

HYDRAULIC ENGINEERING

Subject Code-CE 43003

Section 2-Fluid Machinery

Syllabus

The students should be introduced to the following concepts/principles to understand and learn-by-doing fluid machinery in Hydraulic Engineering:

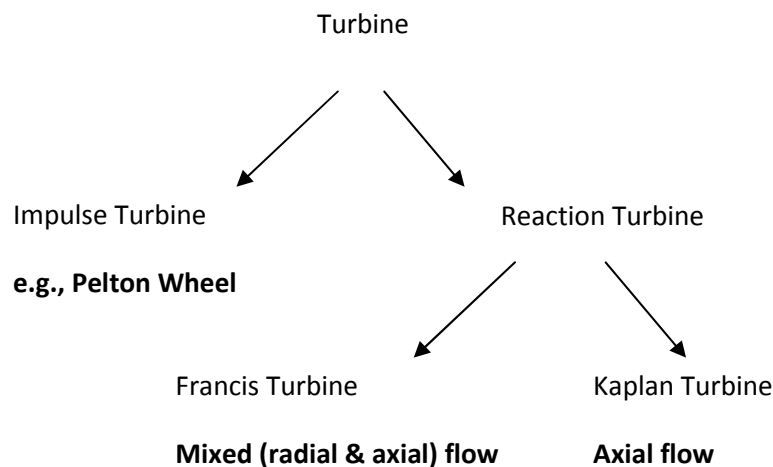
- 1) Types of fluid machinery, classification
- 2) Introduction to turbines
- 3) Introduction to positive displacement pumps, rotodynamic pumps
- 4) Pumps in pipeline systems: characteristics curves, operating conditions
- 5) Characteristics of similar pumps
- 6) Specific speed, cavitation, pump selection, Net Positive Suction Head (NPSH)

1.0 Introduction

The ever increasing demand on energy has been escalated to a new level with the increased population. However, the need for sustainable resource development has been underscored in the contemporary project developments. With the concept of sustainable resource development, to meet the soaring demand on energy, few feasible options have been screened and practiced. The hydropower, conventional renewal source of energy which is clean and considered to have good environmental effect, is one those options that has been used to produce energy in a sustainable manner. In a hydropower project, the fluid is used to transfer energy using a turbine that is a machine that uses the energy carried by the flowing liquid and converts it into mechanical or rotational energy. The mechanical energy is used to run an electric generator that is coupled to the shaft of the turbine. Moreover, in a hydropower project, in the design of pipeline and conveyance systems, flow in a pipe experiences resistance to its movement due to major and minor head, the energy per unit weight, losses due to friction and controls. Due to this resistance, the available head of water in the pipeline systems is reduced. Therefore, there is a need to supply additional energy to overcome head losses due to friction and controls using one of the hydraulic machineries such as a pump that sources hydraulic energy to a flowing fluid by converting the mechanical energy. Furthermore, in a hydropower project, it may also require producing high head or large quantity of flow than what is possible with a single pump. Therefore, there is a need to learn the underlying principles in the operation of turbines and pumps. Thus, the objective of this section of the syllabus is to guide the students, who may be engineered to solve fluid flow problems that involve fluid machineries specifically turbines and pumps, understand and to learn-by-doing fluid machineries in hydraulic engineering.

2.0 Turbines

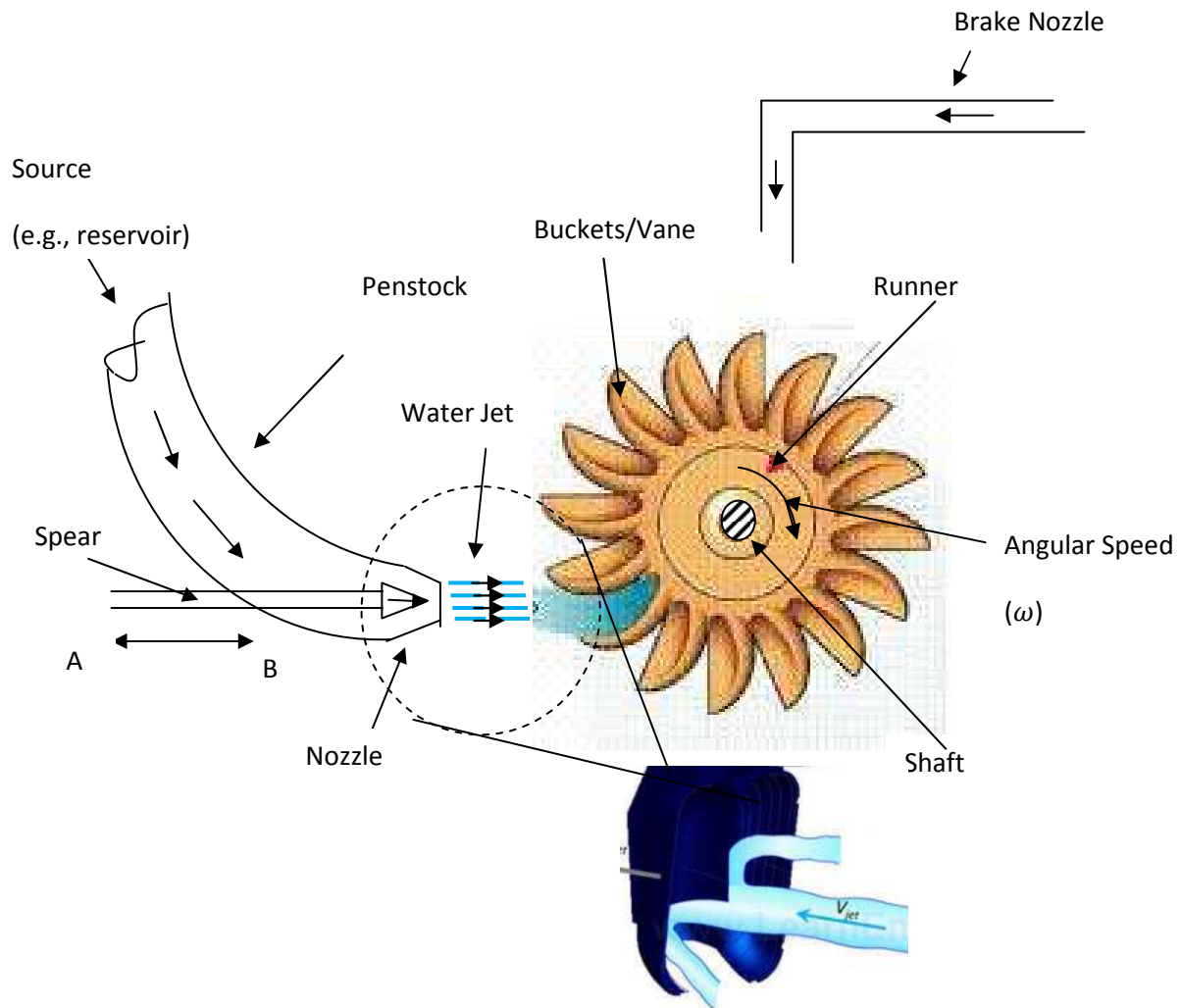
Turbine is a machine that **uses the energy carried by the flowing liquid and converts it into mechanical or rotational energy**. The mechanical energy is used to run an electric generator that is coupled to the shaft of the turbine. Turbines can be classified based on many ways such as based on name of the originator, flow direction, and action of fluid on the moving blades. As shown below, there are two types of turbines based on action of fluid on the moving blades. In an impulse turbine that requires high head and small quantity of flow, all the available potential energy is converted into kinetic energy by discharging water through a nozzle. Therefore, the runner in an impulse turbine utilizes the kinetic energy. Whereas in a reaction turbine that requires low head and high flow rate, both the potential and kinetic energies are utilized by the runner.



2.1 Pelton Wheel

Pelton wheel, which is also known as Pelton turbine, is an impulse turbine. As shown below, in the operation of a Pelton wheel which is sourced through a penstock with a nozzle, before the water jet hits the bucket/vane of the runner to extract the water

energy and convert it to useful work, all the available potential energy is converted into kinetic energy. The spear inside the nozzle, which can move in either (i.e., AB or BA) direction, controls the movement of water through the nozzle.



Learning-by-doing

1) In the above figure, explain the reason for having a brake nozzle.

Ans.

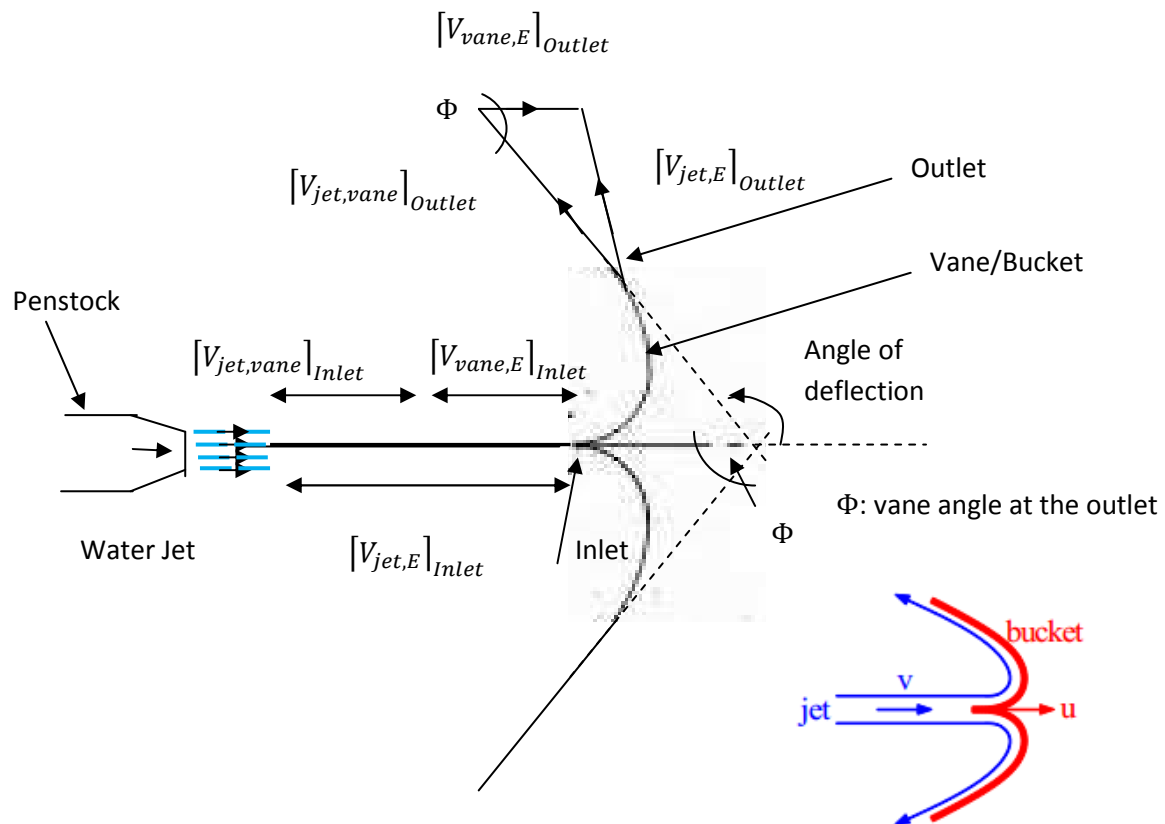
At the closure of nozzle by the movement of spear in the forward direction (i.e., towards turbine), the turbine is sourced with zero amount of water. This means that the turbine should not revolve. However, due to the inertia, the runner continues to

revolve. This may last for a long time. Therefore, to bring the runner to rest immediately, a brake nozzle is provided.

- 2) The spear inside the nozzle is used to control the power supply with time. Do you agree on this? If so, explain briefly.

2.1.1 Fluid Motion in a Pelton Wheel

The below diagram shows the velocities of water associated at the inlet and outlet of a Pelton wheel.



To understand the connectivity between the velocities, for example, consider the velocities of water at the outlet of the turbine. At the outlet, we have the following:

- a) Absolute velocity of the water jet ($V_{jet,E}$)
- b) Velocity of the water jet relative to vane ($V_{jet,vane}$)

- c) Absolute/peripheral velocity of the vane/runner ($V_{vane,E}$). This is a function of angular velocity (ω) of the runner

The above three velocities are connected using the principle of relativity on velocities. In other words, as per the principle of relativity on velocities,

$$[V_{jet,E}]_{outlet} = [V_{jet,vane}]_{outlet} + [V_{vane,E}]_{outlet}$$

Similarly, the relationship between velocities of water at the inlet is given by

$$[V_{jet,E}]_{inlet} = [V_{jet,vane}]_{inlet} + [V_{vane,E}]_{inlet}$$

Where $[V_{vane,E}]_{outlet} = [V_{vane,E}]_{inlet} = u = \text{function}(\omega)$

Learning-by-doing

- 1) Explain the reason for considering $V_{vane,E}$ in horizontal direction
- 2) The **velocity of whirl** is defined as the component of $V_{jet,E}$ in the direction of motion of vane. Write down an expression for velocity of whirl at the outlet and inlet.

Ans.

Considering the above diagram and the definition of velocity of whirl, the velocity of whirl at the outlet ($[V_w]_{outlet}$) is given by $[V_w]_{outlet} = [V_{jet,vane}]_{outlet} \cos \Phi - [V_{vane,E}]_{outlet}$. Similarly, the velocity of whirl at the inlet ($[V_w]_{inlet}$) is given by $[V_w]_{inlet} = [V_{jet,E}]_{inlet}$

- 3) The **velocity of flow** is defined as the component of $V_{jet,E}$ perpendicular to the direction of motion of vane. Write down an expression for velocity of flow at the outlet and inlet.

Ans.

Considering the above diagram and the definition of velocity of flow, the velocity of flow at the outlet ($[V_f]_{outlet}$) is given by $[V_f]_{outlet} = [V_{jet,vane}]_{outlet} \sin \Phi$. Similarly, the velocity of flow at the inlet ($[V_f]_{inlet}$) is given by $[V_f]_{inlet} = 0$

- 4) Determine the **force exerted by the water jet in the direction of motion**. You are given that the area of the water jet is A units.

Ans.

$$\text{Force} = \text{Mass} * \text{Acceleration}$$

$$\text{Force} = \text{Volume} * \rho * \text{Acceleration}$$

$$\text{Force} = \text{Volume} * \rho * \frac{\text{Velocity}}{\text{Time}}$$

$$\text{Force} = \frac{\text{Volume}}{\text{Time}} * \rho * \text{Velocity}$$

$$\text{Force} = \text{Area} * \text{Velocity} * \rho * \text{Velocity} \neq \text{Area} * \rho * \text{Velocity}^2$$

$$\text{Force} = A * [V_{jet,E}]_{inlet} * \rho * [V_w]_{inlet} + A * [V_{jet,E}]_{inlet} * \rho * [V_w]_{outlet}$$

$$\text{Force} = A * [V_{jet,E}]_{inlet} * \rho * \{[V_w]_{inlet} + [V_w]_{outlet}\}$$

- 5) Determine the **work done by the water jet on runner per second**. You are given that the area of the water jet is A units.

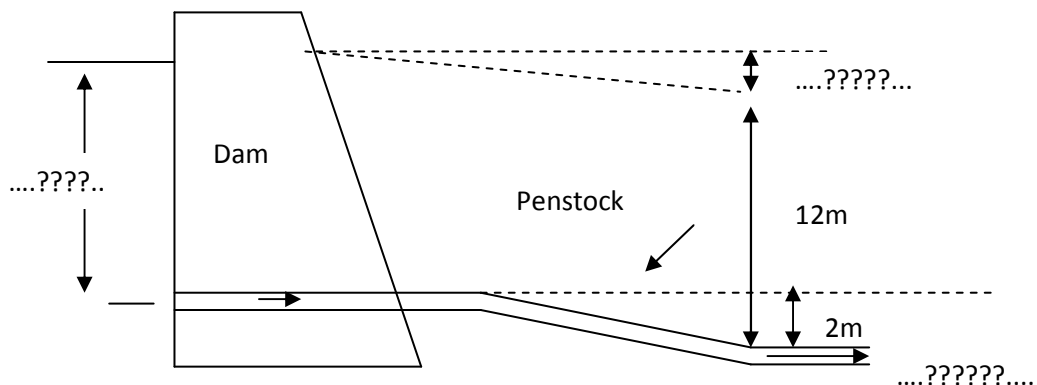
Ans.

$$\text{Work done per second} = \text{Force} * u$$

$$\text{Work done per second} = [A * [V_{jet,E}]_{inlet} * \rho * \{[V_w]_{inlet} + [V_w]_{outlet}\}] * u$$

$$\text{Work done per second} = [A * [V_{jet,E}]_{inlet} * \rho * u * \{[V_w]_{inlet} + [V_w]_{outlet}\}]$$

- 6) For the below given diagram, fill in the blanks



2.1.2 Efficiency of a Pelton Wheel

Efficiency(η) of a Pelton wheel is defined by $\eta = \frac{\text{Shaft Power}}{\text{Water Power}}$

$$\eta = \frac{\text{Shaft Power}}{\text{Water Power}}$$

$$\eta = \frac{\text{Power available at the shaft(say } P)}{\text{Power delivered by the water jet}}$$

$$\eta = \frac{P}{HQ\rho g} \text{ where } H \text{ is the net head on the turbine}$$

$$\eta = \frac{P}{HQ\rho g}$$

In reality, the volume of water actually striking the runner is lesser than what is supplied by the water jet. Therefore, the value of discharge that contributes to strike the runner is lesser than what is supplied by the water jet. Accounting this, the efficiency(η) of a Pelton wheel is defined by

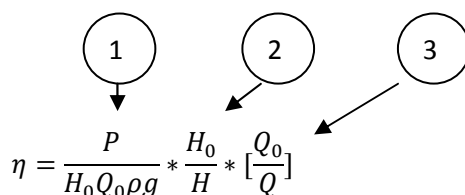
$$\eta = \frac{P}{HQ_0\rho g} * \left[\frac{Q_0}{Q} \right]$$

Even though the power supplied by the water jet at the turbine is $HQ_0\rho g$, in reality, the power that is developed by the runner is lesser. Accounting this, the efficiency(η) of a Pelton wheel is defined by

$$\eta = \frac{P}{H_0Q_0\rho g} * \frac{H_0}{H} * \left[\frac{Q_0}{Q} \right]$$

Where $H_0 = [u/g * \{[V_w]_{inlet} + [V_w]_{outlet}\}]$

Therefore, the efficiency(η) of a Pelton wheel has the following three components:



$$\eta = \frac{P}{H_0Q_0\rho g} * \frac{H_0}{H} * \left[\frac{Q_0}{Q} \right]$$

The component (1) is known as the mechanical efficiency. The reason for the existence of mechanical efficiency is that the power that is developed by the runner is partially lost due to bearing friction. Therefore, the power lost due to bearing friction is $(P - H_0 Q_0 \rho g)$. This loss is formerly known as mechanical loss.

The component (2) is known as the hydraulic efficiency. The reason for the existence of hydraulic efficiency is that the power that is supplied by the water jet at the turbine is not fully developed by the runner. Therefore, the power lost in this transition is $(H Q_0 \rho g - H_0 Q_0 \rho g)$. This loss is formerly known as hydraulic loss.

The component (3) is known as the volumetric efficiency. The reason for the existence of volumetric efficiency is that the volume of water actually striking the runner is lesser than the volume of the water that is supplied by the jet.

Learning-by-doing

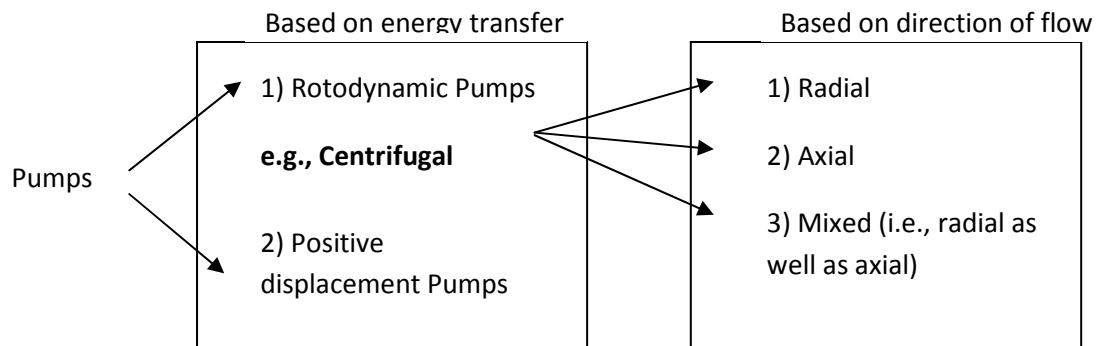
- 1) Determine the power output of a hydropower plant if the efficiency of a generator that is driven by a turbine is 0.92.
- 2) The youtube video that is accessible at <https://www.youtube.com/watch?v=rf9megw2SQA> discusses the working aspect of a Pelton wheel. After listening to the video, write down a paragraph on number of buckets in the runner and its influence on turbine efficiency.

3.0 Pumps

A pump is one of the hydraulic machineries that **sources hydraulic energy to a flowing fluid by converting the mechanical energy**. The addition of energy could increase the following of the flowing fluid:

- a) Pressure energy
- b) Kinetic energy
- c) Pressure and kinetic energy

The classification of pumps could be based on many factors such as direction of flow, working head, and transfer of energy. As shown below, based on transfer of energy, the pumps are classified into two types: positive displacement and rotodynamic. The positive displacement pumps are also known as reciprocating pumps.

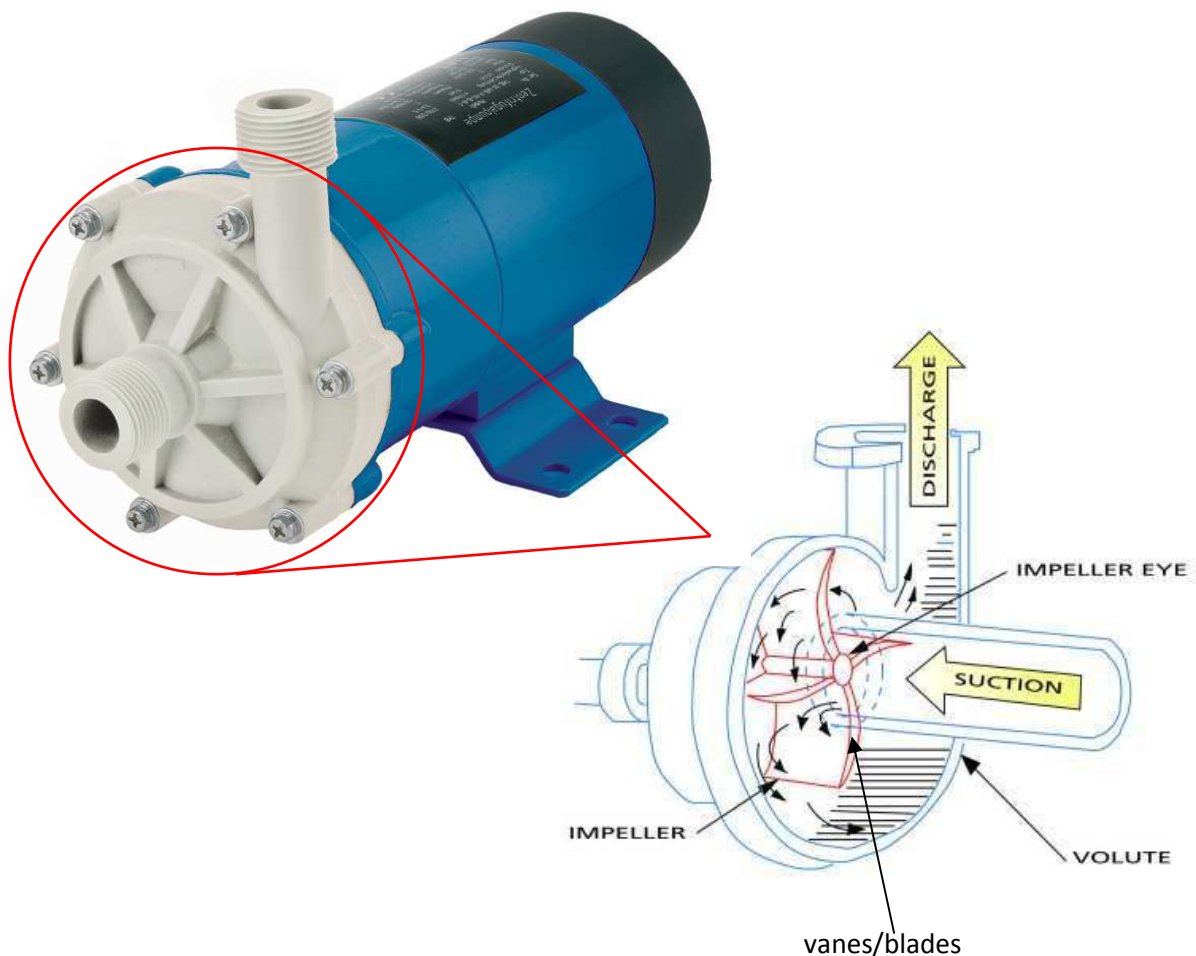


In the operation of a pump, the mechanical action performs two functions. At first, a vacuum is created at the inlet of the pump. This vacuum causes the liquid to flow from the source (e.g., reservoir) into the inlet to the pump. Then, the drawn liquid is delivered to the pump outlet. Due to high capital cost, maintenance cost, greater discharging capacity, etc., centrifugal pumps have become the first choice for industrial uses.

3.1 Centrifugal Pump

A centrifugal pump, which can be used for viscous and non-viscous liquids, is a rotodynamic pump with energy transfer occurs when the flow is in its **radial path**.

Centrifugal pump is one of the pumps that can be operated at very high speeds without the danger of cavitation.



In the operation of a centrifugal pump, the impeller is rotated by an electric motor. The rotation of impeller causes the vanes to impel out the liquid. The liquid that is impelled out by the vanes comes to the out at the outlet.

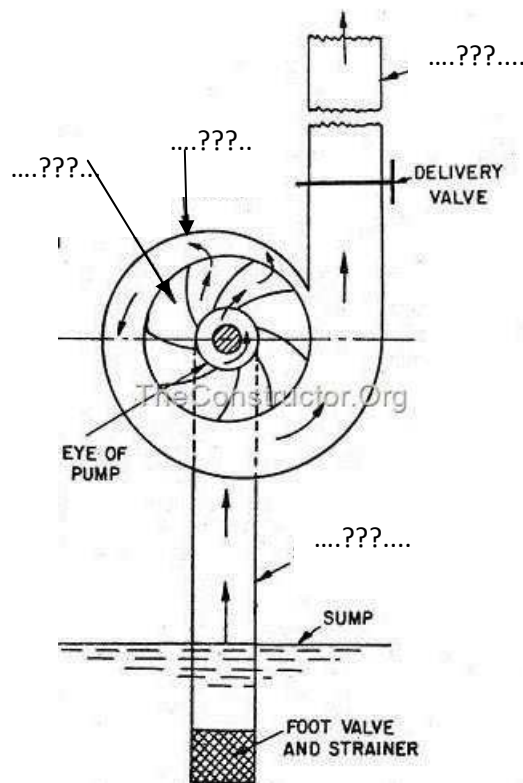
Learning-by-doing

1) What is meant by priming of a centrifugal pump?

Ans.

Before starting the pump, to ensure that the air pockets are not entrapped, the liquids to be pumped fill the suction pipe, casing, the delivery pipe upto delivery valve. This operation of filling is known as priming of a centrifugal pump

2) In the following figure, fill in the blanks.



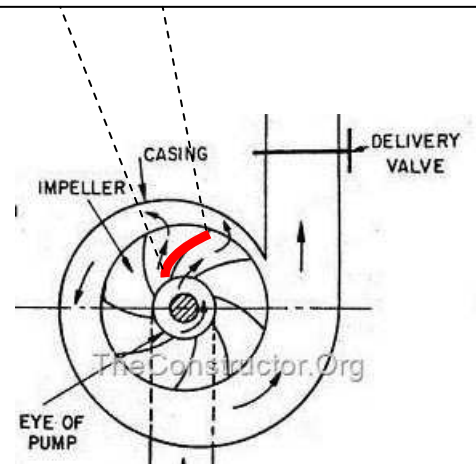
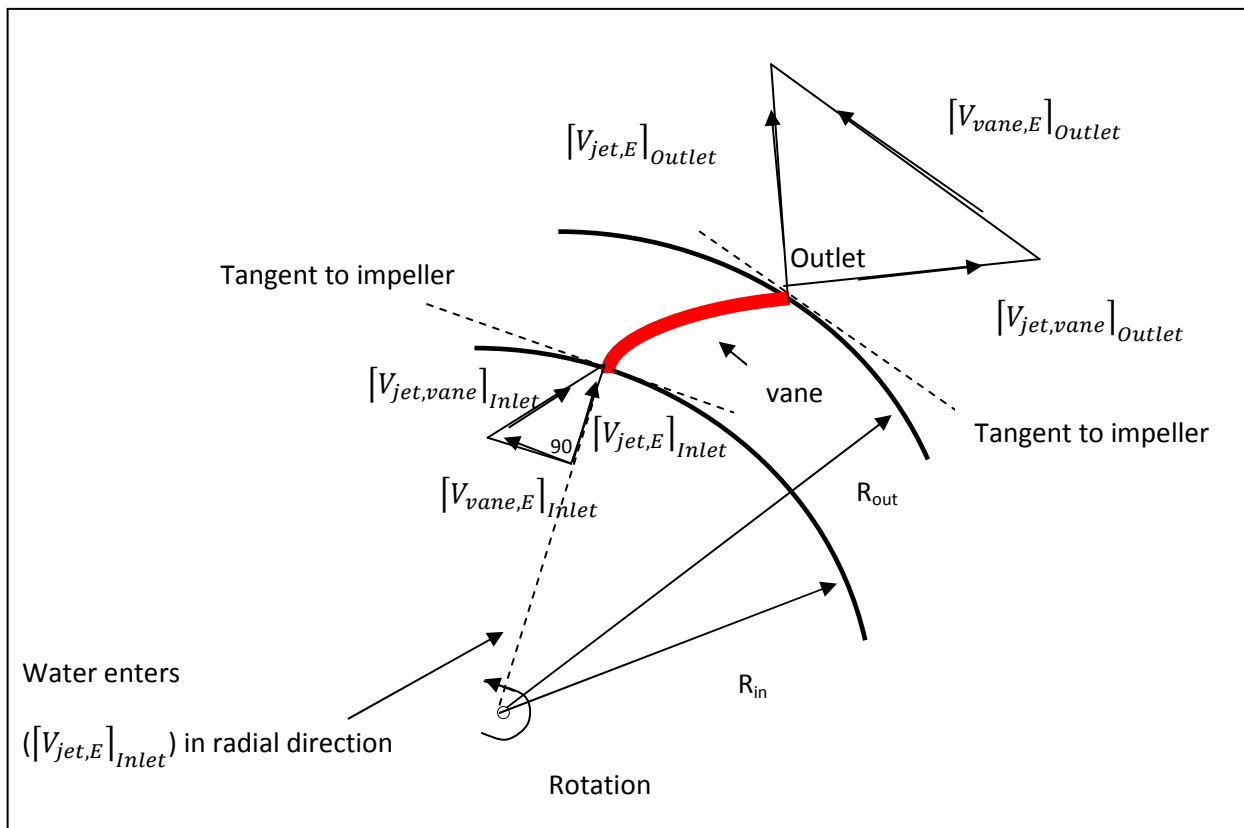
3) In the above figure, write down the function of the following:

- a) Foot valve
- b) Delivery valve
- c) Strainer

- 4) When the pump is to be stopped, the delivery valve should be first closed. What could be the reason?

3.1.1 Fluid Motion in a Centrifugal Pump

The below diagram shows the velocities of water associated at the inlet and outlet of a centrifugal pump.



To understand the connectivity between the velocities, for example, consider the velocities of water at the outlet of the pump. At the outlet, we have the following:

- a) Absolute velocity of the water jet($V_{jet,E}$)
- b) Velocity of the water jet relative to vane($V_{jet,vane}$)
- c) Absolute/peripheral velocity of the vane/runner($V_{vane,E}$). This is a function of angular velocity(ω) of the impeller

The above three velocities are connected using the principle of relativity on velocities. In other words, as per the principle of relativity on velocities,

$$[V_{jet,E}]_{outlet} = [V_{jet,vane}]_{outlet} + [V_{vane,E}]_{outlet}$$

Similarly, the relationship between velocities of water at the inlet is given by

$$[V_{jet,E}]_{inlet} = [V_{jet,vane}]_{inlet} + [V_{vane,E}]_{inlet}$$

Learning-by-doing

- 1) The **velocity of flow** is defined as the component of $V_{jet,E}$ perpendicular to the direction of motion of vane. Write down an expression for velocity of flow at the inlet.

Ans.

The water enters in the radial direction. Therefore, the angle between $[V_{jet,E}]_{inlet}$ and $[V_{vane,E}]_{inlet}$ is 90 degrees. Considering the above diagram and the definition of velocity of flow, the velocity of flow at the inlet ($[V_f]_{inlet}$) is given by $[V_f]_{inlet} = [V_{jet,E}]_{inlet}$.

- 2) Determine the **rate of change of moment of momentum in the direction of motion of vane**.

Ans.

*momentum = mass * velocity*

*moment of momentum = mass * velocity * radius*

Rate of change of moment of momentum

$$= (\text{mass} * \text{velocity} * \text{radius})_{outlet} - (\text{mass} * \text{velocity} * \text{radius})_{inlet}$$

Since the water enters in the radial direction, the **velocity of whirl**, which is defined as the component of $V_{jet,E}$ in the direction of motion of vane, at the inlet is zero. Therefore, $(mass * velocity * radius)_{inlet} = 0$. This simplifies to the following:

$$Rate\ of\ change\ of\ moment\ of\ momentum = (mass * velocity * radius)_{outlet}$$

$$Rate\ of\ change\ of\ moment\ of\ momentum = \rho * [V_w]_{outlet} * R_{out}$$

- 3) Determine the torque and the work done per second per unit weight in the direction of motion of vane.

Ans.

$$Torque = Rate\ of\ change\ of\ moment\ of\ momentum = \rho * [V_w]_{outlet} * R_{out}$$

$$Work\ done\ per\ second = Torque * angular\ velocity$$

$$Work\ done\ per\ second = \rho * [V_w]_{outlet} * R_{out} * \omega$$

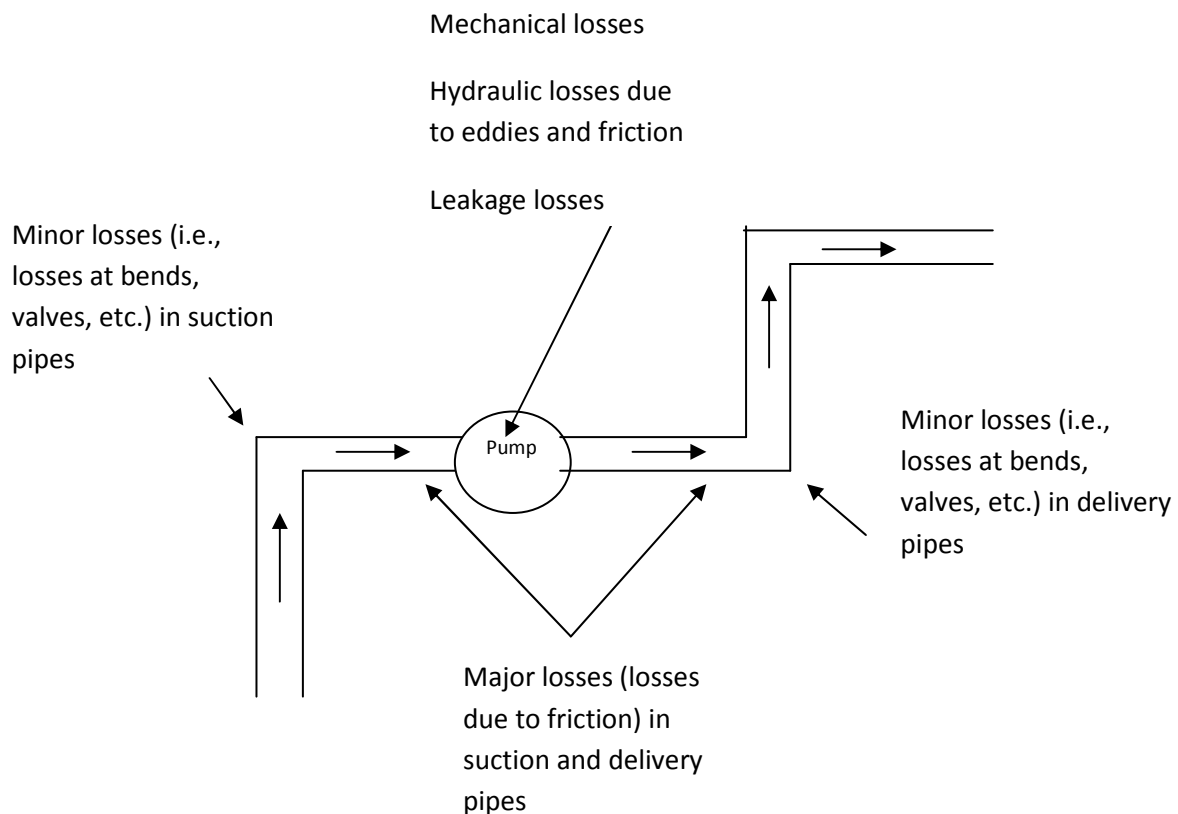
$$Work\ done\ per\ second\ per\ unit\ weight = \frac{[V_w]_{outlet} * R_{out} * \omega}{g}$$

$$Work\ done\ per\ second\ per\ unit\ weight = \frac{[V_w]_{outlet} * u_{out}}{g} \text{ where } u_{out} = R_{out} * \omega$$

- 4) The Euler momentum equation, which is also known as Euler head, for a centrifugal pump is given by $\frac{[V_w]_{outlet} * R_{out} * \omega}{g} - \frac{[V_w]_{inlet} * R_{in} * \omega}{g}$. Determine an expression for Euler head if water enters in the radial direction.

3.1.2 Losses/Gains of Heads in a Centrifugal Pump

A centrifugal pump has to supply head against losses of head. As shown below, the losses of head can be in many forms such as losses in the **pump**, and losses in suction and delivery **pipes**.



The **losses in the pump** are generally due to eddies, leakages, and bearing friction.

The losses due to eddies and friction may arise at the impeller as well as at the vanes of a centrifugal pump. The **losses due to leakage** of liquid, also known as leakage losses, are due to leakage of liquid that happens through the clearances between impeller and casing of a centrifugal pump. The **mechanical losses** are due to bearing friction.

The losses in suction and delivery pipes can be categorized into major and minor losses. The major losses are governed by Darcy-Weisbach equation. The Darcy-Weisbach equation is given by $h_f = \frac{4fLu^2}{2gd}$. In addition to head losses due to friction there are always head losses in pipe lines due to bends, junctions, valves, etc., which are known as minor head losses. Minor head losses are the localized energy losses in pipeline systems. The formula that is commonly used to compute the local head loss is given by $h_L = k \frac{u^2}{2g}$ where h_L is the local head loss and k is a coefficient of constant. Oftentimes, in calculating head losses, minor head losses are neglected as they are insignificant. In practice, in long pipelines of several kilometres the effect of local head losses may be negligible. For short pipelines the local head losses may be greater than those for friction.

3.1.3 Application of Bernoulli's Equation

To understand the relationship between various heads and losses, Bernoulli's equation is applied for the stream line. For the below given figure, the application of Bernoulli's equation from point-1 to point-2 gives

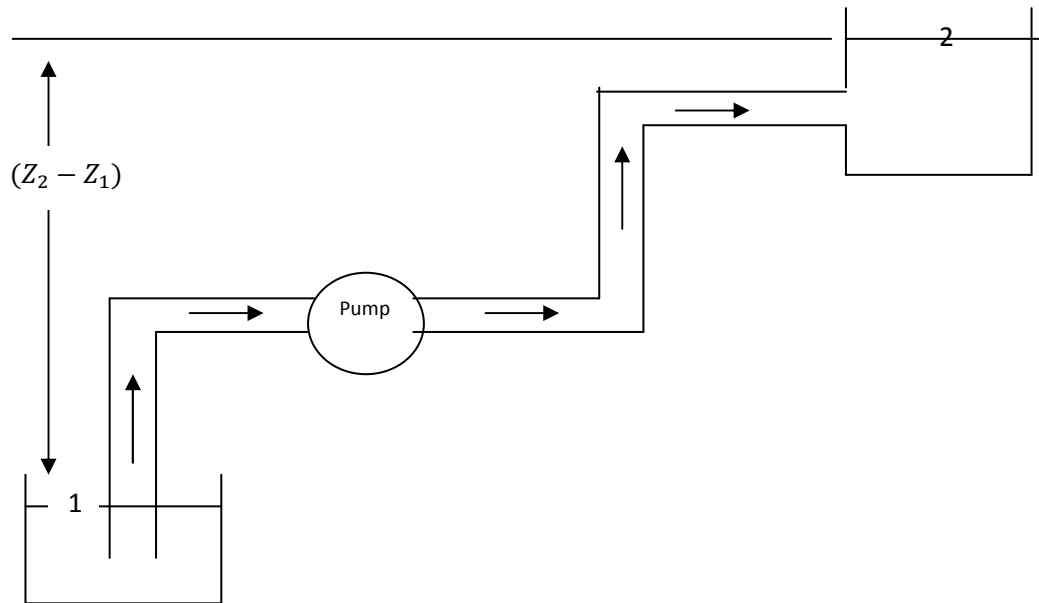
$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 + H_p = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \text{major head losses} + \text{minor losses}$, where H_p is the head supplied by the pump. The subscripts 1 and 2 refer to point-1 and point-2, respectively. The minor losses could be due to losses at the entrance, exit, bends, etc. Oftentimes, in calculating head losses, minor head losses are neglected as they are insignificant. Therefore, substituting for major head losses using Darcy-Weisbach equation gives the following relationship:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 + H_p = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \frac{4f_1L_1V_1^2}{2gD_1} + \frac{4f_2L_2V_2^2}{2gD_2}$$

Further simplification gives

$$H_p = (Z_2 - Z_1) + \frac{4f_1L_1V1^2}{2gD_1} + \frac{4f_2L_2V2^2}{2gD_2}$$

Where $(Z_2 - Z_1)$ is known as the **static head**.



The head (H_p) supplied by the pump is given by

$$H_p = \frac{[V_w]_{outlet} * u_{out}}{g} - \text{head losses in the pump}$$

Neglecting the *head losses in the pump*, the application of Bernoulli's equation from point-1 to point-2 gives,

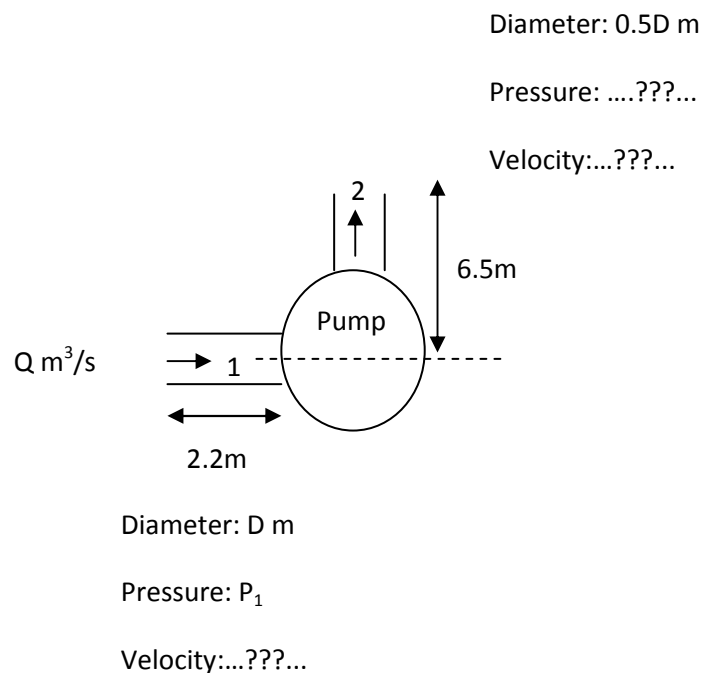
$$H_p = \frac{[V_w]_{outlet} * u_{out}}{g} = (Z_2 - Z_1) + \frac{4f_1L_1V1^2}{2gD_1} + \frac{4f_2L_2V2^2}{2gD_2}$$

Learning-by-doing

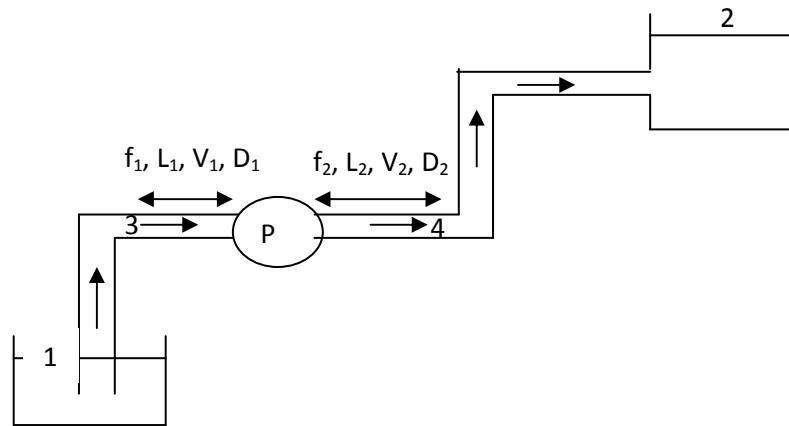
- 1) At the request of villagers, to resolve the existing water crisis in one of the villages in Sri Lanka, the government of Sri Lanka has decided to deliver water from tank A to tank B using a pump whose head discharge relationship is given by $H_p = 83 - 7550Q^2$ where H_p is in meters and Q in m^3/s . The water surface elevation of tank A is 120m. The water surface elevation of tank B is 180m. The suction pipe that connects tank A to pump is 145m in length and 0.35m in diameter. The delivery pipe that connects pump to tank B is 850m in length and 0.25m in diameter. The friction factors (λ) of suction and delivery pipes are 0.024 and 0.022 respectively. As the government of Sri Lanka is keen to produce

resourceful engineers, the government of Sri Lanka has approached some of the students in one of the budding universities in Sri Lanka: South Eastern University of Sri Lanka. As you are specialized in this subject, answer the following questions to help the government of Sri Lanka.

- a) Considering only the major head loss that is due to friction, find out the discharge in the pipe lines
 - b) Find out the power delivered by the pump
 - c) Assume that as a rough rule of thumb, velocities above 0.75m/s are often specified to prevent sedimentation in water distribution network. State whether the velocities in the pipes adhere to the rule of thumb on velocity
- 2) The below figure shows a pump that connects pipes. The head supplied by the pump is $83-7550Q^2$. Neglecting the major losses in the pipes, which is due to friction, fill out the missing information.



- 3) A student has applied the Bernoulli's equation between point-3 and point-4. As per the student, $H_p = \frac{4f_1 L_1 V_1^2}{2gD_1} + \frac{4f_2 L_2 V_2^2}{2gD_2}$. Do you agree on this? If not, state the reason.



3.1.4 Efficiency of a Centrifugal Pump

Efficiency(η) of a centrifugal pump is defined by $\eta = \frac{\text{Power supplied by the pump}}{\text{Shaft power}}$

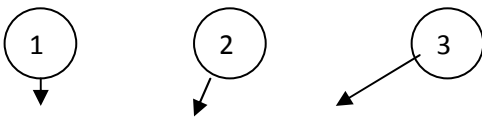
$$\eta = \frac{\text{Power supplied by the pump}}{\text{Shaft power (say } P \text{)}}$$

$$\eta = \frac{\text{Head supplied by the pump (say } H_p \text{) } * Q \rho g}{P}$$

$$\eta = \frac{H_p Q \rho g}{P}$$

$$\eta = \frac{H_p Q \rho g}{P} = \frac{H_p}{\frac{[V_w]_{outlet} * u_{out}}{g}} * \frac{Q}{Q_0} * \frac{\frac{Q_0 * \rho g [V_w]_{outlet} * u_{out}}{g}}{P}$$

In reality, the volume of water actually discharging from the pump is lesser than what is supplied to the impeller of the pump. Therefore, the value of discharge (Q) that comes from the pump is lesser than what is supplied (Q_0) to the impeller of the pump. Even though the power supplied by shaft is P , in reality, the power that is delivered ($\frac{Q_0 * \rho g [V_w]_{outlet} * u_{out}}{g}$) by the impeller to the liquid is less than P . Therefore, the efficiency(η) of a centrifugal pump has the following three components:



$$\eta = \frac{H_p}{\frac{[V_w]_{outlet} * u_{out}}{g}} * \frac{Q}{Q_0} * \frac{\frac{Q_0 * \rho g [V_w]_{outlet} * u_{out}}{g}}{P}$$

The component (1) accounts for the loss of head within the pump. In other words, the head that is delivered by the impeller to the liquid is lost within the pump. This loss of head could be, among others, due to eddies and friction within the pump. Therefore, the head that is supplied by the pump is lesser than what is delivered by the impeller to the liquid. The component (2) is known as the volumetric efficiency. The component (2) accounts for the loss of liquid through the clearances between impeller and casing of a centrifugal pump. The component (3) is known as the mechanical efficiency. The component (3) accounts for the loss of power due to bearing friction.

3.2 Net Positive Suction Head (NPSH)

The term NPSH is commonly used with liquids that can undergo cavitation. Technically,

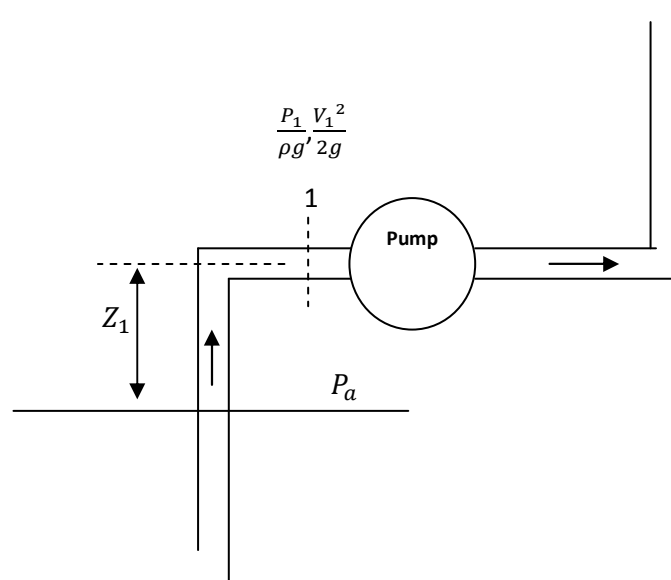
NPSH is defined, as shown below, as $NPSH = \frac{P_1}{\rho g} - \frac{P_v}{\rho g}$ where P_1 is the pressure at pump

inlet (i.e., point-1) and P_v is the vapor pressure of the liquid at a particular

temperature. However, the application of Bernoulli's equation gives $\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 +$

head losses due to friction (h_f) = $\frac{P_a}{\rho g}$ where P_a is the atmospheric pressure. Therefore,

$$NPSH = \frac{P_1}{\rho g} - \frac{P_v}{\rho g} = \frac{P_a}{\rho g} - \frac{V_1^2}{2g} - Z_1 - h_f - \frac{P_v}{\rho g}.$$



3.3 Cavitation

The cavitation, which is generally caused due to high runner speed and high

temperature of the liquid, is a phenomenon that happens when the suction pressure

falls below the vapor pressure of the liquid. The consequences of cavitation are the drop

in efficiency of a pump and noise and vibration. Thoma's cavitation factor (σ) is compared

with critical cavitation factor (σ_c) to determine the occurrence of cavitation in a pump. Thoma's

cavitation factor is defined by $\sigma = \frac{NPSH}{H_p}$ where H_p is defined as per page 18 .The critical

cavitation factor (σ_c) is obtained from published tables or empirical relationships. The cavitation occurs when $\sigma = \frac{NPSH}{H_p} < \sigma_c$. Therefore, the possibility of cavitation increases with the decrease in the value of NPSH.

Learning-by-doing

- 1) The cavitation is a phenomenon that happens when the suction pressure falls below the vapor pressure of the liquid. Derive an expression when the suction pressure just falls below the vapor pressure of the liquid.

3.4 Specific Speed

In the industry, the specific speed (N_s) is used to compare the performances of different pumps. The specific speed of a centrifugal pump is defined as the speed of a geometrically similar pump which would deliver a unit quantity ($1\text{m}^3/\text{s}$) when working under a unit head (1m). Mathematically, the specific speed of a centrifugal pump is defined by $N_s = \frac{N\sqrt{Q}}{H_p^{3/4}}$ where N is the speed of the centrifugal pump, H_p is net head under which pump operates, and Q is the discharge.

Learning-by-doing

- 1) From first principles show that $H_p \propto D^2 N^2$, $Q \propto \frac{H_p^{1.5}}{N^2}$, and $Area \propto \frac{H_p}{N^2}$

Ans.

$$Q = Area * Velocity \text{ of flow}$$

$$\text{tangential velocity}(u) = \frac{\pi DN}{60} \dots u \propto DN$$

$$Velocity \text{ of flow} \propto u \dots Velocity \text{ of flow} \propto DN \dots H_p \propto u^2 \dots H_p \propto D^2 N^2 \dots DN \propto \sqrt{H_p}$$

$$Area \propto D^2 \dots Area \propto \frac{\sqrt{H_p}}{N} * \frac{\sqrt{H_p}}{N} \dots Area \propto \frac{H_p}{N^2}$$

$$Q \propto \frac{H_p}{N^2} \sqrt{H_p} \dots Q \propto \frac{H_p^{1.5}}{N^2}$$

- 2) From first principles show that the specific speed (N_s) of a centrifugal pump is defined by $N_s = \frac{N\sqrt{Q}}{H_p^{3/4}}$

Ans.

As per the previous question, $Q \propto \frac{H_p^{1.5}}{N^2}$. Therefore, $Q = C \frac{H_p^{1.5}}{N^2}$ where C is the proportionality constant. As per the definition of N_s , $Q = C \frac{H_p^{1.5}}{N^2} \dots 1 = C \frac{1^{1.5}}{N_s^2} \dots C = N_s^2$. Therefore, $Q = N_s^2 \frac{H_p^{1.5}}{N^2}$. By rearranging, $N_s^2 = \frac{QN^2}{H_p^{1.5}}$. Therefore, $N_s = \frac{N\sqrt{Q}}{H_p^{0.75}}$.

- 3) Water is required to be pumped out of a well under a head of 75m. The characteristics of an available pump are the following:
- Design speed of 850 r.p.m.
 - Specific speed of 32
 - Rated capacity of 0.14 m³/s

Check whether the available pump can deliver the required task

- 4) Show that $\frac{N\sqrt{Q}}{NPSH^{0.75}}$ is a constant
- 5) Thoma's cavitation factor (σ) is used to determine the occurrence of cavitation in a pump. Thoma's cavitation factor is defined by $\sigma = \frac{NPSH}{H_p}$. Show that

σ is also equal to $\frac{N_s}{N_a}$ where $N_a = \frac{N\sqrt{Q}}{NPSH^{0.75}}$

3.5 Characteristics of Similar Pumps

The models of pumps are tested to better understand the performance of prototypes.

The following conditions are met if the performance of a prototype (p) is correctly predicted by the model (m).

Condition#1 based on specific speed(N_s):

$$|\text{specific speed}(N_s)|_m = |\text{specific speed}(N_s)|_p$$

$$\left| \frac{N\sqrt{Q}}{H^{0.75}} \right|_m = \left| \frac{N\sqrt{Q}}{H^{0.75}} \right|_p$$

Condition#2 based on tangential velocity(u):

$$u = \frac{\pi DN}{60} \dots u \propto DN$$

$$H \propto u^2 \dots H \propto D^2 N^2 \dots \sqrt{H} \propto DN$$

$$\left| \frac{\sqrt{H}}{DN} \right|_m = \left| \frac{\sqrt{H}}{DN} \right|_p$$

Condition#3 based on flow (Q):

$$Q \propto \frac{H}{N^2} \sqrt{H} \dots Q \propto \frac{H^{1.5}}{N^2}$$

$$Q \propto \frac{(DN)^3}{N^2} \dots \text{since } H \propto D^2 N^2$$

$$Q \propto ND^3$$

$$\left| \frac{Q}{ND^3} \right|_m = \left| \frac{Q}{ND^3} \right|_p$$

Condition#4 based on power(P):

$$P = HQ\rho g \dots P \propto HQ$$

$$P \propto D^2 N^2 * ND^3 \dots \text{since } H \propto D^2 N^2 \text{ and } Q \propto ND^3$$

$$\left| \frac{P}{D^5 N^3} \right|_m = \left| \frac{P}{D^5 N^3} \right|_p$$

Learning-by-doing

- 1) Below figures show the prototype and model of a centrifugal pump. The characteristics of the pumps are also given. Using the principle of similarity, determine the

ratio of $\frac{Q_m}{Q_p}$.

$$[H]_p = 18.6\text{m}$$



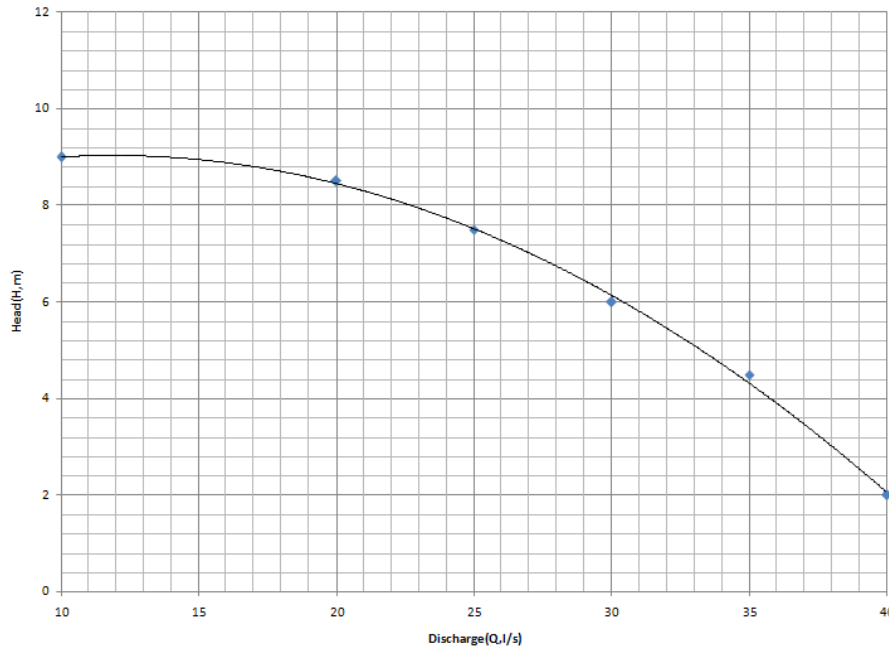
$$\frac{D_m}{D_p} = 0.2$$

$$[H]_m = 6.2\text{m}$$

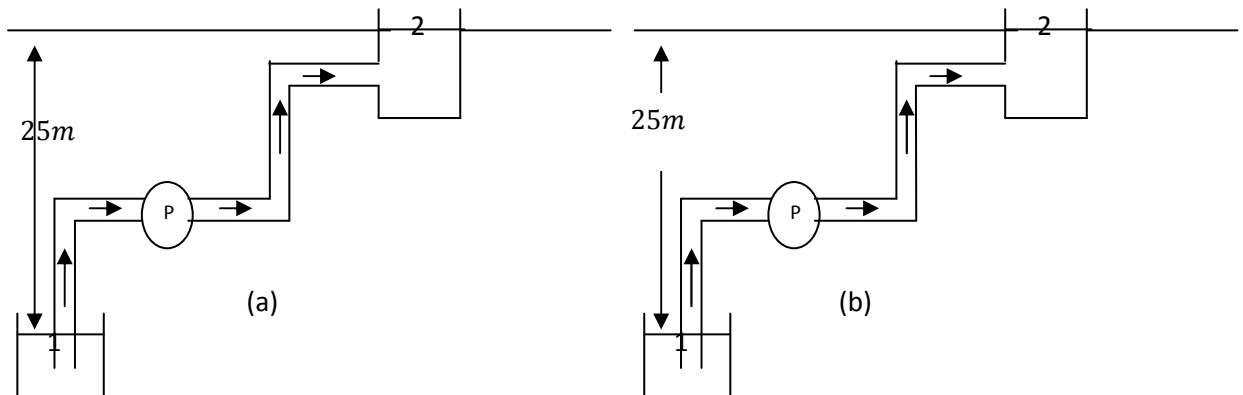
$$[N]_m = 850 \text{ r.p.m.}$$

$$[P]_m = 22000\text{W}$$

- 2) The characteristic curve of a model is given in the below figure. If $\frac{D_m}{D_p} = 0.2$ and $\frac{N_m}{N_p} = 4$, establish the characteristic curve of the prototype.



- 3) As shown below, the water is pumped from tank A to tank B. In (a), the head-discharge relationship of the pump is given by $H_p = 83 - 7550Q^2$. In (b), the head-discharge relationship of the pump is given by $H_p = 63 - 7350Q^2$. Neglecting Major and minor losses in the pipes, determine the ratio of $\frac{N_a}{N_b}$. If the diameter of the pipe is 0.25m, check whether the velocities in the pipes adhere to the rule of thumb on velocity if velocities above 0.75m/s are often specified to prevent sedimentation in a water network.



3.6 Pump Characteristics

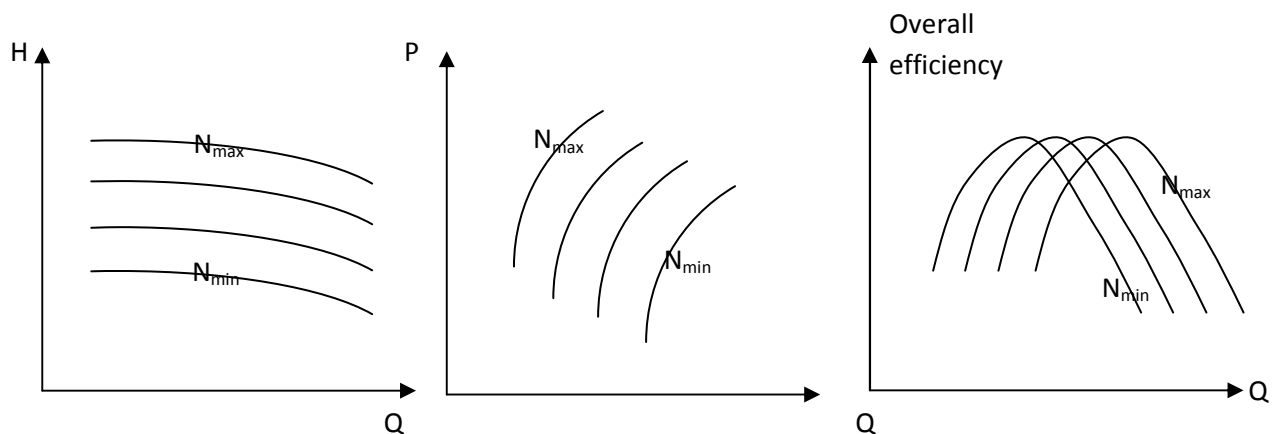
The characteristics curves for pumps are established to predict the behavior of pumps under different conditions that are influenced by variables such as speeds (N), discharges (Q), and heads (H). To establish the characteristics curves, several runs or tests are performed by varying N , Q , H , etc. The typical procedure in establishing the characteristics curves is presented in the below figure. As shown in the below figure, as a first step, the tested are conducted for a given value of N (i.e., N_{\min}) but varying the value of Q (i.e., $Q \leq Q_{\max}$ but $Q \geq Q_{\min}$). Then, as a second step, the tested are conducted for a different value of N (i.e., $N < N_{\max}$ but $N > N_{\min}$) but varying the value of Q (i.e., $Q \leq Q_{\max}$ but $Q \geq Q_{\min}$). In the second step, the tested are conducted for the same set of Q s that were used in the first step. Then, in the third step, the procedure in the second step is repeated many times considering different value of N (i.e., $N < N_{\max}$ but $N > N_{\min}$). Then, as a fourth step, the tested are conducted for given value of N (i.e., N_{\max}) but varying the value of Q (i.e., $Q \leq Q_{\max}$ but $Q \geq Q_{\min}$).

Step#1		
Required many runs		
Run the pump under the following conditions: 1) Speed= N_{\min} (minimum speed) 2) Discharge= Q_{\min} Measure the value of H and P , and determine the overall efficiency	Run the pump under the following conditions: 1) Speed= N_{\min} (minimum speed) 2) Discharge= $Q < Q_{\max}$ but $Q > Q_{\min}$ Measure the value of H and P , and determine the overall efficiency	Run the pump under the following conditions: 1) Speed= N_{\min} (minimum speed) 2) Discharge= Q_{\max} Measure the value of H and P , and determine the overall efficiency

Step#2 and Step#3		
Required many runs		
Run the pump under the following conditions: 1) Speed= $N < N_{\max}$ but $N > N_{\min}$ 2) Discharge= Q_{\min} Measure the value of H and P , and determine the overall efficiency	Run the pump under the following conditions: 1) Speed= $N < N_{\max}$ but $N > N_{\min}$ 2) Discharge= $Q < Q_{\max}$ but $Q > Q_{\min}$ Measure the value of H and P , and determine the overall efficiency	Run the pump under the following conditions: 1) Speed= $N < N_{\max}$ but $N > N_{\min}$ 2) Discharge= Q_{\max} Measure the value of H and P , and determine the overall efficiency

Required many runs		
Run the pump under the following conditions: 1) Speed= N_{\max} (maximum speed) 2) Discharge= Q_{\min} Measure the value of H and P, and determine the overall efficiency	Run the pump under the following conditions: 1) Speed= N_{\max} (maximum speed) 2) Discharge= $Q < Q_{\max}$ but $Q > Q_{\min}$ Measure the value of H and P, and determine the overall efficiency	Run the pump under the following conditions: 1) Speed= N_{\max} (maximum speed) 2) Discharge= Q_{\max} Measure the value of H and P, and determine the overall efficiency

The outcome of the tests performed as per the above figure will help to graph a family of curves that are known as **characteristics curves of a pump**.

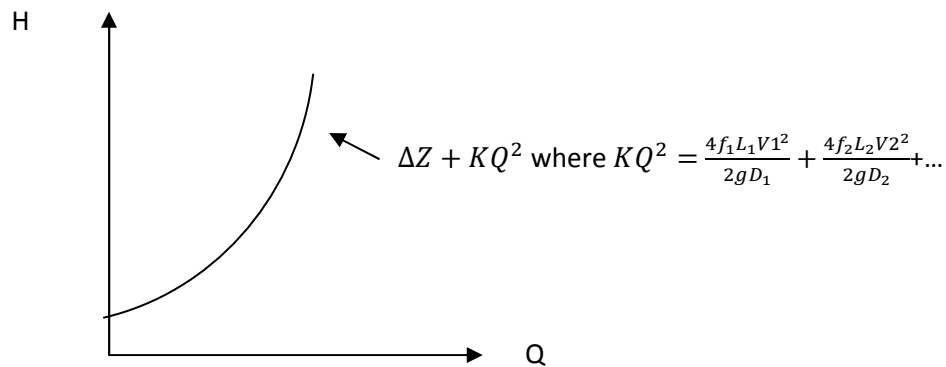


Learning-by-doing

- 1) To establish the characteristics curves, it is required to perform the tests for $Q=[Q_0, Q_1, Q_2, Q_3, Q_4, Q_5]$ and $N=[N_0, N_1, N_2, N_3, N_4]$. Determine the number of tests that need to be performed.

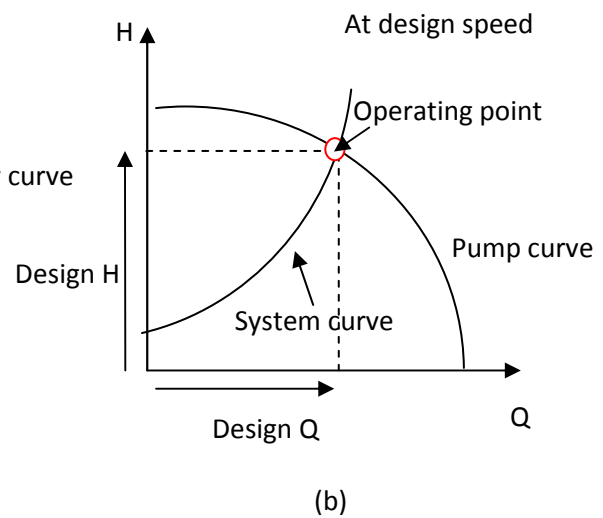
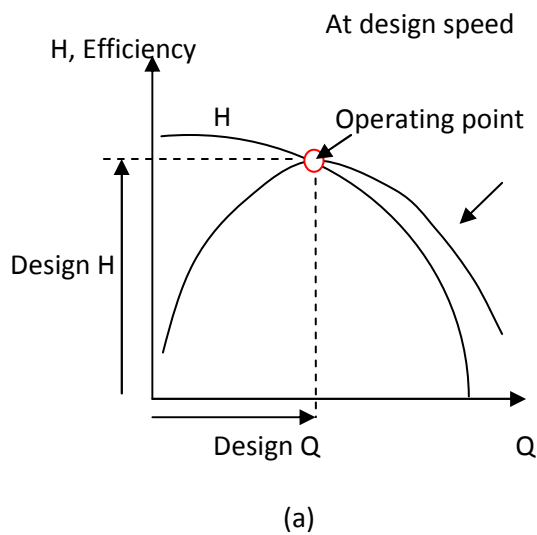
3.7 System Characteristics

The system characteristics curve considers the difference in elevation and the total head loss (i.e., $(Z_2 - Z_1) + \frac{4f_1 L_1 V_1^2}{2gD_1} + \frac{4f_2 L_2 V_2^2}{2gD_2}$). Therefore, the system characteristics curve provides a means of determining the head loss for a given value of Q. If additional losses are introduced by means of valves, a new system curve should be developed as the value of K is changed.



3.8 Operating Condition

For a centrifugal pump to achieve the optimum performance, it has to operate at the design speed. At the optimum performance (i.e., when the efficiency is maximum), the design discharge and the head are obtained from the curve as shown in (a). The operating point could also be obtained using the system curve as shown in (b).



3.9 Pumps in Series

To produce high head than what is possible with a single pump, pumps can be arranged in series by mounting more than one impeller on the same shaft. When pumps are

connected in series, the discharge remains constant. The total head (H_{total}) supplied by “m” pumps that are in series is given by $H_{\text{total}}=mH$ if all the pumps are identical and the head supplied by a single pump is H in meters.

Learning-by-doing

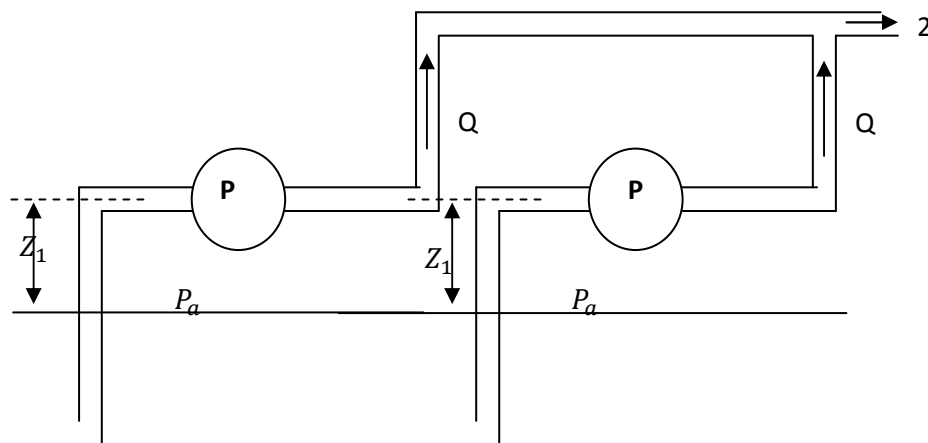
- 1) Water is required to be pumped out of a well under a head of 75m. The characteristics of an available pump are the following:
 - a) Design speed of 850 r.p.m.
 - b) Specific speed of 32
 - c) Rated capacity of $0.14\text{m}^3/\text{s}$

Determine the number of pumps required to be employed in series

- 2) In multi-stage pump, 4 identical pumps are connected in series. The total head supplied by the multi-stage pump is 52m. Determine the specific speed of a single pump if the rate capacity and the design speed are $0.14\text{m}^3/\text{s}$ and 850 r.p.m., respectively.

3.10 Pumps in Parallel

Pumps are connected in parallel to produce large quantity of flow. When pumps are in parallel, each of the pumps work separately as shown in the below figure. The total discharge (Q_{total}) supplied by “m” pumps that are in parallel is given by $Q_{\text{total}} = mQ$ if all the pumps are identical and Q is the discharge capacity of a pump.

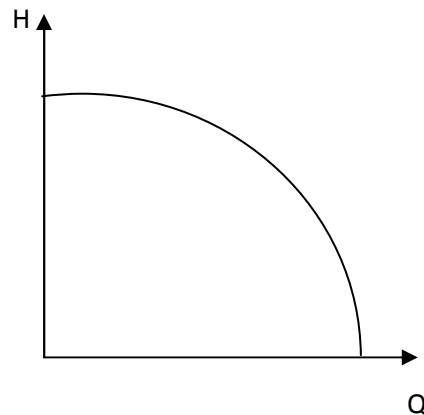


3.11 Pump Selection

Pumps are selected from available types such as slow, medium, and high speed radial flow. These types are defined based on specific speed. The specific speed is a function of discharge, head, and speed. Therefore, the pump selection is mainly governed by discharge, head, and speed. To produce high head than what is possible with a single pump, multi-stage pumps are used.

Learning-by-doing

- 1) The below figure shows the H-Q relationship of a pump. Establish the combined characteristics for two identical pumps operating in parallel and in series.



- 2) To meet the demand on electricity, a businessman has decided to establish an organization to supply electricity to national grid. As the businessman is not well versed in the subject, he has approached an engineering student to gather some information about the working principles of hydraulic machineries specifically on centrifugal pumps. Since you are specialized in this subject, fill in the blanks in the below response.

Dear Kaklo Papuosalai:

I, Kasiko Moto Mithaya, am one of the staff attached to the Institute of Water Resources at Casino Institute of Technology. The director of my institute has requested me to help you on your request.

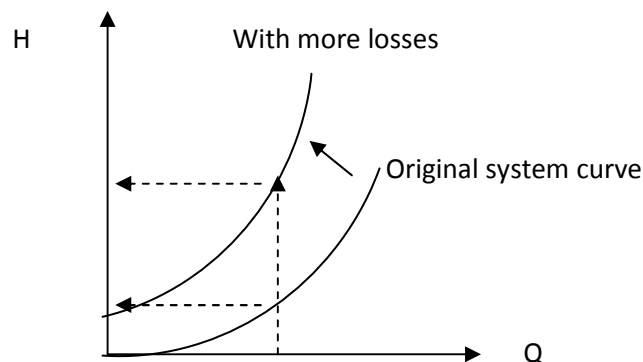
As you may know that a pump is one of the hydraulic machineries that sources energy to a flowing fluid by converting the energy. The classification of pumps could be based on many factors such as direction of flow, working head, and transfer of energy. Based on, the pumps are

classified into two types: positive displacement and The positive displacement pumps are also known as pumps. A centrifugal pump, which can be used for viscous and non-viscous liquids, is a pump with energy transfer occurs when the flow is in its path. Centrifugal pump is one of the pumps that can be operated at very high speeds without the danger of cavitation. In the operation of a centrifugal pump, the impeller is rotated by an electric motor. The rotation of impeller causes the vanes to impel the liquid. The liquid that is impelled out by the vanes comes to the out at the outlet. A centrifugal pump has to supply head against losses of head. The losses of head can be in many forms such as losses in the pump, and losses in suction and pipes. The losses in the pump are generally due to eddies, leakages, and bearing friction. The losses due to eddies and friction may arise at the impeller as well as at the vanes of a centrifugal pump. The losses due to leakage of liquid, also known as leakage losses, are due to of liquid that happens through the clearances between impeller and casing of a centrifugal pump. The mechanical losses are due to friction. The losses in suction and delivery pipes can be categorized into major and minor losses. The losses are governed by Darcy-Weisbach equation. Oftentimes, in calculating head losses, head losses are neglected as they are The efficiency(η) of a centrifugal pump is defined by $\eta = \dots = \eta_1 * \eta_2 * \eta_3$. In η , η_1 accounts for the loss of head within the pump. This loss of head could be, among others, due to eddies and friction within the pump. η_2 that accounts for the loss of liquid through the clearances between impeller and casing of a centrifugal pump, is known as the efficiency. η_3 , which accounts for the loss of power due to bearing friction, is known as the efficiency.

Please do not hesitate to contact me if you need further clarification on working principles of hydraulic machineries specifically on centrifugal pumps.

Regards,
Kasiko Moto Mithaya

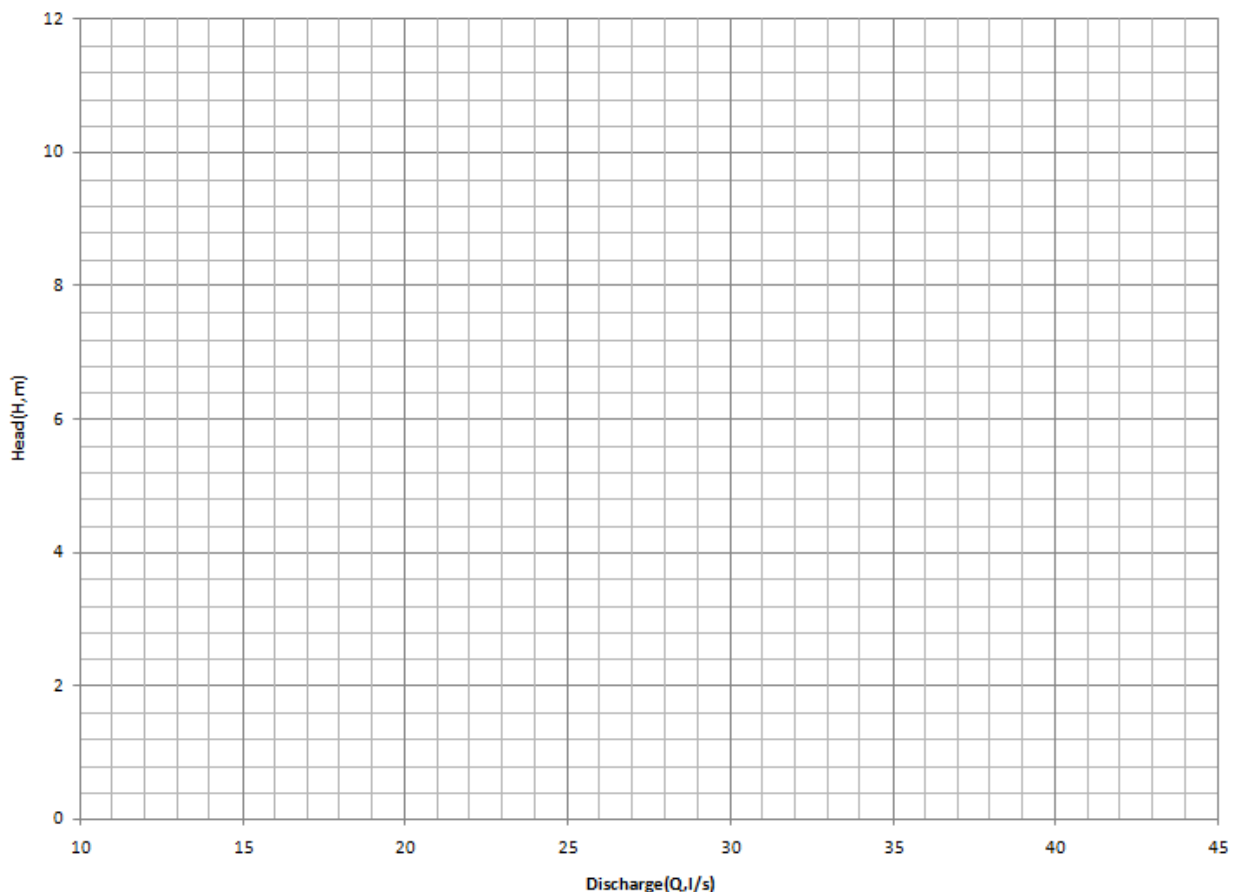
- 3) The below given figure shows the original system curve and the system curve after some additional friction losses due to valve closures. Explain briefly on the design discharge and head.



- 4) The characteristics curve of a pump is given in the below table. This pump has been used to raise water between two tanks. The elevation difference between the tank water surfaces is 3.6m. The total length of the pipe system is 600m. The total length of the pipe includes both the suction and the delivery pipes. The diameter of the pipe is 0.12m. If the friction factor (λ) of the pipe system is 0.002, establish the system curve and determine the design head and the design discharge.

Q(l/s)	10	20	25	30	35	40
H(m)	9	8.5	7.5	6	4.5	2

Ans.



- 5) To meet the demand on electricity, a businessman has decided to establish an organization to supply electricity to national grid. As the businessman is not well versed in the subject, he has approached an engineering student to gather some information about the working principles of hydraulic machineries specifically on Pelton wheel. Since you are specialized in this subject, fill in the blanks in the below response.

Dear Kaklo Papuosalai:

I, Kasiko Moto Mithaya, am one of the staff attached to the Institute of Water Resources at Casino Institute of Technology. The director of my institute has requested me to help you on your request.

As you may know that a turbine is a machine that uses the energy carried by the flowing liquid and converts it into energy. The mechanical energy is used to run an electric generator that is coupled to the shaft of the turbine. Turbines can be classified based on many ways. Based on action of fluid on the moving blades, the turbines are classified into two types. They are and reaction turbine. In an impulse turbine that requires high and small quantity of, before the water jet hits the of the runner to extract the water energy and convert it to useful work, all the available potential energy is converted into energy. Pelton wheel, which is also known as Pelton turbine, is an turbine. Efficiency(η) of a Pelton wheel is defined by $\eta = \dots = \text{mechanical efficiency} * \dots * \text{volumetric efficiency}$. The reason for the existence of efficiency is that the power that is developed by the runner is partially lost due to bearing friction. The power that is lost due to bearing friction is given by The reason for the existence of efficiency is that the power that is supplied by the water jet at the turbine is not fully developed by the runner. Therefore, the loss in this transition is $(H Q_0 \rho g - H_0 Q_0 \rho g)$. The reason for the existence of efficiency is that the volume of water actually striking the runner is than the volume of the water that is supplied by the jet.

Please do not hesitate to contact me if you need further clarification on working principles of hydraulic machineries specifically on Pelton wheel.

Regards,
Kasiko Moto Mithaya

- 6) To drive a generator to develop 1.3MW, Vallibel Power Erathna PLC, the largest public quoted mini hydropower company in Sri Lanka, has decided to use a turbine. The turbine is sourced by a nozzle that is running under a head of 750m. If the efficiency of the generator and the efficiency of the turbine are 0.97 and 0.85, respectively, determine the nozzle diameter. You are given that the coefficient of velocity for the nozzle is 0.96.
- 7) Below you are given an e-mail response on cavitation and NPSH. It has been learnt that the content of the e-mail has some incorrect information. Since you are specialized in this subject, re-write the given e-mail response.

Dear Kaklo Papuosalai:

I, Kasiko Moto Mithaya, am one of the staff attached to the Institute of Water Resources at Casino Institute of Technology. The director of my institute has requested me to help you on your request.

The cavitation, which is generally caused due to high runner speed and high density of the liquid, is a phenomenon that happens when the delivery pressure goes above the saturated pressure of the liquid. The consequences of cavitation are the drop in efficiency of a pump and noise and dehydration. Thoma's normal stress (σ) is compared with critical normal stress (σ_c) to determine the occurrence of cavitation in a pump. The σ is obtained from published tables or empirical relationships. The σ_c depends on NSPH. Technically, NSPH is defined as $NSPH = \frac{V_1}{\rho g} - \frac{P_v}{\rho g}$ where V_1 is the velocity at pump inlet and P_v is the atmospheric pressure of the liquid at a particular elevation.

Please do not hesitate to contact me if you need further clarification on cavitation and NSPH.

Regards,
Kasiko Moto Mithaya

- 8) Below you are given an e-mail response on pump selection and arrangement. Fill in the blanks in the given e-mail response.

Dear Kaklo Papuosalai:

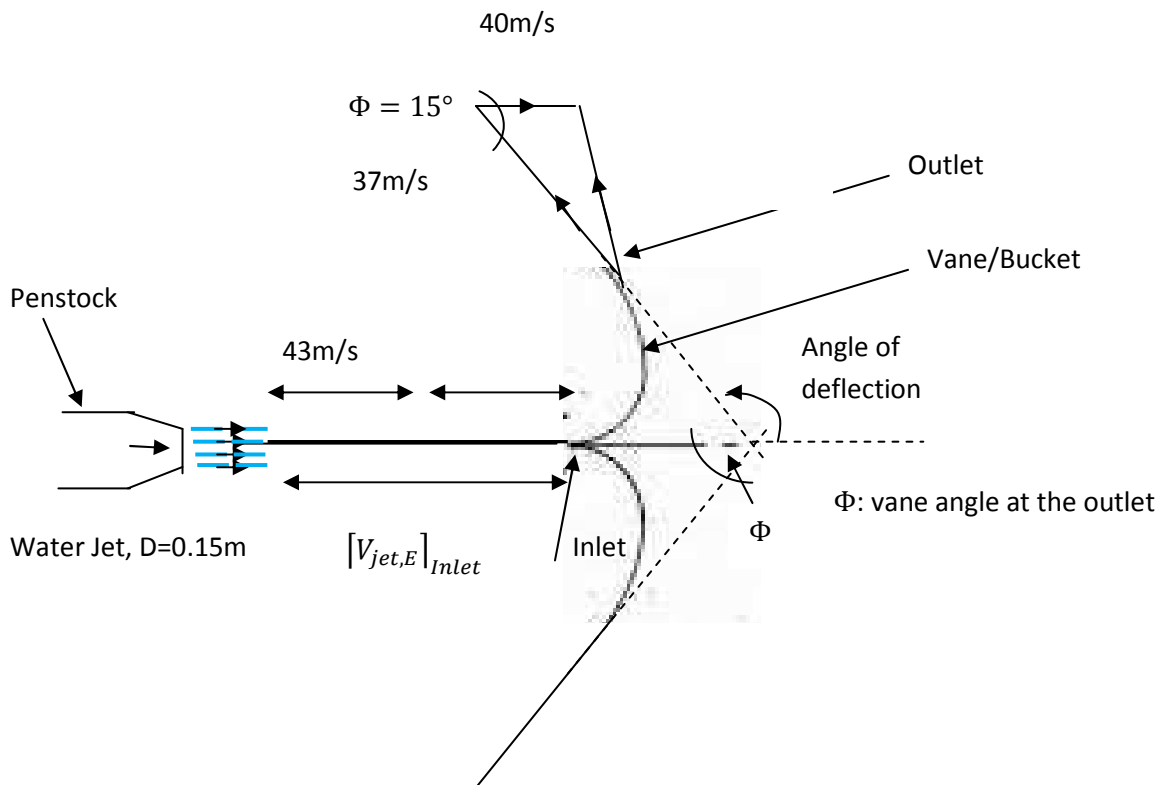
I, Kasiko Moto Mithaya, am one of the staff attached to the Institute of Water Resources at Casino Institute of Technology. The director of my institute has requested me to help you on your request.

In the industry, the specific speed (N_s) is used for the selection of pumps. The specific speed of a pump is defined as the of a geometrically similar pump which would deliver a unit quantity ($1\text{m}^3/\text{s}$) when working under a unit head (1m). Mathematically, the specific speed of a centrifugal pump is defined by $N_s = \dots\dots\dots$ where N is the of the pump, H_p is under which pump operates, and Q is the discharge. Since the specific speed is a function of discharge, head, and speed, the pump selection is mainly governed by discharge, head, and speed. To produce high and high flow than what is possible with a single pump, pumps are used. Pumps are connected in to produce large quantity of flow. When pumps are in parallel, each of the pumps work separately. The total discharge (Q_{total}) supplied by "3" pumps that are in parallel is given by if all the pumps are identical and 10l/s is the discharge capacity of a pump. To produce high than what is possible with a single pump, pumps can be arranged in series by mounting more than one impeller on the same shaft. When pumps are connected in series, the discharge remains The total head (H_{total}) supplied by 3 pumps that are in series is given by if all the pumps are identical and the head supplied by a single pump is 10m.

Please do not hesitate to contact me if you need further clarification on pump selection and arrangement.

Regards,
Kasiko Moto Mithaya

- 9) You are given an e-mail response based on the below diagram which shows the fluid motion of a turbine. Fill in the blanks in the given e-mail response.



Dear Kaklo Papuosalai:

I, Kasiko Moto Mithaya, am one of the staff attached to the Institute of Water Resources at Casino Institute of Technology. The director of my institute has requested me to help you on your request.

The velocity of whirl is defined as the component ofin the direction of motion of vane. Considering the above diagram and the definition of velocity of whirl, the velocity of whirl at the outlet ($[V_w]_{\text{Outlet}}$) is given by $[V_w]_{\text{Outlet}} = \dots\dots\dots$. Similarly, the velocity of whirl at the inlet ($[V_w]_{\text{inlet}}$) is given by $[V_w]_{\text{inlet}} = \dots\dots\dots$. On the other hand, the velocity of flow is defined as the component of perpendicular to the direction of motion of vane. Considering the above diagram and the definition of velocity of flow, the velocity of flow at the outlet ($[V_f]_{\text{Outlet}}$) is given by $[V_f]_{\text{Outlet}} = \dots\dots\dots$. Similarly, the velocity of flow at the inlet ($[V_f]_{\text{inlet}}$) is given by Since the diameter of the water jet is 0.15m, the area of

the water jet isTherefore, the force exerted by the water jet in the direction of motion and the work done by the water jet on runner per second are.....and, respectively.

Please do not hesitate to contact me if you need further clarification on the diagram which shows the fluid motion of a turbine.

Regards,
Kasiko Moto Mithaya

- 10) The characteristic curve of a model is given in the below figure. To solve a particular problem, it is required to operate two identical pumps in series. If $\frac{D_m}{D_p} = 0.2$ and $\frac{N_m}{N_p} = 4$, establish the characteristic curve of the prototype that can be used to solve the same problem.

