Ministry of Higher Education and Scientific Research

University of Technology

Chemical Engineering Department

Second Class

FLUID FLOW

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CHAPTER SIX: NON-NEWTONIAN FLUIDS

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<u>REFERENCES</u>

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CHPTER ONE

Introduction

1.1 Introduction

Chemical engineering has to do with industrial processes in which raw materials are changed or separated into useful products.

The chemical engineer must develop, design, and engineer both the complete process and the equipment used; choose the proper raw materials; operate the plants efficiently, safely, and economically; and see to it that products meet the requirement set by the customers.

A Fluid is any substance that conforms to the shape of its container and it may be defined as a substance that dose not permanently resist distortion and hence, will its shape. *Gases and liquids and vapors* are considered to have the characteristics of fluids and to obey many of the same laws.

In the process industries, many of the materials are in fluid form and must be stored, handled, pumped, and processed, so it is necessary that we become familiar with the principles that govern the flow of fluids and also with the equipment used. Typical fluids encountered include water, acids, air, Co_2, oil, slurries.

If a fluid affected by changes in pressure, it is said to be *"compressible fluid"*, otherwise, it is said to be *"incompressible fluid"*.

Most liquids are incompressible, and gases are can considered to be compressible fluids. However, if gases are subjected to small percentage changes in pressure and temperature, their densities change will be small and they can be considered to be incompressible fluids.

The fluid mechanics can be divided into two branches;

"Fluid static" that means fluid at rest, and

"Fluid dynamics" that means fluid in motion.

1.2 Physical Properties of Fluids

1. **Mass density or density [symbol: ρ (rho)]**

   It is the ratio of mass of fluid to its volume, $\boxed{\rho = \frac{\textbf{Mass of fluid}}{\textbf{Volume of fluid}}}$

   The common units used of density is (kg/m^3), (g/cm^3), (lb/ft^3).

2. **Specific Volume [symbol: υ (upsilon)]**

   It is the ratio of volume of fluid to its mass (or mole); it is the reciprocal of its density, $\boxed{\upsilon = \frac{\textbf{Volume of fluid}}{\textbf{Mass of fluid}}}$

   The common units used of density is (m^3/ kg), (cm^3/g), (ft^3/lb).

3. **Weight density or specific weight [symbol: sp.wt.]**

   It is the ratio of weight of fluid to its volume, $\boxed{sp.wt. = \frac{\textbf{Weight of fluid}}{\textbf{Volume of fluid}}}$

   The common units used of density is (N/m^3), ($dyne/cm^3$), (lb_f/ft^3).

4. Specific gravity [symbol: sp.gr.]

It is the ratio of mass density or (density) of fluid to mass density or (density) of water, Physicists use 39.2°F (4°C) as the standard, but engineers ordinarily use 60°F (15.556°C) $\boxed{\boldsymbol{sp.gr.} = \dfrac{\textbf{Mass density of fluid}}{\textbf{Mass density of water}}}$

The common density used of water is (1000 kg/m^3), (1.0g/cm^3), (62.43 lb/ft^3).

5. Dynamic viscosity [symbol: μ (mu)]

It is the property of a fluid, which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

The common units used of dynamic viscosity is (kg/m.s), (g/cm.s), (lb/ft.s), (poise) (N.s/m^2 ≡ Pa.m), (dyne.s/cm^2). [poise ≡ g/cm.s ≡ dyne.s/cm^2] [poise = 100 c.p]

6. Kinematic viscosity [symbol: ν (nu)]

It is the ratio of the dynamic viscosity to mass density of fluid, $\boxed{\nu = \dfrac{\mu}{\rho}}$

The common units used of kinematics viscosity is (m^2/s), (cm^2/s), (ft^2/s), (stoke).
[stoke ≡ cm^2/s] [stoke = 100 c.stoke]

7. Surface tension [symbol: σ (sigma)]

It is the property of the liquid, which enables it to resist tensile stress. It is due to cohesion between surface molecules of a liquid.

The common units used of Surface tension is (N/m), (dyne/cm), (lb_f/ft).

<u>1.3 Useful Information</u>

1. The shear stress [symbol: τ (tau)]

It is the force per unit surface area that resists the sliding of the fluid layers.

The common units used of shear stress is (N/m^2 ≡ Pa), (dyne/cm^2), (lb_f/ft^2).

2. The pressure [symbol: P]

It is the force per unit cross sectional area normal to the force direction.

The common units used of shear stress is (N/m^2 ≡ Pa), (dyne/cm^2), (lb_f/ft^2) (atm) (bar) (Psi) (torr ≡ mmHg). The pressure difference between two points refers to (ΔP).

The pressure could be expressed as liquid height (or head) (h) where,

$\boxed{\boldsymbol{P} = \boldsymbol{h}\,\rho\,\boldsymbol{g}}$ and $\boxed{\Delta\boldsymbol{P} = \Delta\boldsymbol{h}\,\rho\,\boldsymbol{g}}$

h: is the liquid height (or head), units (m), (cm), (ft).

3. The energy [symbol: E]

Energy is defined as the capacity of a system to perform work or produce heat.

There are many types of energy such as [Internal energy (U), Kinetic energy (K.E), Potential energy (P.E), Pressure energy (Prs.E), and others.

The common units used for energy is (J ≡ N.m), (erg ≡ dyne.cm), (Btu), (lb_f.ft) (cal).

The energy could be expressed in relative quantity per unit mass or mole (J/kg or mol).

The energy could be expressed in head quantity [(m) (cm) (ft)] by dividing the relative energy by acceleration of gravity.

4. The Power [symbol: P]

It is the energy per unit time. The common units used for Power is (W ≡ J/s), (Btu/time), (lb_f.ft/time) (cal/time), (hp).

5. The flow rate

5.1. Volumetric flow rate [symbol: Q]

It is the volume of fluid transferred per unit time.

$\boxed{Q = u\,A}$ where A: is the cross sectional area of flow normal to the flow direction. The common units used for volumetric flow is (m^3/s), (cm^3/s), (ft^3/s).

5.2. Mass flow rate [symbol: $\dot{m}$]

It is the mass of fluid transferred per unit time. $\boxed{\dot{m} = Q\,\rho = u\,A\,\rho}$

The common units used for volumetric flow is (kg/s), (g/s), (lb/s).

5.3. Mass flux or (mass velocity) [symbol: G]

It is the mass flow rate per unit area of flow, $\boxed{G = \dfrac{\dot{m}}{A} = u\,\rho}$

The common units used for mass flux is (kg/m2.s), (g/cm2.s), (lb/ft2.s).

6. Ideal fluid

An ideal fluid is one that is incompressible It is a fluid, and having <u>no</u> viscosity (μ = 0). Ideal fluid is only an imaginary fluid since all the fluids, which exist, have some viscosity.

7. Real fluid

A fluid, which possesses viscosity, is known as real fluid. All the fluids, an actual practice, are real fluids.

<u>1.4 Important Laws</u>

1. Law of conservation of mass

" The mass can neither be created nor destroyed, and it can not be created from nothing"

2. Law of conservation of energy

" The energy can neither be created nor destroyed, though it can be transformed from one form into another"

Newton's Laws of Motion

Newton has formulated three law of motion, which are the basic postulates or assumption on which the whole system of dynamics is based.

3. Newton's first laws of motion

"Every body continues in its state of rest or of uniform motion in a straight line, unless it is acted upon by some external forces"

4. Newton's second laws of motion

"The rate of change in momentum is directly proportional to the impressed force and takes place in the same direction in which the force acts"[momentum = mass × velocity]

5. Newton's third laws of motion

"To every action, there is always an equal and opposite reaction"

6. First law of thermodynamics

"Although energy assumes many forms, the total quantity of energy is constant, and when energy disappears in one form it appears simultaneously in other forms"

1.5 Flow Patterns

The nature of fluid flow is a function of the fluid physical properties, the geometry of the container, and the fluid flow rate. The flow can be characterized either as **Laminar** or as **Turbulent** flow.

Laminar flow is also called "viscous or streamline flow". In this type of flow layers of fluid move relative to each other without any intermixing.

Turbulent flow in this flow, there is irregular random movement of fluid in directions transverse to the main flow.

1.6 Newton's Law of Viscosity and Momentum Transfer

Consider two parallel plates of area (A), distance (dz) apart shown in Figure (1). The space between the plates is filled with a fluid. The lower plate travels with a velocity (u) and the upper plate with a velocity (u-du). The small difference in velocity (du) between the plates results in a resisting force (F) acting over the plate area (A) due to viscous frictional effects in the fluid.

Thus the force (F) must apply to the lower plate to maintain the difference in velocity (du) between the two plates. The force per unit area (F/A) is known as the shear stress (τ).

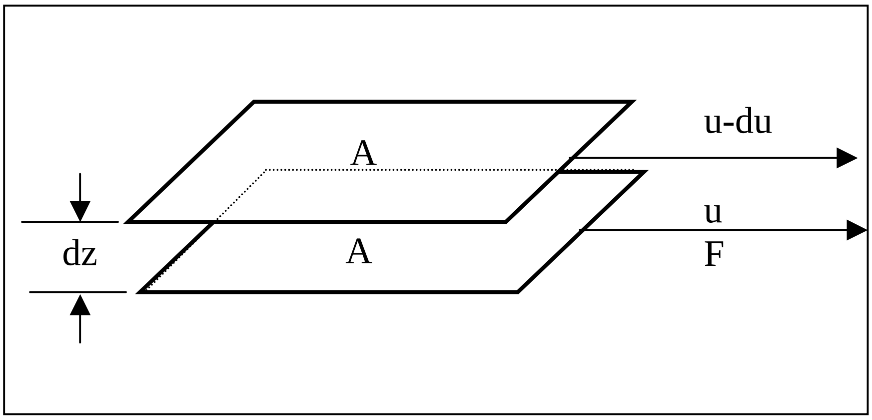


Figure (1) Shear between two plates

Newton's law of viscosity states that:

$$\tau \propto -\frac{du}{dz} \Rightarrow \tau = -\mu \frac{du}{dz}$$

Fluids, which obey this equation, are called "Newtonian Fluids" and Fluids, don't obey this equation, are called "non-Newtonian Fluids".

Note: Newton's law of viscosity holds for Newtonian fluids in laminar flow.

Momentum (shear stress) transfers through the fluid from the region of high velocities to region of low velocities, and the rate of momentum transfer increase with increasing the viscosity of fluids.

1.7 Newtonian and non-Newtonian fluids

The plot of shear stress (τ) against shear rate ($\dot{\gamma} \equiv \frac{du}{dz}$) is different in Newtonian fluids than that in non-Newtonian fluids as shown in Figure (2).

For Newtonian fluids the plot give a straight line from the origin but for non-Newtonian fluids the plot gives different relations than that of Newtonian some of these relations are given in Figure (2).

A- Newtonian fluids

B- non-Newtonian (pseudoplastic)

C- non-Newtonian (dilatant)

D- non-Newtonian (Bingham)

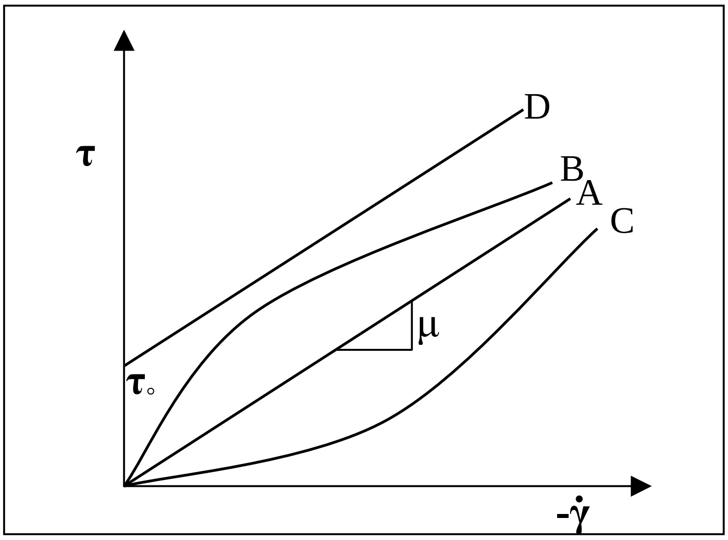


Figure (2): Shear stress (τ) against shear rate ($-\dot{\gamma} \equiv -\frac{du}{dz}$)

Example -1.1-

One liter of certain oil weighs 0.8 kg, calculate the specific weight, density, specific volume, and specific gravity of it.

Solution:

$$sp.wt. = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{(0.8kg)(9.81m/s^2)}{1x10^{-3}m^3} = 7848\frac{N}{m^3}$$

$$\rho = \frac{(0.8kg)}{1x10^{-3}m^3} = 800\frac{kg}{m^3} \qquad \upsilon = \frac{1}{\rho} = 1.25x10^{-3}\frac{m^3}{kg}$$

$$sp.gr. = \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}} = \frac{800kg/m^3}{1000kg/m^3} = 0.8$$

Example -1.2-

Determine the specific gravity of a fluid having viscosity of 4.0 c.poice and kinematic viscosity of 3.6 c.stokes.

Solution:

$$\mu = 4c.p\frac{poice}{100c.p} = 0.04\,poise = 0.04\frac{g}{cm.s} \qquad \nu = 3.6c.s\frac{stoke}{100c.s} = 0.036stoke = 0.04\frac{cm^2}{s}$$

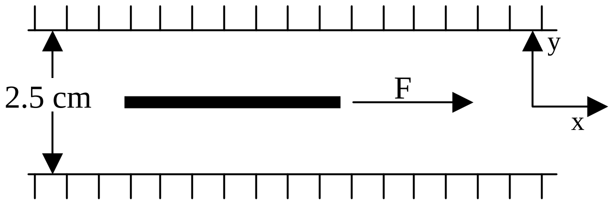
$$\nu = \frac{\mu}{\rho} \Rightarrow \rho = \frac{\mu}{\nu} = \frac{0.04\frac{g}{cm.s}}{0.036cm^2/s} = 1.1111\frac{g}{cc} \qquad \Rightarrow \rho = 1111.1\frac{kg}{m^3} \qquad \Rightarrow sp.gr. = 1.1111$$

Example -1.3-

The space between two large plane surfaces kept 2.5 cm apart is filled with liquid of viscosity 0.0825 kg/m.s. What force is required to drag a thin plate of surface area 0.5 m^2 between the two large surfaces at speed of 0.5 m/s, (i) when the plate is placed in the middle of the two surfaces, and (ii) when the plate is placed 1.5 cm from one of the plates surfaces.

Solution:

(i) Shear stress on the upper side of the plate is



$$\tau_1 = -\mu\frac{du}{dy} = \frac{F_1}{A}$$

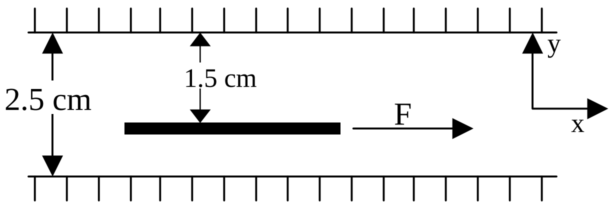
$$\frac{du}{dy} \cong \frac{\Delta u}{\Delta y} = \frac{u|_{y=1.25} - u|_{y=0}}{1.25x10^{-2} - 0} = \frac{0 - 0.5m/s}{1.25x10^{-2}m} = -40s^{-1}$$

$$F_1 = A(-\mu\frac{du}{dy}) = 0.5m^2[-0.0825Pa.s(-40s^{-1})] = 1.65N$$

Likewise on the lower surface $F_2 = A\tau_2 = 1.65N$

The total force required $= F_1 + F_2 = 3.3N$

(ii) Shear stress on the upper side of the plate is



$$\tau_1 = -\mu\frac{du}{dy} = \frac{F_1}{A}$$

$$\frac{du}{dy} \cong \frac{\Delta u}{\Delta y} = \frac{u|_{y=1.5} - u|_{y=0}}{1.5x10^{-2} - 0} = -\frac{100}{3}s^{-1}$$

$$F_1 = A(-\mu\frac{du}{dy}) = 0.5m^2[-0.0825Pa.s(-\frac{100}{3}s^{-1})] = 1.375N$$

$$\tau_2 = -\mu\frac{du}{dy} = \frac{F_2}{A} \quad \text{and} \quad \frac{du}{dy} = \frac{0 - 0.5}{0.01} = -50s^{-1}$$

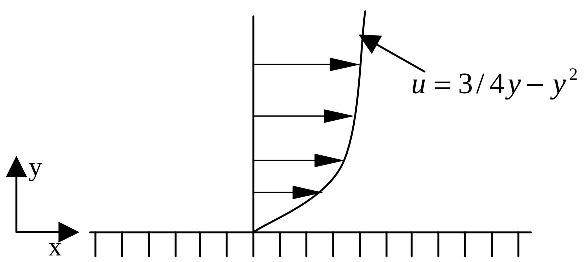
$$F_2 = 0.5m^2[-0.0825Pa.s(-50s^{-1})] = 2.0625N$$

The total force required $= F_1 + F_2 = 3.4375N$

Example -1.4-

The velocity distribution within the fluid flowing over a plate is given by $u = 3/4y - y^2$ where u is the velocity in (m/s) and y is a distance above the plate in (m). Determine the shear stress at y=0 and at y=0.2 m. take that μ=8.4 poise.

Solution:



$$u = 3/4y - y^2 \Rightarrow \frac{du}{dy} = \frac{3}{4} - 2y \Rightarrow \frac{du}{dy}\bigg|_{y=o} = \frac{3}{4}s^{-1}$$

and $\frac{du}{dy}\bigg|_{y=0.2} = \frac{3}{4} - 2(0.2) = 0.35s^{-1}$

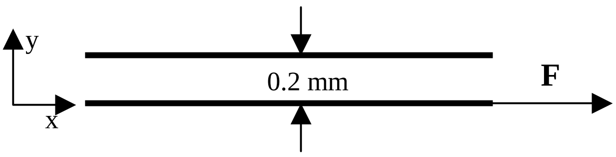
$$\tau = -\mu\frac{du}{dy} = \frac{F}{A};\ \mu = 8.4\frac{g}{cm.c}\left(\frac{100cm}{m}\right)\left(\frac{kg}{1000g}\right)$$

$$\tau|_{y=0} = 0.84Pa.s(3/4s^{-1}) = 0.63Pa \quad \text{and} \quad \tau|_{y=0.2} = 0.84Pa.s(0.35s^{-1}) = 0.294Pa$$

Example -1.5-

A flat plate of area $2x10^4$ cm^2 is pulled with a speed of 0.5 m/s relative to another plate located at a distance of 0.2 mm from it. If the fluid separated the two plates has a viscosity of 1.0 poise, find the force required to maintain the speed.

Solution:



$$\tau = -\mu\frac{du}{dy} = \frac{F}{A} \quad \mu = 1.0\frac{g}{cm.c}\left(\frac{100cm}{m}\right)\left(\frac{kg}{1000g}\right)$$

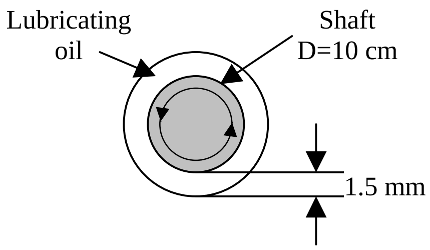
$$\frac{du}{dy} \cong \frac{\Delta u}{\Delta y} = \frac{u_2 - u_1}{y_2 - y_1} = \frac{0 - 0.5m/s}{0.2x10^{-3}m - 0} = -2500s^{-1}$$

$$\tau = 0.1Pa.s(2500s^{-1}) = 250Pa \Rightarrow F = 250Pa(2m^2) = 500N$$

Example -1.6-

A shaft of diameter 10 cm having a clearance of 1.5 mm rotates at 180 rpm in a bearing which is lubricated by an oil of viscosity 100 c.p. Find the intensity of shear of the lubricating oil if the length of the bearing is 20 cm and find the torque.

Solution:



The linear velocity of rotating is

$$u = \pi DN = \frac{\pi(0.1m)180\,rpm}{60\,s/\min} = 0.9425\,m/s$$

$$\mu = 100c.p = 1.0\frac{g}{cm.c}\left(\frac{100cm}{m}\right)\left(\frac{kg}{1000g}\right) = 0.1Pa.s$$

$$\tau = \mu\frac{du}{dy} = \frac{F}{A} = 0.1Pa.s\left(\frac{0.9425m/s}{0.0015m}\right) = 62.83Pa \Rightarrow F = \tau(\pi DL) = 62.83Pa(\pi\,0.1(0.2)) = 3.95\,N$$

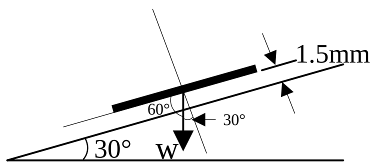
The torque is equivalent to rotating moment

$$\Gamma = F\,\frac{D}{2} = 3.95N\left(\frac{0.1}{2}\right) = 0.1975J$$

Example -1.7-

A plate of size 60 cm x 60 cm slides over a plane inclined to the horizontal at an angle of 30°. It is separated from the plane with a film of oil of thickness 1.5 mm. The plate weighs 25kg and slides down with a velocity of 0.25 m/s. Calculate the dynamic viscosity of oil used as lubricant. What would be its kinematic viscosity if the specific gravity of oil is 0.95.

Solution:



Component of W along the plane =W cos(60) =W sin(30)

= 25 (0.5) = 12.5 kg

F = 12.5 kg (9.81 m/s^2) =122.625 N

τ = F/A = 122.625 N/(0.6 x0.6) m^2 = 340.625 Pa

$$\mu = \frac{\tau}{(du/dy)} = \frac{340.625\ Pa}{(0.25/0.0015)s^{-1}} = 2.044Pa.s = 20.44\ poise$$

$$\nu = \frac{\mu}{\rho} = \frac{2.044\ Pa.s}{950\ kg/m^3} = 0.00215\ m^2/s = 21.5\ stoke$$

Home Work

P.1.1

Two plates are kept separated by a film of oil of 0.025 mm. the top plate moves with a velocity of 50 cm/s while the bottom plate is kept fixed. Find the fluid viscosity of oil if the force required to move the plate is 0.2 kg/m^2. Ans. $\underline{\mu= 9.81\times10^{-5}\ Pa.s}$?

P.1.2

If the equation of a velocity profile over a plate is $u=3\ y^{(2/3)}$ in which the velocity in m/s at a distance y meters above the plate, determine the shear stress at y=0 and y=5 cm. Take μ= 8.4 poise Ans. $\tau_{y=0} = \infty$, $\tau_{y=5} = 4.56$ Pa.s

P.1.3

The equation of a velocity distribution over a plate is $u=1/3\ y - y^2$ in which the velocity in m/s at a distance y meters above the plate, determine the shear stress at y=0 and y=0.1 m. Take μ= 8.35 poise Ans. $\tau_{y=0}$ =2.78, $\tau_{y=0.1}$= 4.56 $dyne/cm^2$

P.1.4

A cylinder of diameter 10 cm rotates concentrically inside another hollow cylinder of inner diameter 10.1 cm. Both cylinders are 20 cm long and stand with their axis vertical. The annular space is filled with oil. If a torque of 100 kg cm is required to rotate the inner cylinder at 100 rpm, determine the viscosity of oil. Ans. μ= 29.82poise

CHPTER TWO

Dimensional Analysis

2.1 Introduction

Any phenomenon is physical sciences and engineering can be described by the *fundamentals dimensions* mass, length, time, and temperature. Till the rapid development of science and technology the engineers and scientists depend upon the experimental data. But the rapid development of science and technology has created new mathematical methods of solving complicated problems, which could not have been solved completely by analytical methods and would have consumed enormous time. This mathematical method of obtaining the equations governing certain natural phenomenon by balancing the fundamental dimensions is called *(Dimensional Analysis)*. Of course, the equation obtained by this method is known as *(Empirical Equation).*

2.2 Fundamentals Dimensions

The various physical quantities used by engineer and scientists can be expressed in terms of fundamentals dimensions are: Mass (M), Length (L), Time (T), and Temperature (θ). All other quantities such as area, volume, acceleration, force, energy, etc., are termed as " derived quantities".

2.3 Dimensional Homogeneity

An equation is called *"dimensionally homogeneous"* if the fundamentals dimensions have identical powers of [L T M] (i.e. length, time, and mass) on both sides. Such an equation be independent of the system of measurement (i.e. metric, English, or S.I.). Let consider the common equation of volumetric flow rate,

$$Q = A\ u$$

$$L^3T^{-1} = L^2\ LT^{-1} = L^3T^{-1}.$$

We see, from the above equation that both right and left hand sides of the equation have the same dimensions, and the equation is therefore dimensionally homogeneous.

Example -2.1-

a) Determine the dimensions of the following quantities in M-L-T system 1- force 2- pressure 3- work 4- power 5- surface tension 6- discharge 7- torque 8- momentum.

b) Check the dimensional homogeneity of the following equations

1- $u = \sqrt{\dfrac{2\,g(\rho_m - \rho)\Delta z}{\rho}}$ 2- $Q = \dfrac{8}{15} cd \tan\dfrac{\theta}{2}\sqrt{2\,g} Z_\circ^{\frac{5}{2}}$

Solution:

a)

1- F = m.g (kg.m/s^2) $\equiv [MLT^{-2}]$

2- P=F/A $\equiv [(MLT^{-2})\ (L^{-2})]$ (Pa) $\equiv [ML^{-1}T^{-2}]$

3- Work = F.L $\equiv [(MLT^{-2})\ (L)]$ (N.m) $\equiv [ML^2T^{-2}]$

4- Power = Work/time $\equiv [(ML^2T^{-2})\ (T^{-1})]$ (W) $\equiv [ML^{-1}T^{-2}]$

5- Surface tension = F/L $\equiv [(MLT^{-2})\ (L^{-1})]$ (N/m) $\equiv [ML^{-1}T^{-2}]$

6- Discharge (Q) m^3/s $\equiv [L^3T^{-1}]$
7- Torque (Γ) = F.L ≡ $[(MLT^{-2})(L)]$ N.m $\equiv [ML^2T^{-2}]$
8- Moment = m.u L)] N.m $\equiv [ML^2T^{-2}]$

b) 1- $u = \sqrt{\dfrac{2\,g(\rho_m - \rho)\Delta z}{\rho}}$

L.H.S. u ≡ $[LT^{-1}]$

R.H.S. u ≡ $\left[\dfrac{LT^{-2}(ML^3)}{ML^{-3}}\right]^{1/2} \equiv [LT^{-1}]$

Since the dimensions on both sides of the equation are same, therefore the equation is dimensionally homogenous.

2- $Q = \dfrac{8}{15} cd \tan\dfrac{\theta}{2}\sqrt{2\,g} Z_{\circ}^{\frac{5}{2}}$

L.H.S. u ≡ $[L^3T^{-1}]$

R.H.S. $(LT^{-2})\ (L)^{5/2} \equiv [L^3T^{-1}]$

This equation is dimensionally homogenous.

2.4 Methods of Dimensional Analysis

Dimensional analysis, which enables the variables in a problem to be grouped into form of *dimensionless groups*. Thus reducing the effective number of variables. The method of dimensional analysis by providing a smaller number of independent groups is most helpful to experimenter.

Many methods of dimensional analysis are available; two of these methods are given here, which are:

1- Rayleigh's method (or Power series)

2- Buckingham's method (or Π-Theorem)

2.4.1 Rayleigh's method (or Power series)

In this method, the functional relationship of some variable is expressed in the form of an exponential equation, which must be dimensionally homogeneous. If (y) is some function of independent variables (x_1, x_2, x_3, ………etc.), then functional relationship may be written as;

$y = f(x_1, x_2, x_3, \ldots\ldots\ldots \text{etc.})$

The dependent variable (y) is one about which information is required; whereas the independent variables are those, which govern the variation of dependent variables.

The Rayleigh's method is based on the following steps:-

1- First of all, write the functional relationship with the given data.
2- Now write the equation in terms of a constant with exponents i.e. powers a, b, c,...
3- With the help of the principle of dimensional homogeneity, find out the values of a, b, c, … by obtaining simultaneous equation and simplify it.
4- Now substitute the values of these exponents in the main equation, and simplify it.

Example -2.2-

If the capillary rise (h) depends upon the specific weight (sp.wt) surface tension (σ) of the liquid and tube radius (r) show that:

$$h = r\phi\left(\frac{\sigma}{(sp.wt.)\ r^2}\right),$$ where ϕ is any function.

Solution:

Capillary rise (h) m ≡ [L]
Specific weight (sp.wt) N/m^3 (MLT^{-2} L^{-3}) ≡ [ML^{-2}T^{-2}]
Surface tension (σ) N/m (MLT^{-2} L^{-1}) ≡ [MT^{-2}]
Tube radius (r) m ≡ [L]

$$h = f\,(sp.wt., \sigma, r)$$

$$h = k\,(sp.wt.^a, \sigma^b, r^c)$$

$$[L] = [ML^{-2}T^{-2}]^a\,[MT^{-2}]^b[L]^c$$

Now by the principle of dimensional homogeneity, equating the power of M, L, T on both sides of the equation

For M $0 = a + b \Rightarrow a = -b$

For L $1 = -2a + c \Rightarrow a = -b$

For T $0 = -2a - 2b \Rightarrow a = -b$

$$h = k\,(sp.wt.^{-b}, \sigma^b, r^{1-2b})$$

$$h = k\,r\left(\frac{\sigma}{sp.wt.\ r^2}\right)^b \qquad \therefore\ h = r\phi\left(\frac{\sigma}{(sp.wt.)\ r^2}\right)$$

Example -2.3-

Prove that the resistance (F) of a sphere of diameter (d) moving at a constant speed (u) through a fluid density (ρ) and dynamic viscosity (μ) may be expressed as:

$$F = \frac{\mu^2}{\rho}\phi\left(\frac{\rho\,u\,d}{\mu}\right),$$ where ϕ is any function.

Solution:

Resistance (F) N ≡ [MLT^{-2}]
Diameter (d) m ≡ [L]
Speed (u) m/s ≡ [LT^{-1}]
Density (ρ) kg/m^3 ≡ [ML^{-3}]
Viscosity (μ) kg/m.s ≡ [ML^{-1} T^{-1}]

$$F = f\,(d, u, \rho, \mu)$$

$$F = k\,(d^a, u^b, \rho^c, \mu^d)$$

$$[MLT^{-2}] = [L]^a\,[LT^{-1}]^b[ML^{-3}]^c[ML^{-1}T^{-1}]^d$$

For M $1 = c + d \Rightarrow c = 1 - b$ -----------------(1)
For L $1 = a + b - 3c - d$ ------------------------(2)
For T $-2 = -b - d \Rightarrow b = 2 - b$ ----------------(3)

By substituting equations (1) and (2) in equation (3) give

$a = 1 - b + 3c + d = 1 - (2 - d) + 3\,(1 - d) + d = 2 - d$

$F = k\,(d^{2-d}, u^{2-d}, \rho^{1-d}, \mu^{d}) = k\,\{(d^2\,u^2\,\rho)\,(\mu / \rho\,u\,d)^d\}$ ---------x $\{(\rho/ \mu^2) / (\rho/ \mu^2)\}$

$F = k\,\{(d^2\,u^2\,\rho^2 / \mu^2)\,(\mu / \rho\,u\,d)^d\,(\mu^2 / \rho)\}$

$$\therefore\;\; F = \frac{\mu^2}{\rho}\,\phi\left(\frac{\rho\,u\,d}{\mu}\right)$$

Example -2.4-

The thrust (P) (قوة الدفع) of a propeller depends upon diameter (D); speed (u) through a fluid density (ρ); revolution per minute (N); and dynamic viscosity (μ) Show that:

$$P = (\rho\,D^2\,u^2)\;f\left((\frac{\mu}{\rho\,D\,u}),(\frac{D\,N}{u})\right)$$, where f is any function.

Solution:

Thrust (P) N $\equiv [MLT^{-2}]$
Diameter (D) m $\equiv [L]$
Speed (u) m/s $\equiv [LT^{-1}]$
Density (ρ) kg/m^3 $\equiv [ML^{-3}]$
Rev. per min. (N) min^{-1} $\equiv [T^{-1}]$
Viscosity (μ) kg/m.s $\equiv [ML^{-1}\,T^{-1}]$

$P = f\,(D, u, \rho, N, \mu)$

$P = k\,(D^a, u^b, \rho^c, N^d, \mu^e)$

$[MLT^{-2}] = [L]^a\,[LT^{-1}]^b[ML^{-3}]^c[T^{-1}][ML^{-1}T^{-1}]^d$

For M $1 = c + e$ $\Rightarrow$ $c = 1 - e$ --------------(1)

For L $1 = a + b - 3c - e$ $\Rightarrow$ $a = 1 - b + 3c + e$ --------------(2)

For T $-2 = -b - d - e$ $\Rightarrow$ $b = 2 - e - d$ --------------(3)

By substituting equations (1) and (3) in equation (2) give

$a = 1 - (2 - e - d) + 3\,(1 - e) + e = 2 - e + d$

$P = k\,(D^{2-e+d}, u^{2-e+d}, \rho^{1-e}, N^d, \mu^e)$

$$P = (\rho\,D^2\,u^2)\,k\left[\left(\frac{\mu}{\rho\,D\,u}\right)^e,\left(\frac{D\,N}{u}\right)\right]$$

$$\therefore\;\; P = (\rho\,D^2\,u^2)\;f\left((\frac{\mu}{\rho\,D\,u}),(\frac{D\,N}{u})\right)$$

Home Work

P.2.1

Show, by dimensional analysis, that the power (P) developed by a hydraulic turbine is given by; $P=(\rho \ N^3 \ D^5) \ f\left((\frac{N^2 \ D^2}{g \ H})\right)$ where (ρ) is the fluid density, (N) is speed of rotation in r.p.m., (D) is the diameter of runner, (H) is the working head, and (g) is the gravitational acceleration.

P.2.2

The resistance (R) experienced by a partially submerged body depends upon the velocity (u), length of the body (L), dynamic viscosity (μ) and density (ρ) of the fluid, and gravitational acceleration (g). Obtain a dimensionless expression for (R).

Ans. $R=(u^2 L^2 \rho) \ f\left((\frac{\mu}{u \ L \ g}),(\frac{L \ g}{u^2})\right)$

P.2.3

Using Rayleigh's method to determine the rational formula for discharge (Q) through a sharp-edged orifice freely into the atmosphere in terms of head (h), diameter (d), density (ρ), dynamic viscosity (μ), and gravitational acceleration (g).

Ans. $Q=(d\sqrt{g \ h}) f\left[(\frac{\mu}{\rho \ d^{\frac{3}{2}} \ g^{\frac{1}{2}}}),(\frac{h}{d})\right]$

2.4.2 Buckingham's method (or Π-Theorem)

It has been observed that the Rayleigh's method of dimensional analysis becomes cumbersome, when a large number of variables are involved. In order to overcome this difficulty, the Buckingham's method may be convenient used. It states that " If there are **(n)** variables in a dimensionally homogeneous equation, and if these variables contain **(m)** fundamental dimensions such as (MLT) they may be grouped into **(n-m)** non-dimensional independent Π-terms".

Mathematically, if a dependent variable X_1 depends upon independent variables (X_2, X_3, X_4, ………. X_n), the functional equation may be written as:

$X_1 = k\,(X_2, X_3, X_4, \ldots\ldots\ldots X_n)$

This equation may be written in its general form as;

$f\,(X_1, X_2, X_3, \ldots\ldots\ldots X_n) = 0$

In this equation, there are n variables. If there are m fundamental dimensions, then according to Buckingham's Π-theorem;

$f_1\,(\Pi_1, \Pi_2, \Pi_3, \ldots\ldots\ldots \Pi_{n-m}) = 0$

The Buckingham's Π-theorem is based on the following steps:

1. First of all, write the functional relationship with the given data.
2. Then write the equation in its general form.
3. Now choose **m** *repeating variables (or recurring set)* and write separate expressions for each Π-term. Every Π-term will contain the repeating variables and one of the remaining variables. **Just** the repeating variables are written in exponential form.
4. With help of the principle of dimensional homogeneity find out the values of powers a, b, c, …… by obtaining simultaneous equations.
5. Now substitute the values of these exponents in the Π-terms.
6. After the Π-terms are determined, write the functional relation in the required form.

Note:-

Any Π-term may be replaced by any power of it, because the power of a non-dimensional term is also non-dimensional e.g. Π_1 may be replaced by Π_1^2, Π_1^3, $\Pi_1^{0.5}$,……. or by $2\Pi_1$, $3\Pi_1$, $\Pi_1/2$, ……etc.

2.4.2.1 Selection of repeating variables

In the previous section, we have mentioned that we should choose **(m)** repeating variables and write separate expressions for each Π-term. Though there is no hard or fast rule for the selection of repeating variables, yet the following points should be borne in mind while selecting the repeating variables:

1. The variables should be such that none of them is dimension les.
2. No two variables should have the same dimensions.
3. Independent variables should, as far as possible, be selected as repeating variables.
4. Each of the fundamental dimensions must appear in at least one of the **m** variables.
5. It must not possible to form a dimensionless group from some or all the variables within the repeating variables. If it were so possible, this dimensionless group would, of course, be one of the Π-term.
6. In general the selected repeating variables should be expressed as the following: **(1)** representing the flow characteristics, **(2)**, representing the geometry and **(3)** representing the physical properties of fluid.
7. In case of that the example is held up, then one of the repeating variables should be changed.

Example -2.5-

By dimensional analysis, obtain an expression for the drag force (F) on a partially submerged body moving with a relative velocity (u) in a fluid; the other variables being the linear dimension (L), surface roughness (e), fluid density (ρ), and gravitational acceleration (g).

Solution:

Drag force (F) N	$\equiv [MLT^{-2}]$
Relative velocity (u) m/s	$\equiv [LT^{-1}]$
Linear dimension (L) m	$\equiv [L]$
Surface roughness (e) m	$\equiv [L]$
Density (ρ) kg/m^3	$\equiv [ML^{-3}]$
Acceleration of gravity (g) m/s^2	$\equiv [ML^{-1}\,T^{-1}]$

$$F = k\,(u, L, e, \rho, g)$$

$$f\,(F, u, L, e, \rho, g) = 0$$

$$n = 6,\ m = 3,\ \Rightarrow\ \Pi = n - m = 6 - 3 = 3$$

No. of repeating variables = m = 3

The selected repeating variables is (u, L, ρ)

$$\Pi_1 = u^{a1}\,L^{b1}\,\rho^{c1}\,F \qquad \text{--------------(1)}$$

$$\Pi_2 = u^{a2}\,L^{b2}\,\rho^{c2}\,e \qquad \text{--------------(2)}$$

$$\Pi_3 = u^{a3}\,L^{b3}\,\rho^{c3}\,g \qquad \text{--------------(3)}$$

For Π_1 equation (1)

$$[M^0\,L^0\,T^0] = [L\,T^{-1}]^{a1}\,[L]^{b1}[ML^{-3}]^{c1}[MLT^{-2}]$$

Now applied dimensional homogeneity

For M $0 = c1 + 1$ $\Rightarrow$ $c1 = -1$

For T $0 = -a1 - 2$ $\Rightarrow$ $a1 = -2$

For L $0 = a1 + b1 - 3c1 + 1$ $\Rightarrow$ $b1 = -2$

$\Pi_1 = u^{-2} L^{-2} \rho^{-1} F \quad \Rightarrow \quad \Pi_1 = \dfrac{F}{u^2 L^2 \rho}$

For Π_2 equation (2)

$[M^0 L^0 T^0] = [L T^{-1}]^{a2} [L]^{b2} [ML^{-3}]^{c2} [L]$

For M $0 = c2$ $\Rightarrow$ $c2 = 0$

For T $0 = -a2$ $\Rightarrow$ $a2 = 0$

For L $0 = a2 + b2 - 3c2 + 1$ $\Rightarrow$ $b2 = -1$

$\Pi_2 = L^{-1} e \quad \Rightarrow \quad \Pi_2 = \dfrac{e}{L}$

For Π_3 equation (3)

$[M^0 L^0 T^0] = [L T^{-1}]^{a3} [L]^{b3} [ML^{-3}]^{c3} [L T^{-2}]$

For M $0 = c3$ $\Rightarrow$ $c3 = 0$

For T $0 = -a3 - 2$ $\Rightarrow$ $a3 = -2$

For L $0 = a3 + b3 - 3c3 + 1$ $\Rightarrow$ $b3 = 1$

$\Pi_3 = u^{-2} L g \quad \Rightarrow \quad \Pi_3 = \dfrac{L g}{u^2}$

$f_1(\Pi_1, \Pi_2, \Pi_3) = 0 \quad \Rightarrow \quad f_1\left(\dfrac{F}{u^2 L^2 \rho}, \dfrac{e}{L}, \dfrac{L g}{u^2}\right) = 0$

$\therefore \; F = u^2 L^2 \rho \; f\left(\dfrac{e}{L}, \dfrac{L g}{u^2}\right)$

Example -2.6-

Prove that the discharge (Q) over a spillway (قناة لتصريف فائض المياه من سد او نهر) is given by the relation $Q = u\, D^2 \; f\left(\dfrac{\sqrt{g\, D}}{u}, \dfrac{H}{D}\right)$ where (u) velocity of flow (D) depth at the throat, (H), head of water, and (g) gravitational acceleration.

Solution:

Discharge (Q) m^3/s $\equiv [L^3 T^{-1}]$

Velocity (u) m/s $\equiv [LT^{-1}]$

Depth (D) m $\equiv [L]$

Head of water (H) m $\equiv [L]$

Acceleration of gravity (g) m/s^2 $\equiv [ML^{-1} T^{-1}]$

$$Q = k\,(u, D, H, g)$$

$$f\,(Q, u, D, H, g) = 0$$

$$n = 5,\ m = 2,\ \Rightarrow\ \Pi = n - m = 5 - 2 = 3$$

No. of repeating variables = m = 2
The selected repeating variables is (u, D)

$$\Pi_1 = u^{a1}\, D^{b1}\, Q \qquad \text{--------------(1)}$$

$$\Pi_2 = u^{a2}\, D^{b2}\, H \qquad \text{--------------(2)}$$

$$\Pi_3 = u^{a3}\, D^{b3}\, g \qquad \text{--------------(3)}$$

For Π_1 equation (1)

$$[M^0\, L^0\, T^0] = [L\, T^{-1}]^{a1}\, [L]^{b1} [L^3 T^{-1}]$$

For T $\quad 0 = -a1 - 1 \quad \Rightarrow \quad a1 = -1$

For L $\quad 0 = a1 + b1 + 3 \quad \Rightarrow \quad b1 = -2$

$$\Pi_1 = u^{-1}\, D^{-2}\, Q \qquad \Rightarrow \quad \Pi_1 = \frac{Q}{u\, D^2}$$

For Π_2 equation (2)

$$[M^0\, L^0\, T^0] = [L\, T^{-1}]^{a2}\, [L]^{b2} [L]$$

For T $\quad 0 = -a2 \quad \Rightarrow \quad a2 = 0$

For L $\quad 0 = a2 + b2 + 1 \quad \Rightarrow \quad b2 = -1$

$$\Pi_2 = D^{-1}\, H \qquad \Rightarrow \quad \Pi_2 = \frac{D}{H}$$

For Π_3 equation (3)

$$[M^0\, L^0\, T^0] = [L\, T^{-1}]^{a3}\, [L]^{b3}\, [L\, T^{-2}]$$

For T $\quad 0 = -a3 - 2 \quad \Rightarrow \quad a3 = -2$

For L $\quad 0 = a3 + b3 + 1 \quad \Rightarrow \quad b3 = 1$

$$\Pi_3 = u^{-2}\, D\, g \qquad \Rightarrow \quad \Pi_3 = \frac{D\, g}{u^2} = \frac{\sqrt{g\, D}}{u}$$

$$f_1\,(\Pi_1, \Pi_2, \Pi_3) = 0 \qquad \Rightarrow \quad f_1\Big(\frac{Q}{u\, D^2}, \frac{D}{H}, \frac{\sqrt{D\, g}}{u}\Big)$$

$$\therefore\ Q = u\, D^2\ f\Big(\frac{\sqrt{g\, D}}{u}, \frac{H}{D}\Big)$$

Example -2.5-

Show that the discharge of a centrifugal pump is given by $Q = N\ D^2\ f(\frac{g\ H}{N^2 D^2}, \frac{\mu}{ND^2\ \rho})$ where (N) is the speed of the pump in r.p.m., (D) the diameter of impeller, (g) gravitational acceleration, (H) manometric head, (μ), (ρ) are the dynamic viscosity and the density of the fluid.

Solution:

Discharge (Q) m^3/s	$\equiv [L^3T^{-1}]$
Pump speed (N) r.p.m.	$\equiv [T^{-1}]$
Diameter of impeller (D) m	$\equiv [L]$
Acceleration of gravity (g) m/s^2	$\equiv [ML^{-1}\ T^{-1}]$
Head of manometer (H) m	$\equiv [L]$
Viscosity (μ) kg/m.s	$\equiv [ML^{-1}\ T^{-1}]$
Density (ρ) kg/m^3	$\equiv [ML^{-3}]$

$$Q = k\ (N, D, g, H, \mu, \rho)$$

$$f\ (Q, N, D, g, H, \mu, \rho) = 0$$

$$n = 7,\ m = 3,\ \Rightarrow\ \Pi = n - m = 7 - 3 = 4$$

No. of repeating variables = m = 3

The selected repeating variables is (N, D, ρ)

$$\Pi_1 = N^{a1}\ D^{b1}\ \rho^{c1}\ Q \qquad \text{--------------(1)}$$

$$\Pi_2 = N^{a2}\ D^{b2}\ \rho^{c2}\ g \qquad \text{--------------(2)}$$

$$\Pi_3 = N^{a3}\ D^{b3}\ \rho^{c3}\ H \qquad \text{--------------(3)}$$

$$\Pi_4 = N^{a4}\ D^{b4}\ \rho^{c4}\ \mu \qquad \text{--------------(4)}$$

For Π_1 equation (1)

$$[M^0\ L^0\ T^0] = [T^{-1}]^{a1}\ [L]^{b1}[ML^{-3}]^{c1}[L^3\ T^{-1}]$$

For M	$0 = c1$	$\Rightarrow$	$c1 = 0$
For T	$0 = -a1 - 1$	$\Rightarrow$	$a1 = -1$
For L	$0 = b1 - 3c1 + 3$	$\Rightarrow$	$b1 = -3$

$$\Pi_1 = N^{-1}\ D^{-3}\ Q \qquad \Rightarrow \quad \Pi_1 = \frac{Q}{N\ D^3}$$

For Π_2 equation (2)

$$[M^0\ L^0\ T^0] = [T^{-1}]^{a2}\ [L]^{b2}[ML^{-3}]^{c2}[LT^{-2}]$$

For M	$0 = c2$	$\Rightarrow$	$c2 = 0$
For T	$0 = -a2 - 2$	$\Rightarrow$	$a2 = -2$

For L $\quad 0 = b2 - 3c2 + 1 \quad \Rightarrow \quad b2 = -1$

$\Pi_2 = N^{-2} D^{-1} g \quad \Rightarrow \quad \Pi_2 = \dfrac{g}{N^2 D}$

For Π_3 equation (3)

$[M^0 L^0 T^0] = [T^{-1}]^{a3} [L]^{b3} [ML^{-3}]^{c3} [L]$

For M $\quad 0 = c3 \quad \Rightarrow \quad c3 = 0$

For T $\quad 0 = -a3 \quad \Rightarrow \quad a3 = 0$

For L $\quad 0 = b3 - 3c3 + 1 \quad \Rightarrow \quad b3 = -1$

$\Pi_3 = D^{-1} H \quad \Rightarrow \quad \Pi_3 = \dfrac{H}{D}$

For Π_3 equation (4)

$[M^0 L^0 T^0] = [T^{-1}]^{a4} [L]^{b4} [ML^{-3}]^{c4} [ML^{-1}T^{-1}]$

For M $\quad 0 = c4 + 1 \quad \Rightarrow \quad c4 = -1$

For T $\quad 0 = -a4 - 1 \quad \Rightarrow \quad a4 = -1$

For L $\quad 0 = b4 - 3c4 - 1 \quad \Rightarrow \quad b4 = -2$

$\Pi_4 = N^{-1} D^{-2} \rho^{-1} \mu \quad \Rightarrow \quad \Pi 4 = \dfrac{\mu}{N D^2 \rho}$

$f_1 (\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 0 \quad \Rightarrow \quad f_1\left(\dfrac{Q}{N D^3}, \dfrac{g}{N^2 D}, \dfrac{H}{D}, \dfrac{\mu}{N D^2 \rho}\right) = 0$

Since the product of two Π-terms is dimensionless, therefore replace the term Π_2 and Π_3 by $\dfrac{gH}{N^2 D^2}$

$\Rightarrow \quad f\left(\dfrac{Q}{ND^3}, \dfrac{gH}{N^2 D^2}, \dfrac{\mu}{ND^2 \rho}\right) \qquad \therefore \; Q = ND^3 f\left(\dfrac{gH}{N^2 D^2}, \dfrac{\mu}{ND^2 \rho}\right)$

Note:

The expression outside the bracket may be multiplied or divided by any amount, whereas the expression inside the bracket should not be multiplied or divided. e.g. π/4 , sin θ, tan θ/2, ….c.

2.5 **Dimensions of some important variables**

Item	Property	Symbol	SI Units	M.L.T.
1-	Velocity	u	m/s	LT^{-1}
2-	Angular velocity	ω	Rad/s, Deg/s	T^{-1}
3-	Rotational velocity	N	Rev/s	T^{-1}
4-	Acceleration	a, g	m/s^2	LT^{-2}
5-	Angular acceleration	α	s^{-2}	T^{-2}
6-	Volumetric flow rate	Q	m^3/s	L^3T^{-1}
7-	Discharge	Q	m^3/s	L^3T^{-1}
8-	Mass flow rate	$\dot{m}$	kg/s	MT^{-1}
9-	Mass (flux) velocity	G	$kg/m^2.s$	$ML^{-2}T^{-1}$
10-	Density	ρ	kg/m^3	ML^{-3}
11-	Specific volume	υ	m^3/kg	L^3M
12-	Specific weight	sp.wt	N/m^3	$ML^{-2}T^{-2}$
13-	Specific gravity	sp.gr	[-]	[-]
14-	Dynamic viscosity	μ	kg/m.s, Pa.s	$ML^{-1}T^{-1}$
15-	Kinematic viscosity	ν	m^2/s	L^2T^{-1}
16-	Force	F	N	MLT^{-2}
17-	Pressure	P	N/m^2≡Pa	$ML^{-1}T^{-2}$
18-	Pressure gradient	**ΔP/L**	**Pa/m**	$ML^{-2}T^{-2}$
19-	Shear stress	τ	N/m^2	$ML^{-1}T^{-2}$
20-	Shear rate	$\dot{\gamma}$	s^{-1}	T^{-1}
21-	Momentum	M	kg.m/s	MLT^{-1}
22-	Work	W	N.m≡J	ML^2T^{-2}
23-	Moment	M	N.m≡J	ML^2T^{-2}
24-	Torque	Γ	N.m≡J	ML^2T^{-2}
25-	Energy	E	J	ML^2T^{-2}
26-	Power	P	J/s≡W	ML^2T^{-3}
27-	Surface tension	σ	N/m	MT^{-2}
28-	Efficiency	η	[-]	[-]
29-	Head	h	m	L
30-	Modulus of elasticity	ε, K	Pa	$ML^{-1}T^{-2}$

English Units	SI Units
$g = 32.741\ ft/s^2$	$g = 9.81\ m/s^2$
$g_c = 32.741\ lb_m.ft/lb_f.s^2$	$g_c = 1.0\ kg.m/N.s^2$

psi ≡ lb_f/in^2 Pa ≡ Pascal = N/m^2 bar = 10^5 Pa

1.0 atm = 1.01325 bar = $1.01325*10^5$ Pa = 101.325 kPa = 14.7 psi = 760 torr (mmHg) ≈ 1.0 kg/cm^2

R = 8.314 ($Pa.m^3$/mol.K) or (J/mol.K) = 82.06 ($atm.cm^3$/mol.K) =10.73 ($psi.ft^3$/lbmol.R) = 1.987 (cal/mol.K) = 1.986 (Btu/lbmol.R) = 1545 (lb_f.ft/lbmol.R)

Home Work

P.2.4

The resisting force (F) of a supersonic plane during flight can be considered as dependent upon the length of the air craft (L), Velocity (u), air dynamic viscosity (μ), air density (ρ), and bulk modulus of elasticity of air (ε). Express, by dimensional analysis, the functional relationship between these variables and the resisting force.

Ans. $F = (\rho \ L^2 \ u^2) \ f \left((\frac{\mu}{Lu \ \rho}) , (\frac{\varepsilon}{u^2 \ \rho}) \right)$

Note: Expressing bulk modulus of elasticity in the form of an equation $\boxed{\varepsilon = -V \frac{dP}{dV}}$ where P is pressure, and V is volume. This mean (ε) is a measure of the increment change in pressure (dP) which takes place when a volume of fluid (V) is changed by an incremental amount (dV). Since arise in pressure always causes a decrease in volume, i.e. (dV) is always negative and so the minus sign comes in the equation to give a positive value of (ε).

where (ρ) is the fluid density, (N) is speed of rotation in r.p.m., (D) is the diameter of runner, (H) is the working head, and (g) is the gravitational acceleration.

P.2.5

The efficiency (η) of a fan depends upon density (ρ), and dynamic viscosity (μ), of the fluid, angular velocity (ω), diameter of the rotator (D), and discharge (Q). Express (η) in terms of dimensionless groups. Ans. $\eta = f \left((\frac{\mu}{\rho \ \omega \ D^2}), (\frac{Q}{\omega \ D3}) \right)$

P.2.6

The pressure drop (ΔP) in a pipe depends upon the mean velocity of flow (u), length of pipe (L), diameter of pipe (d), the fluid density (ρ), and dynamic viscosity (μ), average height of roughness on inside pipe surface (e). By using Buckingham's Π-theorem obtain a dimensionless expression for (ΔP). And show that $h_f = 4 f \frac{L}{d} \frac{u^2}{2g}$ where (h_f) is the head loss due to friction ($\frac{\Delta P}{\rho \ g}$) and ($f$) is the dimensionless fanning friction factor.

P.2.7

The Power (P) required to drive the pump depends upon the diameter (D), the angular velocity (ω), the discharge (Q), and the fluid density (ρ). Drive expression for (P) by dimensional analysis. Ans. $P = \omega^3 \rho \ D^5 \ f \left(, (\frac{\omega \ D}{Q}) \right)$

CHPTER THREE

Fluid Static and Its Applications

3.1 Introduction

Static fluids means that the fluids are at rest.

The pressure in a static fluid is familiar as a surface force exerted by the fluid ageist a unit area of the wall of its container. Pressure also exists at every point within a volume of fluid. It is a scalar quantity; at any given point its magnitude is the same in all directions.

3.2 Pressure in a Fluid

In Figure (1) a stationary column of fluid of height (h_2) and cross-sectional area A, where $A=A_o=A_1=A_2$, is shown. The pressure above the fluid is P_o, it could be the pressure of atmosphere above the fluid. The fluid at any point, say h_1, must support all the fluid above it. It can be shown that the forces at any point in a nonmoving or static fluid must be the same in all directions. Also, for a fluid at rest, the pressure or (force / unit area) in the same at all points with the same elevation. For example, at h_1 from the top, the pressure is the same at all points on the cross-sectional area A_1.

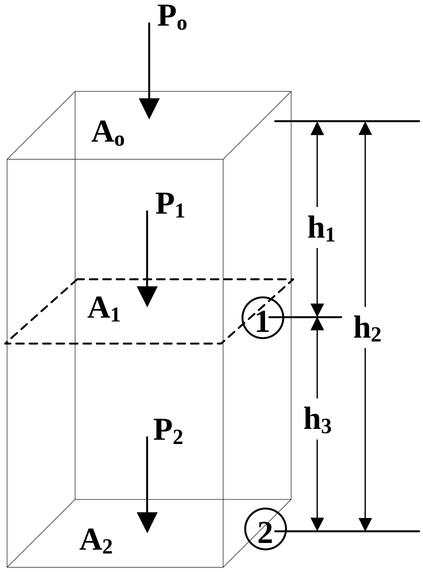


Figure (1): Pressure in a static fluid.

The total mass of fluid for h_2, height and ρ density is: - ($h_2 A \rho$) (kg)

But from Newton's 2nd law in motion the total force of the fluid on area (A) due to the fluid only is: - ($h_2 A \rho g$) (N) i.e. $F = h_2 A \rho g$ (N)

The pressure is defined as ($P = F/A = h_2 \rho g$) (N/m^2 or Pa)

This is the pressure on A_2 due to the weight of the fluid column above it. However to get the total pressure P_2 on A_2, the pressure P_o on the top of the fluid must be added,
i.e. $P_2 = h_2 \rho g + P_o$ (N/m^2 or Pa)

Thus to calculate P_1, $P_1 = h_1 \rho g + P_o$ (N/m^2 or Pa)

The pressure difference between points ① and ② is: -
$P_2 - P_1 = (h_2 \rho g + P_o) - (h_1 \rho g + P_o)$

⇒ $P_2 - P_1 = (h_2 - h_1) \rho g$ SI units

$P_2 - P_1 = (h_2 - h_1) \rho g / g_c$ English units

Since it is vertical height of a fluid that determines the pressure in a fluid, the shape of the vessel does not affect the pressure. For example in Figure (2) the pressure P_1 at the bottom of all three vessels is the same and equal to ($h_1 \rho g + P_o$).

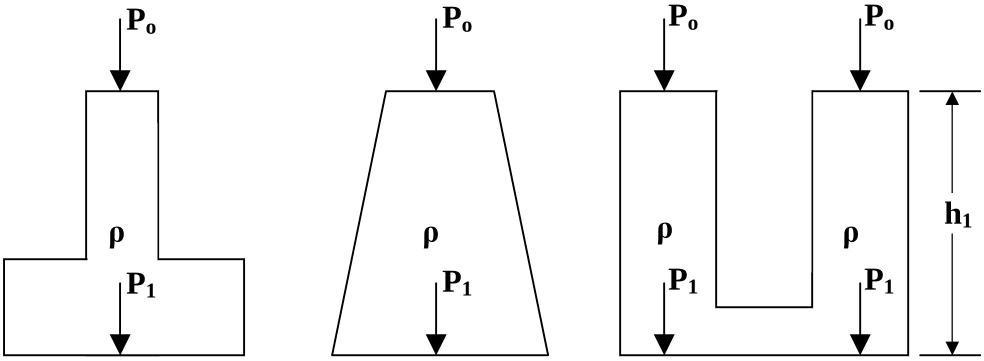


Figure (2): Pressure in vessel of various shapes.

3.3 Absolute and Relative Pressure

The term pressure is sometimes associated with different terms such as *atmospheric*, *gauge*, *absolute*, and *vacuum*. The meanings of these terms have to be understood well before solving problems in hydraulic and fluid mechanics.

1- Atmospheric Pressure

It is the pressure exerted by atmospheric air on the earth due to its weight. This pressure is change as the density of air varies according to the altitudes. Greater the height lesser the density. Also it may vary because of the temperature and humidity of air. Hence for all purposes of calculations the pressure exerted by air at sea level is taken as standard and that is equal to: -

$$1 \text{ atm} = 1.01325 \text{ bar} = 101.325 \text{ kPa} = 10.328 \text{ m } H_2o = 760 \text{ torr (mm Hg)} = 14.7 \text{ psi}$$

2- Gauge Pressure or Positive Pressure

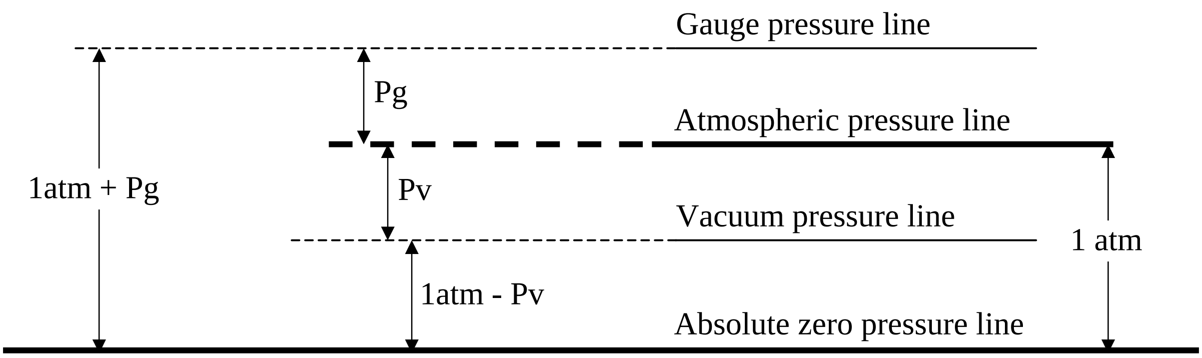
It is the pressure recorded by an instrument. This is always above atmospheric. The zero mark of the dial will have been adjusted to atmospheric pressure.

3- Vacuum Pressure or Negative Pressure

This pressure is caused either artificially or by flow conditions. The pressure intensity will be less than the atmospheric pressure whenever vacuum is formed.

4- Absolute Pressure

Absolute pressure is the algebraic sum of atmospheric pressure and gauge pressure. Atmospheric pressure is usually considered as the datum line and all other pressures are recorded either above or below it.



$$\boxed{\text{Absolute Pressure} = \text{Atmospheric Pressure} + \text{Gauge Pressure}}$$

$$\boxed{\text{Absolute Pressure} = \text{Atmospheric Pressure} - \text{Vacuum Pressure}}$$

For example if the vacuum pressure is 0.3 atm $\Rightarrow$ absolute pressure = 1.0 – 0.3 = 0.7 atm

Note: -

Barometric pressure is the pressure that recorded from the barometer (apparatus used to measure atmospheric pressure).

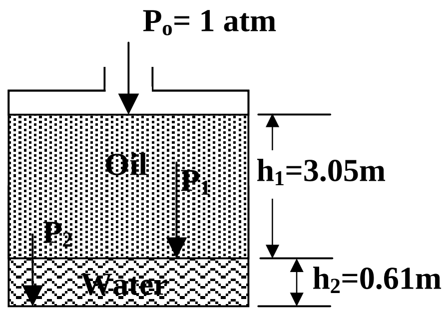
3.4 Head of Fluid

Pressures are given in many different sets of units, such as N/m^2, or Pa, $dyne/cm^2$, psi, lb_f/ft^2. However a common method of expressing pressures is in terms of head (m, cm, mm, in, or ft) of a particular fluid. This height or head of the given fluid will exert the same pressure as the pressures it represents. $P = h \rho g$.

Example -3.1-

A large storage tank contains oil having a density of 917 kg/m^3. The tank is 3.66 m tall and vented (open) to the atmosphere of 1 atm at the top. The tank is filled with oil to a depth of 3.05 m (10 ft) and also contains 0.61 m (2 ft) of water in the bottom of the tank. Calculate the pressure in Pa and psia at 3.05 m from the top of the tank and at the bottom. And calculate the gauge pressure at the bottom of the tank.

Solution:



P_o = 1 atm = 14.696 psia = 1.01325×10^5 Pa

$P_1 = h_1 \rho_{oil} g + P_o$

$= 3.05 \text{ m } (917 \text{ kg/m}^3)\ 9.81 \text{ m/s}^2 + 1.01325 \times 10^5 \text{ Pa}$

$= 1.28762 \times 10^5$ Pa

$P_1 = 1.28762 \times 10^5 \text{ Pa } (14.696 \text{ psia}/1.01325 \times 10^5 \text{ Pa})$

$= 18.675$ psia

or

$P_1 = h_1 \rho_{oil} g + P_o$

$= 10 \text{ ft m } [917 \text{ kg/m}^3 (62.43 \text{ lb/ft}^3/1000 \text{ kg/m}^3)]\ (32.174 \text{ ft/s}^2/32.174 \text{ lb.ft/lb}_f.\text{s}^2)\ 1/144 \text{ ft}^2/\text{in}^2 + 14.696 = 18.675 \text{ psia}$

$P_2 = P_1 + h_2 \rho_{water} g$

$= 1.28762 \times 10^5 \text{ Pa} + 0.61 \text{ m } (1000 \text{ kg/m}^3)\ 9.81 \text{ m/s}^2$

$= 1.347461 \times 10^5$ Pa

$P_2 = 1.347461 \times 10^5 \text{ Pa } (14.696 \text{ psia}/1.01325 \times 10^5 \text{ Pa})$

$= 19.5433$ psia

The gauge pressure = abs – atm

= 33421.1 Pa = 4.9472 psig

Example -3.2-

Convert the pressure of [1 atm =101.325 kPa] to

a- head of water in (m) at 4°C

b- head of Hg in (m) at 0°C

Solution:

a- The density of water at 4°C is approximatly1000 kg/m^3

$h = P / \rho_{water}\, g = 1.01325 \times 10^5$ Pa/(1000 kg/m^3 x 9.81m/s^2) =10.33 m H_2o

b- The density of mercury at 0°C is approximatly13595.5 kg/m^3

$h = P / \rho_{mercury}\, g = 1.01325 \times 10^5$ Pa/(13595.5 kg/m^3 x 9.81m/s^2) =0.76 m Hg

or

$$P = (h\,\rho\, g)_{water} = (h\,\rho\, g)_{mercury} \quad \Rightarrow h_{Hg} = h_{water}\,(\rho_{water} / \rho_{Hg})$$

$$h_{Hg} = 10.33\,(1000 / 13595.5) = 0.76 \text{ m Hg}$$

Example -3.3-

Find the static head of a liquid of sp.gr. 0.8 and pressure equivalent to 5 x 10^4 Pa.

Solution:

$\rho = 0.8\,(1000) = 800$ kg/m^3

$h = P / \rho\, g = 5\times10^4/(800 \times 9.81) = 6.37$ m H_2o

3.5 Measurement of Fluid Pressure

In chemical and other industrial processing plants it is often to measure and control the pressure in vessel or process and/or the liquid level vessel.

The pressure measuring devices are: -

1- Piezometer tube

The piezometer consists a tube open at one end to atmosphere, the other end is capable of being inserted into vessel or pipe of which pressure is to be measured. The height to which liquid rises up in the vertical tube gives the pressure head directly.

i.e. $\boxed{P = h\,\rho\, g}$

Piezometer is used for measuring moderate pressures. It is meant for measuring *gauge pressure* **only** as the end is open to atmosphere. It cannot be used for *vacuum pressures*.

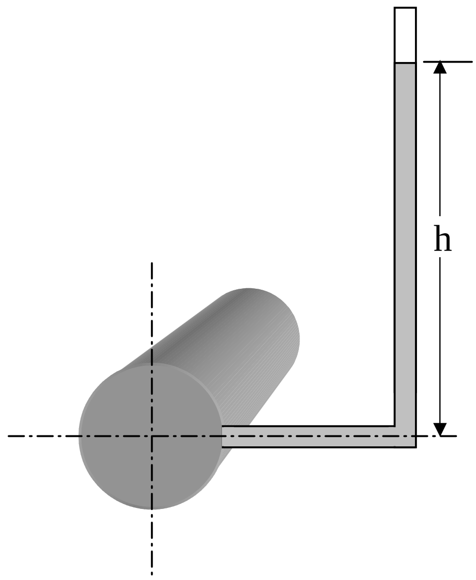


Figure (3): The Piezometer

2- Manometers

The manometer is an improved (modified) form of a piezometer. It can be used for measurement of comparatively *high pressures* and of both *gauge and vacuum pressures*.

Following are the various types of manometers: -

a- Simple manometer

b- The well type manometer

c- Inclined manometer

d- The inverted manometer

e- The two-liquid manometer

a- Simple manometer

It consists of a transparent U-tube containing the fluid A of density (ρ_A) whose pressure is to be measured and an immiscible fluid (B) of higher density (ρ_B). The limbs are connected to the two points between which the pressure difference (P_2 - P_1) is required; the connecting leads should be completely full of fluid A. If P_2 is greater than P_1, the interface between the two liquids in limb ② will be depressed a distance (h_m) (say) below that in limb ①.

The pressure at the level a — a must be the same in each of the limbs and, therefore:

$$P_2 + Z_m \rho_A g = P_1 + (Z_m - h_m) \rho_A g + h_m \rho_B g$$

$$\Rightarrow \qquad \Delta p = P_2 - P_1 = h_m (\rho_B - \rho_A) g$$

If fluid A is a gas, the density ρ_A will normally be small compared with the density of the manometer fluid pm so that:

$$\Delta p = P_2 - P_1 = h_m \rho_B g$$

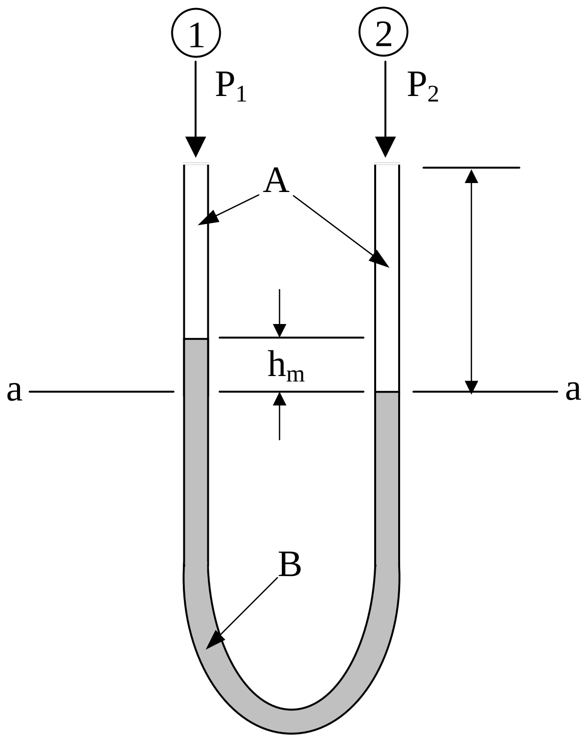


Figure (4): The simple manometer

b- The well-type manometer

In order to avoid the inconvenience of having to read two limbs, and in order to measure low pressures, where accuracy id of much importance, the well-type manometer shown in Figure (5) can be used. If A_w and A_c are the cross-sectional areas of the well and the column and h_m is the increase in the level of the column and h_w the decrease in the level of the well, then:

$$P_2 = P1 + (h_m + h_w) \rho g$$

or: $\qquad \Delta p = P_2 - P_1 = (h_m + h_w) \rho g$

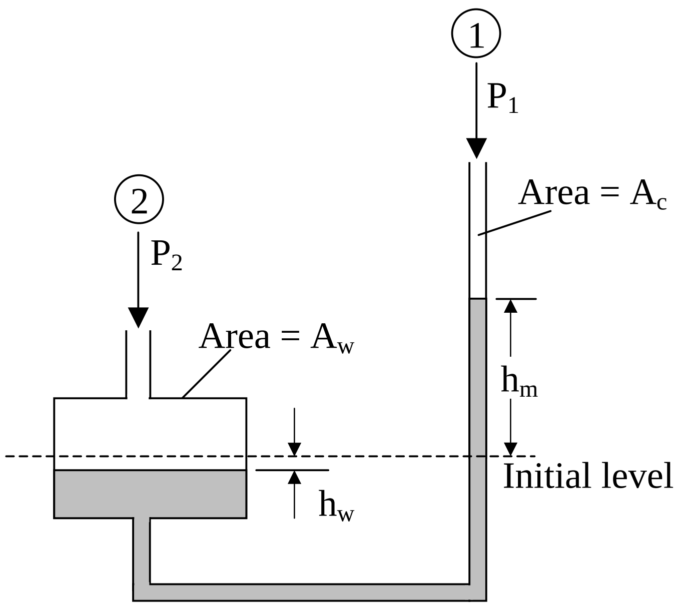


Figure (5): The well-type manometer

The quantity of liquid expelled from the well is equal to the quantity pushed into the column so that:

$$A_w h_w = A_c h_m \;\Rightarrow\; h_w = (A_c/A_w) h_m$$

$$\Rightarrow \; \Delta p = P_2 - P_1 = \rho g h_m (1 + A_c/A_w)$$

If the well is large in comparison to the column then:

i.e. $(A_c/A_w) \rightarrow \approx 0 \Rightarrow \quad \Delta p = P_2 - P_1 = \rho g h_m$

c- The inclined manometer

Shown in Figure (6) enables the sensitivity of the manometers described previously to be increased by measuring the length of the column of liquid. If θ is the angle of inclination of the manometer (typically about 10-20°) and L is the movement of the column of liquid along the limb, then:

$$h_m = L \sin\theta$$

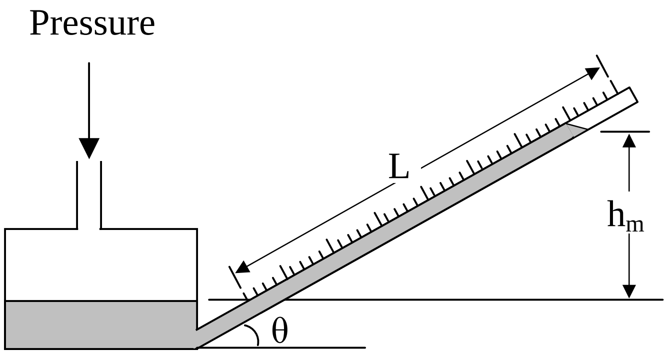


Figure (6): The inclined manometer

If θ = 10°, the manometer reading L is increased by about 5.7 times compared with the reading h_m which would have been obtained from a simple manometer.

d- The inverted manometer

Figure (7) is used for measuring *pressure differences in liquids*. The space above the liquid in the manometer is filled with air, which can be admitted or expelled through the tap A in order to adjust the level of the liquid in the manometer.

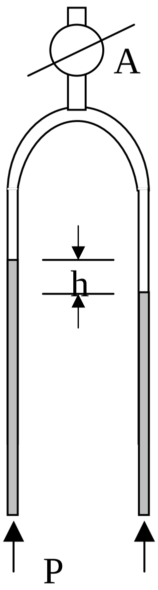


Figure (6): The inverted manometer

e- The two-liquid manometer

Small differences in pressure in gases are often measured with a manometer of the form shown in Figure 6.5. The reservoir at the top of each limb is of a sufficiently large cross-section for the liquid level to remain approximately the on each side of the manometer.

The difference in pressure is then given by:

$$\Delta p = P_2 - P_1 = h_m (\rho_{m1} - \rho_{m2})\, g$$

where ρ_{m1} and ρm2 are the densities of the two manometer liquids. The sensitivity of the instrument is very high if the densities of the two liquids are nearly the same. To obtain accurate readings it is necessary to choose liquids, which give sharp interfaces: paraffin oil and industrial alcohol are commonly used.

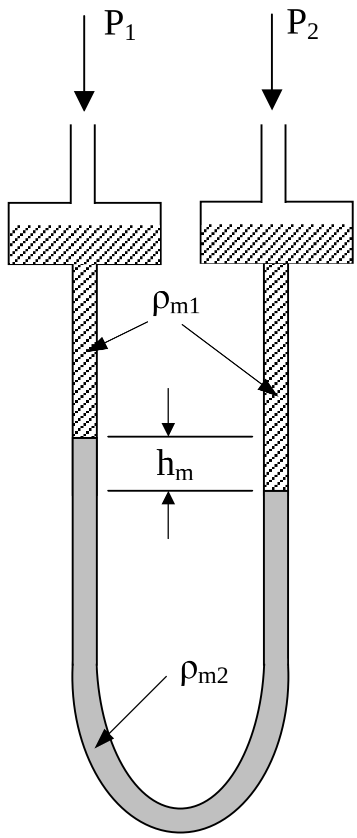


Figure (7): The two-liquid manometer

3- Mechanical Gauges

Whenever a *very high fluid pressure* is to be measured, and a *very great sensitivity* a mechanical gauge is best suited for these purposes. They are also designed to read vacuum pressure. A mechanical gauge is also used for measurement of pressure in boilers or other pipes, where tube manometer cannot be conveniently used.

There are many types of gauge available in the market. But the principle on which all these gauge work is almost the same. The followings are some of the important types of mechanical gauges: -

1- The Bourdon gauge
2- Diaphragm pressure gauge
3- Dead weight pressure gauge

The Bourdon gauge

The pressure to be measured is applied to a curved tube, oval in cross-section, and the deflection of the end of the tube is communicated through a system of levers to a recording needle. This gauge is widely used for *steam* and *compressed gases*, and frequently *forms the indicating element on flow controllers*. The simple form of the gauge is illustrated in Figures (7a) and (7b). Figure (7c) shows a Bourdon type gauge with the sensing element in the form of a helix; this instrument has a very much greater sensitivity and is suitable for very high pressures.

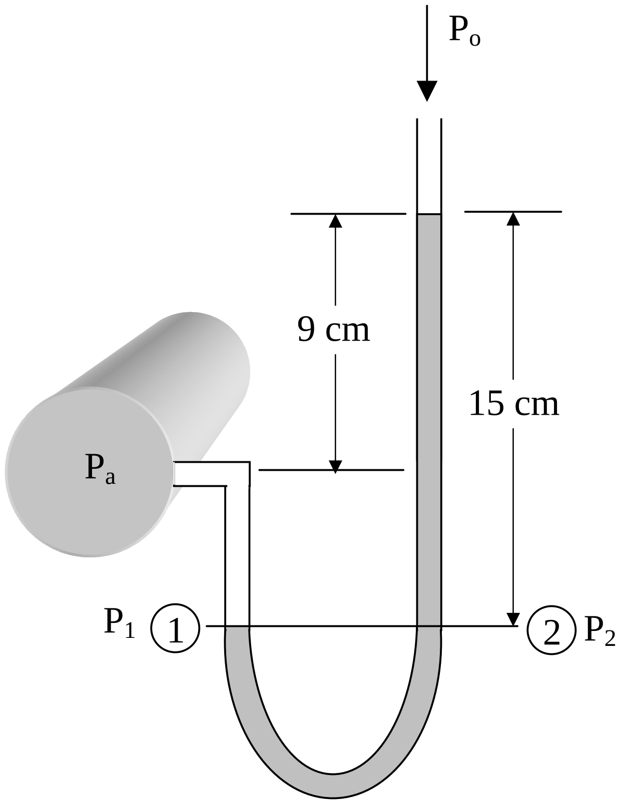
It may be noted that the pressure measuring devices of category (2) all measure a pressure difference ($\Delta p = P_2 - P_1$). In the case of the Bourdon gauge (1) of category (3), the pressure indicated is the difference between that communicated by the system to the tube and the external (ambient) pressure, and this is usually referred to as *the gauge pressure*. It is then necessary to add on the ambient pressure in order to obtain the *(absolute) pressure*.

Figure (7) Bourdon gauge

Gauge pressures are not, however, used in the SI System of units.

Example -3.4-

A simple manometer is used to measure the pressure of oil sp.gr. 0.8 flowing in a pipeline. Its right limb is open to atmosphere and the left limb is connected to the pipe. The center of the pipe is 9.0 cm below the level of the mercury in the right limb. If the difference of the mercury level in the two limbs is 15 cm, determine the absolute and the gauge pressures of the oil in the pipe.



Solution:

$\rho = 0.8\ (1000) = 800\ kg/m^3$

$P_1 = P_2$

$P_1 = (0.15 - 0.0\ 9)m(800\ kg/m^3)9.81 m/s^2 + P_a$

$P_2 = (0.15)\ m\ (13600\ kg/m^3)\ 9.81\ m/s^2 + P_o$

$P_a = 15\ (13600)\ 9.81 + P_o + [(15 - 9)cm\ (800\ kg/m^3)\ 9.81\ m/s^2]$

$= 1.20866 \times 10^5$ Pa (Absolute pressure)

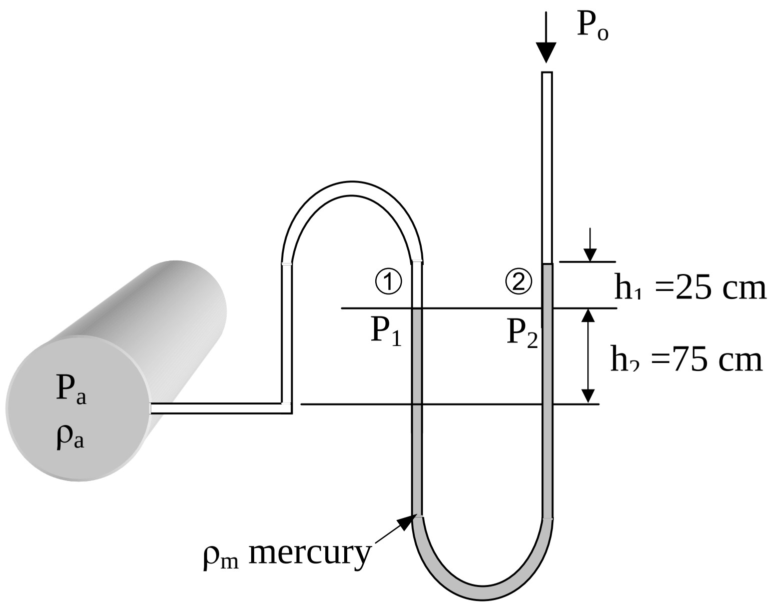
The gauge press. = Abs. press. – Atm. Press.

$= 1.20866 \times 10^5 - 1.0325 \times 10^5$

$= 1.9541 \times 10^4$ Pa

Example -3.5-

The following Figure shows a manometer connected to the pipeline containing oil of sp.gr. 0.8. Determine the absolute pressure of the oil in the pipe, and the gauge pressure.



Solution:

$\rho_a = 0.8\ (1000) = 800\ kg/m^3$

$P_1 = P_2$

$P_1 = P_a - h_2\ \rho_a\ g$

$P_2 = P_o + h_1\ \rho_m\ g$

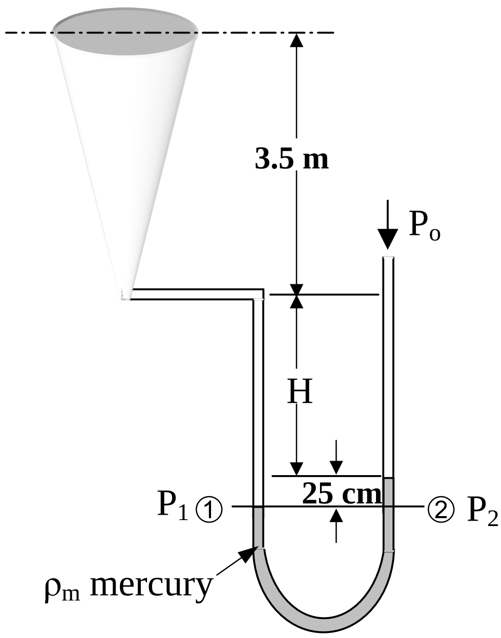
$\Rightarrow\ P_a = P_o + h_1\ \rho_m\ g + h_2\ \rho_a\ g$

$= 1.0325 \times 10^5 + (0.25)\ m\ (13600\ kg/m^3)\ 9.81\ m/s^2 + (0.75)\ m\ (800\ kg/m^3)\ 9.81\ m/s^2$

$= 1.40565 \times 10^5$ Pa

Example -3.6-

A conical vessel is connected to a U-tube having mercury and water as shown in the Figure. When the vessel is empty the manometer reads 0.25 m. find the reading in manometer, when the vessel is full of water.



Solution:

$P_1 = P_2$

$P_1 = (0.25 + H)\,\rho_w\,g + P_o$

$P_2 = 0.25\,\rho_m\,g + P_o$

$\Rightarrow\ (0.25 + H)\,\rho_w\,g + P_o = 0.25\,\rho_m\,g + P_o$

$\Rightarrow\ H = 0.25\,(\rho_m - \rho_w)/\,\rho_w$

$= 0.25\,(12600\,/1000) = 3.15$ m

When the vessel is full of water, let the mercury level in the left limp go down by (x) meter and the mercury level in the right limp go to up by the same amount (x) meter.

i.e. the reading manometer = $(0.25 + 2x)$

$P_1 = P_2$

$P_1 = (0.25 + x + H + 3.5)\,\rho_w\,g + P_o$

$P_2 = (0.25 + 2x)\,\rho_m\,g + P_o$

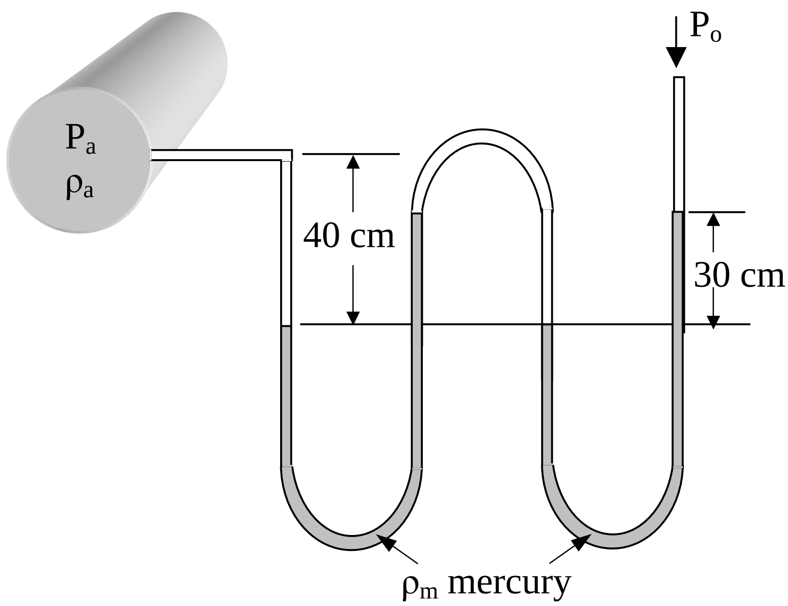
$\Rightarrow\ (0.25 + x + H + 3.5)\,\rho_w\,g + P_o = (0.25 + 2x)\,\rho_m\,g + P_o$

$\Rightarrow\ 6.9 + x = (0.25 + 2x)\,(\rho_m/\,\rho_w) \Rightarrow\ x = 0.1431$ m

The manometer reading = 0.25 + 2 (0.1431) = 0.536 m

Example -3.7-

The following Figure shows a compound manometer connected to the pipeline containing oil of sp.gr. 0.8. Calculate P_a.



Solution:

$\rho_a = 0.8\,(1000) = 800$ kg/m^3

$P_a + 0.4\,\rho_a\,g - 0.3\,\rho_m\,g + 0.3\,\rho_a\,g - 0.3\,\rho_m\,g - P_o = 0$

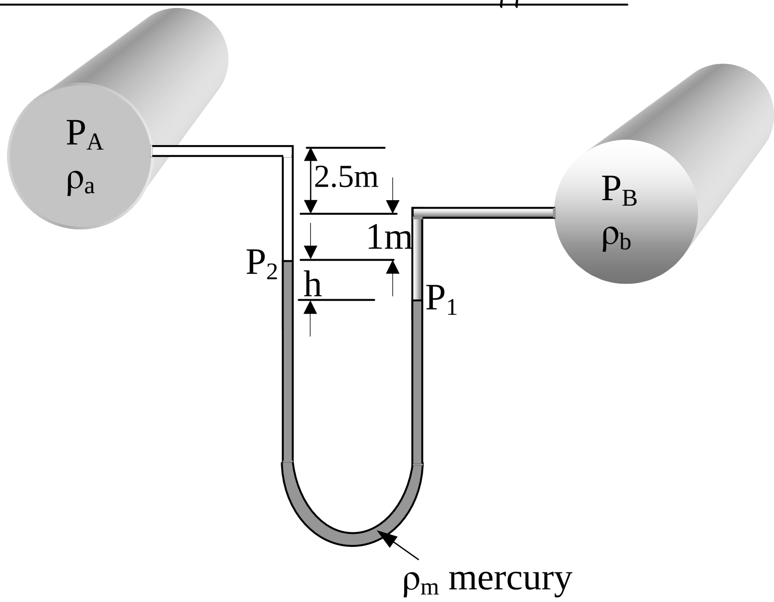
$\Rightarrow\ P_a = P_o + 0.7\,\rho_a\,g - 0.6\,\rho_m\,g$

$= 1.01325 \times 10^5 - 0.7\,(800)\,9.81 + 0.6\,(13600)\,9.81$

$= 1.75881 \times 10^5$ Pa

Example -3.8-

A differential manometer is connected to two pipes as shown in Figure. The pipe A is containing carbon tetrachloride sp.gr. = 1.594 and the pipe B is contain an oil of sp.gr. = 0.8. Find the difference of mercury level if the pressure difference in the two pipes be 0.8 kg/cm^2.



Solution:

$P_1 = P_2$

$P_1 = P_B + (1 + h)\,\rho_b\, g$

$P_2 = P_A + 3.5\,\rho_a\, g + h\,\rho_m\, g$

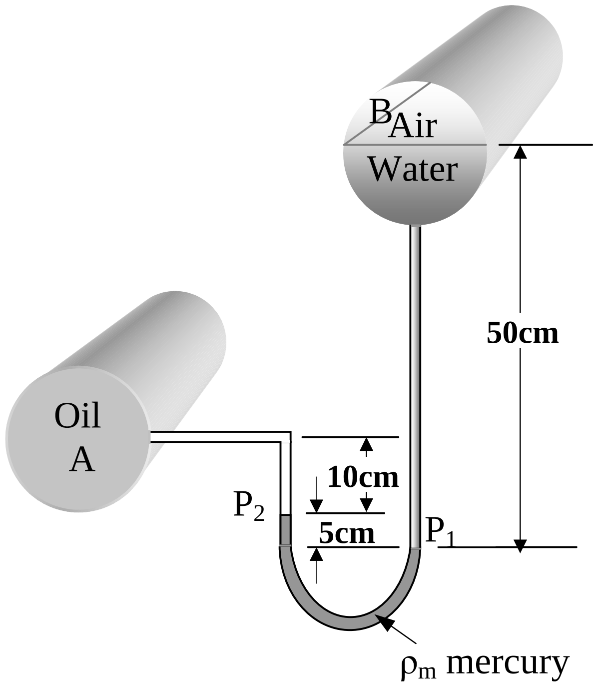
$\Rightarrow\ P_A - P_B = 3.5\,\rho_a\, g + h\,\rho_m\, g - (1 + h)\,\rho_b\, g = (0.8\ kg/cm^2)\ (9.81\ m/s^2)\ (10^4\ cm^2/m^2)$

$\Rightarrow\ 7.848 \times 10^4 = 3.5\ (1594)\ 9.81 + h\ (13600)\ 9.81 - (1+h)\ 800\ (9.81)$

$\Rightarrow h = 25.16$ cm.

Example -3.9-

A differential manometer is connected to two pipes as shown in Figure. At B the air pressure is 1.0 kg/cm^2 (abs), find the absolute pressure at A.



Solution:

$P_1 = P_2$

$P_1 = P_{air} + 0.5\,\rho_w\, g$

$P_2 = P_A + 0.1\,\rho_a\, g + 0.05\,\rho_m\, g$

$\Rightarrow P_A = P_{air} + 0.5\,\rho_w\, g - 0.1\,\rho_a\, g - 0.05\,\rho_m\, g$

$\Rightarrow\ P_{air} = (1.0\ kg/cm^2\ P_B)\ (9.81 m/s^2)\ (10^4\ cm^2/\ m^2)$

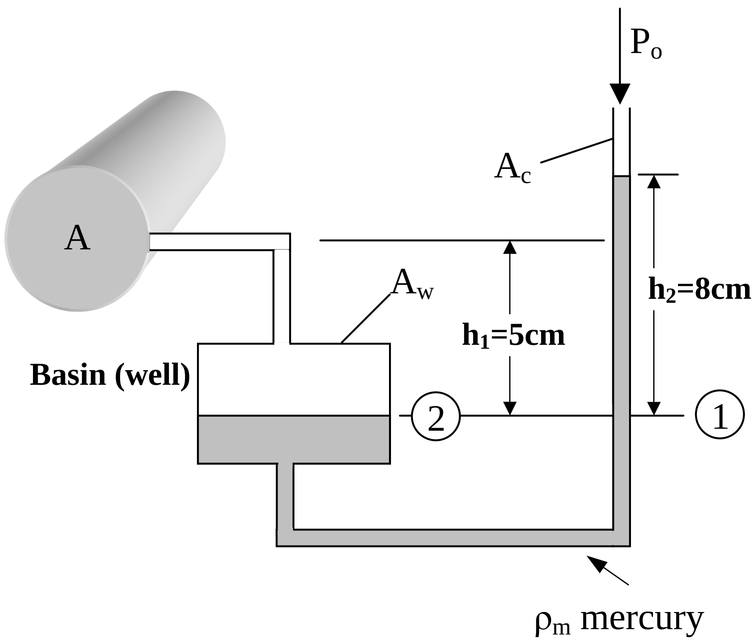
$= 9.81 \times 10^4$ Pa

$\therefore P_A = 9.81 \times 10^4\ Pa + 0.5\ (1000)\ 9.81 - 0.1\ (900)\ 9.81 - 0.05\ (13600)\ 9.81$

$= 9.54513 \times 10^4$ Pa

Example -3.10-

A Micromanometer, having ratio of basin to limb areas as 40, was used to determine the pressure in a pipe containing water. Determine the pressure in the pipe for the manometer reading shown in Figure.



Solution:

$P_1 = P_2$

$P_1 = P_o + h_2\,\rho_m\,g$

$P_2 = P_A + h_1\,\rho_w\,g$

$\Rightarrow\ P_A = P_o + h_2\,\rho_m\,g - h_1\,\rho_w\,g$

$= 1.01325\text{x}10^5 + 0.08\,(13600)\,9.81 - 0.05\,(1000)\,9.81$

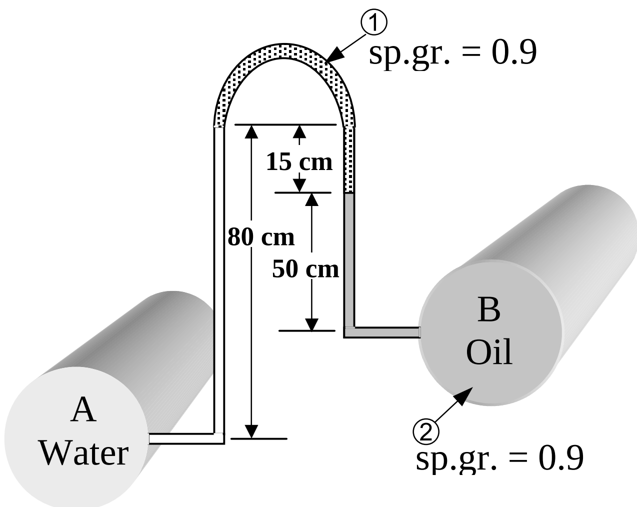
$= 1.11507\ \text{x}10^5$ Pa

Note:

If h_2 and h_1 are the heights from initial level, the ratio (A_w/A_c) will enter in calculation.

Example -3.11-

An inverted manometer, when connected to two pipes A and B, gives the readings as shown in Figure. Determine the pressure in tube B, if the pressure in pipe A 1.0 kg/cm^2.



Solution:

$P_A - 0.8\,\rho_w\,g + 0.15\,\rho_1\,g + 0.5\,\rho_2\,g - P_B = 0$

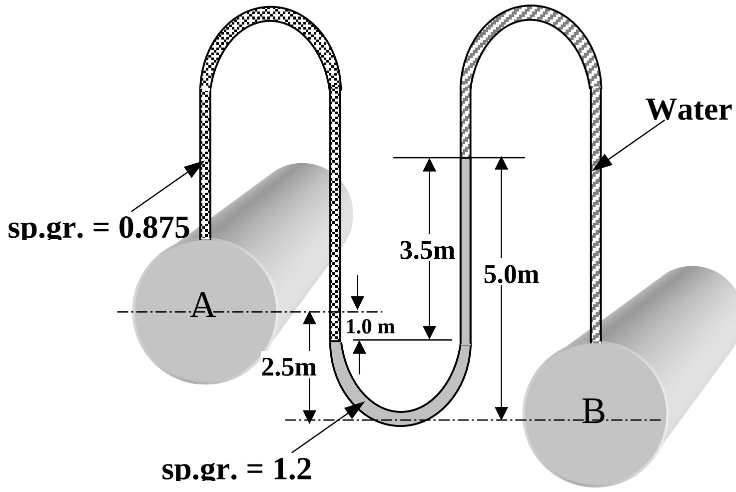
$\Rightarrow\ P_B = P_A - [0.8\,(1000) - 0.15\,(800) - 0.5\,(900)]\,9.81$

$P_A = 1.0\ \text{kg/cm}^2\ \text{x}\ 9.81\ \text{x}\ 10^4 = 9.81\ \text{x}\ 10^4$ Pa

$\therefore P_A = 9.58437\ \text{x}10^4$ Pa

Example -3.12-

Two pipes, one carrying toluene of sp.gr. = 0.875, and the other carrying water are placed at a difference of level of 2.5 m. the pipes are connected by a U-tube manometer carrying liquid of sp.gr. = 1.2. The level of the liquid in the manometer is 3.5 m higher in the right limb than the lower level of toluene in the limb of the manometer. Find the difference of pressure in the two pipes.



Solution:

T ≡ Toluene, W ≡ Water, L ≡ Liquid

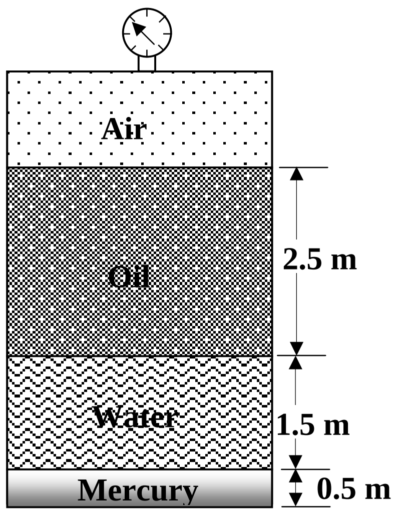
$P_A + 3.5\,\rho_T\,g - 3.5\,\rho_L\,g + 5\,\rho_W\,g - P_B = 0$

$\Rightarrow\ P_A - P_B = [3.5\,(1200) - 3.5\,(875) - 5\,(1000)]\,9.81$
$= -\,3862.5$ Pa

$\Rightarrow\ P_B - P_A = 3862.5$ Pa

Example -3.13-

A closed tank contains 0.5 m of mercury, 1.5 m of water, 2.5 m of oil of sp.gr. = 0.8 and air space above the oil. If the pressure at the bottom of the tank is 2.943 bar gauge, what should be the reading of mechanical gauge at the top of the tank.



Solution:

Pressure due to 0.5 m of mercury

$P_m = 0.5\,(13600)\,9.81 = 0.66708$ bar

Pressure due to 1.5 m of water

$P_w = 1.5\,(1000)\,9.81 = 0.14715$ bar

Pressure due to 2.5 m of oil

$P_O = 2.5\,(800)\,9.81 = 0.19620$ bar

Pressure at the bottom of the tank = $P_m + P_w + P_O + P_{Air}$

$\Rightarrow\ 2.943 = 0.66708\text{ bar} + 0.14715\text{ bar} + 0.19620\text{ bar} + P_{Air}$

$\Rightarrow\ P_{Air} = 1.93257$ bar

Home Work

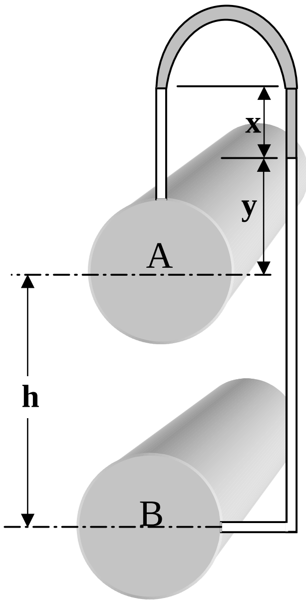
P.3.1

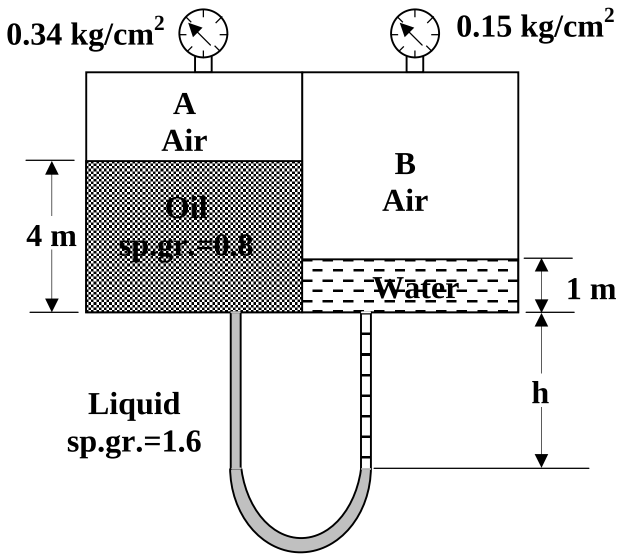
Two pipes A and B carrying water are connected by a connecting tube as shown in Figure,

a- If the manometric liquid is oil of sp.gr. = 0.8, find the difference in pressure intensity at A and B when the difference in level between the two pipes be (h = 2 m) and (x = 40 cm).

b- If mercury is used instead of water in the pipes A and B and the oil used in the manometer has sp.gr. = 1.5, find the difference in pressure intensity at A and B when (h = 50 cm) and (x = 100 cm).

Ans. a- P_B-P_A =18835.2 Pa, b- P_B-P_A =51993 Pa





P.3.2

A closed vessel is divided into two compartments. These compartments contain oil and water as shown in Figure. Determine the value of (h).

Ans. h = 4.5 m

P.3.3

Oil of sp.gr. = 0.9 flows through a vertical pipe (upwards). Two points A and B one above the other 40 cm apart in a pipe are connected by a U-tube carrying mercury. If the difference of pressure between A and B is 0.2 kg/cm^2,

1- Find the reading of the manometer.

2- If the oil flows through a horizontal pipe, find the reading in manometer for the same difference in pressure between A and B.

Ans. 1- R = 0.12913 m, 2- R = 0.1575 m,

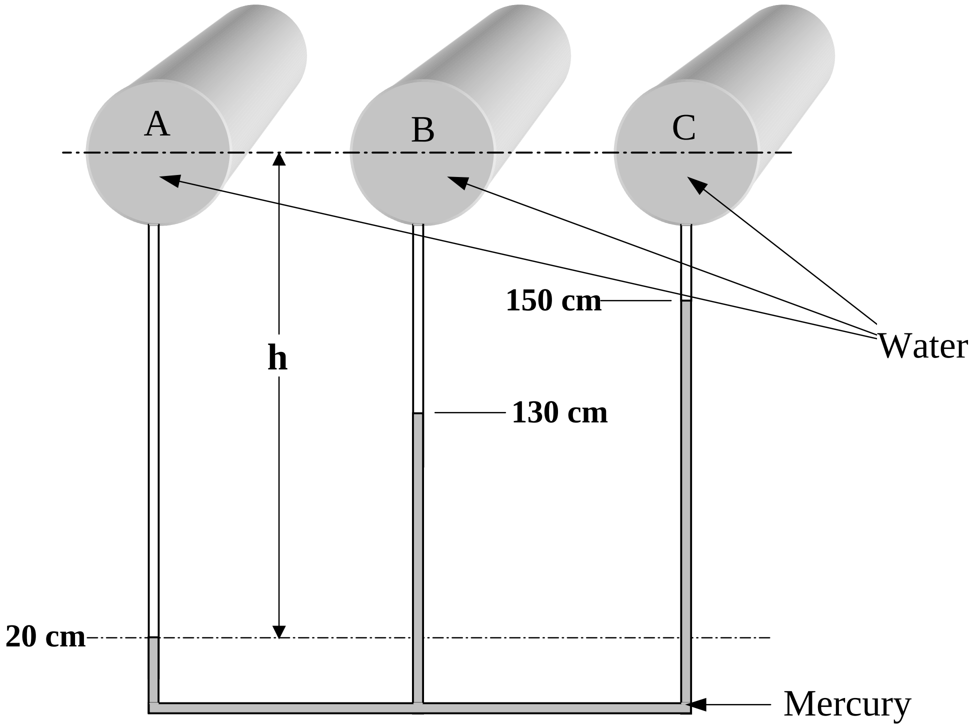
P.3.4

A mercury U-tube manometer is used to measure the pressure drop across an orifice in pipe. If the liquid that flowing through the orifice is brine of sp.gr. 1.26 and upstream pressure is 2 psig and the downstream pressure is (10 in Hg) vacuum, find the reading of manometer.

Ans. R = 394 mm Hg

P.3.5

Three pipes A, B, and C at the same level connected by a multiple differential manometer shows the readings as show in Figure. Find the differential of pressure heads in terms of water column between A and B, between A and C, and between B and C.



Ans. P_A-P_B =1.359666 bar = 13.86 m H_2o

P_A-P_C =1.606878 bar = 16.38 m H_2o

P_B-P_C =0.247212 bar = 2.52 m H_2o

CHPTER FOUR

Fluid Dynamic

4.1 Introduction

In the process industries it is often necessary to pump fluids over long distances from storage to processing units, and there may be a substantial drop in pressure in both the pipeline and in individual units themselves. It is necessary, therefore, to consider the problems concerned with calculating the power requirements for pumping, with designing the most suitable flow system, with estimating the most economical sizes of pipes, with measuring the rate of flow, and frequently with controlling this flow at steady state rate.

It must be realized that when a fluid is flowing over a surface o through a pipe, the velocity at various points in a plane at right angles to the stream velocity is rarely uniform, and the rate change of velocity with distance from the surface will exert a vital influence on the resistance to flow and the rate of mass or heat transfer.

4.2 The Nature of Fluid Flow

When a fluid is flowing through a tube or over a surface, the pattern of flow will vary with *the velocity, the physical properties of fluid*, and *the geometry of the surface*. This problem was first examined by Reynolds in 1883. Reynolds has shown that when the velocity of the fluid is slow, the flow pattern is smooth. However, when the velocity is quite high, an unstable pattern is observed in which eddies or small packets of fluid particles are present moving in all directions and at all angles to the normal line of flow.

The first type of flow at low velocities where the layers of fluid seen to slide by one another without eddies or swirls being present is called "*laminar flow*" and Newton's law of viscosity holds.

The second type of flow at higher velocities where eddies are present giving the fluid a fluctuating nature is called "*turbulent flow*".

4.3 Reynolds Number (Re)

Studies have shown that the transition from laminar to turbulent flow in tubes is not only a function of velocity but also of density (ρ), dynamic viscosity (μ), and the diameter of tube. These variables are combining into the Reynolds number, which is dimensionless group.

$$\boxed{\mathrm{Re} = \frac{\rho u d}{\mu}}$$

where u is the average velocity of fluid, which is defined as the volumetric flow rate divided by the cross-sectional area of the pipe.

$$\boxed{u = \frac{Q}{A} = \frac{Q}{\pi/4\, d^2}}$$

$$\Rightarrow \boxed{\mathrm{Re} = \frac{4\,Q\rho}{\pi\ d\ \mu} = \frac{4\dot{m}}{\pi\ d\ \mu} = \frac{G\ d}{\mu}}$$

Where, Q: volumetric flow rate m^3/s

$\dot{m}$: mass flow rate kg/s

G: mass flux or mass velocity $kg/m^2.s$

for a straight circular pipe when the value of Re is less than 2,100 the flow is always laminar. When the value is over 4,000 the flow be turbulent. In between, which is called the transition region the flow can be laminar or turbulent depending upon the apparatus details.

Example -4.1-

Water at 303 K is flowing at the rate of 10 gal/min in a pipe having an inside diameter I.D. of 2.067 in. calculate the Reynolds number using both English and S.I. units

Solution:

The volumetric flow rate (Q) = 10 gal/min (1.0 ft^3/7.481 gal) (min/60 s) = 0.0223 ft^3/s

Pipe diameter (d) = 2.067 in (ft/12 in) = 0.172 ft

Cross-sectional area (A) = $\pi/4\ d^2 = \pi/4\ (0.172)^2 = 0.0233\ ft^2$

Average velocity (u) =Q/A = (0.0223 ft^3/s) / 0.0233 ft^2 = 0.957 ft/s

At T = 303 K The density of water (ρ = 62.18 lb/ft^3),

The dynamic viscosity ($\mu = 5.38 \times 10^{-4}$) lb/ft.s

$$Re = \frac{\rho\, u d}{\mu} = \frac{62.18\, lb/ft^3 (0.957\ ft/s)(0.172\ ft)}{5.38\times10^{-4}\, lb/ft.s} = 1.902 \times 10^4 \text{ (turbulent)}$$

Using S.I. units

At T = 303 K The density of water (ρ = 996 kg/m^3),

The dynamic viscosity ($\mu = 8.007 \times 10^{-4}$) kg/m.s (or Pa.s)

Pipe diameter (d) = 0.172 ft (m/3.28 ft) = 0.0525m

Average velocity (u) =0.957 ft/s (m/3.28 ft) = 0.2917 m/s

$$Re = \frac{996\ kg/m^3 (0.2917 m/s)(0.0525 m)}{8.007\times10^{-4}\, kg/m.s} = 1.905 \times 10^4 \text{ (turbulent)}$$

4.4 Overall Mass Balance and Continuity Equation

In fluid dynamics, fluids are in motion. Generally, they are moved from place to place by means of mechanical devices such as pumps or blowers, by gravity head, or by pressure, and flow through systems of piping and/or process equipment.

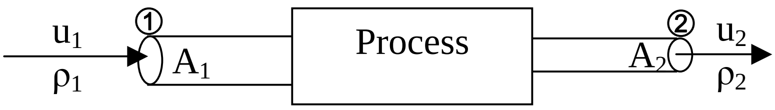
The first step in the solution of flow problems is generally to apply the principles of the conservation of mass to the whole system or any part of the system.

$\boxed{\text{INPUT – OUTPUT = ACCUMULATION}}$

At steady state, the rate of accumulation is zero

$\therefore$ $\boxed{\text{INPUT = OUTPUT}}$

In the following Figure a simple flow system is shown where fluid enters section ① with an average velocity (u_1) and density (ρ_1) through the cross-sectional area (A_1). The fluid leaves section ② with an average velocity (u_2) and density (ρ_1) through the cross-sectional area (A_2).



Thus,

At steady state $\dot{m}_1 = \dot{m}2$

$$Q_1\,\rho_1 = Q_2\,\rho_2$$

$$u_1\,A_1\,\rho_1 = u_2\,A_2\,\rho_2$$

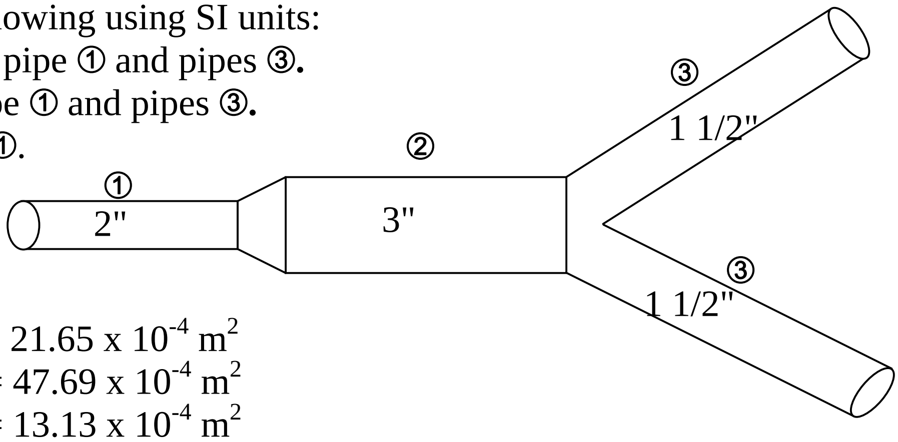
For incompressible fluids at the same temperature [$\rho_1 = \rho_2$]

$$\therefore \boxed{u_1\,A_1 = u_2\,A_2}$$

Example -4.2- **************

A petroleum crude oil having a density of 892 kg/m^3 is flowing, through the piping arrangement shown in the below Figure, at total rate of 1.388 x 10^{-3} m^3/s entering pipe ①. The flow divides equally in each of pipes ③. The steel pipes are schedule 40 pipe. **Table{{{{}}}.** Calculate the following using SI units:

a- The total mass flow rate in pipe ① and pipes ③.
b- The average velocity in pipe ① and pipes ③.
c- The mass velocity in pipe ①.



Solution:

Pipe ① I.D. = 0.0525 m, A1 = 21.65 x 10^{-4} m^2
Pipe ② I.D. = 0.07792 m, A_1 = 47.69 x 10^{-4} m^2
Pipe ③ I.D. = 0.04089 m, A_1 = 13.13 x 10^{-4} m^2

a- the total mass flow rate is the same through pipes ① and ② and is

$\dot{m}_1 = Q_1\,\rho$ = 1.388 x 10^{-3} m^3/s (892 kg/m^3) = 1.238 kg/s

Since the flow divides equally in each pipes ③'

$\Rightarrow\ \dot{m}_3 = \dot{m}_1/2 = 1.238/2 =$ 0.619 kg/s

b- $\dot{m}_1 = Q_1\,\rho = u_1 A_1\,\rho \ \Rightarrow\ u_1 = \dfrac{\dot{m}_1}{A_1\,\rho} = \dfrac{1.238\ kg/s}{(21.65\times10^{-4}\,m^2)(892\ kg/m^3)}$ = 0.641 m/s

$u_3 = \dfrac{\dot{m}_3}{A_3\,\rho} = \dfrac{0.619\ kg/s}{(13.13\times10^{-4}\,m^2)(892\ kg/m^3)}$ = 0.528 m/s

d- $G_1 = u_1\,\rho_1$ = 0.641 m/s (892 kg/m^3) = 572 kg/m^2.s

e- or $G_1 = \dfrac{\dot{m}_1}{A_1} = \dfrac{1.238\ kg/s}{21.65\times10^{-4}\,m^2}$ = 572 kg/m^2.s

4.5 Energy Relationships and Bernoulli's Equation

The total energy of a fluid in motion consists of the following components: -

Internal Energy (U)

This is the energy associated with the physical state of fluid, i.e. the energy of atoms and molecules resulting from their motion and configuration. Internal energy is a function of temperature. It can be written as (U) energy per unit mass of fluid.

Potential Energy (PE)

This is the energy that a fluid has because of its position in the earth's field of gravity. The work required to raise a unit mass of fluid to a height (z) above a datum line is (zg), where (g) is gravitational acceleration. This work is equal to the potential energy per unit mass of fluid above the datum line.

Kinetic Energy (KE)

This is the energy associated with the physical state of fluid motion. The kinetic energy of unit mass of the fluid is (u^2/2), where (u) is the linear velocity of the fluid relative to some fixed body.

Pressure Energy (Prss.E)

This is the energy or work required to introduce the fluid into the system without a change in volume. If (P) is the pressure and (V) is the volume of a mass (m) of fluid, then ($PV/m \equiv P\upsilon$) is the pressure energy per unit mass of fluid. The ratio (V/m) is the fluid density (ρ).

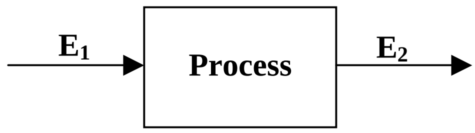
The total energy (E) per unit mass of fluid is given by the equation: -

$$\boxed{E = U + zg + P/\rho + u^2/2}$$

where, each term has the dimension of force times distance per unit mass. In calculation, each term in the equation must be expressed in the same units, such as J/kg, Btu/lb or lb_f.ft/lb. i.e. $(MLT^{-2})(L)(M^{-1}) = [L^2T^{-2}] \equiv \{m^2/s^2, ft^2/s^2\}$.

A flowing fluid is required to do work in order to overcome viscous frictional forces that resist the flow.

The principle of the conservation of energy will be applied to a process of input and output streams for ideal fluid of constant density and without any pump present and no change in temperature.



$E_1 = E_2$

$U_1 + z_1\, g + P_1/\rho + u_1^2/2 = U_2 + z_2\, g + P_2/\rho + u_2^2/2$

$U_1 = U_2$ (no change in temperature)

$P_1/\rho + u_1^2/2 + z_1\, g = P_2/\rho + u_2^2/2 + z_2\, g$

$\Rightarrow P/\rho + u^2/2 + z\, g = \text{constant}$

$\Rightarrow \Delta P/\rho + \Delta u^2/2 + \Delta z\, g = 0$ --------- Bernoulli's equation

4.6 Equations of Motion

According to Newton's second law of motion, *the net force* in x-direction (F_x) acting on a fluid element in x-direction is: -

F_x = (mass) x (acceleration in x-direction)

$F_x = (m)(a_x)$

In the fluid flow the following forces are present: -

1- F_g ---------force due to gravity
2- F_P ---------force due to pressure
3- F_V ---------force due to viscosity
4- F_t ---------force due to turbulence
5- F_c ---------force due to compressibility
6- F_σ ---------force due to surface tension

The net force is could be given by

$$F_x = \underbrace{\underbrace{\underbrace{(F_g)_x + (F_P)_x}_{\text{Euler's equation}} + (F_V)_x}_{\text{Navier-Stokes equation}} + (F_t)_x}_{\text{Reynolds Equation}} + (F_c)_x + (F_\sigma)_x$$

In most of the problems of fluid in motion the forces due to surface tension (F_σ), and the force due to compressibility (F_c) are neglected,

$\Rightarrow \quad F_x = (F_g)_x + (F_P)_x + (F_V)_x + (F_t)_x$

This equation is called "Reynolds equation of motion" which is useful in the analysis of turbulent flow.

In laminar (viscous) flow, the turbulent force becomes insignificant and hence the equation of motion may be written as: -

$F_x = (F_g)_x + (F_P)_x + (F_V)_x$

This equation is called "Navier-Stokes equation of motion" which is useful in the analysis of viscous flow.

If the flowing fluid is ideal and has very small viscosity, the viscous force and viscosity being almost insignificant and the equation will be: -

$F_x = (F_g)_x + (F_P)_x$

This equation is called "Euler's equation of motion".

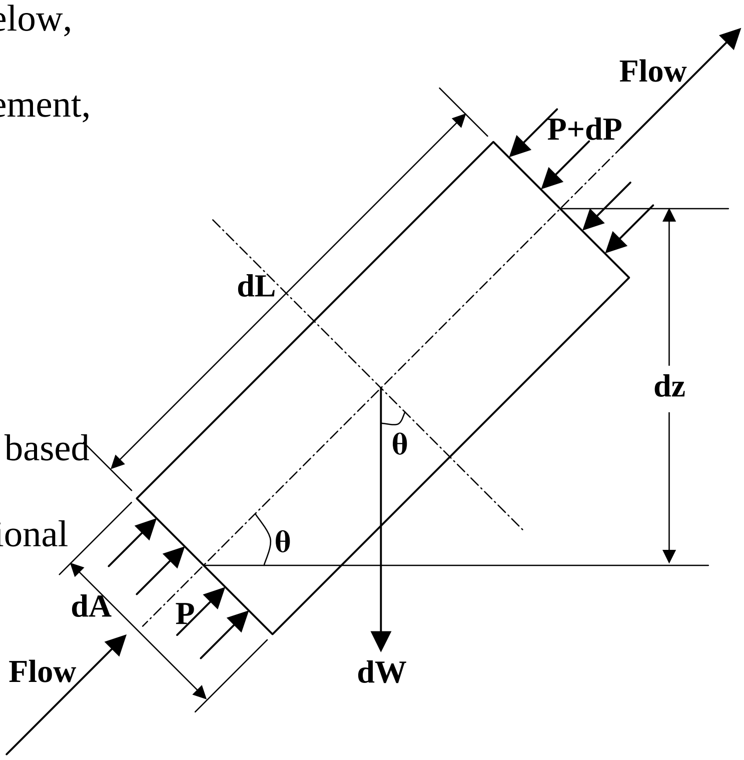
4.6.1 Euler's equation of motion

The Euler's equation for steady state flow on an ideal fluid along a streamline is based on the Newton's second law of motion. The integration of the equation gives Bernoulli's equation in the form of energy per unit mass of the flowing fluid.

Consider a steady flow of an ideal fluid along a streamline. Now consider a small element of the flowing fluid as shown below,

Let:

dA: cross-sectional area of the fluid element,
dL: Length of the fluid element'
dW: Weight of the fluid element'
u: Velocity of the fluid element'
P: Pressure of the fluid element'



The Euler's equation of motion is based on the following assumption: -

1- The fluid is non-viscous (the frictional losses are zero).
2- The fluid is homogenous and Incompressible (the density of fluid is constant).
3- The flow is continuous, steady, and along the streamline (laminar).
4- The velocity of flow is uniform over the section.
5- No energy or force except gravity and pressure forces is involved in the flow.

The forces on the cylindrical fluid element are,

1- Pressure force acting on the direction of flow (PdA)
2- Pressure force acting on the opposite direction of flow [(P+dP)dA]
3- A component of gravity force acting on the opposite direction of flow (dW sin θ)

- The pressure force in the direction of low

$$F_P = PdA - (P+dP)\,dA = -\,dPdA$$

- The gravity force in the direction of flow

$$F_g = -\,dW \sin\theta \qquad \{W = m\,g = \rho\,dA\,dL\,g\}$$
$$= -\,\rho\,g\,dA\,dL \sin\theta \qquad \{\sin\theta = dz/dL\}$$
$$= -\,\rho\,g\,dA\,dz$$

- The net force in the direction of flow

$$F = m\,a \qquad \{m = \rho\,dA\,dL\}$$
$$= \rho\,dA\,dL\,a \qquad \{a = \frac{du}{dt} = \frac{du}{dL} \times \frac{dL}{dt} = u\frac{du}{dL}\}$$
$$= \rho\,dA\,u\,du$$

We have

$$F_x = (F_g)_x + (F_P)_x$$
$$\rho\,dA\,u\,du = -\,dP\,dA - \rho\,g\,dA\,dz \qquad \{\div -\rho\ dA\ z\}$$

$\Rightarrow$ $\boxed{dP/\rho + du^2/2 + dz\,g = 0}$ --------- Euler's equation of motion

Bernoulli's equation could be obtain by integration the Euler's equation

$\int dP/\rho + \int du^2/2 + \int dz\,g = \text{constant}$

$\Rightarrow$ $P/\rho + u^2/2 + z\,g = \text{constant}$

$\Rightarrow$ $\Delta P/\rho + \Delta u^2/2 + \Delta z\,g = 0$ --------- Bernoulli's equation

4.7 Modification of Bernoulli's Equation

1- Correction of the kinetic energy term

The velocity in kinetic energy term is the mean linear velocity in the pipe. To account the effect of the velocity distribution across the pipe [(α) dimensionless correction factor] is used.

For a circular cross sectional pipe:

- $\alpha = 0.5$ for laminar flow
- $\alpha = 1.0$ for turbulent flow

2- Modification for real fluid

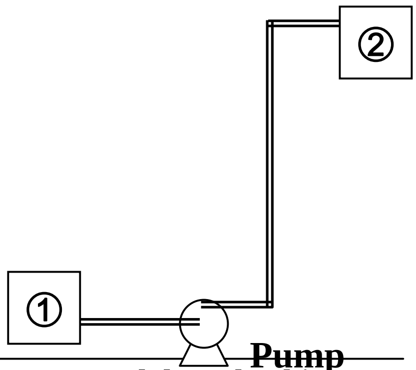
The real fluids are viscous and hence offer resistance to flow. Friction appears wherever the fluid flow is surrounding by solid boundary. Friction can be defined as the amount of mechanical energy irreversibly converted into heat in a flow in stream. As a result of that the total energy is always decrease in the flow direction i.e. ($E_2 < E_1$). Therefore $E_1 = E_2 + F$, where F is the energy losses due to friction.

Thus the modified Bernoulli's equation becomes,

$\boxed{P_1/\rho + u_1^2/2 + z_1\,g = P_2/\rho + u_2^2/2 + z_2\,g + F}$ ---------(J/kg ≡ m^2/s^2)

3- Pump work in Bernoulli's equation

A pump is used in a flow system to increase *the mechanical energy* of the fluid. The increase being used to maintain flow of the fluid. Assume a pump is installed between the stations ① and ② as shown in Figure. The work supplied to the pump is shaft work ($-\,W_s$), the negative sign is due to work added to fluid.



Frictions occurring within the pump are: -

a- Friction by fluid

b- Mechanical friction

Since the shaft work must be discounted by these frictional force (losses) to give net mechanical energy as actually delivered to the fluid by pump (W_p).

Thus, $W_p = \eta\, W_s$ where η, is the efficiency of the pump.

Thus the modified Bernoulli's equation for present of pump between the two selected points ① and ② becomes,

$$\boxed{\frac{P_1}{\rho}+\frac{u_1^2}{2\alpha_1}+g\ z_1+\eta\ Ws=\frac{P_2}{\rho}+\frac{u_2^2}{2\alpha_2}+g\ z_2+F} \text{ ---------(J/kg} \equiv \text{m}^2/\text{s}^2)$$

By dividing each term of this equation by (g), each term will have a length units, and the equation will be: -

$$\boxed{\frac{P_1}{\rho\ g}+\frac{u_1^2}{2\alpha_1 g}+z_1+\frac{\eta\ Ws}{g}=\frac{P_2}{\rho\ g}+\frac{u_2^2}{2\alpha_2 g}+z_2+h_F} \text{ ---------(m)}$$

where $h_F = F/g$ ≡ head losses due to friction.

4.8 Friction in Pipes

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of energy of fluid is lost. This loss of energy is classified on: -

Energy Losses

Major energy losses
(Skin friction)
Due to surface skin
of the pipe

Minor energy losses
(Form friction)

- Sudden expansion or contraction pipe
- Bends, valves and fittings
- An obstruction in pipe

4.8.1 Relation between Skin Friction and Wall Shear Stress

For the flow of a fluid in short length of pipe (dL) of diameter (d), the total frictional force at the wall is the product of shear stress (τ_{rx}) and the surface area of the pipe (π d dL). This frictional force causes a drop in pressure ($-\,dP_{fs}$).

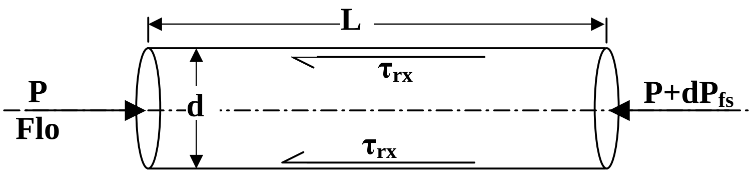
Consider a horizontal pipe as shown in Figure;

Force balance on element (dL)

$\tau(\pi\ d\ dL) = [P-(P+dP_{fs})]\ (\pi/4\ d^2)$

$$\Rightarrow \boxed{-\,dP_{fs} = 4(\tau\ dL/d) = 4\ (\tau/\rho\ u_x^2)\ (dL/d)\ \rho\ u_x^2} \text{ ------------------------(*)}$$

where, $\boxed{(\tau/\rho\ u_x^2) = \Phi = J_f = f/2 = f'/2}$



Φ(or J_f): Basic friction Factor

f: Fanning (or Darcy) friction Factor

f′: Moody friction Factor.

For incompressible fluid flowing in a pipe of constant cross-sectional area, (u) is not a function of pressure or length and equation (*) can be integrated over a length (L) to give the equation of pressure drop due to skin friction:

$\boxed{-\Delta P_{fs} = 4f\,(L/d)\,(\rho u^2/2)}$ ---------------------(Pa)

The energy lost per unit mass F_s is then given by:

$\boxed{F_s = (-\Delta P_{fs}/\rho) = 4f\,(L/d)\,(u^2/2)}$ -----------------(J/kg) or (m^2/s^2)

The head loss due to skin friction (h_{Fs}) is given by:

$\boxed{h_{Fs} = F_s/g = (-\Delta P_{fs}/\rho g) = 4f\,(L/d)\,(u^2/2g)}$ ---------------(m)

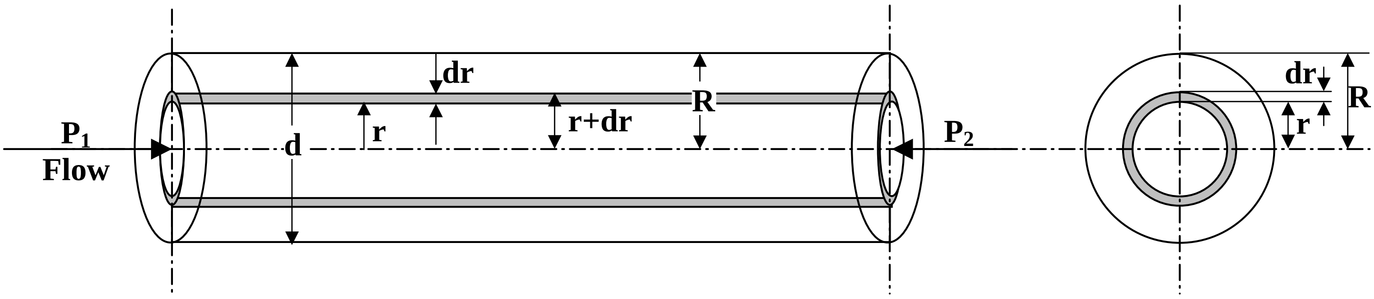
Note: -

- All the above equations could be used for laminar and turbulent flow.
- $\Delta P_{fs} = P_2 - P_1 \Rightarrow -\Delta P_{fs} = P_1 - P_2$ (+ve value)

4.8.2 Evaluation of Friction Factor in Straight Pipes

1. **Velocity distribution in laminar flow**

Consider a horizontal circular pipe of a uniform diameter in which a Newtonian, incompressible fluid flowing as shown in Figure:



Consider the cylinder of radius (r) sliding in a cylinder of radius (r+dr).
Force balance on cylinder of radius (r)

$\tau_{rx}\,(2\pi\, r\, L) = (P_1 - P_2)\,(\pi\, r^2)$

for laminar flow $\qquad \tau_{rx} = -\,\mu\,(du_x/dr)$

$\Rightarrow \quad r\,(P_1 - P_2) = -\,\mu\,(du_x/dr)\,2L \qquad \Rightarrow \quad [(P_2 - P_1)/(2L\,\mu)]\, r\, dr = du_x$

$\Rightarrow \quad [\Delta P_{fs}/(2L\,\mu)]\, r^2/2 = u_x + C$

- **Boundary Condition (1)** (for evaluation of C)

at $r = R \qquad u_x = 0 \qquad \Rightarrow \qquad C = [(\Delta P_{fs}\, R^2)/(4L\,\mu)]$

$\Rightarrow [(\Delta P_{fs}\, r^2)/(4L\,\mu)] = u_x + [(\Delta P_{fs}\, R^2)/(4L\,\mu)]$

$\Rightarrow \boxed{u_x = [(-\Delta P_{fs}\, R^2)/(4L\,\mu)][1 - (r/R)^2]}$ velocity distribution (profile) in laminar flow

- **Boundary Condition (2)** (for evaluation of u_{max})

at r = o $\qquad u_x = u_{max} \qquad \Rightarrow u_{max} = [(-\Delta P_{fs}\, R^2)/(4\,L\,\mu)]$

$\Rightarrow u_{max} = [(-\Delta P_{fs}\, d^2)/(16\,L\,\mu)]$ ----------centerline velocity in laminar flow

$\therefore \quad \boxed{u_x / u_{max} = [1-(r/R)^2]}$ ---------velocity distribution (profile)in laminar flow

2. Average (mean) linear velocity in laminar flow

$Q = u\,A$----------------------- (1)

Where, (u) is the average velocity and (A) is the cross-sectional area = (πR^2)

$dQ = u_x\, dA$ where $u_x = u_{max}[1-(r/R)^2]$, and $dA = 2\pi\, r\, dr$

$\Rightarrow dQ = u_{max}[1-(r/R)^2]\, 2\pi\, r\, dr$

$$\int_0^Q dQ = 2\pi\, u_{max} \int_0^R \left(r - \frac{r^3}{R^2}\right) dr = 2\pi\, u_{max} \left| \frac{r^2}{2} - \frac{r^4}{4R^2} \right|_0^R$$

$\Rightarrow Q = u_{max}/2\ (\pi R^2)$ ----------------------- (2)

By equalization of equations (1) and (2)

$\Rightarrow \boxed{u = u_{max}/2 = [(-\Delta P_{fs}\, R^2)/(8L\, \mu)] = [(-\Delta P_{fs}\, d^2)/(32\, L\, \mu)]}$ average velocity in laminar flow

$\therefore \boxed{-\Delta P_{fs} = (32\, L\, \mu\, u) / d^2}$ Hagen–Poiseuille equation

3. Friction factor in laminar flow

We have $-\Delta P_{fs} = 4f\,(L/d)\,(\rho u^2/2)$----------------------(3)

and also $-\Delta P_{fs} = (32\, L\, \mu\, u) / d^2$ ----------------------(4)

By equalization of these equations [i.e. eqs. (3) and (4)]

$\Rightarrow (32\, L\, \mu\, u) / d^2 = 4f\,(L/d)\,(\rho u^2/2) \Rightarrow f = 16\, \mu /(\rho\, u\, d)$

$\therefore \boxed{f = 16 / Re}$ Fanning or Darcy friction factor in laminar flow.

4. Velocity distribution in turbulent flow

The velocity, at any point in the cross-section of cylindrical pipe, in turbulent flow is proportional to the one-seventh power of the distance from the wall. This may be expressed as follows: -

$\boxed{u_x / u_{max} = [1-(r/R)]^{1/7}}$ Prandtl one-seventh law equation. velocity distribution (profile)in laminar flow

5. Average (mean) linear velocity in Turbulent flow

$Q = u\,A$---------------------- (1)

$dQ = u_x\, dA$ where $u_x = u_{max}\,[1-(r/R)]^{1/7}$, and $dA = 2\pi\, r\, dr$

$\Rightarrow dQ = u_{max}\,[1-(r/R)]^{1/7}\, 2\pi\, r\, dr$

$$\int_0^Q dQ = 2\pi\, u_{max} \int_0^R r\left(1 - \frac{r}{R}\right)^{1/7} dr$$

Let $M = (1-r/R)$ $dM = (-1/R)\, dr$

or $r = R(1-M)$ $dr = -R\, dM$

at $r = 0$ $M=1$

at $r = R$ $M=0$

Rearranging the integration

$$Q = u_{max}\, 2\pi\, R^2 \int_1^0 (1-M) M^{1/7} (-dM) = u_{max}\, 2\pi\, R^2 \int_1^0 (M^{1/7} - M^{8/7})\, dM$$

$$Q = u_{max}\, 2\pi\, R^2 \left| \frac{M^{8/7}}{8/7} - \frac{M^{15/7}}{15/7} \right|_0^1 = u_{max}\, 2\pi\, R^2 \left[\frac{7}{8} - \frac{7}{15}\right]$$

$\Rightarrow Q = 49/60\ u_{max}\ (\pi R^2)$ ----------------------- (5)

By equalization of equations (1) and (5)

$\therefore \boxed{u = 49/60\ u_{max} \approx 0.82\ u_{max}}$ ------------average velocity in turbulent flow

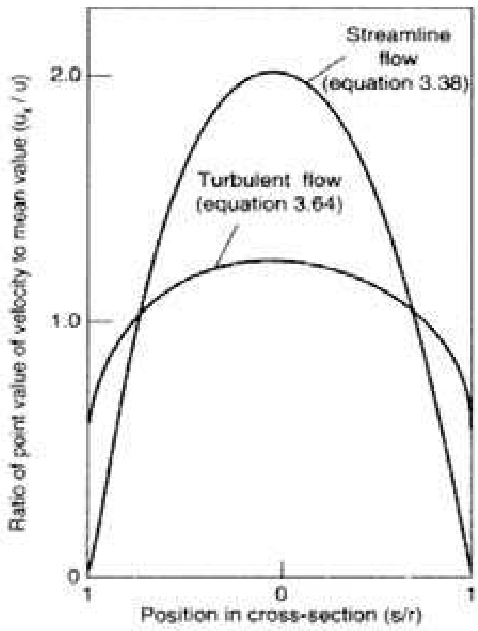


Figure of the shape of velocity profiles for streamline and turbulent flow

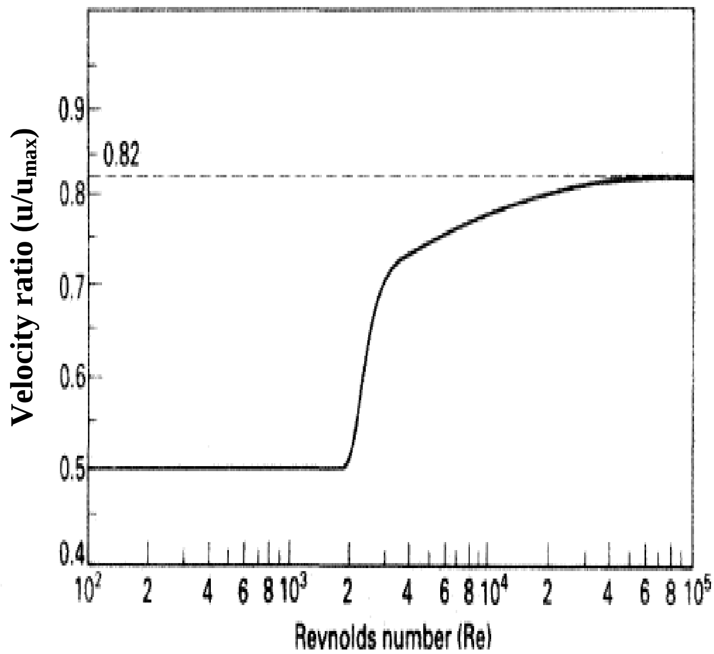


Figure of the Variation of (u/u_{max}) with Reynolds number in a pipe

6. Friction factor in Turbulent flow

A number of expressions have been proposed for calculating friction factor in terms of or function of (Re). Some of these expressions are given here: -

$$f = \frac{0.079}{Re^{0.25}} \qquad \text{for } 2{,}500 < Re < 100{,}000$$

and, $$f^{-0.5} = 4\log(Re\ f^{0.5}) - 0.4 \qquad \text{for } 2{,}500 < Re < 10{,}000{,}000$$

These equations are for ***smooth pipes*** in turbulent flow. For rough pipes, the ratio of (e/d) acts an important role in evaluating the friction factor in turbulent flow as shown in the following equation

$$(f/2)^{-0.5} = -2.5\ln\left[0.27\frac{e}{d} + 0.885\,Re^{-1}(f/2)^{-0.5}\right]$$

Table of the roughness values *e*.

Surface type	ft	mm
Planed wood or finished concrete	0.00015	0.046
Unplaned wood	0.00024	0.073
Unfinished concrete	0.00037	0.11
Cast iron	0.00056	0.17
Brick	0.00082	0.25
Riveted steel	0.0017	0.51
Corrugated metal	0.0055	1.68
Rubble	0.012	3.66

7. Graphical evaluation of friction factor

As with the results of Reynolds number the curves are in three regions (Figure 3.7 vol.I). At low values of Re (Re < 2,000), the friction factor is independent of the surface roughness, but at high values of Re (Re > 2,500) the friction factor vary with the surface roughness. At very high Re, the friction factor become independent of Re and a function of the surface roughness only. Over the transition region of Re from 2,000 to 2,500 the friction factor increased rapidly showing the great increase in friction factor as soon as turbulent motion established.

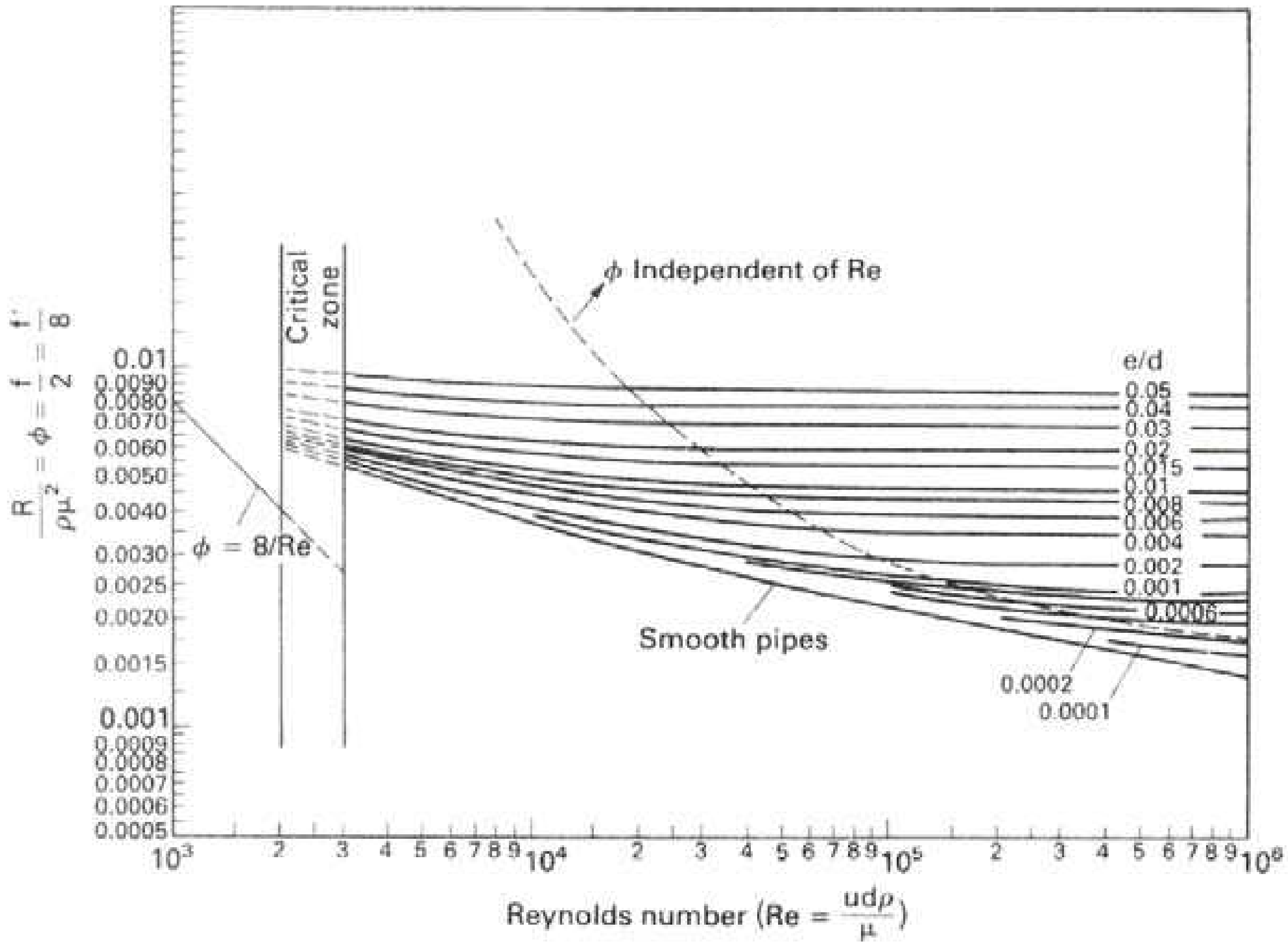
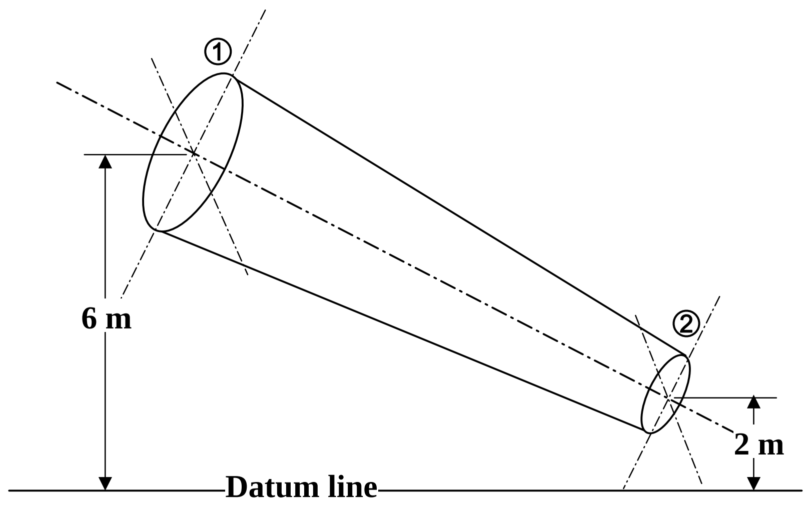


Figure (3.7) Pipe friction chart Φ versus *Re*

Example -4.3-

Water flowing through a pipe of 20 cm I.D. at section ① and 10 cm at section ②. The discharge through the pipe is 35 lit/s. The section ① is 6 m above the datum line and section ② is 2 m above it. If the pressure at section ① is 245 kPa, find the intensity of pressure at section ②. Given that ρ = 1000 kg/m^3, μ= 1.0 mPa.s.



Solution:

Q = 35 lit/s = 0.035 m^3/s

u = Q/A ⇒ u_1= (0.035 m^3/s) / (0.2^2 π/4) m^2 = 1.114 m/s

⇒ u_2= (0.035 m^3/s) / (0.1^2 π/4) m^2 = 4.456 m/s

Re = ρud / μ ⇒ Re_1= (1000 kg/m^3 x 1.114 m/s x 0.2 m) / (0.001Pa.s) = 222,800

Re = ρud / μ ⇒ Re_2= (1000 kg/m^3 x 4.456 m/s x 0.1 m) / (0.001Pa.s) = 445,600

The flow is turbulent along the tube (i.e. $\alpha_1 = \alpha_2$ =1.0)

$$\boxed{\frac{P_1}{\rho}+\frac{u_1^2}{2\alpha_1}+g\ z_1+\cancel{\eta W_s}=\frac{P_2}{\rho}+\frac{u_2^2}{2\alpha_2}+g\ z_2+\cancel{F}}$$

$$\Rightarrow P_2=\rho\left[\frac{P_1}{\rho}+g(z_1-z_2)+\left(\frac{u_1^2}{2\alpha_1}-\frac{u_2^2}{2\alpha_2}\right)\right]=253.3\text{ kPa}$$

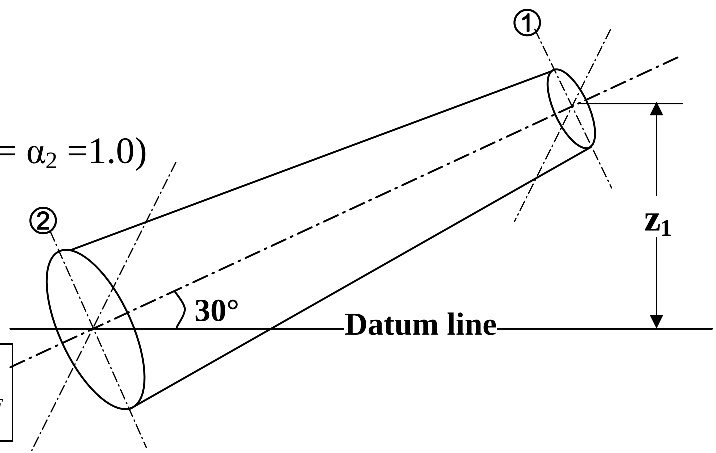
H.W.

If the pipe is smooth and its length is 20 m, find P_2. Ans. P_2 = 246.06 kPa

Example -4.4-

A conical tube of 4 m length is fixed at an inclined angle of 30° with the horizontal-line and its small diameter upwards. The velocity at smaller end is (u_1 = 5 m/s), while (u_2 = 2 m/s) at other end. The head losses in the tub is [0.35 $(u_1-u_2)^2/2g$]. Determine the pressure head at lower end if the flow takes place in down direction and the pressure head at smaller end is 2 m of liquid.

Solution:



No information of the fluid properties.

Then assume the flow is turbulent, (i.e. $\alpha_1 = \alpha_2$ =1.0)

z_1 = L sinθ

=4 sin 30

=2 m

$$\boxed{\frac{P_1}{\rho\,g}+\frac{u_1^2}{2\alpha_1 g}+z_1+\frac{\cancel{\eta W s}}{g}=\frac{P_2}{\rho\,g}+\frac{u_2^2}{2\alpha_2 g}+\cancel{z_2}+h_F}$$

$$\frac{P_2}{\rho\,g}=\frac{P_1}{\rho\,g}+z_1+\frac{u_1^2-u_2^2}{2g}-0.35\frac{(u_1-u_2)}{2g}$$

= 2.0 + 2.0 + (25– 4)/(2 x 9.81) – 0.35$(5–2)^2$ / (2 x 9.81) = 4.9 m

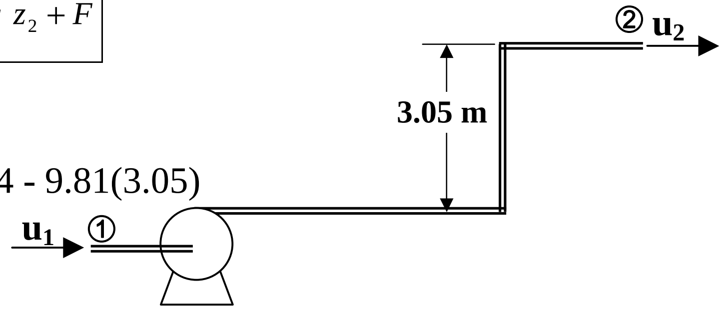
Example -4.5-

Water with density ρ = 998 kg/m^3, is flowing at steady mass flow rate through a uniform-diameter pipe. The entrance pressure of the fluid is 68.9 kPa in the pipe, which connects to a pump, which actually supplies 155.4 J/kg of fluid flowing in the pipe. The exit pipe from the pump is the same diameter as the inlet pipe. The exit section of the pipe is 3.05 m higher than the entrance, and the exit pressure is 137.8 kPa. The Reynolds number in the pipe is above 4,000 in this system. Calculate the frictional loss (F) in the pipe system.

Solution:

Setting the datum line at z_1 thus, $z_1 = 0$, $z_2 = 3.05$ m

$$\boxed{\frac{P_1}{\rho}+\cancel{\frac{u_1^2}{2\alpha_1}}+g\ z_1+\eta\ W_s=\frac{P_2}{\rho}+\cancel{\frac{u_2^2}{2\alpha_2}}+g\ z_2+F}$$



$$\Rightarrow F=\left[\frac{P_1-P_2}{\rho}+\eta\ Ws-g\ z_2\right]$$

= (68.9 - 137.8) x 1000/998 +155.4 - 9.81(3.05)

= 56.5 J/kg or m^2/s^2

Example -4.6

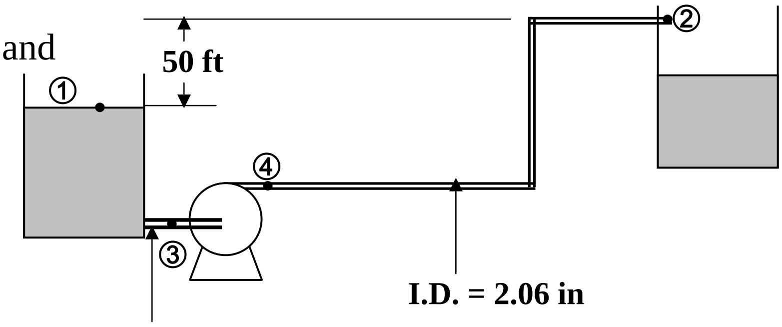
A pump draws 69.1 gal/min of liquid solution having a density of 114.8 lb/ft^3 from an open storage feed tank of large cross-sectional area through a 3.068″I.D. suction pipe. The pump discharges its flow through a 2.067″I.D. line to an open over head tank. The end of the discharge line is 50′ above the level of the liquid in the feed tank. The friction losses in the piping system are F = 10 ft lb_f/lb. what pressure must the pump develop and what is the horsepower of the pump if its efficiency is η=0.65.

Solution:

No information of the type of fluid and then its viscosity, therefore assume the flow is turbulent.

$P_1 = P_2$ = atmospheric press.

$u_1 \approx 0$ large area of the tank



$$\boxed{\cancel{\frac{P_1}{\rho}}+\cancel{\frac{u_1^2}{2\ \alpha_1\ g_c}}+\cancel{\frac{z_1 g}{g_c}}+\eta\ W_s=\cancel{\frac{P_2}{\rho}}+\frac{u_2^2}{2\ \alpha_2\ g_c}+\frac{z_2 g}{g_c}+F}$$

$$\Rightarrow \eta\ Ws=\left[\frac{g\ z_2}{g_c}+\frac{u_2^2}{2\ g_c}+F\right]$$

Q = 69.1 gal/min (ft^3/7.48 gal)(min/60 s) = 0.154 ft^3/s

A_3 (area of suction line) = $\pi/4$ (3.068 in)^2 (ft/12 in)^2 = 0.0513 ft^2

$A_4 = A_2$ (area of discharge line) = $\pi/4$ (2.067 in)^2 (ft/12 in)^2 = 0.0235 ft^2

$u_2 = Q / A_2$ = (0.154 ft^3/s) / (0.0235 ft^2) = 6.55 ft/s

$u_3 = Q / A_3$ = (0.154 ft^3/s) / (0.0513 ft^2) = 3.0 ft/s

$$\Rightarrow\ \eta\ Ws=\frac{32.174\ ft/s^2\times 50\ \ ft}{32.174\ lb\ \ ft/lb_f\ \ s^2}+\frac{(6.55\ \ ft/s)^2}{2\times 32.174 lb\ \ ft/lb_f\ \ s^2}+10\ ft\ lb_f\ /lb = 60.655\ \text{ft}\ lb_f/\text{lb}$$

Ws = ηWs/η = 60.655/0.65 = 93.3 lb_f ft/lb

Mass flow rate $\dot{m}=Q\rho$ = 0.1539 ft^3/s (114.8 lb/ft^3) = 17.65 lb/s

Power required for pump = $\dot{m}\,Ws$ = 17.65 lb/s (93.3 ft lb_f/lb)(hp/550 ft lb_f/s)

= 3.0 hP

To calculate the pressure that must be developed by the pump, Energy Balance equation must be applied over the pump itself (points ③ and ④)

$u_4 = u_2$ = 6.55 ft / s and u_3 = 3 ft / s

$$\boxed{\frac{P_3}{\rho}+\frac{u_3^2}{2\,\alpha_1\,g_c}+\cancel{\frac{z_3 g}{g_c}}+\eta\,W_s=\frac{P_4}{\rho}+\frac{u_4^2}{2\,\alpha_2\,g_c}+\cancel{\frac{z_4 g}{g_c}}+\cancel{F}}$$

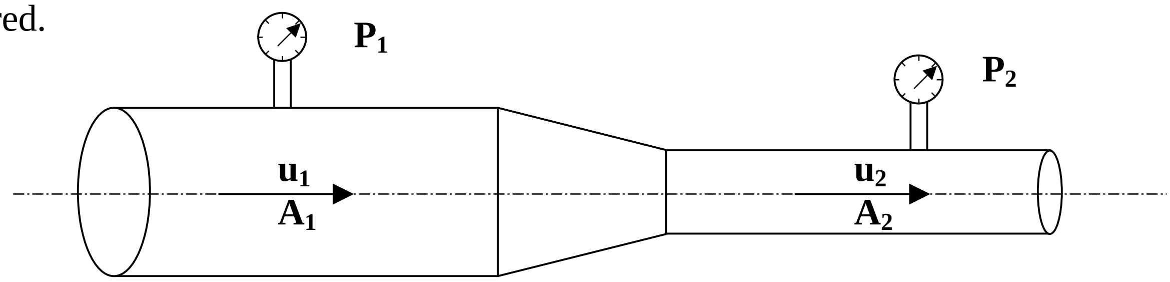
$$\Rightarrow \frac{P_4-P_3}{\rho}=\eta\ Ws+\frac{u_3^2-u_4^2}{2\,g_c}\ =\ 60.655+(-\,0.527)=60.13\text{ ft lb}_f\text{ / lb}$$

$\Rightarrow \Delta P = 60.13$ ft lb_f/lb (114.8) lb/ft^3 $= 69.03$ lb_f/ft^2
$= 47.94$ psi
$= 3.26$ bar

Example -4.7-

A liquid with a constant density (ρ) is flowing at an unknown velocity (u_1) through a horizontal pipe of cross-sectional area (A_1) at a pressure (P_i), and then it passes to a section of the pipe in which the area is reduced gradually to (A_2) and the pressure (P_2). Assume no friction losses, find the velocities (u_1) and (u_2) if the pressure difference (P_1-P_2) is measured.

Solution:



From continuity equation $\dot{m}=\dot{m}_1=\dot{m}_2 \ \Rightarrow\ \rho\, Q = \rho_1\, Q_1 = \rho_2\, Q_2$

And for constant density $\Rightarrow Q = Q_1 = Q_2 \Rightarrow\ u\, A = u_1\, A_1 = u_2\, A_2$

$$\Rightarrow u_2 = u_1\, A_1/A_2$$

$$\boxed{\frac{P_1}{\rho}+\frac{u_1^2}{2\alpha_1}+\cancel{g\,z_1}+\cancel{\eta\,W_s}=\frac{P_2}{\rho}+\frac{u_2^2}{2\alpha_2}+\cancel{g\,z_2}+\cancel{F}}$$

Assume the flow is turbulent ($\alpha_1 = \alpha_2$)

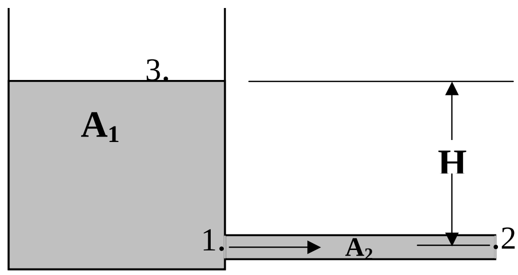
$$\Rightarrow \frac{P_1-P_2}{\rho}=\frac{u_2^2-u_1^2}{2}=\frac{u_1^2\left[\left(\frac{A_1}{A_2}\right)^2-1\right]}{2}\Rightarrow\quad u_1=\sqrt{\left(\frac{P_1-P_2}{\rho}\right)\frac{2}{\left(\frac{A_1}{A_2}\right)^2-1}}\ ,\ u_2=\sqrt{\left(\frac{P_1-P_2}{\rho}\right)\frac{2}{1-\left(\frac{A_1}{A_2}\right)^2}}$$

Example -4.8-

A nozzle of cross-sectional area (A_2) is discharging to the atmosphere and is located in the side of a large tank, in which the open surface of liquid in the tank is (H) above the centerline of the nozzle. Calculate the velocity (u_2) in the nozzle and the volumetric rate of discharge if no friction losses are assumed and the flow is turbulent.

Solution:

Since A_1 is very large compared to A_2 ($\Rightarrow u_1 \approx 0$).
The pressure P_1 is greater than atmosphere pressure by the head of fluid H.
The pressure P_2 which is at nozzle exit, is at atmospheric pressure .



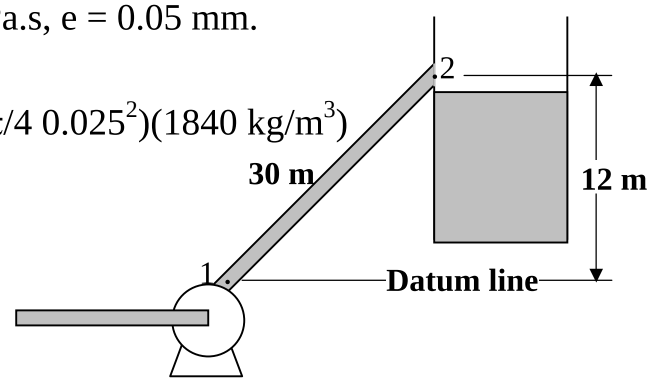
$$\boxed{\frac{P_1}{\rho}+\cancel{\frac{u_1^2}{2\alpha_1}}+\cancel{g\,z_1}+\cancel{\eta W_s}=\frac{P_2}{\rho}+\frac{u_2^2}{2\alpha_2}+\cancel{g\,z_2}+\cancel{F}}\qquad \Rightarrow u_2=\sqrt{\frac{2(P_1-P_2)}{\rho}}$$

but, $(P_1\text{-}P_3) = H\rho g \Rightarrow u_2=\sqrt{2Hg}\Rightarrow Q=A_2\sqrt{2Hg}$

or $$\boxed{\frac{P_2}{\rho}+\frac{u_2^2}{2\alpha_2}+\cancel{g\,z_2}+\cancel{\eta W_s}=\frac{P_3}{\rho}+\cancel{\frac{u_3^2}{2\alpha_3}}+g\,z_3+\cancel{F}\Rightarrow\quad u_2=\sqrt{2Hg}}$$

Example -4.9-

98% H_2So_4 is pumped at 1.25 kg/s through a 25 mm inside diameter pipe, 30 m long, to a reservoir 12 m higher than the feed point. Calculate the pressure drop in the pipeline. Take that ρ = 1840 kg/m^3, μ= 25 mPa.s, e = 0.05 mm.



Solution:

$\dot m = Q\rho = uA\rho \Rightarrow u=\dfrac{\dot m}{\rho\,A}$ ⇒u =(1.25 kg/s) / (π/4 0.025^2)(1840 kg/m^3)

⇒u = 1.38 m/s

Re = (1840 x 1.38 x 0.025)/0.025 = 2546

e/d = 0.05 x 10^{-3} /0.025 = 0.002

From Figure (3.7)- Vol.I

Φ = 0.006 ⇒f = 2 Φ = 0.012

$F_s = (-\Delta P_{fs}/\rho) = 4f\,(L/d)\,(u^2/2) = 4(0.012)\,(30/0.025)(1.38^2/2) = 54.84\ m^2/s^2$

$$\boxed{\frac{P_1}{\rho}+\frac{u_1^2}{2\alpha_1}+\cancel{g\,z_1}+\cancel{\eta W_s}=\frac{P_2}{\rho}+\frac{u_2^2}{2\alpha_2}+g\,z_2+F}$$

If the kinetic energy at point 2 is neglected

$$\frac{P_1-P_2}{\rho}=g\,z_2+F+\frac{u_1^2}{2\,\alpha_1}$$

$F = F_s = 54.84\ m^2/s^2$

$gz_2 = 9.81(12) = 117.72\ m^2/s^2$

$u_1^2/2\alpha_1 = 1.38^2/2 = 0.952\ m^2/s^2$

⇒$(P_1\text{-}P_2)$ = 1840 [117.72+54.84-0.952] = 315.762 kPa

4.8.3 <u>Figure (3.8)- Vol.I</u>

For turbulent flow, it is not possible to determine directly the fluid flow rate through a pipe from Figure (3.7) –Vol.I. For a known pressure drop, the method of solution to this problem is as follows:

$\Phi = J_f = \tau/\rho u^2 \quad \Rightarrow \tau = \Phi\,\rho u^2$ -----------------------------(1)

But from force balance for fluid flow through horizontal pipe

$\tau\,\pi\,dL = -\Delta P_{fs}\,(\pi/4\ d^2)$

$\Rightarrow \tau = -\Delta P_{fs}/L\ (d/4)$--(2)

By equalization of equations (1) and (2)

$\Rightarrow \Phi\rho u^2 = -\Delta P_{fs}/L\ (d/4)$ -------------- x $\rho d^2/\mu^2$

$\Rightarrow \Phi \rho^2 u^2 d^2/\mu^2 = -\Delta P_{fs}/L\,(d^3\rho/4\,\mu^2)$

or $\boxed{\Phi\, Re^2 = -\Delta P_{fs}/L\,(\rho\, d^3/4\,\mu^2)}$ ------------------------(3)

This equation dose not contains the mean linear velocity (u) of fluid. This can be determine through using Figure (3.8)- Vol.I as follows:

1- Calculate the value of ΦRe^2 from equation (3) of (ΔP_{fs}, ρ, d, L, and μ).

2- Read the corresponding value of Re from Figure (3.8) for a known value of (e/d).

3- Calculate U from the extracted value of Re.

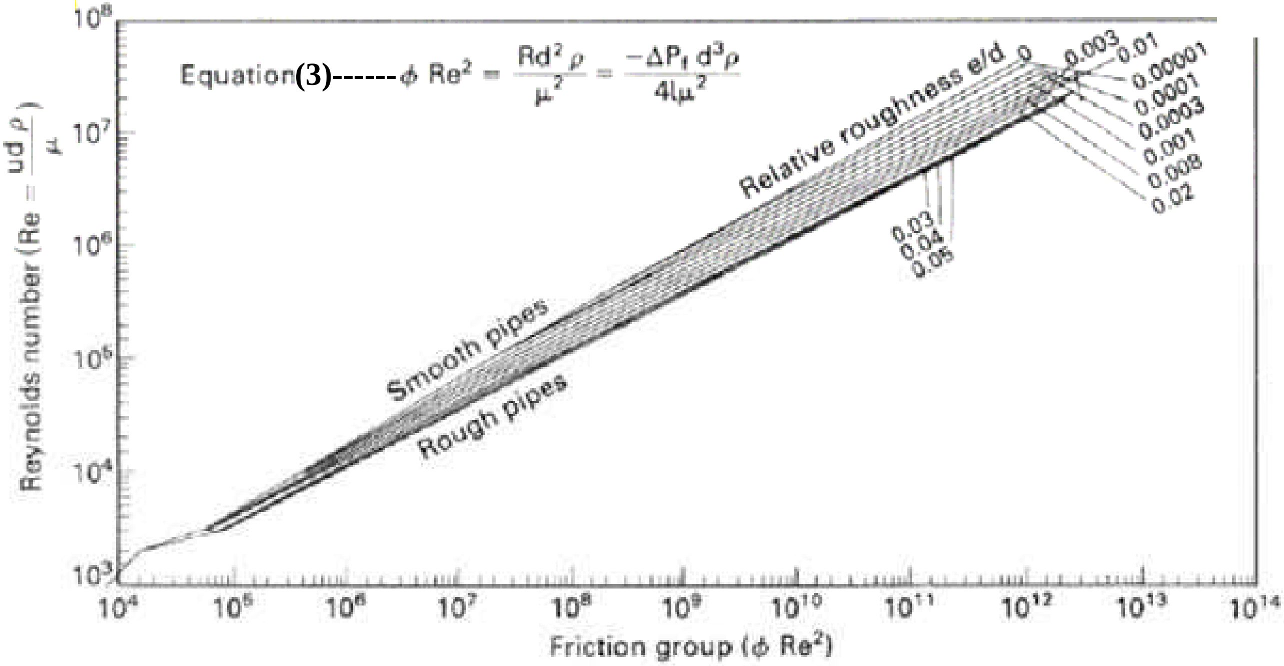


Figure (3.8) Pipe friction chart ΦRe2 versus Re for various values of e/d

Example -4.10-

Calculate the pressure drop in Pa for a fluid flowing through a 30.48 m long commercial steel pipe of I.D. 0.0526 m and a pipe roughness (e = 0.045 mm). The fluid flows at steady transfer rate of 9.085 m^3/h. Take that $\rho = 1200$ kg/m^3, μ= 0.01 Pa.s.

Solution:

$Q = 9.085\ m^3/h \times h/3600s = 2.524 \times 10^{-3}\ m^3/s$

$u = Q/A = (2.524 \times 10^{-3}\ m^3/s) / (\pi/4 \times 0.0526^2) = 1.16$ m/s

$Re = (1200 \times 1.16 \times 0.0526)/0.01 = 7322$

$e/d = 0.000045/0.0526 = 0.000856$

Figure (3.7) $\Phi = 0.0042 \Rightarrow f = 2\,\Phi = 0.084$

$$\frac{\Delta P}{\rho} + \cancel{g\Delta z} + \cancel{\frac{\Delta u^2}{2\alpha}} - \cancel{\eta\, Ws} + F = 0$$

$\Rightarrow -\Delta P_{fs} = \rho\, F_s = 4\, f\, (L/d)(\rho u^2/2) = 4\,(0.0084)\,(30.48/0.0526)\,(1200 \times 1.16^2/2)$
$= 15719$ Pa

Example -4.11-

Repeat the previous example with the following conditions, the volumetric flow rate (i.e. the velocity) is unknown and the pressure drop in the pipe is 15.72 kPa.

Solution:

$\Phi Re^2 = (-\Delta P_{fs}/L)(\rho d^3/4\mu^2) = [(15720)/(30.48)][(1200)(0.0526)^3/(4)(0.01)^2] = 2.252 \times 10^5$

$e/d = 0.000856 \Rightarrow$ Figure (3.8) $Re = 7200 \Rightarrow u = 7200(0.01)/(1200 \times 0.0526) = 1.141 m/s$

Example -4.12-

Repeat the previous example with the following conditions, the diameter of the pipe is unknown and the pressure drop in the pipe is 15.72 kPa and the velocity of the liquid is 1.15 m/s. Estimate the diameter of the pipe.

Solution:

$-\Delta P_{fs} = \rho\, F_s = 4\, f\, (L/d)(\, \rho u^2/2) \Rightarrow d = (4\, \rho\, f\, L\, u^2/2)/\, -\Delta P_{fs}$

$\Rightarrow d = 6.154\, f$ ----------------------------------(1)

$Re = \rho u d/\mu = 138000\, d$ ----------------------(2)

$e/d = 0.000045/d$ -----------------------------(3)

	Φ Figure (3.7)	*f*=2Φ *f*	Eq.(1) d	Eq.(2) Re	Eq.(3) e/d
Assumed →		$\boxed{0.001}$	0.006154	849	0.0073
	0.01	0.02	0.123	16985	0.00036
	0.0037	0.0074	0.045	6284	0.00099
	0.0045	0.009	0.0554	7643	0.0008
	0.0043	0.0086	0.0529	7303	0.00085
⇒d = 0.0529 m	0.0043	0.0086			

Example -4.13-

Sulfuric acid is pumped at 3 kg/s through a 60 m length of smooth 25 mm pipe. Calculate the drop in pressure. If the pressure drop falls to one half, what will new flow rate be? Take that $\rho = 1840\ kg/m^3$, $\mu = 25$ mPa.s.

Solution:

$$u = \frac{Q}{A} = \frac{\dot{m}}{\rho\, A} = \frac{3\, kg/s}{(1840\, kg/m^3)(\pi/4 \times 0.025^2)m^2} = 3.32\, m/s$$

$Re = (1840 \times 3.32 \times 0.025)/0.025 = 6111$

Figure (3.7) for smooth pipe $\Phi = 0.0043 \Rightarrow f = 0.0086$

$-\Delta P_{fs} = \rho\, F_s = 4\, f\, (L/d)(\, \rho u^2/2) = 4(0.0086)(60/0.025)(1840 \times 3.32^2)/2 = 837.209$ kPa

The pressure drop falls to one half (i.e. $-\Delta P_{fs}$ = 837.209 kPa/2 = 418.604kPa

$\Phi Re^2 = (-\Delta P_{fs}/L)(\rho d^3/4\mu^2)$=[(418604)/(60)][(1840)(0.025)^3/(4)(0.025)^2]= 8.02 x 10^4

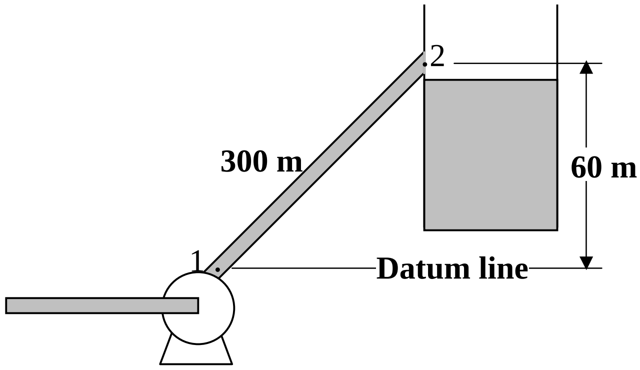
From Figure (3.8) for smooth pipe Re = 3800 $\Rightarrow$u = 2.06m/s

$\dot{m} = \rho\, u\, A =$ 1.865 kg/s.

<u>Example -4.14-</u>

A pump developing a pressure of 800 kPa is used to pump water through a 150 mm pipe, 300 m long to a reservoir 60 m higher. With the valves fully open, the flow rate obtained is 0.05 m^3/s. As a result of corrosion and scalling the effective absolute roughness of the pipe surface increases by a factor of 10 by what percentage is the flow rate reduced. μ= 1 mPa.s

<u>Solution:</u>



The total head of pump developing =($\Delta P/\rho g$)
= 800,000/(1000 x 9.81)= 81.55 mH_2o

The head of potential energy = 60 m

Neglecting the kinetic energy losses (same diameter)

$$\boxed{\frac{\Delta P}{\rho} + g\Delta z + \cancel{\frac{\Delta u^2}{2\alpha}} - \cancel{\eta}\, Ws + F = 0}$$

$\Rightarrow \Delta P/\rho g + \Delta z + h_F = 0$

$\Rightarrow$ h_F =– $\Delta P/\rho g$ – Δz = 81.55 – 60 = 21.55 m

u = Q/A = (0.05 m^3/s)/(π/4 0.15^2) =2.83 m/s

$h_{Fs} = (-\Delta P_{fs}/\rho g) = 4f\,(L/d)\,(u^2/2g)$

$\Rightarrow f = h_{Fs}\, d\, 2g/(4Lu^2)$ = (21.55) (0.15)(9.81)/(2 x 300 x 2.83^2) = 0.0066

Φ = 0.0033, Re = (1000 x 2.83 x 0.15)/0.001 = 4.23 x10^5

From Figure (3.7) e/d = 0.003

Due to corrosion and scalling the roughness increase by factor 10

i.e. $(e/d)_{new}$ = 10 $(e/d)_{old}$ = 0.03

The pump head that supplied is the same

$(-\Delta P_{fs}) = h_{Fs}\, \rho g$ = 21.55 (1000) 9.81 = 211.41 kPa

$\Phi Re^2 = (-\Delta P_{fs}/L)(\rho d^3/4\mu^2)$ = [(211410)/(300)][(1000)(0.15)^3/(4)(0.01)^2] = 6 x 10^8

From Figure (3.8) Re = 2.95 x10^5 $\Rightarrow$u = 1.97m/s

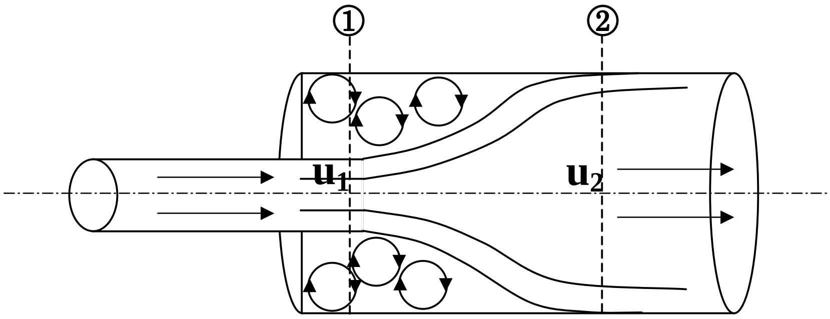
The percentage reduced in flow rate = (2.83 - 1.97)/ 2.83 x 100 % = 30.1 %.

4.8.4 <u>Form Friction</u>

Skin friction loss in flow straight pipe is calculated by using the Fanning friction factor (f). However, if the velocity of the fluid is changed in *direction* <u>or</u> *magnitude*, additional friction losses occur. This results from additional turbulence, which develops because of vertices and other factors.

1- Sudden Expansion (Enlargement) Losses

If the cross section of a pipe enlarges gradually, very little or no extra losses are incurred. If the change is sudden, as that in Figure, it results in additional losses due to eddies formed by the jet expanding in the enlarged suction. This friction loss can be calculated by the following for laminar or turbulent flow in both sections, as:



Continuity equation $u_1A_1 = u_2A_2 \quad \Rightarrow u_2 = u_1(A_1/A_2)$

Momentum balance $F_e = \dfrac{u_1^2}{2\alpha_1} - \dfrac{u_2^2}{2\alpha_2} + u_2^2 - u_1u_2$

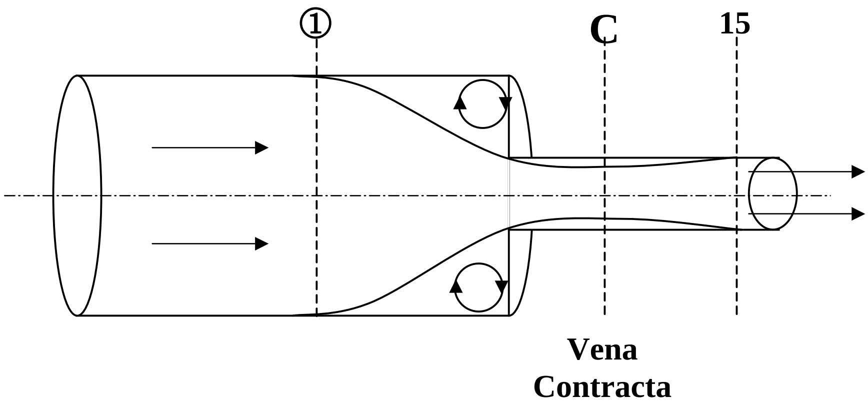
For fully turbulent flow in both sections

$$F_e = \frac{(u_1 - u_2)^2}{2} = \frac{u_1^2}{2}\left[1 - \left(\frac{A_1}{A_2}\right)\right]^2$$

$$\boxed{\therefore F_e = K_e \frac{u_1^2}{2}}; \text{ where } \boxed{K_e = \left[1 - \left(\frac{A_1}{A_2}\right)\right]^2}$$

2- Sudden Contraction Losses

The effective area for the flow gradually decreases as the sudden contraction is approached and then continues to decrease, for a short distance, to what is known as the "Vena Contacta". After the Vena Contracta the flow area gradually approaches that of the smaller pipe, as shown in Figure. When the cross section of the pipe is suddenly reduced, the stream cannot follow around the sharp corner, and additional loses due to eddies occur.



$$\boxed{F_c = K_c \frac{u_2^2}{2}}; \text{ where } \boxed{K_c = 0.55\left[1 - \frac{A_2}{A_1}\right]}$$

3- Losses in Fittings and Valves

Pipe fittings and valves also disturb the normal flow lines in a pipe and cause additional friction losses. In a short pipe with many fittings, the friction losses from these fittings could be greater than in the straight pipe. The friction loss for fittings and valves is:

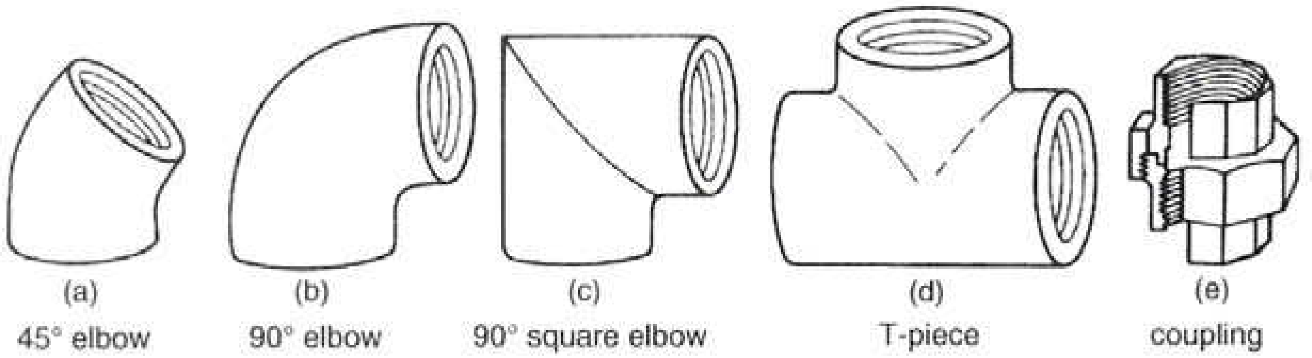
$\boxed{F_f = K_f \frac{u_2}{2}}$; where K_f as in table below.

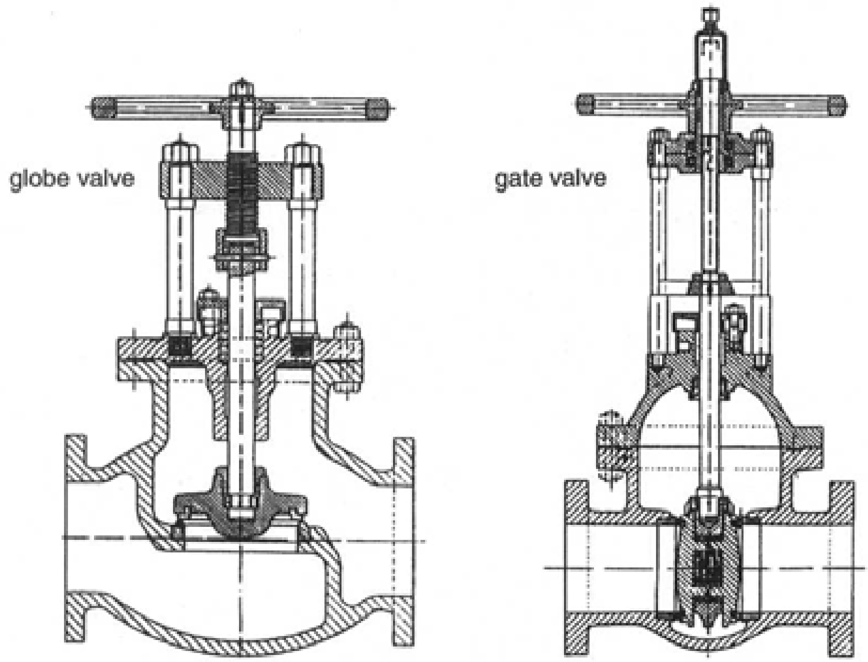
$\boxed{F_f = 4f \frac{Le}{d}\frac{u_2}{2}}$; where $\frac{Le}{d}$ as in table below.

Table of the Friction losses in pipe fittings

Fittings	K_f	Le/d
45° elbows (a)*	15	0.3
90° elbows (standard radius) (b)	30-40	0.6-0.8
90° square elbows (c)	60	1.2
Entry from leg of T-piece (d)	60	1.2
Entry into leg of T-piece (d)	90	1.8
Unions and couplings (e)	Very small	Very small
Globe valves fully open	60-300	1.2-6.0
Gate valves: fully open	7	0.15
3/4 open	40	1
1/2 open	200	4
1/4 open	800	16

*** See Figure below**





Figures of standard pipe fittings and standard valves

4.8.5 Total Friction Losses

The frictional losses from the friction in the straight pipe (skin friction), enlargement losses, contraction losses, and losses in fittings and valves are all incorporated in F term in mechanical energy balance equation (modified Bernoulli's equation), so that,

$$\boxed{F = 4f\frac{L}{d}\frac{u^2}{2} + K_e\frac{u_1^2}{2} + K_c\frac{u_2^2}{2} + K_f\frac{u^2}{2}}$$

If all the velocity u, u_1, and u_2 are the same, then this equation becomes, for this special case;

$$\boxed{F = \left[4f\frac{L}{d} + K_e + K_c + K_f\right]\frac{u^2}{2}}$$

If equivalent length of the straight pipe for the losses in fittings and/or valves, then this equation becomes;

$$\boxed{F = \left[4f\left(\frac{L}{d} + \sum\frac{Le}{d}\right) + K_e + K_c\right]\frac{u^2}{2}}$$

Example -4.15-

630 cm^3/s water at 320 K is pumped in a 40 mm I.D. pipe through a length of 150 m in horizontal direction and up through a vertical height of 10 m. In the pipe there is a control valve which may be taken as equivalent to 200 pipe diameters and also other fittings equivalent to 60 pipe diameters. Also other pipe fittings equivalent to 60 pipe diameters. Also in the line there is a heat exchanger across which there is a loss in head of 1.5 m H_2o. If the main pipe has a roughness of 0.0002 m, what power must supplied to the pump if η = 60%, μ= 0.65 mPa.s.

Solution:

Q = 630 cc/s (m/100 cm)^3 = 6.3×10^{-4} m^3/s

u = (6.3×10^{-4} m^3/s)/(π/4 0.04^2) = 0.5 m/s

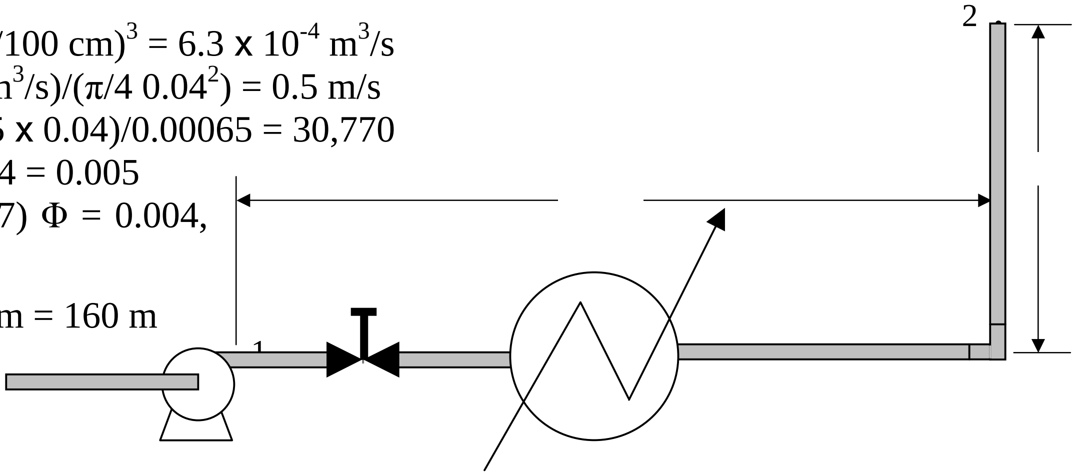
Re = (1000 x 0.5 x 0.04)/0.00065 = 30,770

e/d = 0.0002/0.04 = 0.005

From Figure (3.7) Φ = 0.004,

$\Rightarrow f$ = 0.008

L = 150 m + 10 m = 160 m



$$\frac{\Delta P}{\rho g} + \Delta z + \cancel{\frac{\Delta u^2}{2\alpha g}} - \cancel{\frac{\eta Ws}{g}} + h_F + (h)_{H.Ex.} = 0$$

$$h_F = \left[4f\left(\frac{L}{d} + \sum\frac{Le}{d}\right)\right]\frac{u^2}{2g} = 4\,(0.008)\,(160/0.04 + 200 + 60) \times 0.5^2/(2 \times 9.81) = 1.74 \text{ m}$$

$\Rightarrow$(-ΔP/ρg) = Δh =10 + 1.74 + 1.5 = 13.24 m

⇒ The head required (that must be supplied to water by the pump) is Δh =13.24 m and the power required for the water is $\eta Ws = Q\,\Delta P = Q\,(\Delta h\,\rho g)$

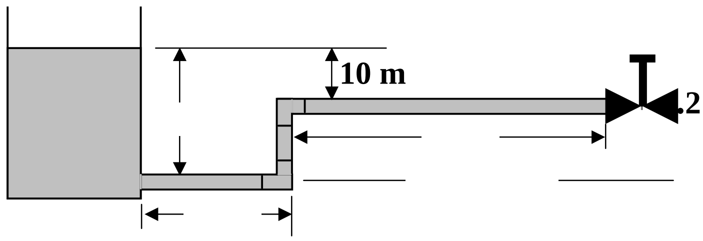
⇒ $\eta Ws = 6.3 \times 10^{-4}\ m^3/s\ (13.24 \times 1000 \times 9.81) = 81.8$ (N.m/s ≡ J/s ≡ W)
The power required for the pump is $(Ws) = \eta Ws / \eta = 81.8 / 0.6 = 136.4$ W.

Example -4.16-

Water in a tank flows through an outlet 25 m below the water level into a 0.15 m I.D. horizontal pipe 30 m long, with 90° elbow at the end leading to vertical pipe of the same diameter 15 m long. This is connected to a second 90° elbow which leads to a horizontal pipe of the same diameter, 60 m long, containing a fully open globe valve and discharge to atmosphere 10 m below the level of the water in the tank. Calculate *the initial rate*. Take that μ= 1 mPa.s, e/d =0.01

Solution:

L = 30 + 15 + 60 = 105 m



$$\boxed{\frac{P_1}{\rho g}+\frac{u_1^2}{2\alpha_1 g}+z_1+\frac{\eta Ws}{g}=\frac{P_2}{\rho g}+\frac{u_2^2}{2\alpha_2 g}+z_2+h_F}$$

$$\Rightarrow\ z_1-z_2=\frac{u_2^2}{2g\alpha_2}+h_F$$

$$\boxed{h_F=\left[4f\left(\frac{L}{d}+\sum\frac{Le}{d}\right)\right]\frac{u^2}{2g}}$$

$= 4f\,[(105/0.15)+2(40)+250]\ u_2^2/(2 \times 9.81)$

Assumed

$= 210\,f\,u_2^2$

Assume turbulent flow (α_2 =1.0)

⇒ $(25\text{-}15) = u_2^2/(2 \times 9.81) + 210\,f\,u_2^2$ $\Rightarrow u_2=\sqrt{\dfrac{10}{0.05+210f}}$ ---(*)

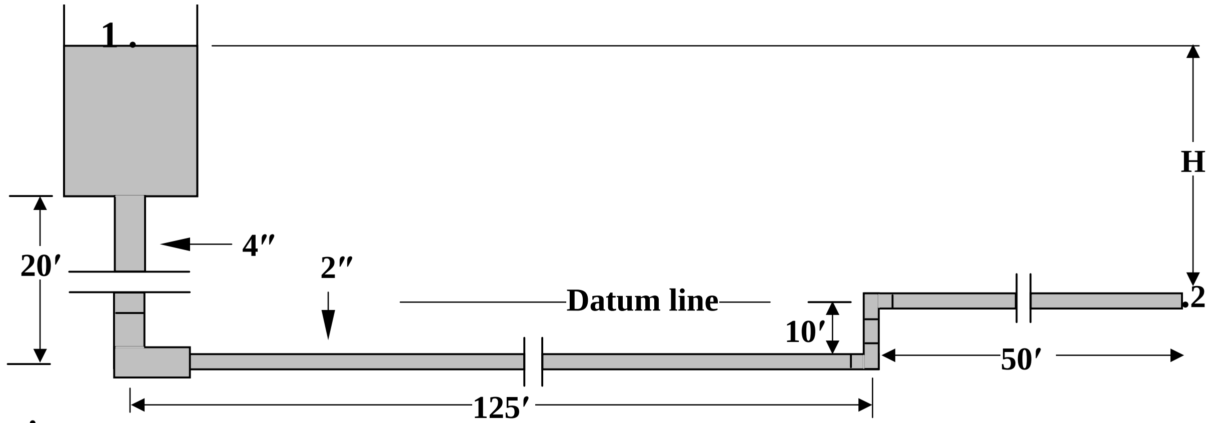
This equation solved by trial and error

	Eq.(*)		Figure (3.7)
f	u_2	Re	Φ
$\boxed{0.01}$	2.16	3.235×10^5	0.0046
0.0092	2.246	3.37×10^5	0.0046

⇒ $u_2(t=0) = 2.246$ m/s, Re = 3.37 x105 (turbulent) ⇒Q=0.04 m^3/s ; m = 40 kg/s

Example -4.17-

An elevated storage tank contains water at 82.2°C as shown in Figure below. It is desired to have a discharge rate at point 2 of 0.223 ft^3/s. What must be the height H in ft of the surface of the water in the tank relative to discharge point? The pipe is schedule 40, e = 1.5 x10^{-4} ft. Take that ρ = 60.52 lb/ft^3, μ= 2.33 x10^{-4} lb/ft.s.



Solution:

$$\boxed{\cancel{\frac{P_1}{\rho}} + \cancel{\frac{u_1^2}{2\alpha_1 g_c}} + \frac{g\, z_1}{g_c} + \cancel{\eta\, W_s} = \cancel{\frac{P_2}{\rho}} + \frac{u_2^2}{2\alpha_2 g_c} + \cancel{\frac{g\, z_2}{g_c}} + F}$$

$\Rightarrow \dfrac{g}{g_c} z_1 = \dfrac{u_2^2}{2\alpha_2 g_c} + F$, where z_1 = H

for schedule 40

$d_{4''}$ = 4.026/12 = 0.3353 ft, $A_{4''}$ = 0.0884 ft^2,

$d_{2''}$ = 2.067/12 = 0.1722 ft, $A_{4''}$ = 0.0233 ft^2,

$u_{4''}$ = (0.223 ft^3/s) / (0.0884 ft^2) = 2.523 ft, $u_{2''}$ = (0.223 ft^3/s) / (0.0233 ft^2) = 9.57 ft,

The F-term for friction losses in the system includes the followings:

1- Contraction losses at tank exit. 2-Friction in 4″ straight pipe.

3- Friction in 4″ elbow. 4-Contraction losses in 4″ to 2″ pipe.

5- Friction in 2″ straight pipe. 6-Friction in the two 2″ elbows.

1- Contraction losses at tank exit. (let tank area =A_1, 4″ pipe area = A_3)

$\boxed{F_c = K_c \dfrac{u_2^2}{2}}$; where $\boxed{K_c = 0.55\left[1 - \dfrac{A_2}{A_1}\right]}$ ≈ 0.5

$\Rightarrow$ F_c = 0.55 (2.523^2 /2 x 32.174) = 0.054 ft.lb$_f$/lb.

2- Friction in 4″ straight pipe.

Re = (60.52 x 2.523 x 0.3353)/ 2.33 x10^{-4} = 2.193 x10^5

e/d = 0.000448 $\Rightarrow$ Figure (3.7) f = 0.0047

$F_{Fs} = 4 f \dfrac{L}{d}\dfrac{u^2}{2g_c}$ =4 (0.0047) (20/0.3353) x 2.523^2/(2 x 32.174) = 0.111 ft.lb$_f$/lb.

3- Friction in 4″ elbow.

$\boxed{F_f = K_f \dfrac{u_2}{2}}$; where K_f = 0.75 $\Rightarrow$ F_f = 0.074 ft.lb$_f$/lb.

4- Contraction losses in 4″ to 2″ pipe.

$\boxed{F_c = K_c \frac{u_2^2}{2}}$; where $\boxed{K_c = 0.55\left[1 - \frac{A_2}{A_1}\right]}$ = 0.55 (1- 0.0233/0.0884) = 0.405

$\Rightarrow$ F_c = 0.405 (9.57^2 /2 x 32.174) = 0.575 ft.lb$_f$/lb.

5- Friction in 2″ straight pipe.

Re = (60.52 x 9.57 x 0.1722)/ 2.33 x10^{-4} = 4.28 x10^5

e/d = 0.00087 $\Rightarrow$ Figure (3.7) f = 0.0048

$F_{Fs} = 4f\frac{L}{d}\frac{u^2}{2g_c}$ =4 (0.0048) (185/0.3353) x 9.57^2/(2 x 32.174) = 29.4 ft.lb$_f$/lb.

6- Friction in the two 2″ elbow.

$\boxed{F_f = 2(K_f \frac{u_2}{2})}$; where K_f = 0.75 $\Rightarrow$ F_f = 2.136 ft.lb$_f$/lb.

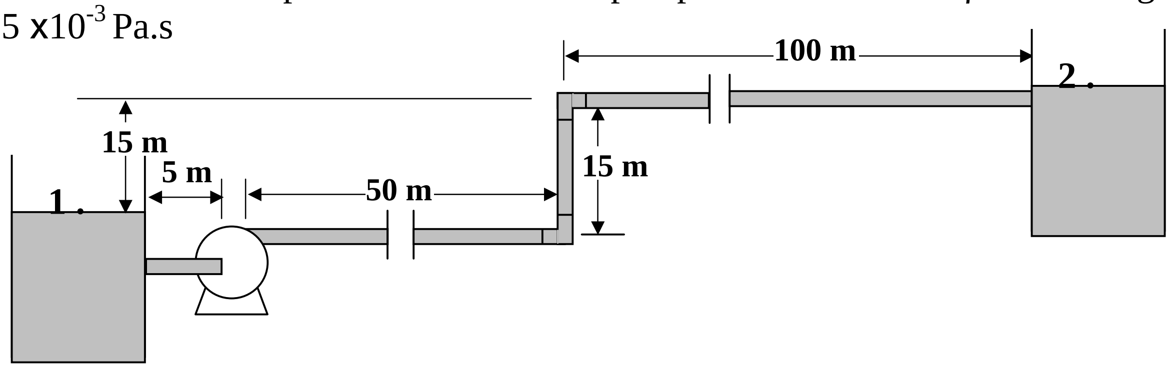
F (total frictional losses) = 0.054 + 0.111 + 0.575 + 29.4 + 2.136 =32.35 ft.lb$_f$/lb

$\Rightarrow$ H g/g$_c$ = (9.57^2 /2 x 32.174) + 32.35= 33.77 ft.lb$_f$/lb

H = 33.77 ft ≈ 10.3m (height of water level above the discharge outlet)

Example -4.18-

Water at 20°C being pumped from a tank to an elevated tank at the rate of 0.005 m^3/s. All the piping in the Figure below is 4″ Schedule 40 pipe. The pump has an efficiency of η= 0.65. calculate the kW power needed for the pump. e = 4.6 x10^{-5} m ρ = 998.2 kg/m^3, μ= 1.005 x10^{-3} Pa.s



Solution:

For 4″ Schedule 40 pipe d = 0.1023 m, A = 8.219 x10^{-3} m^2

u = Q/A = (5 x10^{-3} m^3/s)/ 8.219 x10^{-3} m^2 = 0.6083 m/s

$\boxed{\cancel{\frac{\Delta P}{\rho}} + g\Delta z + \cancel{\frac{\Delta u^2}{2\alpha}} - \eta\, Ws + F = 0}$ $\Rightarrow$ ηWs = F +gΔz

The F-term for friction losses in the system includes the followings:

1- Contraction losses at tank exit. 2-Friction loss in straight pipe.

3- Friction in the two elbows. 4- Expansion loss at the tank entrance.

1- Contraction losses at tank exit.

$\boxed{F_c = K_c \frac{u_2^2}{2}}$; where $\boxed{K_c = 0.55\left[1 - \frac{A_2}{A_1}\right]}$ ≈ 0.5

$\Rightarrow$ F_c = 0.55 (0.6083^2 /2) = 0.102 J/kg or m^2/s^2.

2- Friction loss in straight pipe.

Re = (998.2 x 0.6083 x 0.1023)/ 1.005x10^{-3} = 6.181 x10^4

e/d = 0.00045 ⇒ Figure (3.7) f = 0.0051
L = 5+50+15+100 = 170 m

$F_{Fs} = 4f\dfrac{L}{d}\dfrac{u^2}{2}$ =4 (0.0051) (170/0.1023) x (0.6083^2/2) = 6.272 J/kg or m^2/s^2.

3- Friction in the two elbows.

$\boxed{F_f = 2(K_f\dfrac{u_2}{2})}$; where K_f = 0.75 ⇒ F_f = 0.278 J/kg or m^2/s^2.

4- Expansion loss at the tank entrance.

$\boxed{F_e = K_e\dfrac{u_1^2}{2}}$; where $\boxed{K_e = \left[1-\left(\dfrac{A_1}{A_2}\right)\right]^2}$ ≈ 1.0 ⇒ F_e = 0.185 J/kg or m^2/s^2.

F (total frictional losses) = 0.102 + 6.272 + 0.278 + 0.185 = 6.837 J/kg or m^2/s^2.

⇒ ηWs = 6.837 + 9.81(15) = 153.93 J/kg or m^2/s^2

The power required for pump (Ws) = ηWs/Ws = 153.93/0.65 = 236.8 J/kg or m^2/s^2

The total power required for pump ($\dot{m}$ Ws) = Q ρ Ws

= (5 x10^{-3} m^3/s) 998.2 kg/m^3 (236.8 J/kg) = 1.182 kW.

Example -4.19-

Water at 4.4°C is to flow through a horizontal commercial steel pipe having a length of 305 m at the rate of 150 gal/min. A head of water of 6.1 m is available to overcome the skin friction losses (h_{Fs}). Calculate the pipe diameter. e = 4.6 x10^{-5} m ρ = 1000 kg/m^3, μ= 1.55 x10^{-3} Pa.s.

Solution:

$\boxed{h_{Fs} = \left[4f\left(\dfrac{L}{d}\right)\right]\dfrac{u^2}{2g}}$ = 6.1 m

Q = 150 gal/min (ft^3/ 7.481gal)(min/60s) (m/3.28 ft)^3 = 9.64 x10^{-3} m^3/s

u = Q/A = (9.64 x10^{-3} m^3/s)/(π/4 d^2) ⇒ u = 0.01204 d^{-2}.

⇒ 6.1 = 4 f (305/d)(0.01204 d^{-2})/(2 x9.81)

⇒ f = 676.73 d^5 ⇒ d = (f /676.73)$^{1/5}$ -----------------------(1)
Re = (1000 x (0.01204 d^{-2}) x d)/ 1.55x10^{-3} = 7769.74 d^{-1} -----------------------(2)
e/d = 4.6 x10^{-5} d^{-1} -----------------------(3)
solution by trial and error

	Eq.(1)	Eq.(2)	Fq. (3)	Figure (3.7)
f	d	Re	e/d	f = 2 Φ
Assumed → 0.00378	$\boxed{0.089}$	8.73 x10^4	0.00052	0.0052
0.0052	0.095	8.176 x10^4	0.000484	0.0051
0.0051	0.0945	8.22 x10^4	0.00049	0.0051

⇒ d = 0.0945 m.

Example -4.20-

A petroleum fraction is pumped 2 km from a distillation plant to storage take through a mild steel pipeline, 150 mm I.D. at 0.04 m^3/s rate. What is the pressure drop along the pipe and the power supplied to the pumping unit if it has an efficiency of 50%. The pump impeller is eroded and the pressure at its delivery falls to one half. By how much is the flow rate reduced? Take that: sp.gr. = 0.705, μ= 0.5 m Pa.s e = 0.004 mm.

Solution:

$u = Q/A = (0.04\ m^3/s)/(\pi/4 \times 0.15^2) \Rightarrow u = 2.26\ m/s$

$Re = (705 \times 2.26 \times 0.15)/ 0.5\times10^{-3} = 4.78\times10^5$

$e/d = 0.000027 \Rightarrow$ Figure (3.7) $f = 2\ \Phi \Rightarrow f = 0.0033$

$$\boxed{-\Delta P_{Fs} = \left[4 f\left(\frac{L}{d}\right)\right]\frac{\rho u^2}{2}} = 4\ (0.0033)\ (2000/0.15)\ (705 \times 2.26^2/2) = 316876\ Pa.$$

$$\text{Power} = \frac{Q(-\Delta P)}{\eta} = (0.04\ m^3/s)(316876\ Pa)/0.5 = 25.35\ kW$$

Due to impeller erosion $(-\Delta P)_{new} = (-\Delta P)_{old}/2 = 316876\ Pa/2 = 158438\ Pa$

$\Phi Re^2=(-\Delta P_{fs}/L)(\rho d^3/4\mu^2)=[(158438)/(2000)][(1000)(0.15)^3/(4)(0.5\times10^{-3})^2] = 1.885\times10^8$

$e/d = 0.000027 \Rightarrow$ From Figure (3.8) $Re = 3 \times10^5 \quad \Rightarrow u = 1.42 m/s$

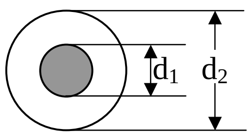
The new volumetric flow rate is now $Q = 1.42\ (\pi/4 \times 0.15^2) = 0.025\ m^3/s$.

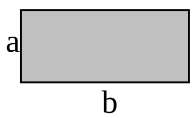
4.9 Friction Losses in Noncircular Conduits

The friction loss in long straight channels or conduits of noncircular cross-section can be estimated by using the same equations employed for circular pipes if the diameter in the Reynolds number and in the friction factor equation is taken as equivalent diameter. The equivalent diameter De or hydraulic diameter defined as four times the cross-sectional area divided by the wetted perimeter of the conduit.

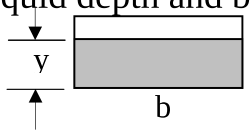
$$De = 4\frac{\text{Cross - sectional area of channel}}{\text{Wetted perimeter of channel}}$$

- For circular cross section.
  $De = 4\ (\pi/4 \times d^2)/\ \pi\ d = d$
- For an annular space with outside diameter d_1 and inside d_2.
  $De = 4\ [\pi/4 \times (d_1^2 - d_2^2)]/\ \pi\ (d_1 + d_2) = d_1+d_2$





- For a rectangular duct of sides a and b.
  $De = 4\ (a.b)\ /\ 2(a + b) = 2ab\ /\ (a + b)$
- For open channels and pattly filled ducts of y-liquid depth and b width
  $De = 4\ (b.y)\ /\ (b + 2y)$



4.10 Selection of Pipe Sizes

In large or complex piping systems, the optimum size of pipe to use for a specific situation depends upon *the relative costs of capital investment*, *power*, *maintenance*, and so on. Charts are available for determining these optimum sizes. However, for small installations approximations are usually sufficient accurate. A table of representative values of ranges of velocity in pipes is shown in the following table: -

Type of fluid	Type of flow	Velocity	
		ft/s	m/s
Nonviscous liquid	Inlet to pump	2 - 3	0.6 – 0.9
	Process line or Pump discharge	5 - 8	1.5 – 2.5
Viscous liquid	Inlet to pump	0.2 – 0.8	0.06 – 0.25
	Process line or Pump discharge	0.5 - 2	0.15 – 0.6
Gas		30 - 120	9 – 36
Steam		30 - 75	9 – 23

4.11 The Boundary Layer

When a fluid flow over a surface, that part of the stream, which is close to the surface, suffers a significant retardation, and a velocity profile develops in the fluid. In the bulk of the fluid away from the boundary layer the flow can be adequately described by the theory of ideal fluids with zero viscosity (μ =0). However in the thin boundary layer, viscosity is important.

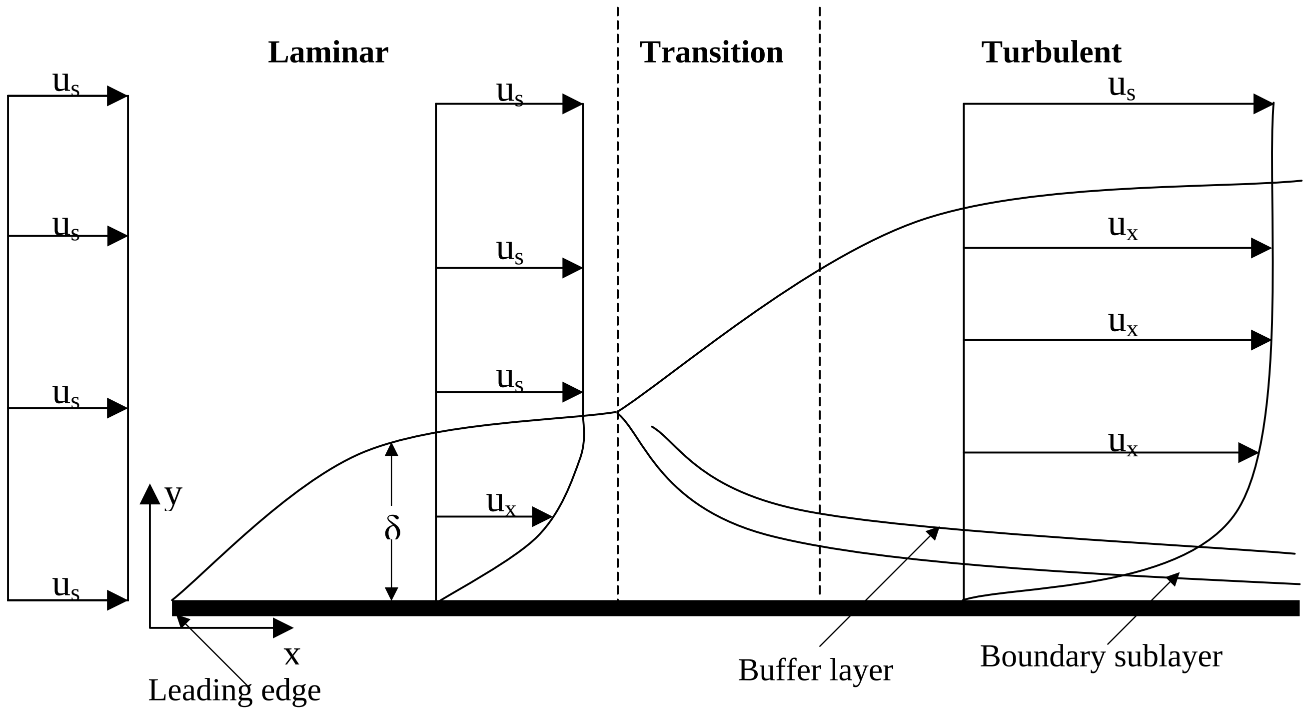


Figure of boundary layer for flow past a flat plate

If the velocity profile of the entrance region of a tube is flat, a certain length of the tube is necessary for the velocity profile to be fully established (developed). This length for the establishment of fully developed flow is called “entrance length”.

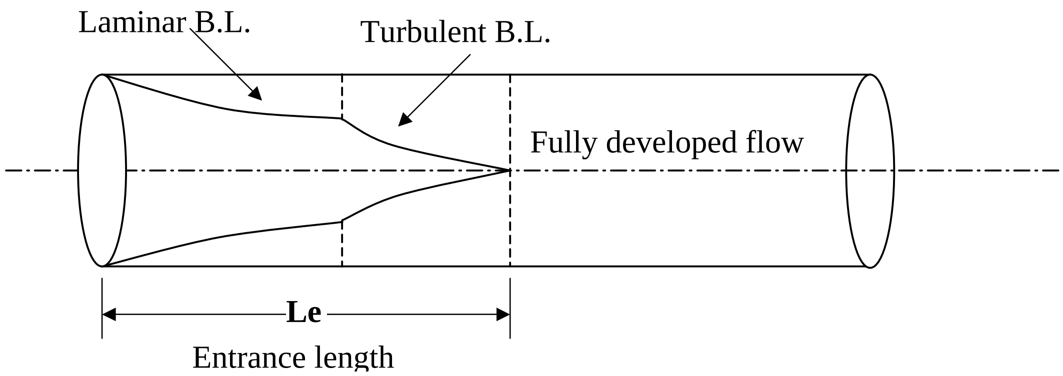


Figure of conditions at entry to pipe.

At the entrance the velocity profile is flat; i.e. the velocity is the same at all positions. As the fluid progresses down the tube, the boundary layer thickness increases until finally they meet at the centerline of the pipe.

For fully developed velocity profile to be formed in laminar flow, the approximate entry length (Le) of pipe having diameter d, is: -

Le/d = 0.0575 Re --------------------laminar

For fully developed velocity profile to be formed in turbulent flow, no relation is available to predict the entry length. As an approximation the entry length (Le) is after 50 diameters downstream of pipe. Thus;

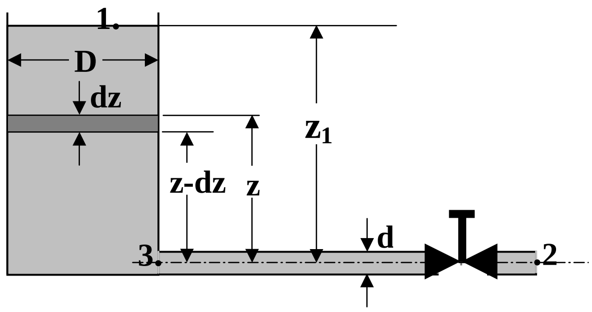
Le/d = 50 ------------------turbulent

4.12 Unsteady State Problems

Example -4.21-

A cylindrical tank, 5 m in diameter, discharges through a horizontal mild steel pipe 100 m long and 225 mm diameter connected to the base of the tank. Find the time taken for the water level in the tank to drop from 3 m to 0.3 m above the bottom. The viscosity of water is 1.0 mNs/m^2, e = 0.05 mm.

Solution:



$$\cancel{\frac{\Delta P}{\rho\, g}} + \Delta z + \frac{\Delta u^2}{2\alpha\, g} - \cancel{\frac{\eta\, Ws}{g}} + h_F = 0$$

$u_1 \approx 0$, $z_2 = 0$ (datum line)

at time = 0 $\Rightarrow z = z_1$

at time = t $\Rightarrow z = z$

$$\Rightarrow z_1 = \frac{u_2^2}{2\alpha_2} + h_F \quad \text{at t = 0}$$

$$\boxed{h_{Fs} = \left[4 f \left(\frac{L}{d}\right)\right]\frac{u^2}{2g}} = 90.61\, f\, u_2^2$$

at time = 0 $\Rightarrow z_1 = (0.051 + 90.61\,f)\,u_2^2 \Rightarrow u_2 = \sqrt{\dfrac{z_1}{0.051+90.61f}}$

at time = t, $\Rightarrow z = (0.051 + 90.61\,f)\,u_2^2 \Rightarrow u_2 = \sqrt{\dfrac{z}{0.051+90.61f}}$ --------------------- (1)

Let the level of liquid in the tank at time (t) is (z)
and the level of liquid in the tank at time (t+dt) is (z–dz)
The volume of liquid discharge during (time =t) to (time = t+dt) is (– dV)
$= (\pi/4\,D^2)\,[z-(z-dz)] = (19.63\,dz)\ m^3$

$Q = dV/dt = -19.63\,(dz/dt)\ m^3/s$ --(2)

But $Q = A\,u_2 = (\pi/4\,d^2)\,u_2 = (0.04\ m^2)\,u_2$ -------------------------------------(3)

Substitute eq.(1) into eq.(3) to give;

$$Q = 0.04\sqrt{\frac{z}{0.051+90.61f}} \qquad \text{----------------------------------(4)}$$

The equalization between eq.(2) and eq.(4) gives;

$$Q = -19.63\frac{dz}{dt} = 0.04\sqrt{\frac{z}{0.051+90.61f}} \Rightarrow \int_0^T dt = \int_3^{0.3} -490.75\sqrt{0.051+90.61f}\,z^{-\frac{1}{2}}dz$$

$$\Rightarrow T = 490.75\sqrt{0.051+90.61f}\int_{0.3}^{3} z^{-\frac{1}{2}}dz \quad = 490.75\sqrt{0.051+90.61f}\left.\frac{z^{\frac{1}{2}}}{1/2}\right|_{0.3}^{3}$$

$$\Rightarrow T = 1169.4\sqrt{0.051+90.61f}$$

$P_3 = P_o + z\rho g$, and $P_2 = P_o$

$\Rightarrow (P_3 - P_2) = (-\Delta P_{Fs})$ the pressure drop along the pipe due to friction

From applied the modified Bernoulli's equation between 3 and 2 $\Rightarrow (-\Delta P_{Fs}) = z\rho g$

But $\Phi Re^2 = (-\Delta P_{fs}/L)(\rho d^3/4\mu^2) = [(z\rho g)/(L)][(\rho d^3/4\mu^2)] = 2.79\times10^8 z$

at z = 3.0 m $\Rightarrow \Phi Re^2 = 8.79\times10^8$

at z = 0.3 m $\Rightarrow \Phi Re^2 = 8.38\times10^7$

e/d = 0.0002 $\Rightarrow$ From Figure (3.8)

$\Rightarrow$at z = 3.0 m Re = 7.0×10^5
$\Rightarrow$at z = 0.3 m Re = 2.2×10^5 Turbulent

e/d = 0.0002 $\Rightarrow$ From Figure (3.7)

$\Rightarrow$at z = 3.0 m Re = 7.0×10^5 $\Rightarrow f = 0.0038$

$\Rightarrow$at z = 0.3 m Re = 2.2×10^5 $\Rightarrow f = 0.004$

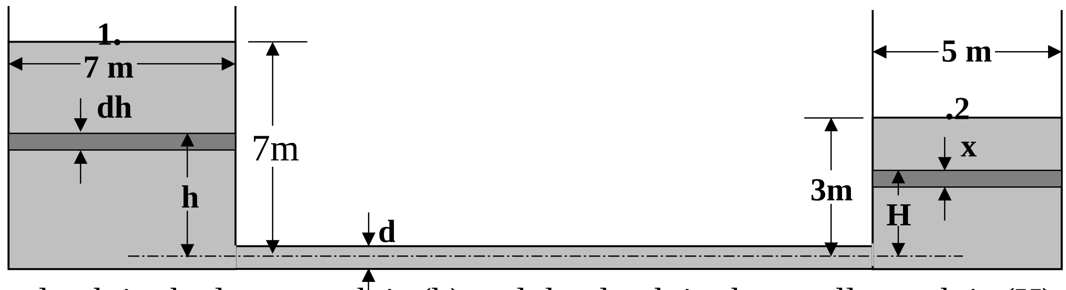
taking a value of f = 0.004, and assume it constant

$\therefore$ T = 752 s

Example -4.21-

Two tanks, the bottoms of which are at the same level, are connected with one another by a horizontal pipe 75 mm diameter and 300 m long. The pipe is bell-mouthed at each end so that losses on entry and exit are negligible. One tank is 7 m diameter and contains water to a depth of 7 m. The other tank is 5 m diameter and contains water to a depth of 3 m. If the tanks are connected to each other by means of the pipe, how long will it take before the water level in the larger tank has fallen to 6 m? Take e =0.05 mm.

Solution:



At any time (t) the depth in the larger tank is (h) and the depth in the smaller tank is (H)

$$\frac{\cancel{\Delta P}}{\cancel{\rho\, g}} + \Delta z + \frac{\cancel{\Delta u^2}}{\cancel{2\alpha\, g}} - \frac{\cancel{\eta\, W_s}}{\cancel{g}} + h_F = 0$$

$\Rightarrow$ $\Delta z = h - H = h_{Fs}$ ------------------------------------(1)

When the level in the large tank fall to (h), the level in the small tank will rise by a height (x) by increasing to reach a height (H).

The volume of the liquid in large tank that discharged to small tank is;

$= \pi/4 \times 7^2\,(7-h) = 38.48\,(7-h)\ \text{m}^3$

and is equal to $= \pi/4 \times 5^2\,(x) = 38.48\,(7-h)$

$\Rightarrow$ $x = 13.72 - 1.96\,h$ ---------------------------(2)

$H = 3 + x = 3 + 13.72 - 1.96\,h$

$\Rightarrow$ $H = 16.72 - 1.96\,h$ ---------------------------(3)

Substitute eq.(3) into eq.(1), to give,

$$h - (16.72 - 1.96\,h) = h_{Fs} = \left[4 f\left(\frac{L}{d}\right)\right]\frac{u^2}{2g}$$

$$2.96\,h - 16.72 = 815.5\, f\, u^2 \quad \Rightarrow u = \sqrt{\frac{0.00363\,h - 0.02}{f}}$$

The level of water in the large at (t = 0) = 7 m

The level of water in the large at (t = t) = h m

The level of water in the large at (t = t+dt) = (h–dh) m

The discharge of liquid during the timed (dt) is,

$Q = dV / dt = \pi/4\ 7^2\,[h-(h-dh)] / dt = \pi/4\ 7^2\,(dh / dt)$ --------------------(4)

But $Q = A\,u = \pi/4\ d^2 \Rightarrow Q = \frac{\pi}{4}(0.075)^2\sqrt{\frac{0.00363\,h - 0.02}{f}}$ ------------------------(5)

By equalization between eq.(4) and eq.(5) $\Rightarrow dt = -8711.11\dfrac{dh}{\sqrt{\dfrac{0.00363 - 0.02}{f}}}$

e/d = 0.05/75 = 0.00067   assume f = 0.004

$$\int_0^T dt = -8711.11\int_7^6 (0.9h-5)^{-0.5}\,dh = 8711.11(\frac{1}{0.9})\frac{(0.9h-5)^{0.5}}{0.5}\Bigg|_6^7 = 19358(1.3^{0.5}-0.4^{0.5})$$

$\Rightarrow$ T = 9777.67 s

Q = [π/4 7^2 (7 – 6)] / 9777.67 = 0.00393 m^3/s average volumetric flow rate

u = Q / A = (0.00393 m^3/s) / (π/4 x 0.075^2) = 0.89 m/s $\Rightarrow$ Re = 6.6552 x10^4

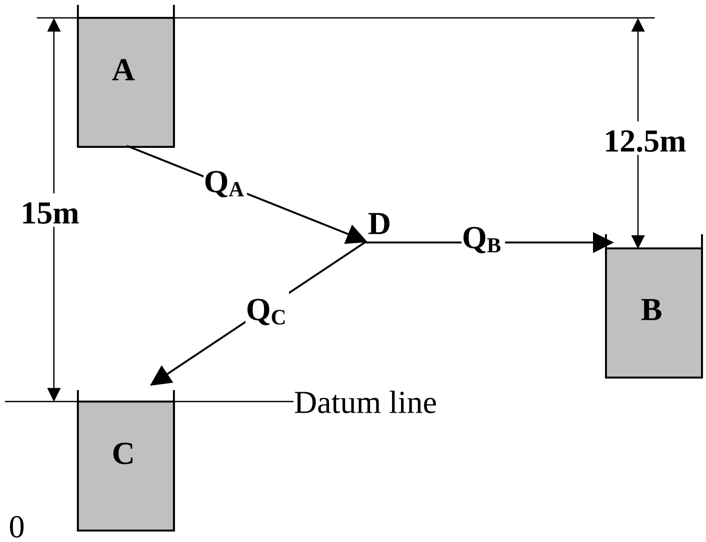
e/d = 0.00067 $\Rightarrow$ From Figure (3.7) $\Rightarrow$ f =0.006

Repeat the integration based on the new value of (f = 0.006) $\Rightarrow$ T = 9777.67 s

Example -4.21-

Water is being discharged, from a reservoir, through a pipe 4 km long and 50 cm I.D. to another reservoir having water level 12.5 m below the first reservoir. It is required to feed a third reservoir, whose level is 15 m below the first reservoir, through a pipe line 1.5 km long to be connected to the pipe at distance of 1.0 km from its entrance. Find the diameter of this new pipe, so that the flow into both the reservoirs may be the same.

Solution:



AD + DB = 4,000 m, its i.d = 50 cm

AD = 1,000 m $\Rightarrow$ DB = 3,000 m

DC = 1,500 m

$Q_A = Q_B + Q_C$

$Q_B = Q_C = Q_A/2$-----------------(1)

A-D

$$-\frac{P_D}{\rho} + (z_D - 15)g + 4f(\frac{L}{d})_A\frac{u_A^2}{2} = 0$$

$$u = \frac{4Q}{\pi\, d^2}$$

$$-\frac{P_D}{\rho} + z_D g - 15g + 4f(\frac{1{,}000}{0.5})(\frac{16Q_A^2}{2\pi^2 d_A^4}) = 0$$

$$-\frac{P_D}{\rho} + z_D g - 147.1 + 830\,Q_A^2 = 0 \text{ ------------------------(2)}$$

A-B

$$-\frac{P_D}{\rho} - (z_D - 15)g + 4f(\frac{3{,}000}{0.5})(\frac{16Q_B^2}{2\pi^2 d_B^4}) = 0$$

$$-\frac{P_D}{\rho} - z_D g + 24.5 + 2{,}490\,Q_B^2 = 0 \text{ ------------------------(3)}$$

A-B

$$-\frac{P_D}{\rho} - z_D g + 4f(\frac{1{,}500}{d_C})(\frac{16Q_C^2}{2\pi^2 d_C^4}) = 0$$

$$-\frac{P_D}{\rho} - z_D g + 38.9\,\frac{Q_C^2}{d_C^5} = 0 \text{ --------------------------------(4)}$$

Substitute eq.(1) into eqs.(3) and (4)

Equation (2) $-\frac{P_D}{\rho}+z_D g-147.1+830\,Q_A^2=0$ -------------------------(2)

Equation (3) $-\frac{P_D}{\rho}-z_D g+24.5+622.5\,Q_A^2=0$ -------------------------(5)

Equation (4) $-\frac{P_D}{\rho}-z_D g+9.72\,\frac{Q_A^2}{d_C^5}=0$ --------------------------------(6)

eq.(2) + eq.(5) ⇒ – 122.6 + 1452.5 Q_A^2 = 0 ⇒ Q_A = 0.29 m^3/s

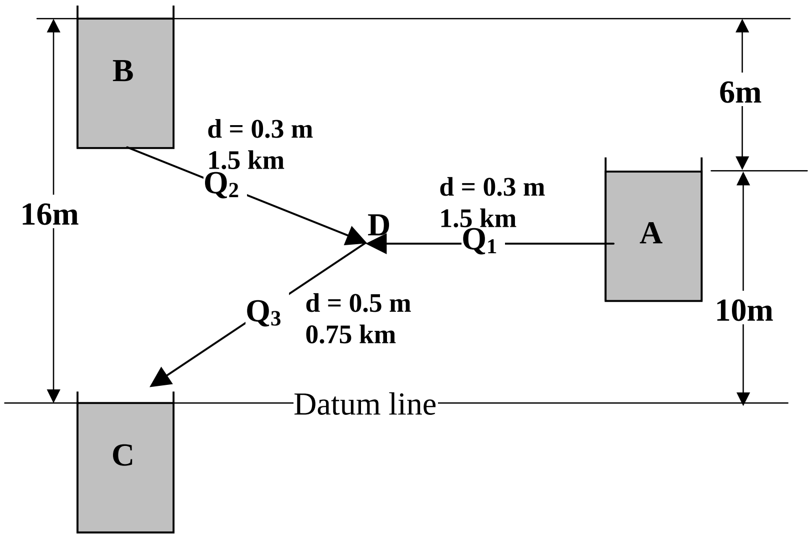
⇒ $Q_B = Q_C$ = (0.29 m^3/s) / 2 = 0.145 m^3/s

eq.(5) – eq.(6) ⇒ 24.5 + 622.5$(0.29)^2$ – 9.72$(0.29)^2$ / d_C^5 = 0 ⇒ d_C^5 = 0.0106 m^5

⇒ d_C = 0.4 m = 40 cm.

Example -4.21-

Two storage tanks, A and B, containing a petroleum product, discharge through pipes each 0.3 m in diameter and 1.5 km long to a junction at D, as shown in Figure. From D the liquid is passed through a 0.5 m diameter pipe to a third storage tank C, 0.75 km away. The surface of the liquid in A is initially 10 m above that in C and the liquid level in B is 6 m higher than that in A. Calculate the initial rate of discharge of liquid into tank C assuming the pipes are of mild steel. The density and viscosity of the liquid are 870 kg/m^3 and 0.7 m Pa.s respectively.



Solution:

Because the pipes are long, the kinetic energy of the fluid and minor losses at the entry to the pipes may be neglected. It may be assumed, as a first approximation, that *f* is the same in each pipe and that the velocities in pipes AD, BD, and DC are u_1, u_2, and u_3 respectively, if the pressure at D is taken as P_D and point D is z_D m above the datum for the calculation of potential energy, the liquid level in C.

Then applying the energy balance equation between D and the liquid level in each of the tanks gives:

A-D $$-\frac{P_D}{\rho}+(z_D-10)g+4f\left(\frac{1{,}500}{0.3}\right)\frac{u_1^2}{2\alpha_1}=0$$

B-D $$-\frac{P_D}{\rho}+(z_D-16)g+4f\left(\frac{1{,}500}{0.3}\right)\frac{u_2^2}{2\alpha_2}=0$$

D-C $$-\frac{P_D}{\rho}-(z_D)g+4f\left(\frac{750}{0.5}\right)\frac{u_3^2}{2\alpha_3}=0$$

Assume turbulent flow in all pipes

$\Rightarrow$ **A-D** $\quad -\dfrac{P_D}{\rho}+z_D g-98.1+10{,}000 f\ u_1^2=0$ --------------------------------(1)

B-D $\quad -\dfrac{P_D}{\rho}+z_D g-156.96+10{,}000 f\ u_2^2=0$ -----------------------------(2)

D-C $\quad -\dfrac{P_D}{\rho}-z_D g+3{,}000 f\ u_3^2=0$ -------------------------------(3)

eq.(1) – eq.(2) $\Rightarrow$ $58.86 + 10{,}000\, f\,(u_1^2 - u_2^2) = 0$ ----------------------(4)

eq.(2) – eq.(3) $\Rightarrow$ $-156.96 + 10{,}000\, f\,(u_2^2 + 0.3u_3^2) = 0$ ---------------(5)

$Q_1 + Q_2 = Q_3 \Rightarrow [(\pi/4\ 0.3^2)\ u_1] + [(\pi/4\ 0.3^2)\ u_2] = [(\pi/4\ 0.5^2)\ u_3]$

$\Rightarrow u_1 + u_2 = 2.78\ u_3$ -------------------------------------(6)

equations (4), (5), and (6) are three equations with 4 unkowns. As first approximation for e/d = 0.0001 to 0.00017 $\Rightarrow f = 0.004$

$\Rightarrow$ eq.(4) become $58.86 + 40\ (u_1^2 - u_2^2) = 0$ -----------------------------(7)

$\Rightarrow$ eq.(5) become $-156.96 + 40\ (u_2^2 - 0.3u_3^2) = 0$ ----------------------(8)

From eq.(7) $u_1^2 = u_2^2 - 1.47$ --------------------------------(9)

$u_3 = (u_1 + u_2) \Rightarrow u_3^2 = (1/2.78)^2\ (u_1^2 + 2u_1 u_2 + u_2^2)$

$\Rightarrow u_3^2 = (1/2.78)^2\ [u_2^2 + (u_2^2 - 1.47) + 2u_2(u_2^2 - 1.47)^{0.5}]$

$\Rightarrow u_3^2 = (1/2.78)^2\ [2u_2^2 - 1.47 + 2u_2(u_2^2 - 1.47)^{0.5}]$ -----------------(10)

Substitute eq.(10) into (8)

$\Rightarrow -156.96 + 40\ \{u_2^2 + 0.3(1/2.78)^2\ [2u_2^2 - 1.47 + 2u_2(u_2^2 - 1.47)^{0.5}]\} = 0$

$\Rightarrow u_2(u_2^2 - 1.47)^{0.5} = (159.24 - 43.2\ u_2^2)/\ 3.2$ squaring the two limits

$\Rightarrow u_2^2(u_2^2 - 1.47)^{0.5} = (49.8 - 13.5\ u_2^2)^2$

$\Rightarrow u_2^4 - 7.41u_2^2 + 13.68 = 0$ $\quad \boxed{x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}}$

$\Rightarrow$ either $u_2^2 = 3.922$ or $u_2^2 = 3.488$

$\Rightarrow u_2 = 1.98$ m/s or $u_2 = 1.87$ m/s

Substituting u_2 into eq.(9)

$\Rightarrow u_1 = 1.56$ m/s or $u_1 = 1.42$ m/s

Substituting u_2, and u_2 into eq.(6)

$\Rightarrow u_3 = 1.3$ m/s or $u_3 = 1.18$ m/s

The lower set values satisfies equation (8)

$\Rightarrow u_1 = 1.42$ m/s, $u_2 = 1.87$ m/s, and $u_3 = 1.18$ m/s

$\Rightarrow Re_1 = 5.3 \times 10^5$, $Re_2 = 6.9 \times 10^5$, and $Re_3 = 7.3 \times 10^5$

From Figure (3.7) $\Rightarrow f_1 = 0.0043$, $f_2 = 0.0036$, and $f_3 = 0.0038$

$\Rightarrow$ the assumption <u>$f = 0.004$ is ok.</u>

$Q_3 = (\pi/4\ 0.5^2)\ u_3 = (\pi/4\ 0.5^2)\ 1.18 = 0.23\ m^3/s$

CHPTER FIVE

Pumping of Liquids

5.1 Introduction

Pupmps are devices for supplying *energy* or *head* to a flowing liquid in order to overcome head losses due to friction and also if necessary, *to raise* liquid to a higher level.

For the pumping of liquids or gases from one vessel to another or through long pipes, some form of mechanical pump is usually employed. **The energy required by the pump** will depend on the height through which the fluid is raised, the pressure required at delivery point, the length and diameter of the pipe, the rate of flow, together with the physical properties of the fluid, particularly its *viscosity* and *density*. The pumping of liquids such as sulphuric acid or petroleum products from bulk store to process buildings, or the pumping of fluids round reaction units and through heat exchangers, are typical illustrations of the use of pumps in the process industries. On the one hand, it may be necessary to inject reactants or catalyst into a reactor at a low, but accurately controlled rate, and on the other to pump cooling water to a power station or refinery at a very high rate. The fluid may be a gas or liquid of low viscosity, or it may be a highly viscous liquid, possibly with non-Newtonian characteristics. It may be clean, or it may contain suspended particles and be very corrosive. All these factors influence the choice of pump.

Because of the wide variety of requirements, many different types are in use including centrifugal, piston, gear, screw, and peristaltic pumps, though in the chemical and petroleum industries the centrifugal type is by far the most important.

5.2 The Total Head (Δh)

The head imparted to a flowing liquid by a pump is known as the total head (Δh). If a pump is placed between points ① and ② in a pipeline, the head for steady flow are related by: -

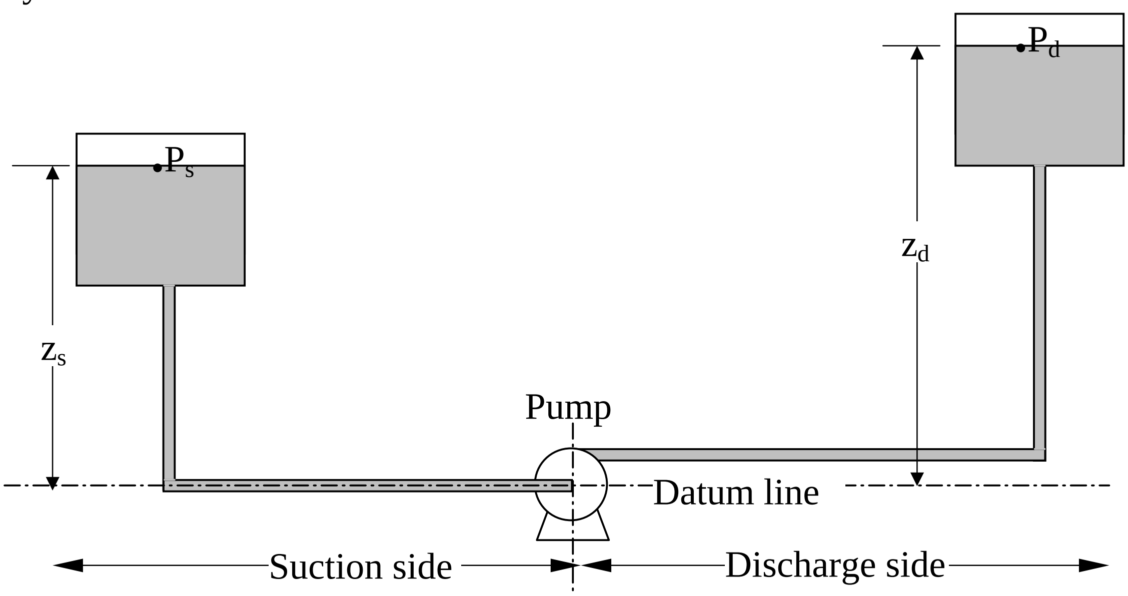


Figure (1) Typical pumping system.

$$\Delta h = \frac{\eta\, Ws}{g} = \left(\frac{P_2}{\rho\, g} + \frac{u_2^2}{2\alpha_2\, g} + z_2\right) - \left(\frac{P_1}{\rho\, g} + \frac{u_1^2}{2\alpha_1\, g} + z_1\right) - h_F$$

$$\Rightarrow \Delta h = \frac{\Delta P}{\rho\, g} + \frac{\Delta\, u^2}{2\alpha\, g} + \Delta z + h_F$$

5.3 System Heads

The important heads to consider in a pumping system are: -

1- Suction head
2- Discharge head
3- Total head
4- Net positive suction head (NPSH)

The following definitions are given in reference to typical pumping system shown in preceding Figure, where the datum line is the centerline of the pump

1- Suction head (h_s)

$$h_s = z_s + \frac{P_s}{\rho\, g} - (h_F)_s$$

2- Discharge head (h_d)

$$h_d = z_d + \frac{P_d}{\rho\, g} + (h_F)_d$$

3- Total head (Δh)

The total head (Δh), which is required to impart to the flowing liquid is the difference between the discharge and suction heads. Thus,

$$\Delta h = h_d - h_s$$

$$\Rightarrow \Delta h = (z_d - z_s) + (\frac{P_d - P_s}{\rho\, g}) + [(h_F)_d + (h_F)_s]$$

where,

$$(h_F)_d = 4 f_d \left(\frac{L}{d} + \sum \frac{Le}{d}\right)_d \frac{u_d^2}{2\, g}$$

$$(h_F)_s = 4 f_s \left(\frac{L}{d} + \sum \frac{Le}{d}\right)_s \frac{u_s^2}{2\, g}$$

The suction head (hs) decreases and the discharge head (hd) increases with increasing liquid flow rate because of the increasing value of the friction head loss terms (hF)s and (hF)d. Thus the total; head (Δh) which the pump is required to impart to the flowing liquid increases with increasing the liquid pumping rate.

Note:

If the liquid level on the suction side is below the centerline of the pump, z_s is negative.

4- Net positive suction head (NPSH)

Available net positive suction head

$$NPSH = z_s + \left(\frac{P_s - P_v}{\rho\, g}\right) - (h_F)_s$$

This equation gives the head available to get the liquid through the suction piping.

P_v is the vapor pressure of the liquid being pumped at the particular temperature in question.

The available net positive suction head (NPSH) can also be written as:

$$NPSH = h_s - \frac{P_v}{\rho\, g}$$

The available net positive suction head (NPSH) in a system should always be positive i.e. the suction head always be capable of overcoming the vapor pressure (Pv) since the frictional head loss (hF)s increases with increasing pumping rate.

At the boiling temperature of the liquid Ps and Pv are equal and the available NPSH becomes [zs-(hF)s]. In this case no suction lift is possible since zs must be positive. If the term (Ps-Pv) is sufficiently large, liquid can b lifted from below the centerline of the pump. In this case zs is negative.

From energy consideration it is immaterial whether the suction pressure is below atmospheric pressure or well above it, as long as the fluid remains liquid. However, <u>if the suction pressure is only slightly greater than the vapor pressure, some liquid may flash to vapor inside the pump</u>, a process called "<u>*Cavitation"*</u>, which greatly <u>reduces the pump capacity</u> and <u>severe erosion</u>.

If the suction pressure is actually less than the vapor pressure, there will be vaporization in the suction line, and no liquid can be drawn into the pump.

<u>To avoid cavitation</u>, the pressure <u>at</u> the pump inlet must exceed the vapor pressure by a certain value, called the " net positive suction head (NPSH)". The required values of NPSH is about 2-3 m H_2o for small pump; but it increases with pump capacity and values up to 15 m H2o are recommended for very large pump.

5.4 <u>Power Requirement</u>

The power requirement to the pump drive from an external source is denoted by (P). It is calculated from Ws by:

$$\boxed{P = \dot{m}Ws = \frac{Q\Delta P}{\eta} = \frac{Q\Delta h\ \rho\, g}{\eta} = \frac{\dot{m}\Delta hg}{\eta}}$$

The mechanical efficiency (η) decreases as the liquid viscosity and hence the frictional losses increase. The mechanical efficiency is also decreased by power losses in gear, Bering, seals, etc.

These losses are not proportional to pump size. Relatively large pumps tend to have the best efficiency whilst small pumps usually have low efficiencies. Furthermore high-speed pumps tend to be more efficient than low-speed pumps. In general, high efficiency pumps have high NPSH requirements.

5.5 <u>Types of Pumps</u>

Pumps can be classified into: -

1- Centrifugal pumps.
2- Positive displacement pumps.

<u>1- Centrifugal pumps</u>

This type depends on giving the liquid a high kinetic energy, which is then converted as efficiently as possible into pressure energy. It used for liquid with very wide ranging properties and suspensions with high solid content including, for example, cement slurries, and may be constructed from a very wide range of corrosion resistant materials. Process industries commonly use centrifugal pumps. The whole pump casing may be constructed from plastics such as polypropylene or it may be fitted with a corrosion resistant lining. Because it operates at high speed, it may be directly coupled to an electric motor and it will give a high flow rate for its size. They are available in sizes about 0.004 to 380 m3/min [1-100,000 gal/min] and for discharge pressures from a few m H2o head to 5,000 kPa.

In this type of pump (Figure 2), the fluid is fed to the center of a rotating impeller and is thrown outward by centrifugal action. As a result of the high speed of rotation the liquid acquires a high kinetic energy and the pressure difference between the suction and delivery sides arises from the interconversion of kinetic and pressure energy.

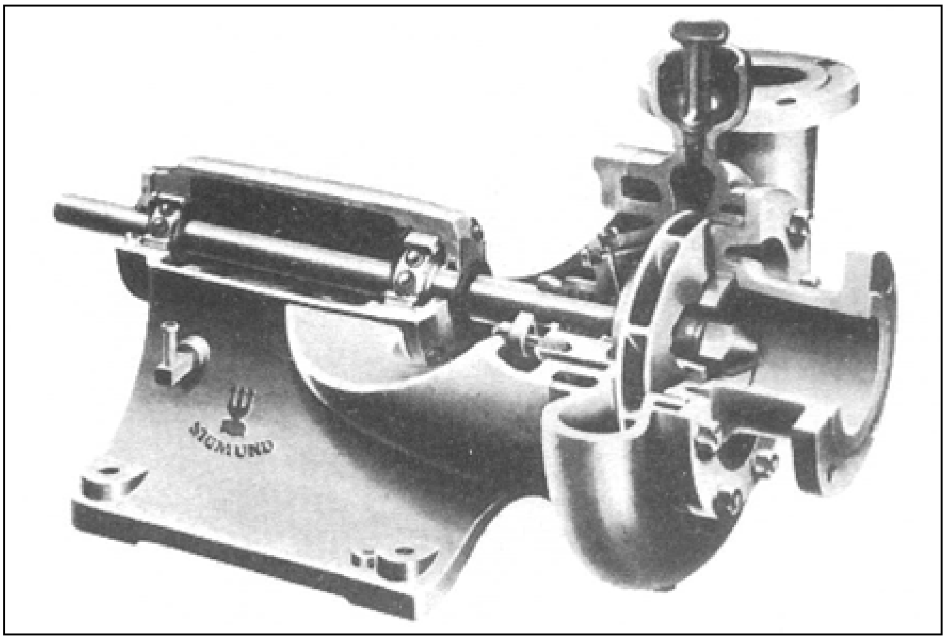


Figure (2) Section of centrifugal pump

The impeller (Figure 3) consists of a series of curved vanes so shaped that the flow within the pump is as smooth as possible. The greater the number of vanes on the impeller, the greater is the control over the direction of motion of the liquid and hence the smaller are the losses due to turbulence and circulation between the vanes. In the open impeller, the vanes are fixed to a central hub, whereas in the closed type the vanes are held between two supporting plates and leakage across the impeller is reduced.

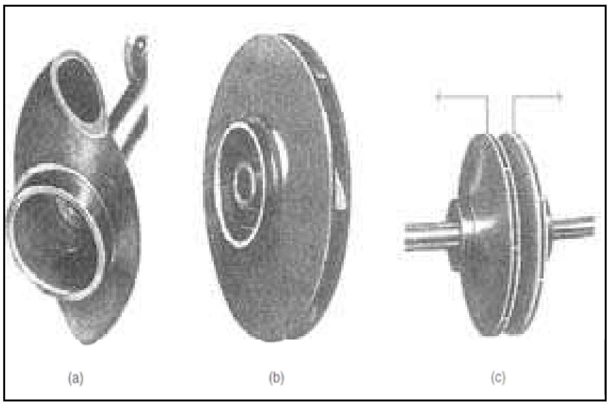


Figure (3) Types of impeller
(a) for pumping suspensions (b) standard closed impeller (c) double impeller

The liquid enters the casing of the pump, normally in an axial direction, and is picked up by the vanes of the impeller. In the simple type of centrifugal pump, the liquid discharges into a volute, a chamber of gradually increasing cross-section with a tangential outlet. A volute type of pump is shown in Figure 4. In the turbine pump

(Figure 4(b)) the liquid flows from the moving vanes of the impeller through a series of fixed vanes forming a diffusion ring. This gives a more gradual change in direction to the fluid and more efficient conversion of kinetic energy into pressure energy than is obtained with the volute type.

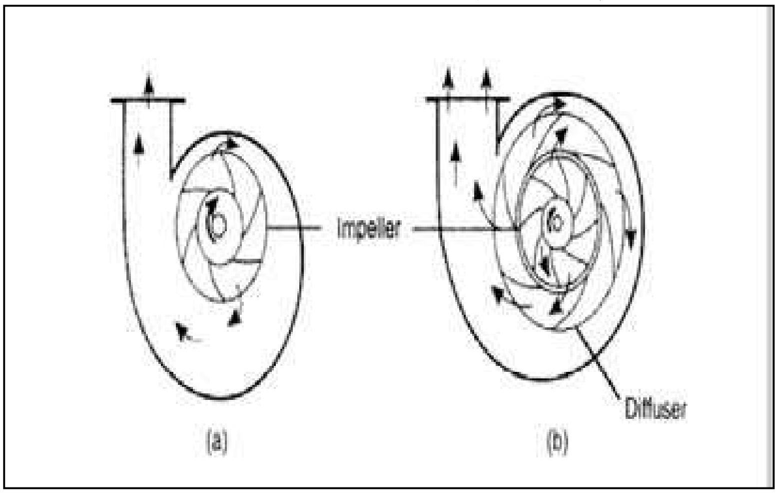


Figure (4) Radial flow pumps
(a) with volute (b) with diffuser vanes

2- Positive Displacement Pumps

In this type, the volume of liquid delivered is directly related to the displacement of the piston and therefore, increases directly with speed and is not appreciably influenced by the pressure. It used for *high pressure and constant rates* this type can be classified into: -

2.1-Reciprocating Pumps, such as

a- The Piston Pump

This pump may be single-acting, with the liquid admitted only to the portion of the cylinder in front of the piston or double-acting, in which case the feed is admitted to both sides of the piston. The majority of pumps are of the single-acting type typically giving a low flow rate of say 0.02 m^3/s at a high pressure of up to 100 Mpa.

b- The Plunger (or Ram) Pump

This pump is the same in principle as the piston type but differs in that the gland is at one end of the cylinder making its replacement easier than with the standard piston type. The piston or ram pump may be used for injections of small quantities of inhibitors to polymerization units or of corrosion inhibitors to high-pressure systems, and also for boiler feed water applications.

c- The Diaphragm Pump

The diaphragm pump has been developed for handling corrosive liquids and those containing suspensions of abrasive solids. It is in two sections separated by a diaphragm of rubber, leather, or plastics material. In one section a plunger or piston operates in a cylinder in which a non-corrosive fluid is displaced. The particularly simple and inexpensive pump results, capable of operating up to 0.2 Mpa.

d- The Metering (or Dosing) Pump

Metering pumps are driven by constant speed electric motors. They are used where *a constant and accurately controlled rate of delivery of a liquid is required*, and they will maintain this constant rate irrespective of changes in the pressure against which they operate. The pumps are usually of the plunger type for low throughput and high-pressure applications; for large volumes and lower pressures a diaphragm is used. In either case, the rate of delivery is controlled by adjusting the stroke of the piston element, and this can be done whilst the pump is in operation. A single-motor driver may operate several individual pumps and in this way give control of the actual flows and of the flow ratio of several streams at the same time. The output may be controlled from zero to maximum flow rate, either manually on the pump or remotely. These pumps may be used for the dosing of works effluents and water supplies, and the feeding of reactants, catalysts, or inhibitors to reactors at controlled rates, and although a simple method for controlling flow rate is provided, high precision standards of construction are required.

2.2-Rotary Pumps, such as

a- The Gear Pump

Gear and lobe pumps operate on the principle of using mechanical means to transfer small elements or "packages" of fluid from the low pressure (inlet) side to the high pressure (delivery) side. There is a wide range of designs available for achieving this end. The general characteristics of the pumps are similar to those of reciprocating piston pumps, but the delivery is more even because the fluid stream is broken down into so much smaller elements. The pumps are capable of delivering to a high pressure, and the pumping rate is approximately proportional to the speed of the pump and is not greatly influenced by the pressure against which it is delivering. Again, it is necessary to provide a pressure relief system to ensure that the safe operating pressure is not exceeded.

b- The Cam Pump

A rotating cam is mounted eccentrically in a cylindrical casing and a very small clearance is maintained between the outer edge of the cam and the casing. As the cam rotates it expels liquid from the space ahead of it and sucks in liquid behind it. The delivery and suction sides of the pump are separated by a sliding valve, which rides on the cam. The characteristics again are similar to those of the gear pump.

c- The Vane Pump

The rotor of the vane pump is mounted off centre in a cylindrical casing. It carries rectangular vanes in a series of slots arranged at intervals round the curved surface of the rotor. The vanes are thrown outwards by centrifugal action and the fluid is carried in the spaces bounded by adjacent vanes, the rotor, and the casing. Most of the wear is on the vanes and these can readily be replaced.

d- The Flexible Vane Pump

The pumps described above will not handle liquids containing solid particles in suspension, and the flexible vane pumps has been developed to overcome this disadvantage. In this case, the rotor (Figure 8.10) is an integral elasomer moulding of a hub with flexible vanes which rotates in a cylindrical casing containing a crescent-shaped block, as in the case of the internal gear pump.

e- The Flow Inducer or Peristaltic Pump

This is a special form of pump in which a length of silicone rubber or other elastic tubing, typically of 3 to 25 mm diameter, is compressed in stages by means of a rotor as shown in Figure 8.11. The tubing is fitted to a curved track mounted concentrically with a rotor carrying three rollers. As the rollers rotate, they flatten the tube against the track at the points of contact. These "flats" move the fluid by positive displacement, and the flow can be precisely controlled by the speed of the motor. These pumps have been particularly useful for biological fluids where all forms of contact must be avoided. They are being increasingly used and are suitable for pumping emulsions, creams, and similar fluids in laboratories and small plants where the freedom from glands, avoidance of aeration, and corrosion resistance are valuable, if not essential. Recent developments^ have produced thick-wall, reinforced moulded tubes which give a pumping performance of up to 0.02 m^3/s at 1 MN/m^2. The control is such that these pumps may conveniently be used as metering pumps for dosage processes.

f- The Mono pump

Another example of a positive acting rotary pump is the single screw-extruder pump typified by the Mono pump, in which a specially shaped helical metal rotor revolves eccentrically within a double-helix, resilient rubber stator of twice the pitch length of the metal rotor. A continuous forming cavity is created as the rotor turns — the cavity progressing towards the discharge, advancing in front of a continuously forming seal line and thus carrying the pumped material with it. The Mono pump gives a uniform flow and is quiet in operation. It will pump against high pressures; the higher the required pressure, the longer are the stator and the rotor and the greater the number of turns. The pump can handle corrosive and gritty liquids and is extensively used for feeding slurries to filter presses. It must never be run dry. The Mono Merlin Wide Throut pump is used for highly viscous liquids.

g- The Screw pumps

A most important class of pump for dealing with highly viscous material is represented by the screw extruder used in the polymer industry. The screw pump is of more general application and will be considered first. The fluid is sheared in the channel between the screw and the wall of the barrel. The mechanism that generates the pressure can be visualized in terms of a model consisting of an open channel covered by a moving plane surface. If a detailed analysis of the flow in a screw pump is to be carried out, then it is also necessary to consider the small but finite leakage flow that can occur between the flight and the wall. With the large pressure generation in a polymer extruder, commonly 100 bar (107 N/m2), the flow through this gap, which is typically about 2 per cent of the barrel internal diameter, can be significant. The pressure drop over a single pitch length may be of the order of 10 bar (106 N/m2), and this will force fluid through the gap. Once in this region the viscous fluid is subject to a high rate of shear (the rotation speed of the screw is often about 2 Hz), and an appreciable part of the total viscous heat generation occurs in this region of an extruder.

5.6 The advantages and disadvantages of the centrifugal pump

The main advantages are:

(1) It is simple in construction and can, therefore, be made in a wide range of materials.
(2) There is a complete absence of valves.
(3) It operates at high speed (up to 100 Hz) and, therefore, can be coupled directly to an electric motor. In general, the higher the speed the smaller the pump and motor for a given duty.
(4) It gives a steady delivery.
(5) Maintenance costs are lower than for any other type of pump.
(6) No damage is done to the pump if the delivery line becomes blocked, provided it is not ran in this condition for a prolonged period.
(7) It is much smaller than other pumps of equal capacity. It can, therefore, be made into a sealed unit with the driving motor, and immersed in the suction tank.
(8) Liquids containing high proportions of suspended solids are readily handled.

The main disadvantages are:

(1) The single-stage pump will not develop a high pressure. Multistage pumps will develop greater heads but they are very much more expensive and cannot readily be made in corrosion-resistant material because of their greater complexity. It is generally better to use very high speeds in order to reduce the number of stages required.
(2) It operates at a high efficiency over only a limited range of conditions: this applies especially to turbine pumps.
(3) It is not usually self-priming.
(4) If a non-return valve is not incorporated in the delivery or suction line, the liquid will run back into the suction tank as soon as the pump stops.
(5) Very viscous liquids cannot be handled efficiently.

5.7 Priming The Pump

The theoretical head developed by a centrifugal pump depends on *the impeller speed*, *the radius of the impeller*, and *the velocity of the fluid leaving the impeller*. If these factors are constant, the developed head is the same for fluids of all densities and is the same for liquids and gases. A centrifugal pump trying to operate on air, then can neither draw liquid upward from an initially empty suction line nor force liquid a full discharge line. Air can be displaced by priming the pump.

For example, if a pump develops a head of 100 ft and is full of water, the increase in pressure is [100 ft (62.3lb/ft^3) (ft^2 / 144 in^2)] = 43 psi (2.9 atm). If full of air the pressure increase is about 0.05 psi (0.0035 atm).

5.8 Operating Characteristics

The operating characteristics of a pump are conveniently shown by plotting the head (h), power (P), efficiency (η), and sometimes required NPSH against the flow (or capacity) (Q) as shown in Figure (5). Theses are known as characteristic curves of the pump. It is important to note that the efficiency reaches a maximum and then falls, whilst the head at first falls slowly with Q but eventually falls off rapidly. The optimum conditions for operation are shown as the duty point, i.e. the point where the head curve cuts the ordinate through the point of maximum efficiency.

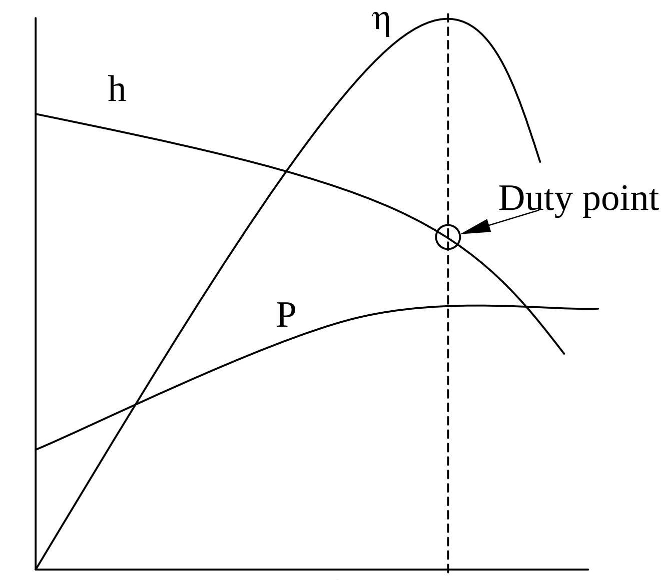


Figure (5) Radial flow pump characteristics

Characteristic curves have a variety of shapes depending on *the geometry of the impeller* and *pump casing*. Pump manufactures normally supply the curves only for operation with water.

In a particular system, a centrifugal pump can only operate at one point on the Δh against Q curve and that is the point where the Δh against Q curve of the pump intersect with the Δh against Q curve of the system as shown in Figure.

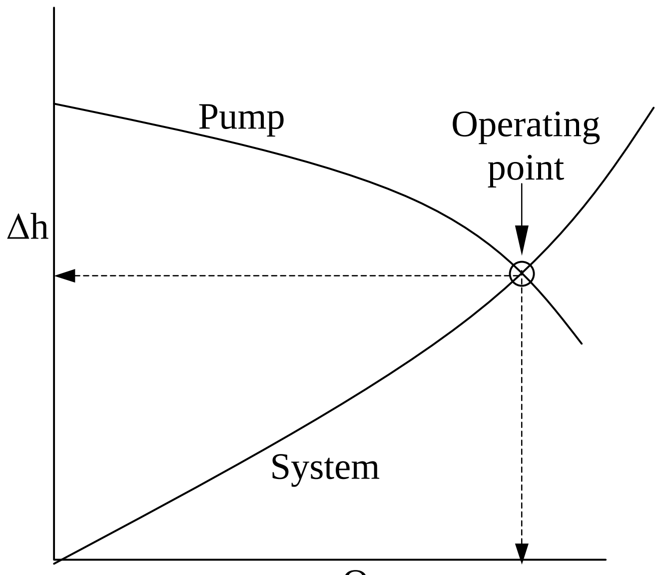


Figure (6) System and pump total head against capacity

The system total head at a particular liquid flow rate

$$\Delta h=(z_d-z_s)+\left(\frac{P_d-P_s}{\rho\,g}\right)+[(h_F)_d+(h_F)_s]$$

where,

$$(h_F)_d=4\,f_d\left[\frac{L}{d}\sum\frac{Le}{d}\right]_d\frac{u_d^2}{2g}$$

$$(h_F)_s=4\,f_s\left[\frac{L}{d}\sum\frac{Le}{d}\right]_s\frac{u_s^2}{2g}$$

For the same pipe <u>type</u> and <u>diameter</u> for suction and discharge lines: -

$$\Delta h=\Delta z+\frac{\Delta P}{\rho\,g}+4\,f\left[\left(\frac{L}{d}+\sum\frac{Le}{d}\right)_d+\left(\frac{L}{d}+\sum\frac{Le}{d}\right)_s\right]\frac{u^2}{2g}$$

but $u=\dfrac{Q}{(\pi/4\,d^2)}$

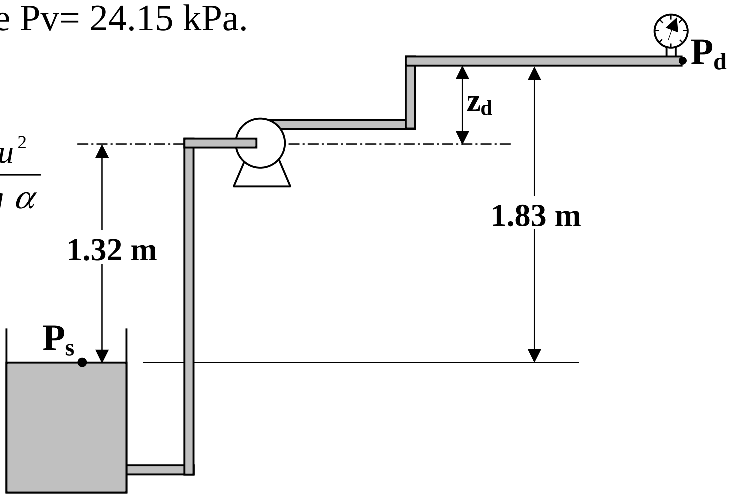
$$\Rightarrow\Delta h=\Delta z+\frac{\Delta P}{\rho\,g}+\frac{4\,f}{2g}\left[\left(\frac{L}{d}+\sum\frac{Le}{d}\right)_d+\left(\frac{L}{d}+\sum\frac{Le}{d}\right)_s\right]\left(\frac{Q}{(\pi/4\,d^2)}\right)^2$$

Example -5.1-

A petroleum product is pumped at a rate of 2.525 x 10-3 m^3/s from a reservoir under atmospheric pressure to 1.83 m height. If the pump 1.32 m height from the reservoir, the discharge line diameter is 4 cm and the pressure drop along its length 3.45 kPa. The gauge pressure reading at the end of the discharge line 345 kPa. The pressure drop along suction line is 3.45 kPa and pump efficiency η=0.6 calculate:-

(i) The total head of the system Δh. **(ii)** The power required for pump. **(iii)** The NPSH

Take that: the density of this petroleum product ρ=879 kg/m^3, the dynamic viscosity μ=6.47 x 10-4 Pa.s, and the vapor pressure Pv= 24.15 kPa.



Solution:

(i)

$$\Delta h = (z_d - z_s) + \left(\frac{P_d - P_s}{\rho\, g}\right) + [(h_F)_d + (h_F)_s] + \frac{\Delta u^2}{2g\,\alpha}$$

$u_s = 0$

$u_d = (2.525 \times 10\text{-}3\ m^3/s)/(\pi/4\ 0.04^2)$

$= 2$ m/s

$Re_d = (879 \times 2 \times 0.04)/\ 6.47 \times 10^{-4}$

$= 1.087 \times 10^5$

The pressure drop in suction line 3.45kPa

$\Rightarrow (h_F)_s = 3.45 \times 10^3/(879 \times 9.81)$

$= 0.4$ m

And in discharge line is also 3.45 kPa $\Rightarrow (h_F)_d = 0.4$ m

The kinetic energy term = $2^2/(2 \times 9.81)$ =0.2 m

The pressure at discharge point = gauge + atmospheric pressure = 345 + 101.325

= 446.325 kPa

The difference in pressure head between discharge and suction points is

$(446.325 – 101.325) \times 10^3 /(879 \times 9.81) = 40$ m

Δz = 1.83 m

$\Rightarrow$ Δh = 40 m + 1.83 m + 0.2 m + 0.4 m + 0.4 m = 42.83 m

(ii)

$$P = \frac{Q\Delta P}{\eta} = \frac{Q\Delta h\ \rho\ g}{\eta} = [(2.525 \times 10^{-3}\ m^3/s)(42.83\ m)(879\ kg/m^3)(9.81\ m/s^2)]/0.6$$

$\Rightarrow$ P = 1.555 kW

(iii)

$$NPSH = z_s + \left(\frac{P_s - P_v}{\rho\, g}\right) - (h_F)_s$$

$= (-\ 1.32) + (1.01325 \times 10^5\ - 24150)/\ (879 \times 9.81) – 0.4$ m

$= 7.23$ m

Example -5.2-

It is required to pump cooling water from storage pond to a condenser in a process plant situated 10 m above the level of the pond. 200 m of 74.2 mm i.d. pipe is available and the pump has the characteristics given below. The head loss in the condenser is equivalent to 16 velocity heads based on the flow in the 74.2 mm pipe. If the friction factor Φ = 0.003, estimate the rate of flow and the power to be supplied to the pump assuming η = 0.5

Q (m^3/s)	0.0028	0.0039	0.005	0.0056	0.0059
Δh (m)	23.2	21.3	18.9	15.2	11.0

Solution:

$$\Delta h = \Delta z + \cancel{\frac{\Delta P}{\rho g}} + \cancel{\frac{\Delta u^2}{2g\alpha}} \left[(h_F)_d + (h_F)_s + (h_F)_{condenser}\right]$$

$$(h_F)_{d+s} = 4f\frac{L}{d}\frac{u^2}{2g} = 4(0.006)(200/0.0742)(u^2/2g) = 3.3\ u^2$$

$$(h_F)_{condenser} = 16\frac{u^2}{2g} = 0.815\ u^2$$

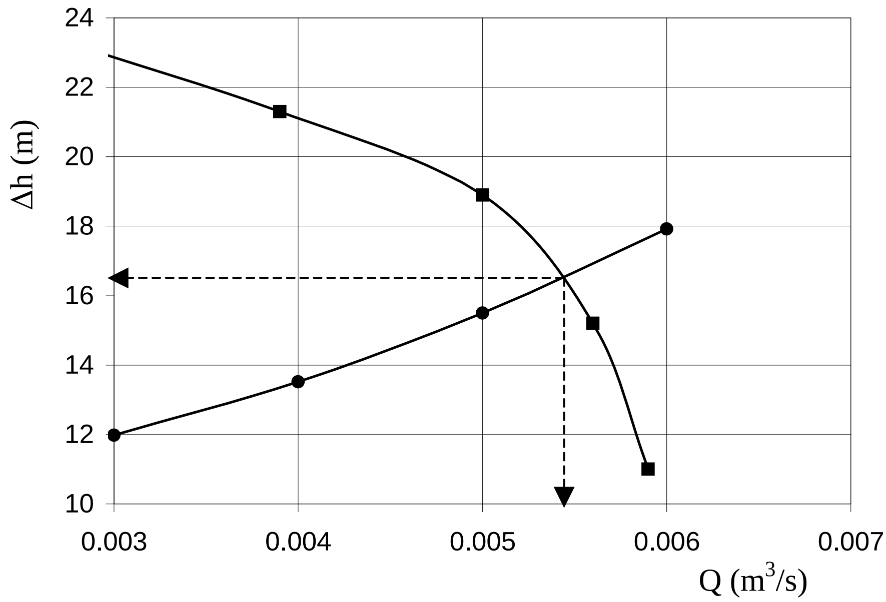
u = Q/A = 321.26 Q

$\Rightarrow \Delta h = 10 + (0.815 + 3.3)(321.26\ Q)^2 = 10 + 2.2 \times 10^5\ Q^2$

To draw the system curve

Q (m^3/s)	0.003	0.004	0.005	0.006
Δh (m)	11.98	13.52	15.5	17.92

From Figure
Q = 0.0054 m^3/s
Δh = 16.4 m



Power required for pump = $\frac{Q\Delta h\ \rho g}{\eta}$ = (0.0054)(16.4)(1000)(9.81)/0.5

= 17.375 kW

Example -5.3-

A centrifugal pump used to take water from reservoir to another through 800 m length and 0.15 m i.d. if the difference in two tank is 8 m, calculate the flow rate of the water and the power required, assume *f* =0.004.

Q (m^3/h)	0	23	46	69	92	115
Δh (m)	17	16	13.5	10.5	6.6	2.0
η	0	0.495	0.61	0.63	0.53	0.1

Solution:

$$\Delta h = \Delta z + \cancel{\frac{\Delta P}{\rho\, g}} + \cancel{\frac{\Delta u^2}{2g\,\alpha}} \left[(h_F)_d + (h_F)_s\right]$$

u = Q/A = 56.59 Q

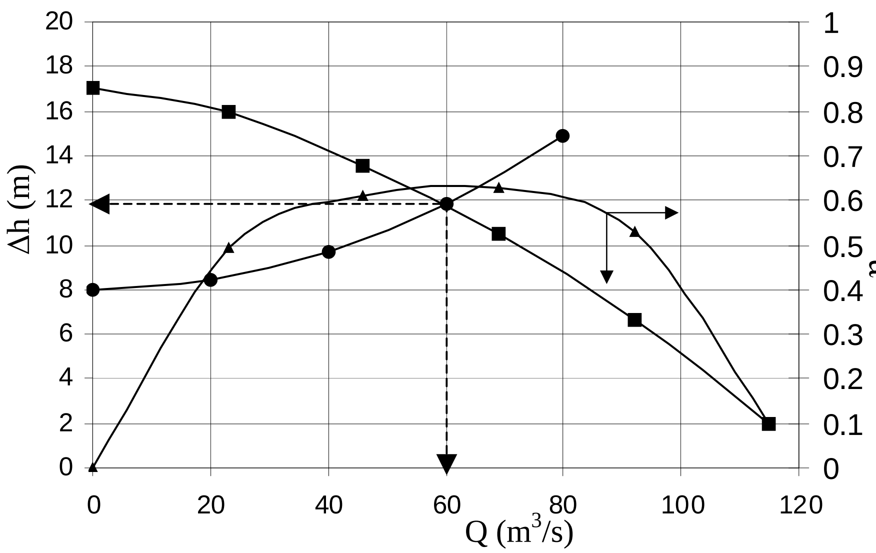
$$(h_F)_{d+s} = 4f\frac{L}{d}\frac{u^2}{2g} = 4(0.004)(800/0.15)(56.59\ Q(h/3600\ s))^2/2g$$

$= 1.0747 \times 10^{-3}\, Q^2$ ----------------------- (Q in m^3/h)

$\Rightarrow \Delta h = 8 + 1.0747 \times 10^{-3}\, Q^2$

To draw the system curve

Q (m^3/h)	0	20	40	60	80
Δh (m)	8.0	8.43	9.72	11.87	14.88



From Figure
Q = 60 m^3/h
Δh = 11.8 m
η = 0.64

Power required for pump = $\dfrac{Q\Delta h\ \rho\, g}{\eta}$ = (60)(1 h/3600 s)(11.8)(1000)(9.81)/0.64

= 3.014 kW

Example -5.4-

A pump take brine solution at a tank and transport it to another in a process plant situated 12 m above the level in the first tank. 250 m of 100 mm i.d. pipe is available sp.gr. of brine is 1.2 and μ = 1.2 cp. The absolute roughness of pipe is 0.04 mm and *f* = 0.0065. Calculate (i) the rate of flow for the pump (ii) the power required for pump if η = 0.65. (iii) if the vapor pressure of water over the brine solution at 86°F is 0.6 psia, calculate the NPSH available, if suction line length is 30 m.

Q (m^3/s)	0.0056	0.0076	0.01	0.012	0.013
Δh (m)	25	24	22	17	13

Solution:

(i) $\Delta h = \Delta z + \cancel{\frac{\Delta P}{\rho\, g}} + \cancel{\frac{\Delta u^2}{2g\, \alpha}} (h_F)_{d+s}$

u = Q/A = 127.33 Q

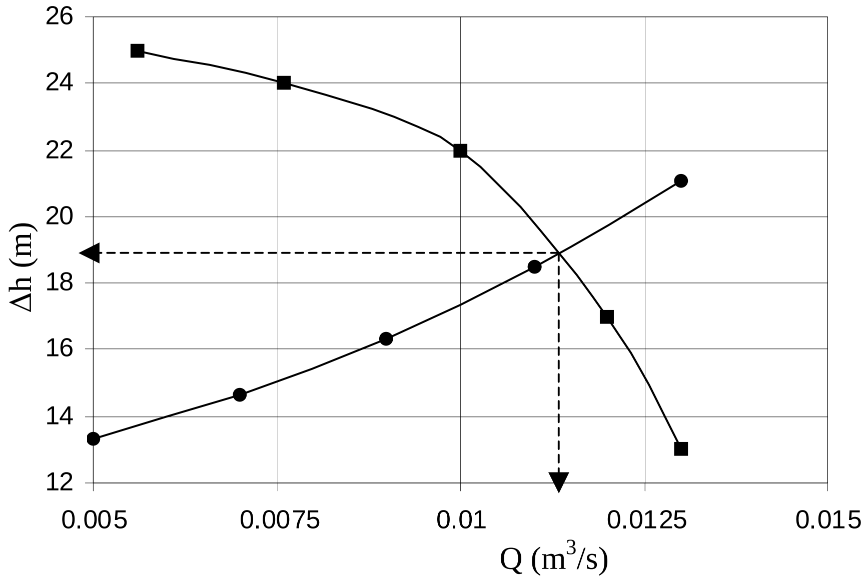
$(h_F)_{d+s} = 4f \frac{L}{d} \frac{u^2}{2g}$ = 4(0.0065)(250/0.1)(127.33 Q)^2/2g

= 53.707 x 10^3 Q^2

⇒ Δh = 12 + 53.707 x 10^3 Q^2

To draw the system curve

Q (m^3/h)	0.005	0.007	0.009	0.011	0.013
Δh (m)	13.34	14.63	16.35	18.5	21.08



From Figure

Q = 0.0114 m^3/s

Δh = 18.9 m

(ii)

Power required for pump = $\frac{Q \Delta h\ \rho\, g}{\eta}$=(00.0114)(18.9)(1200)(9.81)/0.65=3.9 kW

(iii)

$$NPSH = z_s + \left(\frac{P_s - P_v}{\rho\, g} \right) - (h_F)_s$$

u = Q/A = 0.0114/(π/4 0.1^2) =1.45 m/s

For datum line passes through the centerline of the pump (z_s = 0)

$(h_F)_s = 4f \frac{L_s}{d} \frac{u^2}{2g}$ = 4(0.0065)(30/0.1)(1.45)^2/2g = 0.84 m

⇒ NPSH = (101.325 x 10^3 - 0.6psi 101.325 x 10^{3}Pa/14.7psi)/(1200 x 9.81) – 0.84

= 7.416 m

5.9 Centrifugal Pump Relations

The power (P_E) required in ***an ideal centrifugal pump*** can be expected to be a function of the liquid density (ρ), the impeller diameter (D), and the rotational speed of the impeller (N). If the relationship is assumed to be given by the equation,

$P_E = c\, \rho^a N^b D^c$ ----------------------------------(1)

then it can be shown by dimensional analysis that

$P_E = c_1\, \rho\, N^3 D^5$ ----------------------------------(2)

where, c_1 is a constant which depends on the geometry of the system.

The power (P_E) is also proportional to the product of the volumetric flow rate (Q) and the total head (Δh) developed by the pump.

$P_E = c_2\, Q\, \Delta h$ ---------------------------------(3)

where, c_2 is a constant.

The volumetric flow rate (Q) and the total head (Δh) developed by the pump are: -

$Q = c_3\, N\, D^3$ ---------------------------------(4)

$\Delta h = c_4\, N^2\, D^2$ ---------------------------------(5)

where, c_3 and c_4 are constants.

Equation (5) could be written in the following form,

$\Delta h^{3/2} = c_4^{3/2}\, N^3\, D^3$ ---------------------------------(6)

Combine equations (4) and (6) [eq. (4) divided by eq. (6)] to give;

$$\frac{Q}{\Delta h^{\frac{3}{2}}} = \frac{c_3}{c_4^{\frac{3}{2}}}\frac{1}{N} \Rightarrow \frac{QN^2}{\Delta h^{\frac{3}{2}}} = const. \text{ ------------------------------(7)}$$

or, $$\frac{N\sqrt{Q}}{\Delta h^{\frac{3}{4}}} = const. = N_s \text{ ---------------------------------(8)}$$

When the rotational speed of the impeller N is (rpm), the volumetric flow rate Q in (USgalpm) and the total head Δh developed by the pump is in (ft), the constant N_s in equation (8) is known as ***the specific speed of the pump***. The specific speed is used as an index of pump types and always evaluated at the best efficiency point (bep) of the pump. Specific speed vary in the range (400 – 10,000) depends on the impeller type, and has the dimensions of $(L/T^2)^{3/4}$. [British gal=1.2USgal, ft^3=7.48USgal, m^3=264USgal]

5.9.1 Homologous Centrifugal Pumps

Two different size pumps are said to be geometrically similar when the ratios of corresponding dimensions in one pump are equal to those of the other pump. Geometrically similar pumps are said to be homologous. A sets of equations known as the ***affinity laws*** govern the performance of homologous centrifugal pumps at various impeller speeds.

For the tow homologous pumps, equations (4), and (5) are given

$$\frac{Q_1}{Q_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right)^3 \text{ ---------------------------------(9)}$$

$$\frac{\Delta h_1}{\Delta h_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2 \text{ ---------------------------------(10)}$$

Similarly for the tow homologous pumps equation (2)can be written in the form;

$$\frac{P_{E1}}{P_{E2}} = \left(\frac{N_1}{N_2}\right)^3\left(\frac{D_1}{D_2}\right)^5 \text{ ---------------------------------(11)}$$

And by analogy with equation (10),

$$\frac{NPSH_1}{NPSH_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2 \text{---------------------------------(12)}$$

Equations (9), (10), (11), and (12) are the affinity law for homologous centrifugal pumps.

For a particular pump where the impeller of diameter D_1, is replaced by an impeller with a slightly different diameter D_2 the following equations hold

$$\frac{Q_1}{Q_2}=\left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right) \qquad \text{---------------------------------(13)}$$

$$\frac{\Delta h_1}{\Delta h_2}=\left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2 \qquad \text{---------------------------------(14)}$$

$$\frac{P_{E1}}{P_{E2}}=\left(\frac{N_1}{N_2}\right)^3\left(\frac{D_1}{D_2}\right)^3 \qquad \text{---------------------------------(15)}$$

The characteristic performance curves are available for a centrifugal pump operating at a given rotation speed, equations (13), (14), and (15) enable the characteristic performance curves to be plotted for other operating speeds and for other slightly impeller diameters.

Example -5.5-

A volute centrifugal pump with an impeller diameter of 0.02 m has the following performance data when pumping water at the best efficiency point (bep). Impeller speed N = 58.3 rev/s capacity Q = 0.012 m^3/s, total head Δh = 70 m, required NPSH = 18 m, and power = 12,000 W. Evaluate the performance data of an homologous pump with twice the impeller diameter operating at half the impeller speed.

Solution:

Let subscripts 1 and 2 refer to the first and second pump respectively,

$N_1/N_2 = 2$, $\qquad D_1/D_2 = 1/2$

Ratio of capacities

$$\frac{Q_1}{Q_2}=\left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right)^3 = 2\ (1/8) = 1/4$$

$\Rightarrow$ Capacity of the second pump $Q_2 = 4\ Q_1 = 4(0.012) = 0.048\ m^3/s$

Ratio of total heads

$$\frac{\Delta h_1}{\Delta h_2}=\left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2 = 4\ (1/4) = 1$$

$\Rightarrow$ Total head of the second pump $\Delta h_2 = \Delta h_1 = 70$ m

Ratio of powers

$$\frac{P_{E1}}{P_{E2}}=\left(\frac{N_1}{N_2}\right)^3\left(\frac{D_1}{D_2}\right)^5 = 8\ (1/32) = ¼$$

assume $\dfrac{P_{B1}}{P_{B2}}=\dfrac{P_{E1}}{P_{E2}}=\dfrac{1}{4}$

$\Rightarrow$ Break power of the second pump $P_{B2} = 4\ P_{B1} = 4(12{,}000) = 48{,}000$ W

$$\frac{NPSH_1}{NPSH_2}=\left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2 = 4\ (1/4) = 1$$

$\Rightarrow$ NPSH of the second pump $NPSH_2 = NPSH_1 = 18$ m

H.W.

Calculate the specific speed for these two pumps. **Ans.** $N_{s1} = N_{s2} = 816.4$

Note: -

The break power P_B can be defined as the actual power delivered to the pump by prime mover. It is the sum of liquid power and friction power and is given by the equation, $P_B = \frac{P_E}{\eta}$

Example -5.6-

A centrifugal pump was manufactured to couple directly to a 15 hp electric motor running at 1450 rpm delivering 50 liter/min against a total head 20 m. It is desired to replace the motor by a diesel engine with 1,000 rpm speed and couple it directly to the pump. Find the probable discharge and head developed by the pump. Also find the hp of the engine that would be employed.

Solution:

With the same impeller $D_1 = D_2$,
then $Q_1/Q_2 = N_1/N_2$

$\Rightarrow Q_2 = 50\ (1000 / 1450) = 34.5$ liter/min
and $\Delta h_2 = \Delta h_1\ (N_2/N_1)^2 = 20\ (1000/1450)^2 = 9.5$ m
$P_{E2} = P_{E1}\ (N_2/N_1)^3 = 15\ (1000/1450)^3 = 4.9$ hp

H.W.

1- Repeat example 5.6 with Q_1 = 850 lit/min, Δh_1 = 40 m, N = 1450 rpm, and Power = 15 hp.
2- Calculate the pump efficiency (η) for pumping of water, and the specific speed for these two pumps.

Ans. $\eta = 0.497 \approx 0.5$, and $N_{s1} = N_{s2} = 650$

5.10 Centrifugal Pumps in Series and in Parallel

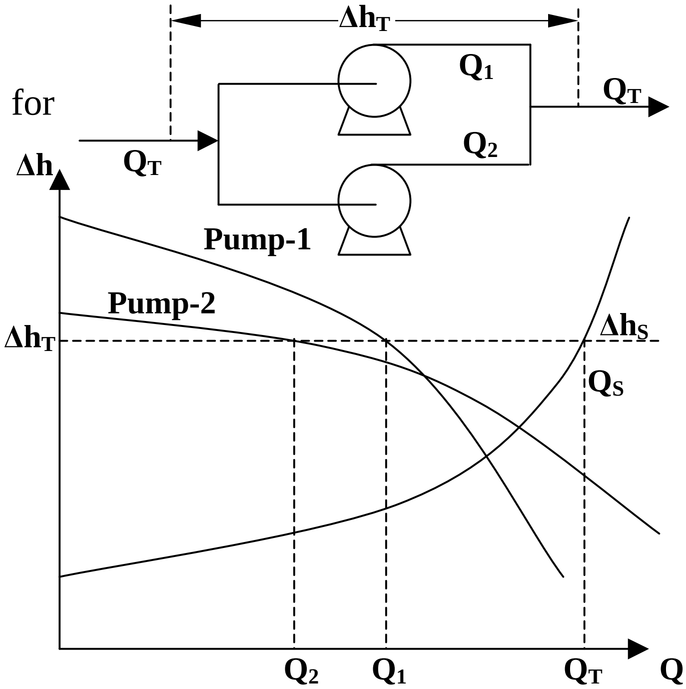
5.10.1 Centrifugal Pumps in Parallel

Consider two centrifugal pumps in ***parallel***. The total head for the pump combination (Δh_T) is the same as the total head for each pump,

$\Delta h_T = \Delta h_1 = \Delta h_2$

$Q_T = Q_1 + Q_2$

The operating characteristics curves for two pumps in parallel are: -



Solution by trail and error

1- Draw Δh versus Q for the two pumps and the system.
2- Draw horizontal Δh_T line and determine Q_1, Q_2, and Q_S.
3- Q_T (Total) = $Q_1 + Q_2 = Q_S$ (system).
4- If $Q_T \neq Q_S$ repeat steps 2, 3, and 4 until $Q_T = Q_S$.

Another procedure for solution

1- The same as above.
2- Draw several horizontal lines (4 to 6) for Δh_T and determine their Q_T.
3- Draw Δh_T versus Q_T.
4- The duty point is the intersection of Δh_T curve with Δh_S curve.

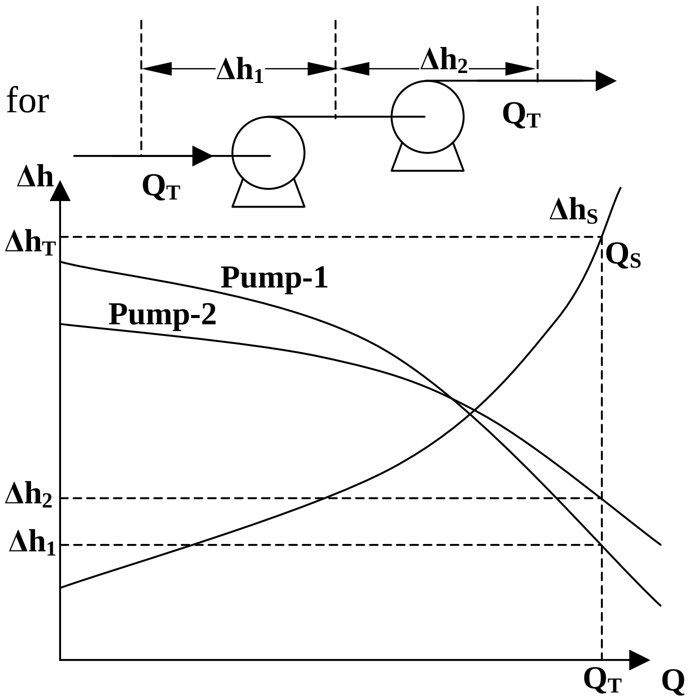
5.10.2 Centrifugal Pumps in Series

Consider two centrifugal pumps in ***series***. The total head for the pump combination (Δh_T) is the sum of the total heads for the two pumps,

$\Delta h_T = \Delta h_1 + \Delta h_2$

$Q_T = Q_1 = Q_2$

The operating characteristics curves for two pumps in series are: -



Solution by trail and error

1- Draw Δh versus Q for the two pumps and the system.
2- Draw vertical Q_T line and determine Δh_1, Δh_2, and Δh_S.
3- Q_T (Total) = $Q_1 + Q_2 = Q_S$ (system).
4- If $\Delta h_T \neq \Delta h_S$ repeat steps 2, 3, and 4 until $\Delta h_T = \Delta h_S$.

Another procedure for solution

1- The same as above.
2- Draw several Vertical lines (4 to 6) for Q_T and determine their Δh_T.
3- Draw Δh_T versus Q_T.
4- The duty point is the intersection of Δh_T curve with Δh_S curve.

Home Work

P.5.1

Show that for homologous pumps, the specific speed (N_S) of them is not depended on the impeller rotational speed (N) and its diameter (D).

P.5.2

Figure 1.5 diagrammatically represents the heads in a liquid flowing through a pipe. Redraw this diagram with a pump placed between points 1 and 2.

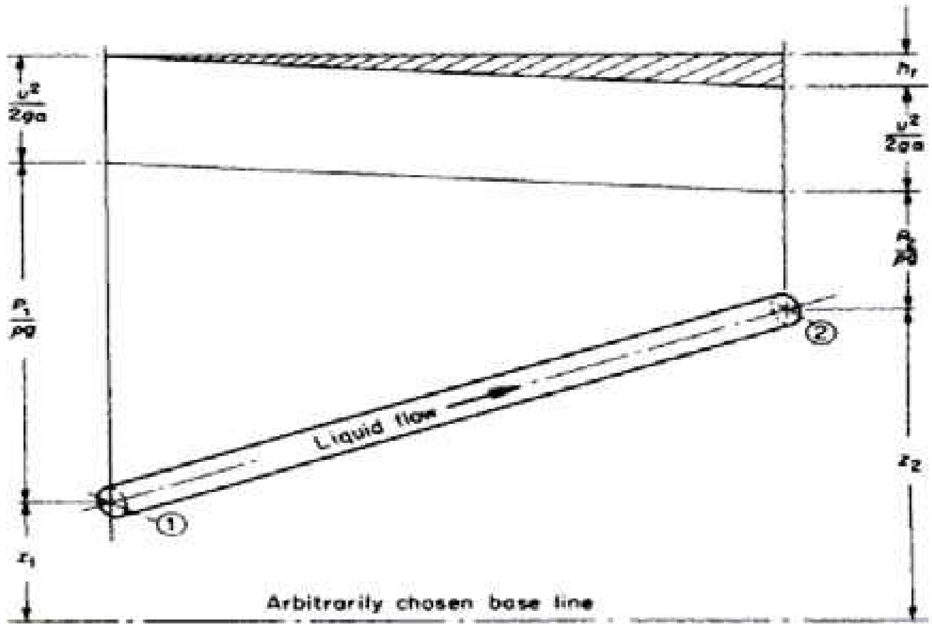


Figure 1.5
Diagrammatic representation of heads in a liquid flowing through a pipe

P.5.3

Calculate the available net positive section head NPSH in a pumping system if the liquid density ρ = 1200 kg/m^3, the liquid dynamic viscosity μ = 0.4 Pa s, the mean velocity u = 1 m/s , the static head on the suction side z_s = 3 m, the inside pipe diameter d_i = 0.0526 m, the gravitational acceleration g = 9.81 m/s^2, and the equivalent length on the suction side $(\sum Le)_s$ = 5.0 m.

The liquid is at its normal boiling point. Neglect entrance and exit losses.

P.5.4

A centrifugal pump is used to pump a liquid in steady turbulent flow through a smooth pipe from one tank to another. Develop an expression for the system total head Δh in terms of the static heads on the discharge and suction sides z_d and z_s respectively, the gas pressures above the tanks on the discharge and suction sides p_d and P_s respectively, the liquid density ρ, the liquid dynamic viscosity μ, the gravitational acceleration g, the total equivalent lengths on the discharge and suction sides $(\sum Le)_d$ and $(\sum Le)_s$ respectively, and the volumetric flow rate Q.

P.5.5

A system total head against mean velocity curve for a particular power law liquid in a particular pipe system can be represented by the equation

$\Delta h = (0.03)(100^n)(u^n) + 4.0$ for $u \leq 1.5$ m/s where, Δh is the total head in m, u is the mean velocity in m/s, and n is the power law index.

A centrifugal pump operates in this particular system with a total head against mean velocity curve represented by the equation

$\Delta h = 8.0 - 0.2u - 1.0u^2$ for $u \leq 1.5$ m/s

(This is a simplification since Δh is also affected by n).

(a) Determine the operating points for the pump for

(i) a Newtonian liquid

(ii) a shear thinning liquid with n = 0.9

(iii) a shear thinning liquid with n = 0.8.

(b) Comment on the effect of slight shear thinning on centrifugal pump operation.

P.5.6

A volute centrifugal pump has the following performance data at the best efficiency point:

Volumetric flow rate	Q = 0.015 m3/s
Total head	Δh = 65 m
Required net positive suction head	NPSH = 16 m
Liquid power	P_E = 14000 W
Impeller speed	N = 58.4 rev/s
Impeller diameter	D = 0.22 m

Evaluate the performance of a homologous pump which operates at an impeller speed of 29.2 rev/s but which develops the same total head Δh and requires the same NPSH.

P.5.7

Two centrifugal pumps are connected in series in a given pumping system. Plot total head Δh against capacity Q pump and system curves and determine the operating points for

(a) only pump 1 running (b) only pump 2 running (c) both pumps running

on the basis of the following data:

operating data for pump 1

Δh_1 m,	50.0	49.5	48.5	48.0	46.5	44.0	42.0	39.5	36.0	32.5	28.5
Q m^3/h,	0	25	50	75	100	125	150	175	200	225	250

operating data for pump 2

Δh_2 m,	40.0	39.5	39.0	38.0	37.0	36.0	34.0	32.0	30.5	28.0	25.5
Q m^3/h,	0	25	50	75	100	125	150	175	200	225	250

data for system

Δh_s m,	35.0	37.0	40.0	43.5	46.5	50.5	54.5	59.5	66.0	72.5	80.0
Q m^3/h,	0	25	50	75	100	125	150	175	200	225	250

P.5.8

Two centrifugal pumps are connected in parallel in a given pumping system. Plot total head Ah against capacity Q pump and system curves for both pumps running on the basis of the following data:

operating data for pump 1

Δh m,	40.0	35.0	30.0	25.0
$Q_1 m^3$/h,	169	209	239	265

operating data for pump 2

Δh m,	0.0	35.0	30.0	25 .0
$Q_2 m^3$/h	0	136	203	267

data for system

Δh m,	20.0	25.0	30.0	35.0
$Q_s m^3/h$,	0	244	372	470

CHPTER SIX

Non-Newtonian Fluids

6.1 Introduction

For Newtonian fluids a plot of shear stress (τ), against shear rate ($-\dot{\gamma} \equiv du_x/dy$) on Cartesian coordinate is a straight line having a slope equal to the dynamic viscosity (μ). For many fluids a plot of shear stress against shear rate does not give a straight line. These are so-called " Non-Newtonian Fluids". Plots of shear stress against shear rate are experimentally determined using viscometer.

The term viscosity has no meaning for a non-Newtonian fluid unless it is related to a particular shear rate $\dot{\gamma}$. ***An apparent viscosity*** (μ_a) can be defined as follows: -

$$\boxed{\mu_a = \frac{\tau}{\dot{\gamma}}}$$

6.2 Types of Non-Newtonian Fluids

There are two types of non-Newtonian fluids: -

1- Time-independent.
2- Time-dependent.

6.2.1 Time-Independent Non-Newtonian Fluids

In this type ***the apparent viscosity*** depends only on the rate of shear at any particular moment and not on the time for which the shear rate is applied.

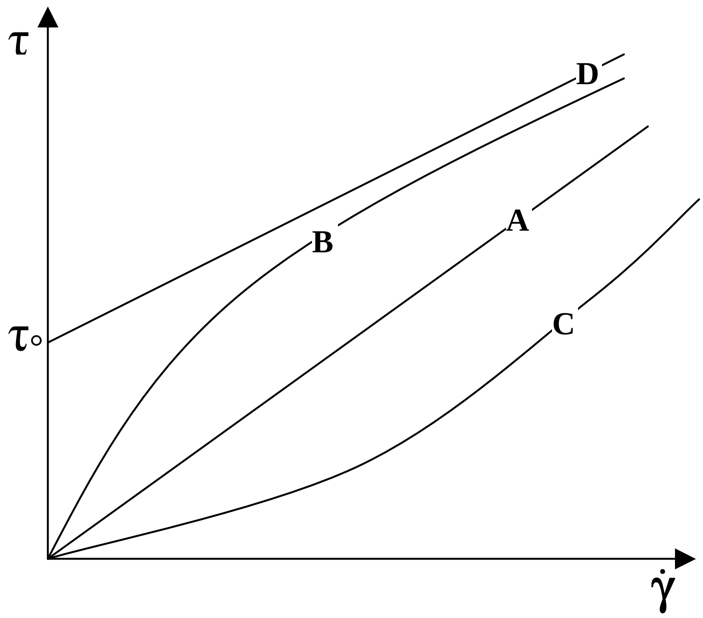
For non-Newtonian fluids the relationship between shear stress and shear rate is more complex and this type can be written as: -

$\boxed{\tau = k(-\dot{\gamma})^n}$ ------------------For power-law fluids

or as $\boxed{\tau = \tau_\circ + k(-\dot{\gamma})}$ ------------------For Bingham plastics fluids

The shape of the flow curve for time-independent fluids in compare with Newtonian fluid is shown in thee Figure, where

A: Newtonian fluids
B: Pseudoplastic fluids [power-law n<1]
Ex. Polymer solution, detergent.
C: Dilatant fluids [power-law n>1]
Ex. Wet beach sand, starch in water.
D: Bingham plastic fluids, it required ($\tau_\circ$) for initial flow
Ex. Chocolate mixture, soap, sewage sludge, toothpaste.



6.2.2 Time-Dependent Non-Newtonian Fluids

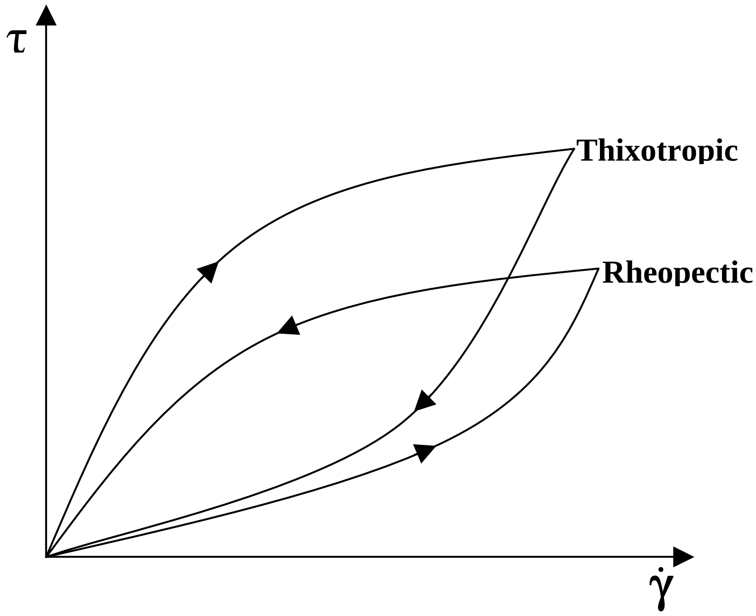
For this type the curves of share stress versus shear rate depend on how long the shear has been active. This type is classified into: -

1- ***Thixotropic Fluids***

Which exhibit a reversible decrease in shear stress and apparent viscosity with time at a constant shear rate. Ex. Paints.

2- ***Rheopectic Fluids***

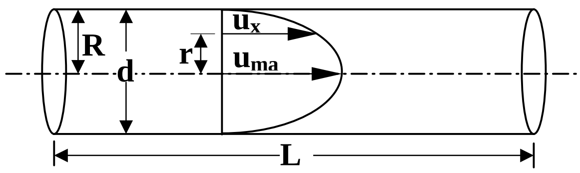
Which exhibit a reversible increase in shear stress and apparent viscosity with time at a constant shear rate. Ex. Gypsum suspensions, bentonite clay.



6.3 Flow Characteristic [8u/d]

The velocity distribution for Newtonian fluid of laminar flow through a circular pipe, as given in chapter four, is given by the following equation;

$$u_x = u_{max}\left[1-\left(\frac{r}{R}\right)^2\right] = 2u\left[1-\left(\frac{r}{R}\right)^2\right]$$



where, u: is the mean (average) linear velocity

u = Q/A

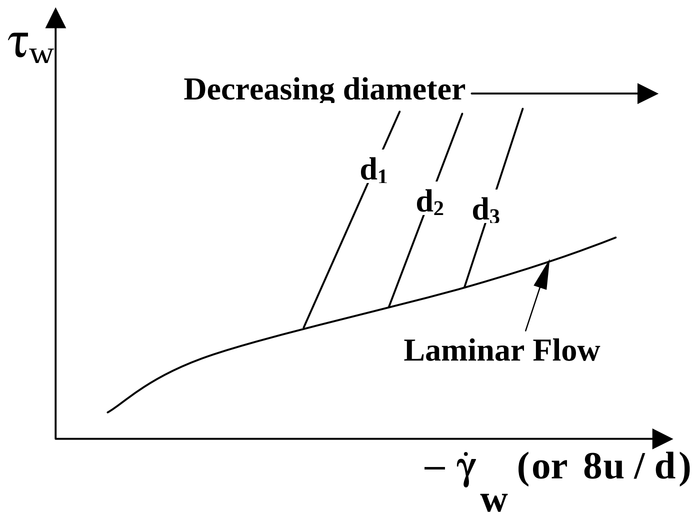
$$\dot{\gamma} = \frac{du_x}{dr} = 2u\left(-\frac{2r}{R}\right) = -4u\frac{r}{R^2}$$

- At pipe walls (r = R) $\dot{\gamma} = \dot{\gamma}_w = \left.\frac{du_x}{dr}\right|_{r=R}$

$\Rightarrow \dot{\gamma}_w = -\frac{4u}{R}$. $\boxed{-\dot{\gamma}_w = \frac{8u}{d}}$ Flow characteristic

For laminar flow

$\tau_w = -\mu\dot{\gamma}_w = \mu\frac{8u}{d}$ ----------at wall



The force balance on an element of fluid of L length is;

$$\tau_w \pi d L = \frac{\pi}{4}d^2\Delta P$$

$\Rightarrow \tau_w = \frac{\Delta P}{4L/d} = \mu\frac{8u}{d}$ -----------(1) this equation for Newtonian fluids

A plot of τ_w or $\Delta P/(4L/d)$ against $-\dot{\gamma}_w$ or (8u/d) is shown in Figure for a typical time independent non-Newtonian fluid flows in a pipe. In laminar flow the plot gives a single line independent of pipe size. In turbulent flow a separate line for each pipe size.

6.4 Flow of Genral Time-Independent Non-Newtonian Fluids

The slope of a log-log plot of shear stress ***at the pipe walls*** against flow characteristic [8u/d] *at any point along the pipe* is the flow behavior index (n′)

$$\mathbf{n'} = \frac{d\ln\tau_w}{d\ln(-\dot{\gamma}_w)} = \frac{d\ln\tau_w}{d\ln(8u/d)} = \frac{d\ln[\Delta P/(4L/d)]}{d\ln(8u/d)} \qquad \text{----------------------(2)}$$

This equation lead to,

$$\tau_w = \frac{\Delta P}{4L/d} = Kp'\left(\frac{8u}{d}\right)^{n'} \qquad \text{----------------------(3)}$$

where, Kp′ and n′ are ***point values*** for a particular value of the flow characteristic (8u/d). or as'

$$\tau_w = \frac{\Delta P}{4L/d} = Kp'\left(\frac{8u}{d}\right)^{n'-1}\frac{8u}{d} \qquad \text{----------------------(4)}$$

By the analogy of equation (4) with equation (1), the following equation can be written for non-Newtonian fluids;

$$\tau_w = \frac{\Delta P}{4L/d} = (\mu_a)_P\left(\frac{8u}{d}\right) \qquad \text{----------------------(5)}$$

where, $(\mu_a)_P$ is apparent viscosity for pipe flow.

$$\therefore (\mu_a)_P = Kp'\left(\frac{8u}{d}\right)^{n'-1} \qquad \text{----------------------(6)}$$

This equation gives a ***point value*** for the apparent viscosity of non-Newtonian fluid flow through a pipe.

Reynolds number for the of non-Newtonian fluids can be written as;

$$Re = \frac{\rho u d}{(\mu_a)_P} = \frac{\rho u d}{Kp'\left(\frac{8u}{d}\right)^{n'-1}} \qquad \text{----------------------(7)}$$

$$\Rightarrow Re = \frac{\rho u^{2-n'} d^{n'}}{m} \qquad \text{----------------------(8)}$$

where, $m = Kp'\,(8^{n'-1})$

Equations (7) or (8) gives a ***point value*** for Re at a particular flow characteristic (8u/d).

A point value of the basic friction factor (Φ or J_f) or fanning friction factor (f) for <u>laminar flow</u> can be obtained from;

$\Phi = J_f = 8 / Re$ or $f = 16 / Re$ --------------(9)

The pressure drop due to skin friction can be calculated in the same way as for Newtonian fluids,

$-\Delta P_{fs} = 4f\,(L/d)\,(\rho u^2/2)$ ----------------------(10)

Equation (10) is used for laminar and turbulent flow, and the fanning friction factor (f) for turbulent flow of general time independent non-Newtonian fluids in ***<u>smooth</u> cylindrical pipes*** can be calculated from;

$f = a / Re^b$ ----------------------(11)

where, a, and b are function of the flow behavior index (n′)

n′	0.2	0.3	0.4	0.6	0.8	1.0	1.4	2.0
a	0.0646	0.0685	0.0714	0.074	0.0761	0.0779	0.0804	0.0826
b	0.349	0.325	0.307	0.281	0.263	0.25	0.231	0.213

There is another equation to calculate (f) for turbulent flow of time-independent non-Newtonian fluids in smooth cylindrical pipes;

$$\frac{1}{f^{1/2}}=\frac{4}{(n')^{0.75}}\log[\mathrm{Re}\ f^{(1-\frac{n'}{2})}]-\frac{0.4}{(n')^{1.2}} \qquad \text{----------------------(12)}$$

Example -6.1-

A general time-independent non-Newtonian liquid of density 961 kg/m^3 flows steadily with an average velocity of 1.523 m/s through a tube 3.048 m long with an inside diameter of 0.0762 m. For these conditions, the pipe flow consistency coefficient Kp' has a value of 1.48 Pa.s$^{0.3}$ [or 1.48 (kg / m.s^2) s$^{0.3}$] and n' a value of 0.3. Calculate the values of the apparent viscosity for pipe flow $(\mu_a)_P$, the Reynolds number Re and the pressure drop across the tube, neglecting end effects.

Solution:

Apparent viscosity $(\mu_a)_P = Kp'(\frac{8u}{d})^{n'-1}$

$= 1.48$ (kg/m) s$^{-1.7}$ [8 (1.523)/0.0762]$^{-0.7}$s$^{-0.7}$

$= 0.04242$ kg/m.s (or Pa .s)

$$\mathrm{Re}=\frac{\rho u d}{(\mu_a)_P}=\frac{\rho u d}{Kp'(\frac{8u}{d})^{n'-1}} = 961\,(1.523)(0.762)\,/\,0.04242$$

$=2629$

$f = a / \mathrm{Re}^b$ from table n' = 0.3, a = 0.0685, b = 0.325

$f = 0.0685 / 2629^{0.325} = 0.005202$

$-\Delta P_{fs} = 4f\,(L/d)\,(\rho u^2/2) = 4(0.005202)\,(3.048 / 0.0762)[961(1.523)^2/2]$

$= 927.65$ Pa.

6.5 Flow of Power-Law Fluids in Pipes

Power-law fluids are those in which the shear stress (τ) is related to the shear rate ($\dot{\gamma}$) by this equation;

$$\tau = k(\dot{\gamma})^n \qquad \text{----------------------(13)}$$

For shear stress at a pipe wall (τ_w) and the shear rate at the pipe wall ($\dot{\gamma}_w$), equation (13) becomes;

$$\tau_w = k(\dot{\gamma}_w)^n \qquad \text{----------------------(14)}$$

Equation (3) gives the relationship between (ΔP) and (8u/d) for general time-independent non-Newtonian fluids.

But for power-law fluids the parameters Kp′ and n′ in equation (3) are no longer point values but remain constant over a range of (8u/d), so that for power-law fluids equation (3) can be written as;

$$\tau_w = \frac{\Delta P}{4L/d} = Kp\left(\frac{8u}{d}\right)^n \qquad \text{----------------------(15)}$$

where,

Kp: is the consistency coefficient for pipe flow.

n: is the power-law inex.

The shear rate at pipe wall for general time-independent non-Newtonian fluids is;

$$\dot{\gamma}_w = \frac{8u}{d}\left(\frac{3n'+1}{4n'}\right) \qquad \text{----------------------(16)}$$

and for <u>power-law fluids</u>;

$$\dot{\gamma}_w = \frac{8u}{d}\left(\frac{3n+1}{4n}\right) \qquad \text{----------------------(17)}$$

Combine equations (14), (15), and (17) to give the relationship between the general consistency coefficient (K) and the consistency coefficient for pipe flow (Kp).

$$Kp = \frac{8u}{d}\left(\frac{3n+1}{4n}\right)^n \qquad \text{----------------------(18)}$$

The apparent viscosity for power-law fluids in pipe flow

$$(\mu_a) = Kp\left(\frac{8u}{d}\right)^{n-1} \qquad \text{----------------------(19)}$$

The Reynolds number for non-Newtonian fluids flow in pipe

$$\mathrm{Re} = \frac{\rho u d}{(\mu_a)_P} \qquad \text{----------------------(20)}$$

For power –law fluids flow in pipes the Re can be written either as;

$$\mathrm{Re} = \frac{\rho u d}{Kp(\frac{8u}{d})^{n-1}} \qquad \text{----------------------(21)}$$

or as;

$$\mathrm{Re} = \frac{\rho u^{2-n} d^n}{m} \qquad \text{----------------------(22)}$$

where, $m = Kp\,(8^{n-1})$ ----------------------(23)

<u>Example -6.2-</u>

A Power-law liquid of density 961 kg/m^3 flows in steady state with an average velocity of 1.523 m/s through a tube 2.67 m length with an inside diameter of 0.0762 m. For a pipe consistency coefficient of 4.46 Pa.s^n [or 4.46 (kg / m.s^2) s$^{0.3}$], calculate the values of the apparent viscosity for pipe flow $(\mu_a)_P$ in Pa.s, the Reynolds number Re, and the pressure drop across the tube for power-law indices n = 0.3, 0.7, 1.0, and 1.5 respectively.

<u>Solution:</u>

Apparent viscosity $(\mu_a)_P = Kp(\frac{8u}{d})^{n-1}$

$= 4.46\ (\text{kg/m})\ s^{n-2}\ [8\ (1.523)/0.0762]^{n-1} s^{n-1}$

$\Rightarrow \quad (\mu_a)_P = 4.46\ (159.9)^{n-1}$ ----------------------------(1)

$$\mathrm{Re} = \frac{\rho u d}{(\mu_a)_P} = \frac{\rho u d}{4.46\,(159.9)^{n-1}} = 961\ (1.523)(0.762)\ /\ 4.46\ (159.9)^{n-1}$$

$\Rightarrow \quad \mathrm{Re} = 25.006/\ (159.9)^{n-1}$ ----------------------------(2)

$-\Delta P_{fs} = 4f\,(L/d)\,(\rho u^2/2) = 4(16/\mathrm{Re})\ (2.67\ /\ 0.0762)[961(1.523)^2/2]$ for laminar

$\Rightarrow \quad -\Delta P_{fs} = 99950.56\ (159.9)^{n-1}$ --------------------------(3)

n	$(\mu_a)_P$ Eq.(1)	Re Eq.(2)	$-\Delta P_{fs}$ Eq.(3)	$(-\Delta P_{fs})_{New}$ / $(-\Delta P_{fs})_{non\text{-}New}$
0.3	0.1278	872.44	2,865	0.0287
0.7	0.9732	114.6	21,809	0.218
1.0	4.46	25.006	999,50.56	1.0
1.5	56.4	1.9776	1,263,890.541	12.7

6.6 Friction Losses Due to Form Friction in Laminar Flow

Since non-Newtonian power-law fluids flowing in conduits are often in laminar flow because of their usually high effective viscosity, loss in sudden changes of diameter (velocity) and in fittings are important in laminar flow.

1- Kinetic Energy in Laminar Flow

Average kinetic energy per unit mass = $u^2/2\alpha$ [m^2/s^2 or J/kg]

$\alpha = 1.0$ -------------in turbulent flow

$$\alpha = \frac{(2n+1)(5n+3)}{3(3n+1)^2}$$ -------------in laminar flow

- For Newtonian fluids (n = 1.0) ⇒ $\alpha = 1/2$ in laminar flow
- For power-law non-Newtonian fluids ($n < 1.0$ **or** $n > 1.0$)

2- Losses in Contraction and Fittings

The frictional pressure losses for non-Newtonian fluids are *very similar* to those for Newtonian fluids at the same generalized Reynolds number in laminar and turbulent flow for **contractions** and also for **fittings** and **valves**.

3- Losses in Sudden Expansion

For a non-Newtonian power-law fluid flow in laminar flow through a sudden expansion from a smaller inside diameter d_1 to a larger inside diameter d_2 of circular cross-sectional area, then the energy losses is

$$F_e = \left[\frac{n+3}{2(5n+3)}\left(\frac{d_1}{d_2}\right)^4 - \left(\frac{d_1}{d_2}\right)^4 + \frac{3(3n+1)}{2(5n+3)}\right]\frac{3n+1}{2n+1}u_1^2$$

6.7 Turbulent Flow and Generalized Friction Factor

The generalized Reynolds number has been defined as

$$\mathbf{Re} = \frac{d^{n'} u^{2-n'} \rho}{m}$$

where, m = Kp′ 8^{n-1} = K 8^{n-1} $(3n+1/4n)^n$

The fanning friction factor is plotted versus the generalized Reynolds number. Since many non-Newtonian power-law fluids have high effective viscosities, they are often in laminar flow. The correction for smooth tube also holds for a rough pipe in laminar flow.

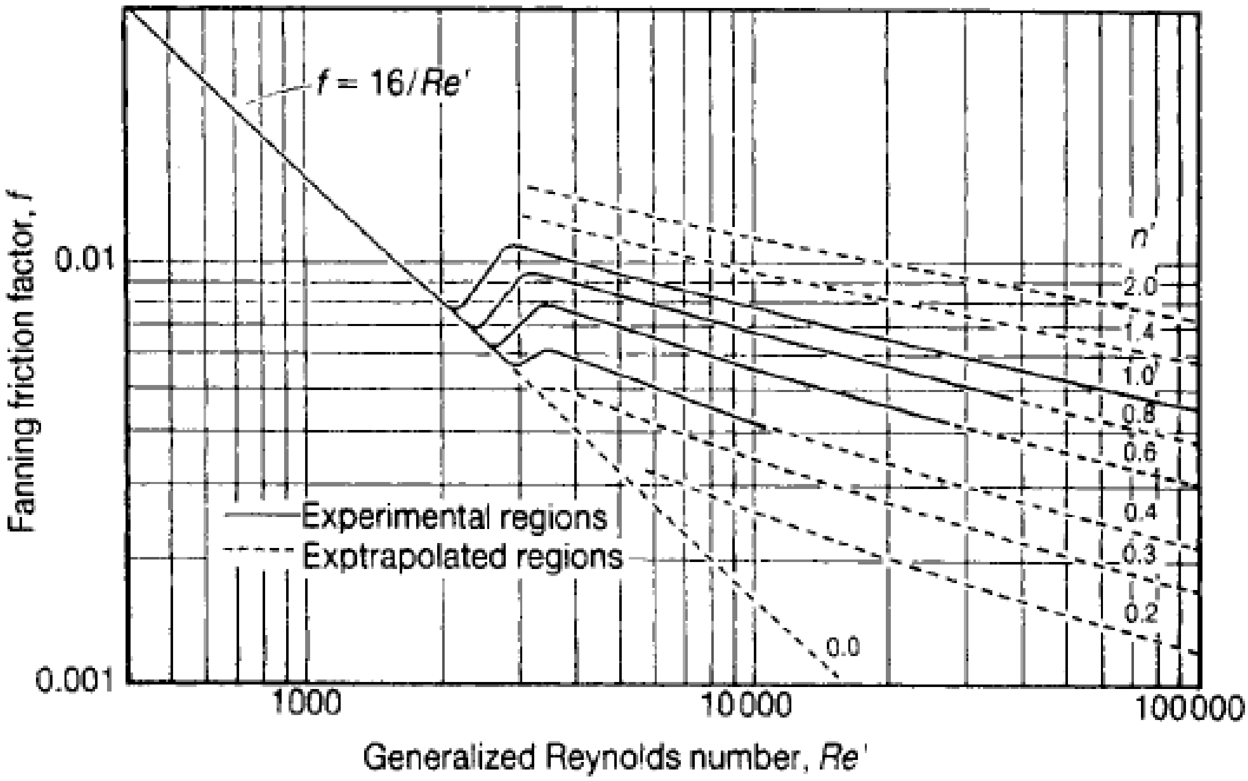


Figure of friction factor chart for purely viscous non-Newtonian fluids

For rough pipes with various values of roughness ratio (e/d), this figure can not be used for turbulent flow, since it is derived for smooth pipes.

Example -6.3-

A pseudoplastic fluid that follows the power-law, having a density of 961 kg/m^3 is flowing in steady state through a smooth circular tube having an inside diameter of 0.0508 m at an average velocity of 6.1 m/s. the flow properties of the fluid are n′ = 0.3, Kp = 2.744 Pa.s^n. Calculate the frictional pressure drop across the tubing of 30.5 m long.

Solution:

$$\mathbf{Re} = \frac{d^{n'} u^{2-n'} \rho}{m} = (1.523)^{0.3}\,(6.1)^{1.7}\,(961) / 2.744\,(8)^{-0.7}$$

$$= 1.328 \times 10^4 \qquad \text{------------ the flow is turbulent}$$

From Figure for Re = 1.328 x10^4, n′ = 0.3 $\Rightarrow f = 0.0032$

$-\Delta P_{fs} = 4f\,(L/d)\,(\rho u^2/2) \quad = 4(0.0032)\,(30.5 / 0.0508)[961(6.1)^2/2]$

$\Rightarrow \quad -\Delta P_{fs} = 134.4$ kPa

Example -6.4-

The laminar flow velocity profile in a pipe for a power-law liquid in steady state flow is given by the equation

$u_x = u\dfrac{3n+1}{n+1}\left[1-\left(\dfrac{2r}{d}\right)^{\frac{n+1}{n}}\right]$, where n is the power-law index and u, is the mean velocity.

Use this equation to drive the following expression

$\dot{\gamma}_w = -\left.\dfrac{du_x}{dr}\right|_{r=\frac{d}{2}} = \left(\dfrac{8u}{d}\right)\left(\dfrac{3n+1}{4n}\right)$ for the velocity gradient at the pipe walls.

Solution:

$$u_x = u\frac{3n+1}{n+1}\left[1-\left(\frac{2r}{d}\right)^{\frac{n+1}{n}}\right]$$

$$\frac{du_x}{dr} = u\frac{3n+1}{n+1}\left[-\frac{n+1}{n}\,\frac{2}{d}\left(\frac{2r}{d}\right)^{\frac{n+1}{n}-1}\right]$$

$$= \frac{2u}{d}\left(\frac{3n+1}{n+1}\right)\left(\frac{2r}{d}\right)^{\frac{1}{n}}$$

$$-\dot{\gamma}_w = -\left.\frac{du_x}{dr}\right|_{r=\frac{d}{2}} = \frac{2u}{d}\left(\frac{3n+1}{n}\right) \qquad \times (4/4)$$

$$\Rightarrow -\dot{\gamma}_w = -\left.\frac{du_x}{dr}\right|_{r=\frac{d}{2}} = \left(\frac{8u}{d}\right)\left(\frac{3n+1}{4n}\right)$$

Home Work

P.6.1

The shear stress in power-law liquids in steady state laminar flow is given by the equation

$\tau_{rx} = K\left(-\frac{du_x}{dr}\right)^n$, show that the velocity distribution is given by the following equation

$u_x = u_{max}\left[1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right]$, where $u_{max} = \frac{n}{n+1}\left(\frac{-\Delta P_{fs}}{2KL}\right)^{\frac{1}{n}} R^{\frac{n+1}{n}}$

Hint: $\tau_{rx}(2\pi rL) = -\Delta P_{fs}(\pi r^2) \Rightarrow \tau_{rx} = \frac{-\ \Delta P_{fs}}{2L} r$

P.6.2

Calculate the frictional pressure gradient $-\Delta P_{fs}/L$ for a time independent non-Newtonian fluid in steady state flow in a cylindrical tube if

the liquid density the	ρ = 1000 kg/m3
inside diameter of the tube	d = 0.08 m
the mean velocity	u = l.0 m / s
the point pipe consistency coefficient	K' = 2 Pa. $s^{0.5}$
and the flow behavior index	n' = 0.5.

P.6.3

Substitute the equation

$\tau_{rx} = K\left(-\frac{du_x}{dr}\right)^n$ into equation $\frac{8u}{d} = \frac{32}{d^3}\int_0^{\frac{d}{2}} r^2\left(-\frac{du_x}{dr}\right)dr$

and integrate to show the shear rate at a pipe wall for power law fluid in steady state flow is

$$-\dot{\gamma}_w = -\left.\frac{du_x}{dr}\right|_{r=\frac{d}{2}} = \left(\frac{8u}{d}\right)\left(\frac{3n+1}{4n}\right)$$

Hint: $\tau_{rx}(2\pi rL) = -\Delta P_{fs}(\pi r^2) \Rightarrow \tau_{rx} = \frac{-\ \Delta P_{fs}}{2L} r$

CHPTER SEVEN

Flow Measurement

7.1 Introduction

It is important to be able to measure and control the amount of material entering and leaving a chemical and other processing plants. Since many of the materials are in the form of fluids, they are flowing in pipes or conduits. Many different types of devices are used to measure the flow of fluids. The flow of fluids is most commonly measured using ***head flow meters***. The operation of these flow meters is based on the Bernoulli's equation.

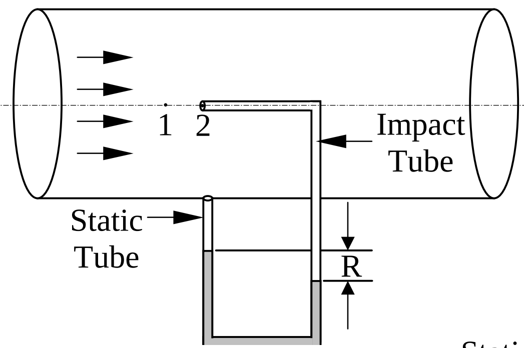
A construction in the flow path is used to increase in the lines flow velocity. This is accompanied by a decrease in pressure intensity or head and since *the resultant pressure drop is a function of the flow rate of fluid*, the latter can be evaluated.

7.2 Flow Measurement Apparatus

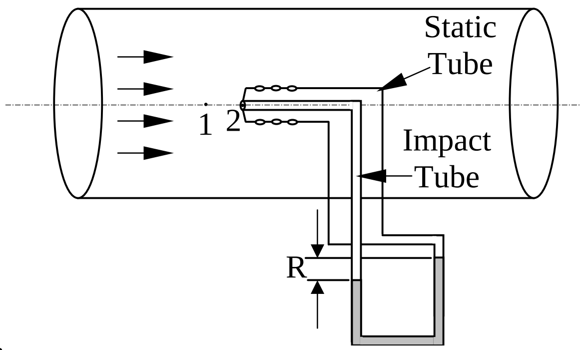
Head flow meters include **orifice, venture meter, flow nozzles, Pitot tubes, and wiers**. They consist of primary element, which causes the pressure or head loss and a secondary element, which measures it.

7.2.1 Pitot Tube

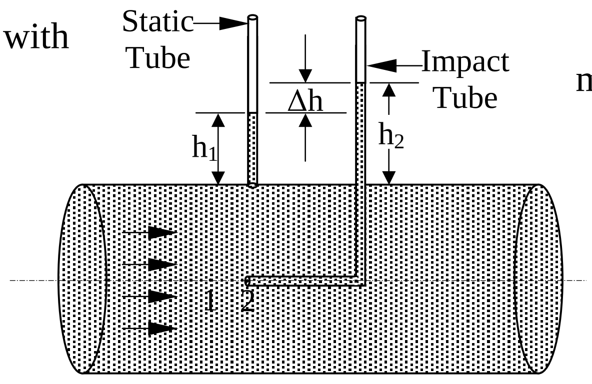
The Pitot tube is used to measure ***the local velocity*** at a given point in the flow stream and not the average velocity in the pipe or conduit. In the Figures below a sketch of this simple device is shown. One tube, ***the impact tube***, has its opening normal to the direction of flow and ***the static tube*** has its opening parallel to the direction of flow.



Simple Pitot tube with manometer



Simple Pitot tube with manometer provided with static pressure holes



Simple Pitot tube

Point 2 called ***stagnation point*** at which the impact pressure is be and $u_2 = 0$.

By applying Bernoulli's equation between points 1 and 2

$$\frac{P_1}{\rho g}+\frac{u_1^2}{2\alpha_1 g}+z_1=\frac{P_2}{\rho g}+\frac{u_2^2}{2\alpha_2 g}+z_2$$

$$\boxed{\Rightarrow u_1=\sqrt{\frac{2(-\Delta P)}{\rho}}=\sqrt{2g\Delta h}=\sqrt{\frac{2R(\rho_m-\rho)g}{\rho}}}$$ where, $\Delta P = R(\rho_m-\rho)g$

The fluid flows into the opening at point 2, pressure builds up, and then remains stationary at this point, called "***Stagnation Point***". The difference in the *stagnation pressure* (impact pressure) at this point (2) and the static pressure measured by the static tube represents the pressure rise associated with the direction of the fluid.

Impact pressure head = Static pressure head + kinetic energy head

Since Bernoulli's equation is used for ideal fluids, therefore for real fluids the last equations of local velocity become:

$$u_x=Cp\sqrt{\frac{2(-\Delta P)}{\rho}}=Cp\sqrt{2g\Delta h}=Cp\sqrt{\frac{2R(\rho_m-\rho)g}{\rho}}$$

where, Cp: dimensionless coefficient to take into account deviations from Bernoulli's equation and general varies between about 0.98 to 1.0.

Since the Pitot tube measures velocity at one point only in the flow, several methods can be used to obtain the average velocity in the pipe;

The first method, the velocity is measured at the exact center of the tube to obtain u_{max}. then by using the Figure, the average velocity can be obtained.

The second method, readings are taken at several known positions in the pipe cross section and then a graphical or numerical integration is performed to obtain the average velocity, from the following equation;

$$u=\frac{\iint_A u_x\,dA}{A}$$ (see Problem 5.16 Vol.I)

Example -7.1-

Find the local velocity of the flow of an oil of sp.gr. =0.8 through a pipe, when the difference of mercury level in differential U-tube manometer connected to the two tapping of the Pitot tube is 10 cm Hg. Take Cp = 0.98.

Solution:

$$u_x=Cp\sqrt{\frac{2R(\rho_m-\rho)g}{\rho}}=0.98\sqrt{\frac{2(0.1)(13600-1000)9.81}{800}}=5.49\,m/s$$

Example -7.2-

A Pitot tube is placed at a center of a 30 cm I.D. pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.84 of the center velocity (i.e. u/u_x =0.94). Find the discharge through the pipe if: -

i- The fluid flow through the pipe is water and the pressure difference between orifice is 6 cm H_2O.

ii- The fluid flow through the pipe is oil of sp.gr. = 0.78 and the reading manometer is 6 cm H_2O. Take Cp = 0.98.

Solution:

i- $u_x=Cp\sqrt{2g\Delta h}=0.98\sqrt{2(9.81)(0.06)}=1.063\,m/s$

u = 0.84 (1.063) = 0.893 m/s, Q = A.u = $\pi/4(0.3)^2$ (0.893) = 0.063 m^3/s

ii- $u_x = Cp\sqrt{\frac{2R(\rho_m - \rho)g}{\rho}} = 0.98\sqrt{\frac{2(0.06)(13600-780)9.81}{780}} = 0.565 m/s$

u = 0.84 (0.565) = 0.475 m/s, Q = A.u = $\pi/4(0.3)^2$ (0.475) = 0.0335 m^3/s

Example -7.3-

A Pitot tube is inserted in the pipe of 30 cm I.D. The static pressure head is 10 cm Hg vacuum, and the stagnation pressure at center of the pipe is 0.981 N/cm^2 gauge. Calculate the discharge of water through the pipe if u/u_{max} = 0.85. Take Cp = 0.98.

Solution:

P_1 = –10 cm Hg (13600) 9.81 (m / 100 cm) = –13.3416 kPa

P_2 = 0.981 N/cm^2 $(m / 100\ cm)^2$ = 9.81 kPa

$\Delta P = P_2 - P_1$ = 9.81- (-13.3416) = 23.1516 kPa

$u_x = Cp\sqrt{\frac{2(-\Delta P)}{\rho}} = 0.98\sqrt{\frac{2(23.1516\times 10^3}{1000}}$ =6.67 m/s

u = 0.85 (6.67) = 5.67 m/s, Q = A.u = $\pi/4(0.3)^2$ (5.67) = 0.4 m^3/s

Example -7.4-

A Pitot tube is used to measure the air flow rate in a circular duct 60 cm I.D. The flowing air temperature is 65.5ºC. The Pitot tube is placed at the center of the duct and the reading R on the manometer is 10.7 mm of water. A static pressure measurement obtained at the Pitot tube position is 205 mm of water above atmospheric. Take Cp = 0.98, $\mu = 2.03 \times 10^{-5}$ Pa.s

a- Calculate the velocity at the center and the average velocity.

b- Calculate the volumetric flow rate of the flowing air in the duct.

Solution:

a-

$P_1 \equiv$ the static pressure

P_1(gauge) = 0.205 (1000) 9.81 = 2011 kPa

P_1(abs) = 2011 + 1.01325 x 10^5 Pa = 1.03336 x 10^5 Pa

ρ_{air} = Mwt. P/(R.T) = 29 (1.03336 x 10^5)/[(8314 Pa.m^3/kmol.K) (65.5 + 273.15)]
= 1.064 kg/m^3

$u_x = Cp\sqrt{\frac{2R(\rho_m - \rho)g}{\rho}} = 0.98\sqrt{\frac{2(0.0107)(1000-1.064)9.81}{1.064}} = 14.04\ m/s = u_{max}$

$Re_{nax} = \rho u_{max} d/\mu$ = 1.064(14.04)0.6/2.03 x 10^{-5} = 4.415x 10^5

From Figure $u/u_{max} = 0.85 \Rightarrow u$ = 0.85 (14.04) = 11.934

b-

Q = A.u = $\pi/4(0.6)^2$ (11.934) = 3.374 m^3/s

H.W.

Problem 5.17 Vol.I

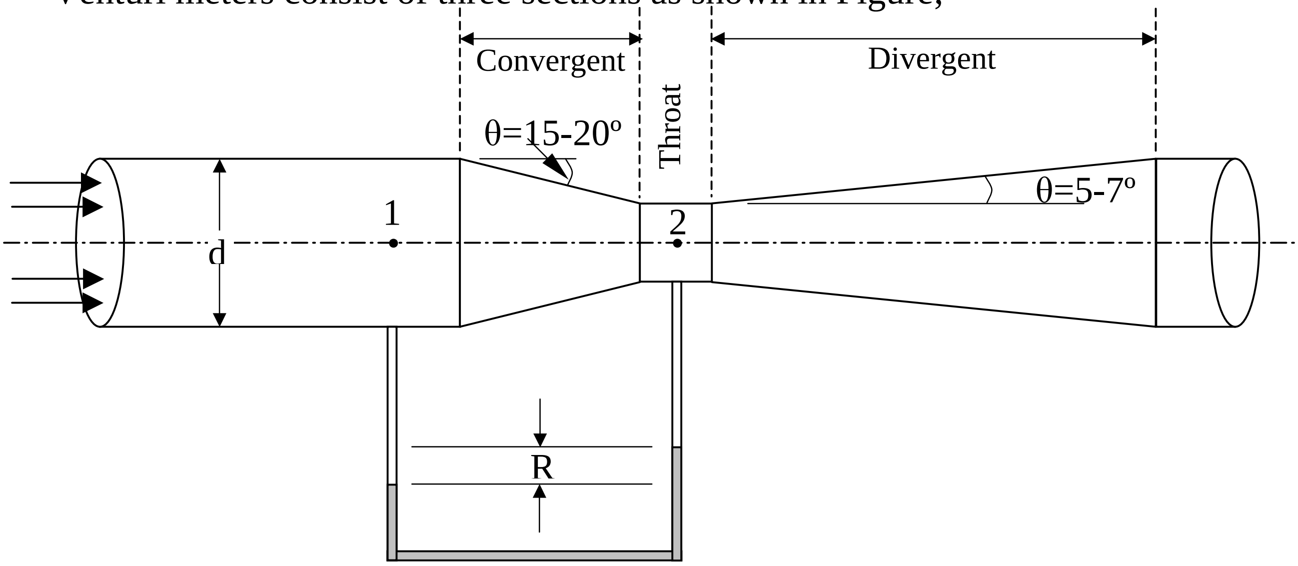
7.2.2 Measurement by Flow Through a Constriction

In measuring devices where the fluid is accelerated by causing it to flow through a constriction, the kinetic energy is thereby increased and the pressure energy therefore decreases. The flow rate is obtained by measuring the pressure difference between the inlet of the meter and a point of reduced pressure.

Venturi meters, ***orifice meters***, and ***flow nozzles*** measure the volumetric flow rate Q or average (mean linear) velocity u. In contrast the Pitot tube measures a point (local) velocity u_x.

7.2.2.1 Venturi Meter

Venturi meters consist of three sections as shown in Figure;



- From continuity equation $A_1u_1 = A_2u_2 \Rightarrow u_1 = (A_2/A_1)\,u_2$
- From Bernoulli's equation between points 1 and 2

$$\frac{P_1}{\rho g}+\frac{u_1^2}{2g}+\cancel{z_1}=\frac{P_2}{\rho g}+\frac{u_2^2}{2g}+\cancel{z_2}$$

$$\Rightarrow \frac{P_1-P_2}{\rho g}=\frac{u_2^2-u_1^2}{2g}=\frac{u_2^2}{2g}\left[1-\left(\frac{A_2}{A_1}\right)^2\right]=\frac{u_2^2}{2g}\left[\frac{A_1^2A_2^2}{A_1^2}\right]$$

$$\Rightarrow \qquad u_2=\sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)\left[\frac{1}{1-(A_2/A_1)^2}\right]}=\sqrt{\frac{2(-\Delta P)}{\rho}}\frac{A_1}{\sqrt{A_1^2-A_2^2}}$$

or $$u_2=\sqrt{(2g\Delta h)\left[\frac{1}{1-(A_2/A_1)^2}\right]}=\sqrt{2g\Delta h}\frac{A_1}{\sqrt{A_1^2-A_2^2}}$$

or $$u_2=\sqrt{\left(\frac{2R(\rho_m-\rho)g}{\rho}\right)\left[\frac{1}{1-(A_2/A_1)^2}\right]}=\sqrt{\frac{2R(\rho_m-\rho)g}{\rho}}\frac{A_1}{\sqrt{A_1^2-A_2^2}}$$

All these equation of velocity ***at throat*** u_2, which derived from Bernoulli's equation are for ideal fluids. Using a coefficient of discharge C_d to take account of the frictional losses in the meter and of the parameters of kinetic energy correction α_1 and α_2. Thus the volumetric flow rate will be obtained by: -

$$Q = u_2 A_2 = C_d \sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)\left[\frac{A_2^2}{1-(A_2/A_1)^2}\right]} = C_d \sqrt{\frac{2(-\Delta P)}{\rho}}\,\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

or $$Q = C_d \sqrt{(2g\Delta h)\left[\frac{A_2^2}{1-(A_2/A_1)^2}\right]} = C_d \sqrt{2g\Delta h}\,\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

or $$Q = C_d \sqrt{\left(\frac{2R(\rho_m-\rho)g}{\rho}\right)\left[\frac{A_2^2}{1-(A_2/A_1)^2}\right]} = C_d \sqrt{\frac{2R(\rho_m-\rho)g}{\rho}}\,\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$\boxed{\dot{m} = Q\,\rho, \qquad G = \rho\, u = \frac{\dot{m}}{A}}$$

For many meters and for Re > 10^4 at point 1

$C_d = 0.98$ for $d_1 < 20$ cm

$C_d = 0.99$ for $d_1 > 20$ cm

Example -7.5-

A horizontal Venturi meter with $d_1 = 20$ cm, and $d_2 = 10$ cm, is used to measure the flow rate of oil of sp.gr. = 0.8, the discharge through venture meter is 60 lit/s. find the reading of (oil-Hg) differential Take $C_d = 0.98$.

Solution:

$Q = u_2 A_2 = 60$ lit/s (m^3/1000lit) = 0.06 m^3/s

$$0.06 = C_d \sqrt{\frac{2R(\rho_m-\rho)g}{\rho}}\,\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} = 0.98\sqrt{\frac{2R(13600-800)9.81}{800}}\,\frac{(\pi/4)^2(0.1)^2(0.2)^2}{\sqrt{(\pi/4)^2\left[(0.2)^4-(0.1)^4\right]}}$$

$\Rightarrow$ R = 0.1815 m Hg = 18.15 cm Hg

Example -7.6-

A horizontal Venturi meter is used to measure the flow rate of water through the piping system of 20 cm I.D, where the diameter of throat in the meter is $d_2 = 10$ cm. The pressure at inlet is 17.658 N/cm^2 gauge and the vacuum pressure of 35 cm Hg at throat. Find the discharge of water. Take $C_d = 0.98$.

Solution:

$P_1 = 17.658$ N/cm^2 $(100 \text{ cm / m})^2 = 176580$ Pa

$P_2 = -35$ mm Hg (m / 100 cm) 9.81 (13600) = –46695.6 Pa

$P_1 - P_2 = 176580-(-46695.6) = 223275.6$ Pa

$$Q = u_2 A_2 = C_d \sqrt{\frac{2\Delta P}{\rho}}\,\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} = 0.98\sqrt{\frac{2(223275.6)}{1000}}\,\frac{(0.2)^2[(\pi/4)(0.1)^2]}{\sqrt{\left[(0.2)^4-(0.1)^4\right]}}$$

$\Rightarrow$ Q = 0.168 m^3/s

Example -7.7-

A Venturi meter is to be fitted to a 25 cm diameter pipe, in which the maximum flow is 7200 lit/min and the pressure head is 6 m of water. What is the maximum diameter of throat, so that there is non-negative head on it?

Solution:

$h_1 = 6$ m H_2O

Since the pressure head at the throat is not to be negative, or maximum it can be zero (i.e. h_2 = zero). Therefore;

$\Delta h = h_1 - h_2 = 6 - 0 = 6$ m H_2O

$Q = u_2 A_2 = 7200$ lit/min (m^3/1000lit) (min / 60 s) = 0.12 m^3/s

$$\Rightarrow 0.12 = C_d\sqrt{2g\Delta h}\,\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}} = 1.0\sqrt{2(9.81)6}\,\frac{(0.25)^2[(\pi/4)(d_2)^2]}{\sqrt{[(0.25)^4-(d_2)^4]}}$$

$$0.225 = \frac{d_2^2}{\sqrt{(0.25)^4-(d_2)^4}} \Rightarrow 0.0507 = \frac{d_2^4}{(0.25)^4-d_2^4}$$

$$\Rightarrow d_2^4 + 0.507d_2^4 - 1.983\times10^{-4} = 0 \qquad \Rightarrow d_2^4 = 1.887237\times10^{-4}$$

$\Rightarrow d_2 = 0.1172$ m = 11.72 cm

Note: -

In case of using **vertical** or **inclined** Venturi meter instead of horizontal one, the same equations for estimation the actual velocity are used.

Example -7.8-

A (30cm x 15cm) Venturi meter is provided in a vertical pipe-line carrying oil of sp.gr. = 0.9. The flow being upwards and the difference in elevations of throat section and entrance section of the venture meter is 30 cm. The differential U-tube mercury manometer shows a gauge deflection of 25 cm. Take C_d = 0.98 and calculate: -

i- The discharge of oil

ii- The pressure difference between the entrance and throat sections.

Solution:

i- $$Q = u_2A_2 = C_d\sqrt{\frac{2R(\rho_m-\rho)g}{\rho}}\,\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}} = 0.98\sqrt{\frac{2(0.25)(12700)9.81}{900}}\left[\frac{0.3^2[\pi/4(0.15)^2]}{\sqrt{0.3^4-0.15^4}}\right]$$

= 0.1488 m^3/s

ii- Applying Bernoulli's equation at points 1 and 2

$$\frac{P_1}{\rho g}+\frac{u_1^2}{2g}+\cancel{z_1}=\frac{P_2}{\rho g}+\frac{u_2^2}{2g}+z_2$$

$$\frac{P_1-P_2}{\rho g} = z_2 + \frac{u_2^2-u_1^2}{2g}$$

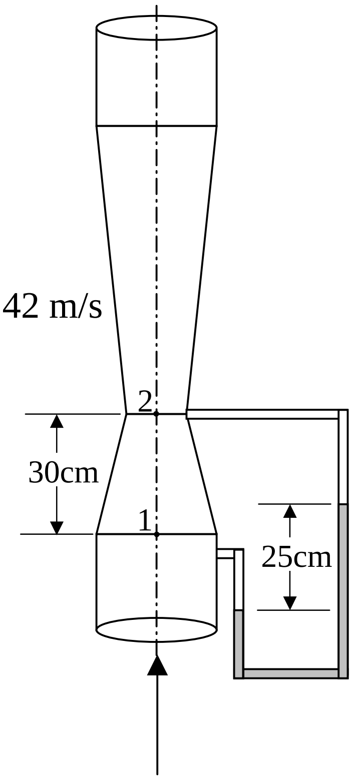
$u_1 = 0.1488/(\pi/4\ 0.3^2) = 2.1$ m/s, $u_2 = 0.1488/(\pi/4\ 0.15^2) = 8.42$ m/s

$\Rightarrow P_1 - P_2 = 900\ (9.81)\ [0.3 + (8.42^2 - 2.1^2)/2(9.81)]$

= 32.5675 kPa

but $P_1 - P_2$ = 0.25 (13600–900)(9.81) = 31.1467 kPa

% error = 4.36 %



Home Work

P.7.1

A Venturi meter with a 15 cm I.D. at inlet and 10 cm I.D. at throat is laid with its axis horizontal and is used for measuring the flow of oil of sp.gr. = 0.9. The oil-mercurry differential manometer shows a gauge difference of 20 cm. If C_d = 0.98, calculate the discharge of oil.

Ans. Q = 0.06393 m^3/s

P.7.2

A horizontal Venturi meter (160mm x 80mm) used to measure the flow of oil of sp.gr. = 0.8. Determine the deflection of oil-mercury gauge, if discharge of oil is 50 lit/s.

Ans. R = 29.6 cm Hg

P.7.3

A Venturi meter has an area ratio (9:1), the larger diameter being 30 cm. During the flow the recorded pressure head in larger section is 6.5 m and that at throst 4.25 m. If C_d = 0.99, compute the discharge through the meter.

Ans. Q = 0.052 m^3/s

P.7.4

A Venturi meter is fitted to 15 cm diameter pipeline conveying water inclined at 60º to the horizontal. The throat diameter is 5 cm and it is placed higher than the inlet side. The difference of pressure between the throat and the inlet which are 0.9 m apart is equivalent to 7.5 cm of mercury. Calculate the discharge if C_d = 0.98.

Ans. Q = 0.00832 m^3/s

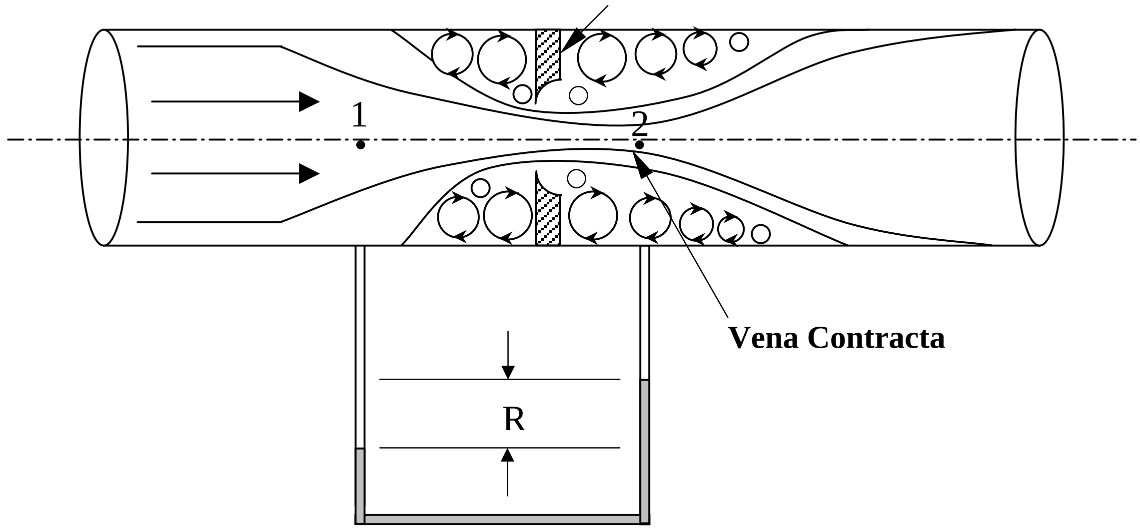
P.7.5

Find the throat diameter of a Venturi meter when fitted to a horizontal pipe 10 cm diameter having a discharge of 20lit/s. The differential U-tube mercury manometer, shows a deflection giving a reading of 60 cm, C_d = 0.98. In case, this Venturi meter is introduced in a vertical pipe, with the water flowing upwards, find the difference in the reading of mercury gauge. The dimensions of pipe and Venturi meter remain unaltered, as well as the discharge through the pipe.

Ans. d_2 = 0.04636 m, and the same reading in case II i.e. 60 cm Hg

7.2.2.2 Orifice Meter

The primary element of an orifice meter is simply a flat plate containing a drilled located in a pipe perpendicular to the direction of fluid flow as shown in Figure;



At point 2 in the pipe the fluid attains its maximum mean linear velocity u_2 and its smallest cross-sectional flow area A_2. This point is known as "***the vena contracta***". It is occurs at about one-half to two pipe diameters downstream from the orifice plate.

Because of relatively the large friction losses from the eddies generated by the expanding jet below vena contracta, the pressure recovery in orifice meter is poor.

- From continuity equation $A_1u_1 = A_2u_2 \Rightarrow u_1 = (A_2/A_1)\, u_2$
- From Bernoulli's equation between points 1 and 2

$$\frac{P_1}{\rho g}+\frac{u_1^2}{2g}+\cancel{z_1}=\frac{P_2}{\rho g}+\frac{u_2^2}{2g}+\cancel{z_2}$$

$$\Rightarrow \frac{P_1-P_2}{\rho g}=\frac{u_2^2-u_1^2}{2g}=\frac{u_2^2}{2g}\left[1-\left(\frac{A_2}{A_1}\right)^2\right]=\frac{u_2^2}{2g}\left[\frac{A_1^2 A_2^2}{A_1^2}\right]$$

But $C_c = A_2/A_o \Rightarrow \quad A_2 = C_cA_o$

C_c: coefficient of contraction [0.6 – 1.0] common value is 0.67

A_2: cross-sectional area at vena contracta

A_o: cross-sectional area of orifice

$$\Rightarrow \quad \frac{P_1-P_2}{\rho}=\frac{u_2^2}{2}\left[1-\left(\frac{C_cA_o}{A_1}\right)^2\right]=\frac{u_2^2}{2}\left[\frac{A_1^2-(C_cA_o)^2}{A_1^2}\right]$$

Using a coefficient of discharge C_d to take into account the frictional losses in the meter and of parameters C_c, α_1, and α_2. Thus the velocity at orifice or the discharge through the meter is;

$$\boxed{Q = C_d\sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)\left[\frac{A_o^2}{1-(A_o/A_1)^2}\right]} = C_d\sqrt{\frac{2(-\Delta P)}{\rho}}\,\frac{A_1A_o}{\sqrt{A_1^2-A_o^2}}}$$

or $$Q = C_d\sqrt{(2g\Delta h)\left[\frac{A_o^2}{1-(A_o/A_1)^2}\right]} = C_d\sqrt{2g\Delta h}\,\frac{A_1A_o}{\sqrt{A_1^2-A_o^2}}$$

or $$Q = C_d\sqrt{\left(\frac{2R(\rho_m-\rho)g}{\rho}\right)\left[\frac{A_o^2}{1-(A_o/A_1)^2}\right]} = C_d\sqrt{\frac{2R(\rho_m-\rho)g}{\rho}}\,\frac{A_1A_o}{\sqrt{A_1^2-A_o^2}}$$

$$\boxed{\dot{m} = Q\,\rho, \qquad G = \rho\, u = \frac{\dot{m}}{A}}$$

$$Re_o = \frac{\rho\, u_o d_d}{\mu}$$

For $Re_o > 10^4$ $C_d = 0.61$

And for $Re_o > 10^4$ C_d From Figure below

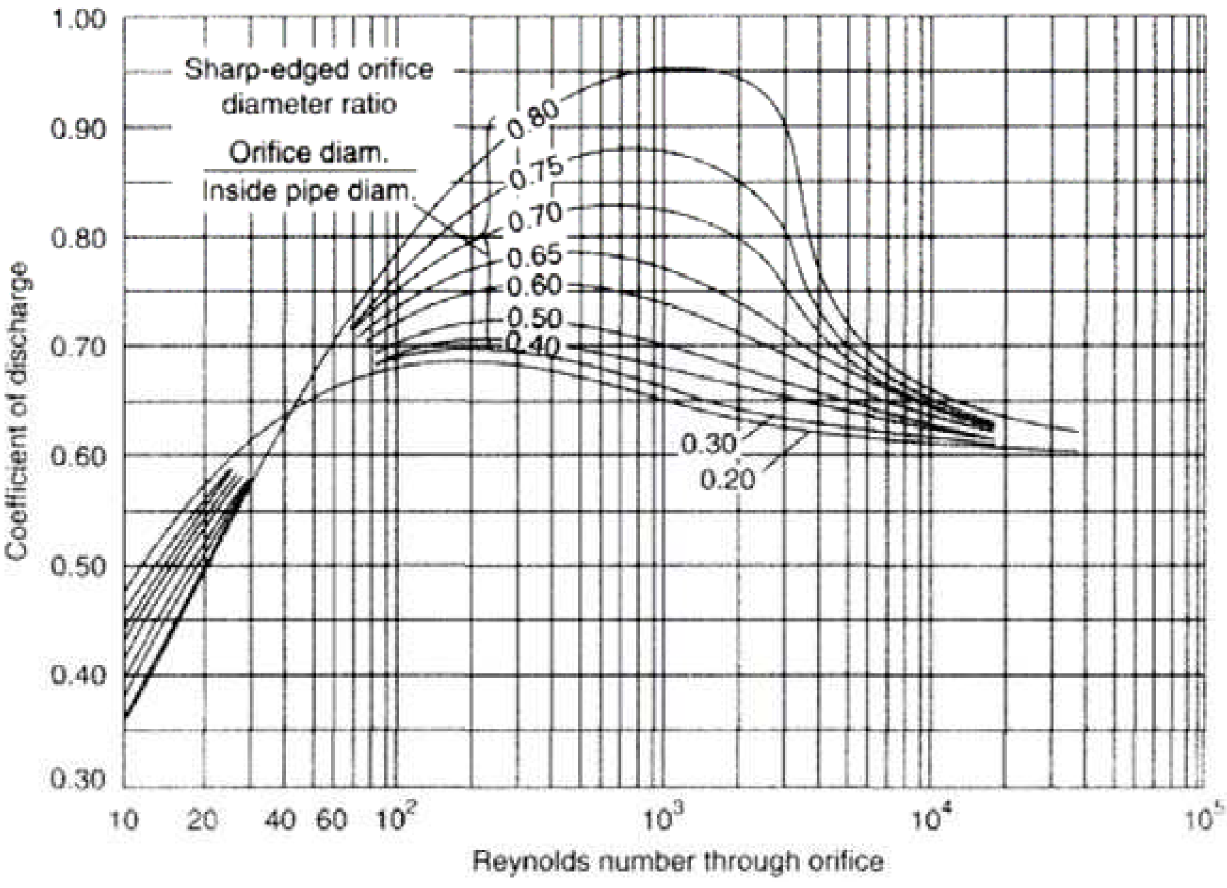


Figure of the discharge coefficient for orifice meter.

The holes in orifice plates may be ***concentric***, ***eccentric*** ***or*** ***segmental*** as shown in Figure. Orifice plates are prone to damage by erosion.

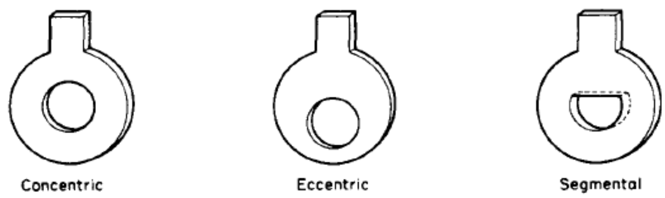


Figure of Concentric. eccentric and segmental orifice plates

Example -7.9-

An orifice meter consisting of 10 cm diameter orifice in a 25 cm diameter pipe has C_d = 0.65. The pipe delivers oil of sp.gr. = 0.8. The pressure difference on the two sides of the orifice plate is measured by mercury oil differential manometer. If the differential gauge is 80 cm Hg, find the rate of flow.

Solution:

$$Q = C_d \sqrt{\frac{2R(\rho_m - \rho)g}{\rho}} \frac{A_1 A_o}{\sqrt{A_1^2 - A_o^2}} = 0.65\sqrt{\frac{2(0.8)(13600-800)9.81}{800}}\left[\frac{(\pi/4)(0.1)^2(0.25)^2}{\sqrt{[(0.25)^4 - (0.1)^4]}}\right]$$

$\Rightarrow$ Q = 0.08196 m^3/s.

Example -7.10-

Water flow through an orifice meter of 25 mm diameter situated in a 75 mm diameter pipe at a rate of 300 cm^3/s, what will be the difference in pressure head across the meter μ = 1.0 mPa.s.

Solution:

Q = 300 x 10^{-6} m^3/s $\Rightarrow$ u = (300 x 10^{-6} m^3/s) / (π/4 0.25^2) = 0.611 m/s

$$Q = C_d \sqrt{2g\Delta h}\frac{A_1 A_o}{\sqrt{A_1^2 - A_o^2}}$$

$$Re_o = \frac{\rho\, u_o d_d}{\mu} \qquad \Rightarrow Re_o = \frac{1000\,(0.611)(0.025)}{1\times 10^{-3}} = 1.528 \times 10^4 \qquad \Rightarrow C_d = 0.61$$

$$300 \times 10^{-6}\ m^3/s = 0.61\sqrt{2(9.81)\Delta h}\left[\frac{(\pi/4)(0.025)^2(0.075)^2}{\sqrt{[(0.075)^4 - (0.025)^4]}}\right]$$

$\Rightarrow \sqrt{\Delta h}$ = 0.2248 $\Rightarrow$ Δh = 0.05 m H_2O = 50 mm H_2O

Example -7.11-

Water flow at between 3000-4000 cm^3/s through a 75 mm diameter pipe and is metered by means of an orifice. Suggest a suitable size of orifice if the pressure difference is to be measured with a simple water manometer. What approximately is the pressure difference recorded at the maximum flow rate? C_d = 0.6.

Solution:

The largest practicable height of a water manometer is 1.0 m

$$Q = C_d \sqrt{2g\Delta h}\frac{A_1 A_o}{\sqrt{A_1^2 - A_o^2}}$$

The maximum flow rate = 4 x 10^{-3} m^3/s

$$4 \times 10^{-3}\ m^3/s = 0.6\sqrt{2(9.81)(1.0)}\left[\frac{(\pi/4)(0.05)^2(d_o)^2}{\sqrt{[(0.05)^4 - (d_o)^4]}}\right]$$

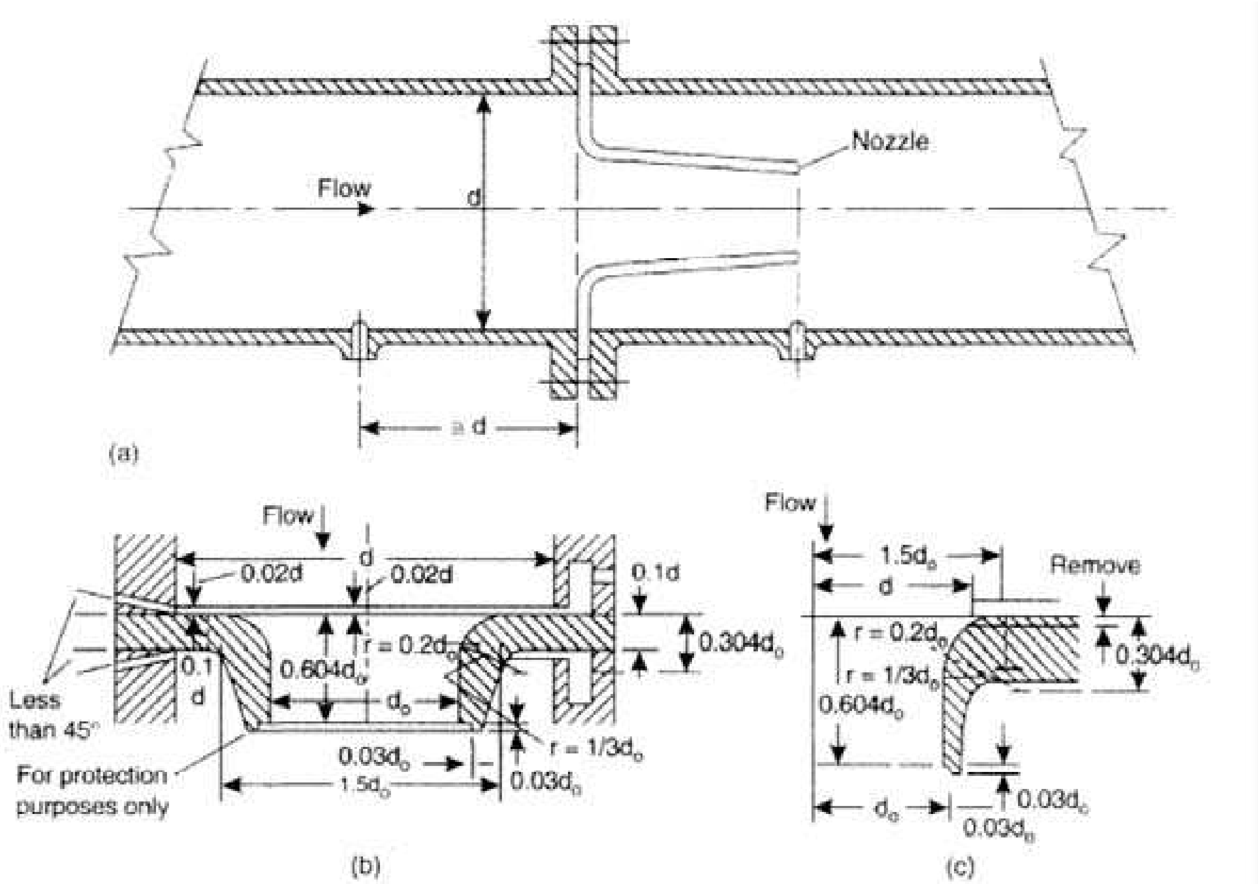
$$\Rightarrow \frac{d_o^2}{\sqrt{(0.05)^4 - d_o^4}} = 0.7665 \quad \Rightarrow \quad d_o^4 = 3.67\times 10^{-6} - 0.5875 d_o^4$$

$\Rightarrow$ d_o = 0.039 m = 39 mm

($P_1 - P_2$) = Δh ρg = 1.0 (1000)(9.81) = 9810 Pa.

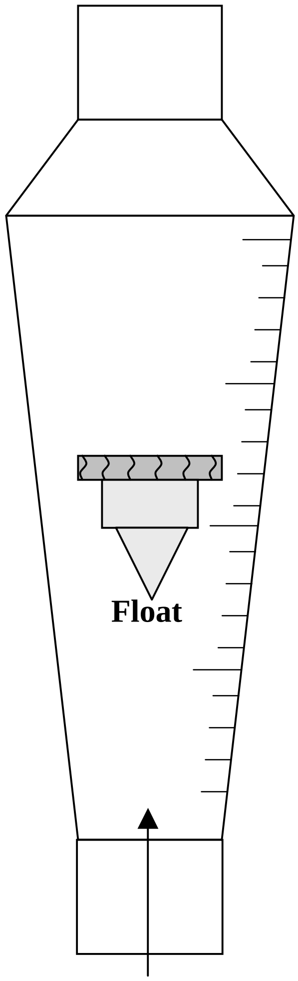
7.2.2.3 The Nozzle

The nozzle is similar to the orifice meter other than that it has a *converging tube* in place of the orifice plate, as shown in below. The velocity of the fluid is gradually increased and the contours are so designed that almost frictionless flow takes place in the converging portion; the outlet corresponds to the *vena contracta* on the orifice meter. The nozzle has a constant high coefficient of discharge (ca. 0.99) over a wide range of conditions because the coefficient of contraction is **unity**, though because the simple nozzle is not fitted with a diverging cone, the head lost is very nearly the same as with an orifice. Although much more costly than the orifice meter, it is extensively used for metering steam. When the ratio of the pressure at the nozzle exit to the upstream pressure is less than the critical pressure ratio ω_c, the flow rate is independent of the downstream pressure and can be calculated from the upstream pressure alone.



Figures of nozzle (a) General arrangement (b) Standard nozzle (A_o/A_1) is less than 0.45. Left half shows construction for corner tappings. Right half shows construction for piezometer ring (c) Standard nozzle where (A_o/A_1) is greater than 0.45

7.2.3 Variable Area Meters - Rotameters



In the previous flow rates the area of constriction or orifice is **constant**, and *the pressure drop is dependent on the rate of the flow* (due to conversions between the pressure energy with kinetic energy).

In the Rotameter *the drop in pressure is* **constant** and *the flow rate is function of the area of constriction*. When the fluid is flowing the float rises until its weight is balanced by the up thrust of the fluid. قوة الطفو او القوة الدافعة للمائع. *Its position then indicting the rate of flow*.

Force balance on the float

Gravity force = up thrust force + Pressure force

$V_f \rho_f g = V_f \rho g + (-\Delta P) A_f$

$$-\Delta P = \frac{V_f g(\rho_f - \rho)}{A_f} \quad \text{i.e. constant}$$

where, V_f ρ_f, and A_f are float volume, float density, and maximum cross- section area of the float.

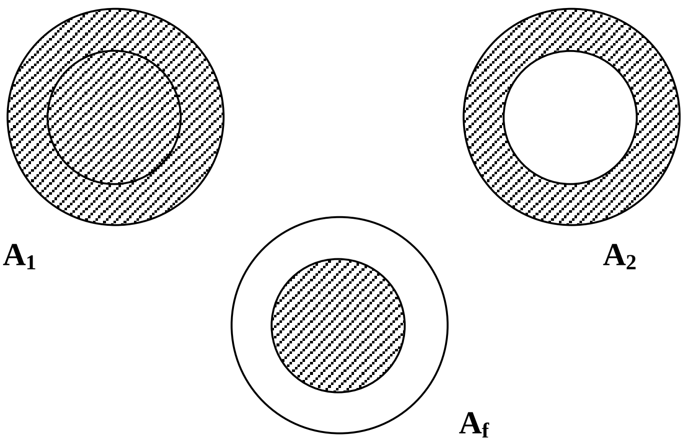
$(-\Delta P)$ is the pressure difference over the float, $(-\Delta P) = P_1 - P_2$.

The area of flow is the annulus formed between the float and the wall of the tube. This meter may thus be considered, as an orifice meter with a variable aperture, and the equation of flow rate already derived are therefore applicable with only minor changes.

$$Q = C_d \sqrt{\frac{2(-\Delta P)}{\rho}} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$Q = C_d \sqrt{\frac{2V_f g(\rho_f - \rho)}{\rho A_f}} \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

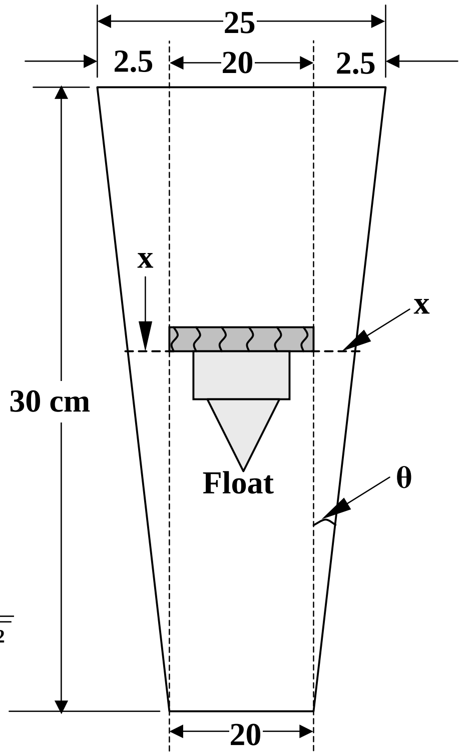
where, A_1 : cross-section area of the tube when the float arrived.
A_1 : cross-section area of the annulus (flow area).



Example -7.12-

A rotameter tube of 0.3 m long with an internal diameter of 25 mm at the top and 20 mm at the bottom. The diameter of float is 20 mm, its sp.gr. is 4.8 and its volume is 6 cm^3. If the coefficient of discharge is 0.7, what will be the flow rate water when the float is half way up the tube?

Solution:



$A_1 = \pi/4\ d_1^2$, $d_1 = d_f + 2x$

To find x

1- $0.25/30 = x/15, \Rightarrow x = 0.125$ cm

or 2- $\tan(\theta) = 0.25 / 30 = x/15, \Rightarrow x = 0.125$ cm

$\Rightarrow d_1 = 2 + 2(0.125) = 2.25$ cm

$\Rightarrow A_1 = \pi/4\ (0.0225)^2 = 3.976 \times 10^{-4}\ m^2$

$A_2 = A_1 - A_f = 3.976 \times 10^{-4} - \pi/4\ (0.02)^2$

$= 8.345 \times 10^{-5}\ m^2$

$$Q = C_d \sqrt{\frac{2V_f\, g(\rho_f - \rho)}{\rho\, A_f}}\ \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

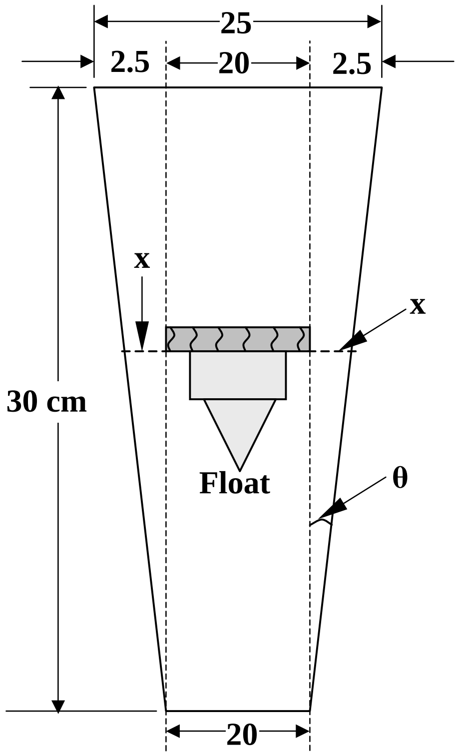
$$= 0.7 \sqrt{\frac{26\times 10^{-6}(9.81)(4.8-1)}{\pi/4(0.02)^2}}\ \frac{(3.976\times 10^{-4})(8.345\times 10^{-5})}{\sqrt{(3.976\times 10^{-4})^2 - (8.345\times 10^{-5})^2}}$$

$= 7.13 \times 10^{-5}\ m^3/s$

Example -7.13-

A rotameter has a tube of 0.3 m long, which has an internal diameter of 25 mm at the top and 20 mm at the bottom. The diameter of float is 20 mm, its effective sp.gr. is 4.8 and its volume is 6.6 cm^3. If the coefficient of discharge is 0.72, what height will the float be when metering water at 100 cm^3/s?

Solution:



$$Q = 10^{-4}\ m^3/s = C_d \sqrt{\frac{2V_f\, g(\rho_f - \rho)}{\rho\, A_f}}\ \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$\Rightarrow \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} = 1.10976\times 10^{-4} = \frac{A_2}{\sqrt{1-(\frac{A_2}{A_1})^2}}$$

assume $1-(\frac{A_2}{A_1})^2 = 1.0$ ------- i.e. $A_2 \longrightarrow 0$

$\Rightarrow A_2 = 1.10976 \times 10^{-4}\ m^2$, $A_1 = A_2 + A_f$

$\Rightarrow A_1 = 4.2513 \times 10^{-4}\ m^2$

Correct the assumption

$\sqrt{1-(\frac{A_2}{A_1})^2} = 0.965$

$\Rightarrow A_2 = 0.965\ (1.10976 \times 10^{-4}\ m^2) = 1.0713 \times 10^{-4}\ m^2$

$A_1 = A_2 + A_f \Rightarrow A_1 = 4.213 \times 10^{-4}\ m^2$

Re-correct the last value

$\sqrt{1-(\frac{A_2}{A_1})^2}$ = 0.967 -------------close enough

$\Rightarrow d_1 = (A_1 / \pi/4)^{0.5}$ = 0.02316 m = 2.316 cm

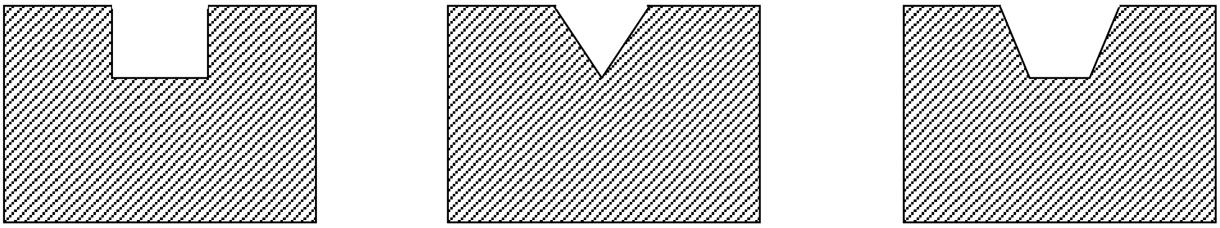
$d_1 = 2x + d_f \Rightarrow x = (0.02316 - 0.02) / 2$ = 0.0016 m = 0.16 cm

$0.25/30 = 0.16/L \Rightarrow L = 19.2$ cm

7.2.4 The Notch or Weir

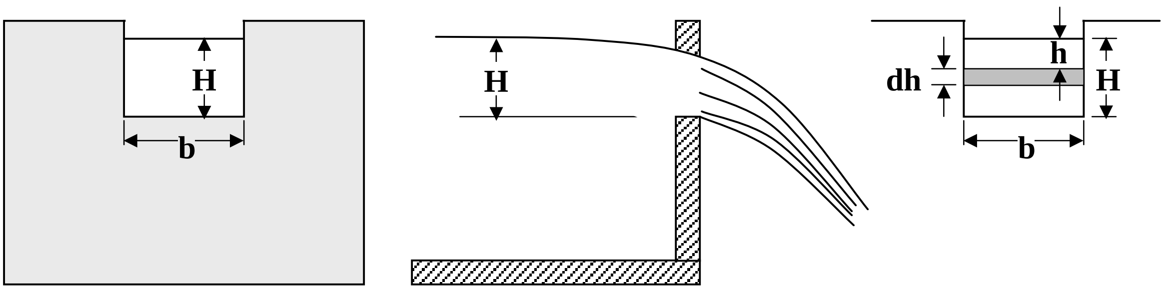
The flow of liquid presenting a free surface (open channels) can be measured by means of a weir. The pressure energy converted into kinetic energy as it flows over the weir, which may or may not cover the full width of the stream, and a calming screen may be fitted before the weir. Then the height of the weir crest gives a measure of the rate of flow. The velocity with which the liquid leaves depends on its initial depth below the surface.

Many shapes of notch are available of which three shapes are given here as shown in Figures,



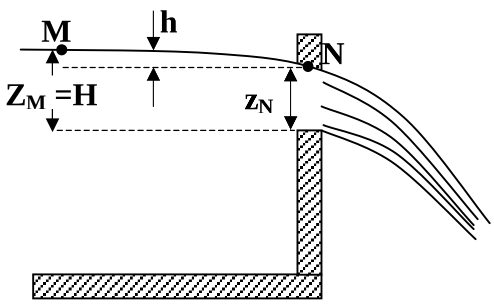
Rectangular notch Triangular notch Trapezoidal notch

7.2.4.1 Rectangular Notch



H: height of liquid above base of the notch
h: depth of liquid from its level
b: width or length of notch
Consider a horizontal strip of liquid of thickness (dh) at depth (h).
The theoretical velocity of liquid flow through the strip = $\sqrt{2gh}$

To prove this equation applies Bernoulli's equation between points M and N as shown in Figure;



$$\frac{P_M}{\rho g}+\frac{u_M^2}{2g}+z_M=\frac{P_N}{\rho g}+\frac{u_N^2}{2g}+z_N$$

The cross sectional area of flow at point M is larger than that at notch (point N), then ($u_M \approx 0$)

$P_M = P_N = P_o$ atmospheric pressure

$$\Rightarrow \quad z_M - z_N = \frac{u_N^2}{2g} \qquad \boxed{\therefore u_N = \sqrt{2gh}}$$

The area of the strip dA = b.dh
The discharge through the strip dQ = u.dA = C_d ($\sqrt{2gh}$)(b.dh)

$$\Rightarrow \int_0^Q dQ = C_d b\sqrt{2g}\int_0^H h^{1/2}dh \qquad \Rightarrow Q = C_d b\sqrt{2g}\,\frac{H^{3/2}}{3/2}$$

$$\boxed{\therefore \quad Q = \frac{2}{3}C_d b\sqrt{2g}H^{3/2}}$$

Example -7.14-

A rectangular notch 2.5 m wide has a constant head of 40 cm, find the discharge over the notch where $C_d = 0.62$

Solution:

$Q = \frac{2}{3}C_d b\sqrt{2g}H^{3/2}$ = 2/3 (0.62) (2.5) $(2 \times 9.81)^{0.5}$ $(0.4)^{3/2}$

Q = 1.16 m^3/s

Example -7.15-

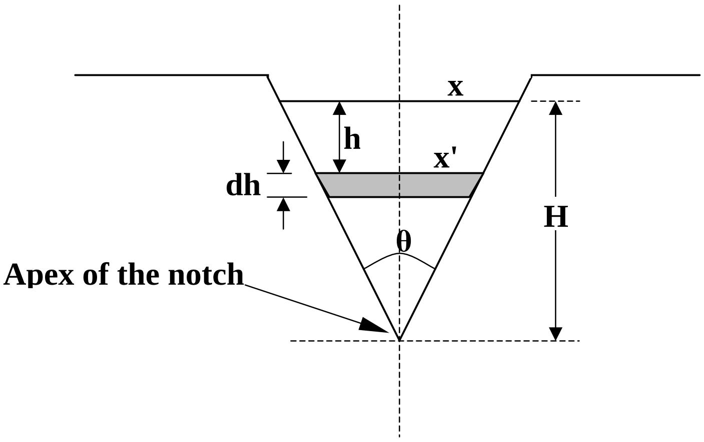
A rectangular notch has a discharge of 21.5 m^3/min, when the head of water is half the length of the notch. Find the length of the notch where $C_d = 0.6$.

Solution:

$Q = \frac{2}{3}C_d b\sqrt{2g}H^{3/2}$ ⇒ 21.5/60 = 2/3 (0.6) (b) $(2 \times 9.81)^{0.5}$ $(0.5\ b)^{3/2}$

⇒ $b^{5/2} = 0.572$ ⇒ $b = (0.572)^{2/5} = 0.8$ m

7.2.4.2 Triangular Notch



A triangular notch is also called a *V-notch*.

H: height of liquid above base of the apex of the notch.

θ: Angle of the notch.

tan (θ/2) = x / H = x' / (H-h)

The width of the notch at liquid surface = 2x = 2H tan(θ/2)

The width of the strip = 2x' = 2(H-h) tan(θ/2)

The area of the strip = 2x' dh = 2(H-h) tan(θ/2)dh

The theoretical velocity of water through the strip = $\sqrt{2gh}$

The discharge over the notch dQ = u. dA = C_d ($\sqrt{2gh}$) [2(H-h) tan(θ/2)dh]

$$\int_0^Q dQ = 2C_d \tan(\theta/2)\sqrt{2g}\int_0^H (Hh^{1/2} - h^{3/2})dh$$

$$Q = 2C_d \tan(\theta/2)\sqrt{2g}\left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H = 2C_d \tan(\theta/2)\sqrt{2g}\left[\frac{2}{3}H^{5/2} - \frac{2}{5}H^{5/2}\right]$$

$$\boxed{\therefore \quad Q = \frac{8}{15}C_d \tan(\theta/2)\sqrt{2g}H^{5/2}}$$

If C_d = 0.6 and θ =90º ⇒ Q = 1.417 $H^{5/2}$

Example -7.16-

During an experiment in a laboratory, 50 liters of water flowing over a right-angled notch was collected in one minute. If the head of still is 50mm. Calculate the coefficient of discharge of the notch.

Solution:

$$Q = \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}H^{5/2} = 50 \text{ lit/min } (m^3/1000\text{lit})(\text{min}/60\text{s}) = 8.334 \times 10^{-4}\ m^3/s$$

$$\Rightarrow C_d = (8.334 \times 10^{-4})/[(8/15)(2\text{x}9.81)^{0.5} \tan(\theta/2)(0.05)^{5/2}] = 0.63$$

Example -7.17-

A rectangular channel 1.5 m wide is used to carry 0.2 m^3/s water. The rate of flow is measured by placing a 90º V-notch weir. If the maximum depth of water is not to exceed 1.2 m, find the position of the apex of the notch from the bed of channel. C_d = 0.6.

Solution:

Q = 1.417 $H^{5/2}$ ⇒ $H^{5/2}$ = (0.2 m^3/s)/1.417 ⇒ H = 0.46 m

The maximum depth of water in channel = 1.2 m

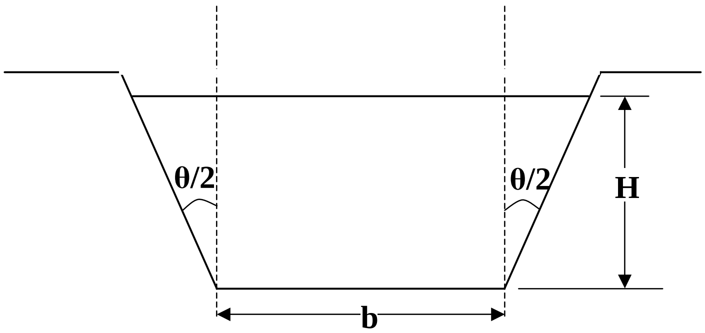
H is the height of water above the apex of notch.

Apex of triangular notch is to be kept at distance = 1.2 –0.46

= 0.74 m from the bed of channel.

7.2.4.3 Trapezoidal Notch

A trapezoidal notch is a combination of a rectangular notch and triangular notch as shown in Figure;



Discharge over the trapezoidal notch,
Q=[Discharge over the rectangular notch + Discharge over the triangular notch]

$$\boxed{Q = \frac{2}{3} C_{d1} b\sqrt{2g}H^{3/2} + \frac{8}{15} C_{d2} \tan(\theta/2)\sqrt{2g}H^{5/2}}$$

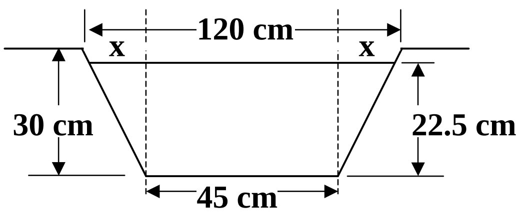
Example -7.18-

A trapezoidal notch 120 cm wide at top and 45 cm at the bottom has 30 cm height. Find the discharge through the notch, if the head of water is 22.5 cm. C_{d1} = C_{d2} = 0.6.

Solution:

x = (120+45)/2 = 37.5 cm

tan(θ/2) = x/30 = 37.5/30 = 1.25



$$Q = \frac{2}{3} C_{d1} b\sqrt{2g}H^{3/2} + \frac{8}{15} C_{d2} \tan(\theta/2)\sqrt{2g}H^{5/2}$$

Q = 2/3(0.6)(0.45)(2 x 9.81)$^{0.5}$ (0.225)$^{3/2}$ +8/15(0.6) (2 x 9.81)$^{0.5}$ (1.25)(0.225)$^{5/2}$
= 0.1276 m^3/s

7.3 Unsteady State Problems

Example -7.19-

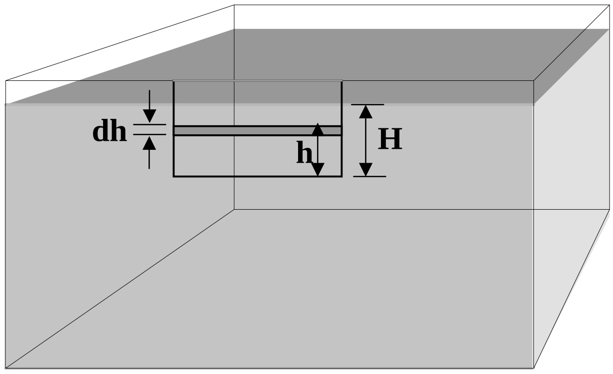
A reservoir 100 m long and 100 m wide is provided with a rectangular notch 2m long. Find the time required to lower the water level in the reservoir from 2 m to 1 m. C_d = 0.6.

Solution:

Let, at some instant, the height of the water above the base of the notch be (h) and the liquid level fall to small height (dh) in time (dt).

The volume of water discharged in time (dt) is:

dV = - A dh, A= 100 x 100 = 10^4 m^2



$$Q = dV/dt = \frac{2}{3}C_d b\sqrt{2g}h^{3/2} = -A\,dh/dt$$

$$\Rightarrow \int_0^T dt = \frac{-A}{(2/3)C_d b\sqrt{2g}}\int_{H_1}^{H_2} h^{-3/2}dh$$

$$T = \frac{3}{2}\frac{-A}{C_d b\sqrt{2g}}\left[\frac{h^{-1/2}}{-1/2}\right]_{H_1}^{H_2}$$

$$= \frac{3A}{C_d b\sqrt{2g}}\left[\frac{1}{\sqrt{H_2}} - \frac{1}{\sqrt{H_1}}\right] = \frac{3\times10^4}{0.6(2)\sqrt{2\times9.81}}\left[\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}}\right] = 1653.1\,\text{sec} = 27\,\text{min},33\,\text{sec}$$

Example -7.20-

A tank 25 m long and 15 m wide is provided with a right-angled V-notch. Find the time required to lower the level in the tank from 1.5 m to 0.5 m. C_d = 0.62.

Solution:

Let, at some instant, the height of the liquid above the apex of the notch be (h) and a small volume of the liquid (dv) flow over the notch in a small interval of time (dt), reducing the liquid level by an amount (dh) in the tank.

dV = - A dh, A= 25 x 15 = 375 m^2

$$Q = dV/dt = \frac{8}{15}C_d\sqrt{2g}\tan(\theta/2)h^{5/2} = -A\,dh/dt$$

$$\Rightarrow \int_0^T dt = \frac{-A}{(8/15)C_d\sqrt{2g}\tan(\theta/2)}\int_{H_1}^{H_2} h^{-5/2}dh$$

$$T = \frac{15}{8}\frac{-A}{C_d\sqrt{2g}\tan(\theta/2)}\left[\frac{h^{-3/2}}{-3/2}\right]_{H_1}^{H_2}$$

$$= \frac{5}{4}\frac{A}{C_d\sqrt{2g}\tan(\theta/2)}\left[\frac{1}{\sqrt{H_2^3}} - \frac{1}{\sqrt{H_1^3}}\right] = \frac{5}{4}\frac{375}{0.6\sqrt{2\times9.81}(1)}\left[\frac{1}{\sqrt{0.5^3}} - \frac{1}{\sqrt{1.5^3}}\right] \cong 390\,\text{sec} = 6\,\text{min},30\,\text{sec}$$

Home Work

P.7.6

A wier 8 m length is to be built across a rectangular channel to discharge a flow of 9 m^3/s. If the maximum depth of water on the upstream side of weir is to be 2 m, what should be the height of the weir? $C_d = 0.62$.

Ans. 1.277 m

P.7.7

A rectangular notch 1 m long and 40 cm high is discharging water. If the same quantity of water be allowed to flow over a 90º V-notch, find the height to which water will rise above the apex of notch. $C_d = 0.62$.

Ans. Q = 464 lit/s, H = 63.1 cm

P.7.8

Water flow over a right angled V-notch under a constant head of 25 cm. 1- Find the discharge. 2- Using principles of geometric similarity find the head required for a flow of 1417.6 lit/s through the same notch. $C_d = 0.62$.

Hint

For similar notch $\frac{Q_1}{Q_2} = \frac{b_1}{b_2}\left[\frac{H_1}{H_2}\right]^{3/2}$ and for the same notch $\frac{Q_1}{Q_2} = \left[\frac{H_1}{H_2}\right]^{3/2}$

Ans. Q = 44.3 lit/s, $H_2 = 1$ m

P.7.9

A sharp-edge 90º V-notch is inserted in the side of a rectangular tank 3 m long and 1.5 m wide. Find how long it will take to reduce the head in tank from 30 cm to 7.5 cm if the water discharges freely over the notch and there is no inflow into the tank. $C_d = 0.62$.

Ans. T = 87s = 1min 27s

CHPTER EIGHT

Flow of Compressible Fluid

8.1 Introduction

All fluids are to some degree compressible, compressibility is sufficiently great to affect flow under normal conditions only for a **gas**. If the pressure of the gas does not change by more than about 20%, [or when the change in density more than 5-10 %] it is usually satisfactory to treat the gas as incompressible fluid with a density equal to that at the mean pressure.

When compressibility is taken into account, the equations of flow become more complex than they are for an incompressible fluid.

The flow of gases through orifices, nozzles, and to flow in pipelines presents in all these cases, the flow may reach a limiting maximum value which independent of the downstream pressure (P_2); this is a phenomenon which does not arise with incompressible fluids.

8.2 Velocity of Propagation of a Pressure Wave

The velocity of propagation is a function of *the bulk modulus of elasticity* (ε), where;

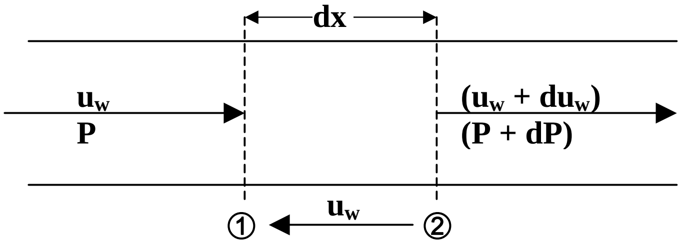
$$\varepsilon = \frac{\textbf{increase of stress within the fluid}}{\textbf{resulting volumetric strain}} = \frac{dP}{-d\upsilon/\upsilon}$$

$$\boxed{\Rightarrow \quad \varepsilon = -\upsilon\frac{dP}{d\upsilon}}$$

where, υ: specific volume (υ = 1/ρ).

Suppose a pressure wave to be transmitted at a velocity u_w over a distance dx in a fluid of cross-sectional area A, from section ② to section ① as shown in Figure;

Now imagine the pressure wave to be brought to rest by causing the fluid to flow at a velocity u_w *in the opposite direction*.



From conservation of mass law;

$$\dot{m}_1 = \dot{m}_2$$

$$\rho u_w A = (\rho + d\rho)(u_w + du_w)A$$

$$\Rightarrow \quad \frac{u_w}{\upsilon}A = \frac{(u_w + du_w)}{(\upsilon + d\upsilon)}A$$

and $\dot{m} = \dfrac{u_w}{\upsilon}A \quad \Rightarrow u_w = \dfrac{\dot{m}}{A}\upsilon \quad \Rightarrow du_w = \dfrac{\dot{m}}{A}d\upsilon$

Newton's 2nd law of motion stated that "The rate of change in momentum of fluid is equal to the net force acting on the fluid between sections ① and ②.

Thus;

$$\dot{m}[(u_w - du_w) - u_w] = A[P - (P + dP)] \quad \Rightarrow \frac{\dot{m}}{A}du_w = -dP$$

but $du_w = \dfrac{\dot{m}}{A}d\upsilon \quad \Rightarrow \dfrac{\dot{m}}{A}(\dfrac{\dot{m}}{A}d\upsilon) = -dP \quad \Rightarrow -\dfrac{dP}{d\upsilon} = \left(\dfrac{\dot{m}}{A}\right)^2$

we have $-\dfrac{dP}{d\upsilon} = \dfrac{\varepsilon}{\upsilon} \quad \Rightarrow \dfrac{\varepsilon}{\upsilon} = \left(\dfrac{\dot{m}}{A}\right)^2 = G^2$

$$\Rightarrow \quad \frac{\varepsilon}{\upsilon} = \left(\frac{u_w A/\upsilon}{A}\right)^2 = \left(\frac{u_w}{\upsilon}\right)^2 \quad \Rightarrow \quad u_w^2 = \upsilon\varepsilon$$

$$\boxed{\therefore \quad u_w = \sqrt{\upsilon\varepsilon}}$$

For ideal gases

$P\upsilon^{\kappa} = const.$ where, $\kappa = 1.0$ for isothermal conditions

$\kappa = \gamma$ for isentropic conditions, $\gamma = \dfrac{c_p}{c_v}$

$$d\left(P\upsilon^k\right) = 0 \quad \Rightarrow \quad \upsilon^k dP + Pk\upsilon^{k-1} d\upsilon = 0 \quad \Rightarrow \quad \upsilon^k dP = -kP\frac{\upsilon^k}{\upsilon} d\upsilon \quad \Rightarrow \quad \frac{dP}{d\upsilon} = -k\frac{P}{\upsilon}$$

$$\Rightarrow -\upsilon\left(\frac{dP}{d\upsilon}\right) = kP = \varepsilon \qquad \boxed{\therefore \quad u_w = \sqrt{kP\upsilon}}$$

- For isothermal conditions k = 1 $\boxed{\Rightarrow \quad u_w = \sqrt{P\upsilon}}$
- For isentropic (adiabatic) conditions k = γ $\boxed{\Rightarrow \quad u_w = \sqrt{\gamma P\upsilon}}$

The value of u_w is found to correspond closely to **<u>the velocity of sound in the fluid</u>** and its correspond to the velocity of the fluid at the end of a pipe uder conditions of maximum flow.

<u>Mach Number</u>

Is the ratio between gas velocity to sonic velocity,

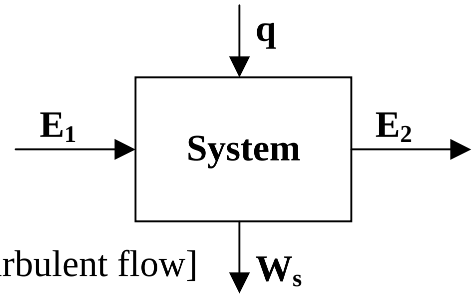
$$\boxed{Ma = \frac{u}{u_w}}$$

where, Ma > 1 supersonic velocity
Ma = 1 sonic velocity
Ma < 1 subsonic velocity

8.3 <u>General Energy Equation for Compressible Fluids</u>

Let E the total energy per unit mass of the fluid where,
E=Internal energy (U)+Pressure energy (Pυ)+Potential energy(zg)+Kinetic energy ($u^2/2$)

Assume the system in the Figure;



Energy balance

$E_1 + q = E_2 + W_s$

$\Rightarrow E_2 - E_1 = q - W_s$

$\Rightarrow \Delta U + \Delta(P\upsilon) + g\Delta(z) + \Delta(u^2/2) = q - W_s$

[α = 1 for compressible fluid since it almost in turbulent flow]

but $\Delta H = \Delta U + \Delta(P\upsilon)$

$\Rightarrow \Delta H + g\Delta(z) + \Delta(u^2/2) = q - W_s$

$dH + gd(z) + ud(u) = dq - dW_s$

but,

$dH = dq + dF + \upsilon dP$

where,

dF: amount of mechanical energy converted into heat

For irreversible process
$dW = Pd\upsilon - dF$ -------useful work
$dU = dq - dW$ -------closed system
$dH = dU + d(P\upsilon)$
$= dq - dW + d(P\upsilon)$
$= dq - (Pd\upsilon - dF) + d(P\upsilon)$
$= dq - Pd\upsilon + dF + Pd\upsilon + \upsilon dP$
$\Rightarrow dH = dq + dF + \upsilon dP$

$\Rightarrow$ u du + g dz + υ dP +dW_s + dF = 0

$$\boxed{\therefore \quad \frac{\Delta u^2}{2} + g\Delta z + \int_{P_1}^{P_2} \upsilon dP + Ws + F = 0}$$

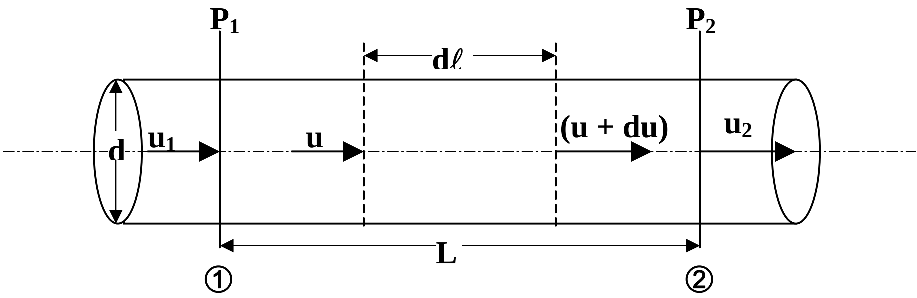
General equation of energy apply to any type of fluid

For compressible fluid flowing through (dℓ) of pipe of constant area

u du + g dz + υ dP +dW_s + 4Φ (dℓ/d) u^2 = 0 ----------------------------(*)

$$\dot{m} = \rho u A \quad \Rightarrow \frac{\dot{m}}{A} = \frac{u}{\upsilon} = G$$

∴ u = G υ ⇒ du = G dυ



Substitute these equations into equation (*), to give

G υ (G dυ) + g dz + υ dP +dW_s + 4Φ (dℓ/d) (G υ)^2 = 0

For horizontal pipe (dz = 0), and no shaft work (W_s =0)

$\Rightarrow$ G^2 υ (dυ) + υ dP + 4Φ (dℓ/d) (G υ)^2 = 0 ----------------------------(**)

Dividing by (υ^2) and integrating over a length L of pipe to give;

$$\boxed{G^2 \ln\left(\frac{\upsilon_2}{\upsilon_1}\right) + \int_{P_1}^{P_2} \frac{dP}{\upsilon} + 4\phi \frac{L}{d} G^2 = 0}$$

Kinetic energy Pressure energy Frictional energy

General equation of energy apply to compressible fluid in horizontal pipe with no shaft work

8.3.1 Isothermal Flow of an Ideal Gas in a Horizontal Pipe

For isothermal conditions of an ideal gas

P υ = constant ⇒ P υ = P_1 υ_1 ⇒ 1/υ = P / (P_1 υ_1)

$$\Rightarrow \int_{P_1}^{P_2} \frac{dP}{\upsilon} = \frac{1}{P_1 \upsilon_1} \int_{P_1}^{P_2} P\, dP = \frac{1}{2\, P_1 \upsilon_1} \left(P_2^2 - P_1^2\right) \quad \text{-----------------------(1)}$$

P_1 υ_1 = P_2 υ_2 ⇒ υ_2 / υ_1 = P_1/ P_2 -----------------------(2)

Substitute equations (1) and (2) into the genral equation of compressibl fluid to give;

$$\boxed{G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2\, P_1 \upsilon_1} + 4\phi \frac{L}{d} G^2 = 0}$$

Let υ_m the mean specific volume at mean pressure P_m, where,

P_m =(P_1 + P_2)/2

P_m υ_m = P_1 υ_1 ⇒ P_m =(P_1 + P_2)/2 = P_1 υ_1/ υ_m

$$\frac{(P_2^2 - P_1^2)}{2\, P_1 \upsilon_1} = \left(\frac{P_2 + P_1}{2}\right)\left(\frac{P_2 - P_1}{P_1 \upsilon_1}\right) = \left(\frac{P_1 \upsilon_1}{\upsilon_m}\right)\left(\frac{P_2 - P_1}{P_1 \upsilon_1}\right)$$

$$\therefore \quad \frac{P_2^2 - P_1^2}{2 P_1 \upsilon_1} = \frac{P_2 - P_1}{\upsilon_m}$$

$$\Rightarrow \quad \boxed{G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{P_2 - P_1}{\upsilon_m} + 4\phi \frac{L}{d} G^2 = 0}$$

If $(P_1 - P_2) / P_1 < 0.2$ the fist term of kinetic energy $[G^2 \ln(P_1/P_2)]$ is negligible.

$$\Rightarrow \quad \boxed{-\Delta P = (P_1 - P_2) = 4\phi \frac{L}{d} G^2 \upsilon_m = 4\phi \frac{L}{d} \rho_m u_m^2 = 4 f \frac{L}{d} \frac{\rho_m u_m^2}{2}}$$

It is used for low-pressure drop.

(i.e. the fluid can be treated as an incompressible fluid at the mean pressure in the pipe.)

8.3.1.1 Maximum Velocity in Isothermal Flow

From equation of isothermal conditions,

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2 P_1 \upsilon_1} + 4\phi \frac{L}{d} G^2 = 0$$

the mass velocity G = 0 when ($P_1 = P_2$)

At some intermediate value of P_2, the flow must therefore be a maximum. To find it, the differentiating the above equation with respect to P_2 for constant P_1 must be obtained.

i.e. ($dG/dP_2 = 0$),

First dividing the above equation by G^2

$$\Rightarrow \frac{(P_2^2 - P_1^2)}{2 P_1 \upsilon_1} \frac{1}{G^2} + \ln\left(\frac{P_1}{P_2}\right) + 4\phi \frac{L}{d} = 0$$

Then differentiating with respect to P_2

$$\frac{2P_2}{2 P_1 \upsilon_1 G^2} + \frac{(P_2^2 - P_1^2)}{2 P_1 \upsilon_1} \left(-2G^{-3}\right) \frac{dG}{dP_2} + \frac{1}{P_1 / P_2} \left(\frac{-P_1}{P_2^2}\right) = 0$$

Rearrangement

$$\frac{P_2}{P_1 \upsilon_1 G^2} + \frac{2}{G^3} \left(\frac{(P_2^2 - P_1^2)}{2 P_1 \upsilon_1}\right) \frac{dG}{dP_2} - \frac{1}{P_2} = 0$$

maximum velocity when ($dG/dP_2 = 0$) where, $P_2 = P_w$, and $G = G_w$

$$\frac{P_w}{P_1 \upsilon_1 G_w^2} = \frac{1}{P_w} \quad \Rightarrow \quad G_w^2 = \frac{P_w^2}{P_1 \upsilon_1}$$

but for isothermal conditions $P_1 \upsilon_1 = P_w \upsilon_w \quad \Rightarrow \quad P_w = P_1 \upsilon_1 / \upsilon_w$

$$\Rightarrow \quad G_w^2 = \frac{P_w}{\upsilon_w} \quad \Rightarrow \quad \left(\frac{u_w}{\upsilon_w}\right)^2 = \frac{P_w}{\upsilon_w}$$

$$\boxed{\therefore \quad u_w = \sqrt{P_w \upsilon_w} = \sqrt{P \upsilon}}$$

i.e. the sonic velocity is the maximum possible velocity.

$$\dot{m}_{\max} = A \rho_w u_w = A \frac{\sqrt{P_w \upsilon_w}}{\upsilon_w} = A \sqrt{\frac{P_w}{\upsilon_w}} \cdots\cdots\cdots \times \sqrt{\frac{P_w}{P_w}}$$

$$\Rightarrow \dot{m}_{\max} = A \sqrt{\frac{P_w^2}{P_w \upsilon_w}} = A P_w \sqrt{\frac{1}{P_w \upsilon_w}} = A P_w \sqrt{\frac{1}{P_1 \upsilon_1}} = A P_w \sqrt{\frac{1}{P_2 \upsilon_2}}$$

To find P_w, the following equation is used,

$$\boxed{\ln\left(\frac{P_1}{P_w}\right)^2 + 1 - \left(\frac{P_1}{P_w}\right)^2 + 8\phi \frac{L}{d} = 0}$$

to get P_w at any given P_1 [**its derivation is H.W.**]

Example -8.1-

Over a 30 m length of a 150 mm vacuum line carrying air at 295 K, the pressure falls from 0.4 kN/m2 to 0.13 kN/m2. If the relative roughness e/d is 0.003 what is the approximate flow rate? Take that $\mu_{\text{air at 295 K}} = 1.8 \times 10^{-5}$ Pa.s

Solution:

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2\,P_1\upsilon_1} + 4\phi\frac{L}{d}G^2 = 0$$

It is required the velocity or G for calculating Re that used to estimate Φ from Figure (3.7)-vol.I. i.e. the solution is by **trial and error** technique.

1- Assume Φ = 0.004

$$\upsilon_1 = \frac{1}{\rho_1} = \frac{RT}{P_1 Mwt} = \frac{8.314\,(\text{Pa.m}^3/\text{mol.K})\ 295\text{K}\ [(10^3\ \text{mol/kmol})]}{(0.4\times10^3\,\text{Pa})\ \ 29\ \text{kg/kmol}}$$

$= 211.434\ m^3/kg$

$$\Rightarrow G^2 \ln\left(\frac{0.4}{0.13}\right) + \frac{(0.13\times10^3 - 0.4\times10^3)}{2(0.4\times10^3)211.434} + 4(0.004)\frac{30}{0.15}G^2 = 0$$

$\Rightarrow \quad 4.324\ G^2 = 0.846 \Rightarrow \quad G = 0.44\ kg/m^2.s$

Re = G d / μ = 3686 ⇒ Φ = 0.005 (Figure 3.7)

2- Assume Φ = 0.005

$\Rightarrow \quad 1.124\ G^2 + 4\ G^2 = 0.846 \Rightarrow \quad G = 0.41\ kg/m^2.s$

Re = G d / μ = 3435 ⇒ Φ = 0.005 (Figure 3.7)

K.E. = $G^2 \ln(P_1/P_2) = (0.41)^2 \ln(0.4/0.13)$	**= 0.189 $kg^2/(m^4.s^2)$**
Press.E. = $(P_2^2 - P_1^2) / (2\ P_1 v_1)$	**= – 0.846 $kg^2/(m^4.s^2)$**
Frc.E. = 4 Φ L/d G^2	**= 0.6724 $kg^2/(m^4.s^2)$**
$[(P_1 - P_2) / P_1]$ %	**= 67.5%**

Example -8.2-

A flow of 50 m^3/s methane, measured at 288 K and 101.3 kPa has to be delivered along a 0.6 m diameter line, 3km long a relative roughness e = 0.0001 m linking a compressor and a processing unit. The methane is to be discharged at the plant at 288 K and 170 kPa, and it leaves the compressor at 297 K. What pressure must be developed at the compressor in order to achieve this flow rate? Take that $\mu_{\text{CH4 at 293 K}} = 0.01 \times 10^{-3}$ Pa.s

Solution:

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2\,P_1\upsilon_1} + 4\phi\frac{L}{d}G^2 = 0$$

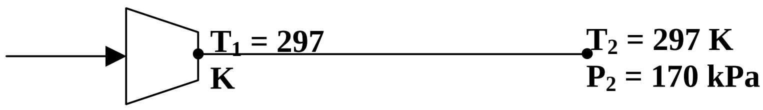
ΔT/L = 11ºC/3000 m = 0.00366ºC/m = 0.0366 ºC/10 m
= 0.366 ºC/100 m = 3.66 ºC/1000 m

$$G = \frac{\dot{m}}{A} = \frac{Q\rho}{A} = \frac{Q}{A\upsilon} \quad ,\upsilon = \frac{RT}{PMwt} = \frac{8.314\,(\text{Pa.m}^3/\text{mol.K})\ 288\text{K}\ [(10^3\ \text{mol/kmol})]}{(101.3\times10^3\,\text{Pa})\ \ 16\ \text{kg/kmol}} = 1.477\,m^3/kg$$

$\Rightarrow \quad G = (50) / [(\pi/4\ 0.6^2)(1.477)] = 119.7\ kg/m^2.s$

Since the difference in temperature is relatively small, therefore the processes could be consider isothermal at (T = T_m),

$T_m = (297 + 288)/2 = 293$ K



$$P_1\upsilon_1 = \frac{RT_m}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\,293\text{K}}{16\text{ kg/kmol}} = 1.5225\times10^5\ Pa.m^3/kg \quad or\left(J/kg \equiv m^2/s^2\right)$$

Re = G d / μ = 119.7(0.6)/0.01 x 10^{-3} = 7.182 x 10^6, e/d = 0.0001 / 0.6 = 0.00016

⇒ Φ = 0.0015 (Figure 3.7)

$$(119.7)^2 \ln\left(\frac{P_1}{170\times10^3}\right) + \frac{(170\times10^3 - P_1^2)}{2(1.5225\times10^5)} + 4(0.0015)\frac{3000}{0.6}(119.7)^2 = 0$$

⇒ ln P_1 – 2.292 x 10^{-10} P_1^2 + 24.58 = 0 $\Rightarrow P_1 = \sqrt{\dfrac{\ln p_1 + 24.58}{2.292\times10^{-10}}}$

Solution by trial and error

P_1 Assumed	200 x 10^3	400.617 x 10^3	404.382 x 10^3	404.432 x 10^3
P_1 Calculated	400.617 x 10^3	404.382 x 10^3	404.432 x 10^3	404.433 x 10^3

⇒ P_1 = 404.433 x 10^3 Pa

K.E. = G^2 ln(P_1/P_2) = 12418 $kg^2/(m^4.s^2)$
Press.E. =– 442253 $kg^2/(m^4.s^2)$
Frc.E. = 429842 $kg^2/(m^4.s^2)$
[(P_1 – P_2) / P_1] % = 58.5%

<u>Example -8.3-</u>

Town gas, having a molecular weight 13 kg/kmol and a kinematic viscosity of 0.25 stoke is flowing through a pipe of 0.25 m I.D. and 5 km long at arate of 0.4 m^3/s <u>and is delivered at atmospheric pressure.</u> Calculate the pressure required to maintain this rate of flow. The volume of occupied by 1 kmol and 101.3 kPa may be taken as 24 m^3. What effect on the pressure required would result if the gas was delivered at a height of 150 m (i) above and (ii) below its point of entry into the pipe? e = 0.0005 m.

<u>Solution:</u>

P_2 = P_1 = 101.3 kPa

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2P_1\upsilon_1} + 4\phi\frac{L}{d}G^2 = 0$$

$$G = \frac{\dot{m}}{A} = \frac{Q\rho}{A} = \frac{Q}{A\upsilon} \qquad ,\upsilon = 24\frac{m^3}{kmol}\left(\frac{1}{13\text{ kg/kmol}}\right) = 1.846 m^3/kg$$

⇒ G = (0.4) / [(π/4 0.25^2)(1.846)] = 4.414 kg/m^2.s

Re = G d / μ = G d / (ρ ν) = 4.414 (0.25) / [(1/1.846) 0.25 x 10^{-4}] = 8.1489 x 10^4,

e/d = 0.0005 / 0.25 = 0.002 ⇒ Φ = 0.0031 (Figure 3.7)

As first approximation the kinetic energy term will be omitted

$$\frac{-\Delta P}{\upsilon_m} = \frac{(P_1 - P_2)}{\upsilon_m} = 4\phi\frac{L}{d}G^2, \qquad \upsilon_2 = 1.846\ m^3/kg,\ \upsilon_1 = RT/(P_1\ Mwt)$$

⇒ υ_1 = (8314) (289) / [(P_1) 13] = 184.826 x $10^3/P_1$

υ_m = (υ_1 + υ_2) / 2 = [(184.826 x $10^3/P_1$) + 1.846]/2 = [92413.3+0.923 P_1]/ P_1

$$\Rightarrow \frac{(P_1 - 101.3\times10^3)P_1}{92413.3 + 0.923P_1} = 4(0.0031)\frac{5000}{0.25}(4.414)^2$$

$$\Rightarrow P_1^2 - 101.3\times10^3 P_1 = 4831.9(92413.3 + 0.923P_1) = 4.4653\times10^8 + 4459.8P_1$$

$$x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$\Rightarrow$ $P_1^2 - 105.76 \times 10^3 P_1 - 4.4653 \times 10^8 = 0$
either $P_1 = 109.825 \times 10^3$ Pa
or $P_1 = -4065.8$ ------neglect

K.E. = $G^2 \ln(P_1/P_2)$ = 1.5744 kg²/(m⁴.s²)
Press.E. =– 4831.9 kg²/(m⁴.s²)
Frc.E. = 4831.9 kg²/(m⁴.s²)
[($P_1 - P_2$) / P_1] % = 7.7 %

$\therefore$ The first approximation is justified

If use the equation of the terms;

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2P_1\upsilon_1} + 4\phi\frac{L}{d}G^2 = 0 \text{------ Neglect the kinetic energy term}$$

$$\frac{(P_2^2 - P_1^2)}{2P_1\upsilon_1} + 4\phi\frac{L}{d}G^2 = 0$$

$$P_1\upsilon_1 = \frac{RT}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 289\text{K}}{13\ \text{kg/kmol}} = 184.8266 \times 10^3 \left(J/kg \equiv m^2/s^2\right)$$

$$\Rightarrow P_1^2 = P_2^2 + 2P_1\upsilon_1\left(4\phi\frac{L}{d}G^2\right) = 0$$

$$= (101.3 \times 10^3)^2 + 2(184.8266 \times 10^3)\,[4(0.0031)(5000/0.25)(4.414)^2]$$

$\Rightarrow P_1^2 = 1.20478 \times 10^{10} \Rightarrow P_1 = 109.762 \times 10^3$ Pa

If the pipe is not horizontal, the term (g dz) must be included in equation (**) or the term ($g\,\Delta z/\upsilon_m^2$) to integration of this equation [i.e. General equation of energy apply to compressible fluid in horizontal pipe with no shaft work]

$\upsilon_m = 1.7644$ m³/kg, $\upsilon_{air} = (8314 \times 289)/(101.3 \times 10^3 \times 29) = 0.8179$ m³/kg
$\rho_m = 0.5668$ kg / m³, $\rho_{air} = 1.223$ kg / m³

As gas is less dense than air, υ_m is replaced by ($\upsilon_{air} - \upsilon_m$) in potential energy term;

$$\frac{g\,\Delta z}{(\upsilon_{air} - \upsilon_m)^2} = \frac{9.81(150)}{(-0.9465)^2} = 1642.55 \quad kg^2/m^4.s^2 \qquad and \quad \frac{g\,\Delta z}{(\upsilon_{air} - \upsilon_m)} = 1555\ Pa$$

(i) Point ② 150 m above point ① ⇒ $P_1 = 109.762 \times 10^3 - 1555 = 108.207 \times 10^3$ Pa

(ii) Point ② 150 m below point ① ⇒ $P_1 = 109.762 \times 10^3 + 1555 = 111.317 \times 10^3$ Pa

Example -8.4-

Nitrogen at 12 MPa pressure fed through 25 mm diameter mild steel pipe to a synthetic ammonia plant at the rate of 1.25 kg/s. What will be the drop in pressure over a 30 m length of pipe for isothermal flow of the gas at 298 K? e = 0.0005 m, μ = 0.02 mPa.s

Solution:

$P_1 = 12$ MPa

First approximation [neglect the kinetic energy]

$$\Rightarrow \quad \frac{(P_2^2 - P_1^2)}{2P_1\upsilon_1} + 4\phi\frac{L}{d}G^2 = 0$$

$$P_1\upsilon_1 = \frac{RT}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 298\text{K}}{28\ \text{kg/kmol}} = 88484.7 \quad \left(J/kg \equiv m^2/s^2\right)$$

$$\Rightarrow P_2^2 = P_1^2 - 2\,P_1\upsilon_1\left(4\phi\frac{L}{d}G^2\right) = 0$$

$$G = \frac{\dot{m}}{A} \quad = \frac{1.25}{\pi/4(0.025)^2} = 2546.48 \quad kg/m^2.s$$

Re = G d / μ = 2546.48 (0.025) / 0.02 x 10^{-3} = 3.183 x 10^6, e/d = 0.0002

⇒ Φ = 0.0017 (Figure 3.7)

$P_2^2 = (12 \times 10^6)^2 - 2(88484.7)\,[4(0.0017)(30/0.025)(2546.48)^2]$

⇒ $P_2 = 11.603 \times 10^6$ Pa

K.E. = $G^2 \ln(P_1/P_2)$	**= 2.1816 x 10^5 $kg^2/(m^4.s^2)$**
Press.E.	**= – 529.492 x 10^5 $kg^2/(m^4.s^2)$**
Frc.E.	**= 529.14 x 10^5 $kg^2/(m^4.s^2)$**
[($P_1 - P_2$) / P_1] %	**= 3.3 %**

∴ the first approximation is justified

Example -8.5-

Hydrogen is pumped from a reservoir at 2 MPa pressure through a clean horizontal mild steel pipe 50 mm diameter and 500 m long. The downstream pressure is also 2 MPa. And the pressure of this gas is raised to 2.6 MPa by a pump at the upstream end of the pipe. The conditions of the flow are isothermal and the temperature of the gas is 293 K. What is the flow rate and what is the effective rate of working of the pump if η = 0.6 e = 0.05 mm, μ = 0.009 mPa.s.

Solution:

First approximation [neglect the kinetic energy]

$$\Rightarrow \quad \frac{(P_2^2 - P_1^2)}{2\,P_1\upsilon_1} + 4\phi\frac{L}{d}G^2 = 0$$

$$P_1\upsilon_1 = \frac{RT}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\,293\text{K}}{2\text{ kg/kmol}} = 121.8\times10^4 \quad (J/kg \equiv m^2/s^2)$$

P_1 = 2.6 MPa, P_2 = 2 MPa, $-\Delta P_f = P_1 - P_2 = 0.6 \times 10^6$ Pa

$\rho_m = 1/\upsilon_m = P_m$ Mwt/RT = (2.3 x 10^6) 2 / (8314 x 293) = 1.89 kg/m^3

$\Phi Re^2 = (-\Delta P_f/L)(\rho_m d^3/4\mu^2) = [(0.6\times10^6)/(500)][(1.89)(0.05)^3/(4)(0.009\times10^{-3})^2] = 8.75\times10^8$

e/d = 0.001 ⇒Figure (3.8) Re = 5.9 x10^5 ⇒ G = 5.9 x10^5 (0.009x10^{-3}) / (0.05)

⇒G = 106.2 kg / m^2.s

Re = 5.9 x10^5, e/d = 0.001 ⇒ Φ = 0.0025 (Figure 3.7)

$$\Rightarrow \quad G^2 = \frac{(P_2^2 - P_1^2)}{(2\,P_1\upsilon_1)(-4\phi\,L/d)} = \frac{(2\times10^6)^2 - (2.6\times10^6)^2}{(2\times121.8\times10^4)[-4(0.0025)(500/0.05)]} = 11330$$

⇒ G = 106.44 kg / m^2.s ------------------------.∴ ok

K.E. = $G^2 \ln(P_1/P_2)$	**= 2.9726 x 10^3 $kg^2/(m^4.s^2)$**
Press.E.	**= – 1133.005 x 10^3 $kg^2/(m^4.s^2)$**
Frc.E.	**= 1132.94736 x 10^3 $kg^2/(m^4.s^2)$**
[($P_1 - P_2$) / P_1] %	**= 3.3 %**

∴ the neglecting the kinetic energy term is OK

$$Power = \frac{\dot{m}P_1\upsilon_1\ln(P_1/P_2)}{\eta} = \frac{0.209(121.8\times10^4)\ln(2.6/2)}{0.6} = 111.3\ kW$$

H.W. drive this equation

Example -8.6-

In the synthetic ammonia plant the hydrogen is fed through a 50 mm diameter steel pipe to the converters. The pressure drop over the 30 m length of pipe is 500 kPa, the pressure at the downstream end being 7.5 MPa. What power is required in order to overcome friction losses in the pipe? Assume isothermal expansion of the gas at 298 K. What error introduced by assuming the gas to be an incompressible fluid of density equal to that at the mean pressure in the pipe? μ = 0.02 mPa.s.

Solution:

$P_2 = 7.5$ MPa, $P_1 = P_2 + (-\Delta P_f) = 7.5$ MPa $+ 0.5$ MPa $= 8$MPa $= 8 \times 10^6$ Pa

The pressure $(P_m) = (P_1 + P_2)/2 = 7.75 \times 10^6$ Pa

$[(P_1 - P_2) / P_1]$ % = 6.25 %

$$\rho_m = \frac{P_m . Mwt}{RT} = \frac{7.75\times10^6(2)}{8314(298)} = 6.256\, kg/m^3$$

For incompressible fluids

$$\frac{-\Delta P}{\rho_m} = 4\phi \frac{L}{d} u^2$$

$$\Rightarrow -\Delta P\,\rho_m = 4\phi \frac{L}{d} u^2 \rho_m^2 = 4\phi \frac{L}{d} G^2 \quad \Rightarrow G^2 = \frac{-\ \Delta P\,\rho_m}{4\phi L/d}$$

Assume Φ = 0.003

⇒ $G^2 = 434{,}444.444$ kg^2/m^4.s^2 ⇒ $G = 659.124$ kg/m^2.s

⇒ $Re = 1.647 \times 10^6$, and Φ = 0.003 ⇒ from Figure (3.7) e/d = 0.00189

⇒ e = 0.09 mm (this value is reasonable for steel)

For compressible fluids

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2\,P_1\upsilon_1} + 4\phi \frac{L}{d} G^2 = 0$$

$$G^2 \ln\left(\frac{8}{7.5}\right) + \frac{(7.5\times10^6)^2 - (8\times10^6)^2}{2[8314(298)/2]} + 4(0.003)\frac{30}{0.05} G^2 = 0$$

⇒ $G^2 = 430{,}593.418$ kg^2/m^4.s^2 ⇒ $G = 656.2$ kg/m^2.s

Very little error is made by the simplifying assumption in this particular case.

$$Power = \frac{\dot{m} P_1 \upsilon_1 \ln(P_1/P_2)}{\eta} = \frac{(656.2\times\pi/4\ 0.005^2)(123.8786\times10^4)\ln(8/7.5)}{0.6} = 171.7\ kW$$

Example -8.7-

A vacuum distillation plant operating at 7 kPa pressure at top has a boil-up rate of 0.125 kg/s of xylene. Calculate the pressure drop along a 150 mm bore vapor pipe used to connect the column to the condenser. And also calculate the maximum flow rate if L = 6 m, e = 0.0003 m, Mwt = 106 kg/kmol, T = 338 K, μ = 0.01 mPa.s.

Solution:

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2\,P_1\upsilon_1} + 4\phi \frac{L}{d} G^2 = 0$$

$G = 0.125 / [\pi/4\ (0.15)^2] = 7.074$ kg/m^2.s

$P_1 = 7$ kPa, P_2 = Pressure at condenser

$$P_1\upsilon_1 = \frac{RT}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 338\text{K}}{106\ \text{kg/kmol}} = 26510.68 \quad \left(J/kg \equiv m^2/s^2\right)$$

Re = G d / μ = 7.074(0.15)/0.01 x 10^{-3} = 1.06 x 10^5, e/d = 0.002

⇒ Φ = 0.003 (Figure 3.7)

$$\Rightarrow \ln\left(\frac{7\times10^3}{P_2}\right) + 3.769\times10^{-7}[P_2^2 - (7\times10^3)^2] + 4(0.003)\frac{6}{0.15} = 0$$

$$\Rightarrow \quad P_2^2 = \left(7\times10^3\right)^2 - \frac{\ln(7\times10^3/P_2)+0.48}{3.769\times10^{-7}} \quad \Rightarrow \quad P_2 = \sqrt{\left(7\times10^3\right)^2 - \frac{\ln(7\times10^3/P_2)+0.48}{3.769\times10^{-7}}}$$

Solution by trial and error

P_2 Assumed	5 x 10^3	6.8435 x 10^3	6.904 x 10^3	6.9057 x 10^3
P_2 Calculated	6.8435 x 10^3	6.904 x 10^3	6.9057 x 10^3	6.9058 x 10^3

⇒ P_2 = 6.9058 x 10^3 Pa

–ΔP = $P_1 – P_2$ = (7 – 6.9058) x 10^3 = 94.2Pa

[($P_1 – P_2$) / P_1] % = 0.665 % we can neglect the K.E. term in this problem

H.W. resolve this example with neglecting the K.E. term

For maximum flow rate calculations

$$\dot{m}_{\max} = A\,P_w\sqrt{1/P_1\upsilon_1} \quad \Rightarrow \quad G_{\max} = P_w\sqrt{1/P_1\upsilon_1}$$

To estimate P_w

$$\ln\left(\frac{P_1}{P_w}\right)^2 + 1 - \left(\frac{P_1}{P_w}\right)^2 + 8\phi\frac{L}{d} = 0$$

Let X ≡ $(P_1/P_w)^2$

⇒ ln(X) + 1 – X + 8 Φ L/d = 0 ⇒ X = 1.96 + ln(X)

Solution by trial and error

X Assumed	1.2	2.14	2.72	2.96	3.074	3.086	3.087
X Calculated	2.14	2.72	2.96	3.074	3.086	3.087	3.087

⇒ X = 3.087 = $(P_1/P_w)^2$ ⇒ $P_w = P_1/(3.087)^{0.5}$ = 3984 Pa

∴ the system does not reach maximum velocity (H.W. explain)

⇒ G_{max} = 3984 / $(26510.68)^{0.5}$ = 24.47 kg/m^2.s

Example -8.8-

A vacuum system is required to handle 10 g/s of vapor (molecular weight 56 kg/kmol) so as to maintain a pressure of 1.5 kN/m^2 in a vessel situated 30 m from the vacuum pump. If the pump is able to maintain a pressure of 0.15 kN/m^2 at its suction point, what diameter of pipe is required? The temperature is 290 K, and isothermal conditions may be assumed in the pipe, whose surface can be taken as smooth. The ideal gas law is followed. Gas viscosity μ = 0.01 mN s/m^2.

Solution:

$$G^2\ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2\,P_1\upsilon_1} + 4\phi\frac{L}{d}G^2 = 0$$

$$G = \frac{\dot{m}}{\pi/4\,d^2} = 10\times10^{-3} / [\pi/4\,(d)^2] = 0.0127\,d^{-2}$$

Re = G d / μ = 1273.25 d^{-1} ----------------------------------(1)

$$P_1\upsilon_1 = \frac{RT}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 290\text{K}}{56\ \text{kg/kmol}} = 43054.64 \quad \left(J/kg \equiv m^2/s^2\right)$$

⇒ $2.3\ G^2 - 52.97 + 120\ \Phi/d\ G^2 = 0$

⇒ $3.733 \times 10^{-4}\ d^{-4} - 52.97 + 0.019\ d^{-3}\ \Phi = 0$

⇒ $d = \left[\dfrac{52.97 - 0.019\,d^{-3}\phi}{3.733 \times 10^{-4}}\right]^{-1/4}$ ----------------------------------(2)

<u>Assume smooth pipe</u>

Solution by trial and error

		Eq.(1)		Figure (3.7)	Eq.(2)
Assume	d = 0.1 ⇒	Re = 1.3 x 10^{-4}	⇒	Φ = 0.0038 ⇒	d = 0.0515
	d = 0.0515 ⇒	Re = 2.5 x 10^{-4}	⇒	Φ = 0.0028 ⇒	d = 0.0516

∴ d = 0.0516 m.

8.3.2 Adiabatic Flow of an Ideal Gas in a Horizontal Pipe

The general energy equation of a steady-state flow system is: -

$dH + g\,dz + u\,du = dq - dW_s$

For adiabatic conditions (dq = 0) and in horizontal pipe (dz = 0) with no shaft work dWs = 0)

$\Rightarrow \quad dH + u\,du = 0$

but $G = \frac{\dot{m}}{A} = \rho u = \frac{u}{\upsilon} \quad \Rightarrow \quad u = \upsilon G$

dH	$= dU + d(P\upsilon)$
$c_p\,dT$	$= c_v\,dT + R\,dT$
$\therefore c_p$	$= c_v + R$

$\Rightarrow \quad dH + G^2\,\upsilon\,d\upsilon = 0$

we have $dH = c_p\,dT$, and $dP\upsilon = RdT \Rightarrow dT = dP\upsilon/R = dP\upsilon/(c_p - c_v)$

$\Rightarrow \quad dH = c_p\,[dP\upsilon/(c_p - c_v)] = (c_p / c_v) /[(c_p - c_v)/ c_v]\,dP\upsilon = [\gamma/(\gamma - 1)]\,dP\upsilon$

$$\boxed{\therefore \frac{\gamma}{\gamma-1}dP\upsilon + G^2\upsilon\,d\upsilon = 0}$$

The integration of this equation gives

$$\boxed{\frac{\gamma}{\gamma-1}P_1\upsilon_1 + \frac{G^2}{2}\upsilon_1^2 = \frac{\gamma}{\gamma-1}P_2\upsilon_2 + \frac{G^2}{2}\upsilon_2^2 = \frac{\gamma}{\gamma-1}P\upsilon + \frac{G^2}{2}\upsilon^2 = K}$$

<u>This equation is used to estimate the downstream pressure P_2</u>

To estimate the downstream specific volume υ_2 the procedure is as follow

$$\frac{\gamma}{\gamma-1}P\upsilon = K - \frac{G^2}{2}\upsilon^2 \Rightarrow P = \left(\frac{\gamma-1}{\gamma}\right)\left[\frac{K}{\upsilon} - \frac{G^2}{2}\upsilon\right]$$

$$\Rightarrow dP = \left(\frac{\gamma-1}{\gamma}\right)\left[-\frac{K}{\upsilon^2} - \frac{G^2}{2}\right]d\upsilon \qquad \div \upsilon$$

$$\Rightarrow \frac{dP}{\upsilon} = \left(\frac{\gamma-1}{\gamma}\right)\left[-\frac{K}{\upsilon^3} - \frac{G^2}{2\upsilon}\right]d\upsilon$$

$$\Rightarrow \int_{P_1}^{P_2}\frac{dP}{\upsilon} = \left(\frac{\gamma-1}{\gamma}\right)\left[\frac{K}{2}\left(\frac{1}{\upsilon_2^2} - \frac{1}{\upsilon_1^2}\right) - \frac{G^2}{2}\ln\left(\frac{\upsilon_2}{\upsilon_1}\right)\right]$$

But, $K = \frac{G^2}{2}\upsilon_1^2 + \frac{\gamma}{\gamma-1}P_1\upsilon_1$

$$\Rightarrow \int_{P_1}^{P_2}\frac{dP}{\upsilon} = \left(\frac{\gamma-1}{\gamma}\right)\left[\frac{G^2}{4}\upsilon_1^2\left(\frac{1}{\upsilon_2^2} - \frac{1}{\upsilon_1^2}\right) + \frac{\gamma}{\gamma-1}\frac{P_1\upsilon_1}{2}\left(\frac{1}{\upsilon_2^2} - \frac{1}{\upsilon_1^2}\right) - \frac{G^2}{2}\ln\left(\frac{\upsilon_2}{\upsilon_1}\right)\right]$$

$$= \frac{\gamma-1}{4\gamma}G^2\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2 - 1 - 2\ln\left(\frac{\upsilon_2}{\upsilon_1}\right)\right] + \frac{P_1\upsilon_1}{2}\left(\frac{1}{\upsilon_2^2} - \frac{1}{\upsilon_1^2}\right)$$

But,

$$G^2\ln\left(\frac{\upsilon_2}{\upsilon_1}\right) + \int_{P_1}^{P_2}\frac{dP}{\upsilon} + 4\phi\frac{L}{d}G^2 = 0$$

The general equation of energy apply to compressible fluid in horizontal pipe with no shaft work

$$\Rightarrow G^2\ln\left(\frac{\upsilon_2}{\upsilon_1}\right)+\frac{\gamma-1}{4\gamma}G^2\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2-1-2\ln\left(\frac{\upsilon_2}{\upsilon_1}\right)\right]+\frac{P_1\upsilon_1}{2}\left(\frac{1}{\upsilon_2^2}-\frac{1}{\upsilon_1^2}\right)+4\phi\frac{L}{d}G^2=0\cdots\cdots\cdots\times\frac{2}{G^2}$$

$$\Rightarrow 2\ln\left(\frac{\upsilon_2}{\upsilon_1}\right)-\frac{\gamma-1}{\gamma}\ln\left(\frac{\upsilon_2}{\upsilon_1}\right)+\frac{\gamma-1}{2\gamma}\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2-1\right]+\frac{P_1}{\upsilon_1G^2}\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2-1\right]+8\phi\frac{L}{d}=0$$

$$\Rightarrow \frac{2\gamma-\gamma+1}{\gamma}\ln\left(\frac{\upsilon_2}{\upsilon_1}\right)+\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2-1\right]\left[\frac{\gamma-1}{2\gamma}+\frac{P_1}{\upsilon_1G^2}\right]+8\phi\frac{L}{d}=0$$

$$\boxed{\Rightarrow \frac{\gamma+1}{\gamma}\ln\left(\frac{\upsilon_2}{\upsilon_1}\right)+\left[\frac{\gamma-1}{2\gamma}+\frac{P_1}{\upsilon_1G^2}\right]\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2-1\right]+8\phi\frac{L}{d}=0}$$

This equation is used to estimate the downstream specific volume υ_2

8.3.2.1 Maximum Velocity in Adiabatic Flow

For constant upstream conditions, the maximum flow through the pipe is found by differentiating (G) with respect to (υ_2) of the last equation and putting ($dG/d\upsilon_2$) equal to zero.

The maximum flow is thus shown to occur when the velocity at downstream end of the pipe is the sonic velocity.

i.e. $\frac{dG}{d\upsilon_2}=0 \quad \Rightarrow u_w=\sqrt{\gamma P_2\upsilon_2} \quad \Rightarrow G_{\max}=\frac{\sqrt{\gamma P_2\upsilon_2}}{\upsilon_2}=\sqrt{\frac{\gamma P_2}{\upsilon_2}}$

Note: -

In isentropic (or adiabatic) flow [$P_1\upsilon_1 \neq P_2\upsilon_2$] where, in these conditions [$P_1\upsilon_1^{\gamma} \neq P_2\upsilon_2^{\gamma}$]

i.e. $u_w=\sqrt{\gamma P_2\upsilon_2}\neq\sqrt{\gamma P_1\upsilon_1}$

Typical values of (γ) for ordinary temperatures and pressures are: -

i- For monatomic gases such as He, Ar (γ = 1.67)
ii- For diatomic gases such as H_2, N_2, CO (γ = 1.4)
iii- For tritomic gases such as CO_2 (γ = 1.3)

Example -8.9-

Air, at a pressure of 10 MN/m^2 and a temperature of 290 K, flows from a reservoir through a mild steel pipe of 10 mm diameter and 30 m long into a second reservoir at a pressure P_2. Plot the mass rate of flow of the air as a function of the pressure P_2. Neglect any effects attributable to differences in level and assume an adiabatic expansion of the air. μ = 0.018 mN s/m^2, γ = 1.36.

Solution:

$$\frac{\gamma+1}{\gamma}\ln\left(\frac{\upsilon_2}{\upsilon_1}\right)+\left[\frac{\gamma-1}{2\gamma}+\frac{P_1}{\upsilon_1G^2}\right]\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2-1\right]+8\phi\frac{L}{d}=0$$

$$\upsilon_1=\frac{RT}{P_1\,Mwt}=\frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\,290\text{K}}{10\times10^6\,Pa\,(29\text{ kg/kmol})}=8.314\times10^{-3}\,m^3/kg$$

$$\Rightarrow 1.735\ln\left(\frac{\upsilon_2}{8.314\times10^{-3}}\right)+\left[0.132+\frac{1.2028\times10^9}{G^2}\right]\left[\left(\frac{8.314\times10^{-3}}{\upsilon_2}\right)^2-1\right]+24000\phi=0$$

$$\Rightarrow \left(\frac{8.314\times10^{-3}}{\upsilon_2}\right)^2 = 1-\frac{1.735\ln\left(\frac{\upsilon_2}{8.314\times10^{-3}}\right)+24000\phi}{0.132+\frac{1.2028\times10^9}{G^2}}$$

$$\Rightarrow \upsilon_2 = \frac{8.314\times10^{-3}}{\sqrt{1-\frac{1.735\ln\left(\frac{\upsilon_2}{8.314\times10^{-3}}\right)+24000\phi}{0.132+\frac{1.2028\times10^9}{G^2}}}} \text{ ----------------------------(1)}$$

Re = Gd/ μ = 555.6 G ----------------------------(2)

$$\frac{\gamma}{\gamma-1}P_1\upsilon_1+\frac{G^2}{2}\upsilon_1^2=\frac{\gamma}{\gamma-1}P_2\upsilon_2+\frac{G^2}{2}\upsilon_2^2 \qquad \Rightarrow \frac{\gamma}{\gamma-1}P_2\upsilon_2=\frac{\gamma}{\gamma-1}P_1\upsilon_1+\frac{G^2}{2}(\upsilon_1^2-\upsilon_2^2)$$

$$\Rightarrow P_2=\frac{P_1\upsilon_1}{\upsilon_2}+\frac{\gamma-1}{2\gamma}\frac{G^2}{\upsilon_2}(\upsilon_1^2-\upsilon_2^2)$$

$$\Rightarrow P_2=\frac{83140}{\upsilon_2}+0.132\frac{G^2}{\upsilon_2}(6.91\times10^{-5}-\upsilon_2^2) \text{ ----------------------------(3)}$$

1- at $P_2 = P_1 \Rightarrow G = 0$

eq.(2) Figure (3.7)

2- assume G = 2000 kg/m^2.s $\Rightarrow$ Re = 1.11 x 10^6 $\Rightarrow$ Φ = 0.0028

Solution by trial and error

υ_2 Assumed	10 x 10^{-3}	9.44 x 10^{-3}
υ_2 Calculated eq.(1)	9.44 x 10^{-3}	9.44 x 10^{-3}

$\Rightarrow$ υ_2 = 9.44 x 10^{-3} m^3/kg $\Rightarrow$ P_2 = 8.8 x 10^6 Pa

3- assume G = 3000 kg/m^2.s $\Rightarrow$ Re = 1.6 x 10^6 $\Rightarrow$ Φ = 0.0028

Solution by trial and error

υ_2 Assumed	10 x 10^{-3}	11.8 x 10^{-3}
υ_2 Calculated eq.(1)	11.8 x 10^{-3}	11.81 x 10^{-3}

$\Rightarrow$ υ_2 = 11.84 x 10^{-3} m^3/kg $\Rightarrow$ P_2 = 7.013 x 10^6 Pa

G (kg/m^2.s)	υ_2(m^3/kg)	P_2(Mpa)
0	8.314 x 10^{-3}	10
2000	9.44 x 10^{-3}	8.8
3000	11.81 x 10^{-3}	7.013
3500	16.5 x 10^{-3}	5.01
4000	25 x 10^{-3}	3.37
4238	39 x 10^{-3}	2.04

Example -8.10-

Nitrogen at 12 MN/m^2 pressure is fed through a 25 mm diameter mild steel pipe to a synthetic ammonia plant at the rate of 0.4 kg/s. **What** will be the drop in pressure over a 30 m length of pipe assuming isothermal expansion of the gas at 300 K? **What** is the average quantity of heat per unit area of pipe surface that must pass through the walls in order to maintain isothermal conditions? **What** would be the pressure drop in the pipe if it were perfectly lagged? **What** would be the maximum flow rate in each case? Or **what** would be the Mach number? $\mu = 0.02$ mNs/m^2, $\gamma = 1.36$, e/d = 0.002.

Solution:

$$G^2 \ln\left(\frac{P_1}{P_2}\right) + \frac{(P_2^2 - P_1^2)}{2 P_1 \upsilon_1} + 4\phi \frac{L}{d} G^2 = 0$$

$$P_1\upsilon_1 = \frac{RT}{Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 300\text{K}}{28\ \text{kg/kmol}} = 89078.6 \quad (J/kg \equiv m^2/s^2)$$

$G = \dfrac{\dot{m}}{\pi/4\, d^2} = 0.4 / [\pi/4\ (0.25)^2] = 814.9$ kg/m^2 .s, Re = G d / μ = 1.02×10^6

e/d = 0.002 $\Rightarrow$ $\Phi = 0.0028$ Figure (3.7)

Neglect the K.E. term

$\Rightarrow$ $P_2^2 = P_1^2 - 2 P_1\upsilon_1(4\,\Phi\,(L/d)\,G^2) = 1.4241 \times 10^{14}$

$\Rightarrow$ $P_2 = 11.93 \times 10^6$ Pa

K.E. = $G^2 \ln(P_1/P_2)$ = 3.885×10^4 kg^2/(m^4.s^2)
Press.E. = -940.24×10^4 kg^2/(m^4.s^2)
Frc.E. = 892.5×10^4 kg^2/(m^4.s^2)
[$(P_1 - P_2) / P_1$] % = 0.583 %

$\therefore$ the neglecting the kinetic energy term is OK

$\Rightarrow$ $-\Delta P = P_1 - P_2 = 0.07 \times 10^6$ Pa

P_1
υ_1 ⟶ **Plant**
u_1 **u_2 =0**

isothermal horizontal no shaft work

$\cancel{dH} + g\,\cancel{dz} + u\,du = dq - \cancel{dW_s}$

$\Rightarrow$ $u\,du = dq$ $\Rightarrow$ $q = \Delta u^2/2 = u_1^2/2$ [since the veloity in the plant is taken as zero]

$\Rightarrow$ $q = (G\,\upsilon_1)^2/2 = [814.9(89078.6/12 \times 10^6)]^2 / 2 = 18.3$ J/kg

The total heat pass through the wall $q_T = \dot{m}\, q$ = 0.4 (18.3) = 7.32 W

Heat flux $q'' = q_T / A$ = q_T / (π d L) = 7.32 / [π (0.025) 30] = 3.1 W/m^2

It is clear that the heat flux is very low value that could be considered the process is adiabatic.

For adiabatic conditions

$$\frac{\gamma+1}{\gamma}\ln\left(\frac{\upsilon_2}{\upsilon_1}\right) + \left[\frac{\gamma-1}{2\gamma} + \frac{P_1}{\upsilon_1 G^2}\right]\left[\left(\frac{\upsilon_1}{\upsilon_2}\right)^2 - 1\right] + 8\phi\frac{L}{d} = 0$$

$$\Rightarrow \upsilon_2 = \frac{\upsilon_1}{\sqrt{1 - \dfrac{\dfrac{\gamma+1}{\gamma}\ln\left(\dfrac{\upsilon_2}{\upsilon_1}\right) + 8\phi\dfrac{L}{d}}{\dfrac{\gamma-1}{2\gamma} + \dfrac{P_1}{G^2\upsilon_1}}}} = \frac{7.423\times10^{-3}}{\sqrt{1 - \dfrac{1.714\ln\left(\dfrac{\upsilon_2}{7.423\times10^{-3}}\right) + 26.88}{0.143 + 2434.336}}}$$

Solution by trial and error

υ_2 Assumed	10 x 10^{-3}	7.5 x 10^{-3}
υ_2 Calculated	7.5 x 10^{-3}	7.46 x 10^{-3}

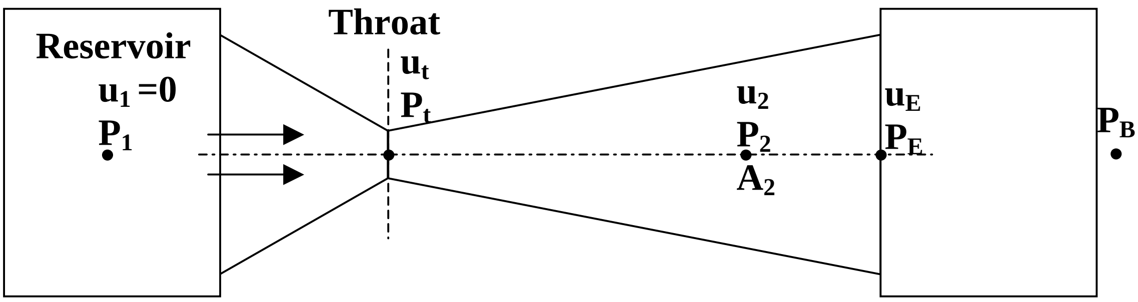
$\Rightarrow \quad \upsilon_2 = 7.46 \text{ x } 10^{-3}$ m^3/kg

$$P_2 = \frac{P_1\upsilon_1}{\upsilon_2} + \frac{\gamma-1}{2\gamma}\frac{G^2}{\upsilon_2}(\upsilon_1^2 - \upsilon_2^2) \;\Rightarrow \qquad P_2 = 11.94 \text{ x } 10^6 \text{ Pa}$$

This value of P_2 in adiabatic conditions is very close to the value in isothermal condition since the actual heat flux is very small.

8.4 <u>Converging-Diverging Nozzles for Gas Flow</u>

Converging-diverging nozzles, sometimes known as “Laval nozzles”, are used for expansion of gases where the pressure drop is large.



P_1: the pressure in the reservoir or initial pressure.
P_2: the pressure at any point in diverging section of the nozzle.
P_E: the pressure at exit of the nozzle.
P_B: the back pressure or the pressure at end.
$P_{critical}$: the pressure at which the velocity of the gas is sonic velocity.

Because the flow rate is large for high-pressure differentials, there is <u>**little time**</u> for heat transfer to take place between the gas and surroundings and the expansion is effectively **isentropic** [adiabatic + reversible]**.**

In these conditions,

$$\frac{\upsilon_2}{\upsilon_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} \quad \Rightarrow \upsilon_2 = \upsilon_1\left(\frac{P_2}{P_1}\right)^{-\frac{1}{\gamma}}$$

$$\frac{\Delta u^2}{2} + g\Delta z + \int_{P_1}^{P_2}\upsilon dP + Ws + F = 0$$ the genral energy equation for any type of fluid.

for gas flow from reservoir (u_1 = 0) at pressure (P_1) in a horizontal direction, with no shaft work, and by assuming F=0 this equation becomes

$$\frac{u_2^2}{2} + \int_{P_1}^{P_2}\upsilon dP = 0$$ and the pressure energy term is,

$$\int_{P_1}^{P_2}\upsilon dP = \upsilon_1 P_1^{\frac{1}{\gamma}}\int_{P_1}^{P_2}P^{\frac{1}{\gamma}}dP = \upsilon_1 P_1^{\frac{1}{\gamma}}\left[\frac{P^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}}\right]_{P_1}^{P_2} = \upsilon_1 P_1^{\frac{1}{\gamma}}\left(\frac{\gamma}{\gamma-1}\right)\left[P_2^{\frac{\gamma-1}{\gamma}} - P_1^{\frac{\gamma-1}{\gamma}}\right]\cdots\cdots\cdots\times\frac{P_1^{\frac{\gamma-1}{\gamma}}}{P_1^{\frac{\gamma-1}{\gamma}}}$$

$$\Rightarrow \int_{P_1}^{P_2} \upsilon dP = \left(\frac{\gamma}{\gamma-1}\right) P_1 \upsilon_1 \left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\boxed{\Rightarrow u_2^2 = -2\int_{P_1}^{P_2} \upsilon dP = \left(\frac{2\gamma}{\gamma-1}\right) P_1 \upsilon_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

To estimate the velocity at any point downstream

we have,

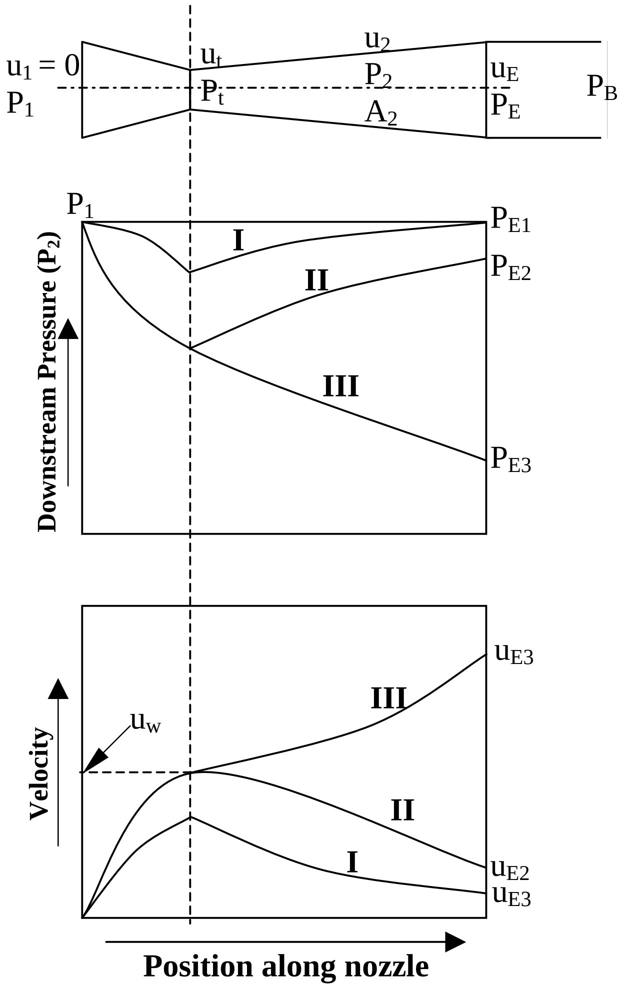
$$\boxed{G_2 = \frac{\dot{m}}{A_2} = \frac{u_2}{\upsilon_2} \quad \Rightarrow A_2 = \dot{m}\frac{\upsilon_2}{u_2}}$$

Cross-sectional area at any point downstream

8.4.1 Maximum Velocity and Critical Pressure Ratio

Critical pressure is the pressure at which the gas reaches sonic velocity [i.e. Ma =1.0].

In converging-diverging nozzles, if the pressure ratio (P_2/P_1) is less than the critical pressure ratio ($P_{critical}/P_1$) (usually, ≈ 0.5) and the velocity at throat is then **equal to** *the velocity of sound*, the effective area for flow presented by nozzle must therefore pass through a minimum. Thus in a converging section the velocity of the gas stream *will never exceed* the sonic velocity, though supersonic velocities *may be obtained* in the diverging section of the converging-diverging nozzle.



Case (I) [P_B high, $P_t > P_{critical}$]

The pressure falls to a minimum at throat [lager than critical pressure] and then rises to a value ($P_{E1}=P_B$). The velocity increase to the maximum at throat [less than sonic velocity] and then decreases to a value of (u_{E1}) at the exit of the nozzle. [**Case (I)** is corresponding to conditions in a venturi meter operating entirely at subsonic velocities]

Case (II) [P_B reduced, $P_B > (P_t = P_{critical})$]

The pressure falls to a critical value at throat where the velocity is sonic. The pressure then rises to a value ($P_{E2}=P_B$) at the exit of the nozzle. The velocity rises to the sonic value at the throat and then falls to a value of (u_{E2}) at the exit of the nozzle.

Case (III) [P_B low, $P_B < (P_t = P_{critical})$]

The pressure falls to a critical value at throat and continues to fall to give an exit pressure ($P_{E3}=P_B$). The velocity rises to the sonic value at the throat and continues to increase to supersonic in the diverging section cone to a value (u_{E3}) at the exit of the nozzle.

With converging-diverging nozzle, the velocity increases beyond the sonic velocity [i.e. reach supersonic velocity] *only if the velocity at the throat is sonic [i.e. critical pressure at throat]* and *the pressure at outlet is lower than the throat pressure*.

8.4.2 The Pressure and Area for Flow

In converging-diverging nozzles, the area required at any point depend upon the ratio of the downstream to upstream pressure (P_2/P_1), and **it is helpful to establish the minimum value of ($A_t = A_2$)**.

$$A_2 = \dot{m}\frac{\upsilon_2}{u_2} \quad \Rightarrow \quad A_2^2 = \dot{m}^2\left(\frac{\upsilon_2}{u_2}\right)^2$$

but $\upsilon_2 = \upsilon_1\left(\frac{P_2}{P_1}\right)^{-\frac{1}{\gamma}}$ and $u_2^2 = \left(\frac{2\gamma}{\gamma-1}\right)P_1\upsilon_1\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]$

$$\Rightarrow A_2^2 = \dot{m}^2\left(\frac{\gamma-1}{2\gamma}\right)\left[\frac{\upsilon_1^2(P_2/P_1)^{-\frac{2}{\gamma}}}{P_1\upsilon_1\left\{1-(P_2/P_1)^{\frac{\gamma-1}{\gamma}}\right\}}\right] \quad \Rightarrow A_2^2 = \left(\frac{\dot{m}^2\upsilon_1(\gamma-1)}{2\gamma P_1}\right)\left[\frac{(r)^{-\frac{2}{\gamma}}}{\left\{1-(r)^{\frac{\gamma-1}{\gamma}}\right\}}\right]; \quad r = \frac{P_2}{P_1}$$

In the flow stream P_1 falls to P_2 at which minimum A_2 which could be obtain by;

$$\left(\frac{dA_2^2}{dr}\right)_{r=rc} = 0$$

$$\left(\frac{dA_2^2}{dr}\right) = 0 \Rightarrow \left(\frac{\dot{m}^2\upsilon_1(\gamma-1)}{2\gamma P_1}\right)\left[\frac{\left(1-r_c^{\frac{\gamma-1}{\gamma}}\right)\left(\frac{-2}{\gamma}\right)\left(r_c^{-\left(\frac{2+\gamma}{\gamma}\right)}\right)-\left(r_c^{-\frac{2}{\gamma}}\right)\left(-\frac{\gamma-1}{\gamma}r_c^{-\frac{1}{\gamma}}\right)}{\left\{1-(r)^{\frac{\gamma-1}{\gamma}}\right\}^2}\right] = 0$$

$$\Rightarrow \left(1-r_c^{\frac{\gamma-1}{\gamma}}\right)\left(\frac{-2}{\gamma}\right)\left(r_c^{\frac{-2-\gamma}{\gamma}}\right)+\left(r_c^{-\frac{2}{\gamma}}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(r_c^{-\frac{1}{\gamma}}\right) = 0 \quad \Rightarrow \quad \left(\frac{-2}{\gamma}\right)\left(r_c^{\frac{-2-\gamma}{\gamma}}\right)+\left(\frac{\gamma+1}{\gamma}\right)\left(r_c^{-\frac{3}{\gamma}}\right) = 0$$

$$\boxed{\Rightarrow \quad r_c = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}; \qquad \gamma = \frac{c_p}{c_v}; \quad r_c = \frac{P_{critical}}{P_1}}$$

if $\gamma = 1.4 \Rightarrow r_c = 0.528$

$$\therefore A_2^2 = \dot{m}^2\frac{(\gamma-1)}{2\gamma}\left(\frac{\upsilon_1}{P_1}\right)\left[\frac{(P_2/P_1)^{-\frac{2}{\gamma}}}{\left\{1-(P_2/P_1)^{\frac{\gamma-1}{\gamma}}\right\}}\right]$$ The area at any point downstream

and $$\Rightarrow \dot{m}^2 = A_2^2\frac{2\gamma}{(\gamma-1)}\frac{P_1}{\upsilon_1}\left(\frac{P_2}{P_1}\right)^{2/\gamma}\left[1-\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}\right]$$ The mass flow rate

and $$\Rightarrow G_2^2 = \frac{\dot{m}^2}{A_2^2} = \frac{2\gamma}{(\gamma-1)}\frac{P_1}{\upsilon_1}\left(\frac{P_2}{P_1}\right)^{2/\gamma}\left[1-\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}\right]$$ The mass velocity

To find the maximum value of (G_2) i.e. $(G_2)_{max}$, set ($dG_2^2/dr = 0$) where, $r = P_2/P_1$ to get the following equation $G_{max} = \sqrt{\gamma P_2/\upsilon_2}$.

Example -8.11-

Air enters at a pressure of 3.5 MPa and a temperature of 500ºC. The air flow rate th rough the nozzle is 1.3 kg/s and it leaves the nozzle at a pressure of 0.7 MPa. The expansion of air may be considered adiabatic. Calculate the area of throat and the exit area. Take $\gamma = 1.4$.

Solution:

$$A_2^2 = \dot{m}^2 \frac{(\gamma-1)}{2\gamma}\left(\frac{\upsilon_1}{P_1}\right)\left[\frac{(P_2/P_1)^{-\frac{2}{\gamma}}}{\left\{1-(P_2/P_1)^{\frac{\gamma-1}{\gamma}}\right\}}\right]$$

$$\upsilon_1 = \frac{RT_1}{P_1 Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 773.15\text{K}}{3.5\times10^6\,Pa(29\ \text{kg/kmol})} = 0.0633 \quad m^3/kg$$

$r = P_2/P_1$, $\quad r_c = P_{critical}/P_1 = P_{critical}/P_1 \quad \Rightarrow \quad r_c = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = 0.528$

$\Rightarrow P_{critical} = P_t = 0.528\ (3.5\ \text{MPa}) = 1.85\ \text{MPa}$

but $P_2 = 0.7$ MPa [i.e. $P_2 < P_t$] $\Rightarrow$ The case is (**III**)

at throat

$$A_t^2 = (1.3)^2 \frac{0.4}{2.8}\left(\frac{0.0633}{3.5\times10^6}\right)\left[\frac{(0.528)^{-\frac{2}{1.4}}}{\left\{1-(0.528)^{\frac{0.4}{1.4}}\right\}}\right] \quad \Rightarrow A_t = 2.55\times10^{-4}\,m^2$$

$\Rightarrow$ the diameter of throat $d_t = 18$ mm

At exit $(P_2/P_1) = 0.7/3.5 = 0.2$

$$\Rightarrow A_t^2 = (1.3)^2 \frac{0.4}{2.8}\left(\frac{0.0633}{3.5\times10^6}\right)\left[\frac{(0.2)^{-\frac{2}{1.4}}}{\left\{1-(0.2)^{\frac{0.4}{1.4}}\right\}}\right] \quad \Rightarrow A_t = 3.436\times10^{-4}\,m^2$$

$\Rightarrow$ the diameter of exit region $d_E = 21$ mm

Or another method

$u_t = u_w = \sqrt{\gamma P_t \upsilon_t}$ $\quad P_t = 1.85$ MPa $\quad \upsilon_t = \upsilon_1 (P_t/P_1)^{-1/\gamma} = 0.0633\ (0.528)^{-1/1.4} = 0.0999 m^3/kg$

$\Rightarrow u_t = \sqrt{1.4(1.85\times10^6)(0.0999)} = 508.666 m/s$ (Sonic velocity)

$$A_t = \dot{m}\frac{\upsilon_t}{u_t} = 1.3\ (0.0999/508.666) = 2.55\times10^{-4}\,m^2$$

Or another method

$$u_2^2 = \left(\frac{2\gamma}{\gamma-1}\right)P_1\upsilon_1\left[1-(P_2/P_1)^{\frac{\gamma-1}{\gamma}}\right] = 1,550,850\left[1-(P_2/P_1)^{\ 0.2857}\right]$$

$u_t^2 = 258671.997 \Rightarrow u_t = 508.6$ m/s

$u_2^2 = 571666.52 \Rightarrow u_2 = 756.086$ m/s

$\upsilon_2 = \upsilon_1 (P_2/P_1)^{-1/\gamma} = 0.0633(0.2)^{-1/1.4} = 0.1998\ m^3/kg$

$A_2 = \dot{m}\ \upsilon_2 / u_2 = 1.3\ (0.198/756.086) = 3.436\times10^{-4}\,m^2$

8.5 Flow Measurement for Compressible Fluid

For horizontal flow with no shaft work and neglecting the frictional energy tem, the net of the general energy will be: -

$$\frac{u_2^2}{2\alpha_2}-\frac{u_1^2}{2\alpha_1}+\int_{P_1}^{P_2}\upsilon dP=0$$

but $\dot{m}_1=\dot{m}_2=\dot{m} \quad \Rightarrow \quad u_1=\frac{\upsilon_1}{\upsilon_2}\frac{A_2}{A_1}u_2$

For isothermal flow

$$\int_{P_1}^{P_2}\upsilon dP=P_1\upsilon_1\ln\frac{P_2}{P_1}$$

$$\Rightarrow u_2^2-\left(\frac{\upsilon_1}{\upsilon_2}\frac{A_2}{A_1}u_2\right)^2\frac{\alpha_2}{\alpha_1}+2\alpha_2P_1\upsilon_1\ln\frac{P_2}{P_1}=0$$

$$\boxed{\Rightarrow u_2^2=\frac{2\alpha_2P_1\upsilon_1\ln(P_2/P_1)}{1-\frac{\alpha_2}{\alpha_1}\left(\frac{\upsilon_1}{\upsilon_2}\frac{A_2}{A_1}\right)^2}} \qquad \text{------------------------------(1)}$$

For adiabatic flow

$$\upsilon=\upsilon_1P_1^{\frac{1}{\gamma}}P^{-\frac{1}{\gamma}}$$

$$\int_{P_1}^{P_2}\upsilon dP=\upsilon_1P_1^{\frac{1}{\gamma}}\int_{P_1}^{P_2}P^{-\frac{1}{\gamma}}dP=\frac{\gamma}{\gamma-1}\upsilon_1P_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right]$$

$$\boxed{\Rightarrow u_2^2=\frac{2\alpha_2P_1\upsilon_1\left(\frac{\gamma}{\gamma-1}\right)\left[(P_2/P_1)^{\frac{\gamma-1}{\gamma}}-1\right]}{1-\frac{\alpha_2}{\alpha_1}\left(\frac{\upsilon_1}{\upsilon_2}\frac{A_2}{A_1}\right)^2}} \qquad \text{------------------------------(2)}$$

It should be noted that equations (1) and (2) apply provided that (P_2/P_1) is greater than the critical pressure ratio (r_c). Where if (P_2/P_1) < (r_c), the flow becomes independent on P_2 and conditions of maximum flow occur.

8.6 Fans, Blowers, and Compression Equipment

Fans and blowers are used for many types of ventilating work such as air-conditioning systems. In large buildings, blowers are often used due to the high delivery pressure needed to overcome the pressure drop in the ventilation system.

Blowers are also used to supply draft air to boilers and furnaces.

Fans are used to move large volumes of air or gas through ducts, supplying air to drying, conveying material suspended in the gas stream, removing fumes, condensing towers and other high flow, low pressure applications.

Fans are used for low pressure where generally the delivery pressure is less than 3.447 kPa (0.5 psi), and blowers are used for higher pressures. However they are usually below delivery pressure of 10.32 kPa (1.5 psi). These units can either be **centrifugal** or the **axial-flow** type.

The axial flow type in which the air or gas enters in an axial direction and leaves in an axial direction.

The centrifugal blowers in which the air or gas enters in the axial direction and being discharge in the radial direction.

Compressors

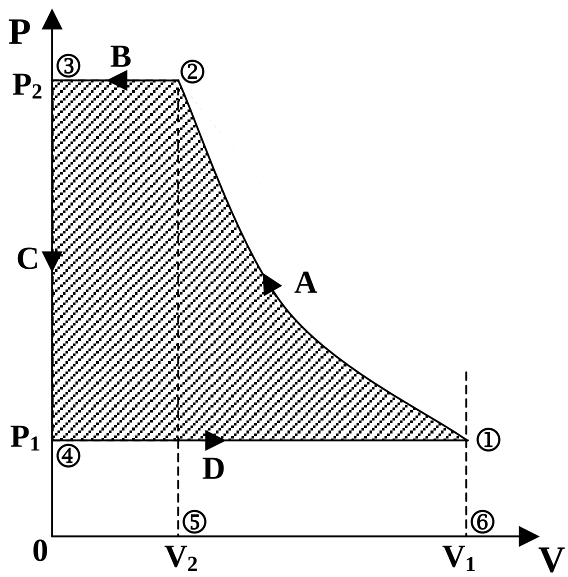
Compressor are used to handle large volume of gas at pressures increase from 10.32 kPa (1.5 psi) to several hundred kPa or (psi). Compressors are classified into: -

1- Cotinuous-flow compressors
   1-a- Centrifugal compressors
   1-b- Axial-flow compressors
2- Positive displacement compressors
   2-a- Rotary compressors
   2-b- Reciprocating compressors

Since a large proportion of the energy of compression appears as heat in the gas, there will normally be a considerable increase in temperature, which may limit the operation of the compressors unless suitable cooling can be effected. For this reason gas compression is often carried out in a number of stages and the gas is cooled between each stage.

8.7 Gas Compression Cycle

Suppose that, after the compression of a volume V_1 of gas at P_1 to a pressure P_2, the whole of the gas is expelled at constant pressure P_2, and a fresh charge of gas is admitted at a pressure P_1. The cycle can be followed as in Figure, where **P** is plotted as **ordinate** against **V** as **abscissa**.



Point ① represents the initial conditions of the gas of pressure and volume of (P_1,V_1).

► A-line ①→② Compression of gas from (P_1,V_1) to (P_2,V_2).

► B-line ②→③ Expulsion of gas at constant pressure P_2.

► C-line ③→④ Sudden reduction in pressure in the cylinder from P_2 to P_1. As the whole of the gas has been expelled.

► D-line ④→① A fresh charge of the gas through the suction stroke of the piston, during which a volume V_1 of gas is admitted at constant pressure P_1.

The Total Work Done Per Cycle

It will be noted that the mass of gas in the cylinder varies during the cycle. The work done by the compressor during each of the cycle is as follows: -

- Step (A): Compression $-\int_{V_1}^{V_2} PdV$ [area ①→②→⑤→⑥]
- Step (B): Expulsion P_2V_2 [area ②→③→**0**→⑤]
- Step (D): Suction $-P_1V_1$ [area ④→**0**→⑥→①]

∴ the total work done per cycle $= -\int_{V_1}^{V_2} PdV + P_2V_2 - P_1V_1$

$= $ [area ①→②→③→④]

$$dPV = P\,dV + V\,dP \quad \Rightarrow \quad P\,dV = dPV - V\,dP$$

$$-\int_{V_1}^{V_2} PdV = \int_{P_1}^{P_2} VdP - \int_{P_1V_1}^{P_2V_2} dPV$$

but $PV = RT$ and $dPV = R\,dT$

$$\Rightarrow \int_{P_1V_1}^{P_2V_2} dPV = R\int_{T_1}^{T_2} dT = RT_2 - RT_1 = P_2V_2 - P_1V_1$$

$$\Rightarrow -\int_{V_1}^{V_2} PdV = \int_{P_1}^{P_2} VdP - (P_2V_2 - P_1V_1)$$

⇒ the total work done per cycle $= \int_{P_1}^{P_2} VdP - \cancel{P_2V_2} + \cancel{P_1V_1} + \cancel{P_2V_2} - \cancel{P_1V_1}$

$$= \int_{P_1}^{P_2} VdP$$

<u>Or</u> The total work done per cycle (W) $= -\int_{V_1}^{V_2} PdV + \Delta PV$

$\Rightarrow \quad dW = -P\,dV + dPV = -\cancel{PdV} + VdP + \cancel{PdV}$

$\Rightarrow \quad dW = dPV \quad \Rightarrow \quad W = \int_{P_1}^{P_2} VdP$

Under isothermal conditions

The work of compression for an ideal gas per cycle $= \int_{P_1}^{P_2} VdP = RT\int_{P_1}^{P_2} dP/P$

$= RT \ln(P_2/P_1) = P_1V_1 \ln(P_2/P_1)$

Under adiabatic conditions

The work of compression for an ideal gas per cycle $= \int_{P_1}^{P_2} VdP = V_1 P_1^{1/\gamma}\int_{P_1}^{P_2} P^{-1/\gamma} dP$

$= P_1V_1\, \gamma/(\gamma - 1)\, [(P_2/P_1)^{(\gamma-1)/\gamma} - 1]$

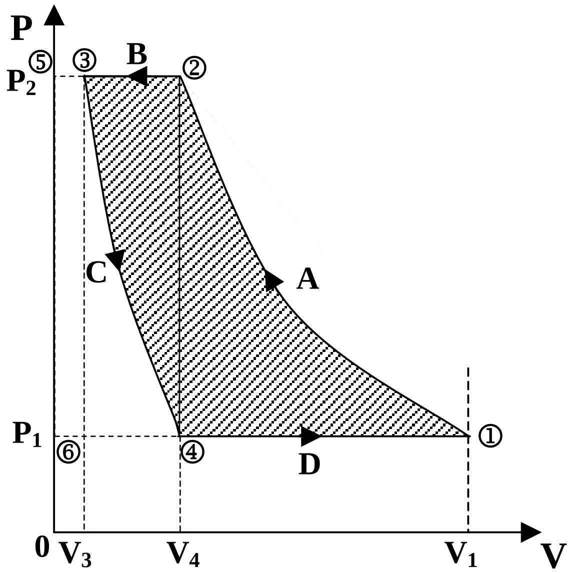
8.7.1 Clearance Volume

It practice, it is not possible to expel the whole of the gas fro the cylinder at the end of the compression; the volume remaining in the cylinder after the forward stroke of the piston is termed "**the clearance volume**".

The volume displaced by the piston is termed "**the swept volume**", and therefore **the total volume of the cylinder** is made up of the clearance volume plus the swept volume.

i.e. Total volume of cylinder = [clearance volume + swept volume]

A typical cycle for a compressor with a finite clearance volume can be followed by reference to the Figure;



A volume V_1 of gas at a pressure P_1 is admitted to the cylinder; its condition is represented by point ①**,**

► A-line ①→② Compression of gas from (P_1,V_1) to (P_2,V_2).

► B-line ②→③ Expulsion of gas at constant pressure P_2, so that the volume remaining in the cylinder is V_3.

► C-line ③→④ Expansion of this residual gas to the lower pressure P_1 and volume V_4 during the return stroke.

► D-line ④→① Introduction of fresh gas into the cylinder at constant pressure P_1.

The Total Work Done Per Cycle

The work done by the compressor during each of the actual cycle is as follows: -

- Step (A): Compression $\quad -\int_{V_1}^{V_2} \boldsymbol{PdV}$

- Step (B): Expulsion $\quad P_2(V_2\text{-}V_3)$

- Step (C): Expansion $\quad -\int_{V_3}^{V_4} \boldsymbol{PdV}$

- Step (D): Suction $\quad -P_1(V_1\text{-}V_4)$

The total work done per cycle is equal to the sum of these four components. It is represented by the selected area [i.e. area ①→②→③→④**]**, which is equal to [area①→②→⑤→⑥**]** less [area ③→④→⑤→⑥**]**

Under isentropic conditions

The work done per cycle $= \int_{P_1}^{P_2} \boldsymbol{VdP} - \int_{P_4}^{P_3} \boldsymbol{VdP}$

$$= \frac{\gamma}{\gamma-1} P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] - \frac{\gamma}{\gamma-1} P_4V_4\left[\left(\frac{P_3}{P_4}\right)^{(\gamma-1)/\gamma} - 1\right]$$

but $(P_1 = P_4)$ and $(P_2 = P_3)$ $\Rightarrow$ $\boxed{W = \frac{\gamma}{\gamma-1} P_1(V_1 - V_4)\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]}$

Now V_4 os not know explicitly, but can be calculated in terms of V_3, the clearance volume, for isentropic conditions

$$V_4 = V_3\,(P_2/P_1)^{1/\gamma}$$

And $V_1 - V_4 = (V_1 - V_3) + V_3 - V_3(P_2/P_1)^{1/\gamma}$

$$= (V_1 - V_3)\,[1 + \{V_3/(V_1 - V_3)\} - \{V_3/(V_1 - V_3)\}\,(P_2/P_1)^{1/\gamma}]$$

where

$(V_1 - V_3) = V_s$: the swept volume

V_3 : the clearance volume

$V_3/(V_1 - V_3) = C$: the clearance

$\Rightarrow \quad V_1 - V_4 = V_s\,[1 + C - C\,(P_2/P_1)^{1/\gamma}]$

∴ The total work done on the fluid per cycle is therefore,

$$\boxed{W = \frac{\gamma}{\gamma - 1} P_1 V_s \left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]\left[1 + C - C\left(\frac{P_2}{P_1}\right)^{1/\gamma}\right]}$$

The factor $\left[1 + C - C\left(\frac{P_2}{P_1}\right)^{1/\gamma}\right]$ is called "**the theoretical volumetric efficiency**", and is a measure of the effect of the clearance on an isentropic compression.

The as is frequently **<u>cooled</u>** during compression so that the work done per cycle is less than that given by the last equation, (γ) is replaced by some smaller quantity (k). the greater the rate of heat removal, the less is the work done.

Notice that the isothermal compression is usually taken as the condition for the least work of compression. The actual work of compression is greater than the theoretical work because of clearance gases, back leakage, and frictional effects, where,

$\eta = W_{theo}/W_{act}$

8.8 Multistage Compressors

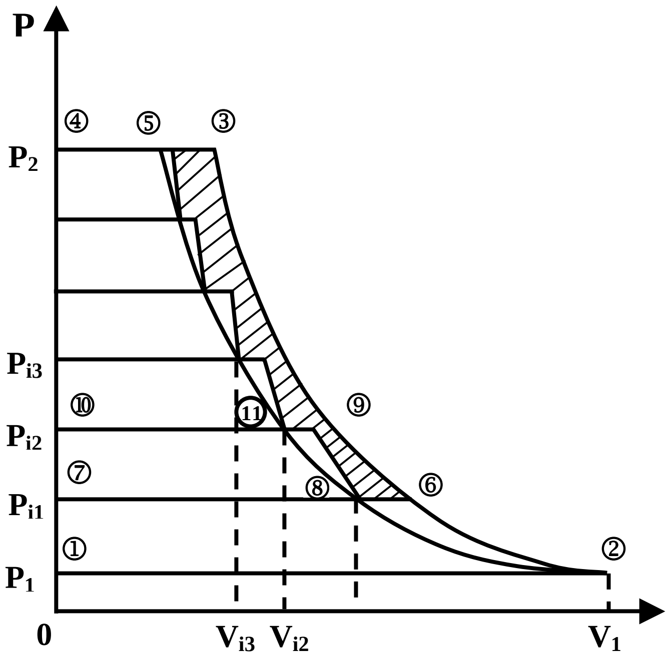
The maximum pressure ratio normally obtained in a single cylinder is (10) but values above (6) are usual. If the required pressure ratio (P_2/P_1) is large, it is not practicable to carry out the whole of the compression in a single cylinder because of the high temperatures, which would be set up, and the adverse effects of clearance volume on the efficiency. Further, lubrication would be difficult due to carbonization of the oil and there would be a risk of causing oil mist explosions in the cylinders when gases containing oxygen were being compressed.

The operation of the multistage compressor can conveniently be followed again on a pressure-volume diagram as shown in the Figure,

The [area ①→②→③→④**]** represents the work done in compressing isentropically from P_1 to P_2 in a single stage. The [area①→②→⑤→④⑥**]** represents the necessary work for an isothermal compression.

Now consider a multistage isentropic compression in which the intermediate pressures are P_{i1}, P_{i2}, P_{i3}, …………etc.

The gas will be assumed to be cooled to its initial temperature in an inter-stage cooler before it enters each cylinder.



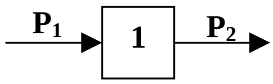
- ► A-line ①→② represents the suction stroke of the first stage where a volume (V_1) of gas is admitted at a pressure (P_1).
- ► B-line ②→⑥ represents an isentropic compression to a pressure (P_{i1}).
- ► C-line ⑥→⑦ represents the delivery of the gas from the first stage at a constant pressure (P_{i1}).
- ► D-line ⑦→⑧ represents the suction stroke of the second stage. The volume of the gas has the reduced in the inter-stage cooler to (V_{i1}), that which would have been obtained as a result of an isothermal compression to (P_{i2}).
- ► E-line ⑧→⑨ represents an isentropic compression in the second stage from a pressure (P_{i1}) to a pressure (P_{i2}).
- ► F-line ⑨→⑩ represents the delivery stroke of the second stage.
- ► G-line ⑩→⑪ represents the suction stroke of the third stage point ⑪ again lyses on the line ②→⑤ that representing an isothermal compression.

It is seen that the overall work done on the gas is intermediate between that for a single stage isothermal compression and that for isentropic compression. The net saving in energy is shown as the shaded area in the last Figure.

The Total Work Done for Multistage Compressors

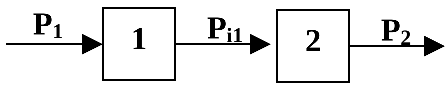
The total work done for compression the gas from P_1 to P_2 in an ideal single stage is,

$$W = \frac{\gamma}{\gamma-1} P_1 V_1 \left[\left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right]$$



The total work done for compression the gas from P_1 to P_2 in an ideal two stages is,

$$W = \frac{\gamma}{\gamma-1} P_1 V_1 \left[\left(\frac{P_{i1}}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right] + \frac{\gamma}{\gamma-1} P_{i1} V_{i1} \left[\left(\frac{P_2}{P_{i1}} \right)^{(\gamma-1)/\gamma} - 1 \right]$$

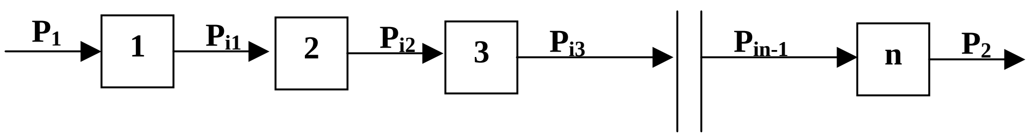


but for perfect inter-stage cooling i.e. at isothermal line $P_1V_1 = P_{i1}V_{i1}$= constant

$$\Rightarrow W = \frac{\gamma}{\gamma-1} P_1 V_1 \left[\left\{ \left(\frac{P_{i1}}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right\} + \left\{ \left(\frac{P_2}{P_{i1}} \right)^{(\gamma-1)/\gamma} - 1 \right\} \right]$$

The total work done for compression the gas from P_1 to P_2 in an ideal n-stages is,

$$W = \frac{\gamma}{\gamma-1} P_1 V_1 \left[\left(\frac{P_{i1}}{P_1} \right)^{(\gamma-1)/\gamma} - 1 \right] + \frac{\gamma}{\gamma-1} P_{i1} V_{i1} \left[\left(\frac{P_{i2}}{P_{i1}} \right)^{(\gamma-1)/\gamma} - 1 \right] + \cdots\cdots\cdots + \frac{\gamma}{\gamma-1} P_{in-1} V_{in-1} \left[\left(\frac{P_2}{P_{in-1}} \right)^{(\gamma-1)/\gamma} - 1 \right]$$



for perfect inter-stage cooling $P_1V_1 = P_{i1}V_{i1} = P_{i2}V_{i2} = \text{------} = P_{in-1}V_{in-1} =$ constant

$$W = \frac{\gamma}{\gamma-1}P_1V_1\left[\left\{\left(\frac{P_{i1}}{P_1}\right)^{(\gamma-1)/\gamma}-1\right\}+\left\{\left(\frac{P_{i2}}{P_{i1}}\right)^{(\gamma-1)/\gamma}-1\right\}+\cdots\cdots+\left\{\left(\frac{P_2}{P_{in-1}}\right)^{(\gamma-1)/\gamma}-1\right\}\right]$$

$$\Rightarrow W = \frac{\gamma}{\gamma-1}P_1V_1\left[\left(\frac{P_{i1}}{P_1}\right)^{(\gamma-1)/\gamma}+\left(\frac{P_{i2}}{P_{i1}}\right)^{(\gamma-1)/\gamma}+\cdots\cdots+\left(\frac{P_2}{P_{in-1}}\right)^{(\gamma-1)/\gamma}-n\right]$$

The optimum values of intermediate pressures P_{i1}, P_{i2}, P_{i3}, -------P_{in-1} are so that **<u>the compression ratio (r) is the same in each stage and equal work is then done in each stage.</u>**

i.e. $\frac{P_{i1}}{P_1} = \frac{P_{i2}}{P_{i1}} = \frac{P_{i3}}{P_{i2}} = \cdots\cdots\cdots = \frac{P_2}{P_{in-1}} = r$

then $\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} = \frac{P_{i1}}{P_1} = \frac{P_{i2}}{P_{i1}} = \frac{P_{i3}}{P_{i2}} = \cdots\cdots\cdots = \frac{P_2}{P_{in-1}} = r$ -------------prove that

$$\Rightarrow W = \frac{\gamma}{\gamma-1}P_1V_1\left[n\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma}-n\right] \qquad \Rightarrow \boxed{W = P_1V_1\frac{n\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma}-1\right]}$$

The effect of clearance volume can now be taken into account. If the clearance in the successive cylinders are C_1, C_2 C_3, -------C_n the theoretical volumetric efficiency of the first cylinder = $\left[1+C_1-C_1\left(P_{i1}/P_1\right)^{1/\gamma}\right]$.

Assuming that the same compression ratio is used in each cylinder, then the theoretical volumetric efficiency of the first stage = $\left[1+C_1-C_1\left(P_2/P_1\right)^{1/n\gamma}\right]$.

If the swept volumes of the cylinders are V_{s1}, V_{s2}, V_{s3}, ----------- the volume of the gas admitted to the first cylinder = $V_{s1}\left[1+C_1-C_1\left(P_2/P_1\right)^{1/n\gamma}\right]$

The same mass of gas passes through each of the cylinders and, therefore, if the inter-stage coolers are assumed perfectly efficient, the ratio of the volumes of gas admitted to successive cylinder is $(P_1/P_2)^{1/n}$ [because lies on the isothermal line].

The volume of gas admitted to the second cylinder

$$= V_{s2}\left[1+C_2-C_2\left(P_2/P_1\right)^{1/n\gamma}\right] \quad = V_{s1}\left[1+C_1-C_1\left(P_2/P_1\right)^{1/n\gamma}\right]\left(P_1/P_2\right)^{1/n}$$

$$\boxed{\Rightarrow \qquad \frac{V_{s1}}{V_{s2}}\frac{\left[1+C_2-C_2\left(P_2/P_1\right)^{1/n\gamma}\right]}{\left[1+C_1-C_1\left(P_2/P_1\right)^{1/n\gamma}\right]}\left(P_2/P_1\right)^{1/n}}$$

In this manner the swept volume of each cylinder can be calculated in terms of V_{s1}, and C_1, C_2, …………., and the cylinder dimensions determined.

Let $V_1 = V_{s1}\left[1+C_1-C_1\left(P_2/P_1\right)^{1/n\gamma}\right]$, $\qquad V_2 = V_{s2}\left[1+C_2-C_2\left(P_2/P_1\right)^{1/n\gamma}\right]$

Where, V_i : represents the volume of gas admitted to stage i.

But for perfectly cooled [i.e. isothermal] $\Rightarrow P_1V_1 = P_{i1}V_1 = P_{i2}V_2 = \text{--------} = P_{in-1}V_n$

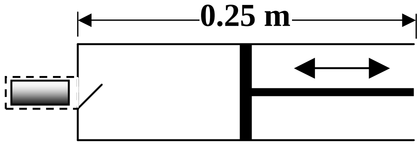
$$\Rightarrow P_1V_{s1}\left[1+C_1-C_1\left(P_2/P_1\right)^{1/n\gamma}\right] = P_{i1}V_{s2}\left[1+C_2-C_2\left(P_2/P_1\right)^{1/n\gamma}\right]$$

$$\textit{but} \quad r = \frac{P_{i1}}{P_1} = \left(\frac{P_2}{P_1}\right)^{1/n}$$

$$\Rightarrow \frac{V_{s1}}{V_{s2}} = \frac{\left[1+C_2-C_2\left(P_2/P_1\right)^{1/n\gamma}\right]}{\left[1+C_1-C_1\left(P_2/P_1\right)^{1/n\gamma}\right]}\left(\frac{P_2}{P_1}\right)^{1/n}$$

Example -8.12-

A single-acting air compressor supplies 0.1 m^3/s of air (at STP) compressed to 380 kPa from 101.3 kPa. If the suction temperature is 289 K, the stroke is 0.25 m, and the speed is 4 Hz, what is the cylinder diameter? Assume the cylinder clearance is 4% and compression and re-expansion are isentropic (γ=1.4). What are the theoretical power requirements for the compression?



Solution:

Stroke (حركة من سلسلة حركات متسلسلة "متوالية ومتشابهة")

Volume of gas per stroke = $(0.1\ m^3/s)/4s^{-1}\ (289/273)$

$= 0.0264\ m^3$

$=(V_1-V_4) \equiv$[volume of gas admitted per cycle]

$P_2/P_1 = 380/101.3 = 3.75$

$(V_1-V_4) = V_s\,[1+ C - C\,(P_2/P_1)^{1/\gamma}]$

$0.0264 = V_s[1+0.04-0.04(3.75)^{1/1.4} \Rightarrow V_s = 0.0283\ m^3 = (V_1 - V_3) \equiv$ volume of cylinder

Cross-section area of cylinder = $V_s/L_{stroke} = 0.0283/0.25 = 0.113\ m^2$

$\Rightarrow$ The diameter of cylinder = $[0.113/(\pi/4)]^{1/2}$ =0.38 m

$$W = \frac{\gamma}{\gamma-1} P_1(V_1 - V_4)\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]$$

for 1kg of gas that compressed [or per cycle]

$$\Rightarrow W = \frac{1.4}{0.4}(101.3\times10^3)(0.0264)\left[(3.75)^{0.4/1.4} - 1\right] = 4278\ J/kg \text{ per stroke}$$

The theoretical power required = 4278 J/kg ($4s^{-1}$) per stroke= 17110 W =17.11kW

Example -8.13-

Air at 290 K is compressed from 101.3 kPa to 2065 kPa in two-stage compressor operating with a mechanical efficiency of 85%. The relation between pressure and volume during the compression stroke and expansion of clearance gas is ($PV^{1.25}$ = constant). The compression ratio in each of the two cylinders is the same, and the inter-stage cooler may be assumed 100% efficient. If the clearance in the two cylinders are 4% and 5%, calculate:

- a- The work of compression per kg of air compressed;
- b- The isothermal efficiency;
- c- The isentropic efficiency;
- d- The ratio of swept volumes in the two cylinders.

Solution:

$P_2/P_1 = 2065/101.3 = 20.4$

$$V_1 = \frac{RT}{P_1 Mwt} = \frac{8314\,(Pa.m^3/kmol.K)\ 290K}{(101.3\times10^3\ Pa)29\ kg/kmol} = 0.82 \quad (m^3/kg)$$

For 100% efficient of cooler at inter-stage, the work of compression in multistage compressor of n-stages is;

$$W = P_1V_1\frac{n\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right] \Rightarrow W = (101.3\times10^3\times0.82)\frac{2(1.25)}{1.25-1}\left[(20.4)^{(0.25)/2.5} - 1\right] = 292.35\frac{kJ}{kg}$$

The work of compressor = W_{act} = W/η =292.3/0.85 = 344 kJ/kg
For isothermal compression = $W_{iso} = P_1V_1 \ln(P_2/P_1)$ = 250.5 kJ/kg
Isothermal efficiency = (W_{iso}/ W_{act}) 100 = 72.8 %
For isentropic compression = $W_{adb} = P_1V_1\, \gamma/(\gamma-1)\, [(P_2/P_1)^{(\gamma-1)/\gamma} -1]$ = 397.4 kJ/kg
Isentropic efficiency = (W_{adb}/ W_{act}) 100 = 115.5 %

$$V_1 = V_{s1}\left[1 + C_1 - C_1(P_2/P_1)^{1/n\gamma}\right]$$

$$\Rightarrow 0.82 = V_{s1}\left[1 + 0.04 - 0.04(20.4)^{1/2.5}\right] \Rightarrow V_{s1} = 0.905\ m^3/kg$$

The swept volume of the second cylinder is given by:

$$V_{s2} = V_{s1}\frac{\left[1 + C_1 - C_1(P_2/P_1)^{1/n\gamma}\right]}{\left[1 + C_2 - C_2(P_2/P_1)^{1/n\gamma}\right]}\left(\frac{P_1}{P_2}\right)^{1/n} \longleftarrow \text{انتبه}$$

$$V_{s2} = \frac{V_1(P_1/P_2)^{1/n}}{\left[1 + C_2 - C_2(P_2/P_1)^{1/n\gamma}\right]} = \frac{0.82(1/20.4)^{1/2}}{\left[1 + 0.05 - 0.05(20.4)^{1/2.5}\right]} = 0.206\ m^3/kg$$

∴ V_{s1}/V_{s2} = 0.905/0.206 = 4.4

Example -8.14-

Calculate the theoretical work in (J/kg) required to compress a diatomic gas initially at T = 200 K adiabatically compressed from a pressure of 10 kPa to 100 kPa in;

1- Single stage compressor;
2- Two equal stages;
3- Three equal stages; Taken that γ = 1.4, Mwt = 28 kg/kmol

Solution:

1- $$W = P_1V_1\frac{\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]$$

P_2/P_1 = 100/10 = 10

$$V_1 = \frac{RT}{P_1 Mwt} = \frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 200\text{K}}{(10\times10^3\ Pa)28\ \text{kg/kmol}} = 5.94 \quad (m^3/kg)$$

$$\Rightarrow W = 10(5.92)\frac{1.4}{0.4}\left[(10)^{0.4/1.4} - 1\right] = 193.44\ kJ/kg$$

2- $$W = P_1V_1\frac{n\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right] \Rightarrow W = 59.4\frac{2(1.4)}{0.4}\left[(10)^{(0.4)/2.8} - 1\right] = 161.95\frac{kJ}{kg}$$

3- $$W = P_1V_1\frac{n\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right] \Rightarrow W = 59.4\frac{3(1.4)}{0.4}\left[(10)^{(0.4)/4.2} - 1\right] = 152.93\frac{kJ}{kg}$$

For 100% efficient of cooler at inter-stage, the work of compression in multistage compressor of n-stages is;

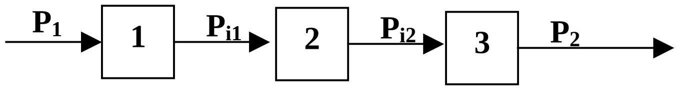
$$W = P_1V_1\frac{n\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right] \Rightarrow W = (101.3\times10^3\times0.82)\frac{2(1.25)}{1.25-1}\left[(20.4)^{(0.25)/2.5} - 1\right] = 292.35\frac{kJ}{kg}$$

Example -8.15-

A three stages compressor is required to compress air from 140 kPa and 283 K to 4000 kPa. Calculate the ideal intermediate pressures, the work required per kg of gas, and the isothermal efficiency of the process. Assume the compression to be adiabatic and perfect the inter-stage cooling to cool the air to the initial temperature. Taken that γ = 1.4.

Solution:

$$\frac{P_{i1}}{P_1}=\frac{P_{i2}}{P_{i1}}=\frac{P_2}{P_{i2}}=r=\left(\frac{P_2}{P_1}\right)^{\frac{1}{3}}=\left(\frac{4000}{140}\right)^{\frac{1}{3}}=3.057$$



$\Rightarrow$ P_{i1}=3.057 (140) =428 kPa

P_{i2}=3.057 (428) =1308.4 kPa

$$W=P_1V_1\frac{n\gamma}{\gamma-1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma}-1\right]$$

$$P_1V_1=\frac{RT}{Mwt}=\frac{8314\,(\text{Pa.m}^3/\text{kmol.K})\ 283\text{K}}{(29\ \text{kg/kmol})}=81.133\quad (kJ/kg)$$

$$\Rightarrow W=81.133\frac{3(1.4)}{0.4}\left[\left(\frac{4000}{140}\right)^{0.4/4.2}-1\right]=320.43\ kJ/kg$$

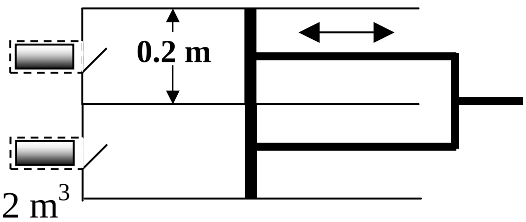
For isothermal compression = W_{iso} = $P_1V_1 \ln(P_2/P_1)$ = 272 kJ/kg

Isothermal efficiency = (W_{iso}/ W) 100 = 84.88 %

Example -8.16-

A twin-cylinder single-acting compressor, working at 5 Hz, delivers air at 515 kPa pressure at the rate of 0.2 m^3/s. If the diameter of the cylinder is 20 cm, the cylinder clearance ratio 5%, and the temperature of the inlet air 283 K, calculate the length of stroke of the piston and delivery temperature (γ=1.4).

Solution:



$T_2/T_1 = (P_2/P_1)^{(\gamma-1)/\gamma}$ $\Rightarrow$ $T_2=283(515/101.3)^{0.4/1.4}$=450K

The volume handled per cylinder = 0.2/2 =0.1 m^3/s

Volume per stroke per cylinder = (0.1 m^3/s) / (5 s^{-1}) = 0.02 m^3

Volume at inlet conditions = (0.02 m^3) (283/450) (515/101.3) = 0.0639 m^3

$V_1 - V_4 = V_s\,[1+ C - C\,(P_2/P_1)^{1/\gamma}]$ $\Rightarrow$ $0.0639 = V_s\,[1+ 0.05 - 0.05\,(515/101.3)^{1/1.4}]$

$\Rightarrow$ V_s = 0.0718 m^3 = $\pi/4\,(0.2)^2\,L_{stroke}$ $\Rightarrow$ L_{stroke} = 2.286 m

Example -8.17-

In a single-acting compressor suction pressure and temperature are 101.3 kPa and 283 K, the final pressure is 380 kPa. If the compression is adiabatic and each new charge is heated 18 K by contact with the clearance gases, calculate the maximum temperature attained in the cylinder (γ=1.4).

Solution: On the first stroke the air enters at 283 K and is compressed adiabatically

$\Rightarrow$ $T_2 = 283\,(380/101.3)^{0.4/1.4} = 415$ K

The clearance volume gases at 413 K which remain in the cylinder are able to raise the next cylinder full of air by 18 K i.e. the air temperature in the next cylinder is [18 + 283 = 301 K] $\Rightarrow$ The exit temperature = 301 $(380/101.3)^{0.4/1.4}$ = 439.2 K

On each subsequent stroke T_{in}=283 K, $T_{cylinder}$ = 301 K, and T_{exit} = 439.2 K.

Example -8.18-

A single-stage double-acting compressor running at 3 Hz is used to compess air from 110 kPa and 282 K to 1150 kPa. If the internal diameter of the cylinder 20 cm, the length of the stroke 25 cm, and the piston clearance 5%. Calculate;

a- The maximum capacity of machine, referred to air at initial conditions;

b- The theoretical power requirements under isentropic conditions.

Solution:

The swept volume per stroke = $2[\pi/4\,(0.2)^2\,(0.25)] = 0.0157\ m^3$

$(V_1 - V_4) = V_s[1 + C - C\,(P_2/P_1)^{1/\gamma}] \Rightarrow (V_1 - V_4) = 0.0157[1 + 0.05 - 0.05\,(1150/110)^{1/1.4}]$

$\Rightarrow \quad (V_1 - V_4) = 0.0123\ m^3$

$$W = P_1(V_1 - V_4)\frac{\gamma}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] \Rightarrow W = 110(0.0123)\frac{1.4}{0.4}\left[\left(\frac{1150}{110}\right)^{0.4/1.4} - 1\right] = 5.775\ kJ/stroke$$

The power required = (3 stroke/s)(5.775 kJ/stroke) = 17.324 kW

Capacity = (3 stroke/s) (0.0123 m^3/stroke) = 0.0369 m^3/s

Example -8.19-

Methane is to be compressed fom atmospheric pressure ton 30 MPa in four stages. Calculate the ideal intermediate pressures and the work required per kg of gas. Assume compression to be isentropic and the gas to behave as an ideal gas and the initial condition at STP (γ=1.4).

$$P_1 \rightarrow \boxed{1} \xrightarrow{P_{i1}} \boxed{2} \xrightarrow{P_{i2}} \boxed{3} \xrightarrow{P_{i3}} \boxed{4} \xrightarrow{P_2}$$

Solution:

$$\frac{P_{i1}}{P_1} = \frac{P_{i2}}{P_{i1}} = \frac{P_{i3}}{P_{i2}} = \frac{P_2}{P_{i3}} = r = \left(\frac{P_2}{P_1}\right)^{\frac{1}{4}} = \left(\frac{30}{0.1013}\right)^{\frac{1}{4}} = 4.148$$

$\Rightarrow \quad P_{i1}$=4.148 (101.3 kPa) = 420.23 kPa

P_{i2}=4.148 (420.23 kPa) = 1743.27 kPa

P_{i3}=4.148 (1743.27 kPa) = 7231.75 kPa

P_2=4.148 (7231.75 kPa) = 30,000 kPa

$$W = P_1V_1\frac{n\gamma}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right]$$

$$(P_1V_1)_{STP} = \frac{RT}{Mwt} = \frac{8314\,(Pa.m^3/kmol.K)\,273K}{(16\ kg/kmol)} = 141.857 \quad (kJ/kg)$$

$$\Rightarrow W = 141.857\frac{4(1.4)}{0.4}\left[\left(\frac{30,000}{101.3}\right)^{0.4/5.6} - 1\right] = 996.06\ kJ/kg$$

CHPTER NINE

Liquid Mixing

9.1 Introduction

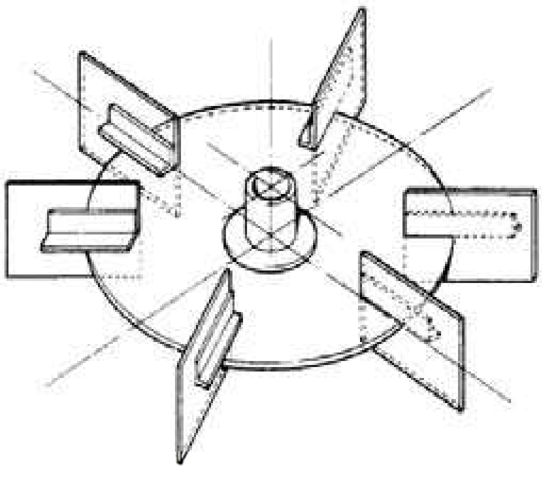
Mixing is one of the most common operations carried out in the chemical, processing. The term " **Mixing**" is applied to the processes used to reduce the degree of non-uniformity, or gradient of a property in the system such as concentrations, viscosity, temperature, and so on. Mixing is achieved by moving material from one region to another. It may be interest simply as a means of achieving a desired degree of homogeneity but it may also be used to promote heat and mass transfer, often where a system is undergoing a chemical reaction.

A rotating agitator generates high velocity streams of liquid, which in turn entrain stagnant or slower moving regions of liquid resulting in uniform mixing by momentum transfer. As viscosity of the liquid is increased, the mixing process becomes more difficult since frictional drag retards the high velocity streams and confines them to immediate vicinity of the rotating agitator.

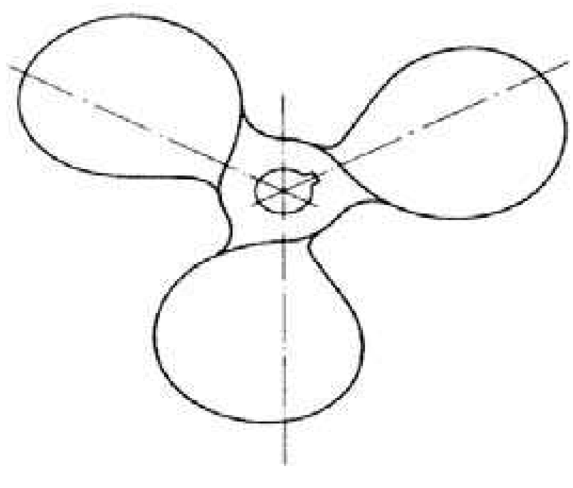
9.2 Types of Agitators

In general, agitators can be classified into the following two types: -

1- Agitators with a small blade area, which rotate at high speeds. These include **turbines** and **marine type propellers.**
2- Agitators with a large blade area, which rotate at low speeds. These include **anchors,** and **Paddles**, and **helical screws.**



Six-blade flat blade turbine



Marine Propeller

The second group is more effective than the first in the mixing of high viscosity-liquids.

For a liquid mixed in a tank with a rotating agitator, the shear rate is greatest in the intermediate vicinity of agitator. In fact the shear rate decreases exponentially with distance from the agitator. Thus the shear stresses and strains vary greatly throughout an agitated liquid in tank. Since the dynamic viscosity of a Newtonian liquid is independent of shear rate at a given temperature, its viscosity will be the same at all points in the tank. In contrast the apparent viscosity of a non-Newtonian liquid varies throughout the tank. This in turn significantly influences the mixing process.

The mean share $\dot{\gamma}_m$ produced by an agitator in a mixing tank is proportional to the rotational speed of the agitator N

i.e. $\dot{\gamma}_m \propto N \quad = \dot{\gamma}_m = KN$

where, K is a dimensionless proportionality constant for a particular system.

It is desirable to produce a particular mixing result in the minimum time (t) and with the minimum input power per unit volume (P_A/V). Thus the efficiency function (E) can be defined as

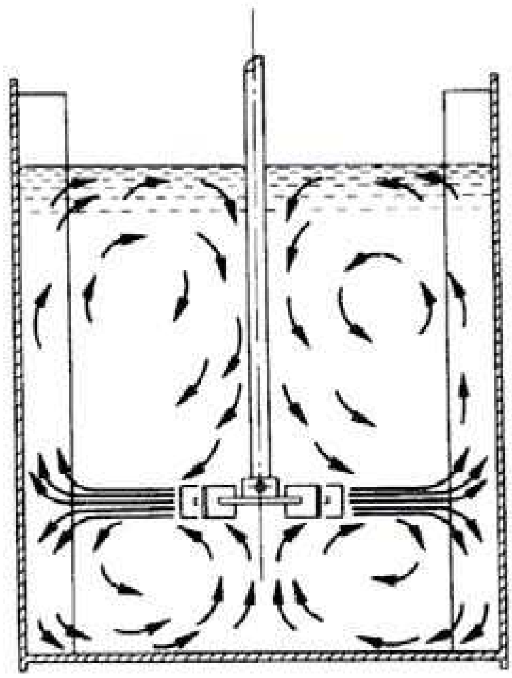
$$E = \left(\frac{1}{P_A/V}\right)\left(\frac{1}{t}\right)$$

9.2.1 Small Blade, High Speed Agitators

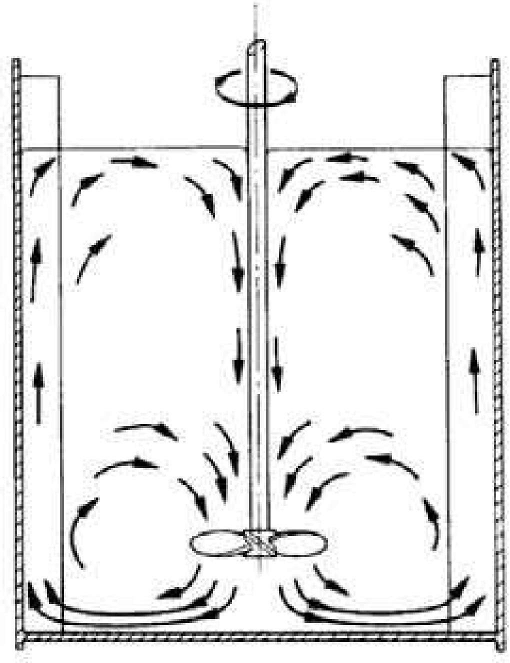
They are used to mix low to medium viscosity liquids. Two of most common types are 6-blade flat blade turbine and the marine type propeller.

Flat blade turbines used to mix liquids in baffled tanks produce radial flow patterns primarily perpendicular to the vessel wall. This type is suitable to mix liquids with dynamic viscosity up to 50 Pa.s.

Marine type Propellers used to mix liquids in baffled tanks produce axial flow patterns primarily parallel to the vessel wall. This type is suitable to mix liquids with dynamic viscosity up to 10 Pa.s.



Radial flow pattern produced by a flat blade turbine



Axial flow pattern produced by a marine

Agitator Tip Speed (TS)

Is commonly used as a measure of the degree of the agitation in a liquid mixing system.

$TS = \pi D_A N$

Where, D_A: diameter of agitator.

N: rotational speed.

Tip speed ranges for turbine agitator are recommended as follows:

TS = 2.5 to 3.3 m/s for low agitation.

TS = 3.3 to 4.1 m/s for medium agitation.

TS = 4.1 to 5.6 m/s for high agitation.

Standard Tank Configuration

A turbine agitator of diameter (D_A) in a cylindrical tank of diameter (D_T) filled with liquid to a height (H_L). The agitator is located at a height (H_A) from the bottom of the tank and the baffles, which are located immediately adjacent to the wall, have a width (b). The agitator has a blade width (a) and a blade length (r) and the blades are mounted on a central disc of diameter (s). A typical turbine mixing system is the standard configuration defined by the following geometrical relationships: -

1- a 6-blade flat blade turbine agitator.
2- $D_A = D_T / 3$
3- $H_A = D_T / 3$
4- $a = D_T / 5$
5- $r = D_T / 8$
6- $H_L = D_T$
7- 4 symmetrical baffles
8- $b = D_T / 10$

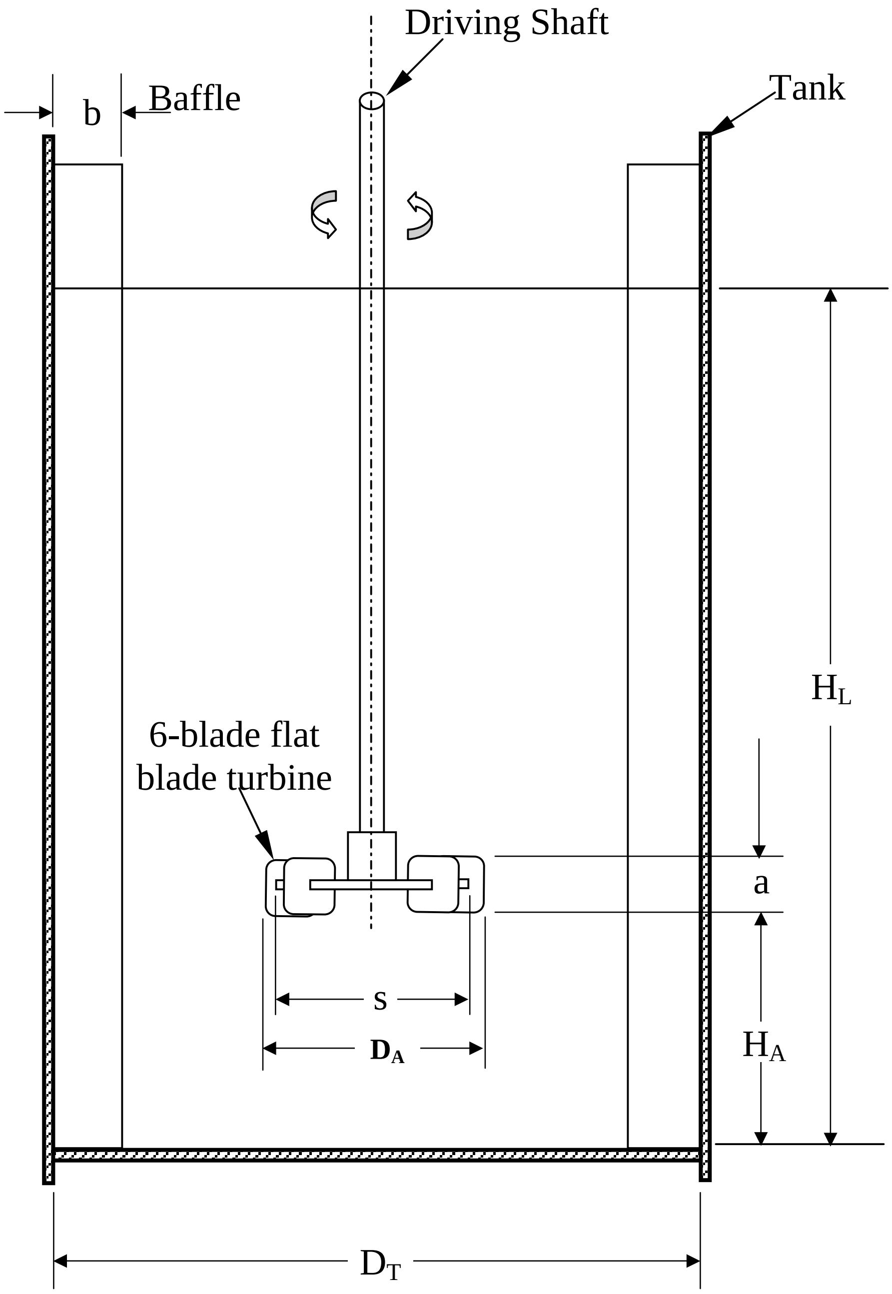


Figure of Standard Tank Configuration

Processing considerations sometimes necessitate deviations from the standard configuration.

Marine Type Propeller

It can be considered as a case-less pump. In this case its volumetric circulating capacity (Q_A) is related to volumetric displacement per revolution (V_D) by the equation;

$Q_A = \eta\, V_D\, N$

where, η: is a dimensionless efficiency factor which is approximately (0.6).

V_D is related to the propeller pitch (P) and the propeller diameter (D_A) by the equation;

$$V_D = \frac{\pi D_A^2 P}{4}$$

Most propellers are square pitch propellers where ($P = D_A$) so that the last equation becomes;

$$V_D = \frac{\pi D_A^3}{4} \qquad \Rightarrow \quad Q_A = \frac{\eta \pi D_A^3 N}{4}$$

A tank turnover rate (I_T) is defined by the equation;

$$I_T = Q_A / V$$

where, V: is the tank volume and I_T: is the number of turnovers per unit time.

To get the best mixing I_T should be at a maximum for a given tank volume (V), this means that the circulating capacity Q_A should have the highest possible value for the minimum consumption of power.

The head developed by the rotating agitator (h_A) can be written as;

$h_A = C_1 N^2 D_A^2$ where, C_1 is a constant.

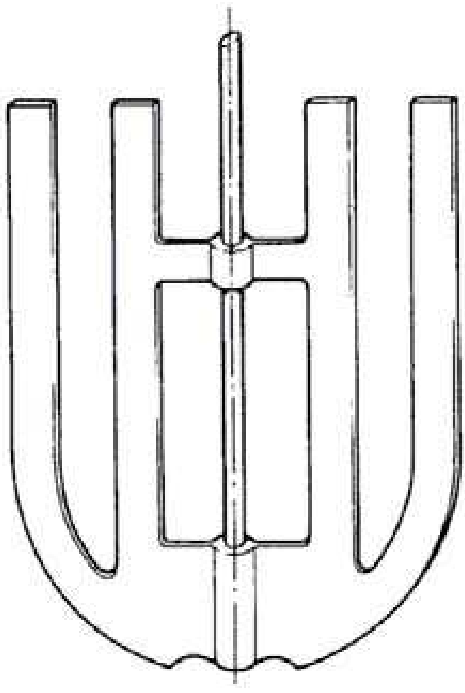
$Q_A/h_A = C\, D_A/N$ where, $C = \eta\pi/(4C_1)$

but $\dot{\gamma}_m = KN$

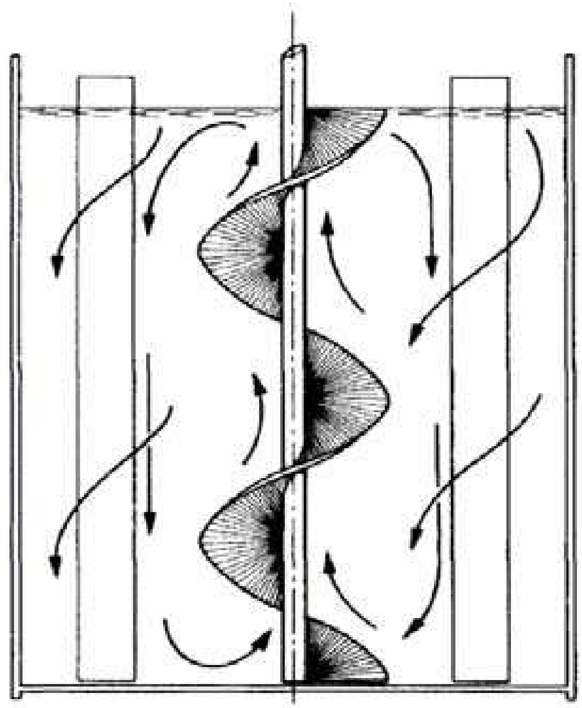
$\dfrac{Q_A}{h_A} = C' \dfrac{D_A}{\dot{\gamma}_m}$ where, $C' = C.K$ = constant

9.2.2 Small Blade, High Speed Agitators

This type of agitators includes **anchors, gates, paddles, helical ribbons, and helical screws**. They are used to mix *relatively high viscosity liquids* and depend on a large blade area to produce liquid movement throughout a tank. Since they are low shear agitators.



Gate type anchor agitator



Flow pattern in a baffled helical screw system

9.3 Dimensionless Groups for Mixing

Some of the various types of forces that may be arise during mixing or agitation will be formulated: -

1- Inertial Force [F_i]

Is associated with the reluctance of a body to change its state of rest or motion.

The inertial force (F_i) = (mass) (acceleration) = m.a

dF_i = dm (du/dt)

but m = ρ V = ρ A L

⇒ dm = ρ dV = ρ A dL

and u = dL/dt

$$\textbf{Or } \dot{m} = \frac{dm}{dt} \Rightarrow dm = \dot{m}\,dt = \rho A u\,dt$$

⇒ dF_i = ρ A dL du/dt = ρ A (dL/dt) du = ρ A u du

$$\Rightarrow F_i = \int_0^{F_i} dF_i = \int_0^{u} \rho A u\,du = \rho A u^2/2$$

In mixing applications;

$A \propto D_A^2$ D_A: diameter of agitator

$u = \pi D_A N$ N: rotational speed

Therefore, the expression for inertial force may be written as;

$$\boxed{F_i \propto \rho D_A^4 N^2}$$

2- Viscous Force [F_v]

The viscous force for Newtonian fluid is given by:

F_v = μ A (du/dy)

In mixing applications;

$A \propto D_A^2$; $du/dy \propto \pi D_A N / D_A$

Therefore, the expression for viscous force may be written as;

$$\boxed{F_v \propto \mu D_A^2 N}$$

3- Gravity Force [F_g]

The inertial force (F_g) = (mass) (gravitational acceleration) = m.g

In mixing applications;

m = ρ V = ρ A L $\propto \rho D_A^3$

$$\boxed{F_g \propto \rho D_A^3 g}$$

4- Surface Tension Force [F_σ]

In mixing applications;

$$\boxed{F_\sigma \propto \sigma D_A}$$

In the design of liquid mixing systems the following dimensionless groups are of importance: -

1- The Power Number (Np)

$$\boxed{Np = \frac{P_A}{\rho N^3 D_A^5}}$$

where, P_A: is the power consumption.

2- <u>The Reynolds Number</u> $(\mathrm{Re})_m$

$$\boxed{(\mathrm{Re})_m = \frac{\text{Inertial Force}}{\text{Viscous Force}}} = \frac{F_i}{F_v} = \frac{\rho D_A^4 N^2}{\mu D_A^2 N}$$

$$\boxed{\Rightarrow \quad (\mathrm{Re})_m = \frac{\rho N D_A^2}{\mu}}$$

3- <u>The Froude Number</u> $(\mathrm{Fr})_m$

This number related to fluid surface [related to vortex system in mixing]

$$\boxed{(Fr)_m = \frac{\text{Inertial Force}}{\text{Gravity Force}}} = \frac{F_i}{F_g} = \frac{\rho D_A^4 N^2}{\rho D_A^3 g}$$

$$\boxed{\Rightarrow \quad (Fr)_m = \frac{N^2 D_A}{g}}$$

4- <u>The Weber Number</u> $(\mathrm{We})_m$

This number related to multiphase fluids [**<u>or</u>** fluid flow with interfacial forces]

$$\boxed{(We)_m = \frac{\text{Inertial Force}}{\text{Surface Tension Force}}} = \frac{F_i}{F_\sigma} = \frac{\rho D_A^4 N^2}{\sigma D_A}$$

$$\boxed{\Rightarrow \quad (We)_m = \frac{\rho N^2 D_A^3}{\sigma}}$$

It can be shown by **<u>dimensional analysis</u>** that the power number (Np) can be related to the Reynolds number $(\mathrm{Re})_m$ and the Froude number $(\mathrm{Fr})_m$ by the equation;

$$\boxed{Np = C(\mathrm{Re})_m^x (Fr)_m^y}$$

where, C is an overall dimensionless shape factor which represents the geometry of the system.

The last equation can also be written in the form;

$$\boxed{\Phi = \frac{Np}{(Fr)_m^y} = C(\mathrm{Re})_m^x}$$

where, Φ is defined as the dimensionless power function.

The Froude number $(\mathrm{Fr})_m$ <u>is usually important only in situations where gross vortexing.</u> Since vortexing is a gravitational effect, the $(\mathrm{Fr})_m$ is not required to describe a baffled liquid mixing systems. In this case the exponent of $(\mathrm{Fr})_m$ (i.e. y) in the last two equations is zero.[$(Fr)^y = (Fr)^0 = 1 \quad \Rightarrow [\Phi = Np]$.

Thus the non-vortexing systems, the equation of power function (Φ) can be written wither as;

$\boxed{\Phi = Np = C(\mathrm{Re})_m^x}$ or as; $\boxed{\log\Phi = \log Np = \log C + x\log(\mathrm{Re})_m}$

The Weber number of mixing $(\mathrm{We})_m$ <u>is only of importance</u> *when separate physical phases are present in the liquid mixing system* as in liquid-liquid extraction.

9.4 Power Curve

A power curve is a plot of the power function (Φ) or the power number (Np) against the Reynolds number of mixing $(Re)_m$ on log-log coordinates. Each **geometrical configuration** has its own power curve and since the plot involves dimensionless groups it is independent of tank size. Thus a power curve that used to correlate power data in a 1 m^3 tank system is also valid for a 1000 m^3 tank system provided that both tank systems have the same **geometrical configuration**.

The Figure below shows the power curve for the standard tank configuration. Since this is a baffled tank (non-vortexing system), the following equation is applied;

$$\log \Phi = \log Np = \log C + x \log(Re)_m \qquad \text{---------------------------} \ (☼)$$

From the Figure it is clear that the power curve for the standard tank configuration is **linear** in the laminar flow region (line-AB) with slope (-1) in this region $[(Re)_m < 10]$. Then the last equation can be written in the following form;

$$\log \Phi = \log Np = \log C - \log(Re)_m$$

which can be rearranged to give;

$$\boxed{P_A = C\,\mu\,N^2\,D_A^3}$$

C is a constant depend on the type of agitator and vessel arrangement and if the tank is with or without baffles. **For the standard tank configuration C = 71** and for marine type 3-blade C = 41. Thus for **the laminar flow**, power (P_A) is directly proportional to *dynamic viscosity* (μ) *for a fixed agitator speed* (N).

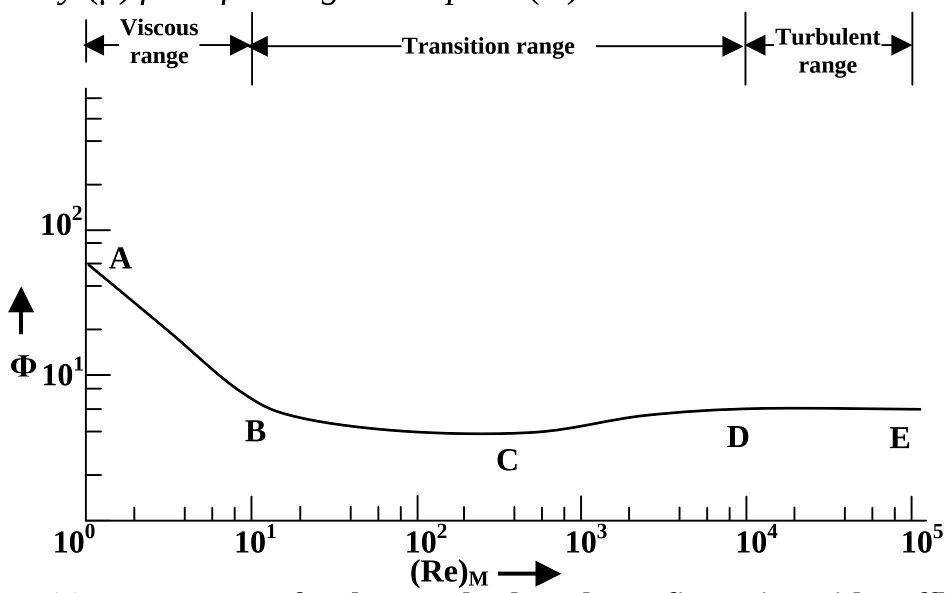


Figure (1): Power Curve for the Standard Tank Configuration with Baffles

For **the transition flow region BCD** which extends up to $(Re)_m$ = 10,000, the constant (C) and the slope (x) in equation (☼) vary continuously.

In **fully turbulent flow** $(Re)_m$ > 10,000, the curve becomes horizontal and the power function (Φ) is independent of Reynolds number of mixing $(Re)_m$.

i.e. $\boxed{\Phi = Np = 6.3 \qquad \text{for } (Re)_m > 10{,}000}$

At point (C) on the power curve, for the standard tank configuration, enough energy is being transferred to the liquid for *vortexing* to start. However the baffles in the tank prevent this.

If the baffles were not present, vortexing would develop and the power curve would be as shown in Figure below;

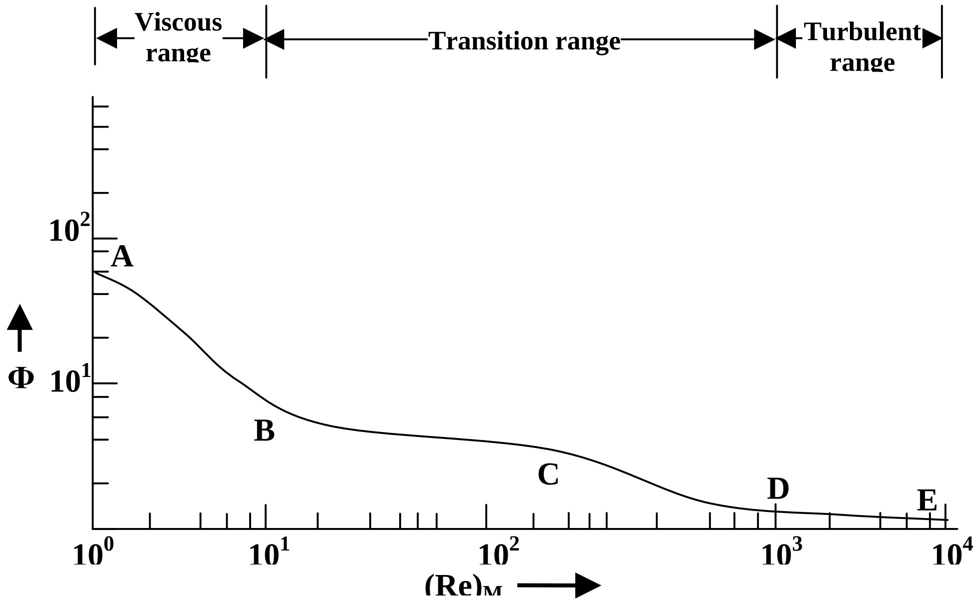


Figure (1): Power Curve for the Standard Tank Configuration without Baffles

The power curve for the baffled system is identical with that for the unbaffled system up to point (C) where [$(Re)_m \approx 300$]. As the $(Re)_m$ increases beyond point (C) in the unbaffled system, vortexing increases and the power falls sharply as shown in the above Figure.

As mentioned previously it can be shown by **<u>dimensional analysis</u>** that the power number (Np) can be related to the Reynolds number $(Re)_m$ and the Froude number $(Fr)_m$ by the equation;

$$Np = C(\mathrm{Re})_m^x (Fr)_m^y$$

$$\Rightarrow \quad \log Np = \log C + x \log(\mathrm{Re})_m + y \log(Fr)_m$$

<u>For the unbaffled system</u>

$\Phi = Np$	for $(Re)_m < 300$
And $\Phi = Np/[(Fr)_m]^y$	for $(Re)_m > 300$

A plot of (Np) against $(Fr)_m$ on log-log coordinates is a straight line of slop y at a constant $(Re)_m$. A number of lines can be plotted for different values of $(Re)_m$. A plot of (y) against $\log(Re)_m$ is also a straight line. If the slope of the line is $(-1/\beta)$ and the intercept at $(Re)_m = 1$ is (α/β) then

$$\boxed{y = \frac{\alpha - \log(\mathrm{Re})_m}{\beta}}$$

$$\boxed{\therefore \quad \Phi = \frac{Np}{(Fr)_m^y} = \frac{Np}{[(Fr)_m]^{\frac{\alpha - \log(\mathrm{Re})_m}{\beta}}}}$$

The values of (α and β) are varying for various vortexing system. For a 6-blade flat blade turbine agitator of 0.1 m diameter [(α = 1) and (β = 40)]

If a power curve is available for particular system geometry, it can be used to calculate the power consumed by an agitator at various rotational speeds, liquid viscosities and densities. The procedure is as follows: -

1- Calculate $(Re)_m$
2- Read power number (Np) or power function (Φ) from the appropriate power curve
3- Calculate the power (P_A) from
either $\boxed{P_A = Np\,\rho\, N^3 D_A^5}$ or $\boxed{P_A = \Phi\,[(Fr)_m]^y\,\rho\, N^3 D_A^5}$

These equations can b used to calculate only the power consumed by the agitator. Electrical and mechanical losses required additional power, which occur in all mixing system.

Example -9.1-

Calculate the theoretical power in Watt for a 3 m diameter, 6-blade flat blade turbine agitator running at 0.2 rev/s in a tank system conforming to the standard tank configuration. The liquid in the tank has a dynamic viscosity of 1 Pa.s, and a liquid density of 1000 kg/m^3.

Solution:

$(Re)_m = \rho N D_A^2 / \mu = (1000)(0.2)(3)^2 / 1 = 1{,}800$

From Figure (1) $\Phi = Np = 4.5$

The theoretical power for mixing

$$P_A = Np\,\rho\,N^3 D_A^5 = 4.5\,(1000)\,(0.2)^3\,(3)^5 = 8{,}748\text{ W}$$

Example -9.2-

Calculate the theoretical power in Watt for a 0.1 m diameter, 6-blade flat blade turbine agitator running at 16 rev/s in a tank system without baffles and conforming to the standard tank configuration. The liquid in the tank has a dynamic viscosity of 0.08 Pa.s, and a liquid density of 900 kg/m^3.

Solution:

$(Re)_m = \rho N D_A^2 / \mu = (900)(16)(0.1)^2 / (0.08) = 1{,}800$

From Figure (2) $\Phi = 2.2$

The theoretical power for mixing

$P_A = \Phi\,[(Fr)_m]^y\,\rho\,N^3 D_A^5$

$$y = \frac{\alpha - \log(Re)_m}{\beta} \quad \Rightarrow y = \frac{1 - \log(1800)}{40} = -0.05638$$

$(Fr)_m = N^2 D_A / g = (16)^2 (0.1) / 9.81 = 2.61$

$[(Fr)_m]^y = [2.61]^{-0.05638} = 0.9479$

$\Rightarrow P_A = 2.2\,(0.9479)\,(900)\,(16)^3\,(0.1)^5 = 76.88\text{ W}$