger. The rate of heat transfer and the tube length are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface temperature is constant and uniform. **3** The inner surfaces of the tubes are smooth. **4** Heat transfer to the surroundings is negligible.

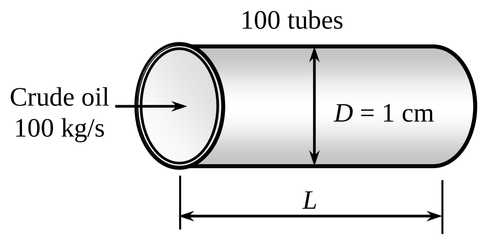
Properties The properties of crude oil are given to be ρ = 950 kg/m^3, c_p = 1.9 kJ/kg·K, k = 0.25 W/m·K, μ = 12 mPa·s.

Analysis The rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (100\text{ kg/s})(1900\text{ J/kg.°C})(40-20)\text{°C} = \mathbf{3.8\times10^6\ W}$$

The water velocity is

$$V = \frac{\dot{m}}{\rho A_c} = \frac{(100/100)\text{ kg/s}}{(950\text{ kg/m}^3)\left[\pi(0.01\text{ m})^2/4\right]} = 13.4\text{ m/s}$$



The Prandtl and Reynolds number are

$$\text{Pr} = \frac{\mu c_p}{k} = \frac{(0.012\text{ kg/m}\cdot\text{s})(1900\text{ J/kg}\cdot\text{K})}{0.25\text{ W/m}\cdot\text{K}} = 91.2$$

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(950\text{ kg/m}^3)(13.4\text{ m/s})(0.01\text{m})}{0.012\text{ kg/m}\cdot\text{s}} = 10{,}610$$

which is greater than 10,000. Therefore, we have turbulent flow. Assuming fully developed flow in the entire tube, the Nusselt number is determined from

$$Nu = \frac{hD}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(10{,}610)^{0.8}(91.2)^{0.4} = 232.4$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.25\text{ W/m.°C}}{0.01\text{ m}}(232.4) = 5811\text{ W/m}^2\text{.°C}$$

Using the average fluid temperature and considering that there are 100 tubes, the length of the tubes is determined as follows:

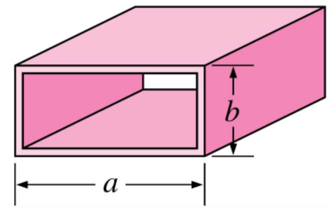
$$\dot{Q} = hA_s(T_s - T_{b,\text{avg}}) \longrightarrow 3.8\times10^6\text{ W} = (5811\text{ W/m}^2\cdot\text{°C})A\left(100 - \frac{20+30}{2}\right)\text{°C} \longrightarrow A_s = 8.719\text{ m}^2$$

$$A_s = \pi DL \longrightarrow 8.719\text{ m}^2 = 100\pi(0.01\text{ m})L \longrightarrow L = \mathbf{2.78\ m}$$

8-138 The rectangular tube surface temperature necessary to heat water to the desired outlet temperature of 80°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Constant tube surface temperature.

Properties The properties of water at $T_b = (T_i + T_e)/2 = 50°C$: $c_p = 4181$ J/kg·K, $k = 0.644$ W/m·K, $\mu = 0.547 \times 10^{-3}$ kg/m·s, and Pr = 3.55 (Table A-15).



Analysis The hydraulic diameter is

$$D_h = \frac{4A_c}{p} = \frac{4ab}{2(a+b)} = 0.03333\text{ m}$$

The Reynolds number, hydrodynamic and thermal entry lengths are

$$\text{Re} = \frac{\rho V_{\text{avg}} D_h}{\mu} = \frac{\dot{m}(4A_c/p)}{A_c \mu} = \frac{4\dot{m}}{p\mu} = \frac{4\dot{m}}{2(a+b)\mu} = \frac{4(0.25\text{ kg/s})}{2(0.075\text{ m})(0.547 \times 10^{-3}\text{ kg/m}\cdot\text{s})}$$
$$= 12200 > 10{,}000$$

$$L_{h,\text{ turb}} \approx L_{t,\text{ turb}} \approx 10 D_h = 0.333\text{ m} < 10\text{ m}$$

Hence, the flow is fully developed turbulent, and using the Dittus-Boelter equation, we have

$$\text{Nu} = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(12200)^{0.8}(3.55)^{0.4} = 70.94$$

Hence, the convection heat transfer coefficient is

$$h = 70.94\left(\frac{0.644\text{ W/m}\cdot\text{K}}{0.03333\text{ m}}\right) = 1371\text{ W/m}^2\cdot\text{K}$$

The tube surface temperature can be determined using

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) \quad \rightarrow \quad T_s = \frac{T_e - T_i \exp[-hA_s/(\dot{m}c_p)]}{1 - \exp[-hA_s/(\dot{m}c_p)]}$$

$$T_s = \frac{80°\text{C} - (20°\text{C})\exp(-1.967)}{1 - \exp(-1.967)} = \mathbf{89.8°C}$$

where

$$\frac{hA_s}{\dot{m}c_p} = \frac{(1371\text{ W/m}^2\cdot\text{K})2(10\text{ m})(0.025\text{ m} + 0.050\text{ m})}{(0.25\text{ kg/s})(4181\text{ J/kg}\cdot\text{K})} = 1.967$$

Discussion Turbulent flow relations developed for circular tube can be used for noncircular tubes with reasonable accuracy.

8-139 Geothermal water is supplied to a city through stainless steel pipes at a specified rate. The electric power consumption and its daily cost are to be determined, and it is to be assessed if the frictional heating during flow can make up for the temperature drop caused by heat loss.

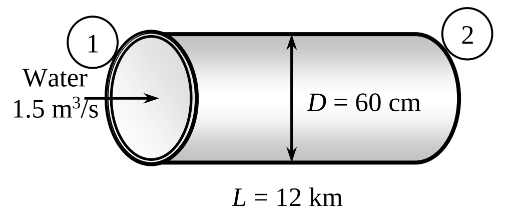
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. **4** The geothermal well and the city are at about the same elevation. **5** The properties of geothermal water are the same as fresh water. **6** The fluid pressures at the wellhead and the arrival point in the city are the same.

Properties The properties of water at 110°C are $\rho = 950.6$ kg/m^3, $\mu = 0.255\times10^{-3}$ kg/m·s, and $c_p = 4.229$ kJ/kg·°C (Table A-9). The roughness of stainless steel pipes is 2×10^{-6} m (Table 8-3).

Analysis (*a*) We take point 1 at the well-head of geothermal resource and point 2 at the final point of delivery at the city, and the entire piping system as the control volume. Both points are at the same elevation ($z_2 = z_2$) and the same velocity ($V_1 = V_2$) since the pipe diameter is constant, and the same pressure ($P_1 = P_2$). Then the energy equation for this control volume simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1+h_{\text{pump,u}} = \frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad h_{\text{pump,u}} = h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. The mean velocity and the Reynolds number are



$$V_{\text{avg}} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{1.5\text{ m}^3/\text{s}}{\pi(0.60\text{ m})^2/4} = 5.305\text{ m/s}$$

$$\text{Re} = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{(950.6\text{ kg/m}^3)(5.305\text{ m/s})(0.60\text{m})}{0.255\times10^{-3}\text{ kg/m}\cdot\text{s}} = 1.186\times10^7$$

which is greater than 10,000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{2\times10^{-6}\text{ m}}{0.60\text{ m}} = 3.33\times10^{-6}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right) \;\rightarrow\; \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{3.33\times10^{-6}}{3.7}+\frac{2.51}{1.187\times10^7\sqrt{f}}\right)$$ It gives $f = 0.00829$. Then the pressure drop, the head loss, and the required power input become

$$\Delta P = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.00829\frac{12{,}000\text{ m}}{0.60\text{ m}}\frac{(950.6\text{ kg/m}^3)(5.305\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right) = 2218\text{ kPa}$$

$$\dot{W}_{\text{elect}} = \frac{\dot{W}_{\text{pump,u}}}{\eta_{\text{pump-motor}}} = \frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}} = \frac{(1.5\text{ m}^3/\text{s})(2218\text{ kPa})}{0.65}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{5118\text{ kW}}$$

Therefore, the pumps will consume 5118 kW of electric power to overcome friction and maintain flow.

(*b*) The daily cost of electric power consumption is determined by multiplying the amount of power used per day by the unit cost of electricity,

$$\text{Amount} = \dot{W}_{\text{elect,in}}\Delta t = (5118\text{ kW})(24\text{ h/day}) = 122{,}832\text{ kWh/day}$$

$$\text{Cost} = \text{Amount}\times\text{Unit cost} = (122{,}832\text{ kWh/day})(\$0.06/\text{kWh}) = \mathbf{\$7370/day}$$

(*c*) The energy consumed by the pump (except the heat dissipated by the motor to the air) is eventually dissipated as heat due to the frictional effects. Therefore, this problem is equivalent to heating the water by a 5118 kW of resistance heater (again except the heat dissipated by the motor). To be conservative, we consider only the useful mechanical energy supplied to the water by the pump. The temperature rise of water due to this addition of energy is

$$\dot{W}_{\text{elect}} = \rho\dot{V}c_p\Delta T \;\rightarrow\; \Delta T = \frac{\eta_{\text{pump-motor}}\dot{W}_{\text{elect,in}}}{\rho\dot{V}c_p} = \frac{0.65\times(5118\text{ kJ/s})}{(950.6\text{ kg/m}^3)(1.5\text{ m}^3/\text{s})(4.229\text{ kJ/kg}\cdot\text{°C})} = \mathbf{0.55°C}$$

Therefore, the temperature of water will rise at least 0.55°C, which is more than the 0.5°C drop in temperature (in reality, the temperature rise will be more since the energy dissipation due to pump inefficiency will also appear as temperature rise of water). Thus we conclude that the frictional heating during flow can more than make up for the temperature drop caused by heat loss.

Discussion The pumping power requirement and the associated cost can be reduced by using a larger diameter pipe. But the cost savings should be compared to the increased cost of larger diameter pipe.

8-140 Geothermal water is supplied to a city through cast iron pipes at a specified rate. The electric power consumption and its daily cost are to be determined, and it is to be assessed if the frictional heating during flow can make up for the temperature drop caused by heat loss.

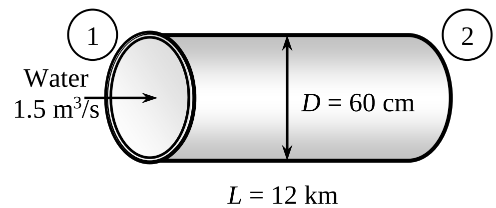
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. **4** The geothermal well and the city are at about the same elevation. **5** The properties of geothermal water are the same as fresh water. **6** The fluid pressures at the wellhead and the arrival point in the city are the same.

Properties The properties of water at 110°C are ρ = 950.6 kg/m^3, μ = 0.255×10^{-3} kg/m⋅s, and c_p = 4.229 kJ/kg⋅°C (Table A-9). The roughness of cast iron pipes is 0.00026 m (Table 8-3).

Analysis (*a*) We take point 1 at the well-head of geothermal resource and point 2 at the final point of delivery at the city, and the entire piping system as the control volume. Both points are at the same elevation ($z_2 = z_2$) and the same velocity ($V_1 = V_2$) since the pipe diameter is constant, and the same pressure ($P_1 = P_2$). Then the energy equation for this control volume simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad h_{\text{pump,u}}=h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. The mean velocity and the Reynolds number are



$$V_{\text{avg}}=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4}=\frac{1.5\ \text{m}^3/\text{s}}{\pi(0.60\ \text{m})^2/4}=5.305\ \text{m/s}$$

$$\text{Re}=\frac{\rho V_{\text{avg}}D}{\mu}=\frac{(950.6\ \text{kg/m}^3)(5.305\ \text{m/s})(0.60\text{m})}{0.255\times10^{-3}\ \text{kg/m}\cdot\text{s}}=1.187\times10^7$$

which is greater than 10,000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00026\ \text{m}}{0.60\ \text{m}}=4.33\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\quad\rightarrow\quad\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{4.33\times10^{-4}}{3.7}+\frac{2.51}{1.187\times10^7\sqrt{f}}\right)$$

It gives f = 0.01623. Then the pressure drop, the head loss, and the required power input become

$$\Delta P=f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2}=0.01623\frac{12{,}000\ \text{m}}{0.60\ \text{m}}\frac{(950.6\ \text{kg/m}^3)(5.305\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)=4342\ \text{kPa}$$

$$\dot{W}_{\text{elect}}=\frac{\dot{W}_{\text{pump,u}}}{\eta_{\text{pump-motor}}}=\frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}}=\frac{(1.5\ \text{m}^3/\text{s})(4342\ \text{kPa})}{0.65}\left(\frac{1\ \text{kW}}{1\ \text{kPa}\cdot\text{m}^3/\text{s}}\right)=\mathbf{10{,}020\ kW}$$

Therefore, the pumps will consume 10,017 W of electric power to overcome friction and maintain flow.

(*b*) The daily cost of electric power consumption is determined by multiplying the amount of power used per day by the unit cost of electricity,

$$\text{Amount}=\dot{W}_{\text{elect,in}}\Delta t=(10{,}020\ \text{kW})(24\ \text{h/day})=240{,}480\ \text{kWh/day}$$

$$\text{Cost}=\text{Amount}\times\text{Unit cost}=(240{,}480\ \text{kWh/day})(\$0.06/\text{kWh})=\mathbf{\$14{,}430/day}$$

(*c*) The energy consumed by the pump (except the heat dissipated by the motor to the air) is eventually dissipated as heat due to the frictional effects. Therefore, this problem is equivalent to heating the water by a 10,020 kW of resistance heater (again except the heat dissipated by the motor). To be conservative, we consider only the useful mechanical energy supplied to the water by the pump. The temperature rise of water due to this addition of energy is

$$\dot{W}_{\text{elect}}=\rho\dot{V}c_p\Delta T\ \rightarrow\ \Delta T=\frac{\eta_{\text{pump-motor}}\dot{W}_{\text{elect,in}}}{\rho\dot{V}c_p}=\frac{0.65\times(10{,}020\ \text{kJ/s})}{(950.6\ \text{kg/m}^3)(1.5\ \text{m}^3/\text{s})(4.229\ \text{kJ/kg}\cdot{}^\circ\text{C})}=\mathbf{1.08^\circ C}$$

Therefore, the temperature of water will rise at least 1.08°C, which is more than the 0.5°C drop in temperature (in reality, the temperature rise will be more since the energy dissipation due to pump inefficiency will also appear as temperature rise of water). Thus we conclude that the frictional heating during flow can more than make up for the temperature drop caused by heat loss.

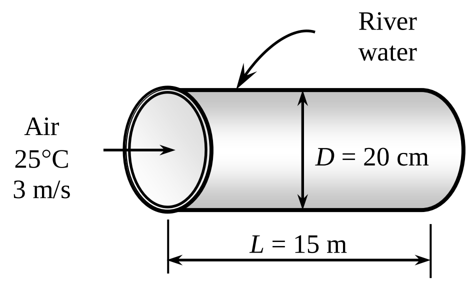
Discussion The pumping power requirement and the associated cost can be reduced by using a larger diameter pipe. But the cost savings should be compared to the increased cost of larger diameter pipe.

8-141 Air enters the underwater section of a duct. The outlet temperature of the air and the fan power needed to overcome the flow resistance are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** The surface of the duct is at the temperature of the water. **5** Air is an ideal gas with constant properties. **6** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 20°C since the mean temperature of air at the inlet will drop somewhat as a result of heat loss through the duct whose surface is at a lower temperature. The properties of air at 1 atm and this temperature are (Table A-15)

$$\rho = 1.204\ \text{kg/m}^3$$
$$k = 0.02514\ \text{W/m.°C}$$
$$\nu = 1.516\times10^{-5}\ \text{m}^2/\text{s}$$
$$c_p = 1007\ \text{J/kg.°C}$$
$$\text{Pr} = 0.7309$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(3\ \text{m/s})(0.2\text{m})}{1.516\times10^{-5}\ \text{m}^2/\text{s}} = 3.958\times10^4$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.2\ \text{m}) = 2\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(3.958\times10^4)^{0.8}(0.7309)^{0.3} = 99.76$$

and $$h = \frac{k}{D_h}Nu = \frac{0.02514\ \text{W/m.°C}}{0.2\ \text{m}}(99.76) = 12.54\ \text{W/m}^2.\text{°C}$$

Next we determine the exit temperature of air,

$$A_s = \pi DL = \pi(0.2\ \text{m})(15\text{m}) = 9.425\ \text{m}^2$$

$$\dot{m} = \rho V_{\text{avg}} A_c = (1.204\ \text{kg/m}^3)(3\ \text{m/s})\left(\frac{\pi(0.2\ \text{m})^2}{4}\right) = (1.204\ \text{kg/m}^3)(0.09425\ \text{m}^3/\text{s}) = 0.1135\ \text{kg/s}$$

and $$T_e = T_s - (T_s - T_i)e^{-hA_s/(\dot{m}c_p)} = 15 - (15-25)e^{-\frac{(12.54)(9.425)}{(0.1135)(1007)}} = \mathbf{18.6°C}$$

The friction factor, pressure drop, and the fan power required to overcome this pressure drop can be determined for the case of fully developed turbulent flow in smooth pipes to be

$$f = (0.790\ln\text{Re} - 1.64)^{-2} = \left[0.790\ln(3.958\times10^4) - 1.64\right]^{-2} = 0.02212$$

$$\Delta P = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.02212\frac{15\ \text{m}}{0.2\ \text{m}}\frac{(1.204\ \text{kg/m}^3)(3\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{Pa}}{1\ \text{N/m}^2}\right) = 8.988\ \text{Pa}$$

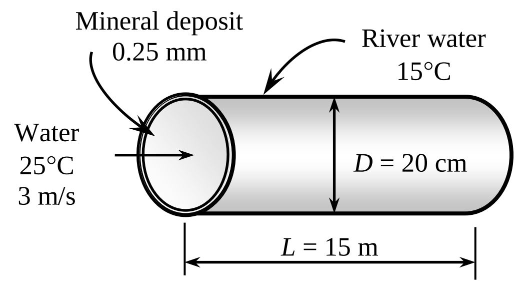
$$\dot{W}_{\text{fan}} = \frac{\dot{W}_{\text{pump,u}}}{\eta_{\text{pump-motor}}} = \frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}} = \frac{(0.09425\ \text{m}^3/\text{s})(8.988\ \text{Pa})}{0.55} = \left(\frac{1\ \text{W}}{1\ \text{Pa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{1.54\ W}$$

8-142 Air enters the underwater section of a duct. The outlet temperature of the air and the fan power needed to overcome the flow resistance are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 20°C since the mean temperature of air at the inlet will drop somewhat as a result of heat loss through the duct whose surface is at a lower temperature. The properties of air at 1 atm and this temperature are (Table A-15)

$$\rho = 1.204 \text{ kg/m}^3$$
$$k = 0.02514 \text{ W/m.°C}$$
$$\nu = 1.516 \times 10^{-5} \text{ m}^2/\text{s}$$
$$c_p = 1007 \text{ J/kg.°C}$$
$$\text{Pr} = 0.7309$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_{avg} D_h}{\nu} = \frac{(3 \text{ m/s})(0.2 \text{m})}{1.516 \times 10^{-5} \text{ m}^2/\text{s}} = 3.958 \times 10^4$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.2 \text{ m}) = 2 \text{ m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number and h from

$$Nu = \frac{hD_h}{k} = 0.023 \,\text{Re}^{0.8}\, \text{Pr}^{0.3} = 0.023(3.959 \times 10^4)^{0.8}(0.7309)^{0.3} = 99.76$$

and $$h = \frac{k}{D_h} Nu = \frac{0.02514 \text{ W/m.°C}}{0.2 \text{ m}}(99.76) = 12.54 \text{ W/m}^2.\text{°C}$$

Next we determine the exit temperature of air,

$$A_s = \pi DL = \pi(0.2 \text{ m})(15 \text{m}) = 9.425 \text{m}^2$$

$$\dot{m} = \rho V_{avg} A_c = (1.204 \text{ kg/m}^3)(3 \text{m/s})\left(\frac{\pi(0.2 \text{m})^2}{4}\right) = (1.204 \text{ kg/m}^3)(0.09425 \text{m}^3/\text{s}) = 0.1135 \text{kg/s}$$

The unit thermal resistance of the mineral deposit is

$$R_{mineral} = \frac{L}{k} = \frac{0.0025 \text{ m}}{3 \text{ W/m.°C}} = 0.00083 \text{ m}^2.\text{°C/W}$$

which is much less than (about 1%) the unit convection resistance,

$$R_{conv} = \frac{1}{h} = \frac{1}{12.54 \text{ W/m}^2.\text{°C}} = 0.0797 \text{ m}^2.\text{°C/W}$$

Therefore, the effect of 0.25 mm thick mineral deposit on heat transfer is negligible. Next we determine the exit temperature of air

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 15 - (15 - 25)e^{-\frac{(12.54)(9.425)}{(0.1135)(1007)}} = \mathbf{18.6°C}$$

The friction factor, pressure drop, and the fan power required to overcome this pressure drop can be determined for the case of fully developed turbulent flow in smooth pipes to be

$$f = (0.790 \ln \text{Re} - 1.64)^{-2} = \left[0.790 \ln(3.958 \times 10^4) - 1.64\right]^{-2} = 0.02212$$

$$\Delta P = f \frac{L}{D} \frac{\rho V_{avg}^2}{2} = 0.02212 \frac{15 \text{ m}}{0.2 \text{ m}} \frac{(1.204 \text{ kg/m}^3)(3 \text{ m/s})^2}{2}\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)\left(\frac{1 \text{ Pa}}{1 \text{ N/m}^2}\right) = 8.988 \text{ Pa}$$

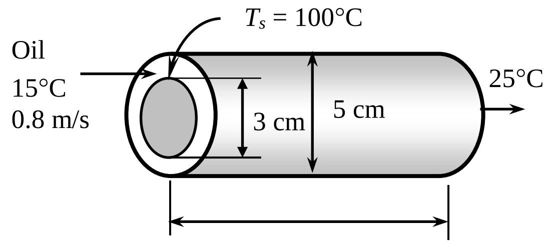
$$\dot{W}_{fan} = \frac{\dot{W}_{pump,u}}{\eta_{pump\text{-}motor}} = \frac{\dot{V}\Delta P}{\eta_{pump\text{-}motor}} = \frac{(0.09425 \text{ m}^3/\text{s})(8.988 \text{ Pa})}{0.55} = \left(\frac{1 \text{ W}}{1 \text{ Pa} \cdot \text{m}^3/\text{s}}\right) = \mathbf{1.54 \ W}$$

8-143 Oil is heated by saturated steam in a double-pipe heat exchanger. The tube length is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces of the tube are smooth.

Properties The properties of oil at the average temperature of (15+25)/2=20°C are (Table A-13)

$$\rho = 888.1\ \text{kg/m}^3$$
$$k = 0.145\ \text{W/m}\cdot{}^\circ\text{C}$$
$$c_p = 1881\ \text{J/kg}\cdot{}^\circ\text{C}$$
$$\mu = 0.8374\ \text{kg/m}\cdot\text{s}$$



Analysis The cross-sectional area of the annulus, the mass flow rate, the rate of heat transfer, and Reynolds number are

$$A_c = \pi\frac{D_o^2 - D_i^2}{4} = \pi\frac{(0.05\ \text{m})^2 - (0.03\ \text{m})^2}{4} = 0.001257\ \text{m}^2$$

$$\dot{m} = \rho A_c V_{\text{avg}} = (888.1\ \text{kg/m}^3)(0.001257\ \text{m}^2)(0.8\ \text{m/s}) = 0.8931\ \text{kg/s}$$

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.8931\ \text{kg/s})(1881\ \text{J/kg.°C})(25-15)\text{°C} = 16{,}799\ \text{W}$$

$$\text{Re} = \frac{\rho V D_h}{\mu} = \frac{\rho V(D_o - D_i)}{\mu} = \frac{(888.1\ \text{kg/m}^3)(0.8\ \text{m/s})(0.02\text{m})}{0.8374\ \text{kg/m}\cdot\text{s}} = 16.97$$

Since the flow is laminar and fully developed, the Nusselt number is determined from Table 8-4 at D_i/D_o =3/5=0.6 to be Nu_i = 5.564. Then the hydraulic diameter of annulus, the heat transfer coefficient, and the logarithmic mean temperature difference are

$$D_h = D_o - D_i = 0.05\ \text{m} - 0.03\ \text{m} = 0.02\ \text{m}$$

$$h_i = \frac{k}{D_h}Nu_i = \frac{0.145\ \text{W/m.°C}}{0.02\ \text{m}}(5.564) = 40.34\ \text{W/m}^2\text{.°C}$$

$$\Delta T_{\text{lm}} = \frac{T_i - T_e}{\ln\left(\dfrac{T_s - T_e}{T_s - T_i}\right)} = \frac{15-25}{\ln\left(\dfrac{100-25}{100-15}\right)} = 79.90\text{°C}$$

The heat transfer surface area is determined from

$$\dot{Q} = hA_s\Delta T_{\text{lm}} \longrightarrow A_s = \frac{\dot{Q}}{h\Delta T_{\text{lm}}} = \frac{16{,}799\ \text{W}}{(40.34\ \text{W/m}^2\text{.°C})(79.90\text{°C})} = 5.212\ \text{m}^2$$

Then the tube length becomes

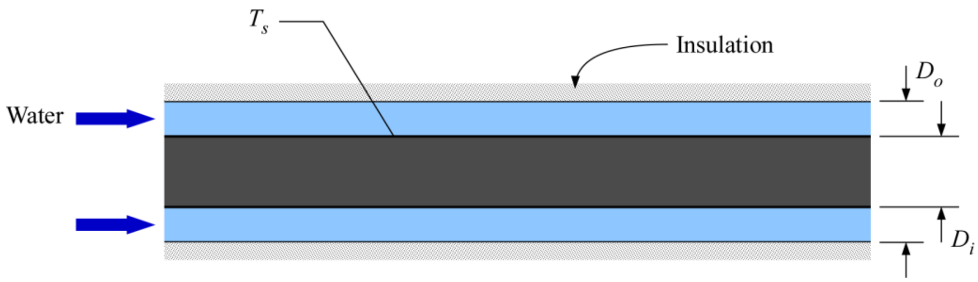
$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D_i} = \frac{5.212\ \text{m}^2}{\pi(0.03\ \text{m})} = \mathbf{55.3\ m}$$

In reality, we need a shorter tube since the effect of entry region is to increase the average heat transfer coefficient and a higher value of heat transfer coefficient translates into a shorter tube length.

8-144E Water flows through a concentric annulus tube with constant inner surface temperature and insulated outer surface, the length of the annulus tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Constant inner tube surface temperature. **4** Insulated outer tube surface. **5** Fully developed flow.

Properties The properties of water at $T_b = (T_i + T_e)/2 = 120$°F: $c_p = 0.999$ Btu/lbm·R, $k = 0.371$ Btu/h·ft·R, $\mu = 3.744 \times 10^{-4}$ lbm/ft·s, $\rho = 61.71$ lbm/ft^3, and Pr = 3.63 (Table A-9E).



Analysis The Reynolds number is

$$\mathrm{Re} = \frac{\rho V_{\mathrm{avg}}(D_o - D_i)}{\mu} = \frac{\dot{m}(D_o - D_i)}{(\pi/4)(D_o^2 - D_i^2)\mu} = \frac{4\dot{m}}{\pi(D_o + D_i)\mu}$$

$$= \frac{4(396/3600\ \mathrm{lbm/s})}{\pi(1+4)\mathrm{ft}/12(3.744\times10^{-4}\ \mathrm{lbm/ft\cdot s})}$$

$$= 898$$

Since Re < 2300, the flow through the annulus is laminar. Assuming fully developed flow, the Nusselt number for the inner tube surface is (from Table 8-4)

$$\mathrm{Nu}_i = \frac{h_i D_h}{k} = 7.37 \qquad \text{for} \qquad D_i / D_o = 0.25$$

Hence, the convection heat transfer coefficient is

$$h_i = 7.37\left(\frac{0.371\ \mathrm{Btu/h\cdot ft\cdot R}}{3/12\ \mathrm{ft}}\right) = 10.94\ \mathrm{Btu/h\cdot ft^2\cdot R}$$

The length of the concentric annulus tube is

$$L = -\frac{\dot{m}c_p}{\pi D_i h_i}\ln\frac{T_s - T_e}{T_s - T_i} = -\frac{(396\ \mathrm{lbm/h})(0.999\ \mathrm{Btu/lbm\cdot R})}{\pi(1/12\ \mathrm{ft})(10.94\ \mathrm{Btu/h\cdot ft^2\cdot R})}\ln\frac{250-172}{250-68} = \mathbf{117\ ft}$$

Discussion Similar to regular tubes, the total rate of heat transfer in the annulus tube can be determined using $\dot{Q} = \dot{m}c_p(T_e - T_i)$.

Fundamentals of Engineering (FE) Exam Problems

8-145 Internal force flows are said to be fully-developed once the ____ at a cross-section no longer changes in the direction of flow.

(a) temperature distribution (b) entropy distribution (c) velocity distribution
(d) pressure distribution (e) none of the above

Answer (c) velocity distribution

8-146 The bulk or mixed temperature of a fluid flowing through a pipe or duct is defined as

(a) $T_b = \frac{1}{A_c}\int_{A_c} T dA_c$ (b) $T_b = \frac{1}{\dot{m}}\int_{A_c} T\rho V dA_c$ (c) $T_b = \frac{1}{\dot{m}}\int_{A_c} h\rho V dA_c$

(d) $T_b = \frac{1}{A_c}\int_{A_c} h dA_c$ (e) $T_b = \frac{1}{\dot{V}}\int_{A_c} T\rho V dA_c$

Answer: (b) $T_b = \frac{1}{\dot{m}}\int_{A_c} T\rho V dA_c$

8-147 Water (μ = 9.0×10^{-4} kg/m·s, ρ = 1000 kg/m^3) enters a 4-cm-diameter, 3-m-long tube whose walls are maintained at 100°C. The water enters this tube with a bulk temperature of 25°C and a volume flow rate of 3 m^3/h. The Reynolds number for this internal flow is

(a) 29,500 (b) 38,200 (c) 72,500 (d) 118,100 (e) 122,900

Answer (a) 29,500

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
rho=1000 [kg/m^3]
mu=0.0009 [kg/m-s]
Vdot=3/3600 [m^3/hr]
D=0.04 [m]
Re=4*Vdot*rho/(pi*D*mu)
```

8-148 Water enters a circular tube whose walls are maintained at constant temperature at a specified flow rate and temperature. For fully developed turbulent flow, the Nusselt number can be determined from Nu = 0.023 $Re^{0.8}$ $Pr^{0.4}$. The correct temperature difference to use in Newton's law of cooling in this case is

(a) the difference between the inlet and outlet water bulk temperature

(b) the difference between the inlet water bulk temperature and the tube wall temperature

(c) the log mean temperature difference

(d) the difference between the average water bulk temperature and the tube temperature

(e) None of the above.

Answer (c) the log mean temperature difference

8-149 Air (c_p = 1007 J/kg·°C) enters a 17-cm-diameter and 4-m-long tube at 65ºC at a rate of 0.08 kg/s and leaves at 15ºC. The tube is observed to be nearly isothermal at 5ºC. The average convection heat transfer coefficient is

(a) 24.5 W/m²·ºC (b) 46.2 W/m²·ºC (c) 53.9 W/m²·ºC (d) 67.6 W/m²·ºC (e) 90.7 W/m²·ºC

Answer (d) 67.6 W/m²·ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_i=65 [C]
T_e=15 [C]
T_s=5 [C]
m_dot=0.08 [kg/s]
D=0.17 [m]
L=4 [m]
c_p=1007 [J/kg-C] "Table A-15"
Q_dot=m_dot*c_p*(T_e-T_i)
A_s=pi*D*L
DELTAT_ln=(T_i-T_e)/ln((T_s-T_e)/(T_s-T_i))
h=Q_dot/(A_s*DELTAT_ln)
"Some Wrong Solutions with Common Mistakes"
DELTAT_am=T_s-(T_i+T_e)/2 "Using arithmetic mean temperature difference"
W1_h=Q_dot/(A_s*DELTAT_am)
```

8-150 Water (c_p = 4180 J/kg·K) enters a 12-cm-diameter and 8.5-m-long tube at 75ºC at a rate of 0.35 kg/s, and is cooled by a refrigerant evaporating outside at -10ºC. If the average heat transfer coefficient on the inner surface is 500 W/m^2·ºC, the exit temperature of water is

(a) 18.4ºC (b) 25.0ºC (c) 33.8ºC (d) 46.5ºC (e) 60.2ºC

Answer (a) 18.4ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.12 [m]
L=8.5 [m]
T_i=75 [C]
T_s=-10 [C]
m_dot=0.35 [kg/s]
h=500 [W/m^2-C]
c_p=4180 [J/kg-C]
A_s=pi*D*L
T_e=T_s-(T_s-T_i)*exp((-h*A_s)/(m_dot*c_p))
```

8-151 Air enters a duct at 20ºC at a rate of 0.08 m^3/s, and is heated to 150ºC by steam condensing outside at 200ºC. The error involved in the rate of heat transfer to the air due to using arithmetic mean temperature difference instead of logarithmic mean temperature difference is

(a) 0% (b) 5.4% (c) 8.1% (d) 10.6% (e) 13.3%

Answer (e) 13.3%

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_i=20 [C]
T_e=150 [C]
T_s=200 [C]
V_dot=0.08 [m^3/s]
DELTAT_am=T_s-(T_i+T_e)/2
DELTAT_ln=(T_i-T_e)/ln((T_s-T_e)/(T_s-T_i))
Error=(DELTAT_am-DELTAT_ln)/DELTAT_ln*Convert(,%)
```

8-152 Engine oil at 60ºC (μ = 0.07399 kg/m·s, ρ = 864 kg/m^3) flows in a 5-cm-diameter tube with a velocity of 2.2 m/s. The pressure drop along a fully developed 6-m long section of the tube is

(a) 5.4 kPa (b) 8.1 kPa (c) 12.5 kPa (d) 18.4 kPa (e) 20.8 kPa

Answer (c) 12.5 kPa

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_oil=60 [C]
D=0.05 [m]
L=6 [m]
V=2.2 [m/s]
"The properties of engine oil at 60 C are (Table A-13)"
rho=864 [kg/m^3]
mu=0.07399 [kg/m-s]
Re=(rho*V*D)/mu "The calculated Re value is smaller than 2300. Therefore the flow is laminar"
f=64/Re
DELTAP=f*L/D*(rho*V^2)/2
```

8-153 Engine oil flows in a 15-cm-diameter horizontal tube with a velocity of 1.3 m/s, experiencing a pressure drop of 12 kPa. The pumping power requirement to overcome this pressure drop is

(a) 190 W (b) 276 W (c) 407 W (d) 655 W (e) 900 W

Answer (b) 276 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.15 [m]
V=1.3 [m/s]
DELTAP=12 [kPa]
A_c=pi*D^2/4
V_dot=V*A_c
W_dot_pump=V_dot*DELTAP
```

8-154 Water enters a 5-mm-diameter and 13-m-long tube at 15ºC with a velocity of 0.3 m/s, and leaves at 45ºC. The tube is subjected to a uniform heat flux of 2000 W/m^2 on its surface. The temperature of the tube surface at the exit is

(a) 48.7ºC (b) 49.4ºC (c) 51.1ºC (d) 53.7ºC (e) 55.2ºC

(For water, use k = 0.615 W/m·°C, Pr = 5.42, ν =0.801×10^{-6} m^2/s)

Answer (a) 48.7ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_i=15 [C]
T_e=45 [C]
D=0.005 [m]
L=13 [m]
V=0.3 [m/s]
q_s=2000 [W/m^2]

"The properties of water at (15+45)/2= 30 C are (Table A-9)"
rho=996 [kg/m^3]
k=0.615 [W/m-C]
mu=0.798E-3 [kg/m-s]
Pr=5.42
Re=(rho*V*D)/mu "The calculated Re value is smaller than 2300. Therefore the flow is laminar."
L_t=0.05*Re*Pr*D "Entry length is much shorter than the total length, and therefore we use fully developed relations"
Nus=4.36 "laminar flow, q_s = constant"
h=k/D*Nus
T_s=T_e+q_s/h

"Some Wrong Solutions with Common Mistakes"
W1_Nus=3.66 "Laminar flow, T_s = constant"
W1_h=k/D*W1_Nus
W1_T_s=T_e+q_s/W1_h
```

8-155 Water enters a 5-mm-diameter and 13-m-long tube at 45ºC with a velocity of 0.3 m/s. The tube is maintained at a constant temperature of 8ºC. The exit temperature of water is

(a) 4.4ºC (b) 8.9ºC (c) 10.6ºC (d) 12.0ºC (e) 14.1ºC

(For water, use k = 0.607 W/m·°C, Pr = 6.14, ν =0.894×10^{-6} m^2/s, c_p = 4180 J/kg·°C, ρ = 997 kg/m^3.)

Answer (b) 8.9ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_i=45 [C]
T_s=8 [C]
D=0.005 [m]
L=13 [m]
V=0.3 [m/s]

"The properties of water at 25 C are (Table A-9)"
rho=997 [kg/m^3]
c_p=4180 [J/kg-C]
k=0.607 [W/m-C]
mu=0.891E-3 [kg/m-s]
Pr=6.14

Re=(rho*V*D)/mu "The calculated Re value is smaller than 2300. Therefore the flow is laminar."
L_t=0.05*Re*Pr*D "Entry length is much shorter than the total length, and therefore we use fully developed
relations"
Nus=3.66 "laminar flow, T_s = constant"
h=k/D*Nus
A_s=pi*D*L
A_c=pi*D^2/4
m_dot=rho*A_c*V
T_e=T_s-(T_s-T_i)*exp((-h*A_s)/(m_dot*c_p))

"Some Wrong Solutions with Common Mistakes"
W1_Nus=4.36 "Laminar flow, q_s = constant"
W1_h=k/D*W1_Nus
W1_T_e=T_s-(T_s-T_i)*exp((-W1_h*A_s)/(m_dot*c_p))
```

8-156 Water enters a 5-mm-diameter and 13-m-long tube at 45ºC with a velocity of 0.3 m/s. The tube is maintained at a constant temperature of 5ºC. The required length of the tube in order for the water to exit the tube at 25ºC is

(a) 1.55 m (b) 1.72 m (c) 1.99 m (d) 2.37 m (e) 2.96 m

(For water, use k = 0.623 W/m·°C, Pr = 4.83, ν =0.724×10^{-6} m^2/s, c_p = 4178 J/kg·°C, ρ = 994 kg/m^3.)

Answer (b) 1.72 m

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_i=45 [C]
T_e=25 [C]
T_s=5 [C]
D=0.005 [m]
V=0.3 [m/s]

"The properties of water at (45+25)/2 = 35 C are (Table A-9)"
rho=994 [kg/m^3]
c_p=4178 [J/kg-C]
k=0.623 [W/m-C]
mu=0.720E-3 [kg/m-s]
Pr=4.83

Re=(rho*V*D)/mu "The calculated Re value is smaller than 2300. Therefore the flow is laminar."
L_t=0.05*Re*Pr*D "We assume that the entire flow remains in the entry region. We will check this after calculating total length of the tube"
Nus=3.66+(0.065*(D/L)*Re*Pr)/(1+0.04*((D/L)*Re*Pr)^(2/3)) "laminar flow, entry region, T_s = constant"
h=k/D*Nus
A_c=pi*D^2/4
m_dot=rho*A_c*V
T_e=T_s-(T_s-T_i)*exp((-h*A_s)/(m_dot*c_p))
A_s=pi*D*L "The total length calculated is shorter than the entry length, and therefore, the earlier entry region assumption is validated."

"Some Wrong Solutions with Common Mistakes"
W1_Nus=3.66 "Laminar flow, T_s = constant, fully developed flow"
W1_h=k/D*W1_Nus
T_e=T_s-(T_s-T_i)*exp((-W1_h*W1_A_s)/(m_dot*c_p))
W1_A_s=pi*D*W1_L
W2_Nus=4.36 "Laminar flow, q_s = constant, fully developed flow"
W2_h=k/D*W2_Nus
T_e=T_s-(T_s-T_i)*exp((-W2_h*W2_A_s)/(m_dot*c_p))
W2_A_s=pi*D*W2_L
```

8-157 Air enters a 7-cm-diameter, 4-m-long tube at 65ºC and leaves at 15ºC. The duct is observed to be nearly isothermal at 5ºC. If the average convection heat transfer coefficient is 20 W/m^2·ºC, the rate of heat transfer from the air is

(a) 491 W (b) 616 W (c) 810 W (d) 907 W (e) 975 W

Answer (a) 491 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_i=65 [C]
T_e=15 [C]
T_s=5 [C]
D=0.07 [m]
L=4 [m]
h=20 [W/m^2-C]
DELTAT_ln=(T_i-T_e)/ln((T_s-T_e)/(T_s-T_i))
A_s=pi*D*L
Q_dot=h*A_s*DELTAT_ln
"Some Wrong Solutions with Common Mistakes"
DELTAT_am=T_s-(T_i+T_e)/2 "Using arithmetic mean temperature difference"
W1_Q_dot=h*A_s*DELTAT_am
```

8-158 Air (c_p = 1000 J/kg·K) enters a 16-cm-diameter and 19-m-long underwater duct at 50°C and 1 atm at an average velocity of 7 m/s, and is cooled by the water outside. If the average heat transfer coefficient is 35 W/m^2·°C and the tube temperature is nearly equal to the water temperature of 5°C, the exit temperature of air is

(a) 6°C (b) 10°C (c) 18°C (d) 25°C (e) 36°C

Answer (b) 10°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
R=0.287 [kPa-m^3/kg-K]
cp=1000 [J/kg-K]
D=0.16 [m]
L=19 [m]
T1=50 [C]
P1=101.3 [kPa]
Vel=7 [m/s]
h=35 [W/m^2-C]
Ts=5 [C]
rho1=P1/(R*(T1+273))
As=pi*D*L
m_dot=rho1*Vel*pi*D^2/4
T2=Ts-(Ts-T1)*exp(-h*As/(m_dot*cp))
"Some Wrong Solutions with Common Mistakes:"
m_dot*cp*(T1-W1_T2)=h*As*((T1+W1_T2)/2-Ts) "Disregarding exponential variation of temperature"
```

8-159 Water enters a 2-cm-diameter, 3-m-long tube whose walls are maintained at 100ºC with a bulk temperature of 25ºC and volume flow rate of 3 m^3/h. Neglecting the entrance effects and assuming turbulent flow, the Nusselt number can be determined from Nu = 0.023 $Re^{0.8}$ $Pr^{0.4}$. The convection heat transfer coefficient in this case is

(a) 4140 W/m²·K (b) 6160 W/m²·K (c) 8180 W/m²·K (d) 9410 W/m²·K (e) 2870 W/m²·K

(For water, use k = 0.610 W/m·°C, Pr = 6.0, $\mu = 9.0\times10^{-4}$ kg/m·s, ρ = 1000 kg/m^3)

Answer (d) 9410 W/m²·K

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
rho=1000 [kg/m^3]
mu=0.0009 [kg/m-s]
Vdot=3/3600 [m^3/hr]
D=0.02 [m]
Pr=6
k=0.61 [W/m-K]
Re=4*rho*Vdot/(pi*D*mu)
Nus=0.023*Re^0.8*Pr^0.4
h=k*Nus/D
```

8-160 Air at 110ºC enters an 18-cm-diameter, 9-m-long duct at a velocity of 4.5 m/s. The duct is observed to be nearly isothermal at 85ºC. The rate of heat loss from the air in the duct is

(a) 760 W (b) 890 W (c) 1210 W (d) 1370 W (e) 1400 W

(For air, use k = 0.03095 W/m·°C, Pr = 0.7111, $\nu = 2.306\times10^{-5}$ m^2/s, c_p = 1009 J/kg·°C.)

Answer (e) 1400 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_i=110 [C]
D=0.18 [m]
L=9 [m]
V=4.5 [m/s]
T_s=85 [C]

"The properties of air at 100 C are (Table A-15)"
rho=0.9458 [kg/m^3]
c_p=1009 [J/kg-C]
k=0.03095 [W/m-C]
nu=2.306E-5 [m^2/s]
Pr=0.7111

Re=(V*D)/nu "The calculated Re value is greater than 10,000. Therefore the flow is turbulent."
L_t=10*D "Entry length is much shorter than the total length, and therefore we use fully developed relations"
Nus=0.023*Re^0.8*Pr^0.3
h=k/D*Nus
A_s=pi*D*L
A_c=pi*D^2/4
m_dot=rho*V*A_c
T_e=T_s-(T_s-T_i)*exp((-h*A_s)/(m_dot*c_p))
Q_dot=m_dot*c_p*(T_i-T_e)

"Some Wrong Solutions with Common Mistakes"
W1_Nus=0.023*Re^0.8*Pr^0.4 "Relation for heating case"
W1_h=k/D*W1_Nus
W1_T_e=T_s-(T_s-T_i)*exp((-W1_h*A_s)/(m_dot*c_p))
W1_Q_dot=m_dot*c_p*(T_i-W1_T_e)
```

8-161 Air at 10ºC enters an 18-m-long rectangular duct of cross section 0.15 m × 0.20 m at a velocity of 4.5 m/s. The duct is subjected to uniform radiation heating throughout the surface at a rate of 400 W/m^2. The wall temperature at the exit of the duct is

(a) 58.8ºC (b) 61.9ºC (c) 64.6ºC (d) 69.1ºC (e) 75.5ºC

(For air, use k = 0.02551 W/m·°C, Pr = 0.7296, $\nu = 1.562\times10^{-5}$ m^2/s, c_p = 1007 J/kg·°C, ρ= 1.184 kg/m^3.)

Answer (c) 64.6ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_i=10 [C]
L=18 [m]
a=0.15 [m]
b=0.20 [m]
V=4.5 [m/s]
q_s=400 [W/m^2]

"The properties of air at 25 C are (Table A-15)"
rho=1.184 [kg/m^3]
c_p=1007 [J/kg-C]
k=0.02551 [W/m-C]
nu=1.562E-5 [m^2/s]
Pr=0.7296

p=2*a+2*b
A_c=a*b
D_h=4*A_c/p
Re=(V*D_h)/nu "The calculated Re value is greater than 10,000. Therefore the flow is turbulent."
L_t=10*D_h "Entry length is much shorter than the total length, and therefore we use fully developed relations"
Nus=0.023*Re^0.8*Pr^0.4
h=k/D_h*Nus
T_s=T_e+q_s/h

"Calculations for air temperature at the duct exit"
m_dot=rho*V*A_c
Q_dot=m_dot*c_p*(T_e-T_i)
A_s=p*L
Q_dot=q_s*A_s
```

8-162 …. 8-164 Design and Essay Problems

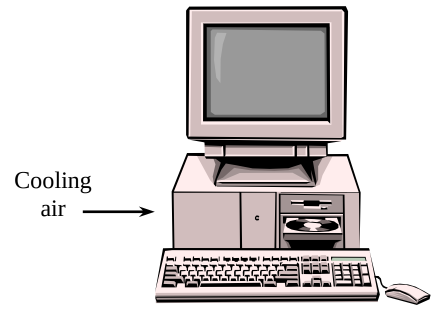
8-164 A computer is cooled by a fan blowing air through the case of the computer. The flow rate of the fan and the diameter of the casing of the fan are to be specified.

Assumptions **1** Steady flow conditions exist. **2** Heat flux is uniformly distributed. **3** Air is an ideal gas with constant properties.

Properties The relevant properties of air are (Tables A-1 and A-15)

$$c_p = 1007 \text{ J/kg.°C}$$

$$R = 0.287 \text{ kPa.m}^3\text{/kg.K}$$



Analysis We need to determine the flow rate of air for the worst case scenario. Therefore, we assume the inlet temperature of air to be 50°C, the atmospheric pressure to be 70.12 kPa, and disregard any heat transfer from the outer surfaces of the computer case. The mass flow rate of air required to absorb heat at a rate of 80 W can be determined from

$$\dot{Q} = \dot{m}c_p(T_{out} - T_{in}) \longrightarrow \dot{m} = \frac{\dot{Q}}{c_p(T_{out} - T_{in})} = \frac{80 \text{ J/s}}{(1007 \text{ J/kg.°C})(60-50)\text{°C}} = 0.007944 \text{ kg/s}$$

In the worst case the exhaust fan will handle air at 60°C. Then the density of air entering the fan and the volume flow rate becomes

$$\rho = \frac{P}{RT} = \frac{70.12 \text{ kPa}}{(0.287 \text{ kPa.m}^3\text{/kg.K})(60+273)\text{K}} = 0.7337 \text{ kg/m}^3$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{0.007944 \text{ kg/s}}{0.7337 \text{ kg/m}^3} = 0.01083 \text{ m}^3\text{/s} = \mathbf{0.6497 \text{ m}^3\text{/min}}$$

For an average velocity of 120 m/min, the diameter of the duct in which the fan is installed can be determined from

$$\dot{V} = A_c V = \frac{\pi D^2}{4} V \longrightarrow D = \sqrt{\frac{4\dot{V}}{\pi V}} = \sqrt{\frac{4(0.6497 \text{ m}^3\text{/min})}{\pi(120 \text{ m/min})}} = 0.083 \text{ m} = \mathbf{8.3 \text{ cm}}$$

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McGraw-Hill Education, 2020

Chapter 9
NATURAL CONVECTION

Physical Mechanisms of Natural Convection

9-1C Natural convection is the mode of heat transfer that occurs between a solid and a fluid which moves under the influence of natural means. Natural convection differs from forced convection in that fluid motion in natural convection is caused by natural effects such as buoyancy.

9-2C The convection heat transfer coefficient is usually higher in forced convection because of the higher fluid velocities involved.

9-3C The hot boiled egg in a spacecraft will cool faster when the spacecraft is on the ground since there is no gravity in space, and thus there will be no natural convection currents which is due to the buoyancy force.

9-4C The buoyancy force is proportional to the density of the medium, and thus is larger in sea water than it is in fresh water. Therefore, the hull of a ship will sink deeper in fresh water because of the smaller buoyancy force acting upwards.

9-5C The upward force exerted by a fluid on a body completely or partially immersed in it is called the buoyancy or "lifting" force. The buoyancy force is proportional to the density of the medium. Therefore, the buoyancy force is the largest in mercury, followed by in water, air, and the evacuated chamber. Note that in an evacuated chamber there will be no buoyancy force because of absence of any fluid in the medium.

9-6C There cannot be any natural convection heat transfer in a medium that experiences no change in volume with temperature.

9-7C The greater the volume expansion coefficient, the greater the change in density with temperature, the greater the buoyancy force, and thus the greater the natural convection currents.

9-8C The Grashof number represents the ratio of the buoyancy force to the viscous force acting on a fluid. The inertial forces in Reynolds number is replaced by the buoyancy forces in Grashof number.

9-9 The volume expansion coefficient is defined as $\beta = \frac{-1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$. For an ideal gas, $P = \rho RT$ or $\rho = \frac{P}{RT}$, and thus

$$\beta = -\frac{1}{\rho}\left(\frac{\partial (P/RT)}{\partial T}\right)_P = \frac{-1}{\rho}\left(\frac{-P}{RT^2}\right) = \frac{1}{\rho T}\left(\frac{P}{RT}\right) = \frac{1}{\rho T}(\rho) = \frac{1}{T}$$

9-10 The volume expansion coefficient of saturated liquid water at 70°C is to be determined using its definition and the values tabulated in Table A-9.

Assumptions Density depends on temperature only and not pressure.

Properties The properties of sat. liq. water are listed in the following table:

T, °C	***ρ*, kg/m³**	***β*, K⁻¹**
65	980.4	
70	977.5	0.578×10^{-3}
75	974.7	

Analysis The volume expansion coefficient is defined as

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

For density varying with temperature at constant pressure, we can approximate

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_1 - \rho_2}{T_1 - T_2} \quad \text{where} \quad T_1 = 65°\text{C}\,, \quad T_2 = 75°\text{C}\,, \quad \text{and} \quad \rho = 977.5\,\text{kg/m}^3$$

Hence, the volume expansion coefficient is calculated to be

$$\beta \approx -\frac{1}{977.5\,\text{kg/m}^3}\frac{(980.4 - 974.7)\,\text{kg/m}^3}{(65-75)\,\text{K}} = \mathbf{5.83 \times 10^{-4}\ K^{-1}}$$

Discussion The calculated volume expansion coefficient is about 1% higher than the value listed in Table A-9 (5.78×10^{-4} K^{-1}).

9-11 Using the given $\rho(T)$ correlation, the volume expansion coefficient of liquid water at 70°C is to be determined.

Assumptions Density depends on temperature only and not pressure.

Properties The volume expansion coefficient of liquid water at 70°C is 5.78×10^{-4} K^{-1} (Table A-9).

Analysis The volume expansion coefficient is defined as

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{d\rho}{dT}\right) = -\frac{1}{\rho}(-0.0736 - 0.0071T)$$

Hence, at T = 70°C the volume expansion coefficient is

$$\beta = -\frac{(-0.0736 - 0.0071T)\ \text{kg/m}^3\cdot\text{K}}{(1000 - 0.0736T - 0.00355T^2)\ \text{kg/m}^3}$$

$$= -\frac{[-0.0736 - 0.0071(70)]\,\text{kg/m}^3\cdot\text{K}}{[1000 - 0.0736(70) - 0.00355(70)^2]\,\text{kg/m}^3}$$

$$= \mathbf{5.84 \times 10^{-4}\ K^{-1}}$$

Discussion The calculated volume expansion coefficient is about 1% higher than the value listed in Table A-9 (5.78×10^{-4} K^{-1}).

9-12 The Grashof numbers for a plate placed in various fluids are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Air behaves as an ideal gas.

Properties The properties of air, liq. water, and engine oil are listed in the following table:

Fluid	T_f **, °C**	ρ**, kg/m³**	μ**, kg/m·s**	β**, K⁻¹**
Air (Table A-15)	90	0.9718	2.139×10^{-5}	2.755×10^{-3}
Liq. water (Table A-9)	90	965.3	0.315×10^{-3}	0.702×10^{-3}
Engine oil (Table A-13)	80	852.0	3.232×10^{-2}	0.700×10^{-3}

For air (ideal gas) $\beta = 1/T_f$

Analysis The Grashof number is given as

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} = \frac{g\beta(T_s - T_\infty)L_c^3}{(\mu/\rho)^2}$$

For air,

$$\mathrm{Gr}_{L,\,\mathrm{air}} = \frac{(9.81\,\mathrm{m/s^2})(2.755\times10^{-3}\ \mathrm{K^{-1}})(150-30)\mathrm{K}(0.1\,\mathrm{m})^3}{(2.139\times10^{-5}/0.9718)^2\ \mathrm{m^4/s^2}} = \mathbf{6.69\times10^6}$$

The Grashof number for liquid water and engine oil are calculated similar to the calculation done for air above

$$\mathrm{Gr}_{L,\,\mathrm{water}} = \mathbf{7.76\times10^9}$$

$$\mathrm{Gr}_{L,\,\mathrm{oil}} = \mathbf{6.68\times10^5}$$

Discussion Higher value of the Grashof number implies increase in buoyancy force over the viscous force, which means increase in natural convection flow.

9-13 The Grashof and Rayleigh numbers for a rod submerged in various fluids are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** Air behaves as an ideal gas. **4** The rod is orientated such that the characteristic length is its diameter.

Properties The properties of air, liq. water, and engine oil are listed in the following table:

Fluid	T_f, °F	ρ, lbm/ft³	μ, lbm/ft·s	Pr	β, R⁻¹
Liq. water (Table A-9E)	120	61.71	3.744×10^{-4}	3.63	0.246×10^{-3}
Liq. ammonia (Table A-11E)	120	35.26	7.444×10^{-5}	1.313	1.74×10^{-3}
Engine oil (Table A-13E)	125	54.24	7.617×10^{-2}	1607	0.389×10^{-3}
Air (Table A-15E)	120	0.06843	1.316×10^{-5}	0.723	1.72×10^{-3}

For air (ideal gas), $\beta = 1/T_f$.

Analysis The Grashof and Rayleigh numbers are given as

$$\mathrm{Gr}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2} = \frac{g\beta(T_s - T_\infty)D^3}{(\mu/\rho)^2} \qquad \text{and} \qquad \mathrm{Ra}_D = \mathrm{Gr}_D\,\mathrm{Pr}$$

(*a*) For liquid water,

$$\mathrm{Gr}_{D,\,\mathrm{water}} = \frac{(32.2\ \mathrm{ft/s^2})(0.246\times10^{-3}\ \mathrm{R^{-1}})(200-40)\mathrm{R}(2/12\ \mathrm{ft})^3}{(3.744\times10^{-4}/61.71)^2\ \mathrm{ft^4/s^2}} = \mathbf{1.59\times10^8}$$

$$\mathrm{Ra}_{D,\,\mathrm{water}} = (1.59\times10^8)(3.63) = \mathbf{5.79\times10^8}$$

The Grashof and Rayleigh numbers for liquid ammonia, engine oil, and air are calculated similar to the calculation done for liquid water above

(*b*) $\mathrm{Gr}_{D,\,\mathrm{ammonia}} = \mathbf{9.31\times10^9}$ and $\mathrm{Ra}_{D,\,\mathrm{ammonia}} = \mathbf{1.22\times10^{10}}$

(*c*) $\mathrm{Gr}_{D,\,\mathrm{oil}} = \mathbf{4.41\times10^3}$ and $\mathrm{Ra}_{D,\,\mathrm{oil}} = \mathbf{7.09\times10^6}$

(*d*) $\mathrm{Gr}_{D,\,\mathrm{air}} = \mathbf{1.11\times10^6}$ and $\mathrm{Ra}_{D,\,\mathrm{air}} = \mathbf{8.02\times10^5}$

Discussion For the rod’s characteristic length to be its diameter, the rod has to be placed horizontally.

Natural Convection over Surfaces

9-14C Rayleigh number is the product of the Grashof and Prandtl numbers.

9-15C No, a hot surface will cool slower when facing down since the warmer air in this position cannot rise and escape easily.

9-16C The heat flux will be higher at the bottom of the plate since the thickness of the boundary layer which is a measure of thermal resistance is the lowest there.

9-17C A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr^{1/4}}$.

9-18 A vertical plate separates the hot water from the cold water. The temperature of the plate surface on the cold water side is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Temperature of each surface is constant. **3** The plate thermal conductivity is constant. **4** Radiation heat transfer is negligible.

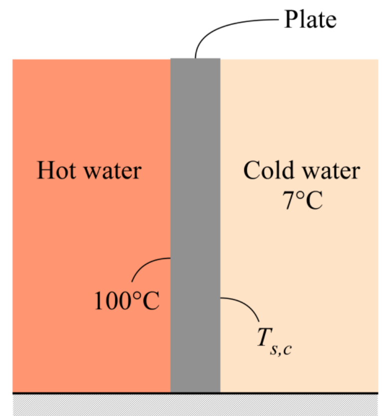
Properties Assuming the surface temperature on the cold water side is $T_{s,c}$ = (100 + 7)°C/2=53.5°C, thus $T_{f,c} = (T_{s,c} + T_{\infty,c})/2$ = (53.5 + 7)°C/2 = 30.25°C. Then, the properties of water at $T_{f,c}$ are k = 0.6033 W/m·K, ρ = 995.5 kg/m^3, μ = 0.0007935 kg/m·s, $\nu = \mu/\rho = 7.971 \times 10^{-7}$ m^2/s, Pr = 5.502, β = 0.0003072 K^{-1} (Table A-9).

The thermal conductivity of the plate is given as k_{plate} = 15 W/m·K.

Analysis The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,c} - T_{\infty,c})L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.0003072\,\text{K}^{-1})(53.5-7)\,\text{K}\,(0.2\,\text{m})^3}{(7.971\times10^{-7}\,\text{m}^2/\text{s})^2}(5.502) = 9.708\times10^9$$

The Nusselt number relation for vertical plate is



$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(9.705\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{5.502}\right)^{9/16}\right]^{8/27}}\right\}^2 = 307.2$$

$$h = \frac{k}{L}\text{Nu} = \frac{0.6033\,\text{W/m}\cdot\text{K}}{0.2\,\text{m}}(307.2) = 926.7\,\text{W/m}^2\cdot\text{K}$$

Thus, the rate of heat transfer balance for conduction through the plate thickness and natural convection is

$$\frac{k_{\text{plate}}}{l}(T_{s,h} - T_{s,c}) = h(T_{s,c} - T_{\infty,c}) \quad \rightarrow \quad T_{s,c} = 43.5°\text{C} \quad \text{(first iteration)}$$

The above solution is repeated iteratively until $T_{s,c}$ converges to $T_{s,c}$ = **46°C**.

Discussion The results from the iterative solution are listed in the following table:

Iter	$T_{s,c}$ [°C]	Ra	Nu
1	53.5	9.708×10^9	307.2
2	43.5	5.915×10^9	264.6
3	47.2	7.194×10^9	280.7
4	45.8	6.694×10^9	274.7
5	46.3	6.870×10^9	276.8
6	**46.1**	6.799×10^9	276.0

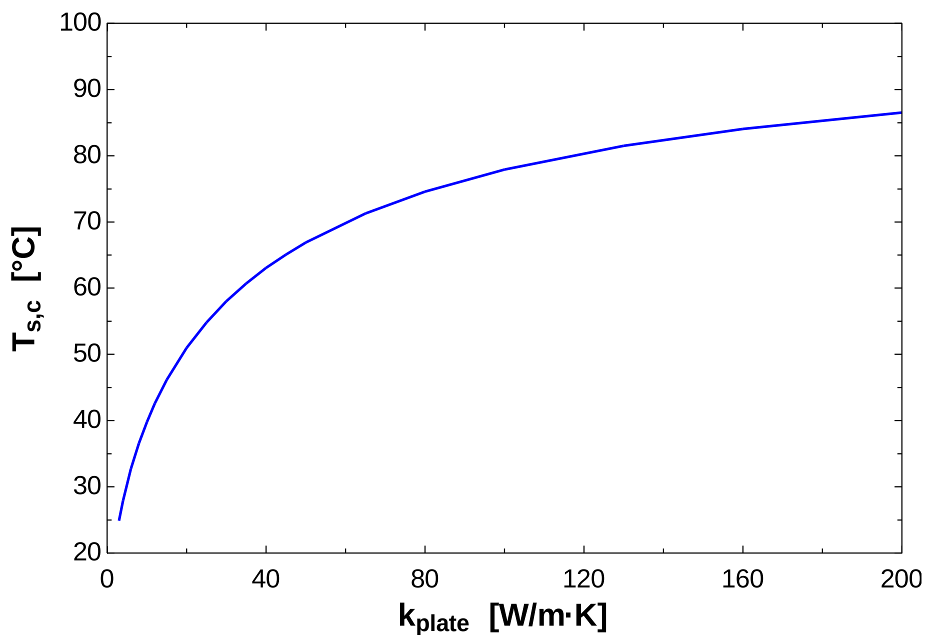
As $T_{s,c}$ changes through the iterations, so does the film temperature used for evaluating the properties.

9-19 Reconsider Prob. 9-18. A vertical plate separates the hot water from the cold water. The effect of k_{plate} on $T_{s,c}$ is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.2 [m]
thickness=0.025 [m]
T_infinity_c=7 [C]
T_s_h=100 [C]

"PROPERTIES"
g=9.81 [m/s^2]
Fluid$='water'
k=Conductivity(Fluid$, T=T_film_c, x=0)
Pr=Prandtl(Fluid$, T=T_film_c, x=0)
rho=Density(Fluid$, T=T_film_c, x=0)
mu=Viscosity(Fluid$, T=T_film_c, x=0)
nu=mu/rho
beta=Volexpcoef(Fluid$, T=T_film_c, x=0)
T_film_c=1/2*(T_s_c+T_infinity_c)

"ANALYSIS"
Ra=(g*beta*(T_s_c-T_infinity_c)*L^3)/nu^2*Pr
Nusselt=(0.825+0.387*Ra^(1/6)/((1+(0.492/Pr)^(9/16))^(8/27)))^2
h=k/L*Nusselt
q_dot=k_plate/thickness*(T_s_h-T_s_c)
q_dot=h*(T_s_c-T_infinity_c)
```

k_{plate} [W/m·K]	$T_{s,c}$ [°C]
3	24.89
4	27.88
6	32.71
8	36.57
10	39.80
12	42.59
15	46.16
20	50.97
25	54.82
30	58.01
35	60.72
40	63.05
45	65.09
50	66.90
65	71.27
80	74.57
100	77.90
130	81.49
160	84.06
200	86.53



Discussion As k_{plate} increases, the thermal resistance of the plate reduces, therefore $T_{s,c}$ increases with increasing value of k_{plate}.

9-20 C&S A vertical ASTM A203 B steel plate with one surface subjected to natural convection with cold air at −70°C. The rate of heat addition to the plate needed to keep the surface from cooling below −30°C is to be determined.

Assumptions **1** Steady state conditions. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm. **4** Thermal radiation is negligible.

Properties The properties of air at the film temperature of $T_f = (T_\infty + T_s)/2 = (-70 - 30)/2 = -50$°C are (Table A–15) Pr = 0.7440, k = 0.01979 W/m·K, $\nu = 9.319 \times 10^{-6}$ m²/s, $\beta = 1/T_f = 1/(-50 + 273\text{ K}) = 0.004484\text{ K}^{-1}$

Analysis The characteristic length of the plate is $L_c = L = 0.5$ m, and the Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.004484\text{ K}^{-1})(-30 + 70\text{ K})(0.5\text{ m})^3}{(9.319 \times 10^{-6}\text{m}^2/\text{s})^2}(0.7440) = 1.8843 \times 10^9$$

The Nusselt number for natural convection in this case is determined from

$$\text{Nu} = \frac{hL}{k} = \left\{0.825 + \frac{0.387\text{ Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$$

So, the natural convection heat transfer coefficient is

$$h = \frac{k}{L}\left\{0.825 + \frac{0.387\text{ Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \frac{0.01917\text{ W/m}\cdot\text{K}}{0.5\text{ m}}\left\{0.825 + \frac{0.387(1.8843 \times 10^9)^{1/6}}{[1 + (0.492/0.7440)^{9/16}]^{8/27}}\right\}^2$$

$$h = 5.955\text{ W/m}^2\cdot\text{K}$$

The heat transfer rate required to keep the plate surface at −30°C can be determined from the Newton's law of cooling:

$$\dot{Q} = hA_s(T_s - T_\infty) = hL^2(T_s - T_\infty) = (5.955\text{ W/m}^2\cdot\text{K})(0.5\text{ m})^2(-30 + 70\text{ K}) = \mathbf{59.6\text{ W}}$$

Discussion With a heat addition rate of about 60 W or higher, the plate surface exposed to natural convection with the cold air can be maintained from being cooled below −30°C.

9-21 C&S A vertical ASTM B152 copper plate with one surface subjected to natural convection with hot air. The rate of heat removal from the plate is 90 W, and the maximum temperature that the air can reach without causing the surface temperature of the copper plate to increase above 260°C is to be determined.

Assumptions **1** Steady state conditions. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm. **4** Thermal radiation is negligible.

Properties The properties of air at the film temperature of T_f = 300°C are (Table A–15) Pr = 0.6935, k = 0.04418 W/m·K, ν = 4.765 × 10^{-5} m²/s, $\beta = 1/T_f$ = 1/(300 + 273 K) = 0.001745 K^{-1}

Analysis The characteristic length of the plate is $L_c = L$ = 0.5 m, and the Rayleigh number is

$$\mathrm{Ra}_L = \frac{g\beta(T_\infty - T_s)L^3}{\nu^2}\mathrm{Pr}$$

The Nusselt number for natural convection in this case is determined from

$$\mathrm{Nu} = \frac{hL}{k} = \left\{0.825 + \frac{0.387\ \mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

So, the natural convection heat transfer coefficient is

$$h = \frac{k}{L}\left\{0.825 + \frac{0.387\ \mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

From energy balance, the rate of heat removal from the plate is equal to the heat transfer rate to the plate by natural convection from the hot air:

$$\dot{Q} = hA_s(T_\infty - T_s) = hL^2(T_\infty - T_s) = 90\ \mathrm{W}$$

By substituting the equation for Ra_L into the equation for h and then substituting the resulting equation in the above Newton's law of cooling equation yields

$$\frac{k}{L}\left\{0.825 + \frac{0.387\ [g\beta(T_\infty - T_s)L^3\mathrm{Pr}/\nu^2]^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2 L^2(T_\infty - T_s) = 90\ \mathrm{W}$$

or

$$\frac{0.04418\ \mathrm{W/m\cdot K}}{0.5\ \mathrm{m}}\left\{0.825 + \frac{0.387\ [(9.81\ \mathrm{m/s^2})(0.001745\ \mathrm{K^{-1}})(T_\infty - T_s)(0.5\ \mathrm{m})^3(0.6935)/(4.765\times 10^{-5}\mathrm{m^2/s})^2]^{1/6}}{[1 + (0.492/0.6935)^{9/16}]^{8/27}}\right\}^2 (0.5\ \mathrm{m})^2(T_\infty - T_s) = 90\ \mathrm{W}$$

The temperature of the hot air can be solved implicitly by trail-and-error with T_s = 260°C, which yields

$$T_\infty = \mathbf{340.9}℃$$

Discussion With a heat removal rate from the copper plate at 90 W, the plate surface exposed to natural convection with the hot air would reach 260°C if the air temperature is at 340.9°C. Thus, the maximum temperature that the air can reach without causing the surface temperature of the copper plate to increase above its maximum use temperature is about 340°C.
With T_∞ = 340.9°C and T_s = 260°C, the film temperature becomes 300.5°C. Thus, 300°C is an appropriate temperature to evaluate the properties of air.

9-22 A vertical plate separates the hot water from the cold air. The surface exposed to the cold air is subjected to radiation heat transfer also. The temperature of the plate surface exposed to the cold air is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Temperature on each surface is constant. **3** The plate thermal conductivity is constant. **4** Local atmospheric pressure is 1 atm. **5** The T_{surr} is the same as the cold air temperature.

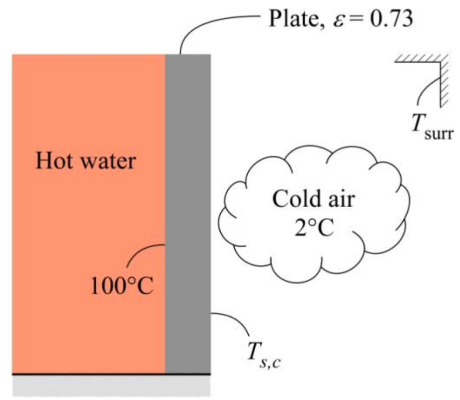
Properties Assuming the surface temperature on the cold air side is $T_{s,c}$ = (100 + 2)°C/2=51°C, thus $T_{f,c} = (T_{s,c} + T_{\infty,c})/2$ = (51 + 2)°C/2 = 26.5°C. Then, the properties of air at $T_{f,c}$ and 1 atm pressure are k = 0.02562 W/m·K, $\nu = 1.576 \times 10^{-5}$ m^2/s, Pr = 0.7277 (Table A-15), and $\beta = 1/T_{f,c}$ = 1/ 299.5 K.

The thermal conductivity and the emissivity of the plate are given as k_{plate} = 1.5 W/m·K and ε_{plate} = 0.73, respectively.

Analysis The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,c} - T_{\infty,c})L_c^{\,3}}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(26.5 + 273\,\text{K})^{-1}(51 - 2)\,\text{K}\,(0.2\,\text{m})^3}{(1.576\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7277) = 3.762\times10^7$$

The Nusselt number relation for vertical plate is



$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1 + \left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(3.762\times10^7)^{1/6}}{\left[1 + \left(\frac{0.492}{0.7277}\right)^{9/16}\right]^{8/27}}\right\}^2$$

$$= 45.89$$

$$h = \frac{k}{L}\text{Nu} = \frac{0.02562\ \text{W/m}\cdot\text{K}}{0.2\ \text{m}}(45.89) = 5.879\ \text{W/m}^2\cdot\text{K}$$

Thus, the rate of heat transfer balance for conduction through the plate thickness l, natural convection and radiation is

$$\frac{k_{plate}}{l}(T_{s,h} - T_{s,c}) = h(T_{s,c} - T_{\infty,c}) + \varepsilon_{plate}\sigma(T_{s,c}^4 - T_{surr}^4) \quad \to \quad T_{s,c} = 84.6°\text{C} \quad \text{(first iteration)}$$

The above solution is repeated iteratively until $T_{s,c}$ converges to $T_{s,c}$ = **83.7°C**.

Discussion The results from the iterative solution are listed in the following table:

Iter	$T_{s,c}$ [°C]	Ra	Nu	h [W/m^2·K]
1	51	3.762×10^7	45.89	5.879
2	84.6	4.934×10^7	49.66	6.670
3	83.7	4.912×10^7	49.60	6.653
4	83.72	4.913×10^7	49.60	6.654
5	**83.71**	4.912×10^7	49.60	6.653

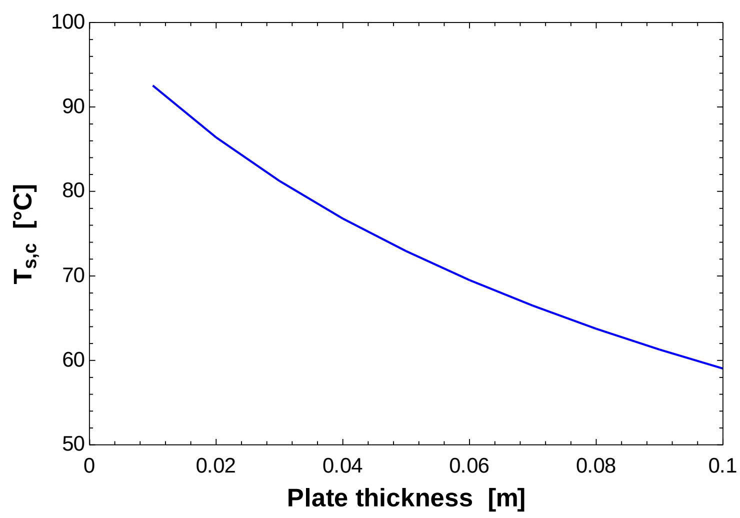
As $T_{s,c}$ changes through the iterations, so does the film temperature used for evaluating the properties.

9-23 Reconsider Prob. 9-22. A vertical plate separates the hot water from the cold air. The surface exposed to the cold air is subjected to radiation heat transfer also. The effect of the plate thickness on $T_{s,c}$ is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.2 [m]
T_infinity_c=2 [C]
T_s_h=100 [C]

"PROPERTIES"
g=9.81 [m/s^2]     "gravitational acceleration"
Fluid$='air'
"Cold air"
k=Conductivity(Fluid$, T=T_film_c)
Pr=Prandtl(Fluid$, T=T_film_c)
rho=Density(Fluid$, T=T_film_c, P=101.3)
mu=Viscosity(Fluid$, T=T_film_c)
nu=mu/rho
beta=Volexpcoef(Fluid$, T=T_film_c)
T_film_c=1/2*(T_s_c+T_infinity_c)
"Plate"
k_plate=1.5 [W/m-K]
epsilon_plate=0.73

"ANALYSIS"
Ra=(g*beta*(T_s_c-T_infinity_c)*L^3)/nu^2*Pr
Nusselt=(0.825+0.387*Ra^(1/6)/((1+(0.492/Pr)^(9/16))^(8/27)))^2
h=k/L*Nusselt
q_dot=k_plate/thickness*(T_s_h-T_s_c)
q_dot=h*(T_s_c-T_infinity_c)+sigma#*epsilon_plate*((T_s_c+273)^4-(T_infinity_c+273)^4)
```

Thickness [m]	$T_{s,c}$ [°C]
0.01	92.54
0.02	86.40
0.03	81.23
0.04	76.79
0.05	72.92
0.06	69.51
0.07	66.47
0.08	63.74
0.09	61.27
0.10	59.03

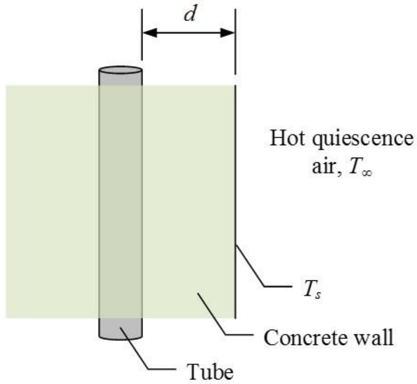


Discussion As the plate thickness increases, the thermal resistance of the plate increases, thus reducing the surface temperature on the cold air side.

9-24 C&S A CPVC tube is embedded in a vertical concrete wall, where the tube surface is 3 cm from the wall surface. The surface of the wall is subjected to convection with hot quiescence air at 140°C. Would the tube comply with the ASME Code for Process Piping?

Assumptions **1** Steady state conditions. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm. **4** Thermal radiation is negligible. **5** No heat generation in the wall. **6** One-dimensional conduction. **7** Constant properties.

Properties The properties of air at the film temperature of $T_f = (T_\infty + T_s)/2 = (140 + 100)/2 = 120°C$ are (Table A–15) Pr = 0.7073, $k = 0.03235$ W/m·K, $\nu = 2.522 \times 10^{-5}$ m²/s, $\beta = 1/T_f = 1/(120 + 273 \text{ K}) = 0.002545 \text{ K}^{-1}$



Analysis The characteristic length of the vertical wall is $L_c = L = 1$ m, and the Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_\infty - T_s)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.002545 \text{ K}^{-1})(140 - 100 \text{ K})(1 \text{ m})^3}{(2.522 \times 10^{-5}\text{m}^2/\text{s})^2}(0.7073) = 1.1105 \times 10^9$$

The Nusselt number for natural convection in this case is determined from

$$\text{Nu} = \frac{hL}{k} = \left\{0.825 + \frac{0.387 \text{ Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$$

So, the natural convection heat transfer coefficient is

$$h = \frac{k}{L}\left\{0.825 + \frac{0.387 \text{ Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \frac{0.03235 \text{ W/m} \cdot \text{K}}{1 \text{ m}}\left\{0.825 + \frac{0.387(1.1105 \times 10^9)^{1/6}}{[1 + (0.492/0.7073)^{9/16}]^{8/27}}\right\}^2$$

$$h = 4.103 \text{ W/m}^2 \cdot \text{K}$$

Applying energy balance, with natural convection at the concrete surface and conduction through the concrete wall,

$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}}$$

$$k_{\text{wall}}\frac{T_s - T_d}{d} = h(T_\infty - T_s)$$

Solving for the temperature T_d in the concrete wall at d = 3 cm, we get

$$T_d = T_s - \frac{hd}{k_{\text{wall}}}(T_\infty - T_s) = 100℃ - \frac{(4.103 \text{ W/m}^2 \cdot \text{K})(0.03 \text{ m})}{1.4 \text{ W/m} \cdot \text{K}}(140 - 100 \text{ ℃}) = \mathbf{96.48}℃ > 93.3℃$$

Discussion With the tube surface located at the depth of 3 cm from the concrete surface, it will be in contact with the concrete at 96.5°C, which is over 3°C higher than the recommended maximum temperature for CPVC (ASME B31.3-2014, Table B-1). Therefore, the tube would not be in compliance.
If the depth is increased to $d \geq 5.72$ cm, the temperature in the concrete wall would be 93.3°C or lower.

9-25 C&S A vertical ASTM A240 410S stainless steel plate with one surface subjected to natural convection with cold gas at −70°C. The type of gas alternates between carbon dioxide and hydrogen. The rate of heat addition to the plate needed to keep the surface from cooling below −30°C, such that it is applicable to both carbon dioxide and hydrogen, is to be determined.

Assumptions **1** Steady state conditions. **2** CO_2 and H_2 are ideal gases. **3** The local atmospheric pressure is 1 atm. **4** Thermal radiation is negligible.

Properties The properties of CO_2 at the film temperature of $T_f = (T_\infty + T_s)/2 = (-70 - 30)/2 = -50$°C are (Table A–16) Pr = 0.8019, $k = 0.01051$ W/m·K, $\nu = 4.699 \times 10^{-6}$ m²/s, $\beta = 1/T_f = 1/(-50 + 273\text{ K}) = 0.004484\text{ K}^{-1}$

The properties of H_2 gas at the film temperature of $T_f = -50$°C are (Table A-16) Pr = 0.6562, $k = 0.1404$ W/m·K, $\nu = 6.624 \times 10^{-5}$ m²/s, $\beta = 1/T_f = 1/(-50 + 273\text{ K}) = 0.004484\text{ K}^{-1}$

Analysis

For CO_2 gas: The characteristic length of the plate is $L_c = L = 0.5$ m, and the Rayleigh number is

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr} = \frac{(9.81\text{ m/s}^2)(0.004484\text{ K}^{-1})(-30 + 70\text{ K})(0.5\text{ m})^3}{(4.699 \times 10^{-6}\text{m}^2/\text{s})^2}(0.8019) = 7.9876 \times 10^9$$

The Nusselt number for natural convection in this case is determined from

$$\mathrm{Nu} = \frac{hL}{k} = \left\{0.825 + \frac{0.387\,\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

So, the natural convection heat transfer coefficient is

$$h = \frac{k}{L}\left\{0.825 + \frac{0.387\,\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2 = \frac{0.01051\text{ W/m}\cdot\text{K}}{0.5\text{ m}}\left\{0.825 + \frac{0.387(7.9876 \times 10^9)^{1/6}}{[1 + (0.492/0.8019)^{9/16}]^{8/27}}\right\}^2$$

$$h = 5.024\text{ W/m}^2\cdot\text{K}$$

The heat transfer rate required to keep the plate surface at −30°C, when exposed to the cold CO_2 gas at −70°C is

$$\dot{Q} = hA_s(T_s - T_\infty) = hL^2(T_s - T_\infty) = (5.024\text{ W/m}^2\cdot\text{K})(0.5\text{ m})^2(-30 + 70\text{ K}) = \mathbf{50.24\text{ W}}$$

For H_2 gas: The characteristic length of the plate is $L_c = L = 0.5$ m, and the Rayleigh number is

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr} = \frac{(9.81\text{ m/s}^2)(0.004484\text{ K}^{-1})(-30 + 70\text{ K})(0.5\text{ m})^3}{(6.624 \times 10^{-5}\text{m}^2/\text{s})^2}(0.6562) = 3.2893 \times 10^7$$

The Nusselt number for natural convection in this case is determined from

$$\mathrm{Nu} = \frac{hL}{k} = \left\{0.825 + \frac{0.387\,\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

So, the natural convection heat transfer coefficient is

$$h = \frac{k}{L}\left\{0.825 + \frac{0.387\,\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2 = \frac{0.1404\text{ W/m}\cdot\text{K}}{0.5\text{ m}}\left\{0.825 + \frac{0.387(3.2893 \times 10^7)^{1/6}}{[1 + (0.492/0.6562)^{9/16}]^{8/27}}\right\}^2$$

$$h = 12.22\text{ W/m}^2\cdot\text{K}$$

The heat transfer rate required to keep the plate surface at −30°C, when exposed to the cold H_2 gas at −70°C is

$$\dot{Q} = hA_s(T_s - T_\infty) = hL^2(T_s - T_\infty) = (12.22\text{ W/m}^2\cdot\text{K})(0.5\text{ m})^2(-30 + 70\text{ K}) = \mathbf{122.2\text{ W}}$$

Discussion Comparing the results for the CO_2 and H_2 gas, a heat addition rate of higher than 122 W is needed to keep the plate surface from cooling below −30°C, such that it is applicable for both gases. The natural convection heat transfer coefficient for the H_2 gas is about 2.4 times higher than that for the CO_2 gas. The H_2 gas has higher thermal conductivity (about 13 times higher than CO_2), while the CO_2 gas is a better thermal insulator.

9-26 A thin vertical plate is subjected to uniform heat flux on one side and exposed to cool air on the other side. The heat flux on the plate is to be determined.

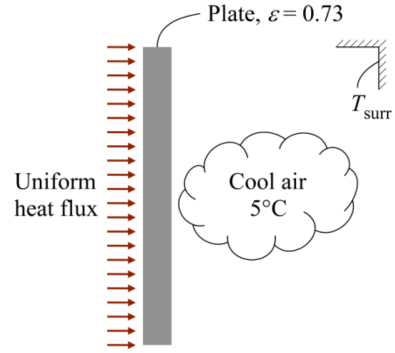
Assumptions **1** Steady operating conditions exist. **2** Constant heat flux on the plate surface. **3** Thermal resistance in the plate is negligible. **4** Local atmospheric pressure is 1 atm. **5** The T_{surr} is the same as the cool air temperature.

Properties The film temperature is determined with the plate midpoint temperature, $T_f = (T_{L/2} + T_\infty)/2 = (55 + 5)°C/2 = 30°C$. Then, the properties of air at $T_f = 30°C$ are $k = 0.02588$ W/m·K, $\nu = 1.608 \times 10^{-5}$ m^2/s, Pr = 0.7282 (Table A-15), and $\beta = 1/T_f = 1/303$ K = 0.0033 K^{-1}.

Analysis The Rayleigh number is

$$\mathrm{Ra} = \frac{g\beta(T_{L/2} - T_\infty)L_c^{\,3}}{\nu^2}\mathrm{Pr} = \frac{(9.81\,\mathrm{m/s^2})(30+273\,\mathrm{K})^{-1}(55-5)\,\mathrm{K}\,(0.5\,\mathrm{m})^3}{(1.608\times10^{-5}\,\mathrm{m^2/s})^2}(0.7282) = 5.699\times10^8$$

The Nusselt number relation for vertical plate is



$$\mathrm{Nu} = \left\{0.825 + \frac{0.387\mathrm{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\mathrm{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(5.699\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7282}\right)^{9/16}\right]^{8/27}}\right\}^2 = 103.7$$

$$h = \frac{k}{L}\mathrm{Nu} = \frac{0.02588\ \mathrm{W/m\cdot K}}{0.5\,\mathrm{m}}(103.7) = 5.368\ \mathrm{W/m^2\cdot K}$$

Thus, the heat flux on the plate surface can be determined from the heat loss by natural convection and radiation on the other side of the plate:

$$\begin{aligned}\dot{q}_s &= h(T_{L/2} - T_\infty) + \varepsilon\sigma(T_{L/2}^4 - T_{\mathrm{surr}}^4)\\ &= (5.368\ \mathrm{W/m^2\cdot K})(55-5)\ \mathrm{K} + (5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4})(0.73)(328^4 - 278^4)\ \mathrm{K^4}\\ &= \mathbf{500\ W/m^2}\end{aligned}$$

Discussion Natural convection contributes to about 54% of the 500 W/m^2 heat flux. For constant surface heat flux, the plate midpoint temperature is used instead of the surface temperature in the evaluation of the fluid properties.

9-27 A thin vertical plate is subjected to uniform heat flux on one side and exposed to hydrogen gas on the other side. The plate midpoint temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Constant heat flux on the plate surface. **3** Thermal resistance in the plate is negligible. **4** Local atmospheric pressure is 1 atm. **5** The film temperature is 50°C (this assumption will be verified).

Properties The properties of H_2 gas at T_f = 50°C are k = 0.1881 W/m·K, $\nu = 1.240 \times 10^{-4}$ m^2/s, Pr = 0.7191 (Table A-16), and $\beta = 1/T_f$ = 1/ 323 K = 0.003096 K^{-1} (Table A-16).

Analysis With the assumption that T_f = 50°C, the plate midpoint temperature is estimated as

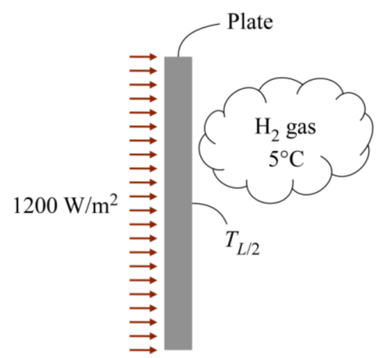
$$T_{L/2} = 2T_f - T_\infty = 95°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{L/2} - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(50+273\,\text{K})^{-1}(95-5)\,\text{K}\,(0.5\,\text{m})^3}{(1.240\times10^{-4}\ \text{m}^2/\text{s})^2}(0.7191) = 1.598\times10^7$$

The Nusselt number relation for vertical plate is

$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.598\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7191}\right)^{9/16}\right]^{8/27}}\right\}^2 = 35.74$$



$$h = \frac{k}{L}\text{Nu} = \frac{0.1881\ \text{W/m}\cdot\text{K}}{0.5\,\text{m}}(35.74) = 13.45\ \text{W/m}^2\cdot\text{K}$$

Thus, the plate midpoint temperature from the heat flux as

$$\dot{q}_s = h(T_{L/2} - T_\infty) \quad \rightarrow \quad T_{L/2} = \frac{\dot{q}_s}{h} + T_\infty$$

$$T_{L/2} = \frac{1200\ \text{W/m}^2}{13.45\ \text{W/m}^2\cdot\text{K}} + 5°\text{C} = \mathbf{94.2°C}$$

Discussion The assumed film temperature T_f = 50°C is an appropriate assumption, since the determined $T_{L/2}$ = 94.2°C would give a film temperature of T_f = 49.6°C. Otherwise, $T_{L/2}$ would have to be solved iteratively.

For constant surface heat flux, the plate midpoint temperature is used instead of the surface temperature in the evaluation of the fluid properties.

9-28 A thin vertical plate is subjected to uniform heat flux on one side and exposed to air on the other side. The plate midpoint temperatures for (*a*) a highly polish copper surface and (*b*) a black oxidized copper surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Constant heat flux on the plate surface. **3** Thermal resistance in the plate is negligible. **4** Local atmospheric pressure is 1 atm. **5** The T_{surr} is the same as the air temperature.

Properties We first assume the film temperature is T_f = 30°C. Then, the properties of air at T_f are k = 0.02588 W/m·K, $\nu = 1.608 \times 10^{-5}$ m^2/s, Pr = 0.7282 (Table A-15), and $\beta = 1/T_f = 1/303$ K $= 0.0033$ K^{-1}.

The emissivity of highly polished copper is $\varepsilon = 0.02$ and of black oxidized copper is $\varepsilon = 0.78$ (TableA-18).

Analysis With the assumption that T_f = 30°C, the plate midpoint temperature is estimated as

$$T_{L/2} = 2T_f - T_\infty = 55°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{L/2} - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(30+273\,\text{K})^{-1}(55-5)\,\text{K}\,(0.5\,\text{m})^3}{(1.608\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7282) = 5.699\times10^8$$

The Nusselt number relation for vertical plate is

$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(5.699\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7282}\right)^{9/16}\right]^{8/27}}\right\}^2 = 103.7$$

$$h = \frac{k}{L}\text{Nu} = \frac{0.02588\ \text{W/m}\cdot\text{K}}{0.5\,\text{m}}(103.7) = 5.368\ \text{W/m}^2\cdot\text{K}$$

(*a*) With the known surface heat flux of 1000 W/m^2 and ε = 0.02 (highly polished copper), the plate midpoint temperature can be determined as

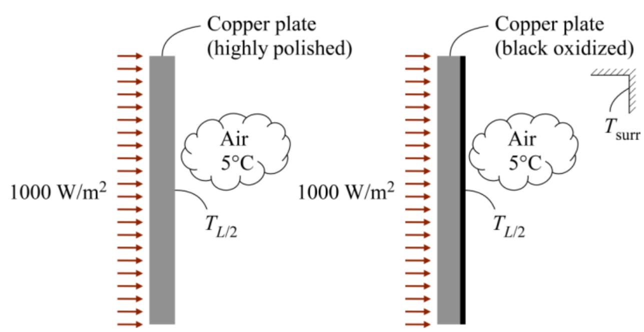
$$\dot{q}_s = h(T_{L/2} - T_\infty) + \varepsilon\sigma(T_{L/2}^4 - T_{\text{surr}}^4)$$

$$1000\ \text{W/m}^2 = (5.368\ \text{W/m}^2\cdot\text{K})(T_{L/2} - 5)\,\text{K} + (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(0.02)[(T_{L/2}+273)^4 - (5+273)^4]\,\text{K}^4$$

Solving for the plate midpoint temperature yields $T_{L/2}$ = 183.4°C (first iteration)

The above solution is repeated iteratively until $T_{L/2}$ converges to $T_{L/2}$ = **147.2°C**. The results from the iterations are as follows:

Iter	$T_{L/2}$ [°C]	Ra	Nu	h [W/m^2·K]
1	55	5.699×10^8	103.7	5.368
2	183.4	8.427×10^8	116.6	7.122
3	141.6	8.423×10^8	116.7	6.778
4	148.3	8.457×10^8	116.8	6.842
5	147.0	8.452×10^8	116.8	6.830
6	**147.2**	8.453×10^8	116.8	6.832



(*b*) With the known surface heat flux of 1000 W/m^2 and ε = 0.78 (black oxidized copper), the plate midpoint temperature can be determined as

$$\dot{q}_s = h(T_{L/2} - T_\infty) + \varepsilon\sigma(T_{L/2}^4 - T_{\text{surr}}^4)$$

$$1000\ \text{W/m}^2 = (5.368\ \text{W/m}^2\cdot\text{K})(T_{L/2} - 5)\,\text{K} + (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(0.78)[(T_{L/2}+273)^4 - (5+273)^4]\,\text{K}^4$$

Solving for the plate midpoint temperature yields $T_{L/2}$ = 92.9°C (first iteration)

The above solution is repeated iteratively until $T_{L/2}$ converges to $T_{L/2}$ = **88.5°C**. The results from the iterations are as follows:

Iter	$T_{L/2}$ [°C]	Ra	Nu	h [W/m^2·K]
1	55	5.699×10^8	103.7	5.368
2	92.9	7.564×10^8	113.0	6.165
3	88.1	7.401×10^8	112.3	6.085
4	88.6	7.419×10^8	112.3	6.093
5	**88.5**	7.415×10^8	112.3	6.092

Discussion For part (*a*), the highly polished copper surface has a low emissivity, which limits the heat loss on the surface by radiation. For part (*b*), the black oxidized copper surface has a higher emissivity, which increases the heat loss on the surface by radiation. Therefore the plate midpoint temperature for part (*a*) is higher than that of part (*b*).

Note that as $T_{L/2}$ changes through the iterations, so does the film temperature used for evaluating the properties.

9-29 Reconsider Prob. 9-28. A thin vertical copper plate is subjected to uniform heat flux on one side and exposed to air on the other side. The effect of the heat flux on the plate midpoint temperature for (*a*) a highly polished copper surface and (*b*) a black oxidized copper surface is to be determined.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.5 [m]
T_infinity=5 [C]
epsilon=0.02     "0.02 for (a) and 0.78 for (b)"

"PROPERTIES"
g=9.81 [m/s^2]     "gravitational acceleration"
Fluid$='air'
"Air"
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
beta=Volexpcoef(Fluid$, T=T_film)
T_film=1/2*(T_0.5L+T_infinity)

"ANALYSIS"
Ra=(g*beta*(T_0.5L-T_infinity)*L^3)/nu^2*Pr
Nusselt=(0.825+0.387*Ra^(1/6)/((1+(0.492/Pr)^(9/16))^(8/27)))^2
h=k/L*Nusselt
q_dot=h*(T_0.5L-T_infinity)+sigma#*epsilon*((T_0.5L+273)^4-(T_infinity+273)^4)
```

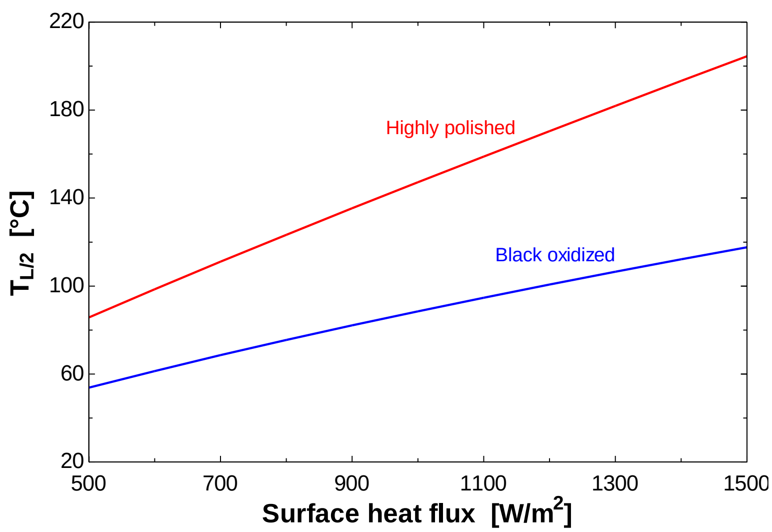
(*a*) Highly polished surface (ε = 0.02)

$\dot{q}_s$ [W/m^2]	$T_{s,c}$ [°C]	Ra
500	85.74	7.314E+08
600	98.57	7.735E+08
700	111.1	8.036E+08
800	123.3	8.243E+08
900	135.3	8.377E+08
1000	147.2	8.453E+08
1100	158.9	8.481E+08
1200	170.5	8.473E+08
1300	181.9	8.434E+08
1400	193.3	8.371E+08
1500	204.5	8.288E+08

(*b*) Black oxidized surface (ε = 0.78)

$\dot{q}_s$ [W/m^2]	$T_{s,c}$ [°C]	Ra
500	53.77	5.598E+08
600	61.34	6.106E+08
700	68.55	6.527E+08
800	75.46	6.877E+08
900	82.11	7.170E+08
1000	88.52	7.416E+08
1100	94.71	7.621E+08
1200	100.7	7.794E+08
1300	106.5	7.938E+08
1400	112.2	8.058E+08
1500	117.7	8.157E+08

Discussion The plate midpoint temperature for the highly polished copper surface is higher than that of the black oxidized copper surface. The highly polished copper surface has very low emissivity, thus the heat loss from the surface is mainly by natural convection. The black oxidized copper surface has significant heat loss by radiation in addition to natural convection, which causes the plate midpoint temperature to be lower.



9-30 A street sign surface is subjected to radiation, the surface temperature of the street sign is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The surface temperature is constant. **4** The street sign is treated as a vertical plate. **5** Air is an ideal gas.

Properties The properties of air (1 atm) at 30°C are given in Table A-15: k = 0.02588 W/m·K, $\nu = 1.608 \times 10^{-5}$ m^2/s, and Pr = 0.7282. Also, $\beta = 1/T_f = 0.0033$ K^{-1}.

Analysis The Rayleigh number is

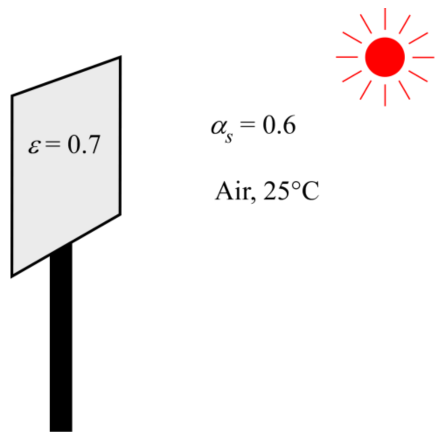
$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.0033\text{ K}^{-1})(T_s - 293)\text{K}(0.2\text{ m})^3}{(1.608\times10^{-5}\text{ m}^2/\text{s})^2}(0.7282) \quad \ldots\ldots (1)$$

Assuming the Rayleigh number is within $10^4 < \text{Ra}_L < 10^9$, the Nusselt number for vertical plate is

$$\text{Nu} = 0.59\text{Ra}_L^{1/4}$$

or

$$h = \left(\frac{0.02588\text{ W/m}\cdot\text{K}}{0.2\text{ m}}\right)0.59\text{Ra}_L^{1/4} \quad \ldots\ldots (2)$$



From energy balance, we obtain

$$\alpha_s \dot{q}_{\text{solar}} = h[T_s - T_\infty] + \varepsilon\sigma[T_s^4 - T_{\text{surr}}^4] \quad \ldots\ldots (3)$$

Equations (1), (2), and (3) can be solved simultaneously to get the surface temperature. Copy the following lines and paste on a blank EES screen to solve the above equation:

```
g=9.81
k=0.02588
L=0.2
Pr=0.7282
q_incindent=200
T_inf=25+273
T_surr=25+273
alpha=0.6
beta=1/(273+30)
epsilon=0.7
nu=1.608e-5
sigma=5.670e-8
Ra_L=g*beta*(T_s-T_inf)*L^3/nu^2*Pr
(h*L/(0.59*k))^4=Ra_L
alpha*q_incindent=h*(T_s-T_inf)+epsilon*sigma*(T_s^4-T_surr^4)
```

Solving by EES software, we get

$$\text{Ra}_L = 9.961\times10^6, \qquad h = 4.289\text{ W/m}^2\cdot\text{K}, \qquad \text{and} \qquad T_s = 311.7\text{ K}$$

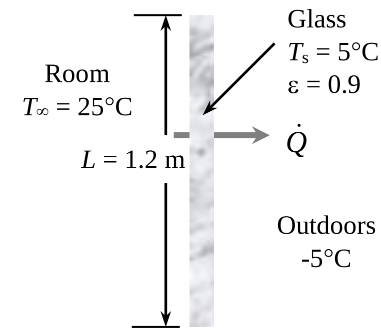
Therefore, the surface temperature of the street sign is

$$T_s = \mathbf{38.7°C}$$

Discussion The assumption that the Rayleigh number is within $10^4 < \text{Ra}_L < 10^9$ turned out to be appropriate. Note that absolute temperatures must be used in calculations involving the radiation heat transfer equation.

9-31 A glass window is considered. The convection heat transfer coefficient on the inner side of the window, the rate of total heat transfer through the window, and the combined natural convection and radiation heat transfer coefficient on the outer surface of the window are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (5+25)/2 = 15°C$ are (Table A-15)

$$k = 0.02476 \text{ W/m.°C}$$
$$\nu = 1.470\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7323$$
$$\beta = \frac{1}{T_f} = \frac{1}{(15+273)\text{K}} = 0.003472 \text{ K}^{-1}$$

Analysis (*a*) The characteristic length in this case is the height of the window, $L_c = L = 1.2$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003472 \text{ K}^{-1})(25-5 \text{ K})(1.2 \text{ m})^3}{(1.470\times10^{-5} \text{ m}^2/\text{s})^2}(0.7323) = 3.989\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(3.989\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7323}\right)^{9/16}\right]^{8/27}}\right\}^2 = 189.7$$

$$h = \frac{k}{L}Nu = \frac{0.02476 \text{ W/m.°C}}{1.2 \text{ m}}(189.7) = \mathbf{3.915\ W/m^2.°C}$$
$$A_s = (1.2 \text{ m})(2 \text{ m}) = 2.4 \text{ m}^2$$

(*b*) The sum of the natural convection and radiation heat transfer from the room to the window is

$$\dot{Q}_{\text{convection}} = hA_s(T_\infty - T_s) = (3.915 \text{ W/m}^2.°\text{C})(2.4 \text{ m}^2)(25-5)°\text{C} = 187.9 \text{ W}$$
$$\dot{Q}_{\text{radiation}} = \varepsilon A_s\sigma(T_{surr}{}^4 - T_s{}^4)$$
$$= (0.9)(2.4 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(25+273 \text{ K})^4 - (5+273 \text{ K})^4] = 234.3 \text{ W}$$
$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{convection}} + \dot{Q}_{\text{radiation}} = 187.9 + 234.3 = \mathbf{422.2\ W}$$

(*c*) The outer surface temperature of the window can be determined from

$$\dot{Q}_{\text{total}} = \frac{kA_s}{t}(T_{s,i} - T_{s,o}) \longrightarrow T_{s,o} = T_{s,i} - \frac{\dot{Q}_{\text{total}}t}{kA_s} = 5°\text{C} - \frac{(422.2 \text{ W})(0.006 \text{ m})}{(0.78 \text{ W/m.°C})(2.4 \text{ m}^2)} = 3.65°\text{C}$$

Then the combined natural convection and radiation heat transfer coefficient on the outer window surface becomes

$$\dot{Q}_{\text{total}} = h_{\text{combined}}A_s(T_{s,o} - T_{\infty,o})$$

or
$$h_{\text{combined}} = \frac{\dot{Q}_{\text{total}}}{A_s(T_{s,o} - T_{\infty,o})} = \frac{422.2 \text{ W}}{(2.4 \text{ m}^2)[3.65-(-5)]°\text{C}} = \mathbf{20.35\ W/m^2.°C}$$

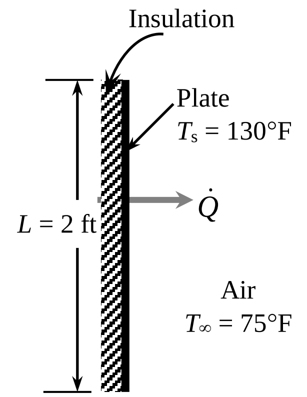
Note that $\Delta T = \dot{Q}R$ and thus the thermal resistance R of a layer is proportional to the temperature drop across that layer. Therefore, the fraction of thermal resistance of the glass is equal to the ratio of the temperature drop across the glass to the overall temperature difference,

$$\frac{R_{\text{glass}}}{R_{\text{total}}} = \frac{\Delta T_{\text{glass}}}{\Delta T_{\text{total}}} = \frac{5-3.65}{25-(-5)} = 0.045 \quad (\text{or } 4.5\%)$$

which is low. Thus it is reasonable to neglect the thermal resistance of the glass.

9-32E A hot plate with an insulated back is considered. The rate of heat loss by natural convection is to be determined for different orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (130+75)/2 = 102.5°F are (Table A-15)

$$k = 0.01535 \text{ Btu/h.ft.°F}$$

$$\nu = 0.1823\times10^{-3} \text{ ft}^2/\text{s}$$

$$\text{Pr} = 0.7256$$

$$\beta = \frac{1}{T_f} = \frac{1}{(102.5+460)\text{R}} = 0.001778 \text{ R}^{-1}$$

Analysis (*a*) When the plate is vertical, the characteristic length is the height of the plate. $L_c = L = 2$ ft. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001778 \text{ R}^{-1})(130-75 \text{ R})(2 \text{ ft})^3}{(0.1823\times10^{-3} \text{ ft}^2/\text{s})^2}(0.7256) = 5.503\times10^8$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(5.503\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7256}\right)^{9/16}\right]^{8/27}}\right\}^2 = 102.6$$

$$h = \frac{k}{L}Nu = \frac{0.01535 \text{ Btu/h.ft.°F}}{2 \text{ ft}}(102.6) = 0.7869 \text{ Btu/h.ft}^2\text{.°F}$$

$$A_s = L^2 = (2 \text{ ft})^2 = 4 \text{ ft}^2$$

and $\dot{Q} = hA_s(T_s - T_\infty) = (0.7869 \text{ Btu/h.ft}^2\text{.°F})(4 \text{ ft}^2)(130-75)\text{°C} = \mathbf{173.1 \text{ Btu/h}}$

(*b*) When the plate is horizontal with hot surface facing up, the characteristic length is determined from

$$L_s = \frac{A_s}{P} = \frac{L^2}{4L} = \frac{L}{4} = \frac{2 \text{ ft}}{4} = 0.5 \text{ ft}.$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001778 \text{ R}^{-1})(130-75 \text{ R})(0.5 \text{ ft})^3}{(0.1823\times10^{-3} \text{ ft}^2/\text{s})^2}(0.7256) = 8.598\times10^6$$

$$Nu = 0.54Ra^{1/4} = 0.54(8.598\times10^6)^{1/4} = 29.24$$

$$h = \frac{k}{L_c}Nu = \frac{0.01535 \text{ Btu/h.ft.°F}}{0.5 \text{ ft}}(29.24) = 0.8975 \text{ Btu/h.ft}^2\text{.°F}$$

and $\dot{Q} = hA_s(T_s - T_\infty) = (0.8975 \text{ Btu/h.ft}^2\text{.°F})(4 \text{ ft}^2)(130-75)\text{°C} = \mathbf{197.4 \text{ Btu/h}}$

(*c*) When the plate is horizontal with hot surface facing down, the characteristic length is again L_c = 0.5 ft and the Rayleigh number is $Ra = 8.598\times10^6$. Then,

$$Nu = 0.27Ra^{1/4} = 0.27(8.598\times10^6)^{1/4} = 14.62$$

$$h = \frac{k}{L_c}Nu = \frac{0.01535 \text{ Btu/h.ft.°F}}{0.5 \text{ ft}}(14.62) = 0.4487 \text{ Btu/h.ft}^2\text{.°F}$$

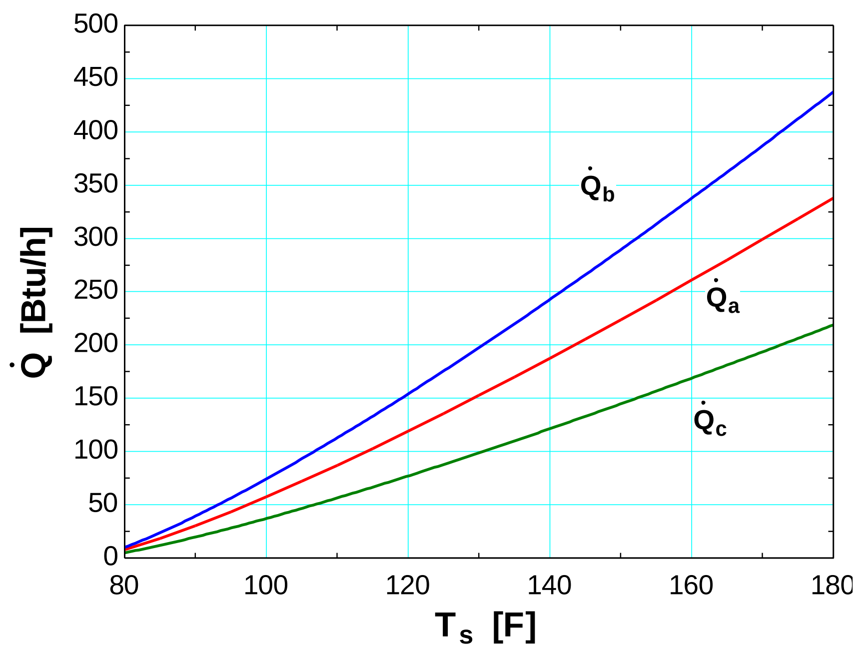
and $\dot{Q} = hA_s(T_s - T_\infty) = (0.4487 \text{ Btu/h.ft}^2\text{.°F})(4 \text{ ft}^2)(130-75)\text{°C} = \mathbf{98.7 \text{ Btu/h}}$

9-33E Prob. 9-32E is reconsidered. The rate of natural convection heat transfer for different orientations of the plate as a function of the plate temperature is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=2 [ft]
T_infinity=75 [F]
T_s=130 [F]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=14.7)
mu=Viscosity(Fluid$, T=T_film)*Convert(lbm/ft-h, lbm/ft-s)
nu=mu/rho
beta=1/(T_film+460)
T_film=1/2*(T_s+T_infinity)
g=32.2 [ft/s^2]

"ANALYSIS"
"(a), plate is vertical"
delta_a=L
Ra_a=(g*beta*(T_s-T_infinity)*delta_a^3)/nu^2*Pr
Nusselt_a=0.59*Ra_a^0.25
h_a=k/delta_a*Nusselt_a
A=L^2
Q_dot_a=h_a*A*(T_s-T_infinity)
"(b), plate is horizontal with hot surface facing up"
delta_b=A/p
p=4*L
Ra_b=(g*beta*(T_s-T_infinity)*delta_b^3)/nu^2*Pr
Nusselt_b=0.54*Ra_b^0.25
h_b=k/delta_b*Nusselt_b
Q_dot_b=h_b*A*(T_s-T_infinity)
"(c), plate is horizontal with hot surface facing down"
delta_c=delta_b
Ra_c=Ra_b
Nusselt_c=0.27*Ra_c^0.25
h_c=k/delta_c*Nusselt_c
Q_dot_c=h_c*A*(T_s-T_infinity)
```

T_s [F]	$\dot{Q}_a$ [Btu/h]	$\dot{Q}_b$ [Btu/h]	$\dot{Q}_c$ [Btu/h]
80	7.714	9.985	4.993
85	18.32	23.72	11.86
90	30.38	39.32	19.66
95	43.47	56.26	28.13
100	57.37	74.26	37.13
105	71.97	93.15	46.58
110	87.15	112.8	56.4
115	102.8	133.1	66.56
120	119	154	77.02
125	135.6	175.5	87.75
130	152.5	197.4	98.72
135	169.9	219.9	109.9
140	187.5	242.7	121.3
145	205.4	265.9	132.9
150	223.7	289.5	144.7
155	242.1	313.4	156.7
160	260.9	337.7	168.8
165	279.9	362.2	181.1
170	299.1	387.1	193.5
175	318.5	412.2	206.1
180	338.1	437.6	218.8

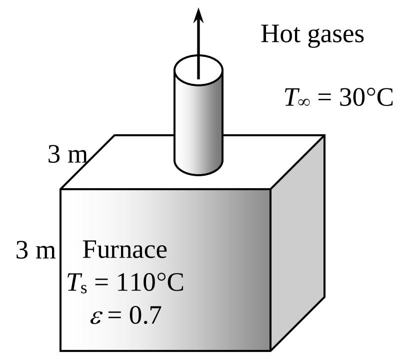


9-34 It is proposed that the side surfaces of a cubic industrial furnace be insulated for $550 in order to reduce the heat loss by 90 percent. The thickness of the insulation and the payback period of the insulation to pay for itself from the energy it saves are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (110+30)/2 = 70°C$ are (Table A-15)

$$k = 0.02881 \text{ W/m.°C}$$
$$\nu = 1.995\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7177$$
$$\beta = \frac{1}{T_f} = \frac{1}{(70+273)\text{K}} = 0.002915 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the height of the furnace, $L_c = L = 3$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.002915 \text{ K}^{-1})(110-30 \text{ K})(3 \text{ m})^3}{(1.995\times10^{-5} \text{ m}^2\text{/s})^2}(0.7177) = 1.114\times10^{11}$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.114\times10^{11})^{1/6}}{\left[1+\left(\frac{0.492}{0.7177}\right)^{9/16}\right]^{8/27}}\right\}^2 = 545.1$$

$$h = \frac{k}{L_c}Nu = \frac{0.02881 \text{ W/m.°C}}{3 \text{ m}}(545.1) = 5.235 \text{ W/m}^2\text{.°C}$$

$$A_s = 4(3 \text{ m})^2 = 36 \text{ m}^2$$

Then the heat loss by combined natural convection and radiation becomes

$$\begin{aligned}\dot{Q} &= hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^4 - T_{surr}^4) \\ &= (5.235 \text{ W/m}^2\text{.°C})(36 \text{ m}^2)(110-30)\text{°C} \\ &\quad + (0.7)(36 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(110+273 \text{ K})^4 - (30+273 \text{ K})^4] \\ &= 33{,}779 \text{ W}\end{aligned}$$

Noting that insulation will reduce the heat losses by 90%, the rate of heat loss after insulation will be

$$\dot{Q}_{\text{saved}} = 0.9\dot{Q}_{\text{no insulation}} = 0.9\times33{,}779 \text{ W} = 30{,}401 \text{ W}$$
$$\dot{Q}_{\text{loss}} = (1-0.9)\dot{Q}_{\text{no insulation}} = 0.1\times33{,}779 \text{ W} = 3378 \text{ W}$$

The furnace operates continuously and thus 8760 h. Then the amount of energy and money the insulation will save becomes

$$\text{Energy saved} = \dot{Q}_{saved}\Delta t = \frac{30.401 \text{ kJ/s}}{0.78}\left(\frac{1 \text{ therm}}{105{,}500 \text{ kJ}}\right)(8760\times3600 \text{ s/yr}) = 11{,}651 \text{ therms/yr}$$

$$\text{Money saved} = \text{(Energy saved)(Unit cost of energy)} = (11{,}651 \text{ therms})(\$1.10/\text{therm}) = \$12{,}816$$

Therefore, the money saved by insulation will pay for the cost of $550 in

550/($12,816/yr)=0.04292 yr = **16 days**.

Insulation will lower the outer surface temperature, the Rayleigh and Nusselt numbers, and thus the convection heat transfer coefficient. For the evaluation of the heat transfer coefficient, we assume the surface temperature in this case to be 50°C. The properties of air at the film temperature of $(T_s+T_\infty)/2 = (50+30)/2 = 40°C$ are (Table A-15)

$k = 0.02662$ W/m.°C

$\nu = 1.702 \times 10^{-5}$ m^2/s

Pr = 0.7255

$$\beta = \frac{1}{T_f} = \frac{1}{(40+273)\text{K}} = 0.003195\ \text{K}^{-1}$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81\ \text{m/s}^2)(0.003195\ \text{K}^{-1})(50-30\ \text{K})(3\ \text{m})^3}{(1.702\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7255) = 4.239\times10^{10}$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(4.239\times10^{10})^{1/6}}{\left[1+\left(\frac{0.492}{0.7255}\right)^{9/16}\right]^{8/27}}\right\}^2 = 400.5$$

$$h = \frac{k}{L}Nu = \frac{0.02662\ \text{W/m.°C}}{3\ \text{m}}(400.5) = 3.554\ \text{W/m}^2.\text{°C}$$

$$A_s = 4\times(3\ \text{m})(3+2t_{insul})\ \text{m}$$

The total rate of heat loss from the outer surface of the insulated furnace by convection and radiation becomes

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$

$$3378\ \text{W} = (3.554\ \text{W/m}^2.\text{°C})A(T_s - 30)\text{°C}$$
$$+ (0.7)A(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)[(T_s + 273\ \text{K})^4 - (30+273\ \text{K})^4]$$

In steady operation, the heat lost by the side surfaces of the pipe must be equal to the heat lost from the exposed surface of the insulation by convection and radiation, which must be equal to the heat conducted through the insulation. Therefore,

$$\dot{Q} = \dot{Q}_{insulation} = kA_s\frac{(T_{\text{furnace}} - T_s)}{t_{ins}} \rightarrow 3378\ \text{W} = (0.038\ \text{W/m.°C})A_s\frac{(110 - T_s)\text{°C}}{t_{insul}}$$

Solving the two equations above by trial-and error (or better yet, an equation solver) gives

T_s = 41.2°C and t_{insul} = 0.0284 m = **2.84 cm**

9-35 A vertical plate with length L is placed in a quiescent air, and the expressions, having the form $\mathrm{Nu} = CRa_L^n$, for the average heat transfer coefficient are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties.

Properties The properties of air at T_f = 20°C are k = 0.02514 W/m·K, $\nu = 1.516 \times 10^{-5}$ m²/s, Pr = 0.7309 (from Table A-15). Also, $\beta = 1/T_f = 0.003413$ K^{-1}.

Analysis The Rayleigh number ($L_c = L$) is

$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr} = \frac{(9.81\ \mathrm{m/s^2})(0.003413\ \mathrm{K^{-1}})\Delta TL^3}{(1.516\times10^{-5}\ \mathrm{m^2/s})^2}(0.7309) = 1.065\times10^8\,\Delta TL^3$$

For $10^4 < \mathrm{Ra}_L < 10^9$, we have

$$\mathrm{Nu} = \frac{hL}{k} = 0.59\mathrm{Ra}_L^{1/4} \quad \rightarrow \quad h = 0.59\frac{k}{L}\mathrm{Ra}_L^{1/4}$$

Substituting the Ra_L yields

$$h = 0.59\left(\frac{0.02514}{L}\right)(1.065\times10^8\,\Delta TL^3)^{1/4} = 1.51(\Delta T/L)^{1/4} \qquad 10^4 < \mathrm{Ra}_L < 10^9$$

For $10^{10} < \mathrm{Ra}_L < 10^{13}$, we have

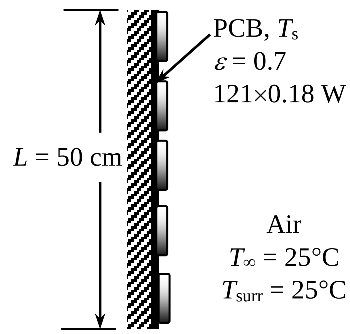
$$\mathrm{Nu} = \frac{hL}{k} = 0.1\mathrm{Ra}_L^{1/3} \quad \rightarrow \quad h = 0.1\frac{k}{L}\mathrm{Ra}_L^{1/3}$$

Substituting the Ra_L yields

$$h = 0.1\left(\frac{0.02514}{L}\right)(1.065\times10^8\,\Delta TL^3)^{1/3} = 1.19\Delta T^{1/3} \qquad 10^{10} < \mathrm{Ra}_L < 10^{13}$$

Discussion The average heat transfer coefficient for laminar conditions ($10^4 < \mathrm{Ra}_L < 10^9$) is dependent on ΔT and L. In turbulent conditions ($10^{10} < \mathrm{Ra}_L < 10^{13}$), the average heat transfer coefficient is not influenced by L.

9-36 A circuit board containing square chips is mounted on a vertical wall in a room. The surface temperature of the chips is to be determined.



Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The heat transfer from the back side of the circuit board is negligible.

Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (35+25)/2 = 30$°C are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$

$$\nu = 1.608\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7282$$

$$\beta = \frac{1}{T_f} = \frac{1}{(30+273)\text{K}} = 0.0033\,\text{K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 35°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the height of the board, $L_c = L = 0.5\,\text{m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.0033\,\text{K}^{-1})(35-25\,\text{K})(0.5\,\text{m})^3}{(1.608\times10^{-5}\text{ m}^2/\text{s})^2}(0.7282) = 1.140\times10^8$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.140\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7282}\right)^{9/16}\right]^{8/27}}\right\}^2 = 63.72$$

$$h = \frac{k}{L}Nu = \frac{0.02588\text{ W/m.°C}}{0.5\,\text{m}}(63.72) = 3.30\text{ W/m}^2.\text{°C}$$

$$A_s = (0.5\,\text{m})^2 = 0.25\,\text{m}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\ 4} - T_{surr}^{\ 4})$$

$$(121\times0.18)\text{ W} = (3.30\text{ W/m}^2.\text{°C})(0.25\,\text{m}^2)(T_s - 25)\text{°C}$$
$$+ (0.7)(0.25\,\text{m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(T_s + 273\,\text{K})^4 - (25+273\,\text{K})^4]$$

Its solution is

T_s = **36.2°C**

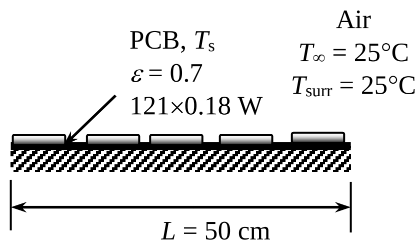
which is sufficiently close to the assumed value in the evaluation of properties and h. Therefore, there is no need to repeat calculations by reevaluating the properties and h at the new film temperature.

Discussion The assumed film temperature of T_f = 30°C is an appropriate assumption, since the determined T_s = 36.2°C would give a film temperature of T_f = 30.6°C. Otherwise, T_s would have to be solved iteratively.

9-37 A circuit board containing square chips is positioned horizontally in a room. The surface temperature of the chips is to be determined for two orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The heat transfer from the back side of the circuit board is negligible.

Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (35+25)/2 = 30$°C are (Table A-15)



$$k = 0.02588 \text{ W/m.°C}$$
$$\nu = 1.608\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7282$$
$$\beta = \frac{1}{T_f} = \frac{1}{(30+273)\text{K}} = 0.0033 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 35°C for the evaluation of the properties and h. The characteristic length for both cases is determined from

$$L_c = \frac{A_s}{p} = \frac{(0.5\text{ m})^2}{2[(0.5\text{ m})+(0.5\text{ m})]} = 0.125\text{ m}.$$

Then, $$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.0033\text{ K}^{-1})(35-25\text{ K})(0.125\text{ m})^3}{(1.608\times10^{-5}\text{ m}^2\text{/s})^2}(0.7282) = 1.781\times10^6$$

(*a*) Chips (hot surface) facing up:

$$Nu = 0.54Ra^{1/4} = 0.54(1.781\times10^6)^{1/4} = 19.73$$
$$h = \frac{k}{L_c}Nu = \frac{0.02588\text{ W/m.°C}}{0.125\text{ m}}(19.73) = 4.08\text{ W/m}^2\text{.°C}$$
$$A_s = (0.5\text{ m})^2 = 0.25\text{ m}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$(121\times0.18)\text{ W} = (4.08\text{ W/m}^2\text{.°C})(0.25\text{ m}^2)(T_s - 25)\text{°C}$$
$$+(0.7)(0.25\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(T_s+273\text{ K})^4 - (25+273\text{ K})^4]$$

Its solution is T_s = **35.2°C**

which is sufficiently close to the assumed value. Therefore, there is no need to repeat calculations.

(*b*) Chips (hot surface) facing down:

$$Nu = 0.27Ra^{1/4} = 0.27(1.781\times10^6)^{1/4} = 9.863$$
$$h = \frac{k}{L_c}Nu = \frac{0.02588\text{ W/m.°C}}{0.125\text{ m}}(9.863) = 2.04\text{ W/m}^2\text{.°C}$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$(121\times0.18)\text{ W} = (2.04\text{ W/m}^2\text{.°C})(0.25\text{ m}^2)(T_s - 25)\text{°C}$$
$$+(0.7)(0.25\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(T_s+273\text{ K})^4 - (25+273\text{ K})^4]$$

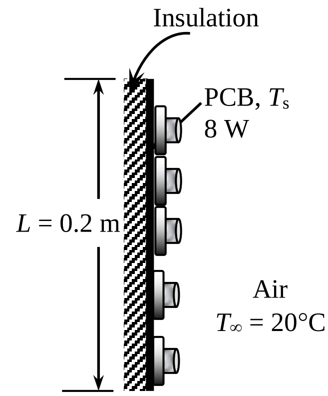
Its solution is T_s = **38.3°C**

which is sufficiently close to the assumed value of 35°C in the evaluation of properties and h. Therefore, there is no need to repeat calculations.

Discussion The assumed film temperature of $T_f = 30$°C is an appropriate assumption, since the determined $T_s = 38.3$°C would give a film temperature of $T_f = 31.7$°C. Otherwise, T_s would have to be solved iteratively.

9-38 A printed circuit board (PCB) is placed in a room. The average temperature of the hot surface of the board is to be determined for different orientations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **3** The heat loss from the back surface of the board is negligible.



Properties We evaluate air properties at a film temperature of $(T_s+T_\infty)/2$ = 32.5°C and 1 atm based on the problem statement. Then, for an air temperature of T_∞ = 20°C, the corresponding surface temperature is T_s = 45°C. The properties of air at 1 atm and 32.5°C are (Table A-15)

$$k = 0.02607 \text{ W/m.°C}$$
$$\nu = 1.631\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7275$$
$$\beta = \frac{1}{T_f} = \frac{1}{(32.5+273)\text{K}} = 0.003273 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown.

(*a*) **Vertical PCB** . We start the solution process by "guessing" the surface temperature to be 45°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the height of the PCB, $L_c = L = 0.2$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003273 \text{ K}^{-1})(45-20 \text{ K})(0.2 \text{ m})^3}{(1.631\times10^{-5} \text{ m}^2\text{/s})^2}(0.7275) = 1.756\times10^7$$

$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.756\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7275}\right)^{9/16}\right]^{8/27}}\right\}^2 = 36.78$$

$$h = \frac{k}{L}Nu = \frac{0.02607 \text{ W/m.°C}}{0.2 \text{ m}}(36.78) = 4.794 \text{ W/m}^2\text{.°C}$$
$$A_s = (0.15 \text{ m})(0.2 \text{ m}) = 0.03 \text{ m}^2$$

Heat loss by both natural convection and radiation heat can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$8 \text{ W} = (4.794 \text{ W/m}^2\text{.°C})(0.03 \text{ m}^2)(T_s - 20)\text{°C} + (0.8)(0.03 \text{ m}^2)(5.67\times10^{-8})\left[(T_s+273)^4 - (20+273 \text{ K})^4\right]$$

Its solution is

$$T_s = \mathbf{46.6°C}$$

which is sufficiently close to the assumed value of 45°C for the evaluation of the properties and h.

(*b*) **Horizontal, hot surface facing up** Again we assume the surface temperature to be 45 °C and use the properties evaluated above. The characteristic length in this case is

$$L_c = \frac{A_s}{p} = \frac{(0.20 \text{ m})(0.15 \text{ m})}{2(0.2 \text{ m}+0.15 \text{ m})} = 0.0429 \text{ m}.$$

Then

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003273 \text{ K}^{-1})(45-20 \text{ K})(0.0429 \text{ m})^3}{(1.631\times10^{-5} \text{ m}^2\text{/s})^2}(0.7275) = 1.728\times10^5$$

$$Nu = 0.54Ra^{1/4} = 0.54(1.728\times10^5)^{1/4} = 11.01$$

$$h = \frac{k}{L_c}Nu = \frac{0.02607 \text{ W/m.°C}}{0.0429 \text{ m}}(11.01) = 6.696 \text{ W/m}^2\text{.°C}$$

Heat loss by both natural convection and radiation heat can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma(T_s^{\ 4} - T_{surr}^{\ 4})$$

$$8\ \text{W} = (6.696\ \text{W/m}^2.°\text{C})(0.03\,\text{m}^2)(T_s - 20)°\text{C} + (0.8)(0.03\,\text{m}^2)(5.67\times10^{-8})[(T_s + 273)^4 - (20 + 273\,\text{K})^4]$$

Its solution is

$$T_s = \mathbf{42.6°C}$$

which is sufficiently close to the assumed value of 45°C in the evaluation of the properties and h.

(c) **Horizontal, hot surface facing down** Again we assume the surface temperature to be 45°C and use the properties evaluated above. The characteristic length in this case is, from part (b), L_c = 0.0429 m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\Pr = \frac{(9.81\,\text{m/s}^2)(0.003273\,\text{K}^{-1})(45 - 20\,\text{K})(0.0429\,\text{m})^3}{(1.631\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7275) = 1.728\times10^5$$

$$Nu = 0.27Ra^{1/4} = 0.27(1.728\times10^5)^{1/4} = 5.505$$

$$h = \frac{k}{L_c}Nu = \frac{0.02607\ \text{W/m.°C}}{0.0429\,\text{m}}(5.505) = 3.3.345\ \text{W/m}^2.°\text{C}$$

Considering both natural convection and radiation heat loses

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma(T_s^{\ 4} - T_{surr}^{\ 4})$$

$$8\ \ \text{W} = (3.345\ \text{W/m}^2.°\text{C})(0.03\,\text{m}^2)(T_s - 20)°\text{C} + (0.8)(0.03\,\text{m}^2)(5.67\times10^{-8})[(T_s + 273)^4 - (20 + 273\,\text{K})^4]$$

Its solution is

$$T_s = \mathbf{50.7°C}$$

which is sufficiently close to the assumed value. Therefore, there is no need to repeat the calculations.

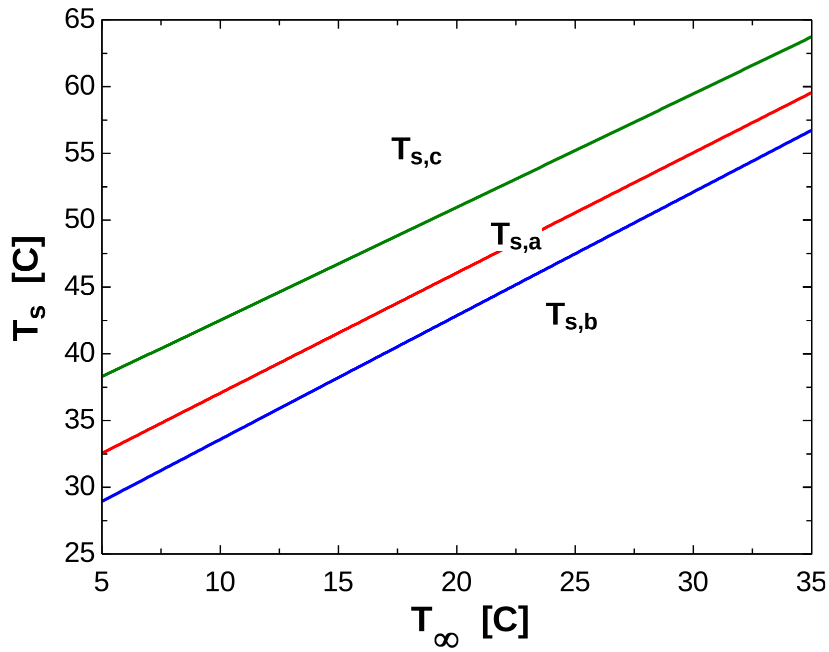
Discussion The assumed film temperature of T_f = 32.5°C is an appropriate assumption, since the determined T_s = 46.6°C, T_s = 42.6°C, and T_s = 50.7°C in parts *a*, *b*, and *c*, respectively would give film temperatures of T_f = 33.3°C, T_f = 31.3°C, and T_f = 35.4°C, respectively. Otherwise, T_s would have to be solved iteratively.

9-39 Prob. 9-38 is reconsidered. The effects of the room temperature and the emissivity of the board on the temperature of the hot surface of the board for different orientations of the board are to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.2 [m]
w=0.15 [m]
T_infinity=20 [C]
Q_dot=8 [W]
epsilon=0.8
T_surr=T_infinity

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
beta=1/(T_film+273)
T_film=1/2*(T_s_a+T_infinity)
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"
g=9.807 [m/s^2] "gravitational acceleration"

"ANALYSIS"
"(a), plate is vertical"
delta_a=L
Ra_a=(g*beta*(T_s_a-T_infinity)*delta_a^3)/nu^2*Pr
Nusselt_a=0.59*Ra_a^0.25
h_a=k/delta_a*Nusselt_a
A=w*L
Q_dot=h_a*A*(T_s_a-T_infinity)+epsilon*A*sigma*((T_s_a+273)^4-(T_surr+273)^4)
"(b), plate is horizontal with hot surface facing up"
delta_b=A/p
p=2*(w+L)
Ra_b=(g*beta*(T_s_b-T_infinity)*delta_b^3)/nu^2*Pr
Nusselt_b=0.54*Ra_b^0.25
h_b=k/delta_b*Nusselt_b
Q_dot=h_b*A*(T_s_b-T_infinity)+epsilon*A*sigma*((T_s_b+273)^4-(T_surr+273)^4)
"(c), plate is horizontal with hot surface facing down"
delta_c=delta_b
Ra_c=Ra_b
Nusselt_c=0.27*Ra_c^0.25
h_c=k/delta_c*Nusselt_c
Q_dot=h_c*A*(T_s_c-T_infinity)+epsilon*A*sigma*((T_s_c+273)^4-(T_surr+273)^4)
```

T_{∞} [F]	$T_{s,a}$ [C]	$T_{s,b}$ [C]	$T_{s,c}$ [C]
5	32.54	28.93	38.29
7	34.34	30.79	39.97
9	36.14	32.65	41.66
11	37.95	34.51	43.35
13	39.75	36.36	45.04
15	41.55	38.22	46.73
17	43.35	40.07	48.42
19	45.15	41.92	50.12
21	46.95	43.78	51.81
23	48.75	45.63	53.51
25	50.55	47.48	55.21
27	52.35	49.33	56.91
29	54.16	51.19	58.62
31	55.96	53.04	60.32
33	57.76	54.89	62.03
35	59.56	56.74	63.74



9-40 A horizontal ASTM A240 410S stainless steel plate has its upper surface subjected to natural convection with cold air. Heat is added to the plate at 70 W, and the lowest temperature that the cold air can reach without causing the plate surface temperature to cool below −30°C is to be determined.

Assumptions **1** Steady state conditions. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm. **4** Thermal radiation is negligible.

Properties The properties of air at the film temperature of $T_f = -50$°C are (Table A-15) Pr = 0.7440, k = 0.01979 W/m·K, ν = 9.319×10^{-6} m²/s, $\beta = 1/T_f = 1/(-50 + 273\text{ K}) = 0.004484\text{ K}^{-1}$

Analysis The characteristic length of the plate is

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6\text{ m}}{4} = 0.15\text{ m}$$

The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr}$$

The Nusselt number for natural convection with horizontal hot surface facing up is determined from

$$\text{Nu} = \frac{hL_c}{k} = 0.15\text{ Ra}_L^{1/3}$$

(Note: The Rayleigh number is assumed to be $10^7 < \text{Ra}_L < 10^{11}$. We will verify the Rayleigh number after finding the T_∞. If $10^4 < \text{Ra}_L < 10^7$, then $\text{Nu} = 0.54\text{ Ra}_L^{1/4}$ would be used.)

So, the natural convection heat transfer coefficient is

$$h = 0.15\frac{k}{L_c}\left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr}\right]^{1/3}$$

From energy balance, the heat rate added to the plate is equal to the heat transfer rate from the plate by natural convection to the cold air:

$$\dot{Q} = hA_s(T_s - T_\infty) = hL^2(T_s - T_\infty) = 70\text{ W}$$

By substituting the equation for Ra_L into the equation for h and then substituting the resulting equation in the above Newton's law of cooling equation yields

$$0.15\frac{k}{L_c}\left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr}\right]^{1/3} L^2(T_s - T_\infty) = 70\text{ W}$$

or

$$0.15\frac{(0.01979\text{ W/m}\cdot\text{K})}{0.15\text{ m}}\left[\frac{(9.81\text{ m/s}^2)(0.004484\text{ K}^{-1})(T_s - T_\infty)(0.15\text{ m})^3}{(9.319\times10^{-6}\text{m}^2/\text{s})^2}(0.7440)\right]^{1/3}(0.6\text{ m})^2(T_s - T_\infty) = 70\text{ W}$$

The temperature of the cold air can be solved implicitly by trail-and-error with T_s = −30°C, which yields

$$T_\infty = \mathbf{-59.4}\text{°C}$$

The Rayleigh number with T_∞ = −59.4°C is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.004484\text{ K}^{-1})(-30 + 59.4\text{ K})(0.15\text{ m})^3}{(9.319\times10^{-6}\text{m}^2/\text{s})^2}(0.7440) = 3.74\times10^7$$

So, the correlation for the Nusselt number is appropriate. (If $\text{Ra}_L < 10^7$, then the calculation for T_∞ would need to repeated with a suitable correlation.)

Discussion With heat being added to the stainless steel plate at 70 W, the plate surface temperature would reach −30°C by natural convection with the cold air at −59.4°C. Thus, the lowest temperature that the air can reach without causing the plate surface temperature to cool below its minimum suitable temperature is −59.4°C.

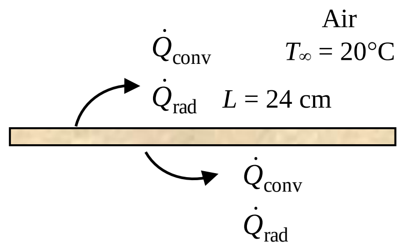
With T_∞ = −59.4°C and T_s = −30°C, the film temperature becomes −44.7°C. Thus, −50°C is a reasonable temperature to evaluate the properties of air and there is no need to repeat the calculations.

9-41 Heat is generated in a horizontal plate while heat is lost from it by convection and radiation. The temperature of the plate when steady operating conditions are reached is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties We assume the surface temperature to be 50°C. Then the properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (50+20)/2 = 35$°C are (Table A-15)

$$k = 0.02625 \text{ W/m.°C}$$
$$\nu = 1.655\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7268$$
$$\beta = \frac{1}{T_f} = \frac{1}{(35+273)\text{K}} = 0.003247 \text{ K}^{-1}$$



Analysis The characteristic length in this case is

$$L_c = \frac{A_s}{p} = \frac{(0.24 \text{ m})(0.20 \text{ m})}{2[(0.24 \text{ m})+(0.20 \text{ m})]} = 0.05455 \text{ m}$$

The Rayleigh number is

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^{\,3}}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003247 \text{ K}^{-1})(50-20 \text{ K})(0.05455 \text{ m})^3}{(1.655\times10^{-5} \text{ m}^2\text{/s})^2}(0.7268) = 411{,}500$$

The Nusselt number relation for the top surface of the plate (horizontal hot surface, facing up) is

$$Nu = 0.54Ra^{0.25} = 0.54(411{,}500)^{0.25} = 13.68$$

Then

$$h = \frac{k}{L_c}Nu = \frac{0.02625 \text{ W/m.°C}}{0.05455 \text{ m}}(13.68) = 6.581 \text{ W/m}^2\text{.°C}$$

and

$$\dot{Q}_{\text{top}} = hA(T_s - T_\infty) = (6.581 \text{ W/m}^2\text{.°C})(0.24\times0.20 \text{ m}^2)(T_s - 20)\text{°C} = 0.3159(T_s - 20)$$

The Nusselt number relation for the bottom surface of the plate (horizontal hot surface, facing down) is

$$Nu = 0.27Ra^{0.25} = 0.27(411{,}500)^{0.25} = 6.838$$

Then

$$h = \frac{k}{L_c}Nu = \frac{0.02625 \text{ W/m.°C}}{0.05455 \text{ m}}(6.838) = 3.291 \text{ W/m}^2\text{.°C}$$

$$\dot{Q}_{\text{bottom}} = hA(T_s - T_\infty) = (3.291 \text{ W/m}^2\text{.°C})(0.24\times0.20 \text{ m}^2)(T_s - 20)\text{°C} = 0.1580(T_s - 20)$$

Considering that radiation heat loss to surroundings occur both from top and bottom surfaces, it may be expressed as

$$\begin{aligned}\dot{Q}_{\text{rad}} &= 2\varepsilon A\sigma(T_s^{\,4} - T_{\text{surr}}^{\,4}) \\ &= (0.9)(2)(0.24\times0.20 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(T_s + 273 \text{ K})^4 - (17+273 \text{ K})^4\right] \\ &= 4.899\times10^{-9}\left[(T_s + 273 \text{ K})^4 - (17+273 \text{ K})^4\right]\end{aligned}$$

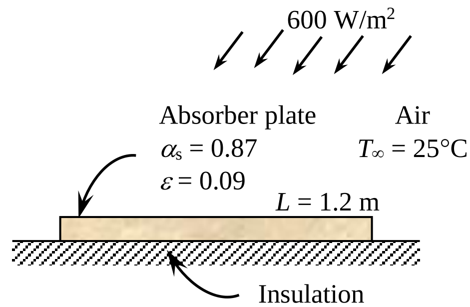
When the heat lost from the plate equals to the heat generated, the steady operating conditions are reached. The surface temperature in this case can be determined by trial-error or using EES to be

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{top}} + \dot{Q}_{\text{bottom}} + \dot{Q}_{\text{rad}}$$
$$20 \text{ W} = 0.3159(T_s - 20) + 0.1580(T_s - 20) + 4.899\times10^{-9}\left[(T_s + 273 \text{ K})^4 - (17+273 \text{ K})^4\right]$$
$$T_s = \mathbf{38.3°C}$$

The surface temperature is about 12°C below the assumed surface temperature of 50°C. For more accuracy, it is recommended to repeat the caluations with a lower value of surface temperature.

9-42 Absorber plates whose back side is heavily insulated is placed horizontally outdoors. Solar radiation is incident on the plate. The equilibrium temperature of the plate is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (115+25)/2 = 70°C$ based on the problem statement are (Table A-15)

$$k = 0.02881 \text{ W/m.°C}$$
$$\nu = 1.995\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7177$$
$$\beta = \frac{1}{T_f} = \frac{1}{(70+273)\text{K}} = 0.002915 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 115°C for the evaluation of the properties and *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is

$$L_c = \frac{A_s}{p} = \frac{(1.2\text{ m})(0.8\text{ m})}{2(1.2\text{ m}+0.8\text{ m})} = 0.24\text{ m}$$

Then,

$$Ra = \frac{g\beta(T_s-T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.002915\text{ K}^{-1})(115-25\text{ K})(0.24\text{ m})^3}{(1.995\times10^{-5}\text{ m}^2/\text{s})^2}(0.7177) = 6.414\times10^7$$

The Nusselt number relation for the horizontal hot surface, facing up is

$$Nu = 0.15Ra^{1/3} = 0.15(6.414\times10^7)^{1/3} = 60.04$$

$$h = \frac{k}{L_c}Nu = \frac{0.02881\text{ W/m.°C}}{0.24\text{ m}}(60.04) = 7.208\text{ W/m}^2\text{.°C}$$

$$A_s = (0.8\text{ m})(1.2\text{ m}) = 0.96\text{ m}^2$$

In steady operation, the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation. Therefore,

$$\dot{Q} = \alpha\dot{q}A_s = (0.87)(600\text{ W/m}^2)(0.96\text{ m}^2) = 501.1\text{ W}$$

$$\dot{Q} = hA_s(T_s-T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{sky}{}^4)$$

$$501.1\text{ W} = (7.208\text{ W/m}^2\text{.°C})(0.96\text{ m}^2)(T_s-25)\text{°C}$$
$$+(0.09)(0.96\text{ m}^2)(5.67\times10^{-8})[(T_s+273)^4 - (10+273\text{ K})^4]$$

Its solution is $T_s = 89.7°C$

which is not very close to the assumed value of 115°C. We repeat the calculations at a new anticipated surface temperature of 95°C. The properties are to be evaluated at the film temperature of (95+25)/2=60°C.

$$k = 0.02808\text{ W/m.°C}$$
$$\nu = 1.896\times10^{-5}\text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7202$$
$$\beta = \frac{1}{T_f} = \frac{1}{(60+273)\text{K}} = 0.003003\text{ K}^{-1}$$

$$Ra = \frac{g\beta(T_s-T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003003\text{ K}^{-1})(95-25\text{ K})(0.24\text{ m})^3}{(1.896\times10^{-5}\text{ m}^2/\text{s})^2}(0.7202) = 5.711\times10^7$$

$$Nu = 0.15Ra^{1/3} = 0.15(5.711\times10^7)^{1/3} = 57.77$$

$$h = \frac{k}{L_c}Nu = \frac{0.02808\text{ W/m.°C}}{0.24\text{ m}}(57.77) = 6.759\text{ W/m}^2\text{.°C}$$

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma (T_s{}^4 - T_{sky}{}^4)$$

$$501.1\,\text{W} = (6.759\ \text{W/m}^2.^\circ\text{C})(0.96\ \text{m}^2)(T_s - 25)^\circ\text{C}$$
$$+ (0.09)(0.96\ \text{m}^2)(5.67\times10^{-8})[(T_s + 273)^4 - (10 + 273\ \text{K})^4]$$

$$T_s = \mathbf{93.5^\circ C}$$

This is close to the assumed surface temperature of 95°C. Therefore, there is no need to repeat the calculations.

If the absorber plate is made of ordinary aluminum which has a solar absorptivity of 0.28 and an emissivity of 0.07, the rate of solar gain becomes

$$\dot{Q} = \alpha \dot{q} A_s = (0.28)(600\ \text{W/m}^2)(0.96\ \text{m}^2) = 161.3\ \text{W}$$

Again noting that in steady operation the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation, and using the convection coefficient determined above for convenience,

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s \sigma (T_s{}^4 - T_{sky}{}^4)$$

$$161.3\,\text{W} = (6.629\ \text{W/m}^2.^\circ\text{C})(0.96\ \text{m}^2)(T_s - 25)^\circ\text{C} + (0.07)(0.96\ \text{m}^2)(5.67\times10^{-8})[(T_s + 273)^4 - (10 + 273\ \text{K})^4]$$

Its solution is T_s = 47.8°C

Repeating the calculations at the new anticipated surface temperature of 55°C and the film temperature of (55+25)/2=40°C, we obtain

h = 5.312 W/m^2.°C and T_s = **53.0°C**

9-43 An absorber plate whose back side is heavily insulated is placed horizontally outdoors. Solar radiation is incident on the plate. The equilibrium temperature of the plate is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

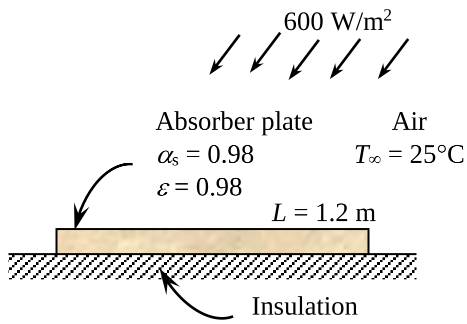
Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (65+25)/2 = 45°C$ based on the problem statement are (Table A-15)

$$k = 0.02699 \text{ W/m.°C}$$

$$\nu = 1.750\times10^{-5} \text{ m}^2/\text{s}$$

$$\Pr = 0.7241$$

$$\beta = \frac{1}{T_f} = \frac{1}{(45+273)\text{K}} = 0.003145 \text{ K}^{-1}$$



Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 65°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is

$$L_c = \frac{A_s}{p} = \frac{(1.2\text{ m})(0.8\text{ m})}{2(1.2\text{ m}+0.8\text{ m})} = 0.24\text{ m}$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.00312\text{ K}^{-1})(65-25\text{ K})(0.24\text{ m})^3}{(1.750\times10^{-5}\text{ m}^2/\text{s})^2}(0.7241) = 4.033\times10^7$$

The Nusselt number relation for the horizontal hot surface, facing up is

$$Nu = 0.15Ra^{1/3} = 0.15(4.033\times10^7)^{1/3} = 51.44$$

$$h = \frac{k}{L_c}Nu = \frac{0.02699\text{ W/m.°C}}{0.24\text{ m}}(51.44) = 5.785\text{ W/m}^2\text{.°C}$$

$$A_s = (0.8\text{ m})(1.2\text{ m}) = 0.96\text{ m}^2$$

In steady operation, the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation. Therefore,

$$\dot{Q} = \alpha\dot{q}A_s = (0.98)(600\text{ W/m}^2)(0.96\text{ m}^2) = 564.5\text{ W}$$

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$

$$564.5\text{ W} = (5.785\text{ W/m}^2\text{.°C})(0.96\text{ m}^2)(T_s - 25)°\text{C}$$
$$+(0.98)(0.96\text{ m}^2)(5.67\times10^{-8})[(T_s+273)^4 - (10+273\text{ K})^4]$$

Its solution is $\quad T_s = \mathbf{64.2°C}$

which is very close to the assumed value. Therefore there is no need to repeat calculations.

For a white painted absorber plate, the solar absorptivity is 0.26 and the emissivity is 0.90. Then the rate of solar gain becomes

$$\dot{Q} = \alpha\dot{q}A_s = (0.26)(600\text{ W/m}^2)(0.96\text{ m}^2) = 149.8\text{ W}$$

Again noting that in steady operation the heat gain by the plate by absorption of solar radiation must be equal to the heat loss by natural convection and radiation, and using the convection coefficient determined above for convenience (actually, we should calculate the new h using data at a lower temperature, and iterating if necessary for better accuracy),

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$

$$149.8\text{ W} = (5.785\text{ W/m}^2\text{.°C})(0.96\text{ m}^2)(T_s - 25)°\text{C}$$
$$+(0.90)(0.96\text{ m}^2)(5.67\times10^{-8})[(T_s+273)^4 - (10+273\text{ K})^4]$$

Its solution is $\quad T_s = 32.1°\text{C}$

Repeating the calculations at the new anticipated surface temperature of 35°C and the film temperature of (35+25)/2=30°C, we obtain

$h = 3.764$ W/m².°C and T_s = **33.6°C**

9-44 C&S A horizontal ASTM A203 B steel plate has its lower surface subjected to natural convection with cold hydrogen gas at −70°C. Thermal radiation exchange occurs between the plate surface and the surrounding. The rate of heat addition to the plate needed to keep the plate surface from cooling below −30°C is to be determined.

Assumptions **1** Steady state conditions. **2** H_2 is an ideal gas. **3** The local atmospheric pressure is 1 atm.

Properties The properties of H_2 gas at the film temperature of $T_f = (T_\infty + T_s)/2 = (-70-30)/2 = -50$°C are (Table A–16) Pr = 0.6562, k = 0.1404 W/m·K, $\nu = 6.624 \times 10^{-5}$ m²/s, $\beta = 1/T_f = 1/(-50 + 273 \text{ K}) = 0.004484 \text{ K}^{-1}$

Analysis The characteristic length of the plate is

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6 \text{ m}}{4} = 0.15 \text{ m}$$

The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.004484 \text{ K}^{-1})(-30 + 70 \text{ K})(0.15 \text{ m})^3}{(6.624 \times 10^{-5} \text{m}^2/\text{s})^2}(0.6562) = 8.881 \times 10^5$$

The Nusselt number for natural convection with horizontal hot surface facing down is determined from

$$\text{Nu} = \frac{hL_c}{k} = 0.27 \text{ Ra}_L^{1/4}$$

So, the natural convection heat transfer coefficient is

$$h = 0.27\frac{k}{L_c}\text{Ra}_L^{1/4} = 0.27\frac{(0.1404 \text{ W/m}\cdot\text{K})}{0.15 \text{ m}}(8.881 \times 10^5)^{1/4} = 7.758 \text{ W/m}^2\cdot\text{K}$$

The heat transfer rate required to keep the plate surface at −30°C can be determined from the total heat loss from the lower plate surface by natural convection and thermal radiation:

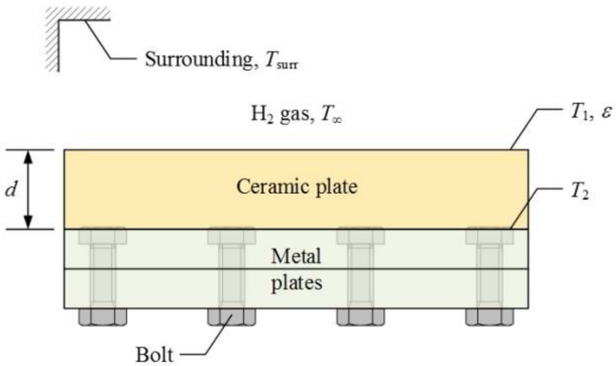
$$\begin{aligned}\dot{Q} &= \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = hA_s(T_s - T_\infty) + \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4)\\ &= (7.758 \text{ W/m}^2\cdot\text{K})(0.6 \text{ m})^2(-30 + 70 \text{ K}) + (0.3)(5.67 \times 10^{-8}\text{W/m}^2\cdot\text{K}^4)(0.6 \text{ m})^2(243^4 - 200^4\text{K}^4)\\ &= 111.72 \text{ W} + 10.95 \text{ W}\\ &= \mathbf{122.7 \text{ W}}\end{aligned}$$

Discussion With a heat addition rate of 123 W or higher, the plate surface exposed to natural convection with the cold H_2 gas and thermal radiation exchange with the surrounding can be maintained from being cooled below −30°C.
Note that with an emissivity of 0.3, the heat loss from the plate surface by thermal radiation is almost 9% of the total heat loss. Due to the generally low natural convection heat transfer coefficient, thermal radiation can contribute significantly in the overall heat transfer.

9-45 C&S Stainless steel bolts (ASTM A437 B4B) are used to secure two horizontal metal plates together. A ceramic plate is used for preventing the bolts in the metal plates from cooling below the minimum suitable temperature of −30°C. The upper surface of the ceramic plate is subjected to natural convection with cold hydrogen gas at −60°C and thermal radiation exchange with the surrounding. The minimum thickness of the plate is to be determined.

Assumptions **1** Steady state conditions. **2** H_2 is an ideal gas. **3** The local atmospheric pressure is 1 atm. **4** Conduction through the plate is one-dimensional. **5** Contact resistance at the interface is negligible.

Properties The properties of H_2 gas at the film temperature of $T_f = (T_\infty + T_s)/2 = (-60-40)/2 = -50$°C are (Table A–16) Pr = 0.6562, $k = 0.1404$ W/m·K, $\nu = 6.624 \times 10^{-5}$ m²/s, $\beta = 1/T_f = 1/(-50 + 273 \text{ K}) = 0.004484 \text{ K}^{-1}$



Analysis The characteristic length of the plate is

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6 \text{ m}}{4} = 0.15 \text{ m}$$

The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_1 - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.004484 \text{ K}^{-1})(-40 + 60 \text{ K})(0.15 \text{ m})^3}{(6.624 \times 10^{-5}\text{m}^2/\text{s})^2}(0.6562) = 4.4405 \times 10^5$$

The Nusselt number for natural convection with horizontal hot surface facing up is determined from

$$\text{Nu} = \frac{hL_c}{k} = 0.54 \text{ Ra}_L^{1/4}$$

So, the natural convection heat transfer coefficient is

$$h = 0.54\frac{k}{L_c}\text{Ra}_L^{1/4} = 0.54\frac{(0.1404 \text{ W/m}\cdot\text{K})}{0.15 \text{ m}}(4.4405 \times 10^5)^{1/4} = 13.05 \text{ W/m}^2\cdot\text{K}$$

Applying energy balance, with natural convection and thermal radiation at the ceramic plate surface and conduction through the ceramic plate,

$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} + \dot{q}_{\text{rad}}$$

$$k_{\text{plate}}\frac{T_2 - T_1}{d} = h(T_1 - T_\infty) + \varepsilon\sigma(T_1^4 - T_{\text{surr}}^4)$$

Solving for the thickness d of the ceramic plate, with $T_2 = -30$°C, yields

$$d = \frac{k_{\text{plate}}(T_2 - T_1)}{h(T_1 - T_\infty) + \varepsilon\sigma(T_1^4 - T_{\text{surr}}^4)}$$

$$= \frac{(1.4 \text{ W/m}\cdot\text{K})(-30 + 40 \text{ K})}{(13.05 \text{ W/m}^2\cdot\text{K})(-40 + 60 \text{ K}) + (0.9)(5.67 \times 10^{-8}\text{W/m}^2\cdot\text{K}^4)(233^4 - 213^4\text{K}^4)} = \mathbf{0.0457 \text{ m}}$$

Discussion The minimum thickness required by the ceramic plate to prevent the surface in contact with the stainless steel bolts from cooling below −30°C is 4.57 cm. With d> 4.57 cm, the ceramic plate surface that is in contact with the bolts would be above the minimum suitable temperature for the ASTM A437 B4B stainless steel bolts.

9-46 The required electrical power to maintain a specified surface temperature of a grill is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Thermal properties are constant.

Properties The properties of air at $T_f = (T_s + T_\infty)/2 = 90°C$ are $k = 0.03024$ W/m·K, $\nu = 2.201 \times 10^{-5}$ m^2/s, Pr = 0.7132 (from Table A-15). Also, $\beta = 1/T_f = 2.755 \times 10^{-3}$ K^{-1}.

Analysis Treating the grill as a horizontal circular plate, the characteristic length is

$$L_c = \frac{A_s}{p} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} = 0.0625\text{ m}$$

The Rayleigh number ($L_c = D/4$) is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.002755\text{ K}^{-1})(150-30)\text{ K}(0.0625\text{ m})^3}{(2.201\times10^{-5}\text{ m}^2/\text{s})^2}(0.7132)$$

$$= 1.166\times10^6$$

Since the grill has a hot upper surface, we use

$$\text{Nu} = 0.54\text{Ra}_L^{1/4} = 0.54(1.166\times10^6)^{1/4} = 17.74$$

$$h = \text{Nu}\frac{k}{L_c} = (17.74)\frac{0.03024\text{ W/m}\cdot\text{K}}{0.0625\text{ m}} = 8.583\text{ W/m}^2\cdot\text{K}$$

The total rate of heat transfer on the grill surface is

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = hA_s(T_s - T_\infty) + \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4)$$

$$= \frac{\pi}{4}(0.25\text{ m})^2[(8.583\text{ W/m}^2\cdot\text{K})(150-30)\text{ K} + 0.8(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(423^4 - 303^4)\text{ K}^4]$$

$$= \mathbf{103\ W}$$

Discussion To maintain a surface temperature of 150°C, the grill needs at least 103 W of electrical power.

9-47 A can of engine oil placed vertically in the trunk of a car and the heat transfer from the ends of the can are negligible, determine the heat transfer rate from the can surface.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Thermal properties are constant. **4** Radiation heat transfer is negligible.

Properties The properties of air at $T_f = (T_s + T_\infty)/2 = 30°\text{C}$ are $k = 0.02588$ W/m·K, $\nu = 1.608 \times 10^{-5}$ m^2/s, Pr = 0.7282 (from Table A-15). Also, $\beta = 1/T_f = 0.0033$ K^{-1}.

Analysis The Rayleigh number ($L_c = L$) is

$$\text{Ra}_L = \frac{g\beta(T_\infty - T_s)L^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.0033\,\text{K}^{-1})(43-17)\text{K}(0.15\,\text{m})^3}{(1.608\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7282)$$
$$= 8.00\times10^6$$

Then

$$\frac{35L}{Gr_L^{1/4}} = \frac{35(0.15\,\text{m})}{(1.099\times10^7)^{1/4}} = 0.0912\,\text{m} < D$$

Since $D \geq 35L/Gr_L^{1/4}$ is satisfied, we can treat this vertical cylinder as a vertical plate, and the Nusselt may be calculated with

$$\text{Nu} = 0.59\text{Ra}_L^{1/4} = 0.59(8.00\times10^6)^{1/4} = 31.38$$

Then, the heat transfer coefficient is

$$h = \frac{k}{L}\text{Nu} = \frac{0.02588\ \text{W/m}\cdot\text{K}}{0.15\,\text{m}}(31.38) = 5.414\ \text{W/m}^2\cdot\text{K}$$

Hence, the rate of heat transfer is

$$\dot{Q} = hA_s(T_\infty - T_s) = h\pi DL(T_\infty - T_s)$$
$$= (5.414\ \text{W/m}^2\cdot\text{K})\pi(0.1\,\text{m})(0.15\,\text{m})(43-17)\ \text{K}$$
$$= \mathbf{6.63\ W}$$

Discussion For vertical cylinder, the characteristic length is its length.

9-48 Flue gases are released to atmosphere using a cylindrical stack. The rates of heat transfer from the stack with and without wind cases are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

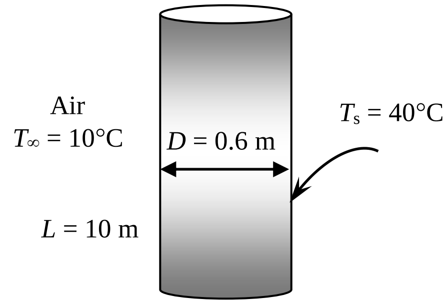
Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (40+10)/2 = 25°C$ are (Table A-15)

$$k = 0.02551 \text{ W/m.°C}$$

$$\nu = 1.562\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7296$$

$$\beta = \frac{1}{T_f} = \frac{1}{(25+273)\text{K}} = 0.003356 \text{ K}^{-1}$$



Analysis (*a*) When there is no wind heat transfer is by natural convection. The characteristic length in this case is the height of the stack, $L_c = L = 10$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003356 \text{ K}^{-1})(40-10 \text{ K})(10 \text{ m})^3}{(1.562\times10^{-5} \text{ m}^2/\text{s})^2}(0.7296) = 2.953\times10^{12}$$

We can treat this vertical cylinder as a vertical plate since

$$\frac{35L}{Gr^{1/4}} = \frac{35(10)}{(2.953\times10^{12}/0.7296)^{1/4}} = 0.246 < 0.6 \quad \text{and thus } D \geq \frac{35L}{Gr^{1/4}}$$ The

Nusselt number is determined from

$$Nu = 0.1Ra^{1/3} = 0.1(2.953\times10^{12})^{1/3} = 1435 \qquad \text{(from Table 9-1)}$$

Then

$$h = \frac{k}{L_c}Nu = \frac{0.02551 \text{ W/m.°C}}{10 \text{ m}}(1435) = 3.660 \text{ W/m}^2.°\text{C}$$

and

$$\dot{Q} = hA(T_s - T_\infty) = (3.660 \text{ W/m}^2.°\text{C})(\pi\times0.6\times10 \text{ m}^2)(40-10)°\text{C} = \mathbf{2070\ W}$$

(*b*) When the stack is exposed to 20 km/h winds, the heat transfer will be by forced convection. We have flow of air over a cylinder and the heat transfer rate is determined as follows:

$$\text{Re} = \frac{VD}{\nu} = \frac{(20\times1000/3600 \text{ m/s})(0.6\text{m})}{1.562\times10^{-5} \text{ m}^2/\text{s}} = 213{,}400$$

$$\text{Nu} = 0.027\,\text{Re}^{0.805}\,\text{Pr}^{1/3} = 0.027(213{,}400)^{0.805}(0.7296)^{1/3} = 473.9 \qquad \text{(from Table 7-1)}$$

$$h = \frac{k}{D}\text{Nu} = \frac{0.02551 \text{ W/m.°C}}{0.6 \text{ m}}(473.9) = 20.15 \text{ W/m}^2.°\text{C}$$

$$\dot{Q} = hA(T_s - T_\infty) = (20.15 \text{ W/m}^2.°\text{C})(\pi\times0.6\times10 \text{ m}^2)(40-10)°\text{C} = \mathbf{11{,}390\ W}$$

Discussion There is more than five-fold increase in heat transfer due to winds.

9-49 Water is boiling in a pan that is placed on top of a stove. The rate of heat loss from the cylindrical side surface of the pan by natural convection and radiation and the ratio of heat lost from the side surfaces of the pan to that by the evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

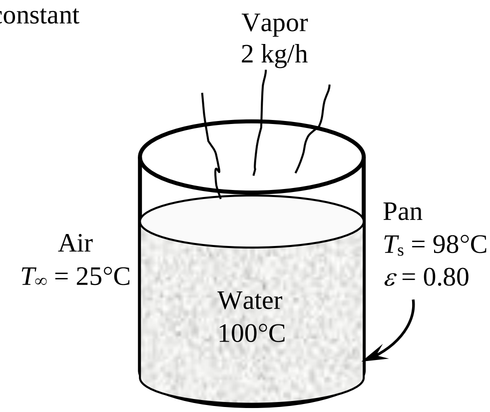
Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (98+25)/2 = 61.5°C are (Table A-15)

$$k = 0.02819 \text{ W/m.°C}$$

$$\nu = 1.910\times10^{-5} \text{ m}^2\text{/s}$$

$$\Pr = 0.7198$$

$$\beta = \frac{1}{T_f} = \frac{1}{(61.5+273)\text{K}} = 0.00299 \text{ K}^{-1}$$



Analysis (*a*) The characteristic length in this case is the height of the pan, $L_c = L = 0.12$ m. Then

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.00299 \text{ K}^{-1})(98-25 \text{ K})(0.12 \text{ m})^3}{(1.910\times10^{-5} \text{ m}^2\text{/s})^2}(0.7198) = 7.299\times10^6$$

We can treat this vertical cylinder as a vertical plate since

$$\frac{35L}{Gr^{1/4}} = \frac{35(0.12)}{(7.299\times10^6 / 0.7198)^{1/4}} = 0.07443 < 0.25 \quad \text{and thus } D \geq \frac{35L}{Gr^{1/4}}$$

Therefore,

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(7.299\times10^6)^{1/6}}{\left[1+\left(\frac{0.492}{0.7198}\right)^{9/16}\right]^{8/27}}\right\}^2 = 28.60$$

$$h = \frac{k}{L}Nu = \frac{0.02819 \text{ W/m.°C}}{0.12 \text{ m}}(28.60) = 6.720 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.25 \text{ m})(0.12 \text{ m}) = 0.09425 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (6.720 \text{ W/m}^2\text{.°C})(0.09425 \text{ m}^2)(98-25)\text{°C} = \mathbf{46.2\ W}$$

(*b*) The radiation heat loss from the pan is

$$\dot{Q}_{rad} = \varepsilon A_s \sigma(T_s{}^4 - T_{surr}{}^4)$$
$$= (0.80)(0.09425 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(98+273 \text{ K})^4 - (25+273 \text{ K})^4\right] = \mathbf{47.3\ W}$$

(*c*) The heat loss by the evaporation of water is

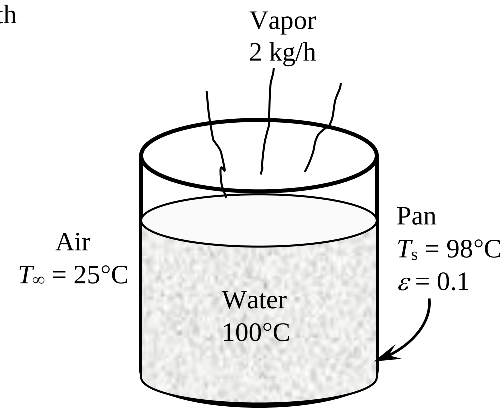
$$\dot{Q} = \dot{m}h_{fg} = (1.5/3600 \text{ kg/s})(2257 \text{ kJ/kg}) = 0.9404 \text{ kW} = 940 \text{ W}$$

Then the ratio of the heat lost from the side surfaces of the pan to that by the evaporation of water then becomes

$$f = \frac{46.2+47.3}{940} = 0.099 = \mathbf{9.9\%}$$

9-50 Water is boiling in a pan that is placed on top of a stove. The rate of heat loss from the cylindrical side surface of the pan by natural convection and radiation and the ratio of heat lost from the side surfaces of the pan to that by the evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (98+25)/2 = 61.5°C$ are (Table A-15)

$$k = 0.02819 \text{ W/m.°C}$$
$$\nu = 1.910\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7198$$
$$\beta = \frac{1}{T_f} = \frac{1}{(61.5+273)\text{K}} = 0.00299 \text{ K}^{-1}$$

Analysis (*a*) The characteristic length in this case is the height of the pan, $L_c = L = 0.12$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.00299 \text{ K}^{-1})(98-25 \text{ K})(0.12 \text{ m})^3}{(1.910\times10^{-5} \text{ m}^2/\text{s})^2}(0.7198) = 7.299\times10^6$$

We can treat this vertical cylinder as a vertical plate since

$$\frac{35L}{Gr^{1/4}} = \frac{35(0.12)}{(7.299\times10^6 / 0.7198)^{1/4}} = 0.07443 < 0.25 \quad \text{and thus } D \ge \frac{35L}{Gr^{1/4}}$$

Therefore,

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(7.299\times10^6)^{1/6}}{\left[1+\left(\frac{0.492}{0.7198}\right)^{9/16}\right]^{8/27}}\right\}^2 = 28.60$$

$$h = \frac{k}{L}Nu = \frac{0.02819 \text{ W/m.°C}}{0.12 \text{ m}}(28.60) = 6.720 \text{ W/m}^2.\text{°C}$$
$$A_s = \pi DL = \pi(0.25 \text{ m})(0.12 \text{ m}) = 0.09425 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (6.720 \text{ W/m}^2.\text{°C})(0.09425 \text{ m}^2)(98-25)\text{°C} = \mathbf{46.2 \text{ W}}$$

(b) The radiation heat loss from the pan is

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4}) \\ &= (0.10)(0.09425 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)\left[(98+273 \text{ K})^4 - (25+273 \text{ K})^4\right] = \mathbf{5.9 \text{ W}}\end{aligned}$$

(c) The heat loss by the evaporation of water is

$$\dot{Q} = \dot{m}h_{fg} = (1.5/3600 \text{ kg/s})(2257 \text{ kJ/kg}) = 0.9404 \text{ kW} = 940 \text{ W}$$

Then the ratio of the heat lost from the side surfaces of the pan to that by the evaporation of water then becomes

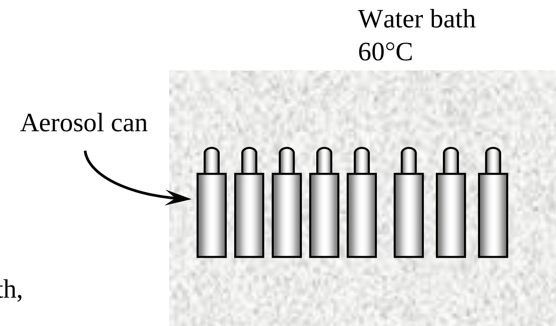
$$f = \frac{46.2+5.9}{940} = 0.055 = \mathbf{5.5\%}$$

9-51 Some cans move slowly in a hot water container made of sheet metal. The rate of heat loss from the four side surfaces of the container and the annual cost of those heat losses are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **3** Heat loss from the top surface is disregarded.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (60+20)/2 = 40°C$ are (Table A-15)

$$k = 0.02662 \text{ W/m}\cdot\text{K}$$
$$\nu = 1.702\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7255$$
$$\beta = \frac{1}{T_f} = \frac{1}{(40+273)\text{K}} = 0.003195 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the height of the bath, $L_c = L = 0.5$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003195 \text{ K}^{-1})(60-20 \text{ K})(0.5 \text{ m})^3}{(1.702\times10^{-5} \text{ m}^2/\text{s})^2}(0.7255) = 3.925\times10^8$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(3.925\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7255}\right)^{9/16}\right]^{8/27}}\right\}^2 = 92.50$$

$$h = \frac{k}{L}Nu = \frac{0.02662 \text{ W/m.°C}}{0.5 \text{ m}}(92.50) = 4.925 \text{ W/m}^2\cdot\text{K}$$
$$A_s = 2[(0.5 \text{ m})(1 \text{ m}) + (0.5 \text{ m})(3.5 \text{ m})] = 4.5 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (4.925 \text{ W/m}^2\cdot\text{K})(4.5 \text{ m}^2)(60-20)°\text{C} = 886.5 \text{ W}$$

The radiation heat loss is

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4}) \\ &= (0.7)(4.5 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)\left[(60+273 \text{ K})^4 - (20+273 \text{ K})^4\right] \\ &= 879.9 \text{ W}\end{aligned}$$

Then the total rate of heat loss becomes

$$\dot{Q}_{total} = \dot{Q}_{\substack{natural \\ convection}} + \dot{Q}_{rad} = 886.5 + 879.9 = \mathbf{1766\ W}$$

The amount and cost of the heat loss during one year is

$$Q_{total} = \dot{Q}_{total}\Delta t = (1.766 \text{ kW})(8760 \text{ h}) = 15{,}470 \text{ kWh}$$
$$\text{Cost} = (15{,}470 \text{ kWh})(\$0.085/\text{kWh}) = \mathbf{\$1315}$$

9-52 Some cans move slowly in a hot water container made of sheet metal. It is proposed to insulate the side and bottom surfaces of the container for $350. The simple payback period of the insulation to pay for itself from the energy it saves is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **3** Heat loss from the top surface is disregarded.

Properties Insulation will drop the outer surface temperature to a value close to the ambient temperature. The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature, which is unknown.

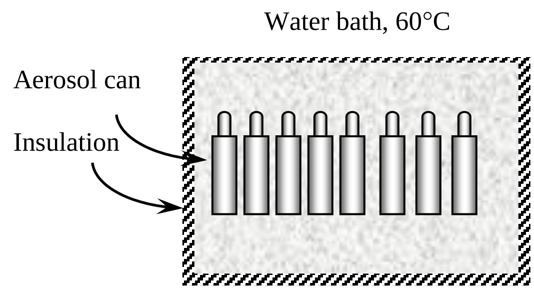
We evaluate air properties at a film temperature of $(T_s+T_\infty)/2$ = 23°C and 1 atm based on the problem statement. Then, for an air temperature of T_∞ = 20°C, the corresponding surface temperature is T_s = 26°C. The properties of air at 1 atm and 23°C are (Table A-15)

$$k = 0.02536 \text{ W/m.°C}$$

$$\nu = 1.543\times10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7301$$

$$\beta = \frac{1}{T_f} = \frac{1}{(23+273)\text{K}} = 0.00338 \text{ K}^{-1}$$



Analysis We start the solution process by "guessing" the outer surface temperature to be 26°C. We will check the accuracy of this guess later and repeat the calculations if necessary with a better guess based on the results obtained. The characteristic length in this case is the height of the tank, $L_c = L = 0.5$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.00338 \text{ K}^{-1})(26-20 \text{ K})(0.5 \text{ m})^3}{(1.543\times10^{-5} \text{ m}^2\text{/s})^2}(0.7301) = 7.622\times10^7$$

$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(7.622\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7301}\right)^{9/16}\right]^{8/27}}\right\}^2 = 56.53$$

$$h = \frac{k}{L}Nu = \frac{0.02536 \text{ W/m.°C}}{0.5 \text{ m}}(56.53) = 2.868 \text{ W/m}^2\text{.°C}$$

$$A_s = 2[(0.5 \text{ m})(1.10 \text{ m}) + (0.5 \text{ m})(3.60 \text{ m})] = 4.7 \text{ m}^2$$

Then the total rate of heat loss from the outer surface of the insulated tank by convection and radiation becomes

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$= (2.868 \text{ W/m}^2\text{.°C})(4.7 \text{ m}^2)(26-20)\text{°C} + (0.1)(4.7 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(26+273 \text{ K})^4 - (20+273 \text{ K})^4]$$
$$= 97.5 \text{ W}$$

In steady operation, the heat lost by the side surfaces of the tank must be equal to the heat lost from the exposed surface of the insulation by convection and radiation, which must be equal to the heat conducted through the insulation. The second conditions requires the surface temperature to be

$$\dot{Q} = \dot{Q}_{insulation} = kA_s\frac{T_{\text{tank}} - T_s}{L} \quad\rightarrow\quad 97.5 \text{ W} = (0.035 \text{ W/m.°C})(4.7 \text{ m}^2)\frac{(60-T_s)\text{°C}}{0.05 \text{ m}}$$

It gives T_s = 30.4°C, which is sufficiently close to the assumed temperature, 26°C. Therefore, there is no need to repeat the calculations. The total amount of heat loss and its cost during one year are

$$Q_{total} = \dot{Q}_{total}\Delta t = (97.5 \text{ W})(8760 \text{ h}) = 853.7 \text{ kWh}$$

$$\text{Cost} = (853.7 \text{ kWh})(\$0.085/\text{kWh}) = \$72.6$$

Then money saved during a one-year period due to insulation becomes

$$\text{Money saved} = \text{Cost}_{\substack{without\\insulation}} - \text{Cost}_{\substack{with\\insulation}} = \$1315 - \$72.6 = \$1242$$

where $1116 is obtained from the solution of Problem 9-29. The insulation will pay for itself in

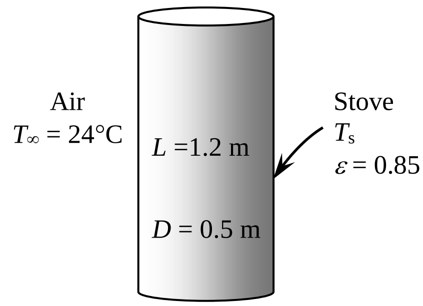
$$\text{Payback period} = \frac{\text{Cost}}{\text{Money saved}} = \frac{\$350}{\$1242/yr} = \mathbf{0.282 \text{ yr} = 103 \text{ days}}$$

Discussion We would definitely recommend the installation of insulation in this case.

9-53 A room is to be heated by a cylindrical coal-burning stove. The surface temperature of the stove and the amount of coal burned during a 14-h period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The temperature of the outer surface of the stove is constant. **5** The heat transfer from the bottom surface is negligible. **6** The heat transfer coefficient at the top surface is the same as that on the side surface.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (130+24)/2 = 77°C$ are (Table A-15)



$$k = 0.02931 \text{ W/m.°C}$$
$$\nu = 2.066\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7161$$
$$\beta = \frac{1}{T_f} = \frac{1}{(77+273)\text{K}} = 0.002857 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the height of the cylindir, $L_c = L = 1.2$ m. Then,

$$\text{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81 \text{ m/s}^2)(0.002857 \text{ K}^{-1})(130-24 \text{ K})(1.2 \text{ m})^3}{(2.066\times10^{-5} \text{ m}^2/\text{s})^2} = 1.203\times10^{10}$$

A vertical cylinder can be treated as a vertical plate when

$$D(=0.50 \text{ m}) \geq \frac{35L}{\text{Gr}^{1/4}} = \frac{35(1.2 \text{ m})}{(1.203\times10^{10})^{1/4}} = 0.1268 \text{ m}$$

which is satisfied. That is, the Nusselt number relation for a vertical plate can be used for side surfaces.

$$\text{Ra} = \text{GrPr} = (1.203\times10^{10})(0.7161) = 8.615\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(8.615\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7161}\right)^{9/16}\right]^{8/27}}\right\}^2 = 241.0$$

$$h = \frac{k}{L}Nu = \frac{0.02931 \text{ W/m.°C}}{1.2 \text{ m}}(241.0) = 5.886 \text{ W/m}^2\text{.°C}$$
$$A_s = \pi DL + \pi D^2/4 = \pi(0.5 \text{ m})(1.2 \text{ m}) + \pi(0.5 \text{ m})^2/4 = 2.081 \text{ m}^2$$

Then the surface temperature of the stove is determined from

$$\dot{Q} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = hA_s(T_s - T_\infty) + \varepsilon\sigma A_s(T_s{}^4 - T_{\text{surr}}{}^4)$$
$$1500 \text{ W} = (5.886 \text{ W/m}^2\text{.°C})(2.081 \text{ m}^2)(T_s - 297)$$
$$+ (0.85)(2.081 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)(T_s{}^4 - 287^4)$$
$$T_s = 350.9 \text{ K} = 77.9°\text{C}$$

This surface temperature is not close to the value assumed for the evaluation of properties and h. We repeat calculations at the anticipated surface temperature of 86°C and the film temperature of (86+24)/2=55°C as follows:

$$k = 0.02772 \text{ W/m.°C}$$
$$\nu = 1.847\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7215$$
$$\beta = \frac{1}{T_f} = \frac{1}{(55+273)\text{K}} = 0.003049 \text{ K}^{-1}$$

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003049 \text{ K}^{-1})(86-24 \text{ K})(1.2 \text{ m})^3}{(1.847\times10^{-5} \text{ m}^2/\text{s})^2}(0.7215) = 6.777\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(6.777\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7215}\right)^{9/16}\right]^{8/27}}\right\}^2 = 223.6$$

$$h = \frac{k}{L}Nu = \frac{0.02772 \text{ W/m.°C}}{1.2 \text{ m}}(223.6) = 5.165 \text{ W/m}^2.\text{°C}$$

$$\dot{Q} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = hA_s(T_s - T_\infty) + \varepsilon\sigma A_s(T_s{}^4 - T_{\text{surr}}{}^4)$$

$$1500 \text{ W} = (5.165 \text{ W/m}^2.\text{°C})(2.081 \text{m}^2)(T_s - 297)$$
$$+ (0.85)(2.081 \text{m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)(T_s{}^4 - 287^4)$$

$$T_s = 353.7 \text{ K} = \mathbf{80.7°C}$$

This is close to the assumed value, and thus there is no need to repeat the calculations. The amount of coal used is determined from

$$Q = \dot{Q}\Delta t = (1.5 \text{ kJ/s})(14\text{h/day} \times 3600 \text{ s/h}) = 75{,}600 \text{ kJ}$$

$$m_{\text{coal}} = \frac{Q/\eta}{HV} = \frac{(75{,}600 \text{ kJ})/0.65}{30{,}000 \text{ kJ/kg}} = \mathbf{3.88 \text{ kg}}$$

9-54 A cylinder with specified length and diameter, the orientation of the cylinder that would achieve higher heat transfer rate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Thermal properties are constant. **4** Radiation heat transfer is negligible.

Properties The properties of air at $T_f = (T_s + T_\infty)/2 = 30°\text{C}$ are $k = 0.02588$ W/m·K, $\nu = 1.608 \times 10^{-5}$ m^2/s, Pr = 0.7282 (from Table A-15). Also, $\beta = 1/T_f = 0.0033\ \text{K}^{-1}$.

Analysis For vertical orientation, the Rayleigh number ($L_c = L$) is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.0033\,\text{K}^{-1})(43-17)\text{K}(0.15\,\text{m})^3}{(1.608\times 10^{-5}\ \text{m}^2/\text{s})^2}(0.7282)$$

$$= 8.00\times 10^6$$

Then

$$\frac{35L}{Gr_L^{1/4}} = \frac{35(0.15\,\text{m})}{(1.099\times 10^7)^{1/4}} = 0.0912\,\text{m} < D$$

Since $D \geq 35L / Gr_L^{1/4}$ is satisfied, we can treat this vertical cylinder as a vertical plate, and the Nusselt may be calculated with

$$\text{Nu}_{\text{vert}} = \left\{0.825 + \frac{0.387\text{Ra}_L^{1/6}}{[1+(0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$$

$$= \left\{0.825 + \frac{0.387(8.00\times 10^6)^{1/6}}{[1+(0.492/0.7282)^{9/16}]^{8/27}}\right\}^2 = 29.39$$

For horizontal orientation, the Rayleigh number ($L_c = D$) is

$$\text{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.0033\,\text{K}^{-1})(43-17)\text{K}(0.1\,\text{m})^3}{(1.608\times 10^{-5}\ \text{m}^2/\text{s})^2}(0.7282)$$

$$= 2.37\times 10^6$$

The Nusselt number for horizontal cylinder is

$$\text{Nu}_{\text{horiz}} = \left\{0.6 + \frac{0.387\text{Ra}_D^{1/6}}{[1+(0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.37\times 10^6)^{1/6}}{[1+(0.559/0.7282)^{9/16}]^{8/27}}\right\}^2 = 18.64$$

Hence, the ratio of heat transfer rate is

$$\frac{\dot{Q}_{\text{vert}}}{\dot{Q}_{\text{horiz}}} = \frac{h_{\text{vert}}(k/L)A_s\Delta T}{h_{\text{horiz}}(k/L)A_s\Delta T} = \frac{\text{Nu}_{\text{vert}}(k/L)A_s\Delta T}{\text{Nu}_{\text{horiz}}(k/L)A_s\Delta T} = \frac{\text{Nu}_{\text{vert}}}{\text{Nu}_{\text{horiz}}} = \frac{29.39}{18.64} = \mathbf{1.58}$$

Discussion For the same ΔT, the rate of heat transfer for the vertical orientation is 58% larger than that for the horizontal orientation.

9-55 A soda can placed horizontally in a refrigerator compartment and the heat transfer from the ends of the can are negligible, determine the heat transfer rate from the can surface.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Thermal properties are constant. **4** Radiation heat transfer is negligible.

Properties The properties of air at $T_f = (T_s + T_\infty)/2 = 20°C$ are $k = 0.02514$ W/m·K, $\nu = 1.516 \times 10^{-5}$ m^2/s, Pr = 0.7309 (from Table A-15). Also, $\beta = 1/T_f = 0.003413$ K^{-1}.

Analysis The Rayleigh number ($L_c = D$) is

$$\text{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.003413\,\text{K}^{-1})(36-4)\text{K}(0.06\,\text{m})^3}{(1.516\times 10^{-5}\,\text{m}^2/\text{s})^2}(0.7309)$$

$$= 7.36\times 10^5$$

The Nusselt number for horizontal cylinder is

$$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}_D^{1/6}}{[1+(0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(7.36\times 10^5)^{1/6}}{[1+(0.559/0.7309)^{9/16}]^{8/27}}\right\}^2 = 13.39$$

Then, the heat transfer coefficient is

$$h = \frac{k}{D}\text{Nu} = \frac{0.02514\,\text{W/m}\cdot\text{K}}{0.06\,\text{m}}(13.39) = 5.61\,\text{W/m}^2\cdot\text{K}$$

Hence, the rate of heat transfer is

$$\dot{Q} = hA_s(T_s - T_\infty) = h\pi DL(T_s - T_\infty)$$

$$= (5.61\,\text{W/m}^2\cdot\text{K})\pi(0.06\,\text{m})(0.15\,\text{m})(36-4)\,\text{K}$$

$$= \mathbf{5.08\,W}$$

Discussion For horizontal cylinder, the characteristic length is its diameter.

9-56 A boiler supplies hot water to a dishwasher through a pipe at 10 g/s. The pipe dimensions are given. The water exits the boiler at 85°C. The pipe section between the boiler and the dishwasher is exposed to natural convection. The water temperature entering the dishwasher is to be determined whether it meets the ANSI/NSF 3 standard.

Assumptions **1** Steady state conditions. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm. **4** Thermal radiation is negligible. **5** Surface temperature of the pipe is constant.

Properties The properties of air at the film temperature of $T_f = (T_\infty + T_s)/2 = (20 + 50)/2 = 35$°C are (Table A–15) Pr = 0.7268, k = 0.02625 W/m·K, $\nu = 1.655 \times 10^{-5}$ m^2/s, $\beta = 1/T_f = = 1/(35 + 273 \text{ K}) = 0.003247 \text{ K}^{-1}$. For water c_p = 4.20 kJ/kg·K

Analysis The characteristic length of the horizontal pipe is D = 0.02 m, and the Rayleigh number is

$$\mathrm{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\mathrm{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003247 \text{ K}^{-1})(50 - 20 \text{ K})(0.02 \text{ m})^3}{(1.655 \times 10^{-5}\text{m}^2/\text{s})^2}(0.7268) = 20285$$

The Nusselt number for natural convection in this case is determined from

$$\mathrm{Nu} = \frac{hD}{k} = \left\{0.6 + \frac{0.387\ \mathrm{Ra}_D^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

So, the natural convection heat transfer coefficient is

$$h = \frac{k}{D}\left\{0.6 + \frac{0.387\ \mathrm{Ra}_D^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2 = \frac{0.02625 \text{ W/m}\cdot\text{K}}{0.02 \text{ m}}\left\{0.6 + \frac{0.387\ (20285)^{1/6}}{[1 + (0.559/0.7268)^{9/16}]^{8/27}}\right\}^2$$

$$h = 6.828 \text{ W/m}^2\cdot\text{K}$$

The rate of heat loss from the hot water flowing in the pipe is equal to the heat transfer rate by natural convection between the pipe surface at the ambient air:

$$\dot{Q} = \dot{m}c_p(T_{\text{in}} - T_{\text{out}}) = hA_s(T_s - T_\infty)$$

where T_{in} is the water temperature exiting the boiler (entering the pipe), and T_{out} is the water temperature entering the dishwasher (exiting the pipe). Solving for T_{out} yields,

$$T_{\text{out}} = T_{\text{in}} - \frac{h(\pi DL)}{\dot{m}c_p}(T_s - T_\infty) = 85℃ - \frac{(6.828 \text{ W/m}^2\cdot\text{K})\pi(0.02 \text{ m})(20 \text{ m})}{(0.01 \text{ kg/s})(4200 \text{ J/kg}\cdot\text{K})}(50 - 20℃) = \mathbf{78.9}℃ < 85℃$$

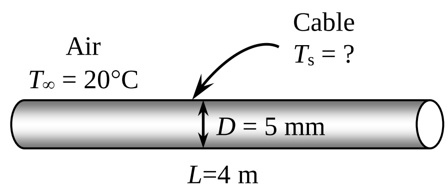
Discussion The hot water entering the dishwasher is about 6°C lower than the temperature required by the ANSI/NSF 3 standard. To increase the water temperature entering the dishwasher, we can add insulation on the pipe to reduce the heat loss from the pipe surface. We can increase the water mass flow rate. We can reduce the pipe distance between the boiler and the dishwasher. We can increase the water temperature coming out from the boiler.
Note that this analysis does not account for the heat loss from the water due to thermal radiation exchange at the pipe surface with the surrounding. If the effect of thermal radiation is included, the water temperature entering the dishwasher would be lower than 78.9°C.

9-57 Heat generated by the electrical resistance of a bare cable is dissipated to the surrounding air. The surface temperature of the cable is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The temperature of the surface of the cable is constant.

Properties We evaluate air properties at a film temperature of $(T_s+T_\infty)/2 = 60°C$ and 1 atm based on the problem statement. Then, for an air temperature of $T_\infty = 20°C$, the corresponding surface temperature is $T_s = 100°C$. The properties of air at 1 atm and 60°C are (Table A-15)

$$k = 0.02808 \text{ W/m.°C}$$
$$\nu = 1.896\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7202$$
$$\beta = \frac{1}{T_f} = \frac{1}{(60+273)\text{K}} = 0.003003 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.005$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003003 \text{ K}^{-1})(100-20 \text{ K})(0.005 \text{ m})^3}{(1.896\times10^{-5} \text{ m}^2/\text{s})^2}(0.7202) = 590.2$$

$$Nu = \left\{0.6 + \frac{0.387 Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(590.2)^{1/6}}{\left[1+\left(0.559/0.7202\right)^{9/16}\right]^{8/27}}\right\}^2 = 2.346$$

$$h = \frac{k}{D}Nu = \frac{0.02808 \text{ W/m.°C}}{0.005 \text{ m}}(2.346) = 13.17 \text{ W/m}^2.\text{°C}$$

$$A_s = \pi DL = \pi(0.005 \text{ m})(4 \text{ m}) = 0.06283 \text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$
$$(60 \text{ V})(1.5 \text{ A}) = (13.17 \text{ W/m}^2.\text{°C})(0.06283 \text{ m}^2)(T_s - 20)\text{°C}$$
$$T_s = 128.8\text{°C}$$

which is not close to the assumed value of 100°C. Repeating calculations for an assumed surface temperature of 120°C, [$T_f = (T_s+T_\infty)/2 = (120+20)/2 = 70°C$]

$$k = 0.02881 \text{ W/m.°C}$$
$$\nu = 1.995\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7177$$
$$\beta = \frac{1}{T_f} = \frac{1}{(70+273)\text{K}} = 0.002915 \text{ K}^{-1}$$

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.002915 \text{ K}^{-1})(120-20 \text{ K})(0.005 \text{ m})^3}{(1.995\times10^{-5} \text{ m}^2/\text{s})^2}(0.7177) = 644.6$$

$$Nu = \left\{0.6 + \frac{0.387 Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(644.6)^{1/6}}{\left[1+\left(0.559/0.7177\right)^{9/16}\right]^{8/27}}\right\}^2 = 2.387$$

$$h = \frac{k}{D}Nu = \frac{0.02881 \text{ W/m.°C}}{0.005 \text{ m}}(2.387) = 13.76 \text{ W/m}^2.\text{°C}$$

$$\dot{Q} = hA_s(T_s - T_\infty)$$
$$(60 \text{ V})(1.5A) = (13.76 \text{ W/m}^2.\text{°C})(0.06283 \text{ m}^2)(T_s - 20)\text{°C}$$
$$T_s = \mathbf{124.1°C}$$

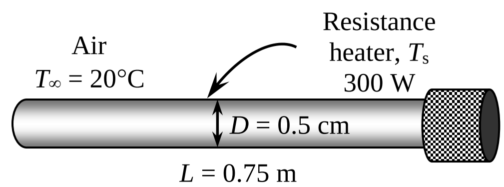
which is sufficiently close to the assumed value of 120°C.

9-58 A cylindrical resistance heater is placed horizontally in a fluid. The outer surface temperature of the resistance wire is to be determined for two different fluids.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer by radiation is ignored. **5** Properties are evaluated at 500°C for air and 40°C for water.

Properties The properties of air at 1 atm and 500°C are (Table A-15)

$$k = 0.05572 \text{ W/m.°C}$$
$$\nu = 7.804\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.6986,$$
$$\beta = \frac{1}{T_f} = \frac{1}{(500+273)\text{K}} = 0.001294 \text{ K}^{-1}$$



The properties of water at 40°C are (Table A-9)

$$k = 0.631 \text{ W/m.°C}$$
$$\nu = \mu/\rho = 0.6582\times10^{-6} \text{ m}^2\text{/s}$$
$$\text{Pr} = 4.32$$
$$\beta = 0.000377 \text{ K}^{-1}$$

Analysis (*a*) The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 1200°C for the calculation of *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the wire, $L_c = D = 0.005$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.001294 \text{ K}^{-1})(1200-20)\text{°C}(0.005 \text{ m})^3}{(7.804\times10^{-5} \text{ m}^2\text{/s})^2}(0.6986) = 214.7$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(214.7)^{1/6}}{\left[1+\left(0.559/0.6986\right)^{9/16}\right]^{8/27}}\right\}^2 = 1.919$$

$$h = \frac{k}{D}Nu = \frac{0.05572 \text{ W/m.°C}}{0.005 \text{ m}}(1.919) = 21.38 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.005 \text{ m})(0.75 \text{ m}) = 0.01178 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) \rightarrow 300 \text{ W} = (21.38 \text{ W/m}^2\text{.°C})(0.01178 \text{ m}^2)(T_s - 20)\text{°C} \rightarrow T_s = \mathbf{1211°C}$$

which is close to the assumed value of 1200°C used in the evaluation of *h*. Therefore, there is no need to repeat calculations.

(*b*) For the case of water, we "guess" the surface temperature to be 40°C. The characteristic length in this case is the outer diameter of the wire, $L_c = D = 0.005$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.000377 \text{ K}^{-1})(40-20 \text{ K})(0.005 \text{ m})^3}{(0.6582\times10^{-6} \text{ m}^2\text{/s})^2}(4.32) = 92{,}197$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(92{,}197)^{1/6}}{\left[1+\left(0.559/4.32\right)^{9/16}\right]^{8/27}}\right\}^2 = 8.986$$

$$h = \frac{k}{D}Nu = \frac{0.631 \text{ W/m.°C}}{0.005 \text{ m}}(8.986) = 1134 \text{ W/m}^2\text{.°C}$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow 300 \text{ W} = (1134 \text{ W/m}^2\text{.°C})(0.01178 \text{ m}^2)(T_s - 20)\text{°C} \longrightarrow T_s = \mathbf{42.5°C}$$

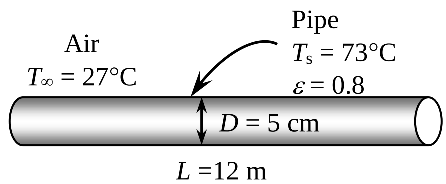
which is sufficiently close to the assumed value of 40°C in the evaluation of the properties and *h*. Therefore, there is no need to repeat calculations.

9-59 A horizontal hot water pipe passes through a large room. The rate of heat loss from the pipe by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The temperature of the outer surface of the pipe is constant.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (73+27)/2 = 50°C$ are (Table A-15)

$$k = 0.02735 \text{ W/m.°C}$$
$$\nu = 1.798\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7228$$
$$\beta = \frac{1}{T_f} = \frac{1}{(50+273)\text{K}} = 0.003096 \text{ K}^{-1}$$



Analysis (*a*) The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.05$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003096 \text{ K}^{-1})(73-27 \text{ K})(0.05 \text{ m})^3}{(1.798\times10^{-5} \text{ m}^2\text{/s})^2}(0.7228) = 3.905\times10^5$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(3.905\times10^5)^{1/6}}{\left[1+(0.559/0.7228)^{9/16}\right]^{8/27}}\right\}^2 = 11.23$$

$$h = \frac{k}{D}Nu = \frac{0.02735 \text{ W/m.°C}}{0.05 \text{ m}}(11.23) = 6.143 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.05 \text{ m})(12 \text{ m}) = 1.885 \text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (6.143 \text{ W/m}^2\text{.°C})(1.885 \text{ m}^2)(73-27)\text{°C} = \mathbf{533 \text{ W}}$$

(*b*) The radiation heat loss from the pipe is

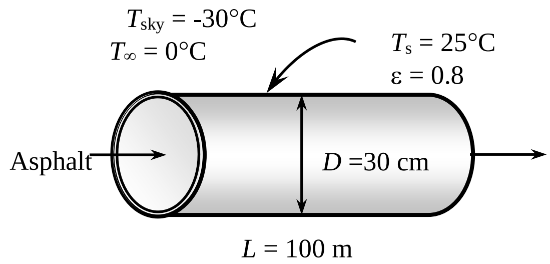
$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)\\ &= (0.8)(1.885 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(73+273 \text{ K})^4 - (27+273 \text{ K})^4\right]\\ &= \mathbf{533 \text{ W}}\end{aligned}$$

9-60 A thick fluid flows through a pipe in calm ambient air. The pipe is heated electrically. The power rating of the electric resistance heater and the cost of electricity during a 15-h period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (25+0)/2 = 12.5°C$ are (Table A-15)

$$k = 0.02458 \text{ W/m.°C}$$
$$\nu = 1.448\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7330$$
$$\beta = \frac{1}{T_f} = \frac{1}{(12.5+273)\text{K}} = 0.003503\text{ K}^{-1}$$



Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.3\text{ m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003503\text{ K}^{-1})(25-0\text{ K})(0.3\text{ m})^3}{(1.448\times10^{-5}\text{ m}^2\text{/s})^2}(0.7330) = 8.106\times10^7$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(8.106\times10^7)^{1/6}}{\left[1+(0.559/0.7330)^{9/16}\right]^{8/27}}\right\}^2 = 53.29$$

$$h = \frac{k}{L_c}Nu = \frac{0.02458\text{ W/m.°C}}{0.3\text{ m}}(53.29) = 4.366\text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.3\text{ m})(100\text{ m}) = 94.25\text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (4.366\text{ W/m}^2\text{.°C})(94.25\text{ m}^2)(25-0)\text{°C} = 10{,}287\text{ W}$$

The radiation heat loss from the cylinder is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$= (0.8)(94.25\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(25+273\text{ K})^4 - (-30+273\text{ K})^4] = 18{,}808\text{ W}$$

Then,

$$\dot{Q}_{total} = \dot{Q}_{\substack{natural\\convection}} + \dot{Q}_{radiation} = 10{,}287 + 18{,}808 = 29{,}094\text{ W} = \mathbf{29.1 kW}$$

The total amount and cost of heat loss during a 15 hour period is

$$Q = \dot{Q}\Delta t = (29.1\text{ kW})(15\text{ h}) = 436.5\text{ kWh}$$

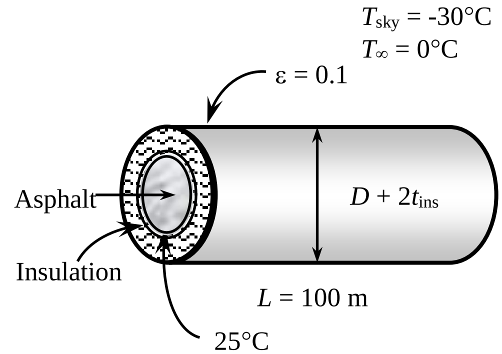
$$\text{Cost} = (436.5\text{ kWh})(\$0.09/\text{kWh}) = \mathbf{\$39.3}$$

9-61 A fluid flows through a pipe in calm ambient air. The pipe is heated electrically. The thickness of the insulation needed to reduce the losses by 85% and the money saved during 15-h are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties Insulation will drop the outer surface temperature to a value close to the ambient temperature, and possible below it because of the very low sky temperature for radiation heat loss. For convenience, we use the properties of air at 1 atm and 5°C (the anticipated film temperature) (Table A-15),

$$k = 0.02401\text{ W/m.°C}$$
$$\nu = 1.382\times10^{-5}\text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7350$$
$$\beta = \frac{1}{T_f} = \frac{1}{(5+273)\text{K}} = 0.003597\text{ K}^{-1}$$



Analysis The rate of heat loss in the previous problem was obtained to be 29,094 W. Noting that insulation will cut down the heat losses by 85%, the rate of heat loss will be

$$\dot{Q} = (1-0.85)\dot{Q}_{\text{no insulation}} = 0.15\times29{,}094\text{ W} = 4364\text{ W}$$

The amount of energy and money insulation will save during a 15-h period is simply determined from

$$Q_{saved,total} = \dot{Q}_{saved}\Delta t = (0.85\times29.094\text{ kW})(15\text{ h}) = 370.9\text{ kWh}$$

$$\text{Money saved} = \text{(Energy saved)(Unit cost of energy)} = (370.9\text{ kWh})(\$0.09/\text{kWh}) = \mathbf{\$33.4}$$

The characteristic length in this case is the outer diameter of the insulated pipe, $L_c = D + 2t_{insul} = 0.3 + 2t_{insul}$ where t_{insul} is the thickness of insulation in m. Then the problem can be formulated for T_s and t_{insul} as follows:

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003597\text{ K}^{-1})[(T_s-273)\text{K}](0.3+2t_{insul})^3}{(1.382\times10^{-5}\text{ m}^2\text{/s})^2}(0.7350)$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/0.7350)^{9/16}\right]^{8/27}}\right\}^2$$

$$h = \frac{k}{L_c}Nu = \frac{0.02401\text{ W/m.°C}}{L_c}Nu$$

$$A_s = \pi D_0 L = \pi(0.3+2t_{insul})(100\text{ m})$$

The total rate of heat loss from the outer surface of the insulated pipe by convection and radiation becomes

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$
$$4364 = hA_s(T_s - 273) + (0.1)A_s(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[T_s^4 - (-30+273\text{ K})^4]$$

In steady operation, the heat lost by the side surfaces of the pipe must be equal to the heat lost from the exposed surface of the insulation by convection and radiation, which must be equal to the heat conducted through the insulation. Therefore,

$$\dot{Q} = \dot{Q}_{insulation} = \frac{2\pi kL(T_{\text{tank}} - T_s)}{\ln(D_o/D)} \rightarrow 4364\text{ W} = \frac{2\pi(0.035\text{ W/m.°C})(100\text{ m})(298-T_s)\text{K}}{\ln[(0.3+2t_{insul})/0.3]}$$

The solution of all of the equations above simultaneously using an equation solver gives

T_s = 281.5 K = 8.5°C and t_{insul} = **0.013 m = 1.3 cm**.

Note that the film temperature is (8.5+0)/2 = 4.25°C which is very close to the assumed value of 5°C. Therefore, there is no need to repeat the calculations using properties at this new film temperature.

9-62 A boiler supplies hot water to an equipment through a pipe at 10 g/s. The pipe dimensions are given. The water enters the equipment at 98°C. The pipe section that is exposed to natural convection is 30 m long. The water temperature exiting the boiler is to be determined whether it meets the ASME Boiler and Pressure Vessel Code.

Assumptions **1** Steady state conditions. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm. **4** Surface temperature of the pipe is constant.

Properties The properties of air at the film temperature of $T_f = (T_\infty + T_s)/2 = (20 + 80)/2 = 50°\text{C}$ are (Table A–15) Pr = 0.7228, $k = 0.02735$ W/m·K, $\nu = 1.798 \times 10^{-5}$ m^2/s, $\beta = 1/T_f = 1/(50 + 273 \text{ K}) = 0.003096 \text{ K}^{-1}$. For water $c_p = 4.20$ kJ/kg·K

Analysis The characteristic length of the horizontal pipe is $D = 0.02$ m, and the Rayleigh number is

$$\text{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003096 \text{ K}^{-1})(80 - 20 \text{ K})(0.02 \text{ m})^3}{(1.798 \times 10^{-5}\text{m}^2/\text{s})^2}(0.7228) = 32595$$

The Nusselt number for natural convection in this case is determined from

$$\text{Nu} = \frac{hD}{k} = \left\{0.6 + \frac{0.387\,\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$$

So, the natural convection heat transfer coefficient is

$$h = \frac{k}{D}\left\{0.6 + \frac{0.387\,\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \frac{0.02735 \text{ W/m}\cdot\text{K}}{0.02 \text{ m}}\left\{0.6 + \frac{0.387\,(32595)^{1/6}}{[1 + (0.559/0.7228)^{9/16}]^{8/27}}\right\}^2$$

$$h = 7.998 \text{ W/m}^2\cdot\text{K}$$

The rate of heat loss from the hot water flowing in the pipe is equal to the heat transfer rate by natural convection and thermal radiation between the pipe surface at the ambient air:

$$\dot{Q} = \dot{m}c_p(T_{\text{in}} - T_{\text{out}}) = hA_s(T_s - T_\infty) + \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4)$$

where T_{in} is the water temperature exiting the boiler (entering the pipe), and T_{out} is the water temperature entering the equipment (exiting the pipe). Solving for T_{in} yields,

$$T_{\text{in}} = \frac{\pi DL}{\dot{m}c_p}[h(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_{\text{surr}}^4)] + T_{\text{out}}$$

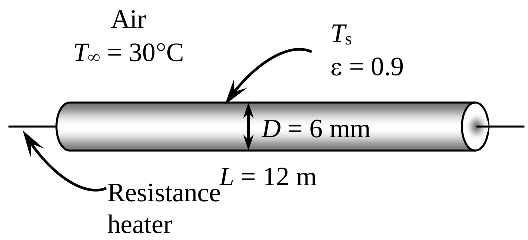
$$T_{\text{in}} = \frac{\pi(0.02 \text{ m})(30 \text{ m})}{(0.01 \text{ kg/s})(4200\ \text{J/kg}\cdot\text{K})}\left[\left(7.998\frac{\text{W}}{\text{m}^2}\cdot\text{K}\right)(80 - 20 \text{ K}) + (0.6)(5.67 \times 10^{-8}\text{W/m}^2\cdot\text{K}^4)(353^4 - 293^4\text{K}^4)\right] = \mathbf{132}℃ > 120℃$$

Discussion The hot water at the boiler outlet is at 132°C, which is 12°C higher than the service restriction of the ASME Boiler and Pressure Vessel Code. Thus, this operation is not in compliance with the ASME Boiler and Pressure Vessel Code (ASME BPVC.IV-2015, HG-101).

9-63 An insulated electric wire is exposed to calm air. The temperature at the interface of the wire and the plastic insulation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (50+30)/2 = 40°C$ based on the problem statement are (Table A-15)



$$k = 0.02662 \text{ W/m.°C}$$
$$\nu = 1.702\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7255$$
$$\beta = \frac{1}{T_f} = \frac{1}{(40+273)\text{K}} = 0.003195 \text{ K}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 50°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the insulated wire $L_c = D = 0.006$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003195 \text{ K}^{-1})(50-30 \text{ K})(0.006 \text{ m})^3}{(1.702\times10^{-5} \text{ m}^2\text{/s})^2}(0.7255) = 339.3$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(339.3)^{1/6}}{\left[1+\left(0.559/0.7255\right)^{9/16}\right]^{8/27}}\right\}^2 = 2.101$$

$$h = \frac{k}{D}Nu = \frac{0.02662 \text{ W/m.°C}}{0.006 \text{ m}}(2.101) = 9.327 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.006 \text{ m})(12 \text{ m}) = 0.2262 \text{ m}^2$$

The rate of heat generation, and thus the rate of heat transfer is

$$\dot{Q} = \mathbf{V}I = (7 \text{ V})(10 \text{ A}) = 70 \text{ W}$$

Considering both natural convection and radiation, the total rate of heat loss can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$
$$70 \text{ W} = (9.327 \text{ W/m}^2\text{.°C})(0.226 \text{ m}^2)(T_s - 30)\text{°C}$$
$$+(0.9)(0.2262 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(T_s+273)^4 - (30+273 \text{ K})^4]$$

Its solution is

$$T_s = 49.9\text{°C}$$

which is very close to the assumed value of 50°C. Then the temperature at the interface of the wire and the plastic cover in steady operation becomes

$$\dot{Q} = \frac{2\pi kL}{\ln(D_2/D_1)}(T_i - T_s) \longrightarrow T_i = T_s + \frac{\dot{Q}\ln(D_2/D_1)}{2\pi kL} = 49.9\text{°C} + \frac{(70 \text{ W})\ln(6/3)}{2\pi(0.20 \text{ W/m.°C})(12 \text{ m})} = \mathbf{53.1°C}$$

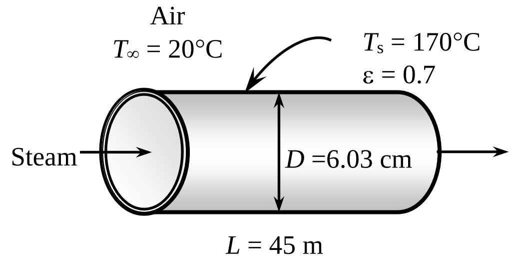
Discussion The assumed film temperature of $T_f = 40°C$ is an appropriate assumption, since the determined $T_s = 49.9°C$ would give a film temperature of $T_f = 39.95°C$. Otherwise, T_s would have to be solved iteratively.

9-64 A steam pipe extended from one end of a plant to the other with no insulation on it. The rate of heat loss from the steam pipe and the annual cost of those heat losses are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (170+20)/2 = 95°C$ are (Table A-15)

$$k = 0.0306 \text{ W/m.°C}$$
$$\nu = 2.254\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7122$$
$$\beta = \frac{1}{T_f} = \frac{1}{(95+273)\text{K}} = 0.002717 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.0603 \text{ m}$. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.002717 \text{ K}^{-1})(170-20 \text{ K})(0.0603 \text{ m})^3}{(2.254\times10^{-5} \text{ m}^2/\text{s})^2}(0.7122) = 1.229\times10^6$$

$$Nu = \left\{0.6 + \frac{0.387 Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.229\times10^6)^{1/6}}{\left[1+(0.559/0.7122)^{9/16}\right]^{8/27}}\right\}^2 = 15.41$$

$$h = \frac{k}{D}Nu = \frac{0.0306 \text{ W/m.°C}}{0.0603 \text{ m}}(15.41) = 7.821 \text{ W/m}^2.°\text{C}$$

$$A_s = \pi DL = \pi(0.0603 \text{ m})(45 \text{ m}) = 8.525 \text{ m}^2$$

Then the total rate of heat transfer by natural convection and radiation becomes

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^4 - T_{surr}^4)$$
$$= (8.525 \text{ W/m}^2.°\text{C})(11.37 \text{ m}^2)(170-20)°\text{C}$$
$$+ (0.7)(8.525 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(170+273 \text{ K})^4 - (20+273 \text{ K})^4]$$
$$= 20{,}539 \text{ W} = \mathbf{20.5 \text{ kW}}$$

The total amount of gas consumption and its cost during a one-year period is

$$Q_{gas} = \frac{\dot{Q}\Delta t}{\eta} = \frac{20.539 \text{ kJ/s}}{0.84}\left(\frac{1 \text{ therm}}{105{,}500 \text{ kJ}}\right)(8760 \text{ h/yr}\times3600 \text{ s/h}) = 7308 \text{ therms/yr}$$

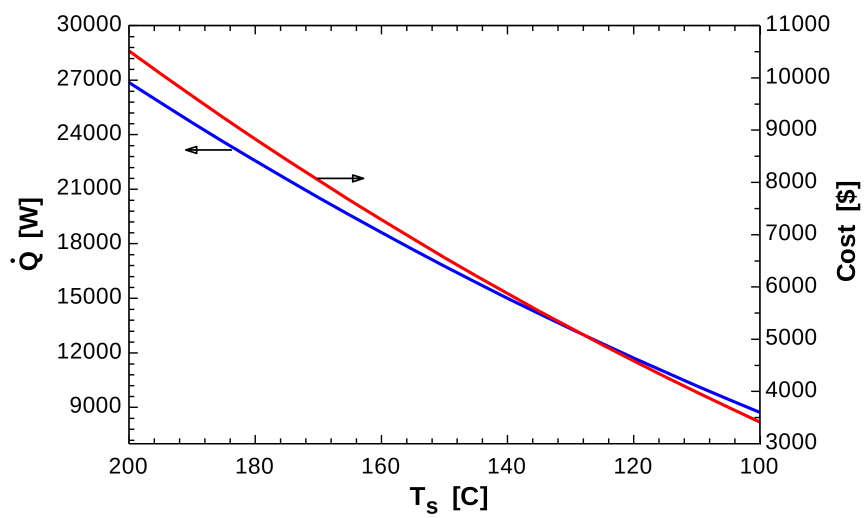
$$\text{Cost} = (7308 \text{ therms/yr})(\$1.10/\text{therm}) = \mathbf{\$8039/yr}$$

9-65 Prob. 9-64 is reconsidered. The effect of the surface temperature of the steam pipe on the rate of heat loss from the pipe and the annual cost of this heat loss is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

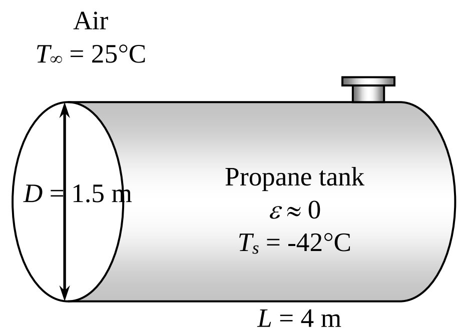
```
"GIVEN"
L=45 [m]
D=0.0603 [m]
T_s=170 [C]
T_infinity=20 [C]
epsilon=0.7
T_surr=T_infinity
eta_furnace=0.84
UnitCost=1.10 [$/therm]
time=24*365 [h]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
beta=1/(T_film+273)
T_film=1/2*(T_s+T_infinity)
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"
g=9.807 [m/s^2] "gravitational acceleration"

"ANALYSIS"
delta=D
Ra=(g*beta*(T_s-T_infinity)*delta^3)/nu^2*Pr
Nusselt=(0.6+(0.387*Ra^(1/6))/(1+(0.559/Pr)^(9/16))^(8/27))^2
h=k/delta*Nusselt
A=pi*D*L
Q_dot=h*A*(T_s-T_infinity)+epsilon*A*sigma*((T_s+273)^4-(T_surr+273)^4)
Q_gas=(Q_dot*time)/eta_furnace*Convert(h, s)*Convert(J, kJ)*Convert(kJ, therm)
Cost=Q_gas*UnitCost
```

T_s[C]	$\dot{Q}$ [W]	Cost [$]
100	8726	3416
105	9445	3697
110	10183	3986
115	10939	4282
120	11714	4585
125	12507	4896
130	13320	5214
135	14153	5540
140	15005	5873
145	15876	6214
150	16768	6563
155	17680	6920
160	18613	7286
165	19567	7659
170	20542	8041
175	21539	8431
180	22558	8830
185	23600	9237
190	24664	9654
195	25751	10080
200	26862	10514



9-66 A cylindrical propane tank is exposed to calm ambient air. The propane is slowly vaporized due to a crack developed at the top of the tank. The time it will take for the tank to empty is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation heat transfer is negligible.



Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (-42+25)/2 = -8.5°C are (Table A-15)

$$k = 0.02299 \text{ W/m.°C}$$

$$\nu = 1.265\times10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7383$$

$$\beta = \frac{1}{T_f} = \frac{1}{(-8.5+273)\text{K}} = 0.003781\,\text{K}^{-1}$$

Analysis The tank gains heat through its cylindrical surface as well as its circular end surfaces. For convenience, we take the heat transfer coefficient at the end surfaces of the tank to be the same as that of its side surface. (The alternative is to treat the end surfaces as a vertical plate, but this will double the amount of calculations without providing much improvement in accuracy since the area of the end surfaces is much smaller and it is circular in shape rather than being rectangular). The characteristic length in this case is the outer diameter of the tank, $L_c = D = 1.5\,\text{m}$. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)D^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.003781\,\text{K}^{-1})[(25-(-42)\,\text{K}](1.5\,\text{m})^3}{(1.265\times10^{-5}\text{ m}^2\text{/s})^2}(0.7383) = 3.869\times10^{10}$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(3.869\times10^{10})^{1/6}}{\left[1+\left(0.559/0.7383\right)^{9/16}\right]^{8/27}}\right\}^2 = 374.1$$

$$h = \frac{k}{D}Nu = \frac{0.02299\text{ W/m.°C}}{1.5\,\text{m}}(374.1) = 5.733\text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL + 2\pi D^2/4 = \pi(1.5\,\text{m})(4\,\text{m}) + 2\pi(1.5\,\text{m})^2/4 = 22.38\,\text{m}^2$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (5.733\text{ W/m}^2\text{.°C})(22.38\,\text{m}^2)[(25-(-42)]\text{°C} = 8598\,\text{W}$$

The total mass and the rate of evaporation of propane are

$$m = \rho \boldsymbol{V} = \rho\frac{\pi D^2}{4}L = (581\,\text{kg/m}^3)\frac{\pi(1.5\,\text{m})^2}{4}(4\,\text{m}) = 4107\text{ kg}$$

$$\dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{8.598\,\text{kJ/s}}{425\,\text{kJ/kg}} = 0.02023\,\text{kg/s}$$

and it will take

$$\Delta t = \frac{m}{\dot{m}} = \frac{4107\text{ kg}}{0.02023\,\text{kg/s}} = 202{,}996\text{ s} = \mathbf{56.4\,hours}$$

for the propane tank to empty.

9-67 Hot water flows in a horizontal pipe with a known inner surface temperature. The pipe outer surface is exposed to cool air. The outer surface temperature of the pipe is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** The pipe thermal conductivity is constant. **4** Radiation heat transfer is negligible. **5** Local atmospheric pressure is 1 atm. **6** The film temperature is 40°C (this assumption will be verified).

Properties The properties of air at the assumed T_f = 40°C are k = 0.02662 W/m·K, $\nu = 1.702 \times 10^{-5}$ m^2/s, Pr = 0.7255 (Table A-15), and $\beta = 1/T_f$ = 1/ 313 K.

Analysis With the assumption that T_f = 40°C, the pipe outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 68°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(40+273\,\text{K})^{-1}(68-12)\,\text{K}\,(0.045\,\text{m})^3}{(1.702\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7255) = 400{,}561$$

The Nusselt number relation for horizontal cylinder is

$$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\dfrac{0.559}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(400{,}561)^{1/6}}{\left[1+\left(\dfrac{0.559}{0.7255}\right)^{9/16}\right]^{8/27}}\right\}^2 = 11.31$$

$$h = \frac{k}{D_o}\text{Nu} = \frac{0.02662\ \text{W/m}\cdot\text{K}}{0.045\,\text{m}}(11.31) = 6.6905\ \text{W/m}^2\cdot\text{K}$$

Thus, the rate of heat transfer balance for conduction through the pipe wall and natural convection on the outer surface is

$$\dot{Q}_{\text{cyl}} = \dot{Q}_{\text{conv}}$$

$$\frac{2\pi L k_{\text{cyl}}}{\ln(D_o/D_i)}(T_{s,i} - T_{s,o}) = hA_s(T_{s,o} - T_\infty) \quad \rightarrow \quad T_{s,o} = \mathbf{67.3°C}$$

where $\quad A_s = \pi D_o L$

Discussion The assumed film temperature of T_f = 40°C is an appropriate assumption, since the determined $T_{s,o}$ = 67.3°C would give a film temperature of T_f = 39.7°C. Otherwise, $T_{s,o}$ would have to be solved iteratively.

9-68 A horizontal CPVC pipe with water flowing inside, while the outer surface is exposed to hot quiescence air at 107°C. The pipe dimensions are given. The convection heat transfer coefficient for the internal flow is given. The temperature of the water is to be determined such that the CPVC pipe temperature does not go above 93°C, which is the recommended maximum temperature by the ASME Code for Process Piping.

Assumptions **1** Steady state conditions. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm. **4** Surface temperature of the pipe is constant. **5** Conduction through the pipe wall is one-dimensional. **6** Thermal radiation is negligible.

Properties The properties of air at the film temperature of $T_f = (T_\infty + T_s)/2 = (107 + 93)/2 = 100°C$ are (Table A–15) Pr = 0.7111, $k = 0.03095$ W/m·K, $\nu = 2.306 \times 10^{-5}$ m²/s, $\beta = 1/T_f = 1/(100 + 273 \text{ K}) = 0.002681 \text{ K}^{-1}$

Analysis The characteristic length of the horizontal pipe is $D = 0.02$ m, and the Rayleigh number is

$$\text{Ra}_D = \frac{g\beta(T_\infty - T_s)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.002681 \text{ K}^{-1})(107 - 93 \text{ K})(0.02 \text{ m})^3}{(2.306 \times 10^{-5}\text{m}^2/\text{s})^2}(0.7111) = 3939$$

The Nusselt number for natural convection in this case is determined from

$$\text{Nu} = \frac{hD}{k} = \left\{0.6 + \frac{0.387\ \text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$$

So, the natural convection heat transfer coefficient is

$$h = \frac{k}{D}\left\{0.6 + \frac{0.387\ \text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \frac{0.03095 \text{ W/m}\cdot\text{K}}{0.02 \text{ m}}\left\{0.6 + \frac{0.387\ (3939)^{1/6}}{[1 + (0.559/0.7111)^{9/16}]^{8/27}}\right\}^2$$

$$h = 5.452 \text{ W/m}^2\cdot\text{K}$$

The heat transfer rate from the hot air to the CPVC pipe outer surface is

$$\dot{Q} = hA_{s,o}(T_\infty - T_s) = h(\pi D_o L)(T_\infty - T_s)$$

Also, the heat transfer rate from the hot air to the water flowing inside the pipe is

$$\dot{Q} = \frac{T_\infty - T_{\text{water}}}{\dfrac{1}{hA_{s,o}} + \dfrac{\ln(D_o/D_i)}{2\pi Lk} + \dfrac{1}{h_{\text{water}}A_{s,i}}} = \frac{T_\infty - T_{\text{water}}}{\dfrac{1}{h(\pi D_o L)} + \dfrac{\ln(D_o/D_i)}{2\pi Lk} + \dfrac{1}{h_{\text{water}}(\pi D_i L)}}$$

Thus,

$$h(\pi D_o L)(T_\infty - T_s) = \frac{T_\infty - T_{\text{water}}}{\dfrac{1}{h(\pi D_o L)} + \dfrac{\ln(D_o/D_i)}{2\pi Lk} + \dfrac{1}{h_{\text{water}}(\pi D_i L)}}$$

Solving for the water temperature, yields

$$T_{\text{water}} = T_\infty - hD_o(T_\infty - T_s)\left[\frac{1}{hD_o} + \frac{\ln(D_o/D_i)}{2k} + \frac{1}{h_{\text{water}}D_i}\right]$$

$$T_{\text{water}} = 107°\text{C} - (5.452 \text{ W/m}^2\cdot\text{K})(0.02 \text{ m})(107 - 93 \text{ K})\left[\frac{1}{(5.452 \text{ W/m}^2\cdot\text{K})(0.02 \text{ m})} + \frac{\ln(20/15)}{2(0.136 \text{ W/m}\cdot\text{K})} + \frac{1}{(50 \text{ W/m}^2\cdot\text{K})(0.015 \text{ m})}\right] = \mathbf{89.35}°\text{C}$$

Discussion To keep the CPVC pipe from heating above the recommended maximum temperature of 93°C by the hot air, the water flowing inside the pipe needs to be at about 89°C or lower. The lower temperature of the water keeps the pipe cool. The highest temperature in the pipe is at the outer surface, since it is exposed to the hot air.

9-69 Hot engine flows in a horizontal pipe with a known inner surface temperature. The pipe outer surface is covered with a layer of insulation. The outer surface temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** Thermal conductivities of pipe and insulation are constant. **4** Contact resistance is negligible. **5** Radiation heat transfer is negligible. **6** Local atmospheric pressure is 1 atm.

Properties We first assume the film temperature is T_f = 50°C. Then, the properties of air at T_f = 50°C are k = 0.02735 W/m·K, $\nu = 1.798 \times 10^{-5}$ m²/s, Pr = 0.7228 (Table A-15), and $\beta = 1/T_f$ = 1/323 K .

The thermal conductivities of the pipe and the insulation are k_{pipe} = 15 W.m·K and k_{ins} = 0.15 W.m·K, respectively.

Analysis With the assumption that T_f = 50°C, the outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 90°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(50+273\,\text{K})^{-1}(90-10)\,\text{K}\,(0.07\,\text{m})^3}{(1.798\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7228) = 1.863\times10^6$$

The Nusselt number relation for horizontal cylinder is

$$\text{Nu} = \left\{0.6 + \frac{0.387\,\text{Ra}^{1/6}}{\left[1+\left(\frac{0.559}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.8633\times10^6)^{1/6}}{\left[1+\left(\frac{0.559}{0.7228}\right)^{9/16}\right]^{8/27}}\right\}^2 = 17.38$$

$$h = \frac{k}{D_o}\text{Nu} = \frac{0.02735\,\text{W/m}\cdot\text{K}}{0.07\,\text{m}}(17.38) = 6.791\,\text{W/m}^2\cdot\text{K}$$

From Chapter 3, the total thermal resistance for the pipe wall and the insulation is

$$R_{\text{total}} = \frac{\ln(D_{\text{interface}}/D_i)}{2\pi k_{\text{pipe}}L} + \frac{\ln(D_o/D_{\text{interface}})}{2\pi k_{\text{ins}}L}$$

$$= \frac{\ln(0.06/0.05)}{2\pi(15\,\text{W/m}\cdot\text{K})(2\,\text{m})} + \frac{\ln(0.07/0.06)}{2\pi(0.15\,\text{W/m}\cdot\text{K})(2\,\text{m})} = 0.08275\,\text{K/W}$$

Thus, the rate of heat transfer balance for conduction through the cylindrical layers and natural convection on the outer surface is

$$\dot{Q}_{\text{cyl}} = \dot{Q}_{\text{conv}}$$

$$\frac{(T_{s,i} - T_{s,o})}{R_{\text{total}}} = h(\pi D_o L)(T_{s,o} - T_\infty) \quad \rightarrow \quad T_{s,o} = 74.15°\text{C} \quad \text{(first iteration)}$$

The above solution is repeated iteratively until $T_{s,o}$ converges to $T_{s,o}$ = **74.8°C**.

Discussion The results from the iterations are as follows:

Iter	$T_{s,o}$ [°C]	Ra	Nu	h [W/m²·K]
1	90	1.863×10^6	17.38	6.791
2	74.15	1.672×10^6	16.86	6.447
3	74.80	1.681×10^6	16.88	6.462
4	**74.77**	1.681×10^6	16.88	6.462

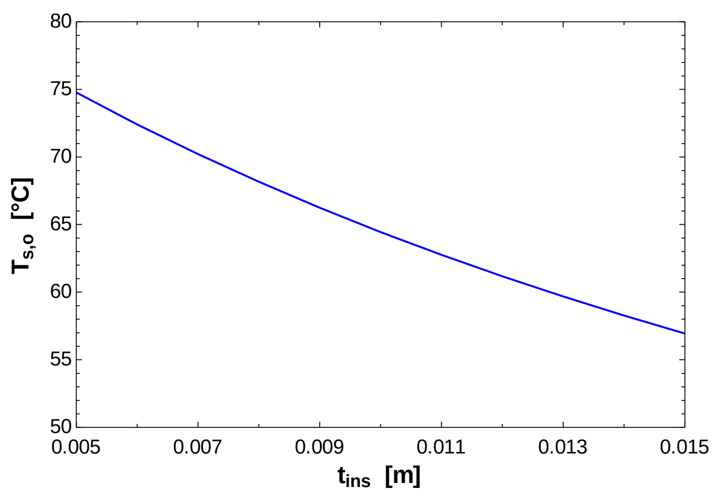
As $T_{s,o}$ changes through the iterations, so does the film temperature used for evaluating the properties.

9-70 Reconsider Prob. 9-69. Hot engine flows in a horizontal pipe with a known inner surface temperature. The pipe outer surface is covered with a layer of insulation. The effect of the insulation layer thickness on the outer surface temperature is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=2 [m]     "Length exposed to natural convection"
D_i=0.05 [m]     "Inner pipe diameter"
t_pipe=5e-3 [m]     "Pipe wall thickness"
T_infinity=10 [C]     "Cool air temperature"
T_s_i=90 [C]     "Inner surface temperature"

"PROPERTIES"
g=9.81 [m/s^2]
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
beta=Volexpcoef(Fluid$, T=T_film)
T_film=1/2*(T_s_o+T_infinity)
"Pipe and insulation"
k_ins=0.15 [W/m-K]
k_pipe=15 [W/m-K]

"ANALYSIS"
"Natural convection"
Gr=(g*beta*(T_s_o-T_infinity)*D_o^3)/nu^2
Ra=(g*beta*(T_s_o-T_infinity)*D_o^3)/nu^2*Pr
Nusselt=(0.6+0.387*Ra^(1/6)/((1+(0.559/Pr)^(9/16))^(8/27)))^2
h=k/D_o*Nusselt
"Heat conduction through cylindrical layers"
D_interface=D_i+t_pipe*2
D_o=D_interface+t_ins*2
R_pipe=ln(D_interface/D_i)/(2*pi#*L*k_pipe)
R_ins=ln(D_o/D_interface)/(2*pi#*L*k_ins)
R_total=R_pipe+R_ins
"Heat balance"
Q_dot=(T_s_i-T_s_o)/R_total     "Heat conduction through cylindrical layers"
Q_dot=h*A_s*(T_s_o-T_infinity)     "Heat loss by natural convection"
A_s=pi#*D_o*L     "Surface area exposed to natural convection"
```

t_{ins} [m]	$T_{s,o}$ [°C]
0.005	74.77
0.006	72.41
0.007	70.21
0.008	68.16
0.009	66.24
0.010	64.45
0.011	62.76
0.012	61.18
0.013	59.68
0.014	58.27
0.015	56.94



Discussion As the insulation layer thickness increases, the heat loss through the pipe is reduced. Therefore the outer surface temperature decreases with increasing thickness of insulation.

9-71 A hot fluid flowing as a fully-developed laminar flow inside a horizontal pipe with constant surface temperature. The pipe outer surface temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** Radiation heat transfer is negligible. **4** Local atmospheric pressure is 1 atm. **5** The film temperature is 50°C (this assumption will be verified).

Properties We first assume the film temperature is T_f = 50°C. Then, the properties of air at T_f = 50°C and 1 atm are k = 0.02735 W/m·K, $\nu = 1.798 \times 10^{-5}$ m^2/s, Pr = 0.7228 (Table A-15), and $\beta = 1/T_f$ = 1/323 K .

Analysis With the assumption that T_f = 50°C, the outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 90°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(50 + 273\,\text{K})^{-1}(90 - 10)\,\text{K}\,(0.045\,\text{m})^3}{(1.798 \times 10^{-5}\ \text{m}^2/\text{s})^2}(0.7228) = 495{,}031$$

At the outer surface, the natural convection Nusselt number relation for horizontal cylinder is

$$\text{Nu} = \left\{0.6 + \frac{0.387\,\text{Ra}^{1/6}}{\left[1 + \left(\dfrac{0.559}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(495{,}031)^{1/6}}{\left[1 + \left(\dfrac{0.559}{0.7228}\right)^{9/16}\right]^{8/27}}\right\}^2 = 11.98$$

$$h_o = \frac{k}{D_o}\text{Nu} = \frac{0.02735\ \text{W/m}\cdot\text{K}}{0.045\,\text{m}}(11.98) = 7.2812\ \text{W/m}^2\cdot\text{K}$$

At the inner surface, the forced convection heat transfer coefficient for laminar fully-developed flow with constant surface temperature is (see Chapter 8)

$$h_i = 3.66\frac{k_{\text{fluid}}}{D_i} = 3.66\frac{0.72\ \text{W/m}\cdot\text{K}}{0.035\,\text{m}} = 75.29\ \text{W/m}^2\cdot\text{K}$$

Thus, the rate of heat transfer balance for internal forced convection at the pipe inner surface and the natural convection at the outer surface is

$$\dot{Q}_{\text{conv},i} = \dot{Q}_{\text{conv},o}$$

$$h_i(\pi D_i L)\Delta T_{\text{avg}} = h_o(\pi D_o L)(T_{s,o} - T_\infty) \quad \to \quad T_{s,o} = \frac{h_i D_i}{h_o D_o}\Delta T_{\text{avg}} + T_\infty$$

$$T_{s,o} = \frac{(75.29\ \text{W/m}^2\cdot\text{K})(0.035\,\text{m})}{(7.2812\ \text{W/m}^2\cdot\text{K})(0.045\,\text{m})}(10°\text{C}) + 10°\text{C} = \mathbf{90.4°C}$$

Discussion The assumed film temperature T_f = 50°C is an appropriate assumption, since the determined $T_{s,o}$ = 90.4°C would give a film temperature of T_f = 50.2°C. Otherwise, $T_{s,o}$ would have to be solved iteratively.

9-72 A hot liquid flowing inside a horizontal pipe with a known mass flow rate and temperature difference of the pipe inlet and outlet. The pipe outer surface temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** Local atmospheric pressure is 1 atm. **4** The film temperature is 35°C (this assumption will be verified). **5** The T_{surr} is the same as the air temperature.

Properties We first assume the film temperature is T_f = 35°C. Then, the properties of air at T_f = 35°C are k = 0.02625 W/m·K, $\nu = 1.655 \times 10^{-5}$ m^2/s, Pr = 0.7268 (Table A-15), and $\beta = 1/T_f = 1/308$ K.

The emissivity of the black oxidized copper pipe outer surface is ε = 0.78 (Table A-18)

Analysis With the assumption that T_f = 35°C, the outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 60°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(35+273\,\text{K})^{-1}(60-10)\,\text{K}\,(0.055\,\text{m})^3}{(1.655\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7268) = 703{,}065$$

At the outer surface, the natural convection Nusselt number relation for horizontal cylinder is

$$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.559}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(703{,}065)^{1/6}}{\left[1+\left(\frac{0.559}{0.7268}\right)^{9/16}\right]^{8/27}}\right\}^2 = 13.21$$

$$h = \frac{k}{D_o}\text{Nu} = \frac{0.02625\,\text{W/m}\cdot\text{K}}{0.055\,\text{m}}(13.21) = 6.3048\,\text{W/m}^2\cdot\text{K}$$

Thus, the outer surface temperature can be solved using

$$\dot{Q} = h(\pi D_o L)(T_{s,o} - T_\infty) + \varepsilon\sigma(\pi D_o L)(T_{s,o}^4 - T_{\text{surr}}^4)$$

$$\dot{m}c_p\Delta T = h(\pi D_o L)(T_{s,o} - T_\infty) + \varepsilon\sigma(\pi D_o L)(T_{s,o}^4 - T_{\text{surr}}^4) \quad \rightarrow \quad T_{s,o} = \mathbf{60.3°C}$$

where $\dot{m} = 0.05\,\text{kg/s}$, $c_p = 1000\,\text{J/kg}\cdot\text{K}$, $\Delta T = 10°\text{C}$

Discussion The assumed film temperature T_f = 35°C is an appropriate assumption, since the determined $T_{s,o}$ = 60.3°C would give a film temperature of T_f = 35.2°C. Otherwise, $T_{s,o}$ would have to be solved iteratively.

9-73 A hot liquid flowing inside a horizontal pipe with a known mass flow rate and temperature difference of the pipe inlet and outlet. The pipe outer surface temperature is to be determined whether or not it is safe from thermal burn hazards.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** Local atmospheric pressure is 1 atm. **4** The film temperature is 35°C (this assumption will be verified). **5** The T_{surr} is the same as the air temperature.

Properties The properties of air at T_f = 25°C and 1 atm pressure are k = 0.02551 W/m·K, $\nu = 1.562 \times 10^{-5}$ m²/s, Pr = 0.7296 (Table A-15), and $\beta = 1/T_f$ = 1/298 K . The emissivity of the black painted surface is given as ε = 0.88.

Analysis With the assumption that T_f = 25°C, the outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 40°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(25+273\,\text{K})^{-1}(40-10)\,\text{K}\,(0.055\,\text{m})^3}{(1.562\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7296) = 491{,}343$$

At the outer surface, the natural convection Nusselt number relation for horizontal cylinder is

$$\text{Nu} = \left\{0.6 + \frac{0.387\,\text{Ra}^{1/6}}{\left[1+\left(\frac{0.559}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(491{,}343)^{1/6}}{\left[1+\left(\frac{0.559}{0.7296}\right)^{9/16}\right]^{8/27}}\right\}^2 = 11.97$$

$$h = \frac{k}{D_o}\text{Nu} = \frac{0.02551\,\text{W/m}\cdot\text{K}}{0.055\,\text{m}}(11.97) = 5.5519\,\text{W/m}^2\cdot\text{K}$$

Thus, the outer surface temperature can be solved using

$$\dot{Q} = h(\pi D_o L)(T_{s,o} - T_\infty) + \varepsilon\sigma(\pi D_o L)(T_{s,o}^4 - T_{\text{surr}}^4)$$

$$\dot{m}c_p\Delta T = h(\pi D_o L)(T_{s,o} - T_\infty) + \varepsilon\sigma(\pi D_o L)(T_{s,o}^4 - T_{\text{surr}}^4) \quad \rightarrow \quad T_{s,o} = \mathbf{39.9°C}$$

where

$$\dot{m} = 0.028\,\text{kg/s}, \quad c_p = 1000\,\text{J/kg}\cdot\text{K}, \quad \Delta T = 10°\text{C}$$

Discussion The black painted surface is sufficient to keep the outer surface temperature below 45°C to prevent thermal burn hazards.

The assumed film temperature T_f = 25°C is appropriate for evaluating the properties of air, since the determined $T_{s,o}$ = 39.9°C would give a film temperature of T_f = 24.95°C.

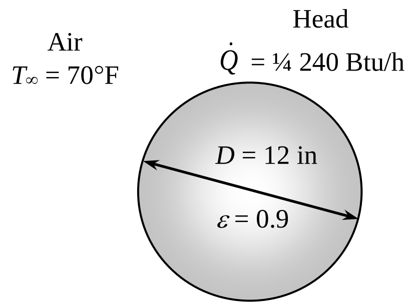
The emissivity of black paint is also listed in Table A-18 as ε = 0.88.

9-74E The average surface temperature of a human head is to be determined when it is not covered.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The head can be approximated as a 12-in.-diameter sphere.

Properties The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 90°F for the evaluation of the properties and *h*. We will check the accuracy of this guess later and repeat the calculations if necessary. The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (90+70)/2 = 80$°F are (Table A-15E)

$$k = 0.01481 \text{ Btu/h.ft.°F}$$
$$\nu = 1.697\times10^{-4} \text{ ft}^2/\text{s}$$
$$\text{Pr} = 0.7290$$
$$\beta = \frac{1}{T_f} = \frac{1}{(80+460)\text{R}} = 0.001852 \text{ R}^{-1}$$



Analysis The characteristic length for a spherical object is $L_c = D = 12/12 = 1$ ft. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001852 \text{ R}^{-1})(90-70 \text{ R})(1 \text{ ft})^3}{(1.697\times10^{-4} \text{ ft}^2/\text{s})^2}(0.7290) = 3.019\times10^7$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+\left(\frac{0.469}{\text{Pr}}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(3.019\times10^7)^{1/4}}{\left[1+\left(\frac{0.469}{0.7290}\right)^{9/16}\right]^{4/9}} = 35.79$$

$$h = \frac{k}{D}Nu = \frac{0.01481 \text{ Btu/h.ft.°F}}{1 \text{ ft}}(35.79) = 0.5300 \text{ Btu/h.ft}^2\text{.°F}$$

$$A_s = \pi D^2 = \pi(1 \text{ ft})^2 = 3.142 \text{ ft}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be written as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$

$$(240/4 \text{ Btu/h}) = (0.5300 \text{ Btu/h.ft}^2\text{.°F})(3.142 \text{ ft}^2)(T_s - 70)\text{°F}$$
$$+ (0.9)(3.142 \text{ ft}^2)(0.1714\times10^{-8} \text{ Btu/h.ft}^2\text{.R}^4)[(T_s + 460 \text{ R})^4 - (70+460 \text{ R})^4]$$

Its solution is

T_s = **82.9°F**

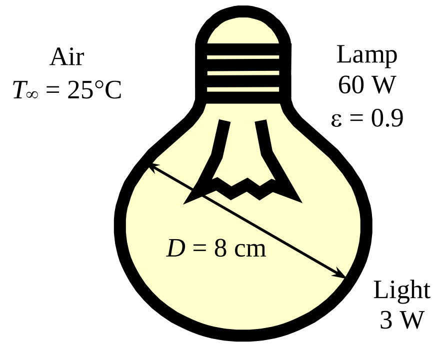
which is sufficiently close to the assumed value in the evaluation of the properties and *h*. Therefore, there is no need to repeat calculations.

9-75 The equilibrium temperature of a light glass bulb in a room is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The light bulb is approximated as an 8-cm-diameter sphere.

Properties The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 170°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (170+25)/2 = 97.5$°C are (Table A-15)

$$k = 0.03077 \text{ W/m.°C}$$
$$\nu = 2.279\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7116$$
$$\beta = \frac{1}{T_f} = \frac{1}{(97.5+273)\text{K}} = 0.002699 \text{ K}^{-1}$$



Analysis The characteristic length in this case is $L_c = D = 0.08$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr}$$
$$= \frac{(9.81 \text{ m/s}^2)(0.002699 \text{ K}^{-1})(170-25 \text{ K})(0.08 \text{ m})^3}{(2.279\times10^{-5} \text{ m}^2\text{/s})^2}(0.7116) = 2.694\times10^6$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+(0.469/\text{Pr})^{9/16}\right]^{4/9}} = 2 + \frac{0.589(2.694\times10^6)^{1/4}}{\left[1+(0.469/0.7116)^{9/16}\right]^{4/9}} = 20.42$$

Then

$$h = \frac{k}{D}Nu = \frac{0.03077 \text{ W/m.°C}}{0.08 \text{ m}}(20.42) = 7.854 \text{ W/m}^2\text{.°C}$$
$$A_s = \pi D^2 = \pi(0.08 \text{ m})^2 = 0.02011 \text{ m}^2$$

Considering both natural convection and radiation, the total rate of heat loss can be written as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$(0.95\times60) \text{ W} = (7.854 \text{ W/m}^2\text{.°C})(0.02011 \text{ m}^2)(T_s - 25)\text{°C}$$
$$+ (0.9)(0.02011 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(T_s+273)^4 - (25+273 \text{ K})^4]$$

Its solution is

T_s = **175°C**

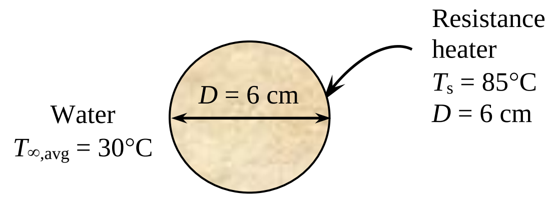
which is sufficiently close to the value assumed in the evaluation of properties and h. Therefore, there is no need to repeat calculations.

9-76 Water in a tank is to be heated by a spherical heater. The heating time is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The temperature of the outer surface of the sphere is constant.

Properties Using the average temperature for water (15+45)/2=30°C as the fluid temperature, the properties of water at the film temperature of $(T_s+T_\infty)/2$ = (85+30)/2 = 57.5°C are (Table A-9)

$$k = 0.6515 \text{ W/m.°C}$$
$$\nu = 0.493\times10^{-6} \text{ m}^2/\text{s}$$
$$\text{Pr} = 3.12$$
$$\beta = 0.501\times10^{-3} \text{ K}^{-1}$$



Also, the properties of water at 30°C are (Table A-15)

$$\rho = 996 \text{ kg/m}^3 \text{ and } c_p = 4178 \text{ J/kg.°C}$$

Analysis The characteristic length in this case is $L_c = D$ = 0.06 m. Then,

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.501\times10^{-3} \text{ K}^{-1})(85-30 \text{ K})(0.06 \text{ m})^3}{(0.493\times10^{-6} \text{ m}^2/\text{s})^2}(3.12) = 7.495\times10^8$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+\left(0.469/\text{Pr}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(7.495\times10^8)^{1/4}}{\left[1+\left(0.469/3.12\right)^{9/16}\right]^{4/9}} = 87.44$$

$$h = \frac{k}{D}Nu = \frac{0.6515 \text{ W/m.°C}}{0.06 \text{ m}}(87.44) = 949.5 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi D^2 = \pi(0.06 \text{ m})^2 = 0.01131 \text{ m}^2$$

The rate of heat transfer by convection is

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) = (949.5 \text{ W/m}^2\text{.°C})(0.01131 \text{m}^2)(85-30) = 590.6 \text{ W}$$

The mass of water in the container is

$$m = \rho \boldsymbol{V} = (996 \text{ kg/m}^3)(0.040 \text{m}^3) = 39.84 \text{ kg}$$

The amount of heat transfer to the water is

$$Q = mc_p(T_2 - T_1) = (39.84 \text{ kg})(4178 \text{ J/kg.°C})(45\text{-}15)\text{°C} = 4.994\times10^6 \text{ J}$$

Then the time the heater should be on becomes

$$\Delta t = \frac{Q}{\dot{Q}} = \frac{4.994\times10^6 \text{ J}}{590.6 \text{ J/s}} = 8456 \text{ s} = \mathbf{2.35\,hours}$$

9-77 Chemical reaction in a spherical tank is releasing heat and the surface temperature is known. The heat rate from the reaction is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperature is constant. **3** Thermal resistance on the tank wall is negligible. **4** Local atmospheric pressure is 1 atm. **5** The T_{surr} is the same as the air temperature.

Properties The properties of air at $T_f = (T_s + T_\infty)/2 = (50 + 20)/2 = 35°C$ are $k = 0.02625$ W/m·K, $\nu = 1.655 \times 10^{-5}$ m^2/s, Pr = 0.7268 (Table A-15), and $\beta = 1/T_f = 1/308$ K.

The emissivity of the tank is given as $\varepsilon = 0.35$.

Analysis The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(35+273\,\text{K})^{-1}(50-20)\,\text{K}\,(2\,\text{m})^3}{(1.655\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7268) = 2.0284\times10^{10}$$

The Nusselt number relation for sphere is

$$\text{Nu} = 2 + \frac{0.589\text{Ra}^{1/4}}{\left[1+\left(\frac{0.469}{\text{Pr}}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(2.0284\times10^{10})^{1/4}}{\left[1+\left(\frac{0.469}{0.7268}\right)^{9/16}\right]^{4/9}} = 173.96$$

$$h = \frac{k}{D}\text{Nu} = \frac{0.02625\,\text{W/m}\cdot\text{K}}{2\,\text{m}}(173.96) = 2.2832\,\text{W/m}^2\cdot\text{K}$$

Thus, the heat rate from the reaction is

$$\dot{Q} = h(\pi D^2)(T_s - T_\infty) + \varepsilon\sigma(\pi D^2)(T_s^4 - T_{\text{surr}}^4)$$

$$\dot{Q} = \pi(2\,\text{m})^2[(2.2832\,\text{W/m}^2\cdot\text{K})(50-20)\,\text{K} + (5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4)(0.35)(323^4 - 293^4)\,\text{K}^4]$$

$$= \mathbf{1737\ W}$$

Discussion At the tank surface, natural convection contributed to about 50% of the 1737 W heat loss. From Table A-18, the emissivity value of 0.35 is a suitable value for lightly oxidized stainless steel surface.

9-78 A spherical tank is filled with a hot liquid and the inner surface temperature is known. The outer surface temperature is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** The tank wall thermal conductivity is constant. **4** Local atmospheric pressure is 1 atm. **5** The T_{surr} is the same as the air temperature. **6** The film temperature is 40°C (this assumption will be verified).

Properties The properties of air at the assumed T_f = 40°C are k = 0.02662 W/m·K, ν = 1.702 × 10^{-5} m^2/s, Pr = 0.7255 (Table A-15), and $\beta = 1/T_f$ = 1/313 K.

The thermal conductivity and the emissivity of the tank are given as k_{sph} = 0.15 W/m·K and ε = 0.35, respectively.

Analysis With the assumption that T_f = 40°C, the tank outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 60°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(40 + 273\,\text{K})^{-1}(60 - 20)\,\text{K}\,(3.06\,\text{m})^3}{(1.702 \times 10^{-5}\,\text{m}^2/\text{s})^2}(0.7255) = 8.9964 \times 10^{10}$$

where

$$D_o = D_i + 2t = 3.06\,\text{m}$$

The Nusselt number relation for sphere is

$$\text{Nu} = 2 + \frac{0.589\text{Ra}^{1/4}}{\left[1 + \left(\dfrac{0.469}{\text{Pr}}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(8.9964 \times 10^{10})^{1/4}}{\left[1 + \left(\dfrac{0.469}{0.7255}\right)^{9/16}\right]^{4/9}} = 251.5$$

$$h = \frac{k}{D_o}\text{Nu} = \frac{0.02662\ \text{W/m}\cdot\text{K}}{3.06\,\text{m}}(251.5) = 2.1879\ \text{W/m}^2\cdot\text{K}$$

Thus, the outer surface temperature can be solved using

$$\dot{Q}_{\text{cond}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

$$\frac{2\pi k_{\text{sph}} D_o D_i}{D_o - D_i}(T_{s,i} - T_{s,o}) = h(\pi D_o^2)(T_{s,o} - T_\infty) + \varepsilon\sigma(\pi D_o^2)(T_{s,o}^4 - T_{\text{surr}}^4) \quad \rightarrow \quad T_{s,o} = \mathbf{61.1°C}$$

Discussion The assumed film temperature T_f = 40°C is an appropriate assumption, since the determined $T_{s,o}$ = 61.1°C would give a film temperature of T_f = 40.6°C. Otherwise, $T_{s,o}$ would have to be solved iteratively.

9-79 A metal spherical tank is filled with chemicals in exothermic reaction and the heat generation is known. The type of surface to be used so that the outer surface temperature is safe from thermal burn is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Surface temperature is constant. **3** Local atmospheric pressure is 1 atm. **4** The T_{surr} is the same as the air temperature.

Properties The properties of air at the assumed T_f = 30°C and 1 atm pressure are k = 0.02588 W/m·K, $\nu = 1.608 \times 10^{-5}$ m^2/s, Pr = 0.7282 (Table A-15), and $\beta = 1/T_f$ = 1/303 K .

The emissivities for the unpainted and painted outer surface are $\varepsilon_{unpaint}$ = 0.20 and ε_{paint} = 0.88, respectively.

Analysis With the assumption that T_f = 30°C, the outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 45°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(30+273\,\text{K})^{-1}(45-15)\,\text{K}\,(3.02\,\text{m})^3}{(1.608\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7282) = 7.5344\times10^{10}$$

where $\quad D_o = D_i + 2t_{\text{tank}} = 3.02\,\text{m}$

The Nusselt number relation for sphere is

$$\text{Nu} = 2 + \frac{0.589\text{Ra}^{1/4}}{\left[1+\left(\frac{0.469}{\text{Pr}}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(7.5344\times10^{10})^{1/4}}{\left[1+\left(\frac{0.469}{0.7282}\right)^{9/16}\right]^{4/9}} = 240.78$$

$$h = \frac{k}{D_o}\text{Nu} = \frac{0.02588\,\text{W/m}\cdot\text{K}}{3.02\,\text{m}}(240.78) = 2.0634\,\text{W/m}^2\cdot\text{K}$$

Thus, the outer surface temperature can be solved using

$$\dot{Q}_{\text{reaction}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

$$\dot{e}_{\text{gen}}(\pi D_i^3/6) = h(\pi D_o^2)(T_{s,o} - T_\infty) + \varepsilon\sigma(\pi D_o^2)(T_{s,o}^4 - T_{\text{surr}}^4)$$

where $\quad \dot{e}_{\text{gen}} = 450\,\text{W/m}^3$

For an unpainted outer surface ($\varepsilon = \varepsilon_{unpaint}$ = 0.20), we have $\quad\rightarrow\quad T_{s,o} = 77.5°\text{C}$ (above 45°C)

For a painted outer surface ($\varepsilon = \varepsilon_{paint}$ = 0.88), we have $\quad\rightarrow\quad T_{s,o} = 44.2°\text{C}$ (below 45°C)

Discussion To prevent thermal burn hazards, the outer surface should be painted with black paint. The black paint increases the radiation heat transfer from the outer surface, thus reducing the surface temperature.

Note that 30°C is an appropriate temperature to evaluate the air properties for the painted surface, since $T_{s,o}$ = 44.2°C would give a film temperature of T_f = 29.6°C.

For the unpainted surface, a more appropriate temperature for evaluating the air properties would be 45°C. By performing an iterative solution, the outer surface temperature for the unpainted surface would converge to about 73°C.

9-80 A metal spherical tank is filled with hot liquid and the inner surface temperature is known. The tank is covered with a layer of insulation and outer surface temperature is to be determined whether it is safe for thermal burn prevention.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** The tank wall thermal conductivity is constant. **4** Local atmospheric pressure is 1 atm. **5** The T_{surr} is the same as the air temperature. **6** The film temperature is 30°C (this assumption will be verified).

Properties The properties of air at the assumed T_f = 30°C and 1 atm pressure are k = 0.02588 W/m·K, ν = 1.608 × 10^{-5} m^2/s, Pr = 0.7282 (Table A-15), and $\beta = 1/T_f$ = 1/303 K. The thermal conductivity of the tank is given as k_{tank} = 15 W/m·K.

The thermal conductivity and the emissivity of the insulation are given as k_{ins} = 0.15 W/m·K and ε = 0.35, respectively.

Analysis With the assumption that T_f = 30°C, the outer surface temperature is estimated as

$$T_{s,o} = 2T_f - T_\infty = 44°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_{s,o} - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(30+273\,\text{K})^{-1}(44-16)\,\text{K}\,(3.16\,\text{m})^3}{(1.608\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7282) = 8.0561\times10^{10}$$

where $D_o = D_i + 2(t_{\text{tank}} + t_{\text{ins}}) = 3.16\,\text{m}$

The Nusselt number relation for sphere is

$$\text{Nu} = 2 + \frac{0.589\text{Ra}^{1/4}}{\left[1+\left(\frac{0.469}{\text{Pr}}\right)^{9/16}\right]^{4/9}} = 2 + \frac{0.589(8.0561\times10^{10})^{1/4}}{\left[1+\left(\frac{0.469}{0.7282}\right)^{9/16}\right]^{4/9}} = 244.8$$

$$h = \frac{k}{D_o}\text{Nu} = \frac{0.02588\,\text{W/m}\cdot\text{K}}{3.16\,\text{m}}(244.8) = 2.0049\,\text{W/m}^2\cdot\text{K}$$

Thus, the outer surface temperature can be solved using

$$\dot{Q}_{\text{cond}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

$$\frac{(T_{s,i} - T_{s,o})}{R_{\text{total}}} = h(\pi D_o^2)(T_{s,o} - T_\infty) + \varepsilon\sigma(\pi D_o^2)(T_{s,o}^4 - T_{\text{surr}}^4) \quad \rightarrow \quad T_{s,o} = \mathbf{43.5°C}$$

where the total thermal resistance for the spherical layers is

$$R_{\text{total}} = \frac{D_{\text{interface}} - D_i}{2\pi k_{\text{tank}} D_{\text{interface}} D_i} + \frac{D_o - D_{\text{interface}}}{2\pi k_{\text{ins}} D_o D_{\text{interface}}} = 0.01559\,\text{K/W}$$

Discussion The insulation layer is able to keep the outer surface temperature 1.5°C below the safe temperature of 45°C. For additional safety precaution, the outer surface temperature can be further reduced by increasing insulation thickness or paint the outer surface to achieve higher emissivity.

The assumed film temperature T_f = 30°C is an appropriate assumption, since the determined $T_{s,o}$ = 43.5°C would give a film temperature of T_f = 29.8°C. Otherwise, $T_{s,o}$ would have to be solved iteratively.

Natural Convection from Finned Surfaces and PCBs

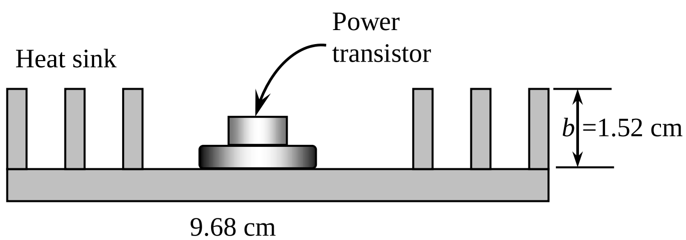
9-81C Finned surfaces are frequently used in practice to enhance heat transfer by providing a larger heat transfer surface area. Finned surfaces are referred to as heat sinks in the electronics industry since they provide a medium to which the waste heat generated in the electronic components can be transferred effectively.

9-82C A heat sink with closely packed fins will have greater surface area for heat transfer, but smaller heat transfer coefficient because of the extra resistance the additional fins introduce to fluid flow through the interfin passages.

9-83C Removing some of the fins on the heat sink will decrease heat transfer surface area, but will increase heat transfer coefficient. The decrease on heat transfer surface area more than offsets the increase in heat transfer coefficient, and thus heat transfer rate will decrease. In the second case, the decrease on heat transfer coefficient more than offsets the increase in heat transfer surface area, and thus heat transfer rate will again decrease.

9-84 An aluminum heat sink of rectangular profile oriented vertically is used to cool a power transistor. The average natural convection heat transfer coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Radiation heat transfer from the sink is negligible. **4** The entire sink is at the base temperature.



Analysis The total surface area of the heat sink is

$$A_{fins} = 2nLb = (2)(6)(0.0762\text{ m})(0.0152\text{ m}) + (2)(0.0048\text{ m})(0.0762\text{ m}) = 0.01463\text{ m}^2$$

$$A_{unfinned} = (4)(0.0145\text{ m})(0.0762\text{ m}) + (0.0317\text{ m})(0.0762\text{ m}) = 0.006835\text{ m}^2$$

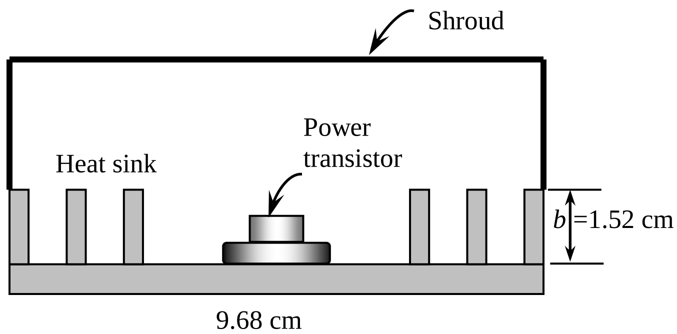
$$A_{total} = A_{fins} + A_{unfinned} = 0.01463 + 0.006835 = 0.021465\text{ m}^2$$

Then the average natural convection heat transfer coefficient becomes

$$\dot{Q} = hA_{total}(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_{total}(T_s - T_\infty)} = \frac{18\text{ W}}{(0.021465\text{ m}^2)(120-22)°\text{C}} = \mathbf{8.56\ W/m^2.°C}$$

9-85 Aluminum heat sinks of rectangular profile oriented vertically are used to cool a power transistor. A shroud is placed very close to the tips of fins. The average natural convection heat transfer coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. 3 Radiation heat transfer from the sink is negligible. **4** The entire sink is at the base temperature.



Analysis The total surface area of the shrouded heat sink is

$$A_{fins} = 2nLb = (2)(6)(0.0762\text{ m})(0.0152\text{ m}) = 0.013898\text{ m}^2$$

$$A_{unfinned} = (4)(0.0145\text{ m})(0.0762\text{ m}) + (0.0317\text{ m})(0.0762\text{ m}) = 0.006835\text{ m}^2$$

$$A_{shroud} = (2)(0.0968\text{ m})(0.0762\text{ m}) = 0.014752\text{ m}^2$$

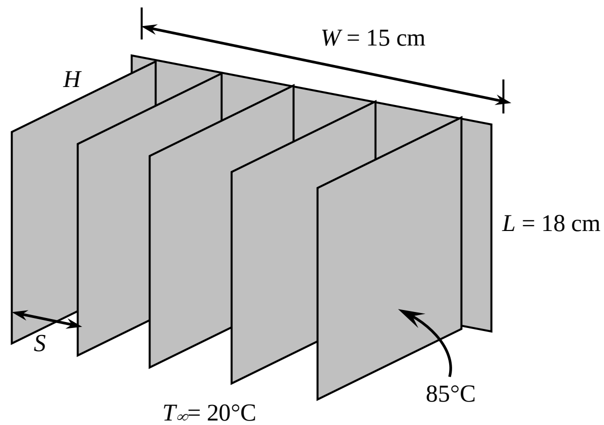
$$A_{total} = A_{fins} + A_{unfinned} + A_{shroud} = 0.013898 + 0.006835 + 0.014752 = 0.035486\text{ m}^2$$

Then the average natural convection heat transfer coefficient becomes

$$\dot{Q} = hA_{total}(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_{total}(T_s - T_\infty)} = \frac{18\text{ W}}{(0.035486\text{ m}^2)(108 - 22)°\text{C}} = \mathbf{5.90\ W/m^2.°C}$$

9-86 A heat sink with equally spaced rectangular fins is to be used to cool a hot surface. The optimum fin spacing and the rate of heat transfer from the heat sink are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The atmospheric pressure at that location is 1 atm.



Properties The properties of air at 1 atm and 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (85+20)/2 = 52.5°C are (Table A-15)

$$k = 0.02753 \text{ W/m.°C}$$

$$\nu = 1.823\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7222$$

$$\beta = \frac{1}{T_f} = \frac{1}{(52.5+273)\text{K}} = 0.003072 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the height of the surface $L_c = L$ = 0.18 m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003072 \text{ K}^{-1})(85-20 \text{ K})(0.18 \text{ m})^3}{(1.823\times10^{-5} \text{ m}^2/\text{s})^2}(0.7222) = 2.483\times10^7$$

The optimum fin spacing is

$$S = 2.714\frac{L}{Ra^{1/4}} = 2.714\frac{0.18 \text{ m}}{(2.483\times10^7)^{1/4}} = 0.006921 \text{ m} = \mathbf{6.921 \text{ mm}}$$

The heat transfer coefficient for this optimum fin spacing case is

$$h = 1.307\frac{k}{S} = 1.307\frac{0.02753 \text{ W/m.°C}}{0.006921 \text{ m}} = 5.199 \text{ W/m}^2.\text{°C}$$

The number of fins and the total heat transfer surface area is

$$n = \frac{w}{S+t} \cong \frac{w}{s} = \frac{0.15}{0.006921} \cong 22 \text{ fins}$$

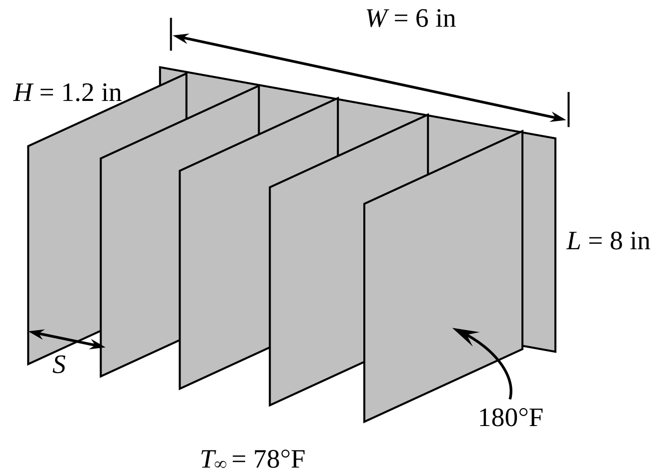
$$A_s = 2nLH = 2\times22\times(0.18 \text{ m})(0.04 \text{m}) = 0.3168 \text{ m}^2$$

Then the rate of natural convection heat transfer becomes

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.199 \text{ W/m}^2.\text{°C})(0.3168 \text{ m}^2)(85-20)\text{°C} = \mathbf{107.1 \text{ W}}$$

9-87E A heat sink with equally spaced rectangular fins is to be used to cool a hot surface. The optimum fin spacing and the rate of heat transfer from the heat sink are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The atmospheric pressure at that location is 1 atm. **4** The thickness t of the fins is very small relative to the fin spacing S so that Eqs. 9-32 and 9-33 for optimum fin spacing are applicable.



Properties The properties of air at 1 atm and 1 atm and the film temperature of $(T_s+T_\infty)/2 = (180+78)/2=129°F$ are (Table A-15E)

$$k = 0.01597 \text{ Btu/h.ft.°F}$$

$$\nu = 0.1975\times10^{-3} \text{ ft}^2\text{/s}$$

$$\Pr = 0.7217$$

$$\beta = \frac{1}{T_f} = \frac{1}{(129+460)\text{ R}} = 0.001698 \text{ R}^{-1}$$

Analysis The characteristic length in this case is the fin height, $L_c = L = 8$ in. Then,

$$Ra = \frac{g\beta(T_1-T_2)L^3}{\nu^2}\Pr = \frac{(32.2 \text{ ft/s}^2)(0.001698 \text{ R}^{-1})(180-78 \text{ R})(8/12 \text{ ft})^3}{(0.1975\times10^{-3} \text{ ft}^2\text{/s})^2}(0.7217) = 3.058\times10^7$$

The optimum fin spacing is

$$S = 2.714\frac{L}{Ra^{1/4}} = 2.714\frac{8/12 \text{ ft}}{(3.058\times10^7)^{1/4}} = 0.02433 \text{ ft} = \mathbf{0.292\,in}$$

The heat transfer coefficient for this optimum spacing case is

$$h = 1.307\frac{k}{S} = 1.307\frac{0.01597 \text{ Btu/h.ft.°F}}{0.02433 \text{ ft}} = 0.8578 \text{ Btu/h.ft}^2\text{.°F}$$

The number of fins and the total heat transfer surface area is

$$n = \frac{w}{S+t} = \frac{6}{0.2916+0.08} = 16 \text{ fins}$$

$$\begin{aligned} A_s &= 2nLH + ntL + 2ntH \\ &= 2\times16\times(8/12 \text{ ft})(1.2/12 \text{ ft}) + 16\times(0.08/12 \text{ ft})(8/12 \text{ ft}) + 2\times16\times(0.08/12 \text{ ft})(1.2/12 \text{ ft}) \\ &= 2.226 \text{ ft}^2 \end{aligned}$$

Then the rate of natural convection heat transfer becomes

$$\dot{Q} = hA_s(T_s - T_\infty) = (0.8578 \text{ Btu/h.ft}^2\text{.°F})(2.226 \text{ ft}^2)(180-78)\text{°F} = \mathbf{194.7\,Btu/h}$$

Discussion If the fin thickness is disregarded, the number of fins and the rate of heat transfer become

$$n = \frac{w}{s+t} \cong \frac{w}{s} = \frac{6}{0.2916} \cong 21 \text{ fins}$$

$$A_s = 2nLH = 2\times21\times(8/12 \text{ ft})(1.2/12 \text{ ft}) = 2.8 \text{ ft}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (0.8578 \text{ Btu/h.ft}^2\text{.°F})(2.8 \text{ ft}^2)(180-78)\text{°F} = \mathbf{245\,Btu/h}$$

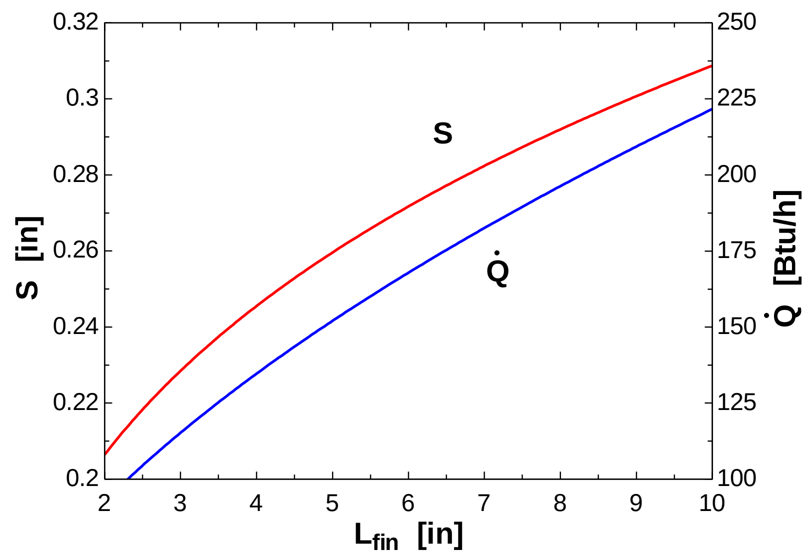
Therefore, the fin tip area is significant in this case.

9-88E Prob. 9-87E is reconsidered. The effect of the length of the fins in the vertical direction on the optimum fin spacing and the rate of heat transfer by natural convection is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
w_s=(6/12) [ft]
H_s=(8/12) [ft]
T_infinity=78 [F]
t_fin=(0.08/12) [ft]
L_fin=8 [in]
H_fin=(1.2/12) [ft]
T_s=180 [F]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_film)
Pr=Prandtl(Fluid$, T=T_film)
rho=Density(Fluid$, T=T_film, P=14.7)
mu=Viscosity(Fluid$, T=T_film)*Convert(lbm/ft-h, lbm/ft-s)
nu=mu/rho
beta=1/(T_film+460)
T_film=1/2*(T_s+T_infinity)
g=32.2 [ft/s^2] "gravitational acceleration"

"ANALYSIS"
L_fin_ft=L_fin*Convert(in, ft)
delta=L_fin_ft
Ra=(g*beta*(T_s-T_infinity)*delta^3)/nu^2*Pr
S_ft=2.714*L_fin_ft/Ra^0.25
S=S_ft*Convert(ft, in)
h=1.307*k/S_ft
n_fin=w_s/(S_ft+t_fin)
A=2*n_fin*L_fin_ft*H_fin+n_fin*t_fin*L_fin_ft+2*n_fin*t_fin*H_fin
Q_dot=h*A*(T_s-T_infinity)
```

L_{fin} [in]	S [in]	$\dot{Q}$ [Btu/h]
2	0.2065	92.73
2.5	0.2183	104.5
3	0.2285	115.3
3.5	0.2375	125.3
4	0.2455	134.7
4.5	0.2529	143.6
5	0.2596	152
5.5	0.2659	160.1
6	0.2717	167.9
6.5	0.2772	175.4
7	0.2824	182.6
7.5	0.2873	189.6
8	0.292	196.3
8.5	0.2964	202.9
9	0.3007	209.3
9.5	0.3048	215.6
10	0.3087	221.7



Natural Convection inside Enclosures

9-89C We would recommend putting the hot fluid into the upper compartment of the container. In this case no convection currents will develop in the enclosure since the lighter (hot) fluid will always be on top of the heavier (cold) fluid.

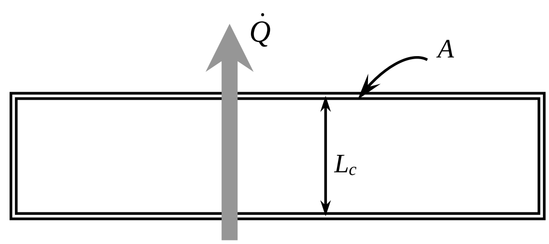
9-90C We would disagree with this recommendation since the air space introduces some thermal resistance to heat transfer. The thermal resistance of air space will be zero only when the convection coefficient approaches infinity, which is never the case. However, when the air space is eliminated, so is its thermal resistance.

9-91C Yes, dividing the air space into two compartments will retard air motion in the air space, and thus slow down heat transfer by natural convection. The vinyl sheet will also act as a radiation shield and reduce heat transfer by radiation.

9-92C The effective thermal conductivity of an enclosure represents the enhancement on heat transfer as result of convection currents relative to conduction. The ratio of the effective thermal conductivity to the ordinary thermal conductivity yields Nusselt number $Nu = k_{eff} / k$.

9-93 Conduction thermal resistance of a medium is expressed as $R = L/(kA)$. Thermal resistance of a rectangular enclosure can be expressed by replacing L with characteristic length of enclosure L_c, and thermal conductivity k with effective thermal conductivity k_{eff} to give

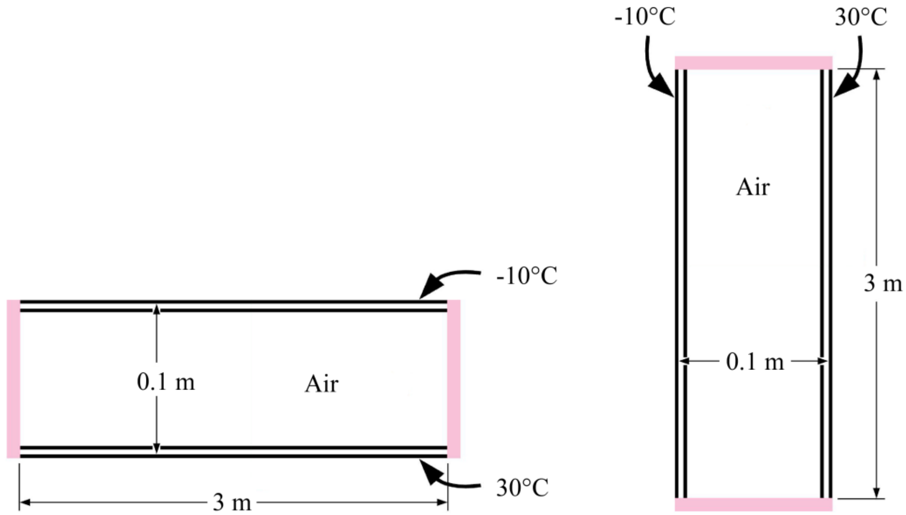
$$R = L_c /(k_{eff} A) = L_c /(kNuA)$$



9-94 A rectangular enclosure consists of two surfaces separated by an air gap, and the ratio of the heat transfer rate for the horizontal orientation (with hotter surface at the bottom) to that of vertical orientation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Thermal properties are constant.

Properties The properties of air at $T_f = (T_s + T_\infty)/2 = 10°\text{C}$ are $k = 0.02439$ W/m·K, $\nu = 1.426 \times 10^{-5}$ m²/s, Pr = 0.7336 (from Table A-15). Also, $\beta = 1/T_f = 3.534 \times 10^{-3}$ K^{-1}.



Analysis The characteristic length for both cases is L_c = 0.1 m. The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.003534\,\text{K}^{-1})(30+10)\,\text{K}(0.1\,\text{m})^3}{(1.426\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7336)$$

$$= 5.003\times10^6$$

The aspect ratio for this rectangular enclosure is H/L = 30 m. The Nusselt numbers for horizontal and vertical orientations are

$$\text{Nu}_{\text{horiz}} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}_L}\right]^+ + \left[\frac{\text{Ra}_L^{1/3}}{18} - 1\right]^+ = 10.94$$

$$\text{Nu}_{\text{vert}} = 0.42 Ra^{1/4}\,\text{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 7.134$$

Hence, the ratio of heat transfer rate is

$$\frac{\dot{Q}_{\text{horiz}}}{\dot{Q}_{\text{vert}}} = \frac{h_{\text{horiz}}(k/L)A_s\Delta T}{h_{\text{vert}}(k/L)A_s\Delta T} = \frac{\text{Nu}_{\text{horiz}}(k/L)A_s\Delta T}{\text{Nu}_{\text{vert}}(k/L)A_s\Delta T} = \frac{\text{Nu}_{\text{horiz}}}{\text{Nu}_{\text{vert}}} = \frac{10.94}{7.134} = \mathbf{1.53}$$

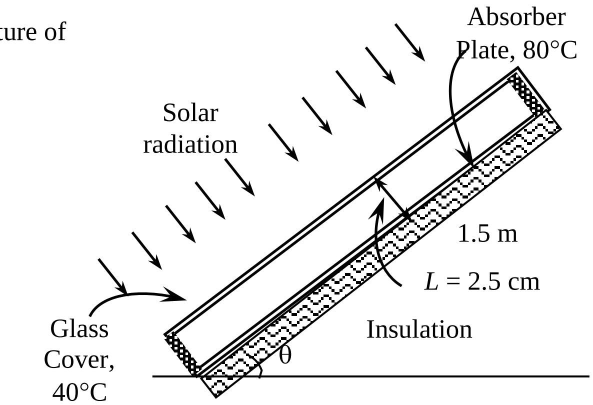
Discussion For the same ΔT, the rate of heat transfer for the horizontal orientation is 53% larger than that for the vertical orientation.

9-95 The absorber plate and the glass cover of a flat-plate solar collector are maintained at specified temperatures. The rate of heat loss from the absorber plate by natural convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat loss by radiation is negligible. **4** The air pressure in the enclusure is 1 atm.

Properties The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2$ = (80+40)/2 = 60°C are (Table A-15)

$$k = 0.02808 \text{ W/m.°C}$$
$$\nu = 1.896\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7202$$
$$\beta = \frac{1}{T_f} = \frac{1}{(60+273)\text{K}} = 0.003003 \text{ K}^{-1}$$



Analysis For $\theta = 0°$, we have horizontal rectangular enclosure. The characteristic length in this case is the distance between the two glasses $L_c = L$ = 0.025 m Then,

$$Ra = \frac{g\beta(T_1 - T_2)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003003 \text{ K}^{-1})(80-40 \text{ K})(0.025 \text{ m})^3}{(1.896\times10^{-5} \text{ m}^2\text{/s})^2}(0.7202) = 3.689\times10^4$$

$$\text{Nu} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}}\right]^+ + \left[\frac{\text{Ra}^{1/3}}{18} - 1\right]^+$$

$$= 1 + 1.44\left[1 - \frac{1708}{3.689\times10^4}\right]^+ + \left[\frac{(3.689\times10^4)^{1/3}}{18} - 1\right]^+ = 3.223$$

Then

$$A_s = H\times W = (1.5 \text{ m})(3 \text{ m}) = 4.5 \text{ m}^2$$

$$\dot{Q} = kNuA_s\frac{T_1 - T_2}{L} = (0.02808 \text{ W/m.°C})(3.223)(4.5 \text{ m}^2)\frac{(80-40)\text{°C}}{0.025 \text{ m}} = \mathbf{652 \text{ W}}$$

For $\theta = 30°$, we obtain

$$\text{Nu} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}\cos\theta}\right]^+\left[1 - \frac{1708(\sin 1.8\theta)^{1.6}}{\text{Ra}\cos\theta}\right] + \left[\frac{(\text{Ra}\cos\theta)^{1/3}}{18} - 1\right]^+$$

$$= 1 + 1.44\left[1 - \frac{1708}{(3.689\times10^4)\cos(30)}\right]^+\left[1 - \frac{1708[\sin(1.8\times30)]^{1.6}}{(3.689\times10^4)\cos(30)}\right] + \left[\frac{[(3.689\times10^4)\cos(30)]^{1/3}}{18} - 1\right]^+$$

$$= 3.074$$

$$\dot{Q} = kNuA_s\frac{T_1 - T_2}{L} = (0.02808 \text{ W/m.°C})(3.074)(4.5 \text{ m}^2)\frac{(80-40)\text{°C}}{0.025 \text{ m}} = \mathbf{621 \text{ W}}$$

For $\theta = 90°$, we have vertical rectangular enclosure. The Nusselt number for this geometry and orientation can be determined from ($Ra = 3.689\times10^4$ - same as that for horizontal case)

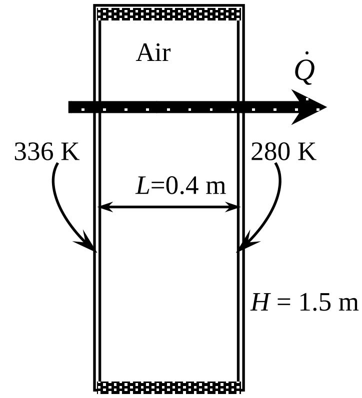
$$Nu = 0.42Ra^{1/4}\text{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(3.689\times10^4)^{1/4}(0.7202)^{0.012}\left(\frac{1.5 \text{ m}}{0.025 \text{ m}}\right)^{-0.3} = 1.69$$

$$\dot{Q} = kNuA_s\frac{T_1 - T_2}{L} = (0.02808 \text{ W/m.°C})(1.69)(4.5 \text{ m}^2)\frac{(80-40)\text{°C}}{0.025 \text{ m}} = \mathbf{344 \text{ W}}$$

Discussion Caution is advised for the vertical case since the condition $H/L < 40$ is not satisfied.

9-96 Two glasses of a double pane window are maintained at specified temperatures. The fraction of heat transferred through the enclosure by radiation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The air pressure in the enclosure is 1 atm.



Properties The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (280+336)/2 = 308$ K = 35°C are (Table A-15E)

$$k = 0.02625 \text{ W/m.°C}$$

$$\nu = 1.655\times10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7268$$

$$\beta = \frac{1}{T_f} = \frac{1}{308\text{ K}} = 0.003247 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the distance between the two glasses, $L_c = L = 0.4$ m. Then,

$$Ra_L = \frac{g\beta(T_1-T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003247\text{ K}^{-1})(336-280\text{ K})(0.4\text{ m})^3}{(1.655\times10^{-5}\text{ m}^2\text{/s})^2}(0.7268) = 3.029\times10^8$$

The aspect ratio of the geometry is $H/L = 1.5/0.4 = 3.75$. For this value of H/L the Nusselt number can be determined from

$$Nu = 0.22\left(\frac{\text{Pr}}{0.2+\text{Pr}}Ra\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4} = 0.22\left(\frac{0.7268}{0.2+0.7268}(3.029\times10^8)\right)^{0.28}\left(\frac{1.5}{0.4}\right)^{-1/4} = 35.00$$

Then,

$$A_s = H\times W = (1.5\text{ m})(3\text{ m}) = 4.5\text{ m}^2$$

$$\dot{Q}_{\text{conv}} = kNuA_s\frac{T_1-T_2}{L} = (0.02625\text{ W/m.°C})(35.00)(4.5\text{ ft}^2)\frac{(336-280)\text{K}}{0.4\text{ m}} = 578.9\text{ W}$$

The effective emissivity is

$$\frac{1}{\varepsilon_{\text{eff}}} = \frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1 = \frac{1}{0.15}+\frac{1}{0.90}-1 = 6.778 \longrightarrow \varepsilon_{\text{eff}} = 0.1475$$

The rate of heat transfer by radiation is

$$\dot{Q}_{\text{rad}} = \varepsilon_{\text{eff}}A_s\sigma(T_1^4-T_2^4)$$
$$= (0.1475)(4.5\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(336\text{ K})^4-(280\text{ K})^4] = 248.4\text{ W}$$

Then the fraction of heat transferred through the enclosure by radiation becomes

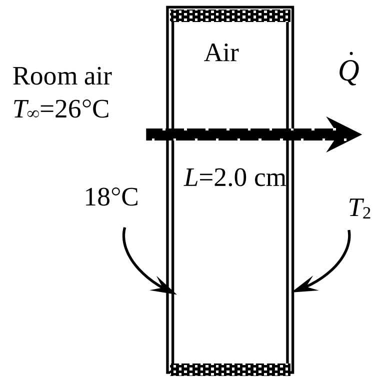
$$f_{\text{rad}} = \frac{\dot{Q}_{\text{rad}}}{\dot{Q}_{\text{conv}}+\dot{Q}_{\text{rad}}} = \frac{248.4}{578.9+248.4} = \mathbf{0.30}$$

9-97 A double pane window with an air gap is considered. The rate of heat transfer through the window by natural convection the temperature of the outer glass layer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The air pressure in the enclosure is 1 atm. **4** Radiation heat transfer is neglected.

Properties For natural convection between the inner surface of the window and the room air, the properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (18+26)/2 = 22°C$ are (Table A-15)

$$k = 0.02529 \text{ W/m.°C}$$
$$\nu = 1.534\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7304$$
$$\beta = \frac{1}{T_f} = \frac{1}{(22+273)\text{K}} = 0.00339 \text{ K}^{-1}$$



For natural convection between the two glass sheets separated by an air gap, the properties of air at 1 atm and the anticipated average temperature of $(T_1+T_2)/2 = (18+0)/2 = 9°C$ are (Table A-15)

$$k = 0.02431 \text{ W/m.°C}, \quad \nu = 1.417\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7339, \quad \beta = \frac{1}{T_f} = \frac{1}{(9+273)\text{K}} = 0.003546 \text{ K}^{-1}$$

Analysis We first calculate the natural convection heat transfer between the room air and the inner surface of the window.

$$L_c = H = 1.5 \text{ m}$$

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)H^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.00339 \text{ K}^{-1})(26-18)\text{K}(1.5 \text{ m})^3}{(1.534\times10^{-5} \text{ m}^2/\text{s})^2}(0.7304) = 2.787\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(2.787\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7304}\right)^{9/16}\right]^{8/27}}\right\}^2 = 169.5$$

$$h = \frac{k}{H}Nu = \frac{0.02529 \text{ W/m.°C}}{1.5 \text{ m}}(169.5) = 2.858 \text{ W/m}^2.\text{°C}$$

$$A_s = H\times W = (1.5 \text{ m})(2.8 \text{ m}) = 4.2 \text{ m}^2$$

$$\dot{Q}_{\text{conv}} = hA_s(T_\infty - T_s) = (2.858 \text{ W/m}^2.\text{°C})(4.2 \text{ m}^2)(26-18)\text{°C} = \mathbf{96.0 \text{ W}}$$

Next, we consider the natural convection between the two glass sheets separated by an air gap.

$$L_c = L = 2.0 \text{ cm}$$

$$\text{Ra} = \frac{g\beta(T_1 - T_2)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003546 \text{ K}^{-1})(18-0)\text{K}(0.020 \text{ m})^3}{(1.417\times10^{-5} \text{ m}^2/\text{s})^2}(0.7339) = 18{,}309$$

$$Nu = 0.42Ra^{1/4}\text{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(18{,}309)^{1/4}(0.7339)^{0.012}\left(\frac{1.5 \text{ m}}{0.020 \text{ m}}\right)^{-0.3} = 1.333$$

Under steady operation, the rate of heat transfer between the room air and the inner surface of the window is equal to the heat transfer through the air gap. Setting these two equal to each other we obtain the temperature of the outer glass sheet

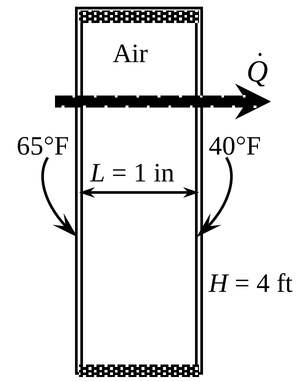
$$\dot{Q} = kNuA_s\frac{T_1 - T_2}{L} \longrightarrow 96.0 \text{ W} = (0.02431 \text{ W/m.°C})(1.333)(4.2 \text{ m}^2)\frac{(18-T_2)\text{°C}}{0.020 \text{ m}} \longrightarrow T_2 = \mathbf{3.9°C}$$

which is sufficiently close to the assumed temperature 0°C. Therefore, there is no need to repeat the calculations.

Discussion The aspect ratio for this geometry is H/L = (1.5 m)/(0.020 m) = 75. There is no Nusselt number relation given in the text covering this value of aspect ratio. One of the closest aspect ratio ranges is given by Eq. 9-54 as $10 < H/L < 40$. We used Eq. 9-54 knowing that there will be some error in the calculation of Nusselt number.

9-98E Two glasses of a double pane window are maintained at specified temperatures. The rate of heat transfer through the window by natural convection and radiation, and the R-value of insulation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The air pressure in the enclosure is 1 atm.



Properties The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (65+40)/2 = 52.5$°F are (Table A-15E)

$$k = 0.01415 \text{ Btu/h.ft.°F}$$
$$\nu = 0.1548\times10^{-3} \text{ ft}^2/\text{s}$$
$$\text{Pr} = 0.7332$$
$$\beta = \frac{1}{T_f} = \frac{1}{(52.5+460)\text{ R}} = 0.001951 \text{ R}^{-1}$$

Analysis (*a*) The characteristic length in this case is the distance between the two glasses, $L_c = L = 1$ in. Then,

$$Ra_L = \frac{g\beta(T_1-T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001951 \text{ R}^{-1})(65-40 \text{ R})(1/12 \text{ ft})^3}{(0.1548\times10^{-3} \text{ ft}^2/\text{s})^2}(0.7332) = 27{,}809$$

The aspect ratio of the geometry is $H/L = 4\times12/1 = 48$ (which is somewhat over 40, but still close enough for an approximate analysis). For these values of H/L and Ra_L, the Nusselt number can be determined from

$$Nu = 0.42Ra^{1/4}\text{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(27{,}809)^{1/4}(0.7332)^{0.012}\left(\frac{4 \text{ ft}}{1/12 \text{ ft}}\right)^{-0.3} = 1.692$$

Then,

$$A_s = H\times W = (4 \text{ ft})(6 \text{ ft}) = 24 \text{ ft}^2$$
$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.01415 \text{ Btu/h.ft.°F})(1.692)(24 \text{ ft}^2)\frac{(65-40)\text{°F}}{(1/12)\text{ft}} = \mathbf{172.4 \text{ Btu/h}}$$

(*b*) The rate of heat transfer by radiation is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_1^4 - T_2^4)$$
$$= (0.82)(24 \text{ ft}^2)(0.1714\times10^{-8} \text{ Btu/h.ft}^2.\text{R}^4)[(65+460 \text{ R})^4 - (40+460 \text{ R})^4] = \mathbf{454.3 \text{ Btu/h}}$$

Then the total rate of heat transfer is

$$\dot{Q}_{total} = \dot{Q}_{convection} + \dot{Q}_{rad} = 172.4 + 454.3 = 626.7 \text{ Btu/h}$$

Then the effective thermal conductivity of the air, which also accounts for the radiation effect and the *R*-value become

$$\dot{Q}_{total} = k_{eff}A_s\frac{T_1-T_2}{L} \longrightarrow k_{eff} = \frac{\dot{Q}L}{A_s(T_1-T_2)} = \frac{(626.7 \text{ Btu/h})(1/12 \text{ ft})}{(24 \text{ ft}^2)(65-40)\text{°F}} = 0.08704 \text{ Btu/h.ft.°F}$$

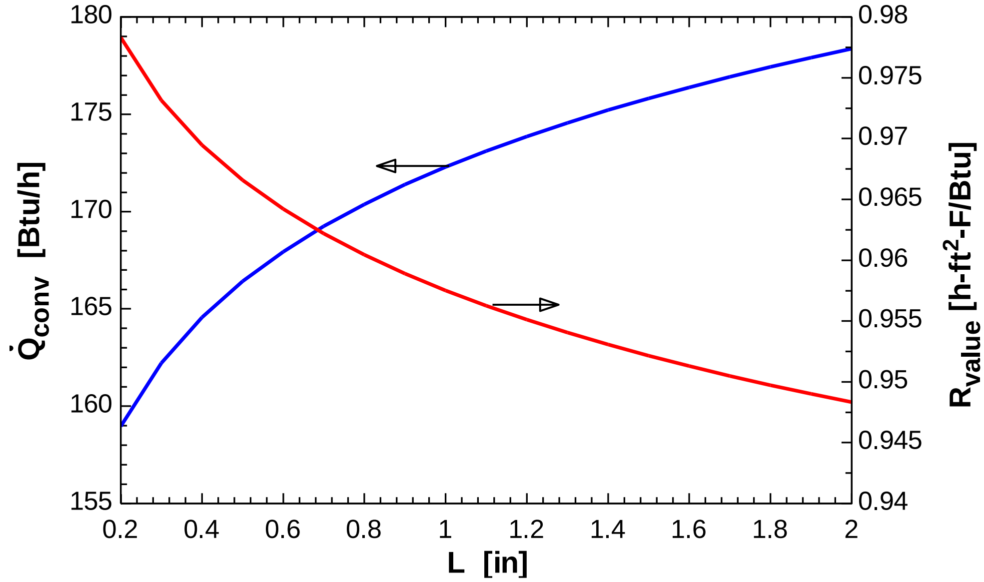
$$R_{value} = \frac{L}{k_{eff}} = \frac{(1/12 \text{ ft})}{0.08704 \text{ Btu/h.ft.°F}} = 0.957 \text{ h.ft}^2.\text{°F/Btu} = \mathbf{R-0.96}$$

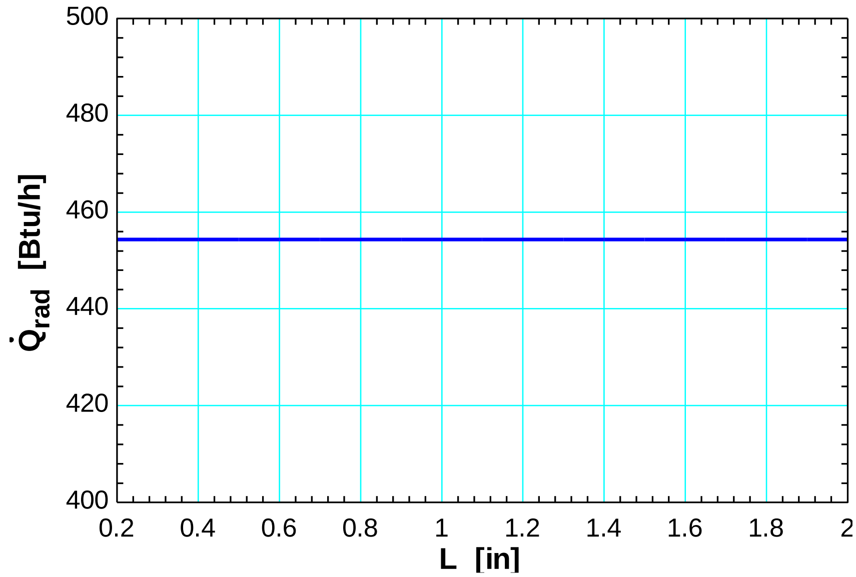
9-99E Prob. 9-98E is reconsidered. The effect of the air gap thickness on the rates of heat transfer by natural convection and radiation, and the *R*-value of insulation is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
H=4 [ft]
W=6 [ft]
L=1 [in]
T_1=65 [F]
T_2=40 [F]
epsilon_eff=0.82

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_ave)
Pr=Prandtl(Fluid$, T=T_ave)
rho=Density(Fluid$, T=T_ave, P=14.7)
mu=Viscosity(Fluid$, T=T_ave)*Convert(lbm/ft-h, lbm/ft-s)
nu=mu/rho
beta=1/(T_ave+460)
T_ave=1/2*(T_1+T_2)
g=32.2 [ft/s^2]
sigma=0.1714E-8 [Btu/h-ft^2-R^4]

"ANALYSIS"
L_ft=L*Convert(in, ft)
Ra=(g*beta*(T_1-T_2)*L_ft^3)/nu^2*Pr
Ratio=H/L_ft
Nusselt=0.42*Ra^0.25*Pr^0.012*(H/L_ft)^(-0.3)
A=H*W
Q_dot_conv=k*Nusselt*A*(T_1-T_2)/L_ft
Q_dot_rad=epsilon_eff*A*sigma*((T_1+460)^4-(T_2+460)^4)
Q_dot_total=Q_dot_conv+Q_dot_rad
Q_dot_total=k_eff*A*(T_1-T_2)/L_ft
R_value=L_ft/k_eff
```

L [in]	$\dot{Q}_{conv}$ [Btu/h]	$\dot{Q}_{rad}$ [Btu/h]	R-value [h.ft^2.F/Btu]
0.2	159	454.3	0.9783
0.3	162.2	454.3	0.9731
0.4	164.6	454.3	0.9695
0.5	166.4	454.3	0.9666
0.6	167.9	454.3	0.9642
0.7	169.2	454.3	0.9622
0.8	170.4	454.3	0.9604
0.9	171.4	454.3	0.9589
1	172.3	454.3	0.9575
1.1	173.1	454.3	0.9563
1.2	173.9	454.3	0.9551
1.3	174.6	454.3	0.9541
1.4	175.2	454.3	0.9531
1.5	175.8	454.3	0.9522
1.6	176.4	454.3	0.9513
1.7	176.9	454.3	0.9505
1.8	177.4	454.3	0.9497
1.9	177.9	454.3	0.949
2	178.4	454.3	0.9483

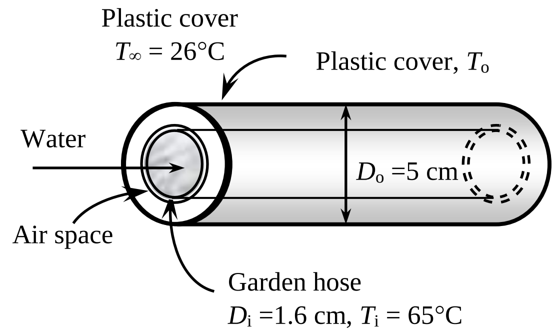

180
175
170
165
160
155
0.98
0.975
0.97
0.965
0.96
0.955
0.95
0.945
0.94
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
L [in]
$\dot{Q}_{conv}$ [Btu/h]
R_{value} [h-ft^2-F/Btu]


500
480
460
440
420
400
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
L [in]
$\dot{Q}_{rad}$ [Btu/h]

9-100 A simple solar collector is built by placing a clear plastic tube around a garden hose. The rate of heat loss from the water in the hose per meter of its length by natural convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat loss by radiation is negligible. **3** The air pressure in the enclosure is 1 atm.

Properties Based on the problem statement, the properties of air at 1 atm and the anticipated average temperature of $(T_i+T_o)/2 = (65+35)/2 = 50°C$ are (Table A-15)



$$k = 0.02735 \text{ W/m.°C}, \quad \nu = 1.798\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7228, \quad \beta = \frac{1}{T_f} = \frac{1}{(50+273)\text{K}} = 0.003096 \text{ K}^{-1}$$

Analysis We assume the plastic tube temperature to be 35°C. We will check this assumption later, and repeat calculations, if necessary. The characteristic length in this case is

$$L_c = \frac{D_o - D_i}{2} = \frac{5-1.6}{2} = 1.7 \text{ cm}$$

Then,

$$Ra = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003096 \text{ K}^{-1})(65-35 \text{ K})(0.017 \text{ m})^3}{(1.798\times10^{-5} \text{ m}^2/\text{s})^2}(0.7228) = 10{,}000$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5}+D_o^{-3/5})^5} = \frac{[\ln(0.05/0.016)]^4}{(0.017 \text{ m})^3\left[(0.016 \text{ m})^{-3/5}+(0.05 \text{ m})^{-3/5}\right]^5} = 0.1821$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$

$$= 0.386(0.02735 \text{ W/m.°C})\left(\frac{0.7228}{0.861+0.7228}\right)^{1/4}[(0.1821)(10{,}000)]^{1/4} = 0.05670 \text{ W/m.°C}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) = \frac{2\pi(0.05670 \text{ W/m.°C})}{\ln(0.05/0.016)}(65 - T_0) \qquad \text{(Eq. 1)}$$

Now we will calculate heat transfer from plastic tube to the ambient air by natural convection. Note that we should find a result close to the value we have already calculated since in steady operation they must be equal to each other. Also note that we neglect radiation heat transfer. We will use the same assumption for the plastic tube temperature (i.e., 35°C). The properties of air at 1 atm and the film temperature of $T_{avg} = (T_s + T_\infty)/2 = (35+26)/2 = 30.5°\text{C}$ are

$$k = 0.02592 \text{ W/m.°C}, \quad \nu = 1.613\times10^{-5} \text{ m}^2/\text{s},$$

$$\text{Pr} = 0.7281, \text{ and } \beta = 1/T_f = 1/(30.5+273)\text{K} = 0.003295 \text{ K}^{-1}$$

The characteristic length in this case is the outer diameter of the solar collector $L_c = D_o = 0.05$ m Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003295 \text{ K}^{-1})(35-26 \text{ K})(0.05 \text{ m})^3}{(1.613\times10^{-5} \text{ m}^2/\text{s})^2}(0.7281) = 1.018\times10^5$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.018\times10^5)^{1/6}}{\left[1+(0.559/0.7281)^{9/16}\right]^{8/27}}\right\}^2 = 7.838$$

$$A_o = \pi D_0 L = \pi(0.05 \text{ m})(1 \text{ m}) = 0.1571 \text{ m}^2 \qquad h_o = \frac{k}{D_o}Nu = \frac{0.02592 \text{ W/m.°C}}{0.05 \text{ m}}(7.838) = 4.063 \text{ W/m}^2\text{.°C}$$

$$\dot{Q} = hA_o(T_o - T_\infty) = (4.063 \text{ W/m}^2\text{.°C})(0.1571 \text{ m}^2)(T_o - 26)°\text{C} \qquad \text{(Eq. 2)}$$

Solving Eq. 1 and Eq. 2 simultaneously, we find

$$T_o = 38.8°\text{C}, \quad \dot{Q} = 8.18 \text{ W}$$

Repeating the calculations at the new average temperature for enclosure analysis and at the new film temperature for convection at the outer surface analysis using the new calculated temperature 38.8°C, we find

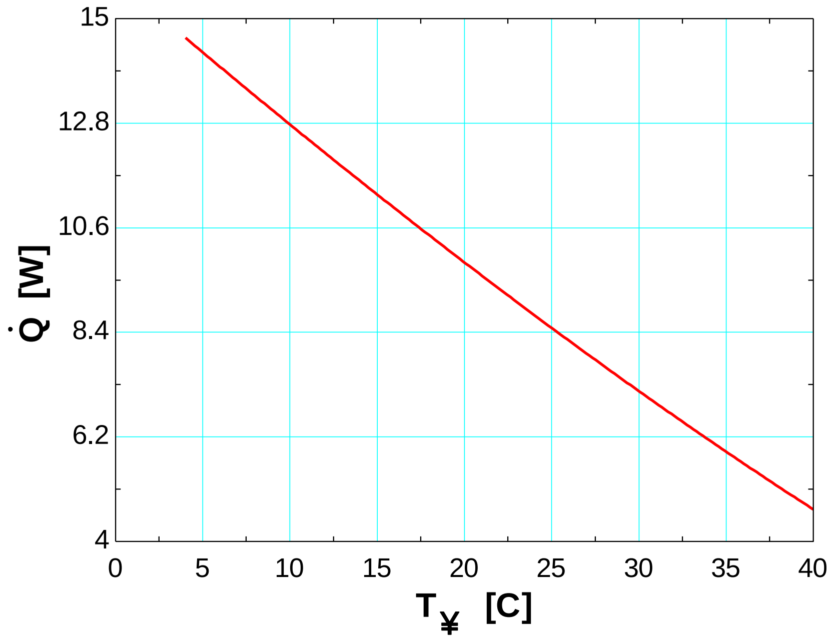
$$T_o = 39.0°\text{C}, \quad \dot{Q} = \mathbf{8.22 \text{ W}}$$

9-101 Prob. 9-100 is reconsidered. The rate of heat loss from the water by natural convection as a function of the ambient air temperature is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D_1=0.016 [m]
D_2=0.05 [m]
T_1=65 "[C]"
T_infinity=26 [C]
Length=1 [m] "unit length of the tube is considered"

"PROPERTIES for enclosure"
Fluid$='air'
k_1=Conductivity(Fluid$, T=T_ave)
Pr_1=Prandtl(Fluid$, T=T_ave)
rho_1=Density(Fluid$, T=T_ave, P=101.3)
mu_1=Viscosity(Fluid$, T=T_ave)
nu_1=mu_1/rho_1
beta_1=1/(T_ave+273)
T_ave=1/2*(T_1+T_2)
g=9.807 [m/s^2]

"ANALYSIS for enclosure"
L=(D_2-D_1)/2
Ra_1=(g*beta_1*(T_1-T_2)*L^3)/nu_1^2*Pr_1
F_cyl=(ln(D_2/D_1))^4/(L^3*(D_1^(-3/5)+D_2^(-3/5))^5)
k_eff=0.386*k_1*(Pr_1/(0.861+Pr_1))^0.25*(F_cyl*Ra_1)^0.25
Q_dot=(2*pi*k_eff)/ln(D_2/D_1)*(T_1-T_2)

"PROPERTIES for convection on the outer surface"
k_2=Conductivity(Fluid$, T=T_film)
Pr_2=Prandtl(Fluid$, T=T_film)
rho_2=Density(Fluid$, T=T_film, P=101.3)
mu_2=Viscosity(Fluid$, T=T_film)
nu_2=mu_2/rho_2
beta_2=1/(T_film+273)
T_film=1/2*(T_2+T_infinity)

"ANALYSIS for convection on the outer surface"
delta=D_2
Ra_2=(g*beta_2*(T_2-T_infinity)*delta^3)/nu_2^2*Pr_2
Nusselt=(0.6+(0.387*Ra_2^(1/6))/(1+(0.559/Pr_2)^(9/16))^(8/27))^2
h=k_2/delta*Nusselt
A=pi*D_2*Length
Q_dot=h*A*(T_2-T_infinity)
```

T_∞ [W]	$\dot{Q}$ [W]
4	14.6
6	13.98
8	13.37
10	12.77
12	12.18
14	11.59
16	11.01
18	10.44
20	9.871
22	9.314
24	8.764
26	8.222
28	7.688
30	7.163
32	6.647
34	6.139
36	5.641
38	5.153
40	4.675



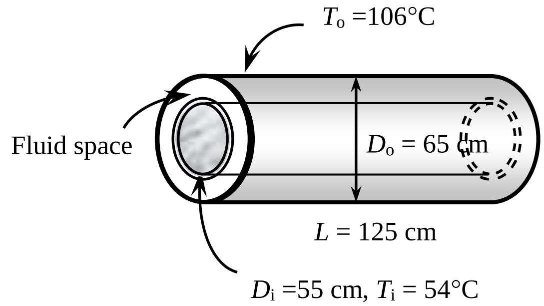
9-102 The space between the two concentric cylinders is filled with water or air. The rate of heat transfer from the outer cylinder to the inner cylinder by natural convection is to be determined for both cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The air pressure in the enclosure is 1 atm. **4** Heat transfer by radiation is negligible.

Properties The properties of water air at the average temperature of $(T_i+T_o)/2 = (54+106)/2 = 80°C$ are (Table A-9)

$$k = 0.670 \text{ W/m.°C}, \quad \nu = 3.653\times10^{-7} \text{ m}^2\text{/s}$$

$$\text{Pr} = 2.22, \quad \beta = 0.653\times10^{-3} \text{ K}^{-1}$$



The properties of air at 1 atm and the average temperature of $(T_i+T_o)/2 = (54+106)/2 = 80°C$ are (Table A-15)

$$k = 0.02953 \text{ W/m.°C}, \quad \nu = 2.097\times10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7154, \quad \beta = \frac{1}{T_f} = \frac{1}{(80+273)\text{K}} = 0.002833 \text{ K}^{-1}$$

Analysis (*a*) The fluid is water:

$$L_c = \frac{D_o - D_i}{2} = \frac{65-55}{2} = 5 \text{ cm.}$$

$$Ra = \frac{g\beta(T_o - T_i)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.653\times10^{-3} \text{ K}^{-1})(106-54)\text{K}(0.05 \text{ m})^3}{(3.653\times10^{-7} \text{ m}^2\text{/s})^2}(2.22) = 6.927\times10^8$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{\left[\ln\frac{D_o}{D_i}\right]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} = \frac{\left[\ln\frac{0.65 \text{ m}}{0.55 \text{ m}}\right]^4}{(0.05 \text{ m})^3\left[(0.55 \text{ m})^{-7/5} + (0.65 \text{ m})^{-7/5}\right]^5} = 0.04136$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$

$$= 0.386(0.670 \text{ W/m.°C})\left(\frac{2.22}{0.861+2.22}\right)^{1/4}\left[(0.04136)(6.927\times10^8)\right]^{1/4} = 17.43 \text{ W/m.°C}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln\left(\frac{D_o}{D_i}\right)}(T_0 - T_i) = \frac{2\pi(17.43 \text{ W/m.°C})}{\ln\left(\frac{0.65 \text{ m}}{0.55 \text{ m}}\right)}(106-54) = 34{,}090 \text{ W} = \mathbf{34.1 \text{ kW}}$$

(*b*) The fluid is air:

$$Ra = \frac{g\beta(T_o - T_i)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.002833 \text{ K}^{-1})(106-54)\text{K}(0.05 \text{ m})^3}{(2.097\times10^{-5} \text{ m}^2\text{/s})^2}(0.7154) = 2.939\times10^5$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{\left[\ln\frac{D_o}{D_i}\right]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} = \frac{\left[\ln\frac{0.65 \text{ m}}{0.55 \text{ m}}\right]^4}{(0.05 \text{ m})^3\left[(0.55 \text{ m})^{-7/5} + (0.65 \text{ m})^{-7/5}\right]^5} = 0.04136$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$

$$= 0.386(0.02953 \text{ W/m.°C})\left(\frac{0.7154}{0.861+0.7154}\right)^{1/4}\left[(0.04136)(2.939\times10^5)\right]^{1/4} = 0.09824 \text{ W/m.°C}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln\left(\frac{D_o}{D_i}\right)}(T_o - T_i) = \frac{2\pi(0.09824 \text{ W/m.°C})}{\ln\left(\frac{0.65 \text{ m}}{0.55 \text{ m}}\right)}(106-54) = \mathbf{192 \text{ W}}$$

9-103 A hot fluid flowing inside a horizontal tube with a known mass flow rate and temperature difference between the tube inlet and outlet. The tube is enclosed in a concentric cylindrical thin cover. The concentric outer cover temperature is to be determined whether it is safe from thermal burn hazards.

Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** Air is an ideal gas with constant properties. **4** Heat loss by radiation is negligible. **5** The air pressure in the enclosure is 1 atm.

Properties The properties of air at the assumed T_{avg} = 80°C and 1 atm pressure are k = 0.02953 W/m·K, $\nu = 2.097 \times 10^{-5}$ m^2/s, Pr = 0.7154 (Table A-15), and $\beta = 1/T_{avg}$ = 1/353 K.

Analysis With the assumption that T_{avg} = 80°C, the outer surface temperature is estimated as

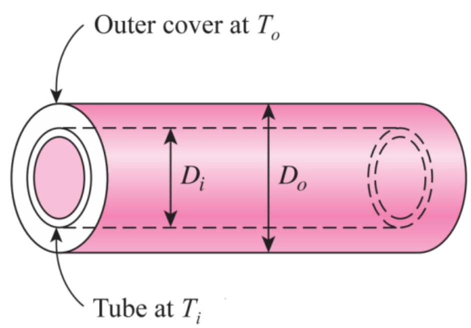
$$T_o = 2T_{avg} - T_i = 40°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(80+273\,\text{K})^{-1}(120-40)\,\text{K}\,(0.0125\,\text{m})^3}{(2.097\times10^{-5}\,\text{m}^2/\text{s})^2}(0.7154) = 7064$$

where

$$L_c = (D_o - D_i)/2 = 0.0125\,\text{m}$$



The effective thermal conductivity is

$$F_{cyl} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5}$$

$$= \frac{\ln(0.05/0.025)^4}{(0.0125\,\text{m})^3[(0.025\,\text{m})^{-3/5} + (0.05\,\text{m})^{-3/5}]^5} = 0.1466$$

$$k_{eff} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{cyl}\text{Ra})^{1/4}$$

$$= 0.386(0.02953\,\text{W/m}\cdot\text{K})\left(\frac{0.7154}{0.861+0.7154}\right)^{1/4}\left[(0.1466)(7064)\right]^{1/4} = 0.05307\,\text{W/m}\cdot\text{K}$$

Thus, the temperature of the outer cylinder can be determined using

$$\dot{Q} = \frac{2\pi k_{eff}}{\ln(D_o/D_i)}(T_i - T_o)$$

$$\dot{m}c_p\Delta T = \frac{2\pi k_{eff}}{\ln(D_o/D_i)}(T_i - T_o) \quad \rightarrow \quad T_o = \mathbf{41°C}$$

where

$$\dot{m} = 0.005\,\text{kg/s}, \quad c_p = 950\,\text{J/kg}\cdot\text{K}, \quad \Delta T = 8°\text{C}, \quad T_i = 120°\text{C}$$

Discussion The air gap between the concentric cylinders is sufficient to keep the outer cover temperature below 45°C to prevent thermal burn hazards.

The assumed average temperature T_{avg} = 80°C is appropriate for evaluating the air properties, since the determined T_o = 41°C would give an average temperature of T_{avg} = 80.5°C.

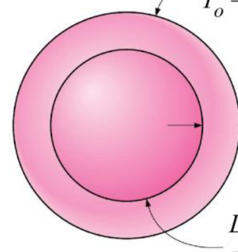
9-104 Two surfaces of a spherical enclosure are maintained at specified temperatures. The rate of heat transfer through the enclosure is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Nitrogen is an ideal gas with constant properties. **3** Radiation heat transfer is negligible

Properties The properties of nitrogen at $T_f = (T_s + T_\infty)/2 = 150°C$ are $k = 0.03416$ W/m·K, $\nu = 2.851 \times 10^{-5}$ m²/s, Pr = 0.7025 (from Table A-16). Also, $\beta = 1/T_f = 0.002364$ K^{-1}.

Analysis The characteristic length in this case is determined from

$$L_c = \frac{D_o - D_i}{2} = \frac{10-5}{2}\text{ cm} = 2.5\text{ cm}$$



The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_o - T_i)L_c^3}{\nu^2}\text{Pr}$$
$$= \frac{(9.81\text{ m/s}^2)(2.364\times10^{-3}\text{ K}^{-1})(200-100)\text{K}(0.025\text{ m})^3}{(2.851\times10^{-5}\text{ m}^2\text{/s})^2}(0.7025)$$
$$= 3.132\times10^4$$

The effective thermal conductivity is

$$F_{\text{sph}} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5} + D_o^{-7/5})^5} = \frac{(0.025\text{ m})}{[(0.1\text{ m})(0.05\text{ m})]^4[(0.05\text{ m})^{-7/5} + (0.1\text{ m})^{-7/5}]^5}$$
$$= 0.006268$$

$$k_{\text{eff}} = 0.74\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{sph}}\text{Ra}_L)^{1/4}k$$
$$= 0.74\left(\frac{0.7025}{0.861+0.7025}\right)^{1/4}[(0.006268)(3.132\times10^4)]^{1/4}(0.03416\text{ W/m}\cdot\text{K})$$
$$= 0.07747\text{ W/m}\cdot\text{K}$$

Then the rate of heat transfer between the spheres becomes

$$\dot{Q} = k_{\text{eff}}\frac{\pi D_i D_o}{L_c}(T_o - T_i) = (0.07747\text{ W/m}\cdot\text{K})\frac{\pi(0.1\text{ m})(0.05\text{ m})}{(0.025\text{ m})}(200-100)\text{ K} = \mathbf{4.87\text{ W}}$$

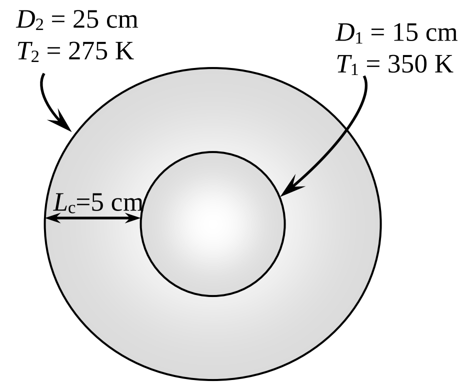
Discussion Note that if $k_{\text{eff}} < k$, then we use $k = k_{\text{eff}}$.

9-105 Two surfaces of a spherical enclosure are maintained at specified temperatures. The rate of heat transfer through the enclosure is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The air pressure in the enclusure is 1 atm.

Properties The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (350+275)/2 = 312.5$ K = 39.5°C are (Table A-15)

$$k = 0.02658 \text{ W/m.°C}$$
$$\nu = 1.697\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7256$$
$$\beta = \frac{1}{T_f} = \frac{1}{312.5\text{ K}} = 0.003200\text{ K}^{-1}$$



Analysis The characteristic length in this case is determined from

$$L_c = \frac{D_2 - D_1}{2} = \frac{25-15}{2} = 5\,\text{cm}.$$

Then,

$$Ra = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003200\text{ K}^{-1})(350-275\text{ K})(0.05\text{ m})^3}{(1.697\times10^{-5}\text{ m}^2/\text{s})^2}(0.7256) = 7.415\times10^5$$

The effective thermal conductivity is

$$F_{\text{sph}} = \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5} = \frac{0.05\text{ m}}{[(0.15\text{ m})(0.25\text{ m})]^4 [(0.15\text{ m})^{-7/5} + (0.25\text{ m})^{-7/5}]^5} = 0.005900$$

$$k_{\text{eff}} = 0.74k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{sph}Ra)^{1/4}$$
$$= 0.74(0.02658\text{ W/m.°C})\left(\frac{0.7256}{0.861+0.7256}\right)^{1/4}[(0.00590)(7.415\times10^5)]^{1/4}$$
$$= 0.1315\text{ W/m.°C}$$

Then the rate of heat transfer between the spheres becomes

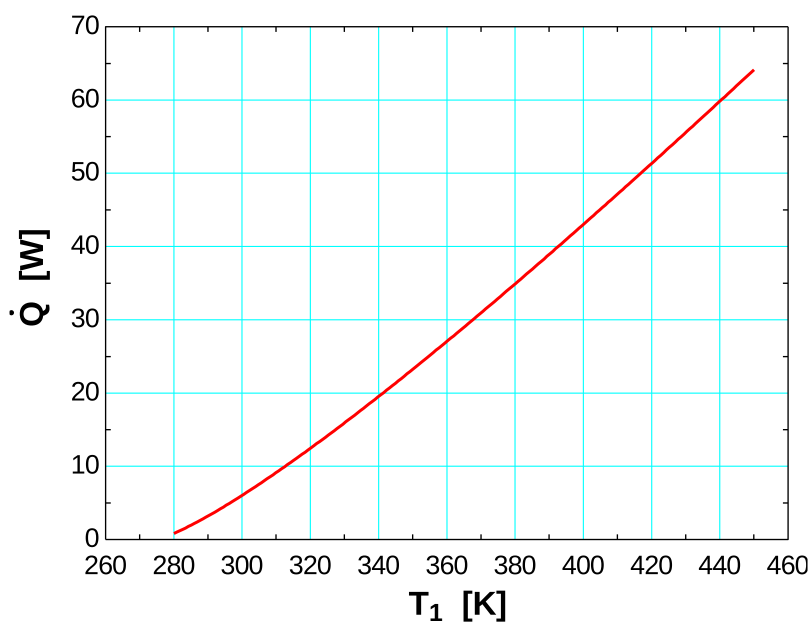
$$\dot{Q} = k_{\text{eff}}\,\pi\left(\frac{D_i D_o}{L_c}\right)(T_i - T_o) = (0.1315\text{ W/m.°C})\pi\left[\frac{(0.15\text{ m})(0.25\text{ m})}{(0.05\text{ m})}\right](350-275)\text{K} = \mathbf{23.3\ W}$$

9-106 Prob. 9-105 is reconsidered. The rate of natural convection heat transfer as a function of the hot surface temperature of the sphere is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

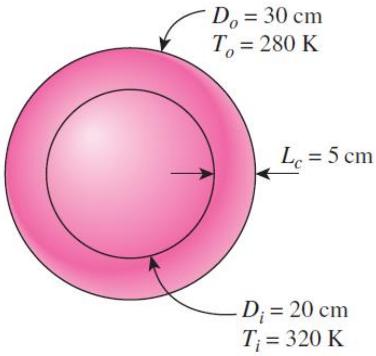
```
"GIVEN"
D_1=0.15 [m]
D_2=0.25 [m]
T_1=350 [K]
T_2=275 [K]

"PROPERTIES"
Fluid$='air'
k=Conductivity(Fluid$, T=T_ave)
Pr=Prandtl(Fluid$, T=T_ave)
rho=Density(Fluid$, T=T_ave, P=101.3)
mu=Viscosity(Fluid$, T=T_ave)
nu=mu/rho
beta=1/T_ave
T_ave=1/2*(T_1+T_2)
g=9.807 [m/s^2]

"ANALYSIS"
L=(D_2-D_1)/2
Ra=(g*beta*(T_1-T_2)*L^3)/nu^2*Pr
F_sph=L/((D_1*D_2)^4*(D_1^(-7/5)+D_2^(-7/5))^5)
k_eff=0.74*k*(Pr/(0.861+Pr))^0.25*(F_sph*Ra)^0.25
Q_dot=k_eff*pi*(D_1*D_2)/L*(T_1-T_2)
```

T_1 [K]	$\dot{Q}$ [W]
250	-
260	-
270	-
280	0.8153
290	3.202
300	6.032
310	9.139
320	12.45
330	15.92
340	19.52
350	23.23
360	27.04
370	30.93
380	34.89
390	38.92
400	43.01
410	47.15
420	51.33
430	55.56
440	59.83
450	64.13



9-107 Two surfaces of a spherical enclosure are maintained at specified temperatures. The rate of heat transfer through the enclosure is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Radiation heat transfer is not considered.



Properties The properties of air at the average temperature of $T_{avg} = (T_i + T_o)/2 = (320 + 280)/2 = 300$ K = 27°C and 1 atm pressure are: $k = 0.02566$ W/m·K, Pr = 0.7290, $\nu = 1.58 \times 10^{-5}$ m²/s (Table A-15), and $\beta = 1/T_{avg} = 1/300$ K.

Analysis We have a spherical enclosure filled with air. The characteristic length in this case is the distance between the two spheres,

$$L_c = (D_o - D_i)/2 = (0.3 - 0.2)/2 = 0.05\text{ m}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(300\text{ K})^{-1}(320-280)\text{ K}(0.05\text{ m})^3}{(1.58\times10^{-5}\text{ m}^2/\text{s})^2}(0.729)$$
$$= 4.775\times10^5$$

The effective thermal conductivity is

$$F_{sph} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5}+D_o^{-7/5})^5} = \frac{0.05\text{ m}}{[(0.2\text{ m})(0.3\text{ m})]^4[(0.2\text{ m})^{-7/5}+(0.3\text{ m})^{-7/5}]^5} = 0.0005229$$

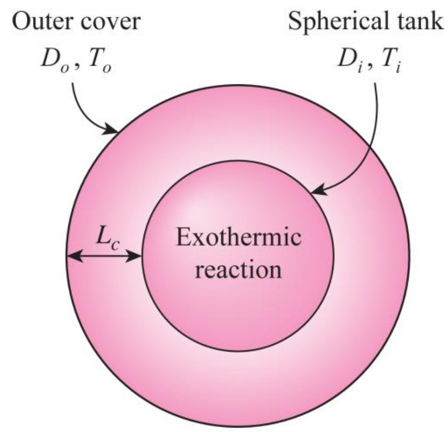
$$k_{\text{eff}} = 0.74k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{sph}\text{Ra})^{1/4}$$
$$= 0.74(0.02566\text{ W/m}\cdot\text{K})\left(\frac{0.729}{0.861+0.729}\right)^{1/4}[(0.0005229)(4.775\times10^5)]^{1/4}$$
$$= 0.1105\text{ W/m}\cdot\text{K}$$

Then, the rate of heat transfer between spheres becomes

$$\dot{Q} = k_{\text{eff}}\,\pi\left(\frac{D_iD_o}{L_c}\right)(T_i - T_o)$$
$$= (0.1105)\pi\left(\frac{0.2\text{ m}\times0.3\text{m}}{0.05\text{m}}\right)(320-280)\text{ K} = \mathbf{16.7\,W}$$

Discussion Note that the air in the spherical enclosure acts like a stationary fluid whose thermal conductivity is k_{eff}/k = 0.1105/0.02566 = 4.3 times that of air as a result of natural convection currents. Also, radiation heat transfer between spheres is usually significant, and should be considered in a complete analysis.

9-108 PtD A metal spherical tank is filled with a solution undergoing an exothermic reaction and the heat generation is known. The tank in enclosed by a concentric outer cover to prevent thermal burn hazards. The temperature of the outer cover is to be determined.



Assumptions **1** Steady operating conditions exist. **2** Surface temperatures are constant. **3** Air is an ideal gas with constant properties. **4** Heat loss by radiation is negligible. **5** The air pressure in the enclosure is 1 atm.

Properties The properties of air at the assumed T_{avg} = 80°C and 1 atm pressure are k = 0.02953 W/m·K, ν = 2.097 × 10^{-5} m^2/s, Pr = 0.7154 (Table A-15), and $\beta = 1/T_f$ = 1/ 353 K.

Analysis With the assumption that T_{avg} = 80°C, the outer surface temperature is estimated as

$$T_o = 2T_{avg} - T_i = 40°\text{C}$$

The Rayleigh number is

$$\text{Ra} = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr}$$
$$= \frac{(9.81\,\text{m/s}^2)(80+273\,\text{K})^{-1}(120-40)\,\text{K}\,(0.05\,\text{m})^3}{(2.097\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7154) = 452{,}112$$

where

$$L_c = (D_o - D_i)/2 = 0.05\,\text{m}$$

The effective thermal conductivity is

$$F_{\text{sph}} = \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5} = \frac{0.05\,\text{m}}{[(3.0\,\text{m})(3.1\,\text{m})]^4[(3.0\,\text{m})^{-7/5} + (3.1\,\text{m})^{-7/5}]^5} = 0.0005117$$

$$k_{\text{eff}} = 0.74k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{sph}\text{Ra})^{1/4}$$
$$= 0.74(0.02953\ \text{W/m}\cdot\text{K})\left(\frac{0.7154}{0.861+0.7154}\right)^{1/4}[(0.0005117)(452{,}112)]^{1/4} = 0.06995\ \text{W/m}\cdot\text{K}$$

Thus, the temperature of the outer cover can be determined using

$$\dot{Q} = k_{\text{eff}}\,\pi\left(\frac{D_i D_o}{L_c}\right)(T_i - T_o)$$

$$\dot{e}_{\text{gen}}(\pi D_i^3/6) = k_{\text{eff}}\,\pi\left(\frac{D_i D_o}{L_c}\right)(T_i - T_o) \quad \rightarrow \quad T_o = \mathbf{39.4°C}$$

where

$$\dot{e}_{\text{gen}} = 233\ \text{W/m}^3$$

Discussion The air gap between the concentric spheres is sufficient to keep the outer cover temperature below 45°C to prevent thermal burn hazards.

The assumed average temperature T_{avg} = 80°C is appropriate for evaluating the air properties, since the determined T_o = 39.4°C would give an average temperature of T_{avg} = 79.7°C.

Combined Natural and Forced Convection

9-109C In combined natural and forced convection, the natural convection is negligible when $Gr/\mathrm{Re}^2 < 0.1$. Otherwise it is not.

9-110C In assisting or transverse flows, natural convection enhances forced convection heat transfer while in opposing flow it hurts forced convection.

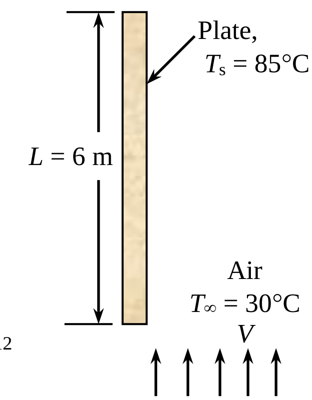
9-111C When neither natural nor forced convection is negligible, it is not correct to calculate each separately and to add them to determine the total convection heat transfer. Instead, the correlation

$$\mathrm{Nu}_{\mathrm{combined}} = \left(\mathrm{Nu}_{\mathrm{forced}}^{n} + \mathrm{Nu}_{\mathrm{natural}}^{n}\right)^{1/n}$$

based on the experimental studies should be used.

9-112 A vertical plate in air is considered. The forced motion velocity above which natural convection heat transfer from the plate is negligible is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The atmospheric pressure at that location is 1 atm.



Properties The properties of air at 1 atm and 1 atm and the film temperature of $(T_s+T_\infty)/2 = (85+30)/2 = 57.5$°C are (Table A-15)

$$\nu = 1.872\times10^{-5}\ \mathrm{m^2/s}$$

$$\beta = \frac{1}{T_f} = \frac{1}{(57.5+273)\mathrm{K}} = 0.003026\ \mathrm{K^{-1}}$$

Analysis The characteristic length is the height of the plate, $L_c = L = 6$ m. The Grashof and Reynolds numbers are

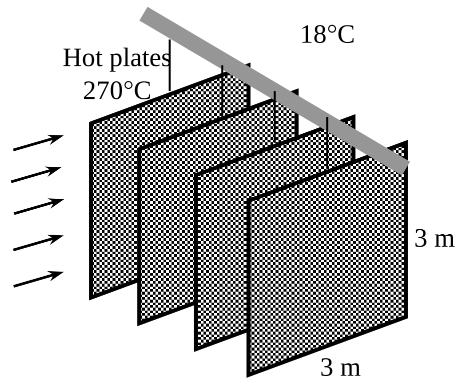
$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81\ \mathrm{m/s^2})(0.003026\ \mathrm{K^{-1}})(85-30\ \mathrm{K})(6\ \mathrm{m})^3}{(1.872\times10^{-5}\ \mathrm{m^2/s})^2} = 1.006\times10^{12}$$

$$\mathrm{Re} = \frac{VL}{\nu} = \frac{V_\infty(6\ \mathrm{m})}{1.872\times10^{-5}\ \mathrm{m^2/s}} = 3.205\times10^5 V$$

and the forced motion velocity above which natural convection heat transfer from this plate is negligible is

$$\frac{Gr}{\mathrm{Re}^2} = 0.1 \longrightarrow \frac{1.006\times10^{12}}{(3.205\times10^5 V)^2} = 0.1 \longrightarrow V = \mathbf{9.90\ m/s}$$

9-113 Thin square plates coming out of the oven in a production facility are cooled by blowing ambient air horizontally parallel to their surfaces. The air velocity above which the natural convection effects on heat transfer are negligible is to be determined.



Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The atmospheric pressure at that location is 1 atm.

Properties The properties of air at 1 atm and 1 atm and the film temperature of $(T_s+T_\infty)/2 = (270+18)/2 = 144$°C are (Table A-15)

$$\nu = 2.791\times10^{-5}\ \text{m}^2/\text{s}$$

$$\beta = \frac{1}{T_f} = \frac{1}{(144+273)\text{K}} = 0.002398\ \text{K}^{-1}$$

Analysis The characteristic length is the height of the plate $L_c = L = 3$ m. The Grashof and Reynolds numbers are

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81\ \text{m/s}^2)(0.002398\ \text{K}^{-1})(270-18\ \text{K})(3\ \text{m})^3}{(2.791\times10^{-5}\ \text{m}^2/\text{s})^2} = 2.055\times10^{11}$$

$$\text{Re} = \frac{VL}{\nu} = \frac{V(3\ \text{m})}{2.791\times10^{-5}\ \text{m}^2/\text{s}} = 1.075\times10^5 V$$

and the forced motion velocity above which natural convection heat transfer from this plate is negligible is

$$\frac{Gr}{\text{Re}^2} = 0.1 \longrightarrow \frac{2.055\times10^{11}}{(1.075\times10^5 V)^2} = 0.1 \longrightarrow V = \mathbf{13.3\ m/s}$$

9-114 The significance of natural convection to the heat transfer process on a vertical rod with water flowing across its outer surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The rod is orientated such that the characteristic length is its length.

Properties The properties of water at $T_f = (T_s + T_\infty)/2 = 80$°C are $\rho = 971.8$ kg/m^3, $\mu = 0.355 \times 10^{-3}$ kg/m·s, and $\beta = 0.653 \times 10^{-3}$ K^{-1} (Table A-9).

Analysis The Reynolds number for the cross flow is

$$\text{R}e = \frac{\rho VD}{\mu} = \frac{(971.8\ \text{kg/m}^3)(0.5\ \text{m/s})(0.150\ \text{m})}{0.355\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 2.05\times10^5$$

For vertical cylinder, the Grashof number with $L_c = L$ is

$$\text{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} = \frac{g\beta(T_s - T_\infty)L_c^3}{(\mu/\rho)^2}$$

$$= \frac{(9.81\ \text{m/s}^2)(0.653\times10^{-3}\ \text{K}^{-1})(120-40)\ \text{K}(1\ \text{m})^3}{(0.355\times10^{-3}/971.8)^2\ \text{m}^4/\text{s}^2} = 3.84\times10^{12}$$

Hence,

$$\frac{\text{Gr}_L}{\text{Re}^2} = \frac{3.84\times10^{12}}{(2.05\times10^5)^2} = \mathbf{91.4}$$

Since $\text{Gr}_L/\text{Re}^2 >> 1$, therefore natural convection effects dominate forced convection effects. Natural convection effects are important to the heat transfer process.

Discussion If $\text{Gr}_L/\text{Re}^2 << 1$, then forced convection effects dominate natural convection effects.

9-115 The significance of natural convection to the heat transfer process on a horizontal rod with water flowing across its outer surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The rod is orientated such that the characteristic length is its diameter.

Properties The properties of water at $T_f = (T_s + T_\infty)/2 = 80°C$ are $\rho = 971.8$ kg/m^3, $\mu = 0.355 \times 10^{-3}$ kg/m·s, and $\beta = 0.653 \times 10^{-3}$ K^{-1} (Table A-9).

Analysis The Reynolds number for the cross flow is

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{(971.8\ \mathrm{kg/m^3})(0.2\ \mathrm{m/s})(0.150\ \mathrm{m})}{0.355\times10^{-3}\ \mathrm{kg/m\cdot s}} = 8.21\times10^4$$

For horizontal cylinder, the Grashof number with $L_c = D$ is

$$\mathrm{Gr}_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2} = \frac{g\beta(T_s - T_\infty)D^3}{(\mu/\rho)^2}$$

$$= \frac{(9.81\ \mathrm{m/s^2})(0.653\times10^{-3}\ \mathrm{K^{-1}})(120-40)\ \mathrm{K}(0.15\ \mathrm{m})^3}{(0.355\times10^{-3}/971.8)^2\ \mathrm{m^4/s^2}} = 1.296\times10^{10}$$

Hence,

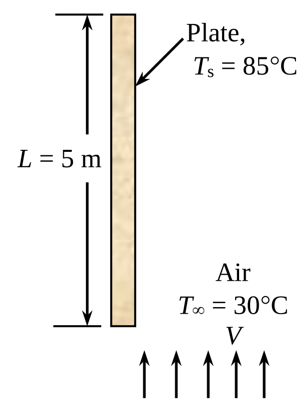
$$\frac{\mathrm{Gr}_D}{\mathrm{Re}^2} = \frac{6.48\times10^8}{(8.21\times10^4)^2} = \mathbf{1.92}$$

Since $\mathrm{Gr}_D/\mathrm{Re}^2 \approx 1$, therefore natural convection and forced convection effects are both important to the heat transfer process.

Discussion When both natural convection and forced convection effects are important, the process is called combined natural and forced convection or mixed convection.

9-116 A vertical plate in air is considered. The forced motion velocity above which natural convection heat transfer from the plate is negligible is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The atmospheric pressure at that location is 1 atm.



Properties The properties of air at 1 atm and 1 atm and the film temperature of $(T_s+T_\infty)/2 = (85+30)/2 = 57.5°C$ are (Table A-15)

$$\nu = 1.872\times10^{-5}\ \mathrm{m^2/s}$$

$$\beta = \frac{1}{T_f} = \frac{1}{(57.5+273)\mathrm{K}} = 0.003026\ \mathrm{K^{-1}}$$

Analysis The characteristic length is the height of the plate, $L_c = L = 5$ m.
The Grashof and Reynolds numbers are

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81\ \mathrm{m/s^2})(0.003026\ \mathrm{K^{-1}})(85-30\ \mathrm{K})(5\ \mathrm{m})^3}{(1.872\times10^{-5}\ \mathrm{m^2/s})^2} = 5.824\times10^{11}$$

$$\mathrm{Re} = \frac{VL}{\nu} = \frac{V_\infty(5\ \mathrm{m})}{1.872\times10^{-5}\ \mathrm{m^2/s}} = 2.671\times10^5 V$$

and the forced motion velocity above which natural convection heat transfer from this plate is negligible is

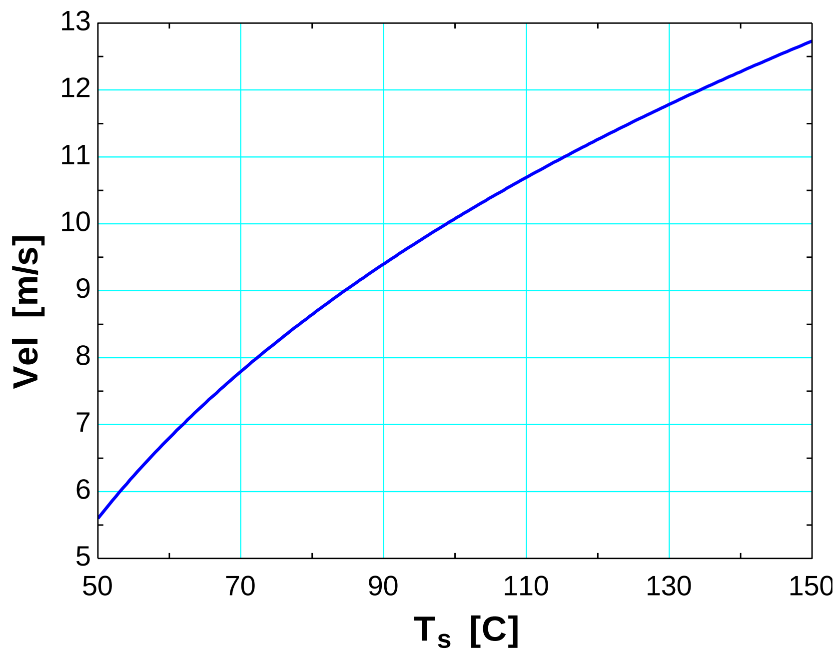
$$\frac{Gr}{\mathrm{Re}^2} = 0.1 \longrightarrow \frac{5.824\times10^{11}}{(2.671\times10^5 V)^2} = 0.1 \longrightarrow V = \mathbf{9.04\ m/s}$$

9-117 Prob. 9-116 is reconsidered. The forced motion velocity above which natural convection heat transfer is negligible as a function of the plate temperature is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=5 [m]
T_s=85 [C]
T_infinity=30 [C]

"PROPERTIES"
Fluid$='air'
rho=Density(Fluid$, T=T_film, P=101.3)
mu=Viscosity(Fluid$, T=T_film)
nu=mu/rho
beta=1/(T_film+273)
T_film=1/2*(T_s+T_infinity)
g=9.807 [m/s^2]

"ANALYSIS"
Gr=(g*beta*(T_s-T_infinity)*L^3)/nu^2
Re=(Vel*L)/nu
Gr/Re^2=0.1
```

T_s [C]	Vel [m/s]
50	5.598
55	6.233
60	6.801
65	7.318
70	7.793
75	8.233
80	8.646
85	9.033
90	9.4
95	9.747
100	10.08
105	10.39
110	10.69
115	10.98
120	11.26
125	11.53
130	11.79
135	12.03
140	12.27
145	12.51
150	12.73

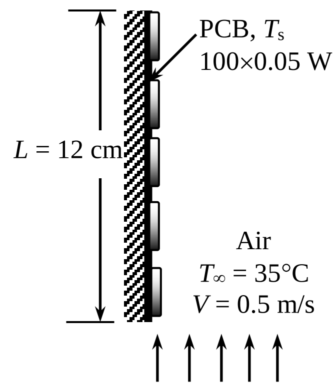


9-118 A circuit board is cooled by a fan that blows air upwards. The average temperature on the surface of the circuit board is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The atmospheric pressure at that location is 1 atm.

Properties Based on the problem statement, the properties of air at 1 atm and 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (60+35)/2 = 47.5°C$ are (Table A-15)

$$k = 0.02717 \text{ W/m.°C}$$
$$\nu = 1.774\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.7235$$
$$\beta = \frac{1}{T_f} = \frac{1}{(47.5+273)\text{K}} = 0.00312 \text{ K}^{-1}$$



Analysis We assume the surface temperature to be 60°C. We will check this assumption later on and repeat calculations with a better assumption, if necessary. The characteristic length in this case is the length of the board in the flow (vertical) direction, $L_c = L = 0.12$ m. Then the Reynolds number becomes

$$\text{Re} = \frac{VL}{\nu} = \frac{(0.5 \text{ m/s})(0.12 \text{ m})}{1.774\times10^{-5} \text{ m}^2/\text{s}} = 3383$$

which is less than critical Reynolds number (5×10^5). Therefore the flow is laminar and the forced convection Nusselt number and h are determined from

$$Nu = \frac{hL}{k} = 0.664\,\text{Re}^{0.5}\,\Pr^{1/3} = 0.664(3383)^{0.5}(0.7235)^{1/3} = 34.67$$

$$h = \frac{k}{L}Nu = \frac{0.02717 \text{ W/m.°C}}{0.12 \text{ m}}(34.67) = 7.85 \text{ W/m}^2.°\text{C}$$

$$A_s = L\times W = (0.12 \text{ m})(0.2 \text{ m}) = 0.024 \text{ m}^2$$

Then, $$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 35°\text{C} + \frac{(100)(0.05 \text{ W})}{(7.85 \text{ W/m}^2.°\text{C})(0.024 \text{ m}^2)} = \mathbf{61.5°C}$$

which is sufficiently close to the assumed value in the evaluation of properties. Therefore, there is no need to repeat calculations.

(b) The Rayleigh number is

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.00312 \text{ K}^{-1})(60-35 \text{ K})(0.12 \text{ m})^3}{(1.774\times10^{-5} \text{ m}^2/\text{s})^2}(0.7235) = 3.041\times10^6$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(3.041\times10^6)^{1/6}}{\left[1+\left(\frac{0.492}{0.7235}\right)^{9/16}\right]^{8/27}}\right\}^2 = 22.42$$

This is an assisting flow and the combined Nusselt number is determined from

$$Nu_{combined} = (Nu_{forced}{}^n + Nu_{natural}{}^n)^{1/n} = (34.67^3 + 22.42^3)^{1/3} = 37.55$$

Then, $$h = \frac{k}{L}Nu_{combined} = \frac{0.02717 \text{ W/m.°C}}{0.12 \text{ m}}(37.55) = 8.502 \text{ W/m}^2.°\text{C}$$

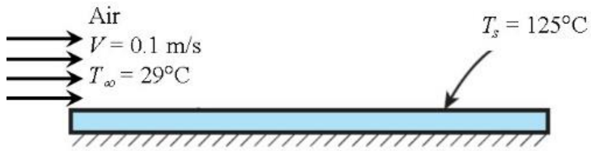
and $$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 35°\text{C} + \frac{(100)(0.05 \text{ W})}{(8.502 \text{ W/m}^2.°\text{C})(0.024 \text{ m}^2)} = \mathbf{59.5°C}$$

which is sufficiently close to the assumed surface temperature value of 60°C used in the evaluation of properties. Therefore, there is no need to repeat calculations. The natural convection lowers the surface temperature in this case by about 2°C.

9-119 A horizontal square plate is being cooled by blowing air over the surface of the plate. Determine the Nusslet number for this case.

Assumptions **1** Steady state conditions. **2** Air is an ideal gas. **3** The local atmospheric pressure is 1 atm. **4** Thermal radiation is negligible. **5** Backside of the plate is insulated.

Properties The properties of air at the film temperature of $T_f = (T_\infty + T_s)/2 = (29 + 125)/2 = 77°C$ are (Table A–15) Pr = 0.7161, $k = 0.02931$ W/m·K, $\nu = 2.066\times 10^{-5}$ m^2/s, $\beta = 1/T_f = 1/(77 + 273 \text{ K}) = 0.002857 \text{ K}^{-1}$



Analysis The characteristic length of the plate is

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.25 \text{ m}}{4} = 0.0625 \text{ m}$$

The Grashof, Rayleigh, and Reynolds numbers are

$$\text{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} = \frac{(9.81 \text{ m/s}^2)(0.002857 \text{ K}^{-1})(125 - 29 \text{ K})(0.0625 \text{ m})^3}{(2.066\times 10^{-5}\text{m}^2/\text{s})^2} = 1.539\times 10^6$$

$$\text{Ra}_L = \text{Gr}_L\,\text{Pr} = (1.539\times 10^6)(0.7161) = 1.102\times 10^6$$

$$\text{Re} = \frac{VL}{\nu} = \frac{(0.1 \text{ m/s})(0.25 \text{ m})}{2.066\times 10^{-5}\text{m}^2/\text{s}} = 1.21\times 10^3$$

Hence,

$$\frac{\text{Gr}_\text{L}}{\text{Re}^2} = \frac{1.539\times 10^6}{(1.21\times 10^3)^2} = 1.05$$

Noting that $\frac{\text{Gr}_L}{Re^2} \approx 1$, both natural convection and forced convection are significant in this case, and we have mixed flow.

The average Nusselt number for natural convection with horizontal hot surface facing up with Ra between 10^4 and 10^7 is determined from

$$\text{Nu}_\text{natural} = 0.54\,\text{Ra}_L^{1/4} = 0.54\,(1.102\times 10^6)^{1/4} = 17.5$$

The average Nusselt number for laminar flow (Re < 5×10^5) forced convection over a flat plate is determined from

$$\text{Nu}_\text{forced} = 0.664\,\text{Re}^{0.5}\text{Pr}^{1/3} = 0.664\,(1.21\times 10^3)^{0.5}(0.7161)^{1/3} = 20.7$$

Finally, the combined Nusselt numbers for the assisting flow is (use n=3.5 for horizontal plate)

$$\text{Nu}_\text{combined} = (\text{Nu}_\text{forced}^{3.5} + \text{Nu}_\text{natural}^{3.5})^{1/3.5} = (20.7^{3.5} + 17.5^{3.5})^{1/3.5} = \mathbf{23.5}$$

Discussion Note that the combined Nusselt number is 34% higher for natural convection Nusselt number and about 14% higher than forced convection Nusselt number. Therefore, both natural and forced convection are important in this case.

Special Topic: Heat Transfer through Windows

9-120C Windows are considered in three regions when analyzing heat transfer through them because the structure and properties of the frame are quite different than those of the glazing. As a result, heat transfer through the frame and the edge section of the glazing adjacent to the frame is two-dimensional. Even in the absence of solar radiation and air infiltration, heat transfer through the windows is more complicated than it appears to be. Therefore, it is customary to consider the windows in three regions when analyzing heat transfer through them: (1) the *center-of-glass*, (2) the *edge-of-glass*, and (3) the *frame* regions. When the heat transfer coefficient for all three regions are known, the overall U-value of the window is determined from

$$U_{\text{window}} = (U_{\text{center}} A_{\text{center}} + U_{\text{edge}} A_{\text{edge}} + U_{\text{frame}} A_{\text{frame}}) / A_{\text{window}}$$

where A_{window} is the window area, and A_{center}, A_{edge}, and A_{frame} are the areas of the center, edge, and frame sections of the window, respectively, and U_{center}, U_{edge}, and U_{frame} are the heat transfer coefficients for the center, edge, and frame sections of the window.

9-121C Of the three similar double pane windows with air gab widths of 5, 10, and 20 mm, the U-factor and thus the rate of heat transfer through the window will be a minimum for the window with 10-mm air gab, as can be seen from Fig. 9-37.

9-122C In an ordinary double pane window, about half of the heat transfer is by radiation. A practical way of reducing the radiation component of heat transfer is to reduce the emissivity of glass surfaces by coating them with low-emissivity (or "low-e") material.

9-123C When a thin polyester film is used to divide the 20-mm wide air of a double pane window space into two 10-mm wide layers, both (*a*) convection and (*b*) radiation heat transfer through the window will be reduced.

9-124C When a double pane window whose air space is flashed and filled with argon gas, (*a*) convection heat transfer will be reduced but (*b*) radiation heat transfer through the window will remain the same.

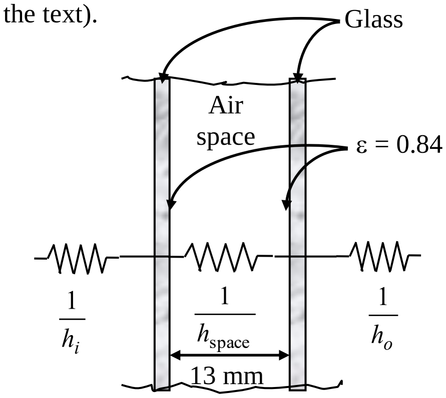
9-125C The heat transfer rate through the glazing of a double pane window is higher at the edge section than it is at the center section because of the two-dimensional effects due to heat transfer through the frame.

9-126C The *U*-factors of windows with aluminum frames will be highest because of the higher conductivity of aluminum. The *U*-factors of wood and vinyl frames are comparable in magnitude.

9-127 The U-factor for the center-of-glass section of a double pane window is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** The thermal resistance of glass sheets is negligible.

Properties The emissivity of clear glass is given to be 0.84. The values of h_i and h_o for winter design conditions are h_i = 8.29 W/m².°C and h_o = 34.0 W/m².°C (from the text).



Analysis Disregarding the thermal resistance of glass sheets, which are small, the U-factor for the center region of a double pane window is determined from

$$\frac{1}{U_{\text{center}}} \cong \frac{1}{h_i} + \frac{1}{h_{\text{space}}} + \frac{1}{h_o}$$

where h_i, h_{space}, and h_o are the heat transfer coefficients at the inner surface of window, the air space between the glass layers, and the outer surface of the window, respectively. The effective emissivity of the air space of the double pane window is

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \frac{1}{1/0.84 + 1/0.84 - 1} = 0.72$$

For this value of emissivity and an average air space temperature of 10°C with a temperature difference across the air space to be 15°C, we read h_{space} = 5.7 W/m².°C from Table 9-3 for 13-mm thick air space. Therefore,

$$\frac{1}{U_{\text{center}}} = \frac{1}{8.29} + \frac{1}{5.7} + \frac{1}{34.0} \longrightarrow U_{\text{center}} = \mathbf{3.07\ W/m^2 \cdot {}^\circ C}$$

Discussion The overall U-factor of the window will be higher because of the edge effects of the frame.

9-128 The overall U-factor of a window is given to be U = 2.76 W/m².°C for 12 km/h winds outside. The new U-factor when the wind velocity outside is doubled is to be determined.

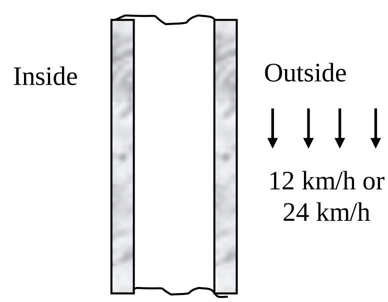
Assumptions Thermal properties of the windows and the heat transfer coefficients are constant.

Properties The heat transfer coefficients at the outer surface of the window are h_o = 22.7 W/m².°C for 12 km/h winds, and h_o = 34.0 W/m².°C for 24 km/h winds (from the text).

Analysis The corresponding convection resistances for the outer surfaces of the window are

$$R_{\text{o,12 km/h}} = \frac{1}{h_{\text{o,12 km/h}}} = \frac{1}{22.7\ \text{W/m}^2.^\circ\text{C}} = 0.044\ \text{m}^2.^\circ\text{C/W}$$

$$R_{\text{o, 24 km/h}} = \frac{1}{h_{\text{o, 24 km/h}}} = \frac{1}{34.0\ \text{W/m}^2.^\circ\text{C}} = 0.029\ \text{m}^2.^\circ\text{C/W}$$



Also, the R-value of the window at 12 km/h winds is

$$R_{\text{window,12 km/h}} = \frac{1}{U_{\text{window,12 km/h}}} = \frac{1}{2.76\ \text{W/m}^2.^\circ\text{C}} = 0.362\ \text{m}^2.^\circ\text{C/W}$$

Noting that all thermal resistances are in series, the thermal resistance of the window for 24 km/h winds is determined by replacing the convection resistance for 12 km/h winds by the one for 24 km/h:

$$R_{\text{window, 24 km/h}} = R_{\text{window,12 km/h}} - R_{\text{o,12 km/h}} + R_{\text{o, 24 km/h}} = 0.362 - 0.044 + 0.029 = 0.347\ \text{m}^2.^\circ\text{C/W}$$

Then the U-factor for the case of 24 km/h winds becomes

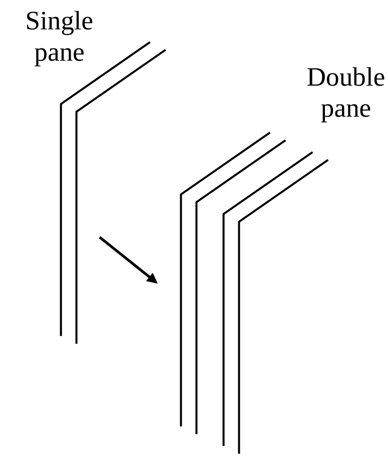
$$U_{\text{window, 24 km/h}} = \frac{1}{R_{\text{window, 24 km/h}}} = \frac{1}{0.347\ \text{m}^2.^\circ\text{C/W}} = \mathbf{2.88\ W/m^2.{}^\circ C}$$

Discussion Note that doubling of the wind velocity increases the U-factor only slightly (about 4%) from 2.76 to 2.88 W/m².°C.

9-129 The existing wood framed single pane windows of an older house in Wichita are to be replaced by double-door type vinyl framed double pane windows with an air space of 6.4 mm. The amount of money the new windows will save the home owner per month is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the windows and the heat transfer coefficients are constant. **4** Infiltration heat losses are not considered.

Properties The U-factors of the windows are 5.57 W/m^2.°C for the old single pane windows, and 3.20 W/m^2.°C. for the new double pane windows (Table 9-6).



Analysis The rate of heat transfer through the window can be determined from

$$\dot{Q}_{\text{window}} = U_{\text{overall}} A_{\text{window}} (T_i - T_o)$$

where T_i and T_o are the indoor and outdoor air temperatures, respectively, U_{overall} is the U-factor (the overall heat transfer coefficient) of the window, and A_{window} is the window area. Noting that the heaters will turn on only when the outdoor temperature drops below 18°C, the rates of heat transfer due to electric heating for the old and new windows are determined to be

$$\dot{Q}_{\text{window, old}} = (5.57\ \text{W/m}^2.°\text{C})(17\ \text{m}^2)(18 - 7.1)°\text{C} = 1032\ \text{W}$$

$$\dot{Q}_{\text{window, new}} = (3.20\ \text{W/m}^2.°\text{C})(17\ \text{m}^2)(18 - 7.1)°\text{C} = 593\ \text{W}$$

$$\dot{Q}_{\text{saved}} = \dot{Q}_{\text{window, old}} - \dot{Q}_{\text{window, new}} = 1032 - 593 = 439\ \text{W}$$

Then the electrical energy and cost savings per month becomes

$$\text{Energy savings} = \dot{Q}_{\text{saved}} \Delta t = (0.439\ \text{kW})(30 \times 24\ \text{h/month}) = 316\ \text{kWh/month}$$

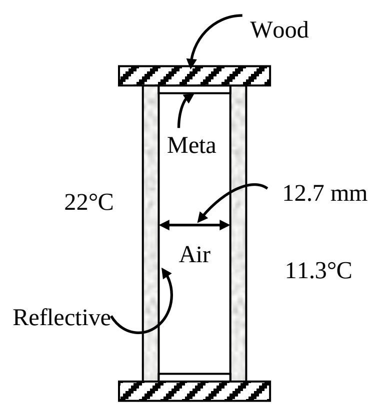
$$\text{Cost savings} = (\text{Energy savings})(\text{Unit cost of energy}) = (316\ \text{kWh/month})(\$0.085/\text{kWh}) = \mathbf{\$26.9/month}$$

Discussion We would obtain the same result if we used the actual indoor temperature (probably 22°C) for T_i instead of the balance point temperature of 18°C.

9-130 The windows of a house in Atlanta are of double door type with wood frames and metal spacers. The average rate of heat loss through the windows in winter is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the windows and the heat transfer coefficients are constant. **4** Infiltration heat losses are not considered.

Properties The U-factor of the window is given in Table 9-6 to be 2.13 W/m^2.°C.



Analysis The rate of heat transfer through the window can be determined from

$$\dot{Q}_{\text{window, avg}} = U_{\text{overall}} A_{\text{window}} (T_i - T_{o,avg})$$

where T_i and T_o are the indoor and outdoor air temperatures, respectively, U_{overall} is the U-factor (the overall heat transfer coefficient) of the window, and A_{window} is the window area. Substituting,

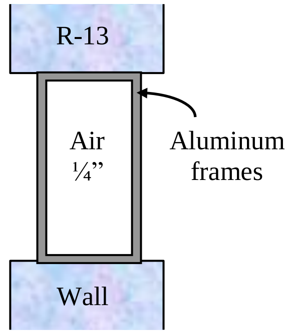
$$\dot{Q}_{\text{window, avg}} = (2.13\ \text{W/m}^2.°\text{C})(14\ \text{m}^2)(22 - 11.3)°\text{C} = \mathbf{319\ W}$$

Discussion This is the "average" rate of heat transfer through the window in winter in the absence of any infiltration.

9-131E The *R*-value of the common double door windows that are double pane with 1/4-in of air space and have aluminum frames is to be compared to the *R*-value of *R*-13 wall. It is also to be determined if more heat is transferred through the windows or the walls.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the windows and the heat transfer coefficients are constant. **4** Infiltration heat losses are not considered.

Properties The U-factor of the window is given in Table 9-6 to be $4.55 \times 0.176 = 0.801$ Btu/h.ft^2.°F.



Analysis The *R*-value of the windows is simply the inverse of its *U*-factor, and is determined to be

$$R_{\text{window}} = \frac{1}{U} = \frac{1}{0.801\,\text{Btu/h}\cdot\text{ft}^2.°\text{F}} = 1.25\,\text{h}\cdot\text{ft}^2.°\text{F/Btu}$$

which is less than 13. Therefore, the *R*-value of a double pane window is **much less** than the *R*-value of an *R*-13 wall.

Now consider a 1-ft^2 section of a wall. The solid wall and the window areas of this section are $A_{\text{wall}} = 0.8$ ft^2 and $A_{\text{window}} = 0.2$ ft^2. Then the rates of heat transfer through the two sections are determined to be

$$\dot{Q}_{\text{wall}} = U_{\text{wall}} A_{\text{wall}} (T_i - T_o) = A_{\text{wall}} \frac{T_i - T_o}{R-\text{value, wall}} = (0.8\,\text{ft}^2) \frac{\Delta T(°\text{F})}{13\,\text{h.ft}^2.°\text{F/Btu}} = 0.0615\Delta T\ \text{Btu/h}$$

$$\dot{Q}_{\text{window}} = U_{\text{window}} A_{\text{window}} (T_i - T_o) = A_{\text{window}} \frac{T_i - T_o}{R-\text{value}} = (0.2\,\text{ft}^2) \frac{\Delta T(°\text{F})}{1.25\,\text{h.ft}^2.°\text{F/Btu}} = 0.160\Delta T\ \text{Btu/h}$$

Therefore, the rate of heat transfer through a double pane window is **much more** than the rate of heat transfer through an *R*-13 wall.

Discussion The ratio of heat transfer through the wall and through the window is

$$\frac{\dot{Q}_{\text{window}}}{\dot{Q}_{\text{wall}}} = \frac{0.160\,\text{Btu/h}}{0.0615\,\text{Btu/h}} = 2.60$$

Therefore, 2.6 times more heat is lost through the windows than through the walls although the windows occupy only 20% of the wall area.

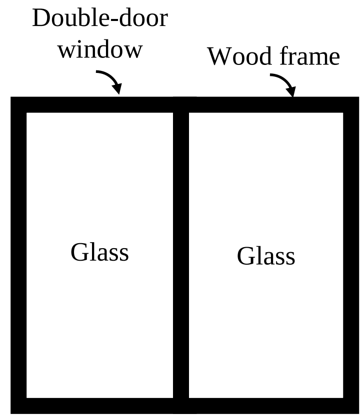
9-132 The rate of heat loss through a double-door wood framed window and the inner surface temperature are to be determined for the cases of single pane, double pane, and low-e triple pane windows.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional. **3** Thermal properties of the windows and the heat transfer coefficients are constant. **4** Infiltration heat losses are not considered.

Properties The U-factors of the windows are given in Table 9-6.

Analysis The rate of heat transfer through the window can be determined from

$$\dot{Q}_{\text{window}} = U_{\text{overall}} A_{\text{window}} (T_i - T_o)$$



where T_i and T_o are the indoor and outdoor air temperatures, respectively, U_{overall} is the U-factor (the overall heat transfer coefficient) of the window, and A_{window} is the window area which is determined to be

$$A_{window} = \text{Height} \times \text{Width} = (1.2\text{ m})(1.8\text{ m}) = 2.16\text{ m}^2$$

The U-factors for the three cases can be determined directly from Table 9-6 to be 5.57, 2.86, and 1.46 W/m^2.°C, respectively. Also, the inner surface temperature of the window glass can be determined from Newton's law,

$$\dot{Q}_{\text{window}} = h_i A_{\text{window}} (T_i - T_{\text{glass}}) \longrightarrow T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}}$$

where h_i is the heat transfer coefficient on the inner surface of the window which is determined from Table 9-5 to be h_i = 8.3 W/m^2.°C. Then the rate of heat loss and the interior glass temperature for each case are determined as follows:

(*a*) Single glazing:

$$\dot{Q}_{\text{window}} = (5.57\text{ W/m}^2\cdot°\text{C})(2.16\text{ m}^2)[20-(-8)°\text{C}] = \mathbf{337\ W}$$

$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}} = 20°\text{C} - \frac{337\text{ W}}{(8.29\text{ W/m}^2\cdot°\text{C})(2.16\text{ m}^2)} = \mathbf{1.2°C}$$

(*b*) Double glazing (13 mm air space):

$$\dot{Q}_{\text{window}} = (2.86\text{ W/m}^2\cdot°\text{C})(2.16\text{ m}^2)[20-(-8)°\text{C}] = \mathbf{173\ W}$$

$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{\text{window}}}{h_i A_{\text{window}}} = 20°\text{C} - \frac{173\text{ W}}{(8.29\text{ W/m}^2\cdot°\text{C})(2.16\text{ m}^2)} = \mathbf{10.3°C}$$

(*c*) Triple glazing (13 mm air space, low-e coated):

$$\dot{Q}_{\text{window}} = (1.46\text{ W/m}^2\cdot°\text{C})(2.16\text{ m}^2)[20-(-8)°\text{C}] = \mathbf{88.3\ W}$$

$$T_{\text{glass}} = T_i - \frac{\dot{Q}_{window}}{h_i A_{window}} = 20 - \frac{88.3\text{ W}}{(8.3\text{ W/m}^2.°\text{C})(2.16\text{ m}^2)} = \mathbf{15.1°C}$$

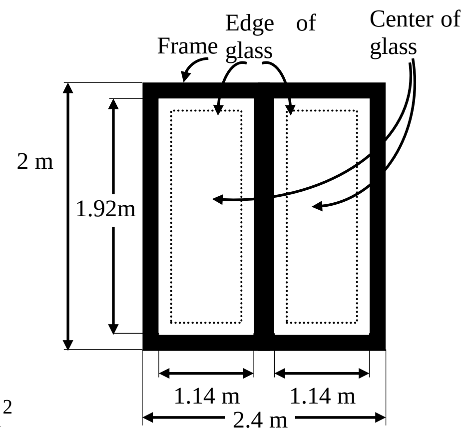
Discussion Note that heat loss through the window will be reduced by 49 percent in the case of double glazing and by 74 percent in the case of triple glazing relative to the single glazing case. Also, in the case of single glazing, the low inner glass surface temperature will cause considerable discomfort in the occupants because of the excessive heat loss from the body by radiation. It is raised from 1.2°C to 10.3°C in the case of double glazing and to 15.1°C in the case of triple glazing.

9-133 The overall U-factor for a double-door type window is to be determined, and the result is to be compared to the value listed in Table 9-6.

Assumptions **1** Steady operating conditions exist. **2** Heat transfer through the window is one-dimensional.

Properties The U-factors for the various sections of windows are given in Table 9-6.

Analysis The areas of the window, the glazing, and the frame are



$$A_{\text{window}} = \text{Height} \times \text{Width} = (2\text{ m})(2.4\text{ m}) = 4.80\text{ m}^2$$

$$A_{\text{glazing}} = 2 \times \text{Height} \times \text{Width} = 2(1.92\text{ m})(1.14\text{ m}) = 4.38\text{ m}^2$$

$$A_{\text{frame}} = A_{\text{window}} - A_{\text{glazing}} = 4.80 - 4.38 = 0.42\text{ m}^2$$

The edge-of-glass region consists of a 6.5-cm wide band around the perimeter of the glazings, and the areas of the center and edge sections of the glazing are determined to be

$$A_{\text{center}} = 2(\text{Height} \times \text{Width}) = 2(1.92 - 0.13\text{ m})(1.14 - 0.13\text{ m}) = 3.62\text{ m}^2$$

$$A_{\text{edge}} = A_{\text{glazing}} - A_{\text{center}} = 4.38 - 3.62 = 0.76\text{ m}^2$$

The U-factor for the frame section is determined from Table 9-4 to be U_{frame} = 2.8 W/m^2.°C. The U-factor for the center and edge sections are determined from Table 9-6 to be U_{center} = 2.78 W/m^2.°C and U_{edge} =3.40 W/m^2.°C. Then the overall U-factor of the entire window becomes

$$\begin{aligned} U_{\text{window}} &= (U_{\text{center}} A_{\text{center}} + U_{\text{edge}} A_{\text{edge}} + U_{\text{frame}} A_{\text{frame}}) / A_{\text{window}} \\ &= (2.78 \times 3.62 + 3.40 \times 0.76 + 2.8 \times 0.42) / 4.80 \\ &= \mathbf{2.88\ W/m^2 \cdot {}^\circ C} \end{aligned}$$

Discussion The overall U-factor listed in Table 9-6 for the specified type of window is 2.86 W/m^2.°C, which is sufficiently close to the value obtained above.

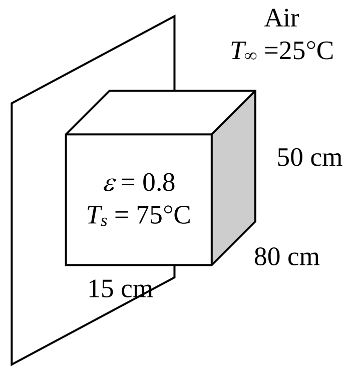
Review Problems

9-134 An electric resistance space heater filled with oil is placed against a wall. The power rating of the heater and the time it will take for the heater to reach steady operation when it is first turned on are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat transfer from the back, bottom, and top surfaces are disregarded. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (75+25)/2 = 50°C$ are (Table A-15)

$$k = 0.02735 \text{ W/m.°C}$$
$$\nu = 1.798\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7228$$
$$\beta = \frac{1}{T_f} = \frac{1}{(50+273)\text{K}} = 0.003096 \text{ K}^{-1}$$



Analysis Heat transfer from the top and bottom surfaces are said to be negligible, and thus the heat transfer area in this case consists of the three exposed side surfaces. The characteristic length is the height of the box, $L_c = L = 0.5$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003096 \text{ K}^{-1})(75-25 \text{ K})(0.5 \text{ m})^3}{(1.798\times10^{-5} \text{ m}^2/\text{s})^2}(0.7228) = 4.244\times10^8$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(4.244\times10^8)^{1/6}}{\left[1+\left(\frac{0.492}{0.7228}\right)^{9/16}\right]^{8/27}}\right\}^2 = 94.68$$

$$h = \frac{k}{L}Nu = \frac{0.02735 \text{ W/m.°C}}{0.5 \text{ m}}(94.68) = 5.179 \text{ W/m}^2.\text{°C}$$
$$A_s = (0.5 \text{ m})(0.8 \text{ m}) + 2(0.15 \text{ m})(0.5 \text{ m}) = 0.55 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.179 \text{ W/m}^2.\text{°C})(0.55 \text{ m}^2)(75-25)\text{°C} = 142.4 \text{ W}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A_s \sigma(T_s{}^4 - T_{surr}{}^4)$$
$$= (0.8)(0.55 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(75+273 \text{ K})^4 - (25+273 \text{ K})^4] = 169.1 \text{ W}$$

Then the total rate of heat transfer, thus the power rating of the heater becomes

$$\dot{Q}_{total} = 142.4 + 169.1 = \mathbf{311.5 \text{ W}}$$

The specific heat of the oil at the average temperature of the oil is 2006 J/kg.°C (Table A-13). Then the amount of heat transfer needed to raise the temperature of the oil to the steady operating temperature and the time it takes become

$$Q = mc_p(T_2 - T_1) = (45 \text{ kg})(2006 \text{ J/kg.°C})(75-25)\text{°C} = 4.514\times10^6 \text{ J}$$

$$Q = \dot{Q}\Delta t \longrightarrow \Delta t = \frac{Q}{\dot{Q}} = \frac{4.514\times10^6 \text{ kJ}}{311.5 \text{ J/s}} = 14{,}490 \text{ s} = \mathbf{4.03 \text{ h}}$$

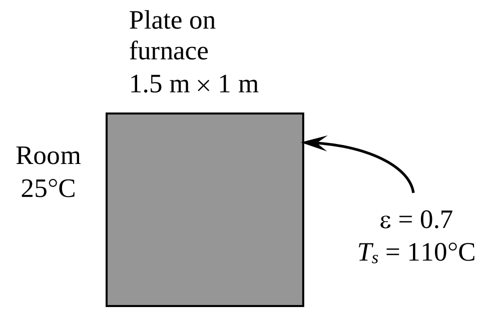
which is not practical. Therefore, the surface temperature of the heater must be allowed to be higher than 75°C.

9-135 A hot part of the vertical front section of a natural gas furnace in a plant is considered. The rate of heat loss from this section and the annual cost of this heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer from other surfaces of the tank is disregarded.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (110+25)/2 = 67.5°C$ are (Table A-15)

$$k = 0.02863 \text{ W/m.°C}$$
$$\nu = 1.97\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7183$$
$$\beta = \frac{1}{T_f} = \frac{1}{(67.5+273)\text{K}} = 0.002937 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the height of that section of furnace, $L_c = L = 1.5$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.002937 \text{ K}^{-1})(110-25 \text{ K})(1.5 \text{ m})^3}{(1.97\times10^{-5} \text{ m}^2\text{/s})^2}(0.7183) = 1.530\times10^{10}$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.530\times10^{10})^{1/6}}{\left[1+\left(\frac{0.492}{0.7183}\right)^{9/16}\right]^{8/27}}\right\}^2 = 289.1$$

$$h = \frac{k}{L}Nu = \frac{0.02863 \text{ W/m.°C}}{1.5 \text{ m}}(289.1) = 5.518 \text{ W/m}^2\text{.°C}$$
$$A_s = (1.5 \text{ m})(1 \text{ m}) = 1.5 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.518 \text{ W/m}^2\text{.°C})(1.5 \text{ m}^2)(110-25)\text{°C} = 703.5 \text{ W}$$

The radiation heat loss is

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s\sigma(T_{surr}^{\ 4} - T_s^{\ 4}) \\ &= (0.7)(1.5 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)[(110+273 \text{ K})^4 - (25+273 \text{ K})^4] \\ &= 811.6 \text{ W}\end{aligned}$$

$$\dot{Q}_{total} = 703.5 + 811.6 = \mathbf{1515 \text{ W}}$$

The amount and cost of natural gas used to overcome this heat loss per year is

$$Q_{gas} = \dot{Q}_{gas}\Delta t = \frac{\dot{Q}_{total}}{0.79}\Delta t = \frac{(1.515 \text{ kJ/s})}{0.79}(310 \text{ days/yr}\times10 \text{ hr/day}\times3600 \text{ s/hr}) = 2.140\times10^7 \text{ kJ}$$

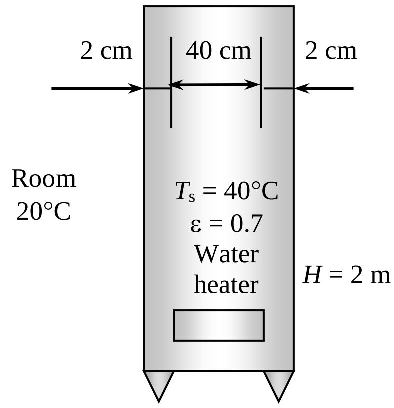
$$\text{Cost} = (2.140\times10^7/105{,}500 \text{ therm})(\$1.20\text{/therm}) = \mathbf{\$243}$$

9-136 An electric hot water heater is located in a small room. A hot water tank insulation kit is available for \$60. The payback period of this insulation to pay for itself from the energy it saves is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer from the top and bottom surfaces of the tank is disregarded. **5** The thermal resistance of the metal sheet is negligible.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (40+20)/2 = 30°C$ are (Table A-15)

$$k = 0.02588 \text{ W/m.°C}$$
$$\nu = 1.608\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7282$$
$$\beta = \frac{1}{T_f} = \frac{1}{(30+273)\text{K}} = 0.0033\,\text{K}^{-1}$$



Analysis The characteristic length in this case is the height of the heater, $L_c = L = 2$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.0033\,\text{K}^{-1})(40-20\,\text{K})(2\,\text{m})^3}{(1.608\times10^{-5}\text{ m}^2\text{/s})^2}(0.7282) = 1.459\times10^{10}$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.459\times10^{10})^{1/6}}{\left[1+\left(\frac{0.492}{0.7282}\right)^{9/16}\right]^{8/27}}\right\}^2 = 285.4$$

$$h = \frac{k}{L}Nu = \frac{0.02588\text{ W/m.°C}}{2\text{ m}}(285.4) = 3.693\text{ W/m}^2\text{.°C}$$
$$A_s = \pi DL = \pi(0.44\text{ m})(2\text{ m}) = 2.765\text{ m}^2$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (3.693\text{ W/m}^2\text{.°C})(2.765\text{ m}^2)(40-20)\text{°C} = 204.2\text{ W}$$

The radiation heat loss is

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s\sigma(T_{surr}{}^4 - T_s{}^4)\\ &= (0.7)(2.765\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(40+273\text{ K})^4 - (20+273\text{ K})^4] = 244.5\text{ W}\end{aligned}$$

and

$$\dot{Q}_{total} = 204.2 + 244.5 = 448.7\text{ W}$$

The reduction in heat loss after adding insulation is

$$\dot{Q} = (0.80)(448.7) = 359.0\text{ W}$$

The amount of heat and money saved per hour is

$$Q_{saved} = \dot{Q}_{saved}\Delta t = (0.3590\text{ kW})(1\text{h}) = 0.3590\text{ kWh}$$
$$\text{Money saved} = (0.3590\text{ kWh})(\$0.08/\text{kWh}) = \$0.02872$$

Then it will take

$$\Delta t = \frac{\$60}{\$0.02872} = 2089\text{ h} = \mathbf{87.0\,days}$$

for the additional insulation to pay for itself from the energy it saves.

9-137 A plate inclined at 30° with the top surface of the plate well insulated. The rate of heat loss from the plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas. **3** Thermal properties are constant. **4** Radiation heat transfer is negligible.

Properties The properties of air at $T_f = (T_s + T_\infty)/2 = 30°C$ are $k = 0.02588$ W/m·K, $\nu = 1.608 \times 10^{-5}$ m^2/s, Pr = 0.7282 (from Table A-15). Also, $\beta = 1/T_f = 0.0033$ K^{-1}.

Analysis The Rayleigh number ($L_c = L$) is

$$\begin{aligned}\mathrm{Ra}_L &= \frac{g\cos\theta\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr}\\ &= \frac{(9.81\,\mathrm{m/s^2})(\cos 30°)(0.0033\,\mathrm{K^{-1}})(60-0)\mathrm{K}(0.5\,\mathrm{m})^3}{(1.608\times 10^{-5}\,\mathrm{m^2/s})^2}(0.7282)\\ &= 5.922\times 10^8\end{aligned}$$

The Nusselt number is calculated using the correlation for vertical plate,

$$\mathrm{Nu} = \left\{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{[1+(0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(5.922\times 10^8)^{1/6}}{[1+(0.492/0.7282)^{9/16}]^{8/27}}\right\}^2 = 104.9$$

The heat transfer coefficient is

$$h = \mathrm{Nu}\frac{k}{L} = (104.9)\frac{0.02588\ \mathrm{W/m\cdot K}}{0.5\,\mathrm{m}} = 5.43\ \mathrm{W/m^2\cdot K}$$

Hence, the rate of heat loss is

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.43\ \mathrm{W/m^2\cdot K})(0.5\,\mathrm{m})(0.5\,\mathrm{m})(60-0)\ \mathrm{K} = \mathbf{81.5\ W}$$

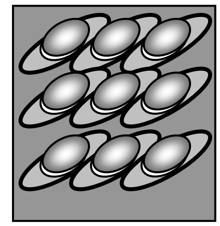
Discussion The inclined plate with well insulated top surface can be treated as an inclined plate with hot surface down.

9-138 A group of 25 transistors are cooled by attaching them to a square aluminum plate and mounting the plate on the wall of a room. The required size of the plate to limit the surface temperature to 50°C is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer from the back side of the plate is negligible.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (50+30)/2 = 40$°C are (Table A-15)

$$k = 0.02662 \text{ W/m.°C}$$
$$\nu = 1.702\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.7255$$
$$\beta = \frac{1}{T_f} = \frac{1}{(40+273)\text{K}} = 0.003195 \text{ K}^{-1}$$



Analysis The Rayleigh number can be determined in terms of the characteristic length (length of the plate) to be

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.003195\text{ K}^{-1})(50-30\text{ K})(L)^3}{(1.702\times10^{-5}\text{ m}^2/\text{s})^2}(0.7255) = 1.571\times10^9 L^3$$

The Nusselt number relation is

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.571\times10^9 L^3)^{1/6}}{\left[1+\left(\frac{0.492}{0.7255}\right)^{9/16}\right]^{8/27}}\right\}^2$$

The heat transfer coefficient is

$$h = \frac{k}{L}Nu = \frac{0.02662\text{ W/m.°C}}{L}Nu$$
$$A_s = L^2$$

Noting that both the surface and surrounding temperatures are known, the rate of convection and radiation heat transfer are expressed as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) = \frac{0.02662\text{ W/m.°C}}{L}NuL^2(50-30)\text{°C}$$
$$\dot{Q}_{\text{rad}} = \varepsilon A_s\sigma(T_s{}^4 - T_{sky}{}^4) = (0.9)L^2(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(50+273)^4 - (30+273)^4]\text{K}^4 = 125.3L^2$$

The rate of total heat transfer is expressed as

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$
$$25\times(1.5\text{ W}) = \frac{0.02662\text{ W/m.°C}}{L}NuL^2(50-30)\text{°C} + 125.3L^2$$

Substituting Nusselt number expression above into this equation and solving for L, the length of the plate is determined to be

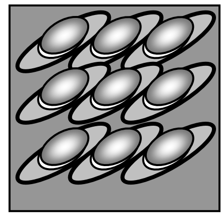
L = **0.426 m**

9-139 A group of 25 transistors are cooled by attaching them to a square aluminum plate and positioning the plate horizontally in a room. The required size of the plate to limit the surface temperature to 50°C is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Any heat transfer from the back side of the plate is negligible.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (50+30)/2 = 40°\text{C}$ are (Table A-15)

$$k = 0.02662 \text{ W/m.°C}$$
$$\nu = 1.702\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.7255$$
$$\beta = \frac{1}{T_f} = \frac{1}{(40+273)\text{K}} = 0.003195 \text{ K}^{-1}$$



Analysis The characteristic length and the Rayleigh number for the horizontal case are determined to be

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4}$$

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003195 \text{ K}^{-1})(50-30 \text{ K})(L/4)^3}{(1.702\times10^{-5} \text{ m}^2/\text{s})^2}(0.7255) = 2.454\times10^7 L^3$$

Noting that both the surface and surrounding temperatures are known, the rate of radiation heat transfer is determined to be

$$\dot{Q}_{\text{rad}} = \varepsilon A_s \sigma(T_s{}^4 - T_{sky}{}^4) = (0.9)L^2(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(50+273)^4 - (30+273)^4]\text{K}^4 = 125.3L^2$$

(a) **Hot surface facing up:** We assume Ra < 10^7 and thus L <0.74 m so that we can determine the Nu number from Eq. 9-22. Then the Nusselt number and the convection heat transfer coefficient become

$$Nu = 0.54Ra^{1/4} = 0.54(2.454\times10^7 L^3)^{1/4} = 38.0L^{3/4}$$

Then,

$$h = \frac{k}{L}Nu = \frac{0.02662 \text{ W/m.°C}}{L/4}(38.0L^{3/4}) = 4.047L^{-1/4} \text{ W/m}^2.\text{°C}$$
$$A_s = L^2$$

The rate of convection heat transfer is

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) = (4.047L^{-1/4})L^2(50-30) = 80.94L^{7/4} \text{ W}$$

Then,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} \longrightarrow 25\times(1.5 \text{ W}) = 80.94L^{7/4} + 125.3L^2 \text{ W}$$

Solving for L, the length of the plate is determined to be

L = **0.407 m**

Note that L < 0.75 m, and therefore the assumption of Ra < 10^7 is verified. That is,

(b) **Hot surface facing down:** The Nusselt number in this case is determined from

$$Nu = 0.27Ra^{1/4} = 0.27(2.454\times10^7 L^3)^{1/4} = 19.0L^{3/4}$$

Then, $$h = \frac{k}{L_c}Nu = \frac{0.02662 \text{ W/m.°C}}{L/4}(19.0L^{3/4}) = 2.023L^{-1/4}$$

The rate of convection heat transfer is

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) = (2.023L^{-1/4})L^2(50-30) = 40.47L^{7/4} \text{ W}$$

Then, $$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} \longrightarrow 25\times(1.5 \text{ W}) = 40.47L^{7/4} + 125.3L^2 \text{ W}$$

Solving for L, the length of the plate is determined to be

L = **0.464 m**

9-140 An $L \times L$ horizontal plate is placed in a quiescent air with the hot surface facing up, and the expressions, having the form $\text{Nu} = CRa_L^n$, for the average heat transfer coefficient are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties.

Properties The properties of air at T_f = 20°C are k = 0.02514 W/m·K, $\nu = 1.516 \times 10^{-5}$ m²/s, Pr = 0.7309 (from Table A-15). Also, $\beta = 1/T_f = 0.003413$ K^{-1}.

Analysis For horizontal plate, the characteristic length is

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4}$$

The Rayleigh number ($L_c = L/4$) is

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\,\text{m/s}^2)(0.003413\,\text{K}^{-1})\Delta T(L/4)^3}{(1.516\times10^{-5}\,\text{m}^2\text{/s})^2}(0.7309)$$

$$= 1.664\times10^6\,\Delta TL^3$$

For $10^4 < \text{Ra}_L < 10^7$, we have

$$\text{Nu} = \frac{hL_c}{k} = \frac{hL}{4k} = 0.54\text{Ra}_L^{1/4} \qquad \rightarrow \qquad h = 2.16\frac{k}{L}\text{Ra}_L^{1/4}$$

Substituting the Ra_L yields

$$h = 2.16\left(\frac{0.02514}{L}\right)(1.664\times10^6\,\Delta TL^3)^{1/4} = 1.95(\Delta T/L)^{1/4} \qquad 10^4 < \text{Ra}_L < 10^7$$

For $10^7 < \text{Ra}_L < 10^{11}$, we have

$$\text{Nu} = \frac{hL_c}{k} = \frac{hL}{4k} = 0.15\text{Ra}_L^{1/3} \qquad \rightarrow \qquad h = 0.6\frac{k}{L}\text{Ra}_L^{1/3}$$

Substituting the Ra_L yields

$$h = 0.6\left(\frac{0.02514}{L}\right)(1.664\times10^6\,\Delta TL^3)^{1/3} = 1.79\Delta T^{1/3} \qquad 10^7 < \text{Ra}_L < 10^{11}$$

Discussion The average heat transfer coefficient for $10^4 < \text{Ra}_L < 10^7$ is dependent on ΔT and L. For $10^{10} < \text{Ra}_L < 10^{13}$, the average heat transfer coefficient is not influenced by L.

9-141 A flat-plate solar collector placed horizontally on the flat roof of a house is exposed to the calm ambient air. The rate of heat loss from the collector by natural convection and radiation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

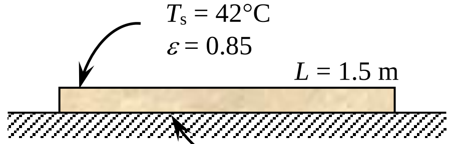
Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (42+8)/2 = 25°C$ are (Table A-15)

$$k = 0.02551 \text{ W/m.°C}$$

$$\nu = 1.562\times10^{-5} \text{ m}^2/\text{s}$$

$$\Pr = 0.7296$$

$$\beta = \frac{1}{T_f} = \frac{1}{(25+273)\text{K}} = 0.003356 \text{ K}^{-1}$$



Analysis The characteristic length in this case is determined from

$$L_c = \frac{A_s}{p} = \frac{(1.5\text{ m})(4.5\text{ m})}{2(1.5\text{ m} + 4.5\text{ m})} = 0.5625\text{ m}$$

Then,

$$Ra = \frac{g\beta(T_\infty - T_s)L_c^3}{\nu^2}\Pr = \frac{(9.81\text{ m/s}^2)(0.003356\text{ K}^{-1})(42-8\text{ K})(0.5625\text{ m})^3}{(1.562\times10^{-5}\text{ m}^2/\text{s})^2}(0.7296) = 5.957\times10^8$$

$$Nu = 0.15Ra^{1/3} = 0.15(5.957\times10^8)^{1/3} = 126.2$$

$$h = \frac{k}{L_c}Nu = \frac{0.02551\text{ W/m.°C}}{0.5625\text{ m}}(126.2) = 5.724\text{ W/m}^2.°\text{C}$$

$$A_s = (1.5\text{ m})(4.5\text{ m}) = 6.75\text{ m}^2$$

and

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = (5.724\text{ W/m}^2.°\text{C})(6.75\text{ m}^2)(42-8)°\text{C} = \mathbf{1314\ W}$$

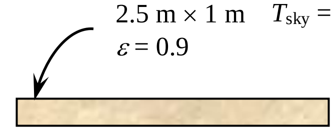
Heat transfer rate by radiation is

$$\begin{aligned}\dot{Q}_{rad} &= \varepsilon A_s\sigma(T_{surr}{}^4 - T_s{}^4)\\ &= (0.85)(6.75\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(42+273\text{ K})^4 - (-15+273\text{ K})^4]\\ &= \mathbf{1762\ W}\end{aligned}$$

9-142 A horizontal skylight made of a single layer of glass on the roof of a house is considered. The rate of heat loss through the skylight is to be determined for two cases.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (-4-10)/2 = –7°C are (Table A-15)



$$k = 0.02311 \text{ W/m.°C}$$
$$\nu = 1.278\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.738$$
$$\beta = \frac{1}{T_f} = \frac{1}{(-7+273)\text{K}} = 0.003759 \text{ K}^{-1}$$

Analysis We assume radiation heat transfer inside the house to be negligible. We start the calculations by "guessing" the glass temperature to be -4°C for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is determined from

$$L_c = \frac{A_s}{p} = \frac{(1\text{ m})(2.5\text{m})}{2(1\text{ m}+2.5\text{ m})} = 0.357 \text{ m}.$$

Then,

$$Ra = \frac{g\beta(T_1-T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003759 \text{ K}^{-1})[-4-(-10)\text{ K})(0.357\text{ m})^3}{(1.278\times10^{-5}\text{ m}^2\text{/s})^2}(0.738) = 4.553\times10^7$$

$$Nu = 0.15Ra^{1/3} = 0.15(4.553\times10^7)^{1/3} = 53.56$$

$$h_o = \frac{k}{L_c}Nu = \frac{0.02311 \text{ W/m.°C}}{0.357 \text{ m}}(53.56) = 3.467 \text{ W/m}^2\text{.°C}$$

$$A_s = (1\text{ m})(2.5\text{ m}) = 2.5 \text{ m}^2$$

Using the assumed value of glass temperature, the radiation heat transfer coefficient is determined to be

$$\begin{aligned} h_{rad} &= \varepsilon\sigma(T_s+T_{sky})(T_s{}^2+T_{sky}{}^2) \\ &= 0.9(5.67\times10^{-8}\text{ W/m}^2\text{.K}^4)[(-4+273)+(-30+273)][(-4+273)^2+(-30+273)^2]\text{K}^3 \\ &= 3.433 \text{ W/m}^2\text{.K} \end{aligned}$$

Then the combined convection and radiation heat transfer coefficient outside becomes

$$h_{o,combined} = h_o + h_{rad} = 3.467 + 3.433 = 6.90 \text{ W/m}^2$$

Again we take the glass temperature to be -4°C for the evaluation of the properties and h for the inner surface of the skylight. The properties of air at 1 atm and the film temperature of T_f = (-4+20)/2 = 8°C are (Table A-15)

$$k = 0.02424 \text{ W/m.°C}$$
$$\nu = 1.408\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7342$$
$$\beta = \frac{1}{T_f} = \frac{1}{(8+273)\text{K}} = 0.003559 \text{ K}^{-1}$$

The characteristic length in this case is also 0.357 m. Then,

$$Ra = \frac{g\beta(T_1-T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003559 \text{ K}^{-1})[20-(-4)\text{ K})(0.357\text{ m})^3}{(1.408\times10^{-5}\text{ m}^2\text{/s})^2}(0.7342) = 1.412\times10^8$$

$$Nu = 0.27Ra^{1/4} = 0.27(1.412\times10^8)^{1/4} = 29.43$$

$$h_i = \frac{k}{L_c}Nu = \frac{0.02424 \text{ W/m.°C}}{0.357 \text{ m}}(29.43) = 1.998 \text{ W/m}^2\text{.°C}$$

Using the thermal resistance network, the rate of heat loss through the skylight is determined to be

$$\dot{Q}_{skylight} = \frac{T_{s,i} - T_{\infty,o}}{R_{conv,i} + R_{cond,glas} + R_{combined,o}}$$

$$= \frac{A_s(T_{room} - T_{out})}{\dfrac{1}{h_i} + \dfrac{t_{\text{glass}}}{k_{glass}} + \dfrac{1}{h}} = \frac{(2.5\text{ m}^2)[20-(-10)]°\text{C}}{\dfrac{1}{1.998\text{ W/m}^2.°\text{C}} + \dfrac{0.005\text{ m}}{0.78\text{ W/m.°C}} + \dfrac{1}{6.90\text{ W/m}^2.°\text{C}}} = 115\text{ W}$$

Using the same heat transfer coefficients for simplicity, the rate of heat loss through the roof in the case of R-5.34 construction is determined to be

$$\dot{Q}_{roof} = \frac{T_{s,i} - T_{\infty,o}}{R_{conv,i} + R_{cond} + R_{combined,o}}$$

$$= \frac{A_s(T_{room} - T_{out})}{\dfrac{1}{h_i} + R_{glass} + \dfrac{1}{h}} = \frac{(2.5\text{ m}^2)[20-(-10)]°\text{C}}{\dfrac{1}{1.998\text{ W/m}^2.°\text{C}} + 5.34\text{ m}^2.°\text{C/W} + \dfrac{1}{6.90\text{ W/m}^2.°\text{C}}} = 12.5\text{ W}$$

Therefore, a house loses 115/12.5 ≅ 9 times more heat through the skylights than it does through an insulated wall of the same size.

Using Newton's law of cooling, the glass temperature corresponding to a heat transfer rate of 115 W is calculated to be –3.3°C, which is sufficiently close to the assumed value of -4°C. Therefore, there is no need to repeat the calculations.

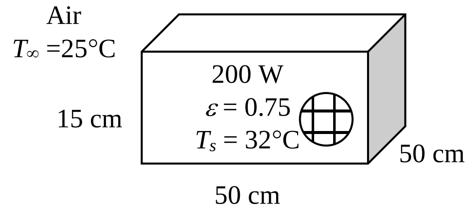
Discussion The assumed film temperature of T_f = -7°C is an appropriate assumption, since the determined T_s = -3.3°C would give a film temperature of T_f = -6.65°C. Otherwise, T_s would have to be solved iteratively.

9-143 An electronic box is cooled internally by a fan blowing air into the enclosure. The fraction of the heat lost from the outer surfaces of the electronic box is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Heat transfer from the base surface is disregarded. **4** The pressure of air inside the enclosure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2$ = (32+15)/2 = 28.5°C are (Table A-15)

$$k = 0.02577 \text{ W/m.°C}$$
$$\nu = 1.594\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7286$$
$$\beta = \frac{1}{T_f} = \frac{1}{(28.5+273)\text{K}} = 0.003317 \text{ K}^{-1}$$



Analysis Heat loss from the horizontal top surface:

The characteristic length in this case is

$$L_c = \frac{A}{P} = \frac{(0.5\text{ m})^2}{2[(0.5\text{ m})+(0.5\text{ m})]} = 0.125\text{ m}.$$

Then,

$$Ra = \frac{g\beta(T_s-T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003317\text{ K}^{-1})(32-25\text{ K})(0.125\text{ m})^3}{(1.594\times10^{-5}\text{ m}^2/\text{s})^2}(0.7286) = 1.275\times10^6$$

$$Nu = 0.54Ra^{1/4} = 0.54(1.275\times10^6)^{1/4} = 18.15$$

$$h = \frac{k}{L_c}Nu = \frac{0.02577\text{ W/m.°C}}{0.125\text{ m}}(18.15) = 3.741\text{ W/m}^2.\text{°C}$$

$$A_{top} = (0.5\text{ m})^2 = 0.25\text{ m}^2$$

and $$\dot{Q}_{top} = hA_{top}(T_s-T_\infty) = (3.741\text{ W/m}^2.\text{°C})(0.25\text{ m}^2)(32-25)\text{°C} = 6.55\text{ W}$$

Heat loss from vertical side surfaces:

The characteristic length in this case is the height of the box $L_c = L$ =0.15 m. Then,

$$Ra = \frac{g\beta(T_s-T_\infty)L^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003317\text{ K}^{-1})(32-25\text{ K})(0.15\text{ m})^3}{(1.594\times10^{-5}\text{ m}^2/\text{s})^2}(0.7286) = 2.204\times10^6$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(2.204\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7286}\right)^{9/16}\right]^{8/27}}\right\}^2 = 20.55$$

$$h = \frac{k}{L}Nu = \frac{0.02577\text{ W/m.°C}}{0.15\text{ m}}(20.55) = 3.530\text{ W/m}^2.\text{°C}$$

$$A_{side} = 4(0.15\text{ m})(0.5\text{ m}) = 0.3\text{ m}^2$$

and $$\dot{Q}_{side} = hA_{side}(T_s-T_\infty) = (3.530\text{ W/m}^2.\text{°C})(0.3\text{ m}^2)(32-25)\text{°C} = 7.41\text{ W}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^4 - T_{surr}^4)$$
$$= (0.75)(0.25+0.3\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)[(32+273\text{ K})^4-(25+273\text{ K})^4] = 17.95\text{ W}$$

Then the fraction of the heat loss from the outer surfaces of the box is determined to be

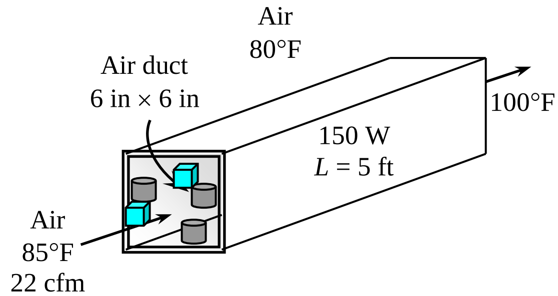
$$f = \frac{(6.55+7.41+17.95)\text{W}}{200\text{ W}} = 0.160 = \mathbf{16.0\%}$$

9-144E The components of an electronic device located in a horizontal duct of rectangular cross section is cooled by forced air. The heat transfer from the outer surfaces of the duct by natural convection and the average temperature of the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation effects are negligible. **5** The thermal resistance of the duct is negligible.

Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (120+80)/2 = 100°F$ are (Table A-15E)

$$k = 0.01529 \text{ Btu/h.ft.°F}$$
$$\nu = 0.1809\times10^{-3} \text{ ft}^2/\text{s}$$
$$\Pr = 0.726$$
$$\beta = 1/T_f = 1/(100+460)\text{R} = 0.001786 \text{ R}^{-1}$$



Analysis (*a*) Using air density at the inlet temperature of 85°F and the specific heat at the average temperature of (85+100))/2 = 92.5°F and 1 atm for the forced air, the mass flow rate of air and the heat transfer rate by forced convection are determined to be

$$\dot{m} = \rho\dot{V} = (0.07284 \text{ lbm/ft}^3)(22 \text{ ft}^3/\text{min}) = 1.602 \text{ lbm/min}$$

$$\dot{Q}_{forced} = \dot{m}c_p(T_{out} - T_{in}) = (1.602\times60 \text{ lbm/h})(0.2404 \text{ Btu/lbm.°F})(100-85)\text{°F} = 346.6 \text{ Btu/h}$$

Noting that radiation heat transfer is negligible, the rest of the 150 W heat generated must be dissipated by natural convection,

$$\dot{Q}_{natural} = \dot{Q}_{total} - \dot{Q}_{forced} = (150\times3.412) - 346.6 = \mathbf{165.2\ Btu/h}$$

(*b*) We start the calculations by "guessing" the surface temperature to be 120°F for the evaluation of the properties and *h*. We will check the accuracy of this guess later.

Horizontal top surface: The characteristic length is

$$L_c = \frac{A_s}{P} = \frac{(5\text{ ft})(6/12\text{ ft})}{2(5\text{ ft} + 6/12\text{ ft})} = 0.2273\text{ ft} .$$

Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\Pr = \frac{(32.2\text{ ft/s}^2)(0.001786\text{ R}^{-1})(120-80\text{ R})(0.2273\text{ ft})^3}{(0.1809\times10^{-3}\text{ ft}^2/\text{s})^2}(0.726) = 5.993\times10^5$$

$$Nu = 0.54Ra^{1/4} = 0.54(5.993\times10^5)^{1/4} = 15.02$$

$$h_{top} = \frac{k}{L_c}Nu = \frac{0.01529\text{ Btu/h.ft.°F}}{0.2273\text{ ft}}(15.02) = 1.011\text{ Btu/h.ft}^2\text{.°F}$$

$$A_{top} = (5\text{ ft})(6/12\text{ ft}) = 2.5\text{ ft}^2 = A_{bottom}$$

Horizontal bottom surface: The Nusselt number for this geometry and orientation can be determined from

$$Nu = 0.27Ra^{1/4} = 0.27(5.993\times10^5)^{1/4} = 7.512$$

$$h_{bottom} = \frac{k}{L_c}Nu = \frac{0.01529\text{ Btu/h.ft.°F}}{0.2273\text{ ft}}(7.512) = 0.5053\text{ Btu/h.ft}^2\text{.°F}$$

Vertical side surfaces: The characteristic length in this case is the height of the duct, $L_c = L = 6$ in. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\Pr = \frac{(32.2\text{ ft/s}^2)(0.001786\text{ R}^{-1})(120-80\text{ R})(0.5\text{ ft})^3}{(0.1809\times10^{-3}\text{ ft}^2/\text{s})^2}(0.726) = 6.379\times10^6$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(6.379\times10^6)^{1/6}}{\left[1+\left(\frac{0.492}{0.726}\right)^{9/16}\right]^{8/27}}\right\}^2 = 27.57$$

$$h_{side} = \frac{k}{L}Nu = \frac{0.01529\text{ Btu/h.ft.°F}}{0.5\text{ ft}}(27.57) = 0.843\text{ Btu/h.ft}^2\text{.°F}$$

$$A_{side} = 2(5\text{ ft})(0.5\text{ ft}) = 5\text{ ft}^2$$

Then the total heat loss from the duct can be expressed as

$$\dot{Q}_{total} = \dot{Q}_{top} + \dot{Q}_{bottom} + \dot{Q}_{side} = [(hA)_{top} + (hA)_{bottom} + (hA)_{side}](T_s - T_\infty)$$

Substituting and solving for the surface temperature,

$$165.2\text{ Btu/h} = [(1.011\times2.5 + 0.5053\times2.5 + 0.843\times5)\text{ Btu/h.°F}](T_s - 80)\text{°F}$$

T_s = **88.1°F**

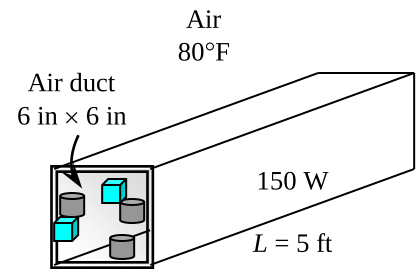
which is not very close to the assumed value of 120°F used in the evaluation of properties and h. The calculations may be repeated by assuming a surface temperature of 90°F for a more accurate result.

9-145E The components of an electronic system located in a horizontal duct of rectangular cross section is cooled by natural convection. The heat transfer from the outer surfaces of the duct by natural convection and the average temperature of the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation effects are negligible. **5** The thermal resistance of the duct is negligible.

Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (160+80)/2 = 120°F$ are (Table A-15E)

$$k = 0.01576 \text{ Btu/h.ft.°F}$$
$$\nu = 0.1923\times10^{-3} \text{ ft}^2\text{/s}$$
$$\Pr = 0.723$$
$$\beta = 1/T_f = 1/(120+460 \text{ R}) = 0.001724 \text{ R}^{-1}$$



Analysis (a) Noting that radiation heat transfer is negligible and no heat is removed by forced convection because of the failure of the fan, the entire 180 W heat generated must be dissipated by natural convection,

$$\dot{Q}_{natural} = \dot{Q}_{total} = \mathbf{150\ W}$$

(b) We start the calculations by "guessing" the surface temperature to be 160°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary.

Horizontal top surface: The characteristic length is $L_c = \dfrac{A_s}{p} = \dfrac{(5 \text{ ft})(6/12 \text{ ft})}{2(5 \text{ ft} + 6/12 \text{ ft})} = 0.2273$ ft. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\Pr = \frac{(32.2 \text{ ft/s}^2)(0.001724 \text{ R}^{-1})(160-80 \text{ R})(0.2273 \text{ ft})^3}{(0.1923\times10^{-3} \text{ ft}^2\text{/s})^2}(0.723) = 1.019\times10^6$$

$$Nu = 0.54Ra^{1/4} = 0.54(1.019\times10^6)^{1/4} = 17.16$$

$$h_{top} = \frac{k}{L_c}Nu = \frac{0.01576 \text{ Btu/h.ft.°F}}{0.2273 \text{ ft}}(17.16) = 1.190 \text{ Btu/h.ft}^2\text{.°F}$$

$$A_{top} = (5 \text{ ft})(6/12 \text{ ft}) = 2.5 \text{ ft}^2 = A_{bottom}$$

Horizontal bottom surface: The Nusselt number for this geometry and orientation can be determined from

$$Nu = 0.27Ra^{1/4} = 0.27(1.019\times10^6)^{1/4} = 8.578$$

$$h_{bottom} = \frac{k}{L_c}Nu = \frac{0.01576 \text{ Btu/h.ft.°F}}{0.2273 \text{ ft}}(8.578) = 0.5948 \text{ Btu/h.ft}^2\text{.°F}$$

Vertical side surfaces: The characteristic length in this case is the height of the duct, $L_c = L = 6$ in. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\Pr = \frac{(32.2 \text{ ft/s}^2)(0.001724 \text{ R}^{-1})(160-80 \text{ R})(0.5 \text{ ft})^3}{(0.1923\times10^{-3} \text{ ft}^2\text{/s})^2}(0.723) = 1.086\times10^7$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(1.086\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.723}\right)^{9/16}\right]^{8/27}}\right\}^2 = 32.03$$

$$h_{side} = \frac{k}{L}Nu = \frac{0.01576 \text{ Btu/h.ft.°F}}{0.5 \text{ ft}}(32.03) = 1.009 \text{ Btu/h.ft}^2\text{.°F}$$

$$A_{side} = 2(5 \text{ ft})(0.5 \text{ ft}) = 5 \text{ ft}^2$$

Then the total heat loss from the duct can be expressed as

$$\dot{Q}_{total} = \dot{Q}_{top} + \dot{Q}_{bottom} + \dot{Q}_{side} = [(hA)_{top} + (hA)_{bottom} + (hA)_{side}](T_s - T_\infty)$$

Substituting and solving for the surface temperature,

$$150\ \text{W}\left(\frac{3.41214\ \text{Btu/h}}{1\ \text{W}}\right)=[(1.190\times 2.5+0.5948\times 2.5+1.009\times 5)\ \text{Btu/h.°F}](T_s-80)\text{°F}$$

T_s = 134°F

which is not sufficiently close to the assumed value of 160°F used in the evaluation of properties and h. Now we repeat the analysis at an improved surface temperature value of 140°F as follows:

$(T_s+T_\infty)/2$ = (140+80)/2 = 110°F (Table A-15E):

$$k=0.01552\ \text{Btu/h.ft.°F}$$

$$\nu=0.1866\times10^{-3}\ \text{ft}^2/\text{s}$$

$$\text{Pr}=0.7245$$

$$\beta=1/T_f=1/(110+460\ \text{R})=0.001754\ \text{R}^{-1}$$

Horizontal top surface:

$$Ra=\frac{g\beta(T_s-T_\infty)L_c^3}{\nu^2}\text{Pr}=\frac{(32.2\ \text{ft/s}^2)(0.001754\ \text{R}^{-1})(140-80\ \text{R})(0.2273\ \text{ft})^3}{(0.1866\times10^{-3}\ \text{ft}^2/\text{s})^2}(0.7245)=0.8280\times10^6$$

$$Nu=0.54Ra^{1/4}=0.54(0.8280\times10^6)^{1/4}=16.29$$

$$h_{top}=\frac{k}{L_c}Nu=\frac{0.01552\ \text{Btu/h.ft.°F}}{0.2273\ \text{ft}}(16.29)=1.112\ \text{Btu/h.ft}^2.\text{°F}$$

Horizontal bottom surface:

$$Nu=0.27Ra^{1/4}=0.27(0.8280\times10^6)^{1/4}=8.145$$

$$h_{bottom}=\frac{k}{L_c}Nu=\frac{0.01552\ \text{Btu/h.ft.°F}}{0.2273\ \text{ft}}(8.145)=0.5561\ \text{Btu/h.ft}^2.\text{°F}$$

Vertical side surfaces:

$$Ra=\frac{g\beta(T_s-T_\infty)L^3}{\nu^2}\text{Pr}=\frac{(32.2\ \text{ft/s}^2)(0.001754\ \text{R}^{-1})(140-80\ \text{R})(0.5\ \text{ft})^3}{(0.1866\times10^{-3}\ \text{ft}^2/\text{s})^2}(0.7245)=0.8814\times10^7$$

$$Nu=\left\{0.825+\frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2=\left\{0.825+\frac{0.387(0.8814\times10^7)^{1/6}}{\left[1+\left(\frac{0.492}{0.7245}\right)^{9/16}\right]^{8/27}}\right\}^2=30.19$$

$$h_{side}=\frac{k}{L}Nu=\frac{0.01552\ \text{Btu/h.ft.°F}}{0.5\ \text{ft}}(30.19)=0.9372\ \text{Btu/h.ft}^2.\text{°F}$$

$$\dot{Q}_{total}=\dot{Q}_{top}+\dot{Q}_{bottom}+\dot{Q}_{side}=[(hA)_{top}+(hA)_{bottom}+(hA)_{side}](T_s-T_\infty)$$

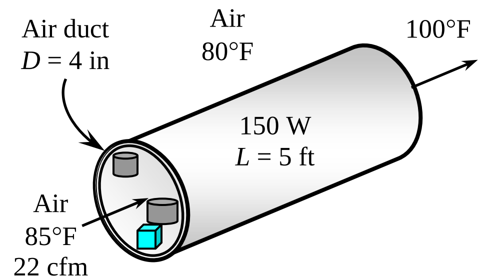
$$150\ \text{W}\left(\frac{3.41214\ \text{Btu/h}}{1\ \text{W}}\right)=[(1.112\times 2.5+0.5561\times 2.5+0.9372\times 5)\ \text{Btu/h.°F}](T_s-80)\text{°F}$$

T_s = **138°F**

This is sufficiently close to the assumed value of 140°F used in the evaluation of properties and h. Therefore, there is no need to repeat the calculations.

Discussion The assumed film temperature of T_f = 110°F is an appropriate assumption, since the determined T_s = 138°F would give a film temperature of T_f = 109°F.

9-146E The components of an electronic system located in a horizontal duct of circular cross section is cooled by forced air. The heat transfer from the outer surfaces of the duct by natural convection and the average temperature of the duct are to be determined.



Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation effects are negligible. **5** The thermal resistance of the duct is negligible.

Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (150+80)/2 = 115°F$ are (Table A-15E)

$$k = 0.01564 \text{ Btu/h.ft.°F}$$
$$\nu = 0.1895\times10^{-3} \text{ ft}^2\text{/s}$$
$$\text{Pr} = 0.7238$$
$$\beta = \frac{1}{T_f} = \frac{1}{(115+460)\text{ R}} = 0.001739 \text{ R}^{-1}$$

Analysis (*a*) Using air density at the inlet temperature of 85°F and the specific heat at the average temperature of (85+100))/2 = 92.5°F and 1 atm for the forced air, the mass flow rate of air and the heat transfer rate by forced convection are determined to be

$$\dot{m} = \rho\dot{V} = (0.07284 \text{ lbm/ft}^3)(22 \text{ ft}^3\text{/min}) = 1.602 \text{ lbm/min}$$
$$\dot{Q}_{forced} = \dot{m}c_p(T_{out} - T_{in}) = (1.602\times60 \text{ lbm/h})(0.2404 \text{ Btu/lbm.°F})(100-85)\text{°F} = 346.6 \text{ Btu/h}$$

Noting that radiation heat transfer is negligible, the rest of the 150 W heat generated must be dissipated by natural convection,

$$\dot{Q}_{natural} = \dot{Q}_{total} - \dot{Q}_{forced} = (150\times3.412) - 346.6 = \mathbf{165\ Btu/h}$$

(b) We start the calculations by "guessing" the surface temperature to be 150°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the duct, $L_c = D = 4$ in. Then,

$$Ra = \frac{g\beta(T_1-T_2)D^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001739 \text{ R}^{-1})(150-80 \text{ R})(4/12 \text{ ft})^3}{(0.1895\times10^{-3} \text{ ft}^2\text{/s})^2}(0.7238) = 2.926\times10^6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.926\times10^6)^{1/6}}{\left[1+\left(0.559/0.7238\right)^{9/16}\right]^{8/27}}\right\}^2 = 19.79$$

$$h = \frac{k}{D}Nu = \frac{0.01564 \text{ Btu/h.ft.°F}}{4/12 \text{ ft}}(19.79) = 0.9285 \text{ Btu/h.ft}^2\text{.°F}$$
$$A_s = \pi DL = \pi(4/12 \text{ ft})(5 \text{ ft}) = 5.236 \text{ ft}^2$$

Then the surface temperature is determined to be

$$\dot{Q} = hA_s(T_s - T_\infty) \quad\rightarrow\quad T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 80\text{°F} + \frac{165 \text{ Btu/h}}{(0.9285 \text{ Btu/h.ft}^2\text{.°F})(5.236 \text{ ft}^2)} = 114\text{°F}$$

which is not close to the assumed value of 150°F used in the evaluation of properties and h. We now repeat the calculations at an improved surface temperature value of 120°F as follows:

$(T_s+T_\infty)/2 = (120+80)/2 = 100°F$ are (Table A-15E):

$$k = 0.01529 \text{ Btu/h.ft.°F}$$
$$\nu = 0.1809\times10^{-3} \text{ ft}^2\text{/s}$$
$$\text{Pr} = 0.726$$
$$\beta = 1/T_f = 1/(100+460)\text{R} = 0.001786 \text{ R}^{-1}$$

$$Ra = \frac{g\beta(T_1 - T_2)D^3}{\nu^2}\Pr = \frac{(32.2\ \text{ft/s}^2)(0.001786\ \text{R}^{-1})(120 - 80\ \text{R})(4/12\ \text{ft})^3}{(0.1809 \times 10^{-3}\ \text{ft}^2/\text{s})^2}(0.726) = 1.890 \times 10^6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1 + (0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.890 \times 10^6)^{1/6}}{\left[1 + (0.559/0.726)^{9/16}\right]^{8/27}}\right\}^2 = 17.46$$

$$h = \frac{k}{D}Nu = \frac{0.01529\ \text{Btu/h.ft.°F}}{4/12\ \text{ft}}(17.46) = 0.8009\ \text{Btu/h.ft}^2.\text{°F}$$

Then the surface temperature is determined to be

$$\dot{Q} = hA_s(T_s - T_\infty) \quad \rightarrow \quad T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 80\text{°F} + \frac{165\ \text{Btu/h}}{(0.8009\ \text{Btu/h.ft}^2.\text{°F})(5.236\ \text{ft}^2)} = \mathbf{119°F}$$

This is very close to the assumed value of 120°F used in the evaluation of properties and *h*. Therefore, there is no need to repeat the calculations.

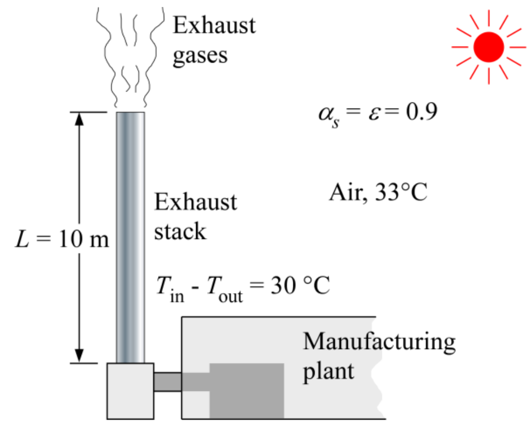
Discussion The assumed film temperature of T_f = 100°F in the second iteration is a good assumption, since the determined T_s = 119°F would give a film temperature of T_f = 99.5°F.

9-147 A 10-m tall exhaust stack discharging exhaust gases at a rate of 0.125 kg/s is subjected to solar radiation and natural convection at the outer surface. The outer surface temperature of the exhaust stack is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Properties are constant. **3** The surface temperature is constant. **4** Air is an ideal gas.

Properties The properties of air at 60°C are k = 0.02808 W/m·K, $\nu = 1.896 \times 10^{-5}$ m²/s, Pr = 0.7202 (from Table A-15). Also, $\beta = 1/T_f = 0.003003$ K^{-1}.

Analysis Assume that the exhaust stack can be treated as a vertical plate, the Rayleigh number is



$$\mathrm{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\mathrm{Pr}$$
$$= \frac{(9.81\,\mathrm{m/s^2})(0.003003\,\mathrm{K^{-1}})(T_s - 306)\mathrm{K}(10\,\mathrm{m})^3}{(1.896\times10^{-5}\,\mathrm{m^2/s})^2}(0.7202) \qquad (1)$$

Assuming the Rayleigh number is within $10^{10} < \mathrm{Ra}_L < 10^{13}$, the Nusselt number for vertical plate is

$$\mathrm{Nu} = 0.1\mathrm{Ra}_L^{1/3} \qquad \text{or} \qquad h = \left(\frac{0.02808\ \mathrm{W/m\cdot K}}{10\ \mathrm{m}}\right)0.1\mathrm{Ra}_L^{1/3} \qquad (2)$$

The outer surface area of the exhaust stack is

$$A_s = \pi DL = \pi(1\,\mathrm{m})(10\,\mathrm{m}) = 31.42\ \mathrm{m^2}$$

The rate of heat loss from the exhaust gases in the exhaust stack can be determined from

$$\dot{Q}_{\mathrm{loss}} = \dot{m}c_p(T_{\mathrm{in}} - T_{\mathrm{out}}) = (0.125\ \mathrm{kg/s})(1600\ \mathrm{J/kg\cdot{}^\circ C})(30)\ {}^\circ\mathrm{C} = 6000\ \mathrm{W}$$

The heat loss on the outer surface of the exhaust stack by radiation and convection can be expressed as

$$\frac{\dot{Q}_{\mathrm{loss}}}{A_s} = h[T_s - T_\infty] + \varepsilon\sigma[T_s^4 - T_{\mathrm{surr}}^4] - \alpha_s\dot{q}_{\mathrm{solar}} \qquad (3)$$

Equations (1), (2), and (3) can be solved simultaneously to get the surface temperature. Copy the following lines and paste on a blank EES screen to solve the above equation:

```
A_s=31.42
D=1
g=9.81
k=0.02808
L=10
Pr=0.7202
q_incident=500
Q_loss=6000
T_inf=33+273
T_surr=33+273
alpha=0.9
beta=1/(273+60)
epsilon=0.9
nu=1.896e-5
sigma=5.670e-8
Ra_L=g*beta*(T_s-T_inf)*L^3/nu^2*Pr
(h*L/(0.1*k))^3=Ra_L
Q_loss/A_s=h*(T_s-T_inf)+epsilon*sigma*(T_s^4-T_surr^4)-alpha*q_incident
```

Solving by EES software, we get

$$\mathrm{Ra}_L = 3.219\times10^{12}, \qquad h = 4.146\ \mathrm{W/m^2\cdot K}, \qquad \text{and} \qquad T_s = 360.5\ \mathrm{K}$$

Therefore, the exhaust stack outer surface temperature is

$$T_s = \mathbf{87.5^\circ C}$$

Now, we need to check if the assumption that the exhaust stack can be treated as a vertical plate is valid:

$$\frac{35L}{Gr_L^{1/4}} = \frac{35L}{(\mathrm{Ra}_L/\mathrm{Pr})^{1/4}} = \frac{35(10\ \mathrm{m})}{(3.219\times10^{12}/0.7202)^{1/4}} = 0.2407 < D$$

Since $D \geq 35L/Gr_L^{1/4}$ is satisfied, we can treat the exhaust stack as a vertical plate.

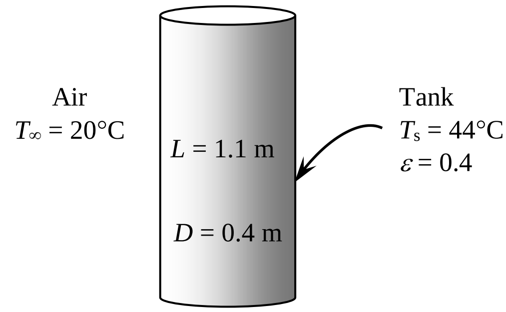
Discussion The assumption that the Rayleigh number is within $10^{10} < \mathrm{Ra}_L < 10^{13}$ turned out to be appropriate. Also, the assumed film temperature of 60 °C is valid, since $(T_s + T_\infty)/2$ = 60.3°C.

9-148 A vertically oriented cylindrical hot water tank is located in a bathroom. The rate of heat loss from the tank by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** The temperature of the outer surface of the tank is constant.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (44+20)/2 = 32°C$ are (Table A-15)

$$k = 0.02603 \text{ W/m}\cdot K$$
$$\nu = 1.627\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7276$$
$$\beta = \frac{1}{T_f} = \frac{1}{(32+273)\text{K}} = 0.003279 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the height of the cylinder, $L_c = L = 1.1\text{ m}$. Then,

$$\text{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81\text{ m/s}^2)(0.003279\text{ K}^{-1})(44-20\text{ K})(1.1\text{ m})^3}{(1.627\times10^{-5}\text{ m}^2/\text{s})^2} = 3.883\times10^9$$

A vertical cylinder can be treated as a vertical plate when

$$D(=0.4\text{ m}) \geq \frac{35L}{\text{Gr}^{1/4}} = \frac{35(1.1\text{ m})}{(3.883\times10^9)^{1/4}} = 0.1542\text{ m}$$

which is satisfied. That is, the Nusselt number relation for a vertical plate can be used for the side surfaces. For the top and bottom surfaces we use the relevant Nusselt number relations. First, for the side surfaces,

$$\text{Ra} = \text{GrPr} = (3.883\times10^9)(0.7276) = 2.825\times10^9$$

$$Nu = \left\{0.825 + \frac{0.387\text{Ra}^{1/6}}{\left[1+\left(\frac{0.492}{\text{Pr}}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.825 + \frac{0.387(2.825\times10^9)^{1/6}}{\left[1+\left(\frac{0.492}{0.7276}\right)^{9/16}\right]^{8/27}}\right\}^2 = 170.2$$

$$h = \frac{k}{L}Nu = \frac{0.02603\text{ W/m}\cdot K}{1.1\text{ m}}(170.2) = 4.027\text{ W/m}^2\cdot K$$

$$A_s = \pi DL = \pi(0.4\text{ m})(1.1\text{ m}) = 1.382\text{ m}^2$$

$$\dot{Q}_{\text{side}} = hA_s(T_s - T_\infty) = (4.027\text{ W/m}^2\cdot K)(1.382\text{ m}^2)(44-20)°\text{C} = 133.6\text{ W}$$

For the top surface,

$$L_c = \frac{A_s}{p} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4} = \frac{0.4\text{ m}}{4} = 0.1\text{ m}$$

$$\text{Ra} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003279\text{ K}^{-1})(44-20\text{ K})(0.1\text{ m})^3}{(1.627\times10^{-5}\text{ m}^2/\text{s})^2}(0.7276) = 2.123\times10^6$$

$$\text{Nu} = 0.54\text{Ra}^{1/4} = 0.54(2.123\times10^6)^{1/4} = 20.61$$

$$h = \frac{k}{L_c}Nu = \frac{0.02603\text{ W/m}\cdot K}{0.1\text{ m}}(20.61) = 5.365\text{ W/m}^2\cdot K$$

$$A_s = \pi D^2/4 = \pi(0.4\text{ m})^2/4 = 0.1257\text{ m}^2$$

$$\dot{Q}_{\text{top}} = hA_s(T_s - T_\infty) = (5.365\text{ W/m}^2\cdot K)(0.1257\text{ m}^2)(44-20)°\text{C} = 16.2\text{ W}$$

For the bottom surface,

$$\text{Nu} = 0.27\text{Ra}^{1/4} = 0.27(2.123\times10^6)^{1/4} = 10.31$$

$$h = \frac{k}{L_c} Nu = \frac{0.02603\ \text{W/m} \cdot K}{0.1\,\text{m}} (10.31) = 2.683\ \text{W/m}^2 \cdot K$$

$$\dot{Q}_{\text{bottom}} = hA_s (T_s - T_\infty) = (2.683\ \text{W/m}^2 \cdot K)(0.1257\ \text{m}^2)(44 - 20)°\text{C} = 8.1\ \text{W}$$

The total heat loss by natural convection is

$$\dot{Q}_{\text{conv}} = \dot{Q}_{\text{side}} + \dot{Q}_{\text{top}} + \dot{Q}_{\text{bottom}} = 133.6 + 16.2 + 8.1 = \mathbf{157.9\ W}$$

The radiation heat loss from the tank is

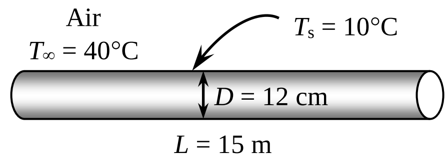
$$\begin{aligned} \dot{Q}_{\text{rad}} &= \varepsilon A_s \sigma (T_s{}^4 - T_{surr}{}^4) \\ &= (0.4)(1.382 + 0.1257 + 0.1257\ \text{m}^2)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)\left[(44 + 273\,\text{K})^4 - (20 + 273\,\text{K})^4\right] \\ &= \mathbf{101.1\ W} \end{aligned}$$

9-149 A cold cylinder is placed horizontally in hot air. The rates of heat transfer from the stack with and without wind cases are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (40+10)/2 = 25°C$ are (Table A-15)

$$k = 0.02551 \text{ W/m.°C}$$
$$\nu = 1.562\times10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.7296$$
$$\beta = \frac{1}{T_f} = \frac{1}{(25+273)\text{K}} = 0.003356 \text{ K}^{-1}$$



Analysis (*a*) When the stack is exposed to 10 m/s winds, the heat transfer will be by forced convection. We have flow of air over a cylinder and the heat transfer rate is determined as follows:

$$\text{Re} = \frac{VD}{\nu} = \frac{(10 \text{ m/s})(0.12\text{m})}{1.562\times10^{-5} \text{ m}^2/\text{s}} = 76{,}825$$

$$\text{Nu} = 0.027\,\text{Re}^{0.805}\,\text{Pr}^{1/3} = 0.027(76{,}825)^{0.805}(0.7296)^{1/3} = 208.2 \qquad \text{(from Table 7-1)}$$

$$h = \frac{k}{D}\text{Nu} = \frac{0.02551 \text{ W/m.°C}}{0.12 \text{ m}}(208.2) = 44.27 \text{ W/m}^2.°\text{C}$$

$$\dot{Q}_{\text{forced conv}} = hA(T_\infty - T_s) = (44.27 \text{ W/m}^2.°\text{C})(\pi\times0.12\times15\text{ m}^2)(40-10)°\text{C} = \mathbf{7510\ W}$$

(b) Without wind the heat transfer will be by natural convection. The characteristic length in this case is the outer diameter of the cylinder, $L_c = D = 0.12$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)D^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003356 \text{ K}^{-1})(40-10 \text{ K})(0.12 \text{ m})^3}{(1.562\times10^{-5} \text{ m}^2/\text{s})^2}(0.7296) = 5.104\times10^6$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(5.104\times10^6)^{1/6}}{\left[1+(0.559/0.7296)^{9/16}\right]^{8/27}}\right\}^2 = 23.28$$

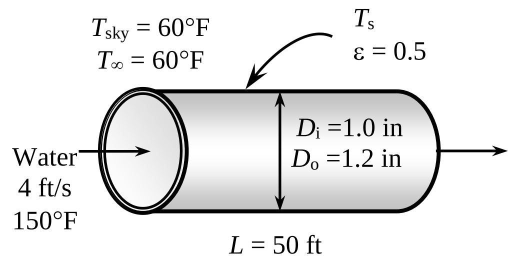
$$h = \frac{k}{D}Nu = \frac{0.02551 \text{ W/m.°C}}{0.12 \text{ m}}(23.28) = 4.949 \text{ W/m}^2.°\text{C}$$

$$\dot{Q}_{\text{nat. conv}} = hA(T_\infty - T_s) = (4.949 \text{ W/m}^2.°\text{C})(\pi\times0.12\times15\text{ m}^2)(40-10)°\text{C} = \mathbf{840\ W}$$

9-150E A hot water pipe passes through a basement. The temperature drop of water in the basement due to heat loss from the pipe is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (150+60)/2 = 105°F are (Table A-15E)



$$k = 0.01541 \text{ Btu/h.ft.°F}$$
$$\nu = 0.1838\times10^{-3} \text{ ft}^2\text{/s}$$
$$\text{Pr} = 0.7253$$
$$\beta = \frac{1}{T_f} = \frac{1}{(105+460)\text{R}} = 0.00177 \text{ R}^{-1}$$

Analysis We expect the pipe temperature to be very close to the water temperature, and start the calculations by "guessing" the average outer surface temperature of the pipe to be 150°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the pipe, $L_c = D_o$ = 1.2 in. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)D_o^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.00177 \text{ R}^{-1})(150-60 \text{ R})(1.2/12 \text{ ft})^3}{(0.1838\times10^{-3} \text{ ft}^2\text{/s})^2}(0.7253) = 1.101\times10^5$$

The natural convection Nusselt number can be determined from

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.101\times10^5)^{1/6}}{\left[1+(0.559/0.7253)^{9/16}\right]^{8/27}}\right\}^2 = 7.998$$

$$h_o = \frac{k}{D_o}Nu = \frac{0.01541 \text{ Btu/h.ft.°F}}{(1.2/12) \text{ ft}}(7.998) = 1.232 \text{ Btu/h.ft}^2\text{.°F}$$
$$A_i = \pi D_i L = \pi(1/12 \text{ ft})(50 \text{ ft}) = 13.09 \text{ ft}^2$$
$$A_o = \pi D_o L = \pi(1.2/12 \text{ ft})(50 \text{ ft}) = 15.708 \text{ ft}^2$$

Using the assumed value of glass temperature, the radiation heat transfer coefficient is determined to be

$$\begin{aligned} h_{rad} &= \varepsilon\sigma(T_s + T_{surr})(T_s^2 + T_{surr}^2) \\ &= (0.5)(0.1714\times10^{-8} \text{ Btu/h.ft}^2\text{.R}^4)[(150+460)+(60+460)][(150+460)^2+(60+460)^2]R^3 \\ &= 0.6222 \text{ Btu/h.ft}^2\text{.R} \end{aligned}$$

Then the combined convection and radiation heat transfer coefficient outside becomes

$$h_{o,combined} = h_o + h_{rad} = 1.232 + 0.6222 = 1.854 \text{ Btu/h.ft}^2\text{.R}$$

and
$$\dot{Q} = \frac{T_{water} - T_\infty}{\frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{4\pi kL} + \frac{1}{h_o A_o}} = \frac{150-60}{\frac{1}{(30)(13.09)} + \frac{\ln(1.2/1)}{4\pi(30)(50)} + \frac{1}{(1.854)(15.708)}} = 2440 \text{ Btu/h}$$

The mass flow rate of water

$$\dot{m} = \rho A_c V = (61.2 \text{ lbm/ft}^3)\left[\pi(1/12 \text{ ft})^2/4\right](4 \text{ ft/s}) = 1.335 \text{ lbm/s} = 4807 \text{ lbm/h}$$

Then the temperature drop of water as it flows through the pipe becomes

$$\dot{Q} = \dot{m}c_p\Delta T \rightarrow \Delta T = \frac{\dot{Q}}{\dot{m}c_p} = \frac{2440 \text{ Btu/h}}{(4807 \text{ lbm/h})(1.0 \text{ Btu/lbm.°F})} = \mathbf{0.51°F}$$

Discussion The outer surface temperature of the pipe can be determined from

$$\dot{Q} = h_o A_o(T_{s,o} - T_\infty) \longrightarrow 2440 \text{ Btu/h} = (1.854 \text{ Btu/h.ft}^2\text{.R})(15.708 \text{ ft}^2)(T_{s,o} - 60) \longrightarrow T_{s,o} = 143.8°\text{F}$$

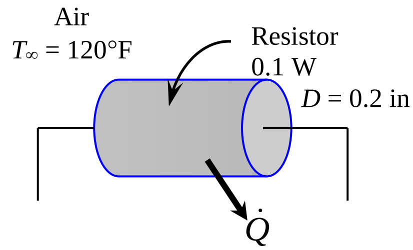
which is sufficiently close to the assumed value of 150°C. Therefore, there is no need to repeat the calculations.

Discussion The assumed film temperature of T_f = 105°F is an appropriate assumption, since the determined T_s = 143.8°F would give a film temperature of T_f = 101.9°F. Otherwise, $T_{s,o}$ would have to be solved iteratively.

9-151E A small cylindrical resistor mounted on the lower part of a vertical circuit board. The approximate surface temperature of the resistor is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm. **4** Radiation effects are negligible. **5** Heat transfer through the connecting wires is negligible.

Properties Based on the problem statement, the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (220+120)/2 =$ 170°F are (Table A-15E)



$$k = 0.01692 \text{ Btu/h.ft.°F}$$
$$\nu = 0.222\times10^{-3} \text{ ft}^2\text{/s}$$
$$\text{Pr} = 0.7161$$
$$\beta = \frac{1}{T_f} = \frac{1}{(170+460)\text{R}} = 0.001587 \text{ R}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 220°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the diameter of resistor, $L_c = D = 0.2$ in. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(32.2 \text{ ft/s}^2)(0.001587 \text{ R}^{-1})(220-120 \text{ R})(0.2/12 \text{ ft})^3}{(0.222\times10^{-3} \text{ ft}^2\text{/s})^2}(0.7161) = 343.8$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(343.8)^{1/6}}{\left[1+\left(0.559/0.7161\right)^{9/16}\right]^{8/27}}\right\}^2 = 2.105$$

$$h = \frac{k}{D}Nu = \frac{0.01692 \text{ Btu/h.ft.°F}}{0.2/12 \text{ ft}}(2.105) = 2.138 \text{ Btu/h.ft}^2\text{.°F}$$

$$A_s = \pi DL + 2\pi D^2/4 = \pi(0.2/12 \text{ ft})(0.3/12 \text{ ft}) + 2\pi(0.2/12 \text{ ft})^2/4 = 0.00175 \text{ ft}^2$$

and

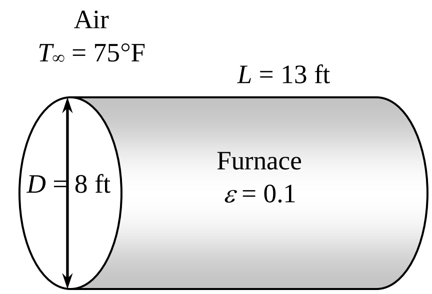
$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow T_s = T_\infty + \frac{\dot{Q}}{hA_s} = 120\text{°F} + \frac{(0.1\times3.412) \text{ Btu/h}}{(2.138 \text{ Btu/h.ft}^2\text{.°F})(0.00175 \text{ ft}^2)} = \mathbf{211°F}$$

which is sufficiently close to the assumed temperature for the evaluation of properties. Therefore, there is no need to repeat calculations.

Discussion The assumed film temperature of T_f = 170°F is an appropriate assumption, since the determined T_s = 211°F would give a film temperature of T_f = 165.5°F. Otherwise, T_s would have to be solved iteratively.

9-152E An industrial furnace that resembles a horizontal cylindrical enclosure whose end surfaces are well insulated. The highest allowable surface temperature of the furnace and the annual cost of this loss to the plant are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.



Properties The properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2$ = (140+75)/2=107.5°F are (Table A-15E)

$$k = 0.01546 \text{ Btu/h.ft.°F}$$
$$\nu = 0.1852\times10^{-3} \text{ ft}^2\text{/s}$$
$$\Pr = 0.7249$$
$$\beta = \frac{1}{T_f} = \frac{1}{(107.5+460)\text{R}} = 0.001762 \text{ R}^{-1}$$

Analysis The solution of this problem requires a trial-and-error approach since the determination of the Rayleigh number and thus the Nusselt number depends on the surface temperature which is unknown. We start the solution process by "guessing" the surface temperature to be 140°F for the evaluation of the properties and h. We will check the accuracy of this guess later and repeat the calculations if necessary. The characteristic length in this case is the outer diameter of the furnace, $L_c = D = 8$ ft. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\Pr = \frac{(32.2 \text{ ft/s}^2)(0.001762 \text{ R}^{-1})(140-75 \text{ R})(8 \text{ ft})^3}{(0.1852\times10^{-3} \text{ ft}^2\text{/s})^2}(0.7249) = 3.991\times10^{10}$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(3.991\times10^{10})^{1/6}}{\left[1+(0.559/0.7249)^{9/16}\right]^{8/27}}\right\}^2 = 376.8$$

$$h = \frac{k}{D}Nu = \frac{0.01546 \text{ Btu/h.ft.°F}}{8 \text{ ft}}(376.8) = 0.7287 \text{ Btu/h.ft}^2\text{.°F}$$

$$A_s = \pi DL = \pi(8 \text{ ft})(13 \text{ ft}) = 326.7 \text{ ft}^2$$

The total rate of heat generated in the furnace is

$$\dot{Q}_{gen} = (0.82)(48 \text{ therms/h})(100{,}000 \text{ Btu/therm}) = 3.936\times10^6 \text{ Btu/h}$$

Noting that 1% of the heat generated can be dissipated by natural convection and radiation,

$$\dot{Q} = (0.01)(3.936\times10^6 \text{ Btu/h}) = 39{,}360 \text{ Btu/h}$$

The total rate of heat loss from the furnace by natural convection and radiation can be expressed as

$$\dot{Q} = hA_s(T_s - T_\infty) + \varepsilon A_s\sigma(T_s{}^4 - T_{surr}{}^4)$$
$$39{,}360 \text{ Btu/h} = (0.7287 \text{ Btu/h.ft}^2\text{.°F})(326.7 \text{ ft}^2)[T_s - (75+460 \text{ R})]$$
$$+ (0.85)(326.7 \text{ m}^2)(0.1714\times10^{-8} \text{ Btu/h.ft}^2\text{.R}^4)[T_s{}^4 - (75+460 \text{ R})^4]$$

Its solution is

$$T_s = 601.8 \text{ R} = \mathbf{141.8°F}$$

which is very close to the assumed value. Therefore, there is no need to repeat calculations. The total amount of heat loss and its cost during a-2800 hour period is

$$Q_{total} = \dot{Q}_{total}\Delta t = (39{,}360 \text{ Btu/h})(2800 \text{ h}) = 1.102\times10^8 \text{ Btu}$$

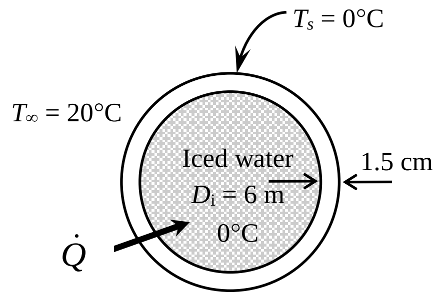
$$\text{Cost} = (1.102\times10^8 / 100{,}000 \text{ therm})(\$1.15/\text{therm}) = \mathbf{\$1267}$$

9-153 A spherical tank made of stainless steel is used to store iced water. The rate of heat transfer to the iced water and the amount of ice that melts during a 24-h period are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Thermal resistance of the tank is negligible. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (0+20)/2 = 10°C$ are (Table A-15)

$$k = 0.02439 \text{ W/m.°C}$$
$$\nu = 1.426\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7336$$
$$\beta = \frac{1}{T_f} = \frac{1}{(10+273)\text{K}} = 0.003534 \text{ K}^{-1}$$



Analysis (*a*) The characteristic length in this case is $L_c = D_o = 6.03$ m. Then,

$$Ra = \frac{g\beta(T_\infty - T_s)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003534 \text{ K}^{-1})(20-0 \text{ K})(6.03 \text{ m})^3}{(1.426\times10^{-5} \text{ m}^2/\text{s})^2}(0.7336) = 5.485\times10^{11}$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+(0.469/\text{Pr})^{9/16}\right]^{4/9}} = 2 + \frac{0.589(5.485\times10^{11})^{1/4}}{\left[1+(0.469/0.7336)^{9/16}\right]^{4/9}} = 394.5$$

$$h = \frac{k}{D_o}Nu = \frac{0.02439 \text{ W/m.°C}}{6.03 \text{ m}}(394.5) = 1.596 \text{ W/m}^2.°\text{C}$$

$$A_s = \pi D_o^2 = \pi(6.03 \text{ m})^2 = 114.2 \text{ m}^2$$

and

$$\dot{Q} = hA_s(T_\infty - T_s) = (1.596 \text{ W/m}^2.°\text{C})(114.2 \text{ m}^2)(20-0)°\text{C} = 3646 \text{ W}$$

Heat transfer by radiation and the total rate of heat transfer are

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^{\,4} - T_{surr}^{\,4})$$
$$= (1)(114.2 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2.\text{K}^4)[(20+273 \text{ K})^4 - (0+273 \text{ K})^4] = 11{,}759 \text{ W}$$
$$\dot{Q}_{total} = 3646 + 11{,}759 = 15{,}404 \text{ W} \cong \mathbf{15.4 \text{ kW}}$$

(*b*) The total amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (15.4 \text{ kJ/s})(24 \text{ h/day}\times3600 \text{ s/h}) = 1.331\times10^6 \text{ kJ/day}$$

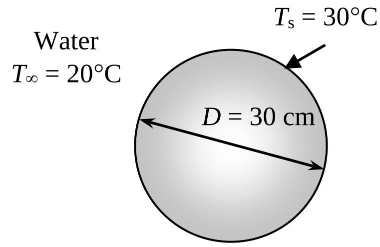
Then the amount of ice that melts during this period becomes

$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{1.331\times10^6 \text{ kJ}}{333.7 \text{ kJ/kg}} = \mathbf{3988 \text{ kg}}$$

9-154 A spherical vessel is completely submerged in a large water-filled tank. The rates of heat transfer from the vessel by natural convection, conduction, and forced convection are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature is constant.

Properties The properties of water at the film temperature of $(T_s+T_\infty)/2 = (30+20)/2 = 25°C$ are (Table A-9)



$$\rho = 997 \text{ kg/m}^3 \qquad k = 0.607 \text{ W/m.°C}$$
$$\mu = 0.891\times10^{-3} \text{ kg/m}\cdot\text{s} \qquad \nu = \mu/\rho = 8.937\times10^{-7} \text{ m}^2\text{/s}$$
$$\text{Pr} = 6.14 \qquad \beta = 0.247\times10^{-3} \text{ K}^{-1}$$

Analysis (*a*) Heat transfer in this case will be by natural convection. The characteristic length in this case is $L_c = D = 0.3$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.247\times10^{-3} \text{ K}^{-1})(30-20 \text{ K})(0.3 \text{ m})^3}{(8.937\times10^{-7} \text{ m}^2\text{/s})^2}(6.14) = 5.029\times10^9$$

$$Nu = 2 + \frac{0.589Ra^{1/4}}{\left[1+(0.469/\text{Pr})^{9/16}\right]^{4/9}} = 2 + \frac{0.589(5.029\times10^9)^{1/4}}{\left[1+(0.469/6.14)^{9/16}\right]^{4/9}} = 144.8$$

Then

$$h = \frac{k}{D}Nu = \frac{0.607 \text{ W/m.°C}}{0.3 \text{ m}}(144.8) = 293.0 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi D^2 = \pi(0.3 \text{ m})^2 = 0.2827 \text{ m}^2$$

The rate of heat transfer is

$$\dot{Q}_{\text{nat. conv}} = hA(T_s - T_\infty) = (293.0 \text{ W/m}^2\text{.°C})(0.2827 \text{ m}^2)(30-20)\text{°C} = \mathbf{828 \text{ W}}$$

(*b*) When buoyancy force is neglected, there will be no convection currents (since β = 0) and the heat transfer will be by conduction. Then Rayleigh number becomes zero (Ra = 0). The Nusselt number in this case is

Nu = 2

Then

$$h = \frac{k}{D}Nu = \frac{0.607 \text{ W/m.°C}}{0.3 \text{ m}}(2) = 4.047 \text{ W/m}^2\text{.°C}$$

$$\dot{Q}_{\text{cond}} = hA(T_s - T_\infty) = (4.047 \text{ W/m}^2\text{.°C})(0.2827 \text{ m}^2)(30-20)\text{°C} = \mathbf{11.4 \text{ W}}$$

(*c*) In this case, the heat transfer from the vessel is by forced convection. The properties of water at the free stream temperature of 20°C are (Table A-9)

$$\rho = 998 \text{ kg/m}^3 \qquad k = 0.598 \text{ W/m.°C}$$
$$\mu_\infty = 1.002\times10^{-3} \text{ kg/m.s} \qquad \nu = \mu_\infty/\rho = 1.004\times10^{-6} \text{ m}^2\text{/s}$$
$$\mu_{s,@30°C} = 0.798\times10^{-3} \text{ kg/m.s} \qquad \text{Pr} = 7.01$$

The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(0.2 \text{ m/s})(0.3 \text{m})}{1.004\times10^{-6} \text{ m}^2\text{/s}} = 59{,}760$$

The Nusselt number is

$$Nu = \frac{hD}{k} = 2 + \left[0.4\text{Re}^{0.5} + 0.06\text{Re}^{2/3}\right]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(59{,}760)^{0.5} + 0.06(59{,}760)^{2/3}\right](7.01)^{0.4}\left(\frac{1.002\times10^{-3}}{0.798\times10^{-3}}\right)^{1/4} = 439.1$$

The heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.598 \text{ W/m.°C}}{0.3 \text{ m}}(439.1) = 875.3 \text{ W/m}^2\text{.°C}$$

The rate of heat transfer is

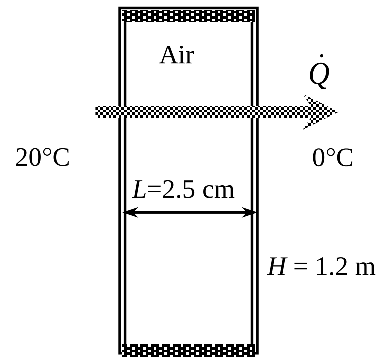
$$\dot{Q}_{\text{forced conv}} = hA(T_s - T_\infty) = (875.3 \text{ W/m}^2\text{.°C})(0.2827 \text{ m}^2)(30-20)\text{°C} = \mathbf{2474 \text{ W}}$$

9-155 A double-pane window consisting of two layers of glass separated by an air space is considered. The rate of heat transfer through the window and the temperature of its inner surface are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Radiation effects are negligible. **4** The pressure of air inside the enclosure is 1 atm.

Properties We expect the average temperature of the air gap to be roughly the average of the indoor and outdoor temperatures, and evaluate The properties of air at 1 atm and the average temperature of $(T_{\infty 1}+T_{\infty 2})/2 = (20+0)/2 = 10°C$ are (Table A-15)

$$k = 0.02439 \text{ W/m.°C}$$
$$\nu = 1.426\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7336$$
$$\beta = \frac{1}{T_f} = \frac{1}{(10+273)\text{K}} = 0.003534 \text{ K}^{-1}$$



Analysis We "guess" the temperature difference across the air gap to be 15°C = 15 K for use in the Ra relation. The characteristic length in this case is the air gap thickness, $L_c = L = 0.025$ m. Then,

$$Ra = \frac{g\beta(T_1-T_2)L^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003534\text{ K}^{-1})(15\text{ K})(0.025\text{ m})^3}{(1.426\times10^{-5}\text{ m}^2\text{/s})^2}(0.7336) = 2.931\times10^4$$

Then the Nusselt number and the heat transfer coefficient are determined to be

$$Nu = 0.42Ra^{1/4}\,\text{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(2.931\times10^4)^{1/4}(0.7336)^{0.012}\left(\frac{1.2\text{ m}}{0.025\text{ m}}\right)^{-0.3} = 1.714$$

$$h_{air} = \frac{k}{L}Nu = \frac{0.02439\text{ W/m.°C}}{0.025\text{ m}}(1.714) = 1.672\text{ W/m}^2\text{.°C}$$

Then the rate of heat transfer through this double pane window is determined to be

$$A_s = H\times W = (1.2\text{ m})(2\text{ m}) = 2.4\text{ m}^2$$

$$\dot{Q} = \frac{T_{\infty,i}-T_{\infty,o}}{R_{conv,i}+R_{cond,glasses}+R_{conv,air}+R_{conv,o}} = \frac{T_\infty - T_{s,i}}{\dfrac{1}{h_iA_s}+\dfrac{2t_{\text{glass}}}{k_{glass}A_s}+\dfrac{1}{h_{air}A_s}+\dfrac{1}{h_oA_s}}$$

$$= \frac{20-0}{\dfrac{1}{(10)(2.4)}+\dfrac{2(0.003)}{(0.78)(2.4)}+\dfrac{1}{(1.672)(2.4)}+\dfrac{1}{(25)(2.4)}} = \mathbf{64.4\ W}$$

Check: The temperature drop across the air gap is determined from

$$\dot{Q} = hA_s\Delta T \;\rightarrow\; \Delta T = \frac{\dot{Q}}{hA_s} = \frac{64.4\text{ W}}{(1.672\text{ W/m}^2\text{.°C})(2.4\text{ m}^2)} = 16.0°\text{C}$$

which is very close to the assumed value of 15°C used in the evaluation of the Ra number.

9-156 A solar collector consists of a horizontal copper tube enclosed in a concentric thin glass tube. Water is heated in the tube, and the annular space between the copper and glass tube is filled with air. The rate of heat loss from the collector by natural convection is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** Radiation effects are negligible. **3** The pressure of air in the enclosure is 1 atm.

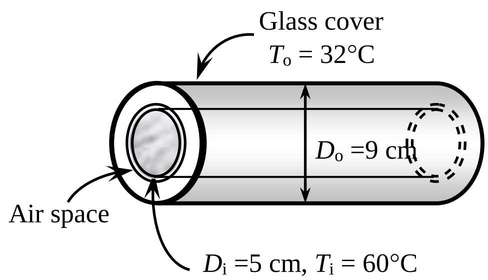
Properties The properties of air at 1 atm and the average temperature of $(T_i+T_o)/2 = (60+32)/2 = 46°C$ are (Table A-15)

$$k = 0.02706 \text{ W/m.°C}$$

$$\nu = 1.760\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7238$$

$$\beta = \frac{1}{T_f} = \frac{1}{(46+273)\text{K}} = 0.003135 \text{ K}^{-1}$$



Analysis The characteristic length in this case is the distance between the two cylinders

$$L_c = \frac{D_o - D_i}{2} = \frac{(9-5)\text{ cm}}{2} = 2\text{ cm}$$

and

$$Ra = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003135 \text{ K}^{-1})(60-32\text{ K})(0.02\text{ m})^3}{(1.760\times10^{-5}\text{ m}^2/\text{s})^2}(0.7238) = 16{,}100$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{\left[\ln\dfrac{D_o}{D_i}\right]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} = \frac{\left[\ln\dfrac{0.09\text{ m}}{0.05\text{ m}}\right]^4}{(0.02\text{ m})^3\left[(0.05\text{ m})^{-7/5} + (0.09\text{ m})^{-7/5}\right]^5} = 0.1303$$

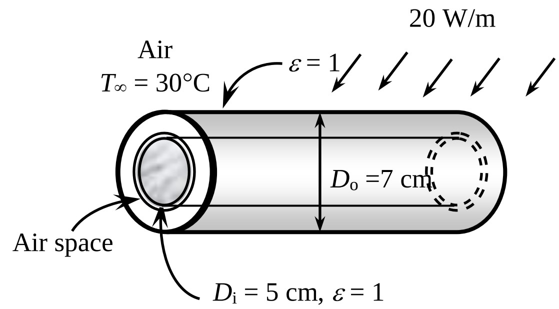
$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$

$$= 0.386(0.02706\text{ W/m.°C})\left(\frac{0.7238}{0.861+0.7238}\right)^{1/4}\left[(0.1303)(16{,}100)\right]^{1/4} = 0.05811\text{ W/m.°C}$$

Then the heat loss from the collector per meter length of the tube becomes

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln\left(\dfrac{D_o}{D_i}\right)}(T_i - T_o) = \frac{2\pi(0.05811\text{ W/m.°C})}{\ln\left(\dfrac{0.09\text{ m}}{0.05\text{ m}}\right)}(60-32)\text{°C} = \mathbf{17.4\text{ W}}$$

9-157 A solar collector consists of a horizontal tube enclosed in a concentric thin glass tube is considered. The pump circulating the water fails. The temperature of the aluminum tube when equilibrium is established is to be determined.



Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The local atmospheric pressure is 1 atm.

Properties We assume a surface temperature of 33°C for glass cover based on the problem statement. Then the properties of air at 1 atm and the anticipated film temperature of $(T_s+T_\infty)/2 = (33+30)/2 = 31.5$°C are (Table A-15)

$$k = 0.02599 \text{ W/m.°C}$$
$$\nu = 1.622\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7278$$
$$\beta = \frac{1}{T_f} = \frac{1}{(31.5+273)\text{K}} = 0.003284 \text{ K}^{-1}$$

Analysis This problem involves heat transfer from the aluminum tube to the glass cover, and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfers will be equal to the rate of heat gain. That is,

$$\dot{Q}_{tube-glass} = \dot{Q}_{glass-ambient} = \dot{Q}_{solar\ gain} = 20 \text{ W (per meter length)}$$

Now we assume the surface temperature of the glass cover to be 33°C. We will check this assumption later on, and repeat calculations with a better assumption, if necessary.

The characteristic length for the outer diameter of the glass cover $L_c = D_o$ =0.07 m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D_o^3}{\nu^2}\text{Pr} = \frac{(9.81 \text{ m/s}^2)(0.003284 \text{ K}^{-1})(33-30 \text{ K})(0.07 \text{ m})^3}{(1.622\times10^{-5} \text{ m}^2/\text{s})^2}(0.7278) = 91{,}700$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(91{,}700)^{1/6}}{\left[1+(0.559/0.7278)^{9/16}\right]^{8/27}}\right\}^2 = 7.626$$

$$A_s = \pi D_o L = \pi(0.07 \text{ m})(1 \text{ m}) = 0.2199 \text{ m}^2$$

$$h = \frac{k}{D_o}Nu = \frac{0.02599 \text{ W/m.°C}}{0.07 \text{ m}}(7.626) = 2.832 \text{ W/m}^2\text{.°C}$$

and,

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) = (2.832 \text{ W/m}^2\text{.°C})(0.2199 \text{ m}^2)(T_{glass} - 30)\text{°C}$$

The radiation heat loss is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^4 - T_{surr}^4) = (1)(0.2199 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(T_{glass}+273 \text{ K})^4 - (20+273 \text{ K})^4\right]$$

The expression for the total rate of heat transfer is

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad}$$
$$20 \text{ W} = (2.832 \text{ W/m}^2\text{.°C})(0.2199 \text{ m}^2)(T_{glass} - 30)\text{°C}$$
$$+ (1)(0.2199 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(T_{glass}+273 \text{ K})^4 - (20+273 \text{ K})^4\right]$$

Its solution is

$$T_{glass} = 33.34\text{°C}$$

which is sufficiently close to the assumed value of 33°C. Therefore, there is no need to repeat the calculations.

Now we will calculate heat transfer through the air layer between aluminum tube and glass cover. We will assume the aluminum tube temperature to be 45°C based on the problem statement and evaluate properties at the average temperature of $(T_i+T_o)/2$ = (45+33.34)/2 = 39.17°C are (Table A-15)

$$k = 0.02656 \text{ W/m.°C}$$
$$\nu = 1.694 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.7257$$
$$\beta = \frac{1}{T_f} = \frac{1}{(39.17 + 273)\text{K}} = 0.003203 \text{ K}^{-1}$$

The characteristic length in this case is the distance between the two cylinders,

$$L_c = (D_o - D_i)/2 = (7-5)/2 \text{ cm} = 1 \text{ cm}$$

Then,

$$Ra = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003203 \text{ K}^{-1})(45 - 33.34 \text{ K})(0.01 \text{ m})^3}{(1.694 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.7257) = 926.5$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{\left[\ln \frac{D_o}{D_i}\right]^4}{L_c^3 (D_i^{-3/5} + D_o^{-3/5})^5} = \frac{\left[\ln \frac{0.07 \text{ m}}{0.05 \text{ m}}\right]^4}{(0.01 \text{ m})^3 \left[(0.05 \text{ m})^{-3/5} + (0.07 \text{ m})^{-3/5}\right]^5} = 0.08085$$

$$k_{\text{eff}} = 0.386k\left(\frac{\Pr}{0.861 + \Pr}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$
$$= 0.386(0.02656 \text{ W/m.°C})\left(\frac{0.7257}{0.861 + 0.7257}\right)^{1/4}\left[(0.08085)(926.5)\right]^{1/4} = 0.02480 \text{ W/m.°C}$$

The heat transfer expression is

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln\left(\frac{D_o}{D_i}\right)}(T_1 - T_2) = \frac{2\pi(0.02480 \text{ W/m.°C})}{\ln\left(\frac{0.07 \text{ m}}{0.05 \text{ m}}\right)}(T_{tube} - 33.34)\text{°C}$$

The radiation heat loss is

$$A_s = \pi D_i L = \pi(0.05 \text{ m})^2(1 \text{ m}) = 0.1571 \text{ m}^2$$
$$\dot{Q}_{rad} = \varepsilon A_s \sigma(T_s^4 - T_{surr}^4)$$
$$= (1)(0.1571 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2.\text{K}^4)\left[(T_{tube} + 273 \text{ K})^4 - (33.34 + 273 \text{ K})^4\right]$$

The expression for the total rate of heat transfer is

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad}$$
$$20 \text{ W} = \frac{2\pi(0.02480 \text{ W/m.°C})}{\ln\left(\frac{0.07 \text{ m}}{0.05 \text{ m}}\right)}(T_{tube} - 33.34)\text{°C}$$
$$+ (1)(0.1571 \text{ m}^2)(5.67 \times 10^{-8} \text{ W/m}^2.\text{K}^4)\left[(T_{tube} + 273 \text{ K})^4 - (33.34 + 273 \text{ K})^4\right]$$

Its solution is

$$T_{tube} = \mathbf{46.3°C},$$

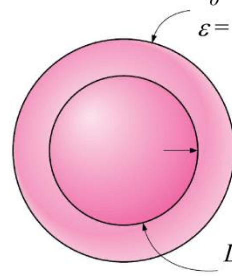
which is sufficiently close to the assumed value of 45°C. Therefore, there is no need to repeat the calculations.

9-158 Two surfaces of a spherical enclosure are maintained at specified temperatures. Both inner and outer surfaces are black, and the rate of heat transfer on the inner surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Nitrogen is an ideal gas with constant properties. **3** The inner and outer surfaces are black.

Properties The properties of nitrogen at $T_f = (T_s + T_\infty)/2 = 150°C$ are $k = 0.03416$ W/m·K, $\nu = 2.851 \times 10^{-5}$ m²/s, Pr = 0.7025 (from Table A-16). Also, $\beta = 1/T_f = 0.002364\ \text{K}^{-1}$.

Analysis The characteristic length in this case is determined from



$$L_c = \frac{D_o - D_i}{2} = \frac{10-5}{2}\ \text{cm} = 2.5\ \text{cm}$$

The Rayleigh number is

$$\text{Ra}_L = \frac{g\beta(T_i - T_o)L_c^3}{\nu^2}\text{Pr}$$
$$= \frac{(9.81\ \text{m/s}^2)(2.364\times10^{-3}\ \text{K}^{-1})(200-100)\text{K}(0.025\ \text{m})^3}{(2.851\times10^{-5}\ \text{m}^2/\text{s})^2}(0.7025)$$
$$= 3.132\times10^4$$

The effective thermal conductivity is

$$F_{\text{sph}} = \frac{L_c}{(D_i D_o)^4 (D_i^{-7/5} + D_o^{-7/5})^5} = \frac{(0.025\ \text{m})}{[(0.1\ \text{m})(0.05\ \text{m})]^4[(0.05\ \text{m})^{-7/5} + (0.1\ \text{m})^{-7/5}]^5}$$
$$= 0.006268$$

$$k_{\text{eff}} = 0.74\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{sph}}\text{Ra}_L)^{1/4}k$$
$$= 0.74\left(\frac{0.7025}{0.861+0.7025}\right)^{1/4}[(0.006268)(3.132\times10^4)]^{1/4}(0.03416\ \text{W/m}\cdot\text{K})$$
$$= 0.07747\ \text{W/m}\cdot\text{K}$$

Then the rate of heat transfer by natural convection is

$$\dot{Q}_{\text{conv}} = k_{\text{eff}}\frac{\pi D_i D_o}{L_c}(T_i - T_o) = (0.07747\ \text{W/m}\cdot\text{K})\frac{\pi(0.1\ \text{m})(0.05\ \text{m})}{(0.025\ \text{m})}(200-100)\ \text{K} = 4.87\ \text{W}$$

The rate of heat transfer by radiation on the inner surface is

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_{s,i}(T_i^4 - T_o^4) = \dot{Q}_{\text{rad}} = \sigma\pi D_i^2(T_i - T_o)$$
$$= (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\pi(0.05\ \text{m})^2(473^4 - 373^4)\ \text{K}^4 = 13.67\ \text{W}$$

Hence, the total rate of heat transfer on the inner surface is

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 4.87\ \text{W} + 13.67\ \text{W} = \mathbf{18.5\ W}$$

Discussion In this problem, radiation heat transfer plays an important role, since the heat transfer rate by radiation is almost 3 times of that by natural convection.

Fundamentals of Engineering (FE) Exam Problems

9-159 Consider a hot boiled egg in a spacecraft that is filled with air at atmospheric pressure and temperature at all times. Disregarding any radiation effect, will the egg cool faster or slower when the spacecraft is in space instead of on the ground?

(a) faster (b) no differencce (c) slower (d) insufficient information

Answer (c) slower [there is no gravity and thus no natural convections currents in space]

9-160 A hot object suspended by a string is to be cooled by natural convection in fluids whose volume changes differently with temperature at constant pressure. In which fluid will the rate of cooling be lowest?

With increasing temperature, a fluid whose volume

(a) increases a lot (b) increases slightly (c) does not change (d) decreases slightly (e) decreases a lot

Answer (c) A fluid whose volume **does not change** [since there will be no natural convection currents in this case]

9-161 A spherical block of dry ice at –79°C is exposed to atmospheric air at 30°C. The general direction in which the air moves in this situation is

(a) horizontal (b) up (c) down

(d) recirculation around the sphere (e) no motion

Answer (c) down

9-162 The primary driving force for natural convection is

(a) shear stress forces (b) buoyancy forces (c) pressure forces

(d) surface tension forces (e) None of them

Answer (b) buoyancy forces

9-163 Consider a horizontal 0.7-m-wide and 0.85-m-long plate in a room at 30ºC. Top side of the plate is insulated while the bottom side is maintained at 0ºC. The rate of heat transfer from the room air to the plate by natural convection is

(a) 36.8 W (b) 43.7 W (c) 128.5 W (d) 92.7 W (e) 69.7 W

(For air, use k = 0.02476 W/m·°C, Pr = 0.7323, $\nu = 1.470\times10^{-5}$ m^2/s)

Answer (b) 43.7 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Width=0.7 [m]
Height=0.85 [m]
T_infinity=30 [C]
T_s=0 [C]

"The properties of air at (0+30)/2 = 15 C are (Table A-15)"
k=0.02476 [W/m-C]
nu=1.470E-5 [m^2/s]
Pr=0.7323
beta=1/T_f
T_f=(T_s+T_infinity)/2+273
g=9.81 [m/s^2]

L=(Width*Height)/(2*(Width+Height))
Ra=(g*beta*(T_infinity-T_s)*L^3)/nu^2*Pr
Nus=0.27*Ra^0.25
h=k/L*Nus
A_s=Width*Height
Q_dot=h*A_s*(T_infinity-T_s)
```

9-164 A 4-m-long section of a 5-cm-diameter horizontal pipe in which a refrigerant flows passes through a room at 20ºC. The pipe is not well insulated and the outer surface temperature of the pipe is observed to be -10ºC. The emissivity of the pipe surface is 0.85 and the surrounding surfaces are at 15ºC. The fraction of heat transferred to the pipe by radiation is

(a) 0.24 (b) 0.30 (c) 0.37 (d) 0.48 (e) 0.58

(For air, use k = 0.02401 W/m·°C, Pr = 0.735, $\nu = 1.382\times10^{-5}$ m^2/s)

Answer (c) 0.37

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
L=4 [m]
D=0.05 [m]
T_infinity=20 [C]
T_s=-10 [C]
T_surr=15 [C]
epsilon=0.85

"The properties of air at (20-10)/2 = 5 C are (Table A-15)"
k=0.02401 [W/m-C]
nu=1.382E-5 [m^2/s]
Pr=0.7350
beta=1/T_f
T_f=(T_s+T_infinity)/2+273
g=9.81 [m/s^2]
sigma=5.67E-8 [W/m^2-K^4]

Ra=(g*beta*(T_infinity-T_s)*D^3)/nu^2*Pr
Nus=((0.6+(0.387*Ra^(1/6))/(1+(0.559/Pr)^(9/16))^(8/27)))^2
h=k/D*Nus
A_s=pi*D*L
Q_dot_conv=h*A_s*(T_infinity-T_s)
Q_dot_rad=epsilon*A_s*sigma*((T_surr+273)^4-(T_s+273)^4)
f_rad=Q_dot_rad/(Q_dot_conv+Q_dot_rad)

"Some Wrong Solutions with Common Mistakes"
W_f_rad=Q_dot_rad/Q_dot_conv "Finding the ratio of radiation to convection"
```

9-165 Consider a 0.2-m-diameter, 1.8-m-long horizontal cylinder in a room at 20ºC. If the outer surface temperature of the cylinder is 40ºC, the natural convection heat transfer coefficient is

(a) 2.9 W/m²·ºC (b) 3.5 W/m²·ºC (c) 4.1 W/m²·ºC (d) 5.2 W/m²·ºC (e) 6.1 W/m²·ºC

(For air, use k = 0.02588 W/m·°C, Pr = 0.7282, $\nu = 1.608\times10^{-5}$ m²/s)

Answer (c) 4.1 W/m²·ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.2 [m]
L=1.8 [m]
T_infinity=20 [C]
T_s=40 [C]

"The properties of air at (20+40)/2 = 30 C are (Table A-15)"
k=0.02588 [W/m-C]
nu=1.608E-5 [m^2/s]
Pr=0.7282
beta=1/T_f
T_f=(T_s+T_infinity)/2+273
g=9.81 [m/s^2]

Ra=(g*beta*(T_s-T_infinity)*D^3)/nu^2*Pr
Nus=((0.6+(0.387*Ra^(1/6))/(1+(0.559/Pr)^(9/16))^(8/27)))^2
h=k/D*Nus
```

9-166 A 4-m-diameter spherical tank contains iced water at 0ºC. The tank is thin-shelled and thus its outer surface temperature may be assumed to be same as the temperature of the iced water inside. Now the tank is placed in a large lake at 20ºC. The rate at which the ice melts is

(a) 0.42 kg/s (b) 0.58 kg/s (c) 0.70 kg/s (d) 0.83 kg/s (e) 0.98 kg/s

(For lake water, use k = 0.580 W/m·°C, Pr = 9.45, $\nu = 0.1307\times10^{-5}$ m²/s, $\beta = 0.138\times10^{-3}$ K⁻¹)

Answer (a) 0.42 kg/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=4 [m]
T_infinity=20 [C]
T_s=0 [C]

"The properties of water at (20+0)/2 = 10 C are (Table A-9)"
k=0.580 [W/m-C]
nu=0.1307E-5 [m^2/s]
Pr=9.45
beta=0.138E-3 [1/K]
g=9.81 [m/s^2]

Ra=(g*beta*(T_infinity-T_s)*D^3)/nu^2*Pr
Nus=2+(0.589*Ra^(1/4))/(1+(0.469/Pr)^(9/16))^(4/9)
h=k/D*Nus
A_s=pi*D^2
Q_dot=h*A_s*(T_infinity-T_s)
h_if=333700 [J/kg]
m_dot_melt=Q_dot/h_if
```

9-167 A vertical double-pane window consists of two sheets of glass separated by a 1.2-cm air gap at atmospheric pressure. The glass surface temperatures across the air gap are measured to be 278 K and 288 K. If it is estimated that the heat transfer by convection through the enclosure is 1.5 times that by pure conduction and that the rate of heat transfer by radiation through the enclosure is about the same magnitude as the convection, the effective emissivity of the two glass surfaces is

(a) 0.35 (b) 0.48 (c) 0.59 (d) 0.84 (e) 0.72

Answer (c) 0.59

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
L=0.012 [m]
T1=288 [C]
T2=278 [C]
Nus=1.5

"The properties of air at (278+288)/2 = 283 K =10 C are (Table A-15)"
k=0.02439 [W/m-K]
sigma=5.67E-8 [W/m^2-K^4]

q_dot_conv=k*Nus*(T1-T2)/L
q_dot_rad=q_dot_conv
q_dot_rad=epsilon_eff*sigma*(T1^4-T2^4)
```

9-168 A horizontal 1.5-m-wide, 4.5-m-long double-pane window consists of two sheets of glass separated by a 3.5-cm gap filled with water. If the glass surface temperatures at the bottom and the top are measured to be 60ºC and 40ºC, respectively, the rate of heat transfer through the window is

(a) 27.6 kW (b) 39.4 kW (c) 59.6 kW (d) 66.4 kW (e) 75.5 kW

(For water, use k = 0.644 W/m·°C, Pr = 3.55, $\nu = 0.554\times10^{-6}$ m^2/s, $\beta = 0.451\times10^{-3}$ K^{-1}. Also, the applicable correlation is $\text{Nu} = 0.069\text{Ra}^{1/3}\,\text{Pr}^{0.074}$).

Answer (d) 66.4 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Width=1.5 [m]
Length=4.5 [m]
L=0.035 [m]
T1=60 [C]
T2=40 [C]

"The properties of water at (60+40)/2 = 50 C are (Table A-9)"
k=0.644 [W/m-C]
nu=0.554E-6 [m^2/s]
Pr=3.55
beta=0.451E-3 [1/K]
g=9.81 [m/s^2]

Ra=(g*beta*(T1-T2)*L^3)/nu^2*Pr
Nus=0.069*Ra^(1/3)*Pr^(0.074)
A_s=Width*Length
Q_dot=k*Nus*A_s*(T1-T2)/L

"Some Wrong Solutions with Common Mistakes"
W1_Nus=0.068*Ra^(1/3) "Relation for air gap, Eq. 9-45"
W1_Q_dot=k*W1_Nus*A_s*(T1-T2)/L
W2_Nus=0.195*Ra^(1/4) "Relation for air gap, Eq. 9-44"
W2_Q_dot=k*W2_Nus*A_s*(T1-T2)/L
```

9-169 A vertical 0.9-m-high and 1.5-m-wide double-pane window consists of two sheets of glass separated by a 2.0-cm air gap at atmospheric pressure. If the glass surface temperatures across the air gap are measured to be 20ºC and 30ºC, the rate of heat transfer through the window is

(a) 16.3 W (b) 21.7 W (c) 24.0 W (d) 31.3 W (e) 44.6 W

(For air, use k = 0.02551 W/m·°C, Pr = 0.7296, $\nu = 1.562\times10^{-5}$ m^2/s. Also, the applicable correlation is $\mathrm{Nu} = 0.42\mathrm{Ra}^{1/4}\,\mathrm{Pr}^{0.012}(H/L)^{-0.3}$)

Answer (b) 21.7 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
H=0.9 [m]
W=1.5 [m]
L=0.020 [m]
T1=30 [C]
T2=20 [C]

"The properties of air at (20+30)/2 = 25 C are (Table A-15)"
k=0.02551 [W/m-C]
nu=1.562E-5 [m^2/s]
Pr=0.7296
beta=1/T_ave
T_ave=(T1+T2)/2+273
g=9.81 [m/s^2]

Ra=(g*beta*(T1-T2)*L^3)/nu^2*Pr
Nus=0.42*Ra^(1/4)*Pr^0.012*(H/L)^(-0.3)
A_s=H*W
Q_dot=k*Nus*A_s*(T1-T2)/L
```

9-170 Two concentric cylinders of diameters D_i = 30 cm and D_o = 40 cm and L = 5 m are separated by air at 1 atm pressure. Heat is generated within the inner cylinder uniformly at a rate of 1100 W/m^3 and the inner surface temperature of the outer cylinder is 300 K. The steady-state outer surface temperature of the inner cylinder is

(a) 402 K (b) 415 K (c) 429 K (d) 442 K (e) 456 K

(For air, use k = 0.03095 W/m·°C, Pr = 0.7111, $\nu = 2.306\times10^{-5}$ m^2/s.)

Answer (c) 429 K

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D_i=0.30 [m]
D_o=0.40 [m]
L=5 [m]
g_dot=1100 [W/m^3]
T_o=300 [K]

"The properties of air at 100 C are (Table A-15)"
k=0.03095 [W/m-K]
nu=2.306E-5 [m^2/s]
Pr=0.7111
beta=1/T_ave
T_ave=100 [C]+273 [K]
g=9.81 [m/s^2]

L_c=(D_o-D_i)/2
Ra=(g*beta*(T_i-T_o)*L_c^3)/nu^2*Pr
F_cyl=(ln(D_o/D_i))^4/(L_c^3*(D_i^(-3/5)+D_o^(-3/5))^5)
k_eff=k*0.386*(Pr/(0.861+Pr))^0.25*(F_cyl*Ra)^0.25

Vol=pi*D_i^2/4*L
Q_dot=g_dot*Vol
Q_dot=(2*pi*k_eff)/ln(D_o/D_i)*(T_i-T_o)
```

9-171 ….. 9-174 Design and Essay Problems

Solutions Manual for

Heat and Mass Transfer: Fundamentals & Applications

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Chapter 10
BOILING AND CONDENSATION

Boiling Heat Transfer

10-1C Boiling is the liquid-to-vapor phase change process that occurs at a solid-liquid interface when the surface is heated above the saturation temperature of the liquid. The formation and rise of the bubbles and the liquid entrainment coupled with the large amount of heat absorbed during liquid-vapor phase change at essentially constant temperature are responsible for the very high heat transfer coefficients associated with nucleate boiling.

10-2C The different boiling regimes that occur in a vertical tube during flow boiling are forced convection of liquid, bubbly flow, slug flow, annular flow, transition flow, mist flow, and forced convection of vapor.

10-3C Both boiling and evaporation are liquid-to-vapor phase change processes, but evaporationoccurs at the *liquid-vapor interface* when the vapor pressure is less than the saturation pressure of the liquid at a given temperature, and it involves no bubble formation or bubble motion. Boiling, on the other hand, occurs at the *solid-liquid interface* when a liquid is brought into contact with a surface maintained at a temperature T_s sufficiently above the saturation temperature T_{sat} of the liquid.

10-4C Boiling is called *pool boiling* in the absence of bulk fluid flow, and *flow boiling* (or *forced convection boiling*) in the presence of it. In pool boiling, the fluid is stationary, and any motion of the fluid is due to natural convection currents and the motion of the bubbles due to the influence of buoyancy.

10-5C The boiling curve is given in Figure 10-6 in the text. In the *natural convection boiling* regime, the fluid motion is governed by natural convection currents, and heat transfer from the heating surface to the fluid is by natural convection. In the *nucleate boiling* regime, bubbles form at various preferential sites on the heating surface, and rise to the top. In the *transition boiling* regime, part of the surface is covered by a vapor film. In the *film boiling* regime, the heater surface is completely covered by a continuous stable vapor film, and heat transfer is by combined convection and radiation.

10-6C In the *film boiling* regime, the heater surface is completely covered by a continuous stable vapor film, and heat transfer is by combined convection and radiation. In the *nucleate boiling* regime, the heater surface is covered by the liquid. The boiling heat flux in the stable film boiling regime can be higher or lower than that in the nucleate boiling regime, as can be seen from the boiling curve.

10-7C The boiling curve is given in Figure 10-6 in the text. The burnout point in the curve is point C. The *burnout* during boiling is caused by the heater surface being blanketed by a continuous layer of vapor film at increased heat fluxes, and the resulting rise in heater surface temperature in order to maintain the same heat transfer rate across a low-conducting vapor film. Any attempt to increase the heat flux beyond $\dot{q}_{\max}$ will cause the operation point on the boiling curve to jump suddenly from point C to point E. However, the surface temperature that corresponds to point E is beyond the melting point of most heater materials, and burnout occurs. The burnout point is avoided in the design of boilers in order to avoid the disastrous explosions of the boilers.

10-8C Pool boiling heat transfer can be increased *permanently* by increasing the number of nucleation sites on the heater surface by *coating* the surface with a thin layer (much less than 1 mm) of very porous material, or by *forming cavities* on the surface mechanically to facilitate continuous vapor formation. Such surfaces are reported to enhance heat transfer in the nucleate boiling regime by a factor of up to 10, and the critical heat flux by a factor of 3. The use of finned surfaces is also known to enhance nucleate boiling heat transfer and the critical heat flux.

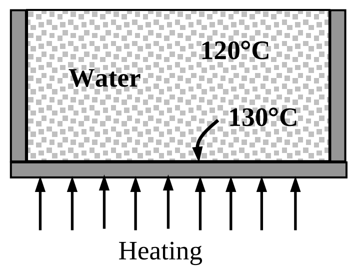
10-9C Yes. Otherwise we can create energy by alternately vaporizing and condensing a substance.

10-10 Water is boiled at T_{sat} = 120°C in a mechanically polished stainless steel pressure cooker whose inner surface temperature is maintained at T_s = 130°C. The heat flux on the surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the heater and the boiler are negligible.

Properties The properties of water at the saturation temperature of 120°C are (Tables 10-1 and A-9)

$$\rho_l = 943.4\ \text{kg/m}^3 \quad h_{fg} = 2203\times10^3\ \text{J/kg}$$
$$\rho_v = 1.121\ \text{kg/m}^3 \quad \mu_l = 0.232\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\sigma = 0.0550\ \text{N/m} \quad c_{pl} = 4244\ \text{J/kg}\cdot°\text{C}$$
$$\text{Pr}_l = 1.44$$



Also, $C_{sf} = 0.0130$ and n = 1.0 for the boiling of water on amechanically polished stainless steel surface (Table 10-3). Note that we expressed the properties in units specified under Eq. 10-2 in connection with their definitions in order to avoid unit manipulations.

Analysis The excess temperature in this case is $\Delta T = T_s - T_{sat} = 130 - 120 = 10°\text{C}$ which is relatively low (less than 30°C). Therefore, nucleate boiling will occur. The heat flux in this case can be determined from Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{\text{sat}})}{C_{sf}h_{fg}\,\text{Pr}_l^n}\right)^3$$
$$= (0.232\times10^{-3})(2203\times10^3)\left[\frac{9.81(943.4\text{-}1.121)}{0.0550}\right]^{1/2}\left(\frac{4244(130-120)}{0.0130(2203\times10^3)1.44}\right)^3$$
$$= 228{,}400\ \text{W/m}^2 = \mathbf{228.4\ kW/m^2}$$

10-11 The nucleate pool boiling heat transfer rate per unit length and the rate of evaporation per unit length of water being boiled by a rod that is maintained at 10°C above the saturation temperature are to be determined.

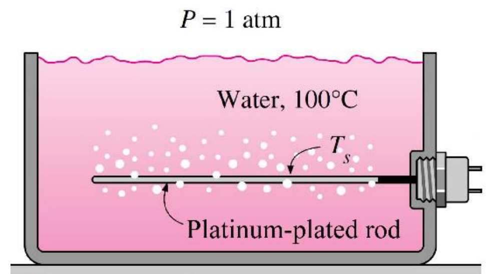
Assumptions **1** Steady operating condition exists. **2** Heat losses from the boiler are negligible.

Properties The properties of water at the saturation temperature of 100°C are σ = 0.0589 N/m (Table 10-1) and, from Table A-9,

ρ_l = 957.9 kg/m^3 h_{fg} = 2257 × 10^3 J/kg

ρ_v = 0.5978 kg/m^3 μ_l = 0.282 × 10^{-3} kg/m·s

Pr$_l$ = 1.75 c_{pl} = 4217 J/kg·K



Also, C_{sf} = 0.0130 and n = 1.0 for the boiling of water on platinum surface (Table 10-3).

Analysis The excess temperature in this case is $\Delta T = T_s - T_{\text{sat}}$ = 10°C, which is less than 30°C for water from Fig. 10-6. Therefore, nucleate boiling will occur. The heat flux in this case can be determined from the Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left[\frac{c_{pl}(T_s-T_{\text{sat}})}{C_{sf}h_{fg}\,\text{Pr}_l^n}\right]^3$$

$$= (0.282\times10^{-3})(2257\times10^3)\left[\frac{9.81(957.9-0.5978)}{0.0589}\right]^{1/2}\left[\frac{4217(10)}{(0.013)(2257\times10^3)(1.75)}\right]^3$$

$$= 1.408\times10^5\ \text{W/m}^2$$

Finally, the nucleate pool boiling heat transfer rate per unit length is

$$\dot{Q}_{\text{boiling}}/L = \pi D\dot{q}_{\text{nucleate}} = \pi(0.010\ \text{m})(1.408\times10^5\ \text{W/m}^2) = \mathbf{4420\ W/m}$$

The rate of evaporation per unit length is

$$\frac{\dot{m}_{\text{evaporation}}}{L} = \frac{(\dot{Q}_{\text{boiling}}/L)}{h_{fg}} = \frac{4420\ \text{J/s}\cdot\text{m}}{2257\times10^3\ \text{J/kg}} = \mathbf{1.96\times10^{-3}\ kg/s\cdot m}$$

Discussion The value for the rate of evaporation per unit length indicates that 1 m of the platinum-plated rod would boil water at a rate of about 2 grams per second.

10-12 C&S Water is boiled at 361 kPa by a horizontal ASTM B371 brass rod. The rod is supplied with a heat transfer rate of 20 kW. The supplied heat transfer rate is to be determined whether it would heat the rod surface temperature above the maximum use temperature.

Assumptions **1** Steady state conditions. **2** Heat losses from the boiler are negligible.

Properties The properties of water at the saturation temperature of 140°C (at 361 kPa) are from Table A-9,

c_{pl} = 4286 J/kg·K, h_{fg}= 2145 kJ/kg, $\Pr_l$ = 1.24
ρ_l = 921.7 kg/m^3, ρ_v = 1.965 kg/m^3, μ_l = 0.197 × 10^{-3} kg/m·s

The surface tension is from Table 10-1 at 140°C, σ = 0.0509 N/m.
Also, C_{sf} = 0.0060 and n = 1.0 for the boiling of water on a brass surface (Table 10-3).

Analysis The excess temperature in this case is $\Delta T = T_s - T_{\text{sat}}$ = 149 – 140 = 9°C, which is less than 30°C for water. Therefore, it is in the nucleate boiling regime. The nucleate boiling heat flux is

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left[\frac{c_{pl}(T_s - T_{\text{sat}})}{C_{sf}h_{fg}\Pr_l^n}\right]^3$$

$$\dot{q}_{\text{nucleate}} = (0.197\times10^{-3})(2145000)\left[\frac{(9.81)(921.7-1.965)}{0.0509}\right]^{1/2}\left[\frac{(4286)(9)}{(0.0060)(2145000)(1.24)^{1.0}}\right]^3$$

$$\dot{q}_{\text{nucleate}} = 2.512\times10^6 \text{W/m}^2$$

The highest heat transfer rate that can be supplied from the ASTM B371 brass rod to boil the water at the excess temperature of $\Delta T = T_s - T_{\text{sat}}$ = 9°C, without heating the rod surface above 149°C is

$$\dot{Q} = A\dot{q}_{\text{total}} = (\pi DL)\dot{q}_{\text{total}} = \pi(0.005\text{ m})(0.15\text{ m})(2.512\times10^6\text{W/m}^2) = 5919\text{ W} < 20{,}000\text{ W}$$

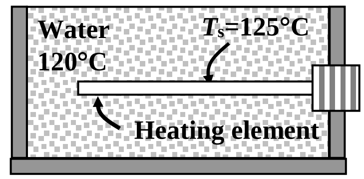
Discussion The highest heat transfer rate that can be supplied to the ASTM B371 rod to boil water is 5919 W. If the ASTM B371 rod is to be supplied with a heat transfer rate of 20 kW, this would require the excess temperature to increase above 9°C. Thus, the rod surface temperature would be higher than the maximum use temperature established by the ASME Code for Process Piping. For a nucleate boiling heat transfer rate of 20 kW, the rod surface temperature would be at 153.5°C.

10-13 Water is boiled at a saturation (or boiling) temperature of T_{sat} = 120°C by a brass heating element whose temperature is not to exceed T_s = 125°C. The highest rate of steam production is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the boiler are negligible. **3** The boiling regime is nucleate boiling since $\Delta T = T_s - T_{sat} = 125 - 120 = 5°C$ which is in the nucleate boiling range of 5 to 30°C for water.

Properties The properties of water at the saturation temperature of 120°C are (Tables 10-1 and A-9)

$$\rho_l = 943.4 \text{ kg/m}^3$$
$$\rho_v = 1.12 \text{ kg/m}^3$$
$$\sigma = 0.0550 \text{ N/m}$$
$$\text{Pr}_l = 1.44$$
$$h_{fg} = 2203 \times 10^3 \text{ J/kg}$$
$$\mu_l = 0.232 \times 10^{-3} \text{ kg} \cdot \text{m/s}$$
$$c_{pl} = 4244 \text{ J/kg} \cdot °\text{C}$$



Also, $C_{sf} = 0.0060$ and n = 1.0 for the boiling of water on a brass surface (Table 10-3). Note that we expressed the properties in units specified under Eq. 10-2 in connection with their definitions in order to avoid unit manipulations.

Analysis Assuming nucleate boiling, the heat flux in this case can be determined from Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left(\frac{c_{p,l}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \text{Pr}_l^n} \right)^3$$

$$= (0.232 \times 10^{-3})(2203 \times 10^3) \left[\frac{9.81(943.4 - 1.12)}{0.0550} \right]^{1/2} \left(\frac{4244(125 - 120)}{0.0060(2203 \times 10^3)1.44} \right)^3$$

$$= 290{,}300 \text{ W/m}^2$$

The surface area of the heater is

$$A_s = \pi DL = \pi(0.02 \text{ m})(0.65 \text{m}) = 0.04084 \text{ m}^2$$

Then the rate of heat transfer during nucleate boiling becomes

$$\dot{Q}_{\text{boiling}} = A_s \dot{q}_{\text{nucleate}} = (0.04084 \text{ m}^2)(290{,}300 \text{ W/m}^2) = 11{,}856 \text{ W}$$

(b) The rate of evaporation of water is determined from

$$\dot{m}_{\text{evaporation}} = \frac{\dot{Q}_{\text{boiling}}}{h_{fg}} = \frac{11{,}856 \text{ J/s}}{2203 \times 10^3 \text{ J/kg}} \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) = \mathbf{19.4 \text{ kg/h}}$$

Therefore, steam can be produced at a rate of about 20 kg/h by this heater.

10-14 C&S Water is boiled at 1254 kPa by a heating element sheathed in an ASTM B75 copper tube. The tube is immersed in the water horizontally. The highest evaporation rate of water that can be achieved by the heater is to be determined.

Assumptions **1** Steady state conditions. **2** Heat losses from the boiler are negligible.

Properties The saturation temperature of water at 1254 kPa is 190°C (Table A-9), and properties of water at the saturation temperature are from Table A-9,

c_{pl} = 4460 J/kg·K, h_{fg}= 1979 kJ/kg, $\Pr_l$ = 0.947
ρ_l = 876.4 kg/m^3, ρ_v = 6.388 kg/m^3, $\mu_l = 0.142 \times 10^{-3}$ kg/m·s

The surface tension is from Table 10-1 at 190°C, σ = 0.03995 N/m.
Also, C_{sf} = 0.0130 and n = 1.0 for the boiling of water on a polished copper surface (Table 10-3).

Analysis The excess temperature in this case is $\Delta T = T_s - T_{\text{sat}}$ = 204 – 190 = 14°C, which is less than 30°C for water. Therefore, it is in the nucleate boiling regime. The Rohsenow equation for nucleate boiling heat flux is

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2} \left[\frac{c_{pl}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \Pr_l^n}\right]^3$$

$$\dot{q}_{\text{nucleate}} = (0.142 \times 10^{-3})(1979000) \left[\frac{(9.81)(876.4 - 6.388)}{0.03995}\right]^{1/2} \left[\frac{(4460)(14)}{(0.0130)(1979000)(0.947)^{1.0}}\right]^3$$

$$\dot{q}_{\text{nucleate}} = 2.186 \times 10^6 \text{W/m}^2$$

Thus, the highest evaporation rate of water that can be achieved by the heater, without heating the ASTM B75 tube above 204°C, is

$$\dot{m}_{\text{evap}} = \frac{\dot{Q}_{\text{boil}}}{h_{fg}} = \frac{(\pi DL)\dot{q}_{\text{nucleate}}}{h_{fg}} = \frac{\pi(0.005 \text{ m})(0.10 \text{ m})(2.186 \times 10^6 \text{W/m}^2)}{1979000 \text{ J/kg}} = 0.00174 \text{ kg/s} = 1.74 \text{ g/s}$$

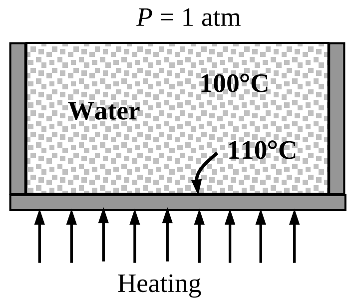
Discussion The highest evaporation rate that can be achieved by the heater, with ASTM B75 copper tube as the sheath, is about 1.74 g/s. If the water evaporation rate is higher than 1.74 g/s, the surface temperature of the ASTM B75 tube would heat above the maximum use temperature established by the ASME Code for Process Piping.

10-15 Water is boiled at 1 atm pressure and thus at a saturation (or boiling) temperature of T_{sat} = 100°C in a mechanically polished stainless steel pan whose inner surface temperature is maintained at T_s = 110°C. The rate of heat transfer to the water and the rate of evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the heater and the boiler are negligible.

Properties The properties of water at the saturation temperature of 100°C are (Tables 10-1 and A-9)

$$\rho_l = 957.9 \text{ kg/m}^3$$
$$\rho_v = 0.60 \text{ kg/m}^3$$
$$\sigma = 0.0589 \text{ N/m}$$
$$\text{Pr}_l = 1.75$$
$$h_{fg} = 2257\times10^3 \text{ J/kg}$$
$$\mu_l = 0.282\times10^{-3} \text{ kg}\cdot\text{m/s}$$
$$c_{pl} = 4217 \text{ J/kg}\cdot\text{°C}$$



Also, $C_{sf} = 0.0130$ and n = 1.0 for the boiling of water on a mechanically polished stainless steel surface (Table 10-3). Note that we expressed the properties in units specified under Eq. 10-2 in connection with their definitions in order to avoid unit manipulations.

Analysis The excess temperature in this case is $\Delta T = T_s - T_{sat} = 110 - 100 = 10°\text{C}$ which is relatively low (less than 30°C). Therefore, nucleate boiling will occur. The heat flux in this case can be determined from Rohsenow relation to be

$$\dot{q}_{nucleate} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{sat})}{C_{sf}h_{fg}\text{Pr}_l^n}\right)^3$$
$$= (0.282\times10^{-3})(2257\times10^3)\left[\frac{9.81(957.9-0.60)}{0.0589}\right]^{1/2}\left(\frac{4217(110-100)}{0.0130(2257\times10^3)1.75}\right)^3$$
$$= 140{,}700 \text{ W/m}^2$$

The surface area of the bottom of the pan is

$$A_s = \pi D^2/4 = \pi(0.30 \text{ m})^2/4 = 0.07069 \text{ m}^2$$

Then the rate of heat transfer during nucleate boiling becomes

$$\dot{Q}_{boiling} = A_s\dot{q}_{nucleate} = (0.07069 \text{ m}^2)(140{,}700 \text{ W/m}^2) = \mathbf{9945\ W}$$

(*b*) The rate of evaporation of water is determined from

$$\dot{m}_{evaporation} = \frac{\dot{Q}_{boiling}}{h_{fg}} = \frac{9945 \text{ J/s}}{2257\times10^3 \text{ J/kg}} = \mathbf{0.00441 kg/s}$$

That is, water in the pan will boil at a rate of 4.4 grams per second.

10-16 C&S Water is boiled at 3344 kPa by an ASME SA-240 stainless steel plate. The plate surface temperature is to be determined whether it complies with the ASME Boiler and Pressure Vessel Code if it is used to boil the water at an evaporation rate of 4 kg/s.

Assumptions **1** Steady state conditions. **2** Heat losses from the boiler are negligible. **3** Boiling regime is nucleate.

Properties The saturation temperature of water at 3344 kPa is 240°C (Table A-9), and properties of water at the saturation temperature are from Table A-9,

c_{pl} = 4760 J/kg·K, h_{fg}= 1767 kJ/kg, Pr_l = 0.836
ρ_l = 813.7 kg/m^3, ρ_v = 16.73 kg/m^3, $\mu_l = 0.111 \times 10^{-3}$ kg/m·s

The surface tension is from Table 10-1 at 240°C, σ = 0.0284 N/m.
Also, C_{sf} = 0.0130 and n = 1.0 for the boiling of water on a mechanically polished stainless steel surface (Table 10-3).

Analysis The evaporation rate of water is

$$\dot{m}_{\text{evap}} = \frac{\dot{Q}_{\text{boil}}}{h_{fg}} = \frac{(\pi D^2/4)\dot{q}_{\text{nucleate}}}{h_{fg}} = 4\text{ kg/s}$$

Thus,

$$\dot{q}_{\text{nucleate}} = \frac{\dot{m}_{\text{evap}} h_{fg}}{\pi D^2/4} = \frac{(4\text{ kg/s})(1767000\text{ J/kg})}{\pi\,(0.5\text{ m})^2/4} = 3.5997 \times 10^7 \text{W/m}^2$$

The Rohsenow equation for nucleate boiling heat flux is

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2} \left[\frac{c_{pl}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \mathrm{Pr}_l^n}\right]^3$$

$$3.5997 \times 10^7 = (0.111 \times 10^{-3})(1767000) \left[\frac{(9.81)(813.7 - 16.73)}{0.0284}\right]^{1/2} \left[\frac{(4760)(T_s - 240)}{(0.0130)(1767000)(0.836)^{1.0}}\right]^3$$

Solving for the surface temperature yields,

$$T_s = 268.4\text{°C} > 260\text{°C}$$

Note: The nucleate boiling assumption is appropriate. With T_s = 268.4°C, the excess temperature is $\Delta T = T_s - T_{\text{sat}}$ = 268.4 – 240 = 28.4°C, which is less than 30°C for water.

Discussion To boil the water at an evaporation rate of 4 kg/s would require the ASME SA-240 plate surface temperature to be 268.4°C. This is above the maximum use temperature limited by the ASME Boiler and Pressure Vessel Code. To comply with the code, the evaporation rate of water will need to be reduced to 1.39 kg/s or lower.

10-17 A polished copper tube is used to generate 1.5 kg/s of steam at 270 kPa. The surface temperature of the tube, with the interest to minimize the excess temperature, is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The boiling regime is nucleate boiling since ΔT_{excess} is to be minimized (this assumption will be verified).

Properties At 270 kPa, the saturation temperature of water is T_{sat} = 130°C. The properties of water at T_{sat} = 130°C are σ = 0.05295 N/m (Tables 10-1) and, from Table A-9,

$$\rho_l = 934.6\ \text{kg/m}^3 \qquad h_{fg} = 2174\times10^3\ \text{J/kg}$$
$$\rho_v = 1.496\ \text{kg/m}^3 \qquad \mu_l = 0.213\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\text{Pr}_l = 1.33 \qquad c_{pl} = 4263\ \text{J/kg}\cdot\text{K}$$

Also, C_{sf} = 0.013 and n = 1 for the boiling of water on a mechanically polished copper surface (Table 10-3).

Analysis The heat flux can be determined from the rate of vaporization to be

$$\dot{q}_{\text{nucleate}} = \frac{\dot{m}_{\text{vapor}} h_{fg}}{A_s} = \frac{\dot{m}_{\text{vapor}} h_{fg}}{\pi DL} = \frac{(1.5\ \text{kg/s})(2174\times10^3\ \text{J/kg})}{\pi(0.05\ \text{m})(15\ \text{m})} = 1.384\times10^6\ \text{W/m}^2$$

In the interest of minimizing the excess temperature, the boiling regime would be nucleate boiling. The heat flux can be expressed using the Rohsenow relation,

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{\text{sat}})}{C_{sf} h_{fg}\,\text{Pr}_l^n}\right)^3$$

$$1.384\times10^6\ \text{W/m}^2 = (0.213\times10^{-3})(2174\times10^3)\left[\frac{9.81(934.6-1.496)}{0.05295}\right]^{1/2}\left[\frac{4263(T_s-130)}{0.0130(2174\times10^3)1.33}\right]^3$$

Solving for T_s yields

$$T_s = \mathbf{147^\circ C}$$

Discussion With T_s = 147°C, the excess temperature would be since $\Delta T_{\text{excess}} = T_s - T_{\text{sat}}$ = 17°C, which is in the nucleate boiling range of 5 to 30°C for water. Thus the assumption of nucleate boiling regime is verified.

10-18 C&S Water is boiled at 3344 kPa by a heating element sheathed in an ASTM B725 nickel-copper alloy tube. The tube is immersed in the water horizontally. Determine whether the tube complies with the ASME Code for Process Piping if it is used to boil the water at an evaporation rate of 1 kg/s. If not, the highest evaporation rate of water that can be achieved by the heater is to be determined.

Assumptions **1** Steady state conditions. **2** Heat losses from the boiler are negligible. **3** Boiling regime is nucleate.

Properties The saturation temperature of water at 3344 kPa is 240°C (Table A-9), and properties of water at the saturation temperature are from Table A-9,

c_{pl} = 4760 J/kg·K, h_{fg}= 1767 kJ/kg, Pr_l = 0.836
ρ_l = 813.7 kg/m^3, ρ_v = 16.73 kg/m^3, μ_l = 0.111 × 10^{-3} kg/m·s

The surface tension is from Table 10-1 at 240°C, σ = 0.0284 N/m.
Also, C_{sf} = 0.0060 and n = 1.0 for the boiling of water on a nickel surface (Table 10-3). ASTM B725 nickel-copper alloy is majority nickel in material composition.

Analysis The evaporation rate of water is

$$\dot{m}_{\text{evap}} = \frac{\dot{Q}_{\text{boil}}}{h_{fg}} = \frac{(\pi DL)\dot{q}_{\text{nucleate}}}{h_{fg}} = 1\ \text{kg/s}$$

Thus,

$$\dot{q}_{\text{nucleate}} = \frac{\dot{m}_{\text{evap}} h_{fg}}{\pi DL} = \frac{(1\ \text{kg/s})(1767000\ \text{J/kg})}{\pi(0.005\ \text{m})(0.2\ \text{m})} = 5.6245 \times 10^8 \text{W/m}^2$$

The Rohsenow equation for nucleate boiling heat flux is

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left[\frac{c_{pl}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \mathrm{Pr}_l^n}\right]^3$$

$$5.6245 \times 10^8 = (0.111 \times 10^{-3})(1767000)\left[\frac{(9.81)(813.7 - 16.73)}{0.0284}\right]^{1/2}\left[\frac{(4760)(T_s - 240)}{(0.0060)(1767000)(0.836)^{1.0}}\right]^3$$

Solving for the surface temperature yields,

$$T_s = 272.8\text{°C} > 260\text{°C}$$

The highest boiling heat flux that can be achieved by the heater, without heating the ASTM B725 tube above 260°C, is

$$\dot{q}_{\text{nucleate}} = (0.111 \times 10^{-3})(1767000)\left[\frac{(9.81)(813.7 - 16.73)}{0.0284}\right]^{1/2}\left[\frac{(4760)(260 - 240)}{(0.0060)(1767000)(0.836)^{1.0}}\right]^3$$

$$\dot{q}_{\text{nucleate}} = 1.2752 \times 10^8 \text{W/m}^2$$

Note: The nucleate boiling assumption is appropriate. With T_s = 260°C, the excess temperature is $\Delta T = T_s - T_{\text{sat}}$ = 260 – 240 = 20°C, which is less than 30°C for water.

Thus, the highest evaporation rate of water that can be achieved by the heater, without heating the ASTM B725 tube above 260°C, is

$$\dot{m}_{\text{evap}} = \frac{\dot{Q}_{\text{boil}}}{h_{fg}} = \frac{(\pi DL)\dot{q}_{\text{nucleate}}}{h_{fg}} = \frac{\pi(0.005\ \text{m})(0.20\ \text{m})(1.2752 \times 10^8 \text{W/m}^2)}{1767000\ \text{J/kg}} = 0.2267\ \text{kg/s}$$

Discussion To boil the water at an evaporation rate of 1 kg/s would require the ASTM B725 tube surface temperature to be 272.8°C. This is above the maximum use temperature set by the ASME Code for Process Piping. To comply with the code, the evaporation rate of water will need to be reduced to 0.2267 kg/s or lower.

10-19 The nucleate pool boiling heat transfer coefficient of water being boiled by a horizontal platinum-plated rod is to be determined.

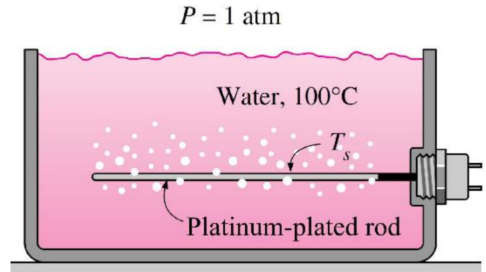
Assumptions **1** Steady operating condition exists. **2** Heat losses from the boiler are negligible.

Properties The properties of water at the saturation temperature of 100°C are σ = 0.0589 N/m (Table 10-1) and, from Table A-9,

ρ_l = 957.9 kg/m^3 $\quad$ h_{fg} = 2257 × 10^3 J/kg

ρ_v = 0.5978 kg/m^3 $\quad$ μ_l = 0.282 × 10^{-3} kg/m·s

$\Pr_l$ = 1.75 $\quad$ c_{pl} = 4217 J/kg·K



Also, C_{sf} = 0.0130 and n = 1.0 for the boiling of water on platinum surface (Table 10-3).

Analysis The excess temperature in this case is $\Delta T = T_s - T_{sat}$ = 10°C, which is less than 30°C for water from Fig. 10-6. Therefore, nucleate boiling will occur. The heat flux in this case can be determined from the Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left[\frac{c_{pl}(T_s-T_{\text{sat}})}{C_{sf}h_{fg}\Pr_l^n}\right]^3$$

$$= (0.282\times10^{-3})(2257\times10^3)\left[\frac{9.81(957.9-0.5978)}{0.0589}\right]^{1/2}\left[\frac{4217(10)}{(0.013)(2257\times10^3)(1.75)}\right]^3$$

$$= 1.408\times10^5\ \text{W/m}^2$$

Using the Newton's law of cooling, the boiling heat transfer coefficient is

$$\dot{q}_{\text{nucleate}} = h(T_s - T_{\text{sat}}) \qquad \rightarrow \qquad h = \frac{\dot{q}_{\text{nucleate}}}{T_s - T_{\text{sat}}}$$

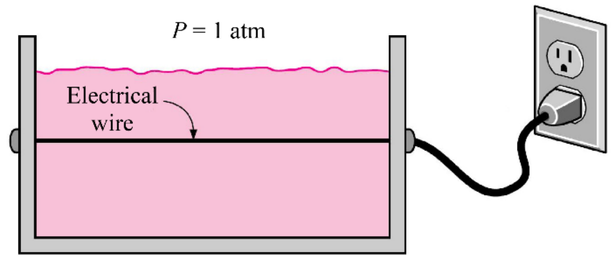
$$h = \frac{1.408\times10^5\ \text{W/m}^2}{(110-100)\ \text{K}} = \mathbf{14{,}100\ W/m^2\cdot K}$$

Discussion Heat transfer coefficient on the order of 10^4 W/m^2·K can be obtained in nucleate boiling with a temperature difference of just 10°C.

10-20 The nucleate boiling heat transfer coefficient and the value of C_{sf} for water being boiled by a long electrical wire are to be determined.

Assumptions **1** Steady operating condition exists. **2** Heat losses from the boiler are negligible.

Properties The properties of water at the saturation temperature of 100°C are σ = 0.0589 N/m (Table 10-1) and, from Table A-9,



ρ_l = 957.9 kg/m^3 $\quad$ h_{fg} = 2257 × 10^3 J/kg

ρ_v = 0.5978 kg/m^3 $\quad$ μ_l = 0.282 × 10^{-3} kg/m·s

Pr$_l$ = 1.75 $\quad$ c_{pl} = 4217 J/kg·K

Also, n = 1.0 is given.

Analysis The excess temperature in this case is $\Delta T = T_s - T_{sat}$ = 28°C, which is less than 30°C for water from Fig. 10-6. Therefore, nucleate boiling will occur. The nucleate boiling heat transfer coefficient can be determined using

$$\dot{q}_{\text{boiling}} = h(T_s - T_{\text{sat}}) \qquad \rightarrow \qquad h = \frac{\dot{q}_{\text{boiling}}}{T_s - T_{\text{sat}}}$$

Also, we know

$$\dot{Q}_{\text{boiling}} / L = \pi D \dot{q}_{\text{boiling}} = 4100 \text{ W/m}$$

$$\dot{q}_{\text{boiling}} = \frac{4100 \text{ W/m}}{\pi D} = \frac{4100 \text{ W/m}}{\pi(0.001\,\text{m})} = 1.305 \times 10^6 \text{ W/m}^2$$

Hence, the nucleate boiling heat transfer coefficient is

$$h = \frac{1.305 \times 10^6 \text{ W/m}}{(128 - 100)\text{ K}} = \mathbf{46{,}600\ W/m^2 \cdot K}$$

The value of the experimental constant C_{sf} can be determined from the Rohsenow relation to be

$$\dot{q}_{\text{boiling}} = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left[\frac{c_{pl}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \text{Pr}_l^n} \right]^3$$

or

$$C_{sf} = \left\{ \frac{\mu_l h_{fg}}{\dot{q}_{\text{boiling}}} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left[\frac{c_{pl}(T_s - T_{\text{sat}})}{h_{fg} \text{Pr}_l^n} \right]^3 \right\}^{1/3}$$

$$C_{sf} = \left\{ \frac{(0.282 \times 10^{-3})(2257 \times 10^3)}{1.305 \times 10^6} \left[\frac{9.81(957.9 - 0.5978)}{0.0589} \right]^{1/2} \left[\frac{4217(128 - 100)}{(2257 \times 10^3)(1.75)} \right]^3 \right\}^{1/3}$$

$$= \mathbf{0.0173}$$

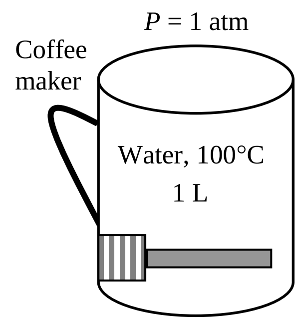
Discussion The boiling heat transfer coefficient of 46,600 W/m^2·K is within the range suggested by Table 1-5 for boiling and condensation (2500 to 100,000 W/m^2·K).

10-21 Water is boiled at sea level (1 atm pressure) and thus at a saturation (or boiling) temperature of T_{sat} = 100°C by a stainless steel heating element. The surface temperature of the heating element and its power rating are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the coffee maker are negligible. **3** The boiling regime is nucleate boiling (this assumption will be checked later).

Properties The properties of water at the saturation temperature of 100°C are (Tables 10-1 and A-9)

$$\rho_l = 957.9\ \text{kg/m}^3$$
$$\rho_v = 0.60\ \text{kg/m}^3$$
$$\sigma = 0.0589\ \text{N/m}$$
$$\Pr_l = 1.75$$
$$h_{fg} = 2257\times 10^3\ \text{J/kg}$$
$$\mu_l = 0.282\times 10^{-3}\ \text{kg}\cdot\text{m/s}$$
$$c_{pl} = 4217\ \text{J/kg}\cdot°\text{C}$$



Also, C_{sf} = 0.0130 and n = 1.0 for the boiling of water on a stainless steel surface (Table 10-3). Note that we expressed the properties in units specified under Eq. 10-2 connection with their definitions in order to avoid unit manipulations.

Analysis The density of water at room temperature is very nearly 1 kg/L, and thus the mass of 1 L water at 18°C is nearly 1 kg. The rate of energy transfer needed to evaporate half of this water in 32 min and the heat flux are

$$Q = \dot{Q}\Delta t = mh_{fg} \rightarrow \dot{Q} = \frac{mh_{fg}}{\Delta t} = \frac{(0.5\ \text{kg})(2257\ \text{kJ/kg})}{(32\times 60\ \text{s})} = 0.5878\ \text{kW}$$

$$A_s = \pi DL = \pi(0.004\ \text{m})(0.30\ \text{m}) = 0.003770\ \text{m}^2$$

$$\dot{q} = \dot{Q}/A_s = (0.5878\ \text{kW})/(0.003770\ \text{m}^2) = 155.92\ \text{kW/m}^2 = 155{,}920\ \text{W/m}^2$$

The Rohsenow relation which gives the nucleate boiling heat flux for a specified surface temperature can also be used to determine the surface temperature when the heat flux is given.

Assuming nucleate boiling, the temperature of the inner surface of the pan is determined from Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{\text{sat}})}{C_{sf}h_{fg}\Pr_l^n}\right)^3$$

$$155{,}920 = (0.282\times 10^{-3})(2257\times 10^3)\left[\frac{9.81(957.9-0.60)}{0.0589}\right]^{1/2}\left(\frac{4217(T_s-100)}{0.0130(2257\times 10^3)1.75}\right)^3$$

It gives

$$T_s = \mathbf{110.3°C}$$

which is in the nucleate boiling range (5 to 30°C above surface temperature). Therefore, the nucleate boiling assumption is valid.

The specific heat of water at the average temperature of (14+100)/2 = 57°C is c_p = 4.184 kJ/kg·°C. Then the time it takes for the entire water to be heated from 14°C to 100°C is determined to be

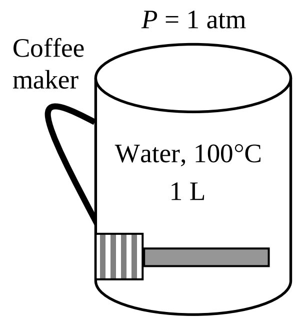
$$Q = \dot{Q}\Delta t = mc_p\Delta T \rightarrow \Delta t = \frac{mc_p\Delta T}{\dot{Q}} = \frac{(1\ \text{kg})(4.184\ \text{kJ/kg}\cdot°\text{C})(100-14)°\text{C}}{0.5878\ \text{kJ/s}} = 612\ \text{s} = \mathbf{10.2\ min}$$

10-22 Water is boiled at sea level (1 atm pressure) and thus at a saturation (or boiling) temperature of T_{sat} = 100°C by a copper heating element. The surface temperature of the heating element and its power rating are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the coffee maker are negligible. **3** The boiling regime is nucleate boiling (this assumption will be checked later).

Properties The properties of water at the saturation temperature of 100°C are (Tables 10-1 and A-9)

$$\rho_l = 957.9 \text{ kg/m}^3$$
$$\rho_v = 0.60 \text{ kg/m}^3$$
$$\sigma = 0.0589 \text{ N/m}$$
$$\text{Pr}_l = 1.75$$
$$h_{fg} = 2257 \times 10^3 \text{ J/kg}$$
$$\mu_l = 0.282 \times 10^{-3} \text{ kg} \cdot \text{m/s}$$
$$c_{pl} = 4217 \text{ J/kg} \cdot °\text{C}$$



Also, $C_{sf} = 0.0130$ and n = 1.0 for the boiling of water on a copper surface (Table 10-3). Note that we expressed the properties in units specified under Eq. 10-2 connection with their definitions in order to avoid unit manipulations.

Analysis The density of water at room temperature is very nearly 1 kg/L, and thus the mass of 1 L water at 18°C is nearly 1 kg. The rate of energy transfer needed to evaporate half of this water in 32 min and the heat flux are

$$Q = \dot{Q}\Delta t = mh_{fg} \rightarrow \dot{Q} = \frac{mh_{fg}}{\Delta t} = \frac{(0.5 \text{ kg})(2257 \text{ kJ/kg})}{(32 \times 60 \text{ s})} = 0.5878 \text{ kW}$$

$$A_s = \pi DL = \pi(0.004 \text{ m})(0.30 \text{ m}) = 0.003770 \text{ m}^2$$

$$\dot{q} = \dot{Q} / A_s = (0.5878 \text{ kW})/(0.003770 \text{ m}^2) = 155.92 \text{ kW/m}^2 = 155{,}920 \text{ W/m}^2$$

The Rohsenow relation which gives the nucleate boiling heat flux for a specified surface temperature can also be used to determine the surface temperature when the heat flux is given.

Assuming nucleate boiling, the temperature of the inner surface of the pan is determined from Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left(\frac{c_{p,l}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \text{Pr}_l^n} \right)^3$$

$$155{,}920 = (0.282 \times 10^{-3})(2257 \times 10^3) \left[\frac{9.81(957.9 - 0.60)}{0.0589} \right]^{1/2} \left(\frac{4217(T_s - 100)}{0.0130(2257 \times 10^3)1.75} \right)^3$$

It gives

$$T_s = \mathbf{110.3°C}$$

which is in the nucleate boiling range (5 to 30°C above surface temperature). Therefore, the nucleate boiling assumption is valid.

The specific heat of water at the average temperature of (14+100)/2 = 57°C is c_p = 4.184 kJ/kg·°C. Then the time it takes for the entire water to be heated from 14°C to 100°C is determined to be

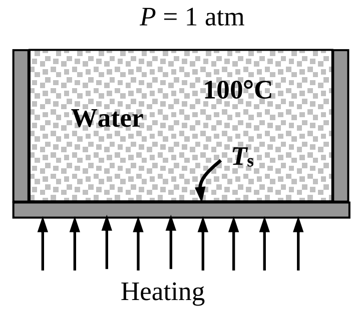
$$Q = \dot{Q}\Delta t = mc_p \Delta T \rightarrow \Delta t = \frac{mc_p \Delta T}{\dot{Q}} = \frac{(1 \text{ kg})(4.184 \text{ kJ/kg} \cdot °\text{C})(100 - 14)°\text{C}}{0.5878 \text{ kJ/s}} = 612 \text{ s} = \mathbf{10.2 \text{ min}}$$

10-23 Water is boiled at sea level (1 atm pressure) and thus at a saturation (or boiling) temperature of T_{sat} = 100°C in a teflon-pitted stainless steel pan placed on an electric burner. The water level drops by 10 cm in 30 min during boiling. The inner surface temperature of the pan is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the pan are negligible. **3** The boiling regime is nucleate boiling (this assumption will be checked later).

Properties The properties of water at the saturation temperature of 100°C are (Tables 10-1 and A-9)

$$\rho_l = 957.9 \text{ kg/m}^3$$
$$\rho_v = 0.60 \text{ kg/m}^3$$
$$\sigma = 0.0589 \text{ N/m}$$
$$\text{Pr}_l = 1.75$$
$$h_{fg} = 2257\times10^3 \text{ J/kg}$$
$$\mu_l = 0.282\times10^{-3} \text{ kg}\cdot\text{m/s}$$
$$c_{pl} = 4217 \text{ J/kg}\cdot°\text{C}$$



Also, $C_{sf} = 0.0058$ and n = 1.0 for the boiling of water on a teflon-pitted stainless steel surface (Table 10-3). Note that we expressed the properties in units specified under Eq. 10-2 connection with their definitions in order to avoid unit manipulations.

Analysis The rate of heat transfer to the water and the heat flux are

$$\dot{m}_{\text{evap}} = \frac{m_{\text{evap}}}{\Delta t} = \frac{\rho \Delta V}{\Delta t} = \frac{(957.9 \text{ kg/m}^3)(\pi\times(0.2\text{ m})^2\ /4\times 0.10\text{ m})}{15\times 60\text{ s}} = 0.003344 \text{ kg/s}$$

$$\dot{Q} = \dot{m}_{\text{evap}} h_{fg} = (0.03344 \text{ kg/s})(2257 \text{ kJ/kg}) = 7.547 \text{ kW}$$

$$A_s = \pi D^2 / 4 = \pi(0.20\text{ m})^2 / 4 = 0.03142 \text{ m}^2$$

$$\dot{q} = \dot{Q} / A_s = (7547 \text{ W})/(0.03142 \text{ m}^2) = 240{,}200 \text{ W/m}^2$$

The Rohsenow relation which gives the nucleate boiling heat flux for a specified surface temperature can also be used to determine the surface temperature when the heat flux is given. Assuming nucleate boiling, the temperature of the inner surface of the pan is determined from Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg} \left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2} \left(\frac{c_{p,l}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \text{Pr}_l^n}\right)^3$$

$$240{,}200 = (0.282\times10^{-3})(2257\times10^3)\left[\frac{9.81(957.9-0.60)}{0.0589}\right]^{1/2}\left(\frac{4217(T_s-100)}{0.0058(2257\times10^3)1.75}\right)^3$$

It gives

T_s = **105.3°C**

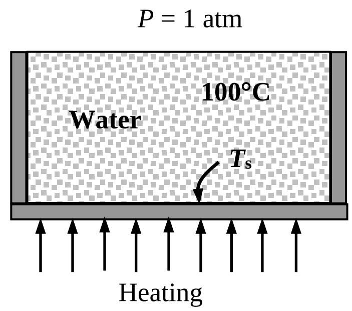
which is in the nucleate boiling range (5 to 30°C above surface temperature). Therefore, the nucleate boiling assumption is valid.

10-24 Water is boiled at sea level (1 atm pressure) and thus at a saturation (or boiling) temperature of T_{sat} = 100°C in a polished copper pan placed on an electric burner. The water level drops by 10 cm in 30 min during boiling. The inner surface temperature of the pan is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the pan are negligible. **3** The boiling regime is nucleate boiling (this assumption will be checked later).

Properties The properties of water at the saturation temperature of 100°C are (Tables 10-1 and A-9)

$$\rho_l = 957.9 \text{ kg/m}^3$$
$$\rho_v = 0.60 \text{ kg/m}^3$$
$$\sigma = 0.0589 \text{ N/m}$$
$$\text{Pr}_l = 1.75$$
$$h_{fg} = 2257\times10^3 \text{ J/kg}$$
$$\mu_l = 0.282\times10^{-3} \text{ kg}\cdot\text{m/s}$$
$$c_{pl} = 4217 \text{ J/kg}\cdot°\text{C}$$



Also, $C_{sf} = 0.0130$ and n = 1.0 for the boiling of water on a copper surface (Table 10-3). Note that we expressed the properties in units specified under Eq. 10-2 connection with their definitions in order to avoid unit manipulations.

Analysis The rate of heat transfer to the water and the heat flux are

$$\dot{m}_{\text{evap}} = \frac{m_{\text{evap}}}{\Delta t} = \frac{\rho\Delta V}{\Delta t} = \frac{(957.9 \text{ kg/m}^3)(\pi\times(0.2\text{ m})^2\ /4\times0.10\text{ m})}{15\times60\text{ s}} = 0.003344 \text{ kg/s}$$

$$\dot{Q} = \dot{m}_{\text{evap}} h_{fg} = (0.03344 \text{ kg/s})(2257 \text{ kJ/kg}) = 7.547 \text{ kW}$$

$$A_s = \pi D^2/4 = \pi(0.20\text{ m})^2/4 = 0.03142 \text{ m}^2$$

$$\dot{q} = \dot{Q}/A_s = (7547 \text{ W})/(0.03142 \text{ m}^2) = 240{,}200 \text{ W/m}^2$$

The Rohsenow relation which gives the nucleate boiling heat flux for a specified surface temperature can also be used to determine the surface temperature when the heat flux is given. Assuming nucleate boiling, the temperature of the inner surface of the pan is determined from Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{\text{sat}})}{C_{sf} h_{fg}\text{Pr}_l^n}\right)^3$$

$$240{,}200 = (0.282\times10^{-3})(2257\times10^3)\left[\frac{9.81(957.9-0.60)}{0.0589}\right]^{1/2}\left(\frac{4217(T_s-100)}{0.0130(2257\times10^3)1.75}\right)^3$$

It gives

T_s = **111.9°C**

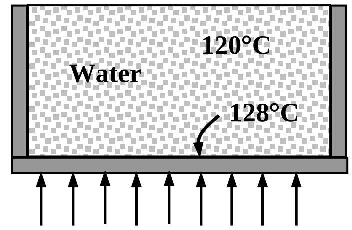
which is in the nucleate boiling range (5 to 30°C above surface temperature). Therefore, the nucleate boiling assumption is valid.

10-25 Water is boiled at T_{sat} = 120°C in a mechanically polished stainless steel pressure cooker whose inner surface temperature is maintained at T_s = 128°C. The boiling heat transfer coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the heater and the boiler are negligible.

Properties The properties of water at the saturation temperature of 120°C are (Tables 10-1 and A-9)

$$\rho_l = 943.4\ \text{kg/m}^3 \qquad h_{fg} = 2203\times10^3\ \text{J/kg}$$
$$\rho_v = 1.121\ \text{kg/m}^3 \qquad \mu_l = 0.232\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\sigma = 0.0550\ \text{N/m} \qquad c_{pl} = 4244\ \text{J/kg}\cdot°\text{C}$$
$$\text{Pr}_l = 1.44$$



Also, $C_{sf} = 0.0130$ and n = 1.0 for the boiling of water on a mechanically polished stainless steel surface (Table 10-3). Note that we expressed the properties in units specified under Eq. 10-2 in connection with their definitions in order to avoid unit manipulations.

Analysis The excess temperature in this case is $\Delta T = T_s - T_{sat} = 128 - 120 = 8°\text{C}$ which is relatively low (less than 30°C). Therefore, nucleate boiling will occur. The heat flux in this case can be determined from Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s - T_{\text{sat}})}{C_{sf} h_{fg}\,\text{Pr}_l^n}\right)^3$$

$$= (0.232\times10^{-3})(2203\times10^3)\left[\frac{9.81(943.4 - 1.121)}{0.0550}\right]^{1/2}\left(\frac{4244(128-120)}{0.0130(2203\times10^3)1.44}\right)^3$$

$$= 116{,}900\ \text{W/m}^2$$

The boiling heat transfer coefficient is

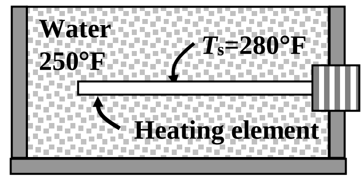
$$\dot{q}_{\text{nucleate}} = h(T_s - T_{\text{sat}}) \longrightarrow h = \frac{\dot{q}_{\text{nucleate}}}{T_s - T_{\text{sat}}} = \frac{116{,}900\ \text{W/m}^2}{(128-120)°\text{C}} = 14{,}610\ \text{W/m}^2\cdot°\text{C} = \mathbf{14.6\ kW/m^2\cdot C}$$

10-26E Water is boiled at a temperature of T_{sat} = 250°F by a nickel-plated heating element whose surface temperature is maintained at T_s = 280°F. The boiling heat transfer coefficient, the electric power consumed, and the rate of evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the boiler are negligible. **3** The boiling regime is nucleate boiling since $\Delta T = T_s - T_{sat} = 280 - 250 = 30°\text{ F}$ which is in the nucleate boiling range of 9 to 55°F for water.

Properties The properties of water at the saturation temperature of 250°F are (Tables 10-1 and A-9E)

$$\rho_l = 58.82\text{ lbm/ft}^3$$
$$\rho_v = 0.0723\text{ lbm/ft}^3$$
$$\sigma = 0.003755\text{ lbf/ft} = 0.1208\text{ lbm/s}^2$$
$$\text{Pr}_l = 1.43$$
$$h_{fg} = 946\text{ Btu/lbm}$$
$$\mu_l = 1.544\times10^{-4}\text{ lbm/ft}\cdot\text{s} = 0.556\text{ lbm/ft}\cdot\text{h}$$
$$c_{pl} = 1.015\text{ Btu/lbm}\cdot°\text{F}$$



Also, g = 32.2 ft/s^2 and C_{sf} = 0.0060 and n = 1.0 for the boiling of water on a nickel plated surface (Table 10-3). Note that we expressed the properties in units that will cancel each other in boiling heat transfer relations.

Analysis (*a*) Assuming nucleate boiling, the heat flux can be determined from Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s - T_{sat})}{C_{sf}h_{fg}\text{Pr}_l^n}\right)^3$$
$$= (0.556)(946)\left[\frac{32.2(58.82 - 0.0723)}{0.1208}\right]^{1/2}\left(\frac{1.015(280-250)}{0.0060(946)1.43}\right)^3$$
$$= 3{,}475{,}221\text{ Btu/h}\cdot\text{ft}^2$$

Then the convection heat transfer coefficient becomes

$$\dot{q} = h(T_s - T_{sat}) \rightarrow h = \frac{\dot{q}}{T_s - T_{sat}} = \frac{3{,}475{,}221\text{ Btu/h}\cdot\text{ft}^2}{(280-250)°\text{F}} = \mathbf{115{,}840\text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F}}$$

(*b*) The electric power consumed is equal to the rate of heat transfer to the water, and is determined from

$$\dot{W}_e = \dot{Q} = \dot{q}A_s = (\pi DL)\dot{q} = (\pi\times0.25/12\text{ ft}\times2\text{ ft})(3{,}475{,}221\text{ Btu/h}\cdot\text{ft}^2) = 454{,}905\text{ Btu/h}$$
$$= \mathbf{133.3\text{ kW}} \quad (\text{since } 1\text{ kW} = 3412\text{ Btu/h})$$

(*c*) Finally, the rate of evaporation of water is determined from

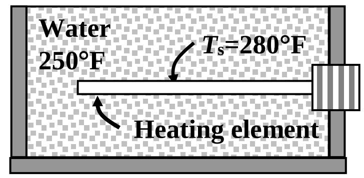
$$\dot{m}_{\text{evaporation}} = \frac{\dot{Q}_{\text{boiling}}}{h_{fg}} = \frac{454{,}905\text{ Btu/h}}{946\text{ Btu/lbm}} = \mathbf{480.9\text{ lbm/h}}$$

10-27E Water is boiled at a temperature of T_{sat} = 250°F by a platinum-plated heating element whose surface temperature is maintained at T_s = 280°F. The boiling heat transfer coefficient, the electric power consumed, and the rate of evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the boiler are negligible. **3** The boiling regime is nucleate boiling since $\Delta T = T_s - T_{sat} = 280 - 250 = 30°F$ which is in the nucleate boiling range of 9 to 55°F for water.

Properties The properties of water at the saturation temperature of 250°F are (Tables 10-1 and A-9E)

$$\rho_l = 58.82 \text{ lbm/ft}^3$$
$$\rho_v = 0.0723 \text{ lbm/ft}^3$$
$$\sigma = 0.003755 \text{ lbf/ft} = 0.1208 \text{ lbm/s}^2$$
$$\Pr_l = 1.43$$
$$h_{fg} = 946 \text{ Btu/lbm}$$
$$\mu_l = 1.544\times10^{-4} \text{ lbm/ft}\cdot\text{s} = 0.556 \text{ lbm/ft}\cdot\text{h}$$
$$c_{pl} = 1.015 \text{ Btu/lbm}\cdot°\text{F}$$



Also, g = 32.2 ft/s^2 and C_{sf} = 0.0130 and n = 1.0 for the boiling of water on a platinum plated surface (Table 10-3). Note that we expressed the properties in units that will cancel each other in boiling heat transfer relations.

Analysis (*a*) Assuming nucleate boiling, the heat flux can be determined from Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s - T_{sat})}{C_{sf} h_{fg} \Pr_l^n}\right)^3$$
$$= (0.556)(946)\left[\frac{32.2(58.82 - 0.0723)}{0.1208}\right]^{1/2}\left(\frac{1.015(280-250)}{0.0130(0.1208\times10^3)1.43}\right)^3$$
$$= 341{,}670 \text{ Btu/h}\cdot\text{ft}^2$$

Then the convection heat transfer coefficient becomes

$$\dot{q} = h(T_s - T_{sat}) \rightarrow h = \frac{\dot{q}}{T_s - T_{sat}} = \frac{341{,}670 \text{ Btu/h}\cdot\text{ft}^2}{(280-250)°\text{F}} = \mathbf{11{,}390\ Btu/h\cdot ft^2\cdot°F}$$

(*b*) The electric power consumed is equal to the rate of heat transfer to the water, and is determined from

$$\dot{W}_e = \dot{Q} = \dot{q}A_s = (\pi DL)\dot{q} = (\pi\times0.25/12 \text{ ft}\times 2 \text{ ft})(341{,}670 \text{ Btu/h}\cdot\text{ft}^2) = 44{,}724 \text{ Btu/h}$$
$$= \mathbf{13.1\ kW} \quad (\text{since } 1 \text{ kW} = 3412 \text{ Btu/h})$$

(*c*) Finally, the rate of evaporation of water is determined from

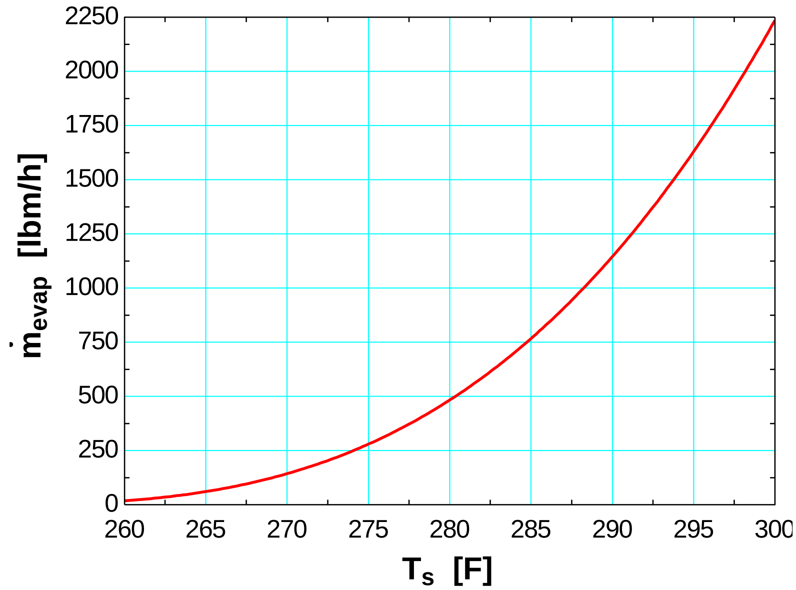
$$\dot{m}_{\text{evaporation}} = \frac{\dot{Q}_{\text{boiling}}}{h_{fg}} = \frac{44{,}724 \text{ Btu/h}}{946 \text{ Btu/lbm}} = \mathbf{47.3\ lbm/h}$$

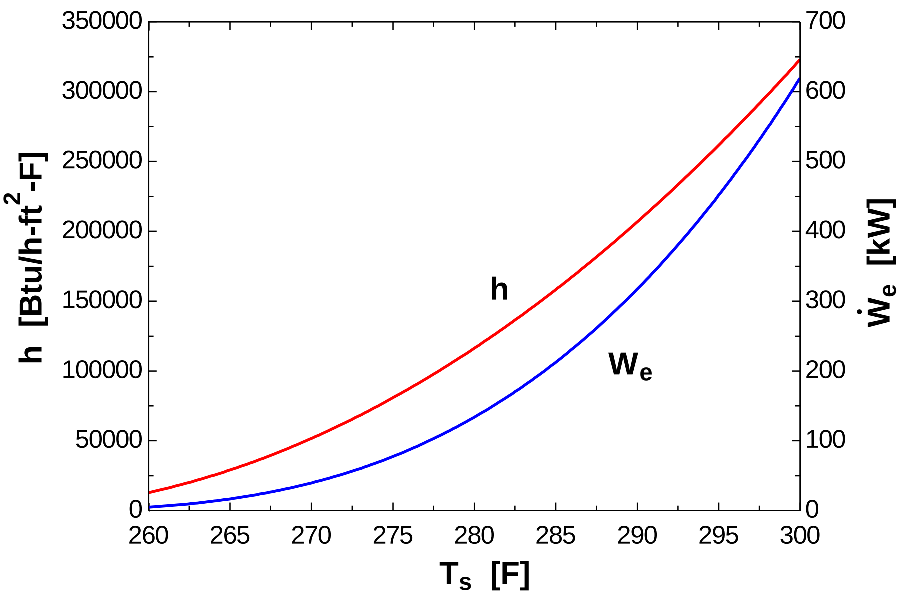
10-28E Prob. 10-26E is reconsidered. The effect of surface temperature of the heating element on the boiling heat transfer coefficient, the electric power, and the rate of evaporation of water is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_sat=250 [F]
L=2 [ft]
D=0.25/12 [ft]
T_s=280 [F]

"PROPERTIES"
Fluid$='steam_IAPWS'
P_sat=pressure(Fluid$, T=T_sat, x=1)
rho_l=density(Fluid$, T=T_sat, x=0)
rho_v=density(Fluid$, T=T_sat, x=1)
sigma=SurfaceTension(Fluid$, T=T_sat)*Convert(lbf/ft, lbm/s^2)
mu_l=Viscosity(Fluid$,T=T_sat, x=0)
Pr_l=Prandtl(Fluid$, T=T_sat, P=P_sat+1) "P=P_sat+1 is used to get liquid state"
C_l=CP(Fluid$, T=T_sat, x=0)
h_f=enthalpy(Fluid$, T=T_sat, x=0)
h_g=enthalpy(Fluid$, T=T_sat, x=1)
h_fg=h_g-h_f
C_sf=0.0060 "from Table 10-3 of the text"
n=1 "from Table 10-3 of the text"
g=32.2 [ft/s^2]

"ANALYSIS"
"(a)"
q_dot_nucleate=mu_l*h_fg*(((g*(rho_l-rho_v))/sigma)^0.5)*((C_l*(T_s-T_sat))/(C_sf*h_fg*Pr_l^n))^3
q_dot_nucleate=h*(T_s-T_sat)
"(b)"
W_dot_e=q_dot_nucleate*A*Convert(Btu/h, kW)
A=pi*D*L
"(c)"
m_dot_evap=Q_dot_boiling/h_fg
Q_dot_boiling=W_dot_e*Convert(kW, Btu/h)
```

T_s [F]	h [Btu/h.ft^2.F]	$\dot{W}_e$ [kW]	$\dot{m}_{evap}$ [lbm/h]
260	12919	4.956	17.89
262	18603	8.564	30.91
264	25321	13.6	49.08
266	33073	20.3	73.27
268	41857	28.9	104.3
270	51676	39.65	143.1
272	62528	52.77	190.5
274	74413	68.51	247.3
276	87332	87.11	314.4
278	101285	108.8	392.7
280	116271	133.8	483
282	132290	162.4	586.1
284	149343	194.8	703
286	167430	231.2	834.5
288	186550	272	981.5
290	206704	317.2	1145
292	227891	367.2	1325
294	250111	422.2	1524
296	273366	482.4	1741
298	297653	548.1	1978
300	322974	619.5	2236





10-29 Hot mechanically polished stainless steel ball bearings are cooled by submerging them in water at 1 atm. The rate of heat removed from a ball bearing at the instant it is submerged in the water is to be determined.

Assumptions **1** Steady operating conditions exist at the instant of submersion. **2** Surface temperature is uniform. **3** The boiling regime is nucleate boiling since $\Delta T_{\text{excess}} = T_s - T_{\text{sat}}$ = 25°C, which is in the nucleate boiling range of 5 to 30°C for water.

Properties At 1 atm, the saturation temperature of water is T_{sat} = 100°C. The properties of water at T_{sat} = 100°C are σ = 0.0589 N/m (Tables 10-1) and, from Table A-9,

$$\rho_l = 957.9\ \text{kg/m}^3 \qquad h_{fg} = 2257\times10^3\ \text{J/kg}$$

$$\rho_v = 0.5978\ \text{kg/m}^3 \qquad \mu_l = 0.282\times10^{-3}\ \text{kg/m}\cdot\text{s}$$

$$\text{Pr}_l = 1.75 \qquad c_{pl} = 4217\ \text{J/kg}\cdot\text{K}$$

Also, C_{sf} = 0.0130 and n = 1 for the boiling of water on a mechanically polished stainless steel surface (Table 10-3).

Analysis The instant a ball bearing is submerged in the water, with ΔT_{excess} = 25°C, nucleate boiling would occur. The heat flux can be determined from Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{\text{sat}})}{C_{sf}h_{fg}\text{Pr}_l^n}\right)^3$$

$$= (0.282\times10^{-3})(2257\times10^3)\left[\frac{9.81(957.9-0.5978)}{0.0589}\right]^{1/2}\left[\frac{4217(125-100)}{0.0130(2257\times10^3)1.75}\right]^3$$

$$= 2.1998\times10^6\ \text{W/m}^2$$

The heat transfer surface area is

$$A_s = \pi D^2 = \pi(0.05\ \text{m})^2 = 0.007854\ \text{m}^2$$

The rate of heat removed from a ball bearing at the instant it is submerged in the water is

$$\dot{Q}_{\text{boiling}} = A_s\dot{q}_{\text{nucleate}} = (0.007854\ \text{m}^2)(2.1998\times10^6\ \text{W/m}^2) = \mathbf{17.3\ kW}$$

Discussion Note that a 5-cm-diameter stainless steel ball can release 17.3 kW of heat at the instant it is submerged in water. This high value of heat rate removal, even with a temperature difference of only 25°C, is a result from nucleate boiling.

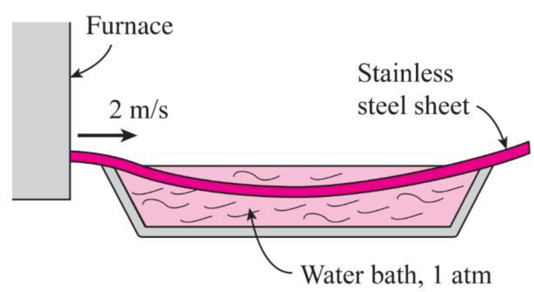
10-30 PtD A long hot mechanically polished stainless steel sheet is being cooled in a water bath. The temperature of the stainless steel sheet leaving the water bath is to be determined whether or not it has the risk of thermal burn hazard.

Assumptions **1** Steady operating conditions exist. **2** Surface temperature is uniform. **3** The boiling regime is nucleate boiling since $\Delta T_{\text{excess}} = T_s - T_{\text{sat}} = 25°\text{C}$, which is in the nucleate boiling range of 5 to 30°C for water.

Properties The specific heat and the density of stainless steel are given as $c_{p,ss}$ = 450 J/kg·K and ρ_{ss} = 7900 kg/m^3, respectively.

At 1 atm, the saturation temperature of water is T_{sat} = 100°C. The properties of water at T_{sat} = 100°C are σ = 0.0589 N/m (Tables 10-1) and, from Table A-9,

$$\rho_l = 957.9\ \text{kg/m}^3 \qquad h_{fg} = 2257\times10^3\ \text{J/kg}$$
$$\rho_v = 0.5978\ \text{kg/m}^3 \qquad \mu_l = 0.282\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\text{Pr}_l = 1.75 \qquad c_{pl} = 4217\ \text{J/kg}\cdot\text{K}$$



Also, C_{sf} = 0.013 and n = 1 for the boiling of water on a mechanically polished stainless steel surface (Table 10-3).

Analysis The mass of the stainless steelsheet being conveyed enters and exits the water bath at a rate of

$$\dot{m} = \rho_{ss} Vwt$$

The rate of heat that needs to be removed from the sheet so that it leaves the water bath below 45°C is

$$\dot{Q}_{\text{removed}} = \dot{m}c_{p,ss}(T_{\text{in}} - T_{\text{out}})$$

Then,

$$\begin{aligned}\dot{Q}_{\text{removed}} &= \rho_{ss} Vwtc_{p,ss}(T_{\text{in}} - T_{\text{out}}) \\ &= (7900\ \text{kg/m}^3)(2\ \text{m/s})(0.5\ \text{m})(0.005\ \text{m})(450\ \text{J/kg}\cdot\text{K})(125-45)\ \text{K} \\ &= 1.422\times10^6\ \text{W}\end{aligned}$$

With ΔT_{excess} = 25°C, nucleate boiling would occur in the water bath. The heat flux can be determined from Rohsenow relation to be

$$\begin{aligned}\dot{q}_{\text{nucleate}} &= \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s - T_{\text{sat}})}{C_{sf} h_{fg}\,\text{Pr}_l^n}\right)^3 \\ &= (0.282\times10^{-3})(2257\times10^3)\left[\frac{9.81(957.9-0.5978)}{0.0589}\right]^{1/2}\left[\frac{4217(125-100)}{0.0130(2257\times10^3)1.75}\right]^3 \\ &= 2.1998\times10^6\ \text{W/m}^2\end{aligned}$$

The heat transfer surface area of the sheet submerged in the water bath is

$$A_s = 2(1\ \text{m})(0.5\ \text{m}) + 2(1\ \text{m})(0.005\ \text{m}) = 1.01\ \text{m}^2$$

The rate of heat that could be removed from the sheet in the water bath is

$$\dot{Q}_{\text{boiling}} = A_s\dot{q}_{\text{nucleate}} = (1.01\ \text{m}^2)(2.1998\times10^6\ \text{W/m}^2) = \mathbf{2.222\times10^6\ W} > \dot{Q}_{\text{removed}} = 1.422\times10^6\ \text{W}$$

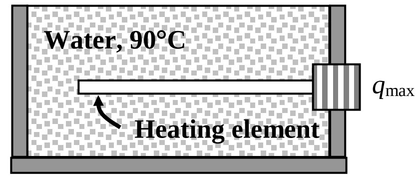
Discussion The rate of heat that could be removed from the stainless steel sheet in the water bath via nucleate boiling is greater than the heat that needs to be removed from the sheet so that it leaves the water bath below 45°C. Thus, there is no risk of thermal burn on the stainless steel sheet as it leaves the water bath.

Note that this analysis is simplified to steady state conditions, but the actual cooling process is transient.

10-31 Water is boiled at the saturation (or boiling) temperature of T_{sat} = 90°C by a horizontal brass heating element. The maximum heat flux in the nucleate boiling regime is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the boiler are negligible.

Properties The properties of water at the saturation temperature of 90°C are (Tables 10-1 and A-9)



$$\rho_l = 965.3\ \text{kg/m}^3 \qquad h_{fg} = 2283\times10^3\ \text{J/kg}$$
$$\rho_v = 0.4235\ \text{kg/m}^3 \qquad \mu_l = 0.315\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\sigma = 0.0608\ \text{N/m} \qquad c_{pl} = 4206\ \text{J/kg}\cdot°\text{C}$$
$$\text{Pr}_l = 1.96$$

Also, $C_{sf} = 0.0060$ and n = 1.0 for the boiling of water on a brass heating (Table 10-3). Note that we expressed the properties in units specified under Eqs. 10-2 and 10-3 in connection with their definitions in order to avoid unit manipulations.

Analysis For a large horizontal heating element, the coefficient C_{cr} is determined form Table 10-4 to be

$$L^* = L\ [g\ (\rho_l - \rho_v)\ /\sigma]^2 = (0.008/2)[\ 9.81(965.3 - 0.4235)\ /\ (0.0608)]^2 = 1.58 > 1.2$$

C_{cr} = 0.12 (sinceL^* = 1.58 > 1.2 and thus large cylinder).

The maximum or critical heat flux is determined from

$$\begin{aligned}\dot{q}_{\max} &= C_{cr} h_{fg} [\sigma g \rho_v^2 (\rho_l - \rho_v)]^{1/4} \\ &= 0.12(2283\times10^3)[0.0608\times9.81\times(0.4235)^2(965.3 - 0.4235)]^{1/4} \\ &= 873{,}200\ \text{W/m}^2 = \mathbf{873.2\ kW/m^2}\end{aligned}$$

10-32 The electrical current at which a nickel wire would be in danger of burnout in nucleate boiling is to be determined.

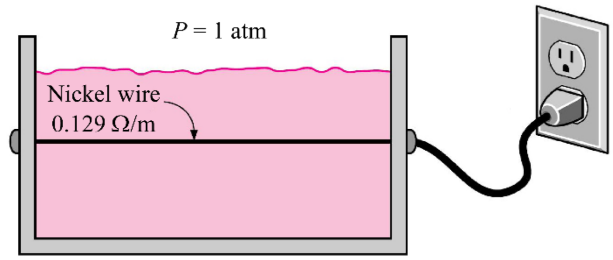
Assumptions **1** Steady operating condition exists. **2** Heat losses from the boiler are negligible.

Properties The properties of water at the saturation temperature of 100°C are σ = 0.0589 N/m (Table 10-1) and, from Table A-9,

ρ_l = 957.9 kg/m^3

h_{fg} = 2257 × 10^3 J/kg

ρ_v = 0.5978 kg/m^3



Analysis The danger of burnout occurs when the heat flux is at maximum in nucleate boiling, which can be determined using

$$\dot{q}_{\max} = C_{cr} h_{fg}[\sigma g \rho_v^2(\rho_l - \rho_v)]^{1/4}$$

Using Table 10-4, the parameter L^* and the constant C_{cr} are determined to be

$$L^* = L\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2} = (0.0005)\left[\frac{9.81(957.9 - 0.5978)}{0.0589}\right]^{1/2} = 0.1997$$

which correspond to

$$C_{cr} = 0.12L^{*-0.25} = 0.12(0.1997)^{-0.25} = 0.1795$$

Hence, the maximum heat flux is

$$\begin{aligned}\dot{q}_{\max} &= (0.1795)(2257 \times 10^3)[(0.0589)(9.81)(0.5978)^2(957.9 - 0.5978)]^{1/4} \\ &= 1.519 \times 10^6 \text{ W/m}^2\end{aligned}$$

We know that

$$\dot{q} = \frac{I^2 R}{A_s} = \frac{I^2 R}{\pi D L}$$

Thus, the electrical current at which the wire would be in danger of burnout is

$$I = \left[\frac{\dot{q}_{\max}\pi D}{(R/L)}\right]^{1/2} = \left[\frac{(1.519 \times 10^6 \text{ W/m}^2)\pi(0.001\,\text{m})}{0.129\,\Omega/\text{m}}\right]^{1/2} = \mathbf{192\ A}$$

Discussion The electrical current at which burnout could occur will decrease if the resistance of the wire increases.

10-33 Water is boiled at a temperature of T_{sat} = 100°C by hot gases flowing through a tube submerged in water. The maximum rate of vaporization is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the boiler are negligible.

Properties The properties of water at T_{sat} = 100°C are σ = 0.0589 N/m (Tables 10-1) and, from Table A-9, ρ_l = 957.9 kg/m^3, ρ_v = 0.5978 kg/m^3, h_{fg} = 2257 × 10^3 J/kg.

Analysis The maximum rate of vaporization occurs at the maximum heat flux.

For a horizontal cylindrical heating element, the coefficient C_{cr} is determined from Table 10-4 to be

$$L^* = \frac{D}{2}\left(\frac{g(\rho_l-\rho_v)}{\sigma}\right)^{1/2} = (0.050/2)\left[\frac{9.81(957.9-0.5978)}{0.0589}\right]^{1/2} = 9.98 > 1.2$$

$$C_{cr} = 0.12 \quad (\text{since } L^* > 1.2 \text{ and thus large cylinder})$$

Then the maximum heat flux is determined from

$$\dot{q}_{\max} = C_{cr} h_{fg}[\sigma g \rho_v^2(\rho_l-\rho_v)]^{1/4}$$
$$= 0.12(2257\times10^3)[0.0589\times9.81\times(0.5978)^2(957.9-0.5978)]^{1/4} = 1.0155\times10^6 \text{ W/m}^2$$

The heat transfer surface area is

$$A_s = \pi DL = \pi(0.05\text{ m})(10\text{ m}) = 1.5708\text{ m}^2$$

Then, the rate of heat transfer during nucleate boiling becomes

$$\dot{Q}_{\text{boiling}} = A_s\dot{q}_{\max} = (1.5708\text{ m}^2)(1.0155\times10^6 \text{ W/m}^2) = 1.5952\times10^6 \text{ W}$$

The maximum rate of vaporization of water is determined from

$$\dot{m}_{\text{vapor}} = \frac{\dot{Q}_{\text{boiling}}}{h_{fg}} = \frac{1.5952\times10^6 \text{ J/s}}{2257\times10^3 \text{ J/kg}} = \mathbf{0.707\ kg/s}$$

Discussion The rate of vaporization can be increased by increasing the tube diameter, thereby increasing the heat transfer surface area.

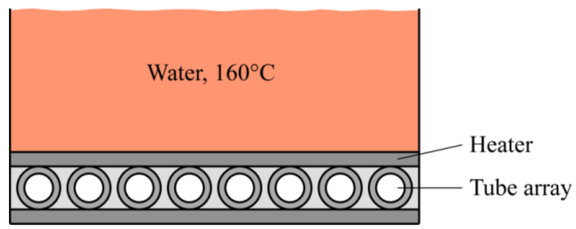
10-34 Water is boiled at a temperature of T_{sat} = 160°C by a 3 m × 3 m horizontal flat heater heated by hot gases flowing through an array of tubes embedded in it. The maximum rate of vaporization is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the heater are negligible.

Properties The properties of water at T_{sat} = 160°C are σ = 0.0466 N/m (Tables 10-1) and, from Table A-9, ρ_l = 907.4 kg/m^3, ρ_v = 3.256 kg/m^3, h_{fg} = 2083 × 10^3 J/kg.

Analysis The maximum rate of vaporization occurs at the maximum heat flux.

For a horizontal flat heating element, the coefficient C_{cr} is determined from Table 10-4 to be



$$L^* = L\left(\frac{g(\rho_l - \rho_v)}{\sigma}\right)^{1/2} = (3)\left[\frac{9.81(907.4 - 3.256)}{0.0466}\right]^{1/2} = 1309 > 27$$

$$C_{cr} = 0.149 \quad (\text{since } L^* > 27 \text{ and thus large flat heater})$$

Then the maximum heat flux is determined from

$$\begin{aligned}\dot{q}_{\max} &= C_{cr} h_{fg}[\sigma g \rho_v^2(\rho_l - \rho_v)]^{1/4} \\ &= 0.149(2083\times10^3)[0.0466\times9.81\times(3.256)^2(907.4-3.256)]^{1/4} \\ &= 2.5252\times10^6 \text{ W/m}^2\end{aligned}$$

The heat transfer surface area is

$$A_s = L\times L = 3\,\text{m}\times3\,\text{m} = 9\,\text{m}^2$$

Then, the rate of heat transfer during nucleate boiling becomes

$$\dot{Q}_{\text{boiling}} = A_s\dot{q}_{\max} = (9\,\text{m}^2)(2.5252\times10^6 \text{ W/m}^2) = 2.2727\times10^7 \text{ W}$$

The maximum rate of vaporization of water is determined from

$$\dot{m}_{\text{vapor}} = \frac{\dot{Q}_{\text{boiling}}}{h_{fg}} = \frac{2.2727\times10^7 \text{ J/s}}{2083\times10^3 \text{ J/kg}} = \mathbf{10.9\ kg/s}$$

Discussion The rate of vaporization can be increased by increasing the surface area of the plate.

10-35 Water is to be boiled at 1 atm by a spherical heater and a square horizontal flat heater of the same surface area, and which heater geometry would produce higher maximum rate of vaporization is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the heaters are negligible.

Properties At 1 atm, the saturation temperature of water is T_{sat} = 100°C. The properties of water at T_{sat} = 100°C are σ = 0.0589 N/m (Tables 10-1) and, from Table A-9, ρ_l = 957.9 kg/m^3, ρ_v = 0.5978 kg/m^3, h_{fg} = 2257 × 10^3 J/kg.

Analysis The maximum rate of vaporization occurs at the maximum heat flux.

For a spherical heating element, the coefficient C_{cr} is determined from Table 10-4 to be

$$L^* = \frac{D}{2}\left(\frac{g(\rho_l - \rho_v)}{\sigma}\right)^{1/2} = (1/2)\left[\frac{9.81(957.9 - 0.5978)}{0.0589}\right]^{1/2} = 199.7 > 4.26$$

$$C_{cr} = 0.11 \quad (\text{since } L^* > 4.26 \text{ and thus large sphere})$$

Then the maximum heat flux is determined from

$$\begin{aligned}\dot{q}_{\max} &= C_{cr} h_{fg}[\sigma g \rho_v^2(\rho_l - \rho_v)]^{1/4} \\ &= 0.11(2257\times10^3)[0.0589\times9.81\times(0.5978)^2(957.9-0.5978)]^{1/4} = 9.3092\times10^5 \text{ W/m}^2\end{aligned}$$

The heat transfer surface area for a sphere is

$$A_s = \pi D^2 = \pi(1\,\text{m})^2 = 3.142\,\text{m}^2$$

The maximum rate of vaporization of water is determined from

$$\dot{m}_{\text{vapor}} = \frac{\dot{Q}_{\text{boiling}}}{h_{fg}} = \frac{A_s \dot{q}_{\max}}{h_{fg}} = \frac{(3.142\,\text{m}^2)(9.3092\times10^5 \text{ W/m}^2)}{2257\times10^3 \text{ J/kg}} = \mathbf{1.296\ kg/s} \quad \text{(spherical heater)}$$

The width of a square with the same surface area as the spherical heater is

$$A_s = L^2 = 3.142\,\text{m}^2 \rightarrow L = 1.7726\,\text{m}$$

For a horizontal flat heating element, the coefficient C_{cr} is determined from Table 10-4 to be

$$L^* = L\left(\frac{g(\rho_l - \rho_v)}{\sigma}\right)^{1/2} = (1.7726)\left[\frac{9.81(957.9 - 0.5978)}{0.0589}\right]^{1/2} = 707.8 > 27$$

$$C_{cr} = 0.149 \quad (\text{since } L^* > 27 \text{ and thus large flat heater})$$

Then the maximum heat flux is determined from

$$\begin{aligned}\dot{q}_{\max} &= C_{cr} h_{fg}[\sigma g \rho_v^2(\rho_l - \rho_v)]^{1/4} \\ &= 0.149(2257\times10^3)[0.0589\times9.81\times(0.5978)^2(957.9-0.5978)]^{1/4} = 1.261\times10^6 \text{ W/m}^2\end{aligned}$$

The maximum rate of vaporization of water is determined from

$$\dot{m}_{\text{vapor}} = \frac{\dot{Q}_{\text{boiling}}}{h_{fg}} = \frac{A_s \dot{q}_{\max}}{h_{fg}} = \frac{(3.142\,\text{m}^2)(1.261\times10^6 \text{ W/m}^2)}{2257\times10^3 \text{ J/kg}} = \mathbf{1.755\ kg/s} \quad \text{(square heater)}$$

Discussion For the same surface area, the square heater would produce about 35% higher in the maximum rate of vaporization than the spherical heater. This is because, for the same surface area, the square heater has a C_{cr} coefficient that is about 35% higher than that of the spherical heater.

10-36 Water is boiled at a temperature of T_{sat} = 180°C by a 3 m × 3 m nickel plated flat heater that is heated by hot gases flowing through an array of tubes embedded in it. The surface temperature that produced the maximum rate of steam generation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the heater are negligible. **3** The boiling regime is nucleate boiling.

Properties The properties of water at T_{sat} = 180°C are σ = 0.0422 N/m (Tables 10-1) and, from Table A-9,

$$\rho_l = 887.3\ \text{kg/m}^3 \qquad h_{fg} = 2015\times10^3\ \text{J/kg}$$
$$\rho_v = 5.153\ \text{kg/m}^3 \qquad \mu_l = 0.150\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\text{Pr}_l = 0.983 \qquad c_{pl} = 4410\ \text{J/kg}\cdot\text{K}$$

Also, C_{sf} = 0.0060 and n = 1 for the boiling of water on a nickel surface (Table 10-3).

Analysis The maximum rate of steam generation occurs at the maximum heat flux.

For a horizontal flat heating element, the coefficient C_{cr} is determined from Table 10-4 to be

$$L^* = L\left(\frac{g(\rho_l-\rho_v)}{\sigma}\right)^{1/2} = (3)\left[\frac{9.81(887.3-5.153)}{0.0422}\right]^{1/2} = 1359 > 27$$
$$C_{cr} = 0.149 \quad (\text{since } L^* > 27 \text{ and thus large flat heater})$$

Then the maximum heat flux is determined from

$$\dot{q}_{\max} = C_{cr}h_{fg}[\sigma g\rho_v^2(\rho_l-\rho_v)]^{1/4}$$
$$= 0.149(2015\times10^3)[0.0422\times9.81\times(5.153)^2(887.3-5.153)]^{1/4} = 2.9794\times10^6\ \text{W/m}^2$$

The heat flux for nucleate boiling can be expressed using the Rohsenow relation to be

$$\dot{q}_{\max} = \dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{\text{sat}})}{C_{sf}h_{fg}\text{Pr}_l^n}\right)^3$$

$$2.9794\times10^6\ \text{W/m}^2 = (0.150\times10^{-3})(2015\times10^3)\left[\frac{9.81(887.3-5.153)}{0.0422}\right]^{1/2}\left[\frac{4410(T_s-180)}{0.0060(2015\times10^3)0.983}\right]^3$$

Solving for T_s yields

$$T_s = \mathbf{187.5°C}$$

The convection heat transfer coefficient can be determined as

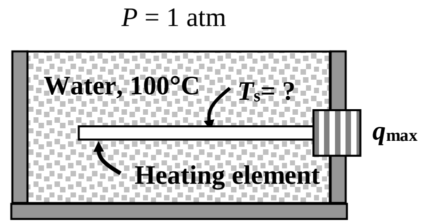
$$\dot{q} = h(T_s-T_{\text{sat}}) \rightarrow h = \frac{2.9794\times10^6\ \text{W/m}^2}{(187.5-180)\ \text{K}} = \mathbf{3.97\times10^5\ W/m^2\cdot K}$$

Discussion Note that a heat transfer coefficient of about 400 kW/m^2·K can be achieved in nucleate boiling with a temperature difference of less than 10°C.

10-37 Water is boiled at 1 atm pressure and thus at a saturation (or boiling) temperature of T_{sat} = 100°C by a mechanically polished stainless steel heating element. The maximum heat flux in the nucleate boiling regime and the surface temperature of the heater for that case are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the boiler are negligible.

Properties The properties of water at the saturation temperature of 100°C are (Tables 10-1 and A-9)



$$\rho_l = 957.9 \text{ kg/m}^3 \qquad h_{fg} = 2257\times10^3 \text{ J/kg}$$
$$\rho_v = 0.60 \text{ kg/m}^3 \qquad \mu_l = 0.282\times10^{-3} \text{ kg}\cdot\text{m/s}$$
$$\sigma = 0.0589 \text{ N/m} \qquad c_{pl} = 4217 \text{ J/kg}\cdot°\text{C}$$
$$\text{Pr}_l = 1.75$$

Also, $C_{sf} = 0.0130$ and n = 1.0 for the boiling of water on a mechanically polished stainless steel surface (Table 10-3). Note that we expressed the properties in units specified under Eqs. 10-2 and 10-3 in connection with their definitions in order to avoid unit manipulations.

Analysis For a large horizontal heating element, the coefficient C_{cr} is determined form Table 10-4 to be

$L^* = L\,[g\,(\rho_l - \rho_v)\,/\sigma]^2$ = (0.02/2)[9.81(957.9 – 0.60) / (0.0589)]^2 = 3.99 > 1.2

C_{cr} = 0.12 (sinceL^* = 3.99> 1.2 and thus large cylinder).

The maximum or critical heat flux is determined from

$$\begin{aligned}\dot{q}_{max} &= C_{cr} h_{fg} [\sigma g \rho_v^2 (\rho_l - \rho_v)]^{1/4} \\ &= 0.12(2257\times10^3)[0.0589\times 9.81\times (0.6)^2 (957.9 - 0.60)]^{1/4} \\ &= \mathbf{1{,}017{,}000\ W/m^2}\end{aligned}$$

The Rohsenow relation which gives the nucleate boiling heat flux for a specified surface temperature can also be used to determine the surface temperature when the heat flux is given. Substituting the maximum heat flux into the Rohsenow relation together with other properties gives

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \Pr_l^n}\right)^3$$

$$1{,}017{,}000 = (0.282\times10^{-3})(2257\times10^3)\left[\frac{9.81(957.9-0.60)}{0.0589}\right]^{1/2}\left(\frac{4217(T_s - 100)}{0.0130(2257\times10^3)1.75}\right)^3$$

It gives

$$T_s = \mathbf{119.3°C}$$

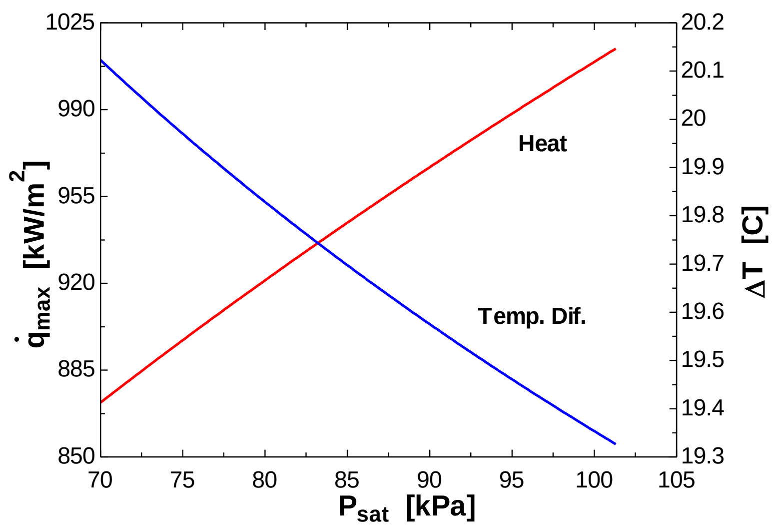
Therefore, the temperature of the heater surface will be only 19.3°C above the boiling temperature of water when burnout occurs.

10-38 Prob. 10-37 is reconsidered. Theeffect of local atmospheric pressure on the maximum heat flux and the temperature difference $T_s - T_{sat}$ is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.02 [m]
P_sat=101.3 [kPa]

"PROPERTIES"
Fluid$='steam_IAPWS'
T_sat=temperature(Fluid$, P=P_sat, x=1)
rho_l=density(Fluid$, T=T_sat, x=0)
rho_v=density(Fluid$, T=T_sat, x=1)
sigma=SurfaceTension(Fluid$, T=T_sat)
mu_l=Viscosity(Fluid$,T=T_sat, x=0)
Pr_l=Prandtl(Fluid$, T=T_sat, P=P_sat+1[kPa])
c_l=CP(Fluid$, T=T_sat, x=0)
h_f=enthalpy(Fluid$, T=T_sat, x=0)
h_g=enthalpy(Fluid$, T=T_sat, x=1)
h_fg=h_g-h_f
C_sf=0.0130 "from Table 10-3 of the text"
n=1 "from Table 10-3 of the text"
C_cr=0.12 "from Table 10-4 of the text"
g=9.81 [m/s^2] “gravitational acceleraton"
"ANALYSIS"
q_dot_max=C_cr*h_fg*(sigma*g*rho_v^2*(rho_l-rho_v))^0.25
q_dot_nucleate=q_dot_max
q_dot_nucleate=mu_l*h_fg*(((g*(rho_l-rho_v))/sigma)^0.5)*((c_l*(T_s-T_sat))/(C_sf*h_fg*Pr_l^n))^3
DELTAT=T_s-T_sat
```

P_{sat} [kPa]	$\dot{q}_{max}$ [kW/m^2]	ΔT [C]
70	871.9	20.12
71.65	880.3	20.07
73.29	888.6	20.02
74.94	896.8	19.97
76.59	904.9	19.92
78.24	912.8	19.88
79.88	920.7	19.83
81.53	928.4	19.79
83.18	936.1	19.74
84.83	943.6	19.7
86.47	951.1	19.66
88.12	958.5	19.62
89.77	965.8	19.58
91.42	973	19.54
93.06	980.1	19.5
94.71	987.2	19.47
96.36	994.1	19.43
98.01	1001	19.4
99.65	1008	19.36
101.3	1015	19.33



10-39 A 10 cm × 10 cm flat heater is used for vaporizing refrigerant-134a at 350 kPa. The surface temperature of the heater is given as 25°C and the heater is subjected to a heat flux of 0.35 MW/m². The coefficient C_{sf} is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the heater are negligible. **3** The boiling regime is nucleate boiling since $\Delta T_{\text{excess}} = T_s - T_{\text{sat}} = 20°\text{C}$.

Properties At 350 kPa, the saturation temperature of R134a is 5°C (Table A-10). The properties of R134a at T_{sat} = 5°C are from Table A-10,

$$\rho_l = 1278 \text{ kg/m}^3 \qquad h_{fg} = 194.8\times10^3 \text{ J/kg}$$
$$\rho_v = 17.12 \text{ kg/m}^3 \qquad \mu_l = 2.589\times10^{-4} \text{ kg/m}\cdot\text{s}$$
$$\text{Pr}_l = 3.802 \qquad c_{pl} = 1358 \text{ J/kg}\cdot\text{K}$$
$$\sigma = 0.01084 \text{ N/m}$$

Also, n = 1.7 for the boiling of R134a is given.

Analysis The heat flux for nucleate boiling can be expressed using the Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{\text{sat}})}{C_{sf}h_{fg}\text{Pr}_l^n}\right)^3$$

Using the Rohsenow relation to solve for C_{sf} yields

$$C_{sf} = \frac{c_{p,l}(T_s-T_{\text{sat}})}{h_{fg}\text{Pr}_l^n}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/6}\left(\frac{\mu_l h_{fg}}{\dot{q}_{\text{nucleate}}}\right)^{1/3}$$
$$= \frac{1358(25-5)}{(194.8\times10^3)(3.802^{1.7})}\left[\frac{9.81(1278-17.12)}{0.01084}\right]^{1/6}\left(\frac{(2.589\times10^{-4})(194.8\times10^3)}{0.35\times10^6}\right)^{1/3} = \mathbf{0.00772}$$

For a horizontal flat heating element, the coefficient C_{cr} is determined from Table 10-4 to be

$$L^* = L\left(\frac{g(\rho_l-\rho_v)}{\sigma}\right)^{1/2} = (0.1)\left[\frac{9.81(1278-17.12)}{0.01084}\right]^{1/2} = 106.8 > 27$$
$$C_{cr} = 0.149 \quad (\text{since } L^* > 27 \text{ and thus large flat heater})$$

The maximum heat flux in the nucleate boiling regime can be determined from

$$\dot{q}_{\max} = C_{cr}h_{fg}[\sigma g\rho_v^2(\rho_l-\rho_v)]^{1/4}$$
$$= 0.149(194.8\times10^3)[0.01084\times9.81\times(17.12)^2(1278-17.12)]^{1/4}$$
$$= 4.087\times10^5 \text{ W/m}^2 > 0.35 \text{ MW/m}^2$$

Discussion Since $\dot{q}_{\max}$> 0.35 MW/m², the Rohsenow relation for nucleate boiling is appropriate for this analysis.

10-40 Water is boiled at a temperature of T_{sat} = 150°C by hot gases flowing through a mechanically polished stainless steel pipe submerged in water whose outer surface temperature is maintained at T_s = 160°C. The rate of heat transfer to the water, the rate of evaporation, the ratio of critical heat flux to current heat flux, and the pipe surface temperature at critical heat flux conditions are to be determined.

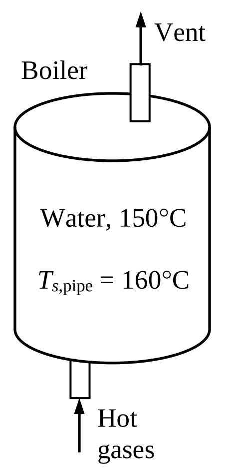
Assumptions **1** Steady operating conditions exist. **2** Heat losses from the boiler are negligible. **3** The boiling regime is nucleate boiling since $\Delta T = T_s - T_{\text{sat}} = 160 - 150 = 10°\text{C}$ which is in the nucleate boiling range of 5 to 30°C for water.

Properties The properties of water at the saturation temperature of 150°C are (Tables 10-1 and A-9)

$$\begin{aligned} &\rho_l = 916.6 \text{ kg/m}^3 && h_{fg} = 2114\times10^3 \text{ J/kg} \\ &\rho_v = 2.55 \text{ kg/m}^3 && \mu_l = 0.183\times10^{-3} \text{ kg}\cdot\text{m/s} \\ &\sigma = 0.0488 \text{ N/m} && c_{pl} = 4311 \text{ J/kg}\cdot°\text{C} \\ &\text{Pr}_l = 1.16 \end{aligned}$$

Also, C_{sf} = 0.0130 and n = 1.0 for the boiling of water on a mechanically polished stainless steel surface (Table 10-3). Note that we expressed the properties in units specified under Eq. 10-2 in connection with their definitions in order to avoid unit manipulations.

Analysis (*a*) Assuming nucleate boiling, the heat flux can be determined from Rohsenow relation to be



$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{\text{sat}})}{C_{sf}h_{fg}\text{Pr}_l^n}\right)^3$$

$$= (0.183\times10^{-3})(2114\times10^3)\left[\frac{9.81(916.6-2.55)}{0.0488}\right]^{1/2}\left(\frac{4311(160-150)}{0.0130(2114\times10^3)1.16}\right)^3$$

$$= 410{,}090 \text{ W/m}^2$$

The heat transfer surface area is

$$A_s = \pi DL = \pi(0.05\text{ m})(25\text{ m}) = 3.927\text{ m}^2$$

Then the rate of heat transfer during nucleate boiling becomes

$$\dot{Q}_{\text{boiling}} = A_s\dot{q}_{\text{nucleate}} = (3.927\text{ m}^2)(410{,}090\text{ W/m}^2) = 1{,}610{,}400\text{ W} = \mathbf{1610\ kW}$$

(*b*) The rate of evaporation of water is determined from

$$\dot{m}_{\text{evaporation}} = \frac{\dot{Q}_{\text{boiling}}}{h_{fg}} = \frac{1610\text{ kJ/s}}{2114\text{ kJ/kg}} = \mathbf{0.762\ kg/s}$$

(*c*) For a horizontal cylindrical heating element, the coefficient C_{cr} is determined from Table 10-4 to be

$$L^* = L\left(\frac{g(\rho_l-\rho_v)}{\sigma}\right)^{1/2} = (0.025)\left(\frac{9.8(916.6-2.55)}{0.0488}\right)^{1/2} = 10.7 > 1.2$$

$$C_{cr} = 0.12 \quad (\text{since } L^* > 1.2 \text{ and thus large cylinder})$$

Then the maximum or critical heat flux is determined from

$$\dot{q}_{\max} = C_{cr}h_{fg}[\sigma g\rho_v^2(\rho_l-\rho_v)]^{1/4}$$

$$= 0.12(2114\times10^3)[0.0488\times9.81\times(2.55)^2(916.6-2.55)]^{1/4} = 1{,}852{,}000\text{ W/m}^2$$

Therefore,

$$\frac{\dot{q}_{\max}}{\dot{q}_{\text{current}}} = \frac{1{,}852{,}000}{410{,}090} = \mathbf{4.52}$$

(*d*) The surface temperature of the pipe at the burnout point is determined from Rohsenow relation at the critical heat flux value to be

$$\dot{q}_{\text{nucleate},cr} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_{s,cr}-T_{\text{sat}})}{C_{sf}h_{fg}\text{Pr}_l^n}\right)^3$$

$$1{,}852{,}000 = (0.183\times10^{-3})(2114\times10^3)\left[\frac{9.81(916.6-2.55)}{0.0488}\right]^{1/2}\left(\frac{4311(T_{s,cr}-150)}{0.0130(2114\times10^3)1.16}\right)^3$$

$$T_{s,cr} = \mathbf{166.5°C}$$

10-41 Water is boiled at a temperature of T_{sat} = 155°C by hot gases flowing through a mechanically polished stainless steel pipe submerged in water whose outer surface temperature is maintained at T_s = 160°C. The rate of heat transfer to the water, the rate of evaporation, the ratio of critical heat flux to current heat flux, and the pipe surface temperature at critical heat flux conditions are to be determined.

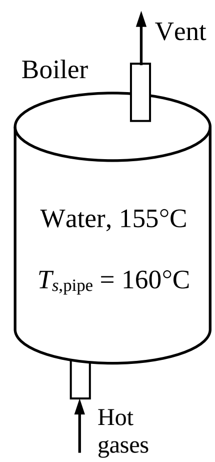
Assumptions **1** Steady operating conditions exist. **2** Heat losses from the boiler are negligible. **3** The boiling regime is nucleate boiling since $\Delta T = T_s - T_{sat} = 160 - 155 = 5°C$ which is in the nucleate boiling range of 5 to 30°C for water.

Properties The properties of water at the saturation temperature of 155°C are (Tables 10-1 and A-9)

$$\rho_l = 912.0\ \text{kg/m}^3 \qquad h_{fg} = 2099 \times 10^3\ \text{J/kg}$$
$$\rho_v = 2.901\ \text{kg/m}^3 \qquad \mu_l = 0.177 \times 10^{-3}\ \text{kg}\cdot\text{m/s}$$
$$\sigma = 0.0477\ \text{N/m} \qquad c_{pl} = 4326\ \text{J/kg}\cdot°\text{C}$$
$$\text{Pr}_l = 1.125$$

Also, $C_{sf} = 0.0130$ and n = 1.0 for the boiling of water on a mechanically polished stainless steel surface (Table 10-3). Note that we expressed the properties in units specified under Eq. 10-2 in connection with their definitions in order to avoid unit manipulations.

Analysis (*a*) Assuming nucleate boiling, the heat flux can be determined from Rohsenow relation to be



$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \text{Pr}_l^n}\right)^3$$
$$= (0.177\times10^{-3})(2099\times10^3)\left[\frac{9.81(912.0-2.901)}{0.0477}\right]^{1/2}\left(\frac{4326(160-155)}{0.0130(2099\times10^3)1.125}\right)^3$$
$$= 56{,}197\ \text{W/m}^2$$

The heat transfer surface area is

$$A_s = \pi DL = \pi(0.05\ \text{m})(25\ \text{m}) = 3.927\ \text{m}^2$$

Then the rate of heat transfer during nucleate boiling becomes

$$\dot{Q}_{\text{boiling}} = A_s \dot{q}_{\text{nucleate}} = (3.927\ \text{m}^2)(56{,}197\ \text{W/m}^2) = \mathbf{220{,}700\ W}$$

(*b*) The rate of evaporation of water is determined from

$$\dot{m}_{\text{evaporation}} = \frac{\dot{Q}_{\text{boiling}}}{h_{fg}} = \frac{220.7\ \text{kJ/s}}{2099\ \text{kJ/kg}} = \mathbf{0.105\ kg/s}$$

(*c*) For a horizontal cylindrical heating element, the coefficient C_{cr} is determined from Table 10-4 to be

$$L^* = L\left(\frac{g(\rho_l - \rho_v)}{\sigma}\right)^{1/2} = (0.025)\left(\frac{9.81(912.0-2.901)}{0.0477}\right)^{1/2} = 10.8 > 1.2$$
$$C_{cr} = 0.12 \quad \text{(since } L^* > 1.2 \text{ and thus large cylinder)}$$

Then the maximum or critical heat flux is determined from

$$\dot{q}_{\max} = C_{cr} h_{fg}[\sigma g \rho_v^2(\rho_l - \rho_v)]^{1/4}$$
$$= 0.12(2099\times10^3)[0.0477\times9.81\times(2.901)^2(912.0-2.901)]^{1/4} = 1.948\times10^6\ \text{W/m}^2$$

Therefore,

$$\frac{\dot{q}_{\max}}{\dot{q}_{\text{current}}} = \frac{1.948\times10^6\ \text{W/m}^2}{56{,}197\ \text{W/m}^2} = \mathbf{34.7}$$

(*d*) The surface temperature of the pipe at the burnout point is determined from Rohsenow relation at the critical heat flux value to be

$$1.948\times10^6 = \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \text{Pr}_l^n}\right)^3$$
$$= (0.177\times10^{-3})(2099\times10^3)\left[\frac{9.81(912.0-2.901)}{0.0477}\right]^{1/2}\left(\frac{4326(T_{s,cr}-155)}{0.0130(2099\times10^3)1.125}\right)^3$$
$$T_{s,cr} = \mathbf{171.3°C}$$

10-42 Steam is generated by a 1 m × 1 m flat heater boiling water at 1 atm with an excess temperature above 300°C. The range of the steam generation rate that can be achieved by the heater is to be determined.

Assumptions **1** Steady operating conditions exists. **2** Heat losses from the heater are negligible. **3** The boiling regime is film boiling since ΔT_{excess}> 300°C, which is much larger than 30°C.

Properties At 1 atm, the saturation temperature of water is T_{sat} = 100°C. The properties of water at T_{sat} = 100°C are σ = 0.0589 N/m (Tables 10-1) and, from Table A-9, ρ_l = 957.9 kg/m^3, ρ_v = 0.5978 kg/m^3, h_{fg} = 2257 × 10^3 J/kg.

Analysis For a horizontal flat heating element, the coefficient C_{cr} is determined from Table 10-4 to be

$$L^* = L\left(\frac{g(\rho_l - \rho_v)}{\sigma}\right)^{1/2} = (1)\left[\frac{9.81(957.9 - 0.5978)}{0.0589}\right]^{1/2} = 399.4 > 27$$

$$C_{cr} = 0.149 \quad (\text{since } L^* > 27 \text{ and thus large flat heater})$$

The range of steam generation rate not exceeding the burnout point can be determined from the minimum and maximum boiling heat fluxes.

The minimum rate of vaporization occurs at the minimum heat flux, which can be determined from

$$\dot{q}_{\min} = 0.09\rho_v h_{fg}\left[\frac{\sigma g(\rho_l - \rho_v)}{(\rho_l + \rho_v)^2}\right]^{1/4}$$

$$= 0.09(0.5978)(2257\times10^3)\left[\frac{(0.0589)(9.81)(957.9 - 0.5978)}{(957.9 + 0.5978)^2}\right]^{1/4} = 19021\ \text{W/m}^2$$

The maximum rate of vaporization occurs at the maximum heat flux, which can be determined from

$$\dot{q}_{\max} = C_{cr} h_{fg}[\sigma g \rho_v^2(\rho_l - \rho_v)]^{1/4}$$

$$= 0.149(2257\times10^3)[0.0589\times9.81\times(0.5978)^2(957.9 - 0.5978)]^{1/4} = 1.261\times10^6\ \text{W/m}^2$$

The heat transfer surface area is

$$A_s = L\times L = 1\,\text{m}\times1\,\text{m} = 1\,\text{m}^2$$

Then, the rate of heat transfer during boiling is

$$\dot{Q}_{\text{boiling}} = A_s\dot{q}$$

Thus, the range of the steam generation rate in the film boiling regime is

$$\frac{\dot{Q}_{\text{boiling,min}}}{h_{fg}} \le \dot{m}_{\text{vapor}} \le \frac{\dot{Q}_{\text{boiling,max}}}{h_{fg}}$$

$$\frac{A_s\dot{q}_{\min}}{h_{fg}} \le \dot{m}_{\text{vapor}} \le \frac{A_s\dot{q}_{\max}}{h_{fg}}$$

$$\frac{(1\,\text{m}^2)(19021\ \text{W/m}^2)}{2257\times10^3\ \text{J/kg}} \le \dot{m}_{\text{vapor}} \le \frac{(1\,\text{m}^2)(1.261\times10^6\ \text{W/m}^2)}{2257\times10^3\ \text{J/kg}}$$

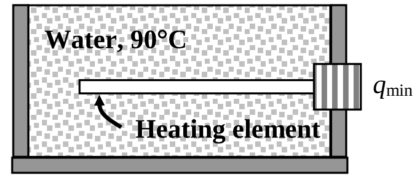
or $\quad \mathbf{0.00843\ kg/s \le \dot{m}_{vapor} \le 0.559\ kg/s}$

Discussion The maximum rate of steam generation is more than 66 times larger than the minimum rate of steam generation.

10-43 Water is boiled at T_{sat} = 90°C in a brass heating element. The surface temperature of the heater is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the heater and the boiler are negligible.

Properties The properties of water at the saturation temperature of 90°C are (Tables 10-1 and A-9)



$$\rho_l = 965.3\ \text{kg/m}^3 \qquad h_{fg} = 2283\times10^3\ \text{J/kg}$$
$$\rho_v = 0.4235\ \text{kg/m}^3 \qquad \mu_l = 0.315\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\sigma = 0.0608\ \text{N/m} \qquad c_{pl} = 4206\ \text{J/kg}\cdot{}^\circ\text{C}$$
$$\text{Pr}_l = 1.96$$

Also, $C_{sf} = 0.0060$ and n = 1.0 for the boiling of water on a brass heating (Table 10-3).

Analysis The minimum heat flux is determined from

$$\dot{q}_{\min} = 0.09\rho_v h_{fg}\left[\frac{\sigma g(\rho_l-\rho_v)}{(\rho_l+\rho_v)^2}\right]^{1/4}$$
$$= 0.09(0.4235)(2283\times10^3)\left[\frac{(0.0608)(9.81)(965.3-0.4235)}{(965.3+0.4235)^2}\right]^{1/4} = 13{,}715\ \text{W/m}^2$$

The surface temperature can be determined from Rohsenow equation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{\text{sat}})}{C_{sf}h_{fg}\,\text{Pr}_l^n}\right)^3$$
$$13{,}715\ \text{W/m}^2 = (0.315\times10^{-3})(2283\times10^3)\left[\frac{9.81(965.3\text{-}0.4235)}{0.0608}\right]^{1/2}\left(\frac{4206(T_s-90)}{0.0060(2283\times10^3)1.96}\right)^3$$
$$T_s = \mathbf{92.3^\circ C}$$

10-44 C&S Water is boiled at 1 atm by a heating element sheathed in an ASTM B165 tube. The tube is immersed in the water horizontally. The highest evaporation rate of water that can be achieved by the heater is to be determined.

Assumptions **1** Steady state conditions. **2** Heat losses from the boiler are negligible. **3** The water is boiled at 1 atm.

Properties The properties of water at the saturation temperature of 100°C are (Table A-9) h_{fg} = 2257 kJ/kg and ρ_l = 957.9 kg/m^3. The properties of vapor at the film temperature of T_f = (100 + 260)°C/2 = 180°C are (Table A-16)

c_{pv} = 1924.2 J/kg·K, k_v = 0.0314 W/m·K, ρ_v = 0.48596 kg/m^3, μ_v = 1.5724 × 10^{-5} kg/m·s

Analysis The excess temperature in this case is $\Delta T = T_s - T_{\text{sat}}$ = 260 – 100 = 160°C, which is larger than 30°C for water. Therefore, it is in the film boiling regime. The film boiling heat flux is

$$\dot{q}_{\text{film}} = C_{film}\left[\frac{g\,k_v^3\rho_v(\rho_l-\rho_v)\left[h_{fg}+0.4\,c_{pv}(T_s-T_{\text{sat}})\right]}{\mu_v\,D\,(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}})$$

where C_{film} = 0.62for horizontal cylinders

$$\dot{q}_{\text{film}} = 0.62\left[\frac{(9.81)(0.0314)^3(0.48596)(957.9-0.48596)[2257000+0.4(1924.2)(160)]}{(1.5724\times10^{-5})(0.005)(160)}\right]^{1/4}(160)$$

$$\dot{q}_{\text{film}} = 40113\ \text{W/m}^2$$

The highest heat transfer rate for the film boiling that can be achieved by the nickel-copper tube, without heating the tube surface above 260°C, is

$$\dot{Q}_{\text{boil}} = A\dot{q}_{\text{film}} = (\pi DL)\dot{q}_{\text{film}} = \pi(0.005\ \text{m})(0.15\ \text{m})(40113\ \text{W/m}^2) = 94.514\ \text{W}$$

Thus, the highest evaporation rate of water that can be achieved by the heater is

$$\dot{m}_{\text{evap}} = \frac{\dot{Q}_{\text{boil}}}{h_{fg}} = \frac{94.514\ \text{J/s}}{2257000\ \text{J/kg}} = 4.188\times10^{-5}\text{kg/s} = 0.04188\ \text{g/s}$$

Discussion The highest evaporation rate that can be achieved by the heater in the film boiling regime, with ASTM B165 nickel-copper alloy tube as the sheath, is about 0.042 g/s. The heat transfer rate to boil water at this evaporation rate is 94.5 W. If the heat transfer rate is higher than 94.5 W, the surface temperature of the ASTM B165 tube would heat above the maximum use temperature established by the ASME Code for Process Piping. Note that in this problem since T_s< 300°C radiation is negligible.

10-45 The power dissipation per unit length of a metal rod submerged horizontally in water, when electric current is passed through it, is to be determined.

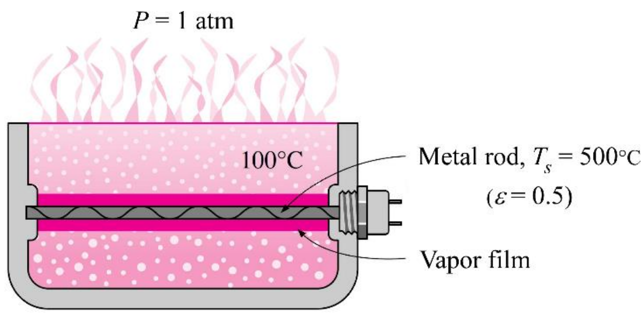
Assumptions **1** Steady operating condition exists. **2** Heat losses from the boiler are negligible.

Properties The properties of water at the saturation temperature of 100°C are h_{fg} = 2257 kJ/kg (Table A-2) and ρ_l = 957.9 kg/m^3 (Table A-9).

The properties of vapor at the film temperature of $T_f = (T_{sat} + T_s)/2$ = 300°C are, from Table A-16,

ρ_v = 0.3831 kg/m^3 $\quad$ c_{pv} = 1997 J/kg·K

$\mu_v = 2.045 \times 10^{-5}$ kg/m·s $\quad$ k_v = 0.04345 W/m·K



Analysis The excess temperature in this case is $\Delta T = T_s - T_{sat}$ = 400°C, which is much larger than 30°C for water from Fig. 10-6. Therefore, film boiling will occur. The film boiling heat flux in this case can be determined from

$$\dot{q}_{\text{film}} = C_{\text{film}}\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4c_{pv}(T_s-T_{\text{sat}})]}{\mu_v D(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}})$$

$$=0.62\left[\frac{9.81(0.04345)^3(0.3831)(957.9-0.3831)[2257\times10^3+0.4(1997)(400)]}{(2.045\times10^{-5})(0.002)(400)}\right]^{1/4}(400)$$

$$=1.152\times10^5\ \text{W/m}^2$$

The radiation heat flux is determined from

$$\dot{q}_{\text{rad}} = \varepsilon\sigma(T_s^4-T_{\text{sat}}^4) = (0.5)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(773^4-373^4)\,\text{K}^4 = 9573\,\text{W/m}^2$$

Then the total heat flux becomes

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}} = 1.152\times10^5\ \text{W/m}^2 + \frac{3}{4}(9573\,\text{W/m}^2) = 1.224\times10^5\ \text{W/m}^2$$

Finally, the power dissipation per unit length of the metal rod is

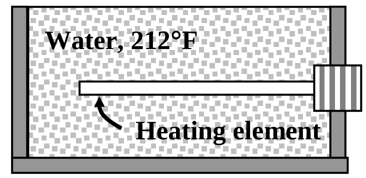
$$\dot{Q}_{\text{total}}/L = \pi D\dot{q}_{\text{total}} = \pi(0.002\,\text{m})(1.224\times10^5\ \text{W/m}^2) = \mathbf{769\ W/m}$$

Discussion The contribution of radiation to the total heat flux is about 8%.

10-46E Water is boiled at 1 atm pressure and thus at a saturation (or boiling) temperature of T_{sat} = 212°F by a horizontal polished copper heating element whose surface temperature is maintained at T_s = 788°F. The rate of heat transfer to the water per unit length of the heater is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the boiler are negligible.

Properties The properties of water at the saturation temperature of 212°F are $\rho_l = 59.82\ \text{lbm/ft}^3$ and $h_{fg} = 970\ \text{Btu/lbm}$ (Table A-9E). The properties of the vapor at the film temperature of $T_f = (T_{sat} + T_s)/2 = (212+788)/2 = 500°\text{F}$ are (Table A-16E)



$$\rho_v = 0.02571\ \text{lbm/ft}^3$$
$$\mu_v = 1.267\times10^{-5}\ \text{lbm/ft}\cdot\text{s} = 0.04561\ \text{lbm/ft}\cdot\text{h}$$
$$c_{pv} = 0.4707\ \text{Btu/lbm}\cdot°\text{F}$$
$$k_v = 0.02267\ \text{Btu/h}\cdot\text{ft}\cdot°\text{F}$$

Also, g = 32.2 ft/s^2 = 32.2×(3600)^2 ft/h^2. Note that we expressed the properties in units that will cancel each other in boiling heat transfer relations. Also note that we used vapor properties at 1 atm pressure from Table A-16E instead of the properties of saturated vapor from Table A-9E since the latter are at the saturation pressure of 680 psia (46 atm).

Analysis The excess temperature in this case is $\Delta T = T_s - T_{sat} = 788-212 = 576°\text{F}$, which is much larger than 30°C or 54°F. Therefore, film boiling will occur. The film boiling heat flux in this case can be determined to be

$$\dot{q}_{\text{film}} = 0.62\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4c_{pv}(T_s-T_{sat})]}{\mu_v D(T_s-T_{sat})}\right]^{1/4}(T_s-T_{sat})$$
$$= 0.62\left[\frac{32.2(3600)^2(0.02267)^3(0.02571)(59.82-0.02571)[970+0.4\times0.4707(788-212)]}{(0.04561)(0.5/12)(788-212)}\right]^{1/4}(788-212)$$
$$= 18{,}600\ \text{Btu/h}\cdot\text{ft}^2$$

The radiation heat flux is determined from

$$\dot{q}_{\text{rad}} = \varepsilon\sigma(T_s^4 - T_{sat}^4)$$
$$= (0.2)(0.1714\times10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)\left[(788+460\ \text{R})^4-(212+460\ \text{R})^4\right]$$
$$= 761.7\ \text{Btu/h}\cdot\text{ft}^2$$

Note that heat transfer by radiation is very small in this case because of the low emissivity of the surface and the relatively low surface temperature of the heating element. Then the total heat flux becomes

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}} = 18{,}600 + \frac{3}{4}\times761.7 = 19{,}171\ \text{Btu/h}\cdot\text{ft}^2$$

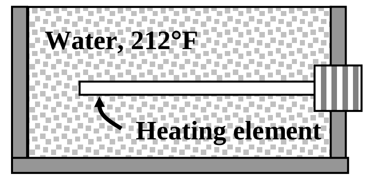
Finally, the rate of heat transfer from the heating element to the water is determined by multiplying the heat flux by the heat transfer surface area,

$$\dot{Q}_{\text{total}} = A_s\dot{q}_{\text{total}} = (\pi DL)\dot{q}_{\text{total}}$$
$$= (\pi\times0.5/12\ \text{ft}\times1\ \text{ft})(19{,}171\ \text{Btu/h}\cdot\text{ft}^2)$$
$$= \mathbf{2509\ Btu/h}$$

10-47E Water is boiled at 1 atm pressure and thus at a saturation (or boiling) temperature of T_{sat} = 212°F by a horizontal polished copper heating element whose surface temperature is maintained at T_s = 988°F. The rate of heat transfer to the water per unit length of the heater is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the boiler are negligible.

Properties The properties of water at the saturation temperature of 212°F are $\rho_l = 59.82\ \text{lbm/ft}^3$ and $h_{fg} = 970\ \text{Btu/lbm}$ (Table A-9E). The properties of the vapor at the film temperature of $T_f = (T_{sat} + T_s)/2 = (212+988)/2 = 600°\text{F}$ are, by interpolation, (Table A-16E)



$$\rho_v = 0.02395\ \text{lbm/ft}^3$$
$$\mu_v = 1.416\times10^{-5}\ \text{lbm/ft}\cdot\text{s} = 0.05099\ \text{lbm/ft}\cdot\text{h}$$
$$c_{pv} = 0.4799\ \text{Btu/lbm}\cdot°\text{F}$$
$$k_v = 0.02640\ \text{Btu/h}\cdot\text{ft}\cdot°\text{F}$$

Also, g = 32.2 ft/s^2 = 32.2×(3600)^2 ft/h^2. Note that we expressed the properties in units that will cancel each other in boiling heat transfer relations. Also note that we used vapor properties at 1 atm pressure from Table A-16E instead of the properties of saturated vapor from Table A-9E since the latter are at the saturation pressure of 1541 psia (105 atm).

Analysis The excess temperature in this case is $\Delta T = T_s - T_{sat} = 988 - 212 = 776°\text{F}$, which is much larger than 30°C or 54°F. Therefore, film boiling will occur. The film boiling heat flux in this case can be determined from

$$\begin{aligned}\dot{q}_{\text{film}} &= 0.62\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4C_{pv}(T_s-T_{sat})]}{\mu_v D(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}})\\ &= 0.62\left[\frac{32.2(3600)^2(0.0264)^3(0.02395)(59.82-0.02395)[970+0.4\times0.4799(988-212)]}{(0.05099)(0.5/12)(988-212)}\right]^{1/4}(988-212)\\ &= 25{,}147\ \text{Btu/h}\cdot\text{ft}^2\end{aligned}$$

The radiation heat flux is determined from

$$\begin{aligned}\dot{q}_{\text{rad}} &= \varepsilon\sigma(T_s^4 - T_{\text{sat}}^4)\\ &= (0.2)(0.1714\times10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)\left[(988+460\ \text{R})^4-(212+460\ \text{R})^4\right]\\ &= 1437\ \text{Btu/h}\cdot\text{ft}^2\end{aligned}$$

Note that heat transfer by radiation is very small in this case because of the low emissivity of the surface and the relatively low surface temperature of the heating element. Then the total heat flux becomes

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}} = 25{,}147 + \frac{3}{4}\times1437 = 26{,}225\ \text{Btu/h}\cdot\text{ft}^2$$

Finally, the rate of heat transfer from the heating element to the water is determined by multiplying the heat flux by the heat transfer surface area,

$$\begin{aligned}\dot{Q}_{\text{total}} &= A_s\dot{q}_{\text{total}} = (\pi DL)\dot{q}_{\text{total}}\\ &= (\pi\times0.5/12\ \text{ft}\times1\ \text{ft})(26{,}225\ \text{Btu/h}\cdot\text{ft}^2)\\ &= \mathbf{3433\ Btu/h}\end{aligned}$$

10-48 C&S Water is boiled at 1 atm by a horizontal ASTM B335 nickel alloy rod. The highest heat transfer rate that can be supplied from the rod to the water, without heating the rod surface above the maximum use temperature, is to be determined.

Assumptions **1** Steady state conditions. **2** Heat losses from the boiler are negligible. **3** The water is boiled at 1 atm.

Properties The properties of water at the saturation temperature of 100°C are (Table A-9) h_{fg} = 2257 kJ/kg and ρ_l = 957.9 kg/m^3. The properties of vapor at the film temperature of T_f = (100 + 427)°C/2 = 263.5°C are (Table A-16)

c_{pv} = 1974.4 J/kg·K, k_v = 0.03973 W/m·K, ρ_v = 0.41263 kg/m^3, μ_v = 1.9008 × 10^{-5} kg/m·s

Analysis The excess temperature in this case is $\Delta T = T_s - T_{\text{sat}}$ = 427 – 100 = 327°C, which is much larger than 30°C for water from Fig. 10-6. Therefore, it is in the film boiling regime. The film boiling heat flux is

$$\dot{q}_{\text{film}} = C_{film}\left[\frac{g\, k_v^3 \rho_v(\rho_l - \rho_v)\left[h_{fg} + 0.4\, c_{pv}(T_s - T_{\text{sat}})\right]}{\mu_v\, D\,(T_s - T_{\text{sat}})}\right]^{1/4}(T_s - T_{\text{sat}})$$

where C_{film} = 0.62for horizontal cylinders

$$\dot{q}_{\text{film}} = 0.62\left[\frac{(9.81)(0.03973)^3(0.41263)(957.9 - 0.41263)[2257000 + 0.4(1974.4)(327)]}{(1.9008\times 10^{-5})(0.005)(327)}\right]^{1/4}(327)$$

$$\dot{q}_{\text{film}} = 75928\ \text{W/m}^2$$

Since T_s > 300°C radiation becomes significant. The heat flux from radiation heat transfer is

$$\dot{q}_{\text{rad}} = \varepsilon\sigma(T_s^4 - T_{\text{sat}}^4) = (0.3)(5.67\times 10^{-8}\text{W/m}^2)[(427 + 273\text{K})^4 - (100 + 273\text{K})^4] = 3754.8\ \text{W/m}^2$$

The total heat flux is

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}} = 75928\ \text{W/m}^2 + \frac{3}{4}(3754.8\ \text{W/m}^2) = 78744\ \text{W/m}^2$$

The highest heat transfer rate that can be supplied from the nickel alloy rod to the water, without heating the rod surface above 427°C, is

$$\dot{Q} = A\dot{q}_{\text{total}} = (\pi DL)\dot{q}_{\text{total}} = \pi(0.005\ \text{m})(0.10\ \text{m})(78744\ \text{W/m}^2) = 123.7\ \text{W}$$

Discussion The highest heat transfer rate that can be supplied to the ASTM B335 rod to boil water in the film boiling regime is 123.7 W. If the heat transfer rate is higher than 123.7 W, the surface temperature of the ASTM B335 rod would heat above 427°C, the maximum use temperature established by the ASME Code for Process Piping.

10-49 The initial heat transfer rate from a hot metal sphere that is suddenly submerged in a water bath is to be determined.

Assumptions **1** Steady operating condition exists. **2** The metal sphere has uniform initial surface temperature.

Properties The properties of water at the saturation temperature of 100°C are h_{fg} = 2257 kJ/kg (Table A-2) and ρ_l = 957.9 kg/m^3 (Table A-9). The properties of vapor at the film temperature of $T_f = (T_{sat} + T_s)/2$ = 400°C are, from Table A-16,

$\rho_v = 0.3262$ kg/m^3 $c_{pv} = 2066$ J/kg·K

$\mu_v = 2.446 \times 10^{-5}$ kg/m·s $k_v = 0.05467$ W/m·K

Analysis The excess temperature in this case is $\Delta T = T_s - T_{sat}$ = 600°C, which is much larger than 30°C for water from Fig. 10-6. Therefore, film boiling will occur. The film boiling heat flux in this case can be determined from

$$\begin{aligned}
\dot{q}_{\text{film}} &= C_{\text{film}}\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4c_{pv}(T_s-T_{\text{sat}})]}{\mu_v D(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}}) \\
&= 0.67\left[\frac{9.81(0.05467)^3(0.3262)(957.9-0.3262)[2257\times10^3+0.4(2066)(600)]}{(2.446\times10^{-5})(0.02)(600)}\right]^{1/4}(600) \\
&= 1.052\times10^5\ \text{W/m}^2
\end{aligned}$$

The radiation heat flux is determined from

$$\begin{aligned}
\dot{q}_{\text{rad}} &= \varepsilon\sigma(T_s^4 - T_{\text{sat}}^4) \\
&= (0.75)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(973^4-373^4)\ \text{K}^4 \\
&= 3.729\times10^4\ \text{W/m}^2
\end{aligned}$$

Then the total heat flux becomes

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}} = 1.052\times10^5\ \text{W/m}^2 + \frac{3}{4}(3.729\times10^4\ \text{W/m}^2) = 1.332\times10^5\ \text{W/m}^2$$

Finally, the initial heat transfer rate from the submerged metal sphere is

$$\dot{Q}_{\text{total}} = \dot{q}_{\text{total}}\pi D^2 = (1.332\times10^5\ \text{W/m}^2)\pi(0.02\ \text{m})^2 = \mathbf{167\ W}$$

Discussion The contribution of radiation to the total heat flux is about 21%, which is significant and cannot be neglected.

10-50 The initial heat transfer rate from a hot steel rod that is suddenly submerged in a water bath is to be determined.

Assumptions **1** Steady operating condition exists. **2** The steel rod has uniform initial surface temperature.

Properties The properties of water at the saturation temperature of 100°C are h_{fg} = 2257 kJ/kg (Table A-2) and ρ_l = 957.9 kg/m^3 (Table A-9). The properties of vapor at the film temperature of $T_f = (T_{sat} + T_s)/2$ = 300°C are, from Table A-16,

ρ_v = 0.3831 kg/m^3 $\quad$ c_{pv} = 1997 J/kg·K

$\mu_v = 2.045 \times 10^{-5}$ kg/m·s $\quad$ k_v = 0.04345 W/m·K

Analysis The excess temperature in this case is $\Delta T = T_s - T_{sat}$ = 400°C, which is much larger than 30°C for water from Fig. 10-6. Therefore, film boiling will occur. The film boiling heat flux in this case can be determined from

$$\begin{aligned}\dot{q}_{\text{film}} &= C_{\text{film}}\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4c_{pv}(T_s-T_{\text{sat}})]}{\mu_v D(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}})\\ &= 0.62\left[\frac{9.81(0.04345)^3(0.3831)(957.9-0.3831)[2257\times10^3+0.4(1997)(400)]}{(2.045\times10^{-5})(0.02)(400)}\right]^{1/4}(400)\\ &= 6.476\times10^4\ \text{W/m}^2\end{aligned}$$

The radiation heat flux is determined from

$$\begin{aligned}\dot{q}_{\text{rad}} &= \varepsilon\sigma(T_s^4-T_{\text{sat}}^4)\\ &= (0.9)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(773^4-373^4)\ \text{K}^4\\ &= 1.723\times10^4\ \text{W/m}^2\end{aligned}$$

Then the total heat flux becomes

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}} = 6.476\times10^4\ \text{W/m}^2 + \frac{3}{4}(1.723\times10^4\ \text{W/m}^2) = 7.768\times10^4\ \text{W/m}^2$$

Finally, the initial heat transfer rate from the submerged steel rod is

$$\dot{Q}_{\text{total}} = \dot{q}_{\text{total}}\pi DL = (7.768\times10^4\ \text{W/m}^2)\pi(0.02\ \text{m})(0.2\ \text{m}) = \mathbf{976\ W}$$

Discussion The contribution of radiation to the total heat flux is about 17%, which is significant and cannot be neglected.

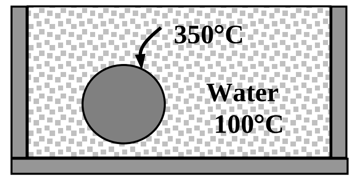
10-51 Water is boiled at T_{sat} = 100°C by a spherical platinum heating element immersed in water. The surface temperature is T_s = 350°C. The rate of heat transfer is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the heater and the boiler are negligible.

Properties The properties of water at the saturation temperature of 100°C are (Table A-9)

$$h_{fg} = 2257\times10^3 \text{ J/kg}$$
$$\rho_l = 957.9 \text{ kg/m}^3$$

The properties of water vapor at (350+100)/2 = 225°C are (Table A-16)



$$\rho_v = 0.444 \text{ kg/m}^3$$
$$\mu_v = 1.749\times10^{-5} \text{ kg/m}\cdot\text{s}$$
$$c_{pv} = 1951 \text{ J/kg}\cdot°\text{C}$$
$$k_v = 0.03581 \text{ W/m}\cdot°\text{C}$$

Analysis The film boiling occurs since the temperature difference between the surface and the fluid. The heat flux in this case can be determined from

$$\dot{q}_{\text{film}} = 0.67\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)\left[h_{fg}+0.4c_{pv}(T_s-T_{\text{sat}})\right]}{\mu_v D(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}})$$
$$= 0.67\left[\frac{(9.81)(0.03581)^3(0.444)(957.9-0.444)\left[2257\times10^3+0.4(1951)(350-100)\right]}{(1.749\times10^{-5})(0.15)(350-100)}\right]^{1/4}(350-100)$$
$$= 25{,}207 \text{ W/m}^2$$

The radiation heat transfer is

$$\dot{q}_{\text{rad}} = \varepsilon\sigma(T_s^4-T_{\text{sat}}^4) = (0.10)(5.67\times10^{-8})\left[(350+273)^4-(100+273)^4\right] = 745 \text{ W/m}^2$$

The total heat flux is

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}} = 25{,}207 + \frac{3}{4}(745) = 25{,}766 \text{ W/m}^2$$

Then the total rate of heat transfer becomes

$$\dot{Q}_{\text{total}} = A\dot{q}_{\text{total}} = \pi(0.15)^2(25{,}766 \text{ W/m}^2) = \mathbf{1821 W}$$

10-52 Cylindrical stainless steel rods are heated to 700°C and then suddenly quenched in water at 1 atm. The convection heat transfer coefficient and the total rate of heat removed from a rod at the instant it is submerged in the water are to be determined.

Assumptions **1** Steady operating conditions exist at the instant of submersion. **2** Surface temperature is uniform. **3** The boiling regime is film boiling since $\Delta T_{\text{excess}} = T_s - T_{\text{sat}}$ = 600°C, which is much larger than 30°C.

Properties At 1 atm, the saturation temperature of water is T_{sat} = 100°C. The properties of water at T_{sat} = 100°C are $h_{fg} = 2257 \times 10^3$ J/kg and $\rho_l = 957.9$ kg/m^3 (Table A-9). The properties of vapor at the film temperature of $T_f = (T_{\text{sat}} + T_s)/2 = (100 + 700)/2 = 400$°C are, from Table A-16,

$$\rho_v = 0.3262 \text{ kg/m}^3 \qquad c_{pv} = 2066 \text{ J/kg}\cdot\text{K}$$
$$\mu_v = 2.446\times10^{-5} \text{ kg/m}\cdot\text{s} \qquad k_v = 0.05467 \text{ W/m}\cdot\text{K}$$

Analysis The film boiling heat flux can be determined from

$$\dot{q}_{\text{film}} = C_{\text{film}}\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4c_{pv}(T_s-T_{\text{sat}})]}{\mu_v D(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}})$$

$$where\ C_{film} = 0.62\ (horizontal\ cylinders)$$

$$= 0.62\left[\frac{9.81(0.05467)^3(0.3262)(957.9-0.3262)[2257\times10^3+0.4(2066)(700-100)]}{(2.446\times10^{-5})(0.025)(700-100)}\right]^{1/4}(700-100)$$
$$= 92097 \text{ W/m}^2$$

Thus, the convection heat transfer coefficient is

$$\dot{q}_{\text{film}} = h(T_s - T_{\text{sat}}) \rightarrow h = \frac{92097 \text{ W/m}^2}{(700-100)\text{ K}} = \mathbf{153.5\ W/m^2\cdot K}$$

The radiation heat flux is determined from

$$\dot{q}_{\text{rad}} = \varepsilon\sigma(T_s^4 - T_{\text{sat}}^4)$$
$$= (0.3)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(973^4 - 373^4)\text{ K}^4$$
$$= 14917 \text{ W/m}^2$$

Then the total heat flux becomes

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}} = 92097 \text{ W/m}^2 + \frac{3}{4}(14917 \text{ W/m}^2) = 1.0328\times10^5 \text{ W/m}^2$$

The total rate of heat removed from a rod at the instant it is submerged in the water is

$$\dot{Q}_{\text{total}} = (\pi DL)\dot{q}_{\text{total}} = \pi(0.025\text{ m})(0.25\text{ m})(1.0328\times10^5 \text{ W/m}^2) = \mathbf{2028\ W}$$

Discussion Convection heat transfer coefficient in film boiling is generally lower than that of nucleate boiling, because the excess temperature of film boiling is much larger than that of nucleate boiling.

10-53 PtD A long cylindrical stainless steel rod with mechanically polished surface is being quenched in a water bath. The temperature of the rod leaving the water bath is to be determined whether or not it has the risk of thermal burn hazard.

Assumptions **1** Steady operating conditions exist. **2** Surface temperature is uniform. **3** The boiling regime is film boiling since $\Delta T_{\text{excess}} = T_s - T_{\text{sat}}$ = 700°C − 100°C = 600°C, which is much larger than 30°C.

Properties The specific heat and the density of stainless steel are given as $c_{p,ss}$ = 450 J/kg·K and ρ_{ss} = 7900 kg/m^3, respectively.

At 1 atm, the saturation temperature of water is T_{sat} = 100°C. The properties of water at T_{sat} = 100°C are h_{fg} = 2257 × 10^3 J/kg and ρ_l = 957.9 kg/m^3 (Table A-9). The properties of vapor at the film temperature of $T_f = (T_{\text{sat}} + T_s)/2$ = (100 + 700) = 400°C are, from Table A-16,

$$\rho_v = 0.3262 \text{ kg/m}^3 \qquad c_{pv} = 2066 \text{ J/kg}\cdot\text{K}$$
$$\mu_v = 2.446\times10^{-5} \text{ kg/m}\cdot\text{s} \qquad k_v = 0.05467 \text{ W/m}\cdot\text{K}$$

Analysis With ΔT_{excess} = 600°C, film boiling would occur in the water bath. The heat flux can be determined from

$$\dot{q}_{\text{film}} = C_{\text{film}}\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4c_{pv}(T_s-T_{\text{sat}})]}{\mu_v D(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}})$$

$$\textit{where } C_{film} = 0.62\,(\textit{horizontal cylinders})$$

$$= 0.62\left[\frac{9.81(0.05467)^3(0.3262)(957.9-0.3262)[2257\times10^3+0.4(2066)(700-100)]}{(2.446\times10^{-5})(0.025)(700-100)}\right]^{1/4}(700-100)$$
$$= 92097 \text{ W/m}^2$$

The radiation heat flux is determined from

$$\dot{q}_{\text{rad}} = \varepsilon\sigma(T_s^4 - T_{\text{sat}}^4) = (0.3)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(973^4 - 373^4)\text{ K}^4 = 14917 \text{ W/m}^2$$

Then the total heat flux becomes

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}} = 92097 \text{ W/m}^2 + \frac{3}{4}(14917 \text{ W/m}^2) = 1.0328\times10^5 \text{ W/m}^2$$

The rate of heat that could be removed from the rod in the water bath is

$$\dot{Q}_{\text{total}} = A_s\dot{q}_{\text{total}} = (\pi DL)\dot{q}_{\text{total}} = \pi(0.025\text{ m})(3\text{ m})(1.0328\times10^5 \text{ W/m}^2) = 24335 \text{ W}$$

The mass of the stainless steelrod being conveyed enters and exits the water bath at a rate of

$$\dot{m} = \rho_{ss}V(\pi D^2/4)$$

The rate of heat that needs to be removed from the rod so that it leaves the water bath below 45°C can be determined using

$$\dot{Q}_{\text{total}} = \dot{m}c_{p,ss}(T_{\text{in}} - T_{\text{out}}) = \rho_{ss}V(\pi D^2/4)c_{p,ss}(T_{\text{in}} - T_{\text{out}})$$

Thus, the speed of the rod conveying through the water bath is

$$V = \frac{\dot{Q}_{\text{total}}}{\rho_{ss}(\pi D^2/4)c_{p,ss}(T_{\text{in}} - T_{\text{out}})}$$
$$= \frac{24335 \text{ W}}{(7900 \text{ kg/m}^3)[\pi(0.025\text{ m})^2/4](450 \text{ J/kg}\cdot\text{K})(700-45)\text{K}} = 0.0213 \text{ m/s} = \mathbf{76.7\ m/hr}$$

Discussion To ensure that the stainless steel rod leaves the water bath below 45°C, in order to prevent thermal burn hazard, the speed of the rod conveying through the water bath should be about 77 m/hr or slower.

Note that this analysis is simplified to steady state conditions, but the actual quenching process is transient.

10-54 A cylindrical heater is used for boiling water at 1 atm. The film boiling convection heat transfer coefficient at the burnout point is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the heater are negligible. **3** The boiling regime is film boiling.

Properties At 1 atm, the saturation temperature of water is T_{sat} = 100°C. The properties of water at T_{sat} = 100°C are σ = 0.0589 N/m (Tables 10-1) and, from Table A-9, ρ_l = 957.9 kg/m^3, $\rho_{v,sat}$ = 0.5978 kg/m^3, h_{fg} = 2257 × 10^3 J/kg.

The properties of vapor at the film temperature of T_f = 1150°C are, from Table A-16,

$$\rho_v = 0.1543\ \text{kg/m}^3 \qquad c_{pv} = 2571\ \text{J/kg}\cdot\text{K}$$
$$\mu_v = 5.283\times10^{-5}\ \text{kg/m}\cdot\text{s} \qquad k_v = 0.1588\ \text{W/m}\cdot\text{K}$$

Analysis For a cylindrical heating element, the coefficient C_{cr} is determined from Table 10-4 to be

$$L^* = \frac{D}{2}\left(\frac{g(\rho_l-\rho_{v,\text{sat}})}{\sigma}\right)^{1/2} = (0.01/2)\left[\frac{9.81(957.9-0.5978)}{0.0589}\right]^{1/2} = 1.997 > 1.2$$
$$C_{cr} = 0.12 \quad (\text{since}\ L^* > 1.2\ \text{and thus large cylinder})$$

The burnout point occurs at the maximum heat flux, which is

$$\dot{q}_{\max} = C_{cr}h_{fg}[\sigma g\rho_{v,\text{sat}}^2(\rho_l-\rho_{v,\text{sat}})]^{1/4}$$
$$= 0.12(2257\times10^3)[0.0589\times9.81\times(0.5978)^2(957.9-0.5978)]^{1/4} = 1.0155\times10^6\ \text{W/m}^2$$

To determine the film boiling convection heat transfer coefficient, the knowledge of T_s is needed, which can be determined from the heat transfer in the film boiling region:

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}} = C_{\text{film}}\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4c_{pv}(T_s-T_{sat})]}{\mu_v D(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}}) + \frac{3}{4}\varepsilon\sigma(T_s^4-T_{\text{sat}}^4)$$
$$\textit{where}\ C_{film} = 0.62\ (\textit{horizontal cylinders})$$

Substituting the values,

$$1.0155\times10^6 = 0.62\left[\frac{9.81(0.1588)^3(0.1543)(957.9-0.1543)[2257\times10^3+0.4(2571)(\text{T}_s-100)]}{(5.283\times10^{-5})(0.01)(\text{T}_s-100)}\right]^{1/4}(\text{T}_s-100)$$
$$+\frac{3}{4}(0.3)(5.67\times10^{-8})[(\text{T}_s+273)^4-(373)^4]$$

Solving for the surface temperature yield T_s = 2231°C

The film boiling heat flux is

$$\dot{q}_{\text{film}} = C_{\text{film}}\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4c_{pv}(T_s-T_{sat})]}{\mu_v D(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}})$$
$$= 0.62\left[\frac{9.81(0.1588)^3(0.1543)(957.9-0.1543)[2257\times10^3+0.4(2571)(2231-100)]}{(5.283\times10^{-5})(0.01)(2231-100)}\right]^{1/4}(2231-100)$$
$$= 5.1419\times10^5\ \text{W/m}^2$$

Thus, the film boiling convection heat transfer coefficient is

$$h = \frac{\dot{q}_{\text{film}}}{T_s-T_{\text{sat}}} = \frac{5.1419\times10^5\ \text{W/m}^2}{(2231-100)\ \text{K}} = \mathbf{241.3\ W/m^2\cdot K}$$

Discussion Note that the film temperature T_f = (2231 + 100)/2 = 1166°C, is close to the assumed value of 1150°C for the evaluation of vapor properties. Therefore, 1150°C is a reasonable film temperature for the vapor properties.

10-55 A cylindrical heater is used for boiling water at 100°C. The boiling convection heat transfer coefficients at the maximum heat flux for nucleate boiling and film boiling are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the heater are negligible.

Properties The properties of water at T_{sat} = 100°C are σ = 0.0589 N/m (Tables 10-1) and, from Table A-9,

$$\rho_l = 957.9 \text{ kg/m}^3 \qquad h_{fg} = 2257\times 10^3 \text{ J/kg}$$
$$\rho_{v,\text{sat}} = 0.5978 \text{ kg/m}^3 \qquad \mu_l = 0.282\times 10^{-3} \text{ kg/m}\cdot\text{s}$$
$$\text{Pr}_l = 1.75 \qquad c_{pl} = 4217 \text{ J/kg}\cdot\text{K}$$

Also, C_{sf} = 0.0130 and n = 1 for the boiling of water on a mechanically polished stainless steel surface (Table 10-3). The properties of vapor at the film temperature of T_f = 1150°C are, from Table A-16,

$$\rho_v = 0.1543 \text{ kg/m}^3 \qquad c_{pv} = 2571 \text{ J/kg}\cdot\text{K}$$
$$\mu_v = 5.283\times 10^{-5} \text{ kg/m}\cdot\text{s} \qquad k_v = 0.1588 \text{ W/m}\cdot\text{K}$$

Analysis For a cylindrical heating element, the coefficient C_{cr} is determined from Table 10-4 to be

$$L^* = \frac{D}{2}\left(\frac{g(\rho_l - \rho_{v,\text{sat}})}{\sigma}\right)^{1/2} = (0.003/2)\left[\frac{9.81(957.9-0.5978)}{0.0589}\right]^{1/2} = 0.599 < 1.2$$
$$C_{cr} = 0.12L^{*-0.25} = 0.1364 \quad (\text{since } L^* < 1.2 \text{ and thus small cylinder})$$

The maximum heat flux can be determined as

$$\dot{q}_{\max} = C_{cr} h_{fg}[\sigma g \rho_{v,\text{sat}}^2(\rho_l - \rho_{v,\text{sat}})]^{1/4}$$
$$= 0.1364(2257\times 10^3)[0.0589\times 9.81\times(0.5978)^2(957.9-0.5978)]^{1/4} = 1.1543\times 10^6 \text{ W/m}^2$$

(a) The surface temperature T_s for nucleate boiling at $\dot{q}_{\max}$ can be solved as

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_{v,\text{sat}})}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s - T_{\text{sat}})}{C_{sf} h_{fg}\text{Pr}_l^n}\right)^3$$

Substituting the values,

$$1.1543\times 10^6 = (0.282\times 10^{-3})(2257\times 10^3)\left[\frac{9.81(957.9-0.5978)}{0.0589}\right]^{1/2}\left[\frac{4217(T_s - 100)}{0.0130(2257\times 10^3)1.75}\right]^3$$

$\therefore T_s$ = 120.2°C

Thus, the nucleate boiling convection heat transfer coefficient is

$$h_{\text{nucleate}} = \frac{\dot{q}_{\text{nucleate}}}{T_s - T_{\text{sat}}} = \frac{1.1543\times 10^6 \text{ W/m}^2}{(120.2-100)\text{ K}} = \mathbf{57{,}144 \text{ W/m}^2\cdot\text{K}}$$

(b) The surface temperature T_s for film boiling at $\dot{q}_{\max}$ can be solved as

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}} = C_{\text{film}}\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4c_{pv}(T_s - T_{sat})]}{\mu_v D(T_s - T_{\text{sat}})}\right]^{1/4}(T_s - T_{\text{sat}}) + \frac{3}{4}\varepsilon\sigma(T_s^4 - T_{\text{sat}}^4)$$

Substituting the values,

$$1.1543\times 10^6 = 0.62\left[\frac{9.81(0.1588)^3(0.1543)(957.9-0.1543)[2257\times 10^3 + 0.4(2571)(T_s - 100)]}{(5.283\times 10^{-5})(0.003)(T_s - 100)}\right]^{1/4}$$
$$\times(T_s - 100) + \frac{3}{4}(0.3)(5.67\times 10^{-8})[(T_s + 273)^4 - (373)^4]$$

$\therefore T_s = 2192°C$

The film boiling heat flux is

$$\dot{q}_{film} = C_{film}\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4c_{pv}(T_s-T_{sat})]}{\mu_v D(T_s-T_{sat})}\right]^{1/4}(T_s-T_{sat})$$

$$\text{where} C_{film} = 0.62 \text{ (horizontal cylinders)}$$

$$= 0.62\left[\frac{9.81(0.1588)^3(0.1543)(957.9-0.1543)[2257\times10^3+0.4(2571)(2192-100)]}{(5.283\times10^{-5})(0.003)(2192-100)}\right]^{1/4}(2192-100)$$

$$= 6.8366\times10^5 \text{ W/m}^2$$

Thus, the film boiling convection heat transfer coefficient is

$$h = \frac{\dot{q}_{film}}{T_s - T_{sat}} = \frac{6.8366\times10^5 \text{ W/m}^2}{(2192-100)\text{ K}} = \mathbf{327\ W/m^2\cdot K}$$

Discussion The nucleate boiling convection heat transfer coefficient is about 175 times higher than that of film boiling. This is because the vapor film surrounding the heater surface during film boiling impedes convection heat transfer.

Note that the film temperature T_f = (2192 + 100)/2 = 1146°C, is close to the assumed value of 1150°C used in film boiling for the evaluation of vapor properties.

10-56 Water is boiled at 1 atm pressure and thus at a saturation (or boiling) temperature of T_{sat} = 100°C by a horizontal nickel plated copper heating element. The maximum (critical) heat flux and the temperature jump of the wire when the operating point jumps from nucleate boiling to film boiling regime are to be determined.

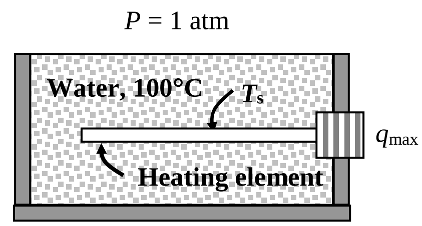
Assumptions **1** Steady operating conditions exist. **2** Heat losses from the boiler are negligible.

Properties The properties of water at the saturation temperature of 100°C are (Tables 10-1 and A-9)

$$\rho_l = 957.9 \text{ kg/m}^3 \qquad h_{fg} = 2257\times10^3 \text{ J/kg}$$
$$\rho_v = 0.5978 \text{ kg/m}^3 \qquad \mu_l = 0.282\times10^{-3} \text{ kg}\cdot\text{m/s}$$
$$\sigma = 0.0589 \text{ N/m} \qquad c_{pl} = 4217 \text{ J/kg}\cdot°\text{C}$$
$$\text{Pr}_l = 1.75$$

Also, $C_{sf} = 0.0060$ and n = 1.0 for the boiling of water on a nickel plated surface (Table 10-3). Note that we expressed the properties in units specified under Eqs. 10-2 and 10-3 in connection with their definitions in order to avoid unit manipulations. The vapor properties at the anticipated film temperature of $T_f = (T_s+T_{sat})/2$ of 1000°C (will be checked) (Table A-16)

$$\rho_v = 0.1725 \text{ kg/m}^3 \qquad c_{pv} = 2471 \text{ J/kg}\cdot°\text{C}$$
$$k_v = 0.1362 \text{ W/m}\cdot°\text{C} \qquad \mu_v = 4.762\times10^{-5} \text{ kg/m}\cdot\text{s}$$



Analysis (a) For a horizontal heating element, the coefficient C_{cr} is determined from Table 10-4 to be

$$L^* = L\left(\frac{g(\rho_l-\rho_v)}{\sigma}\right)^{1/2} = (0.002)\left(\frac{9.81(957.9-0.5978)}{0.0589}\right)^{1/2} = 0.7986 < 1.2$$

$$C_{cr} = 0.12L^{*-0.25} = 0.12(0.7986)^{-0.25} = 0.1269$$

Then the maximum or critical heat flux is determined from

$$\dot{q}_{\max} = C_{cr}h_{fg}[\sigma g\rho_v^2(\rho_l-\rho_v)]^{1/4}$$
$$= 0.1269(2257\times10^3)[0.0589\times9.81\times(0.5978)^2(957.9-0.5978)]^{1/4} = \mathbf{1{,}074{,}000\ W/m^2}$$

The Rohsenow relation which gives the nucleate boiling heat flux for a specified surface temperature can also be used to determine the surface temperature when the heat flux is given. Substituting the maximum heat flux into the Rohsenow relation together with other properties gives

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{\text{sat}})}{C_{sf}h_{fg}\text{Pr}_l^n}\right)^3$$

$$1{,}074{,}000 = (0.282\times10^{-3})(2257\times10^3)\left[\frac{9.81(957.9\text{-}0.5978)}{0.0589}\right]^{1/2}\left(\frac{4217(T_s-100)}{0.0060(2257\times10^3)1.75}\right)^3$$

It gives $T_s = 109.1°\text{C}$

(b) Heat transfer in the film boiling region can be expressed as

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}} = 0.62\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4c_{pv}(T_s-T_{sat})]}{\mu_v D(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}}) + \frac{3}{4}\varepsilon\sigma(T_s^4-T_{\text{sat}}^4)$$

Substituting,

$$1{,}074{,}000 = 0.62\left[\frac{9.81(0.1362)^3(0.1725)(957.9-0.1725)[2257\times10^3+0.4\times2471(T_s-100)]}{(4.762\times10^{-5})(0.004)(T_s-100)}\right]^{1/4}$$
$$\times(T_s-100) + \frac{3}{4}(0.3)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(T_s+273)^4-(100+273)^4\right]$$

Solving for the surface temperature gives T_s = 2200°C. Therefore, the temperature jump of the wire when the operating point jumps from nucleate boiling to film boiling is

Temperature jump: $\Delta T = T_{s,\text{film}} - T_{s,\text{crit}} = 2200 - 109 = \mathbf{2091°C}$

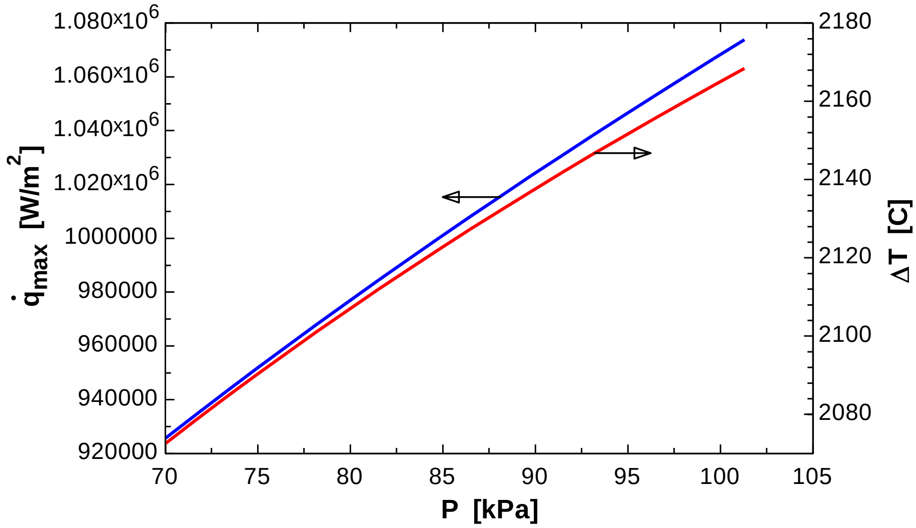
Note that the film temperature is (2200+100)/2=1150°C, which is close enough to the assumed value of 1000°C for the evaluation of vapor properties.

10-57 Prob. 10-56 is reconsidered. The effects of the local atmospheric pressure and the emissivity of the wire on the critical heat flux and the temperature rise of wire are to be investigated.

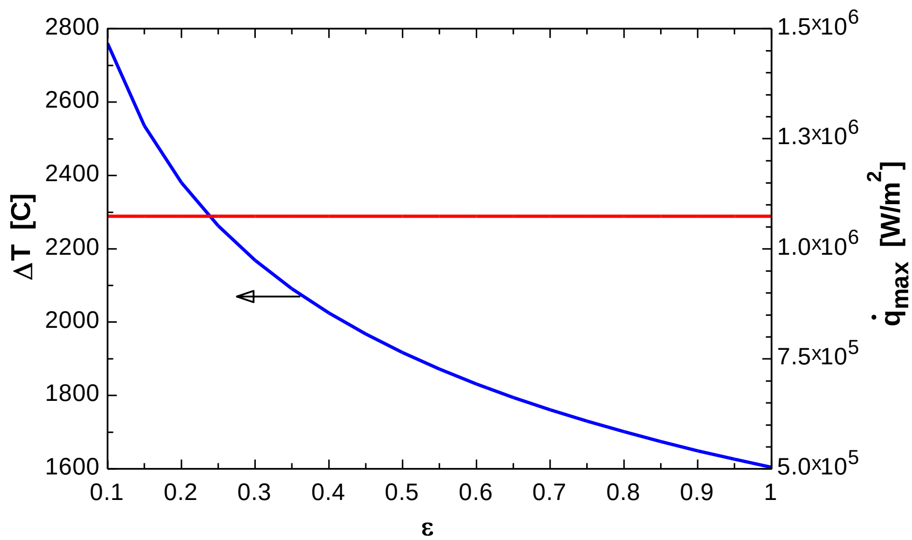
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.3 [m]
D=0.004 [m]
epsilon=0.3
P=101.3 [kPa]

"PROPERTIES"
Fluid$='steam_IAPWS'
T_sat=temperature(Fluid$, P=P, x=1)
rho_l=density(Fluid$, T=T_sat, x=0)
rho_v=density(Fluid$, T=T_sat, x=1)
sigma=SurfaceTension(Fluid$, T=T_sat)
mu_l=Viscosity(Fluid$,T=T_sat, x=0)
Pr_l=Prandtl(Fluid$, T=T_sat, P=P+1)
c_l=CP(Fluid$, T=T_sat, x=0)*Convert(kJ/kg-C, J/kg-C)
h_f=enthalpy(Fluid$, T=T_sat, x=0)
h_g=enthalpy(Fluid$, T=T_sat, x=1)
h_fg=(h_g-h_f)*Convert(kJ/kg, J/kg)
C_sf=0.0060 "from Table 10-3 of the text"
n=1 "from Table 10-3 of the text"

T_vapor=1000-273 "[C], assumed vapor temperature in the film boiling region"
rho_v_f=density(Fluid$, T=T_vapor, P=P) "f stands for film"
c_v_f=CP(Fluid$, T=T_vapor, P=P)*Convert(kJ/kg-C, J/kg-C)
k_v_f=Conductivity(Fluid$, T=T_vapor, P=P)
mu_v_f=Viscosity(Fluid$,T=T_vapor, P=P)

g=9.81 [m/s^2] "gravitational acceleraton"
sigma_rad=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"

"ANALYSIS"
"(a)"
"C_cr is to be determined from Table 10-4 of the text"
C_cr=0.12*L_star^(-0.25)
L_star=D/2*((g*(rho_l-rho_v))/sigma)^0.5
q_dot_max=C_cr*h_fg*(sigma*g*rho_v^2*(rho_l-rho_v))^0.25
q_dot_nucleate=q_dot_max
q_dot_nucleate=mu_l*h_fg*(((g*(rho_l-rho_v))/sigma)^0.5)*((c_l*(T_s_crit-T_sat))/(C_sf*h_fg*Pr_l^n))^3
"(b)"
q_dot_total=q_dot_film+3/4*q_dot_rad "Heat transfer in the film boiling region"
q_dot_total=q_dot_nucleate
q_dot_film=0.62*((g*k_v_f^3*rho_v_f*(rho_l-rho_v_f)*(h_fg+0.4*c_v_f*(T_s_film-
T_sat)))/(mu_v_f*D*(T_s_film-T_sat)))^0.25*(T_s_film-T_sat)
q_dot_rad=epsilon*sigma_rad*((T_s_film+273)^4-(T_sat+273)^4)
DELTAT=T_s_film-T_s_crit
```

P [kPa]	$\dot{q}_{max}$ [kW/m^2]	ΔT [C]
70	925656	2073
71.65	934417	2079
73.29	943050	2084
74.94	951559	2090
76.59	959948	2096
78.24	968222	2101
79.88	976385	2107
81.53	984439	2112
83.18	992389	2117
84.83	1000237	2122
86.47	1007987	2127
88.12	1015642	2132
89.77	1023205	2137
91.42	1030677	2142
93.06	1038062	2146
94.71	1045363	2151
96.36	1052581	2155
98.01	1059719	2160
99.65	1066778	2164
101.3	1073762	2168



ε	$\dot{q}_{max}$ [kW/m^2]	ΔT [C]
0.1	1073762	2760
0.15	1073762	2535
0.2	1073762	2380
0.25	1073762	2262
0.3	1073762	2168
0.35	1073762	2091
0.4	1073762	2025
0.45	1073762	1967
0.5	1073762	1917
0.55	1073762	1872
0.6	1073762	1831
0.65	1073762	1794
0.7	1073762	1761
0.75	1073762	1730
0.8	1073762	1701
0.85	1073762	1674
0.9	1073762	1649
0.95	1073762	1626
1	1073762	1604



Condensation Heat Transfer

10-58C Condensation is a vapor-to-liquid phase change process. It occurs when the temperature of a vapor is reduced *below* its saturation temperature T_{sat}. This is usually done by bringing the vapor into contact with a solid surface whose temperature T_s is *below* the saturation temperature T_{sat} of the vapor.

10-59C In *film condensation*, the condensate wets the surface and forms a liquid film on the surface which slides down under the influence of gravity. The thickness of the liquid film increases in the flow direction as more vapor condenses on the film. This is how condensation normally occurs in practice. In *dropwise condensation,* the condensed vapor forms droplets on the surface instead of a continuous film, and the surface is covered by countless droplets of varying diameters. Dropwise condensation is a much more effective mechanism of heat transfer.

10-60C The presence of noncondensable gases in the vapor has a detrimental effect on condensation heat transfer. Even small amounts of a noncondensable gas in the vapor cause significant drops in heat transfer coefficient during condensation.

10-61C The modified latent heat of vaporization h^*_{fg} is the amount of heat released as a unit mass of vapor condenses at a specified temperature, plus the amount of heat released as the condensate is cooled further to some average temperature between T_{sat} and T_s. It is defined as $h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s)$ where c_{pl} is the specific heat of the liquid at the average film temperature.

10-62C During film condensation on a vertical plate, heat flux at the top will be higher since the thickness of the film at the top, and thus its thermal resistance, is lower.

10-63C The condensation heat transfer coefficient for the tubes will be the highest for the case of horizontal side by side (case b) since (1) for long tubes, the horizontal position gives the highest heat transfer coefficients, and (2) for tubes in a vertical tier, the average thickness of the liquid film at the lower tubes is much larger as a result of condensate falling on top of them from the tubes directly above, and thus the average heat transfer coefficient at the lower tubes in such arrangements is smaller.

10-64C In condensate flow, the wetted perimeter is defined as the length of the surface-condensate interface at a cross-section of condensate flow. It differs from the ordinary perimeter in that the latter refers to the entire circumference of the condensate at some cross-section.

10-65 The hydraulic diameter D_h for all 4 cases are expressed in terms of the boundary layer thickness δ as follows:

(*a*) Vertical plate: $$D_h = \frac{4A_c}{p} = \frac{4w\delta}{w} = 4\delta$$

(*b*) Tilted plate: $$D_h = \frac{4A_c}{p} = \frac{4w\delta}{w} = 4\delta$$

(*c*)Vertical cylinder: $$D_h = \frac{4A_c}{p} = \frac{4\pi D\delta}{\pi D} = 4\delta$$

(*d*) Horizontal cylinder: $$D_h = \frac{4A_c}{p} = \frac{4(2L\delta)}{2L} = 4\delta$$

(*e*) Sphere: $$D_h = \frac{4A_c}{p} = \frac{4\pi D\delta}{\pi D} = 4\delta$$

Therefore, the Reynolds number for all 5 cases can be expressed as

$$\mathrm{Re} = \frac{4\dot{m}}{p\mu_l} = \frac{4A_c \rho_l V_l}{p\mu_l} = \frac{D_h \rho_l V_l}{\mu_l} = \frac{4\delta\rho_l V_l}{\mu_l}$$

10-66 The local heat transfer coefficients at the middle and at the bottom of a vertical plate undergoing film condensation are to be determined.

Assumptions **1** Steady operating condition exists. **2** The plate surface has uniform temperature. **3** The flow is laminar.

Properties The properties of water at the saturation temperature of 100°C are h_{fg} = 2257 kJ/kg (Table A-2) and ρ_v = 0.5978 kg/m^3 (Table A-9). The properties of liquid water at the film temperature of $T_f = (T_{\rm sat} + T_s)/2$ = 90°C are, from Table A-9,

ρ_l = 965.3 kg/m^3 $\quad$ c_{pl} = 4206 J/kg·K

$\mu_l = 0.315 \times 10^{-3}$ kg/m·s $\quad$ k_l = 0.675 W/m·K

$\nu_l = \mu_l / \rho_l = 0.326 \times 10^{-6}$ m^2/s

Analysis The modified latent heat of vaporization is

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68c_{pl}(T_{\rm sat} - T_s) \\ &= 2257\times10^3 + 0.68(4206)(100-80) \\ &= 2314\times10^3 \text{ J/kg} \end{aligned}$$

The local heat transfer coefficient can be calculated using

$$\begin{aligned} h_x &= \left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}k_l^3}{4\mu_l(T_{\rm sat} - T_s)x}\right]^{1/4} \\ &= \left[\frac{(9.81)(965.3)(965.3-0.5978)(2314\times10^3)(0.675)^3}{4(0.315\times10^{-3})(100-80)x}\right]^{1/4} \\ &= 4008\left(\frac{1}{x}\right)^{1/4} \text{ W/m}^2\cdot\text{K} \end{aligned}$$

The local heat transfer coefficient at the middle of the plate (x = 0.1 m) is

$$h_x = 4008\left(\frac{1}{x}\right)^{1/4} \text{ W/m}^2\cdot\text{K} = 4008\left(\frac{1}{0.1}\right)^{1/4} \text{ W/m}^2\cdot\text{K} = \mathbf{7130\ W/m^2\cdot K}$$

The local heat transfer coefficient at the bottom of the plate (x = 0.2 m) is

$$h_x = 4008\left(\frac{1}{x}\right)^{1/4} \text{ W/m}^2\cdot\text{K} = 4008\left(\frac{1}{0.2}\right)^{1/4} \text{ W/m}^2\cdot\text{K} = \mathbf{5990\ W/m^2\cdot K}$$

Discussion The assumption that the flow is laminar is verified to be appropriate:

$$\text{Re} \cong \frac{4g\rho_l^2}{3\mu_l^2}\left(\frac{k_l}{h_{x=L}}\right)^3 = \frac{4(9.81)(965.3)^2}{3(0.315\times10^{-3})^2}\left(\frac{0.675}{5990}\right)^3 = 176 < 1800$$

10-67 The necessary surface temperature of the plate used to condensate saturated water vapor at a desired condensation rate is to be determined.

Assumptions **1** Steady operating condition exists. **2** The plate surface has uniform temperature. **3** The film temperature is 90°C.

Properties Based on the problem statement, we take film temperature to be $T_f = (T_{sat} + T_s)/2 = 90°C$ and the surface temperature to be $T_s = 80°C$. The properties of liquid water at the film temperature of $T_f = 90°C$ are, from Table A-9,

$\rho_l = 965.3$ kg/m^3 $\qquad$ $c_{pl} = 4206$ J/kg·K

$\mu_l = 0.315 \times 10^{-3}$ kg/m·s $\qquad$ $k_l = 0.675$ W/m·K

$\nu_l = \mu_l / \rho_l = 0.326 \times 10^{-6}$ m^2/s

The properties of water at the saturation temperature of T_{sat}= 100°C are h_{fg} = 2257 kJ/kg (Table A-2) and ρ_v = 0.5978 kg/m^3 (Table A-9).

Analysis The calculation of the modified latent heat of vaporization requires the knowledge of the T_s. Hence, we assume T_s = 80°C, and iterate the solution, if necessary, until good agreement with the calculated value of T_s is achieved:

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68 c_{pl}(T_{sat} - T_s) \\ &= 2257 \times 10^3 + 0.68(4206)(100 - 80) \\ &= 2314 \times 10^3 \text{ J/kg} \end{aligned}$$

The Reynolds number is

$$\text{Re} = \frac{4\dot{m}}{p\mu_l} = \frac{4(0.016 \text{ kg/s})}{(0.5\text{ m})(0.315 \times 10^{-3} \text{ kg/m}\cdot\text{s})} = 406.3$$

which is between 30 and 1800, and thus the flow is wavy-laminar. The heat transfer coefficient is

$$\begin{aligned} h = h_{\text{vert, wavy}} &= \frac{\text{Re}\, k_l}{1.08\,\text{Re}^{1.22} - 5.2}\left(\frac{g}{\nu_l^2}\right)^{1/3} \\ &= \frac{(406.3)(0.675 \text{ W/m}\cdot\text{K})}{1.08(406.3)^{1.22} - 5.2}\left[\frac{9.81 \text{ m/s}^2}{(0.326 \times 10^{-6} \text{ m}^2\text{/s})^2}\right]^{1/3} \\ &= 7558 \text{ W/m}^2\cdot\text{K} \end{aligned}$$

Hence, the surface temperature can be calculated using

$$hA_s(T_{sat} - T_s) = \dot{m}h^*_{fg} \quad \rightarrow \quad T_s = T_{sat} - \frac{\dot{m}h^*_{fg}}{hA_s}$$

$$T_s = 100°\text{C} - \frac{(0.016 \text{ kg/s})(2314 \times 10^3 \text{ J/kg})}{(7558 \text{ W/m}^2\cdot\text{K})(0.5\text{ m})^2} = \mathbf{80.4°C}$$

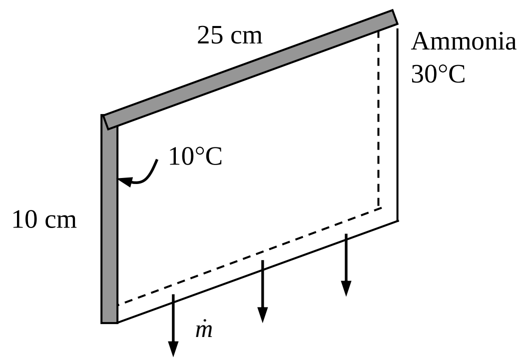
Discussion The assumed T_s = 80°C and T_f = 90°C are good, thus the solution does not require iteration.

10-68 Saturated ammonia at a saturation temperature of T_{sat} = 30°C condenses on vertical plates which are maintained at 10°C. The average heat transfer coefficient and the rate of condensation of ammonia are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The plate is isothermal. **3** The condensate flow is wavy-laminar over the entire plate (this assumption will be verified). **4** The density of vapor is much smaller than the density of liquid, $\rho_v << \rho_l$.

Properties The properties of ammonia at the saturation temperature of 30°C are h_{fg} = 1144×10^3 J/kg and ρ_v = 9.055 kg/m^3. The properties of liquid ammonia at the film temperature of $T_f = (T_{sat} + T_s)/2$ = (30 + 10)/2 = 20°C are (Table A-11),

$$\rho_l = 610.2 \text{ kg/m}^3$$
$$\mu_l = 1.519\times10^{-4} \text{ kg/m}\cdot\text{s}$$
$$\nu_l = \mu_l / \rho_l = 2.489\times10^{-7} \text{ m}^2\text{/s}$$
$$c_{pl} = 4745 \text{ J/kg}\cdot°\text{C}$$
$$k_l = 0.4927 \text{ W/m}\cdot°\text{C}$$



Analysis The modified latent heat of vaporization is

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68 c_{pl}(T_{sat} - T_s) \\ &= 1144\times10^3 \text{ J/kg} + 0.68\times 4745 \text{ J/kg}\cdot°\text{C}(30-10)°\text{C} \\ &= 1209\times10^3 \text{ J/kg} \end{aligned}$$

Assuming wavy-laminar flow, the Reynolds number is determined from

$$\text{Re} = \text{Re}_{\text{vertical,wavy}} = \left[4.81 + \frac{3.70 L k_l (T_{sat} - T_s)}{\mu_l h^*_{fg}}\left(\frac{g}{\nu_l^2}\right)^{1/3}\right]^{0.820}$$

$$= \left[4.81 + \frac{3.70\times(0.1\text{ m})\times(0.4927 \text{ W/m}\cdot°\text{C})\times(30-10)°\text{C}}{(1.519\times10^{-4} \text{ kg/m}\cdot\text{s})(1209\times10^3 \text{ J/kg})}\left(\frac{9.8 \text{ m/s}^2}{(2.489\times10^{-7} \text{ m}^2/\text{s})^2}\right)^{1/3}\right]^{0.82} = 307.0$$

which is between 30 and 1800, and thus our assumption of wavy laminar flow is verified. Then the condensation heat transfer coefficient is determined to be

$$h = h_{\text{vertical,wavy}} = \frac{\text{Re}\, k_l}{1.08\,\text{Re}^{1.22} - 5.2}\left(\frac{g}{\nu_l^2}\right)^{1/3}$$

$$= \frac{307\times(0.4927 \text{ W/m}\cdot°\text{C})}{1.08(307)^{1.22} - 5.2}\left(\frac{9.8 \text{ m/s}^2}{(2.489\times10^{-7} \text{ m}^2/\text{s})^2}\right)^{1/3} = \mathbf{7032\ W/m^2\cdot°C}$$

The total heat transfer surface area of the plates is

$$A_s = W\times L = 30(0.25\text{ m})(0.10\text{ m}) = 0.75 \text{ m}^2$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat} - T_s) = (7032 \text{ W/m}^2\cdot°\text{C})(0.75 \text{ m}^2)(30-10)°\text{C} = 105{,}480 \text{ W}$$

(*b*) The rate of condensation of steam is determined from

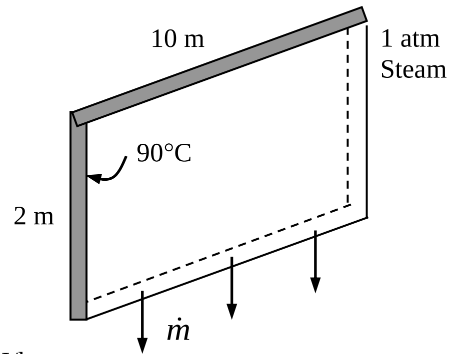
$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{105{,}480 \text{ J/s}}{1209\times10^3 \text{ J/kg}} = \mathbf{0.0872\ kg/s}$$

10-69 Saturated steam at atmospheric pressure thus at a saturation temperature of T_{sat} = 100°C condenses on a vertical plate which is maintained at 90°C by circulating cooling water through the other side. The rate of heat transfer to the plate and the rate of condensation of steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The plate is isothermal. **3** The condensate flow is wavy-laminar over the entire plate (this assumption will be verified). **4** The density of vapor is much smaller than the density of liquid, $\rho_v << \rho_l$.

Properties The properties of water at the saturation temperature of 100°C are $h_{fg} = 2257\times10^3$ J/kg and $\rho_v = 0.60$ kg/m^3. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2 = (100 + 90)/2 = 95$°C are (Table A-9),

$$\rho_l = 961.5 \text{ kg/m}^3$$
$$\mu_l = 0.297\times10^{-3} \text{ kg/m}\cdot\text{s}$$
$$\nu_l = \mu_l / \rho_l = 0.309\times10^{-6} \text{ m}^2\text{/s}$$
$$c_{pl} = 4212 \text{ J/kg}\cdot°\text{C}$$
$$k_l = 0.677 \text{ W/m}\cdot°\text{C}$$



Analysis The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68 c_{pl}(T_{sat} - T_s)$$
$$= 2257\times10^3 \text{ J/kg} + 0.68\times4212 \text{ J/kg}\cdot°\text{C}(100-90)°\text{C} = 2{,}286\times10^3 \text{ J/kg}$$

Assuming wavy-laminar flow, the Reynolds number is determined from

$$\text{Re} = \text{Re}_{\text{vertical,wavy}} = \left[4.81 + \frac{3.70 L k_l (T_{sat} - T_s)}{\mu_l h^*_{fg}}\left(\frac{g}{\nu_l^2}\right)^{1/3}\right]^{0.820}$$

$$= \left[4.81 + \frac{3.70\times(2 \text{ m})\times(0.677 \text{ W/m}\cdot°\text{C})\times(100-90)°\text{C}}{(0.297\times10^{-3} \text{ kg/m}\cdot\text{s})(2286\times10^3 \text{ J/kg})}\left(\frac{9.81 \text{ m/s}^2}{(0.309\times10^{-6} \text{ m}^2\text{/s})^2}\right)^{1/3}\right]^{0.82} = 798.0$$

which is between 30 and 1800, and thus our assumption of wavy laminar flow is verified. Then the condensation heat transfer coefficient is determined to be

$$h = h_{\text{vertical,wavy}} = \frac{\text{Re}\, k_l}{1.08\,\text{Re}^{1.22} - 5.2}\left(\frac{g}{\nu_l^2}\right)^{1/3}$$

$$= \frac{798\times(0.677 \text{ W/m}\cdot°\text{C})}{1.08(798)^{1.22} - 5.2}\left(\frac{9.81 \text{ m/s}^2}{(0.309\times10^{-6} \text{ m}^2\text{/s})^2}\right)^{1/3} = 6757 \text{ W/m}^2\cdot°\text{C}$$

The heat transfer surface area of the plate is

$$A_s = W\times L = (2 \text{ m})(10 \text{ m}) = 20 \text{ m}^2$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat} - T_s) = (6757 \text{ W/m}^2\cdot°\text{C})(20 \text{ m}^2)(100-90)°\text{C} = 1{,}351{,}500 \text{ W} = \mathbf{1352\ kW}$$

(*b*) The rate of condensation of steam is determined from

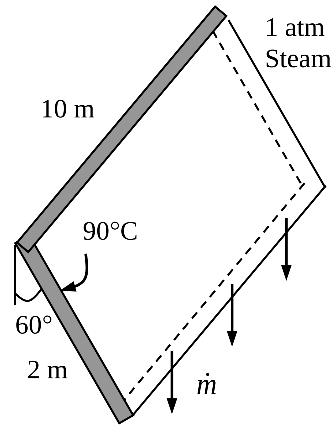
$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{1{,}351{,}500 \text{ J/s}}{2286\times10^3 \text{ J/kg}} = \mathbf{0.591\ kg/s}$$

10-70 Saturated steam at a saturation temperature of T_{sat} = 100°C condenses on a plate which is tilted 60° from the vertical and maintained at 90°C by circulating cooling water through the other side. The rate of heat transfer to the plate and the rate of condensation of the steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The plate is isothermal. **3** The condensate flow is wavy-laminar over the entire plate (this assumption will be verified). **4** The density of vapor is much smaller than the density of liquid, $\rho_v << \rho_l$.

Properties The properties of water at the saturation temperature of 100°C are $h_{fg} = 2257\times10^3$ J/kg and $\rho_v = 0.60$ kg/m^3. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2 = (100 + 90)/2 = 95$°C are (Table A-9),

$$\rho_l = 961.5 \text{ kg/m}^3$$
$$\mu_l = 0.297\times10^{-3} \text{ kg/m}\cdot\text{s}$$
$$\nu_l = \mu_l / \rho_l = 0.309\times10^{-6} \text{ m}^2\text{/s}$$
$$c_{pl} = 4212 \text{ J/kg}\cdot°\text{C}$$
$$k_l = 0.677 \text{ W/m}\cdot°\text{C}$$



Analysis The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s)$$
$$= 2257\times10^3 \text{ J/kg} + 0.68\times4212 \text{ J/kg}\cdot°\text{C}(100-90)°\text{C}$$
$$= 2{,}286\times10^3 \text{ J/kg}$$

Assuming wavy-laminar flow, the Reynolds number is determined from the vertical plate relation by replacing g by $g\cos\theta$ where θ = 60° to be

$$\text{Re} = \text{Re}_{\text{tilted,wavy}} = \left[4.81 + \frac{3.70Lk_l(T_{sat}-T_s)}{\mu_l h^*_{fg}}\left(\frac{g\cos 60}{\nu_l^2}\right)^{1/3}\right]^{0.820}$$

$$= \left[4.81 + \frac{3.70\times(2\text{ m})\times(0.677\text{ W/m}\cdot°\text{C})\times(100-90)°\text{C}}{(0.297\times10^{-3}\text{ kg/m}\cdot\text{s})(2286\times10^3\text{ J/kg})}\left(\frac{(9.81\text{ m/s}^2)\cos 60}{(0.309\times10^{-6}\text{ m}^2\text{/s})^2}\right)^{1/3}\right]^{0.82} = 660.5$$

which is between 30 and 1800, and thus our assumption of wavy laminar flow is verified. Then the condensation heat transfer coefficient is determined from

$$h = h_{\text{tilted,wavy}} = \frac{\text{Re}k_l}{1.08\text{Re}^{1.22} - 5.2}\left(\frac{g\cos\theta}{\nu_l^2}\right)^{1/3}$$

$$= \frac{660.5\times(0.677\text{ W/m}\cdot°\text{C})}{1.08(660.5)^{1.22} - 5.2}\left(\frac{(9.81\text{ m/s}^2)\cos 60}{(0.309\times10^{-6}\text{ m}^2\text{/s})^2}\right)^{1/3} = 5593\text{ W/m}^2\cdot°\text{C}$$

The heat transfer surface area of the plate is $A_s = W\times L = (2\text{ m})(10\text{ m}) = 20\text{ m}^2$.

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat} - T_s) = (5593\text{ W/m}^2\cdot°\text{C})(20\text{ m}^2)(100-90)°\text{C} = 1{,}118{,}600\text{ W} = \mathbf{1119\text{ kW}}$$

(*b*) The rate of condensation of steam is determined from

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{1{,}118{,}600\text{ J/s}}{2286\times10^3\text{ J/kg}} = \mathbf{0.489\text{ kg/s}}$$

Discussion Using the heat transfer coefficient determined in the previous problem for the vertical plate, we could also determine the heat transfer coefficient from $h_{\text{inclined}} = h_{\text{vert}}(\cos\theta)^{1/4}$. It would give 5682 W/m^2·°C, which is 1.6% different than the value determined above.

10-71 Saturated ammonia vapor condenses on a vertical ASTM A240 410S stainless steel plate at 190 kPa. The highest rate of condensation that can be produced by the plate, without cooling the plate below the minimum suitable temperature set by the ASME Code for Process Piping, is to be determined.

Assumptions **1** Steady state conditions. **2** The plate is isothermal. **3** The condensate flow is wavy-laminar over the plate. **4** The vapor density is much smaller than the liquid density.

Properties The saturation temperature of ammonia at 190 kPa is −20°C (Table A-11), and the properties of saturated ammonia are from Table A-11:

At the saturation temperature, T_{sat} = −20°C,
h_{fg}= 1329 kJ/kg, ρ_v = 1.603 kg/m^3.

At the film temperature, T_f = (−20 − 30)°C/2 = −25°C,
c_{pl} = 4489 J/kg·K, k_l = 0.5968 W/m·K, ρ_l = 671.5 kg/m^3, μ_l = 2.492 × 10^{-4} kg/m·s
$\nu_l = \mu_l / \rho_l$ = 3.711 × 10^{-7} m^2/s

Analysis The modified latent heat of vaporization is

$$h_{fg}^* = h_{fg} + 0.68\, c_{pl}(T_{\text{sat}} - T_s) = 1329000 + 0.68(4489)(-20+30) = 1359500 \text{ J/kg}$$

For wavy-laminar flow, the Reynolds number is

$$\text{Re} = \left[4.81 + \frac{3.70\, L\, k_l(T_{\text{sat}} - T_s)}{\mu_l h_{fg}^*}\left(\frac{g}{\nu_l^2}\right)^{1/3}\right]^{0.820}$$

$$\text{Re} = \left[4.81 + \frac{3.70(2)(0.5968)(-20+30)}{(2.492\times 10^{-4})(1359500)}\left(\frac{9.81}{(3.711\times 10^{-7})^2}\right)^{1/3}\right]^{0.820} = 1151$$

The Reynolds number is 30 < Re < 1800, so the assumption for wavy-laminar flow is verified. The condensation heat transfer coefficient is then

$$h = h_{\text{vert,wavy}} = \frac{\text{Re}\, k_l}{1.08\,\text{Re}^{1.22} - 5.2}\left(\frac{g}{\nu_l^2}\right)^{1/3} = \frac{(1151)(0.5968)}{1.08(1151)^{1.22} - 5.2}\left(\frac{9.81}{(3.711\times 10^{-7})^2}\right)^{1/3} = 4863 \text{ W/m}^2\cdot\text{K}$$

The rate of heat transfer on the plate during the condensation process is

$$\dot{Q} = hA_s(T_{\text{sat}} - T_s) = (4863 \text{ W/m}^2\cdot\text{K})(2 \text{ m}\times 1.5 \text{ m})(-20+30)\text{K} = 1.4589\times 10^5 \text{W}$$

The highest rate of condensation that can be produced by the plate, without cooling it below −30°C is

$$\dot{m}_{\text{condensate}} = \frac{\dot{Q}}{h_{fg}^*} = \frac{1.4589\times 10^5 \text{J/s}}{1359500 \text{ J/kg}} = 0.1073 \text{ kg/s}$$

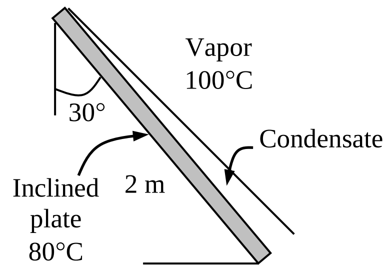
Discussion The highest condensation rate that can be achieved from the vertical ASTM A240 410S stainless steel plate is 0.107 kg/s. If the condensation rate is above 0.107 kg/s, it would increase the heat transfer rate on the plate. This would require the plate surface temperature to decrease below the minimum suitable temperature of −30°C set by the ASME Code for Process Piping.

10-72 Saturated steam at a saturation temperature of T_{sat} = 100°C condenses on a plate which is tilted 30° from the vertical and maintained at 80°C by circulating cooling water through the other side. The rate of heat transfer to the plate and the rate of condensation of the steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The plate is isothermal. **3** The condensate flow is wavy-laminar over the entire plate (this assumption will be verified). **4** The density of vapor is much smaller than the density of liquid, $\rho_v << \rho_l$.

Properties The properties of water at the saturation temperature of 100°C are h_{fg} = 2257×10³ J/kg and ρ_v = 0.60 kg/m³. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2$ = (100 + 80)/2 = 90°C are (Table A-9),

$$\rho_l = 965.3\ \text{kg/m}^3$$
$$\mu_l = 0.315\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\nu_l = \mu_l/\rho_l = 0.326\times10^{-6}\ \text{m}^2/\text{s}$$
$$c_{pl} = 4206\ \text{J/kg}\cdot°\text{C}$$
$$k_l = 0.675\ \text{W/m}\cdot°\text{C}$$



Analysis The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s)$$
$$= 2257\times10^3\ \text{J/kg} + 0.68\times4206\ \text{J/kg}\cdot°\text{C}(100-80)°\text{C} = 2,314\times10^3\ \text{J/kg}$$

Assuming wavy-laminar flow, the Reynolds number is determined from the vertical plate relation by replacing g by $g\cos\theta$ where θ = 30° to be

$$\text{Re} = \text{Re}_{\text{tilted,wavy}} = \left[4.81 + \frac{3.70Lk_l(T_{sat}-T_s)}{\mu_l h^*_{fg}}\left(\frac{g\cos\theta}{\nu_l^2}\right)^{1/3}\right]^{0.820}$$
$$= \left[4.81 + \frac{3.70\times(2\ \text{m})\times(0.675\ \text{W/m}\cdot°\text{C})\times(100-80)°\text{C}}{(0.315\times10^{-3}\ \text{kg/m}\cdot\text{s})(2314\times10^3\ \text{J/kg})}\left(\frac{(9.81\ \text{m/s}^2)\cos30}{(0.326\times10^{-6}\ \text{m}^2/\text{s})^2}\right)^{1/3}\right]^{0.82} = 1237$$

which is between 30 and 1800, and thus our assumption of wavy laminar flow is verified. Then the condensation heat transfer coefficient is determined from

$$h = h_{\text{tilted,wavy}} = \frac{\text{Re}k_l}{1.08\text{Re}^{1.22} - 5.2}\left(\frac{g\cos\theta}{\nu_l^2}\right)^{1/3}$$
$$= \frac{1237\times(0.675\ \text{W/m}\cdot°\text{C})}{1.08(1237)^{1.22} - 5.2}\left(\frac{(9.81\ \text{m/s}^2)\cos30}{(0.326\times10^{-6}\ \text{m}^2/\text{s})^2}\right)^{1/3} = \mathbf{5623\ W/m^2\cdot°C}$$

The heat transfer surface area of the plate is:

$$A = w\times L = (2\ \text{m})(2\ \text{m}) = 4\ \text{m}^2.$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA(T_{sat} - T_s) = (5623\ \text{W/m}^2\cdot°\text{C})(4\ \text{m}^2)(100-80)°\text{C} = 449,900\ \text{W}$$

(*b*) The rate of condensation of steam is determined from

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{449,900\ \text{J/s}}{2314\times10^3\ \text{J/kg}} = \mathbf{0.194\ kg/s}$$

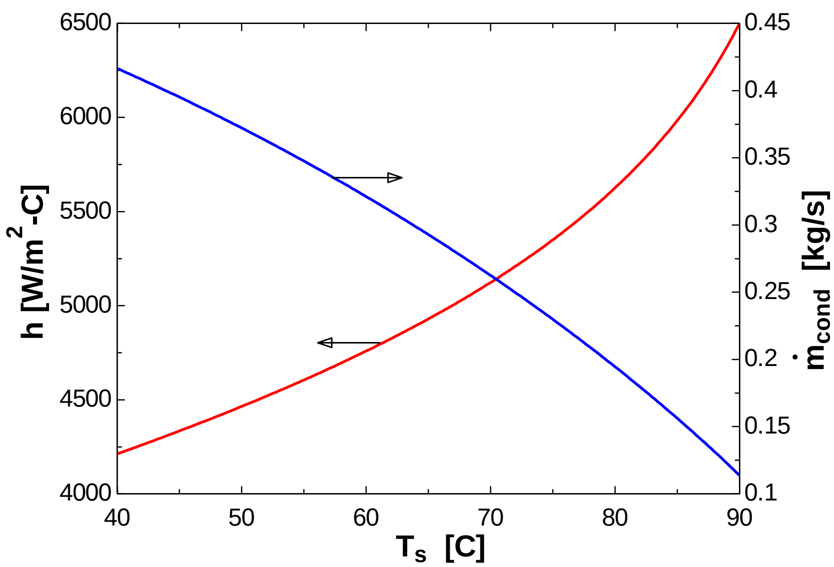
Discussion We could also determine the heat transfer coefficient from $h_{\text{inclined}} = h_{\text{vert}}(\cos\theta)^{1/4}$.

10-73 Prob. 10-72 is reconsidered. The effects of plate temperature and the angle of the plate from the vertical on the average heat transfer coefficient and the rate at which the condensate drips off are to be investigated.

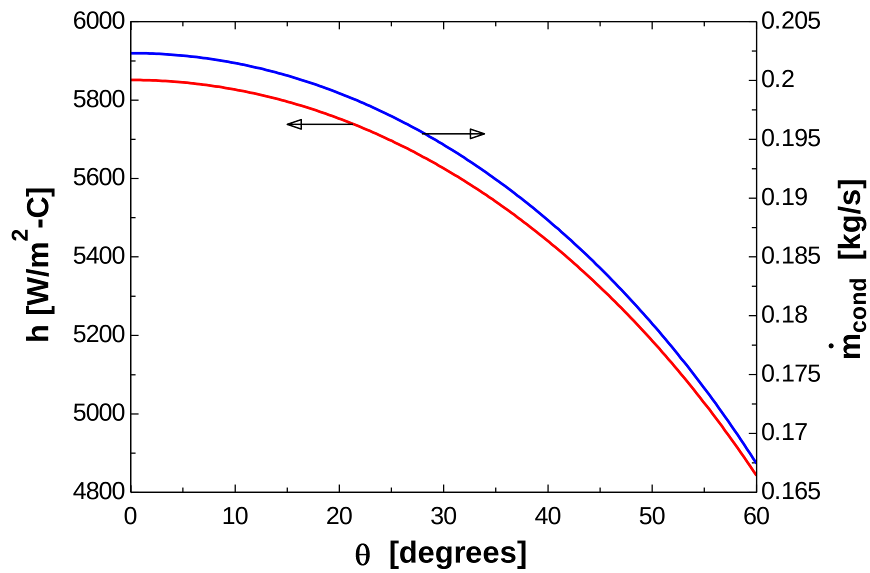
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_sat=100 [C]
L=2 [m]
theta=30 [degrees]
T_s=80 [C]

"PROPERTIES"
Fluid$='steam_IAPWS'
T_f=1/2*(T_sat+T_s)
P_sat=pressure(Fluid$, T=T_sat, x=1)
rho_l=density(Fluid$, T=T_f, x=0)
mu_l=Viscosity(Fluid$,T=T_f, x=0)
nu_l=mu_l/rho_l
c_l=CP(Fluid$, T=T_f, x=0)*Convert(kJ/kg-C, J/kg-C)
k_l=Conductivity(Fluid$, T=T_f, P=P_sat+1)
h_f=enthalpy(Fluid$, T=T_sat, x=0)
h_g=enthalpy(Fluid$, T=T_sat, x=1)
h_fg=(h_g-h_f)*Convert(kJ/kg, J/kg)
g=9.81 [m/s^2]

"ANALYSIS"
"(a)"
h_fg_star=h_fg+0.68*c_l*(T_sat-T_s)
Re=(4.81+(3.7*L*k_l*(T_sat-T_s))/(mu_l*h_fg_star)*((g*Cos(theta))/nu_l^2)^(1/3))^0.820
h=(Re*k_l)/(1.08*Re^1.22-5.2)*((g*Cos(theta))/nu_l^2)^(1/3)
Q_dot=h*A*(T_sat-T_s)
A=L^2
"(b)"
m_dot_cond=Q_dot/h_fg_star
```

T_s [C]	h [W/m^2.C]	$\dot{m}_{cond}$ [kg/s]
40	4212	0.4165
42.5	4272	0.406
45	4334	0.3951
47.5	4398	0.3839
50	4464	0.3722
52.5	4533	0.3601
55	4605	0.3476
57.5	4680	0.3346
60	4759	0.3212
62.5	4841	0.3073
65	4929	0.2929
67.5	5023	0.2779
70	5123	0.2625
72.5	5231	0.2465
75	5350	0.2298
77.5	5480	0.2125
80	5626	0.1945
82.5	5791	0.1758
85	5984	0.1561
87.5	6213	0.1355
90	6500	0.1138



θ [degrees]	h [W/m²·C]	$\dot{m}_{cond}$ [kg/s]
0	5851	0.2023
3	5849	0.2022
6	5842	0.202
9	5831	0.2016
12	5816	0.2011
15	5796	0.2004
18	5771	0.1996
21	5742	0.1986
24	5708	0.1974
27	5670	0.196
30	5626	0.1945
33	5577	0.1928
36	5522	0.1909
39	5462	0.1889
42	5396	0.1866
45	5323	0.1841
48	5243	0.1813
51	5156	0.1783
54	5061	0.175
57	4957	0.1714
60	4842	0.1674

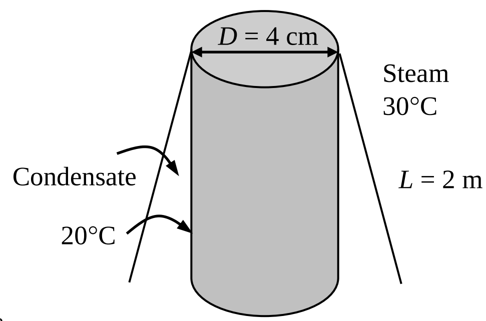


10-74 Saturated steam condenses outside of vertical tube. The rate of heat transfer to the coolant, the rate of condensation and the thickness of the condensate layer at the bottom are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube is isothermal. **3** The tube can be treated as a vertical plate. **4** The condensate flow is wavy-laminar over the entire tube (this assumption will be verified). **5** Nusselt's analysis can be used to determine the thickness of the condensate film layer. **6** The density of vapor is much smaller than the density of liquid, $\rho_v << \rho_l$.

Properties The properties of water at the saturation temperature of 30°C are h_{fg} = 2431×10^3 J/kg and ρ_v = 0.03 kg/m^3. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2 = (30 + 20)/2 = 25$°C are (Table A-9),

$$\rho_l = 997.0 \text{ kg/m}^3$$
$$\mu_l = 0.891\times10^{-3} \text{ kg/m}\cdot\text{s}$$
$$\nu_l = \mu_l / \rho_l = 0.894\times10^{-6} \text{ m}^2/\text{s}$$
$$c_{pl} = 4180 \text{ J/kg}\cdot°\text{C}$$
$$k_l = 0.607 \text{ W/m}\cdot°\text{C}$$



Analysis (*a*)The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s)$$
$$= 2431\times10^3 \text{ J/kg} + 0.68\times 4180 \text{ J/kg}\cdot°\text{C}(30-20)°\text{C} = 2459\times10^3 \text{ J/kg}$$

Assuming wavy-laminar flow, the Reynolds number is determined from

$$\text{Re} = \text{Re}_{\text{vertical,wavy}} = \left[4.81 + \frac{3.70 L k_l (T_{sat} - T_s)}{\mu_l h^*_{fg}}\left(\frac{g}{\nu_l^2}\right)^{1/3}\right]^{0.820}$$

$$= \left[4.81 + \frac{3.70\times(2\text{ m})\times(0.607 \text{ W/m}\cdot°\text{C})\times(30-20)°\text{C}}{(0.891\times10^{-3} \text{ kg/m}\cdot\text{s})(2459\times10^3 \text{ J/kg})}\left(\frac{9.81 \text{ m/s}^2}{(0.894\times10^{-6} \text{ m}^2/\text{s})^2}\right)^{1/3}\right]^{0.82} = 157.3$$

which is between 30 and 1800, and thus our assumption of wavy laminar flow is verified. Then the condensation heat transfer coefficient is determined to be

$$h = h_{\text{vertical,wavy}} = \frac{\text{Re}\, k_l}{1.08\text{Re}^{1.22} - 5.2}\left(\frac{g}{\nu_l^2}\right)^{1/3}$$

$$= \frac{157.3\times(0.607 \text{ W/m}\cdot°\text{C})}{1.08(157.3)^{1.22} - 5.2}\left(\frac{9.81 \text{ m/s}^2}{(0.894\times10^{-6} \text{ m}^2/\text{s})^2}\right)^{1/3} = 4302 \text{ W/m}^2\cdot°\text{C}$$

The heat transfer surface area of the tube is $A_s = \pi DL = \pi(0.04\text{ m})(2\text{ m}) = 0.2513\text{ m}^2$. Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat} - T_s) = (4302 \text{ W/m}^2\cdot°\text{C})(0.2513\text{ m}^2)(30-20)°\text{C} = \mathbf{10{,}811\ W}$$

(*b*) The rate of condensation of steam is determined from

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{10{,}811 \text{ J/s}}{2459\times10^3 \text{ J/kg}} = \mathbf{4.40\times10^{-3}\ kg/s}$$

(*c*) Combining equations $\delta_L = k_l / h_l$ and $h = (4/3)h_L$, the thickness of the liquid film at the bottom of the tube is determined to be

$$\delta_L = \frac{4k_l}{3h} = \frac{4(0.607 \text{ W/m}\cdot°\text{C})}{3(4302 \text{ W/m}^2\cdot°\text{C})} = 0.188\times10^{-3} \text{ m} = \mathbf{0.2\ mm}$$

Discussion The assumption of wavy laminar flow is verified since Reynolds number is between 30 and 1800. The assumption that the tube diameter is large relative to the thickness of the liquid film at the bottom of the tube is verified since the thickness of the liquid film is 0.2 mm, which is much smaller than the diameter of the tube (4 cm).

10-75 The rate of condensation and the heat transfer rate for a vertical pipe, with specified surface temperature, are to be determined.

Assumptions **1** Steady operating condition exists. **2** The surface has uniform temperature. **3** The pipe can be treated as a vertical plate. **4** The condensate flow is wavy-laminar over the entire tube (this assumption will be verified). **5** Nusselt's analysis can be used to determine the thickness of the condensate film layer. **6** The density of vapor is much smaller than the density of liquid, $\rho_v << \rho_l$.

Properties The properties of water at the saturation temperature of 100°C are h_{fg} = 2257 kJ/kg (Table A-2) and ρ_v = 0.5978 kg/m^3 (Table A-9). The properties of liquid water at the film temperature of $T_f = (T_{\text{sat}} + T_s)/2$ = 90°C are, from Table A-9,

ρ_l = 965.3 kg/m^3 $\quad$ c_{pl} = 4206 J/kg·K

μ_l = 0.315 × 10^{-3} kg/m·s $\quad$ k_l = 0.675 W/m·K

$\nu_l = \mu_l / \rho_l$ = 0.326 × 10^{-6} m^2/s

Analysis The modified latent heat of vaporization is

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68 c_{pl}(T_{\text{sat}} - T_s) \\ &= 2257\times10^3 + 0.68(4206)(100-80) \\ &= 2314\times10^3 \text{ J/kg} \end{aligned}$$

Assuming wavy-laminar flow, the Reynolds number is determined from

$$\begin{aligned} \text{Re}_{\text{vert, wavy}} &= \left[4.81 + \frac{3.70 L k_l (T_{\text{sat}} - T_s)}{\mu_l h^*_{fg}}\left(\frac{g}{\nu_l^2}\right)^{1/3}\right]^{0.820} \\ &= \left[4.81 + \frac{3.70(1)(0.675)(100-80)}{(0.315\times10^{-3})(2314\times10^3)}\left(\frac{9.81}{(0.326\times10^{-6})^2}\right)^{1/3}\right]^{0.820} = 729.7 \end{aligned}$$

which is between 30 and 1800, and thus our assumption of wavy laminar flow is verified. Then the condensation heat transfer coefficient is determined to be

$$\begin{aligned} h = h_{\text{vert, wavy}} &= \frac{\text{Re}\, k_l}{1.08\,\text{Re}^{1.22} - 5.2}\left(\frac{g}{\nu_l^2}\right)^{1/3} = \frac{(729.7)(0.675 \text{ W/m}\cdot\text{K})}{1.08(729.7)^{1.22} - 5.2}\left[\frac{9.81 \text{ m/s}^2}{(0.326\times10^{-6} \text{ m}^2\text{/s})^2}\right]^{1/3} \\ &= 6633 \text{ W/m}^2\cdot\text{K} \end{aligned}$$

Then the rate of heat transfer during this condensation process becomes

$$\begin{aligned} \dot{Q} &= \pi D L h (T_{\text{sat}} - T_s) \\ &= \pi(0.1\text{ m})(1\text{ m})(6633 \text{ W/m}^2\cdot\text{K})(100-80)\text{ K} \\ &= \mathbf{4.168\times10^4\ W} \end{aligned}$$

The rate of condensation of steam is determined from

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{4.168\times10^4 \text{ W}}{2314\times10^3 \text{ J/kg}} = \mathbf{0.018\ kg/s}$$

Discussion Combining equations $\delta_L = k_l / h_L$ and $h = (4/3)h_L$, the thickness of the liquid film at the bottom of the tube is determined to be

$$\delta_L = \frac{4k_l}{3h} = \frac{4(0.675 \text{ W/m}\cdot\text{K})}{3(6633 \text{ W/m}^2\cdot\text{K})} = 0.136 \text{ mm} << 100 \text{ mm}$$

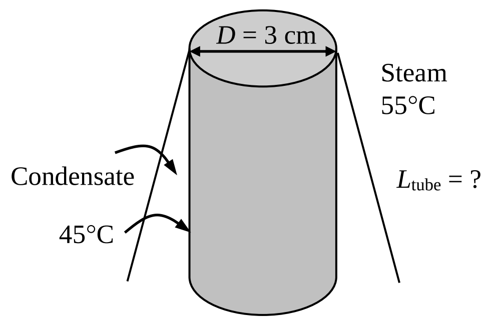
Since $\delta_L << D$, the pipe can be treated as a vertical plate.

10-76 Saturated steam at a saturation temperature of T_{sat} = 55°C condenses on the outer surface of a vertical tube which is maintained at 45°C. The required tube length to condense steam at a rate of 10 kg/h is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube is isothermal. **3** The vertical tube can be treated as a vertical plate. **4** The density of vapor is much smaller than the density of liquid, $\rho_v << \rho_l$.

Properties The properties of water at the saturation temperature of 55°C are h_{fg} = 2371×10^3 J/kg and ρ_v = 0.1045 kg/m^3. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2$ = (55 + 45)/2 = 50°C are (Table A-9),

$$\rho_l = 988.1\ \text{kg/m}^3$$
$$\mu_l = 0.547\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\nu_l = \mu_l/\rho_l = 0.554\times10^{-6}\ \text{m}^2/\text{s}$$
$$c_{pl} = 4181\ \text{J/kg}\cdot°\text{C}$$
$$k_l = 0.644\ \text{W/m}\cdot°\text{C}$$



Analysis The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s)$$
$$= 2371\times10^3\ \text{J/kg} + 0.68\times4181\ \text{J/kg}\cdot°\text{C}(55-45)°\text{C} = 2399\times10^3\ \text{J/kg}$$

The Reynolds number is determined from its definition to be

$$\text{Re} = \frac{4\dot{m}}{p\mu_l} = \frac{4(10/3600\ \text{kg/s})}{\pi(0.03\ \text{m})(0.547\times10^{-3}\ \text{kg/m}\cdot\text{s})} = 215.5$$

which is between 30 and 1800. Therefore the condensate flow is wavy laminar, and the condensation heat transfer coefficient is determined from

$$h = h_{\text{vertical,wavy}} = \frac{\text{Re}\,k_l}{1.08\text{Re}^{1.22} - 5.2}\left(\frac{g}{\nu_l^2}\right)^{1/3}$$
$$= \frac{215.5\times(0.644\ \text{W/m}\cdot°\text{C})}{1.08(215.5)^{1.22} - 5.2}\left(\frac{9.81\ \text{m/s}^2}{(0.554\times10^{-6}\ \text{m}^2/\text{s})^2}\right)^{1/3} = 5644\ \text{W/m}^2\cdot°\text{C}$$

The rate of heat transfer during this condensation process is

$$\dot{Q} = \dot{m}h^*_{fg} = (10/3600\ \text{kg/s})(2399\times10^3\ \text{J/kg}) = 6{,}664\ \text{W}$$

Heat transfer can also be expressed as

$$\dot{Q} = hA_s(T_{sat} - T_s) = h(\pi DL)(T_{sat} - T_s)$$

Then the required length of the tube becomes

$$L = \frac{\dot{Q}}{h(\pi D)(T_{sat} - T_s)} = \frac{6664\ \text{W}}{(5844\ \text{W/m}^2\cdot°\text{C})\pi(0.03\ \text{m})(55-45)°\text{C}} = \mathbf{1.21\ m}$$

Discussion Combining equations $\delta_L = k_l/h_l$ and $h = (4/3)h_L$, the thickness of the liquid film at the bottom of the tube is determined to be

$$\delta_L = \frac{4k_l}{3h} = \frac{4(0.644\ \text{W/m}\cdot°\text{C})}{3(5844\ \text{W/m}^2\cdot°\text{C})} = 0.147\times10^{-3}\ \text{m} = 0.15\ \text{mm}$$

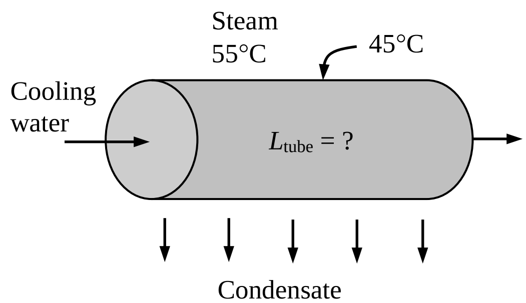
The assumption that the tube diameter is large relative to the thickness of the liquid film at the bottom of the tube is verified since the thickness of the liquid film is 0.15 mm, which is much smaller than the diameter of the tube (3 cm). Also, the assumption of wavy laminar flow is verified since Reynolds number is between 30 and 1800.

10-77 Saturated steam at a saturation temperature of T_{sat} = 55°C condenses on the outer surface of a horizontal tube which is maintained at 45°C. The required tube length to condense steam at a rate of 10 kg/h is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube is isothermal.

Properties The properties of water at the saturation temperature of 55°C are $h_{fg} = 2371\times10^3$ J/kg and ρ_v = 0.1045 kg/m³. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2 = (55 + 45)/2 = 50$°C are (Table A-9),

$$\rho_l = 988.1 \text{ kg/m}^3$$
$$\mu_l = 0.547\times10^{-3} \text{ kg/m}\cdot\text{s}$$
$$\nu_l = \mu_l/\rho_l = 0.554\times10^{-6} \text{ m}^2\text{/s}$$
$$c_{pl} = 4181 \text{ J/kg}\cdot°\text{C}$$
$$k_l = 0.644 \text{ W/m}\cdot°\text{C}$$



Analysis The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s)$$
$$= 2371\times10^3 \text{ J/kg} + 0.68\times4181 \text{ J/kg}\cdot°\text{C}(55-45)°\text{C} = 2399\times10^3 \text{ J/kg}$$

Noting that the tube is horizontal, the condensation heat transfer coefficient is determined from

$$h = h_{horizontal} = 0.729\left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat} - T_s)D}\right]^{1/4}$$
$$= 0.729\left[\frac{(9.81 \text{ m/s}^2)(988.1 \text{ kg/m}^3)(988.1 - 0.10 \text{ kg/m}^3)(2399\times10^3 \text{ J/kg})(0.644 \text{ W/m}\cdot°\text{C})^3}{(0.547\times10^{-3} \text{ kg/m}\cdot\text{s})(55-45)°\text{C}(0.03 \text{ m})}\right]^{1/4}$$
$$= 10{,}135 \text{ W/m}^2\cdot°\text{C}$$

The rate of heat transfer during this condensation process is

$$\dot{Q} = \dot{m}h^*_{fg} = (10/3600 \text{ kg/s})(2399\times10^3 \text{ J/kg}) = 6{,}664 \text{ W}$$

Heat transfer can also be expressed as

$$\dot{Q} = hA_s(T_{sat} - T_s) = h(\pi DL)(T_{sat} - T_s)$$

Then the required length of the tube becomes

$$L = \frac{\dot{Q}}{h(\pi D)(T_{sat} - T_s)} = \frac{6664 \text{ W}}{(10{,}135 \text{ W/m}^2\cdot°\text{C})\pi(0.03 \text{ m})(55-45)°\text{C}} = \mathbf{0.70 \text{ m}}$$

10-78 Saturated vapor condenses on the outer surface of a 1.5-m-long vertical tube at 60°C that is maintained with a surface temperature of 40°C. The rate of heat transfer to the tube and the required tube diameter to condense 12 kg/h of steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Isothermal tube surface. **3** The vertical tube can be treated as a vertical plate (this assumption will be verified). **4** The condensate flow is wavy-laminar over the entire plate (this assumption will be verified). **5** The density of vapor is much smaller than the density of liquid, $\rho_v << \rho_l$.

Properties The properties of water at the saturation temperature of 60°C are h_{fg} = 2359 × 10^3 J/kg and ρ_v = 0.1304 kg/m^3 (Table A-9). The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2 = (60 + 40)/2 = 50$°C are (Table A-9)

$$\rho_l = 988.1\ \text{kg/m}^3$$
$$\mu_l = 0.547\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\nu_l = \mu_l/\rho_l = 0.554\times10^{-6}\ \text{m}^2/\text{s} \quad 0$$
$$c_{pl} = 4181\ \text{J/kg}\cdot\text{K}$$
$$k_l = 0.644\ \text{W/m}\cdot\text{K}$$

Analysis The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s) = 2359\times10^3\ \text{J/kg} + 0.68(4181\ \text{J/kg}\cdot\text{K})(60-40)\ \text{K} = 2.4159\times10^6\ \text{J/kg}$$

Assuming wavy-laminar flow, the Reynolds number is determined from

$$\text{Re} = \text{Re}_{\text{vertical,wavy}} = \left[4.81 + \frac{3.70Lk_l(T_{sat}-T_s)}{\mu_l h^*_{fg}}\left(\frac{g}{\nu_l^2}\right)^{1/3}\right]^{0.820}$$
$$= \left[4.81 + \frac{(3.70)(1.5\ \text{m})(0.644\ \text{W/m}\cdot\text{K})(60-40)\ \text{K}}{(0.547\times10^{-3}\ \text{kg/m}\cdot\text{s})(2.4159\times10^6\ \text{J/kg})}\left(\frac{9.81\ \text{m/s}^2}{(0.554\times10^{-6}\ \text{m}^2/\text{s})^2}\right)^{1/3}\right]^{0.820} = 454.73$$

which is between 30 and 1800, and thus our assumption of wavy laminar flow is verified. Then the condensation heat transfer coefficient is determined to be

$$h = h_{\text{vertical,wavy}} = \frac{\text{Re}\,k_l}{1.08\,\text{Re}^{1.22} - 5.2}\left(\frac{g}{\nu_l^2}\right)^{1/3}$$
$$= \frac{(454.73)(0.644\ \text{W/m}\cdot\text{K})}{1.08(454.73)^{1.22} - 5.2}\left[\frac{9.81\ \text{m/s}^2}{(0.554\times10^{-6}\ \text{m}^2/\text{s})^2}\right]^{1/3} = 4937.5\ \text{W/m}^2\cdot\text{K}$$

The rate of heat transfer to the tube during this condensation process is

$$\dot{Q} = \dot{m}h^*_{fg} = (12/3600\ \text{kg/s})(2.4159\times10^6\ \text{J/kg}) = \mathbf{8053\ W}$$

Heat transfer can also be expressed as

$$\dot{Q} = hA_s(T_{sat} - T_s) = h(\pi DL)(T_{sat} - T_s)$$

Then the required diameter of the tube becomes

$$D = \frac{\dot{Q}}{h(\pi L)(T_{sat} - T_s)} = \frac{8053\ \text{W}}{(4937.5\ \text{W/m}^2\cdot\text{K})\pi(1.5\ \text{m})(60-40)\ \text{K}} = \mathbf{0.0173\ m}$$

To verify the assumption that vertical tube can be treated as a vertical plate ($D >> \delta$), calculate δ from

$$\delta = (4k_l)/3h = 4(0.664\ \text{W/m}\cdot\text{K})/(3)(4937.5\ \text{W/m}^2\cdot\text{K}) = 1.79\times10^{-4}\ \text{m} << D = 0.0173\ \text{m}$$

Thus our assumption of $D >> \delta$ is verified.

Discussion With diameter known, the Reynolds number can also be verified to be wavy- laminar flow

$$\text{Re} = \frac{4\dot{m}}{\pi D\mu_l} = \frac{4(12/3600\ \text{kg/s})}{\pi(0.0173\ \text{m})(0.547\times10^{-3}\ \text{kg/m}\cdot\text{s})} = 448$$

Note that the Re value obtained based on equation 10-27 will not exactly match the above calculated value of Re since equation 10-27 is based on experimental data and several approximations. However, the two values are very close (within 2%).

10-79 Saturated vapor condenses on the outer surface of a 1.5-m-long horizontal tube at 60°C that is maintained with a surface temperature of 40°C. The rate of heat transfer to the tube and the required tube diameter to condense 12 kg/h of steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Isothermal tube surface.

Properties The properties of water at the saturation temperature of 60°C are h_{fg} = 2359 × 10³ J/kg and ρ_v = 0.1304 kg/m³ (Table A-9). The properties of liquid water at the film temperature of $T_f = (T_{\text{sat}} + T_s)/2 = (60 + 40)/2 = 50$°C are (Table A-9)

$$\begin{aligned}
\rho_l &= 988.1\,\text{kg/m}^3\\
\mu_l &= 0.547\times10^{-3}\,\text{kg/m}\cdot\text{s}\\
\nu_l &= \mu_l/\rho_l = 0.554\times10^{-6}\,\text{m}^2/\text{s}\\
c_{pl} &= 4181\,\text{J/kg}\cdot\text{K}\\
k_l &= 0.644\ \text{W/m}\cdot\text{K}
\end{aligned}$$

Analysis The modified latent heat of vaporization is

$$\begin{aligned}
h^*_{fg} &= h_{fg} + 0.68c_{pl}(T_{\text{sat}} - T_s)\\
&= 2359\times10^3\ \text{J/kg} + 0.68(4181\,\text{J/kg}\cdot\text{K})(60-40)\,\text{K} = 2.4159\times10^6\ \text{J/kg}
\end{aligned}$$

The rate of heat transfer to the tube during this condensation process is

$$\dot{Q} = \dot{m}h^*_{fg} = (12/3600\,\text{kg/s})(2.4159\times10^6\ \text{J/kg}) = \mathbf{8053\ W}$$

Heat transfer can also be expressed as

$$\dot{Q} = hA_s(T_{\text{sat}} - T_s) = h(\pi DL)(T_{\text{sat}} - T_s) \rightarrow h = \frac{\dot{Q}}{(\pi DL)(T_{\text{sat}} - T_s)}$$

Noting that the tube is horizontal, the condensation heat transfer coefficient is determined from

$$h = h_{\text{horiz}} = 0.729\left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}k_l^3}{\mu_l(T_{\text{sat}} - T_s)D}\right]^{1/4} = \frac{\dot{Q}}{(\pi DL)(T_{\text{sat}} - T_s)}$$

Solving for the required tube diameter,

$$\begin{aligned}
D &= \left\{0.729\frac{(\pi L)(T_{\text{sat}} - T_s)}{\dot{Q}}\left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}k_l^3}{\mu_l(T_{\text{sat}} - T_s)}\right]^{1/4}\right\}^{-4/3}\\
&= \left\{0.729\frac{\pi(1.5\,\text{m})(60-40)\text{K}}{8053\,\text{J/s}}\right.\\
&\quad\left.\times\left[\frac{(9.81\,\text{m/s}^2)(988.1\,\text{kg/m}^3)(988.1 - 0.1304)\text{kg/m}^3(2.4159\times10^6\ \text{J/kg})(0.644\ \text{W/m}\cdot\text{K})^3}{(0.547\times10^{-3}\,\text{kg/m}\cdot\text{s})(60-40)\text{K}}\right]^{1/4}\right\}^{-4/3}\\
&= \mathbf{0.00694\ m}
\end{aligned}$$

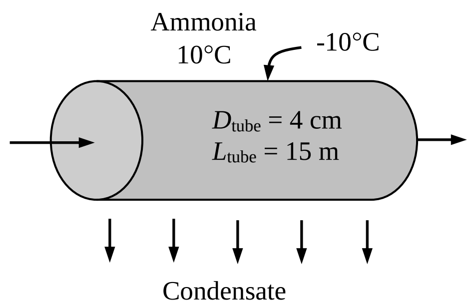
Discussion When placed vertically, the required tube diameter is about 2.5 times larger than that of a horizontal tube. Due to the higher heat transfer coefficient for a horizontal tube, in comparison with a vertical tube, the horizontal tube requires a smaller diameter for the same length to achieve the same rate of condensation.

10-80 Saturated ammonia vapor at a saturation temperature of T_{sat} = 10°C condenses on the outer surface of a horizontal tube which is maintained at -10°C. The rate of heat transfer from the ammonia and the rate of condensation of ammonia are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube is isothermal.

Properties The properties of ammonia at the saturation temperature of 10°C are h_{fg} = 1226×10^3 J/kg and ρ_v = 4.870 kg/m^3 (Table A-11). The properties of liquid ammonia at the film temperature of $T_f = (T_{sat} + T_s)/2$ = (10 + (-10))/2 = 0°C are (Table A-11),

$$\rho_l = 638.6\ \text{kg/m}^3$$
$$\mu_l = 1.896\times10^{-4}\ \text{kg/m}\cdot\text{s}$$
$$\nu_l = 0.2969\times10^{-6}\ \text{m}^2/\text{s}$$
$$c_{pl} = 4617\ \text{J/kg}\cdot°\text{C}$$
$$k_l = 0.5390\ \text{W/m}\cdot°\text{C}$$



Analysis The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s)$$
$$= 1226\times10^3\ \text{J/kg} + 0.68\times4617\ \text{J/kg}\cdot°\text{C}[10-(-10)]°\text{C} = 1288\times10^3\ \text{J/kg}$$

Noting that the tube is horizontal, the condensation heat transfer coefficient is determined from

$$h = h_{\text{horizontal}} = 0.729\left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat} - T_s)D}\right]^{1/4}$$

$$= 0.729\left[\frac{(9.81\ \text{m/s}^2)(638.6\ \text{kg/m}^3)(638.6 - 4.870\ \text{kg/m}^3)(1288\times10^3\ \text{J/kg})(0.5390\ \text{W/m}\cdot°\text{C})^3}{(1.896\times10^{-4}\ \text{kg/m}\cdot\text{s})[10-(-10)]°\text{C}(0.02\ \text{m})}\right]^{1/4}$$

$$= 7390\ \text{W/m}^2.°\text{C}$$

The heat transfer surface area of the tube is

$$A_s = \pi DL = \pi(0.04\ \text{m})(15\ \text{m}) = 1.885\ \text{m}^2$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat} - T_s) = (7390\ \text{W/m}^2.°\text{C})(1.885\ \text{m}^2)[10-(-10)]°\text{C} = \mathbf{278{,}600\ W}$$

(*b*) The rate of condensation of ammonia is determined from

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{278{,}600\ \text{J/s}}{1288\times10^3\ \text{J/kg}} = \mathbf{0.216\ kg/s}$$

10-81 C&S Saturated R-134a vapor condenses on a horizontal ASTM A268 TP443 stainless steel tube at 133 kPa. Determine whether the tube would comply with the ASME Code for Process Piping if the condensation rate of R-134a on the tube surface is 3 g/s.

Assumptions **1** Steady state conditions. **2** The tube is isothermal.

Properties The saturation temperature of R-134a at 133 kPa is −20°C (Table A-10), and the properties of saturated R-134a are from Table A-10:

At the saturation temperature, T_{sat} = −20°C,
h_{fg}= 213.0 kJ/kg, ρ_v = 6.787 kg/m^3.

At the film temperature, T_f = (−20 − 30)°C/2 = −25°C,
c_{pl} = 1283 J/kg·K, k_l = 0.1047 W/m·K, ρ_l = 1374 kg/m^3, μ_l = 3.882 × 10^{-4} kg/m·s

Analysis The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68\, c_{pl}(T_{\text{sat}} - T_s) = 213000 + 0.68(1283)(-20+30) = 221724\ \text{J/kg}$$

For condensation on a horizontal tube, the heat transfer coefficient is

$$h = 0.729\left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}k_l^3}{\mu_l(T_{\text{sat}} - T_s)D}\right]^{1/4}$$

$$h = 0.729\left[\frac{(9.81)(1374)(1374 - 6.787)(221724)(0.1047)^3}{(3.882\times 10^{-4})(-20+30)(0.015)}\right]^{1/4} = 2184\ \text{W/m}^2\cdot\text{K}$$

The rate of heat transfer on the tube surface during the condensation process is

$$\dot{Q} = hA_s(T_{\text{sat}} - T_s) = (2184\ \text{W/m}^2\cdot\text{K})\pi(0.015\ \text{m})(0.50\ \text{m})(-20+30)\text{K} = 514.6\ \text{W}$$

The highest rate of condensation on the tube surface, without cooling it below −30°C, is

$$\dot{m}_{\text{condensate}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{514.6\ \text{J/s}}{221724\ \text{J/kg}} = 0.00232\ \text{kg/s} = 2.32\ \text{g/s} < 3\ \text{g/s}$$

Discussion If the condensation rate of R-134a on the ASTM A268 TP443 tube surface is 3 g/s, the tube surface temperature would be below the minimum suitable temperature of −30°C. This would violate the ASME Code for Process Piping. Since the highest rate of condensation on the tube surface, without cooling it below −30°C, is 2.32 g/s.

10-82 PtD A spherical tank containing cold fluid is causing condensation of moist air on the outer surface. The rate of moisture condensation is to be determined whether or not risk of electrical hazard exists.

Assumptions **1** Steady operating conditions exist. **2** Isothermal tank surface. **3** Film condensation occurs on the tank surface.

Properties The properties of water at the saturation temperature of 25°C are h_{fg} = 2442 × 10^3 J/kg and ρ_v = 0.0231 kg/m^3 (Table A-9). The properties of liquid water at the film temperature of $T_f = (T_{\text{sat}} + T_s)/2 = (25 + 5)/2 = 15$°C are (Table A-9)

$$\rho_l = 999.1\ \text{kg/m}^3$$
$$\mu_l = 1.138 \times 10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\nu_l = \mu_l / \rho_l = 1.139 \times 10^{-6}\ \text{m}^2/\text{s}$$
$$c_{pl} = 4185\ \text{J/kg}\cdot\text{K}$$
$$k_l = 0.589\ \text{W/m}\cdot\text{K}$$

Analysis The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{\text{sat}} - T_s)$$
$$= 2442 \times 10^3\ \text{J/kg} + 0.68(4185\ \text{J/kg}\cdot\text{K})(25 - 5)\ \text{K} = 2.4989 \times 10^6\ \text{J/kg}$$

The film condensation heat transfer coefficient for a sphere is determined from

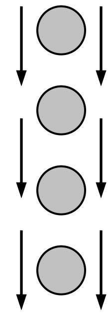
$$h = h_{\text{horiz}} = 0.815\left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}k_l^3}{\mu_l(T_{\text{sat}} - T_s)D}\right]^{1/4}$$
$$= 0.815\left[\frac{(9.81\ \text{m/s}^2)(999.1\ \text{kg/m}^3)(999.1 - 0.0231)\text{kg/m}^3(2.4989\times10^6\ \text{J/kg})(0.589\ \text{W/m}\cdot\text{K})^3}{(1.138\times10^{-3}\ \text{kg/m}\cdot\text{s})(25-5)\text{K}(3\ \text{m})}\right]^{1/4}$$
$$= 2384.1\ \text{W/m}^2\cdot\text{K}$$

Thus, the rate of film condensation is

$$\dot{m} = \frac{\dot{Q}}{h^*_{fg}} = \frac{h(\pi D^2)(T_{\text{sat}} - T_s)}{h^*_{fg}}$$
$$= \frac{(2384.1\ \text{W/m}^2\cdot\text{K})\pi(3\ \text{m})^2(25-5)\ \text{K}}{2.4989\times10^6\ \text{J/kg}} = \mathbf{0.5395\ kg/s} > 0.5\ \text{kg/s}$$

Discussion The rate of condensation from the tank surface is greater than the capability of the system in removing the condensate. Thus, there is a risk of excess condensate coming in contact with the high voltage device and cause electrical hazard. To prevent electrical hazard, the preventive system should be capable of removing more than 0.54 kg/s of condensate.

10-83 There is film condensation on the outer surfaces of *N* horizontal tubes arranged in a vertical tier. The value of *N* for which the average heat transfer coefficient for the entire tier be equal to half of the value for a single horizontal tube is to be determined.



Assumptions Steady operating conditions exist.

Analysis The relation between the heat transfer coefficients for the two cases is given to be

$$h_{\text{horizontal, N tubes}} = \frac{h_{\text{horizontal, 1 tube}}}{N^{1/4}}$$

Therefore,

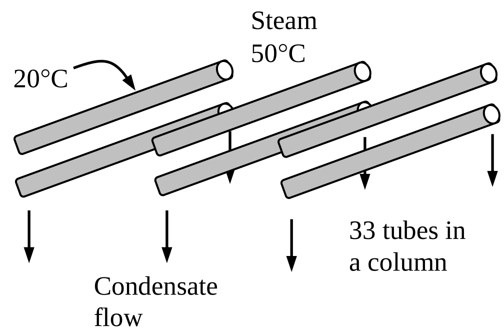
$$\frac{h_{\text{horizontal, N tubes}}}{h_{\text{horizontal, 1 tube}}} = \frac{1}{2} = \frac{1}{N^{1/4}} \longrightarrow N = \mathbf{16}$$

10-84 Saturated steam at a saturation temperature of T_{sat} = 50°C condenses on the outer surfaces of a tube bank with 33 tubes in each column maintained at 20°C. The average heat transfer coefficient and the rate of condensation of steam on the tubes are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tubes are isothermal.

Properties The properties of water at the saturation temperature of 50°C are h_{fg} = 2383×10^3 J/kg and ρ_v = 0.0831 kg/m^3. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2$ = (50 + 20)/2 = 35°C are (Table A-9),

$$\rho_l = 994.0\text{ kg/m}^3$$
$$\mu_l = 0.720\times10^{-3}\text{ kg/m}\cdot\text{s}$$
$$\nu_l = \mu_l/\rho_l = 0.724\times10^{-6}\text{ m}^2\text{/s}$$
$$c_{pl} = 4178\text{ J/kg}\cdot°\text{C}$$
$$k_l = 0.623\text{ W/m}\cdot°\text{C}$$



Analysis (*a*) The modified latent heat of vaporization is

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68c_{pl}(T_{sat} - T_s) \\ &= 2383\times10^3\text{ J/kg} + 0.68\times4178\text{ J/kg}\cdot°\text{C}(50-20)°\text{C} \\ &= 2468\times10^3\text{ J/kg} \end{aligned}$$

The heat transfer coefficient for condensation on a single horizontal tube is

$$\begin{aligned} h = h_{horizontal} &= 0.729\left[\frac{g\rho_l(\rho_l-\rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat}-T_s)D}\right]^{1/4} \\ &= 0.729\left[\frac{(9.81\text{ m/s}^2)(994\text{ kg/m}^3)(994-0.08\text{ kg/m}^3)(2468\times10^3\text{ J/kg})(0.623\text{ W/m}\cdot°\text{C})^3}{(0.720\times10^{-3}\text{ kg/m}\cdot\text{s})(50-20)°\text{C}(0.015\text{ m})}\right]^{1/4} \\ &= 8425\text{ W/m}^2\cdot°\text{C} \end{aligned}$$

Then the average heat transfer coefficient for a 33-tube high vertical tier becomes

$$h_{\text{horiz, N tubes}} = \frac{1}{N^{1/4}}h_{\text{horiz, 1 tube}} = \frac{1}{33^{1/4}}(8425\text{ W/m}^2\cdot°\text{C}) = \mathbf{3515\text{ W/m}^2\cdot°\text{C}}$$

The surface area for all 33 tubes per unit length is

$$A_s = N_{total}\pi DL = 33\pi(0.015\text{ m})(1\text{m}) = 1.555\text{ m}^2$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat} - T_s) = (3515\text{ W/m}^2\cdot°\text{C})(1.555\text{ m}^2)(50-20)°\text{C} = 164{,}000\text{ W}$$

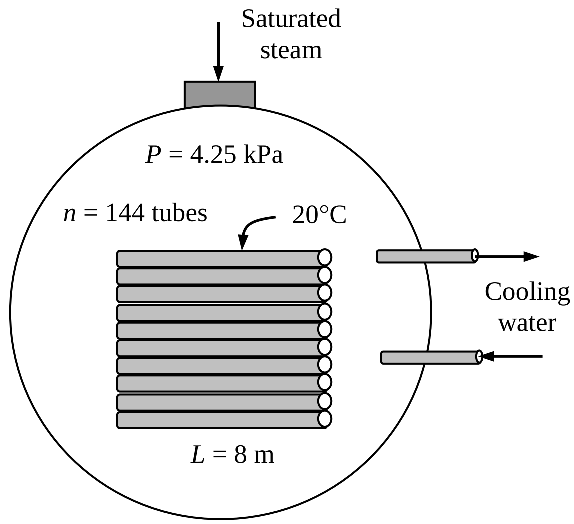
(*b*) The rate of condensation of steam is determined from

$$\dot{m}_{condensation} = \frac{\dot{Q}}{h^*_{fg}} = \frac{164{,}000\text{ J/s}}{2468\times10^3\text{ J/kg}} = \mathbf{0.0664\text{ kg/s}}$$

10-85 Saturated steam at a pressure of 4.25 kPa and thus at a saturation temperature of T_{sat} = 30°C (Table A-9) condenses on the outer surfaces of 144 horizontal tubes arranged in a 12×12 square array maintained at 20°C by circulating cooling water. The rate of heat transfer to the cooling water and the rate of condensation of steam on the tubes are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tubes are isothermal.

Properties The properties of water at the saturation temperature of 30°C are $h_{fg} = 2431\times10^3$ J/kg and $\rho_v = 0.03$ kg/m^3. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2 = (30 + 20)/2 = 25°C$ are (Table A-9),



$$\rho_l = 997.0\ \text{kg/m}^3$$
$$\mu_l = 0.891\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\nu_l = \mu_l / \rho_l = 0.894\times10^{-6}\ \text{m}^2/\text{s}$$
$$c_{pl} = 4180\ \text{J/kg}\cdot°\text{C}$$
$$k_l = 0.607\ \text{W/m}\cdot°\text{C}$$

Analysis (*a*) The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s)$$
$$= 2431\times10^3\ \text{J/kg} + 0.68\times4180\ \text{J/kg}\cdot°\text{C}(30-20)°\text{C} = 2{,}459\times10^3\ \text{J/kg}$$

The heat transfer coefficient for condensation on a single horizontal tube is

$$h = h_{horizontal} = 0.729\left[\frac{g\rho_l(\rho_l-\rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat}-T_s)D}\right]^{1/4}$$
$$= 0.729\left[\frac{(9.81\ \text{m/s}^2)(997\ \text{kg/m}^3)(997-0.03\ \text{kg/m}^3)(2459\times10^3\ \text{J/kg})(0.607\ \text{W/m}\cdot°\text{C})^3}{(0.891\times10^{-3}\ \text{kg/m}\cdot\text{s})(30-20)°\text{C}(0.03\ \text{m})}\right]^{1/4}$$
$$= 8674\ \text{W/m}^2.°\text{C}$$

Then the average heat transfer coefficient for a 10-pipe high vertical tier becomes

$$h_{\text{horiz, N tubes}} = \frac{1}{N^{1/4}}h_{\text{horiz, 1 tube}} = \frac{1}{12^{1/4}}(8674\ \text{W/m}^2\cdot°\text{C}) = 4661\ \text{W/m}^2\cdot°\text{C}$$

The surface area for all 144 tubes is

$$A_s = N_{total}\pi DL = 144\pi(0.03\,\text{m})(8\,\text{m}) = 108.6\,\text{m}^2$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat} - T_s) = (4661\ \text{W/m}^2.°\text{C})(108.6\ \text{m}^2)(30-20)°\text{C} = 5{,}060{,}000\ \text{W} = \mathbf{5060\ kW}$$

(*b*) The rate of condensation of steam is determined from

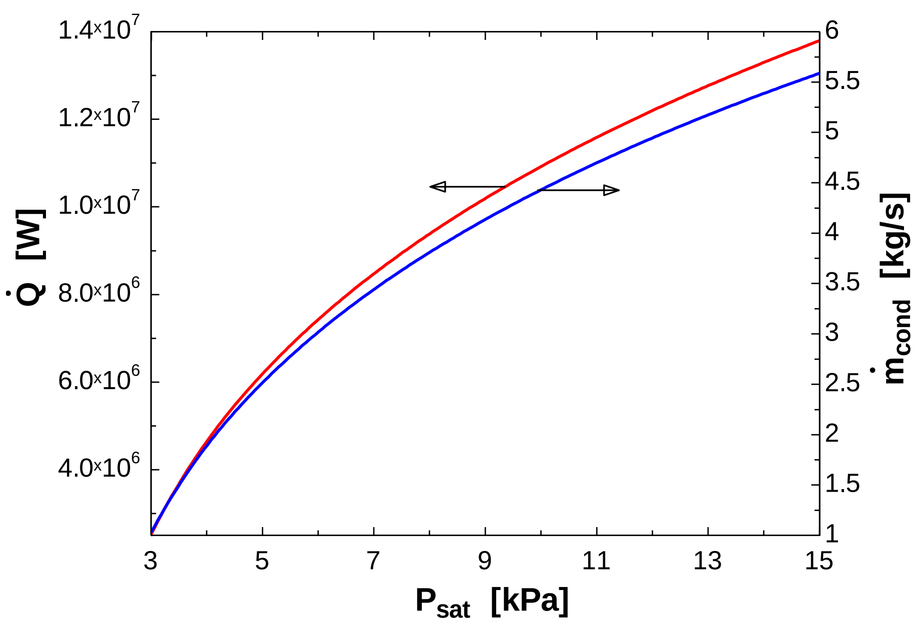
$$\dot{m}_{condensation} = \frac{\dot{Q}}{h^*_{fg}} = \frac{5{,}060{,}000\ \text{J/s}}{2459\times10^3\ \text{J/kg}} = \mathbf{2.06\ kg/s}$$

10-86 Prob. 10-85 is reconsidered. The effect of the condenser pressure on the rate of heat transfer and the rate of condensation of the steam is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
P_sat=4.25 [kPa]
n_tube=144
N=12
L=8 [m]
D=0.03 [m]
T_s=20 [C]

"PROPERTIES"
Fluid$='steam_IAPWS'
T_sat=temperature(Fluid$, P=P_sat, x=1)
T_f=1/2*(T_sat+T_s)
h_f=enthalpy(Fluid$, T=T_sat, x=0)
h_g=enthalpy(Fluid$, T=T_sat, x=1)
h_fg=(h_g-h_f)*Convert(kJ/kg, J/kg)
rho_v=density(Fluid$, T=T_sat, x=1)
rho_l=density(Fluid$, T=T_f, x=0)
mu_l=Viscosity(Fluid$,T=T_f, x=0)
nu_l=mu_l/rho_l
c_l=CP(Fluid$, T=T_f, x=0)*Convert(kJ/kg-C, J/kg-C)
k_l=Conductivity(Fluid$, T=T_f, P=P_sat+1)
g=9.81 [m/s^2]

"ANALYSIS"
h_fg_star=h_fg+0.68*c_l*(T_sat-T_s)
h_1tube=0.729*((g*rho_l*(rho_l-rho_v)*h_fg_star*k_l^3)/(mu_l*(T_sat-T_s)*D))^0.25
h=1/N^0.25*h_1tube
Q_dot=h*A*(T_sat-T_s)
A=n_tube*pi*D*L
m_dot_cond=Q_dot/h_fg_star
```

P_{sat} [kPa]	$\dot{Q}$ [W]	$\dot{m}_{cond}$ [kg/s]
3	2523819	1.028
4	4642913	1.889
5	6185636	2.515
6	7426047	3.017
7	8473057	3.441
8	9383654	3.809
9	10192213	4.136
10	10921191	4.43
11	11586164	4.698
12	12198509	4.945
13	12766595	5.174
14	13296988	5.387
15	13794833	5.587

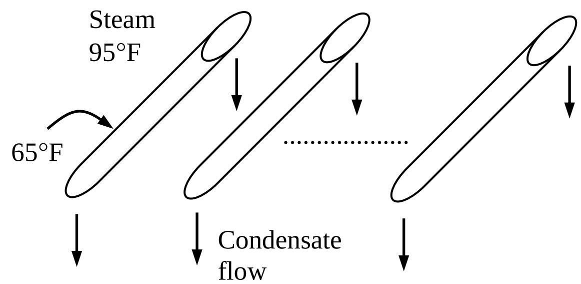


10-87E Saturated steam at a saturation temperature of T_{sat} = 95°F condenses on the outer surfaces of horizontal pipes which are maintained at 65°F by circulating cooling water. The rate of heat transfer to the cooling water and the rate of condensation per unit length of a single horizontal pipe are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The pipe is isothermal. **3** There is no interference between the pipes (no drip of the condensate from one tube to another).

Properties The properties of water at the saturation temperature of 95°F are h_{fg} = 1040 Btu/lbm and ρ_v = 0.0025 lbm/ft³. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2$ = (95 + 65)/2 = 80°F are (Table A-9E),

$$\rho_l = 62.22 \text{ lbm/ft}^3$$
$$\mu_l = 5.764\times10^{-4} \text{ lbm/ft}\cdot\text{s} = 2.075 \text{ lbm/ft}\cdot\text{h}$$
$$\nu_l = \mu_l / \rho_l = 0.03335 \text{ ft}^2\text{/h}$$
$$c_{pl} = 0.999 \text{ Btu/lbm}\cdot°\text{F}$$
$$k_l = 0.352 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}$$



Analysis The modified latent heat of vaporization is

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68c_{pl}(T_{sat} - T_s) \\ &= 1040 \text{ Btu/lbm} + 0.68\times(0.999 \text{ Btu/lbm}\cdot°\text{F})(95-65)°\text{F} \\ &= 1060 \text{ Btu/lbm} \end{aligned}$$

Noting that we have condensation on a horizontal tube, the heat transfer coefficient is determined from

$$\begin{aligned} h = h_{\text{horiz}} &= 0.729\left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat} - T_s)D}\right]^{1/4} \\ &= 0.729\left[\frac{(32.2 \text{ ft/s}^2)(62.22 \text{ lbm/ft}^3)(62.22 - 0.0025 \text{ lbm/ft}^3)(1060 \text{ Btu/lbm})(0.352 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F})^3}{[(1 \text{ h/3600 s})^2](2.075 \text{ lbm/ft}\cdot\text{h})(95-65)°\text{F}(0.8/12 \text{ ft})}\right]^{1/4} \\ &= 1501 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F} \end{aligned}$$

The heat transfer surface area of the tube per unit length is

$$A_s = \pi DL = \pi(0.8/12 \text{ ft})(1 \text{ ft}) = 0.2094 \text{ ft}^2$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat} - T_s) = (1501 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(0.2094 \text{ ft}^2)(95-65)°\text{F} = \mathbf{9431\ Btu/h}$$

(*b*) The rate of condensation of steam is determined from

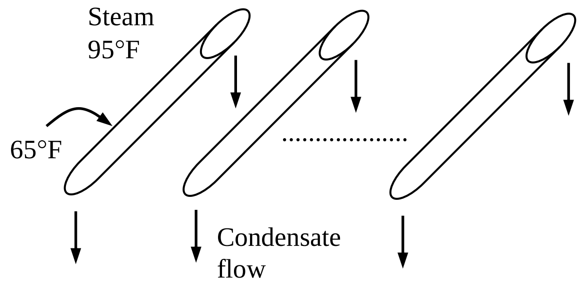
$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{9431 \text{ Btu/h}}{1060 \text{ Btu/lbm}} = \mathbf{8.90\ lbm/h}$$

10-88E Saturated steam at a saturation temperature of T_{sat} = 95°F condenses on the outer surfaces of 20 horizontal pipes which are maintained at 65°F by circulating cooling water and arranged in a rectangular array of 4 pipes high and 5 pipes wide. The rate of heat transfer to the cooling water and the rate of condensation per unit length of the pipes are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The pipes are isothermal.

Properties The properties of water at the saturation temperature of 95°F are h_{fg} = 1040 Btu/lbm and ρ_v = 0.0025 lbm/ft^3. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2$ = (95 + 65)/2 = 80°F are (Table A-9E),

$$\rho_l = 62.22 \text{ lbm/ft}^3$$
$$\mu_l = 5.764\times10^{-4} \text{ lbm/ft}\cdot\text{s} = 2.075 \text{ lbm/ft}\cdot\text{h}$$
$$\nu_l = \mu_l/\rho_l = 0.03335 \text{ ft}^2\text{/h}$$
$$c_{pl} = 0.999 \text{ Btu/lbm}\cdot°\text{F}$$
$$k_l = 0.352 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}$$



Analysis The modified latent heat of vaporization is

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68c_{pl}(T_{sat} - T_s) \\ &= 1040 \text{ Btu/lbm} + 0.68\times(0.999 \text{ Btu/lbm}\cdot°\text{F})(95-65)°\text{F} \\ &= 1060 \text{ Btu/lbm} \end{aligned}$$

Noting that we have condensation on a horizontal tube, the heat transfer coefficient is determined from

$$\begin{aligned} h = h_{horiz} &= 0.729\left[\frac{g\rho_l(\rho_l-\rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat}-T_s)D}\right]^{1/4} \\ &= 0.729\left[\frac{(32.2 \text{ ft/s}^2)(62.22 \text{ lbm/ft}^3)(62.22-0.0025 \text{ lbm/ft}^3)(1060 \text{ Btu/lbm})(0.352 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F})^3}{[(1 \text{ h/3600 s})^2](2.075 \text{ lbm/ft}\cdot\text{h})(95-65)°\text{F}(0.8/12 \text{ ft})}\right]^{1/4} \\ &= 1501 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F} \end{aligned}$$

Then the average heat transfer coefficient for a 4-pipe high vertical tier becomes

$$h_{\text{horiz, N tubes}} = \frac{1}{N^{1/4}}h_{\text{horiz,1 tube}} = \frac{1}{4^{1/4}}(1501 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F}) = 1061 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F}$$

The surface area for all 32 pipes per unit length of the pipes is

$$A_s = N_{total}\pi DL = 32\pi(0.8/12 \text{ ft})(1 \text{ ft}) = 6.702 \text{ ft}^2$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat} - T_s) = (1061 \text{ Btu/h.ft}^2\cdot°\text{F})(6.702 \text{ ft}^2)(95-65)°\text{F} = \mathbf{213{,}300 \text{ Btu/h}}$$

(*b*) The rate of condensation of steam is determined from

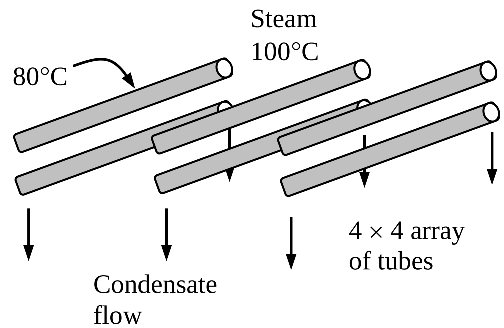
$$\dot{m}_{condensation} = \frac{\dot{Q}}{h^*_{fg}} = \frac{213{,}300 \text{ Btu/h}}{1060 \text{ Btu/lbm}} = \mathbf{201.3 \text{ lbm/h}}$$

10-89 Saturated steam at a saturation temperature of T_{sat} = 100°C condenses on the outer surfaces of a tube bank maintained at 80°C. The rate of condensation of steam on the tubes are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tubes are isothermal.

Properties The properties of water at the saturation temperature of 100°C are h_{fg} = 2257×10^3 J/kg and ρ_v = 0.5978 kg/m^3 (Table A-9). The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2$ = (100 + 80)/2 = 90°C are (Table A-9),

$$\begin{aligned}\rho_l &= 965.3\ \text{kg/m}^3\\ \mu_l &= 0.315\times10^{-3}\ \text{kg/m}\cdot\text{s}\\ c_{pl} &= 4206\ \text{J/kg}\cdot°\text{C}\\ k_l &= 0.675\ \text{W/m}\cdot°\text{C}\end{aligned}$$



Analysis (*a*) The modified latent heat of vaporization is

$$\begin{aligned}h^*_{fg} &= h_{fg} + 0.68c_{pl}(T_{sat} - T_s)\\ &= 2357\times10^3\ \text{J/kg} + 0.68\times4206\ \text{J/kg}\cdot°\text{C}(100-80)°\text{C}\\ &= 2314\times10^3\ \text{J/kg}\end{aligned}$$

The heat transfer coefficient for condensation on a single horizontal tube is

$$\begin{aligned}h = h_{horizontal} &= 0.729\left[\frac{g\rho_l(\rho_l-\rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat}-T_s)D}\right]^{1/4}\\ &= 0.729\left[\frac{(9.81\ \text{m/s}^2)(965.3\ \text{kg/m}^3)(965.3-0.5978\ \text{kg/m}^3)(2314\times10^3\ \text{J/kg})(0.675\ \text{W/m}\cdot°\text{C})^3}{(0.315\times10^{-3}\ \text{kg/m}\cdot\text{s})(100-80)°\text{C}(0.05\ \text{m})}\right]^{1/4}\\ &= 8736\ \text{W/m}^2\cdot°\text{C}\end{aligned}$$

Then the average heat transfer coefficient for a 4-pipe high vertical tier becomes

$$h_{\text{horiz, N tubes}} = \frac{1}{N^{1/4}}h_{\text{horiz, 1 tube}} = \frac{1}{4^{1/4}}(8736\ \text{W/m}^2\cdot°\text{C}) = 6177\ \text{W/m}^2\cdot°\text{C}$$

The surface area for all 16 tubes is

$$A_s = N_{total}\pi DL = 16\pi(0.05\ \text{m})(2\ \text{m}) = 5.027\ \text{m}^2$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat}-T_s) = (6177\ \text{W/m}^2\cdot°\text{C})(5.027\ \text{m}^2)(100-80)°\text{C} = 621{,}000\ \text{W}$$

The rate of condensation of steam is determined from

$$\dot{m}_{condensation} = \frac{\dot{Q}}{h^*_{fg}} = \frac{621{,}000\ \text{J/s}}{2314\times10^3\ \text{J/kg}} = 0.2684\ \text{kg/s} = \mathbf{966\ kg/h}$$

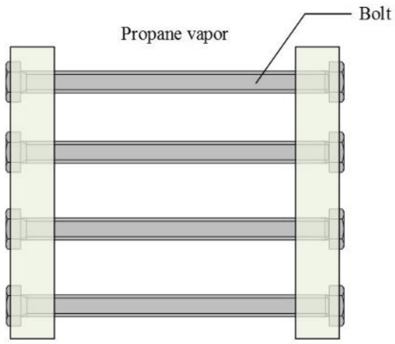
10-90 Saturated propane vapor condenses on four ASTM A437 B4B stainless steel bolts that are arranged in a vertical tier. The condensation occurs at 344 kPa. The highest rate of condensation of propane on the bolts, without cooling the them below the minimum suitable temperature set by the ASME Code for Process Piping, is to be determined.

Assumptions **1** Steady state conditions. **2** The bolts are isothermal.

Properties The saturation temperature of saturated propane at 344 kPa is −10°C (Table A-12), and the properties of saturated propane are from Table A-12:

At the saturation temperature, T_{sat} = −10°C,
h_{fg}= 387.8 kJ/kg, ρ_v = 7.635 kg/m^3.

At the film temperature, T_f = (−10 − 30)°C/2 = −20°C,
c_{pl} = 2368 J/kg·K, k_l = 0.1163 W/m·K, ρ_l = 554.7 kg/m^3, μ_l = 1.551 × 10^{-4} kg/m·s



Analysis The modified latent heat of vaporization is

$$h_{fg}^* = h_{fg} + 0.68\, c_{pl}(T_{\text{sat}} - T_s) = 387800 + 0.68(2368)(-10+30) = 420005 \text{ J/kg}$$

For average condensation heat transfer coefficient for the horizontal bolts arranged in a vertical tier of four is

$$h_N = \frac{0.729}{N^{1/4}}\left[\frac{g\rho_l(\rho_l-\rho_v)h_{fg}^* k_l^3}{\mu_l(T_{\text{sat}}-T_s)D}\right]^{1/4}$$

$$h_N = \frac{0.729}{4^{1/4}}\left[\frac{(9.81)(554.7)(554.7-7.635)(420005)(0.1163)^3}{(1.551\times10^{-4})(-10+30)(0.013)}\right]^{1/4} = 1362 \text{ W/m}^2\cdot\text{K}$$

The rate of heat transfer on the bolts during the condensation process is

$$\dot{Q} = h_N A_s(T_{\text{sat}}-T_s) = (1362 \text{ W/m}^2\cdot\text{K})\pi(4\times0.013\text{ m}\times0.15\text{ m})(-10+30)\text{K} = 667.5 \text{ W}$$

where $A_s = N\pi DL$

The highest rate of condensation on the bolts, without cooling them below −30°C, is

$$\dot{m}_{\text{condensate}} = \frac{\dot{Q}}{h_{fg}^*} = \frac{667.5 \text{ J/s}}{420005 \text{ J/kg}} = 0.00159 \text{ kg/s} = 1.59 \text{ g/s}$$

Discussion The highest condensation rate of propane that can occur on the ASTM A437 B4B bolts is 1.59 g/s. If the condensation rate is above that value, it would increase the heat transfer rate on the bolts. This means that the surface temperature of the bolts would be at below the minimum suitable temperature of −30°C set by the ASME Code for Process Piping.

10-91 Saturated water vapor at a pressure of 12.4 kPa condenses on a rectangular array of 100 horizontal tubes at 30°C. The condensation rate per unit length of is to be determined.

Assumptions **1** Steady operating condition exists. **2** The tube surfaces are isothermal.

Properties The properties of water at the saturation temperature of 50°C corresponding to 12.4 kPa are h_{fg} = 2383 kJ/kg and ρ_v = 0.0831 kg/m^3 (Table A-9). The properties of liquid water at the film temperature of $T_f = (T_{\text{sat}} + T_s)/2$ = 40°C are, from Table A-9,

ρ_l = 992.1 kg/m^3 $\qquad$ c_{pl} = 4179 J/kg·K

$\mu_l = 0.653 \times 10^{-3}$ kg/m·s $\qquad$ k_l = 0.631 W/m·K

Analysis The modified latent heat of vaporization is

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68c_{pl}(T_{\text{sat}} - T_s) \\ &= 2383\times10^3 + 0.68(4179)(50-30) \\ &= 2440\times10^3 \text{ J/kg} \end{aligned}$$

The heat transfer coefficient for condensation on a single horizontal tube is

$$\begin{aligned} h_{\text{horiz,1 tube}} &= 0.729\left[\frac{g\rho_l(\rho_l-\rho_v)h^*_{fg}k_l^3}{\mu_l(T_{\text{sat}}-T_s)D}\right]^{1/4} \\ &= 0.729\left[\frac{(9.81)(992.1)(992.1-0.0831)(2440\times10^3)(0.631)^3}{(0.653\times10^{-3})(50-30)(0.008)}\right]^{1/4} \\ &= 11{,}250 \text{ W/m}^2\cdot\text{K} \end{aligned}$$

Then the average heat transfer coefficient for a 5-tube high vertical tier becomes

$$h = h_{\text{horiz, }N\text{ tubes}} = \frac{1}{N^{1/4}}h_{\text{horiz,1 tube}} = \frac{1}{5^{1/4}}(11{,}250 \text{ W/m}^2\cdot\text{K}) = 7523 \text{ W/m}^2\cdot\text{K}$$

The rate of heat transfer per unit length during this condensation process becomes

$$\begin{aligned} \dot{Q}/L &= N_{\text{total}}\pi Dh(T_{\text{sat}}-T_s) \\ &= (100)\pi(0.008\text{ m})(7523 \text{ W/m}^2\cdot\text{K})(50-30)\text{ K} \\ &= 3.781\times10^5 \text{ W/m} \end{aligned}$$

The rate of condensation per unit length is

$$\frac{\dot{m}_{\text{condensation}}}{L} = \frac{\dot{Q}/L}{h^*_{fg}} = \frac{3.781\times10^5 \text{ W/m}}{2440\times10^3 \text{ J/kg}} = \mathbf{0.155\,kg/s\cdot m}$$

Discussion Therefore, water vapor condenses at a rate of 155 g/s per meter length of the tubes.

10-92 Saturated water vapor at a pressure of 12.4 kPa condenses on an array of 100 horizontal tubes at 30°C. The condensation rates for (*a*) a rectangular array of 5 tubes high and 20 tubes wide and (*b*) a square array of 10 tubes high and 10 tubes wide are to be determined.

Assumptions **1** Steady operating condition exists. **2** The tube surfaces are isothermal.

Properties The properties of water at the saturation temperature of 50°C corresponding to 12.4 kPa are h_{fg} = 2383 kJ/kg and ρ_v = 0.0831 kg/m^3 (Table A-9). The properties of liquid water at the film temperature of $T_f = (T_{\text{sat}} + T_s)/2$ = 40°C are, from Table A-9,

ρ_l = 992.1 kg/m^3 $\quad\quad$ c_{pl} = 4179 J/kg·K

$\mu_l = 0.653 \times 10^{-3}$ kg/m·s $\quad\quad$ k_l = 0.631 W/m·K

Analysis The modified latent heat of vaporization is

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68c_{pl}(T_{\text{sat}} - T_s) \\ &= 2383\times10^3 + 0.68(4179)(50-30) \\ &= 2440\times10^3 \text{ J/kg} \end{aligned}$$

The heat transfer coefficient for condensation on a single horizontal tube is

$$\begin{aligned} h_{\text{horiz, 1 tube}} &= 0.729\left[\frac{g\rho_l(\rho_l-\rho_v)h^*_{fg}k_l^3}{\mu_l(T_{\text{sat}}-T_s)D}\right]^{1/4} \\ &= 0.729\left[\frac{(9.81)(992.1)(992.1-0.0831)(2440\times10^3)(0.631)^3}{(0.653\times10^{-3})(50-30)(0.008)}\right]^{1/4} \\ &= 11{,}250 \text{ W/m}^2\cdot\text{K} \end{aligned}$$

(*a*) For a rectangular array, the average heat transfer coefficient for a 5-tube high vertical tier becomes

$$h = h_{\text{horiz, } N \text{ tubes}} = \frac{1}{N^{1/4}}h_{\text{horiz, 1 tube}} = \frac{1}{5^{1/4}}(11{,}250 \text{ W/m}^2\cdot\text{K}) = 7523 \text{ W/m}^2\cdot\text{K}$$

The rate of heat transfer during this condensation process becomes

$$\dot{Q} = N_{\text{total}}\pi DLh(T_{\text{sat}} - T_s) = (100)\pi(0.008 \text{ m})(1 \text{ m})(7523 \text{ W/m}^2\cdot\text{K})(50-30) \text{ K} = 3.781\times10^5 \text{ W}$$

The rate of condensation is

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{3.781\times10^5 \text{ W}}{2440\times10^3 \text{ J/kg}} = \mathbf{0.155\ kg/s} \qquad \text{(rectangular array)}$$

(*b*) For a square array, the average heat transfer coefficient for a 10-tube high vertical tier becomes

$$h = h_{\text{horiz, } N \text{ tubes}} = \frac{1}{N^{1/4}}h_{\text{horiz, 1 tube}} = \frac{1}{10^{1/4}}(11{,}250 \text{ W/m}^2\cdot\text{K}) = 6326 \text{ W/m}^2\cdot\text{K}$$

The rate of heat transfer during this condensation process becomes

$$\dot{Q} = N_{\text{total}}\pi DLh(T_{\text{sat}} - T_s) = (100)\pi(0.008 \text{ m})(1 \text{ m})(6326 \text{ W/m}^2\cdot\text{K})(50-30) \text{ K} = 3.180\times10^5 \text{ W}$$

The rate of condensation is

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{3.180\times10^5 \text{ W}}{2440\times10^3 \text{ J/kg}} = \mathbf{0.130\ kg/s} \qquad \text{(square array)}$$

Discussion The condensation rate of the rectangular array tube bank is about 20% higher than that of the square array tube bank:

$$\frac{\dot{m}_{\text{condensation}}\text{(rectangular)}}{\dot{m}_{\text{condensation}}\text{(square)}} = \frac{0.155 \text{ kg/s}}{0.130 \text{ kg/s}} = 1.19$$

10-93 Saturated refrigerant-134a vapor is condensed as it is flowing through a tube. With a given vapor flow rate at the entrance, the flow rate of the vapor at the exit is to be determined.

Assumptions **1** Steady operating condition exists. **2** The tube surfaces are isothermal.

Properties The properties of refrigerant-134a at the saturation temperature of 35°C corresponding to 888 kPa are h_{fg} = 168.2 kJ/kg, ρ_v = 43.41 kg/m^3, and $\mu_v = 1.327 \times 10^{-5}$ kg/m·s (Table A-10). The properties of liquid refrigerant-134a at the film temperature of $T_f = (T_{\text{sat}} + T_s)/2$ = 25°C are, from Table A-10,

ρ_l = 1207 kg/m^3 $\qquad$ c_{pl} = 1427 J/kg·K

$\mu_l = 2.012 \times 10^{-4}$ kg/m·s $\qquad$ k_l = 0.0833 W/m·K

Analysis The modified latent heat of vaporization for film condensation inside horizontal tube is

$$\begin{aligned}h^*_{fg} &= h_{fg} + \frac{3}{8}c_{pl}(T_{\text{sat}} - T_s)\\ &= 168.2\times10^3 + \frac{3}{8}(1427)(35-15)\\ &= 178.9\times10^3 \text{ J/kg}\end{aligned}$$

The Reynolds number associated with film condensation inside a horizontal tube is

$$\text{Re}_{\text{vapor}} = \left(\frac{\rho_v V_v D}{\mu_v}\right)_{\text{inlet}} = \frac{4\dot{m}_{v,\text{inlet}}}{\pi D\mu_v} = \frac{4(0.003\text{ kg/s})}{\pi(0.012\text{ m})(1.327\times10^{-5}\text{ kg/m}\cdot\text{s})} = 24{,}000 < 35{,}000$$

Hence, the heat transfer coefficient for film condensation inside a horizontal tube can be determined using

$$\begin{aligned}h = h_{\text{internal}} &= 0.555\left[\frac{g\rho_l(\rho_l-\rho_v)k_l^3 h^*_{fg}}{\mu_l(T_{\text{sat}}-T_s)D}\right]^{1/4}\\ &= 0.555\left[\frac{(9.81)(1207)(1207-43.41)(0.0833)^3(178.9\times10^3)}{(2.012\times10^{-4})(35-15)(0.012)}\right]^{1/4}\\ &= 1293\text{ W/m}^2\cdot\text{K}\end{aligned}$$

The rate of heat transfer during this condensation process becomes

$$\begin{aligned}\dot{Q} &= \pi DLh(T_{\text{sat}} - T_s)\\ &= \pi(0.012\text{ m})(0.25\text{ m})(1293\text{ W/m}^2\cdot\text{K})(35-15)\text{ K}\\ &= 243.7\text{ W}\end{aligned}$$

Then, the rate of condensation can be calculated as

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{243.7\text{ W}}{178.9\times10^3\text{ J/kg}} = 0.00136\text{ kg/s}$$

Applying the conservation of mass, the flow rate of vapor leaving the tube can be determined as

$$\dot{m}_{v,\text{inlet}} = \dot{m}_{v,\text{outlet}} + \dot{m}_{\text{condensation}} \quad\rightarrow\quad \dot{m}_{v,\text{outlet}} = \dot{m}_{v,\text{inlet}} - \dot{m}_{\text{condensation}}$$

$$\dot{m}_{v,\text{outlet}} = 0.003\text{ kg/s} - 0.00136\text{ kg/s} = \mathbf{0.00164\text{ kg/s}}$$

Discussion About 45% of the refrigerant-134a vapor that entered the tube is condensed inside it.

10-94 C&S Saturated ammonia vapor condenses inside a horizontal ASTM A268 TP443 stainless steel tube at 190 kPa. Determine whether the tube would comply with the minimum suitable temperature of the ASME Code for Process Piping, if the ammonia vapor exits the tube at a flow rate of 0.5 g/s.

Assumptions **1** Steady state conditions. **2** The tube is isothermal.

Properties The saturation temperature of ammonia at 190 kPa is −20°C (Table A-11), and the properties of saturated ammonia are from Table A-11:

At the saturation temperature, $T_{sat} = -20°C$,
$h_{fg} = 1329$ kJ/kg, $\rho_v = 1.603$ kg/m^3, $\mu_v = 8.669 \times 10^{-6}$ kg/m·s

At the film temperature, $T_f = (-20 - 30)°C/2 = -25°C$,
$c_{pl} = 4489$ J/kg·K, $k_l = 0.5968$ W/m·K, $\rho_l = 671.5$ kg/m^3, $\mu_l = 2.492 \times 10^{-4}$ kg/m·s

Analysis The modified latent heat of vaporization for film condensation inside a horizontal tube is

$$h^*_{fg} = h_{fg} + \frac{3}{8}c_{pl}(T_{\text{sat}} - T_s) = 1329000 + \frac{3}{8}(4489)(-20+30) = 1345834 \text{ J/kg}$$

The vapor Reynolds number associated with the film condensation inside the tube is

$$\text{Re}_{\text{vapor}} = \left(\frac{\rho_v V_v D}{\mu_v}\right) = \frac{4\dot{m}_{v,\text{in}}}{\pi D \mu_v} = \frac{4(0.005 \text{ kg/s})}{\pi(0.025 \text{ m})(8.669 \times 10^{-6}\text{kg/m}\cdot\text{s})} = 29375 < 35000$$

Thus, the heat transfer coefficient for film condensation inside the horizontal tube can be determine as

$$h = 0.555\left[\frac{g\rho_l(\rho_l - \rho_v)k_l^3}{\mu_l(T_{\text{sat}} - T_s)D}h^*_{fg}\right]^{1/4}$$

$$h = 0.555\left[\frac{(9.81)(671.5)(671.5 - 1.603)(0.5968)^3}{(2.492 \times 10^{-4})(-20+30)(0.025)}(1345834)\right]^{1/4} = 6621.7 \text{ W/m}^2\cdot\text{K}$$

The rate of heat transfer during the condensation process is

$$\dot{Q} = hA_s(T_{\text{sat}} - T_s) = (6622 \text{ W/m}^2\cdot\text{K})\pi(0.025\ m)(1 \text{ m})(-20+30)\text{K} = 5201 \text{ W}$$

The highest rate of condensation inside the tube that can occur without cooling the tube wall below −30°C is

$$\dot{m}_{\text{condensate}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{5201 \text{ J/s}}{1345834 \text{ J/kg}} = 0.00386 \text{ kg/s} = 3.86 \text{ g/s}$$

So, the minimum flow rate of ammonia vapor at the tube exit is

$$\dot{m}_{v,\text{out}} = \dot{m}_{v,\text{in}} - \dot{m}_{\text{condensate}} = 5 \text{ g/s} - 3.86 \text{ g/s} = 1.14 \text{ g/s} > 0.5\ g/s$$

Discussion If the ammonia vapor flow rate at the tube exit is 0.5 g/s, this means that the rate of condensation of ammonia inside the tube is 4.5 g/s. This rate of condensation is greater than 3.86 g/s, which occurs at a tube wall temperature of −30°C. For a condensation rate of ammonia greater than 3.86 g/s to occur inside the horizontal tube, the tube wall temperature would need to be below −30°C, thus violating the ASME Code for Process Piping. For the ASTM A268 TP443 tube to comply with the ASME code, the flow rate of ammonia vapor at the tube exit needs to be 1.14 g/s or higher.

10-95 Saturated ammonia vapor is condensed as it flows through a tube. With a given vapor flow rate at the exit, the flow rate of the vapor at the inlet is to be determined.

Assumptions **1** Steady operating condition exists. **2** The tube surfaces are isothermal. **3** The Reynolds number of the vapor at the inlet is less than 35,000 (this assumption will be verified).

Properties The properties of ammonia at the saturation temperature of 25°C corresponding to 1003 kPa are h_{fg} = 1166 kJ/kg, ρ_v = 7.809 kg/m^3, and μ_v = 1.037 × 10^{-5} kg/m·s (Table A-11). The properties of liquid ammonia at the film temperature of T_f = $(T_{\text{sat}} + T_s)/2$ = 15°C are, from Table A-11,

ρ_l = 617.5 kg/m^3

c_{pl} = 4709 J/kg·K

μ_l = 1.606 × 10^{-4} kg/m·s

k_l = 0.5042 W/m·K

Analysis The modified latent heat of vaporization for film condensation inside horizontal tube is

$$\begin{aligned} h^*_{fg} &= h_{fg} + \frac{3}{8}c_{pl}(T_{\text{sat}} - T_s) \\ &= 1166 \times 10^3 + \frac{3}{8}(4709)(25 - 5) \\ &= 1201 \times 10^3 \text{ J/kg} \end{aligned}$$

Assuming Re$_{\text{vapor}}$< 35,000 and the heat transfer coefficient for film condensation inside a horizontal tube can be determined using

$$\begin{aligned} h = h_{\text{internal}} &= 0.555\left[\frac{g\rho_l(\rho_l - \rho_v)k_l^3 h^*_{fg}}{\mu_l(T_{\text{sat}} - T_s)D}\right]^{1/4} \\ &= 0.555\left[\frac{(9.81)(617.5)(617.5 - 7.809)(0.5042)^3(1201\times10^3)}{(1.606\times10^{-4})(25-5)(0.025)}\right]^{1/4} \\ &= 5091 \text{ W/m}^2\cdot\text{K} \end{aligned}$$

The rate of heat transfer during this condensation process becomes

$$\begin{aligned} \dot{Q} &= \pi DLh(T_{\text{sat}} - T_s) \\ &= \pi(0.025\text{ m})(0.5\text{ m})(5091\text{ W/m}^2\cdot\text{K})(25-5)\text{ K} \\ &= 3998\text{ W} \end{aligned}$$

Then, the rate of condensation can be calculated as

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{3998\text{ W}}{1201\times10^3\text{ J/kg}} = 0.00333\text{ kg/s}$$

Applying the conservation of mass, the flow rate of vapor leaving the tube can be determined as

$$\dot{m}_{v,\text{ inlet}} = \dot{m}_{v,\text{ outlet}} + \dot{m}_{\text{condensation}}$$

$$\dot{m}_{v,\text{ inlet}} = 0.002\text{ kg/s} + 0.00333\text{ kg/s} = \mathbf{0.00533\ kg/s}$$

Discussion The Reynolds number associated with film condensation inside a horizontal tube is

$$\text{Re}_{\text{vapor}} = \left(\frac{\rho_v V_v D}{\mu_v}\right)_{\text{inlet}} = \frac{4\dot{m}_{v,\text{ inlet}}}{\pi D\mu_v} = \frac{4(0.00533\text{ kg/s})}{\pi(0.025\text{ m})(1.037\times10^{-5}\text{ kg/m}\cdot\text{s})} = 26{,}200 < 35{,}000$$

Thus, the Re$_{\text{vapor}}$< 35,000 assumption is appropriate for this problem.

10-96 A copper tube transporting cold coolant has a surface temperature of 5°C and condenses moist air at 25°C. The rate of condensation is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Isothermal tube surface.

Properties The required property of water at the saturation temperature T_{sat} = 25°C is h_{fg} = 2442 × 10^3 J/kg (Table A-9). The required property of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2 = (25 + 5)/2$ = 15°C is c_{pl} = 4185 J/kg·K (Table A-9).

Analysis The modified latent heat of vaporization is

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68 c_{pl}(T_{sat} - T_s) \\ &= 2442 \times 10^3 \text{ J/kg} + 0.68(4185 \text{ J/kg}\cdot\text{K})(25-5) \text{ K} = 2.4989 \times 10^6 \text{ J/kg} \end{aligned}$$

The dropwise condensation heat transfer coefficient, for 22°C <T_{sat}< 100°C, is determined to be

$$\begin{aligned} h = h_{\text{dropwise}} &= 51{,}104 + 2044 T_{sat} \\ &= 51{,}104 + 2044(25°\text{C}) = 102{,}204 \text{ W/m}^2\cdot\text{K} \end{aligned}$$

The rate of heat transfer to the tube during this condensation process is

$$\begin{aligned} \dot{Q} &= hA_s(T_{sat} - T_s) = h(\pi DL)(T_{sat} - T_s) \\ &= (102{,}204 \text{ W/m}^2\cdot\text{K})\pi(0.025 \text{ m})(10 \text{ m})(25-5) \text{ K} \\ &= 1.6054 \times 10^6 \text{ W} \end{aligned}$$

Thus, the rate of condensation is

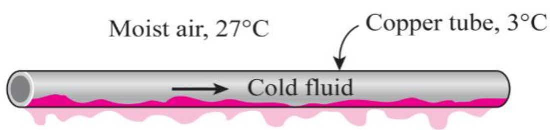
$$\dot{m} = \frac{\dot{Q}}{h^*_{fg}} = \frac{1.6054 \times 10^6 \text{ J/s}}{2.4989 \times 10^6 \text{ J/kg}} = \mathbf{0.6424 \text{ kg/s}}$$

Discussion The heat transfer rate during the dropwise condensation can be increased by increasing the surface area of the tube, and thereby increasing the rate of condensation as well.

10-97 A copper tube that is used for transporting cold fluid is causing condensation of moist air on its outer surface. The rate of moisture condensation is to be determined so that a system for removing the condensate can be sized to alleviate the risk of electrical hazard.

Assumptions **1** Steady operating conditions exist. **2** Isothermal tube surface.

Properties The required property of water at the saturation temperature T_{sat} = 27°C is h_{fg} = 2438 × 10^3 J/kg (Table A-9). The required property of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2 = (27 + 3)/2$ = 15°C is c_{pl} = 4185 J/kg·K (Table A-9).



Analysis Since dropwise condensation can occur, and it will have higher rate of heat transfer and therefore higher condensation rate than film condensation. The system must be able to handle the dropwise condensation rate.

The modified latent heat of vaporization is

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68c_{pl}(T_{sat} - T_s) \\ &= 2438\times10^3 \text{ J/kg} + 0.68(4185 \text{ J/kg}\cdot\text{K})(27-3)\text{ K} = 2.5063\times10^6 \text{ J/kg} \end{aligned}$$

The dropwise condensation heat transfer coefficient, for 22°C < T_{sat} < 100°C, is determined to be

$$\begin{aligned} h = h_{dropwise} &= 51{,}104 + 2044T_{sat} \\ &= 51{,}104 + 2044(27°\text{C}) = 106{,}292 \text{ W/m}^2\cdot\text{K} \end{aligned}$$

The rate of heat transfer during the condensation process is

$$\begin{aligned} \dot{Q} &= hA_s(T_{sat} - T_s) \\ &= h(\pi DL)(T_{sat} - T_s) \\ &= (106{,}292 \text{ W/m}^2\cdot\text{K})\pi(0.025\text{ m})(10\text{ m})(27-3)\text{ K} \\ &= 2.0036\times10^6 \text{ W} \end{aligned}$$

Thus, the rate of dropwise condensation is

$$\dot{m} = \frac{\dot{Q}}{h^*_{fg}} = \frac{2.0036\times10^6 \text{ J/s}}{2.5063\times10^6 \text{ J/kg}} = \mathbf{0.799\ kg/s}$$

Discussion In order to alleviate the risk of electrical hazard, the system must be able to remove the condensate at a rate of 0.8 kg/s. Note that dropwise condensation rate are higher than film condensation rate (by 10 times or more), thus a system capable of removing the condensate at a rate of 0.8 kg/s can also handle the condensate from film condensation.

10-98 Steam condenses at 60°C on a copper tube that is maintained with a surface temperature of 40°C. The condensation rates during film condensation and dropwise condensation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Isothermal tube surface.

Properties The properties of water at the saturation temperature of 60°C are h_{fg} = 2359 × 10^3 J/kg and ρ_v = 0.1304 kg/m^3 (Table A-9). The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2 = (60 + 40)/2 = 50$°C are (Table A-9)

$$\begin{aligned}
\rho_l &= 988.1\ \text{kg/m}^3 \\
\mu_l &= 0.547\times10^{-3}\ \text{kg/m}\cdot\text{s} \\
\nu_l &= \mu_l/\rho_l = 0.554\times10^{-6}\ \text{m}^2/\text{s} \\
c_{pl} &= 4181\ \text{J/kg}\cdot\text{K} \\
k_l &= 0.644\ \text{W/m}\cdot\text{K}
\end{aligned}$$

Analysis The modified latent heat of vaporization is

$$\begin{aligned}
h^*_{fg} &= h_{fg} + 0.68c_{pl}(T_{\text{sat}} - T_s) \\
&= 2359\times10^3\ \text{J/kg} + 0.68(4181\ \text{J/kg}\cdot\text{K})(60-40)\ \text{K} = 2.4159\times10^6\ \text{J/kg}
\end{aligned}$$

(a) The film condensation heat transfer coefficient is determined from

$$\begin{aligned}
h_{\text{film}} = h_{\text{horiz}} &= 0.729\left[\frac{g\rho_l(\rho_l-\rho_v)h^*_{fg}k_l^3}{\mu_l(T_{\text{sat}}-T_s)D}\right]^{1/4} \\
&= 0.729\left[\frac{(9.81\ \text{m/s}^2)(988.1\ \text{kg/m}^3)(988.1-0.1304)\text{kg/m}^3(2.4159\times10^6\ \text{J/kg})(0.644\ \text{W/m}\cdot\text{K})^3}{(0.547\times10^{-3}\ \text{kg/m}\cdot\text{s})(60-40)\text{K}(0.025\ \text{m})}\right]^{1/4} \\
&= 8937.7\ \text{W/m}^2\cdot\text{K}
\end{aligned}$$

Thus, the rate of film condensation is

$$\begin{aligned}
\dot m_{\text{film}} &= \frac{\dot Q}{h^*_{fg}} = \frac{h_{\text{film}}(\pi DL)(T_{\text{sat}}-T_s)}{h^*_{fg}} \\
&= \frac{(8937.7\ \text{W/m}^2\cdot\text{K})\pi(0.025\ \text{m})(15\ \text{m})(60-40)\ \text{K}}{2.4159\times10^6\ \text{J/kg}} = \mathbf{0.08717\ kg/s}
\end{aligned}$$

(b) The dropwise condensation heat transfer coefficient, for 22°C < T_{sat} < 100°C, is determined from

$$\begin{aligned}
h_{\text{dropwise}} &= 51{,}104 + 2044T_{\text{sat}} \\
&= 51{,}104 + 2044(60°\text{C}) = 173{,}744\ \text{W/m}^2\cdot\text{K}
\end{aligned}$$

Thus, the rate of dropwise condensation is

$$\begin{aligned}
\dot m_{\text{dropwise}} &= \frac{\dot Q}{h^*_{fg}} = \frac{h_{\text{dropwise}}(\pi DL)(T_{\text{sat}}-T_s)}{h^*_{fg}} \\
&= \frac{(173{,}744\ \text{W/m}^2\cdot\text{K})\pi(0.025\ \text{m})(15\ \text{m})(60-40)\ \text{K}}{2.4159\times10^6\ \text{J/kg}} = \mathbf{1.695\ kg/s}
\end{aligned}$$

Discussion The dropwise condensation rate is 19.4 times greater than that of film condensation. This is because the convection heat transfer coefficient for dropwise condensation is greater than that of film condensation at the same factor, $h_{\text{dropwise}}/h_{\text{film}}$ = 19.4.

With dropwise condensation, there is no liquid film to impede heat transfer. Therefore, a much higher heat transfer coefficient can be achieved in dropwise condensation than in film condensation.

Special Topic: Non-Boiling Two-Phase Flow Heat Transfer

10-99 The flow quality of a non-boiling two-phase flow in a tube with $\dot{m}_l / \dot{m}_g = 300$ is to be determined.

Assumptions **1** Steady operating condition exists. **2** Two-phase flow is non-boiling and it does not involve phase change. **3** Fluid properties are constant.

Analysis The flow quality is given as

$$x = \frac{\dot{m}_g}{\dot{m}_l + \dot{m}_g}$$

Hence, the equation can be rearranged as

$$x = \frac{\dot{m}_g}{\dot{m}_l + \dot{m}_g} = \frac{\dot{m}_g / \dot{m}_g}{(\dot{m}_l + \dot{m}_g) / \dot{m}_g} = \frac{1}{\dot{m}_l / \dot{m}_g + 1}$$

Thus, the flow quality is

$$x = \frac{1}{300 + 1} = \mathbf{0.00332}$$

Discussion The flow quality is a dimensionless parameter.

10-100 The flow quality and the mass flow rates of the gas and the liquid for a non-boiling two-phase flow, where $V_{sl} = 3V_{sg}$, are to be determined.

Assumptions **1** Steady operating condition exists. **2** Two-phase flow is non-boiling and it does not involve phase change. **3** Fluid properties are constant.

Properties The densities of the gas and liquid are given to be ρ_g = 8.5 kg/m^3 and ρ_l = 855 kg/m^3, respectively.

Analysis The mass flow rate of gas can be calculated using

$$\dot{m}_g = \rho_g V_{sg} A_c = \rho_g V_{sg} \pi \frac{D^2}{4}$$

$$= (8.5\ \text{kg/m}^3)(0.8\ \text{m/s})\pi \frac{(0.102\ \text{m})^2}{4} = \mathbf{0.0556\ kg/s}$$

Then, the mass flow rate of liquid is (with $V_{sl} = 3V_{sg}$)

$$\dot{m}_l = \rho_l V_{sl} A_c = 3\rho_l V_{sg} \pi \frac{D^2}{4}$$

$$= 3(855\ \text{kg/m}^3)(0.8\ \text{m/s})\pi \frac{(0.102\ \text{m})^2}{4} = \mathbf{16.8\ kg/s}$$

Thus, the flow quality is

$$x = \frac{\dot{m}_g}{\dot{m}_l + \dot{m}_g} = \frac{0.0556}{16.8 + 0.0556} = \mathbf{0.00330}$$

Discussion The total mass flow rate of gas and liquid for this two-phase flow is simply

$$\dot{m}_{\text{tot}} = \dot{m}_l + \dot{m}_g = 16.8\ \text{kg/s} + 0.0556\ \text{kg/s} = 16.86\ \text{kg/s}$$

10-101 The mass flow rate of air and the superficial velocities of air and engine oil for a non-boiling two-phase flow in a tube are to be determined.

Assumptions **1** Steady operating condition exists. **2** Two-phase flow is non-boiling and it does not involve phase change. **3** Fluid properties are constant.

Properties The densities of air and engine oil at the bulk mean temperature T_b = 140°C are ρ_g = 0.8542 kg/m^3 (Table A-15) and ρ_l = 816.8 kg/m^3 (Table A-13), respectively.

Analysis The flow quality is given as

$$x = \frac{\dot{m}_g}{\dot{m}_l + \dot{m}_g} = \frac{\dot{m}_g / \dot{m}_g}{(\dot{m}_l + \dot{m}_g)/\dot{m}_g} = \frac{1}{\dot{m}_l / \dot{m}_g + 1}$$

or

$$\frac{\dot{m}_l}{\dot{m}_g} + 1 = \frac{1}{x} \qquad \rightarrow \qquad \frac{\dot{m}_l}{\dot{m}_g} = \frac{1-x}{x}$$

With known liquid (engine oil) mass flow rate and flow quality, the gas (air) mass flow rate is determined using

$$\dot{m}_g = \frac{x}{1-x}\dot{m}_l = \frac{2.1\times 10^{-3}}{1-2.1\times 10^{-3}}(0.9\text{ kg/s}) = \mathbf{0.00189\ kg/s}$$

From the gas and liquid mass flow rates, the superficial gas and liquid velocities can be calculated:

$$V_{sg} = \frac{\dot{m}_g}{\rho_g A} = \frac{4\dot{m}_g}{\rho_g \pi D^2} = \frac{4(0.00189\text{ kg/s})}{(0.8542\text{ kg/m}^3)\pi(0.025\text{ m})^2} = \mathbf{4.51\ m/s}$$

$$V_{sl} = \frac{\dot{m}_l}{\rho_l A} = \frac{4\dot{m}_l}{\rho_l \pi D^2} = \frac{4(0.9\text{ kg/s})}{(816.8\text{ kg/m}^3)\pi(0.025\text{ m})^2} = \mathbf{2.25\ m/s}$$

Discussion The superficial velocity of air is twice that of the engine oil.

10-102 Starting with the two-phase non-boiling heat transfer correlation, the expression that is appropriate for the case when only water is flowing in the tube is to be determined.

Assumptions **1** Steady operating condition exists. **2** Two-phase flow is non-boiling and it does not involve phase change. **3** Fluid properties are constant.

Analysis The two-phase non-boiling heat transfer correlation is given as

$$h_{tp} = h_l F_p \left[1 + 0.55\left(\frac{x}{1-x}\right)^{0.1}\left(\frac{1-F_p}{F_p}\right)^{0.4}\left(\frac{Pr_g}{Pr_l}\right)^{0.25}\left(\frac{\mu_l}{\mu_g}\right)^{0.25}\left(I^*\right)^{0.25}\right]$$

where

$$F_p = (1-\alpha) + \alpha\left[\frac{2}{\pi}\tan^{-1}\left(\sqrt{\frac{\rho_g (V_g - V_l)^2}{gD(\rho_l - \rho_g)}}\right)\right]^2 \qquad \text{and} \qquad x = \frac{\dot{m}_g}{\dot{m}_l + \dot{m}_g}$$

For the situation when the air flow is shut off and only water is flowing in the pipe, we have $\dot{m}_g = 0$ and $\alpha = 0$. Hence, we get

$$F_p = (1-0) + 0 = 1 \qquad \text{and} \qquad x = 0$$

Thus, the two-phase non-boiling heat transfer correlation becomes

$$h_{tp,\ \alpha=0} = h_l (1)\left[1 + 0.55\left(\frac{0}{1-0}\right)^{0.1}\left(\frac{1-1}{1}\right)^{0.4}\left(\frac{Pr_g}{Pr_l}\right)^{0.25}\left(\frac{\mu_l}{\mu_g}\right)^{0.25}\left(I^*\right)^{0.25}\right] = h_l (1)\ [1+0] = h_l$$

The liquid phase heat transfer coefficient is calculated using:

$$h_l = 0.027\,\mathrm{Re}_l^{4/5}\,\mathrm{Pr}_l^{1/3}\left(\frac{k_l}{D}\right)\left(\frac{\mu_l}{\mu_s}\right)^{0.14}$$

where the *in situ* liquid Reynolds number is

$$\mathrm{Re}_l = \frac{4\dot{m}_l}{\pi\sqrt{1-\alpha}\,\mu_l D} = \frac{4\dot{m}_l}{\pi\sqrt{1-0}\,\mu_l D} = \frac{4\dot{m}_l}{\pi\mu_l D}$$

Therefore, we have

$$h_{tp,\ \alpha=0} = h_l = 0.027\,\mathrm{Re}_l^{4/5}\,\mathrm{Pr}_l^{1/3}\left(\frac{k_l}{D}\right)\left(\frac{\mu_l}{\mu_s}\right)^{0.14}$$

Discussion When only water is flowing in the tube, the two-phase non-boiling heat transfer correlation is reduced to a familiar equation for internal forced convection.

10-103 Air-water slug flows through a 25.4-mm diameter horizontal tube in microgravity condition. Using the non-boiling two-phase heat transfer correlation, the two-phase heat transfer coefficient (h_{tp}) is to be determined

Assumptions **1** Steady operating condition exists. **2** Two-phase flow is non-boiling and it does not involve phase change. **3** Fluid properties are constant.

Properties The properties of water (liquid) are given to be $\mu_l = 85.5\times10^{-5}$ kg/m·s, $\mu_s = 73.9\times10^{-5}$ kg/m·s, $\rho_l = 997$ kg/m^3, $k_l = 0.613$ W/m·K, and $\Pr_l = 5.0$. The properties of air (gas) are given to be $\mu_g = 18.5\times10^{-6}$ kg/m·s, $\rho_g = 1.16$ kg/m^3, and $\Pr_g = 0.71$.

Analysis From the superficial gas and liquid velocities, and void fraction, the gas and liquid velocities can be calculated as

$$V_g = \frac{V_{sg}}{\alpha} = \frac{0.3\text{ m/s}}{0.27} = 1.11\text{ m/s}$$

$$V_l = \frac{V_{sl}}{1-\alpha} = \frac{0.544\text{ m/s}}{1-0.27} = 0.745\text{ m/s}$$

The gas and liquid mass flow rates are calculated as

$$\dot m_g = \rho_g V_{sg} A_c = \rho_g V_{sg}\pi\frac{D^2}{4} = (1.16\text{ kg/m}^3)(0.3\text{ m/s})\pi\frac{(0.0254\text{ m})^2}{4} = 1.76\times10^{-4}\text{ kg/s}$$

$$\dot m_l = \rho_l V_{sl} A_c = \rho_l V_{sl}\pi\frac{D^2}{4} = (997\text{ kg/m}^3)(0.544\text{ m/s})\pi\frac{(0.0254\text{ m})^2}{4} = 0.275\text{ kg/s}$$

Using the gas and liquid mass flow rates, the quality is determined to be

$$x = \frac{\dot m_g}{\dot m_l + \dot m_g} = \frac{1.76\times10^{-4}}{0.275+1.76\times10^{-4}} = 6.40\times10^{-4}$$

The flow pattern factor (F_p) can be calculated using

$$F_p = (1-\alpha) + \alpha\left[\frac{2}{\pi}\tan^{-1}\left(\sqrt{\frac{\rho_g\,(V_g - V_l)^2}{gD\,(\rho_l-\rho_g)}}\right)\right]^2$$

$$= (1-0.27)+(0.27)\left[\frac{2}{\pi}\tan^{-1}\left(\sqrt{\frac{(1.16\text{ kg/m}^3)(1.11\text{ m/s}-0.745\text{ m/s})^2}{(9.81\text{ m/s}^2)(0.0254\text{ m})\,(997\text{ kg/m}^3-1.16\text{ kg/m}^3)}}\right)\right]^2 = 0.730$$

The liquid phase heat transfer coefficient is calculated using:

$$h_l = 0.027\,\text{Re}_l^{4/5}\,\text{Pr}_l^{1/3}\left(\frac{k_l}{D}\right)\left(\frac{\mu_l}{\mu_s}\right)^{0.14}$$

$$= 0.027(18870)^{4/5}(5.0)^{1/3}\left(\frac{0.613\text{ W/m}\cdot\text{K}}{0.0254\text{ m}}\right)\left(\frac{85.5\times10^{-5}\text{ kg/m}\cdot\text{s}}{73.9\times10^{-5}\text{ kg/m}\cdot\text{s}}\right)^{0.14} = 2995\text{ W/m}^2\cdot\text{K}$$

where the *in situ* liquid Reynolds number is

$$\text{Re}_l = \frac{4\dot m_l}{\pi\sqrt{1-\alpha}\,\mu_l D} = \frac{4(0.275\text{ kg/s})}{\pi\sqrt{1-0.27}\,(85.5\times10^{-5}\text{ kg/m}\cdot\text{s})(0.0254\text{ m})} = 18870$$

The inclination factor (I^*) has a value of one for horizontal tube ($\theta = 0$). Thus, using the general two-phase heat transfer correlation, the value for h_{tp} is estimated to be

$$\frac{h_{tp}}{h_l} = F_p\left[1+0.55\left(\frac{x}{1-x}\right)^{0.1}\left(\frac{1-F_p}{F_p}\right)^{0.4}\left(\frac{Pr_g}{Pr_l}\right)^{0.25}\left(\frac{\mu_l}{\mu_g}\right)^{0.25}\left(I^*\right)^{0.25}\right]$$

$$= (0.730)\left[1+0.55\left(\frac{6.40\times10^{-4}}{1-6.40\times10^{-4}}\right)^{0.1}\left(\frac{1-0.730}{0.730}\right)^{0.4}\left(\frac{0.71}{5.0}\right)^{0.25}\left(\frac{85.5\times10^{-5}\text{ kg/m}\cdot\text{s}}{18.5\times10^{-6}\text{ kg/m}\cdot\text{s}}\right)^{0.25}\right] = 0.9369$$

or $\quad h_{tp} = 0.9369 h_l = 0.9369(2995\text{ W/m}^2\cdot\text{K}) = \mathbf{2810\text{ W/m}^2\cdot\text{K}}$

Discussion The non-boiling two-phase heat transfer coefficient (h_{tp}) is about 7% lower than the liquid phase heat transfer coefficient (h_l).

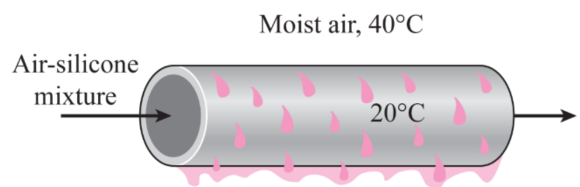
10-104 A two-phase flow of air and silicone (Dow Corning 200® Fluid, 5 cs) is being transported in an 11.7-mm diameter horizontal tube, where condensation occurs on the tube outer surface. The overall convection heat transfer coefficient is to be determined.

Assumptions **1** Steady operating condition exists. **2** Two-phase flow inside tube is non-boiling and does not involve phase change. **3** Fluid properties are constant. **4** The thermal resistance of the tube wall is negligible. **5** Isothermal tube surface. **6** Film condensation occurs on the tube outer surface.

Properties Inside the tube: The properties of liquid silicone (liquid) are given to be $\mu_l = 45.7\times10^{-4}$ kg/m·s, $\rho_l = 913$ kg/m³, $k_l = 0.117$ W/m·K, $\sigma = 19.7\times10^{-3}$ N/m and $\text{Pr}_l = 64$. The properties of air (gas) are given to be $\mu_g = 18.4\times10^{-6}$ kg/m·s, $\rho_g = 1.19$ kg/m³, $\text{Pr}_g = 0.71$.

Outside the tube: The properties of water at the saturation temperature of 40°C are $h_{fg} = 2407\times10^3$ J/kg and $\rho_v = 0.0512$ kg/m³ (Table A-9). The properties of liquid water at the film temperature of $T_f = (T_{\text{sat}} + T_s)/2 = (40 + 20)/2 = 30$°C are (Table A-9),

$$\begin{aligned}
\rho_{l,f} &= 996.0\ \text{kg/m}^3\\
\mu_{l,f} &= 0.798\times10^{-3}\ \text{kg/m}\cdot\text{s}\\
\nu_{l,f} &= \mu_l/\rho_l = 0.801\times10^{-6}\ \text{m}^2/\text{s}\\
c_{pl,f} &= 4178\ \text{J/kg}\cdot\text{K}\\
k_{l,f} &= 0.615\ \text{W/m}\cdot\text{K}\\
\text{Pr}_f &= 5.42
\end{aligned}$$



Analysis Inside the tube: The flow pattern factor (F_P) can be calculated using

$$\begin{aligned}
F_P &= (1-\alpha) + \alpha\left[\frac{2}{\pi}\tan^{-1}\left(\sqrt{\frac{\rho_g(V_g - V_l)^2}{gD(\rho_l - \rho_g)}}\right)\right]^2\\
&= (1-0.011) + (0.011)\left[\frac{2}{\pi}\tan^{-1}\left(\sqrt{\frac{(1.19\ \text{kg/m}^3)(13.5\ \text{m/s} - 9.34\ \text{m/s})^2}{(9.81\ \text{m/s}^2)(0.0117\ \text{m})(913\ \text{kg/m}^3 - 1.19\ \text{kg/m}^3)}}\right)\right]^2\\
&= 0.9898
\end{aligned}$$

The inclination factor (I^*) for horizontal tube ($\theta = 0°$) is calculated to be

$$I^* = 1 + \frac{(\rho_i - \rho_g)gD^2}{\sigma}\left|\sin\theta\right| = 1$$

The liquid phase heat transfer coefficient is calculated using the Seder and Tate (1936) equation:

$$\begin{aligned}
h_l &= 0.027\,\text{Re}_l^{0.8}\,\text{Pr}^{1/3}\left(\frac{\mu_l}{\mu_s}\right)^{0.14}\left(\frac{k_l}{D}\right)\\
&= 0.027(21718)^{0.8}(64)^{1/3}\left(\frac{45.7}{39.8}\right)^{0.14}\left(\frac{0.117\ \text{W/m}\cdot\text{K}}{0.0117\ \text{m}}\right)\\
&= 3246\ \text{W/m}^2\cdot\text{K}
\end{aligned}$$

where the *in situ* liquid Reynolds number is

$$\text{Re}_l = \frac{4\dot{m}_l}{\pi\sqrt{1-\alpha}\,\mu_l D} = \frac{4(0.907\ \text{kg/s})}{\pi\sqrt{1-0.011}(45.7\times10^{-4}\ \text{kg/m}\cdot\text{s})(0.0117\ \text{m})} = 21718$$

Using the general two-phase heat transfer correlation, Eq. (10-38), the two-phase heat transfer coefficient (h_{tp}) is estimated to be

$$h_{tp} = h_l F_P \left[1 + 0.55\left(\frac{x}{1-x}\right)^{0.1}\left(\frac{1-F_P}{F_P}\right)^{0.4}\left(\frac{Pr_g}{Pr_l}\right)^{0.25}\left(\frac{\mu_l}{\mu_g}\right)^{0.25}\left(I^*\right)^{0.25}\right]$$

$$= (3246\ \text{W/m}^2\cdot\text{K})(0.9898)\left[1 + 0.55\left(\frac{2.08\times10^{-5}}{1-2.08\times10^{-5}}\right)^{0.1}\left(\frac{1-0.9898}{0.9898}\right)^{0.4}\left(\frac{0.71}{64}\right)^{0.25}\left(\frac{45.7\times10^{-4}}{18.4\times10^{-6}}\right)^{0.25}\right]$$

$$h_i = h_{tp} = 3337\ \text{W/m}^2\cdot\text{K}$$

Outside the tube: The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl,f}(T_{\text{sat}} - T_s) = 2407\times10^3\ \text{J/kg} + 0.68\times(4182\ \text{J/kg}\cdot\text{K})(40-20)\text{K} = 2464\times10^3\ \text{J/kg}$$

The heat transfer coefficient for condensation on a single horizontal tube is

$$h_o = h_{\text{horizontal}} = 0.729\left[\frac{g\rho_{l,f}(\rho_{l,f}-\rho_v)h^*_{fg}k^3_{l,f}}{\mu_{l,f}(T_{\text{sat}}-T_s)D}\right]^{1/4}$$

$$= 0.729\left[\frac{(9.81\ \text{m/s}^2)(996\ \text{kg/m}^3)(996-0.05\ \text{kg/m}^3)(2464\times10^3\ \text{J/kg})(0.615\ \text{W/m}\cdot\text{K})^3}{(0.798\times10^{-3}\ \text{kg/m}\cdot\text{s})(40-20)\text{K}(0.0117\ \text{m})}\right]^{1/4}$$

$$= 9584\ \text{W/m}^2\cdot\text{K}$$

Noting that the thermal resistance of the tube is negligible, the overall heat transfer coefficient becomes

$$U = \frac{1}{1/h_i + 1/h_o} = \frac{1}{1/3337 + 1/9584} = \mathbf{2475\ W/m^2\cdot K}$$

Discussion The condensation heat transfer coefficient is almost 3 times higher than the non-boiling heat transfer coefficient. This is expected, as heat transfer coefficient involving phase change is much higher than that without phase change.

10-105 Air-water mixture is flowing in a 5° inclined tube with diameter of 25.4 mm, and the mixture superficial gas and liquid velocities are 1 m/s and 2 m/s, respectively. The two-phase heat transfer coefficient (h_{tp}) is to be determined.

Assumptions **1** Steady operating condition exists. **2** Two-phase flow is non-boiling and it does not involve phase change. **3** Fluid properties are constant.

Properties The properties of water (liquid) at bulk mean temperature $T_b = (T_i + T_e)/2 = 45°C$ are, from Table A-9, $\mu_l = 0.596\times10^{-3}$ kg/m·s, $\rho_l = 990.1$ kg/m^3, $k_l = 0.637$ W/m·K, and $Pr_l = 3.91$. The properties of air (gas) at bulk mean temperature $T_b = 45°C$ are, from Table A-15, $\mu_g = 1.941\times10^{-5}$ kg/m·s, $\rho_g = 1.109$ kg/m^3, and $Pr_g = 0.7241$. Also, at $T_s = 80°C$ we get $\mu_s = 0.355\times10^{-3}$ kg/m·s from Table A-9.

Analysis From the superficial gas and liquid velocities, and void fraction, the gas and liquid velocities can be calculated as

$$V_g = \frac{V_{sg}}{\alpha} = \frac{1\text{ m/s}}{0.33} = 3.030\text{ m/s} \qquad V_l = \frac{V_{sl}}{1-\alpha} = \frac{2\text{ m/s}}{1-0.33} = 2.985\text{ m/s}$$

The gas and liquid mass flow rates are calculated as

$$\dot{m}_g = \rho_g V_{sg} A_c = \rho_g V_{sg}\pi\frac{D^2}{4} = (1.109\text{ kg/m}^3)(1\text{ m/s})\pi\frac{(0.0254\text{ m})^2}{4} = 5.619\times10^{-4}\text{ kg/s}$$

$$\dot{m}_l = \rho_l V_{sl} A_c = \rho_l V_{sl}\pi\frac{D^2}{4} = (990.1\text{ kg/m}^3)(2\text{ m/s})\pi\frac{(0.0254\text{ m})^2}{4} = 1.003\text{ kg/s}$$

Using the gas and liquid mass flow rates, the quality is determined to be

$$x = \frac{\dot{m}_g}{\dot{m}_l + \dot{m}_g} = \frac{5.619\times10^{-4}}{1.003 + 5.619\times10^{-4}} = 5.599\times10^{-4}$$

The flow pattern factor (F_p) can be calculated using

$$F_p = (1-\alpha) + \alpha\left[\frac{2}{\pi}\tan^{-1}\left(\sqrt{\frac{\rho_g(V_g - V_l)^2}{gD(\rho_l - \rho_g)}}\right)\right]^2$$

$$= (1-0.33) + (0.33)\left[\frac{2}{\pi}\tan^{-1}\left(\sqrt{\frac{(1.109\text{ kg/m}^3)(3.03\text{ m/s} - 2.985\text{ m/s})^2}{(9.81\text{ m/s}^2)(0.0254\text{ m})(990.1\text{ kg/m}^3 - 1.109\text{ kg/m}^3)}}\right)\right]^2 = 0.670$$

The inclination factor (I^*) for $\theta = 5°$ is calculated to be

$$I^* = 1 + \frac{(\rho_l - \rho_g)gD^2}{\sigma}|\sin\theta| = 1 + \frac{(990.1\text{ kg/m}^3 - 1.109\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.0254\text{ m})^2}{0.068\text{ N/m}}|\sin 5°| = 9.023$$

The liquid phase heat transfer coefficient is calculated using:

$$h_l = 0.027\,\text{Re}_l^{4/5}\,\text{Pr}_l^{1/3}\left(\frac{k_l}{D}\right)\left(\frac{\mu_l}{\mu_s}\right)^{0.14}$$

$$= 0.027(103100)^{4/5}(3.91)^{1/3}\left(\frac{0.637\text{ W/m}\cdot\text{K}}{0.0254\text{ m}}\right)\left(\frac{0.596\times10^{-3}\text{ kg/m}\cdot\text{s}}{0.355\times10^{-3}\text{ kg/m}\cdot\text{s}}\right)^{0.14} = 11754\text{ W/m}^2\cdot\text{K}$$

where the *in situ* liquid Reynolds number is

$$\text{Re}_l = \frac{4\dot{m}_l}{\pi\sqrt{1-\alpha}\,\mu_l D} = \frac{4(1.003\text{ kg/s})}{\pi\sqrt{1-0.33}(0.596\times10^{-3}\text{ kg/m}\cdot\text{s})(0.0254\text{ m})} = 103100$$

Thus, using the general two-phase heat transfer correlation, the value for h_{tp} is estimated to be

$$\frac{h_{tp}}{h_l} = F_p\left[1 + 0.55\left(\frac{x}{1-x}\right)^{0.1}\left(\frac{1-F_p}{F_p}\right)^{0.4}\left(\frac{Pr_g}{Pr_l}\right)^{0.25}\left(\frac{\mu_l}{\mu_g}\right)^{0.25}\left(I^*\right)^{0.25}\right]$$

$$= (0.670)\left[1 + 0.55\left(\frac{5.599\times10^{-4}}{1-5.599\times10^{-4}}\right)^{0.1}\left(\frac{1-0.670}{0.670}\right)^{0.4}\left(\frac{0.7241}{3.91}\right)^{0.25}\left(\frac{59.6}{1.941}\right)^{0.25}(9.023)^{0.25}\right] = 1.021$$

or $\qquad h_{tp} = 1.021h_l = 1.021(11754\text{ W/m}^2\cdot\text{K}) = \mathbf{12{,}000\ W/m^2\cdot K}$

Discussion The inclination factor is $I^* = 1$ when the tube is at horizontal position, since sin(0°) = 0. The two-phase heat transfer coefficient for horizontal tube would be $h_{tp} = 10300$ W/m^2·K, which is about 14% lower than that of 5° inclined tube.

10-106 Mixture of petroleum and natural gas is being transported in a pipeline that is located in a terrain that caused it to have an average inclination angle of 10°. The two-phase heat transfer coefficient is to be determined.

Assumptions **1** Steady operating condition exists. **2** Two-phase flow is non-boiling and it does not involve phase change. **3** Fluid properties are constant.

Properties The properties of petroleum (liquid) are given to be μ_l = 297.5×10^{-4} kg/m·s, μ_s = 238×10^{-4} kg/m·s, ρ_l = 853 kg/m^3, k_l = 0.163 W/m·K, σ = 0.020 N/m, and Pr$_l$ = 405. The properties of natural gas are given to be μ_g = 9.225×10^{-6} kg/m·s, ρ_g = 9.0 kg/m^3, and Pr$_g$ = 0.80.

Analysis From the gas and liquid mass flow rates, the superficial gas and liquid velocities can be calculated:

$$V_{sg} = \frac{\dot m_g}{\rho_g A} = \frac{4\dot m_g}{\rho_g \pi D^2} = \frac{4(0.055\text{ kg/s})}{(9.0\text{ kg/m}^3)\pi(0.102\text{ m})^2} = 0.748\text{ m/s}$$

$$V_{sl} = \frac{\dot m_l}{\rho_l A} = \frac{4\dot m_l}{\rho_l \pi D^2} = \frac{4(16\text{ kg/s})}{(853\text{ kg/m}^3)\pi(0.102\text{ m})^2} = 2.296\text{ m/s}$$

Using the superficial velocities and void fraction, the gas and liquid velocities are found to be

$$V_g = \frac{V_{sg}}{\alpha} = \frac{0.748\text{ m/s}}{0.22} = 3.400\text{ m/s} \qquad V_l = \frac{V_{sl}}{1-\alpha} = \frac{2.296\text{ m/s}}{1-0.22} = 2.944\text{ m/s}$$

Using the gas and liquid mass flow rates, the quality is determined to be

$$x = \frac{\dot m_g}{\dot m_l + \dot m_g} = \frac{0.055}{16+0.055} = 3.426\times10^{-3}$$

The flow pattern factor (F_p) can be calculated using

$$F_p = (1-\alpha) + \alpha\left[\frac{2}{\pi}\tan^{-1}\left(\sqrt{\frac{\rho_g\,(V_g - V_l)^2}{gD\,(\rho_l - \rho_g)}}\right)\right]^2$$

$$= (1-0.22) + (0.22)\left[\frac{2}{\pi}\tan^{-1}\left(\sqrt{\frac{(9.0\text{ kg/m}^3)(3.40\text{ m/s} - 2.944\text{ m/s})^2}{(9.81\text{ m/s}^2)(0.102\text{ m})(853\text{ kg/m}^3 - 9.0\text{ kg/m}^3)}}\right)\right]^2 = 0.7802$$

The liquid phase heat transfer coefficient is calculated using:

$$h_l = 0.027\,\text{Re}_l^{4/5}\,\text{Pr}_l^{1/3}\left(\frac{k_l}{D}\right)\left(\frac{\mu_l}{\mu_s}\right)^{0.14}$$

$$= 0.027(7601)^{4/5}(405)^{1/3}\left(\frac{0.163\text{ W/m}\cdot\text{K}}{0.102\text{ m}}\right)\left(\frac{297.5\times10^{-4}\text{ kg/m}\cdot\text{s}}{238\times10^{-4}\text{ kg/m}\cdot\text{s}}\right)^{0.14} = 419.1\text{ W/m}^2\cdot\text{K}$$

where the *in situ* liquid Reynolds number is

$$\text{Re}_l = \frac{4\dot m_l}{\pi\sqrt{1-\alpha}\,\mu_l D} = \frac{4(16\text{ kg/s})}{\pi\sqrt{1-0.22}\,(297.5\times10^{-4}\text{ kg/m}\cdot\text{s})(0.102\text{ m})} = 7601$$

The inclination factor (I^*) for θ = 10° is calculated to be

$$I^* = 1 + \frac{(\rho_l - \rho_g)gD^2}{\sigma}|\sin\theta| = 1 + \frac{(853\text{ kg/m}^3 - 9.0\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.102\text{ m})^2}{0.020\text{ N/m}}|\sin 10°| = 748.9$$

Thus, using the general two-phase heat transfer correlation, the value for h_{tp} is estimated to be

$$\frac{h_{tp}}{h_l} = F_p\left[1 + 0.55\left(\frac{x}{1-x}\right)^{0.1}\left(\frac{1-F_p}{F_p}\right)^{0.4}\left(\frac{Pr_g}{Pr_l}\right)^{0.25}\left(\frac{\mu_l}{\mu_g}\right)^{0.25}\left(I^*\right)^{0.25}\right]$$

$$= (0.7802)\left[1 + 0.55\left(\frac{3.426\times10^{-3}}{1-3.426\times10^{-3}}\right)^{0.1}\left(\frac{1-0.7802}{0.7802}\right)^{0.4}\left(\frac{0.80}{405}\right)^{0.25}\left(\frac{29750}{9.225}\right)^{0.25}(748.9)^{0.25}\right] = 1.999$$

or $\qquad h_{tp} = 1.999h_l = 1.999(419.1\text{ W/m}^2\cdot\text{K}) = \mathbf{838\text{ W/m}^2\cdot\text{K}}$

Discussion Since $V_g > V_l$, there will be slippage between the gas and liquid phases. When $V_g \neq V_l$, slippage between the gas and liquid phases exists. When $V_g = V_l$, slippage between the gas and liquid phases is negligible, and the flow is called homogeneous two-phase flow.

10-107 Air-water mixture is flowing in a tube with diameter of 25.4 mm, and the mixture superficial gas and liquid velocities are 1 m/s and 2 m/s, respectively. The two-phase heat transfer coefficient (h_{tp}) for (*a*) horizontal tubee (θ = 0°) and (*b*) vertical tube (θ = 90°), are to be determined and compared.

Assumptions **1** Steady operating condition exists. **2** Two-phase flow is non-boiling and it does not involve phase change. **3** Fluid properties are constant.

Properties The properties of water (liquid) at bulk mean temperature $T_b = (T_i + T_e)/2$ = 45°C are, from Table A-9, $\mu_l = 0.596\times10^{-3}$ kg/m·s, ρ_l = 990.1 kg/m^3, k_l = 0.637 W/m·K, and $\Pr_l$ = 3.91. The properties of air (gas) at bulk mean temperature T_b = 45°C are, from Table A-15, $\mu_g = 1.941\times10^{-5}$ kg/m·s, ρ_g = 1.109 kg/m^3, and $\Pr_g$ = 0.7241. Also, at T_s = 80°C we get $\mu_s = 0.355\times10^{-3}$ kg/m·s from Table A-9.

Analysis From the superficial gas and liquid velocities, and void fraction, the gas and liquid velocities can be calculated as

$$V_g = \frac{V_{sg}}{\alpha} = \frac{1\text{ m/s}}{0.33} = 3.030\text{ m/s}$$

$$V_l = \frac{V_{sl}}{1-\alpha} = \frac{2\text{ m/s}}{1-0.33} = 2.985\text{ m/s}$$

The gas and liquid mass flow rates are calculated as

$$\dot{m}_g = \rho_g V_{sg} A_c = \rho_g V_{sg}\pi\frac{D^2}{4} = (1.109\text{ kg/m}^3)(1\text{ m/s})\pi\frac{(0.0254\text{ m})^2}{4} = 5.619\times10^{-4}\text{ kg/s}$$

$$\dot{m}_l = \rho_l V_{sl} A_c = \rho_l V_{sl}\pi\frac{D^2}{4} = (990.1\text{ kg/m}^3)(2\text{ m/s})\pi\frac{(0.0254\text{ m})^2}{4} = 1.003\text{ kg/s}$$

Using the gas and liquid mass flow rates, the quality is determined to be

$$x = \frac{\dot{m}_g}{\dot{m}_l + \dot{m}_g} = \frac{5.619\times10^{-4}}{1.003 + 5.619\times10^{-4}} = 5.599\times10^{-4}$$

The flow pattern factor (F_p) can be calculated using

$$F_p = (1-\alpha) + \alpha\left[\frac{2}{\pi}\tan^{-1}\left(\sqrt{\frac{\rho_g(V_g - V_l)^2}{gD(\rho_l - \rho_g)}}\right)\right]^2$$

$$= (1-0.33) + (0.33)\left[\frac{2}{\pi}\tan^{-1}\left(\sqrt{\frac{(1.109\text{ kg/m}^3)(3.03\text{ m/s} - 2.985\text{ m/s})^2}{(9.81\text{ m/s}^2)(0.0254\text{ m})(990.1\text{ kg/m}^3 - 1.109\text{ kg/m}^3)}}\right)\right]^2$$

$$= 0.670$$

The liquid phase heat transfer coefficient is calculated using:

$$h_l = 0.027\,\mathrm{Re}_l^{4/5}\,\Pr_l^{1/3}\left(\frac{k_l}{D}\right)\left(\frac{\mu_l}{\mu_s}\right)^{0.14}$$

$$= 0.027(103100)^{4/5}(3.91)^{1/3}\left(\frac{0.637\text{ W/m}\cdot\text{K}}{0.0254\text{ m}}\right)\left(\frac{0.596\times10^{-3}\text{ kg/m}\cdot\text{s}}{0.355\times10^{-3}\text{ kg/m}\cdot\text{s}}\right)^{0.14}$$

$$= 11754\text{ W/m}^2\cdot\text{K}$$

where the *in situ* liquid Reynolds number is

$$\mathrm{Re}_l = \frac{4\dot{m}_l}{\pi\sqrt{1-\alpha}\,\mu_l D} = \frac{4(1.003\text{ kg/s})}{\pi\sqrt{1-0.33}(0.596\times10^{-3}\text{ kg/m}\cdot\text{s})(0.0254\text{ m})} = 103100$$

(*a*) The inclination factor (I^*) for horizontal tube (θ = 0°) is $I^* = 1$. Thus, using the general two-phase heat transfer correlation, the value for h_{tp} is estimated to be

$$\frac{h_{tp,\,\text{horiz}}}{h_l} = F_p \left[1 + 0.55 \left(\frac{x}{1-x} \right)^{0.1} \left(\frac{1-F_p}{F_p} \right)^{0.4} \left(\frac{Pr_g}{Pr_l} \right)^{0.25} \left(\frac{\mu_l}{\mu_g} \right)^{0.25} \left(I^* \right)^{0.25} \right]$$

$$= (0.670) \left[1 + 0.55 \left(\frac{5.599 \times 10^{-4}}{1 - 5.599 \times 10^{-4}} \right)^{0.1} \left(\frac{1-0.670}{0.670} \right)^{0.4} \left(\frac{0.7241}{3.91} \right)^{0.25} \left(\frac{59.6}{1.941} \right)^{0.25} \right]$$

$$= 0.8727$$

or

$$h_{tp,\,\text{horiz}} = 0.87271 h_l = 0.8727(11754 \text{ W/m}^2 \cdot \text{K}) = \mathbf{10{,}300 \text{ W/m}^2 \cdot K}$$

(b) The inclination factor (I^*) for vertical tube (θ = 90°) is calculated to be

$$I^* = 1 + \frac{(\rho_l - \rho_g) g D^2}{\sigma} \left| \sin\theta \right|$$

$$= 1 + \frac{(990.1 \text{ kg/m}^3 - 1.109 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.0254 \text{ m})^2}{0.068 \text{ N/m}} \left| \sin 90° \right|$$

$$= 93.05$$

Thus, using the general two-phase heat transfer correlation, the value for h_{tp} is estimated to be

$$\frac{h_{tp,\,\text{vert}}}{h_l} = F_p \left[1 + 0.55 \left(\frac{x}{1-x} \right)^{0.1} \left(\frac{1-F_p}{F_p} \right)^{0.4} \left(\frac{Pr_g}{Pr_l} \right)^{0.25} \left(\frac{\mu_l}{\mu_g} \right)^{0.25} \left(I^* \right)^{0.25} \right]$$

$$= (0.670) \left[1 + 0.55 \left(\frac{5.599 \times 10^{-4}}{1 - 5.599 \times 10^{-4}} \right)^{0.1} \left(\frac{1-0.670}{0.670} \right)^{0.4} \left(\frac{0.7241}{3.91} \right)^{0.25} \left(\frac{59.6}{1.941} \right)^{0.25} (93.05)^{0.25} \right]$$

$$= 1.30$$

or

$$h_{tp,\,\text{vert}} = 1.30 h_l = 1.30(11754 \text{ W/m}^2 \cdot \text{K}) = \mathbf{15{,}300 \text{ W/m}^2 \cdot K}$$

Discussion The two-phase heat transfer coefficient of vertical pipe is about 49% higher than that of horizontal pipe:

$$\frac{h_{tp,\,\text{vert}}}{h_{tp,\,\text{horiz}}} = \frac{15{,}300}{10{,}300} = 1.49$$

10-108 Air-water mixture flows through a 0.0254 m stainless steel pipe at specified flow condition. Using the concept of Reynolds analogy, the two phase convective heat transfer coefficient is to be determined.

Assumptions **1** Steady state operating conditions exist.**2** Two phase flow is non-boiling in nature and does not undergo any phase change.**3** Fluid properties are constant.

Properties Use the following thermo physical properties for water and air:

$\rho_l = 997.1\ \mathrm{kg/m^3}, \mu_l = 8.9\times10^{-4}\mathrm{kg/m\cdot s},\ \mu_s = 4.66\times10^{-4}\ \mathrm{kg/m\cdot s}, \mathrm{Pr}_l = 6.26, \mathrm{k}_l = \ 0.595\ \mathrm{W/m\cdot K},$
$\sigma = 0.0719\ \mathrm{N/m}, \rho_g = 2.35\ \mathrm{kg/m^3}, \mathrm{and} \mu_g = 1.84\times10^{-5}\ \mathrm{kg/m\cdot s}$

Analysis We first need to determine superficial Reynolds number and the actual velocities for each phase.

$$Re_{sl} = \frac{\rho_l V_{sl} D}{\mu_l} = \frac{997.1(\mathrm{kg/m^3})\times0.3(\mathrm{m/s})\times0.0254(\mathrm{m})}{8.9\times10^{-4}(\mathrm{kg/m\cdot s})} = 8537$$

$$Re_{sg} = \frac{\rho_g V_{sg} D}{\mu_g} = \frac{2.35(\mathrm{kg/m^3})\times23(\mathrm{m/s})\times0.0254(\mathrm{m})}{1.84\times10^{-5}(\mathrm{kg/m\cdot s})} = 74612$$

The actual phase velocities are calculated from the known values of superficial velocities and void fraction.

$$V_l = \frac{V_{sl}}{1-\alpha} = \frac{0.3(\mathrm{m/s})}{1-0.86} = 2.14\ \mathrm{m/s}\ \text{ and }\ V_g = \frac{V_{sg}}{\alpha} = \frac{23(\mathrm{m/s})}{0.86} = 26.74\ \mathrm{m/s}$$

The mass flow rate of each phase is,

$$\dot{m}_l = \rho_l V_{sl}\frac{\pi}{4}D^2 = 997.1(\mathrm{kg/m^3})\times0.3(\mathrm{m/s})\times\frac{\pi}{4}\times0.0254^2(\mathrm{m^2}) = 0.15\ \mathrm{kg/s}$$

$$\dot{m}_g = \rho_g V_{sg}\frac{\pi}{4}D^2 = 2.35(\mathrm{kg/m^3})\times23(\mathrm{m/s})\times\frac{\pi}{4}\times0.0254^2(\mathrm{m^2}) = 0.027\ \mathrm{kg/s}$$

Thus the total mass flow rate is, $\dot{m} = \dot{m}_l + \dot{m}_g = 0.15 + 0.027 = 0.177\ \mathrm{kg/s}$

Since this is a vertical upward flow of air water we can use the Reynolds analogy given by Equation (10-40).

$$\frac{h_{tp}}{h_l} = F_p^{\ m}\left(\frac{\dot{m}_l}{\dot{m}}\right)\left(\frac{\rho_{tp}}{\rho_l}\right)^n \phi_l^{\ p}$$

The single phase heat transfer coefficient is calculated as,

$$h_l = 0.027\,\mathrm{Re}_l^{4/5}\,\mathrm{Pr}_l^{1/3}\,(k_l / D)\,(\mu_l / \mu_s)^{0.14}$$

The in-situ Reynolds number required in single phase heat transfer equation is calcutaed as,

$$\mathrm{Re}_l = \frac{4\dot{m}_l}{\mu_l D\pi\sqrt{1-\alpha}} = \frac{4\times0.15(\mathrm{kg/s})}{8.9\times10^{-4}(\mathrm{kg/m.s})\times0.0254(\mathrm{m})\times\pi\sqrt{1-0.86}} = 22579$$

$$h_l = 0.027\times22579^{4/5}\times6.26^{1/3}\times\frac{0.595(\mathrm{W/m\cdot K})}{0.0254(\mathrm{m})}\times\left(\frac{8.9\times10^{-4}}{4.66\times10^{-4}}\right)^{0.14} = 3878\ \mathrm{W/m^2\cdot K}$$

$$\mathrm{F_p} = (1-\alpha) + \alpha\left(\frac{2}{\pi}\tan^{-1}\left(\sqrt{\frac{\rho_g(V_g - V_l)^2}{gD(\rho_l - \rho_g)}}\right)\right)^2$$

$$\mathrm{F_p} = (1-0.86) + 0.86\left(\frac{2}{\pi}\tan^{-1}\left(\sqrt{\frac{2.35(\mathrm{kg/m^3})(26.74-2.14)^2}{9.81\times0.0254(\mathrm{m})\times(997.1-2.35)(\mathrm{kg/m^3})}}\right)\right)^2$$

$$\therefore \mathrm{F_p} = 0.621$$

The two phase density required in Reynolds analogy equation is calculated as,

$$\rho_{tp} = (1-\alpha)\rho_l + \alpha\rho_g = (1-0.86)\times 997.1(\text{kg/m}^3) + 0.86\times 2.35(\text{kg/m}^3) = 141.6\,\text{kg/m}^3$$

Single phase pressure drop for turbulent pipe flow (with superficial Reynolds number of Re_{sl} = 8537) is calculated by first calculating the single phase friction factor from either the Moody chart (Fig. A-20) or the Colebrook equation (Eq.8-76) as follows,

$$\frac{1}{\sqrt{f_l}} = -2\log\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{\text{Re}_{sl}\sqrt{f_l}}\right) = -2\log\left(\frac{0.002/25.4}{3.7} + \frac{2.51}{8537\sqrt{f_l}}\right) \rightarrow f_l = 0.0323$$

Where from Table 8-3 for stainless steel pipe, the roughness is ε = 0.002.

Single phase pressure drop is then calculated as,

$$(dP/dL)_{f,l} = \frac{f_l \rho_l V_{sl}^2}{2D} = \frac{0.0323\times 997.1(\text{kg/m}^3)\times 0.3^2(\text{m}^2)}{2\times 0.0254(\text{m})} = 57.06\,\text{Pa/m}$$

Thus the two-phase friction multiplier is,

$$\phi_l = \sqrt{\frac{(dP/dL)_{f,tp}}{(dP/dL)_{f,l}}} = \sqrt{\frac{2700(\text{Pa/m})}{57.06(\text{Pa/m})}} = 6.87$$

Thus the two-phase heat transfer coefficient calculated using Reynolds analogy (Eq. 10-40) is,

$$\frac{h_{tp}}{h_l} = F_p{}^m\left(\frac{\dot{m}_l}{\dot{m}}\right)\left(\frac{\rho_{tp}}{\rho_l}\right)^n \phi_l{}^p = 0.621^{0.5}\left(\frac{0.15}{0.177}\right)\left(\frac{141.6(\text{kg/m}^3)}{997.1(\text{kg/m}^3)}\right)^{-0.5} 6.87^{0.2}$$

$$\frac{h_{tp}}{h_l} = 2.605$$

Thus the two phase heat transfer coefficient is: h_{tp} = **10102 W/m²·K**

Discussion The use of Reynolds analogy strongly depends on the correct estimation of the two-phase pressure drop. Most of the correlations available for two-phase pressure drop fail to correctly estimate the two-phase pressure drop and hence based on the calculated value of two-phase pressure drop, the Reynolds analogy may not predict the two- phase heat transfer coefficient correctly.

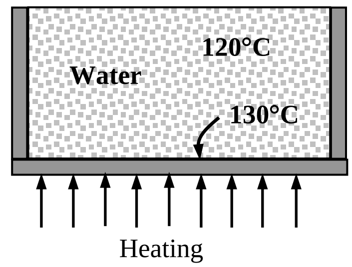
Review Problems

10-109 Water is boiled at T_{sat} = 120°C in a mechanically polished stainless steel pressure cooker whose inner surface temperature is maintained at T_s = 130°C. The time it will take for the tank to empty is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the heater and the boiler are negligible.

Properties The properties of water at the saturation temperature of 120°C are (Tables 10-1 and A-9)

$$\rho_l = 943.4\ \text{kg/m}^3 \quad h_{fg} = 2203\times10^3\ \text{J/kg}$$
$$\rho_v = 1.121\ \text{kg/m}^3 \quad \mu_l = 0.232\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\sigma = 0.0550\ \text{N/m} \quad c_{pl} = 4244\ \text{J/kg}\cdot°\text{C}$$
$$\Pr_l = 1.44$$



Also, $C_{sf} = 0.0130$ and n = 1.0 for the boiling of water on a mechanically polished stainless steel surface (Table 10-3). Note that we expressed the properties in units specified under Eq. 10-2 in connection with their definitions in order to avoid unit manipulations.

Analysis The excess temperature in this case is $\Delta T = T_s - T_{sat} = 130-120 = 10°\text{C}$ which is relatively low (less than 30°C). Therefore, nucleate boiling will occur. The heat flux in this case can be determined from Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{sat})}{C_{sf}h_{fg}\Pr_l^n}\right)^3$$
$$= (0.232\times10^{-3})(2203\times10^3)\left[\frac{9.81(943.4-1.121)}{0.0550}\right]^{1/2}\left(\frac{4244(130-120)}{0.0130(2203\times10^3)1.44}\right)^3$$
$$= 228{,}400\ \text{W/m}^2$$

The rate of heat transfer is

$$\dot{Q} = A\dot{q}_{\text{nucleate}} = \frac{1}{4}\pi(0.20\ \text{m})^2(228{,}400\ \text{W/m}^2) = 7174\ \text{W}$$

The rate of evaporation is

$$\dot{m}_{\text{evap}} = \frac{\dot{Q}}{h_{fg}} = \frac{7174\ \text{W}}{2203\times10^3\ \text{kJ/kg}} = 0.003256\ \text{kg/s}$$

Noting that the tank is half-filled, the mass of the water and the time it will take for the tank to empty are

$$m = \frac{1}{2}\rho_l \boldsymbol{V} = \frac{1}{2}(943.4\ \text{kg/m}^3)\left[\pi(0.20\ \text{m})^2 / 4\times(0.30\ \text{m})\right] = 4.446\ \text{kg}$$

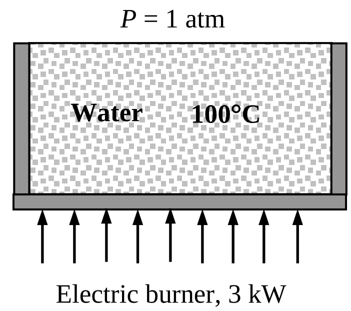
$$t = \dot{m}_{\text{evap}} = \frac{m}{\dot{m}_{\text{evap}}} = \frac{4.446\ \text{kg}}{0.003256\ \text{kg/s}} = 1365\ \text{s} = \mathbf{22.8\ min}$$

10-110 Water is boiled at sea level (1 atm pressure) and thus at a saturation (or boiling) temperature of T_{sat} = 100°C in a mechanically polished AISI 304 stainless steel pan placed on top of a 3-kW electric burner. Only 60% of the heat (1.8 kW) generated is transferred to the water. The inner surface temperature of the pan and the temperature difference across the bottom of the pan are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the boiler are negligible. **3** The boiling regime is nucleate boiling (this assumption will be checked later). **4** Heat transfer through the bottom of the pan is one-dimensional.

Properties The properties of water at the saturation temperature of 100°C are (Tables 10-1 and A-9)

$$\rho_l = 957.9 \text{ kg/m}^3$$
$$\rho_v \approx 0.60 \text{ kg/m}^3$$
$$\sigma = 0.0589 \text{ N/m}$$
$$\text{Pr}_l = 1.75$$
$$h_{fg} = 2257 \times 10^3 \text{ J/kg}$$
$$\mu_l = 0.282 \times 10^{-3} \text{ kg}\cdot\text{m/s}$$
$$c_{pl} = 4217 \text{ J/kg}\cdot K$$



Also, k_{steel} = 14.9 W/m·K (Table A-3), $C_{sf} = 0.0130$ and n = 1.0 for the boiling of water on a mechanically polished stainless steel surface (Table 10-3). Note that we expressed the properties in units specified under Eq. 10-2 in connection with their definitions in order to avoid unit manipulations.

Analysis The rate of heat transfer to the water and the heat flux are

$$\dot{Q} = 0.60 \times 3 \text{ kW} = 1.8 \text{ kW} = 1800 \text{ W}$$
$$A_s = \pi D^2/4 = \pi(0.30 \text{ m})^2/4 = 0.07069 \text{ m}^2$$
$$\dot{q} = \dot{Q}/A_s = (1800 \text{ W})/(0.07069 \text{ m}^2) = 25.46 \text{ W/m}^2$$

Then temperature difference across the bottom of the pan is determined directly from the steady one-dimensional heat conduction relation to be

$$\dot{q} = k_{\text{steel}}\frac{\Delta T}{L} \rightarrow \Delta T = \frac{\dot{q}L}{k_{\text{steel}}} = \frac{(25{,}460 \text{ W/m}^2)(0.006 \text{ m})}{14.9 \text{ W/m}\cdot K} = \mathbf{10.3°C}$$

The Rohsenow relation which gives the nucleate boiling heat flux for a specified surface temperature can also be used to determine the surface temperature when the heat flux is given.

Assuming nucleate boiling, the temperature of the inner surface of the pan is determined from Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s - T_{\text{sat}})}{C_{sf} h_{fg} \text{Pr}_l^n}\right)^3$$

$$25{,}460 = (0.282\times10^{-3})(2257\times10^3)\left[\frac{9.81(957.9-0.60)}{0.0589}\right]^{1/2}\left(\frac{4217(T_s - 100)}{0.0130(2257\times10^3)1.75}\right)^3$$

It gives

$$T_s = \mathbf{105.7°C}$$

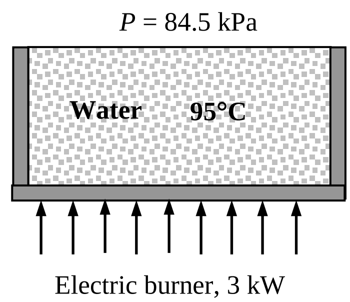
which is in the nucleate boiling range (5 to 30°C above surface temperature). Therefore, the nucleate boiling assumption is valid.

10-111 Water is boiled at 84.5 kPa pressure and thus at a saturation (or boiling) temperature of T_{sat} = 95°C in a mechanically polished AISI 304 stainless steel pan placed on top of a 3-kW electric burner. Only 60% of the heat (1.8 kW) generated is transferred to the water. The inner surface temperature of the pan and the temperature difference across the bottom of the pan are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the boiler are negligible. **3** The boiling regime is nucleate boiling (this assumption will be checked later). **4** Heat transfer through the bottom of the pan is one-dimensional.

Properties The properties of water at the saturation temperature of 95°C are (Tables 10-1 and A-9)

$$\rho_l = 961.5 \text{ kg/m}^3$$
$$\rho_v \approx 0.50 \text{ kg/m}^3$$
$$\sigma = 0.0599 \text{ N/m}$$
$$\text{Pr}_l = 1.85$$
$$h_{fg} = 2270\times10^3 \text{ J/kg}$$
$$\mu_l = 0.297\times10^{-3} \text{ kg}\cdot\text{m/s}$$
$$c_{pl} = 4212 \text{ J/kg}\cdot K$$



Also, k_{steel} = 14.9 W/m·K (Table A-3), $C_{sf} = 0.0130$ and n = 1.0 for the boiling of water on a mechanically polished stainless steel surface (Table 10-3). Note that we expressed the properties in units specified under Eq. 10-2 in connection with their definitions in order to avoid unit manipulations.

Analysis The rate of heat transfer to the water and the heat flux are

$$\dot{Q} = 0.60\times 3 \text{ kW} = 1.8 \text{ kW} = 1800 \text{ W}$$
$$A_s = \pi D^2/4 = \pi(0.30 \text{ m})^2/4 = 0.07069 \text{ m}^2$$
$$\dot{q} = \dot{Q}/A_s = (1800 \text{ W})/(0.07069 \text{ m}^2) = 25{,}460 \text{ W/m}^2 = 25.46 \text{ kW/m}^2$$

Then temperature difference across the bottom of the pan is determined directly from the steady one-dimensional heat conduction relation to be

$$\dot{q} = k_{\text{steel}}\frac{\Delta T}{L} \rightarrow \Delta T = \frac{\dot{q}L}{k_{\text{steel}}} = \frac{(25{,}460 \text{ W/m}^2)(0.006 \text{ m})}{14.9 \text{ W/m}\cdot K} = \mathbf{10.3°C}$$

The Rohsenow relation which gives the nucleate boiling heat flux for a specified surface temperature can also be used to determine the surface temperature when the heat flux is given.

Assuming nucleate boiling, the temperature of the inner surface of the pan is determined from Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{\text{sat}})}{C_{sf}h_{fg}\text{Pr}_l^n}\right)^3$$

$$25{,}460 = (0.297\times10^{-3})(2270\times10^3)\left[\frac{9.81(961.5-0.50)}{0.0599}\right]^{1/2}\left(\frac{4212(T_s-95)}{0.0130(2270\times10^3)1.85}\right)^3$$

It gives

$$T_s = \mathbf{100.9°C}$$

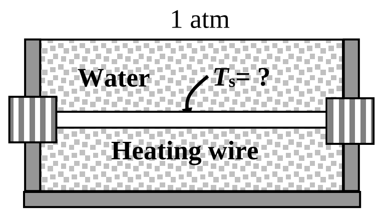
which is in the nucleate boiling range (5 to 30°C above surface temperature). Therefore, the nucleate boiling assumption is valid.

10-112 Water is boiled at 1 atm pressure and thus at a saturation temperature of T_{sat} = 100°C by a nickel electric heater whose diameter is 2 mm. The highest temperature at which this heater can operate without burnout is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the water are negligible.

Properties The properties of water at the saturation temperature of 100°C are (Tables 10-1 and A-9)

$$\rho_l = 957.9 \text{ kg/m}^3 \qquad h_{fg} = 2257\times10^3 \text{ J/kg}$$
$$\rho_v \approx 0.60 \text{ kg/m}^3 \qquad \mu_l = 0.282\times10^{-3} \text{ kg}\cdot\text{m/s}$$
$$\sigma = 0.0589 \text{ N/m} \qquad c_{pl} = 4217 \text{ J/kg}\cdot K$$
$$\Pr_l = 1.75$$



Also, $C_{sf} = 0.0060$ and n = 1.0 for the boiling of water on a nickel surface (Table 10-3).

Analysis The maximum rate of heat transfer without the burnout is simply the critical heat flux. For a horizontal heating wire, the coefficient C_{cr} is determined from Table 10-4 to be

$$L^* = L\left(\frac{g(\rho_l-\rho_v)}{\sigma}\right)^{1/2} = (0.001)\left(\frac{9.81(957.9-0.60)}{0.0589}\right)^{1/2} = 0.399 < 1.2$$
$$C_{cr} = 0.12L^{*-0.25} = 0.12(0.399)^{-0.25} = 0.151$$

Then the maximum or critical heat flux is determined from

$$\begin{aligned}\dot{q}_{\max} &= C_{cr}h_{fg}[\sigma g\rho_v^2(\rho_l-\rho_v)]^{1/4}\\ &= 0.151(2257\times10^3)[0.0589\times9.8\times(0.6)^2(957.9-0.60)]^{1/4}\\ &= 1,280,000 \text{ W/m}^2\end{aligned}$$

Rohsenow relation which gives the nucleate boiling heat flux for a specified surface temperature can also be used to determine the surface temperature when the heat flux is given. Substituting the maximum heat flux into Rohsenow relation together with other properties gives

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{\text{sat}})}{C_{sf}h_{fg}\Pr_l^n}\right)^3$$

$$1,280,000 = (0.282\times10^{-3})(2257\times10^3)\left[\frac{9.81(957.9-0.60)}{0.0589}\right]^{1/2}\left(\frac{4217(T_s-100)}{0.0060(2257\times10^3)1.75}\right)^3$$

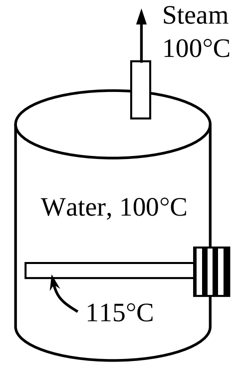
It gives the maximum temperature to be:

$$T_s = \mathbf{109.6°C}$$

10-113 Water is boiled at T_{sat} = 100°C by a chemically etched stainless steel electric heater whose surface temperature is maintained at T_s = 115°C. The rate of heat transfer to the water, the rate of evaporation of water, and the maximum rate of evaporation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Heat losses from the heater and the boiler are negligible.

Properties The properties of water at the saturation temperature of 100°C are (Tables 10-1 and A-9)



$$\rho_l = 957.9\ \text{kg/m}^3$$
$$\rho_v = 0.60\ \text{kg/m}^3$$
$$\sigma = 0.0589\ \text{N/m}$$
$$\text{Pr}_l = 1.75$$
$$h_{fg} = 2257\times10^3\ \text{J/kg}$$
$$\mu_l = 0.282\times10^{-3}\ \text{kg}\cdot\text{m/s}$$
$$c_{pl} = 4217\ \text{J/kg}\cdot°\text{C}$$

Also, $C_{sf} = 0.0130$ and n = 1.0 for the boiling of water on a chemically etched stainless steel surface (Table 10-3). Note that we expressed the properties in units specified under Eq. 10-2 in connection with their definitions in order to avoid unit manipulations.

Analysis (*a*) The excess temperature in this case is $\Delta T = T_s - T_{\text{sat}} = 115 - 100 = 15°\text{C}$ which is relatively low (less than 30°C). Therefore, nucleate boiling will occur. The heat flux in this case can be determined from Rohsenow relation to be

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l-\rho_v)}{\sigma}\right]^{1/2}\left(\frac{c_{p,l}(T_s-T_{\text{sat}})}{C_{sf}h_{fg}\text{Pr}_l^n}\right)^3$$
$$= (0.282\times10^{-3})(2257\times10^3)\left[\frac{9.81(957.9-0.60)}{0.0589}\right]^{1/2}\left(\frac{4217(115-100)}{0.0130(2257\times10^3)1.75}\right)^3$$
$$= 474{,}900\ \text{W/m}^2$$

The surface area of the bottom of the heater is $A_s = \pi DL = \pi(0.002\ \text{m})(0.8\text{m}) = 0.005027\ \text{m}^2$.

Then the rate of heat transfer during nucleate boiling becomes

$$\dot{Q}_{\text{boiling}} = A_s\dot{q}_{\text{nucleate}} = (0.005027\ \text{m}^2)(474{,}900\ \text{W/m}^2) = \mathbf{2387\ W}$$

The rate of evaporation of water is determined from

$$\dot{m}_{\text{evaporation}} = \frac{\dot{Q}_{\text{boiling}}}{h_{fg}} = \frac{2387\ \text{J/s}}{2257\times10^3\ \text{J/kg}} = \mathbf{1.058\times10^{-3}\ kg/s = 3.81\,kg/h}$$

(*b*) For a horizontal heating wire, the coefficient C_{cr} is determined from Table 10-4 to be

$$L^* = L\left(\frac{g(\rho_l-\rho_v)}{\sigma}\right)^{1/2} = (0.001)\left(\frac{9.81(957.9-0.60)}{0.0589}\right)^{1/2} = 0.399 < 1.2$$
$$C_{cr} = 0.12L^{*-0.25} = 0.12(0.399)^{-0.25} = 0.151$$

Then the maximum or critical heat flux is determined from

$$\dot{q}_{\max} = C_{cr}h_{fg}[\sigma g\rho_v^2(\rho_l-\rho_v)]^{1/4} = 0.151(2257\times10^3)[0.0589\times9.8\times(0.6)^2(957.9-0.60)]^{1/4}$$
$$= 1{,}280{,}000\ \text{W/m}^2 = \mathbf{1280\ kW/m^2}$$

10-114 The initial boiling heat transfer coefficient and the total heat transfer coefficient, when a heated steel rod was submerged in a water bath, are to be determined.

Assumptions **1** Steady operating condition exists. **2** The steel rod has uniform initial surface temperature.

Properties The properties of water at the saturation temperature of 100°C are h_{fg} = 2257 kJ/kg (Table A-2) and ρ_l = 957.9 kg/m^3 (Table A-9). The properties of vapor at the film temperature of $T_f = (T_{sat} + T_s)/2$ = 300°C are, from Table A-16,

ρ_v = 0.3831 kg/m^3 $\qquad$ c_{pv} = 1997 J/kg·K

$\mu_v = 2.045 \times 10^{-5}$ kg/m·s $\qquad$ k_v = 0.04345 W/m·K

Analysis The excess temperature in this case is $\Delta T = T_s - T_{sat}$ = 400°C, which is much larger than 30°C for water from Fig. 10-6. Therefore, film boiling will occur. The film boiling heat flux in this case can be determined from

$$\dot{q}_{\text{film}} = C_{\text{film}}\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4c_{pv}(T_s-T_{\text{sat}})]}{\mu_v D(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}})$$

$$= 0.62\left[\frac{9.81(0.04345)^3(0.3831)(957.9-0.3831)[2257\times10^3+0.4(1997)(400)]}{(2.045\times10^{-5})(0.02)(400)}\right]^{1/4}(400)$$

$$= 6.476\times10^4\ \text{W/m}^2$$

Using the Newton's law of cooling, the boiling heat transfer coefficient is

$$\dot{q}_{\text{film}} = h_{\text{film}}(T_s-T_{\text{sat}}) \quad\rightarrow\quad h_{\text{film}} = \frac{\dot{q}_{\text{film}}}{T_s-T_{\text{sat}}}$$

$$h_{\text{film}} = \frac{6.476\times10^4\ \text{W/m}^2}{(500-100)\ \text{K}} = \mathbf{162\ W/m^2\cdot K}$$

The radiation heat transfer coefficient can be determined using

$$\dot{q}_{\text{rad}} = \varepsilon\sigma(T_s^4-T_{\text{sat}}^4) = h_{\text{rad}}(T_s-T_{\text{sat}}) \quad\rightarrow\quad h_{\text{rad}} = \frac{\varepsilon\sigma(T_s^4-T_{\text{sat}}^4)}{T_s-T_{\text{sat}}}$$

$$h_{\text{rad}} = \frac{\varepsilon\sigma(T_s^4-T_{\text{sat}}^4)}{T_s-T_{\text{sat}}} = \frac{(0.9)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(773^4-373^4)\ \text{K}^4}{(500-100)\ \text{K}} = 43.08\ \text{W/m}^2\cdot\text{K}$$

Then, the total heat transfer coefficient can be determined using

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}} \quad\rightarrow\quad h_{\text{total}}(T_s-T_{\text{sat}}) = h_{\text{film}}(T_s-T_{\text{sat}}) + \frac{3}{4}h_{\text{rad}}(T_s-T_{\text{sat}})$$

or

$$h_{\text{total}} = h_{\text{film}} + \frac{3}{4}h_{\text{rad}}$$

$$= 162\ \text{W/m}^2\cdot\text{K} + \frac{3}{4}(43.08\ \text{W/m}^2\cdot\text{K})$$

$$= \mathbf{194\ W/m^2\cdot K}$$

Discussion The boiling heat transfer coefficient (h_{film}) is 3.76 times the radiation heat transfer coefficient (h_{rad}).

10-115 The boiling heat transfer coefficient and the total heat transfer coefficient for water being boiled by a cylindrical metal rod are to be determined.

Assumptions **1** Steady operating condition exists. **2** Heat losses from the boiler are negligible.

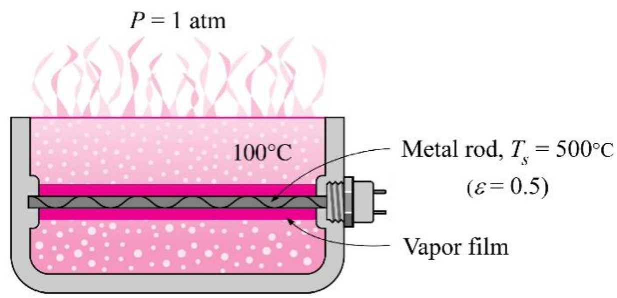
Properties The properties of water at the saturation temperature of 100°C are h_{fg} = 2257 kJ/kg (Table A-2) and ρ_l = 957.9 kg/m^3 (Table A-9). The properties of vapor at the film temperature of $T_f = (T_{sat} + T_s)/2$ = 300°C are, from Table A-16,

ρ_v = 0.3831 kg/m^3

c_{pv} = 1997 J/kg·K

$\mu_v = 2.045 \times 10^{-5}$ kg/m·s

k_v = 0.04345 W/m·K



Analysis The excess temperature in this case is $\Delta T = T_s - T_{sat}$ = 400°C, which is much larger than 30°C for water from Fig 10-6. Therefore, film boiling will occur. The film boiling heat flux in this case can be determined from

$$\dot{q}_{\text{film}} = C_{\text{film}}\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4c_{pv}(T_s-T_{\text{sat}})]}{\mu_v D(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}})$$

$$= 0.62\left[\frac{9.81(0.04345)^3(0.3831)(957.9-0.3831)[2257\times10^3+0.4(1997)(400)]}{(2.045\times10^{-5})(0.002)(400)}\right]^{1/4}(400)$$

$$= 1.152\times10^5\ \text{W/m}^2$$

Using the Newton's law of cooling, the boiling heat transfer coefficient is

$$\dot{q}_{\text{film}} = h_{\text{film}}(T_s - T_{\text{sat}}) \quad\rightarrow\quad h_{\text{film}} = \frac{\dot{q}_{\text{film}}}{T_s - T_{\text{sat}}}$$

$$h_{\text{film}} = \frac{1.152\times10^5\ \text{W/m}^2}{(500-100)\ \text{K}} = \mathbf{288\ W/m^2\cdot K}$$

The radiation heat transfer coefficient can be determined using

$$\dot{q}_{\text{rad}} = \varepsilon\sigma(T_s^4 - T_{\text{sat}}^4) = h_{\text{rad}}(T_s - T_{\text{sat}}) \quad\rightarrow\quad h_{\text{rad}} = \frac{\varepsilon\sigma(T_s^4 - T_{\text{sat}}^4)}{T_s - T_{\text{sat}}}$$

$$h_{\text{rad}} = \frac{\varepsilon\sigma(T_s^4 - T_{\text{sat}}^4)}{T_s - T_{\text{sat}}} = \frac{(0.5)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(773^4-373^4)\ \text{K}^4}{(500-100)\ \text{K}} = 23.93\ \text{W/m}^2\cdot\text{K}$$

Then, the total heat transfer coefficient can be determined using

$$\dot{q}_{\text{total}} = \dot{q}_{\text{film}} + \frac{3}{4}\dot{q}_{\text{rad}} \quad\rightarrow\quad h_{\text{total}}(T_s - T_{\text{sat}}) = h_{\text{film}}(T_s - T_{\text{sat}}) + \frac{3}{4}h_{\text{rad}}(T_s - T_{\text{sat}})$$

or

$$h_{\text{total}} = h_{\text{film}} + \frac{3}{4}h_{\text{rad}}$$

$$= 288\ \text{W/m}^2\cdot\text{K} + \frac{3}{4}(23.93\ \text{W/m}^2\cdot\text{K})$$

$$= \mathbf{306\ W/m^2\cdot K}$$

Discussion The boiling heat transfer coefficient (h_{film}) is about 12 times the radiation heat transfer coefficient (h_{rad}).

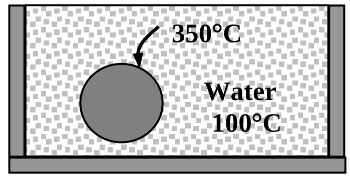
10-116 Water is boiled at T_{sat} = 100°C by a spherical platinum heating element immersed in water. The surface temperature is T_s = 350°C. The boiling heat transfer coefficient is to be determined.

Assumptions 1 Steady operating conditions exist. **2** Heat losses from the heater and the boiler are negligible.

Properties The properties of water at the saturation temperature of 100°C are (Table A-9)

$$h_{fg} = 2257\times10^3 \text{ J/kg}$$
$$\rho_l = 957.9 \text{ kg/m}^3$$

The properties of water vapor at (350+100)/2 = 225°C are (Table A-16)



$$\rho_v = 0.444 \text{ kg/m}^3$$
$$\mu_v = 1.749\times10^{-5} \text{ kg/m}\cdot\text{s}$$
$$c_{pv} = 1951 \text{ J/kg}\cdot°\text{C}$$
$$k_v = 0.03581 \text{ W/m}\cdot°\text{C}$$

Analysis The film boiling occurs since the temperature difference between the surface and the fluid. The heat flux in this case can be determined from

$$\begin{aligned}\dot{q}_{\text{film}} &= 0.67\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)\left[h_{fg}+0.4c_{pv}(T_s-T_{\text{sat}})\right]}{\mu_v D(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}})\\ &= 0.67\left[\frac{(9.81)(0.03581)^3(0.444)(957.9-0.444)\left[2257\times10^3+0.4(1951)(350-100)\right]}{(1.749\times10^{-5})(0.08)(350-100)}\right]^{1/4}(350-100)\\ &= 32{,}062 \text{ W/m}^2\end{aligned}$$

The boiling heat transfer coefficient is

$$\dot{q}_{\text{film}} = h(T_s-T_{\text{sat}}) \longrightarrow h = \frac{\dot{q}_{\text{film}}}{T_s-T_{\text{sat}}} = \frac{32{,}062 \text{ W/m}^2}{(350-100)°\text{C}} = \mathbf{128\ W/m^2\cdot C}$$

10-117 The initial boiling heat transfer coefficient and the total heat transfer coefficient, when heated steel ball bearings are submerged in a water bath, are to be determined.

Assumptions **1** Steady operating condition exists. **2** The steel ball bearings have uniform initial surface temperature.

Properties The properties of water at the saturation temperature of 100°C are h_{fg} = 2257 kJ/kg (Table A-2) and ρ_l = 957.9 kg/m^3 (Table A-9). The properties of vapor at the film temperature of $T_f = (T_{\text{sat}} + T_s)/2$ = 400°C are, from Table A-16,

ρ_v = 0.3262 kg/m^3 $\quad$ c_{pv} = 2066 J/kg·K

μ_v = 2.446 × 10^{-5} kg/m·s $\quad$ k_v = 0.05467 W/m·K

Analysis The excess temperature in this case is $\Delta T = T_s - T_{\text{sat}}$ = 600°C, which is much larger than 30°C for water in Fig. 10-6. Therefore, film boiling will occur. The film boiling heat flux in this case can be determined from

$$\dot{q}_{\text{film}} = C_{\text{film}}\left[\frac{gk_v^3\rho_v(\rho_l-\rho_v)[h_{fg}+0.4c_{pv}(T_s-T_{\text{sat}})]}{\mu_v D(T_s-T_{\text{sat}})}\right]^{1/4}(T_s-T_{\text{sat}})$$

$$=0.67\left[\frac{9.81(0.05467)^3(0.3262)(957.9-0.3262)[2257\times10^3+0.4(2066)(600)]}{(2.446\times10^{-5})(0.02)(600)}\right]^{1/4}(600)$$

$$=1.052\times10^5 \text{ W/m}^2$$

Using the Newton's law of cooling, the boiling heat transfer coefficient is

$$\dot{q}_{\text{film}} = h_{\text{film}}(T_s-T_{\text{sat}}) \quad\rightarrow\quad h_{\text{film}}=\frac{\dot{q}_{\text{film}}}{T_s-T_{\text{sat}}}$$

$$h_{\text{film}}=\frac{1.052\times10^5 \text{ W/m}^2}{(700-100)\text{ K}}=\mathbf{175\ W/m^2\cdot K}$$

The radiation heat transfer coefficient can be determined using

$$\dot{q}_{\text{rad}}=\varepsilon\sigma(T_s^4-T_{\text{sat}}^4)=h_{\text{rad}}(T_s-T_{\text{sat}}) \quad\rightarrow\quad h_{\text{rad}}=\frac{\varepsilon\sigma(T_s^4-T_{\text{sat}}^4)}{T_s-T_{\text{sat}}}$$

$$h_{\text{rad}}=\frac{\varepsilon\sigma(T_s^4-T_{\text{sat}}^4)}{T_s-T_{\text{sat}}}=\frac{(0.75)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(973^4-373^4)\text{ K}^4}{(700-100)\text{ K}}=62.15\text{ W/m}^2\cdot\text{K}$$

Then, the total heat transfer coefficient can be determined using

$$\dot{q}_{\text{total}}=\dot{q}_{\text{film}}+\frac{3}{4}\dot{q}_{\text{rad}} \quad\rightarrow\quad h_{\text{total}}(T_s-T_{\text{sat}})=h_{\text{film}}(T_s-T_{\text{sat}})+\frac{3}{4}h_{\text{rad}}(T_s-T_{\text{sat}})$$

or

$$h_{\text{total}}=h_{\text{film}}+\frac{3}{4}h_{\text{rad}}$$

$$=175\text{ W/m}^2\cdot\text{K}+\frac{3}{4}(62.15\text{ W/m}^2\cdot\text{K})$$

$$=\mathbf{222\ W/m^2\cdot K}$$

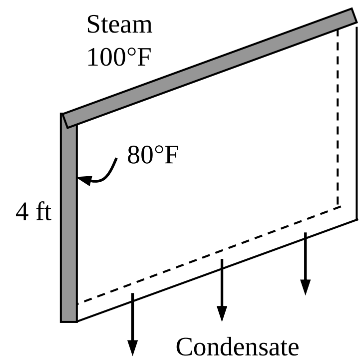
Discussion The boiling heat transfer coefficient (h_{film}) is 2.82 times the radiation heat transfer coefficient (h_{rad}).

10-118E Steam at a saturation temperature of T_{sat} = 100°F condenses on a vertical plate which is maintained at 80°F. The rate of heat transfer to the plate and the rate of condensation of steam per ft width of the plate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The plate is isothermal. **3** The condensate flow is wavy-laminar over the entire plate (this assumption will be verified). **4** The density of vapor is much smaller than the density of liquid, $\rho_v << \rho_l$.

Properties The properties of water at the saturation temperature of 100°F are h_{fg} = 1037 Btu/lbm and ρ_v = 0.00286 lbm/ft^3. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2$ = (100 + 80)/2 = 90°F are (Table A-9E),

$$\rho_l = 62.12 \text{ lbm/ft}^3$$
$$\mu_l = 5.117\times10^{-4} \text{ lbm/ft}\cdot\text{s} = 1.842 \text{ lbm/ft}\cdot\text{h}$$
$$\nu_l = \mu_l/\rho_l = 0.02965 \text{ ft}^2\text{/h}$$
$$c_{pl} = 0.999 \text{ Btu/lbm}\cdot°\text{F}$$
$$k_l = 0.358 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}$$



Analysis The modified latent heat of vaporization is

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68c_{pl}(T_{sat} - T_s) \\ &= 1037 \text{ Btu/lbm} + 0.68\times(0.999 \text{ Btu/lbm}\cdot°\text{F})(100-80)°\text{F} \\ &= 1051 \text{ Btu/lbm} \end{aligned}$$

Assuming wavy-laminar flow, the Reynolds number is determined from

$$\text{Re} = \text{Re}_{\text{vertical,wavy}} = \left[4.81 + \frac{3.70Lk_l(T_{sat}-T_s)}{\mu_l h^*_{fg}}\left(\frac{g}{\nu_l^2}\right)^{1/3}\right]^{0.820}$$

$$= \left[4.81 + \frac{3.70\times(4 \text{ ft})\times(0.358 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F})\times(100-80)°\text{F}}{(1.842 \text{ lbm/ft}\cdot\text{h})(1051 \text{ Btu/lbm})}\left(\frac{32.2 \text{ ft/s}^2}{(0.02965 \text{ ft}^2/\text{h})^2}\frac{(3600 \text{ s})^2}{(1 \text{ h})^2}\right)^{1/3}\right]^{0.82} = 145$$

which is between 30 and 1800, and thus our assumption of wavy laminar flow is verified. Then the condensation heat transfer coefficient is determined from

$$h = h_{\text{vertical,wavy}} = \frac{\text{Re}\, k_l}{1.08\,\text{Re}^{1.22} - 5.2}\left(\frac{g}{\nu_l^2}\right)^{1/3}$$

$$= \frac{145\times(0.358 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F})}{1.08(145)^{1.22} - 5.2}\left(\frac{32.2 \text{ ft/s}^2}{(0.02965 \text{ ft}^2/\text{h})^2}\frac{(3600 \text{ s})^2}{(1 \text{ h})^2}\right)^{1/3} = 875 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F}$$

The heat transfer surface area of the plate is

$$A_s = W \times L = (4 \text{ ft})(1 \text{ ft}) = 4 \text{ ft}^2$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat} - T_s) = (875 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(4 \text{ ft}^2)(100-80)°\text{F} = \mathbf{70{,}000\ Btu/h}$$

The rate of condensation of steam is determined from

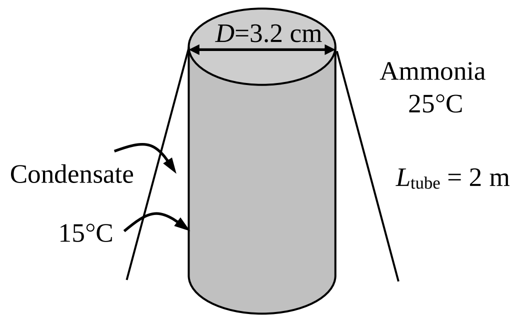
$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{70{,}000 \text{ Btu/h}}{1051 \text{ Btu/lbm}} = \mathbf{66.6\ lbm/h}$$

10-119 Saturated ammonia at a saturation temperature of T_{sat} = 25°C condenses on the outer surface of vertical tube which is maintained at 15°C by circulating cooling water. The rate of heat transfer to the coolant and the rate of condensation of ammonia are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube is isothermal. **3** The tube can be treated as a vertical plate. **4** The condensate flow is turbulent over the entire tube (this assumption will be verified). **5** The density of vapor is much smaller than the density of liquid, $\rho_v << \rho_l$.

Properties The properties of ammonia at the saturation temperature of 25°C are h_{fg} = 1166×10^3 J/kg and ρ_v = 7.809 kg/m^3. The properties of liquid ammonia at the film temperature of $T_f = (T_{sat} + T_s)/2$ = (25 + 15)/2 = 20°C are (Table A-11),

$$\rho_l = 610.2 \text{ kg/m}^3$$
$$\mu_l = 1.519\times10^{-4} \text{ kg/m}\cdot\text{s}$$
$$\nu_l = \mu_l/\rho_l = 0.2489\times10^{-6} \text{ m}^2\text{/s}$$
$$c_{pl} = 4745 \text{ J/kg}\cdot°\text{C}$$
$$k_l = 0.4927 \text{ W/m}\cdot°\text{C}$$
$$\text{Pr}_l = 1.463$$



Analysis (*a*) The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s)$$
$$= 1166\times10^3 \text{ J/kg} + 0.68\times4745 \text{ J/kg}\cdot°\text{C}(25-15)°\text{C} = 1198\times10^3 \text{ J/kg}$$

Assuming turbulent flow, the Reynolds number is determined from

$$\text{Re} = \text{Re}_{\text{vertical,turb}} = \left[\frac{0.0690Lk_l\,\text{Pr}^{0.5}(T_{sat}-T_s)}{\mu_l h^*_{fg}}\left(\frac{g}{\nu_l^2}\right)^{1/3} - 151\,\text{Pr}^{0.5} + 253\right]^{4/3}$$

$$= \left[\frac{0.0690\times2\times0.4927(1.463)^{0.5}(25-15)}{(1.519\times10^{-4}\text{ kg/m.s})(1198\times10^3\text{ J/kg})}\left(\frac{9.81}{(0.2489\times10^{-6}\text{ m}^2/\text{s})^2}\right)^{1/3} - 151(1.463)^{0.5} + 253\right]^{4/3}$$
$$= 2142$$

which is greater than 1800, and thus our assumption of turbulent flow is verified. Then the condensation heat transfer coefficient is determined from

$$h = h_{\text{vertical,turbulent}} = \frac{\text{Re}\,k_l}{8750 + 58\,\text{Pr}^{-0.5}(\text{Re}^{0.75} - 253)}\left(\frac{g}{\nu_l^2}\right)^{1/3}$$

$$= \frac{2142\times(0.4927\text{ W/m}\cdot°\text{C})}{8750 + 58\times1.463^{-0.5}(2142^{0.75} - 253)}\left(\frac{9.81\text{ m/s}^2}{(0.2489\times10^{-6}\text{ m}^2/\text{s})^2}\right)^{1/3} = 4873 \text{ W/m}^2\cdot°\text{C}$$

The heat transfer surface area of the tube is $A_s = \pi DL = \pi(0.032\text{ m})(2\text{ m}) = 0.2011\text{ m}^2$. Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat} - T_s) = (4873\text{ W/m}^2\cdot°\text{C})(0.2011\text{ m}^2)(25-15)°\text{C} = \mathbf{9800\ W}$$

(*b*) The rate of condensation of ammonia is determined from

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{9800\text{ J/s}}{1198\times10^3\text{ J/kg}} = \mathbf{8.180\times10^{-3}\ kg/s}$$

Discussion Combining equations $\delta_L = k_l/h_l$ and $h = (4/3)h_L$, the thickness of the liquid film at the bottom of the tube is determined to be

$$\delta_L = \frac{4k_l}{3h} = \frac{4(0.4927\text{ W/m}\cdot°\text{C})}{3(4873\text{ W/m}^2\cdot°\text{C})} = 0.135\times10^{-3}\text{ m} = 0.135\text{ mm}$$

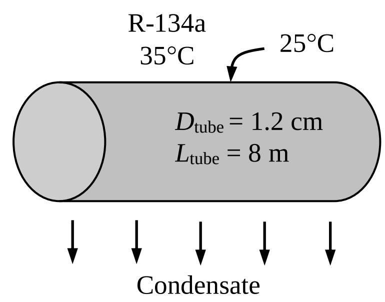
The assumption that the tube diameter is large relative to the thickness of the liquid film at the bottom of the tube is verified since the thickness of the liquid film is 0.135 mm, which is much smaller than the diameter of the tube (3.2 cm). Also, the assumption of turbulent flow is verified since Reynolds number is greater than 1800.

10-120 Saturated refrigerant-134a vapor condenses on the outside of a horizontal tube maintained at a specified temperature. The rate of condensation of the refrigerant is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube is isothermal.

Properties The properties of refrigerant-134a at the saturation temperature of 35°C are h_{fg} = 168.2×10^3 J/kg and ρ_v = 43.41 kg/m^3. The properties of liquid R-134a at the film temperature of $T_f = (T_{sat} + T_s)/2 = (35 + 25)/2 = 30$°C are (Table A-10),

$$\rho_l = 1188 \text{ kg/m}^3$$
$$\mu_l = 1.888\times10^{-4} \text{ kg/m.s}$$
$$c_{pl} = 1448 \text{ J/kg.°C}$$
$$k_l = 0.0808 \text{ W/m.°C}$$



Analysis The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s)$$
$$= 168.2\times10^3 \text{ J/kg} + 0.68\times1448 \text{ J/kg}\cdot\text{°C}(35-25)\text{°C} = 178.0\times10^3 \text{ J/kg}$$

The heat transfer coefficient for condensation on a single horizontal tube is

$$h = h_{\text{horizontal}} = 0.729\left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat} - T_s)D}\right]^{1/4}$$
$$= 0.729\left[\frac{(9.81 \text{ m/s}^2)(1188 \text{ kg/m}^3)(1188 - 43.41 \text{ kg/m}^3)(178.0\times10^3 \text{ J/kg})(0.0808 \text{ W/m}\cdot\text{°C})^3}{(1.888\times10^{-4} \text{ kg/m}\cdot\text{s})(35-25)\text{°C}(0.012 \text{ m})}\right]^{1/4}$$
$$= 1988 \text{ W/m}^2\cdot\text{°C}$$

The heat transfer surface area of the pipe is

$$A_s = \pi DL = \pi(0.012 \text{ m})(8 \text{ m}) = 0.3016 \text{ m}^2$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat} - T_s) = (1988 \text{ W/m}^2\cdot\text{°C})(0.3016 \text{ m}^2)(35-25)\text{°C} = 5996 \text{ W}$$

The rate of condensation of steam is determined from

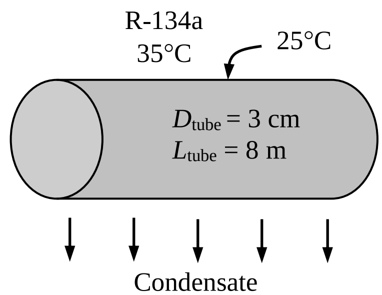
$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{5996 \text{ J/s}}{178.0\times10^3 \text{ J/kg}} = 0.03368 \text{ kg/s} = \mathbf{2.02 \text{ kg/min}}$$

10-121 Saturated refrigerant-134a vapor condenses on the outside of a horizontal tube maintained at a specified temperature. The rate of condensation of the refrigerant is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube is isothermal.

Properties The properties of refrigerant-134a at the saturation temperature of 35°C are $h_{fg} = 168.2\times10^3$ J/kg and $\rho_v = 43.41$ kg/m^3. The properties of liquid R-134a at the film temperature of $T_f = (T_{sat} + T_s)/2 = (35 + 25)/2 = 30$°C are (Table A-10),

$$\rho_l = 1188 \text{ kg/m}^3$$
$$\mu_l = 1.888\times10^{-4} \text{ kg/m.s}$$
$$\nu_l = \mu_l/\rho_l = 0.1590\times10^{-6} \text{ m}^2\text{/s}$$
$$c_{pl} = 1448 \text{ J/kg.°C}$$
$$k_l = 0.0808 \text{ W/m.°C}$$



Analysis The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s)$$
$$= 168.2\times10^3 \text{ J/kg} + 0.68\times1448 \text{ J/kg}\cdot\text{°C}(35-25)\text{°C} = 178.0\times10^3 \text{ J/kg}$$

The heat transfer coefficient for condensation on a single horizontal tube is

$$h = h_{\text{horizontal}} = 0.729\left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat} - T_s)D}\right]^{1/4}$$
$$= 0.729\left[\frac{(9.81 \text{ m/s}^2)(1188 \text{ kg/m}^3)(1188 - 43.41 \text{ kg/m}^3)(178.0\times10^3 \text{ J/kg})(0.0808 \text{ W/m}\cdot\text{°C})^3}{(1.888\times10^{-4} \text{ kg/m}\cdot\text{s})(35-25)\text{°C}(0.03 \text{ m})}\right]^{1/4}$$
$$= 1581 \text{ W/m}^2\cdot\text{°C}$$

The heat transfer surface area of the pipe is

$$A_s = \pi DL = \pi(0.03 \text{ m})(8 \text{ m}) = 0.7540 \text{ m}^2$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat} - T_s) = (1581 \text{ W/m}^2\cdot\text{°C})(0.7540 \text{ m}^2)(35-25)\text{°C} = 11{,}920 \text{ W}$$

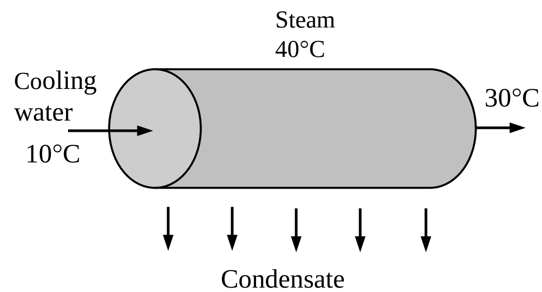
The rate of condensation of steam is determined from

$$\dot{m}_{\text{condensation}} = \frac{\dot{Q}}{h^*_{fg}} = \frac{11{,}920 \text{ J/s}}{178.0\times10^3 \text{ J/kg}} = 0.06697 \text{ kg/s} = \mathbf{4.02\ kg/min}$$

10-122 Steam at a saturation temperature of T_{sat} = 40°C condenses on the outside of a thin horizontal tube. Heat is transferred to the cooling water that enters the tube at 10°C and exits at 30°C. The rate of condensation of steam, the average overall heat transfer coefficient, and the tube length are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube can be taken to be isothermal at the bulk mean fluid temperature in the evaluation of the condensation heat transfer coefficient. **3** Liquid flow through the tube is fully developed. **4** The thickness and the thermal resistance of the tube is negligible.

Properties The properties of water at the saturation temperature of 40°C are $h_{fg} = 2407\times10^3$ J/kg and ρ_v = 0.05 kg/m^3. The properties of liquid water at the film temperature of $T_f = (T_{sat}+T_s)/2 = (40+20)/2$ = 30°C and at the bulk fluid temperature of $T_b = (T_{in}+T_{out})/2 = (10$ + 30)/2 = 20°C are (Table A-9),



At 30°C :

$$\rho_l = 996.0\ \text{kg/m}^3$$
$$\mu_l = 0.798\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\nu_l = \mu_l/\rho_l = 0.801\times10^{-6}\ \text{m}^2/\text{s}$$
$$c_{pl} = 4178\ \text{J/kg}\cdot°\text{C}$$
$$k_l = 0.615\ \text{W/m}\cdot°\text{C}$$
$$\text{Pr} = 5.42$$

At 20°C :

$$\rho_l = 998.0\ \text{kg/m}^3$$
$$\mu_l = 1.002\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$\nu_l = \mu_l/\rho_l = 1.004\times10^{-6}\ \text{m}^2/\text{s}$$
$$c_{pl} = 4182\ \text{J/kg}\cdot°\text{C}$$
$$k_l = 0.598\ \text{W/m}\cdot°\text{C}$$
$$\text{Pr} = 7.01$$

Analysis The mass flow rate of water and the rate of heat transfer to the water are

$$\dot{m}_{water} = \rho V A_c = (998\ \text{kg/m}^3)(2\ \text{m/s})[\pi(0.03\ \text{m})^2/4] = 1.411\ \text{kg/s}$$

$$\dot{Q} = \dot{m}c_p(T_{out}-T_{in}) = (1.411\ \text{kg/s})(4182\ \text{J/kg}\cdot°\text{C})(30-10)°\text{C} = \mathbf{118{,}000\ W}$$

The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat}-T_s) = 2407\times10^3\ \text{J/kg} + 0.68\times4182\ \text{J/kg}\cdot°\text{C}(40-20)°\text{C} = 2464\times10^3\ \text{J/kg}$$

The heat transfer coefficient for condensation on a single horizontal tube is

$$h_o = h_{horizontal} = 0.729\left[\frac{g\rho_l(\rho_l-\rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat}-T_s)D}\right]^{1/4}$$

$$= 0.729\left[\frac{(9.81\ \text{m/s}^2)(996\ \text{kg/m}^3)(996-0.05\ \text{kg/m}^3)(2464\times10^3\ \text{J/kg})(0.615\ \text{W/m}\cdot°\text{C})^3}{(0.798\times10^{-3}\ \text{kg/m}\cdot\text{s})(40-20)°\text{C}(0.03\ \text{m})}\right]^{1/4} = 7572\ \text{W/m}^2\cdot°\text{C}$$

The average heat transfer coefficient for flow inside the tube is determined as follows:

$$\text{Re} = \frac{V_{avg}D}{\nu} = \frac{(2\ \text{m/s})(0.03\text{m})}{1.004\times10^{-6}} = 59{,}761$$

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(59{,}761)^{0.8}(7.01)^{0.4} = 332.0$$

$$h_i = \frac{k\text{Nu}}{D} = \frac{(0.598\ \text{W/m}\cdot°\text{C})\times332.0}{0.03\ \text{m}} = 6618\ \text{W/m}^2\cdot°\text{C}$$

Noting that the thermal resistance of the tube is negligible, the overall heat transfer coefficient becomes

$$U = \frac{1}{1/h_i + 1/h_o} = \frac{1}{1/6618 + 1/7572} = \mathbf{3531\ W/m^2.°C}$$

The logarithmic mean temperature difference is:

$$\Delta T_{lm} = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i/\Delta T_o)} = \frac{30-10}{\ln(30/10)} = 18.20°\text{C}$$

The tube length is determined from

$$\dot{Q} = UA_s\Delta T_{lm} \rightarrow L = \frac{\dot{Q}}{U(\pi D)\Delta T_{lm}} = \frac{118{,}000\ \text{W}}{(3531\ \text{W/m}^2\cdot°\text{C})\pi(0.03\ \text{m})(18.20°\text{C})} = \mathbf{19.5\ m}$$

Note that the flow is turbulent, and thus the entry length in this case is 10D = 0.3 m is much shorter than the total tube length. This verifies our assumption of fully developed flow.

10-123 Saturated steam condenses on a suspended silver sphere which is initially at 25°C. The time needed for the temperature of the sphere to rise to 50°C and the amount of steam condenses are to be determined.

Assumptions **1** The temperature of the sphere changes uniformly and thus the lumped system analysis is applicable. **2** The average condensation heat transfer coefficient evaluated for the average temperature can be used for the entire process. **3** Constant properties at room temperature can be used for the silver ball.

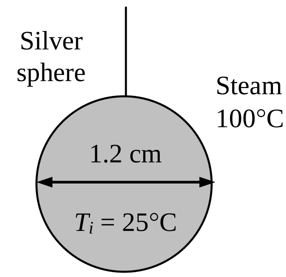
Properties The properties of water at the saturation temperature of 100°C are h_{fg} = 2257×10^3 J/kg and ρ_v = 0.60 kg/m^3. The properties of the silver ball at room temperature and the properties of liquid water at the average film temperature of $T_f = (T_{sat} + T_{s,avg})/2 = (100 + 37.5)/2 = 69°C \approx 70°C$ are (Tables A-3 and A-9),

Silver Ball:

$$\rho = 10{,}500 \text{ kg/m}^3$$
$$\alpha = 174\times10^{-6} \text{ m}^2/\text{s}$$
$$c_p = 235 \text{ J/kg}\cdot°\text{C}$$
$$k_l = 429 \text{ W/m}\cdot°\text{C}$$

Liquid Water:

$$\rho_l = 977.5 \text{ kg/m}^3$$
$$\mu_l = 0.404\times10^{-3} \text{ kg/m}\cdot\text{s}$$
$$c_{pl} = 4190 \text{ J/kg}\cdot°\text{C}$$
$$k_l = 0.663 \text{ W/m}\cdot°\text{C}$$



Analysis The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s)$$
$$= 2257\times10^3 \text{ J/kg} + 0.68\times4190 \text{ J/kg}\cdot°\text{C}(100-37.5)°\text{C} = 2435\times10^3 \text{ J/kg}$$

Noting that the tube is horizontal, the condensation heat transfer coefficient is determined from

$$h = h_{sph} = 0.815\left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat} - T_s)D}\right]^{1/4}$$
$$= 0.815\left[\frac{(9.81 \text{ m/s}^2)(977.5 \text{ kg/m}^3)(977.5 - 0.60 \text{ kg/m}^3)(2435\times10^3 \text{ J/kg})(0.663 \text{ W/m}\cdot°\text{C})^3}{(0.404\times10^{-3} \text{ kg/m}\cdot\text{s})(100-37.5)°\text{C}(0.012 \text{ m})}\right]^{1/4}$$
$$= 9916 \text{ W/m}^2\cdot°\text{C}$$

The characteristic length and the Biot number for the lumped system analysis is (see Chap. 4)

$$L_c = \frac{V}{A} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.012 \text{ m}}{6} = 0.002 \text{ m}$$
$$Bi = \frac{hL_c}{k} = \frac{(9916 \text{ W/m}^2\cdot°\text{C})(0.002 \text{ m})}{(429 \text{ W/m}\cdot°\text{C})} = 0.0462 < 0.1$$

The lumped system analysis is applicable since Bi < 0.1. Then the time needed for the temperature of the sphere to rise from 25 to 50°C is determined to be

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{9916 \text{ W/m}^2\cdot°\text{C}}{(10{,}500 \text{ kg/m}^3)(235 \text{ J/kg}\cdot°\text{C})(0.002 \text{ m})} = 2.009 \text{ s}^{-1}$$
$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{50-100}{25-100} = e^{-2.009t} \longrightarrow t = \mathbf{0.202\ s}$$

The total heat transfer to the ball and the amount of steam that condenses become

$$m_{sphere} = \rho V = \rho\frac{\pi D^3}{6} = (10{,}500 \text{ kg/m}^3)\frac{\pi(0.012 \text{ m})^3}{6} = 0.009500 \text{ kg}$$
$$Q = mc_p[T(t) - T_i]_{sphere} = (0.009500 \text{ kg})(235 \text{ J/kg}\cdot°\text{C})(50-25)°\text{C} = 55.81 \text{ J}$$
$$\dot{m}_{condensation} = \frac{\dot{Q}}{h^*_{fg}} = \frac{55.81 \text{ J/s}}{2435\times10^3 \text{ J/kg}} = \mathbf{2.29\times10^{-5}\ kg/s}$$

10-124 Steam at a saturation temperature of T_{sat} = 100°C condenses on a suspended silver sphere which is initially at 25°C. The time needed for the temperature of the sphere to rise to 50°C and the amount of steam condenses during this process are to be determined.

Assumptions **1** The temperature of the sphere changes uniformly and thus the lumped system analysis is applicable. **2** The average condensation heat transfer coefficient evaluated for the average temperature can be used for the entire process. **3** Constant properties at room temperature can be used for the silver ball.

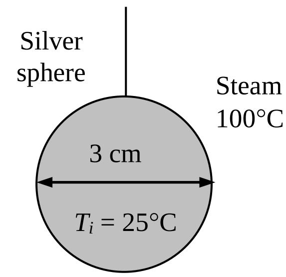
Properties The properties of water at the saturation temperature of 100°C are $h_{fg} = 2257\times10^3$ J/kg and $\rho_v = 0.60$ kg/m^3. The properties of the silver ball at room temperature and the properties of liquid water at the average film temperature of $T_f = (T_{sat} + T_{s,avg})/2 = (100 + 37.5)/2 = 69°C \approx 70°C$ 70°C are (Tables A-3 and A-9),

Copper Ball:

$$\rho = 8933\ \text{kg/m}^3$$
$$\alpha = 117\times10^{-6}\ \text{m}^2/\text{s}$$
$$c_p = 385\ \text{J/kg}\cdot°\text{C}$$
$$k_l = 401\ \text{W/m}\cdot°\text{C}$$

Liquid Water:

$$\rho_l = 977.5\ \text{kg/m}^3$$
$$\mu_l = 0.404\times10^{-3}\ \text{kg/m}\cdot\text{s}$$
$$c_{pl} = 4190\ \text{J/kg}\cdot°\text{C}$$
$$k_l = 0.663\ \text{W/m}\cdot°\text{C}$$



Analysis The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s)$$
$$= 2257\times10^3\ \text{J/kg} + 0.68\times4190\ \text{J/kg}\cdot°\text{C}(100-37.5)°\text{C} = 2435\times10^3\ \text{J/kg}$$

Noting that the tube is horizontal, the condensation heat transfer coefficient is determined from

$$h = h_{sph} = 0.815\left[\frac{g\rho_l(\rho_l-\rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat}-T_s)D}\right]^{1/4}$$
$$= 0.815\left[\frac{(9.81\ \text{m/s}^2)(977.5\ \text{kg/m}^3)(977.5-0.60\ \text{kg/m}^3)(2435\times10^3\ \text{J/kg})(0.663\ \text{W/m}\cdot°\text{C})^3}{(0.404\times10^{-3}\ \text{kg/m}\cdot\text{s})(100-37.5)°\text{C}\ (0.03\ \text{m})}\right]^{1/4}$$
$$= 7886\ \text{W/m}^2\cdot°\text{C}$$

The characteristic length and the Biot number for the lumped system analysis is (see Chap. 4)

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.03\ \text{m}}{6} = 0.005\ \text{m}$$
$$Bi = \frac{hL_c}{k} = \frac{(7886\ \text{W/m}^2\cdot°\text{C})(0.005\ \text{m})}{(401\ \text{W/m}\cdot°\text{C})} = 0.098 < 0.1$$

The lumped system analysis is applicable since Bi < 0.1. Then the time needed for the temperature of the sphere to rise from 25 to 50°C is determined to be

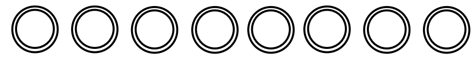
$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{7886\ \text{W/m}^2\cdot°\text{C}}{(8933\ \text{kg/m}^3)(385\ \text{J/kg}\cdot°\text{C})(0.005\ \text{m})} = 0.4586\ \text{s}^{-1}$$
$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{50-100}{25-100} = e^{-0.4586t} \longrightarrow t = \mathbf{0.884\ s}$$

The total heat transfer to the ball and the amount of steam that condenses become

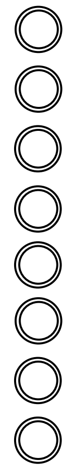
$$m_{sphere} = \rho V = \rho\frac{\pi D^3}{6} = (8933\ \text{kg/m}^3)\frac{\pi(0.03\ \text{m})^3}{6} = 0.1263\ \text{kg}$$
$$Q = mc_p[T(t)-T_i]_{sphere} = (0.1263\ \text{kg})(385\ \text{J/kg}\cdot°\text{C})(50-25)°\text{C} = 1216\ \text{J}$$
$$\dot m_{condensation} = \frac{\dot Q}{h^*_{fg}} = \frac{1216\ \text{J/s}}{2435\times10^3\ \text{J/kg}} = \mathbf{4.99\times10^{-4}\ kg/s}$$

10-125 There is film condensation on the outer surfaces of 8horizontal tubes arranged in a horizontal or vertical tier. The ratio of the condensation rate for the cases of the tubes being arranged in a horizontal tier versus in a vertical tier is to be determined.

Assumptions Steady operating conditions exist.



Horizontal tier



Vertical tier

Analysis The heat transfer coefficients for the two cases are related to the heat transfer coefficient on a single horizontal tube by

Horizontal tier:

$$h_{\text{horizontal tier of N tubes}} = h_{\text{horizontal, 1 tube}}$$

Vertical tier:

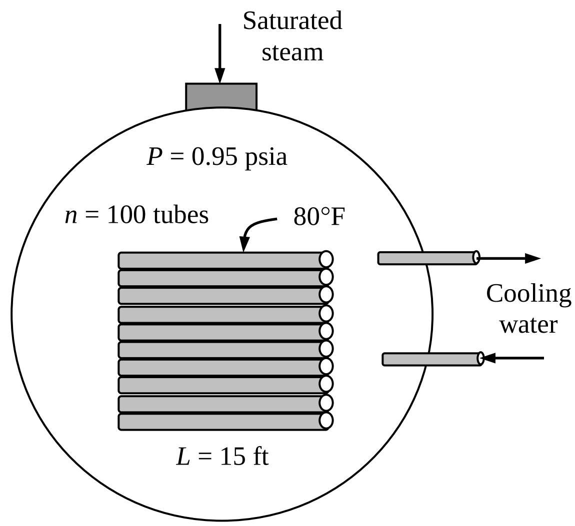
$$h_{\text{vertical tier of N tubes}} = \frac{h_{\text{horizontal, 1 tube}}}{N^{1/4}}$$

Therefore,

$$\begin{aligned}
\text{Ratio} &= \frac{\dot{m}_{\text{horizontal tier of N tubes}}}{\dot{m}_{\text{vertical tier of N tubes}}} \\
&= \frac{h_{\text{horizontal tier of N tubes}}}{h_{\text{vertical tier of N tubes}}} \\
&= \frac{h_{\text{horizontal, 1 tube}}}{h_{\text{horizontal, 1 tube}} / N^{1/4}} \\
&= N^{1/4} \\
&= 8^{1/4} = \mathbf{1.68}
\end{aligned}$$

10-126E Saturated steam at a saturation pressure of 0.95 psia and thus at a saturation temperature of T_{sat} = 100°F (Table A-9E) condenses on the outer surfaces of 100 horizontal tubes which are maintained at 80°F by circulating cooling water and arranged in a 10 × 10 square array. The rate of heat transfer to the cooling water and the rate of condensation of steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tubes are isothermal.



Properties The properties of water at the saturation temperature of 100°F are h_{fg} = 1037 Btu/lbm and ρ_v = 0.00286 lbm/ft³. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2 = (100 + 80)/2 = 90$°F are (Table A-9E),

$$\rho_l = 62.12 \text{ lbm/ft}^3$$
$$\mu_l = 5.117\times10^{-4} \text{ lbm/ft}\cdot\text{s} = 1.842 \text{ lbm/ft}\cdot\text{h}$$
$$\nu_l = \mu_l/\rho_l = 0.02965 \text{ ft}^2/\text{h}$$
$$c_{pl} = 0.999 \text{ Btu/lbm}\cdot°\text{F}$$
$$k_l = 0.358 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}$$

Analysis (*a*) The modified latent heat of vaporization is

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68c_{pl}(T_{sat} - T_s) \\ &= 1037 \text{ Btu/lbm} + 0.68\times(0.999 \text{ Btu/lbm}\cdot°\text{F})(100-80)°\text{F} \\ &= 1051 \text{ Btu/lbm} \end{aligned}$$

The heat transfer coefficient for condensation on a single horizontal tube is

$$\begin{aligned} h = h_{horiz} &= 0.729\left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat} - T_s)D}\right]^{1/4} \\ &= 0.729\left[\frac{(32.2 \text{ ft/s}^2)(62.12 \text{ lbm/ft}^3)(62.12 - 0.00286 \text{ lbm/ft}^3)(1051 \text{ Btu/lbm})(0.358 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F})^3}{[(1 \text{ h}/3600 \text{ s})^2](1.842 \text{ lbm/ft}\cdot\text{h})(100-80)°\text{F}(1.2/12 \text{ ft})}\right]^{1/4} \\ &= 1562 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F} \end{aligned}$$

Then the average heat transfer coefficient for a 4-tube high vertical tier becomes

$$h_{\text{horiz, N tubes}} = \frac{1}{N^{1/4}}h_{\text{horiz, 1 tube}} = \frac{1}{10^{1/4}}(1562 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F}) = 878.3 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F}$$

The surface area for all 100 tubes is

$$A_s = N_{total}\pi DL = 100\pi(1.2/12 \text{ ft})(15 \text{ ft}) = 471.2 \text{ ft}^2$$

Then the rate of heat transfer during this condensation process becomes

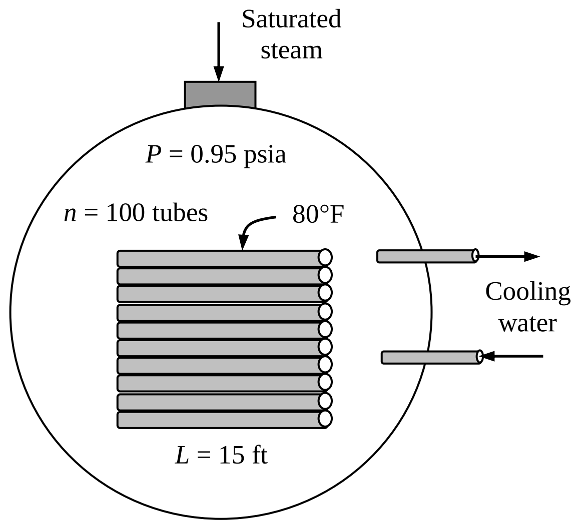
$$\dot{Q} = hA_s(T_{sat} - T_s) = (878.3 \text{ Btu/h.ft}^2\cdot°\text{F})(471.2 \text{ ft}^2)(100-80)°\text{F} = \mathbf{8{,}278{,}000 \text{ Btu/h}}$$

(*b*) The rate of condensation of steam is determined from

$$\dot{m}_{condensation} = \frac{\dot{Q}}{h^*_{fg}} = \frac{8{,}278{,}000 \text{ Btu/h}}{1051 \text{ Btu/lbm}} = \mathbf{7879 \text{ lbm/h}}$$

10-127E Saturated steam at a saturation pressure of 0.95 psia and thus at a saturation temperature of T_{sat} = 100°F (Table A-9E) condenses on the outer surfaces of 100 horizontal tubes which are maintained at 80°F by circulating cooling water and arranged in a 10 × 10 square array. The rate of heat transfer to the cooling water and the rate of condensation of steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tubes are isothermal.



Properties The properties of water at the saturation temperature of 100°F are h_{fg} = 1037 Btu/lbm and ρ_v = 0.00286 lbm/ft^3. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2 = (100 + 80)/2 = 90$°F are (Table A-9E),

$$\rho_l = 62.12 \text{ lbm/ft}^3$$
$$\mu_l = 5.117\times10^{-4} \text{ lbm/ft}\cdot\text{s} = 1.842 \text{ lbm/ft}\cdot\text{h}$$
$$\nu_l = \mu_l/\rho_l = 0.02965 \text{ ft}^2/\text{h}$$
$$c_{pl} = 0.999 \text{ Btu/lbm}\cdot°\text{F}$$
$$k_l = 0.358 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}$$

Analysis (*a*) The modified latent heat of vaporization is

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68c_{pl}(T_{sat} - T_s) \\ &= 1037 \text{ Btu/lbm} + 0.68\times(0.999 \text{ Btu/lbm}\cdot°\text{F})(100-80)°\text{F} \\ &= 1051 \text{ Btu/lbm} \end{aligned}$$

The heat transfer coefficient for condensation on a single horizontal tube is

$$\begin{aligned} h = h_{horiz} &= 0.729\left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat} - T_s)D}\right]^{1/4} \\ &= 0.729\left[\frac{(32.2 \text{ ft/s}^2)(62.12 \text{ lbm/ft}^3)(62.12 - 0.00286 \text{ lbm/ft}^3)(1051 \text{ Btu/lbm})(0.358 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F})^3}{[(1 \text{ h}/3600 \text{ s})^2](1.842 \text{ lbm/ft}\cdot\text{h})(100-80)°\text{F}(2.0/12 \text{ ft})}\right]^{1/4} \\ &= 1375 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F} \end{aligned}$$

Then the average heat transfer coefficient for a 4-tube high vertical tier becomes

$$h_{\text{horiz, N tubes}} = \frac{1}{N^{1/4}}h_{\text{horiz, 1 tube}} = \frac{1}{10^{1/4}}(1375 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F}) = 773.0 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F}$$

The surface area for all 100 tubes is

$$A_s = N_{total}\pi DL = 100\pi(2/12 \text{ ft})(15 \text{ ft}) = 785.4 \text{ ft}^2$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat} - T_s) = (773.0 \text{ Btu/h.ft}^2\cdot°\text{F})(785.4 \text{ ft}^2)(100-80)°\text{F} = \mathbf{12{,}142{,}000 \text{ Btu/h}}$$

(*b*) The rate of condensation of steam is determined from

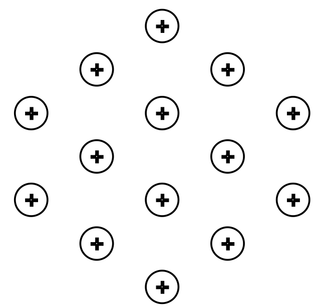
$$\dot{m}_{condensation} = \frac{\dot{Q}}{h^*_{fg}} = \frac{12{,}142{,}000 \text{ Btu/h}}{1051 \text{ Btu/lbm}} = \mathbf{11{,}560 \text{ lbm/h}}$$

10-128 Ammonia is liquefied in a horizontal condenser at 37°C by a coolant at 20°C. The average value of overall heat transfer coefficient and the tube length are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tubes are isothermal. **3** The thermal resistance of the tube walls is negligible.

Properties The properties of ammonia at the saturation temperature of 310 K (37°C) are h_{fg} = 1113×10^3 J/kg and ρ_v = 11.09 kg/m^3 (Table A-11). We assume a tube outer surface temperature of 31°C. The properties of liquid ammonia at the film temperature of $T_f = (T_{sat} + T_s)/2 = (37 + 31)/2 = 34$°C are (Table A-11)

$$\rho_l = 589.0 \text{ kg/m}^3$$
$$\mu_l = 1.303\times10^{-4} \text{ kg/m}\cdot\text{s}$$
$$c_{pl} = 4867 \text{ J/kg}\cdot°\text{C}$$
$$k_l = 0.4602 \text{ W/m}\cdot°\text{C}$$



The thermal conductivity of copper is 401 W/m·°C (Table A-3).

Analysis (*a*) The modified latent heat of vaporization is

$$h^*_{fg} = h_{fg} + 0.68c_{pl}(T_{sat} - T_s)$$
$$= 1113\times10^3 \text{ J/kg} + 0.68\times4867 \text{ J/kg}\cdot°\text{C}(37-31)°\text{C} = 1133\times10^3 \text{ J/kg}$$

The heat transfer coefficient for condensation on a single horizontal tube is

$$h = h_{horizontal} = 0.729\left[\frac{g\rho_l(\rho_l - \rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat}-T_s)D}\right]^{1/4}$$
$$= 0.729\left[\frac{(9.81 \text{ m/s}^2)(589.0 \text{ kg/m}^3)(589.0-11.09 \text{ kg/m}^3)(1133\times10^3 \text{ J/kg})(0.4602 \text{ W/m}\cdot°\text{C})^3}{(1.303\times10^{-4} \text{ kg/m}\cdot\text{s})(37-31)°\text{C}(0.038 \text{ m})}\right]^{1/4}$$
$$= 7693 \text{ W/m}^2\cdot°\text{C}$$

Noting that there are two 2-pipe high, two 3-pipe high, and one 4-pipe high vertical tiers in the tube-layout, the average heat transfer coefficient is to be determined as follows

$$h_1 = \frac{1}{N^{1/4}}h_{horiz,1\,tube} = \frac{1}{2^{1/4}}(7693 \text{ W/m}^2\cdot°\text{C}) = 6469 \text{ W/m}^2\cdot°\text{C}$$
$$h_2 = \frac{1}{N^{1/4}}h_{horiz,1\,tube} = \frac{1}{3^{1/4}}(7693 \text{ W/m}^2\cdot°\text{C}) = 5845 \text{ W/m}^2\cdot°\text{C}$$
$$h_1 = \frac{1}{N^{1/4}}h_{horiz,1\,tube} = \frac{1}{4^{1/4}}(7693 \text{ W/m}^2\cdot°\text{C}) = 5440 \text{ W/m}^2\cdot°\text{C}$$
$$h_o = \frac{2\times2h_1 + 2\times3h_2 + 1\times4h_3}{2\times2+2\times3+1\times4} = \frac{4\times6469+6\times5845+4\times5440}{14} = 5908 \text{ W/m}^2\cdot°\text{C}$$

Let us check if the assumed value for the rube temperature was reasonable

$$h_iA_i\Delta T_i = h_oA_o\Delta T_o$$
$$(4000)\pi(0.030)L(T_{tube}-20) = (5908)\pi(0.038)L(37-T_{tube}) \longrightarrow T_{tube} = 31.1°\text{C}$$

which is very close to the assumed value of 31°C. Therefore, the assumption was good. The overall heat transfer coefficient based on the outer surface is determined from

$$U_o = \left(\frac{D_o}{D_ih_i} + \frac{D_o\ln(D_o/D_i)}{2k} + \frac{1}{h_o}\right)^{-1} = \left(\frac{0.038}{0.030\times4000} + \frac{0.038\ln(3.8/3.0)}{2(401)} + \frac{1}{5908}\right)^{-1} = \mathbf{2012\ W/m^2\cdot°C}$$

(*b*) The rate of heat transfer is

$$\dot{Q} = \dot{m}_{condensation}h^*_{fg} = (900/3600 \text{ kg/s})(1133\times10^3 \text{ J/kg}) = 2.833\times10^5 \text{ W}$$

Then the tube length may be determined from

$$\dot{Q} = U_oA_o\Delta T$$
$$2.833\times10^5 \text{ W} = (2012 \text{ W/m}^2\cdot\text{C})(14)\pi(0.038 \text{ m})L(37-20) \longrightarrow L = \mathbf{4.96\ m}$$

10-129 Saturated ammonia vapor at a saturation temperature of T_{sat} = 25°C condenses on the outer surfaces of a tube bank in which cooling water flows. The rate of condensation of ammonia, the overall heat transfer coefficient, and the tube length are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tubes are isothermal. **3** The thermal resistance of the tube walls is negligible.

Properties The properties of ammonia at the saturation temperature of 25°C are h_{fg} = 1166×10³ J/kg and ρ_v = 7.809 kg/m³ (Table A-11). We assume that the tube temperature is 20°C. Then, the properties of liquid ammonia at the film temperature of $T_f = (T_{sat} + T_s)/2$ = (25 + 20)/2 = 22.5°C are (Table A-11)

$$\rho_l = 606.5 \text{ kg/m}^3$$
$$\mu_l = 1.479\times10^{-4} \text{ kg/m}\cdot\text{s}$$
$$c_{pl} = 4765 \text{ J/kg}\cdot°\text{C}$$
$$k_l = 0.4869 \text{ W/m}\cdot°\text{C}$$

The water properties at the average temperature of (14+17)/2 = 15.5°C are (Table A-9)

$$\rho = 999.0 \text{ kg/m}^3$$
$$c_p = 4185 \text{ J/kg}\cdot°\text{C}$$
$$\mu = 1.124\times10^{-3} \text{ kg/m}\cdot\text{s}$$
$$k = 0.590 \text{ W/m}\cdot°\text{C}$$
$$\text{Pr} = 7.98$$

Analysis (*a*) The modified latent heat of vaporization is

$$\begin{aligned} h^*_{fg} &= h_{fg} + 0.68c_{pl}(T_{sat} - T_s) \\ &= 1166\times10^3 \text{ J/kg} + 0.68\times4765 \text{ J/kg}\cdot°\text{C}(25-20)°\text{C} \\ &= 1182\times10^3 \text{ J/kg} \end{aligned}$$

The heat transfer coefficient for condensation on a single horizontal tube is

$$\begin{aligned} h = h_{horizontal} &= 0.729\left[\frac{g\rho_l(\rho_l-\rho_v)h^*_{fg}k_l^3}{\mu_l(T_{sat}-T_s)D}\right]^{1/4} \\ &= 0.729\left[\frac{(9.81 \text{ m/s}^2)(606.5 \text{ kg/m}^3)(606.5-7.809 \text{ kg/m}^3)(1182\times10^3 \text{ J/kg})(0.4869 \text{ W/m}\cdot°\text{C})^3}{(1.479\times10^{-4} \text{ kg/m}\cdot\text{s})(25-20)°\text{C}(0.025 \text{ m})}\right]^{1/4} \\ &= 9280 \text{ W/m}^2\cdot°\text{C} \end{aligned}$$

Then the average heat transfer coefficient for a 4-pipe high vertical tier becomes

$$h_o = h_{\text{horiz, N tubes}} = \frac{1}{N^{1/4}}h_{\text{horiz, 1 tube}} = \frac{1}{4^{1/4}}(9280 \text{ W/m}^2\cdot°\text{C}) = 6562 \text{ W/m}^2\cdot°\text{C}$$

The rate of heat transfer in the condenser is

$$\dot{m} = 16\rho A_c \boldsymbol{V} = 16(999 \text{ kg/m}^3)\pi(0.25)(0.025 \text{ m})^2(2 \text{ m/s}) = 15.69 \text{ kg/s}$$
$$\dot{Q} = \dot{m}c_p(T_{out} - T_{in}) = (15.69 \text{ kg/s})(4185 \text{ J/kg}\cdot°\text{C})(17-14) = 1.970\times10^5 \text{ W}$$

Then the rate of condensation becomes

$$\dot{m}_{cond} = \frac{\dot{Q}}{h^*_{fg}} = \frac{1.970\times10^5 \text{ W}}{1182\times10^3 \text{ J/kg}} = \mathbf{0.167 \text{ kg/s}}$$

(*b*) For the calculation of the heat transfer coefficient on the inner surfaces of the tubes, we first determine the Reynolds number

$$\text{Re} = \frac{VD\rho}{\mu} = \frac{(2 \text{ m/s})(0.025\text{m})(999.0\text{kg/m}^3)}{1.124\times10^{-3} \text{ kg/m}\cdot\text{s}} = 44{,}440$$

which is greater than 10,000. Therefore, the flow is turbulent. Assuming fully developed flow, the Nusselt number and the heat transfer coefficient are determined to be

$$Nu = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(44{,}440)^{0.8}(7.98)^{0.4} = 275.9$$

$$h_i = \frac{k}{D}Nu = \frac{(0.590\ \text{W/m}\cdot{}^\circ\text{C})}{0.025\,\text{m}}(275.9) = 6511\ \text{W/m}^2\cdot{}^\circ\text{C}$$

Let us check if the assumed value for the rube temperature was reasonable

$$h_i\Delta T_i = h_o\Delta T_o$$

$$(6511)(T_{\text{tube}} - 15.5) = (6562)(25 - T_{\text{tube}})$$

$$T_{\text{tube}} = 20.3^\circ\text{C}$$

which is sufficiently close to the assumed value of 20°C. Disregarding thermal resistance of the tube walls, the overall heat transfer coefficient is determined from

$$U = \left(\frac{1}{h_i} + \frac{1}{h_o}\right)^{-1} = \left(\frac{1}{6511} + \frac{1}{6562}\right)^{-1} = \mathbf{3268\ W/m^2\cdot{}^\circ C}$$

(*c*) The tube length may be determined from

$$\dot{Q} = UA\Delta T$$

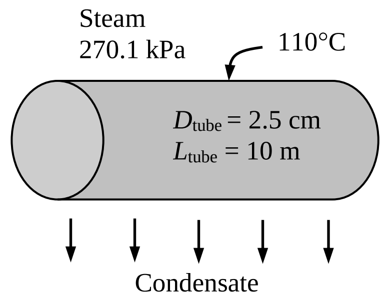
$$1.970\times10^5\ \text{W} = (3268\ \text{W/m}^2\cdot\text{C})(16)\pi(0.025\ \text{m})L\left[25 - \frac{1}{2}(14+17)\right]$$

$$L = \mathbf{5.05\ m}$$

10-130 Saturated steam at 270.1 kPa pressure and thus at a saturation temperature of T_{sat} = 130°C (Table A-9) condenses inside a horizontal tube which is maintained at 110°C. The average heat transfer coefficient and the rate of condensation of steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube is isothermal. **3** The vapor velocity is low so that Re_{vapor}< 35,000.

Properties The properties of water at the saturation temperature of 130°C are h_{fg} = 2174×10³ J/kg and ρ_v = 1.50 kg/m³. The properties of liquid water at the film temperature of $T_f = (T_{sat} + T_s)/2$ = (130 + 110)/2 = 120°C are (Table A-9),



$$\rho_l = 943.4\ \text{kg/m}^3$$
$$\mu_l = 0.232\times10^{-3}\ \text{kg/m.s}$$
$$\nu_l = \mu_l / \rho_l = 0.246\times10^{-6}\ \text{m}^2/\text{s}$$
$$c_{pl} = 4244\ \text{J/kg.°C}$$
$$k_l = 0.683\ \text{W/m.°C}$$

Analysis The condensation heat transfer coefficient is determined from

$$h = h_{internal} = 0.555\left[\frac{g\rho_l(\rho_l-\rho_v)k_l^3}{\mu_l(T_{sat}-T_s)D}\left(h_{fg}+\frac{3}{8}c_{pl}(T_{sat}-T_s)\right)\right]^{1/4}$$
$$= 0.555\left[\frac{(9.81\ \text{m/s}^2)(943.4\ \text{kg/m}^3)(943.4-1.50)\ \text{kg/m}^3)(0.683\ \text{W/m}\cdot°\text{C})^3}{(0.232\times10^{-3}\ \text{kg/m}\cdot\text{s})(130-110)°\text{C}\,(0.025\ \text{m})}\right.$$
$$\left.\times\left(2174\times10^3\ \text{J/kg}+\frac{3}{8}(4244\ \text{J/kg}\cdot°\text{C})(130-110)°\text{C}\right)\right]^{1/4}$$
$$= \mathbf{8413\ W/m^2\cdot°C}$$

The heat transfer surface area of the pipe is

$$A_s = \pi DL = \pi(0.025\ \text{m})(10\ \text{m}) = 0.7854\ \text{m}^2$$

Then the rate of heat transfer during this condensation process becomes

$$\dot{Q} = hA_s(T_{sat}-T_s) = (8413\ \text{W/m}^2\cdot°\text{C})(0.7854\ \text{m}^2)(130-110)°\text{C} = 132{,}151\ \text{W}$$

The rate of condensation of steam is determined from

$$\dot{m}_{condensation} = \frac{\dot{Q}}{h_{fg}} = \frac{132{,}151\ \text{J/s}}{2174\times10^3\ \text{J/kg}} = \mathbf{0.0608\ kg/s}$$

Fundamentals of Engineering (FE) Exam Problems

10-131 When boiling a saturated liquid, one must be careful while increasing the heat flux to avoid "burnout." Burnout occurs when the boiling transitions from _____ boiling.

(a) convection to nucleate (b) convection to film (c) film to nucleate

(d) nucleate to film (e) none of them

Answer (d) nucleate to film

10-132 Heat transfer coefficients for a vapor condensing on a surface can be increased by promoting

(a) film condensation (b) dropwise condensation (c) rolling action (d) none of them

Answer (b) dropwise condensation

10-133 At a distance x down a vertical, isothermal flat plate on which a saturated vapor is condensing in a continuous film, the thickness of the liquid condensate layer is δ. The heat transfer coefficient at this location on the plate is given by

(a) k_l / δ (b) δh_f (c) δh_{fg} (d) δh_g (e) none of them

Answer (a) k_l / δ

10-134 When a saturated vapor condenses on a vertical, isothermal flat plate in a continuous film, the rate of heat transfer is proportional to

(a) $(T_s - T_{\text{sat}})^{1/4}$ (b) $(T_s - T_{\text{sat}})^{1/2}$ (c) $(T_s - T_{\text{sat}})^{3/4}$ (d) $(T_s - T_{\text{sat}})$ (e) $(T_s - T_{\text{sat}})^{2/3}$

Answer (c) $(T_s - T_{\text{sat}})^{3/4}$

10-135 Saturated water vapor is condensing on a 0.5 m^2 vertical flat plate in a continuous film with an average heat transfer coefficient of 5 $kW/m^2 \cdot K$. The temperature of the water is 80ºC (h_{fg} = 2309 kJ/kg) and the temperature of the plate is 60ºC. The rate at which condensate is being formed is

(a) 0.022 kg/s (b) 0.048 kg/s (c) 0.077 kg/s (d) 0.16 kg/s (e) 0.32 kg/s

Answer (a) 0.022 kg/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
hfg=2309 [kJ/kg]
dT=20 [C]
A=0.5 [m^2]
h=5 [kJ/m^2-K-s]
mdot=h*A*dT/hfg
```

10-136 Steam condenses at 50ºC on a 1.2-m-high and 2.4-m-wide vertical plate that is maintained at 30ºC. The condensation heat transfer coefficient is

(a) 4260 $W/m^2 \cdot K$ (b) 4780 $W/m^2 \cdot K$ (c) 5510 $W/m^2 \cdot K$ (d) 6260 $W/m^2 \cdot K$ (e) 6940 $W/m^2 \cdot K$

(For water, use ρ_l = 992.1 kg/m^3, $\mu_l = 0.653 \times 10^{-3}$ kg/m·s, k_l = 0.631 W/m·°C, c_{pl} = 4179 J/kg·°C, $h_{fg\,@\,Tsat}$ = 2383 kJ/kg)

Answer (b) 4780 $W/m^2 \cdot K$

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_sat=50 [C]
T_s=30 [C]
L=1.2 [m]
w=2.4 [m]

h_fg=2383E3 [J/kg] "at 50 C from Table A-9"
"The properties of water at (50+30)/2=40 C are (Table A-9)"
rho_l=992.1 [kg/m^3]
mu_l=0.653E-3 [kg/m-s]
nu_l=mu_l/rho_l
c_p_l=4179 [J/kg-C]
k_l=0.631 [W/m-C]
g=9.81 [m/s^2]

h_fg_star=h_fg+0.68*c_p_l*(T_sat-T_s)
Re=(4.81+(3.70*L*k_l*(T_sat-T_s))/(mu_l*h_fg_star)*(g/nu_l^2)^(1/3))^0.820
"Re is between 30 and 1800, and therefore there is wavy laminar flow"
h=(Re*k_l)/(1.08*Re^1.22-5.2)*(g/nu_l^2)^(1/3)
```

10-137 An air conditioner condenser in an automobile consists of 2 m^2 of tubular heat exchange area whose surface temperature is 30ºC. Saturated refrigerant 134a vapor at 50ºC (h_{fg} = 152 kJ/kg) condenses on these tubes. What heat transfer coefficient must exist between the tube surface and condensing vapor to produce 1.5 kg/min of condensate?

(a) 95 W/m^2·K (b) 640 W/m^2·K (c) 727 W/m^2·K (d) 799 W/m^2·K (e) 960 W/m^2·K

Answer (a) 95 W/m^2·K

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
hfg=152000 [J/kg]
dT=20 [C]
A=2 [m^2]
mdot=(1.5/60) [kg/s]
Q=mdot*hfg
Q=h*A*dT
```

10-138 Saturated water vapor at 40°C is to be condensed as it flows through a tube at a rate of 0.2 kg/s. The condensate leaves the tube as a saturated liquid at 40°C. The rate of heat transfer from the tube is

(a) 34 kJ/s (b) 268 kJ/s (c) 453 kJ/s (d) 481 kJ/s (e) 515 kJ/s

Answer (d) 481 kJ/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T1=40 [C]
m_dot=0.2 [kg/s]
h_f=ENTHALPY(Steam_IAPWS,T=T1,x=0)
h_g=ENTHALPY(Steam_IAPWS,T=T1,x=1)
h_fg=h_g-h_f
Q_dot=m_dot*h_fg

"Wrong Solutions:"
W1_Q=m_dot*h_f  "Using hf"
W2_Q=m_dot*h_g "Using hg"
W3_Q=h_fg "not using mass flow rate"
W4_Q=m_dot*(h_f+h_g) "Adding hf and hg"
```

10-139 Steam condenses at 50ºC on the outer surface of a horizontal tube with an outer diameter of 6 cm. The outer surface of the tube is maintained at 30ºC. The condensation heat transfer coefficient is

(a) 5493 W/m^2·ºC (b) 5921 W/m^2·ºC (c) 6796 W/m^2·ºC (d) 7040 W/m^2·ºC (e) 7350 W/m^2·ºC

(For water, use ρ_l= 992.1 kg/m^3, μ_l= 0.653×10^{-3} kg/m·s, k_l= 0.631 W/m·°C, c_{pl}= 4179 J/kg·°C, $h_{fg\ @\ Tsat}$ = 2383 kJ/kg)

Answer (c) 6796 W/m^2·ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_sat=50 [C]
T_s=30 [C]
D=0.06 [m]
h_fg=2383E3 [J/kg] "at 50 C from Table A-9"
rho_v=0.0831 [kg/m^3]
"The properties of water at (50+30)/2=40 C are (Table A-9)"
rho_l=992.1 [kg/m^3]
mu_l=0.653E-3 [kg/m-s]
c_p_l=4179 [J/kg-C]
k_l=0.631 [W/m-C]
g=9.81 [m/s^2]
h_fg_star=h_fg+0.68*c_p_l*(T_sat-T_s)
h=0.729*((g*rho_l*(rho_l-rho_v)*h_fg_star*k_l^3)/(mu_l*(T_sat-T_s)*D))^0.25
```

10-140 Steam condenses at 50ºC on a tube bank consisting of 20 tubes arranged in a rectangular array of 4 tubes high and 5 tubes wide. Each tube has a diameter of 3 cm and a length of 5 m and the outer surfaces of the tubes are maintained at 30ºC. The rate of condensation of steam is

(a) 0.12 kg/s (b) 0.28 kg/s (c) 0.31 kg/s (d) 0.45 kg/s (e) 0.62 kg/s

(For water, use ρ_l = 992.1 kg/m^3, μ_l = 0.653×10^{-3} kg/m·s, k_l = 0.631 W/m·°C, c_{pl} = 4179 J/kg·°C, $h_{fg\ @\ Tsat}$ = 2383 kJ/kg)

Answer (c) 0.31 kg/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_sat=50 [C]
T_s=30 [C]
D=0.03 [m]
L=5 [m]
N=4
N_total=5*N

h_fg=2383E3 [J/kg] "at 50 C from Table A-9"
rho_v=0.0831 [kg/m^3] "at 50 C from Table A-9"
"The properties of water at (50+30)/2=40 C are (Table A-9)"
rho_l=992.1 [kg/m^3]
mu_l=0.653E-3 [kg/m-s]
c_p_l=4179 [J/kg-C]
k_l=0.631 [W/m-C]
g=9.81 [m/s^2]

h_fg_star=h_fg+0.68*c_p_l*(T_sat-T_s)
h_1tube=0.729*((g*rho_l*(rho_l-rho_v)*h_fg_star*k_l^3)/(mu_l*(T_sat-T_s)*D*N))^0.25
h_Ntubes=1/N^0.25*h_1tube
A_s=N_total*pi*D*L
Q_dot=h_Ntubes*A_s*(T_sat-T_s)
m_dot_cond=Q_dot/h_fg_star
```

10-141 ... 10-146 Design and Essay Problems

Solutions Manual for

Heat and Mass Transfer: Fundamentals & Applications
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Chapter 11
HEAT EXCHANGERS

Types of Heat Exchangers

11-1C Heat exchangers are classified according to the flow type as parallel flow, counter flow, and cross-flow arrangement. In parallel flow, both the hot and cold fluids enter the heat exchanger at the same end and move in the same direction. In counter-flow, the hot and cold fluids enter the heat exchanger at opposite ends and flow in opposite direction. In cross-flow, the hot and cold fluid streams move perpendicular to each other.

11-2C A heat exchanger is classified as being compact if β > 700 m^2/m^3 or (200 ft^2/ft^3) where β is the ratio of the heat transfer surface area to its volume which is called the area density. The area density for double-pipe heat exchanger can not be in the order of 700. Therefore, it can not be classified as a compact heat exchanger.

11-3C Regenerative heat exchanger involves the alternate passage of the hot and cold fluid streams through the same flow area. The static type regenerative heat exchanger is basically a porous mass which has a large heat storage capacity, such as a ceramic wire mash. Hot and cold fluids flow through this porous mass alternately. Heat is transferred from the hot fluid to the matrix of the regenerator during the flow of the hot fluid and from the matrix to the cold fluid. Thus the matrix serves as a temporary heat storage medium. The dynamic type regenerator involves a rotating drum and continuous flow of the hot and cold fluid through different portions of the drum so that any portion of the drum passes periodically through the hot stream, storing heat and then through the cold stream, rejecting this stored heat. Again the drum serves as the medium to transport the heat from the hot to the cold fluid stream.

11-4C In the shell and tube exchangers, baffles are commonly placed in the shell to force the shell side fluid to flow across the shell to enhance heat transfer and to maintain uniform spacing between the tubes. Baffles disrupt the flow of fluid, and an increased pumping power will be needed to maintain flow. On the other hand, baffles eliminate dead spots and increase heat transfer rate.

11-5C Using six-tube passes in a shell and tube heat exchanger increases the heat transfer surface area, and the rate of heat transfer increases. But it also increases the manufacturing costs.

11-6C Using so many tubes increases the heat transfer surface area which in turn increases the rate of heat transfer.

11-7C In counter-flow heat exchangers, the hot and the cold fluids move parallel to each other but both enter the heat exchanger at opposite ends and flow in opposite direction. In cross-flow heat exchangers, the two fluids usually move perpendicular to each other. The cross-flow is said to be unmixed when the plate fins force the fluid to flow through a particular interfin spacing and prevent it from moving in the transverse direction. When the fluid is free to move in the transverse direction, the cross-flow is said to be mixed.

The Overall Heat Transfer Coefficient

11-8C Heat is first transferred from the hot liquid to the wall by convection, through the wall by conduction and from the wall to the cold liquid again by convection.

11-9C When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, which is usually the case, the thermal resistance of the tube is negligible.

11-10C The heat transfer surface areas are $A_i = \pi D_1 L$ and $A_o = \pi D_2 L$. When the thickness of inner tube is small, it is reasonable to assume $A_i \cong A_o \cong A_s$.

11-11C The effect of fouling on a heat transfer is represented by a fouling factor R_f. Its effect on the heat transfer coefficient is accounted for by introducing a thermal resistance R_f / A_s. The fouling increases with increasing temperature and decreasing velocity.

11-12C None.

11-13C When one of the convection coefficients is much smaller than the other $h_i << h_o$, and $A_i \approx A_0 \approx A_s$. Then we have ($1/h_i >> 1/h_o$) and thus $U_i = U_0 = U \cong h_i$.

11-14C The most common type of fouling is the precipitation of solid deposits in a fluid on the heat transfer surfaces. Another form of fouling is corrosion and other chemical fouling. Heat exchangers may also be fouled by the growth of algae in warm fluids. This type of fouling is called the biological fouling. Fouling represents additional resistance to heat transfer and causes the rate of heat transfer in a heat exchanger to decrease, and the pressure drop to increase.

11-15C When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, the thermal resistance of the tube is negligible and the inner and the outer surfaces of the tube are almost identical ($A_i \cong A_o \cong A_s$). Then the overall heat transfer coefficient of a heat exchanger can be determined to from $U = (1/h_i + 1/h_o)^{-1}$

11-16E The overall heat transfer coefficients based on the outer and inner surfaces for a heat exchanger are to be determined.

Assumptions **1** Steady operating condition exists. **2** Thermal properties are constant.

Properties The conductivity of the tube material is given to be 0.5 Btu/hr·ft·°F.

Analysis The overall heat transfer coefficient based on the outer surface is

$$\frac{1}{U_o A_o}=\frac{1}{h_i A_i}+\frac{1}{2\pi kL}\ln(D_o/D_i)+\frac{1}{h_o A_o}$$

$$\frac{1}{U_o}=\frac{A_o}{h_i A_i}+\frac{A_o}{2\pi kL}\ln\left(\frac{D_o}{D_i}\right)+\frac{A_o}{h_o A_o}=\frac{1}{h_i}\frac{D_o}{D_i}+\frac{D_o}{2k}\ln\left(\frac{D_o}{D_i}\right)+\frac{1}{h_o}$$

Thus

$$U_o=\left[\frac{1}{h_i}\frac{D_o}{D_i}+\frac{D_o}{2k}\ln\left(\frac{D_o}{D_i}\right)+\frac{1}{h_o}\right]^{-1}$$

$$=\left[\left(\frac{1}{50}\right)\left(\frac{3}{2}\right)+\frac{3/12}{2(0.5)}\ln\left(\frac{3}{2}\right)+\frac{1}{10}\right]^{-1}\ \text{Btu/hr}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

$$=\mathbf{4.32\ Btu/hr\cdot ft^2\cdot{}^\circ F}$$

The overall heat transfer coefficient based on the inner surface is

$$\frac{1}{U_i A_i}=\frac{1}{h_i A_i}+\frac{1}{2\pi kL}\ln(D_o/D_i)+\frac{1}{h_o A_o}$$

$$\frac{1}{U_i}=\frac{A_i}{h_i A_i}+\frac{A_i}{2\pi kL}\ln\left(\frac{D_o}{D_i}\right)+\frac{A_i}{h_o A_o}=\frac{1}{h_i}+\frac{D_i}{2k}\ln\left(\frac{D_o}{D_i}\right)+\frac{1}{h_o}\frac{D_i}{D_o}$$

Thus

$$U_i=\left[\frac{1}{h_i}+\frac{D_i}{2k}\ln\left(\frac{D_o}{D_i}\right)+\frac{1}{h_o}\frac{D_i}{D_o}\right]^{-1}$$

$$=\left[\left(\frac{1}{50}\right)+\frac{2/12}{2(0.5)}\ln\left(\frac{3}{2}\right)+\frac{1}{10}\left(\frac{2}{3}\right)\right]^{-1}\ \text{Btu/hr}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

$$=\mathbf{6.48\ Btu/hr\cdot ft^2\cdot{}^\circ F}$$

Discussion The two overall heat transfer coefficients differ significantly with U_i larger than U_o by a factor of 1.5. The overall heat transfer coefficient ratio can be expressed as

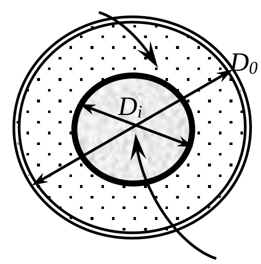
$$\frac{1}{U_o A_o}=\frac{1}{U_i A_i}\qquad\rightarrow\qquad\frac{U_i}{U_o}=\frac{A_o}{A_i}=\frac{D_o}{D_i}=1.5$$

11-17 Refrigerant-134a is cooled by water in a double-pipe heat exchanger. The overall heat transfer coefficient is to be determined.

Assumptions **1** The thermal resistance of the inner tube is negligible since the tube material is highly conductive and its thickness is negligible. **2** Both the water and refrigerant-134a flow are fully developed. **3** Properties of the water and refrigerant-134a are constant.

Properties The properties of water at 20°C are (Table A-9)

$$\rho = 998 \text{ kg/m}^3$$
$$\nu = \mu/\rho = 1.004\times10^{-6} \text{ m}^2\text{/s}$$
$$k = 0.598 \text{ W/m.°C}$$
$$\text{Pr} = 7.01$$



Analysis The hydraulic diameter for annular space is

$$D_h = D_o - D_i = 0.025 - 0.01 = 0.015\text{ m}$$

The average velocity of water in the tube and the Reynolds number are

$$V_{avg} = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho\left(\pi\dfrac{D_o^{\,2} - D_i^{\,2}}{4}\right)} = \frac{0.3\text{ kg/s}}{(998\text{ kg/m}^3)\left(\pi\dfrac{(0.025\text{ m})^2 - (0.01\text{ m})^2}{4}\right)} = 0.729\text{ m/s}$$

$$\text{Re} = \frac{V_{avg}D_h}{\nu} = \frac{(0.729\text{ m/s})(0.015\text{m})}{1.004\times10^{-6}\text{ m}^2\text{ / s}} = 10{,}890$$

which is greater than 4000. Therefore flow is turbulent. Assuming fully developed flow,

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(10{,}890)^{0.8}(7.01)^{0.4} = 85.0$$

and

$$h_o = \frac{k}{D_h}Nu = \frac{0.598\text{ W/m.°C}}{0.015\text{ m}}(85.0) = 3390\text{ W/m}^2\text{.°C}$$

Then the overall heat transfer coefficient becomes

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{4100\text{ W/m}^2\text{.°C}} + \dfrac{1}{3390\text{ W/m}^2\text{.°C}}} = \mathbf{1856\ W/m^2.°C}$$

Discussion This problem can also be solved using Gnielinski equation. First the friction factor is determined from the first Petukhov equation.

$$f = (0.790\ln\text{Re} - 1.64)^{-2} = [0.790\ln(10{,}890) - 1.64]^{-2} = 0.03074$$

$$Nu = \frac{hD_h}{k} = \frac{(f/8)(\text{Re} - 1000)\text{Pr}}{1 + 12.7(f/8)^{0.5}(\text{Pr}^{2/3} - 1)} = \frac{(0.03074/8)(10{,}890 - 1000)(7.01)}{1 + 12.7(0.03074/8)^{0.5}(7.01^{2/3} - 1)} = 86.04$$

$$h_o = \frac{k}{D_h}Nu = \frac{0.598\text{ W/m.°C}}{0.015\text{ m}}(86.04) = 3430\text{ W/m}^2\text{.°C}$$

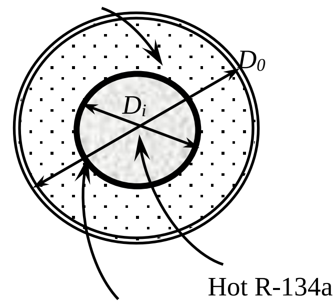
$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{4100\text{ W/m}^2\text{.°C}} + \dfrac{1}{3430\text{ W/m}^2\text{.°C}}} = \mathbf{1868\ W/m^2.°C}$$

The result is very close to that obtained by using the modified Colburn equation for the Nusselt number. Therefore, different heat transfer correlations can be used to solve for the heat transfer coefficient.

11-18 Refrigerant-134a is cooled by water in a double-pipe heat exchanger. The overall heat transfer coefficient is to be determined.

Assumptions **1** The thermal resistance of the inner tube is negligible since the tube material is highly conductive and its thickness is negligible. **2** Both the water and refrigerant-134a flows are fully developed. **3** Properties of the water and refrigerant-134a are constant. **4** The limestone layer can be treated as a plain layer since its thickness is very small relative to its diameter.

Properties The properties of water at 20°C are (Table A-9)



$$\rho = 998 \text{ kg/m}^3$$
$$\nu = \mu / \rho = 1.004 \times 10^{-6} \text{ m}^2\text{/s}$$
$$k = 0.598 \text{ W/m.°C}$$
$$\text{Pr} = 7.01$$

Analysis The hydraulic diameter for annular space is

$$D_h = D_o - D_i = 0.025 - 0.01 = 0.015 \text{ m}$$

The average velocity of water in the tube and the Reynolds number are

$$V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho\left(\pi \dfrac{D_o^2 - D_i^2}{4}\right)} = \frac{0.3 \text{ kg/s}}{(998 \text{ kg/m}^3)\left(\pi \dfrac{(0.025 \text{ m})^2 - (0.01 \text{ m})^2}{4}\right)} = 0.729 \text{ m/s}$$

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(0.729 \text{ m/s})(0.015 \text{m})}{1.004 \times 10^{-6} \text{ m}^2\text{ / s}} = 10{,}890$$

which is greater than 10,000. Therefore flow is turbulent. Assuming fully developed flow,

$$Nu = \frac{hD_h}{k} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} = 0.023(10{,}890)^{0.8}(7.01)^{0.4} = 85.0$$

and

$$h_o = \frac{k}{D_h} Nu = \frac{0.598 \text{ W/m.°C}}{0.015 \text{ m}}(85.0) = 3390 \text{ W/m}^2\text{.°C}$$

Disregarding the curvature effects, the overall heat transfer coefficient is determined to be

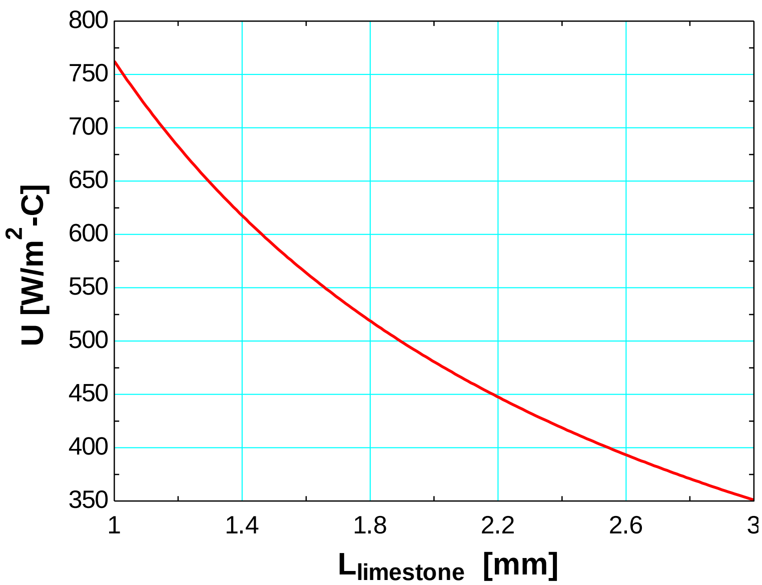
$$U = \frac{1}{\dfrac{1}{h_i} + \left(\dfrac{L}{k}\right)_{\text{limeston}} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{4100 \text{ W/m}^2\text{.°C}} + \dfrac{0.002 \text{ m}}{1.3 \text{ W/m.°C}} + \dfrac{1}{3390 \text{ W/m}^2\text{.°C}}} = \mathbf{481 \text{ W/m}^2\text{.°C}}$$

11-19 Prob. 11-18 is reconsidered. The overall heat transfer coefficient as a function of the limestone thickness is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D_i=0.010 [m]
D_o=0.025 [m]
T_w=20 [C]
h_i=4100 [W/m^2-C]
m_dot=0.3 [kg/s]
L_limestone=2 [mm]
k_limestone=1.3 [W/m-C]

"PROPERTIES"
k=conductivity(Water, T=T_w, P=100)
Pr=Prandtl(Water, T=T_w, P=100)
rho=density(Water, T=T_w, P=100)
mu=viscosity(Water, T=T_w, P=100)
nu=mu/rho

"ANALYSIS"
D_h=D_o-D_i
Vel=m_dot/(rho*A_c)
A_c=pi*(D_o^2-D_i^2)/4
Re=(Vel*D_h)/nu
"Re is calculated to be greater than 10,000. Therefore, the flow is turbulent."
Nusselt=0.023*Re^0.8*Pr^0.4
h_o=k/D_h*Nusselt
U=1/(1/h_i+(L_limestone*Convert(mm, m))/k_limestone+1/h_o)
```

$L_{limestone}$ [mm]	U [W/m^2-C]
1	762.4
1.1	720.2
1.2	682.4
1.3	648.3
1.4	617.5
1.5	589.5
1.6	564
1.7	540.5
1.8	518.9
1.9	499
2	480.6
2.1	463.4
2.2	447.5
2.3	432.6
2.4	418.7
2.5	405.6
2.6	393.3
2.7	381.8
2.8	370.9
2.9	360.6
3	350.9

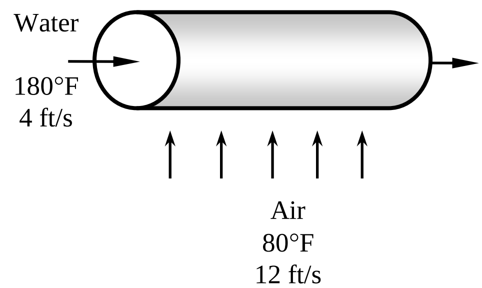


11-20E Water is cooled by air in a cross-flow heat exchanger. The overall heat transfer coefficient is to be determined.

Assumptions **1** The thermal resistance of the inner tube is negligible since the tube material is highly conductive and its thickness is negligible. **2** Both the water and air flow are fully developed. **3** Properties of the water and air are constant.

Properties The properties of water at 180°F are (Table A-9E)

$$k = 0.388\ \text{Btu/h.ft.°F}$$
$$\nu = 3.825\times10^{-6}\ \text{ft}^2/\text{s}$$
$$\text{Pr} = 2.15$$



The properties of air at 80°F are (Table A-15E)

$$k = 0.01481\ \text{Btu/h.ft.°F}$$
$$\nu = 1.697\times10^{-4}\ \text{ft}^2/\text{s}$$
$$\text{Pr} = 0.7290$$

Analysis The overall heat transfer coefficient can be determined from

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o}$$

The Reynolds number of water is

$$\text{Re} = \frac{V_{avg} D_h}{\nu} = \frac{(4\ \text{ft/s})[0.75/12\ \text{ft}]}{3.825\times10^{-6}\ \text{ft}^2/\text{s}} = 65{,}360$$

which is greater than 10,000. Therefore the flow of water is turbulent. Assuming the flow to be fully developed, the Nusselt number is determined from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(65{,}360)^{0.8}(2.15)^{0.4} = 222$$

and $$h_i = \frac{k}{D_h}Nu = \frac{0.388\ \text{Btu/h.ft.°F}}{0.75/12\ \text{ft}}(222) = 1378\ \text{Btu/h.ft}^2\text{.°F}$$

The Reynolds number of air is

$$\text{Re} = \frac{VD}{\nu} = \frac{(12\ \text{ft/s})[3/(4\times12)\ \text{ft}]}{1.697\times10^{-4}\ \text{ft}^2/\text{s}} = 4420$$

The flow of air is across the cylinder. The proper relation for Nusselt number in this case is

$$Nu = \frac{hD}{k} = 0.3 + \frac{0.62\,\text{Re}^{0.5}\,\text{Pr}^{1/3}}{\left[1+(0.4/\text{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\text{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$= 0.3 + \frac{0.62(4420)^{0.5}(0.7290)^{1/3}}{\left[1+(0.4/0.7290)^{2/3}\right]^{1/4}}\left[1+\left(\frac{4420}{282{,}000}\right)^{5/8}\right]^{4/5} = 34.86$$

and $$h_o = \frac{k}{D}Nu = \frac{0.01481\ \text{Btu/h.ft.°F}}{0.75/12\ \text{ft}}(34.86) = 8.26\ \text{Btu/h.ft}^2\text{.°F}$$

Then the overall heat transfer coefficient becomes

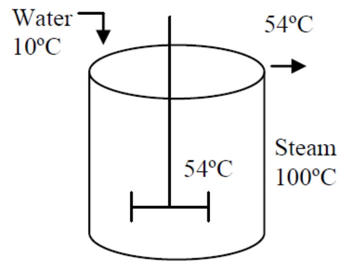
$$U = \frac{1}{\dfrac{1}{h_i}+\dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{1378\ \text{Btu/h.ft}^2\text{.°F}}+\dfrac{1}{8.26\ \text{Btu/h.ft}^2\text{.°F}}} = \mathbf{8.21\ Btu/h.ft^2.°F}$$

11-21 A water stream is heated by a jacketted-agitated vessel, fitted with a turbine agitator. The mass flow rate of water is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The properties of water at 54°C are (Table A-9)

k = 0.648 W/m·K
ρ = 985.8 kg/m^3
μ = 0.513 × 10^{-3} kg/m·s
Pr = 3.31



The specific heat of water at the average temperature of (10 + 54) / 2 = 32°C is 4178 J/kg·K (Table A-9)

Analysis We first determine the heat transfer coefficient on the inner wall of the vessel from the given correlation for the Nusselt number. To do so we need calculate the flow Reynolds number which is given by

$$\text{Re} = \frac{V_a D_t \rho}{\mu} = \frac{(0.62832)(0.6\text{ m})(985.8\text{ kg/m}^3)}{0.513\times10^{-3}\text{kg/m·s}} = 7.2444\times10^5$$

where D_t is the tank diameter, V_a is the tangential velocity of the agitator, and D_a is the agitator diameter,

$$V_a = (\text{agitator speed})\left(\frac{D_a}{2}\right) = (60\ \frac{\text{rev}}{\text{min}})(\frac{1\text{ min}}{60\text{ s}})(\frac{2\pi\text{rad}}{\text{rev}})\left(\frac{0.2\text{ m}}{2}\right) = 0.62832\text{ m/s}$$

The Nusselt number at the vessel inner wall is

$$\text{Nu} = 0.76\text{Re}^{2/3}\text{Pr}^{1/3} = 0.76(7.2444\times10^5)^{2/3}(3.31)^{1/3} = 9136$$

The heat transfer coefficient on the vessel inner wall (jacket) is

$$h_j = \frac{k}{D_t}\text{Nu} = \frac{0.648\text{ W/m·K}}{0.6\text{ m}}(9136) = 9866.9\text{ W/m}^2\text{·K}$$

The heat transfer coefficient on the outer side is determined as follows

$$h_o = 13{,}100\left(T_g - T_w\right)^{-0.25}\text{W/m}^2\text{·K}$$

From heat transfer balance at the vessel wall

$$h_o\left(T_g - T_w\right) = h_j(T_w - T_l)$$

where the steam temperature is $T_g = 100$°C and the water temperature is $T_l = 54$°C.

$$\begin{aligned} 13{,}100\left(T_g - T_w\right)^{-0.25}\left(T_g - T_w\right) &= h_j(T_w - T_l) \\ 13{,}100(100 - T_w)^{-0.25}(100 - T_w) &= 9866.9(T_w - 54) \\ 13{,}100(100 - T_w)^{0.75} &= 9866.9(T_w - 54) \\ \Rightarrow\quad T_w &= 70.71\text{°C} \end{aligned}$$

So,

$$h_o = 13{,}100(100 - 70.71)^{-0.25} = 5631.1\text{ W/m}^2\text{·K}$$

Neglecting the wall resistance and the thickness of the wall, the overall heat transfer coefficient can be written as

$$U = \left(\frac{1}{h_j} + \frac{1}{h_o}\right)^{-1} = \left(\frac{1}{9866.9} + \frac{1}{5631.1}\right)^{-1} = 3585.1\text{ W/m}^2\text{·K}$$

From an energy balance

$$\dot{m}_w c(T_{\text{out}} - T_{\text{in}}) = UA\Delta T_{\text{lm}}$$

where

$$\Delta T_{\text{lm}} = \frac{\left(T_g - T_{\text{in}}\right) - \left(T_g - T_{\text{out}}\right)}{\ln\left(\dfrac{T_g - T_{\text{in}}}{T_g - T_{\text{out}}}\right)} = \frac{(100-10)-(100-54)}{\ln\left(\dfrac{100-10}{100-54}\right)} = 65.56\text{°C}$$

$$\dot{m}_w = \frac{UA\Delta T_{\text{lm}}}{c(T_{\text{out}} - T_{\text{in}})} = \frac{(3585.1\text{ W/m}^2\text{·K})(\pi\times0.6\times0.6\text{ m}^2)(65.56)\text{K}}{(4178\ \text{J/kg·K})(54-10)\text{K}} = \mathbf{1.446\ kg/s}$$

Discussion When calculating the Reynolds number at the vessel inner wall, the water velocity is assumed to be equal to the tangential velocity of the agitator. In reality, the flow condition is much more complex, where the agitator can cause secondary flow to occur near the inner wall.

11-22E The overall heat transfer coefficient of a heat exchanger and the percentage change in the overall heat transfer coefficient due to scale built-up are to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat transfer coefficients and the fouling factors are constant and uniform.

Analysis When operating at design and clean conditions, the overall heat transfer coefficient is given as

$$U_{\text{w/o scale}} = 50 \text{ Btu/hr} \cdot \text{ft}^2 \cdot {}^\circ\text{F}$$

(*a*) After a period of use, the overall heat transfer coefficient due to the scale built-up is

$$\begin{aligned}\frac{1}{U_{\text{w/ scale}}} &= \frac{1}{U_{\text{w/o scale}}} + R_f \\ &= \frac{1}{50 \text{ Btu/hr} \cdot \text{ft}^2 \cdot {}^\circ\text{F}} + 0.002 \text{ hr} \cdot \text{ft}^2 \cdot {}^\circ\text{F/Btu} \\ &= 0.022 \text{ hr} \cdot \text{ft}^2 \cdot {}^\circ\text{F/Btu}\end{aligned}$$

or

$$U_{\text{w/ scale}} = \mathbf{45.5 \text{ Btu/hr} \cdot ft^2 \cdot {}^\circ F}$$

(*b*) The percentage change in the overall heat transfer coefficient due to the scale built-up is

$$\frac{U_{\text{w/o scale}} - U_{\text{w/ scale}}}{U_{\text{w/o scale}}} \times 100 = \frac{50 - 45.5}{50} \times 100 = \mathbf{9\%}$$

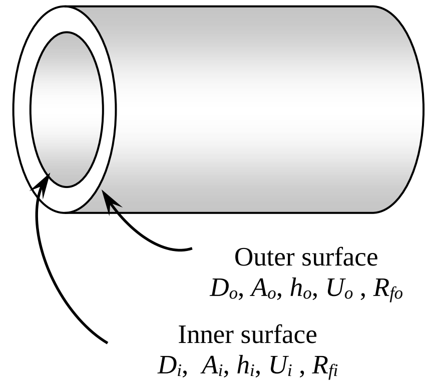
Discussion The scale built-up caused a 9% decrease in the overall heat transfer coefficient of the heat exchanger.

11-23 Water flows through the tubes in a boiler. The overall heat transfer coefficient of this boiler based on the inner surface area is to be determined.

Assumptions **1** Water flow is fully developed. **2** Properties of the water are constant.

Properties The properties of water at 110°C are (Table A-9)

$$\nu = \mu / \rho = 0.268 \times 10^{-6} \text{ m}^2/\text{s}$$
$$k = 0.682 \text{ W/m}^2.\text{K}$$
$$\text{Pr} = 1.58$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(3.5 \text{ m/s})(0.01\text{m})}{0.268 \times 10^{-6} \text{ m}^2 / \text{s}} = 130{,}600$$

which is greater than 10,000. Therefore, the flow is turbulent.
Assuming fully developed flow,

$$Nu = \frac{hD_h}{k} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} = 0.023(130{,}600)^{0.8}(1.58)^{0.4} = 341.9$$

and

$$h = \frac{k}{D_h} Nu = \frac{0.682 \text{ W/m.°C}}{0.01 \text{ m}}(341.9) = 23{,}320 \text{ W/m}^2.\text{°C}$$

The total resistance of this heat exchanger is then determined from

$$\begin{aligned} R = R_{total} &= R_i + R_{wall} + R_o = \frac{1}{h_i A_i} + \frac{\ln(D_o / D_i)}{2\pi kL} + \frac{1}{h_o A_o} \\ &= \frac{1}{(23{,}320 \text{ W/m}^2.\text{°C})[\pi(0.01 \text{ m})(7 \text{ m})]} + \frac{\ln(1.4/1)}{[2\pi(14.2 \text{ W/m.°C})(7 \text{ m})]} \\ &\quad + \frac{1}{(7200 \text{ W/m}^2.\text{°C})[\pi(0.014 \text{ m})(7 \text{ m})]} \\ &= 0.001185 \text{°C/W} \end{aligned}$$

and

$$R = \frac{1}{U_i A_i} \longrightarrow U_i = \frac{1}{RA_i} = \frac{1}{(0.001185\text{°C/W})[\pi(0.01\text{m})(7\text{m})]} = \mathbf{3838 \ W/m^2.°C}$$

11-24 Water is flowing through the tubes in a boiler. The overall heat transfer coefficient of this boiler based on the inner surface area is to be determined.

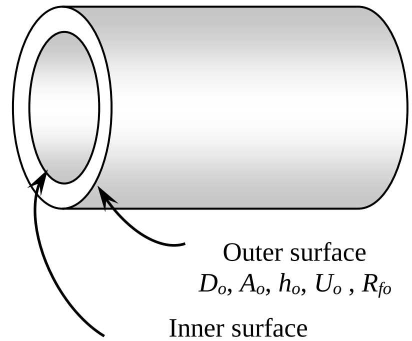
Assumptions **1** Water flow is fully developed. **2** Properties of water are constant. **3** The heat transfer coefficient and the fouling factor are constant and uniform.

Properties The properties of water at 110°C are (Table A-9)

$$\nu = \mu / \rho = 0.268\times10^{-6}\ \text{m}^2/\text{s}$$

$$k = 0.682\ \text{W/m}^2.\text{K}$$

$$\text{Pr} = 1.58$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(3.5\ \text{m/s})(0.01\text{m})}{0.268\times10^{-6}\ \text{m}^2/\text{s}} = 130{,}600$$

which is greater than 10,000. Therefore, the flow is turbulent.
Assuming fully developed flow,

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(130{,}600)^{0.8}(1.58)^{0.4} = 341.9$$

and

$$h = \frac{k}{D_h} Nu = \frac{0.682\ \text{W/m.°C}}{0.01\,\text{m}}(341.9) = 23{,}320\ \text{W/m}^2.\text{°C}$$

The thermal resistance of heat exchanger with a fouling factor of $R_{f,i} = 0.0005\,\text{m}^2.\text{°C/W}$ is determined from

$$R = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{1}{h_o A_o}$$

$$R = \frac{1}{(23{,}320\ \text{W/m}^2.\text{°C})[\pi(0.01\ \text{m})(7\ \text{m})]} + \frac{0.0005\ \text{m}^2.\text{°C/W}}{[\pi(0.01\ \text{m})(7\ \text{m})]}$$
$$+ \frac{\ln(1.4/1)}{2\pi(14.2\ \text{W/m.°C})(7\ \text{m})} + \frac{1}{(7200\ \text{W/m}^2.\text{°C})[\pi(0.014\ \text{m})(7\ \text{m})]}$$
$$= 0.003459\text{°C/W}$$

Then,

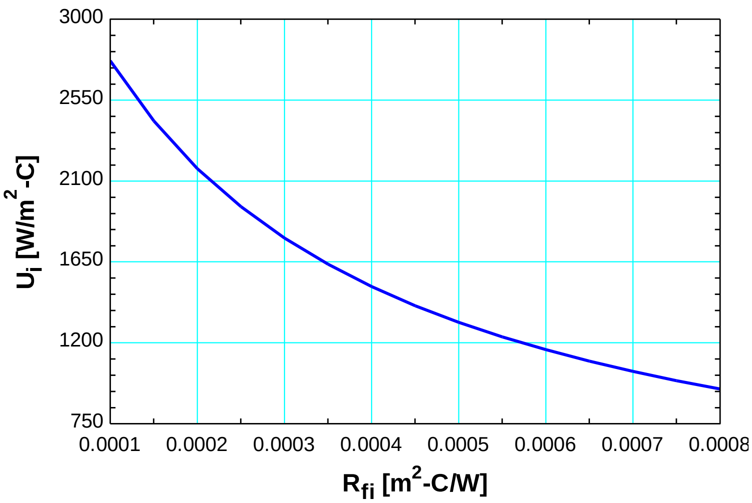
$$R = \frac{1}{U_i A_i} \longrightarrow U_i = \frac{1}{RA_i} = \frac{1}{(0.003459\text{°C/W})[\pi(0.01\text{m})(7\,\text{m})]} = \mathbf{1315\ W/m^2.°C}$$

11-25 Prob. 11-24 is reconsidered. The overall heat transfer coefficient based on the inner surface as a function of fouling factor is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_w=110 [C]
Vel=3.5 [m/s]
L=7 [m]
k_pipe=14.2 [W/m-C]
D_i=0.010 [m]
D_o=0.014 [m]
h_o=7200 [W/m^2-C]
R_f_i=0.0005 [m^2-C/W]

"PROPERTIES"
k=conductivity(Water, T=T_w, P=300)
Pr=Prandtl(Water, T=T_w, P=300)
rho=density(Water, T=T_w, P=300)
mu=viscosity(Water, T=T_w, P=300)
nu=mu/rho

"ANALYSIS"
Re=(Vel*D_i)/nu  "Re is calculated to be greater than 10,000. Therefore, the flow is turbulent."
Nusselt=0.023*Re^0.8*Pr^0.4
h_i=k/D_i*Nusselt
A_i=pi*D_i*L
A_o=pi*D_o*L
R=1/(h_i*A_i)+R_f_i/A_i+ln(D_o/D_i)/(2*pi*k_pipe*L)+1/(h_o*A_o)
U_i=1/(R*A_i)
```

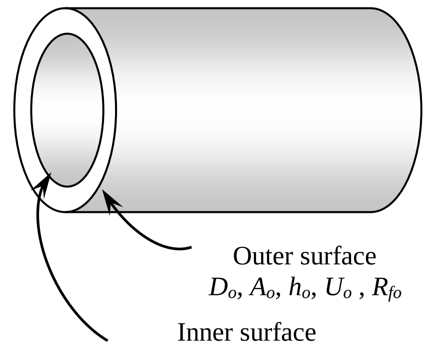
$R_{f,i}$ [m^2-C/W]	U_i [W/m^2-C]
0.0001	2769
0.00015	2433
0.0002	2169
0.00025	1957
0.0003	1782
0.00035	1636
0.0004	1513
0.00045	1406
0.0005	1314
0.00055	1233
0.0006	1161
0.00065	1098
0.0007	1040
0.00075	989
0.0008	942.4



11-26 The heat transfer coefficients and the fouling factors on tube and shell side of a heat exchanger are given. The thermal resistance and the overall heat transfer coefficients based on the inner and outer areas are to be determined.

Assumptions **1** The heat transfer coefficients and the fouling factors are constant and uniform.

Analysis (*a*) The total thermal resistance of the heat exchanger per unit length is



$$R = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{\ln(D_o / D_i)}{2\pi kL} + \frac{R_{fo}}{A_o} + \frac{1}{h_o A_o}$$

$$R = \frac{1}{(800\ \text{W/m}^2.°\text{C})[\pi(0.012\ \text{m})(1\text{m})]} + \frac{(0.0005\ \text{m}^2.°\text{C/W})}{[\pi(0.012\ \text{m})(1\text{m})]}$$

$$+ \frac{\ln(1.6/1.2)}{2\pi(380\ \text{W/m.°C})(1\text{m})} + \frac{(0.0002\ \text{m}^2.°\text{C/W})}{[\pi(0.016\ \text{m})(1\text{m})]}$$

$$+ \frac{1}{(240\ \text{W/m}^2.°\text{C})[\pi(0.016\ \text{m})(1\text{m})]}$$

$$= \mathbf{0.1334°C/W}$$

(*b*) The overall heat transfer coefficient based on the inner and the outer surface areas of the tube per length are

$$R = \frac{1}{UA} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o}$$

$$U_i = \frac{1}{RA_i} = \frac{1}{(0.1334\ °\text{C/W})[\pi(0.012\ \text{m})(1\text{m})]} = \mathbf{199\ W/m^2.°C}$$

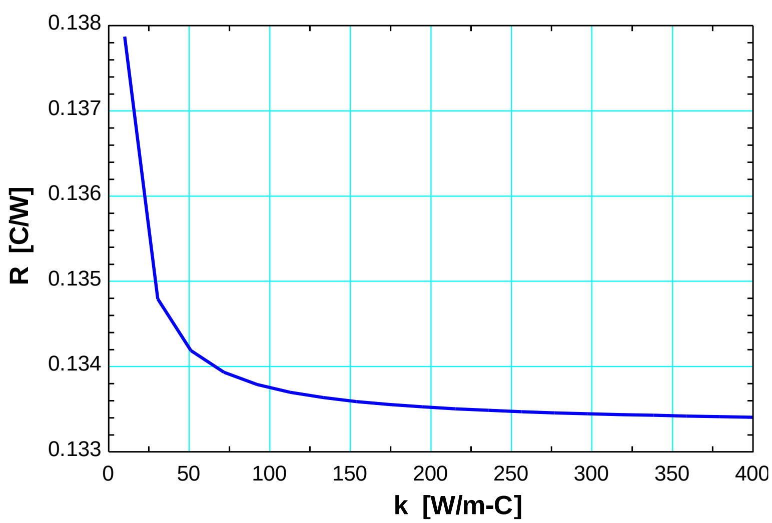
$$U_o = \frac{1}{RA_o} = \frac{1}{(0.1334\ °\text{C/W})[\pi(0.016\ \text{m})(1\text{m})]} = \mathbf{149\ W/m^2.°C}$$

11-27 Prob. 11-26 is reconsidered. The effects of pipe conductivity and heat transfer coefficients on the thermal resistance of the heat exchanger are to be investigated.

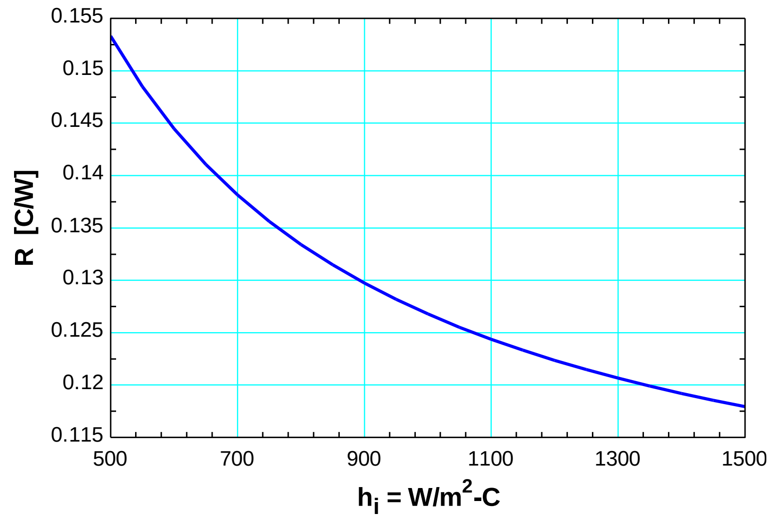
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
k=380 [W/m-C]
D_i=0.012 [m]
D_o=0.016 [m]
D_2=0.03 [m]
h_i=800 [W/m^2-C]
h_o=240 [W/m^2-C]
R_f_i=0.0005 [m^2-C/W]
R_f_o=0.0002 [m^2-C/W]

"ANALYSIS"
R=1/(h_i*A_i)+R_f_i/A_i+ln(D_o/D_i)/(2*pi*k*L)+R_f_o/A_o+1/(h_o*A_o)
L=1 [m] "a unit length of the heat exchanger is considered"
A_i=pi*D_i*L
A_o=pi*D_o*L
U_i=1/(R*A_i)
U_o=1/(R*A_o)
```

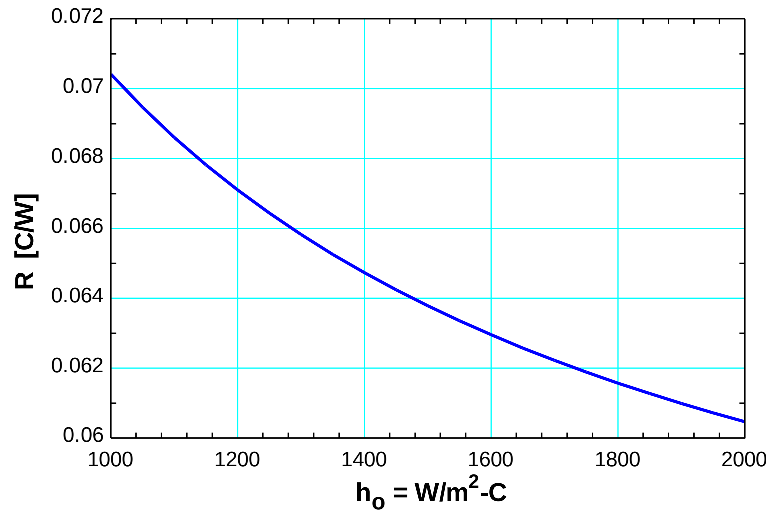
k [W/m-C]	R [C/W]
10	0.1379
30.53	0.1348
51.05	0.1342
71.58	0.1339
92.11	0.1338
112.6	0.1337
133.2	0.1336
153.7	0.1336
174.2	0.1336
194.7	0.1335
215.3	0.1335
235.8	0.1335
256.3	0.1335
276.8	0.1335
297.4	0.1334
317.9	0.1334
338.4	0.1334
358.9	0.1334
379.5	0.1334
400	0.1334



h_i [W/m^2-C]	R [C/W]
500	0.1533
550	0.1485
600	0.1445
650	0.1411
700	0.1381
750	0.1356
800	0.1334
850	0.1315
900	0.1297
950	0.1282
1000	0.1268
1050	0.1255
1100	0.1244
1150	0.1233
1200	0.1224
1250	0.1215
1300	0.1207
1350	0.1199
1400	0.1192
1450	0.1185
1500	0.1179



h_o [W/m^2-C]	R [C/W]
1000	0.07041
1050	0.06947
1100	0.06861
1150	0.06782
1200	0.0671
1250	0.06644
1300	0.06582
1350	0.06526
1400	0.06473
1450	0.06424
1500	0.06378
1550	0.06335
1600	0.06295
1650	0.06258
1700	0.06222
1750	0.06189
1800	0.06157
1850	0.06127
1900	0.06099
1950	0.06072
2000	0.06047



11-28 Hot oil entering a double pipe heat exchanger is cooled by cold water at 20°C. For the known values of oil and water flow rates and fouling factors, the overall heat transfer coefficient on inner and outer surface of the copper tube are to be determined.

Assumptions **1**Steady state conditions exist. **2** Heat exchanger is well insulated. **3** The flow of oil and water is both hydrodynamically and thermally fully developed. **4** Properties of oil and water are constant.

Properties The properties of oil are to be evaluated at the average inlet and exit temperatures of 150°C and 50°C or 100 °C. The properties of oil at an average temperature of 100°C are (Table A-13)

$$\rho = 840\ \text{kg/m}^3,\ c_p = 2220\ \text{J/kg}\cdot\text{K},\ k = 0.1367\ \text{W/m}\cdot\text{K},\ \mu = 0.01718\ \text{kg/m}\cdot\text{s and }\ \text{Pr} = 279.1$$

The properties of water evaluated at an average temperature at the average inlet and exit temperatures of 20°C and 70°C or 45°C are (Table A-9)

$$\rho = 990.1\ \text{kg/m}^3,\ c_p = 4180\ \text{J/kg}\cdot\text{K},\ k = 0.637\ \text{W/m}\cdot\text{K},\ \mu = 0.596\times10^{-3}\ \text{kg/m}\cdot\text{s and }\ \text{Pr} = 3.91$$

Analysis The overall heat transfer coefficient is determined as,

$$\frac{1}{UA_s} = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

The internal and outer surface areas of the heat exchanger tube are,

$$A_i = \pi D_i L = \pi(0.02\ \text{m})(1.5\text{m}) = 0.0942\ \text{m}^2$$

$$A_o = \pi D_o L = \pi(0.0225\ \text{m})(1.5\text{m}) = 0.106\ \text{m}^2$$

We first need to determine the convection heat transfer coefficients at inner and outer pipe surfaces. Based on the mass flow rate and density of oil the average velocity of the oil and the Reynolds number is calculated as follows,

$$V = \frac{4\dot{m}_h}{\rho\pi D_i^2} = \frac{4\,(2\ \text{kg/s})}{(840\ \text{kg/m}^3)\pi\,(0.02\,\text{m})^2} = 7.58\ \text{m/s}$$

The Reynolds number for the flow of oil is,

$$\text{Re} = \frac{\rho V D_i}{\mu} = \frac{(840\ \text{kg/m}^3)(7.58\text{m/s})(0.02\text{m})}{0.01718\ \text{kg/m}\cdot\text{s}} = 7412$$

The mass flow rate of cooling water can be determined from the heat balance such that the heat rejected by the hot engine oil is equal to the heat absorbed by the cooling water

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

$$(444\ \text{W/K})\ (150 - 50)°C = \dot{m}_c\,(4180\text{kJ/kg}\cdot\text{K})\ (70 - 20)°\text{C}$$

$$\dot{m}_c = 2.124\ \text{kg/s}$$

Thus the mass flow rate of cooling water is 2.124 kg/s. The hydraulic diameter of the annular space on the shell side is calculated as,

$$D_h = (D_o - D_i) = (0.06 - 0.0225)\,m = 0.0375\ \text{m}$$

The average velocity of the cooling water is

$$V = \frac{4\dot{m}}{\rho\pi D_h^2} = \frac{4(2.124\ \text{kg/s})}{(990.1\text{kg/m}^3)\,\pi(0.0375\,\text{m})^2} = 1.942\ \text{m/s}$$

The Reynolds number for the flow of water is,

$$\text{Re} = \frac{\rho V D_h}{\mu} = \frac{(990.1\text{kg/m}^3)(1.942\text{m/s})(0.0375\text{m})}{0.596\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 120{,}980$$

For both oil and water since $3\times10^3 < \text{Re} < 5\times10^6$ and $0.5 \le \text{Pr} \le 2000$ we can use from Chapter 9, the Gnielinski (1976) correlation to find the Nusselt number and hence the convective heat transfer coefficient

$$Nu = \frac{(f/8)(\text{Re} - 1000)\text{Pr}}{1 + 12.7\sqrt{(f/8)}\left(\text{Pr}^{2/3} - 1\right)}$$

where the friction factor f may be calculated from the explicit *first Pteukhov equation* [Pteukhov (1970)] as,

$$f = (0.79\ln \text{Re} - 1.64)^{-2}$$

For oil, $f = (0.79\ln(7412) - 1.64)^{-2} = 0.03429$

For water, $f = (0.79\ln(120980) - 1.64)^{-2} = 0.01728$

The Nusselt number for oil and water side are calculate as

For oil,

$$Nu = \frac{(f/8)(\mathrm{Re}-1000)\mathrm{Pr}}{1+12.7\sqrt{(f/8)}(\mathrm{Pr}^{2/3}-1)} = \frac{0.00428(6412)\times 279.1}{1+12.7\times 0.0654(41.7)} = 214.94$$

and

$$h = \frac{k}{D}Nu = \frac{0.1367\ \mathrm{W/m\cdot K}}{0.02\ \mathrm{m}}(214.94) = 1469.11\ \mathrm{W/m^2\cdot K}$$

For water,

$$Nu = \frac{(f/8)(\mathrm{Re}-1000)\mathrm{Pr}}{1+12.7\sqrt{(f/8)}(\mathrm{Pr}^{2/3}-1)} = \frac{0.00216(119980)\times 3.91}{1+12.7\times 0.0464(1.482)} = 540.91$$

and

$$h = \frac{k}{D_h}Nu = \frac{0.637\mathrm{W/m\cdot K}}{0.0375\ \mathrm{m}}(540.91) = 9188.25\ \mathrm{W/m^2\cdot K}$$

Thus the overall heat transfer coefficient is determined as,

$$\frac{1}{UA_s} = \frac{1}{(1469.1\ \mathrm{W/m^2\cdot K})(0.0942\ \mathrm{m^2})} + \frac{0.00015\ \mathrm{m^2\cdot K/W}}{0.0942\ \mathrm{m^2}} + \frac{\ln(0.0225/0.02)}{2\pi\pi(25\ \mathrm{W/m\cdot K})\ (1.5\ \mathrm{m})} +$$

$$\frac{0.0001\mathrm{m^2\cdot K/W}}{0.106\ \mathrm{m^2}} + \frac{1}{(9188.25\ \mathrm{W/m^2\cdot K})(0.106\ \mathrm{m^2})} = 0.01083\ \mathrm{K/W}$$

Using the relationship

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o}$$

Thus the overall heat transfer coefficient based on inner surface of the tube is

U_i = **979.4 W/m²·K.**

The overall heat transfer coefficient based on outer surface of the tube is

U_o = **871.09 W/m²·K.**

Analysis of Heat Exchangers

11-29C The heat exchangers usually operate for long periods of time with no change in their operating conditions, and then they can be modeled as steady-flow devices. As such , the mass flow rate of each fluid remains constant and the fluid properties such as temperature and velocity at any inlet and outlet remain constant. The kinetic and potential energy changes are negligible. The specific heat of a fluid can be treated as constant in a specified temperature range. Axial heat conduction along the tube is negligible. Finally, the outer surface of the heat exchanger is assumed to be perfectly insulated so that there is no heat loss to the surrounding medium and any heat transfer thus occurs is between the two fluids only.

11-30C That relation is valid under steady operating conditions, constant specific heats, and negligible heat loss from the heat exchanger.

11-31C The mass flow rate of the cooling water can be determined from $\dot{Q} = (\dot{m}c_p\Delta T)_{\text{cooling water}}$. The rate of condensation of the steam is determined from $\dot{Q} = (\dot{m}h_{fg})_{\text{steam}}$, and the total thermal resistance of the condenser is determined from $R = \dot{Q}/\Delta T$.

11-32C The product of the mass flow rate and the specific heat of a fluid is called the heat capacity rate and is expressed as $C = \dot{m}c_p$. When the heat capacity rates of the cold and hot fluids are equal, the temperature change is the same for the two fluids in a heat exchanger. That is, the temperature rise of the cold fluid is equal to the temperature drop of the hot fluid. A heat capacity of infinity for a fluid in a heat exchanger is experienced during a phase-change process in a condenser or boiler.

11-33C When the heat capacity rates of the cold and hot fluids are identical, the temperature rise of the cold fluid will be equal to the temperature drop of the hot fluid.

11-34 Hot and cold fluid streams have same heat capacity rates. It is to be proved that the temperature profiles of the hot and cold fluids are parallel to each other at any given section of the heat exchanger.

Assumption **1** Heat exchanger is well insulated. **2** Fluid properties do not change with heat exchanger length.

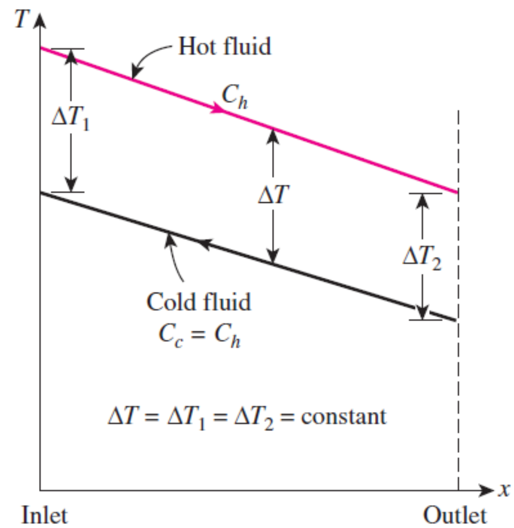
Analysis Assuming heat exchanger to be well insulated and the heat transfer occurs only between the hot and cold fluid, the heat transfer across the differential section of the heat exchanger can be expressed as,

$$\delta\dot{Q} = -\dot{m}_h c_{ph} dT_h = \dot{m}_c c_{pc} dT_c$$

Thus the rate of heat loss from the hot fluid at any section of the heat exchanger is equal to the rate of heat gain by the cold fluid in that section.

However, in a counter flow heat exchanger, the temperature of both hot and cold stream decreases in the direction of heat exchanger length. Thus we have,

$$\delta\dot{Q} = -\dot{m}_h c_{ph} dT_h = -\dot{m}_c c_{pc} dT_c$$



The above energy balance can be written as,

$$dT_h = -\frac{\delta\dot{Q}}{\dot{m}_h c_{ph}} \text{ and } dT_c = -\frac{\delta\dot{Q}}{\dot{m}_c c_{pc}}$$

Thus we get,

$$dT_h - dT_c = -\frac{\delta\dot{Q}}{\dot{m}_h c_{ph}} + \frac{\delta\dot{Q}}{\dot{m}_c c_{pc}} = -\delta\dot{Q}\left(\frac{1}{\dot{m}_h c_{ph}} - \frac{1}{\dot{m}_h c_{ph}}\right) = d\Delta T$$

For same heat capacity rates of both hot and cold fluid streams, we have,

$$dT_h - dT_c = d\Delta T = 0$$

This implies that the temperature difference between the hot and cold fluid stream at any given section remains constant. Hence, the temperature profile of two fluid streams (hot and cold) that have same heat capacity rates is parallel to each other at every section of the counter flow heat exchanger as shown in the figure.

The Log Mean Temperature Difference Method

11-35C ΔT_{lm} is called the log mean temperature difference, and is expressed as

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

where

$\Delta T_1 = T_{h,in} - T_{c,in}$ $\quad \Delta T_2 = T_{h,out} - T_{c,out}$ $\quad$ for parallel-flow heat exchangers

and

$\Delta T \ = T_{h,in} - T_{c,out}$ $\quad \Delta T_2 = T_{h,out} - T_{c,in}$ $\quad$ for counter-flow heat exchangers

11-36C The temperature difference between the two fluids decreases from ΔT_1 at the inlet to ΔT_2 at the outlet, and arithmetic mean temperature difference is defined as $\Delta T_{\text{am}} = \dfrac{\Delta T_1 + \Delta T_2}{2}$. The logarithmic mean temperature difference ΔT_{lm} is obtained by tracing the actual temperature profile of the fluids along the heat exchanger, and is an exact representation of the average temperature difference between the hot and cold fluids. It truly reflects the exponential decay of the local temperature difference. The logarithmic mean temperature difference is always less than the arithmetic mean temperature.

11-37C ΔT_{lm} cannot be greater than both ΔT_1 and ΔT_2 because ΔT_{ln} is always less than or equal to ΔT_{m} (arithmetic mean) which can not be greater than both ΔT_1 and ΔT_2.

11-38C In the parallel-flow heat exchangers the hot and cold fluids enter the heat exchanger at the same end, and the temperature of the hot fluid decreases and the temperature of the cold fluid increases along the heat exchanger. But the temperature of the cold fluid can never exceed that of the hot fluid. In case of the counter-flow heat exchangers the hot and cold fluids enter the heat exchanger from the opposite ends and the outlet temperature of the cold fluid may exceed the outlet temperature of the hot fluid.

11-39C First heat transfer rate is determined from $\dot{Q} = \dot{m}c_p[T_{in} - T_{out}]$, ΔT_{ln} from $\Delta T_{lm} = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$, correction factor from the figures, and finally the surface area of the heat exchanger from $\dot{Q} = UAFDT_{lm,CF}$

11-40C The factor F is called as correction factor which depends on the geometry of the heat exchanger and the inlet and the outlet temperatures of the hot and cold fluid streams. It represents how closely a heat exchanger approximates a counter-flow heat exchanger in terms of its logarithmic mean temperature difference. F cannot be greater than unity.

11-41C In this case it is not practical to use the LMTD method because it requires tedious iterations. Instead, the effectiveness-NTU method should be used.

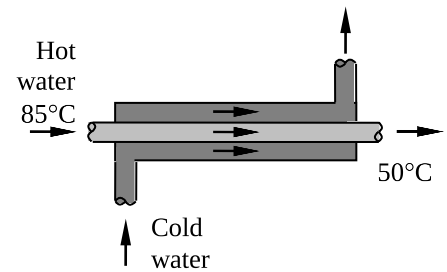
11-42C The ΔT_{lm} will be greatest for double-pipe counter-flow heat exchangers.

11-43 Water is heated in a double-pipe, parallel-flow uninsulated heat exchanger by geothermal water. The rate of heat transfer to the cold water and the log mean temperature difference for this heat exchanger are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heat of hot water is given to be 4.25 kJ/kg.°C.

Analysis The rate of heat given up by the hot water is



$$\begin{aligned}\dot{Q}_h &= [\dot{m}c_p(T_{in}-T_{out})]_{\text{hot water}}\\ &= (1.4\,\text{kg/s})(4.25\,\text{kJ/kg.°C})(85°\text{C}-50°\text{C})\\ &= 208.3\,\text{kW}\end{aligned}$$

The rate of heat picked up by the cold water is

$$\dot{Q}_c = (1-0.03)\dot{Q}_h = (1-0.03)(208.3\,\text{kW}) = \mathbf{202.0\,kW}$$

The log mean temperature difference is

$$\dot{Q} = UA\Delta T_{lm} \longrightarrow \Delta T_{lm} == \frac{\dot{Q}}{UA} = \frac{202.0\,\text{kW}}{(1.15\,\text{kW/m}^2\cdot°\text{C})(4\,\text{m}^2)} = \mathbf{43.9°C}$$

11-44 A stream of hydrocarbon is cooled by water in a double-pipe counterflow heat exchanger. The overall heat transfer coefficient is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

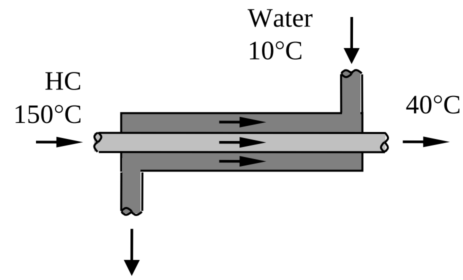
Properties The specific heats of hydrocarbon and water are given to be 2.2 and 4.18 kJ/kg.°C, respectively.

Analysis The rate of heat transfer is

$$\dot{Q} = [\dot{m}c_p(T_{out}-T_{in})]_{\text{HC}} = (720/3600\,\text{kg/s})(2.2\,\text{kJ/kg.°C})(150°\text{C}-40°\text{C}) = 48.4\,\text{kW}$$

The outlet temperature of water is

$$\begin{aligned}\dot{Q} &= [\dot{m}c_p(T_{out}-T_{in})]_{\text{w}}\\ 48.4\,\text{kW} &= (540/3600\,\text{kg/s})(4.18\,\text{kJ/kg.°C})(T_{\text{w,out}}-10°\text{C})\\ T_{\text{w,out}} &= 87.2\,°\text{C}\end{aligned}$$



The logarithmic mean temperature difference is

$$\Delta T_1 = T_{h,in} - T_{c,out} = 150°\text{C} - 87.2°\text{C} = 62.8°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 40°\text{C} - 10°\text{C} = 30°\text{C}$$

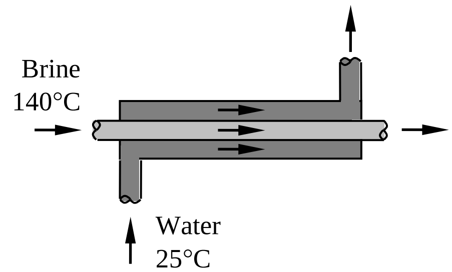
and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{62.8-30}{\ln(62.8/30)} = 44.4°\text{C}$$

The overall heat transfer coefficient is determined from

$$\begin{aligned}\dot{Q} &= UA\Delta T_{lm}\\ 48.4\,\text{kW} &= U(\pi\times 0.025\times 6.0)(44.4°\text{C})\\ U &= \mathbf{2.31\,kW/m^2\cdot K}\end{aligned}$$

11-45 Water is heated in a double-pipe parallel-flow heat exchanger by geothermal water. The required length of tube is to be determined.



Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heats of water and geothermal fluid are given to be 4.18 and 4.31 kJ/kg.°C, respectively.

Analysis The rate of heat transfer in the heat exchanger is

$$\dot{Q} = [\dot{m}c_p(T_{out} - T_{in})]_{\text{water}} = (0.2\text{ kg/s})(4.18\text{kJ/kg.°C})(60°\text{C} - 25°\text{C}) = 29.26\text{ kW}$$

Then the outlet temperature of the geothermal water is determined from

$$\dot{Q} = [\dot{m}c_p(T_{in} - T_{out})]_{\text{geot.water}} \longrightarrow T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}c_p} = 140°\text{C} - \frac{29.26\text{ kW}}{(0.3\text{ kg/s})(4.31\text{kJ/kg.°C})} = 117.4°\text{C}$$

The logarithmic mean temperature difference is

$$\Delta T_1 = T_{h,in} - T_{c,in} = 140°\text{C} - 25°\text{C} = 115°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,out} = 117.4°\text{C} - 60°\text{C} = 57.4°\text{C}$$

and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{115 - 57.4}{\ln(115/57.4)} = 82.9°\text{C}$$

The surface area of the heat exchanger is determined from

$$\dot{Q} = UA_s\Delta T_{lm} \longrightarrow A_s = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{29.26\text{ kW}}{(0.55\text{ kW/m}^2)(82.9°\text{C})} = 0.642\text{ m}^2$$

Then the length of the tube required becomes

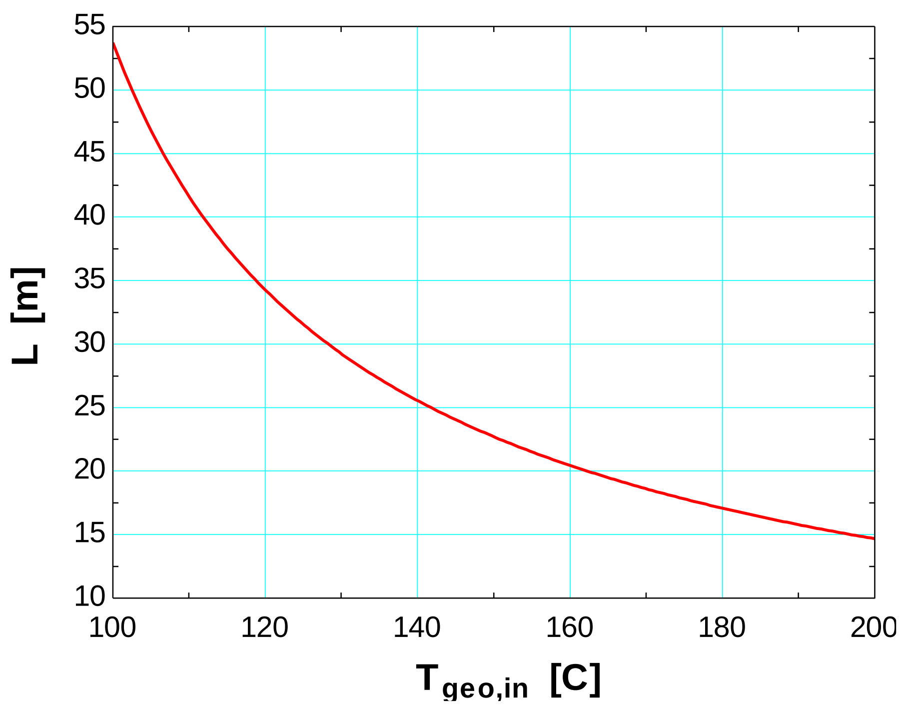
$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{0.642\text{ m}^2}{\pi(0.008\text{ m})} = \mathbf{25.5\text{ m}}$$

11-46  Prob. 11-45 is reconsidered. The effects of temperature and mass flow rate of geothermal water on the length of the tube are to be investigated.

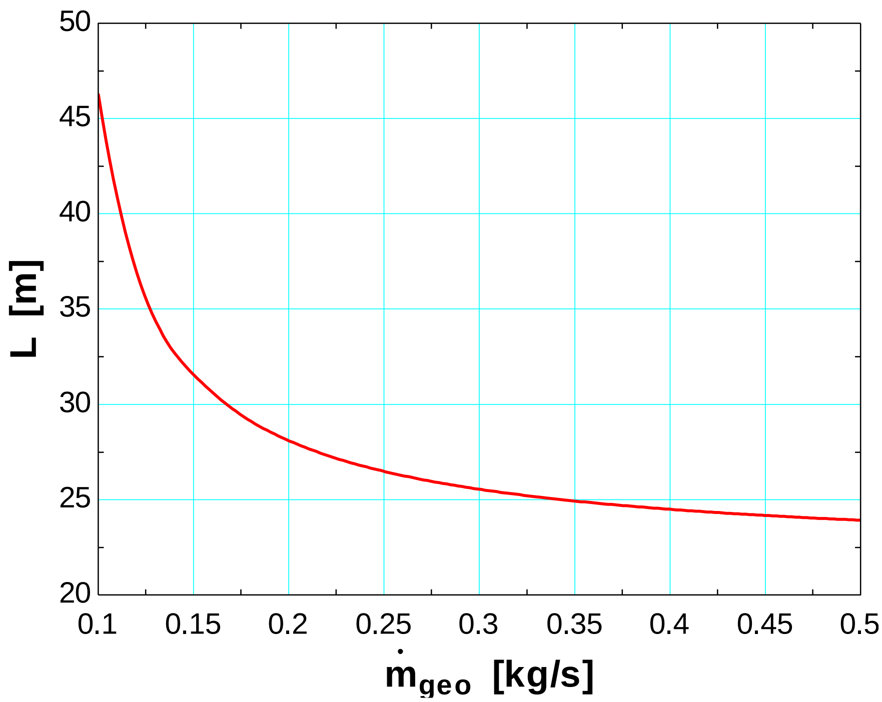
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_w_in=25 [C]
T_w_out=60 [C]
m_dot_w=0.2 [kg/s]
c_p_w=4.18 [kJ/kg-C]
T_geo_in=140 [C]
m_dot_geo=0.3 [kg/s]
c_p_geo=4.31 [kJ/kg-C]
D=0.008 [m]
U=0.55 [kW/m^2-C]

"ANALYSIS"
Q_dot=m_dot_w*c_p_w*(T_w_out-T_w_in)
Q_dot=m_dot_geo*c_p_geo*(T_geo_in-T_geo_out)
DELTAT_1=T_geo_in-T_w_in
DELTAT_2=T_geo_out-T_w_out
DELTAT_lm=(DELTAT_1-DELTAT_2)/ln(DELTAT_1/DELTAT_2)
Q_dot=U*A*DELTAT_lm
A=pi*D*L
```

$T_{geo,in}$ [C]	L [m]
100	53.73
105	46.81
110	41.62
115	37.56
120	34.27
125	31.54
130	29.24
135	27.26
140	25.54
145	24.04
150	22.7
155	21.51
160	20.45
165	19.48
170	18.61
175	17.81
180	17.08
185	16.4
190	15.78
195	15.21
200	14.67



$\dot{m}_{geo}$ [kg/s]	L [m]
0.1	46.31
0.125	35.52
0.15	31.57
0.175	29.44
0.2	28.1
0.225	27.16
0.25	26.48
0.275	25.96
0.3	25.54
0.325	25.21
0.35	24.93
0.375	24.69
0.4	24.49
0.425	24.32
0.45	24.17
0.475	24.04
0.5	23.92



11-47 Glycerin is heated by ethylene glycol in a thin-walled double-pipe parallel-flow heat exchanger. The rate of heat transfer, the outlet temperature of the glycerin, and the mass flow rate of the ethylene glycol are to be determined.

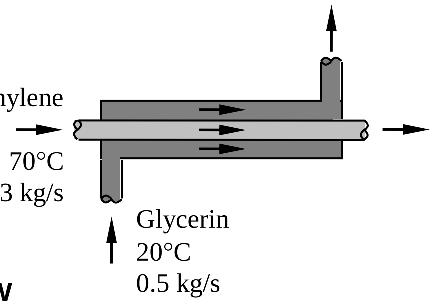
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant. **6** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Properties The specific heats of glycerin and ethylene glycol are given to be 2.4 and 2.5 kJ/kg.°C, respectively.

Analysis (*a*) The temperature differences at the two ends are

$$\Delta T_1 = T_{h,in} - T_{c,in} = 70°\text{C} - 20°\text{C} = 50°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,out} = T_{h,out} - (T_{h,out} - 15°\text{C}) = 15°\text{C}$$



and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{50-15}{\ln(50/15)} = 29.1°\text{C}$$

Then the rate of heat transfer becomes

$$\dot{Q} = UA_s\Delta T_{lm} = (240\text{ W/m}^2.°\text{C})(3.2\text{ m}^2)(29.1°\text{C}) = 22{,}330\text{ W} = \mathbf{22.33\text{ kW}}$$

(*b*) The outlet temperature of the glycerin is determined from

$$\dot{Q} = [\dot{m}c_p(T_{out} - T_{in})]_{\text{glycerin}} \longrightarrow T_{out} = T_{in} + \frac{\dot{Q}}{\dot{m}c_p} = 20°\text{C} + \frac{22.33\text{ kW}}{(0.5\text{ kg/s})(2.4\text{ kJ/kg.°C})} = \mathbf{38.6°C}$$

(*c*) Then the mass flow rate of ethylene glycol becomes

$$\dot{Q} = [\dot{m}c_p(T_{in} - T_{out})]_{\text{ethylene glycol}}$$

$$\dot{m}_{\text{ethylene glycol}} = \frac{\dot{Q}}{c_p(T_{in} - T_{out})} = \frac{22.33\text{ kJ/s}}{(2.5\text{ kJ/kg.°C})[70°\text{C} - (38.6+15)°\text{C}]} = \mathbf{0.545\text{ kg/s}}$$

11-48 The heat transfer rate of a heat exchanger containing 400 tubes with specified inner and outer diameters and length is to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible. **5** Thermal resistance of the tubes is negligible.

Analysis The overall heat transfer coefficient based on the outer surface is

$$\frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{1}{h_o A_o} \qquad \rightarrow \qquad \frac{1}{U_o} = \frac{A_o}{h_i A_i} + \frac{1}{h_o} = \frac{\pi D_o L}{\pi D_i L h_i} + \frac{1}{h_o} = \frac{D_o}{D_i}\frac{1}{h_i} + \frac{1}{h_o}$$

or

$$U_o = \left[\frac{D_o}{D_i}\frac{1}{h_i} + \frac{1}{h_o}\right]^{-1} = \left[\left(\frac{25}{23}\right)\left(\frac{1}{3410}\right) + \frac{1}{6820}\right]^{-1} \text{ W/m}^2\cdot\text{K} = 2149 \text{ W/m}^2\cdot\text{K}$$

The heat transfer rate is

$$\begin{aligned}\dot{Q} &= U_o A_o \Delta T_{\text{lm}} \\ &= n U_o \pi D_o L \Delta T_{\text{lm}} \\ &= (400)(2149 \text{ W/m}^2\cdot\text{K})\pi(0.025\text{ m})(3.7\text{ m})(23\text{ K}) \\ &= \mathbf{5.75\times10^6\ W}\end{aligned}$$

Discussion If the inner to outer diameter ratio is neglected, the overall heat transfer coefficient based on the outer surface area becomes

$$U_o = \left[\frac{1}{h_i} + \frac{1}{h_o}\right]^{-1} = 2273 \text{ W/m}^2\cdot\text{K}$$

which is about 6% larger than the original value of $U_o = 2149 \text{ W/m}^2\cdot\text{K}$.

11-49E The required number of tubes and length of tubes for a single pass heat exchanger to heat 100,000 lbm of water in an hour from 60°F to 100°F are to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible. **5** Thermal resistance of the tubes is negligible.

Properties The density and specific heat of water are given to be 62.3 lbm/ft^3 and c_{pc} = 1 Btu/lbm·°F, respectively.

Analysis From the equation for mass flow rate, we have

$$\dot{m}_c = n\rho AV \quad \rightarrow \quad n = \frac{\dot{m}_c}{\rho AV}$$

$$n = \frac{\dot{m}_c}{\rho AV} = \frac{100{,}000\ \text{lbm/hr}}{(62.3\ \text{lbm/ft}^2)(\pi/4)(1.2/12\ \text{ft})^2(4\ \text{ft/s})(3600\ \text{s/hr})} = 14.19$$

Hence, the number of tubes required to heat 100,000 lbm of water in an hour is

$$n = \mathbf{15\ tubes}$$

The overall heat transfer coefficient based on the inner surface is

$$\frac{1}{U_i A_i} = \frac{1}{h_i A_i} + \frac{1}{h_o A_o} \quad \rightarrow \quad \frac{1}{U_i} = \frac{A_i}{h_i A_i} + \frac{A_i}{h_o A_o} = \frac{1}{h_i} + \frac{D_i}{D_o}\frac{1}{h_o}$$

where

$$D_o = D_i + 2t = 1.44\ \text{in.}$$

Hence,

$$U_i = \left[\frac{1}{h_i} + \frac{D_i}{D_o}\frac{1}{h_o}\right]^{-1} = \left[\frac{1}{480} + \left(\frac{1.2}{1.44}\right)\frac{1}{2000}\right]^{-1} \text{Bth/hr}\cdot\text{ft}^2\cdot{}^\circ\text{F} = 400\ \text{Bth/hr}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

The log mean temperature difference is

$$\Delta T_{\text{lm,}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{(230-60)-(230-100)}{\ln[(230-60)/(230-100)]}{}^\circ\text{F} = 149.1^\circ\text{F}$$

Using the equation for the heat transfer rate, we have

$$\dot{Q} = U_i A_i \Delta T_{\text{lm}} = nU_i \pi D_i L \Delta T_{\text{lm}} \quad \rightarrow \quad L = \frac{\dot{Q}}{nU_i \pi D_i \Delta T_{\text{lm}}} = \frac{\dot{m}_c c_{pc}(T_{c,\text{out}} - T_{c,\text{in}})}{nU_i \pi D_i \Delta T_{\text{lm}}}$$

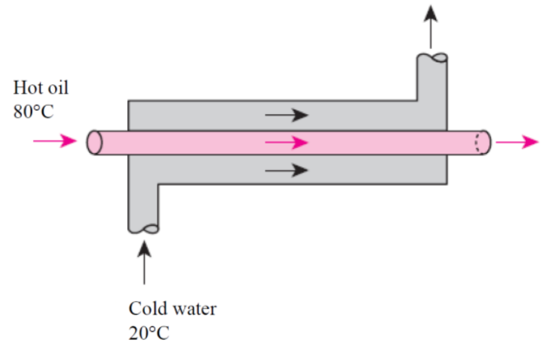
$$L = \frac{(100{,}000\ \text{lbm/hr})(1\ \text{Btu/lbm}\cdot{}^\circ\text{F})(100-60)^\circ\text{F}}{(15)(400\ \text{Bth/hr}\cdot\text{ft}^2\cdot{}^\circ\text{F})\pi(1.2/12\ \text{ft})(149.1^\circ\text{F})} = \mathbf{14.2\ ft}$$

Discussion For process involving condensation, we have $T_{h,\text{in}} = T_{h,\text{out}}$.

11-50 Hot engine oil and cooling water flows through a parallel heat exchanger at specified temperatures. The mass flow rate of the cooling water, log mean temperature difference and the area of heat exchanger are to be determined.

Assumption **1** Thermal resistance of the thin walled copper tube is negligible. **2** Thermal properties of oil and water are constant. **3** Heat exchanger is well insulated.

Analysis The heat transfer rate though the heat exchanger is



$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out})$$
$$= (0.3\,\text{kg/s})(2048\,\text{J/kg}\cdot\text{K})(80-40)^{\circ}\text{C}$$
$$= 24.576\ \text{kW}$$

(a) From energy balance,

Rate of heat loss by engine oil = Rate of heat gain by cooling water

Thus we have,

$$\dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

$$\dot{m}_c = \dot{m}_h \frac{c_{ph}(T_{h,in} - T_{h,out})}{c_{pc}(T_{c,out} - T_{c,in})} = (0.3\,\text{kg/s})\frac{(2048\,\text{J/kg}\cdot\text{K})(80-40)^{\circ}\text{C}}{(4180\text{J/kg}\cdot\text{K})(32-20)^{\circ}\text{C}} = \mathbf{0.489\ kg/s}$$

(b) The log mean temperature difference is calculated based on the inlet and exit temperatures of the hot and cold fluids. The temperature difference between the two fluids at inlet and exit of the heat exchanger are calculated as follows.

$$\Delta T_1 = T_{h,in} - T_{c,in} = 80 - 20 = 60\ ^{o}C$$

and

$$\Delta T_2 = T_{h,out} - T_{c,out} = 40 - 32 = 8^{o}C$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{60-8}{\ln(60/8)} = \mathbf{25.8\ ^{o}C}$$

(c) The heat transfer rate in the heat exchanger is also calculated as,

$$\dot{Q} = UA_s\Delta T_{lm}$$

The overall heat transfer coefficient is calculated as,

$$U = \left(\frac{1}{h_i} + \frac{1}{h_o}\right)^{-1} = \left(\frac{1}{750} + \frac{1}{350}\right)^{-1} = 238.6\,\text{W/m}^2\cdot\text{K}$$

Thus the surface area of the heat exchanger is,

$$A_s = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{24576\,\text{W}}{(238.6\,\text{W/m}^2\cdot\text{K})\,(25.8^{\circ}\text{C})} = \mathbf{3.99\ m^2}$$

11-51 The hot and cold fluid streams at specified temperature and mass flow rate enter a parallel heat exchanger. For the known values of convection heat transfer coefficients and fouling factors, the overall heat transfer coefficient, the exit temperature of the hot fluid and the surface area of the heat exchanger are to be determined.

Assumptions **1** Thermal resistance due to pipe thickness is negligible. **2** Thermal properties of the hot and cold fluids are constant. **3** Heat exchanger is well insulated.

Analysis (a) The overall heat transfer coefficient is calculated based on the given heat transfer coefficients and fouling factors at the tube inlet and outlet surface. Since the tube is of negligible thickness, the inner and outer surface areas of the tube may be assumed to be equal.

$$\frac{1}{U}=\frac{1}{h_i}+R_{f,i}+R_{f,o}+\frac{1}{h_o}=\frac{1}{300\text{ W/m}^2\cdot\text{K}}+(0.0003\text{ m}^2\cdot\text{K/W})+(0.0001\text{ m}^2\cdot\text{K/W})+\frac{1}{800\text{ W/m}^2\cdot\text{K}}$$

$$U=\mathbf{200.67\ W/m^2\cdot K}$$

(b) Rate of heat loss from hot fluid is

$$\dot{Q}=\dot{m}_h c_{ph}(T_{h,in}-T_{h,out})=(3\text{ kg/s})(1150\text{ J/kg}\cdot\text{K})\,(150^\circ\text{C}-T_{h,out})$$

Rate of heat gain by the cold fluid is,

$$\dot{Q}=\dot{m}_c c_{pc}(T_{c,out}-T_{c,in})=(0.5\text{ kg/s})\,(4180\text{ J/kg}\cdot\text{K})\,(70-30)^\circ\text{C}=83.6\text{ kW}$$

From energy balance we have

$$83.6\text{ kW}=(3450\text{ W/K})\,(150^\circ\text{C}-T_{h,out})$$

Thus we get the exit temperature of the hot side fluid to be

$$T_{h,out}=150-\frac{83.6\times10^3\text{ W}}{3450\text{ W/K}}=\mathbf{125.8\ ^\circ C}$$

(c) Using the concept of log mean temperature difference, the heat transfer rate from the heat exchanger can also be calculated as

$$\dot{Q}=UA_s\Delta T_{lm}$$

where

$$\Delta T_{lm}=\frac{\Delta T_1-\Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$\Delta T_1=T_{h,in}-T_{c,in}=150-30=120\ ^\circ\text{C}$$

and

$$\Delta T_2=T_{h,out}-T_{c,out}=125.8-70=55.8\ ^\circ\text{C}$$

Thus the equation for log mean temperature difference is written as

$$\Delta T_{lm}=\frac{120-55.8}{\ln(120/55.8)}=83.84\ ^\circ\text{C}$$

Thus the surface area of the heat exchanger is,

$$A_s=\frac{\dot{Q}}{U\Delta T_{lm}}=\frac{83.6\times10^3\text{ W}}{(200.67\text{ W/m}^2\cdot\text{K})\,(83.84\ ^\circ\text{C})}=\mathbf{4.97\,m^2}$$

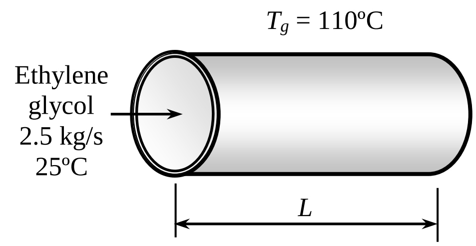
11-52 Ethylene glycol is heated in a tube while steam condenses on the outside tube surface. The tube length is to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the tubes are smooth. **3** Heat transfer to the surroundings is negligible.

Properties The properties of ethylene glycol are given to be ρ = 1109 kg/m^3, c_p = 2428 J/kg·K, k = 0.253 W/m·K, μ = 0.01545 kg/m·s, Pr = 148.5. The thermal conductivity of copper is given to be 386 W/m·K.

Analysis The rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (2.5\text{ kg/s})(2428\text{ J/kg.°C})(40-25)\text{°C} = 91{,}050\text{ W}$$



The fluid velocity is

$$V = \frac{\dot{m}}{\rho A_c} = \frac{2.5\text{ kg/s}}{(1109\text{ kg/m}^3)\left[\pi(0.02\text{ m})^2/4\right]} = 7.176\text{ m/s}$$

The Reynolds number is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(1109\text{ kg/m}^3)(7.176\text{ m/s})(0.02\text{m})}{0.01545\text{ kg/m}\cdot\text{s}} = 10{,}302$$

which is greater than 10,000. Therefore, we have fully developed flow and evaluate the Nusselt number from turbulent flow relation:

$$Nu = \frac{hD}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(10{,}302)^{0.8}(148.5)^{0.4} = 275.9$$

Heat transfer coefficient on the inner surface is

$$h_i = \frac{k}{D}Nu = \frac{0.253\text{ W/m.°C}}{0.02\text{ m}}(275.9) = 3490\text{ W/m}^2\text{.°C}$$

Assuming a wall temperature of 100°C, the heat transfer coefficient on the outer surface is determined to be

$$h_o = 9200(T_g - T_w)^{-0.25} = 9200(110-100)^{-0.25} = 5174\text{ W/m}^2\text{.°C}$$

Let us check if the assumption for the wall temperature holds:

$$h_i A_i(T_w - T_{b,\text{avg}}) = h_o A_o(T_g - T_w)$$

$$h_i \pi D_i L(T_w - T_{b,\text{avg}}) = h_o \pi D_o L(T_g - T_w)$$

$$3490\times 0.02(T_w - 32.5) = 5174\times 0.025(110 - T_w) \longrightarrow T_w = 82.84\text{°C}$$

Now we assume a wall temperature of 80°C:

$$h_o = 9200(T_g - T_w)^{-0.25} = 9200(110-80)^{-0.25} = 3931\text{ W/m}^2\text{.°C}$$

Again checking, $3490\times 0.02(T_w - 30) = 3931\times 0.025(110 - T_w) \longrightarrow T_w = 77.8\text{°C}$

which is sufficiently close to the assumed value of 80°C. Now that both heat transfer coefficients are available, we use thermal resistance concept to find overall heat transfer coefficient based on the outer surface area as follows:

$$U_o = \frac{1}{\dfrac{D_o}{h_i D_i} + \dfrac{D_o \ln(D_2/D_1)}{2k_{\text{copper}}} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{0.025}{(3490)(0.02)} + \dfrac{(0.025)\ln(2.5/2)}{2(386)} + \dfrac{1}{3931}} = 1613\text{ W/m}^2\cdot\text{C}$$

The rate of heat transfer can be expressed as

$$\dot{Q} = U_o A_o \Delta T_{\ln}$$

where the logarithmic mean temperature difference is

$$\Delta T_{lm} = \frac{(T_g - T_e) - (T_g - T_i)}{\ln\left(\dfrac{T_g - T_e}{T_g - T_i}\right)} = \frac{(110-40)-(110-25)}{\ln\left(\dfrac{110-40}{110-25}\right)} = 77.26\text{°C}$$

Substituting, the tube length is determined to be

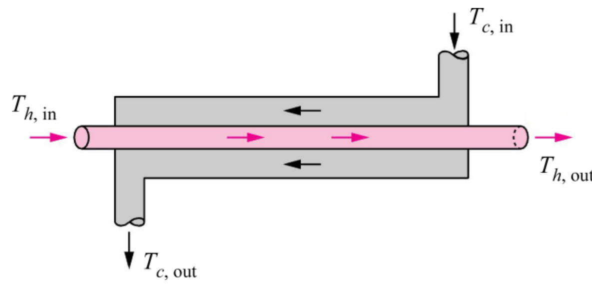
$$\dot{Q} = U_o A_o \Delta T_{lm} \longrightarrow 91{,}050 = (1613)\pi(0.025)L(77.26) \longrightarrow L = \mathbf{9.30\text{ m}}$$

11-53 A counter-flow heat exchanger has a specified overall heat transfer coefficient operating at design and clean conditions. After a period of use built-up scale gives a fouling factor, (*a*) the rate of heat transfer in the heat exchanger and (*b*) the mass flow rates of both hot and cold fluids are to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat transfer coefficients and the fouling factors are constant and uniform. **3** Fluid properties are constant.

Properties The specific heat of both hot and cold fluids is given as 4.2 kJ/kg·K.

Analysis When operating at design and clean conditions, the overall heat transfer coefficient is given as



$$U_{\text{w/o scale}} = 284 \text{ W/m}^2\cdot\text{K}$$

(*a*) After a period of use, the overall heat transfer coefficient due to the scale built-up is

$$\frac{1}{U_{\text{w/ scale}}} = \frac{1}{U_{\text{w/o scale}}} + R_f = \frac{1}{284 \text{ W/m}^2\cdot\text{K}} + 0.0004 \text{ m}^2\cdot\text{K/W} = 0.00392 \text{ m}^2\cdot\text{K/W}$$

or

$$U_{\text{w/ scale}} = 255 \text{ W/m}^2\cdot\text{K}$$

The log mean temperature difference is

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{(93-38)-(71-27)}{\ln[(93-38)/(71-27)]}\,°\text{C} = 49.3°\text{C}$$

Then, the rate of heat transfer in the heat exchanger is

$$\dot{Q} = UA_s\Delta T_{\text{lm}} = (255 \text{ W/m}^2\cdot\text{K})(93 \text{ m}^2)(49.3 \text{ K}) = \mathbf{1.17\times 10^6 \text{ W}}$$

(*b*) The mass flow rate of the hot fluid is

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,\text{in}} - T_{h,\text{out}}) \quad \rightarrow \quad \dot{m}_h = \frac{\dot{Q}}{c_{ph}(T_{h,\text{in}} - T_{h,\text{out}})}$$

$$\dot{m}_h = \frac{1.17\times 10^6 \text{ J/s}}{(4200 \text{ J/kg}\cdot\text{K})(93-71) \text{ K}} = \mathbf{12.7 \text{ kg/s}}$$

The mass flow rate of the cold fluid is

$$\dot{m}_c = \frac{\dot{Q}}{c_{pc}(T_{c,\text{out}} - T_{c,\text{in}})} = \frac{1.17\times 10^6 \text{ J/s}}{(4200 \text{ J/kg}\cdot\text{K})(38-27) \text{ K}} = \mathbf{25.3 \text{ kg/s}}$$

Discussion The scale built-up caused a decrease in the overall heat transfer coefficient of the heat exchanger, which reduces the heat removal capability of the heat exchanger.

11-54 A double-pipe counter-flow heat exchanger is used to cool a hot fluid such that the fluid flowing into a pipe system is below the temperature limit for polypropylene lining, 107°C. The highest level of fouling factor that the heat exchanger can tolerate before it becomes unable to cool the hot fluid to the temperature limit at the outlet is to be determined.

Assumptions **1** Steady state conditions. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Properties of fluids are constant.

Analysis The LMTD for a counter flow heat exchanger is

$$\Delta T_{\text{lm}} = \frac{(T_{h,\text{in}} - T_{c,\text{out}}) - (T_{h,\text{out}} - T_{c,\text{in}})}{\ln\left(\frac{T_{h,\text{in}} - T_{c,\text{out}}}{T_{h,\text{out}} - T_{c,\text{in}}}\right)} = \frac{(120-30)-(107-10)}{\ln\left(\frac{120-30}{107-10}\right)} = 93.46\text{°C}$$

The overall heat transfer coefficient for the heat exchanger is

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} + R_f$$

The heat transfer rate in the heat exchanger is

$$\dot{Q} = C_h(T_{h,\text{in}} - T_{h,\text{out}})$$

$$\dot{Q} = UA_s\Delta T_{\text{lm}}$$

This gives

$$\frac{1}{U} = \frac{A_s\Delta T_{\text{lm}}}{\dot{Q}} = \frac{(\pi DL)\Delta T_{\text{lm}}}{C_h(T_{h,\text{in}} - T_{h,\text{out}})}$$

Thus,

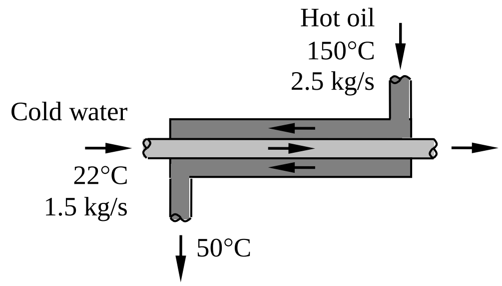
$$R_f = \frac{(\pi DL)\Delta T_{\text{lm}}}{C_h(T_{h,\text{in}} - T_{h,\text{out}})} - \frac{1}{h_i} - \frac{1}{h_o} = \frac{(\pi)(0.025\text{ m})(5\text{ m})(93.46\text{°C})}{\left(1000\frac{\text{W}}{\text{K}}\right)(120-107)\text{°C}} - \frac{1}{1000\frac{\text{W}}{\text{m}^2}\cdot\text{K}} - \frac{1}{1500\frac{\text{W}}{\text{m}^2}\cdot\text{K}} = \mathbf{0.00116\ m^2{\cdot}K/W}$$

Discussion The highest level of fouling factor that the heat exchanger can tolerate before it becomes unable to cool the hot fluid at the heat exchanger outlet to 107°C (the maximum temperature for polypropylene lining set by the ASME Code for Process Piping) is 0.00116 m^2·K/W. A fouling factor greater than this value would diminish the performance of the heat exchanger resulting in higher $T_{h,\text{out}}$.

11-55 Oil is cooled by water in a thin-walled double-pipe counter-flow heat exchanger. The overall heat transfer coefficient of the heat exchanger is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant. **6** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Properties The specific heats of water and oil are given to be 4.18 and 2.20 kJ/kg.°C, respectively.



Analysis The rate of heat transfer from the water to the oil is

$$\dot{Q} = [\dot{m}c_p(T_{in} - T_{out})]_{\text{oil}}$$
$$= (2.5\text{ kg/s})(2.2\text{kJ/kg.°C})(150°\text{C} - 50°\text{C})$$
$$= 550\text{ kW}$$

The outlet temperature of the water is determined from

$$\dot{Q} = [\dot{m}c_p(T_{out} - T_{in})]_{\text{water}} \longrightarrow T_{out} = T_{in} + \frac{\dot{Q}}{\dot{m}c_p}$$

$$= 22°\text{C} + \frac{550\text{ kW}}{(1.5\text{ kg/s})(4.18\text{kJ/kg.°C})} = 109.7°\text{C}$$

The logarithmic mean temperature difference is

$$\Delta T_1 = T_{h,in} - T_{c,out} = 150°\text{C} - 109.7°\text{C} = 40.3°\text{C}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 50°\text{C} - 22°\text{C} = 28°\text{C}$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{40.3 - 28}{\ln(40.3/28)} = 33.8°\text{C}$$

Then the overall heat transfer coefficient becomes

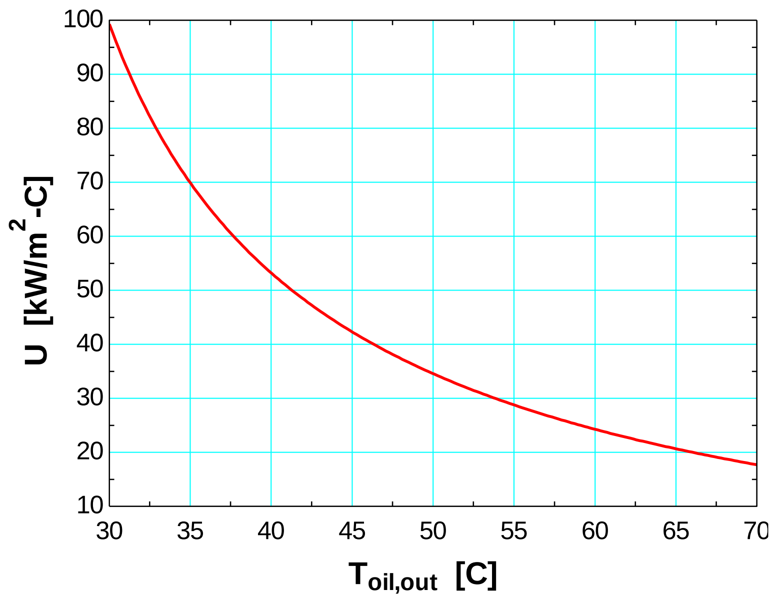
$$U = \frac{\dot{Q}}{A_s \Delta T_{lm}} = \frac{550\text{ kW}}{\pi(0.025\text{ m})(6\text{ m})(33.8°\text{C})} = \mathbf{34.6\ kW/m^2.°C}$$

11-56 Prob. 11-55 is reconsidered. The effects of oil exit temperature and water inlet temperature on the overall heat transfer coefficient of the heat exchanger are to be investigated.

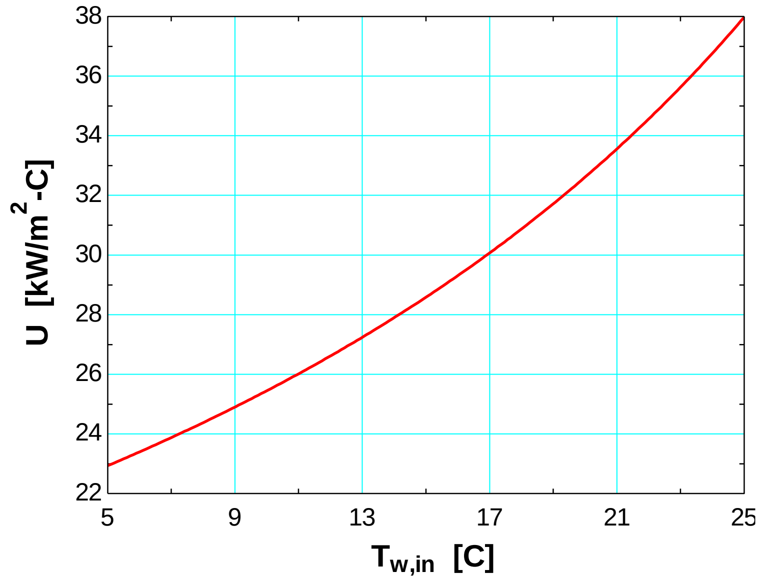
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_oil_in=150 [C]
T_oil_out=50 [C]
m_dot_oil=2.5 [kg/s]
c_p_oil=2.20 [kJ/kg-C]
T_w_in=22 [C]
m_dot_w=1.5 [kg/s]
C_p_w=4.18 [kJ/kg-C]
D=0.025 [m]
L=6 [m]

"ANALYSIS"
Q_dot=m_dot_oil*c_p_oil*(T_oil_in-T_oil_out)
Q_dot=m_dot_w*c_p_w*(T_w_out-T_w_in)
DELTAT_1=T_oil_in-T_w_out
DELTAT_2=T_oil_out-T_w_in
DELTAT_lm=(DELTAT_1-DELTAT_2)/ln(DELTAT_1/DELTAT_2)
Q_dot=U*A*DELTAT_lm
A=pi*D*L
```

$T_{oil,out}$ [C]	U [kW/m^2-C]
30	99.27
32.5	82.18
35	69.89
37.5	60.57
40	53.21
42.5	47.25
45	42.3
47.5	38.13
50	34.56
52.5	31.47
55	28.77
57.5	26.38
60	24.26
62.5	22.37
65	20.65
67.5	19.11
70	17.7



$T_{w,in}$ [C]	U [kW/m^{2}C]
5	22.93
6	23.39
7	23.88
8	24.38
9	24.9
10	25.45
11	26.02
12	26.61
13	27.24
14	27.89
15	28.58
16	29.31
17	30.07
18	30.87
19	31.72
20	32.61
21	33.56
22	34.56
23	35.63
24	36.77
25	37.98



11-57 Engine oil is heated by condensing steam in a condenser. The rate of heat transfer and the length of the tube required are to be determined.

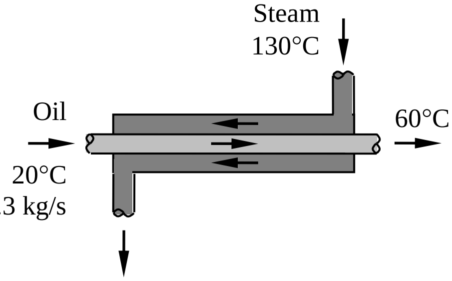
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant. **6** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Properties The specific heat of engine oil is given to be 2.1 kJ/kg.°C. The heat of condensation of steam at 130°C is given to be 2174 kJ/kg.

Analysis The rate of heat transfer in this heat exchanger is

$$\dot{Q} = [\dot{m}c_p(T_{out} - T_{in})]_{\text{oil}} = (0.3\,\text{kg/s})(2.1\text{kJ/kg.°C})(60°\text{C} - 20°\text{C}) = \mathbf{25.2\,kW}$$

The temperature differences at the two ends of the heat exchanger are



$$\Delta T_1 = T_{h,in} - T_{c,out} = 130°\text{C} - 60°\text{C} = 70°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 130°\text{C} - 20°\text{C} = 110°\text{C}$$

and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{70 - 110}{\ln(70/110)} = 88.5°\text{C}$$

The surface area is

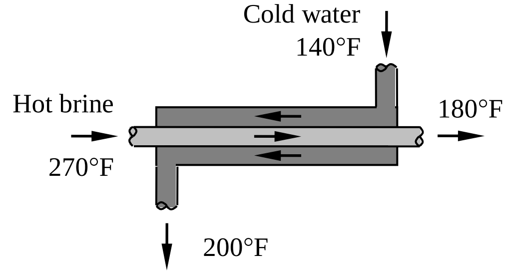
$$A_s = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{25.2\,\text{kW}}{(0.65\,\text{kW/m}^2.°\text{C})(88.5°\text{C})} = 0.44\,\text{m}^2$$

Then the length of the tube required becomes

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{0.44\,\text{m}^2}{\pi(0.02\,\text{m})} = \mathbf{7.0\,m}$$

11-58E Water is heated by geothermal water in a double-pipe counter-flow heat exchanger. The mass flow rate of each fluid and the total thermal resistance of the heat exchanger are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.



Properties The specific heats of water and geothermal fluid are given to be 1.0 and 1.03 Btu/lbm.°F, respectively.

Analysis The mass flow rate of each fluid is determined from

$$\dot{Q} = [\dot{m}c_p(T_{out} - T_{in})]_{\text{water}}$$

$$\dot{m}_{\text{water}} = \frac{\dot{Q}}{c_p(T_{out} - T_{in})} = \frac{40 \text{ Btu/s}}{(1.0 \text{ Btu/lbm.°F})(200\text{°F} - 140\text{°F})} = \mathbf{0.667\ lbm/s}$$

$$\dot{Q} = [\dot{m}c_p(T_{out} - T_{in})]_{\text{geo. water}}$$

$$\dot{m}_{\text{geo. water}} = \frac{\dot{Q}}{c_p(T_{out} - T_{in})} = \frac{40 \text{ Btu/s}}{(1.03 \text{ Btu/lbm.°F})(270\text{°F} - 180\text{°F})} = \mathbf{0.431 lbm/s}$$

The temperature differences at the two ends of the heat exchanger are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 270\text{°F} - 200\text{°F} = 70\text{°F}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 180\text{°F} - 140\text{°F} = 40\text{°F}$$

and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{70 - 40}{\ln(70/40)} = 53.61\text{°F}$$

Then

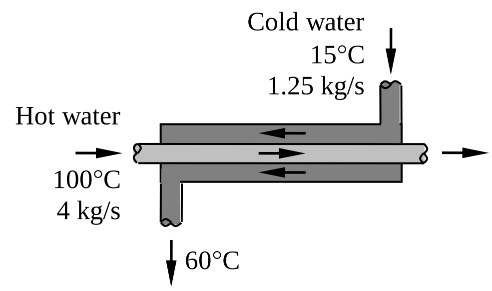
$$\dot{Q} = UA_s \Delta T_{lm} \longrightarrow UA_s = \frac{\dot{Q}}{\Delta T_{lm}} = \frac{40 \text{ Btu/s}}{53.61\text{°F}} = 0.7462 \text{ Btu/s.}^{\text{o}}\text{ F}$$

$$U = \frac{1}{RA_s} \longrightarrow R = \frac{1}{UA_s} = \frac{1}{0.7462 \text{ Btu/s.°F}} = \mathbf{1.34\ s \cdot °F/Btu}$$

11-59 Cold water is heated by hot water in a double-pipe counter-flow heat exchanger. The rate of heat transfer and the heat transfer surface area of the heat exchanger are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant. **6** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Properties The specific heats of cold and hot water are given to be 4.18 and 4.19 kJ/kg.°C, respectively.



Analysis The rate of heat transfer in this heat exchanger is

$$\begin{aligned}\dot{Q} &= [\dot{m}c_p(T_{out} - T_{in})]_{\text{cold water}} \\ &= (1.25\text{ kg/s})(4.18\text{kJ/kg.°C})(60°\text{C} - 15°\text{C}) \\ &= \mathbf{235.1 kW}\end{aligned}$$

The outlet temperature of the hot water is determined from

$$\dot{Q} = [\dot{m}c_p(T_{in} - T_{out})]_{\text{hot water}}$$

$$\longrightarrow T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}c_p} = 100°\text{C} - \frac{235.1\text{ kW}}{(4\text{ kg/s})(4.19\text{ kJ/kg.°C})} = 86.0°\text{C}$$

The temperature differences at the two ends of the heat exchanger are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 100°\text{C} - 60°\text{C} = 40°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 86.0°\text{C} - 15°\text{C} = 71°\text{C}$$

and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{40 - 71}{\ln(40/71)} = 54.0°\text{C}$$

Then the surface area of this heat exchanger becomes

$$\dot{Q} = UA_s\Delta T_{lm} \longrightarrow A_s = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{235.1\text{ kW}}{(0.880\text{ kW/m}^2.°\text{C})(54.0°\text{C})} = \mathbf{4.95\ m^2}$$

11-60 A double-pipe counter-flow heat exchanger is used to cool a hot fluid such that the fluid flowing into a pipe system is below the temperature limit for EPDM O-rings, 150°C. The hot fluid temperature at the heat exchanger outlet is to be determined. If the outlet temperature is above 150°C, the length of the heat exchanger necessary to cool the hot fluid below 150°C as it exits the heat exchanger is to be determined.

Assumptions **1** Steady state conditions. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Effects of fouling are negligible. **4** Properties of fluids are constant.

Analysis The heat transfer rate in the heat exchanger is

$$\dot{Q} = UA_s\Delta T_{\text{lm}}$$

$$\dot{Q} = \dot{m}_h c_{ph}\left(T_{h,\text{in}} - T_{h,\text{out}}\right)$$

From energy balance, we have

$$UA_s\Delta T_{\text{lm}} = C_h\left(T_{h,\text{in}} - T_{h,\text{out}}\right)$$

This gives

$$U(\pi DL)\frac{\left(T_{h,\text{in}} - T_{c,\text{out}}\right) - \left(T_{h,\text{out}} - T_{c,\text{in}}\right)}{\ln\left(\dfrac{T_{h,\text{in}} - T_{c,\text{out}}}{T_{h,\text{out}} - T_{c,\text{in}}}\right)} = C_h\left(T_{h,\text{in}} - T_{h,\text{out}}\right)$$

The outlet temperature $T_{h,\text{out}}$ can be determined implicitly, or by trial-and-error,

$$(2000\ \text{W/m}^2\cdot\text{K})(\pi)(0.025\ \text{m})(5\ \text{m})\frac{(210-80) - \left(T_{h,\text{out}} - 10\right)}{\ln\left(\dfrac{210-80}{T_{h,\text{out}} - 10}\right)} = (3000\ \text{W/K})\left(210 - T_{h,\text{out}}\right)\text{K}$$

which yields,

$$T_{h,\text{out}} = \mathbf{171.9}℃ > 150℃$$

Thus, the hot fluid exiting the heat exchanger is hotter than 150°C. To determine the length of the heat exchanger so that $T_{h,\text{out}}$ = 150°C or lower, we again use

$$UA_s\Delta T_{\text{lm}} = C_h\left(T_{h,\text{in}} - T_{h,\text{out}}\right)$$

This gives

$$L = \frac{C_h}{U(\pi D)\Delta T_{\text{lm}}}\left(T_{h,\text{in}} - T_{h,\text{out}}\right) = \frac{3000\ \text{W/K}}{(2000\ \text{W/m}^2\cdot\text{K})(\pi)(0.025\ \text{m})(134.94\ ℃)}(210-150)℃ = \mathbf{8.492\ m}$$

where

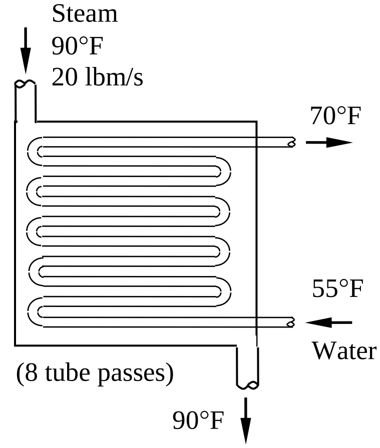
$$\Delta T_{\text{lm}} = \frac{\left(T_{h,\text{in}} - T_{c,\text{out}}\right) - \left(T_{h,\text{out}} - T_{c,\text{in}}\right)}{\ln\left(\dfrac{T_{h,\text{in}} - T_{c,\text{out}}}{T_{h,\text{out}} - T_{c,\text{in}}}\right)} = \frac{(210-80) - (150-10)}{\ln\left(\dfrac{210-80}{150-10}\right)} = 134.94℃$$

Discussion With L = 5 m, the heat exchanger provided insufficient surface area to cool the hot fluid to 150°C or lower, at the heat exchanger outlet. To remove sufficient heat from the hot fluid before it flows into the pipe system at 150°C or lower, the length of the heat exchanger needs to be at least 8.5-m long.

11-61E Steam is condensed by cooling water in a condenser. The rate of heat transfer, the rate of condensation of steam, and the mass flow rate of cold water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant. **6** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Properties We take specific heat of water are given to be 1.0 Btu/lbm.°F.
The heat of condensation of steam at 90°F is 1043 Btu/lbm.



Analysis (*a*) The log mean temperature difference is determined from

$$\Delta T_1 = T_{h,in} - T_{c,out} = 90°\text{F} - 70°\text{F} = 20°\text{F}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 90°\text{F} - 55°\text{F} = 35°\text{F}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{20-35}{\ln(20/35)} = 26.8°\text{F}$$

The heat transfer surface area is

$$A_s = 8n\pi DL = 8\times 60\times \pi(3/48\text{ ft})(5\text{ ft}) = 471.2\text{ ft}^2$$

and

$$\dot{Q} = UA_s\Delta T_{lm} = (600\text{ Btu/h.ft}^2.°\text{F})(471.2\text{ ft}^2)(26.8°\text{F})$$
$$= \mathbf{7.579\times 10^6\ Btu/h} = \mathbf{2105\ Btu/s}$$

(*b*) The rate of condensation of the steam is

$$\dot{Q} = (\dot{m}h_{fg})_{steam} \longrightarrow \dot{m}_{steam} = \frac{\dot{Q}}{h_{fg}} = \frac{2105\text{ Btu/s}}{1043\text{ Btu/lbm}} = \mathbf{2.02\ lbm/s}$$

(*c*) Then the mass flow rate of cold water becomes

$$\dot{Q} = [\dot{m}c_p(T_{out} - T_{in})]_{\text{cold water}}$$

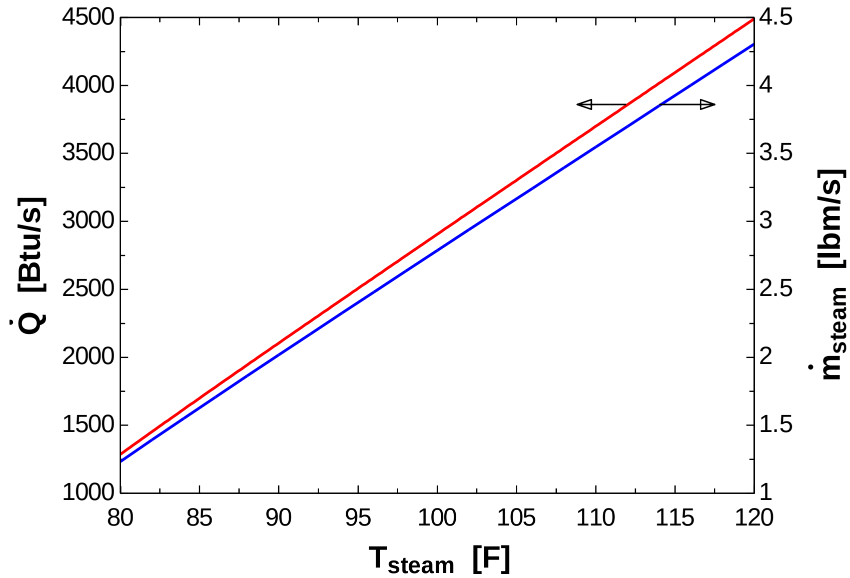
$$\dot{m}_{\text{cold water}} = \frac{\dot{Q}}{c_p(T_{out} - T_{in})} = \frac{2105\text{ Btu/s}}{(1.0\text{ Btu/lbm.°F})(70°\text{F} - 55°\text{F}]} = \mathbf{140\ lbm/s}$$

11-62E Prob. 11-61E is reconsidered. The effect of the condensing steam temperature on the rate of heat transfer, the rate of condensation of steam, and the mass flow rate of cold water is to be investigated.

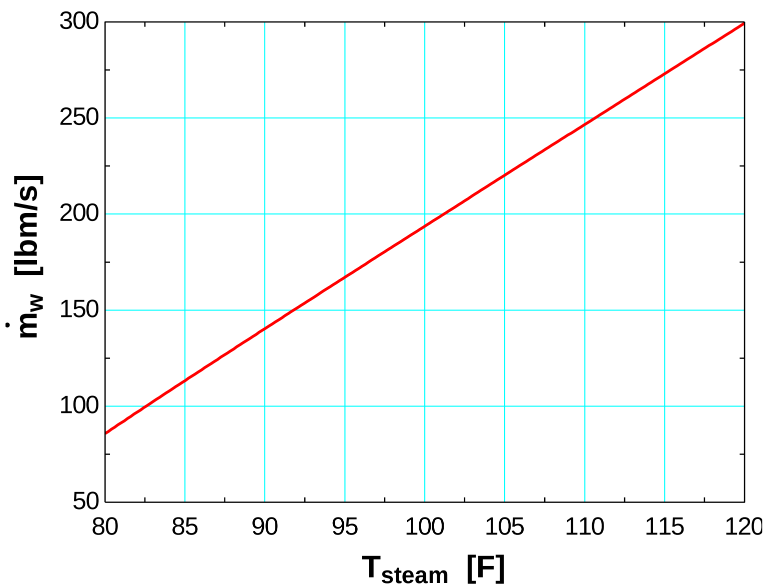
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
N_pass=8
N_tube=60
T_steam=90 [F]
h_fg_steam=1043 [Btu/lbm]
T_w_in=55 [F]
T_w_out=70 [F]
c_p_w=1.0 [Btu/lbm-F]
D=3/4*1/12 [ft]
L=5 [ft]
U=600 [Btu/h-ft^2-F]

"ANALYSIS"
"(a)"
DELTAT_1=T_steam-T_w_out
DELTAT_2=T_steam-T_w_in
DELTAT_lm=(DELTAT_1-DELTAT_2)/ln(DELTAT_1/DELTAT_2)
A=N_pass*N_tube*pi*D*L
Q_dot=U*A*DELTAT_lm*Convert(Btu/h, Btu/s)
"(b)"
Q_dot=m_dot_steam*h_fg_steam
"(c)"
Q_dot=m_dot_w*c_p_w*(T_w_out-T_w_in)
```

T_{steam} [F]	$\dot{Q}$ [Btu/s]	$\dot{m}_{steam}$ [lbm/s]	$\dot{m}_w$ [lbm/s]
80	1286	1.233	85.72
82	1453	1.393	96.85
84	1618	1.551	107.8
86	1781	1.708	118.7
88	1944	1.863	129.6
90	2105	2.018	140.3
92	2266	2.173	151.1
94	2427	2.326	161.8
96	2587	2.48	172.4
98	2746	2.633	183.1
100	2906	2.786	193.7
102	3065	2.938	204.3
104	3224	3.091	214.9
106	3382	3.243	225.5
108	3541	3.395	236.1
110	3699	3.547	246.6
112	3858	3.699	257.2
114	4016	3.85	267.7
116	4174	4.002	278.3
118	4332	4.154	288.8
120	4490	4.305	299.4



4500
4000
3500
3000
2500
2000
1500
1000
4.5
4
3.5
3
2.5
2
1.5
1
80
85
90
95
100
105
110
115
120
$\dot{Q}$ [Btu/s]
$\dot{m}_{steam}$ [lbm/s]
T_{steam} [F]

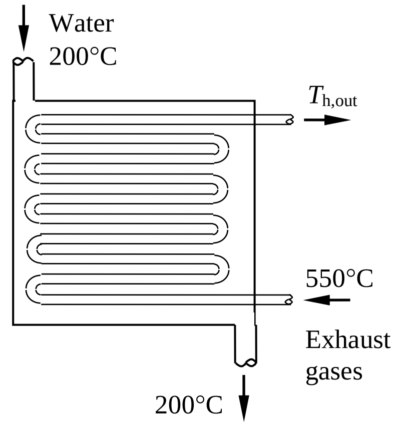


300
250
200
150
100
50
80
85
90
95
100
105
110
115
120
$\dot{m}_w$ [lbm/s]
T_{steam} [F]

11-63 Water is evaporated by hot exhaust gases in an evaporator. The rate of heat transfer, the exit temperature of the exhaust gases, and the rate of evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The heat of vaporization of water at 200°C is given to be h_{fg} = 1941 kJ/kg and specific heat of exhaust gases is given to be c_p = 1051 J/kg.°C.



Analysis The temperature differences between the water and the exhaust gases at the two ends of the evaporator are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 550°C - 200°C = 350°C$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = (T_{h,out} - 200)°C$$

and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{350 - (T_{h,out} - 200)}{\ln\left[350/(T_{h,out} - 200)\right]}$$

Then the rate of heat transfer can be expressed as

$$\dot{Q} = UA_s \Delta T_{lm} = (1.780 \text{ kW/m}^2.°C)(0.5 \text{ m}^2)\frac{350 - (T_{h,out} - 200)}{\ln\left[350/(T_{h,out} - 200)\right]} \qquad (1)$$

The rate of heat transfer can also be expressed as in the following forms

$$\dot{Q} = [\dot{m}c_p(T_{h,in} - T_{h,out})]_{\substack{\text{exhaust}\\\text{gases}}} = (0.25 \text{ kg/s})(1.051 \text{ kJ/kg.°C})(550°C - T_{h,out}) \qquad (2)$$

$$\dot{Q} = (\dot{m}h_{fg})_{water} = \dot{m}_{water}(1941 \text{ kJ/kg}) \qquad (3)$$

We have three equations with three unknowns. Using an equation solver such as EES, the unknowns are determined to be

$$\dot{Q} = \mathbf{88.85 \text{ kW}}$$

$$T_{h,out} = \mathbf{211.8°C}$$

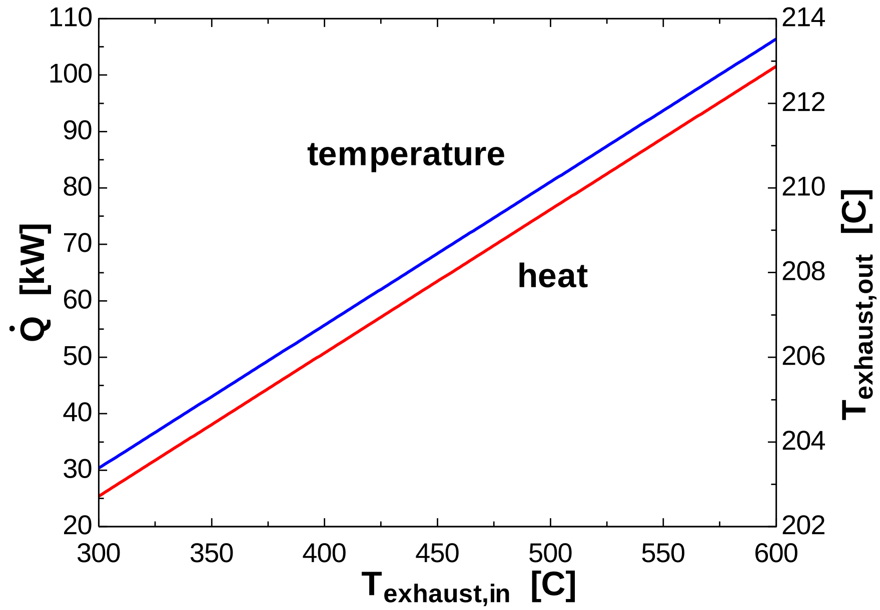
$$\dot{m}_{water} = \mathbf{0.0458 \text{ kg/s}}$$

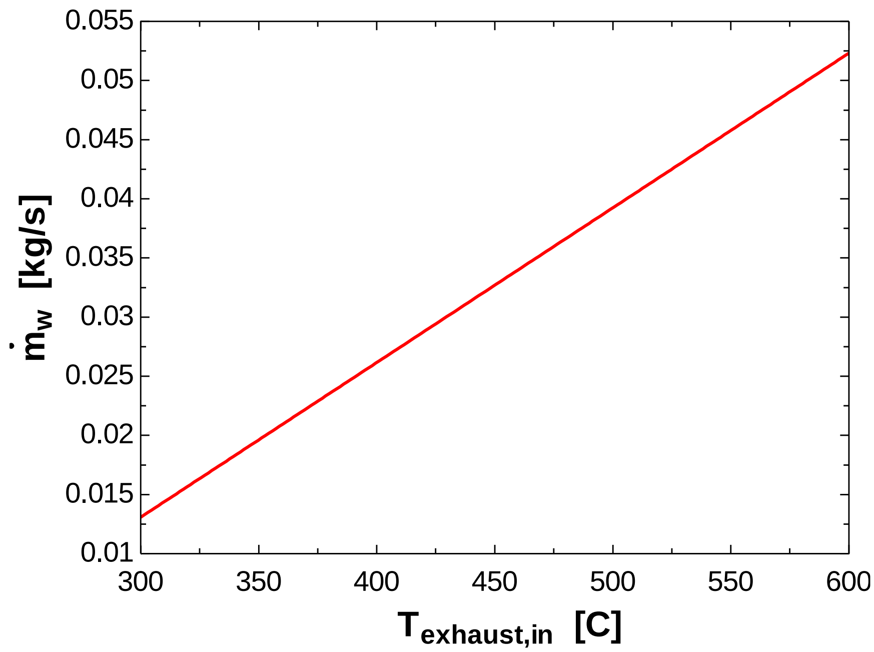
11-64 Prob. 11-63 is reconsidered. The effect of the exhaust gas inlet temperature on the rate of heat transfer, the exit temperature of exhaust gases, and the rate of evaporation of water is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_exhaust_in=550 [C]
c_p_exhaust=1.051 [kJ/kg-C]
m_dot_exhaust=0.25 [kg/s]
T_w=200 [C]
h_fg_w=1941 [kJ/kg]
A=0.5 [m^2]
U=1.780 [kW/m^2-C]

"ANALYSIS"
DELTAT_1=T_exhaust_in-T_w
DELTAT_2=T_exhaust_out-T_w
DELTAT_lm=(DELTAT_1-DELTAT_2)/ln(DELTAT_1/DELTAT_2)
Q_dot=U*A*DELTAT_lm
Q_dot=m_dot_exhaust*c_p_exhaust*(T_exhaust_in-T_exhaust_out)
Q_dot=m_dot_w*h_fg_w
```

$T_{exhaust,in}$ [C]	$\dot{Q}$ [kW]	$T_{exhaust,out}$ [C]	$\dot{m}_w$ [kg/s]
300	25.39	203.4	0.01308
320	30.46	204.1	0.0157
340	35.54	204.7	0.01831
360	40.62	205.4	0.02093
380	45.7	206.1	0.02354
400	50.77	206.8	0.02616
420	55.85	207.4	0.02877
440	60.93	208.1	0.03139
460	66.01	208.8	0.03401
480	71.08	209.5	0.03662
500	76.16	210.1	0.03924
520	81.24	210.8	0.04185
540	86.32	211.5	0.04447
560	91.39	212.2	0.04709
580	96.47	212.8	0.0497
600	101.5	213.5	0.05232


temperature
heat
$\dot{Q}$ [kW]
$T_{exhaust,out}$ [C]
$T_{exhaust,in}$ [C]
110
100
90
80
70
60
50
40
30
20
214
212
210
208
206
204
202
300
350
400
450
500
550
600

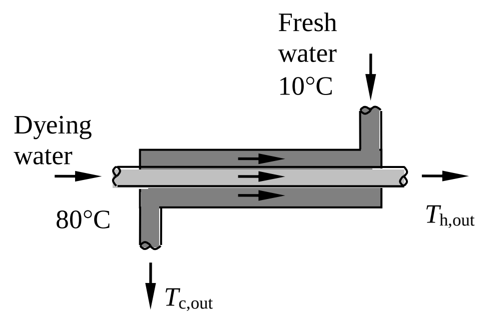

$\dot{m}_w$ [kg/s]
$T_{exhaust,in}$ [C]
0.055
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
300
350
400
450
500
550
600

11-65 The waste dyeing water is to be used to preheat fresh water. The outlet temperatures of each fluid and the mass flow rate are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heats of waste dyeing water and the fresh water are given to be c_p = 4295 J/kg.°C and c_p = 4180 J/kg.°C, respectively.

Analysis The temperature differences between the dyeing water and the fresh water at the two ends of the heat exchanger are



$$\Delta T_1 = T_{h,in} - T_{c,out} = 80 - T_{c,out}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = T_{h,out} - 10$$

and

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{(80 - T_{c,out}) - (T_{h,out} - 10)}{\ln\left[(80 - T_{c,out})/(T_{h,out} - 10)\right]}$$

Then the rate of heat transfer can be expressed as

$$\dot{Q} = UA_s \Delta T_{lm}$$

$$35\text{ kW} = (0.625\text{ kW/m}^2.°\text{C})(1.65\text{ m}^2)\frac{(80 - T_{c,out}) - (T_{h,out} - 10)}{\ln\left[(80 - T_{c,out})/(T_{h,out} - 10)\right]} \qquad (1)$$

The rate of heat transfer can also be expressed as

$$\dot{Q} = [\dot{m}c_p(T_{h,in} - T_{h,out})]_{\substack{\text{dyeing}\\\text{water}}} \longrightarrow 35\text{ kW} = \dot{m}(4.295\text{ kJ/kg.°C})(80°\text{C} - T_{h,out}) \qquad (2)$$

$$\dot{Q} = [\dot{m}c_p(T_{h,in} - T_{h,out})]_{\text{water}} \longrightarrow 35\text{ kW} = \dot{m}(4.18\text{ kJ/kg.°C})(T_{c,out} - 10°\text{C}) \qquad (3)$$

We have three equations with three unknowns. Using an equation solver such as EES, the unknowns are determined to be

$$T_{c,out} = \mathbf{46.6°C}$$

$$T_{h,out} = \mathbf{44.4°C}$$

$$\dot{m} = \mathbf{0.229\ kg/s}$$

11-66 Counterflow double pipe heat exchanger with a surface area of 7.5 m^2 and U = 450 W/m^2·K is used to heat the engine oil using water at 100°C. It is to be determined if fouling has occurred in the heat exchanger over a period of time.

Assumptions **1** Steady state conditions exist. **2** Heat exchanger is well insulated. **3** Fluid properties remain constant.

Properties Heat capacity of engine oil is evaluated at an average temperature of (25 +55)°C/2 = 40°C from Table A-13: c_p = 1964 J/kg·K.

Analysis From the energy balance between hot water and engine oil we have,

Heat lost by water = Heat gained by engine oil

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in}) = \dot{m}_h c_{ph}(T_{h,out} - T_{h,in})$$

$$\therefore \dot{Q} = (2.5\,\text{kg/s})(1964\,\text{J/kg}\cdot\text{K})\,(55 - 25)^{\text{o}}\text{C} = 147.3\ \text{kW}$$

Thus, the exit temperature of hot water is,

$$T_{h,out} = T_{h,in} - \frac{\dot{Q}}{\dot{m}_h c_{ph}} = 100^o C - \frac{147.3\,\text{kW}}{(1.75\,\text{kg/s})(4206\,\text{J/kg}\cdot\text{K})} = 80\ ^{\text{o}}\text{C}$$

The heat transfer rate is calculated as

$$\dot{Q} = UA_s \Delta T_{lm}$$

Now the logarithmic temperature difference is calculated as

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)},$$

$$\Delta T_1 = T_{h,in} - T_{c,out} = 100 - 55 = 45^o C$$

and

$$\Delta T_2 = T_{h,out} - T_{c,in} = 80 - 25 = 55^o C$$

$$\Delta T_{lm} = \frac{45 - 55}{\ln(45/55)} = 49.8\ ^{\text{o}}\text{C}$$

Therefore the actual overall heat transfer coefficient is

$$U = \frac{\dot{Q}}{A_s \Delta T_{lm}} = \frac{147.3k\text{W}}{(7.5\text{m}^2)(49.8^{\text{o}}\text{C})} = 394.4\ \text{W/m}^2\cdot\text{K}$$

Since the actual overall heat transfer coefficient (394.4 W/m^2·K) is less than the designed value of the overall heat transfer coefficient (450 W/m^2·K), it can be concluded that fouling has occurred in the heat exchanger. The thermal resistance caused by fouling is then,

$$\frac{1}{U} = \frac{1}{U_{design}} + R_f \Rightarrow R_f = \frac{1}{U} - \frac{1}{U_{design}} = \frac{1}{394.4\,\text{W/m}^2\cdot\text{K}} - \frac{1}{450\,\text{W/m}^2\cdot\text{K}} = \mathbf{0.00031\ m^2 \cdot K/W}$$

Discussion The fouling developed in the heat exchanger although cannot be prevented; it can be mitigated and controlled periodically using techniques such as chemical cleaning, reversal of flow direction and use of turbulence promoters.

11-67 Feed water at specified temperature and mass flow rate is to be heated by superheated steam in a counter flow heat exchanger. For the known values of convection heat transfer coefficient surface area of the heat exchanger in counter flow and parallel flow arrangement is to be determined.

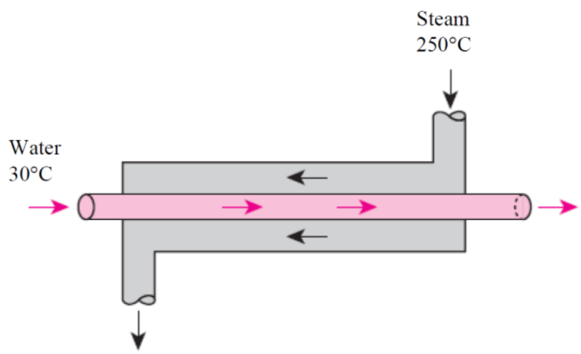
Assumptions **1** Steady state operating conditions exist. **2** Heat exchanger is well insulated. **3** Fouling on the steam side is assumed to be negligible. **4** Properties of steam and water stay constant. **5** Thermal resistance due to the pipe wall thickness is neglected.

Properties The specific heat of water is determined at an average temperature of (30 + 70)°C/2 = 50°C from Table A-9 to be c_p = 4181 J/kg·K.

Analysis For a 70°C drop in steam temperature, the steam exit temperature is $T_{h,out} = 250 - 70 = 180^oC$. The exit temperature of the water is to be maintained at a minimum temperature of 70°C.

(a) The rate of heat transfer in the heat exchanger is calculated as,

$$\dot{Q} = UA_s\Delta T_{lm} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$



where $\Delta T_{lm} = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$

$$\Delta T_1 = T_{h,in} - T_{c,out} = 250 - 70 = 180^oC$$

and $\Delta T_2 = T_{h,out} - T_{c,in} = 180 - 30 = 150^oC$

$$\Delta T_{lm} = \frac{180 - 150}{\ln(180/150)} = 164.5^oC$$

Now the overall heat transfer coefficient (U) is determined as,

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} + R_{f,i}$$

The fouling resistance on the feed water side is obtained from Table 11-2. Since the average temperature of the feed water as it flows through heat exchanger is 50°C, we take the average value of fouling factor above and below 50°C as 0.00015 m^2·K/W.

$$\therefore \frac{1}{U} = \frac{1}{1250\,\text{W/m}^2\cdot\text{K}} + \frac{1}{850\text{W/m}^2\cdot\text{K}} + 0.00015\,\text{m}^2\cdot\text{K/W} \;\rightarrow\; U = 470.26\ \text{W/m}^2\cdot\text{K}$$

The counter flow heat exchanger area to maintain the feed water exit temperature to a minimum of 70°C is calculated as,

$$A_s = \frac{\dot{m}_c C_{pc}(T_{c,out} - T_{c,in})}{U\Delta T_{lm}} = \frac{(3.47\,\text{kg/s})(4181\text{J/kg}\cdot\text{K})(70-30)^o\text{C}}{(470.26\,\text{W/m}^2\cdot\text{K})\,(164.5^o\,\text{C})} = 7.5\ \text{m}^2$$

This is the heat exchanger area required to maintain the feed water exit temperature to a minimum of 70°C.

(b) In case of parallel flow arrangement, the log mean temperature difference is calculated as,

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)},\ \Delta T_1 = T_{h,in} - T_{c,in} = 250 - 30 = 220^oC$$

and

$$\Delta T_2 = T_{h,out} - T_{c,out} = 180 - 70 = 110^oC$$

$$\Delta T_{lm} = \frac{220 - 110}{\ln(220/110)} = 158.7^oC$$

Overall heat transfer coefficient remains the same for parallel flow arrangement. The parallel flow heat exchanger area to maintain the feed water exit temperature to a minimum of 70°C is calculated as,

$$A_s = \frac{\dot{m}_c c_{pc}(T_{c,out} - T_{c,in})}{U\Delta T_{lm}} = \frac{(3.47\,\text{kg/s})\,(4181\text{J/kg}\cdot\text{K})\,(70-30)^o\text{C}}{(470.26\,\text{W/m}^2\cdot\text{K})\,(158.7^o\,\text{C})} = 7.78\,\text{m}^2$$

Discussion The parallel flow arrangement requires about 3.6% higher heat transfer area. Fouling on the steam side is not considered in this problem. However, over a long run the superheated steam may cause fouling on the tubes outer surface leading to a decrease in the overall heat transfer coefficient and hence requiring a higher heat exchanger area.

11-68 C&S A double-pipe heat exchanger is used to cool water such that it flows into a system of CPVC pipes at a temperature not exceeding 93.3°C (maximum temperature for CPCV pipes recommended by the ASME Code for Process Piping). The flow configuration should be used, whether parallel flow or counter flow, is to be determined.

Assumptions **1** Steady state conditions. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Effects of fouling are negligible. **4** Properties of fluids are constant.

Properties The specific heat of water (hot side) at the average mean temperature T_m = (105 + 93.3)°C/2 = 99.2°C is c_{ph} = 4216 J/kg·K (Table A-9).

Analysis The overall heat transfer coefficient for the heat exchanger is

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{3600\ \text{W/m}^2\cdot\text{K}} + \frac{1}{4500\ \text{W/m}^2\cdot\text{K}}$$

$$U = \left(\frac{1}{3600\ \text{W/m}^2\cdot\text{K}} + \frac{1}{4500\ \text{W/m}^2\cdot\text{K}}\right)^{-1} = 2000\ \text{W/m}^2\cdot\text{K}$$

The heat transfer rate in the heat exchanger is

$$\dot{Q} = \dot{m}_h c_{ph}\left(T_{h,\text{in}} - T_{h,\text{out}}\right)$$

$$\dot{Q} = UA_s\Delta T_{\text{lm}}$$

<u>Parallel flow</u>

The LMTD for a parallel flow configuration is

$$\Delta T_{\text{lm,PF}} = \frac{\left(T_{h,\text{in}} - T_{c,\text{in}}\right) - \left(T_{h,\text{out}} - T_{c,\text{out}}\right)}{\ln\left(\dfrac{T_{h,\text{in}} - T_{c,\text{in}}}{T_{h,\text{out}} - T_{c,\text{out}}}\right)} = \frac{(105 - 10) - (93.3 - 80)}{\ln\left(\dfrac{105 - 10}{93.3 - 80}\right)} = 41.55\text{°C}$$

This gives

$$U(\pi D L_{\text{PF}})\Delta T_{\text{lm,PF}} = \dot{m}_h c_{ph}\left(T_{h,\text{in}} - T_{h,\text{out}}\right)$$

The parallel flow heat exchanger length is

$$L_{\text{PF}} = \frac{\dot{m}_h c_{ph}}{U\pi D}\left(\frac{T_{h,\text{in}} - T_{h,\text{out}}}{\Delta T_{\text{lm,PF}}}\right) = \frac{(0.75\ \text{kg/s})(4216\ \text{J/kg}\cdot\text{K})(105 - 93.3)\text{°C}}{(2000\ \text{W/m}^2\cdot\text{K})(\pi)\ (0.025\ \text{m})(41.55\ \text{°C})} = \mathbf{5.67\ m} > 5\ \text{m}$$

If it is a parallel flow configuration, it would require more than 5 m of the heat exchanger length to cool $T_{h,\text{out}}$ below 93.3°C.

<u>Counter flow</u>

The LMTD for a counter flow configuration is

$$\Delta T_{\text{lm,CF}} = \frac{\left(T_{h,\text{in}} - T_{c,\text{out}}\right) - \left(T_{h,\text{out}} - T_{c,\text{in}}\right)}{\ln\left(\dfrac{T_{h,\text{in}} - T_{c,\text{out}}}{T_{h,\text{out}} - T_{c,\text{in}}}\right)} = \frac{(105 - 80) - (93.3 - 10)}{\ln\left(\dfrac{105 - 80}{93.3 - 10}\right)} = 48.44\text{°C}$$

This gives

$$U(\pi D L_{\text{CF}})\Delta T_{\text{lm,CF}} = \dot{m}_h c_{ph}\left(T_{h,\text{in}} - T_{h,\text{out}}\right)$$

The counter flow heat exchanger length is

$$L_{\text{CF}} = \frac{\dot{m}_h c_{ph}}{U\pi D}\left(\frac{T_{h,\text{in}} - T_{h,\text{out}}}{\Delta T_{\text{lm,CF}}}\right) = \frac{(0.75\ \text{kg/s})(4216\ \text{J/kg}\cdot\text{K})(105 - 93.3)\text{°C}}{(2000\ \text{W/m}^2\cdot\text{K})(\pi)\ (0.025\ \text{m})(48.44\ \text{°C})} = \mathbf{4.86\ m} < 5\ \text{m}$$

If it is a counter flow configuration, it would require 4.86 m of the heat exchanger length to cool $T_{h,\text{out}}$ below 93.3°C. Therefore, a counter flow configuration should be employed.

Discussion With L = 5 m, a double-pipe parallel-flow heat exchanger would not provide sufficient surface area to cool the hot water to 93.3°C, at the heat exchanger outlet. A double-pipe parallel-flow heat exchanger would require at least 5.67 m in length.

With L = 5 m, a double-pipe counter-flow heat exchanger would provide sufficient surface area to cool the hot water to 93.3°C, at the heat exchanger outlet. A double-pipe counter-flow heat exchanger would require at least 4.86 m in length. Thus, counter flow is the preferred configuration for this scenario.

11-69 C&S A double-pipe heat exchanger is used to cool a hot fluid such that the fluid flowing into a pipe system is below the temperature limit for HNBR O-rings, 150°C. The fouling factors for a parallel flow and a counter flow configuration are to be determined and compared.

Assumptions **1** Steady state conditions. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Properties of fluids are constant.

Analysis The overall heat transfer coefficient for the heat exchanger is

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} + R_f$$

The heat transfer rate in the heat exchanger is

$$\dot{Q} = C_h\left(T_{h,\text{in}} - T_{h,\text{out}}\right)$$

$$\dot{Q} = UA_s\Delta T_{\text{lm}}$$

This gives

$$\frac{1}{U} = \frac{A_s\Delta T_{\text{lm}}}{\dot{Q}} = \frac{(\pi DL)\Delta T_{\text{lm}}}{C_h\left(T_{h,\text{in}} - T_{h,\text{out}}\right)}$$

Thus,

$$R_f = \frac{(\pi DL)\Delta T_{\text{lm}}}{C_h\left(T_{h,\text{in}} - T_{h,\text{out}}\right)} - \frac{1}{h_i} - \frac{1}{h_o}$$

Parallel flow

The LMTD for a parallel flow configuration is

$$\Delta T_{\text{lm,PF}} = \frac{\left(T_{h,\text{in}} - T_{c,\text{in}}\right) - \left(T_{h,\text{out}} - T_{c,\text{out}}\right)}{\ln\left(\dfrac{T_{h,\text{in}} - T_{c,\text{in}}}{T_{h,\text{out}} - T_{c,\text{out}}}\right)} = \frac{(180-10)-(150-30)}{\ln\left(\dfrac{180-10}{150-30}\right)} = 143.55℃$$

Thus,

$$R_{f,\text{PF}} = \frac{(\pi DL)\Delta T_{\text{lm,PF}}}{C_h\left(T_{h,\text{in}} - T_{h,\text{out}}\right)} - \frac{1}{h_i} - \frac{1}{h_o} = \frac{(\pi)(0.025\text{ m})(5\text{ m})(143.55\text{ ℃})}{\left(1000\dfrac{\text{W}}{\text{K}}\right)(180-150)\text{℃}} - \frac{1}{1000\dfrac{\text{W}}{\text{m}^2}\cdot\text{K}} - \frac{1}{1500\dfrac{\text{W}}{\text{m}^2}\cdot\text{K}}$$
$$= \mathbf{0.000212\ m^2{\cdot}K/W}$$

Counter flow

The LMTD for a counter flow configuration is

$$\Delta T_{\text{lm,CF}} = \frac{\left(T_{h,\text{in}} - T_{c,\text{out}}\right) - \left(T_{h,\text{out}} - T_{c,\text{in}}\right)}{\ln\left(\dfrac{T_{h,\text{in}} - T_{c,\text{out}}}{T_{h,\text{out}} - T_{c,\text{in}}}\right)} = \frac{(180-30)-(150-10)}{\ln\left(\dfrac{180-30}{150-10}\right)} = 144.94℃$$

Thus,

$$R_{f,\text{CF}} = \frac{(\pi DL)\Delta T_{\text{lm,CF}}}{C_h\left(T_{h,\text{in}} - T_{h,\text{out}}\right)} - \frac{1}{h_i} - \frac{1}{h_o} = \frac{(\pi)(0.025\text{ m})(5\text{ m})(144.94\text{ ℃})}{\left(1000\dfrac{\text{W}}{\text{K}}\right)(180-150)\text{℃}} - \frac{1}{1000\dfrac{\text{W}}{\text{m}^2}\cdot\text{K}} - \frac{1}{1500\dfrac{\text{W}}{\text{m}^2}\cdot\text{K}}$$
$$= \mathbf{0.000231\ m^2{\cdot}K/W} > R_{f,\text{PF}}$$

Discussion The counter flow configuration can tolerate higher level of fouling factor than the parallel flow configuration. The counter flow configuration can have almost 9% more level of fouling than the parallel flow configuration before it becomes unable to cool the hot fluid at the heat exchanger outlet to 150°C (the maximum temperature for HNBR O-rings set by the ASME Boiler and Pressure Vessel Code).

11-70 During an experiment, the inlet and exit temperatures of water and oil and the mass flow rate of water are measured. The overall heat transfer coefficient based on the inner surface area is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

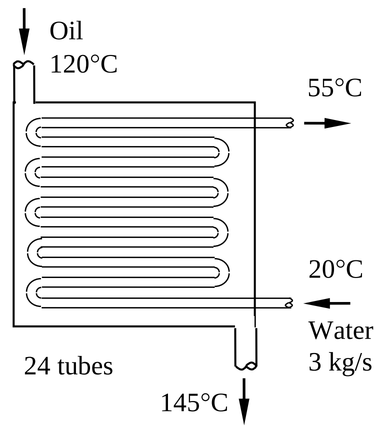
Properties The specific heats of water and oil are given to be 4180 and 2150 J/kg.°C, respectively.

Analysis The rate of heat transfer from the oil to the water is

$$\dot{Q} = [\dot{m}c_p(T_{out} - T_{in})]_{\text{water}} = (3\,\text{kg/s})(4.18\text{kJ/kg.°C})(55°\text{C} - 20°\text{C}) = 438.9\,\text{kW}$$

The heat transfer area on the tube side is

$$A_i = n\pi D_i L = 24\pi(0.012\,\text{m})(2\,\text{m}) = 1.8\,\text{m}^2$$



The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 120°\text{C} - 55°\text{C} = 65°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 45°\text{C} - 20°\text{C} = 25°\text{C}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{65 - 25}{\ln(65/25)} = 41.9°\text{C}$$

$$\left.\begin{array}{l} P = \dfrac{t_2 - t_1}{T_1 - t_1} = \dfrac{55 - 20}{120 - 20} = 0.35 \\[2ex] R = \dfrac{T_1 - T_2}{t_2 - t_1} = \dfrac{120 - 45}{55 - 20} = 2.14 \end{array}\right\} F = 0.70$$

Then the overall heat transfer coefficient becomes

$$\dot{Q} = U_i A_i F \Delta T_{lm,CF} \longrightarrow U_i = \frac{\dot{Q}}{A_i F \Delta T_{lm,CF}} = \frac{438.9\,\text{kW}}{(1.8\,\text{m}^2)(0.70)(41.9°\text{C})} = \mathbf{8.31 kW/m^2.°C}$$

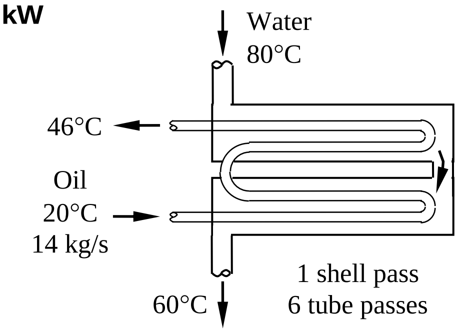
11-71 Oil is heated by water in a 1-shell pass and 6-tube passes heat exchanger. The rate of heat transfer and the heat transfer surface area are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heat of oil is given to be 2.0 kJ/kg.°C.

Analysis The rate of heat transfer in this heat exchanger is

$$\dot{Q} = [\dot{m}c_p(T_{out} - T_{in})]_{\text{oil}} = (14\text{ kg/s})(2.0\text{kJ/kg.°C})(46°\text{C} - 20°\text{C}) = \mathbf{728\,kW}$$



The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 80°\text{C} - 46°\text{C} = 34°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 60°\text{C} - 20°\text{C} = 40°\text{C}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{34-40}{\ln(34/40)} = 36.92°\text{C}$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{46-20}{80-20} = 0.43 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{80-60}{46-20} = 0.77 \end{aligned}\right\} F = 0.94$$

Then the heat transfer surface area on the tube side becomes

$$\dot{Q} = UA_sF\Delta T_{lm,CF} \longrightarrow A_s = \frac{\dot{Q}}{UF\Delta T_{lm,CF}} = \frac{728\text{ kW}}{(1.0\text{ kW/m}^2.°\text{C})(0.94)(36.92°\text{C})} = \mathbf{21.0\,m^2}$$

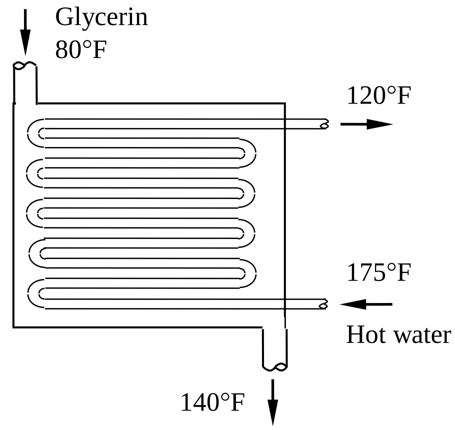
11-72E Glycerin is heated by hot water in a 1-shell pass and 8-tube passes heat exchanger. The rate of heat transfer for the cases of fouling and no fouling are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Heat transfer coefficients and fouling factors are constant and uniform. **5** The thermal resistance of the inner tube is negligible since the tube is thin-walled and highly conductive.

Properties The specific heats of glycerin and water are given to be 0.60 and 1.0 Btu/lbm.°F, respectively.

Analysis (*a*) The tubes are thin walled and thus we assume the inner surface area of the tube to be equal to the outer surface area. Then the heat transfer surface area of this heat exchanger becomes

$$A_s = n\pi DL = 8\pi(0.5/12\text{ ft})(400\text{ ft}) = 418.9\text{ ft}^2$$



The temperature differences at the two ends of the heat exchanger are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 175°\text{F} - 140°\text{F} = 35°\text{F}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 120°\text{F} - 80°\text{F} = 40°\text{F}$$

and $$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{35-40}{\ln(35/40)} = 37.44°\text{F}$$

The correction factor is

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{120-175}{80-175} = 0.58 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{80-140}{120-175} = 1.09 \end{aligned}\right\} F = 0.50$$

In case of no fouling, the overall heat transfer coefficient is determined from

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{50\text{ Btu/h.ft}^2.°\text{F}} + \dfrac{1}{4\text{ Btu/h.ft}^2.°\text{F}}} = 3.704\text{ Btu/h.ft}^2.°\text{F}$$

Then the rate of heat transfer becomes

$$\dot{Q} = UA_s F\Delta T_{lm,CF} = (3.704\text{ Btu/h.ft}^2.°\text{F})(418.9\text{ft}^2)(0.50)(37.44°\text{F}) = \mathbf{29{,}040\text{ Btu/h}}$$

(*b*) The thermal resistance of the heat exchanger with a fouling factor is

$$R = \frac{1}{h_i A_i} + \frac{R_{fi}}{A_i} + \frac{1}{h_o A_o}$$

$$= \frac{1}{(50\text{ Btu/h.ft}^2.°\text{F})(418.9\text{ ft}^2)} + \frac{0.002\text{ h.ft}^2.°\text{F/Btu}}{418.9\text{ ft}^2} + \frac{1}{(4\text{ Btu/h.ft}^2.°\text{F})(418.9\text{ ft}^2)}$$

$$= 0.0006493\text{ h.°F/Btu}$$

The overall heat transfer coefficient in this case is

$$R = \frac{1}{UA_s} \longrightarrow U = \frac{1}{RA_s} = \frac{1}{(0.0006493\text{ h.°F/Btu})(418.9\text{ ft}^2)} = 3.676\text{ Btu/h.ft}^2.°\text{F}$$

Then rate of heat transfer becomes

$$\dot{Q} = UA_s F\Delta T_{lm,CF} = (3.676\text{ Btu/h.ft}^2.°\text{F})(418.9\text{ft}^2)(0.50)(37.44°\text{F}) = \mathbf{28{,}830\text{ Btu/h}}$$

11-73 Water is heated by ethylene glycol in a 2-shell passes and 12-tube passes heat exchanger. The rate of heat transfer and the heat transfer surface area on the tube side are to be determined.

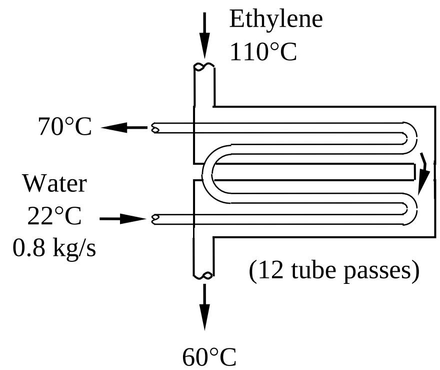
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heats of water and ethylene glycol are given to be 4.18 and 2.68 kJ/kg.°C, respectively.

Analysis The rate of heat transfer in this heat exchanger is :

$$\dot{Q} = [\dot{m}c_p(T_{out} - T_{in})]_{\text{water}} = (0.8\text{ kg/s})(4.18\text{kJ/kg.°C})(70°\text{C} - 22°\text{C}) = \mathbf{160.5\ kW}$$

The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are



$$\Delta T_1 = T_{h,in} - T_{c,out} = 110°\text{C} - 70°\text{C} = 40°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 60°\text{C} - 22°\text{C} = 38°\text{C}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{40-38}{\ln(40/38)} = 39°\text{C}$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{70-22}{110-22} = 0.55 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{110-60}{70-22} = 1.04 \end{aligned}\right\} F = 0.92$$

Then the heat transfer surface area on the tube side becomes

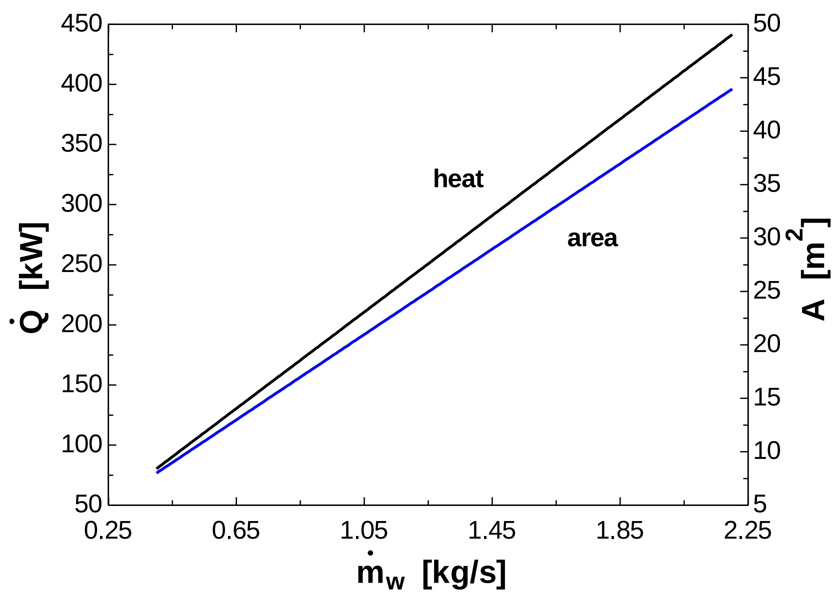
$$\dot{Q} = U_i A_i F \Delta T_{lm,CF} \longrightarrow A_i = \frac{\dot{Q}}{U_i F \Delta T_{lm,CF}} = \frac{160.5\text{ kW}}{(0.28\text{ kW/m}^2.°\text{C})(0.92)(39°\text{C})} = \mathbf{16.0\ m^2}$$

11-74 Prob. 11-73 is reconsidered. The effect of the mass flow rate of water on the rate of heat transfer and the tube-side surface area is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

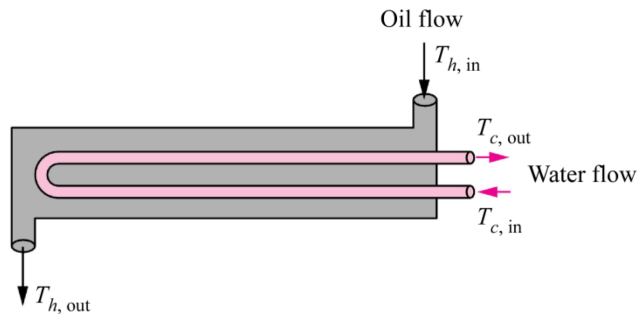
```
"GIVEN"
T_w_in=22 [C]
T_w_out=70 [C]
m_dot_w=0.8 [kg/s]
c_p_w=4.18 [kJ/kg-C]
T_glycol_in=110 [C]
T_glycol_out=60 [C]
c_p_glycol=2.68 [kJ/kg-C]
U=0.28 [kW/m^2-C]

"ANALYSIS"
Q_dot=m_dot_w*c_p_w*(T_w_out-T_w_in)
Q_dot=m_dot_glycol*c_p_glycol*(T_glycol_in-T_glycol_out)
DELTAT_1=T_glycol_in-T_w_out
DELTAT_2=T_glycol_out-T_w_in
DELTAT_lm_CF=(DELTAT_1-DELTAT_2)/ln(DELTAT_1/DELTAT_2)
P=(T_w_out-T_w_in)/(T_glycol_in-T_w_in)
R=(T_glycol_in-T_glycol_out)/(T_w_out-T_w_in)
F=0.92 "from Fig. 11-19b of the text at the calculated P and R"
Q_dot=U*A*F*DELTAT_lm_CF
```

$\dot{m}_w$ [kg/s]	$\dot{Q}$ [kW]	A [m^2]
0.4	80.26	7.99
0.5	100.3	9.988
0.6	120.4	11.99
0.7	140.4	13.98
0.8	160.5	15.98
0.9	180.6	17.98
1	200.6	19.98
1.1	220.7	21.97
1.2	240.8	23.97
1.3	260.8	25.97
1.4	280.9	27.97
1.5	301	29.96
1.6	321	31.96
1.7	341.1	33.96
1.8	361.2	35.96
1.9	381.2	37.95
2	401.3	39.95
2.1	421.3	41.95
2.2	441.4	43.95



11-75E A 1-shell and 2-tube heat exchanger has specified overall heat transfer coefficient, inlet and outlet temperatures, and mass flow rates, (*a*) the log mean temperature difference and (*b*) the surface area of the heat exchanger are to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.



Properties The specific heat of water is given to be c_{pc} = 1.0 Btu/lbm·°F.

Analysis (*a*) Using Fig. 11-19*a*, the correction factor can be determined to be

$$\left.\begin{array}{l} P = \dfrac{t_2 - t_1}{T_1 - t_1} = \dfrac{100-80}{180-80} = 0.2 \\[2ex] R = \dfrac{T_1 - T_2}{t_2 - t_1} = \dfrac{180-120}{100-80} = 3.0 \end{array}\right\} F \approx 0.94 \qquad \text{(Fig. 11-19}a\text{)}$$

The log mean temperature difference for the counter-flow arrangement is

$$\Delta T_{\text{lm,CF}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{(180-100)-(120-80)}{\ln[(180-100)/(120-80)]}\ ^\circ\text{C} = 57.7^\circ\text{F}$$

Hence, the log mean temperature difference is

$$\Delta T_{\text{lm,}} = F\Delta T_{\text{lm,CF}} = 0.94(57.7^\circ\text{F}) = \mathbf{54.2^\circ F}$$

(*b*) The surface area of the heat exchanger can be determined using

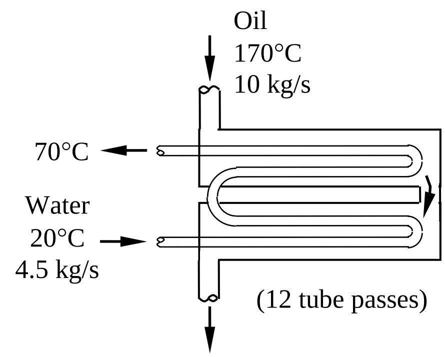
$$\dot{Q} = UA_s F\Delta T_{\text{lm,CF}} \qquad \rightarrow \qquad A_s = \frac{\dot{Q}}{UF\Delta T_{\text{lm,CF}}} = \frac{\dot{m}_c c_{pc}(T_{c,\text{out}} - T_{c,\text{in}})}{UF\Delta T_{\text{lm,CF}}}$$

$$A_s = \frac{(20{,}000\text{ lbm/hr})(1.0\text{ Btu/lbm}\cdot{}^\circ\text{F})(100-80)^\circ\text{F}}{(40\text{ Btu/hr}\cdot\text{ft}^2\cdot{}^\circ\text{F})(0.94)(57.7^\circ\text{F})} = \mathbf{184\ ft^2}$$

Discussion The surface area of the heat exchanger can also be determined using the effectiveness-NTU method.

11-76 Water is heated by hot oil in a 2-shell passes and 12-tube passes heat exchanger. The heat transfer surface area on the tube side is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.



Properties The specific heats of water and oil are given to be 4.18 and 2.3 kJ/kg.°C, respectively.

Analysis The rate of heat transfer in this heat exchanger is

$$\begin{aligned}\dot{Q} &= [\dot{m}c_p(T_{out} - T_{in})]_{\text{water}} \\ &= (4.5\text{ kg/s})(4.18\text{kJ/kg.°C})(70°\text{C} - 20°\text{C}) \\ &= 940.5\text{ kW}\end{aligned}$$

The outlet temperature of the oil is determined from

$$\dot{Q} = [\dot{m}c_p(T_{in} - T_{out})]_{\text{oil}} \longrightarrow T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}c_p} = 170°\text{C} - \frac{940.5\text{ kW}}{(10\text{ kg/s})(2.3\text{kJ/kg.°C})} = 129°\text{C}$$

The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 170°\text{C} - 70°\text{C} = 100°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 129°\text{C} - 20°\text{C} = 109°\text{C}$$

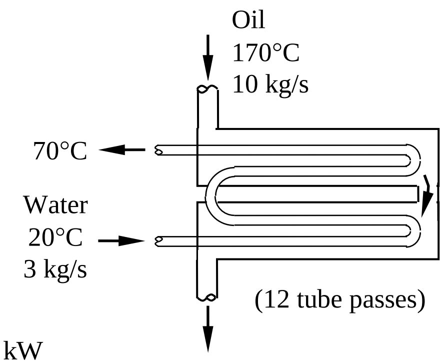
$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{100 - 109}{\ln(100/109)} = 104.4°\text{C}$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{70 - 20}{170 - 20} = 0.33 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{170 - 129}{70 - 20} = 0.82 \end{aligned}\right\} F = 1.0$$

Then the heat transfer surface area on the tube side becomes

$$\dot{Q} = UA_s F\Delta T_{lm,CF} \longrightarrow A_s = \frac{\dot{Q}}{UF\Delta T_{lm,CF}} = \frac{940.5\text{ kW}}{(0.350\text{ kW/m}^2.°\text{C})(1.0)(104.4°\text{C})} = \mathbf{25.7\ m^2}$$

11-77 Water is heated by hot oil in a 2-shell passes and 12-tube passes heat exchanger. The heat transfer surface area on the tube side is to be determined.



Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heats of water and oil are given to be 4.18 and 2.3 kJ/kg.°C, respectively.

Analysis The rate of heat transfer in this heat exchanger is

$$\dot{Q} = [\dot{m}c_p(T_{out} - T_{in})]_{\text{water}} = (3\,\text{kg/s})(4.18\text{kJ/kg.°C})(70°\text{C} - 20°\text{C}) = 627\,\text{kW}$$

The outlet temperature of the oil is determined from

$$\dot{Q} = [\dot{m}c_p(T_{in} - T_{out})]_{\text{oil}} \longrightarrow T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}c_p} = 170°\text{C} - \frac{627\,\text{kW}}{(10\,\text{kg/s})(2.3\text{kJ/kg.°C})} = 142.7°\text{C}$$

The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 170°\text{C} - 70°\text{C} = 100°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 142.7°\text{C} - 20°\text{C} = 122.7°\text{C}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{100 - 122.7}{\ln(100/122.7)} = 111.0°\text{C}$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{70 - 20}{170 - 20} = 0.33 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{170 - 142.7}{70 - 20} = 0.55 \end{aligned}\right\} F = 1.0$$

Then the heat transfer surface area on the tube side becomes

$$\dot{Q} = U_i A_i F \Delta T_{lm,CF} \longrightarrow A_i = \frac{\dot{Q}}{U_i F \Delta T_{lm,CF}} = \frac{627\,\text{kW}}{(0.350\,\text{kW/m}^2.°\text{C})(1.0)(111.0°\text{C})} = \mathbf{16.1\,m^2}$$

11-78 Ethyl alcohol is heated by water in a 2-shell passes and 8-tube passes heat exchanger. The heat transfer surface area of the heat exchanger is to be determined.

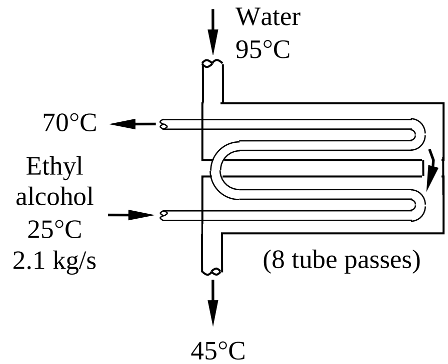
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** There is no fouling. **5** Fluid properties are constant.

Properties The specific heats of water and ethyl alcohol are given to be 4.19 and 2.67 kJ/kg.°C, respectively.

Analysis The rate of heat transfer in this heat exchanger is

$$\dot{Q} = [\dot{m}c_p(T_{out} - T_{in})]_{\text{ethyl alcohol}} = (2.1\,\text{kg/s})(2.67\,\text{kJ/kg.°C})(70°\text{C} - 25°\text{C}) = 252.3\,\text{kW}$$

The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are



$$\Delta T_1 = T_{h,in} - T_{c,out} = 95°\text{C} - 70°\text{C} = 25°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 45°\text{C} - 25°\text{C} = 20°\text{C}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{25 - 20}{\ln(25/20)} = 22.4°\text{C}$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{70 - 25}{95 - 25} = 0.64 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{95 - 45}{70 - 25} = 1.1 \end{aligned}\right\} F = 0.82$$

Then the heat transfer surface area on the tube side becomes

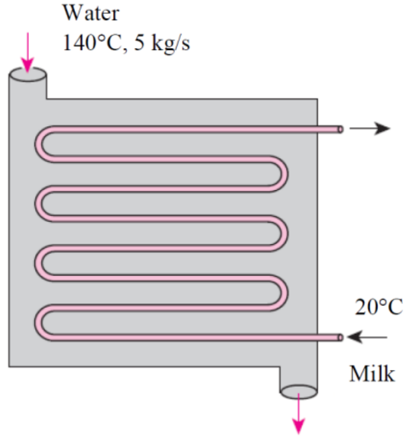
$$\dot{Q} = U_i A_i F \Delta T_{lm,CF} \longrightarrow A_i = \frac{\dot{Q}}{U_i F \Delta T_{lm,CF}} = \frac{252.3\,\text{kW}}{(0.950\,\text{kW/m}^2.°\text{C})(0.82)(22.4°\text{C})} = \mathbf{14.5\,m^2}$$

11-79 One shell and 10 tube pass heat exchanger is used in a milk pasteurizing process. The width of the shell (tube length in each pass) is to be determined in order to ensure complete pasteurization of milk.

Assumptions **1** Steady state conditions exist. **2** Thermal properties of milk and hot water remain constant. **3** heat exchanger is well insulated. **4** No fouling inside the heat exchanger.

Properties The specific heat of water is evaluated from Table A-9 at an average temperature of (140 + 80)°C/2 = 110 °C. $c_{ph} = 4229 \text{ J/kg}\cdot\text{K}$

Analysis From energy balance between the cold milk and hot water we have



Heat lost by water = Heat gained by milk.

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

$$\dot{Q} = (5\text{ kg/s})(4229\text{ J/kg}\cdot\text{K})(140 - 80)^\circ\text{C} = 1268.7\text{ kW}$$

The actual rate of heat transfer can also be expressed as

$$\dot{Q} = UA_s F\Delta T_{lm,CF}$$

The overall heat transfer coefficient is determined from the convective heat transfer coefficient values on milk and water side.

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} = \frac{1}{450\text{ W/m}^2\cdot\text{K}} + \frac{1}{1100\text{ W/m}^2\cdot\text{K}} \rightarrow U = 319.35\text{ W/m}^2\cdot\text{K}$$

The log mean temperature difference is determined as

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$\Delta T_1 = T_{h,in} - T_{c,out} = 140 - 70 = 70^o C$$

and $$\Delta T_2 = T_{h,out} - T_{c,in} = 80 - 20 = 60^o C$$

$$\Delta T_{lm,CF} = \frac{70 - 60}{\ln(70/60)} = 64.87^o C$$

Since this is a multiple pass heat exchanger we need to determine the correction factor F from the temperature ratios P and R.

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{70 - 20}{140 - 20} = 0.417 \quad \text{and} \quad R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{140 - 80}{70 - 20} = 1.2$$

From Figure 11-19(a), for the corresponding values of P and R we get,

$F = 0.865$

Thus the actual rate of heat transfer is,

$$\dot{Q} = (319.35\text{ W/m}^2\cdot\text{K})(A_s m^2)(0.865)(64.87^\circ\text{C})$$

$$\therefore A_s = \frac{1268.7\times 10^3\text{ W}}{(319.35\text{W/m}^2\cdot\text{K})(0.865)(64.87^o C)} = 70.79\text{ m}^2$$

The surface area of the heat exchanger tubes is,

$$A_s = \pi DL \times n \times p$$

where 'n' and 'p' are the number of tubes and tube passes, respectively. Therefore the length of each tube is,

$$L = \frac{70.79\text{ m}^2}{\pi(0.02\text{ m})(30)(10)} = \mathbf{3.75\text{ m}}$$

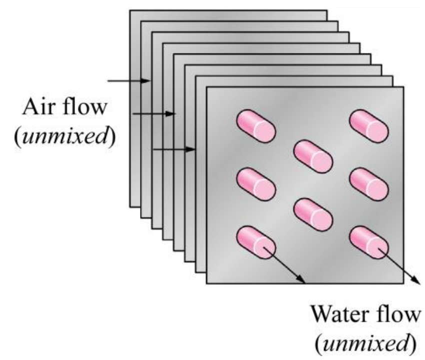
Discussion In this problem it is assumed that the effect of fouling on the overall heat transfer coefficient is negligible. However, considering that the fouling will deteriorate the performance of the heat exchanger over a period of time, a suitable factor of safety must be considered in deciding upon the length of heat exchanger.

11-80E A single-pass cross-flow heat exchanger is used to cool jacket water using air. The log mean temperature difference for the heat exchanger is to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heats of both water and air are given to be c_{ph} = 1.0 Btu/lbm·°F and c_{pc} = 0.245 Btu/lbm·°F, respectively.

Analysis The rate of heat transfer in the heat exchanger is



$$\begin{aligned}\dot{Q} &= \dot{m}_h c_{ph}(T_{h,\text{in}} - T_{h,\text{out}}) \\ &= (92{,}000\ \text{lbm/hr})(1.0\ \text{Btu/lbm}\cdot{}^\circ\text{F})(190-140)^\circ\text{F} \\ &= 4.6\times 10^6\ \text{Btu/hr}\end{aligned}$$

Since heat transfer from the hot fluid is equal to the heat transfer to the cold fluid, we have

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,\text{out}} - T_{c,\text{in}}) \quad \rightarrow \quad T_{c,\text{out}} = \frac{\dot{Q}}{\dot{m}_c c_{pc}} + T_{c,\text{in}}$$

$$T_{c,\text{out}} = \frac{4.6\times 10^6\ \text{Btu/hr}}{(400{,}000\ \text{lbm/hr})(0.245\ \text{Btu/lbm}\cdot{}^\circ\text{F})} + 90^\circ\text{F} = 136.9^\circ\text{F}$$

Thus, the log mean temperature difference for the counter-flow arrangement is

$$\Delta T_{\text{lm,CF}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{(190-136.9)-(140-90)}{\ln[(190-136.9)/(140-90)]}\ {}^\circ\text{F} = 51.5^\circ\text{F}$$

Using Fig. 11-19*c*, the correction factor can be determined to be

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{140-190}{90-190} = 0.50 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{90-136.9}{140-190} = 0.94 \end{aligned}\right\} F \approx 0.92 \qquad \text{(Fig. 11-19}c\text{)}$$

The log mean temperature difference is

$$\Delta T_{\text{lm,}} = F\Delta T_{\text{lm,CF}} = 0.92(51.5^\circ\text{F}) = \mathbf{47.4^\circ F}$$

Discussion The correction factor (F) represents how closely the cross-flow heat exchanger approximates a counter-flow heat exchanger in terms of its logarithmic mean temperature difference.

11-81 A single-pass cross-flow heat exchanger with both fluids unmixed, the value of the overall heat transfer coefficient is to be determined.

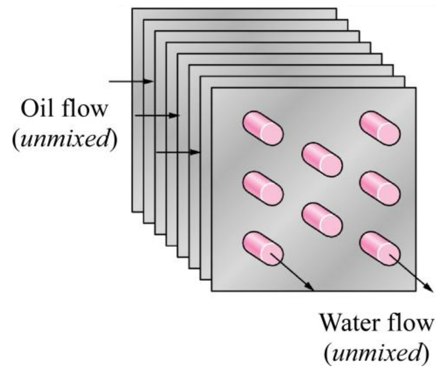
Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The properties of oil are given to be c_{ph} = 1.93 kJ/kg·K and ρ = 870 kg/m^3.

Analysis The mass flow rate of oil (hot fluid) is

$$\dot{m}_h = \rho\dot{\boldsymbol{V}} = (870 \text{ kg/m}^3)(0.19 \text{ m}^3/\text{min})(1/60 \text{ min/s}) = 2.755 \text{ kg/s}$$

Using energy balance on the hot fluid, we have



$$\begin{aligned}\dot{Q} &= \dot{m}_h c_{ph}(T_{h,\text{in}} - T_{h,\text{out}})\\ &= (2.755 \text{ kg/s})(1930 \text{ J/kg}\cdot\text{K})(38-29) \text{ K}\\ &= 4.785\times10^4 \text{ W}\end{aligned}$$

Using Fig. 11-19*c*, the correction factor can be determined to be

$$\left.\begin{aligned}P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{33-16}{38-16} = 0.77\\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{38-29}{33-16} = 0.53\end{aligned}\right\} F \approx 0.85 \qquad \text{(Fig. 11-19}c\text{)}$$

The log mean temperature difference for the counter-flow arrangement is

$$\Delta T_{\text{lm,CF}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{(38-33)-(29-16)}{\ln[(38-33)/(29-16)]} \text{°C} = 8.372\text{°C}$$

Thus, the overall heat transfer coefficient can be determined using

$$\dot{Q} = UA_s F\Delta T_{\text{lm,CF}} \qquad \rightarrow \qquad U = \frac{\dot{Q}}{A_s F\Delta T_{\text{lm,CF}}}$$

$$U = \frac{4.785\times10^4 \text{ W}}{(20 \text{ m}^2)(0.85)(8.372 \text{ K})} = \mathbf{336\ W/m^2\cdot K}$$

Discussion Cross-flow heat exchangers are commonly found in automobile radiators.

The Effectiveness-NTU Method

11-82C The effectiveness of a heat exchanger is defined as the ratio of the actual heat transfer rate to the maximum possible heat transfer rate and represents how closely the heat transfer in the heat exchanger approaches to maximum possible heat transfer. Since the actual heat transfer rate can not be greater than maximum possible heat transfer rate, the effectiveness can not be greater than one. The effectiveness of a heat exchanger depends on the geometry of the heat exchanger as well as the flow arrangement.

11-83C For a specified fluid pair, inlet temperatures and mass flow rates, the counter-flow heat exchanger will have the highest effectiveness.

11-84C Once the effectiveness ε is known, the rate of heat transfer and the outlet temperatures of cold and hot fluids in a heat exchanger are determined from

$$\dot{Q} = \varepsilon\dot{Q}_{\max} = \varepsilon C_{\min}(T_{h,in} - T_{c,in})$$

$$\dot{Q} = \dot{m}_c c_{p,c}(T_{c,out} - T_{c,in})$$

$$\dot{Q} = \dot{m}_h c_{p,h}(T_{h,in} - T_{h,out})$$

11-85C The heat transfer in a heat exchanger will reach its maximum value when the hot fluid is cooled to the inlet temperature of the cold fluid. Therefore, the temperature of the hot fluid cannot drop below the inlet temperature of the cold fluid at any location in a heat exchanger.

11-86C The heat transfer in a heat exchanger will reach its maximum value when the cold fluid is heated to the inlet temperature of the hot fluid. Therefore, the temperature of the cold fluid cannot rise above the inlet temperature of the hot fluid at any location in a heat exchanger.

11-87C The fluid with the lower mass flow rate will experience a larger temperature change. This is clear from the relation

$$\dot{Q} = \dot{m}_c c_p \Delta T_{cold} = \dot{m}_h c_p \Delta T_{hot}$$

11-88C The maximum possible heat transfer rate is in a heat exchanger is determined from

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in})$$

where $C_{\min}$ is the smaller heat capacity rate. The value of $\dot{Q}_{\max}$ does not depend on the type of heat exchanger.

11-89C When the capacity ratio is equal to zero and the number of transfer units value is greater than 5, a counter-flow heat exchanger has an effectiveness of one. In this case the exit temperature of the fluid with smaller capacity rate will equal to inlet temperature of the other fluid. For a parallel-flow heat exchanger the answer would be the same.

11-90C The increase of effectiveness with NTU is not linear. The effectiveness increases rapidly with NTU for small values (up to abo ut NTU = 1.5), but rather slowly for larger values. Therefore, the effectiveness will not double when the length of heat exchanger is doubled.

11-91C A heat exchanger has the smallest effectiveness value when the heat capacity rates of two fluids are identical. Therefore, reducing the mass flow rate of cold fluid by half will increase its effectiveness.

11-92C The longer heat exchanger is more likely to have a higher effectiveness.

11-93C The NTU of a heat exchanger is defined as $NTU = \dfrac{UA_s}{C_{\min}} = \dfrac{UA_s}{(\dot{m}c_p)_{\min}}$ where U is the overall heat transfer coefficient and A_s is the heat transfer surface area of the heat exchanger. For specified values of U and $C_{\min}$, the value of NTU is a measure of the heat exchanger surface area A_s. Because the effectiveness increases slowly for larger values of NTU, a large heat exchanger cannot be justified economically. Therefore, a heat exchanger with a very large NTU is not necessarily a good one to buy.

11-94C The value of effectiveness increases slowly with a large values of NTU (usually larger than 3). Therefore, doubling the size of the heat exchanger will not save much energy in this case since the increase in the effectiveness will be very small.

11-95C The value of effectiveness increases rapidly with small values of NTU (up to about 1.5). Therefore, tripling the NTU will cause a rapid increase in the effectiveness of the heat exchanger, and thus saves energy. I would support this proposal.

11-96 Hot water coming from the engine of an automobile is cooled by air in the radiator. The outlet temperature of the air and the rate of heat transfer are to be determined.

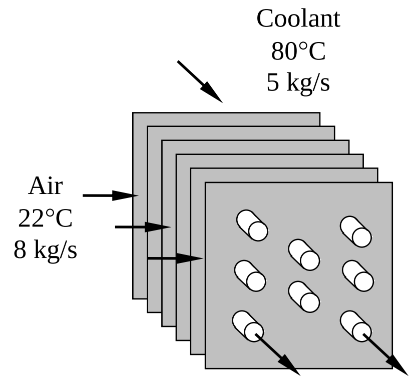
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The specific heats of water and air are given to be 4.00 and 1.00 kJ/kg.°C, respectively.

Analysis (*a*) The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = (5\,\text{kg/s})(4.00\text{kJ/kg.°C}) = 20\,\text{kW/°C}$$

$$C_c = \dot{m}_c c_{pc} = (8\,\text{kg/s})(1.00\text{kJ/kg.°C}) = 8\,\text{kW/°C}$$



Therefore

$$C_{\min} = C_c = 8\,\text{kW/°C}$$

which is the smaller of the two heat capacity rates. Noting that the heat capacity rate of the air is the smaller one, the outlet temperature of the air is determined from the effectiveness relation to be

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_{\min}(T_{a,out} - T_{c,in})}{C_{\min}(T_{h,in} - T_{c,in})}$$

$$0.4 = \frac{(T_{a,out} - 22)°\text{C}}{(80 - 22)°\text{C}} \longrightarrow T_{a,out} = \mathbf{45.2°C}$$

(*b*) The rate of heat transfer is determined from

$$\dot{Q} = C_{air}(T_{a,out} - T_{a,in}) = (8\,\text{kW/°C})(50°\text{C - }22°\text{C}) = \mathbf{224\,kW}$$

11-97 Inlet and outlet temperatures of the hot and cold fluids in a double-pipe heat exchanger are given. It is to be determined whether this is a parallel-flow or counter-flow heat exchanger and the effectiveness of it.

Analysis This is a counter-flow heat exchanger because in the parallel-flow heat exchangers the outlet temperature of the cold fluid (55°C in this case) cannot exceed the outlet temperature of the hot fluid, which is (40°C in this case). Noting that the mass flow rates of both hot and cold oil streams are the same, we have $C_{\min} = C_{\max}$. Then the effectiveness of this heat exchanger is determined from

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h(T_{h,in} - T_{h,out})}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{C_h(T_{h,in} - T_{h,out})}{C_h(T_{h,in} - T_{c,in})} = \frac{80°\text{C} - 40°\text{C}}{80°\text{C} - 15°\text{C}} = \mathbf{0.615}$$

11-98E Inlet and outlet temperatures of the hot and cold fluids in a double-pipe heat exchanger are given. It is to be determined the fluid, which has the smaller heat capacity rate and the effectiveness of the heat exchanger.

Analysis Hot water has the smaller heat capacity rate since it experiences a greater temperature change. The effectiveness of this heat exchanger is determined from

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h(T_{h,in} - T_{h,out})}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{C_h(T_{h,in} - T_{h,out})}{C_h(T_{h,in} - T_{c,in})} = \frac{190°\text{F} - 100°\text{F}}{190°\text{F} - 70°\text{F}} = \mathbf{0.75}$$

11-99 Glycerin is heated by ethylene glycol in a heat exchanger. Mass flow rates and inlet temperatures are given. The rate of heat transfer and the outlet temperatures are to be determined.

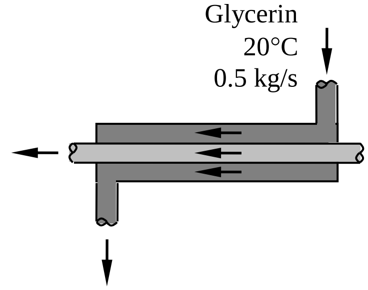
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform. **5** The thickness of the tube is negligible.

Properties The specific heats of the glycerin and ethylene glycol are given to be 2.4 and 2.5 kJ/kg.°C, respectively.

Analysis (*a*) The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = (0.5\,\text{kg/s})(2400\,\text{J/kg.°C}) = 1200\ \text{W/°C}$$

$$C_c = \dot{m}_c c_{pc} = (0.5\,\text{kg/s})(2500\,\text{J/kg.°C}) = 1250\ \text{W/°C}$$



Therefore,

$$C_{\min} = C_h = 1200\ \text{W/°C}$$

and

$$c = \frac{C_{\min}}{C_{\max}} = \frac{1200}{1250} = 0.96$$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (1200\ \text{W/°C})(60°\text{C} - 20°\text{C}) = 48{,}000\ \text{W} = 48.0\,\text{kW}$$

The NTU of this heat exchanger is

$$NTU = \frac{UA_s}{C_{\min}} = \frac{(380\ \text{W/m}^2.°\text{C})(6.5\,\text{m}^2)}{1200\ \text{W/°C}} = 2.058$$

Effectiveness of this heat exchanger corresponding to c = 0.96 and NTU = 2.058 is determined using the proper relation in Table 11-4

$$\varepsilon = \frac{1-\exp[-NTU(1+c)]}{1+c} = \frac{1-\exp[-2.058(1+0.96)]}{1+0.96} = 0.5012$$

Then the actual rate of heat transfer becomes

$$\dot{Q} = \varepsilon\dot{Q}_{\max} = (0.5012)(48.0\,\text{kW}) = \mathbf{24.06\,kW}$$

(*b*) Finally, the outlet temperatures of the cold and the hot fluid streams are determined from

$$\dot{Q} = C_c(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{C_c} = 20°\text{C} + \frac{24.06\,\text{kW}}{1.25\,\text{kW}/°\text{C}} = \mathbf{39.2°C}$$

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) \longrightarrow T_{h,out} = T_{h,in} - \frac{\dot{Q}}{C_h} = 60°\text{C} - \frac{24.06\,\text{kW}}{1.20\,\text{kW/°C}} = \mathbf{40.0°C}$$

11-100 A thin-walled concentric tube counter-flow heat exchanger has specified mass flow rates and inlet temperatures, (*a*) the heat transfer rate for the heat exchanger, (*b*) the outlet temperatures of the cold and hot fluids, and (*c*) the fouling factor after a period of operation are to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heats of the hot and cold fluids are given to be c_{ph} = 4188 J/kg·K and c_{pc} = 4178 J/kg·K, respectively.

Analysis (*a*) The heat capacity rates are

$$C_c = \dot{m}_c c_{pc} = (5\text{ kg/s})(4178\text{ J/kg}\cdot\text{K}) = 20890\text{ W/K}$$

$$C_h = \dot{m}_h c_{ph} = (2.5\text{ kg/s})(4188\text{ J/kg}\cdot\text{K}) = 10470\text{ W/K}$$

The capacity ratio is

$$c = \frac{C_{\min}}{C_{\max}} = \frac{C_h}{C_c} = \frac{10470\text{ W/K}}{20890\text{ W/K}} = 0.5012$$

The NTU of the heat exchanger is

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{(1000\text{ W/m}^2\cdot\text{K})(23\text{ m}^2)}{10470\text{ W/K}} = 2.197$$

Using the equation listed in Table 11-4, the heat transfer effectiveness is

$$\varepsilon = \frac{1-\exp[-\text{NTU}(1-c)]}{1-c\exp[-\text{NTU}(1-c)]} = \frac{1-\exp[-2.197(1-0.5012)]}{1-(0.5012)\exp[-2.197(1-0.5012)]} = 0.7997$$

The heat transfer rate for the heat exchanger is

$$\dot{Q} = C_{\min}\varepsilon(T_{h,\text{in}} - T_{c,\text{in}}) = (10470\text{ W/K})(0.7997)(100-20)\text{ K} = \mathbf{6.70\times10^5\ W}$$

(*b*) The outlet temperatures of the cold and hot fluids are

$$\dot{Q} = C_c(T_{c,\text{out}} - T_{c,\text{in}}) \quad\rightarrow\quad T_{c,\text{out}} = \frac{\dot{Q}}{C_c} + T_{c,\text{in}} = \frac{6.70\times10^5\text{ W}}{20890\text{ W/K}} + 20°\text{C} = \mathbf{52.1°C}$$

and

$$\dot{Q} = C_h(T_{h,\text{in}} - T_{h,\text{out}}) \quad\rightarrow\quad T_{h,\text{out}} = T_{h,\text{in}} - \frac{\dot{Q}}{C_h} = 100°\text{C} - \frac{6.70\times10^5\text{ W}}{10470\text{ W/K}} = \mathbf{36.0°C}$$

(*c*) The overall heat transfer coefficient at clean conditions is U_{clean} = 1000 W/m²·K. After a period of operation, the overall heat transfer coefficient is reduced to U_{dirty} = 500 W/m²·K. Hence, the fouling factor can be determined to be

$$\frac{1}{U_{\text{dirty}}} = \frac{1}{U_{\text{clean}}} + R_f \quad\rightarrow\quad R_f = \frac{1}{U_{\text{dirty}}} - \frac{1}{U_{\text{clean}}}$$

$$R_f = \left(\frac{1}{500} - \frac{1}{1000}\right)\text{m}^2\cdot\text{K/W} = \mathbf{0.001\ m^2\cdot K/W}$$

Discussion Using Figure 11-27*b*, the heat transfer effectiveness is approximately $\varepsilon \approx 78\%$.

11-101 Water is heated by solar-heated hot air in a heat exchanger. The mass flow rates and the inlet temperatures are given. The outlet temperatures of the water and the air are to be determined.

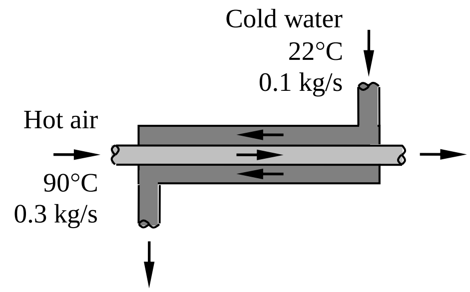
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and air are given to be 4.18 and 1.01 kJ/kg.°C, respectively.

Analysis The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = (0.3\text{kg/s})(1010\text{ J/kg.°C}) = 303\text{ W/°C}$$

$$C_c = \dot{m}_c c_{pc} = (0.1\text{kg/s})(4180\text{ J/kg.°C}) = 418\text{ W/°C}$$



Therefore,

$$C_{\min} = C_c = 303\text{ W/°C}$$

and

$$c = \frac{C_{\min}}{C_{\max}} = \frac{303}{418} = 0.725$$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (303\text{ W/°C})(90°\text{C - }22°\text{C}) = 20{,}604\text{ kW}$$

The heat transfer surface area is

$$A_s = \pi DL = (\pi)(0.012\text{ m})(12\text{ m}) = 0.45\text{ m}^2$$

Then the NTU of this heat exchanger becomes

$$NTU = \frac{UA_s}{C_{\min}} = \frac{(80\text{ W/m}^2.°\text{C})\,(0.45\text{ m}^2)}{303\text{ W/°C}} = 0.119$$

The effectiveness of this counter-flow heat exchanger corresponding to c = 0.725 and NTU = 0.119 is determined using the relation in Table 11-4 to be

$$\varepsilon = \frac{1-\exp[-NTU(1-c)]}{1-c\exp[-NTU(1-c)]} = \frac{1-\exp[-0.119(1-0.725)]}{1-0.725\exp[-0.119(1-0.725)]} = 0.108$$

Then the actual rate of heat transfer becomes

$$\dot{Q} = \varepsilon\dot{Q}_{\max} = (0.108)(20{,}604\text{ W}) = 2225.2\text{ W}$$

Finally, the outlet temperatures of the cold and hot fluid streams are determined to be

$$\dot{Q} = C_c(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{C_c} = 22°\text{C} + \frac{2225.2\text{ W}}{418\text{ W / °C}} = \mathbf{27.3°C}$$

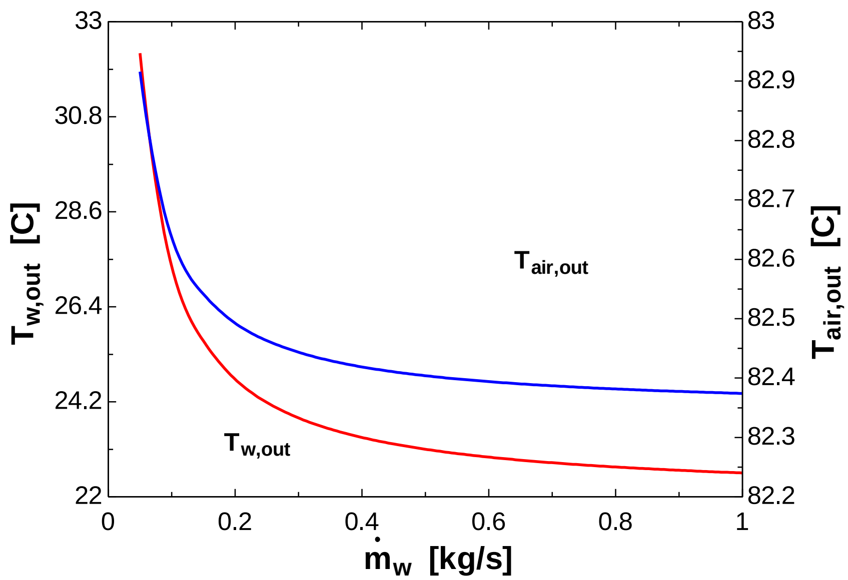
$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) \longrightarrow T_{h,out} = T_{h,in} - \frac{\dot{Q}}{C_h} = 90°\text{C} - \frac{2225.2\text{ W}}{303\text{ W/°C}} = \mathbf{82.7°C}$$

11-102 Prob. 11-101 is reconsidered. The effects of the mass flow rate of water and the tube length on the outlet temperatures of water and air are to be investigated.

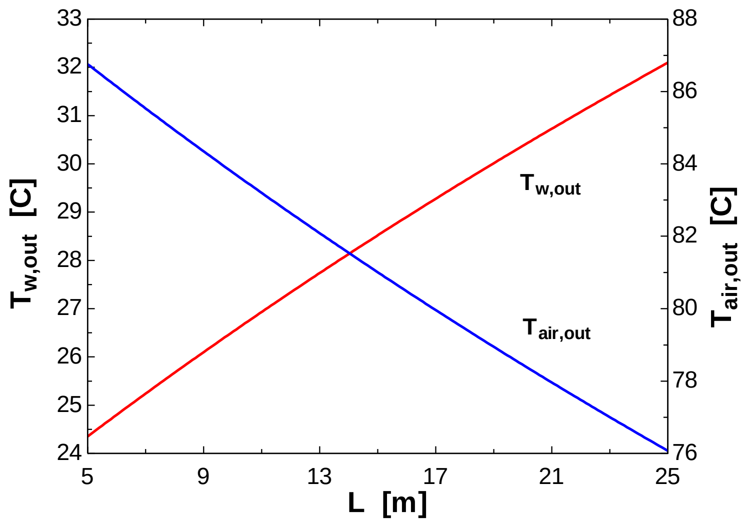
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_air_in=90 [C]
m_dot_air=0.3 [kg/s]
c_p_air=1.01 [kJ/kg-C]
T_w_in=22 [C]
m_dot_w=0.1 [kg/s]
c_p_w=4.18 [kJ/kg-C]
U=0.080 [kW/m^2-C]
L=12 [m]
D=0.012 [m]

"ANALYSIS"
"With EES, it is easier to solve this problem using LMTD method than NTU method. Below, we use LMTD method. Both methods give the same results."
DELTAT_1=T_air_in-T_w_out
DELTAT_2=T_air_out-T_w_in
DELTAT_lm=(DELTAT_1-DELTAT_2)/ln(DELTAT_1/DELTAT_2)
A=pi*D*L
Q_dot=U*A*DELTAT_lm
Q_dot=m_dot_air*c_p_air*(T_air_in-T_air_out)
Q_dot=m_dot_w*c_p_w*(T_w_out-T_w_in)
```

$\dot{m}_w$ [kg/s]	$T_{w,out}$ [C]	$T_{air,out}$ [C]
0.05	32.27	82.92
0.1	27.34	82.64
0.15	25.6	82.54
0.2	24.72	82.49
0.25	24.19	82.46
0.3	23.83	82.44
0.35	23.57	82.43
0.4	23.37	82.42
0.45	23.22	82.41
0.5	23.1	82.4
0.55	23	82.4
0.6	22.92	82.39
0.65	22.85	82.39
0.7	22.79	82.39
0.75	22.74	82.38
0.8	22.69	82.38
0.85	22.65	82.38
0.9	22.61	82.38
0.95	22.58	82.38
1	22.55	82.37



L [m]	$T_{w,out}$ [C]	$T_{air,out}$ [C]
5	24.35	86.76
6	24.8	86.14
7	25.24	85.53
8	25.67	84.93
9	26.1	84.35
10	26.52	83.77
11	26.93	83.2
12	27.34	82.64
13	27.74	82.09
14	28.13	81.54
15	28.52	81.01
16	28.9	80.48
17	29.28	79.96
18	29.65	79.45
19	30.01	78.95
20	30.37	78.45
21	30.73	77.96
22	31.08	77.48
23	31.42	77
24	31.76	76.53
25	32.1	76.07



11-103 A double-pipe heat exchanger is used to cool a hot fluid such that when it flows into a pipe system it is below the temperature limit for polypropylene lining, 107°C. (a) The effectiveness for the parallel- and counter-flow configurations are to be determined. (b) The flow configuration should be used, whether parallel flow or counter flow, is to be determined.

Assumptions **1** Steady state conditions. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Properties of fluids are constant.

Analysis The overall heat transfer coefficient for the heat exchanger is

$$U = \left(\frac{1}{h_i} + \frac{1}{h_o} + R_f\right)^{-1} = \left(\frac{1}{1400 \text{ W/m}^2\cdot\text{K}} + \frac{1}{1100 \text{ W/m}^2\cdot\text{K}} + 0.0002 \text{ m}^2\cdot\text{K/W}\right)^{-1} = 548.43 \text{ W/m}^2\cdot\text{K}$$

The heat capacity rates for the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = (0.4 \text{ kg/s})(3800 \text{ J/kg}\cdot\text{K}) = 1520 \text{ W/K} = C_{\min}$$

$$C_c = \dot{m}_c c_{pc} = (0.5 \text{ kg/s})(4200 \text{ J/kg}\cdot\text{K}) = 2100 \text{ W/K}$$

The capacity ratio is

$$c = \frac{C_{min}}{C_{max}} = \frac{C_h}{C_c} = \frac{1520 \text{ W/K}}{2100 \text{ W/K}} = 0.72381$$

The maximum possible heat transfer rate for the heat exchanger is

$$\dot{Q}_{\max} = C_{\min}\left(T_{h,\text{in}} - T_{c,\text{in}}\right) = (1520 \text{ W/K})(200 - 10)\text{K} = 288800 \text{ W}$$

The NTU is

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{(548.43 \text{ W/m}^2\cdot\text{K})(2.5 \text{ m}^2)}{1520 \text{ W/K}} = 0.90202$$

<u>Parallel flow</u>

(a) The effectiveness–NTU relation for parallel-flow heat exchanger is

$$\varepsilon_{\text{PF}} = \frac{1 - \exp[-\text{NTU}(1+c)]}{1+c} = \frac{1 - \exp[-(0.90202)(1 + 0.72381)]}{1 + 0.72381} = 0.4576$$

(b) The outlet temperature of the hot fluid for parallel-flow configuration is

$$\varepsilon_{\text{PF}} = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}} = \frac{200 - T_{h,\text{out}}}{200 - 10} = 0.4576$$

$$T_{h,\text{out}} = 200 - 0.4576(200 - 10) = \mathbf{113.1}\text{°C} > 107\text{°C}$$

<u>Counter flow</u>

(a) The effectiveness–NTU relation for counter-flow heat exchanger is

$$\varepsilon_{\text{CF}} = \frac{1 - \exp[-\text{NTU}(1-c)]}{1 - c\exp[-\text{NTU}(1-c)]} = \frac{1 - \exp[-0.90202(1 - 0.72381)]}{1 - (0.72381)\exp[-0.90202(1 - 0.72381)]} = 0.5060$$

(b) The outlet temperature of the hot fluid for counter-flow configuration is

$$\varepsilon_{\text{CF}} = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}} = \frac{200 - T_{h,\text{out}}}{200 - 10} = 0.50601$$

$$T_{h,\text{out}} = 200 - 0.5060(200 - 10) = \mathbf{103.9}\text{°C} < 107\text{°C}$$

Discussion The effectiveness for the counter-flow configuration is higher than the parallel-flow configuration. This led to lower $T_{h,\text{out}}$ with counter-flow configuration. The $T_{h,\text{out}}$ for the parallel-flow configuration exceeds the maximum use temperature for polypropylene lining, while the $T_{h,\text{out}}$ for the counter-flow configuration is below that limit. Therefore, to ensure that the hot fluid temperature exits the heat exchanger at 107°C or lower, the counter-flow configuration should be used.

11-104 Cold water is heated by hot water in a heat exchanger. The net rate of heat transfer and the heat transfer surface area of the heat exchanger are to be determined.

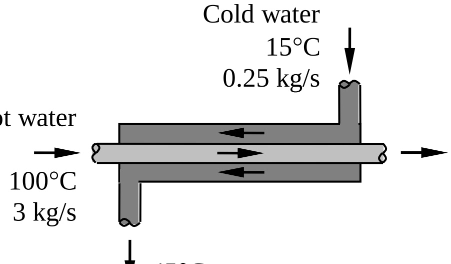
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform. **5** The thickness of the tube is negligible.

Properties The specific heats of the cold and hot water are given to be 4.18 and 4.19 kJ/kg.°C, respectively.

Analysis The heat capacity rates of the hot and cold fluids are

$$C_c = \dot{m}_c c_{pc} = (0.25\,\text{kg/s})(4180\,\text{J/kg.°C}) = 1045\,\text{W/°C}$$

$$C_h = \dot{m}_h c_{ph} = (3\,\text{kg/s})(4190\,\text{J/kg.°C}) = 12{,}570\,\text{W/°C}$$



Therefore,

$$C_{\min} = C_c = 1045\text{ W/°C}$$

and

$$c = \frac{C_{\min}}{C_{\max}} = \frac{1045}{12{,}570} = 0.083$$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (1045\text{ W/°C})(100°\text{C} - 15°\text{C}) = 88{,}825\text{ W}$$

The actual rate of heat transfer is

$$\dot{Q} = C_c(T_{c,out} - T_{c,in}) = (1045\,\text{W/°C})(45°\text{C} - 15°\text{C}) = \mathbf{31{,}350\,W}$$

Then the effectiveness of this heat exchanger becomes

$$\varepsilon = \frac{Q}{Q_{\max}} = \frac{31{,}350}{88{,}825} = 0.35$$

The NTU of this heat exchanger is determined using the relation in Table 11-5 to be

$$NTU = \frac{1}{c-1}\ln\left(\frac{\varepsilon - 1}{\varepsilon c - 1}\right) = \frac{1}{0.083-1}\ln\left(\frac{0.35-1}{0.35\times 0.083-1}\right) = 0.438$$

Then the surface area of the heat exchanger is determined from

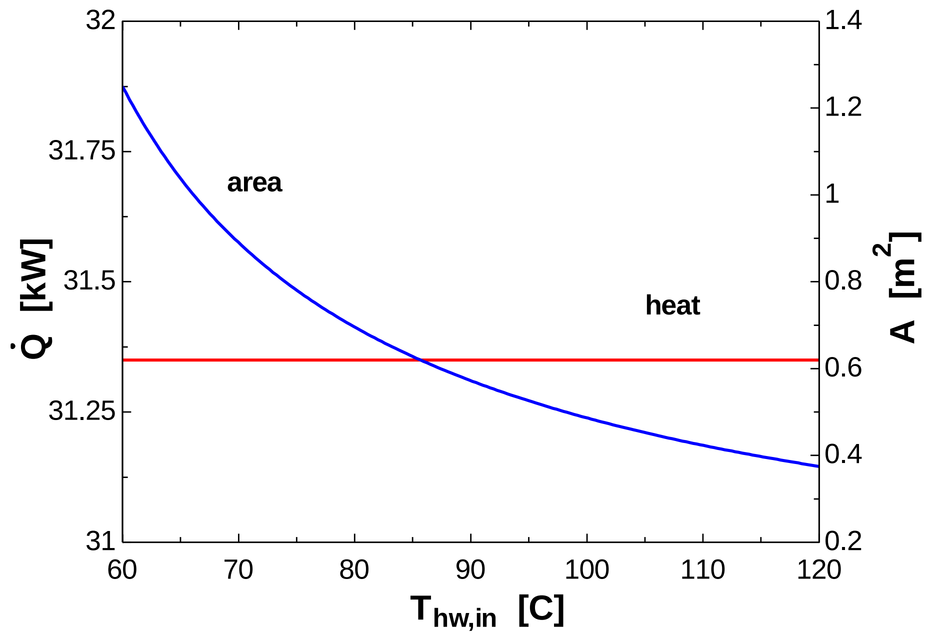
$$NTU = \frac{UA}{C_{\min}} \longrightarrow A = \frac{NTU\ C_{\min}}{U} = \frac{(0.438)(1045\text{ W/°C})}{950\text{ W/m}^2\text{.°C}} = \mathbf{0.482\,m^2}$$

11-105 Prob. 11-104 is reconsidered. The effects of the inlet temperature of hot water and the heat transfer coefficient on the rate of heat transfer and the surface area are to be investigated.

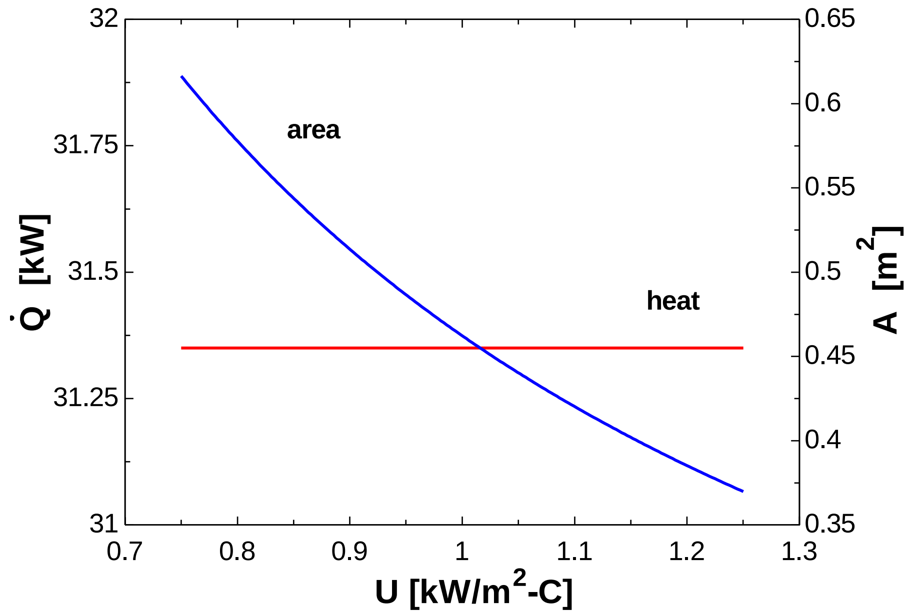
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_cw_in=15 [C]
T_cw_out=45 [C]
m_dot_cw=0.25 [kg/s]
c_p_cw=4.18 [kJ/kg-C]
T_hw_in=100 [C]
m_dot_hw=3 [kg/s]
c_p_hw=4.19 [kJ/kg-C]
U=0.95 [kW/m^2-C]

"ANALYSIS"
"With EES, it is easier to solve this problem using LMTD method than NTU method. Below, we use LMTD method. Both methods give the same results."
DELTAT_1=T_hw_in-T_cw_out
DELTAT_2=T_hw_out-T_cw_in
DELTAT_lm=(DELTAT_1-DELTAT_2)/ln(DELTAT_1/DELTAT_2)
Q_dot=U*A*DELTAT_lm
Q_dot=m_dot_hw*c_p_hw*(T_hw_in-T_hw_out)
Q_dot=m_dot_cw*c_p_cw*(T_cw_out-T_cw_in)
```

$T_{hw,\,in}$ [C]	$\dot{Q}$ [kW]	A [m^2]
60	31.35	1.25
65	31.35	1.038
70	31.35	0.8903
75	31.35	0.7807
80	31.35	0.6957
85	31.35	0.6279
90	31.35	0.5723
95	31.35	0.5259
100	31.35	0.4865
105	31.35	0.4527
110	31.35	0.4234
115	31.35	0.3976
120	31.35	0.3748



U [kW/m^2-C]	$\dot{Q}$ [kW]	A [m^2]
0.75	31.35	0.6163
0.8	31.35	0.5778
0.85	31.35	0.5438
0.9	31.35	0.5136
0.95	31.35	0.4865
1	31.35	0.4622
1.05	31.35	0.4402
1.1	31.35	0.4202
1.15	31.35	0.4019
1.2	31.35	0.3852
1.25	31.35	0.3698



11-106 C&S A double-pipe counter-flow heat exchanger is used to cool a hot fluid such that when it flows into a pipe system it is below the temperature limit for polypropylene pipes, 99°C. The heat transfer surface area of the heat exchanger is to be determined.

Assumptions **1** Steady state conditions. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Properties of fluids are constant.

Analysis The overall heat transfer coefficient for the heat exchanger is

$$U = \left(\frac{1}{h_i} + \frac{1}{h_o} + R_f\right)^{-1} = \left(\frac{1}{1500\ \text{W/m}^2\cdot\text{K}} + \frac{1}{1000\ \text{W/m}^2\cdot\text{K}} + 0.0001\ \text{m}^2\cdot\text{K/W}\right)^{-1} = 566\ \text{W/m}^2\cdot\text{K}$$

The heat capacity rates for the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = (0.5\ \text{kg/s})(3800\ \text{J/kg}\cdot\text{K}) = 1900\ \text{W/K} = C_{\min}$$

$$C_c = \dot{m}_c c_{pc} = (0.75\ \text{kg/s})(4200\ \text{J/kg}\cdot\text{K}) = 3150\ \text{W/K}$$

The capacity ratio is

$$c = \frac{C_{min}}{C_{max}} = \frac{C_h}{C_c} = \frac{1900\ \text{W/K}}{3150\ \text{W/K}} = 0.6032$$

The maximum possible heat transfer rate for the heat exchanger is

$$\dot{Q}_{\max} = C_{\min}(T_{h,\text{in}} - T_{c,\text{in}}) = (1900\ \text{W/K})(150 - 10)\text{K} = 266000\ \text{W}$$

With the outlet temperature for the hot fluid at 99°C, the effectiveness of the heat exchanger becomes

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{h,\text{in}} - T_{c,\text{in}}} = \frac{150 - 99}{150 - 10} = 0.3643$$

The effectiveness–NTU relation for counter-flow heat exchanger is

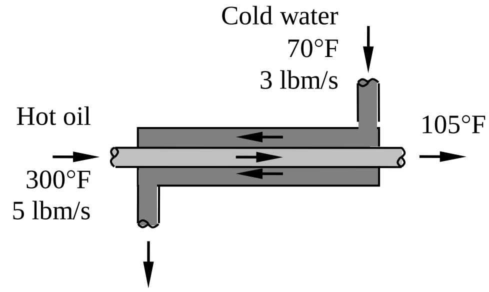
$$\text{NTU} = \frac{1}{c-1}\ln\left(\frac{\varepsilon - 1}{\varepsilon c - 1}\right) = \frac{1}{0.6032 - 1}\ln\left[\frac{0.3643 - 1}{(0.3643)(0.6032) - 1}\right] = 0.5164$$

Thus, the heat transfer surface area of the heat exchanger is determined from the NTU,

$$A_s = \text{NTU}\frac{C_{\min}}{U} = (0.5164)\frac{1900\ \text{W/K}}{566\ \text{W/m}^2\cdot\text{K}} = \mathbf{1.73\ m^2}$$

Discussion To ensure that the hot fluid temperature exits the heat exchanger at 99°C or lower, the heat exchanger needs to have a heat transfer surface area of at least 1.73 m^2. If the heat transfer surface area is below 1.73 m^2, the outlet temperature for the hot fluid would exceed the recommended maximum temperature for polypropylene pipes.

11-107E Oil is cooled by water in a double-pipe heat exchanger. The overall heat transfer coefficient of this heat exchanger is to be determined using both the LMTD and NTU methods.



Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The thickness of the tube is negligible since it is thin-walled.

Properties The specific heats of the water and oil are given to be 1.0 and 0.525 Btu/lbm.°F, respectively.

Analysis (*a*) The rate of heat transfer is

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,in-}T_{h,out}) = (5\,\text{lbm/s})(0.525\,\text{Btu/lbm.°F})(300-105°\text{F}) = 511.9\,\text{Btu/s}$$

The outlet temperature of the cold fluid is

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{\dot{m}_c c_{pc}} = 70°\text{F} + \frac{511.9\,\text{Btu/s}}{(3\,\text{lbm/s})(1.0\,\text{Btu/lbm.°F})} = 240.6°\text{F}$$

The temperature differences between the two fluids at the two ends of the heat exchanger are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 300°\text{F} - 240.6°\text{F} = 59.4°\text{F}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 105°\text{F} - 70°\text{F} = 35°\text{F}$$

The logarithmic mean temperature difference is

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{59.4-35}{\ln(59.4/35)} = 46.1°\text{F}$$

Then the overall heat transfer coefficient becomes

$$\dot{Q} = UA_s\Delta T_{lm} \longrightarrow U = \frac{\dot{Q}}{A_s\Delta T_{lm}} = \frac{511.9\,\text{Btu/s}}{\pi(5/12\,\text{m})(200\,\text{ft})(46.1°\text{F})} = \mathbf{0.0424\,Btu/s.ft^2.°F}$$

(*b*) The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = (5\,\text{lbm/s})(0.525\,\text{Btu/lbm.°F}) = 2.625\,\text{Btu/s.°F}$$

$$C_c = \dot{m}_c c_{pc} = (3\,\text{lbm/s})(1.0\,\text{Btu/lbm.°F}) = 3.0\,\text{Btu/s.°F}$$

Therefore, $C_{\min} = C_h = 2.625\,\text{Btu/s.°F}$ and $c = \dfrac{C_{\min}}{C_{\max}} = \dfrac{2.625}{3.0} = 0.875$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (2.625\,\text{Btu/s.°F})(300°\text{F - }70°\text{F}) = 603.75\,\text{Btu/s}$$

The actual rate of heat transfer and the effectiveness are

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) = (2.625\,\text{Btu/s.°F})(300°\text{F -}105°\text{F}) = 511.9\,\text{Btu/s}$$

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{511.9}{603.75} = 0.85$$

The NTU of this heat exchanger is determined using the relation in Table 11-5 to be

$$NTU = \frac{1}{c-1}\ln\left(\frac{\varepsilon-1}{\varepsilon c-1}\right) = \frac{1}{0.875-1}\ln\left(\frac{0.85-1}{0.85\times0.875-1}\right) = 4.28$$

The heat transfer surface area of the heat exchanger is

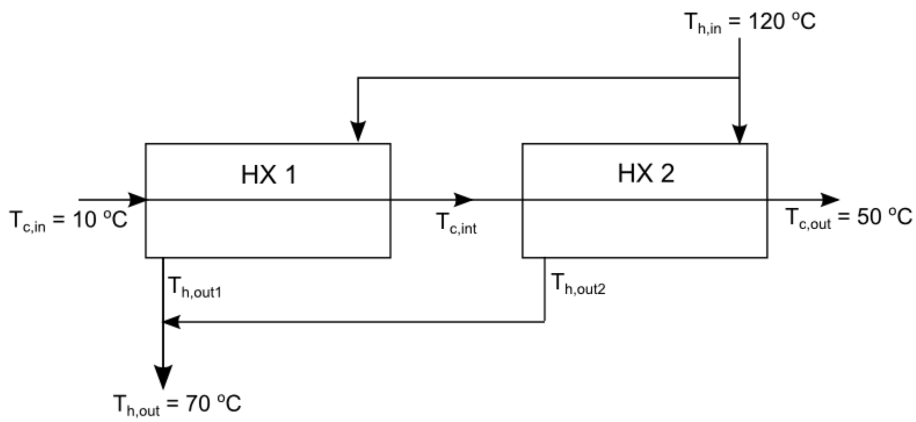
$$A_s = \pi DL = \pi(5/12\,\text{ft})(200\,\text{ft}) = 261.8\,\text{ft}^2$$

and $$NTU = \frac{UA_s}{C_{\min}} \longrightarrow U = \frac{NTU\,C_{\min}}{A_s} = \frac{(4.28)(2.625\,\text{Btu/s.°F})}{261.8\,\text{ft}^2} = \mathbf{0.0429\,Btu/s.ft^2.°F}$$

11-108 For the given set of flow conditions, the effectiveness, NTU and surface area of a large heat exchanger and its proposed replacement of two small heat exchangers arranged in series is to be determined. Based on the surface area and construction cost, economic choice of the heat exchanger is to be made.

Assumptions **1** Steady state conditions exist. **2** Fluid properties remain constant. **3** Heat exchanger is well insulated. **4** Negligible fouling resistance.

Properties Specific heat of glycerin is calculated at an inlet and exit average temperature of (10 + 50)°C/2 = 30°C from Table A-13. c_{pc} = 2447 J/kg·K.



Analysis Let's first consider the **large single heat exchanger**. The energy balance within the heat exchanger gives

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

$$\therefore \dot{m}_h c_{ph} \times (120 - 70)^{\circ}\text{C} = (1.5\,\text{kg/s})(2447\ \text{J/kg}\cdot\text{K})\,(50 - 10)^{\circ}\text{C} = 146.82\ \text{kW}$$

Thus the heat capacity rate of the hot fluid is

$$C_h = \dot{m}_h c_{ph} = 2936.4\ \text{W/K}$$

And that for the cold fluid is

$$C_c = \dot{m}_c c_{pc} = (1.5\,\text{kg/s})(2447\ \text{J/kg}\cdot\text{K}) = 3670.5\ \text{W/K}.$$

Thus

$$C_{min} = C_h.$$

Thus the effectiveness of the large single heat exchanger is,

(a) $$\varepsilon = \frac{\dot{Q}}{Q_{\max}} = \frac{C_c(T_{c,out} - T_{c,in})}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{(3670.5\text{W/K})(50-10)^{\circ}\text{C}}{(2936.4\text{W/K})(120-10)^{\circ}\text{C}} = \mathbf{0.455}$$

The capacity ratio of the large single heat exchanger is,

$$c = \frac{C_{\min}}{C_{\max}} = \frac{2936.4\,\text{W/K}}{3670.5\,\text{W/K}} = 0.8$$

The number of transfer units (NTU) for single heat exchanger is determined from the counter flow relation in Table 11-5.

(b) $$NTU = \frac{1}{c-1}\ln\left(\frac{\varepsilon - 1}{\varepsilon c - 1}\right) = \frac{1}{0.8-1}\ln\left(\frac{0.455-1}{0.455\times 0.8 - 1}\right) = \mathbf{0.77}$$

The heat transfer is area is now calculated as,

(c) $$A_s = \frac{NTU\ C_{\min}}{U} = \frac{(0.77)(2936.4\,\text{W/K})}{950\,\text{W/m}^2\cdot\text{K}} = \mathbf{2.38}\ \text{m}^2$$

Now let's consider the **two small heat exchangers** connected in series.

Given that the flow of water is split between the two heat exchangers such that 60% would go to the first heat exchanger and the remaining 40% would go to the second heat exchanger. The heat capacity rate of the 1st heat exchanger is C_{h1} = 0.6 (2936.4) = 1761.8 W/K while that of the 2nd heat exchanger is C_{h2} = 0.4 (2936.4) = 1174.6 W/K . The heat capacity rate of the cold side remains unchanged i.e. 3670.5 W/K.

The capacity ratio of the 1st heat exchangers is

$$c_1 = \frac{C_{\min}}{C_{\max}} = \frac{1761.8}{3670.5} = 0.48$$

The capacity ratio of the 2nd heat exchangers is

$$c_2 = \frac{C_{\min}}{C_{\max}} = \frac{1174.6}{3670.5} = 0.32$$

The effectiveness of 1st heat exchanger is,

$$\varepsilon_1 = \frac{\dot{Q}}{Q_{\max}} = \frac{C_c(T_{c,out} - T_{c,in})}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{(3670.5\,\text{W/K})(T_{c,\text{int}} - 10)^\circ\text{C}}{(1761.8\,\text{W/K})(120 - 10)^\circ\text{C}} = 0.0189(T_{c,\text{int}} - 10) \quad (1)$$

The effectiveness of the 2nd heat exchanger is,

$$\varepsilon_2 = \frac{\dot{Q}}{Q_{\max}} = \frac{C_c(T_{c,out} - T_{c,\text{int}})}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{(3670.5\,\text{W/K})(50 - T_{c,\text{int}})^\circ\text{C}}{(1174.6\,\text{W/K})(120 - T_{c,\text{int}})^\circ\text{C}} = 3.125\frac{(50 - T_{c,\text{int}})}{(120 - T_{c,\text{int}})} \quad (2)$$

Since the total water flow rate is split into 60% in the first heat exchanger and 40% in the second heat exchanger, the average exit temperature of the hot water from an energy balance is,

$$\dot{m}_{h,out1}c_{ph}\ T_{h,out1} + \dot{m}_{h,out1}c_{ph}\ T_{h,out1} = \dot{m}_{h,out}c_{ph}\ T_{h,out}$$

where c_{ph} is constant evaluated at the average inlet and outlet temperature of the hot stream

$$0.6\dot{m}_{h,out}c_{ph}\ T_{h,out1} + 0.4\dot{m}_{h,out}c_{ph}\ T_{h,out1} = \dot{m}_{h,out}c_{ph}\ T_{h,out}$$

$$\therefore\quad 0.6T_{h,out1} + 0.4T_{h,out2} = 70^o C$$

Further, the energy balance on the 1st heat exchanger gives

$$\dot{m}_h c_{ph,1}(T_{h,in} - T_{h,out1}) = \dot{m}_c c_{pc}(T_{c,\text{int}} - T_{c,in})$$
$$\therefore (1761.8\,\text{W/K})(120 - T_{h,out1})^\circ\text{C} = (3670.5\,\text{W/K})(T_{c,\text{int}} - 10)^\circ\text{C} \quad (3)$$

For 2nd heat exchanger, the energy balance is,

$$\dot{m}_h C_{ph,1}(T_{h,in} - T_{h,out2}) = \dot{m}_c C_{pc}(T_{c,out} - T_{c,\text{int}})$$
$$\therefore (1174.6\,\text{W/K})(120 - T_{h,out2}) = (3670.5\,\text{W/K})(50 - T_{c,\text{int}}) \quad (4)$$

The number of transfer units for 1st heat exchanger is

$$NTU_1 = \frac{1}{c_1 - 1}\ln\left(\frac{\varepsilon_1 - 1}{c_1\varepsilon_1 - 1}\right) \quad (5)$$

The number of transfer units for 2nd heat exchanger is

$$NTU_2 = \frac{1}{c_2 - 1}\ln\left(\frac{\varepsilon_2 - 1}{c_2\varepsilon_2 - 1}\right) \quad (6)$$

Further, since the surface area of each heat exchanger is same i.e., $A_{s1} = A_{s2}$ we get,

$$\frac{NTU_1 C_{ph,1}}{U} = \frac{NTU_2 C_{ph,2}}{U} \quad (7)$$

Solving Equations (1) to (7) simultaneously in EES or any other software the exit temperature of hot fluid in each heat exchanger and the intermediate temperature of the cold fluid is,

$$T_{h,out1} = 70.82\ {}^oC, T_{h,out2} = 68.77\ {}^oC, T_{c,\text{int}} = 33.61\ {}^oC$$

(a) The effectiveness of 1st and 2nd heat exchanger is **0.447** and **0.593**, respectively.

(b) The number of transfer units (NTU) of 1st and 2nd heat exchanger is **0.675** and **1.012**, respectively.

(c) The surface area of each small heat exchanger is **1.252 m^2**.

(d) The heat exchanger surface area of 1.252 m^2 is for one heat exchanger. Thus for two heat exchangers arranged in series the total surface area is 2.5 m^2. This area is about 5% higher than that of a single large heat exchanger. Moreover, the construction cost of a small heat exchanger is about 15% higher than a large heat exchanger per unit surface area. This translates to about 2% increase in the construction cost. Hence, it is recommended to use one large heat exchanger instead of two heat exchangers arranged in series.

11-109E A 1-shell and 2-tube type heat exchanger has a specified overall heat transfer coefficient, (*a*) the heat transfer effectiveness and (*b*) the actual heat transfer rate in the heat exchanger are to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.

Analysis (*a*) The heat capacity rates are given as

$$C_{\min} = 20{,}000\ \text{Btu/hr}\cdot{}^\circ\text{F} \qquad \text{and} \qquad C_{\max} = 40{,}000\ \text{Btu/hr}\cdot{}^\circ\text{F}$$

The capacity ratio is

$$c = \frac{C_{\min}}{C_{\max}} = \frac{20{,}000\ \text{Btu/hr}\cdot{}^\circ\text{F}}{40{,}000\ \text{Btu/hr}\cdot{}^\circ\text{F}} = 0.5$$

The NTU of the heat exchanger is

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{(300\ \text{Btu/hr}\cdot\text{ft}^2\cdot{}^\circ\text{F})(100\ \text{ft}^2)}{20{,}000\ \text{Btu/hr}\cdot{}^\circ\text{F}} = 1.5$$

Using the equation listed in Table 11-4, the heat transfer effectiveness is

$$\varepsilon = 2\left\{1 + c + \sqrt{1+c^2}\,\frac{1+\exp[-\text{NTU}\sqrt{1+c^2}\,]}{1-\exp[-\text{NTU}\sqrt{1+c^2}\,]}\right\}^{-1}$$

$$= 2\left\{1 + 0.5 + \sqrt{1+0.5^2}\,\frac{1+\exp[-1.5\sqrt{1+0.5^2}\,]}{1-\exp[-1.5\sqrt{1+0.5^2}\,]}\right\}^{-1} = \mathbf{0.639 = 63.9\%}$$

(*b*) The maximum possible heat transfer rate is

$$\dot{Q}_{\max} = C_{\min}(T_{h,\text{in}} - T_{c,\text{in}}) = (20{,}000\ \text{Btu/hr}\cdot{}^\circ\text{F})(200-90)^\circ\text{F} = 2.20\times10^6\ \text{Btu/hr}$$

Hence, the actual heat transfer rate in the heat exchanger is

$$\dot{Q} = \varepsilon\dot{Q}_{\max} = (0.639)(2.20\times10^6\ \text{Btu/hr}) = \mathbf{1.41\times10^6\ Btu/hr}$$

Discussion Using Figure 11-27*c*, the heat transfer effectiveness is verified to be $\varepsilon \approx 64\%$.

11-110 Cold water is being heated in a 1-shell and 2-tube heat exchanger, the outlet temperatures of the cold water and hot water are to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heats of the cold water and hot water are given to be c_{pc} = 4178 J/kg·K and c_{ph} = 4188 J/kg·K, respectively.

Analysis The heat capacity rates are

$$C_c = \dot{m}_c c_{pc} = (5000\text{ kg/h})(1/3600\text{ h/s})(4178\text{ J/kg}\cdot\text{K}) = 5802.8\text{ W/K}$$

$$C_h = \dot{m}_h c_{ph} = (10{,}000\text{ kg/h})(1/3600\text{ h/s})(4188\text{ J/kg}\cdot\text{K}) = 11{,}633\text{ W/K}$$

The capacity ratio is

$$c = \frac{C_{\min}}{C_{\max}} = \frac{C_c}{C_h} = \frac{5802.8\text{ W/K}}{11{,}633\text{ W/K}} = 0.499$$

The NTU of the heat exchanger is

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{11{,}600\text{ W/K}}{5802.8\text{ W/K}} = 1.999$$

Using the equation listed in Table 11-4, the heat transfer effectiveness is

$$\varepsilon = 2\left\{1 + c + \sqrt{1+c^2}\,\frac{1+\exp[-\text{NTU}\sqrt{1+c^2}\,]}{1-\exp[-\text{NTU}\sqrt{1+c^2}\,]}\right\}^{-1}$$

$$= 2\left\{1 + 0.499 + \sqrt{1+0.499^2}\,\frac{1+\exp[-1.999\sqrt{1+0.499^2}\,]}{1-\exp[-1.999\sqrt{1+0.499^2}\,]}\right\}^{-1} = 0.6933$$

The outlet temperature of the cold water is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c(T_{c,\text{out}} - T_{c,\text{in}})}{C_{\min}(T_{h,\text{in}} - T_{c,\text{in}})} = \frac{C_c(T_{c,\text{out}} - T_{c,\text{in}})}{C_c(T_{h,\text{in}} - T_{c,\text{in}})} \quad \longrightarrow \quad T_{c,\text{out}} = \varepsilon(T_{h,\text{in}} - T_{c,\text{in}}) + T_{c,\text{in}}$$

$$T_{c,\text{out}} = \varepsilon(T_{h,\text{in}} - T_{c,\text{in}}) + T_{c,\text{in}} = (0.6933)(80-20)°\text{C} + 20°\text{C} = \mathbf{61.9°C}$$

The outlet temperature of the hot water is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h(T_{h,\text{in}} - T_{h,\text{out}})}{C_{\min}(T_{h,\text{in}} - T_{c,\text{in}})} = \frac{C_h(T_{h,\text{in}} - T_{h,\text{out}})}{C_c(T_{h,\text{in}} - T_{c,\text{in}})}$$

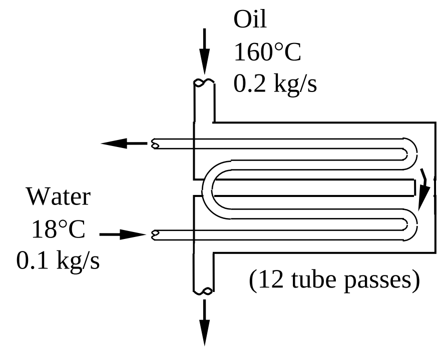
$$T_{h,\text{out}} = T_{h,\text{in}} - \frac{C_c}{C_h}\varepsilon(T_{h,\text{in}} - T_{c,\text{in}}) = 80°\text{C} - (0.499)(0.6933)(80-20)°\text{C} = \mathbf{59.2°C}$$

Discussion Using Figure 11-27*c*, the heat transfer effectiveness is approximately $\varepsilon \approx 69\%$.

11-111 Hot oil is to be cooled by water in a heat exchanger. The mass flow rates and the inlet temperatures are given. The rate of heat transfer and the outlet temperatures are to be determined. √

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The thickness of the tube is negligible since it is thin-walled. **5** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and oil are given to be 4.18 and 2.2 kJ/kg.°C, respectively.



Analysis The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = (0.2\,\text{kg/s})(2200\,\text{J/kg.°C}) = 440\text{ W/°C}$$

$$C_c = \dot{m}_c c_{pc} = (0.1\,\text{kg/s})(4180\,\text{J/kg.°C}) = 418\text{ W/°C}$$

Therefore,

$$C_{\min} = C_c = 418\text{ W/°C}$$

and

$$c = \frac{C_{\min}}{C_{\max}} = \frac{418}{440} = 0.95$$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (418\text{ W/°C})(160\text{°C} - 18\text{°C}) = 59.36\,\text{kW}$$

The heat transfer surface area is

$$A_s = n(\pi DL) = (12)(\pi)(0.018\,\text{m})(3\,\text{m}) = 2.04\,\text{m}^2$$

The NTU of this heat exchanger is

$$NTU = \frac{UA_s}{C_{\min}} = \frac{(340\text{ W/m}^2.\text{°C})\,(2.04\,\text{m}^2)}{418\text{ W/°C}} = 1.659$$

Then the effectiveness of this heat exchanger corresponding to c = 0.95 and NTU = 1.659 is determined from Fig. 11-27d to be

$$\varepsilon = 0.61$$

Then the actual rate of heat transfer becomes

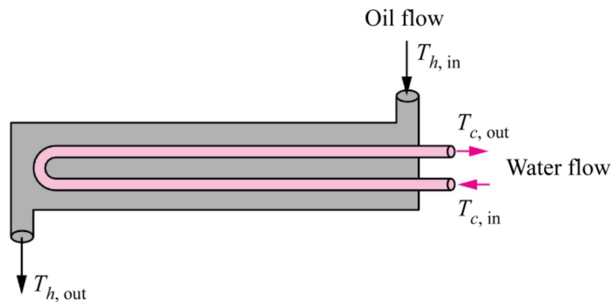
$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.61)(59.36\,\text{kW}) = \mathbf{36.2\,kW}$$

Finally, the outlet temperatures of the cold and hot fluid streams are determined to be

$$\dot{Q} = C_c(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{C_c} = 18\text{°C} + \frac{36.2\,\text{kW}}{0.418\,\text{kW / °C}} = \mathbf{104.6°C}$$

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) \longrightarrow T_{h,out} = T_{h,in} - \frac{\dot{Q}}{C_h} = 160\text{°C} - \frac{36.2\,\text{kW}}{0.44\,\text{kW/°C}} = \mathbf{77.7°C}$$

11-112E A 1-shell and 2-tube heat exchanger has specified overall heat transfer coefficient, inlet and outlet temperatures, and mass flow rates, (*a*) the NTU value and (*b*) the surface area of the heat exchanger are to be determined.



Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heat of water is given to be c_{pc} = 1.0 Btu/lbm·°F.

Analysis (*a*) The heat capacity rate for the cold fluid (water) is

$$C_c = \dot{m}_c c_{pc} = (20{,}000 \text{ lbm/hr})(1.0 \text{ Btu/lbm}\cdot°\text{F}) = 20{,}000 \text{ Btu/hr}\cdot°\text{F}$$

Using energy balance, we have

$$C_c(T_{c,\text{out}} - T_{c,\text{in}}) = C_h(T_{h,\text{in}} - T_{h,\text{out}}) \qquad \rightarrow \qquad \frac{C_c}{C_h} = \frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{c,\text{out}} - T_{c,\text{in}}} = \frac{180-120}{100-80} = 3.0$$

or

$$c = \frac{C_h}{C_c} = \frac{C_{\min}}{C_{\max}} = \frac{1}{3} = 0.3333$$

The heat transfer effectiveness is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c(T_{c,\text{out}} - T_{c,\text{in}})}{C_{\min}(T_{h,\text{in}} - T_{c,\text{in}})} = \frac{C_c(T_{c,\text{out}} - T_{c,\text{in}})}{C_h(T_{h,\text{in}} - T_{c,\text{in}})} = (3.0)\frac{100-80}{180-80} = 0.60$$

From Table 11-4, the NTU value can be determined from

$$\varepsilon = 2\left\{1 + c + \sqrt{1+c^2}\,\frac{1+\exp[-\text{NTU}\sqrt{1+c^2}\,]}{1-\exp[-\text{NTU}\sqrt{1+c^2}\,]}\right\}^{-1}$$

Copy the following lines and paste on a blank EES screen to solve the above equation:

```
c=1/3
epsilon=0.60
epsilon=2*(1+c+sqrt(1+c^2)*(1+exp(-NTU*sqrt(1+c^2)))/(1-exp(-NTU*sqrt(1+c^2))))^(-1)
```

Solving by EES software, we get

$$\text{NTU} = \mathbf{1.11}$$

(*b*) The surface area of the heat exchanger can be determined using

$$\text{NTU} = \frac{UA_s}{C_{\min}} \qquad \rightarrow \qquad A_s = \text{NTU}\frac{C_{\min}}{U} = \text{NTU}\frac{cC_c}{U}$$

$$A_s = \text{NTU}\frac{cC_c}{U} = (1.11)\frac{(1/3)(20{,}000 \text{ Btu/hr}\cdot°\text{F})}{40 \text{ Btu/hr}\cdot\text{ft}^2\cdot°\text{F}} = \mathbf{185\ ft^2}$$

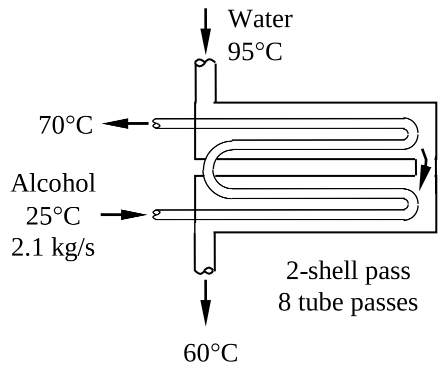
Discussion Using Figure 11-27*c*, the NTU value is found to be approximately NTU ≈ 1.2.

11-113 Ethyl alcohol is heated by water in a shell-and-tube heat exchanger. The heat transfer surface area of the heat exchanger is to be determined using both the LMTD and NTU methods.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the ethyl alcohol and water are given to be 2.67 and 4.19 kJ/kg.°C, respectively.

Analysis (*a*) The temperature differences between the two fluids at the two ends of the heat exchanger are



$$\Delta T_1 = T_{h,in} - T_{c,out} = 95°\text{C} - 70°\text{C} = 25°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 60°\text{C} - 25°\text{C} = 35°\text{C}$$

The logarithmic mean temperature difference and the correction factor are

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{25-35}{\ln(25/35)} = 29.7°\text{C}$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{70-25}{95-25} = 0.64 \\ R &= \frac{T_2 - T_1}{t_1 - t_1} = \frac{95-60}{70-25} = 0.78 \end{aligned}\right\} F = 0.93$$

The rate of heat transfer is determined from

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in}) = (2.1\,\text{kg/s})(2.67\,\text{kJ/kg.°C})(70°\text{C} - 25°\text{C}) = 252.3\,\text{kW}$$

The surface area of heat transfer is

$$\dot{Q} = UA_s\Delta T_{lm} \longrightarrow A_s = \frac{\dot{Q}}{UF\Delta T_{lm}} = \frac{252.3\,\text{kW}}{0.8\,\text{kW/m}^2\text{.°C})(0.93)(29.7°\text{C})} = \mathbf{11.4\,m^2}$$

(*b*) The rate of heat transfer is

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in}) = (2.1\,\text{kg/s})(2.67\,\text{kJ/kg.°C})(70°\text{C} - 25°\text{C}) = 252.3\,\text{kW}$$

The mass flow rate of the hot fluid is

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) \rightarrow \dot{m}_h = \frac{\dot{Q}}{c_{ph}(T_{h,in} - T_{h,out})} = \frac{252.3\,\text{kW}}{(4.19\,\text{kJ/kg.°C})(95°\text{C} - 60°\text{C})} = 1.72\,\text{kg/s}$$

The heat capacity rates of the hot and the cold fluids are

$$C_h = \dot{m}_h c_{ph} = (1.72\,\text{kg/s})(4.19\,\text{kJ/kg.°C}) = 7.21\,\text{kW/°C}$$

$$C_c = \dot{m}_c c_{pc} = (2.1\,\text{kg/s})(2.67\,\text{kJ/kg.°C}) = 5.61\,\text{kW/°C}$$

Therefore, $C_{\min} = C_c = 5.61\,\text{W/°C}$ and $c = \dfrac{C_{\min}}{C_{\max}} = \dfrac{5.61}{7.21} = 0.78$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (5.61\,\text{W/°C})(95°\text{C} - 25°\text{C}) = 392.7\,\text{kW}$$

The effectiveness of this heat exchanger is $\varepsilon = \dfrac{Q}{Q_{\max}} = \dfrac{252.3}{392.7} = 0.64$

The NTU of this heat exchanger corresponding to this emissivity and c = 0.78 is determined from 11-27d to be NTU = 1.7. Then the surface area of heat exchanger is determined to be

$$NTU = \frac{UA_s}{C_{\min}} \longrightarrow A_s = \frac{NTU\ C_{\min}}{U} = \frac{(1.7)(5.61\,\text{kW/°C})}{0.8\,\text{kW/m}^2\text{.°C}} = \mathbf{11.9\,m^2}$$

The small difference between the two results is due to the reading error of the chart.

11-114 Cold water is heated by hot oil in a shell-and-tube heat exchanger. The rate of heat transfer is to be determined using both the LMTD and NTU methods.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and oil are given to be 4.18 and 2.2 kJ/kg.°C, respectively.

Analysis (*a*) The LMTD method in this case involves iterations, which involves the following steps:

1) Choose $T_{h,out}$

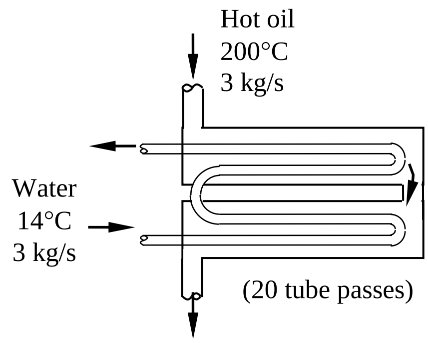
2) Calculate $\dot{Q}$ from $\dot{Q} = \dot{m}_h c_p (T_{h,out} - T_{h,in})$

3) Calculate $T_{h,out}$ from $\dot{Q} = \dot{m}_h c_p (T_{h,out} - T_{h,in})$

4) Calculate $\Delta T_{lm,CF}$

5) Calculate $\dot{Q}$ from $\dot{Q} = UA_s F \Delta T_{lm,CF}$

6) Compare to the $\dot{Q}$ calculated at step 2, and repeat until reaching the same result



Result: **651 kW**

(*b*) The heat capacity rates of the hot and the cold fluids are

$$C_h = \dot{m}_h c_{ph} = (3\,\text{kg/s})(2.2\text{kJ/kg.°C}) = 6.6\,\text{kW/°C}$$

$$C_c = \dot{m}_c c_{pc} = (3\,\text{kg/s})(4.18\text{kJ/kg.°C}) = 12.54\,\text{kW/°C}$$

Therefore,

$$C_{\min} = C_h = 6.6\,\text{kW/°C}$$

and

$$c = \frac{C_{\min}}{C_{\max}} = \frac{6.6}{12.54} = 0.53$$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (6.6\,\text{kW/°C})(200\text{°C} - 14\text{°C}) = 1228\,\text{kW}$$

The NTU of this heat exchanger is

$$NTU = \frac{UA_s}{C_{\min}} = \frac{(0.3\,\text{kW/m}^2.\text{°C})\,(20\,\text{m}^2)}{6.6\,\text{kW/°C}} = 0.91$$

Then the effectiveness of this heat exchanger corresponding to c = 0.53 and NTU = 0.91 is determined from Fig. 11-27d to be

$$\varepsilon = 0.53$$

The actual rate of heat transfer then becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.53)(1228\ \text{kW}) = \mathbf{651 kW}$$

11-115 C&S A shell-and-tube heat exchanger is used to heat water for a commercial warewashing equipment by geothermal brine. The number of passes for the tubes inside the shell is to be determined so that the heated water is within the temperature range required by the ANIS/NSF 3 standard.

Assumptions **1** Steady state conditions. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Properties of fluids are constant.

Analysis The overall heat transfer coefficient for the heat exchanger is

$$U = \left(\frac{1}{h_i} + \frac{1}{h_o} + R_f\right)^{-1} = \left(\frac{1}{2700\ \text{W/m}^2\cdot\text{K}} + \frac{1}{1050\ \text{W/m}^2\cdot\text{K}} + 0.0002\ \text{m}^2\cdot\text{K/W}\right)^{-1} = 656.71\ \text{W/m}^2\cdot\text{K}$$

The heat capacity rate for the cold fluid is

$$C_c = \dot{m}_c c_{pc} = (0.5\ \text{kg/s})(4200\ \text{J/kg}\cdot\text{K}) = 2100\ \text{W/K} = C_{\min}$$

The heat transfer rate in the heat exchanger is

$$\dot{Q} = C_c\left(T_{c,\text{out}} - T_{c,\text{in}}\right)$$

$$\dot{Q} = C_h\left(T_{h,\text{in}} - T_{h,\text{out}}\right)$$

Thus, the heat capacity rate for the hot fluid is

$$C_h = C_c\frac{\left(T_{c,\text{out}} - T_{c,\text{in}}\right)}{\left(T_{h,\text{in}} - T_{h,\text{out}}\right)} = (2100\ \text{W/K})\frac{(86-48)°\text{C}}{(98-90)°\text{C}} = 9975\ \text{W/K}$$

The capacity ratio is

$$c = \frac{C_{min}}{C_{max}} = \frac{C_c}{C_h} = \frac{2100\ \text{W/K}}{9975\ \text{W/K}} = 0.21053$$

The maximum possible heat transfer rate for the heat exchanger is

$$\dot{Q}_{\max} = C_{\min}\left(T_{h,\text{in}} - T_{c,\text{in}}\right) = (2100\ \text{W/K})(98-48)\text{K} = 105000\ \text{W}$$

The heat exchanger effectiveness is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{T_{c,\text{out}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}} = \frac{86-48}{98-48} = 0.76$$

The effectiveness–NTU relation for one-shell pass heat exchanger is

$$\text{NTU} = -\frac{1}{\sqrt{1+c^2}}\ln\left(\frac{2/\varepsilon - 1 - c - \sqrt{1+c^2}}{2/\varepsilon - 1 - c + \sqrt{1+c^2}}\right)$$

$$\text{NTU} = -\frac{1}{\sqrt{1+(0.21053)^2}}\ln\left[\frac{2/0.76 - 1 - 0.21053 - \sqrt{1+(0.21053)^2}}{2/0.76 - 1 - 0.21053 + \sqrt{1+(0.21053)^2}}\right] = 1.7728$$

The heat transfer surface area is

$$A_s = \text{NTU}\frac{C_{\min}}{U} = (1.7728)\frac{2100\ \text{W/K}}{656.71\ \text{W/m}^2\cdot\text{K}} = 5.669\ \text{m}^2$$

The heat transfer surface area is determined from the no. of tubes, no. of tube passes, tube length per pass, and tube diameter:

$$A_s = \pi DL \times \text{No. tubes} \times \text{No. tube passes}$$

Thus,

$$\text{No. tube passes} = \frac{A_s}{\pi DL \times \text{No. tubes}} = \frac{5.669}{\pi\,(0.0025\ \text{m})(5\ \text{m})(4)} = 3.61 = 4\ \text{tube passes}$$

Discussion To heat water by geothermal brine from 48 to 86°C (between 82 and 90°C required by the ANIS/NSF 3 standard), the shell-and-tube heat exchanger would need four tube passes.

11-116E In a one shell pass and eight tube pass heat exchanger, water is to be heated using hot air at 600°F. For the given value of convection heat transfer coefficient on the outer surface of the tubes and the fouling resistances, the heat exchanger area is to be determined using LMTD and $\varepsilon - NTU$ methods.

Assumptions **1** Steady state conditions exist. **2** Fluid properties remain constant. **3** Heat exchanger is well insulated.

Properties Calculate the thermal properties of water at an average temperature of (150 + 70)°F/2 = 110°F from Table 9-E. c_{pc} = 0.999 Btu/1bm·°F ≈ 1 Btu/1bm·°F

Analysis The heat gained by water from hot air is,

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in}) = (50{,}000\ \text{lbm/h})\,(1\ \text{Btu/lbm}\cdot{}^\circ F)(150 - 70)^\circ\text{F} = 4\times10^6\ \text{Btu/h}$$

From energy balance we have, heat lost by air = heat gained by water.

$$\dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

$$\dot{m}_h = \frac{\dot{m}_c c_{pc}(T_{c,out} - T_{c,in})}{c_{ph}(T_{h,in} - T_{h,out})} = \frac{4\times10^6\ \text{Btu/h}}{(0.25\ \text{Btu/lbm}\cdot{}^\circ\text{F})(600 - 300)^\circ\text{F}} = 53{,}333.33\ \text{lbm/h}$$

Now the logarithmic temperature difference for a counter flow heat exchanger is calculated as

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$\Delta T_1 = T_{h,in} - T_{c,out} = 600 - 150 = 450\ ^\text{o}\text{F}$$

and

$$\Delta T_2 = T_{h,out} - T_{c,in} = 300 - 70 = 230^\text{o}\ \text{F}$$

$$\Delta T_{lm,CF} = \frac{450 - 230}{\ln(450/230)} = 327.78\ ^\text{o}\text{F}$$

In order to determine correction factor we first need to find the temperature ratios P and R as follows:

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{300 - 600}{70 - 600} = 0.566 \quad \text{and} \quad R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{70 - 150}{300 - 600} = 0.266$$

From Figure 11-19 (a), the correction factor is

F = 0.96.

The heat transfer rate is

$$\dot{Q} = UA_s F\Delta T_{lm,CF}$$

The overall heat transfer coefficient is to be calculated from the given value of convection heat transfer coefficient on the outer surface of the tubes and accounting for the possible fouling resistance on both water and air side.

$$\frac{1}{U} = \frac{1}{h} + R_{f,water} + R_{f,air}$$

$$\therefore \frac{1}{U} = \frac{1}{30\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}} + (0.0015\ \text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F/Btu}) + (0.001\ \text{h}\cdot\text{ft}^2\cdot{}^\circ\text{F/Btu}) \rightarrow \text{U} = 27.9\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F}$$

Therefore, the surface area of the heat exchanger is,

$$A_s = \frac{\dot{Q}}{UF\Delta T_{lm,CF}} = \frac{4\times10^6\ \text{Btu/h}}{(27.9\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(0.96)(327.78^\text{o}\ \text{F})} = \mathbf{455.5\ ft^2}$$

To use $\varepsilon - NTU$ method we first need to calculate the capacity ratio and effectiveness.

The heat capacity rate of the cold fluid is

$$C_c = \dot{m}_c c_{pc} = (50{,}000\ \text{lbm/h})(1\text{Btu/lbm}\cdot{}^\circ\text{F}) = 50{,}000\ \text{Btu/h}\cdot{}^\circ\text{F}$$

The heat capacity rate of the hot fluid is

$$C_h = \dot{m}_h c_{ph} = (53333.33\ \text{lbm/h})(0.25\ \text{Btu/lbm}\cdot{}^\circ\text{F}) = 13333.33\ \text{Btu/h}\cdot{}^\circ\text{F}$$

The capacity ratio is

$$c = \frac{C_{\min}}{C_{\max}} = \frac{13333.33}{50{,}000} = 0.266$$

The effectiveness of the heat exchanger is,

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h(T_{h,in} - T_{h,out})}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{(600-300)}{(600-70)} = 0.566$$

The NTU of the heat exchanger is calculated using expressions given in Table 11-5.

$$NTU = -\frac{1}{\sqrt{1+c^2}} \ln\left(\frac{2/\varepsilon - 1 - c - \sqrt{1+c^2}}{2/\varepsilon - 1 - c + \sqrt{1+c^2}}\right)$$

$$\therefore NTU = -\frac{1}{\sqrt{1+0.266^2}} \ln\left(\frac{2/0.566 - 1 - 0.266 - \sqrt{1+0.266^2}}{2/0.566 - 1 - 0.266 + \sqrt{1+0.266^2}}\right) = -\frac{1}{1.034} \ln\left(\frac{1.233}{3.301}\right) = 0.952$$

From definition of NTU we get,

$$NTU = \frac{UA_s}{C_{\min}} \rightarrow A_s = \frac{NTU \times C_{\min}}{U} = \frac{(0.952)\ (13333.33\ \text{Btu/h} \cdot {}^\circ\text{F})}{27.9\ \text{Btu/h} \cdot \text{ft}^2 \cdot {}^\circ\text{F})} = \mathbf{454.95\ ft^2}$$

Discussion The surface area of the heat exchanger calculated using LMTD and NTU methods is within 0.1% of each other.

11-117 C&S A shell-and-tube (two-shell passes) heat exchanger is used to heat water for a commercial warewashing equipment by geothermal brine. The number of tube passes inside each shell is to be determined so that the heated water is within the temperature range required by the ANIS/NSF 3 standard.

Assumptions **1** Steady state conditions. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Properties of fluids are constant.

Analysis The overall heat transfer coefficient for the heat exchanger is

$$U = \left(\frac{1}{h_i} + \frac{1}{h_o} + R_f\right)^{-1} = \left(\frac{1}{2700\ \mathrm{W/m^2{\cdot}K}} + \frac{1}{450\ \mathrm{W/m^2{\cdot}K}} + 0.0002\ \mathrm{m^2{\cdot}K/W}\right)^{-1} = 358.09\ \mathrm{W/m^2{\cdot}K}$$

The heat capacity rate for the cold fluid is

$$C_c = \dot{m}_c c_{pc} = (0.5\ \mathrm{kg/s})(4200\ \mathrm{J/kg\cdot K}) = 2100\ \mathrm{W/K} = C_{\min}$$

The heat transfer rate in the heat exchanger is

$$\dot{Q} = C_c\left(T_{c,\mathrm{out}} - T_{c,\mathrm{in}}\right)$$

$$\dot{Q} = C_h\left(T_{h,\mathrm{in}} - T_{h,\mathrm{out}}\right)$$

Thus, the heat capacity rate for the hot fluid is

$$C_h = C_c \frac{\left(T_{c,\mathrm{out}} - T_{c,\mathrm{in}}\right)}{\left(T_{h,\mathrm{in}} - T_{h,\mathrm{out}}\right)} = (2100\ \mathrm{W/K})\frac{(86-20)}{(98-90)} = 17325\ \mathrm{W/K}$$

The capacity ratio is

$$c = \frac{C_c}{C_h} = \frac{2100\ \mathrm{W/K}}{17325\mathrm{W/K}} = 0.12121$$

The maximum possible heat transfer rate for the heat exchanger is

$$\dot{Q}_{\max} = C_{\min}\left(T_{h,\mathrm{in}} - T_{c,\mathrm{in}}\right) = (2100\ \mathrm{W/K})(98-20)\mathrm{K} = 163800\ \mathrm{W}$$

The heat exchanger effectiveness for 2-shell passes is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{T_{c,\mathrm{out}} - T_{c,\mathrm{in}}}{T_{h,\mathrm{in}} - T_{c,\mathrm{in}}} = \frac{86-20}{98-20} = 0.84615$$

This follows that the effectiveness for 1-shell pass is

$$\varepsilon_1 = \left[\left(\frac{\varepsilon c - 1}{\varepsilon - 1}\right)^{1/n} - 1\right]\left[\left(\frac{\varepsilon c - 1}{\varepsilon - 1}\right)^{1/n} - c\right]^{-1}$$

$$\varepsilon_1 = \left\{\left[\frac{(0.84615)(0.12121) - 1}{0.84615 - 1}\right]^{1/2} - 1\right\}\left\{\left[\frac{(0.84615)(0.12121) - 1}{0.84615 - 1}\right]^{1/2} - 0.12121\right\}^{-1} = 0.61692$$

The effectiveness–NTU relation for 1-shell pass is

$$\mathrm{NTU}_1 = -\frac{1}{\sqrt{1+c^2}}\ln\left(\frac{2/\varepsilon_1 - 1 - c - \sqrt{1+c^2}}{2/\varepsilon_1 - 1 - c + \sqrt{1+c^2}}\right)$$

$$\mathrm{NTU}_1 = -\frac{1}{\sqrt{1+(0.12121)^2}}\ln\left[\frac{2/0.61692 - 1 - 0.12121 - \sqrt{1+(0.12121)^2}}{2/0.61692 - 1 - 0.12121 + \sqrt{1+(0.12121)^2}}\right] = 1.0255$$

The NTU for the 2-shell passes becomes

$$\text{NTU} = n\text{NTU}_1 = (2)(1.0255) = 2.051$$

The total heat transfer surface area for 2-shell passes is

$$A_s = \text{NTU}\frac{C_{\min}}{U} = (2.051)\frac{2100\ \text{W/K}}{358.09\ \text{W/m}^2\cdot\text{K}} = 12.028\ \text{m}^2$$

The heat transfer surface area is determined from the no. of shell passes, no. of tube passes, tube length per pass, and tube diameter:

$$A_s = \pi DL \times \text{No. shell passes} \times \text{No. tube passes}$$

Thus, the number of tube passes in each shell is

$$\text{No. tube passes} = \frac{A_s}{\pi DL \times \text{No. shell passes}} = \frac{12.028\ \text{m}^2}{\pi(0.025\ \text{m})(4\ \text{m})(2)} = 19.14 = \mathbf{20\ tube\ passes\ per\ shell}$$

Discussion To heat water by geothermal brine from 20 to 86°C (between 82 and 90°C required by the ANIS/NSF 3 standard), the shell-and-tube (2-shell) heat exchanger would need 20 tube passes in each shell.

11-118 A shell-and-tube (2-shell passes, 12-tube passes in each shell) heat exchanger is used to cool a hot fluid such that when it flows into a pipe system it is below the maximum use temperature for ASTM B75 copper tube, 204°C (ASME Code for Process Piping).

Assumptions **1** Steady state conditions. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Properties of fluids are constant. **4** Negligible fouling factor.

Analysis The overall heat transfer coefficient for the heat exchanger is

$$U = \left(\frac{1}{h_i} + \frac{1}{h_o}\right)^{-1} = \left(\frac{1}{3000\ \text{W/m}^2\cdot\text{K}} + \frac{1}{2300\ \text{W/m}^2\cdot\text{K}}\right)^{-1} = 1301.9\ \text{W/m}^2\cdot\text{K}$$

The heat capacity rate for the cold fluid is

$$C_c = \dot{m}_c c_{pc} = (1.4\ \text{kg/s})(3000\ \ \text{J/kg}\cdot\text{K}) = 4200\ \text{W/K} = C_{\min}$$

The heat transfer rate in the heat exchanger is

$$\dot{Q} = C_c\left(T_{c,\text{out}} - T_{c,\text{in}}\right)$$

$$\dot{Q} = C_h\left(T_{h,\text{in}} - T_{h,\text{out}}\right)$$

Thus, the heat capacity rate for the hot fluid is

$$C_h = C_c\frac{\left(T_{c,\text{out}} - T_{c,\text{in}}\right)}{\left(T_{h,\text{in}} - T_{h,\text{out}}\right)} = (4200\ \text{W/K})\frac{(200-50)}{(250-204)} = 13696\ \text{W/K}$$

The capacity ratio is

$$c = \frac{C_c}{C_h} = \frac{4200\ \text{W/K}}{13696\ \text{W/K}} = 0.30666$$

The maximum possible heat transfer rate for the heat exchanger is

$$\dot{Q}_{\max} = C_{\min}\left(T_{h,\text{in}} - T_{c,\text{in}}\right) = (4200\ \text{W/K})(250-50)\text{K} = 840000\ \text{W}$$

The heat exchanger effectiveness for 2-shell passes is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{T_{c,\text{out}} - T_{c,\text{in}}}{T_{h,\text{in}} - T_{c,\text{in}}} = \frac{200-50}{250-50} = 0.75$$

This follows that the effectiveness for 1-shell pass is

$$\varepsilon_1 = \left[\left(\frac{\varepsilon c - 1}{\varepsilon - 1}\right)^{1/n} - 1\right]\left[\left(\frac{\varepsilon c - 1}{\varepsilon - 1}\right)^{1/n} - c\right]^{-1}$$

$$\varepsilon_1 = \left\{\left[\frac{(0.75)(0.30666) - 1}{0.75 - 1}\right]^{1/2} - 1\right\}\left\{\left[\frac{(0.75)(0.30666) - 1}{0.75 - 1}\right]^{1/2} - 0.30666\right\}^{-1} = 0.52129$$

The effectiveness–NTU relation for 1-shell pass is

$$\text{NTU}_1 = -\frac{1}{\sqrt{1+c^2}}\ln\left(\frac{2/\varepsilon_1 - 1 - c - \sqrt{1+c^2}}{2/\varepsilon_1 - 1 - c + \sqrt{1+c^2}}\right)$$

$$\mathrm{NTU}_1 = -\frac{1}{\sqrt{1+(0.30666)^2}}\ln\left[\frac{2/0.52129 - 1 - 0.30666 - \sqrt{1+(0.30666)^2}}{2/0.52129 - 1 - 0.30666 + \sqrt{1+(0.30666)^2}}\right] = 0.84083$$

The NTU for the 2-shell passes becomes

$$\mathrm{NTU} = n\mathrm{NTU}_1 = (2)(0.84083) = 1.6817$$

The total heat transfer surface area for 2-shell passes is

$$A_s = \mathrm{NTU}\frac{C_{\min}}{U} = (1.6817)\frac{4200\text{ W/K}}{1301.9\text{ W/m}^2\cdot\text{K}} = 5.4253\text{ m}^2$$

The heat transfer surface area is determined from the no. of shell passes, no. of tube passes, tube length per pass, and tube diameter:

$$A_s = \pi DL \times \text{No. shell passes} \times \text{No. tube passes}$$

Thus, the tube length per pass is

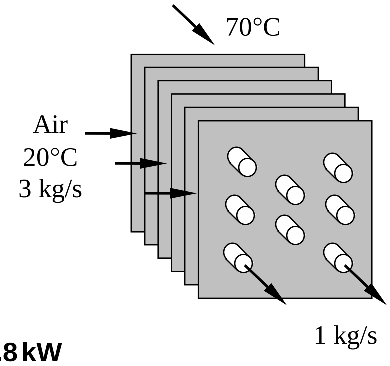
$$L = \frac{A_s}{\pi D \times \text{No. shell passes} \times \text{No. tube passes}} = \frac{5.4253\text{ m}^2}{\pi(0.025\text{ m})(2\,)(12)} = \mathbf{2.88\text{ m per tube pass}}$$

Discussion To cool the hot fluid from 250 to 204°C (the maximum use temperature for ASTM B75 copper tubes), the shell-and-tube (2-shell passes, 12-tube passes in each shell) heat exchanger would need a tube length per pass of almost 3 m.

11-119 Air is heated by a hot water stream in a cross-flow heat exchanger. The maximum heat transfer rate and the outlet temperatures of the cold and hot fluid streams are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The specific heats of water and air are given to be 4.19 and 1.005 kJ/kg.°C.



Analysis The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = (1\,\text{kg/s})(4190\,\text{J/kg.°C}) = 4190\ \text{W/°C}$$

$$C_c = \dot{m}_c c_{pc} = (3\,\text{kg/s})(1005\,\text{J/kg.°C}) = 3015\ \text{W/°C}$$

Therefore

$$C_{\min} = C_c = 3015\ \text{W/°C}$$

which is the smaller of the two heat capacity rates. Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (3015\ \text{W/°C})(70\text{°C} - 20\text{°C}) = 150{,}750\ \text{W} = \mathbf{150.8\,kW}$$

The outlet temperatures of the cold and the hot streams in this limiting case are determined to be

$$\dot{Q} = C_c(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{C_c} = 20\text{°C} + \frac{150.75\,\text{kW}}{3.015\,\text{kW/°C}} = \mathbf{70°C}$$

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) \longrightarrow T_{h,out} = T_{h,in} - \frac{\dot{Q}}{C_h} = 70\text{°C} - \frac{150.75\,\text{kW}}{4.19\,\text{kW/°C}} = \mathbf{34.0°C}$$

11-120 A cross-flow heat exchanger with both fluids unmixed has a specified overall heat transfer coefficient, and the exit temperature of the cold fluid is to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.

Analysis The heat capacity rates are given as

$$C_h = C_{\min} = 40{,}000 \text{ W/K} \quad \text{and} \quad C_c = C_{\max} = 80{,}000 \text{ W/K}$$

The capacity ratio is

$$c = \frac{C_{\min}}{C_{\max}} = \frac{C_h}{C_c} = \frac{40{,}000 \text{ W/K}}{80{,}000 \text{ W/K}} = 0.5$$

The NTU of the heat exchanger is

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{(200 \text{ W/m}^2 \cdot \text{K})(400 \text{ m}^2)}{40{,}000 \text{ W/K}} = 2.0$$

Using the equation listed in Table 11-4, the heat transfer effectiveness is

$$\varepsilon = 1 - \exp\left\{ \frac{\text{NTU}^{0.22}}{c} [\exp(-c\,\text{NTU}^{0.78}) - 1] \right\}$$

$$= 1 - \exp\left(\frac{2.0^{0.22}}{0.5} \{\exp[-(0.5)(2.0)^{0.78}] - 1\} \right) = 0.7388$$

From the definition of heat transfer effectiveness,

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c (T_{c,\text{out}} - T_{c,\text{in}})}{C_{\min}(T_{h,\text{in}} - T_{c,\text{in}})} = \frac{C_c (T_{c,\text{out}} - T_{c,\text{in}})}{C_h (T_{h,\text{in}} - T_{c,\text{in}})}$$

or

$$T_{c,\text{out}} = \frac{C_h}{C_c} \varepsilon (T_{h,\text{in}} - T_{c,\text{in}}) + T_{c,\text{in}} = (0.5)(0.7388)(80 - 20)°\text{C} + 20°\text{C} = \mathbf{42.2°C}$$

Discussion Using Figure 11-27*e*, the heat transfer effectiveness is approximately $\varepsilon \approx 73\%$.

11-121 Water is heated by hot air in a heat exchanger. The mass flow rates and the inlet temperatures are given. The heat transfer surface area of the heat exchanger on the water side is to be determined.

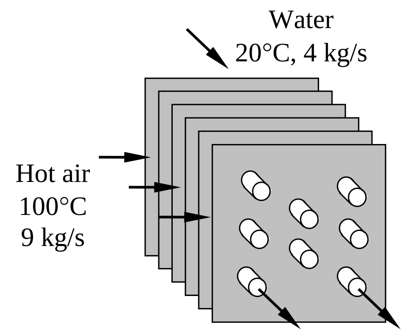
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and air are given to be 4.18 and 1.01kJ/kg.°C, respectively.

Analysis The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = (4\text{ kg/s})(4.18\text{kJ/kg.°C}) = 16.72\text{ kW/°C}$$

$$C_c = \dot{m}_c c_{pc} = (9\text{ kg/s})(1.01\text{kJ/kg.°C}) = 9.09\text{ kW/°C}$$



Therefore,

$$C_{\min} = C_c = 9.09\text{ kW/°C}$$

and

$$C = \frac{C_{\min}}{C_{\max}} = \frac{9.09}{16.72} = 0.544$$

Then the NTU of this heat exchanger corresponding to $c = 0.544$ and $\varepsilon = 0.65$ is determined from Fig. 11-27 to be

NTU = 1.5

Then the surface area of this heat exchanger becomes

$$\text{NTU} = \frac{UA_s}{C_{\min}} \longrightarrow A_s = \frac{\text{NTU}\,C_{\min}}{U} = \frac{(1.5)(9.09\text{ kW/°C})}{0.260\text{ kW/m}^2.\text{°C}} = \mathbf{52.4\ m^2}$$

11-122 Water is heated by a hot water stream in a heat exchanger. The maximum outlet temperature of the cold water and the effectiveness of the heat exchanger are to be determined.

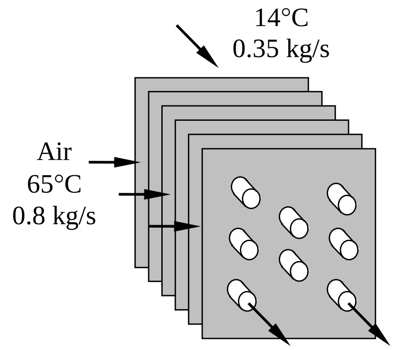
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The specific heats of water and air are given to be 4.18 and 1.0 kJ/kg.°C.

Analysis The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = (0.8\,\text{kg/s})(1.0\,\text{kJ/kg.°C}) = 0.8\,\text{kW/°C}$$

$$C_c = \dot{m}_c c_{pc} = (0.35\,\text{kg/s})(4.18\,\text{kJ/kg.°C}) = 1.463\,\text{kW/°C}$$



Therefore

$$C_{\min} = C_h = 0.8\,\text{kW/°C}$$

which is the smaller of the two heat capacity rates. Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (0.8\,\text{kW/°C})(65\text{°C} - 14\text{°C}) = 40.80\,\text{kW}$$

The maximum outlet temperature of the cold fluid is determined to be

$$\dot{Q}_{\max} = C_c(T_{c,out,\max} - T_{c,in}) \longrightarrow T_{c,out,\max} = T_{c,in} + \frac{\dot{Q}_{\max}}{C_c} = 14\text{°C} + \frac{40.80\,\text{kW}}{1.463\,\text{kW/°C}} = \mathbf{41.9°C}$$

The actual rate of heat transfer and the effectiveness of the heat exchanger are

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) = (0.8\,\text{kW/°C})(65\text{°C} - 25\text{°C}) = 32\,\text{kW}$$

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{32\,\text{kW}}{40.8\,\text{kW}} = \mathbf{0.784}$$

11-123 Oil in an engine is being cooled by air in a cross-flow heat exchanger, where both fluids are unmixed; (*a*) the heat transfer effectiveness and (*b*) the outlet temperature of the oil are to be determined.

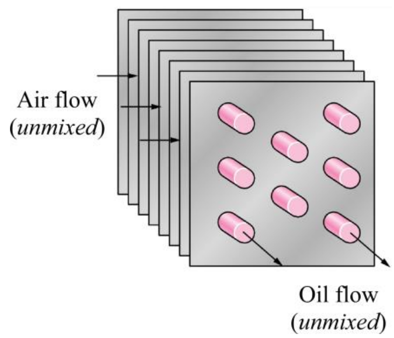
Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heats of oil and air are given to be c_{ph} = 2047 J/kg·K and c_{pc} = 1007 J/kg·K, respectively.

Analysis (*a*) The heat capacity rates are

$$C_c = \dot{m}_c c_{pc} = (0.21\,\text{kg/s})(1007\,\text{J/kg}\cdot\text{K}) = 211.5\,\text{W/K}$$

$$C_h = \dot{m}_h c_{ph} = (0.026\,\text{kg/s})(2047\,\text{J/kg}\cdot\text{K}) = 53.22\,\text{W/K}$$



The capacity ratio is

$$c = \frac{C_{\min}}{C_{\max}} = \frac{C_h}{C_c} = \frac{53.22\,\text{W/K}}{211.5\,\text{W/K}} = 0.2516$$

The NTU of the heat exchanger is

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{(53\,\text{W/m}^2\cdot\text{K})(1\,\text{m}^2)}{53.22\,\text{W/K}} = 0.9959$$

Using the equation listed in Table 11-4, the heat transfer effectiveness is

$$\varepsilon = 1 - \exp\left\{\frac{\text{NTU}^{0.22}}{c}[\exp(-c\,\text{NTU}^{0.78}) - 1]\right\}$$

$$= 1 - \exp\left(\frac{0.9959^{0.22}}{0.2516}\{\exp[-(0.2516)(0.9959)^{0.78}] - 1\}\right) = \mathbf{0.586}$$

(*b*) The outlet temperature of the cold water can be determined using

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h(T_{h,\text{in}} - T_{h,\text{out}})}{C_{\min}(T_{h,\text{in}} - T_{c,\text{in}})} = \frac{C_h(T_{h,\text{in}} - T_{h,\text{out}})}{C_h(T_{h,\text{in}} - T_{c,\text{in}})}$$

$$T_{h,\text{out}} = T_{h,\text{in}} - \varepsilon(T_{h,\text{in}} - T_{c,\text{in}}) = 75°\text{C} - (0.586)(75 - 30)°\text{C} = \mathbf{48.6°C}$$

Discussion Using Figure 11-27*b*, the heat transfer effectiveness is approximately $\varepsilon \approx 60\%$.

11-124 Water is heated by hot air in a cross-flow heat exchanger. Mass flow rates and inlet temperatures are given. The rate of heat transfer and the outlet temperatures are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform. **5** The thickness of the tube is negligible.

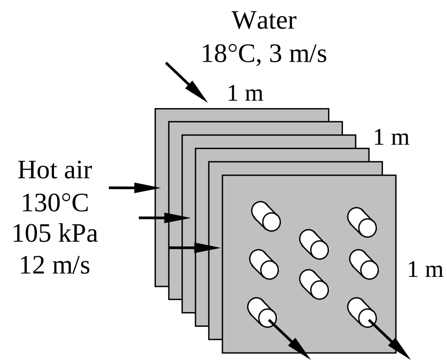
Properties The specific heats of the water and air are given to be 4.18 and 1.01 kJ/kg.°C, respectively.

Analysis The mass flow rates of the hot and the cold fluids are

$$\dot{m}_c = \rho V A_c = (1000\,\text{kg/m}^3)(3\,\text{m/s})[80\pi(0.03\,\text{m})^2/4] = 169.6\,\text{kg/s}$$

$$\rho_{air} = \frac{P}{RT} = \frac{105\,\text{kPa}}{(0.287\,\text{kPa.m}^3/\text{kg.K}) \times (130+273\,\text{K})} = 0.908\,\text{kg/m}^3$$

$$\dot{m}_h = \rho V A_c = (0.908\,\text{kg/m}^3)(12\,\text{m/s})(1\text{m})^2 = 10.90\,\text{kg/s}$$



The heat transfer surface area and the heat capacity rates are

$$A_s = n\pi DL = 80\pi(0.03\,\text{m})(1\text{m}) = 7.540\,\text{m}^2$$

$$C_c = \dot{m}_c c_{pc} = (169.6\,\text{kg/s})(4.18\text{kJ/kg.°C}) = 708.9\,\text{kW/°C}$$

$$C_h = \dot{m}_h c_{ph} = (10.9\,\text{kg/s})(1.010\,\text{kJ/kg.°C}) = 11.01\,\text{kW/°C}$$

Therefore,

$$C_{\min} = C_c = 11.01\,\text{kW/°C}$$

and

$$c = \frac{C_{\min}}{C_{\max}} = \frac{11.01}{708.9} = 0.01553$$

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (11.01\text{kW/°C})(130°\text{C} - 18°\text{C}) = 1233\,\text{kW}$$

The NTU of this heat exchanger is

$$NTU = \frac{UA_s}{C_{\min}} = \frac{(130\text{ W/m}^2.°\text{C})\,(7.540\,\text{m}^2)}{11{,}010\text{ W/°C}} = 0.08903$$

Noting that this heat exchanger involves mixed cross-flow, the fluid with $C_{\min}$ is mixed, $C_{\max}$ unmixed, effectiveness of this heat exchanger corresponding to c = 0.01553 and NTU =0.08903 is determined using the proper relation in Table 11-4 to be

$$\varepsilon = 1 - \exp\left[-\frac{1}{c}(1-e^{-cNTU})\right] = 1 - \exp\left[-\frac{1}{0.01553}(1-e^{-0.01553\times 0.08903})\right] = 0.08513$$

Then the actual rate of heat transfer becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.08513)(1233\,\text{kW}) = \mathbf{105.0\,kW}$$

Finally, the outlet temperatures of the cold and the hot fluid streams are determined from

$$\dot{Q} = C_c(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{C_c} = 18°\text{C} + \frac{105.0\,\text{kW}}{708.9\,\text{kW / °C}} = \mathbf{18.15°C}$$

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) \longrightarrow T_{h,out} = T_{h,in} - \frac{\dot{Q}}{C_h} = 130°\text{C} - \frac{105.0\,\text{kW}}{11.01\,\text{kW/°C}} = \mathbf{120.5°C}$$

11-125 Exhaust gases are used in a recuperative cross flow heat exchanger to heat air. For the given values of flow condition, heat transfer coefficients and the fouling resistance, the air exit temperature and the area of heat exchanger is to be determined.

Assumptions **1** Steady state conditions exist. **2** Heat exchanger is well insulated. **3** Fluid properties are constant.

Analysis (a) For the given flow conditions the energy balance between the two fluid streams gives,

Heat lost by exhaust gas = Heat gained by air

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

$$\therefore T_{c,out} = \frac{\dot{m}_h c_{ph}(T_{h,in} - T_{h,out})}{\dot{m}_c c_{pc}} + T_{c,in} = \frac{(7.5\,\text{kg/s})(1069(\text{J/kg}\cdot\text{K})\,(500 - 320)°\text{C}}{(15\,\text{kg/s})(1069\,\text{J/kg}\cdot\text{K})} + 30°C = \mathbf{120^o\,C} < \mathbf{150°C}$$

Hence the air cannot be heated to the desired temperature of 150°C.

(b) The overall heat transfer coefficient is calculated as,

$$\frac{1}{U} = \frac{1}{h_i} + R_{f,i} + \frac{1}{h_o} + R_{f,o} = \frac{1}{750\,\text{W/m}^2\cdot\text{K}} + (0.0004\,\text{m}^2\cdot\text{K/W}) + \frac{1}{300\,\text{W/m}^2\cdot\text{K}} + (0.0004\,\text{m}^2\cdot\text{K/W})$$

Therefore, the overall heat transfer coefficient is

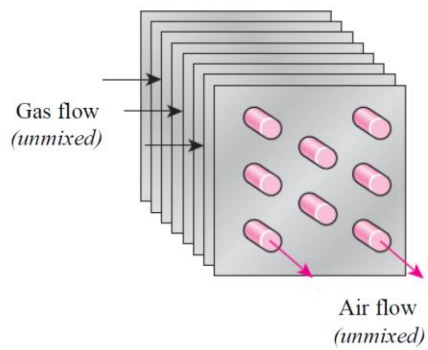
U = 182.92 W/m^2·K.

The heat capacity rate of the cold fluid (air) is

$$C_c = \dot{m}_c c_{pc} = (15\,\text{kg/s})(1069\,\text{J/kg}\cdot\text{K}) = 16035\,\text{W/K} \rightarrow \text{C}_{\text{max}}$$

The heat capacity rate of the hot fluid (exhaust gas) is,

$$C_h = \dot{m}_h c_{ph} = (7.5\,\text{kg/s})\,(1069\,\text{J/kg}\cdot\text{K}) = 8017.5\,\text{W/K} \rightarrow \text{C}_{\text{min}}$$



Thus the capacity ratio is

$$c = \frac{C_{\min}}{C_{\max}} = \frac{8017.5}{16035} = 0.5$$

The effectiveness of the heat exchanger is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h(T_{h,in} - T_{h,out})}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{(500 - 320)^o C}{(500 - 30)^o C} = 0.383$$

Now the number of transfer units (NTU) for both fluids unmixed are obtained from Figure 11-27(e),

NTU = 0.6

From the definition of NTU we get,

$$A_s = \frac{NTU \times C_{\min}}{U} = \frac{0.6(8017.5\,\text{W/K})}{182.9\,\text{W/m}^2\cdot\text{K}} = \mathbf{26.30\,m^2}$$

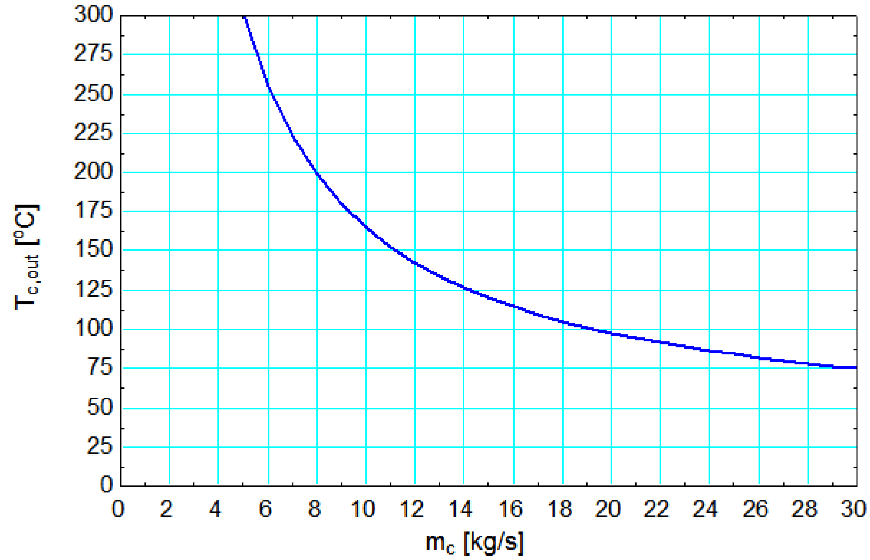
(c) The air mass flow rate in order to attain the desired exit temperature of 150°C is obtained form the energy balance of part (a).

$$\dot{m}_c = \frac{\dot{m}_h c_{ph}(T_{h,in} - T_{h,out})}{c_{pc}(T_{c,out} - T_{c,in})} = \frac{(7.5\,\text{kg/s})(1069(\text{J/kg}\cdot\text{K})\,(500 - 320)°\text{C}}{(1069\,\text{J/kg}\cdot\text{K})(T_{c,out} - 30°C)} \qquad (1)$$

For the desired $T_{c,out}$ = 150°C from the above equation we get

$\dot{m}_c$ = 11.25 kg/s

(d) Plotting equation 1 for the given range of the exit air temperature results in the following figure.



Change in air exit temperature with change in the air mass flow rate.

Discussion The plot shows that the air exit temperature is very sensitive to low air mass flow rates. It is also found that the effectiveness of the heat exchanger does not change with change in the air mass flow rate and hence increase in the air exit temperature. Since, for a cross flow heat exchanger with both fluids unmixed, the NTU is a function of effectiveness only. This implies that the NTU and hence the heat exchanger area will remain unchanged even with decrease in the air mass flow rate. However, this conclusion is in limit for $C_{min} = C_h$.

11-126 Single pass cross flow heat exchanger uses water (mixed) to heat methanol (unmixed) initially at 10°C. The heat exchanger area corresponding to the effectiveness of 0.5 is to be determined. Further, the effect of mass flow rate of water on the heat transfer rate, methanol exit temperature, overall heat transfer coefficient and the effectiveness of the heat exchanger is to be investigated.

Assumptions **1** Steady state conditions exist. **2** Heat exchanger is well insulated. **3** Fluid properties remain constan. **4** No fouling within the heat exchanger.

Analysis The effectiveness of heat exchanger is given to be 0.5.

$$\therefore 0.5 = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c(T_{c,out} - T_{c,in})}{C_{\min}(T_{h,in} - T_{c,in})}$$

Given that the total heat transfer rate is 250 kW, we find the maximum possible heat transfer as,

$$\dot{Q}_{\max} = 250\,kW/0.5 = 500\text{ kW}$$

For the known values of $T_{h,in}$, $T_{c,in}$ and $\dot{Q}_{max}$, we get

$$C_{min} = (500 \times 10^3\text{ W})/(90 - 10)°\text{C} = 6250\text{ W/K}.$$

Assuming the methanol to have the minimum heat capacity rate,

$$C_c = C_{min} = 6250\text{ W/K}.$$

Now, the mass flow rate of methanol is

$$\dot{m}_c = C_c / c_{pc} = (6250\text{ W/K})/(2577\text{J/kg}\cdot\text{K}) = 2.425\text{ kg/s}$$

The mass flow rate of water can be calculated from the energy balance between water and methanol.

Heat lost by water = Heat gained by methanol

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

$$250 \times 10^3\text{ W} = (2.425\text{kg/s})(2577\text{ J/kg}\cdot\text{K}) \times (T_{c,out} - 10)^{\text{o}}\text{C}$$

$$\therefore T_{c,out} = 50\ ^oC$$

Thus the mass flow rate of water is,

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out})$$

$$250 \times 10^3\text{ W} = \dot{m}_h (4193\text{ J/kg}\cdot\text{K})(90 - 60)^{\text{o}}\text{C}$$

$$\therefore \dot{m}_h = \mathbf{1.99\ kg/s}$$

The heat capacity of the water is

$$C_h = \dot{m}_h c_{ph} = 8333\text{ W/K} \rightarrow C_{\max}$$

Since C_h is C_{max} our assumption of $C_c = C_{min}$ is correct.

Thus the capacity ratio is

$$c = \frac{C_{\min}}{C_{\max}} = \frac{6250}{8333} = 0.75$$

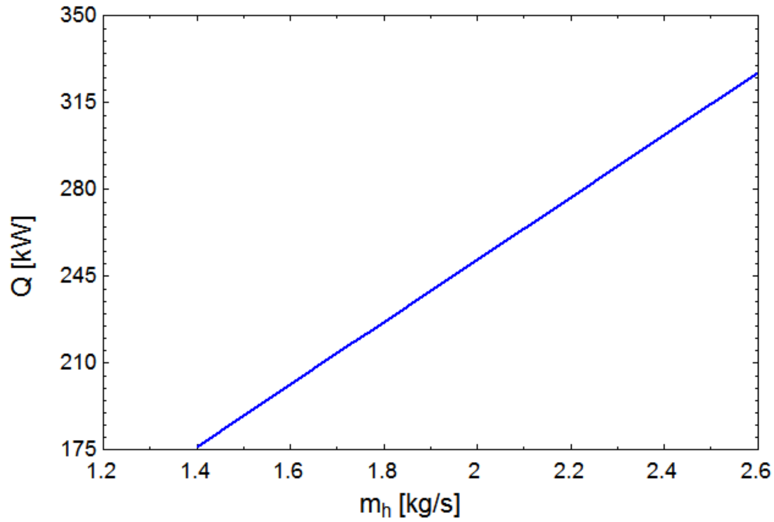
Now the number of transfer units (NTU) are calculated from the equation in Table 11-5 for C_{max} (mixed) and C_{min} (unmixed).

$$NTU = -\ln\left[1 + \frac{\ln(1 - \varepsilon\, c)}{c}\right] = -\ln\left[1 + \frac{\ln(1 - 0.5 \times 0.75)}{0.75}\right] = 0.985$$

Now from the definition of NTU we can find the surface area as

$$A_s = \frac{NTU\ C_{\min}}{U} = \frac{0.985\,(6250\text{ W/K})}{650\text{ W/m}^2\cdot\text{K}} = \mathbf{9.47\ m^2}$$

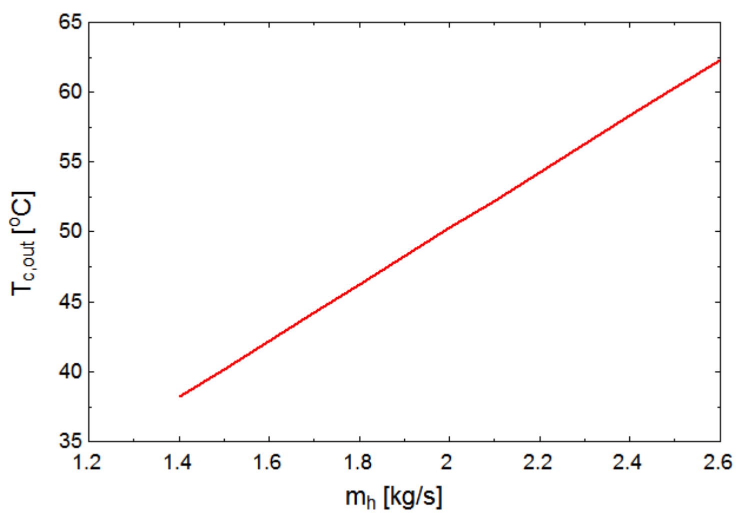
(b)



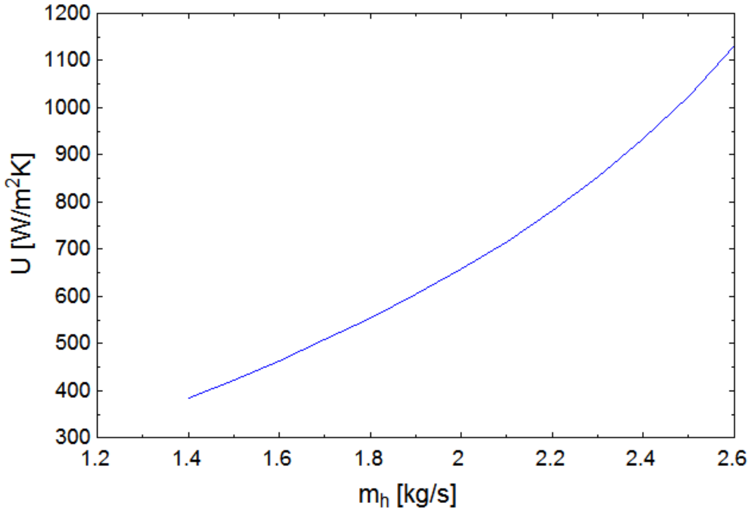
Variation of the total heat transfer rate with change in water mass flow rate

As shown in the figure above, a variation of ±30% in the water mass flow rate causes the heat transfer rate to vary by 85%. The effect of ±30% variation in the mass flow rate of water on $T_{c,out}$, U and ε is shown in the following figures. The methanol exit temperature and effectiveness of heat exchanger is observed to vary linearly with change in water mass flow rate. The overall heat transfer coefficient also increases with increase in the water mass flow rate as a consequence of increase in the convection heat transfer coefficient.

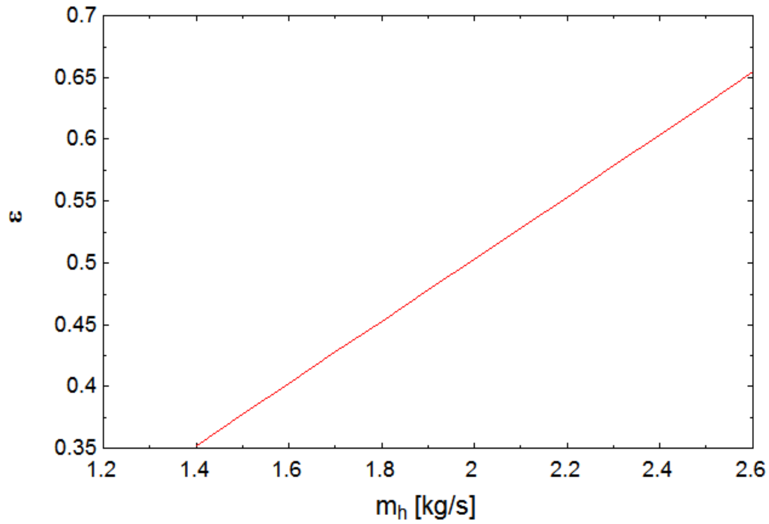
The capacity ratio '*c*' is always lower than unity except for the lowest water mass flow rate of 1.4 kg/s. In this case the heat capacity of methanol is more than that of the water.



Variation of the methanol exit temperature witch change in water mass flow rate.



Variation of overall heat transfer coefficient with change in water mass flow rate.



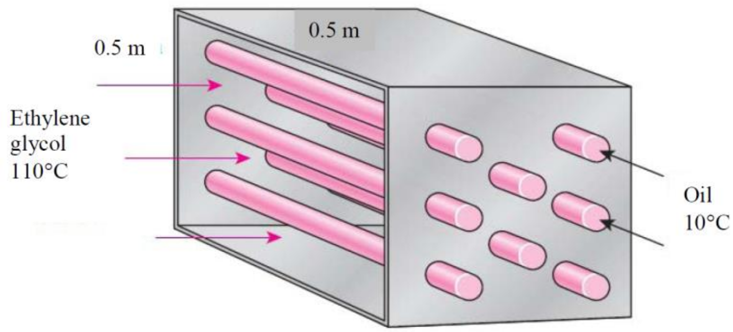
Variation of the heat exchanger effectiveness with change in water mass flow rate

Discussion In such problems multivariable optimization techniques can be used to decide upon the most optimum water flow rate that will give higher heat transfer rates with maximum economy.

11-127 Ethylene glycol flows over a staggered tube bank to heat the engine oil flowing through tubes. For the known geometry of the tube bank determine the mass flow rate of ethylene glycol and the number of tube rows.

Assumption **1** Steady state conditions exist. **2** Heat exchanger is well insulated. **3** Properties are constant. **4** No fouling inside the heat exchanger.

Properties Evaluate the properties of oil at an average temperature of (10 + 70)°C /2 = 40°C from Table A-13: c_p = 1964 J/kg·K



Analysis (a) From energy balance between ethylene glycol and engine oil we get,

Heat lost by ethylene glycol = Heat gained by engine oil.

$$\dot{Q} = \dot{m}_h c_{ph}(T_{c,in} - T_{c,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

$$\therefore \dot{m}_h = \frac{\dot{m}_c c_{pc}(T_{c,out} - T_{c,in})}{c_{ph}(T_{h,in} - T_{h,out})} \rightarrow \dot{m}_h = \frac{(4.05 \text{ kg/s})(1964 \text{ J/kg}\cdot\text{K})(70 - 10)^{\circ}\text{C}}{(2742 \text{ J/kg}\cdot\text{K})(110 - 90)^{\circ}\text{C}} = \mathbf{8.7\,kg/s}$$

Thus the mass flow rate of ethylene glycol is **8.7 kg/s**.

(b) For the given tube bank geometry,

$$S_D = \sqrt{S_L^2 + (S_T/2)^2} = \sqrt{0.035^2 + 0.0175^2} = 0.0391$$

For staggered arrangement since $S_D > S_T$, we have $2A_D > A_T$.

Thus the maximum velocity across staggered tube banks is calculated from Equation (7-40).

$$V_{\max} = \frac{S_T}{S_T - D_o} V \rightarrow V_{\max} = \frac{S_T}{S_T - D_o}\frac{\dot{m}_h}{\rho A}$$

$$V_{\max} = \frac{0.035}{(0.035 - 0.025)}\frac{8.7 \text{ kg/s}}{(1062 \text{ kg/m}^3)(0.5 \times 0.5)\text{m}^2} = 0.114 \text{ m/s}$$

Reynolds number based on maximum velocity is,

$$\text{Re}_D = \frac{\rho V_{\max} D_o}{\mu} = \frac{(1062 \text{ kg/m}^3)(0.114 \text{ m/s})(0.025 \text{ m})}{2.499 \times 10^{-3} \text{ kg/m}\cdot\text{s}} = 1211$$

For staggered arrangement we initially assume that the number of tube rows are greater than 16. Thus from Table 7-2 we get,

$$Nu_D = 0.35\left(\frac{S_T}{S_L}\right)^{0.2} \text{Re}_D^{0.6}\,\text{Pr}^{0.36}(\text{Pr}/\text{Pr}_s)^{0.25} \quad where\ S_T = S_L$$

$$\therefore Nu_D = 0.35(1211)^{0.6}(26.12)^{0.36}(26.12/96.97)^{0.25} = 57.76$$

Hence the heat transfer coefficient on the ethylene glycol side is,

$$h_o = \frac{Nu_D k}{D_o} = \frac{57.76(0.262 \text{ W/m}\cdot\text{K})}{0.025 \text{ m}} = 605.3 \text{ W/m}^2\cdot\text{K}$$

The overall heat transfer coefficient is

$$\frac{1}{U} = \frac{D_o}{h_i D_i} + \frac{D_o \ln(D_o / D_i)}{2k} + \frac{1}{h_o}$$

$$= \frac{0.025\,\text{m}}{(2500\,\text{W/m}^2\cdot\text{K})(0.023\,\text{m})} + \frac{(0.025\,m)\ln(0.025/0.023)}{2(250\,\text{W/m}\cdot\text{K})} + \frac{1}{605.3\,\text{W/m}^2\cdot\text{K}}$$

$$\therefore \frac{1}{U} = (4.34\times10^{-4} + 4.169\times10^{-6} + 1.652\times10^{-3})\ \text{m}^2\cdot\text{K/W}$$

The overall heat transfer coefficient is

$$U = 478.4\ \text{W/m}^2\cdot\text{K}$$

The heat capacity of ethylene glycol is

$$C_h = \dot{m}_h c_{ph} = (8.7\ \text{kg/s})(2742\ \text{J/kg}\cdot\text{K}) = 23855.4\ \text{W/K} \rightarrow \text{C}_{\text{max}}$$

The heat capacity of engine oil is

$$\text{C}_\text{c} = \dot{\text{m}}_\text{c}\text{c}_\text{pc} = (4.05\,\text{kg/s})(1964\ \text{J/kg}\cdot\text{K}) = 7954.2\ \text{W/K} \rightarrow \text{C}_\text{min}$$

Thus the capacity ratio is

$$c = \frac{C_{\text{min}}}{C_{\text{max}}} = \frac{7954.2}{23855.4} = 0.333$$

The effectiveness of heat exchanger is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\text{max}}} = \frac{C_c(T_{c,out} - T_{c,in})}{C_{\text{min}}(T_{h,in} - T_{c,in})} = \frac{(70-10)^o C}{(110-10)^o C} = 0.6$$

The number of transfer units (NTU) of heat exchanger is calculated from Table 11-5 with C_{max} mixed and C_{min} unmixed as

$$NTU = -\ln\left[1 + \frac{\ln(1-\varepsilon c)}{c}\right] = -\ln\left[1 + \frac{\ln(1-0.6\times0.333)}{0.333}\right] = 1.106$$

From the definition of NTU, we find the surface area of the heat exchanger as,

$$A_s = \frac{NTU\ C_{\text{min}}}{U} = \frac{(1.106)\ (7954.2\,\text{W/K})}{478.4\,\text{W/m}^2\cdot\text{K}} = 18.38\,\text{m}^2$$

Also the surface area of heat exchanger is expressed as

$$A_s = \pi D_o L \times N_L \times N_T$$

The number of transverse rows are

$$N_T = 0.5/S_T = 14$$

Thus the number of tube rows are

$$N_L = \frac{A_s}{\pi D_o L N_T} = \frac{18.38\,\text{m}^2}{\pi(0.025\,\text{m})(0.5\text{m})(14)} \approx \mathbf{33}$$

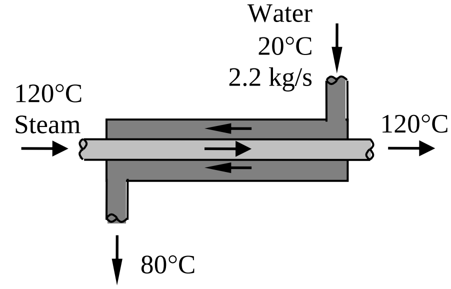
Discussion Initial assumption of number tube rows more than 16 is correct. In this problem the convection heat transfer coefficient was assumed. However, it is recommended to calculate the 'h' value for oil and verify if the assumption is correct.

11-128 Water is heated by steam condensing in a condenser. The required length of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heat of the water is given to be 4.18 kJ/kg.°C. The heat of vaporization of water at 120°C is given to be 2203 kJ/kg.

Analysis (*a*) The temperature differences between the steam and the water at the two ends of the condenser are



$$\Delta T_1 = T_{h,in} - T_{c,out} = 120°\text{C} - 80°\text{C} = 40°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 120°\text{C} - 20°\text{C} = 100°\text{C}$$

The logarithmic mean temperature difference is

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{40 - 100}{\ln(40/100)} = 65.48°\text{C}$$

The rate of heat transfer is determined from

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in}) = (2.2\text{ kg/s})(4.18\text{kJ/kg.°C})(80°\text{C} - 20°\text{C}) = 551.8\text{ kW}$$

The surface area of heat transfer is

$$\dot{Q} = UA_s\Delta T_{lm} \longrightarrow A_s = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{551.8\text{ kW}}{(0.7\text{ kW/m}^2.°\text{C})(65.48°\text{C})} = 12.04\text{ m}^2$$

The length of tube required then becomes

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{12.04\text{ m}^2}{\pi(0.025\text{ m})} = \mathbf{153.3\text{ m}}$$

(*b*) The maximum rate of heat transfer rate is

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (2.2\text{ kg/s})(4.18\text{kJ/kg.°C})(120°\text{C - }20°\text{C}) = 919.6\text{ kW}$$

Then the effectiveness of this heat exchanger becomes

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{551.8\text{ kW}}{919.6\text{ kW}} = 0.600$$

The NTU of this heat exchanger is determined using the relation in Table 11-5 to be

$$\text{NTU} = -\ln(1-\varepsilon) = -\ln(1-0.600) = 0.9163$$

The surface area is

$$NTU = \frac{UA_s}{C_{\min}} \longrightarrow A_s = \frac{NTU\ C_{\min}}{U} = \frac{(0.9163)(2.2\text{ kg/s})(4.18\text{kJ/kg.°C})}{0.7\text{ kW/m}^2.°\text{C}} = 12.04\text{ m}^2$$

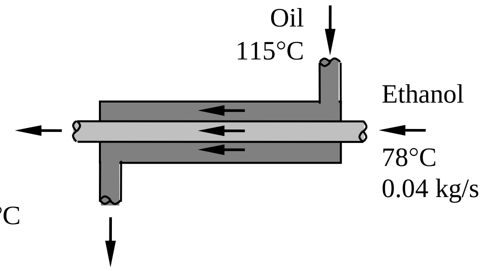
Finally, the length of tube required is

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{12.04\text{ m}^2}{\pi(0.025\text{ m})} = \mathbf{153.3\text{ m}}$$

11-129 Ethanol is vaporized by hot oil in a double-pipe parallel-flow heat exchanger. The outlet temperature and the mass flow rate of oil are to be determined using the LMTD and NTU methods.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heat of oil is given to be 2.2 kJ/kg.°C. The heat of vaporization of ethanol at 78°C is given to be 846 kJ/kg.



Analysis (*a*) The rate of heat transfer is

$$\dot{Q} = \dot{m}h_{fg} = (0.04\text{ kg/s})(846\text{ kJ/kg}) = 33.84\text{ kW}$$

The log mean temperature difference is

$$\dot{Q} = UA_s\Delta T_{lm} \longrightarrow \Delta T_{lm} = \frac{\dot{Q}}{UA_s} = \frac{33{,}840\text{ W}}{(320\text{ W/m}^2.°\text{C})(6.2\text{ m}^2)} = 17.06°\text{C}$$

The outlet temperature of the hot fluid can be determined as follows

$$\Delta T_1 = T_{h,in} - T_{c,in} = 115°\text{C} - 78°\text{C} = 37°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,out} = T_{h,out} - 78°\text{C}$$

and $$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{37 - (T_{h,out} - 78)}{\ln[37/(T_{h,out} - 78)]} = 17.06°\text{C}$$

whose solution is $T_{h,out} = \mathbf{84.0°C}$

Then the mass flow rate of the hot oil becomes

$$\dot{Q} = \dot{m}c_p(T_{h,in} - T_{h,out}) \longrightarrow \dot{m} = \frac{\dot{Q}}{c_p(T_{h,in} - T_{h,out})} = \frac{33{,}840\text{ W}}{(2200\text{ J/kg.°C})(115°\text{C} - 84.0°\text{C})} = \mathbf{0.496\ kg/s}$$

(*b*) The heat capacity rate $C = \dot{m}c_p$ of a fluid condensing or evaporating in a heat exchanger is infinity, and thus

$$c = C_{\min}/C_{\max} = 0\ .$$

The effectiveness in this case is determined from $\varepsilon = 1 - e^{-NTU}$

where $$NTU = \frac{UA_s}{C_{\min}} = \frac{(320\text{ W/m}^2.°\text{C})(6.2\text{ m}^2)}{(\dot{m},\text{ kg/s})(2200\text{ J/kg.°C})}$$

and $$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in})$$

$$\varepsilon = \frac{Q}{Q_{\max}} = \frac{C_{\min}(T_{h,in} - T_{c,in})}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{115 - T_{h,out}}{115 - 78}$$

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) = 33{,}840\text{ W}$$

$$\dot{Q} = \dot{m} \times 2200(115 - T_{h,out}) = 33{,}840\text{ W} \qquad (1)$$

Also $$\frac{115 - T_{h,out}}{115 - 78} = 1 - e^{-\frac{6.2\times 320}{\dot{m}\times 2200}} \qquad (2)$$

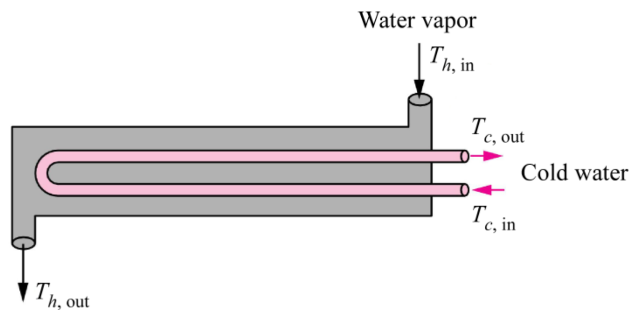
Solving (1) and (2) simultaneously gives

$$\dot{m}_h = \mathbf{0.496\ kg/s} \text{ and } T_{h,out} = \mathbf{84.0°C}$$

11-130 Saturated water vapor condenses in a 1-shell and 2-tube heat exchanger, (*a*) the heat transfer effectiveness, (*b*) the outlet temperature of the cold water, and (*c*) the heat transfer rate for the heat exchanger are to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heat of the cold water is given to be c_{pc} = 4179 J/kg·K.



Analysis (*a*) The minimum heat capacity rate is from the cold fluid, since for the hot fluid,

$$C_h = C_{\max} \to \infty$$

So, we have

$$C_c = C_{\min} = \dot{m}_c c_{pc} = (0.5\ \text{kg/s})(4179\ \text{J/kg}\cdot\text{K})$$
$$= 2090\ \text{W/K}$$

The heat capacity ratio in condensation process is

$$c = \frac{C_c}{C_h} = \frac{C_{\min}}{C_{\max}} \to 0$$

The NTU of the heat exchanger is

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{(2000\ \text{W/m}^2\cdot\text{K})(0.5\ \text{m}^2)}{2090\ \text{W/K}} = 0.4785$$

Using the equation listed in Table 11-4, the heat transfer effectiveness is

$$\varepsilon = 1 - \exp(-\text{NTU}) = 1 - \exp(-0.4785) = \mathbf{0.380}$$

(*b*) The outlet temperature of the cold water can be determined using

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c(T_{c,\text{out}} - T_{c,\text{in}})}{C_{\min}(T_{h,\text{in}} - T_{c,\text{in}})} = \frac{C_c(T_{c,\text{out}} - T_{c,\text{in}})}{C_c(T_{h,\text{in}} - T_{c,\text{in}})}$$

$$T_{c,\text{out}} = \varepsilon(T_{h,\text{in}} - T_{c,\text{in}}) + T_{c,\text{in}} = (0.380)(100-15)°\text{C} + 15°\text{C} = \mathbf{47.3°C}$$

(*c*) The heat transfer rate for the heat exchanger is

$$\dot{Q} = C_c(T_{c,\text{out}} - T_{c,\text{in}}) = (2090\ \text{W/K})(47.3-15)\ \text{K} = \mathbf{6.75\times10^4\ W}$$

Discussion The rate of heat transfer in the heat exchanger can also be calculated using

$$\dot{Q} = C_{\min}\varepsilon(T_{h,\text{in}} - T_{c,\text{in}})$$

11-131 Steam is condensed by cooling water in a shell-and-tube heat exchanger. The rate of heat transfer and the rate of condensation of steam are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform. **5** The thickness of the tube is negligible.

Properties The specific heat of the water is given to be 4.18 kJ/kg.°C. The heat of condensation of steam at 30°C is given to be 2430 kJ/kg.

Analysis (*a*) The heat capacity rate of a fluid condensing in a heat exchanger is infinity. Therefore,

$$C_{\min} = C_c = \dot{m}_c c_{pc} = (2200/3600\,\text{kg/s})(4.18\text{kJ/kg.°C}) = 2.554\,\text{kW/°C}$$

and

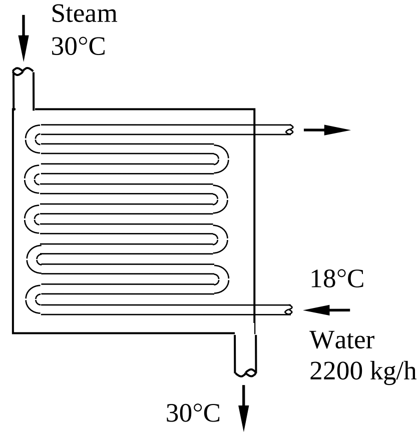
$$c = 0$$

Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (2.554\,\text{kW/°C})(30\text{°C} - 18\text{°C}) = 30.65\,\text{kW}$$

and

$$A_s = 8n\pi DL = 8\times 50\pi(0.015\,\text{m})(2\,\text{m}) = 37.7\,\text{m}^2$$



The NTU of this heat exchanger

$$NTU = \frac{UA_s}{C_{\min}} = \frac{(3\,\text{kW/m}^2.\text{°C})\,(37.7\,\text{m}^2)}{2.554\,\text{kW/°C}} = 44.27$$

Then the effectiveness of this heat exchanger corresponding to $c = 0$ and NTU = 44.27 is determined using the proper relation in Table 11-5

$$\varepsilon = 1 - \exp(-\text{NTU}) = 1 - \exp(-44.27) = 1$$

Then the actual heat transfer rate becomes

$$\dot{Q} = \varepsilon\dot{Q}_{\max} = (1)(30.65\text{kW}) = \mathbf{30.65\,kW}$$

(*b*) Finally, the rate of condensation of the steam is determined from

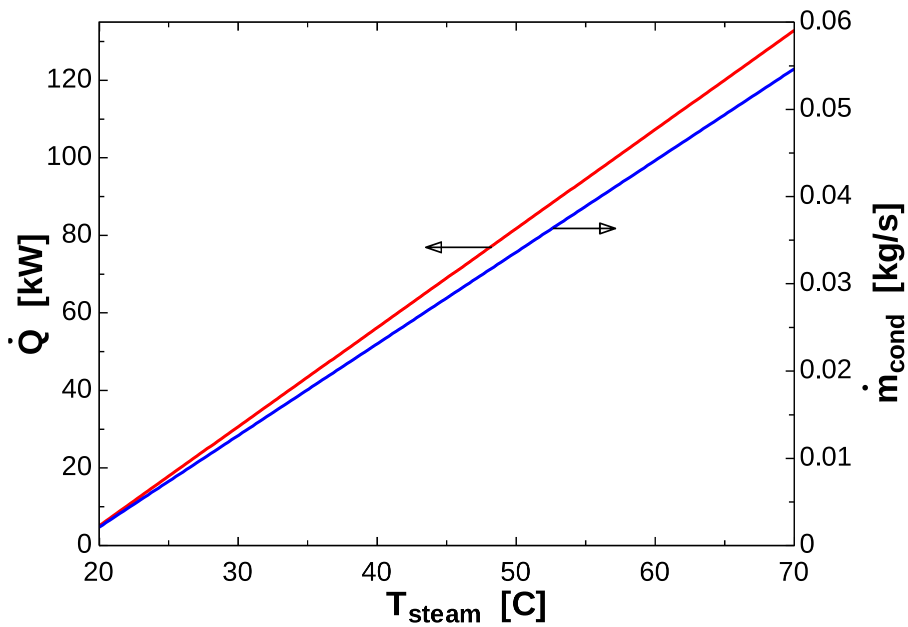
$$\dot{Q} = \dot{m}h_{fg} \longrightarrow \dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{30.65\,\text{kJ/s}}{2431\,\text{kJ/kg}} = \mathbf{0.0126\,kg/s}$$

11-132 Prob. 11-131 is reconsidered. The effects of the condensing steam temperature and the tube diameter on the rate of heat transfer and the rate of condensation of steam are to be investigated.

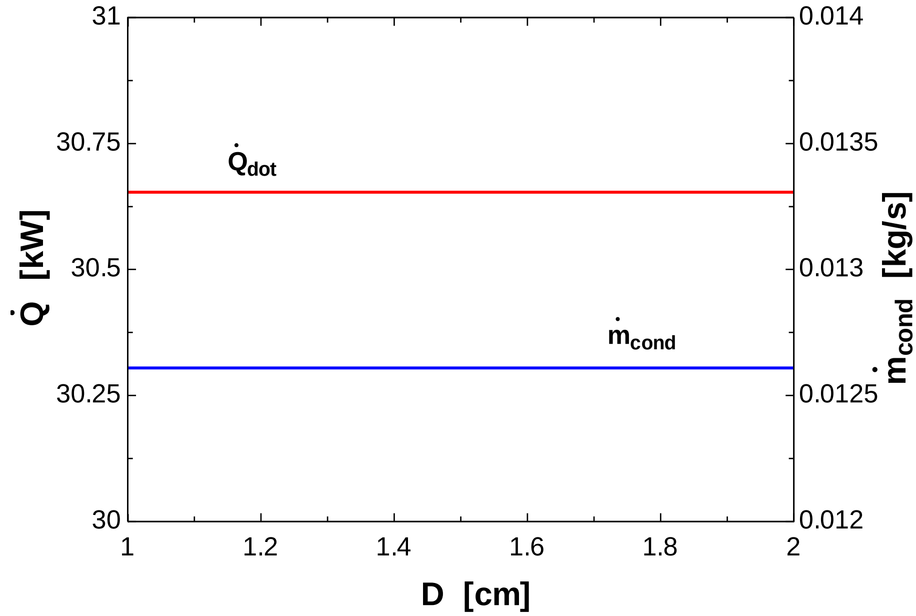
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
N_pass=8
N_tube=50
T_steam=30 [C]
h_fg_steam=2431 [kJ/kg]
T_w_in=18 [C]
m_dot_w=2200[kg/h]*Convert(kg/h, kg/s)
c_p_w=4.18 [kJ/kg-C]
D=1.5 [cm]
L=2 [m]
U=3 [kW/m^2-C]

"ANALYSIS"
"With EES, it is easier to solve this problem using LMTD method than NTU method. Below, we use NTU method. Both methods give the same results."
C_min=m_dot_w*c_p_w
c=0 "since the heat capacity rate of a fluid condensing is infinity"
Q_dot_max=C_min*(T_steam-T_w_in)
A=N_pass*N_tube*pi*D*L*Convert(cm, m)
NTU=(U*A)/C_min
epsilon=1-exp(-NTU) "from Table 11-4 of the text with c=0"
Q_dot=epsilon*Q_dot_max
Q_dot=m_dot_cond*h_fg_steam
```

T_{steam} [C]	$\dot{Q}$ [kW]	$\dot{m}_{cond}$ [kg/s]
20	5.109	0.002102
22.5	11.5	0.004729
25	17.88	0.007355
27.5	24.27	0.009982
30	30.65	0.01261
32.5	37.04	0.01524
35	43.43	0.01786
37.5	49.81	0.02049
40	56.2	0.02312
42.5	62.58	0.02574
45	68.97	0.02837
47.5	75.36	0.031
50	81.74	0.03362
52.5	88.13	0.03625
55	94.51	0.03888
57.5	100.9	0.04151
60	107.3	0.04413
62.5	113.7	0.04676
65	120.1	0.04939
67.5	126.4	0.05201
70	132.8	0.05464



D [cm]	$\dot{Q}$ [kW]	$\dot{m}_{cond}$ [kg/s]
1	30.65	0.01261
1.05	30.65	0.01261
1.1	30.65	0.01261
1.15	30.65	0.01261
1.2	30.65	0.01261
1.25	30.65	0.01261
1.3	30.65	0.01261
1.35	30.65	0.01261
1.4	30.65	0.01261
1.45	30.65	0.01261
1.5	30.65	0.01261
1.55	30.65	0.01261
1.6	30.65	0.01261
1.65	30.65	0.01261
1.7	30.65	0.01261
1.75	30.65	0.01261
1.8	30.65	0.01261
1.85	30.65	0.01261
1.9	30.65	0.01261
1.95	30.65	0.01261
2	30.65	0.01261



11-133E The hot water exiting the condenser is to be cooled by passing it through a heat exchanger immersed in large lake. Using $\varepsilon - NTU$ method, exit temperature of the water from immersed heat exchanger is to be determined.

Assumptions **1** Steady state conditions exist. **2** Fluid properties remain constant. **3** Lake water is an infinite medium.

Analysis Since the problem statement requires use of $\varepsilon - NTU$ method we first calculate the heat capacity rates of cold and hot fluids. The cold side i.e. lake water is an infinite medium and its temperature remain virtually unchanged. Thus the cold side heat capacity C_c is $C_{max,}$ since $C_{max} \rightarrow \infty$, then c = 0. The hot side heat capacity (condenser exit water) is

$$C_h = \dot{m}_h c_{ph} = (\rho A_c V) c_{ph}$$

$$\therefore C_h = (62.4\,\text{lbm/ft}^3)[\frac{\pi}{4}\left(\frac{1}{12}\right)^2 \text{ft}^2](9\,\text{ft/s}) \times 1(\text{Btu/lbm}\cdot°\text{F}) = 3.06\,\text{Btu/s}\cdot°\text{F}$$

From the definition of NTU we have

$$NTU = \frac{UA_s}{C_{\min}} = \frac{U\pi DL}{C_{\min}} = \frac{(250\,\text{Btu/h}\cdot\text{ft}^2\cdot°\text{F})[\pi\,(1/12)\text{ft}(500\,\text{ft})]}{3.06\,\text{Btu/s}\cdot°\text{F}}\frac{1\text{h}}{3600\text{s}} = 2.97$$

Now form the equation in Table 11-4, the effectiveness is calculated as

$$\varepsilon = 1 - \exp(-NTU) = 1 - \exp(-2.97) = 0.948$$

The heat transfer rate is,

$$\dot{Q} = \varepsilon\ \dot{Q}_{\max} = \varepsilon\, C_{\min}(T_{h,in} - T_{c,in})$$

$$\therefore \dot{Q} = (0.948)\,(3.06\,\text{Btu/s}\cdot°\text{F})\,(100 - 45)°F = 159.55\,\text{Btu/s}$$

Thus from energy balance we can find the exit temperature of the water from immersed heat exchanger as,

$$\dot{Q} = C_h\,(T_{h,in} - T_{h,out}) = (3.06\,\text{Btu/s}\cdot°\text{F}) \times (100 - T_{h,out})°\text{F}$$

$$\therefore T_{h,out} = \mathbf{47.9\ {}^{o}F}$$

Discussion In case of the heat exchangers immersed in ponds or lakes, the immersion depth plays a key role in maintaining the fluid exit temperature. For shallow depths, the lake water temperature is subject to change with change in environmental conditions and hence influences the fluid exit temperature.

11-134 Saturated steam flows over tubes in a shell and tube heat exchanger at specified conditions. Effectiveness of the heat exchanger, length of the tube and the rate of steam condensation are to be determined.

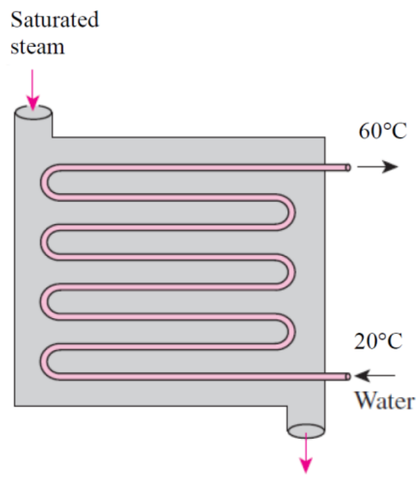
Assumptions **1** Steady state conditions exist. **2** Fluid properties are constant. **3** Heat exchanger is well insulated.

Properties The properties of water are evaluated at an average temperature of (60 + 20)°C/2 = 40°C from Table A-9:

ρ = 992.1 kg/m^3, μ = 0.653 × 10^{-3} kg/m·s, c_p = 4179 J/kg·K, k = 0.631 W/m·K, and Pr = 4.32

The properties of steam are calculated at saturation temperature corresponding to the given saturation pressure. At a saturationpressure of 270.1 kPa from Table A-9, the saturation temperature of the steam is 130°C. At 130°C, the enthalpy ofvaporization is 2174 kJ/kg.

Analysis (*a*) Since the steam undergoes a condensation process the temperature of steam remains unchanged at 130°C. Hencethe $C_{\max}$ is infinity and thus $C_{\min}$ is for the cold fluid. Thus the capacity ratio is



$$c = \frac{C_{\min}}{C_{\max}} \to 0$$

The effectiveness is calculated as

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c\left(T_{c,\text{out}} - T_{c,\text{in}}\right)}{C_{\min}\left(T_{h,\text{in}} - T_{c,\text{in}}\right)} = \frac{60-20}{130-20} = \mathbf{0.36364}$$

(*b*) $C_{\min}$ is calculated as

$$C_{\min} = \dot{m}_c c_{pc} = 4(0.25\ \text{kg/s})(4179\ \text{J/kg·K}) = 4179\ \text{W/K}$$

For a phase change process, the number of transfer units (NTU) is calculated as

$$\text{NTU} = -\ln(1-\varepsilon) = -\ln(1-0.36364) = 0.45199$$

The number of transfer units is defined as

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{Un\pi DL}{C_{\min}}$$

Here we first need to calculate the overall heat transfer coefficient. For the given flow condition of water through tubes we get

$$\text{Re} = \frac{4\dot{m}}{\pi D\mu} = \frac{4(0.25\ \text{kg/s})}{\pi(0.0125\ \text{m})(0.653\times10^{-3}\text{kg/m·s})} = 38997$$

Since Re > 10,000, the flow is fully turbulent. Assuming the flow to be fully developed the Nusselt number can be determinedfrom Dittus-Bolter equation:

$$\text{Nu} = \frac{h_i D}{k} = 0.023\text{Re}^{0.8}\text{Pr}^{0.4} = 0.023(38997)^{0.8}(4.32)^{0.4} = 194.42$$

The convection heat transfer coefficient for water through the tubes is

$$h_i = \text{Nu}\frac{k}{D} = \frac{(194.42)(0.631\ \text{W/m·K})}{0.0125\ \text{m}} = 9814.3\ \text{W/m}^2\text{·K}$$

Thus the overall heat transfer coefficient is

$$U = \left(\frac{1}{h_i} + \frac{1}{h_o} + R_{f,i} + R_{f,o}\right)^{-1} = \left(\frac{1}{9814.3\text{W/m}^2\text{·K}} + \frac{1}{1500\text{W/m}^2\text{·K}} + 0.00015\text{m}^2\text{·K/W} + 0.0001\text{m}^2\text{·K/W}\right)^{-1}$$
$$= 981.78\ \text{W/m}^2\text{·K}$$

Therefore the length of heat exchanger tube is calculated as

$$L = \frac{\text{NTU}C_{\min}}{Un\pi D} = \frac{(0.45199)(4179\ \text{W/K})}{(981.78\ \text{W/m}^2\text{·K})(4\pi)(0.0125\ \text{m})} = \mathbf{12.25\ m}$$

(*c*) From the energy balance we have,

$$\dot{m}_h h_{fg} = \dot{m}_c c_{pc}\left(T_{c,\text{out}} - T_{c,\text{in}}\right)$$

$$\dot{m}_h = \frac{\dot{m}_c c_{pc}}{h_{fg}}\left(T_{c,\text{out}} - T_{c,\text{in}}\right) = \frac{4(0.25\ \text{kg/s})(4179\ \text{J/kg·K})}{2174000\ \text{J/kg}}(60-20)\text{K} = \mathbf{0.07689\ kg/s = 276.8\ kg/h}$$

Discussion To increase the rate of steam condensation, we can increase the water flow rate while maintaining the temperature difference between the water inlet and exit.

If there are no fouling resistances, the overall heat transfer coefficient would be higher. This would reduce the heat exchanger length.

11-135 Cold water is used to condense saturated refrigerant R134a in a shell and tube heat exchanger. For a given overall heat transfer coefficient the heat exchanger area and mass flow rate of cooling water is to be determined. In case of fouling when the overall heat transfer coefficient drops down, new flow rate of cooling water for complete condensation of refrigerant is to be determined.

Assumptions **1** Steady state conditions exist. **2** Heat exchanger is well insulated. **3** Fluid properties remain constant. **4** Thermal resistance due to heat exchanger tube walls is negligible.

Properties Thermo physical properties of the cooling water are evaluated at an average temperature of (10 + 40)°C/2 = 25°C from Table A-9. $c_p = 4180\text{ J/kg}\cdot\text{K}$.

Properties of refrigerant R134a are calculated at 1318.6 kPa from Table A-10.

$$T_{sat} = T_{h,in} = T_{h,out} = 50^{\circ}\text{C},\ h_{fg} = 151.8\times10^3\text{ J/kg}\cdot\text{K}$$

Analysis (a) From energy balance we have,

The required rate of heat transfer for complete condensation is,

$$\dot{Q} = \dot{m}_h h_{fg} = (2.5\text{ kg/s})(151.8\times10^3\text{ J/kg}\cdot\text{K}) = 3.795\times10^5\text{ W}$$

From heat balance, the heat lost by refrigerant = heat gained by cooling water.

$$\dot{Q} = \dot{m}_h h_{fg} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

$$\therefore \dot{m}_c = \frac{\dot{m}_h h_{fg}}{c_{pc}(T_{c,out} - T_{c,in})} = \frac{3.795\times10^5\text{ W}}{(4180\text{ J/kg}\cdot\text{K})(40 - 10)^{\circ}\text{C}} = \mathbf{3.02\ kg/s}$$

The heat capacity of the cold water is

$$C_c = \dot{m}_c c_{pc} = (3.02\text{ kg/s})(4180\text{ J/kg}\cdot\text{K}) = 12624\text{ W/K}$$

Since the saturated refrigerant undergoes a phase change process, $C_h \rightarrow \infty$ and hence the capacity ratio c = 0.

The effectiveness of the heat exchanger is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\dot{m}_h h_{fg}}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{3.795\times10^5\text{ W}}{(12624\text{ W/K})(50-10)^{o}C} = 0.75$$

Thus the number of transfer units (NTU) are

$$NTU = -\ln(1-\varepsilon) = -\ln(1-0.75) = 1.38$$

From the definition of NTU we find heat exchanger surface area as

$$A_s = \frac{NTU\ C_{\min}}{U} = \frac{1.38\,(12624\text{ W/K})}{3500\text{ W/m}^{2}\cdot\text{K}} = \mathbf{4.98\ m^2}$$

(b) In case of fouling, the new heat transfer coefficient is 2800 W/m^2·K (0.2 × 3500 W/m^2·K).

Based on this new heat transfer coefficient we have the rate of heat transfer as

$$\dot{Q} = UA_s\Delta T_{lm}$$

Now the logarithmic temperature difference is calculated as

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$\Delta T_1 = T_{h,in} - T_{c,out} = 50 - 40 = 10^{o}C$$

and

$$\Delta T_2 = T_{h,out} - T_{c,in} = 50 - 10 = 40^{o}C$$

$$\Delta T_{lm} = \frac{10-40}{\ln(10/40)} = 21.64\ ^{\circ}\text{C}$$

Therefore the rate of heat transfer is

$$\dot{Q} = UA_s\Delta T_{lm} = (2800\text{ W/m}^2\cdot\text{K})(4.98\text{m}^2)(21.64^{\circ}\text{C}) = 3.0174\times10^5\text{ W}$$

Thus the reduction in heat transfer due to fouling is $(3.795\times10^5\text{ W} - 3.0174\times10^5\text{ W})$ = 0.777×10^5 W and hence the refrigerant will not condense completely to saturated liquid at the heat exchanger exit. In order to overcome this situation, the mass flow rate of cooling water has to be increased to account for this heat transfer reduction. Thus the new mass flow rate of cooling water is,

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

$$\therefore \dot{m}_c = \frac{\dot{Q}}{c_{pc}(T_{c,out} - T_{c,in})} = \frac{(3.795 + 0.777)\times 10^5 \text{ W}}{(4182 \text{ J/kg}\cdot\text{K})(40 - 10)^{\text{o}}\text{C}} = \mathbf{3.64\,kg/s}$$

The mass flow rate of cooling water that accounts for fouling in the heat exchanger is **3.64 kg/s**.

Discussion Fouling in heat exchanger causes the mass flow rate of cooling water to be increased by about 21%. In case of large scale systems, this significant increase in mass flow rates will require additional pumping making the heat exchange process uneconomical.

Selection of the Heat Exchangers

11-136C In the case of automotive and aerospace industry, where weight and size considerations are important, and in situations where the space availability is limited, we choose the smaller heat exchanger.

11-137C The first thing we need to do is determine the life expectancy of the system. Then we need to evaluate how much the larger will save in pumping cost, and compare it to the initial cost difference of the two units. If the larger system saves more than the cost difference in its lifetime, it should be preferred.

11-138C 1) Calculate heat transfer rate, 2) select a suitable type of heat exchanger, 3) select a suitable type of cooling fluid, and its temperature range, 4) calculate or select U, and 5) calculate the size (surface area) of heat exchanger

11-139 Oil is to be cooled by water in a heat exchanger. The heat transfer rating of the heat exchanger is to be determined and a suitable type is to be proposed.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heat of the oil is given to be 2.2 kJ/kg.°C.

Analysis The heat transfer rate of this heat exchanger is

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in}) = (10\text{ kg/s})(2.2\text{kJ/kg.°C})(120°\text{C} - 40°\text{C}) = \mathbf{1760\,kW}$$

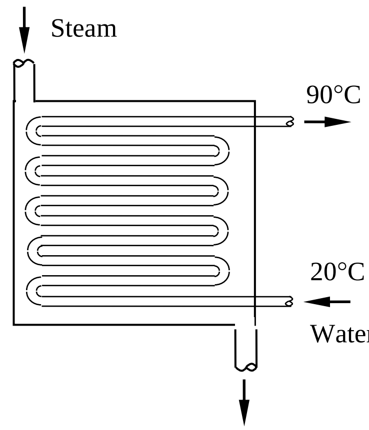
We propose a compact heat exchanger (like the car radiator) if air cooling is to be used, or a tube-and-shell or plate heat exchanger if water cooling is to be used.

11-140 Water is to be heated by steam in a shell-and-tube process heater. The number of tube passes need to be used is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heat of the water is given to be 4.19 kJ/kg.°C.

Analysis The mass flow rate of the water is



$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

$$\dot{m} = \frac{\dot{Q}}{c_{pc}(T_{c,out} - T_{c,in})}$$

$$= \frac{600 \text{ kW}}{(4.19 \text{ kJ/kg.°C})(90°\text{C} - 20°\text{C})}$$

$$= 2.046 \text{ kg/s}$$

The total cross-section area of the tubes corresponding to this mass flow rate is

$$\dot{m} = \rho V A_c \rightarrow A_c = \frac{\dot{m}}{\rho V} = \frac{2.046 \text{ kg/s}}{(1000 \text{ kg/m}^3)(3 \text{ m/s})} = 6.82\times10^{-4} \text{ m}^2$$

Then the number of tubes that need to be used becomes

$$A_s = n\frac{\pi D^2}{4} \longrightarrow n = \frac{4A_s}{\pi D^2} = \frac{4(6.82\times10^{-4} \text{ m}^2)}{\pi(0.01 \text{ m})^2} = 8.68 \cong \mathbf{9}$$

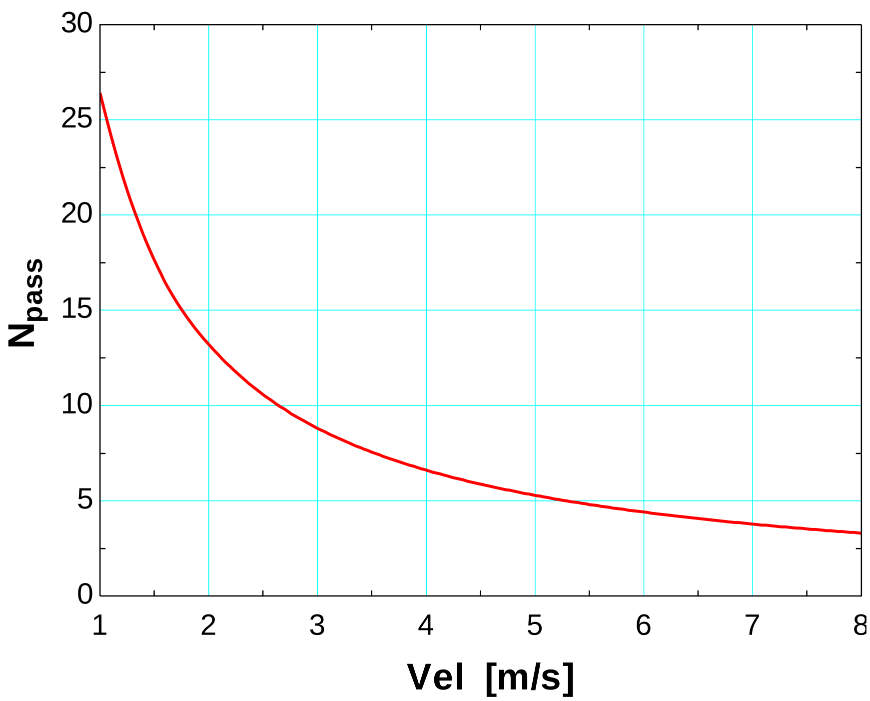
Therefore, we need to use at least 9 tubes entering the heat exchanger.

11-141  Prob. 11-140 is reconsidered. The number of tube passes as a function of water velocity is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
c_p_w=4.19 [kJ/kg-C]
T_w_in=20 [C]
T_w_out=90 [C]
Q_dot=600 [kW]
D=0.01 [m]
Vel=3 [m/s]

"PROPERTIES"
rho=density(water, T=T_ave, P=100)
T_ave=1/2*(T_w_in+T_w_out)

"ANALYSIS"
Q_dot=m_dot_w*c_p_w*(T_w_out-T_w_in)
m_dot_w=rho*A_c*Vel
A_c=N_pass*pi*D^2/4
```

Vel [m/s]	N_{pass}
1	26.42
1.5	17.62
2	13.21
2.5	10.57
3	8.808
3.5	7.55
4	6.606
4.5	5.872
5	5.285
5.5	4.804
6	4.404
6.5	4.065
7	3.775
7.5	3.523
8	3.303



11-142 Cooling water is used to condense the steam in a power plant. The total length of the tubes required in the condenser is to be determined and a suitable HX type is to be proposed.

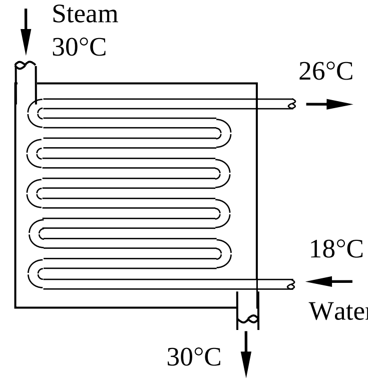
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heat of the water is given to be 4.18 kJ/kg.°C. The heat of condensation of steam at 30°C is given to be 2431 kJ/kg.

Analysis The temperature differences between the steam and the water at the two ends of condenser are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 30°\text{C} - 26°\text{C} = 4°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 30°\text{C} - 18°\text{C} = 12°\text{C}$$



and the logarithmic mean temperature difference is

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{4-12}{\ln(4/12)} = 7.28°\text{C}$$

The heat transfer surface area is

$$\dot{Q} = UA_s \Delta T_{lm} \longrightarrow A_s = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{125\times10^6 \text{ W}}{(3500 \text{ W/m}^2.°\text{C})(7.28°\text{C})} = 4906 \text{ m}^2$$

The total length of the tubes required in this condenser then becomes

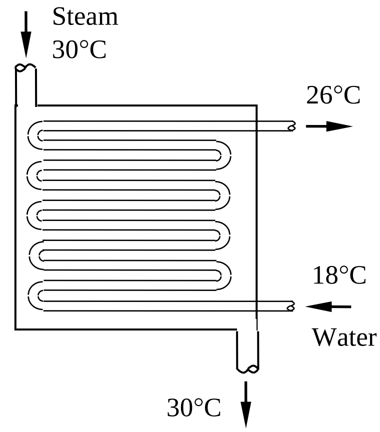
$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{4906 \text{ m}^2}{\pi(0.02 \text{ m})} = 78{,}078 \text{ m} = \mathbf{78.1 km}$$

A multi-pass shell-and-tube heat exchanger is suitable in this case.

11-143 Cold water is heated by hot water in a heat exchanger. The net rate of heat transfer and the heat transfer surface area of the heat exchanger are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the cold and hot water are given to be 4.18 and 4.19 kJ/kg.°C, respectively.



Analysis The temperature differences between the steam and the water at the two ends of condenser are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 30°\text{C} - 26°\text{C} = 4°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 30°\text{C} - 18°\text{C} = 12°\text{C}$$

and the logarithmic mean temperature difference is

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{4 - 12}{\ln(4/12)} = 7.28°\text{C}$$

The heat transfer surface area is

$$\dot{Q} = UA_s \Delta T_{lm} \longrightarrow A_s = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{50 \times 10^6 \text{ W}}{(3500 \text{ W/m}^2.°\text{C})(7.28°\text{C})} = 1962 \text{ m}^2$$

The total length of the tubes required in this condenser then becomes

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{1962 \text{ m}^2}{\pi (0.02 \text{ m})} = 31{,}231 \text{ m} = \mathbf{31.2\ km}$$

A multi-pass shell-and-tube heat exchanger is suitable in this case.

11-144 Petroleum based organic vapor is to be cooled inside a shell and tube heat exchanger. For the specified flow rates of each fluid, the number of tubes is to be found.

Assumptions **1** Steady state conditions exist. **2** Heat exchanger is insulated. **3** Negligible thermal resistance due to pipe wall thickness and negligible fouling. **4** Constant fluid properties.

Analysis From the energy balance between the organic vapor and cooling water we have

Heat lost by organic vapor = Heat gained by cooling water

$$\dot{Q} = \dot{m}_h h_{fg} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

$$\therefore (5\,\text{kg/s})(580\times10^3\ \text{J/kg}) = (25\,\text{kg/s})(4187\,\text{J/kg}\cdot\text{K})(T_{c,out} - 15)^\text{o}\text{C} = 29\times10^5\ \text{W}$$

Therefore the exit temperature of the cooling water is

$$T_{c,out} = 42.70\ ^\text{o}\text{C}$$

Now the logarithmic temperature difference is calculated as

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$\Delta T_1 = T_{h,in} - T_{c,out} = 75 - 42.7 = 32.3^o C$$

and

$$\Delta T_2 = T_{h,out} - T_{c,in} = 75 - 15 = 60^o C$$

$$\Delta T_{lm} = \frac{32.3 - 60}{\ln(32.3/60)} = 44.73\ ^\text{o}\text{C}$$

The heat transfer rate is calculated as

$$\dot{Q} = \dot{m}_h h_{fg} = UA_s\Delta T_{lm} = U(n\pi DL)\Delta T_{lm}$$

$$29\times10^5\ \text{W} = (550\ \text{W/m}^2\cdot\text{K})\ \text{n}\ \pi\ (0.018\,\text{m})(5\,\text{m})(44.73^\text{o}\,\text{C})$$

$$\therefore n = \mathbf{417\ tubes}$$

11-145E Hot water flowing through the shell side at known mass flow rate is cooled by water flowing through tubes at specified mass flow rate and temperature. For the given space constraint, number of tube passes, number of tubes per pass and the length of each tube are to be determined.

Assumptions **1** Steady state operating conditions exist. **2** Heat exchanger is well insulated. **3** Fluid properties are constant. **4** No fouling inside heat exchanger.

Properties The specific heats of water on shell side and tube side are taken to be 1 Btu/lbm·°F from Table A-9E since for the range of temperatures in this problem the specific heat of water is almost constant and close to 1 Btu/lbm·°F. Also from Table A-9E the density of water on tube side is evaluated at an average temperature of (110 +80)°F/2 = 95°F is 62.06 lbm/ft^3.

Analysis As given in the problem statement we first calculate the tube length based on one tube pass and check if it satisfies the given tube length constraint. The exit temperature of the hot water is calculated by doing an energy balance. The heat gained by cold water is equal to the heat lost by hot water. Thus we have,

$$\dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

$$(30000\ lbm/h)(1\,\text{Btu/lbm}\cdot°\text{F})(175 - T_{h,out})°F = (40000\ \text{lbm/h})(1\text{Btu/lbm}\cdot°\text{F})(110-80)°\text{F}$$

$$\therefore T_{h,out} = 135°\text{F}$$

The total heat transfer within the heat exchanger is,

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in}) = (40000\ \text{lbm/h})(1\text{Btu/lbm}\cdot°\text{F})(110-80)°\text{F} = 12\times10^5\ \text{Btu/h}$$

The heat transfer within a heat exchanger can also be calculated using overall heat transfer coefficient and the log mean temperature difference.

$$\dot{Q} = UA_s\Delta T_{lm}$$

where

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$\Delta T_1 = T_{h,in} - T_{c,out} = 175 - 110 = 65^oF$$

and $$\Delta T_2 = T_{h,out} - T_{c,in} = 135 - 80 = 55^oF$$

$$\Delta T_{lm} = \frac{65-55}{\ln(65/55)} = 59.9\ ^o\text{F}$$

Thus the surface area of the heat exchanger is found to be

$$A_s = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{12\times10^5\ \text{Btu/h}}{(220\,\text{Btu/h}\cdot\text{ft}^2\cdot°\text{F})(59.9°\text{F})} = 91.06\,\text{ft}^2$$

This is the total surface area of the heat exchanger based on 'n' number of tubes. We first need to find number of tubes in the heat exchanger based on the inside pipe diameter of the tubes and average water velocity.

$$\dot{m}_c = \rho A_c V \Rightarrow A_c = \frac{40000\,\text{lbm/h})}{(62.06\text{lbm/ft}^3)(1.5\text{ft/s})} \times \frac{1\text{h}}{3600\text{s}} = 0.1193\,\text{ft}^2$$

This cross sectional area is the total cross sectional area for 'n' tubes. Thus the number of tubes are,

$$A_c = n\frac{\pi D_i^2}{4} \Rightarrow n = \frac{4\,(0.1193\text{ft}^2)}{\pi\,(0.75/12)^2\ ft^2} = 38.88$$

Thus for $n \approx 39$ tubes and surface area of 91.06 ft^2, the length of each tube is calculated as,

$$A_s = n\pi D_i L \Rightarrow L = \frac{A_s}{n\pi D_i} = \frac{91.06\,\text{ft}^2}{39\pi\,(0.75/12)\text{ft}} = \mathbf{11.9\,ft}$$

The length of tube is greater than the given length constraint of 8 ft. Hence it is required to use more than one tube pass. For multiple pass heat exchangers we need to find the correction factor 'F' that accounts for the reduction in LMTD during each pass. The correction factor 'F' requires calculation of temperature ratios P and R from Equations 11-27 and 11-28, respectively.

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{110-80}{175-80} = 0.316 \quad \text{and} \quad R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{175-135}{110-80} = 1.33$$

Thus from Figure 11-19(a) we get

$F = 0.95$.

Based on this correction factor, the total surface area of the tubes is recalculated as,

$$A_s = \frac{\dot{Q}}{UF\Delta T_{lm}} = \frac{12\times 10^5\ \text{Btu/h}}{(220\ \text{Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(0.95)(59.9^\circ\text{F})} = 95.85\ \text{ft}^2$$

For two tube pass heat exchanger, the length of each tube per pass is calculated as,

$$A_s = 2n\pi D_i L \Rightarrow L = \frac{A_s}{2n\pi D_i} = \frac{95.85\ \text{ft}^2}{2(39)\,\pi\,(0.75/12)\text{ft}} = 6.26\ \text{ft}$$

This tube length is now within the given space constraints. Hence the final heat exchanger design specifications are,

Number of tubes per pass = **39**

Number of tube pass = **2**

Length of tube per pass = **6.26 ft**

11-146 Saturated liquid benzene is to be cooled using cold water at 15°C. For the given value of overall heat transfer coefficient, the surface area of the heat exchanger is to be determined for four different configurations.

Assumptions **1** Steady state conditions exist. **2** Heat exchanger is well insulated. **3** Fluid properties remain constant. **4** There is no fouling inside the heat exchanger.

Analysis Heat balance between saturated liquid benzene solution and cooling water gives,

Heat lost by liquid benzene = Heat gained by water

$$\dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

$$T_{c,out} = \frac{\dot{m}_h c_{ph}(T_{h,in} - T_{h,out})}{\dot{m}_c c_{pc}} + T_{c,in} = \frac{(5\,\text{kg/s})\,(1839\,\text{J/kg}\cdot\text{K})\,(75\text{ - }45)^{\circ}\text{C}}{3.5(\text{kg/s})\times 4187(\text{J/kg}\cdot\text{K})} + 15^{\circ}\text{C} = 33.82^{\circ}\text{C}$$

In order to use effectiveness-NTU method we first need to determine the heat capacity rates, capacity ratio and effectiveness of the heat exchanger.

The heat capacity rate of cold fluid (water) is

$$C_c = \dot{m}_c c_{pc} = (3.5\,\text{kg/s})(4187\,\text{J/kg}\cdot\text{K}) = 14654.5\ \text{W/K}$$

The heat capacity rate of the hot fluid (liquid benzene) is

$$C_h = \dot{m}_h c_{ph} = (5\,\text{kg/s})(1839\,\text{J/kg}\cdot\text{K}) = 9195\ \text{W/K}$$

Thus the capacity ratio is

$$c = \frac{C_{\min}}{C_{\max}} = \frac{9195}{14654.5} = 0.627$$

(a) For parallel flow arrangement,

$$\therefore \varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h(T_{h,in} - T_{h,out})}{C_{\min}(T_{h,in} - T_{c,in})} = \frac{75-45}{75-15} = 0.5$$

Therefore, for the known values of effectiveness and capacity ratio we can find the number of transfer units (NTU) from the relation given in Table 11-5.

$$NTU = -\frac{\ln[1-\varepsilon(1+c)]}{1+c} = -\frac{\ln[1-0.5(1+0.627)]}{1+0.627} = 1.032$$

From the definition of NTU we find the surface area as

$$A_s = \frac{NTU\ C_{\min}}{U} = \frac{1.032\,(9195\ \text{W/K})}{750\ \text{W/m}^2\cdot\text{K}} = \mathbf{12.65\ m^2}$$

Thus for a parallel flow arrangement with 12.65 m^2 surface area, the tube length is

$$L = \frac{A_s}{\pi D} = \frac{12.65(\text{m}^2)}{\pi \times 0.02(\text{m})} = 201.33\ \text{m}$$

(b) For counter flow arrangement, with ε = 0.5 from part (a), the number of transfer units (NTU) using the relation from Table 11-5 is found to be,

$$NTU = \frac{1}{c-1}\ln\left(\frac{\varepsilon-1}{c\varepsilon-1}\right) = \frac{1}{0.627-1}\ln\left(\frac{0.5-1}{0.627\times 0.5-1}\right) = 0.85$$

From the definition of NTU we find the surface area as

$$A_s = \frac{NTU\ C_{\min}}{U} = \frac{0.85\,(9195\ \text{W/K})}{750\ \text{W/m}^2\cdot\text{K}} = \mathbf{10.42\ m^2}$$

Thus for a counter flow arrangement with 10.42 m^2 surface area, the tube length is

$$L = \frac{A_s}{\pi D} = \frac{10.42(\text{m}^2)}{\pi \times 0.02(\text{m})} = 165.85\ \text{m}$$

(c) For two shell passes and 40 tube passes, the calculated effectiveness of 0.5 from part (a) is for 2 shell passes.

$$\therefore \varepsilon_2 = 0.5$$

Now using expressions from Table 11-5 we have

$$F = \left(\frac{c\varepsilon_2 - 1}{\varepsilon_2 - 1}\right)^{1/2} = \left(\frac{0.627 \times 0.5 - 1}{0.5 - 1}\right)^{0.5} = 1.172$$

The effectiveness of heat exchanger in case of one shell pass would be

$$\varepsilon_1 = \frac{F-1}{F-c} = \frac{1.172-1}{1.172-0.627} = 0.315$$

Based on this effectiveness for 1 shell pass we calculate the NTU_1 for 1 shell and 40 tube passes as follows.

$$NTU_1 = -\frac{1}{\sqrt{1+c^2}}\ln\left(\frac{2/\varepsilon_1 - 1 - c - \sqrt{1+c^2}}{2/\varepsilon_1 - 1 - c + \sqrt{1+c^2}}\right) = -\frac{1}{\sqrt{1+0.627^2}}\ln\left(\frac{2/0.315 - 1 - 0.627 - \sqrt{1+0.627^2}}{2/0.315 - 1 - 0.627 + \sqrt{1+0.627^2}}\right) = 0.433$$

Since for multiple passes, $NTU_2 = n(NTU_1)$ i.e., NTU is assumed to be distributed equally during each pass.

For two shell pass we have

$NTU_2 = 2\ (0.433) = 0.866$.

From the definition of NTU we find the surface area as

$$A_s = \frac{NTU_2\ C_{\min}}{U} = \frac{0.866\ (9195\ \text{W/K})}{750\ \text{W/m}^2\cdot\text{K}} = \mathbf{10.62\ m^2}$$

Thus for a two shell and 40 tube pass flow arrangement with 10.62 m^2 surface area, the tube length is

$$L = \frac{A_s}{n \times \pi D} = \frac{10.62(\text{m}^2)}{\pi \times 0.02(\text{m}) \times 40} = 4.22\ \text{m}$$

(d) For a cross flow heat exchanger with liquid benzene (mixed) and cooling water (unmixed), from Table 11-5 and ε = 0.5 from part (a), we get,

$$NTU = -\frac{\ln\left[c\ln(1-\varepsilon)+1\right]}{c} = -\frac{\ln\left[0.627\ln(1-0.5)+1\right]}{0.627} = 0.909$$

From the definition of NTU we find the surface area as

$$A_s = \frac{NTU\ C_{\min}}{U} = \frac{0.909\ (9195\ \text{W/K})}{750\ \text{W/m}^2\cdot\text{K}} = \mathbf{11.14\ m^2}$$

Discussion Although the heat exchanger surface area in case of counter flow arrangement is less than the shell and tube heat exchanger, the length of pipe required in this arrangement is impractical. Since in most of the cases available space is a major constraint, shell and tube heat exchanger may be preferred.

Special Topic: The Human Cardiovascular System as a Counter-Current Heat Exchanger

11-147C The cardiovascular counter-current mechanism is a method that allows for the blood returning to the human body core to be warmed back to core body temperature after losing heat near the skin. An artery with warmer blood is paired with a vein with cooler blood (although in some instances the temperature can be cooler in the artery and warmer in the vein). There is significant heat transfer from the artery to the vein to warm the blood. Remember that at the extremities, the blood temperature will be close to the environment temperature under "reasonable" scenarios.

11-148C There are two major differences. The first difference is that the fluid is the same throughout the "warm" side and the "cool" side. The second major difference is that the "warm" side's output is the input for the "cool" side. Therefore, the temperature differences are generally not as large as in conventional engineering heat exchangers. A minor difference is that most heat exchangers have multiple looping passes although it is possible to have single-pass exhcangers. The cardiovascular system is always paired with single passes. It is also common that one of the fluids surrounds the channel(s) with the other fluid; in the cardiovascular system both fluids are contained within their own channel and there is no direct bathing from between these fluids.

11-149C Clothing would insulate the skin blood vessels from the environment. Therefore, the skin temperature may be closer to the core body temperature instead of closer to the environmental temperature. This would effectively reduce the load on the heat exchanger. Extreme environmental conditions may cause the skin blood vessels to be significantly colder or warmer than the core body temperature. If the skin is relatively cold (< 10°C), the counter-current mechanisms would have a significant load needed to warm blood back to core temperature. If the skin is very hot (>45°C), then the cardiovascular counter-current heat exchanger would act to cool venous blood, instead of warm venous blood.

11-150 For the given dimensions of artery, vein and blood vessels total thermal resistance of the cardiovascular system is to be determined.

Assumptions **1** Steady state conditions exist. **2** Heat transfer coefficients of artery and vein are constant and uniform.

Analysis The total thermal resistance of the cardiovascular system can be formulated as,

$$R = \left(\frac{1}{hA}\right)_{artery} + \left(\frac{1}{hA}\right)_{vein} + \frac{\ln(D_{vein} / D_{artery})}{2\pi kL}$$

$$R = \frac{1}{(300\,\text{W/m}^2\cdot\text{K})\pi\,(0.0005\,\text{m})\,(0.05\,\text{m})} + \frac{1}{(190\,\text{W/m}^2\cdot\text{K})\pi\,(0.0006\,\text{m})\,(0.05\,\text{m})} + \frac{\ln(600/500)}{2\pi\,(0.670\,\text{W/m}\cdot\text{K})(0.05\,\text{m})} = \mathbf{99\,K/W}$$

11-151 Using the values provided in Prob. 11-150 and the calculated value of the total resistance, calculate the arterial and venous overall heat transfer coefficient.

Assumptions **1** Steady state conditions exist. **2** Heat transfer coefficients of artery and vein are constant and uniform.

Analysis The overall heat transfer coefficient for the arterial and venous is calculated from the following equations

$$Artery: U_i = \frac{1}{A_i R} = \frac{1}{\pi(0.0005\text{ m})(0.05\text{ m})\left(99\ \frac{\text{K}}{\text{W}}\right)} = \mathbf{128.61\,\frac{W}{m^2\cdot K}}$$

$$Vein: U_o = \frac{1}{A_i R} = \frac{1}{\pi(0.0006\text{ m})(0.05\text{ m})\left(99\ \frac{\text{K}}{\text{W}}\right)} = \mathbf{107.18\,\frac{W}{m^2\cdot K}}$$

where the value of total resistance of the cardiovascular system (R = 99 K/W) is obtained from the results of Prob. 11-150.

11-152 Reconsider Prob. 11-150 but with fouling resistance due to physiological inhomogeneities for both artery and vein.

Assumption **1** Steady state conditions exist. **2** Heat transfer coefficients of artery and vein are constant and uniform.

Analysis The total thermal resistance of the cardiovascular system can be formulated as,

$$R = \left(\frac{1}{hA}\right)_{artery} + \left(\frac{1}{hA}\right)_{vein} + \frac{\ln(D_{vein}/D_{artery})}{2\pi kL} + \left(\frac{R_f}{A}\right)_{artery} + \left(\frac{R_f}{A}\right)_{vein}$$

$$R = \frac{1}{(300\text{ W/m}^2\cdot\text{K})\pi(0.0005\text{ m})(0.05\text{ m})} + \frac{1}{(190\text{ W/m}^2\cdot\text{K})\pi(0.0006\text{ m})(0.05\text{ m})} + \frac{\ln(600/500)}{2\pi(0.670\text{ W/m}\cdot\text{K})(0.05\text{ m})}$$
$$+ \frac{0.0005\text{ m}^2\cdot\text{K/W}}{\pi\ (0.0005\text{ m})(0.05\text{ m})} + \frac{0.0003\text{ m}^2\cdot\text{K/W}}{\pi\,(0.0006\text{ m})(0.05\text{ m})} = \mathbf{108.7\ K/W}$$

11-153 The cardiovascular system used as a counter-current heat exchanger is used to warm venous blood. For the known temperature and mass flow rate of arterial blood, the overall blood vessel length is to be determined.

Assumption **1** Steady operating conditions. **2** Fluid properties remain constant **3** Negligible changes in the kinetic and potential energies.

Analysis The heat capacity rate of the hot and cold side of the heat exchanger is calculated as

$$C_h = \dot{m}_h c_{ph} = (0.005\,\text{kg/s})(3475\ \text{J/kg}\cdot\text{K}) = 17.38\,\text{W/K} \rightarrow C_{\max}$$

$$C_c = \dot{m}_c c_{pc} = (0.002\,\text{kg/s})(3475\,\text{J/kg}\cdot\text{K}) = 6.95\,\text{W/K} \rightarrow C_{\min}$$

Thus the capacity ratio is

$$c = \frac{C_{\min}}{C_{\max}} = \frac{6.95\,\text{W/K}}{17.38\ \text{W/K}} = 0.4$$

Now, the theoretical maximum heat transfer rate is calculated as

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in})$$

$$\therefore \dot{Q}_{\max} = (6.95\,\text{W/K})\,(37 - 28)°\text{C} = 62.55\,\text{W}$$

The actual heat transfer rate is,

$$\dot{Q}_{\max} = C_c(T_{c,out} - T_{c,in})$$

$$\therefore \dot{Q}_{\max} = (6.95\,\text{W/K})\,(35 - 28)^{\text{o}}\,\text{C} = 48.65\,\text{W}$$

From the definition of the effectiveness of the heat exchanger we get

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{48.65\,\text{W}}{62.55\,\text{W}} = 0.778$$

Now the number of transfer units can be obtained from the ε-NTU correlations in Table 11-5.

$$NTU = \frac{1}{c-1}\ln\left(\frac{\varepsilon - 1}{c\varepsilon - 1}\right) = \frac{1}{0.4-1}\ln\left(\frac{0.778-1}{0.4\times 0.778-1}\right) = 1.89$$

From the definition of NTU, we find the surface area of the heat exchanger as

$$A_s = \frac{NTU\ C_{\min}}{U} = \frac{1.89\,(6.95\,\text{W/K})}{125\,\text{W/m}^2\cdot\text{K}} = 0.105\,\text{m}^2$$

Hence, the length of the heat exchanger is

$$L = \frac{A_s}{\pi D} = \frac{0.105\ \text{m}^2}{\pi\,(0.05\text{m})} = \mathbf{0.668\,m}$$

11-154 The cardiovascular system used as a counter-current heat exchanger is used to warm venous blood. For the known values of overall heat transfer coefficient and inlet and exit temperatures of blood, the mass flow rate of venous and arterial blood is to be determined.

Assumptions **1** Steady state conditions exist. **2** Fluid (arterial and venous blood) properties are constant.

Analysis Since we do not know the mass flow rate of hot (arterial) and cold (venous) blood, we can find the heat transfer rate from,

$$\dot{Q} = UA_s \Delta T_{lm}$$

The log mean temperature difference for a counter-flow heat exchanger is determined as follows.

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

$$\Delta T_1 = T_{h,in} - T_{c,out} = 37 - 34 = 3^o C$$

and $$\Delta T_2 = T_{h,out} - T_{c,in} = 27 - 25 = 2^o C$$

$$\Delta T_{lm} = \frac{3-2}{\ln(3/2)} = 2.47^o C$$

Thus the actual rate of heat transfer is

$$\dot{Q} = UA_s \Delta T_{lm} = (100\ \mathrm{W/m^2 \cdot K})\,(0.15\mathrm{m^2})(2.47\ ^\circ\mathrm{C}) = 37.1\ \mathrm{W}$$

Now, the mass flow rate of hot (arterial) and cold (venous) blood can be determined from energy balance.

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in})$$

The mass flow rate of hot (arterial) blood is

$$\dot{m}_h = \frac{\dot{Q}}{c_{ph}(T_{h,in} - T_{h,out})} = \frac{37.1\mathrm{W}}{(3475\,\mathrm{J/kg \cdot K})\,(37-27)^o C} = \mathbf{1.06\,g/s}$$

The mass flow rate of cold (venous) blood is

$$\dot{m}_c = \frac{\dot{Q}}{c_{pc}(T_{cout} - T_{c,in})} = \frac{37.1\mathrm{W}}{(3475\,\mathrm{J/kg \cdot K})\,(34-25)^o C} = \mathbf{1.18\,g/s}$$

Review Problems

11-155 The inlet and outlet temperatures of the cold and hot fluids in a double-pipe heat exchanger are given. It is to be determined whether this is a parallel-flow or counter-flow heat exchanger.

Analysis In parallel-flow heat exchangers, the temperature of the cold water can never exceed that of the hot fluid. In this case $T_{\text{cold out}}$ = 50°C which is greater than $T_{\text{hot out}}$ = 45°C. Therefore this must be a counter-flow heat exchanger.

11-156 It is to be shown that when $\Delta T_1 = \Delta T_2$ for a heat exchanger, the ΔT_{lm} relation reduces to $\Delta T_{\text{lm}} = \Delta T_1 = \Delta T_2$.

Analysis When $\Delta T_1 = \Delta T_2$, we obtain

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{0}{0}$$

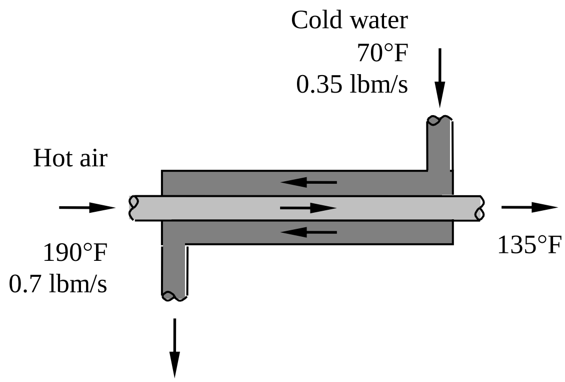
This case can be handled by applying L'Hospital's rule (taking derivatives of nominator and denominator separately with respect to ΔT_1 or ΔT_2). That is,

$$\Delta T_{lm} = \frac{d(\Delta T_1 - \Delta T_2) / d\Delta T_1}{d[\ln(\Delta T_1 / \Delta T_2)] / d\Delta T_1} = \frac{1}{1/\Delta T_1} = \Delta T_1 = \Delta T_2$$

11-157E Water is heated by solar-heated hot air in a double-pipe counter-flow heat exchanger. The required length of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and air are given to be 1.0 and 0.24 Btu/lbm.°F, respectively.



Analysis The rate of heat transfer in this heat exchanger is

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = (0.7\text{ lbm/s})(0.24\text{ Btu/lbm.°F})(190°\text{F} - 135°\text{F}) = 9.24\text{ Btu/s}$$

The outlet temperature of the cold water is

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{\dot{m}_c c_{pc}} = 70°\text{F} + \frac{9.24\text{ Btu/s}}{(0.35\text{ lbm/s})(1.0\text{ Btu/lbm.°F})} = 96.4°\text{F}$$

The temperature differences at the two ends are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 190°\text{F} - 96.4°\text{F} = 93.6°\text{F}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 135°\text{F} - 70°\text{F} = 65°\text{F}$$

The logarithmic mean temperature difference is

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{93.6 - 65}{\ln(93.6/65)} = 78.43°\text{F}$$

The heat transfer surface area on the outer side of the tube is determined from

$$\dot{Q} = UA_s \Delta T_{lm} \longrightarrow A_s = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{9.24\text{ Btu/s}}{(20/3600\text{ Btu/s.ft}^2\text{.°F})(78.43°\text{F})} = 21.21\text{ ft}^2$$

Then the length of the tube required becomes

$$A_s = \pi D L \longrightarrow L = \frac{A_s}{\pi D} = \frac{21.21\text{ ft}^2}{\pi(0.5/12\text{ ft})} = \mathbf{162.0\text{ ft}}$$

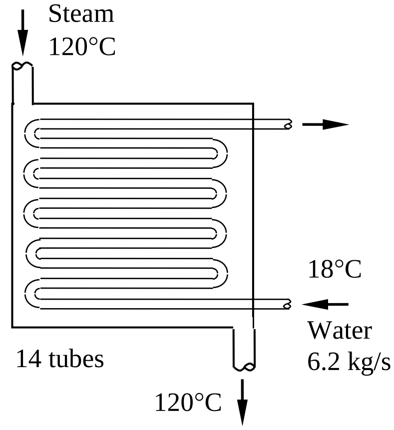
11-158 A shell-and-tube heat exchanger is used to heat water with geothermal steam condensing. The rate of heat transfer, the rate of condensation of steam, and the overall heat transfer coefficient are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The heat of vaporization of geothermal water at 120°C is given to be h_{fg} = 2203 kJ/kg and specific heat of water is given to be c_p = 4180 J/kg.°C.

Analysis (*a*) The outlet temperature of the water is

$$T_{\text{c,out}} = T_{\text{h,out}} - 46 = 120°\text{C} - 46°\text{C} = 74°\text{C}$$



Then the rate of heat transfer becomes

$$\dot{Q} = [\dot{m}c_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}}$$
$$= (6.2\text{ kg/s})(4.18\text{kJ/kg.°C})(74°\text{C} - 18°\text{C})$$
$$= \mathbf{1451 kW}$$

(*b*) The rate of condensation of steam is determined from

$$\dot{Q} = (\dot{m}h_{fg})_{\substack{\text{geothermal}\\ \text{steam}}}$$

$$1451\text{ kW} = \dot{m}(2203\text{ kJ/kg}) \longrightarrow \dot{m} = \mathbf{0.659\ kg/s}$$

(*c*) The heat transfer area is

$$A_i = n\pi D_i L = 14\pi(0.024\text{ m})(3.2\text{m}) = 3.378\text{ m}^2$$

The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are

$$\Delta T_1 = T_{\text{h,in}} - T_{\text{c,out}} = 120°\text{C} - 74°\text{C} = 46°\text{C}$$
$$\Delta T_2 = T_{\text{h,out}} - T_{\text{c,in}} = 120°\text{C} - 18°\text{C} = 102°\text{C}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{46 - 102}{\ln(46/102)} = 70.3°\text{C}$$

$$\left.\begin{array}{l} P = \dfrac{t_2 - t_1}{T_1 - t_1} = \dfrac{74 - 18}{120 - 18} = 0.55 \\ R = \dfrac{T_1 - T_2}{t_2 - t_1} = \dfrac{120 - 120}{74 - 18} = 0 \end{array}\right\} F = 1$$

Then the overall heat transfer coefficient is determined to be

$$\dot{Q} = U_i A_i F \Delta T_{\text{lm,CF}} \longrightarrow U_i = \frac{\dot{Q}}{A_i F \Delta T_{\text{lm,CF}}} = \frac{1{,}451{,}000\text{ W}}{(3.378\text{ m}^2)(1)(70.3°\text{C})} = \mathbf{6110\ W/m^2.°C}$$

11-159 Hot water is cooled by cold water in a 1-shell pass and 2-tube passes heat exchanger. The mass flow rates of both fluid streams are to be determined.

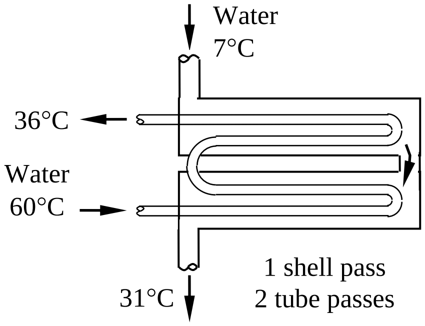
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant. **5** There is no fouling.

Properties The specific heats of both cold and hot water streams are taken to be 4.18 kJ/kg.°C.

Analysis The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 60°\text{C} - 31°\text{C} = 29°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 36°\text{C} - 7°\text{C} = 29°\text{C}$$



Since $\Delta T_1 = \Delta T_2$, we have $\Delta T_{lm,CF} = 29°\text{C}$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{31-60}{7-60} = 0.45 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{7-31}{36-60} = 1.0 \end{aligned}\right\} F = 0.88 \ \text{(Fig. 11-19)}$$

The rate of heat transfer in this heat exchanger is

$$\dot{Q} = UA_s F \Delta T_{lm,CF} = (950 \text{ W/m}^2.°\text{C})(15 \text{ m}^2)(0.88)(29°\text{C}) = 3.64 \times 10^5 \text{ W} = 364 \text{ kW}$$

The mass flow rates of fluid streams are

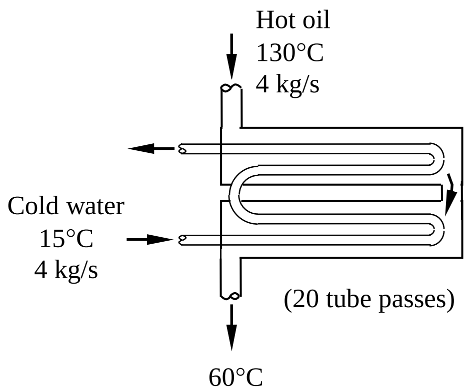
$$\dot{m}_c = \frac{\dot{Q}}{c_p (T_{out} - T_{in})} = \frac{364 \text{ kW}}{(4.18 \text{ kJ/kg.°C})(60°\text{C} - 36°\text{C})} = \mathbf{3.63 \text{ kg/s}}$$

$$\dot{m}_h = \frac{\dot{Q}}{c_p (T_{in} - T_{out})} = \frac{364 \text{ kW}}{(4.18 \text{ kJ/kg.°C})(31°\text{C} - 7°\text{C})} = \mathbf{3.63 \text{ kg/s}}$$

11-160 Water is heated by hot oil in a multi-pass shell-and-tube heat exchanger. The rate of heat transfer and the heat transfer surface area on the outer side of the tube are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the water and oil are given to be 4.18 and 2.2 kJ/kg.°C, respectively.



Analysis (*a*)The rate of heat transfer in this heat exchanger is

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,in} - T_{h,out}) = (4\text{ kg/s})(2.2\text{kJ/kg.°C})(130°\text{C} - 60°\text{C}) = \mathbf{616\text{ kW}}$$

(*b*) The outlet temperature of the cold water is

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{\dot{m}_c c_{pc}} = 15°\text{C} + \frac{616\text{ kW}}{(4\text{ kg/s})(4.18\text{kJ/kg.°C})} = 51.8°\text{C}$$

The temperature differences at the two ends are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 130°\text{C} - 51.8°\text{C} = 78.2°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 60°\text{C} - 15°\text{C} = 45°\text{C}$$

The logarithmic mean temperature difference is

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{78.2 - 45}{\ln(78.2/45)} = 60.1°\text{C}$$

and

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{51.8 - 15}{130 - 15} = 0.32 \\ R &= \frac{T_2 - T_1}{t_2 - t_1} = \frac{130 - 60}{51.8 - 15} = 1.90 \end{aligned}\right\} F = 0.97$$

The heat transfer surface area on the outer side of the tube is then determined from

$$\dot{Q} = UA_s F\Delta T_{lm} \longrightarrow A_s = \frac{\dot{Q}}{UF\Delta T_{lm}} = \frac{616\text{ kW}}{(0.22\text{ kW/m}^2\text{.°C})(0.97)(60.1°\text{C})} = \mathbf{48.0\text{ m}^2}$$

11-161 Water is used to cool a process stream in a shell and tube heat exchanger. The tube length is to be determined for one tube pass and four tube pass cases.

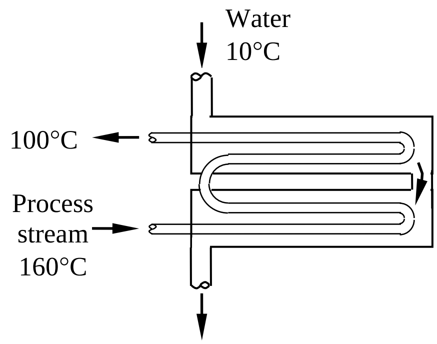
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The properties of process stream and water are given in problem statement.

Analysis (a) The rate of heat transfer is

$$\dot{Q} = \dot{m}_h c_h (T_{h,in} - T_{h,out}) = (47 \text{ kg/s})(3.5 \text{kJ/kg}\cdot°\text{C})(160 - 100)°\text{C} = 9870 \text{ kW}$$

The outlet temperature of water is determined from



$$\dot{Q} = \dot{m}_c c_c (T_{c,out} - T_{c,in})$$

$$T_{c,out} = T_{c,in} + \frac{\dot{Q}}{\dot{m}_c C_c} = 10°\text{C} + \frac{9870 \text{ kW}}{(66 \text{ kg/s})(4.18 \text{kJ/kg}\cdot°\text{C})} = 45.8°\text{C}$$

The logarithmic mean temperature difference is

$$\Delta T_1 = T_{h,in} - T_{c,out} = 160°\text{C} - 45.8°\text{C} = 114.2°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 100°\text{C} - 10°\text{C} = 90°\text{C}$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} = \frac{114.2 - 90}{\ln\left(\dfrac{114.2}{90}\right)} = 101.6°\text{C}$$

The Reynolds number is

$$V = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{N_{tube} \rho \pi D^2 / 4} = \frac{(47 \text{ kg/s})}{(100)(950 \text{ kg/m}^3)\pi(0.025 \text{ m})^2 / 4} = 1.008 \text{ m/s}$$

$$\text{Re} = \frac{VD\rho}{\mu} = \frac{(1.008 \text{ m/s})(0.025\text{m})(950 \text{ kg/m}^3)}{0.002 \text{ kg/m}\cdot\text{s}} = 11{,}968$$

which is greater than 10,000. Therefore, we have turbulent flow. We assume fully developed flow and evaluate the Nusselt number from

$$\text{Pr} = \frac{\mu c_p}{k} = \frac{(0.002 \text{ kg/m}\cdot\text{s})(3500 \text{J/kg}\cdot°\text{C})}{0.50 \text{ W/m}\cdot°\text{C}} = 14$$

$$Nu = \frac{hD}{k} = 0.023 \text{Re}^{0.8} \text{ Pr}^{0.3} = 0.023(11{,}968)^{0.8}(14)^{0.3} = 92.9$$

Heat transfer coefficient on the inner surface of the tubes is

$$h_i = \frac{k}{D} Nu = \frac{0.50 \text{ W/m.°C}}{0.025 \text{ m}}(92.9) = 1858 \text{ W/m}^2\text{.°C}$$

Disregarding the thermal resistance of the tube wall the overall heat transfer coefficient is determined from

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{1858} + \dfrac{1}{4000}} = 1269 \text{ W/m}^2\cdot°\text{C}$$

The correction factor for one shell pass and one tube pass heat exchanger is $F = 1$. The tube length is determined to be

$$\dot{Q} = UAF\Delta T_{lm}$$

$$9870 \text{ kW} = (1.269 \text{ kW/m}^2\cdot\text{C})[100\pi(0.025 \text{ m})L](1)(101.6°\text{C})$$

$$L = \mathbf{9.75 \text{ m}}$$

(b) For 1 shell pass and 4 tube passes, there are 100/4=25 tubes per pass and this will increase the velocity fourfold. We repeat the calculations for this case as follows:

$$V = 4\times1.008 = 4.032 \text{ m/s}$$

$$\text{Re} = 4\times11{,}968 = 47{,}872$$

$$Nu = \frac{hD}{k} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.3} = 0.023(47{,}872)^{0.8}(14)^{0.3} = 281.6$$

$$h_i = \frac{k}{D} Nu = \frac{0.50\ \mathrm{W/m.°C}}{0.025\,\mathrm{m}}(281.6) = 5632\ \mathrm{W/m^2.°C}$$

$$U = \frac{1}{\dfrac{1}{h_i}+\dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{5632}+\dfrac{1}{4000}} = 2339\ \mathrm{W/m^2\cdot °C}$$

The correction factor is determined from Fig. 11-19:

$$\left.\begin{aligned} P &= \frac{t_2-t_1}{T_1-t_1} = \frac{100-160}{10-160} = 0.4 \\ R &= \frac{T_1-T_2}{t_2-t_1} = \frac{10-45.8}{100-160} = 0.60 \end{aligned}\right\} F = 0.96$$

The tube length is determined to be

$$\dot{Q} = UAF\Delta T_{lm}$$

$$9870\,\mathrm{kW} = (2.339\,\mathrm{kW/m^2\cdot C})\left[100\pi(0.025\,\mathrm{m})L\right](0.96)(101.6°\mathrm{C})$$

$$L = \mathbf{5.51m}$$

11-162 A hydrocarbon stream is heated by a water stream in a 2-shell passes and 4-tube passes heat exchanger. The rate of heat transfer and the mass flow rates of both fluid streams and the fouling factor after usage are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

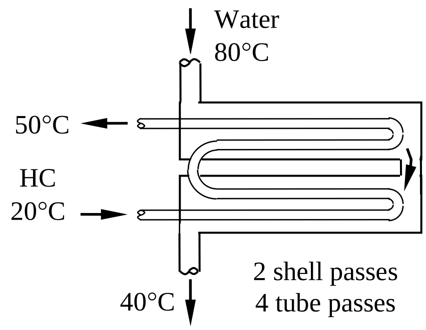
Properties The specific heat of HC is given to be 2 kJ/kg.°C. The specific heat of water is taken to be 4.18 kJ/kg.°C.

Analysis (a) The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 80°C - 50°C = 30°C$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 40°C - 20°C = 20°C$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{30-20}{\ln(30/20)} = 24.66°C$$



$$\left. \begin{array}{l} P = \dfrac{t_2 - t_1}{T_1 - t_1} = \dfrac{50-20}{80-20} = 0.5 \\ R = \dfrac{T_1 - T_2}{t_2 - t_1} = \dfrac{80-40}{50-20} = 1.33 \end{array} \right\} F = 0.90 \text{ (Fig. 11-19)}$$

The overall heat transfer coefficient of the heat exchanger is

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{1600} + \dfrac{1}{2500}} = 975.6 \text{ W/m}^2 \cdot °C$$

The rate of heat transfer in this heat exchanger is

$$\dot{Q} = UA_s F \Delta T_{lm,CF} = (975.6 \text{ W/m}^2.°C)[160\pi(0.02 \text{ m})(1.5 \text{ m})](0.90)(24.66°C) = 3.265 \times 10^5 \text{ W} = \mathbf{326.5 \text{ kW}}$$

The mass flow rates of fluid streams are

$$\dot{m}_c = \frac{\dot{Q}}{c_p (T_{out} - T_{in})} = \frac{326.5 \text{ kW}}{(2.0 \text{ kJ/kg.°C})(50°C - 20°C)} = \mathbf{5.44 \text{ kg/s}}$$

$$\dot{m}_h = \frac{\dot{Q}}{c_p (T_{in} - T_{out})} = \frac{326.5 \text{ kW}}{(4.18 \text{ kJ/kg.°C})(80°C - 40°C)} = \mathbf{1.95 \text{ kg/s}}$$

(b) The rate of heat transfer in this case is

$$\dot{Q} = [\dot{m} c_p (T_{out} - T_{in})]_c = (5.44 \text{ kg/s})(2.0 \text{ kJ/kg.°C})(45°C - 20°C) = 272 \text{ kW}$$

This corresponds to a 17% decrease in heat transfer. The outlet temperature of the hot fluid is

$$\dot{Q} = [\dot{m} c_p (T_{in} - T_{out})]_h$$

$$272 \text{ kW} = (1.95 \text{ kg/s})(4.18 \text{ kJ/kg.°C})(80°C - T_{h,out})$$

$$T_{h,out} = 46.6°C$$

The logarithmic temperature difference is

$$\Delta T_1 = T_{h,in} - T_{c,out} = 80°C - 45°C = 35°C$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 46.6°C - 20°C = 26.6°C$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{35-26.6}{\ln(35/26.6)} = 30.61°C$$

$$\left. \begin{array}{l} P = \dfrac{t_2 - t_1}{T_1 - t_1} = \dfrac{45-20}{80-20} = 0.42 \\ R = \dfrac{T_1 - T_2}{t_2 - t_1} = \dfrac{80-46.6}{45-20} = 1.34 \end{array} \right\} F = 0.97 \text{ (Fig. 11-19)}$$

The overall heat transfer coefficient is

$$\dot{Q} = UA_s F\Delta T_{lm,CF}$$

$$272{,}000\text{ W} = U\left[160\pi(0.02\text{ m})(1.5\text{m})\right](0.97)(30.61°\text{C})$$

$$U = 607.5\text{ W/m}^2.°\text{C}$$

The fouling factor is determined from

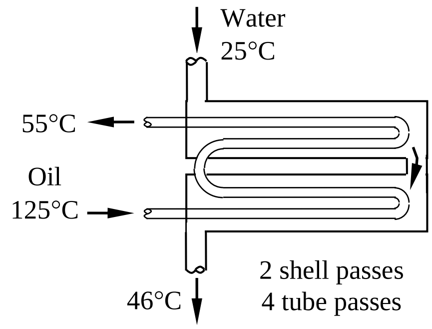
$$R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}} = \frac{1}{607.5} - \frac{1}{975.6} = \mathbf{6.21 \times 10^{-4}\ m^2 \cdot °C/W}$$

11-163 Oil is cooled by water in a 2-shell passes and 4-tube passes heat exchanger. The mass flow rate of water and the surface area are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant. **5** There is no fouling.

Properties The specific heat of oil is given to be 2 kJ/kg.°C. The specific heat of water is taken to be 4.18 kJ/kg.°C.

Analysis The logarithmic mean temperature difference for counter-flow arrangement and the correction factor F are



$$\Delta T_1 = T_{h,in} - T_{c,out} = 125°\text{C} - 46°\text{C} = 79°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 55°\text{C} - 25°\text{C} = 30°\text{C}$$

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{79-30}{\ln(79/30)} = 50.61°\text{C}$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{55-125}{25-125} = 0.7 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{25-46}{55-125} = 0.3 \end{aligned}\right\} F = 0.97 \text{ (Fig. 11-19)}$$

The rate of heat transfer is

$$\dot{Q} = \dot{m}_h c_h (T_{h,in} - T_{h,out}) = (10 \text{ kg/s})(2.0 \text{ kJ/kg}\cdot°\text{C})(125-55)°\text{C} = 1400 \text{ kW}$$

The mass flow rate of water is

$$\dot{m}_w = \frac{\dot{Q}}{c_p (T_{out} - T_{in})} = \frac{1400 \text{ kW}}{(4.18 \text{ kJ/kg.°C})(46°\text{C} - 25°\text{C})} = \mathbf{15.9 \text{ kg/s}}$$

The surface area of the heat exchanger is determined to be

$$\dot{Q} = UAF\Delta T_{lm}$$

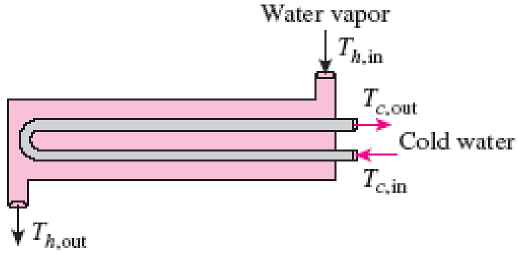
$$1400 \text{ kW} = (0.9 \text{ kW/m}^2 \cdot \text{C}) A_s (0.97)(50.61°\text{C})$$

$$A_s = \mathbf{31.7 \text{ m}^2}$$

11-164 Saturated water vapor condenses in a 1-shell and 2-tube heat exchanger, the outlet temperature of the cold water and the heat transfer rate for the heat exchanger are to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heat of the cold water is given to be c_{pc} = 4179 J/kg·K.



Analysis The log mean temperature difference for the counter-flow arrangement is

$$\Delta T_{\text{lm,CF}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{(100 - T_{c,\text{out}}) - (100 - 15)}{\ln[(100 - T_{c,\text{out}})/(100 - 15)]}$$

The heat transfer rate can be written as

$$\dot{Q} = UA_s F\Delta T_{\text{lm,CF}} = (2000\ \text{W/m}^2 \cdot \text{K})(0.5\ \text{m}^2)\frac{(100 - T_{c,\text{out}}) - (100 - 15)}{\ln[(100 - T_{c,\text{out}})/(100 - 15)]}\ \text{K} \qquad (1)$$

where F = 1 for condensation process. From energy balance, the heat transfer rate can also be written as

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,\text{out}} - T_{c,\text{in}}) = (0.5\ \text{kg/s})(4179\ \text{J/kg} \cdot \text{K})(T_{c,\text{out}} - 15)\ \text{K} \qquad (2)$$

The outlet temperature of the cold water and the heat transfer rate can be determined by solving Eqs. (1) and (2) simultaneously. Copy the following lines and paste on a blank EES screen:

```
Q_dot=(2000)*(0.5)*((100-T_co)-(100-15))/ln((100-T_co)/(100-15))
Q_dot=(0.5)*(4179)*(T_co-15)
```

Solving by EES software, we get

$$\dot{Q} = \mathbf{67600\ W}$$

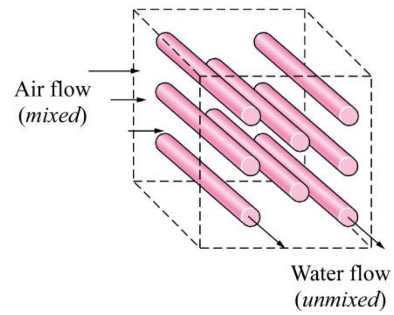
$$T_{c,\text{out}} = \mathbf{47.3°C}$$

Discussion The value of the correction factor is F = 1 for process involving phase-change (boiling or condensation).

11-165 A single-pass cross-flow heat exchanger uses hot air (mixed) to heat water (unmixed), and the required surface area of the heat exchanger is to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heat of water at the average temperature of 55°C is c_{pc} = 4183 J/kg·K (Table A-9); the specific heat of air at the average temperature of 160°C is c_{ph} = 1016 J/kg·K (Table A-15).



Analysis Using Fig. 11-19*d*, the correction factor can be determined to be

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{80-30}{220-30} = 0.26 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{220-100}{80-30} = 2.4 \end{aligned}\right\} F \approx 0.92 \qquad \text{(Fig. 11-19}d\text{)}$$

Using energy balance on the cold fluid, we have

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,\text{out}} - T_{c,\text{in}}) = (3\,\text{kg/s})(4183\,\text{J/kg}\cdot\text{K})(80-30)\,\text{K} = 6.275\times10^5\ \text{W}$$

The log mean temperature difference for the counter-flow arrangement is

$$\Delta T_{\text{lm,CF}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{(220-80)-(100-30)}{\ln[(220-80)/(100-30)]}\,°\text{C} = 101°\text{C}$$

Thus, the surface area can be determined using

$$\dot{Q} = UA_s F\Delta T_{\text{lm,CF}} \qquad \rightarrow \qquad A_s = \frac{\dot{Q}}{UF\Delta T_{\text{lm,CF}}}$$

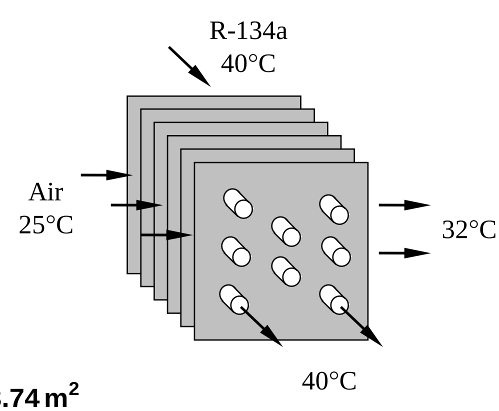
$$A_s = \frac{6.275\times10^5\ \text{W}}{(200\ \text{W/m}^2\cdot\text{K})(0.92)(101\,\text{K})} = \mathbf{33.7\,m^2}$$

Discussion If there is fouling, it will reduce the rate of heat transfer of the heat exchanger.

11-166 Refrigerant-134a is condensed by air in the condenser of a room air conditioner. The heat transfer area on the refrigerant side is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heat of air is given to be 1.005 kJ/kg.°C.



Analysis The temperature differences at the two ends are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 40°\text{C} - 32°\text{C} = 8°\text{C}$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 40°\text{C} - 25°\text{C} = 15°\text{C}$$

The logarithmic mean temperature difference is

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{8-15}{\ln(8/15)} = 11.1°\text{C}$$

The heat transfer surface area on the outer side of the tube is determined from

$$\dot{Q} = UA_s\Delta T_{lm} \longrightarrow A_s = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{(22{,}500/3600)\ \text{kW}}{(0.150\ \text{kW/m}^2.°\text{C})(11.1°\text{C})} = \mathbf{3.74\,m^2}$$

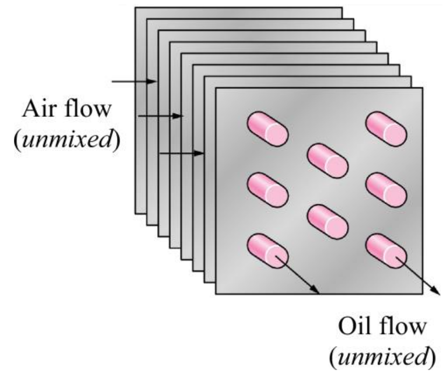
11-167 Oil in an engine is being cooled by air in a cross-flow heat exchanger, where both fluids are unmixed; with a specified correction factor, the outlet temperatures of the oil and air are to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heats of oil and air are given to be c_{ph} = 2047 J/kg·K and c_{pc} = 1007 J/kg·K, respectively.

Analysis On the shell side (air),

$$(\dot{m}_c c_{pc})_{\text{shell side}} = (0.21\text{ kg/s})(1007\text{ J/kg}\cdot\text{K}) = 211.5\text{ W/K}$$



On the tube side (oil),

$$(\dot{m}_h c_{ph})_{\text{tube side}} = (0.026\text{ kg/s})(2047\text{ J/kg}\cdot\text{K}) = 53.22\text{ W/K}$$

Then, we have

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}_h c_{ph})_{\text{tube side}}}{(\dot{m}_c c_{pc})_{\text{shell side}}} = \frac{53.22\text{ W/K}}{211.5\text{ W/K}} = 0.2516$$

With R = 0.25 and F = 0.96, using Fig. 11-19*c* yields

$$P = \frac{t_2 - t_1}{T_1 - t_1} \approx 0.60$$

The outlet temperature of the oil is

$$P = \frac{t_2 - t_1}{T_1 - t_1} \quad \rightarrow \quad T_{h,\text{out}} = t_2 = P(T_1 - t_1) + t_1 = (0.6)(30 - 75)°\text{C} + 75°\text{C} = \mathbf{48.0°C}$$

The outlet temperature of the air is

$$R = \frac{T_1 - T_2}{t_2 - t_1} \quad \rightarrow \quad T_{c,\text{out}} = T_2 = T_1 - R(t_2 - t_1) = 30°\text{C} - (0.2516)(48 - 75)°\text{C} = \mathbf{36.8°C}$$

Discussion The outlet temperatures can be determined using the effectiveness-NTU method without knowing the value of the correction factor (F).

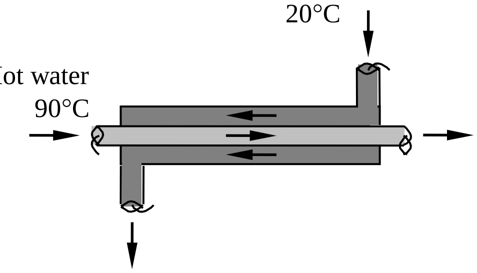
11-168 A water-to-water counter-flow heat exchanger is considered. The outlet temperature of the cold water, the effectiveness of the heat exchanger, the mass flow rate of the cold water, and the heat transfer rate are to be determined.

.***Assumptions*** **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of both the cold and the hot water are given to be 4.18 kJ/kg.°C.

Analysis (*a*) The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = 1.5\dot{m}_c (4.18\,\text{kJ/kg.°C}) = 6.27\dot{m}_c$$
$$C_c = \dot{m}_c c_{pc} = \dot{m}_c (4.18\,\text{kJ/kg.°C}) = 4.18\dot{m}_c$$



Therefore,

$$C_{\min} = C_c = 4.18\dot{m}_c$$

and

$$C = \frac{C_{\min}}{C_{\max}} = \frac{4.18\dot{m}_c}{6.27\dot{m}_c} = 0.667$$

The rate of heat transfer can be expressed as

$$\dot{Q} = C_c (T_{\text{c,out}} - T_{\text{c,in}}) = (4.18\dot{m}_c)(T_{\text{c,out}} - 20)$$
$$\dot{Q} = C_h (T_{\text{h,in}} - T_{\text{h,out}}) = (6.27\dot{m}_c)\left[90 - (T_{\text{c,out}} + 15)\right] = (6.27\dot{m}_c)(75 - T_{\text{c,out}})$$

Setting the above two equations equal to each other we obtain the outlet temperature of the cold water

$$\dot{Q} = 4.18\dot{m}_c (T_{c,out} - 20) = 6.27\dot{m}_c (75 - T_{c,out})$$
$$4.18 (T_{c,out} - 20) = 6.27 (75 - T_{c,out}) \longrightarrow T_{c,out} = \mathbf{53.0°C}$$

(*b*) The effectiveness of the heat exchanger is determined from

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c (T_{\text{c,out}} - T_{\text{c,in}})}{C_c (T_{\text{h,in}} - T_{\text{c,in}})} = \frac{4.18\dot{m}_c (53.0 - 20)}{4.18\dot{m}_c (90 - 20)} = \mathbf{0.471}$$

(*c*) The NTU of this heat exchanger is determined from

$$NTU = \frac{1}{c-1}\ln\left(\frac{\varepsilon - 1}{\varepsilon c - 1}\right) = \frac{1}{0.667 - 1}\ln\left(\frac{0.471 - 1}{0.471 \times 0.667 - 1}\right) = 0.780$$

Then, from the definition of NTU, we obtain the mass flow rate of the cold fluid:

$$NTU = \frac{UA_s}{C_{\min}} \longrightarrow 0.780 = \frac{2.200\,\text{kW/°C}}{4.18\dot{m}_c} \longrightarrow \dot{m}_c = \mathbf{0.675\,kg/s}$$

(*d*) The rate of heat transfer is determined from

$$\dot{Q} = \dot{m}_c c_{pc} (T_{\text{c,out}} - T_{\text{c,in}}) = (0.675\,\text{kg/s})(4.18\,\text{kJ/kg.°C})(53 - 20)°\text{C} = \mathbf{93.1kW}$$

11-169 Water is heated by geothermal water in a double-pipe counter-flow heat exchanger. The mass flow rate of the geothermal water and the outlet temperatures of both fluids are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the geothermal water and the cold water are given to be 4.25 and 4.18 kJ/kg.°C, respectively.

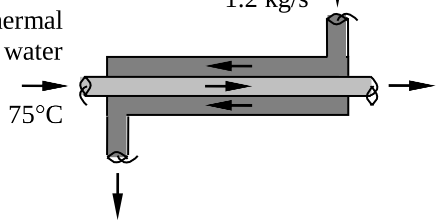
Analysis The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = \dot{m}_h (4.25\,\text{kJ/kg.°C}) = 4.25\dot{m}_h$$

$$C_c = \dot{m}_c c_{pc} = (1.2\,\text{kg/s})(4.18\,\text{kJ/kg.°C}) = 5.016\,\text{kW/°C}$$

$$C_{\min} = C_c = 5.016\,\text{kW/°C}$$

and $$c = \frac{C_{\min}}{C_{\max}} = \frac{5.016}{4.25\dot{m}_h} = \frac{1.1802}{\dot{m}_h}$$



The NTU of this heat exchanger is

$$NTU = \frac{UA_s}{C_{\min}} = \frac{(0.480\,\text{kW/m}^2.\text{°C})(25\,\text{m}^2)}{5.016\,\text{kW/°C}} = 2.392$$

Using the effectiveness relation, we find the capacity ratio

$$\varepsilon = \frac{1-\exp[-\text{NTU}(1-c)]}{1-c\exp[-\text{NTU}(1-c)]} \longrightarrow 0.823 = \frac{1-\exp[-2.392(1-c)]}{1-c\exp[-2.392(1-c)]} \longrightarrow c = 0.494$$

Then the mass flow rate of geothermal water is determined from

$$c = \frac{1.1802}{\dot{m}_h} \longrightarrow 0.494 = \frac{1.1802}{\dot{m}_h} \longrightarrow \dot{m}_h = \mathbf{2.39\,kg/s}$$

The maximum heat transfer rate is

$$\dot{Q}_{\max} = C_{\min}(T_{\text{h,in}} - T_{\text{c,in}}) = (5.016\,\text{kW/°C})(75\text{°C} - 17\text{°C}) = 290.9\,\text{kW}$$

Then the actual rate of heat transfer rate becomes

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = (0.823)(290.9\,\text{kW}) = 239.4\,\text{kW}$$

The outlet temperatures of the geothermal and cold waters are determined to be

$$\dot{Q} = C_c(T_{\text{c,out}} - T_{\text{c,in}}) \longrightarrow 239.4\,\text{kW} = (5.016\,\text{kW/°C})(T_{\text{c,out}} - 17) \longrightarrow T_{\text{c,out}} = \mathbf{64.7°C}$$

$$\dot{Q} = \dot{m}_h c_{ph}(T_{\text{h,in}} - T_{\text{h,out}})$$

$$239.4\,\text{kW} = (2.39\,\text{kg/s})(4.25\,\text{kJ/kg.°C})(75 - T_{\text{h,out}}) \longrightarrow T_{\text{h,out}} = \mathbf{51.4°C}$$

11-170 A cross-flow heat exchanger with both fluids unmixed has a specified overall heat transfer coefficient, (*a*) the exit temperature of the hot fluid and (*b*) the rate of heat transfer in the heat exchanger are to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.

Analysis (*a*) The heat capacity rates are given as

$$C_h = C_{\min} = 40{,}000 \text{ W/K} \qquad \text{and} \qquad C_c = C_{\max} = 80{,}000 \text{ W/K}$$

The capacity ratio is

$$c = \frac{C_{\min}}{C_{\max}} = \frac{C_h}{C_c} = \frac{40{,}000 \text{ W/K}}{80{,}000 \text{ W/K}} = 0.5$$

The NTU of the heat exchanger is

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{(200 \text{ W/m}^2\cdot\text{K})(400 \text{ m}^2)}{40{,}000 \text{ W/K}} = 2.0$$

Using the equation listed in Table 11-4, the heat transfer effectiveness is

$$\begin{aligned}\varepsilon &= 1-\exp\left\{\frac{\text{NTU}^{0.22}}{c}[\exp(-c\,\text{NTU}^{0.78})-1]\right\}\\ &= 1-\exp\left(\frac{2.0^{0.22}}{0.5}\{\exp[-(0.5)(2.0)^{0.78}]-1\}\right)\\ &= 0.7388\end{aligned}$$

From the definition of heat transfer effectiveness,

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_h(T_{h,\text{in}}-T_{h,\text{out}})}{C_{\min}(T_{h,\text{in}}-T_{c,\text{in}})} = \frac{C_h(T_{h,\text{in}}-T_{h,\text{out}})}{C_h(T_{h,\text{in}}-T_{c,\text{in}})}$$

$$T_{h,\text{out}} = T_{h,\text{in}} - \varepsilon(T_{h,\text{in}}-T_{c,\text{in}}) = 80°\text{C} - (0.7388)(80°\text{C}-20°\text{C}) = \mathbf{35.7°C}$$

(*b*) The rate of heat transfer in the heat exchanger is

$$\dot{Q} = C_h(T_{h,\text{in}}-T_{h,\text{out}}) = (40{,}000 \text{ W/K})(80°\text{C}-35.7°\text{C}) = \mathbf{1.77\times10^6\ W}$$

Discussion The rate of heat transfer in the heat exchanger can also be calculated using

$$\dot{Q} = C_c(T_{c,\text{out}}-T_{c,\text{in}})$$

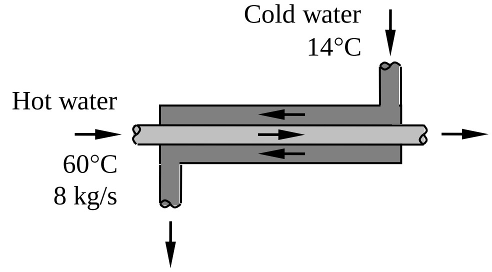
11-171 A water-to-water heat exchanger is proposed to preheat the incoming cold water by the drained hot water in a plant to save energy. The heat transfer rating of the heat exchanger and the amount of money this heat exchanger will save are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The specific heat of the hot water is given to be 4.18 kJ/kg.°C.

Analysis The maximum rate of heat transfer is

$$\begin{aligned}\dot{Q}_{\max} &= \dot{m}_h c_{ph}(T_{h,in} - T_{c,in})\\ &= (8/60\ \text{kg/s})(4.18\,\text{kJ/kg.°C})(60°\text{C} - 14°\text{C})\\ &= 25.6\ \text{kW}\end{aligned}$$



Noting that the heat exchanger will recover 72% of it, the actual heat transfer rate becomes

$$\dot{Q} = \varepsilon\dot{Q}_{\max} = (0.72)(25.6\ \text{kJ/s}) = \mathbf{18.43\ kW}$$

which is the heat transfer rating. The operating hours per year are

The annual operating hours = (8 h/day)(5 days/week)(52 week/year) = 2080 h/year

The energy saved during the entire year will be

Energy saved = (heat transfer rate)(operating time)

= (18.43 kJ/s)(2080 h/year)(3600 s/h)

= 1.38x10^8 kJ/year

Then amount of fuel and money saved will be

$$\text{Fuel saved} = \frac{\text{Energy saved}}{\text{Furnace efficiency}} = \frac{1.38\times10^8\ \text{kJ/year}}{0.78}\left(\frac{1\ \text{therm}}{105{,}500\ \text{kJ}}\right) = 1677\ \text{therms/year}$$

Money saved = (fuel saved)(the price of fuel)

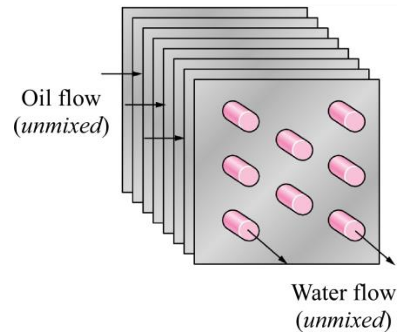
= (1677 therms/year)($1.00/therm)

= **$1677/year**

11-172 A single-pass cross-flow heat exchanger with both fluids unmixed, (*a*) the NTU value and (*b*) the value of the overall heat transfer coefficient are to be determined.

Assumptions **1** Steady operating condition exists. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible. **3** Fluid properties are constant. **4** Changes in the kinetic and potential energies of fluid streams are negligible.

Properties The properties of oil are given to be c_{ph} = 1.93 kJ/kg·K and ρ = 870 kg/m^3.



Analysis (*a*) The mass flow rate of oil (hot fluid) is

$$\dot{m}_h = \rho\dot{\boldsymbol{V}} = (870\ \text{kg/m}^3)(0.19\ \text{m}^3/\text{min})(1/60\ \text{min/s}) = 2.755\ \text{kg/s}$$

The heat capacity rate for the hot fluid is

$$C_h = \dot{m}_h c_{ph} = (2.755\ \text{kg/s})(1930\ \text{J/kg}\cdot\text{K}) = 5317\ \text{W/K}$$

Using energy balance, we have

$$C_c(T_{c,\text{out}} - T_{c,\text{in}}) = C_h(T_{h,\text{in}} - T_{h,\text{out}})$$

$$\rightarrow \qquad \frac{C_c}{C_h} = \frac{T_{h,\text{in}} - T_{h,\text{out}}}{T_{c,\text{out}} - T_{c,\text{in}}} = \frac{38-29}{33-16} = 0.5294$$

or $\qquad c = \dfrac{C_c}{C_h} = \dfrac{C_{\min}}{C_{\max}} = 0.5294$

The heat transfer effectiveness is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c(T_{c,\text{out}} - T_{c,\text{in}})}{C_{\min}(T_{h,\text{in}} - T_{c,\text{in}})} = \frac{C_c(T_{c,\text{out}} - T_{c,\text{in}})}{C_c(T_{h,\text{in}} - T_{c,\text{in}})} = \frac{33-16}{38-16} = 0.7727$$

From Table 11-4, the NTU value can be determined from

$$\varepsilon = 1 - \exp\left\{\frac{\text{NTU}^{0.22}}{c}[\exp(-c\ \text{NTU}^{0.78}) - 1]\right\}$$

Copy the following lines and paste on a blank EES screen to solve the above equation:

```
c=0.5294
epsilon=0.7727
epsilon=1-exp(NTU^0.22/c*(exp(-c*NTU^0.78)-1))
```

Solving by EES software, we get

NTU = **2.39**

(*b*) The value of the overall heat transfer coefficient is

$$\text{NTU} = \frac{UA_s}{C_{\min}} \qquad \rightarrow \qquad U = \text{NTU}\frac{C_{\min}}{A_s} = \text{NTU}\frac{cC_h}{A_s}$$

$$U = (2.39)\frac{(0.5294)(5317\ \text{W/K})}{20\ \text{m}^2} = \mathbf{336\ W/m^2\cdot K}$$

Discussion Using Figure 11-27*c*, the NTU value is found to be approximately NTU ≈ 2.4.

11-173 Air is to be heated by hot oil in a cross-flow heat exchanger with both fluids unmixed. The effectiveness of the heat exchanger, the mass flow rate of the cold fluid, and the rate of heat transfer are to be determined.

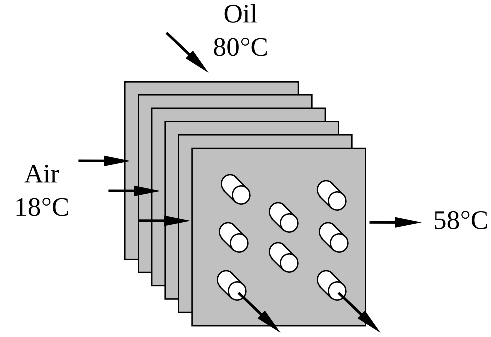
Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** The overall heat transfer coefficient is constant and uniform.

Properties The specific heats of the air and the oil are given to be 1.006 and 2.15 kJ/kg.°C, respectively.

Analysis (*a*) The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = 0.5\dot{m}_c(2.15\,\text{kJ/kg.°C}) = 1.075\dot{m}_c$$

$$C_c = \dot{m}_c c_{pc} = \dot{m}_c(1.006\,\text{kJ/kg.°C}) = 1.006\dot{m}_c$$



Therefore,

$$C_{\min} = C_c = 1.006\dot{m}_c$$

and

$$c = \frac{C_{\min}}{C_{\max}} = \frac{1.006\dot{m}_c}{1.075\dot{m}_c} = 0.936$$

The effectiveness of the heat exchanger is determined from

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{C_c(T_{\text{c,out}} - T_{\text{c,in}})}{C_c(T_{\text{h,in}} - T_{\text{c,in}})} = \frac{58-18}{80-18} = \mathbf{0.645}$$

(*b*) The NTU of this heat exchanger is expressed as

$$NTU = \frac{UA_s}{C_{\min}} = \frac{(0.750\,\text{kW/°C})}{1.006\dot{m}_c} = \frac{0.7455}{\dot{m}_c}$$

The NTU of this heat exchanger can also be determined from

$$NTU = -\frac{\ln\left[c\ln(1-\varepsilon)+1\right]}{c} = -\frac{\ln\left[0.936\times\ln(1-0.645)+1\right]}{0.936} = 3.724$$

Then the mass flow rate of the air is determined to be

$$\text{NTU} = \frac{UA_s}{C_{\min}} \longrightarrow 3.724 = \frac{(0.750\,\text{kW/°C})}{1.006\dot{m}_c} \longrightarrow \dot{m}_c = \mathbf{0.20\,kg/s}$$

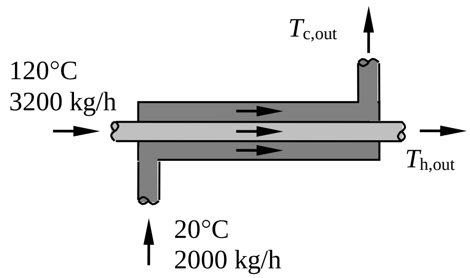
(*c*) The rate of heat transfer is determined from

$$\dot{Q} = \dot{m}_c c_{pc}(T_{\text{c,out}} - T_{\text{c,in}}) = (0.20\,\text{kg/s})(1.006\,\text{kJ/kg.°C})(58\text{-}18)\text{°C} = \mathbf{8.05\,kW}$$

11-174 The inlet conditions of hot and cold fluid streams in a heat exchanger are given. The outlet temperatures of both streams are to be determined using LMTD and the effectiveness-NTU methods.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant.

Properties The specific heats of hot and cold fluid streams are given to be 2.0 and 4.2 kJ/kg.°C, respectively.



Analysis (a) The rate of heat transfer can be expressed as

$$\dot{Q} = \dot{m}c_p(T_{h,in} - T_{h,out}) = (3200/3600\text{ kg/s})(2.0\text{kJ/kg.°C})(120 - T_{h,out}) = 1.778(120 - T_{h,out}) \qquad (1)$$

$$\dot{Q} = \dot{m}c_p(T_{c,out} - T_{c,in}) = (2000/3600\text{ kg/s})(4.2\text{kJ/kg.°C})(T_{c,out} - 20) = 2.333(T_{c,out} - 20) \qquad (2)$$

The heat transfer can also be expressed using the logarithmic mean temperature difference as

$$\Delta T_1 = T_{h,in} - T_{c,in} = 120°\text{C} - 20°\text{C} = 100°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,out}$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} = \frac{100 - (T_{h,out} - T_{c,out})}{\ln\left(\dfrac{100}{T_{h,out} - T_{c,out}}\right)}$$

$$\dot{Q} = UA\Delta T_{lm} = \frac{\dot{Q}_{hc,m}}{A\Delta T_{lm}}$$

$$= (2.0\text{ kW/m}^2\cdot°\text{C})(0.50\text{ m}^2)\frac{100 - (T_{h,out} - T_{c,out})}{\ln\left(\dfrac{100}{T_{h,out} - T_{c,out}}\right)} = \frac{100 - (T_{h,out} - T_{c,out})}{\ln\left(\dfrac{100}{T_{h,out} - T_{c,out}}\right)} \qquad (3)$$

Now we have three expressions for heat transfer with three unknowns: $\dot{Q}$, $T_{h,out}$, $T_{c,out}$. Solving them using an equation solver such as EES, we obtain

$$\dot{Q} = 63.45\text{ kW}$$

$$T_{h,out} = \mathbf{84.3°C}$$

$$T_{c,out} = \mathbf{47.2°C}$$

(b) The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = (3200/3600\text{ kg/s})(2.0\text{kJ/kg.°C}) = 1.778\text{ kW/°C}$$

$$C_c = \dot{m}_c c_{pc} = (2000/3600\text{ kg/s})(4.2\text{kJ/kg.°C}) = 2.333\text{ kW/°C}$$

Therefore

$$C_{\min} = C_h = 1.778\text{ kW/°C}$$

which is the smaller of the two heat capacity rates. The heat capacity ratio and the NTU are

$$c = \frac{C_{\min}}{C_{\max}} = \frac{1.778}{2.333} = 0.7621$$

$$NTU = \frac{UA}{C_{\min}} = \frac{(2.0\,\text{kW/m}^2\cdot\text{C})(0.50\,\text{m}^2)}{1.778\,\text{kW/°C}} = 0.5624$$

The effectiveness of this parallel-flow heat exchanger is

$$\varepsilon = \frac{1-\exp\left[-NTU(1+c)\right]}{1+c} = \frac{1-\exp\left[-(0.5624)(1+0.7621)\right]}{1+0.7621} = 0.3568$$

The maximum heat transfer rate is

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (1.778\,\text{kW/°C})(120°\text{C} - 20°\text{C}) = 177.8\,\text{kW}$$

The actual heat transfer rate is

$$\dot{Q} = \varepsilon\dot{Q}_{\max} = (0.3568)(177.8) = 63.44\,\text{kW}$$

Then the outlet temperatures are determined to be

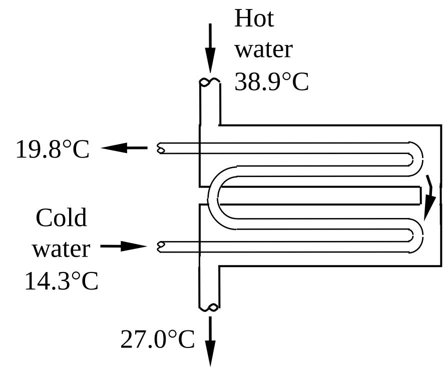
$$\dot{Q} = C_c(T_{c,out} - T_{c,in}) \longrightarrow T_{c,out} = T_{c,in} + \frac{\dot{Q}}{C_c} = 20°\text{C} + \frac{63.44\,\text{kW}}{2.333\,\text{kW/°C}} = \mathbf{47.2°C}$$

$$\dot{Q} = C_h(T_{h,in} - T_{h,out}) \longrightarrow T_{h,out} = T_{h,in} - \frac{\dot{Q}}{C_h} = 120°\text{C} - \frac{63.44\,\text{kW}}{1.778\,\text{kW/°C}} = \mathbf{84.3°C}$$

Discussion The results obtained by two methods are same as expected. However, the effectiveness-NTU method is easier for this type of problems.

11-175 The inlet and exit temperatures and the volume flow rates of hot and cold fluids in a heat exchanger are given. The rate of heat transfer to the cold water, the overall heat transfer coefficient, the fraction of heat loss, the heat transfer efficiency, the effectiveness, and the NTU of the heat exchanger are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Changes in the kinetic and potential energies of fluid streams are negligible. **3** Fluid properties are constant.



Properties The densities of hot water and cold water at the average temperatures of (38.9+27.0)/2 = 33.0°C and (14.3+19.8)/2 = 17.1°C are 994.8 and 998.6 kg/m^3, respectively. The specific heat at the average temperature is 4178 J/kg.°C for hot water and 4184 J/kg.°C for cold water (Table A-9).

Analysis (a) The mass flow rates are

$$\dot{m}_h = \rho_h \dot{V}_h = (994.8\ \text{kg/m}^3)(0.0025/60\ \text{m}^3/\text{s}) = 0.04145\ \text{kg/s}$$

$$\dot{m}_c = \rho_c \dot{V}_c = (998.6\ \text{kg/m}^3)(0.0045/60\ \text{m}^3/\text{s}) = 0.07490\ \text{kg/s}$$

The rates of heat transfer from the hot water and to the cold water are

$$\dot{Q}_h = [\dot{m}c_p(T_{in} - T_{out})]_\text{h} = (0.04145\ \text{kg/s})(4178\ \text{kJ/kg.°C})(38.9°\text{C} - 27.0°\text{C}) = 2061\ \text{W}$$

$$\dot{Q}_c = [\dot{m}c_p(T_{out} - T_{in})]_\text{c} = (0.07490\ \text{kg/s})(4184\ \text{kJ/kg.°C})(19.8°\text{C} - 14.3°\text{C}) = \mathbf{1724\ W}$$

(b) The logarithmic mean temperature difference and the overall heat transfer coefficient are

$$\Delta T_1 = T_{h,in} - T_{c,out} = 38.9°\text{C} - 19.8°\text{C} = 19.1°\text{C}$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 27.0°\text{C} - 14.3°\text{C} = 12.7°\text{C}$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{19.1 - 12.7}{\ln\left(\frac{19.1}{12.7}\right)} = 15.68°\text{C}$$

$$U = \frac{\dot{Q}_{hc,m}}{A\Delta T_{lm}} = \frac{(1724 + 2061)/2\ \text{W}}{(0.056\ \text{m}^2)(15.68°\text{C})} = \mathbf{2155\ W/m^2 \cdot C}$$

Note that we used the average of two heat transfer rates in calculations.

(c) The fraction of heat loss and the heat transfer efficiency are

$$f_{loss} = \frac{\dot{Q}_h - \dot{Q}_c}{\dot{Q}_h} = \frac{2061 - 1724}{2061} = 0.164 = \mathbf{16.4\%}$$

$$\eta = \frac{\dot{Q}_c}{\dot{Q}_h} = \frac{1724}{2061} = 0.836 = \mathbf{83.6\%}$$

(d) The heat capacity rates of the hot and cold fluids are

$$C_h = \dot{m}_h c_{ph} = (0.04145\ \text{kg/s})(4178\ \text{kJ/kg.°C}) = 173.2\ \text{W/°C}$$

$$C_c = \dot{m}_c c_{pc} = (0.07490\ \text{kg/s})(4184\ \text{kJ/kg.°C}) = 313.4\ \text{W/°C}$$

Therefore

$$C_{\min} = C_h = 173.2\ \text{W/°C}$$

which is the smaller of the two heat capacity rates. Then the maximum heat transfer rate becomes

$$\dot{Q}_{\max} = C_{\min}(T_{h,in} - T_{c,in}) = (173.2\ \text{W/°C})(38.9°\text{C} - 14.3°\text{C}) = 4261\ \text{W}$$

The effectiveness of the heat exchanger is

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{(1724 + 2061)/2\ \text{kW}}{4261\ \text{kW}} = 0.444 = \mathbf{44.4\%}$$

One again we used the average heat transfer rate. We could have used the smaller or greater heat transfer rates in calculations. The NTU of the heat exchanger is determined from

$$NTU = \frac{UA}{C_{\min}} = \frac{(2155\ \text{W/m}^2 \cdot \text{C})(0.056\ \text{m}^2)}{173.2\ \text{W/°C}} = \mathbf{0.697}$$

Fundamentals of Engineering (FE) Exam Problems

11-176 Saturated water vapor at 40°C is to be condensed as it flows through the tubes of an air-cooled condenser at a rate of 0.2 kg/s. The condensate leaves the tubes as a saturated liquid at 40°C. The rate of heat transfer to air is

(a) 34 kJ/s (b) 268 kJ/s (c) 453 kJ/s (d) 481 kJ/s (e) 515 kJ/s

Answer (d) 481 kJ/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T1=40 [C]
m_dot=0.2 [kg/s]
h_f=ENTHALPY(Steam_IAPWS,T=T1,x=0)
h_g=ENTHALPY(Steam_IAPWS,T=T1,x=1)
h_fg=h_g-h_f
Q_dot=m_dot*h_fg
"Wrong Solutions:"
W1_Q=m_dot*h_f  "Using hf"
W2_Q=m_dot*h_g "Using hg"
W3_Q=h_fg "not using mass flow rate"
W4_Q=m_dot*(h_f+h_g) "Adding hf and hg"
```

11-177 Consider a double-pipe heat exchanger with a tube diameter of 10 cm and negligible tube thickness. The total thermal resistance of the heat exchanger was calculated to be 0.025 ºC/W when it was first constructed. After some prolonged use, fouling occurs at both the inner and outer surfaces with the fouling factors 0.00045 $m^2 \cdot$ºC/W and 0.00015 $m^2 \cdot$ºC/W, respectively. The percentage decrease in the rate of heat transfer in this heat exchanger due to fouling is

(a) 2.3% (b) 6.8% (c) 7.1% (d) 7.6% (e) 8.5%

Answer (c) 7.1%

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.10 [m]
R_old=0.025 [C/W]
R_f_i=0.00045 [m^2-C/W]
R_f_o=0.00015 [m^2-C/W]
L=1 [m] "Consider a unit length"
A=pi*D*L
R_fouling=R_f_i/A+R_f_o/A
R_new=R_old+R_fouling
U_old=1/(R_old*A)
U_new=1/(R_new*A)
PercentDecrease=(U_old-U_new)/U_old*Convert(, %)
"Some Wrong Solutions with Common Mistakes"
W1_PercentDecrease=R_fouling/R_old*Convert(, %) "Comparing fouling resistance to old resistance"
W2_R_fouling=R_f_i+R_f_o "Treating fouling factors as fouling resistances"
W2_R_new=R_old+W2_R_fouling
W2_U_new=1/(W2_R_new*A)
W2_PercentDecrease=(U_old-W2_U_new)/U_old*Convert(, %)
```

11-178 Hot water coming from the engine is to be cooled by ambient air in a car radiator. The aluminum tubes in which the water flows have a diameter of 4 cm and negligible thickness. Fins are attached on the outer surface of the tubes in order to increase the heat transfer surface area on the air side. The heat transfer coefficients on the inner and outer surfaces are 2000 and 150 W/m^2·ºC, respectively. If the effective surface area on the finned side is 12 times the inner surface area, the overall heat transfer coefficient of this heat exchanger based on the inner surface area is

(a) 760 W/m^2·ºC (b) 832 W/m^2·ºC (c) 947 W/m^2·ºC (d) 1075 W/m^2·ºC (e) 1210 W/m^2·ºC

Answer (c) 947 W/m^2·ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.04 [m]
h_i=2000 [W/m^2-C]
h_o=150 [W/m^2-C]
A_i=1 [m^2]
A_o=12 [m^2]
1/(U_i*A_i)=1/(h_i*A_i)+1/(h_o*A_o) "Wall resistance is negligible"

"Some Wrong Solutions with Common Mistakes"
W1_U_i=h_i "Using h_i as the answer"
W2_U_o=h_o "Using h_o as the answer"
W3_U_o=1/2*(h_i+h_o) "Using the average of h_i and h_o as the answer"
```

11-179 A heat exchanger is used to heat cold water entering at 12°C at a rate of 1.2 kg/s by hot air entering at 90°C at rate of 2.5 kg/s. The highest rate of heat transfer in the heat exchanger is

(a) 82 kW (b) 156 kW (c) 195 kW (d) 224 kW (e) 391 kW

Answer (c) 195 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
cp_c=4.18 [kJ/kg-C]
cp_h=1.0 [kJ/kg-C]
Tc_in=12 [C]
Th_in=90 [C]
m_c=1.2 [kg/s]
m_h=2.5 [kg/s]
"From Q_max relation, Q_max=C_min(Th,in-Tc,in)"
Cc=m_c*cp_c
Ch=m_h*cp_h
C_min=min(Cc, Ch)
Q_max=C_min*(Th_in-Tc_in)

"Some Wrong Solutions with Common Mistakes:"
C_max=max(Cc, Ch)
W1Q_max=C_max*(Th_in-Tc_in) "Using Cmax"
```

11-180 Cold water (c_p = 4.18 kJ/kg·ºC) enters a heat exchanger at 15ºC at a rate of 0.5 kg/s where it is heated by hot air (c_p = 1.0 kJ/kg·ºC) that enters the heat exchanger at 50ºC at a rate of 1.8 kg/s. The maximum possible heat transfer rate in this heat exchanger is

(a) 51.1 kW (b) 63.0 kW (c) 66.8 kW (d) 73.2 kW (e) 80.0 kW

Answer (b) 63.0 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_c_in=15 [C]
m_dot_c=0.5 [kg/s]
c_p_c=4.18 [kJ/kg-C]
T_h_in=50 [C]
m_dot_h=1.8 [kg/s]
c_p_h=1.0 [kJ/kg-C]
C_c=m_dot_c*c_p_c
C_h=m_dot_h*c_p_h
C_min=min(C_c, C_h)
Q_dot_max=C_min*(T_h_in-T_c_in)

"Some Wrong Solutions with Common Mistakes"
W1_C_min=C_c "Using the greater heat capacity in the equation"
W1_Q_dot_max=W1_C_min*(T_h_in-T_c_in)
```

11-181 Hot oil (c_p = 2.1 kJ/kg·°C) at 110°C and 12 kg/s is to be cooled in a heat exchanger by cold water (c_p = 4.18 kJ/kg·°C) entering at 10°C and at a rate of 2 kg/s. The lowest temperature that oil can be cooled in this heat exchanger is

(a) 10°C (b) 24°C (c) 47°C (d) 61°C (e) 77°C

Answer (e) 77°C

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
cp_c=4.18 [kJ/kg-C]
cp_h=2.1 [kJ/kg-C]
Tc_in=10 [C]
Th_in=110 [C]
m_c=2 [kg/s]
m_h=12 [kg/s]
"From Q_max relation, Q_max=C_min(Th,in-Tc,in)"
Cc=m_c*cp_c
Ch=m_h*cp_h
C_min=min(Cc, Ch)
Q_max=C_min*(Th_in-Tc_in)
Q_max=Ch*(Th_in-Th_out)

“Some Wrong Solutions with Common Mistakes:”
C_max=max(Cc, Ch)
W1Q_max=C_max*(Th_in-Tc_in) “Using Cmax”
W1Q_max=Ch*(Th_in-W1Th_out)
```

11-182 Cold water (c_p = 4.18 kJ/kg·ºC) enters a counter-flow heat exchanger at 18ºC at a rate of 0.7 kg/s where it is heated by hot air (c_p = 1.0 kJ/kg·ºC) that enters the heat exchanger at 50ºC at a rate of 1.6 kg/s and leaves at 25ºC. The maximum possible outlet temperature of the cold water is

(a) 25.0ºC (b) 32.0ºC (c) 35.5ºC (d) 39.7ºC (e) 50.0ºC

Answer (c) 35.5ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_c_in=18 [C]
m_dot_c=0.7 [kg/s]
c_p_c=4.18 [kJ/kg-C]
T_h_in=50 [C]
T_h_out=25 [C]
m_dot_h=1.6 [kg/s]
c_p_h=1.0 [kJ/kg-C]
C_c=m_dot_c*c_p_c
C_h=m_dot_h*c_p_h
C_min=min(C_c, C_h)
Q_dot_max=C_min*(T_h_in-T_c_in)
Q_dot_max=C_c*(T_c_out_max-T_c_in)
"Some Wrong Solutions with Common Mistakes"
W1_C_min=C_c "Using the greater heat capacity in the equation"
W1_Q_dot_max=W1_C_min*(T_h_in-T_c_in)
W1_Q_dot_max=C_c*(W1_T_c_out_max-T_c_in)
W2_T_c_out_max=T_h_in "Using T_h_in as the answer"
W3_T_c_out_max=T_h_out "Using T_h_in as the answer"
```

11-183 A heat exchanger is used to condense steam coming off the turbine of a steam power plant by cold water from a nearby lake. The cold water (c_p = 4.18 kJ/kg·ºC) enters the condenser at 16ºC at a rate of 42 kg/s and leaves at 25ºC while the steam condenses at 45ºC. The condenser is not insulated and it is estimated that heat at a rate of 8 kW is lost from the condenser to the surrounding air. The rate at which the steam condenses is

(a) 0.228 kg/s (b) 0.318 kg/s (c) 0.426 kg/s (d) 0.525 kg/s (e) 0.663 kg/s

Answer (e) 0.663 kg/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_c_in=16 [C]
T_c_out=25 [C]
m_dot_c=42 [kg/s]
c_p_c=4.18 [kJ/kg-C]
T_h=45 [C]
Q_dot_lost=8 [kW]
Q_dot_c=m_dot_c*c_p_c*(T_c_out-T_c_in) "Heat picked up by the cold fluid"
Q_dot_h=Q_dot_c+Q_dot_lost "Heat given up by the hot fluid"
h_fg=2395 [kJ/kg] "Table A-9"
m_dot_cond=Q_dot_h/h_fg

"Some Wrong Solutions with Common Mistakes"
W1_m_dot_cond=Q_dot_c/h_fg "Ignoring heat loss from the heat exchanger"
```

11-184 An air handler is a large unmixed heat exchanger used for comfort control in large buildings. In one such application, chilled water (c_p = 4.2 kJ/kg·K) enters an air handler at 5ºC and leaves at 12ºC with a flow rate of 1000 kg/h. This cold water cools 5000 kg/h of air (c_p = 1.0 kJ/kg·K) which enters the air handler at 25ºC. If these streams are in counter-flow and the water stream conditions remain fixed, the minimum temperature at the air outlet is

(a) 5ºC (b) 12ºC (c) 19ºC (d) 22°C (e) 25ºC

Answer (c) 19ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
cp_c=4.2 [kJ/kg-K]
T_c_in=5 [C]
T_c_out=12 [C]
m_dot_c=1000/3600 "[kg/s]"
m_dot_h=5000/3600 "[kg/s]"
cp_h=1.0 [kJ/kg-K]
T_h_in=25 [C]
Q_dot=m_dot_c*cp_c*(T_c_out-T_c_in)
Q_dot=m_dot_h*cp_h*(T_h_in-T_h_out)
```

11-185 An air handler is a large unmixed heat exchanger used for comfort control in large buildings. In one such application, chilled water (c_p = 4.2 kJ/kg·K) enters an air handler at 5ºC and leaves at 12ºC with a flow rate of 1000 kg/hr. This cold water cools air (c_p = 1.0 kJ/kg·K) from 25ºC to 15ºC. The rate of heat transfer between the two streams is

(a) 8.2 kW (b) 23.7 kW (c) 33.8 kW (d) 44.8 kW (e) 52.8 kW

Answer (a) 8.2 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
cp_c=4.2 [kJ/kg-K]
T_c_in=5 [C]
T_c_out=12 [C]
m_dot_c=1000/3600 "[kg/s]"
cp_h=1.0 [kJ/kg-K]
T_h_in=25 [C]
T_h_out=15 [C]
Q_dot=m_dot_c*cp_c*(T_c_out-T_c_in)
```

11-186 In a parallel-flow, liquid-to-liquid heat exchanger, the inlet and outlet temperatures of the hot fluid are 150ºC and 90ºC while that of the cold fluid are 30ºC and 70ºC, respectively. For the same overall heat transfer coefficient, the percentage decrease in the surface area of the heat exchanger if counter-flow arrangement is used is

(a) 3.9% (b) 9.7% (c) 14.5% (d) 19.7% (e) 24.6%

Answer (e) 24.6%

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_h_in=150 [C]
T_h_out=90 [C]
T_c_in=30 [C]
T_c_out=70 [C]
"Parallel flow arrangement"
DELTAT_1_p=T_h_in-T_c_in
DELTAT_2_p=T_h_out-T_c_out
DELTAT_lm_p=(DELTAT_1_p-DELTAT_2_p)/ln(DELTAT_1_p/DELTAT_2_p)
"Counter flow arrangement"
DELTAT_1_c=T_h_in-T_c_out
DELTAT_2_c=T_h_out-T_c_in
DELTAT_lm_c=(DELTAT_1_c-DELTAT_2_c)/ln(DELTAT_1_c/DELTAT_2_c)
PercentDecrease=(DELTAT_lm_c-DELTAT_lm_p)/DELTAT_lm_p*Convert(, %)
"From Q_dot = U*A_s *DELTAT_lm, for the same Q_dot and U, DELTAT_lm and A_s are inversely proportional."

"Some Wrong Solutions with Common Mistakes"
W_PercentDecrease=(DELTAT_lm_c-DELTAT_lm_p)/DELTAT_lm_c*Convert(, %) "Dividing the difference by DELTAT_lm_c "
```

11-187 A counter-flow heat exchanger is used to cool oil (c_p = 2.20 kJ/kg·ºC) from 110ºC to 85ºC at a rate of 0.75 kg/s by cold water (c_p = 4.18 kJ/kg·ºC) that enters the heat exchanger at 20ºC at a rate of 0.6 kg/s. If the overall heat transfer coefficient is 800 W/m²·ºC, the heat transfer area of the heat exchanger is

(a) 0.745 m² (b) 0.760 m² (c) 0.775 m² (d) 0.790 m² (e) 0.805 m²

Answer (a) 0.745 m²

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_h_in=110 [C]
T_h_out=85 [C]
m_dot_h=0.75 [kg/s]
c_p_h=2.20 [kJ/kg-C]
T_c_in=20 [C]
m_dot_c=0.6 [kg/s]
c_p_c=4.18 [kJ/kg-C]
U=0.800 [kW/m^2-C]
Q_dot=m_dot_h*c_p_h*(T_h_in-T_h_out)
Q_dot=m_dot_c*c_p_c*(T_c_out-T_c_in)
DELTAT_1=T_h_in-T_c_out
DELTAT_2=T_h_out-T_c_in
DELTAT_lm=(DELTAT_1-DELTAT_2)/ln(DELTAT_1/DELTAT_2)
Q_dot=U*A_s*DELTAT_lm
```

11-188 The radiator in an automobile is a cross-flow heat exchanger (UA_s = 10 kW/K) that uses air (c_p = 1.00 kJ/kg·K) to cool the engine coolant fluid (c_p = 4.00 kJ/kg·K). The engine fan draws 30ºC air through this radiator at a rate of 12 kg/s while the coolant pump circulates the engine coolant at a rate of 5 kg/s. The coolant enters this radiator at 80ºC. Under these conditions, what is the number of transfer units (NTU) of this radiator?

(a) 2.0 (b) 2.5 (c) 3.0 (d) 3.5 (e) 4.0

Answer (b) 2.5

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
UA=30 [kW/K]
m_dot_a=12 [kg/s]
c_p_a=1.0 [kJ/kg-K]
m_dot_c=5 [kg/s]
c_p_c=4.0 [kJ/kg-K]
C_a=m_dot_a*c_p_a
C_c=m_dot_c*c_p_c
C_min=C_a
NTU=UA/C_min
```

11-189 In a parallel-flow, water-to-water heat exchanger, the hot water enters at 75ºC at a rate of 1.2 kg/s and cold water enters at 20ºC at a rate of 0.9 kg/s. The overall heat transfer coefficient and the surface area for this heat exchanger are 750 W/m^2·ºC and 6.4 m^2, respectively. The specific heat for both the hot and cold fluid may be taken to be 4.18 kJ/kg·ºC. For the same overall heat transfer coefficient and the surface area, the increase in the effectiveness of this heat exchanger if counter-flow arrangement is used is

(a) 0.09 (b) 0.11 (c) 0.14 (d) 0.17 (e) 0.19

Answer (a) 0.09

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_h_in=75 [C]
m_dot_h=1.2 [kg/s]
T_c_in=20 [C]
m_dot_c=0.9 [kg/s]
c_p=4.18 [kJ/kg-C]
U=0.750 [kW/m^2-C]
A_s=6.4 [m^2]
C_h=m_dot_h*c_p
C_c=m_dot_c*c_p
C_min=min(C_c, C_h)
C_max=max(C_c, C_h)
c=C_min/C_max
NTU=(U*A_s)/C_min
epsilon_p=(1-exp((-NTU)*(1+c)))/(1+c)
epsilon_c=(1-exp((-NTU)*(1-c)))/(1-c*exp((-NTU)*(1-c)))
Increase_epsilon=epsilon_c-epsilon_p
```

11-190 In a parallel-flow heat exchanger, the NTU is calculated to be 2.5. The lowest possible effectiveness for this heat exchanger is

(a) 10% (b) 27% (c) 41% (d) 50% (e) 92%

Answer (d) 50%

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
NTU=2.5
c=1 "The effectiveness is lowest when c = 1"
epsilon=(1-exp((-NTU)*(1+c)))/(1+c)

"Some Wrong Solutions with Common Mistakes"
W_epsilon=1-exp(-NTU) "Finding maximum effectiveness when c=0"
```

11-191 Cold water (c_p = 4.18 kJ/kg·ºC) enters a counter-flow heat exchanger at 10ºC at a rate of 0.35 kg/s where it is heated by hot air (c_p = 1.0 kJ/kg·ºC) that enters the heat exchanger at 50ºC at a rate of 1.9 kg/s and leaves at 25ºC. The effectiveness of this heat exchanger is

(a) 0.50 (b) 0.63 (c) 0.72 (d) 0.81 (e) 0.89

Answer (d) 0.81

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_c_in=10 [C]
m_dot_c=0.35 [kg/s]
c_p_c=4.18 [kJ/kg-C]
T_h_in=50 [C]
T_h_out=25 [C]
m_dot_h=1.9 [kg/s]
c_p_h=1.0 [kJ/kg-C]
C_c=m_dot_c*c_p_c
C_h=m_dot_h*c_p_h
C_min=min(C_c, C_h)
Q_dot_max=C_min*(T_h_in-T_c_in)
Q_dot=m_dot_h*c_p_h*(T_h_in-T_h_out)
epsilon=Q_dot/Q_dot_max

"Some Wrong Solutions with Common Mistakes"
W1_C_min=C_h "Using the greater heat capacity in the equation"
W1_Q_dot_max=W1_C_min*(T_h_in-T_c_in)
W1_epsilon=Q_dot/W1_Q_dot_max
```

11-192 Steam is to be condensed on the shell side of a 2-shell-passes and 8-tube-passes condenser, with 20 tubes in each pass. Cooling water enters the tubes at a rate of 2 kg/s. If the heat transfer area is 14 m^2 and the overall heat transfer coefficient is 1800 $W/m^2 \cdot °C$, the effectiveness of this condenser is

(a) 0.70 (b) 0.80 (c) 0.90 (d) 0.95 (e) 1.0

Answer (d) 0.95

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
cp_c=4.18 [kJ/kg-C]
m_c=2 [kg/s]
A=14
U=1.8 [kW/m^2-K]
"From NTU and Effectivenss relations for counterflow HX:"
C_min=m_c*cp_c
NTU=U*A/C_min
Eff=1-Exp(-NTU)
```

11-193 Water is boiled at 150ºC in a boiler by hot exhaust gases (c_p = 1.05 kJ/kg·ºC) that enter the boiler at 540ºC at a rate of 0.4 kg/s and leaves at 200ºC. The surface area of the heat exchanger is 0.64 m^2. The overall heat transfer coefficient of this heat exchanger is

(a) 880 $W/m^2 \cdot ºC$ (b) 1120 $W/m^2 \cdot ºC$ (c) 1350 $W/m^2 \cdot ºC$ (d) 2120 $W/m^2 \cdot ºC$ (e) 1840 $W/m^2 \cdot ºC$

Answer (c) 1350 $W/m^2 \cdot ºC$

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_w=150 [C]
T_h_in=540 [C]
T_h_out=200 [C]
m_dot_h=0.4 [kg/s]
c_p_h=1.05 [kJ/kg-C]
A_s=0.64 [m^2]
C_h=m_dot_h*c_p_h
C_min=C_h
Q_dot_max=C_min*(T_h_in-T_w)
Q_dot=C_h*(T_h_in-T_h_out)
epsilon=Q_dot/Q_dot_max
NTU=-ln(1-epsilon)
U=(NTU*C_min)/A_s
```

11-194 An air-cooled condenser is used to condense isobutane in a binary geothermal power plant. The isobutane is condensed at 85ºC by air (c_p = 1.0 kJ/kg·ºC) that enters at 22ºC at a rate of 18 kg/s. The overall heat transfer coefficient and the surface area for this heat exchanger are 2.4 kW/m^2·ºC and 2.6 m^2, respectively. The outlet temperature of the air is

(a) 35.6ºC (b) 40.5ºC (c) 52.1ºC (d) 58.5ºC (e) 62.8ºC

Answer (b) 40.5ºC

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T_h=85 [C]
T_c_in=22 [C]
m_dot_c=18 [kg/s]
c_p_c=1.0 [kJ/kg-C]
U=2.4 [kW/m^2-C]
A_s=2.6 [m^2]
C_c=m_dot_c*c_p_c
C_min =C_c
NTU=(U*A_s)/C_min
epsilon=1-exp(-NTU)
Q_dot_max=C_min*(T_h-T_c_in)
Q_dot=epsilon*Q_dot_max
Q_dot=m_dot_c*c_p_c*(T_c_out-T_c_in)
```

11-195 . . . 11-201 Design and Essay Problems

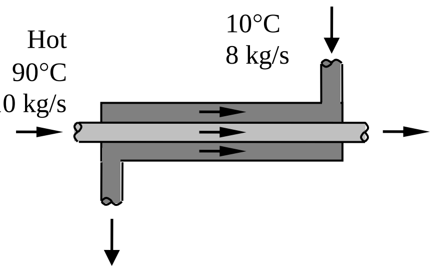
11-201 A counter flow double-pipe heat exchanger is used for cooling a liquid stream by a coolant. The rate of heat transfer and the outlet temperatures of both fluids are to be determined. Also, a replacement proposal is to be analyzed.

Assumptions **1** Steady operating conditions exist. **2** The heat exchanger is well-insulated so that heat loss to the surroundings is negligible and thus heat transfer from the hot fluid is equal to the heat transfer to the cold fluid. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant. **5** There is no fouling.

Properties The specific heats of hot and cold fluids are given to be 3.15 and 4.2 kJ/kg.°C, respectively.

Analysis (a) The overall heat transfer coefficient is

$$U = \frac{600}{\dfrac{1}{\dot{m}_c^{0.8}} + \dfrac{2}{\dot{m}_h^{0.8}}} = \frac{600}{\dfrac{1}{8^{0.8}} + \dfrac{2}{10^{0.8}}} = 1185 \text{ W/m}^2.\text{K}$$



The rate of heat transfer may be expressed as

$$\dot{Q} = \dot{m}_c c_c (T_{c,out} - T_{c,in}) = (8)(4200)(T_{c,out} - 10) \qquad (1)$$

$$\dot{Q} = \dot{m}_h c_h (T_{h,in} - T_{h,out}) = (10)(3150)(90 - T_{h,out}) \qquad (2)$$

It may also be expressed using the logarithmic mean temperature difference as

$$\dot{Q} = UA\Delta T_{lm} = UA\frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = (1185)(9)\frac{(90 - T_c) - (T_h - 10)}{\ln\left(\dfrac{90 - T_c}{T_h - 10}\right)} \qquad (3)$$

We have three equations with three unknowns, solving an equation solver such as EES, we obtain

$$\dot{Q} = \mathbf{6.42 \times 10^5\ W}, \quad T_{c,out} = \mathbf{29.1°C}, \quad T_{h,out} = \mathbf{69.6°C}$$

(b) The overall heat transfer coefficient for each unit is

$$U = \frac{600}{\dfrac{1}{\dot{m}_c^{0.8}} + \dfrac{2}{\dot{m}_h^{0.8}}} = \frac{600}{\dfrac{1}{4^{0.8}} + \dfrac{2}{5^{0.8}}} = 680.5 \text{ W/m}^2.\text{K}$$

Then

$$\dot{Q} = \dot{m}_c c_c (T_{c,out} - T_{c,in}) = (2 \times 4)(4200)(T_{c,out} - 10) \qquad (1)$$

$$\dot{Q} = \dot{m}_h c_h (T_{h,in} - T_{h,out}) = (2 \times 5)(3150)(90 - T_{h,out}) \qquad (2)$$

$$\dot{Q} = UA\Delta T_{lm} = UA\frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = (680.5)(2 \times 5)\frac{(90 - T_c) - (T_h - 10)}{\ln\left(\dfrac{90 - T_c}{T_h - 10}\right)} \qquad (3)$$

Once again, we have three equations with three unknowns, solving an equation solver such as EES, we obtain

$$\dot{Q} = \mathbf{4.5 \times 10^5\ W}, \quad T_{c,out} = \mathbf{23.4°C}, \quad T_{h,out} = \mathbf{75.7°C}$$

Discussion Despite a higher heat transfer area, the new heat transfer is about 30% lower. This is due to much lower *U*, because of the halved flow rates. So, the vendor's recommendation is not acceptable. The vendor's unit will do the job provided that they are connected in series. Then the two units will have the same *U* as in the existing unit.

Solutions Manual for

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Chapter 12
FUNDAMENTALS OF THERMAL RADIATION

Electromagnetic and Thermal Radiation

12-1C Electromagnetic waves are caused by accelerated charges or changing electric currents giving rise to electric and magnetic fields. Sound waves are caused by disturbances. Electromagnetic waves can travel in vacuum, sound waves cannot.

12-2C Electromagnetic waves are characterized by their frequency ν and wavelength λ. These two properties in a medium are related by $\lambda = c/\nu$ where *c* is the speed of light in that medium.

12-3C Thermal radiation is the radiation emitted as a result of vibrational and rotational motions of molecules, atoms and electrons of a substance, and it extends from about 0.1 to 100 μm in wavelength. Unlike the other forms of electromagnetic radiation, thermal radiation is emitted by bodies because of their temperature.

12-4C Microwaves in the range of 10^2 to 10^5 μm are very suitable for use in cooking as they are reflected by metals, transmitted by glass and plastics and absorbed by food (especially water) molecules. Thus the electric energy converted to radiation in a microwave oven eventually becomes part of the internal energy of the food with no conduction and convection thermal resistances involved. In conventional cooking, on the other hand, conduction and convection thermal resistances slow down the heat transfer, and thus the heating process.

12-5C Visible light is a kind of electromagnetic wave whose wavelength is between 0.40 and 0.76 μm. It differs from the other forms of electromagnetic radiation in that it triggers the sensation of seeing in the human eye.

12-6C Light (or visible) radiation consists of narrow bands of colors from violet to red. The color of a surface depends on its ability to reflect certain wavelength. For example, a surface that reflects radiation in the wavelength range 0.63-0.76 μm while absorbing the rest appears red to the eye. A surface that reflects all the light appears white while a surface that absorbs the entire light incident on it appears black. The color of a surface at room temperature is not related to the radiation it emits.

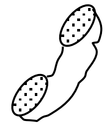
12-7C Because the snow reflects almost all of the visible and ultraviolet radiation, and the skin is exposed to radiation both from the sun and from the snow.

12-8C Infrared radiation lies between 0.76 and 100 μm whereas ultraviolet radiation lies between the wavelengths 0.01 and 0.40 μm. The human body does not emit any radiation in the ultraviolet region since bodies at room temperature emit radiation in the infrared region only.

12-9C Radiation in opaque solids is considered surface phenomena since only radiation emitted by the molecules in a very thin layer of a body at the surface can escape the solid.

12-10 A cordless telephone operates at a frequency of 8.5×10^8 Hz. The wavelength of these telephone waves is to be determined.

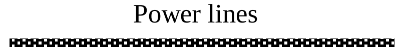
Analysis The wavelength of the telephone waves is



$$\lambda = \frac{c}{v} = \frac{2.998\times10^8 \text{ m/s}}{8.5\times10^8 \text{ Hz}(1/\text{s})} = 0.353\,\text{m} = \mathbf{353\,mm}$$

12-11 Electricity is generated and transmitted in power lines at a frequency of 50 Hz. The wavelength of the electromagnetic waves is to be determined.

Analysis The wavelength of the electromagnetic waves is



$$\lambda = \frac{c}{v} = \frac{2.998\times10^8 \text{ m/s}}{50\,\text{Hz}(1/\text{s})} = \mathbf{5.996\times10^6\ m}$$

12-12 The speeds of light in air, water, and glass are to be determined.

Analysis The speeds of light in air, water and glass are

Air: $$c = \frac{c_0}{n} = \frac{3.0\times10^8 \text{ m/s}}{1} = \mathbf{3.0\times10^8\ m/s}$$

Water: $$c = \frac{c_0}{n} = \frac{3.0\times10^8 \text{ m/s}}{1.33} = \mathbf{2.26\times10^8\ m/s}$$

Glass: $$c = \frac{c_0}{n} = \frac{3.0\times10^8 \text{ m/s}}{1.5} = \mathbf{2.0\times10^8\ m/s}$$

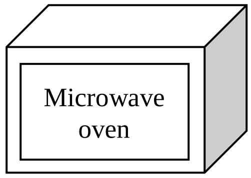
12-13 A radio station is broadcasting radiowaves at a wavelength of 150 m. The frequency of these waves is to be determined.

Analysis The frequency of the waves is determined from

$$\lambda = \frac{c}{v} \longrightarrow v = \frac{c}{\lambda} = \frac{2.998\times10^8 \text{ m/s}}{150\,\text{m}} = \mathbf{2.00\times10^6\ Hz}$$

12-14 A microwave oven operates at a frequency of 2.2×10^9 Hz. The wavelength of these microwaves and the energy of each microwave are to be determined.

Analysis The wavelength of these microwaves is



$$\lambda = \frac{c}{v} = \frac{2.998\times10^8 \text{ m/s}}{2.2\times10^9 \text{ Hz(1/s)}} = 0.136 \text{ m} = \mathbf{136\ mm}$$

Then the energy of each microwave becomes

$$e = hv = \frac{hc}{\lambda} = \frac{(6.625\times10^{-34} \text{ Js})(2.998\times10^8 \text{ m/s})}{0.136 \text{ m}} = \mathbf{1.46\times10^{-24}\ J}$$

12-15 The photon energies of a radio wave and a γ-ray, and the photon energy ratio of the γ-ray to the radio wave are to be determined.

Assumptions **1** The medium is air and index of refraction is unity.

Properties The speed of light in a medium with a refraction index of 1 is c = 2.9979 × 10^8 m/s. The Planck's constant is h = 6.626069 × 10^{-34} J·s.

Analysis The photon energy of an electromagnetic wave is

$$e = \frac{hc}{\lambda}$$

The photon energy of the radio wave is

$$e_{\text{radio}} = \frac{hc}{\lambda} = \frac{(6.626069\times10^{-34} \text{ J}\cdot\text{s})(2.9979\times10^8 \text{ m/s})}{10^7 \ \mu\text{m}} = \mathbf{1.986\times10^{-26}\ J}$$

The photon energy of the γ-ray is

$$e_{\gamma\text{-ray}} = \frac{hc}{\lambda} = \frac{(6.626069\times10^{-34} \text{ J}\cdot\text{s})(2.9979\times10^8 \text{ m/s})}{10^{-7} \ \mu\text{m}} = \mathbf{1.986\times10^{-12}\ J}$$

The photon energy ratio of the γ-ray to the radio wave is

$$\frac{e_{\gamma\text{-ray}}}{e_{\text{radio}}} = \frac{\lambda_{\text{radio}}}{\lambda_{\gamma\text{-ray}}} = \frac{10^7 \ \mu\text{m}}{10^{-7} \ \mu\text{m}} = \mathbf{10^{14}}$$

Discussion There is 10^{14} times more energy in a γ-ray wave than a radio wave.

12-16 The photon energies of an electromagnetic wave in air, water, and glass are to be determined.

Assumptions **1** The refraction index of each medium is uniform.

Properties The speed of light in a vacuum is c_0 = 2.9979 × 10^8 m/s, and the Planck's constant is h = 6.626069 × 10^{-34} J·s.

Analysis The photon energy of an electromagnetic wave is

$$e = \frac{hc}{\lambda} = \frac{hc_0}{\lambda n}$$

Thus,

Air: $$e = \frac{hc_0}{\lambda n} = \frac{(6.626069 \times 10^{-34}\ \text{J}\cdot\text{s})(2.9979 \times 10^8\ \text{m/s})}{(0.5 \times 10^{-6}\ \text{m})(1.0)}\left(\frac{1\,\text{eV}}{1.6022 \times 10^{-19}\ \text{J}}\right) = \mathbf{2.48\ eV}$$

Water: $$e = \frac{hc_0}{\lambda n} = \frac{(6.626069 \times 10^{-34}\ \text{J}\cdot\text{s})(2.9979 \times 10^8\ \text{m/s})}{(0.5 \times 10^{-6}\ \text{m})(1.33)}\left(\frac{1\,\text{eV}}{1.6022 \times 10^{-19}\ \text{J}}\right) = \mathbf{1.86\ eV}$$

Glass: $$e = \frac{hc_0}{\lambda n} = \frac{(6.626069 \times 10^{-34}\ \text{J}\cdot\text{s})(2.9979 \times 10^8\ \text{m/s})}{(0.5 \times 10^{-6}\ \text{m})(1.5)}\left(\frac{1\,\text{eV}}{1.6022 \times 10^{-19}\ \text{J}}\right) = \mathbf{1.65\ eV}$$

Discussion Medium with higher index of refraction causes the speed of wave propagation to decrease. As the speed of the wave propagation decreases, so does the wave energy.

12-17 The photon energies of violet color and red color are to be determined.

Assumptions **1** The medium is air and index of refraction is unity.

Properties The speed of light in a medium with a refraction index of 1 is c = 2.9979 × 10^8 m/s. The Planck's constant is h = 6.626069 × 10^{-34} J·s.

Analysis The photon energy of an electromagnetic wave is

$$e = \frac{hc}{\lambda}$$

Thus, the photon energy of each color is

Violet: $$e = \frac{hc}{\lambda} = \frac{(6.626069 \times 10^{-34}\ \text{J}\cdot\text{s})(2.9979 \times 10^8\ \text{m/s})}{(0.40 \times 10^{-6}\ \text{m})} = \mathbf{4.966 \times 10^{-19}\ J}$$

Red: $$e = \frac{hc}{\lambda} = \frac{(6.626069 \times 10^{-34}\ \text{J}\cdot\text{s})(2.9979 \times 10^8\ \text{m/s})}{(0.76 \times 10^{-6}\ \text{m})} = \mathbf{2.614 \times 10^{-19}\ J}$$

Discussion Violet color propagates higher level (1.9 times higher) of photon energy than red color because of its shorter wavelength. An electromagnetic wave with a shorter wavelength means that it has higher frequency, and therefore higher photon energy.

Blackbody Radiation

12-18C A blackbody is a perfect emitter and absorber of radiation. A blackbody does not actually exist. It is an idealized body that emits the maximum amount of radiation that can be emitted by a surface at a given temperature.

12-19C *Spectral blackbody emissive power* is the amount of radiation energy emitted by a blackbody at an absolute temperature T per unit time, per unit surface area and per unit wavelength about wavelength λ. The integration of the spectral blackbody emissive power over the entire wavelength spectrum gives the *total blackbody emissive power*,

$$E_b(T) = \int_0^\infty E_{b\lambda}(T)d\lambda = \sigma T^4$$

The spectral blackbody emissive power varies with wavelength, the total blackbody emissive power does not.

12-20C We defined the blackbody radiation function f_λ because the integration $\int_0^\infty E_{b\lambda}(T)d\lambda$ cannot be performed. The blackbody radiation function f_λ represents the fraction of radiation emitted from a blackbody at temperature *T* in the wavelength range from $\lambda = 0$ to λ. This function is used to determine the fraction of radiation in a wavelength range between λ_1 and λ_2.

12-21C The larger the temperature of a body, the larger the fraction of the radiation emitted in shorter wavelengths. Therefore, the body at 1500 K will emit more radiation in the shorter wavelength region. The body at 1000 K emits more radiation at 20 μm than the body at 1500 K since $\lambda T = \text{constant}$.

12-22 The maximum thermal radiation that can be emitted by a surface is to be determined.

Analysis The maximum thermal radiation that can be emitted by a surface is determined from Stefan-Boltzman law to be

$$E_b(T) = \sigma T^4 = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1000\ \text{K})^4 = \mathbf{56{,}700\ W/m^2}$$

12-23 A thin-walled tube's inner surface is coated with polypropylene lining. The maximum amount of total radiation emission rate per unit length of the tube that can be achieved without exceeding the maximum use temperature for polypropylene lining is to be determined.

Assumptions **1** The tube behaves as blackbody. **2** Uniform surface temperature. **3** Thin-walled tube has negligible thermal resistance for conduction.

Analysis The maximum total blackbody emissive power that can be achieved without exceeding the maximum use temperature for polypropylene lining is at 107°C. So, from the Stefan-Boltzmann law,

$$E_b(T) = \sigma T^4 = (5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4)(107 \text{ K} + 273 \text{ K})^4 = 1182.3 \text{ W/m}^2$$

The corresponding total radiation emission rate per unit length of the tube is

$$\dot{Q}_{\text{rad}} = A_s E_b(T) = \pi D L E_b(T)$$

$$\frac{\dot{Q}_{\text{rad}}}{L} = \pi D E_b(T) = \pi(0.05 \text{ m})(1182.3 \text{ W/m}^2) = \mathbf{185.7 \text{ W/m}}$$

Discussion The maximum amount of total radiation emission rate per unit length from the tube is 185.7 W/m. Any amount higher than that would require the tube surface temperature to exceed 107°C, which is the maximum use temperature for polypropylene lining.

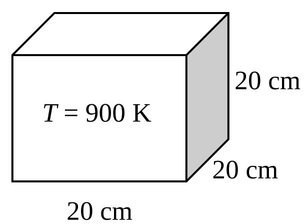
12-24 An isothermal cubical body is suspended in the air. The rate at which the cube emits radiation energy and the spectral blackbody emissive power are to be determined.

Assumptions The body behaves as a black body.

Analysis (*a*) The total blackbody emissive power is determined from Stefan-Boltzman Law to be

$$A_s = 6a^2 = 6(0.2^2) = 0.24 \text{ m}^2$$

$$E_b(T) = \sigma T^4 A_s = (5.67 \times 10^{-8} \text{ W/m}^2.\text{K}^4)(900 \text{ K})^4 (0.24 \text{ m}^2) = \mathbf{8928 \text{ W}}$$



(*b*) The spectral blackbody emissive power at a wavelength of 4 μm is determined from Plank's distribution law,

$$E_{b\lambda} = \frac{C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right]} = \frac{3.74177 \times 10^8 \text{ W} \cdot \mu\text{m}^4/\text{m}^2}{(4 \mu\text{m})^5\left[\exp\left(\frac{1.43878 \times 10^4 \ \mu\text{m} \cdot \text{K}}{(4 \mu\text{m})(900 \text{K})}\right) - 1\right]}$$

$$= 6841 \text{ W/m}^2 \cdot \mu\text{m} = \mathbf{6.84 \text{kW/m}^2 \cdot \mu m}$$

12-25 The peak spectral blackbody emissive power for a match flame and moonlight is to be determined.

Assumptions **1** The match and the moon behave as black bodies.

Analysis Using the combination of Planck's law and Wien's displacement law, the peak spectral blackbody emissive power can be determined:

$$E_{b\lambda}(\lambda_1, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T)-1]} = \frac{3.74177\times10^8}{\lambda^5[\exp(1.43878\times10^4/\lambda T)-1]}\ \text{W/m}^2\cdot\mu\text{m}$$

and

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K}$$

Then,

$$E_{b\lambda_{\max}}(T) = \frac{3.74177\times10^8 T^5}{(2897.8)^5[\exp(1.43878\times10^4/2897.8)-1]}\ \text{W/m}^2\cdot\mu\text{m}$$

$$= 1.278\times10^{-11}T^5\ \text{W/m}^2\cdot\mu\text{m}$$

For a match flame (T = 1700 K), the peak spectral blackbody emissive power is

$$E_{b\lambda_{\max}}(1700) = 1.278\times10^{-11}(1700)^5\ \text{W/m}^2\cdot\mu\text{m}$$

$$= \mathbf{1.81\times10^5\ W/m^2\cdot\mu m}$$

For moonlight (T = 4000 K), the peak spectral blackbody emissive power is

$$E_{b\lambda_{\max}}(4000) = 1.278\times10^{-11}(4000)^5\ \text{W/m}^2\cdot\mu\text{m}$$

$$= \mathbf{1.31\times10^7\ W/m^2\cdot\mu m}$$

Discussion The peak spectral blackbody emissive power by moonlight is about 72 times higher than that by a match flame.

12-26 An ASTM A479 904L stainless steel bar has a maximum use temperature of 260°C set by the ASME Code for Process Piping. (*a*) The spectral blackbody emissive powers at the limits of the thermal radiation spectrum for the maximum use temperature are to be determined and (*b*) The maximum spectral blackbody emissive power that the bar can reach without exceeding its maximum use temperature is to be determined.

Assumptions **1** The bar behaves as blackbody. **2** Uniform surface temperature.

Analysis The thermal radiation electromagnetic spectrum lies between λ = 0.1 μm and λ = 100 μm. The spectral blackbody emissive power is determined from Planck's law,

$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]}$$

(*a*) So, at λ = 0.1 μm and the maximum use temperature 260°C,

$$E_{b\lambda} = \frac{3.74177 \times 10^8 \mathrm{W \cdot \mu m^4/m^2}}{(0.1\ \mu\mathrm{m})^5 \left[\exp\left(\frac{1.43878 \times 10^4 \mu\mathrm{m \cdot K}}{(0.1\ \mu\mathrm{m})(260 + 273)\mathrm{K}}\right) - 1\right]} = \mathbf{2.186 \times 10^{-104} W/m^2{\cdot}\mu m \approx 0}$$

At λ = 100 μm and the maximum use temperature 260°C,

$$E_{b\lambda} = \frac{3.74177 \times 10^8 \mathrm{W \cdot \mu m^4/m^2}}{(100\ \mu\mathrm{m})^5 \left[\exp\left(\frac{1.43878 \times 10^4 \mu\mathrm{m \cdot K}}{(100\ \mu\mathrm{m})(260 + 273)\mathrm{K}}\right) - 1\right]} = \mathbf{0.1207\ W/m^2{\cdot}\mu m}$$

(*b*) The wavelength for the maximum spectral blackbody emissive power at the maximum use temperature can be determined from the Wien's displacement law,

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\mathrm{m{\cdot}K}$$

So,

$$\lambda_{\text{max power}} = \frac{2897.8\ \mu\mathrm{m{\cdot}K}}{(260 + 273)\mathrm{K}} = 5.4368\ \mu\mathrm{m}$$

The maximum spectral blackbody emissive power that the bar can reach without exceeding its maximum use temperature is

$$E_{b\lambda,\text{ max power}} = \frac{3.74177 \times 10^8 \mathrm{W \cdot \mu m^4/m^2}}{(5.4368\ \mu\mathrm{m})^5 \left[\exp\left(\frac{1.43878 \times 10^4 \mu\mathrm{m \cdot K}}{2897.8\ \mu\mathrm{m{\cdot}K}}\right) - 1\right]} = \mathbf{553.5\ W/m^2{\cdot}\mu m}$$

Discussion The maximum spectral blackbody emissive power for the bar is 553.5 W/m^2·μm at λ = 5.4368 μm and 260°C. Any amount higher than that would require the temperature of the bar to exceed the maximum use temperature set by the ASME Code for Process Piping. Note that λ = 5.4368 μm is within the electromagnetic spectrum for thermal radiation.

12-27 The blackbody temperature and the total emissive power at a given wavelength and its corresponding emissive power are to be determined.

Assumptions **1** Blackbody radiation.

Analysis (a) Using the Planck's law find the blackbody radiation

$$E_{b\lambda}(\lambda_1, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]}$$

$$10^8 \text{ W/m}^3 = \frac{3.74177 \times 10^8 \text{ W} \cdot \mu\text{m}^4/\text{m}^2}{(0.7 \times 10^{-6} \ \mu\text{m})^5 \left\{\exp[1.43878 \times 10^4 \ (\mu\text{m} \cdot \text{K})/(0.7 \times 10^{-6} \ \mu\text{m})T \text{ (K)}] - 1\right\}}$$

Solve for T

T = **1215 K**

(b) The blackbody total emitted energy at this temperature is

$$E_b(T) = \sigma T^4 = (5.67 \times 10^{-8} \text{ W/m}^2.\text{K}^4)(1215 \text{ K})^4 = \mathbf{123{,}563 \text{ W/m}^2}$$

12-28 An ASTM A240 410S stainless steel plate has a maximum use temperature of 649°C. The plate temperature is to be determined whether it exceeds the specified maximum use temperature, when the maximum spectral blackbody emissive power from the plate surface is 9000 W/m^2·μm.

Assumptions **1** The plate behaves as blackbody. **2** Uniform surface temperature.

Analysis The wavelength for the maximum spectral blackbody emissive power can be determined from the Planck's law, along with the Wien's displacement law,

$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]}$$

and

$$(\lambda T)_{\text{max power}} = 2897.8 \text{ μm·K}$$

The maximum spectral blackbody emissive power is

$$E_{b\lambda,\text{ max power}} = \frac{C_1}{\lambda^5\left[\exp\left(C_2/(\lambda T)_{\text{max power}}\right) - 1\right]}$$

Thus, the wavelength at the maximum spectral blackbody emissive power is

$$\lambda_{\text{max power}} = \left\{\frac{C_1}{\left(E_{b\lambda,\text{ max power}}\right)\left[\exp\left(C_2/(\lambda T)_{\text{max power}}\right) - 1\right]}\right\}^{1/5}$$

$$\lambda_{\text{max power}} = \left\{\frac{3.74177 \times 10^8 \text{W} \cdot \text{μm}^4/\text{m}^2}{(9000 \text{ W/m}^2 \cdot \text{μm})\left[\exp\left(\frac{1.43878 \times 10^4 \text{μm} \cdot \text{K}}{2897.8 \text{ μm·K}}\right) - 1\right]}\right\}^{1/5} = 3.1125 \text{ μm}$$

From the Wien's displacement law, the plate surface temperature is

$$(\lambda T)_{\text{max power}} = 2897.8 \text{ μm·K}$$

$$T = \frac{2897.8 \text{ μm·K}}{3.1125 \text{ μm}} = 931 \text{ K} = \mathbf{658}℃ > 649℃$$

Discussion With a maximum spectral blackbody emissive power of 9000 W/m^2·μm, the plate surface temperature would need to be 685°C, which exceeds the maximum use temperature. Thus, the ASTM A240 410S stainless steel plate is not suitable for this process with the given maximum spectral blackbody emissive power.

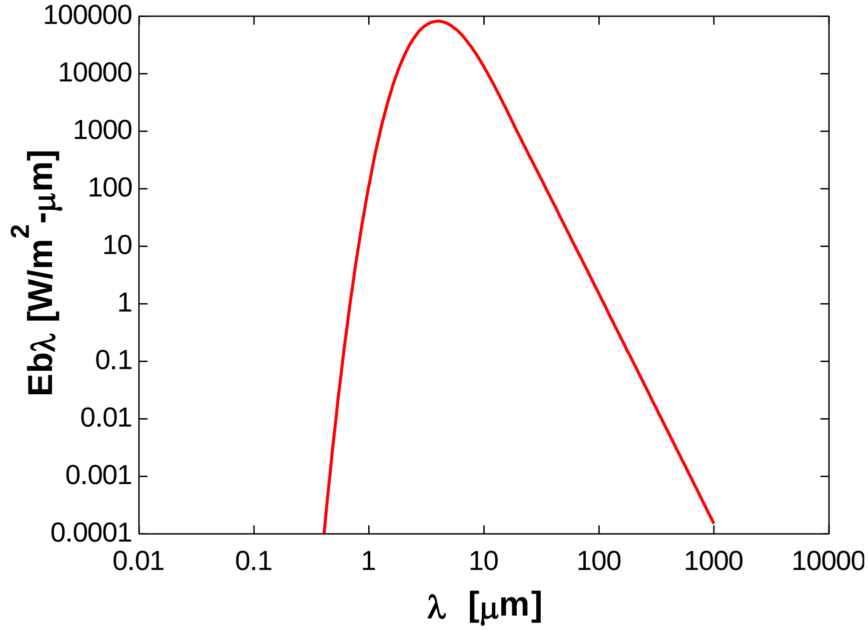
12-29  The spectral blackbody emissive power of the sun versus wavelength in the range of 0.01 μm to 1000 μm is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T=5780 [K]
lambda=0.01[micrometer]

"ANALYSIS"
E_b_lambda=C_1/(lambda^5*(exp(C_2/(lambda*T))-1))
C_1=3.742E8 [W-micrometer^4/m^2]
C_2=1.439E4 [micrometer-K]
```

λ [μm]	$E_{b,\lambda}$ [W/m^2-μm]
0.01	0
10.11	12684
20.21	846.3
30.31	170.8
40.41	54.63
50.51	22.52
60.62	10.91
70.72	5.905
80.82	3.469
90.92	2.17
…	…
…	…
909.1	0.0002198
919.2	0.0002103
929.3	0.0002013
939.4	0.0001928
949.5	0.0001847
959.6	0.000177
969.7	0.0001698
979.8	0.0001629
989.9	0.0001563
1000	0.0001501



12-30 A small body is placed inside an evacuated spherical chamber with constant surface temperature. The radiation incident on the small body surface is to be determined for (*a*) black chamber surface and (*b*) well-polished chamber surface.

Assumptions **1** The small body surface is much smaller than the chamber surface. **2** The chamber surface temperature is isothermal.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴.

Analysis The spherical chamber with isothermal surface temperature forms a blackbody cavity regardless of the radiation properties of the chamber surface. The small body inside the chamber is too small to interfere with the blackbody nature of the cavity.

Therefore, the radiation incident on any part of the small body surface is equal to the radiation emitted by a blackbody at the surface temperature of the chamber.

(*a*) For chamber surface coated in black:

$$E_b = \sigma T_s^4$$

$$E_b = (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(500\ \text{K})^4 = \mathbf{3544\ W/m^2}$$

(*b*) For a well-polished chamber surface, the radiation incident on the small body surface is **3544 W/m²**, the same as that of part (*a*), since it is independent of the radiation properties of the chamber surface.

Discussion The radiation incident on the small body surface depends only on the chamber surface temperature, and is independent of the conditions of the chamber surface.

The blackbody assumption is only valid when the surface area of the small body is much smaller than that of the chamber. This allows the radiation emitted by the chamber surface to go through multiple reflections and become diffuse.

12-31 A black ball is suspended in air. The surface temperature that is necessary to heat 10 kg of air from 20 to 30°C is to be determined.

Assumptions **1** The ball behaves as a blackbody.

Properties The specific heat of air at (20 + 30)°C / 2 = 25°C is c_v = 718 J/kg·K (Table A-1). The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴.

Analysis For a blackbody, the total emissive power is determined from the Stefan-Boltzmann law as

$$E_b = \sigma T_s^4$$

The heat energy release from the ball in the form of radiation is then

$$Q_{\text{rad}} = E_b A_s \Delta t = \sigma T_s^4 (\pi D^2) \Delta t$$

The energy required to heat 10 kg of air by ΔT = 10°C is

$$Q = m c_v \Delta T$$

Thus, $$m c_v \Delta T = \sigma T_s^4 (\pi D^2) \Delta t \quad \rightarrow \quad T_s = \left[\frac{m c_v \Delta T}{\sigma (\pi D^2) \Delta t} \right]^{1/4}$$

$$T_s = \left[\frac{(10\ \text{kg})(718\ \text{J/kg} \cdot \text{K})(10\ \text{K})}{(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)\pi (0.25\ \text{m})^2 (5 \times 60\ \text{s})} \right]^{1/4} = 383\ \text{K} = \mathbf{110°C}$$

Discussion With a surface temperature of 110°C, the ball can release enough energy in the form of electromagnetic waves to heat 10 kg of the air by 10°C in 5 minutes.

12-32 C&S A tube placed in a vacuum has its inner surface coated with PTFE lining. The total radiation emission rate from the tube outer surface is known. The tube inner surface temperature is to be determined whether it exceeds the maximum use temperature for PTFE lining.

Assumptions **1** The tube behaves as blackbody. **2** Uniform surface temperature. **3** One-dimensional conduction through the tube wall. **4** Irradiation on the tube from the surroundings is negligible.

Analysis The total blackbody emissive power from the tube outer surface at the surface temperature T_2 is determined from the Stefan-Boltzmann law,

$$E_b(T) = \sigma T_2^4$$

The corresponding total radiation emission rate per unit length of the tube is

$$\dot{Q}_{\text{rad}} = A_s E_b(T) = \pi D_2 L E_b(T)$$

$$\frac{\dot{Q}_{\text{rad}}}{L} = \pi D_2 \sigma T_2^4 = 680 \text{ W/m}$$

So, the outer surface temperature of the tube is

$$T_2 = \left(\frac{1}{\pi D_2 \sigma}\frac{\dot{Q}_{\text{rad}}}{L}\right)^{0.25} = \left[\frac{680 \text{ W/m}}{\pi(0.05 \text{ m})(5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4)}\right]^{0.25} = 525.7 \text{ K}$$

From energy balance,

$$\dot{Q}_{\text{rad}} = \dot{Q}_{\text{cond}}$$

The conduction through the tube wall is given as

$$\frac{\dot{Q}_{\text{cond}}}{L} = 2\pi k \frac{T_1 - T_2}{\ln(D_2/D_1)}$$

Thus, the tube inner surface temperature is

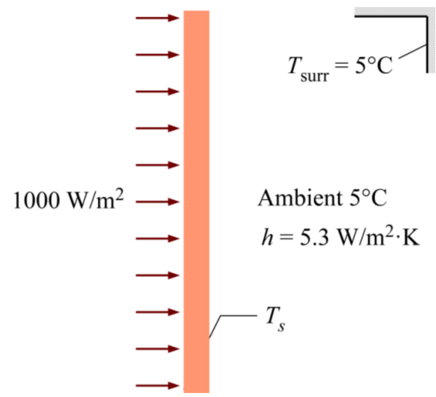
$$T_1 = T_2 + \frac{\ln(D_2/D_1)}{2\pi k}\frac{\dot{Q}_{\text{cond}}}{L} = 525.7 \text{ K} + \frac{\ln\left(\frac{5 \text{ cm}}{3 \text{ cm}}\right)}{2\pi(5 \text{ W/m}\cdot\text{K})}(680 \text{ W/m}) = 536.8 \text{ K} = \mathbf{264}℃ > 260℃$$

Discussion With a total radiation emission rate per unit length of the tube from the tube outer surface at 680 W/m, the tube inner surface temperature would be 264°C. This exceeds the maximum use temperature for PTFE lining stipulated by the ASME Code for Process Piping. So, the PTFE lining is not suitable for the tube inner surface at the given radiation emission rate from the outer surface.

12-33 A thin vertical plate, modeled as blackbody, is subjected to uniform heat flux on one side and exposed to radiation and natural convection on the other side. The plate surface temperature is to be determined.

Assumptions **1** The plate emits radiation as a blackbody. **2** Thermal properties are constant. **3** Plate surface temperature is uniform. **4** Heat loss from plate's side surface is negligible. **5** The surroundings are treated as an isothermal surface, $T_{surr} = T_\infty$.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴.



Analysis The heat emitted from the plate, as a blackbody, is

$$\dot{q}_{\text{emit}} = \sigma T_s^4$$

The radiation incident on the plate (blackbody) from the surroundings is

$$\dot{q}_{\text{incident}} = \sigma T_{\text{surr}}^4$$

The heat transfer from the plate to the surroundings by natural convection is

$$\dot{q}_{\text{conv}} = h(T_s - T_\infty)$$

By performing energy balance on the plate we have

$$\dot{q} = \dot{q}_{\text{emit}} - \dot{q}_{\text{incident}} + \dot{q}_{\text{conv}}$$

$$\dot{q} = \sigma T_s^4 - \sigma T_{\text{surr}}^4 + h(T_s - T_\infty)$$

$$1000 \text{ W/m}^2 = (5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(T_s^4 - 278^4)\,\text{K}^4 + (5.3 \text{ W/m}^2\cdot\text{K})(T_s - 278)\,\text{K}$$

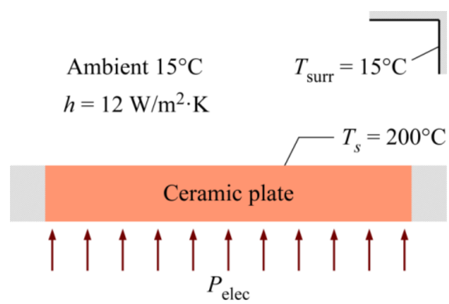
$$\rightarrow \quad T_s = 357 \text{ K} = \mathbf{84°C}$$

Discussion The net rate of radiation heat transfer between the plate and the surroundings is $\dot{q}_{\text{rad}} = \sigma(T_s^4 - T_{\text{surr}}^4) = 582 \text{ W/m}^2$, which is about 58% of the total heat loss from the plate. Natural convection only contributed about 42% of the total heat loss from the plate.

12-34 A circular plate that is modeled as a blackbody is heated by an electrical heater with an efficiency of 80%. The electric power required to keep the plate surface temperature at 200°C is to be determined.

Assumptions **1** The plate emits radiation as a blackbody. **2** Thermal properties are constant. **3** Plate surface temperature is uniform. **4** Heat loss from plate's side surface is negligible. **5** The surroundings are treated as an isothermal surface, $T_{surr} = T_\infty$.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴.



Analysis The heat emitted from the plate, as a blackbody, is

$$\dot{Q}_{\text{emit}} = \sigma A_s T_s^4 = \pi\frac{(0.30\text{ m})^4}{4}(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(473^4)\text{K}^4 = 18.06 \text{ W}$$

The radiation incident on the plate (blackbody) from the surroundings is

$$\dot{Q}_{\text{incident}} = \sigma A_s T_{\text{surr}}^4 = \pi\frac{(0.30\text{ m})^4}{4}(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(288^4)\text{K}^4 = 2.48 \text{ W}$$

The heat transfer from the plate to the surroundings by natural convection is

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) = \pi\frac{(0.30\text{ m})^4}{4}(12 \text{ W/m}^2\cdot\text{K})(200-15)\text{K} = 14.12 \text{ W}$$

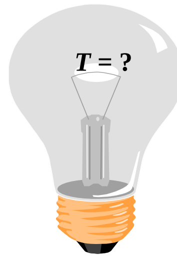
Thus, by performing energy balance on the plate we have

$$\eta P_{\text{elec}} = \dot{Q}_{\text{emit}} - \dot{Q}_{\text{incident}} + \dot{Q}_{\text{conv}}$$

$$P_{\text{elec}} = \frac{\dot{Q}_{\text{emit}} - \dot{Q}_{\text{incident}} + \dot{Q}_{\text{conv}}}{\eta} = \frac{18.06 - 2.48 + 14.12}{0.80}\text{ W} = \mathbf{37.13\ W}$$

Discussion The net rate of radiation heat transfer between the plate and the surroundings is $\dot{Q}_{\text{rad}} = \dot{Q}_{\text{emit}} - \dot{Q}_{\text{incident}} = \sigma A_s(T_s^4 - T_{\text{surr}}^4)$. Since the plate is treated as a blackbody, the emissivity is unity (ε = 1).

12-35 An incandescent light bulb emits 15% of its energy at wavelengths shorter than 0.8 μm. The temperature of the filament is to be determined.



Assumptions The filament behaves as a black body.

Analysis From the Table 12-2 for the fraction of the radiation, we read

$$f_\lambda = 0.15 \longrightarrow \lambda T = 2445\ \mu\text{mK}$$

For the wavelength range of $\lambda_1 = 0.0\ \mu\text{m}$ to $\lambda_2 = 0.8\ \mu\text{m}$

$$\lambda = 0.8\ \mu\text{m} \longrightarrow \lambda T = 2445\ \mu\text{mK} \longrightarrow T = \mathbf{3056\ K}$$

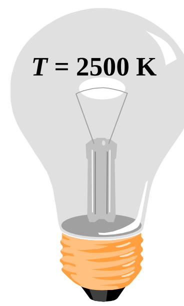
12-36 The temperature of the filament of an incandescent light bulb is given. The fraction of visible radiation emitted by the filament and the wavelength at which the emission peaks are to be determined.

Assumptions The filament behaves as a black body.

Analysis The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40\ \mu\text{m}$ to $\lambda_2 = 0.76\ \mu\text{m}$. Noting that T = 2500 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 12-2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(2500\ \text{K}) = 1000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.000321$$

$$\lambda_2 T = (0.76\ \mu\text{m})(2500\ \text{K}) = 1900\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.053035$$



Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 0.053035 - 0.000321 = \mathbf{0.052714} \quad \text{(or 5.2\%)}$$

The wavelength at which the emission of radiation from the filament is maximum is

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K} \longrightarrow \lambda_{\text{max power}} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{2500\ \text{K}} = \mathbf{1.16\ \mu m}$$

Discussion Note that the radiation emitted from the filament peaks in the infrared region.

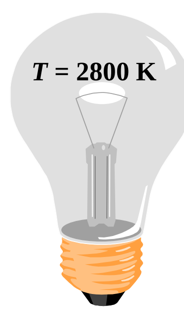
12-37 The temperature of the filament of an incandescent light bulb is given. The fraction of visible radiation emitted by the filament and the wavelength at which the emission peaks are to be determined.

Assumptions The filament behaves as a black body.

Analysis The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40\ \mu\text{m}$ to $\lambda_2 = 0.76\ \mu\text{m}$. Noting that T = 2800 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 12-2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(2800\ \text{K}) = 1120\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0014088$$

$$\lambda_2 T = (0.76\ \mu\text{m})(2800\ \text{K}) = 2128\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.088590$$



Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 0.088590 - 0.0014088 = \mathbf{0.08718} \quad \text{(or 8.7\%)}$$

The wavelength at which the emission of radiation from the filament is maximum is

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K} \longrightarrow \lambda_{\text{max power}} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{2800\ \text{K}} = \mathbf{1.035\ \mu m}$$

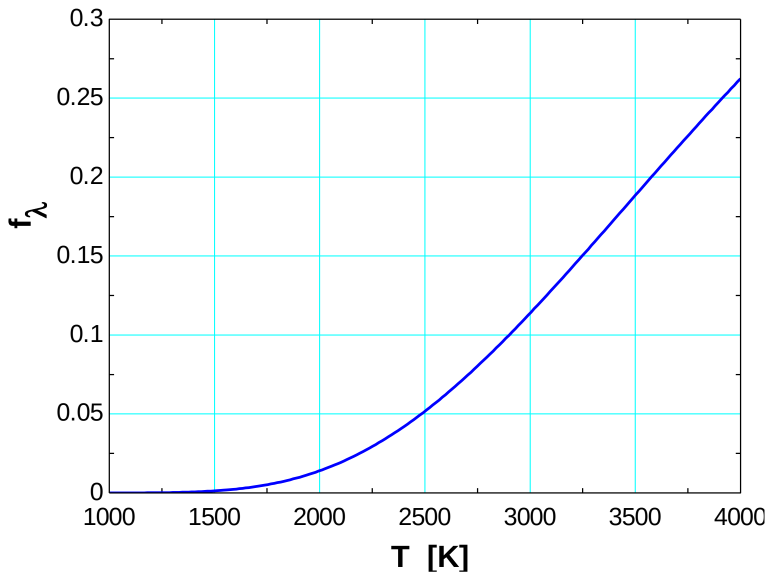
12-38 Prob. 12-37 is reconsidered. The effect of temperature on the fraction of radiation emitted in the visible range is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T=2800 [K]
lambda_1=0.40 [micrometer]
lambda_2=0.76 [micrometer]

"ANALYSIS"
E_b_lambda=C_1/(lambda^5*(exp(C_2/(lambda*T))-1))
C_1=3.742E8 [W-micrometer^4/m^2]
C_2=1.439E4 [micrometer-K]
f_lambda=integral(E_b_lambda, lambda, lambda_1, lambda_2)/E_b
E_b=sigma*T^4
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"
```

T [K]	f_λ
1000	0.000007353
1200	0.0001032
1400	0.0006403
1600	0.002405
1800	0.006505
2000	0.01404
2200	0.02576
2400	0.04198
2600	0.06248
2800	0.08671
3000	0.1139
3200	0.143
3400	0.1732
3600	0.2036
3800	0.2336
4000	0.2623



12-39E The radiation energy emitted by the black surface per unit area (at 2060 °F) for λ ≥ 4 μm is to be determined.

***Assumptions* 1** The surface behaves as a black body.

Analysis The blackbody radiation function corresponding to λ_1 = 4 μm is determined from Table 12-2 to be

$$\lambda_1 T = (4.0\ \mu\text{m})(2060 + 460)(1/1.8)\ \text{K} = 5600\ \mu\text{m}\cdot\text{K} \quad \rightarrow \quad f_{\lambda_1} = 0.701046$$

Then, the radiation energy emitted is determined using

$$\begin{aligned} E_{b,\ \lambda_1-\infty}(T) &= \int_{\lambda_1}^{\infty} E_{b\lambda}(\lambda,\ T) = \sigma T^4 f_{\lambda_1-\infty}(T) \\ &= \sigma T^4 [f_\infty(T) - f_{\lambda 1}(T)] \\ &= (0.1714\times 10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(2520\ \text{R})^4(1-0.701046) \\ &= \mathbf{20{,}700\ Btu/h\cdot ft^2} \end{aligned}$$

Discussion The total radiation energy emitted by this black surface is simply

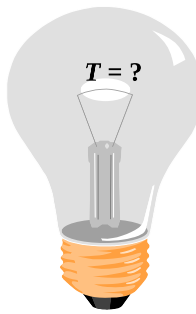
$$E_b(T) = \sigma T^4 = 69{,}100\ \text{Btu/h}\cdot\text{ft}^2$$

12-40 Radiation emitted by a light source is maximum in the blue range. The temperature of this light source and the fraction of radiation it emits in the visible range are to be determined.

Assumptions The light source behaves as a black body.

Analysis The temperature of this light source is

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K} \longrightarrow T = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{0.47\ \mu\text{m}} = \mathbf{6166\ K}$$



The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40\ \mu\text{m}$ to $\lambda_2 = 0.76\ \mu\text{m}$. Noting that T = 6166 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 12-2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(6166\ \text{K}) = 2466\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.15440$$

$$\lambda_2 T = (0.76\ \mu\text{m})(6166\ \text{K}) = 4686\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.59144$$

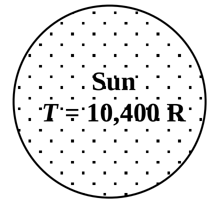
Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 0.59144 - 0.15440 \cong \mathbf{0.437} \qquad \text{(or 43.7\%)}$$

12-41E The sun is at an effective surface temperature of 10,400 R. The rate of infrared radiation energy emitted by the sun is to be determined.

Assumptions The sun behaves as a black body.

Analysis Noting that T = 10,400 R = 5778 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 12-2 to be



$$\lambda_1 T = (0.76\ \mu\text{m})(5778\,\text{K}) = 4391.3\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.547370$$

$$\lambda_2 T = (100\ \mu\text{m})(5778\,\text{K}) = 577{,}800\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 1.0$$

Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 1.0 - 0.547 = 0.453 \qquad \text{(or 45.3\%)}$$

The total blackbody emissive power of the sun is determined from Stefan-Boltzmann Law to be

$$E_b = \sigma T^4 = (0.1714\times10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(10{,}400\ \text{R})^4 = 2.005\times10^7\ \text{Btu/h.ft}^2$$

Then,

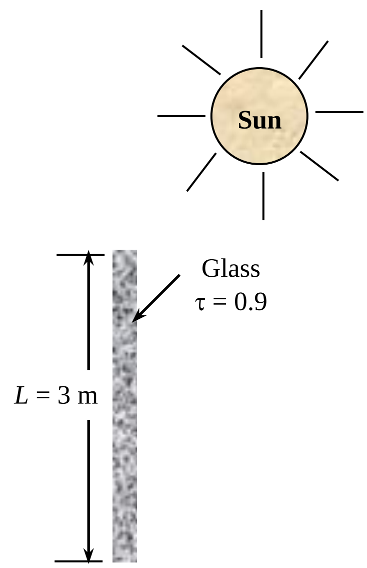
$$E_{\text{infrared}} = (0.453)E_b = (0.453)(2.005\times10^7\ \text{Btu/h.ft}^2) = \mathbf{9.08\times10^6\ Btu/h.ft^2}$$

12-42 A glass window transmits 90% of the radiation in a specified wavelength range and is opaque for radiation at other wavelengths. The rate of radiation transmitted through this window is to be determined for two cases.

Assumptions The sources behave as a black body.

Analysis The surface area of the glass window is

$$A_s = 9\ \text{m}^2$$



(a) For a blackbody source at 5800 K, the total blackbody radiation emission is

$$E_b(T) = \sigma T^4 A_s = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(5800\ \text{K})^4(9\ \text{m}^2) = 5.775\times10^8\ \text{W}$$

The fraction of radiation in the range of 0.3 to 3.0 μm is

$$\lambda_1 T = (0.30\ \mu\text{m})(5800\,\text{K}) = 1740\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.03345$$

$$\lambda_2 T = (3.0\ \mu\text{m})(5800\,\text{K}) = 17{,}400\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.97875$$

$$\Delta f = f_{\lambda_2} - f_{\lambda_1} = 0.97875 - 0.03345 = 0.9453$$

Noting that 90% of the total radiation is transmitted through the window,

$$E_{\text{transmit}} = 0.90\Delta f E_b(T)$$

$$= (0.90)(0.9453)(5.775\times10^5\ \text{kW}) = \mathbf{491{,}300\ kW}$$

(b) For a blackbody source at 1000 K, the total blackbody emissive power is

$$E_b(T) = \sigma T^4 A_s = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1000\ \text{K})^4(9\ \text{m}^2) = 510{,}300\ \text{W}$$

The fraction of radiation in the visible range of 0.3 to 3.0 μm is

$$\lambda_1 T = (0.30\ \mu\text{m})(1000\,\text{K}) = 300\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0000$$

$$\lambda_2 T = (3.0\ \mu\text{m})(1000\,\text{K}) = 3000\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.273232$$

$$\Delta f = f_{\lambda_2} - f_{\lambda_1} = 0.273232 - 0$$

and

$$E_{\text{transmit}} = 0.90\Delta f E_b(T) = (0.90)(0.273232)(510.3\,\text{kW}) = \mathbf{125.5\ kW}$$

12-43 The radiation energy emitted within the visible light region by daylight and candlelight is to be determined.

Assumptions **1** The sun and the candlelight behave as black bodies.

Analysis The visible range of the electromagnetic spectrum extends from λ_1 = 0.40 μm to λ_2 = 0.76 μm. For daylight (T = 5800 K), the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 12-2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(5800\ \text{K}) = 2320\ \mu\text{m}\cdot\text{K} \quad \rightarrow \quad f_{\lambda_1,\,\text{daylight}} = 0.124509$$

$$\lambda_2 T = (0.76\ \mu\text{m})(5800\ \text{K}) = 4408\ \mu\text{m}\cdot\text{K} \quad \rightarrow \quad f_{\lambda_2,\,\text{daylight}} = 0.550015$$

Then the fraction of radiation emitted between these two wavelengths (for daylight) becomes

$$f_{\lambda_1-\lambda_2,\,\text{daylight}} = 0.550015 - 0.124509 = 0.4255$$

Hence, the radiation energy emitted (for daylight) is determined using

$$E_{b,\ \lambda_1-\lambda_2}(T) = \int_{\lambda_1}^{\lambda_2} E_{b\lambda}(\lambda,\ T) = \sigma T^4 f_{\lambda_1-\lambda_2,\,\text{daylight}} = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(5800\ \text{K})^4(0.4255)$$

$$= \mathbf{2.73\times10^7\ W/m^2}$$

For candlelight (T = 1800 K), the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 12-2 to be

$$\lambda_1 T = (0.40\ \mu\text{m})(1800\ \text{K}) = 720\ \mu\text{m}\cdot\text{K} \quad \rightarrow \quad f_{\lambda_1,\,\text{candle}} = 0.0000096$$

$$\lambda_2 T = (0.76\ \mu\text{m})(1800\ \text{K}) = 1368\ \mu\text{m}\cdot\text{K} \quad \rightarrow \quad f_{\lambda_2,\,\text{candle}} = 0.006885$$

Then the fraction of radiation emitted between these two wavelengths (for candlelight) becomes

$$f_{\lambda_1-\lambda_2,\,\text{candle}} = 0.006885 - 0.0000096 = 0.006875$$

Hence, the radiation energy emitted (for candlelight) is determined using

$$E_{b,\ \lambda_1-\lambda_2}(T) = \int_{\lambda_1}^{\lambda_2} E_{b\lambda}(\lambda,\ T) = \sigma T^4 f_{\lambda_1-\lambda_2,\,\text{daylight}} = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1800\ \text{K})^4(0.006875)$$

$$= \mathbf{4090\ W/m^2}$$

Discussion The total radiation energy emitted by daylight is almost 7000 times higher than that by candlelight.

12-44 C&S An ASTM B335 nickel alloy rod has a maximum use temperature of 427°C. The highest radiation energy per unit area that can be emitted from the rod, over a wavelength band from 2 to 10 μm, without exceeding the maximum use temperature is to be determined.

Assumptions **1** The rod behaves as blackbody. **2** Uniform surface temperature.

Analysis The highest radiation energy per unit area that can be emitted from the rod, over a wavelength band from 2 to 10 μm, without exceeding the maximum use temperature is at 427°C or 700 K. The blackbody radiation functions corresponding to the wavelength band from 2 to 10 μm are

$$\lambda_1 T = (2)(700) = 1400 \text{ μm·K} \quad \rightarrow \quad f_{\lambda_1} = 0.007790$$

$$\lambda_2 T = (10)(700) = 7000 \text{ μm·K} = \quad \rightarrow \quad f_{\lambda_2} = 0.808109$$

The radiation energy per unit area emitted from the rod, over a wavelength band from 2 to 10 μm, at the maximum use temperature of 700 K is

$$E_{b,\lambda_1-\lambda_2} = \int_{\lambda_1}^{\lambda_2} E_{b\lambda}(\lambda, T)\, d\lambda = \sigma T^4 \left(f_{\lambda_2} - f_{\lambda_1}\right)$$

Thus,

$$E_{b,\lambda_1-\lambda_2} = (5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4)(700 \text{ K})^4 (0.808109 - 0.007790) = \mathbf{10895\ W/m^2}$$

Discussion The highest radiation energy per unit area that can be emitted from the rod, over a wavelength band from 2 to 10 μm is 10.9 kW/m^2. Any radiation energy emitted from the rod that is higher than 10.9 kW/m^2, over a wavelength band from 2 to 10 μm, would require the rod surface temperature to exceed the maximum use temperature specified by the ASME Code for Process Piping.

12-45 The percentage of solar energy for different wavelengths assuming the sun's surface temperature is 5800 K.

Assumptions **1** Blackbody radiation.

Analysis (a) The visible range is between λ_1 = 0.40 μm and λ_2 = 0.76 μm

with

$$\lambda_1 T = (0.40 \text{ μm})(5800 \text{ K}) = 2320 \text{ μm·K, from Table 12-2} \rightarrow f_{\lambda_1} = 0.124509$$

Similarly with

$$\lambda_2 T = (0.76 \text{ μm})(5800 \text{ K}) = 4408 \text{ μm·K, from Table 12-2} \rightarrow f_{\lambda_2} = 0.550019$$

∴ The percentage of solar energy contained in the visible range is

$$f_{\lambda_2} - f_{\lambda_1} = 0.550019 - 0.124509 = 0.426 = \mathbf{42.6\%}$$

(b) The percentage of solar energy at wavelengths shorter than visible is

$$f_{\lambda_1} = 0.124509 = \mathbf{12.5\%}$$

(c) The percentage of solar energy at wavelengths longer than visible is

$$f_{\lambda_2-\infty} = 1 - f_{\lambda_2} = 1 - 0.550019 = 0.449981 \approx \mathbf{45\%}$$

Discussion Approximately 45% of solar energy is infrared, about 43% visible, and about 13% is ultraviolet.

Radiation Intensity

12-46C A solid angle represents an opening in space, whereas a plain angle represents an opening in a plane. For a sphere of unit radius, the solid angle about the origin subtended by a given surface on the sphere is equal to the area of the surface. For a cicle of unit radius, the plain angle about the origin subtended by a given arc is equal to the length of the arc. The value of a solid angle associated with a sphere is 4π.

12-47C Irradiation G is the radiation flux incident on a surface from all directions. For diffusely incident radiation, irradiation on a surface is related to the intensity of incident radiation by $G = \pi I_i$ (or $G_\lambda = \pi I_{\lambda,i}$ for spectral quantities).

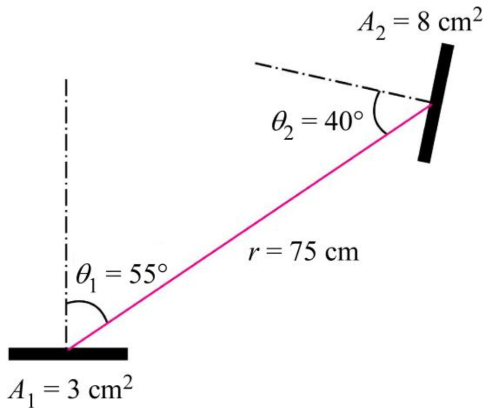
12-48C Radiosity J is the rate at which radiation energy leaves a unit area of a surface by emission and reflection in all directions.. For a diffusely emitting and reflecting surface, radiosity is related to the intensity of emitted and reflected radiation by $J = \pi I_{e+r}$ (or $J_\lambda = \pi I_{\lambda,e+r}$ for spectral quantities).

12-49C When the variation of a spectral radiation quantity with wavelength is known, the correcponding total quantity is determined by integrating that quantity with respect to wavelength from $\lambda = 0$ to $\lambda = \infty$.

12-50C The intensity of emitted radiation $I_e(\theta, \phi)$ is defined as the rate at which radiation energy $d\dot{Q}_e$ is emitted in the (θ, ϕ) direction per unit area normal to this direction and per unit solid angle about this direction. For a diffusely emitting surface, the emissive power is related to the intensity of emitted radiation by $E = \pi I_e$ (or $E_\lambda = \pi I_{\lambda,e}$ for spectral quantities).

12-51 A surface (A_2) is subjected to radiation emitted by another surface (A_1). The intensity of the radiation emitted by A_1 is to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.



Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from

$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} = \frac{(8\,\text{cm}^2)\cos 40°}{(75\,\text{cm})^2} = 10.89\times10^{-4}\ \text{sr}$$

The rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is given as

$$\dot{Q}_{1-2} = I_1(A_1\cos\theta_1)\omega_{2-1}$$

Hence, the intensity of the radiation emitted by A_1 can be determined with

$$I_1 = \frac{\dot{Q}_{1-2}}{(A_1\cos\theta_1)\omega_{1-2}} = \frac{274\times10^{-6}\ \text{W}}{(3\times10^{-4}\ \text{m}^2)(\cos 55°)(10.89\times10^{-4}\ \text{sr})} = \mathbf{1460\ W/m^2\cdot sr}$$

The temperature of A_1 is determined using

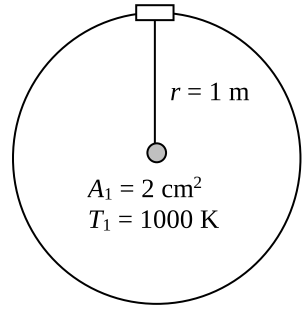
$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} \qquad \rightarrow \qquad T_1 = \left(\frac{I_1\pi}{\sigma}\right)^{1/4}$$

$$T_1 = \left[\frac{(1460\ \text{W/m}^2\cdot\text{sr})(\pi\ \text{sr})}{5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4}\right]^{1/4} = \mathbf{533\,K}$$

Discussion If A_2 were directly above A_1 at a distance 75 cm, then $\theta_1 = 0°$ and $\theta_2 = 90°$. That means the rate of radiation energy emitted by A_1 is $\dot{Q}_{1-2} = 0$, since $\omega_{1-2} = 0$.

12-52 Radiation is emitted from a small circular surface located at the center of a sphere. Radiation energy streaming through a hole located on top of the sphere and the side of sphere are to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.



Analysis (*a*) Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from

$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2}{r^2} = \frac{\pi(0.005\text{ m})^2}{(1\text{ m})^2} = 7.854\times10^{-5}\text{ sr}$$

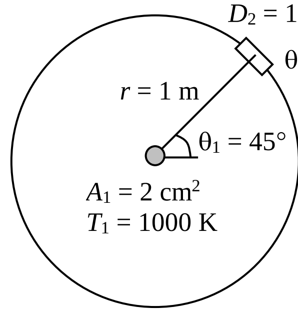
since A_2 were positioned normal to the direction of viewing.

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle ω_{2-1}. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(1000\text{ K})^4}{\pi} = 18{,}048\text{ W/m}^2\cdot\text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle ω_{2-1} is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle ω_{2-1}. That is,

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (18{,}048\text{ W/m}^2\cdot\text{sr})(2\times10^{-4}\cos0°\text{ m}^2)(7.854\times10^{-5}\text{ sr})\\ &= \mathbf{2.835\times10^{-4}\ W}\end{aligned}$$



where θ_1 = 0°. Therefore, the radiation emitted from surface A_1 will strike surface A_2 at a rate of 2.835×10^{-4} W.

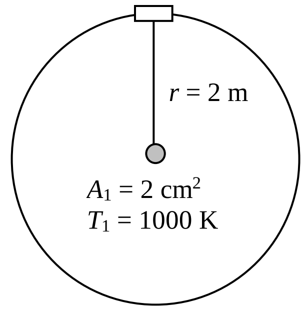
(*b*) In this orientation, θ_1 = 45° and θ_2 = 0°. Repeating the calculation we obtain the rate of radiation to be

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (18{,}048\text{ W/m}^2\cdot\text{sr})(2\times10^{-4}\cos45°\text{ m}^2)(7.854\times10^{-5}\text{ sr})\\ &= \mathbf{2.005\times10^{-4}\ W}\end{aligned}$$

12-53 Radiation is emitted from a small circular surface located at the center of a sphere. Radiation energy streaming through a hole located on top of the sphere and the side of sphere are to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis (a) Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from to be



$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2}{r^2} = \frac{\pi(0.005\text{ m})^2}{(2\text{ m})^2} = 1.963\times10^{-5}\text{ sr}$$

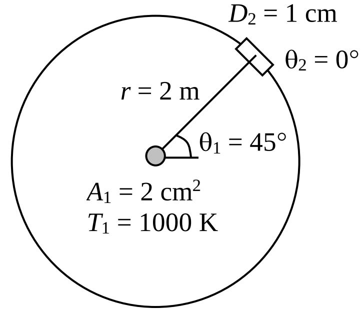
since A_2 were positioned normal to the direction of viewing.

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(1000\text{ K})^4}{\pi} = 18{,}048\text{ W/m}^2\cdot\text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (18{,}048\text{ W/m}^2\cdot\text{sr})(2\times10^{-4}\cos0°\text{ m}^2)(1.963\times10^{-5}\text{ sr})\\ &= \mathbf{7.086\times10^{-5}\ W}\end{aligned}$$



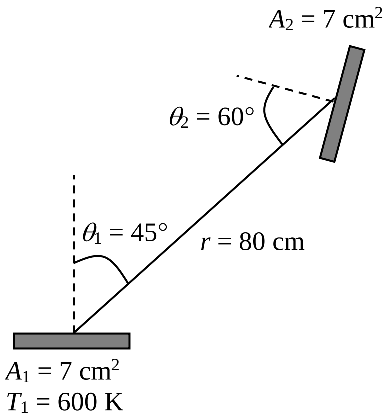
where θ_1 = 0°. Therefore, the radiation emitted from surface A_1 will strike surface A_2 at a rate of 2.835×10^{-4} W.

(b) In this orientation, θ_1 = 45° and θ_2 = 0°. Repeating the calculation we obtain the rate of radiation as

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (18{,}048\text{ W/m}^2\cdot\text{sr})(2\times10^{-4}\cos45°\text{ m}^2)(1.963\times10^{-5}\text{ sr})\\ &= \mathbf{5.010\times10^{-5}\ W}\end{aligned}$$

12-54 A surface is subjected to radiation emitted by another surface. The solid angle subtended and the rate at which emitted radiation is received are to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.



Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from

$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} = \frac{(7\text{ cm}^2)\cos 60°}{(80\text{ cm})^2} = \mathbf{5.469 \times 10^{-4}\ sr}$$

since the normal of A_2 makes 60° with the direction of viewing. Note that solid angle subtended by A_2 would be maximum if A_2 were positioned normal to the direction of viewing. Also, the point of viewing on A_1 is taken to be a point in the middle, but it can be any point since A_1 is assumed to be very small.

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(800\text{ K})^4}{\pi} = 7393\text{ W/m}^2\cdot\text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1} \\ &= (7393\text{ W/m}^2\cdot\text{sr})(7\times10^{-4}\cos 45°\text{ m}^2)(5.469\times10^{-4}\text{ sr}) \\ &= \mathbf{2.001\times10^{-3}\ W}\end{aligned}$$

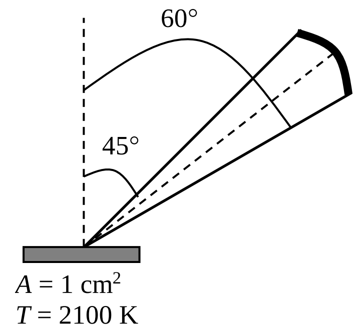
Therefore, the radiation emitted from surface A_1 will strike surface A_2 at a rate of 2.001×10^{-3} W.

If A_2 were directly above A_1 at a distance 80 cm, θ_1 = 0° and the rate of radiation energy emitted by A_1 becomes zero.

12-55 A small surface emits radiation. The rate of radiation energy emitted through a band is to be determined.

Assumptions Surface *A* emits diffusely as a blackbody.

Analysis The rate of radiation emission from a surface per unit surface area in the direction (θ,ϕ) is given as



$$dE = \frac{d\dot{Q}_e}{dA} = I_e(\theta,\phi)\cos\theta\sin\theta d\theta d\phi$$

The total rate of radiation emission through the band between 60° and 45° can be expressed as

$$E = \int_{\phi=0}^{2\pi}\int_{\theta=45}^{60} I_e(\theta,\phi)\cos\theta\sin\theta\ d\theta\ d\phi = I_b\frac{\pi}{4} = \frac{\sigma T^4}{\pi}\frac{\pi}{4} = \frac{\sigma T^4}{4}$$

since the blackbody radiation intensity is constant (I_b = constant), and

$$\int_{\phi=0}^{2\pi}\int_{\theta=45}^{60}\cos\theta\sin\theta\ d\theta\ d\phi = 2\pi\int_{\theta=45}^{60}\cos\theta\sin\theta\ d\theta = \pi(\sin^2 60 - \sin^2 45) = \pi/4$$

Approximating a small area as a differential area, the rate of radiation energy emitted from an area of 1 cm^2 in the specified band becomes

$$\dot{Q}_e = EdA = \frac{\sigma T^4}{4}dA = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(2100\ \text{K})^4}{4}(1\times10^{-4}\ \text{m}^2) = \mathbf{27.6\ W}$$

12-56 The intensity of solar radiation incident on earth's surface is given. The peak value for the intensity of incident solar radiation and the solar irradiation on earth's surface are to be determined.

Assumptions **1** The intensity is not dependent of the azimuth angle ϕ.

Analysis The intensity of solar radiation incident on earth's surface peaks at the zenith angle of θ = 0°, hence

$$I_{i,\max} = 100\cos 0° = \mathbf{100\ W/m^2\cdot sr}$$

The solar irradiation on earth's surface is

$$G = \int_{\text{hemisphere}} dG = \int_0^{2\pi}\int_0^{\pi/2} I_i(\theta)\cos\theta\sin\theta\, d\theta\, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi/2} 100\cos^2\theta\ \sin\theta\, d\theta\, d\phi$$

$$= 100(2\pi)\int_0^{\pi/2}\cos^2\theta\ \sin\theta\ d\theta$$

$$= 200\pi\left[-\frac{(\cos\theta)^3}{3}\right]_0^{\pi/2}$$

$$= \frac{200\pi}{3}$$

$$= \mathbf{209\ W/m^2}$$

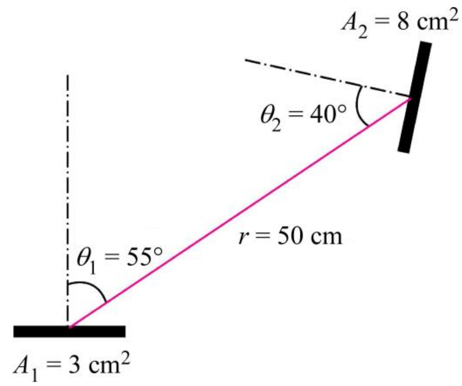
Discussion The intensity of incident solar radiation is at minimum when the sun is at the horizon, where the zenith angle is $\theta \approx 90°$.

12-57 A surface (A_2) is subjected to radiation emitted by another surface (A_1). The rate at which emitted radiation is received and the irradiation on A_2 are to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from

$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} = \frac{(8\,\text{cm}^2)\cos 40°}{(50\,\text{cm})^2} = 24.51\times10^{-4}\ \text{sr}$$



since the normal of A_2 makes 40° with the direction of viewing. Note that solid angle subtended by A_2 would be maximum if A_2 were positioned normal to the direction of viewing. Also, the point of viewing on A_1 is taken to be a point in the middle, but it can be any point since A_1 is assumed to be very small.

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{5.67\times10^4\ \text{W/m}^2}{\pi\ \text{sr}} = 18048\ \text{W/m}^2\cdot\text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\begin{aligned}\dot{Q}_{1-2} &= I_1(A_1\cos\theta_1)\omega_{2-1}\\ &= (18048\ \text{W/m}^2\cdot\text{sr})(3\times10^{-4}\ \text{m}^2)(\cos 55°)(24.51\times10^{-4}\ \text{sr})\\ &= \mathbf{76.1\times10^{-4}\ W}\end{aligned}$$

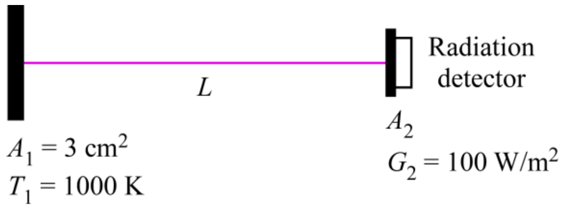
Irradiation is the rate at which radiation is incident upon the surface per unit surface area. Hence, the irradiation on A_2.is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{76.1\times10^{-4}\ \text{W}}{8\times10^{-4}\ \text{m}^2} = \mathbf{9.51\ W/m^2}$$

Discussion The total rate of radiation emission from surface A_1 is $\dot{Q}_e = A_1 E_b(T_1)$ = 17.01 W. Therefore, the fraction of emitted radiation that strikes A_2 is 76.1 × 10^{-4} / 17.01 = 0.045 percent.

12-58 A radiation detector (A_2) is placed normal to the direction of viewing from another surface (A_1), and is measuring a specified amount of irradiation. The distance between the radiation detector and the radiation emitting surface is to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.



Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2} \qquad (1)$$

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} \qquad (2)$$

Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\dot{Q}_{1-2} = I_1(A_1 \cos\theta_1)\omega_{2-1} \qquad (3)$$

Irradiation is the rate at which radiation is incident upon the surface per unit surface area. Hence, the irradiation measured by the radiation detector A_2.is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} \qquad (4)$$

Substituting Eqs. (1) to (3) into Eq. (4) yields

$$G_2 = \frac{\sigma T_1^4 (A_1 \cos\theta_1)(A_2 \cos\theta_2)}{\pi A_2 L^2} \quad \rightarrow \quad L = \left[\frac{\sigma T_1^4 A_1 \cos\theta_1 \cos\theta_2}{\pi G_2}\right]^{1/2}$$

Since the radiation detector is placed normal to the direction of viewing from A_1, we have $\theta_1 = \theta_2 = 0°$. Hence the distance L is

$$L = \left[\frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1000\ \text{K})^4(3\times10^{-4}\ \text{m}^2)}{\pi(100\ \text{W/m}^2)}\right]^{1/2} = \mathbf{0.233\,m}$$

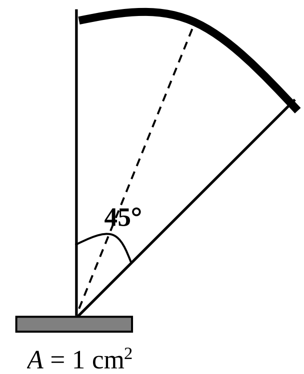
Discussion The solid angle subtended by A_2 is at maximum, since the radiation detector is positioned normal to the direction of viewing ($\theta_2 = 0°$).

12-59 A small surface is subjected to uniform incident radiation. The rates of radiation emission through two specified bands are to be determined.

Assumptions The intensity of incident radiation is constant.

Analysis (*a*) The rate at which radiation is incident on a surface per unit surface area in the direction (θ,ϕ) is given as

$$dG = \frac{d\dot{Q}_i}{dA} = I_i(\theta,\phi)\cos\theta \sin\theta d\theta d\phi$$



The total rate of radiation emission through the band between 0° and 45° can be expressed as

$$G_1 = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{45} I_i(\theta,\phi)\cos\theta \sin\theta\ d\theta\ d\phi = I_i\frac{\pi}{2}$$

since the incident radiation is constant (I_i = constant), and

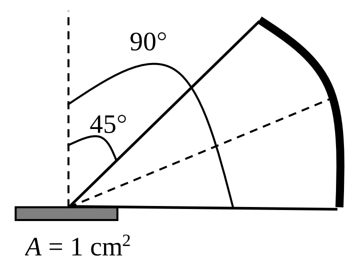
$$\int_{\phi=0}^{2\pi}\int_{\theta=0}^{45}\cos\theta\sin\theta\ d\theta\ d\phi = 2\pi\int_{\theta=0}^{45}\cos\theta\sin\theta\ d\theta = \pi(\sin^2 45 - \sin^2 0) = \pi/2$$

Approximating a small area as a differential area, the rate of radiation energy emitted from an area of 1 cm^2 in the specified band becomes

$$\dot{Q}_{i,1} = G_1 dA = 0.5\pi I_i dA = 0.5\pi(2.2\times10^4\ \text{W/m}^2\cdot\text{sr})(1\times10^{-4}\ \text{m}^2) = \mathbf{3.46\ W}$$

(*b*) Similarly, the total rate of radiation emission through the band between 45° and 90° can be expressed as

$$G_1 = \int_{\phi=0}^{2\pi}\int_{\theta=45}^{90} I_i(\theta,\phi)\cos\theta \sin\theta\ d\theta\ d\phi = I_i\frac{\pi}{2}$$



since

$$\int_{\phi=0}^{2\pi}\int_{\theta=45}^{90}\cos\theta\sin\theta\ d\theta\ d\phi = 2\pi\int_{\theta=45}^{90}\cos\theta\sin\theta\ d\theta$$
$$= \pi(\sin^2 90 - \sin^2 45) = \pi/2$$

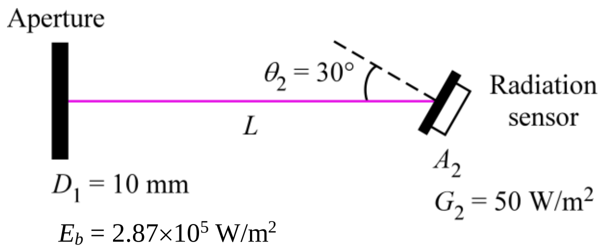
and

$$\dot{Q}_{i,2} = G_2 dA = 0.5\pi I_i dA = 0.5\pi(2.2\times10^4\ \text{W/m}^2\cdot\text{sr})(1\times10^{-4}\ \text{m}^2) = \mathbf{3.46\ W}$$

Discussion Note that the viewing area for the band 0° - 45° is much smaller, but the radiation energy incident through it is equal to the energy streaming through the remaining area.

12-60 A radiation sensor is placed with a 30° tilt off the normal direction of viewing from an aperture through which radiation is emitted as a blackbody. The distance between the sensor and the aperture is to be determined.

Assumptions **1** The aperture emits diffusely as a blackbody. **2** Both aperture and sensor can be approximated as differential surfaces since both are very small compared to the square of the distance between them.



Analysis Approximating both aperture (A_1) and radiation sensor (A_2) as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2} \qquad (1)$$

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b}{\pi} \qquad (2)$$

Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\dot{Q}_{1-2} = I_1 (A_1 \cos\theta_1)\omega_{2-1} \qquad (3)$$

where $A_1 = \pi D_1^2 / 4$

Irradiation is the rate at which radiation is incident upon the surface per unit surface area. Hence, the irradiation measured by the radiation detector A_2.is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} \qquad (4)$$

Substituting Eqs. (1) to (3) into Eq. (4) yields

$$G_2 = \frac{E_b (A_1 \cos\theta_1)(A_2 \cos\theta_2)}{\pi A_2 L^2} \quad \rightarrow \quad L = \left[\frac{E_b \pi D_1^2 \cos\theta_1 \cos\theta_2}{4\pi G_2}\right]^{1/2}$$

With $\theta_1 = 0°$ and $\theta_2 = 30°$, the distance between the sensor and the aperture is

$$L = \left[\frac{(2.87\times10^5 \text{ W/m}^2)(0.01\text{ m})^2 \cos 30°}{4(50 \text{ W/m}^2)}\right]^{1/2} = \mathbf{0.353\,m}$$

Discussion If the radiation sensor is positioned normal to the direction of viewing ($\theta_2 = 0°$) with $L = 0.353$ m, it would measure an irradiation of $G_2 = 58$ W/m^2.

12-61 A radiometer is placed normal to the direction of viewing from a circular plate (blackbody) and is measuring a specified amount of irradiation. The temperature of the plate is to be determined.

Assumptions **1** The plate emits diffusely as a blackbody. **2** Both plate and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both objects as differential surfaces, the solid angle subtended by the radiometer A_2 when viewed from the plate A_1 can be determined from

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2} \qquad (1)$$

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} \qquad (2)$$

Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle ω_{2-1} is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle ω_{2-1}. That is,

$$\dot{Q}_{1-2} = I_1(A_1 \cos\theta_1)\omega_{2-1} \qquad (3)$$

Irradiation is the rate at which radiation is incident upon the surface per unit surface area. Hence, the irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} \qquad (4)$$

Substituting Eqs. (1) to (3) into Eq. (4) yields

$$G_2 = \frac{\sigma T_1^4 (A_1 \cos\theta_1)(A_2 \cos\theta_2)}{\pi A_2 L^2} \quad \rightarrow \quad T_1 = \left[\frac{G_2 \pi L^2}{\sigma(\pi D_1^2/4)(\cos\theta_1)(\cos\theta_2)}\right]^{1/4}$$

Since the radiometer is placed normal to the direction of viewing from the plate ($\theta_1 = \theta_2 = 0°$), we have temperature of the plate as

$$T_1 = \left[\frac{(85\ \text{W/m}^2)(0.50\ \text{m})^2}{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(0.02\ \text{m})^2/4}\right]^{1/4} = \mathbf{1391\ K}$$

Discussion If the temperature of the plate is maintained constant while the distance between the plate and the radiometer decreases, then the irradiation detected by the radiometer would increase.

12-62 PtD A radiometer is used to monitor the surface temperature of an engine to prevent fire hazard in the event of an oil leakage. The engine surface temperature is to be determined from the irradiation measured by the radiometer.

Assumptions **1** The engine surface emits diffusely as a blackbody. **2** Both target surface and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them. **3** Engine surface temperature is uniform.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴.

Analysis The solid angle subtended by the radiometer when viewed from the engine surface is

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2}$$

The intensity of the radiation emitted by the target surface of the engine A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi}$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1 \cos\theta_1)\omega_{2-1}}{A_2} = \frac{\sigma T_1^4 A_1(\cos\theta_1)(\cos\theta_2)}{\pi L^2}$$

Since the radiometer is placed normal to the direction of viewing from the target surface ($\theta_1 = \theta_2 = 0°$), we have

$$G_2 = \frac{\sigma T_1^4 A_1}{\pi L^2} \quad \rightarrow \quad T_1 = \left(\frac{G_2 \pi L^2}{\sigma A_1}\right)^{1/4}$$

The engine surface temperature is

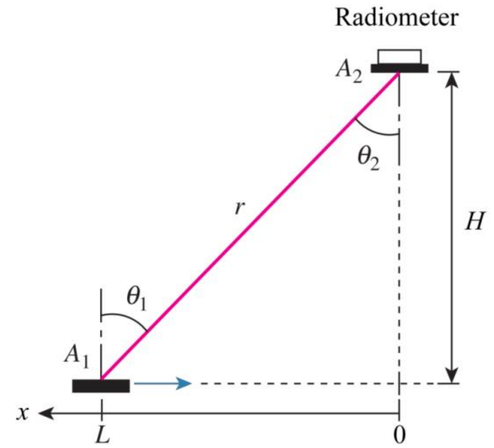
$$T_1 = \left[\frac{(0.1\text{ W/m}^2)(1\text{ m})^2 \pi}{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(1\times10^{-4}\text{ m}^2)}\right]^{1/4} = 485\text{ K} = \mathbf{212°C} > 180°\text{C}$$

Discussion The irradiation measured by the radiometer indicates that the engine surface temperature is higher than 180°C. Therefore there is a risk of fire hazard in the event of an oil leakage.

12-63 A radiometer is used to measure the position of an approaching hot object. The position of the object when the irradiation on the radiometer is 80% corresponding to the object position of $x = 0$ is to be determined.

Assumptions **1** The approaching object emits diffusely as a blackbody. **2** Both object and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them

Analysis Approximating both objects as differential surfaces, the solid angle subtended by the radiometer A_2 when viewed from the approaching object A_1 can be determined from



$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} = \frac{A_2 H}{r^2 r} = \frac{A_2 H}{(H^2 + x^2)^{3/2}} \qquad (1)$$

Note that

$$r = (H^2 + x^2)^{1/2}$$

and

$$\cos\theta_1 = \cos\theta_2 = \frac{H}{r} = \frac{H}{(H^2 + x^2)^{1/2}}$$

Then, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle ω_{2-1} is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle ω_{2-1}. That is

$$\dot{Q}_{1-2} = I_1 (A_1 \cos\theta_1)\omega_{2-1} = I_1 A_1 \frac{H}{(H^2 + x^2)^{1/2}} \omega_{2-1} \qquad (2)$$

Substituting Eq. (1) into (2) yields

$$\dot{Q}_{1-2} = I_1 A_1 A_2 \frac{H^2}{(H^2 + x^2)^2}$$

Hence, the irradiation measured by the radiometer A_2 is

$$G_{2,x} = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1 A_1 H^2}{(H^2 + x^2)^2}$$

Hence, the position L at which the sensor is measuring 80% of the irradiation corresponding to the position of the object directly under the radiometer at $x = 0$ can be determined as

$$\frac{G_{2,L}}{G_{2,0}} = 0.80 = \left[\frac{I_1 A_1 H^2}{(H^2 + L^2)^2}\right]\left[\frac{(H^2)^2}{I_1 A_1 H^2}\right] = \left(\frac{H^2}{H^2 + L^2}\right)^2$$

$$L = \left(\frac{H^2}{0.80^{0.5}} - H^2\right)^{0.5} = \left[\frac{(0.5\text{ m})^2}{0.80^{0.5}} - (0.5\text{ m})^2\right]^{0.5} = \mathbf{0.172\,m}$$

Discussion Knowing the relationship of $G_{2,L}$ / $G_{2,0}$ with the position of the approaching object, engineers can use it to implement specific treatment, such as spray painting the object when it reaches a position on a production line.

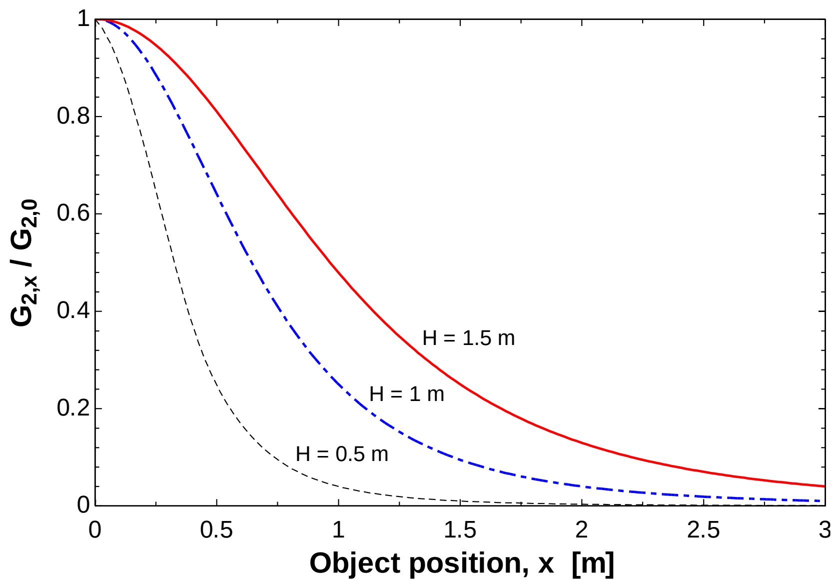
12-64 A radiometer is used to measure the position of an approaching hot object. The effect of the approaching object position on the irradiation measured by the radiometer is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
H_1=0.5 [m]
H_2=1.0 [m]
H_3=1.5 [m]

"ANALYSIS"
G_ratio1=(H_1^2/(H_1^2+x^2))^2
G_ratio2=(H_2^2/(H_2^2+x^2))^2
G_ratio3=(H_3^2/(H_3^2+x^2))^2
```

	$G_{2,x} / G_{2,0}$		
x [m]	H = 0.5 m	1 m	1.5 m
0	1	1	1
0.2	0.7432	0.9246	0.9654
0.4	0.3718	0.7432	0.8716
0.6	0.1680	0.5407	0.7432
0.8	0.0789	0.3718	0.6061
1.0	0.0400	0.2500	0.4793
1.2	0.02188	0.1680	0.3718
1.4	0.01280	0.1141	0.2856
1.6	0.007915	0.07890	0.2188
1.8	0.005131	0.05562	0.1680
2.0	0.003460	0.04000	0.12960
2.2	0.002412	0.02932	0.10070
2.4	0.001730	0.02188	0.07890
2.6	0.001272	0.01661	0.06236
2.8	0.0009550	0.01280	0.04973
3.0	0.0007305	0.01000	0.04000



Discussion When the placement of the radiometer is nearest to the x-axis (H = 0.5 m), $G_{2,x} / G_{2,0}$ increases most rapidly as the object approaches to x = 0. Therefore, the radiometer should be placed appropriately based on its response to the approaching object.

12-65 A radiometer A_2 is placed normal to the direction of viewing from the plate A_1 at a distance L. The irradiation on the radiometer, if the distance L is doubled, is to be determined.

Assumptions **1** The plate emits diffusely as a blackbody. **2** Both plate and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both objects as differential surfaces, the solid angle subtended by the radiometer A_2 when viewed from the plate A_1 can be determined from

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2} \qquad (1)$$

The radiation emitted by A_1 that strikes A_2 is equivalent to the radiation emitted by A_1 through the solid angle $\omega_{2\text{-}1}$. The intensity of the radiation emitted by A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} \qquad (2)$$

Therefore, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle ω_{2-1} is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle ω_{2-1}. That is,

$$\dot{Q}_{1-2} = I_1(A_1 \cos\theta_1)\omega_{2-1} \qquad (3)$$

Irradiation is the rate at which radiation is incident upon the surface per unit surface area. Hence, the irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} \qquad (4)$$

Substituting Eqs. (1) to (3) into Eq. (4) yields

$$G_2 = \frac{\sigma T_1^4 (A_1 \cos\theta_1)(A_2 \cos\theta_2)}{\pi\, A_2\, L^2}$$

The irradiation on the radiometer at L and $2L$ are

$$G_{2,L} = \frac{\sigma T_1^4 A_1(\cos\theta_1)(\cos\theta_2)}{\pi\, L^2} \quad \text{and} \quad G_{2,2L} = \frac{\sigma T_1^4 A_1(\cos\theta_1)(\cos\theta_2)}{\pi\,(2L)^2}$$

Thus,

$$\frac{G_{2,2L}}{G_{2,L}} = \left[\frac{\sigma T_1^4 A_1(\cos\theta_1)(\cos\theta_2)}{\pi(2L)^2}\right]\left[\frac{\pi L^2}{\sigma T_1^4 A_1(\cos\theta_1)(\cos\theta_2)}\right] = \mathbf{\frac{1}{4}} \quad \rightarrow \quad G_{2,2L} = \frac{1}{4} G_{2,L}$$

Discussion Doubling the distance L would quarter the irradiation on the radiometer.

12-66 A blackbody plate is subjected to uniform heat flux at the bottom and the top surface is exposed to ambient surrounding. A radiometer is placed above the plate and the irradiation detected by the radiometer is to be determined.

Assumptions **1** The plate emits diffusely as a blackbody. **2** Both plate and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them. **3** Plate surface temperature is uniform. **4** Heat loss from plate's side surface is negligible. **5** The surroundings are treated as an isothermal surface, $T_{surr} = T_\infty$.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴.

Analysis By performing energy balance on the plate, we can determine the surface temperature of the plate:

$$\dot{q} = \dot{q}_{\text{emit}} - \dot{q}_{\text{incident}} + \dot{q}_{\text{conv}}$$

$$\dot{q} = \sigma T_1^4 - \sigma T_{\text{surr}}^4 + h(T_1 - T_\infty)$$

$$1000 \text{ W/m}^2 = (5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(T_1^4 - 278^4)\text{ K}^4 + (5 \text{ W/m}^2\cdot\text{K})(T_1 - 278)\text{ K}$$

$$\rightarrow \quad T_1 = 358 \text{ K}$$

The solid angle subtended by the radiometer when viewed from the plate is

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2\cos\theta_2}{L^2}$$

The intensity of the radiation emitted by the plate A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi}$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1\cos\theta_1)\omega_{2-1}}{A_2} = \frac{\sigma T_1^4 A_1(\cos\theta_1)(\cos\theta_2)}{\pi L^2}$$

Since the radiometer is placed normal to the direction of viewing from the plate ($\theta_1 = \theta_2 = 0°$), we have

$$G_2 = \frac{\sigma T_1^4 A_1}{\pi L^2} = \frac{(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(358^4)\text{ K}^4(5\times10^{-4}\text{ m}^2)}{\pi(1\text{ m})^2} = \mathbf{0.148\ W/m^2}$$

Discussion If the radiometer is placed at a distance farther away from the plate, it would detect a smaller value of irradiation G_2.

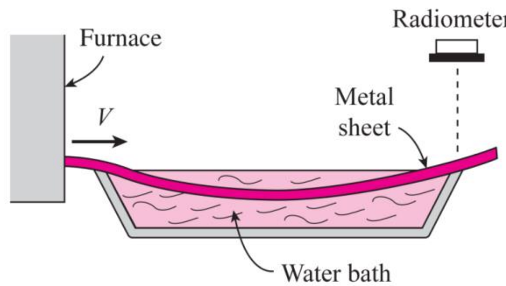
12-67 PtD A radiometer is used to monitor the surface temperature of a metal sheet exiting a water bath. The irradiation detected by the radiometer when the surface temperature is unsafe to touch is to be determined.

Assumptions **1** The metal sheet surface emits diffusely as a blackbody. **2** Both target surface and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them. **3** Metal sheet temperature at the water bath exit is uniform.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴.

Analysis The solid angle subtended by the radiometer when viewed from the metal sheet surface is

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2}$$



The intensity of the radiation emitted by the target surface of the metal sheet A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi}$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1 \cos\theta_1)\omega_{2-1}}{A_2} = \frac{\sigma T_1^4 A_1(\cos\theta_1)(\cos\theta_2)}{\pi L^2}$$

Since the radiometer is placed normal to the direction of viewing from the target surface ($\theta_1 = \theta_2 = 0°$), we have

$$G_2 = \frac{\sigma T_1^4 A_1}{\pi L^2}$$

The irradiation measured by the radiometer when the metal sheet temperature is at 45°C or higher is

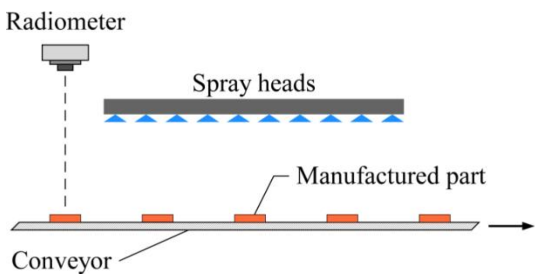
$$G_2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(318\ \text{K})^4(1\times10^{-4}\ \text{m}^2)}{\pi(0.5\ \text{m})^2} = \mathbf{0.0738\ W/m^2}$$

Discussion When the radiometer detects an irradiation of 0.0738 W/m² or higher, an alarm should be triggered warning people that the metal sheet is unsafe to touch.

12-68 PtD A radiometer is used to monitor temperatures of manufactured parts. The irradiation detected by the radiometer when the temperature of a part is unsafe to touch is to be determined.

Assumptions **1** The manufactured parts emit diffusely as blackbody. **2** Both part surface and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them. **3** Part surface temperature is uniform.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4.



Analysis The solid angle subtended by the radiometer when viewed from the manufactured parts is

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2}$$

The intensity of the radiation emitted by a manufactured part A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi}$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1 \cos\theta_1)\omega_{2-1}}{A_2} = \frac{\sigma T_1^4 A_1 (\cos\theta_1)(\cos\theta_2)}{\pi L^2}$$

Since the radiometer measures the radiation emitted by a manufactured part when it is normal in the direction of viewing ($\theta_1 = \theta_2 = 0°$), we have

$$G_2 = \frac{\sigma T_1^4 A_1}{\pi L^2}$$

The irradiation measured by the radiometer when a manufactured part temperature is at 45°C or higher is

$$G_2 = \frac{(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(318\ \text{K})^4 (10 \times 10^{-4}\ \text{m}^2)}{\pi (1\ \text{m})^2} = \mathbf{0.185\ W/m^2}$$

Discussion When the radiometer detects an irradiation of 0.185 W/m^2 or higher, the spray heads should release mist to cool the parts to prevent thermal burn hazards at the end of the conveyor.

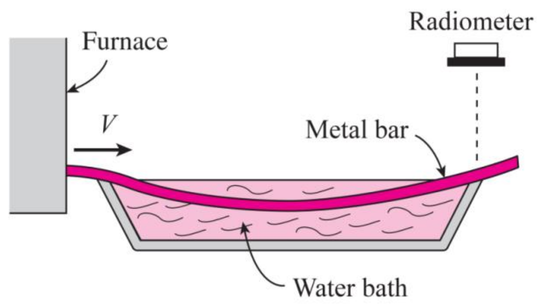
12-69 A radiometer is used to monitor temperature of a metal bar leaving a quenching process. The speed of the metal bar is to be determined from the irradiation detected by the radiometer.

Assumptions **1** The metal bar emits diffusely as blackbody. **2** Both target surface and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them. **3** The metal bar temperature at the water bath exit is uniform.

Properties The specific heat and the density of metal bar are given as $c_{p,ss}$ = 450 J/kg·K and ρ_{ss} = 7900 kg/m^3, respectively. The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4.

Analysis The solid angle subtended by the radiometer when viewed from the metal bar surface is

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2}$$



The intensity of the radiation emitted by the target surface of the metal bar A_1 is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi}$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1\cos\theta_1)\omega_{2-1}}{A_2} = \frac{\sigma T_1^4 A_1(\cos\theta_1)(\cos\theta_2)}{\pi L^2}$$

Since the radiometer is placed normal to the direction of viewing from the target surface ($\theta_1 = \theta_2 = 0°$), we have

$$G_2 = \frac{\sigma T_1^4 A_1}{\pi L^2} \quad \rightarrow \quad T_1 = \left(\frac{G_2 \pi L^2}{\sigma A_1}\right)^{1/4}$$

$$T_1 = \left[\frac{(0.015\ \text{W/m}^2)\pi(1\ \text{m})^2}{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1\times10^{-4}\ \text{m}^2)}\right]^{1/4} = 302\ \text{K} = 29°\text{C} = T_{\text{out}} < 45°\text{C}$$

The mass of the metal bar being conveyed enters and exits the water bath at a rate of

$$\dot{m} = \rho_{ss} V w t$$

The rate of heat removed from the metal bar is

$$\dot{Q}_{\text{removed}} = \dot{m} c_{p,ss}(T_{\text{in}} - T_{\text{out}}) = \rho_{ss} V w t c_{p,ss}(T_{\text{in}} - T_{\text{out}})$$

The speed of the bar is

$$V = \frac{\dot{Q}_{\text{removed}}}{\rho_{ss} w t c_{p,ss}(T_{\text{in}} - T_{\text{out}})} = \frac{500\times10^3\ \text{J/s}}{(7900\ \text{kg/m}^3)(0.030\ \text{m})(0.015\ \text{m})(450\ \text{J/kg}\cdot\text{K})(700-29)\ \text{K}} = \mathbf{0.466\ m/s}$$

Discussion At a speed of about 0.47 m/s, the metal bar can be cooled down to below 45°C for prevention from thermal burn hazards.

12-70 A radiometer is used to monitor the surface temperature of a CPCV pipe. The surface temperature of the CPVC pipe is to be determined from the irradiation measured by the radiometer and whether it complies with the ASME Code for Process Piping.

Assumptions **1** The CPVC pipe surface behaves as blackbody. **2** Irradiation on the pipe surface from the surroundings is negligible. **3** Both target surface and radiometer can be approximated as differential surfaces since both are very small compared to the square distance between them. **4** Uniform surface temperature.

Analysis The solid angle subtended by the radiometer when viewed from the CPVC pipe surface is

$$\omega_{2-1} = \frac{A_2 \cos\theta_2}{L^2}$$

The intensity of the radiation emitted from the target surface A_1 of the CPVC pipe is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi}$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1\cos\theta_1)\omega_{2-1}}{A_2} = \frac{\sigma T_1^4}{\pi}\frac{(A_1\cos\theta_1)(\cos\theta_2)}{L^2}$$

With the radiometer placed normal to the direction of viewing from the target surface, we have $\theta_1 = \theta_2 = 0°$,

$$G_2 = \frac{\sigma T_1^4 A_1}{\pi L^2}$$

With an irradiation of 500 μW/m^2 measured by the radiometer, the surface temperature of the CPVC pipe is

$$T_1 = \left(\frac{\pi L^2 G_2}{\sigma A_1}\right)^{1/4} = \left[\frac{\pi(1\text{ m})^2(500\times10^{-6}\text{W/m}^2)}{(5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4)(1\times10^{-6}\text{m}^2)}\right]^{1/4} = 408\text{ K} = \mathbf{135}℃ > 93.3℃$$

Discussion With an irradiation of 500 μW/m^2 measured by the radiometer, the CPVC pipe surface temperature is determined to be 135°C. Thus, it exceeds the maximum use temperature of 93.3°C specified by the ASME Code for Process Piping, and it does not comply with the code.

12-71 C&S A radiometer is used to monitor the surface temperature of a stainless steel sheet. The surface temperature of the sheet is to be determined from the irradiation measured by the radiometer and whether it complies with the ASME Code for Process Piping.

Assumptions **1** The sheet surface is gray and diffuse. **2** Irradiation on the surface of the sheet from the surroundings is negligible. **3** Both target surface and radiometer can be approximated as differential surfaces since both are very small compared to the square distance between them. **4** Uniform surface temperature.

Property The emissivity of the stainless steel sheet is given as 0.4.

Analysis The solid angle subtended by the radiometer when viewed from the stainless steel sheet surface is

$$\omega_{2-1} = \frac{A_2 \cos\theta_2}{L^2}$$

The intensity of the radiation emitted from the target surface A_1 of the stainless steel is

$$I_1 = \frac{\varepsilon E_b(T_1)}{\pi} = \frac{\varepsilon\sigma T_1^4}{\pi}$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1\cos\theta_1)\omega_{2-1}}{A_2} = \frac{\varepsilon\sigma T_1^4}{\pi}\frac{(A_1\cos\theta_1)(\cos\theta_2)}{L^2}$$

With the radiometer placed normal to the direction of viewing from the target surface, we have $\theta_1 = \theta_2 = 0°$,

$$G_2 = \frac{\varepsilon\sigma T_1^4 A_1}{\pi L^2}$$

With an irradiation of 0.2 W/m^2 measured by the radiometer, the surface temperature of the stainless steel sheet is

$$T_1 = \left(\frac{\pi L^2 G_2}{\varepsilon\sigma A_1}\right)^{1/4} = \left[\frac{\pi(0.7\text{ m})^2(0.2\text{ W/m}^2)}{(0.4)(5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4)(5\times10^{-6}\text{m}^2)}\right]^{1/4} = 1284\text{ K} = \mathbf{1011}°\text{C} > 649°\text{C}$$

Discussion With an irradiation of 0.2 W/m^2 measured by the radiometer, the stainless steel sheet surface temperature is determined to be 1011°C. Thus, it exceeds the maximum use temperature of 649°C specified by the ASME Code for Process Piping, and it does not comply with the code.

Radiation Properties

12-72C A body whose surface properties are independent of wavelength is said to be a graybody. The emissivity of a blackbody is one for all wavelengths, the emissivity of a graybody is between zero and one. A surface whose properties change with wavelength and direction is called a diffuse gray surface.

12-73C The emissivity ε is the ratio of the radiation emitted by the surface to the radiation emitted by a blackbody at the same temperature. The fraction of radiation absorbed by the surface is called the absorptivity α,

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} \quad \text{and} \quad \alpha = \frac{\text{absorbed radiation}}{\text{incident radiation}} = \frac{G_{abs}}{G}$$

When the surface temperature is equal to the temperature of the source of radiation, the total hemispherical emissivity of a surface at temperature T is equal to its total hemispherical absorptivity for radiation coming from a blackbody at the same temperature $\varepsilon_\lambda(T) = \alpha_\lambda(T)$.

12-74C The fraction of irradiation reflected by the surface is called reflectivity ρ and the fraction transmitted is called the transmissivity τ

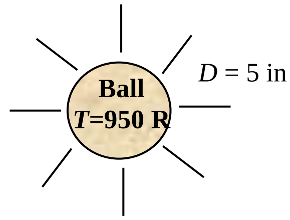
$$\rho = \frac{G_{ref}}{G} \quad \text{and} \quad \tau = \frac{G_{tr}}{G}$$

Surfaces are assumed to reflect in a perfectly spectral or diffuse manner for simplicity. In spectral (or mirror like) reflection, the angle of reflection equals the angle of incidence of the radiation beam. In diffuse reflection, radiation is reflected equally in all directions.

12-75C The heating effect which is due to the non-gray characteristic of glass, clear plastic, or atmospheric gases is known as the greenhouse effect since this effect is utilized primarily in greenhouses. The combustion gases such as CO_2 and water vapor in the atmosphere transmit the bulk of the solar radiation but absorb the infrared radiation emitted by the surface of the earth, acting like a heat trap. There is a concern that the energy trapped on earth will eventually cause global warming and thus drastic changes in weather patterns.

12-76C Glass has a transparent window in the wavelength range 0.3 to 3 μm and it is not transparent to the radiation which has wavelength range greater than 3 μm. Therefore, because the microwaves are in the range of 10^2 to 10^5 μm, the harmful microwave radiation cannot escape from the glass door.

12-77E A spherical ball emits radiation at a certain rate. The average emissivity of the ball is to be determined at the given temperature.



Analysis The surface area of the ball is

$$A = \pi D^2 = \pi(5/12\text{ ft})^2 = 0.5454\text{ ft}^2$$

Then the average emissivity of the ball at this temperature is determined to be

$$E = \varepsilon A\sigma T^4 \longrightarrow \varepsilon = \frac{E}{A\sigma T^4} = \frac{480\text{ Btu/h}}{(0.5454\text{ ft}^2)(0.1714\times10^{-8}\text{ Btu/h.ft}^2\cdot\text{R}^4)(950\text{ R})^4} = \mathbf{0.630}$$

12-78 A radiation sensor (A_2) is placed normal to the direction of viewing from another surface (A_1). An optical filter with specified spectral transmissivity is placed in front of the sensor. The irradiation that is measured by the sensor is to be determined.

Assumptions **1** Surface A_1 emits diffusely as a blackbody. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis For T = 1000 K, the blackbody radiation functions corresponding to $\lambda_1 T$ is determined from Table 12-2 to be

$$\lambda_1 T = (2\ \mu\text{m})(1000\text{ K}) = 2000\ \mu\text{m}\cdot\text{K} \qquad \rightarrow \qquad f_{\lambda_1} = 0.066728$$

Hence, the transmissivity of the optical filter is

$$\tau = \tau_1(f_{\lambda_1}) + \tau_2(1 - f_{\lambda_1}) = 0(0.066728) + 0.5(1 - 0.066728) = 0.4666$$

The rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\dot{Q}_{1-2} = I_1(A_1\cos\theta_1)\omega_{2-1}\,\tau$$

where

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2\cos\theta_2}{L^2} \qquad \text{and} \qquad I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi}$$

Since the radiation sensor is placed normal to the direction of viewing from A_1, we have $\theta_1 = \theta_2 = 0°$, hence

$$\dot{Q}_{1-2} = \frac{\sigma T_1^4 A_1 A_2}{L^2\pi}\tau$$

Irradiation is the rate at which radiation is incident upon the surface per unit surface area. Hence, the irradiation measured by the radiation sensor A_2 is

$$G_{\text{tr}} = G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{\sigma T_1^4 A_1}{L^2\pi}\tau$$

$$= \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(1000\text{ K})^4(5\times10^{-4}\text{ m}^2)}{(0.5\text{ m})^2\pi}(0.4666)$$

$$= \mathbf{16.8\ W/m^2}$$

Discussion If the optical filter is removed, the irradiation measured by the radiation sensor would be

$$G = \frac{G_{\text{tr}}}{0.4666} = \frac{16.8\text{ W/m}^2}{0.4666} = 36\text{ W/m}^2$$

12-79 PtD A radiometer is used to monitor the surface temperature of a tank surface to prevent thermal burn hazard. The tank surface temperature is to be determined from the irradiation measured by the radiometer.

Assumptions **1** Both target surface and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them. **2** Tank surface temperature and properties are uniform. **3** Kirchhoff's law is applicable. **4** The tank wall is opaque.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴.

Analysis From Kirchhoff's law, the emissivity of the plate is

$$\varepsilon = \alpha = 0.85$$

The tank wall is opaque and the reflectivity of the tank is

$$\rho = 1 - \alpha = 1 - 0.85 = 0.15$$

The solid angle subtended by the radiometer when viewed from the target surface of the tank A_1 is

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2}$$

The intensity of the radiation emitted and reflected (radiosity) by the target surface A_1 is

$$I_1 = \frac{J}{\pi} = \frac{E + G_{\text{ref}}}{\pi} = \frac{\varepsilon\sigma T_1^4 + \rho G_1}{\pi}$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1\cos\theta_1)\omega_{2-1}}{A_2} = \frac{(\varepsilon\sigma T_1^4 + \rho G)A_1(\cos\theta_1)(\cos\theta_2)}{\pi L^2}$$

Since the radiometer is placed normal to the direction of viewing from the target surface ($\theta_1 = \theta_2 = 0°$), we have

$$G_2 = \frac{(\varepsilon\sigma T_1^4 + \rho G)A_1}{\pi L^2} \quad \to \quad T_1 = \left[\left(\frac{G_2 \pi L^2}{A_1} - \rho G_1\right)\left(\frac{1}{\varepsilon\sigma}\right)\right]^{0.25}$$

$$T_1 = \left\{\left[\frac{(0.085\text{ W/m}^2)\pi(0.5\text{ m})^2}{1\times10^{-4}\text{ m}^2} - (0.15)(390\text{ W/m}^2)\right]\left[\frac{1}{(0.85)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)}\right]\right\}^{0.25}$$

$$= 335\text{ K}$$

$$= \mathbf{62°C} > 45°\text{C}$$

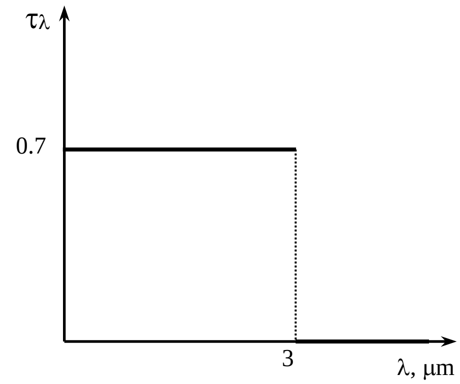
Discussion The irradiation measured by the radiometer indicates that the tank surface temperature is higher than 45°C. Therefore there is a risk of thermal burn hazard.

12-80 The variation of transmissivity of the glass window of a furnace at a specified temperature with wavelength is given. The fraction and the rate of radiation coming from the furnace and transmitted through the window are to be determined.

Assumptions The window glass behaves as a black body.

Analysis The fraction of radiation at wavelengths smaller than 3 μm is

$$\lambda T = (3\ \mu\text{m})(1200\ \text{K}) = 3600\ \mu\text{mK} \longrightarrow f_\lambda = 0.403607$$



The fraction of radiation coming from the furnace and transmitted through the window is

$$\begin{aligned}\tau(T) &= \tau_1 f_\lambda + \tau_2(1-f_\lambda) \\ &= (0.7)(0.403607)+(0)(1-0.403607) \\ &= \mathbf{0.2825}\end{aligned}$$

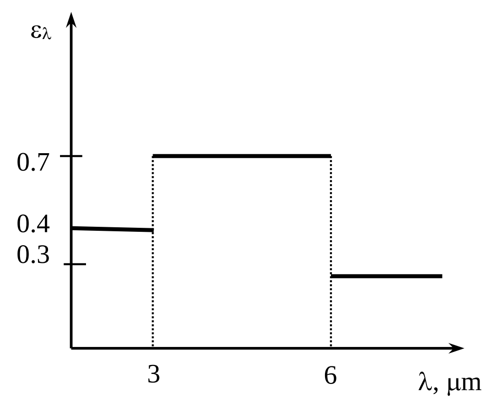
Then the rate of radiation coming from the furnace and transmitted through the window becomes

$$G_{tr} = \tau A\sigma T^4 = 0.2825(0.40\times 0.40\ \text{m}^2)(5.67\times 10^{-8}\ \text{W/m}^2.\text{K}^4)(1200\ \text{K})^4 = \mathbf{5315\ W}$$

12-81 The variation of emissivity of a surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

Analysis The average emissivity of the surface can be determined from

$$\begin{aligned}\varepsilon(T) &= \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b_\lambda}(T)d\lambda}{\sigma T^4} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b_\lambda}(T)d\lambda}{\sigma T^4} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b_\lambda}(T)d\lambda}{\sigma T^4} \\ &= \varepsilon_1 f_{0\text{-}\lambda_1} + \varepsilon_2 f_{\lambda_1\text{-}\lambda_2} + \varepsilon_3 f_{\lambda_2\text{-}\infty} \\ &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3(1-f_{\lambda_2})\end{aligned}$$



where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, determined from

$$\lambda_1 T = (3\ \mu\text{m})(1000\ \text{K}) = 3000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.273232$$

$$\lambda_2 T = (6\ \mu\text{m})(1000\ \text{K}) = 6000\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.737818$$

$$f_{0-\lambda_1} = f_{\lambda 1} - f_0 = f_{\lambda_1} \text{ since } f_0 = 0 \text{ and } f_{\lambda_2-\infty} = f_\infty - f_{\lambda_2} \text{ since } f_\infty = 1.$$

and,

$$\varepsilon = (0.4)0.273232 + (0.7)(0.737818 - 0.273232) + (0.3)(1-0.737818) = \mathbf{0.5132}$$

Then the emissive power of the surface becomes

$$E = \varepsilon\sigma T^4 = 0.5132(5.67\times 10^{-8}\ \text{W/m}^2.\text{K}^4)(1000\ \text{K})^4 = 29{,}100\ \text{W/m}^2 = \mathbf{29.1 kW/m^2}$$

12-82 The spectral emissivity for an ASTM A240 410S stainless steel plate is given. The highest rate of radiation emission that the plate can achieve without exceeding the maximum use temperature specified by the ASME code is to be determined.

Assumptions **1** The plate is diffuse. **2** Uniform surface temperature.

Analysis The average emissivity of the plate surface can be determined from

$$\varepsilon(T) = \frac{\varepsilon_1}{\sigma T^4}\int_0^{\lambda_1} E_{b\lambda}\, d\lambda + \frac{\varepsilon_2}{\sigma T^4}\int_{\lambda_1}^{\lambda_2} E_{b\lambda}\, d\lambda + \frac{\varepsilon_3}{\sigma T^4}\int_{\lambda_2}^{\infty} E_{b\lambda}\, d\lambda$$

$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2\left(f_{\lambda_2} - f_{\lambda_1}\right) + \varepsilon_3\left(1 - f_{\lambda_2}\right)$$

The highest rate of radiation emission that the plate can achieve without violating the ASME code is at 649°C or 922 K. At 922 K, the blackbody radiation functions corresponding to the wavelengths are

$$\lambda_1 T = (3.25\ \mu\text{m})(922\ \text{K}) = 2997\ \mu\text{m·K} \approx 3000\ \mu\text{m·K} \qquad \longrightarrow \qquad f_{\lambda_1} = 0.273232$$

$$\lambda_2 T = (10.3\ \mu\text{m})(922\ \text{K}) = 9497\ \mu\text{m·K} \approx 9500\ \mu\text{m·K} \qquad \longrightarrow \qquad f_{\lambda_2} = 0.903085$$

So, the average emissivity of the plate surface is

$$\varepsilon(T) = 0.8(0.273232) + 0.6(0.903085 - 0.273232) + 0.3(1 - 0.903085) = 0.6256$$

Thus, the highest rate of radiation emission that the plate can achieve without exceeding the maximum use temperature is

$$E = \varepsilon\sigma T^4 = (0.6256\ \text{W/m}^2\cdot\text{K}^4)(5.67\times10^{-8})(922\ \text{K})^4 = \mathbf{25633\ W/m^2}$$

Discussion The highest rate of radiation emission that can be achieved from that plate is 25.6 kW/m^2. Any radiation emission that is higher than 25.6 kW/m^2 would require the surface temperature to exceed the maximum use temperature specified by the ASME code.

12-83 The spectral reflectivity for an ASTM A992 carbon steel plate is given. The highest rate of radiation emission that the plate can achieve without exceeding the maximum use temperature specified by the ASME code is to be determined.

Assumptions **1** The plate is diffuse. **2** The spectral form of Kirchhoff's law applies, $\varepsilon_\lambda = \alpha_\lambda$. **3** The plate is opaque. **4** Uniform surface temperature.

Analysis For an opaque plate, the absorptivity and reflectivity are related as

$$\alpha_\lambda + \rho_\lambda = 1$$

With the spectral form of Kirchhoff's law ($\varepsilon_\lambda = \alpha_\lambda$), we have

$$\varepsilon_\lambda = 1 - \rho_\lambda$$

Thus, the spectral emissivity for the plate becomes

$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = 0.8, & 0 \le \lambda \le 4\ \mu\text{m} \\ \varepsilon_2 = 0.5, & 4\ \mu\text{m} \le \lambda \le 15\ \mu\text{m} \\ \varepsilon_3 = 0.2, & 15\ \mu\text{m} \le \lambda < \infty \end{cases}$$

The average emissivity of the plate surface can be determined from

$$\varepsilon(T) = \frac{\varepsilon_1}{\sigma T^4}\int_0^{\lambda_1} E_{b\lambda}\, d\lambda + \frac{\varepsilon_2}{\sigma T^4}\int_{\lambda_1}^{\lambda_2} E_{b\lambda}\, d\lambda + \frac{\varepsilon_3}{\sigma T^4}\int_{\lambda_2}^{\infty} E_{b\lambda}\, d\lambda$$

$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2\left(f_{\lambda_2} - f_{\lambda_1}\right) + \varepsilon_3\left(1 - f_{\lambda_2}\right)$$

The highest rate of radiation emission that the plate can achieve without violating the ASME code is at 427°C or 700 K. At 700 K, the blackbody radiation functions corresponding to the wavelengths are

$$\lambda_1 T = (4\ \mu\text{m})(700\ \text{K}) = 2800\ \mu\text{m·K} \quad \longrightarrow \quad f_{\lambda_1} = 0.227897$$

$$\lambda_2 T = (15\ \mu\text{m})(700\ \text{K}) = 10500\ \mu\text{m·K} \quad \longrightarrow \quad f_{\lambda_2} = 0.923710$$

So, the average emissivity of the plate surface is

$$\varepsilon(T) = 0.8(0.227897) + 0.5(0.923710 - 0.227897) + 0.2(1 - 0.923710) = 0.5455$$

Thus, the highest rate of radiation emission that the plate can achieve without exceeding the maximum use temperature is

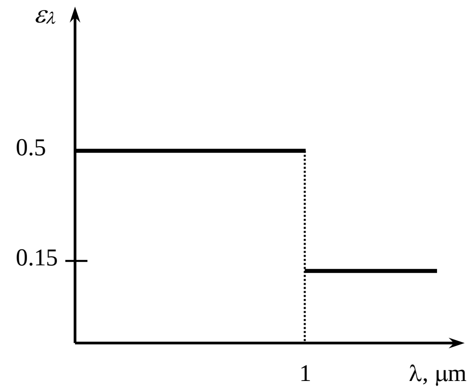
$$E = \varepsilon\sigma T^4 = (0.5455)(5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K}^4)(700\ \text{K})^4 = \mathbf{7426\ W/m^2}$$

Discussion The highest rate of radiation emission that can be achieved from the plate is 7426 W/m^2. Any radiation emission that is higher than 7426 W/m^2 would require the surface temperature to exceed the maximum use temperature specified by the ASME code.

12-84 The variation of emissivity of a tungsten filament with wavelength is given. The average emissivity, absorptivity, and reflectivity of the filament are to be determined for two temperatures.

Analysis (*a*) T = 1500 K

$$\lambda_1 T = (1\ \mu\text{m})(1500\ \text{K}) = 1500\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.013754$$



The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) \\ &= (0.5)(0.013754) + (0.15)(1 - 0.013754) \\ &= \mathbf{0.155}\end{aligned}$$

From Kirchhoff's law,

$$\varepsilon = \alpha = \mathbf{0.155} \quad \text{(at 1500 K)}$$

and $\quad \alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.155 = \mathbf{0.845}$

(*b*) T = 2500 K

$$\lambda_1 T = (1\ \mu\text{m})(2500\ \text{K}) = 2500\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.161688$$

Then

$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1}) = (0.5)(0.161688) + (0.15)(1 - 0.161688) = \mathbf{0.207}$$

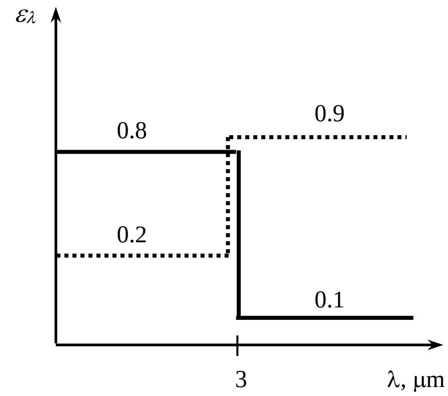
From Kirchhoff's law,

$$\varepsilon = \alpha = \mathbf{0.207} \quad \text{(at 2500 K)}$$

and $\quad \alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.207 = \mathbf{0.793}$

12-85 The variations of emissivity of two surfaces are given. The average emissivity, absorptivity, and reflectivity of each surface are to be determined at the given temperature.

Analysis For the first surface:



$$\lambda_1 T = (3\ \mu\text{m})(3000\,\text{K}) = 9000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.890029$$

The average emissivity of this surface is

$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1})$$
$$= (0.2)(0.890029) + (0.9)(1 - 0.890029)$$
$$= \mathbf{0.28}$$

The absorptivity and reflectivity are determined from Kirchhoff's law

$$\varepsilon = \alpha = \mathbf{0.28} \quad \text{(at 3000 K)}$$

$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.28 = \mathbf{0.72}$$

For the second surface:

$$\lambda_1 T = (3\ \mu\text{m})(3000\,\text{K}) = 9000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.890029$$

The average emissivity of this surface is

$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (1 - f_{\lambda_1})$$
$$= (0.8)(0.890029) + (0.1)(1 - 0.890029)$$
$$= \mathbf{0.72}$$

Then,

$$\varepsilon = \alpha = \mathbf{0.72} \quad \text{(at 3000 K)}$$

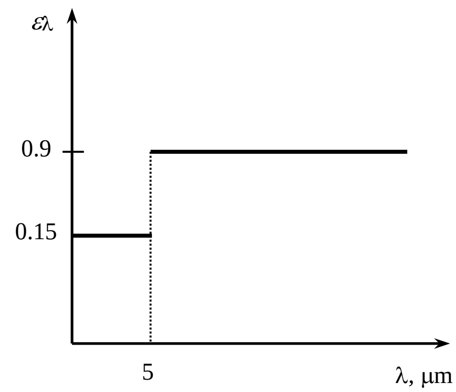
$$\alpha + \rho = 1 \rightarrow \rho = 1 - \alpha = 1 - 0.72 = \mathbf{0.28}$$

Discussion The second surface is more suitable to serve as a solar absorber since its absorptivity for short wavelength radiation (typical of radiation emitted by a high-temperature source such as the sun) is high, and its emissivity for long wavelength radiation (typical of emitted radiation from the absorber plate) is low.

12-86 The variation of emissivity of a surface with wavelength is given. The average emissivity and absorptivity of the surface are to be determined for two temperatures.

Analysis (*a*) For T = 5800 K:

$$\lambda_1 T = (5\ \mu\text{m})(5800\ \text{K}) = 29{,}000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.994715$$



The average emissivity of this surface is

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2(1 - f_{\lambda_1})\\ &= (0.15)(0.994715) + (0.9)(1 - 0.994715)\\ &= \mathbf{0.154}\end{aligned}$$

(*b*) For T = 300 K:

$$\lambda_1 T = (5\mu\text{m})(300\ \text{K}) = 1500\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.013754$$

and

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2(1 - f_{\lambda_1})\\ &= (0.15)(0.013754) + (0.9)(1 - 0.013754)\\ &= \mathbf{0.89}\end{aligned}$$

The absorptivities of this surface for radiation coming from sources at 5800 K and 300 K are, from Kirchhoff's law,

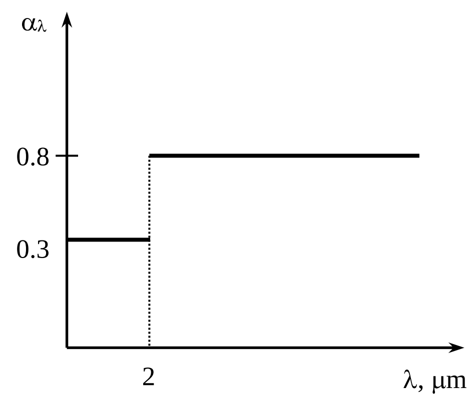
$$\alpha = \varepsilon = \mathbf{0.154} \quad \text{(at 5800 K)}$$

$$\alpha = \varepsilon = \mathbf{0.89} \quad \text{(at 300 K)}$$

12-87 The variation of absorptivity of a surface with wavelength is given. The average absorptivity, reflectivity, and emissivity of the surface are to be determined at given temperatures.

Analysis For T = 2500 K:

$$\lambda_1 T = (2\ \mu\text{m})(2500\ \text{K}) = 5000\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.633747$$



The average absorptivity of this surface is

$$\begin{aligned}\alpha(T) &= \alpha_1 f_{\lambda_1} + \alpha_2(1 - f_{\lambda_1})\\ &= (0.3)(0.633747) + (0.8)(1 - 0.633747)\\ &= \mathbf{0.483}\end{aligned}$$

Then the reflectivity of this surface becomes

$$\alpha + \rho = 1 \longrightarrow \rho = 1 - \alpha = 1 - 0.483 = \mathbf{0.517}$$

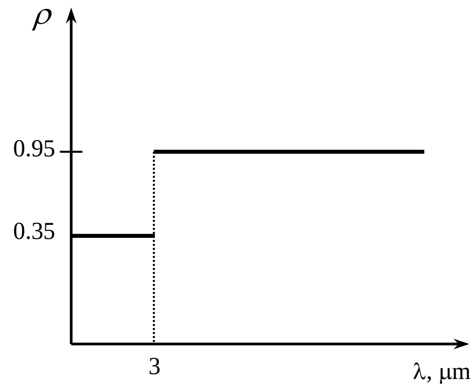
Using Kirchhoff's law, $\alpha = \varepsilon$, the average emissivity of this surface at T = 3000 K is determined to be

$$\lambda T = (2\mu\text{m})(3000\ \text{K}) = 6000\ \mu\text{mK} \longrightarrow f_{\lambda} = 0.737818$$

$$\begin{aligned}\varepsilon(T) &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2(1 - f_{\lambda_1})\\ &= (0.3)(0.737818) + (0.8)(1 - 0.737818)\\ &= \mathbf{0.431}\end{aligned}$$

12-88 The variation of reflectivity of a surface with wavelength is given. The average reflectivity, emissivity, and absorptivity of the surface are to be determined for two source temperatures.

Analysis The average reflectivity of this surface for solar radiation (T = 5800 K) is determined to be



$$\lambda T = (3\ \mu\text{m})(5800\ \text{K}) = 17400\ \mu\text{mK} \rightarrow f_\lambda = 0.978746$$

$$\begin{aligned}\rho(T) &= \rho_1 f_{0-\lambda_1}(T) + \rho_2 f_{\lambda_1-\infty}(T) \\ &= \rho_1 f_{\lambda_1} + \rho_2(1 - f_{\lambda_1}) \\ &= (0.35)(0.978746) + (0.95)(1 - 0.978746) \\ &= \mathbf{0.362}\end{aligned}$$

Noting that this is an opaque surface, $\tau = 0$

At T = 5800 K: $\alpha + \rho = 1 \longrightarrow \alpha = 1 - \rho = 1 - 0.362 = \mathbf{0.638}$

Repeating calculations for radiation coming from surfaces at T = 300 K,

$$\lambda T = (3\ \mu\text{m})(300\ \text{K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0001685$$

$$\rho(T) = (0.35)(0.0001685) + (0.95)(1 - 0.0001685) = \mathbf{0.95}$$

At T = 300 K: $\alpha + \rho = 1 \longrightarrow \alpha = 1 - \rho = 1 - 0.95 = \mathbf{0.05}$

and $\varepsilon = \alpha = \mathbf{0.05}$

The temperature of the aluminum plate is close to room temperature, and thus emissivity of the plate will be equal to its absorptivity at room temperature. That is,

$$\varepsilon = \varepsilon_{room} = 0.05$$
$$\alpha = \alpha_s = 0.638$$

which makes it suitable as a solar collector. ($\alpha_s = 1$ and $\varepsilon_{\text{room}} = 0$ for an ideal solar collector)

12-89 The spectral reflectivity for an ASTM A992 carbon steel plate is given. Solar irradiation on the plate, and the net radiation heat flux from the plate are given. Determine whether the carbon steel plate is suitable for the given conditions, so that it complies with the ASME code.

Assumptions **1** The plate is diffuse. **2** The spectral form of Kirchhoff's law applies, $\varepsilon_\lambda = \alpha_\lambda$. **3** The plate is opaque. **4** Uniform surface temperature.

Analysis For an opaque plate, the absorptivity and reflectivity are related as

$$\alpha_\lambda + \rho_\lambda = 1$$

Thus, the spectral emissivity for the plate becomes

$$\alpha_\lambda = \begin{cases} \alpha_1 = 0.55, & 0 \le \lambda \le 1\ \mu\text{m} \\ \alpha_2 = 0.35, & 1\ \mu\text{m} \le \lambda \le 6\ \mu\text{m} \\ \alpha_3 = 0.20, & 6\ \mu\text{m} \le \lambda < \infty \end{cases}$$

The average absorptivity of the plate can be determined from

$$\alpha(T) = \frac{\alpha_1}{\sigma T^4}\int_0^{\lambda_1} E_{b\lambda}\,d\lambda + \frac{\alpha_2}{\sigma T^4}\int_{\lambda_1}^{\lambda_2} E_{b\lambda}\,d\lambda + \frac{\alpha_3}{\sigma T^4}\int_{\lambda_2}^{\infty} E_{b\lambda}\,d\lambda$$

$$\alpha(T) = \alpha_1 f_{\lambda_1} + \alpha_2\left(f_{\lambda_2} - f_{\lambda_1}\right) + \alpha_3\left(1 - f_{\lambda_2}\right)$$

The blackbody radiation functions corresponding to the wavelengths for the absorptivity of the plate to the solar irradiation at 5800 K are

$$\lambda_1 T = (1\ \mu\text{m})(5800\ \text{K}) = 5800\ \mu\text{m·K} \quad \longrightarrow \quad f_{\lambda_1} = 0.720158$$

$$\lambda_2 T = (6\ \mu\text{m})(5800\ \text{K}) = 34800\ \mu\text{m·K} \quad \longrightarrow \quad f_{\lambda_2} = 0.996601$$

So, the average absorptivity of the plate to solar irradiation is

$$\alpha(T) = 0.55(0.720158) + 0.35(0.996601 - 0.720158) + 0.20(1 - 0.996601) = 0.4935$$

With the spectral form of Kirchhoff's law, we have

$$\varepsilon_\lambda = \alpha_\lambda$$

The average emissivity of the plate surface can be determined from

$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2\left(f_{\lambda_2} - f_{\lambda_1}\right) + \varepsilon_3\left(1 - f_{\lambda_2}\right)$$

At the plate maximum use temperature of 700 K, the blackbody radiation functions corresponding to the wavelengths are

$$\lambda_1 T = (1\ \mu\text{m})(700\ \text{K}) = 700\ \mu\text{m·K} \quad \longrightarrow \quad f_{\lambda_1} = 0.000008$$

$$\lambda_2 T = (6\ \mu\text{m})(700\ \text{K}) = 4200\ \mu\text{m·K} \quad \longrightarrow \quad f_{\lambda_2} = 0.516014$$

So, the average emissivity of the plate at the maximum use temperature of 700 K is

$$\varepsilon(T) = 0.55(0.000008) + 0.35(0.516014 - 0.000008) + 0.20(1 - 0.516014) = 0.2774$$

The net radiation heat flux leaving the plate surface at T_s = 700 K is

$$\dot{q}_{\text{rad}} = E - G_{\text{abs}} = \varepsilon\sigma T_s^4 - \alpha G_{\text{solar}}$$

$$\dot{q}_{\text{rad}} = (0.2774)(5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4)(700\ \text{K})^4 - (0.4935)(1000\ \text{W/m}^2) = \mathbf{3283\ W/m^2} < 6000\ \text{W/m}^2$$

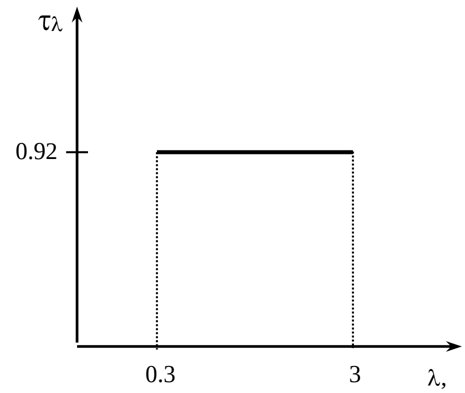
Discussion The net radiation heat flux leaving the plate surface at the maximum use temperature of 700 K is less than 6000 W/m^2. For a net radiation heat flux of 6000 W/m^2 to leave the plate surface would require a higher surface temperature that would exceed the maximum use temperature specified by the ASME code. Therefore, the plate is not suitable to be used in the given conditions.

12-90 The variation of transmissivity of a glass is given. The average transmissivity of the pane at two temperatures and the amount of solar radiation transmitted through the pane are to be determined.

Analysis For T=5800 K:

$$\lambda_1 T_1 = (0.3\ \mu\text{m})(5800\ \text{K}) = 1740\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.033454$$

$$\lambda_2 T_1 = (3\ \mu\text{m})(5800\ \text{K}) = 17{,}400\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.978746$$



The average transmissivity of this surface is

$$\tau(T) = \tau_1(f_{\lambda_2} - f_{\lambda_1}) = (0.92)(0.978746 - 0.033454) = \mathbf{0.870}$$

For T=300 K:

$$\lambda_1 T_2 = (0.3\ \mu\text{m})(300\ \text{K}) = 90\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0$$

$$\lambda_2 T_2 = (3\ \mu\text{m})(300\ \text{K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.0001685$$

Then,

$$\tau(T) = \tau_1(f_{\lambda_2} - f_{\lambda_1}) = (0.92)(0.0001685 - 0.0) = \mathbf{0.00016 \approx 0}$$

The amount of solar radiation transmitted through this glass is

$$G_{\text{tr}} = \tau G_{\text{incident}} = 0.870(650\ \text{W/m}^2) = \mathbf{566\ \ W/m^2}$$

12-91 An opaque horizontal plate that is well insulated on the edges and the lower surface has a constant temperature of 500 K and α = 0.51, (a) the total hemispherical emissivity and (b) the radiosity of the plate surface are to be determined.

Assumptions **1** The plate has a uniform temperature. **2** The plate is well insulated on the edges and the lower surface.

Properties The total hemispherical absorptivity of the plate is given to be 0.51.

Analysis (a) The average emissivity of the surface can be determined from

$$\varepsilon(T) = \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b\lambda}\, d\lambda}{E_b} + \frac{\varepsilon_2 \int_{\lambda_1}^{\infty} E_{b\lambda}\, d\lambda}{E_b} = \varepsilon_1\, f_{0-\lambda_1} + \varepsilon_2\, f_{\lambda_1-\infty} = \varepsilon_1\, f_{\lambda_1} + \varepsilon_2\,(1 - f_{\lambda_1})$$

For T = 500 K, the blackbody radiation functions corresponding to $\lambda_1 T$ is determined from Table 12-2 to be

$$\lambda_1 T = (4\ \mu\text{m})(500\ \text{K}) = 2000\ \mu\text{m}\cdot\text{K} \quad \rightarrow \quad f_{\lambda_1} = 0.066728$$

Hence, the total hemispherical emissivity is

$$\varepsilon(T) = 0.4(0.066728) + 0.8(1 - 0.066728) = \mathbf{0.773}$$

(b) The radiosity of the plate surface can be determined from the following expression:

$$J = E + G_{\text{ref}} = \varepsilon E_b + \rho G$$

Since the plate is opaque (τ = 0), the reflectivity is then, $\rho = 1 - \alpha$. Hence,

$$\begin{aligned} J &= \varepsilon\sigma T^4 + (1-\alpha)G \\ &= (0.773)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(500\ \text{K})^4 + (1-0.51)(5600\ \text{W/m}^2) \\ &= \mathbf{5480\ W/m^2} \end{aligned}$$

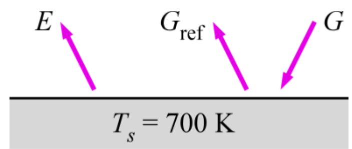
Discussion Both emissive power (E) and reflected irradiation (G_{ref}) contributed equal amount to the radiosity, with $E = G_{\text{ref}}$ = 2740 W/m^2.

12-92 An opaque horizontal plate that is well insulated on the edges and the lower surface experiences irradiation, the total emissivity and absorptivity of the plate are to be determined.

Assumptions **1** The plate has a uniform temperature. **2** The plate is well insulated on the edges and the lower surface.

Analysis The total emissivity of the plate can be determined using

$$\varepsilon = \frac{E}{E_b} = \frac{E}{\sigma T_s^4} = \frac{5000 \text{ W/m}^2}{(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(700 \text{ K})^4} = \mathbf{0.367}$$



The total absorptivity of the plate is determined using

$$\alpha+\rho+\tau=1 \quad \rightarrow \quad \alpha=1-\rho \qquad \text{(for opaque surface, } \tau = 0)$$

The reflectivity of the plate is

$$\rho = \frac{G_{\text{ref}}}{G} = \frac{500}{3000} = 0.167$$

Hence, the total absorptivity of the plate is

$$\alpha = 1-0.167 = \mathbf{0.833}$$

Discussion The plate has a total absorptivity that is about 5 times the reflectivity.

12-93 Irradiation is on a semi-transparent medium. The medium's absorptivity, reflectivity, transmissivity, and emissivity are to be determined.

Assumptions **1** Properties are constant. **2** Kirchhoff's law is applicable.

Analysis The irradiation on the medium is

$$G = G_{\text{abs}} + G_{\text{ref}} + G_{\text{tr}}$$

The irradiation absorbed by the medium is

$$G_{\text{abs}} = G - G_{\text{ref}} - G_{\text{tr}} = (520-160-130) \text{ W/m}^2 = 230 \text{ W/m}^2$$

The absorptivity is

$$\alpha = \frac{G_{\text{abs}}}{G} = \frac{230}{520} = \mathbf{0.442}$$

The reflectivity is

$$\rho = \frac{G_{\text{ref}}}{G} = \frac{160}{520} = \mathbf{0.308}$$

The transmissivity is

$$\tau = \frac{G_{\text{tr}}}{G} = \frac{130}{520} = \mathbf{0.25}$$

From Kirchhoff's law, the emissivity of the medium is

$$\varepsilon = \alpha = \mathbf{0.442}$$

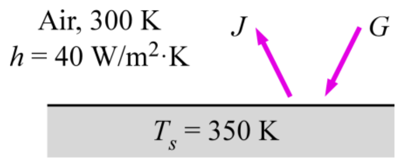
Discussion Having determined α and ρ, the transmissivity can also be determined using $\tau = 1 - \alpha - \rho = 0.25$.

12-94 An opaque horizontal plate that is well insulated on the edges and the lower surface is uniformly irradiated from above, (*a*) the irradiation on the plate, (*b*) the total reflectivity of the plate, (*c*) the emissive power of the plate, and (*d*) the total emissivity of the plate are to be determined.

Assumptions **1** The plate has a uniform temperature. **2** The plate is well insulated on the edges and the lower surface.

Properties The total absorptivity of the plate is given to be 0.40.

Analysis (*a*) Applying energy balance on the surface,



$$G = J + \dot{q}_{\text{conv}} = J + h(T_s - T_\infty)$$
$$= 4000\ \text{W/m}^2 + (40\ \text{W/m}^2 \cdot \text{K})(350 - 300)\ \text{K}$$
$$= \mathbf{6000\ W/m^2}$$

(*b*) The total reflectivity of the plate is determined using

$$\alpha + \rho + \tau = 1 \quad \rightarrow \quad \rho = 1 - \alpha - \tau \quad \text{(for opaque surface, } \tau = 0\text{)}$$
$$\rho = 1 - 0.40 - 0 = \mathbf{0.60}$$

(*c*) The emissive power of the plate is

$$J = E + G_{\text{ref}} = E + \rho G \quad \rightarrow \quad E = J - \rho G$$
$$E = 4000\ \text{W/m}^2 - (0.60)(6000\ \text{W/m}^2) = \mathbf{400\ W/m^2}$$

(*d*) The total emissivity of the plate is

$$\varepsilon = \frac{E}{E_b} = \frac{E}{\sigma T_s^4} = \frac{400\ \text{W/m}^2}{(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(350\ \text{K})^4} = \mathbf{0.470}$$

Discussion The emissivity may also be determined by applying energy balance on the plate:

$$E = J - \rho G \quad \text{or} \quad \varepsilon\sigma T_s^4 = J - \rho G \quad \rightarrow \quad \varepsilon = \frac{J - \rho G}{\sigma T_s^4}$$

12-95 An opaque plate is being heated uniformly at the bottom and the top surface is exposed to natural convection and irradiated uniformly. The radiosity of the plate is to be determined.

Assumptions **1** The plate has a uniform temperature. **2** Heat loss from plate's side surface is negligible. **3** The surroundings are treated as an isothermal surface, $T_{surr} = T_\infty$. **4** Kirchhoff's law is applicable.

Properties The emissivity of the plate is given as $\varepsilon = 0.67$.

Analysis By performing energy balance on the plate surface gives

$$G + \dot{q}_{\text{elec}} = J + \dot{q}_{\text{conv}} = J + h(T_s - T_\infty) \qquad (1)$$

The reflectivity of the plate can be determined from

$$\alpha + \rho + \tau = 1$$

From Kirchhoff's law ($\varepsilon = \alpha$) and for an opaque medium ($\tau = 0$) we have

$$\begin{aligned} \rho &= 1 - \alpha - \tau \\ &= 1 - \varepsilon \\ &= 1 - 0.67 \\ &= 0.33 \end{aligned}$$

The radiosity of the plate is given as

$$J = E + G_{\text{ref}} = E + \rho G \quad \rightarrow \quad G = \frac{J - E}{\rho} \qquad (2)$$

Substituting Eq. (2) into (1) gives

$$\frac{J - E}{\rho} + \dot{q}_{\text{elec}} = J + h(T_s - T_\infty) \quad \rightarrow \quad J = \frac{\rho h(T_s - T_\infty) - \rho \dot{q}_{\text{elec}} + E}{1 - \rho}$$

Thus,

$$\begin{aligned} J &= \frac{\rho h(T_s - T_\infty) - \rho \dot{q}_{\text{elec}} + \varepsilon \sigma T_s^4}{1 - \rho} \\ &= \frac{(0.33)(7\ \text{W/m}^2 \cdot \text{K})(80 - 7)\ \text{K} - (0.33)(1000\ \text{W/m}^2) + (0.67)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(353\ \text{K})^4}{1 - 0.33} \\ &= \mathbf{639.6\ W/m^2} \end{aligned}$$

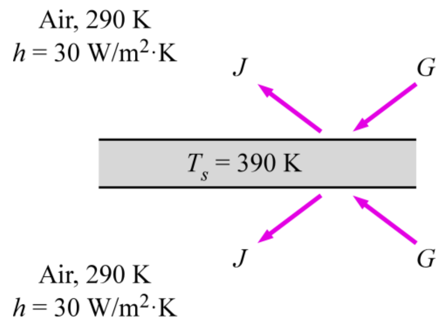
Discussion Radiosity contributed to about 56% of the total heat loss from the plate surface.

12-96 A horizontal non-opaque plate is experiencing uniform irradiation on the both upper and lower surfaces. The irradiation and emissivity of the plate are to be determined.

Assumptions **1** Steady operating condition exists. **2** The plate has a uniform temperature. **3** The convection heat transfer coefficient is uniform.

Properties The absorptivity of the plate is given to be 0.527.

Analysis Applying energy balance on the plate, we have



$$2G = 2J + 2\dot{q}_{\text{conv}} \quad \rightarrow \quad G = J + h(T_s - T_\infty)$$

$$G = 4000 \text{ W/m}^2 + (30 \text{ W/m}^2 \cdot \text{K})(390 - 290) \text{ K} = \mathbf{7000 \text{ W/m}^2}$$

Applying the definition of radiosity, we have

$$J = E + G_{\text{ref}} + G_{\text{tr}} = E + \rho G + \tau G = E + (\rho + \tau)G$$

Also, we have

$$\alpha + \rho + \tau = 1 \quad \text{or} \quad \rho + \tau = 1 - \alpha$$

Hence,

$$J = E + (1-\alpha)G \quad \text{or} \quad E = J - (1-\alpha)G$$

Then, the emissivity of the plate is

$$\varepsilon = \frac{E}{E_b}$$
$$= \frac{J - (1-\alpha)G}{\sigma T_s^4}$$
$$= \frac{[4000 - (1 - 0.527)7000] \text{ W/m}^2}{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(390 \text{ K})^4} = \mathbf{0.525}$$

Discussion Since $\alpha \approx \varepsilon \approx 0.53$, the plate can be considered as a gray surface.

12-97 Irradiation is on a semi-transparent plate. A radiometer is placed above the plate and the irradiation detected by the radiometer is to be determined.

Assumptions **1** Both plate and radiometer can be approximated as differential surfaces since both are very small compared to the square of the distance between them. **2** Plate surface temperature and properties are uniform. **3** Heat loss from plate's side surface is negligible. **4** Kirchhoff's law is applicable.

Properties The Stefan-Boltzmann constant is $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴.

Analysis The irradiation on the plate is

$$G = G_{\text{abs}} + G_{\text{ref}} + G_{\text{tr}} \quad \rightarrow \quad G_{\text{abs}} = G - G_{\text{ref}} - G_{\text{tr}}$$

The irradiation absorbed by the medium is

$$G_{\text{abs}} = G - 0.3G - 0.5G = 500(1 - 0.3 - 0.5)\ \text{W/m}^2 = 100\ \text{W/m}^2$$

The absorptivity and reflectivity of the plate are

$$\alpha = \frac{G_{\text{abs}}}{G} = \frac{100}{500} = 0.2 \quad \text{and} \quad \rho = 0.3\ \text{(given)}$$

From Kirchhoff's law, the emissivity of the plate is

$$\varepsilon = \alpha = 0.2$$

The solid angle subtended by the radiometer when viewed from the plate is

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2}$$

The intensity of the radiation emitted and reflected (radiosity) by the plate A_1 is

$$I_1 = \frac{J}{\pi} = \frac{E + G_{\text{ref}}}{\pi} = \frac{\varepsilon\sigma T_1^4 + \rho G}{\pi}$$

$$= \frac{(0.2)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(350^4)\ \text{K}^4 + (0.3)(500\ \text{W/m}^2)}{\pi} = 101.91\ \text{W/m}^2$$

The irradiation measured by the radiometer A_2 is

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{I_1(A_1 \cos\theta_1)\omega_{2-1}}{A_2} = \frac{I_1 A_1 (\cos\theta_1)(\cos\theta_2)}{L^2}$$

Since the radiometer is placed normal to the direction of viewing from the plate ($\theta_1 = \theta_2 = 0°$), we have

$$G_2 = \frac{I_1 A_1}{L^2} = \frac{(101.91\ \text{W/m}^2)(2 \times 10^{-4}\ \text{m}^2)}{(0.5\ \text{m})^2} = \mathbf{0.0815\ \ W/m^2}$$

Discussion The emissive power of the plate E contributed about 53% of the radiosity of the plate.

Atmospheric and Solar Radiation

12-98C The reason for different seasons is the tilt of the earth which causes the solar radiation to travel through a longer path in the atmosphere in winter, and a shorter path in summer. Therefore, the solar radiation is attenuated much more strongly in winter.

12-99C Because of different wavelengths of solar radiation and radiation originating from surrounding bodies, the surfaces usually have quite different absorptivities. Solar radiation is concentrated in the short wavelength region and the surfaces in the infrared region.

12-100C There is heat loss from both sides of the bridge (top and bottom surfaces of the bridge) which reduces temperature of the bridge surface to very low values. The relatively warm earth under a highway supply heat to the surface continuously, making the water on it less likely to freeze.

12-101C The amount of solar radiation incident on earth will decrease by a factor of

$$\text{Reduction factor} = \frac{\sigma T_{sun}{}^4}{\sigma T_{sun,new}{}^4} = \frac{5762^4}{2000^4} = 68.9$$

(or to 1.5% of what it was). Also, the fraction of radiation in the visible range would be much smaller.

12-102C The solar constant represents the rate at which solar energy is incident on a surface normal to sun's rays at the outer edge of the atmosphere when the earth is at its mean distance from the sun. It value is $G_s = 1353\ \text{W/m}^2$. The solar constant is used to estimate the effective surface temperature of the sun from the requirement that

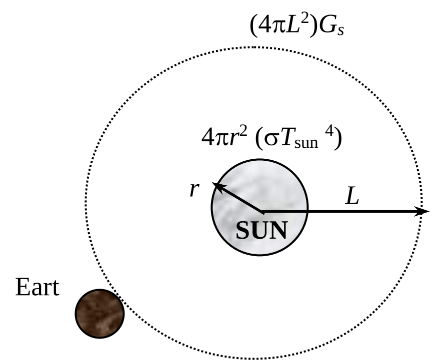
$$(4\pi L^2)G_{s1} = (4\pi r^2)\sigma T_{sun}{}^4$$

where L is the mean distance between the sun and the earth and r is the radius of the sun. If the distance between the earth and the sun doubled, the value of G_s drops to one-fourth since

$$4\pi(2L)^2 G_{s2} = (4\pi r^2)\sigma T_{sun}{}^4$$

$$16\pi L^2 G_{s2} = (4\pi r^2)\sigma T_{sun}{}^4$$

$$16\pi L^2 G_{s2} = 4\pi L^2 G_{s1} \longrightarrow \mathbf{G_{s2} = \frac{G_{s1}}{4}}$$



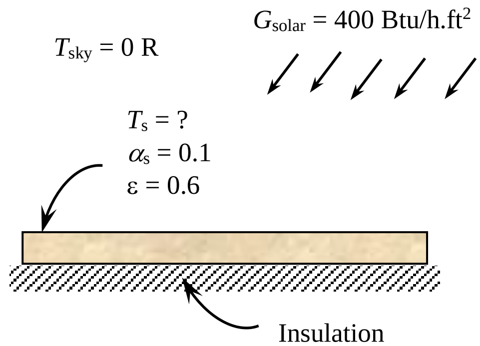
12-103C Air molecules scatter blue light much more than they do red light. This molecular scattering in all directions is what gives the sky its bluish color. At sunset, the light travels through a thicker layer of atmosphere, which removes much of the blue from the natural light, letting the red dominate.

12-104C The gas molecules and the suspended particles in the atmosphere emit radiation as well as absorbing it. Although this emission is far from resembling the distribution of radiation from a blackbody, it is found convenient in radiation calculations to treat the atmosphere as a blackbody at some lower fictitious temperature that emits an equivalent amount of radiation energy. This fictitious temperature is called the effective sky temperature T_{sky}.

12-105E A surface is exposed to solar and sky radiation. The equilibrium temperature of the surface is to be determined.

Properties The solar absorptivity and emissivity of the surface are given to $\alpha_s = 0.10$ and $\varepsilon = 0.6$.

Analysis The equilibrium temperature of the surface in this case is



$$\dot{q}_{net,rad} = \alpha_s G_{solar} - \varepsilon\sigma(T_s^{\,4} - T_{sky}^{\,4}) = 0$$

$$\alpha_s G_{solar} = \varepsilon\sigma(T_s^{\,4} - T_{sky}^{\,4})$$

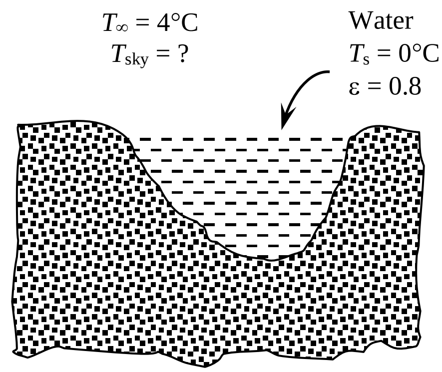
$$0.10(400 \text{ Btu/h.ft}^2) = 0.6(0.1714\times10^{-8} \text{ Btu/h.ft}^2\cdot\text{R}^4)\left[T_s^{\,4} - (0 \text{ R})^4\right]$$

$$T_s = \mathbf{444\,R}$$

12-106 Water is observed to have frozen one night while the air temperature is above freezing temperature. The effective sky temperature is to be determined.

Properties The emissivity of water is $\varepsilon = 0.95$ (Table A-18).

Analysis Assuming the water temperature to be 0°C, the value of the effective sky temperature is determined from an energy balance on water to be



$$h(T_{\text{air}} - T_{\text{surface}}) = \varepsilon\sigma(T_s^4 - T_{\text{sky}}^4)$$

and

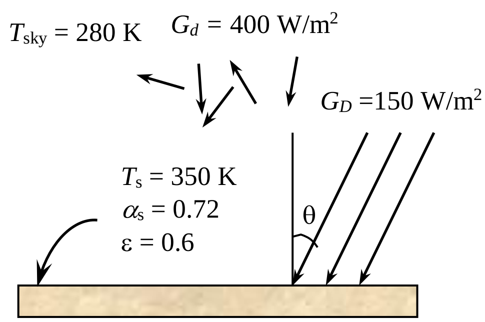
$$(18 \text{ W/m}^2\cdot°\text{C})(4°\text{C} - 0°\text{C}) = 0.95(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)\left[(273 \text{ K})^4 - T_{sky}^{\,4}\right]$$

$$\longrightarrow T_{\text{sky}} = \mathbf{254.8\,K}$$

Therefore, the effective sky temperature must have been below 255 K.

12-107 A surface is exposed to solar and sky radiation. The net rate of radiation heat transfer is to be determined.

Properties The solar absorptivity and emissivity of the surface are given to α_s = 0.72 and ε = 0.6.



Analysis The total solar energy incident on the surface is

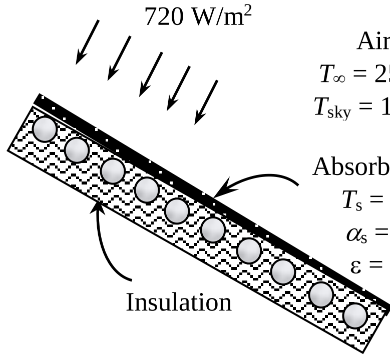
$$G_{solar} = G_D \cos\theta + G_d$$
$$= (350 \text{ W/m}^2)\cos 30° + (400 \text{ W/m}^2)$$
$$= 703.1 \text{ W/m}^2$$

Then the net rate of radiation heat transfer in this case becomes

$$\dot{q}_{net,rad} = \alpha_s G_{solar} - \varepsilon\sigma(T_s{}^4 - T_{sky}{}^4)$$
$$= 0.72(703.1 \text{ W/m}^2) - 0.6(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)\left[(350 \text{ K})^4 - (280 \text{ K})^4\right]$$
$$= \mathbf{205\ W/m^2} \quad \text{(to the surface)}$$

12-108 The absorber plate of a solar collector is exposed to solar and sky radiation. The net rate of solar energy absorbed by the absorber plate is to be determined.

Properties The solar absorptivity and emissivity of the surface are given to α_s = 0.87 and ε = 0.09.



Analysis The net rate of solar energy delivered by the absorber plate to the water circulating behind it can be determined from an energy balance to be

$$\dot{q}_{net} = \dot{q}_{gain} - \dot{q}_{loss}$$

$$\dot{q}_{net} = \alpha_s G_{solar} - \left[\varepsilon\sigma(T_s^4 - T_{sky}^4) + h(T_s - T_{air})\right]$$

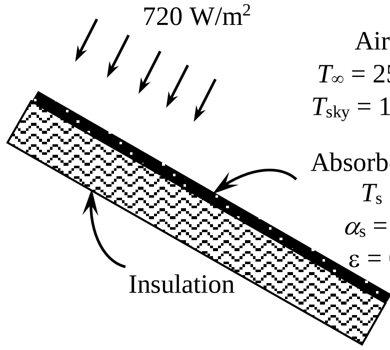
Then,

$$\dot{q}_{net} = 0.87(720\text{ W/m}^2) - 0.09(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)\left[(70+273\text{ K})^4 - (15+273\text{ K})^4\right]$$
$$-(10\text{ W/m}^2\cdot\text{K})(70°\text{C} - 25°\text{C})$$
$$= \mathbf{141\,W/m^2}$$

Therefore, heat is gained by the plate and transferred to water at a rate of 36.5 W per m² surface area.

12-109 The absorber surface of a solar collector is exposed to solar and sky radiation. The equilibrium temperature of the absorber surface is to be determined if the backside of the plate is insulated.

Properties The solar absorptivity and emissivity of the surface are given to α_s = 0.87 and ε = 0.09.



Analysis The backside of the absorbing plate is insulated (instead of being attached to water tubes), and thus

$$\dot{q}_{net} = 0$$

$$\alpha_s G_{solar} = \varepsilon\sigma(T_s^4 - T_{sky}^4) + h(T_s - T_{air})$$

$$(0.87)(720\text{ W/m}^2) = (0.09)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(T_s)^4 - (288\text{ K})^4\right] + (10\text{ W/m}^2\cdot\text{K})(T_s - 298\text{ K})$$

$$T_s = \mathbf{356\,K}$$

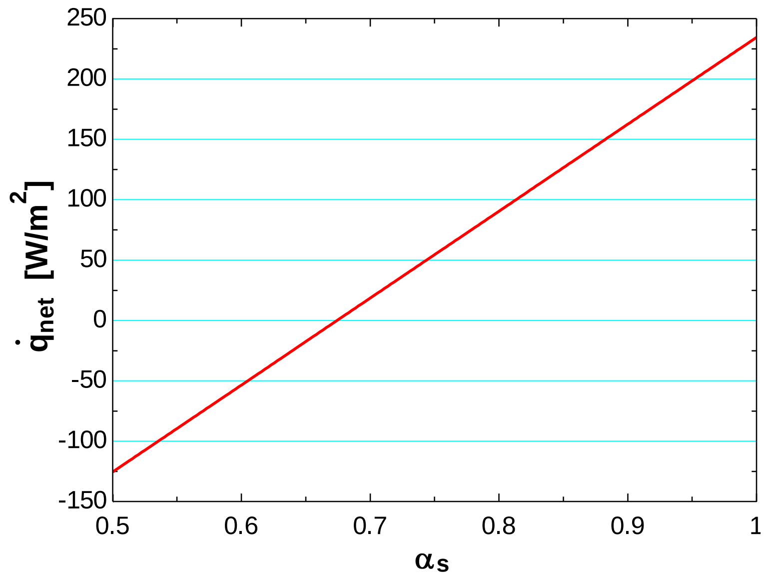
12-110 Prob. 12-108 is reconsidered. The the net rate of solar energy transferred to water as a function of the absorptivity of the absorber plate is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
alpha_s=0.87
epsilon=0.09
G_solar=720 [W/m^2]
T_air=25+273 "[K]”
T_sky=15+273 "[K]"
T_s=70+273 "[K]"
h=10 [W/m^2-C]
sigma=5.67E-8 [W/m^2-K^4] “Stefan-Boltzmann constant"

"ANALYSIS"
q_dot_net=q_dot_gain-q_dot_loss "energy balance"
q_dot_gain=alpha_s*G_solar
q_dot_loss=epsilon*sigma*(T_s^4-T_sky^4)+h*(T_s-T_air)
```

α_s	$\dot{q}_{net}$ [W/m^2]
0.5	-125.5
0.525	-107.5
0.55	-89.52
0.575	-71.52
0.6	-53.52
0.625	-35.52
0.65	-17.52
0.675	0.4751
0.7	18.48
0.725	36.48
0.75	54.48
0.775	72.48
0.8	90.48
0.825	108.5
0.85	126.5
0.875	144.5
0.9	162.5
0.925	180.5
0.95	198.5
0.975	216.5
1	234.5



Special Topic: Solar Heat Gain through Windows

12-111C (*a*) The spectral distribution of solar radiation beyond the earth's atmosphere resembles the energy emitted by a black body at 5982°C, with about 39 percent in the visible region (0.4 to 0.7 μm), and the 52 percent in the near infrared region (0.7 to 3.5 μm). (*b*) At a solar altitude of 41.8°, the total energy of direct solar radiation incident at sea level on a clear day consists of about 3 percent ultraviolet, 38 percent visible, and 59 percent infrared radiation.

12-112C A window that transmits visible part of the spectrum while absorbing the infrared portion is ideally suited for minimizing the air-conditioning load since such windows provide maximum daylighting and minimum solar heat gain. The ordinary window glass approximates this behavior remarkably well.

12-113C A low-e coating on the inner surface of a window glass reduces both the (*a*) heat loss in winter and (*b*) heat gain in summer. This is because the radiation heat transfer to or from the window is proportional to the emissivity of the inner surface of the window. In winter, the window is colder and thus radiation heat loss from the room to the window is low. In summer, the window is hotter and the radiation transfer from the window to the room is low.

12-114C A device that blocks solar radiation and thus reduces the solar heat gain is called a shading device. External shading devices are more effective in reducing the solar heat gain since they intercept sun's rays before they reach the glazing. The solar heat gain through a window can be reduced by as much as 80 percent by exterior shading. *Light colored* shading devices maximize the back reflection and thus minimize the solar gain. *Dark colored* shades, on the other hand, minimize the back refection and thus maximize the solar heat gain.

12-115C The SC (shading coefficient) of a device represents the solar heat gain relative to the solar heat gain of a reference glazing, typically that of a standard 3 mm (1/8 in) thick double-strength clear window glass sheet whose SHGC is 0.87. The shading coefficient of a 3-mm thick *clear glass* is SC = 1.0 whereas SC = 0.88 for 3-mm thick *heat absorbing glass*.

12-116C The **solar heat gain coefficient (**SHGC) is defined as the fraction of incident solar radiation that enters through the glazing. The solar heat gain of a glazing relative to the solar heat gain of a reference glazing, typically that of a standard 3 mm (1/8 in) thick double-strength clear window glass sheet whose SHGC is 0.87, is called the **shading coefficient**. They are related to each other by

$$\mathrm{SC} = \frac{\text{Solar heat gain of product}}{\text{Solar heat gain of reference glazing}} = \frac{\mathrm{SHGC}}{\mathrm{SHGC}_{\mathrm{ref}}} = \frac{\mathrm{SHGC}}{0.87} = 1.15 \times \mathrm{SHGC}$$

For single pane clear glass window, SHGC = 0.87 and SC = 1.0.

12-117 A building at 40º N latitude has double pane heat absorbing type windows that are equipped with light colored venetian blinds. The total solar heat gains of the building through the south windows at solar noon in April for the cases of with and without the blinds are to be determined.

Assumptions The calculations are performed for an "average" day in April, and may vary from location to location.

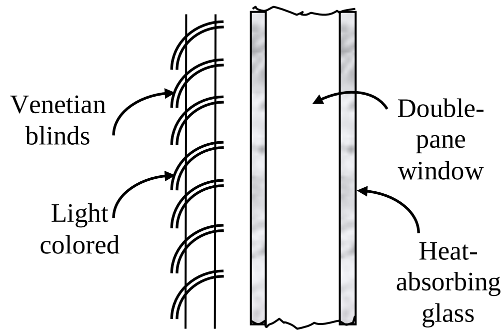
Properties The shading coefficient of a double pane heat absorbing type windows is SC = 0.58 (Table 12-5). It is given to be SC = 0.30 in the case of blinds. The solar radiation incident at a South-facing surface at 12:00 noon in April is 559 W/m^2 (Table 12-4).

Analysis The solar heat gain coefficient (SHGC) of the windows without the blinds is determined from Eq.12-57 to be

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times0.58 = 0.5046$$

Then the rate of solar heat gain through the window becomes

$$\begin{aligned}\dot{Q}_{\text{solar gain, no blinds}} &= SHGC\times A_{\text{glazing}}\times\dot{q}_{\text{solar, incident}}\\ &= 0.5046(76\text{ m}^2)(559\text{ W/m}^2)\\ &= \mathbf{21{,}440\text{ W}}\end{aligned}$$



In the case of windows equipped with venetian blinds, the SHGC and the rate of solar heat gain become

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times0.30 = 0.261$$

Then the rate of solar heat gain through the window becomes

$$\begin{aligned}\dot{Q}_{\text{solar gain, no blinds}} &= SHGC\times A_{\text{glazing}}\times\dot{q}_{\text{solar, incident}}\\ &= 0.261(76\text{ m}^2)(559\text{ W/m}^2)\\ &= \mathbf{11{,}090\text{ W}}\end{aligned}$$

Discussion Note that light colored venetian blinds significantly reduce the solar heat, and thus air-conditioning load in summers.

12-118 A house has double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers. It is to be determined if the house is losing more or less heat than it is gaining from the sun through an east window in a typical day in January.

Assumptions **1** The calculations are performed for an "average" day in January. **2** Solar data at 40° latitude can also be used for a location at 39° latitude.

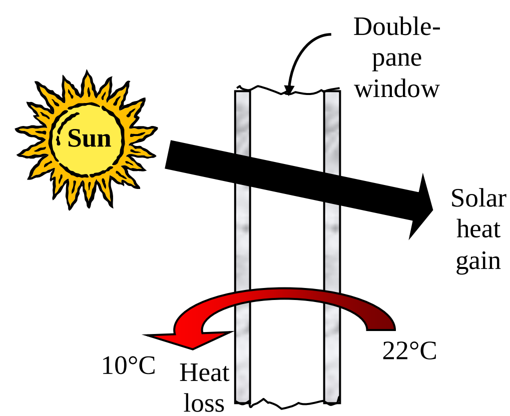
Properties The shading coefficient of a double pane window with 3-mm thick clear glass is SC = 0.88 (Table 12-5). The overall heat transfer coefficient for double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers is 4.55 W/m^2.°C. (Table 9-6). The total solar radiation incident at an East-facing surface in January during a typical day is 1863 Wh/m^2 (Table 12-4).

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq. 12-57 to be

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times0.88 = 0.7656$$

Then the solar heat gain through the window per unit area becomes

$$\begin{aligned}Q_{\text{solar gain}} &= SHGC\times A_{\text{glazing}}\times q_{\text{solar, daily total}}\\ &= 0.7656(1\text{ m}^2)(1863\text{ Wh/m}^2)\\ &= \mathbf{1426\text{ Wh} = 1.426\text{ kWh}}\end{aligned}$$



The heat loss through a unit area of the window during a 24-h period is

$$\begin{aligned}Q_{\text{loss, window}} &= \dot{Q}_{\text{loss, window}}\Delta t = U_{\text{window}}A_{\text{window}}(T_i - T_{0,\text{ave}})(1\text{ day})\\ &= (4.55\text{ W/m}^2\cdot{}^\circ\text{C})(1\text{ m}^2)(22-10)^\circ\text{C}(24\text{ h})\\ &= \mathbf{1310\text{ Wh} = 1.31\text{ kWh}}\end{aligned}$$

Therefore, the house is loosing **less** heat than it is gaining through the East windows during a typical day in January.

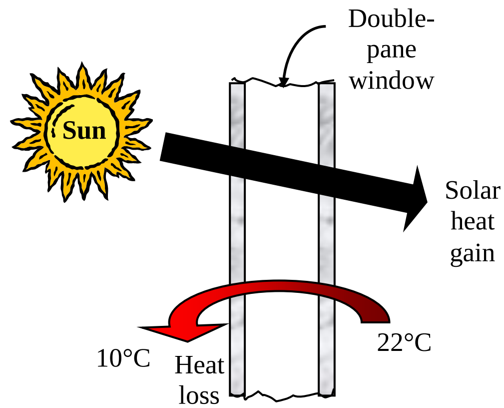
12-119 A house has double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers. It is to be determined if the house is losing more or less heat than it is gaining from the sun through a South window in a typical day in January.

Assumptions **1** The calculations are performed for an "average" day in January. **2** Solar data at 40° latitude can also be used for a location at 39° latitude.

Properties The shading coefficient of a double pane window with 3-mm thick clear glass is SC = 0.88 (Table 12-5). The overall heat transfer coefficient for double door type windows that are double pane with 6.4 mm of air space and aluminum frames and spacers is 4.55 W/m^2.°C (Table 9-6). The total solar radiation incident at a South-facing surface in January during a typical day is 5897 Wh/m^2 (Table 12-5).

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq. 12-57 to be

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times 0.88 = 0.7656$$



Then the solar heat gain through the window per unit area becomes

$$\begin{aligned} Q_{\text{solar gain}} &= SHGC\times A_{\text{glazing}}\times q_{\text{solar, daily total}} \\ &= 0.7656(1\,\text{m}^2)(5897\,\text{Wh/m}^2) \\ &= \mathbf{4515\ Wh} = \mathbf{4.515\ kWh} \end{aligned}$$

The heat loss through a unit area of the window during a 24-h period is

$$\begin{aligned} Q_{\text{loss, window}} &= \dot{Q}_{\text{loss, window}}\Delta t = U_{\text{window}}A_{\text{window}}(T_i - T_{0,\text{ave}})(1\,\text{day}) \\ &= (4.55\,\text{W/m}^2\cdot{}^\circ\text{C})(1\,\text{m}^2)(22-10)^\circ\text{C}(24\,\text{h}) \\ &= \mathbf{1310\ Wh} = \mathbf{1.31\,kWh} \end{aligned}$$

Therefore, the house is **loosing** much less heat than it is gaining through the South windows during a typical day in January.

12-120 A building located near 40º N latitude has equal window areas on all four sides. The side of the building with the highest solar heat gain in summer is to be determined.

Assumptions The shading coefficients of windows on all sides of the building are identical.

Analysis The reflective films should be installed on the side that receives the most incident solar radiation in summer since the window areas and the shading coefficients on all four sides are identical. The incident solar radiation at different windows in July are given to be (Table 12-5)

Month	Time	The daily total solar radiation incident on the surface, Wh/m^2			
		North	East	South	West
July	Daily total	1621	4313	2552	4313

Therefore, the reflective film should be installed on the **East** or **West** windows (instead of the South windows) in order to minimize the solar heat gain and thus the cooling load of the building.

12-121E A house has 1/8-in thick single pane windows with aluminum frames on a West wall. The rate of net heat gain (or loss) through the window at 3 PM during a typical day in January is to be determined.

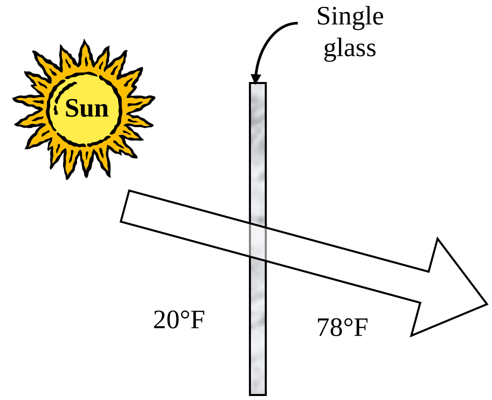
Assumptions **1** The calculations are performed for an "average" day in January. **2** The frame area relative to glazing area is small so that the glazing area can be taken to be the same as the window area.

Properties The shading coefficient of a 1/8-in thick single pane window is SC = 1.0 (Table 12-5). The overall heat transfer coefficient for 1/8-in thick single pane windows with aluminum frames is 6.63 W/m^2.°C = 1.17 Btu/h.ft^2.°F (Table 9-6). The total solar radiation incident at a West-facing surface at 3 PM in January during a typical day is 557 W/m^2 = 177 Btu/h.ft^2 (Table 12-4).

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq. 12-57 to be

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times1.0 = 0.87$$

The window area is: $A_{\text{window}} = (9\text{ ft})(15\text{ ft}) = 135\text{ ft}^2$



Then the rate of solar heat gain through the window at 3 PM becomes

$$\begin{aligned}\dot{Q}_{\text{solar gain, 3 PM}} &= SHGC\times A_{\text{glazing}}\times\dot{q}_{\text{solar, 3 PM}}\\ &= 0.87(135\text{ ft}^2)(177\text{ Btu/h.ft}^2)\\ &= 20{,}789\text{ Btu/h}\end{aligned}$$

The rate of heat loss through the window at 3 PM is

$$\begin{aligned}\dot{Q}_{\text{loss, window}} &= U_{\text{window}}A_{\text{window}}(T_i - T_0)\\ &= (1.17\text{ Btu/h}\cdot\text{ft}^2\cdot{}^\circ\text{F})(135\text{ ft}^2)(78-20)^\circ\text{F}\\ &= 9161\text{ Btu/h}\end{aligned}$$

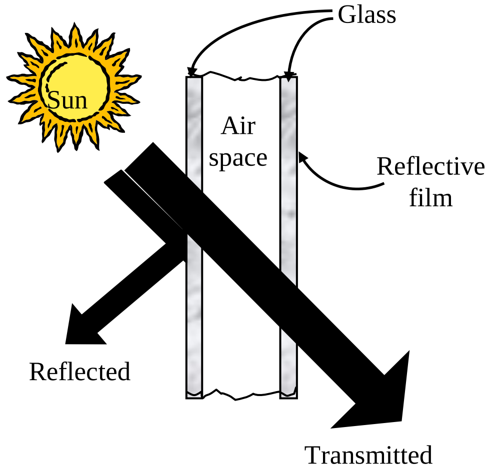
The house will be gaining heat at 3 PM since the solar heat gain is larger than the heat loss. The rate of net heat gain through the window is

$$\dot{Q}_{\text{net}} = \dot{Q}_{\text{solar gain, 3 PM}} - \dot{Q}_{\text{loss, window}} = 20{,}789 - 9161 = \mathbf{11{,}630\ Btu/h}$$

Discussion The actual heat gain will be less because of the area occupied by the window frame.

12-122 The net annual cost savings due to installing reflective coating on the West windows of a building and the simple payback period are to be determined.

Assumptions **1** The calculations given below are for an average year. **2** The unit costs of electricity and natural gas remain constant.



Analysis Using the daily averages for each month and noting the number of days of each month, the total solar heat flux incident on the glazing during summer and winter months are determined to be

$$Q_{\text{solar, summer}} = 4.24\times30 + 4.16\times31 + 3.93\times31 + 3.48\times30 = 482 \text{ kWh/year}$$

$$Q_{\text{solar, winter}} = 2.94\times31 + 2.33\times30 + 2.07\times31 + 2.35\times31 + 3.03\times28 + 3.62\times31 + 4.00\times30 = 615 \text{ kWh/year}$$

Then the decrease in the annual cooling load and the increase in the annual heating load due to reflective film become

$$\text{Cooling load decrease} = Q_{\text{solar, summer}} A_{\text{glazing}} (\text{SHGC}_{\text{without film}} - \text{SHGC}_{\text{with film}})$$
$$= (482 \text{ kWh/year})(60 \text{ m}^2)(0.766-0.35)$$
$$= 12{,}031 \text{ kWh/year}$$

$$\text{Heating load increase} = Q_{\text{solar, winter}} A_{\text{glazing}} (\text{SHGC}_{\text{without film}} - \text{SHGC}_{\text{with film}})$$
$$= (615 \text{ kWh/year})(60 \text{ m}^2)(0.766-0.35)$$
$$= 15{,}350 \text{ kWh/year} = 523.7 \text{ therms/year}$$

since 1 therm = 29.31 kWh. The corresponding decrease in cooling costs and increase in heating costs are

Decrease in cooling costs = (Cooling load decrease)(Unit cost of electricity)/COP
= (12,031 kWh/year)($0.15/kWh)/3.2 = $564/year

Increase in heating costs = (Heating load increase)(Unit cost of fuel)/Efficiency
= (523.7 therms/year)($0.90/therm)/0.90 = $524/year

Then the net annual cost savings due to reflective films become

Cost Savings = Decrease in cooling costs - Increase in heating costs = $564 - 524 = **$40/year**

The implementation cost of installing films is

Implementation Cost = ($15/m^2)(60 m^2) = $900

This gives a simple payback period of

$$\text{Simple payback period} = \frac{\text{Implementation cost}}{\text{Annual cost savings}} = \frac{\$900}{\$40/\text{year}} = \mathbf{22.5 \text{ years}}$$

Discussion The reflective films will pay for themselves in this case in about 23 years, which is unacceptable to most manufacturers since they are not usually interested in any energy conservation measure which does not pay for itself within 3 years.

12-123 A house located at 40ºN latitude has ordinary double pane windows. The total solar heat gain of the house at 9:00, 12:00, and 15:00 solar time in July and the total amount of solar heat gain per day for an average day in January are to be determined.

Assumptions The calculations are performed for an average day in a given month.

Properties The shading coefficient of a double pane window with 6-mm thick glasses is SC = 0.82 (Table 12-5). The incident radiation at different windows at different times are given as (Table 12-4)

Month	Time	Solar radiation incident on the surface, W/m^2			
		North	East	South	West
July	9:00	117	701	190	114
July	12:00	138	149	395	149
July	15:00	117	114	190	701
January	Daily total	446	1863	5897	1863

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq.12-57 to be

SHGC = 0.87×SC = 0.87×0.82 = 0.7134

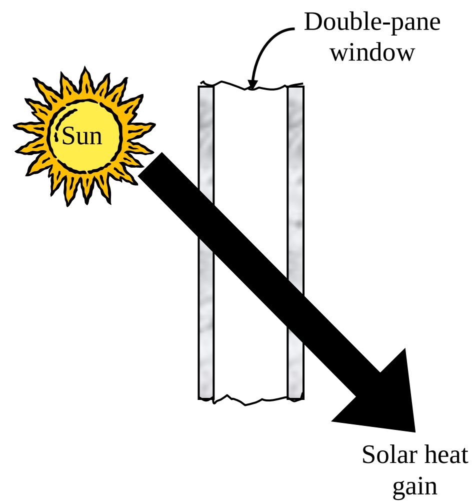
The rate of solar heat gain is determined from

$$\dot{Q}_{\text{solar gain}} = SHGC \times A_{\text{glazing}} \times \dot{q}_{\text{solar, incident}}$$
$$= 0.7134 \times A_{\text{glazing}} \times \dot{q}_{\text{solar, incident}}$$

Then the rates of heat gain at the 4 walls at 3 different times in July become

North wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.7134 \times (4\text{ m}^2) \times (117\text{ W/m}^2) = \mathbf{334\ W}$$
$$\dot{Q}_{\text{solar gain, 12:00}} = 0.7134 \times (4\text{ m}^2) \times (138\text{ W/m}^2) = \mathbf{394\ W}$$
$$\dot{Q}_{\text{solar gain, 15:00}} = 0.7134 \times (4\text{ m}^2) \times (117\text{ W/m}^2) = \mathbf{334\ W}$$



East wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.7134 \times (6\text{ m}^2) \times (701\text{ W/m}^2) = \mathbf{3001\ W}$$
$$\dot{Q}_{\text{solar gain, 12:00}} = 0.7134 \times (6\text{ m}^2) \times (149\text{ W/m}^2) = \mathbf{638\ W}$$
$$\dot{Q}_{\text{solar gain, 15:00}} = 0.7134 \times (6\text{ m}^2) \times (114\text{ W/m}^2) = \mathbf{488\ W}$$

South wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.7134 \times (8\text{ m}^2) \times (190\text{ W/m}^2) = \mathbf{1084\ W}$$
$$\dot{Q}_{\text{solar gain, 12:00}} = 0.7134 \times (8\text{ m}^2) \times (395\text{ W/m}^2) = \mathbf{2254\ W}$$
$$\dot{Q}_{\text{solar gain, 15:00}} = 0.7134 \times (8\text{ m}^2) \times (190\text{ W/m}^2) = \mathbf{1084\ W}$$

West wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.7134 \times (6\text{ m}^2) \times (114\text{ W/m}^2) = \mathbf{488\ W}$$
$$\dot{Q}_{\text{solar gain, 12:00}} = 0.7134 \times (6\text{ m}^2) \times (149\text{ W/m}^2) = \mathbf{638\ W}$$
$$\dot{Q}_{\text{solar gain, 15:00}} = 0.7134 \times (6\text{ m}^2) \times (701\text{ W/m}^2) = \mathbf{3001\ W}$$

Similarly, the solar heat gain of the house through all of the windows in January is determined to be

January:

$$\dot{Q}_{\text{solar gain, North}} = 0.7134 \times (4\text{ m}^2) \times (446\text{ Wh/m}^2 \cdot \text{day}) = 1273\text{ Wh/day}$$
$$\dot{Q}_{\text{solar gain, East}} = 0.7134 \times (6\text{ m}^2) \times (1863\text{ Wh/m}^2 \cdot \text{day}) = 7974\text{ Wh/day}$$
$$\dot{Q}_{\text{solar gain, South}} = 0.7134 \times (8\text{ m}^2) \times (5897\text{ Wh/m}^2 \cdot \text{day}) = 33{,}655\text{ Wh/day}$$
$$\dot{Q}_{\text{solar gain, West}} = 0.7134 \times (6\text{ m}^2) \times (1863\text{ Wh/m}^2 \cdot \text{day}) = 7974\text{ Wh/day}$$

Therefore, for an average day in January,

$$\dot{Q}_{\text{solar gain per day}} = 1273 + 7974 + 33{,}655 + 7974 = 58{,}876\text{ Wh/day} \cong \mathbf{58.9\ kWh/day}$$

12-124 A house located at 40º N latitude has gray-tinted double pane windows. The total solar heat gain of the house at 9:00, 12:00, and 15:00 solar time in July and the total amount of solar heat gain per day for an average day in January are to be determined.

Assumptions The calculations are performed for an average day in a given month.

Properties The shading coefficient of a gray-tinted double pane window with 6-mm thick glasses is SC = 0.58 (Table 12-5). The incident radiation at different windows at different times are given as (Table 12-4)

Month	Time	Solar radiation incident on the surface, W/m^2			
		North	East	South	West
July	9:00	117	701	190	114
July	12:00	138	149	395	149
July	15:00	117	114	190	701
January	Daily total	446	1863	5897	1863

Analysis The solar heat gain coefficient (SHGC) of the windows is determined from Eq.11-57 to be

$$\text{SHGC} = 0.87\times\text{SC} = 0.87\times0.58 = 0.5046$$

The rate of solar heat gain is determined from

$$\dot{Q}_{\text{solar gain}} = SHGC\times A_{\text{glazing}}\times\dot{q}_{\text{solar, incident}} = 0.5046\times A_{\text{glazing}}\times\dot{q}_{\text{solar, incident}}$$

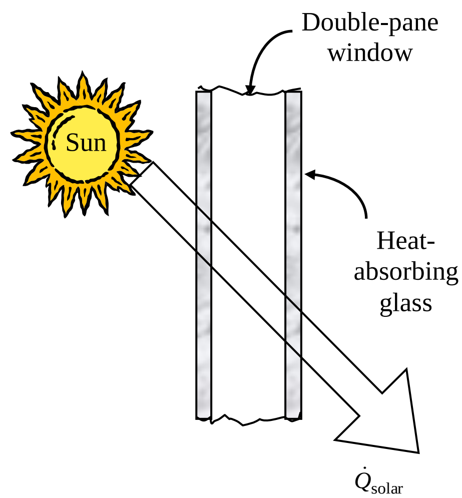
Then the rates of heat gain at the 4 walls at 3 different times in July become

North wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.5046\times(4\text{ m}^2)\times(117\text{ W/m}^2) = \mathbf{236\ W}$$

$$\dot{Q}_{\text{solar gain, 12:00}} = 0.5046\times(4\text{ m}^2)\times(138\text{ W/m}^2) = \mathbf{279\ W}$$

$$\dot{Q}_{\text{solar gain, 15:00}} = 0.5046\times(4\text{ m}^2)\times(117\text{ W/m}^2) = \mathbf{236\ W}$$



East wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.5046\times(6\text{ m}^2)\times(701\text{ W/m}^2) = \mathbf{2122\ W}$$

$$\dot{Q}_{\text{solar gain, 12:00}} = 0.5046\times(6\text{ m}^2)\times(149\text{ W/m}^2) = \mathbf{451\ W}$$

$$\dot{Q}_{\text{solar gain, 15:00}} = 0.5046\times(6\text{ m}^2)\times(114\text{ W/m}^2) = \mathbf{345\ W}$$

South wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.5046\times(8\text{ m}^2)\times(190\text{ W/m}^2) = \mathbf{767\ W}$$

$$\dot{Q}_{\text{solar gain, 12:00}} = 0.5046\times(8\text{ m}^2)\times(395\text{ W/m}^2) = \mathbf{1595\ W}$$

$$\dot{Q}_{\text{solar gain, 15:00}} = 0.5046\times(8\text{ m}^2)\times(190\text{ W/m}^2) = \mathbf{767\ W}$$

West wall:

$$\dot{Q}_{\text{solar gain, 9:00}} = 0.5046\times(6\text{ m}^2)\times(114\text{ W/m}^2) = \mathbf{345\ W}$$

$$\dot{Q}_{\text{solar gain, 12:00}} = 0.5046\times(6\text{ m}^2)\times(149\text{ W/m}^2) = \mathbf{451\ W}$$

$$\dot{Q}_{\text{solar gain, 15:00}} = 0.5046\times(6\text{ m}^2)\times(701\text{ W/m}^2) = \mathbf{2122\ W}$$

Similarly, the solar heat gain of the house through all of the windows in January is determined to be

January:

$$\dot{Q}_{\text{solar gain, North}} = 0.5046\times(4\text{ m}^2)\times(446\text{ Wh/m}^2\cdot\text{day}) = 900\text{ Wh/day}$$

$$\dot{Q}_{\text{solar gain,East}} = 0.5046\times(6\text{ m}^2)\times(1863\text{ Wh/m}^2\cdot\text{day}) = 5640\text{ Wh/day}$$

$$\dot{Q}_{\text{solar gain, South}} = 0.5046\times(8\text{ m}^2)\times(5897\text{ Wh/m}^2\cdot\text{day}) = 23{,}805\text{ Wh/day}$$

$$\dot{Q}_{\text{solar gain,West}} = 0.5046\times(6\text{ m}^2)\times(1863\text{ Wh/m}^2\cdot\text{day}) = 5640\text{ Wh/day}$$

Therefore, for an average day in January,

$$\dot{Q}_{\text{solar gain per day}} = 900+5640+23{,}805+5640 = 35{,}985\text{ Wh/day} = \mathbf{35.895\,kWh/day}$$

Review Problems

12-125 A hole is drilled in a spherical cavity. The maximum rate of radiation energy streaming through the hole is to be determined.

Analysis The maximum rate of radiation energy streaming through the hole is the blackbody radiation, and it can be determined from

$$E = A\sigma T^4 = \pi(0.0025\text{ m})^2(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(600\text{ K})^4 = \mathbf{0.144\ W}$$

The result would not change for a different diameter of the cavity.

12-126 The wavelengths at maximum emission of radiation for both daylight and incandescent light are to be determined.

Assumptions **1** The sun and the incandescent light filament behave as black bodies.

Analysis The wavelength at maximum emission of radiation can be determined using the Wien's displacement law:

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K}$$

For daylight,

$$\lambda_{\text{max power}} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{T} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{5800\text{ K}} = \mathbf{0.50\ \mu m} \qquad \text{(daylight)}$$

For incandescent liht,

$$\lambda_{\text{max power}} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{T} = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{2800\text{ K}} = \mathbf{1.04\ \mu m} \qquad \text{(incandescent light)}$$

Discussion For daylight, the peak of emissive power is at 0.50 μm, which is within the visible spectrum. On the other hand, the peak of emissive power for incandescent light (1.04 μm) is outside the visible spectrum.

12-127 The fraction of the incident solar radiation that is absorbed by the human skin is to be determined.

Assumptions **1** The sun behaves as a blackbody.

Analysis For solar radiation (T = 5800 K), the blackbody radiation functions corresponding to λ_1 = 0.517 μm to λ_2 = 1.552 μm are determined from Table 12-2 to be

$$\lambda_1 T = (0.517\ \mu\text{m})(5800\text{ K}) = 3000\ \mu\text{m}\cdot\text{K} \quad \to \quad f_{\lambda_1} = 0.273232$$

$$\lambda_2 T = (1.552\ \mu\text{m})(5800\text{ K}) = 9000\ \mu\text{m}\cdot\text{K} \quad \to \quad f_{\lambda_2} = 0.890029$$

The fraction of radiation emitted for $0 \le \lambda < \lambda_1$ is

$$f_{0-\lambda_1} = 0.273232$$

The fraction of radiation emitted for $\lambda_1 \le \lambda < \lambda_2$ is

$$f_{\lambda_1-\lambda_2} = 0.890029 - 0.273232 = 0.616797$$

The fraction of radiation emitted for $\lambda_2 \le \lambda < \infty$ is

$$f_{\lambda_2-\infty} = 1 - 0.890029 = 0.109971$$

Thus, the fraction of the incident solar radiation that is absorbed by the human skin is

$$1.0(f_{0-\lambda_1}) + 0.5(f_{\lambda_1-\lambda_2}) + 1.0(f_{\lambda_2-\infty}) = 1.0(0.273232) + 0.5(0.616797) + 1.0(0.109971)$$

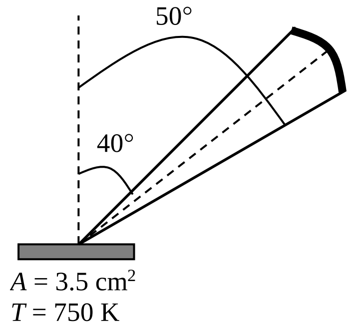
$$= \mathbf{0.6916}$$

Discussion The calculation shows that human skin absorbs about 69% of the incident solar radiation.

12-128 A small surface emits radiation. The rate of radiation energy emitted through a band is to be determined.

Assumptions Surface *A* emits diffusely as a blackbody.

Analysis The rate of radiation emission from a surface per unit surface area in the direction (θ,ϕ) is given as



$$dE = \frac{d\dot{Q}_e}{dA} = I_e(\theta,\phi)\cos\theta\sin\theta\, d\theta\, d\phi$$

The total rate of radiation emission through the band between 40° and 50° can be expressed as

$$E = \int_{\phi=0}^{2\pi}\int_{\theta=40}^{50} I_e(\theta,\phi)\cos\theta\sin\theta\ d\theta\ d\phi = I_b(0.1736\pi)$$

$$= \frac{\sigma T^4}{\pi}(0.1736\pi) = 0.1736\sigma T^4$$

since the blackbody radiation intensity is constant (I_b = constant), and

$$\int_{\phi=0}^{2\pi}\int_{\theta=40}^{50}\cos\theta\sin\theta\ d\theta\ d\phi = 2\pi\int_{\theta=40}^{50}\cos\theta\sin\theta\ d\theta = \pi(\sin^2 50 - \sin^2 40) = 0.1736\pi$$

Approximating a small area as a differential area, the rate of radiation energy emitted from an area of 3.5 cm^2 in the specified band becomes

$$\dot{Q}_e = EdA = 0.1736\sigma T^4 dA = 0.1736\times(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(750\ \text{K})^4(3.5\times10^{-4}\ \text{m}^2) = \mathbf{1.09\ W}$$

12-129 The intensity of radiation emitting from a surface is given. The emissive power from the surface into the surrounding hemisphere and the rate of radiation emission from the surface are to be determined.

***Assumptions* 1** The intensity is a function of both the azimuth angle ϕ and the zenith angle θ.

Analysis The emissive power from surface *A* can be determined by integration as

$$E = \int_{\text{hemisphere}} dE = \int_0^{2\pi}\int_0^{\pi/2} I_e(\theta,\ \phi)\cos\theta\sin\theta\, d\theta\, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi/2} 100\phi\cos^2\theta\ \sin\theta\, d\theta\, d\phi$$

$$= \int_0^{2\pi} 100\phi\left[-\frac{(\cos\theta)^3}{3}\right]_0^{\pi/2} d\phi$$

$$= \int_0^{2\pi}\frac{100\phi}{3}\ d\phi$$

$$= \left[\frac{100\phi^2}{6}\right]_0^{2\pi}$$

$$= \frac{200\pi^2}{3} = \mathbf{658\ W/m^2}$$

Then, the rate of radiation emission from the surface is

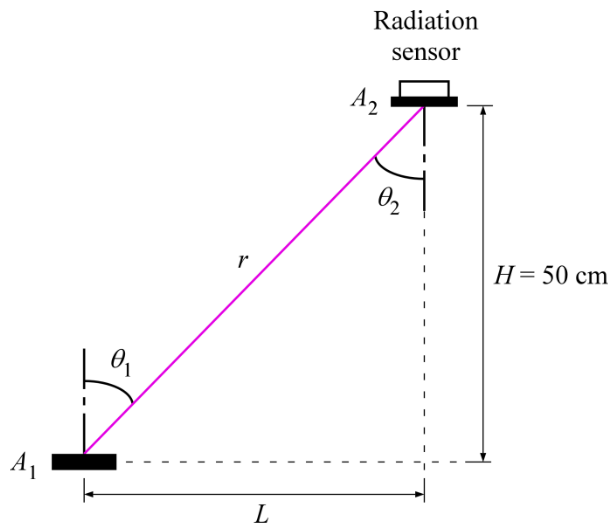
$$\dot{Q}_e = AE = (3\times10^{-4}\ \text{m}^2)(658\ \text{W/m}^2) = \mathbf{0.197\ W}$$

Discussion The rate of radiation emission from the black surface, which is a diffuse emitter, is simply $\dot{Q}_e = A\sigma T^4$.

12-130 A radiation sensor is measuring radiation rate emitted by another surface (A_1). The distance at which the sensor is measuring two-thirds of the radiation rate corresponding to the position of A_1 directly under the sensor is to be determined.

Assumptions **1** The surface A_1 emits diffusely as a blackbody. **2** Both surface A_1 and sensor can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from



$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos\theta_2}{r^2} = \frac{A_2 H}{r^2 r} = \frac{A_2 H}{(H^2 + L^2)^{3/2}} \quad (1)$$

Note that

$$r = (H^2 + L^2)^{1/2}$$

and

$$\cos\theta_1 = \cos\theta_2 = \frac{H}{r} = \frac{H}{(H^2 + L^2)^{1/2}}$$

Then, the rate of radiation energy emitted by A_1 in the direction of θ_1 through the solid angle $\omega_{2\text{-}1}$ is determined by multiplying I_1 by the area of A_1 normal to θ_1 and the solid angle $\omega_{2\text{-}1}$. That is,

$$\dot{Q}_{1-2} = I_1(A_1 \cos\theta_1)\omega_{2-1} = I_1 A_1 \frac{H}{(H^2 + L^2)^{1/2}}\omega_{2-1} \quad (2)$$

Substituting Eq. (1) into (2) yields

$$\dot{Q}_{1-2} = I_1 A_1 A_2 \frac{H^2}{(H^2 + L^2)^2}$$

Also, when the surface A_1 is positioned directly under the sensor at $L = 0$, we have

$$\dot{Q}_{1-2,\,0} = I_1 A_1 A_2 \frac{H^2}{H^4} = I_1 A_1 A_2 \frac{1}{H^2}$$

Hence, the distance L at which the sensor is measuring two-thirds of the radiation rate emitted from surface A_1 corresponding to the position directly under the sensor at $L = 0$ can be determined as

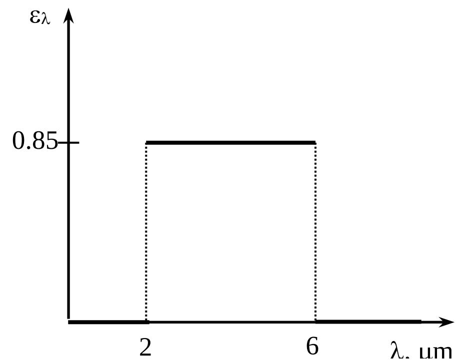
$$\frac{\dot{Q}_{1-2}}{\dot{Q}_{1-2,\,0}} = \frac{I_1 A_1 A_2 \dfrac{H^2}{(H^2 + L^2)^2}}{I_1 A_1 A_2 \dfrac{1}{H^2}} = \frac{2}{3} \quad \rightarrow \quad \frac{\dot{Q}_{1-2}}{\dot{Q}_{1-2,\,0}} = \left(\frac{H^2}{H^2 + L^2}\right)^2 = \frac{2}{3}$$

$$L = 0.4741H = 0.4741(0.5\,\text{m}) = \mathbf{0.237\,m}$$

Discussion In this orientation of the radiation sensor and surface A_1, the $(\dot{Q}_{1-2}/\dot{Q}_{1-2,\,0})$ ratio is simply expressed as

$$\frac{\dot{Q}_{1-2}}{\dot{Q}_{1-2,\,0}} = \left(\frac{H^2}{H^2 + L^2}\right)^2 = \left(\frac{H}{r}\right)^4$$

12-131 The variation of emissivity of an opaque surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.



Analysis The average emissivity of the surface can be determined from

$$\varepsilon(T) = \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 12-2 to be

$$\lambda_1 T = (2\ \mu\text{m})(1200\ \text{K}) = 2400\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.140256$$

$$\lambda_2 T = (6\ \mu\text{m})(1200\ \text{K}) = 7200\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.819217$$

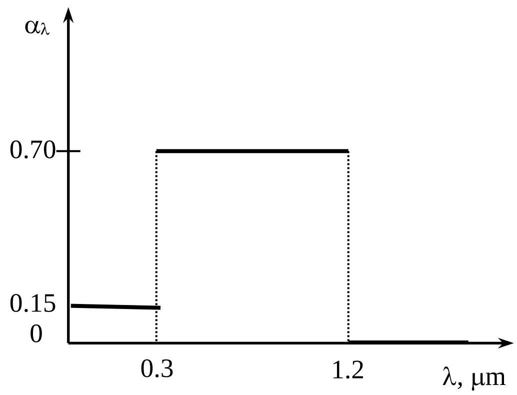
and

$$\varepsilon = (0.0)(0.140256) + (0.85)(0.819217 - 0.140256) + (0.0)(1 - 0.819217) = \mathbf{0.5771}$$

Then the emissive flux of the surface becomes

$$E = \varepsilon \sigma T^4 = (0.5771)(5.67 \times 10^{-8}\ \text{W/m}^2.\text{K}^4)(1200\ \text{K})^4 = \mathbf{67{,}850\ W/m^2}$$

12-132 The variation of absorptivity of a surface with wavelength is given. The average absorptivity of the surface is to be determined for two source temperatures.



Analysis (*a*) $\underline{T = 1000\text{ K}}$. The average absorptivity of the surface can be determined from

$$\alpha(T) = \alpha_1 f_{0\text{-}\lambda_1} + \alpha_2 f_{\lambda_1\text{-}\lambda_2} + \alpha_3 f_{\lambda_2\text{-}\infty}$$
$$= \alpha_1 f_{\lambda_1} + \alpha_2 (f_{\lambda_2} - f_{\lambda_1}) + \alpha_3 (1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, determined from

$$\lambda_1 T = (0.3\ \mu\text{m})(1000\text{ K}) = 300\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0$$
$$\lambda_2 T = (1.2\ \mu\text{m})(1000\text{ K}) = 1200\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.002134$$
$$f_{0-\lambda_1} = f_{\lambda 1} - f_0 = f_{\lambda_1} \text{ since } f_0 = 0 \text{ and } f_{\lambda_2-\infty} = f_\infty - f_{\lambda_2} \text{ since } f_\infty = 1.$$

and

$$\alpha = (0.15)0.0 + (0.70)(0.002134 - 0.0) + (0.0)(1 - 0.002134) = \mathbf{0.00149}$$

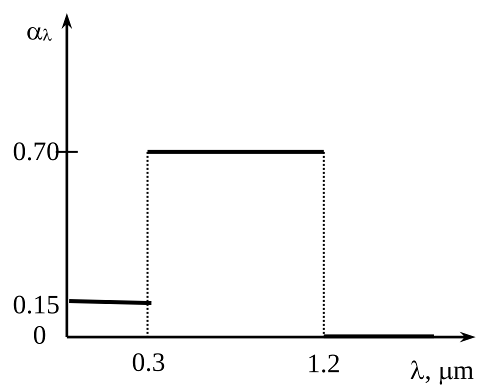
(*a*) $\underline{T = 3000\text{ K}}$.

$$\lambda_1 T = (0.3\ \mu\text{m})(3000\text{ K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.000169$$
$$\lambda_2 T = (1.2\ \mu\text{m})(3000\text{ K}) = 3600\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.403607$$
$$\alpha = (0.15)0.000169 + (0.70)(0.403607 - 0.000169) + (0.0)(1 - 0.403607) = \mathbf{0.282}$$

12-133 The variation of absorptivity of a surface with wavelength is given. The surface receives solar radiation at a specified rate. The solar absorptivity of the surface and the rate of absorption of solar radiation are to be determined.

Analysis For solar radiation, T = 5800 K. The solar absorptivity of the surface is

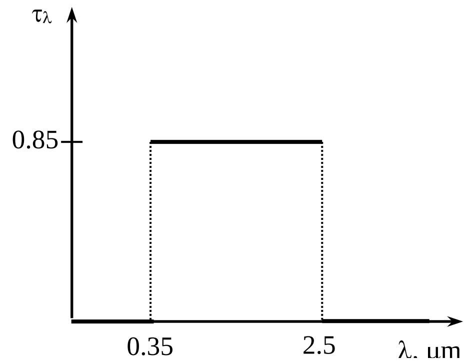
$$\lambda_1 T = (0.3\ \mu\text{m})(5800\text{ K}) = 1740\ \mu\text{mK} \rightarrow f_{\lambda_1} = 0.033454$$
$$\lambda_2 T = (1.2\ \mu\text{m})(5800\text{ K}) = 6960\ \mu\text{mK} \rightarrow f_{\lambda_2} = 0.805713$$
$$\alpha = (0.15)0.033454 + (0.70)(0.805713 - 0.033454)$$
$$+ (0.0)(1 - 0.805713)$$
$$= \mathbf{0.5456}$$



The rate of absorption of solar radiation is determined from

$$E_{\text{absorbed}} = \alpha I = 0.5456(470\text{ W/m}^2) = \mathbf{256\ W/m^2}$$

12-134 The variation of transmissivity of glass with wavelength is given. The transmissivity of the glass for solar radiation and for light are to be determined.



Analysis For solar radiation, T = 5800 K. The average transmissivity of the surface can be determined from

$$\tau(T) = \tau_1 f_{\lambda_1} + \tau_2 (f_{\lambda_2} - f_{\lambda_1}) + \tau_3 (1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 12-2 to be

$$\lambda_1 T = (0.35\ \mu\text{m})(5800\ \text{K}) = 2030\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.071852$$

$$\lambda_2 T = (2.5\ \mu\text{m})(5800\ \text{K}) = 14{,}500\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.966440$$

and

$$\tau = (0.0)(0.071852) + (0.85)(0.966440 - 0.071852) + (0.0)(1 - 0.966440) = \mathbf{0.760}$$

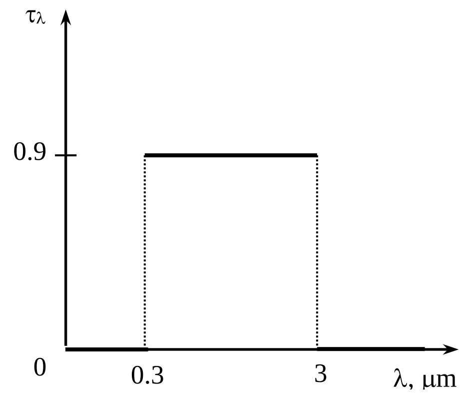
For light, we take T = 300 K. Repeating the calculations at this temperature we obtain

$$\lambda_1 T = (0.35\ \mu\text{m})(300\ \text{K}) = 105\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.00$$

$$\lambda_2 T = (2.5\ \mu\text{m})(300\ \text{K}) = 750\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.000012$$

$$\tau = (0.0)(0.00) + (0.85)(0.000012 - 0.00) + (0.0)(1 - 0.000012) = \mathbf{0.00001}$$

12-135 The spectral transmissivity of a glass cover used in a solar collector is given. Solar radiation is incident on the collector. The solar flux incident on the absorber plate, the transmissivity of the glass cover for radiation emitted by the absorber plate, and the rate of heat transfer to the cooling water are to be determined.



Analysis (*a*) For solar radiation, T = 5800 K. The average transmissivity of the surface can be determined from

$$\tau(T)=\tau_1 f_{\lambda_1} + \tau_2(f_{\lambda_2} - f_{\lambda_1}) + \tau_3(1 - f_{\lambda_2})$$

where f_{λ_1} and f_{λ_2} are blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$. These functions are determined from Table 12-2 to be

$$\lambda_1 T = (0.3\ \mu\text{m})(5800\ \text{K}) = 1740\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.033454$$

$$\lambda_2 T = (3\ \mu\text{m})(5800\ \text{K}) = 17{,}400\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.978746$$

and

$$\tau = (0.0)(0.033454) + (0.9)(0.978746 - 0.033454) + (0.0)(1 - 0.978746) = 0.851$$

Since the absorber plate is black, all of the radiation transmitted through the glass cover will be absorbed by the absorber plate and therefore, the solar flux incident on the absorber plate is same as the radiation absorbed by the absorber plate:

$$E_{\text{abs. plate}} = \tau I = 0.851(950\ \text{W/m}^2) = \mathbf{808.5\ W/m^2}$$

(*b*) For radiation emitted by the absorber plate, we take T = 300 K, and calculate the transmissivity as follows:

$$\lambda_1 T = (0.3\ \mu\text{m})(300\ \text{K}) = 90\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0$$

$$\lambda_2 T = (3\ \mu\text{m})(300\ \text{K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.000169$$

$$\tau = (0.0)(0.0) + (0.9)(0.000169 - 0.0) + (0.0)(1 - 0.000169) = \mathbf{0.00015}$$

(*c*) The rate of heat transfer to the cooling water is the difference between the radiation absorbed by the absorber plate and the radiation emitted by the absorber plate, and it is determined from

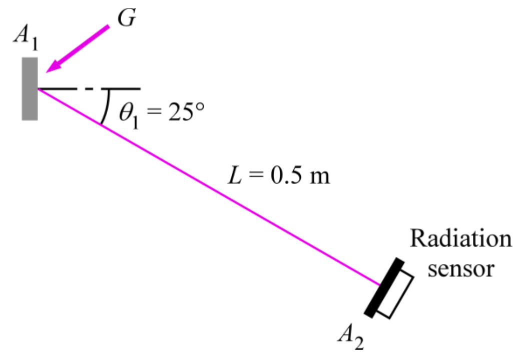
$$\dot{Q}_{\text{water}} = (\tau_{\text{solar}} - \tau_{\text{room}})I = (0.851 - 0.00015)(950\ \text{W/m}^2) = \mathbf{808.3\ W/m^2}$$

12-136 In a configuration involving a small opaque surface A_1 and a radiation sensor, the rate at which radiation emitted from A_1 that is intercepted by the sensor is to be determined.

Assumptions **1** Surface A_1 is an opaque, diffuse emitter and reflector. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis Approximating both A_1 and A_2 as differential surfaces, the solid angle subtended by A_2 when viewed from A_1 can be determined from

$$\omega_{2-1} \cong \frac{A_{n,2}}{L^2} = \frac{A_2 \cos\theta_2}{L^2} = \frac{A_2}{L^2} = \frac{1\times10^{-4}\ \text{m}^2}{(0.5\ \text{m})^2} = 4\times10^{-4}\ \text{sr} \qquad (\text{for } \theta_2 = 0°)$$



The radiosity of surface A_1 is expressed as

$$\begin{aligned} J &= E + G_{\text{ref}} \\ &= E + \rho G \\ &= \varepsilon E_b + (1-\alpha)G \end{aligned}$$

where

$$\alpha + \rho + \tau = 1 \quad \rightarrow \quad \rho = 1 - \alpha \qquad (\text{for opaque surface, } \tau = 0)$$

Hence, the radiosity can be calculated as

$$\begin{aligned} J &= \varepsilon \sigma T_1^4 + (1-\alpha)G \\ &= (0.7)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(400\ \text{K})^4 + (0.5)(2000\ \text{W/m}^2) \\ &= 2016\ \text{W/m}^2 \end{aligned}$$

Since surface A_1 is a diffuse emitter and reflector, the sum of the emitted and reflected intensities is

$$I_{e+r} = \frac{J}{\pi} = \frac{2016\ \text{W/m}^2}{\pi} = 641.7\ \text{W/m}^2\cdot\text{sr}$$

Therefore, the rate at which radiation emitted from A_1 that is intercepted by the sensor is

$$\begin{aligned} \dot{Q}_{1-2} &= I_{e+r}(A_1 \cos\theta_1)\omega_{2-1} \\ &= (641.7\ \text{W/m}^2\cdot\text{sr})(3\times10^{-4}\ \text{m}^2)\cos 25°(4\times10^{-4}\ \text{sr}) \\ &= \mathbf{6.98\times10^{-5}\ W} \end{aligned}$$

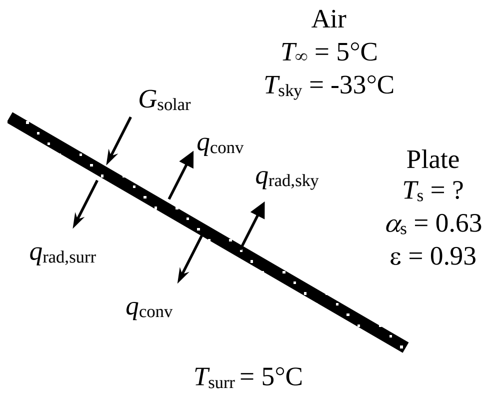
Discussion From the radiation rate intercepted by the sensor, the irradiation on the sensor can be calculated to be

$$G_2 = \frac{\dot{Q}_{1-2}}{A_2} = \frac{6.98\times10^{-5}\ \text{W}}{1\times10^{-4}\ \text{m}^2} = 0.698\ \text{W/m}^2$$

12-137 Solar radiation is incident on front surface of a plate. The equilibrium temperature of the plate is to be determined.

Assumptions The plate temperature is uniform.

Properties The solar absorptivity and emissivity of the surface are given to $\alpha_s = 0.63$ and $\varepsilon = 0.93$.



Analysis The solar radiation is

$$G_{solar} = G_{direct}\cos\alpha + G_{diffuse}$$
$$= (300\ \text{W/m}^2)\cos(30°) + 250\ \text{W/m}^2 = 509.8\ \text{W/m}^2$$

The front surface is exposed to solar and sky radiation and convection while the back surface is exposed to convection and radiation with the surrounding surfaces. An energy balance can be written as

$$\dot{q}_{in} = \dot{q}_{out}$$
$$\alpha_s G_{solar} + \varepsilon\sigma T_{sky}^{\ 4} + \varepsilon\sigma T_{surr}^{\ 4} = 2\varepsilon\sigma T_s^{\ 4} + 2h(T_s - T_{air})$$

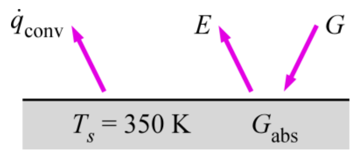
Substituting,

$$(0.63)(509.8) + (0.93)(5.67\times10^{-8})(-33+273)^4 + (0.93)(5.67\times10^{-8})(5+273)^4$$
$$= 2(0.93)(5.67\times10^{-8})T_s^{\ 4} + 2(20\ \text{W/m}^2\cdot\text{K})(T_s - 278) \longrightarrow T_s = 281.7\ \text{K} = \mathbf{8.7°C}$$

12-138 A horizontal opaque flat plate is well insulated on the edges and the lower surface is experiencing irradiation and heat loss by convection. The absorptivity, reflectivity, and emissivity of the plate are to be determined.

Assumptions **1** Steady operating condition exists. **2** The plate has a uniform temperature. **3** The plate is well insulated on the edges and the lower surface.

Analysis The irradiation on the plate is



$$G = \frac{5000\text{ W}}{5\text{ m}^2} = 1000\text{ W/m}^2$$

The irradiation absorbed by the plate is

$$G_{\text{abs}} = \frac{4000\text{ W}}{5\text{ m}^2} = 800\text{ W/m}^2$$

The convection heat flux is

$$\dot{q}_{\text{conv}} = \frac{500\text{ W}}{5\text{ m}^2} = 100\text{ W/m}^2$$

Applying energy balance on the surface, the emissive power is

$$E = G_{\text{abs}} - \dot{q}_{\text{conv}} = 800\text{ W/m}^2 - 100\text{ W/m}^2 = 700\text{ W/m}^2$$

Hence, the absorptivity of the plate is

$$\alpha = \frac{G_{\text{abs}}}{G} = \frac{800\text{ W/m}^2}{1000\text{ W/m}^2} = \mathbf{0.80}$$

Then, the reflectivity of the plate is determined using

$$\alpha + \rho + \tau = 1 \quad \rightarrow \quad \rho = 1 - \alpha \qquad \text{(for opaque surface, } \tau = 0\text{)}$$

$$\rho = 1 - 0.80 = \mathbf{0.20}$$

Finally, the emissivity of the plate is

$$\varepsilon = \frac{E}{E_b} = \frac{E}{\sigma T_s^4} = \frac{700\text{ W/m}^2}{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(350\text{ K})^4} = \mathbf{0.823}$$

Discussion Since the plate is opaque, that means it is reflecting $G_{\text{ref}} = G - G_{\text{abs}} = 200\text{ W/m}^2$ of the irradiation.

Fundamentals of Engineering (FE) Exam Problems

12-139 Consider a surface at -10ºC in an environment at 25ºC. The maximum rate of heat that can be emitted from this surface by radiation is

(a) 152 W/m^2 (b) 176 W/m^2 (c) 211 W/m^2 (d) 271 W/m^2 (e) 324 W/m^2

Answer (d) 271 W/m^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=-10 [C]
T_infinity=25 [C]
sigma=5.67E-8 [W/m^2-K^4]
E_b=sigma*(T+273)^4
"Some Wrong Solutions with Common Mistakes"
W1_E_b=sigma*T^4 "Using C unit for temperature"
W2_E_b=sigma*((T_infinity+273)^4-(T+273)^4) "Finding radiation exchange between the surface and the environment"
```

12-140 Consider a surface at 900 K. The spectral blackbody emissive power at a wavelength of 50 μm is

(a) 3.2 W/m^2·μm (b) 9.6 W/m^2·μm (c) 24 W/m^2·μm (d) 76 W/m^2·μm (e) 108 W/m^2·μm

Answer (a) 3.2 W/m^2·μm

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=900 [K]
lambda=50 [micrometer]
C1=3.742E8 [W-micrometer^4/m^2]
C2=1.439E4 [micrometer-K]
E_b_lambda=C1/(lambda^5*(exp(C2/(lambda*T))-1))
```

12-141 The wavelength at which the blackbody emissive power reaches its maximum value at 300 K is

(a) 5.1 μm (b) 9.7 μm (c) 15.5 μm (d) 38.0 μm (e) 73.1 μm

Answer (b) 9.7 μm

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=300 [K]
lambda*T=2897.8 [micrometer-K] "Wien's displacement law"
```

12-142 A surface absorbs 10% of radiation at wavelengths less than 3 μm and 50% of radiation at wavelengths greater than 3 μm. The average absorptivity of this surface for radiation emitted by a source at 3000 K is

(a) 0.14 (b) 0.22 (c) 0.30 (d) 0.38 (e) 0.42

Answer (a) 0.14

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Abs1=0.1
Abs2=0.5
T=3000
Wave= 3
LT=Wave*T
F1=0.890029 "The radiation fraction corresponding to lamda-T = 9000, from Table 12-2"
Abs =F1*Abs1+(1-F1)*Abs2
```

12-143 A surface at 300ºC has an emissivity of 0.7 in the wavelength range of 0-4.4 μm and 0.3 over the rest of the wavelength range. At a temperature of 300ºC, 19 percent of the blackbody emissive power is in wavelength range up to 4.4 μm. The total emissivity of this surface is

(a) 0.300 (b) 0.376 (c) 0.624 (d) 0.70 (e) 0.50

Answer (b) 0.376

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
f=0.19
e1=0.7
e2=0.3
e=f*e1+(1-f)*e2
```

12-144 Consider a 4-cm-diameter and 6-cm-long cylindrical rod at 1200 K. If the emissivity of the rod surface is 0.75, the total amount of radiation emitted by all surfaces of the rod in 20 min is

(a) 88 kJ (b) 118 kJ (c) 6661 kJ (d) 1064 kJ (e) 1418 kJ

Answer (d) 1064 kJ

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.04 [m]
L=0.06 [m]
T=1200 [K]
epsilon=0.75
time=20*60 [s]
sigma=5.67E-8 [W/m^2-K^4]
A_s=2*pi*D^2/4+pi*D*L
q_dot_emission=epsilon*sigma*T^4
Q_emission=Q_dot_emission*A_s*time
"Some Wrong Solutions with Common Mistakes"
W1_Q_emission=q_dot_emission "Using rate of emission as the answer"
W2_A_s=pi*D*L "Ignoring bottom and top surfaces of the rod"
W2_Q_emission=q_dot_emission*W2_A_s*time
W3_q_dot_emission=sigma*T^4 "Assuming the surface to be a blackbody"
W3_Q_emission=W3_q_dot_emission*A_s*time
```

12-145 Solar radiation is incident on a semi-transparent body at a rate of 500 W/m^2. If 150 W/m^2 of this incident radiation is reflected back and 225 W/m^2 is transmitted across the body, the absorptivity of the body is

(a) 0 (b) 0.25 (c) 0.30 (d) 0.45 (e) 1

Answer (b) 0.25

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
G=500 [W/m^2]
G_ref=150 [W/m^2]
G_tr=225 [W/m^2]
G_abs=G-G_ref-G_tr
alpha=G_abs/G
"Some Wrong Solutions with Common Mistakes"
W1_alpha=G_ref/G "Definition for reflectivity"
W2_alpha=G_tr/G "Definition for transmissivity"
```

12-146 Solar radiation is incident on an opaque surface at a rate of 400 W/m^2. The emissivity of the surface is 0.65 and the absorptivity to solar radiation is 0.85. The convection coefficient between the surface and the environment at 25ºC is 6 W/m^2·ºC. If the surface is exposed to atmosphere with an effective sky temperature of 250 K, the equilibrium temperature of the surface is

(a) 281 K (b) 298 K (c) 303 K (d) 317 K (e) 339 K

Answer (d) 317 K

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
G_solar=400 [W/m^2]
epsilon=0.65
alpha_s=0.85
h=6 [W/m^2-C]
T_infinity=25[C]+273 [K]
T_sky=250 [K]
sigma=5.67E-8 [W/m^2-K^4]
E_in=E_out
E_in=alpha_s*G_solar+epsilon*sigma*T_sky^4
E_out=epsilon*sigma*T_s^4+h*(T_s-T_infinity)
"Some Wrong Solutions with Common Mistakes"
W1_E_in=W1_E_out "Ignoring atmospheric radiation"
W1_E_in=alpha_s*G_solar
W1_E_out=epsilon*sigma*W1_T_s^4+h*(W1_T_s-T_infinity)
W2_E_in=W2_E_out  "Ignoring convection heat transfer"
W2_E_in=alpha_s*G_solar+epsilon*sigma*T_sky^4
W2_E_out=epsilon*sigma*W2_T_s^4
```

12-147 A surface is exposed to solar radiation. The direct and diffuse components of solar radiation are 480 and 250 W/m^2, and the direct radiation makes a 35º angle with the normal of the surface. The solar absorptivity and the emissivity of the surface are 0.24 and 0.41, respectively. If the surface is observed to be at 315 K and the effective sky temperature is 256 K, the net rate of radiation heat transfer to the surface is

(a) -79 W/m^2 (b) -22 W/m^2 (c) 25 W/m^2 (d) 154 W/m^2 (e) 643 W/m^2

Answer (c) 25 W/m^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
G_direct=480 [W/m^2]
G_diffuse=250 [W/m^2]
theta=35 [degrees]
alpha_s=0.24
epsilon=0.41
T_s=315 [K]
T_sky=256 [K]
sigma=5.67E-8 [W/m^2-K^4]
G_solar=G_direct*cos(theta)+G_diffuse
q_dot_net=alpha_s*G_solar+epsilon*sigma*(T_sky^4-T_s^4)
"Some Wrong Solutions with Common Mistakes"
W1_q_dot_net=G_solar "Using solar radiation as the answer"
W2_q_dot_net=alpha_s*G_solar "Using absorbed solar radiation as the answer"
W3_q_dot_net=epsilon*sigma*(T_sky^4-T_s^4) "Ignoring solar radiation"
```

12-148 ….. 12-149 Design and Essay Problems

Solutions Manual for

Heat and Mass Transfer: Fundamentals & Applications
6th Edition
Yunus A. Çengel, Afshin J. Ghajar
McGraw-Hill Education, 2020

Chapter 13
RADIATION HEAT TRANSFER

View Factors

13-1C The view factor $F_{i \to j}$ represents the fraction of the radiation leaving surface i that strikes surface j directly. The view factor from a surface to itself is non-zero for concave surfaces.

13-2C The pair of view factors $F_{i \to j}$ and $F_{j \to i}$ are related to each other by the reciprocity rule $A_i F_{ij} = A_j F_{ji}$ where A_i is the area of the surface i and A_j is the area of the surface j. Therefore,

$$A_1 F_{12} = A_2 F_{21} \longrightarrow F_{12} = \frac{A_2}{A_1} F_{21}$$

13-3C The summation rule for an enclosure and is expressed as $\sum_{j=1}^{N} F_{i \to j} = 1$ where N is the number of surfaces of the enclosure. It states that the sum of the view factors from surface i of an enclosure to all surfaces of the enclosure, including to itself must be equal to unity.

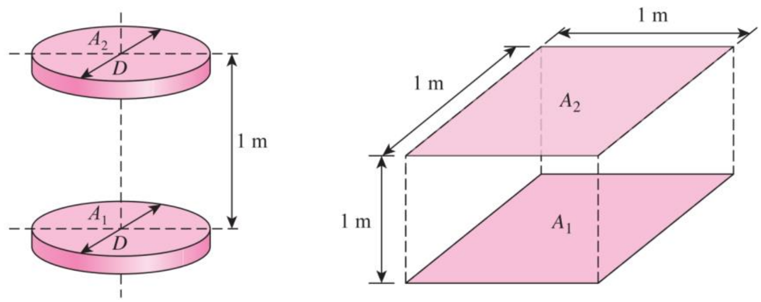
The superposition rule is stated as the view factor from a surface *i* to a surface *j* is equal to the sum of the view factors from surface *i* to the parts of surface *j*, $F_{1 \to (2,3)} = F_{1 \to 2} + F_{1 \to 3}$.

13-4C The cross-string method is applicable to geometries which are very long in one direction relative to the other directions. By attaching strings between corners the Crossed-Strings Method is expressed as

$$F_{i \to j} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2 \times \text{Length of surface } i}$$

13-5 Two coaxial parallel circular disks (D = 1 m) and two aligned parallel square plates (1 m × 1 m) have the same distance of 1 m apart. The view factors of the two geometries are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors.



Analysis For the coaxial parallel disks, from Table 13-1 with i = 1, j = 2, we have

$$R_1 = R_2 = R = \frac{D}{2L} = \frac{1}{2}$$

$$S = 1 + \frac{1 + R_2^2}{R_1^2} = 2 + \frac{1}{R^2} = 2 + 4(L/D)^2 = 6$$

$$F_{12,\text{disk}} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_2}{D_1}\right)^2\right]^{1/2}\right\} = \frac{1}{2}[S - (S^2 - 4)^{1/2}] = \frac{1}{2}[6 - (6^2 - 4)^{1/2}] = \mathbf{0.1716}$$

For the parallel plates, from Table 13-1 with i = 1, j = 2, we have

$$\bar{X} = X/L = 1 \quad \text{and} \quad \bar{Y} = Y/L = 1$$

$$F_{12,\text{plate}} = \frac{2}{\pi \bar{X}\bar{Y}}\left\{\ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}}\right.$$
$$\left. + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$$

$$F_{12,\text{plate}} = \frac{2}{\pi}\left[\ln\left(\frac{4}{3}\right)^{1/2} + 2\left(2^{1/2}\tan^{-1}\frac{1}{2^{1/2}} - \tan^{-1}(1)\right)\right] = \mathbf{0.1998}$$

The view factor of the aligned parallel square plates is greater than that of the coaxial parallel disks, $F_{12,\text{plate}} > F_{12,\text{disk}}$.

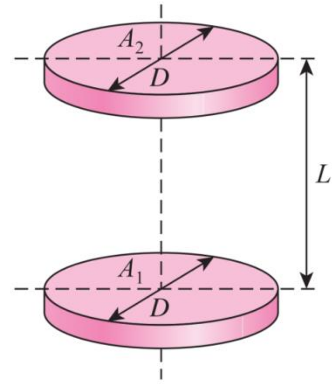
Discussion $F_{12,\text{plate}}$ is expected to be greater than $F_{12,\text{disk}}$ because the area of a 1 m × 1 m plate is greater than the area of a disk with a diameter of 1 m. At the same spacing apart, the geometry with the larger area is expected to have larger view factor.

13-6 Two coaxial parallel circular disks spaced apart at a distance L. The parameter that would increase the view factor F_{12} by a factor of 5 is to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis For coaxial parallel disks, from Table 13-1 with $r_i = r_j$, we have

$$F_{12} = 1 + \frac{1 - \sqrt{4R^2 + 1}}{2R^2} \quad \text{where} \quad R = \frac{r}{L} = \frac{D}{2L}$$



So,

$$F_{12} = 1 + \frac{1 - \sqrt{4[D/(2L)]^2 + 1}}{2[D/(2L)]^2} = 1 + \frac{1 - \sqrt{(D/L)^2 + 1}}{(1/2)(D/L)^2}$$

For $F_{12} = 0.1$, and solving for D/L_1, we have

$$0.1 = 1 + \frac{1 - \sqrt{(D/L_1)^2 + 1}}{(1/2)(D/L_1)^2} \quad \rightarrow \quad \frac{D}{L_1} = 0.70273$$

To increase the view factor by a factor of five to $F_{12} = 0.5$, we have

$$0.5 = 1 + \frac{1 - \sqrt{(D/L_2)^2 + 1}}{(1/2)(D/L_2)^2} \quad \rightarrow \quad \frac{D}{L_2} = 2.8284$$

For fixed diameter of the disks, we have

$$\frac{L_2}{L_1} = \frac{D}{L_1}\frac{L_2}{D} = \frac{0.70273}{2.8284} = \mathbf{0.2485}$$

Discussion In order to increase the view factor by a factor of five, the distance between the disks needs to be reduced to about a quarter of its initial distance, $L_2 = 0.2485L_1$.

13-7 A row of cylinders is spaced between two large parallel plates. The view factor between the plate and the row of cylinders is to be determined.

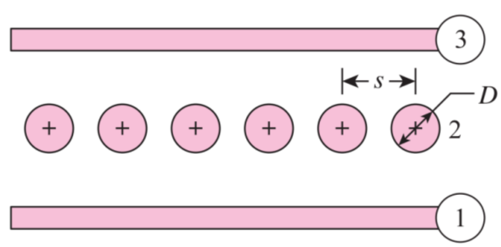
Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis From symmetry, the view factor between the plate (top or bottom) and the row of cylinders is the same, and from Table 13-2, we get

$$F_{12} = F_{32} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{0.5} + \left(\frac{D}{s}\right)\left\{\tan^{-1}\left[\left(\frac{s}{D}\right)^2 - 1\right]^{0.5}\right\}$$

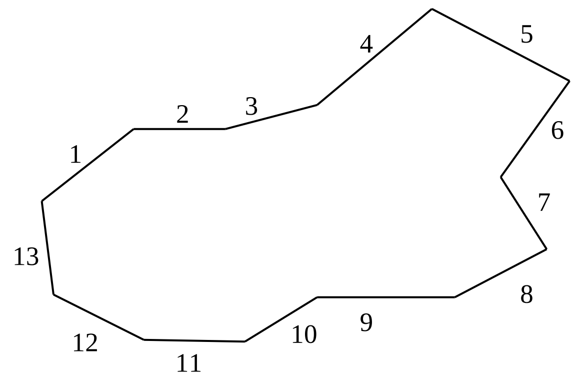
$$= 1 - \left[1 - \left(\frac{3.5}{5}\right)^2\right]^{0.5} + \left(\frac{3.5}{5}\right)\left\{\tan^{-1}\left[\left(\frac{5}{3.5}\right)^2 - 1\right]^{0.5}\right\}$$

$$= \mathbf{0.8426}$$



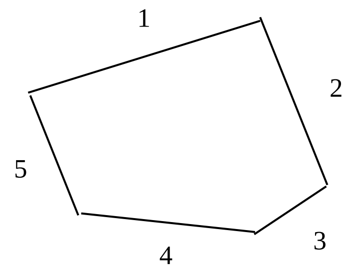
Discussion If the spacing between the cylinders is the same as the diameter ($s = D$), then the view factor would be $F_{12} = 1$. Note that the equation is only valid for $s \geq D$.

13-8 An enclosure consisting of thirteen surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.



Analysis A thirteen surface enclosure (N = 13) involves $N^2 = 13^2 = \mathbf{169}$ view factors and we need to determine $\frac{N(N-1)}{2} = \frac{13(13-1)}{2} = 78$ view factors directly. The remaining 169 - 78 = **91** of the view factors can be determined by the application of the reciprocity and summation rules.

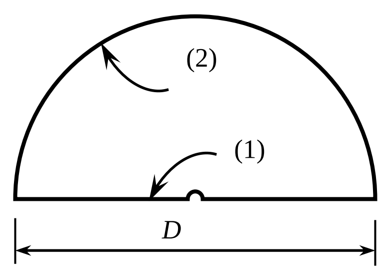
13-9 An enclosure consisting of five surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.



Analysis A five surface enclosure (N=5) involves $N^2 = 5^2 = \mathbf{25}$ view factors and we need to determine $\frac{N(N-1)}{2} = \frac{5(5-1)}{2} = 10$ view factors directly. The remaining 25-10 = **15** of the view factors can be determined by the application of the reciprocity and summation rules.

13-10 A semispherical furnace is considered. The view factor from the dome of this furnace to its flat base is to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.



Analysis We number the surfaces as follows:

(1): circular base surface

(2): dome surface

Surface (1) is flat, and thus $F_{11} = 0$.

$$\text{Summation rule}: \ F_{11} + F_{12} = 1 \rightarrow F_{12} = 1$$

$$\text{reciprocity rule}: \ A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{A_2}(1) = \frac{\frac{\pi D^2}{4}}{\frac{\pi D^2}{2}} = \frac{1}{2} = \mathbf{0.5}$$

13-11 The view factor from the conical side surface to a hole located at the center of the base of a conical enclosure is to be determined.

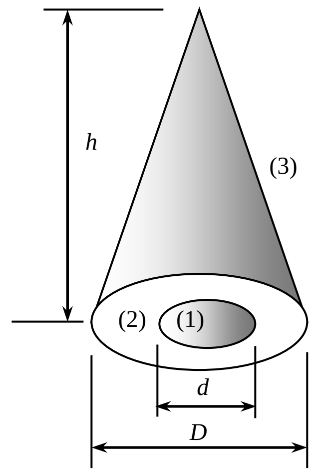
Assumptions The conical side surface is diffuse emitter and reflector.

Analysis We number different surfaces as

the hole located at the center of the base (1)

the base of conical enclosure (2)

conical side surface (3)



Surfaces 1 and 2 are flat, and they have no direct view of each other. Therefore,

$$F_{11} = F_{22} = F_{12} = F_{21} = 0$$

$$\text{summation rule: } F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1$$

$$\text{reciprocity rule: } A_1 F_{13} = A_3 F_{31} \longrightarrow \frac{\pi d^2}{4}(1) = \frac{\pi D h}{2} F_{31} \longrightarrow F_{31} = \mathbf{\frac{d^2}{2Dh}}$$

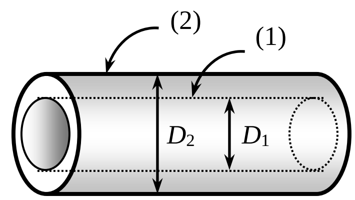
13-12 The four view factors associated with an enclosure formed by two very long concentric cylinders are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.

Analysis We number different surfaces as

the outer surface of the inner cylinder (1)

the inner surface of the outer cylinder (2)



No radiation leaving surface 1 strikes itself and thus $F_{11} = \mathbf{0}$

All radiation leaving surface 1 strikes surface 2 and thus $F_{12} = \mathbf{1}$

$$\text{reciprocity rule: } A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{\pi D_1 h}{\pi D_2 h}(1) = \mathbf{\frac{D_1}{D_2}}$$

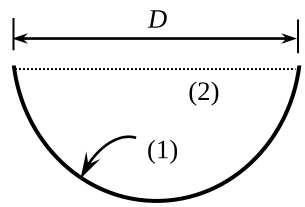
$$\text{summation rule: } F_{21} + F_{22} = 1 \longrightarrow F_{22} = 1 - F_{21} = \mathbf{1 - \frac{D_1}{D_2}}$$

13-13 View factors from the very long grooves shown in the figure to the surroundings are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.

Analysis (*a*) We designate the circular dome surface by (1) and the imaginary flat top surface by (2). Noting that (2) is flat,

$$F_{22} = 0$$

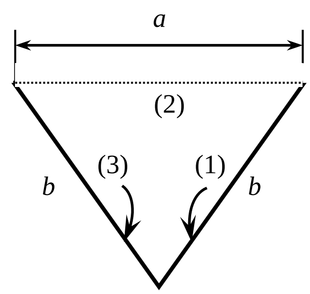


$$\text{summation rule}: \; F_{21} + F_{22} = 1 \longrightarrow F_{21} = 1$$

$$\text{reciprocity rule}: A_1 F_{12} = A_2 F_{21} \longrightarrow F_{12} = \frac{A_2}{A_1} F_{21} = \frac{D}{\frac{\pi D}{2}}(1) = \frac{2}{\pi} = \mathbf{0.64}$$

(*b*) We designate the two identical surfaces of length *b* by (1) and (3), and the imaginary flat top surface by (2). Noting that (2) is flat,

$$F_{22} = 0$$



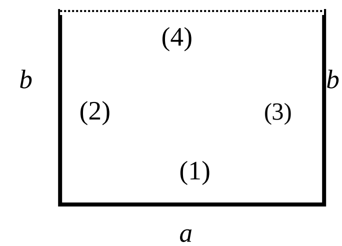
$$\text{summation rule}: \; F_{21} + F_{22} + F_{23} = 1 \longrightarrow F_{21} = F_{23} = 0.5 \quad \text{(symmetry)}$$

$$\text{summation rule}: F_{22} + F_{2\to(1+3)} = 1 \longrightarrow F_{2\to(1+3)} = 1$$

$$\text{reciprocity rule}: A_2 F_{2\to(1+3)} = A_{(1+3)} F_{(1+3)\to 2}$$

$$\longrightarrow F_{(1+3)\to 2} = F_{(1+3)\to surr} = \frac{A_2}{A_{(1+3)}}(1) = \mathbf{\frac{a}{2b}}$$

(*c*) We designate the bottom surface by (1), the side surfaces by (2) and (3), and the imaginary top surface by (4). Surface 4 is flat and is completely surrounded by other surfaces. Therefore, $F_{44} = 0$ and $F_{4\to(1+2+3)} = 1$.



$$\text{reciprocity rule}: A_4 F_{4\to(1+2+3)} = A_{(1+2+3)} F_{(1+2+3)\to 4}$$

$$\longrightarrow F_{(1+2+3)\to 4} = F_{(1+2+3)\to surr} = \frac{A_4}{A_{(1+2+3)}}(1) = \mathbf{\frac{a}{a+2b}}$$

13-14 A cylindrical enclosure is considered. (*a*) The expression for the view factor between the base and the side surface F_{13} in terms of K and (*b*) the value of the view factor F_{13} for $L = D$ are to be determined.

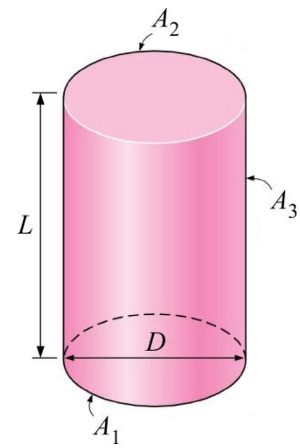
Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis (*a*) The surfaces are designated as follows: Base surface as A_1, top surface as A_2, and side surface as A_3

Applying the summation rule to A_1, we have

$$F_{11} + F_{12} + F_{13} = 1 \qquad \text{(where } F_{11} = 0\text{)}$$

or $$F_{13} = 1 - F_{12} \qquad (1)$$



For coaxial parallel disks, from Table 13-1, with $i = 1$, $j = 2$,

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_2}{D_1}\right)^2\right]^{1/2}\right\} = \frac{1}{2}[S - (S^2 - 4)^{1/2}] \qquad (2)$$

$$S = 1 + \frac{1 + R_2^2}{R_1^2} = 2 + \frac{1}{R^2} = 2 + \frac{4}{(D/L)^2} = 2 + 4K^2 \qquad (3)$$

where

$$R_1 = R_2 = R = \frac{D}{2L} = \frac{1}{2K}$$

Substituting Eq. (3) into Eq. (2), we get

$$\begin{aligned} F_{12} &= \frac{1}{2}\{2 + 4K^2 - [(2 + 4K^2)^2 - 4]^{1/2}\} \\ &= \frac{1}{2}[2 + 4K^2 - (16K^4 + 16K^2)^{1/2}] \\ &= \frac{1}{2}[2 + 4K^2 - 4K(K^2 + 1)^{1/2}] \\ &= 1 + 2K^2 - 2K(K^2 + 1)^{1/2} \end{aligned}$$

Substituting the above expression for F_{12} into Eq. (1) yields the expression for F_{13}:

$$F_{13} = 1 - [1 + 2K^2 - 2K(K^2 + 1)^{1/2}]$$

Hence,

$$F_{13} = 2K(K^2 + 1)^{1/2} - 2K^2$$

(*b*) The value of the view factor F_{13} for $L = D$ (i.e., $K = 1$) is

$$F_{13} = 2(1)(1^2 + 1)^{1/2} - 2(1)^2 = 2\sqrt{2} - 2 = \mathbf{0.828}$$

Discussion If the cylinder has a length and diameter of $L = 2D$, then from the expression for F_{13} we have

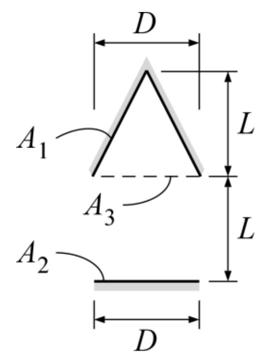
$$F_{13} = 2(2)(2^2 + 1)^{1/2} - 2(2)^2 = \mathbf{0.944}$$

13-15 A circular cone is positioned on a common axis with a circular disk, the values of F_{11} and F_{12} for specified L and D are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** The surface A_1 is treated as a single surface.

Analysis The area for A_1, A_2, and A_3 are

$$A_1 = \frac{\pi D}{2}\left[L^2 + \left(\frac{D}{2}\right)^2\right]^{1/2} = \frac{\sqrt{5}D^2\pi}{4} \qquad \text{and} \qquad A_2 = A_3 = \frac{\pi D^2}{4}$$



The surfaces A_1 and A_3 can be treated as an enclosure, and using the summation rule yields

$$F_{11} + F_{13} = 1$$

Applying reciprocity relation between A_1 and A_3, we have

$$A_1F_{13} = A_3F_{31} \quad \rightarrow \quad F_{11} = 1 - F_{13} = 1 - (A_3/A_1)F_{31}$$

Note that

$$F_{31} + F_{33} = 1 \quad \rightarrow \quad F_{31} = 1 \quad (\text{since } F_{33} = 0)$$

Hence, the view factor F_{11} is

$$F_{11} = 1 - (A_3/A_1) = 1 - \frac{\pi D^2}{\sqrt{5}D^2\pi} = \mathbf{0.553}$$

The radiation leaving A_2 is intercepted by A_1 and A_3 equally,

$$F_{21} = F_{23}$$

Applying reciprocity relation between A_1 and A_2, we have

$$A_1F_{12} = A_2F_{21} \quad \rightarrow \quad F_{12} = (A_2/A_1)F_{21} = (A_2/A_1)F_{23} = \frac{F_{23}}{\sqrt{5}}$$

For coaxial parallel disks, the view factor F_{23} is evaluated from Table 13-1 as

$$F_{23} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_3}{D_2}\right)^2\right]^{1/2}\right\} = \frac{1}{2}[S - (S^2 - 4)^{1/2}] = \frac{1}{2}[6 - (6^2 - 4)^{1/2}] = 0.1716$$

where

$$R_1 = R_2 = R = \frac{D}{2L} = \frac{1}{2}$$

$$S = 1 + \frac{1 + R_2^2}{R_1^2} = 2 + \frac{1}{R^2} = 2 + 4 = 6$$

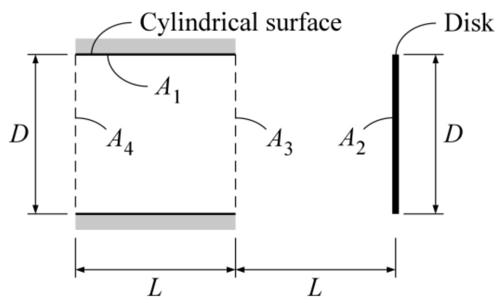
Hence, the view factor F_{12} is

$$F_{12} = \frac{F_{23}}{\sqrt{5}} = \frac{0.1716}{\sqrt{5}} = \mathbf{0.0767}$$

Discussion As long as $L = D$, the view factors are constant at $F_{11} = 0.553$ and $F_{12} = 0.0767$ (i.e., F_{11} and F_{12} are independent of L and D).

13-16 A cylindrical surface and a disk are oriented coaxially a distance L apart. The view factor F_{12} between them is to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.



Analysis The end surfaces A_3 and A_4 are treated as hypothetical surfaces. The summation rule for surfaces facing the disk can be expressed as

$$F_{23} = F_{21} + F_{24} \quad \rightarrow \quad F_{21} = F_{23} - F_{24} \qquad (1)$$

From reciprocity relation, we have

$$A_2 F_{21} = A_1 F_{12} \qquad (2)$$

Substituting Eq. (2) in to Eq. (1) and rearranging give

$$(A_1 / A_2) F_{12} = F_{23} - F_{24} \quad \rightarrow \quad F_{12} = (A_2 / A_1)(F_{23} - F_{24}) \qquad (3)$$

The view factors F_{23} and F_{24} can be determined from the relation in Table 13-1 by treating the two surfaces as coaxial parallel disks of identical diameters:

$$F_{i\rightarrow j} = F_{j\rightarrow i} = 1 + \frac{1 - \sqrt{4R^2 + 1}}{2R^2} \quad \text{where } R = \frac{r}{L}$$

For surface 3 ($L = 2D$):

$$R = \frac{r}{L} = \frac{D/2}{L} = \frac{D/2}{2D} = 0.25$$

and

$$F_{23} = 1 + \frac{1 - \sqrt{4 \times 0.25^2 + 1}}{2 \times 0.25^2} = 0.05573$$

For surface 4 ($L = 4D$): $R = \frac{r}{L} = \frac{D/2}{L} = \frac{D/2}{4D} = 0.125$

and

$$F_{24} = 1 + \frac{1 - \sqrt{4 \times 0.125^2 + 1}}{2 \times 0.125^2} = 0.01515$$

Substituting these values of F_{23} and F_{24} into Eq. (3), and noting that $L = 2D$, the view factor between the cylindrical surface and the disk is determined to be

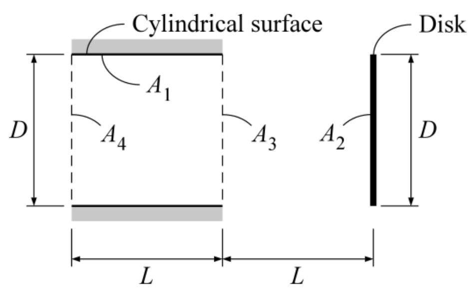
$$F_{12} = \frac{A_2}{A_1}(F_{23} - F_{24}) = \frac{\pi D^2/4}{\pi D L}(F_{23} - F_{24}) = \frac{\pi D^2/4}{\pi D(2D)}(F_{23} - F_{24})$$

$$= \frac{1}{8}(F_{23} - F_{24}) = \frac{1}{8}(0.05573 - 0.01515) = \mathbf{0.00507}$$

Discussion The view factors F_{23} and F_{24} can also be determined using Fig. 13-7, but this would involve reading error.

13-17 A cylindrical surface and a disk are oriented coaxially a distance L apart. The view factor F_{12} between them is to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.



Analysis The end surfaces A_3 and A_4 are treated as hypothetical surfaces. The summation rule for surfaces facing the disk can be expressed as

$$F_{23} = F_{21} + F_{24} \quad \rightarrow \quad F_{21} = F_{23} - F_{24} \qquad (1)$$

From reciprocity relation, we have

$$A_2 F_{21} = A_1 F_{12} \qquad (2)$$

Substituting Eq. (2) in to Eq. (1) and rearranging give

$$(A_1 / A_2) F_{12} = F_{23} - F_{24} \quad \rightarrow \quad F_{12} = (A_2 / A_1)(F_{23} - F_{24}) \qquad (3)$$

The view factors F_{23} and F_{24} can be determined from the relation in Table 13-1 by treating the two surfaces as coaxial parallel disks of identical diameters:

$$F_{i \to j} = F_{j \to i} = 1 + \frac{1 - \sqrt{4R^2 + 1}}{2R^2} \quad \text{where } R = \frac{r}{L}$$

For surface 3 ($L = D$):

$$R = \frac{r}{L} = \frac{D/2}{L} = \frac{D/2}{D} = 0.5$$

and

$$F_{23} = 1 + \frac{1 - \sqrt{4 \times 0.5^2 + 1}}{2 \times 0.5^2} = 0.1716$$

For surface 4 ($L = 2D$):

$$R = \frac{r}{L} = \frac{D/2}{L} = \frac{D/2}{2D} = 0.25$$

and

$$F_{24} = 1 + \frac{1 - \sqrt{4 \times 0.25^2 + 1}}{2 \times 0.25^2} = 0.05573$$

Substituting these values of F_{23} and F_{24} into Eq. (3), and noting that $L = D$, the view factor between the cylindrical surface and the disk is determined to be

$$F_{12} = \frac{A_2}{A_1}(F_{23} - F_{24}) = \frac{\pi D^2 / 4}{\pi D L}(F_{23} - F_{24}) = \frac{1}{4}(0.1716 - 0.05573) = \mathbf{0.0290}$$

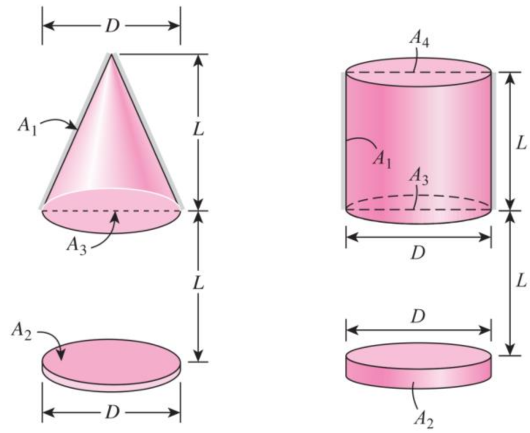
Discussion The view factors F_{23} and F_{24} can also be determined using Fig. 13-7, but this would involve reading error.

13-18 A circular cone is positioned on a common axis with a disk and a cylindrical surface oriented coaxially with a disk. The view factors of the two geometries are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis For the *circular cone and disk* geometry: The area for A_1, A_2, and A_3 are

$$A_1 = \frac{\pi D}{2}\left[L^2 + \left(\frac{D}{2}\right)^2\right]^{1/2} = \frac{\sqrt{5}D^2\pi}{4} \quad \text{and} \quad A_2 = A_3 = \frac{\pi D^2}{4}$$



The radiation leaving A_2 is intercepted by A_1 and A_3 equally,

$$F_{21} = F_{23}$$

Applying reciprocity relation between A_1 and A_2, we have

$$A_1F_{12} = A_2F_{21} \quad \rightarrow \quad F_{12} = (A_2/A_1)F_{21} = (A_2/A_1)F_{23} = \frac{F_{23}}{\sqrt{5}}$$

For coaxial parallel disks, the view factor F_{23} is evaluated from Table 13-1 as

$$F_{23} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_3}{D_2}\right)^2\right]^{1/2}\right\} = \frac{1}{2}[S-(S^2-4)^{1/2}] = \frac{1}{2}[6-(6^2-4)^{1/2}] = 0.1716$$

where $\quad R_1 = R_2 = R = \dfrac{D}{2L} = \dfrac{1}{2} \quad$ and $\quad S = 1 + \dfrac{1+R_2^2}{R_1^2} = 2 + \dfrac{1}{R^2} = 2+4 = 6$

Hence, the view factor F_{12} is

$$F_{12} = \frac{F_{23}}{\sqrt{5}} = \frac{0.1716}{\sqrt{5}} = \mathbf{0.0767} \quad \text{(circular cone and disk)}$$

For the *circular surface and disk* geometry: The end surfaces A_3 and A_4 are treated as hypothetical surfaces. From the summation rule for surfaces facing the disk and the reciprocity relation, we have

$$F_{23} = F_{21} + F_{24} \quad \rightarrow \quad F_{21} = F_{23} - F_{24} \quad \text{and} \quad A_2F_{21} = A_1F_{12}$$

Hence

$$(A_1/A_2)F_{12} = F_{23} - F_{24} \quad \rightarrow \quad F_{12} = (A_2/A_1)(F_{23} - F_{24}) \qquad (1)$$

The view factors F_{23} and F_{24} can be determined from the relation in Table 13-1 by treating the two surfaces as coaxial parallel disks of identical diameters:

$$F_{i\to j} = F_{j\to i} = 1 + \frac{1-\sqrt{4R^2+1}}{2R^2} \quad \text{where} \quad R = \frac{r}{L}$$

For surface 3: $\quad F_{23} = 1 + \dfrac{1-\sqrt{4\times 0.5^2+1}}{2\times 0.5^2} = 0.1716 \quad$ where $\quad R = \dfrac{r}{L} = \dfrac{D/2}{L} = \dfrac{D/2}{D} = 0.5$

For surface 4: $\quad F_{24} = 1 + \dfrac{1-\sqrt{4\times 0.25^2+1}}{2\times 0.25^2} = 0.05573 \quad$ where $\quad R = \dfrac{r}{2L} = \dfrac{D/2}{2L} = \dfrac{D/2}{2D} = 0.25$

Using Eq. (1), with $L = D$, we get

$$F_{12} = \frac{A_2}{A_1}(F_{23} - F_{24}) = \frac{\pi D^2/4}{\pi DL}(F_{23} - F_{24}) = \frac{1}{4}(0.1716 - 0.05573) = \mathbf{0.0290} \quad \text{(circular surface and disk)}$$

Discussion The F_{12} for the circular cone and disk geometry is greater than the F_{12} for the circular surface and disk geometry. This is expected as surface 1 of the circular cone and disk geometry is angled toward the disk, but such is not the case for the circular surface and disk geometry.

13-19 The view factors from the base of a cube to each of the other five surfaces are to be determined.

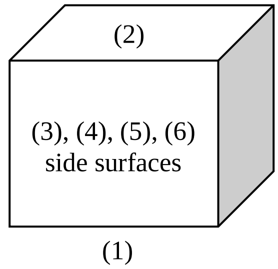
Assumptions The surfaces are diffuse emitters and reflectors.

Analysis Noting that $L_1 / D = L_2 / D = 1$, from Fig. 13-6 we read

$$F_{12} = 0.2$$

Because of symmetry, we have

$$F_{12} = F_{13} = F_{14} = F_{15} = F_{16} = \mathbf{0.2}$$

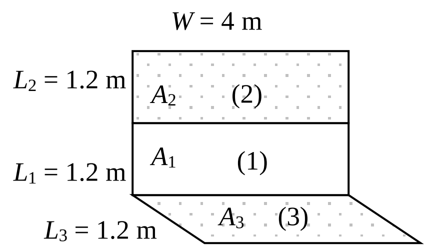


13-20 The view factors between the rectangular surfaces shown in the figure are to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis From Fig. 13-6,

$$\left.\begin{array}{l} \dfrac{L_3}{W} = \dfrac{1.2}{4} = 0.3 \\ \dfrac{L_1}{W} = \dfrac{1.2}{4} = 0.3 \end{array}\right\} F_{31} = 0.26$$



and

$$\left.\begin{array}{l} \dfrac{L_3}{W} = \dfrac{1.2}{4} = 0.3 \\ \dfrac{L_1 + L_2}{W} = \dfrac{2.4}{4} = 0.6 \end{array}\right\} F_{3\to(1+2)} = 0.32$$

We note that $A_1 = A_3$. Then the reciprocity and superposition rules gives

$$\mathrm{A}_1 F_{13} = A_3 F_{31} \longrightarrow F_{13} = F_{31} = \mathbf{0.26}$$

$$F_{3\to(1+2)} = F_{31} + F_{32} \longrightarrow \quad 0.32 = 0.26 + F_{32} \longrightarrow F_{32} = 0.06$$

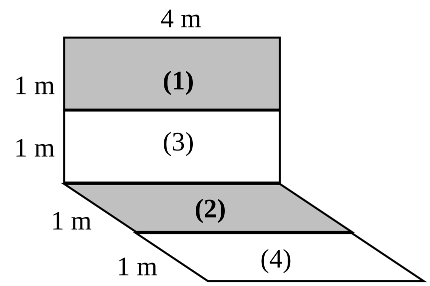
Finally, $A_2 = A_3 \longrightarrow F_{23} = F_{32} = \mathbf{0.06}$

13-21 The view factors between the rectangular surfaces shown in the figure are to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis We designate the different surfaces as follows:

shaded part of perpendicular surface by (1),
bottom part of perpendicular surface by (3),
shaded part of horizontal surface by (2), and
front part of horizontal surface by (4).



(*a*) From Fig.13-6

$$\left.\begin{array}{l}\dfrac{L_2}{W}=\dfrac{1}{4}=0.25\\ \dfrac{L_1}{W}=\dfrac{1}{4}=0.25\end{array}\right\}F_{23}=0.26\ \text{ and }\ \left.\begin{array}{l}\dfrac{L_2}{W}=\dfrac{2}{4}=0.5\\ \dfrac{L_1}{W}=\dfrac{1}{4}=0.25\end{array}\right\}F_{2\to(1+3)}=0.33$$

$$\text{superposition rule: } F_{2\to(1+3)}=F_{21}+F_{23}\longrightarrow F_{21}=F_{2\to(1+3)}-F_{23}=0.33-0.26=0.07$$

$$\text{reciprocity rule: } A_1=A_2\longrightarrow A_1F_{12}=A_2F_{21}\longrightarrow F_{12}=F_{21}=\mathbf{0.07}$$

(*b*) From Fig.13-6,

$$\left.\begin{array}{l}\dfrac{L_2}{W}=\dfrac{1}{4}=0.25\\ \dfrac{L_1}{W}=\dfrac{2}{4}=0.5\end{array}\right\}F_{(4+2)\to 3}=0.16\quad \text{and}\quad \left.\begin{array}{l}\dfrac{L_2}{W}=\dfrac{2}{4}=0.5\\ \dfrac{L_1}{W}=\dfrac{2}{4}=0.5\end{array}\right\}F_{(4+2)\to(1+3)}=0.24$$

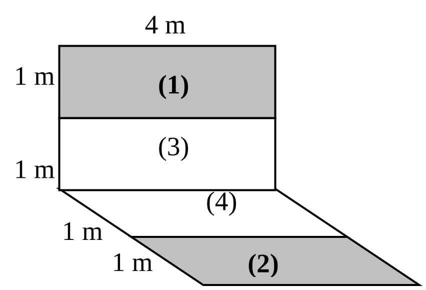
$$\text{superposition rule: } F_{(4+2)\to(1+3)}=F_{(4+2)\to 1}+F_{(4+2)\to 3}\longrightarrow F_{(4+2)\to 1}=0.24-0.16=0.08$$

$$\text{reciprocity rule: } A_{(4+2)}F_{(4+2)\to 1}=A_1F_{1\to(4+2)}$$

$$\longrightarrow F_{1\to(4+2)}=\frac{A_{(4+2)}}{A_1}F_{(4+2)\to 1}=\frac{8}{4}(0.08)=0.16$$

$$\text{superposition rule: } F_{1\to(4+2)}=F_{14}+F_{12}$$

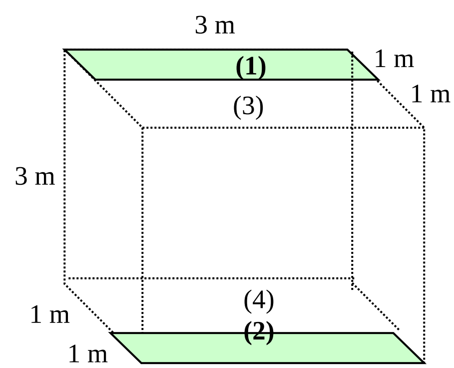
$$\longrightarrow F_{14}=0.16-0.08=\mathbf{0.08}$$



since F_{12} = 0.07 (from part *a*). Note that F_{14} in part (*b*) is equivalent to F_{12} in part (*a*).

(*c*) We designate

shaded part of top surface by (1),
remaining part of top surface by (3),
remaining part of bottom surface by (4), and
shaded part of bottom surface by (2).



From Fig.13-5,

$$\left.\begin{array}{l}\dfrac{L_2}{D}=\dfrac{3}{3}=1\\ \dfrac{L_1}{D}=\dfrac{2}{3}=0.67\end{array}\right\}F_{(2+4)\to(1+3)}=0.15\quad \text{and}\quad \left.\begin{array}{l}\dfrac{L_2}{D}=\dfrac{3}{3}=1\\ \dfrac{L_1}{D}=\dfrac{1}{3}=0.33\end{array}\right\}F_{14}=0.082$$

$$\text{superposition rule: } F_{(2+4)\to(1+3)}=F_{(2+4)\to 1}+F_{(2+4)\to 3}$$

$$\text{symmetry rule: } F_{(2+4)\to 1}=F_{(2+4)\to 3}$$

Substituting symmetry rule gives

$$F_{(2+4)\to 1} = F_{(2+4)\to 3} = \frac{F_{(2+4)\to(1+3)}}{2} = \frac{0.15}{2} = 0.075$$

reciprocity rule : $A_1 F_{1\to(2+4)} = A_{(2+4)} F_{(2+4)\to 1} \longrightarrow (3)F_{1\to(2+4)} = (6)(0.075) \longrightarrow F_{1\to(2+4)} = 0.15$

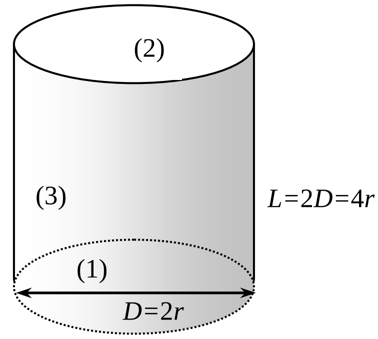
superposition rule: $F_{1\to(2+4)} = F_{12} + F_{14} \longrightarrow 0.15 = F_{12} + 0.082 \longrightarrow F_{12} = 0.15 - 0.082 = \mathbf{0.068}$

13-22 A cylindrical enclosure is considered. The view factor from the side surface of this cylindrical enclosure to its base surface is to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis We designate the surfaces as follows:

Base surface by (1),
top surface by (2), and
side surface by (3).



Then from Fig. 13-7

$$\left.\begin{array}{l}\dfrac{L}{r_1}=\dfrac{4r_1}{r_1}=4\\[2ex] \dfrac{r_2}{L}=\dfrac{r_2}{4r_2}=0.25\end{array}\right\} F_{12}=F_{21}=0.05$$

$$\text{summation rule}: F_{11}+F_{12}+F_{13}=1$$

$$0+0.05+F_{13}=1 \longrightarrow F_{13}=0.95$$

$$\text{reciprocity rule}: A_1F_{13}=A_3F_{31} \longrightarrow F_{31}=\frac{A_1}{A_3}F_{13}=\frac{\pi r_1^2}{2\pi r_1 L}F_{13}=\frac{\pi r_1^2}{8\pi r_1^2}F_{13}=\frac{1}{8}(0.95)=\mathbf{0.119}$$

Discussion This problem can be solved more accurately by using the view factor relation from Table 13-1 to be

$$R_1=\frac{r_1}{L}=\frac{r_1}{4r_1}=0.25$$

$$R_2=\frac{r_2}{L}=\frac{r_2}{4r_2}=0.25$$

$$S=1+\frac{1+R_2^2}{R_1^2}=1+\frac{1+0.25^2}{0.25^2}=18$$

$$F_{12}=\tfrac{1}{2}\left\{S-\left[S^2-4\left(\frac{R_2}{R_1}\right)^2\right]^{0.5}\right\}=\tfrac{1}{2}\left\{18-\left[18^2-4\left(\frac{1}{1}\right)^2\right]^{0.5}\right\}=0.056$$

$$F_{13}=1-F_{12}=1-0.056=0.944$$

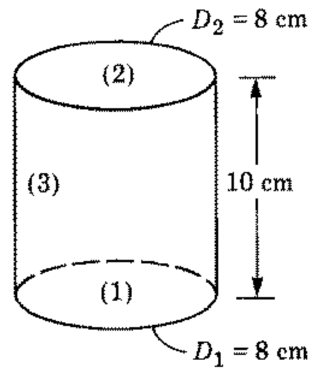
$$\text{reciprocity rule}: A_1F_{13}=A_3F_{31} \longrightarrow F_{31}=\frac{A_1}{A_3}F_{13}=\frac{\pi r_1^2}{2\pi r_1 L}F_{13}=\frac{\pi r_1^2}{8\pi r_1^2}F_{13}=\frac{1}{8}(0.944)=\mathbf{0.118}$$

13-23 For a right circular cylinder the view factors F_{13} and F_{33} are to be determined.

Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis Neither result is given in the book tables or figures. To obtain F_{13}, we can make use of Fig. 13-7 or Table 13-1 for the two parallel disks.

For



$$\frac{L}{r_1} = \frac{10}{4} = 2.5 \quad \text{and} \quad \frac{r_2}{L} = \frac{4}{10} = 0.4$$

From Fig. 13-7 or compute from item 2 of Table 13-1

$$F_{12} = 0.12$$

From the summation rule, Eq. 13-12

$$\sum_{j=1}^{N} F_{ij} = 1$$

or

$$F_{11} + F_{12} + F_{13} = 1$$

where

$$F_{11} = 0$$

$\therefore$ $F_{13} = 1 - F_{12} = 1 - 0.12 = \mathbf{0.88}$

To obtain F_{33}, the summation rule (Eq. 13-12) requires

$$F_{33} + F_{31} + F_{32} = 1 = F_{33} + 2F_{31}$$

Since the cylinder has identical views from the top and bottom disks. To obtain F_{31} use the reciprocity relation (Eq. 13-10).

$$A_3F_{31} = A_1F_{13}$$

where

$$A_3 = \pi D_1 L = 80\ \pi\,\text{cm}^2$$

and

$$A_1 = \pi D_1^2 = 16\ \pi\,\text{cm}^2$$

$$F_{31} = (A_1/A_3)\,F_{13} = (16\ \pi\,\text{cm}^2/\ 80\ \pi\,\text{cm}^2)\,(0.88) = 0.176$$

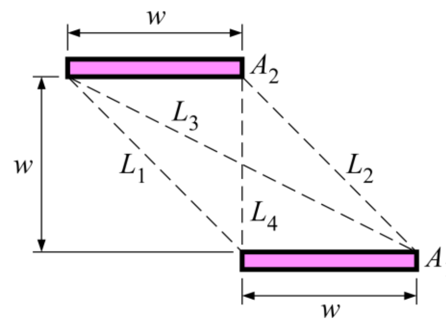
Finally, from the summation rule eq. above

$$F_{33} = 1 - 2F_{31} = 1 - 2\,(0.176) = \mathbf{0.65}$$

13-24 The expression for the view factor F_{12} of two infinitely long parallel plates is to be determined using the Hottel's crossed-strings method.

Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis From the Hottel's crossed-strings method, we have



$$F_{i\to j} = \frac{\Sigma\,(\text{Crossed strings}) - \Sigma\,(\text{Uncrossed strings})}{2\times\,(\text{Length of surface } i)}$$

For uncrossed strings, we have

$$L_1 = L_2 = (w^2 + w^2)^{1/2} = (w^2 + w^2)^{1/2} = \sqrt{2}w$$

For crossed strings, we have

$$L_3 = (w^2 + 4w^2)^{1/2} = \sqrt{5}w \qquad \text{and} \qquad L_4 = w$$

Applying the Hottel's crossed-strings method, we get F_{12} as

$$F_{12} = \frac{(L_3 + L_4) - (L_1 + L_2)}{2w}$$

$$= \frac{(\sqrt{5}w + w) - (\sqrt{2}w + \sqrt{2}w)}{2w}$$

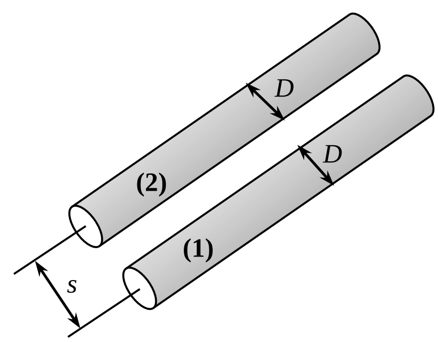
$$= \mathbf{0.204}$$

Discussion The Hottel's crossed-string method is applicable only to surfaces that are very long, such that they can be considered to be two-dimensional and radiation interaction through the end surfaces is negligible.

13-25 The view factor between the two infinitely long parallel cylinders located a distance s apart from each other is to be determined.

Assumptions The surfaces are diffuse emitters and reflectors.

Analysis Using the crossed-strings method, the view factor between two cylinders facing each other for $s/D > 3$ is determined to be



$$F_{1-2} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2\times \text{Length of surface } i}$$

$$= \frac{2\sqrt{s^2 + D^2} - 2s}{2\,(\pi D/2)}$$

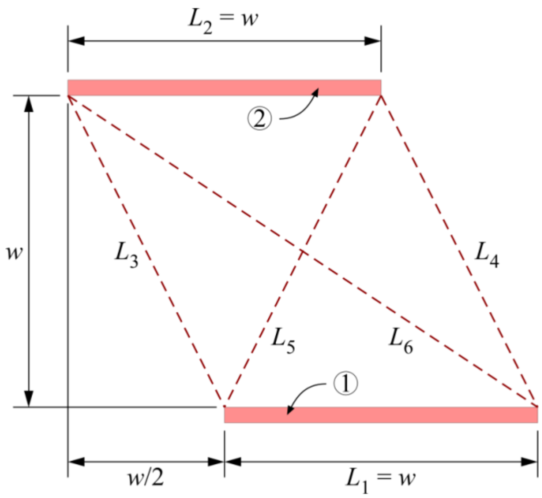
or $$F_{1-2} = \frac{2\left(\sqrt{s^2 + D^2} - s\right)}{\pi D}$$

13-26 The expressions for the view factors F_{12} and F_{21} of two infinitely long parallel plates are to be determined using the Hottel's crossed-strings method.

Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis From the Hottel's crossed-strings method, we have

$$F_{i\to j} = \frac{\Sigma\ (\text{Crossed strings}) - \Sigma\ (\text{Uncrossed strings})}{2\times\ (\text{Length of surface } i)}$$



where

$$L_1 = L_2 = w$$

$$L_3 = L_4 = L_5 = \sqrt{w^2 + (w/2)^2} = \frac{\sqrt{5}}{2}w$$

$$L_6 = \sqrt{w^2 + \left(\frac{3}{2}w\right)^2} = \frac{\sqrt{13}}{2}w$$

Applying the Hottel's crossed-strings method, we get F_{12} as

$$F_{12} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2w}$$

$$= \frac{(\sqrt{5}w/2 + \sqrt{13}w/2) - (\sqrt{5}w/2 + \sqrt{5}w/2)}{2w}$$

$$= \mathbf{0.3424}$$

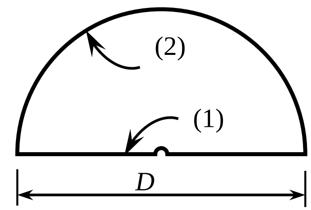
Since surface 1 and surface 2 have the same width, thus the Hottel's crossed-strings method will give

$$F_{12} = F_{21} = \mathbf{0.3424}$$

Discussion The Hottel's crossed-string method is applicable only to surfaces that are very long, such that they can be considered to be two-dimensional and radiation interaction through the end surfaces is negligible.

13-27 Two view factors associated with three very long ducts with different geometries are to be determined.

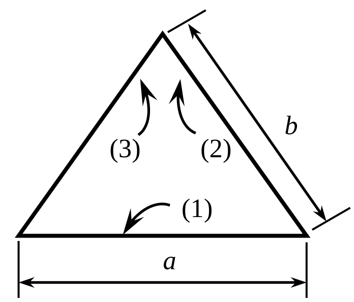
Assumptions **1** The surfaces are diffuse emitters and reflectors. **2** End effects are neglected.



Analysis (*a*) Surface (1) is flat, and thus $F_{11} = 0$.

$$\text{summation rule: } F_{11} + F_{12} = 1 \rightarrow F_{12} = \mathbf{1}$$

$$\text{reciprocity rule: } A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{Ds}{\left(\frac{\pi D}{2}\right)s}(1) = \frac{2}{\pi} = \mathbf{0.64}$$



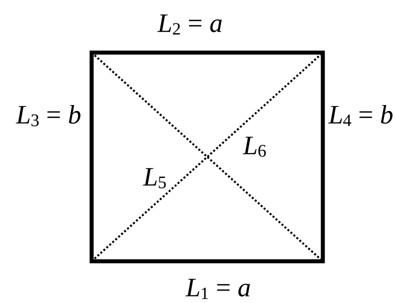
(*b*) Noting that surfaces 2 and 3 are symmetrical and thus $F_{12} = F_{13}$, the summation rule gives

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow 0 + F_{12} + F_{13} = 1 \longrightarrow F_{12} = \mathbf{0.5}$$

Also by using the equation obtained in Example 13-4,

$$F_{12} = \frac{L_1 + L_2 - L_3}{2L_1} = \frac{a + b - b}{2a} = \frac{a}{2a} = \frac{1}{2} = \mathbf{0.5}$$

$$\text{reciprocity rule: } A_1 F_{12} = A_2 F_{21} \longrightarrow F_{21} = \frac{A_1}{A_2} F_{12} = \frac{a}{b}\left(\frac{1}{2}\right) = \frac{\mathbf{a}}{\mathbf{2b}}$$



(*c*) Applying the crossed-string method gives

$$F_{12} = F_{21} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}$$

$$= \frac{2\sqrt{a^2 + b^2} - 2b}{2a} = \frac{\sqrt{\mathbf{a}^2 + \mathbf{b}^2} - \mathbf{b}}{\mathbf{a}}$$

Radiation Heat Transfer between Surfaces

13-28C The analysis of radiation exchange between black surfaces is relatively easy because of the absence of reflection. The rate of radiation heat transfer between two surfaces in this case is expressed as $\dot{Q} = A_1 F_{12} \sigma (T_1^4 - T_2^4)$ where A_1 is the surface area, F_{12} is the view factor, and T_1 and T_2 are the temperatures of two surfaces.

13-29C Radiosity is the total radiation energy leaving a surface per unit time and per unit area. Radiosity includes the emitted radiation energy as well as reflected energy. Radiosity and emitted energy are equal for blackbodies since a blackbody does not reflect any radiation.

13-30C Radiation surface resistance is given as $R_i = \dfrac{1-\varepsilon_i}{A_i \varepsilon_i}$ and it represents the resistance of a surface to the emission of radiation. It is zero for black surfaces. The space resistance is the radiation resistance between two surfaces and is expressed as $R_{ij} = \dfrac{1}{A_i F_{ij}}$

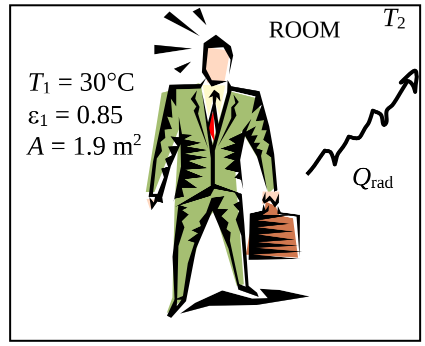
13-31C Some surfaces encountered in numerous practical heat transfer applications are modeled as being adiabatic as the back sides of these surfaces are well insulated and net heat transfer through these surfaces is zero. When the convection effects on the front (heat transfer) side of such a surface is negligible and steady-state conditions are reached, the surface must lose as much radiation energy as it receives. Such a surface is called reradiating surface. In radiation analysis, the surface resistance of a reradiating surface is taken to be zero since there is no heat transfer through it.

13-32C The two methods used in radiation analysis are the matrix and network methods. In matrix method, equations 13-34 and 13-35 give N linear algebraic equations for the determination of the N unknown radiosities for an N -surface enclosure. Once the radiosities are available, the unknown surface temperatures and heat transfer rates can be determined from these equations respectively. This method involves the use of matrices especially when there are a large number of surfaces. Therefore this method requires some knowledge of linear algebra.

The network method involves drawing a surface resistance associated with each surface of an enclosure and connecting them with space resistances. Then the radiation problem is solved by treating it as an electrical network problem where the radiation heat transfer replaces the current and the radiosity replaces the potential. The network method is not practical for enclosures with more than three or four surfaces due to the increased complexity of the network.

13-33 The rate of heat loss from a person by radiation in a large room whose walls are maintained at a uniform temperature is to be determined for two cases.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Properties The emissivity of the person is given to be $\varepsilon_1 = 0.85$.

Analysis (*a*) Noting that the view factor from the person to the walls $F_{12} = 1$, the rate of heat loss from that person to the walls at a large room which are at a temperature of 295 K is

$$\begin{aligned}\dot{Q}_{12} &= \varepsilon_1 F_{12} A_1 \sigma (T_1^{\ 4} - T_2^{\ 4}) \\ &= (0.85)(1)(1.9\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(303\ \text{K})^4 - (295\ \text{K})^4] \\ &= \mathbf{78.3\ W}\end{aligned}$$

(*b*) When the walls are at a temperature of 260 K,

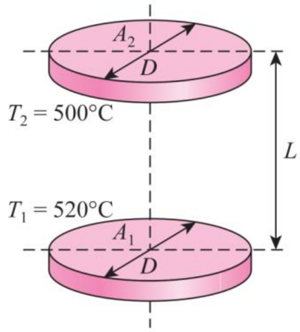
$$\begin{aligned}\dot{Q}_{12} &= \varepsilon_1 F_{12} A_1 \sigma (T_1^{\ 4} - T_2^{\ 4}) \\ &= (0.85)(1)(1.9\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(303\ \text{K})^4 - (260\ \text{K})^4] \\ &= \mathbf{353\ W}\end{aligned}$$

13-34 Two coaxial parallel circular disks spaced apart at a distance of $L = D$. The radiation heat transfer coefficient between the disks is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis The net radiation heat flux between the two circular disks is

$$\dot{q}_{12} = F_{12}\sigma(T_1^4 - T_2^4)$$
$$= F_{12}\sigma(T_1 + T_2)(T_1^2 + T_2^2)(T_1 - T_2)$$



Also, expressed in terms of the radiation heat transfer coefficient, we have

$$\dot{q}_{12} = h_{\text{rad}}(T_1 - T_2)$$

Hence, the radiation heat transfer coefficient can be expressed as

$$h_{\text{rad}} = F_{12}\sigma(T_1 + T_2)(T_1^2 + T_2^2)$$

For coaxial parallel disks, from Table 13-1 with $r_i = r_j$, we have

$$F_{12} = 1 + \frac{1 - \sqrt{4R^2 + 1}}{2R^2} \quad \text{where} \quad R = \frac{r}{L} = \frac{D}{2L} = 0.5$$

So,

$$F_{12} = 1 + \frac{1 - \sqrt{4(0.5)^2 + 1}}{2(0.5)^2} = 0.1716$$

Thus,

$$h_{\text{rad}} = F_{12}\sigma(T_1 + T_2)(T_1^2 + T_2^2)$$
$$= (0.1716)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(793 + 773)\,\text{K}\,(793^2 + 773^2)\,\text{K}^2$$
$$= \mathbf{18.69\ W/m^2 \cdot K}$$

Discussion The radiation heat transfer coefficient h_{rad} is dependent upon the view factor and the temperatures of both disks.

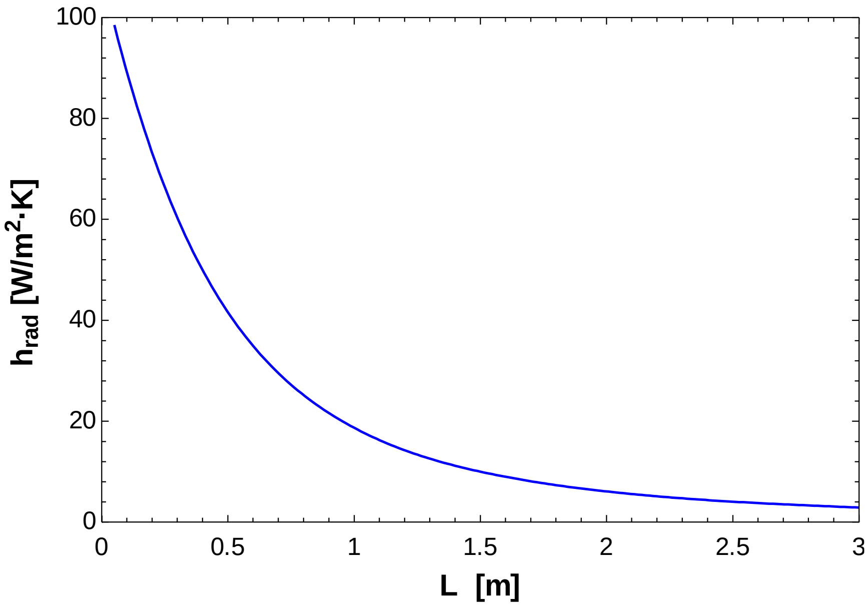
13-35 Two coaxial parallel circular disks spaced apart at a distance L. The effect of the distance L on the radiation heat transfer coefficient between the disks is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_1=520 [C]
T_2=500 [C]
D=1 [m]

"ANALYSIS"
R=D/(2*L)
F_12=1+(1-(4*R^2+1)^0.5)/(2*R^2)
h_rad=F_12*sigma#*((T_1+273)+(T_2+273))*((T_1+273)^2+(T_2+273)^2)
```

L [m]	F_{12}	h_{rad} [W/m^2·K]
0.05	0.9049	98.53
0.10	0.8190	89.18
0.15	0.7416	80.75
0.20	0.6721	73.18
0.25	0.6096	66.38
0.30	0.5536	60.28
0.35	0.5034	54.81
0.40	0.4584	49.91
0.45	0.4181	45.52
0.50	0.3820	41.59
0.75	0.2500	27.22
1.0	0.1716	18.68
1.5	0.09167	9.982
2.0	0.05573	6.068
2.5	0.03709	4.038
3.0	0.02633	2.867



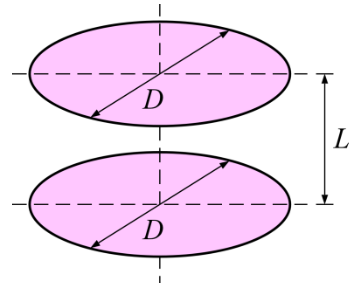
Discussion By reducing the spacing L between the parallel disks the radiation heat transfer coefficient h_{rad} increases. As L decreases below the diameter of the disks D, h_{rad} increases drastically.

13-36 Two coaxial parallel disks of equal diameter 1 m are originally placed at a distance of 1 m apart. The new distance between the disks such that there is a 75% reduction in radiation heat transfer rate from the original distance of 1 m is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis For coaxial parallel disks, from Table 13-1, with i = 1, j = 2,

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_2}{D_1}\right)^2\right]^{1/2}\right\} = \frac{1}{2}[S - (S^2 - 4)^{1/2}] \qquad (1)$$



$$S = 1 + \frac{1+R_2^2}{R_1^2} = 2 + \frac{1}{R^2} = 2 + 4(L/D)^2 \qquad (2)$$

where $R_1 = R_2 = R = \dfrac{D}{2L}$

Substituting Eq. (2) into Eq. (1), we get

$$F_{12} = \frac{1}{2}(2 + 4(L/D)^2 - \{[2 + 4(L/D)^2]^2 - 4\}^{1/2})$$

$$= \frac{1}{2}\{2 + 4(L/D)^2 - [16(L/D)^4 + 16(L/D)^2]^{1/2}\}$$

$$= \frac{1}{2}\{2 + 4(L/D)^2 - 4(L/D)[(L/D)^2 + 1]^{1/2}\}$$

$$F_{12} = 1 + 2(L/D)^2 - 2(L/D)[(L/D)^2 + 1]^{1/2}$$

Hence, for D = 1 m we have

$$F_{12} = 1 + 2L^2 - 2L[L^2 + 1]^{1/2} \qquad (3)$$

From Eq. (3), the view factor at the original distance L = 1 m (with D = l m) is

$$F_{12,\text{old}} = 1 + 2(1)^2 - 2(1)[(1)^2 + 1]^{1/2} = 3 - 2\sqrt{2} = 0.1716$$

The rate of radiation heat transfer between the two surfaces is

$$\dot{Q}_{12} = AF_{12}\sigma(T_1^4 - T_2^4)$$

The percentage of reduction in radiation heat transfer rate can be expressed as

$$\%\,\text{Change} = \frac{\dot{Q}_{12,\text{old}} - \dot{Q}_{12,\text{new}}}{\dot{Q}_{12,\text{old}}} = \frac{F_{12,\text{old}} - F_{12,\text{new}}}{F_{12,\text{old}}}$$

For the two surfaces to experience 75% reduction in radiation heat transfer rate, the new view factor should be

$$F_{12,\text{new}} = 0.25F_{12,\text{old}} = 0.0429$$

Then, substituting the value of $F_{12,\text{new}}$ into Eq. (3), the new distance L_{new} such that the two surfaces experience 75% reduction in radiation heat transfer rate can be calculated as

$$F_{12,\text{new}} = 1 + 2L_{\text{new}}^2 - 2L_{\text{new}}[L_{\text{new}}^2 + 1]^{1/2}$$

$$0.0429 = 1 + 2L_{\text{new}}^2 - 2L_{\text{new}}[L_{\text{new}}^2 + 1]^{1/2}$$

Hence, $L_{\text{new}} = \mathbf{2.31\,m}$

Discussion Increasing the distance between the disks would decrease the view factor F_{12}, thereby reducing the radiation heat transfer rate $\dot{Q}_{12}$.

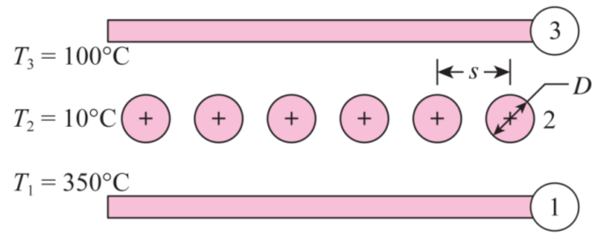
13-37 A row of tubes is spaced between two large parallel plates. The net radiation heat flux leaving the bottom plate is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis From energy balance, the net radiation heat flux leaving the bottom plate (surface 1) is

$$\dot{q}_1 = \dot{q}_{12} + \dot{q}_{13} = F_{12}\sigma(T_1^4 - T_2^4) + F_{13}\sigma(T_1^4 - T_3^4)$$

With $F_{13} = 1 - F_{12}$, we have



$$\dot{q}_1 = F_{12}\sigma(T_1^4 - T_2^4) + (1 - F_{12})\sigma(T_1^4 - T_3^4)$$

The view factor between the bottom plate and the row of tubes (s = 2D), from Table 13-2, is

$$F_{12} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{0.5} + \left(\frac{D}{s}\right)\left\{\tan^{-1}\left[\left(\frac{s}{D}\right)^2 - 1\right]^{0.5}\right\}$$

$$= 1 - (1 - 0.5^2)^{0.5} + (0.5)[\tan^{-1}(2^2 - 1)^{0.5}]$$

$$= 0.6576$$

Thus,

$$\dot{q}_1 = \sigma[F_{12}(T_1^4 - T_2^4) + (1 - F_{12})(T_1^4 - T_3^4)]$$

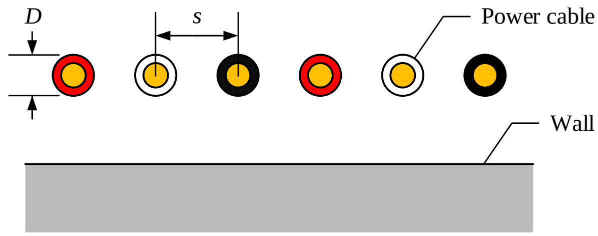
$$= (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)[(0.6576)(623^4 - 283^4)\,\text{K}^4 + (1 - 0.6576)(623^4 - 373^4)\,\text{K}^4]$$

$$= \mathbf{7927\ W/m^2}$$

Discussion The view factor between the bottom plate and the circular tubes F_{12} is independent of the distance between them.

13-38 C&S A row of long cylindrical power cables, shielded with PVC insulation, are placed in parallel with a large wall at high temperature. The surface temperature of the power cables is to be determined whether it is below the operation temperature specified by the ASTM standard.

Assumptions **1** Steady state conditions. **2** Uniform surface temperatures. **3** Surfaces behaves as blackbody.



Analysis The view factor from the wall to the row of power cables is

$$F_{12} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{1/2} + \frac{D}{s}\tan^{-1}\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}$$

$$F_{12} = 1 - \left[1 - \left(\frac{1}{2}\right)^2\right]^{1/2} + \frac{1}{2}\tan^{-1}\left(\frac{2^2 - 1^2}{1^2}\right)^{1/2} = 0.6576$$

The radiation heat transfer per unit area from the wall to the power cables is

$$\dot{q}_{12} = F_{12}\sigma(T_1^4 - T_2^4)$$

Solving for the surface temperature of the power cables yields

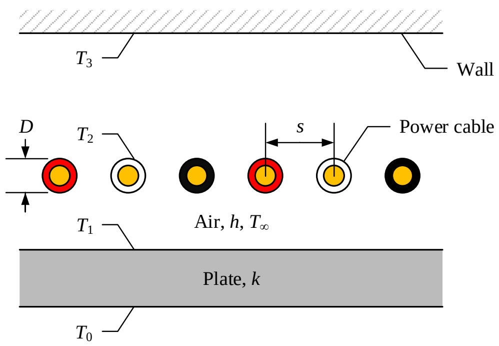
$$T_2 = \left(T_1^4 - \frac{\dot{q}_{12}}{F_{12}\sigma}\right)^{1/4} = \left[(373\text{ K})^4 - \frac{200\text{ W/m}^2}{(0.6576)(5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4)}\right]^{1/4} = 344\text{ K} = \mathbf{71}℃ > 60℃$$

Discussion With the given conditions, the surfaces of the power cables (PVC surface) would be 71°C. This is 11°C higher than the 60°C operation temperature limit specified by the ASTM D2219-11 standard. Therefore, the PVC insulation for the power cables would not comply with the ASTM standard.

13-39 A row of long cylindrical power cables, shielded with polyethylene insulation, are placed in parallel with a large plate. Multimode heat transfer involving conduction, convection, and radiation occur. The surface temperature of the power cables is to be determined whether it is below the operation temperature specified by the ASTM standard.

Assumptions **1** Steady state conditions. **2** Uniform surface temperatures. **3** Surfaces behaves as blackbody. **4** One-dimensional conduction through the plate.

Properties Plate thermal conductivity is given as k = 105 W/m·K.



Analysis The view factor from the plate's upper surface to the row of power cables is

$$F_{12} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{1/2} + \frac{D}{s}\tan^{-1}\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}$$

$$F_{12} = 1 - \left[1 - \left(\frac{1}{2}\right)^2\right]^{1/2} + \frac{1}{2}\tan^{-1}\left(\frac{2^2 - 1^2}{1^2}\right)^{1/2} = 0.6576$$

The view factors F_{12} and F_{13} are related from the summation rule as

$$F_{12} + F_{13} = 1 \qquad \text{or} F_{13} = 1 - F_{12} = 0.3424$$

Applying energy balance on the plate,

$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} + \dot{q}_{\text{rad}}$$

where

$$\dot{q}_{\text{cond}} = k\frac{T_0 - T_1}{L}$$

$$\dot{q}_{\text{conv}} = h(T_1 - T_\infty)$$

$$\dot{q}_{\text{rad}} = F_{12}\sigma(T_1^4 - T_2^4) + F_{13}\sigma(T_1^4 - T_3^4)$$

Solving for the surface temperature of the power cables T_2,

$$k\frac{T_0 - T_1}{L} = h(T_1 - T_\infty) + F_{12}\sigma(T_1^4 - T_2^4) + F_{13}\sigma(T_1^4 - T_3^4)$$

Thus,

$$T_2 = \left\{T_1^4 - \frac{1}{F_{12}\sigma}\left[k\frac{T_0 - T_1}{L} - h(T_1 - T_\infty) - F_{13}\sigma(T_1^4 - T_3^4)\right]\right\}^{1/4}$$

$$T_2 = \left\{(480\text{ K})^4 - \frac{1}{(0.6576)(5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4)}\left[\left(15\frac{\text{W}}{\text{m}}\cdot\text{K}\right)\left(\frac{227-207}{0.05\text{ m}}\right) - \left(20\frac{\text{W}}{\text{m}^2}\cdot\text{K}\right)(207-20)\text{K} - (0.3424)\left(5.67\times10^{-8}\frac{\text{W}}{\text{m}^2}\cdot\text{K}^4\right)(480^4 - 293^4)\text{K}^4\right]\right\}^{1/4}$$

$$T_2 = 357\text{ K} = \mathbf{84}℃ > 75℃$$

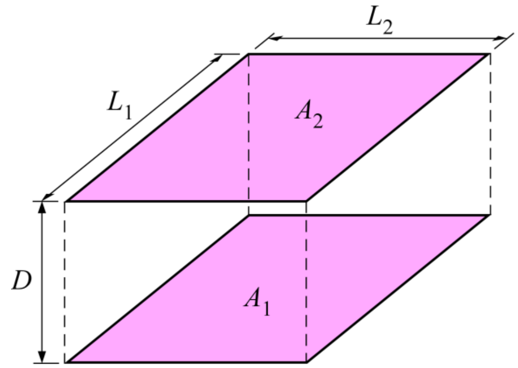
Discussion With the given conditions, the surfaces of the power cables (polyethylene surface) would be 84°C. This is 9°C higher than the 75°C operation temperature limit specified by the ASTM D1351-14 standard. Therefore, the polyethylene insulation for the power cables would not comply with the ASTM standard.

13-40E Two black parallel rectangles are spaced apart by a distance of 1 ft, the temperature of the bottom rectangle is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis For D = 1 ft, L_1 = 3 ft and L_2 = 5 ft, we have

$$\frac{L_1}{D} = \frac{3}{1} = 3 \quad \text{and} \quad \frac{L_2}{D} = \frac{5}{1} = 5$$



From Fig. 13-5, we get

$$F_{12} \approx 0.60$$

The rate of radiation heat transfer between the two rectangles is

$$\dot{Q}_{12} = AF_{12}\sigma(T_1^4 - T_2^4) = L_1 L_2 F_{12}\sigma(T_1^4 - T_2^4)$$

Hence

$$T_1 = \left[\frac{\dot{Q}_{12}}{L_1 L_2 F_{12}\sigma} + T_2^4\right]^{1/4}$$

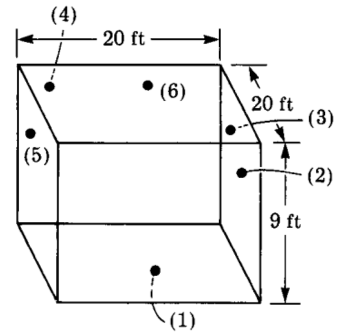
$$= \left[\frac{180000 \text{ Btu/h}}{(3 \text{ ft})(5 \text{ ft})(0.60)(0.1714\times 10^{-8} \text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)} + (60+460)^4 \text{ R}^4\right]^{1/4}$$

$$T_1 = \mathbf{1851\ R}$$

Discussion If T_1 and T_2 are constant, increasing the distance between the two rectangles will decrease the view factor F_{12}, thereby decreasing the radiation heat transfer rate received by the top rectangle.

13-41E For a square room with specified dimensions and floor, wall and ceiling temperatures, determine the net radiation heat transfer (a) from floor to walls and (b) from floor to ceiling.

***Assumptions* 1** All surfaces are assumed black.



Analysis

(a) $\dot{Q}_{1\to 2,3,4,5} = 4A_1 F_{12}\,\sigma\left(T_1^4 - T_2^4\right)$

Since for a square

$$F_{12} = F_{13} = F_{14} = F_{15}$$

From Fig. 13-6 or Table 13-1 with

$$L_1 / W = 20/20 = 1$$

and

$$L_2 / W = 9/20 \approx 0.45 \quad \to \quad F_{12} \approx 0.14$$

$$\dot{Q}_{1\to 2,3,4,5} = 4\,(20 \times 20)\text{ ft}^2\,(0.14)\,(0.1714\times 10^{-8}\text{ Btu/h·ft·R}^4) \times [(560)^4 - (520)^4]\text{R}^4$$

$\dot{Q}_{1\to 2,3,4,5}$ = **9686 Btu/h** (Floor to four walls)

(b) $\dot{Q}_{1\to 6} = A_1 F_{16}\,\sigma\left(T_1^4 - T_6^4\right)$

From Fig. 13-5 with

$$\frac{L_1}{D} = \frac{20}{9} = 2.23$$

and

$$\frac{L_2}{D} = \frac{20}{9} = 2.23 \quad \to \quad F_{16} \approx 0.45$$

$$\dot{Q}_{1\to 6} = (20 \times 20)\text{ ft}^2\,(0.45)\,(0.1714\times 10^{-8}\text{ Btu/h·ft·R}^4) \times [(560)^4 - (500)^4]\text{R}^4$$

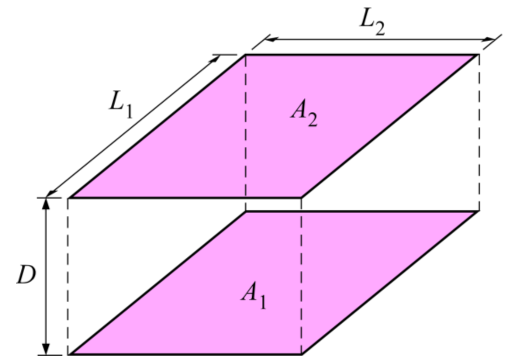
$\dot{Q}_{1\to 6}$ = **11,059 Btu/h** (Floor to ceiling)

13-42 Two aligned parallel rectangles are apart by a distance of 2 m. The percentage of change in radiation heat transfer rate when the rectangles are moved apart from 2 m to 8 m is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis For D = 2 m, L_1 = 6 m and L_2 = 8 m, we have

$$\frac{L_1}{D} = \frac{6}{2} = 3 \quad \text{and} \quad \frac{L_2}{D} = \frac{8}{2} = 4$$



From Fig. 13-5, we get

$$F_{12} \approx 0.58 \qquad \text{(for } D = 2\text{m)}$$

For D = 8 m, L_1 = 6 m and L_2 = 8 m, we have

$$\frac{L_1}{D} = \frac{6}{8} = 0.75 \quad \text{and} \quad \frac{L_2}{D} = \frac{8}{8} = 1$$

From Fig. 13-5, we get

$$F_{12} \approx 0.165 \qquad \text{(for } D = 8 \text{ m)}$$

The rate of radiation heat transfer between the two rectangles is

$$\dot{Q}_{12} = AF_{12}\sigma(T_1^4 - T_2^4)$$

Hence, the percentage of change in radiation heat transfer rate is

$$\begin{aligned} \%\,\text{Change} &= \frac{\dot{Q}_{12}(D = 2\text{ m}) - \dot{Q}_{12}(D = 8\text{ m})}{\dot{Q}_{12}(D = 2\text{ m})} = \frac{F_{12}(D = 2\text{ m}) - F_{12}(D = 8\text{ m})}{F_{12}(D = 2\text{ m})} \\ &= \frac{0.58 - 0.165}{0.58} \\ &= \mathbf{0.716\ (or\ 71.6\%)} \end{aligned}$$

Discussion By moving the distance between the two parallel rectangles from 2 m to 8 m, there is about 72% reduction in radiation heat transfer rate.

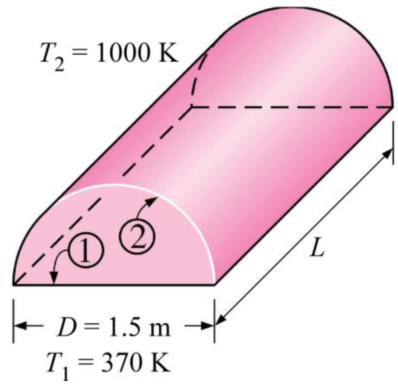
13-43 The base and the dome of a long semi-cylindrical dryer are maintained at uniform temperatures. The drying rate per unit length experienced by the base surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered. **4** The dryer is well insulated from heat loss to the surrounding.

Properties The latent heat of vaporization for water is h_{fg} = 2257 kJ/kg (Table A-2)

Analysis The view factor from the dome to the base is determined from

$$F_{11} + F_{12} = 1 \qquad \rightarrow \qquad F_{12} = 1 \qquad \text{(where } F_{11} = 0\text{)}$$



Hence, from reciprocity relation, we get

$$F_{21} = \frac{A_1}{A_2}F_{12} = \frac{DL}{\pi DL/2} = \frac{2}{\pi}$$

Applying energy balance on the base surface, we have

$$\dot{Q}_{21} = \dot{Q}_{\text{evap}} = \dot{m}h_{fg}$$

Hence,

$$\dot{m}h_{fg} = A_2 F_{21}\sigma(T_2^4 - T_1^4) = \frac{\pi DL}{2}F_{21}\sigma(T_2^4 - T_1^4)$$

or

$$\frac{\dot{m}}{L} = \frac{\pi D}{2h_{fg}}F_{21}\sigma(T_2^4 - T_1^4) = \frac{\pi D}{2h_{fg}}\frac{2}{\pi}\sigma(T_2^4 - T_1^4)$$

$$= \frac{D}{h_{fg}}\sigma(T_2^4 - T_1^4)$$

$$= \frac{(1.5\text{ m})}{(2257\times10^3\text{ J/kg})}(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(1000^4 - 370^4)\text{ K}^4$$

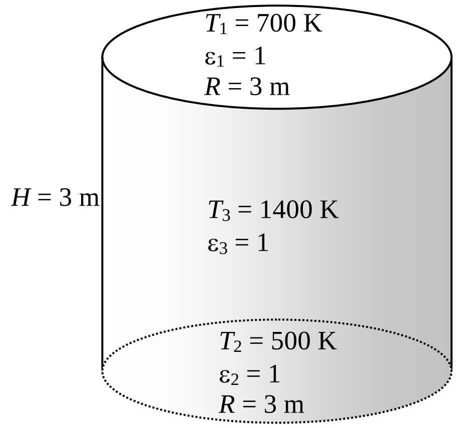
$$= \mathbf{0.0370\ kg/s\cdot m}$$

Discussion The view factor from the dome to the base is constant $F_{21} = 2/\pi$, which implies that the view factor is independent of the dryer dimensions.

13-44 The base, top, and side surfaces of a furnace of cylindrical shape are black, and are maintained at uniform temperatures. The net rate of radiation heat transfer to or from the top surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.

Properties The emissivity of all surfaces are ε = 1 since they are black.



Analysis We consider the top surface to be surface 1, the base surface to be surface 2 and the side surfaces to be surface 3. The cylindrical furnace can be considered to be three-surface enclosure. We assume that steady-state conditions exist. Since all surfaces are black, the radiosities are equal to the emissive power of surfaces, and the net rate of radiation heat transfer from the top surface can be determined from

$$\dot{Q} = A_1 F_{12}\sigma(T_1^{\ 4} - T_2^{\ 4}) + A_1 F_{13}\sigma(T_1^{\ 4} - T_3^{\ 4})$$

and $\quad A_1 = \pi R^2 = \pi(3\,\text{m})^2 = 28.27\,\text{m}^2$

The view factor from the base to the top surface of the cylinder is $F_{12} = 0.38$ (From Figure 13-7). The view factor from the base to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.38 = 0.62$$

Substituting,

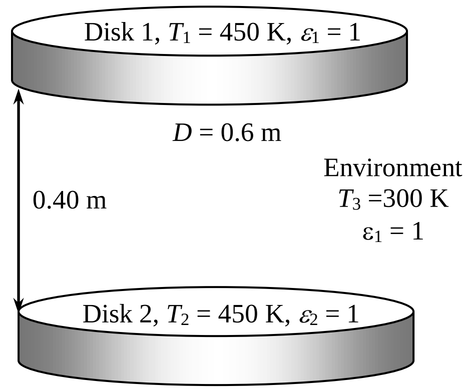
$$\begin{aligned}
\dot{Q} &= A_1 F_{12}\sigma(T_1^{\ 4} - T_2^{\ 4}) + A_1 F_{13}\sigma(T_1^{\ 4} - T_3^{\ 4}) \\
&= (28.27\,\text{m}^2)(0.38)(5.67\times 10^{-8}\ \text{W/m}^2.\text{K}^4)(700\,\text{K}^4 - 500\,\text{K}^4) \\
&\quad + (28.27\,\text{m}^2)(0.62)(5.67\times 10^{-8}\ \text{W/m}^2.\text{K}^4)(700\,\text{K}^4 - 1400\,\text{K}^4) \\
&= 1.0818\times 10^5\ \text{W} - 3.579\times 10^6\ \text{W} = -3.471\times 10^6\ \text{W} = \mathbf{-3471\,kW}
\end{aligned}$$

Discussion The negative sign indicates that net heat transfer is to the top surface.

13-45 Two parallel disks whose back sides are insulated are black, and are maintained at a uniform temperature. The net rate of radiation heat transfer from the disks to the environment is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of all surfaces are ε = 1 since they are black.



Analysis Both disks possess same properties and they are black. Noting that environment can also be considered to be blackbody, we can treat this geometry as a three surface enclosure. We consider the two disks to be surfaces 1 and 2 and the environment to be surface 3. Then from Figure 13-7, we read

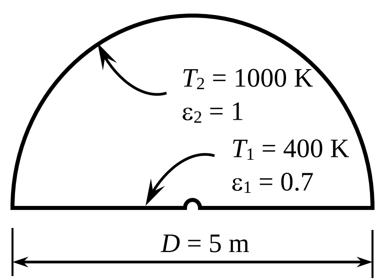
$$F_{12} = F_{21} = 0.26$$
$$F_{13} = 1 - 0.26 = 0.74 \quad \text{(summation rule)}$$

The net rate of radiation heat transfer from the disks into the environment then becomes

$$\dot{Q}_3 = \dot{Q}_{13} + \dot{Q}_{23} = 2\dot{Q}_{13}$$
$$\dot{Q}_3 = 2F_{13}A_1\sigma(T_1^{\,4} - T_3^{\,4})$$
$$= 2(0.74)[\pi(0.3\,\text{m})^2](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(450\,\text{K})^4 - (300\,\text{K})^4]$$
$$= \mathbf{781\,W}$$

13-46 The base and the dome of a hemispherical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer from the dome to the base surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Analysis The view factor is first determined from

$$F_{11} = 0 \quad \text{(flat surface)}$$
$$F_{11} + F_{12} = 1 \rightarrow F_{12} = 1 \quad \text{(summation rule)}$$

Noting that the dome is black, net rate of radiation heat transfer from dome to the base surface can be determined from

$$\dot{Q}_{21} = -\dot{Q}_{12} = -\varepsilon A_1 F_{12}\sigma(T_1^{\,4} - T_2^{\,4})$$
$$= -(0.7)[\pi(5\,\text{m})^2/4](1)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(400\,\text{K})^4 - (1000\,\text{K})^4]$$
$$= 7.594\times10^5\ \text{W}$$
$$= \mathbf{759\,kW}$$

The positive sign indicates that the net heat transfer is from the dome to the base surface, as expected.

13-47 The base and the dome of a long semi-cylindrical dryer are maintained at uniform temperatures. The length of the dryer necessary to dry the materials at 0.1 kg/s is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered. **4** The dryer is well insulated from heat loss to the surrounding.

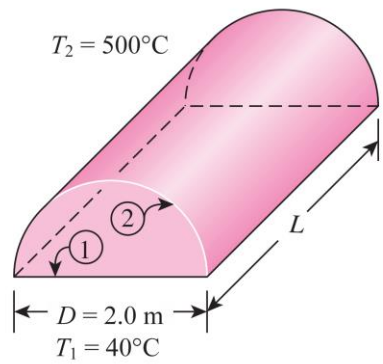
Properties The latent heat of vaporization for water at 40°C is h_{fg} = 2407 kJ/kg (Table A-9)

Analysis The view factor from the dome to the base is determined from

$$F_{11} + F_{12} = 1 \quad \rightarrow \quad F_{12} = 1 \quad (\text{with } F_{11} = 0)$$

Hence, from reciprocity relation, we get

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{DL}{\pi DL/2} = \frac{2}{\pi}$$



Applying energy balance on the base (surface 1), we have

$$\dot{Q}_{21} = \dot{Q}_{\text{evap}} = \dot{m}h_{fg}$$

Hence,

$$\dot{m}h_{fg} = A_2 F_{21}\sigma(T_2^4 - T_1^4) = \frac{\pi DL}{2}\frac{2}{\pi}\sigma(T_2^4 - T_1^4)$$

which gives

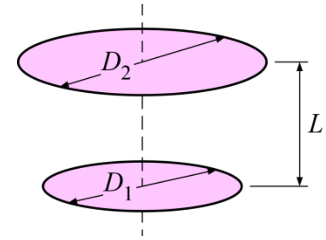
$$L = \frac{\dot{m}h_{fg}}{D\sigma(T_2^4 - T_1^4)}$$

$$= \frac{(0.1\,\text{kg/s})(2407\times 10^3\ \text{J/kg})}{(2\,\text{m})(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(773^4 - 313^4)\,\text{K}^4} = \mathbf{6.11\,m}$$

Discussion The view factor from the dome to the base is constant $F_{21} = 2/\pi$, which implies that the view factor is independent of the dryer dimensions.

13-48 Two parallel black disks are positioned coaxially, where the lower disk is heated electrically. The temperature of the upper disk is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered.



Analysis For coaxial parallel disks, from Table 13-1, with i = 1, j = 2,

$$R_1 = \frac{D_1/2}{L} = \frac{0.2/2}{0.25} = 0.4 \quad \text{and} \quad R_2 = \frac{D_2/2}{L} = \frac{0.4/2}{0.25} = 0.8$$

$$S = 1 + \frac{1+R_2^2}{R_1^2} = 1 + \frac{1+(0.8)^2}{(0.4)^2} = 11.25$$

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_2}{D_1}\right)^2\right]^{1/2}\right\} = \frac{1}{2}\left\{11.25 - \left[(11.25)^2 - 4\left(\frac{0.4}{0.2}\right)^2\right]^{1/2}\right\} = 0.3676$$

Then, using the summation rule,

$$F_{12} + F_{1\text{surr}} = 1 \quad \rightarrow \quad F_{1\text{surr}} = 1 - F_{12} = 0.6324$$

The net radiation heat transfer rate leaving the lower surface can be expressed as

$$\dot{Q}_{\text{elec}} = \dot{Q}_{12} + \dot{Q}_{1\text{surr}} = A_1 F_{12}\sigma(T_1^4 - T_2^4) + A_1 F_{1\text{surr}}\sigma(T_1^4 - T_{\text{surr}}^4)$$

$$\dot{Q}_{\text{elec}} = A_1\sigma[F_{12}(T_1^4 - T_2^4) + F_{1\text{surr}}(T_1^4 - T_{\text{surr}}^4)]$$

Hence

$$T_2 = \left[T_1^4 - \frac{\dot{Q}_{\text{elec}}}{A_1 F_{12}\sigma} + \frac{F_{1\text{surr}}}{F_{12}}(T_1^4 - T_{\text{surr}}^4)\right]^{1/4} = \left[T_1^4 - \frac{4\dot{Q}_{\text{elec}}}{\pi D_1^2 F_{12}\sigma} + \frac{F_{1\text{surr}}}{F_{12}}(T_1^4 - T_{\text{surr}}^4)\right]^{1/4}$$

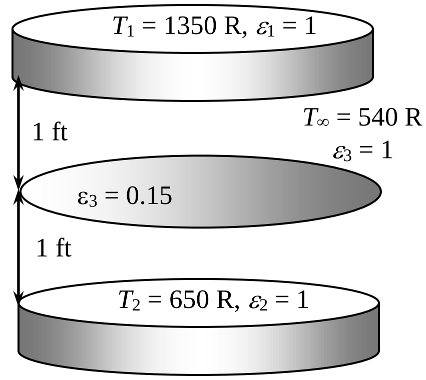
$$T_2 = \left[(500\text{ K})^4 - \frac{4(100\text{ W})}{\pi(0.2\text{ m})^2(0.3676)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)} + \frac{0.6324}{0.3676}(500^4 - 300^4)\text{ K}^4\right]^{1/4}$$

$$T_2 = \mathbf{241\ K}$$

Discussion The view factor F_{12} can also be determined using Fig. 13-7 to be

$$F_{12} \approx 0.36 \quad \text{with} \quad L/r_1 = 2.5 \quad \text{and} \quad r_2/L = 0.8$$

13-49E A radiation shield is placed between two parallel disks which are maintained at uniform temperatures. The net rate of radiation heat transfer through the shields is to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 1$ and $\varepsilon_3 = 0.15$.

Analysis From Fig. 13-7 we have $F_{32} = F_{13} = 0.52$. Then $F_{34} = 1 - 0.52 = 0.48$. The disk in the middle is surrounded by black surfaces on both sides. Therefore, heat transfer between the top surface of the middle disk and its black surroundings can expressed as

$$\dot{Q}_3 = \varepsilon A_3 \sigma [F_{31}(T_3^4 - T_1^4)] + \varepsilon A_3 \sigma [F_{32}(T_3^4 - T_2^4)]$$
$$= 0.15(7.069\text{ ft}^2)(0.1714\times10^{-8}\text{ Btu/h.ft}^2\cdot\text{R}^4)\{0.52[(T_3^4 - (1350\text{ R})^4] + 0.48[T_3^4 - (540\text{ R})^4]\}$$

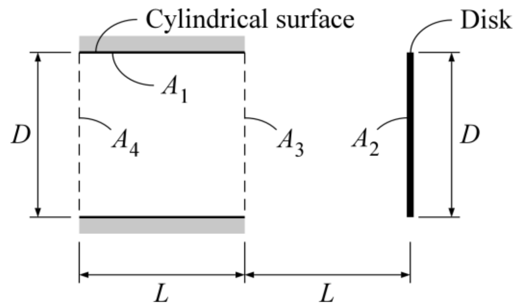
where $A_3 = \pi(3\text{ ft})^2/4 = 7.069\text{ ft}^2$. Similarly, for the bottom surface of the middle disk, we have

$$-\dot{Q}_3 = \varepsilon A_3 \sigma [F_{32}(T_2^4 - T_4^4)] + \varepsilon A_3 \sigma [F_{34}(T_3^4 - T_4^4)]$$
$$= 0.15(7.069\text{ ft}^2)(0.1714\times10^{-8}\text{ Btu/h.ft}^2\cdot\text{R}^4)\{0.48[(T_3^4 - (650\text{ R})^4] + 0.52[T_3^4 - (540\text{ R})^4]\}$$

Combining the equations above, the rate of heat transfer between the disks through the radiation shield (the middle disk) is determined to be

$\dot{Q} = \mathbf{1490\ Btu/h}$ and T_3 = 987 R

13-50 A hot cylindrical surface is placed coaxially with a disk at a distance L apart. The radiation heat transfer rate from the cylindrical surface to the disk is to be determined.



Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered. **4** Outer surface of the cylinder is well insulated.

Analysis The end surfaces A_3 and A_4 are treated as hypothetical surfaces. Applying the summation rule, we have

$$F_{23} = F_{21} + F_{24} \quad \rightarrow \quad F_{21} = F_{23} - F_{24} \qquad (1)$$

From reciprocity relation, we have

$$A_2 F_{21} = A_1 F_{12} \qquad (2)$$

Substituting Eq. (2) in to Eq. (1), we get

$$(A_1 / A_2) F_{12} = F_{23} - F_{24} \quad \rightarrow \quad F_{12} = (A_2 / A_1)(F_{23} - F_{24}) \qquad (3)$$

The view factors F_{23} and F_{24} can be determined by treating them as view factors for coaxial parallel disks using Table 13-1:

With $\quad R_2 = \frac{r_2}{L} = \frac{D/2}{L} = 0.5 \quad$ and $\quad R_3 = R_2 = \frac{r_3}{L} = \frac{D/2}{L} = 0.5$

We get

$$S = 1 + \frac{1+R_3^2}{R_2^2} = 1 + \frac{1+(0.5)^2}{(0.5)^2} = 6$$

$$F_{23} = \frac{1}{2}\left\{ S - \left[S^2 - 4\left(\frac{D_3}{D_2}\right)^2 \right]^{1/2} \right\} = \frac{1}{2}[6 - (6^2 - 4)^{1/2}] = 0.1716$$

With $\quad R_2 = \frac{D/2}{2L} = 0.25 \quad$ and $\quad R_2 = R_4 = 0.25$

We get

$$S = 1 + \frac{1+R_4^2}{R_2^2} = 1 + \frac{1+(0.25)^2}{(0.25)^2} = 18$$

$$F_{24} = \frac{1}{2}\left\{ S - \left[S^2 - 4\left(\frac{D_4}{D_2}\right)^2 \right]^{1/2} \right\} = \frac{1}{2}[18 - (18^2 - 4)^{1/2}] = 0.05573$$

Substituting the values of F_{23} and F_{24} into Eq. (3), we have

$$F_{12} = (A_2 / A_1)(F_{23} - F_{24}) = (A_2 / A_1)(0.1716 - 0.05573) = 0.1159(A_2 / A_1)$$

The rate of heat transfer by radiation is then

$$\begin{aligned}
\dot{Q}_{12} &= A_1 F_{12} \sigma (T_1^4 - T_2^4) \\
&= 0.1159 A_1 (A_2 / A_1) \sigma (T_1^4 - T_2^4) \\
&= 0.1159 A_2 \sigma (T_1^4 - T_2^4) \\
&= 0.1159 (\pi D^2 / 4) \sigma (T_1^4 - T_2^4) \\
&= 0.1159 \frac{\pi (0.2\ \text{m})^2}{4} (5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)(1000^4 - 300^4)\ \text{K}^4 \\
&= \mathbf{205\ W}
\end{aligned}$$

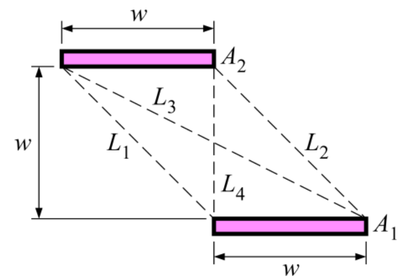
Discussion The view factors F_{23} and F_{24} can also be determined using Fig. 13-7.

13-51 The radiation heat flux between two infinitely long parallel plates of specified surface temperatures is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered. **4** The surface temperatures are uniform.

Analysis From the Hottel's crossed-strings method, we have

$$F_{i\to j} = \frac{\Sigma\ (\text{Crossed strings}) - \Sigma\ (\text{Uncrossed strings})}{2\times\ (\text{Length of surface } i)}$$



For uncrossed strings, we have

$$L_1 = L_2 = (w^2 + w^2)^{1/2} = (w^2 + w^2)^{1/2} = \sqrt{2}w$$

For crossed strings, we have

$$L_3 = (w^2 + 4w^2)^{1/2} = \sqrt{5}w \qquad \text{and} \qquad L_4 = w$$

Applying the Hottel's crossed-strings method, we get F_{12} as

$$\begin{aligned} F_{12} &= \frac{(L_3 + L_4) - (L_1 + L_2)}{2w} \\ &= \frac{(\sqrt{5}w + w) - (\sqrt{2}w + \sqrt{2}w)}{2w} \\ &= 0.204 \end{aligned}$$

The radiation heat flux between the two surfaces is

$$\begin{aligned} \dot{q}_{12} &= F_{12}\sigma(T_1^4 - T_2^4) \\ &= (0.204)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(700^4 - 300^4)\ \text{K}^4 \\ &= \mathbf{2680\ W/m^2} \end{aligned}$$

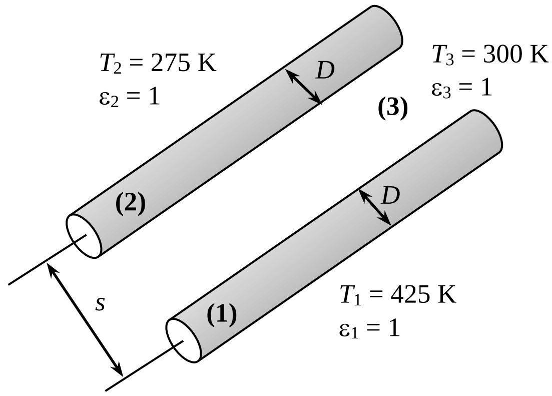
Discussion The Hottel's crossed-string method is applicable only to surfaces that are very long, such that they can be considered to be two-dimensional and radiation interaction through the end surfaces is negligible.

13-52 Two long parallel cylinders are maintained at specified temperatures. The rates of radiation heat transfer between the cylinders and between the hot cylinder and the surroundings are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are black. **3** Convection heat transfer is not considered.

Analysis We consider the hot cylinder to be surface 1, cold cylinder to be surface 2, and the surroundings to be surface 3. Using the crossed-strings method, the view factor between two cylinders facing each other is determined to be

$$F_{1-2} = \frac{\sum \text{Crossed strings} - \sum \text{Uncrossed strings}}{2 \times \text{Length of surface } i} = \frac{2\sqrt{s^2 + D^2} - 2s}{2(\pi D/2)}$$



or

$$F_{1-2} = \frac{2\left(\sqrt{s^2 + D^2} - s\right)}{\pi D} = \frac{2\left(\sqrt{0.3^2 + 0.20^2} - 0.5\right)}{\pi(0.20)} = 0.444$$

The view factor between the hot cylinder and the surroundings is

$$F_{13} = 1 - F_{12} = 1 - 0.444 = 0.556 \text{ (summation rule)}$$

The rate of radiation heat transfer between the cylinders per meter length is

$$A = \pi DL/2 = \pi(0.20\text{ m})(1\text{ m})/2 = 0.3142\text{ m}^2$$

$$\dot{Q}_{12} = AF_{12}\sigma(T_1^{\,4} - T_2^{\,4})$$
$$= (0.3142\text{ m}^2)(0.444)(5.67\times10^{-8}\text{ W/m}^2.°\text{C})(425^4 - 275^4)\text{K}^4$$
$$= \mathbf{212.8\ W}$$

Note that half of the surface area of the cylinder is used, which is the only area that faces the other cylinder. The rate of radiation heat transfer between the hot cylinder and the surroundings per meter length of the cylinder is

$$A_1 = \pi DL = \pi(0.20\text{ m})(1\text{ m}) = 0.6283\text{ m}^2$$

$$\dot{Q}_{13} = A_1 F_{13}\sigma(T_1^{\,4} - T_3^{\,4})$$
$$= (0.6283\text{ m}^2)(0.556)(5.67\times10^{-8}\text{ W/m}^2.°\text{C})(425^4 - 300^4)\text{K}^4$$
$$= \mathbf{485.8\ W}$$

13-53 C & S A long cylindrical power cable is shielded with polyethylene insulation is placed in parallel with a long cylindrical metal rod at a known temperature. The surface temperature of the polyethylene insulation is to be determined whether it is below the operation temperature specified by the ASTM standard.

Assumptions **1** Steady state conditions. **2** Uniform surface temperatures. **3** Surfaces are opaque, gray and diffuse. **4** Kirchhoff's law is applicable. **5** The surrounding is a large blackbody enclosure

Properties The emissivities of the metal rod and the polyethylene insulation are given as 0.33 and 0.95, respectively.

AnalysisWith the metal rod as surface 1, the power cable as surface 2, and the surrounding as surface 3. The view factor from the metal rod to the power cable is

$$F_{12} = \frac{\sum(\text{Crossed strings}) - \sum(\text{Uncrossed strings})}{2\times(\text{Length of surface 1})}$$

$$F_{12} = \frac{2\sqrt{s^2+D^2}-2s}{2(\pi D)} = \frac{\sqrt{(0.2\text{ m})^2+(0.01\text{ m})^2}-0.2\text{ m}}{\pi\ (0.01\text{ m})} = 0.007953$$

where, D is the outer dimeter of the cylinders, and s is the distance between the two cylinders. From the summation rule, the view factor F_{13} is

$$F_{13} = 1 - F_{12} = 0.99205$$

The radiation heat transfer per unit area from the metal rod can be expressed as

$$\dot{q}_1 = F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)(1)$$

and

$$\sigma T_1^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}[F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)](2)$$

The radiosity of the surrounding (a blackbody) is

$$J_3 = \sigma T_3^4 = (5.67\times10^{-8})(300)^4 = 459.27\ \text{W/m}^2$$

Solving equations (1) and (2) simultaneously for J_1 and J_2,

$$445\ \text{W/m}^2 = (0.007953)(J_1 - J_2) + (0.99205)(J_1 - 459.27\ \text{W/m}^2)$$

$$(5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4)(423\text{ K})^4 = J_1 + \frac{1-0.33}{0.33}[(0.007953)(J_1 - J_2) + 0.99205(J_1 - 459.27\ \text{W/m}^2)]$$

yields

$$J_1 = 911.92\ \text{W/m}^2 \text{and} J_2 = 1417.2\ \text{W/m}^2$$

The surface temperature of the power cable T_2 can be determined from

$$\sigma T_2^4 = J_2 + \frac{1-\varepsilon_2}{\varepsilon_2}[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)]$$

Using the reciprocity rule,

$$A_1F_{12} = A_2F_{21} \qquad \Longrightarrow \qquad F_{12} = F_{21} = 0.007953 \qquad \text{and} F_{23} = 1 - F_{21} = 0.99205$$

Thus,

$$T_2 = \left(\frac{1}{\sigma}\right)^{1/4} \left\{J_2 + \frac{1-\varepsilon_2}{\varepsilon_2}\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right]\right\}^{1/4}$$

$$T_2 = \left(\frac{1}{5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K}^4}\right)^{1/4} \left\{1417.2\ \text{W/m}^2 + \frac{1 - 0.95}{0.95}\left[(0.007953)(1417.2 - 911.92)\text{W/m}^2 + (0.99205)(1417.2\ - 459.27)\text{W/m}^2\right]\right\}^{1/4}$$

$$T_2 = 401\ K = \mathbf{128}℃ > 75℃$$

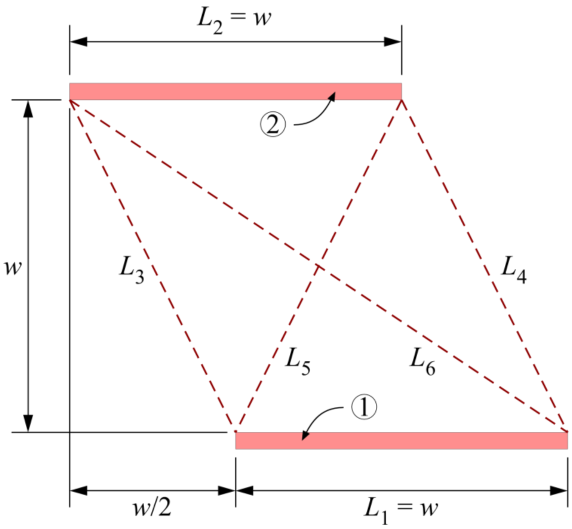
Discussion With the given conditions, the surface of the power cable (polyethylene insulation surface) would be 128°C. This is 53°C higher than the 75°C operation temperature limit specified by the ASTM D1351-14 standard. Therefore, the polyethylene insulation for the power cable would not comply with the ASTM standard.

13-54 The radiation heat flux between two infinitely long parallel plates of specified surface temperatures is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered. **4** The surface temperatures are uniform.

Analysis From the Hottel's crossed-strings method, we have

$$F_{i\to j} = \frac{\sum(\text{Crossed strings}) - \sum(\text{Uncrossed strings})}{2\times(\text{Length of surface } i)}$$



where

$$L_1 = L_2 = w$$

$$L_3 = L_4 = L_5 = \sqrt{w^2 + (w/2)^2} = \frac{\sqrt{5}}{2}w$$

$$L_6 = \sqrt{w^2 + \left(\frac{3}{2}w\right)^2} = \frac{\sqrt{13}}{2}w$$

Applying the Hottel's crossed-strings method, we get F_{12} as

$$F_{12} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2w}$$

$$= \frac{(\sqrt{5}w/2 + \sqrt{13}w/2) - (\sqrt{5}w/2 + \sqrt{5}w/2)}{2w}$$

$$= 0.3424$$

The radiation heat flux between the two surfaces is

$$\dot{q}_{12} = F_{12}\sigma(T_1^4 - T_2^4)$$

$$= (0.3424)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(773^4 - 323^4)\ \text{K}^4$$

$$= \mathbf{6720\ W/m^2}$$

Discussion The Hottel's crossed-string method is applicable only to surfaces that are very long, such that they can be considered to be two-dimensional and radiation interaction through the end surfaces is negligible.

13-55 A large ceramic plate (blackbody) and a large ASTM 992 carbon steel plate are in parallel near each other. The temperature of the ceramic plate and the net radiation heat flux between the two plates are known. The temperature of the steel plate is to be determined whether it is below the maximum use temperature of 427°C.

Assumptions **1** Steady state conditions. **2** Uniform surface temperatures on both plates. **3** Surfaces are opaque, gray and diffuse. **4** Kirchhoff's law is applicable.

Properties The reflectivity of the steel plate is given as $\rho = 0.68$.

Analysis For large parallel plates, the view factor is $F_{12} = 1$. So, the net radiation heat transfer per unit area between the two plates is

$$\dot{q}_{12} = J_1 - J_2$$

The radiosity from the ceramic plate (blackbody) J_1 and the radiosity from the steel plate J_2 are

$$J_1 = \sigma T_1^4$$

$$J_2 = \varepsilon_2 \sigma T_2^4 + \rho_2 J_1$$

The steel plate is opaque ($\tau = 0$), and with Kirchhoff's law ($\alpha = \varepsilon$), yields

$$\alpha_2 + \rho_2 + \tau_2 = 1 \qquad \Rightarrow \qquad \varepsilon_2 = 1 - \rho_2$$

So,

$$J_2 = (1 - \rho_2)\sigma T_2^4 + \rho_2 \sigma T_1^4$$

Thus,

$$\dot{q}_{12} = \sigma T_1^4 - (1 - \rho_2)\sigma T_2^4 - \rho_2 \sigma T_1^4$$

The cooling mechanism removes heat from the steel plate at 2000 W/m^2, and from energy balance we have

$$\dot{q}_{12} = \dot{q}_{\text{remove}} = \sigma T_1^4 - (1 - \rho_2)\sigma T_2^4 - \rho_2 \sigma T_1^4$$

Solving for the temperature of the steel plate T_2,

$$T_2^4 = \left[\frac{\sigma T_1^4 - \rho_2 \sigma T_1^4 - \dot{q}_{\text{remove}}}{(1 - \rho_2)\sigma}\right]^{1/4} = \left[\frac{(1 - \rho_2)\sigma T_1^4 - \dot{q}_{\text{remove}}}{(1 - \rho_2)\sigma}\right]^{1/4}$$

$$T_2 = \left[\frac{(1 - 0.68)\left(5.67 \times 10^{-8}\,\dfrac{\text{W}}{\text{m}^2} \cdot \text{K}^4\right)(973\ \text{K})^4 - 2000\ \text{W/m}^2}{(1 - 0.68)\left(5.67 \times 10^{-8}\,\dfrac{\text{W}}{\text{m}^2} \cdot \text{K}^4\right)}\right]^{1/4} = 942\ \text{K}$$

$$= \mathbf{669}\,^\circ\text{C} > 427^\circ\text{C}$$

To keep the steel plate from heating above 427°C, the cooling mechanism needs to remove at least,

$$\dot{q}_{\text{remove}} = (1 - \rho_2)\sigma(T_1^4 - T_2^4) = (1 - 0.68)(5.67 \times 10^{-8}\text{W/m}^2 \cdot \text{K}^4)(973^4 - 700^4)\text{K}^4 = 11906\ \text{W/m}^2$$

Discussion A cooling mechanism that removes 2000 W/m^2 from the net radiation heat transfer between the two plates is not sufficient to keep the steel plate below 427°C. To keep the steel plate from heating above 427°C, the cooling mechanism needs to remove at least 11906 W/m^2 of heat due to the net radiation heat transfer between the two plates.

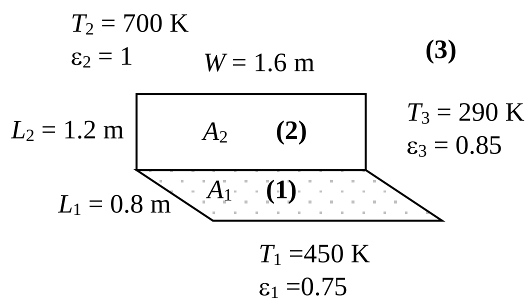
13-56 Two perpendicular rectangular surfaces with a common edge are maintained at specified temperatures. The net rate of radiation heat transfers between the two surfaces and between the horizontal surface and the surroundings are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of the horizontal rectangle and the surroundings are ε = 0.75 and ε = 0.85, respectively.

Analysis We consider the horizontal rectangle to be surface 1, the vertical rectangle to be surface 2 and the surroundings to be surface 3. This system can be considered to be a three-surface enclosure. The view factor from surface 1 to surface 2 is determined from

$$\left.\begin{aligned}\frac{L_1}{W} &= \frac{0.8}{1.6} = 0.5\\ \frac{L_2}{W} &= \frac{1.2}{1.6} = 0.75\end{aligned}\right\} F_{12} = 0.27 \quad \text{(Fig. 13-6)}$$



The surface areas are

$$A_1 = (0.8\text{ m})(1.6\text{ m}) = 1.28\text{ m}^2$$

$$A_2 = (1.2\text{ m})(1.6\text{ m}) = 1.92\text{ m}^2$$

$$A_3 = 2\times\frac{1.2\times0.8}{2} + \sqrt{0.8^2+1.2^2}\times1.6 = 3.268\text{ m}^2$$

Note that the surface area of the surroundings is determined assuming that surroundings forms flat surfaces at all openings to form an enclosure. Then other view factors are determined to be

$$A_1F_{12} = A_2F_{21} \longrightarrow (1.28)(0.27) = (1.92)F_{21} \longrightarrow F_{21} = 0.18 \qquad \text{(reciprocity rule)}$$

$$F_{11}+F_{12}+F_{13} = 1 \longrightarrow 0+0.27+F_{13} = 1 \longrightarrow F_{13} = 0.73 \qquad \text{(summation rule)}$$

$$F_{21}+F_{22}+F_{23} = 1 \longrightarrow 0.18+0+F_{23} = 1 \longrightarrow F_{23} = 0.82 \qquad \text{(summation rule)}$$

$$A_1F_{13} = A_3F_{31} \longrightarrow (1.28)(0.73) = (3.268)F_{31} \longrightarrow F_{31} = 0.29 \qquad \text{(reciprocity rule)}$$

$$A_2F_{23} = A_3F_{32} \longrightarrow (1.92)(0.82) = (3.268)F_{32} \longrightarrow F_{32} = 0.48 \qquad \text{(reciprocity rule)}$$

We now apply Eq. 13-35 to each surface to determine the radiosities.

Surface 1:

$$\sigma T_1^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1-J_2)+F_{13}(J_1-J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(450\text{ K})^4 = J_1 + \frac{1-0.75}{0.75}\left[0.27(J_1-J_2)+0.73(J_1-J_3)\right]$$

Surface 2: $\sigma T_2^4 = J_2 \longrightarrow (5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(700\text{ K})^4 = J_2$

Surface 3:

$$\sigma T_3^4 = J_3 + \frac{1-\varepsilon_3}{\varepsilon_3}\left[F_{31}(J_3-J_1)+F_{32}(J_3-J_2)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(290\text{ K})^4 = J_3 + \frac{1-0.85}{0.85}\left[0.29(J_3-J_1)+0.48(J_3-J_2)\right]$$

Solving the above equations, we find

$$J_1 = 2937\text{ W/m}^2,\quad J_2 = 13{,}614\text{ W/m}^2,\quad J_3 = 1501\text{ W/m}^2$$

Then the net rate of radiation heat transfers between the two surfaces and between the horizontal surface and the surroundings are determined to be

$$\dot{Q}_{21} = -\dot{Q}_{12} = -A_1F_{12}(J_1-J_2) = -(1.28\text{ m}^2)(0.27)(2937-13{,}614)\text{W/m}^2 = \mathbf{3690\ W}$$

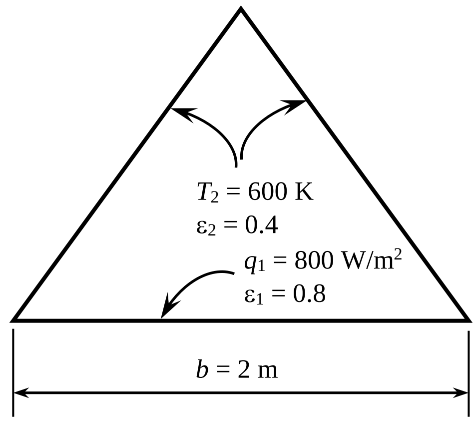
$$\dot{Q}_{13} = A_1F_{13}(J_1-J_3) = (1.28\text{ m}^2)(0.73)(2937-1501)\text{W/m}^2 = \mathbf{1342\ W}$$

13-57 A furnace shaped like a long equilateral-triangular duct is considered. The temperature of the base surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** End effects are neglected.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.8$ and $\varepsilon_2 = 0.4$.

Analysis This geometry can be treated as a two surface enclosure since two surfaces have identical properties. We consider base surface to be surface 1 and other two surface to be surface 2. Then the view factor between the two becomes $F_{12} = 1$. The temperature of the base surface is determined from



$$\dot{Q}_{12} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$800\text{ W} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(T_1)^4 - (600\text{ K})^4]}{\dfrac{1-0.8}{(1\text{ m}^2)(0.8)} + \dfrac{1}{(1\text{ m}^2)(1)} + \dfrac{1-0.4}{(2\text{ m}^2)(0.4)}}$$

$$T_1 = \mathbf{630\ K}$$

Note that $A_1 = 1\text{ m}^2$ and $A_2 = 2\text{ m}^2$.

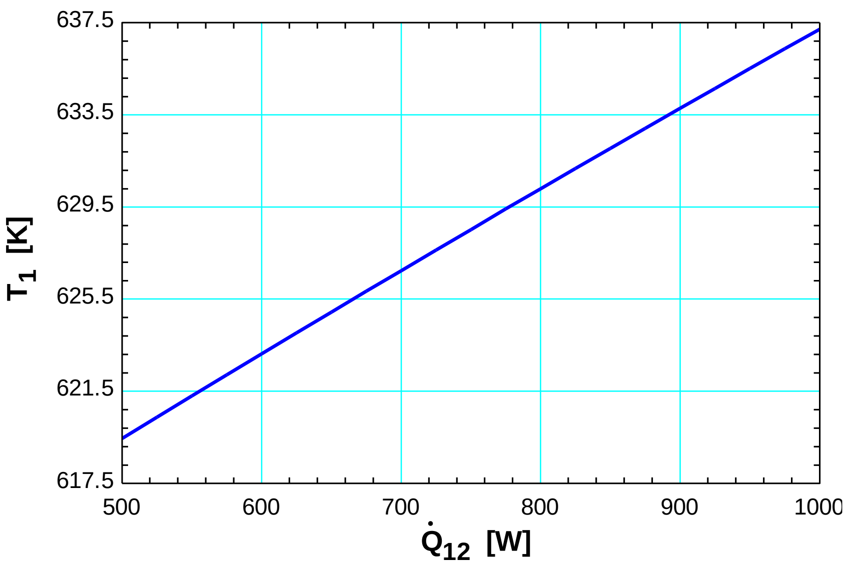
13-58 Prob. 13-57 is reconsidered. The effects of the rate of the heat transfer at the base surface and the temperature of the side surfaces on the temperature of the base surface are to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

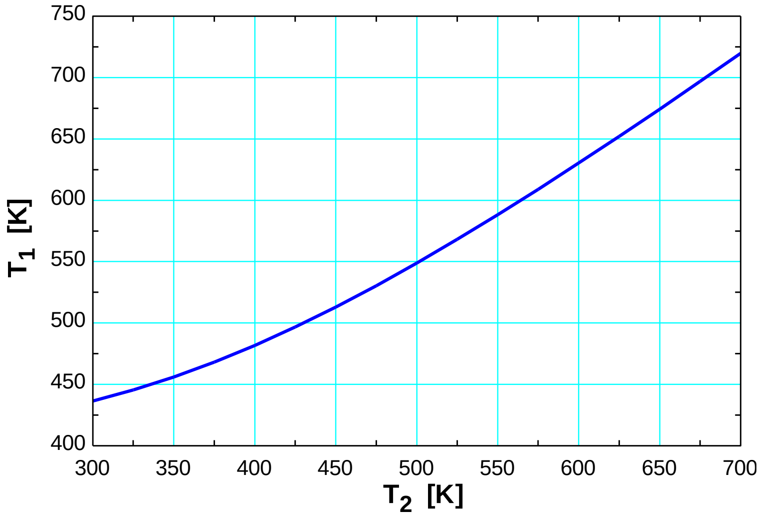
```
"GIVEN"
a=2 [m]
epsilon_1=0.8
epsilon_2=0.4
Q_dot_12=800 [W]
T_2=600 [K]
sigma=5.67E-8 [W/m^2-K^4]

"ANALYSIS"
"Consider the base surface to be surface 1, the side surfaces to be surface 2"
Q_dot_12=(sigma*(T_1^4-T_2^4))/((1-epsilon_1)/(A_1*epsilon_1)+1/(A_1*F_12)+(1-
epsilon_2)/(A_2*epsilon_2))
F_12=1
A_1=1 "[m^2], since rate of heat supply is given per meter square area"
A_2=2*A_1
```

$\dot{Q}_{12}$ [W]	T_1 [K]
500	619.4
525	620.4
550	621.3
575	622.2
600	623.1
625	624
650	624.9
675	625.8
700	626.7
725	627.6
750	628.5
775	629.4
800	630.3
825	631.2
850	632
875	632.9
900	633.8
925	634.6
950	635.5
975	636.4
1000	637.2



T_2 [K]	T_1 [K]
300	436.5
325	445.5
350	456
375	468.1
400	481.7
425	496.7
450	512.9
475	530.4
500	548.8
525	568.1
550	588.2
575	609
600	630.3
625	652.1
650	674.3
675	696.9
700	719.7

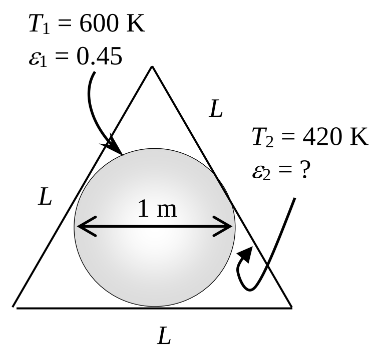


13-59 A solid sphere is placed in an evacuated equilateral triangular enclosure. The view factor from the enclosure to the sphere and the emissivity of the enclosure are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of sphere is given to be $\varepsilon_1 = 0.45$.

Analysis (*a*) We take the sphere to be surface 1 and the surrounding enclosure to be surface 2. The view factor from surface 2 to surface 1 is determined from reciprocity relation:



$$A_1 = \pi D^2 = \pi(1\,\text{m})^2 = 3.142\,\text{m}^2$$

$$L = D\sqrt{6} = (1\,\text{m})^2\sqrt{6} = 2.449\,\text{m}$$

$$A_2 = 4\frac{L^2\sqrt{3}}{4} = L^2\sqrt{3} = (2.449\,\text{m})^2\sqrt{3} = 10.39\,\text{m}^2$$

$$A_1F_{12} = A_2F_{21}$$

$$(3.142)(1) = (10.39)F_{21}$$

$$F_{21} = \mathbf{0.3023}$$

We note that the tetrahedron has four equal surfaces.

(b) The net rate of radiation heat transfer can be expressed for this two-surface enclosure to yield the emissivity of the enclosure:

$$\dot{Q} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$3100\text{ W} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(600\text{ K})^4 - (420\text{ K})^4\right]}{\dfrac{1-0.45}{(3.142\text{ m}^2)(0.45)} + \dfrac{1}{(3.142\text{ m}^2)(1)} + \dfrac{1-\varepsilon_2}{(5.196\text{ m}^2)\varepsilon_2}}$$

$$\varepsilon_2 = \mathbf{0.7515}$$

13-60 A long semi-cylindrical duct with specified temperature on the side surface is considered. The temperature of the base surface for a specified heat transfer rate is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

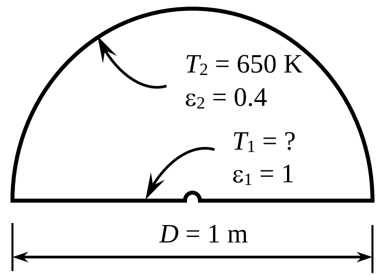
Properties The emissivity of the side surface is ε = 0.4.

Analysis We consider the base surface to be surface 1, the side surface to be surface 2. This system is a two-surface enclosure, and we consider a unit length of the duct. The surface areas and the view factor are determined as

$$A_1 = (1.0\text{ m})(1.0\text{ m}) = 1.0\text{ m}^2$$

$$A_2 = \pi DL/2 = \pi(1.0\text{ m})(1\text{ m})/2 = 1.571\text{ m}^2$$

$$F_{11} + F_{12} = 1 \longrightarrow 0 + F_{12} = 1 \longrightarrow F_{12} = 1 \quad \text{(summation rule)}$$



The temperature of the base surface is determined from

$$\dot{Q}_{12} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$1200\text{ W} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot K^4)[T_1^{\,4} - (650\text{ K})^4]}{\dfrac{1}{(1.0\text{ m}^2)(1)} + \dfrac{1-0.4}{(1.571\text{ m}^2)(0.4)}}$$

$$T_1 = \mathbf{684.8\,K}$$

13-61 A hemisphere with specified base and dome temperatures and heat transfer rate is considered. The emissivity of the dome is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

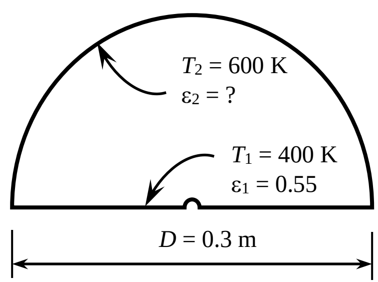
Properties The emissivity of the base surface is ε = 0.55.

Analysis We consider the base surface to be surface 1, the dome surface to be surface 2. This system is a two-surface enclosure. The surface areas and the view factor are determined as

$$A_1 = \pi D^2/4 = \pi(0.3\text{ m})^2/4 = 0.07069\text{ m}^2$$

$$A_2 = \pi D^2/2 = \pi(0.3\text{ m})^2/2 = 0.1414\text{ m}^2$$

$$F_{11} + F_{12} = 1 \longrightarrow 0 + F_{12} = 1 \longrightarrow F_{12} = 1 \quad \text{(summation rule)}$$



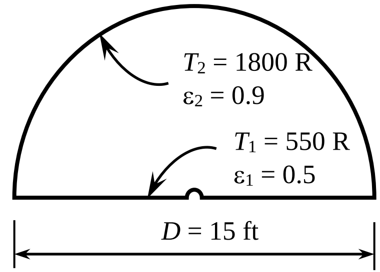
The emissivity of the dome is determined from

$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$65\text{ W} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(400\text{ K})^4 - (600\text{ K})^4]}{\dfrac{1-0.55}{(0.07069\text{ m}^2)(0.55)} + \dfrac{1}{(0.07069\text{ m}^2)(1)} + \dfrac{1-\varepsilon_2}{(0.1414\text{ m}^2)\varepsilon_2}} \longrightarrow \varepsilon_2 = \mathbf{0.0981}$$

13-62E The base and the dome of a long semicylindrical duct are maintained at uniform temperatures. The net rate of radiation heat transfer from the dome to the base surface is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.9$.

Analysis The view factor from the base to the dome is first determined from

$$F_{11} = 0 \ \text{(flat surface)}$$

$$F_{11} + F_{12} = 1 \rightarrow F_{12} = 1 \ \text{(summation rule)}$$

The net rate of radiation heat transfer from dome to the base surface can be determined from

$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} = -\frac{(0.1714\times10^{-8}\ \text{Btu/h.ft}^2\cdot\text{R}^4)[(550\ \text{R})^4 - (1800\ \text{R})^4]}{\dfrac{1-0.5}{(15\ \text{ft}^2)(0.5)} + \dfrac{1}{(15\ \text{ft}^2)(1)} + \dfrac{1-0.9}{\left[\dfrac{\pi(15\ \text{ft})(1\ \text{ft})}{2}\right](0.9)}}$$

$$= \mathbf{129{,}200\ Btu/h} \ \text{per ft length}$$

The positive sign indicates that the net heat transfer is from the dome to the base surface, as expected.

13-63 Radiation heat transfer occurs between a sphere and a circular disk. The view factors and the net rate of radiation heat transfer for the existing and modified cases are to be determined.

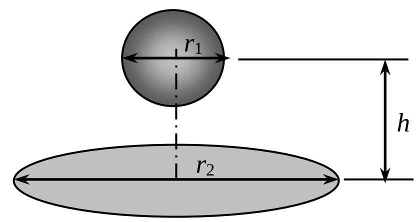
Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of sphere and disk are given to be $\varepsilon_1 = 0.9$ and $\varepsilon_2 = 0.5$, respectively.

Analysis (*a*) We take the sphere to be surface 1 and the disk to be surface 2. The view factor from surface 1 to surface 2 is determined from

$$F_{12} = 0.5\left\{1-\left[1+\left(\frac{r_2}{h}\right)^2\right]^{-0.5}\right\} = 0.5\left\{1-\left[1+\left(\frac{1.2\text{ m}}{0.60\text{ m}}\right)^2\right]^{-0.5}\right\} = \mathbf{0.2764}$$

The view factor from surface 2 to surface 1 is determined from reciprocity relation:



$$A_1 = 4\pi r_1^2 = 4\pi(0.3\text{ m})^2 = 1.131\text{ m}^2$$

$$A_2 = \pi r_2^2 = \pi(1.2\text{ m})^2 = 4.524\text{ m}^2$$

$$A_1F_{12} = A_2F_{21}$$

$$(1.131)(0.2764) = (4.524)F_{21}$$

$$F_{21} = \mathbf{0.0691}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{Q} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1}+\dfrac{1}{A_1F_{12}}+\dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(873\text{ K})^4-(473\text{ K})^4\right]}{\dfrac{1-0.9}{(1.131\text{ m}^2)(0.9)}+\dfrac{1}{(1.131\text{ m}^2)(0.2764)}+\dfrac{1-0.5}{(4.524\text{ m}^2)(0.5)}} = \mathbf{8550\ W}$$

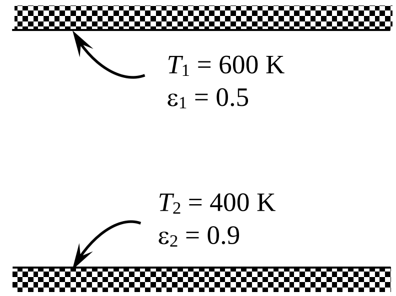
(c) The best values are $\varepsilon_1 = \varepsilon_2 = 1$ and $h = r_1 = 0.3\text{ m}$. Then the view factor becomes

$$F_{12} = 0.5\left\{1-\left[1+\left(\frac{r_2}{h}\right)^2\right]^{-0.5}\right\} = 0.5\left\{1-\left[1+\left(\frac{1.2\text{ m}}{0.30\text{ m}}\right)^2\right]^{-0.5}\right\} = 0.3787$$

The net rate of radiation heat transfer in this case is

$$\dot{Q} = A_1F_{12}\sigma\left(T_1^4 - T_2^4\right) = (1.131\text{ m}^2)(0.3787)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(873\text{ K})^4-(473\text{ K})^4\right] = \mathbf{12{,}890\ W}$$

13-64 Two very large parallel plates are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities ε of the plates are given to be 0.5 and 0.9.

Analysis The net rate of radiation heat transfer between the two surfaces per unit area of the plates is determined directly from

$$\frac{\dot{Q}_{12}}{A_s} = \frac{\sigma(T_1^{\ 4} - T_2^{\ 4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\ \text{K})^4 - (400\ \text{K})^4]}{\dfrac{1}{0.5} + \dfrac{1}{0.9} - 1} = \mathbf{2793\ W/m^2}$$

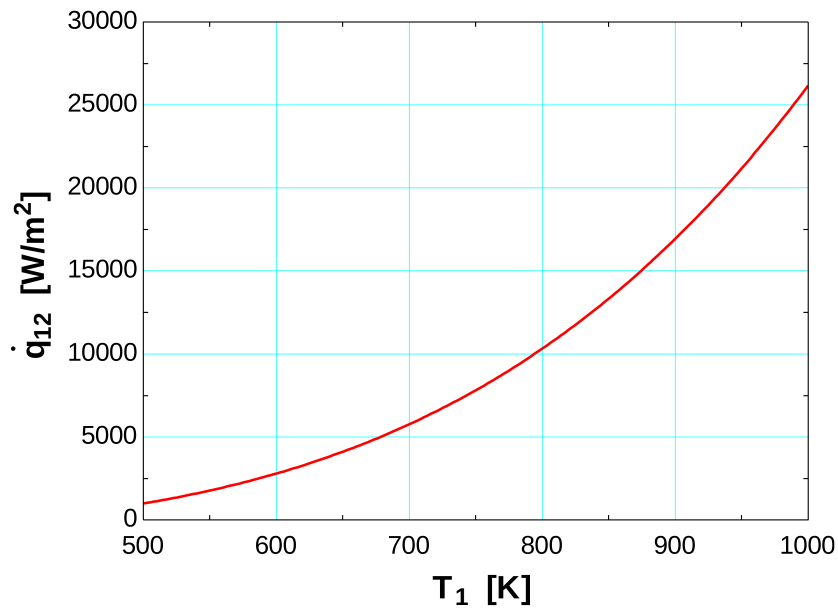
13-65 Prob. 13-64 is reconsidered. The effects of the temperature and the emissivity of the hot plate on the net rate of radiation heat transfer between the plates are to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

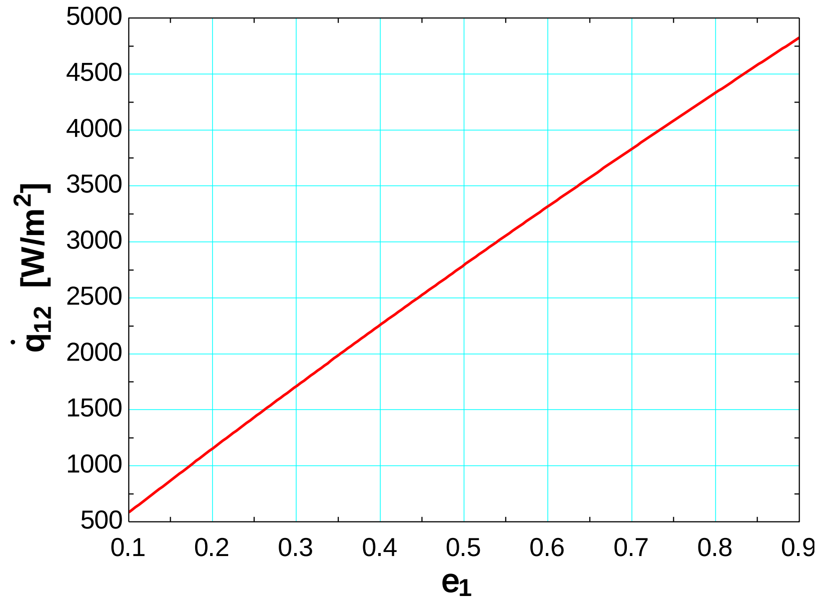
```
"GIVEN"
T_1=600 [K]
T_2=400 [K]
epsilon_1=0.5
epsilon_2=0.9
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"

"ANALYSIS"
q_dot_12=(sigma*(T_1^4-T_2^4))/(1/epsilon_1+1/epsilon_2-1)
```

T_1 [K]	$\dot{q}_{12}$ [W/m^2]
500	991.1
525	1353
550	1770
575	2248
600	2793
625	3411
650	4107
675	4888
700	5761
725	6733
750	7810
775	9001
800	10313
825	11754
850	13332
875	15056
900	16934
925	18975
950	21188
975	23584
1000	26170



ε_1	$\dot{q}_{12}$ [W/m^2]
0.1	583.2
0.15	870
0.2	1154
0.25	1434
0.3	1712
0.35	1987
0.4	2258
0.45	2527
0.5	2793
0.55	3056
0.6	3317
0.65	3575
0.7	3830
0.75	4082
0.8	4332
0.85	4580
0.9	4825



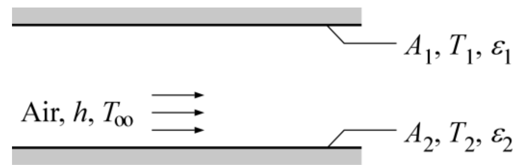
13-66 Air is flowing between two infinitely large parallel plates. The convection heat transfer coefficient associated with the air is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** The surface temperatures are uniform.

Properties The emissivity of the upper plate is given as $\varepsilon_1 = 0.7$. The lower plate surface is black, $\varepsilon_2 = 1$.

Analysis For infinitely large parallel plates, the rate of radiation heat transfer is (from Table 13-3),

$$\dot{Q}_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$



Applying energy balance on the lower plate, we have

$$\dot{Q}_{12} = \dot{Q}_{\text{conv}}$$

$$hA(T_2 - T_\infty) = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

$$h = \frac{\sigma}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \frac{(T_1^4 - T_2^4)}{(T_2 - T_\infty)}$$

$$= \frac{(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)}{\frac{1}{0.7} + 1 - 1} \frac{(500^4 - 330^4)\ \text{K}^4}{(330 - 290)\ \text{K}}$$

$$h = \mathbf{50.3\ W/m^2 \cdot K}$$

Discussion The calculated value of the convection heat transfer coefficient (h = 50.3 W/m^2·K) is typical for forced convection of gases (see Table 1-5)

13-67 C & S A large ASTM B152 copper plate and a large ceramic plate are placed in parallel near each other. The temperature of the ceramic plate and the net radiation heat flux between the two plates are known. The temperature of the copper plate is to be determined whether it is below the maximum use temperature of 260°C.

Assumptions **1** Steady state conditions. **2** Uniform surface temperatures on both plates. **3** Surfaces are opaque, gray and diffuse.

Properties The emissivities of the ceramic and copper plates are given as $\varepsilon_1 = 0.92$ and $\varepsilon_2 = 0.15$, respectively.

Analysis For large parallel plates, the net rate of radiation heat transfer between the two plates is

$$\dot{Q}_{12} = \frac{\sigma A(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

or in heat flux as

$$\dot{q}_{12} = \frac{\dot{Q}_{12}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

Solving for the temperature of the copper plate T_2,

$$T_2 = \left[T_1^4 - \frac{\dot{q}_{12}}{\sigma}\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)\right]^{1/4}$$

$$T_2 = \left[(793\text{ K})^4 - \frac{2000\text{ W/m}^2}{5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4}\left(\frac{1}{0.92} + \frac{1}{0.15} - 1\right)\right]^{1/4} = 630\text{ K} = \mathbf{357}℃ > 260℃$$

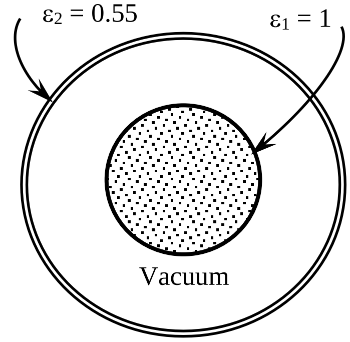
Discussion With the given temperature of the ceramic plate and the net radiation heat flux between the two plates, the copper plate temperature would be 97°C higher than the maximum use temperature. Thus, the copper plate would not comply with the ASME Code for Process Piping.

13-68 Two very long concentric cylinders are maintained at uniform temperatures. The net rate of radiation heat transfer between the two cylinders is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 1$ and $\varepsilon_2 = 0.55$.

Analysis The net rate of radiation heat transfer between the two cylinders per unit length of the cylinders is determined from



$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

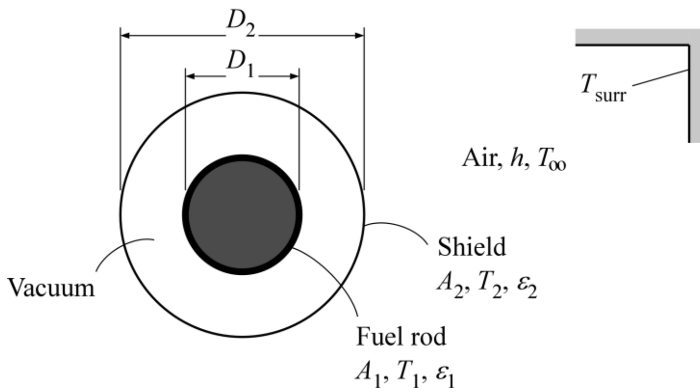
$$= \frac{[\pi(0.35\text{ m})(1\text{m})](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(950\text{ K})^4 - (500\text{ K})^4]}{\dfrac{1}{1} + \dfrac{1-0.55}{0.55}\left(\dfrac{3.5}{5}\right)}$$

$$= 29{,}810\text{ W} = \mathbf{29.81 kW}$$

13-69 A long cylindrical black surface fuel rod is shielded by a concentric surface that has a uniform temperature. The surface temperature of the fuel rod is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The fuel rod surface is black. 3 The shield is opaque, diffuse, and gray. **4** The fuel rod and shield form an infinitely long concentric cylinder.

Properties The emissivity of the shield is given as $\varepsilon_2 = 0.05$. The fuel rod surface is black, $\varepsilon_1 = 1$.



Analysis For infinitely long concentric cylinder, the rate of radiation heat transfer at the fuel rod surface is (from Table 13-3),

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{1} + \dfrac{1-0.05}{0.05}\left(\dfrac{25}{50}\right)} = 0.09524A_1\sigma(T_1^4 - T_2^4)$$

Applying energy balance on the shield, we have the following expression:

$$\dot{Q}_{12} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = hA_2(T_2 - T_\infty) + \varepsilon_2 A_2\sigma(T_2^4 - T_{\text{surr}}^4)$$

Hence

$$T_1^4 = \frac{h(T_2 - T_\infty) + \varepsilon_2\sigma(T_2^4 - T_{\text{surr}}^4)}{0.09524\sigma}\left(\frac{A_2}{A_1}\right) + T_2^4$$

$$= \frac{h(T_2 - T_\infty) + \varepsilon_2\sigma(T_2^4 - T_{\text{surr}}^4)}{0.09524\sigma}\left(\frac{D_2}{D_1}\right) + T_2^4$$

or

$$T_1 = \left[\frac{h(T_2 - T_\infty) + \varepsilon_2\sigma(T_2^4 - T_{\text{surr}}^4)}{0.09524\sigma}\left(\frac{D_2}{D_1}\right) + T_2^4\right]^{1/4}$$

$$T_1 = \left[\frac{(15)(320-300) + (0.05)(5.67\times10^{-8})(320^4 - 300^4)}{0.09524(5.67\times10^{-8})}\left(\frac{50}{25}\right)\text{K}^4 + (320\text{ K})^4\right]^{1/4}$$

$$T_1 = \mathbf{594\ K}$$

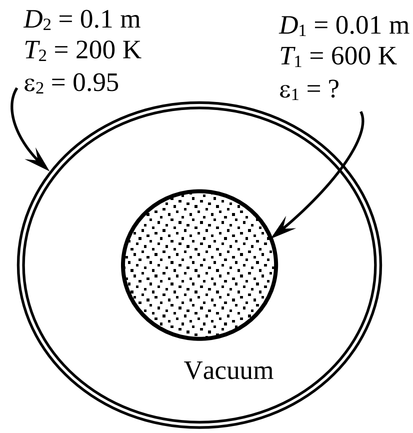
Discussion The use of absolute temperatures is necessary for calculations involving radiation heat transfer.

13-70 A long cylindrical rod coated with a new material is placed in an evacuated long cylindrical enclosure which is maintained at a uniform temperature. The emissivity of the coating on the rod is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Properties The emissivity of the enclosure is given to be $\varepsilon_2 = 0.95$.

Analysis The emissivity of the coating on the rod is determined from



$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$$

$$12\text{ W} = \frac{[\pi(0.01\text{ m})(1\text{m})](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(600\text{ K})^4 - (200\text{ K})^4]}{\dfrac{1}{\varepsilon_1} + \dfrac{1-0.95}{0.95}\left(\dfrac{1}{10}\right)}$$

which gives

$\varepsilon_1 = \mathbf{0.0527}$

13-71 Liquid NH_3 flows in an insulated tube that is protected by a concentric shield. The surrounding temperature is to be determined so that the NH_3 is maintained in the liquid state.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** The ambient temperature is equal to the temperature of the surrounding surfaces, $T_\infty = T_{\text{surr}}$.

Properties The emissivity of both surfaces is given to be $\varepsilon = \varepsilon_1 = \varepsilon_2 = 0.33$.

Analysis The net rate of radiation heat transfer between the two concentric cylinders is

$$\dot{Q}_{12} = \frac{A_1\sigma(T_2^4 - T_1^4)}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)}$$

Radiation heat transfer rate from the outer sphere to the surrounding is

$$\dot{Q}_{\text{rad}} = \varepsilon A_2\sigma(T_{\text{surr}}^4 - T_2^4) = \varepsilon A_2\sigma(T_\infty^4 - T_2^4)$$

The natural convection heat transfer rate from the outer surface is

$$\dot{Q}_{\text{conv}} = hA_2(T_\infty - T_2)$$

Performing the energy balance on the outer surface, we have

$$\dot{Q}_{12} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

$$\frac{A_1\sigma(T_2^4 - T_1^4)}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)} = hA_2(T_\infty - T_2) + \varepsilon A_2\sigma(T_\infty^4 - T_2^4)$$

$$\frac{D_1\sigma(T_2^4 - T_1^4)}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)} = hD_2(T_\infty - T_2) + \varepsilon D_2\sigma(T_\infty^4 - T_2^4)$$

Thus,

$$\frac{(0.04\text{ m})(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(283^4 - 273^4)\text{ K}^4}{\dfrac{1}{0.33} + \dfrac{1-0.33}{0.33}\left(\dfrac{0.04\text{ m}}{0.08\text{ m}}\right)} = (3\text{ W/m}^2\cdot\text{K})(0.08\text{ m})(T_\infty - 283)\text{ K}$$

$$+ (0.33)(0.08\text{ m})(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(T_\infty^4 - 283^4)\text{ K}^4$$

The surrounding air temperature T_∞ can be solved by trial-and-error to yield

T_∞ = **11.3°C**.

Discussion By monitoring the temperatures of the surrounding air and the shield surface, the vaporization of the liquid NH_3 flowing inside the tube can be prevented.

13-72 Hot fluid flowing inside a long tube and the tube is enclosed in a concentric cylindrical thin cover. The emissivity of the inside tube is to be determined so that the outer surface temperature is kept below 45°C to prevent thermal burn hazards.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** The ambient temperature is equal to the temperature of the surrounding surfaces, $T_\infty = T_{\text{surr}}$.

Properties The emissivity of outer cylindrical cover is given to be $\varepsilon_2 = 0.6$.

Analysis The net rate of radiation heat transfer between the two concentric cylinders is

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)}$$

Radiation heat transfer rate from the outer sphere to the surrounding is

$$\dot{Q}_{\text{rad}} = \varepsilon_2 A_2 \sigma(T_2^{\,4} - T_{\text{surr}}^4)$$

The natural convection heat transfer rate from the outer surface is

$$\dot{Q}_{\text{conv}} = hA_2(T_2 - T_\infty)$$

Performing the energy balance on the outer surface, we have

$$\dot{Q}_{12} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}}$$

$$\frac{A_1\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)} = \varepsilon_2 A_2 \sigma(T_2^{\,4} - T_{\text{surr}}^4) + hA_2(T_2 - T_\infty)$$

Thus,

$$\begin{aligned}
\varepsilon_1 &= \left[\frac{A_1\sigma(T_1^{\,4} - T_2^{\,4})}{\varepsilon_2 A_2 \sigma(T_2^{\,4} - T_{\text{surr}}^4) + hA_2(T_2 - T_\infty)} - \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)\right]^{-1} \\
&= \left[\frac{D_1\sigma(T_1^{\,4} - T_2^{\,4})}{\varepsilon_2 D_2 \sigma(T_2^{\,4} - T_{\text{surr}}^4) + hD_2(T_2 - T_\infty)} - \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)\right]^{-1} \\
&= \left[\frac{(0.025\text{ m})(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(423^4 - 318^4)\text{ K}^4}{(0.6)(0.05\text{ m})(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(318^4 - 293^4)\text{ K}^4 + (8\text{ W/m}^2\cdot\text{K})(0.05\text{ m})(318-293)\text{ K}}\right. \\
&\qquad \left. - \frac{1-0.6}{0.6}\left(\frac{0.025\text{ m}}{0.05\text{ m}}\right)\right]^{-1}
\end{aligned}$$

$$\varepsilon_1 = \mathbf{0.573}$$

Discussion In order to keep the outer surface at 45°C, the emissivity of the inner tube should be 0.573 or lower.

13-73 C&S A long cylindrical fuel rod is enclosed by a concentric tube. Convection and radiation occur on the tube outer surface. The temperature of the tube surface is to be determined whether it is below the maximum use temperature for ASTM A249 904L stainless steel tube.

Assumptions **1** Steady state conditions. **2** Uniform surface temperatures on fuel rod and tube. **3** The tube has a thin wall with negligible thermal resistance for conduction. **4** Surfaces are gray and diffuse.

Properties The emissivities of the fuel rod and ASTM A249 904L tube are given as ε_1 = 0.97 and ε_2 = 0.33, respectively.

Analysis For long concentric cylinders, the radiation heat transfer from the fuel rod to the ASTM A249 904L tube is

$$\dot{Q}_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)}$$

Apply energy balance on the tube, yields

$$\dot{Q}_{12} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

$$\frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)} = hA_2(T_2 - T_\infty) + \sigma\varepsilon_2 A_2(T_2^4 - T_{\text{surr}}^4)$$

$$\frac{\sigma D_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)} = hD_2(T_2 - T_\infty) + \sigma\varepsilon_2 D_2(T_2^4 - T_{\text{surr}}^4)$$

Substituting the known values and solve for T_2,

$$\frac{\left(5.67\times10^{-8}\,\dfrac{\text{W}}{\text{m}^2}\cdot\text{K}^4\right)(0.025\text{ m})(823^4 - T_2^4)\text{K}^4}{\dfrac{1}{0.97} + \dfrac{1-0.33}{0.33}\left(\dfrac{25\text{ mm}}{50\text{ mm}}\right)}$$

$$= (15\text{ W}\cdot\text{K/m}^2)(0.050\text{ m})(T_2 - 293)\text{K} + \left(5.67\times10^{-8}\,\frac{\text{W}}{\text{m}^2}\cdot\text{K}^4\right)(0.33)(0.050\text{ m})(T_2^4 - 293^4)\text{K}^4$$

$$\Rightarrow \quad T_2 = 541\text{ K} = \mathbf{268}°\text{C} > 260°\text{C}$$

Discussion With the given conditions, the surface temperature of the tube would exceed the maximum use temperature of 260°C. One approach that can be used to reduce the tube surface temperature is by increasing the convection heat transfer coefficient of the tube with the ambient air.

13-74 Cryogenic fluid flows inside a metal tube. The metal tube is enclosed by a concentric polypropylene tube. Radiation occurs on the polypropylene tube outer surface. The lowest temperature that the inner metal tube can go without cooling the polypropylene tube below its minimum temperature limit is to be determined.

Assumptions **1** Steady state conditions. **2** Uniform surface temperatures on both tubes. **3** Both tubes have thin walls with negligible thermal resistance for conduction. **4** Surfaces are opaque, gray and diffuse.

Properties The emissivities of the inner metal tube and the outer polypropylene tube are given as ε_1 = 0.5 and ε_2 = 0.97, respectively.

Analysis For long concentric cylinders, the radiation heat transfer from the metal tube surface to the polypropylene tube surface is

$$\dot{Q}_{12} = \frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)}$$

Apply energy balance on the polypropylene tube, yields

$$\dot{Q}_{12} = \dot{Q}_{\text{rad}}$$

$$\frac{\sigma A_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)} = \sigma\varepsilon_2 A_2(T_2^4 - T_{\text{surr}}^4)$$

$$\frac{\sigma D_1(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)} = \sigma\varepsilon_2 D_2(T_2^4 - T_{\text{surr}}^4)$$

Solving for the inner metal tube surface temperature T_1,

$$T_1 = \left\{T_2^4 + \left[\varepsilon_2\frac{D_2}{D_1}(T_2^4 - T_{\text{surr}}^4)\right]\left[\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)\right]\right\}^{1/4}$$

$$T_1 = \left\{(255\text{ K})^4 + \left[(0.97)\left(\frac{15\text{ mm}}{10\text{ mm}}\right)[(255\text{ K})^4 - (273\text{ K})^4)\right]\left[\frac{1}{0.5} + \frac{1-0.97}{0.97}\left(\frac{10\text{ mm}}{15\text{ mm}}\right)\right]\right\}^{1/4}$$

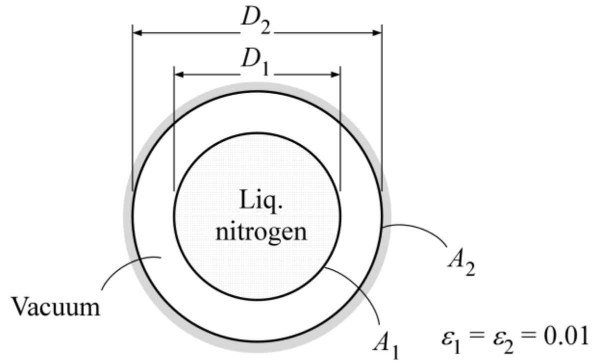
$$T_1 = 135\text{ K} = \mathbf{-138}°\text{C}$$

Discussion To avoid the polypropylene tube from cooling below the minimum temperature limit specified by the ASME Code for Process Piping, the inner metal tube that is used to transport cryogenic fluid needs to stay above 135 K or −138°C.

13-75 Liquid nitrogen is stored in a spherical tank this is enclosed by a concentric spherical surface at 273 K. The rate of vaporization for the liquid nitrogen is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Heat transfer by radiation only.

Properties The emissivity of the two surfaces is given as $\varepsilon_1 = \varepsilon_2 = 0.01$. The latent heat of vaporization for nitrogen is h_{fg} = 198.6 kJ/kg (Table A-2).



Analysis For concentric spheres, the rate of radiation heat transfer at the inner surface is (from Table 13-3),

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)^2}$$

Hence,

$$\dot{Q}_{12} = \frac{\pi(1\,\text{m})^2(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(80^4 - 273^4)\,\text{K}^4}{\dfrac{1}{0.01} + \dfrac{1-0.01}{0.01}\left(\dfrac{1}{1.6}\right)^2} = -7.082\ \text{W}$$

The rate of vaporization can be determined using

$$-\dot{Q}_{12} = \dot{m}h_{fg}$$

$$\dot{m} = -\frac{\dot{Q}_{12}}{h_{fg}} = -\frac{-7.082\ \text{W}}{198.6\times10^3\ \text{J/kg}}$$

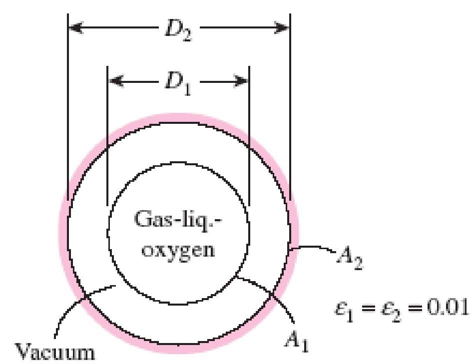
$$\dot{m} = \mathbf{3.57\times10^{-5}\ kg/s}$$

Discussion The rate of vaporization can be reduced by placing a radiation shield midway between the inner and outer spherical surfaces.

13-76 Two phase gas-liquid oxygen is stored in a spherical tank this is enclosed by a concentric spherical surface at 273 K. The heat transfer rate at the spherical tank surface is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Heat transfer by radiation only.

Properties The emissivity of the two surfaces is given as $\varepsilon_1 = \varepsilon_2 = 0.01$. The normal boiling point of oxygen is −183°C (Table A-2).



Analysis For concentric spheres, the rate of radiation heat transfer at the inner surface is (from Table 13-3),

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)^2}$$

Note that the spherical tank surface has the same temperature as the oxygen at normal boiling point, T_1 = −183°C = 90 K. Hence,

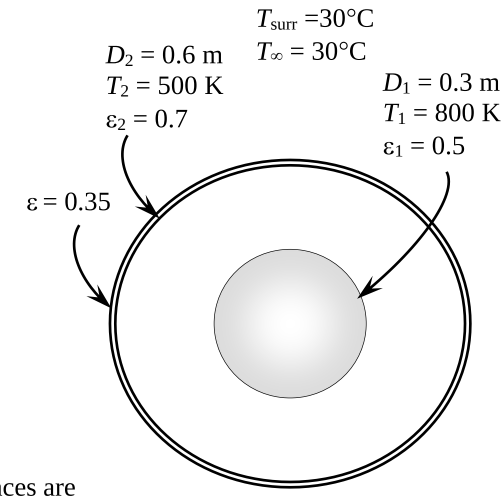
$$\dot{Q}_{12} = \frac{\pi(1\,\text{m})^2(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(90^4 - 273^4)\,\text{K}^4}{\dfrac{1}{0.01} + \dfrac{1-0.01}{0.01}\left(\dfrac{1}{1.6}\right)^2} = \mathbf{-7.05\ W}$$

Discussion The negative value of $\dot{Q}_{12}$ indicates that heat is being added to the oxygen. As long as the oxygen is maintained in the two-phase gas-liquid state, its temperature will remain constant at the normal boiling point.

13-77 Two concentric spheres are maintained at uniform temperatures. The net rate of radiation heat transfer between the two spheres and the convection heat transfer coefficient at the outer surface are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$ and $\varepsilon_2 = 0.7$.



Analysis The net rate of radiation heat transfer between the two spheres is

$$\dot{Q}_{12} = \frac{A_1\sigma\left(T_1^{\ 4} - T_2^{\ 4}\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1^{\ 2}}{r_2^{\ 2}}\right)}$$

$$= \frac{\left[\pi(0.3\,\text{m})^2\right]\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left[(800\,\text{K})^4 - (500\,\text{K})^4\right]}{\dfrac{1}{0.5} + \dfrac{1-0.7}{0.7}\left(\dfrac{0.15\,\text{m}}{0.3\,\text{m}}\right)^2}$$

$$= \mathbf{2641\,W}$$

Radiation heat transfer rate from the outer sphere to the surrounding surfaces are

$$\dot{Q}_{rad} = \varepsilon F A_2 \sigma (T_2^{\ 4} - T_{surr}^{\ \ 4})$$
$$= (0.35)(1)[\pi(0.6\,\text{m})^2](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(500\,\text{K})^4 - (30+273\,\text{K})^4]$$
$$= 1214\ \text{W}$$

The convection heat transfer rate at the outer surface of the cylinder is determined from requirement that heat transferred from the inner sphere to the outer sphere must be equal to the heat transfer from the outer surface of the outer sphere to the environment by convection and radiation. That is,

$$\dot{Q}_{conv} = \dot{Q}_{12} - \dot{Q}_{rad} = 2641 - 1214 = 1427\ \text{W}$$

Then the convection heat transfer coefficient becomes

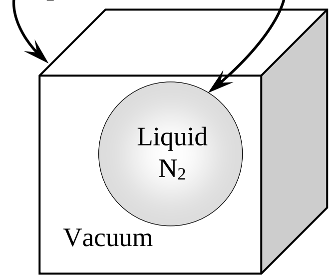
$$\dot{Q}_{conv.} = hA_2\left(T_2 - T_\infty\right)$$
$$1427\ \text{W} = h\left[\pi(0.6\,\text{m})^2\right](500\,\text{K - }303\,\text{K})$$
$$h = \mathbf{6.40\ W/m^2\cdot{}^\circ C}$$

13-78 A spherical tank filled with liquid nitrogen is kept in an evacuated cubic enclosure. The net rate of radiation heat transfer to the liquid nitrogen is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** The thermal resistance of the tank is negligible.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.8$.

Analysis We take the sphere to be surface 1 and the surrounding cubic enclosure to be surface 2. Noting that $F_{12} = 1$, for this two-surface enclosure, the net rate of radiation heat transfer to liquid nitrogen can be determined from



$$\dot{Q}_{21} = -\dot{Q}_{12} = -\frac{A_1\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{A_1}{A_2}\right)}$$

$$= -\frac{\left[\pi(2\text{ m})^2\right]\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left[(100\text{ K})^4 - (240\text{ K})^4\right]}{\dfrac{1}{0.1} + \dfrac{1-0.8}{0.8}\left[\dfrac{\pi(2\text{ m})^2}{6(3\text{ m})^2}\right]}$$

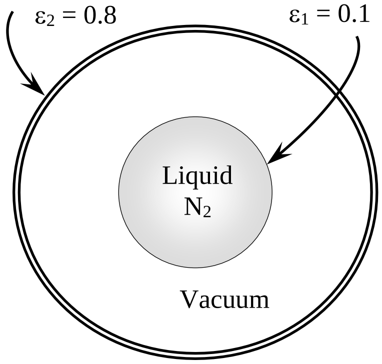
$$= \mathbf{228\ W}$$

13-79 A spherical tank filled with liquid nitrogen is kept in an evacuated spherical enclosure. The net rate of radiation heat transfer to the liquid nitrogen is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** The thermal resistance of the tank is negligible.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.1$ and $\varepsilon_2 = 0.8$.

Analysis The net rate of radiation heat transfer to liquid nitrogen can be determined from



$$\dot{Q}_{12} = \frac{A_1\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1^{\,2}}{r_2^{\,2}}\right)}$$

$$= \frac{\left[\pi(2\text{ m})^2\right]\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left[(240\text{ K})^4 - (100\text{ K})^4\right]}{\dfrac{1}{0.1} + \dfrac{1-0.8}{0.8}\left[\dfrac{(1\text{ m})^2}{(1.5\text{ m})^2}\right]}$$

$$= \mathbf{227\ W}$$

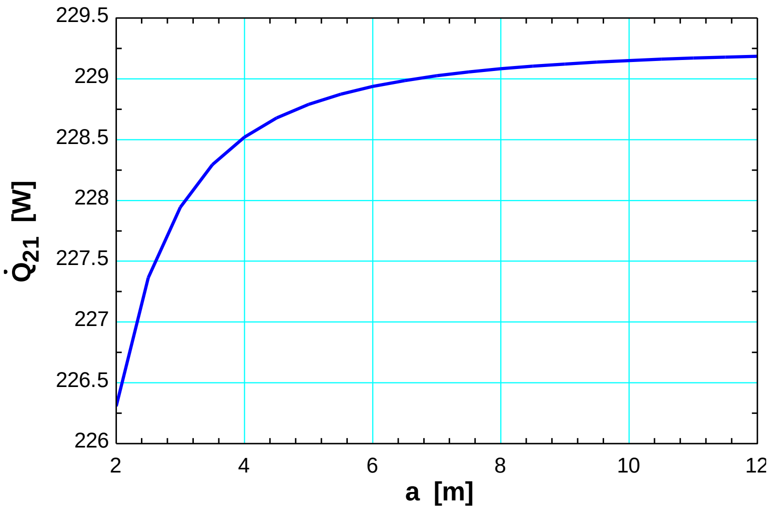
13-80 Prob. 13-78 is reconsidered. The effects of the side length and the emissivity of the cubic enclosure, and the emissivity of the spherical tank on the net rate of radiation heat transfer are to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

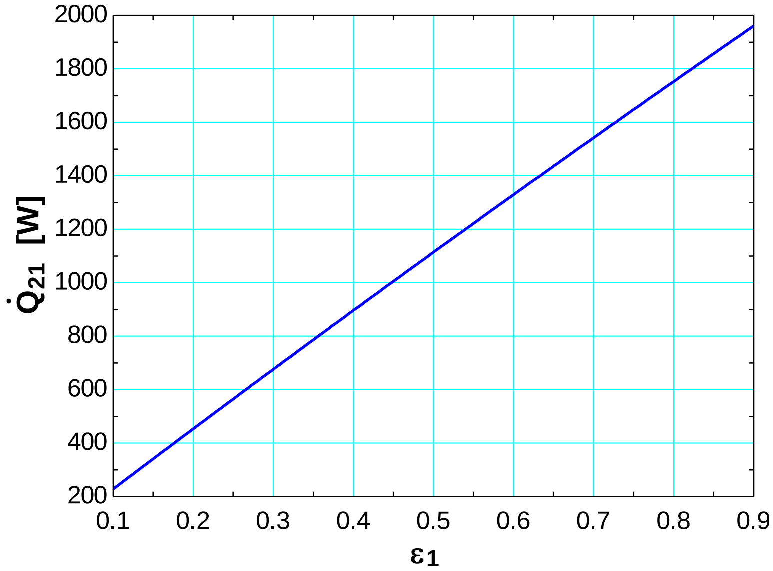
```
"GIVEN"
D=2 [m]
a=3 [m]
T_1=100 [K]
T_2=240 [K]
epsilon_1=0.1
epsilon_2=0.8
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"

"ANALYSIS"
"Consider the sphere to be surface 1, the surrounding cubic enclosure to be surface 2"
Q_dot_12=(A_1*sigma*(T_1^4-T_2^4))/(1/epsilon_1+(1-epsilon_2)/epsilon_2*(A_1/A_2))
Q_dot_21=-Q_dot_12
A_1=pi*D^2
A_2=6*a^2
```

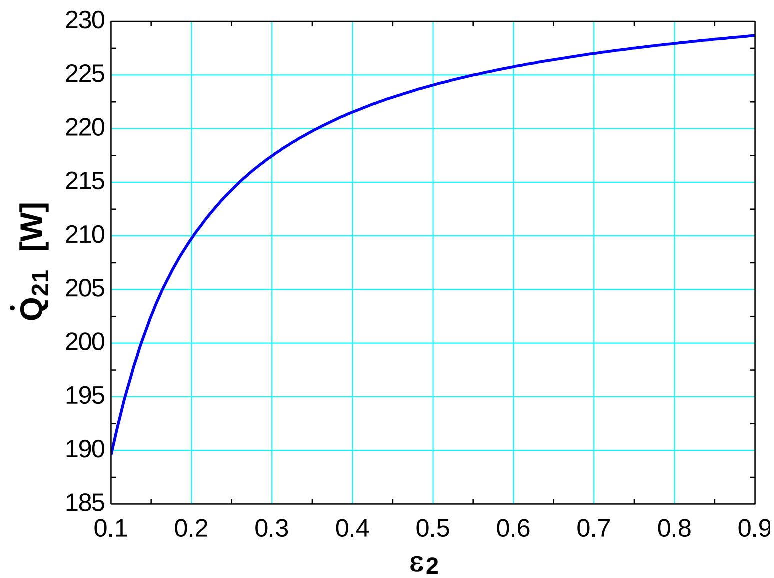
a [m]	$\dot{Q}_{21}$ [W]
2	226.3
2.5	227.4
3	227.9
3.5	228.3
4	228.5
4.5	228.7
5	228.8
5.5	228.9
6	228.9
6.5	229
7	229
7.5	229.1
8	229.1
8.5	229.1
9	229.1
9.5	229.1
10	229.1
10.5	229.2
11	229.2
11.5	229.2
12	229.2



ε_1	$\dot{Q}_{21}$ [W]
0.1	227.9
0.15	340.9
0.2	453.3
0.25	565
0.3	676
0.35	786.4
0.4	896.2
0.45	1005
0.5	1114
0.55	1222
0.6	1329
0.65	1436
0.7	1542
0.75	1648
0.8	1753
0.85	1857
0.9	1961



ε_2	$\dot{Q}_{21}$ [W]
0.1	189.6
0.15	202.6
0.2	209.7
0.25	214.3
0.3	217.5
0.35	219.8
0.4	221.5
0.45	222.9
0.5	224.1
0.55	225
0.6	225.8
0.65	226.4
0.7	227
0.75	227.5
0.8	227.9
0.85	228.3
0.9	228.7



13-81 PtD Cold fluid stored in a spherical tank enclosed in a concentric outer cover. The gap of the vacuumed enclosure is to be determined so that the outer surface temperature is not below the dew point.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Radiation heat transfer between the outer surface and the surrounding is negligible.

Properties The emissivity of both surfaces is given to be $\varepsilon = \varepsilon_1 = \varepsilon_2 = 0.6$.

Analysis The net rate of radiation heat transfer between the two spheres is

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)^2}$$

The natural convection heat transfer rate from the outer surface is

$$\dot{Q}_{\text{conv}} = hA_2(T_2 - T_\infty)$$

Performing the energy balance on the outer surface, we have

$$\dot{Q}_{12} = \dot{Q}_{\text{conv}}$$

$$\frac{A_1\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)^2} = hA_2(T_2 - T_\infty) \quad \rightarrow \quad \frac{D_1^2\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)^2} = hD_2^2(T_2 - T_\infty)$$

Hence,

$$D_2 = \left[\frac{D_1^2\,\varepsilon\sigma(T_1^{\,4} - T_2^{\,4})}{h(T_2 - T_\infty)} - (1-\varepsilon)D_1^2\right]^{0.5}$$

$$= \left[\frac{(3\,\text{m})^2(0.6)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(278^4 - 283^4)\,\text{K}^4}{(3\,\text{W/m}^2\cdot\text{K})(283-286)\,\text{K}} - (1-0.6)(3\,\text{m})^2\right]^{0.5}$$

$$= 3.38\,\text{m}$$

Thus, the gap of the vacuumed enclosure is

$$L_{\text{gap}} = \frac{D_2 - D_1}{2} = \frac{3.38-3}{2}\,\text{m} = \mathbf{0.19\,m}$$

Discussion To keep the outer surface temperature above the dew point of 10°C, the vacuumed gap should be greater than 19 cm. Gap size below 19 cm will bring the outer surface temperature to below 10°C and condensation could occur to cause electrical hazards.

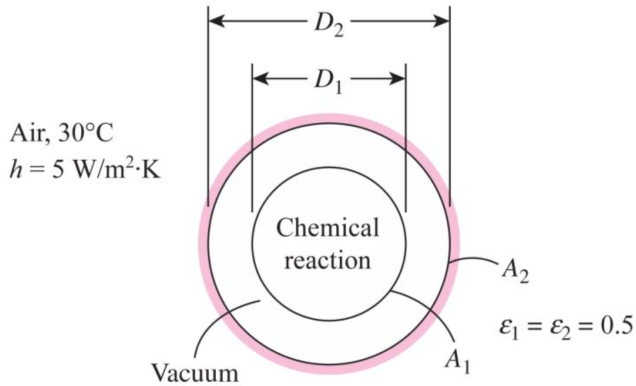
13-82 PtD A spherical tank is filled with chemical in an exothermic reaction that heats up the surface temperature. The tank in enclosed by a concentric outer cover to prevent thermal burn hazards. The temperature of the outer cover is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** The ambient temperature is equal to the temperature of the surrounding surfaces, $T_\infty = T_{surr}$.

Properties The emissivity of both surfaces is given to be $\varepsilon = \varepsilon_1 = \varepsilon_2 = 0.5$.

Analysis The net rate of radiation heat transfer between the two spheres is

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)^2}$$



Radiation heat transfer rate from the outer sphere to the surrounding is

$$\dot{Q}_{rad} = \varepsilon A_2 \sigma(T_2^4 - T_{surr}^4)$$

The natural convection heat transfer rate from the outer surface is

$$\dot{Q}_{conv} = hA_2(T_2 - T_\infty)$$

Performing an energy balance on the outer surface, we have

$$\dot{Q}_{12} = \dot{Q}_{rad} + \dot{Q}_{conv}$$

$$\frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)^2} = \varepsilon A_2 \sigma(T_2^4 - T_{surr}^4) + hA_2(T_2 - T_\infty)$$

$$\frac{D_1^2\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon} + \dfrac{1-\varepsilon}{\varepsilon}\left(\dfrac{D_1}{D_2}\right)^2} = \varepsilon D_2^2 \sigma(T_2^4 - T_{surr}^4) + hD_2^2(T_2 - T_\infty)$$

Hence,

$$\frac{(3\,\text{m})^2(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(393^4 - T_2^4)\,\text{K}^4}{\dfrac{1}{0.5} + \dfrac{1-0.5}{0.5}\left(\dfrac{3\,\text{m}}{3.1\,\text{m}}\right)^2} = (0.5)(3.1\,\text{m})^2(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(T_2^4 - 303^4)\,\text{K}^4$$

$$+(5\ \text{W/m}^2\cdot\text{K})(3.1\,\text{m})^2(T_2 - 303)\,\text{K}$$

The outer cover temperature T_2 can be solved by trial-and-error to yield T_2 = **55.7°C** > 45°C.

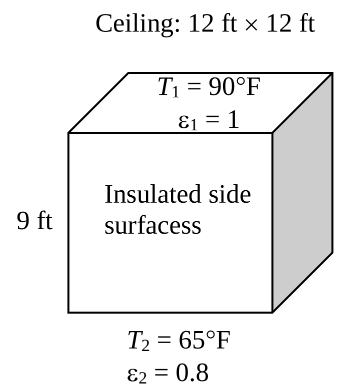
Discussion The outer cover temperature is above the safe temperature of 45°C, and that is a potential thermal burn hazard. This hazard can be alleviated by lowering the emissivity of both surfaces to 0.23 or lower. Another approach is adding radiation shields between the concentric surfaces to lower the radiation heat transfer from the inner surface to the outer surface (see section 13-5 of the text for additional information).

13-83E A room is heated by electric resistance heaters placed on the ceiling which is maintained at a uniform temperature. The rate of heat loss from the room through the floor is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** There is no heat loss through the side surfaces.

Properties The emissivities are $\varepsilon = 1$ for the ceiling and $\varepsilon = 0.8$ for the floor. The emissivity of insulated (or reradiating) surfaces is also 1.

Analysis The room can be considered to be three-surface enclosure with the ceiling surface 1, the floor surface 2 and the side surfaces surface 3. We assume steady-state conditions exist. Since the side surfaces are reradiating, there is no heat transfer through them, and the entire heat lost by the ceiling must be gained by the floor. Then the rate of heat loss from the room through its floor can be determined from



$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1} + R_2}$$

where

$$E_{b1} = \sigma T_1^{\ 4} = (0.1714 \times 10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(90 + 460\ \text{R})^4 = 157\ \text{Btu/h.ft}^2$$

$$E_{b2} = \sigma T_2^{\ 4} = (0.1714 \times 10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(65 + 460\ \text{R})^4 = 130\ \text{Btu/h.ft}^2$$

and

$$A_1 = A_2 = (12\ \text{ft})^2 = 144\ \text{ft}^2$$

The view factor from the floor to the ceiling of the room is $F_{12} = 0.27$ (From Figure 13-5). The view factor from the ceiling or the floor to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.27 = 0.73$$

since the ceiling is flat and thus $F_{11} = 0$. Then the radiation resistances which appear in the equation above become

$$R_2 = \frac{1 - \varepsilon_2}{A_2 \varepsilon_2} = \frac{1 - 0.8}{(144\ \text{ft}^2)(0.8)} = 0.00174\ \text{ft}^{-2}$$

$$R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{(144\ \text{ft}^2)(0.27)} = 0.02572\ \text{ft}^{-2}$$

$$R_{13} = R_{23} = \frac{1}{A_1 F_{13}} = \frac{1}{(144\ \text{ft}^2)(0.73)} = 0.009513\ \text{ft}^{-2}$$

Substituting,

$$\dot{Q}_{12} = \frac{(157 - 130)\ \text{Btu/h.ft}^2}{\left(\dfrac{1}{0.02572\ \text{ft}^{-2}} + \dfrac{1}{2(0.009513\ \text{ft}^{-2})}\right)^{-1} + 0.00174\ \text{ft}^{-2}} = \mathbf{2130\ Btu/h}$$

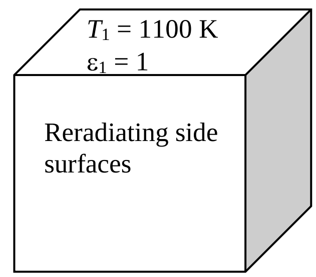
13-84 The floor and the ceiling of a cubical furnace are maintained at uniform temperatures. The net rate of radiation heat transfer between the floor and the ceiling is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of all surfaces are $\varepsilon = 1$ since they are black or reradiating.

Analysis We consider the ceiling to be surface 1, the floor to be surface 2 and the side surfaces to be surface 3. The furnace can be considered to be three-surface enclosure. We assume that steady-state conditions exist. Since the side surfaces are reradiating, there is no heat transfer through them, and the entire heat lost by the ceiling must be gained by the floor. The view factor from the ceiling to the floor of the furnace is $F_{12} = 0.2$. Then the rate of heat loss from the ceiling can be determined from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}}\right)^{-1}}$$



where

$$E_{b1} = \sigma T_1^{\,4} = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1100\ \text{K})^4 = 83{,}015\ \text{W/m}^2$$

$$E_{b2} = \sigma T_2^{\,4} = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(550\ \text{K})^4 = 5188\ \text{W/m}^2$$

and

$$A_1 = A_2 = (4\,\text{m})^2 = 16\,\text{m}^2$$

$$R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{(16\,\text{m}^2)(0.2)} = 0.3125\,\text{m}^{-2}$$

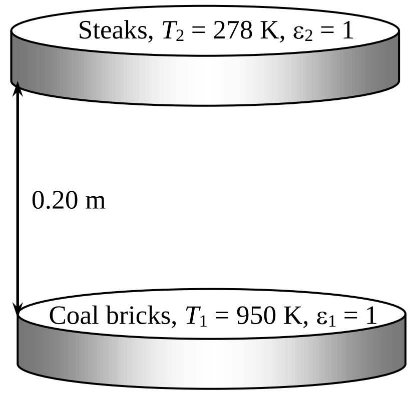
$$R_{13} = R_{23} = \frac{1}{A_1 F_{13}} = \frac{1}{(16\,\text{m}^2)(0.8)} = 0.078125\,\text{m}^{-2}$$

Substituting,

$$\dot{Q}_{12} = \frac{(83{,}015 - 5188)\ \text{W/m}^2}{\left(\dfrac{1}{0.3125\,\text{m}^{-2}} + \dfrac{1}{2(0.078125\,\text{m}^{-2})}\right)^{-1}} = 7.47\times10^5\ \text{W} = \mathbf{747\,kW}$$

13-85 A circular grill is considered. The bottom of the grill is covered with hot coal bricks, while the wire mesh on top of the grill is covered with steaks. The initial rate of radiation heat transfer from coal bricks to the steaks is to be determined for two cases.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Properties The emissivities are $\varepsilon = 1$ for all surfaces since they are black or reradiating.

Analysis We consider the coal bricks to be surface 1, the steaks to be surface 2 and the side surfaces to be surface 3. First we determine the view factor between the bricks and the steaks (Table 13-1),

$$R_i = R_j = \frac{r_i}{L} = \frac{0.15\,\text{m}}{0.20\,\text{m}} = 0.75$$

$$S = 1 + \frac{1 + R_j^{\,2}}{R_i^{\,2}} = \frac{1 + 0.75^2}{0.75^2} = 3.7778$$

$$F_{12} = F_{ij} = \frac{1}{2}\left\{ S - \left[S^2 - 4\left(\frac{R_j}{R_i}\right)^2 \right]^{1/2} \right\} = \frac{1}{2}\left\{ 3.7778 - \left[3.7778^2 - 4\left(\frac{0.75}{0.75}\right)^2 \right]^{1/2} \right\} = 0.2864$$

(It can also be determined from Fig. 13-7).

Then the initial rate of radiation heat transfer from the coal bricks to the stakes becomes

$$\begin{aligned}
\dot{Q}_{12} &= F_{12} A_1 \sigma (T_1^{\,4} - T_2^{\,4}) \\
&= (0.2864)[\pi (0.3\,\text{m})^2/4](5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(950\,\text{K})^4 - (278\,\text{K})^4] \\
&= \mathbf{928\ W}
\end{aligned}$$

When the side opening is closed with aluminum foil, the entire heat lost by the coal bricks must be gained by the stakes since there will be no heat transfer through a reradiating surface. The grill can be considered to be three-surface enclosure. Then the rate of heat loss from the coal bricks can be determined from

$$\dot{Q}_1 = \frac{E_{b1} - E_{b2}}{\left(\dfrac{1}{R_{12}} + \dfrac{1}{R_{13} + R_{23}} \right)^{-1}}$$

where

$$E_{b1} = \sigma T_1^{\,4} = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(950\,\text{K})^4 = 46{,}183\ \text{W/m}^2$$

$$E_{b2} = \sigma T_2^{\,4} = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(5 + 273\,\text{K})^4 = 339\ \text{W/m}^2$$

and $A_1 = A_2 = \dfrac{\pi (0.3\,\text{m})^2}{4} = 0.07069\,\text{m}^2$

$$R_{12} = \frac{1}{A_1 F_{12}} = \frac{1}{(0.07069\,\text{m}^2)(0.2864)} = 49.39\,\text{m}^{-2}$$

$$R_{13} = R_{23} = \frac{1}{A_1 F_{13}} = \frac{1}{(0.07069\,\text{m}^2)(1 - 0.2864)} = 19.82\,\text{m}^{-2}$$

Substituting, $$\dot{Q}_{12} = \frac{(46{,}183 - 339)\ \text{W/m}^2}{\left(\dfrac{1}{49.39\,\text{m}^{-2}} + \dfrac{1}{2(19.82\,\text{m}^{-2})} \right)^{-1}} = \mathbf{2085\ W}$$

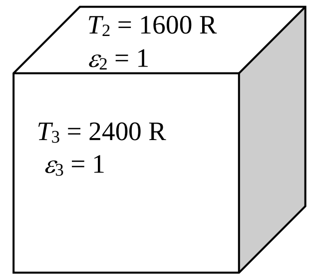
13-86E Top and side surfaces of a cubical furnace are black, and are maintained at uniform temperatures. Net radiation heat transfer rate to the base from the top and side surfaces are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities are given to be $\varepsilon = 0.4$ for the bottom surface and 1 for other surfaces.

Analysis We consider the base surface to be surface 1, the top surface to be surface 2 and the side surfaces to be surface 3. The cubical furnace can be considered to be three-surface enclosure. The areas and blackbody emissive powers of surfaces are

$$A_1 = A_2 = (10\text{ ft})^2 = 100\text{ ft}^2 \qquad A_3 = 4(10\text{ ft})^2 = 400\text{ ft}^2$$



$$E_{b1} = \sigma T_1^4 = (0.1714\times10^{-8}\text{ Btu/h.ft}^2.\text{R}^4)(800\text{ R})^4 = 702\text{ Btu/h.ft}^2$$

$$E_{b2} = \sigma T_2^4 = (0.1714\times10^{-8}\text{ Btu/h.ft}^2.\text{R}^4)(1600\text{ R})^4 = 11{,}233\text{ Btu/h.ft}^2$$

$$E_{b3} = \sigma T_3^4 = (0.1714\times10^{-8}\text{ Btu/h.ft}^2.\text{R}^4)(2400\text{ R})^4 = 56{,}866\text{ Btu/h.ft}^2$$

The view factor from the base to the top surface of the cube is $F_{12} = 0.2$. From the summation rule, the view factor from the base or top to the side surfaces is

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.2 = 0.8$$

since the base surface is flat and thus $F_{11} = 0$. Then the radiation resistances become

$$R_1 = \frac{1-\varepsilon_1}{A_1\varepsilon_1} = \frac{1-0.4}{(100\text{ ft}^2)(0.4)} = 0.015\text{ ft}^{-2} \qquad R_{12} = \frac{1}{A_1F_{12}} = \frac{1}{(100\text{ ft}^2)(0.2)} = 0.0500\text{ ft}^{-2}$$

$$R_{13} = \frac{1}{A_1F_{13}} = \frac{1}{(100\text{ ft}^2)(0.8)} = 0.0125\text{ ft}^{-2}$$

Note that the side and the top surfaces are black, and thus their radiosities are equal to their emissive powers. The radiosity of the base surface is determined

$$\frac{E_{b1}-J_1}{R_1} + \frac{E_{b2}-J_1}{R_{12}} + \frac{E_{b3}-J_1}{R_{13}} = 0$$

Substituting,

$$\frac{702-J_1}{0.015} + \frac{11{,}233-J_1}{0.05} + \frac{56{,}866-J_1}{0.0125} = 0 \longrightarrow J_1 = 28{,}925\text{ W/m}^2$$

(*a*) The net rate of radiation heat transfer between the base and the side surfaces is

$$\dot{Q}_{31} = \frac{E_{b3}-J_1}{R_{13}} = \frac{(56{,}866-28{,}915)\text{ Btu/h.ft}^2}{0.0125\text{ ft}^{-2}} = \mathbf{2.235\times10^6\ Btu/h}$$

(*b*) The net rate of radiation heat transfer between the base and the top surfaces is

$$\dot{Q}_{12} = \frac{J_1-E_{b2}}{R_{12}} = \frac{(28{,}925-11{,}233)\text{ Btu/h.ft}^2}{0.05\text{ ft}^{-2}} = \mathbf{3.538\times10^5\ Btu/h}$$

The net rate of radiation heat transfer to the base surface is finally determined from

$$\dot{Q}_1 = \dot{Q}_{21} + \dot{Q}_{31} = -3.538\times10^5 + 2.235\times10^6 = \mathbf{1.882\times10^6\ Btu/h}$$

Discussion The same result can be found form

$$\dot{Q}_1 = \frac{J_1-E_{b1}}{R_1} = \frac{(28{,}925-702)\text{ Btu/h.ft}^2}{0.015\text{ ft}^{-2}} = 1.882\times10^6\text{ Btu/h}$$

The result is the same as expected.

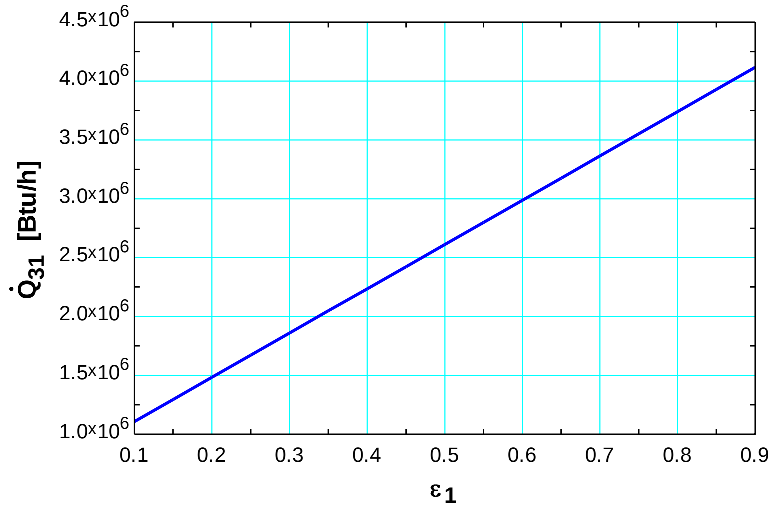
13-87E Prob. 13-86E is reconsidered. The effect of base surface emissivity on the net rates of radiation heat transfer between the base and the side surfaces, between the base and top surfaces, and to the base surface is to be investigated.

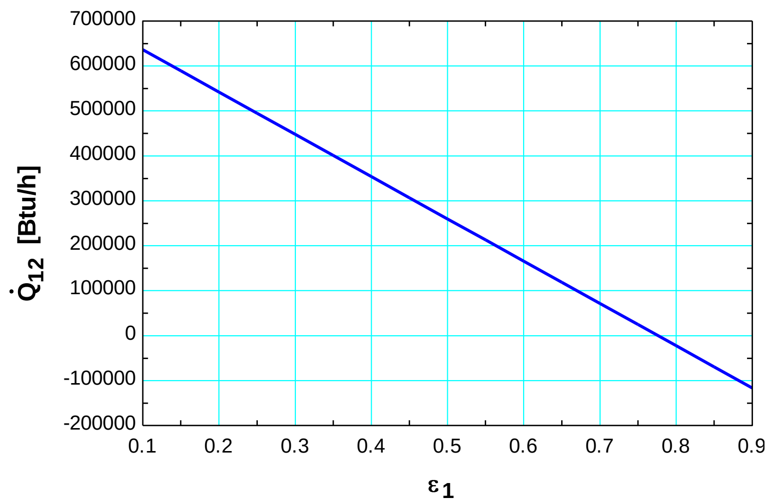
Analysis The problem is solved using EES, and the solution is given below.

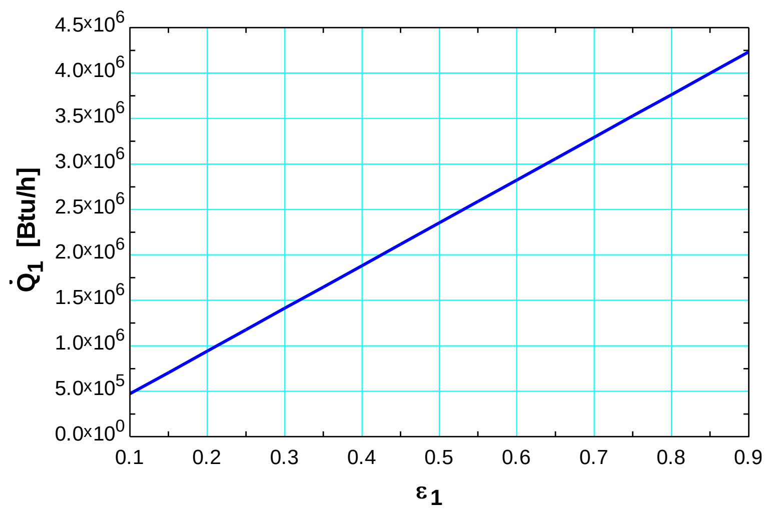
```
"GIVEN"
a=10 [ft]
epsilon_1=0.4
T_1=800 [R]
T_2=1600 [R]
T_3=2400 [R]

"ANALYSIS"
sigma=0.1714E-8 [Btu/h-ft^2-R^4] "Stefan-Boltzmann constant"
"Consider the base surface 1, the top surface 2, and the side surface 3"
E_b1=sigma*T_1^4
E_b2=sigma*T_2^4
E_b3=sigma*T_3^4
A_1=a^2
A_2=A_1
A_3=4*a^2
F_12=0.2 "view factor from the base to the top of a cube"
F_11+F_12+F_13=1 "summation rule"
F_11=0 "since the base surface is flat"
R_1=(1-epsilon_1)/(A_1*epsilon_1) "surface resistance"
R_12=1/(A_1*F_12) "space resistance"
R_13=1/(A_1*F_13) "space resistance"
(E_b1-J_1)/R_1+(E_b2-J_1)/R_12+(E_b3-J_1)/R_13=0 "J_1 : radiosity of base surface"
"(a)"
Q_dot_31=(E_b3-J_1)/R_13
"(b)"
Q_dot_12=(J_1-E_b2)/R_12
Q_dot_21=-Q_dot_12
Q_dot_1=Q_dot_21+Q_dot_31
```

ε_1	$\dot{Q}_{31}$ [Btu/h]	$\dot{Q}_{12}$ [Btu/h]	$\dot{Q}_1$ [Btu/h]
0.1	1.106E+06	636061	470376
0.15	1.295E+06	589024	705565
0.2	1.483E+06	541986	940753
0.25	1.671E+06	494948	1.176E+06
0.3	1.859E+06	447911	1.411E+06
0.35	2.047E+06	400873	1.646E+06
0.4	2.235E+06	353835	1.882E+06
0.45	2.423E+06	306798	2.117E+06
0.5	2.612E+06	259760	2.352E+06
0.55	2.800E+06	212722	2.587E+06
0.6	2.988E+06	165685	2.822E+06
0.65	3.176E+06	118647	3.057E+06
0.7	3.364E+06	71610	3.293E+06
0.75	3.552E+06	24572	3.528E+06
0.8	3.741E+06	-22466	3.763E+06
0.85	3.929E+06	-69503	3.998E+06
0.9	4.117E+06	-116541	4.233E+06


4.5x10^6
4.0x10^6
3.5x10^6
3.0x10^6
2.5x10^6
2.0x10^6
1.5x10^6
1.0x10^6
$\dot{Q}_{31}$ [Btu/h]
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
ε_1


700000
600000
500000
400000
300000
200000
100000
0
-100000
-200000
$\dot{Q}_{12}$ [Btu/h]
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
ε_1


4.5x10^6
4.0x10^6
3.5x10^6
3.0x10^6
2.5x10^6
2.0x10^6
1.5x10^6
1.0x10^6
5.0x10^5
0.0x10^0
$\dot{Q}_1$ [Btu/h]
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
ε_1

Radiation Shields and the Radiation Effect

13-88C Radiation heat transfer between two surfaces can be reduced greatly by inserting a thin, high reflectivity(low emissivity) sheet of material between the two surfaces. Such highly reflective thin plates or shells are known as radiation shields. Multilayer radiation shields constructed of about 20 shields per cm. thickness separated by evacuated space are commonly used in cryogenic and space applications to minimize heat transfer. Radiation shields are also used in temperature measurements of fluids to reduce the error caused by the radiation effect.

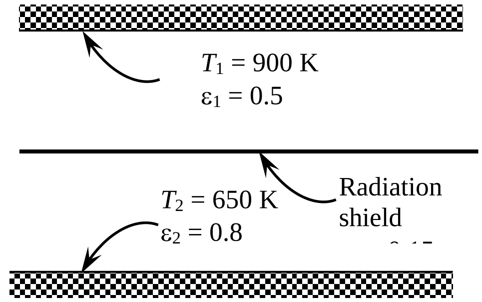
13-89C The influence of radiation on heat transfer or temperature of a surface is called the radiation effect. The radiation exchange between the sensor and the surroundings may cause the thermometer to indicate a different reading for the medium temperature. To minimize the radiation effect, the sensor should be coated with a material of high reflectivity (low emissivity).

13-90C A person who feels fine in a room at a specified temperature may feel chilly in another room at the same temperature as a result of radiation effect if the walls of second room are at a considerably lower temperature. For example most people feel comfortable in a room at 22°C if the walls of the room are also roughly at that temperature. When the wall temperature drops to 5°C for some reason, the interior temperature of the room must be raised to at least 27°C to maintain the same level of comfort. Also, people sitting near the windows of a room in winter will feel colder because of the radiation exchange between the person and the cold windows.

13-91 A thin aluminum sheet is placed between two very large parallel plates that are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined for the cases of with and without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.8$, and $\varepsilon_3 = 0.15$.



Analysis The net rate of radiation heat transfer with a thin aluminum shield per unit area of the plates is

$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_{3,1}}+\frac{1}{\varepsilon_{3,2}}-1\right)}$$

$$= \frac{(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(900\ \text{K})^4 - (650\ \text{K})^4]}{\left(\frac{1}{0.5}+\frac{1}{0.8}-1\right)+\left(\frac{1}{0.15}+\frac{1}{0.15}-1\right)}$$

$$= \mathbf{1857\ W/m^2}$$

The net rate of radiation heat transfer between the plates in the case of no shield is

$$\dot{Q}_{12,\textit{no}\text{ shield}} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)} = \frac{(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(900\ \text{K})^4 - (650\ \text{K})^4]}{\left(\frac{1}{0.5}+\frac{1}{0.8}-1\right)} = 12{,}035\ \text{W/m}^2$$

Then the ratio of radiation heat transfer for the two cases becomes

$$\frac{\dot{Q}_{12,\text{one shield}}}{\dot{Q}_{12,\text{no shield}}} = \frac{1857\ \text{W}}{12{,}035\ \text{W}} \cong \mathbf{\frac{1}{6}}$$

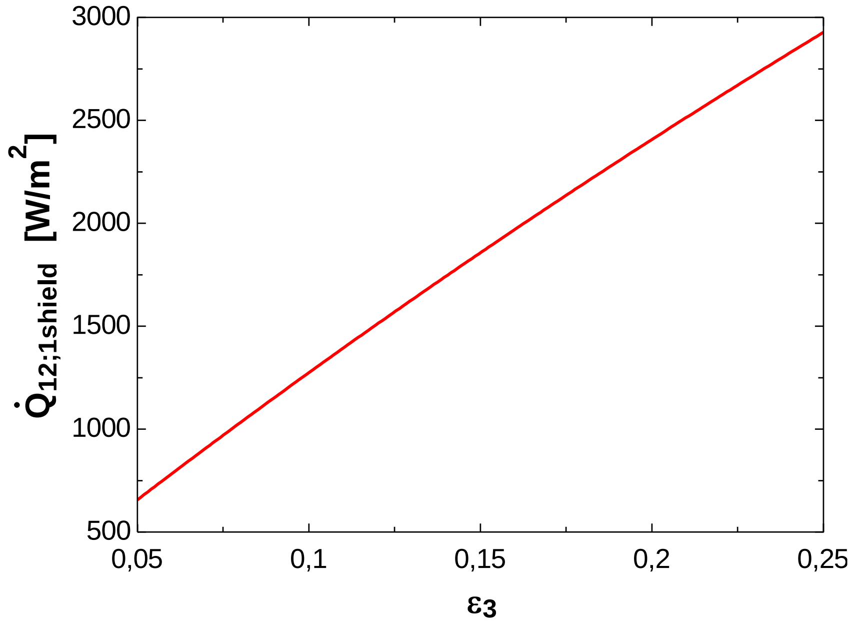
13-92  Prob. 13-91 is reconsidered. The net rate of radiation heat transfer between the two plates as a function of the emissivity of the aluminum sheet is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
epsilon_3=0.15
T_1=900 [K]
T_2=650 [K]
epsilon_1=0.5
epsilon_2=0.8

"ANALYSIS"
sigma=5.67E-8 [W/m^2-K^4]
Q_dot_12_1shield=(sigma*(T_1^4-T_2^4))/((1/epsilon_1+1/epsilon_2-1)+(1/epsilon_3+1/epsilon_3-1))
```

ε_3	$\dot{Q}_{12,1shield}$ [W/m^2]
0.05	656.5
0.06	783
0.07	908.1
0.08	1032
0.09	1154
0.1	1274
0.11	1394
0.12	1511
0.13	1628
0.14	1743
0.15	1857
0.16	1969
0.17	2081
0.18	2191
0.19	2299
0.2	2407
0.21	2513
0.22	2619
0.23	2723
0.24	2826
0.25	2928



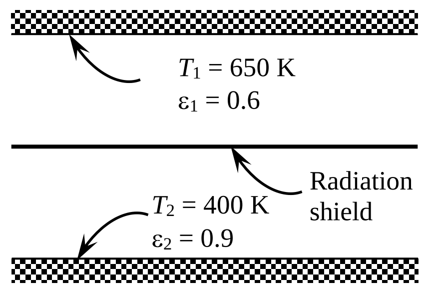
13-93 A radiation shield is placed between two large parallel plates which are maintained at uniform temperatures. The emissivity of the radiation shield is to be determined if the radiation heat transfer between the plates is reduced to 15% of that without the radiation shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.6$ and $\varepsilon_2 = 0.9$.

Analysis First, the net rate of radiation heat transfer between the two large parallel plates per unit area without a shield is

$$\dot{Q}_{12,\text{no shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1}+\dfrac{1}{\varepsilon_2}-1} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(650\ \text{K})^4-(400\ \text{K})^4]}{\dfrac{1}{0.6}+\dfrac{1}{0.9}-1} = 4877\ \text{W/m}^2$$



The radiation heat transfer in the case of one shield is

$$\dot{Q}_{12,\text{one shield}} = 0.15\times\dot{Q}_{12,\text{no shield}}$$
$$= 0.15\times4877\ \text{W/m}^2 = 731.6\ \text{W/m}^2$$

Then the emissivity of the radiation shield becomes

$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1}+\dfrac{1}{\varepsilon_2}-1\right)+\left(\dfrac{1}{\varepsilon_{3,1}}+\dfrac{1}{\varepsilon_{3,2}}-1\right)}$$

$$731.6\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(650\ \text{K})^4-(400\ \text{K})^4]}{\left(\dfrac{1}{0.6}+\dfrac{1}{0.9}-1\right)+\left(\dfrac{2}{\varepsilon_3}-1\right)}$$

which gives $\varepsilon_3 = \mathbf{0.18}$

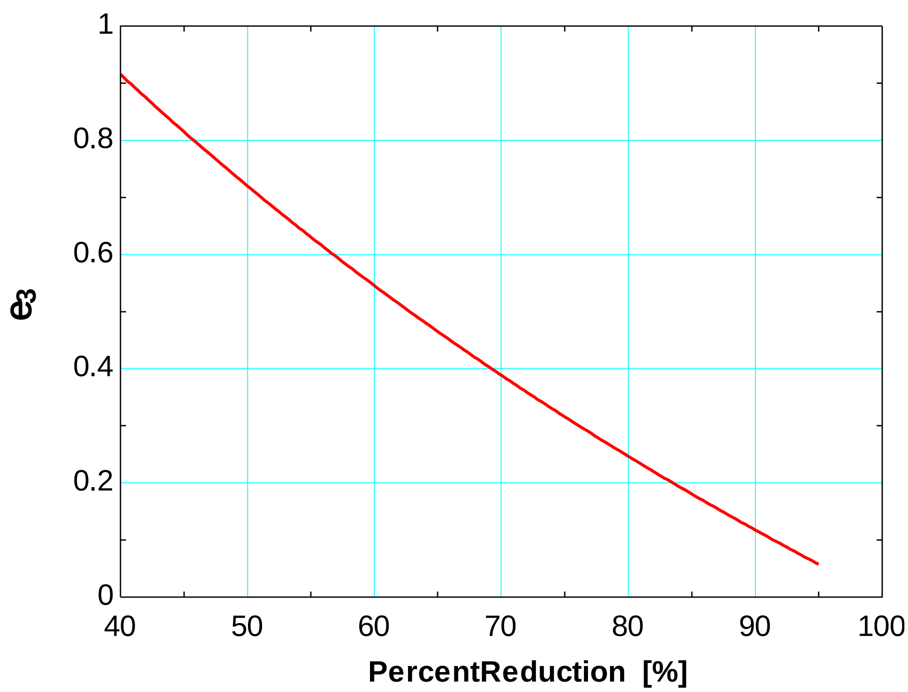
13-94 Prob. 13-93 is reconsidered. The effect of the percent reduction in the net rate of radiation heat transfer between the plates on the emissivity of the radiation shields is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_1=650 [K]
T_2=400 [K]
epsilon_1=0.6
epsilon_2=0.9
PercentReduction=85 “[%]”

"ANALYSIS"
sigma=5.67E-8 [W/m^2-K^4] “Stefan-Boltzmann constant"
Q_dot_12_NoShield=(sigma*(T_1^4-T_2^4))/(1/epsilon_1+1/epsilon_2-1)
Q_dot_12_1shield=(sigma*(T_1^4-T_2^4))/((1/epsilon_1+1/epsilon_2-1)+(1/epsilon_3+1/epsilon_3-1))
Q_dot_12_1shield=(1-PercentReduction/100)*Q_dot_12_NoShield
```

Percent Reduction [%]	ε_3
40	0.9153
45	0.8148
50	0.72
55	0.6304
60	0.5455
65	0.4649
70	0.3885
75	0.3158
80	0.2466
85	0.1806
90	0.1176
95	0.05751



13-95 C&S A large ASTM B152 copper plate and a large ceramic plate are placed in parallel near each other. The temperature of the ceramic plate and the net radiation heat flux between the two plates are known. The emissivity of a radiation shield placed between the two plates is to be determined so that the copper plate surface does not exceed 260°C.

Assumptions **1** Steady state conditions. **2** Uniform surface temperatures on both plates. **3** Surfaces are opaque, gray and diffuse. **4** Uniform emissivity on the radiation shield surfaces.

Properties The emissivities of the ceramic and copper plates are given as $\varepsilon_1 = 0.91$ and $\varepsilon_2 = 0.95$, respectively.

Analysis For large parallel plates with a radiation shield in between them, the net rate of radiation heat transfer between the two plates is

$$\dot{Q}_{12,\text{ 1-shield}} = \frac{\sigma A(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{3,1}} + \frac{1}{\varepsilon_{3,2}} - 1\right)}$$

or in heat flux as (with $\varepsilon_{3,1} = \varepsilon_{3,2}$)

$$\dot{q}_{12,\text{ 1-shield}} = \frac{\dot{Q}_{12,\text{ 1-shield}}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{2}{\varepsilon_3} - 1\right)}$$

Solving for the emissivity of the radiation shield ε_3, with $T_2 = 533$ K (maximum use T for the copper plate),

$$\varepsilon_3 = 2\left[\frac{\sigma(T_1^4 - T_2^4)}{\dot{q}_{12,\text{ 1-shield}}} - \frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_2} + 2\right]^{-1}$$

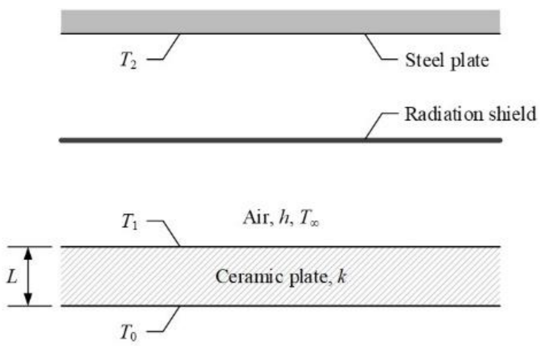
$$\varepsilon_3 = 2\left[\frac{(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4)(873^4 - 533^4)\text{K}^4}{1000\ \text{W/m}^2} - \frac{1}{0.91} - \frac{1}{0.95} + 2\right]^{-1} = \mathbf{0.071}$$

Discussion A radiation shield with an emissivity of 0.071 or lower would keep the copper plate from heating above the maximum use temperature of 260°C. Without the radiation shield, the copper plate surface would be at 592°C, exceeding the maximum use temperature.

13-96 C&S A large ASTM A992 carbon steel plate and a large ceramic plate are placed in parallel near each other. Multimode heat transfer involving conduction, convection, and radiation occur. The emissivity of a radiation shield placed between the two plates is to be determined so that the steel plate surface does not exceed 427°C.

Assumptions **1** Steady state conditions. **2** Uniform surface temperatures on both plates. **3** Surfaces are opaque, gray and diffuse. **4** One-dimensional conduction through the ceramic plate. **5** Uniform emissivity on the radiation shield surfaces.

Properties The emissivities of the ceramic and steel plates are given as ε_1 = 0.93 and ε_2 = 0.75, respectively. The ceramic plate thermal conductivity is given as k = 10 W/m·K.



Analysis Applying energy balance on the ceramic plate,

$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} + \dot{q}_{\text{rad}}$$

where

$$\dot{q}_{\text{cond}} = k\frac{T_0 - T_1}{L} = (10\ \text{W/m}\cdot\text{K})\frac{(800-700)\text{K}}{0.1\ \text{m}} = 10000\ \text{W/m}^2$$

$$\dot{q}_{\text{conv}} = h(T_1 - T_\infty) = (12\ \text{W/m}^2\cdot\text{K})(700-20)\text{K} = 8160\ \text{W/m}^2$$

$$\dot{q}_{\text{rad}} = \dot{q}_{12,\ \text{1-shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_{3,1}}+\frac{1}{\varepsilon_{3,2}}-1\right)}$$

Thus (with $\varepsilon_{3,1} = \varepsilon_{3,2}$),

$$\dot{q}_{\text{cond}} = \dot{q}_{\text{conv}} + \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_3}-1\right)}$$

Solving for the emissivity of the radiation shield ε_3, with T_2 = 700 K (maximum use T for the steel plate),

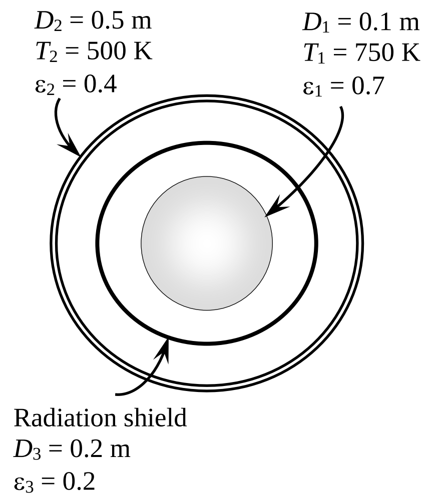
$$\varepsilon_3 = 2\left[\frac{\sigma(T_1^4 - T_2^4)}{\dot{q}_{\text{cond}} - \dot{q}_{\text{conv}}} - \frac{1}{\varepsilon_1} - \frac{1}{\varepsilon_2} + 2\right]^{-1}$$

$$\varepsilon_3 = 2\left[\frac{(5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4)(973^4 - 700^4)\text{K}^4}{(10000-8160)\text{W/m}^2} - \frac{1}{0.93} - \frac{1}{0.75} + 2\right]^{-1} = \mathbf{0.10}$$

Discussion A radiation shield with an emissivity of 0.1 or lower would keep the carbon steel plate from heating above the maximum use temperature of 427°C. Without the radiation shield, the carbon steel plate surface would be at 687°C, exceeding the maximum use temperature.

13-97 A coaxial radiation shield is placed between two coaxial cylinders which are maintained at uniform temperatures. The net rate of radiation heat transfer between the two cylinders is to be determined and compared with that without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.7$, $\varepsilon_2 = 0.4$. and $\varepsilon_3 = 0.2$.

Analysis The surface areas of the cylinders and the shield per unit length are

$$A_{\text{pipe,inner}} = A_1 = \pi D_1 L = \pi(0.1\,\text{m})(1\,\text{m}) = 0.314\,\text{m}^2$$

$$A_{\text{pipe,outer}} = A_2 = \pi D_2 L = \pi(0.5\,\text{m})(1\,\text{m}) = 1.571\,\text{m}^2$$

$$A_{\text{shield}} = A_3 = \pi D_3 L = \pi(0.2\,\text{m})(1\,\text{m}) = 0.628\,\text{m}^2$$

The net rate of radiation heat transfer between the two cylinders with a shield per unit length is

$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{13}} + \dfrac{1-\varepsilon_{3,1}}{A_3\varepsilon_{3,1}} + \dfrac{1-\varepsilon_{3,2}}{A_3\varepsilon_{3,2}} + \dfrac{1}{A_3F_{32}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(750\ \text{K})^4 - (500\ \text{K})^4]}{\dfrac{1-0.7}{(0.314)(0.7)} + \dfrac{1}{(0.314)(1)} + 2\dfrac{1-0.2}{(0.628)(0.2)} + \dfrac{1}{(0.628)(1)} + \dfrac{1-0.4}{(1.571)(0.4)}}$$

$$= \mathbf{726\ W}$$

If there was no shield,

$$\dot{Q}_{12,\text{no shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)} = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(750\ \text{K})^4 - (500\ \text{K})^4]}{\dfrac{1}{0.7} + \dfrac{1-0.4}{0.4}\left(\dfrac{0.1}{0.5}\right)} = \mathbf{8329\ W}$$

Then their ratio becomes

$$\frac{\dot{Q}_{12,\text{one shield}}}{\dot{Q}_{12,\text{no shield}}} = \frac{726\ \text{W}}{8329\ \text{W}} = \mathbf{0.0872}$$

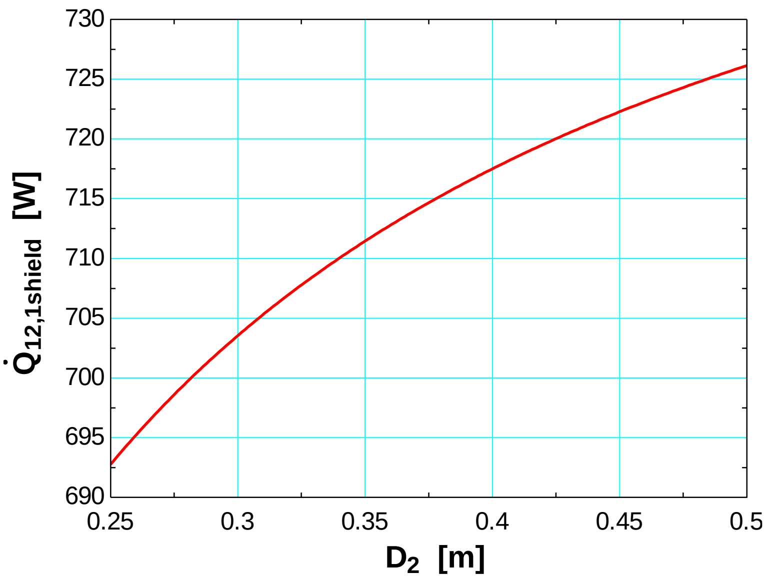
13-98  Prob. 13-97 is reconsidered. The effects of the diameter of the outer cylinder and the emissivity of the radiation shield on the net rate of radiation heat transfer between the two cylinders are to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

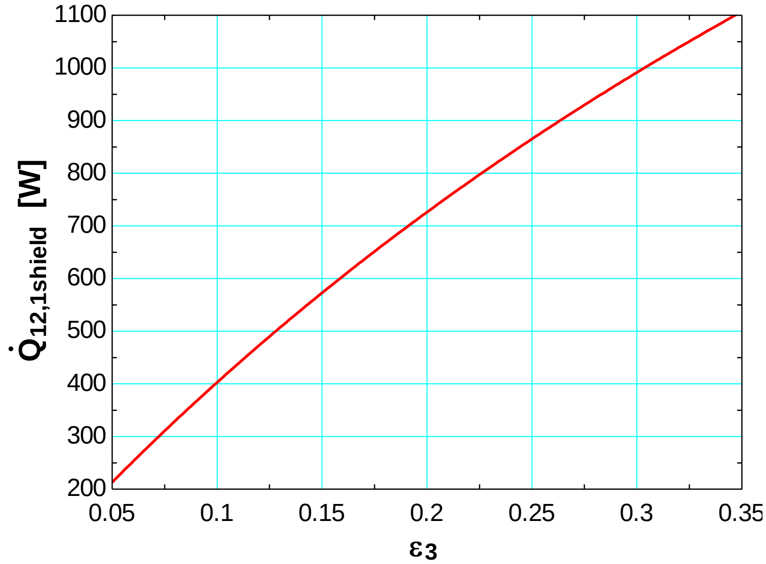
```
"GIVEN"
D_1=0.10 [m]
D_2=0.50 [m]
D_3=0.20 [m]
epsilon_1=0.7
epsilon_2=0.4
epsilon_3=0.2
T_1=750 [K]
T_2=500 [K]

"ANALYSIS"
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"
L=1 [m] "a unit length of the cylinders is considered"
A_1=pi*D_1*L
A_2=pi*D_2*L
A_3=pi*D_3*L
F_13=1
F_32=1
Q_dot_12_1shield=(sigma*(T_1^4-T_2^4))/((1-epsilon_1)/(A_1*epsilon_1)+1/(A_1*F_13)+(1-
epsilon_3)/(A_3*epsilon_3)+(1-epsilon_3)/(A_3*epsilon_3)+1/(A_3*F_32)+(1-epsilon_2)/(A_2*epsilon_2))
```

D_2 [m]	$\dot{Q}_{12,1shield}$ [W]
0.25	692.8
0.275	698.6
0.3	703.5
0.325	707.8
0.35	711.4
0.375	714.7
0.4	717.5
0.425	720
0.45	722.3
0.475	724.3
0.5	726.1



ε_3	$\dot{Q}_{12,1shield}$ [W]
0.05	213.1
0.07	291.5
0.09	366.5
0.11	438.3
0.13	507
0.15	572.9
0.17	636
0.19	696.7
0.21	755
0.23	811.1
0.25	865
0.27	917
0.29	967.1
0.31	1015
0.33	1062
0.35	1107



13-99 C&S A long cylindrical fuel rod is enclosed by a concentric stainless steel tube. The surface temperature of the fuel rod and the radiation heat transfer rate from the fuel rod to the stainless steel tube are known. (*a*) The emissivity of a concentric radiation shield placed between the fuel rod and the stainless steel tube is to be determined so that the stainless steel tube does not exceed 260°C. (*b*) The temperature of the stainless steel tube, if there is no radiation shield, is to be determined.

Assumptions **1** Steady state conditions. **2** Uniform surface temperatures on fuel rod and tube. **3** Surfaces are gray and diffuse. **4** Uniform emissivity on the radiation shield surfaces.

Properties The emissivities of the fuel rod and ASTM A249 904L tube are given as ε_1 = 0.97 and ε_2 = 0.33, respectively.

Analysis (*a*) For long concentric cylinders with a radiation shield in between, the net rate of radiation heat transfer is

$$\dot{Q}_{12,\text{ 1-shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{13}} + \dfrac{1-\varepsilon_{3,1}}{A_3\varepsilon_{3,1}} + \dfrac{1-\varepsilon_{3,2}}{A_3\varepsilon_{3,2}} + \dfrac{1}{A_3F_{32}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

Note that surfaces 1, 2, and 3 are the fuel rod, the stainless steel tube and the radiation shield, respectively. The surface areas are

$$A_1 = \pi D_1 L, A_2 = \pi D_2 L, \qquad A_3 = \pi D_3 L$$

Where

$$D_1 = 0.03\text{ m}, \qquad D_2 = 0.06\text{ m}, \qquad D_3 = 0.045\text{ m}$$

With the view factors $F_{13} = F_{32} = 1$ and $\varepsilon_{3,1} = \varepsilon_{3,2}$,

$$\frac{\dot{Q}_{12,\text{ 1-shield}}}{L} = \frac{\sigma\pi(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{D_1\varepsilon_1} + \dfrac{1}{D_1} + 2\left(\dfrac{1-\varepsilon_3}{D_3\varepsilon_3}\right) + \dfrac{1}{D_3} + \dfrac{1-\varepsilon_2}{D_2\varepsilon_2}} = 120\text{ W/m}$$

Solving for the emissivity of the radiation shield ε_3, with T_2 = 533 K (maximum use T for the stainless steel tube),

$$\varepsilon_3 = 2\left[\frac{\sigma\pi D_3(T_1^4 - T_2^4)}{\dot{Q}_{12,\text{ 1-shield}}/L} - \frac{1-\varepsilon_1}{\varepsilon_1}\left(\frac{D_3}{D_1}\right) - \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_3}{D_2}\right) - \frac{D_3}{D_1} + 1\right]^{-1}$$

$$\varepsilon_3 = 2\left[\frac{\pi(5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4)(0.045\text{ m})(823^4 - 533^4)\text{K}^4}{120\text{ W/m}} - \frac{1-0.97}{0.97}\left(\frac{4.5\text{ cm}}{3\text{ cm}}\right) - \frac{1-0.33}{0.33}\left(\frac{4.5\text{ cm}}{6\text{ cm}}\right) - \frac{4.5\text{ cm}}{3\text{ cm}} + 1\right]^{-1} = \mathbf{0.086}$$

(*b*) For long concentric cylinders without radiation shield, the net rate of radiation heat transfer is

$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

With the view factor $F_{12} = 1$,

$$\frac{\dot{Q}_{12}}{L} = \frac{\sigma \pi D_1 (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)} = 120 \text{ W/m}$$

Solving for the temperature of the stainless steel tube T_2,

$$T_2 = \left\{T_1^4 - \left(\frac{\dot{Q}_{12}/L}{\sigma \pi D_1}\right)\left[\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)\right]\right\}^{1/4}$$

$$T_2 = \left\{(823 \text{ K})^4 - \left[\frac{120 \text{ W/m}}{(5.67\times 10^{-8} \text{W/m}^2\cdot\text{K}^4)\pi(0.03 \text{ m})}\right]\left[\frac{1}{0.97} + \frac{1-0.33}{0.33}\left(\frac{3 \text{ cm}}{6 \text{ cm}}\right)\right]\right\}^{1/4} = 802 \text{ K} = \mathbf{529}℃ > 260℃$$

Discussion A radiation shield with an emissivity of 0.086 or lower would keep the stainless steel tube from heating above the maximum use temperature of 260°C. Without the radiation shield, the stainless steel tube would be at 529°C, exceeding the maximum use temperature.

13-100 Two very large plates are maintained at uniform temperatures. The number of thin aluminum sheets that will reduce the net rate of radiation heat transfer between the two plates to one-fifth is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

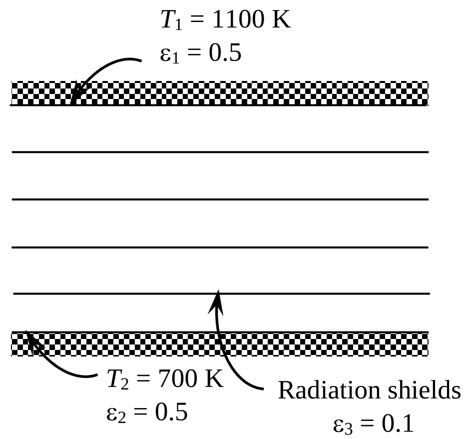
Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.5$, $\varepsilon_2 = 0.5$, and $\varepsilon_3 = 0.1$.

Analysis The net rate of radiation heat transfer between the plates in the case of no shield is

$$\dot{Q}_{12,no\,\text{shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1100\ \text{K})^4 - (700\ \text{K})^4]}{\left(\dfrac{1}{0.5} + \dfrac{1}{0.5} - 1\right)}$$

$$= 23{,}134\ \text{W/m}^2$$



The number of sheets that need to be inserted in order to reduce the net rate of heat transfer between the two plates to one-fifth can be determined from

$$\dot{Q}_{12,\text{shields}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + N_{\text{shield}}\left(\dfrac{1}{\varepsilon_{3,1}} + \dfrac{1}{\varepsilon_{3,2}} - 1\right)}$$

$$\frac{1}{5}(23{,}134\ \text{W/m}^2) = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1100\ \text{K})^4 - (700\ \text{K})^4]}{\left(\dfrac{1}{0.5} + \dfrac{1}{0.5} - 1\right) + N_{\text{shield}}\left(\dfrac{1}{0.1} + \dfrac{1}{0.1} - 1\right)}$$

$$N_{\text{shield}} = 1.48 \cong \mathbf{2}$$

That is, only two sheets are more than enough to reduce heat transfer to one-fifth.

The number of sheets that need to be inserted in order to reduce the net rate of heat transfer between the two plates to one-fifth can be determined from

$$\dot{Q}_{12,\text{shields}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + N_{\text{shield}}\left(\dfrac{1}{\varepsilon_{3,1}} + \dfrac{1}{\varepsilon_{3,2}} - 1\right)}$$

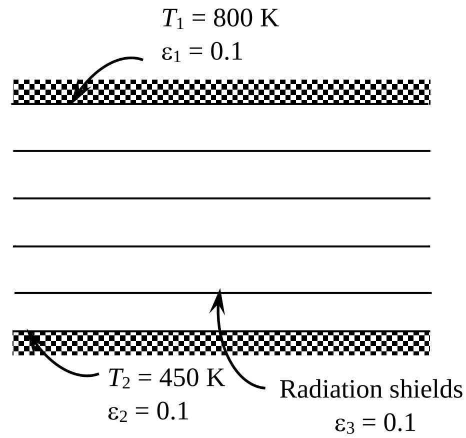
$$\frac{1}{5}(11{,}159\ \text{W/m}^2) = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(1000\ \text{K})^4 - (800\ \text{K})^4]}{\left(\dfrac{1}{0.5} + \dfrac{1}{0.5} - 1\right) + N_{\text{shield}}\left(\dfrac{1}{0.1} + \dfrac{1}{0.1} - 1\right)}$$

$$N_{\text{shield}} = 0.632 \cong \mathbf{1}$$

That is, only one sheet with a low emissivity is more than enough to reduce heat transfer to one-fifth.

13-101 Five identical thin aluminum sheets are placed between two very large parallel plates which are maintained at uniform temperatures. The net rate of radiation heat transfer between the two plates is to be determined and compared with that without the shield.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.



Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.1$ and $\varepsilon_3 = 0.1$.

Analysis Since the plates and the sheets have the same emissivity value, the net rate of radiation heat transfer with 5 thin aluminum shield can be determined from

$$\dot{Q}_{12,5\text{ shield}} = \frac{1}{N+1}\dot{Q}_{12,\text{no shield}} = \frac{1}{N+1}\frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1}+\dfrac{1}{\varepsilon_2}-1\right)}$$

$$= \frac{1}{5+1}\frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(800\text{ K})^4-(450\text{ K})^4]}{\left(\dfrac{1}{0.1}+\dfrac{1}{0.1}-1\right)}$$

$$= \mathbf{183\ W/m^2}$$

The net rate of radiation heat transfer without the shield is

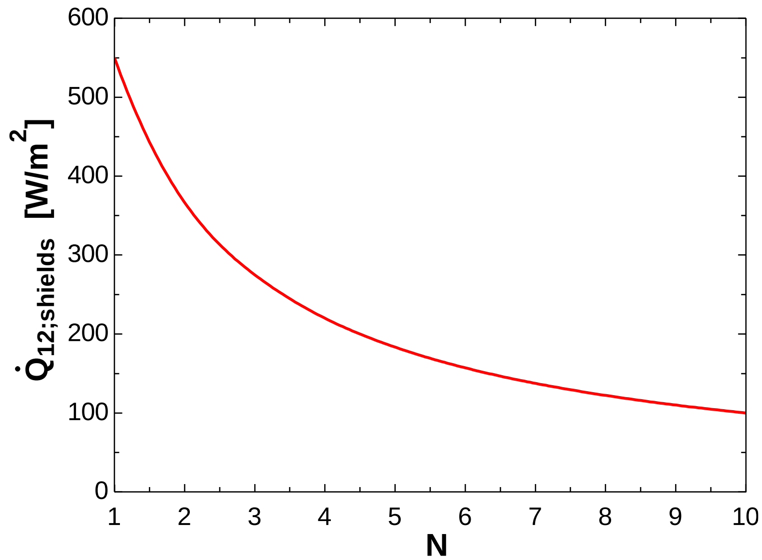
$$\dot{Q}_{12,5\text{ shield}} = \frac{1}{N+1}\dot{Q}_{12,\text{ no shield}} \longrightarrow \dot{Q}_{12,\text{ no shield}} = (N+1)\dot{Q}_{12,5\text{ shield}} = 6\times183\text{ W} = \mathbf{1098\ W}$$

13-102 Prob. 13-101 is reconsidered. The effects of the number of the aluminum sheets and the emissivities of the plates on the net rate of radiation heat transfer between the two plates are to be investigated.

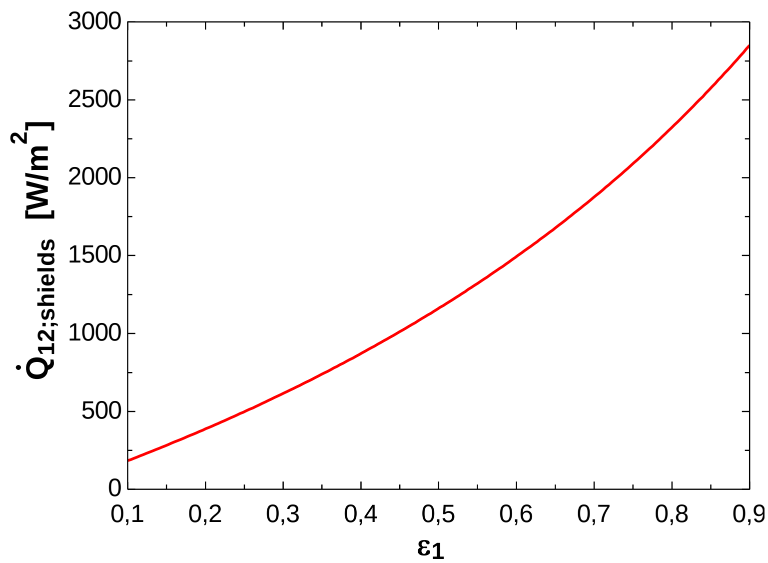
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
N=5
epsilon_3=0.1
epsilon_1=0.1
epsilon_2=epsilon_1
T_1=800 [K]
T_2=450 [K]
"ANALYSIS"
sigma=5.67E-8 [W/m^2-K^4] "Stefan-Boltzmann constant"
Q_dot_12_shields=1/(N+1)*Q_dot_12_NoShield
Q_dot_12_NoShield=(sigma*(T_1^4-T_2^4))/(1/epsilon_1+1/epsilon_2-1)
```

N	$\dot{Q}_{12,shields}$ [W/m^2]
1	550
2	366.7
3	275
4	220
5	183.3
6	157.1
7	137.5
8	122.2
9	110
10	100



ε_1	$\dot{Q}_{12,shields}$ [W/m^2]
0.1	183.3
0.15	282.4
0.2	387
0.25	497.6
0.3	614.7
0.35	738.9
0.4	870.8
0.45	1011
0.5	1161
0.55	1321
0.6	1493
0.65	1677
0.7	1876
0.75	2090
0.8	2322
0.85	2575
0.9	2850

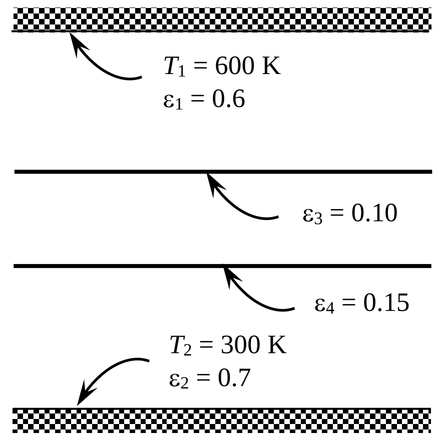


13-103 Two thin radiation shields are placed between two large parallel plates that are maintained at uniform temperatures. The net rate of radiation heat transfer between the plates with and without the shields, and the temperatures of radiation shields are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.6$, $\varepsilon_2 = 0.7$, $\varepsilon_3 = 0.10$, and $\varepsilon_4 = 0.15$.

Analysis The net rate of radiation heat transfer without the shields per unit area of the plates is



$$\dot{Q}_{12,\text{no shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\ \text{K})^4 - (300\ \text{K})^4]}{\dfrac{1}{0.6} + \dfrac{1}{0.7} - 1}$$

$$= \mathbf{3288\ W/m^2}$$

The net rate of radiation heat transfer with two thin radiation shields per unit area of the plates is

$$\dot{Q}_{12,\text{two-shields}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + \left(\dfrac{1}{\varepsilon_3} + \dfrac{1}{\varepsilon_3} - 1\right) + \left(\dfrac{1}{\varepsilon_4} + \dfrac{1}{\varepsilon_4} - 1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\ \text{K})^4 - (300\ \text{K})^4]}{\left(\dfrac{1}{0.6} + \dfrac{1}{0.7} - 1\right) + \left(\dfrac{1}{0.10} + \dfrac{1}{0.10} - 1\right) + \left(\dfrac{1}{0.15} + \dfrac{1}{0.15} - 1\right)}$$

$$= \mathbf{206\ W/m^2}$$

The equilibrium temperatures of the radiation shields are determined from

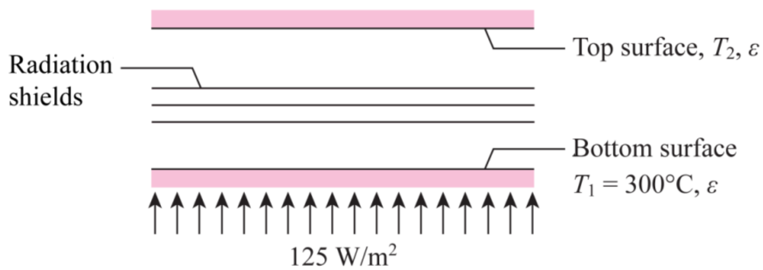
$$\dot{Q}_{13} = \frac{\sigma(T_1^4 - T_3^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_3} - 1\right)} \longrightarrow 206\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(600\ \text{K})^4 - T_3^4]}{\left(\dfrac{1}{0.6} + \dfrac{1}{0.10} - 1\right)} \longrightarrow T_3 = \mathbf{549\ K}$$

$$\dot{Q}_{42} = \frac{\sigma(T_4^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_4} + \dfrac{1}{\varepsilon_2} - 1\right)} \longrightarrow 206\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[T_4^4 - (300\ \text{K})^4]}{\left(\dfrac{1}{0.15} + \dfrac{1}{0.7} - 1\right)} \longrightarrow T_4 = \mathbf{429\ K}$$

13-104 An engine cover is made of two parallel plates. The number of radiation shields necessary to keep the top plate below 150°C, to prevent fire hazards in the event of oil leakage, is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of all the surfaces is given to be ε = 0.3.



Analysis The net radiation heat flux between the two parallel plates engine cover is

$$\dot{q}_{12} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dfrac{1}{\varepsilon} + \dfrac{1}{\varepsilon} - 1}$$

The temperature of the top plate of the engine cover is

$$T_2 = \left[T_1^{\,4} - \frac{\dot{q}_{12}}{\sigma}\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right)\right]^{1/4}$$

$$= \left[(573\text{ K})^4 - \left(\frac{125\text{ W/m}^2}{5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4}\right)\left(\frac{1}{0.3} + \frac{1}{0.3} - 1\right)\right]^{1/4} = 556\text{ K} = 283°\text{C} > 150°\text{C}$$

So, without radiation shield, the top plate temperature is above the safe temperature of 150°C. The number of radiation shields needed to reduce the top plate temperature to 150°C can be determined using,

$$\dot{q}_{12} = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{(N+1)\left(\dfrac{1}{\varepsilon} + \dfrac{1}{\varepsilon} - 1\right)}$$

Hence,

$$N = \frac{\sigma(T_1^{\,4} - T_2^{\,4})}{\dot{q}_{12}\left(\dfrac{1}{\varepsilon} + \dfrac{1}{\varepsilon} - 1\right)} - 1$$

$$= \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(573^4 - 423^4)\text{ K}^4}{(125\text{ W/m}^2)\left(\dfrac{1}{0.3} + \dfrac{1}{0.3} - 1\right)} - 1$$

$$= 5.07$$

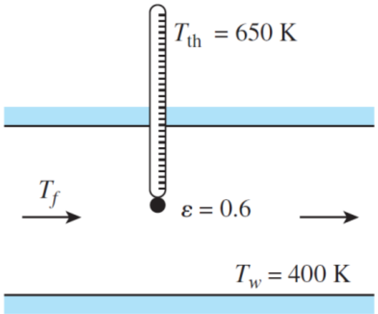
Thus, placing 6 radiation shields will reduce the top plate to below 150°C.

Discussion In order to keep the top plate of the engine cover below 150°C, to prevent fire hazards in the event of oil leakage, the heat flux through the plates can be reduced using radiation shields. By placing 6 or more radiation shields in parallel between the two plates, the top plate temperature can be reduced to below 150°C.

13-105 The temperature of air in a duct is measured. Accounting for the radiation effect, and the actual air temperature is to be determined.

Assumptions The surfaces are opaque, diffuse, and gray.

Properties The emissivity of thermocouple is given to be $\varepsilon = 0.6$



Analysis The actual air temperature is determined from

$$T_f = T_{\text{th}} + \frac{\varepsilon_1 \sigma (T_{\text{th}}^4 - T_w^4)}{h}$$

$$= (650\text{ K}) + \frac{(0.6)\left(5.67 \times 10^{-8} \dfrac{\text{W}}{\text{m}^2} \cdot \text{K}^4\right)[(650\text{ K})^4 - (400\text{ K})^4]}{80 \dfrac{\text{W}}{\text{m}^2} \cdot \text{K}}$$

$$= \mathbf{715\ K}$$

Discussion The walls of the duct are at a considerably lower temperature than the air in it, and thus we expect the thermocouple to show a reading lower than the actual air temperature as a result of the radiation effect.Note that the radiation effect causes a difference of 65°C (or 65 K since °C = K for temperature differences) in temperature reading in this case.

Radiation Exchange with Absorbing and Emitting Gases

13-106C A nonparticipating medium is completely transparent to thermal radiation, and thus it does not emit, absorb, or scatter radiation. A participating medium, on the other hand, emits and absorbs radiation throughout its entire volume.

13-107C Spectral transmissivity of a medium of thickness L is the ratio of the intensity of radiation leaving the medium to that entering the medium, and is expressed as $\tau_\lambda = \frac{I_{\lambda,L}}{I_{\lambda,0}} = e^{-\kappa_\lambda L}$ and $\tau_\lambda = 1 - \alpha_\lambda$.

13-108C Gases emit and absorb radiation at a number of narrow wavelength bands. The emissivity-wavelength charts of gases typically involve various peaks and dips together with discontinuities, and show clearly the band nature of absorption and the strong nongray characteristics. This is in contrast to solids, which emit and absorb radiation over the entire spectrum.

13-109C Using Kirchhoff's law, the spectral emissivity of a medium of thickness L in terms of the spectral absorption coefficient is expressed as $\varepsilon_\lambda = \alpha_\lambda = 1 - e^{-\kappa_\lambda L}$.

13-110 An equimolar mixture of CO_2 and O_2 gases at 800 K and a total pressure of 0.5 atm is considered. The emissivity of the gas is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis Volumetric fractions are equal to pressure fractions. Therefore, the partial pressure of CO_2 is

$$P_c = y_{\mathrm{CO_2}} P = 0.5(0.5\,\text{atm}) = 0.25\,\text{atm}$$

Then,

$$P_c L = (0.25\,\text{atm})(1.2\,\text{m}) = 0.30\,\text{m}\cdot\text{atm} = 0.98\,\text{ft}\cdot\text{atm}$$

The emissivity of CO_2 corresponding to this value at the gas temperature of T_g = 800 K and 1 atm is, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.15$$

This is the base emissivity value at 1 atm, and it needs to be corrected for the 0.5 atm total pressure. The pressure correction factor is, from Fig. 13-37,

$$C_c = 0.90$$

Then the effective emissivity of the gas becomes

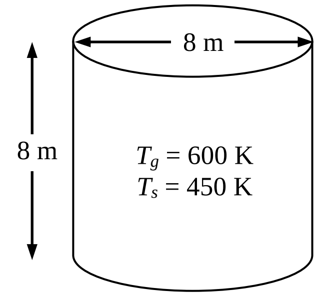
$$\varepsilon_g = C_c \varepsilon_{c,1\,\text{atm}} = 0.90 \times 0.15 = \mathbf{0.135}$$

13-111 A mixture of CO_2 and N_2 gases at 600 K and a total pressure of 1 atm are contained in a cylindrical container. The rate of radiation heat transfer between the gas and the container walls is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The mean beam length is, from Table 13-4

$$L = 0.60D = 0.60(8\text{ m}) = 4.8\text{ m}$$



Then,

$$P_c L = (0.15\,\text{atm})(4.8\text{m}) = 0.72\text{ m}\cdot\text{atm} = 2.36\,\text{ft}\cdot\text{atm}$$

The emissivity of CO_2 corresponding to this value at the gas temperature of T_g = 600 K and 1 atm is, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.16$$

For a source temperature of T_s = 450 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L\frac{T_s}{T_g} = (0.15\,\text{atm})(4.8\text{m})\frac{450\text{ K}}{600\text{ K}} = 0.54\text{ m}\cdot\text{atm} = 1.77\text{ ft}\cdot\text{atm}$$

The emissivity of CO_2 corresponding to this value at a temperature of T_s = 450 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.14$$

The absorptivity of CO_2 is determined from

$$\alpha_c = C_c\left(\frac{T_g}{T_s}\right)^{0.65}\varepsilon_{c,1\,\text{atm}} = (1)\left(\frac{600\text{ K}}{450\text{ K}}\right)^{0.65}(0.14) = 0.17$$

The surface area of the cylindrical surface is

$$A_s = \pi DH + 2\frac{\pi D^2}{4} = \pi(8\text{ m})(8\text{ m}) + 2\frac{\pi(8\text{ m})^2}{4} = 301.6\text{ m}^2$$

Then the net rate of radiation heat transfer from the gas mixture to the walls of the furnace becomes

$$\begin{aligned}\dot{Q}_{\text{net}} &= A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)\\ &= (301.6\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[0.16(600\text{ K})^4 - 0.17(450\text{ K})^4]\\ &= \mathbf{2.35\times10^5\ W}\end{aligned}$$

13-112 A mixture of H_2O and N_2 gases at 600 K and a total pressure of 1 atm are contained in a cylindrical container. The rate of radiation heat transfer between the gas and the container walls is to be determined.

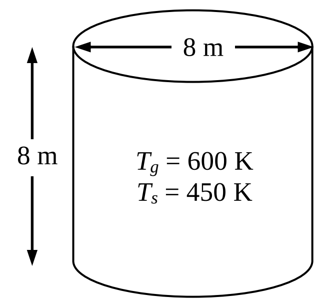
Assumptions All the gases in the mixture are ideal gases.

Analysis The mean beam length is, from Table 13-4

$$L = 0.60D = 0.60(8 \text{ m}) = 4.8 \text{ m}$$

Then,

$$P_w L = (0.15\,\text{atm})(4.8\,\text{m}) = 0.72\,\text{m}\cdot\text{atm} = 2.36\,\text{ft}\cdot\text{atm}$$



The emissivity of H_2O corresponding to this value at the gas temperature of T_g = 600 K and 1 atm is, from Fig. 13-36,

$$\varepsilon_{w\,1\,\text{atm}} = 0.36$$

For a source temperature of T_s = 450 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_w L \frac{T_s}{T_g} = (0.15\,\text{atm})(4.8\,\text{m})\frac{450\text{ K}}{600\text{ K}} = 0.54\,\text{m}\cdot\text{atm} = 1.77\,\text{ft}\cdot\text{atm}$$

The emissivity of H_2O corresponding to this value at a temperature of T_s = 450 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{w,1\,\text{atm}} = 0.34$$

The absorptivity of H_2O is determined from

$$\alpha_w = C_w\left(\frac{T_g}{T_s}\right)^{0.65}\varepsilon_{w,1\,\text{atm}} = (1)\left(\frac{600\text{ K}}{450\text{ K}}\right)^{0.45}(0.34) = 0.39$$

The surface area of the cylindrical surface is

$$A_s = \pi DH + 2\frac{\pi D^2}{4} = \pi(8\,\text{m})(8\,\text{m}) + 2\frac{\pi(8\,\text{m})^2}{4} = 301.6\,\text{m}^2$$

Then the net rate of radiation heat transfer from the gas mixture to the walls of the furnace becomes

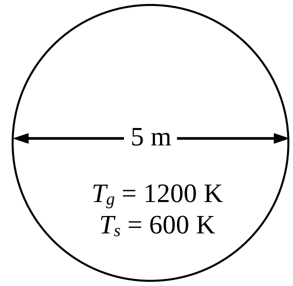
$$\begin{aligned}\dot{Q}_{\text{net}} &= A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)\\ &= (301.6\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.36(600\,\text{K})^4 - 0.39(450\,\text{K})^4]\\ &= \mathbf{5.244\times10^5\ W}\end{aligned}$$

13-113 A mixture of CO_2 and N_2 gases at 1200 K and a total pressure of 1 atm are contained in a spherical furnace. The net rate of radiation heat transfer between the gas mixture and furnace walls is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The mean beam length is, from Table 13-4

$$L = 0.65D = 0.65(5\text{ m}) = 3.25\text{ m}$$



The mole fraction is equal to pressure fraction. Then,

$$P_c L = (0.15\,\text{atm})(3.25\,\text{m}) = 0.4875\,\text{m}\cdot\text{atm} = 1.60\,\text{ft}\cdot\text{atm}$$

The emissivity of CO_2 corresponding to this value at the gas temperature of T_g = 1200 K and 1 atm is, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.17$$

For a source temperature of T_s = 600 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.15\,\text{atm})(3.25\,\text{m})\frac{600\,\text{K}}{1200\,\text{K}} = 0.244\,\text{m}\cdot\text{atm} = 0.800\,\text{ft}\cdot\text{atm}$$

The emissivity of CO_2 corresponding to this value at a temperature of T_s = 600 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.12$$

The absorptivity of CO_2 is determined from

$$\alpha_c = C_c\left(\frac{T_g}{T_s}\right)^{0.65}\varepsilon_{c,1\,\text{atm}} = (1)\left(\frac{1200\,\text{K}}{600\,\text{K}}\right)^{0.65}(0.12) = 0.1883$$

The surface area of the sphere is

$$A_s = \pi D^2 = \pi(5\,\text{m})^2 = 78.54\,\text{m}^2$$

Then the net rate of radiation heat transfer from the gas mixture to the walls of the furnace becomes

$$\begin{aligned}\dot{Q}_{\text{net}} &= A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)\\ &= (78.54\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.17(1200\,\text{K})^4 - 0.1883(600\,\text{K})^4]\\ &= \mathbf{1.46\times10^6\ W}\end{aligned}$$

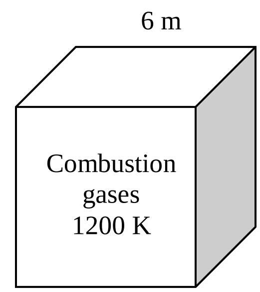
13-114 The temperature, pressure, and composition of a gas mixture is given. The emissivity of the mixture is to be determined.

Assumptions **1** All the gases in the mixture are ideal gases. **2** The emissivity determined is the mean emissivity for radiation emitted to all surfaces of the cubical enclosure.

Analysis The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2}P = 0.10(1\,\text{atm}) = 0.10\,\text{atm}$$

$$P_w = y_{H_2O}P = 0.09(1\,\text{atm}) = 0.09\,\text{atm}$$



The mean beam length for a cube of side length 6 m for radiation emitted to all surfaces is, from Table 13-4,

$$L = 0.66(6\text{ m}) = 3.96\text{ m}$$

Then,

$$P_c L = (0.10\,\text{atm})(3.96\,\text{m}) = 0.396\,\text{m}\cdot\text{atm} = 1.30\,\text{ft}\cdot\text{atm}$$

$$P_w L = (0.09\,\text{atm})(3.96\,\text{m}) = 0.36\,\text{m}\cdot\text{atm} = 1.18\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 1200 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.17 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.16$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 1200 K is, from Fig. 13-38,

$$\left.\begin{array}{l} P_c L + P_w L = 1.30 + 1.18 = 2.48 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.09}{0.09 + 0.10} = 0.474 \end{array}\right\} \Delta\varepsilon = 0.049$$

Then the effective emissivity of the combustion gases becomes

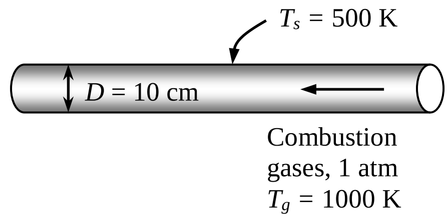
$$\varepsilon_g = C_c\varepsilon_{c,1\,\text{atm}} + C_w\varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1\times 0.17 + 1\times 0.16 - 0.049 = \mathbf{0.281}$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm.

13-115 The temperature, pressure, and composition of combustion gases flowing inside long tubes are given. The rate of heat transfer from combustion gases to tube wall is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The mean beam length for an infinite cicrcular cylinder is, from Table 13-4,



$$L = 0.95(0.10\text{ m}) = 0.095\text{ m}$$

Then,

$$P_c L = (0.12\text{ atm})(0.095\text{m}) = 0.0114\text{ m}\cdot\text{atm} = 0.037\text{ ft}\cdot\text{atm}$$
$$P_w L = (0.18\text{ atm})(0.095\text{m}) = 0.0171\text{ m}\cdot\text{atm} = 0.056\text{ ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 1000 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.055 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.039$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 1000 K is, from Fig. 13-38,

$$\left.\begin{array}{l} P_c L + P_w L = 0.037 + 0.056 = 0.093 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.18}{0.18 + 0.12} = 0.6 \end{array}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\,\text{atm}} + C_w \varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1\times 0.055 + 1\times 0.039 - 0.0 = 0.094$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm. For a source temperature of T_s = 500 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L\frac{T_s}{T_g} = (0.12\text{ atm})(0.095\text{ m})\frac{500\text{ K}}{1000\text{ K}} = 0.005700\text{ m}\cdot\text{atm} = 0.019\text{ ft}\cdot\text{atm}$$
$$P_w L\frac{T_s}{T_g} = (0.18\text{ atm})(0.095\text{ m})\frac{500\text{ K}}{1000\text{ K}} = 0.00855\text{ m}\cdot\text{atm} = 0.028\text{ ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 500 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.038 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.045$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c\left(\frac{T_g}{T_s}\right)^{0.65}\varepsilon_{c,1\,\text{atm}} = (1)\left(\frac{1000\text{ K}}{500\text{ K}}\right)^{0.65}(0.038) = 0.0596$$
$$\alpha_w = C_w\left(\frac{T_g}{T_s}\right)^{0.45}\varepsilon_{w,1\,\text{atm}} = (1)\left(\frac{1000\text{ K}}{500\text{ K}}\right)^{0.45}(0.045) = 0.0615$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 13-38 at $T = T_s$ = 500 K instead of T_g = 1000 K. There is no chart for 500 K in the figure, but we can read $\Delta\varepsilon$ values at 400 K and 800 K, and interpolate. At $P_w/(P_w + P_c)$ = 0.6 and $P_cL + P_wL$ = 0.093 we read $\Delta\varepsilon$ = 0.0. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.0596 + 0.0615 - 0.0 = 0.121$$

The surface area of the pipe is

$$A_s = \pi DL = \pi(0.10\text{ m})(6\text{ m}) = 1.885\text{ m}^2$$

Then the net rate of radiation heat transfer from the combustion gases to the walls of the tube becomes

$$\begin{aligned}\dot{Q}_\text{net} &= A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= (1.885\text{ m}^2)(5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)[0.094(1000\text{ K})^4 - 0.121(500\text{ K})^4] \\ &= \mathbf{9238\ W}\end{aligned}$$

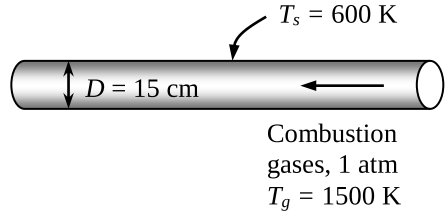
13-116 The temperature, pressure, and composition of combustion gases flowing inside long tubes are given. The rate of heat transfer from combustion gases to tube wall is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2} P = 0.06(1\,\text{atm}) = 0.06\,\text{atm}$$

$$P_w = y_{H_2O} P = 0.09(1\,\text{atm}) = 0.09\,\text{atm}$$



The mean beam length for an infinite cicrcular cylinder is, from Table 13-4,

$$L = 0.95(0.15\text{ m}) = 0.1425\text{ m}$$

Then,

$$P_c L = (0.06\,\text{atm})(0.1425\,\text{m}) = 0.00855\,\text{m}\cdot\text{atm} = 0.028\,\text{ft}\cdot\text{atm}$$

$$P_w L = (0.09\,\text{atm})(0.1425\,\text{m}) = 0.0128\,\text{m}\cdot\text{atm} = 0.042\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 1500 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.034 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.016$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 1500 K is, from Fig. 13-38,

$$\left.\begin{array}{l} P_c L + P_w L = 0.028 + 0.042 = 0.07 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.09}{0.09 + 0.06} = 0.6 \end{array}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\,\text{atm}} + C_w \varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1\times 0.034 + 1\times 0.016 - 0.0 = 0.05$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm. For a source temperature of T_s = 600 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.06\,\text{atm})(0.1425\,\text{m})\frac{600\text{ K}}{1500\text{ K}} = 0.00342\,\text{m}\cdot\text{atm} = 0.011\,\text{ft}\cdot\text{atm}$$

$$P_w L \frac{T_s}{T_g} = (0.09\,\text{atm})(0.1425\,\text{m})\frac{600\text{ K}}{1500\text{ K}} = 0.00513\,\text{m}\cdot\text{atm} = 0.017\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 600 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.031 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.027$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\,\text{atm}} = (1)\left(\frac{1500\text{ K}}{600\text{ K}}\right)^{0.65}(0.031) = 0.056$$

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,1\,\text{atm}} = (1)\left(\frac{1500\text{ K}}{600\text{ K}}\right)^{0.45}(0.027) = 0.041$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 13-38 at $T = T_s$ = 600 K instead of T_g = 1500 K. There is no chart for 600 K in the figure, but we can read $\Delta\varepsilon$ values at 400 K and 800 K, and take their average. At $P_w/(P_w + P_c)$ = 0.6 and $P_cL + P_wL$ = 0.07 we read $\Delta\varepsilon$ = 0.0. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.056 + 0.041 - 0.0 = 0.097$$

The surface area of the pipe per m length of tube is

$$A_s = \pi DL = \pi(0.15\,\text{m})(1\,\text{m}) = 0.4712\,\text{m}^2$$

Then the net rate of radiation heat transfer from the combustion gases to the walls of the furnace becomes

$$\dot{Q}_{\text{net}} = A_s \sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)$$
$$= (0.4712\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.05(1500\text{ K})^4 - 0.097(600\text{ K})^4] = \mathbf{6427\ W}$$

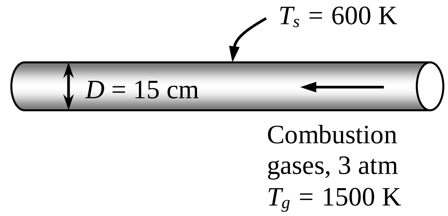
13-117 The temperature, pressure, and composition of combustion gases flowing inside long tubes are given. The rate of heat transfer from combustion gases to tube wall is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2} P = 0.06(1\,\text{atm}) = 0.06\,\text{atm}$$

$$P_w = y_{H_2O} P = 0.09(1\,\text{atm}) = 0.09\,\text{atm}$$



The mean beam length for an infinite cicrcular cylinder is, from Table 13-4,

$$L = 0.95(0.15\text{ m}) = 0.1425\text{ m}$$

Then,

$$P_c L = (0.06\,\text{atm})(0.1425\,\text{m}) = 0.00855\,\text{m}\cdot\text{atm} = 0.028\,\text{ft}\cdot\text{atm}$$

$$P_w L = (0.09\,\text{atm})(0.1425\,\text{m}) = 0.0128\,\text{m}\cdot\text{atm} = 0.042\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 1500 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.034 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.016$$

These are base emissivity values at 1 atm, and they need to be corrected for the 3 atm total pressure. Noting that $(P_w+P)/2$ = (0.09+3)/2 = 1.545 atm, the pressure correction factors are, from Fig. 13-37,

$C_c = 1.5$ and $C_w = 1.8$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 1500 K is, from Fig. 13-38,

$$\left.\begin{array}{l} P_c L + P_w L = 0.028 + 0.042 = 0.07 \\ \dfrac{P_w}{P_w + P_c} = \dfrac{0.09}{0.09 + 0.06} = 0.6 \end{array}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\,\text{atm}} + C_w \varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1.5\times 0.034 + 1.8\times 0.016 - 0.0 = 0.080$$

For a source temperature of T_s = 600 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.06\,\text{atm})(0.1425\,\text{m})\frac{600\,\text{K}}{1500\,\text{K}} = 0.00342\,\text{m}\cdot\text{atm} = 0.011\,\text{ft}\cdot\text{atm}$$

$$P_w L \frac{T_s}{T_g} = (0.09\,\text{atm})(0.1425\,\text{m})\frac{600\,\text{K}}{1500\,\text{K}} = 0.00513\,\text{m}\cdot\text{atm} = 0.017\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 600 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.031 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.027$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\,\text{atm}} = (1.5)\left(\frac{1500\,\text{K}}{600\,\text{K}}\right)^{0.65} (0.031) = 0.084$$

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,1\,\text{atm}} = (1.8)\left(\frac{1500\,\text{K}}{600\,\text{K}}\right)^{0.45} (0.027) = 0.073$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 13-38 at $T = T_s$ = 600 K instead of T_g = 1500 K. There is no chart for 600 K in the figure, but we can read $\Delta\varepsilon$ values at 400 K and 800 K, and take their average. At $P_w/(P_w+ P_c)$ = 0.6 and $P_cL + P_wL$ = 0.07 we read $\Delta\varepsilon$ = 0.0. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.084 + 0.073 - 0.0 = 0.157$$

The surface area of the pipe per m length of tube is

$$A_s = \pi D L = \pi(0.15\,\text{m})(1\,\text{m}) = 0.4712\,\text{m}^2$$

Then the net rate of radiation heat transfer from the combustion gases to the walls of the furnace becomes

$$\begin{aligned} \dot{Q}_{\text{net}} &= A_s \sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= (0.4712\,\text{m}^2)(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.08(1500\,\text{K})^4 - 0.157(600\,\text{K})^4] = \mathbf{10{,}280\ W} \end{aligned}$$

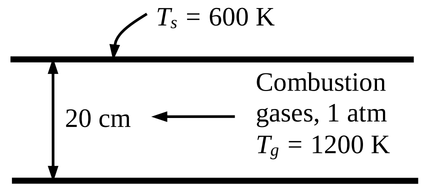
13-118 The temperature, pressure, and composition of combustion gases flowing inside long tubes are given. The rate of heat transfer from combustion gases to tube wall is to be determined.

Assumptions All the gases in the mixture are ideal gases.

Analysis The volumetric analysis of a gas mixture gives the mole fractions y_i of the components, which are equivalent to pressure fractions for an ideal gas mixture. Therefore, the partial pressures of CO_2 and H_2O are

$$P_c = y_{CO_2} P = 0.10(1\,\text{atm}) = 0.10\,\text{atm}$$

$$P_w = y_{H_2O} P = 0.10(1\,\text{atm}) = 0.10\,\text{atm}$$



The mean beam length for this geometry is, from Table 13-4,

$$L = 3.6V/A_s = 1.8D = 1.8(0.20\text{ m}) = 0.36\text{ m}$$

where D is the distance between the plates. Then,

$$P_c L = P_w L = (0.10\,\text{atm})(0.36\,\text{m}) = 0.036\,\text{m}\cdot\text{atm} = 0.118\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the gas temperature of T_g = 1200 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.080 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.055$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 1200 K is, from Fig. 13-38,

$$\left.\begin{aligned} &P_c L + P_w L = 0.118 + 0.118 = 0.236 \\ &\frac{P_w}{P_w + P_c} = \frac{0.10}{0.10 + 0.10} = 0.5 \end{aligned}\right\} \Delta\varepsilon = 0.0025$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\,\text{atm}} + C_w \varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1\times 0.080 + 1\times 0.055 - 0.0025 = 0.1325$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm. For a source temperature of T_s = 600 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = P_w L \frac{T_s}{T_g} = (0.10\,\text{atm})(0.36\,\text{m})\frac{600\text{ K}}{1200\text{ K}} = 0.018\,\text{m}\cdot\text{atm} = 0.059\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 600 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.060 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.067$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\,\text{atm}} = (1)\left(\frac{1200\text{ K}}{600\text{ K}}\right)^{0.65}(0.060) = 0.090$$

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,1\,\text{atm}} = (1)\left(\frac{1200\text{ K}}{600\text{ K}}\right)^{0.45}(0.067) = 0.092$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 13-38 at $T = T_s$ = 600 K instead of T_g = 1200 K. There is no chart for 600 K in the figure, but we can read $\Delta\varepsilon$ values at 400 K and 800 K, and take their average. At $P_w/(P_w + P_c)$ = 0.5 and $P_cL + P_wL$ = 0.236 we read $\Delta\varepsilon$ = 0.00125. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.090 + 0.092 - 0.00125 = 0.1808$$

Then the net rate of radiation heat transfer from the gas to each plate per unit surface area becomes

$$\begin{aligned} \dot{Q}_{\text{net}} &= A_s \sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= (1\,\text{m}^2)(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.1325(1200\text{ K})^4 - 0.1808(600\text{ K})^4] \\ &= \mathbf{1.42\times 10^4\ W} \end{aligned}$$

Special Topic: Heat Transfer from the Human Body

13-119C (*a*) Heat is lost through the skin by convection, radiation, and evaporation. (*b*) The body loses both sensible heat by convection and latent heat by evaporation from the lungs, but there is no heat transfer in the lungs by radiation.

13-120C Sensible heat is the energy associated with a temperature change. The sensible heat loss from a human body increases as (*a*) the skin temperature increases, (*b*) the environment temperature decreases, and (*c*) the air motion (and thus the convection heat transfer coefficient) increases.

13-121C Latent heat is the energy released as water vapor condenses on cold surfaces, or the energy absorbed from a warm surface as liquid water evaporates. The latent heat loss from a human body increases as (*a*) the skin wettedness increases and (*b*) the relative humidity of the environment decreases. The rate of evaporation from the body is related to the rate of latent heat loss by $\dot{Q}_{\text{latent}} = \dot{m}_{\text{vapor}} h_{fg}$ where h_{fg} is the latent heat of vaporization of water at the skin temperature.

13-122C The insulating effect of clothing is expressed in the unit **clo** with 1 clo = 0.155 m^2.°C/W = 0.880 ft^2.°F.h/Btu. Clothing serves as insulation, and thus reduces heat loss from the body by convection, radiation, and evaporation by serving as a resistance against heat flow and vapor flow. Clothing decreases heat gain from the sun by serving as a radiation shield.

13-123C Yes, roughly one-third of the metabolic heat generated by a person who is resting or doing light work is dissipated to the environment by convection, one-third by evaporation, and the remaining one-third by radiation.

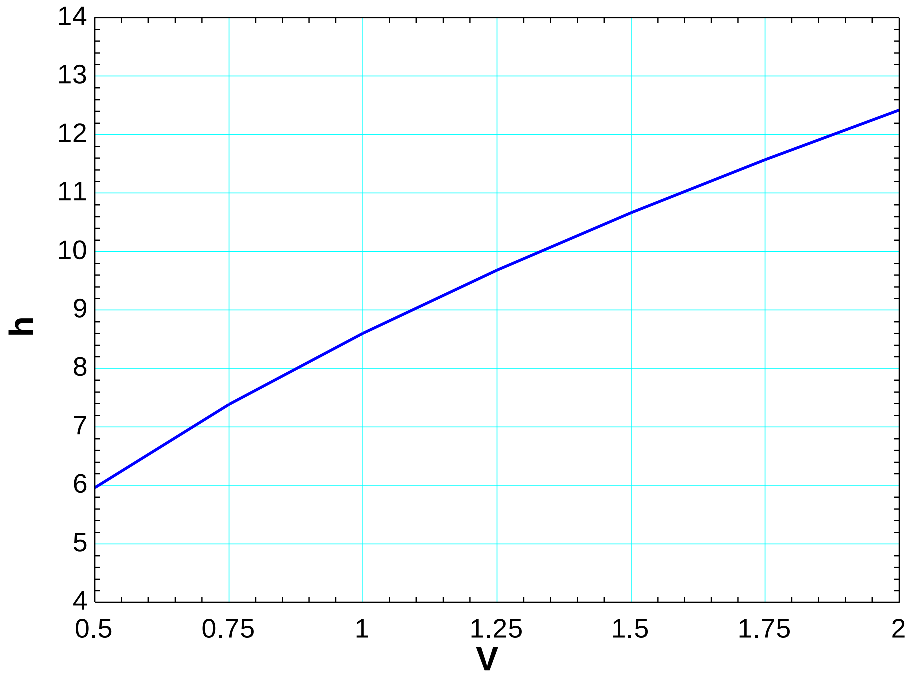
13-124C The *operative temperature* $T_{\text{operative}}$ is the average of the mean radiant and ambient temperatures weighed by their respective convection and radiation heat transfer coefficients, and is expressed as

$$T_{\text{operative}} = \frac{h_{\text{conv}} T_{\text{ambient}} + h_{\text{rad}} T_{\text{surr}}}{h_{\text{conv}} + h_{\text{rad}}} \cong \frac{T_{\text{ambient}} + T_{\text{surr}}}{2}$$

When the convection and radiation heat transfer coefficients are equal to each other, the operative temperature becomes the arithmetic average of the ambient and surrounding surface temperatures. Another environmental index used in thermal comfort analysis is the effective temperature, which combines the effects of temperature and humidity.

13-125 The convection heat transfer coefficient for a clothed person while walking in still air at a velocity of 0.5 to 2 m/s is given by $h = 8.6V^{0.53}$ where V is in m/s and h is in W/m².°C. The convection coefficients in that range vary from 5.96 W/m².°C at 0.5 m/s to 12.42 W/m².°C at 2 m/s. Therefore, at low velocities, the radiation and convection heat transfer coefficients are comparable in magnitude. But at high velocities, the convection coefficient is much larger than the radiation heat transfer coefficient.

Velocity, m/s	$h = 8.6V^{0.53}$ W/m².°C
0.50	5.96
0.75	7.38
1.00	8.60
1.25	9.68
1.50	10.66
1.75	11.57
2.00	12.42



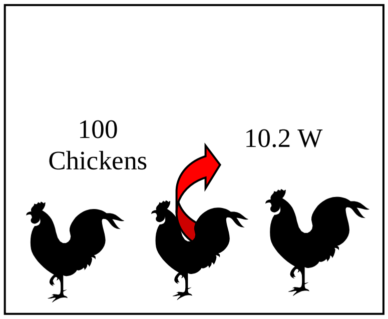
13-126 There are 100 chickens in a breeding room. The rate of total heat generation and the rate of moisture production in the room are to be determined.

Assumptions All the moisture from the chickens is condensed by the air-conditioning system.

Properties The latent heat of vaporization of water is given to be 2430 kJ/kg. The average metabolic rate of chicken during normal activity is 10.2 W (3.78 W sensible and 6.42 W latent).

Analysis The total rate of heat generation of the chickens in the breeding room is

$$\dot{Q}_{\text{gen, total}} = \dot{q}_{\text{gen, total}}(\text{No. of chickens})$$
$$= (10.2\ \text{W/chicken})(100\ \text{chickens}) = \mathbf{1020\ W}$$



The latent heat generated by the chicken and the rate of moisture production are

$$\dot{Q}_{\text{gen, latent}} = \dot{q}_{\text{gen, latent}}(\text{No. of chickens})$$
$$= (6.42\ \text{W/chicken})(100\ \text{chickens}) = 642\ \text{W}$$
$$= 0.642\ \text{kW}$$

$$\dot{m}_{\text{moisture}} = \frac{\dot{Q}_{\text{gen, latent}}}{h_{fg}} = \frac{0.642\ \text{kJ/s}}{2430\ \text{kJ/kg}} = 0.000264\ \text{kg/s} = \mathbf{0.264\ g/s}$$

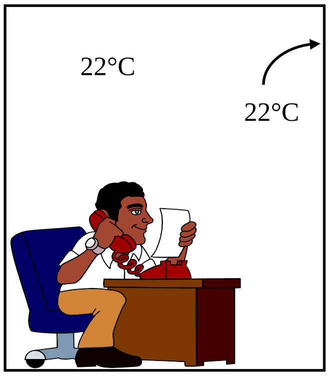
13-127 The average mean radiation temperature during a cold day drops to 18°C. The required rise in the indoor air temperature to maintain the same level of comfort in the same clothing is to be determined.

Assumptions **1** Air motion in the room is negligible. **2** The average clothing and exposed skin temperature remains the same. **3** The latent heat loss from the body remains constant. **4** Heat transfer through the lungs remain constant.

Properties The emissivity of the person is 0.95 (from Appendix tables). The convection heat transfer coefficient from the body in still air or air moving with a velocity under 0.2 m/s is h_{conv} = 3.1 W/m^2·°C (Table 13-5).

Analysis The total rate of heat transfer from the body is the sum of the rates of heat loss by convection, radiation, and evaporation,

$$\dot{Q}_{\text{body, total}} = \dot{Q}_{\text{sensible}} + \dot{Q}_{\text{latent}} + \dot{Q}_{\text{lungs}} = (\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}) + \dot{Q}_{\text{latent}} + \dot{Q}_{\text{lungs}}$$



Noting that heat transfer from the skin by evaporation and from the lungs remains constant, the sum of the convection and radiation heat transfer from the person must remain constant.

$$\begin{aligned}\dot{Q}_{\text{sensible,old}} &= hA_s(T_s - T_{\text{air, old}}) + \varepsilon A_s \sigma (T_s^4 - T_{\text{surr,old}}^4) \\ &= hA_s(T_s - 22) + 0.95 A_s \sigma [(T_s + 273)^4 - (22+273)^4] \\ \dot{Q}_{\text{sensible,new}} &= hA_s(T_s - T_{\text{air, new}}) + \varepsilon A_s \sigma (T_s^4 - T_{\text{surr,new}}^4) \\ &= hA_s(T_s - T_{\text{air, new}}) + 0.95 A_s \sigma [(T_s + 273)^4 - (18+273)^4]\end{aligned}$$

Setting the two relations above equal to each other, canceling the surface area A_s, and simplifying gives

$$-22h - 0.95\sigma(22+273)^4 = -hT_{\text{air, new}} - 0.95\sigma(18+273)^4$$

$$3.1(T_{\text{air, new}} - 22) + 0.95 \times 5.67 \times 10^{-8}(291^4 - 295^4) = 0$$

Solving for the new air temperature gives

$T_{\text{air, new}}$ = **29.0°C**

Therefore, the air temperature must be raised to 29°C to counteract the increase in heat transfer by radiation.

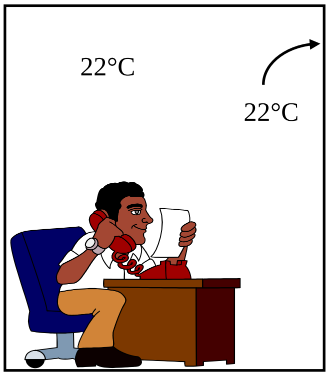
13-128 The average mean radiation temperature during a cold day drops to 10°C. The required rise in the indoor air temperature to maintain the same level of comfort in the same clothing is to be determined.

Assumptions **1** Air motion in the room is negligible. **2** The average clothing and exposed skin temperature remains the same. **3** The latent heat loss from the body remains constant. **4** Heat transfer through the lungs remain constant.

Properties The emissivity of the person is 0.95 (from Appendix tables). The convection heat transfer coefficient from the body in still air or air moving with a velocity under 0.2 m/s is h_{conv} = 3.1 W/m^2·°C (Table 13-5).

Analysis The total rate of heat transfer from the body is the sum of the rates of heat loss by convection, radiation, and evaporation,

$$\dot{Q}_{\text{body, total}} = \dot{Q}_{\text{sensible}} + \dot{Q}_{\text{latent}} + \dot{Q}_{\text{lungs}} = (\dot{Q}_{\text{conv}} + Q_{\text{rad}}) + \dot{Q}_{\text{latent}} + \dot{Q}_{\text{lungs}}$$



Noting that heat transfer from the skin by evaporation and from the lungs remains constant, the sum of the convection and radiation heat transfer from the person must remain constant.

$$\dot{Q}_{\text{sensible,old}} = hA_s(T_s - T_{\text{air, old}}) + \varepsilon A_s\sigma(T_s^4 - T_{\text{surr,old}}^4)$$
$$= hA_s(T_s - 22) + 0.95A_s\sigma[(T_s + 273)^4 - (22+273)^4]$$
$$\dot{Q}_{\text{sensible,new}} = hA_s(T_s - T_{\text{air, new}}) + \varepsilon A_s\sigma(T_s^4 - T_{\text{surr,new}}^4)$$
$$= hA_s(T_s - T_{\text{air, new}}) + 0.95A_s\sigma[(T_s + 273)^4 - (10+273)^4]$$

Setting the two relations above equal to each other, canceling the surface area A_s, and simplifying gives

$$-22h - 0.95\sigma(22+273)^4 = -hT_{\text{air, new}} - 0.95\sigma(10+273)^4$$
$$3.1(T_{\text{air, new}} - 22) + 0.95\times 5.67\times 10^{-8}(283^4 - 295^4) = 0$$

Solving for the new air temperature gives

$T_{\text{air, new}}$ = **42.1°C**

Therefore, the air temperature must be raised to 42.11°C to counteract the increase in heat transfer by radiation.

13-129 Chilled air is to cool a room by removing the heat generated in a large insulated classroom by lights and students. The required flow rate of air that needs to be supplied to the room is to be determined.

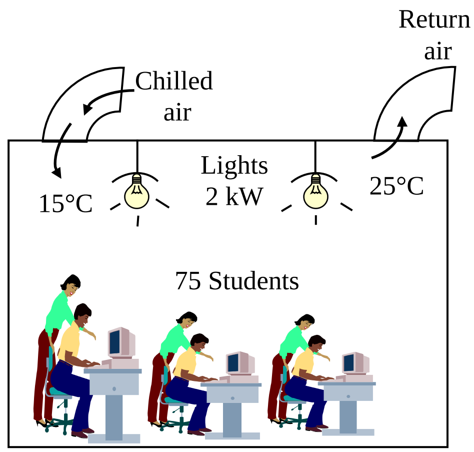
Assumptions **1** The moisture produced by the bodies leave the room as vapor without any condensing, and thus the classroom has no latent heat load. **2** Heat gain through the walls and the roof is negligible.

Properties The specific heat of air at room temperature is 1.00 kJ/kg·°C (Table A-15). The average rate of metabolic heat generation by a person sitting or doing light work is 115 W (70 W sensible, and 45 W latent).

Analysis The rate of sensible heat generation by the people in the room and the total rate of sensible internal heat generation are

$$\dot{Q}_{\text{gen, sensible}} = \dot{q}_{\text{gen, sensible}}(\text{No. of people})$$
$$= (70\text{ W/person})(75\text{ persons}) = 5250\text{ W}$$

$$\dot{Q}_{\text{total,sensible}} = \dot{Q}_{\text{gen, sensible}} + \dot{Q}_{\text{lighting}}$$
$$= 5250 + 2000 = 7250\text{ W}$$



Then the required mass flow rate of chilled air becomes

$$\dot{m}_{\text{air}} = \frac{\dot{Q}_{\text{total,sensible}}}{c_p \Delta T}$$
$$= \frac{7.25\text{ kJ/s}}{(1.0\text{ kJ/kg}\cdot{}^\circ\text{C})(25-15)^\circ\text{C}} = \mathbf{0.725\ kg/s}$$

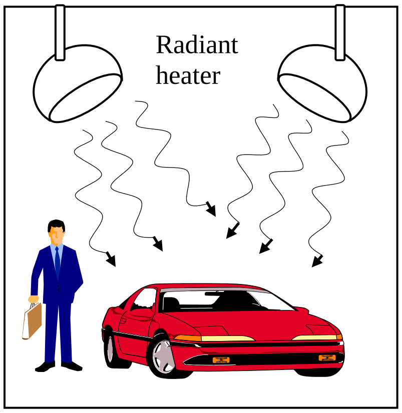
Discussion The latent heat will be removed by the air-conditioning system as the moisture condenses outside the cooling coils.

13-130 A car mechanic is working in a shop heated by radiant heaters in winter. The lowest ambient temperature the worker can work in comfortably is to be determined.

Assumptions **1** The air motion in the room is negligible, and the mechanic is standing. **2** The average clothing and exposed skin temperature of the mechanic is 33°C.

Properties The emissivity and absorptivity of the person is given to be 0.95. The convection heat transfer coefficient from a standing body in still air or air moving with a velocity under 0.2 m/s is h_{conv} = 4.0 W/m²·°C (Table 13-5).

Analysis The equivalent thermal resistance of clothing is



$$R_{\text{cloth}} = 0.7\ \text{clo} = 0.7 \times 0.155\ \text{m}^2.°\text{C/W} = 0.1085\ \text{m}^2.°\text{C/W}$$

Radiation from the heaters incident on the person and the rate of sensible heat generation by the person are

$$\dot{Q}_{\text{rad, incident}} = 0.05 \times \dot{Q}_{\text{rad, total}} = 0.05(4\ \text{kW}) = 0.2\ \text{kW} = 200\ \text{W}$$

$$\dot{Q}_{\text{gen, sensible}} = 0.5 \times \dot{Q}_{\text{gen, total}} = 0.5(350\ \text{W}) = 175\ \text{W}$$

Under steady conditions, and energy balance on the body can be expressed as

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_{\text{gen}} = 0$$

$$\dot{Q}_{\text{rad from heater}} - \dot{Q}_{\text{conv+rad from body}} + \dot{Q}_{\text{gen, sensible}} = 0$$

or

$$\alpha \dot{Q}_{\text{rad, incident}} - h_{\text{conv}} A_s (T_s - T_{\text{surr}}) - \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4) + \dot{Q}_{\text{gen, sensible}} = 0$$

$$0.95(200\ \text{W}) - (4.0\ \text{W/m}^2 \cdot \text{K})(1.8\ \text{m}^2)(306 - T_{\text{surr}})$$
$$- 0.95(1.8\ \text{m}^2)(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)[(306\ \text{K})^4 - T_{\text{surr}}^4] + 175\ \text{W} = 0$$

Solving the equation above gives

$$T_{\text{surr}} = 284.8\ \text{K} = \mathbf{11.8°C}$$

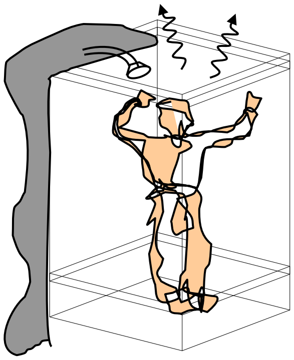
Therefore, the mechanic can work comfortably at temperatures as low as 12°C.

13-131E An average person produces 0.50 lbm of moisture while taking a shower. The contribution of showers of a family of four to the latent heat load of the air-conditioner per day is to be determined.

Assumptions All the water vapor from the shower is condensed by the air-conditioning system.

Properties The latent heat of vaporization of water is given to be 1050 Btu/lbm.

Analysis The amount of moisture produced per day is



m_{vapor} = (Moisture produced per person)(No. of persons)
= (0.5 lbm/person)(4 persons/day) = 2 lbm/day

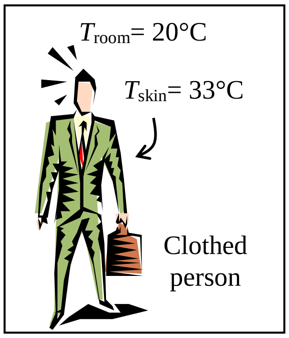
Then the latent heat load due to showers becomes

$Q_{\text{latent}} = m_{\text{vapor}} h_{fg}$ = (2 lbm/day)(1050 Btu/lbm) = **2100 Btu/day**

13-132 A man wearing summer clothes feels comfortable in a room at 20°C. The room temperature at which this man would feel thermally comfortable when unclothed is to be determined.

Assumptions **1** Steady conditions exist. **2** The latent heat loss from the person remains the same. **3** The heat transfer coefficients remain the same. **4** The air in the room is still (there are no winds or running fans). **5** The surface areas of the clothed and unclothed person are the same.

Analysis At low air velocities, the convection heat transfer coefficient for a standing man is given in Table 13-5 to be 4.0 W/m^2.°C. The radiation heat transfer coefficient at typical indoor conditions is 4.7 W/m^2.°C. Therefore, the heat transfer coefficient for a standing person for combined convection and radiation is



$$h_{combined} = h_{conv} + h_{rad} = 4.0 + 4.7 = 8.7 \text{ W/m}^2.°\text{C}$$

The thermal resistance of the clothing is given to be

$$R_{cloth} = 1.1 \text{ clo} = 1.1 \times 0.155 \text{ m}^2.°\text{C/W} = 0.171 \text{ m}^2.°\text{C/W}$$

Noting that the surface area of an average man is 1.8 m^2, the sensible heat loss from this person when clothed is determined to be

$$\dot{Q}_{sensible,clothed} = \frac{A_s(T_{skin} - T_{ambient})}{R_{cloth} + \dfrac{1}{h_{combined}}} = \frac{(1.8 \text{ m}^2)(33-20)°\text{C}}{0.171 \text{ m}^2.°\text{C/W} + \dfrac{1}{8.7 \text{ W/m}^2.°\text{C}}} = 82 \text{ W}$$

From heat transfer point of view, taking the clothes off is equivalent to removing the clothing insulation or setting R_{cloth} = 0. The heat transfer in this case can be expressed as

$$\dot{Q}_{sensible,unclothed} = \frac{A_s(T_{skin} - T_{ambient})}{\dfrac{1}{h_{combined}}} = \frac{(1.8 \text{ m}^2)(33 - T_{ambient})°\text{C}}{\dfrac{1}{8.7 \text{ W/m}^2.°\text{C}}}$$

To maintain thermal comfort after taking the clothes off, the skin temperature of the person and the rate of heat transfer from him must remain the same. Then setting the equation above equal to 82 W gives

$$T_{ambient} = \mathbf{27.8°C}$$

Therefore, the air temperature needs to be raised from 22 to 27.8°C to ensure that the person will feel comfortable in the room after he takes his clothes off. Note that the effect of clothing on latent heat is assumed to be negligible in the solution above. We also assumed the surface area of the clothed and unclothed person to be the same for simplicity, and these two effects should counteract each other.

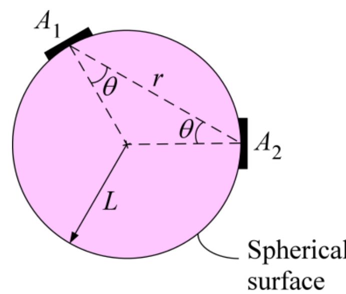
Review Problems

13-133 Two diffuse surfaces A_1 and A_2 placed at an specified orientation, (*a*) the expression for the view factor F_{12} in terms of A_2 and L, and (*b*) the value of the view factor F_{12} when A_2 = 0.02 m^2 and L = 1 m are to be determined.

Assumptions **1** The surfaces A_1 and A_2 are diffuse. **2** Both A_1 and A_2 can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

Analysis (*a*) The view factor for surfaces A_1 and A_2 can be determined using the integral

$$F_{12} = \frac{1}{A_1}\int_{A_2}\int_{A_1} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1 dA_2$$
$$= \frac{1}{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} A_1 dA_2$$
$$= \frac{1}{A_1} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} A_1 A_2$$
$$= \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} A_2$$



From orientation of the two surfaces, we have

$$\theta_1 = \theta_2 \quad \rightarrow \quad \cos\theta_1 = \cos\theta_2 \qquad (1)$$

and

$$r = 2L\cos\theta_1 \qquad (2)$$

Substituting Eqs. (1) and (2) into the expression for F_{12}, we get

$$F_{12} = \frac{(\cos\theta_1)^2}{\pi(2L\cos\theta_1)^2} A_2 = \frac{A_2}{4L^2\pi}$$

(*b*) The value of the view factor F_{12} when A_2 = 0.02 m^2 and L = 1 m is

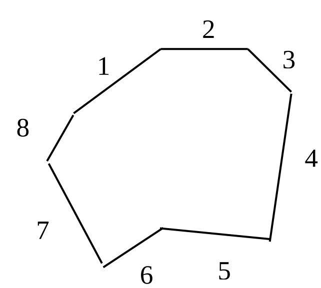
$$F_{12} = \frac{A_2}{4L^2\pi} = \frac{0.02\text{ m}^2}{4(1\text{ m})^2\pi} = \mathbf{1.59 \times 10^{-3}}$$

Discussion The view factor F_{21} can simply be determined with the reciprocity relation as

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{4L^2\pi}$$

13-134 An enclosure consisting of eight surfaces is considered. The number of view factors this geometry involves and the number of these view factors that can be determined by the application of the reciprocity and summation rules are to be determined.

Analysis An eight surface enclosure (N = 8) involves $N^2 = 8^2 = \mathbf{64}$ view factors and we need to determine $\frac{N(N-1)}{2} = \frac{8(8-1)}{2} = 28$ view factors directly. The remaining 64 - 28 = **36** of the view factors can be determined by the application of the reciprocity and summation rules.



13-135 A cylindrical enclosure is considered. (*a*) The expression for the view from the side surface to itself F_{33} in terms of K and (*b*) the value of the view factor F_{33} for $L = D$ are to be determined.

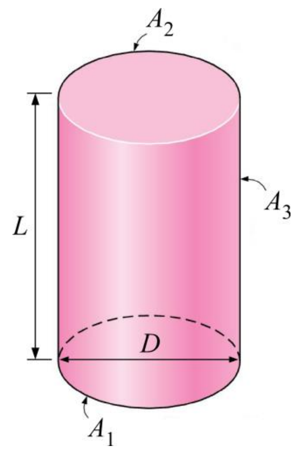
Assumptions **1** The surfaces are diffuse emitters and reflectors.

Analysis (*a*) The surfaces are designated as follows: Base surface as A_1, top surface as A_2, and side surface as A_3

Applying the summation rule to A_1, we have

$$F_{11} + F_{12} + F_{13} = 1 \qquad \text{(where } F_{11} = 0\text{)}$$

or $$F_{13} = 1 - F_{12} \tag{1}$$



For coaxial parallel disks, from Table 13-1, with $i = 1$, $j = 2$,

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_2}{D_1}\right)^2\right]^{1/2}\right\} = \frac{1}{2}[S - (S^2 - 4)^{1/2}] \tag{2}$$

$$S = 1 + \frac{1 + R_2^2}{R_1^2} = 2 + \frac{1}{R^2} = 2 + \frac{4}{(D/L)^2} = 2 + 4K^2 \tag{3}$$

where $$R_1 = R_2 = R = \frac{D}{2L} = \frac{1}{2K}$$

Substituting Eq. (3) into Eq. (2), we get

$$\begin{aligned} F_{12} &= \frac{1}{2}\{2 + 4K^2 - [(2 + 4K^2)^2 - 4]^{1/2}\} \\ &= \frac{1}{2}[2 + 4K^2 - (16K^4 + 16K^2)^{1/2}] \\ &= \frac{1}{2}[2 + 4K^2 - 4K(K^2 + 1)^{1/2}] \\ &= 1 + 2K^2 - 2K(K^2 + 1)^{1/2} \end{aligned}$$

Substituting the above expression for F_{12} into Eq. (1) yields the expression for F_{13}:

$$F_{13} = 1 - [1 + 2K^2 - 2K(K^2 + 1)^{1/2}] = 2K(K^2 + 1)^{1/2} - 2K^2 \tag{4}$$

Then, applying the summation rule to A_3, we have

$$F_{31} + F_{32} + F_{33} = 1 \qquad \text{(where } F_{31} = F_{32}\text{)}$$

or $$F_{33} = 1 - 2F_{31}$$

Applying reciprocity relation, we have

$$F_{33} = 1 - 2F_{13}(A_1 / A_3)$$

where $$F_{31} = F_{13}(A_1 / A_3)$$

Also, we know that

$$A_1 = \pi D^2 / 4 \qquad \text{and} \qquad A_3 = \pi DL \qquad \rightarrow \qquad \frac{A_1}{A_3} = \frac{D}{4L}$$

Hence, the expression for F_{33} becomes

$$F_{33} = 1 - 2\frac{D}{4L}F_{13} = 1 - \frac{1}{2K}F_{13} \tag{5}$$

Finally, substituting Eq. (4) into Eq. (5) yields the expression for F_{33}:

$$F_{33} = 1 - \frac{1}{2K}[2K(K^2 + 1)^{1/2} - 2K^2]$$

Hence, $$F_{33} = 1 + K - (K^2 + 1)^{1/2}$$

(*b*) The value of the view factor F_{33} for $L = D$ (i.e., $K = 1$) is

$$F_{33} = 1 + K - (K^2 + 1)^{1/2} = 1 + 1 - (1^2 + 1)^{1/2} = 2 - \sqrt{2} = \mathbf{0.589}$$

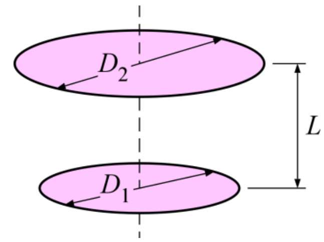
Discussion If the cylinder has a length and diameter of $L = 2D$, then from the expression for F_{33} we have

$$F_{33} = 1 + 2 - (2^2 + 1)^{1/2} = \mathbf{0.764}$$

13-136 Two parallel black disks are positioned coaxially, where the lower disk is heated electrically. The temperature of the upper disk is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surfaces are black. **3** Convection heat transfer is not considered. **4** Radiation heat transfer only between the two disks. **5** No heat loss to the surrounding.

Analysis For coaxial parallel disks, from Table 13-1, with i = 1, j = 2,



$$R_1 = \frac{D_1/2}{L} = \frac{0.2/2}{0.25} = 0.4 \qquad \text{and} \qquad R_2 = \frac{D_2/2}{L} = \frac{0.4/2}{0.25} = 0.8$$

$$S = 1 + \frac{1+R_2^2}{R_1^2} = 1 + \frac{1+(0.8)^2}{(0.4)^2} = 11.25$$

$$F_{12} = \frac{1}{2}\left\{S - \left[S^2 - 4\left(\frac{D_2}{D_1}\right)^2\right]^{1/2}\right\} = \frac{1}{2}\left\{11.25 - \left[(11.25)^2 - 4\left(\frac{0.4}{0.2}\right)^2\right]^{1/2}\right\} = 0.3676$$

The net radiation heat transfer rate leaving the lower surface can be expressed as

$$\dot{Q}_{\text{elec}} = \dot{Q}_{12} = A_1 F_{12}\sigma(T_1^4 - T_2^4) \qquad \to \qquad T_2 = \left(T_1^4 - \frac{4\dot{Q}_{\text{elec}}}{\pi D_1^2 F_{12}\sigma}\right)^{1/4}$$

$$T_2 = \left[(500\text{ K})^4 - \frac{4(20\text{ W})}{\pi(0.2\text{ m})^2(0.3676)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)}\right]^{1/4} = \mathbf{423\ K}$$

Discussion The view factor F_{12} can also be determined using Fig. 13-7 to be

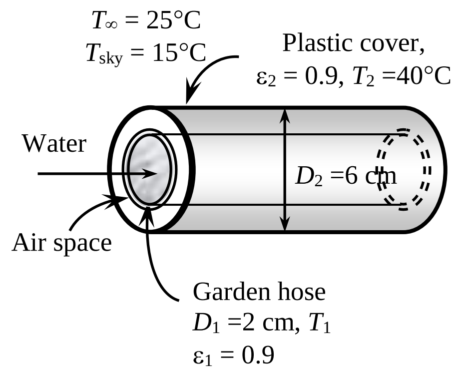
$$F_{12} \approx 0.36 \qquad \text{with} \qquad L/r_1 = 2.5 \qquad \text{and} \qquad r_2/L = 0.8$$

13-137 A simple solar collector is built by placing a clear plastic tube around a garden hose. The rate of heat loss from the water in the hose by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.9$. The properties of air are at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (40+25)/2 = 32.5°C$ are (Table A-15)

$$k = 0.02607 \text{ W/m.°C}$$
$$\nu = 1.632\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7275$$
$$\beta = \frac{1}{(32.5+273)\text{ K}} = 0.003273 \text{ K}^{-1}$$



Analysis Under steady conditions, the heat transfer rate from the water in the hose equals to the rate of heat loss from the clear plastic tube to the surroundings by natural convection and radiation. The characteristic length in this case is the diameter of the plastic tube, $L_c = D_{plastic} = D_2 = 0.06\text{ m}$.

$$Ra = \frac{g\beta(T_s - T_\infty)D_2^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003273\text{ K}^{-1})(40-25)\text{K}(0.06\text{ m})^3}{(1.632\times10^{-5}\text{ m}^2/\text{s})^2}(0.7275) = 2.842\times10^5$$

$$\text{Nu} = \left\{0.6 + \frac{0.387\,\text{Ra}_D^{1/6}}{\left[1+(0.559/\text{Pr})^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(2.842\times10^5)^{1/6}}{\left[1+(0.559/0.7241)^{9/16}\right]^{8/27}}\right\}^2 = 10.30$$

$$h = \frac{k}{D_2}Nu = \frac{0.02607\text{ W/m.°C}}{0.06\text{ m}}(10.30) = 4.475\text{ W/m}^2.\text{°C}$$

$$A_{plastic} = A_2 = \pi D_2 L = \pi(0.06\text{ m})(1\text{m}) = 0.1885\text{ m}^2$$

Then the rate of heat transfer from the outer surface by natural convection becomes

$$\dot{Q}_{conv} = hA_2(T_s - T_\infty) = (4.475\text{ W/m}^2.\text{°C})(0.1885\text{ m}^2)(40-25)\text{°C} = \mathbf{12.7\text{ W}}$$

The rate of heat transfer by radiation from the outer surface is determined from

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \varepsilon A_2\sigma(T_s^{\,4} - T_{sky}^{\,4}) \\ &= (0.90)(0.1885\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(40+273\text{ K})^4 - (15+273\text{ K})^4] \\ &= \mathbf{26.1\text{W}}\end{aligned}$$

Finally,

$$\dot{Q}_{total,loss} = 12.7 + 26.1 = 38.8\text{ W}$$

Discussion Note that heat transfer is mostly by radiation.

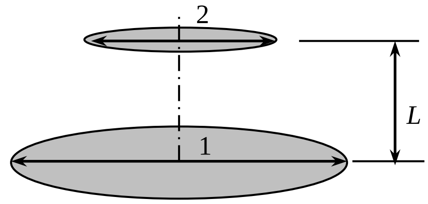
13-138 Radiation heat transfer occurs between two concentric disks. The view factors and the net rate of radiation heat transfer for two cases are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of disk 1 and 2 are given to be $\varepsilon_a = 0.6$ and $\varepsilon_b = 0.8$, respectively.

Analysis (*a*) The view factor from surface 1 to surface 2 is determined using Fig. 13-7 as

$$\frac{L}{r_1} = \frac{0.10}{0.20} = 0.5, \quad \frac{r_2}{L} = \frac{0.10}{0.10} = 1 \longrightarrow F_{12} = \mathbf{0.19}$$



Using reciprocity rule,

$$A_1 = \pi (0.2\,\mathrm{m})^2 = 0.1257\,\mathrm{m}^2$$

$$A_2 = \pi (0.1\,\mathrm{m})^2 = 0.0314\,\mathrm{m}^2$$

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{0.1257\,\mathrm{m}^2}{0.0314\,\mathrm{m}^2}(0.19) = \mathbf{0.76}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{Q} = \frac{\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} = \frac{(5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4})\left[(1073\,\mathrm{K})^4 - (573\,\mathrm{K})^4\right]}{\dfrac{1-0.6}{(0.1257\,\mathrm{m}^2)(0.6)} + \dfrac{1}{(0.1257\,\mathrm{m}^2)(0.19)} + \dfrac{1-0.8}{(0.0314\,\mathrm{m}^2)(0.8)}} = \mathbf{1250\ W}$$

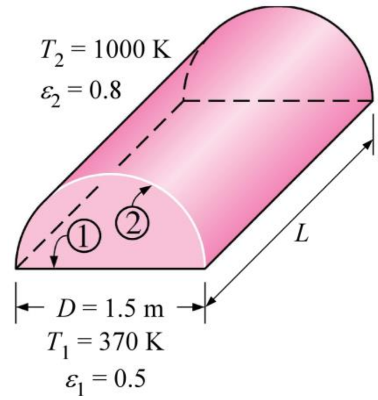
(c) When the space between the disks is completely surrounded by a refractory surface, the net rate of radiation heat transfer can be determined from

$$\dot{Q} = \frac{A_1\sigma\left(T_1^{\,4} - T_2^{\,4}\right)}{\dfrac{A_1 + A_2 - 2A_1F_{12}}{A_2 - A_1F_{12}^2} + \left(\dfrac{1}{\varepsilon_1} - 1\right) + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{(0.1257\,\mathrm{m}^2)(5.67\times10^{-8}\ \mathrm{W/m^2\cdot K^4})\left[(1073\,\mathrm{K})^4 - (573\,\mathrm{K})^4\right]}{\dfrac{0.1257 + 0.0314 - 2(0.1257)(0.19)}{0.0314 - (0.1257)(0.19)^2} + \left(\dfrac{1}{0.6} - 1\right) + \dfrac{0.1257}{0.0314}\left(\dfrac{1}{0.8} - 1\right)} = \mathbf{1510\ W}$$

Discussion The rate of heat transfer in part (c) is 21 percent higher than that in part (b).

13-139 The base and the dome of a long semi-cylindrical dryer are maintained at uniform temperatures. The drying rate per unit length experienced by the base surface is to be determined.



Assumptions **1** Steady operating conditions exist. **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered. **4** The dryer is well insulated from heat loss to the surrounding.

Properties The latent heat of vaporization for water is h_{fg} = 2257 kJ/kg (Table A-2)

Analysis The view factor from the dome to the base is determined from

$$F_{11} + F_{12} = 1 \quad \rightarrow \quad F_{12} = 1 \quad \text{(where } F_{11} = 0\text{)}$$

Hence, from reciprocity relation, we get

$$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{DL}{\pi DL/2} = \frac{2}{\pi}$$

Applying energy balance on the base surface, we have

$$\dot{Q}_{21} = \dot{Q}_{\text{evap}} = \dot{m}h_{fg} = \frac{\sigma(T_2^4 - T_1^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_2 F_{21}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}} \quad \rightarrow \quad \dot{m} = \frac{(\sigma/h_{fg})(T_2^4 - T_1^4)}{\dfrac{1-\varepsilon_1}{DL\varepsilon_1} + \dfrac{2}{\pi DLF_{21}} + \dfrac{2(1-\varepsilon_2)}{\pi DL\varepsilon_2}}$$

Hence

$$\begin{aligned}
\frac{\dot{m}}{L} &= \frac{D(\sigma/h_{fg})(T_2^4 - T_1^4)}{\dfrac{1-\varepsilon_1}{\varepsilon_1} + \dfrac{2}{\pi F_{21}} + \dfrac{2(1-\varepsilon_2)}{\pi\varepsilon_2}} \\
&= \frac{D(\sigma/h_{fg})(T_2^4 - T_1^4)}{\dfrac{1-\varepsilon_1}{\varepsilon_1} + 1 + \dfrac{2(1-\varepsilon_2)}{\pi\varepsilon_2}} \\
&= \frac{(1.5\text{ m})(1000^4 - 370^4)\text{ K}^4}{\dfrac{1-0.5}{0.5} + 1 + \dfrac{2(1-0.8)}{0.8\pi}} \left(\frac{5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4}{2257\times10^3\text{ J/kg}} \right) \\
&= \mathbf{0.0171\ kg/s \cdot m}
\end{aligned}$$

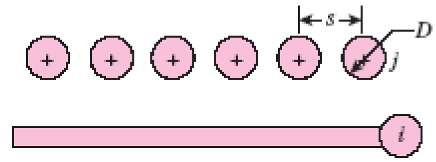
Discussion The view factor from the dome to the base is constant $F_{21} = 2/\pi$, which implies that the view factor is independent of the dryer dimensions.

13-140 Radiation heat transfer occurs between a tube-bank and a wall. The view factors, the net rate of radiation heat transfer, and the temperature of tube surface are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** The tube wall thickness and convection from the outer surface are negligible.

Properties The emissivities of the wall and tube bank are given to be $\varepsilon_i = 0.8$ and $\varepsilon_j = 0.9$, respectively.

Analysis (*a*) We take the wall to be surface *i* and the tube bank to be surface *j*. The view factor from surface *i* to surface *j* is determined from



$$F_{ij} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{0.5} + \left(\frac{D}{s}\right)\left\{\tan^{-1}\left[\left(\frac{s}{D}\right)^2 - 1\right]^{0.5}\right\}$$

$$= 1 - \left[1 - \left(\frac{1.5}{3}\right)^2\right]^{0.5} + \left(\frac{1.5}{3}\right)\left\{\tan^{-1}\left[\left(\frac{3}{1.5}\right)^2 - 1\right]^{0.5}\right\} = \mathbf{0.658}$$

The view factor from surface *j* to surface *i* is determined from reciprocity relation. Taking *s* to be the width of the wall

$$A_i F_{ij} = A_j F_{ji} \longrightarrow F_{ji} = \frac{A_i}{A_j} F_{ij} = \frac{sL}{\pi DL} F_{ij} = \frac{s}{\pi D} F_{ij} = \frac{3}{\pi(1.5)}(0.658) = \mathbf{0.419}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{q} = \frac{\sigma\left(T_i^4 - T_j^4\right)}{\left(\dfrac{1-\varepsilon_i}{\varepsilon_i}\right)\dfrac{1}{A_i} + \dfrac{1}{A_i F_{ij}} + \left(\dfrac{1-\varepsilon_j}{\varepsilon_j}\right)\dfrac{1}{A_j}} = \frac{\sigma\left(T_i^4 - T_j^4\right)}{\dfrac{1-\varepsilon_i}{\varepsilon_i} + \dfrac{1}{F_{ij}} + \left(\dfrac{1-\varepsilon_j}{\varepsilon_j}\right)\dfrac{A_i}{A_j}}$$

$$= \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(1173\text{ K})^4 - (333\text{ K})^4\right]}{\dfrac{1-0.8}{0.8} + \dfrac{1}{0.658} + \left(\dfrac{1-0.9}{0.9}\right)\dfrac{(0.03\text{ m})}{\pi(0.015\text{ m})}} = \mathbf{57{,}900\ W/m^2}$$

(c) Under steady conditions, the rate of radiation heat transfer from the wall to the tube surface is equal to the rate of convection heat transfer from the tube wall to the fluid. Denoting T_w to be the wall temperature,

$$\dot{q}_{rad} = \dot{q}_{conv}$$

$$\frac{\sigma\left(T_i^4 - T_w^4\right)}{\dfrac{1-\varepsilon_i}{\varepsilon_i} + \dfrac{1}{F_{ij}} + \left(\dfrac{1-\varepsilon_j}{\varepsilon_j}\right)\dfrac{A_i}{A_j}} = hA_j(T_w - T_j)$$

$$\frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(1173\text{ K})^4 - T_w^4\right]}{\dfrac{1-0.8}{0.8} + \dfrac{1}{0.658} + \left(\dfrac{1-0.9}{0.9}\right)\dfrac{(0.03\text{ m})}{\pi(0.015\text{ m})}} = (2000\text{ W/m}^2\cdot\text{K})\left[\frac{\pi(0.015\text{ m})}{0.03\text{ m}}\right]\left[T_w - (40+273\text{ K})\right]$$

Solving this equation by an equation solver such as EES, we obtain

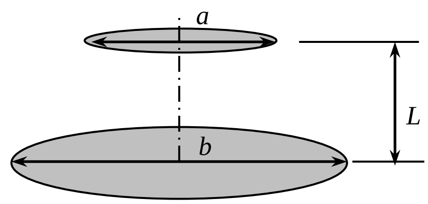
$T_w = 331.4\text{ K}$ **= 58.4°C**

13-141 Radiation heat transfer occurs between two parallel coaxial disks. The view factors and the rate of radiation heat transfer for the existing and modified cases are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of disk *a* and *b* are given to be $\varepsilon_a = 0.60$ and $\varepsilon_b = 0.8$, respectively.

Analysis (*a*) The view factor from surface a to surface b is determined as follows



$$A = \frac{a}{2L} = \frac{0.20}{2(0.10)} = 1$$

$$B = \frac{b}{2L} = \frac{0.40}{2(0.10)} = 2$$

$$C = 1 + \frac{1+A^2}{B^2} = 1 + \frac{1+1^2}{2^2} = 1.5$$

$$F_{ab} = 0.5\left(\frac{B}{A}\right)^2\left\{C - \left[C^2 - 4\left(\frac{A}{B}\right)^2\right]^{0.5}\right\} = 0.5\left(\frac{2}{1}\right)^2\left\{1.5 - \left[1.5^2 - 4\left(\frac{1}{2}\right)^2\right]^{0.5}\right\} = \mathbf{0.764}$$

The view factor from surface *b* to surface *a* is determined from reciprocity relation:

$$A_a = \frac{\pi a^2}{4} = \frac{\pi(0.2\text{ m})^2}{4} = 0.0314\text{ m}^2$$

$$A_b = \frac{\pi b^2}{4} = \frac{\pi(0.4\text{ m})^2}{4} = 0.1257\text{ m}^2$$

$$A_a F_{ab} = A_b F_{ba}$$

$$(0.0314)(0.764) = (0.1257)F_{ba} \longrightarrow F_{ba} = \mathbf{0.191}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{Q}_{ab} = \frac{\sigma\left(T_a^{\ 4} - T_b^{\ 4}\right)}{\dfrac{1-\varepsilon_a}{A_a\varepsilon_a} + \dfrac{1}{A_aF_{ab}} + \dfrac{1-\varepsilon_b}{A_b\varepsilon_b}} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(873\text{ K})^4 - (473\text{ K})^4\right]}{\dfrac{1-0.6}{(0.0314\text{ m}^2)(0.6)} + \dfrac{1}{(0.0314\text{ m}^2)(0.764)} + \dfrac{1-0.8}{(0.1257\text{ m}^2)(0.8)}} = \mathbf{464\ W}$$

(c) In this case we have $\varepsilon_c = 0.7$, $A_c \to \infty$, $F_{ac} = F_{bc} = 1$ and $\dot{Q}_{ac} = \dot{Q}_{cb} = \dot{Q}_{bc}$. An energy balance gives

$$\frac{\sigma\left(T_a^{\ 4} - T_c^{\ 4}\right)}{\dfrac{1-\varepsilon_a}{A_a\varepsilon_a} + \dfrac{1}{A_aF_{ac}} + \dfrac{1-\varepsilon_c}{A_c\varepsilon_c}} = \frac{\sigma\left(T_c^{\ 4} - T_b^{\ 4}\right)}{\dfrac{1-\varepsilon_c}{A_c\varepsilon_c} + \dfrac{1}{A_cF_{cb}} + \dfrac{1-\varepsilon_b}{A_b\varepsilon_b}}$$

$$\frac{T_a^{\ 4} - T_c^{\ 4}}{\dfrac{1-\varepsilon_a}{A_a\varepsilon_a} + \dfrac{1}{A_aF_{ac}} + 0} = \frac{T_c^{\ 4} - T_b^{\ 4}}{0 + \dfrac{1}{A_bF_{bc}} + \dfrac{1-\varepsilon_b}{A_b\varepsilon_b}}$$

$$\frac{(873)^4 - T_c^{\ 4}}{\dfrac{1-0.6}{(0.0314\text{ m}^2)(0.6)} + \dfrac{1}{(0.0314\text{ m}^2)(1)}} = \frac{T_c^{\ 4} - 473^4}{\dfrac{1}{(0.1257\text{ m}^2)(1)} + \dfrac{1-0.8}{(0.1257\text{ m}^2)(0.8)}}$$

$$\longrightarrow T_c = 605\text{ K}$$

Then

$$\dot{Q}_{bc} = \dot{Q}_{ac} = \frac{\sigma\left(T_a^{\ 4} - T_c^{\ 4}\right)}{\dfrac{1-\varepsilon_a}{A_a\varepsilon_a} + \dfrac{1}{A_aF_{ac}} + \dfrac{1-\varepsilon_c}{A_c\varepsilon_c}} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(873\text{ K})^4 - (605\text{ K})^4\right]}{\dfrac{1-0.6}{(0.0314\text{ m}^2)(0.6)} + \dfrac{1}{(0.0314\text{ m}^2)(1)} + 0} = \mathbf{477\ W}$$

Discussion The rate of heat transfer is higher in part (c) because the large disk c is able to collect all radiation emitted by disk a. It is not acting as a shield.

13-142 Radiation heat transfer occurs between two square parallel plates. The view factors, the rate of radiation heat transfer and the temperature of a third plate to be inserted are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

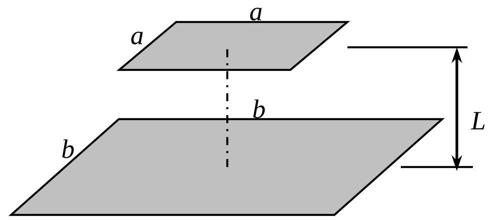
Properties The emissivities of plate a, b, and c are given to be $\varepsilon_a = 0.8$, $\varepsilon_b = 0.4$, and $\varepsilon_c = 0.1$, respectively.

Analysis (*a*) The view factor from surface a to surface b is determined as follows

$$A = \frac{a}{L} = \frac{20}{40} = 0.5,\ \ B = \frac{b}{L} = \frac{60}{40} = 1.5$$

$$F_{ab} = \frac{1}{2A}\left\{\left[(B+A)^2+4\right]^{0.5} - \left[(B-A)^2+4\right]^{0.5}\right\} = \frac{1}{2(0.5)}\left\{\left[(1.5+0.5)^2+4\right]^{0.5} - \left[(1.5-0.5)^2+4\right]^{0.5}\right\} = \mathbf{0.592}$$

The view factor from surface b to surface a is determined from reciprocity relation:



$$A_a = (0.2\text{ m})(0.2\text{ m}) = 0.04\text{ m}^2$$

$$A_b = (0.6\text{ m})(0.6\text{ m}) = 0.36\text{ m}^2$$

$$A_a F_{ab} = A_b F_{ba}$$

$$(0.04)(0.592) = (0.36)F_{ba} \longrightarrow F_{ba} = \mathbf{0.0658}$$

(b) The net rate of radiation heat transfer between the surfaces can be determined from

$$\dot{Q}_{ab} = \frac{\sigma\left(T_a^{\ 4} - T_b^{\ 4}\right)}{\dfrac{1-\varepsilon_a}{A_a\varepsilon_a} + \dfrac{1}{A_a F_{ab}} + \dfrac{1-\varepsilon_b}{A_b\varepsilon_b}} = \frac{(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)\left[(1073\text{ K})^4 - (473\text{ K})^4\right]}{\dfrac{1-0.8}{(0.04\text{ m}^2)(0.8)} + \dfrac{1}{(0.04\text{ m}^2)(0.592)} + \dfrac{1-0.4}{(0.36\text{ m}^2)(0.4)}} = \mathbf{1374\ W}$$

(c) In this case we have

$$A = \frac{a}{L} = \frac{0.2\text{ m}}{0.2\text{ m}} = 1,\ \ C = \frac{c}{L} = \frac{2.0\text{ m}}{0.2\text{ m}} = 10$$

$$F_{ac} = \frac{1}{2A}\left\{\left[(C+A)^2+4\right]^{0.5} - \left[(C-A)^2+4\right]^{0.5}\right\} = \frac{1}{2(0.5)}\left\{\left[(10+0.5)^2+4\right]^{0.5} - \left[(10-0.5)^2+4\right]^{0.5}\right\} = 0.981$$

$$B = \frac{b}{L} = \frac{0.6\text{ m}}{0.2\text{ m}} = 3,\ \ C = \frac{c}{L} = \frac{2.0\text{ m}}{0.2\text{ m}} = 10$$

$$F_{bc} = \frac{1}{2A}\left\{\left[(C+B)^2+4\right]^{0.5} - \left[(C-B)^2+4\right]^{0.5}\right\}$$

$$= \frac{1}{2(3)}\left\{\left[(10+3)^2+4\right]^{0.5} - \left[(10-3)^2+4\right]^{0.5}\right\} = 0.979$$

$$A_b F_{bc} = A_c F_{cb}$$

$$(0.36)(0.979) = (4.0)F_{cb} \longrightarrow F_{ba} = 0.0881$$

An energy balance gives

$$\dot{Q}_{ac} = \dot{Q}_{cb}$$

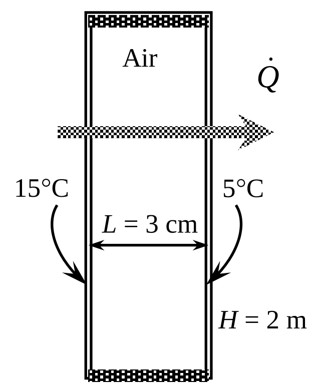
$$\frac{\sigma\left(T_a^{\ 4} - T_c^{\ 4}\right)}{\dfrac{1-\varepsilon_a}{A_a\varepsilon_a} + \dfrac{1}{A_a F_{ac}} + \dfrac{1-\varepsilon_c}{A_c\varepsilon_c}} = \frac{\sigma\left(T_c^{\ 4} - T_b^{\ 4}\right)}{\dfrac{1-\varepsilon_c}{A_c\varepsilon_c} + \dfrac{1}{A_c F_{cb}} + \dfrac{1-\varepsilon_b}{A_b\varepsilon_b}}$$

$$\frac{(1073\text{ K})^4 - T_c^{\ 4}}{\dfrac{1-0.8}{(0.04\text{ m}^2)(0.8)} + \dfrac{1}{(0.04\text{ m}^2)(0.981)} + \dfrac{1-0.1}{(4\text{ m}^2)(0.1)}} = \frac{T_c^{\ 4} - (473\text{ K})^4}{\dfrac{1-0.1}{(4\text{ m}^2)(0.1)} + \dfrac{1}{(4\text{ m}^2)(0.0881)} + \dfrac{1-0.4}{(0.36\text{ m}^2)(0.4)}}$$

Solving the equation with an equation solver such as EES, we obtain T_c = 754 K **= 481°C**

13-143 A double-pane window consists of two sheets of glass separated by an air space. The rates of heat transfer through the window by natural convection and radiation are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats. **4** Heat transfer through the window is one-dimensional and the edge effects are negligible.



Properties The emissivities of glass surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.9$. The properties of air at 0.3 atm and the average temperature of $(T_1+T_2)/2 = (15+5)/2 = 10$°C are (Table A-15)

$$k = 0.02439 \text{ W/m.°C}$$

$$\nu = \nu_{1atm} / 0.3 = 1.426\times10^{-5} /0.3 = 4.753\times10^{-5} \text{ m}^2\text{/s}$$

$$\text{Pr} = 0.7336$$

$$\beta = \frac{1}{(10+273)\text{ K}} = 0.003534 \text{ K}^{-1}$$

Analysis The characteristic length in this case is the distance between the glasses, $L_c = L = 0.03\text{ m}$

$$Ra = \frac{g\beta(T_1 - T_2)L^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003534\text{ K}^{-1})(15-5)\text{K}(0.03\text{ m})^3}{(4.753\times10^{-5}\text{ m}^2\text{/s})^2}(0.7336) = 3040$$

$$Nu = 0.197Ra^{1/4}\left(\frac{H}{L}\right)^{-1/9} = 0.197(3040)^{1/4}\left(\frac{2}{0.05}\right)^{-1/9} = 0.971$$

Note that heat transfer through the air space is less than that by pure conduction as a result of partial evacuation of the space. Then the rate of heat transfer through the air space becomes

$$A_s = (2\text{ m})(5\text{ m}) = 10\text{ m}^2$$

$$\dot{Q}_{conv} = kNuA_s\frac{T_1 - T_2}{L} = (0.02439 \text{ W/m.°C})(0.971)(10\text{ m}^2)\frac{(15-5)\text{°C}}{0.03\text{ m}} = \mathbf{78.9\ W}$$

The rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_s\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{(10\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(15+273\text{ K})^4 - (5+273\text{ K})^4]}{\frac{1}{0.9} + \frac{1}{0.9} - 1} = \mathbf{421\,W}$$

Then the rate of total heat transfer becomes

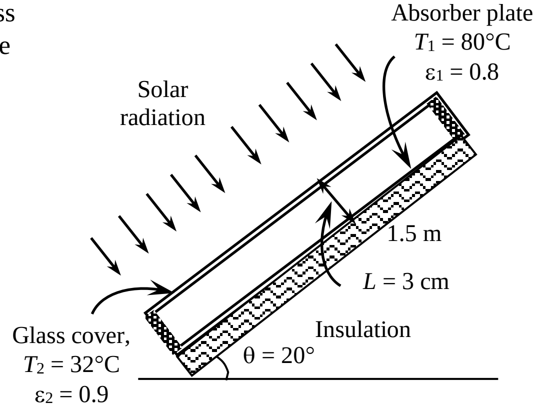
$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 79 + 421 = \mathbf{500\ W}$$

Discussion Note that heat transfer through the window is mostly by radiation.

13-144 A solar collector is considered. The absorber plate and the glass cover are maintained at uniform temperatures, and are separated by air. The rate of heat loss from the absorber plate by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant properties.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.9$ for glass and $\varepsilon_2 = 0.8$ for the absorber plate. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (80+32)/2 = 56$°C are (Table A-15)



$$k = 0.02779 \text{ W/m.°C}$$
$$\nu = 1.857 \times 10^{-5} \text{ m}^2/\text{s}$$
$$\Pr = 0.7212$$
$$\beta = \frac{1}{T_f} = \frac{1}{(56+273)\text{K}} = 0.003040 \text{ K}^{-1}$$

Analysis For $\theta = 0°$, we have horizontal rectangular enclosure. The characteristic length in this case is the distance between the two glasses $L_c = L = 0.03$ m Then,

$$Ra = \frac{g\beta(T_1-T_2)L^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.00304 \text{ K}^{-1})(80-32 \text{ K})(0.03 \text{ m})^3}{(1.857\times10^{-5} \text{ m}^2/\text{s})^2}(0.7212) = 8.083\times10^4$$

$$A_s = H \times W = (1.5 \text{ m})(3 \text{ m}) = 4.5 \text{ m}^2$$

$$\text{Nu} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}\cos\theta}\right]^+ \left[1 - \frac{1708(\sin 1.8\theta)^{1.6}}{\text{Ra}\cos\theta}\right] + \left[\frac{(\text{Ra}\cos\theta)^{1/3}}{18} - 1\right]^+$$

$$= 1 + 1.44\left[1 - \frac{1708}{(8.083\times10^4)\cos(20)}\right]^+ \left[1 - \frac{1708[\sin(1.8\times20)]^{1.6}}{(8.083\times10^4)\cos(20)}\right] + \left[\frac{[(8.083\times10^4)\cos(20)]^{1/3}}{18} - 1\right]^+$$

$$= 3.747$$

$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.02779 \text{ W/m.°C})(3.747)(4.5 \text{ m}^2)\frac{(80-32)\text{°C}}{0.03 \text{ m}} = \mathbf{750 \text{ W}}$$

Neglecting the end effects, the rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_s\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{(4.5 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(80+273 \text{ K})^4 - (32+273 \text{ K})^4]}{\frac{1}{0.8} + \frac{1}{0.9} - 1} = \mathbf{1289 \text{ W}}$$

Discussion The rates of heat loss by natural convection for the horizontal and vertical cases would be as follows (Note that the Ra number remains the same):

Horizontal:

$$\text{Nu} = 1 + 1.44\left[1 - \frac{1708}{\text{Ra}}\right]^+ + \left[\frac{\text{Ra}^{1/3}}{18} - 1\right]^+ = 1 + 1.44\left[1 - \frac{1708}{8.083\times10^4}\right]^+ + \left[\frac{(8.083\times10^4)^{1/3}}{18} - 1\right]^+ = 3.812$$

$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.02779 \text{ W/m.°C})(3.812)(6 \text{ m}^2)\frac{(80-32)\text{°C}}{0.03 \text{ m}} = \mathbf{1017 \text{ W}}$$

Vertical:

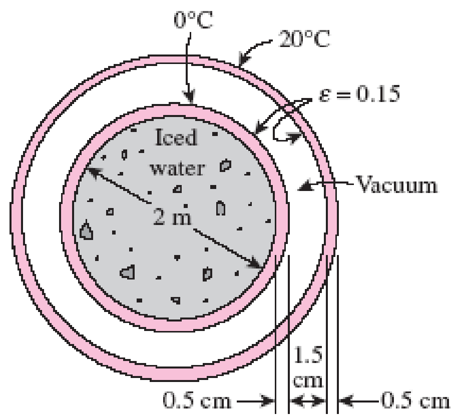
$$Nu = 0.42Ra^{1/4}\Pr^{0.012}\left(\frac{H}{L}\right)^{-0.3} = 0.42(8.083\times10^4)^{1/4}(0.7212)^{0.012}\left(\frac{2 \text{ m}}{0.03 \text{ m}}\right)^{-0.3} = 2.001$$

$$\dot{Q} = kNuA_s\frac{T_1-T_2}{L} = (0.02779 \text{ W/m.°C})(2.001)(6 \text{ m}^2)\frac{(80-32)\text{°C}}{0.03 \text{ m}} = \mathbf{534 \text{ W}}$$

13-145 A double-walled spherical tank is used to store iced water. The air space between the two walls is evacuated. The rate of heat transfer to the iced water and the amount of ice that melts a 24-h period are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Properties The emissivities of both surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.15$.



Analysis (*a*) Assuming the conduction resistance s of the walls to be negligible, the rate of heat transfer to the iced water in the tank is determined to be

$$A_1 = \pi D_1^2 = \pi (2.01\,\text{m})^2 = 12.69\,\text{m}^2$$

$$\dot{Q}_{12} = \frac{A_1 \sigma (T_2^4 - T_1^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)^2}$$

$$= \frac{(12.69\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(20+273\,\text{K})^4 - (0+273\,\text{K})^4]}{\dfrac{1}{0.15} + \dfrac{1-0.15}{0.15}\left(\dfrac{2.01}{2.04}\right)^2}$$

$$= \mathbf{107.4\ W}$$

(*b*) The amount of heat transfer during a 24-hour period is

$$Q = \dot{Q}\Delta t = (0.1074\,\text{kJ/s})(24\times3600\,\text{s}) = 9279\,\text{kJ}$$

The amount of ice that melts during this period then becomes

$$Q = mh_{if} \longrightarrow m = \frac{Q}{h_{if}} = \frac{9279\,\text{kJ}}{333.7\,\text{kJ/kg}} = \mathbf{27.8\,kg}$$

13-146 A solar collector consists of a horizontal copper tube enclosed in a concentric thin glass tube. The annular space between the copper and the glass tubes is filled with air at 1 atm. The rate of heat loss from the collector by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant specific heats.

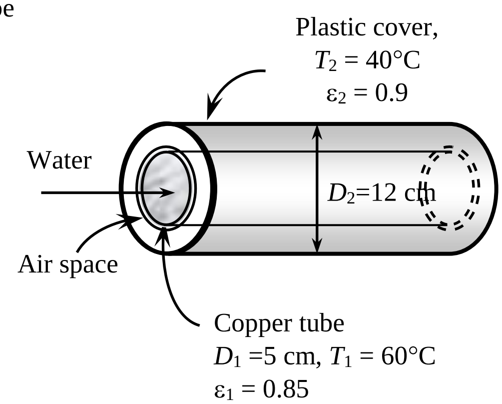
Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.85$ for the tube surface and $\varepsilon_2 = 0.9$ for glass cover. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (60+40)/2 = 50°C$ are (Table A-15)

$$k = 0.02735 \text{ W/m.°C}$$

$$\nu = 1.798\times10^{-5} \text{ m}^2/\text{s}$$

$$\text{Pr} = 0.7228$$

$$\beta = \frac{1}{(50+273)\text{ K}} = 0.003096 \text{ K}^{-1}$$



Analysis The characteristic length in this case is

$$L_c = \frac{1}{2}(D_2 - D_1) = \frac{1}{2}(0.12\text{ m} - 0.05\text{ m}) = 0.07\text{ m}$$

$$Ra = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003096\text{ K}^{-1})(60-40)\text{K}(0.035\text{ m})^3}{(1.798\times10^{-5}\text{ m}^2/\text{s})^2}(0.7228) = 5.823\times10^4$$

The effective thermal conductivity is

$$F_{\text{cyl}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5}+D_o^{-3/5})^5} = \frac{[\ln(0.12/0.05)]^4}{(0.035\text{ m})^3\left[(0.05\text{ m})^{-3/5}+(0.12\text{ m})^{-3/5}\right]^5} = 0.1678$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra})^{1/4}$$

$$= 0.386(0.02735\text{ W/m.°C})\left(\frac{0.7228}{0.861+0.7228}\right)^{1/4}\left[(0.1678)(5.823\times10^4)\right]^{1/4} = 0.08626\text{ W/m.°C}$$

Then the rate of heat transfer between the cylinders becomes

$$\dot{Q}_{\text{conv}} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) = \frac{2\pi(0.08626\text{ W/m.°C})}{\ln(0.12/0.05)}(60-40)°\text{C} = \mathbf{12.4\ W}$$

The rate of heat transfer by radiation is determined from

$$\dot{Q}_{\text{rad}} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)} = \frac{[\pi(0.05\text{ m})(1\text{m})](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(60+273\text{ K})^4-(40+273\text{ K})^4]}{\frac{1}{0.85}+\frac{1-0.9}{0.9}\left(\frac{5}{12}\right)}$$

$$= \mathbf{19.7\ W}$$

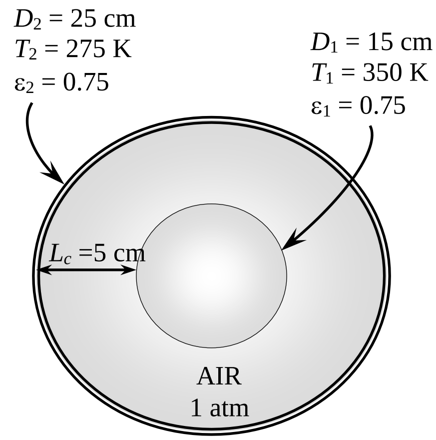
Finally,

$$\dot{Q}_{total,loss} = 12.4 + 19.7 = 32.1\text{ W} \quad \text{(per m length)}$$

13-147 Two concentric spheres which are maintained at uniform temperatures are separated by air at 1 atm pressure. The rate of heat transfer between the two spheres by natural convection and radiation is to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Air is an ideal gas with constant properties.

Properties The emissivities of the surfaces are given to be $\varepsilon_1 = \varepsilon_2 = 0.75$. The properties of air at 1 atm and the average temperature of $(T_1+T_2)/2 = (350+275)/2 = 312.5$ K = 39.5°C are (Table A-15)



$$k = 0.02658 \text{ W/m.°C}$$
$$\nu = 1.697\times10^{-5} \text{ m}^2/\text{s}$$
$$\text{Pr} = 0.7256$$
$$\beta = \frac{1}{312.5 \text{ K}} = 0.0032 \text{ K}^{-1}$$

Analysis (*a*) Noting that $D_i = D_1$ and $D_o = D_2$, the characteristic length is

$$L_c = \frac{1}{2}(D_o - D_i) = \frac{1}{2}(0.25\text{ m} - 0.15\text{ m}) = 0.05\text{ m}$$

Then

$$Ra = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)(0.003200\text{ K}^{-1})(350-275\text{ K})(0.05\text{ m})^3}{(1.697\times10^{-5}\text{ m}^2/\text{s})^2}(0.7256) = 7.415\times10^5$$ The effective thermal conductivity is

$$F_{\text{sph}} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5}+D_o^{-7/5})^5} = \frac{0.05\text{ m}}{\left[(0.15\text{ m})(0.25\text{ m})\right]^4\left[(0.15\text{ m})^{-7/5}+(0.25\text{ m})^{-7/5}\right]^5} = 0.005900$$

$$k_{\text{eff}} = 0.74k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{sph}Ra)^{1/4}$$
$$= 0.74(0.02658\text{ W/m.°C})\left(\frac{0.7256}{0.861+0.7256}\right)^{1/4}\left[(0.00590)(7.415\times10^5)\right]^{1/4}$$
$$= 0.1315\text{ W/m.°C}$$

Then the rate of heat transfer between the spheres becomes

$$\dot{Q} = k_{\text{eff}}\,\pi\left(\frac{D_iD_o}{L_c}\right)(T_i - T_o) = (0.1315\text{ W/m.°C})\pi\left[\frac{(0.15\text{ m})(0.25\text{ m})}{(0.05\text{ m})}\right](350-275)\text{K} = \mathbf{23.2\ W}$$

(*b*) The rate of heat transfer by radiation is determined from

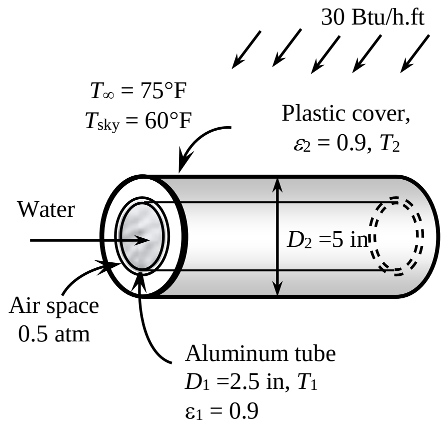
$$A_1 = \pi D_1^2 = \pi(0.15\text{ m})^2 = 0.0707\text{ m}^2$$

$$\dot{Q}_{12} = \frac{A_1\sigma(T_2^4 - T_1^4)}{\dfrac{1}{\varepsilon_1}+\dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)^2} = \frac{(0.0707\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(350\text{ K})^4-(275\text{ K})^4]}{\dfrac{1}{0.75}+\dfrac{1-0.75}{0.75}\left(\dfrac{0.15}{0.25}\right)^2} = \mathbf{25.6\ W}$$

13-148E The circulating pump of a solar collector that consists of a horizontal tube and its glass cover fails. The equilibrium temperature of the tube is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The tube and its cover are isothermal. **3** Air is an ideal gas. **4** The surfaces are opaque, diffuse, and gray for infrared radiation. **5** The glass cover is transparent to solar radiation.

Properties The properties of air should be evaluated at the average temperature. But we do not know the exit temperature of the air in the duct, and thus we cannot determine the bulk fluid and glass cover temperatures at this point, and thus we cannot evaluate the average temperatures. Therefore, we will assume the glass temperature to be 85°F, and use properties at an anticipated average temperature of (75+85)/2 =80°F (Table A-15E),



$$k = 0.01481 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}$$
$$\nu = 1.697\times10^{-4} \text{ ft}^2/s$$
$$\text{Pr} = 0.7290$$
$$\beta = \frac{1}{T_{\text{ave}}} = \frac{1}{540 \text{ R}}$$

Analysis We have a horizontal cylindrical enclosure filled with air at 0.5 atm pressure. The problem involves heat transfer from the aluminum tube to the glass cover and from the outer surface of the glass cover to the surrounding ambient air. When steady operation is reached, these two heat transfer rates must equal the rate of heat gain. That is,

$$\dot{Q}_{\text{tube-glass}} = \dot{Q}_{\text{glass-ambient}} = \dot{Q}_{\text{solar gain}} = 30 \text{ Btu/h} \qquad \text{(per foot of tube)}$$

The heat transfer surface area of the glass cover is

$$A_o = A_{glass} = (\pi D_o W) = \pi(5/12 \text{ ft})(1 \text{ ft}) = 1.309 \text{ ft}^2 \quad \text{(per foot of tube)}$$

To determine the Rayleigh number, we need to know the surface temperature of the glass, which is not available. Therefore, solution will require a trial-and-error approach. Assuming the glass cover temperature to be 85°F, the Rayleigh number, the Nusselt number, the convection heat transfer coefficient, and the rate of natural convection heat transfer from the glass cover to the ambient air are determined to be

$$\text{Ra}_{Do} = \frac{g\beta(T_o - T_\infty)D_o^3}{\nu^2}\text{Pr}$$
$$= \frac{(32.2 \text{ ft/s}^2)[1/(540 \text{ R})](85-75 \text{ R})(5/12 \text{ ft})^3}{(1.697\times10^{-4} \text{ ft}^2/\text{s})^2}(0.7290) = 1.092\times10^6$$

$$\text{Nu} = \left\{0.6 + \frac{0.387\,\text{Ra}_D^{1/6}}{\left[1+\left(0.559/\text{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1.092\times10^6)^{1/6}}{\left[1+\left(0.559/0.7290\right)^{9/16}\right]^{8/27}}\right\}^2$$
$$= 14.95$$

$$h_o = \frac{k}{D_0}\text{Nu} = \frac{0.01481 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}}{5/12 \text{ ft}}(14.95) = 0.5315 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F}$$

$$\dot{Q}_{o,\text{conv}} = h_o A_o(T_o - T_\infty) = (0.5315 \text{ Btu/h}\cdot\text{ft}^2\cdot°\text{F})(1.309 \text{ ft}^2)(85-75)°\text{F} = 6.96 \text{ Btu/h}$$

Also,

$$\dot{Q}_{o,\text{rad}} = \varepsilon_o\sigma A_o(T_o^4 - T_{\text{sky}}^4)$$
$$= (0.9)(0.1714\times10^{-8} \text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(1.309 \text{ ft}^2)\left[(545 \text{ R})^4 - (520 \text{ R})^4\right]$$
$$= 30.5 \text{ Btu/h}$$

Then the total rate of heat loss from the glass cover becomes

$$\dot{Q}_{o,\text{total}} = \dot{Q}_{o,\text{conv}} + \dot{Q}_{o,\text{rad}} = 7.0 + 30.5 = 37.5 \text{ Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 85°F for the glass cover is high. Repeating the calculations with lower temperatures (including the evaluation of properties), the glass cover temperature corresponding to 30 Btu/h is determined to be 81.5°F.

The temperature of the aluminum tube is determined in a similar manner using the natural convection and radiation relations for two horizontal concentric cylinders. The characteristic length in this case is the distance between the two cylinders, which is

$$L_c = (D_o - D_i)/2 = (5-2.5)/2 = 1.25\text{ in} = 1.25/12\text{ ft}$$

Also,

$$A_i = A_{tube} = (\pi D_i W) = \pi(2.5/12\text{ ft})(1\text{ ft}) = 0.6545\ \text{ft}^2 \quad \text{(per foot of tube)}$$

We start the calculations by assuming the tube temperature to be 118.5°F, and thus an average temperature of (81.5+118.5)/2 = 100°F=560 R. Using properties at 100°F,

$$\text{Ra}_L = \frac{g\beta(T_i - T_o)L^3}{\nu^2}\text{Pr} = \frac{(32.2\text{ ft/s}^2)[1/(560\text{ R})](118.5-81.5\text{ R})(1.25/12\text{ ft})^3}{[(1.809\times10^{-4}\ \text{ft}^2/\text{s})/0.5]^2}(0.726) = 1.334\times10^4$$

The effective thermal conductivity is

$$F_{\text{cyc}} = \frac{[\ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5} = \frac{[\ln(5/2.5)]^4}{(1.25/12\text{ ft})^3[(2.5/12\text{ ft})^{-3/5} + (5/12\text{ ft})^{-3/5}]^5} = 0.1466$$

$$k_{\text{eff}} = 0.386k\left(\frac{\text{Pr}}{0.861+\text{Pr}}\right)^{1/4}(F_{\text{cyc}}\text{Ra}_L)^{1/4}$$

$$= 0.386(0.01529\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})\left(\frac{0.726}{0.861+0.726}\right)(0.1466\times1.334\times10^4)^{1/4}$$

$$= 0.03227\text{ Btu/h}\cdot\text{ft}\cdot°\text{F}$$

Then the rate of heat transfer between the cylinders by convection becomes

$$\dot{Q}_{i,\text{conv}} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o) = \frac{2\pi(0.03227\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})}{\ln(5/2.5)}(118.5-81.5)°\text{F} = 10.8\text{ Btu/h}$$

Also,

$$\dot{Q}_{i,\text{rad}} = \frac{\sigma A_i(T_i^4 - T_o^4)}{\dfrac{1}{\varepsilon_i} + \dfrac{1-\varepsilon_o}{\varepsilon_o}\left(\dfrac{D_i}{D_o}\right)}$$

$$= \frac{(0.1714\times10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)(0.6545\text{ ft}^2)\left[(578.5\ \text{R})^4 - (541.5\text{ R})^4\right]}{\dfrac{1}{0.9} + \dfrac{1-0.9}{0.9}\left(\dfrac{2.5\text{ in}}{5\text{ in}}\right)} = 25.0\text{ Btu/h}$$

Then the total rate of heat loss from the glass cover becomes

$$\dot{Q}_{i,\text{total}} = \dot{Q}_{i,\text{conv}} + \dot{Q}_{i,\text{rad}} = 10.8 + 25.0 = 35.8\text{ Btu/h}$$

which is more than 30 Btu/h. Therefore, the assumed temperature of 118.5°F for the tube is high. By trying other values, the tube temperature corresponding to 30 Btu/h is determined to be **113°F**. Therefore, the tube will reach an equilibrium temperature of 113°F when the pump fails.

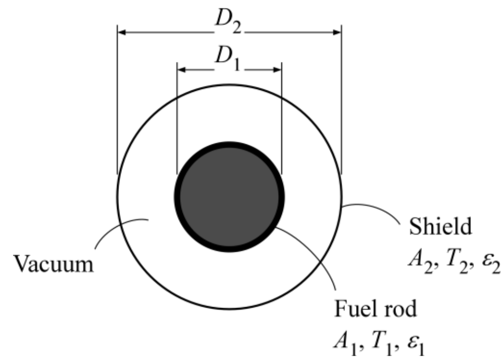
13-149 A long cylindrical black surface fuel rod is shielded by a concentric surface that has a uniform temperature. The fuel rod generates 0.5 MW/m^3 of heat per unit length. The surface temperature of the fuel rod is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The fuel rod surface is black. 3 The shield is opaque, diffuse, and gray. **4** The fuel rod and shield formed an infinitely long concentric cylinder.

Properties The emissivity of the shield is given as ε_2 = 0.05. The fuel rod surface is black, ε_1 = 1.

Analysis The heat generation of the fuel rod is

$$\dot{e}_{\text{gen}} = \frac{\dot{E}_{\text{gen}}}{V} = \frac{4\dot{E}_{\text{gen}}}{\pi D_1^2 L} \quad \rightarrow \quad \dot{E}_{\text{gen}} = \dot{e}_{\text{gen}} V = \dot{e}_{\text{gen}} \frac{\pi D_1^2 L}{4}$$



Hence, the total heat generation rate per unit length (L = 1 m) by the fuel rod is

$$\dot{E}_{\text{gen}} = \dot{e}_{\text{gen}} \frac{\pi D_1^2 L}{4} = (0.5\times 10^6 \text{ W/m}^3)\frac{\pi(0.025\text{ m})^2(1\text{ m})}{4} = 245.4\text{ W}$$

For infinitely long concentric cylinder, the rate of radiation heat transfer at the fuel rod surface is (from Table 13-3),

$$\dot{E}_{\text{gen}} = \dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1-\varepsilon_2}{\varepsilon_2}\left(\dfrac{D_1}{D_2}\right)}$$

$$T_1 = \left\{\left[\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)\right]\frac{\dot{E}_{\text{gen}}}{A_1\sigma} + T_2^4\right\}^{1/4} = \left\{\left[\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{D_1}{D_2}\right)\right]\frac{\dot{E}_{\text{gen}}}{\pi D_1 L\sigma} + T_2^4\right\}^{1/4}$$

$$= \left\{\left[\frac{1}{1} + \frac{1-0.05}{0.05}\left(\frac{25}{50}\right)\right]\frac{245.4\text{ W}}{\pi(0.025\text{ m})(1\text{ m})(5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)} + (320\text{ K})^4\right\}^{1/4}$$

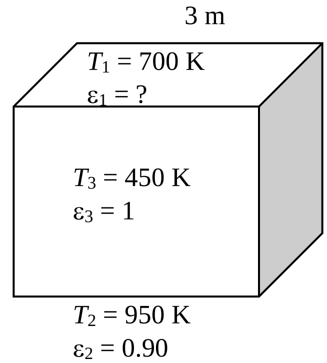
$$T_1 = \mathbf{876\ K}$$

Discussion The use of absolute temperatures is necessary for calculations involving radiation heat transfer.

13-150 A cylindrical furnace with specified top and bottom surface temperatures and specified heat transfer rate at the bottom surface is considered. The emissivity of the top surface and the net rates of heat transfer between the top and the bottom surfaces, and between the bottom and the side surfaces are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivity of the bottom surface is 0.90.



Analysis We consider the top surface to be surface 1, the base surface to be surface 2, and the side surface to be surface 3. This system is a three-surface enclosure. The view factor from the base to the top surface of the cube is from Fig. 13-5 $F_{12} = 0.2$. The view factor from the base or the top to the side surfaces is determined by applying the summation rule to be

$$F_{11} + F_{12} + F_{13} = 1 \longrightarrow F_{13} = 1 - F_{12} = 1 - 0.2 = 0.8$$

since the base surface is flat and thus $F_{11} = 0$. Other view factors are

$$F_{21} = F_{12} = 0.20, \qquad F_{23} = F_{13} = 0.80, \qquad F_{31} = F_{32} = 0.20$$

We now apply Eq. 13-35 to each surface

Surface 1:

$$\sigma T_1^{\ 4} = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1 - J_2) + F_{13}(J_1 - J_3)\right]$$

$$(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(700\ \text{K})^4 = J_1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left[0.20(J_1 - J_2) + 0.80(J_1 - J_3)\right]$$

Surface 2:

$$\sigma T_2^{\ 4} = J_2 + \frac{1-\varepsilon_2}{\varepsilon_2}\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right]$$

$$(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(950\ \text{K})^4 = J_2 + \frac{1-0.90}{0.90}\left[0.20(J_2 - J_1) + 0.80(J_2 - J_3)\right]$$

Surface 3:

$$\sigma T_3^{\ 4} = J_3$$

$$(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(450\ \text{K})^4 = J_3$$

We now apply Eq. 13-34 to surface 2

$$\dot{Q}_2 = A_2\left[F_{21}(J_2 - J_1) + F_{23}(J_2 - J_3)\right] = (9\ \text{m}^2)\left[0.20(J_2 - J_1) + 0.80(J_2 - J_3)\right]$$

Solving the above four equations, we find

$$\varepsilon_1 = \mathbf{0.44}, \quad J_1 = 11{,}736\ \text{W/m}^2, \quad J_2 = 41{,}985\ \text{W/m}^2, \quad J_3 = 2325\ \text{W/m}^2$$

The rate of heat transfer between the bottom and the top surface is

$$A_1 = A_2 = (3\ \text{m})^2 = 9\ \text{m}^2$$

$$\dot{Q}_{21} = A_2 F_{21}(J_2 - J_1) = (9\ \text{m}^2)(0.20)(41{,}985 - 11{,}736)\text{W/m}^2 = \mathbf{54.4\ kW}$$

The rate of heat transfer between the bottom and the side surface is

$$A_3 = 4A_1 = 4(9\ \text{m}^2) = 36\ \text{m}^2$$

$$\dot{Q}_{23} = A_2 F_{23}(J_2 - J_3) = (9\ \text{m}^2)(0.8)(41{,}985 - 2325)\text{W/m}^2 = \mathbf{285.6\ kW}$$

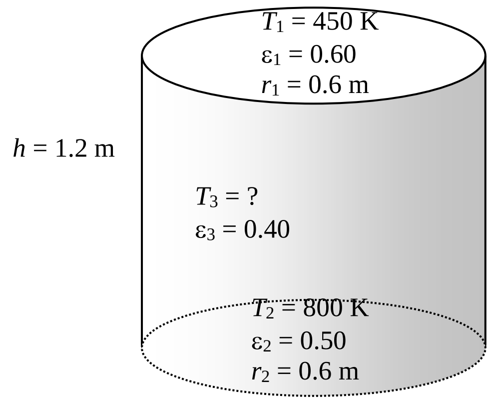
Discussion The sum of these two heat transfer rates are 54.4 + 285.6 = 340 kW, which is equal to 340 kW heat supply rate from surface 2.

13-151 A cylindrical furnace with specified top and bottom surface temperatures and specified heat transfer rate at the bottom surface is considered. The temperature of the side surface and the net rates of heat transfer between the top and the bottom surfaces, and between the bottom and the side surfaces are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of the top, bottom, and side surfaces are 0.60, 0.50, and 0.40, respectively.

Analysis We consider the top surface to be surface 1, the bottom surface to be surface 2, and the side surface to be surface 3. This system is a three-surface enclosure. The view factor from surface 1 to surface 2 is determined from



$$\left.\begin{array}{l}\dfrac{L}{r}=\dfrac{1.2}{0.6}=2\\[2ex]\dfrac{r}{L}=\dfrac{0.6}{1.2}=0.5\end{array}\right\}F_{12}=0.17\ \ \text{(Fig. 13-7)}$$

The surface areas are

$$A_1=A_2=\pi D^2/4=\pi(1.2\text{ m})^2/4=1.131\text{ m}^2$$

$$A_3=\pi DL=\pi(1.2\text{ m})(1.2\text{ m})=4.524\text{ m}^2$$

Then other view factors are determined to be

$$F_{12}=F_{21}=0.17$$

$$F_{11}+F_{12}+F_{13}=1\longrightarrow 0+0.17+F_{13}=1\longrightarrow F_{13}=0.83\ \ \text{(summation rule)},\quad F_{23}=F_{13}=0.83$$

$$A_1F_{13}=A_3F_{31}\longrightarrow(1.131)(0.83)=(4.524)F_{31}\longrightarrow F_{31}=0.21\ \ \text{(reciprocity rule)},\quad F_{32}=F_{31}=0.21$$

We now apply Eq. 13-35 to each surface

Surface 1:

$$\sigma T_1^{\,4}=J_1+\frac{1-\varepsilon_1}{\varepsilon_1}\left[F_{12}(J_1-J_2)+F_{13}(J_1-J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(450\text{ K})^4=J_1+\frac{1-0.60}{0.60}\left[0.17(J_1-J_2)+0.83(J_1-J_3)\right]$$

Surface 2:

$$\sigma T_2^{\,4}=J_2+\frac{1-\varepsilon_2}{\varepsilon_2}\left[F_{21}(J_2-J_1)+F_{23}(J_2-J_3)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)(800\text{ K})^4=J_2+\frac{1-0.50}{0.50}\left[0.17(J_2-J_1)+0.83(J_2-J_3)\right]$$

Surface 3:

$$\sigma T_3^{\,4}=J_3+\frac{1-\varepsilon_3}{\varepsilon_3}\left[F_{31}(J_3-J_1)+F_{32}(J_3-J_2)\right]$$

$$(5.67\times10^{-8}\text{ W/m}^2.\text{K}^4)T_3^{\,4}=J_3+\frac{1-0.40}{0.40}\left[0.21(J_3-J_1)+0.21(J_3-J_2)\right]$$

We now apply Eq. 13-34 to surface 2

$$\dot{Q}_2=A_2\left[F_{21}(J_2-J_1)+F_{23}(J_2-J_3)\right]=(1.131\text{ m}^2)\left[0.17(J_2-J_1)+0.83(J_2-J_3)\right]$$

Solving the above four equations, we find

$$T_3=\mathbf{831\ K},\quad J_1=10{,}477\text{ W/m}^2,\quad J_2=21{,}986\text{ W/m}^2,\quad J_3=22{,}852\text{ W/m}^2$$

The rate of heat transfer between the bottom and the top surface is

$$\dot{Q}_{21}=A_2F_{21}(J_2-J_1)=(1.131\text{ m}^2)(0.17)(21{,}986-10{,}477)\text{W/m}^2=\mathbf{2213\ W}$$

The rate of heat transfer between the bottom and the side surface is

$$\dot{Q}_{23}=A_2F_{23}(J_2-J_3)=(1.131\text{ m}^2)(0.83)(21{,}986-22{,}852)\text{W/m}^2=\mathbf{-813\ W}$$

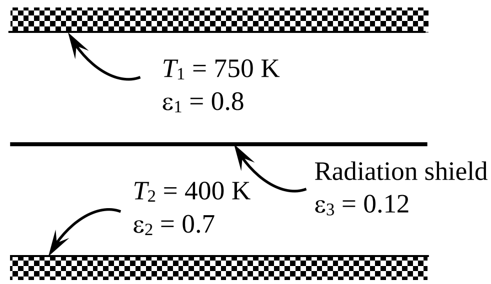
Discussion The sum of these two heat transfer rates are 2213 +(-813) = 1400 W, which is the heat supply rate from surface 2. This must be satisfied to maintain the surfaces at the specified temperatures under steady operation. Note that the difference is due to round-off error.

13-152 A thin aluminum sheet is placed between two very large parallel plates that are maintained at uniform temperatures. The net rate of radiation heat transfer between the plates and the temperature of the radiation shield are to be determined.

Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray. **3** Convection heat transfer is not considered.

Properties The emissivities of surfaces are given to be $\varepsilon_1 = 0.8$, $\varepsilon_2 = 0.7$, and $\varepsilon_3 = 0.12$.

Analysis The net rate of radiation heat transfer with a thin aluminum shield per unit area of the plates is



$$\dot{Q}_{12,\text{one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_{3,1}}+\frac{1}{\varepsilon_{3,2}}-1\right)}$$

$$= \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(750\ \text{K})^4-(400\ \text{K})^4]}{\left(\frac{1}{0.8}+\frac{1}{0.7}-1\right)+\left(\frac{1}{0.12}+\frac{1}{0.12}-1\right)}$$

$$= \mathbf{951\ W/m^2}$$

The equilibrium temperature of the radiation shield is determined from

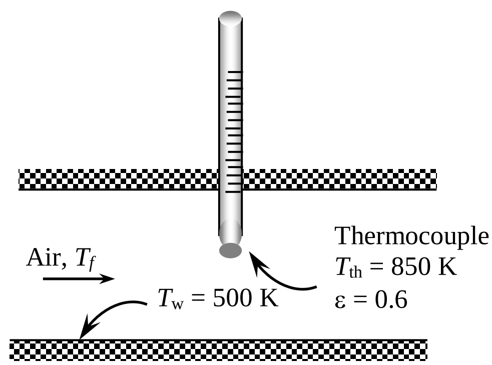
$$\dot{Q}_{13} = \frac{\sigma(T_1^4 - T_3^4)}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right)} \longrightarrow 951\ \text{W/m}^2 = \frac{(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(750\ \text{K})^4-T_3^4]}{\left(\frac{1}{0.8}+\frac{1}{0.12}-1\right)} \longrightarrow T_3 = \mathbf{644\ K}$$

13-153 The temperature of air in a duct is measured by a thermocouple. The radiation effect on the temperature measurement is to be quantified, and the actual air temperature is to be determined.

Assumptions The surfaces are opaque, diffuse, and gray.

Properties The emissivity of thermocouple is given to be ε=0.6.

Analysis The actual temperature of the air can be determined from



$$T_f = T_{th} + \frac{\varepsilon_{th}\sigma(T_{th}^4 - T_w^4)}{h}$$

$$= 850\ \text{K} + \frac{(0.6)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(850\ \text{K})^4-(500\ \text{K})^4]}{75\ \text{W/m}^2\cdot{}^\circ\text{C}}$$

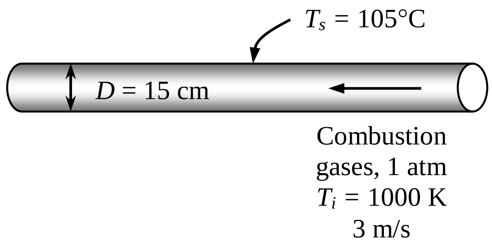
$$= \mathbf{1058\ K}$$

13-154 Combustion gases flow inside a tube in a boiler. The rates of heat transfer by convection and radiation and the rate of evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Combustion gases are assumed to have the properties of air, which is an ideal gas with constant properties.

Properties The properties of air at 1000 K = 727°C and 1 atm are (Table A-15)

$$\rho = 0.3535\ \text{kg/m}^3$$
$$k = 0.06704\ \text{W/m.°C}$$
$$\nu = 1.185\times10^{-4}\ \text{m}^2\text{/s}$$
$$c_p = 1140\ \text{J/kg.°C}$$
$$\text{Pr} = 0.7107$$



Analysis (*a*) The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}}D}{\nu} = \frac{(3\ \text{m/s})(0.15\text{m})}{1.185\times10^{-4}\ \text{m}^2\text{/s}} = 3797$$

which is higher than 2300, but much less than 10,000. We assume laminar flow. The Nusselt number in this case is

$$Nu = \frac{hD_h}{k} = 3.66$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.06704\ \text{W/m.°C}}{0.15\ \text{m}}(3.66) = 1.636\ \text{W/m}^2\text{.°C}$$

Next we determine the exit temperature of air

$$A = \pi DL = \pi(0.15\ \text{m})(6\ \text{m}) = 2.827\ \text{m}^2$$
$$A_c = \pi D^2/4 = \pi(0.15\ \text{m})^2/4 = 0.01767\ \text{m}^2$$
$$\dot{m} = \rho VA_c = (0.3535\ \text{kg/m}^3)(3\ \text{m/s})(0.01767\ \text{m}^2) = 0.01874\ \text{kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 105 - (105 - 727)e^{-\frac{(1.636)(2.827)}{(0.01874)(1140)}} = 605.9\text{°C}$$

Then the rate of heat transfer by convection becomes

$$\dot{Q}_{\text{conv}} = \dot{m}c_p(T_i - T_e) = (0.01874\ \text{kg/s})(1140\ \text{J/kg.°C})(727 - 605.9)\text{°C} = \mathbf{2587\ W}$$

Next, we determine the emissivity of combustion gases. First, the mean beam length for an infinite circular cylinder is, from Table 13-4,

$$L = 0.95(0.15\ \text{m}) = 0.1425\ \text{m}$$

Then,

$$P_cL = (0.08\ \text{atm})(0.1425\ \text{m}) = 0.0114\ \text{m}\cdot\text{atm} = 0.037\ \text{ft}\cdot\text{atm}$$
$$P_wL = (0.16\ \text{atm})(0.1425\ \text{m}) = 0.0228\ \text{m}\cdot\text{atm} = 0.075\ \text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the average gas temperature of T_g=(T_g+T_g)/2 = (727+606)/2 = 667°C = 940 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.055 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.050$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at $T = T_g$ = 940 K is, from Fig. 13-38,

$$\left.\begin{aligned} P_cL + P_wL &= 0.037 + 0.075 = 0.112 \\ \frac{P_w}{P_w + P_c} &= \frac{0.16}{0.16 + 0.08} = 0.67 \end{aligned}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c\varepsilon_{c,1\,\text{atm}} + C_w\varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1\times0.055 + 1\times0.050 - 0.0 = 0.105$$

Note that the pressure correction factor is 1 for both gases since the total pressure is 1 atm. For a source temperature of T_s = 105°C = 378 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.08\,\text{atm})(0.1425\,\text{m})\frac{378\,\text{K}}{940\,\text{K}} = 0.00458\,\text{m}\cdot\text{atm} = 0.015\,\text{ft}\cdot\text{atm}$$

$$P_w L \frac{T_s}{T_g} = (0.16\,\text{atm})(0.1425\,\text{m})\frac{378\,\text{K}}{940\,\text{K}} = 0.00916\,\text{m}\cdot\text{atm} = 0.030\,\text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 378 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.034 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.055$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\,\text{atm}} = (1)\left(\frac{940\,\text{K}}{378\,\text{K}}\right)^{0.65}(0.034) = 0.0615$$

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,1\,\text{atm}} = (1)\left(\frac{940\,\text{K}}{378\,\text{K}}\right)^{0.45}(0.055) = 0.0829$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 13-38 at $T = T_s$ = 378 K instead of T_g = 940 K. We use the chart for 400 K. At $P_w/(P_w + P_c)$ = 0.67 and $P_cL + P_wL$ = 0.112 we read $\Delta\varepsilon$ = 0.0. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.0615 + 0.0829 - 0.0 = 0.144$$

The emissivity of the inner surfaces of the tubes is 0.9. Then the net rate of radiation heat transfer from the combustion gases to the walls of the tube becomes

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \frac{\varepsilon_s + 1}{2} A_s \sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= \frac{0.9+1}{2}(2.827\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.105(940\,\text{K})^4 - 0.144(378\,\text{K})^4] \\ &= \mathbf{12{,}040\ W}\end{aligned}$$

(*b*) The heat of vaporization of water at 1 atm is 2257 kJ/kg (Table A-9). Then rate of evaporation of water becomes

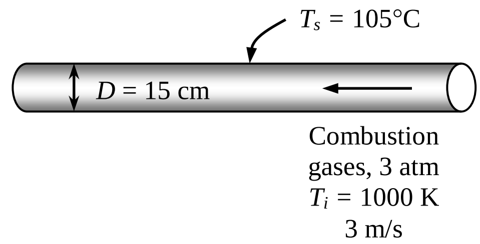
$$\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = \dot{m}_{\text{evap}} h_{fg} \longrightarrow \dot{m}_{\text{evap}} = \frac{\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}}{h_{fg}} = \frac{(2587 + 12{,}040)\ \text{W}}{2257\times10^3\ \text{J/kg}} = \mathbf{0.00648\,kg/s}$$

13-155 Combustion gases flow inside a tube in a boiler. The rates of heat transfer by convection and radiation and the rate of evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Combustion gases are assumed to have the properties of air, which is an ideal gas with constant properties.

Properties The properties of air at 1000 K = 727°C and 3 atm are (Table A-15)

$$\rho = 0.3535\ \text{kg/m}^3$$
$$k = 0.06704\ \text{W/m.°C}$$
$$\nu = \nu = (1.185\times10^{-4}\ \text{m}^2/\text{s})/3 = 0.395\times10^{-4}\ \text{m}^2/\text{s}$$
$$c_p = 1140\ \text{J/kg.°C}$$
$$\Pr = 0.7107$$



Analysis (*a*) The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D}{\nu} = \frac{(3\ \text{m/s})(0.15\text{m})}{0.395\times10^{-4}\ \text{m}^2/\text{s}} = 11{,}392$$

which is greater than 10,000. The flow is turbulent. The entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.15\ \text{m}) = 1.5\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\Pr^{0.3} = 0.023(11{,}392)^{0.8}(0.7107)^{0.3} = 36.52$$

Heat transfer coefficient is

$$h = \frac{k}{D} Nu = \frac{0.06704\ \text{W/m.°C}}{0.15\ \text{m}}(36.52) = 16.32\ \text{W/m}^2.\text{°C}$$

Next we determine the exit temperature of air

$$A = \pi DL = \pi(0.15\ \text{m})(6\ \text{m}) = 2.827\ \text{m}^2$$
$$A_c = \pi D^2/4 = \pi(0.15\ \text{m})^2/4 = 0.01767\ \text{m}^2$$
$$\dot{m} = \rho V A_c = (0.3535\ \text{kg/m}^3)(3\ \text{m/s})(0.01767\ \text{m}^2) = 0.01874\ \text{kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 105 - (105 - 727)e^{-\frac{(16.32)(2.827)}{(0.01874)(1140)}} = 176.8\text{°C}$$

Then the rate of heat transfer by convection becomes

$$\dot{Q}_{\text{conv}} = \dot{m}c_p(T_i - T_e) = (0.01874\ \text{kg/s})(1140\ \text{J/kg.°C})(727 - 176.8)\text{°C} = \mathbf{11{,}760\ W}$$

Next, we determine the emissivity of combustion gases. First, the mean beam length for an infinite circular cylinder is, from Table 13-4,

$$L = 0.95(0.15\ \text{m}) = 0.1425\ \text{m}$$

Then,
$$P_c L = (0.08\ \text{atm})(0.1425\ \text{m}) = 0.0114\ \text{m}\cdot\text{atm} = 0.037\ \text{ft}\cdot\text{atm}$$
$$P_w L = (0.16\ \text{atm})(0.1425\ \text{m}) = 0.0228\ \text{m}\cdot\text{atm} = 0.075\ \text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at the average gas temperature of T_g=(T_g+T_g)/2 = (727+177)/2 = 452°C = 725 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.053 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.068$$

These are the base emissivity values at 1 atm, and they need to be corrected for the 3 atm total pressure. Noting that (P_w+P)/2 = (0.16+3)/2 = 1.58 atm, the pressure correction factors are, from Fig. 13-37,

$$C_c = 1.5 \quad \text{and} \quad C_w = 1.8$$

Both CO_2 and H_2O are present in the same mixture, and we need to correct for the overlap of emission bands. The emissivity correction factor at T = T_g = 725 K is, from Fig. 13-38,

$$\left.\begin{aligned} P_c L + P_w L &= 0.037 + 0.075 = 0.112 \\ \frac{P_w}{P_w + P_c} &= \frac{0.16}{0.16 + 0.08} = 0.67 \end{aligned}\right\} \Delta\varepsilon = 0.0$$

Then the effective emissivity of the combustion gases becomes

$$\varepsilon_g = C_c \varepsilon_{c,1\,\text{atm}} + C_w \varepsilon_{w,1\,\text{atm}} - \Delta\varepsilon = 1.5 \times 0.053 + 1.8 \times 0.068 - 0.0 = 0.202$$

For a source temperature of T_s = 105°C = 378 K, the absorptivity of the gas is again determined using the emissivity charts as follows:

$$P_c L \frac{T_s}{T_g} = (0.08\ \text{atm})(0.1425\ \text{m})\frac{378\ \text{K}}{725\ \text{K}} = 0.00594\ \text{m}\cdot\text{atm} = 0.020\ \text{ft}\cdot\text{atm}$$

$$P_w L \frac{T_s}{T_g} = (0.16\ \text{atm})(0.1425\ \text{m})\frac{378\ \text{K}}{725\ \text{K}} = 0.0119\ \text{m}\cdot\text{atm} = 0.039\ \text{ft}\cdot\text{atm}$$

The emissivities of CO_2 and H_2O corresponding to these values at a temperature of T_s = 378 K and 1atm are, from Fig. 13-36,

$$\varepsilon_{c,1\,\text{atm}} = 0.040 \quad \text{and} \quad \varepsilon_{w,1\,\text{atm}} = 0.065$$

Then the absorptivities of CO_2 and H_2O become

$$\alpha_c = C_c \left(\frac{T_g}{T_s}\right)^{0.65} \varepsilon_{c,1\,\text{atm}} = (1.5)\left(\frac{725\ \text{K}}{378\ \text{K}}\right)^{0.65}(0.040) = 0.0916$$

$$\alpha_w = C_w \left(\frac{T_g}{T_s}\right)^{0.45} \varepsilon_{w,1\,\text{atm}} = (1.8)\left(\frac{725\ \text{K}}{378\ \text{K}}\right)^{0.45}(0.065) = 0.1568$$

Also $\Delta\alpha = \Delta\varepsilon$, but the emissivity correction factor is to be evaluated from Fig. 13-38 at $T = T_s$ = 378 K instead of T_g = 725 K. We use the chart for 400 K. At $P_w/(P_w + P_c)$ = 0.67 and $P_cL + P_wL$ = 0.112 we read $\Delta\varepsilon$ = 0.0. Then the absorptivity of the combustion gases becomes

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha = 0.0916 + 0.1568 - 0.0 = 0.248$$

The emissivity of the inner surfaces of the tubes is 0.9. Then the net rate of radiation heat transfer from the combustion gases to the walls of the tube becomes

$$\begin{aligned}\dot{Q}_{\text{rad}} &= \frac{\varepsilon_s + 1}{2} A_s \sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4) \\ &= \frac{0.9+1}{2}(2.827\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[0.202(725\ \text{K})^4 - 0.248(378\ \text{K})^4] \\ &= \mathbf{7727\ W}\end{aligned}$$

(*b*) The heat of vaporization of water at 1 atm is 2257 kJ/kg (Table A-9). Then rate of evaporation of water becomes

$$\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = \dot{m}_{\text{evap}} h_{fg} \longrightarrow \dot{m}_{\text{evap}} = \frac{\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}}{h_{fg}} = \frac{(11{,}760 + 7727)\ \text{W}}{2257\times10^3\ \text{J/kg}} = \mathbf{0.00863\ kg/s}$$

Fundamentals of Engineering (FE) Exam Problems

13-156 Consider an infinitely long three-sided triangular enclosure with side lengths 5 cm, 3, cm, and 4 cm. The view factor from the 5 cm side to the 4 cm side is

(a) 0.3 (b) 0.4 (c) 0.5 (d) 0.6 (e) 0.7

Answer (d) 0.6

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
w1=5 [cm]
w2=3 [cm]
w3=4 [cm]
F_13=(w1+w3-w2)/(2*w1) "from Table 13-2"
"Some Wrong Solutions with Common Mistakes"
W_F_13=(w1+w2-w3)/(2*w1) "Using incorrect form of the equation"
```

13-157 Consider a 15-cm-diameter sphere placed within a cubical enclosure with a side length of 15 cm. The view factor from any of the square cube surface to the sphere is

(a) 0.09 (b) 0.26 (c) 0.52 (d) 0.78 (e) 1

Answer (c) 0.52

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.15 [m]
s=0.15 [m]
A1=pi*D^2
A2=6*s^2
F_12=1
A1*F_12=A2*F_21 "Reciprocity relation"
"Some Wrong Solutions with Common Mistakes"
W_F_21=F_21/6 "Dividing the result by 6"
```

13-158 A 90-cm-diameter flat black disk is placed in the center of the top surface of a 1 m × 1 m × 1 m black box. The view factor from the entire interior surface of the box to the interior surface of the disk is

(a) 0.07 (b) 0.13 (c) 0.26 (d) 0.32 (e) 0.50

Answer (b) 0.13

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
d=0.9 [m]
A1=pi*d^2/4 [m^2]
A2=5*1*1 [m^2]
F12=1
F21=A1*F12/A2
```

13-159 Consider two concentric spheres with diameters 12 cm and 18 cm, forming an enclosure. The view factor from the inner surface of the outer sphere to the inner sphere is

(a) 0 (b) 0.18 (c) 0.44 (d) 0.56 (e) 0.67

Answer (c) 0.44

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D1=0.12 [m]
D2=0.18 [m]
A1=pi*D1^2
A2=pi*D2^2
F_12=1
A1*F_12=A2*F_21 "Reciprocity relation"
"Some Wrong Solutions with Common Mistakes"
W1_F_21=F_12 "Using F_12 as the answer"
D1*F_12=D2*W2_F_21 "Using diameters instead of areas"
W3_F_21=1-F_21 "Evaluation of F_22"
```

13-160 The number of view factors that need to be evaluated directly for a 10-surface enclosure is

(a) 1 (b) 10 (c) 22 (d) 34 (e) 45

Answer (e) 45

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
N=10
n_viewfactors=1/2*N*(N-1)
```

13-161 Consider two concentric spheres forming an enclosure with diameters 12 and 18 cm and surface temperatures 300 and 500 K, respectively. Assuming that the surfaces are black, the net radiation exchange between the two spheres is

(a) 21 W (b) 140 W (c) 160 W (d) 1275 W (e) 3084 W

Answer (b) 140 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D1=0.12 [m]
D2=0.18 [m]
T1=300 [K]
T2=500 [K]
sigma=5.67E-8 [W/m^2-K^4]
A1=pi*D1^2
F_12=1
Q_dot=A1*F_12*sigma*(T2^4-T1^4)
"Some Wrong Solutions with Common Mistakes"
W1_Q_dot=F_12*sigma*(T2^4-T1^4) "Ignoring surface area"
W2_Q_dot=A1*F_12*sigma*T1^4 "Emissive power of inner surface"
W3_Q_dot=A1*F_12*sigma*T2^4 "Emissive power of outer surface"
```

13-162 Consider a vertical 2-m-diameter cylindrical furnace whose surfaces closely approximate black surfaces. The base, top, and side surfaces of the furnace are maintained at 400 K, 600 K, and 1200 K, respectively. If the view factor from the base surface to the top surface is 0.2, the net radiation heat transfer between the base and the side surfaces is

(a) 73 kW (b) 126 kW (c) 215 kW (d) 292 kW (e) 344 kW

Answer (d) 292 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=2 [m]
T1=400 [K]
T2=600 [K]
T3=1200 [K]
F_12=0.2
A1=pi*D^2/4
A2=A1
F_13=1-F_12
sigma=5.67E-8 [W/m^2-K^4]
Q_dot_13=A1*F_13*sigma*(T1^4-T3^4)
"Some Wrong Solutions with Common Mistakes"
W_Q_dot_13=A1*F_12*sigma*(T1^4-T3^4) "Using the view factor between the base and top surfaces"
```

13-163 Consider a vertical 2-m-diameter cylindrical furnace whose surfaces closely approximate black surfaces. The base, top, and side surfaces of the furnace are maintained at 400 K, 600 K, and 900 K, respectively. If the view factor from the base surface to the top surface is 0.2, the net radiation heat transfer from the bottom surface is

(a) -93.6 kW (b) -86.1 kW (c) 0 kW (d) 86.1 kW (e) 93.6 kW

Answer (a) -93.6 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=2 [m]
T1=400 [K]
T2=600 [K]
T3=900 [K]
A1=pi*D^2/4
A2=A1
F_12=0.2
F_13=1-F_12
sigma=5.67E-8 [W/m^2-K^4]
Q_dot_12=A1*F_13*sigma*(T1^4-T2^4)
Q_dot_13=A1*F_13*sigma*(T1^4-T3^4)
Q_dot_1=Q_dot_12+Q_dot_13
"Some Wrong Solutions with Common Mistakes"
W1_Q_dot_1=-Q_dot_1 "Using wrong sign"
W2_Q_dot_1=Q_dot_12-Q_dot_13 "Substracting heat transfer terms"
W3_Q_dot_1=Q_dot_13-Q_dot_12 "Substracting heat transfer terms"
```

13-164 A solar flux of 1400 W/m^2 directly strikes a space vehicle surface which has a solar absortivity of 0.4 and thermal emissivity of 0.6. The equilibrium temperature of this surface in space at 0 K is

(a) 300 K (b) 360 K (c) 410 K (d) 467 K (e) 510 K

Answer (b) 360 K

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
a=0.4
e=0.6
Q=1400 [W/m^2]
a*Q=e*sigma#*T^4
```

13-165 A 70-cm-diameter flat black disk is placed at the center of the ceiling a 1 m × 1 m × 1 m black box. If the temperature of the box is 620ºC and the temperature of the disk is 27ºC, the rate of heat transfer by radiation between the interior of the box and the disk is

(a) 2 kW (b) 5 kW (c) 8 kW (d) 11 kW (e) 14 kW

Answer (e) 14 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
d=0.7 [m]
A1=pi*d^2/4 [m^2]
A2=5*1*1 [m^2]
F12=1
T2=893 [K]
T1=300 [K]
F21=A1*F12/A2
Q=A2*F21*sigma#*(T2^4-T1^4)
```

13-166 Consider two infinitely long concentric cylinders with diameters 20 and 25 cm. The inner surface is maintained at 700 K and has an emissivity of 0.40 while the outer surface is black. If the rate of radiation heat transfer from the inner surface to the outer surface is 2400 W per unit area of the inner surface, the temperature of the outer surface is

(a) 605 K (b) 538 K (c) 517 K (d) 451 K (e) 415 K

Answer (a) 605 K

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D1=0.20 [m]
D2=0.25 [m]
T1=700 [K]
epsilon_1=0.40
epsilon_2=1
Q_dot_12=2400 [W/m^2]
sigma=5.67E-8 [W/m^2-K^4]
Q_dot_12=epsilon_1*sigma*(T1^4-T2^4)
"Some Wrong Solutions with Common Mistakes"
Q_dot_12=(sigma*(T1^4-W1_T2^4))/((1/epsilon_1)+(1-epsilon_1)/epsilon_1*D1/D2) "Incorrect equation"
A1=pi*D1*1[m] "Finding the area for a unit length of the inner cylinder"
Q_dot_12=A1*epsilon_1*sigma*(T1^4-W2_T2^4)
```

13-167 Consider a surface at 0ºC that may be assumed to be a blackbody in an environment at 25ºC. If 300 W/m^2 radiation is incident on the surface, the radiosity of this black surface is

(a) 0 W/m^2 (b) 15 W/m^2 (c) 132 W/m^2 (d) 300 W/m^2 (e) 315 W/m^2

Answer (e) 315 W/m^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=0 [C]
T_infinity=25 [C]
G=300 [W/m^2]
sigma=5.67E-8 [W/m^2-K^4]
J=sigma*(T+273)^4 "J=E_b for a blackbody"
"Some Wrong Solutions with Common Mistakes"
W1_J=sigma*T^4 "Using C unit for temperature"
W2_J=sigma*((T_infinity+273)^4-(T+273)^4) "Finding radiation exchange between the surface and the environment"
W3_J=G "Using the incident radiation as the answer"
W4_J=J-G "Finding the difference between the emissive power and incident radiation"
```

13-168 Consider a gray and opaque surface at 0ºC in an environment at 25ºC. The surface has an emissivity of 0.8. If the radiation incident on the surface is 240 W/m^2, the radiosity of the surface is

(a) 38 W/m^2 (b) 132 W/m^2 (c) 240 W/m^2 (d) 300 W/m^2 (e) 315 W/m^2

Answer (d) 300 W/m^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=0 [C]
T_infinity=25 [C]
epsilon=0.80
G=240 [W/m^2]
sigma=5.67E-8 [W/m^2-K^4]
J=epsilon*sigma*(T+273)^4+(1-epsilon)*G
"Some Wrong Solutions with Common Mistakes"
W1_J=sigma*(T+273)^4 "Radiosity for a black surface"
W2_J=epsilon*sigma*T^4+(1-epsilon)*G "Using C unit for temperature"
W3_J=sigma*((T_infinity+273)^4-(T+273)^4) "Finding radiation exchange between the surface and the environment"
W4_J=G "Using the incident radiation as the answer"
```

13-169 Consider a 3-m × 3-m × 3-m cubical furnace. The base surface is black and has a temperature of 400 K. The radiosities for the top and side surfaces are calculated to be 7500 W/m^2 and 3200 W/m^2, respectively. If the temperature of the side surfaces is 485 K, the emissivity of the side surfaces is

(a) 0.37 (b) 0.55 (c) 0.63 (d) 0.80 (e) 0.89

Answer (e) 0.89

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
s=3 [m]
T1=400 [K]
epsilon_1=1
J2=7500 [W/m^2]
J3=3200 [W/m^2]
T3=485 [K]
sigma=5.67E-8 [W/m^2-K^4]
F_31=0.2
F_32=F_31
J1=sigma*T1^4
sigma*T3^4=J3+(1-epsilon_3)/epsilon_3*(F_31*(J3-J1)+F_32*(J3-J2))
```

13-170 The base surface of a cubical furnace with a side length of 3 m has an emissivity of 0.80 and is maintained at 500 K. If the top and side surfaces also have an emissivity of 0.80 and are maintained at 900 K, the net rate of radiation heat transfer from the top and side surfaces to the bottom surface is

(a) 194 kW (b) 233 kW (c) 288 kW (d) 312 kW (e) 242 kW

Answer (b) 233 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Sigma=5.67
T1=500
T2=900
A1=3*3
A2=5*A1
Eps1=0.8
Eps2=0.8
F12=1
Q=sigma*((T1/100)^4-(T2/100)^4)/(((1-Eps1)/(A1*Eps1)+1/(A1*F12)+(1-Eps2)/(A2*Eps2)))
"Some Wrong Solutions with Common Mistakes:"
W1_Q=A1*Eps1*sigma*((T1/100)^4-(T2/100)^4)
```

13-171 Two grey surfaces that form an enclosure exchange heat with one another by thermal radiation. Surface 1 has a temperature of 400 K, an area of 0.2 m^2, and a total emissivity of 0.4. Surface 2 has a temperature of 600 K, an area of 0.3 m^2, and a total emissivity of 0.6. If the view factor F_{12} is 0.3, the rate of radiation heat transfer between the two surfaces is

(a) 135 W (b) 223 W (c) 296 W (d) 342 W (e) 422 W

Answer (b) 223 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
A1=0.2 [m^2]
T1=400 [K]
e1=0.4
A2=0.3 [m^2]
T2=600 [K]
e2=0.6
F12=0.3
R1=(1-e1)/(A1*e1)
R2=1/(A1*F12)
R3=(1-e2)/(A2*e2)
Q=sigma#*(T2^4-T1^4)/(R1+R2+R3)
```

13-172 The surfaces of a two-surface enclosure exchange heat with one another by thermal radiation. Surface 1 has a temperature of 400 K, an area of 0.2 m^2, and a total emissivity of 0.4. Surface 2 is black, has a temperature of 800 K, and an area of 0.3 m^2. If the view factor F_{12} is 0.3, the rate of radiation heat transfer between the two surfaces is

(a) 340 W (b) 560 W (c) 780 W (d) 900 W (e) 1160 W

Answer (d) 900 W

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
A1=0.2 [m^2]
T1=400 [K]
e1=0.4
A2=0.3 [m^2]
T2=800 [K]
F12=0.3
R1=(1-e1)/(A1*e1)
R2=1/(A1*F12)
Q=sigma#*(T2^4-T1^4)/(R1+R2)
```

13-173 Two concentric spheres are maintained at uniform temperatures T_1 = 45ºC and T_2 = 280ºC and have emissivities ε_1 = 0.25 and ε_2 = 0.7, respectively. If the ratio of the diameters is D_1/D_2 = 0.30, the net rate of radiation heat transfer between the two spheres per unit surface area of the inner sphere is

(a) 86 W/m^2 (b) 1169 W/m^2 (c) 1181 W/m^2 (d) 2510 W/m^2 (e) 3306 W/m^2

Answer (b) 1169 W/m^2

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T1=45 [C]
T2=280 [C]
epsilon_1=0.25
epsilon_2=0.70
D1\D2=0.30
sigma=5.67E-8 [W/m^2-K^4]
Q_dot=((sigma*((T2+273)^4-(T1+273)^4)))/((1/epsilon_1)+(1-epsilon_2)/epsilon_2*D1\D2^2)
"Some Wrong Solutions with Common Mistakes"
W1_Q_dot=((sigma*(T2^4-T1^4)))/((1/epsilon_1)+(1-epsilon_2)/epsilon_2*D1\D2^2) "Using C unit for temperature"
W2_Q_dot=epsilon_1*sigma*((T2+273)^4-(T1+273)^4) "The equation when the outer sphere is black"
W3_Q_dot=epsilon_2*sigma*((T2+273)^4-(T1+273)^4) "The equation when the inner sphere is black"
```

13-174 Consider a 3-m × 3-m × 3-m cubical furnace. The base surface of the furnace is black and has a temperature of 550 K. The radiosities for the top and side surfaces are calculated to be 7500 W/m^2 and 3200 W/m^2, respectively. The net rate of radiation heat transfer to the bottom surface is

(a) 10 kW (b) 54 kW (c) 61 kW (d) 113 kW (e) 248 kW

Answer (a) 10 kW

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
s=3 [m]
T1=550 [K]
epsilon_1=1
J2=7500 [W/m^2]
J3=3200 [W/m^2]
sigma=5.67E-8 [W/m^2-K^4]
A1=s^2
F_12=0.2
F_13=0.8
J1=sigma*T1^4
Q_dot_1=A1*(F_12*(J1-J2)+F_13*(J1-J3))
"Some Wrong Solutions with Common Mistakes"
W1_Q_dot_1=(F_12*(J1-J2)+F_13*(J1-J3)) "Not multiplying with area"
W2_A1=6*s^2 "Using total area"
W2_Q_dot_1=W2_A1*(F_12*(J1-J2)+F_13*(J1-J3))
```

13-175 Two very large parallel plates are maintained at uniform temperatures T_1 = 750 K and T_2 = 500 K and have emissivities ε_1 = 0.85 and ε_2 = 0.7, respectively. If a thin aluminum sheet with the same emissivity on both sides is to be placed between the plates in order to reduce the net rate of radiation heat transfer between the plates by 90 percent, the emissivity of the aluminum sheet must be

(a) 0.07 (b) 0.10 (c) 0.13 (d) 0.16 (e) 0.19

Answer (c) 0.13

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T1=750 [K]
T2=500 [K]
epsilon_1=0.85
epsilon_2=0.70
f=0.9
sigma=5.67E-8 [W/m^2-K^4]
Q_dot_noshield=(sigma*(T1^4-T2^4))/((1/epsilon_1)+(1/epsilon_2)-1)
Q_dot_1shield=(1-f)*Q_dot_noshield
Q_dot_1shield=(sigma*(T1^4-T2^4))/((1/epsilon_1)+(1/epsilon_2)-1+(1/epsilon_3)+(1/epsilon_3)-1)
```

13-176 ….. 13-178 Design and Essay Problems

Solutions Manual for

Heat and Mass Transfer: Fundamentals & Applications

6th Edition

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Chapter 14
MASS TRANSFER

Analogy between Heat and Mass Transfer

14-1C The *concentration* of a commodity is defined as the amount of that commodity per unit volume. The *concentration gradient dC/dx* is defined as the change in the concentration *C* of a commodity per unit length in the direction of flow *x*. The *diffusion rate* of the commodity is expressed as

$$\dot{Q} = -k_{\text{diff}} A \frac{dC}{dx}$$

where *A* is the area normal to the direction of flow and k_{diff} is the *diffusion coefficient* of the medium, which is a measure of how fast a commodity diffuses in the medium.

14-2C Examples of different kinds of diffusion processes:

(a) *Liquid-to-gas*: A gallon of gasoline left in an open area will eventually evaporate and diffuse into air.

(b) *Solid-to-liquid*: A spoon of sugar in a cup of tea will eventually dissolve and move up.

(c) *Solid-to gas*: A moth ball left in a closet will sublimate and diffuse into the air.

(d) *Gas-to-liquid*: Air dissolves in water.

14-3C (*a*) *Temperature difference* is the driving force for heat transfer, (*b*) *voltage difference* is the driving force for electric current flow, and (*c*) *concentration difference* is the driving force for mass transfer.

14-4C In the relation $\dot{Q} = -kA(dT/dx)$, the quantities $\dot{Q}$, *k*, *A*, and *T* represent the following in heat conduction and mass diffusion:

$\dot{Q}$ = Rate of heat transfer in heat conduction, and rate of mass transfer in mass diffusion.

k = Thermal conductivity in heat conduction, and mass diffusivity in mass diffusion.

A = Area normal to the direction of flow in both heat and mass transfer.

T = Temperature in heat conduction, and concentration in mass diffusion.

14-5C *Bulk fluid flow* refers to the transportation of a fluid on a macroscopic level from one location to another in a flow section by a mover such as a fan or a pump. *Mass flow* requires the presence of two regions at different chemical compositions, and it refers to the movement of a chemical species from a high concentration region towards a lower concentration one relative to the other chemical species present in the medium. Mass transfer cannot occur in a homogeneous medium.

14-6C (*a*) *Homogenous reactions* in mass transfer represent the generation of a species within the medium. Such reactions are analogous to internal heat generation in heat transfer. (*b*) *Heterogeneous reactions* in mass transfer represent the generation of a species at the surface as a result of chemical reactions occurring at the surface. Such reactions are analogous to specified surface heat flux in heat transfer.

Mass Diffusion

14-7C The molecular weights of CO_2 and N_2O gases are the same (both are 44). Therefore, the mass and mole fractions of each of these two gases in a gas mixture will be the same.

14-8C (*a*) T (*b*) F (*c*) F (*d*) T (*e*) F

14-9C In the Fick's law of diffusion relations expressed as $\dot{m}_{\text{diff, A}} = -\rho A D_{\text{AB}} \dfrac{dw_{\text{A}}}{dx}$ and $\dot{N}_{\text{diff, A}} = -CAD_{\text{AB}} \dfrac{dy_{\text{A}}}{dx}$, the diffusion coefficients D_{AB} are the same.

14-10C The mass diffusivity of a gas mixture (*a*) increases with increasing temperature and (*a*) decreases with increasing pressure.

14-11C In a binary ideal gas mixture of species A and B, the diffusion coefficient of A in B is equal to the diffusion coefficient of B in A. Therefore, the mass diffusivity of air in water vapor will be equal to the mass diffusivity of water vapor in air since the air and water vapor mixture can be treated as ideal gases.

14-12C Solids, in general, have different diffusivities in each other. At a given temperature and pressure, the mass diffusivity of copper in aluminum will not be the equal to the mass diffusivity of aluminum in copper.

14-13C We would carry out the hardening process of steel by carbon at high temperature since mass diffusivity increases with temperature, and thus the hardening process will be completed in a short time.

14-14 The maximum mass fraction of calcium bicarbonate in water at 300 K is to be determined.

Assumptions The small amounts of gases in air are ignored, and dry air is assumed to consist of N_2 and O_2 only.

Properties The solubility of [$Ca(HCO_3)_2$] in 100 kg of water at 300 K is 16.75 kg (Table 14-5).

Analysis The maximum mass fraction is determined from

$$w_{\text{(CaHCO3)2}} = \frac{m_{\text{(CaHCO3)2}}}{m_{total}} = \frac{m_{\text{(CaHCO3)2}}}{m_{\text{(CaHCO3)2}} + m_w} = \frac{16.75\ \text{kg}}{(16.75+100)\ \text{kg}} = \mathbf{0.143}$$

14-15 The molar fractions of the constituents of moist air are given. The mass fractions of the constituents are to be determined.

Assumptions The small amounts of gases in air are ignored, and dry air is assumed to consist of N_2 and O_2 only.

Properties The molar masses of N_2, O_2, and H_2O are 28.0, 32.0, and 18.0 kg/kmol, respectively (Table A-1)

Analysis The molar mass of moist air is determined to be

$$M = \sum y_i M_i = 0.78 \times 28.0 + 0.20 \times 32.0 + 0.02 \times 18 = 28.6 \text{ kg/kmol}$$

Moist air
78% N_2
20% O_2
2% H_2O
(Mole fractions)

Then the mass fractions of constituent gases are determined from Eq. 14-10 to be

$$N_2: \quad w_{N_2} = y_{N_2} \frac{M_{N_2}}{M} = (0.78)\frac{28.0}{28.6} = \mathbf{0.764}$$

$$O_2: \quad w_{O_2} = y_{O_2} \frac{M_{O_2}}{M} = (0.20)\frac{32.0}{28.6} = \mathbf{0.224}$$

$$H_2O: \quad w_{H_2O} = y_{H_2O} \frac{M_{H_2O}}{M} = (0.02)\frac{18.0}{28.6} = \mathbf{0.012}$$

Therefore, the mass fractions of N_2, O_2, and H_2O in dry air are 76.4%, 22.4%, and 1.2%, respectively.

14-16 The mole numbers of the constituents of a gas mixture at a specified pressure and temperature are given. The mass fractions and the partial pressures of the constituents are to be determined.

Assumptions The gases behave as ideal gases.

Properties The molar masses of N_2, O_2 and CO_2 are 28, 32, and 44 kg/kmol, respectively (Table A-1)

Analysis When the mole fractions of a gas mixture are known, the mass fractions can be determined from

$$w_i = \frac{m_i}{m_m} = \frac{N_i M_i}{N_m M_m} = y_i \frac{M_i}{M_m}$$

65% N_2
20% O_2
15% CO_2
290 K
250 kPa

The apparent molar mass of the mixture is

$$M = \sum y_i M_i = 0.65 \times 28.0 + 0.20 \times 32.0 + 0.15 \times 44.0 = 31.2 \text{ kg/kmol}$$

Then the mass fractions of the gases are determined from

$$N_2: \quad w_{N_2} = y_{N_2} \frac{M_{N_2}}{M} = (0.65)\frac{28.0}{31.2} = \mathbf{0.583} \quad (\text{or } 58.3\%)$$

$$O_2: \quad w_{O_2} = y_{O_2} \frac{M_{O_2}}{M} = (0.20)\frac{32.0}{31.2} = \mathbf{0.205} \quad (\text{or } 20.5\%)$$

$$CO_2: \quad w_{CO_2} = y_{CO_2} \frac{M_{CO_2}}{M_m} = (0.15)\frac{44}{31.2} = \mathbf{0.212} \quad (\text{or } 21.2\%)$$

Noting that the total pressure of the mixture is 250 kPa and the pressure fractions in an ideal gas mixture are equal to the mole fractions, the partial pressures of the individual gases become

$$P_{N_2} = y_{N_2} P = (0.65)(250 \text{ kPa}) = 162.5 \text{kPa}$$

$$P_{O_2} = y_{O_2} P = (0.20)(250 \text{ kPa}) = \mathbf{50 kPa}$$

$$P_{CO_2} = y_{CO_2} P = (0.15)(250 \text{ kPa}) = \mathbf{37.5 kPa}$$

14-17E The masses of the constituents of a gas mixture are given. The mass fractions, mole fractions, and the molar mass of the mixture are to be determined.

Assumptions None.

Properties The molar masses of N_2, O_2, and CO_2 are 28, 32, and 44 lbm/lbmol, respectively (Table A-1E)

Analysis (*a*) The total mass of the gas mixture is determined to be

$$m = \sum m_i = m_{O_2} + m_{N_2} + m_{CO_2} = 7 + 8 + 10 = 25 \text{ lbm}$$

Then the mass fractions of constituent gases are determined to be

7 lbm O_2
8 lbm N_2
10 lbm CO_2

$$N_2: \quad w_{N_2} = \frac{m_{N_2}}{m} = \frac{8}{25} = \mathbf{0.32}$$

$$O_2: \quad w_{O_2} = \frac{m_{O_2}}{m} = \frac{7}{25} = \mathbf{0.28}$$

$$CO_2: \quad w_{CO_2} = \frac{m_{CO_2}}{m} = \frac{10}{25} = \mathbf{0.40}$$

(*b*) To find the mole fractions, we need to determine the mole numbers of each component first,

$$N_2: \quad N_{N_2} = \frac{m_{N_2}}{M_{N_2}} = \frac{8 \text{ lbm}}{28 \text{ lbm/lbmol}} = \mathbf{0.286\ lbmol}$$

$$O_2: \quad N_{O_2} = \frac{m_{O_2}}{M_{O_2}} = \frac{7 \text{ lbm}}{32 \text{ lbm/lbmol}} = \mathbf{0.219\ lbmol}$$

$$CO_2: \quad N_{CO_2} = \frac{m_{CO_2}}{M_{CO_2}} = \frac{10 \text{ lbm}}{44 \text{ lbm/lbmol}} = \mathbf{0.227\ lbmol}$$

Thus,

$$N_m = \sum N_i = N_{N_2} + N_{O_2} + N_{CO_2} = 0.286 + 0.219 + 0.227 = 0.732 \text{ lbmol}$$

Then the mole fraction of gases are determined to be

$$N_2: \quad y_{N_2} = \frac{N_{N_2}}{N_m} = \frac{0.286}{0.732} = \mathbf{0.391}$$

$$O_2: \quad y_{O_2} = \frac{N_{O_2}}{N_m} = \frac{0.219}{0.732} = \mathbf{0.299}$$

$$CO_2: \quad y_{CO_2} = \frac{N_{CO_2}}{N_m} = \frac{0.227}{0.732} = \mathbf{0.310}$$

(*c*) The molar mass of the mixture is determined from

$$M = \frac{m_m}{N_m} = \frac{25 \text{ lbm}}{0.732 \text{ lbmol}} = \mathbf{34.2\ lbm/lbmol}$$

14-18 The mass fraction of $PM_{2.5}$ in ambient air is given. The level of $PM_{2.5}$ is to be determined whether it is above the limit set by the NAAQS.

Assumptions **1** Air at 1 atm pressure.

Properties The density of air at 0°C is 1.292 kg/m^3 (Table A-15).

Analysis The mass fraction of a species in a mixture is

$$w_i = \frac{m_i}{m} = \frac{\rho_i}{\rho}$$

Thus, the partial density of $PM_{2.5}$ is

$$\rho_{\mathrm{PM}_{2.5}} = w_{\mathrm{PM}_{2.5}}\,\rho = w_{\mathrm{PM}_{2.5}}\left(\rho_{\mathrm{PM}_{2.5}} + \rho_{\mathrm{air}}\right)$$

$$\frac{1}{w_{\mathrm{PM}_{2.5}}} = 1 + \frac{\rho_{\mathrm{air}}}{\rho_{\mathrm{PM}_{2.5}}}$$

$$\rho_{\mathrm{PM}_{2.5}} = \rho_{\mathrm{air}}\left(\frac{w_{\mathrm{PM}_{2.5}}}{1 - w_{\mathrm{PM}_{2.5}}}\right)$$

$$\rho_{\mathrm{PM}_{2.5}} = (1.292\ \mathrm{kg/m^3})\left(\frac{0.00062}{1 - 0.00062}\right) = 0.000802\ \mathrm{kg/m^3} = \mathbf{802000\ \mu g/m^3} > 35\ \mu\mathrm{g/m^3}$$

Discussion With a mass fraction of $PM_{2.5}$ at 0.62 mg/kg in an ambient air at 0°C, the level of $PM_{2.5}$ would be 802000 μg/m^3. At this level, it is about 23 thousand times above the level set by the NAAQS (35 μg/m^3), which is harmful to public health and the environment.

14-19 The mole fractions of the constituents of a gas mixture are given. The mass of each gas and apparent gas constant of the mixture are to be determined.

Assumptions None.

Properties The molar masses of H_2 and N_2 are 2.0 and 28.0 kg/kmol, respectively (Table A-1)

Analysis The mass of each gas is

$$\mathrm{H_2}:\quad m_{\mathrm{H_2}} = N_{\mathrm{H_2}} M_{\mathrm{H_2}} = (10\ \mathrm{kmol}) \times (2\ \mathrm{kg/kmol}) = \mathbf{20\,kg}$$

$$\mathrm{N_2}:\quad m_{\mathrm{N_2}} = N_{\mathrm{N_2}} M_{\mathrm{N_2}} = (2\ \mathrm{kmol}) \times (28\ \mathrm{kg/kmol}) = \mathbf{56\,kg}$$

10 kmol H_2
2 kmol N_2

The molar mass of the mixture and its apparent gas constant are determined to be

$$M = \frac{m_m}{N_m} = \frac{20 + 56\ \mathrm{kg}}{10 + 2\ \mathrm{kmol}} = 6.333\,\mathrm{kg/kmol}$$

$$R = \frac{R_u}{M} = \frac{8.314\ \mathrm{kJ/kmol \cdot K}}{6.333\ \mathrm{kg/kmol}} = \mathbf{1.313\ kJ/kg{\cdot}K}$$

14-20 C & S The partial pressure ratio of CO and air is given. The CO level is to be determined whether it is above 35 ppm.

Assumptions **1** The gases behave as ideal gases.

Analysis The mole fraction of a species in a mixture is

$$y_i = \frac{N_i}{N} = \frac{C_i}{C}$$

For ideal gas mixture at a given total pressure, the mole fraction is equal the volume fraction

$$y_i = \frac{N_i}{N} = \frac{\frac{PV_i}{R_uT}}{\frac{PV}{R_uT}} = \frac{V_i}{V}$$

Also, the pressure fraction in an ideal gas mixture is

$$y_i = \frac{P_i}{P}$$

So, the volume fraction of CO, for a partial pressure ratio (CO to air) of 0.01:0.99, is

$$y_{\mathrm{CO}} = \frac{V_{\mathrm{CO}}}{V} = \frac{P_{\mathrm{CO}}}{P} = \frac{P_{\mathrm{CO}}}{P_{\mathrm{CO}} + P_{\mathrm{air}}} = \frac{0.01}{0.01 + 0.99}$$

$$\frac{V_{\mathrm{CO}}}{V} = \frac{0.01}{0.01 + 0.99} = 0.01 = \mathbf{10000\ ppm} > 35\ \mathrm{ppm}$$

Discussion For partial pressures of CO and air in the ratio of 0.01:0.99, the level of CO would be 10000 ppm. At this level, the CO is more than 285 times above the level set by the NAAQS, which is harmful to public health and the environment.

14-21 C&S The mole fraction of Pb in the ambient air of a residential place is given. The level of Pb is to be determined whether it is above the limit set by the NAAQS.

Assumptions **1** Air at 1 atm pressure.

Properties The density of air at 20°C is 1.204 kg/m^3 (Table A-15). The molar mass of air is 28.97 kg/kmol (Table A-1).

Analysis The mass fraction and mole fraction of a species in a mixture is related to each other as

$$w_i = y_i \frac{M_i}{M}$$

So, the mass fraction of Pb is

$$w_{\mathrm{Pb}} = \frac{y_{\mathrm{Pb}} M_{\mathrm{Pb}}}{y_{\mathrm{Pb}} M_{\mathrm{Pb}} + y_{\mathrm{air}} M_{\mathrm{air}}} = \frac{y_{\mathrm{Pb}} M_{\mathrm{Pb}}}{y_{\mathrm{Pb}} M_{\mathrm{Pb}} + (1 - y_{\mathrm{Pb}}) M_{\mathrm{air}}}$$

$$w_{\mathrm{Pb}} = \frac{(0.000005)(207.2\ \mathrm{kg/kmol})}{(0.000005)(207.2\ \mathrm{kg/kmol}) + (1 - 0.000005)(28.97\ \mathrm{kg/kmol})} = 0.00003576$$

The mass fraction of a species in a mixture is

$$w_i = \frac{m_i}{m} = \frac{\rho_i}{\rho}$$

Thus, the partial density of Pb is

$$\rho_{\mathrm{Pb}} = w_{\mathrm{Pb}}\, \rho = w_{\mathrm{Pb}} (\rho_{\mathrm{Pb}} + \rho_{\mathrm{air}})$$

$$\frac{1}{w_{\mathrm{Pb}}} = 1 + \frac{\rho_{\mathrm{air}}}{\rho_{\mathrm{Pb}}}$$

$$\rho_{\mathrm{Pb}} = \rho_{\mathrm{air}} \left(\frac{w_{\mathrm{Pb}}}{1 - w_{\mathrm{Pb}}} \right)$$

$$\rho_{\mathrm{Pb}} = (1.204\ \mathrm{kg/m^3}) \left(\frac{0.00003576}{1 - 0.00003576} \right) = 0.00004306\ \mathrm{kg/m^3} = \mathbf{43060\ \mu g/m^3} > 0.15\ \mathrm{\mu g/m^3}$$

Discussion With a mole fraction of Pb at 0.005 mol/kmol in the ambient air of a residential place at 20°C, the level of Pb would be 43060 μg/m^3. At this level, it is more than 287 thousand times above the level set by the NAAQS (0.15 μg/m^3), which is harmful to public health and the environment.

14-22 C&S The molar concentration of PM_{10} soot particulates in ambient air is given. The level of PM_{10} soot particulates is to be determined whether it is above the limit set by the NAAQS.

Assumptions **1** Air at 1 atm pressure.

Properties The density of air at 20°C is 1.204 kg/m³ (Table A-15). The molar mass of air is 28.97 kg/kmol (Table A-1).

Analysis The molar concentration of air is

$$C_{\text{air}} = \frac{\rho_{\text{air}}}{M_{\text{air}}} = \frac{1.204\ \text{kg/m}^3}{28.97\ \text{kg/kmol}} = 0.04156\ \text{kmol/m}^3$$

The mole fraction of soot in the mixture is

$$y_{\text{soot}} = \frac{C_{\text{soot}}}{C_{\text{soot}} + C_{\text{air}}} = \frac{0.00001\ \text{kmol/m}^3}{(0.00001 + 0.04156)\ \text{kmol/m}^3} = 0.0002406$$

The mass fraction and mole fraction of a species in a mixture is related to each other as

$$w_i = \frac{\rho_i}{\rho} = y_i \frac{M_i}{M}$$

So, the mass fraction of soot in the mixture is

$$w_{\text{soot}} = y_{\text{soot}} \frac{M_{\text{soot}}}{M} = \frac{y_{\text{soot}} M_{\text{soot}}}{y_{\text{soot}} M_{\text{soot}} + (1 - y_{\text{soot}}) M_{\text{air}}}$$

$$w_{\text{soot}} = \frac{(0.0002406)(12.01\ \text{kg/kmol})}{(0.0002406)(12.01\ \text{kg/kmol}) + (1 - 0.0002406)(28.97\ \text{kg/kmol})} = 0.00009976$$

Thus, the mass concentration (partial density) of soot is

$$\rho_{\text{soot}} = w_{\text{soot}}\, \rho = w_{\text{soot}} (\rho_{\text{soot}} + \rho_{\text{air}})$$

$$\frac{1}{w_{\text{soot}}} = 1 + \frac{\rho_{\text{air}}}{\rho_{\text{soot}}}$$

$$\rho_{\text{soot}} = \rho_{\text{air}} \left(\frac{w_{\text{soot}}}{1 - w_{\text{soot}}} \right)$$

$$\rho_{\text{soot}} = (1.204\ \text{kg/m}^3) \left(\frac{0.00009976}{1 - 0.00009976} \right) = 0.0001201\ \text{kg/m}^3 = \mathbf{120100\ \mu g/m^3} > 150\ \mu\text{g/m}^3$$

Discussion With a molar concentration of PM_{10} soot particulates at 0.01 mol/m³ in an ambient air at 20°C, the level of PM_{10} soot particulates would be 120100 μg/m³. At this level, it is more than 800 times above the level set by the NAAQS (150 μg/m³), which is harmful to public health and the environment.

14-23 C&S The mass fraction of SO_2 in ambient air is given. The level of SO_2 is to be determined whether it is above the limit set by the NAAQS.

Assumptions **1** Air at 1 atm pressure. **2** Gas mixture behaves as ideal gas.

Properties The density of air at 20°C is 1.204 kg/m^3 (Table A-15). The molar masses of air and SO_2 are 28.97 and 64.06 kg/kmol, respectively (Table A-1).

Analysis The mass fraction and mole fraction of a species in a mixture is related to each other as

$$w_i = y_i \frac{M_i}{M}$$

So, the mole fraction of SO_2 in the ambient air is

$$w_{\mathrm{SO_2}} = \frac{y_{\mathrm{SO_2}} M_{\mathrm{SO_2}}}{y_{\mathrm{SO_2}} M_{\mathrm{SO_2}} + y_{\mathrm{air}} M_{\mathrm{air}}} = \frac{y_{\mathrm{SO_2}} M_{\mathrm{SO_2}}}{y_{\mathrm{SO_2}} M_{\mathrm{SO_2}} + (1 - y_{\mathrm{SO_2}}) M_{\mathrm{air}}}$$

$$\frac{1}{w_{\mathrm{SO_2}}} = 1 + \left(\frac{1 - y_{\mathrm{SO_2}}}{y_{\mathrm{SO_2}}}\right)\left(\frac{M_{\mathrm{air}}}{M_{\mathrm{SO_2}}}\right)$$

$$y_{\mathrm{SO_2}} = \left[1 + \left(\frac{1 - w_{\mathrm{SO_2}}}{w_{\mathrm{SO_2}}}\right)\left(\frac{M_{\mathrm{SO_2}}}{M_{\mathrm{air}}}\right)\right]^{-1}$$

$$y_{\mathrm{SO_2}} = \left[1 + \left(\frac{1 - 0.0005}{0.0005}\right)\left(\frac{64.06\ \mathrm{kg/kmol}}{28.97\ \mathrm{kg/kmol}}\right)\right]^{-1} = 0.0002262$$

For ideal gas mixture at a given total pressure, the mole fraction is equal the volume fraction,

$$y_i = \frac{N_i}{N} = \frac{\frac{PV_i}{R_u T}}{\frac{PV}{R_u T}} = \frac{V_i}{V}$$

Thus,

$$y_{\mathrm{SO_2}} = \frac{V_{\mathrm{SO_2}}}{V} = 0.0002262 = \mathbf{226.2\ ppm} > 0.5\ \mathrm{ppm}$$

Discussion With a mass fraction of SO_2 at 0.5 mg/kg in an ambient air at 20°C, the level of SO_2 would be 226 ppm. At this level, it is more than 450 times above the level set by the NAAQS (0.5 ppm), which is harmful to the environment.

14-24 The binary diffusion coefficients of CO_2 in air at various temperatures and pressures are to be determined.

Assumptions The mixture is sufficiently dilute so that the diffusion coefficient is independent of mixture composition.

Properties The binary diffusion coefficients of CO_2 in air at 1 atm pressure are given in Table 14-1 to be 0.74×10^{-5}, 2.63×10^{-5}, and 5.37×10^{-5} m^2/s at temperatures of 200 K, 400 K, and 600 K, respectively.

Analysis Noting that the binary diffusion coefficients of gases are inversely proportional to pressure, the diffusion coefficients at given pressures are determined from

$$D_{AB}(T,P) = D_{AB}(T, 1\,\text{atm})/P$$

where P is in atm.

(a) At 200 K and 1 atm: D_{AB} (200 K, 1 atm) = $\mathbf{0.74\times10^{-5}}$ **m²/s** (since P = 1 atm).

(b) At 400 K and 0.5 atm: D_{AB}(400 K, 0.5 atm)=D_{AB}(400 K, 1 atm)/0.5=(2.63×10^{-5})/0.5 = $\mathbf{5.26\times10^{-5}}$ **m²/s**

(c) At 600 K and 5 atm: D_{AB}(600 K, 5 atm)=D_{AB}(600 K, 1 atm)/5=(5.37×10^{-5})/5 = $\mathbf{1.07\times10^{-5}}$ **m²/s**

14-25 The binary diffusion coefficient of O_2 in N_2 at various temperature and pressures are to be determined.

Assumptions The mixture is sufficiently dilute so that the diffusion coefficient is independent of mixture composition.

Properties The binary diffusion coefficient of O_2 in N_2 at T_1 = 273 K and P_1 = 1 atm is given in Table 14-2 to be 1.8×10^{-5} m^2/s.

Analysis Noting that the binary diffusion coefficient of gases is proportional to 3/2 power of temperature and inversely proportional to pressure, the diffusion coefficients at other pressures and temperatures can be determined from

$$\frac{D_{\text{AB},1}}{D_{\text{AB},2}} = \frac{P_2}{P_1}\left(\frac{T_1}{T_2}\right)^{3/2} \rightarrow D_{\text{AB},2} = D_{\text{AB},1}\frac{P_1}{P_2}\left(\frac{T_2}{T_1}\right)^{3/2}$$

(a) At 200 K and 1 atm: $D_{\text{AB},2} = (1.8\times10^{-5}\ \text{m}^2/\text{s})\dfrac{1\,\text{atm}}{1\,\text{atm}}\left(\dfrac{200\ \text{K}}{273\ \text{K}}\right)^{3/2} = \mathbf{1.13\times10^{-5}\ m^2/s}$

(b) At 400 K and 0.5 atm: $D_{\text{AB},2} = (1.8\times10^{-5}\ \text{m}^2/\text{s})\dfrac{1\,\text{atm}}{0.5\,\text{atm}}\left(\dfrac{400\ \text{K}}{273\ \text{K}}\right)^{3/2} = \mathbf{6.38\times10^{-5}\ m^2/s}$

(c) At 600 K and 5 atm: $D_{\text{AB},2} = (1.8\times10^{-5}\ \text{m}^2/\text{s})\dfrac{1\,\text{atm}}{5\,\text{atm}}\left(\dfrac{600\ \text{K}}{273\ \text{K}}\right)^{3/2} = \mathbf{1.17\times10^{-5}\ m^2/s}$

14-26 The binary diffusion coefficient of (*a*) CO_2 in N_2, (*b*) CO_2 in O_2, and (*c*) CO_2 in H_2 at 320 K and 2 atm are to be determined.

Assumptions **1** The mixture is sufficiently dilute so that the diffusion coefficient is independent of mixture composition.

Properties From Table 14-2, we find the following binary diffusion coefficients:

CO_2 in N_2:	$D_{AB,1} = 1.6 \times 10^{-5}$ m²/s	at T_1 = 293 K and P_1 = 1 atm
CO_2 in O_2:	$D_{AB,1} = 1.4 \times 10^{-5}$ m²/s	at T_1 = 273 K and P_1 = 1 atm
CO_2 in H_2:	$D_{AB,1} = 5.5 \times 10^{-5}$ m²/s	at T_1 = 273 K and P_1 = 1 atm

Analysis Noting that the binary diffusion coefficient of gases is proportional to 3/2 power of temperature and inversely proportional to pressure, the diffusion coefficients at other pressures and temperatures can be determined from

$$\frac{D_{AB,\,1}}{D_{AB,\,2}} = \frac{P_2}{P_1}\left(\frac{T_1}{T_2}\right)^{3/2} \quad \rightarrow \quad D_{AB,\,2} = D_{AB,\,1}\frac{P_1}{P_2}\left(\frac{T_2}{T_1}\right)^{3/2}$$

(*a*) For CO_2 in N_2 at 320 K and 2 atm:

$$D_{AB,\,2} = (1.6\times10^{-5}\text{ m}^2/\text{s})\frac{1\,\text{atm}}{2\,\text{atm}}\left(\frac{320\text{ K}}{293\text{ K}}\right)^{3/2} = \mathbf{0.913\times10^{-5}\ m^2/s}$$

(*b*) For CO_2 in O_2 at 320 K and 2 atm:

$$D_{AB,\,2} = (1.4\times10^{-5}\text{ m}^2/\text{s})\frac{1\,\text{atm}}{2\,\text{atm}}\left(\frac{320\text{ K}}{273\text{ K}}\right)^{3/2} = \mathbf{0.888\times10^{-5}\ m^2/s}$$

(*c*) For CO_2 in H_2 at 320 K and 2 atm:

$$D_{AB,\,2} = (5.5\times10^{-5}\text{ m}^2/\text{s})\frac{1\,\text{atm}}{2\,\text{atm}}\left(\frac{320\text{ K}}{273\text{ K}}\right)^{3/2} = \mathbf{3.49\times10^{-5}\ m^2/s}$$

Discussion The binary diffusion coefficient is also known as mass diffusivity. The mass diffusivity D_{AB} in mass diffusion equation is analogous to the thermal diffusivity α in heat diffusion equation.

14-27 The diffusion coefficient of hydrogen in steel is given as a function of temperature. The diffusion coefficients at various temperatures are to be determined.

Analysis The diffusion coefficient of hydrogen in steel is given as

$$D_{AB} = 1.65\times10^{-6}\exp(-4630/T) \qquad \text{m}^2/\text{s}$$

Using this relation, the diffusion coefficients at various temperatures are determined to be

300 K: $D_{AB} = 1.65\times10^{-6}\exp(-4630/300) = 3.27\times10^{-13}\text{ m}^2/\text{s}$

500 K: $D_{AB} = 1.65\times10^{-6}\exp(-4630/500) = 1.57\times10^{-10}\text{ m}^2/\text{s}$

1000 K: $D_{AB} = 1.65\times10^{-6}\exp(-4630/1000) = 1.61\times10^{-8}\text{ m}^2/\text{s}$

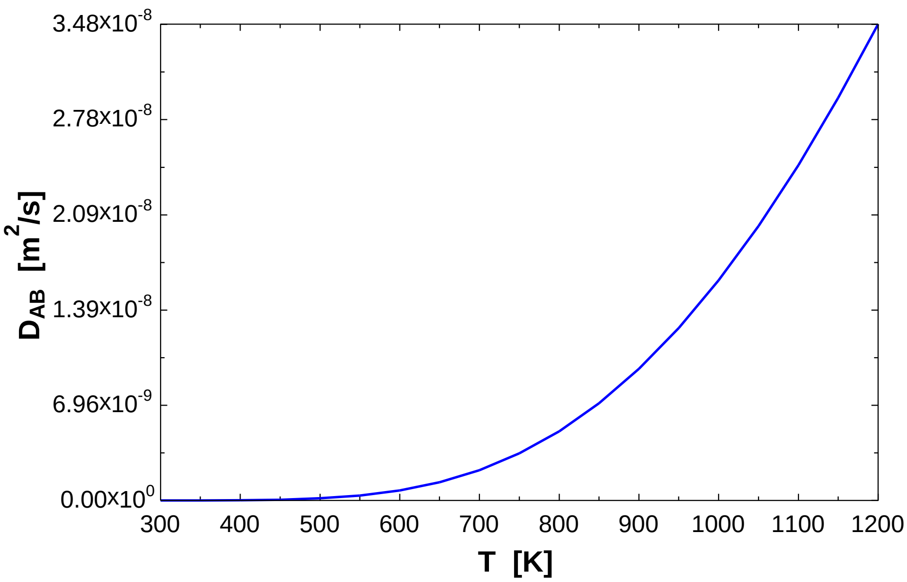
1500 K: $D_{AB} = 1.65\times10^{-6}\exp(-4630/1500) = 7.53\times10^{-8}\text{ m}^2/\text{s}$

14-28 Prob. 14-27 is reconsidered. The diffusion coefficient as a function of the temperature is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"The diffusion coeffcient of hydrogen in steel as a function of temperature"
D_AB=1.65E-6*exp(-4630/T)
```

T [K]	D_{AB} [m^2/s]
300	3.272E-13
350	2.967E-12
400	1.551E-11
450	5.611E-11
500	1.570E-10
550	3.643E-10
600	7.348E-10
650	1.330E-09
700	2.213E-09
750	3.439E-09
800	5.058E-09
850	7.110E-09
900	9.622E-09
950	1.261E-08
1000	1.610E-08
1050	2.007E-08
1100	2.452E-08
1150	2.944E-08
1200	3.482E-08



Boundary Conditions

14-29C Temperature is necessarily a *continuous* function, but concentration, in general, is not. Therefore, the mole fraction of water vapor in air will, in general, be different from the mole fraction of water in the lake (which is nearly 1).

14-30C When prescribing a boundary condition for mass transfer at a solid-gas interface, we need to specify the side of the surface (whether the solid or the gas side). This is because concentration, in general, is not a continuous function, and there may be large differences in concentrations on the gas and solid sides of the boundary. We did not do this in heat transfer because temperature is a continuous function.

14-31C Three boundary conditions for mass transfer (on mass basis) that correspond to specified temperature, specified heat flux, and convection boundary conditions in heat transfer are expressed as follows:

1) $w(0) = w_0$ (specified concentration - corresponds to specified temperature)

2) $-\rho D_{\mathrm{AB}} \left. \dfrac{dw_A}{dx} \right|_{x=0} = J_{A,0}$ (specified mass flux - corresponds to specified heat flux)

3) $j_{\mathrm{A},s} = -D_{\mathrm{AB}} \left. \dfrac{\partial w_{\mathrm{A}}}{\partial x} \right|_{x=0} = h_{\mathrm{mass}}(w_{A,s} - w_{A,\infty})$ (mass convection - corresponds to heat convection)

14-32C An impermeable surface is a surface that does not allow any mass to pass through. Mathematically it is expressed (at $x = 0$) as

$$\left. \frac{dw_A}{dx} \right|_{x=0} = 0$$

An impermeable surface in mass transfer corresponds to an insulated surface in heat transfer.

14-33C Using solubility data of a solid in a specified liquid, the mass fraction w of the solid A in the liquid at the interface at a specified temperature can be determined from

$$w_A = \frac{m_{\mathrm{solid}}}{m_{\mathrm{solid}} + m_{\mathrm{liquid}}}$$

where m_{solid} is the maximum amount of solid dissolved in the liquid of mass m_{liquid} at the specified temperature.

14-34C Using Henry's constant data for a gas dissolved in a liquid, the mole fraction of the gas dissolved in the liquid at the interface at a specified temperature can be determined from Henry's law expressed as

$$y_{\mathrm{i,\,liquid\,side}}(0) = \frac{P_{\mathrm{i,\,gas\,side}}(0)}{H}$$

where H is *Henry's constant* and $P_{\mathrm{i,\,gas\,side}}(0)$ is the partial pressure of the gas i at the gas side of the interface. This relation is applicable for dilute solutions (gases that are weakly soluble in liquids).

14-35C The mole fraction of the water vapor at the surface of a lake when the temperature of the lake surface and the atmospheric pressure are specified can be determined from

$$y_{\text{vapor}} = \frac{P_{\text{vapor}}}{P} = \frac{P_{\text{sat@T}}}{P_{\text{atm}}}$$

where P_{vapor} is equal to the saturation pressure of water at the lake surface temperature.

14-36C The molar concentration C_i of the gas species *i* in the solid at the interface $C_{\text{i, solid side}}(0)$ is proportional to the *partial pressure* of the species *i* in the gas $P_{\text{i, gas side}}(0)$ on the gas side of the interface, and is determined from

$$C_{\text{i, solid side}}(0) = \text{S} \times P_{\text{i, gas side}}(0) \quad (\text{kmol/m}^3)$$

where S is the *solubility* of the gas in that solid at the specified temperature.

14-37C The permeability is a measure of the ability of a gas to penetrate a solid. The permeability of a gas in a solid, P, is related to the solubility of the gas by P = SD_{AB} where D_{AB} is the diffusivity of the gas in the solid.

14-38 The mole fraction of CO_2 dissolved in water at the surface of water at 300 K is to be determined.

Assumptions **1** Both the CO_2 and water vapor are ideal gases. **2** Air at the lake surface is saturated.

Properties The saturation pressure of water at 300 K = 27°C is 3.60 kPa (Table A-9). The Henry's constant for CO_2 in water at 300 K is 1710 bar (Table 14-6).

Analysis The air at the water surface will be saturated. Therefore, the partial pressure of water vapor in the air at the lake surface will simply be the saturation pressure of water at 27°C,

$$P_{\text{vapor}} = P_{\text{sat@27°C}} = 3.60 \text{ kPa}$$

Assuming both the air and vapor to be ideal gases, the partial pressure and mole fraction of dry air in the air at the surface of the lake are determined to be

$$P_{\text{dry air}} = P - P_{\text{vapor}} = 100 - 3.60 = 96.4 \text{ kPa}$$

The partial pressure of CO2 is

$$P_{\text{CO2}} = y_{\text{CO2}} P_{\text{dry air}} = (0.006)(96.4) = 0.5784 \text{ kPa} = 0.005784 \text{ bar}$$

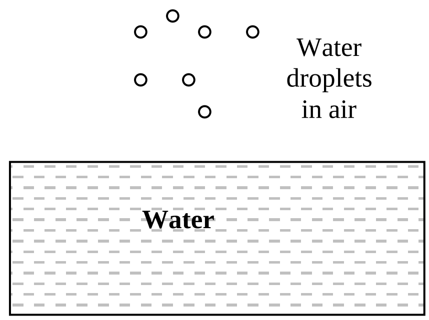
$$y_{\text{CO2}} = \frac{P_{\text{CO2}}}{H} = \frac{0.005784 \text{ bar}}{1710 \text{ bar}} = \mathbf{3.38 \times 10^{-6}}$$

14-39E Water is sprayed into air, and the falling water droplets are collected in a container. The mass and mole fractions of air dissolved in the water are to be determined.

Assumptions **1** Both the air and water vapor are ideal gases. **2** Air is saturated since water is constantly sprayed into it. **3** Air is weakly soluble in water and thus Henry's law is applicable.

Properties The saturation pressure of water at 80°F is 0.5073 psia (Table A-9E). Henry's constant for air dissolved in water at 80ºF (300 K) is given in Table 14-6 to be H = 74,000 bar. Molar masses of dry air and water are 29 and 18 lbm / lbmol, respectively (Table A-1E).

Analysis Noting that air is saturated, the partial pressure of water vapor in the air will simply be the saturation pressure of water at 80°F,



$$P_{\text{vapor}} = P_{\text{sat@80°F}} = 0.5073\,\text{psia}$$

Then the partial pressure of dry air becomes

$$P_{\text{dry air}} = P - P_{\text{vapor}} = 14.3 - 0.5073 = 13.79\text{psia}$$

From Henry's law, the mole fraction of air in the water is determined to be

$$y_{\text{dry air,liquidside}} = \frac{P_{\text{dry air,gasside}}}{H} = \frac{13.79\text{psia}(1\text{atm}/14.696\,\text{psia})}{74{,}000\text{bar}(1\text{atm}/1.01325\text{bar})} = \mathbf{1.29 \times 10^{-5}}$$

which is very small, as expected. The mass and mole fractions of a mixture are related to each other by

$$w_i = \frac{m_i}{m_m} = \frac{N_i M_i}{N_m M_m} = y_i \frac{M_i}{M_m}$$

where the apparent molar mass of the liquid water - air mixture is

$$\begin{aligned} M_m &= \sum y_i M_i = y_{\text{liquid water}} M_{\text{water}} + y_{\text{dry air}} M_{\text{dry air}} \\ &\cong 1 \times 29.0 + 0 \times 18.0 \cong 29.0\,\text{kg/kmol} \end{aligned}$$

Then the mass fraction of dissolved air in liquid water becomes

$$w_{\text{dryair, liquidside}} = y_{\text{dryair, liquidside}}(0)\frac{M_{\text{dryair}}}{M_m} = 1.29 \times 10^{-5}\,\frac{29}{29} = \mathbf{1.29 \times 10^{-5}}$$

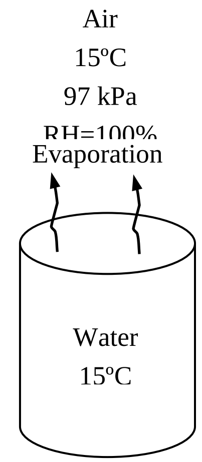
Discussion The mass and mole fractions of dissolved air in this case are identical because of the very small amount of air in water.

14-40 A glass of water is left in a room. The mole fraction of the water vapor in the air and the mole fraction of air in the water are to be determined when the water and the air are in thermal and phase equilibrium.

Assumptions **1** Both the air and water vapor are ideal gases. **2** Air is saturated since the humidity is 100 percent. **3** Air is weakly soluble in water and thus Henry's law is applicable.

Properties The saturation pressure of water at 15°C is 1.7051 kPa (Table A-9). Henry's constant for air dissolved in water at 15ºC (288 K) is given in Table 14-6 to be H = 59,600 bar (determined by extrapolation). Molar masses of dry air and water are 29 and 18 kg/kmol, respectively (Table A-1).

Analysis (*a*) Noting that air is saturated, the partial pressure of water vapor in the air will simply be the saturation pressure of water at 15°C,



$$P_{\text{vapor}} = P_{sat\,@15°C} = 1.7051\,\text{kPa}$$

Assuming both the air and vapor to be ideal gases, the mole fraction of water vapor in the air is determined to be

$$y_{\text{vapor}} = \frac{P_{\text{vapor}}}{P} = \frac{1.7051\text{ kPa}}{97\text{ kPa}} = \mathbf{0.0176}$$

(*b*) Noting that the total pressure is 97 kPa, the partial pressure of dry air is

$$P_{dry\,air} = P - P_{vapor} = 97 - 1.7051 = 95.3\,\text{kPa} = 0.953\text{ bar}$$

From Henry's law, the mole fraction of air in the water is determined to be

$$y_{\text{dry air,liquidside}} = \frac{P_{\text{dry air,gas side}}}{H} = \frac{0.953\text{ bar}}{59{,}600\text{ bar}} = \mathbf{1.60 \times 10^{-5}}$$

Discussion The amount of air dissolved in water is very small, as expected.

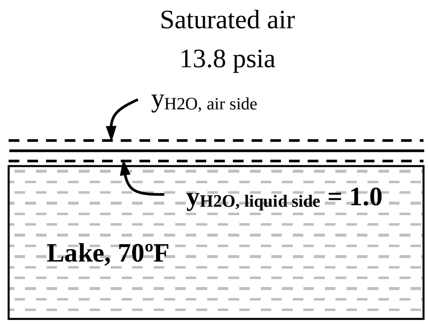
14-41E The mole fraction of the water vapor at the surface of a lake and the mole fraction of water in the lake are to be determined and compared.

Assumptions **1** Both the air and water vapor are ideal gases. **2** Air is weakly soluble in water and thus Henry's law is applicable.

Properties The saturation pressure of water at 70°F is 0.3632 psia (Table A-9E). Henry's constant for air dissolved in water at 70ºF (294 K) is given in Table 14-6 to be H = 66,800 bar.

Analysis The air at the water surface will be saturated. Therefore, the partial pressure of water vapor in the air at the lake surface will simply be the saturation pressure of water at 70°F,

$$P_{\text{vapor}} = P_{\text{sat@70°F}} = 0.3632\ \text{psia}$$



Assuming both the air and vapor to be ideal gases, the mole fraction of water vapor in the air at the surface of the lake is determined from Eq. 14-11 to be

$$y_{\text{vapor}} = \frac{P_{\text{vapor}}}{P} = \frac{0.3632\ \text{psia}}{13.8\ \text{psia}} = \mathbf{0.0263\ (or\ 2.63\ percent)}$$

The partial pressure of dry air just above the lake surface is

$$P_{\text{dry air}} = P - P_{\text{vapor}} = 13.8 - 0.3632 = 13.44\ \text{psia}$$

Then the mole fraction of air in the water becomes

$$y_{\text{dry air,liquidside}} = \frac{P_{\text{dry air,gasside}}}{H} = \frac{13.44\ \text{psia}(1\ \text{atm}/14.696\ \text{psia})}{66{,}800\ \text{bar}(1\ \text{atm}/1.01325\ \text{bar})} = 1.39\times10^{-5}$$

which is very small, as expected. Therefore, the mole fraction of water in the lake near the surface is

$$y_{\text{water,liquidside}} = 1 - y_{\text{dry air, liquidside}} = 1 - 1.39\times10^{-5} = \mathbf{0.99999}$$

Discussion The concentration of air in water just below the air-water interface is 1.39 moles per 100,000 moles. The amount of air dissolved in water will decrease with increasing depth.

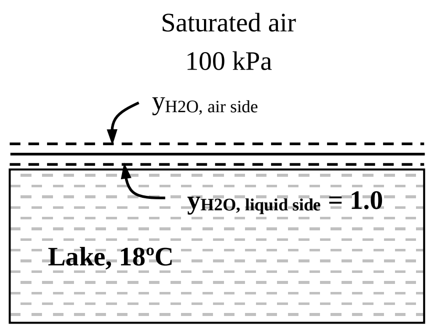
14-42 The mole fraction of the water vapor at the surface of a lake at a specified temperature is to be determined.

Assumptions **1** Both the air and water vapor are ideal gases. **2** Air at the lake surface is saturated.

Properties The saturation pressure of water at 18°C is 2.065 kPa (Table A-9).

Analysis The air at the water surface will be saturated. Therefore, the partial pressure of water vapor in the air at the lake surface will simply be the saturation pressure of water at 18°C,

$$P_{\text{vapor}} = P_{\text{sat@18°C}} = 2.065\ \text{kPa}$$



Assuming both the air and vapor to be ideal gases, the partial pressure and mole fraction of dry air in the air at the surface of the lake are determined to be

$$P_{\text{dry air}} = P - P_{\text{vapor}} = 100 - 2.065 = 97.935\ \text{kPa}$$

$$y_{\text{dry air}} = \frac{P_{\text{dry air}}}{P} = \frac{97.935\ \text{kPa}}{100\ \text{kPa}} = \mathbf{0.979}\ \ (\text{or } 97.9\%)$$

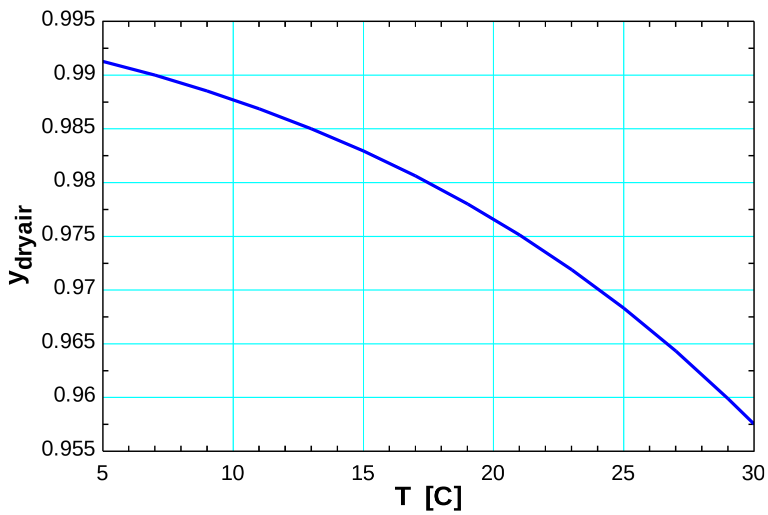
Therefore, the mole fraction of dry air is 97.9 percent just above the air-water interface.

14-43 Prob. 14-42 is reconsidered. The mole fraction of dry air at the surface of the lake as a function of the lake temperature is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T=18 [C]
P_atm=100 [kPa]

"PROPERTIES"
Fluid$='steam_IAPWS'
P_sat=Pressure(Fluid$, T=T, x=1)

"ANALYSIS"
P_vapor=P_sat
P_dryair=P_atm-P_vapor
y_dryair=P_dryair/P_atm
```

T [C]	$y_{dry\ air}$
5	0.9913
7	0.99
9	0.9885
11	0.9869
13	0.985
15	0.9829
17	0.9806
19	0.978
21	0.9751
23	0.9719
25	0.9683
27	0.9643
29	0.9599
30	0.9575



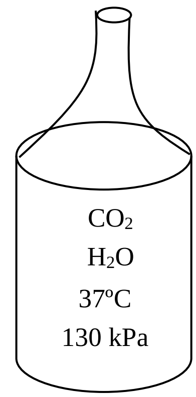
14-44 A carbonated drink in a bottle is considered. Assuming the gas space above the liquid consists of a saturated mixture of CO_2 and water vapor and treating the drink as a water, determine the mole fraction of the water vapor in the CO_2 gas and the mass of dissolved CO_2 in a 200 ml drink are to be determined when the water and the CO_2 gas are in thermal and phase equilibrium.

Assumptions **1** The liquid drink can be treated as water. **2** Both the CO_2 and the water vapor are ideal gases. **3** The CO_2 gas and water vapor in the bottle from a saturated mixture. **4** The CO_2 is weakly soluble in water and thus Henry's law is applicable.

Properties The saturation pressure of water at 37°C is 6.33 kPa (Table A-9). Henry's constant for CO_2 issolved in water at 37ºC (310 K) is given in Table 14-6 to be H = 2170 bar. Molar masses of CO_2 and water are 44 and 18 kg/kmol, respectively (Table A-1).

Analysis (*a*) Noting that the CO_2 gas in the bottle is saturated, the partial pressure of water vapor in the air will simply be the saturation pressure of water at 37°C,

$$P_{\text{vapor}} = P_{sat@37°C} = 6.33\text{kPa}$$



Assuming both CO_2 and vapor to be ideal gases, the mole fraction of water vapor in the CO_2 gas becomes

$$y_{\text{vapor}} = \frac{P_{\text{vapor}}}{P} = \frac{6.33\text{kPa}}{130\text{kPa}} = \mathbf{0.0487} \quad \text{(or 4.87\%)}$$

(*b*) Noting that the total pressure is 130 kPa, the partial pressure of CO_2 is

$$P_{\text{CO}_2\text{ gas}} = P - P_{vapor} = 130 - 6.33 = 123.7\text{kPa} = 1.237\text{bar}$$

From Henry's law, the mole fraction of CO_2 in the drink is determined to be

$$y_{\text{CO}_2\text{,liquidside}} = \frac{P_{\text{CO}_2\text{,gas side}}}{H} = \frac{1.237\text{ bar}}{2170\text{ bar}} = 5.70\times10^{-4}$$

Then the mole fraction of water in the drink becomes

$$y_{\text{water, liquidside}} = 1 - y_{\text{CO}_2\text{, liquidside}} = 1 - 5.70\times10^{-4} = 0.9994$$

The mass and mole fractions of a mixture are related to each other by

$$w_i = \frac{m_i}{m_m} = \frac{N_i M_i}{N_m M_m} = y_i \frac{M_i}{M_m}$$

where the apparent molar mass of the drink (liquid water - CO_2 mixture) is

$$M_m = \sum y_i M_i = y_{\text{liquid water}} M_{\text{water}} + y_{\text{CO}_2} M_{\text{CO}_2} = 0.9994\times18.0 + (5.70\times10^{-4})\times44 = 18.00\text{kg/kmol}$$

Then the mass fraction of dissolved CO_2 gas in liquid water becomes

$$w_{\text{CO}_2\text{, liquidside}} = y_{\text{CO}_2\text{, liquidside}}(0)\frac{M_{\text{CO}_2}}{M_m} = 5.70\times10^{-4}\,\frac{44}{18.00} = 0.00139$$

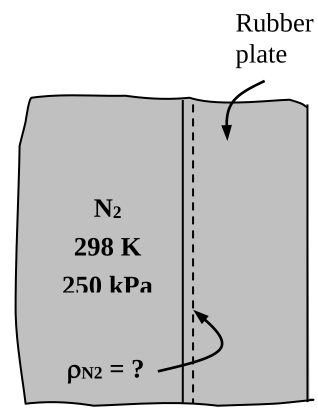
Therefore, the mass of dissolved CO_2 in a 200 ml ≈ 200 g drink is

$$m_{\text{CO}_2} = w_{\text{CO}_2} m_m = 0.00139(200\text{ g}) = \mathbf{0.278\text{ g}}$$

14-45 A rubber plate is exposed to nitrogen. The molar and mass density of nitrogen in the rubber at the interface is to be determined.

Assumptions Rubber and nitrogen are in thermodynamic equilibrium at the interface.

Properties The molar mass of nitrogen is M = 28.0 kg/kmol (Table A-1). The solubility of nitrogen in rubber at 298 K is 0.00156 kmol/m^3·bar (Table 14-7).



Analysis Noting that 250 kPa = 2.5 bar, the molar density of nitrogen in the rubber at the interface is determined from Eq. 14-20 to be

$$C_{N_2,\text{solid side}}(0) = \mathcal{S} \times P_{N_2,\text{gas side}}$$
$$= (0.00156\ \text{kmol/m}^3.\text{bar})(2.5\ \text{bar})$$
$$= \mathbf{0.0039\ kmol/m^3}$$

It corresponds to a mass density of

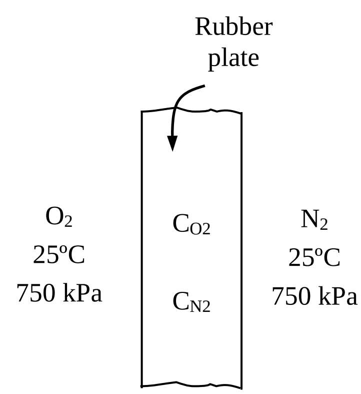
$$\rho_{N_2,\text{solid side}}(0) = C_{N_2,\text{solid side}}(0)M_{N_2}$$
$$= (0.0039\ \text{kmol/m}^3)(28\ \text{kmol/kg})$$
$$= \mathbf{0.1092\ kg/m^3}$$

That is, there will be 0.0039 kmol (or 0.1092 kg) of N_2 gas in each m^3 volume of rubber adjacent to the interface.

14-46 A rubber wall separates O_2 and N_2 gases. The molar concentrations of O_2 and N_2 in the wall are to be determined.

Assumptions The O_2 and N_2 gases are in phase equilibrium with the rubber wall.

Properties The molar mass of oxygen and nitrogen are 32.0 and 28.0 kg/kmol, respectively (Table A-1). The solubility of oxygen and nitrogen in rubber at 298 K are 0.00312 and 0.00156kmol/m^3·bar, respectively (Table 14-7).



Analysis Noting that 750 kPa = 7.5 bar, the molar densities of oxygen and nitrogen in the rubber wall are determined from Eq. 14-20 to be

$$C_{O_2,\text{solid side}}(0) = \mathcal{S} \times P_{O_2,\text{gas side}}$$
$$= (0.00312\ \text{kmol/m}^3.\text{bar})(7.5\ \text{bar})$$
$$= \mathbf{0.0234\ kmol/m^3}$$
$$C_{N_2,\text{solid side}}(0) = \mathcal{S} \times P_{N_2,\text{gas side}}$$
$$= (0.00156\ \text{kmol/m}^3.\text{bar})(7.5\ \text{bar})$$
$$= \mathbf{0.0117\ kmol/m^3}$$

That is, there will be 0.0234 kmol of O_2 and 0.0117 kmol of N_2 gas in each m^3 volume of the rubber wall.

14-47 Hydrogen gas is stored in a spherical nickel vessel that is in an atmospheric air surrounding. The concentrations of hydrogen at the inner and outer surfaces are to be determined.

Assumptions **1** Hydrogen is in thermodynamic equilibrium with the nickel wall.

Properties The molar mass for H_2 is 2.016 kg/kmol (Table A-1). The solubility of H_2 in nickel at 358 K is $\mathscr{S}$= 0.00901 kmol/m^3·bar (Table 14-7).

Analysis At the inner surface, there is 100% H_2, so the H_2 concentration is

$$C_{\mathrm{H_2},1} = \mathscr{S}_{\mathrm{H_2}} y_{\mathrm{H_2}} P_1$$
$$= (0.00901\,\mathrm{kmol/m^3\cdot bar})(1)(7.5\,\mathrm{bar})$$
$$= \mathbf{0.0676\ kmol/m^3}$$

$$\rho_{\mathrm{H_2},1} = MC_{\mathrm{H_2},1}$$
$$= (2.016\,\mathrm{kg/kmol})(0.0676\,\mathrm{kmol/m^3})$$
$$= \mathbf{0.136\ kg/m^3}$$

At the outer surface, there is 0% H_2, so the H_2 concentration is

$$C_{\mathrm{H_2},2} = \mathscr{S}_{\mathrm{H_2}} y_{\mathrm{H_2}} P_2 = 0 \quad \text{and} \quad \rho_{\mathrm{H_2},2} = MC_{\mathrm{H_2},2} = 0$$

Discussion The mole fraction of hydrogen in the atmosphere is extremely low (0.000055%). Thus, at the outer surface of the nickel vessel the hydrogen concentration is practically zero.

14-48 Pure N_2 gas is flowing through a rubber pipe that is in an atmospheric air (79% N_2 & 21% O_2) surrounding. The concentrations of N_2 and O_2 at the inner and outer surfaces are to be determined.

Assumptions **1** Nitrogen and oxygen are in thermodynamic equilibrium with the rubber wall.

Properties The molar masses for N_2 and O_2 are 28.01 kg/kmol and 32.0 kg/kmol, respectively (Table A-1). The solubility of N_2 and O_2 in rubber at 298 K are $\mathscr{S}_{N2}$ = 0.00156 kmol/m^3·bar and $\mathscr{S}_{O2}$ = 0.00312 kmol/m^3·bar, respectively (Table 14-7).

Analysis At the inner surface, there is 100% N_2 and 0% O_2.

For nitrogen:

$$\begin{aligned} C_{N_2,1} &= \mathscr{S}_{N_2} y_{N_2} P_1 \\ &= (0.00156 \text{ kmol/m}^3 \cdot \text{bar})(1)(2.0 \text{ bar}) \\ &= \mathbf{0.00312 \text{ kmol/m}^3} \end{aligned}$$

For oxygen:

$$\begin{aligned} C_{O_2,1} &= \mathscr{S}_{O_2} y_{O_2} P_1 \\ &= (0.00312 \text{ kmol/m}^3 \cdot \text{bar})(0)(2.0 \text{ bar}) \\ &= \mathbf{0} \end{aligned}$$

At the outer surface, there is 79% N_2 and 21% O_2.

For nitrogen:

$$\begin{aligned} C_{N_2,2} &= \mathscr{S}_{N_2} y_{N_2} P_2 \\ &= (0.00156 \text{ kmol/m}^3 \cdot \text{bar})(0.79)(1.013 \text{ bar}) \\ &= \mathbf{0.00125 \text{ kmol/m}^3} \end{aligned}$$

For oxygen:

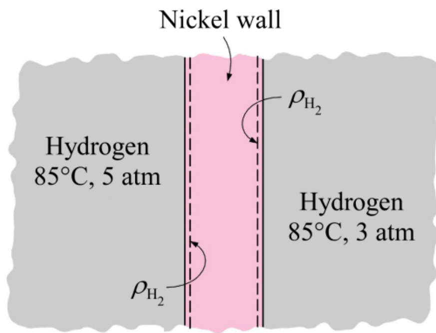
$$\begin{aligned} C_{O_2,2} &= \mathscr{S}_{O_2} y_{O_2} P_2 \\ &= (0.00312 \text{ kmol/m}^3 \cdot \text{bar})(0.21)(1.013 \text{ bar}) \\ &= \mathbf{0.000664 \text{ kmol/m}^3} \end{aligned}$$

Discussion The higher solubility of O_2 over N_2 in rubber means that at the same mole fraction and pressure, the concentration of O_2 in rubber would be higher than N_2.

14-49 A nickel wall separates H_2 gas at different pressures. (*a*) The mass densities of H_2 in the nickel wall and (*b*) outside the nickel wall are to be determined.

Assumptions **1** Nickel and hydrogen are in thermodynamic equilibrium at the interface. **2** Hydrogen an ideal gas.

Properties The molar mass of H_2 is M = 2.016 kg/kmol (Table A-1). The solubility of H_2 in nickel at 85ºC = 358K is 0.00901 kmol/m^3·bar (Table 14-7).



Analysis (*a*) The mass density of H_2 (for 5 atm) in the nickel at the interface is determined using

$$\begin{aligned}\rho_{H_2,\text{solid side}} &= (0.00901\ \text{kmol/m}^3\cdot\text{bar})MP_{H_2,\text{gas side}}\\ &= (0.00901\ \text{kmol/m}^3\cdot\text{bar})(2.016\ \text{kg/kmol})(5\ \text{atm})(1.01325\ \text{bar/atm})\\ &= \mathbf{0.0920\ kg/m^3}\end{aligned}$$

Then, the mass density of H_2 (for 3 atm) in the nickel at the interface is

$$\begin{aligned}\rho_{H_2,\text{solid side}} &= (0.00901\ \text{kmol/m}^3\cdot\text{bar})MP_{H_2,\text{gas side}}\\ &= (0.00901\ \text{kmol/m}^3\cdot\text{bar})(2.016\ \text{kg/kmol})(3\ \text{atm})(1.01325\ \text{bar/atm})\\ &= \mathbf{0.0552\ kg/m^3}\end{aligned}$$

(*b*) The mass density of H_2 (for 5 atm) outside the nickel is determined using

$$\rho_{H_2} = \frac{P_{H_2}M}{R_uT} = \frac{(5\ \text{atm})(101.325\ \text{kPa/atm})(2.016\ \text{kg/kmol})}{(8.314\ \text{kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(273+85)\ \text{K}} = \mathbf{0.343\ kg/m^3}$$

Then, the mass density of H_2 (for 3 atm) outside the rubber is

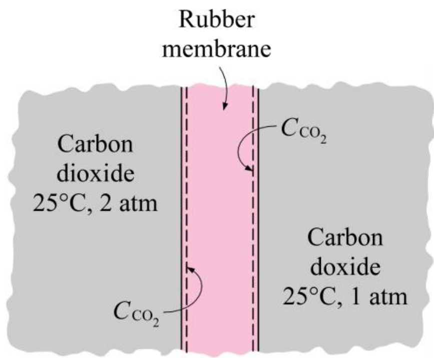
$$\rho_{H_2} = \frac{P_{H_2}M}{R_uT} = \frac{(3\ \text{atm})(101.325\ \text{kPa/atm})(2.016\ \text{kg/kmol})}{(8.314\ \text{kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(273+85)\ \text{K}} = \mathbf{0.206\ kg/m^3}$$

Discussion Note that the densities of H_2 outside the nickel wall are quite different from those in the nickel wall.

14-50 A rubber membrane separates CO_2 gas at different pressures. (*a*) The molar densities of CO_2 in the membrane and (*b*) outside the membrane are to be determined.

Assumptions **1** Rubber and nitrogen are in thermodynamic equilibrium at the interface. **2** Carbon dioxide is an ideal gas.

Properties The molar mass of CO_2 is M = 44.01 kg/kmol (Table A-1). The solubility of CO_2 in rubber at 25ºC = 298 K is 0.04015 kmol/m^3·bar (Table 14-7).



Analysis (*a*) The molar density of CO_2 (for 2 atm) in the rubber at the interface is determined using

$$\begin{aligned} C_{CO_2,\text{solid side}} &= (0.04015\ \text{kmol/m}^3\cdot\text{bar})P_{CO_2,\text{gas side}} \\ &= (0.04015\ \text{kmol/m}^3\cdot\text{bar})(2\ \text{atm})(1.01325\ \text{bar/atm}) \\ &= \mathbf{0.0814\ kmol/m^3} \end{aligned}$$

Then, the molar density of CO_2 (for 1 atm) in the rubber at the interface is

$$\begin{aligned} C_{CO_2,\text{solid side}} &= (0.04015\ \text{kmol/m}^3\cdot\text{bar})P_{CO_2,\text{gas side}} \\ &= (0.04015\ \text{kmol/m}^3\cdot\text{bar})(1\ \text{atm})(1.01325\ \text{bar/atm}) \\ &= \mathbf{0.0407\ kmol/m^3} \end{aligned}$$

(*b*) The molar density of CO_2 (for 2 atm) outside the rubber is determined using

$$C_{CO_2} = \frac{P_{CO_2}}{R_u T} = \frac{(2\ \text{atm})(101.325\ \text{kPa/atm})}{(8.314\ \text{kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(273+25)\ \text{K}} = \mathbf{0.0818\ kmol/m^3}$$

Then, the molar density of CO_2 (for 1 atm) outside the rubber is

$$C_{CO_2} = \frac{P_{CO_2}}{R_u T} = \frac{(1\ \text{atm})(101.325\ \text{kPa/atm})}{(8.314\ \text{kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(273+25)\ \text{K}} = \mathbf{0.409\ kmol/m^3}$$

Discussion Due to its relatively high solubility in rubber, in comparison to O_2 and N_2 (see Table 14-7), the molar concentrations of CO_2 in the solid side and the gas side are almost equal.

14-51 A nickel vessel with specified dimensions is used to contain hydrogen. The rate gas loss from the vessel and the fraction of the hydrogen lost after one year of storage are to be determined.

Assumptions **1** Mass diffusion is steady and one-dimensional. **2** There are no chemical reactions in the rubber plug that result in the generation or depletion of carbon dioxide.

Properties The binary diffusion coefficient for H_2 in the nickel at 358 K is $D_{AB} = 1.2\times10^{-12}$ m^2/s (Table 14–3b). The molar mass of H_2 is M = 2.016 kg/kmol (Table A-1). The solubility of H_2 in the rubber at 358 K is 0.00901 kmol/m^3·bar (Table 14-7).

Analysis The mass density of H_2 (for 3 bar) in the nickel at the interface is determined from

$$\begin{aligned}\rho_{A,1} &= \mathscr{S}P_{A,1}\\ &= (0.00901\,\text{kmol/m}^3\cdot\text{bar})(3\,\text{bar})\left(\frac{2.016\,\text{kg}}{1\,\text{kmol}}\right)\\ &= 0.05449\,\text{kg/m}^3\end{aligned}$$

On the opposite side, the mass density of H_2 is zero, $\rho_{A,2} = 0$. Then the rate of carbon dioxide gas loss through the rubber plug becomes

$$\begin{aligned}\dot{m}_{\text{diff}} &= D_{AB}A\frac{\rho_{A,1}-\rho_{A,2}}{L} = D_{AB}\left(\frac{\pi D^2}{4}\right)\frac{\rho_{A,1}-\rho_{A,2}}{L}\\ &= (1.2\times10^{-12}\text{ m}^2\text{/s})(1600\times10^{-4}\text{ m}^2)\frac{(0.05449-0)\text{ kg/m}^3}{0.002\text{ m}}\\ &= \mathbf{5.23\times10^{-12}\ kg/s} = 1.65\times10^{-4}\text{ kg/year}\end{aligned}$$

This corresponds to about 0.165 gram of H_2 per year. The mass of H_2 in the vessel is

$$m = \frac{PV}{RT} = \frac{(300\,\text{kPa})(0.005\,\text{m}^3)}{(4.124\,\text{kJ/kg}\cdot\text{K})(358\,\text{K})} = 0.001016\,\text{kg}$$

The fraction of H_2 lost after one year of storage is then

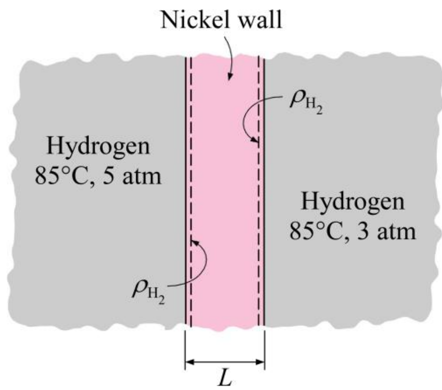
$$f = \frac{1.65\times10^{-4}\text{ kg}}{0.001016\text{ kg}} = \mathbf{0.162}\quad(\text{or }16.2\%)$$

Discussion A loss of 16.2% per year may be significant, and therefore, alternatives should be considered for long-term storage.

14-52 A nickel wall separates H_2 gas at different pressures. The molar diffusion rate per unit area through the nickel wall is to be determined.

Assumptions **1** Mass diffusion is steady and one-dimensional. **2** There are no chemical reactions in the nickel wall that result in the generation or depletion of hydrogen.

Properties The binary diffusion coefficient for hydrogen in the nickel at 85°C = 358 K is $D_{AB} = 1.2\times10^{-12}$ m²/s (Table 14–3b). The solubility of H_2 in nickel at 85°C = 358 K is 0.00901 kmol/m³·bar (Table 14-7).



Analysis The molar density of H_2 (for 5 atm) in the nickel at the interface is determined using

$$
\begin{aligned}
C_{A,1} &= (0.00901\,\text{kmol/m}^3\cdot\text{bar})P_{A,1} \\
&= (0.00901\,\text{kmol/m}^3\cdot\text{bar})(5\,\text{atm})(1.01325\,\text{bar/atm}) \\
&= 0.0456\,\text{kmol/m}^3
\end{aligned}
$$

Then, the molar density of H_2 (for 3 atm) in the nickel at the interface is

$$
\begin{aligned}
C_{A,2} &= (0.00901\,\text{kmol/m}^3\cdot\text{bar})P_{A,2} \\
&= (0.00901\,\text{kmol/m}^3\cdot\text{bar})(3\,\text{atm})(1.01325\,\text{bar/atm}) \\
&= 0.0274\,\text{kmol/m}^3
\end{aligned}
$$

The molar diffusion rate per unit area of hydrogen through the nickel wall can readily be determined using

$$
\begin{aligned}
\bar{j}_{\text{diff}} &= \frac{\dot{N}_{\text{diff}}}{A} = D_{AB}\frac{C_{A,1}-C_{A,2}}{L} \\
&= (1.2\times10^{-12}\ \text{m}^2/\text{s})\frac{(0.0456-0.0274)\ \text{kmol/m}^3}{0.0001\,\text{m}} \\
&= \mathbf{2.18\times10^{-10}\ kmol/s\cdot m^2}
\end{aligned}
$$

Discussion The molar mass of H_2 is M = 2.016 kg/kmol (Table A-1). Hence, the mass diffusion rate per unit area of hydrogen through the nickel wall is

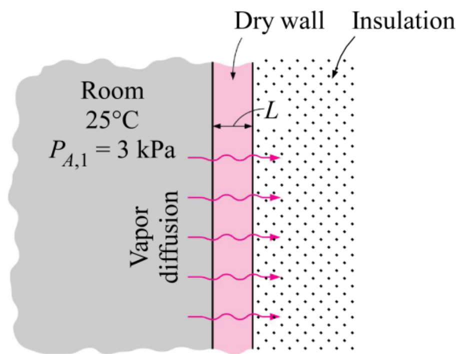
$$
j_{\text{diff}} = (2.18\times10^{-10}\ \text{kmol/s}\cdot\text{m}^2)(2.016\,\text{kg/kmol}) = 4.40\times10^{-10}\ \text{kg/s}\cdot\text{m}^2
$$

14-53 A dry wall separates air in a room with vapor pressure of 3 kPa from air with negligible vapor pressure in the insulation adjoining the wall. The mass diffusion rate of water vapor through the wall is to be determined.

Assumptions **1** Mass diffusion is steady and one-dimensional. **2** Constant properties. **3** Condensation in the wall is negligible.

Properties The molar mass of water vapor is M = 18.015 kg/kmol (Table A-1).

Analysis The molar density of water vapor in the dry wall at the interface is determined using



$$
\begin{aligned}
C_{A,1} &= (0.007 \text{ kmol/m}^3 \cdot \text{bar})P_{A,1} \\
&= (0.007 \text{ kmol/m}^3 \cdot \text{bar})(3 \text{ kPa})(0.01 \text{ bar/kPa}) \\
&= 0.00021 \text{ kmol/m}^3
\end{aligned}
$$

On the opposite side, the molar density of water vapor is zero, since the vapor pressure is negligible,

$$C_{A,2} = 0$$

The molar diffusion rate of water vapor through the wall can be determined using

$$
\begin{aligned}
\dot{N}_{\text{diff}} &= D_{AB} A \frac{C_{A,1} - C_{A,2}}{L} \\
&= (0.2 \times 10^{-9} \text{ m}^2\text{/s})(3 \times 10 \text{ m}^2) \frac{(0.00021 - 0) \text{ kmol/m}^3}{0.012 \text{ m}} \\
&= 1.05 \times 10^{-10} \text{ kmol/s}
\end{aligned}
$$

Hence the mass diffusion rate of water vapor through the wall is

$$\dot{m}_{\text{diff}} = \dot{N}_{\text{diff}} M = (1.05 \times 10^{-10} \text{ kmol/s})(18.015 \text{ kg/kmol}) = \mathbf{1.89 \times 10^{-9} \text{ kg/s}}$$

Discussion At 25°C, the saturation pressure of water is 3169 Pa (from Table 14-9). With the given vapor pressure inside the room being 3 kPa, the relative humidity of the air is

$$\phi = \frac{P_v}{P_{\text{sat}}} = \frac{3000 \text{ Pa}}{3169 \text{ Pa}} = 0.947 = \mathbf{94.7\%}$$

Steady Mass Diffusion through a Wall

14-54C During one-dimensional mass diffusion of species *A* through a plane wall, the species *A* content of the wall will remain constant during steady mass diffusion, but will change during transient mass diffusion.

14-55C The relations for steady one-dimensional heat conduction and mass diffusion through a plane wall are expressed as follows:

Heat conduction: $$\dot{Q}_{cond} = -kA\frac{T_1 - T_2}{L}$$

Mass diffusion: $$\dot{m}_{\text{diff, A,wall}} = \rho D_{\text{AB}} A \frac{w_{\text{A},1} - w_{\text{A},2}}{L} = D_{\text{AB}} A \frac{\rho_{\text{A},1} - \rho_{\text{A},2}}{L}$$

where *A* is the normal area and *L* is the thickness of the wall, and the other variables correspond to each other as follows:

rate of heat conduction $\dot{Q}_{\text{cond}} \longleftrightarrow \dot{m}_{\text{diff, A,wall}}$ rate of mass diffusion

thermal conductivity $k \longleftrightarrow D_{\text{AB}}$ mass diffusivity

temperature $T \longleftrightarrow \rho_{\text{A}}$ density of *A*

14-56C (*a*) T, (*b*) F, (*c*) T, (*d*) F

14-57 A thin plastic membrane separates hydrogen from air. The diffusion rate of hydrogen by diffusion through the membrane under steady conditions is to be determined.

Assumptions **1** Mass diffusion is *steady* and *one-dimensional* since the hydrogen concentrations on both sides of the membrane are maintained constant. Also, there is symmetry about the center plane of the membrane. **2** There are no chemical reactions in the membrane that results in the generation or depletion of hydrogen.

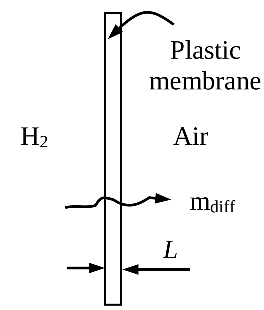
Properties The binary diffusion coefficient of hydrogen in the plastic membrane at the operation temperature is given to be 5.3×10^{-10} m^2/s. The molar mass of hydrogen is M = 2 kg/kmol (Table A-1).

Analysis (*a*) We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the plastic membrane to be a *stationary* medium since there is no diffusion of plastic molecules ($\dot{N}_B = 0$) and the concentration of the hydrogen in the membrane is extremely low (C_A << 1). Then the molar flow rate of hydrogen through the membrane by diffusion per unit area is determined from

$$\bar{j}_{\text{diff}} = \frac{\dot{N}_{\text{diff}}}{A} = D_{AB}\frac{C_{A,1}-C_{A,2}}{L}$$

$$= (5.3\times10^{-10}\ \text{m}^2/\text{s})\frac{(0.045-0.002)\,\text{kmol/m}^3}{2\times10^{-3}\,\text{m}}$$

$$= 1.14\times10^{-8}\ \text{kmol/m}^2.\text{s}$$



The mass flow rate is determined by multiplying the molar flow rate by the molar mass of hydrogen,

$$\dot{m}_{\text{diff}} = M\,\bar{j}_{\text{diff}} = (2\,\text{kg/kmol})(1.14\times10^{-8}\,\text{kmol/m}^2.\text{s})$$

$$= \mathbf{2.28\times10^{-8}\ kg/m^2.s}$$

(*b*) Repeating the calculations for a 0.5-mm thick membrane gives

$$\bar{j}_{\text{diff}} = \frac{\dot{N}_{\text{diff}}}{A} = D_{AB}\frac{C_{A,1}-C_{A,2}}{L}$$

$$= (5.3\times10^{-10}\ \text{m}^2/\text{s})\frac{(0.045-0.002)\,\text{kmol/m}^3}{0.5\times10^{-3}\,\text{m}}$$

$$= 4.56\times10^{-8}\ \text{kmol/m}^2.\text{s}$$

and

$$\dot{m}_{\text{diff}} = M\,\bar{j}_{\text{diff}} = (2\,\text{kg/kmol})(4.56\times10^{-8}\,\text{kmol/m}^2.\text{s}) = \mathbf{9.12\times10^{-8}\ kg/m^2.s}$$

The mass flow rate through the entire membrane can be determined by multiplying the mass flux value above by the membrane area.

14-58 A circular plastic plug with specified dimensions is used to contain ammonia inside a vessel. The diffusion rate of ammonia through plug is to be determined whether it is safe or not.

Assumptions **1** Mass diffusion is steady and one-dimensional. **2** There are no chemical reactions in the plug that result in the generation and depletion of ammonia.

Properties The diffusion coefficient of ammonia in the plug is given as D_{AB} = 1.3 × 10^{-10} m^2/s. The molar mass of ammonia is M = 17.03 kg/kmol (Table A-1).

Analysis The ammonia concentration in the atmosphere and at the outer surface of the plug is zero, $C_{A,2}$ = 0. Thus, the rate of ammonia diffusion through the plug is

$$\begin{aligned}\dot{N}_{\text{diff}} &= D_{AB} A \frac{C_{A,1} - C_{A,2}}{L} = D_{AB}(\pi D^2/4)\frac{C_{A,1} - C_{A,2}}{L} \\ &= (1.3\times10^{-10}\text{ m}^2/\text{s})\frac{\pi}{4}(0.10\text{ m})^2\left(\frac{30\text{ mol/L}}{0.002\text{ m}}\right)\left(\frac{1\text{ kmol/m}^3}{1\text{ mol/L}}\right) \\ &= 1.532\times10^{-8}\text{ kmol/s}\end{aligned}$$

Thus, the ammonia mass diffusion rate is

$$\begin{aligned}\dot{m}_{\text{diff}} &= \dot{N}_{\text{diff}} M \\ &= (1.532\times10^{-8}\text{ kmol/s})(17.03\text{ kg/kmol}) \\ &= \mathbf{2.61\times10^{-7}\text{ kg/s}} \\ &= \mathbf{0.261\text{ mg/s}} > 0.2\text{ mg/s}\end{aligned}$$

Discussion The rate of ammonia being released by diffusion through the plug is greater than the rate that the ventilation system can handle; therefore the plug cannot safely contain the ammonia inside the vessel.

The plug should be replaced with another that has lower diffusion coefficient, smaller diameter, or larger thickness.

14-59 Natural gas with 8% hydrogen content is transported in an above ground pipeline. The highest rate of hydrogen loss through the pipe at steady conditions is to be determined.

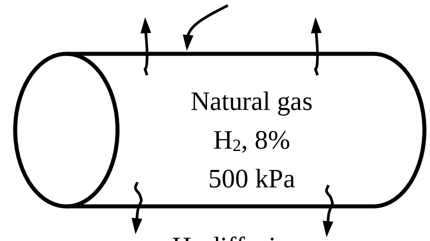
Assumptions **1** Mass diffusion is *steady* and *one-dimensional* since the hydrogen concentrations inside the pipe is constant, and in the atmosphere it is negligible. Also, there is symmetry about the centerline of the pipe. **2** There are no chemical reactions in the pipe that results in the generation or depletion of hydrogen. **3** Both H_2 and CH_4 are ideal gases.

Properties The binary diffusion coefficient of hydrogen in the steel pipe at the operation temperature is given to be 2.9×10^{-13} m^2/s. The molar masses of H_2 and CH_4 are 2 and 16 kg/kmol, respectively (Table A-1). The solubility of hydrogen gas in steel is given as $w_{H_2} = 2.09\times10^{-4}\exp(-3950/T)P_{H_2}^{0.5}$. The density of steel pipe is 7854 kg/m^3 (Table A-3).

Analysis We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the steel pipe to be a *stationary* medium since there is no diffusion of steel molecules ($\dot{N}_B = 0$) and the concentration of the hydrogen in the steel pipe is extremely low ($C_A << 1$). The molar mass of the H_2 and CH_4 mixture in the pipe is

$$M = \sum y_i M_i = (0.08)(2) + (0.92)(16) = 14.88\,\text{kg/kmol}$$

Noting that the mole fraction of hydrogen is 0.08, the partial pressure of hydrogen is



$$y_{H_2} = \frac{P_{H_2}}{P} \to P_{H_2} = (0.08)(500\,\text{kPa}) = 40\,\text{kPa} = 0.4\,\text{bar}$$

Then the mass fraction of hydrogen becomes

$$\begin{aligned} w_{H_2} &= 2.09\times10^{-4}\exp(-3950/T)P_{H_2}{}^{0.5} \\ &= 2.09\times10^{-4}\exp(-3950/293)(0.4)^{0.5} \\ &= 1.85\times10^{-10} \end{aligned}$$

The hydrogen concentration in the atmosphere is practically zero, and thus in the limiting case the hydrogen concentration at the outer surface of pipe can be taken to be zero. Then the highest rate of hydrogen loss through a 100 m long section of the pipe at steady conditions is determined to be

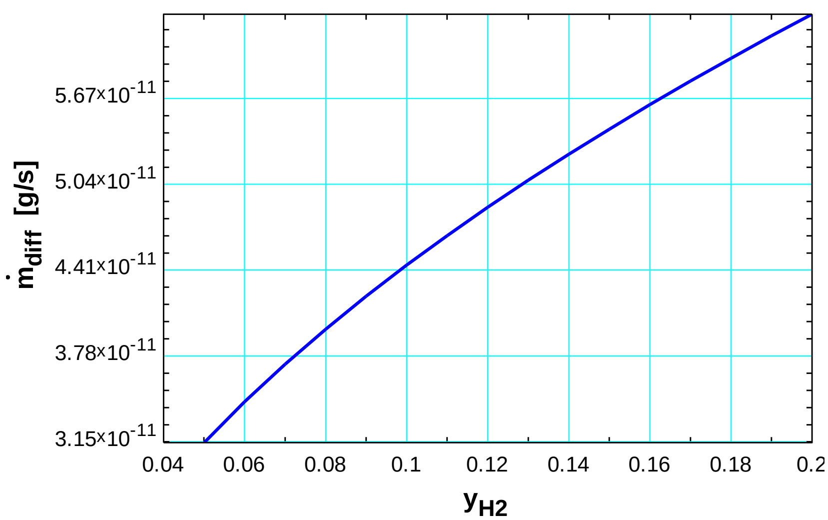
$$\begin{aligned} \dot{m}_{\text{diff, A,cyl}} &= 2\pi L\rho D_{AB}\frac{w_{A,1} - w_{A,2}}{\ln(r_2/r_1)} \\ &= 2\pi(100\text{m})(7854\,\text{kg/m}^3)(2.9\times10^{-13})\frac{1.85\times10^{-10} - 0}{\ln(1.51/1.50)} \\ &= \mathbf{3.98\times10^{-14}\ kg/s} \end{aligned}$$

14-60 Prob. 14-59 is reconsidered. The highest rate of hydrogen loss as a function of the mole fraction of hydrogen in natural gas is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
thickness=0.01 [m]
D_i=3 [m]
L=100 [m]
P=500 [kPa]
y_H2=0.08
T=293 [K]
D_AB=2.9E-13 [m^2/s]

"PROPERTIES"
MM_H2=molarmass(H2)
MM_CH4=molarmass(CH4)
R_u=8.314 [kPa-m^3/kmol-K]
rho=7854 [kg/m^3]

"ANALYSIS"
MM=y_H2*MM_H2+(1-y_H2)*MM_CH4
P_H2=y_H2*P*Convert(kPa, bar)
w_H2=2.09E-4*exp(-3950/T)*P_H2^0.5
m_dot_diff=2*pi*L*rho*D_AB*w_H2/ln(r_2/r_1)*Convert(kg/s, g/s)
r_1=D_i/2
r_2=r_1+thickness
```

y_{H2}	$\dot{m}_{diff}$ [g/s]
0.05	3.144E-11
0.06	3.444E-11
0.07	3.720E-11
0.08	3.977E-11
0.09	4.218E-11
0.1	4.446E-11
0.11	4.663E-11
0.12	4.871E-11
0.13	5.070E-11
0.14	5.261E-11
0.15	5.446E-11
0.16	5.624E-11
0.17	5.797E-11
0.18	5.966E-11
0.19	6.129E-11
0.2	6.288E-11



14-61 Pure H_2 gas is flowing through an iron pipe. The rate at which H_2 leaks out by diffusion is to be determined for a known concentration at the inner surface.

Assumptions **1** Mass diffusion is *steady* and *one-dimensional* since the hydrogen concentration in the pipe and thus at the inner surface of the pipe is practically constant, and the hydrogen concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the centerline of the pipe. **2** There are no chemical reactions in the pipe that results in the generation or depletion of hydrogen.

Properties The binary diffusion coefficient for hydrogen in the iron at 25°C is $D_{AB} = 2.6 \times 10^{-13}$ m^2/s (Table 14-3*b*). The molar mass of hydrogen is M = 2.016 kg/kmol (Table A-1).

Analysis We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the container to be a *stationary* medium since there is no diffusion of iron molecules ($\dot{N}_B = 0$) and the concentration of the hydrogen in the pipe is extremely low ($C_A << 1$). Then the molar flow rate of hydrogen through the pipe wall by diffusion can be determined to be

$$\begin{aligned}\dot{N}_{\text{diff}} &= 2\pi L D_{AB}\frac{C_{A,1}-C_{A,2}}{\ln(r_2/r_1)}\\ &= 2\pi(10\text{ m})(2.6\times10^{-13}\text{ m}^2/\text{s})\frac{(0.1-0)\text{ kmol/m}^3}{\ln(35/25)}\\ &= 4.855\times10^{-12}\text{ kmol/s}\end{aligned}$$

The mass diffusion rate of hydrogen through the pipe wall is determined by multiplying the molar flow rate with the molar mass of hydrogen,

$$\begin{aligned}\dot{m}_{\text{diff}} &= M\dot{N}_{\text{diff}}\\ &= (4.855\times10^{-12}\text{ kmol/s})(2.016\text{ kg/kmol})\\ &= \mathbf{9.79\times10^{-12}\ kg/s}\end{aligned}$$

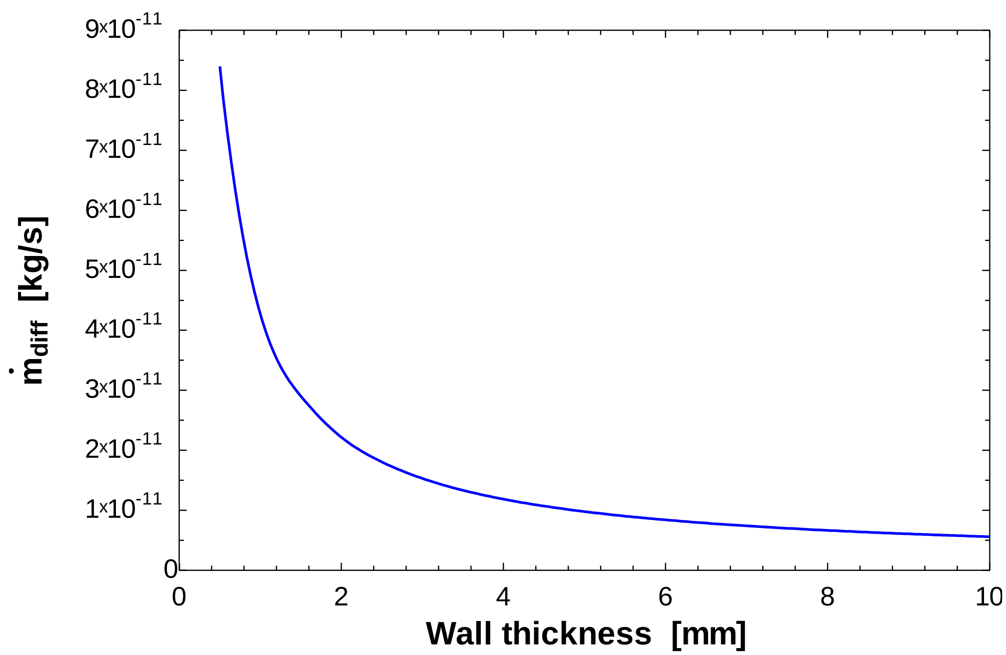
Discussion The hydrogen will leak out through the pipe wall by diffusion at a rate of 9.79×10^{-12} kg/s. The leakage can be reduced by increasing the wall thickness of the pipe.

14-62 Pure H_2 gas is flowing through an iron pipe. The effect of wall thickness on the rate of H_2 diffusion through the pipe wall is to be evaluated.

Analysis The problem is solved using EES, and the solution is given below:

```
"GIVEN"
M=2.016 [kmol/kg]
D_1=0.025 [m]
D_2=D_1+2*t*1e-3
L=10 [m]
D_AB=2.6e-13 [m^2/s]
C_A_1=0.1 [kmol/m^3]
C_A_2=0 [kmol/m^3]

"ANALYSIS"
N_dot_diff=2*pi#*L*D_AB*(C_A_1-C_A_2)/ln(D_2/D_1)
N_dot_diff=m_dot_diff/M
```

t [mm]	$\dot{m}_{\text{diff}}$ [kg/s]
0.5	8.397E-11
1.0	4.279E-11
1.5	2.906E-11
2.0	2.219E-11
2.5	1.806E-11
3.0	1.531E-11
3.5	1.334E-11
4.0	1.186E-11
4.5	1.071E-11
5.0	9.788E-12
6.0	8.401E-12
7.0	7.406E-12
8.0	6.657E-12
9.0	6.073E-12
10	5.603E-12



Discussion Knowing how the wall thickness affects the rate of hydrogen diffusion through the pipe wall can help engineers to design systems that minimize gas leakage from diffusion. As the wall thickness increases, the rate of mass diffusion through pipe wall decreases, at first drastically and then gradually toward zero.

14-63 Helium gas stored inside a cylindrical Pyrex tank, and a sensor detects a leakage of the gas at 1.8×10^{-6} g/h. The concentration of helium at the inner surface of the Pyrex tank is to be determined.

Assumptions **1** Mass diffusion is *steady* and *one-dimensional* since the helium concentration in the tank and thus at the inner surface of the tank is practically constant, and the helium concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the centerline. **2** There are no chemical reactions in the tank that results in the generation or depletion of helium. **3** Both ends of the cylindrical tank are impermeable.

Properties The binary diffusion coefficient for helium in the Pyrex tank at 20°C is $D_{AB} = 4.5 \times 10^{-15}$ m²/s (Table 14-3*b*). The molar mass of helium is $M = 4.003$ kg/kmol (Table A-1).

Analysis The mass diffusion rate through the tank wall is given

$$\begin{aligned}\dot{m}_{\text{diff}} &= (1.8\times10^{-6}\ \text{g/h})(1/1000\ \text{kg/g})(1/3600\ \text{h/s})\\ &= 5\times10^{-13}\ \text{kg/s}\end{aligned}$$

We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B =$ constant), and the container to be a *stationary* medium since there is no diffusion of Pyrex molecules ($\dot{N}_B = 0$) and the concentration of the helium in the tank is extremely low ($C_A << 1$). Then the molar flow rate of helium through the pipe wall by diffusion can be determined to be

$$\begin{aligned}\dot{N}_{\text{diff}} &= \frac{\dot{m}_{\text{diff}}}{M}\\ &= 2\pi L D_{AB}\frac{C_{A,1}-C_{A,2}}{\ln(r_2/r_1)}\end{aligned}$$

Solving for the concentration of helium at the inner surface $C_{A,1}$ we have

$$\begin{aligned}C_{A,1} &= \frac{\dot{m}_{\text{diff}}}{M}\frac{\ln(r_2/r_1)}{2\pi L D_{AB}}\\ &= \left(\frac{5\times10^{-13}\ \text{kg/s}}{4.003\ \text{kg/kmol}}\right)\left[\frac{\ln(12.5/12)}{2\pi(2\ \text{m})(4.5\times10^{-15}\ \text{m}^2/\text{s})}\right]\\ &= \mathbf{0.0902\ kmol/m^3}\end{aligned}$$

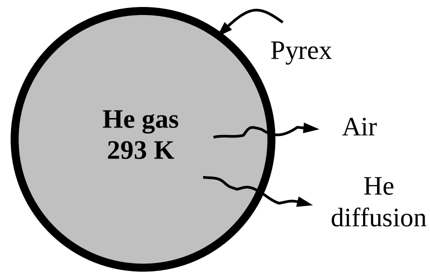
Discussion The concentration of helium at the inner surface of the tank can be expressed in mass basis as $\rho_{A,1} = MC_{A,1} =$ 0.361 kg/m³.

14-64 Pressurized helium gas is stored in a spherical container. The diffusion rate of helium through the container is to be determined.

Assumptions **1** Mass diffusion is *steady* and *one-dimensional* since the helium concentration in the tank and thus at the inner surface of the container is practically constant, and the helium concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the center of the container. **2** There are no chemical reactions in the pyrex shell that results in the generation or depletion of helium.

Properties The binary diffusion coefficient of helium in the pyrex at the specified temperature is 4.5×10^{-15} m^2/s (Table 14-3b). The molar mass of helium is M = 4 kg/kmol (Table A-1).

Analysis We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the container to be a *stationary* medium since there is no diffusion of pyrex molecules ($\dot{N}_B = 0$) and the concentration of the helium in the container is extremely low (C_A << 1). Then the molar flow rate of helium through the shell by diffusion can readily be determined from Eq. 14-28 to be



$$\dot{N}_{\text{diff}} = 4\pi r_1 r_2 D_{AB}\frac{C_{A,1}-C_{A,2}}{r_2-r_1}$$

$$= 4\pi(1.47\text{ m})(1.50\text{ m})(4.5\times10^{-15}\text{ m}^2/\text{s})\frac{(0.00069-0)\text{ kmol/m}^3}{1.50-1.47}$$

$$= 2.868\times10^{-15}\text{ kmol/s}$$

The mass flow rate is determined by multiplying the molar flow rate by the molar mass of helium,

$$\dot{m}_{\text{diff}} = M\dot{N}_{\text{diff}} = (4\text{ kg/kmol})(2.868\times10^{-15}\text{ kmol/s}) = \mathbf{1.15\times10^{-14}\text{ kg/s}}$$

Therefore, helium will leak out of the container through the shell by diffusion at a rate of 1.15×10^{-14} kg/s or 0.00036 g/year.

Discussion Note that the concentration of helium in the pyrex at the inner surface depends on the temperature and pressure of the helium in the tank, and can be determined as explained in the previous example. Also, the assumption of zero helium concentration in pyrex at the outer surface is reasonable since there is only a trace amount of helium in the atmosphere (0.5 parts per million by mole numbers).

14-65 A metal spherical vessel is used to contain hydrogen gas. The diffusion rate of hydrogen through the vessel is to be determined whether it is safe or not with respect to the room's ventilation system.

Assumptions **1** Mass diffusion is steady and one-dimensional. **2** There are no chemical reactions in the vessel that result in the generation and depletion of hydrogen.

Properties The solubility of hydrogen in the vessel is given as $\mathscr{S}_{AB}$ = 0.005 kmol/m^3·bars. The molar mass of hydrogen is M = 2.016 kg/kmol (Table A-1).

Analysis The hydrogen concentration in the atmosphere and at the outer surface of the vessel is zero, $C_{A,2} = 0$ (or $P_{A,2} = 0$). Thus, the rate of hydrogen diffusion through the vessel is

$$\begin{aligned}
\dot{N}_{\text{diff}} &= 4\pi r_1 r_2 D_{AB} \frac{C_{A,1} - C_{A,2}}{r_2 - r_1} \\
&= 4\pi r_1 r_2 D_{AB} \mathscr{S}_{AB} \frac{P_{A,1} - P_{A,2}}{r_2 - r_1} \\
&= 4\pi (2.5\text{ m})(2.503\text{ m})(1.5\times10^{-12}\text{ m}^2/\text{s})(0.005\text{ kmol/m}^3\cdot\text{bar})\left(\frac{20\text{ bar}}{2.503\text{ m} - 2.5\text{ m}}\right) \\
&= 3.932\times10^{-9}\text{ kmol/s}
\end{aligned}$$

Thus, the hydrogen mass diffusion rate through the vessel is

$$\begin{aligned}
\dot{m}_{\text{diff}} &= \dot{N}_{\text{diff}} M \\
&= (3.932\times10^{-9}\text{ kmol/s})(2.016\text{ kg/kmol}) \\
&= \mathbf{7.93\times10^{-9}\ kg/s} \\
&= \mathbf{7.93\ \mu g/s} > 5\ \mu\text{g/s}
\end{aligned}$$

Discussion The rate of hydrogen leakage by diffusion through the vessel is greater than the rate that the ventilation system can handle; therefore the vessel cannot safely contain the hydrogen.

To prevent hazards from hydrogen leakage, a vessel with lower diffusion coefficient and solubility of hydrogen should be used. Or the ventilation system for the room should be upgraded.

14-66 Helium gas is stored in a spherical fused silica container. The diffusion rate of helium through the container and the pressure drop in the tank in one week as a result of helium loss are to be determined.

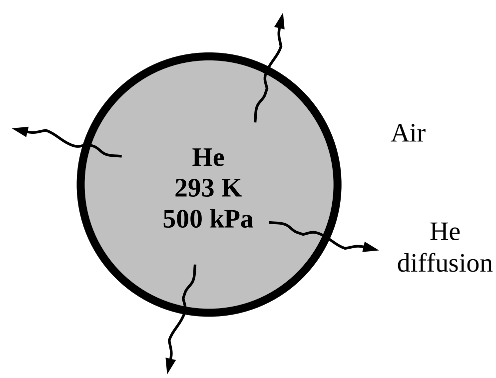
Assumptions **1** Mass diffusion is *steady* and *one-dimensional* since the helium concentration in the tank and thus at the inner surface of the container is practically constant, and the helium concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the midpoint of the container. **2** There are no chemical reactions in the fused silica that results in the generation or depletion of helium. **3** Helium is an ideal gas. **4** The helium concentration at the inner surface of the container is at the highest possible level (the solubility).

Properties The solubility of helium in fused silica (SiO_2) at 293 K and 500 kPa is 0.00045 kmol /m^3.bar (Table 14-7). The diffusivity of helium in fused silica at 293 K (actually, at 298 K) is 4×10^{-14} m^2/s (Table 14-3b). The molar mass of helium is M = 4 kg/kmol (Table A-1).

Analysis (*a*) We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the container to be a *stationary* medium since there is no diffusion of silica molecules ($\dot{N}_B = 0$) and the concentration of the helium in the container is extremely low ($C_A << 1$). The molar concentration of helium at the inner surface of the container is determined from the solubility data to be

$$C_{A,1} = S \times P_{\text{He}} = (0.00045\,\text{kmol/m}^3.\text{bar})(5\,\text{bar}) = 2.25\times10^{-3}\,\text{kmol/m}^3 = 0.00225\,\text{kmol/m}^3$$

The helium concentration in the atmosphere and thus at the outer surface is taken to be zero since the tank is well ventilated. Then the molar flow rate of helium through the tank by diffusion becomes



$$\dot{N}_{\text{diff}} = 4\pi r_1 r_2 D_{AB}\frac{C_{A,1} - C_{A,2}}{r_2 - r_1}$$
$$= 4\pi(1\,\text{m})(1.01\,\text{m})(4\times10^{-14}\,\text{m}^2/\text{s})\frac{(0.00225-0)\text{kmol/m}^3}{(1.01-1)\text{m}}$$
$$= 1.14\times10^{-13}\,\text{kmol/s}$$

The mass flow rate is determined by multiplying the molar flow rate by the molar mass of helium,

$$\dot{m}_{\text{diff}} = M\dot{N}_{\text{diff}} = (4\,\text{kg/kmol})(1.14\times10^{-13}\,\text{kmol/s}) = \mathbf{4.57\times10^{-13}\,kg/s}$$

(*b*) Noting that the molar flow rate of helium is 1.14 $\times10^{-13}$ kmol / s, the amount of helium diffused through the shell in 1 week becomes

$$N_{\text{diff}} = N_{\text{diff}}\,\Delta t = (1.14\times10^{-13}\,\text{kmol/s})(7\times24\times3600\,\text{s/week})$$
$$= 6.895\times10^{-8}\,\text{kmol/week}$$

The volume of the spherical tank and the initial amount of helium gas in the tank are

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1\text{m})^3 = 4.189\,\text{m}^3$$
$$N_{\text{initial}} = \frac{PV}{R_u T} = \frac{(500\,\text{kPa})(4.189\,\text{m}^3)}{(8.314\,\text{kPa m}^3/\text{kmol K})(293\,\text{K})} = 0.85977\ \text{kmol}$$

Then the number of moles of helium remaining in the tank after one week becomes

$$N_{\text{final}} = N_{\text{initial}} - N_{\text{diff}} = 0.85977 - 6.895\times10^{-8} \cong 0.85977\,\text{kPa}$$

which is the practically the same as the initial value. Therefore, the amount of helium that leaves the tank by diffusion is negligible, and the final pressure in the tank is the same as the initial pressure of $P_2 = P_1$ = **500 kPa.**

14-67 Pure N_2 gas is flowing through a rubber pipe. The rate at which N_2 leaks out by diffusion is to be determined for the cases of vacuum and atmospheric air outside.

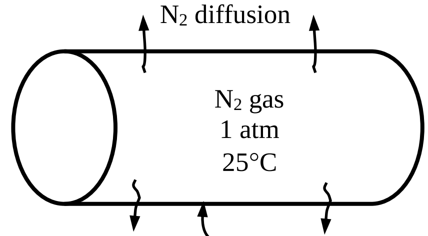
Assumptions **1** Mass diffusion is *steady* and *one-dimensional* since the nitrogen concentration in the pipe and thus at the inner surface of the pipe is practically constant, and the nitrogen concentration in the atmosphere also remains constant. Also, there is symmetry about the centerline of the pipe. **2** There are no chemical reactions in the pipe that results in the generation or depletion of nitrogen. **3** Both the nitrogen and air are ideal gases.

Properties The diffusivity and solubility of nitrogen in rubber at 25°C are 1.5×10^{-10} m^2/s and 0.00156 kmol/m^3.bar, respectively (Tables 14-3 and 14-7).

Analysis We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the container to be a *stationary* medium since there is no diffusion of rubber molecules ($\dot{N}_B = 0$) and the concentration of the nitrogen in the container is extremely low (C_A << 1). The partial pressures of oxygen and nitrogen in the air are

$$P_{N_2} = y_{N_2}P = (0.79)(100\,\text{kPa}) = 79\,\text{kPa} = 0.79\,\text{bar}$$

$$P_{O_2} = y_{O_2}P = (0.21)(100\,\text{kPa}) = 21\,\text{kPa} = 0.21\,\text{bar}$$



When solubility data is available, the molar flow rate of a gas through a solid can be determined by replacing the molar concentration by $C_{A,\,\text{solid side}}(0) = \mathcal{S}_{AB}P_{A,\,\text{gas side}}(0)$ For a cylindrical pipe the molar rate of diffusion can be expressed in terms of solubility as

$$\dot{N}_{\text{diff, A,cyl}} = 2\pi L D_{AB}\mathcal{S}_{AB}\,\frac{P_{A,1}-P_{A,2}}{\ln(r_2/r_1)}$$

where $\mathcal{S}_{AB}$ is the *solubility* and $P_{A,1}$ and $P_{A,2}$ are the partial pressures of gas *A* on the two sides of the wall.

(*a*) The pipe is in vacuum and thus $P_{A,2} = 0$:

$$\dot{N}_{\text{diff, A,cyl}} = 2\pi(10\,\text{m})(1.5\times10^{-10}\,\text{m}^2/\text{s})(0.00156\,\text{kmol/m}^3\cdot\text{s}\cdot\text{bar})\frac{(1-0)\text{bar}}{\ln(0.032/0.03)}$$

$$= \mathbf{2.278\times10^{-10}\ kmol/s}$$

(*b*) The pipe is in atmospheric air and thus $P_{A,2}$ = 0.79 bar:

$$\dot{N}_{\text{diff, A,cyl}} = 2\pi(10\,\text{m})(1.5\times10^{-10}\,\text{m}^2/\text{s})(0.00156\,\text{kmol/m}^3\cdot\text{s}\cdot\text{bar})\frac{(1-0.79)\text{bar}}{\ln(0.032/0.03)}$$

$$= \mathbf{4.784\times10^{-11}\ kmol/s}$$

Discussion In the case of a vacuum environment, the diffusion rate of nitrogen from the pipe is about 5 times the rate in atmospheric air. This is expected since mass diffusion is proportional to the concentration difference.

14-68 A balloon is filled with helium gas. The initial rates of diffusion of helium, oxygen, and nitrogen through the balloon and the mass fraction of helium that escapes during the first 5 h are to be determined.

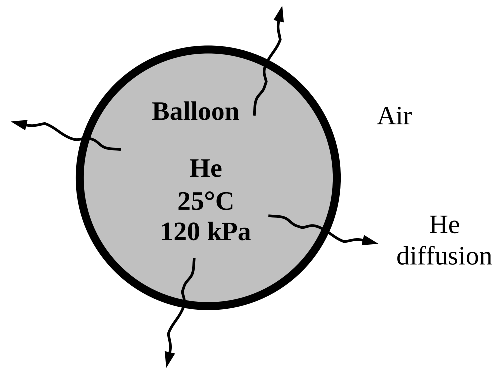
Assumptions **1** The pressure of helium inside the balloon remains nearly constant. **2** Mass diffusion is *steady* for the time period considered. **3** Mass diffusion is *one-dimensional* since the helium concentration in the balloon and thus at the inner surface is practically constant, and the helium concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the midpoint of the balloon. **4** There are no chemical reactions in the balloon that results in the generation or depletion of helium. **5** Both the helium and the air are ideal gases. **7** The curvature effects of the balloon are negligible so that the balloon can be treated as a plane layer.

Properties The permeability of rubber to helium, oxygen, and nitrogen at 25°C are given to be 9.4×10^{-13}, 7.05×10^{-13}, and 2.6×10^{-13} kmol/m.s.bars, respectively. The molar mass of helium is M = 4 kg/kmol and its gas constant is R = 2.0709 kPa.m^3/kg.K (Table A-1).

Analysis We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the balloon to be a *stationary* medium since there is no diffusion of rubber molecules ($\dot{N}_B = 0$) and the concentration of the helium in the balloon is extremely low (C_A << 1). The partial pressures of oxygen and nitrogen in the air are

$$P_{N_2} = y_{N_2}P = (0.79)(100\text{ kPa}) = 79\text{ kPa} = 0.79\text{ bar}$$

$$P_{O_2} = y_{O_2}P = (0.21)(100\text{ kPa}) = 21\text{ kPa} = 0.21\text{ bar}$$



The partial pressure of helium in the air is negligible. Since the balloon is filled with pure helium gas at 120 kPa, the initial partial pressure of helium in the balloon is 120 kPa, and the initial partial pressures of oxygen and nitrogen are zero.

When permeability data is available, the molar flow rate of a gas through a solid wall of thickness L under steady one-dimensional conditions can be determined from Eq. 14-29,

$$\dot{N}_{\text{diff, A,wall}} = \text{P}_{AB}A\frac{P_{A,1}-P_{A,2}}{L} \quad \text{(kmol/s)}$$

where P_{AB} is the permeability and $P_{A,1}$ and $P_{A,2}$ are the partial pressures of gas A on the two sides of the wall (Note that the balloon can be treated as a plain layer since its thickness is very small compared to its diameter). Noting that the surface area of the balloon is $A = \pi D^2 = \pi(0.15\text{ m})^2 = 0.07069\text{ m}^2$, the initial rates of diffusion of helium, oxygen, and nitrogen at 25ºC are determined to be

$$\dot{N}_{\text{diff,He}} = \text{P}_{AB}A\frac{P_{He,1}-P_{He,2}}{L} = (9.4\times10^{-13}\text{ kmol/m.s.bar})(0.07069\text{ m}^2)\frac{(1.2-0)\text{bar}}{0.2\times10^{-3}\text{ m}} = \mathbf{0.399\times10^{-9}\ kmol/s}$$

$$\dot{N}_{\text{diff},O_2} = \text{P}_{AB}A\frac{P_{O_2,1}-P_{O_2,2}}{L} = (7.05\times10^{-13}\text{ kmol/m.s.bar})(0.07069\text{ m}^2)\frac{(0-0.21)\text{bar}}{0.2\times10^{-3}\text{ m}} = \mathbf{-0.523\times10^{-10}\ kmol/s}$$

$$\dot{N}_{\text{diff},N_2} = \text{P}_{AB}A\frac{P_{N_2,1}-P_{N_2,2}}{r_2-r_1} = (2.6\times10^{-13}\text{ kmol/m.s.bar})(0.07069\text{ m}^2)\frac{(0-0.79)\text{bar}}{0.2\times10^{-3}\text{ m}} = \mathbf{-0.726\times10^{-10}\ kmol/s}$$

The initial mass flow rate of helium and the amount of helium that escapes during the first 5 hours are

$$\dot{m}_{diff,\text{He}} = M\dot{N}_{diff,\text{He}} = (4\text{ kg/kmol})(0.399\times10^{-9}\text{ kmol/s}) = 1.59\times10^{-9}\text{ kg/s}$$

$$m_{\text{diff, He}} = \dot{m}_{\text{diff, He}}\Delta t = (1.59\times10^{-9}\text{ kg/s})(5\times3600\text{ s}) = \mathbf{2.87\times10^{-5}\ kg = 0.0287\ g}$$

The initial mass of helium in the balloon is

$$m_{\text{initial}} = \frac{PV}{RT} = \frac{(120\text{ kPa})[4\pi(0.075\text{ m})^3/3]}{(2.077\text{ kPa.m}^3/\text{kg}\cdot\text{K})(298\text{ K})} = 3.43\times10^{-4}\text{ kg} = 0.343\text{ g}$$

Therefore, the fraction of helium that escapes the balloon during the first 5 h is

$$\text{Fraction} = \frac{m_{\text{diff, He}}}{m_{\text{initial}}} = \frac{0.0287\text{ g}}{0.343\text{ g}} = \mathbf{0.0837}\ \text{(or}\ \mathbf{8.4\%)}$$

Discussion This is a significant amount of helium gas that escapes the balloon, and explains why the helium balloons do not last long. Also, our assumption of constant pressure for the helium in the balloon is obviously not very accurate since 8.4% of helium is lost during the process.

14-69 A balloon is filled with helium gas. A relation for the variation of pressure in the balloon with time as a result of mass transfer through the balloon material is to be obtained, and the time it takes for the pressure in the balloon to drop from 120 to 100 kPa is to be determined.

Assumptions **1** The pressure of helium inside the balloon remains nearly constant. **2** Mass diffusion is *transient* since the conditions inside the balloon change with time. **3** Mass diffusion is *one-dimensional* since the helium concentration in the balloon and thus at the inner surface is practically constant, and the helium concentration in the atmosphere and thus at the outer surface is practically zero. Also, there is symmetry about the midpoint of the balloon. **4** There are no chemical reactions in the balloon material that results in the generation or depletion of helium. **5** Helium is an ideal gas. **6** The diffusion of air into the balloon is negligible. **7** The volume of the balloon is constant. **8** The curvature effects of the balloon are negligible so that the balloon material can be treated as a plane layer.

Properties The permeability of rubber to helium at 25°C is given to be 9.4×10^{-13} kmol/m.s.bar. The molar mass of helium is M = 4 kg/kmol and its gas constant is R = 2.077 kPa.m^3/kg.K (Table A-1).

Analysis We can consider the total molar concentration to be constant ($C = C_A + C_B \cong C_B$ = constant), and the balloon to be a *stationary* medium since there is no diffusion of rubber molecules ($\dot{N}_B = 0$) and the concentration of the helium in the balloon is extremely low (C_A << 1). The partial pressure of helium in the air is negligible. Since the balloon is filled with pure helium gas at 120 kPa, the initial partial pressure of helium in the balloon is 120 kPa.

When permeability data is available, the molar flow rate of a gas through a solid wall of thickness L under steady one-dimensional conditions can be determined from Eq. 14-29,

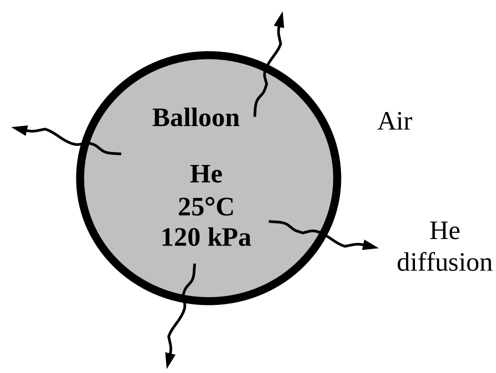
$$\dot{N}_{\text{diff, A,wall}} = \wp_{AB} A \frac{P_{A,1} - P_{A,2}}{L} = \wp_{AB} A \frac{P}{L} \qquad \text{(kmol/s)}$$

where $\wp_{AB}$ is the permeability and $P_{A,1}$ and $P_{A,2}$ are the partial pressures of helium on the two sides of the wall (note that the balloon can be treated as a plain layer since its thickness very small compared to its diameter, and $P_{A,1}$ is simply the pressure P of helium inside the balloon).

Noting that the amount of helium in the balloon can be expressed as $N = P\mathbf{V}/R_uT$ and taking the temperature and volume to be constants,

$$N = \frac{P\mathbf{V}}{R_uT} \rightarrow \frac{dN}{dt} = \frac{\mathbf{V}}{R_uT}\frac{dP}{dt} \rightarrow \frac{dP}{dt} = \frac{R_uT}{\mathbf{V}}\frac{dN}{dt} \qquad (1)$$

Conservation of mass dictates that the mass flow rate of helium from the balloon be equal to the rate of change of mass inside the balloon,



$$\frac{dN}{dt} = -\dot{N}_{\text{diff, A,wall}} = -\wp_{AB} A \frac{P}{L} \qquad (2)$$

Substituting (2) into (1),

$$\frac{dP}{dt} = \frac{R_uT}{\mathbf{V}}\frac{dN}{dt} = -\frac{R_uT}{\mathbf{V}}\wp_{AB} A \frac{P}{L} = -\frac{R_uT\wp_{AB}A}{\mathbf{V}L}P$$

Separating the variables and integrating gives

$$\frac{dP}{P} = -\frac{R_uT\wp_{AB}A}{\mathbf{V}L}dt \rightarrow \ln P\Big|_{P_0}^{P} = -\frac{R_uT\wp_{AB}A}{\mathbf{V}L}\, t\Big|_0^t \rightarrow \ln\frac{P}{P_0} = -\frac{R_uT\wp_{AB}A}{\mathbf{V}L}t$$

Rearranging, the desired relation for the variation of pressure in the balloon with time is determined to be

$$P = P_0 \exp\left(-\frac{R_uT\wp_{AB}A}{\mathbf{V}L}t\right) = P_0 \exp\left(-\frac{3R_uT\wp_{AB}}{rL}t\right) \qquad \text{since, for a sphere, } \frac{A}{\mathbf{V}} = \frac{4\pi r^2}{4\pi r^3/3} = \frac{3}{r}$$

Then the time it takes for the pressure inside the balloon to drop from 120 kPa to 100 kPa becomes

$$\frac{100\text{ kPa}}{120\text{ kPa}} = \exp\left(-\frac{3(0.08314\text{ bar}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(298\text{ K})(9.4\times10^{-13}\text{ kmol/m}\cdot\text{s}\cdot\text{bar})}{(0.075\text{ m})(0.2\times10^{-3}\text{ m})}t\right) \rightarrow t = 39{,}140\text{ s} = \mathbf{10.9\text{ h}}$$

Therefore, the balloon will lose 20% of its pressure in about 11 h.

Water Vapor Migration in Buildings

14-70C Excess moisture changes the *dimensions* of wood, and cyclic changes in dimensions weaken the joints, and can jeopardize the structural integrity of building components, causing "squeaking" at the minimum. Excess moisture can also cause *rotting* in woods, *mold* growth on wood surfaces, *corrosion* and *rusting* in metals, and *peeling of paint* on the interior and exterior wall surfaces.

14-71C The condensation or freezing of water vapor in the wall increases the thermal conductivity of the insulation material, and thus increases the rate of heat transfer through the wall. Similarly, the thermal conductivity of the soil increases with increasing amount of moisture.

14-72C Vapor barriers are materials that are impermeable to moisture such as sheet metals, heavy metal foils, and thick plastic layers, and they completely *eliminate* the vapor migration. Vapor retarders such as reinforced plastics or metals, thin foils, plastic films, treated papers and coated felts, on the other hand, *slow down* the flow of moisture through the structures. Vapor retarders are commonly used in residential buildings to control the vapor migration through the walls.

14-73C Insulations on *chilled water lines* are always wrapped with *vapor barrier jackets* to eliminate the possibility of vapor entering the insulation. This is because moisture that migrates through the insulation to the cold surface will condense and remain there indefinitely with no possibility of vaporizing and moving back to the outside.

14-74C A tank that contains moist air at 3 atm is located in moist air that is at 1 atm. The driving force for moisture transfer is the vapor pressure difference, and thus it is possible for the water vapor to flow into the tank from surroundings if the vapor pressure in the surroundings is greater than the vapor pressure in the tank.

14-75C When the temperature, total pressure, and the relative humidity are given, the vapor pressure can be determined from the psychrometric chart or the relation $P_v = \phi P_{\text{sat}}$ where P_{sat} is the saturation (or boiling) pressure of water at the specified temperature and ϕ is the relative humidity.

14-76C The mass flow rate of water vapor through a wall of thickness L in therms of the partial pressure of water vapor on both sides of the wall and the permeability of the wall to the water vapor can be expressed as

$$\dot{m}_{\text{diff, A,wall}} = \text{M}\text{P}_{\text{AB}} A \frac{P_{\text{A},1} - P_{\text{A},2}}{L}$$

where M is the molar mass of vapor, P_{AB} is the permeability, A is the normal area, and P_{A} is the partial pressure of the vapor.

14-77 A glass of milk left on top of a counter is tightly sealed by a sheet of 0.009-mm thick aluminum foil. The drop in the level of the milk in the glass in 12 h due to vapor migration through the foil is to be determined.

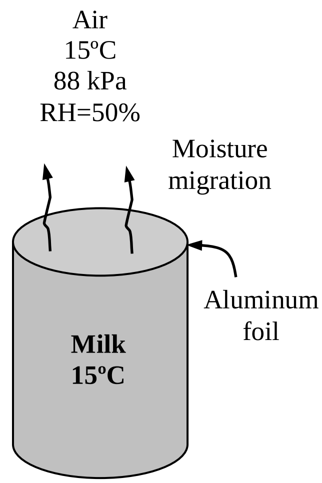
Assumptions **1** Steady operating conditions exist. **2** Mass transfer through the foil is one-dimensional. **3** The vapor permeability of the foil is constant.

Properties The permeance of the foil to water vapor is given to be 2.9×10^{-12} kg/s.m^2.Pa. The saturation pressure of water at 15ºC is 1705 Pa (Table 14-9). We take the density of milk to be 1000 kg/m^3.

Analysis The mass flow rate of water vapor through a plain layer of thickness L and normal area A is given as (Eq. 14-31)

$$\dot{m}_v = \mathsf{P}A\frac{P_{v,1}-P_{v,2}}{L} = \mathsf{P}A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{L} = \mathsf{M}A(\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2})$$

where P is the vapor permeability and M = P/L is the permeance of the material, ϕ is the relative humidity and P_{sat} is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the states of the air on the two sides of the foil.



The diffusion area of the foil is $A = \pi r^2 = \pi(0.06\text{ m})^2 = 0.0113\text{ m}^2$.

Substituting, the mass flow rate of water vapor through the foil becomes

$$\begin{aligned}\dot{m}_v &= (2.9\times10^{-12}\text{ kg/s.m}^2\text{.Pa})(0.0113\text{ m}^2)[1(1705\text{ Pa})-0.5(1705\text{ Pa})]\\ &= 2.79\times10^{-11}\text{ kg/s}\end{aligned}$$

Then the total amount of moisture that flows through the foil during a 12-h period becomes

$$m_{v,12-h} = \dot{m}_v\Delta t = (2.79\times10^{-11}\text{ kg/s})(12\times3600\text{ s}) = 1.21\times10^{-6}\text{ kg}$$

$$V = m/\rho = (1.21\times10^{-6}\text{ kg})/(1000\text{ kg/m}^3) = 1.21\times10^{-9}\text{ m}^3$$

Then the drop in the level of the milk becomes

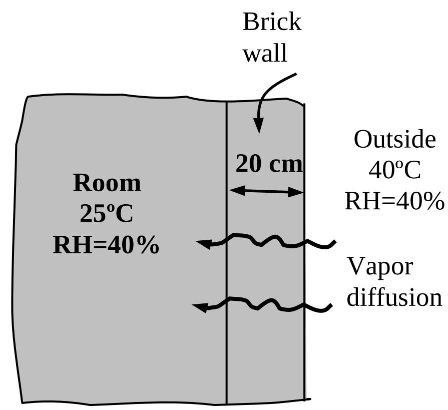
$$\Delta h = \frac{V}{A} = \frac{1.21\times10^{-9}\text{ m}^3}{0.0113\text{ m}^2} = 1.1\times10^{-7}\text{ m} = \mathbf{0.0011\,mm}$$

Discussion The drop in the level of the milk in 12 h is very small, and thus it is not noticeable.

14-78 The wall of a house is made of a 20-cm thick brick. The amount of moisture flowing through the wall in 24-h is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Mass transfer through the wall is one-dimensional. **3** The vapor permeability of the wall is constant.

Properties The permeance of 100 mm thick wall is 46×10^{-12} kg/s.m^2.Pa (Table 14-10). The saturation pressures of water are 3169 Pa at 25ºC, and 7384 Pa at 40ºC (Table 14-9).



Analysis The permeability of the wall is

$$P = ML = (46\times10^{-12}\ \text{kg/s}\cdot\text{m}^2\cdot\text{Pa})(0.10\,\text{m})$$
$$= 46\times10^{-13}\ \text{kg/s}\cdot\text{m}\cdot\text{Pa}$$

The mass flow rate of water vapor through a plain layer of thickness L and normal area A is given as (Eq. 14-31)

$$\dot{m}_v = \mathsf{P}A\frac{P_{v,1}-P_{v,2}}{L} = \mathsf{P}A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{L}$$

where P is the vapor permeability, ϕ is the relative humidity and P_{sat} is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the states of the air on the two sides of the roof. Substituting, the mass flow rate of water vapor through the wall is determined to be

$$\dot{m}_v = (46\times10^{-13}\,\text{kg/s.m.Pa})\frac{[0.40(7384\,\text{Pa})-0.40(3169\,\text{Pa})]}{0.20\,\text{m}} = 3.88\times10^{-8}\ \text{kg/s}\cdot\text{m}^2$$

Then the total amount of moisture that flows through the roof during a 24-h period becomes

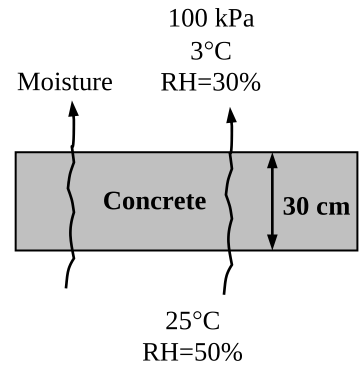
$$m_{v,24-h} = \dot{m}_v\Delta t = (3.88\times10^{-8}\ \text{kg/s}\cdot\text{m}^2)(24\times3600\,\text{s}) = 0.00335\ \text{kg} = \mathbf{3.35\,g}$$

Discussion The moisture migration through the wal can be reduced significantly by covering the roof with a vapor barrier or vapor retarder.

14-79 The roof of a house is made of a 30-cm thick concrete layer. The amount of water vapor that will diffuse through a 15 m × 8 m section of the roof in 24-h is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Mass transfer through the roof is one-dimensional. **3** The vapor permeability of the roof is constant.

Properties The permeability of the roof to water vapor is given to be 24.7×10^{-12} kg/s.m.Pa. The saturation pressures of water are 768 Pa at 3ºC, and 3169 Pa at 25ºC (Table 14-9).



Analysis The mass flow rate of water vapor through a plain layer of thickness L and normal area A is given as (Eq. 14-31)

$$\dot{m}_v = \mathsf{P}A\frac{P_{v,1}-P_{v,2}}{L} = \mathsf{P}A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{L}$$

where P is the vapor permeability, ϕ is the relative humidity and P_{sat} is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the states of the air on the two sides of the roof. Substituting, the mass flow rate of water vapor through the roof is determined to be

$$\dot{m}_v = (24.7\times10^{-12}\ \text{kg/s.m.Pa})(15\times 8\ \text{m}^2)\frac{[0.50(3169\ \text{Pa}) - 0.30(768\ \text{Pa})]}{(0.30\ \text{m})} = 1.34\times10^{-5}\ \text{kg/s}$$

Then the total amount of moisture that flows through the roof during a 24-h period becomes

$$m_{v,24-h} = \dot{m}_v\Delta t = (1.34\times10^{-5}\ \text{kg/s})(24\times 3600\ \text{s}) = \mathbf{1.16\ kg}$$

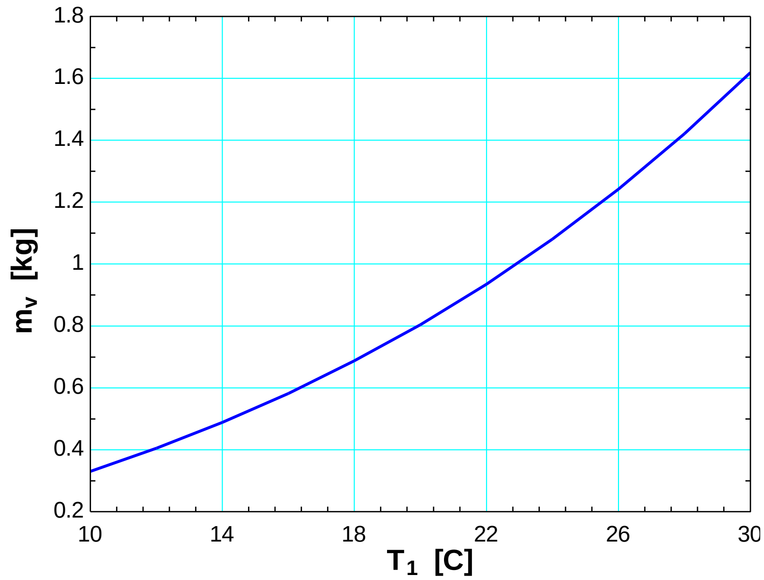
Discussion The moisture migration through the roof can be reduced significantly by covering the roof with a vapor barrier or vapor retarder.

14-80 Prob. 14-79 is reconsidered. The effects of temperature and relative humidity of air inside the house on the amount of water vapor that will migrate through the roof are to be investigated.

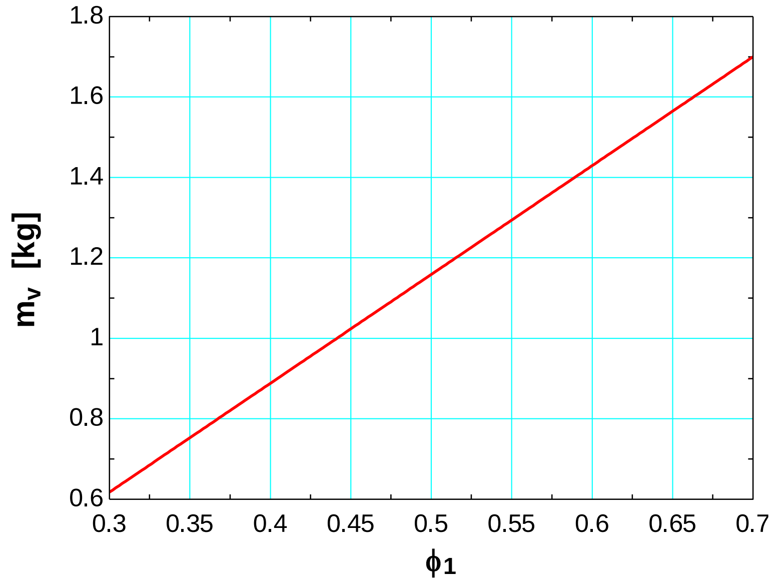
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
A=15*8 [m^2]
L=0.30 [m]
T_1=25 [C]
phi_1=0.50
P_atm=100 [kPa]
time=24*3600 [s]
T_2=3 [C]
phi_2=0.30
Permeability=24.7E-12 [kg/s-m-Pa]

"PROPERTIES"
Fluid$='steam_IAPWS'
P_sat1=Pressure(Fluid$, T=T_1, x=1)*Convert(kPa, Pa)
P_sat2=Pressure(Fluid$, T=T_2, x=1)*Convert(kPa, Pa)

"ANALYSIS"
m_dot_v=Permeability*A*(phi_1*P_sat1-phi_2*P_sat2)/L
m_v=m_dot_v*time
```

T_1 [C]	m_v [kg]
10	0.33
12	0.4046
14	0.4883
16	0.5821
18	0.6871
20	0.8043
22	0.9349
24	1.08
26	1.242
28	1.421
30	1.619



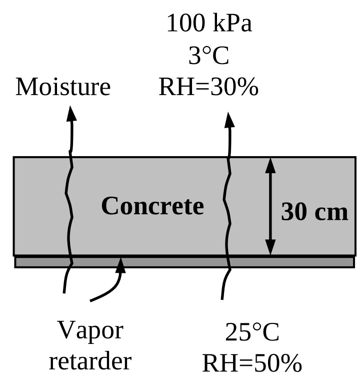
ϕ_1	m_v [kg]
0.3	0.6176
0.32	0.6717
0.34	0.7259
0.36	0.78
0.38	0.8341
0.4	0.8882
0.42	0.9423
0.44	0.9965
0.46	1.051
0.48	1.105
0.5	1.159
0.52	1.213
0.54	1.267
0.56	1.321
0.58	1.375
0.6	1.429
0.62	1.484
0.64	1.538
0.66	1.592
0.68	1.646
0.7	1.7



14-81 The roof of a house is made of a 30-cm thick concrete layer painted with a vapor retarder paint. The amount of water vapor that will diffuse through a 15 m × 8 m section of the roof in 24-h is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Mass transfer through the roof is one-dimensional. **3** The vapor permeabilities of the roof and of the vapor barrier are constant.

Properties The permeability of concrete to water vapor and the permeance of the vapor retarder to water vapor are given to be 24.7×10^{-12} kg/s.m.Pa and 26×10^{-12} kg/s.m^2.Pa, respectively. The saturation pressures of water are 768 Pa at 3ºC, and 3169 Pa at 25ºC (Table 14-9).



Analysis The mass flow rate of water vapor through a two-layer plain roof of normal area A is given as (Eqs. 14-33 and 14-35)

$$\dot{m}_v = A\frac{P_{v,1}-P_{v,2}}{R_{v,\text{total}}} = A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{R_{v,\text{total}}}$$

where $R_{\text{v,total}}$ is the total vapor resistance of the medium, ϕ is the relative humidity and P_{sat} is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the air on the two sides of the roof. The total vapor resistance of the roof is

$$R_{v,\text{total}} = R_{v,\text{roof}} + R_{v,\text{film}} = \frac{L}{P}+\frac{1}{M} = \frac{0.30\text{ m}}{24.7\times10^{-12}\text{ kg/s.m.Pa}} + \frac{1}{26\times10^{-12}\text{ kg/s.m}^2\text{.Pa}}$$

$$= 5.061\times10^{10}\text{ s.m}^2\text{.Pa/kg}$$

Substituting, the mass flow rate of water vapor through the roof is determined to be

$$\dot{m}_v = A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{R_{v,\text{total}}} = (15\times 8\text{ m}^2)\frac{0.50(3169\text{ Pa})-0.30(768\text{ Pa})}{5.061\times10^{10}\text{ s.m}^2\text{.Pa/kg}} = 3.211\times10^{-6}\text{ kg/s}$$

Then the total amount of moisture that flows through the roof during a 24-h period becomes

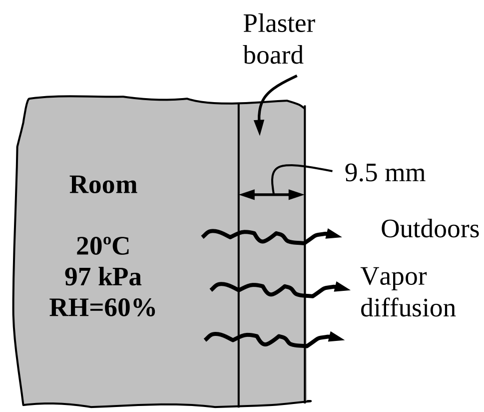
$$m_{v,24-h} = \dot{m}_v\Delta t = (3.211\times10^{-6}\text{ kg/s})(24\times3600\text{ s}) = \mathbf{0.277\ kg = 277\ g}$$

14-82 The inside wall of a house is finished with 9.5-mm thick gypsum wallboard. The maximum amount of water vapor that will diffuse through a 3 m × 8 m section of the wall in 24-h is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Mass transfer through the wall is one-dimensional. **3** The vapor permeability of the wall is constant. **4** The vapor pressure at the outer side of the wallboard is zero.

Properties The permeance of the 9.5 mm thick gypsum wall board to water vapor is given to be 2.86×10^{-9} kg/s.m^2.Pa. (Table 14-10). The saturation pressure of water at 20ºC is 2339 Pa (Table 14-9).

Analysis The mass flow rate of water vapor through a plain layer of thickness L and normal area A is given as (Eq. 14-31)



$$\dot{m}_v = \mathsf{P}A\frac{P_{v,1} - P_{v,2}}{L}$$

$$= \mathsf{P}A\frac{\phi_1 P_{\text{sat},1} - \phi_2 P_{\text{sat},2}}{L} = \mathsf{M}A(\phi_1 P_{\text{sat},1} - \phi_2 P_{\text{sat},2})$$

where P is the vapor permeability and M = P/L is the permeance of the material, ϕ is the relative humidity and P_{sat} is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the air on the two sides of the wall.

Noting that the vapor pressure at the outer side of the wallboard is zero ($\phi_2 = 0$) and substituting, the mass flow rate of water vapor through the wall is determined to be

$$\dot{m}_v = (2.86 \times 10^{-9}\ \text{kg/s.m}^{\prime}\text{.Pa})(3 \times 8\ \text{m}^2)[0.60(2339\ \text{Pa}) - 0] = 9.63 \times 10^{-5}\ \text{kg/s}$$

Then the total amount of moisture that flows through the wall during a 24-h period becomes

$$m_{v,24-h} = \dot{m}_v \Delta t = (9.63 \times 10^{-5}\ \text{kg/s})(24 \times 3600\ \text{s}) = \mathbf{8.32\ kg}$$

Discussion This is the maximum amount of moisture that can migrate through the wall since we assumed the vapor pressure on one side of the wall to be zero.

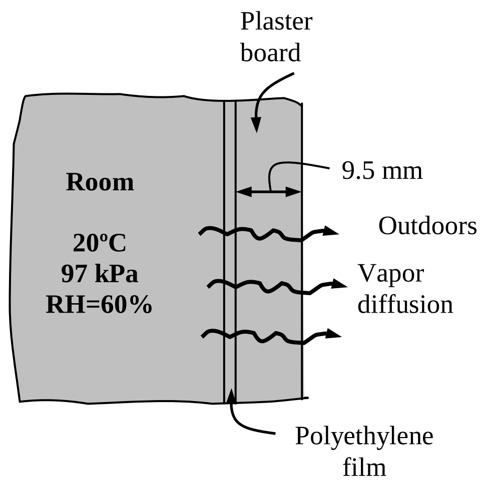
14-83 The inside wall of a house is finished with 9.5-mm thick gypsum wallboard with a 0.051-mm thick polyethylene film on one side. The maximum amount of water vapor that will diffuse through a 3 m × 8 m section of the wall in 24-h is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Mass transfer through the wall is one-dimensional. **3** The vapor permeabilities of the wall and of the vapor barrier are constant. **4** The vapor pressure at the outer side of the wallboard is zero.

Properties The permeances of the 9.5 mm thick gypsum wall board and of the 0.051-mm thick polyethylene film are given to be 2.86×10^{-9} and 9.1×10^{-12} kg/s.m^2.Pa, respectively (Table 14-10). The saturation pressure of water at 20ºC is 2339 Pa (Table 14-9).

Analysis The mass flow rate of water vapor through a two-layer plain wall of normal area A is given as (Eqs. 14-33 and 14-35)

$$\dot{m}_v = A\frac{P_{v,1}-P_{v,2}}{R_{v,\text{total}}} = A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{R_{v,\text{total}}}$$



where $R_{v,\text{total}}$ is the total vapor resistance of the medium, ϕ is the relative humidity and P_{sat} is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the air on the two sides of the wall. The total vapor resistance of the wallboard is

$$\begin{aligned}R_{v,\text{total}} &= R_{v,\text{wall}} + R_{v,\text{film}}\\ &= \frac{1}{2.86\times10^{-9}\ \text{kg/s.m}^2\text{.Pa}} + \frac{1}{9.1\times10^{-12}\ \text{kg/s.m}^2\text{.Pa}}\\ &= 1.10\times10^{11}\ \text{s.m}^2\text{.Pa/kg}\end{aligned}$$

Noting that the vapor pressure at the outer side of the wallboard is zero ($\phi_2 = 0$) and substituting, the mass flow rate of water vapor through the wall is determined to be

$$\dot{m}_v = A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{R_{v,\text{total}}} = (3\times8\ \text{m}^2)\frac{0.60(2339\ \text{Pa})-0}{1.10\times10^{11}\ \text{s.m}^2\text{.Pa/kg}} = 3.06\times10^{-7}\ \text{kg/s}$$

Then the total amount of moisture that flows through the wall during a 24-h period becomes

$$m_{v,24-h} = \dot{m}_v\Delta t = (3.06\times10^{-7}\ \text{kg/s})(24\times3600\ \text{s}) = \mathbf{0.0264\ kg = 26.4\ g}$$

Discussion This is the maximum amount of moisture that can migrate through the wall since we assumed the vapor pressure on one side of the wall to be zero. Note that the vapor barrier reduced the amount of vapor migration to a negligible level.

Transient Mass Diffusion

14-84C The diffusion of a solid species into another solid of finite thickness can usually be treated as a diffusion process in a semi-infinite medium regardless of the shape and thickness of the solid since the diffusion process affects a very thin layer at the surface.

14-85C When the density of a species *A* in a semi-infinite medium is known initially and at the surface, the concentration of the species *A* at a specified location and time can be determined from

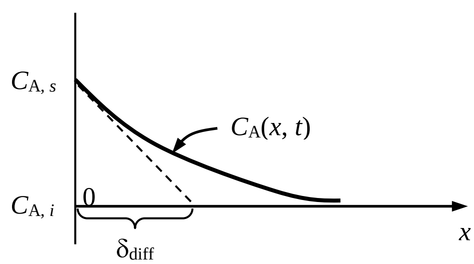
$$\frac{C_A(x,t)-C_{A,i}}{C_{A,s}-C_{A,i}}=\operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

where $C_{A,i}$ is the initial concentration of species *A* at time $t = 0$, $C_{A,s}$ is the concentration at the inner side of the exposed surface of the medium, and erfc is the complementary error function.

14-86C The penetration depth is defined as the location where the tangent to the concentration profile at the surface ($x = 0$) intercepts the $C_A = C_{A,i}$ line, and it represents the depth of diffusion at a given time. The penetration depth can be determined to be

$$\delta_{diff} = \sqrt{\pi D_{AB}t}$$

where D_{AB} is the diffusion coefficient and *t* is the time.



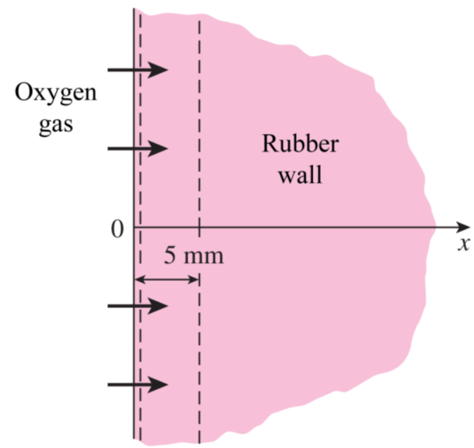
14-87 A thick natural rubber wall is exposed to pure oxygen gas on one side of the surface. The time required for the oxygen concentration to reach 5% at x = 5 mm of the concentration at the surface is to be determined.

Assumptions **1** The natural rubber wall can be modeled as a semi-infinite medium. **2** The O_2 concentration in the rubber is initially zero. **3** The concentration of O_2 at the rubber surface is constant.

Properties The diffusion coefficient of O_2 in natural rubber at 25°C = 298 K is $D_{AB} = 2.1 \times 10^{-10}$ m²/s (Table 14-3b).

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. The solution can be expressed as

$$\frac{C_A(x,t) - C_{A,i}}{C_{A,s} - C_{A,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}\, t}}\right)$$



Substituting the specified quantities gives

$$\frac{C_A(x,t)}{C_{A,s}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}\, t}}\right)$$

$$0.05 = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}\, t}}\right)$$

From Table 4-4, we have 0.05 = erfc(1.386), hence

$$\frac{x}{2\sqrt{D_{AB}\, t}} = 1.386$$

Thus,

$$t = \left[\frac{x}{(1.386)2}\right]^2 \frac{1}{D_{AB}}$$

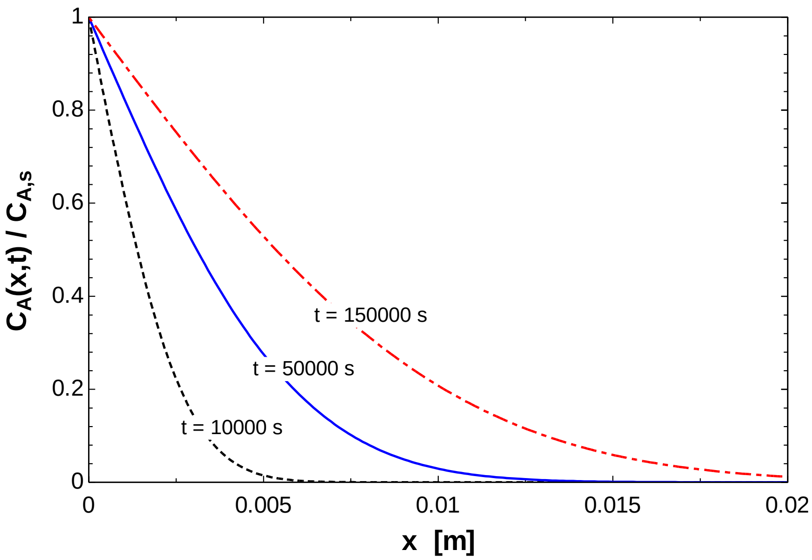
$$= \left[\frac{0.005\,\text{m}}{(1.386)2}\right]^2 \frac{1}{2.1\times10^{-10}\ \text{m}^2/\text{s}} = 15493\,\text{s} = \mathbf{4.30\ h}$$

Discussion It takes 4 hours and 18 minutes for the O_2 concentration at 5 mm from the surface to reach 5% of the O_2 concentration at the surface.

14-88 A thick natural rubber wall is exposed to pure oxygen gas on one side of the surface. The oxygen concentrations varying with the distance from the surface are to be evaluated at different time elapsed.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
c_i=0
c_s=1
D_AB=2.1e-10 [m^2/s]
time_1=10000 [s]
time_2=50000 [s]
time_3=150000 [s]

"ANALYSIS"
c_1=SemiInf1(c_i,c_s,D_AB,x,time_1)
c_2=SemiInf1(c_i,c_s,D_AB,x,time_2)
c_3=SemiInf1(c_i,c_s,D_AB,x,time_3)
```



		$C_A(x,t)/C_{A,s}$	
x [m]	t = 10000 s	50000 s	150000 s
0	1	1	1
0.001	0.6256	0.8273	0.8997
0.002	0.3291	0.6625	0.8011
0.003	0.1432	0.5127	0.7055
0.004	0.05096	0.3827	0.6143
0.005	0.01470	0.2752	0.5287
0.006	0.003415	0.1904	0.4497
0.007	0.0006363	0.1266	0.3778
0.008	9.477E-05	0.08086	0.3135
0.009	1.125E-05	0.04953	0.2568
0.010	1.064E-06	0.02910	0.2077
0.012	4.759E-09	0.008829	0.1306
0.014	8.415E-12	0.002250	0.07776
0.016	5.847E-15	0.0004803	0.04382
0.018	1.572E-18	8.568E-05	0.02334
0.020	0	1.275E-05	0.01174

Discussion As the elapsed time increases the curves are approaching a linear profile that resembles the steady state profile.

14-89 A piece of steel undergoing a decarburization process, the depth below the surface of the steel at which the concentration of carbon is reduced to 40% from its initial value as a result of the decarburization process for (*a*) an hour and (*b*) ten hours are to be determined.

Assumptions Carbon penetrates into a very thin layer beneath the surface of the component, and thus the component can be modeled as a semi-infinite medium regardless of its thickness or shape.

Properties The relevant properties are given in the problem statement.

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. The solution can be expressed as

$$\frac{C_A(x,t)-C_{A,i}}{C_{A,s}-C_{A,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$

Substituting the specified quantities gives

$$\frac{0.4C_{A,i}-C_{A,i}}{0-C_{A,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$

$$0.6 = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$

From Table 4-4, we have 0.6 = erfc(0.371), hence

$$\frac{x}{2\sqrt{D_{AB}\,t}} = 0.371$$

(*a*) The depth of the steel after t = 1 h = 3600 s is

$$\begin{aligned} x &= 2\sqrt{D_{AB}\,t}\,(0.371) \\ &= 2\sqrt{(1\times10^{-7}\ \text{cm}^2/\text{s})(3600\,\text{s})}\,(0.371) = \mathbf{0.0141\,cm} \end{aligned}$$

(*b*) The depth of the steel after t = 10 h = 36000 s is

$$\begin{aligned} x &= 2\sqrt{D_{AB}\,t}\,(0.371) \\ &= 2\sqrt{(1\times10^{-7}\ \text{cm}^2/\text{s})(36000\,\text{s})}\,(0.371) = \mathbf{0.0445\,cm} \end{aligned}$$

Discussion The value for the complimentary error function can be determined using the EES software:

0.6=erfc(eta) → eta=0.3708

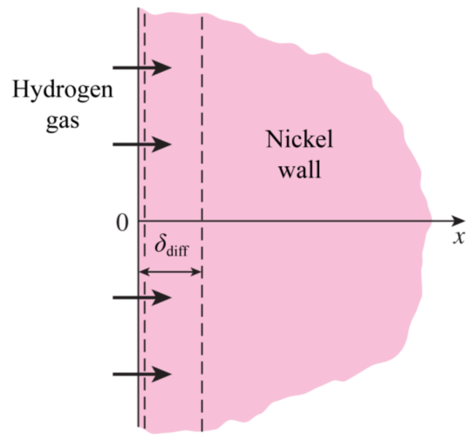
14-90 A thick nickel wall is exposed to pure hydrogen gas on one side of the surface. The hydrogen concentration at the penetration depth, in percentage of its concentration at the surface, is to be determined.

Assumptions **1** The nickel wall can be modeled as a semi-infinite medium. **2** The H_2 concentration in the nickel is initially zero. **3** The concentration of H_2 at the rubber surface is constant.

Properties The diffusion coefficient of H_2 in nickel at 165°C = 438 K is $D_{AB} = 1.0 \times 10^{-11}$ m²/s (Table 14-3b).

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. The solution can be expressed as

$$\frac{C_A(x,t)-C_{A,i}}{C_{A,s}-C_{A,i}} = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$



The penetration depth is given as

$$x = \delta_{\text{diff}} = \sqrt{\pi D_{AB}\,t}$$

Substituting gives

$$\frac{C_A(x,t)}{C_{A,s}} = \operatorname{erfc}\left(\frac{\sqrt{\pi D_{AB}\,t}}{2\sqrt{D_{AB}\,t}}\right) = \operatorname{erfc}\left(\frac{\sqrt{\pi}}{2}\right) = \operatorname{erfc}(0.8862)$$

From Table 4-4, we have erfc(0.8862) = 0.2101, hence

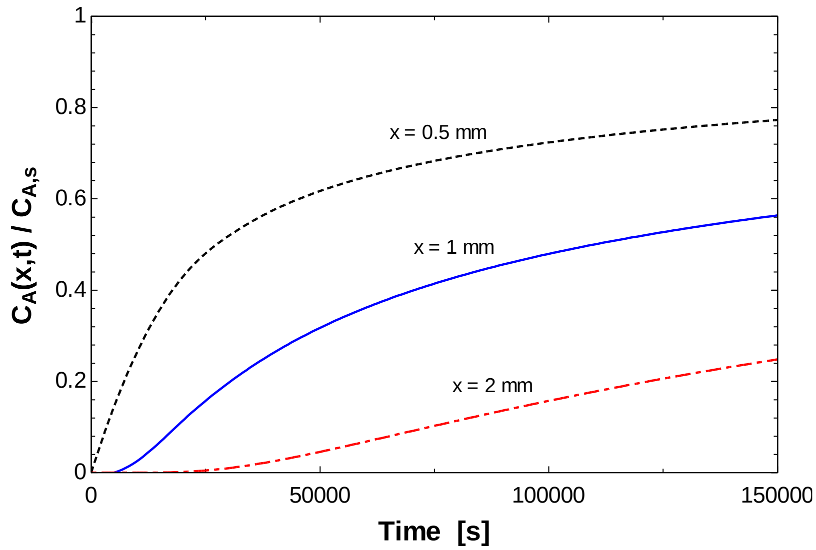
$$\frac{C_A(x,t)}{C_{A,s}} = 0.2101 = \mathbf{21\%}$$

Discussion The hydrogen concentration, $C_A(x,t)/C_{A,s}$, at the penetration depth is independent of x and t.

14-91 A thick nickel wall is exposed to pure hydrogen gas on one side of the surface. The hydrogen concentrations varying with time are to be evaluated at different distances from the surface.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
c_i=0
c_s=1
x_1=0.0005
x_2=0.001
x_3=0.002
D_AB=1.0e-11 [m^2/s]
k=D_AB
alpha=D_AB

"ANALYSIS"
c_1=SemiInf1(c_i,c_s,D_AB,x_1,time)
c_2=SemiInf1(c_i,c_s,D_AB,x_2,time)
c_3=SemiInf1(c_i,c_s,D_AB,x_3,time)
```



		$C_A(x,t)/C_{A,s}$	
t [s]	x = 0.5 mm	1 mm	2 mm
0	0	0	0
10000	0.2636	0.02535	7.744E-06
20000	0.4292	0.1138	0.001565
30000	0.5186	0.1967	0.009823
40000	0.5762	0.2636	0.02535
50000	0.6171	0.3173	0.04550
60000	0.6481	0.3613	0.06789
70000	0.6726	0.3980	0.09097
80000	0.6926	0.4292	0.1138
90000	0.7094	0.4561	0.1360
100000	0.7237	0.4795	0.1573
110000	0.7360	0.5002	0.1775
120000	0.7469	0.5186	0.1967
130000	0.7565	0.5351	0.2148
140000	0.7651	0.5501	0.2320
150000	0.7728	0.5637	0.2482

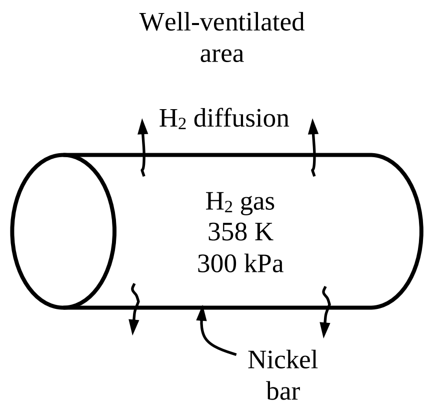
Discussion Hydrogen concentration increases with time as it diffuses through the nickel wall.

14-92 A long cylindrical nickel bar saturated with hydrogen is taken into an area that is free of hydrogen. The length of time for the hydrogen concentration at the center of the bar to drop by half is to be determined.

Assumptions **1** The bar can be treated as an infinitely long cylinder since it is very long and there is symmetry about the centerline. **2** The initial hydrogen concentration in the steel bar is uniform. **3** The hydrogen concentration at the surface remains constant at zero at all times. **4** The Fourier number is $\tau > 0.2$ so that the one-term transient solutions are valid.

Properties The molar mass of hydrogen H_2 is M = 2 kg / kmol (Table A-1). The solubility of hydrogen in nickel at 358 K is 0.00901 kmol / m^3.bar (Table 14-7). The diffusion coefficient of hydrogen in nickel at 298 K is $D_{AB} = 1.2\times10^{-12}$ m^2/s (Table 14-3b).

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in an infinitely long cylinder with specified surface temperature, and thus can be solved accordingly. Noting that 300 kPa = 3 bar, the molar density of hydrogen in the nickel bar before it is taken out of the storage room is



$$\begin{aligned} C_{H_2,\text{solid side}}(0) &= S \times P_{H_2,\text{gas side}} \\ &= (0.00901\,\text{kmol/m}^3.\text{bar})(3\,\text{bar}) \\ &= 0.027\,\text{kmol/m}^3 \end{aligned}$$

The molar concentration of hydrogen at the center of the bar can be calculated from

$$\frac{C_{H_2,o} - C_{H_2,\infty}}{C_{H_2,i} - C_{H_2,\infty}} = A_1 e^{-\lambda_1^2 \tau}$$

The Biot number in this case can be taken to be infinity since the bar is in a well-ventilated area during the transient case. The constants A_1 and λ_1 for the infinite Bi are determined from Table 4-2 to be 1.6021 and 2.4048, respectively. Noting that the concentration of hydrogen at the outer surface is zero, and the concentration of hydrogen at the center of the bar is one half of the initial concentration, the Fourier number, τ, can be determined from

$$\frac{(0.027/2) - 0}{0.027 - 0} = 1.6021 e^{-(2.4048)^2 \tau} \longrightarrow \tau = 0.2014$$

Using the definition of the Fourier number, the time required to drop the concentration of hydrogen by half is determined to be

$$\tau = \frac{D_{AB} t}{r_o^2} \longrightarrow t = \frac{\tau r_o^2}{D_{AB}} = \frac{(0.2014)(0.025)^2}{1.2\times10^{-12}} = 1.049\times10^8\ \text{s} = 1214\,\text{days} = \mathbf{3.33\,years}$$

Therefore, it will take years for this nickel bar to be free of hydrogen.

14-93 Chemical resistant gloves of 0.67 mm thickness are used for handling tetrachloroethylene solution. The time it will take for the solution to penetrate to the inner glove surface at 1% of the concentration at the outer surface is to be determined.

Assumptions **1** The glove wall can be modeled as a semi-infinite medium. **2** The solution concentration in the glove is initially zero. **3** The concentration of the solution at the outer glove surface is constant.

Properties The diffusion coefficient of tetrachloroethylene in the glove is given as $D_{AB} = 3.0 \times 10^{-8}$ m²/s.

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. The solution can be expressed as

$$\frac{C_A(x,t) - C_{A,i}}{C_{A,s} - C_{A,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$

Substituting the specified quantities gives

$$\frac{C_A(x,t)}{C_{A,s}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$

$$0.01 = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$

From Table 4-4, we have 0.01 = erfc(1.82), hence

$$\frac{x}{2\sqrt{D_{AB}\,t}} = 1.82$$

Thus,

$$t = \left[\frac{x}{(1.82)2}\right]^2 \frac{1}{D_{AB}}$$

$$= \left[\frac{0.00067\text{ m}}{(1.82)2}\right]^2 \frac{1}{3.0\times10^{-8}\text{ m}^2/\text{s}} = \mathbf{1.13\,s}$$

Discussion It only takes the tetrachloroethylene solution about 1 second for 1% of the concentration at the exposed outer glove surface to penetrate to the inner surface. Therefore, this type of glove is not suitable for handling tetrachloroethylene solution.

Thicker gloves with lower diffusion coefficient of tetrachloroethylene should be used for the safety of the employees.

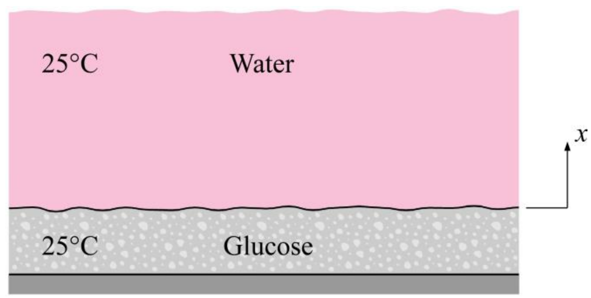
14-94 A layer of glucose is submerged under a deep layer of water. (*a*) The time required for the glucose concentration at x = 1 cm to reach 1% of its concentration at the glucose-water interface, and (*b*) the penetration depth are to be determined.

Assumptions **1** The water layer can be modeled as a semi-infinite medium. **2** The glucose concentration in the water is initially zero. **3** The concentration at the glucose-water interface is constant.

Properties The diffusion coefficient of glucose in water at 25°C = 298 K is $D_{AB} = 0.69 \times 10^{-9}$ m²/s (Table 14-3a).

Analysis (*a*) This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. The solution can be expressed as

$$\frac{C_A(x,t)-C_{A,i}}{C_{A,s}-C_{A,i}} = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$



Substituting the specified quantities gives

$$\frac{C_A(x,t)}{C_{A,s}} = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$

$$0.01 = \operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$

From Table 4-4, we have 0.01 = erfc(1.82), hence

$$\frac{x}{2\sqrt{D_{AB}\,t}} = 1.82$$

Thus,

$$\begin{aligned} t &= \left[\frac{x}{(1.82)2}\right]^2 \frac{1}{D_{AB}} = \left[\frac{0.01\,\text{m}}{(1.82)2}\right]^2 \frac{1}{0.69\times10^{-9}\ \text{m}^2/\text{s}} \\ &= 10940\,\text{s} \\ &= \mathbf{3.04\ h} \end{aligned}$$

(*b*) The penetration depth of glucose in the water is

$$\begin{aligned} \delta_{\text{diff}} &= \sqrt{\pi D_{AB}\,t} \\ &= \sqrt{\pi(0.69\times10^{-9}\ \text{m}^2/\text{s})(10940\,\text{s})} \\ &= 0.00487\,\text{m} \\ &= \mathbf{4.87\,mm} \end{aligned}$$

Discussion It took more than three hours for the glucose concentration in the water layer at 1 cm from the glucose-water interface to reach 1% of its concentration at the interface.

14-95 C&S A layer of benzene covers the surface of water in a reservoir. The time duration for the benzene concentration to reach 0.005 mg/L at the depth of x = 0.1 m, measured from the interface, is to be determined.

Assumptions **1** The water is modeled as a stationary semi-infinite medium. **2** Benzene concentration in the water is initially zero. **3** Benzene concentration at the interface is constant.

Properties The diffusion coefficient of benzene in water at 20°C (293 K) is $D_{AB} = 1.0 \times 10^{-9}$ m^2/s (Table 14-3a).

Analysis The maximum contaminant level for benzene in drinking water is $\rho_A(x, t)$ = 0.005 mg/L = 5×10^{-6} kg/m^3. For transient mass diffusion in a semi-infinite medium,

$$\frac{\rho_A(x,t)-\rho_{A,i}}{\rho_{A,s}-\rho_{A,i}} = \mathrm{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$

With $\rho_{A,\,s}$ = 5 kg/m^3 and $\rho_{A,\,i}$ = 0,

$$\frac{5\times10^{-6}}{5} = 1\times10^{-6} = \mathrm{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$

The argument whose complementary error function is 1×10^{-6} is determined from Table 4-4 to be 3.459. That is 1×10^{-6} = erfc(3.459), hence

$$\frac{x}{2\sqrt{D_{AB}\,t}} = 3.459$$

The time duration for the benzene concentration at the depth of x = 0.1 m to reach 0.005 mg/L is

$$t = \frac{1}{D_{AB}}\left[\frac{x}{(2)(3.459)}\right]^2 = \left(\frac{1}{1\times10^{-9}\mathrm{m^2/s}}\right)\left[\frac{0.1\ \mathrm{m}}{(2)(3.459)}\right]^2 = 2.0895\times10^5\mathrm{s} = \mathbf{58.04\ hours}$$

Discussion It takes a little over 58 hours for the benzene to reach the maximum contaminant level (0.005 mg/L) in the water at 10 cm from the interface. Note that the water is assumed as a stationary semi-infinite medium in this analysis. However, in reality water is generally non-stationary, and therefore the contaminant will diffuse much quicker in the water.

14-96 A layer of benzene covers the floor of a tiny room. The time duration for the benzene concentration to reach 5 ppm at the height of x = 1.5 m, measured from the interface, is to be determined.

Assumptions **1** The air is modeled as a stationary semi-infinite medium. **2** Benzene concentration in the air is initially zero. **3** Benzene concentration at the interface is constant. **4** Gas mixture behave as an ideal gas.

Properties The diffusion coefficient of benzene in air at 25°C (298 K) is $D_{AB} = 0.88 \times 10^{-5}$ m^2/s (Table 14-2). The molar mass of benzene is given as 78.11 kg/kmol. The molar mass of air is 28.97 kg/kmol (Table A-1).

Analysis For ideal gas mixture at a given total pressure, the mole fraction is equal the volume fraction,

$$y_i = \frac{N_i}{N} = \frac{\frac{PV_i}{R_u T}}{\frac{PV}{R_u T}} = \frac{V_i}{V}$$

The maximum short-term exposure limit for benzene is 5 ppm, so

$$y_{\text{benzene}} = \frac{V_{\text{benzene}}}{V} = 5 \times 10^{-6}$$

The mass fraction and mole fraction of a species in a mixture is related to each other as

$$w_i = \frac{\rho_i}{\rho} = y_i \frac{M_i}{M}$$

So, the mass fraction of benzene at 5 ppm is

$$w_{\text{benzene}} = y_{\text{benzene}} \frac{M_{\text{benzene}}}{M} = \frac{y_{\text{benzene}} M_{\text{benzene}}}{y_{\text{benzene}} M_{\text{benzene}} + (1 - y_{\text{benzene}}) M_{\text{air}}}$$

$$w_A(x,t) = w_{\text{benzene}} = \frac{(5 \times 10^{-6})(78.11 \text{ kg/kmol})}{(5 \times 10^{-6})(78.11 \text{ kg/kmol}) + (1 - 5 \times 10^{-6})(28.97 \text{ kg/kmol})} = 1.3481 \times 10^{-5}$$

For transient mass diffusion in a semi-infinite medium,

$$\frac{w_A(x,t) - w_{A,i}}{w_{A,s} - w_{A,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}\, t}}\right)$$

With $w_{A,\,s}$ = 0.02 and $w_{A,\,i}$ = 0,

$$\frac{1.3481 \times 10^{-5}}{0.02} = 6.7405 \times 10^{-4} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}\, t}}\right)$$

The argument whose complementary error function is 6.7405×10^{-4} is determined from Table 4-4 to be 2.404. That is 6.7405×10^{-4}= erfc(2.404), hence

$$\frac{x}{2\sqrt{D_{AB}\, t}} = 2.404$$

The time duration for the benzene concentration at the height of x = 1.5 m to reach 5 ppm is

$$t = \frac{1}{D_{AB}}\left[\frac{x}{(2)(2.404)}\right]^2 = \left(\frac{1}{0.88 \times 10^{-5} \text{m}^2/\text{s}}\right)\left[\frac{1.5 \text{ m}}{(2)(2.404)}\right]^2 = 11060 \text{ s} = \mathbf{3.07 \text{ hours}}$$

Discussion It takes a little over 3 hours for the benzene to reach the maximum short-term exposure limit of 5 ppm at 1.5 m from the interface. If the spill is quickly cleaned up, then level of benzene reaching the maximum short-term exposure limit of OSHA will not occur.

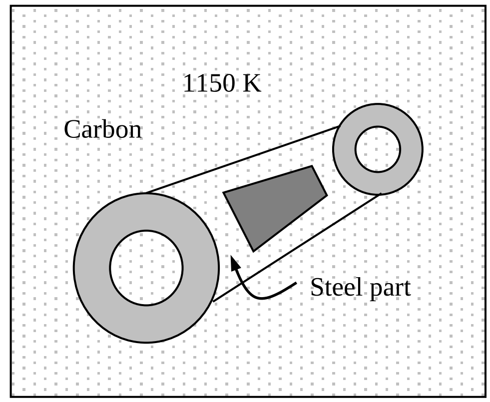
14-97 A steel component is to be surface hardened by packing it in a carbonaceous material in a furnace at 1150 K. The length of time the component should be kept in the furnace is to be determined.

Assumptions **1** Carbon penetrates into a very thin layer beneath the surface of the component, and thus the component can be modeled as a semi-infinite medium regardless of its thickness or shape. **2** The initial carbon concentration in the steel component is uniform. **3** The carbon concentration at the surface remains constant.

Properties The relevant properties are given in the problem statement.

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature, and thus can be solved accordingly. Using mass fraction for concentration since the data is given in that form, the solution can be expressed as

$$\frac{w_A(x,t)-w_{A,i}}{w_{A,s}-w_{A,i}}=\operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$



Substituting the specified quantities gives

$$\frac{0.0032-0.0010}{0.011-0.0010}=0.22=\operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

The argument whose complementary error function is 0.22 is determined from Table 4-4 to be 0.8674. That is,

$$\frac{x}{2\sqrt{D_{AB}t}}=0.8674$$

Then solving for the time t gives

$$t=\frac{x^2}{4D_{AB}(0.8674)^2}=\frac{(0.0006\,\text{m})^2}{4\times(7.2\times10^{-12}\,\text{m}^2/\text{s})(0.8674)^2}=16{,}615\text{ s}=\mathbf{4.62\ h}$$

Therefore, the steel component must be held in the furnace for 4.62 h to achieve the desired level of hardening.

Discussion The diffusion coefficient of carbon in steel increases exponentially with temperature, and thus this process is commonly done at high temperatures to keep the diffusion time to a reasonable level.

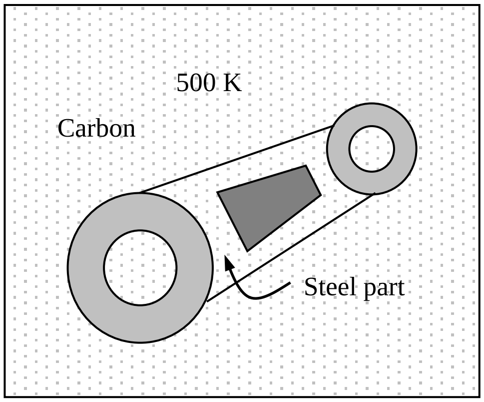
14-98 A steel component is to be surface hardened by packing it in a carbonaceous material in a furnace at 500 K. The length of time the component should be kept in the furnace is to be determined.

Assumptions **1** Carbon penetrates into a very thin layer beneath the surface of the component, and thus the component can be modeled as a semi-infinite medium regardless of its thickness or shape. **2** The initial carbon concentration in the steel component is uniform. **3** The carbon concentration at the surface remains constant.

Properties The relevant properties are given in the problem statement.

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature, and thus can be solved accordingly. Using mass fraction for concentration since the data is given in that form, the solution can be expressed as

$$\frac{w_A(x,t)-w_{A,i}}{w_{A,s}-w_{A,i}}=\mathrm{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$



Substituting the specified quantities gives

$$\frac{0.0032-0.0010}{0.011-0.0010}=0.22=\mathrm{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

The argument whose complementary error function is 0.22 is determined from Table 4-4 to be 0.8674. That is,

$$\frac{x}{2\sqrt{D_{AB}t}}=0.8674$$

Solving for the time t gives

$$t=\frac{x^2}{4D_{AB}(0.8674)^2}=\frac{(0.0006\,\mathrm{m})^2}{4\times(2.1\times10^{-20}\,\mathrm{m^2/s})(0.8674)^2}=\mathbf{5.696\times10^{12}\,s=180{,}620\ \ years}$$

Therefore, the steel component must be held in the furnace forever to achieve the desired level of hardening.

Discussion The diffusion coefficient of carbon in steel increases exponentially with temperature, and thus this process is commonly done at high temperatures to keep the diffusion time to a reasonable level.

14-99 A pond is to be oxygenated by forming a tent over the water surface and filling the tent with oxygen gas. The molar concentration of oxygen at a depth of 0.8 cm from the surface after 24 h is to be determined.

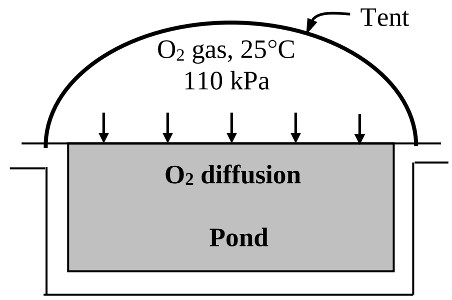
Assumptions **1** The oxygen in the tent is saturated with water vapor. **2** Oxygen penetrates into a thin layer at the pond surface, and thus the pond can be modeled as a semi-infinite medium. **3** Both the water vapor and oxygen are ideal gases. **4** The initial oxygen content of the pond is zero.

Properties The diffusion coefficient of oxygen in water at 25ºC is D_{AB} = 2.4 ×10^{-9} m^2/s (Table 14-3a). Henry's constant for oxygen dissolved in water at 300 K (≅ 25ºC) is given in Table 14-6 to be H = 43,600 bar. The saturation pressure of water at 25ºC is 3.17 kPa (Table 14-9).

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature, and thus can be solved accordingly. The vapor pressure in the tent is the saturation pressure of water at 25ºC since the oxygen in the tent is saturated, and thus the partial pressure of oxygen in the tank is

$$P_{O_2} = P - P_v = 110 - 3.17 = 106.83\,\text{kPa}$$

Then the mole fraction of oxygen in the water at the pond surface becomes



$$y_{O_2,\text{liquid side}}(0) = \frac{P_{O_2,\text{gas side}}(0)}{H} = \frac{1.0683\,\text{bar}}{43{,}600\,\text{bar}} = 2.45\times10^{-5}$$

The molar concentration of oxygen at a depth of 0.8 cm from the surface after 12 h can be determined from

$$\frac{y_{O_2}(x,t) - y_{O_2,i}}{y_{O_2,s} - y_{O_2,i}} = \text{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

Substituting,

$$\frac{y_{O_2}(x,t) - 0}{2.45\times10^{-5} - 0} = \text{erfc}\left(\frac{0.008\text{ m}}{2\sqrt{(2.4\times10^{-9}\text{ m}^2/\text{s})(24\times3600\text{s})}}\right) = 0.6944 \longrightarrow y_{O_2}(0.008\text{ m}, 24\text{ h}) = \mathbf{1.70\times10^{-5}}$$

Therefore, there will be 17 moles of oxygen per million at a depth of 0.8 cm from the surface in 24 h.

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14-100 A piece of steel was exposed to a carburizing atmosphere for an hour, and the percentage of mass concentration of carbon at 0.2 mm and 0.4 mm below the surface are to be determined.

Assumptions Carbon penetrates into a very thin layer beneath the surface of the component, and thus the component can be modeled as a semi-infinite medium regardless of its thickness or shape.

Properties The relevant properties are given in the problem statement.

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. Using mass fraction for concentration since the data are given in that form, the solution can be expressed as

$$\frac{w_A(x,t)-w_{A,i}}{w_{A,s}-w_{A,i}}=\operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$

Substituting the specified quantities (for x = 0.0002 m and t = 1 h = 3600 s) gives

$$\frac{w_A(x,t)-0.002}{0.007-0.002}=\operatorname{erfc}\left(\frac{0.0002\text{ m}}{2\sqrt{(1\times10^{-11}\text{ m}^2/\text{s})(3600\text{ s})}}\right)$$

$$\frac{w_A(x,t)-0.002}{0.007-0.002}=\operatorname{erfc}(0.527)$$

Thus, mass concentration of carbon at x = 0.2 mm and t = 1 h is

$$w_A(0.2\text{ mm},1\text{ h})=(0.007-0.002)(0.4561)+0.002=0.00428=\mathbf{0.428\%}$$

Similarly, substituting the specified quantities (for x = 0.0004 m and t = 1 h = 3600 s) gives

$$\frac{w_A(x,t)-0.002}{0.007-0.002}=\operatorname{erfc}(1.054)$$

Thus, mass concentration of carbon at x = 0.4 mm and t = 1 h is

$$w_A(0.4\text{ mm},1\text{ h})=(0.007-0.002)(0.136)+0.002=0.00268=\mathbf{0.268\%}$$

Discussion The values for the complimentary error function can be determined from Table 4-4 or using the EES software:

z=erfc(0.527) → z=0.4561 and z=erfc(1.054) → z=0.136

14-101 During an aeration process, the pond surface has its oxygen density suddenly increased to 9 kg/m^3. The oxygen density at 5 cm below the pond surface after 100 hours is to be determined.

Assumptions **1** The pond can be modeled as a semi-infinite medium. **2** Both air and water are stationary.

Properties The diffusion coefficient of oxygen in water at 25°C is $D_{AB} = 2.4\times10^{-9}$ m^2/s (Table 14-3a).

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. The solution can be expressed as

$$\frac{\rho_A(x,t)-\rho_{A,i}}{\rho_{A,s}-\rho_{A,i}} = \mathrm{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$

Substituting the specified quantities (x = 5 cm and t = 100 h = 360000 s) gives

$$\frac{\rho_A(x,t)-2\ \mathrm{kg/m^3}}{(9\ \mathrm{kg/m^3}-2\ \mathrm{kg/m^3})} = \mathrm{erfc}\left(\frac{0.05\ \mathrm{m}}{2\sqrt{(2.4\times10^{-9}\ \mathrm{m^2/s})(360000\ \mathrm{s})}}\right)$$

$$\frac{\rho_A(x,t)-2\ \mathrm{kg/m^3}}{(9\ \mathrm{kg/m^3}-2\ \mathrm{kg/m^3})} = \mathrm{erfc}(0.8505)$$

From Table 4-4, we have erfc(0.8505) = 0.229, hence

$$\rho_A(5\ \mathrm{cm}, 100\ \mathrm{h}) = (9\ \mathrm{kg/m^3}-2\ \mathrm{kg/m^3})(0.229)+2\ \mathrm{kg/m^3}$$
$$= \mathbf{3.60\ kg/m^3}$$

Discussion After 100 h, the oxygen density at 5 cm below the pond surface is increased by 80%. This shows that mass diffusion through a stationary layer is very slow.

Diffusion in a Moving Medium

14-102C The **diffusion velocity** at a location is the average velocity of a group of molecules at that location moving under the influence of concentration gradient. The average velocity of a species in a moving medium is equal to the sum of the bulk flow velocity and the diffusion velocity. Therefore, the diffusion velocity can increase of decrease the average velocity, depending on the direction of diffusion relative to the direction of bulk flow. The velocity of a species in the moving medium relative to a fixed reference point **will be zero** when the diffusion velocity of the species and the bulk flow velocity are equal in magnitude and opposite in direction.

14-103C The **mass-average velocity** of a medium at some location is the average velocity of the mass at that location relative to an external reference point. The **molar-average velocity** of a medium at some location is the average velocity of the molecules at that location, regardless of their mass, relative to an external reference point. If one of these velocities are zero, the other will not necessarily be zero. The mass-average and molar-average velocities of a binary mixture will be the same when the molar masses of the two constituents are equal to each other. The mass and mole fractions of each species in this case will be the same.

14-104C The **mass-average velocity** of a medium at some location is the average velocity of the mass at that location relative to an external reference point. It is the velocity that would be measured by a velocity sensor such as a pitot tube, a turbine device, or a hot wire anemometer inserted into the flow. The **diffusion velocity** at a location is the average velocity of a group of molecules at that location moving under the influence of concentration gradient. A **stationary medium** is a medium whose mass average velocity is zero. A **moving medium** is a medium that involves a bulk fluid motion caused by an external force.

14-105C (*a*) T, (*b*) T, (*c*) F, (*d*) F

14-106C The diffusion of a vapor through a stationary gas column is called the **Stefan flow**. The **Stefan's law** can be expressed as

$$\bar{j}_A = \dot{N}_A / A = \frac{CD_{AB}}{L} \ln \frac{1 - y_{A,L}}{1 - y_{A,o}}$$

where C is the average concentration of the mixture, D_{AB} is the diffusion coefficient of A in B, L is the height of the gas column, $y_{A,L}$ is the molar concentration of a species at $x = L$, and $y_{A,o}$ is the molar concentration of the species A at $x = 0$.

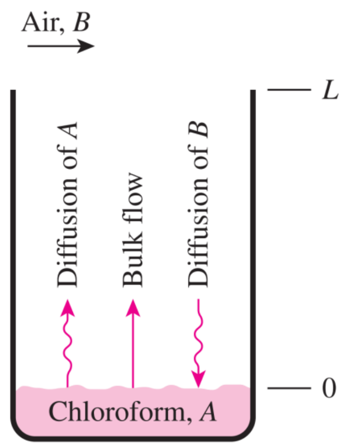
14-107 The amount of chloroform that diffuses from a Stefan tube at a specified temperature and pressure over a specified time period is measured. The mass diffusivity of chloroform in air is to be determined.

Assumptions **1** Chloroform vapor and atmospheric air are ideal gases. **2** The amount of air dissolved in liquid chloroform is negligible. **3** Temperatures of air and chloroform remain constant at 25°C.

Properties The relevant properties are given in the problem statement.

Analysis The vapor pressure at the air-chloroform interface is the vapor pressure of chloroform at 25°C, $P_{A,0}$ = 0.263 atm, and the mole fraction of chloroform vapor (species A) at the interface is determined from

$$y_{A,0} = \frac{P_{A,0}}{P} = \frac{0.263\,\text{atm}}{1\,\text{atm}} = 0.263$$



The total molar density throughout the tube remains constant because of the constant temperature and pressure conditions and is determined to be

$$C = \frac{P}{RT} = \frac{101.325\,\text{kPa}}{(8.314\,\text{kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(273+25)\,\text{K}} = 0.0409\,\text{kmol/m}^3$$

The diffusion rate is given to be 222 g per 10 hours. Then the diffusion rate per unit interface area is

$$\frac{\dot{N}_A}{A} = \frac{m}{MAt} = \frac{4m}{M\pi D^2 t}$$
$$= \frac{4(222\times10^{-3}\text{ kg})}{(119.39\text{ kg/kmol})\pi(0.05\text{ m})^2(10\times3600\text{ s})}$$
$$= 2.63\times10^{-5}\text{ kmol/s}\cdot\text{m}^2$$

The distance between the chloroform surface and the top of the tube is initially 7 cm, and will be 7.44 cm after 10 hours. Therefore, we can take the average height to be (7 + 7.44)/2 = 7.22 cm.

Finally, the mass diffusivity of chloroform in air is determined using

$$\frac{\dot{N}_A}{A} = \frac{CD_{AB}}{L}\ln\left(\frac{1-y_{A,L}}{1-y_{A,0}}\right)$$

$$2.63\times10^{-5}\text{ kmol/s}\cdot\text{m}^2 = \frac{(0.0409\text{ kmol/m}^3)D_{AB}}{0.0722\text{ m}}\ln\left(\frac{1-0}{1-0.263}\right)$$

which gives

$$D_{AB} = \mathbf{1.52\times10^{-4}\ m^2/s}$$

Discussion The Stefan tube is sometimes also known as Arnold diffusion cell.

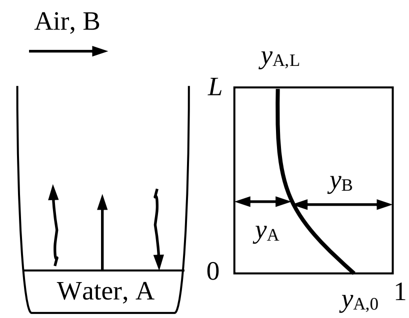
14-108E The amount of water that evaporates from a Stefan tube at a specified temperature and pressure over a specified time period is measured. The diffusion coefficient of water vapor in air is to be determined.

Assumptions **1** Water vapor and atmospheric air are ideal gases. **2** The amount of air dissolved in liquid water is negligible. **3** Heat is transferred to the water from the surroundings to make up for the latent heat of vaporization so that the temperature of water remains constant at 80°F.

Properties The saturation pressure of water at 80°F is 0.5073 psia (Table A-9E).

Analysis The vapor pressure at the air-water interface is the saturation pressure of water at 80°F, and the mole fraction of water vapor (species A) is determined from

$$y_{\text{vapor,o}} = y_{A,o} = \frac{P_{\text{vapor,o}}}{P} = \frac{0.5073\,\text{psia}}{13.8\,\text{psia}} = 0.0368$$



Dry air is blown on top of the tube and thus $y_{\text{vapor},L} = y_{A,L}=0$. Also, the total molar density throughout the tube remains constant because of the constant temperature and pressure conditions, and is determined to be

$$C = \frac{P}{R_u T} = \frac{13.8\ \text{psia}}{(10.73\text{psia.ft}^3/\text{lbmol}\cdot\text{R})(540\text{R})} = 0.00238\ \text{lbmol/ft}^3$$

The cross-sectional area of the valve is

$$A = \pi D^2/4 = \pi(1/12\,\text{ft})^2/4 = 5.45\times10^{-3}\ \text{ft}^2$$

The evaporation rate is given to be 0.0025 lbm per 10 days. Then the molar flow rate of vapor is determined to be

$$\dot{N}_A = \dot{N}_{\text{vapor}} = \frac{m_{\text{vapor}}}{M_{\text{vapor}}} = \frac{0.0025\,\text{lbm}}{(10\times24\times3600\text{s})(18\,\text{lbm/lbmol})} = 1.61\times10^{-10}\ \text{lbm/s}$$

Finally, substituting the information above into Eq. 14-59 we get

$$\frac{\dot{N}_A}{A} = \frac{CD_{AB}}{L}\ln\left(\frac{1-y_{A,L}}{1-y_{A,o}}\right) \longrightarrow \frac{1.61\times10^{-10}\ \text{lbm/s}}{5.45\times10^{-3}\ \text{ft}^2} = \frac{(0.00238\ \text{lbm/ft}^3)D_{\text{AB}}}{10/12\ \text{ft}}\ln\left(\frac{1-0}{1-0.0368}\right)$$

It gives

D_{AB} = $\mathbf{2.76\times10^{-4}\ ft^2/s}$

for the binary diffusion coefficient of water vapor in air at 80°F and 13.8 psia.

14-109 Benzene undergoes evaporation in 10 test tubes. The total evaporation rate of benzene to the surrounding air is to be determined whether or not the level of benzene could result in health risks.

Assumptions **1** Benzene vapor and atmospheric air are ideal gases. **2** The amount of air dissolved in liquid benzene is negligible. **3** Temperatures of air and benzene remain constant at 25°C.

Properties The molar mass of benzene is given as M = 78.11 kg/kmol. The diffusion coefficient of benzene in air at 25°C is $D_{AB} = 0.88 \times 10^{-5}$ m²/s (Table 14-2).

Analysis The vapor pressure at the air-benzene interface is the vapor pressure of benzene at 25°C, $P_{A,0}$ = 10 kPa, and the mole fraction of benzene vapor (species A) at the interface is determined from

$$y_{A,0} = \frac{P_{A,0}}{P} = \frac{10\text{ kPa}}{101.325\text{ kPa}} = 0.09869$$

The total molar density throughout the tube remains constant because of the constant temperature and pressure conditions and is determined to be

$$C = \frac{P}{R_u T} = \frac{101.325\text{ kPa}}{(8.314\text{ kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(273+25)\text{ K}} = 0.0409\text{ kmol/m}^3$$

Then, the diffusion rate per unit interface area for one test tube is

$$\begin{aligned}
\frac{\dot{N}_A}{A} &= \frac{CD_{AB}}{L}\ln\left(\frac{1-y_{A,L}}{1-y_{A,0}}\right) \\
&= \frac{(0.0409\text{ kmol/m}^3)(0.88\times10^{-5}\text{ m}^2/\text{s})}{0.01\text{ m}}\ln\left(\frac{1-0}{1-0.09869}\right) \\
&= 3.7398\times10^{-6}\text{ kmol/s}\cdot\text{m}^2
\end{aligned}$$

The evaporation rate of benzene from one test tube is

$$\dot{N}_A = \frac{\dot{m}}{M} \quad \rightarrow \quad \dot{m} = M\pi\frac{D^2}{4}\frac{\dot{N}_A}{A}$$

$$\begin{aligned}
\dot{m} &= (78.11\text{ kg/kmol})\frac{\pi}{4}(0.025\text{ m})^2(3.7398\times10^{-6}\text{ kmol/s}\cdot\text{m}^2) \\
&= (1.434\times10^{-7}\text{ kg/s})(1000\text{ g/kg})(3600\text{ s/hr}) \\
&= 1.434\times10^{-7}\text{ kg/s} = 0.516\text{ g/h}
\end{aligned}$$

Thus, the total rate of benzene being evaporated from ten test tubes to the surrounding air is

$$\dot{m}_{\text{total}} = 10(0.516\text{ g/h}) = \mathbf{5.16\ g/h} > 3\text{ g/h}$$

Discussion The rate of benzene evaporation to the surrounding air is greater than the rate that the HVAC system can handle. That means benzene vapor will be accumulating and eventually reach a level that is hazardous to health.

To prevent health hazards from benzene exposure, the liquid benzene should be handled under a fume hood.

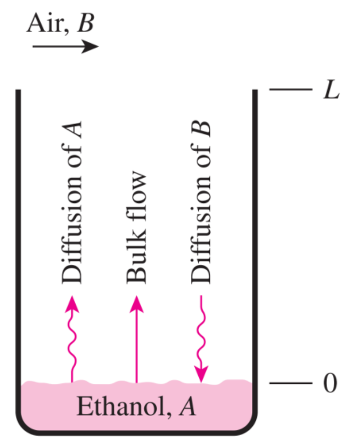
14-110 The amount of ethanol that evaporates from a Stefan tube at a specified temperature and pressure over a specified time period is measured. The mass diffusivity of ethanol in air is to be determined.

Assumptions **1** Ethanol vapor and atmospheric air are ideal gases. **2** The amount of air dissolved in liquid ethanol is negligible. **3** Temperatures of air and ethanol remain constant at 24°C.

Properties The relevant properties are given in the problem statement.

Analysis The vapor pressure at the air-ethanol interface is the vapor pressure of ethanol at 24°C, $P_{A,0}$ = 0.0684 atm, and the mole fraction of ethanol vapor (species A) at the interface is determined from

$$y_{A,0} = \frac{P_{A,0}}{P} = \frac{0.0684\text{ atm}}{1\text{ atm}} = 0.0684$$



The total molar density throughout the tube remains constant because of the constant temperature and pressure conditions and is determined to be

$$C = \frac{P}{RT} = \frac{101.325\text{ kPa}}{(8.314\text{ kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(273+24)\text{ K}} = 0.04103\text{ kmol/m}^3$$

Then the diffusion rate per unit interface area is

$$\frac{\dot{N}_A}{A} = \frac{m}{MAt} = \frac{\rho \boldsymbol{V}}{MAt}$$
$$= \frac{(789\text{ kg/m}^3)(0.0445\text{ cm}^3)(10^{-6}\text{ m}^3/\text{cm}^3)}{(46\text{ kg/kmol})(0.8\text{ cm}^2)(10^{-4}\text{ m}^2/\text{cm}^2)(10\times 3600\text{ s})}$$
$$= 2.65\times 10^{-7}\text{ kmol/s}\cdot\text{m}^2$$

The distance between the chloroform surface and the top of the tube is initially 10 cm, and will be 25 cm after 10 hours. Therefore, we can take the average height to be (10 + 25)/2 = 17.5 cm.

Finally, the mass diffusivity of ethanol in air is determined using

$$\frac{\dot{N}_A}{A} = \frac{CD_{AB}}{L}\ln\left(\frac{1-y_{A,L}}{1-y_{A,0}}\right)$$

$$2.65\times 10^{-7}\text{ kmol/s}\cdot\text{m}^2 = \frac{(0.04103\text{ kmol/m}^3)D_{AB}}{0.175\text{ m}}\ln\left(\frac{1-0}{1-0.0684}\right)$$

which gives

$$D_{AB} = \mathbf{1.60\times 10^{-5}\ m^2/s}$$

Discussion The Stefan tube is sometimes also known as Arnold diffusion cell.

14-111 Methanol undergoes evaporation in a vertical tube. (*a*) The evaporation rate of methanol is to be determined, and (*b*) the mole fraction of methanol vapor as a function of the tube height is to be plotted.

Assumptions **1** Methanol vapor and atmospheric air are ideal gases. **2** The amount of air dissolved in liquid methanol is negligible. **3** Temperatures of air and methanol remain constant at 25°C.

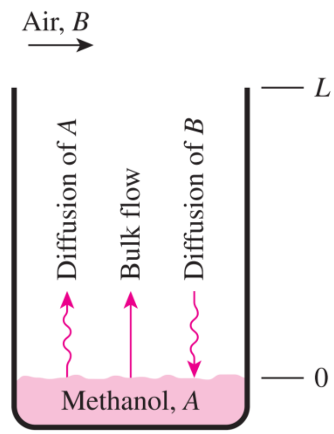
Properties The molar mass of methanol is given as M = 32 kg/kmol. The diffusion coefficient of methanol in air is given as $D_{AB} = 1.62 \times 10^{-5}$ m^2/s.

Analysis (*a*) The vapor pressure at the air-methanol interface is the vapor pressure of methanol at 25°C, $P_{A,0}$ = 17 kPa, and the mole fraction of methanol vapor (species A) at the interface is determined from

$$y_{A,0} = \frac{P_{A,0}}{P} = \frac{17\text{ kPa}}{101.325\text{ kPa}} = 0.1678$$

The total molar density throughout the tube remains constant because of the constant temperature and pressure conditions and is determined to be

$$C = \frac{P}{R_u T} = \frac{101.325\text{ kPa}}{(8.314\text{ kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(273+25)\text{ K}} = 0.0409\text{ kmol/m}^3$$



Then the diffusion rate per unit interface area is

$$\frac{\dot{N}_A}{A} = \frac{CD_{AB}}{L}\ln\left(\frac{1-y_{A,L}}{1-y_{A,0}}\right)$$

$$= \frac{(0.0409\text{ kmol/m}^3)(1.62\times10^{-5}\text{ m}^2/\text{s})}{0.30\text{ m}}\ln\left(\frac{1-0}{1-0.1678}\right)$$

$$= 4.0568\times10^{-7}\text{ kmol/s}\cdot\text{m}^2$$

The evaporation rate of the methanol in kg/h can be determined with

$$\dot{N}_A = \frac{\dot{m}}{M} \quad\rightarrow\quad \dot{m} = M\frac{\dot{N}_A}{A}A$$

$$\dot{m} = (32\text{ kg/kmol})(4.0568\times10^{-7}\text{ kmol/s}\cdot\text{m}^2)(0.8\text{ cm}^2)(10^{-4}\text{ m}^2/\text{cm}^2)$$

$$= 1.039\times10^{-9}\text{ kg/s}$$

$$= \mathbf{3.74\times10^{-6}\text{ kg/h}}$$

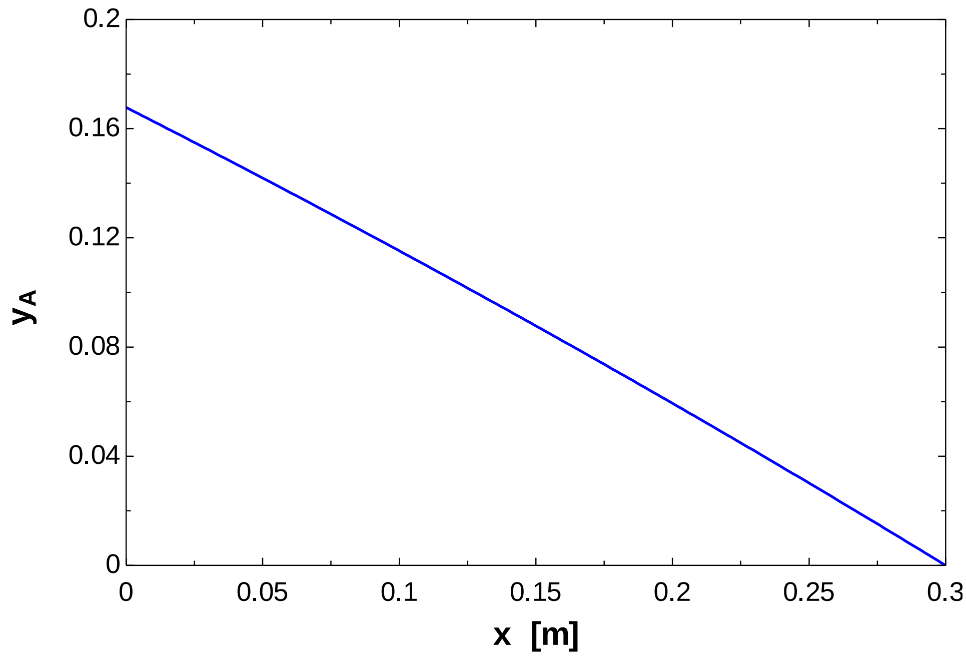
Discussion With ρ = 791 kg/m^3, that means the evaporation rate in terms of volume is 4.73×10^{-9} m^3/h.

(*b*) This part is solved using EES, and the solution is given below:

```
"GIVEN"
P_A_0=17 [kPa]
P=101.325 [kPa]
y_A_L=0
L=0.3 [m]

"ANALYSIS"
y_A_0=P_A_0/P
(1-y_A)/(1-y_A_0)=((1-0)/(1-y_A_0))^(x/L)
```

x [mm]	y_A
0	0.1678
0.01	0.1627
0.02	0.1575
0.03	0.1524
0.04	0.1471
0.05	0.1419
0.10	0.1152
0.15	0.08774
0.20	0.05938
0.25	0.03015
0.30	0



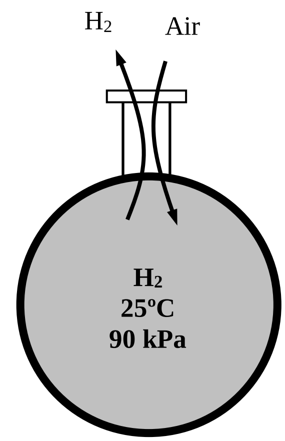
14-112 A hydrogen tank is maintained at atmospheric temperature and pressure by venting to the atmosphere through the charging valve. The initial mass flow rate of hydrogen out of the tank is to be determined.

Assumptions **1** Steady operating conditions at initial conditions exist. **2** Hydrogen and atmospheric air are ideal gases. **3** No chemical reactions occur in the valve. **4** Air concentration in the tank and hydrogen concentration in the atmosphere are negligible so that the mole fraction of the hydrogen is 1 in the tank, and 0 in the atmosphere (we will check this assumption later).

Properties The molar mass of hydrogen is M = 2 kg/kmol (Table A-1). The diffusion coefficient of hydrogen in air (or air in hydrogen) at 1 atm and 25ºC is $D_{AB} = 7.2 \times 10^{-5}$ m³/s (Table 14-2). However, the pressure in the tank is 90 kPa = 0.88 atm. The diffusion coefficient at 25ºC and 0.88 atm is determined from

$$D_{AB} = \frac{D_{AB,1\,\text{atm}}}{P\ (\text{in atm})} = \frac{7.2\times10^{-5}}{0.88} = 8.18\times10^{-5}\ \text{m}^2/\text{s}$$

Analysis This is a typical equimolar counterdiffusion process since the problem involves two large reservoirs of ideal gas mixtures connected to each other by a channel, and the concentrations of species in each reservoir (the pipeline and the atmosphere) remain constant. The cross-sectional area of the valve is



$$A = \pi D^2/4 = \pi(0.03\text{m})^2/4 = 7.069\times10^{-4}\ \text{m}^2$$

Noting that the pressure of hydrogen is 90 kPa at the bottom of the charging valve (x = 0) and 0 kPa at the top (x = L), its molar flow rate is determined from Eq. 14-64 to be

$$\dot{N}_{H_2} = \dot{N}_{diff,A} = \frac{D_{AB}A}{R_u T}\frac{P_{A,0}-P_{A,L}}{L}$$
$$= \frac{(8.18\times10^{-5}\ \text{m}^2/\text{s})(7.069\times10^{-4}\ \text{m}^2)}{(8.314\,\text{kPa.m³/kmol.K})(298\ \text{K})}\frac{(90-0)\text{kPa}}{0.1\,\text{m}}$$
$$= 2.098\times10^{-8}\ \text{kmol/s}$$

Then the mass flow rate of hydrogen becomes

$$\dot{m}_{H_2} = (\dot{N}M)_{H_2} = (2.081\times10^{-8}\ \text{kmol/s})(2\,\text{kg/kmol}) = \mathbf{4.2\times10^{-8}\ kg/s}$$

Discussion This is the highest mass flow rate. It will decrease during the process as air diffuses into the tank and the concentration of hydrogen in tank drops.

14-113 Prob. 14-112 is reconsidered. The mass flow rate of hydrogen lost as a function of the diameter of the charging valve is to be plotted.

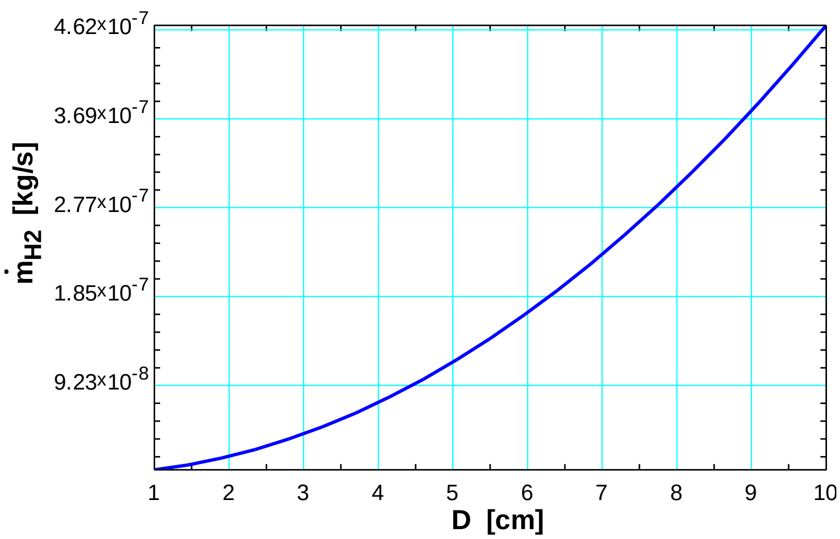
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
thickness=0.02 [m]
T=(25+273) [K]
P_atm=90 [kPa]
D=3 [cm]
extension=0.08 [m]
L=0.10 [m]

"PROPERTIES"
MM_H2=Molarmass(H2)
D_AB_1atm=7.2E-5 [m^2/s] "from Table 14-2 of the text at 1 atm and 25 C"
D_AB=D_AB_1atm*P_1atm/(P_atm*Convert(kPa, atm)) "at 90 kPa and 25 C"
P_1atm=1 [atm]
R_u=8.314 [kPa-m^3/kmol-K]

"ANALYSIS"
A=pi*D^2/4*Convert(cm^2, m^2)
N_dot_H2=(D_AB*A)/(R_u*T)*(P_atm-0)/L
m_dot_H2=N_dot_H2*MM_H2
```

D [cm]	$\dot{m}_{H2}$ [kg/s]
1	4.662E-09
1.45	9.803E-09
1.9	1.683E-08
2.35	2.575E-08
2.8	3.655E-08
3.25	4.925E-08
3.7	6.383E-08
4.15	8.030E-08
4.6	9.865E-08
5.05	1.189E-07
5.5	1.410E-07
5.95	1.651E-07
6.4	1.910E-07
6.85	2.188E-07
7.3	2.485E-07
7.75	2.800E-07
8.2	3.135E-07
8.65	3.488E-07
9.1	3.861E-07
9.55	4.252E-07
10	4.662E-07

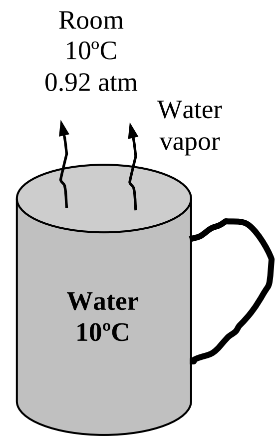


14-114 A pitcher that is half filled with water is left in a room with its top open. The time it takes for the entire water in the pitcher to evaporate is to be determined.

Assumptions **1** Water vapor and atmospheric air are ideal gases. **2** The amount of air dissolved in liquid water is negligible. **3** Heat is transferred to the water from the surroundings to make up for the latent heat of vaporization so that the temperature of water remains constant at 10°C.

Properties The saturation pressure of water at 10°C is 1.2276 kPa (Table A-9). The density of water in the pitcher can be taken to be 1000 kg/m³. The diffusion coefficient of water vapor in air at 10ºC (= 283 K) and 0.92 atm can be determined from

$$D_{AB} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(283\text{ K})^{2.072}}{0.92} = 2.444\times10^{-5}\text{ m}^2/\text{s}$$



Analysis The flow area, which is the cross-sectional area of the pitcher, is

$$A = \pi D^2/4 = \pi(0.08\text{m})^2/4 = 5.026\times10^{-3}\text{ m}^2$$

The vapor pressure at the air-water interface is the saturation pressure of water at 10ºC, which is 1.2276 kPa. The air at the top of the pitcher ($x = L$) can be assumed to be dry ($P_{A,L} = 0$). The distance between the water surface and the top of the pitcher is initially 15 cm, and will be 30 cm at the end of the process when all the water is evaporated. Therefore, we can take the average height of the air column above the water surface to be (15+30)/2 = 22.5 cm. Then the molar flow rate is determined from

$$\dot{N}_A = \frac{D_{AB}A}{R_u T}\left(\frac{P_{A,o} - P_{A,L}}{L}\right)$$
$$= \frac{(2.444\times10^{-5}\text{ m}^2/\text{s})(5.026\times10^{-3}\text{ m}^2)}{(8.314\text{kPa.m}^3/\text{kmol.K})(283\text{ K})}\frac{(1.2276-0)\text{ kPa}}{0.225\text{m}}$$
$$= 2.848\times10^{-10}\text{ kmol/s}$$

The initial mass of water in the pitcher is

$$m_{water} = \rho\frac{\pi D^2}{4}L = (1000\text{kg/m}^3)\frac{\pi(0.08\text{m})^2}{4}(0.15\text{m}) = 0.754\text{kg}$$

Then the time required to evaporate the water completely becomes

$$\dot{N}_{\text{vapor}} = \frac{m_{\text{vapor}}}{\Delta t\times M_{\text{vapor}}}$$

$$\Delta t = \frac{m_{\text{vapor}}}{\dot{N}_{\text{vapor}}\times M_{\text{vapor}}} = \frac{0.754\text{kg}}{(2.848\times10^{-10}\text{ kmol/s})(18\text{ kg/kmol})} = \mathbf{1.47\times10^8\text{ s}}$$

which is equivalent to **1702 days**. Therefore, it will take the water in the pitcher about 4.7 years to evaporate completely.

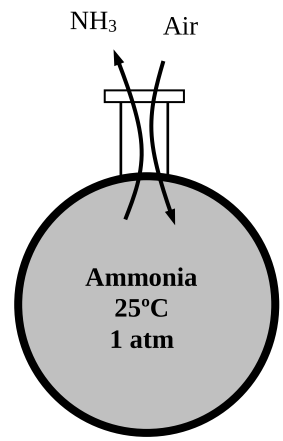
14-115 A large ammonia tank is vented to the atmosphere. The rate of loss of ammonia and the rate of air infiltration into the tank are to be determined.

Assumptions **1** Ammonia vapor and atmospheric air are ideal gases. **2** The amount of air dissolved in liquid ammonia is negligible. **3** Heat is transferred to the ammonia from the surroundings to make up for the latent heat of vaporization so that the temperature of ammonia remains constant at 25°C.

Properties The molar mass of ammonia is M = 17 kg/kmol, and the molar mass of air is M = 29 kg/kmol (Table A-1). The diffusion coefficient of ammonia in air (or air in ammonia) at 1 atm and 25ºC is D_{AB} =2.6 ×10^{-5} m^2/s (Table 14-2).

Analysis This is a typical equimolar counterdiffusion process since the problem involves two large reservoirs of ideal gas mixtures connected to each other by a channel, and the concentrations of species in each reservoir (the tank and the atmosphere) remain constant. The flow area, which is the cross-sectional area of the tube, is

$$A = \pi D^2/4 = \pi(0.015\,\text{m})^2/4 = 1.767\times10^{-4}\ \text{m}^2$$



Noting that the pressure of ammonia is 1 atm = 101.3 kPa at the bottom of the tube (x = 0) and 0 at the top ($x = L$), its molar flow rate is determined from Eq. 14-64 to be

$$\dot{N}_{\text{ammonia}} = \dot{N}_{\text{diff, A}} = \frac{D_{AB}A}{R_uT}\frac{P_{A,o}-P_{A,L}}{L}$$

$$= \frac{(2.6\times10^{-5}\ \text{m}^2/\text{s})(1.767\times10^{-4}\ \text{m}^2)}{(8.314\ \text{kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(298\text{K})}\frac{(101.3-0)\,\text{kPa}}{2\ \text{m}}$$

$$= \mathbf{9.39\times10^{-11}\ kmol/s}$$

Therefore, the mass flow rate of ammonia through the tube is

$$\dot{m}_{\text{NH}_3} = (\dot{N}M)_{\text{NH}_3} = (9.39\times10^{-11}\ \text{kmol/s})(17\,\text{kg/kmol}) = \mathbf{1.60\times10^{-9}\ kg/s}$$

which corresponds to 0.0504 kg per year.

Note that $\dot{N}_B = -\dot{N}_A$ during an equimolar counter diffusion process. Therefore, the molar flow rate of air into the ammonia tank is equal to the molar flow rate of ammonia out of the tank. Then the mass flow rate of air into the pipeline becomes

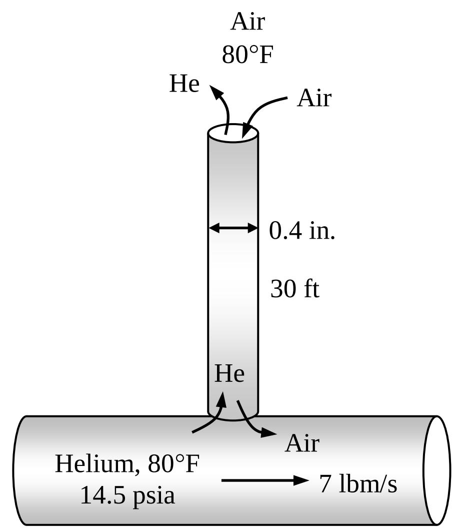
$$\dot{m}_{\text{air}} = (\dot{N}M)_{\text{air}} = (-9.39\times10^{-11}\ \text{kmol/s})(29\,\text{kg/kmol}) = \mathbf{-2.72\times10^{-9}\ kg/s}$$

14-116EE The pressure in a helium pipeline is maintained constant by venting to the atmosphere through a long tube. The mass flow rates of helium and air, and the net flow velocity at the bottom of the tube are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Helium and atmospheric air are ideal gases. **3** No chemical reactions occur in the tube. **4** Air concentration in the pipeline and helium concentration in the atmosphere are negligible so that the mole fraction of the helium is 1 in the pipeline, and 0 in the atmosphere (we will check this assumption later).

Properties The diffusion coefficient of helium in air (or air in helium) at normal atmospheric conditions is $D_{AB} = 7.75 \times 10^{-4}$ ft^2/s (Table 14-2). The molar mass of helium is M = 4 lbm / lbmol, and the molar mass of air is 29 lbm / lbmol (Table A-1E).

Analysis This is a typical equimolar counterdiffusion process since the problem involves two large reservoirs of ideal gas mixtures connected to each other by a channel, and the concentrations of species in each reservoir (the pipeline and the atmosphere) remain constant.



(*a*) The flow area, which is the cross-sectional area of the tube, is

$$A = \pi D^2/4 = \pi(0.4/12\,\text{ft})^2/4 = 8.727 \times 10^{-4}\ \text{ft}^2$$

Noting that the pressure of helium is 14.5 psia at the bottom of the tube (x = 0) and 0 at the top ($x = L$), its molar flow rate is

$$\dot{N}_{\text{helium}} = \dot{N}_{\text{diff, A}} = \frac{D_{AB}A}{R_u T}\frac{P_{A,0} - P_{A,L}}{L}$$

$$= \frac{(7.75 \times 10^{-4}\ \text{ft}^2/\text{s})(8.727 \times 10^{-4}\ \text{ft}^2)}{(10.73\,\text{psia.ft}^3/\text{lbmol.R})(540\ \text{R})}\frac{(14.5 - 0)\ \text{psia}}{30\ \text{ft}}$$

$$= 5.642 \times 10^{-11}\ \text{lbmol/s}$$

Therefore, the mass flow rate of helium through the tube is

$$\dot{m}_{\text{helium}} = (\dot{N}M)_{\text{helium}} = (5.642 \times 10^{-11}\ \text{lbmol/s})(4\ \text{lbm/lbmol}) = \mathbf{2.26 \times 10^{-10}\ lbm/s}$$

which corresponds to 0.00712 lbm per year.

(*b*) Noting that $\dot{N}_B = -\dot{N}_A$ during an equimolar counterdiffusion process, the molar flow rate of air into the helium pipeline is equal to the molar flow rate of helium. Thus the mass flow rate of air into the pipeline is

$$\dot{m}_{\text{air}} = (\dot{N}M)_{\text{air}} = (-5.642 \times 10^{-11}\ \text{lbmol/s})(29\ \text{lbm/lbmol}) = \mathbf{-1.64 \times 10^{-9}\ lbm/s}$$

The mass fraction of air in helium pipeline is

$$w_{\text{air}} = \frac{|\dot{m}_{\text{air}}|}{\dot{m}_{\text{total}}} = \frac{1.64 \times 10^{-9}\ \text{lbm/s}}{(7 - 2.26 \times 10^{-10} + 1.64 \times 10^{-9})\ \text{lbm/s}} = 2.34 \times 10^{-10} \approx 0$$

which validates our original assumption of negligible air in the pipeline.

(*c*) The net mass flow rate through the tube is

$$\dot{m}_{net} = \dot{m}_{helium} + \dot{m}_{air} = 2.26 \times 10^{-10} - 1.64 \times 10^{-9} = -1.41 \times 10^{-9}\ \text{lbm/s}$$

The mass fraction of air at the bottom of the tube is very small, as shown above, and thus the density of the mixture at x = 0 can simply be taken to be the density of helium which is

$$\rho \cong \rho_{\text{helium}} = \frac{P}{RT} = \frac{14.5\ \text{psia}}{(2.681\,\text{psia.ft}^3/\text{lbm.R})(540\ \text{R})} = 0.01002\ \text{lbm/ft}^3$$

Then the average flow velocity at the bottom part of the tube becomes

$$V = \frac{\dot{m}_{net}}{\rho A} = \frac{-1.41 \times 10^{-9}\ \text{lbm/s}}{(0.01002\ \text{lbm/ft}^3)(8.727 \times 10^{-4}\ \text{ft}^2)} = \mathbf{-1.62 \times 10^{-4}\ ft/s}$$

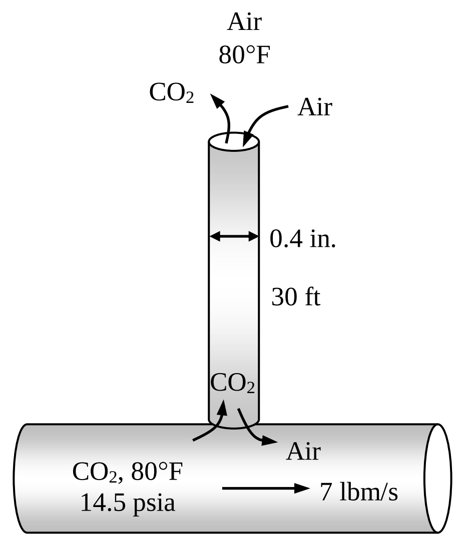
Discussion This flow rate is difficult to measure by even the most sensitive velocity measurement devices. The negative sign indicates flow in the negative x direction (towards the pipeline).

14-117E The pressure in a carbon dioxide pipeline is maintained constant by venting to the atmosphere through a long tube. The mass flow rates of carbon dioxide and air, and the net flow velocity at the bottom of the tube are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Carbon dioxide and atmospheric air are ideal gases. **3** No chemical reactions occur in the tube. **4** Air concentration in the pipeline and carbon dioxide concentration in the atmosphere are negligible so that the mole fraction of the carbon dioxide is 1 in the pipeline, and 0 in the atmosphere (we will check this assumption later).

Properties The diffusion coefficient of carbon dioxide in air (or air in carbon dioxide) at normal atmospheric conditions is $D_{AB} = 1.72\times10^{-4}$ ft²/s (Table 14-2). The molar mass of carbon dioxide is $M = 44$ lbm / lbmol, and the molar mass of air is 29 lbm / lbmol (Table A-1E).

Analysis This is a typical equimolar counterdiffusion process since the problem involves two large reservoirs of ideal gas mixtures connected to each other by a channel, and the concentrations of species in each reservoir (the pipeline and the atmosphere) remain constant.



(*a*) The flow area, which is the cross-sectional area of the tube, is

$$A = \pi D^2/4 = \pi(0.4/12\,\text{ft})^2/4 = 8.727\times10^{-4}\ \text{ft}^2$$

Noting that the pressure of carbon dioxide is 14.5 psia at the bottom of the tube ($x = 0$) and 0 at the top ($x = L$), its molar flow rate is determined from Eq. 14-64 to be

$$\dot N_{CO_2} = \dot N_{\text{diff, A}} = \frac{D_{AB}A}{R_uT}\frac{P_{A,0}-P_{A,L}}{L}$$
$$= \frac{(1.72\times10^{-4}\ \text{ft}^2/\text{s})(8.727\times10^{-4}\ \text{ft}^2)}{(10.73\,\text{psia.ft}^3/\text{lbmol.R})(540\ \text{R})}\frac{(14.5-0)\ \text{psia}}{30\ \text{ft}}$$
$$= 1.252\times10^{-11}\ \text{lbmol/s}$$

Therefore, the mass flow rate of carbon dioxide through the tube is

$$\dot m_{CO_2} = (\dot NM)_{CO_2} = (1.252\times10^{-11}\ \text{lbmol/s})(44\ \text{lbm/lbmol}) = \mathbf{5.51\times10^{-10}\ lbm/s}$$

which corresponds to 0.0174 lbm per year.

(*b*) Noting that $\dot N_B = -\dot N_A$ during an equimolar counter diffusion process, the molar flow rate of air into the CO_2 pipeline is equal to the molar flow rate of CO_2. Thus the mass flow rate of air into the pipeline is

$$\dot m_{\text{air}} = (\dot NM)_{\text{air}} = (-1.252\times10^{-11}\ \text{lbmol/s})(29\ \text{lbm/lbmol}) = \mathbf{-3.63\times10^{-10}\ lbm/s}$$

The mass fraction of air in carbon dioxide pipeline is

$$w_{\text{air}} = \frac{|\dot m_{\text{air}}|}{\dot m_{\text{total}}} = \frac{3.63\times10^{-10}\ \text{lbm/s}}{(7+3.63\times10^{-10}-5.51\times10^{-10})\ \text{lbm/s}} = 5.19\times10^{-11} \approx 0$$

which validates our original assumption of negligible air in the pipeline.

(*c*) The net mass flow rate through the tube is

$$\dot m_{net} = \dot m_{CO_2} + \dot m_{air} = 5.51\times10^{-10} - 3.63\times10^{-10} = 1.88\times10^{-10}\ \text{lbm/s}$$

The mass fraction of air at the bottom of the tube is very small, as shown above, and thus the density of the mixture at $x = 0$ can simply be taken to be the density of carbon dioxide which is

$$\rho \cong \rho_{CO_2} = \frac{P}{RT} = \frac{14.5\ \text{psia}}{(0.2438\,\text{psia.ft}^3/\text{lbm.R})(540\ \text{R})} = 0.110\ \text{lbm/ft}^3$$

Then the average flow velocity at the bottom part of the tube becomes

$$V = \frac{\dot m_{\text{net}}}{\rho A} = \frac{1.88\times10^{-10}\ \text{lbm/s}}{(0.110\ \text{lbm/ft}^3)(8.727\times10^{-4}\ \text{ft}^2)} = \mathbf{1.96\times10^{-6}\ ft/s}$$

Discussion This flow rate is difficult to measure by even the most sensitive velocity measurement devices.

Mass Convection

14-118C The region of the fluid near the surface in which concentration gradients exist is called the **concentration boundary layer**. In **flow over a plate**, the thickness of the concentration boundary layer δ_c for a species *A* at a specified location on the surface is defined as the normal distance *y* from the surface at which

$$\frac{\rho_{A,s}-\rho_A}{\rho_{A,s}-\rho_\infty}=0.99$$

where $\rho_{A,s}$ and $\rho_{A,\infty}$ are the densities of species *A* at the surface (on the fluid side) and the free stream, respectively.

14-119C The dimensionless **Schmidt number** is defined as the ratio of momentum diffusivity to mass diffusivity $\text{Sc}=\nu/D_{AB}$, and it represents the relative magnitudes of momentum and mass diffusion at molecular level in the velocity and concentration boundary layers, respectively. The Schmidt number corresponds to the *Prandtl number* in heat transfer. A Schmidt number of *unity* indicates that momentum and mass transfer by diffusion are comparable, and velocity and concentration boundary layers almost coincide with each other.

14-120C The dimensionless **Lewis number** is defined as the ratio of thermal diffusivity to mass diffusivity $(\text{Le}=\alpha/D_{AB})$, and it represents the relative magnitudes of heat and mass diffusion at molecular level in the thermal and concentration boundary layers, respectively. A Lewis number of unity indicates that heat and mass diffuse at the same rate, and the thermal and concentration boundary layers coincide.

14-121C The normalized velocity, thermal, and concentration boundary layers coincide during flow over a plate when the molecular diffusivity of momentum, heat, and mass are identical. That is, $\nu=\alpha=D_{AB}$ or Pr = Sc = Le = 1.

14-122C Mass convection is expressed on a mass basis in an analogous manner to heat transfer as

$$\dot{m}_{\text{conv}}=h_{\text{mass}}A_s(\rho_{A,s}-\rho_{A,\infty})$$

where h_{mass} is the average mass transfer coefficient in m/s, A_s is the surface area in m^2, and $\rho_{A,s}$ and $\rho_{A,\infty}$ are the densities of species *A* at the surface (on the fluid side) and the free stream, respectively.

14-123C The dimensionless **Sherwood number** is defined as $\text{Sh}=h_{\text{mass}}L/D_{AB}$ where *L* is the characteristic length, h_{mass} is the mass transfer coefficient and D_{AB} is the mass diffusivity. The Sherwood number represents the effectiveness of mass convection at the surface, and serves as the dimensionless mass transfer coefficient. The Sherwood number corresponds to the *Nusselt number* in heat transfer. A Sherwood number of unity for a plain fluid layer indicates mass transfer by pure diffusion in a fluid.

14-124C Yes, the Grasshof number evaluated using density difference instead of temperature difference can also be used in natural convection heat transfer calculations. In natural convection heat transfer, the term $\Delta\rho/\rho$ is replaced by $\beta\Delta T$ for convenience in calculations.

14-125C The relation *f* Re / 2= Nu = Sh is known as the **Reynolds analogy**. It is valid under the conditions that the Prandtl, Schmidt, and Lewis numbers are equal to units. That is, $\nu=\alpha=D_{AB}$ or Pr = Sc = Le = 1. Reynolds analogy enables us to determine the seemingly unrelated friction, heat transfer, and mass transfer coefficients when only one of them is known or measured.

14-126C The relation $f / 2 = \text{St Pr}^{2/3} = \text{St}_{mass}\text{Sc}^{2/3}$ is known as the **Chilton-Colburn analogy**. Here St is the Stanton number, Pr is the Prandtl number, St_{mass} is the Stanton number in mass transfer, and Sc is the Schmidt number. The relation is valid for $0.6 < \text{Pr} < 60$ and $0.6 < \text{Sc} < 3000$. Its importance in engineering is that Chilton-Colburn analogy enables us to determine the seemingly unrelated friction, heat transfer, and mass transfer coefficients when only one of them is known or measured.

14-127C Using the analogy between heat and mass transfer, the mass transfer coefficient can be determined from the relations for heat transfer coefficient using the **Chilton-Colburn Analogy** expressed as

$$\frac{h_{\text{heat}}}{h_{\text{mass}}} = \rho c_p \left(\frac{\text{Sc}}{\text{Pr}}\right)^{2/3} = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{2/3} = \rho c_p \text{Le}^{2/3}$$

Once the heat transfer coefficient h_{heat} is available, the mass transfer coefficient h_{heat} can be obtained from the relation above by substituting the values of the properties.

14-128C The relation $h_{heat} = \rho\, c_p\, h_{mass}$ is the result of the Lewis number Le = 1, and is known as the **Lewis relation**. It is valid for air-water vapor mixtures in the temperature range encountered in heating and air-conditioning applications. The Lewis relation is commonly used in air-conditioning practice. It asserts that the wet-bulb and adiabatic saturation temperatures of moist air are nearly identical. The Lewis relation can be used for heat and mass transfer in turbulent flow even when the Lewis number is not unity.

14-129C A convection mass transfer is referred to as the **low mass flux** when the flow rate of species undergoing mass flow is low relative to the total flow rate of the liquid or gas mixture so that the mass transfer between the fluid and the surface does not affect the *flow velocity*. The evaporation of water into air from lakes, rivers, etc. can be treated as a low mass-flux process since the mass fraction of water vapor in the air in such cases is just a few percent.

14-130 Carbon dioxide and air are separated by a flat rubber plate. The mass concentration gradient of carbon dioxide at the plate surface on the air side is to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of CO_2 in the air is low. **2** There are no chemical reactions in the plate that results in the generation or depletion of CO_2. **3** Mass diffusion of CO_2 through the plate is being convected to the air on the other side of the plate.

Properties The molar mass for CO_2 is 44.01 kg/kmol (Table A-1). The binary diffusion coefficient for CO_2 in air at 298 K is $D_{AB} = 1.6 \times 10^{-5}$ m^2/s (Table 14-2).

Analysis With the CO_2 diffuses through the rubber plate being convected by air on the other side, we can write the mass transfer at the air-side plate surface as

$$j_{A,\text{diff}} = j_{A,\text{conv}} = j_{A,s}$$

$$j_{A,s} = -D_{AB} \left.\frac{d\rho_A}{dy}\right|_s = h_{\text{mass}}(\rho_{A,s} - \rho_{A,\infty})$$

Thus, mass concentration gradient at the air-side plate surface is

$$\left.\frac{d\rho_A}{dy}\right|_s = -\frac{j_{A,s}}{D_{AB}} = -\frac{\bar{j}_{A,s} M}{D_{AB}}$$

$$= -\frac{4.43 \times 10^{-11}\ \text{kmol/s} \cdot \text{m}^2}{1.6 \times 10^{-5}\ \text{m}^2/\text{s}}(44.01\,\text{kg/kmol})$$

$$= \mathbf{-1.219 \times 10^{-4}\ kg/m^4}$$

Discussion The negative sign of the concentration gradient implies that mass transfers from higher concentration to lower concentration.

14-131 A film of water on a concrete is undergoing mass convection to air. The mass fraction gradient of water at the surface is to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low. **2** Water is at the same temperature as air.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 15ºC and 1 atm, for which $\nu = 1.47 \times 10^{-5}$ m²/s and $\rho = 1.225$ kg/m³ (Table A-15). The saturation pressure of water at 15°C is 1.705 kPa (Table A-9). The mass diffusivity of water vapor in air at 15° (288 K) is determined from Eq. 14-15 to be

$$D_{AB} = D_{\mathrm{H_2O\text{-}air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(288\,\mathrm{K})^{2.072}}{1\,\mathrm{atm}} = 2.33\times10^{-5}\ \mathrm{m^2/s}$$

Analysis The air at the water surface will be saturated and that the saturation pressure of water at 15°C is 1.705 kPa, the mass fraction of water vapor in the air at the surface and at the free stream conditions are, from Eq. 14-10 (molar mass of air and water is obtained from Table A-1),

$$w_{A,s} = y_{A,s}\frac{M_A}{M} = \frac{P_{sat}}{P}\frac{M_A}{M_{air}} = \frac{(1.705\mathrm{kPa})}{101.325\mathrm{kPa}}\left(\frac{18.015\mathrm{kg/kmol}}{28.97\mathrm{kg/kmol}}\right) = 0.01046$$

$$w_{A,\infty} = y_{A,\infty}\frac{M_A}{M_{air}} = \frac{\phi P_{sat}}{P}\frac{M_A}{M_{air}} = \frac{(0.35)(1.705\mathrm{kPa})}{101.325\mathrm{kPa}}\left(\frac{18.015\mathrm{kg/kmol}}{28.97\mathrm{kg/kmol}}\right) = 0.003662$$

The mass transfer conditions on the water surface is given as

$$j_{A,s} = -D_{AB}\left.\frac{dw_A}{dy}\right|_s = h_{\mathrm{mass}}(w_{A,s} - w_{A,\infty})$$

Thus, the mass fraction gradient is

$$\left.\frac{dw_A}{dy}\right|_s = -\frac{h_{\mathrm{mass}}(w_{A,s} - w_{A,\infty})}{D_{AB}}$$

$$= -\frac{(0.03\,\mathrm{m/s})(0.01046 - 0.003662)}{2.33\times10^{-5}\ \mathrm{m^2/s}}$$

$$= \mathbf{-8.75\ m^{-1}}$$

Discussion Factors affecting the mass fraction gradient at the water surface are the mass transfer convection coefficient, mass diffusivity of water vapor in air, and the difference in concentrations at the surface and at the free stream.

14-132 The average Reynolds number, Schmidt number, Sherwood number, and friction coefficient for (*a*) an evaporation process and (*b*) a sublimation process are to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable. **2** Air and naphthalene vapors behave as ideal gases. **3** Process is isothermal.

Properties The mass diffusivities are

$$D_{AB} = D_{\mathrm{H_2O-Air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(298\ \mathrm{K})^{2.072}}{1\ \mathrm{atm}} = 2.5\times10^{-5}\ \mathrm{m^2/s}$$

$$D_{AB} = D_{\mathrm{Naph-Air}} = 0.61\times10^{-5}\ \mathrm{m^2/s} \qquad \text{(given)}$$

The kinematic viscosity of air at 298K = 25°C is $\nu = 1.562\times10^{-5}$ m^2/s (Table A-15).

Analysis (*a*) For the evaporation process (water-air), we have

$$\mathrm{Re} = \frac{VL_c}{\nu} = \frac{(2\ \mathrm{m/s})(2\ \mathrm{m})}{1.562\times10^{-5}\ \mathrm{m^2/s}} = \mathbf{2.56\times10^5}$$

$$\mathrm{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562\times10^{-5}\ \mathrm{m^2/s}}{2.5\times10^{-5}\ \mathrm{m^2/s}} = \mathbf{0.625}$$

$$\mathrm{Sh} = \frac{h_{\mathrm{mass}}L_c}{D_{AB}} = \frac{(0.015\ \mathrm{m/s})(2\ \mathrm{m})}{2.5\times10^{-5}\ \mathrm{m^2/s}} = \mathbf{1200}$$

Finally, from the Chilton-Colburn analogy,

$$f = 2\mathrm{St}_{\mathrm{mass}}\mathrm{Sc}^{2/3} = 2\frac{h_{\mathrm{mass}}}{V}\mathrm{Sc}^{2/3} = 2\frac{0.015\ \mathrm{m/s}}{2\ \mathrm{m/s}}(0.625)^{2/3} = \mathbf{0.01097}$$

(*b*) For the sublimation process (naphthalene-air), we have

$$\mathrm{Re} = \frac{VL_c}{\nu} = \frac{(2\ \mathrm{m/s})(2\ \mathrm{m})}{1.562\times10^{-5}\ \mathrm{m^2/s}} = \mathbf{2.56\times10^5}$$

$$\mathrm{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562\times10^{-5}\ \mathrm{m^2/s}}{0.61\times10^{-5}\ \mathrm{m^2/s}} = \mathbf{2.56}$$

$$\mathrm{Sh} = \frac{h_{\mathrm{mass}}L_c}{D_{AB}} = \frac{(0.015\ \mathrm{m/s})(2\ \mathrm{m})}{0.61\times10^{-5}\ \mathrm{m^2/s}} = \mathbf{4920}$$

Finally, from the Chilton-Colburn analogy,

$$f = 2\mathrm{St}_{\mathrm{mass}}\mathrm{Sc}^{2/3} = 2\frac{h_{\mathrm{mass}}}{V}\mathrm{Sc}^{2/3} = 2\frac{0.015\ \mathrm{m/s}}{2\ \mathrm{m/s}}(2.56)^{2/3} = \mathbf{0.0281}$$

Discussion Note that both evaporation and sublimation processes have the same Reynolds number, since in both cases the free stream fluid is air at 298 K and 1 atm.

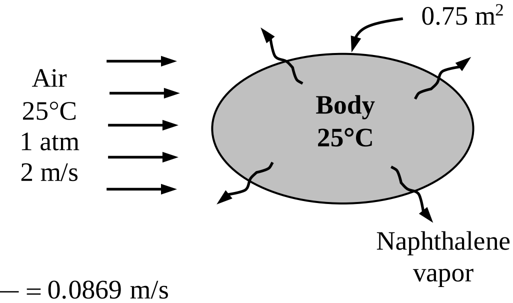
14-133 Air is blown over a body covered with a layer of naphthalene, and the rate of sublimation is measured. The heat transfer coefficient under the same flow conditions over the same geometry is to be determined.

Assumptions **1** The concentration of naphthalene in the air is very small, and the low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable (will be verified). **2** Both air and naphthalene vapor are ideal gases.

Properties The molar mass of naphthalene is 128.2 kg/kmol. Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 25°C and 1 atm, at which $\rho = 1.184\ \text{kg/m}^3$, $c_p = 1007\ \text{J/kg}\cdot\text{K}$, and $\alpha = 2.141\times10^{-5}\ \text{m}^2/\text{s}$ (Table A-15).

Analysis The incoming air is free of naphthalene, and thus the mass fraction of naphthalene at free stream conditions is zero, $w_{A,\infty} = 0$. Noting that the vapor pressure of naphthalene at the surface is 11 Pa, the surface mass fraction is determined to be

$$w_{A,s} = \frac{P_{A,s}}{P}\left(\frac{M_A}{M_{air}}\right) = \frac{11\text{Pa}}{101{,}325\text{Pa}}\left(\frac{128.2\text{kg/kmol}}{29\text{kg/kmol}}\right) = 4.8\times10^{-4}$$



which confirms that the low mass flux approximation is valid.
The rate of evaporation of naphthalene in this case is

$$\dot{m}_{\text{evap}} = \frac{m}{\Delta t} = \frac{0.1\text{kg}}{(45\times60\text{s})} = 3.703\times10^{-5}\ \text{kg/s}$$

Then the mass convection coefficient becomes

$$h_{mass} = \frac{\dot{m}}{\rho A(w_{A,s} - w_{A,\infty})} = \frac{3.703\times10^{-5}\ \text{kg/s}}{(1.184\text{kg/m}^3)(0.75\text{m}^2)(4.8\times10^{-4} - 0)} = 0.0869\ \text{m/s}$$

Using the analogy between heat and mass transfer, the average heat transfer coefficient is determined from Eq. 14-89 to be

$$h_{\text{heat}} = \rho c_p h_{\text{mass}}\left(\frac{\alpha}{D_{AB}}\right)^{2/3}$$

$$= (1.184\text{kg/m}^3)(1007\ \text{J/kg.K})(0.0869\text{m/s})\left(\frac{2.141\times10^{-5}\ \text{m}^2/\text{s}}{0.61\times10^{-5}\ \text{m}^2/\text{s}}\right)^{2/3}$$

$$= \mathbf{239\ W/m^2\cdot{}^\circ C}$$

Discussion Naphthalene has been commonly used in heat transfer studies to determine convection heat transfer coefficients because of the convenience it offers.

14-134 Using a known expression for local convection heat transfer coefficient, the average mass convection coefficient over a plate is to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Water is at the same temperature as air.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 25°C and 1 atm, for which c_p = 1007 J/kg·K, ρ = 1.184 kg/m^3, and α = 2.141×10^{-5} m^2/s (Table A-15). The mass diffusivity of water vapor in air at 298 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\mathrm{H_2O-Air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(298\,\mathrm{K})^{2.072}}{1\,\mathrm{atm}} = 2.5\times10^{-5}\ \mathrm{m^2/s}$$

Analysis The average convection heat transfer coefficient can be determined for L = 1 m as

$$\begin{aligned} h_{\mathrm{heat}} &= \frac{1}{L}\int_0^L h_x\,dx = \frac{1}{L}\int_0^L 0.5 + 12x - 0.7x^3\,dx \\ &= \frac{1}{L}\left[0.5x + 6x^2 - 0.175x^4\right]_0^L = 0.5 + 6L - 0.175L^3 \\ &= 0.5 + 6(1\,\mathrm{m}) - 0.175(1\,\mathrm{m})^3 = 6.325\ \mathrm{W/m^2\cdot K} \end{aligned}$$

Applying the Chilton-Colburn analogy,

$$\frac{h_{\mathrm{heat}}}{h_{\mathrm{mass}}} = \rho c_p\left(\frac{\alpha}{D_{AB}}\right)^{2/3} \quad\rightarrow\quad h_{\mathrm{mass}} = \frac{h_{\mathrm{heat}}}{\rho c_p}\left(\frac{D_{AB}}{\alpha}\right)^{2/3}$$

Hence, the average mass convection coefficient is

$$h_{\mathrm{mass}} = \frac{6.325\ \mathrm{W/m^2\cdot K}}{(1.184\ \mathrm{kg/m^3})(1007\ \mathrm{J/kg\cdot K})}\left(\frac{2.5\times10^{-5}}{2.141\times10^{-5}}\right)^{2/3} = \mathbf{5.88\times10^{-3}\ m/s}$$

Discussion Using the Lewis relation, the average mass convection coefficient can be estimated to be

$$h_{\mathrm{mass}} \cong h_{\mathrm{heat}}/(\rho c_p) = 5.30\times10^{-3}\ \mathrm{m/s}$$

which is about 10% lower than 5.88×10^{-3} m/s.

14-135 Ethyl alcohol is spread over a flat table where dry air is blowing over it. The average mass transfer coefficient is to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable. **2** The critical Reynolds number for flow over a flat plate is 500,000.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 25°C and 1 atm: $\nu = 1.562 \times 10^{-5}$ m^2/s (Table A-15). The mass diffusivity of ethyl alcohol in air at 25°C is $D_{AB} = 1.2 \times 10^{-5}$ m^2/s (Table 14-2).

Analysis The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(1\text{ m/s})(1\text{m})}{1.562\times10^{-5}\text{ m}^2/\text{s}} = 64{,}020$$

which is less than 500,000, and thus the flow is laminar. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562\times10^{-5}\text{ m}^2/\text{s}}{1.2\times10^{-5}\text{ m}^2/\text{s}} = 1.3017$$

Therefore, the Sherwood number in this case is determined from Table 14-13 (for laminar flow over a flat plate with Sc > 0.6) to be

$$\text{Sh} = 0.664\,\text{Re}^{0.5}\,\text{Sc}^{1/3} = 0.664(64{,}020)^{0.5}(1.3017)^{1/3} = 183.44$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(183.44)(1.2\times10^{-5}\text{ m}^2/\text{s})}{1\text{ m}} = \mathbf{0.00220\ m/s}$$

Discussion Note that the Nusselt number relations in heat transfer can be used to determine the Sherwood number in mass transfer by replacing Prandtl number by the Schmidt number.

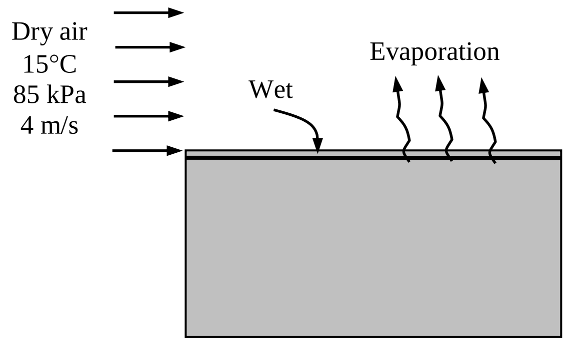
14-136 A wet flat plate is dried by blowing air over it. The mass transfer coefficient is to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The critical Reynolds number for flow over a flat plate is 500,000.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 15°C and 85 kPa = 85/101.325 = 0.839 atm, for which (Table A-15)

$$\nu = \nu_{1\,\text{atm}} / P(\text{atm}) = (1.47\times10^{-5}\ \text{m}^2/\text{s})/0.839\,\text{atm} = 1.75\times10^{-5}\ \text{m}^2/\text{s}$$

Analysis The mass diffusivity of water vapor in air at 288 K is determined from Eq. 14-15 to be



$$\begin{aligned} D_{AB} &= D_{\text{H}_2\text{O-air}} \\ &= 1.87\times10^{-10}\ \frac{T^{2.072}}{P} \\ &= 1.87\times10^{-10}\ \frac{(288\ \text{K})^{2.072}}{0.839\ \text{atm}} \\ &= 2.78\times10^{-5}\ \text{m}^2/\text{s} \end{aligned}$$

The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(3\ \text{m/s})(2\ \text{m})}{1.75\times10^{-5}\ \text{m}^2/\text{s}} = 342{,}857$$

which is less than 500,000, and thus the flow is laminar. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.75\times10^{-5}\ \text{m}^2/\text{s}}{2.78\times10^{-5}\ \text{m}^2/\text{s}} = 0.629$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664\,\text{Re}^{0.5}\,\text{Sc}^{1/3} = 0.664(342{,}857)^{0.5}(0.629)^{1/3} = 333.1$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{mass} = \frac{\text{Sh}D_{AB}}{L} = \frac{(333.1)(2.78\times10^{-5}\ \text{m}^2/\text{s})}{2\ \text{m}} = \mathbf{0.00463\ m/s}$$

14-137 A thin slab of solid salt is being dragged through seawater. The mass convection rate of salt being dissolved in seawater is to be determined.

Assumptions **1** The analogy between heat and mass transfer is applicable. **2** The critical Reynolds number for flow over a flat plate is 500,000.

Properties The relevant properties are given in the problem statement.

Analysis The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(0.6\text{ m/s})(0.15\text{ m})}{1.022\times10^{-6}\text{ m}^2/\text{s}} = 88063$$

which is less than 500,000 and thus the flow is laminar. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.022\times10^{-6}\text{ m}^2/\text{s}}{1.2\times10^{-9}\text{ m}^2/\text{s}} = 851.7$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664\,\text{Re}^{0.5}\,\text{Sc}^{1/3} = 0.664(88063)^{0.5}(851.7)^{1/3} = 1868$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(1868)(1.2\times10^{-9}\text{ m}^2/\text{s})}{0.15\text{ m}} = 1.494\times10^{-5}\text{ m/s}$$

Hence, the mass convection rate of salt being dissolved in the seawater is

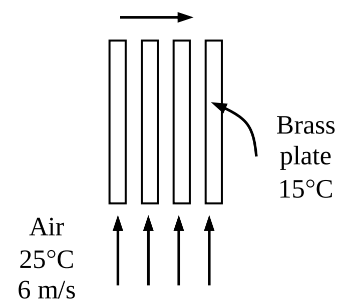
$$\begin{aligned}\dot{m}_{\text{conv}} &= h_{\text{mass}}A_s(\rho_{A,s} - \rho_{A,\infty})\\ &= (1.494\times10^{-5}\text{ m/s})2(0.15\times0.15\text{ m}^2)(35000-31)\text{ kg/m}^3\\ &= \mathbf{0.0235\,kg/s}\end{aligned}$$

Discussion In the analysis of this problem, the mass convection from the edges of the salt slab is considered negligible. This is a reasonable assumption as the salt slab is thin and the mass convection mainly occurs on the top and bottom surfaces.

14-138 Wet steel plates are to be dried by blowing air parallel to their surfaces. The rate of evaporation from both sides of a plate is to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** The plates are far enough from each other so that they can be treated as flat plates. **4** The air is dry so that the amount of moisture in the air is negligible.

Properties The molar masses of air and water are M = 29 and M = 18 kg/kmol, respectively (Table A-1).



Because of low mass flux conditions, we can use dry air properties for the mixture. The properties of the air at 1 atm and at the film temperature of (15 + 25) = 20ºC are (Table A-15)

$\nu = 1.516\times10$P^{-5P} mP^{2P}/s

$\rho = 1.204$ kg /mP^{3P}

cB$_{pB}$ = 1007 J / kg K

Pr = 0.7309

The saturation pressure of water at 15ºC is 1.705 kPa (Table A-9). The mass diffusivity of water vapor in air at 20ºC = 293 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(293\text{K})^{2.072}}{1\text{atm}} = 2.42\times10^{-5}\ \text{m}^2/\text{s}$$

Analysis The Reynolds number for flow over the flat plate is

$$\text{Re} = \frac{VL}{\nu} = \frac{(6\ \text{m/s})(0.5\text{m})}{1.516\times10^{-5}\ \text{m}^2/\text{s}} = 197{,}890$$

which is less than 500,000, and thus the air flow is laminar over the entire plate. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.516\times10^{-5}\ \text{m}^2/\text{s}}{2.42\times10^{-5}\ \text{m}^2/\text{s}} = 0.626$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664\,\text{Re}_L^{\ 0.5}\text{Sc}^{1/3} = 0.664(197{,}890)^{0.5}(0.626)^{1/3} = 252.7$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(252.7)(2.42\times10^{-5}\ \text{m}^2/\text{s})}{0.5\ \text{m}} = 0.01223\ \text{m/s}$$

Noting that the air at the water surface will be saturated and that the saturation pressure of water at 15ºC is 1.705 kPa, the mass fraction of water vapor in the air at the surface of the plate is, from Eq. 14-10,

$$w_{A,s} = y_{A,s}\frac{M_A}{M} = \frac{P_{sat}}{P}\frac{M_A}{M_{air}} = \frac{(1.705\text{kPa})}{101.325\text{kPa}}\left(\frac{18\text{kg/kmol}}{29\text{kg/kmol}}\right) = 0.01044$$

and $\quad w_{A,\infty} = 0$

Then the rate of mass transfer to the air becomes

$$\begin{aligned}\dot{m}_{\text{evap.}} &= h_{\text{mass}}\rho A(w_{A,s} - w_{A,\infty})\\ &= (0.01223\ \text{m/s})(1.204\text{kg/m}^3)(2\times0.5\ \text{m}\times0.5\text{m})(0.01044-0)\\ &= \mathbf{7.69\times10^{-5}\ kg/s}\end{aligned}$$

Discussion This is the upper limit for the evaporation rate since the air is assumed to be completely dry.

14-139E Air is blown over a square pan filled with water. The rate of evaporation of water and the rate of heat transfer to the pan to maintain the water temperature constant are to be determined.

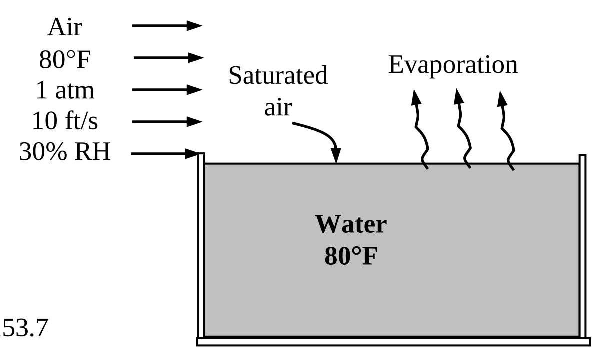
Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 80°F). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** Water is at the same temperature as the air.

Properties The molar masses of air and water are M = 29 and M = 18 lbm/lbmol, respectively (Table A-1E). Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 80°F and 1 atm, for which ν = 1.697 × 10P-4P ftP2P/s, and ρ = 0.0735 lbm/ftP3P (Table A-15E). The saturation pressure of water at 80ºF is 0.5073 psia, and the heat of vaporization is 1048 Btu/lbm. The mass diffusivity of water vapor in air at 80ºF = 540 R = 300 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\mathrm{H_2O\text{-}air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(300\,\mathrm{K})^{2.072}}{1\mathrm{atm}} = 2.54\times10^{-5}\ \mathrm{m^2/s} = 2.734\times10^{-4}\ \mathrm{ft^2/s}$$

Analysis The Reynolds number for flow over the free surface is

$$\mathrm{Re} = \frac{VL}{\nu} = \frac{(10\ \mathrm{ft/s})(15/12\ \mathrm{ft})}{1.697\times10^{-4}\ \mathrm{ft^2/s}} = 73{,}660$$



which is less than 500,000, and thus the flow is laminar over the entire surface. The Schmidt number in this case is

$$\mathrm{Sc} = \frac{\nu}{D_{AB}} = \frac{1.697\times10^{-4}\ \mathrm{ft^2/s}}{2.734\times10^{-4}\ \mathrm{ft^2/s}} = 0.6207$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\mathrm{Sh} = 0.664\,\mathrm{Re_L}^{0.5}\mathrm{Sc}^{1/3} = 0.664(73{,}660)^{0.5}(0.6207)^{1/3} = 153.7$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\mathrm{mass}} = \frac{\mathrm{Sh}D_{AB}}{L} = \frac{(153.7)(2.734\times10^{-4}\ \mathrm{ft^2/s})}{15/12\ \mathrm{ft}} = 0.0336\ \mathrm{ft/s}$$

Noting that the air at the water surface will be saturated and that the saturation pressure of water at 80ºF is 0.5073 psia (= 0.0345 atm), the mass fraction of water vapor in the air at the surface and at the free stream conditions are, from Eq. 14-10,

$$w_{A,s} = y_{A,s}\frac{M_A}{M} = \frac{P_{sat}}{P}\frac{M_A}{M_{air}} = \frac{(0.3)(0.5073\,\mathrm{psia})}{14.7\,\mathrm{psia}}\left(\frac{18\,\mathrm{lbm/lbmol}}{29\,\mathrm{lbm/lbmol}}\right) = 0.00643$$

$$w_{A,\infty} = y_{A,\infty}\frac{M_A}{M_{air}} = \frac{\phi P_{sat}}{P}\frac{M_A}{M_{air}} = \frac{(1.0)(0.5073\,\mathrm{psia})}{14.7\,\mathrm{psia}}\left(\frac{18\,\mathrm{lbm/lbmol}}{29\,\mathrm{lbm/lbmol}}\right) = 0.02142$$

Then the rate of mass transfer to the air becomes

$$\dot{m}_{\mathrm{evap}} = h_{\mathrm{mass}}\rho A_s(w_{A,s} - w_{A,\infty}) = (0.0336\ \mathrm{ft/s})(0.074\ \mathrm{lbm/ft^3})(15/12\ \mathrm{ft})^2(0.02142 - 0.00642) = \mathbf{5.83\times10^{-5}\ lbm/s}$$

Noting that the latent heat of vaporization of water at 80ºF is hB$_{fg}$B = 1048 Btu/ lbm, the required rate of heat supply to the water to maintain its temperature constant is

$$\dot{Q} = \dot{m}_{evap}\,h_{fg} = (5.83\times10^{-5}\ \mathrm{lbm/s})(1048\ \mathrm{Btu/lbm}) = \mathbf{0.0611\ Btu/s = 220\ Btu/h}$$

Discussion If no heat is supplied to the pan, the heat of vaporization of water will come from the water, and thus the water temperature will have to drop below the air temperature.

14-140E Air is blown over a square pan filled with water. The rate of evaporation of water and the rate of heat transfer to the pan to maintain the water temperature constant are to be determined.

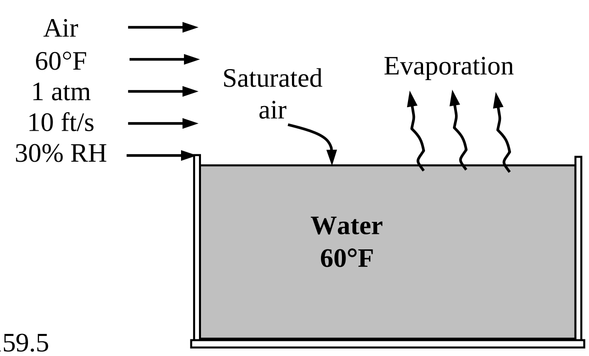
Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 60°F). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** Water is at the same temperature as air.

Properties The molar masses of air and water are M = 29 and M = 18 lbm/lbmol, respectively (Table A-1E). Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 60°F and 1 atm, for which ν = 1.588 × 10P^{-4P} ftP^{2P}/s, and ρ = 0.07633 lbm / ftP^{3P} (Table A-15E). The saturation pressure of water at 60ºF is 0.2563 psia, and the heat of vaporization is 1060 Btu/lbm. The mass diffusivity of water vapor in air at 60ºF = 520 R = 288.9 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\mathrm{H_2O\text{-}air}} = 1.87\times 10^{-10}\frac{T^{2.072}}{P} = 1.87\times 10^{-10}\frac{(288.9\,\mathrm{K})^{2.072}}{1\mathrm{atm}} = 2.35\times 10^{-5}\ \mathrm{m^2/s} = 2.53\times 10^{-4}\ \mathrm{ft^2/s}$$

Analysis The Reynolds number for flow over the free surface is

$$\mathrm{Re} = \frac{VL}{\nu} = \frac{(10\ \mathrm{ft/s})(15/12\ \mathrm{ft})}{1.588\times 10^{-4}\ \mathrm{ft^2/s}} = 78{,}715$$



which is less than 500,000, and thus the flow is laminar over the entire surface. The Schmidt number in this case is

$$\mathrm{Sc} = \frac{\nu}{D_{AB}} = \frac{1.588\times 10^{-4}\ \mathrm{ft^2/s}}{2.53\times 10^{-4}\ \mathrm{ft^2/s}} = 0.6277$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\mathrm{Sh} = 0.664\,\mathrm{Re_L}^{0.5}\mathrm{Sc}^{1/3} = 0.664(78{,}715)^{0.5}(0.6277)^{1/3} = 159.5$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\mathrm{mass}} = \frac{\mathrm{Sh}D_{AB}}{L} = \frac{(159.5)(2.53\times 10^{-4}\ \mathrm{ft^2/s})}{15/12\ \mathrm{ft}} = 0.0323\ \mathrm{ft/s}$$

Noting that the air at the water surface will be saturated and that the saturation pressure of water at 60ºF is 0.2563 psia, the mass fraction of water vapor in the air at the surface and at the free stream conditions are, from Eq. 14-10,

$$w_{A,s} = y_{A,s}\frac{M_A}{M} = \frac{P_{sat}}{P}\frac{M_A}{M_{air}} = \frac{(0.3)(0.2563\,\mathrm{psia})}{14.7\,\mathrm{psia}}\left(\frac{18\,\mathrm{lbm/lbmol}}{29\,\mathrm{lbm/lbmol}}\right) = 0.00325$$

$$w_{A,\infty} = y_{A,\infty}\frac{M_A}{M_{air}} = \frac{\phi P_{sat}}{P}\frac{M_A}{M_{air}} = \frac{(1.0)(0.2563\,\mathrm{psia})}{14.7\,\mathrm{psia}}\left(\frac{18\,\mathrm{lbm/lbmol}}{29\,\mathrm{lbm/lbmol}}\right) = 0.01082$$

Then the rate of mass transfer to the air becomes

$$\dot{m}_{\mathrm{evap}} = h_{\mathrm{mass}}\rho A(w_{A,s} - w_{A,\infty}) = (0.0323\ \mathrm{ft/s})(0.07633\ \mathrm{lbm/ft^3})(15/12\ \mathrm{ft})^2(0.01082 - 0.00325) = \mathbf{2.82\times 10^{-5}\ lbm/s}$$

Noting that the latent heat of vaporization of water at 60ºF is hB$_{fg}$ = 1060 Btu/ lbm, the required rate of heat supply to the water to maintain its temperature constant is

$$\dot{Q} = \dot{m}_{\mathrm{evap}}h_{fg} = (2.82\times 10^{-5}\ \mathrm{lbm/s})(1060\,\mathrm{Btu/lbm}) = \mathbf{0.0299\,Btu/s = 108\,Btu/h}$$

Discussion If no heat is supplied to the pan, the heat of vaporization of water will come from the water, and thus the water temperature will have to drop below the air temperature.

14-141 A wet concrete patio is to be dried by winds. The time it takes for the patio to dry is to be determined.

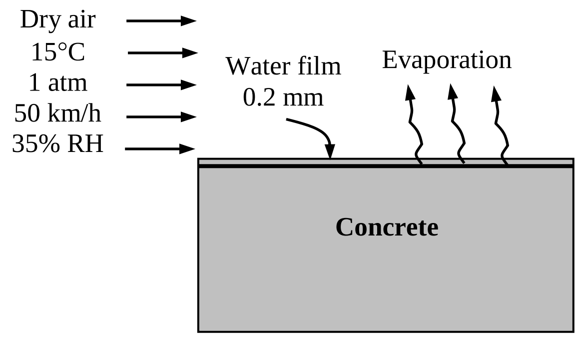
Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** Water is at the same temperature as air.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 15ºC and 1 atm, for which $\nu = 1.47\times10^{-5}\ \text{m}^2/\text{s}$ and $\rho = 1.225\ \text{kg/m}^3$ (Table A-15). The saturation pressure of water at 15ºC is 1.705 kPa. The mass diffusivity of water vapor in air at 15ºC = 288 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(288\ \text{K})^{2.072}}{1\,\text{atm}} = 2.33\times10^{-5}\ \text{m}^2/\text{s}$$

Analysis The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(50\ \text{km/h})(5\ \text{m})}{1.47\times10^{-5}\ \text{m}^2/\text{s}}\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right) = 4.724\times10^6$$



which is more than 500,000, and thus the flow is turbulent over most of the surface. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.47\times10^{-5}\ \text{m}^2/\text{s}}{2.33\times10^{-5}\ \text{m}^2/\text{s}} = 0.631$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\begin{aligned}\text{Sh} &= 0.037\,\text{Re}^{0.8}\,\text{Sc}^{1/3}\\ &= 0.037(4.724\times10^6)^{0.8}(0.631)^{1/3} = 6934\end{aligned}$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{mass} = \frac{\text{Sh}D_{AB}}{L} = \frac{(6934)(2.33\times10^{-5}\ \text{m}^2/\text{s})}{5\,\text{m}} = 0.0323\text{m/s}$$

Noting that the air at the water surface will be saturated and that the saturation pressure of water at 15ºC is 1.705 kPa, the mass fraction of water vapor in the air at the surface and at the free stream conditions are, from Eq. 14-10,

$$w_{A,s} = y_{A,s}\frac{M_A}{M} = \frac{P_{sat}}{P}\frac{M_A}{M_{air}} = \frac{(1.705\text{kPa})}{101.325\text{kPa}}\left(\frac{18\,\text{kg/kmol}}{29\,\text{kg/kmol}}\right) = 0.01044$$

$$w_{A,\infty} = y_{A,\infty}\frac{M_A}{M_{air}} = \frac{\phi P_{sat}}{P}\frac{M_A}{M_{air}} = \frac{(0.35)(1.705\text{kPa})}{101.325\text{kPa}}\left(\frac{18\,\text{kg/kmol}}{29\,\text{kg/kmol}}\right) = 0.003655$$

Then the rate of mass transfer to the air becomes

$$\begin{aligned}\dot{m}_{\text{evap.}} &= h_{\text{mass}}\rho A(w_{A,s} - w_{A,\infty})\\ &= (0.0323\text{m/s})(1.225\text{kg/m}^3)(5\text{m}\times5\text{m})(0.01044 - 0.003655)\\ &= 0.00671\text{kg/s}\end{aligned}$$

The total mass of water on the concrete patio is

$$m_{water} = \rho\boldsymbol{V} = (1000\text{kg/m}^3)(5\ \text{m}\times5\ \text{m}\times0.2\times10^{-3}\ \text{m}) = 5\ \text{kg}$$

Then the time required to evaporate the water on the concrete patio becomes

$$\Delta t = \frac{m_{water}}{\dot{m}_{evap}} = \frac{5\ \text{kg}}{0.00671\ \text{kg/s}} = 745\ \text{s} = \mathbf{12.4\,min}$$

14-142 PtD Liquid benzene is spread over a highway where wind is blowing over it. The mass transfer rate by convection is to be determined whether or not residents in the vicinity should be evacuated.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable. **2** The critical Reynolds number for flow over a flat plate is 500,000.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 25°C and 1 atm: $\nu = 1.562 \times 10^{-5}$ m^2/s, $\rho = 1.184$ kg/m^3 (Table A-15), for air: $M = 28.97$ kg/kmol (Table A-1). The molar mass of benzene is given as $M = 78.11$ kg/kmol, and the mass diffusivity of benzene in air at 25°C is $D_{AB} = 0.88 \times 10^{-5}$ m^2/s (Table 14-2).

Analysis The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(10\text{ m/s})(10\text{m})}{1.562\times 10^{-5}\text{ m}^2\text{/s}} = 6.402\times 10^6$$

which is greater than 500,000, and thus the flow is turbulent. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562\times 10^{-5}\text{ m}^2\text{/s}}{0.88\times 10^{-5}\text{ m}^2\text{/s}} = 1.775$$

Therefore, the Sherwood number in this case is determined from Table 14-13 (for turbulent flow over a flat plate with Sc > 0.6) to be

$$\text{Sh} = 0.037\,\text{Re}^{0.8}\,\text{Sc}^{1/3} = 0.037(6.402\times 0^6)^{0.8}(1.775)^{1/3} = 12483$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(12483)(0.88\times 10^{-5}\text{ m}^2\text{/s})}{10\text{ m}} = 0.01099\text{ m/s}$$

At the liquid benzene surface, the mass fraction of benzene vapor in air is

$$w_{A,s} = y_{A,s}\frac{M_A}{M} = \frac{P_{A,s}}{P}\frac{M_A}{M_{air}} = \left(\frac{10\text{ kPa}}{101.325\text{ kPa}}\right)\left(\frac{78.11\text{ kg/kmol}}{28.97\text{ kg/kmol}}\right) = 0.2661$$

At the free stream, the mass fraction of benzene vapor is zero

$$w_{A,\infty} = 0$$

Thus, the mass transfer rate of benzene to the air by convection is

$$\begin{aligned}\dot{m} &= h_{\text{mass}}\rho A(w_{A,s} - w_{A,\infty})\\ &= (0.01099\text{ m/s})(1.184\text{ kg/m}^3)\frac{\pi}{4}(10\text{ m})^2(0.2661 - 0)\\ &= 0.272\text{ kg/s}\\ &= \mathbf{979\ kg/h} > 500\text{ kg/h}\end{aligned}$$

Discussion The mass transfer rate of benzene to the air is estimated to be 979 kg/h, which is greater than 500 kg/hr. Since the mass transfer rate of benzene is greater than the safe level, the residents should be evacuated.

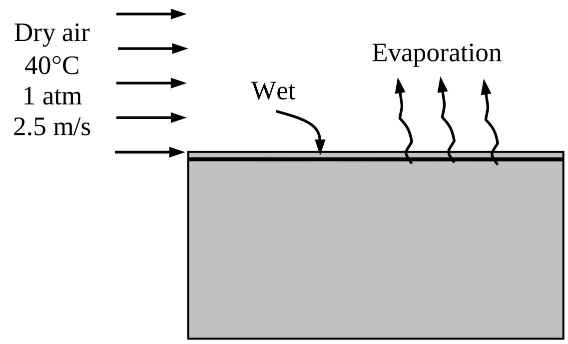
14-143 A wet flat plate is dried by blowing air over it. The mass transfer coefficient is to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The critical Reynolds number for flow over a flat plate is 500,000.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 40°C and 1 atm, for which (Table A-15)

$$\nu = 1.702\times10^{-5}\ \mathrm{m^2/s}$$

Analysis The mass diffusivity of water vapor in air at 313 K is determined from Eq. 14-15 to be



$$\begin{aligned} D_{AB} &= D_{\mathrm{H_2O\text{-}air}} \\ &= 1.87\times10^{-10}\frac{T^{2.072}}{P} \\ &= 1.87\times10^{-10}\frac{(313\,\mathrm{K})^{2.072}}{1\,\mathrm{atm}} \\ &= 2.77\times10^{-5}\ \mathrm{m^2/s} \end{aligned}$$

The Reynolds number of the flow is

$$\mathrm{Re} = \frac{VL}{\nu} = \frac{(2.5\,\mathrm{m/s})(8\,\mathrm{m})}{1.702\times10^{-5}\ \mathrm{m^2/s}} = 1{,}175{,}100$$

which is greater than 500,000, and thus we have combined laminar and turbulent flow. The Schmidt number in this case is

$$\mathrm{Sc} = \frac{\nu}{D_{AB}} = \frac{1.702\times10^{-5}\ \mathrm{m^2/s}}{2.77\times10^{-5}\ \mathrm{m^2/s}} = 0.614$$

Therefore, the Sherwood number in this case is determined using the analogy between the heat and mass transfer to be

$$\mathrm{Sh} = (0.037\,\mathrm{Re}^{0.8} - 871)\mathrm{Sc}^{1/3} = (0.037\times1{,}175{,}100^{0.8} - 871)(0.614)^{1/3} = 1517$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{mass} = \frac{\mathrm{Sh}D_{AB}}{L} = \frac{(1517)(2.77\times10^{-5}\ \mathrm{m^2/s})}{8\ \mathrm{m}} = \mathbf{0.00525\ m/s}$$

14-144E A spherical naphthalene ball is suspended in a room where it is subjected to forced air flow. The average mass transfer coefficient between the naphthalene and the air is to be determined.

Assumptions **1** The concentration of naphthalene in the air is very small, and the low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable (will be verified). **2** Both air and naphthalene vapor are ideal gases. **3** Both the ball and the room are at the same temperature.

Properties The Schmidt number of naphthalene in air at room temperature is given to be 2.35. Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 80°F and 1 atm from Table A-15E,

$$k = 0.01481 \text{ Btu/h.ft.°F}$$

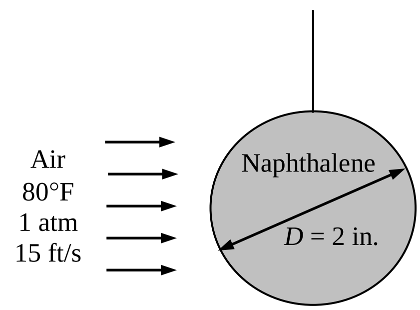
$$\mu = 1.247\times10^{-5} \text{ lbm/ft.s}$$

$$\nu = 1.697\times10^{-4} \text{ ft}^2\text{/s}$$

$$\text{Pr} = 0.7290$$

Analysis Noting that the Schmidt number for naphthalene in air is 2.35, the mass diffusivity of naphthalene in air is determined from

$$\text{Sc} = \frac{\nu}{D_{AB}} \longrightarrow D_{AB} = \frac{\nu}{\text{Sc}} = \frac{1.697\times10^{-4} \text{ ft}^2\text{/s}}{2.35} = 7.22\times10^{-5} \text{ ft}^2\text{/s}$$



The Reynolds number of the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(15 \text{ ft/s})(2/12 \text{ ft})}{(1.697\times10^{-4} \text{ ft}^2\text{/s})} = 14{,}732$$

Noting that $\mu_\infty = \mu_s$ for air in this case since the air and the ball are assumed to be at the same temperature, the Sherwood number can be determined from the forced heat convection relation for a sphere by replacing Pr by the Sc number to be

$$\text{Sh} = \frac{h_{mass}D}{D_{AB}} = 2 + \left[0.4\,\text{Re}^{1/2} + 0.06\,\text{Re}^{2/3}\right]\text{Sc}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$
$$= 2 + \left[0.4(14{,}732)^{1/2} + 0.06(14{,}732)^{2/3}\right](2.35)^{0.4}$$
$$= 121$$

Then the mass transfer coefficient becomes

$$h_{mass} = \frac{\text{Sh}D_{AB}}{D} = \frac{(121)(7.22\times10^{-5} \text{ ft}^2\text{/s})}{(2/12) \text{ ft}} = \mathbf{0.0524 \text{ ft/s}}$$

Discussion Note that the Nusselt number relations in heat transfer can be used to determine the Sherwood number in mass transfer by replacing Prandtl number by the Schmidt number.

14-145 A raindrop is falling freely in atmospheric air. The terminal velocity of the raindrop at which the drag force equals the weight of the drop and the average mass transfer coefficient are to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The raindrop is spherical in shape. **3** The reduction in the diameter of the raindrop due to evaporation when the terminal velocity is reached is negligible.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture. The properties of air at 1 atm and the free-stream temperature of 25ºC (and the dynamic viscosity at the surface temperature of 9ºC) are (Table A-15)

$$\rho = 1.184\ \text{kg/m}^3 \qquad \mu_\infty = 1.849\times10^{-5}\ \text{kg/m.s}$$

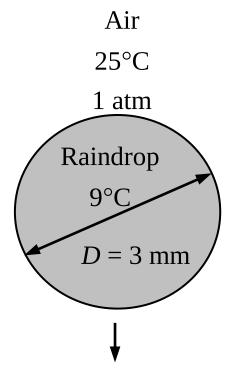
$$\nu = 1.562\times10^{-5}\ \text{m}^2/\text{s} \qquad \mu_{s,@9°\text{C}} = 1.759\times10^{-5}\ \text{kg/m.s}$$

At 1 atm and the film temperature of (25+9)/2 = 17ºC = 290 K, the kinematic viscosity of air is, from Table A-15, $\nu = 1.488\times10^{-5}\ \text{m}^2/\text{s}$, while the mass diffusivity of water vapor in air is, Eq. 14-15,

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(290\ \text{K})^{2.072}}{1\,\text{atm}} = 2.37\times10^{-5}\ \text{m}^2/\text{s}$$

Analysis The weight of the raindrop before any evaporation occurs is

$$F_D = mg = \rho V g = (1000\ \text{kg/m}^3)\left[\frac{\pi(0.003\,\text{m})^3}{6}\right](9.8\ \text{m/s}^2) = 1.38\times10^{-4}\ \text{N}$$



The drag force is determined from $F_D = C_D A_N \dfrac{\rho u_\infty^2}{2}$ where drag coefficient C_D is to be determined using Fig. 10-20 which requires the Reynolds number. Since we do not know the velocity we cannot determine the Reynolds number. Therefore, the solution requires a trial-error approach. We choose a velocity and perform calculations to obtain the drag force. After a couple trial we choose a velocity of 8 m/s. Then the Reynolds number becomes

$$\text{Re} = \frac{VD}{\nu} = \frac{(8\,\text{m/s})(0.003\text{m})}{1.562\times10^{-5}\ \text{m}^2/\text{s}} = 1536$$

The corresponding drag coefficient from Fig. 7-17 is 0.5. Then,

$$F_D = C_D A_N \frac{\rho u_\infty^2}{2} = (0.5)\left[\frac{\pi(0.003\,\text{m})^2}{4}\right]\frac{(1.184\ \text{kg/m}^3)(8\ \text{m/s})^2}{2} = 1.34\times10^{-4}$$

which is sufficiently close to the value calculated before. Therefore, the terminal velocity of the raindrop is ***V* = 8 m/s.** The Schmidt number is

$$\text{Sc} = \frac{\nu}{D_{\text{AB}}} = \frac{1.488\times10^{-5}\ \text{m}^2/\text{s}}{2.37\times10^{-5}\ \text{m}^2/\text{s}} = 0.628$$

Then the Sherwood number can be determined from the forced heat convection relation for a sphere by replacing Pr by the Sc number to be

$$\text{Sh} = \frac{h_{mass}D}{D_{AB}} = 2 + \left[0.4\,\text{Re}^{1/2} + 0.06\,\text{Re}^{2/3}\right]\text{Sc}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(1536)^{1/2} + 0.06(1536)^{2/3}\right](0.628)^{0.4}\left(\frac{1.849\times10^{-5}}{1.759\times10^{-5}}\right)^{1/4} = 21.9$$

Then the mass transfer coefficient becomes

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{D} = \frac{(21.9)(2.37\times10^{-5}\ \text{m}^2/\text{s})}{0.003\text{m}} = \mathbf{0.173\ m/s}$$

14-146E The liquid layer on the inner surface of a circular pipe is dried by blowing air through it. The mass transfer coefficient is to be determined.

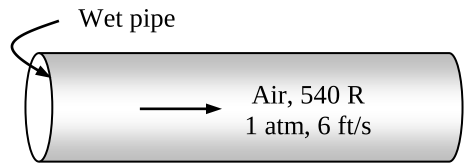
Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 540 R). **2** The flow is fully developed.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 540 R and 1 atm, for which $\nu = 1.697\times10^{-4}\ \text{ft}^2/\text{s}$ (Table A-15E). The mass diffusivity of water vapor in air at 540 R is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P}$$

$$= 1.87\times10^{-10}\frac{(540/1.8)^{2.072}}{1} = 2.54\times10^{-5}\ \text{m}^2/\text{s}$$

$$= 2.73\times10^{-4}\ \text{ft}^2/\text{s}$$



The Reynolds number of the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(6\ \text{ft/s})(0.7/12\ \text{ft})}{1.697\times10^{-4}\ \text{ft}^2/\text{s}} = 2062$$

which is less than 2300 and thus the flow is laminar. Therefore, based on the analogy between heat and mass transfer, the Nusselt and the Sherwood numbers in this case are Nu = Sh = 3.66. Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{D} = \frac{(3.66)(2.73\times10^{-4}\ \text{ft}^2/\text{s})}{0.7/12\text{ft}} = \mathbf{0.017ft/s}$$

Discussion The mass transfer rate (or the evaporation rate) in this case can be determined by defining logarithmic mean concentration difference in an analogous manner to the logarithmic mean temperature difference.

14-147 The liquid layer on the inner surface of a circular pipe is dried by blowing air through it. The mass transfer coefficient is to be determined.

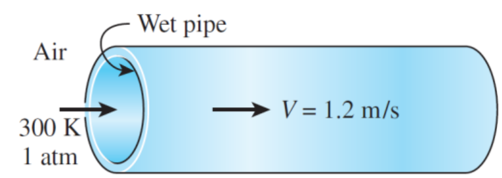
Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The flow is fully developed.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 300 K and 1 atm, for which $\nu = 1.58 \times 10^{-5}$ m^2/s (Table A-15). The mass diffusivity of water vapor in the air at 300 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\mathrm{H_2O-air}} = 1.87 \times 10^{-10}\frac{T^{2.072}}{P} = 1.87 \times 10^{-10}\frac{(300\text{ K})^{2.072}}{1\text{ atm}} = 2.54 \times 10^{-5}\,\mathrm{m^2/s}$$

Analysis The Reynolds number for this internal flow is

$$Re_L = \frac{VD}{\nu} = \frac{(1.2\text{ m/s})(0.015\text{m})}{1.58 \times 10^{-5}\mathrm{m^2/s}} = 1139$$



which is less than 2300 and thus the flow is laminar. Therefore, based on the analogy between heat and mass transfer, the Nusselt and the Sherwood numbers in this case are Nu = Sh = 3.66. Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\mathrm{Sh}D_{AB}}{L} = \frac{(3.66)(2.54 \times 10^{-5}\,\frac{\mathrm{m^2}}{\mathrm{s}})}{0.015\text{ m}} = \mathbf{0.00620\ m/s}$$

Discussion The mass transfer rate (or the evaporation rate) in this case can be determined by defining the logarithmic mean concentration difference in an analogous manner to the logarithmic mean temperature difference.

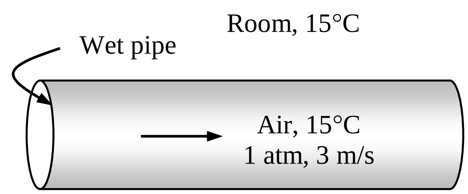
14-148 The liquid layer on the inner surface of a circular pipe is dried by blowing air through it. The mass transfer coefficient is to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The flow is fully developed.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 15°C and 1 atm, for which $\nu = 1.47 \times 10^{-5}\ \text{m}^2/\text{s}$ (Table A-15). The mass diffusivity of water vapor in air at 288 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87 \times 10^{-10}\frac{T^{2.072}}{P}$$

$$= 1.87 \times 10^{-10}\frac{(288\ \text{K})^{2.072}}{1} = 2.332 \times 10^{-5}\ \text{m}^2/\text{s}$$



Analysis The Reynolds number of the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(3\ \text{m/s})(0.12\text{m})}{1.47 \times 10^{-5}\ \text{m}^2/\text{s}} = 24{,}490$$

which is greater than 10,000 and thus the flow is turbulent. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.47 \times 10^{-5}\ \text{m}^2/\text{s}}{2.332 \times 10^{-5}\ \text{m}^2/\text{s}} = 0.6302$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.023\,\text{Re}^{0.8}\,\text{Sc}^{0.4} = 0.023(24{,}490)^{0.8}(0.6302)^{0.4} = 62.05$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{mass} = \frac{\text{Sh}D_{AB}}{D} = \frac{(62.05)(2.332 \times 10^{-5}\ \text{m}^2/\text{s})}{0.12\ \text{m}} = \mathbf{0.0121\ m/s}$$

14-149 Prob. 14-148 is reconsidered. The mass transfer coefficient as a function of the air velocity is to be plotted.

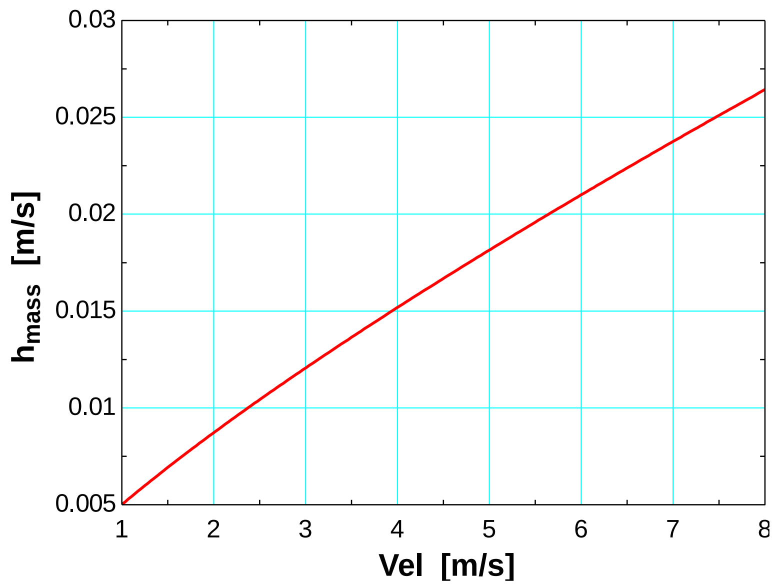
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.12 [m]
L=14 [m]
P=101.3 [kPa]
T=(15+273) [K]
Vel=3 [m/s]

"PROPERTIES"
Fluid$='air'
rho=Density(Fluid$, T=T, P=P)
mu=Viscosity(Fluid$, T=T)
nu=mu/rho
D_AB=1.87E-10*T^2.072/(P*Convert(kPa, atm)) "from the text"

"ANALYSIS"
Re=Vel*D/nu
"Re is calculated to be greater than 10,000, and thus the flow is turbulent."
Sc=nu/D_AB
Sh=0.023*Re^0.8*Sc^0.4
h_mass=Sh*D_AB/D
```

Vel [m/s]	h_{mass} [m/s]
1	0.005008
1.5	0.006927
2	0.008719
2.5	0.01042
3	0.01206
3.5	0.01364
4	0.01518
4.5	0.01668
5	0.01815
5.5	0.01959
6	0.021
6.5	0.02239
7	0.02375
7.5	0.0251
8	0.02643



14-150 The inner surface of a pipe for transporting water contains lead. The level of lead in the water from the pipe is to be determined.

Assumptions **1** The analogy between heat and mass transfer is applicable. **2** The flow is at steady state. **3** The flow is fully developed. **4** Mass concentration of lead on the pipe surface is much greater than in the water flow, $\rho_{A,s} >> \rho_{A,\infty}$.

Properties The dynamic viscosity and density for water at 20°C are 1.002×10^{-3} kg/m·s and 998 kg/m^3, respectively (Table A-9).

Analysis The Reynolds number of the flow is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(998\ \text{kg/m}^3)(0.1\ \text{m/s})(0.02\ \text{m})}{1.002 \times 10^{-3}\text{kg/m}\cdot\text{s}} = 1992 < 2300$$

So, the flow inside the pipe is laminar. Based on the analogy between heat and mass transfer, the Nusselt and the Sherwood numbers in this case are Nu = Sh = 3.66. Hence,

$$\text{Sh} = \frac{h_{\text{mass}} D}{D_{AB}} = 3.66$$

The mass transfer coefficient becomes

$$h_{\text{mass}} = 3.66\frac{D_{AB}}{D} = 3.66\left(\frac{3.6 \times 10^{-10}\text{m}^2/\text{s}}{0.02\ \text{m}}\right) = 6.588 \times 10^{-8}\text{m/s}$$

The rate of mass convection of lead from the inner pipe surface to the water is

$$\dot{m}_{\text{conv}} = h_{\text{mass}} A_s(\rho_{A,s} - \rho_{A,\infty}) = h_{\text{mass}}(\pi D L)(\rho_{A,s} - \rho_{A,\infty})$$

At the pipe inner surface, the mass concentration of lead in water is $\rho_{A,s}$ = 5 g/m^3; and for the water flow $\rho_{A,\infty} \approx 0$,

$$\dot{m}_{\text{conv}} = (6.588 \times 10^{-8}\text{m/s})\pi(0.02\ \text{m})(100\ \text{m})(5 - 0)\ \text{g/m}^3 = 2.0697 \times 10^{-6}\text{g/s} = 0.0020697\ \text{mg/s}$$

The volumetric flow rate of water in the pipe is

$$VA = V\pi\left(\frac{\text{D}^2}{4}\right) = (0.1\ \text{m/s})\pi\frac{(0.02\ \text{m})^2}{4} = 3.1416 \times 10^{-5}\text{m}^3/\text{s} = 0.031416\ \text{L/s}$$

The level of lead in the water flowing from the pipe is

$$\text{Lead level in water} = \frac{\dot{m}_{\text{conv}}}{VA} = \frac{0.0020697\ \text{mg/s}}{0.031416\ \text{L/s}} = \mathbf{0.0659\ mg/L} > 0.015\ \text{mg/L}$$

Discussion Under the given conditions, the lead level in the water flowing out of the pipe is more than 4 times the 0.015 mg/L limit set by the NPDWR. Therefore, the water is not safe for public use. Additional steps of treatment are necessary to reduce the lead contamination in the water.

14-151 C&S The inner surface of a tube for transporting water contains known concentration of copper. The level of copper in the water from the tube is to be determined.

Assumptions **1** The analogy between heat and mass transfer is applicable. **2** The flow is at steady state. **3** The flow is fully developed. **4** Mass concentration of copper on the tube surface is much greater than in the water flow, $\rho_{A,s} >> \rho_{A,\infty}$.

Properties The dynamic viscosity and density for water at 20°C are 1.002×10^{-3} kg/m·s and 998 kg/m^3, respectively (Table A-9).

Analysis The Reynolds number of the flow is

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{(998\ \mathrm{kg/m^3})(0.11\ \mathrm{m/s})(0.02\ \mathrm{m})}{1.002 \times 10^{-3}\mathrm{kg/m \cdot s}} = 2191 < 2300$$

So, the flow inside the tube is laminar. Based on the analogy between heat and mass transfer, the Nusselt and the Sherwood numbers in this case are Nu = Sh = 3.66. Hence,

$$\mathrm{Sh} = \frac{h_{\mathrm{mass}} D}{D_{AB}} = 3.66$$

The mass transfer coefficient becomes

$$h_{\mathrm{mass}} = 3.66 \frac{D_{AB}}{D} = 3.66 \left(\frac{1.5 \times 10^{-9}\mathrm{m^2/s}}{0.02\ \mathrm{m}} \right) = 2.745 \times 10^{-7}\mathrm{m/s}$$

The rate of mass convection of copper from the inner tube surface to the water is

$$\dot{m}_{\mathrm{conv}} = h_{\mathrm{mass}} A_s (\rho_{A,s} - \rho_{A,\infty}) = h_{\mathrm{mass}} (\pi D L)(\rho_{A,s} - \rho_{A,\infty})$$

At the tube inner surface, the mass concentration of copper in water is $\rho_{A,s}$ = 50 g/m^3; and for the water flow $\rho_{A,\infty} \approx 0$,

$$\dot{m}_{\mathrm{conv}} = (2.745 \times 10^{-7}\mathrm{m/s})\pi(0.02\ \mathrm{m})(100\ \mathrm{m})(50 - 0)\ \mathrm{g/m^3} = 8.624 \times 10^{-5}\mathrm{g/s} = 0.08624\ \mathrm{mg/s}$$

The volumetric flow rate of water in the tube is

$$VA = V\pi \left(\frac{\mathrm{D}^2}{4} \right) = (0.11\ \mathrm{m/s})\pi \frac{(0.02\ \mathrm{m})^2}{4} = 3.456 \times 10^{-5}\mathrm{m^3/s} = 0.03456\mathrm{L/s}$$

The level of copper in the water flowing from the tube is

$$\text{Copper level in water} = \frac{\dot{m}_{\mathrm{conv}}}{VA} = \frac{0.08624\ \mathrm{mg/s}}{0.03456\mathrm{L/s}} = \mathbf{2.49\ mg/L} > 1.3\mathrm{mg/L}$$

Discussion Under the given conditions, the copper level in the water flowing out of the tube is almost twice the 1.3 mg/L limit set by the NPDWR. Therefore, the water is not safe for public use. Additional steps of treatment are necessary to reduce the copper contamination in the water.

Simultaneous Heat and Mass Transfer

14-152C In steady operation, the mass transfer process does not have to involve heat transfer. However, a mass transfer process that involves phase change (evaporation, sublimation, condensation, melting etc.) must involve heat transfer. For example, the evaporation of water from a lake into air (mass transfer) requires the transfer of latent heat of water at a specified temperature to the liquid water at the surface (heat transfer).

14-153C During evaporation from a water body to air, the latent heat of vaporization will be equal to *convection* heat transfer from the air when *conduction* from the lower parts of the water body to the surface is negligible, and temperature of the surrounding surfaces is at about the temperature of the water surface so that the *radiation* heat transfer is negligible.

14-154C It is possible for a shallow body of water to freeze during a cool and dry night even when the ambient air and surrounding surface temperatures never drop to 0°C. This is because when the air is not saturated ($\phi < 100$ percent), there will be a difference between the concentration of water vapor at the water-air interface (which is always saturated) and some distance above it. Concentration difference is the driving force for mass transfer, and thus this concentration difference will drive the water into the air. But the water must vaporize first, and it must absorb the latent heat of vaporization from the water. The temperature of water near the surface must drop as a result of the sensible heat loss, possibly below the freezing point.

14-155E In a hot summer day, a bottle of drink is to be cooled by wrapping it in a wet cloth, and blowing air to it. The temperature of the drink in the bottle when steady conditions are reached is to be determined.

Assumptions **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 80°F). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** Radiation effects are negligible.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2$ which cannot be determined at this point because of the unknown surface temperature *T*BsB. We know that $T_s < T_\infty$ and, for the purpose of property evaluation, we take *T*BsB to be 60°F. Then the properties of water at 60°F and the properties of dry air at the average temperature of (60+80)/2 = 70°F and 1 atm are (Tables A-9E and A-15E)

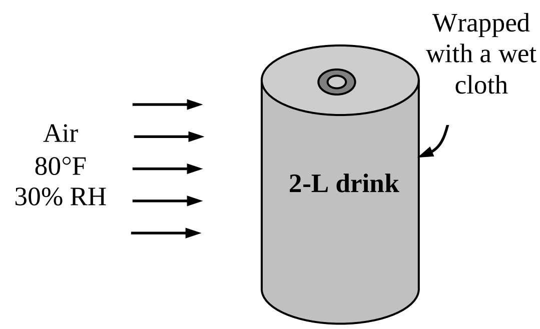
$$\text{Water at 60°F: } h_{fg} = 1060 \text{ Btu/lbm},\ P_v = 0.2563 \text{ psia. Also, at 80°F, } P_{sat@80°F} = 0.5073 \text{ psia}$$

$$\text{Dry air at 70°F: } c_p = 0.24 \text{ Btu/lbm}\cdot°\text{F},\ \alpha = 0.8093 \text{ ft}^2/\text{h} = 2.25\times10^{-4} \text{ ft}^2/\text{s}$$

Also, the molar masses of water and air are 18 and 29 lbm/lbmol, respectively (Table A-1E), and the mass diffusivity of water vapor in air at 70°F (= 294.4 K) is

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(294.4\text{K})^{2.072}}{1\text{atm}} = 2.44\times10^{-5} \text{ m}^2/\text{s} = 2.63\times10^{-4} \text{ ft}^2/\text{s}$$

Analysis The surface temperature of the jug can be determined by rearranging Chilton-Colburn equation as



$$T_s = T_\infty - \frac{h_{fg}}{c_p \text{Le}^{2/3}}\frac{M_v}{M}\frac{P_{v,s} - P_{v,\infty}}{P}$$

where the Lewis number is

$$\text{Le} = \frac{\alpha}{D_{AB}} = \frac{2.25\times10^{-4} \text{ ft}^2/\text{s}}{2.63\times10^{-4} \text{ ft}^2/\text{s}} = 0.856$$

Note that we could take the Lewis number to be 1 for simplicity, but we chose to incorporate it for better accuracy.

The air at the surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (0.2563 psia). The vapor pressure of air far from the surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat}@T_\infty} = (0.3)P_{\text{sat}@80°\text{F}} = (0.3)(0.5073\text{psia}) = 0.152\text{psia}$$

Noting that the atmospheric pressure is 1 atm = 14.7 psia, substituting the known quantities gives

$$T_s = 80°\text{F} - \frac{1060 \text{ Btu/lbm}}{(0.24 \text{ Btu/lbm.°F})(0.856)^{2/3}}\left(\frac{18 \text{ lbm/lbmol}}{29 \text{ lbm/lbmol}}\right)\frac{(0.2563 - 0.152)\text{psia}}{14.7 \text{ psia}} = \mathbf{58.4°F}$$

Therefore, the temperature of the drink can be lowered to 58.4°F by this process.

Discussion Note that the value obtained is very close to the assumed value of 60°F for the surface temperature. Therefore, there is no need to repeat the calculations with properties at the new surface temperature of 58.4°F

14-156 Air is blown over a jug made of porous clay to cool it by simultaneous heat and mass transfer. The temperature of the water in the jug when steady conditions are reached is to be determined.

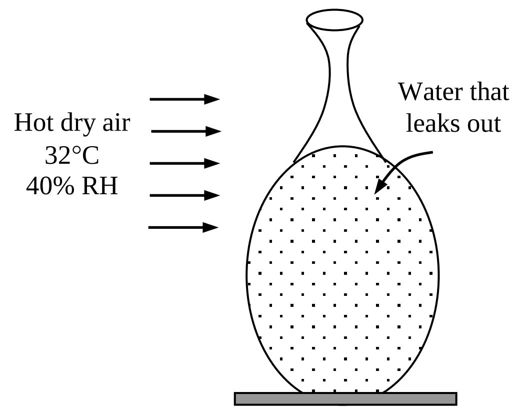
Assumptions **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** Radiation effects are negligible.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2$ which cannot be determined at this point because of the unknown surface temperature *T*BsB. We know that $T_s < T_\infty$ and, for the purpose of property evaluation, we take *T*BsB to be 20°C. Then, the properties of water at 20°C and the properties of dry air at the average temperature of 26°C and 1 atm are (Tables A-9 and A-15)

$$\text{Water at } 20°\text{C}: \; h_{fg} = 2454 \text{ kJ/kg}, \; P_v = 2.34 \text{ kPa. Also, at } 32°\text{C}, P_{sat@32°\text{C}} = 4.76 \text{ kPa}$$

$$\text{Dry air at } 26°\text{C}: \; c_p = 1.007 \text{ kJ/kg}\cdot°\text{C}, \; \alpha = 2.154\times10^{-5} \text{ m}^2/\text{s}$$

Also, the mass diffusivity of water vapor in air at 26°C is $D_{H_2O\text{-air}} = 2.518\times10^{-5}$ m^2/s (Table 14-4), and the molar masses of water and air are 18 and 29 kg/kmol, respectively (Table A-1).



Analysis The surface temperature of the jug can be determined by rearranging Chilton-Colburn equation as

$$T_s = T_\infty - \frac{h_{fg}}{c_p \text{Le}^{2/3}} \frac{M_v}{M} \frac{P_{v,s} - P_{v,\infty}}{P}$$

where the Lewis number is

$$\text{Le} = \frac{\alpha}{D_{AB}} = \frac{2.154\times10^{-5} \text{ m}^2/\text{s}}{2.518\times10^{-5} \text{ m}^2/\text{s}} = 0.8554$$

Note that we could take the Lewis number to be 1 for simplicity, but we chose to incorporate it for better accuracy.

The air at the surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (2.34 kPa). The vapor pressure of air far from the surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat@}T_\infty} = (0.40)P_{\text{sat@32°C}} = (0.40)(4.76 \text{ kPa}) = 1.904 \text{ kPa}$$

Noting that the atmospheric pressure is 1 atm = 101.3 Pa, substituting the known quantities gives

$$T_s = 32°\text{C} - \frac{2454 \text{ kJ/kg}}{(1.007 \text{ kJ/kg.°C})(0.8554)^{2/3}} \frac{18 \text{ kg/kmol}}{29 \text{ kg/kmol}} \frac{(2.34 - 1.904) \text{ kPa}}{101.3 \text{ kPa}} = \mathbf{24.8°C}$$

Therefore, the temperature of the drink can be lowered to 24.8°C by this process.

Discussion The accuracy of this result can be improved by repeating the calculations with dry air properties evaluated at (32+24.8)/2 = 28.4°C and water properties at 24.8°C.

14-157 Prob. 14-156 is reconsidered. The water temperature as a function of the relative humidity of air is to be plotted.

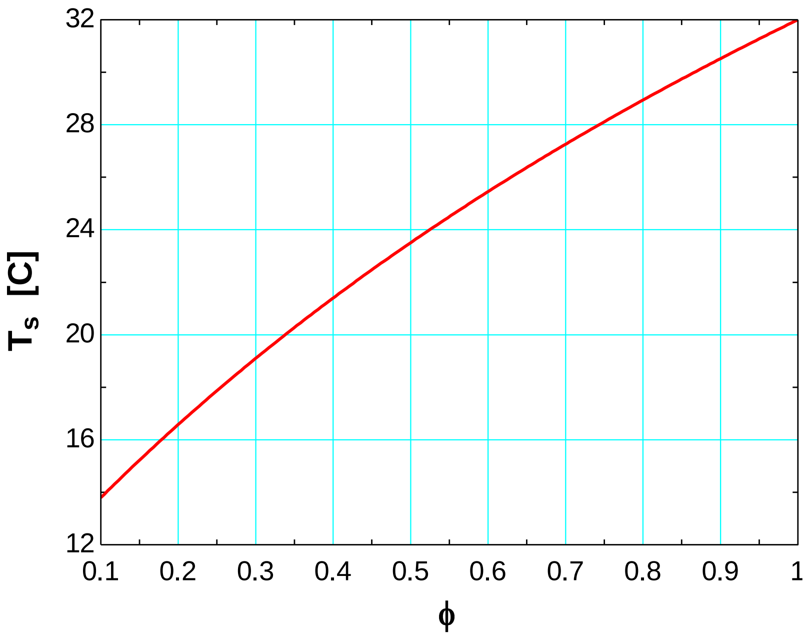
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
P=101.3 [kPa]
T_infinity=32 [C]
phi=0.40

"PROPERTIES"
Fluid$='steam_IAPWS'
h_f=enthalpy(Fluid$, T=T_s, x=0)
h_g=enthalpy(Fluid$, T=T_s, x=1)
h_fg=h_g-h_f
P_sat_s=Pressure(Fluid$, T=T_s, x=0)
P_sat_infinity=Pressure(Fluid$, T=T_infinity, x=0)
c_p_air=CP(air, T=T_ave)
T_ave=1/2*(T_infinity+T_s)
alpha=2.18E-5 [m^2/s] "from the text"
D_AB=2.50E-5 [m^2/s] "from the text"
MM_H2O=molarmass(H2O)
MM_air=molarmass(air)

"ANALYSIS"
Le=alpha/D_AB
P_v_infinity=phi*P_sat_infinity
P_v_s=P_sat_s
T_s=T_infinity-h_fg/(c_p_air*Le^(2/3))*MM_H2O/MM_air*(P_v_s-P_v_infinity)/P
```

ϕ	T_s [C]
0.1	13.79
0.15	15.22
0.2	16.58
0.25	17.87
0.3	19.1
0.35	20.28
0.4	21.4
0.45	22.48
0.5	23.51
0.55	24.5
0.6	25.45
0.65	26.37
0.7	27.26
0.75	28.11
0.8	28.94
0.85	29.74
0.9	30.52
0.95	31.27
1	32



14-158 A soaked sponge is experiencing dry air flow over its surface. The temperature difference, $T_\infty - T_s$, is to be determined if it is soaked with (*a*) water and (*b*) ammonia.

Assumptions **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fractions of water and ammonia vapor in the air are low. **2** Air and water and ammonia vapors at specified conditions are ideal gases. **3** Radiation effects are negligible.

Properties The properties of air at 1 atm and 20°C (Table A-15):

Air: c_p = 1.007 kJ/kg·K, $\alpha = 2.074 \times 10^{-5}$ m²/s, M = 28.97 kg/kmol (Table A-1)

The properties of water at 10°C (Table A-9):

Water: h_{fg} = 2478 kJ/kg, $P_{v,s}$ = 1.2276 kPa, M_v = 18.015 kg/kmol (Table A-1)

The properties of ammonia at −40°C (Table A-11):

NH_3: h_{fg} = 1389 kJ/kg, $P_{v,s}$ = 71.66 kPa, M_v = 17.03 kg/kmol (Table A-1)

Analysis (*a*) For mass transfer between water vapor and air, the Lewis number is

$$\text{Le} = \frac{\alpha}{D_{AB}} = \frac{2.074\times10^{-5}\ \text{m}^2/\text{s}}{2.42\times10^{-5}\ \text{m}^2/\text{s}} = 0.8570$$

The temperature difference ($T_\infty - T_s$) can be determined from the Chilton-Colburn analogy, Eq. 14-92:

$$\begin{aligned} T_\infty - T_s &= \frac{h_{fg}}{c_p \text{Le}^{2/3}} \frac{M_v}{M} \frac{P_{v,s} - P_{v,\infty}}{P} \\ &= \frac{(2478\ \text{kJ/kg})}{(1.007\ \text{kJ/kg}\cdot\text{K})(0.8570)^{2/3}}\left(\frac{18.015}{28.97}\right)\left(\frac{1.2276-0}{101.325}\right) \\ &= \mathbf{20.5\ K} \end{aligned}$$

(*b*) For mass transfer between ammonia and air, the Lewis number is

$$\text{Le} = \frac{\alpha}{D_{AB}} = \frac{2.074\times10^{-5}\ \text{m}^2/\text{s}}{2.6\times10^{-5}\ \text{m}^2/\text{s}} = 0.7977$$

The temperature difference ($T_\infty - T_s$) can be determined from the Chilton-Colburn analogy, Eq. 14-92:

$$\begin{aligned} T_\infty - T_s &= \frac{h_{fg}}{c_p \text{Le}^{2/3}} \frac{M_v}{M} \frac{P_{v,s} - P_{v,\infty}}{P} \\ &= \frac{(1389\ \text{kJ/kg})}{(1.007\ \text{kJ/kg}\cdot\text{K})(0.7977)^{2/3}}\left(\frac{17.03}{28.97}\right)\left(\frac{71.66-0}{101.325}\right) \\ &= \mathbf{667\ K} \end{aligned}$$

Discussion The much higher temperature difference ($T_\infty - T_s$) experienced on the ammonia-soaked sponge means that ammonia can achieve greater evaporative cooling than water. Indeed, because of ammonia's vaporization properties, it has been used as a refrigerant.

14-159 A water-soaked 10 cm × 10 cm square sponge is experiencing heat transfer by convection and radiation. (*a*) The rate of evaporation of water from the sponge and (*b*) the net radiation heat transfer rate are to be determined.

Assumptions **1** The analogy between heat and mass transfer is applicable. **2** Steady state condition exists. **3** Constant properties. **4** Water vapor behaves as ideal gas. **5** The bottom surface of the sponge is well insulated.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the average temperature of $(T_\infty+T_s)/2 = (20+30)°C/2 = 25°C$. The properties of dry air at 25°C and 1 atm are (Table A-15)

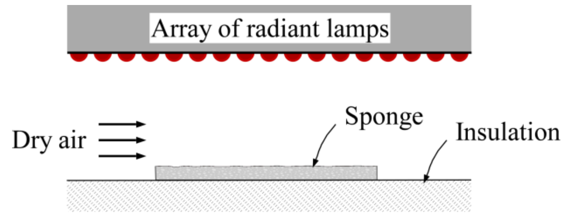
Dry air: $c_p = 1007$ J/kg·K, $\rho = 1.184$ kg/m^3, and $\alpha = 2.141\times10^{-5}$ m^2/s.

Then the properties of water at 30°C are (Table A-9)

Water: $\rho_{A,s} = \rho_v = 0.0304$ kg/m^3 and $h_{fg} = 2431$ kJ/kg.

The mass diffusivity of water vapor in air at 25°C is

$D_{AB} = 2.50\times10^{-5}$ m^2/s (Table 14-4).



Analysis (*a*) Applying the Chilton-Colburn analogy,

$$\frac{h_{\text{heat}}}{h_{\text{mass}}} = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{2/3} \quad \rightarrow \quad h_{\text{mass}} = \frac{h_{\text{heat}}}{\rho c_p}\left(\frac{D_{AB}}{\alpha}\right)^{2/3}$$

Hence, the average mass convection coefficient is

$$h_{\text{mass}} = \frac{30 \text{ W/m}^2\cdot\text{K}}{(1.184\text{ kg/m}^3)(1007\text{ J/kg}\cdot\text{K})}\left(\frac{2.5\times10^{-5}}{2.141\times10^{-5}}\right)^{2/3} = 27.9\times10^{-3}\text{ m/s}$$

The evaporation rate is then

$$\begin{aligned} \dot{m}_v &= \dot{m}_{\text{conv}} = h_{\text{mass}} A_s(\rho_{A,s} - \rho_{A,\infty}) \\ &= (27.9\times10^{-3}\text{ m/s})(0.1\text{ m})^2(0.0304-0)\text{ kg/m}^3 \\ &= \mathbf{8.48\times10^{-6}\ kg/s} \end{aligned}$$

(*b*) Performing energy balance on the sponge, considering the processes of evaporation, convection and radiation, we have

$$\dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = \dot{Q}_{\text{evap}} \quad \rightarrow \quad \dot{Q}_{\text{rad}} = \dot{Q}_{\text{evap}} - \dot{Q}_{\text{conv}} = \dot{m}_v h_{fg} - h_{heat} A_s(T_\infty - T_s)$$

$$\begin{aligned} \dot{Q}_{\text{rad}} &= (8.48\times10^{-6}\text{ kg/s})(2431\times10^3\text{ J/kg}) - (30\text{ W/m}^2\cdot\text{K})(0.1\text{ m})^2(20-30)\text{ K} \\ &= 20.6\text{ W} - (-3\text{ W}) \\ &= \mathbf{23.6\ W} \end{aligned}$$

Discussion Note that the heat transfer by evaporation is about 7 times larger than the heat transfer by convection. Also, for dry air flow the density of water vapor at the free stream is negligible.

14-160 A thin layer of liquid water on a concrete surface is experiencing simultaneous heat and mass transfer. The conduction heat flux through the concrete is to be determined.

Assumptions **1** The analogy between heat and mass transfer is applicable. **2** Steady state condition exists. **3** Constant properties. **4** Water vapor behaves as ideal gas. **5** The bottom surface of the concrete is well insulated.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the average temperature of $(T_\infty+T_s)/2 = (30+20)°C/2 = 25°C$. The properties of dry air at 25°C and 1 atm are (Table A-15)

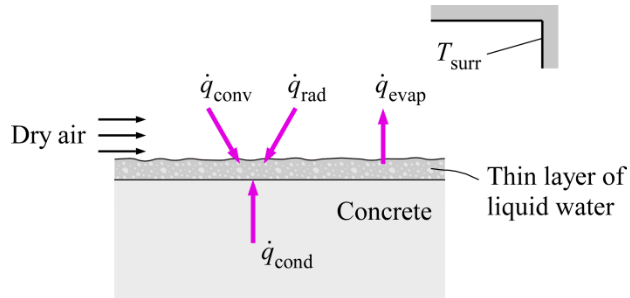
Dry air: $c_p = 1007$ J/kg·K, $\rho = 1.184$ kg/m^3, and $\alpha = 2.141\times10^{-5}$ m^2/s.

Then the properties of water at 20°C are (Table A-9)

Water: $\rho_{A,s} = \rho_v = 0.0173$ kg/m^3 and $h_{fg} = 2454$ kJ/kg.

The mass diffusivity of water vapor in air at 25°C is

$D_{AB} = 2.50\times10^{-5}$ m^2/s (Table 14-4).



Analysis Applying the Chilton-Colburn analogy,

$$\frac{h_{heat}}{h_{mass}} = \rho c_p \left(\frac{\alpha}{D_{AB}}\right)^{2/3} \quad \rightarrow \quad h_{mass} = \frac{h_{heat}}{\rho c_p}\left(\frac{D_{AB}}{\alpha}\right)^{2/3}$$

Hence, the average mass convection coefficient is

$$h_{mass} = \frac{50\ \text{W/m}^2\cdot\text{K}}{(1.184\ \text{kg/m}^3)(1007\ \text{J/kg}\cdot\text{K})}\left(\frac{2.5\times10^{-5}}{2.141\times10^{-5}}\right)^{2/3} = 46.5\times10^{-3}\ \text{m/s}$$

The evaporation rate per unit area is then

$$\begin{aligned} j_v &= \dot{m}_{conv}/A_s = h_{mass}(\rho_{A,s} - \rho_{A,\infty}) \\ &= (46.5\times10^{-3}\ \text{m/s})(0.0173 - 0)\ \text{kg/m}^3 \\ &= 8.045\times10^{-4}\ \text{kg/s}\cdot\text{m}^2 \end{aligned}$$

Then, the heat flux for each of the heat transfer process is

Evaporation: $\dot{q}_{evap} = j_v h_{fg} = (8.045\times10^{-4}\ \text{kg/s}\cdot\text{m}^2)(2454\times10^3\ \text{J/kg}) = 1974\ \text{W/m}^2$

Convection: $\dot{q}_{conv} = h_{heat}(T_\infty - T_s) = (50\ \text{W/m}^2\cdot\text{K})(30-20)\ \text{K} = 500\ \text{W/m}^2$

Radiation: $\dot{q}_{rad} = \varepsilon\sigma(T_{surr} - T_s) = (0.95)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(303^4 - 293^4)\ \text{K}^4 = 57.03\ \text{W/m}^2$

Performing energy balance on liquid water layer, considering the processes of evaporation, convection, radiation and conduction, we have

$$\dot{q}_{conv} + \dot{q}_{rad} + \dot{q}_{cond} = \dot{q}_{evap}$$

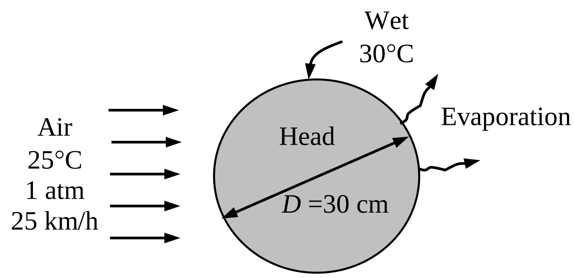
$$\begin{aligned} \dot{q}_{cond} &= \dot{q}_{evap} - \dot{q}_{conv} - \dot{q}_{rad} \\ &= 1974\ \text{W} - 500\ \text{W} - 57.03\ \text{W} \\ &= \mathbf{1417\ W} \end{aligned}$$

Discussion The positive value of the conduction heat flux through the concrete indicates that heat flux by conduction is going into the liquid water layer.

14-161 A person is standing outdoors in windy weather. The rates of heat loss from the head by radiation, forced convection, and evaporation are to be determined for the cases of the head being wet and dry.

Assumptions **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The head can be approximated as a sphere of 30 cm diameter maintained at a uniform temperature of 30°C. **4** The surrounding surfaces are at the same temperature as the ambient air.

Properties The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture. The properties of air at the free stream temperature of 25°C and 1 atm are, from Table A-15,



$$k = 0.02551 \text{ W/m}\cdot\text{C}, \quad \text{Pr} = 0.7296$$

$$\mu = 1.849\times10^{-5} \text{ kg/m}\cdot\text{s} \quad \nu = 1.562\times10^{-5} \text{ m}^2/\text{s}$$

Also, $\mu_s = \mu_{@30°\text{C}} = 1.872\times10^{-5} \text{ kg/m}\cdot\text{s}$.

The mass diffusivity of water vapor in air at the average temperature of (25 + 30)/2 = 27.5°C = 300.5 K is, from Eq. 14-15,

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(300.5\text{K})^{2.072}}{1\text{atm}} = 2.55\times10^{-5} \text{ m}^2/\text{s}$$

The saturation pressure of water at 25°C is $P_{\text{sat@25°C}} = 3.169 \text{ kPa}$. Properties of water at 30°C are $h_{fg} = 2431 \text{ kJ/kg}$ and $P_v = 4.246 \text{ kPa}$ (Table A-9).

The gas constants of dry air and water are R_{air} = 0.287 kPa.m^3/kg.K and R_{water} = 0.4615 kPa.m^3/kg.K (Table A-1). Also, the emissivity of the head is given to be 0.95.

Analysis (*a*) When the head is dry, heat transfer from the head is by forced convection and radiation only. The radiation heat transfer is

$$\dot{Q}_{\text{rad}} = \varepsilon A_s\sigma(T_s^4 - T_{\text{surr}}^4) = (0.95)[\pi(0.3\text{ m})^2](5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)[(30+273\text{ K})^4 - (25+273\text{ K})^4] = 8.3\text{ W}$$

The Reynolds number for flow over the head is

$$\text{Re} = \frac{VD}{\nu} = \frac{(25/3.6 \text{ m/s})(0.3\text{m})}{1.562\times10^{-5} \text{ m}^2/\text{s}} = 133{,}380$$

Then the Nusselt number and the heat transfer coefficient become

$$\text{Nu} = 2 + \left[0.4\,\text{Re}^{1/2} + 0.06\,\text{Re}^{2/3}\right]\text{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(133{,}380)^{1/2} + 0.06(133{,}380)^{2/3}\right](0.7296)^{0.4}\left(\frac{1.849\times10^{-5}}{1.872\times10^{-5}}\right)^{1/4} = 268$$

$$h = \frac{k}{D}Nu = \frac{0.02551 \text{ W/m}\cdot°\text{C}}{0.3 \text{ m}}(268) = 22.8 \text{ W/m}^2\cdot°\text{C}$$

Then the rate of convection heat transfer from the head becomes

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) = (22.8\text{W/m}^2.°\text{C})[\pi(0.3\text{m})^2](30-25)°\text{C} = 32.2\text{W}$$

Therefore,

$$\dot{Q}_{\text{total,dry}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 32.2 + 8.3 = \mathbf{40.5\,W}$$

(*b*) When the head is wet, there is additional heat transfer mechanism by evaporation. The Schmidt number is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562\times10^{-5} \text{ m}^2/\text{s}}{2.55\times10^{-5} \text{ m}^2/\text{s}} = 0.613$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\mathrm{Sh} = 2 + \left[0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}\right]\mathrm{Sc}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$= 2 + \left[0.4(133{,}380)^{1/2} + 0.06(133{,}380)^{2/3}\right](0.613)^{0.4}\left(\frac{1.849\times10^{-5}}{1.872\times10^{-5}}\right)^{1/4} = 250$$

$$h_{\text{mass}} = \frac{\mathrm{Sh}D_{AB}}{L} = \frac{(250)(2.55\times10^{-5}\ \mathrm{m^2/s})}{0.3\,\mathrm{m}} = 0.0213\,\mathrm{m/s}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (4.246 kPa at 30°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat@}T_\infty} = (0.30)P_{\text{sat@25°C}} = (0.30)(3.169\,\mathrm{kPa}) = 0.9507\,\mathrm{kPa}$$

Treating the water vapor and the air as ideal gases, the vapor densities at the water-air interface and far from the surface are determined to be

At the surface: $$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{4.246\,\mathrm{kPa}}{(0.4615\,\mathrm{kPa\cdot m^3/kg\cdot K})(30+273)\,\mathrm{K}} = 0.0304\,\mathrm{kg/m^3}$$

Away from the surface: $$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{0.9507\,\mathrm{kPa}}{(0.4615\,\mathrm{kPa\cdot m^3/kg\cdot K})(25+273)\,\mathrm{K}} = 0.0069\,\mathrm{kg/m^3}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\dot{m}_v = h_{\text{mass}}A_s(\rho_{v,s} - \rho_{v,\infty}) = (0.0213\,\mathrm{m/s})[\pi(0.3\,\mathrm{m})^2](0.0304 - 0.0069)\,\mathrm{kg/m^3}$$
$$= 0.0001415\,\mathrm{kg/s}$$

and

$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.0001415\,\mathrm{kg/s})(2431\mathrm{kJ/kg}) = 0.344\,\mathrm{kW} = 344\,\mathrm{W}$$

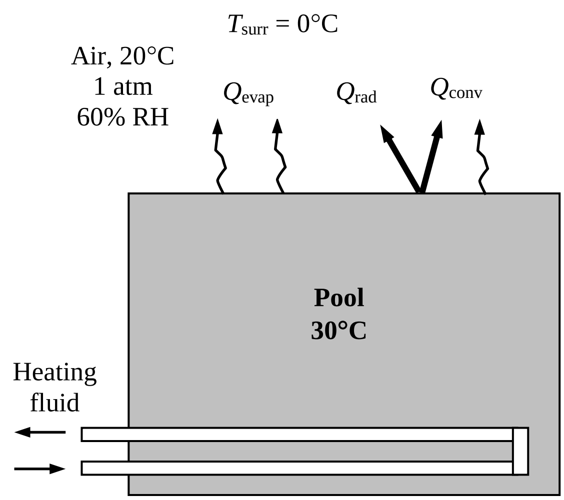
Then the total rate of heat loss from the wet head to the surrounding air and surfaces becomes

$$\dot{Q}_{\text{total,wet}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} + \dot{Q}_{\text{evap}} = 32.2 + 8.3 + 344 = \mathbf{385\,W}$$

Discussion Note that the heat loss from the head can be increased by more than 9 times in this case by wetting the head and allowing heat transfer by evaporation.

14-162 The heating system of a heated swimming pool is being designed. The rates of heat loss from the top surface of the pool by radiation, natural convection, and evaporation are to be determined.

Assumptions **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The entire water body in the pool is maintained at a uniform temperature of 30°C. **4** The air motion around the pool is negligible so that there are no forced convection effects.



Properties The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2$ = (20+30)/2 = 25°C = 298 K. The properties of dry air at 298 K and 1 atm are, from Table A-15,

$$k = 0.02551\,\text{W/m}\cdot°\text{C},\ \Pr = 0.7296$$

$$\alpha = 2.141\times10^{-5}\ \text{m}^2/\text{s} \quad \nu = 1.562\times10^{-5}\ \text{m}^2/\text{s}$$

The mass diffusivity of water vapor in air at the average temperature of 298 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P}$$

$$= 1.87\times10^{-10}\frac{(298\text{K})^{2.072}}{1\text{atm}} = 2.50\times10^{-5}\ \text{m}^2/\text{s}$$

The saturation pressure of water at 20°C is $P_{\text{sat@20°C}} = 2.339\ \text{kPa}$.

Properties of water at 30°C are

$$h_{fg} = 2431\ \text{kJ/kg}\ \text{and}\ P_v = 4.246\ \text{kPa}\ \text{(Table A-9)}.$$

The gas constants of dry air and water are R_{air} = 0.287 kPa.m^3/kg.K and R_{water} = 0.4615 kPa.m^3/kg.K (Table A-1). The emissivity of water is 0.95 (Table A-18).

Analysis (*a*) The surface area of the pool is

$$A = (25\,\text{m})(25\,\text{m}) = 625\,\text{m}^2$$

Heat transfer from the top surface of the pool by radiation is

$$\dot{Q}_{\text{rad}} = \varepsilon A\sigma(T_s^4 - T_{\text{surr}}^4) = (0.95)(625\,\text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(30+273\,\text{K})^4 - (0+273\,\text{K})^4\right] = \mathbf{96{,}770\ W}$$

(*b*) The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (4.246 kPa at 30°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat@}T_\infty} = (0.60)P_{\text{sat@20°C}} = (0.60)(2.339\,\text{kPa}) = 1.40\,\text{kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

At the surface:

$$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{4.246\,\text{kPa}}{(0.4615\,\text{kPa.m}^3/\text{kg}\cdot\text{K})(30+273)\,\text{K}} = 0.0304\,\text{kg/m}^3$$

$$\rho_{a,s} = \frac{P_{a,s}}{R_a T_s} = \frac{(101.325-4.246)\,\text{kPa}}{(0.287\,\text{kPa.m}^3/\text{kg}\cdot\text{K})(30+273)\,\text{K}} = 1.1164\,\text{kg/m}^3$$

$$\rho_s = \rho_{v,s} + \rho_{a,s} = 0.0304 + 1.1164 = 1.1468\,\text{kg/m}^3$$

Away from the surface:

$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{1.40\,\text{kPa}}{(0.4615\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273)\,\text{K}} = 0.0104\,\text{kg/m}^3$$

$$\rho_{a,\infty} = \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325-1.40)\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273)\,\text{K}} = 1.1883\,\text{kg/m}^3$$

$$\rho_\infty = \rho_{v,\infty} + \rho_{a,\infty} = 0.0104 + 1.1883 = 1.1987\,\text{kg/m}^3$$

Note that $\rho_\infty > \rho_s$, and thus this corresponds to hot surface facing up. The perimeter of the top surface of the pool is p = 1.5(25+ 25) = 75 m. Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{625\,\text{m}^2}{75\,\text{m}} = 8.333\,\text{m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{ave}}\nu^2} = \frac{(9.81\,\text{m/s}^2)(1.1987 - 1.1468\,\text{kg/m}^3)(8.333\,\text{m})^3}{[(1.1987 + 1.1468)/2\,\text{kg/m}^3](1.562\times10^{-5}\,\text{m}^2/\text{s})^2} = 1.03\times10^{12}$$

Recognizing that this is a natural convection problem with hot horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be

$$\text{Nu} = 0.15(\text{Gr Pr})^{1/3} = 0.15(1.03\times10^{12}\times0.7296)^{1/3} = 1364$$

and $$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(1364)(0.02551\,\text{W/m}\cdot{}^\circ\text{C})}{8.333\,\text{m}} = 4.174\,\text{W/m}^2\cdot{}^\circ\text{C}$$

Then natural convection heat transfer rate becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}}A_s(T_s - T_\infty) = (4.174\,\text{W/m}^2\cdot{}^\circ\text{C})(625\,\text{m}^2)(30-20)^\circ\text{C} = \mathbf{26{,}090\,W}$$

(c) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562\times10^{-5}\,\text{m}^2/\text{s}}{2.50\times10^{-5}\,\text{m}^2/\text{s}} = 0.625$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\text{Sh} = 0.15(\text{GrSc})^{1/3} = 0.15(1.03\times10^{12}\times0.625)^{1/3} = 1295$$

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(1295)(2.50\times10^{-5}\,\text{m}^2/\text{s})}{8.333\,\text{m}} = 0.003886\,\text{m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\dot{m}_v = h_{\text{mass}}A_s(\rho_{v,s} - \rho_{v,\infty}) = (0.003886\,\text{m/s})(625\,\text{m}^2)(0.0304 - 0.0104)\text{kg/m}^3$$
$$= 0.04857\,\text{kg/s} = 174.9\,\text{kg/h}$$

and $$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.04857\,\text{kg/s})(2{,}431{,}000\,\text{J/kg}) = \mathbf{118{,}070\,W}$$

Then the total rate of heat loss from the open top surface of the pool to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 96{,}770 + 26{,}090 + 118{,}070 = \mathbf{240{,}930\,W}$$

Therefore, if the pool is heated electrically, a 241 kW resistance heater will be needed to make up for the heat losses from the top surface.

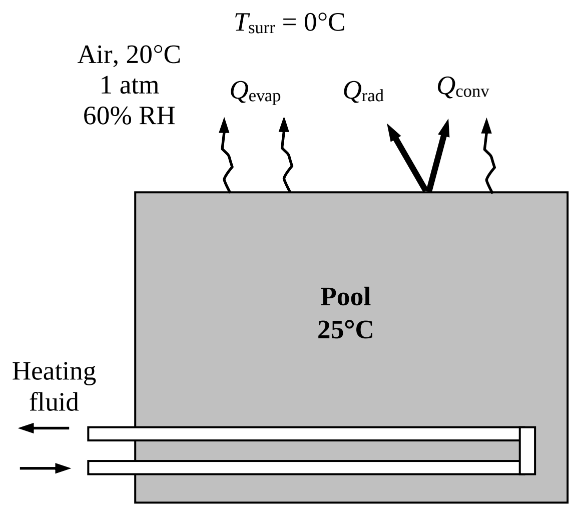
14-163 The heating system of a heated swimming pool is being designed. The rates of heat loss from the top surface of the pool by radiation, natural convection, and evaporation are to be determined.

Assumptions **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The entire water body in the pool is maintained at a uniform temperature of 25°C. **4** The air motion around the pool is negligible so that there are no forced convection effects.

Properties The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2 = (20+25)/2 = 22.5°C =$ 295.5 K. The properties of dry air at 22.5°C and 1 atm are, from Table A-15,

$$k = 0.02533\ \text{W/m}\cdot°\text{C},\quad \Pr = 0.7303$$

$$\alpha = 2.108\times10^{-5}\ \text{m}^2/\text{s}\quad \nu = 1.539\times10^{-5}\ \text{m}^2/\text{s}$$



The mass diffusivity of water vapor in air at the average temperature of 295.5 K is, from Eq. 14-15,

$$D_{AB} = D_{H_2O\text{-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P}$$

$$= 1.87\times10^{-10}\frac{(295.5\text{K})^{2.072}}{1\text{atm}}$$

$$= 2.46\times10^{-5}\ \text{m}^2/\text{s}$$

The saturation pressure of water at 20°C is $P_{sat@20°C} = 2.339\ \text{kPa}$. Properties of water at 25°C are $h_{fg} = 2442\ \text{kJ/kg}$ and $P_v = 3.169\ \text{kPa}$ (Table A-9). The gas constants of dry air and water are R_{air} = 0.287 kPa.m^3/kg.K and R_{water} = 0.4615 kPa.m^3/kg.K (Table A-1). The emissivity of water is 0.95 (Table A-18).

Analysis (*a*) The surface area of the pool is

$$A_s = (25\ \text{m})(25\text{m}) = 625\ \text{m}^2$$

Heat transfer from the top surface of the pool by radiation is

$$\dot{Q}_{rad} = \varepsilon A\sigma(T_s^4 - T_{surr}^4) = (0.95)(625\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[(25+273\ \text{K})^4 - (0+273\ \text{K})^4\right] = \mathbf{78{,}490\ W}$$

(*b*) The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (3.169 kPa at 25°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{sat@T_\infty} = (0.60)P_{sat@20°C} = (0.60)(2.339\ \text{kPa}) = 1.40\ \text{kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

At the surface:

$$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{3.169\ \text{kPa}}{(0.4615\ \text{kPa.m}^3/\text{kg}\cdot\text{K})(25+273)\ \text{K}} = 0.0230\ \text{kg/m}^3$$

$$\rho_{a,s} = \frac{P_{a,s}}{R_a T_s} = \frac{(101.325-3.169)\ \text{kPa}}{(0.287\ \text{kPa.m}^3/\text{kg}\cdot\text{K})(25+273)\ \text{K}} = 1.1477\ \text{kg/m}^3$$

$$\rho_s = \rho_{v,s} + \rho_{a,s} = 0.0230 + 1.1477 = 1.1707\ \text{kg/m}^3$$

Away from the surface:

$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{1.40\ \text{kPa}}{(0.4615\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273)\ \text{K}} = 0.0104\ \text{kg/m}^3$$

$$\rho_{a,\infty} = \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325-1.40)\ \text{kPa}}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273)\ \text{K}} = 1.1883\ \text{kg/m}^3$$

$$\rho_\infty = \rho_{v,\infty} + \rho_{a,\infty} = 0.0104 + 1.1883 = 1.1987\ \text{kg/m}^3$$

Note that $\rho_\infty > \rho_s$, and thus this corresponds to hot surface facing up. The perimeter of the top surface of the pool is p = 1.5(25+ 25) = 75 m. Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{625\,\text{m}^2}{75\,\text{m}} = 8.333\,\text{m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{avg}}\nu^2} = \frac{(9.81\,\text{m/s}^2)(1.1987 - 1.1707\,\text{kg/m}^3)(8.333\,\text{m})^3}{[(1.1987 + 1.1707)/2\,\text{kg/m}^3](1.539\times10^{-5}\,\text{m}^2/\text{s})^2} = 5.664\times10^{11}$$

Recognizing that this is a natural convection problem with hot horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be

$$\text{Nu} = 0.15(\text{Gr}\,\text{Pr})^{1/3} = 0.15(5.664\times10^{11}\times0.73)^{1/3} = 1117$$

and $$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(1117)(0.02533\,\text{W/m}\cdot°\text{C})}{8.333\,\text{m}} = 3.397\ \text{W/m}^2\cdot°\text{C}$$

Then natural convection heat transfer rate becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}}A_s(T_s - T_\infty) = (3.397\ \text{W/m}^2\cdot°\text{C})(625\,\text{m}^2)(25-20)°\text{C} = \mathbf{10{,}620\ W}$$

(*c*) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.539\times10^{-5}\ \text{m}^2/\text{s}}{2.46\times10^{-5}\ \text{m}^2/\text{s}} = 0.626$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\text{Sh} = 0.15(\text{GrSc})^{1/3} = 0.15(5.664\times10^{11}\times0.626)^{1/3} = 1062$$

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(1062)(2.46\times10^{-5}\ \text{m}^2/\text{s})}{8.333\,\text{m}} = 0.003134\,\text{m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned}\dot{m}_v &= h_{\text{mass}}A_s(\rho_{v,s} - \rho_{v,\infty}) = (0.003134\,\text{m/s})(625\text{m}^2)(0.0230 - 0.0104)\text{kg/m}^3\\ &= 0.02468\,\text{kg/s} = 88.85\,\text{kg/h}\end{aligned}$$

and $$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.02468\,\text{kg/s})(2{,}442{,}000\,\text{J/kg}) = \mathbf{60{,}270\ W}$$

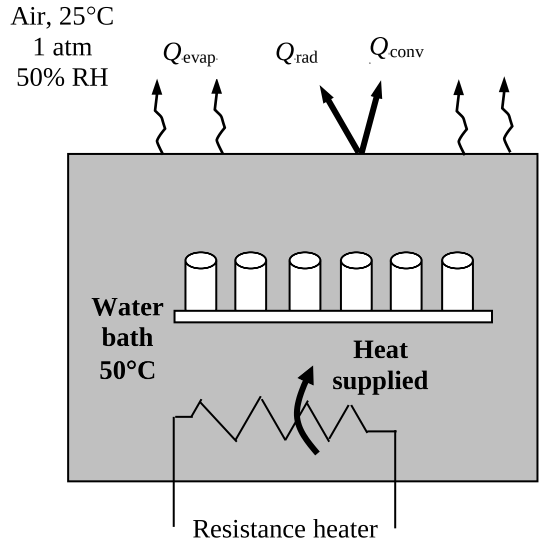
Then the total rate of heat loss from the open top surface of the pool to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 78{,}490 + 10{,}620 + 60{,}270 = \mathbf{149{,}380\ W}$$

Therefore, if the pool is heated electrically, a 149 kW resistance heater will be needed to make up for the heat losses from the top surface.

14-164 Glass bottles are washed in hot water in an uncovered rectangular glass washing bath. The rates of heat loss from the top and side surfaces of the bath by radiation, natural convection, and evaporation as well as the rates of heat and water mass that need to be supplied to the water are to be determined.

Assumptions **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The entire water body and the metal container are maintained at a uniform temperature of 50°C. **4** Heat losses from the bottom surface are negligible. **5** The air motion around the bath is negligible so that there are no forced convection effects.



Properties The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2$ = (25+50)/2 = 37.5°C. The properties of dry air at 37.5°C and 1 atm are, from Table A-15,

$$k = 0.02643\,\text{W/m}\cdot°\text{C}, \qquad \Pr = 0.7262$$

$$\alpha = 2.312\times10^{-5}\,\text{m}^2/\text{s} \qquad \nu = 1.679\times10^{-5}\,\text{m}^2/\text{s}$$

The mass diffusivity of water vapor in air at the average temperature of 310.5 K is, from Eq. 14-15,

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(310.5\,\text{K})^{2.072}}{1\text{atm}} = 2.73\times10^{-5}\,\text{m}^2/\text{s}$$

The saturation pressure of water at 25°C is $P_{\text{sat@25°C}} = 3.169\,\text{kPa}$. Properties of water at 50°C are $h_{fg} = 2383\,\text{kJ/kg}$ and $P_v = 12.35\,\text{kPa}$ (Table A-9). The specific heat of water at the average temperature of (15+50)/2 = 32.5°C is *c*B$_{\text{pB}}$ = 4.178 kJ/kg.°C.

The gas constants of dry air and water are *R*B$_{\text{airB}}$ = 0.287 kPa.mP$^{\text{3P}}$/kg.K and *R*B$_{\text{waterB}}$ = 0.4615 kPa.mP$^{\text{3P}}$/kg.K (Table A-1). Also, the emissivities of water and the sheet metal are given to be 0.61 and 0.95, respectively, and the specific heat of glass is given to be 1.0 kJ/kg.°C.

Analysis (*a*) The mass flow rate of glass bottles through the water bath in steady operation is

$$\dot m_{\text{bottle}} = m_{\text{bottle}} \times \text{Bottle flow rate} = (0.150\,\text{kg/bottle})(800\,\text{bottles/min}) = 120\,\text{kg/min} = 2\,\text{kg/s}$$

Then the rate of heat removal by the bottles as they are heated from 25 to 50°C is

$$\dot Q_{\text{bottle}} = \dot m_{\text{bottle}} c_p \Delta T = (2\,\text{kg/s})(1\text{kJ/kg.°C})(50-25)°\text{C} = 50{,}000\,\text{W}$$

The amount of water removed by the bottles is

$$\dot m_{\text{water,out}} = (\text{Flow rate of bottles})(\text{Water removed per bottle})$$

$$= (800\,\text{bottles / min})(0.6\text{g/bottle}) = 480\,\text{g/min} = \mathbf{8\times10^{-3}\,kg/s} = \mathbf{28.8\,kg/h}$$

Noting that the water removed by the bottles is made up by fresh water entering at 15°C, the rate of heat removal by the water that sticks to the bottles is

$$\dot Q_{\text{water removed}} = \dot m_{\text{water removed}} c_p \Delta T = (8\times10^{-3}\,\text{kg/s})(4178\,\text{J/kg}\cdot°\text{C})(50-15)°\text{C} = 1170\,\text{W}$$

Therefore, the total amount of heat removed by the wet bottles is

$$\dot Q_{\text{total, removed}} = \dot Q_{\text{glass removed}} + \dot Q_{\text{water removed}} = 50{,}000 + 1170 = \mathbf{51{,}170\,W}$$

(*b*) The rate of heat loss from the top surface of the water bath is the sum of the heat losses by radiation, natural convection, and evaporation. Then the radiation heat loss from the top surface of water to the surrounding surfaces is

$$\dot Q_{\text{rad, top}} = \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4) = (0.95)(8\,\text{m}^2)(5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4)[(50+273\,\text{K})^4 - (15+273\,\text{K})^4] = \mathbf{1726\,W}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (12.35 kPa at 50°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat@}T_\infty} = (0.50)P_{\text{sat@25°C}} = (0.50)(3.169\,\text{kPa}) = 1.585\,\text{kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

At the surface:

$$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{12.35\,\text{kPa}}{(0.4615\,\text{kPa.m}^3/\text{kg}\cdot\text{K})(50+273\,\text{K})} = 0.0829\,\text{kg/m}^3$$

$$\rho_{a,s} = \frac{P_{a,s}}{R_a T_s} = \frac{(101.325-12.35)\,\text{kPa}}{(0.287\,\text{kPa.m}^3/\text{kg}\cdot\text{K})(50+273\,\text{K})} = 0.9598\,\text{kg/m}^3$$

$$\rho_s = \rho_{v,s} + \rho_{a,s} = 0.0829 + 0.9598 = 1.0427\,\text{kg/m}^3$$

Away from the surface:

$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{1.585\,\text{kPa}}{(0.4615\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(25+273\,\text{K})} = 0.0115\,\text{kg/m}^3$$

$$\rho_{a,\infty} = \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325-1.585)\,\text{kPa}}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(25+273\,\text{K})} = 1.1662\,\text{kg/m}^3$$

$$\rho_\infty = \rho_{v,\infty} + \rho_{a,\infty} = 0.0115 + 1.1662 = 1.1777\,\text{kg/m}^3$$

Note that $\rho_\infty > \rho_s$, and thus this corresponds to hot surface facing up. The area of the top surface of the water bath is A_s = 2 m × 4 m = 8 m^2 and its perimeter is p = 2(2 + 4) = 12 m. Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{8\,\text{m}^2}{12\,\text{m}} = 0.667\,\text{m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{avg}}\nu^2} = \frac{(9.81\,\text{m/s}^2)(1.1777-1.0427\,\text{kg/m}^3)(0.667\,\text{m})^3}{[(1.1777+1.0427)/2\,\text{kg/m}^3](1.679\times10^{-5}\,\text{m}^2/\text{s})^2} = 1.26\times10^9$$

Recognizing that this is a natural convection problem with hot horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be

$$\text{Nu} = 0.15(\text{Gr Pr})^{1/3} = 0.15(1.26\times10^9 \times 0.7262)^{1/3} = 146$$

and $$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(146)(0.02643\,\text{W/m}\cdot{}^\circ\text{C})}{0.667\,\text{m}} = 5.79\,\text{W/m}^2\cdot{}^\circ\text{C}$$

Then the natural convection heat transfer rate becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}}A_s(T_s - T_\infty) = (5.79\,\text{W/m}^2\cdot{}^\circ\text{C})(8\,\text{m}^2)(50-25)^\circ\text{C} = \mathbf{1158\,W}$$

Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.679\times10^{-5}\,\text{m}^2/\text{s}}{2.73\times10^{-5}\,\text{m}^2/\text{s}} = 0.615$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\text{Sh} = 0.15(\text{GrSc})^{1/3} = 0.15(1.27\times10^9 \times 0.615)^{1/3} = 138$$

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(138)(2.73\times10^{-5}\,\text{m}^2/\text{s})}{0.667\,\text{m}} = 0.00565\,\text{m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned}\dot{m}_v &= h_{\text{mass}}A_s(\rho_{v,s} - \rho_{v,\infty}) \\ &= (0.00568\,\text{m/s})(8\,\text{m}^2)(0.0829-0.0115)\text{kg/m}^3 \\ &= 0.00324\,\text{kg/s} = 11.7\,\text{kg/h}\end{aligned}$$

and $$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.00324\,\text{kg/s})(2383\text{kJ/kg}) = 7.72\,\text{kW} = \mathbf{7720\,W}$$

The total rate of heat loss from the open top surface of the bath to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 1726 + 1158 + 7720 = \mathbf{10{,}604\ W}$$

Therefore, if the water bath is heated electrically, a 10.6 kW resistance heater will be needed just to make up for the heat loss from the top surface.

(*c*) The side surfaces are vertical plates, and treating the air as dry air for simplicity, heat transfer from them by natural convection is determined to be

$$\text{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81\ \text{m/s}^2)(1/310.5\ \text{K})(50-25)\ \text{K})(1\ \text{m})^3}{(1.679\times10^{-5}\ \text{m}^2/\text{s})^2} = 2.80\times10^9$$

$$\text{Nu} = 0.1(\text{Gr Pr})^{1/3} = 0.1(2.80\times10^9 \times 0.7262)^{1/3} = 127$$

$$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(127)(0.02643\ \text{W/m}\cdot{}^\circ\text{C})}{1\ \text{m}} = 3.36\ \text{W/m}^2\cdot{}^\circ\text{C}$$

$$\dot{Q}_{\text{conv, side}} = h_{\text{conv}} A_s (T_s - T_\infty) = (3.36\ \text{W/m}^2\cdot{}^\circ\text{C})(12\times1\ \text{m}^2)(50-25)^\circ\text{C} = 1007\ \text{W}$$

The radiation heat loss from the side surfaces of the bath to the surrounding surfaces is

$$\dot{Q}_{\text{rad,side}} = \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4)$$
$$= (0.61)(12\ \text{m}\times1\ \text{m})(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(50+273\ \text{K})^4 - (15+273\ \text{K})^4] = 1662\ \text{W}$$

and $$\dot{Q}_{\text{total,side}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 1007 + 1662 = \mathbf{2669\ W}$$

(*d*) The rate at which water must be supplied to the maintain steady operation is equal to the rate of water removed by the bottles plus the rate evaporation,

$$\dot{m}_{\text{make-up}} = \dot{m}_{\text{removed}} + \dot{m}_{\text{evap}} = 0.00800 + 0.00324 = \mathbf{0.01124\ kg/s = 40.5\ kg/h}$$

Noting that the entire make-up water enters the bath 15°C, the rate of heat supply to preheat the make-up water to 50°C is

$$\dot{Q}_{\text{preheating water}} = \dot{m}_{\text{make-up water}} c_p \Delta T = (0.01124\ \text{kg/s})(4178\ \text{J/kg}\cdot{}^\circ\text{C})(50-15)^\circ\text{C} = 1644\ \text{W}$$

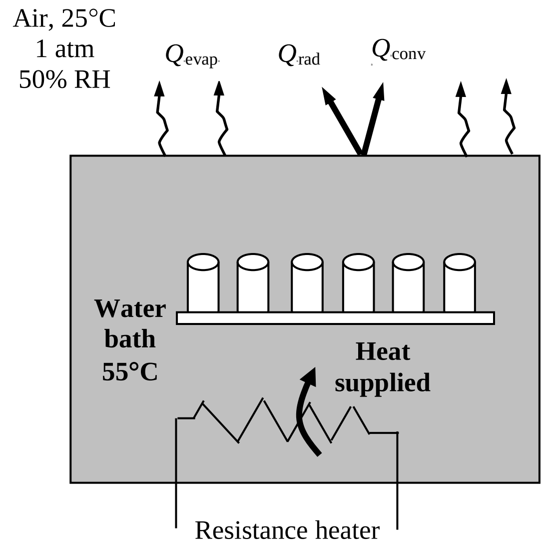
Then the rate of required heat supply for the bath becomes the sum of heat losses from the top and side surfaces, plus the heat needed for preheating the make-up water and the bottles,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{bottle}} + \left(\dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}}\right)_{\text{top}} + \left(\dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}}\right)_{\text{side}} + \dot{Q}_{\text{makeup water}}$$
$$= 51{,}170 + 10{,}604 + 2669 + 1644 = \mathbf{66{,}087\ W}$$

Therefore, the heater must be able to supply heat at a rate of 66.1 kW to maintain steady operating conditions.

14-165 Glass bottles are washed in hot water in an uncovered rectangular glass washing bath. The rates of heat loss from the top and side surfaces of the bath by radiation, natural convection, and evaporation as well as the rates of heat and water mass that need to be supplied to the water are to be determined.

Assumptions **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The entire water body and the metal container are maintained at a uniform temperature of 55°C. **4** Heat losses from the bottom surface are negligible. **5** The air motion around the bath is negligible so that there are no forced convection effects.



Properties The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2$ = (25+55)/2 = 40°C. The properties of dry air at 40°C and 1 atm are, from Table A-15,

$$k = 0.02662\ \text{W/m}\cdot\text{°C}, \quad \text{Pr} = 0.7255$$

$$\alpha = 2.346\times10^{-5}\ \text{m}^2/\text{s} \quad \nu = 1.700\times10^{-5}\ \text{m}^2/\text{s}$$

The mass diffusivity of water vapor in air at the average temperature of 313 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(313\text{K})^{2.072}}{1\text{atm}} = 2.77\times10^{-5}\ \text{m}^2/\text{s}$$

The saturation pressure of water at 25°C is $P_{\text{sat@25°C}} = 3.169\ \text{kPa}$. Properties of water at 55°C are $h_{fg} = 2371\ \text{kJ/kg}$ and $P_v = 15.76\ \text{kPa}$ (Table A-9). The specific heat of water at the average temperature of (15+55)/2 = 35°C is c_p = 4.178 kJ/kg.°C.

The gas constants of dry air and water are R_{air} = 0.287 kPa.m^3/kg.K and R_{water} = 0.4615 kPa.m^3/kg.K (Table A-1). Also, the emissivities of water and the sheet metal are given to be 0.61 and 0.95, respectively, and the specific heat of glass is 1.0 kJ/kg.°C.

Analysis (*a*) The mass flow rate of glass bottles through the water bath in steady operation is

$$\dot{m}_{\text{bottle}} = m_{\text{bottle}}\times\text{Bottle flow rate} = (0.150\ \text{kg/bottle})(800\ \text{bottles/min}) = 120\ \text{kg/min} = 2\ \text{kg/s}$$

Then the rate of heat removal by the bottles as they are heated from 25 to 55°C is

$$\dot{Q}_{\text{bottle}} = \dot{m}_{\text{bottle}}c_p\Delta T = (2\ \text{kg/s})(1\text{kJ/kg.°C})(55-25)\text{°C} = 60{,}000\ \text{W}$$

The amount of water removed by the bottles is

$$\dot{m}_{\text{water,out}} = (\text{Flow rate of bottles})(\text{Water removed per bottle})$$

$$= (800\ \text{bottles / min})(0.6\text{g/bottle}) = 480\ \text{g/min} = \mathbf{8\times10^{-3}\ kg/s} = \mathbf{28.8\ kg/h}$$

Noting that the water removed by the bottles is made up by fresh water entering at 15°C, the rate of heat removal by the water that sticks to the bottles is

$$\dot{Q}_{\text{water removed}} = \dot{m}_{\text{water removed}}c_p\Delta T = (8\times10^{-3}\ \text{kg/s})(4178\text{J/kg}\cdot\text{°C})(55-15)\text{°C} = 1337\ \text{W}$$

Therefore, the total amount of heat removed by the wet bottles is

$$\dot{Q}_{\text{total, removed}} = \dot{Q}_{\text{glass removed}} + \dot{Q}_{\text{water removed}} = 60{,}000 + 1337 = \mathbf{61{,}337\ W}$$

(*b*) The rate of heat loss from the top surface of the water bath is the sum of the heat losses by radiation, natural convection, and evaporation. Then the radiation heat loss from the top surface of water to the surrounding surfaces is

$$\dot{Q}_{\text{rad, top}} = \varepsilon A\sigma(T_s^4 - T_{\text{surr}}^4) = (0.95)(8\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(55+273\ \text{K})^4 - (15+273\ \text{K})^4] = 2023\ \text{W}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (15.76 kPa at 55°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat@}T_\infty} = (0.50)P_{\text{sat@25°C}} = (0.50)(3.169\ \text{kPa}) = 1.585\ \text{kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

At the surface:

$$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{15.76 \text{ kPa}}{(0.4615 \text{ kPa.m}^3/\text{kg}\cdot\text{K})(55+273 \text{ K})} = 0.1041 \text{ kg/m}^3$$

$$\rho_{a,s} = \frac{P_{a,s}}{R_a T_s} = \frac{(101.325-15.76) \text{ kPa}}{(0.287 \text{ kPa.m}^3/\text{kg}\cdot\text{K})(55+273 \text{ K})} = 0.9090 \text{ kg/m}^3$$

$$\rho_s = \rho_{v,s} + \rho_{a,s} = 0.1041 + 0.9090 = 1.0131 \text{ kg/m}^3$$

Away from the surface:

$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{1.585 \text{ kPa}}{(0.4615 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(25+273 \text{ K})} = 0.0115 \text{ kg/m}^3$$

$$\rho_{a,\infty} = \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325-1.585) \text{ kPa}}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(25+273 \text{ K})} = 1.1662 \text{ kg/m}^3$$

$$\rho_\infty = \rho_{v,\infty} + \rho_{a,\infty} = 0.0115 + 1.1662 = 1.1777 \text{ kg/m}^3$$

Note that $\rho_\infty > \rho_s$, and thus this corresponds to hot surface facing up. The area of the top surface of the water bath is A_s = 2 m × 4 m = 8 m^2 and its perimeter is p = 2(2 + 4) = 12 m. Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{8 \text{ m}^2}{12 \text{ m}} = 0.667 \text{ m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{avg}}\nu^2} = \frac{(9.81 \text{ m/s}^2)(1.1777-1.0131 \text{ kg/m}^3)(0.667 \text{ m})^3}{[(1.1777+1.0131)/2 \text{ kg/m}^3](1.702\times10^{-5} \text{ m}^2/\text{s})^2} = 1.52\times10^9$$

Recognizing that this is a natural convection problem with hot horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be

$$\text{Nu} = 0.15(\text{Gr Pr})^{1/3} = 0.15(1.52\times10^9 \times 0.726)^{1/3} = 155$$

and $$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(155)(0.02662 \text{ W/m}\cdot°\text{C})}{0.667 \text{ m}} = 6.19 \text{ W/m}^2\cdot°\text{C}$$

Then the natural convection heat transfer rate becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}} A_s (T_s - T_\infty) = (6.19 \text{ W/m}^2\cdot°\text{C})(8 \text{ m}^2)(55-25)°\text{C} = 1486 \text{ W}$$

Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.702\times10^{-5} \text{ m}^2/\text{s}}{2.77\times10^{-5} \text{ m}^2/\text{s}} = 0.614$$

The Sherwood number and the mass transfer coefficients are determined to be

$$\text{Sh} = 0.15(\text{GrSc})^{1/3} = 0.15(1.52\times10^9 \times 0.614)^{1/3} = 147$$

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(147)(2.77\times10^{-5} \text{ m}^2/\text{s})}{0.667 \text{ m}} = 0.00610 \text{ m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned}\dot{m}_v &= h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) \\ &= (0.00610 \text{ m/s})(8 \text{ m}^2)(0.1041 - 0.0115)\text{kg/m}^3 \\ &= 0.00452 \text{ kg/s} = 16.3 \text{ kg/h}\end{aligned}$$

and $$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (0.00452 \text{ kg/s})(2371 \text{kJ/kg}) = 10.7 \text{ kW} = 10{,}700 \text{ W}$$

Then the total rate of heat loss from the open top surface of the bath to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 2023 + 1486 + 10{,}700 = \mathbf{14{,}209\ W}$$

Therefore, if the water bath is heated electrically, a 14 kW resistance heater will be needed just to make up for the heat loss from the top surface.

(*c*) The side surfaces are vertical plates, and treating the air as dry air for simplicity, heat transfer from them by natural convection is determined to be

$$\text{Gr} = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81\ \text{m/s}^2)(1/313\ \text{K})(55-25)\ \text{K})(1\ \text{m})^3}{(1.702\times10^{-5}\ \text{m}^2/\text{s})^2} = 3.25\times10^9$$

$$\text{Nu} = 0.1(\text{Gr Pr})^{1/3} = 0.1(3.25\times10^9 \times 0.7255)^{1/3} = 133$$

$$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(133)(0.02662\ \text{W/m}\cdot°\text{C})}{1\ \text{m}} = 3.54\ \text{W/m}^2\cdot°\text{C}$$

$$\dot{Q}_{\text{conv, side}} = h_{\text{conv}} A_s (T_s - T_\infty) = (3.54\ \text{W/m}^2\cdot°\text{C})(12\times1\ \text{m}^2)(55-25)°\text{C} = 1275\ \text{W}$$

The radiation heat loss from the side surfaces of the bath to the surrounding surfaces is

$$\dot{Q}_{\text{rad,side}} = \varepsilon A_s \sigma (T_s^4 - T_{\text{surr}}^4)$$
$$= (0.61)(12\ \text{m}\times1\ \text{m})(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)[(55+273\ \text{K})^4 - (15+273\ \text{K})^4] = 1948\ \text{W}$$

and $$\dot{Q}_{\text{total,side}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} = 1275 + 1948 = \mathbf{3223\ W}$$

(*d*) The rate at which water must be supplied to the maintain steady operation is equal to the rate of water removed by the bottles plus the rate evaporation,

$$\dot{m}_{\text{make-up}} = \dot{m}_{\text{removed}} + \dot{m}_{\text{evap}} = 0.00800 + 0.00452 = \mathbf{0.01252\ kg/s = 45.1\ kg/h}$$

Noting that the entire make-up water enters the bath 15°C, the rate of heat supply to preheat the make-up water to 55°C is

$$\dot{Q}_{\text{preheating water}} = \dot{m}_{\text{make-up water}} c_p \Delta T = (0.01252\ \text{kg/s})(4178\ \text{J/kg}\cdot°\text{C})(55-15)°\text{C} = 2092\ \text{W}$$

Then the rate of required heat supply for the bath becomes the sum of heat losses from the top and side surfaces, plus the heat needed for preheating the make-up water and the bottles,

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{bottle}} + \left(\dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}}\right)_{\text{top}} + \left(\dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}}\right)_{\text{side}} + \dot{Q}_{\text{makeup water}}$$
$$= 61{,}337 + 14{,}209 + 3223 + 2092 = \mathbf{80{,}860\ W}$$

Therefore, the heater must be able to supply heat at a rate of 80.9 kW to maintain steady operating conditions.

Review Problems

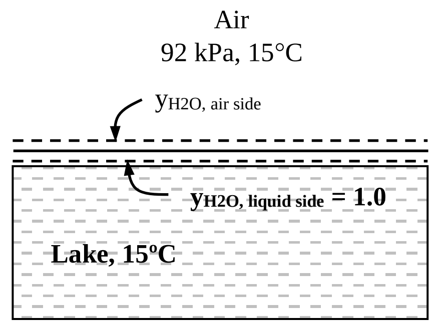
14-166 The mole fraction of the water vapor at the surface of a lake and the mole fraction of water in the lake are to be determined and compared.

Assumptions **1** Both the air and water vapor are ideal gases. **2** Air is weakly soluble in water and thus Henry's law is applicable.

Properties The saturation pressure of water at 15°C is 1.705 kPa (Table A-9).

Analysis The air at the water surface will be saturated. Therefore, the partial pressure of water vapor in the air at the lake surface will simply be the saturation pressure of water at 15°C,

$$P_{\text{vapor}} = P_{\text{sat@15°C}} = 1.705\,\text{kPa}$$



Assuming both the air and vapor to be ideal gases, the mole fraction of water vapor in the air at the surface of the lake is determined from Eq. 14-11 to be

$$y_{\text{vapor}} = \frac{P_{\text{vapor}}}{P} = \frac{1.705\,\text{kPa}}{92\,\text{kPa}} = \mathbf{0.0185}\ \ (\text{or}\,18.5\,\text{percent})$$

Water contains some dissolved air, but the amount is negligible. Therefore, we can assume the entire lake to be liquid water. Then its mole fraction becomes

$$y_{\text{water,liquidside}} \cong 1.0 \quad (\text{or}\,100\%)$$

Discussion Note that the concentration of water on a molar basis is 100 percent just beneath the air–water interface and 1.85 percent just above it, even though the air is assumed to be saturated (so this is the highest value at 15°C). Therefore, huge discontinuities can occur in the concentrations of a species across phase boundaries.

14-167 The ideal gas relation can be expressed as $PV = NR_uT = mRT$ where R_u is the universal gas constant, whose value is the same for all gases, and R is the gas constant whose value is different for different gases. The molar and mass densities of an ideal gas mixture can be expressed as

$$PV = NR_uT \;\rightarrow\; C = \frac{N}{V} = \frac{P}{R_uT} = \text{constant}$$

and $$PV = mRT \;\rightarrow\; \rho = \frac{m}{V} = \frac{P}{RT} \neq \text{constant}$$

Therefore, for an ideal gas mixture maintained at a constant temperature and pressure, the molar concentration C of the mixture remains constant but this is not necessarily the case for the density ρ of mixture.

14-168E The masses of the constituents of a gas mixture at a specified temperature and pressure are given. The partial pressure of each gas and the volume of the mixture are to be determined.

Assumptions The gas mixture and its constituents are ideal gases.

Properties The molar masses of CO_2 and CH_4 are 44 and 16 lbm/lbmol, respectively (Table A-1E)

Analysis The mole numbers of each gas and of the mixture are

1 lbm CO_2
3 lbm CH_4
550 R
25 psia

$$CO_2: \quad N_{CO_2} = \frac{m_{CO_2}}{M_{CO_2}} = \frac{1\text{ lbm}}{44\text{ lbm/lbmol}} = 0.0227\text{ lbmol}$$

$$CH_4: \quad N_{CH_4} = \frac{m_{CH_4}}{M_{CH_4}} = \frac{3\text{ lbm}}{16\text{ lbm/lbmol}} = 0.1875\text{ lbmol}$$

$$N_{\text{total}} = N_{CO_2} + N_{CH_4} = 0.0227 + 0.1875 = 0.2102\text{ lbmol}$$

Using the ideal gas relation for the mixture and for the constituents, the volume of the mixture and the partial pressures of the constituents are determined to be

$$V = \frac{NR_uT}{P} = \frac{(0.2102\text{ lbmol})(10.73\text{ psia}\cdot\text{ft}^3/\text{lbmol}\cdot\text{R})(550\text{ R})}{25\text{ psia}} = \mathbf{49.62\ ft^3}$$

$$P_{CO_2} = \frac{N_{CO_2}R_uT}{V} = \frac{(0.0227\text{ lbmol})(10.73\text{ psia}\cdot\text{ft}^3/\text{lbmol}\cdot\text{R})(550\text{ R})}{49.62\text{ ft}^3} = \mathbf{2.70\ psia}$$

$$P_{CH_4} = \frac{N_{CH_4}R_uT}{V} = \frac{(0.1875\text{ lbmol})(10.73\text{ psia}\cdot\text{ft}^3/\text{lbmol}\cdot\text{R})(550\text{ R})}{49.62\text{ ft}^3} = \mathbf{22.3\ psia}$$

Discussion Note that each constituent of a gas mixture occupies the same volume (the volume of the container), and that the total pressure of a gas mixture is equal to the sum of the partial pressures of its constituents. That is, $P_{\text{total}} = P_{CO_2} + P_{CH_4} = 2.70 + 22.3 = 25\text{ psia}$.

14-169 Dry air flows over a water body at constant pressure and temperature until it is saturated. The molar analysis of the saturated air and the density of air before and after the process are to be determined.

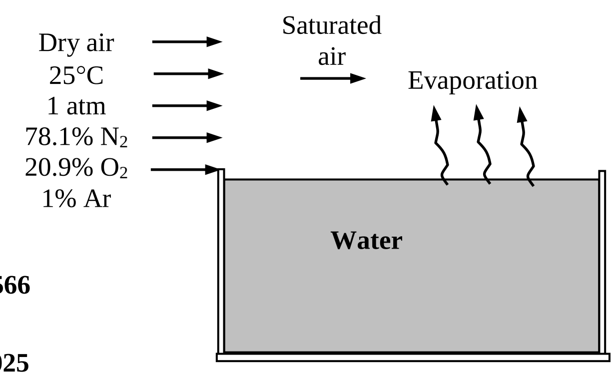
Assumptions The air and the water vapor are ideal gases.

Properties The molar masses of N_2, O_2, Ar, and H_2O are 28.0, 32.0, 39.95 and 18 kg / kmol, respectively (Table A-1). The molar analysis of dry air is given to be 78.1 percent N_2, 20.9 percent O_2, and 1 percent Ar. The saturation pressure of water at 25°C is 3.169 kPa (Table A-9). Also, 1 atm = 101.325 kPa.

Analysis (*a*) Noting that the total pressure remains constant at 101.32 kPa during this process, the partial pressure of air becomes

$$P = P_{\text{air}} + P_{\text{vapor}}$$

$$P_{\text{air}} = P - P_{\text{vapor}} = 101.325 - 3.169 = 98.156\,\text{kPa}$$



Then the molar analysis of the saturated air becomes

$$y_{H_2O} = \frac{P_{H_2O}}{P} = \frac{3.169}{101.325} = \mathbf{0.0313}$$

$$y_{N_2} = \frac{P_{N_2}}{P} = \frac{y_{N_2,\text{dry}} P_{\text{dry air}}}{P} = \frac{0.781(98.156\,\text{kPa})}{101.325} = \mathbf{0.7566}$$

$$y_{O_2} = \frac{P_{O_2}}{P} = \frac{y_{O_2,\text{dry}} P_{\text{dry air}}}{P} = \frac{0.209(98.156\,\text{kPa})}{101.325} = \mathbf{0.2025}$$

$$y_{\text{Ar}} = \frac{P_{\text{Ar}}}{P} = \frac{y_{\text{Ar,dry}} P_{\text{dry air}}}{P} = \frac{0.01(98.156\,\text{kPa})}{101.325} = \mathbf{0.0097}$$

(*b*) The molar masses of dry and saturated air are

$$M_{\text{dry air}} = \sum y_i M_i = 0.781\times 28.0 + 0.209\times 32.0 + 0.01\times 39.95 = 29.0\,\text{kg/kmol}$$

$$M_{\text{sat air}} = \sum y_i M_i = 0.7566\times 28.0 + 0.2025\times 32.0 + 0.0097\times 39.9 + 0.0313\times 18 = 28.62\,\text{kg/kmol}$$

Then the densities of dry and saturated air are determined from the ideal gas relation to be

$$\rho_{\text{dry air}} = \frac{P}{(R_u / M_{\text{dry air}})T} = \frac{101.325\,\text{kPa}}{[(8.314\,\text{kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})/29.0\,\text{kg/kmol}](25+273)\text{K}} = \mathbf{1.186\,kg/m^3}$$

$$\rho_{\text{sat air}} = \frac{P}{(R_u / M_{\text{sat air}})T} = \frac{101.325\,\text{kPa}}{[(8.314\,\text{kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})/28.62\,\text{kg/kmol}](25+273)\text{K}} = \mathbf{1.170\,kg/m^3}$$

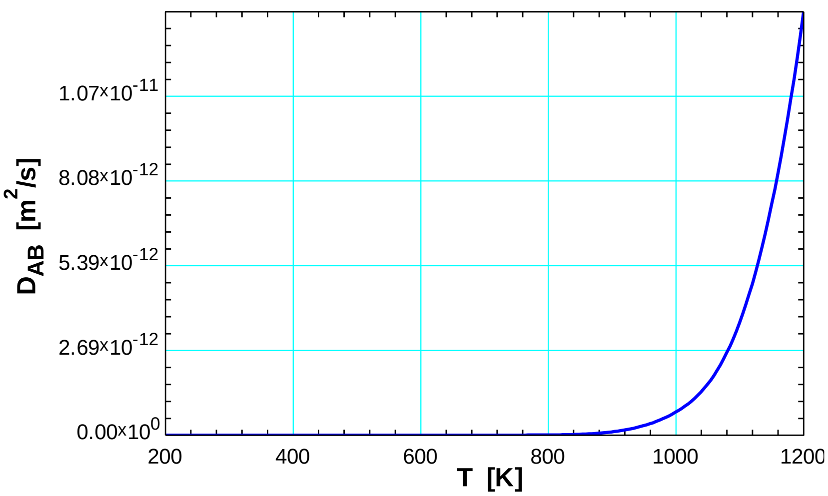
Discussion We conclude that the density of saturated air is less than that of the dry air, as expected. This is due to the molar mass of water being less than that of dry air.

14-170 Using the relation $D_{AB} = 2.67\times10^{-5}\exp(-17{,}400/T)$ the diffusion coefficient of carbon in steel is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

D_AB=2.67E-5*exp(-17400/T)

T, K	D_{AB}, m^2 / s
200	4.394E-43
300	1.728E-30
400	3.425E-24
500	2.056E-20
600	6.792E-18
700	4.278E-16
800	9.563E-15
900	1.071E-13
1000	7.409E-13
1100	3.604E-12
1200	1.346E-11



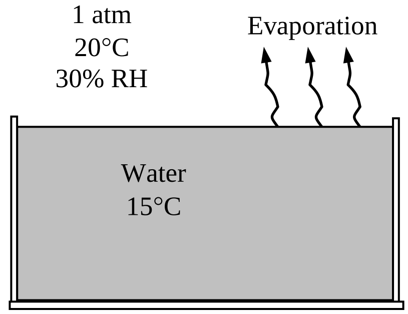
14-171 A circular pan filled with water is cooled naturally. The rate of evaporation of water, the rate of heat transfer by natural convection, and the rate of heat supply to the water needed to maintain its temperature constant are to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 25°C). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** Radiation heat transfer is negligible. **4** Both air and water vapor are ideal gases.

Properties The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2$ = (15+20)/2 = 17.5°C = 290.5 K. The properties of dry air at 17.5°C and 1 atm are, from Table A-15,

$$k = 0.02495\ \text{W/m}\cdot°\text{C},\quad \text{Pr} = 0.7316$$

$$\alpha = 2.042\times10^{-5}\ \text{m}^2/\text{s}\quad \nu = 1.493\times10^{-5}\ \text{m}^2/\text{s}$$



The mass diffusivity of water vapor in air at the average temperature of 290.5 K is, from Eq. 14-15,

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P}$$

$$= 1.87\times10^{-10}\frac{(290.5\,\text{K})^{2.072}}{1\text{atm}} = 2.37\times10^{-5}\ \text{m}^2/\text{s}$$

The saturation pressure of water at 20°C is $P_{\text{sat@20°C}} = 2.339\ \text{kPa}$. Properties of water at 15°C are $h_{fg} = 2466\ \text{kJ/kg}$ and $P_v = 1.7051\ \text{kPa}$ (Table A-9). The specific heat of water at the average temperature of (15+20)/2 = 17.5°C is c_p = 4.184 kJ/kg.°C. The gas constants of dry air and water are R_{air} = 0.287 kPa.m^3/kg.K and R_{water} = 0.4615 kPa.m^3/kg.K (Table A-1).

Analysis (*a*) The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (1.7051 kPa at 15°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat@}T_\infty} = (0.30)P_{\text{sat@20°C}} = (0.30)(2.339\ \text{kPa}) = 0.7017\ \text{kPa}$$

Treating the water vapor and the air as ideal gases and noting that the total atmospheric pressure is the sum of the vapor and dry air pressures, the densities of the water vapor, dry air, and their mixture at the water-air interface and far from the surface are determined to be

At the surface:

$$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{1.7051\ \text{kPa}}{(0.4615\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(15+273)\ \text{K}} = 0.01283\ \text{kg/m}^3$$

$$\rho_{a,s} = \frac{P_{a,s}}{R_a T_s} = \frac{(101.325-1.7051)\ \text{kPa}}{(0.287\ \text{kPa.m}^3/\text{kg}\cdot\text{K})(15+273)\ \text{K}} = 1.2052\ \text{kg/m}^3$$

$$\rho_s = \rho_{v,s} + \rho_{a,s} = 0.01283 + 1.2052 = 1.21803\ \text{kg/m}^3$$

Away from the surface:

$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{0.7017\ \text{kPa}}{(0.4615\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273)\ \text{K}} = 0.00519\ \text{kg/m}^3$$

$$\rho_{a,\infty} = \frac{P_{a,\infty}}{R_a T_\infty} = \frac{(101.325-0.7017)\ \text{kPa}}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273\ \text{K})} = 1.1966\ \text{kg/m}^3$$

$$\rho_\infty = \rho_{v,\infty} + \rho_{a,\infty} = 0.00519 + 1.1966 = 1.2018\ \text{kg/m}^3$$

Note that $\rho_\infty < \rho_s$, and thus this corresponds to hot surface facing down. The area of the top surface of the water $A_s = \pi r_o^2$ and its perimeter is $p = 2\pi r_o$. Therefore, the characteristic length is

$$L = \frac{A_s}{p} = \frac{\pi r_o^2}{2\pi r_o} = \frac{r_o}{2} = \frac{0.15\ \text{m}}{2} = 0.075\ \text{m}$$

Then using densities (instead of temperatures) since the mixture is not homogeneous, the Grashoff number is determined to be

$$\text{Gr} = \frac{g(\rho_\infty - \rho_s)L^3}{\rho_{\text{ave}}\nu^2} = \frac{(9.81\,\text{m/s}^2)(1.21803 - 1.2018\,\text{kg/m}^3)(0.075\,\text{m})^3}{[(1.2180 + 1.2018)/2\,\text{kg/m}^3](1.493\times10^{-5}\,\text{m}^2/\text{s})^2} = 2.49\times10^5$$

Recognizing that this is a natural convection problem with cold horizontal surface facing up, the Nusselt number and the convection heat transfer coefficients are determined to be (Table 14-13)

$$\text{Nu} = 0.27(\text{Gr Pr})^{1/4} = 0.27(2.49\times10^5 \times 0.7316)^{1/4} = 5.58$$

and

$$h_{\text{conv}} = \frac{\text{Nu}k}{L} = \frac{(5.58)(0.02495\ \text{W/m}\cdot{}^\circ\text{C})}{0.075\,\text{m}} = 1.86\ \text{W/m}^2\cdot{}^\circ\text{C}$$

Then the rate of heat transfer from the air to the water by forced convection becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}} A_s (T_\infty - T_s) = (1.86\ \text{W/m}^2\cdot{}^\circ\text{C})[\pi(0.15\,\text{m})^2](20-15)^\circ\text{C} = \mathbf{0.66\ W}\quad \text{(to water)}$$

(*b*) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.493\times10^{-5}\ \text{m}^2/\text{s}}{2.37\times10^{-5}\ \text{m}^2/\text{s}} = 0.630$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.27(\text{GrSc})^{1/4} = 0.27(2.53\times10^5 \times 0.629)^{1/4} = 5.39$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(5.39)(2.37\times10^{-5}\ \text{m}^2/\text{s})}{0.075\text{m}} = 0.00170\,\text{m/s}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\dot{m}_v = h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) = (0.00170\,\text{m/s})[\pi(0.15\,\text{m})^2](0.01283 - 0.00519)\ \text{kg/m}^3$$
$$= 9.18\times10^{-7}\ \text{kg/s} = \mathbf{0.0033\,kg/h}$$

and

$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (9.17\times10^{-7}\ \text{kg/s})(2466\,\text{kJ/kg}) = 0.00226\,\text{kW} = 2.26\,\text{W}$$

(*c*) The net rate of heat transfer to the water needed to maintain its temperature constant at 15°C is

$$\dot{Q}_{\text{net}} = \dot{Q}_{\text{evap}} + \dot{Q}_{\text{conv}} = 2.26 + (-0.66) = \mathbf{1.6\,W}$$

Discussion Note that if no heat is supplied to the water (by a resistance heater, for example), the temperature of the water in the pan would drop until the heat gain by convection equals the heat loss by evaporation.

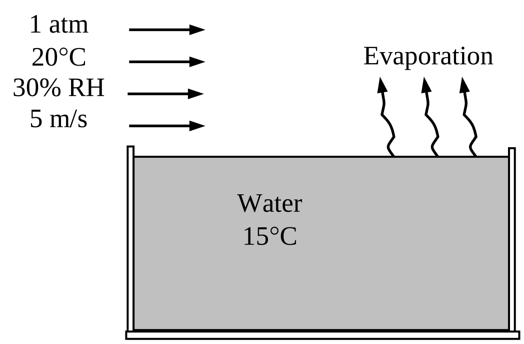
14-172 Air is blown over a circular pan filled with water. The rate of evaporation of water, the rate of heat transfer by convection, and the rate of energy supply to the water to maintain its temperature constant are to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 15°C). **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** Radiation heat transfer is negligible. **4** Both air and water vapor are ideal gases.

Properties The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2$ = (15+20)/2 = 17.5°C = 290.5 K. The properties of dry air at 17.5°C and 1 atm are, from Table A-15,

$$k = 0.02496 \text{ W/m}\cdot°\text{C}, \quad \text{Pr} = 0.7316$$

$$\alpha = 2.042\times10^{-5} \text{ m}^2/\text{s} \quad \nu = 1.493\times10^{-5} \text{ m}^2/\text{s}$$



The mass diffusivity of water vapor in air at the average temperature of 290.5 K is, from Eq. 14-15,

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P}$$

$$= 1.87\times10^{-10}\frac{(290.5\text{K})^{2.072}}{1\text{atm}} = 2.37\times10^{-5} \text{ m}^2/\text{s}$$

The saturation pressure of water at 20°C is $P_{\text{sat@20°C}} = 2.339 \text{ kPa}$.

Properties of water at 15°C are $h_{fg} = 2466 \text{ kJ/kg}$ and $P_v = 1.7051 \text{ kPa}$ (Table A-9). Also, the gas constants of water is R_{water} = 0.4615 kPa.m^3/kg.K (Table A-1).

Analysis (*a*) Taking the radius of the pan r_0 = 0.15 m to be the characteristic length, the Reynolds number for flow over the pan is

$$\text{Re} = \frac{VL}{\nu} = \frac{(5 \text{ m/s})(0.15 \text{ m})}{1.493\times10^{-5} \text{ m}^2/\text{s}} = 50{,}234$$

which is less than 500,000, and thus the flow is laminar over the entire surface. The Nusselt number and the heat transfer coefficient are

$$\text{Nu} = 0.664\,\text{Re}_L^{0.5}\text{Pr}^{1/3} = 0.664(50{,}234)^{0.5}(0.7316)^{1/3} = 134.1$$

$$h_{\text{heat}} = \frac{\text{Nu}k}{L} = \frac{(134.1)(0.02496 \text{ W/m}\cdot°\text{C})}{0.15\text{m}} = 22.31 \text{W/m}^2\cdot°\text{C}$$

Then the rate of heat transfer from the air to the water by forced convection becomes

$$\dot{Q}_{\text{conv}} = h_{\text{conv}}A_s(T_\infty - T_s) = (22.31 \text{ W/m}^2\cdot°\text{C})[\pi(0.15\text{m})^2](20-15)°\text{C} = \mathbf{7.9 \text{ W}} \quad \text{(to water)}$$

(*b*) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.493\times10^{-5} \text{ m}^2/\text{s}}{2.37\times10^{-5} \text{ m}^2/\text{s}} = 0.630$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.664\,\text{Re}_L^{0.5}\text{Sc}^{1/3} = 0.664\,(50{,}234)^{0.5}\,(0.630)^{1/3} = 127.6$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(127.6)(2.37\times10^{-5} \text{ m}^2/\text{s})}{0.15 \text{ m}} = 0.02016 \text{ m/s}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (1.7051 kPa at 15°C). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\text{sat@}T_\infty} = (0.30)P_{\text{sat@20°C}} = (0.30)(2.339 \text{ kPa}) = 0.7017 \text{ kPa}$$

Treating the water vapor and the air as ideal gases, the vapor densities at the water-air interface and far from the surface are determined to be

At the surface: $$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{1.7051\,\text{kPa}}{(0.4615\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(15+273)\,\text{K}} = 0.01283\,\text{kg/m}^3$$

Away from the surface: $$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} = \frac{0.7017\,\text{kPa}}{(0.4615\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(20+273)\,\text{K}} = 0.00519\,\text{kg/m}^3$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\dot{m}_v = h_{\text{mass}} A_s (\rho_{v,s} - \rho_{v,\infty}) = (0.02016\text{ m/s})[\pi(0.15\text{ m})^2](0.01283 - 0.00519)\text{ kg/m}^3$$
$$= 1.089\times10^{-5}\text{ kg/s} = \mathbf{0.03919\,kg/h}$$

and

$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (1.089\times10^{-5}\text{ kg/s})(2466\text{ kJ/kg}) = 0.02685\text{ kW} = 26.9\text{ W}$$

(*c*) The net rate of heat transfer to the water needed to maintain its temperature constant at 15°C is

$$\dot{Q}_{\text{net}} = \dot{Q}_{\text{evap}} + \dot{Q}_{\text{conv}} = 26.9 + (-7.9) = \mathbf{19.0\ W}$$

Discussion Note that if no heat is supplied to the water (by a resistance heater, for example), the temperature of the water in the pan would drop until the heat gain by convection equals the heat loss by evaporation.

Also, the rate of evaporation could be determined almost as accurately using mass fractions of vapor instead of vapor fractions and the average air density from the relation $\dot{m}_{\text{evap}} = h_{\text{mass}} \rho A (w_{A,s} - w_{A,\infty})$.

14-173 Henry's law is expressed as

$$y_{\text{i, liquid side}}(0) = \frac{P_{\text{i, gas side}}(0)}{H}$$

Henry's constant H increases with temperature, and thus the fraction of gas i in the liquid $y_{\text{i, liquid side}}$ decreases. Therefore, heating a liquid will drive off the dissolved gases in a liquid.

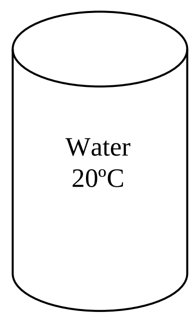
14-174 A glass of water is left in a room. The mole fraction of the water vapor in the air at the water surface and far from the surface as well as the mole fraction of air in the water near the surface are to be determined when the water and the air are at the same temperature.

Assumptions **1** Both the air and water vapor are ideal gases. **2** Air is weakly soluble in water and thus Henry's law is applicable.

Properties The saturation pressure of water at 20°C is 2.339 kPa (Table A-9). Henry's constant for air dissolved in water at 20ºC (293 K) is given in Table 14-6 to be H = 65,600 bar. Molar masses of dry air and water are 29 and 18 kg/kmol, respectively (Table A-1).

Analysis (*a*) Noting that the relative humidity of air is 70%, the partial pressure of water vapor in the air far from the water surface will be

$$P_{\text{v,room air}} = \phi P_{\text{sat @ 20°C}} = (0.7)(2.339\,\text{kPa}) = 1.637\,\text{kPa}$$



Assuming both the air and vapor to be ideal gases, the mole fraction of water vapor in the room air is

$$y_{\text{vapor}} = \frac{P_{\text{vapor}}}{P} = \frac{1.637\,\text{kPa}}{100\,\text{kPa}} = \mathbf{0.0164} \quad \mathbf{(or\ 1.64\%)}$$

(*b*) Noting that air at the water surface is saturated, the partial pressure of water vapor in the air near the surface will simply be the saturation pressure of water at 20°C, $P_{\text{v,interface}} = P_{sat\,@\,20°\text{C}} = 2.339\,\text{kPa}$. Then the mole fraction of water vapor in the air at the interface becomes

$$y_{\text{v, surface}} = \frac{P_{\text{v, surface}}}{P} = \frac{2.339\,\text{kPa}}{100\,\text{kPa}} = \mathbf{0.0234} \quad \mathbf{(or\ 2.34\%)}$$

(*c*) Noting that the total pressure is 100 kPa, the partial pressure of dry air at the water surface is

$$P_{\text{air, surface}} = P - P_{\text{v, surface}} = 100 - 2.339 = 97.661\ \text{kPa}$$

From Henry's law, the mole fraction of air in the water is determined to be

$$y_{\text{dry air,liquid side}} = \frac{P_{\text{dry air,gas side}}}{H} = \frac{(97.661/101.325)\,\text{bar}}{65{,}600\ \text{bar}} = \mathbf{1.47\times10^{-5}} = \mathbf{0.0015\%}$$

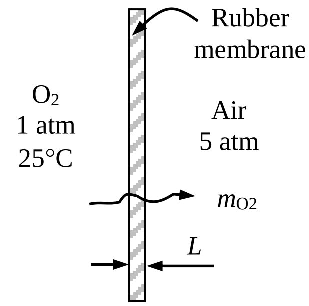
Discussion The water cannot remain at the room temperature when the air is not saturated. Therefore, some water will evaporate and the water temperature will drop until a balance is reached between the rate of heat transfer to the water and the rate of evaporation.

14-175 A 0.1-mm thick soft rubber membrane separates pure O_2 from air. The mass flow rate of O_2 through the membrane per unit area and the direction of flow are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Mass transfer through the membrane is one-dimensional. **3** The permeability of the membrane is constant.

Properties The mass diffusivity of oxygen in rubber at 298 K is $D_{AB} = 2.1\times10^{-10}$ m²/s (Table 14-3b). The solubility of oxygen in rubber at 298 K is 0.00312 kmol / m³.bar (Table 14-7). The molar mass of oxygen is 32 kg / kmol (Table A-1).

Analysis The molar fraction of oxygen in air is 0.21. Therefore, the partial pressure of oxygen in the air is



$$y_{O_2} = \frac{P_{O_2,2}}{P} \rightarrow P_{O_2,2} = y_{O_2}P = 0.21\times(5\text{ atm}) = 1.05\text{ atm}$$

The partial pressure of oxygen on the other side is simply $P_{O_2,1} = 1\text{atm}$. Then the molar flow rate of oxygen through the membrane by diffusion can readily be determined to be

$$\dot{N}_{\text{diff, A,wall}} = D_{AB}S\frac{P_{A,1}-P_{A,2}}{L}$$

$$= (2.1\times10^{-10}\text{ m}^2/\text{s})(0.00312\,\text{kmol/m}^3.\text{bar})\frac{(1-1.05)\text{atm}}{0.15\times10^{-3}\text{ m}}\left(\frac{1.01325\text{ bar}}{1\text{ atm}}\right)$$

$$= -2.21\times10^{-10}\,\text{kmol/m}^2\cdot\text{s}$$

Then the mass flow rate of oxygen gas through the membrane becomes

$$\dot{m}_{\text{diff}} = M\dot{N}_{\text{diff}} = (32\text{ kg/kmol})(-2.21\times10^{-10}\text{ kmol/m}^2\cdot\text{s}) = \mathbf{-7.08\times10^{-9}\ kg/m^2\cdot s}$$

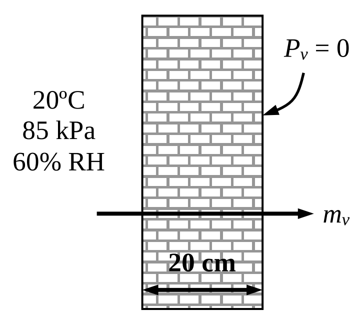
The negative sign indicates that the direction of the flow will be from the air outside to the pure oxygen inside.

14-176 The walls of a house are made of 20-cm thick bricks. The maximum amount of water vapor that will diffuse through a 3 m × 5 m section of the wall in 24-h is to be determined.

Assumptions **1** Steady operating conditions exist. **2** Mass transfer through the wall is one-dimensional. **3** The vapor permeability of the wall is constant. **4** The vapor pressure at the outer side of the wall is zero.

Properties The permeance of the brick wall is given to be 23×10^{-12} kg/s.m².Pa. The saturation pressure of water at 20ºC is 2339 Pa (Table 14-9).

Analysis The mass flow rate of water vapor through a plain layer of thickness L and normal area A is given by (Eq. 14-31)



$$\dot{m}_v = \mathsf{P}A\frac{P_{v,1}-P_{v,2}}{L} = \mathsf{P}A\frac{\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2}}{L} = \mathsf{M}A(\phi_1 P_{\text{sat},1}-\phi_2 P_{\text{sat},2})$$

where P is the vapor permeability and M = P/L is the permeance of the material, ϕ is the relative humidity and P_{sat} is the saturation pressure of water at the specified temperature. Subscripts 1 and 2 denote the air on the two sides of the wall.

Noting that the vapor pressure at the outer side of the wallboard is zero (ϕ_2 = 0) and substituting, the mass flow rate of water vapor through the wall is determined to be

$$\dot{m}_v = (23\times10^{-12}\,\text{kg/s.m}^2.\text{Pa})(3\times5\,\text{m}^2)[0.60(2339\text{ Pa})-0] = 4.842\times10^{-7}\text{ kg/s}$$

Then the total amount of moisture that flows through the wall during a 24-h period becomes

$$m_{v,24-h} = \dot{m}_v\Delta t = (4.842\times10^{-7}\text{ kg/s})(24\times3600\text{ s}) = \mathbf{0.0418\,kg = 41.8\,g}$$

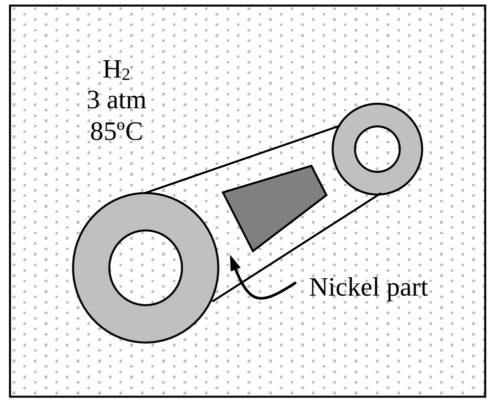
Discussion This is the maximum amount of moisture that can migrate through the wall since we assumed the vapor pressure on one side of the wall to be zero.

14-177 A nickel part is put into a room filled with hydrogen. The ratio of hydrogen concentrations at the surface of the part and at a depth of 2-mm from the surface after 24 h is to be determined.

Assumptions **1** Hydrogen penetrates into a thin layer beneath the surface of the nickel component, and thus the component can be modeled as a semi-infinite medium regardless of its thickness or shape. **2** The initial hydrogen concentration in the nickel part is zero.

Properties The molar mass of hydrogen H_2 is M = 2 kg/kmol (Table A-1). The solubility of hydrogen in nickel at 358 K (=85ºC) is 0.00901 kmol/m³.bar (Table 14-7). The mass diffusivity of hydrogen in nickel at 358 K is D_{AB} =1.2×10^{-12} m^2/s (Table 14-3b). Also, 1 atm = 1.01325 bar.

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature, and thus can be solved accordingly. Using mass fraction for concentration since the data is given in that form, the solution can be expressed as



$$\frac{w_A(x,t)-w_{A,i}}{w_{A,s}-w_{A,i}}=\operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)$$

The molar density of hydrogen in the nickel at the interface is determined from Eq. 14-20 to be

$$\begin{aligned}C_{H_2,\text{solid side}}(0) &= S\times P_{H_2,\text{gas side}}\\ &= (0.00901\,\text{kmol/mμ.bar})(3\times 1.01325\,\text{bar})\\ &= 0.0274\,\text{kmol/mμ}\end{aligned}$$

The argument of the complementary error function is

$$\xi=\frac{x}{2\sqrt{D_{AB}t}}=\frac{2\times 10^{-3}\text{ m}}{2\sqrt{(1.2\times 10^{-12}\text{ m}^2/\text{s})(24\times 3600\text{s})}}=3.106$$

The corresponding value of the complementary error function is determined from Table 4-4 to be

$$\operatorname{erfc}\left(\frac{x}{2\sqrt{D_{AB}t}}\right)=\operatorname{erfc}(3.105)=0.000015$$

Substituting the known quantities,

$$\frac{C_A(x,t)-0}{0.0274-0}=0.000015\ \rightarrow\ C_A(x,t)=\mathbf{4.1\times 10^{-7}\ kmol/m^3}$$

Therefore, the hydrogen concentration in the steel component at a depth of 2 mm in 24 h is very small.

14-178 A tanker truck carrying liquid herbicide overturned and caused a spill over a field. The depth of the soil at which plant and insect life is likely to be affected by the spill is to be determined.

Assumptions The herbicide-soaked soil can be modeled as a semi-infinite medium.

Properties The relevant properties are given in the problem statement.

Analysis This problem is analogous to the one-dimensional transient heat conduction problem in a semi-infinite medium with specified surface temperature discussed in chapter 4, and thus can be solved accordingly. The solution can be expressed as

$$\frac{w_A(x,t)-w_{A,i}}{w_{A,s}-w_{A,i}}=\text{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$

Substituting the specified quantities gives

$$\frac{0.001-0}{1-0}=\text{erfc}\left(\frac{x}{2\sqrt{D_{AB}\,t}}\right)$$

From Table 4-4, we have 0.001 = erfc(2.33), hence

$$\frac{x}{2\sqrt{D_{AB}\,t}}=2.33$$

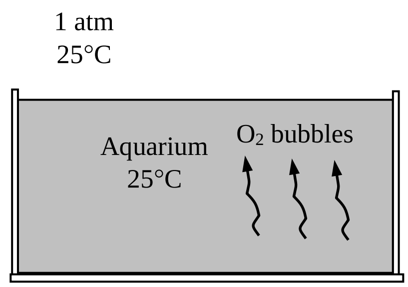
$$\begin{aligned}x&=2\sqrt{D_{AB}\,t}\,(2.33)\\&=2\sqrt{(2\times10^{-8}\text{ m}^2/\text{s})(1800\text{ s})}\,(2.33)\\&=0.0280\text{ m}\\&=\mathbf{2.8\,cm}\end{aligned}$$

Discussion The spill will likely affect life down to about 3 cm from the soil surface.

14-179 An aquarium is oxygenated by forcing oxygen to the bottom of it, and letting the oxygen bubbles rise. The penetration depth of oxygen in the water during the rising time is to be determined.

Assumptions **1** Convection effects in the water are negligible. **2** The pressure and temperature of the oxygen bubbles remain constant.

Properties The mass diffusivity of oxygen in liquid water at 298 K is D_{AB} = 2.4 ×10^{-9} m^2 /s (Table 14-3b).



Analysis The penetration depth can be determined directly from its definition (Eq. 14-38) to be

$$\begin{aligned}\delta_{\text{diff}}&=\sqrt{\pi D_{AB}t}=\sqrt{\pi(2.4\times10^{-9}\text{ m}^2/\text{s})(4\text{s})}\\&=1.73\times10^{-4}\text{ m}=\mathbf{0.173\,mm}\end{aligned}$$

Therefore, oxygen will penetrate the water only a fraction of a milimeter.

14-180 A sphere of crystalline sodium chloride (NaCl) was suspended in a stirred tank filled with water. The average mass transfer coefficient is to be determined.

Assumptions **1** The properties of NaCl are constant.

Properties The density of NaCl and its solubility in water at 20°C are given to be 2160 kg/m^3 and 320 kg/m^3, respectively.

Analysis The initial diameter of the sphere is

$$m_1 = \rho \boldsymbol{V} = \rho \frac{\pi D^3}{6} \longrightarrow 0.100\text{ kg} = (2160\text{ kg/m}^3)\frac{\pi D_1^3}{6} \longrightarrow D_1 = 0.04455\text{ m}$$

The final diameter of the sphere is

$$m_2 = \rho \boldsymbol{V} = \rho \frac{\pi D^3}{6} \longrightarrow (0.90)(0.100\text{ kg}) = (2160\text{ kg/m}^3)\frac{\pi D_2^3}{6} \longrightarrow D_2 = 0.04301\text{ m}$$

The rate of mass change is

$$\dot{m} = \frac{m_1 - m_2}{\Delta t} = \frac{(0.100 - 0.090)\text{ kg}}{10 \times 60\text{ s}} = 1.667 \times 10^{-5}\text{ kg/s}$$

The average surface area is

$$A_s = \frac{\pi D_1^2 + \pi D_2^2}{2} = \frac{\pi(0.04455\text{ m})^2 + \pi(0.04301\text{ m})^2}{2} = 6.023 \times 10^{-3}\text{ m}^2$$

The mass transfer coefficient is determined from

$$h_{\text{mass}} = \frac{\dot{m}}{A_s \Delta\rho_A} = \frac{1.667 \times 10^{-5}\text{ kg/s}}{(6.023 \times 10^{-3}\text{ m}^2)(320 - 0)\text{kg/m}^3} = \mathbf{8.65 \times 10^{-6}\text{ m/s}}$$

14-181 Liquid toluene evaporates into air from the open-top of a cylindrical container. The concentration of toluene at a certain location is to be determined.

Properties The molar mass of toluene is 92 kg/kmol. The diffusion coefficient of toluene at 25°C is given to be $D_{AB} = 0.084 \times 10^{-4}\ \text{m}^2/\text{s}$.

Analysis The vapor pressure of toluene is

$$P_{A,0} = \frac{10\ \text{mmHg}}{760\ \text{mmHg}}(101{,}325\ \text{kPa}) = 1333\ \text{Pa}$$

The rate of evaporation can be expressed by

$$\frac{0.080\ \text{kg/day}}{24 \times 3600\ \text{s/day}} = \dot{N}_A \frac{\pi(0.3\ \text{m})^2}{4}(92\ \text{kg/kmol}) \longrightarrow \dot{N}_A = 1.424 \times 10^{-7}\ \text{kmol/m}^2 \cdot \text{s}$$

The diffusion coefficient at 6.4°C is determined from

$$D_{AB} = (0.084 \times 10^{-4}\ \text{m}^2/\text{s})\left(\frac{6.4+273}{25+273}\right)^{1.5} = 7.63 \times 10^{-6}\ \text{m}^2/\text{s}$$

The vapor pressure of toluene at 10 mm above the surface is determined from

$$\dot{N}_A = \frac{D_{AB}P}{LR_uT}\ln\left(\frac{P-P_{A,L}}{P-P_{A,0}}\right)$$

$$1.424 \times 10^{-7}\ \text{kmol/m}^2 \cdot \text{s} = \frac{(7.63 \times 10^{-6}\ \text{m}^2/\text{s})(101{,}325\ \text{Pa})}{(0.010\ \text{m})(8314\ \text{Pa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(6.4+273\ \text{K})}\ln\left(\frac{101{,}325 - P_{A,L}}{101{,}325 - 1333}\right)$$

$$P_{A,L} = 904.3\ \text{Pa}$$

Then the concentration of toluene is determined to be

$$C_{A,L} = \frac{P_{A,L}}{R_uT}M = \frac{904.3\ \text{Pa}}{(8314\ \text{Pa} \cdot \text{m}^3/\text{kmol} \cdot \text{K})(6.4+273\ \text{K})}(92\ \text{kg/kmol}) = 0.0358\ \text{kg/m}^3 = \mathbf{35.8\ g/m^3}$$

14-182 Liquid methanol is accidentally spilt on a 1 m × 1 m laboratory bench while a fan is providing a 20 m/s air flow parallel over the bench surface. The evaporation rate of methanol in molar basis is to be determined.

Assumptions **1** The analogy between heat and mass transfer is applicable. **2** The critical Reynolds number for flow over a flat plate is 500,000. **3** Methanol vapor is an ideal gas.

Properties The kinematic viscosity of air at 25°C and 1 atm is $\nu = 1.562\times10^{-5}$ m²/s (Table A-15). The diffusion coefficient of methanol vapor in air at 25°C = 298 K is $D_{AB} = 1.6\times10^{-5}$ m²/s.

Analysis The Reynolds number of the flow is

$$\text{Re} = \frac{VL}{\nu} = \frac{(20\text{ m/s})(1\text{ m})}{1.562\times10^{-5}\text{ m}^2\text{/s}} = 1.2804\times10^6$$

which is greater than 500,000 and thus the flow is turbulent. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562\times10^{-5}\text{ m}^2\text{/s}}{1.6\times10^{-5}\text{ m}^2\text{/s}} = 0.9763$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.037\,\text{Re}^{0.8}\,\text{Sc}^{1/3} = 0.037(1.2804\times10^6)^{0.8}(0.9763)^{1/3} = 2822$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{L} = \frac{(2822)(1.6\times10^{-5}\text{ m}^2\text{/s})}{1\text{ m}} = 0.04515\text{ m/s}$$

The concentration of methanol vapor at the air-methanol interface can be determined using

$$C_{A,s} = \frac{P_A}{R_u T} = \frac{4\text{ kPa}}{(8.314\text{ kPa}\cdot\text{m}^3\text{/kmol}\cdot\text{K})(273+25)\text{ K}} = 0.001614\text{ kmol/m}^3$$

Hence, the evaporation rate of methanol in molar basis is

$$\begin{aligned}\dot{N}_{\text{conv}} &= h_{\text{mass}}A_s(C_{A,s} - C_{A,\infty}) \\ &= (0.04515\text{ m/s})(1\text{ m}^2)(0.001614 - 0)\text{ kmol/m}^3 \\ &= \mathbf{7.29\times10^{-5}\ kmol/s}\end{aligned}$$

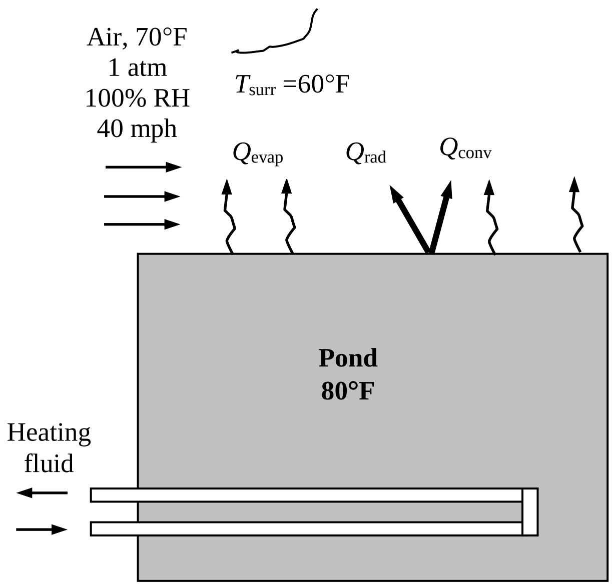
Discussion Methanol has a molar mass of M = 32.04 kg/kmol. Hence he evaporation rate in mass basis is

$$\dot{m}_{\text{conv}} = \dot{N}_{\text{conv}}M = (7.29\times10^{-5}\text{ kmol/s})(32.04\text{ kg/kmol}) = 2.33\times10^{-3}\text{ kg/s}$$

14-183E The top section of a solar pond is maintained at a constant temperature. The rates of heat loss from the top surface of the pond by radiation, natural convection, and evaporation are to be determined.

Assumptions **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 80°F). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The water in the pool is maintained at a uniform temperature of 80°F. **4** The critical Reynolds number for flow over a flat surface is 500,000.

Properties The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2 = (70+80)/2 = 75°F$.



The properties of dry air at 75°F and 1 atm are, from Table A-15E,

$$k = 0.01469\ \text{Btu/h}\cdot\text{ft}\cdot°\text{F}$$
$$\Pr = 0.7298$$
$$\alpha = 2.288\times10^{-4}\ \text{ft}^2/\text{s}$$
$$\nu = 0.167\times10^{-3}\ \text{ft}^2/\text{s}$$

The saturation pressure of water at 70°F is $P_{sat@70°F} = 0.3632$ psia. Properties of water at 80°F are $h_{fg} = 1048$ Btu/lbm and $P_v = 0.5073$ psia (Table A-9). The gas constant of water is R_{water} = 0.5957 psia.ft^3/lbm.R (Table A-1E). The emissivity of water is 0.95 (Table A-18). The mass diffusivity of water vapor in air at the average temperature of 75°F = 535 R = 297.2 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{H_2O\text{-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(297.2\text{K})^{2.072}}{1\text{atm}} = 2.49\times10^{-5}\ \text{m}^2/\text{s} = 2.68\times10^{-4}\ \text{ft}^2/\text{s}$$

Analysis (*a*) The pond surface can be treated as a flat surface. The Reynolds number for flow over a flat surface is

$$\text{Re} = \frac{VL}{\nu} = \frac{(40\times5280/3600\text{ft/s})(100\ \text{ft})}{0.167\times10^{-3}\ \text{ft}^2/\text{s}} = 3.51\times10^7$$

which is much larger than the critical Reynolds number of 500,000. Therefore, the air flow over the pond surface is turbulent, and the Nusselt number and the heat transfer coefficient are determined to be

$$\text{Nu} = 0.037\,\text{Re}_L^{0.8}\Pr^{1/3} = 0.037(3.51\times10^7)^{0.8}(0.7298)^{1/3} = 36{,}212$$

$$h_{heat} = \frac{\text{Nu}k}{L} = \frac{(36{,}212)(0.01469\ \text{Btu/h}\cdot\text{ft}\cdot°\text{F})}{100\ \text{ft}} = 5.32\ \text{Btu/h}\cdot\text{ft}^2\cdot°\text{F}$$

Then the rate of heat transfer from the air to the water by forced convection becomes

$$\dot{Q}_{conv} = h_{conv}A_s(T_\infty - T_s) = (5.32\text{Btu/h}\cdot\text{ft}^2\cdot°\text{F})(10{,}000\,\text{ft}^2)(80-70)°\text{F} = \mathbf{532{,}000\,Btu/h}\quad\text{(to water)}$$

(*b*) Noting that the emissivity of water is 0.95 and the surface area of the pool is $A_s = (100\,\text{ft})(100\,\text{ft}) = 10{,}000\,\text{ft}^2$, heat transfer from the top surface of the pool by radiation is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^4 - T_{surr}^4) = (0.95)(10{,}000\,\text{ft}^2)(0.1714\times10^{-8}\ \text{Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)[(540\,\text{R})^4 - (520\,\text{R})^4] = \mathbf{194{,}000\,Btu/h}$$

(*c*) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{0.167\times10^{-3}\ \text{ft}^2/\text{s}}{2.68\times10^{-4}\ \text{ft}^2/\text{s}} = 0.623$$

Then utilizing the analogy between heat and mass convection, the Sherwood number is determined by replacing Pr number by the Schmidt number to be

$$\text{Sh} = 0.037\,\text{Re}_L^{0.8}\text{Sc}^{1/3} = 0.037(3.51\times10^7)^{0.8}(0.623)^{1/3} = 34{,}350$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{D} = \frac{(34{,}350)(2.68\times10^{-4}\ \text{ft}^2/\text{s})}{100\,\text{ft}} = 0.0921\,\text{ft/s}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature ($P_{v,s}$ = 0.5073 psia at 80°F). The humidity of air is given to be 100%, and thus the air far from the water surface is also saturated. Therefore, $P_{v,\infty} = P_{\text{sat@70°F}} = 0.3632\ \text{psia}$.

Treating the water vapor as an ideal gas, the vapor densities at the water-air interface and far from the surface are determined to be

At the surface:
$$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{0.5073\ \text{psia}}{(0.5957\ \text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(80+460)\ \text{R}} = 0.00158\ \text{lbm/ft}^3$$

Away from the surface:
$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} == \frac{0.3632\ \text{psia}}{(0.5957\ \text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(70+460)\ \text{R}} = 0.00115\ \text{lbm/ft}^3$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\begin{aligned}\dot{m}_v &= h_{\text{mass}} A_s(\rho_{v,s} - \rho_{v,\infty}) = (0.0921\ \text{ft/s})(10{,}000\ \text{ft}^2)(0.00158 - 0.00115)\ \text{lbm/ft}^3\\ &= 0.396\ \text{lbm/s} = 1426\ \text{lbm/h}\end{aligned}$$

and

$$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (1426\ \text{lbm/h})(1048\ \text{Btu/lbm}) = \mathbf{1{,}494{,}000\ Btu/h}$$

Discussion All of the quantities calculated above represent heat loss for the pond, and the total rate of heat loss from the open top surface of the pond to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 194{,}000 + 532{,}000 + 1{,}494{,}000 = 2{,}220{,}000\ \text{Btu/h}$$

This heat loss will come from the deeper parts of the pond, and thus the pond will start cooling unless it gains heat from the sun or another heat source. Note that the evaporative heat losses dominate. Also, the rate of evaporation could be determined almost as accurately using mass fractions of vapor instead of vapor fractions and the average air density from the relation

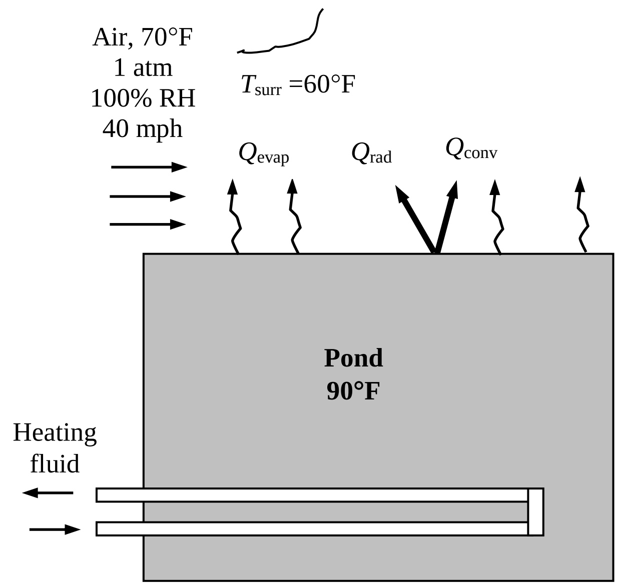
$$\dot{m}_{\text{evap}} = h_{\text{mass}}\rho A_s(w_{A,s} - w_{A,\infty}).$$

14-184E The top section of a solar pond is maintained at a constant temperature. The rates of heat loss from the top surface of the pond by radiation, natural convection, and evaporation are to be determined.

Assumptions **1** The low mass flux conditions exist so that the Chilton-Colburn analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 90°F). **2** Both air and water vapor at specified conditions are ideal gases (the error involved in this assumption is less than 1 percent). **3** The water in the pool is maintained at a uniform temperature of 90°F. **4** The critical Reynolds number for flow over a flat surface is 500,000.

Properties The air-water vapor mixture is assumed to be dilute, and thus we can use dry air properties for the mixture at the average temperature of $(T_\infty + T_s)/2 =$ (70+90)/2 = 80°F. The properties of dry air at 80°F and 1 atm are, from Table A-15E,

$$k = 0.01481 \text{ Btu/h}\cdot\text{ft}\cdot°\text{F}$$
$$\Pr = 0.7290$$
$$\alpha = 2.328\times10^{-4}\ \text{ft}^2/\text{h}$$
$$\nu = 1.697\times10^{-4}\ \text{ft}^2/\text{s}$$



The saturation pressure of water at 70°F is $P_{sat@70°F} = 0.3632$ psia. Properties of water at 90°F are $h_{fg} = 1043$ Btu/lbm and $P_v = 0.6988$ psia (Table A-9). The gas constant of water is R_{water} = 0.5957 psia.ft^3/lbm.R (Table A-1E). The emissivity of water is 0.95 (Table A-18). The mass diffusivity of water vapor in air at the average temperature of 80°F = 540 R = 300 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(300\text{K})^{2.072}}{1\text{atm}} = 2.54\times10^{-5}\ \text{m}^2/\text{s} = 2.73\times10^{-4}\ \text{ft}^2/\text{s}$$

Analysis (*a*) The pond surface can be treated as a flat surface. The Reynolds number for flow over a flat surface is

$$\text{Re} = \frac{VL}{\nu} = \frac{(40\times5280/3600\text{ft/s})(100\text{ ft})}{1.697\times10^{-4}\ \text{ft}^2/\text{s}} = 3.46\times10^7$$

which is much larger than the critical Reynolds number of 500,000. Therefore, the air flow over the pond surface is turbulent, and the Nusselt number and the heat transfer coefficient are determined to be

$$\text{Nu} = 0.037\,\text{Re}_L^{0.8}\Pr^{1/3} = 0.037(3.46\times10^7)^{0.8}(0.7290)^{1/3} = 35{,}785$$

$$h_{heat} = \frac{\text{Nu}k}{L} = \frac{(35{,}785)(0.01481\text{ Btu/h}\cdot\text{ft}\cdot°\text{F})}{100\text{ ft}} = 5.30\ \text{Btu/h}\cdot\text{ft}^2\cdot°\text{F}$$

Then the rate of heat transfer from the air to the water by forced convection becomes

$$\dot{Q}_{conv} = h_{conv}A_s(T_\infty - T_s) = (5.30\text{Btu/h}\cdot\text{ft}^2\cdot°\text{F})(10{,}000\text{ft}^2)(90-70)°\text{F} = \mathbf{1{,}060{,}000\,Btu/h}\quad\text{(to water)}$$

(*b*) Noting that the emissivity of water is 0.95 and the surface area of the pool is $A_s = (100\text{ ft})(100\text{ft}) = 10{,}000\text{ ft}^2$, heat transfer from the top surface of the pool by radiation is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^4 - T_{surr}^4) = (0.95)(10{,}000\text{ ft}^2)(0.1714\times10^{-8}\text{ Btu/h}\cdot\text{ft}^2\cdot\text{R}^4)[(550\text{ R})^4-(520\text{ R})^4] = \mathbf{299{,}400\,Btu/h}$$

(*c*) Utilizing the analogy between heat and mass convection, the mass transfer coefficient is determined the same way by replacing Pr by Sc. The Schmidt number is determined from its definition to be

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.697\times10^{-4}\ \text{ft}^2/\text{s}}{2.73\times10^{-4}\ \text{ft}^2/\text{s}} = 0.622$$

Then utilizing the analogy between heat and mass convection, the Sherwood number is determined by replacing Pr number by the Schmidt number to be

$$\text{Sh} = 0.037\,\text{Re}_L^{0.8}\text{Sc}^{1/3} = 0.037(3.46\times10^7)^{0.8}(0.622)^{1/3} = 33{,}940$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\text{mass}} = \frac{\text{Sh}D_{AB}}{D} = \frac{(33{,}940)(2.73\times10^{-4}\ \text{ft}^2/\text{s})}{100\,\text{ft}} = 0.0927\,\text{ft/s}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature ($P_{v,s}$ = 0.6988 psia at 90°F). The humidity of air is given to be 100%, and thus the air far from the water surface is also saturated. Therefore, $P_{v,\infty} = P_{\text{sat@70°F}} = 0.3632\ \text{psia}$.

Treating the water vapor as an ideal gas, the vapor densities at the water-air interface and far from the surface are determined to be

At the surface: $$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{0.6988\ \text{psia}}{(0.5957\ \text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(90+460)\ \text{R}} = 0.00213\ \text{lbm/ft}^3$$

Away from the surface: $$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} == \frac{0.3632\ \text{psia}}{(0.5957\ \text{psia}\cdot\text{ft}^3/\text{lbm}\cdot\text{R})(70+460)\ \text{R}} = 0.00115\ \text{lbm/ft}^3$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\dot{m}_v = h_{\text{mass}} A_s(\rho_{v,s} - \rho_{v,\infty}) = (0.0927\ \text{ft/s})(10{,}000\ \text{ft}^2)(0.00213 - 0.00115)\ \text{lbm/ft}^3$$
$$= 0.908\ \text{lbm/s} = 3269\ \text{lbm/h}$$

and $$\dot{Q}_{\text{evap}} = \dot{m}_v h_{fg} = (3269\ \text{lbm/h})(1043\ \text{Btu/lbm}) = \mathbf{3{,}410{,}000\ Btu/h}$$

Discussion All of the quantities calculated above represent heat loss for the pond, and the total rate of heat loss from the open top surface of the pond to the surrounding air and surfaces is

$$\dot{Q}_{\text{total, top}} = \dot{Q}_{\text{rad}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{evap}} = 299{,}400 + 1{,}060{,}000 + 3{,}410{,}000 = 4{,}769{,}400\ \text{Btu/h}$$

This heat loss will come from the deeper parts of the pond, and thus the pond will start cooling unless it gains heat from the sun or another heat source. Note that the evaporative heat losses dominate. Also, the rate of evaporation could be determined almost as accurately using mass fractions of vapor instead of vapor fractions and the average air density from the relation

$$\dot{m}_{\text{evap}} = h_{\text{mass}}\rho A(w_{A,s} - w_{A,\infty}).$$

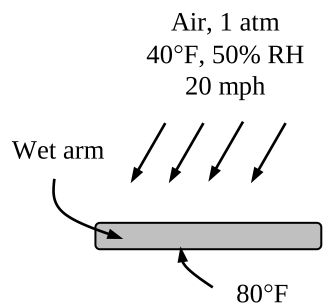
14-185E A swimmer extends his wet arms into the windy air outside. The rate at which water evaporates from both arms and the corresponding rate of heat transfer by evaporation are to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 60°F). **2** The arm can be modeled as a long cylinder.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the average temperature of (40 + 80)/2 = 60°F and 1 atm, for which $\nu = 1.588\times10^{-4}$ ft^2/s , and $\rho = 0.07633$ lbm / ft^3 (Table A-15E). The saturation pressure of water at 40ºF is 0.1217 psia. Also, at 80ºF, the saturation pressure is 0.5073 psia and the heat of vaporization is 1048 Btu/lbm (Table A-9E). The gas constant of water is R = 0.5957 psia.ft^3/lbm.R (Table A-1E). The mass diffusivity of water vapor in air at 60ºF = 520 R = 288.9 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\mathrm{H_2O\text{-}air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P} = 1.87\times10^{-10}\frac{(288.9\,\mathrm{K})^{2.072}}{1\mathrm{atm}} = 2.35\times10^{-5}\ \mathrm{m^2/s} = 2.53\times10^{-4}\ \mathrm{ft^2/s}$$

Analysis The Reynolds number for flow over a cylinder is



$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(20\times5280/3600\ \mathrm{ft/s})(3/12\ \mathrm{ft})}{1.588\times10^{-4}\ \mathrm{ft^2/s}} = 46{,}180$$

The Schmidt number in this case is

$$\mathrm{Sc} = \frac{\nu}{D_{AB}} = \frac{1.588\times10^{-4}\ \mathrm{ft^2/s}}{2.53\times10^{-4}\ \mathrm{ft^2/s}} = 0.628$$

Then utilizing the analogy between heat and mass convection, the Sherwood number is determined from Table 14-13 or by replacing Pr number by the Schmidt number in Eq. 7-35, the result is

$$\mathrm{Sh} = 0.3 + \frac{0.62\,\mathrm{Re}^{0.5}\mathrm{Sc}^{1/3}}{\left[1+(0.4/\mathrm{Sc})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5} = 0.3 + \frac{0.62(46{,}180)^{0.5}(0.628)^{1/3}}{\left[1+(0.4/0.628)^{2/3}\right]^{1/4}}\left[1+\left(\frac{46{,}180}{282{,}000}\right)^{5/8}\right]^{4/5} = 125$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\mathrm{mass}} = \frac{\mathrm{Sh}D_{AB}}{D} = \frac{(125)(2.53\times10^{-4}\ \mathrm{ft^2/s})}{3/12\,\mathrm{ft}} = 0.1265\,\mathrm{ft/s}$$

The air at the water surface is saturated, and thus the vapor pressure at the surface is simply the saturation pressure of water at the surface temperature (0.5073 psia at 80°F). The vapor pressure of air far from the water surface is determined from

$$P_{v,\infty} = \phi P_{\mathrm{sat@}T_\infty} = (0.50)P_{\mathrm{sat@40°F}} = (0.50)(0.1217\ \mathrm{psia}) = 0.0609\ \mathrm{psia}$$

Treating the water vapor as an ideal gas, the vapor densities at the water-air interface and far from the surface are determined to be

At the surface:
$$\rho_{v,s} = \frac{P_{v,s}}{R_v T_s} = \frac{0.5073\ \mathrm{psia}}{(0.5957\ \mathrm{psia\cdot ft^3/lbm\cdot R})(80+460)\ \mathrm{R}} = 0.00158\ \mathrm{lbm/ft^3}$$

Away from the surface:
$$\rho_{v,\infty} = \frac{P_{v,\infty}}{R_v T_\infty} == \frac{0.0609\ \mathrm{psia}}{(0.5957\ \mathrm{psia\cdot ft^3/lbm\cdot R})(40+460)\ \mathrm{R}} = 0.000204\ \mathrm{lbm/ft^3}$$

Then the evaporation rate and the rate of heat transfer by evaporation become

$$\dot m_v = h_{\mathrm{mass}}A_s(\rho_{v,s}-\rho_{v,\infty}) = (0.1265\ \mathrm{ft/s})[2\times\pi(3/12\ \mathrm{ft})(2\,\mathrm{ft})](0.00158-0.000204)\ \mathrm{lbm/ft^3}$$
$$= 5.47\times10^{-4}\ \mathrm{lbm/s} = \mathbf{1.97\ lbm/h}$$

and
$$\dot Q_{\mathrm{evap}} = \dot m_v h_{fg} = (5.47\times10^{-4}\ \mathrm{lbm/s})(1048\ \mathrm{Btu/lbm}) = \mathbf{0.573\ Btu/s}$$

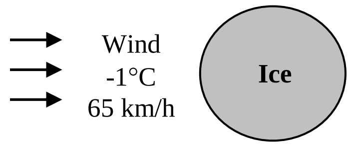
Discussion The rate of evaporation could be determined almost as accurately using mass fractions of vapor instead of vapor fractions and the average air density from the relation $\dot m_{\mathrm{evap}} = h_{\mathrm{mass}}\rho A(w_{A,s}-w_{A,\infty})$.

14-186 A sphere of ice is exposed to wind. The ice evaporation rate is to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 300 K). **2** The flow is fully developed.

Properties The properties are given in problem statement.

Analysis The Reynolds and Schmidt numbers are



$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(65/3.6\ \mathrm{m/s})(0.05\mathrm{m})}{1.32\times10^{-7}\ \mathrm{m^2/s}} = 6.84\times10^{6}$$

$$\mathrm{Sc} = \frac{\nu}{D_{AB}} = \frac{1.32\times10^{-7}\ \mathrm{m^2/s}}{2.5\times10^{-5}\ \mathrm{m^2/s}} = 5.28\times10^{-3}$$

The Sherwood number is

$$\mathrm{Sh} = \left[4 + 1.21(\mathrm{Re}\,Sc)^{2/3}\right]^{0.5} = \left[4 + 1.21\left[(6.84\times10^{6})(5.28\times10^{-3})\right]^{2/3}\right]^{0.5} = 36.4$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{\mathrm{mass}} = \frac{\mathrm{Sh}D_{AB}}{D} = \frac{(36.4)(2.5\times10^{-5}\ \mathrm{m^2/s})}{0.05\mathrm{m}} = 0.0182\ \mathrm{m/s}$$

The evaporation rate is determined as follows:

$$\dot{N} = h_{\mathrm{mass}}\Delta C = h_{\mathrm{mass}}\left(\frac{P_{A,0}}{R_uT} - \frac{P_{A,L}}{R_uT}\right) = h_{\mathrm{mass}}\frac{P_v}{R_uT}(1-0.15)$$

$$= (0.0182\ \mathrm{m/s})\frac{0.56\ \mathrm{kPa}}{(8.314\ \mathrm{kPa\cdot m^3/kmol\cdot kg})(272\ \mathrm{K})}(1-0.15) = 3.83\times10^{-6}\ \mathrm{kmol/m^2\cdot s}$$

$$\dot{m}_{\mathrm{evap}} = \dot{N}MA = (3.83\times10^{-6}\ \mathrm{kmol/m^2\cdot s})(18\ \mathrm{kg/kmol})\left[\pi(0.05\ \mathrm{m})^2\right]$$

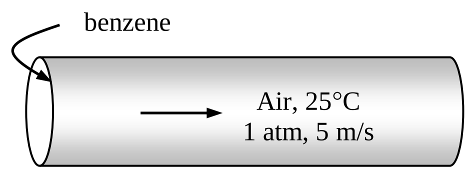
$$= 5.42\times10^{-7}\ \mathrm{kg/s} = \mathbf{1.95\ g/h}$$

14-187 Benzene-free air flows in a tube whose inner surface is coated with pure benzene. The average mass transfer coefficient, the molar concentration of benzene in the outlet air, and the evaporation rate of benzene are to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable. **2** The flow is fully developed.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 25°C and 1 atm, for which $\nu = 1.562\times10^{-5}\ \text{m}^2/\text{s}$ (Table A-15). The mass diffusivity of benzene in air at 298 K is $D_{AB} = 0.88\times10^{-5}\ \text{m}^2/\text{s}$ (Table 14-2). The molar mass of benzene is 78 kg/kmol.

Analysis (a) The Reynolds number of the flow is



$$\text{Re} = \frac{VD}{\nu} = \frac{(5\ \text{m/s})(0.05\text{m})}{1.562\times10^{-5}\ \text{m}^2/\text{s}} = 16{,}005$$

which is greater than 10,000 and thus the flow is turbulent. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.562\times10^{-5}\ \text{m}^2/\text{s}}{0.88\times10^{-5}\ \text{m}^2/\text{s}} = 1.775$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.023\,\text{Re}^{0.8}\,\text{Sc}^{0.4} = 0.023(16{,}005)^{0.8}(1.775)^{0.4} = 66.8$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{mass} = \frac{\text{Sh}D_{AB}}{D} = \frac{(66.8)(0.88\times10^{-5}\ \text{m}^2/\text{s})}{0.05\ \text{m}} = \mathbf{0.0118\ m/s}$$

(b) The molar concentration of benzene in the outlet air is determined as follows

$$C_s = \frac{P_v}{R_u T} = \frac{13\ \text{kPa}}{(8.314\ \text{kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(25+273\ \text{K})} = 5.25\times10^{-3}\ \text{kmol/m}^3$$

$$VA_c(C_{out} - C_{in}) = h_{mass}A\left[\frac{(C_s - C_{in}) - (C_s - C_{out})}{\ln\left(\dfrac{C_s - C_{in}}{C_s - C_{out}}\right)}\right]$$

$$(5)(\pi\times0.05^2/4)(C_{out} - 0) = (0.0118)(\pi\times0.05\times6)\left[\frac{(5.25\times10^{-3} - 0) - (5.25\times10^{-3} - C_{out})}{\ln\left(\dfrac{5.25\times10^{-3} - 0}{5.25\times10^{-3} - C_{out}}\right)}\right]$$

$$\longrightarrow C_{out} = \mathbf{3.56\times10^{-3}\ kmol/m^3}$$

(c) The evaporation rate of benzene is determined from

$$\dot{m}_{evap} = MVA_c(C_{out} - C_{in})$$

$$= (78\ \text{kg/kmol})(5\ \text{m/s})\frac{\pi\times(0.05\ \text{m})^2}{4}(3.56\times10^{-3}\ \text{kmol/m}^3 - 0)$$

$$= 2.73\times10^{-3}\ \text{kg/s} = \mathbf{9.81 kg/h}$$

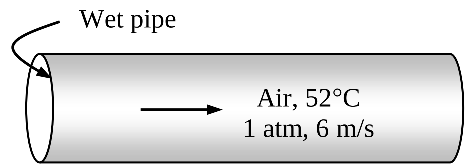
14-188 The liquid layer on the inner surface of a circular pipe is dried by blowing air through it. The average mass transfer coefficient, log-mean driving force for mass transfer (in molar concentration units, the evaporation rate, and the tube length are to be determined.

Assumptions **1** The low mass flux model and thus the analogy between heat and mass transfer is applicable since the mass fraction of vapor in the air is low (about 2 percent for saturated air at 325 K). **2** The flow is fully developed.

Properties Because of low mass flux conditions, we can use dry air properties for the mixture at the specified temperature of 52°C and 1 atm, for which $\nu = 1.818\times10^{-5}\ \text{m}^2/\text{s}$ (Table A-15). The mass diffusivity of water vapor in air at 52+273 = 325 K is determined from Eq. 14-15 to be

$$D_{AB} = D_{\text{H}_2\text{O-air}} = 1.87\times10^{-10}\frac{T^{2.072}}{P}$$

$$= 1.87\times10^{-10}\frac{(325\ \text{K})^{2.072}}{1} = 3.00\times10^{-5}\ \text{m}^2/\text{s}$$



Analysis (a) The Reynolds number of the flow is

$$\text{Re} = \frac{VD}{\nu} = \frac{(6\ \text{m/s})(0.05\text{m})}{1.818\times10^{-5}\ \text{m}^2/\text{s}} = 16{,}500$$

which is greater than 10,000 and thus the flow is turbulent. The Schmidt number in this case is

$$\text{Sc} = \frac{\nu}{D_{AB}} = \frac{1.818\times10^{-5}\ \text{m}^2/\text{s}}{3.00\times10^{-5}\ \text{m}^2/\text{s}} = 0.606$$

Therefore, the Sherwood number in this case is determined from Table 14-13 to be

$$\text{Sh} = 0.023\,\text{Re}^{0.8}\,\text{Sc}^{0.4} = 0.023(16{,}500)^{0.8}(0.606)^{0.4} = 44.54$$

Using the definition of Sherwood number, the mass transfer coefficient is determined to be

$$h_{mass} = \frac{\text{Sh}D_{AB}}{D} = \frac{(44.54)(3.00\times10^{-5}\ \text{m}^2/\text{s})}{0.05\text{m}} = \mathbf{0.0267\ m/s}$$

(b) The log-mean driving force for mass transfer (in molar concentration units) is determined as follows

$$C_w = \frac{P_v}{R_uT} = \frac{13.6\ \text{kPa}}{(8.314\ \text{kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(325\ \text{K})} = 5.03\times10^{-3}\ \text{kmol/m}^3$$

$$C_{in} = \frac{P_v}{R_uT} = \frac{0.2\times13.6\ \text{kPa}}{(8.314\ \text{kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(325\ \text{K})} = 1.01\times10^{-3}\ \text{kmol/m}^3$$

$$C_{out} = \frac{P_v}{R_uT} = \frac{10.0\ \text{kPa}}{(8.314\ \text{kPa}\cdot\text{m}^3/\text{kmol}\cdot\text{K})(325\ \text{K})} = 3.70\times10^{-3}\ \text{kmol/m}^3$$

$$\Delta C = \frac{(C_w - C_{in}) - (C_w - C_{out})}{\ln\left(\dfrac{C_w - C_{in}}{C_w - C_{out}}\right)}$$

$$= \frac{(5.03\times10^{-3} - 1.01\times10^{-3}) - (5.03\times10^{-3} - 3.70\times10^{-3})}{\ln\left(\dfrac{5.03\times10^{-3} - 1.01\times10^{-3}}{5.03\times10^{-3} - 3.70\times10^{-3}}\right)} = \mathbf{2.43\times10^{-3}\ kmol/m^3}$$

(c) The evaporation rate is determined from

$$\dot{m}_{evap} = MVA_c(C_{out} - C_{in}) = (18\ \text{kg/kmol})(6\ \text{m/s})\frac{\pi(0.05\ \text{m})^2}{4}(3.70\times10^{-3} - 1.01\times10^{-3})\ \text{kmol/m}^3$$

$$= 5.70\times10^{-4}\ \text{kg/s} = \mathbf{2.05\ kg/h}$$

(d) The tube length is determined from

$$\frac{\dot{m}_{evap}}{M} = h_{mass}A\Delta C \rightarrow \frac{5.70\times10^{-4}\ \text{kg/s}}{18\ \text{kg/kmol}} = (0.0267\ \text{m/s})\pi(0.05\ \text{m})L(2.43\times10^{-3}\ \text{kmol/m}^3) \rightarrow L = \mathbf{3.11m}$$

Fundamentals of Engineering (FE) Exam Problems

14-189 The basic equation describing the diffusion of one medium through another stationary medium is

(a) $j_A = -CD_{AB} \dfrac{d(C_A/C)}{dx}$ (b) $j_A = -D_{AB} \dfrac{d(C_A/C)}{dx}$

c) $j_A = -k \dfrac{d(C_A/C)}{dx}$ (d) $j_A = -k \dfrac{dT}{dx}$ e) None of them

Answer (a) $j_A = -CD_{AB} \dfrac{d(C_A/C)}{dx}$

14-190 For the absorption of a gas, like carbon dioxide, into a liquid, like water, Henry's law states that partial pressure of the gas is proportional to the mole fraction of the gas in the liquid-gas solution with the constant of proportionality being Henry's constant. A bottle of soda pop (CO_2-H_2O) at room temperature has a Henry's constant of 17,100 kPa. If the pressure in this bottle is 140 kPa and the partial pressure of the water vapor in the gas volume at the top of the bottle is neglected, the concentration of the CO_2 in the liquid H_2O is

(a) 0.004 mol-CO_2/mol (b) 0.008 mol-CO_2/mol (c) 0.012 mol-CO_2/mol
(d) 0.024 mol-CO_2/mol (e) 0.035 mol-CO_2/mol

Answer (b) 0.008 mol-CO_2/mol

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
H=17.1 [MPa]
P=0.140 [MPa]
y=P/H
```

14-191 A rubber object is in contact with nitrogen (N_2) at 298 K and 250 kPa. The solubility of nitrogen gas in rubber is 0.00156 kmol/m^3·bar. The mass density of nitrogen at the interface is

(a) 0.049 kg/m^3 (b) 0.064 kg/m^3 (c) 0.077 kg/m^3 (d) 0.092 kg/m^3 (e) 0.109 kg/m^3

Answer (e) 0.109 kg/m^3

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=298 [K]
P_N2_gasside=250 [kPa]*Convert(kPa, bar)
S=0.00156 [kmol/m^3-bar] "Table 14-7"
C_N2_solidside=S*P_N2_gasside
M_N2=MolarMass(N2)
rho_N2_solidside=C_N2_solidside*M_N2
```

14-192 Nitrogen gas at high pressure and 298 K is contained in a 2-m × 2-m × 2-m cubical container made of natural rubber whose walls are 3 cm thick. The concentration of nitrogen in the rubber at the inner and outer surfaces are 0.067 kg/m^3 and 0.009 kg/m^3, respectively. The diffusion coefficient of nitrogen through rubber is 1.5×10^{-10} m^2/s. The mass flow rate of nitrogen by diffusion through the cubical container is

(a) 8.1×10^{-10} kg/s (b) 3.2×10^{-10} kg/s (c) 3.8×10^{-9} kg/s (d) 7.0×10^{-9} kg/s (e) 1.60×10^{-8} kg/s

Answer (d) 7.0×10^{-9} kg/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
s=2 [m]
L=0.03 [m]
rho_A_1=0.067 [kg/m^3]
rho_A_2=0.009 [kg/m^3]
A=6*s^2
D_AB=1.5E-10 [m^2/s] "Table 14-3b"
m_dot_diff=D_AB*A*(rho_A_1-rho_A_2)/L
```

14-193 A recent attempt to circumnavigate the world in a balloon used a helium filled balloon whose volume was 7240 m^3 and surface area was 1800 m^2. The skin of this balloon is 2 mm thick and is made of a material whose helium diffusion coefficient is 1×10^{-9} m^2/s. The molar concentration of the helium at the inner surface of the balloon skin is 0.2 kmol/m^3 and the molar concentration at the outer surface is extremely small. The rate at which helium is lost from this balloon is

(a) 0.26 kg/h (b) 1.5 kg/h (c) 2.6 kg/h (d) 3.8 kg/h (e) 5.2 kg/h

Answer (c) 2.6 kg/h

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
Ci=0.2 [kmol/m^3]
Co=0 [kmol/m^3]
D=1E-9 [m^2/s]
L=0.002 [m]
M=4 [kg/kmol]
A=1800 [m^2]
Ndot=D*A*(Ci-Co)/L
Mdot=Ndot*M*3600
```

14-194 Carbon at 1273 K is contained in a 7-cm-inner-diameter cylinder made of iron whose thickness is 1.2 mm. The concentration of carbon in the iron at the inner surface is 0.5 kg/m^3 and the concentration of carbon in the iron at the outer surface is negligible. The diffusion coefficient of carbon through iron is 3×10^{-11} m^2/s. The mass flow rate carbon by diffusion through the cylinder shell per unit length of the cylinder is

(a) 2.8×10^{-9} kg/s (b) 5.4×10^{-9} kg/s (c) 8.8×10^{-9} kg/s (d) 1.6×10^{-8} kg/s (e) 5.2×10^{-8} kg/s

Answer (a) 2.8×10^{-9} kg/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=1273 [K]
D_i=0.07 [m]
D_o=D_i+2*0.0012 [m]
rho_A_1=0.5 [kg/m^3]
rho_A_2=0 [kg/m^3]
D_AB=3.0E-11 [m^2/s] "Table 14-3b"
r_1=D_i/2
r_2=D_o/2
L=1 [m]
m_dot_diff=2*pi*L*D_AB*(rho_A_1-rho_A_2)/ln(r_2/r_1)
```

14-195 The surface of an iron component is to be hardened by carbon. The diffusion coefficient of carbon in iron at 1000ºC is given to be 3×10^{-11} m^2/s. If the penetration depth of carbon in iron is desired to be 1.0 mm, the hardening process must take at least

(a) 1.10 h (b) 1.47 h (c) 1.86 h (d) 2.50 h (e) 2.95 h

Answer (e) 2.95 h

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D_AB=3E-11 [m^2/s]
delta_diff=1E-3 [m]
delta_diff=sqrt(pi*D_AB*t)
t_hour=t*Convert(s, h)
```

14-196 A natural gas (methane, CH_4) storage facility uses 3 cm diameter by 6 m long vent tubes on its storage tanks to keep the pressure in these tanks at atmospheric value. If the diffusion coefficient for methane in air is 0.2×10^{-4} m^2/s and the temperature of the tank and environment is 300 K, the rate at which natural gas is lost from a tank through one vent tube is

(a) 13×10^{-5} kg/day (b) 3.2×10^{-5} kg/day (c) 8.7×10^{-5} kg/day (d) 5.3×10^{-5} kg/day (e) 0.12×10^{-5} kg/day

Answer (a) 13×10^{-5} kg/day

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
d=0.03 [m]
L=6 [m]
D_AB=0.2E-4 [m^2/s]
P=101 [kPa]
T=300 [K]
M=16 [kg/kmol]
A=pi*d^2/4
Ndot=(D_AB*A/(R#*T))*(P/L)
Mdot=Ndot*M*Convert(day, s)
```

14-197 Saturated water vapor at 25ºC (P_{sat} = 3.17 kPa) flows in a pipe that passes through air at 25ºC with a relative humidity of 40 percent. The vapor is vented to the atmosphere through a 9-mm-internal diameter tube that extends 10 m into the air. The diffusion coefficient of vapor through air is 2.5×10^{-5} m^2/s. The amount of water vapor lost to the atmosphere through this individual tube by diffusion is

(a) 1.7×10^{-6} kg (b) 2.3×10^{-6} kg (c) 3.8×10^{-6} kg (d) 5.0×10^{-6} kg (e) 7.1×10^{-6} kg

Answer (b) 2.3×10^{-6} kg

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=25 [C]
phi=0.40
D=0.009 [m]
L=10 [m]
D_AB=2.5E-5 [m^2/s] "Table 14-2"
P_A_0=pressure(steam_iapws, x=1, T=T) "pressure of vapor at x=0"
P_A_L=phi*P_A_0 "pressure of vapor at x=L=10 m"
A=pi*D^2/4
R_u=8.314[kPa-m^3/kmol-K]
N_dot_vapor=(D_AB*A)/(R_u*T)*(P_A_0-P_A_L)/L
MM=MolarMass(H2O)
m_dot_vapor=N_dot_vapor*MM
time=24*3600 [s]
m_vapor=m_dot_vapor*time
"Some Wrong Solutions with Common Mistakes"
W_P_A_L=0 "Taking the vapor pressure at air side zero"
W_N_dot_vapor=(D_AB*A)/(R_u*T)*(P_A_0-W_P_A_L)/L
W_m_dot_vapor=W_N_dot_vapor*MM
W_m_vapor=W_m_dot_vapor*time
```

14-198 When the ____ is unity, one can expect the momentum and mass transfer by diffusion to be the same.

(a) Grashof (b) Reynolds (c) Lewis (d) Schmidt (e) Sherwood

Answer (d) Schmidt

14-199 Air flows in a 4-cm-diameter wet pipe at 20ºC and 1 atm with an average velocity of 4 m/s in order to dry the surface. The Nusselt number in this case can be determined from $\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4}$ where Re = 10,550 and Pr = 0.731. Also, the diffusion coefficient of water vapor in air is 2.42×10^{-5} m^2/s. Using the analogy between heat and mass transfer, the mass transfer coefficient inside the pipe for fully developed flow becomes

(a) 0.0918 m/s (b) 0.0408 m/s (c) 0.0366 m/s (d) 0.0203 m/s (e) 0.0022 m/s

Answer (d) 0.0203 m/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
D=0.04 [m]
T=20[C]+273 [K]
P=1 [atm]
V=4 [m/s]
Re=10550
Pr=0.731
D_AB=2.42E-5 [m^2/s]
Nus=0.023*Re^0.8*Pr^0.4 "Table 14-13"
Sh=Nus
h_mass=(Sh*D_AB)/D
"Some Wrong Solutions with Common Mistakes"
W_Sh=3.66 "Considering laminar flow"
W_h_mass=(W_Sh*D_AB)/D
```

14-200 Air flows through a wet pipe at 298 K and 1 atm, and the diffusion coefficient of water vapor in air is 2.5×10^{-5} m^2/s. If the heat transfer coefficient is determined to be 80 W/m$^2\cdot$°C, the mass transfer coefficient is

(a) 0.0022 m/s (b) 0.036 m/s (c) 0.074 m/s (d) 0.092 m/s (e) 0.13 m/s

Answer (c) 0.074 m/s

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen.

```
T=298 [K]
P=1 [atm]
h_heat=80 [W/m^2-C]
D_AB=2.5E-5 [m^2/s] "Table 14-2"
rho=1.184 [kg/m^3]
c_p=1007 [J/kg-C]
alpha=2.141E-5 [m^2/s]
h_heat=h_mass*rho*c_p*(alpha/D_AB)^(2/3)
```

14-201 .... 14-204 Design and Essay Problems