

## CHAPTER TWO SOLUTIONS

1.      (a)  $12\ \mu\text{s}$       (d) 3.5 Gbits      (g) 39 pA  
         (b) 750 mJ      (e) 6.5 nm      (h)  $49\ \text{k}\Omega$   
         (c)  $1.13\ \text{k}\Omega$       (f) 13.56 MHz      (i) 11.73 pA

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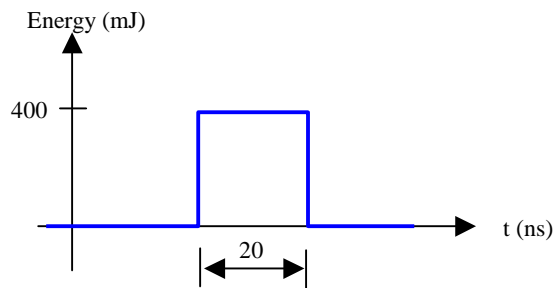
2.      (a) 1 MW      (e) 33  $\mu$ J      (i) 32 mm  
         (b) 12.35 mm      (f) 5.33 nW  
         (c) 47. kW      (g) 1 ns  
         (d) 5.46 mA      (h) 5.555 MW

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3. Motor power = 175 Hp
- (a) With 100% efficient mechanical to electrical power conversion,  
 $(175 \text{ Hp})[1 \text{ W} / (1/745.7 \text{ Hp})] = 130.5 \text{ kW}$
- (b) Running for 3 hours,  
 $\text{Energy} = (130.5 \times 10^3 \text{ W})(3 \text{ hr})(60 \text{ min/hr})(60 \text{ s/min}) = 1.409 \text{ GJ}$
- (c) A single battery has 430 kW-hr capacity. We require  
 $(130.5 \text{ kW})(3 \text{ hr}) = 391.5 \text{ kW-hr}$  therefore one battery is sufficient.

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4. The 400-mJ pulse lasts 20 ns.  
(a) To compute the peak power, we assume the pulse shape is square:



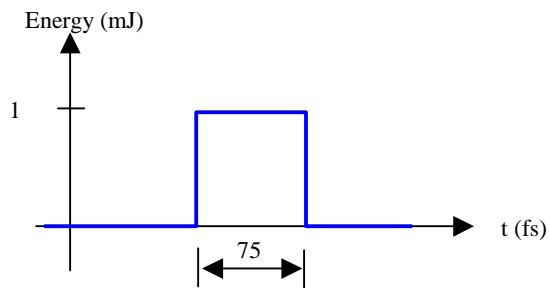
Then  $P = 400 \times 10^{-3} / 20 \times 10^{-9} = 20 \text{ MW.}$

- (b) At 20 pulses per second, the average power is

$$P_{\text{avg}} = (20 \text{ pulses})(400 \text{ mJ/pulse}) / (1 \text{ s}) = 8 \text{ W.}$$

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5. The 1-mJ pulse lasts 75 fs.  
(c) To compute the peak power, we assume the pulse shape is square:



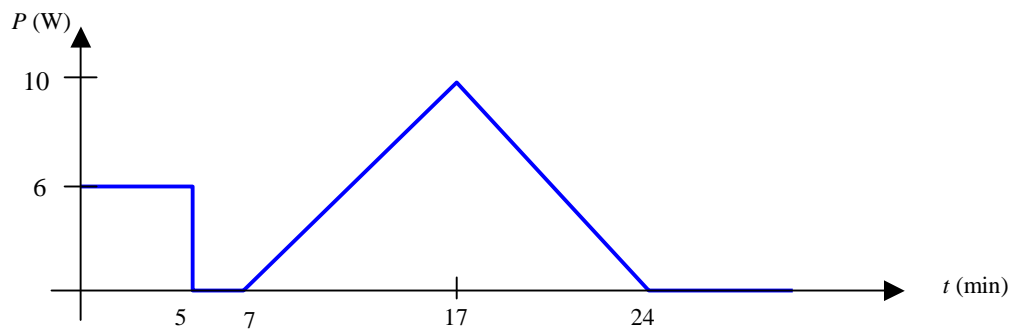
Then  $P = 1 \times 10^{-3} / 75 \times 10^{-15} = 13.33 \text{ GW}.$

- (d) At 100 pulses per second, the average power is

$$P_{\text{avg}} = (100 \text{ pulses})(1 \text{ mJ/pulse})/(1 \text{ s}) = 100 \text{ mW}.$$

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6. The power drawn from the battery is (not quite drawn to scale):



- (a) Total energy (in J) expended is

$$[6(5) + 0(2) + 0.5(10)(10) + 0.5(10)(7)]60 = 6.9 \text{ kJ.}$$

- (b) The average power in Btu/hr is

$$(6900 \text{ J}/24 \text{ min})(60 \text{ min}/1 \text{ hr})(1 \text{ Btu}/1055 \text{ J}) = 16.35 \text{ Btu/hr.}$$

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7. Total charge  $q = 18t^2 - 2t^4$  C.

(a)  $q(2 \text{ s}) = 40 \text{ C.}$

(b) To find the maximum charge within  $0 \leq t \leq 3 \text{ s}$ , we need to take the first and second derivatives:

$$\begin{aligned} dq/dt &= 36t - 8t^3 = 0, \text{ leading to roots at } 0, \pm 2.121 \text{ s} \\ d^2q/dt^2 &= 36 - 24t^2 \end{aligned}$$

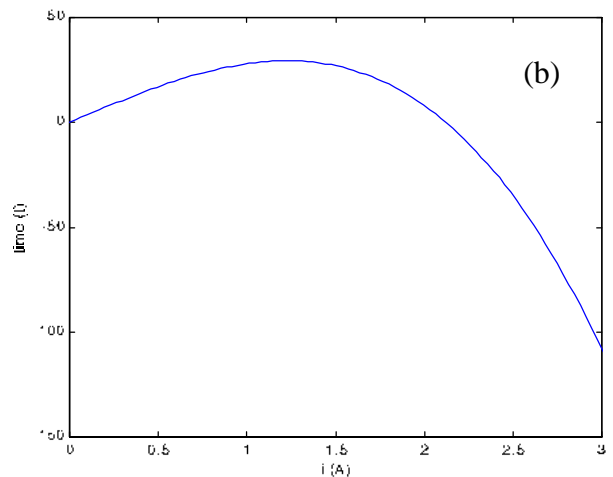
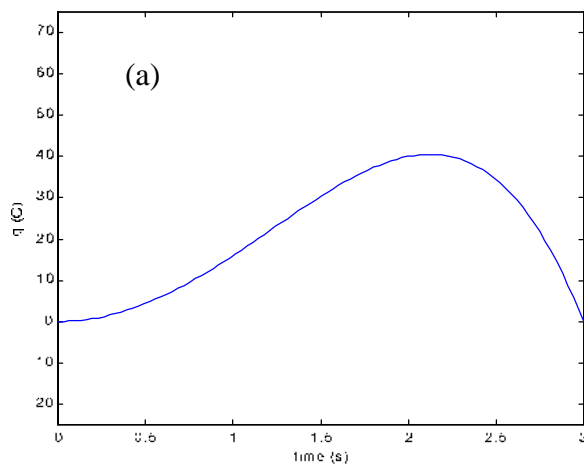
substituting  $t = 2.121 \text{ s}$  into the expression for  $d^2q/dt^2$ , we obtain a value of  $-14.9$ , so that this root represents a maximum.

Thus, we find a maximum charge  $q = 40.5 \text{ C}$  at  $t = 2.121 \text{ s}$ .

(c) The rate of charge accumulation at  $t = 0.8 \text{ s}$  is

$$dq/dt|_{t=0.8} = 36(0.8) - 8(0.8)^3 = 24.7 \text{ C/s.}$$

(d) See Fig. (a) and (b).



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8. Referring to Fig. 2.6c,

$$i_1(t) = \begin{cases} -2 + 3e^{-5t} \text{ A}, & t < 0 \\ -2 + 3e^{3t} \text{ A}, & t > 0 \end{cases}$$

Thus,

(a)  $i_1(-0.2) = 6.155 \text{ A}$

(b)  $i_1(0.2) = 3.466 \text{ A}$

(c) To determine the instants at which  $i_1 = 0$ , we must consider  $t < 0$  and  $t > 0$  separately:

for  $t < 0$ ,  $-2 + 3e^{-5t} = 0$  leads to  $t = -0.2 \ln(2/3) = +2.027 \text{ s}$  (impossible)

for  $t > 0$ ,  $-2 + 3e^{3t} = 0$  leads to  $t = (1/3) \ln(2/3) = -0.135 \text{ s}$  (impossible)

Therefore, the current is *never* negative.

(d) The total charge passed left to right in the interval  $0.08 < t < 0.1 \text{ s}$  is

$$\begin{aligned} q(t) &= \int_{-0.08}^{0.1} i_1(t) dt \\ &= \int_{-0.08}^0 [-2 + 3e^{-5t}] dt + \int_0^{0.1} [-2 + 3e^{3t}] dt \\ &= -2 + 3e^{-5t} \Big|_{-0.08}^0 + -2 + 3e^{3t} \Big|_0^{0.1} \\ &= 0.1351 + 0.1499 \\ &= 285 \text{ mC} \end{aligned}$$



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9. Referring to Fig. 2.28,

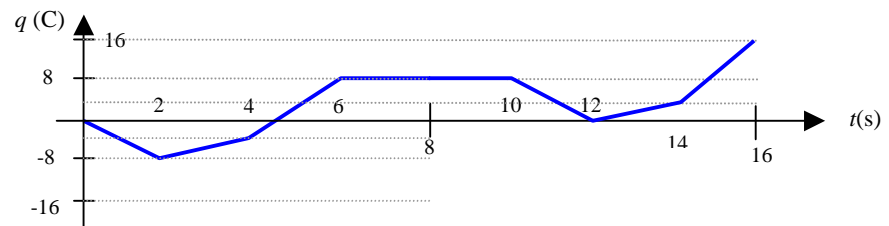
(a) The average current over one period (10 s) is

$$i_{\text{avg}} = [-4(2) + 2(2) + 6(2) + 0(4)]/10 = \boxed{800 \text{ mA}}$$

(b) The total charge transferred over the interval  $1 < t < 12$  s is

$$q_{\text{total}} = \int_1^{12} i(t) dt = -4(2) + 2(2) + 6(2) + 0(4) - 4(2) = \boxed{0 \text{ C}}$$

(d) See Fig. below



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10. (a)  $P_{\text{abs}} = (+3.2 \text{ V})(-2 \text{ mA}) = -6.4 \text{ mW}$  (or  $+6.4 \text{ mW}$  *supplied*)

(b)  $P_{\text{abs}} = (+6 \text{ V})(-20 \text{ A}) = -120 \text{ W}$  (or  $+120 \text{ W}$  *supplied*)

(d)  $P_{\text{abs}} = (+6 \text{ V})(2 i_x) = (+6 \text{ V})[(2)(5 \text{ A})] = +60 \text{ W}$

(e)  $P_{\text{abs}} = (4 \sin 1000t \text{ V})(-8 \cos 1000t \text{ mA}) \Big|_{t=2 \text{ ms}} = +12.11 \text{ W}$

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11.  $i = 3te^{-100t}$  mA and  $v = [6 - 600t] e^{-100t}$  mV

(a) The power absorbed at  $t = 5$  ms is

$$P_{\text{abs}} = [(6 - 600t)e^{-100t} \cdot 3te^{-100t}]_{t=5\text{ms}} \mu\text{W}$$
$$= 0.01655 \mu\text{W} = \boxed{16.55 \text{ nW}}$$

(b) The energy delivered over the interval  $0 < t < \infty$  is

$$\int_0^{\infty} P_{\text{abs}} dt = \int_0^{\infty} 3t(6 - 600t)e^{-200t} dt \quad \mu\text{J}$$

Making use of the relationship

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}} \quad \text{where } n \text{ is a positive integer and } a > 0,$$

we find the energy delivered to be

$$= 18/(200)^2 - 1800/(200)^3$$

$$\boxed{= 0}$$

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$$12. \quad (a) \quad P_{\text{abs}} = (40i)(3e^{-100t}) \Big|_{t=8 \text{ ms}} = 360 \left[ e^{-100t} \right]_{t=8 \text{ ms}} = \boxed{72.68 \text{ W}}$$

$$(b) \quad P_{\text{abs}} = \left( 0.2 \frac{di}{dt} \right) i \Big|_{t=8 \text{ ms}} = -180 \left[ e^{-100t} \right]_{t=8 \text{ ms}} = \boxed{-36.34 \text{ W}}$$

$$\begin{aligned} (c) \quad P_{\text{abs}} &= \left( 30 \int_0^t i dt + 20 \right) (3e^{-100t}) \Big|_{t=8 \text{ ms}} \\ &= \left( 90e^{-100t} \int_0^t 3e^{-100t'} dt' + 60e^{-100t} \right) \Big|_{t=8 \text{ ms}} = \boxed{27.63 \text{ W}} \end{aligned}$$

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13. (a) The short-circuit current is the value of the current at  $V = 0$ .

Reading from the graph, this corresponds to approximately 3.0 A.

- (b) The open-circuit voltage is the value of the voltage at  $I = 0$ .

Reading from the graph, this corresponds to roughly 0.4875 V, estimating the curve as hitting the x-axis 1 mm behind the 0.5 V mark.

- (c) We see that the maximum current corresponds to zero voltage, and likewise, the maximum voltage occurs at zero current. The maximum power point, therefore, occurs somewhere between these two points. By trial and error,

$P_{\max}$  is roughly  $(375 \text{ mV})(2.5 \text{ A}) = 938 \text{ mW}$ , or just under 1 W.

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14. Note that in the table below, only the  $-4\text{-A}$  source and the  $-3\text{-A}$  source are actually “absorbing” power; the remaining sources are supplying power to the circuit.

Source	Absorbed Power	Absorbed Power
2-V source	$(2\text{ V})(-2\text{ A})$	- 4 W
8-V source	$(8\text{ V})(-2\text{ A})$	- 16 W
-4-A source	$(10\text{ V})[-(-4\text{ A})]$	40 W
10-V source	$(10\text{ V})(-5\text{ A})$	- 50 W
-3-A source	$(10\text{ V})[-(-3\text{ A})]$	30 W

The 5 power quantities sum to  $-4 - 16 + 40 - 50 + 30 = 0$ , as demanded from conservation of energy.

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15. We are told that  $V_x = 1\text{ V}$ , and from Fig. 2.33 we see that the current flowing through the dependent source (and hence through each element of the circuit) is  $5V_x = 5\text{ A}$ . We will compute *absorbed* power by using the current flowing *into* the positive reference terminal of the appropriate voltage (passive sign convention), and we will compute *supplied* power by using the current flowing *out of* the positive reference terminal of the appropriate voltage.

(a) The power absorbed by element “A” =  $(9\text{ V})(5\text{ A}) = 45\text{ W}$

(b) The power supplied by the 1-V source =  $(1\text{ V})(5\text{ A}) = 5\text{ W}$ , and  
the power supplied by the dependent source =  $(8\text{ V})(5\text{ A}) = 40\text{ W}$

(c) The sum of the supplied power =  $5 + 40 = 45\text{ W}$   
The sum of the absorbed power is  $45\text{ W}$ , so

yes, the sum of the power supplied = the sum of the power absorbed, as we expect from the principle of conservation of energy.

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16. We are asked to determine the voltage  $v_s$ , which is identical to the voltage labeled  $v_1$ . The only remaining reference to  $v_1$  is in the expression for the current flowing through the dependent source,  $5v_1$ .

This current is equal to  $-i_2$ .

Thus,

$$5 v_1 = -i_2 = -5 \text{ mA}$$

Therefore  $v_1 = -1 \text{ mV}$

and so

$$v_s = v_1 = -1 \text{ mV}$$



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17. The battery delivers an energy of 460.8 W-hr over a period of 8 hrs.

(a) The power delivered to the headlight is therefore  $(460.8 \text{ W-hr})/(8 \text{ hr}) = 57.6 \text{ W}$

(b) The current through the headlight is equal to the power it absorbs from the battery divided by the voltage at which the power is supplied, or

$$I = (57.6 \text{ W})/(12 \text{ V}) = 4.8 \text{ A}$$

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18. The supply voltage is 110 V, and the maximum dissipated power is 500 W. The fuses are specified in terms of current, so we need to determine the maximum current that can flow through the fuse.

$$P = VI \quad \text{therefore } I_{\max} = P_{\max}/V = (500 \text{ W})/(110 \text{ V}) = 4.545 \text{ A}$$

If we choose the 5-A fuse, it will allow up to  $(110 \text{ V})(5 \text{ A}) = 550 \text{ W}$  of power to be delivered to the application (we must assume here that the fuse absorbs zero power, a reasonable assumption in practice). This exceeds the specified maximum power.

If we choose the 4.5-A fuse instead, we will have a maximum current of 4.5 A. This leads to a maximum power of  $(110)(4.5) = 495 \text{ W}$  delivered to the application.

Although 495 W is less than the maximum power allowed, this fuse will provide adequate protection for the application circuitry. If a fault occurs and the application circuitry attempts to draw too much power, 1000 W for example, the fuse will blow, no current will flow, and the application circuitry will be protected. However, if the application circuitry tries to draw its maximum rated power (500 W), the fuse will also blow. In practice, most equipment will not draw its maximum rated power continuously- although to be safe, we typically assume that it will.

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19. (a)  $P_{\text{abs}} = i^2 R = [20e^{-12t}]^2 (1200) \mu\text{W}$

$$= [20e^{-1.2}]^2 (1200) \mu\text{W}$$

$$= 43.54 \text{ mW}$$

(b)  $P_{\text{abs}} = v^2/R = [40 \cos 20t]^2 / 1200 \text{ W}$

$$= [40 \cos 2]^2 / 1200 \text{ W}$$

$$= 230.9 \text{ mW}$$

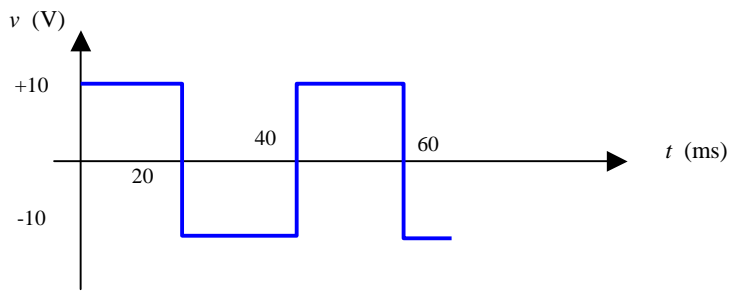
*keep in mind we  
are using radians*

(c)  $P_{\text{abs}} = v i = 8t^{1.5} \text{ W}$

$$= 253.0 \text{ mW}$$

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20. It's probably best to begin this problem by sketching the voltage waveform:



(a)  $v_{\max} = +10 \text{ V}$

(b)  $v_{\text{avg}} = [(+10)(20 \times 10^{-3}) + (-10)(20 \times 10^{-3})] / (40 \times 10^{-3}) = 0$

(c)  $i_{\text{avg}} = v_{\text{avg}} / R = 0$

(d)  $p_{\text{abs}}|_{\max} = \frac{v_{\max}^2}{R} = (10)^2 / 50 = 2 \text{ W}$

(e)  $p_{\text{abs}}|_{\text{avg}} = \frac{1}{40} \left[ \frac{(+10)^2}{R} \cdot 20 + \frac{(-10)^2}{R} \cdot 20 \right] = 2 \text{ W}$

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21. We are given that the conductivity  $\sigma$  of copper is  $5.8 \times 10^7$  S/m.

(a) 50 ft of #18 (18 AWG) copper wire, which has a diameter of 1.024 mm, will have a resistance of  $l/(\sigma A)$  ohms, where  $A$  = the cross-sectional area and  $l = 50$  ft.

Converting the dimensional quantities to meters,

$$l = (50 \text{ ft})(12 \text{ in/ft})(2.54 \text{ cm/in})(1 \text{ m/100 cm}) = 15.24 \text{ m}$$

and

$$r = 0.5(1.024 \text{ mm})(1 \text{ m/1000 mm}) = 5.12 \times 10^{-4} \text{ m}$$

so that

$$A = \pi r^2 = \pi (5.12 \times 10^{-4} \text{ m})^2 = 8.236 \times 10^{-7} \text{ m}^2$$

$$\text{Thus, } R = (15.24 \text{ m}) / [(5.8 \times 10^7)(8.236 \times 10^{-7})] = \boxed{319.0 \text{ m}\Omega}$$

(b) We assume that the conductivity value specified also holds true at  $50^\circ\text{C}$ .

The cross-sectional area of the foil is

$$A = (33 \text{ }\mu\text{m})(500 \text{ }\mu\text{m})(1 \text{ m}/10^6 \text{ }\mu\text{m})(1 \text{ m}/10^6 \text{ }\mu\text{m}) = 1.65 \times 10^{-8} \text{ m}^2$$

So that

$$R = (15 \text{ cm})(1 \text{ m}/100 \text{ cm}) / [(5.8 \times 10^7)(1.65 \times 10^{-8})] = \boxed{156.7 \text{ m}\Omega}$$

A 3-A current flowing through this copper in the direction specified would lead to the dissipation of

$$I^2 R = (3)^2 (156.7 \text{ mW}) = \boxed{1.410 \text{ W}}$$

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22. Since we are informed that the same current must flow through each component, we begin by defining a current  $I$  flowing out of the positive reference terminal of the voltage source.

The power supplied by the voltage source is  $V_s I$ .

The power absorbed by resistor  $R_1$  is  $I^2 R_1$ .

The power absorbed by resistor  $R_2$  is  $I^2 R_2$ .

Since we know that the total power supplied is equal to the total power absorbed, we may write:

$$V_s I = I^2 R_1 + I^2 R_2$$

or

$$V_s = I R_1 + I R_2$$

$$V_s = I (R_1 + R_2)$$

By Ohm's law,

$$I = V_{R_2} / R_2$$

so that

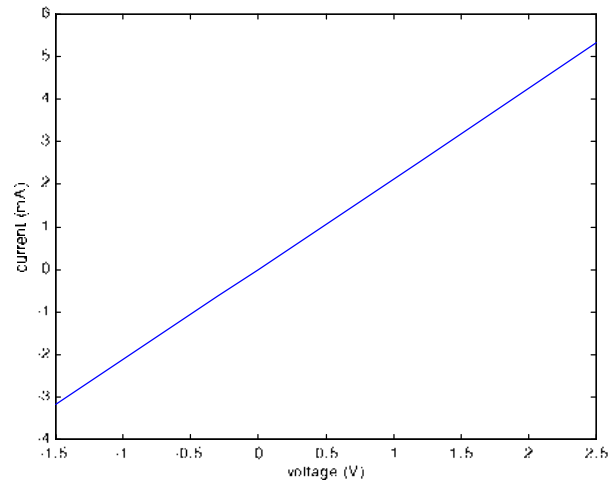
$$V_s = \frac{V_{R_2}}{R_2} (R_1 + R_2)$$

Solving for  $V_{R_2}$  we find

$$V_{R_2} = V_s \frac{R_2}{(R_1 + R_2)} \quad \text{Q.E.D.}$$

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23. (a)

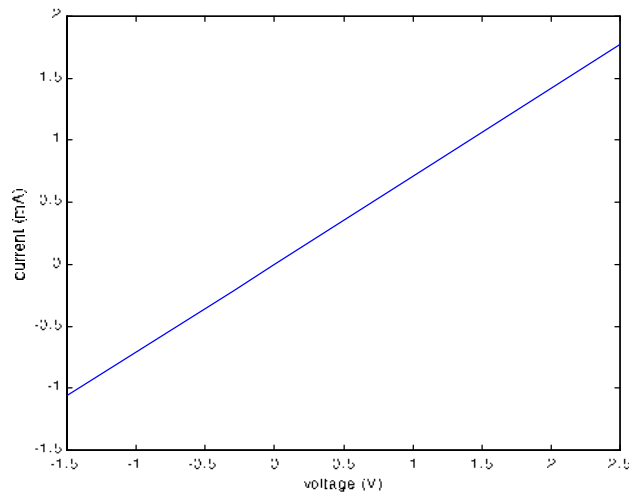


(b) We see from our answer to part (a) that this device has a reasonably linear characteristic (a not unreasonable degree of experimental error is evident in the data). Thus, we choose to estimate the resistance using the two extreme points:

$$R_{\text{eff}} = [(2.5 - (-1.5))/[5.23 - (-3.19)]] \text{ k}\Omega = 475 \Omega$$

Using the last two points instead, we find  $R_{\text{eff}} = 469 \Omega$ , so that we can state with some certainty at least that a reasonable estimate of the resistance is approximately 470  $\Omega$ .

(c)



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24. Top Left Circuit:  $I = (5/10) \text{ mA} = 0.5 \text{ mA}$ , and  $P_{10k} = V^2/10 \text{ mW} = 2.5 \text{ mW}$
- Top Right Circuit:  $I = -(5/10) \text{ mA} = -0.5 \text{ mA}$ , and  $P_{10k} = V^2/10 \text{ mW} = 2.5 \text{ mW}$
- Bottom Left Circuit:  $I = (-5/10) \text{ mA} = -0.5 \text{ mA}$ , and  $P_{10k} = V^2/10 \text{ mW} = 2.5 \text{ mW}$
- Bottom Right Circuit:  $I = -(-5/10) \text{ mA} = 0.5 \text{ mA}$ , and  $P_{10k} = V^2/10 \text{ mW} = 2.5 \text{ mW}$



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25. The voltage  $v_{\text{out}}$  is given by

$$\begin{aligned}v_{\text{out}} &= -10^{-3} v_{\pi} (1000) \\&= -v_{\pi}\end{aligned}$$

Since  $v_{\pi} = v_s = 0.01 \cos 1000t$  V, we find that

$$v_{\text{out}} = -v_{\pi} = \boxed{-0.001 \cos 1000t \text{ V}}$$

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26. 18 AWG wire has a resistance of  $6.39 \, \Omega / 1000 \text{ ft}$ .

Thus, we require  $1000 (53) / 6.39 = 8294 \text{ ft of wire}$ .  
(Or 1.57 miles. Or, 2.53 km).

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27. We need to create a 470- $\Omega$  resistor from 28 AWG wire, knowing that the ambient temperature is 108°F, or 42.22°C.

Referring to Table 2.3, 28 AWG wire is 65.3 m $\Omega$ /ft at 20°C, and using the equation provided we compute

$$R_2/R_1 = (234.5 + T_2)/(234.5 + T_1) = (234.5 + 42.22)/(234.5 + 20) = 1.087$$

We thus find that 28 AWG wire is  $(1.087)(65.3) = 71.0$  m $\Omega$ /ft.

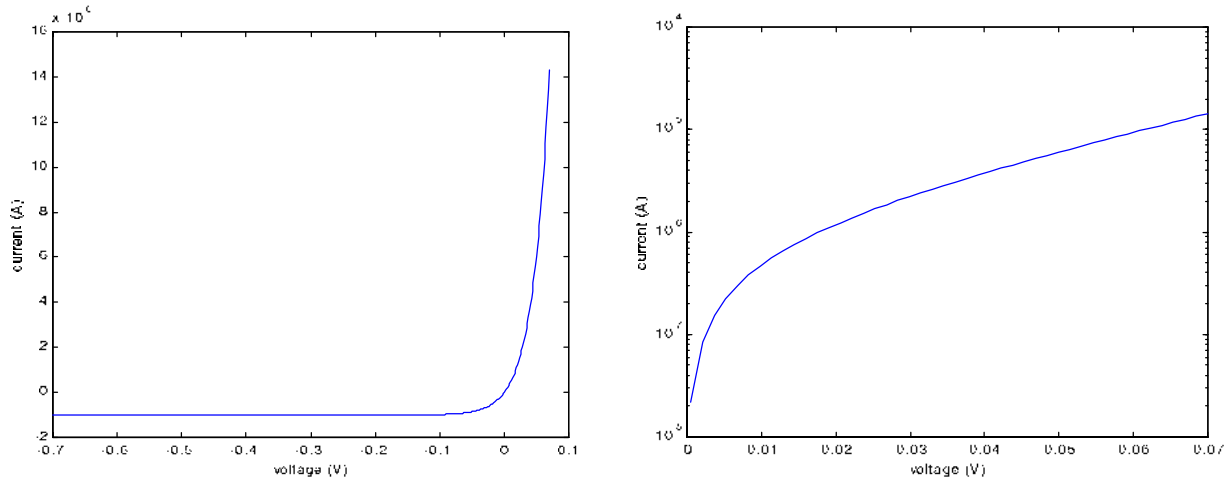
Thus, to repair the transmitter we will need

$$(470 \Omega)/(71.0 \times 10^{-3} \Omega/\text{ft}) = \boxed{6620 \text{ ft}} (1.25 \text{ miles, or } 2.02 \text{ km}).$$

Note: This seems like a lot of wire to be washing up on shore. We may find we don't have enough. In that case, perhaps we should take our cue from Eq. [6], and try to squash a piece of the wire flat so that it has a very small cross-sectional area.....

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28. (a) We need to plot the negative and positive voltage ranges separately, as the positive voltage range is, after all, exponential!



- (b) To determine the resistance of the device at  $V = 550$  mV, we compute the corresponding current:

$$I = 10^{-6} [e^{39(0.55)} - 1] = 2.068 \text{ A}$$

Thus,  $R(0.55 \text{ V}) = 0.55/2.068 = \boxed{266 \text{ m}\Omega}$

- (c)  $R = 1 \text{ }\Omega$  corresponds to  $V = I$ . Thus, we need to solve the transcendental equation

$$I = 10^{-6} [e^{39I} - 1]$$

Using a scientific calculator or the tried-and-true trial and error approach, we find that

$$\boxed{I = 325.5 \text{ mA}}$$

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29. We require a  $10\text{-}\Omega$  resistor, and are told it is for a portable application, implying that size, weight or both would be important to consider when selecting a wire gauge. We have 10,000 ft of each of the gauges listed in Table 2.3 with which to work. Quick inspection of the values listed eliminates 2, 4 and 6 AWG wire as their respective resistances are too low for only 10,000 ft of wire.

Using 12-AWG wire would require  $(10\text{ }\Omega) / (1.59\text{ m}\Omega/\text{ft}) = 6290\text{ ft}$ .

Using 28-AWG wire, the narrowest available, would require

$$(10\text{ }\Omega) / (65.3\text{ m}\Omega/\text{ft}) = 153\text{ ft.}$$

Would the 28-AWG wire weight less? Again referring to Table 2.3, we see that the cross-sectional area of 28-AWG wire is  $0.0804\text{ mm}^2$ , and that of 12-AWG wire is  $3.31\text{ mm}^2$ . The volume of 12-AWG wire required is therefore  $6345900\text{ mm}^3$ , and that of 28-AWG wire required is only  $3750\text{ mm}^3$ .

The best (but not the only) choice for a portable application is clear: 28-AWG wire!

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30. Our target is a 100- $\Omega$  resistor. We see from the plot that at  $N_D = 10^{15} \text{ cm}^{-3}$ ,  $\mu_n \sim 2 \times 10^3 \text{ cm}^2/\text{V-s}$ , yielding a resistivity of 3.121  $\Omega\text{-cm}$ .

At  $N_D = 10^{18} \text{ cm}^{-3}$ ,  $\mu_n \sim 230 \text{ cm}^2/\text{V-s}$ , yielding a resistivity of 0.02714  $\Omega\text{-cm}$ .

Thus, we see that the lower doping level clearly provides material with higher resistivity, requiring less of the available area on the silicon wafer.

Since  $R = \rho L/A$ , where we know  $R = 10 \text{ }\Omega$  and  $\rho = 3.121 \text{ }\Omega\text{-cm}$ , we need only define the resistor geometry to complete the design.

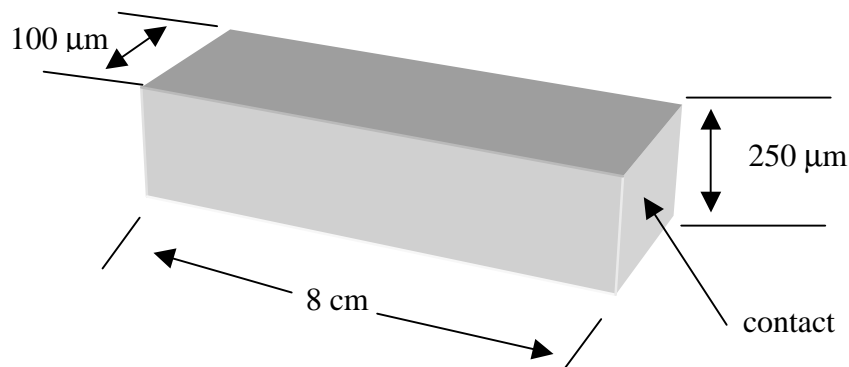
We typically form contacts primarily on the surface of a silicon wafer, so that the wafer thickness would be part of the factor  $A$ ;  $L$  represents the distance between the contacts. Thus, we may write

$$R = 3.121 L / (250 \times 10^{-4} Y)$$

where  $L$  and  $Y$  are dimensions on the surface of the wafer.

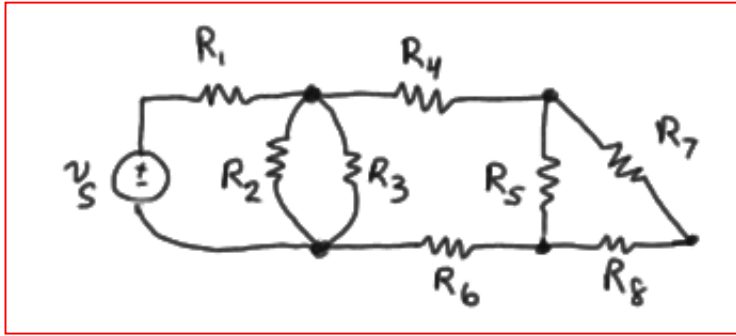
If we make  $Y$  small (*i.e.* a narrow width as viewed from the top of the wafer), then  $L$  can also be small. Seeking a value of 0.080103 then for  $L/Y$ , and choosing  $Y = 100 \text{ }\mu\text{m}$  (a large dimension for silicon devices), we find a contact-to-contact length of  $L = 8 \text{ cm}$ ! While this easily fits onto a 6" diameter wafer, we could probably do a little better. We are also assuming that the resistor is to be cut from the wafer, and the ends made the contacts, as shown below in the figure.

Design summary (one possibility):  $N_D = 10^{15} \text{ cm}^{-3}$   
 $L = 8 \text{ cm}$   
 $Y = 100 \text{ }\mu\text{m}$



## CHAPTER THREE SOLUTIONS

1.



## CHAPTER THREE SOLUTIONS

2. (a) six nodes; (b) nine branches.



## CHAPTER THREE SOLUTIONS

3. (a) Four nodes; (b) five branches; (c) path, yes – loop, no.

## CHAPTER THREE SOLUTIONS

4. (a) Five nodes; (b) seven branches; (c) path, yes – loop, no.

## CHAPTER THREE SOLUTIONS

5. (a) 3 A; (b)  $-3$  A; (c) 0.

## CHAPTER THREE SOLUTIONS

6. By KCL, we may write:

$$5 + i_y + i_z = 3 + i_x$$

---

$$(a) i_x = 2 + i_y + i_z = 2 + 2 + 0 = 4 \text{ A}$$

$$(b) i_y = 3 + i_x - 5 - i_z$$

$$i_y = -2 + 2 - 2 i_y$$

Thus, we find that  $i_y = 0$ .

(c) *This situation is impossible*, since  $i_x$  and  $i_z$  are in opposite directions. The only possible value (zero), is also disallowed, as KCL will not be satisfied ( $5 \neq 3$ ).

### CHAPTER THREE SOLUTIONS

7. Focusing our attention on the bottom left node, we see that  $i_x = 1 \text{ A.}$

Focusing our attention next on the top right node, we see that  $i_y = 5 \text{ A.}$

## CHAPTER THREE SOLUTIONS

8. (a)  $v_y = 1(3v_x + i_z)$

$v_x = 5$  V and given that  $i_z = -3$  A, we find that

$$v_y = 3(5) - 3 = 12 \text{ V}$$

(b)  $v_y = 1(3v_x + i_z) = -6 = 3v_x + 0.5$

Solving, we find that  $v_x = (-6 - 0.5)/3 = -2.167$  V.

### CHAPTER THREE SOLUTIONS

9. (a)  $i_x = v_1/10 + v_1/10 = 5$

$$2v_1 = 50$$

so  $v_1 = 25 \text{ V.}$

By Ohm's law, we see that  $i_y = v_2/10$

also, using Ohm's law in combination with KCL, we may write

$$i_x = v_2/10 + v_2/10 = i_y + i_y = 5 \text{ A}$$

Thus,  $i_y = 2.5 \text{ A.}$

(b) From part (a),  $i_x = 2 v_1/10$ . Substituting the new value for  $v_1$ , we find that

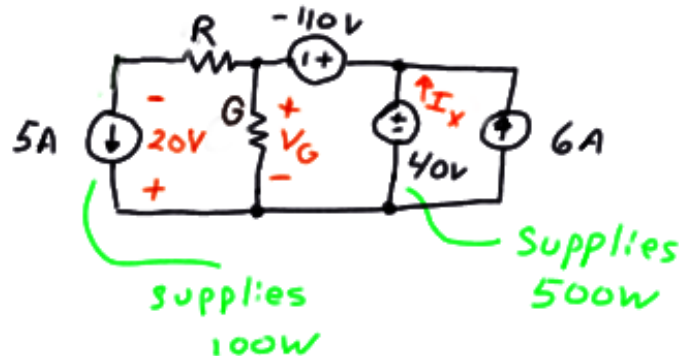
$$i_x = 6/10 = 600 \text{ mA.}$$

Since we have found that  $i_y = 0.5 i_x$ ,  $i_y = 300 \text{ mA.}$

(c) no value – this is impossible.

## CHAPTER THREE SOLUTIONS

10. We begin by making use of the information given regarding the power generated by the 5-A and the 40-V sources. The 5-A source supplies 100 W, so it must therefore have a terminal voltage of 20 V. The 40-V source supplies 500 W, so it must therefore provide a current of 12.5 A. These quantities are marked on our schematic below:



(1) By KVL,  $-40 - 110 + R(5) = 0$

Thus,  $R = 30 \, \Omega$ .

(2) By KVL,  $-V_G - (-110) + 40 = 0$

So  $V_G = 150 \, \text{V}$

Now that we know the voltage across the unknown conductance  $G$ , we need only to find the current flowing through it to find its value by making use of Ohm's law.

KCL provides us with the means to find this current: The current flowing into the "+" terminal of the  $-110\text{-V}$  source is  $12.5 + 6 = 18.5 \, \text{A}$ .

Then,  $I_x = 18.5 - 5 = 13.5 \, \text{A}$

By Ohm's law,  $I_x = G \cdot V_G$

So  $G = 13.5/150$  or  $G = 90 \, \text{mS}$



### CHAPTER THREE SOLUTIONS

11. (a)  $-1 + 2 + 10i - 3.5 + 10i = 0$

Solving,  $i = 125 \text{ mA}$

(b)  $+10 + 1i - 2 + 2i + 2 - 6 + i = 0$

Solving, we find that  $4i = -4$  or  $i = -1 \text{ A}$ .

## CHAPTER THREE SOLUTIONS

12. (a) By KVL,  $-2 + v_x + 8 = 0$

so that  $v_x = -6 \text{ V.}$

(b) By KCL at the top right node,

$$I_S + 4 v_x = 4 - v_x/4$$

So  $I_S = 29.5 \text{ A.}$

(c) By KCL at the top left node,

$$i_{in} = 1 + I_S + v_x/4 - 6$$

or  $i_{in} = 23 \text{ A}$

(d) The power provided by the dependent source is  $8(4v_x) = -192 \text{ W.}$

## CHAPTER THREE SOLUTIONS

13. (a) Working from left to right,

$$v_1 = 60 \text{ V}$$

$$v_2 = 60 \text{ V}$$

$$i_2 = 60/20 = 3 \text{ A}$$

$$i_4 = v_1/4 = 60/4 = 15 \text{ A}$$

$$v_3 = 5i_2 = 15 \text{ V}$$

$$\text{By KVL, } -60 + v_3 + v_5 = 0$$

$$v_5 = 60 - 15 = 45 \text{ V}$$

$$v_4 = v_5 = 45$$

$$i_5 = v_5/5 = 45/5 = 9 \text{ A}$$

$$i_3 = i_4 + i_5 = 15 + 9 = 24 \text{ A}$$

$$i_1 = i_2 + i_3 = 3 + 24 = 27$$

$v_1 = 60 \text{ V}$	$i_1 = 27 \text{ A}$
$v_2 = 60 \text{ V}$	$i_2 = 3 \text{ A}$
$v_3 = 15 \text{ V}$	$i_3 = 24 \text{ A}$
$v_4 = 45 \text{ V}$	$i_4 = 15 \text{ A}$
$v_5 = 45 \text{ V}$	$i_5 = 9 \text{ A}$

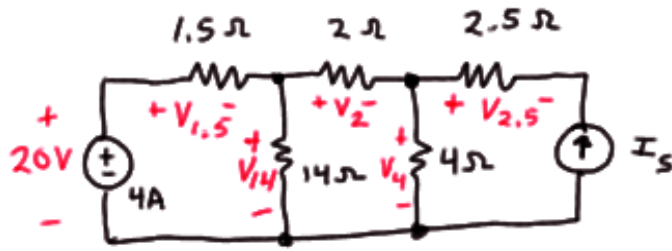
- (b) It is now a simple matter to compute the power absorbed by each element:

$p_1$	$= -v_1 i_1$	$= -(60)(27)$	$= -1.62 \text{ kW}$
$p_2$	$= v_2 i_2$	$= (60)(3)$	$= 180 \text{ W}$
$p_3$	$= v_3 i_3$	$= (15)(24)$	$= 360 \text{ W}$
$p_4$	$= v_4 i_4$	$= (45)(15)$	$= 675 \text{ W}$
$p_5$	$= v_5 i_5$	$= (45)(9)$	$= 405 \text{ W}$

and it is a simple matter to check that these values indeed sum to zero as they should.

## CHAPTER THREE SOLUTIONS

14. Refer to the labeled diagram below.



Beginning from the left, we find

$$p_{20V} = -(20)(4) = -80 \text{ W}$$

$$v_{1.5} = 4(1.5) = 6 \text{ V} \quad \text{therefore} \quad p_{1.5} = (v_{1.5})^2 / 1.5 = 24 \text{ W.}$$

$$v_{14} = 20 - v_{1.5} = 20 - 6 = 14 \text{ V} \quad \text{therefore} \quad p_{14} = 14^2 / 14 = 14 \text{ W.}$$

$$i_2 = v_2 / 2 = v_{1.5} / 1.5 - v_{14} / 14 = 6 / 1.5 - 14 / 14 = 3 \text{ A}$$

$$\text{Therefore } v_2 = 2(3) = 6 \text{ V and } p_2 = 6^2 / 2 = 18 \text{ W.}$$

$$v_4 = v_{14} - v_2 = 14 - 6 = 8 \text{ V} \quad \text{therefore} \quad p_4 = 8^2 / 4 = 16 \text{ W}$$

$$i_{2.5} = v_{2.5} / 2.5 = v_2 / 2 - v_4 / 4 = 3 - 2 = 1 \text{ A}$$

$$\text{Therefore } v_{2.5} = (2.5)(1) = 2.5 \text{ V and so } p_{2.5} = (2.5)^2 / 2.5 = 2.5 \text{ W.}$$

$$I_{2.5} = -I_s, \text{ therefore } I_s = -1 \text{ A.}$$

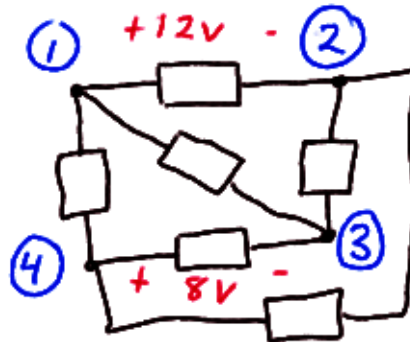
$$\text{KVL allows us to write } -v_4 + v_{2.5} + v_{I_s} = 0$$

$$\text{so } V_{I_s} = v_4 - v_{2.5} = 8 - 2.5 = 5.5 \text{ V and } p_{I_s} = -V_{I_s} I_s = 5.5 \text{ W.}$$

*A quick check assures us that these power quantities sum to zero.*

## CHAPTER THREE SOLUTIONS

15. Sketching the circuit as described,



(a)  $v_{14} = 0$ .

$v_{13} = v_{43}$	$= 8 \text{ V}$
$v_{23} = -v_{12} - v_{34} = -12 + 8$	$= -4 \text{ V}$
$v_{24} = v_{23} + v_{34} = -4 - 8$	$= -12 \text{ V}$

(b)  $v_{14} = 6 \text{ V}$ .

$v_{13} = v_{14} + v_{43} = 6 + 8$	$= 14 \text{ V}$
$v_{23} = v_{13} - v_{12} = 14 - 12$	$= 2 \text{ V}$
$v_{24} = v_{23} + v_{34} = 2 - 8$	$= -6 \text{ V}$

(c)  $v_{14} = -6 \text{ V}$ .

$v_{13} = v_{14} + v_{43} = -6 + 8$	$= 2 \text{ V}$
$v_{23} = v_{13} - v_{12} = 2 - 12$	$= -10 \text{ V}$
$v_{24} = v_{23} + v_{34} = -10 - 8$	$= -18 \text{ V}$

## CHAPTER THREE SOLUTIONS

16. (a) By KVL,  $-12 + 5000I_D + V_{DS} + 2000I_D = 0$

Therefore,  $V_{DS} = 12 - 7(1.5) = 1.5 \text{ V.}$

(b) By KVL,  $-V_G + V_{GS} + 2000I_D = 0$

Therefore,  $V_{GS} = V_G - 2(2) = -1 \text{ V.}$

## CHAPTER THREE SOLUTIONS

17. Applying KVL around this series circuit,

$$-120 + 30i_x + 40i_x + 20i_x + v_x + 20 + 10i_x = 0$$

where  $v_x$  is defined across the unknown element X, with the “+” reference on top. Simplifying, we find that  $100i_x + v_x = 100$

To solve further we require specific information about the element X and its properties.

---

- (a) if X is a 100- $\Omega$  resistor,

$$v_x = 100i_x \text{ so we find that } 100i_x + 100i_x = 100.$$

Thus

$$i_x = 500 \text{ mA and } p_x = v_x i_x = 25 \text{ W.}$$

- (b) If X is a 40-V independent voltage source such that  $v_x = 40 \text{ V}$ , we find that

$$i_x = (100 - 40) / 100 = 600 \text{ mA and } p_x = v_x i_x = 24 \text{ W}$$

- (c) If X is a dependent voltage source such that  $v_x = 25i_x$ ,

$$i_x = 100/125 = 800 \text{ mA and } p_x = v_x i_x = 16 \text{ W.}$$

- (d) If X is a dependent voltage source so that  $v_x = 0.8v_1$ , where  $v_1 = 40i_x$ , we have

$$100i_x + 0.8(40i_x) = 100$$

$$\text{or } i_x = 100/132 = 757.6 \text{ mA and } p_x = v_x i_x = 0.8(40)(0.7576)^2 = 18.37 \text{ W.}$$

- (e) If X is a 2-A independent current source, arrow up,

$$100(-2) + v_x = 100$$

$$\text{so that } v_x = 100 + 200 = 300 \text{ V and } p_x = v_x i_x = -600 \text{ W}$$

## CHAPTER THREE SOLUTIONS

18. (a) We first apply KVL:

$$-20 + 10i_1 + 90 + 40i_1 + 2v_2 = 0$$

where  $v_2 = 10i_1$ . Substituting,

$$70 + 70 i_1 = 0$$

or  $i_1 = -1 \text{ A.}$

- (b) Applying KVL,

$$-20 + 10i_1 + 90 + 40i_1 + 1.5v_3 = 0 \quad [1]$$

where

$$v_3 = -90 - 10i_1 + 20 = -70 - 10 i_1$$

alternatively, we could write

$$v_3 = 40i_1 + 1.5v_3 = -80i_1$$

Using either expression in Eq. [1], we find  $i_1 = 1 \text{ A.}$

- (c) Applying KVL,

$$-20 + 10i_1 + 90 + 40i_1 - 15 i_1 = 0$$

Solving,  $i_1 = -2 \text{ A.}$



### CHAPTER THREE SOLUTIONS

19. Applying KVL, we find that

$$-20 + 10i_1 + 90 + 40i_1 + 1.8v_3 = 0 \quad [1]$$

Also, KVL allows us to write

$$v_3 = 40i_1 + 1.8v_3$$

$$v_3 = -50i_1$$

So that we may write Eq. [1] as

$$50i_1 - 1.8(50)i_1 = -70$$

or  $i_1 = -70/-40 = 1.75$  A.

Since  $v_3 = -50i_1 = -87.5$  V, no further information is required to determine its value.

The 90-V source is absorbing  $(90)(i_1) = 157.5$  W of power and the dependent source is absorbing  $(1.8v_3)(i_1) = -275.6$  W of power.

Therefore, *none of the conditions specified in (a) to (d) can be met by this circuit.*

## CHAPTER THREE SOLUTIONS

20. (a) Define the charging current  $i$  as flowing clockwise in the circuit provided. By application of KVL,

$$-13 + 0.02i + Ri + 0.035i + 10.5 = 0$$

We know that we need a current  $i = 4$  A, so we may calculate the necessary resistance

$$R = [13 - 10.5 - 0.055(4)] / 4 = 570 \text{ m}\Omega$$

(b) The total power delivered to the battery consists of the power absorbed by the  $0.035\text{-}\Omega$  resistance ( $0.035i^2$ ), and the power absorbed by the  $10.5\text{-V}$  ideal battery ( $10.5i$ ). Thus, we need to solve the quadratic equation

$$0.035i^2 + 10.5i = 25$$

which has the solutions  $i = -302.4$  A and  $i = 2.362$  A.

In order to determine which of these two values should be used, we must recall that the idea is to charge the battery, implying that it is absorbing power, or that  $i$  as defined is positive. Thus, we choose  $i = 2.362$  A, and, making use of the expression developed in part (a), we find that

$$R = [13 - 10.5 - 0.055(2.362)] / 2.362 = 1.003 \text{ }\Omega$$

(c) To obtain a voltage of  $11$  V across the battery, we apply KVL:

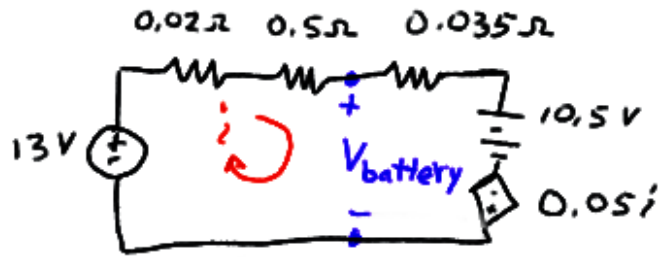
$$0.035i + 10.5 = 11 \quad \text{so that } i = 14.29 \text{ A}$$

From part (a), this means we need

$$R = [13 - 10.5 - 0.055(14.29)] / 14.29 = 119.9 \text{ m}\Omega$$

## CHAPTER THREE SOLUTIONS

21. Drawing the circuit described, we also define a clockwise current  $i$ .



By KVL, we find that

$$-13 + (0.02 + 0.5 + 0.035)i + 10.5 - 0.05i = 0$$

or that  $i = (13 - 10.5)/0.505 = 4.950 \text{ A}$

and  $V_{\text{battery}} = 13 - (0.02 + 0.5)i = 10.43 \text{ V.}$

## CHAPTER THREE SOLUTIONS

22. Applying KVL about this simple loop circuit (the dependent sources are still linear elements, by the way, as they depend only upon a sum of voltages)

$$-40 + (5 + 25 + 20)i - (2v_3 + v_2) + (4v_1 - v_2) = 0 \quad [1]$$

where we have defined  $i$  to be flowing in the clockwise direction, and  $v_1 = 5i$ ,  $v_2 = 25i$ , and  $v_3 = 20i$ .

Performing the necessary substitution, Eq. [1] becomes

$$50i - (40i + 25i) + (20i - 25i) = 40$$

so that  $i = 40/-20 = -2$  A

Computing the absorbed power is now a straightforward matter:

$p_{40V}$	$= (40)(-i)$	$= 80$ W
$p_{5W}$	$= 5i^2$	$= 20$ W
$p_{25W}$	$= 25i^2$	$= 100$ W
$p_{20W}$	$= 20i^2$	$= 80$ W
$p_{\text{depsrc1}}$	$= (2v_3 + v_2)(-i) = (40i + 25i)$	$= -260$ W
$p_{\text{depsrc2}}$	$= (4v_1 - v_2)(-i) = (20i - 25i)$	$= -20$ W

and we can easily verify that these quantities indeed sum to zero as expected.

## CHAPTER THREE SOLUTIONS

23. We begin by defining a clockwise current  $i$ .

(a)  $i = 12/(40 + R)$  mA, with  $R$  expressed in  $k\Omega$ .

We want  $i^2 \cdot 25 = 2$

or 
$$\left( \frac{12}{40 + R} \right)^2 \cdot 25 = 2$$

Rearranging, we find a quadratic expression involving  $R$ :

$$R^2 + 80R - 200 = 0$$

which has the solutions  $R = -82.43 \text{ k}\Omega$  and  $R = 2.426 \text{ k}\Omega$ . Only the latter is a physical solution, so

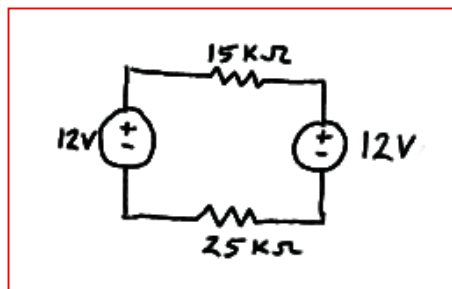
$$R = 2.426 \text{ k}\Omega.$$

(b) We require  $i \cdot 12 = 3.6$  or  $i = 0.3 \text{ mA}$

From the circuit, we also see that  $i = 12/(15 + R + 25) \text{ mA}$ .

Substituting the desired value for  $i$ , we find that the required value of  $R$  is  $R = 0$ .

(c)



## CHAPTER THREE SOLUTIONS

24. By KVL,

$$-12 + (1 + 2.3 + R_{\text{wire segment}}) i = 0$$

The wire segment is a 3000-ft section of 28-AWG solid copper wire. Using Table 2.3, we compute its resistance as

$$(16.2 \text{ m}\Omega/\text{ft})(3000 \text{ ft}) = 48.6 \Omega$$

which is certainly *not* negligible compared to the other resistances in the circuit!

Thus,

$$i = 12/(1 + 2.3 + 48.6) = \boxed{231.2 \text{ mA}}$$

### CHAPTER THREE SOLUTIONS

25. We can apply Ohm's law to find an expression for  $v_o$ :

$$v_o = 1000(-g_m v_\pi)$$

We do not have a value for  $v_\pi$ , but KVL will allow us to express that in terms of  $v_o$ , which we *do* know:

$$-10 \times 10^{-3} \cos 5t + (300 + 50 \times 10^3) i = 0$$

where  $i$  is defined as flowing clockwise.

$$\begin{aligned} \text{Thus, } v_\pi = 50 \times 10^3 i &= 50 \times 10^3 (10 \times 10^{-3} \cos 5t) / (300 + 50 \times 10^3) \\ &= 9.940 \times 10^{-3} \cos 5t \text{ V} \end{aligned}$$

and we by substitution we find that

$$\begin{aligned} v_o &= 1000(-25 \times 10^{-3})(9.940 \times 10^{-3} \cos 5t) \\ &= \boxed{-248.5 \cos 5t \text{ mV}} \end{aligned}$$

## CHAPTER THREE SOLUTIONS

26. By KVL, we find that

$$-3 + 100 I_D + V_D = 0$$

Substituting  $I_D = 3 \times 10^{-6} (e^{V_D / 27 \times 10^{-3}} - 1)$ , we find that

$$-3 + 300 \times 10^{-6} (e^{V_D / 27 \times 10^{-3}} - 1) + V_D = 0$$

This is a transcendental equation. Using a scientific calculator or a numerical software package such as MATLAB<sup>®</sup>, we find

$$V_D = \boxed{246.4 \text{ mV}}$$

Let's assume digital assistance is unavailable. In that case, we need to "guess" a value for  $V_D$ , substitute it into the right hand side of our equation, and see how close the result is to the left hand side (in this case, zero).

GUESS	RESULT
<b>0</b>	-3
<b>1</b>	$3.648 \times 10^{12}$
<b>0.5</b>	$3.308 \times 10^4$
<b>0.25</b>	0.4001
<b>0.245</b>	-0.1375
<b>0.248</b>	0.1732
<b>0.246</b>	-0.0377

oops

better

At this point, the error is getting much smaller, and our confidence is increasing as to the value of  $V_D$ .



## CHAPTER THREE SOLUTIONS

27. Define a voltage  $v_x$ , “+” reference on the right, across the dependent current source. Note that in fact  $v_x$  appears across each of the four elements. We first convert the 10 mS conductance into a 100- $\Omega$  resistor, and the 40-mS conductance into a 25- $\Omega$  resistor.

(a) Applying KCL, we sum the currents flowing into the right-hand node:

$$5 - v_x / 100 - v_x / 25 + 0.8 i_x = 0 \quad [1]$$

This represents one equation in two unknowns. A second equation to introduce at this point is

$i_x = v_x / 25$  so that Eq. [1] becomes

$$5 - v_x / 100 - v_x / 25 + 0.8 (v_x / 25) = 0$$

Solving for  $v_x$ , we find  $v_x = 277.8$  V. It is a simple matter now to compute the power absorbed by each element:

$P_{5A}$	$= -5 v_x$	$= -1.389 \text{ kW}$
$P_{100\Omega}$	$= (v_x)^2 / 100$	$= 771.7 \text{ W}$
$P_{25\Omega}$	$= (v_x)^2 / 25$	$= 3.087 \text{ kW}$
$P_{\text{dep}}$	$= -v_x(0.8 i_x) = -0.8 (v_x)^2 / 25$	$= -2.470 \text{ kW}$

A quick check assures us that the calculated values sum to zero, as they should.

(b) Again summing the currents into the right-hand node,

$$5 - v_x / 100 - v_x / 25 + 0.8 i_y = 0 \quad [2]$$

where  $i_y = 5 - v_x / 100$

Thus, Eq. [2] becomes

$$5 - v_x / 100 - v_x / 25 + 0.8(5) - 0.8 (i_y) / 100 = 0$$

Solving, we find that  $v_x = 155.2$  V and  $i_y = 3.448$  A

So that

$P_{5A}$	$= -5 v_x$	$= -776.0 \text{ W}$
$P_{100\Omega}$	$= (v_x)^2 / 100$	$= 240.9 \text{ W}$
$P_{25\Omega}$	$= (v_x)^2 / 25$	$= 963.5 \text{ W}$
$P_{\text{dep}}$	$= -v_x(0.8 i_y)$	$= -428.1 \text{ W}$

A quick check assures us that the calculated values sum to 0.3, which is reasonably close to zero (small roundoff errors accumulate here).

### CHAPTER THREE SOLUTIONS

28. Define a voltage  $v$  with the “+” reference at the top node. Applying KCL and summing the currents flowing out of the top node,

$$v/5,000 + 4 \times 10^{-3} + 3i_1 + v/20,000 = 0 \quad [1]$$

This, unfortunately, is one equation in two unknowns, necessitating the search for a second suitable equation. Returning to the circuit diagram, we observe that

$$i_1 = 3i_1 + v/2,000$$

or 
$$i_1 = -v/40,000 \quad [2]$$

Upon substituting Eq. [2] into Eq. [1], Eq. [1] becomes,

$$v/5,000 + 4 \times 10^{-3} - 3v/40,000 + v/20,000 = 0$$

Solving, we find that

$$v = -22.86 \text{ V}$$

and

$$i_1 = 571.4 \text{ } \mu\text{A}$$

Since  $i_x = i_1$ , we find that  $i_x = 571.4 \text{ } \mu\text{A}$ .

## CHAPTER THREE SOLUTIONS

29. Define a voltage  $v_x$  with its “+” reference at the center node. Applying KCL and summing the currents into the center node,

$$8 - v_x/6 + 7 - v_x/12 - v_x/4 = 0$$

Solving,  $v_x = 30$  V.

It is now a straightforward matter to compute the power absorbed by each element:

$P_{8A}$	$= -8 v_x$	$= -240$ W
$P_{6\Omega}$	$= (v_x)^2 / 6$	$= 150$ W
$P_{8A}$	$= -7 v_x$	$= -210$ W
$P_{12\Omega}$	$= (v_x)^2 / 12$	$= 75$ W
$P_{4\Omega}$	$= (v_x)^2 / 4$	$= 225$ W

and a quick check verifies that the computed quantities sum to zero, as expected.

### CHAPTER THREE SOLUTIONS

30. (a) Define a voltage  $v$  across the  $1\text{-k}\Omega$  resistor with the “+” reference at the top node. Applying KCL at this top node, we find that

$$80 \times 10^{-3} - 30 \times 10^{-3} = v/1000 + v/4000$$

Solving,

$$v = (50 \times 10^{-3})(4 \times 10^3 / 5 \times 10^3) = 40 \text{ V}$$

and

$$P_{4\text{k}\Omega} = v^2/4000 = \boxed{400 \text{ mW}}$$

- (b) Once again, we first define a voltage  $v$  across the  $1\text{-k}\Omega$  resistor with the “+” reference at the top node. Applying KCL at this top node, we find that

$$80 \times 10^{-3} - 30 \times 10^{-3} - 20 \times 10^{-3} = v/1000$$

Solving,

$$v = 30 \text{ V}$$

and

$$P_{20\text{mA}} = v \cdot 20 \times 10^{-3} = \boxed{600 \text{ mW}}$$

- (c) Once again, we first define a voltage  $v$  across the  $1\text{-k}\Omega$  resistor with the “+” reference at the top node. Applying KCL at this top node, we find that

$$80 \times 10^{-3} - 30 \times 10^{-3} - 2i_x = v/1000$$

where

$$i_x = v/1000$$

so that

$$80 \times 10^{-3} - 30 \times 10^{-3} = 2v/1000 + v/1000$$

and

$$v = 50 \times 10^{-3} (1000)/3 = 16.67 \text{ V}$$

Thus,

$$P_{\text{dep}} = v \cdot 2i_x = \boxed{555.8 \text{ mW}}$$

- (d) We note that  $i_x = 60/1000 = 60 \text{ mA}$ . KCL stipulates that (viewing currents into and out of the top node)

$$80 - 30 + i_s = i_x = 60$$

Thus,  $i_s = 10 \text{ mA}$

$$\text{and } P_{60\text{V}} = 60(-10) \text{ mW} = \boxed{-600 \text{ mW}}$$

### CHAPTER THREE SOLUTIONS

31. (a) To cancel out the effects of both the 80-mA and 30-mA sources,  $i_S$  must be set to

$$i_S = -50 \text{ mA.}$$

(b) Define a current is flowing out of the “+” reference terminal of the independent voltage source. Interpret “no power” to mean “zero power.”

Summing the currents flowing into the top node and invoking KCL, we find that

$$80 \times 10^{-3} - 30 \times 10^{-3} - v_S / 1 \times 10^3 + i_S = 0$$

Simplifying slightly, this becomes

$$50 - v_S + 10^3 i_S = 0 \quad [1]$$

We are seeking a value for  $v_S$  such that  $v_S \cdot i_S = 0$ . Clearly, setting  $v_S = 0$  will achieve this. From Eq. [1], we also see that setting  $v_S = 50 \text{ V}$  will work as well.

### CHAPTER THREE SOLUTIONS

32. Define a voltage  $v_9$  across the  $9\text{-}\Omega$  resistor, with the “+” reference at the top node.

(a) Summing the currents into the right-hand node and applying KCL,

$$5 + 7 = v_9 / 3 + v_9 / 9$$

Solving, we find that  $v_9 = 27\text{ V}$ . Since  $i_x = v_9 / 9$ ,  $i_x = 3\text{ A}$ .

(b) Again, we apply KCL, this time to the top left node:

$$2 - v_8 / 8 + 2i_x - 5 = 0$$

Since we know from part (a) that  $i_x = 3\text{ A}$ , we may calculate  $v_8 = 24\text{ V}$ .

(c)  $p_{5A} = (v_9 - v_8) \cdot 5 = 15\text{ W}$ .

### CHAPTER THREE SOLUTIONS

33. Define a voltage  $v_x$  across the 5-A source, with the “+” reference on top.

Applying KCL at the top node then yields

$$5 + 5v_1 - v_x / (1 + 2) - v_x / 5 = 0 \quad [1]$$

where  $v_1 = 2[v_x / (1 + 2)] = 2 v_x / 3$ .

Thus, Eq. [1] becomes

$$5 + 5(2 v_x / 3) - v_x / 3 - v_x / 5 = 0$$

or  $75 + 50 v_x - 5 v_x - 3 v_x = 0$ , which, upon solving, yields  $v_x = -1.786$  V.

The power absorbed by the 5- $\Omega$  resistor is then simply  $(v_x)^2/5 = 638.0$  mW.

## CHAPTER THREE SOLUTIONS

34. Despite the way it may appear at first glance, this is actually a simple node-pair circuit. Define a voltage  $v$  across the elements, with the “+” reference at the top node.

Summing the currents leaving the top node and applying KCL, we find that

$$2 + 6 + 3 + v/5 + v/5 + v/5 = 0$$

or  $v = -55/3 = -18.33$  V. The power supplied by each source is then computed as:

$$\begin{aligned} p_{2A} &= -v(2) = 36.67 \text{ W} \\ p_{6A} &= -v(6) = 110 \text{ W} \\ p_{3A} &= -v(3) = 55 \text{ W} \end{aligned}$$

We can check our results by first determining the power absorbed by each resistor, which is simply  $v^2/5 = 67.22$  W for a total of 201.67 W, which is the total power supplied by all sources.



### CHAPTER THREE SOLUTIONS

35. Defining a voltage  $V_x$  across the 10-A source with the “+” reference at the top node, KCL tells us that  $10 = 5 + I_{1\Omega}$ , where  $I_{1\Omega}$  is defined flowing downward through the 1- $\Omega$  resistor.

Solving, we find that  $I_{1\Omega} = 5$  A, so that  $V_x = (1)(5) = 5$  V.

So, we need to solve

$$V_x = 5 = 5(0.5 + R_{\text{segment}})$$

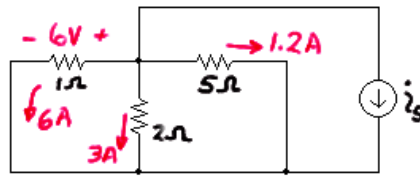
with  $R_{\text{segment}} = 500$  m $\Omega$ .

From Table 2.3, we see that 28-AWG solid copper wire has a resistance of 65.3 m $\Omega$ /ft. Thus, the total number of miles needed of the wire is

$$\frac{500 \text{ m}\Omega}{(65.3 \text{ m}\Omega/\text{ft})(5280 \text{ ft/mi})} = 1.450 \times 10^{-3} \text{ miles}$$

## CHAPTER THREE SOLUTIONS

36. Since  $v = 6$  V, we know the current through the  $1\text{-}\Omega$  resistor is 6 A, the current through the  $2\text{-}\Omega$  resistor is 3 A, and the current through the  $5\text{-}\Omega$  resistor is  $6/5 = 1.2$  A, as shown below:



By KCL,  $6 + 3 + 1.2 + i_s = 0$  or  $i_s = -10.2$  A.

### CHAPTER THREE SOLUTIONS

37. (a) Applying KCL,  $1 - i - 3 + 3 = 0$  so  $i = 1$  A.

(b) The rightmost source should be labeled 3.5 A to satisfy KCL.

Then, looking at the left part of the circuit, we see  $1 + 3 = 4$  A flowing into the unknown current source, which, by virtue of KCL, must therefore be a 4-A current source. Thus, KCL at the node labeled with the “+” reference of the voltage  $v$  gives

$$4 - 2 + 7 - i = 0 \quad \text{or} \quad i = 9 \text{ A}$$

## CHAPTER THREE SOLUTIONS

38. (a) We may redraw the circuit as



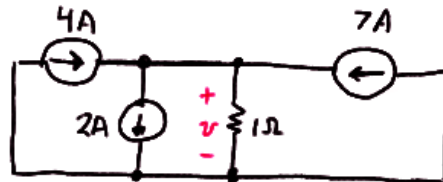
Then, we see that  $v = (1)(1) = 1 \text{ V}$ .

- (b) The current source at the far right should be labeled 3.5 A, or KCL is violated.

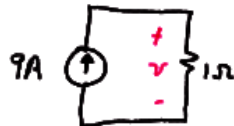
In that case, we may combine all sources to the right of the  $1\text{-}\Omega$  resistor into a single 7-A current source. On the left, the two 1-A sources in series reduce to a single 1-A source.

The new 1-A source and the 3-A source combine to yield a 4-A source in series with the unknown current source which, by KCL, must be a 4-A current source.

At this point we have reduced the circuit to



Further simplification is possible, resulting in



From which we see clearly that  $v = (9)(1) = 9 \text{ V}$ .

### CHAPTER THREE SOLUTIONS

39. (a) Combine the 12-V and 2-V series connected sources to obtain a new  $12 - 2 = 10$  V source, with the “+” reference terminal at the top. The result is two 10-V sources in parallel, which is permitted by KVL. Therefore,

$$i = 10/1000 = 10 \text{ mA.}$$

(b) No current flows through the 6-V source, so we may neglect it for this calculation. The 12-V, 10-V and 3-V sources are connected in series as a result, so we replace them with a  $12 + 10 - 3 = 19$  V source as shown

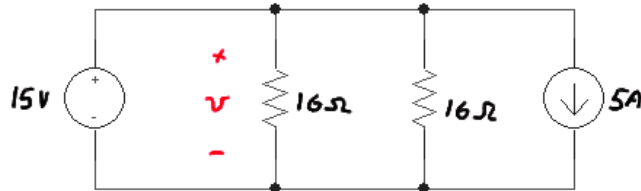


Thus,  $i = 19/5 = 3.8 \text{ A.}$

### CHAPTER THREE SOLUTIONS

40. We first combine the 10-V and 5-V sources into a single 15-V source, with the “+” reference on top. The 2-A and 7-A current sources combine into a  $7 - 2 = 5$  A current source (arrow pointing down); although these two current sources may not appear to be in parallel at first glance, they actually are.

Redrawing our circuit,

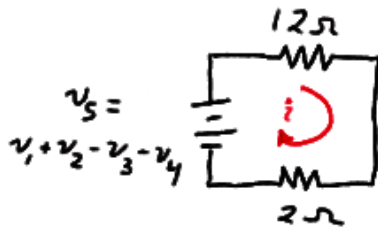


we see that  $v = 15$  V (note that we can completely ignore the 5-A source here, since we have a voltage source directly across the resistor). Thus,

$$p_{16\Omega} = v^2/16 = \boxed{14.06 \text{ W.}}$$

## CHAPTER THREE SOLUTIONS

41. We can combine the voltage sources such that



$$i = v_s / 14$$

(a)  $v_s = 10 + 10 - 6 - 6 = 20 - 12 = 8$

Therefore

$$i = 8/14 = \boxed{571.4 \text{ mA.}}$$

(b)  $v_s = 3 + 2.5 - 3 - 2.5 = 0$

Therefore  $i = \boxed{0.}$

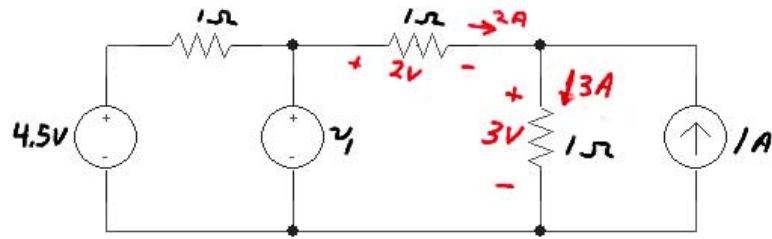
(c)  $v_s = -3 + 1.5 - (-0.5) - 0 = -1 \text{ V}$

Therefore

$$i = -1/14 = \boxed{-71.43 \text{ mA.}}$$

### CHAPTER THREE SOLUTIONS

42. We first simplify as shown, making use of the fact that we are told  $i_x = 2\text{ A}$  to find the voltage across the middle and right-most  $1\text{-}\Omega$  resistors as labeled.



By KVL, then, we find that  $v_1 = 2 + 3 = 5\text{ V}$ .



### CHAPTER THREE SOLUTIONS

43. We see that to determine the voltage  $v$  we will need  $v_x$  due to the presence of the dependent current source. So, let's begin with the right-hand side, where we find that

$$v_x = 1000(1 - 3) \times 10^{-3} = -2 \text{ V}.$$

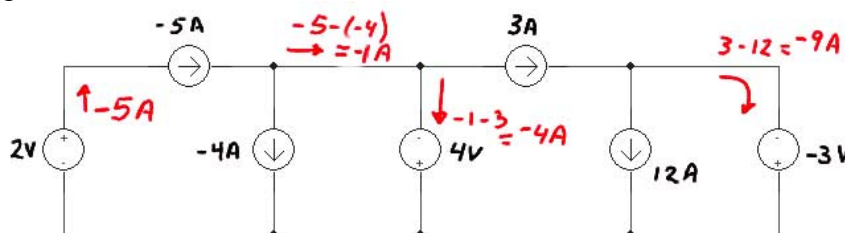
Returning to the left-hand side of the circuit, and summing currents into the top node, we find that

$$(12 - 3.5) \times 10^{-3} + 0.03 v_x = v/10 \times 10^3$$

or  $v = -515 \text{ V}.$

## CHAPTER THREE SOLUTIONS

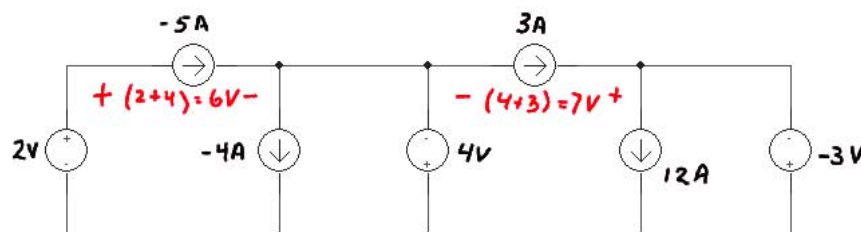
44. (a) We first label the circuit with a focus on determining the current flowing through each voltage source:



Then the power absorbed by each voltage source is

$P_{2V}$	$= -2(-5)$	$= 10 \text{ W}$
$P_{4V}$	$= -(-4)(4)$	$= 16 \text{ W}$
$P_{-3V}$	$= -(-9)(-3)$	$= 27 \text{ W}$

For the current sources,



So that the absorbed power is

$P_{-5A}$	$= +(-5)(6)$	$= -30 \text{ W}$
$P_{-4A}$	$= -(-4)(4)$	$= 16 \text{ W}$
$P_{3A}$	$= -(3)(7)$	$= -21 \text{ W}$
$P_{12A}$	$= -(12)(-3)$	$= 36 \text{ W}$

A quick check assures us that these absorbed powers sum to zero as they should.

- (b) We need to change the 4-V source such that the voltage across the  $-5\text{-A}$  source drops to zero. Define  $V_x$  across the  $-5\text{-A}$  source such that the “+” reference terminal is on the left. Then,

$$-2 + V_x - V_{\text{needed}} = 0$$

or  $V_{\text{needed}} = -2 \text{ V}.$

## CHAPTER THREE SOLUTIONS

45. We begin by noting several things:
- (1) The bottom resistor has been shorted out;
  - (2) the far-right resistor is only connected by one terminal and therefore does not affect the equivalent resistance *as seen from the indicated terminals*;
  - (3) All resistors to the right of the top left resistor have been shorted.

Thus, from the indicated terminals, we only see the single 1-k $\Omega$  resistor, so that

$$R_{eq} = 1 \text{ k}\Omega.$$

## CHAPTER THREE SOLUTIONS

46. (a) We see  $1\Omega \parallel (1\Omega + 1\Omega) \parallel (1\Omega + 1\Omega + 1\Omega)$   
 $= 1\Omega \parallel 2\Omega \parallel 3\Omega$   
 $= 545.5 \text{ m}\Omega$

(b)  $1/R_{\text{eq}} = 1 + 1/2 + 1/3 + \dots 1/N$

Thus,  $R_{\text{eq}} = [1 + 1/2 + 1/3 + \dots 1/N]^{-1}$

## CHAPTER THREE SOLUTIONS

47. (a)  $5 \text{ k}\Omega = 10 \text{ k}\Omega \parallel 10 \text{ k}\Omega$

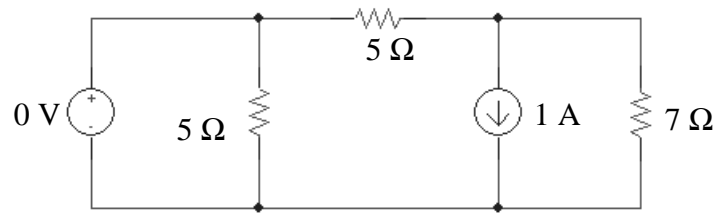
(b)  $57\,333 \text{ }\Omega = 47 \text{ k}\Omega + 10 \text{ k}\Omega + 1 \text{ k}\Omega \parallel 1 \text{ k}\Omega \parallel 1 \text{ k}\Omega$

(c)  $29.5 \text{ k}\Omega = 47 \text{ k}\Omega \parallel 47 \text{ k}\Omega + 10 \text{ k}\Omega \parallel 10 \text{ k}\Omega + 1 \text{ k}\Omega$

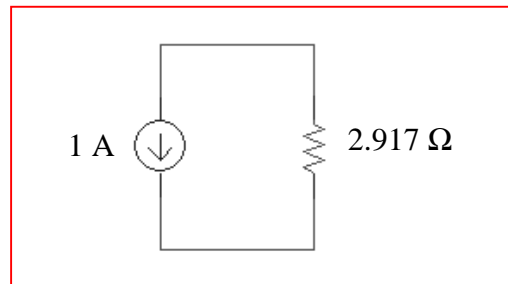
### CHAPTER THREE SOLUTIONS

48. (a) no simplification is possible using only source and/or resistor combination techniques.

(b) We first simplify the circuit to



and then notice that the 0-V source is shorting out one of the 5-Ω resistors, so a further simplification is possible, noting that  $5\ \Omega \parallel 7\ \Omega = 2.917\ \Omega$ :

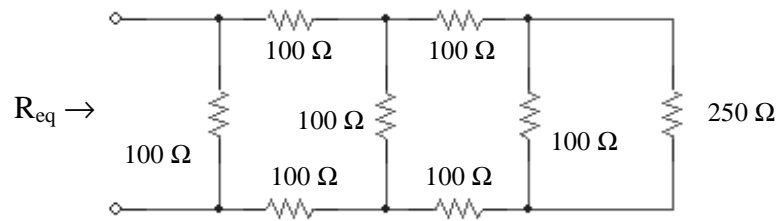


## CHAPTER THREE SOLUTIONS

$$\begin{aligned} 49. \quad R_{eq} &= 1 \text{ k}\Omega + 2 \text{ k}\Omega \parallel 2 \text{ k}\Omega + 3 \text{ k}\Omega \parallel 3 \text{ k}\Omega + 4 \text{ k}\Omega \parallel 4 \text{ k}\Omega \\ &= 1 \text{ k}\Omega + 1 \text{ k}\Omega + 1.5 \text{ k}\Omega + 2 \text{ k}\Omega \\ &= 5.5 \text{ k}\Omega. \end{aligned}$$

## CHAPTER THREE SOLUTIONS

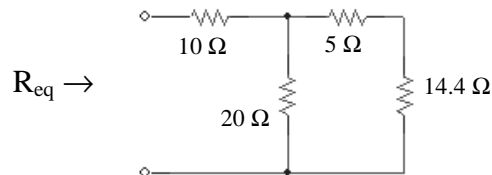
50. (a) Working from right to left, we first see that we may combine several resistors as  $100\ \Omega + 100\ \Omega \parallel 100\ \Omega + 100\ \Omega = 250\ \Omega$ , yielding the following circuit:



Next, we see  $100\ \Omega + 100\ \Omega \parallel 250\ \Omega + 100\ \Omega = 271.4\ \Omega$ ,  
and subsequently  $100\ \Omega + 100\ \Omega \parallel 271.4\ \Omega + 100\ \Omega = 273.1\ \Omega$ ,  
and, finally,

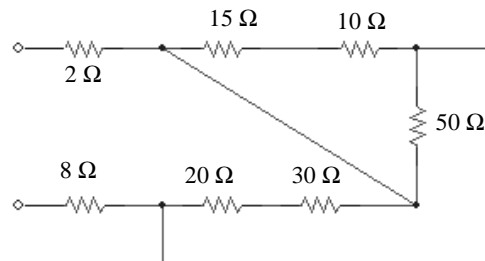
$$R_{eq} = 100\ \Omega \parallel 273.1\ \Omega = 73.20\ \Omega.$$

- (b) First, we combine  $24\ \Omega \parallel (50\ \Omega + 40\ \Omega) \parallel 60\ \Omega = 14.4\ \Omega$ , which leaves us with

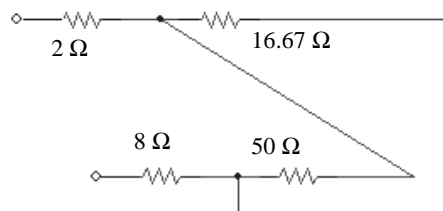


Thus,  $R_{eq} = 10\ \Omega + 20\ \Omega \parallel (5 + 14.4\ \Omega) = 19.85\ \Omega.$

- (c) First combine the 10- $\Omega$  and 40- $\Omega$  resistors and redraw the circuit:



We now see we have  $(10\ \Omega + 15\ \Omega) \parallel 50\ \Omega = 16.67\ \Omega$ . Redrawing once again,



where the equivalent resistance is seen to be  $2\ \Omega + 50\ \Omega \parallel 16.67\ \Omega + 8\ \Omega = 22.5\ \Omega.$



### CHAPTER THREE SOLUTIONS

51. (a)  $R_{eq} = [(40\ \Omega + 20\ \Omega) \parallel 30\ \Omega + 80\ \Omega] \parallel 100\ \Omega + 10\ \Omega = 60\ \Omega.$

(b)  $R_{eq} = 80\ \Omega = [(40\ \Omega + 20\ \Omega) \parallel 30\ \Omega + R] \parallel 100\ \Omega + 10\ \Omega$   
 $70\ \Omega = [(60\ \Omega \parallel 30\ \Omega) + R] \parallel 100\ \Omega$   
 $1/70 = 1/(20 + R) + 0.01$   
 $20 + R = 233.3\ \Omega$  therefore  $R = 213.3\ \Omega.$

(c)  $R = [(40\ \Omega + 20\ \Omega) \parallel 30\ \Omega + R] \parallel 100\ \Omega + 10\ \Omega$   
 $R - 10\ \Omega = [20 + R] \parallel 100$   
 $1/(R - 10) = 1/(R + 20) + 1/100$   
 $3000 = R^2 + 10R - 200$

Solving, we find  $R = -61.79\ \Omega$  or  $R = 51.79\ \Omega$ .

Clearly, the first is not a physical solution, so

$R = 51.79\ \Omega.$

## CHAPTER THREE SOLUTIONS

52. (a)  $25\ \Omega = 100\ \Omega \parallel 100\ \Omega \parallel 100\ \Omega$

(b)  $60\ \Omega = [(100\ \Omega \parallel 100\ \Omega) + 100\ \Omega] \parallel 100\ \Omega$

(c)  $40\ \Omega = (100\ \Omega + 100\ \Omega) \parallel 100\ \Omega \parallel 100\ \Omega$

### CHAPTER THREE SOLUTIONS

53.  $R_{eq} = [(5\ \Omega \parallel 20\ \Omega) + 6\ \Omega] \parallel 30\ \Omega + 2.5\ \Omega = 10\ \Omega$   
The source therefore provides a total of 1000 W and a current of  $100/10 = 10\text{ A}$ .

$$P_{2.5\Omega} = (10)^2 \cdot 2.5 = \boxed{250\text{ W}}$$

$$V_{30\Omega} = 100 - 2.5(10) = 75\text{ V}$$

$$P_{30\Omega} = 75^2/30 = \boxed{187.5\text{ W}}$$

$$I_{6\Omega} = 10 - V_{30\Omega}/30 = 10 - 75/30 = 7.5\text{ A}$$

$$P_{6\Omega} = (7.5)^2 \cdot 6 = \boxed{337.5\text{ W}}$$

$$V_{5\Omega} = 75 - 6(7.5) = 30\text{ V}$$

$$P_{5\Omega} = 30^2/5 = \boxed{180\text{ W}}$$

$$V_{20\Omega} = V_{5\Omega} = 30\text{ V}$$

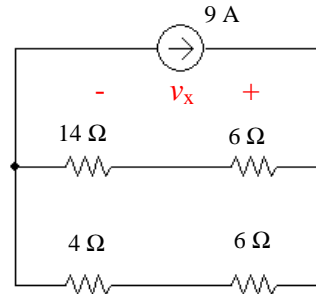
$$P_{20\Omega} = 30^2/20 = \boxed{45\text{ W}}$$

We check our results by verifying that the absorbed powers in fact add to 1000 W.

## CHAPTER THREE SOLUTIONS

54. To begin with, the 10- $\Omega$  and 15- $\Omega$  resistors are in parallel ( $= 6\ \Omega$ ), and so are the 20- $\Omega$  and 5- $\Omega$  resistors ( $= 4\ \Omega$ ).

Also, the 4-A, 1-A and 6-A current sources are in parallel, so they can be combined into a single  $4 + 6 - 1 = 9\text{ A}$  current source as shown:



Next, we note that  $(14\ \Omega + 6\ \Omega) \parallel (4\ \Omega + 6\ \Omega) = 6.667\ \Omega$  so that

$$v_x = 9(6.667) = 60\text{ V}$$

and

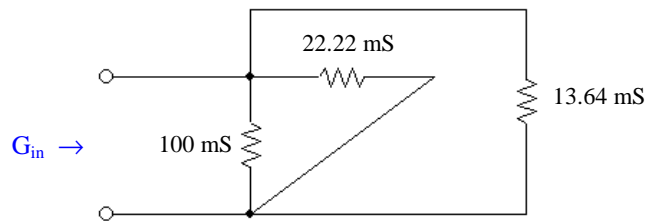
$$i_x = -60/10 = -6\text{ A}.$$

## CHAPTER THREE SOLUTIONS

55. (a) Working from right to left, and borrowing  $x \parallel y$  notation from resistance calculations to indicate the operation  $xy/(x + y)$ ,

$$\begin{aligned} G_{in} &= \{[(6 \parallel 2 \parallel 3) + 0.5] \parallel 1.5 \parallel 2.5 + 0.8\} \parallel 4 \parallel 5 \text{ mS} \\ &= \{[(1) + 0.5] \parallel 1.5 \parallel 2.5 + 0.8\} \parallel 4 \parallel 5 \text{ mS} \\ &= \{1.377\} \parallel 4 \parallel 5 \\ &= 0.8502 \text{ mS} = \boxed{850.2 \text{ mS}} \end{aligned}$$

- (b) The 50-mS and 40-mS conductances are in series, equivalent to  $(50(40)/90) = 22.22 \text{ mS}$ . The 30-mS and 25-mS conductances are also in series, equivalent to  $13.64 \text{ mS}$ . Redrawing for clarity,



we see that  $G_{in} = 10 + 22.22 + 13.64 = \boxed{135.9 \text{ mS}}$ .

## CHAPTER THREE SOLUTIONS

56. The bottom four resistors between the 2- $\Omega$  resistor and the 30-V source are shorted out. The 10- $\Omega$  and 40- $\Omega$  resistors are in parallel ( $= 8\ \Omega$ ), as are the 15- $\Omega$  and 60- $\Omega$  ( $=12\ \Omega$ ) resistors. These combinations are in series.

Define a clockwise current  $I$  through the 1- $\Omega$  resistor:

$$I = (150 - 30)/(2 + 8 + 12 + 3 + 1 + 2) = 4.286\text{ A}$$

$$P_{1\Omega} = I^2 \cdot 1 = 18.37\text{ W}$$

To compute  $P_{10\Omega}$ , consider that since the 10- $\Omega$  and 40- $\Omega$  resistors are in parallel, the same voltage  $V_x$  (“+” reference on the left) appears across both resistors. The current  $I = 4.286\text{ A}$  flows into this combination. Thus,  $V_x = (8)(4.286) = 34.29\text{ V}$  and

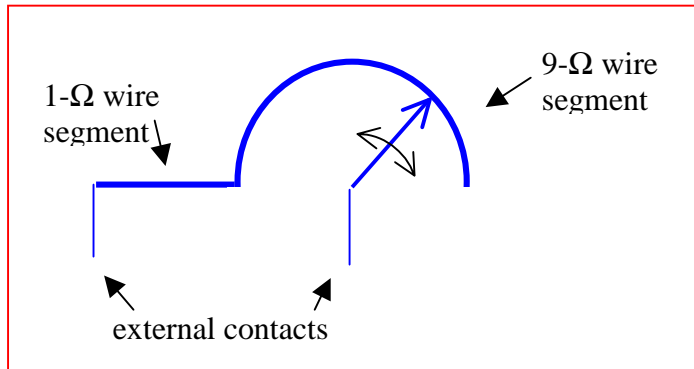
$$P_{10\Omega} = (V_x)^2 / 10 = 117.6\text{ W}.$$

$P_{13\Omega} = 0$  since no current flows through that resistor.

## CHAPTER THREE SOLUTIONS

57. One possible solution of many:

The basic concept is as shown



If we use 28-AWG soft copper wire, we see from Table 2.3 that 9-Ω would require 138 feet, which is somewhat impractical. Referring to p. 4-48 of the *Standard Handbook for Electrical Engineers* (this should be available in most engineering/science libraries), we see that 44-AWG soft copper wire has a resistance of 2590 Ω per 1000 ft, or 0.08497 Ω/cm.

Thus, 1-Ω requires 11.8 cm of 44-AWG wire, and 9-Ω requires 105.9 cm. We decide to make the wiper arm and leads out of 28-AWG wire, which will add a slight resistance to the total value, but a negligible amount.

The radius of the wiper arm should be  $(105.9 \text{ cm})/\pi = 33.7 \text{ cm}$ .

## CHAPTER THREE SOLUTIONS

58. One possible solution of many:

$$\begin{aligned}v_S &= 2(5.5) = 11 \text{ V} \\ R_1 &= R_2 = 1 \text{ k}\Omega.\end{aligned}$$



## CHAPTER THREE SOLUTIONS

59. One possible solution of many:

$$\begin{aligned}i_S &= 11 \text{ mA} \\ R_1 &= R_2 = 1 \text{ k}\Omega.\end{aligned}$$

### CHAPTER THREE SOLUTIONS

60.  $p_{15\Omega} = (v_{15})^2 / 15 \times 10^3 \text{ A}$

$$v_{15} = 15 \times 10^3 (-0.3 v_1)$$

where  $v_1 = [4 (5) / (5 + 2)] \cdot 2 = 5.714 \text{ V}$

Therefore  $v_{15} = -25714 \text{ V}$  and  $p_{15} = 44.08 \text{ kW}$ .

### CHAPTER THREE SOLUTIONS

61. Replace the top  $10\text{ k}\Omega$ ,  $4\text{ k}\Omega$  and  $47\text{ k}\Omega$  resistors with  $10\text{ k}\Omega + 4\text{ k}\Omega \parallel 47\text{ k}\Omega = 13.69\text{ k}\Omega$ .

Define  $v_x$  across the  $10\text{ k}\Omega$  resistor with its “+” reference at the top node: then

$$v_x = 5 \cdot (10\text{ k}\Omega \parallel 13.69\text{ k}\Omega) / (15\text{ k}\Omega + 10 \parallel 13.69\text{ k}\Omega) = 1.391\text{ V}$$

$$i_x = v_x / 10\text{ mA} = 139.1\text{ }\mu\text{A}$$

$$v_{15} = 5 - 1.391 = 3.609\text{ V} \text{ and } p_{15} = (v_{15})^2 / 15 \times 10^3 = 868.3\text{ }\mu\text{W}.$$

## CHAPTER THREE SOLUTIONS

62. We may combine the 12-A and 5-A current sources into a single 7-A current source with its arrow oriented upwards. The left three resistors may be replaced by a  $3 + 6 \parallel 13 = 7.105 \, \Omega$  resistor, and the right three resistors may be replaced by a  $7 + 20 \parallel 4 = 10.33 \, \Omega$  resistor.

By current division,  $i_y = 7 (7.105)/(7.105 + 10.33) = 2.853 \, \text{A}$

We must now return to the original circuit. The current into the  $6 \, \Omega$ ,  $13 \, \Omega$  parallel combination is  $7 - i_y = 4.147 \, \text{A}$ . By current division,

$$i_x = 4.147 \cdot 13 / (13 + 6) = 2.837 \, \text{A}$$

$$\text{and } p_x = (4.147)^2 \cdot 3 = 51.59 \, \text{W}$$

### CHAPTER THREE SOLUTIONS

63. The controlling voltage  $v_1$ , needed to obtain the power into the 47-k $\Omega$  resistor, can be found separately as that network does not depend on the left-hand network. The right-most 2 k $\Omega$  resistor can be neglected.

By current division, then, in combination with Ohm's law,

$$v_1 = 3000[5 \times 10^{-3} (2000) / (2000 + 3000 + 7000)] = 2.5 \text{ V}$$

Voltage division gives the voltage across the 47-k $\Omega$  resistor:

$$0.5v_1 \frac{47}{47 + 100 \parallel 20} = \frac{0.5(2.5)(47)}{47 + 16.67} = 0.9228 \text{ V}$$

$$\text{So that } p_{47\text{k}\Omega} = (0.9228)^2 / 47 \times 10^3 = 18.12 \text{ }\mu\text{W}$$

## CHAPTER THREE SOLUTIONS

64. The temptation to write an equation such as

$$v_1 = 10 \frac{20}{20 + 20}$$

must be fought!

Voltage division only applies to resistors connected in series, meaning that the *same* current must flow through *each* resistor. In this circuit,  $i_1 \neq 0$ , so we do not have the same current flowing through both 20 k $\Omega$  resistors.

## CHAPTER THREE SOLUTIONS

$$\begin{aligned}
 65. \quad (a) \quad v_2 &= V_s \frac{R_2 \parallel (R_3 + R_4)}{R_1 + [R_2 \parallel (R_3 + R_4)]} \\
 &= V_s \frac{R_2 (R_3 + R_4) / (R_2 + R_3 + R_4)}{R_1 + R_2 (R_3 + R_4) / (R_2 + R_3 + R_4)} \\
 &= V_s \frac{R_2 (R_3 + R_4)}{R_1 (R_2 + R_3 + R_4) + R_2 (R_3 + R_4)}
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad v_1 &= V_s \frac{R_1}{R_1 + [R_2 \parallel (R_3 + R_4)]} \\
 &= V_s \frac{R_1}{R_1 + R_2 (R_3 + R_4) / (R_2 + R_3 + R_4)} \\
 &= V_s \frac{R_1 (R_2 + R_3 + R_4)}{R_1 (R_2 + R_3 + R_4) + R_2 (R_3 + R_4)}
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad i_4 &= \left( \frac{v_1}{R_1} \right) \left( \frac{R_2}{R_2 + R_3 + R_4} \right) \\
 &= V_s \frac{R_1 (R_2 + R_3 + R_4) R_2}{R_1 [R_1 (R_2 + R_3 + R_4) + R_2 (R_3 + R_4) (R_2 + R_3 + R_4)]} \\
 &= V_s \frac{R_2}{R_1 (R_2 + R_3 + R_4) + R_2 (R_3 + R_4)}
 \end{aligned}$$

### CHAPTER THREE SOLUTIONS

66. (a) With the current source open-circuited, we find that

$$v_1 = -40 \frac{500}{500 + 3000 \parallel 6000} = \boxed{-8 \text{ V}}$$

- (b) With the voltage source short-circuited, we find that

$$i_2 = (3 \times 10^{-3}) \frac{1/3000}{1/500 + 1/3000 + 1/6000} = \boxed{400 \text{ mA}}$$

$$i_3 = (3 \times 10^{-3}) \frac{500}{500 + 3000 \parallel 6000} = \boxed{600 \text{ mA}}$$



### CHAPTER THREE SOLUTIONS

67. (a) The current through the  $5\text{-}\Omega$  resistor is  $10/5 = 2\text{ A}$ . Define  $R$  as  $3 \parallel (4 + 5) = 2.25\text{ }\Omega$ . The current through the  $2\text{-}\Omega$  resistor then is given by

$$I_s \frac{1}{1 + (2 + R)} = \frac{I_s}{5.25}$$

The current through the  $5\text{-}\Omega$  resistor is

$$\frac{I_s}{5.25} \left( \frac{3}{3 + 9} \right) = 2\text{ A}$$

so that  $I_s = 42\text{ A}$ .

- (b) Given that  $I_s$  is now  $50\text{ A}$ , the current through the  $5\text{-}\Omega$  resistor becomes

$$\frac{I_s}{5.25} \left( \frac{3}{3 + 9} \right) = 2.381\text{ A}$$

Thus,  $v_x = 5(2.381) = 11.90\text{ V}$

$$(c) \frac{v_x}{I_s} = \frac{\left[ \frac{5I_s}{5.25} \left( \frac{3}{3 + 9} \right) \right]}{I_s} = 0.2381$$

### CHAPTER THREE SOLUTIONS

68. First combine the 1 k $\Omega$  and 3 k $\Omega$  resistors to obtain 750  $\Omega$ .  
By current division, the current through resistor  $R_x$  is

$$I_{R_x} = 10 \times 10^{-3} \frac{2000}{2000 + R_x + 750}$$

and we know that  $R_x \cdot I_{R_x} = 9$

$$\text{so } 9 = \frac{20 R_x}{2750 + R_x}$$

$$9 R_x + 24750 = 20 R_x \quad \text{or } R_x = 2250 \text{ W. Thus,}$$

$$P_{R_x} = 9^2 / R_x = 36 \text{ mW.}$$

### CHAPTER THREE SOLUTIONS

69. Define  $R = R_3 \parallel (R_4 + R_5)$

$$\begin{aligned}\text{Then } v_R &= V_s \left( \frac{R}{R + R_2} \right) \\ &= V_s \left( \frac{R_3 (R_4 + R_5) / (R_3 + R_4 + R_5)}{R_3 (R_4 + R_5) / (R_3 + R_4 + R_5) + R_2} \right) \\ &= V_s \left( \frac{R_3 (R_4 + R_5)}{R_2 (R_3 + R_4 + R_5) + R_3 (R_4 + R_5)} \right)\end{aligned}$$

Thus,

$$\begin{aligned}v_5 &= v_R \left( \frac{R_5}{R_4 + R_5} \right) \\ &= V_s \left( \frac{R_3 R_5}{R_2 (R_3 + R_4 + R_5) + R_3 (R_4 + R_5)} \right)\end{aligned}$$

### CHAPTER THREE SOLUTIONS

70. Define  $R_1 = 10 + 15 \parallel 30 = 20 \, \Omega$  and  $R_2 = 5 + 25 = 30 \, \Omega$ .

---

$$(a) I_x = I_1 \cdot 15 / (15 + 30) = 4 \text{ mA}$$

$$(b) I_1 = I_x \cdot 45 / 15 = 36 \text{ mA}$$

$$(c) I_2 = I_S R_1 / (R_1 + R_2) \text{ and } I_1 = I_S R_2 / (R_1 + R_2)$$

So  $I_1/I_2 = R_2/R_1$

Therefore

$$I_1 = R_2 I_2 / R_1 = 30(15)/20 = 22.5 \text{ mA}$$

$$\text{Thus, } I_x = I_1 \cdot 15 / 45 = 7.5 \text{ mA}$$

$$(d) I_1 = I_S R_2 / (R_1 + R_2) = 60(30) / 50 = 36 \text{ A}$$

$$\text{Thus, } I_x = I_1 \cdot 15 / 45 = 12 \text{ A.}$$

### CHAPTER THREE SOLUTIONS

71.  $v_{\text{out}} = -g_m v_\pi (100 \text{ k}\Omega \parallel 100 \text{ k}\Omega) = -4.762 \times 10^3 g_m v_\pi$

where  $v_\pi = (3 \sin 10t) \cdot 15/(15 + 0.3) = 2.941 \sin 10t$

Thus,  $v_{\text{out}} = -56.02 \sin 10t \text{ V}$

## CHAPTER THREE SOLUTIONS

72.  $v_{\text{out}} = -1000g_m v_\pi$

where  $v_\pi = 3 \sin 10t \frac{15 \parallel 3}{(15 \parallel 3) + 0.3} = 2.679 \sin 10t \text{ V}$

therefore

$$v_{\text{out}} = -(2.679)(1000)(38 \times 10^{-3}) \sin 10t = \boxed{-101.8 \sin 10t \text{ V.}}$$

## CHAPTER FOUR SOLUTIONS

$$1. \quad (a) \quad \begin{bmatrix} 0.1 & -0.3 & -0.4 \\ -0.5 & 0.1 & 0 \\ -0.2 & -0.3 & 0.4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 6 \end{bmatrix}$$

Solving this matrix equation using a scientific calculator,  $v_2 = -8.387 \text{ V}$

(b) Using a scientific calculator, the determinant is equal to  $32$ .

## CHAPTER FOUR SOLUTIONS

$$2. \quad (a) \quad \begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 3 \\ 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} = \begin{bmatrix} 27 \\ -16 \\ -6 \end{bmatrix}$$

Solving this matrix equation using a scientific calculator,

$$\begin{aligned} v_A &= 19.57 \\ v_B &= 18.71 \\ v_C &= -11.29 \end{aligned}$$

(b) Using a scientific calculator,

$$\begin{vmatrix} 1 & 1 & 1 \\ -1 & 2 & 3 \\ 2 & 0 & 4 \end{vmatrix} = 16$$



## CHAPTER FOUR SOLUTIONS

3. The bottom node has the largest number of branch connections, so we choose that as our reference node. This also makes  $v_P$  easier to find, as it will be a nodal voltage. Working from left to right, we name our nodes 1, P, 2, and 3.

$$\text{NODE 1:} \quad 10 = v_1/20 + (v_1 - v_P)/40 \quad [1]$$

$$\text{NODE P:} \quad 0 = (v_P - v_1)/40 + v_P/100 + (v_P - v_2)/50 \quad [2]$$

$$\text{NODE 2:} \quad -2.5 + 2 = (v_2 - v_P)/50 + (v_2 - v_3)/10 \quad [3]$$

$$\text{NODE 3:} \quad 5 - 2 = v_3/200 + (v_3 - v_2)/10 \quad [4]$$

Simplifying,

$$60v_1 - 20v_P = 8000 \quad [1]$$

$$-50v_1 + 110v_P - 40v_2 = 0 \quad [2]$$

$$-v_P + 6v_2 - 5v_3 = -25 \quad [3]$$

$$-200v_2 + 210v_3 = 6000 \quad [4]$$

Solving,

$$v_P = 171.6 \text{ V}$$

## CHAPTER FOUR SOLUTIONS

4. The logical choice for a reference node is the bottom node, as then  $v_x$  will automatically become a nodal voltage.

$$\text{NODE 1:} \quad 4 = v_1/100 + (v_1 - v_2)/20 + (v_1 - v_x)/50 \quad [1]$$

$$\text{NODE } x: \quad 10 - 4 - (-2) = (v_x - v_1)/50 + (v_x - v_2)/40 \quad [2]$$

$$\text{NODE 2:} \quad -2 = v_2/25 + (v_2 - v_x)/40 + (v_2 - v_1)/20 \quad [3]$$

Simplifying,

$$4 = 0.0800v_1 - 0.0500v_2 - 0.0200v_x \quad [1]$$

$$8 = -0.0200v_1 - 0.0250v_2 + 0.0450v_x \quad [2]$$

$$-2 = -0.0500v_1 + 0.1150v_2 - 0.0250v_x \quad [3]$$

Solving,

$$v_x = 397.4 \text{ V.}$$

## CHAPTER FOUR SOLUTIONS

5. Designate the node between the 3- $\Omega$  and 6- $\Omega$  resistors as node X, and the right-hand node of the 6- $\Omega$  resistor as node Y. The bottom node is chosen as the reference node.
- 

(a) Writing the two nodal equations, then

$$\text{NODE X:} \quad -10 = (v_X - 240)/3 + (v_X - v_Y)/6 \quad [1]$$

$$\text{NODE Y:} \quad 0 = (v_Y - v_X)/6 + v_Y/30 + (v_Y - 60)/12 \quad [2]$$

$$\text{Simplifying,} \quad -180 + 1440 = 9 v_X - 3 v_Y \quad [1]$$

$$10800 = -360 v_X + 612 v_Y \quad [2]$$

$$\text{Solving,} \quad v_X = 181.5 \text{ V} \quad \text{and} \quad v_Y = 124.4 \text{ V}$$

$$\text{Thus,} \quad v_1 = 240 - v_X = \boxed{58.50 \text{ V}} \quad \text{and} \quad v_2 = v_Y - 60 = \boxed{64.40 \text{ V}}$$

(b) The power absorbed by the 6- $\Omega$  resistor is

$$(v_X - v_Y)^2 / 6 = \boxed{543.4 \text{ W}}$$

## CHAPTER FOUR SOLUTIONS

6. Only one nodal equation is required: At the node where three resistors join,

$$0.02v_1 = (v_x - 5 i_2) / 45 + (v_x - 100) / 30 + (v_x - 0.2 v_3) / 50 \quad [1]$$

This, however, is one equation in four unknowns, the other three resulting from the presence of the dependent sources. Thus, we require three additional equations:

$$i_2 = (0.2 v_3 - v_x) / 50 \quad [2]$$

$$v_1 = 0.2 v_3 - 100 \quad [3]$$

$$v_3 = 50i_2 \quad [4]$$

Simplifying,

$$v_1 - 0.2v_3 = -100 \quad [3]$$

$$-v_3 + 50 i_2 = 0 \quad [4]$$

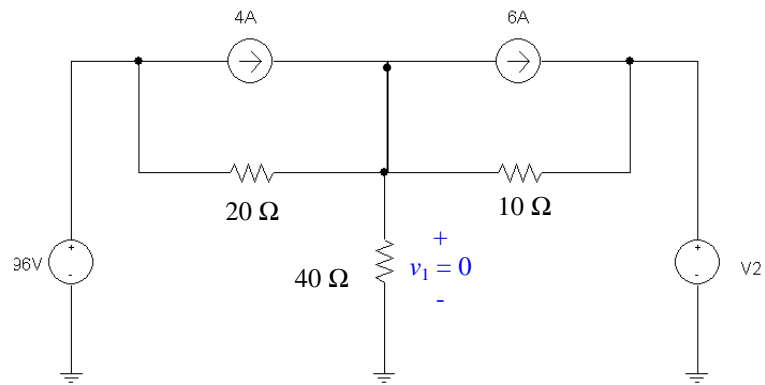
$$-v_x + 0.2v_3 - 50 i_2 = 0 \quad [2]$$

$$0.07556v_x - 0.02v_1 - 0.004v_3 - 0.111i_2 = 33.33 \quad [1]$$

Solving, we find that  $v_1 = -103.8 \text{ V}$  and  $i_2 = -377.4 \text{ mA}$ .

## CHAPTER FOUR SOLUTIONS

7. If  $v_1 = 0$ , the dependent source is a short circuit and we may redraw the circuit as:



At NODE 1:  $4 - 6 = v_1 / 40 + (v_1 - 96) / 20 + (v_1 - V_2) / 10$

Since  $v_1 = 0$ , this simplifies to

$$-2 = -96 / 20 - V_2 / 10$$

so that  $V_2 = -28 \text{ V.}$

## CHAPTER FOUR SOLUTIONS

8. We choose the bottom node as ground to make calculation of  $i_5$  easier. The left-most node is named “1”, the top node is named “2”, the central node is named “3” and the node between the 4- $\Omega$  and 6- $\Omega$  resistors is named “4.”

$$\text{NODE 1:} \quad -3 = v_1/2 + (v_1 - v_2)/1 \quad [1]$$

$$\text{NODE 2:} \quad 2 = (v_2 - v_1)/1 + (v_2 - v_3)/3 + (v_2 - v_4)/4 \quad [2]$$

$$\text{NODE 3:} \quad 3 = v_3/5 + (v_3 - v_4)/7 + (v_3 - v_2)/3 \quad [3]$$

$$\text{NODE 4:} \quad 0 = v_4/6 + (v_4 - v_3)/7 + (v_4 - v_2)/4 \quad [4]$$

Rearranging and grouping terms,

$$3v_1 - 2v_2 = -6 \quad [1]$$

$$-12v_1 + 19v_2 - 4v_3 - 3v_4 = 24 \quad [2]$$

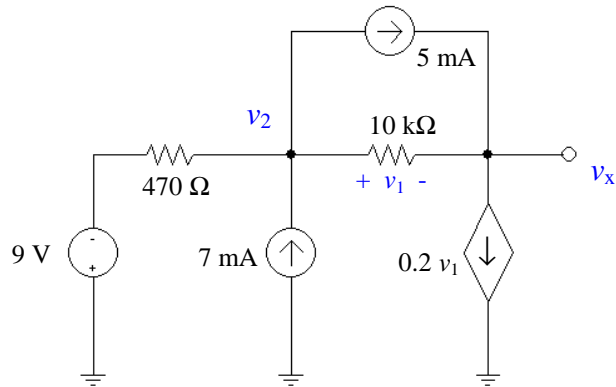
$$-35v_2 + 71v_3 - 15v_4 = 315 \quad [3]$$

$$-42v_2 - 24v_3 + 94v_4 = 0 \quad [4]$$

Solving, we find that  $v_3 = 6.760$  V and so  $i_5 = v_3/5 = 1.352$  A.

## CHAPTER FOUR SOLUTIONS

9. We can redraw this circuit and eliminate the 2.2-k $\Omega$  resistor as no current flows through it:



$$\text{At NODE 2: } 7 \times 10^{-3} - 5 \times 10^{-3} = (v_2 + 9)/470 + (v_2 - v_x)/10 \times 10^{-3} \quad [1]$$

$$\text{At NODE } x: 5 \times 10^{-3} - 0.2v_1 = (v_x - v_2)/10 \times 10^{-3} \quad [2]$$

The additional equation required by the presence of the dependent source and the fact that its controlling variable is not one of the nodal voltages:

$$v_1 = v_2 - v_x \quad [3]$$

Eliminating the variable  $v_1$  and grouping terms, we obtain:

$$10,470 v_2 - 470 v_x = -89,518$$

and

$$1999 v_2 - 1999 v_x = 50$$

Solving, we find  $v_x = -8.086 \text{ V.}$

## CHAPTER FOUR SOLUTIONS

10. We need concern ourselves with the bottom part of this circuit only. Writing a single nodal equation,

$$-4 + 2 = v/50$$

We find that

$$v = -100 \text{ V.}$$



## CHAPTER FOUR SOLUTIONS

11. We choose the center node for our common terminal, since it connects to the largest number of branches. We name the left node “A”, the top node “B”, the right node “C”, and the bottom node “D”. We next form a supernode between nodes A and B.

$$\text{At the supernode:} \quad 5 = (V_A - V_B)/10 + V_A/20 + (V_B - V_C)/12.5 \quad [1]$$

$$\text{At node C:} \quad V_C = 150 \quad [2]$$

$$\text{At node D:} \quad -10 = V_D/25 + (V_D - V_A)/10 \quad [3]$$

$$\text{Our supernode-related equation is } V_B - V_A = 100 \quad [4]$$

Simplifying and grouping terms,

$$0.15 V_A + 0.08 V_B - 0.08 V_C - 0.1 V_D = 5 \quad [1]$$

$$V_C = 150 \quad [2]$$

$$-25 V_A + 35 V_D = -2500 \quad [3]$$

$$-V_A + V_B = 100 \quad [4]$$

Solving, we find that  $V_D = -63.06 \text{ V}$ . Since  $v_4 = -V_D$ ,

$$v_4 = 63.06 \text{ V.}$$

## CHAPTER FOUR SOLUTIONS

12. Choosing the bottom node as the reference terminal and naming the left node “1”, the center node “2” and the right node “3”, we next form a supernode about nodes 1 and 2, encompassing the dependent voltage source.

$$\text{At the supernode,} \quad 5 - 8 = (v_1 - v_2)/2 + v_3/2.5 \quad [1]$$

$$\text{At node 2,} \quad 8 = v_2/5 + (v_2 - v_1)/2 \quad [2]$$

$$\text{Our supernode equation is } v_1 - v_3 = 0.8 v_A \quad [3]$$

$$\text{Since } v_A = v_2, \text{ we can rewrite [3] as } v_1 - v_3 = 0.8 v_2$$

Simplifying and collecting terms,

$$0.5 v_1 - 0.5 v_2 + 0.4 v_3 = -3 \quad [1]$$

$$-0.5 v_1 + 0.7 v_2 = 8 \quad [2]$$

$$v_1 - 0.8 v_2 - v_3 = 0 \quad [3]$$

---

(a) Solving for  $v_2 = v_A$ , we find that  $v_A = 25.91 \text{ V}$

(b) The power absorbed by the  $2.5\text{-}\Omega$  resistor is  $(v_3)^2/2.5 = (-0.4546)^2/2.5 = 82.66 \text{ mW}.$

## CHAPTER FOUR SOLUTIONS

13. Selecting the bottom node as the reference terminal, we name the left node “1”, the middle node “2” and the right node “3.”

$$\text{NODE 1:} \quad 5 = (v_1 - v_2)/20 + (v_1 - v_3)/50 \quad [1]$$

$$\text{NODE 2:} \quad v_2 = 0.4 v_1 \quad [2]$$

$$\text{NODE 3:} \quad 0.01 v_1 = (v_3 - v_2)/30 + (v_3 - v_1)/50 \quad [3]$$

Simplifying and collecting terms, we obtain

$$0.07 v_1 - 0.05 v_2 - 0.02 v_3 = 5 \quad [1]$$

$$0.4 v_1 - v_2 = 0 \quad [2]$$

$$-0.03 v_1 - 0.03333 v_2 + 0.05333 v_3 = 0 \quad [3]$$

Since our choice of reference terminal makes the controlling variable of both dependent sources a nodal voltage, we have no need for an additional equation as we might have expected.

Solving, we find that  $v_1 = 148.2 \text{ V}$ ,  $v_2 = 59.26 \text{ V}$ , and  $v_3 = 120.4 \text{ V}$ .

The power supplied by the dependent current source is therefore

$$(0.01 v_1) \cdot v_3 = 177.4 \text{ W.}$$

## CHAPTER FOUR SOLUTIONS

14. At node x:  $v_x/4 + (v_x - v_y)/2 + (v_x - 6)/1 = 0$  [1]  
At node y:  $(v_y - kv_x)/3 + (v_y - v_x)/2 = 2$  [2]

Our additional constraint is that  $v_y = 0$ , so we may simplify Eqs. [1] and [2]:

$$14 v_x = 48 \quad [1]$$

$$-2k v_x - 3 v_x = 12 \quad [2]$$

Since Eq. [1] yields  $v_x = 48/14 = 3.429 \text{ V}$ , we find that

$$k = (12 + 3 v_x)/(-2 v_x) = \boxed{-3.250}$$

## CHAPTER FOUR SOLUTIONS

15. Choosing the bottom node joining the 4- $\Omega$  resistor, the 2-A current source and the 4-V voltage source as our reference node, we next name the other node of the 4- $\Omega$  resistor node “1”, and the node joining the 2- $\Omega$  resistor and the 2-A current source node “2.” Finally, we create a supernode with nodes “1” and “2.”

At the supernode:  $-2 = v_1/4 + (v_2 - 4)/2$  [1]

Our remaining equations:  $v_1 - v_2 = -3 - 0.5i_1$  [2]

and  $i_1 = (v_2 - 4)/2$  [3]

Equation [1] simplifies to  $v_1 + 2v_2 = 0$  [1]

Combining Eqs. [2] and [3],  $4v_1 - 3v_2 = -8$  [4]

Solving these last two equations, we find that  $v_2 = 727.3$  mV. Making use of Eq. [3], we therefore find that

$$i_1 = -1.636 \text{ A.}$$

## CHAPTER FOUR SOLUTIONS

16. We first number the nodes as 1, 2, 3, 4, and 5 moving left to right. We next select node 5 as the reference terminal. To simplify the analysis, we form a supernode from nodes 1, 2, and 3.

*At the supernode,*

$$-4 - 8 + 6 = v_1/40 + (v_1 - v_3)/10 + (v_3 - v_1)/10 + v_2/50 + (v_3 - v_4)/20 \quad [1]$$

Note that since both ends of the 10- $\Omega$  resistor are connected to the supernode, the related terms cancel each other out, and so could have been ignored.

*At node 4:*  $v_4 = 200$  [2]

*Supernode KVL equation:*  $v_1 - v_3 = 400 + 4v_{20}$  [3]

Where the controlling voltage  $v_{20} = v_3 - v_4 = v_3 - 200$  [4]

Thus, Eq. [1] becomes  $-6 = v_1/40 + v_2/50 + (v_3 - 200)/20$  or, more simply,

$$4 = v_1/40 + v_2/50 + v_3/20 \quad [1']$$

and Eq. [3] becomes  $v_1 - 5v_3 = -400$  [3']

Eqs. [1'], [3'], and [5] are not sufficient, however, as we have four unknowns. At this point we need to seek an additional equation, possibly in terms of  $v_2$ . Referring to the circuit,

$$v_1 - v_2 = 400 \quad [5]$$

Rewriting as a matrix equation,

$$\begin{bmatrix} 1/40 & 1/50 & 1/20 \\ 1 & 0 & -5 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -400 \\ 400 \end{bmatrix}$$

Solving, we find that

$v_1 = 145.5$  V,  $v_2 = -254.5$  V, and  $v_3 = 109.1$  V. Since  $v_{20} = v_3 - 200$ , we find that

$v_{20} = -90.9$  V.

## CHAPTER FOUR SOLUTIONS

17. We begin by naming the top left node “1”, the top right node “2”, the bottom node of the 6-V source “3” and the top node of the 2- $\Omega$  resistor “4.” The reference node has already been selected, and designated using a ground symbol.

By inspection,  $v_2 = 5 \text{ V}$ .

Forming a supernode with nodes 1 & 3, we find

$$\text{At the supernode:} \quad -2 = v_3/1 + (v_1 - 5)/10 \quad [1]$$

$$\text{At node 4:} \quad 2 = v_4/2 + (v_4 - 5)/4 \quad [2]$$

$$\text{Our supernode KVL equation:} \quad v_1 - v_3 = 6 \quad [3]$$

Rearranging, simplifying and collecting terms,

$$v_1 + 10 v_3 = -20 + 5 = -15 \quad [1]$$

and

$$v_1 - v_3 = 6 \quad [2]$$

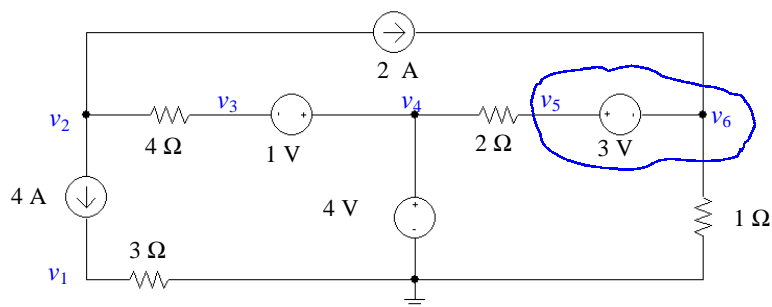
Eq. [3] may be directly solved to obtain  $v_4 = 4.333 \text{ V}$ .

Solving Eqs. [1] and [2], we find that

$$v_1 = 4.091 \text{ V} \quad \text{and} \quad v_3 = -1.909 \text{ V}.$$

## CHAPTER FOUR SOLUTIONS

18. We begin by selecting the bottom node as the reference, naming the nodes as shown below, and forming a supernode with nodes 5 & 6.



By inspection,  $v_4 = 4 \text{ V}$ .

By KVL,  $v_3 - v_4 = 1$  so  $v_3 = -1 + v_4 = -1 + 4$  or  $v_3 = 3 \text{ V}$ .

At the supernode,  $2 = v_6/1 + (v_5 - 4)/2$  [1]

At node 1,  $4 = v_1/3$  therefore,  $v_1 = 12 \text{ V}$ .

At node 2,  $-4 - 2 = (v_2 - 3)/4$

Solving, we find that  $v_2 = -21 \text{ V}$

Our supernode KVL equation is  $v_5 - v_6 = 3$  [2]

Solving Eqs. [1] and [2], we find that

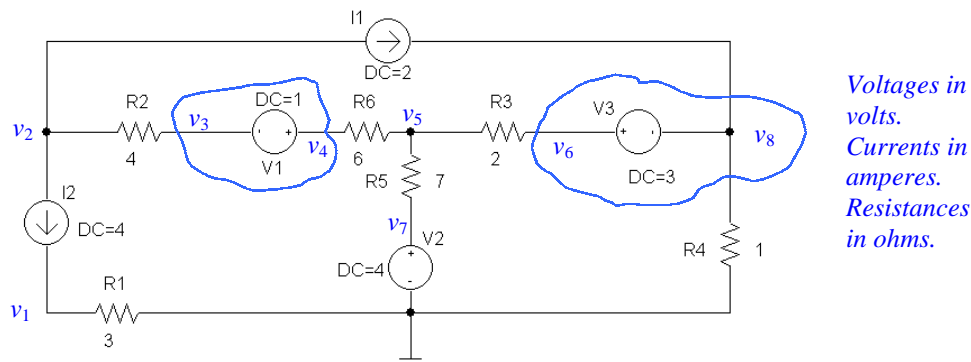
$$v_5 = 4.667 \text{ V} \quad \text{and} \quad v_6 = 1.667 \text{ V}.$$

The power supplied by the 2-A source therefore is  $(v_6 - v_2)(2) = 45.33 \text{ W}$ .



## CHAPTER FOUR SOLUTIONS

19. We begin by selecting the bottom node as the reference, naming each node as shown below, and forming two different supernodes as indicated.



By inspection,  $v_7 = 4 \text{ V}$  and  $v_1 = (3)(4) = 12 \text{ V}$ .

At node 2:  $-4 - 2 = (v_2 - v_3)/4$  or  $v_2 - v_3 = -24$  [1]

At the 3-4 supernode:  
 $0 = (v_3 - v_2)/4 + (v_4 - v_5)/6$  or  $-6v_2 + 6v_3 + 4v_4 - 4v_5 = 0$  [2]

At node 5:  
 $0 = (v_5 - v_4)/6 + (v_5 - 4)/7 + (v_5 - v_6)/2$  or  $-14v_4 + 68v_5 - 42v_6 = 48$  [3]

At the 6-8 supernode:  $2 = (v_6 - v_5)/2 + v_8/1$  or  $-v_5 + v_6 + 2v_8 = 4$  [4]

3-4 supernode KVL equation:  $v_3 - v_4 = -1$  [5]

6-8 supernode KVL equation:  $v_6 - v_8 = 3$  [6]

Rewriting Eqs. [1] to [6] in matrix form,

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -6 & 6 & 4 & -4 & 0 & 0 \\ 0 & 0 & -14 & 68 & -42 & 0 \\ 0 & 0 & 0 & -1 & 1 & 2 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_8 \end{bmatrix} = \begin{bmatrix} -24 \\ 0 \\ 48 \\ 4 \\ -1 \\ 3 \end{bmatrix}$$

Solving, we find that

$$v_2 = -68.9 \text{ V}, v_3 = -44.9 \text{ V}, v_4 = -43.9 \text{ V}, v_5 = -7.9 \text{ V}, v_6 = 700 \text{ mV}, v_8 = -2.3 \text{ V}.$$

The power generated by the 2-A source is therefore  $(v_8 - v_6)(2) = 133.2 \text{ W}$ .

## CHAPTER FOUR SOLUTIONS

20. With the reference terminal already specified, we name the bottom terminal of the 3-mA source node “1,” the right terminal of the bottom 2.2-k $\Omega$  resistor node “2,” the top terminal of the 3-mA source node “3,” the “+” reference terminal of the 9-V source node “4,” and the “-” terminal of the 9-V source node “5.”

Since we know that 1 mA flows through the top 2.2-k $\Omega$  resistor,  $v_5 = -2.2$  V.

Also, we see that  $v_4 - v_5 = 9$ , so that  $v_4 = 9 - 2.2 = 6.8$  V.

Proceeding with nodal analysis,

$$\text{At node 1: } -3 \times 10^{-3} = v_1 / 10 \times 10^3 + (v_1 - v_2) / 2.2 \times 10^3 \quad [1]$$

$$\text{At node 2: } 0 = (v_2 - v_1) / 2.2 \times 10^3 + (v_2 - v_3) / 4.7 \times 10^3 \quad [2]$$

$$\text{At node 3: } 1 \times 10^3 + 3 \times 10^3 = (v_3 - v_2) / 4.7 \times 10^3 + v_3 / 3.3 \times 10^3 \quad [3]$$

Solving,  $v_1 = -8.614$  V,  $v_2 = -3.909$  V and  $v_3 = 6.143$  V.

Note that we could also have made use of the supernode approach here.

## CHAPTER FOUR SOLUTIONS

21. Moving from left to right, we name the bottom three meshes, mesh “1”, mesh “2,” and mesh “3.” In each of these three meshes we define a clockwise current. The remaining mesh current is clearly 8 A. We may then write:

$$\text{MESH 1:} \quad 12 i_1 - 4 i_2 = 100$$

$$\text{MESH 2:} \quad -4 i_1 + 9 i_2 - 3 i_3 = 0$$

$$\text{MESH 3:} \quad -3 i_2 + 18 i_3 = -80$$

Solving this system of three (independent) equations in three unknowns, we find that

$$i_2 = i_x = 2.791 \text{ A.}$$

## CHAPTER FOUR SOLUTIONS

22. We define four clockwise mesh currents. The top mesh current is labeled  $i_4$ . The bottom left mesh current is labeled  $i_1$ , the bottom right mesh current is labeled  $i_3$ , and the remaining mesh current is labeled  $i_2$ . Define a voltage " $v_{4A}$ " across the 4-A current source with the "+" reference terminal on the left.

By inspection,  $i_3 = 5 \text{ A}$  and  $i_a = i_4$ .

$$\text{MESH 1: } -60 + 2i_1 - 2i_4 + 6i_4 = 0 \quad \text{or} \quad 2i_1 + 4i_4 = 60 \quad [1]$$

$$\text{MESH 2: } -6i_4 + v_{4A} + 4i_2 - 4(5) = 0 \quad \text{or} \quad 4i_2 - 6i_4 + v_{4A} = 30 \quad [2]$$

$$\text{MESH 4: } 2i_4 - 2i_1 + 5i_4 + 3i_4 - 3(5) - v_{4A} = 0 \quad \text{or} \quad -2i_1 + 10i_4 - v_{4A} = 15 \quad [3]$$

At this point, we are short an equation. Returning to the circuit diagram, we note that

$$i_2 - i_4 = 4 \quad [4]$$

Collecting these equations and writing in matrix form, we have

$$\begin{bmatrix} 2 & 0 & 4 & 0 \\ 0 & 4 & -6 & 1 \\ -2 & 0 & 10 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_4 \\ v_{4A} \end{bmatrix} = \begin{bmatrix} 60 \\ 20 \\ 15 \\ 4 \end{bmatrix}$$

Solving,  $i_1 = 16.83 \text{ A}$ ,  $i_2 = 10.58 \text{ A}$ ,  $i_4 = 6.583 \text{ A}$  and  $v_{4A} = 17.17 \text{ V}$ .

Thus, the power dissipated by the  $2\text{-}\Omega$  resistor is

$$(i_1 - i_4)^2 \cdot (2) = 210.0 \text{ W}$$

## CHAPTER FOUR SOLUTIONS

23. We begin our analysis by defining three clockwise mesh currents. We will call the top mesh current  $i_3$ , the bottom left mesh current  $i_1$ , and the bottom right mesh current  $i_2$ .

By inspection,  $i_1 = 5 \text{ A}$  [1] and  $i_2 = -0.01 v_1$  [2]

MESH 3:  $50 i_3 + 30 i_3 - 30 i_2 + 20 i_3 - 20 i_1 = 0$   
or  $-20 i_1 - 30 i_2 + 100 i_3 = 0$  [3]

These three equations are insufficient, however, to solve for the unknowns. It would be nice to be able to express the dependent source controlling variable  $v_1$  in terms of the mesh currents. Returning to the diagram, it can be seen that KVL around mesh 1 will yield

$-v_1 + 20 i_1 - 20 i_3 + 0.4 v_1 = 0$   
or  $v_1 = 20 i_1 / 0.6 - 20 i_3 / 0.6$  or  $v_1 = (20(5) / 0.6 - 20 i_3 / 0.6)$  [4]

Substituting Eq. [4] into Eq. [2] and then the modified Eq. [2] into Eq. [3], we find

$$-20(5) - 30(-0.01)(20(5)/0.6 - 20 i_3 / 0.6) + 100 i_3 = 0$$

Solving, we find that  $i_3 = (100 - 50) / 90 = 555.6 \text{ mA}$

Thus,  $v_1 = 148.1 \text{ V}$ ,  $i_2 = -1.481 \text{ A}$ , and the power generated by the dependent voltage source is

$$0.4 v_1 (i_2 - i_1) = -383.9 \text{ W.}$$

## CHAPTER FOUR SOLUTIONS

24. We begin by defining four clockwise mesh currents  $i_1$ ,  $i_2$ ,  $i_3$  and  $i_4$ , in the meshes of our circuit, starting at the left-most mesh. We also define a voltage  $v_{\text{dep}}$  across the dependent current source, with the “+” on the top.

By inspection,  $i_1 = 2\text{ A}$  and  $i_4 = -5\text{ A}$ .

$$\text{At Mesh 2: } 10 i_2 - 10(2) + 20 i_2 + v_{\text{dep}} = 0 \quad [1]$$

$$\text{At Mesh 3: } -v_{\text{dep}} + 25 i_3 + 5 i_3 - 5(-5) = 0 \quad [2]$$

Collecting terms, we rewrite Eqs. [1] and [2] as

$$30 i_2 + v_{\text{dep}} = 20 \quad [1]$$

$$30 i_3 - v_{\text{dep}} = -25 \quad [2]$$

This is only two equations but three unknowns, however, so we require an additional equation. Returning to the circuit diagram, we note that it is possible to express the current of the dependent source in terms of mesh currents. (We might also choose to obtain an expression for  $v_{\text{dep}}$  in terms of mesh currents using KVL around mesh 2 or 3.)

$$\text{Thus, } 1.5i_x = i_3 - i_2 \text{ where } i_x = i_1 - i_2 \text{ so } -0.5 i_2 - i_3 = -3 \quad [3]$$

In matrix form,

$$\begin{bmatrix} 30 & 0 & 1 \\ 0 & 30 & -1 \\ -0.5 & -1 & 0 \end{bmatrix} \begin{bmatrix} i_2 \\ i_3 \\ v_{\text{dep}} \end{bmatrix} = \begin{bmatrix} 20 \\ -25 \\ -3 \end{bmatrix}$$

Solving, we find that  $i_2 = -6.333\text{ A}$  so that  $i_x = i_1 - i_2 = 8.333\text{ A}$ .

## CHAPTER FOUR SOLUTIONS

25. We define a clockwise mesh current  $i_1$  in the bottom left mesh, a clockwise mesh current  $i_2$  in the top left mesh, a clockwise mesh current  $i_3$  in the top right mesh, and a clockwise mesh current  $i_4$  in the bottom right mesh.

$$\text{MESH 1:} \quad -0.1 v_a + 4700 i_1 - 4700 i_2 + 4700 i_1 - 4700 i_4 = 0 \quad [1]$$

$$\text{MESH 2:} \quad 9400 i_2 - 4700 i_1 - 9 = 0 \quad [2]$$

$$\text{MESH 3:} \quad 9 + 9400 i_3 - 4700 i_4 = 0 \quad [3]$$

$$\text{MESH 4:} \quad 9400 i_4 - 4700 i_1 - 4700 i_3 + 0.1 i_x = 0 \quad [4]$$

The presence of the two dependent sources has led to the introduction of two additional unknowns ( $i_x$  and  $v_a$ ) besides our four mesh currents. In a perfect world, it would simplify the solution if we could express these two quantities in terms of the mesh currents.

Referring to the circuit diagram, we see that  $i_x = i_2$  (easy enough) and that  $v_a = 4700 i_3$  (also straightforward). Thus, substituting these expressions into our four mesh equations and creating a matrix equation, we arrive at:

$$\begin{bmatrix} 9400 & -4700 & -470 & -4700 \\ -4700 & 9400 & 0 & 0 \\ 0 & 0 & 9400 & -4700 \\ -4700 & 0.1 & -4700 & 9400 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \\ -9 \\ 0 \end{bmatrix}$$

Solving,

$$i_1 = 239.3 \mu\text{A}, i_2 = 1.077 \text{ mA}, i_3 = -1.197 \text{ mA} \text{ and } i_4 = -478.8 \mu\text{A}.$$

## CHAPTER FOUR SOLUTIONS

26. We define a clockwise mesh current  $i_3$  in the upper right mesh, a clockwise mesh current  $i_1$  in the lower left mesh, and a clockwise mesh current  $i_2$  in the lower right mesh.

$$\text{MESH 1:} \quad -6 + 6 i_1 - 2 = 0 \quad [1]$$

$$\text{MESH 2:} \quad 2 + 15 i_2 - 12 i_3 - 1.5 = 0 \quad [2]$$

$$\text{MESH 3:} \quad i_3 = 0.1 v_x \quad [3]$$

Eq. [1] may be solved directly to obtain  $i_1 = 1.333 \text{ A.}$

It would help in the solution of Eqs. [2] and [3] if we could express the dependent source controlling variable  $v_x$  in terms of mesh currents. Referring to the circuit diagram, we see that  $v_x = (1)(i_1) = i_1$ , so Eq. [3] reduces to

$$i_3 = 0.1 v_x = 0.1 i_1 = 133.3 \text{ mA.}$$

As a result, Eq. [1] reduces to  $i_2 = [-0.5 + 12(0.1333)]/15 = 73.31 \text{ mA.}$



## CHAPTER FOUR SOLUTIONS

27. (a) Define a mesh current  $i_2$  in the second mesh. Then KVL allows us to write:

$$\text{MESH 1: } -9 + R i_1 + 47000 i_1 - 47000 i_2 = 0 \quad [1]$$

$$\text{MESH 2: } 67000 i_2 - 47000 i_1 - 5 = 0 \quad [2]$$

Given that  $i_1 = 1.5 \text{ mA}$ , we may solve Eq. [2] to find that

$$i_2 = \frac{5 + 47(1.5)}{67} \text{ mA} = 1.127 \text{ mA}$$

and so

$$R = \frac{9 - 47(1.5) + 47(1.127)}{1.5 \times 10^{-3}} = -5687 \, \Omega.$$

- (b) This value of  $R$  is unique; no other value will satisfy **both** Eqs. [1] **and** [2].

## CHAPTER FOUR SOLUTIONS

28. Define three clockwise mesh currents  $i_1$ ,  $i_2$  and  $i_3$ . The bottom 1-k $\Omega$  resistor can be ignored, as no current flows through it.

$$\text{MESH 1:} \quad -4 + (2700 + 1000 + 5000) i_1 - 1000 i_2 = 0 \quad [1]$$

$$\text{MESH 2:} \quad (1000 + 1000 + 4400 + 3000) i_2 - 1000 i_1 - 4400 i_3 + 2.2 - 3 = 0 \quad [2]$$

$$\text{MESH 3:} \quad (4400 + 4000 + 3000) i_3 - 4400 i_2 - 1.5 = 0 \quad [3]$$

Combining terms,

$$8700 i_1 - 1000 i_2 = 4 \quad [1]$$

$$-1000 i_1 + 9400 i_2 - 4400 i_3 = 0.8 \quad [2]$$

$$-4400 i_2 + 11400 i_3 = 1.5 \quad [3]$$

Solving,

$$i_1 = 487.6 \text{ mA}, i_2 = 242.4 \text{ mA and } i_3 = 225.1 \text{ mA}.$$

The power absorbed by each resistor may now be calculated:

$P_{5k}$	=	$5000 (i_1)^2$	=	1.189 mW
$P_{2.7k}$	=	$2700 (i_1)^2$	=	641.9 $\mu$ W
$P_{1k\text{top}}$	=	$1000 (i_1 - i_2)^2$	=	60.12 $\mu$ W
$P_{1k\text{middle}}$	=	$1000 (i_2)^2$	=	58.76 $\mu$ W
$P_{1k\text{bottom}}$	=	0	=	0
$P_{4.4k}$	=	$4400 (i_2 - i_3)^2$	=	1.317 $\mu$ W
$P_{3k\text{top}}$	=	$3000 (i_3)^2$	=	152.0 $\mu$ W
$P_{4k}$	=	$4000 (i_3)^2$	=	202.7 $\mu$ W
$P_{3k\text{bottom}}$	=	$3000 (i_2)^2$	=	176.3 $\mu$ W

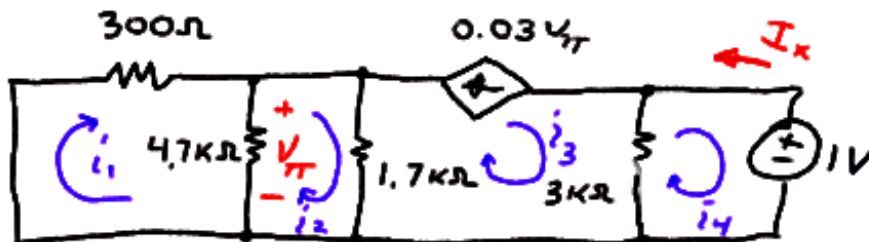
Check: The sources supply a total of

$$4(487.6) + (3 - 2.2)(242.4) + 1.5(225.1) = 2482 \text{ } \mu\text{W}.$$

The absorbed powers add to 2482  $\mu$ W.

## CHAPTER FOUR SOLUTIONS

29. (a) We begin by naming four mesh currents as depicted below:



Proceeding with mesh analysis, then, keeping in mind that  $I_x = -i_4$ ,

$$\text{MESH 1:} \quad (4700 + 300) i_1 - 4700 i_2 = 0 \quad [1]$$

$$\text{MESH 2:} \quad (4700 + 1700) i_2 - 4700 i_1 - 1700 i_3 = 0 \quad [2]$$

Since we have a current source on the perimeter of mesh 3, we do not require a KVL equation for that mesh. Instead, we may simply write

$$i_3 = -0.03 v_\pi \quad [3a] \quad \text{where} \quad v_\pi = 4700(i_1 - i_2) \quad [3b]$$

$$\text{MESH 4:} \quad 3000 i_4 - 3000 i_3 + 1 = 0 \quad [4]$$

Simplifying and combining Eqs. 3a and 3b,

$$\begin{aligned} 5000 i_1 - 4700 i_2 &= 0 \\ -4700 i_1 + 6400 i_2 - 1700 i_3 &= 0 \\ -141 i_1 + 141 i_2 - i_3 &= 0 \\ -3000 i_3 + 3000 i_4 &= -1 \end{aligned}$$

Solving, we find that  $i_4 = -333.3 \text{ mA}$ , so  $I_x = 333.3 \text{ } \mu\text{A}$ .

(b) At node “ $\pi$ ”:  $0.03 v_\pi = v_\pi / 300 + v_\pi / 4700 + v_\pi / 1700$

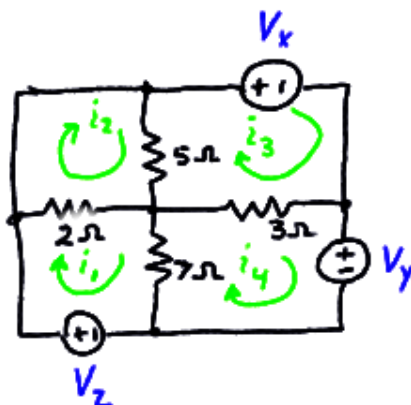
Solving, we find that  $v_\pi = 0$ , therefore no current flows through the dependent source.

Hence,  $I_x = 333.3 \text{ } \mu\text{A}$  as found in part (a).

(c)  $V_s / I_x$  has units of resistance. It can be thought of as the resistance “seen” by the voltage source  $V_s$ .... more on this in Chap. 5....

## CHAPTER FOUR SOLUTIONS

30. We begin by naming each mesh and the three undefined voltage sources as shown below:



$$\text{MESH 1:} \quad -V_z + 9i_1 - 2i_2 \quad -7i_4 = 0$$

$$\text{MESH 2:} \quad -2i_1 + 7i_2 - 5i_3 = 0$$

$$\text{MESH 3:} \quad V_x - 5i_2 + 8i_3 - 3i_4 = 0$$

$$\text{MESH 4:} \quad V_y - 7i_1 - 3i_3 + 10i_4 = 0$$

Rearranging and setting  $i_1 - i_2 = 0$ ,  $i_2 - i_3 = 0$ ,  $i_1 - i_4 = 0$  and  $i_4 - i_3 = 0$ ,

$$\begin{aligned} 9i_1 - 2i_2 - 7i_4 &= V_z \\ -2i_1 + 7i_2 - 5i_3 &= 0 \\ -5i_2 + 8i_3 - 3i_4 &= -V_x \\ -7i_1 - 3i_3 + 10i_4 &= -V_y \end{aligned}$$

Since  $i_1 = i_2 = i_3 = i_4$ , these equations produce:

$$\begin{aligned} V_z &= 0 \\ 0 &= 0 \\ -V_x &= 0 \\ -V_y &= 0 \end{aligned}$$

This is a unique solution. Therefore, the request that nonzero values be found cannot be satisfied.

## CHAPTER FOUR SOLUTIONS

31. The “supermesh” concept is not required (or helpful) in solving this problem, as there are no current sources shared between meshes. Starting with the left-most mesh and moving right, we define four clockwise mesh currents  $i_1$ ,  $i_2$ ,  $i_3$  and  $i_4$ . By inspection, we see that  $i_1 = 2$  mA.

$$\text{MESH 2:} \quad -10 + 5000i_2 + 4 + 1000i_3 = 0 \quad [1]$$

$$\text{MESH 3:} \quad -1000i_3 + 6 + 10,000 - 10,000i_4 = 0 \quad [2]$$

$$\text{MESH 4:} \quad i_4 = -0.5i_2 \quad [3]$$

Reorganising, we find

$$5000 i_2 + 1000 i_3 = 6 \quad [1]$$

$$9000 i_3 - 10,000 i_4 = -6 \quad [2]$$

$$0.5 i_2 + i_4 = 0 \quad [3]$$

We could either substitute Eq. [3] into Eq. [2] to reduce the number of equations, or simply go ahead and solve the system of Eqs. [1-3]. Either way, we find that

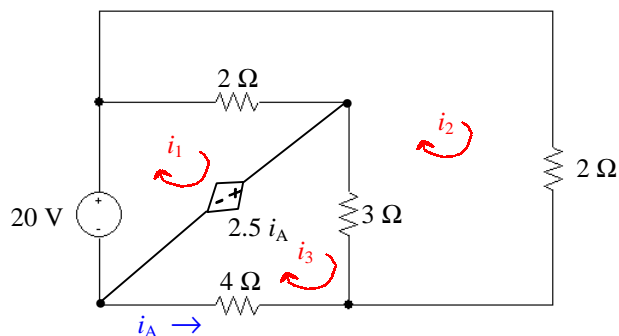
$$i_1 = 2 \text{ mA}, i_2 = 1.5 \text{ mA}, i_3 = -1.5 \text{ mA} \text{ and } i_4 = -0.75 \text{ mA}.$$

The power generated by each source is:

$P_{2\text{mA}}$	$= 5000(i_1 - i_2)(i_1)$	$= 5 \text{ mW}$
$P_{4\text{V}}$	$= 4 (-i_2)$	$= -6 \text{ mW}$
$P_{6\text{V}}$	$= 6 (-i_3)$	$= 9 \text{ mW}$
$P_{\text{depV}}$	$= 1000 i_3 (i_3 - i_2)$	$= 4.5 \text{ mW}$
$P_{\text{depI}}$	$= 10,000(i_3 - i_4)(0.5 i_2)$	$= -5.625 \text{ mW}$

## CHAPTER FOUR SOLUTIONS

32. This circuit does not require the supermesh technique, as it does not contain any current sources. Redrawing the circuit so its planar nature and mesh structure are clear,



$$\text{MESH 1:} \quad -20 + 2 i_1 - 2 i_2 + 2.5 i_A = 0 \quad [1]$$

$$\text{MESH 2:} \quad 2 i_2 + 3 i_2 - 3 i_3 + 2 i_2 - 2 i_1 = 0 \quad [2]$$

$$\text{MESH 3:} \quad -2.5 i_A + 7 i_3 - 3 i_2 = 0 \quad [3]$$

Combining terms and making use of the fact that  $i_A = -i_3$ ,

$$2 i_1 - 2 i_2 - 2.5 i_3 = 20 \quad [1]$$

$$-2 i_1 + 7 i_2 - 3 i_3 = 0 \quad [2]$$

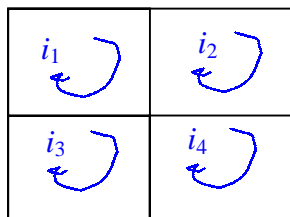
$$-3 i_2 + 9.5 i_3 = 0 \quad [3]$$

Solving,  $i_1 = 18.55 \text{ A}$ ,  $i_2 = 6.129 \text{ A}$ , and  $i_3 = 1.935 \text{ A}$ . Since  $i_A = -i_3$ ,

$i_A = -1.935 \text{ A.}$

## CHAPTER FOUR SOLUTIONS

33. Define four mesh currents



By inspection,  $i_1 = -4.5$  A.

We form a supermesh with meshes 3 and 4 as defined above.

$$\text{MESH 2:} \quad 2.2 + 3 i_2 + 4 i_2 + 5 - 4 i_3 = 0 \quad [1]$$

$$\text{SUPERMESH:} \quad 3 i_4 + 9 i_4 - 9 i_1 + 4 i_3 - 4 i_2 + 6 i_3 + i_3 - 3 = 0 \quad [2]$$

$$\text{Supermesh KCL equation:} \quad i_4 - i_3 = 2 \quad [3]$$

Simplifying and combining terms, we may rewrite these three equations as:

$$7 i_2 - 4 i_3 = -7.2 \quad [1]$$

$$-4 i_2 + 11 i_3 + 12 i_4 = -37.5 \quad [2]$$

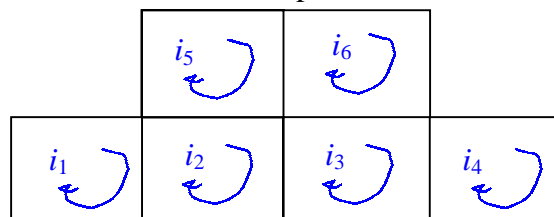
$$-i_3 + i_4 = 2 \quad [3]$$

Solving, we find that  $i_2 = -2.839$  A,  $i_3 = -3.168$  A, and  $i_4 = -1.168$  A.

The power supplied by the 2.2-V source is then  $2.2 (i_1 - i_2) = -3.654$  W.

## CHAPTER FOUR SOLUTIONS

34. We begin by defining six mesh currents as depicted below:



- We form a supermesh with meshes 1 and 2 since they share a current source.
- We form a *second* supermesh with meshes 3 and 4 since they also share a current source.

1, 2 Supermesh:

$$(4700 + 1000 + 10,000) i_1 - 2200 i_5 + (2200 + 1000 + 4700) i_2 - 1000 i_3 = 0 \quad [1]$$

3, 4 Supermesh:

$$(4700 + 1000 + 2200) i_3 - 1000 i_2 - 2200 i_6 + (4700 + 10,000 + 1000) i_4 = 0 \quad [2]$$

$$\text{MESH 5:} \quad (2200 + 4700) i_5 - 2200 i_2 + 3.2 - 1.5 = 0 \quad [3]$$

$$\text{MESH 6:} \quad 1.5 + (4700 + 4700 + 2200) i_6 - 2200 i_3 = 0 \quad [4]$$

$$\text{1, 2 Supermesh KCL equation:} \quad i_1 - i_2 = 3 \times 10^{-3} \quad [5]$$

$$\text{3, 4 Supermesh KCL equation:} \quad i_4 - i_3 = 2 \times 10^{-3} \quad [6]$$

We can simplify these equations prior to solution in several ways. Choosing to retain six equations,

$$15,700 i_1 + 7900 i_2 - 1000 i_3 - 2200 i_5 = 0 \quad [1]$$

$$- 1000 i_2 + 7900 i_3 + 15,700 i_4 - 2200 i_6 = 0 \quad [2]$$

$$- 2200 i_2 + 6900 i_5 = -1.7 \quad [3]$$

$$- 2200 i_3 + 11,600 i_6 = -1.5 \quad [4]$$

$$i_1 - i_2 = 3 \times 10^{-3} \quad [5]$$

$$- i_3 + i_4 = 2 \times 10^{-3} \quad [6]$$

Solving, we find that  $i_4 = 540.8 \text{ mA}$ . Thus, the voltage across the 2-mA source is

$$(4700 + 10,000 + 1000) (540.8 \times 10^{-6}) = \boxed{8.491 \text{ V}}$$



## CHAPTER FOUR SOLUTIONS

35. We define a mesh current  $i_a$  in the left-hand mesh, a mesh current  $i_1$  in the top right mesh, and a mesh current  $i_2$  in the bottom right mesh.

The left-most mesh can be analysed separately to determine the controlling voltage  $v_a$ , as KCL assures us that no current flows through either the  $1\text{-}\Omega$  or  $6\text{-}\Omega$  resistor.

Thus,  $-1.8 + 3 i_a - 1.5 + 2 i_a = 0$ , which may be solved to find  $i_a = 0.66\text{ A}$ . Hence,  $v_a = 3 i_a = 1.98\text{ V}$ .

Forming one supermesh from the remaining two meshes, we may write:

$$-3 + 2.5 i_1 + 3 i_2 + 4 i_2 = 0$$

and the supermesh KCL equation:  $i_2 - i_1 = 0.05 v_a = 0.05(1.98) = 99 \times 10^{-3}$

Thus, we have two equations to solve:

$$\begin{aligned} 2.5 i_1 + 7 i_2 &= 3 \\ -i_1 + i_2 &= 99 \times 10^{-3} \end{aligned}$$

Solving, we find that  $i_1 = 242.8\text{ mA}$  and the voltage across the  $2.5\text{-}\Omega$  resistor (arbitrarily assuming the left terminal is the “+” reference) is  $2.5 i_1 = 607\text{ mV}$ .

## CHAPTER FOUR SOLUTIONS

36. UNDEFINED RESISTOR VALUE IN FIGURE. Set to 10 mΩ.

There are only three meshes in this circuit, as the bottom 22-mΩ resistor is not connected at its left terminal. Thus, we define three mesh currents,  $i_1$ ,  $i_2$ , and  $i_3$ , beginning with the left-most mesh.

We next create a supermesh from meshes 1 and 2 (note that mesh 3 is independent, and can be analysed separately).

Thus,  $-11.8 + 10 \times 10^{-3} i_1 + 22 \times 10^{-3} i_2 + 10 \times 10^{-3} i_2 + 17 \times 10^{-3} i_1 = 0$

and applying KCL to obtain an equation containing the current source,

$$i_1 - i_2 = 100$$

Combining terms and simplifying, we obtain

$$\begin{array}{rcl} 27 \times 10^{-3} i_1 + 32 \times 10^{-3} i_2 & = & 11.8 \\ i_1 - i_2 & = & 100 \end{array}$$

Solving, we find that  $i_1 = 254.2 \text{ A}$  and  $i_2 = 154.2 \text{ A}$ .

The final mesh current is easily found:  $i_3 = 13 \times 10^3 / (14 + 11.6 + 15) = 320.2 \text{ A}$ .

## CHAPTER FOUR SOLUTIONS

37. MESH 1:  $-7 + i_1 - i_2 = 0$  [1]  
MESH 2:  $i_2 - i_1 + 2i_2 + 3i_2 - 3i_3 = 0$  [2]  
MESH 3:  $3i_3 - 3i_2 + xi_3 + 2i_3 - 7 = 0$  [3]

Grouping terms, we find that

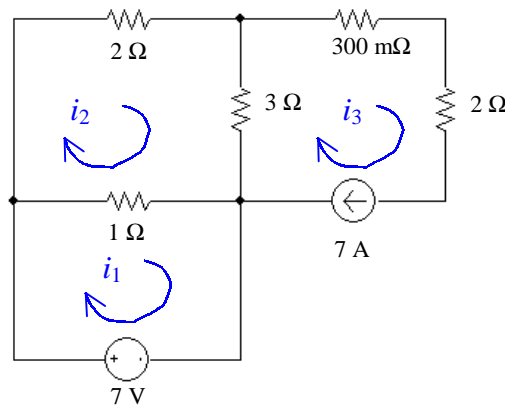
$$\begin{array}{rclcl} i_1 - i_2 & = & 7 & [1] \\ -i_1 + 6i_2 - 3i_3 & = & 0 & [2] \\ -3i_2 + (5 + x)i_3 & = & 7 & [3] \end{array}$$

This, unfortunately, is four unknowns but only three equations. However, we have not yet made use of the fact that we are trying to obtain  $i_2 = 2.273$  A. Solving these “four” equations, we find that

$$x = (7 + 3 i_2 - 5 i_3) / i_3 = 4.498 \Omega.$$

## CHAPTER FOUR SOLUTIONS

38. We begin by redrawing the circuit as instructed, and define three mesh currents:



By inspection,  $i_3 = 7 \text{ A}$ .

$$\text{MESH 1: } -7 + i_1 - i_2 = 0 \quad \text{or} \quad i_1 - i_2 = 7 \quad [1]$$

$$\text{MESH 2: } (1 + 2 + 3) i_2 - i_1 - 3(7) = 0 \quad \text{or} \quad -i_1 + 6i_2 = 21 \quad [2]$$

There is no need for supermesh techniques for this situation, as the only current source lies on the outside perimeter of a mesh- it is not shared between meshes.

Solving, we find that  $i_1 = 12.6 \text{ A}$ ,  $i_2 = 5.6 \text{ A}$  and  $i_3 = 7 \text{ A}$ .

## CHAPTER FOUR SOLUTIONS

39. (a) We are asked for a voltage, and have one current source and one voltage source. Nodal analysis is probably best then- the nodes can be named so that the desired voltage is a nodal voltage, or, at worst, we have one supernode equation to solve.

Name the top left node “1” and the top right node “x”; designate the bottom node as the reference terminal. Next, form a supernode with nodes “1” and “x.”

$$\text{At the supernode:} \quad 11 = v_1/2 + v_x/9 \quad [1]$$

$$\text{and the KVL Eqn:} \quad v_1 - v_x = 22 \quad [2]$$

$$\text{Rearranging,} \quad 11(18) = 9 v_1 + 2 v_x \quad [1]$$

$$22 = v_1 - v_x \quad [2]$$

$$\text{Solving,} \quad v_x = 0$$

(b) We are asked for a voltage, and so may suspect that nodal analysis is preferable; with two current sources and only one voltage source (easily dealt with using the supernode technique), nodal analysis does seem to have an edge over mesh analysis here.

Name the top left node “x,” the top right node “y” and designate the bottom node as the reference node. Forming a supernode from nodes “x” and “y,”

$$\text{At the supernode:} \quad 6 + 9 = v_x/10 + v_y/20 \quad [1]$$

$$\text{and the KVL Eqn:} \quad v_y - v_x = 12 \quad [2]$$

$$\text{Rearranging,} \quad 15(20) = 2 v_x + v_y \quad [1]$$

$$\text{and} \quad 12 = -v_x + v_y \quad [2]$$

$$\text{Solving, we find that} \quad v_x = 96 \text{ V.}$$

(c) We are asked for a voltage, but would have to subtract two nodal voltages (not much harder than invoking Ohm’s law). On the other hand, the dependent current source depends on the desired unknown, which would lead to the need for another equation if invoking mesh analysis. Trying nodal analysis,

$$0.1 v_x = (v_1 - 50)/2 + v_x/4 \quad [1]$$

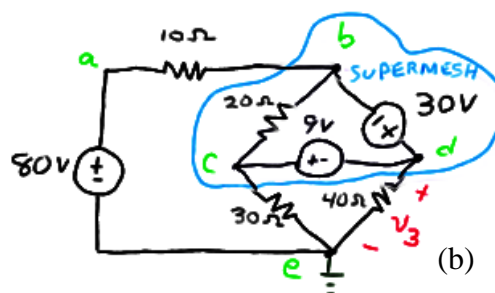
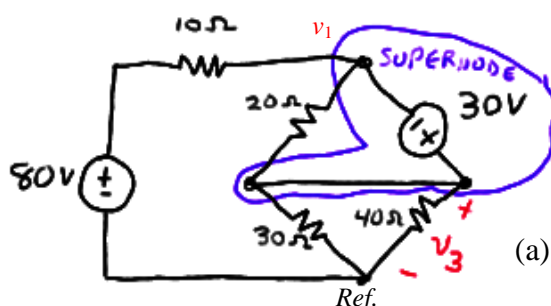
referring to the circuit we see that  $v_x = v_1 - 100$ . Rearranging so that we may eliminate  $v_1$  in Eq. [1], we obtain  $v_1 = v_x + 100$ . Thus, Eq. [1] becomes

$$0.1 v_x = (v_x + 100 - 50)/2 + v_x/4$$

$$\text{and a little algebra yields} \quad v_x = -38.46 \text{ V.}$$

# CHAPTER FOUR SOLUTIONS

40.



(a) We begin by noting that it is a voltage that is required; no current values are requested. This is a three-mesh circuit, or a four-node circuit, depending on your perspective. Either approach requires three equations.... Except that applying the supernode technique reduces the number of needed equations by one.

At the 1, 3 supernode:

$$0 = (v_1 - 80)/10 + (v_1 - v_3)/20 + (v_3 - v_1)/20 + v_3/40 + v_3/30$$

and

$$v_3 - v_1 = 30$$

We simplify these two equations and collect terms, yielding

$$0.1 v_1 + 0.05833 v_3 = 8$$

$$-v_1 + v_3 = 30$$

Solving, we find that  $v_3 = 69.48 \text{ V}$

Both ends of the resistor are connected to the supernode, so we could actually just ignore it...

(b) Mesh analysis would be straightforward, requiring 3 equations and a (trivial) application of Ohm's law to obtain the final answer. Nodal analysis, on the other hand, would require only two equations, and the desired voltage will be a nodal voltage.

At the b, c, d supernode:  $0 = (v_b - 80)/10 + v_d/40 + v_c/30$

and:  $v_d - v_b = 30$

$$v_c - v_d = 9$$

Simplify and collect terms:  $0.1 v_b + 0.03333 v_c + 0.025 v_d = 80$

$$-v_b + v_d = 30$$

$$v_c - v_d = 9$$

Solving,  $v_d (= v_3) = 67.58 \text{ V}$

(c) We are now faced with a dependent current source whose value depends on a mesh current. Mesh analysis in this situation requires 1 supermesh, 1 KCL equation and Ohm's law. Nodal analysis requires 1 supernode, 1 KVL equation, 1 other nodal equation, and one equation to express  $i_1$  in terms of nodal voltages. Thus, mesh analysis has an edge here. Define the left mesh as "1," the top mesh as "2", and the bottom mesh as "3."

$$\text{Mesh 1: } -80 + 10 i_1 + 20 i_1 - 20 i_2 + 30 i_1 - 30 i_3 = 0$$

$$\text{2, 3 supermesh: } 20 i_2 - 20 i_1 - 30 + 40 i_3 + 30 i_3 - 30 i_1 = 0$$

$$\text{and: } i_2 - i_3 = 5 i_1$$

$$\text{Rewriting, } 60 i_1 - 20 i_2 - 30 i_3 = 80$$

$$-50 i_1 + 20 i_2 + 70 i_3 = 30$$

$$5 i_1 - i_2 + i_3 = 0$$

Solving,  $i_3 = 4.727 \text{ A}$

so

$$v_3 = 40 i_3 = 189 \text{ V.}$$

## CHAPTER FOUR SOLUTIONS

41. This circuit consists of 3 meshes, and no dependent sources. Therefore 3 simultaneous equations and 1 subtraction operation would be required to solve for the two desired currents. On the other hand, if we use nodal analysis, forming a supernode about the 30-V source would lead to  $5 - 1 - 1 = 3$  simultaneous equations as well, plus several subtraction and division operations to find the currents. Thus, mesh analysis has a slight edge here.

Define three clockwise mesh currents:  $i_a$  in the left-most mesh,  $i_b$  in the top right mesh, and  $i_c$  in the bottom right mesh. Then our mesh equations will be:

$$\text{Mesh } a: \quad -80 + (10 + 20 + 30) i_a - 20 i_b - 30 i_c = 0 \quad [1]$$

$$\text{Mesh } b: \quad -30 + (12 + 20) i_b - 12 i_c - 20 i_a = 0 \quad [2]$$

$$\text{Mesh } c: \quad (12 + 40 + 30) i_c - 12 i_b - 30 i_a = 0 \quad [3]$$

Simplifying and collecting terms,

$$60 i_a - 20 i_b - 30 i_c = 80 \quad [1]$$

$$-20 i_a + 32 i_b - 12 i_c = 30 \quad [2]$$

$$-30 i_a - 12 i_b + 82 i_c = 0 \quad [3]$$

Solving, we find that  $i_a = 3.549$  A,  $i_b = 3.854$  A, and  $i_c = 1.863$  A. Thus,

$$i_1 = i_a = \boxed{3.549 \text{ A}} \quad \text{and} \quad i_2 = i_a - i_c = \boxed{1.686 \text{ A.}}$$

## CHAPTER FOUR SOLUTIONS

42. Approaching this problem using nodal analysis would require 3 separate nodal equations, plus one equation to deal with the dependent source, plus subtraction and division steps to actually find the current  $i_{10}$ . Mesh analysis, on the other hand, will require 2 mesh/supermesh equations, 1 KCL equation, and one subtraction step to find  $i_{10}$ . Thus, mesh analysis has a clear edge. Define three clockwise mesh currents:  $i_1$  in the bottom left mesh,  $i_2$  in the top mesh, and  $i_3$  in the bottom right mesh.

$$\text{MESH 1:} \quad i_1 = 5 \text{ mA by inspection} \quad [1]$$

$$\begin{aligned} \text{SUPERMESH:} \quad i_1 - i_2 &= 0.4 i_{10} \\ i_1 - i_2 &= 0.4(i_3 - i_2) \\ i_1 - 0.6 i_2 - 0.4 i_3 &= 0 \end{aligned} \quad [2]$$

$$\text{MESH 3:} \quad -5000 i_1 - 10000 i_2 + 35000 i_3 = 0 \quad [3]$$

$$\begin{aligned} \text{Simplify:} \quad 0.6 i_2 + 0.4 i_3 &= 5 \times 10^{-3} \quad [2] \\ -10000 i_2 + 35000 i_3 &= 25 \quad [3] \end{aligned}$$

Solving, we find  $i_2 = 6.6 \text{ mA}$  and  $i_3 = 2.6 \text{ mA}$ . Since  $i_{10} = i_3 - i_2$ , we find that

$$i_{10} = -4 \text{ mA.}$$



## CHAPTER FOUR SOLUTIONS

43. For this circuit problem, nodal analysis will require 3 simultaneous nodal equations, then subtraction/ division steps to obtain the desired currents. Mesh analysis requires 1 mesh equation, 1 supermesh equation, 2 simple KCL equations and one subtraction step to determine the currents. If either technique has an edge in this situation, it's probably mesh analysis. Thus, define four clockwise mesh equations:  $i_a$  in the bottom left mesh,  $i_b$  in the top left mesh,  $i_c$  in the top right mesh, and  $i_d$  in the bottom right mesh.

At the  $a, b, c$  supermesh:  $-100 + 6 i_a + 20 i_b + 4 i_c + 10 i_c - 10 i_d = 0$  [1]

Mesh d:  $100 + 10 i_d - 10 i_c + 24 i_d = 0$  [2]

KCL:  $-i_a + i_b = 2$  [3]

and  $-i_b + i_c = 3 i_3 = 3 i_a$  [4]

Collecting terms & simplifying,

$$6 i_a + 20 i_b + 14 i_c - 10 i_d = 100 \quad [1]$$

$$-10 i_c + 34 i_d = -100 \quad [2]$$

$$-i_a + i_b = 2 \quad [3]$$

$$-3 i_a - i_b + i_c = 0 \quad [4]$$

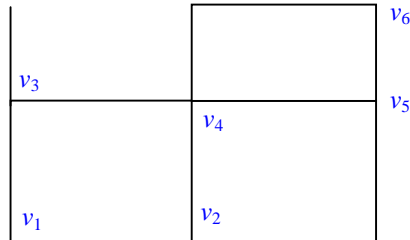
Solving,

$i_a = 0.1206$  A,  $i_b = 2.121$  A,  $i_c = 2.482$  A, and  $i_d = -2.211$  A. Thus,

$$i_3 = i_a = \boxed{120.6 \text{ mA}} \quad \text{and} \quad i_{10} = i_c - i_d = \boxed{4.693 \text{ A.}}$$

## CHAPTER FOUR SOLUTIONS

44. With 7 nodes in this circuit, nodal analysis will require the solution of three simultaneous nodal equations (assuming we make use of the supernode technique) and one KVL equation. Mesh analysis will require the solution of three simultaneous mesh equations (one mesh current can be found by inspection), plus several subtraction and multiplication operations to finally determine the voltage at the central node. Either will probably require a comparable amount of algebraic manoeuvres, so we go with nodal analysis, as the desired unknown is a direct result of solving the simultaneous equations. Define the nodes as:



NODE 1:  $-2 \times 10^{-3} = (v_1 - 1.3) / 1.8 \times 10^3 \rightarrow v_1 = -2.84 \text{ V}.$

2, 4 Supernode:

$$2.3 \times 10^{-3} = (v_2 - v_5) / 1 \times 10^3 + (v_4 - 1.3) / 7.3 \times 10^3 + (v_4 - v_5) / 1.3 \times 10^3 + v_4 / 1.5 \times 10^3$$

KVL equation:  $-v_2 + v_4 = 5.2$

Node 5:  $0 = (v_5 - v_2) / 1 \times 10^3 + (v_5 - v_4) / 1.3 \times 10^3 + (v_5 - 2.6) / 6.3 \times 10^3$

Simplifying and collecting terms,

$$14.235 v_2 + 22.39 v_4 - 25.185 v_5 = 35.275 \quad [1]$$

$$-v_2 + v_4 = 5.2 \quad [2]$$

$$-8.19 v_2 - 6.3 v_4 + 15.79 v_5 = 3.38 \quad [3]$$

Solving, we find the voltage at the central node is  $v_4 = 3.460 \text{ V}.$

## CHAPTER FOUR SOLUTIONS

45. Mesh analysis yields current values directly, so use that approach. We therefore define four clockwise mesh currents, starting with  $i_1$  in the left-most mesh, then  $i_2$ ,  $i_3$  and  $i_4$  moving towards the right.

Mesh 1:  $-0.8i_x + (2 + 5) i_1 - 5 i_2 = 0$  [1]

Mesh 2:  $i_2 = 1$  A by inspection [2]

Mesh 3:  $(3 + 4) i_3 - 3(1) - 4(i_4) = 0$  [3]

Mesh 4:  $(4 + 3) i_4 - 4 i_3 - 5 = 0$  [4]

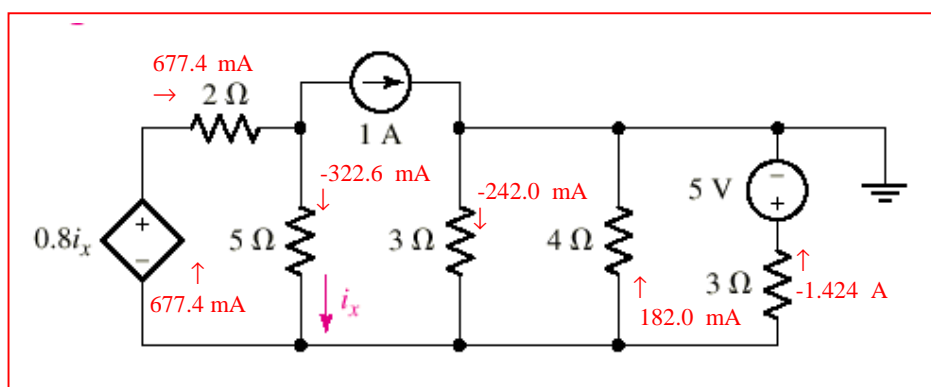
Simplify and collect terms, noting that  $i_x = i_1 - i_2 = i_1 - 1$

$-0.8(i_1 - 1) + 7 i_1 - 5(1) = 0$  yields  $i_1 = 677.4$  mA

Thus, [3] and [4] become:  $7 i_3 - 4 i_4 = 3$  [3]

$-4 i_3 + 7 i_4 = 5$  [4]

Solving, we find that  $i_3 = 1.242$  A and  $i_4 = 1.424$  A. A map of individual branch currents can now be drawn:



## CHAPTER FOUR SOLUTIONS

46. If we choose to perform mesh analysis, we require 2 simultaneous equations (there are four meshes, but one mesh current is known, and we can employ the supermesh technique around the left two meshes). In order to find the voltage across the 2-mA source we will need to write a KVL equation, however. Using nodal analysis is less desirable in this case, as there will be a large number of nodal equations needed. Thus, we define four clockwise mesh currents  $i_1$ ,  $i_2$ ,  $i_3$  and  $i_4$  starting with the left-most mesh and moving towards the right of the circuit.

$$\begin{array}{ll} \text{At the 1,2 supermesh:} & 2000 i_1 + 6000 i_2 - 3 + 5000 i_2 = 0 \quad [1] \\ \text{and} & i_1 - i_2 = 2 \times 10^{-3} \quad [2] \end{array}$$

by inspection,  $i_4 = -1$  mA. However, this as well as any equation for mesh four are unnecessary: we already have two equations in two unknowns and  $i_1$  and  $i_2$  are sufficient to enable us to find the voltage across the current source.

$$\begin{array}{ll} \text{Simplifying, we obtain} & 2000 i_1 + 11000 i_2 = 3 \quad [1] \\ & 1000 i_1 - 1000 i_2 = 2 \quad [2] \end{array}$$

Solving,  $i_1 = 1.923$  mA and  $i_2 = -76.92$   $\mu$ A.

Thus, the voltage across the 2-mA source (“+” reference at the top of the source) is

$$v = -2000 i_1 - 6000 (i_1 - i_2) = \boxed{-15.85 \text{ V.}}$$

## CHAPTER FOUR SOLUTIONS

47. Nodal analysis will require 2 nodal equations (one being a “supernode” equation), 1 KVL equation, and subtraction/division operations to obtain the desired current. Mesh analysis simply requires 2 “supermesh” equations and 2 KCL equations, with the desired current being a mesh current. Thus, we define four clockwise mesh currents  $i_a$ ,  $i_b$ ,  $i_c$ ,  $i_d$  starting with the left-most mesh and proceeding to the right of the circuit.

At the  $a, b$  supermesh:  $-5 + 2 i_a + 2 i_b + 3 i_b - 3 i_c = 0$  [1]

At the  $c, d$  supermesh:  $3 i_c - 3 i_b + 1 + 4 i_d = 0$  [2]

and  $i_a - i_b = 3$  [3]  
 $i_c - i_d = 2$  [4]

Simplifying and collecting terms, we obtain

$$2 i_a + 5 i_b - 3 i_c = 5 \quad [1]$$

$$-3 i_b + 3 i_c + 4 i_d = -1 \quad [2]$$

$$i_a - i_b = 3 \quad [3]$$

$$i_c - i_d = 2 \quad [4]$$

Solving, we find  $i_a = 3.35$  A,  $i_b = 350$  mA,  $i_c = 1.15$  A, and  $i_d = -850$  mA. As  $i_1 = i_b$ ,

$i_1 = 350$  mA.

## CHAPTER FOUR SOLUTIONS

48. Define a voltage  $v_x$  at the top node of the current source  $I_2$ , and a clockwise mesh current  $i_b$  in the right-most mesh.

We want 6 W dissipated in the  $6\text{-}\Omega$  resistor, which leads to the requirement  $i_b = 1\text{ A}$ . Applying nodal analysis to the circuit,

$$I_1 + I_2 = (v_x - v_1)/6 = 1$$

so our requirement is  $I_1 + I_2 = 1$ . There is no constraint on the value of  $v_1$  other than we are told to select a nonzero value.

Thus, we choose  $I_1 = I_2 = 500\text{ mA}$  and  $v_1 = 3.1415\text{ V}$ .

## CHAPTER FOUR SOLUTIONS

49. Inserting the new 2-V source with “+” reference at the bottom, and the new 7-mA source with the arrow pointing down, we define four clockwise mesh currents  $i_1$ ,  $i_2$ ,  $i_3$ ,  $i_4$  starting with the left-most mesh and proceeding towards the right of the circuit.

Mesh 1:  $(2000 + 1000 + 5000) i_1 - 6000 i_2 - 2 = 0$  [1]

2, 3 Supermesh:

$$2 + (5000 + 5000 + 1000 + 6000) i_2 - 6000 i_1 + (3000 + 4000 + 5000) i_3 - 5000 i_4 = 0 \quad [2]$$

and  $i_2 - i_3 = 7 \times 10^{-3}$  [3]

Mesh 4:  $i_4 = -1 \text{ mA}$  by inspection [4]

Simplifying and combining terms,

$$8000 i_1 - 6000 i_2 = 2 \quad [1]$$

$$1000 i_2 - 1000 i_3 = 7 \quad [4]$$

$$-6000 i_1 + 17000 i_2 + 12000 i_3 = -7 \quad [2]$$

Solving, we find that

$$i_1 = 2.653 \text{ A}, i_2 = 3.204 \text{ A}, i_3 = -3.796 \text{ A}, i_4 = -1 \text{ mA}$$

## CHAPTER FOUR SOLUTIONS

50. Define node 1 as the top left node, and node 2 as the node joining the three  $2\text{-}\Omega$  resistors. Place the “+” reference terminal of the  $2\text{-V}$  source at the right. The right-most  $2\text{-}\Omega$  resistor has therefore been shorted out. Applying nodal analysis then,

$$\text{Node 1:} \quad -5 i_1 = (v_1 - v_2)/2 \quad [1]$$

$$\text{Node 2:} \quad 0 = (v_2 - v_1)/2 + v_2/2 + (v_2 - 2)/2 \quad [2]$$

$$\text{and,} \quad i_1 = (v_2 - 2)/2 \quad [3]$$

Simplifying and collecting terms,

$$v_1 + v_2 = 10 \quad [1]$$

$$-v_1 + 3 v_2 = 2 \quad [2]$$

Solving, we find that  $v_1 = 3.143 \text{ V}$  and  $v_2 = 1.714 \text{ V}$ .

Defining clockwise mesh currents  $i_a$ ,  $i_b$ ,  $i_c$ ,  $i_d$  starting with the left-most mesh and proceeding right, we may easily determine that

$$i_a = -5 i_1 = 714.3 \text{ mA}$$

$$i_b = -142.9 \text{ mA}$$

$$i_c = i_1 - 2 = -2.143 \text{ A}$$

$$i_d = 3 + i_c = 857.1 \text{ mA}$$



## CHAPTER FOUR SOLUTIONS

### 51. Hand analysis:

Define three clockwise mesh currents:  $i_1$  in the bottom left mesh,  $i_2$  in the top mesh, and  $i_3$  in the bottom right mesh.

$$\text{MESH 1: } i_1 = 5 \text{ mA by inspection} \quad [1]$$

$$\begin{aligned} \text{SUPERMESH: } i_1 - i_2 &= 0.4 i_{10} \\ i_1 - i_2 &= 0.4(i_3 - i_2) \\ i_1 - 0.6 i_2 - 0.4 i_3 &= 0 \end{aligned} \quad [2]$$

$$\text{MESH 3: } -5000 i_1 - 10000 i_2 + 35000 i_3 = 0 \quad [3]$$

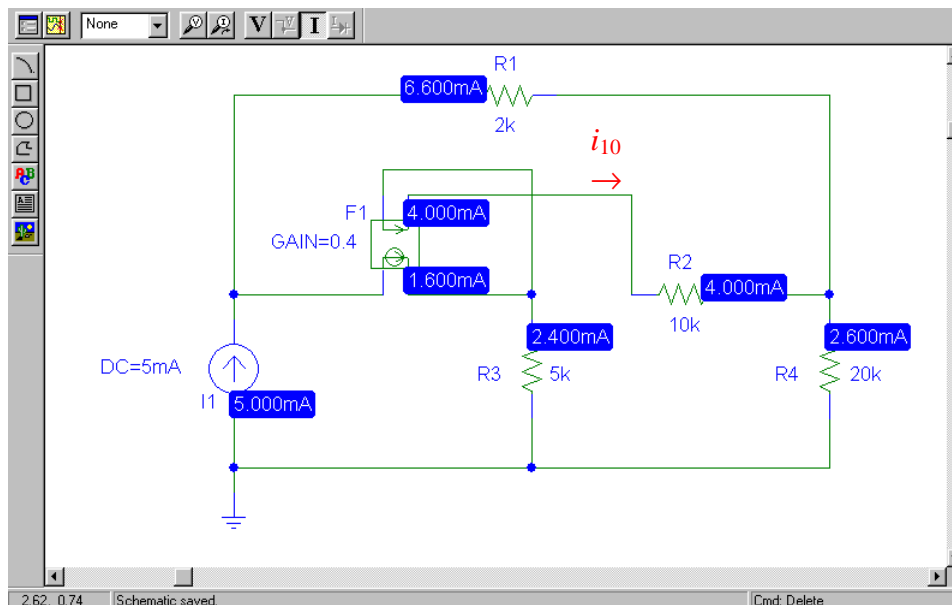
$$\text{Simplify: } 0.6 i_2 + 0.4 i_3 = 5 \times 10^{-3} \quad [2]$$

$$-10000 i_2 + 35000 i_3 = 25 \quad [3]$$

Solving, we find  $i_2 = 6.6 \text{ mA}$  and  $i_3 = 2.6 \text{ mA}$ . Since  $i_{10} = i_3 - i_2$ , we find that

$$i_{10} = -4 \text{ mA.}$$

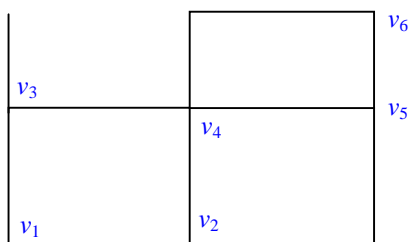
### PSpice simulation results:



**Summary:** The current entering the right-hand node of the 10-k $\Omega$  resistor R2 is equal to 4.000 mA. Since this current is  $-i_{10}$ ,  $i_{10} = -4.000 \text{ mA}$  as found by hand.

## CHAPTER FOUR SOLUTIONS

52. **Hand analysis:**  
Define the nodes as:



NODE 1:  $-2 \times 10^{-3} = (v_1 - 1.3) / 1.8 \times 10^3 \rightarrow v_1 = -2.84 \text{ V}.$

2, 4 Supernode:

$$2.3 \times 10^{-3} = (v_2 - v_5) / 1 \times 10^3 + (v_4 - 1.3) / 7.3 \times 10^3 + (v_4 - v_5) / 1.3 \times 10^3 + v_4 / 1.5 \times 10^3$$

KVL equation:  $-v_2 + v_4 = 5.2$

Node 5:  $0 = (v_5 - v_2) / 1 \times 10^3 + (v_5 - v_4) / 1.3 \times 10^3 + (v_5 - 2.6) / 6.3 \times 10^3$

Simplifying and collecting terms,

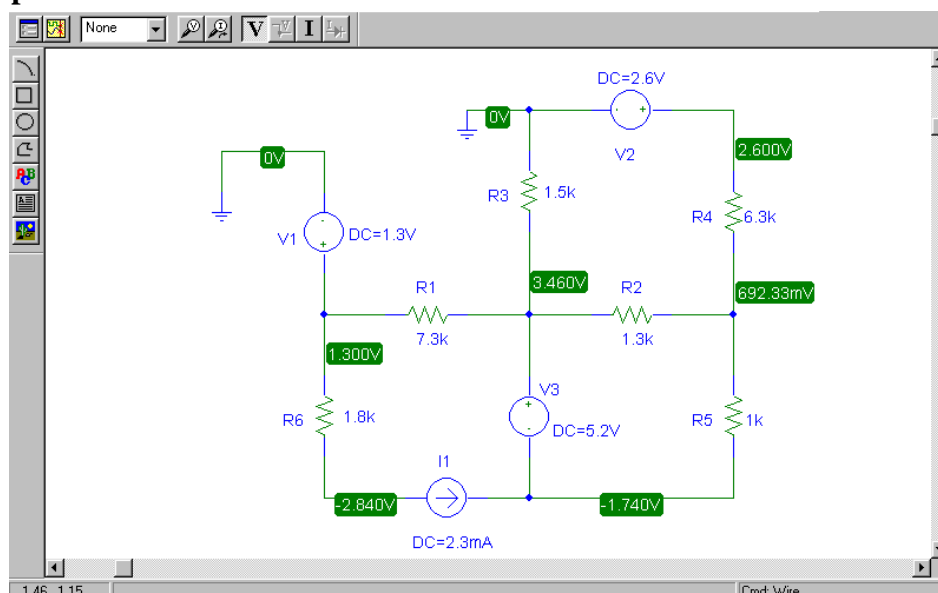
$$14.235 v_2 + 22.39 v_4 - 25.185 v_5 = 35.275 \quad [1]$$

$$-v_2 + v_4 = 5.2 \quad [2]$$

$$-8.19 v_2 - 6.3 v_4 + 15.79 v_5 = 3.38 \quad [3]$$

Solving, we find the voltage at the central node is  $v_4 = 3.460 \text{ V}.$

**PSpice simulation results:**



**Summary:** The voltage at the center node is found to be  $3.460 \text{ V}$ , which is in agreement with our hand calculation.

## CHAPTER FOUR SOLUTIONS

**53. Hand analysis:**

At the 1,2 supermesh:  $2000 i_1 + 6000 i_2 - 3 + 5000 i_2 = 0$  [1]

and  $i_1 - i_2 = 2 \times 10^{-3}$  [2]

by inspection,  $i_4 = -1$  mA. However, this as well as any equation for mesh four are unnecessary: we already have two equations in two unknowns and  $i_1$  and  $i_2$  are sufficient to enable us to find the voltage across the current source.

Simplifying, we obtain  $2000 i_1 + 11000 i_2 = 3$  [1]

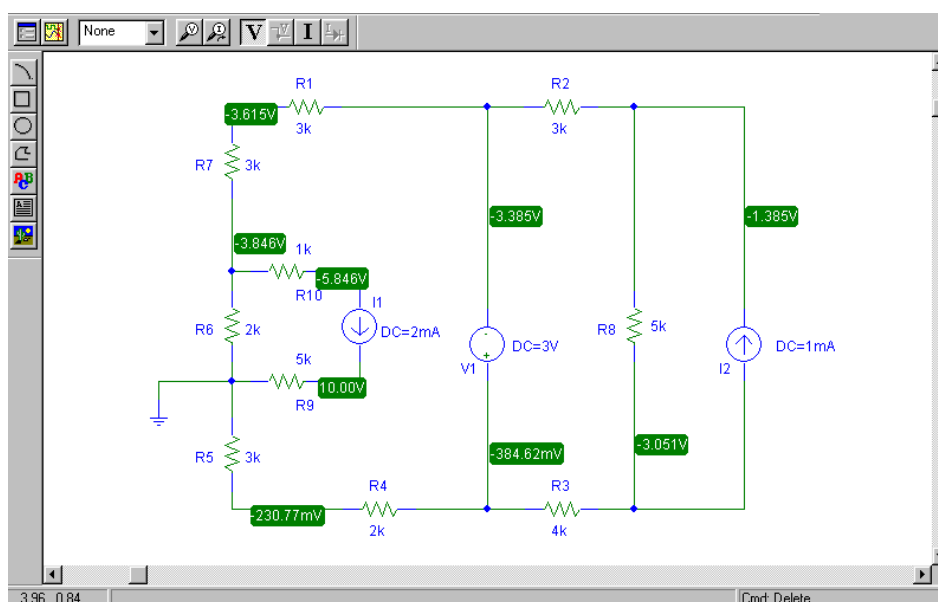
$1000 i_1 - 1000 i_2 = 2$  [2]

Solving,  $i_1 = 1.923$  mA and  $i_2 = -76.92$   $\mu$ A.

Thus, the voltage across the 2-mA source (“+” reference at the top of the source) is

$$v = -2000 i_1 - 6000 (i_1 - i_2) = \boxed{-15.85 \text{ V.}}$$

**PSpice simulation results:**



**Summary:** Again arbitrarily selecting the “+” reference as the top node of the 2-mA current source, we find the voltage across it is  $-5.846 - 10 = -15.846$  V, in agreement with our hand calculation.

## CHAPTER FOUR SOLUTIONS

54. **Hand analysis:**

Define a voltage  $v_x$  at the top node of the current source  $I_2$ , and a clockwise mesh current  $i_b$  in the right-most mesh.

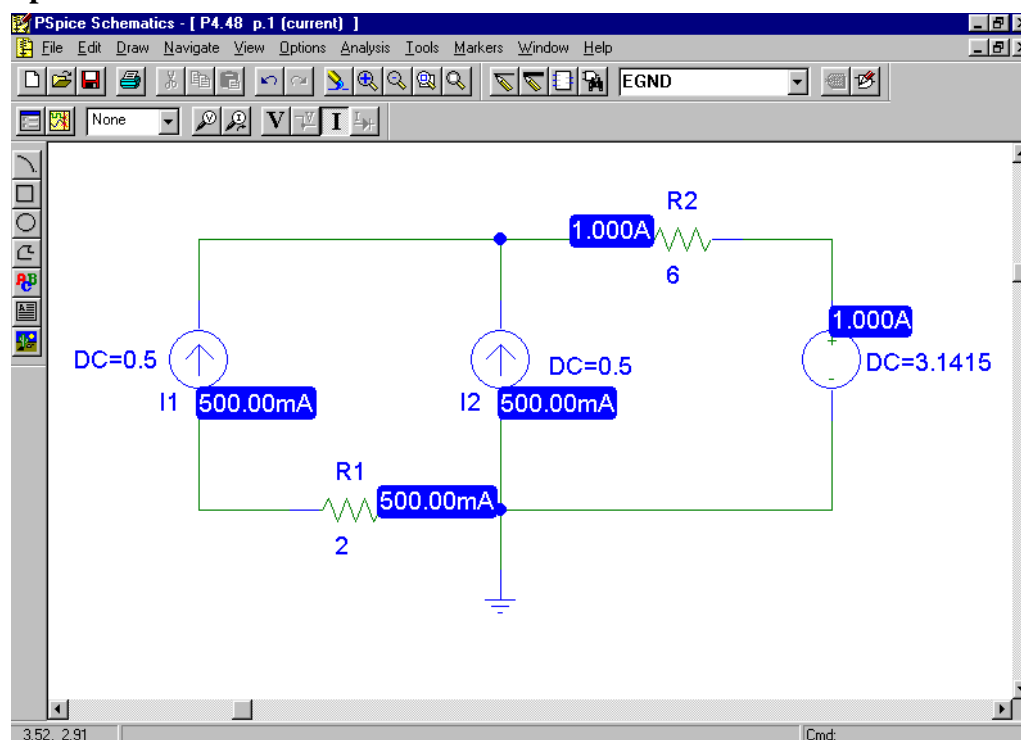
We want 6 W dissipated in the 6- $\Omega$  resistor, which leads to the requirement  $i_b = 1$  A. Applying nodal analysis to the circuit,

$$I_1 + I_2 = (v_x - v_1)/6 = 1$$

so our requirement is  $I_1 + I_2 = 1$ . There is no constraint on the value of  $v_1$  other than we are told to select a nonzero value.

Thus, we choose  $I_1 = I_2 = 500$  mA and  $v_1 = 3.1415$  V.

**PSpice simulation results:**



**Summary:** We see from the labeled schematic above that our choice for  $I_1$ ,  $I_2$  and  $V_1$  lead to 1 A through the 6- $\Omega$  resistor, or 6 W dissipated in that resistor, as desired.

## CHAPTER FOUR SOLUTIONS

### 55. Hand analysis:

Define node 1 as the top left node, and node 2 as the node joining the three  $2\text{-}\Omega$  resistors. Place the “+” reference terminal of the  $2\text{-V}$  source at the right. The right-most  $2\text{-}\Omega$  resistor has therefore been shorted out. Applying nodal analysis then,

$$\text{Node 1:} \quad -5 i_1 = (v_1 - v_2)/2 \quad [1]$$

$$\text{Node 2:} \quad 0 = (v_2 - v_1)/2 + v_2/2 + (v_2 - 2)/2 \quad [2]$$

$$\text{and,} \quad i_1 = (v_2 - 2)/2 \quad [3]$$

Simplifying and collecting terms,

$$v_1 + v_2 = 10 \quad [1]$$

$$-v_1 + 3 v_2 = 2 \quad [2]$$

Solving, we find that  $v_1 = 3.143 \text{ V}$  and  $v_2 = 1.714 \text{ V}$ .

Defining clockwise mesh currents  $i_a$ ,  $i_b$ ,  $i_c$ ,  $i_d$  starting with the left-most mesh and proceeding right, we may easily determine that

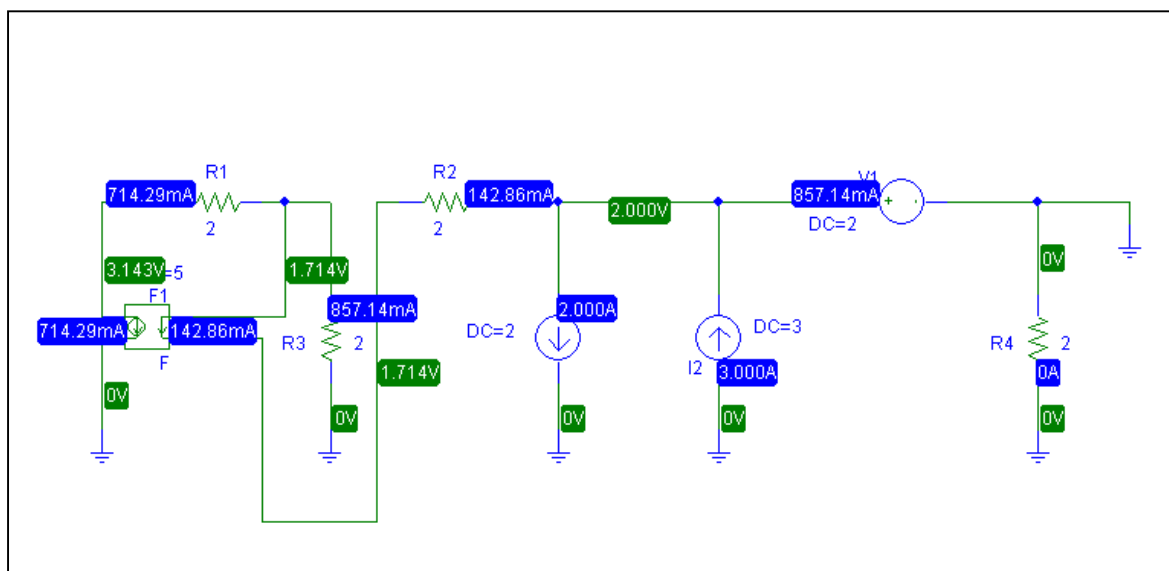
$$i_a = -5 i_1 = 714.3 \text{ mA}$$

$$i_b = -142.9 \text{ mA}$$

$$i_c = i_1 - 2 = -2.143 \text{ A}$$

$$i_d = 3 + i_c = 857.1 \text{ mA}$$

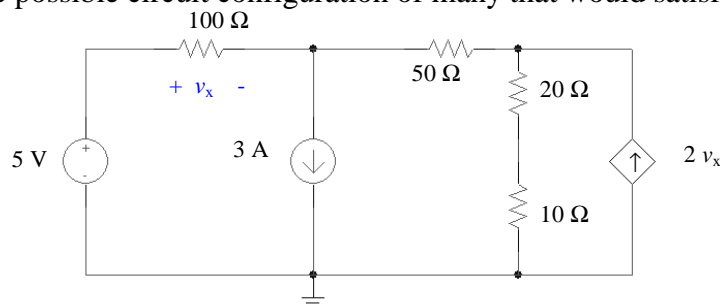
### PSpice simulation results:



**Summary:** The simulation results agree with the hand calculations.

## CHAPTER FOUR SOLUTIONS

56. (a) One possible circuit configuration of many that would satisfy the requirements:



At node 1:  $-3 = (v_1 - 5)/100 + (v_1 - v_2)/50$  [1]

At node 2:  $2v_x = (v_2 - v_1)/50 + v_2/30$  [2]

and,  $v_x = 5 - v_1$  [3]

Simplifying and collecting terms,

$150v_1 - 100v_2 = -14750$  [1]

$2970v_1 + 80v_2 = 15000$  [2]

Solving, we find that  $v_1 = 1.036\text{ V}$  and  $v_2 = 149.1\text{ V}$ .

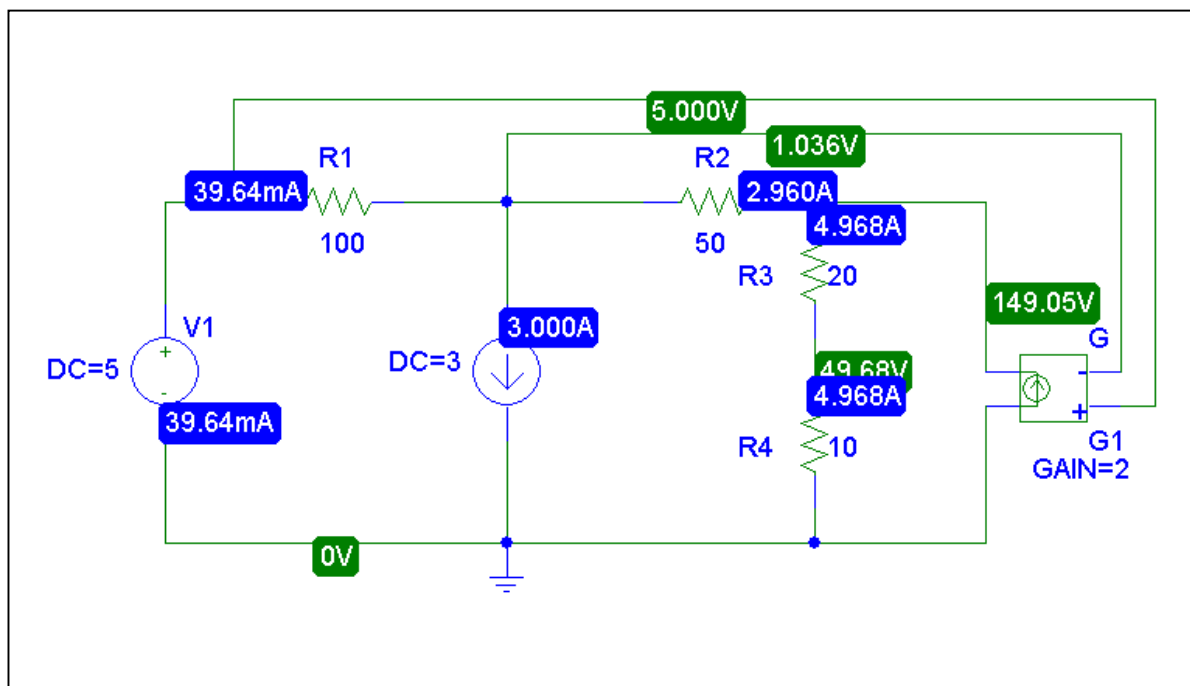
The current through the  $100\text{-}\Omega$  resistor is simply  $(5 - v_1)/100 = 39.64\text{ mA}$

The current through the  $50\text{-}\Omega$  resistor is  $(v_1 - v_2)/50 = -2.961\text{ A}$ ,

and the current through the  $20\text{-}\Omega$  and  $10\text{-}\Omega$  series combination is  $v_2/30 = 4.97\text{ A}$ .

Finally, the dependent source generates a current of  $2v_x = 7.928\text{ A}$ .

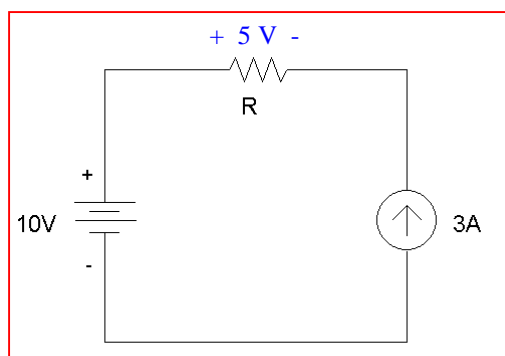
- (b) PSpice simulation results



**Summary:** The simulated results agree with the hand calculations.

## CHAPTER FOUR SOLUTIONS

57. One possible solution of many:



Choose  $R$  so that  $3R = 5$ ; then the voltage across the current source will be 5 V, and so will the voltage across the resistor  $R$ .

$R = 5/3 \Omega$ . To construct this from  $1\text{-}\Omega$  resistors, note that

$$5/3 \Omega = 1 \Omega + 2/3 \Omega = 1 \Omega + 1 \Omega \parallel 1\Omega \parallel 1\Omega + 1\Omega \parallel 1\Omega \parallel 1\Omega$$

\* Solution to Problem 4.57

.OP

V1 1 0 DC 10

I1 0 4 DC 3

R1 1 2 1

R2 2 3 1

R3 2 3 1

R4 2 3 1

R5 3 4 1

R6 3 4 1

R7 3 4 1

.END

```

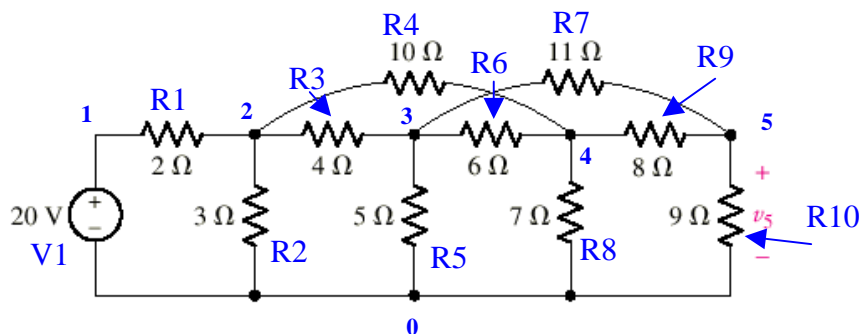
****  SMALL SIGNAL BIAS SOLUTION    TEMPERATURE =  27.000 DEG C
*****
NODE  VOLTAGE  NODE  VOLTAGE  NODE  VOLTAGE  NODE  VOLTAGE
(  1) 10.0000 (  2)  7.0000 (  3)  6.0000 (  4)  5.0000

VOLTAGE SOURCE CURRENTS
NAME      CURRENT

V1        -3.000E+00
    
```

## CHAPTER FOUR SOLUTIONS

58. We first name each node, resistor and voltage source:



We next write an appropriate input deck for SPICE:

```
* Solution to Problem 4.58
```

```
.OP
```

```
V1 1 0 DC 20
```

```
R1 1 2 2
```

```
R2 2 0 3
```

```
R3 2 3 4
```

```
R4 2 4 10
```

```
R5 3 0 5
```

```
R6 3 4 6
```

```
R7 3 5 11
```

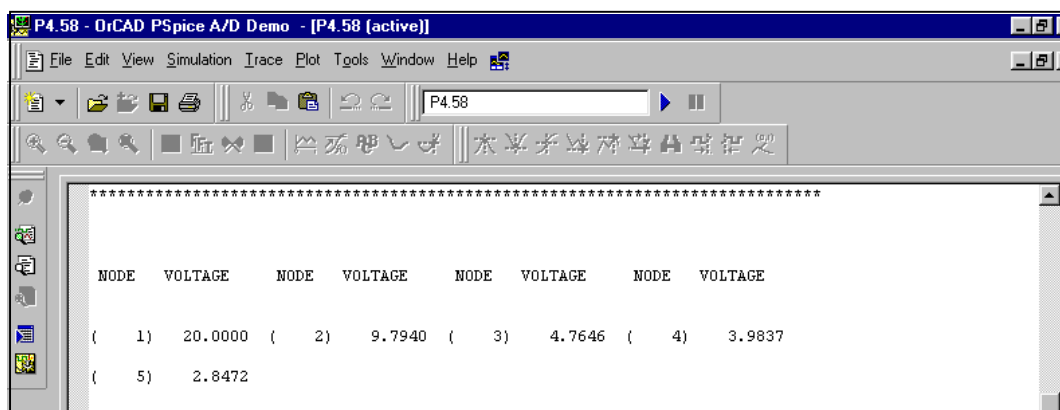
```
R8 4 0 7
```

```
R9 4 5 8
```

```
R10 5 0 9
```

```
.END
```

And obtain the following output:

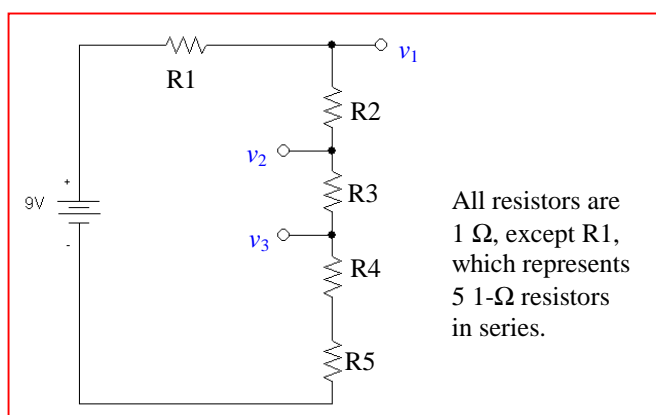


We see from this simulation result that the voltage  $v_5 = 2.847$  V.



## CHAPTER FOUR SOLUTIONS

59. One possible solution of many:



Verify:

$$v_1 = 9(4/9) = 4 \text{ V}$$

$$v_2 = 9(3/9) = 3 \text{ V}$$

$$v_3 = 9(2/9) = 2 \text{ V}$$

SPICE INPUT DECK:

\* Solution to Problem 4.59

.OP

V1 1 0 DC 9

R1 1 2 5

R2 2 3 1

R3 3 4 1

R4 4 5 1

R5 5 0 1

.END

\*\*\*\* 07/29/01 21:36:26 \*\*\*\*\* Evaluation PSpice (Nov 1999) \*\*\*\*\*

\* Solution to Problem 4.59

\*\*\*\* SMALL SIGNAL BIAS SOLUTION TEMPERATURE = 27.000 DEG C

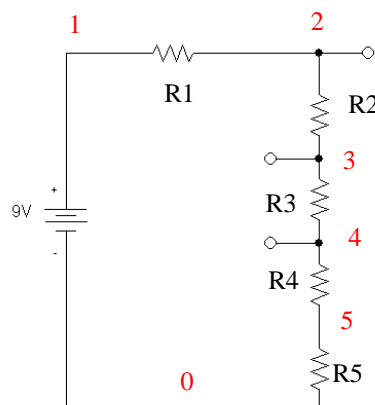
\*\*\*\*\*

\*\*\*\*\*

NODE	VOLTAGE	NODE	VOLTAGE	NODE	VOLTAGE	NODE	VOLTAGE
( 1 )	9.0000	( 2 )	4.0000	( 3 )	3.0000	( 4 )	2.0000
( 5 )	1.0000						

( 1 ) 9.0000 ( 2 ) 4.0000 ( 3 ) 3.0000 ( 4 ) 2.0000

( 5 ) 1.0000



## CHAPTER FOUR SOLUTIONS

60. (a) If only two bulbs are not lit (and thinking of each bulb as a resistor), the bulbs must be in parallel otherwise, the burned out bulbs, acting as short circuits, would prevent current from flowing to the “good” bulbs.

(b) In a parallel connected circuit, each bulb “sees” 115 VAC. Therefore, the individual bulb current is  $1 \text{ W} / 115 \text{ V} = 8.696 \text{ mA}$ . The resistance of each “good” bulb is  $V/I = 13.22 \text{ k}\Omega$ . A simplified, electrically-equivalent model for this circuit would be a 115 VAC source connected in parallel to a resistor  $R_{eq}$  such that

$$1/R_{eq} = 1/13.22 \times 10^3 + 1/13.22 \times 10^3 + \dots + 1/13.22 \times 10^3 \quad (400 - 2 = 398 \text{ terms})$$

or  $R_{eq} = 33.22 \text{ }\Omega$ . We expect the source to provide 398 W.

\* Solution to Problem 4.60

.OP

V1 1 0 AC 115 60  
R1 1 0 33.22

.AC LIN 1 60 60  
.PRINT AC VM(1)IM(V1)

.END

\*\*\*\* 07/29/01 21:09:32 \*\*\*\*\* Evaluation PSpice (Nov 1999) \*\*\*\*\*

\* Solution to Problem 4.60

\*\*\*\* SMALL SIGNAL BIAS SOLUTION TEMPERATURE = 27.000 DEG C

\*\*\*\*\*

NODE VOLTAGE	NODE VOLTAGE	NODE VOLTAGE	NODE VOLTAGE
( 1) 0.0000			

VOLTAGE SOURCE CURRENTS	
NAME	CURRENT

V1	0.000E+00
----	-----------

TOTAL POWER DISSIPATION	0.00E+00 WATTS
-------------------------	----------------

This calculated power is not the value sought. It is an artifact of the use of an ac source, which requires that we perform an ac analysis. The supplied power is then separately computed as  $(1.15 \times 10^2)(3.462) = 398.1 \text{ W}$ .

\*\*\*\* 07/29/01 21:09:32 \*\*\*\*\* Evaluation PSpice (Nov 1999)

\* Solution to Problem 4.60

\*\*\*\* AC ANALYSIS TEMPERATURE = 27.000 DEG C

\*\*\*\*\*

FREQ	VM(1)	IM(V1)
6.000E+01	1.150E+02	3.462E+00

- (c) The inherent series resistance of the wire connections leads to a voltage drop which increases the further one is from the voltage source. Thus, the furthest bulbs actually have less than 115 VAC across them, so they draw slightly less current and glow more dimly.

## CHAPTER FIVE SOLUTIONS

1. Define percent error as  $100 [e^x - (1 + x)] / e^x$

$x$	$1 + x$	$e^x$	% error
0.001	1.001	1.001	$5 \times 10^{-5}$
0.005	1.005	1.005	$1 \times 10^{-3}$
0.01	1.01	1.010	$5 \times 10^{-3}$
0.05	1.05	1.051	0.1
0.10	1.10	1.105	0.5
0.50	1.50	1.649	9
1.00	2.00	2.718	26
5.00	6.00	148.4	96

Of course, “reasonable” is a very subjective term. However, if we choose  $x < 0.1$ , we ensure that the error is less than 1%.

## CHAPTER FIVE SOLUTIONS

2.  $i_A, v_B$  “on”,  $v_C = 0$ :  $i_x = 20$  A  
 $i_A, v_C$  “on”,  $v_B = 0$ :  $i_x = -5$  A  
 $i_A, v_B, v_C$  “on” :  $i_x = 12$  A

so, we can write

$$\begin{aligned} i_x' + i_x'' + i_x''' &= 12 \\ i_x' + i_x'' &= 20 \\ i_x' &+ i_x''' = -5 \end{aligned}$$

In matrix form,

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_x' \\ i_x'' \\ i_x''' \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ -5 \end{bmatrix}$$

- (a) with  $i_A$  on only, the response  $i_x = i_x' = 3$  A.  
 (b) with  $v_B$  on only, the response  $i_x = i_x'' = 17$  A.  
 (c) with  $v_C$  on only, the response  $i_x = i_x''' = -8$  A.  
 (d)  $i_A$  and  $v_C$  doubled,  $v_B$  reversed:  $2(3) + 2(-8) + (-1)(17) = -27$  A.

## CHAPTER FIVE SOLUTIONS

3. One source at a time:

The contribution from the 24-V source may be found by shorting the 45-V source and open-circuiting the 2-A source. Applying voltage division,

$$v_x' = 24 \frac{20}{10 + 20 + 45 \parallel 30} = 24 \frac{20}{10 + 20 + 18} = 10 \text{ V}$$

We find the contribution of the 2-A source by shorting both voltage sources and applying current division:

$$v_x'' = 20 \left[ 2 \frac{10}{10 + 20 + 18} \right] = 8.333 \text{ V}$$

Finally, the contribution from the 45-V source is found by open-circuiting the 2-A source and shorting the 24-V source. Defining  $v_{30}$  across the 30- $\Omega$  resistor with the “+” reference on top:

$$0 = v_{30}/20 + v_{30}/(10 + 20) + (v_{30} - 45)/45$$

solving,  $v_{30} = 11.25 \text{ V}$  and hence  $v_x''' = -11.25(20)/(10 + 20) = -7.5 \text{ V}$

Adding the individual contributions, we find that  $v_x = v_x' + v_x'' + v_x''' = 10.83 \text{ V}$ .

## CHAPTER FIVE SOLUTIONS

4. The contribution of the 8-A source is found by shorting out the two voltage sources and employing simple current division:

$$i_3' = -8 \frac{50}{50 + 30} = -5 \text{ A}$$

The contribution of the voltage sources may be found collectively or individually. The contribution of the 100-V source is found by open-circuiting the 8-A source and shorting the 60-V source. Then,

$$i_3'' = \frac{100}{(50 + 30) \parallel 60 \parallel 30} = 6.25 \text{ A}$$

The contribution of the 60-V source is found in a similar way as  $i_3''' = -60/30 = -2 \text{ A}$ .

The total response is  $i_3 = i_3' + i_3'' + i_3''' = -750 \text{ mA}$ .

## CHAPTER FIVE SOLUTIONS

5. (a) By current division, the contribution of the 1-A source  $i_2'$  is  $i_2' = 1 (200) / 250 = 800 \text{ mA}$ .

The contribution of the 100-V source is  $i_2'' = 100 / 250 = 400 \text{ mA}$ .

The contribution of the 0.5-A source is found by current division once the 1-A source is open-circuited and the voltage source is shorted. Thus,

$$i_2''' = 0.5 (50) / 250 = 100 \text{ mA}$$

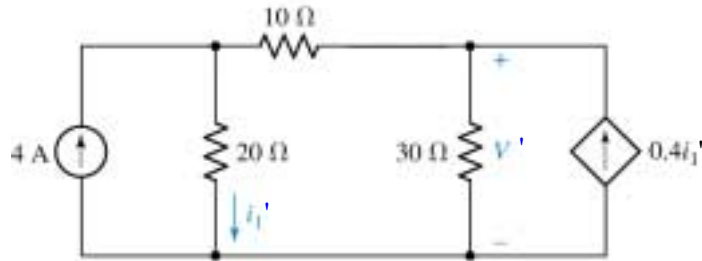
Thus,  $i_2 = i_2' + i_2'' + i_2''' = 1.3 \text{ A}$

(b)  $P_{1A} = (1) [(200)(1 - 1.3)] = 60 \text{ W}$   
 $P_{200} = (1 - 1.3)^2 (200) = 18 \text{ W}$   
 $P_{100V} = -(1.3)(100) = -130 \text{ W}$   
 $P_{50} = (1.3 - 0.5)^2 (50) = 32 \text{ W}$   
 $P_{0.5A} = (0.5) [(50)(1.3 - 0.5)] = 20 \text{ W}$

Check:  $60 + 18 + 32 + 20 = +130$ .

## CHAPTER FIVE SOLUTIONS

6. We find the contribution of the 4-A source by shorting out the 100-V source and analysing the resulting circuit:



$$4 = V_1' / 20 + (V_1' - V') / 10 \quad [1]$$

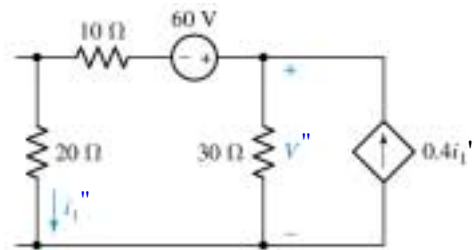
$$0.4 i_1' = V_1' / 30 + (V' - V_1') / 10 \quad [2]$$

where  $i_1' = V_1' / 20$

Simplifying & collecting terms, we obtain  $30 V_1' - 20 V' = 800$  [1]

$$-7.2 V_1' + 8 V' = 0 \quad [2]$$

Solving, we find that  $V' = 60$  V. Proceeding to the contribution of the 60-V source, we analyse the following circuit after defining a clockwise mesh current  $i_a$  flowing in the left mesh and a clockwise mesh current  $i_b$  flowing in the right mesh.



$$30 i_a - 60 + 30 i_a - 30 i_b = 0 \quad [1]$$

$$i_b = -0.4 i_1'' = +0.4 i_a \quad [2]$$

Solving, we find that  $i_a = 1.25$  A and so  $V'' = 30(i_a - i_b) = 22.5$  V.

Thus,  $V = V' + V'' = 82.5$  V.



## CHAPTER FIVE SOLUTIONS

7. (a) Linearity allows us to consider this by viewing each source as being scaled by 25/ 10. This means that the response ( $v_3$ ) will be scaled by the same factor:

$$25 i_A' / 10 + 25 i_B' / 10 = 25 v_3' / 10$$
$$\therefore v_3 = 25 v_3' / 10 = 25(80) / 10 = \boxed{200 \text{ V}}$$

(b)

$i_A' = 10 \text{ A}, i_B' = 25 \text{ A}$	$\rightarrow v_4' = 100 \text{ V}$
$i_A'' = 10 \text{ A}, i_B'' = 25 \text{ A}$	$\rightarrow v_4'' = -50 \text{ V}$
$i_A = 20 \text{ A}, i_B = -10 \text{ A}$	$\rightarrow v_4 = ?$

We can view this in a somewhat abstract form: the currents  $i_A$  and  $i_B$  multiply the same circuit parameters regardless of their value; the result is  $v_4$ .

Writing in matrix form,  $\begin{bmatrix} 10 & 25 \\ 25 & 10 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 100 \\ -50 \end{bmatrix}$ , we can solve to find

$a = -4.286$  and  $b = 5.714$ , so that  $20a - 10b$  leads to  $v_4 = \boxed{-142.9 \text{ V}}$

## CHAPTER FIVE SOLUTIONS

8. With the current source open-circuited and the 7-V source shorted, we are left with  $100\text{k} \parallel (22\text{k} + 4.7\text{k}) = 21.07\text{ k}\Omega$ .

$$\text{Thus, } V_{3\text{V}} = 3 (21.07) / (21.07 + 47) = 0.9286\text{ V.}$$

In a similar fashion, we find that the contribution of the 7-V source is:

$$V_{7\text{V}} = 7 (31.97) / (31.97 + 26.7) = 3.814\text{ V}$$

Finally, the contribution of the current source to the voltage V across it is:

$$V_{5\text{mA}} = (5 \times 10^{-3}) (47\text{k} \parallel 100\text{k} \parallel 26.7\text{k}) = 72.75\text{ V.}$$

$$\text{Adding, we find that } V = 0.9286 + 3.814 + 72.75 = 77.49\text{ V.}$$

## CHAPTER FIVE SOLUTIONS

9. We must find the current through the 500-k $\Omega$  resistor using superposition, and then calculate the dissipated power.

The contribution from the current source may be calculated by first noting that  $1\text{M} \parallel 2.7\text{M} \parallel 5\text{M} = 636.8\text{ k}\Omega$ . Then,

$$i_{60\mu\text{A}} = 60 \times 10^{-6} \left( \frac{3}{0.5 + 3 + 0.6368} \right) = 43.51\text{ }\mu\text{A}$$

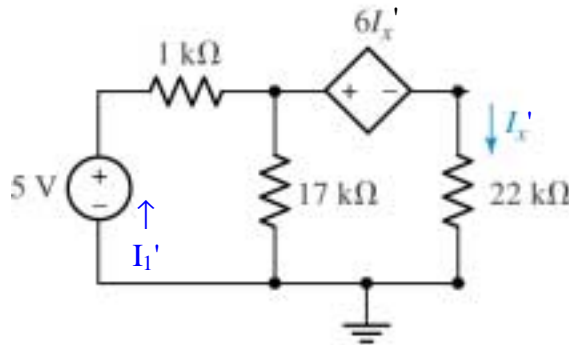
The contribution from the voltage source is found by first noting that  $2.7\text{M} \parallel 5\text{M} = 1.753\text{ M}\Omega$ . The total current flowing from the voltage source (with the current source open-circuited) is  $-1.5 / (3.5 \parallel 1.753 + 1)\text{ }\mu\text{A} = -0.6919\text{ }\mu\text{A}$ . The current flowing through the 500-k $\Omega$  resistor due to the voltage source acting alone is then

$$i_{1.5\text{V}} = 0.6919 (1.753) / (1.753 + 3.5)\text{ mA} = 230.9\text{ nA}.$$

The total current through the 500-k $\Omega$  resistor is then  $i_{60\mu\text{A}} + i_{1.5\text{V}} = 43.74\text{ }\mu\text{A}$  and the dissipated power is  $(43.74 \times 10^{-9})^2 (500 \times 10^3) = 956.6\text{ }\mu\text{W}$ .

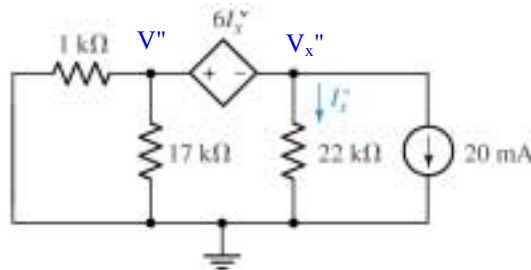
## CHAPTER FIVE SOLUTIONS

10. We first determine the contribution of the voltage source:



Via mesh analysis, we write:  $5 = 18000 I_1' - 17000 I_x'$   
 $-6 I_x' = -17000 I_x' + 39000 I_x'$

Solving, we find  $I_1' = 472.1 \text{ mA}$  and  $I_x' = 205.8 \text{ mA}$ , so  $V' = 17 \times 10^3 (I_1' - I_x') = 4.527 \text{ V}$ . We proceed to find the contribution of the current source:



Via supernode:  $-20 \times 10^{-3} = V_x'' / 22 \times 10^3 + V'' / 0.9444 \times 10^3$  [1]  
 and  $V'' - V_x'' = 6 I_x''$  or  $V'' - V_x'' = 6 V_x'' / 22 \times 10^3$  [2]

Solving, we find that  $V'' = -18.11 \text{ V}$ . Thus,  $V = V' + V'' = -13.58 \text{ V}$ .

The maximum power is  $V^2 / 17 \times 10^3 = V^2 / 17 \text{ mW} = 250 \text{ mW}$ , so

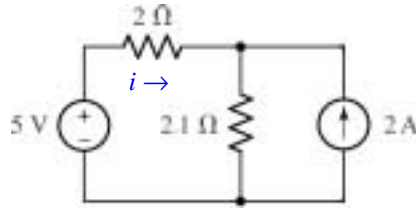
$V = \sqrt{(17)(250)} = 65.19 = V' - 13.58$ . Solving, we find  $V'_{\max} = 78.77 \text{ V}$ .

The 5-V source may then be increased by a factor of  $78.77 / 4.527$ , so that its maximum positive value is **87 V**; past this value, and the resistor will overheat.

## CHAPTER FIVE SOLUTIONS

11. It is impossible to identify the individual contribution of each source to the power dissipated in the resistor; superposition cannot be used for such a purpose.

Simplifying the circuit, we may at least determine the total power dissipated in the resistor:



Via superposition in one step, we may write

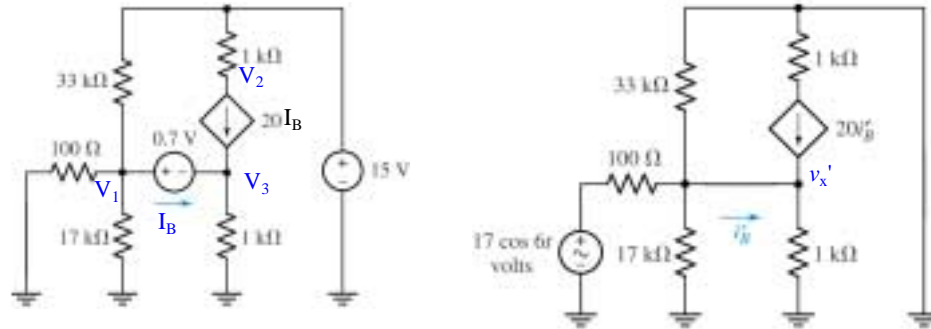
$$i = \frac{5}{2 + 2.1} - 2 \frac{2.1}{2 + 2.1} = 195.1 \text{ mA}$$

Thus,

$$P_{2\Omega} = i^2 \cdot 2 = 76.15 \text{ mW}$$

## CHAPTER FIVE SOLUTIONS

12. We will analyse this circuit by first considering the combined effect of both dc sources (left), and then finding the effect of the single ac source acting alone (right).



$$1, 3 \text{ supernode: } V_1/100 + V_1/17 \times 10^3 + (V_1 - 15)/33 \times 10^3 + V_3/10^3 = 20 I_B \quad [1]$$

$$\text{and: } V_1 - V_3 = 0.7 \quad [2]$$

$$\text{Node 2: } -20 I_B = (V_2 - 15)/1000 \quad [3]$$

We require one additional equation if we wish to have  $I_B$  as an unknown:

$$20 I_B + I_B = V_3/1000 \quad [4]$$

Simplifying and collecting terms,

$$10.08912 V_1 + V_3 - 20 \times 10^3 I_B = 0.4545 \quad [1]$$

$$V_1 - V_3 = 0.7 \quad [2]$$

$$V_2 + 20 \times 10^3 I_B = 15 \quad [3]$$

$$-V_3 + 21 \times 10^3 I_B = 0 \quad [4]$$

Solving, we find that  $I_B = -31.04 \mu\text{A}$ .

To analyse the right-hand circuit, we first find the Thévenin equivalent to the left of the wire marked  $i_B'$ , noting that the 33-k $\Omega$  and 17-k $\Omega$  resistors are now in parallel. We find that  $V_{TH} = 16.85 \cos 6t \text{ V}$  by voltage division, and  $R_{TH} = 100 \parallel 17k \parallel 33k = 99.12 \Omega$ . We may now proceed:

$$20 i_B' = v_x' / 1000 + (v_x' - 16.85 \cos 6t) / 99.12 \quad [1]$$

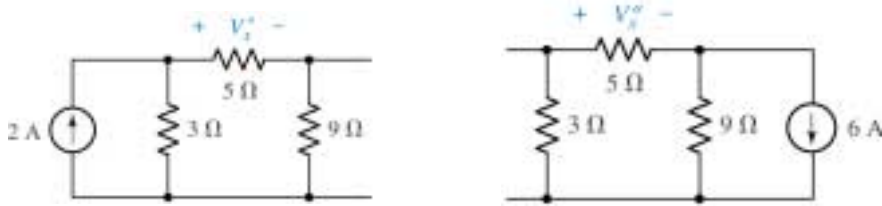
$$20 i_B' + i_B'' = v_x' / 1000 \quad [2]$$

Solving, we find that  $i_B' = 798.6 \cos 6t \text{ mA}$ . Thus, adding our two results, we find the complete current is

$$i_B = i_B' + I_B = -31.04 + 798.6 \cos 6t \mu\text{A}.$$

## CHAPTER FIVE SOLUTIONS

13.



We first consider the effect of the 2-A source separately, using the left circuit:

$$V_{x'} = 5 \left[ 2 \frac{3}{3+14} \right] = 1.765 \text{ V}$$

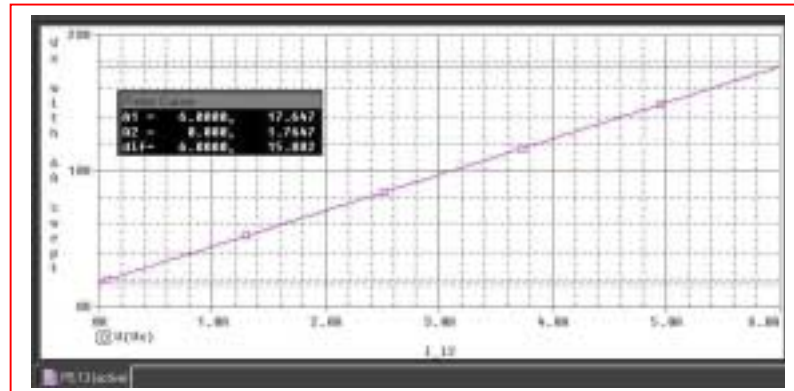
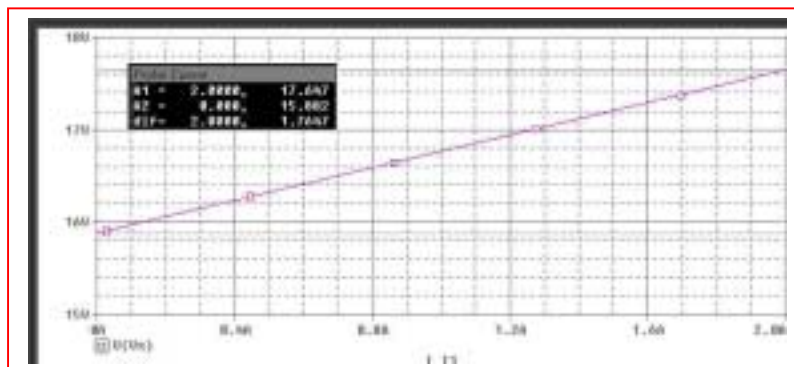
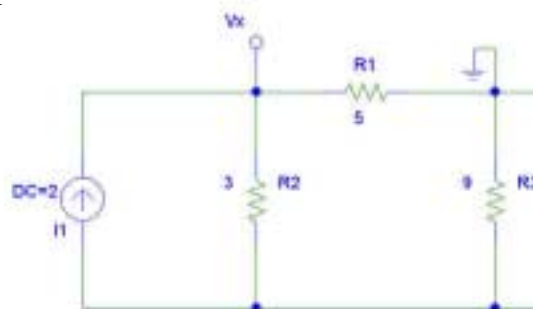
Next we consider the effect of the 6-A source on its own using the right circuit:

$$V_{x''} = 5 \left[ 6 \frac{9}{9+8} \right] = 15.88 \text{ V}$$

Thus,  $V_x = V_{x'} + V_{x''} = 17.65 \text{ V}$ .

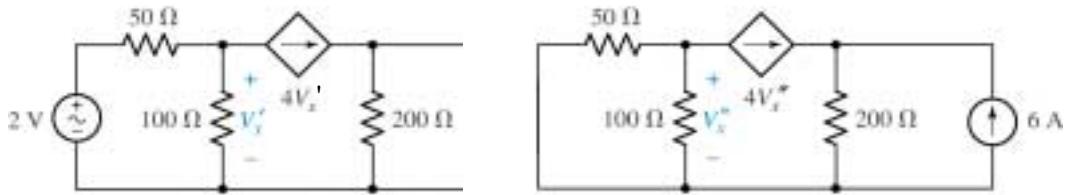
(b) PSpice verification (DC Sweep)

The DC sweep results below confirm that  $V_{x'} = 1.765 \text{ V}$



## CHAPTER FIVE SOLUTIONS

14.



(a) Beginning with the circuit on the left, we find the contribution of the 2-V source to  $V_x$ :

$$-4V'_x = \frac{V'_x}{100} + \frac{V'_x - 2}{50}$$

which leads to  $V'_x = 9.926 \text{ mV}$ .

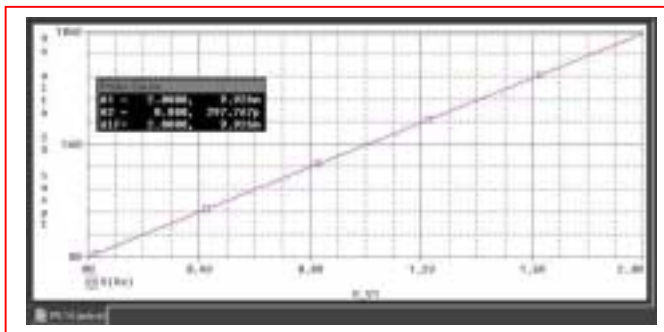
The circuit on the right yields the contribution of the 6-A source to  $V_x$ :

$$-4V''_x = \frac{V''_x}{100} + \frac{V''_x}{50}$$

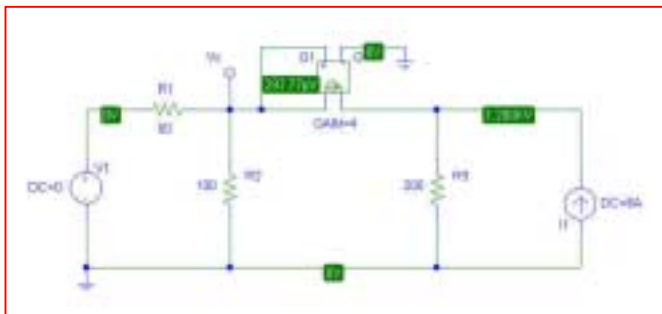
which leads to  $V''_x = 0$ .

Thus,  $V_x = V'_x + V''_x = 9.926 \text{ mV}$ .

(b) PSpice verification.



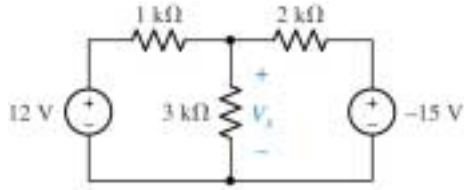
As can be seen from the two separate PSpice simulations, our hand calculations are correct; the pV-scale voltage in the second simulation is a result of numerical inaccuracy.





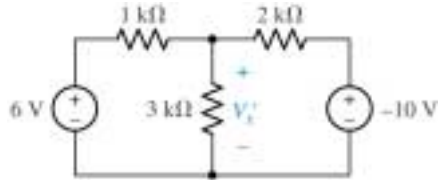
## CHAPTER FIVE SOLUTIONS

15.



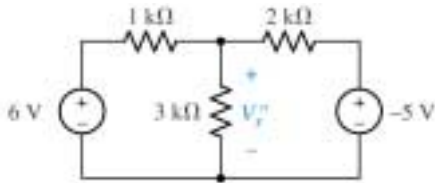
$$\frac{V_x - 12}{1} + \frac{V_x}{3} + \frac{V_x + 15}{2} = 0$$

so  $V_x = 2.455 \text{ V}$



$$\frac{V'_x - 6}{1} + \frac{V'_x}{3} + \frac{V'_x + 10}{2} = 0$$

so  $V'_x = 0.5455 \text{ V}$



$$\frac{V''_x - 6}{1} + \frac{V''_x}{3} + \frac{V''_x + 5}{2} = 0$$

so  $V''_x = 1.909 \text{ V}$

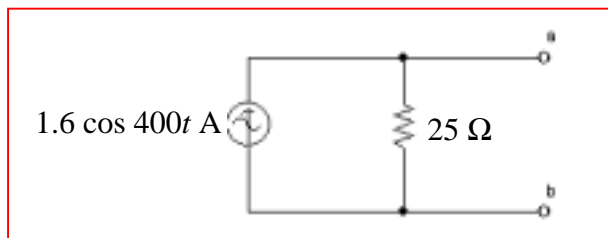
Adding, we find that  $V'_x + V''_x = 2.455 \text{ V} = V_x$  as promised.

## CHAPTER FIVE SOLUTIONS

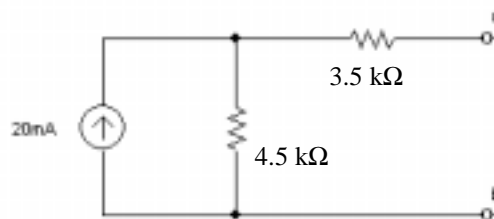
16. (a)  $[120 \cos 400t] / 60 = 2 \cos 400t \text{ A}$ .  $60 \parallel 120 = 40 \Omega$ .

$[2 \cos 400t] (40) = 80 \cos 400t \text{ V}$ .  $40 + 10 = 50 \Omega$ .

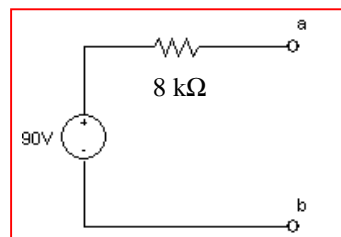
$[80 \cos 400t] / 50 = 1.6 \cos 400t \text{ A}$ .  $50 \parallel 50 = 25 \Omega$ .



(b)  $2k \parallel 3k + 6k = 7.2 \text{ k}\Omega$ .  $7.2k \parallel 12k = 4.5 \text{ k}\Omega$



$(20)(4.5) = 90 \text{ V}$ .



## CHAPTER FIVE SOLUTIONS

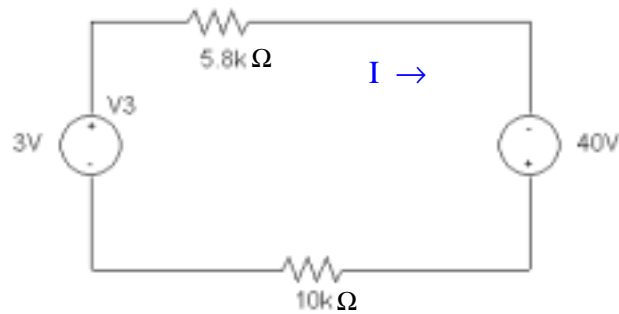
17. We can ignore the 1-k $\Omega$  resistor, at least when performing a source transformation on this circuit, as the 1-mA source will pump 1 mA through *whatever* value resistor we place there. So, we need only combine the 1 and 2 mA sources (which are in parallel once we replace the 1-k $\Omega$  resistor with a 0- $\Omega$  resistor). The current through the 5.8-k $\Omega$  resistor is then simply given by voltage division:

$$i = 3 \times 10^{-3} \frac{4.7}{4.7 + 5.8} = 1.343 \text{ mA}$$

The power dissipated by the 5.8-k $\Omega$  resistor is then  $i^2 \cdot 5.8 \times 10^3 = 10.46 \text{ mW}$ .

## CHAPTER FIVE SOLUTIONS

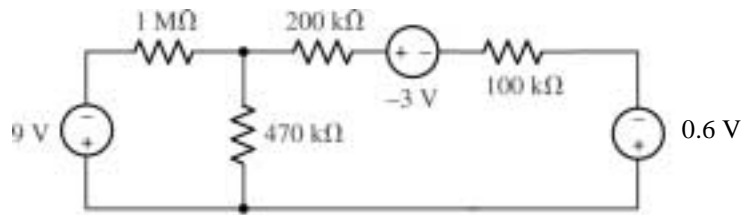
18. We may ignore the  $10\text{-k}\Omega$  and  $9.7\text{-k}\Omega$  resistors, as  $3\text{-V}$  will appear across them regardless of their value. Performing a quick source transformation on the  $10\text{-k}\Omega$  resistor/  $4\text{-mA}$  current source combination, we replace them with a  $40\text{-V}$  source in series with a  $10\text{-k}\Omega$  resistor:



$I = 43 / 15.8 \text{ mA} = 2.722 \text{ mA}$ . Therefore,  $P_{5.8\Omega} = I^2 \cdot 5.8 \times 10^3 = 42.97 \text{ mW}$ .

## CHAPTER FIVE SOLUTIONS

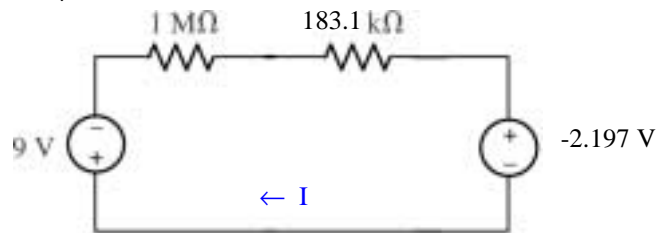
19.  $(100 \text{ k}\Omega)(6 \text{ mA}) = 0.6 \text{ V}$



$$470 \text{ k} \parallel 300 \text{ k} = 183.1 \text{ k}\Omega$$

$$(-3 - 0.6) / 300 \times 10^3 = -12 \text{ }\mu\text{A}$$

$$(183.1 \text{ k}\Omega)(-12 \text{ }\mu\text{A}) = -2.197 \text{ V}$$

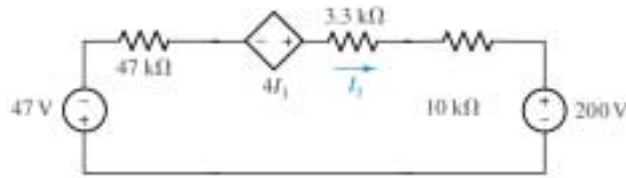


Solving,  $9 + 1183.1 \times 10^3 I - 2.197 = 0$ , so  $I = -5.750 \text{ }\mu\text{A}$ . Thus,

$$P_{1\text{M}\Omega} = I^2 \cdot 10^6 = \boxed{33.06 \text{ }\mu\text{W}}.$$

## CHAPTER FIVE SOLUTIONS

20. (1)(47) = 47 V. (20)(10) = 200 V. Each voltage source “+” corresponds to its corresponding current source’s arrow head.



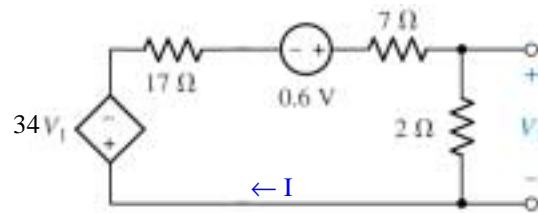
Using KVL on the simplified circuit above,

$$47 + 47 \times 10^3 I_1 - 4 I_1 + 13.3 \times 10^3 I_1 + 200 = 0$$

Solving, we find that  $I_1 = -247 / (60.3 \times 10^3 - 4) = -4.096 \text{ mA}$ .

## CHAPTER FIVE SOLUTIONS

21.  $(2 V_1)(17) = 34 V_1$



Analysing the simplified circuit above,

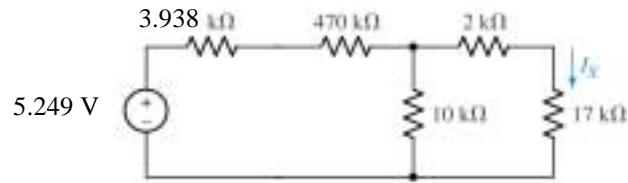
$$34 V_1 - 0.6 + 7 I + 2 I + 17 I = 0 \quad [1] \quad \text{and} \quad V_1 = 2 I \quad [2]$$

Substituting, we find that  $I = 0.6 / (68 + 7 + 2 + 17) = 6.383 \text{ mA}$ . Thus,

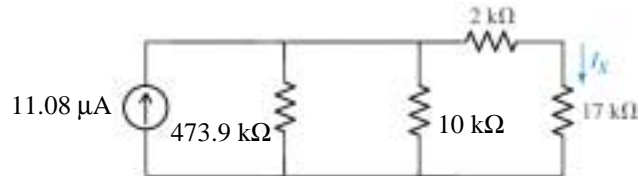
$$V_1 = 2 I = \boxed{12.77 \text{ mV}}$$

## CHAPTER FIVE SOLUTIONS

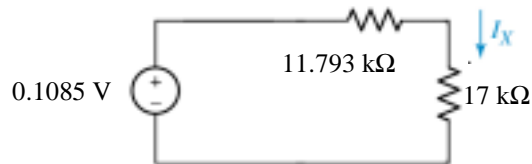
22.  $12/9000 = 1.333 \text{ mA}$ .  $9\text{k} \parallel 7\text{k} = 3.938 \text{ k}\Omega$ .  $\rightarrow (1.333 \text{ mA})(3.938 \text{ k}\Omega) = 5.249 \text{ V}$ .



$$5.249 / 473.938 \times 10^3 = 11.08 \text{ }\mu\text{A}$$



$$473.9 \text{ k} \parallel 10 \text{ k} = 9.793 \text{ k}\Omega. (11.08 \text{ mA})(9.793 \text{ k}\Omega) = 0.1085 \text{ V}$$

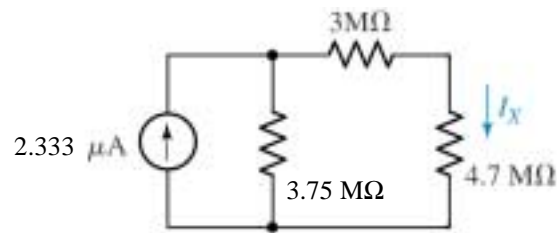


$$I_x = 0.1085 / 28.793 \times 10^3 = \boxed{3.768 \text{ }\mu\text{A}}.$$



## CHAPTER FIVE SOLUTIONS

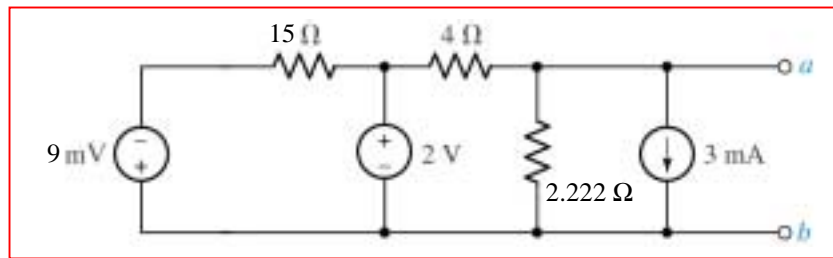
23. First,  $(-7 \mu\text{A})(2 \text{ M}\Omega) = -14 \text{ V}$ , “+” reference down.  $2 \text{ M}\Omega + 4 \text{ M}\Omega = 6 \text{ M}\Omega$ .  
 $+14 \text{ V} / 6 \text{ M}\Omega = 2.333 \mu\text{A}$ , arrow pointing up;  $6 \text{ M} \parallel 10 \text{ M} = 3.75 \text{ M}\Omega$ .



$$(2.333)(3.75) = 8.749 \text{ V. } R_{\text{eq}} = 6.75 \text{ M}\Omega$$
$$\therefore I_x = 8.749 / (6.75 + 4.7) \mu\text{A} = \boxed{764.1 \text{ nA.}}$$

## CHAPTER FIVE SOLUTIONS

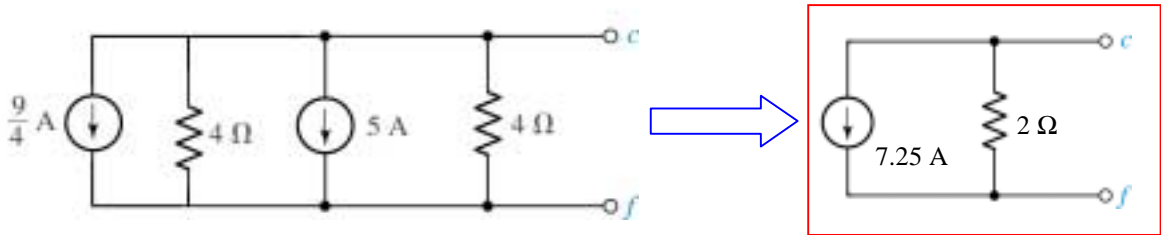
24. To begin, note that  $(1 \text{ mA})(9 \Omega) = 9 \text{ mV}$ , and  $5 \parallel 4 = 2.222 \Omega$ .



The above circuit may not be further simplified using only source transformation techniques.

## CHAPTER FIVE SOLUTIONS

25. Label the “-” terminal of the 9-V source node **x** and the other terminal node **x'**. The 9-V source will force the voltage across these two terminals to be  $-9$  V regardless of the value of the current source and resistor to its left. These two components may therefore be neglected from the perspective of terminals **a** & **b**. Thus, we may draw:



## CHAPTER FIVE SOLUTIONS

26. Beware of the temptation to employ superposition to compute the dissipated power- *it won't work!*

Instead, define a current  $I$  flowing into the bottom terminal of the  $1\text{-M}\Omega$  resistor. Using superposition to compute this current,

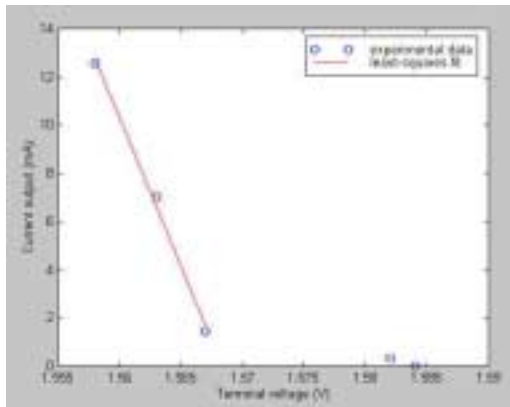
$$I = 1.8 / 1.840 + 0 + 0 \text{ } \mu\text{A} = 978.3 \text{ nA.}$$

Thus,

$$P_{1\text{M}\Omega} = (978.3 \times 10^{-9})^2 (10^6) = 957.1 \text{ nW.}$$

## CHAPTER FIVE SOLUTIONS

27. Let's begin by plotting the experimental results, along with a least-squares fit to part of the data:



**Least-squares fit results:**

Voltage (V)	Current (mA)
1.567	1.6681
1.563	6.599
1.558	12.763

We see from the figure that we cannot draw a very good line through all data points representing currents from 1 mA to 20 mA. We have therefore chosen to perform a linear fit for the three lower voltages only, as shown. Our model will not be as accurate at 1 mA; there is no way to know if our model will be accurate at 20 mA, since that is beyond the range of the experimental data.

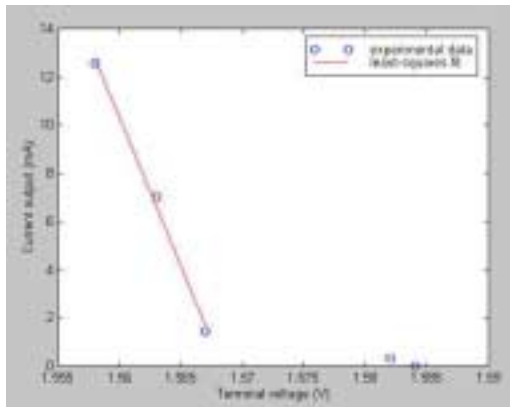
Modeling this system as an ideal voltage source in series with a resistance (representing the internal resistance of the battery) and a varying load resistance, we may write the following two equations based on the linear fit to the data:

$$\begin{aligned} 1.567 &= V_{\text{src}} - R_s (1.6681 \times 10^{-3}) \\ 1.558 &= V_{\text{src}} - R_s (12.763 \times 10^{-3}) \end{aligned}$$

Solving,  $V_{\text{src}} = 1.568 \text{ V}$  and  $R_s = 811.2 \text{ m}\Omega$ . It should be noted that depending on the line fit to the experimental data, these values can change somewhat, particularly the series resistance value.

## CHAPTER FIVE SOLUTIONS

28. Let's begin by plotting the experimental results, along with a least-squares fit to part of the data:



### Least-squares fit results:

Voltage (V)	Current (mA)
1.567	1.6681
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We see from the figure that we cannot draw a very good line through all data points representing currents from 1 mA to 20 mA. We have therefore chosen to perform a linear fit for the three lower voltages only, as shown. Our model will not be as accurate at 1 mA; there is no way to know if our model will be accurate at 20 mA, since that is beyond the range of the experimental data.

Modeling this system as an ideal current source in parallel with a resistance  $R_p$  (representing the internal resistance of the battery) and a varying load resistance, we may write the following two equations based on the linear fit to the data:

$$1.6681 \times 10^{-3} = I_{src} - 1.567 / R_p$$

$$12.763 \times 10^{-3} = I_{src} - 1.558 / R_p$$

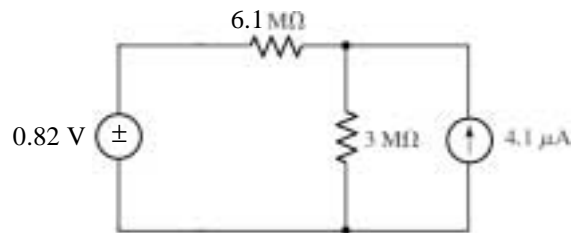
Solving,  $I_{src} = 1.933 \text{ A}$  and  $R_s = 811.2 \text{ m}\Omega$ . It should be noted that depending on the line fit to the experimental data, these values can change somewhat, particularly the series resistance value.

## CHAPTER FIVE SOLUTIONS

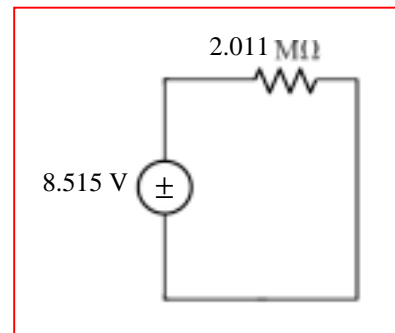
29. Reference terminals are required to avoid ambiguity: *depending on the sources with which we begin the transformation process, we will obtain entirely different answers.* Working from left to right in this case,

$$2\ \mu\text{A} - 1.8\ \mu\text{A} = 200\ \text{nA, arrow up.}$$
$$1.4\ \text{M}\Omega + 2.7\ \text{M}\Omega = 4.1\ \text{M}\Omega$$

An additional transformation back to a voltage source yields  $(200\ \text{nA})(4.1\ \text{M}\Omega) = 0.82\ \text{V}$  in series with  $4.1\ \text{M}\Omega + 2\ \text{M}\Omega = 6.1\ \text{M}\Omega$ , as shown below:

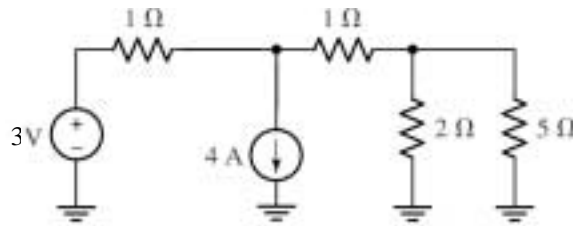


Then,  $0.82\ \text{V} / 6.1\ \text{M}\Omega = 134.4\ \text{nA}$ , arrow up.  
 $6.1\ \text{M}\Omega \parallel 3\ \text{M}\Omega = 2.011\ \text{M}\Omega$   
 $4.1\ \mu\text{A} + 134.4\ \text{nA} = 4.234\ \text{mA}$ , arrow up.  
 $(4.234\ \mu\text{A})(2.011\ \text{M}\Omega) = 8.515\ \text{V}.$

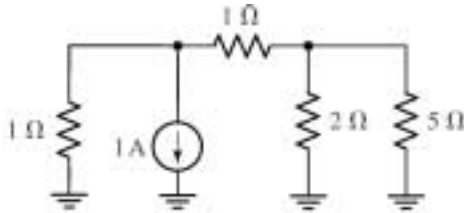


## CHAPTER FIVE SOLUTIONS

30. To begin, we note that the 5-V and 2-V sources are in series:



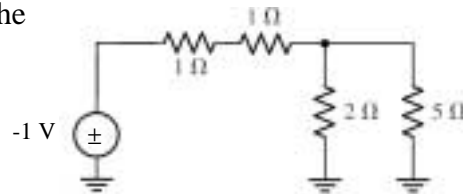
Next, noting that  $3\text{ V} / 1\ \Omega = 3\text{ A}$ , and  $4\text{ A} - 3\text{ A} = +1\text{ A}$  (arrow down), we obtain:



The left-hand resistor and the current source are easily transformed into a 1-V source in series with a 1-Ω resistor:

By voltage division, the voltage across the 5-Ω resistor in the circuit to the right is:

$$(-1) \frac{2 \parallel 5}{2 \parallel 5 + 2} = -0.4167\text{ V.}$$



Thus, the power dissipated by the 5-Ω resistor is  $(-0.4167)^2 / 5 = 34.73\text{ mW}$ .



## CHAPTER FIVE SOLUTIONS

31. (a)  $R_{TH} = 25 \parallel (10 + 15) = 25 \parallel 25 = 12.5 \Omega.$

$$V_{TH} = V_{ab} = 50 \left( \frac{25}{10 + 15 + 25} \right) + 100 \left( \frac{15 + 10}{15 + 10 + 25} \right) = 75 \text{ V.}$$

(b) If  $R_{ab} = 50 \Omega$ ,

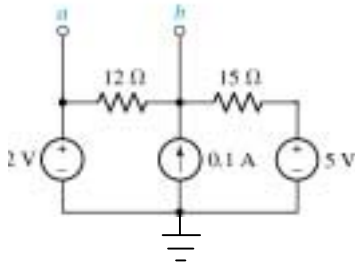
$$P_{50\Omega} = \left[ 75 \left( \frac{50}{50 + 12.5} \right) \right]^2 \left( \frac{1}{50} \right) = 72 \text{ W}$$

(c) If  $R_{ab} = 12.5 \Omega$ ,

$$P_{12.5\Omega} = \left[ 75 \left( \frac{12.5}{12.5 + 12.5} \right) \right]^2 \left( \frac{1}{12.5} \right) = 112.5 \text{ W}$$

## CHAPTER FIVE SOLUTIONS

32. (a) Removing terminal **c**, we need write only one nodal equation:



$$0.1 = \frac{V_b - 2}{12} + \frac{V_b - 5}{15}, \text{ which may be solved to}$$

yield  $V_b = 4 \text{ V}$ . Therefore,  $V_{ab} = V_{TH} = 2 - 4 = -2 \text{ V}$ .

$R_{TH} = 12 \parallel 15 = 6.667 \Omega$ . We may then calculate  $I_N$  as  $I_N = V_{TH} / R_{TH}$

$$= -300 \text{ mA (arrow pointing upwards).}$$

- (b) Removing terminal **a**, we again find  $R_{TH} = 6.667 \Omega$ , and only need write a single nodal equation; in fact, it is identical to that written for the circuit above, and we once again find that  $V_b = 4 \text{ V}$ . In this case,  $V_{TH} = V_{bc} = 4 - 5 = -1 \text{ V}$ , so  $I_N = -1 / 6.667 = -150 \text{ mA (arrow pointing upwards)}$ .

## CHAPTER FIVE SOLUTIONS

33. (a) Shorting out the 88-V source and open-circuiting the 1-A source, we see looking into the terminals x and x' a 50- $\Omega$  resistor in parallel with 10  $\Omega$  in parallel with (20  $\Omega$  + 40  $\Omega$ ), so

$$R_{TH} = 50 \parallel 10 \parallel (20 + 40) = 7.317 \Omega$$

Using superposition to determine the voltage  $V_{xx'}$  across the 50- $\Omega$  resistor, we find

$$\begin{aligned} V_{xx'} = V_{TH} &= \left[ 88 \frac{50 \parallel (20 + 40)}{10 + [50 \parallel (20 + 40)]} \right] + (1)(50 \parallel 10) \left[ \frac{40}{40 + 20 + (50 \parallel 10)} \right] \\ &= \left[ 88 \frac{27.27}{37.27} \right] + (1)(8.333) \left[ \frac{40}{40 + 20 + 8.333} \right] = 69.27 \text{ V} \end{aligned}$$

- (b) Shorting out the 88-V source and open-circuiting the 1-A source, we see looking into the terminals y and y' a 40- $\Omega$  resistor in parallel with [20  $\Omega$  + (10  $\Omega$   $\parallel$  50  $\Omega$ )]:

$$R_{TH} = 40 \parallel [20 + (10 \parallel 50)] = 16.59 \Omega$$

Using superposition to determine the voltage  $V_{yy'}$  across the 1-A source, we find

$$\begin{aligned} V_{yy'} = V_{TH} &= (1)(R_{TH}) + \left[ 88 \frac{27.27}{10 + 27.27} \right] \left( \frac{40}{20 + 40} \right) \\ &= 59.52 \text{ V} \end{aligned}$$

## CHAPTER FIVE SOLUTIONS

34. (a) Select terminal **b** as the reference terminal, and define a nodal voltage  $V_1$  at the top of the 200- $\Omega$  resistor. Then,

$$0 = \frac{V_1 - 20}{40} + \frac{V_1 - V_{TH}}{100} + \frac{V_1}{200} \quad [1]$$

$$1.5 i_1 = (V_{TH} - V_1)/100 \quad [2]$$

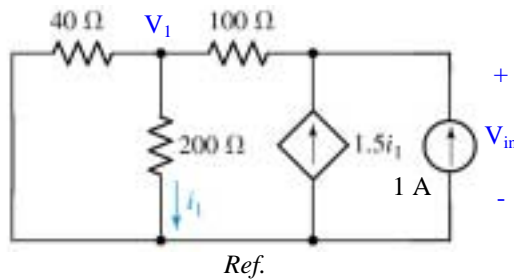
where  $i_1 = V_1/200$ , so Eq. [2] becomes  $150 V_1/200 + V_1 - V_{TH} = 0$  [2]

Simplifying and collecting terms, these equations may be re-written as:

$$(0.25 + 0.1 + 0.005) V_1 - 0.1 V_{TH} = 5 \quad [1]$$

$$(1 + 15/20) V_1 - V_{TH} = 0 \quad [2]$$

Solving, we find that  $V_{TH} = 38.89 \text{ V}$ . To find  $R_{TH}$ , we short the voltage source and inject 1 A into the port:



$$0 = \frac{V_1 - V_{in}}{100} + \frac{V_1}{40} + \frac{V_1}{200} \quad [1]$$

$$1.5 i_1 + 1 = \frac{V_{in} - V_1}{100} \quad [2]$$

$$i_1 = V_1/200 \quad [3]$$

Combining Eqs. [2] and [3] yields  $1.75 V_1 - V_{in} = -100$  [4]

Solving Eqs. [1] & [4] then results in  $V_{in} = 177.8 \text{ V}$ , so that  $R_{TH} = V_{in}/1 \text{ A} = 177.8 \Omega$ .

(b) Adding a 100- $\Omega$  load to the original circuit or our Thévenin equivalent, the voltage across the load is

$$V_{100\Omega} = V_{TH} \left( \frac{100}{100 + 177.8} \right) = 14.00 \text{ V}, \text{ and so } P_{100\Omega} = (V_{100\Omega})^2 / 100 = 1.96 \text{ W}.$$

## CHAPTER FIVE SOLUTIONS

35. We inject a current of 1 A into the port (arrow pointing up), select the bottom terminal as our reference terminal, and define the nodal voltage  $V_x$  across the 200- $\Omega$  resistor.

Then,  $1 = V_1/100 + (V_1 - V_x)/50$  [1]

$$-0.1 V_1 = V_x/200 + (V_x - V_1)/50 \quad [2]$$

which may be simplified to

$$3 V_1 - 2 V_x = 100 \quad [1]$$

$$16 V_1 + 5 V_x = 0 \quad [2]$$

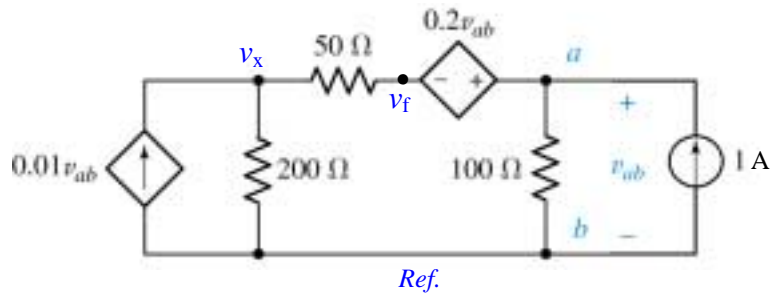
Solving, we find that  $V_1 = 10.64$  V, so  $R_{TH} = V_1/(1 \text{ A}) = 10.64 \Omega$ .

Since there are no independent sources present in the original network,  $I_N = 0$ .

## CHAPTER FIVE SOLUTIONS

36. With no independent sources present,  $V_{TH} = 0$ .

We decide to inject a 1-A current into the port:



Node 'x':  $0.01 v_{ab} = v_x / 200 + (v_x - v_f) / 50$  [1]

Supernode:  $1 = v_{ab} / 100 + (v_f - v_x)$  [2]

and:  $v_{ab} - v_f = 0.2 v_{ab}$  [3]

Rearranging and collecting terms,

$$-2 v_{ab} + 5 v_x - 4 v_f = 0 \quad [1]$$

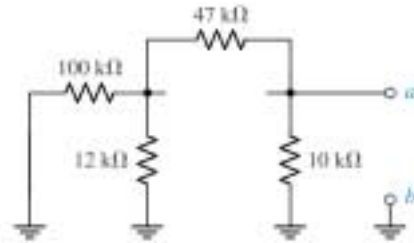
$$v_{ab} - 2 v_x + 2 v_f = 100 \quad [2]$$

$$0.8 v_{ab} - v_f = 0 \quad [3]$$

Solving, we find that  $v_{ab} = 192.3$  V, so  $R_{TH} = v_{ab} / (1 \text{ A}) = 192.3 \Omega$ .

## CHAPTER FIVE SOLUTIONS

37. We first find  $R_{TH}$  by shorting out the voltage source and open-circuiting the current source.



Looking into the terminals **a** & **b**, we see  
 $R_{TH} = 10 \parallel [47 + (100 \parallel 12)]$   
 $= 8.523 \, \Omega.$

Returning to the original circuit, we decide to perform nodal analysis to obtain  $V_{TH}$ :

$$-12 \times 10^3 = (V_1 - 12) / 100 \times 10^3 + V_1 / 12 \times 10^3 + (V_1 - V_{TH}) / 47 \times 10^3 \quad [1]$$

$$12 \times 10^3 = V_{TH} / 10 \times 10^3 + (V_{TH} - V_1) / 47 \times 10^3 \quad [2]$$

Rearranging and collecting terms,

$$0.1146 V_1 - 0.02128 V_{TH} = -11.88 \quad [1]$$

$$-0.02128 V_1 + 0.02128 V_{TH} = 12 \quad [2]$$

Solving, we find that  $V_{TH} = 83.48 \, V.$

## CHAPTER FIVE SOLUTIONS

38. (a)  $R_{TH} = 4 + 2 \parallel 2 + 10 = 15 \Omega.$   
(b) same as above:  $15 \Omega.$



## CHAPTER FIVE SOLUTIONS

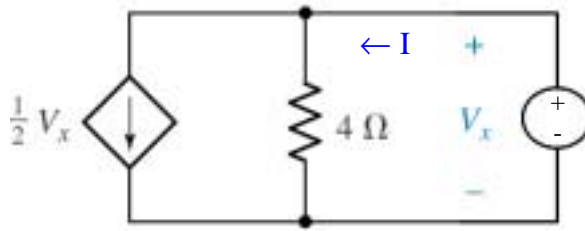
39. For Fig. 5.78a,  $I_N = 12 / \sim 0 \rightarrow \infty \text{ A in parallel with } \sim 0 \, \Omega.$

For Fig. 5.78b,  $V_{TH} = (2)(\sim \infty) \rightarrow \infty \text{ V in series with } \sim \infty \, \Omega.$

## CHAPTER FIVE SOLUTIONS

40. With no independent sources present,  $V_{TH} = 0$ .

Connecting a 1-V source to the port and measuring the current that flows as a result,



$$I = 0.5 V_x + 0.25 V_x = 0.5 + 0.25 = 0.75 \text{ A.}$$

$$R_{TH} = 1/I = 1.333 \Omega.$$

The Norton equivalent is 0 A in parallel with  $1.333 \Omega$ .

## CHAPTER FIVE SOLUTIONS

41. Performing nodal analysis to determine  $V_{TH}$ ,

$$100 \times 10^{-3} = V_x / 250 + V_{oc} / 7.5 \times 10^3 \quad [1]$$

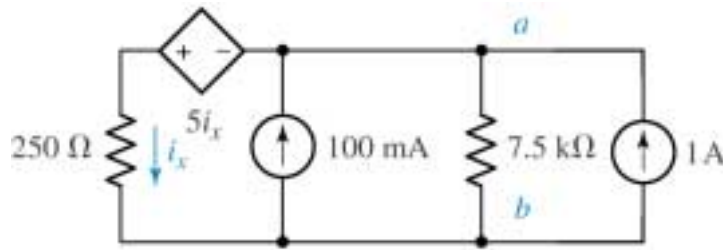
$$\text{and } V_x - V_{oc} = 5 i_x$$

where  $i_x = V_x / 250$ . Thus, we may write the second equation as

$$0.98 V_x - V_{oc} = 0 \quad [2]$$

Solving, we find that  $V_{oc} = V_{TH} = 23.72 \text{ V}$ .

In order to determine  $R_{TH}$ , we inject 1 A into the port:



$$V_{ab} / 7.5 \times 10^3 + V_x / 250 = 1 \quad [1]$$

$$\text{and } V_x - V_{ab} = 5 i_x = 5 V_x / 250 \quad \text{or}$$

$$-V_{ab} + (1 - 5 / 250) V_x = 0 \quad [2]$$

Solving, we find that  $V_{ab} = 237.2 \text{ V}$ . Since  $R_{TH} = V_{ab} / (1 \text{ A})$ ,  $R_{TH} = 237.2 \Omega$ .

Finally,  $I_N = V_{TH} / R_{TH} = 100 \text{ mA}$ .

## CHAPTER FIVE SOLUTIONS

42. We first note that  $V_{TH} = V_x$ , so performing nodal analysis,

$$-5 V_x = V_x / 19 \quad \text{which has the solution } V_x = 0 \text{ V.}$$

Thus,  $V_{TH}$  (and hence  $I_N$ ) = 0. (Assuming  $R_{TH} \neq 0$ )

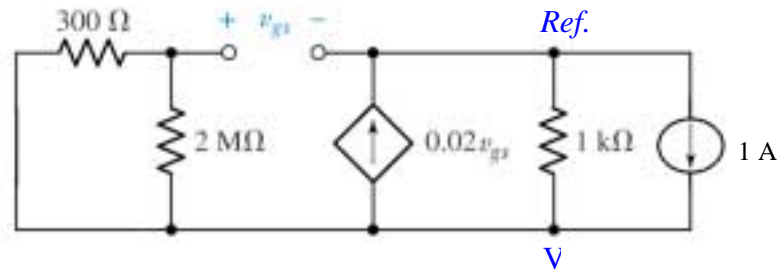
To find  $R_{TH}$ , we inject 1 A into the port, noting that  $R_{TH} = V_x / 1 \text{ A}$ :

$$-5 V_x + 1 = V_x / 19$$

Solving, we find that  $V_x = 197.9 \text{ mV}$ , so that  $R_{TH} = R_N = 197.9 \text{ mV}$ .

## CHAPTER FIVE SOLUTIONS

43. Shorting out the voltage source, we redraw the circuit with a 1-A source in place of the 2-k $\Omega$  resistor:



Noting that  $300\ \Omega \parallel 2\ \text{M}\Omega \approx 300\ \Omega$ ,

$$0 = (v_{gs} - V) / 300 \quad [1]$$

$$1 - 0.02 v_{gs} = V / 1000 + (V - v_{gs}) / 300 \quad [2]$$

Simplifying & collecting terms,

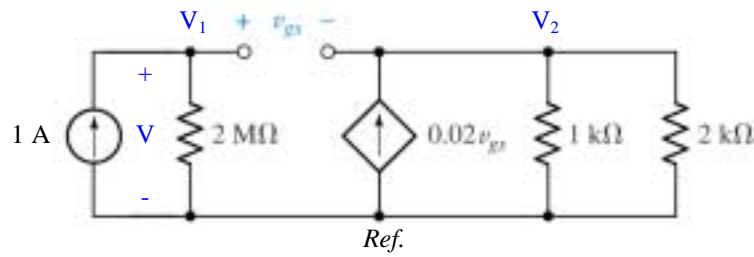
$$v_{gs} - V = 0 \quad [1]$$

$$0.01667 v_{gs} + 0.00433 V = 1 \quad [2]$$

Solving, we find that  $v_{gs} = V = 47.62\ \text{V}$ . Hence,  $R_{TH} = V / 1\ \text{A} = 47.62\ \Omega$ .

## CHAPTER FIVE SOLUTIONS

44. We replace the source  $v_s$  and the  $300\text{-}\Omega$  resistor with a  $1\text{-A}$  source and seek its voltage:

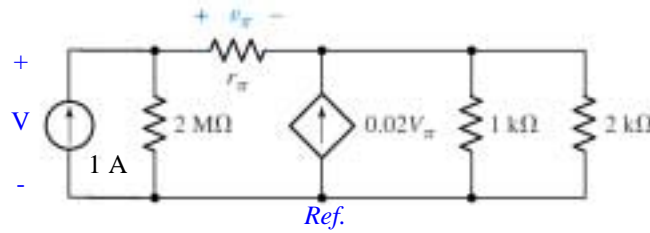


By nodal analysis,  $1 = V_1 / 2 \times 10^6$  so  $V_1 = 2 \times 10^6 \text{ V}$ .

Since  $V = V_1$ , we have  $R_{\text{in}} = V / 1 \text{ A} = 2 \text{ M}\Omega$ .

## CHAPTER FIVE SOLUTIONS

45. Removing the voltage source and the 300- $\Omega$  resistor, we replace them with a 1-A source and seek the voltage that develops across its terminals:



We select the bottom node as our reference terminal, and define nodal voltages  $V_1$  and  $V_2$ . Then,

$$1 = V_1 / 2 \times 10^6 + (V_1 - V_2) / r_\pi \quad [1]$$

$$0.02 v_\pi = (V_2 - V_1) / r_\pi + V_2 / 1000 + V_2 / 2000 \quad [2]$$

where  $v_\pi = V_1 - V_2$

Simplifying & collecting terms,

$$(2 \times 10^6 + r_\pi) V_1 - 2 \times 10^6 V_2 = 2 \times 10^6 r_\pi \quad [1]$$

$$-(2000 + 40 r_\pi) V_1 + (2000 + 43 r_\pi) V_2 = 0 \quad [2]$$

Solving, we find that  $V_1 = V = 2 \times 10^6 \left( \frac{666.7 + 14.33 r_\pi}{2 \times 10^6 + 666.7 + 14.33 r_\pi} \right)$ .

Thus,

$$R_{TH} = 2 \times 10^6 \parallel (666.7 + 14.33 r_\pi) \, \Omega.$$

## CHAPTER FIVE SOLUTIONS

46. Such a scheme probably would lead to maximum or at least near-maximum power transfer to our home. Since we pay the utility company based on the power we use, however, this might not be such a hot idea...

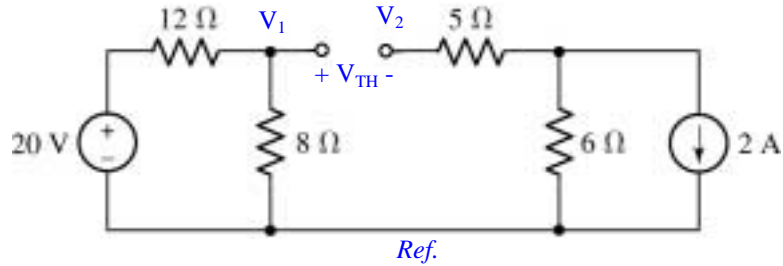


## CHAPTER FIVE SOLUTIONS

47. We need to find the Thévenin equivalent resistance of the circuit connected to  $R_L$ , so we short the 20-V source and open-circuit the 2-A source; by inspection, then

$$R_{TH} = 12 \parallel 8 + 5 + 6 = 15.8 \, \Omega$$

Analyzing the original circuit to obtain  $V_1$  and  $V_2$  with  $R_L$  removed:



$$V_1 = 20 \cdot 8 / 20 = 8 \, \text{V}; \quad V_2 = -2(6) = -12 \, \text{V}.$$

We define  $V_{TH} = V_1 - V_2 = 8 + 12 = 20 \, \text{V}$ . Then,

$$P_{R_L}|_{\max} = \frac{V_{TH}^2}{4 R_L} = \frac{400}{4(15.8)} = 6.329 \, \text{W}$$

## CHAPTER FIVE SOLUTIONS

48. (a)  $R_{TH} = 25 \parallel (10 + 15) = 12.5 \, \Omega$

Using superposition,  $V_{ab} = V_{TH} = 50 \frac{25}{15 + 10 + 25} + 100 \frac{15 + 10}{50} = 75 \, \text{V}.$

(b) Connecting a 50- $\Omega$  resistor,

$$P_{\text{load}} = \frac{V_{TH}^2}{R_{TH} + R_{\text{load}}} = \frac{75^2}{12.5 + 50} = 90 \, \text{W}$$

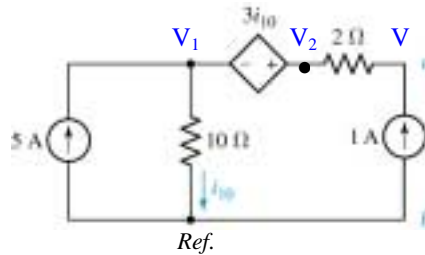
(c) Connecting a 12.5- $\Omega$  resistor,

$$P_{\text{load}} = \frac{V_{TH}^2}{4 R_{TH}} = \frac{75^2}{4(12.5)} = 112.5 \, \text{W}$$

## CHAPTER FIVE SOLUTIONS

49. (a) By inspection, we see that  $i_{10} = 5$  A, so  
 $V_{TH} = V_{ab} = 2(0) + 3 i_{10} + 10 i_{10} = 13 i_{10} = 13(5) = 65$  V.

To find  $R_{TH}$ , we connect a 1-A source between terminals **a** & **b**:



$$5 = V_1 / 10 + (V_2 - V) / 2 \quad [1] \quad \rightarrow \quad V_1 + 5 V_2 - 5 V = 50 \quad [1]$$

$$1 = (V - V_2) / 2 \quad [2] \quad \rightarrow \quad -V_2 + V = 2 \quad [2]$$

$$\text{and } V_2 - V_1 = 3 i_{10} \quad [3]$$

where  $i_{10} = V_1 / 10 \quad \rightarrow \quad -13 V_1 + 10 V_2 = 0 \quad [3]$

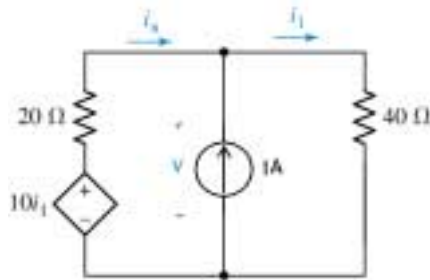
Solving, we find that  $V = 80$  V, so that  $R_{TH} = V / 1 \text{ A} = 80 \Omega$ .

$$(b) P_{\max} = \frac{V_{TH}^2}{4 R_{TH}} = \frac{65^2}{4(80)} = 13.20 \text{ W}$$

## CHAPTER FIVE SOLUTIONS

50.

- (a) Replacing the resistor  $R_L$  with a 1-A source, we seek the voltage that develops across its terminals with the independent voltage source shorted:



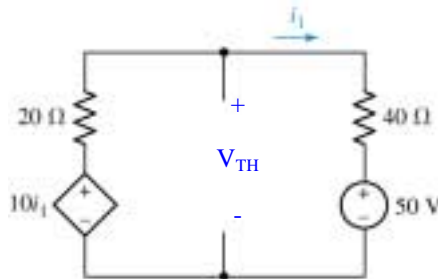
$$-10i_1 + 20i_x + 40i_1 = 0 \quad [1] \Rightarrow 30i_1 + 20i_x = 0 \quad [1]$$

$$\text{and } i_1 - i_x = 1 \quad [2] \Rightarrow i_1 - i_x = 1 \quad [2]$$

Solving,  $i_1 = 400 \text{ mA}$

$$\text{So } V = 40i_1 = 16 \text{ V and } R_{TH} = \frac{V}{1 \text{ A}} = 16 \Omega$$

- (b) Removing the resistor  $R_L$  from the original circuit, we seek the resulting open-circuit voltage:



$$0 = \frac{V_{TH} - 10i_1}{20} + \frac{V_{TH} - 50}{40} \quad [1]$$

$$\text{where } i_1 = \frac{V_{TH} - 50}{40}$$

$$\text{so [1] becomes } 0 = \frac{V_{TH}}{20} - \frac{1}{2} \left( \frac{V_{TH} - 50}{40} \right) + \left( \frac{V_{TH} - 50}{40} \right)$$

$$0 = \frac{V_{TH}}{20} + \frac{V_{TH} - 50}{80}$$

$$0 = 4V_{TH} + V_{TH} - 50$$

$$5V_{TH} = 50$$

$$\text{or } V_{TH} = 10 \text{ V}$$

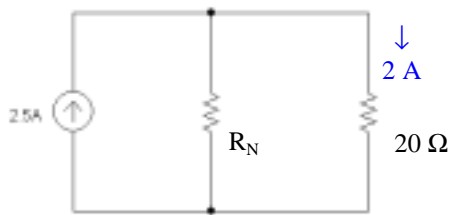
Thus, if  $R_L = R_{TH} = 16 \Omega$ ,

$$V_{R_L} = V_{TH} \frac{R_L}{R_L + R_{TH}} = \frac{V_{TH}}{2} = 5 \text{ V}$$

## CHAPTER FIVE SOLUTIONS

51.

(a)  $I_N = 2.5 \text{ A}$



$$20i^2 = 80$$

$$i = 2 \text{ A}$$

By current division,

$$2 = 2.5 \frac{R_N}{R_N + 20}$$

Solving,  $R_N = R_{TH} = 80 \Omega$

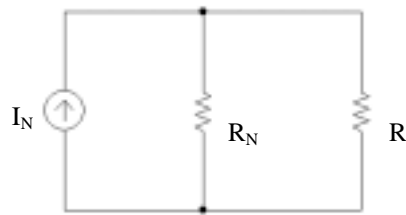
Thus,  $V_{TH} = V_{OC} = 2.5 \times 80 = 200 \text{ V}$

(b)  $P_{\max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{200^2}{4 \times 80} = 125 \text{ W}$

(c)  $R_L = R_{TH} = 80 \Omega$

## CHAPTER FIVE SOLUTIONS

52.



10 W to  $250\Omega$  corresp to 200 mA.

20 W to  $80\Omega$  corresp to 500 mA.

By Voltage  $\div$ ,  $I_R = I_N \frac{R_N}{R + R_N}$

So  $0.2 = I_N \frac{R_N}{250 + R_N}$  [1]

$0.5 = I_N \frac{R_N}{80 + R_N}$  [2]

Solving,  $I_N = 1.7 \text{ A}$  and  $R_N = 33.33\Omega$

(a) If  $v_L i_L$  is a maximum,

$R_L = R_N = 33.33\Omega$

$i_L = 1.7 \times \frac{33.33}{33.33 + 33.33} = 850 \text{ mA}$

$v_L = 33.33 i_L = 28.33 \text{ V}$

(b) If  $v_L$  is a maximum

$V_L = I_N (R_N \parallel R_L)$

So  $v_L$  is a maximum when  $R_N \parallel R_L$  is a maximum, which occurs at  $R_L = \infty$ .

Then  $i_L = 0$  and  $v_L = 1.7 \times R_N = 56.66 \text{ V}$

(c) If  $i_L$  is a maximum

$i_L = I_N \frac{R_N}{R_N + R_L}$ ; max when  $R_L = 0\Omega$

So  $i_L = 1.7 \text{ A}$

$v_L = 0 \text{ V}$

## CHAPTER FIVE SOLUTIONS

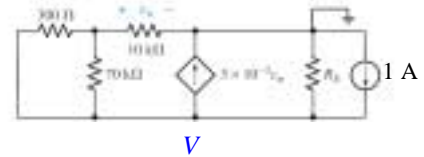
53. There is no conflict with our derivation concerning maximum power. While a dead short across the battery terminals will indeed result in maximum current draw from the battery, and power is indeed proportional to  $i^2$ , the power delivered to the load is  $i^2 R_{LOAD} = i^2(0) = 0$  watts. This is the *minimum*, not the maximum, power that the battery can deliver to a load.

## CHAPTER FIVE SOLUTIONS

54. Remove  $R_E$ :  $R_{TH} = R_E \parallel R_{in}$

bottom node:  $1 - 3 \times 10^{-3} v_\pi = \frac{V - v_\pi}{300} + \frac{V - v_\pi}{70 \times 10^3}$  [1]

at other node:  $0 = \frac{v_\pi}{10 \times 10^3} + \frac{v_\pi - V}{300} + \frac{v_\pi - V}{70 \times 10^3}$  [2]



Simplifying and collecting terms,

$$210 \times 10^5 = 70 \times 10^3 V + 300V + 63000 v_\pi - 70 \times 10^3 v_\pi - 300 v_\pi$$

$$\text{or } 70.3 \times 10^3 V - 7300 v_\pi = 210 \times 10^5 \quad [1]$$

$$0 = 2100 v_\pi + 70 \times 10^3 v_\pi - 70 \times 10^3 V + 300 v_\pi - 300V$$

$$\text{or } -69.7 \times 10^3 V + 72.4 \times 10^3 v_\pi = 0 \quad [2]$$

$$\text{solving, } V = 331.9V \quad \text{So } R_{TH} = R_E \parallel 331.9\Omega$$

Next, we determine  $v_{TH}$  using mesh analysis:

$$-v_s + 70.3 \times 10^3 i_1 - 70 \times 10^3 i_2 = 0 \quad [1]$$

$$80 \times 10^3 i_2 - 70 \times 10^3 i_1 + R_E i_3 = 0 \quad [2]$$

$$\text{and: } i_3 - i_2 = 3 \times 10^{-3} v_\pi \quad [3]$$

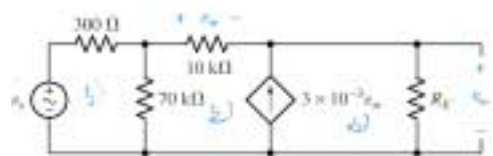
$$\text{or } i_3 - i_2 = 3 \times 10^{-3} (10 \times 10^3) i_2$$

$$\text{or } i_3 - i_2 = 30 i_2$$

or

$$-31 i_2 + i_3 = 0 \quad [3]$$

$$\text{Solving: } \begin{bmatrix} 70.3 \times 10^3 & -70 \times 10^3 & 0 \\ -70 \times 10^3 & 80 \times 10^3 & R_E \\ 0 & -31 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \\ 0 \end{bmatrix}$$



We seek  $i_3$ :

$$i_3 = \frac{-21.7 \times 10^3 v_s}{7.24 \times 10^6 + 21.79 \times 10^3 R_E}$$

$$\text{So } V_{OC} = V_{TH} = R_E i_3 = \frac{-21.7 \times 10^3 R_E}{7.24 \times 10^6 + 21.79 \times 10^3 R_E} v_s$$

$$P_{8\Omega} = 8 \left[ \frac{V_{TH}}{R_{TH} + 8} \right]^2 = \left[ \frac{-21.7 \times 10^3 R_E}{7.24 \times 10^6 + 21.79 \times 10^3 R_E} \right]^2 \frac{8 v_s^2}{\left[ \frac{331.9 R_E}{331.9 + R_E} \right]^2}$$

$$= \frac{11.35 \times 10^6 (331.9 + R_E)^2}{(7.24 \times 10^6 + 21.79 \times 10^3 R_E)^2} v_s^2$$

This is maximized by setting  $R_E = \infty$ .



## CHAPTER FIVE SOLUTIONS

55. Thévenize the left-hand network, assigning the nodal voltage  $V_x$  at the free end of right-most 1-k $\Omega$  resistor.

A single nodal equation:  $40 \times 10^{-3} = \frac{V_x|_{oc}}{7 \times 10^3}$

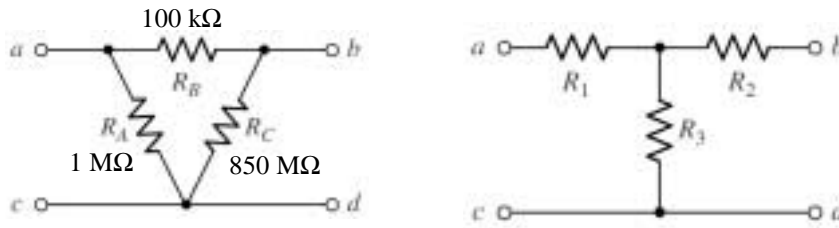
So  $V_{TH} = V_x|_{oc} = 280 \text{ V}$

$R_{TH} = 1 \text{ k} + 7 \text{ k} = 8 \text{ k}\Omega$

Select  $R_1 = R_{TH} = 8 \text{ k}\Omega$ .

## CHAPTER FIVE SOLUTIONS

56.



$$D = R_A + R_B + R_C = 1 + 850 + 0.1 = 851.1 \times 10^6$$

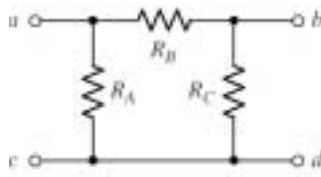
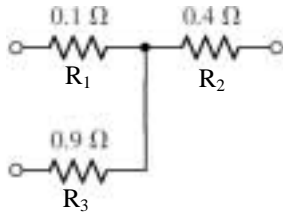
$$R_1 = \frac{R_A R_B}{D} = \frac{10^6 \times 10^5}{851.1 \times 10^6} = 117.5 \Omega$$

$$R_2 = \frac{R_B R_C}{D} = \frac{10^5 \times 850 \times 10^6}{851.1 \times 10^6} = 99.87 k\Omega$$

$$R_3 = \frac{R_C R_A}{D} = \frac{850 \times 10^6 \times 10^6}{851.1 \times 10^6} = 998.7 k\Omega$$

## CHAPTER FIVE SOLUTIONS

57.



$$\begin{aligned} N &= R_1 R_2 + R_2 R_3 + R_3 R_1 \\ &= 0.1 \times 0.4 + 0.4 \times 0.9 + 0.9 \times 0.1 \\ &= 0.49\ \Omega \end{aligned}$$

$$R_A = \frac{N}{R_2} = 1.225\ \Omega$$

$$R_B = \frac{N}{R_3} = 544.4\ \text{m}\Omega$$

$$R_C = \frac{N}{R_1} = 4.9\ \Omega$$

## CHAPTER FIVE SOLUTIONS

58.

$$\Delta_1 : 1 + 6 + 3 = 10 \Omega$$

$$\frac{6 \times 1}{10} = 0.6, \frac{6 \times 3}{10} = 1.8, \frac{3 \times 1}{10} = 0.3$$

$$\Delta_2 : 5 + 1 + 4 = 10 \Omega$$

$$\frac{5 \times 1}{10} = 0.5, \frac{1 \times 4}{10} = 0.4, \frac{5 \times 4}{10} = 2$$

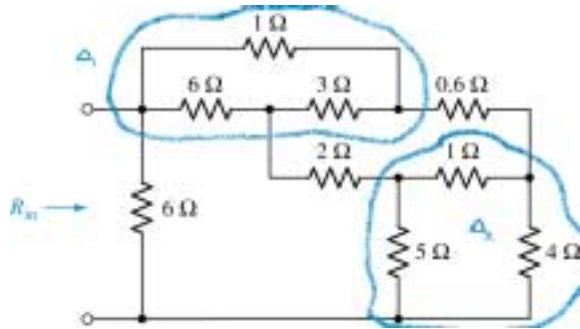
$$1.8 + 2 + 0.5 = 4.3 \Omega$$

$$0.3 + 0.6 + 0.4 = 1.3 \Omega$$

$$1.3 \parallel 4.3 = 0.9982 \Omega$$

$$0.9982 + 0.6 + 2 = 3.598 \Omega$$

$$3.598 \parallel 6 = \boxed{2.249 \Omega}$$



## CHAPTER FIVE SOLUTIONS

59.

$$6 \times 2 + 2 \times 3 + 3 \times 6 = 36 \Omega^2$$

$$\frac{36}{6} = 6 \Omega, \quad \frac{36}{2} = 18 \Omega, \quad \frac{36}{3} = 12 \Omega$$

$$12 \parallel 4 = 3 \Omega, \quad 6 \parallel 12 \Omega = 4 \Omega$$

$$4 + 3 + 18 = 25 \Omega$$

$$3 \times \frac{18}{25} = 2.16 \Omega$$

$$4 \times \frac{18}{25} = 2.88 \Omega$$

$$4 \times \frac{3}{25} = 0.48 \Omega$$

$$9.48 \times 2.16 + 9.48 \times 2.88 + 2.88 \times 2.16 = 54 \Omega^2$$

$$\frac{54}{2.88} = 18.75 \Omega, \quad \frac{54}{9.48} = 5.696 \Omega$$

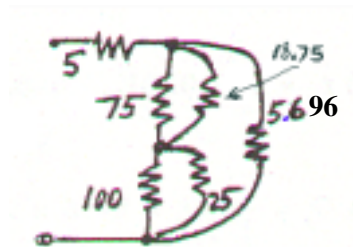
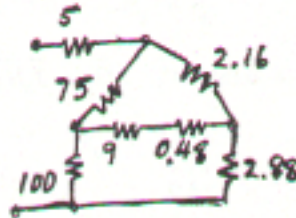
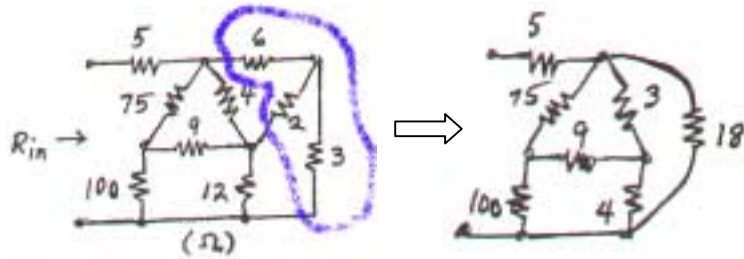
$$\frac{54}{2.16} = 25 \Omega$$

$$75 \parallel 18.75 = 15 \Omega$$

$$100 \parallel 25 = 20 \Omega$$

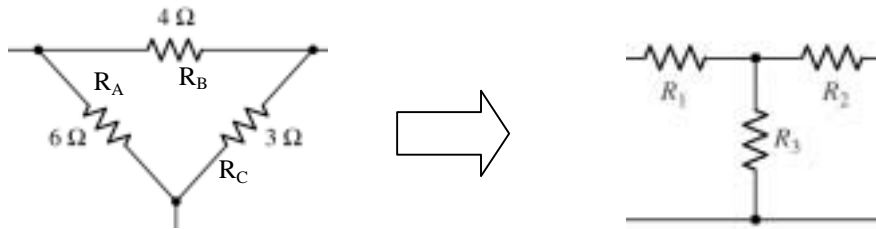
$$(15 + 20) \parallel 5.696 = 4.899 \Omega$$

$$\therefore R_{in} = 5 + 4.899 = \boxed{9.899 \Omega}$$



## CHAPTER FIVE SOLUTIONS

60. We begin by converting the  $\Delta$ -connected network consisting of the 4-, 6-, and 3- $\Omega$  resistors to an equivalent Y-connected network:



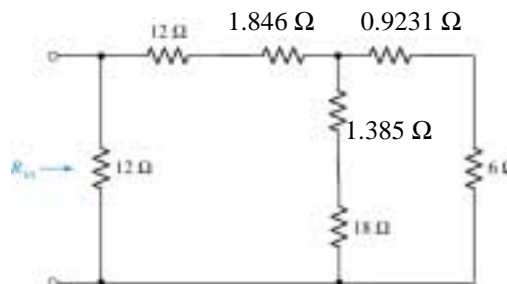
$$D = 6 + 4 + 3 = 13 \Omega$$

$$R_1 = \frac{R_A R_B}{D} = \frac{6 \times 4}{13} = 1.846 \Omega$$

$$R_2 = \frac{R_B R_C}{D} = \frac{4 \times 3}{13} = 923.1 \text{ m}\Omega$$

$$R_3 = \frac{R_C R_A}{D} = \frac{3 \times 6}{13} = 1.385 \Omega$$

Then network becomes:



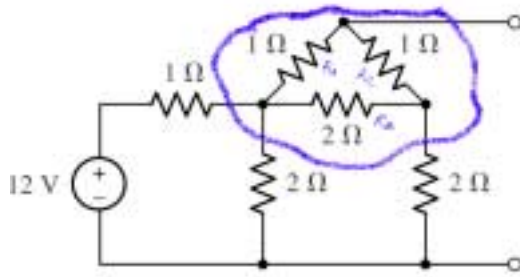
Then we may write

$$R_{in} = 12 \parallel [1.846 + (0.9231 \parallel 6)]$$

$$= 7.347 \Omega$$

## CHAPTER FIVE SOLUTIONS

61.

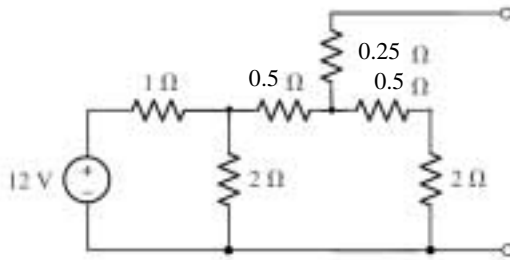


$$1 + 1 + 2 = 4\Omega$$

$$R_1 = \frac{1 \times 2}{4} = \frac{1}{2}\Omega$$

$$R_2 = \frac{2 \times 1}{4} = \frac{1}{2}\Omega$$

$$R_3 = \frac{1 \times 1}{4} = 0.25\Omega$$



Next, we convert the Y-connected network on the left to a  $\Delta$ -connected network:

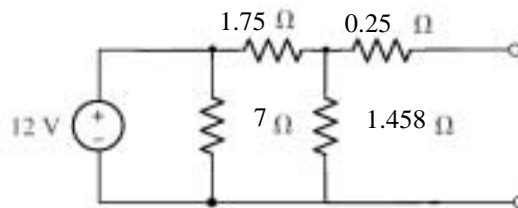
$$1 \times 0.5 + 0.5 \times 2 + 2 \times 1 = 3.5\Omega^2$$

$$R_A = \frac{3.5}{0.5} = 7\Omega$$

$$R_B = \frac{3.5}{2} = 1.75\Omega$$

$$R_C = \frac{3.5}{1} = 3.5\Omega$$

After this procedure, we have a 3.5- $\Omega$  resistor in parallel with the 2.5- $\Omega$  resistor. Replacing them with a 1.458- $\Omega$  resistor, we may redraw the circuit:



This circuit may be easily analysed to find:

$$V_{oc} = \frac{12 \times 1.458}{1.75 + 1.458} = 5.454\text{ V}$$

$$R_{TH} = 0.25 + 1.458 \parallel 1.75$$

$$= 1.045\Omega$$

## CHAPTER FIVE SOLUTIONS

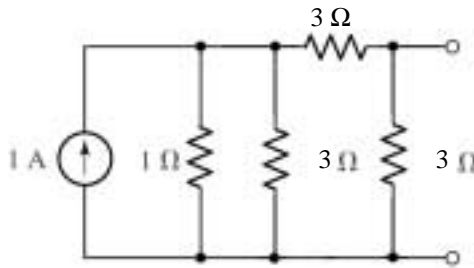
62. We begin by converting the Y-network to a  $\Delta$ -connected network:

$$N = 1.1 + 1.1 + 1.1 = 3\Omega^2$$

$$R_A = \frac{3}{1} = 3\Omega$$

$$R_B = \frac{3}{1} = 3\Omega$$

$$R_C = \frac{3}{1} = 3\Omega$$



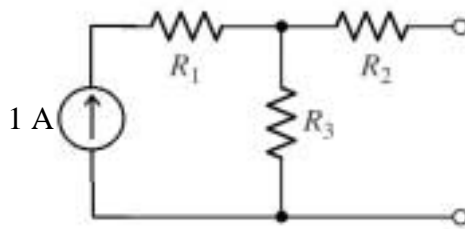
Next, we note that  $1\parallel 3 = 0.75\Omega$ , and hence have a simple  $\Delta$ -network. This is easily converted to a Y-connected network:

$$0.75 + 3 + 3 = 6.75\Omega$$

$$R_1 = \frac{0.75 \times 3}{6.75} = 0.3333\Omega$$

$$R_2 = \frac{3 \times 3}{6.75} = 1.333\Omega$$

$$R_3 = \frac{3 \times 0.75}{6.75} = 0.3333\Omega$$



Analysing this final circuit,

$$R_N = 1.333 + 0.3333$$

$$= 1.667\Omega$$

$$I_N = I_{SC} = 1 \times \frac{1/3}{1/3 + 1 + 1/3}$$

$$= \frac{1}{1 + 3 + 1} = \frac{1}{5}$$

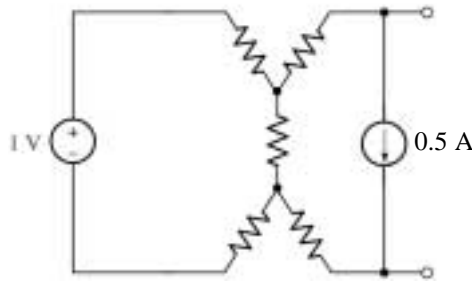
$$= 0.2\text{ A}$$

$$= 200\text{ mA}$$



## CHAPTER FIVE SOLUTIONS

63. Since 1 V appears across the resistor associated with  $I_1$ , we know that  $I_1 = 1 \text{ V} / 10 \Omega = 100 \text{ mA}$ . From the perspective of the open terminals, the  $10\text{-}\Omega$  resistor in parallel with the voltage source has no influence if we replace the “dependent” source with a fixed 0.5-A source:



Then, we may write:

$$-1 + (10 + 10 + 10) i_a - 10 (0.5) = 0$$

so that  $i_a = 200 \text{ mA}$ .

We next find that  $V_{TH} = V_{ab} = 10(-0.5) + 10(i_a - 0.5) + 10(-0.5) = -13 \text{ V}$ .

To determine  $R_{TH}$ , we first recognise that with the 1-V source shorted,  $I_1 = 0$  and hence the dependent current source is dead. Thus, we may write  $R_{TH}$  from inspection:

$$R_{TH} = 10 + 10 + 10 \parallel 20 = 26.67 \Omega.$$

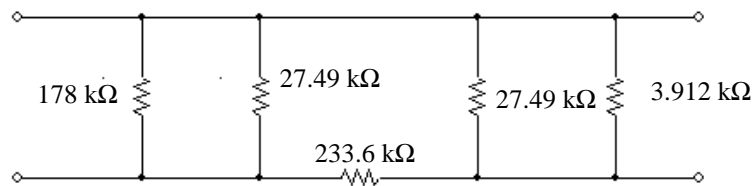
## CHAPTER FIVE SOLUTIONS

64. (a) We begin by splitting the 1-k $\Omega$  resistor into two 500- $\Omega$  resistors in series. We then have two related Y-connected networks, each with a 500- $\Omega$  resistor as a leg. Converting those networks into  $\Delta$ -connected networks,

$$\Sigma = (17)(10) + (1)(4) + (4)(17) = 89 \times 106 \Omega^2$$

$$89/0.5 = 178 \text{ k}\Omega; \quad 89/17 = 5.236 \text{ k}\Omega; \quad 89/4 = 22.25 \text{ k}\Omega$$

Following this conversion, we find that we have two 5.235 k $\Omega$  resistors in parallel, and a 178-k $\Omega$  resistor in parallel with the 4-k $\Omega$  resistor. Noting that  $5.235 \text{ k} \parallel 5.235 \text{ k} = 2.618 \text{ k}\Omega$  and  $178 \text{ k} \parallel 4 \text{ k} = 3.912 \text{ k}\Omega$ , we may draw the circuit as:



We next attack the Y-connected network in the center:

$$\Sigma = (22.25)(22.25) + (22.25)(2.618) + (2.618)(22.25) = 611.6 \times 106 \Omega^2$$

$$611.6/22.25 = 27.49 \text{ k}\Omega; \quad 611.6/2.618 = 233.6 \text{ k}\Omega$$

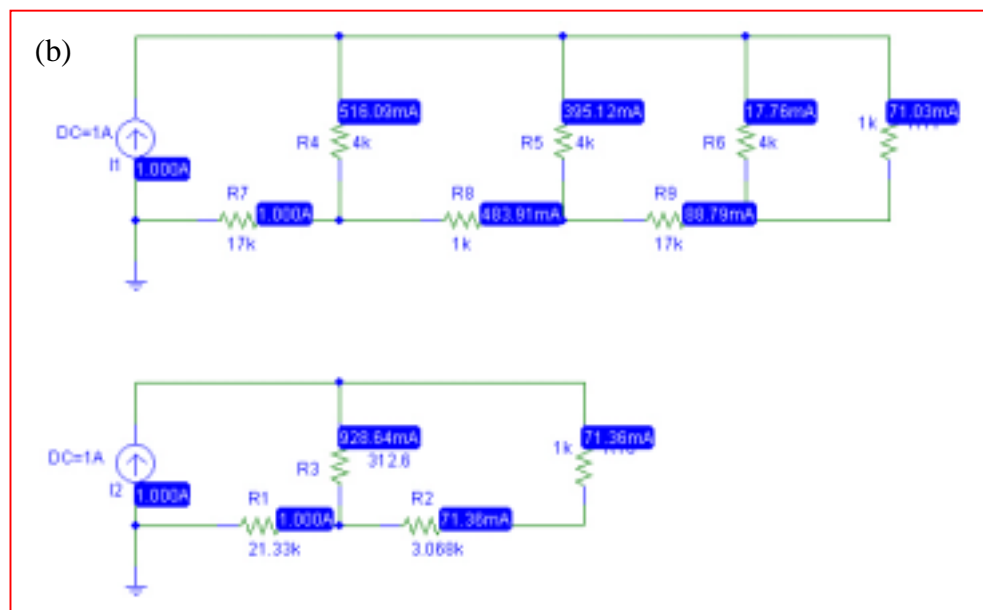
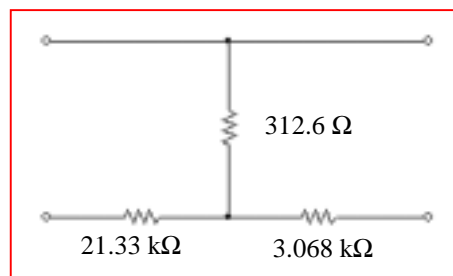
Noting that  $178 \text{ k} \parallel 27.49 \text{ k} = 23.81 \text{ k}\Omega$  and  $27.49 \parallel 3.912 = 3.425 \text{ k}\Omega$ , we are left with a simple  $\Delta$ -connected network. To convert this to the requested Y-network,

$$\Sigma = 23.81 + 233.6 + 3.425 = 260.8 \text{ k}\Omega$$

$$(23.81)(233.6)/260.8 = 21.33 \text{ k}\Omega$$

$$(233.6)(3.425)/260.8 = 3.068 \text{ k}\Omega$$

$$(3.425)(23.81)/260.8 = 312.6 \Omega$$



## CHAPTER FIVE SOLUTIONS

65. (a) Although this network may be simplified, it is not possible to replace it with a three-resistor equivalent.
- (b) See (a).

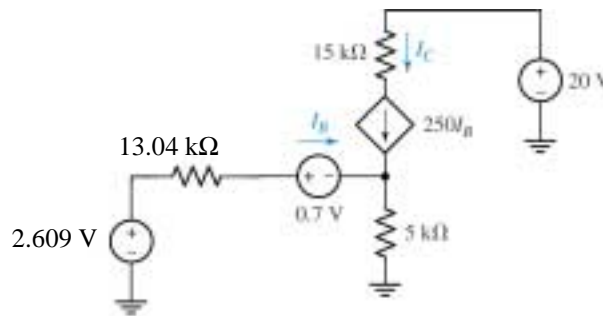
## CHAPTER FIVE SOLUTIONS

66. First, replace network to left of the 0.7-V source with its Thévenin equivalent:

$$V_{TH} = 20 \times \frac{15}{100 + 15} = 2.609 \text{ V}$$

$$R_{TH} = 100k \parallel 15k = 13.04k\Omega$$

Redraw:



Analysing the new circuit to find  $I_B$ , we note that  $I_C = 250 I_B$ :

$$-2.609 + 13.04 \times 10^3 I_B + 0.7 + 5000(I_B + 250I_B) = 0$$

$$I_B = \frac{2.609 - 0.7}{13.04 \times 10^3 + 251 \times 5000} = 1.505 \mu\text{A}$$

$$I_C = 250I_B = 3.764 \times 10^{-4} \text{ A}$$

$$= \boxed{376.4 \mu\text{A}}$$

## CHAPTER FIVE SOLUTIONS

67. (a) Define a nodal voltage  $V_1$  at the top of the current source  $I_S$ , and a nodal voltage  $V_2$  at the top of the load resistor  $R_L$ . Since the load resistor can safely dissipate 1 W, and we know that

$$P_{R_L} = \frac{V_2^2}{1000}$$

then  $V_2|_{\max} = 31.62 \text{ V}$ . This corresponds to a load resistor (and hence lamp) current of 32.62 mA, so we may treat the lamp as a  $10.6\text{-}\Omega$  resistor.

Proceeding with nodal analysis, we may write:

$$I_S = V_1/200 + (V_1 - 5 V_x)/200 \quad [1]$$

$$0 = V_2/1000 + (V_2 - 5 V_x)/10.6 \quad [2]$$

$$V_x = V_1 - 5 V_x \quad \text{or} \quad V_x = V_1/6 \quad [3]$$

Substituting Eq. [3] into Eqs. [1] and [2], we find that

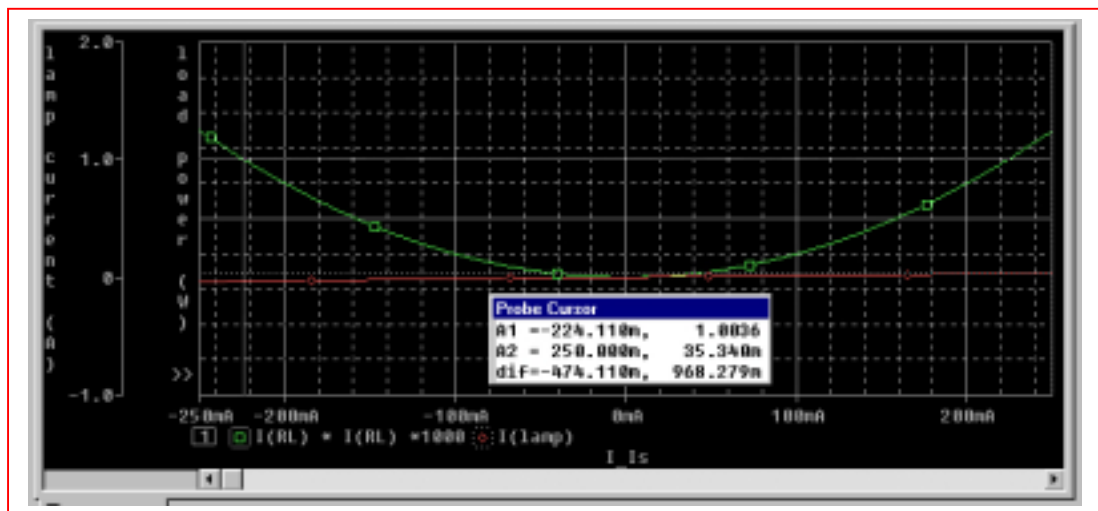
$$7 V_1 = 1200 I_S \quad [1]$$

$$-5000 V_1 + 6063.6 V_2 = 0 \quad [2]$$

Substituting  $V_2|_{\max} = 31.62 \text{ V}$  into Eq. [2] then yields  $V_1 = 38.35 \text{ V}$ , so that

$$I_S|_{\max} = (7)(38.35)/1200 = 223.7 \text{ mA}.$$

- (b) PSpice verification.



The lamp current does not exceed 36 mA in the range of operation allowed (*i.e.* a load power of  $< 1 \text{ W}$ .) The simulation result shows that the load will dissipate slightly more than 1 W for a source current magnitude of 224 mA, as predicted by hand analysis.

## CHAPTER FIVE SOLUTIONS

68. Short out all but the source operating at  $10^4$  rad/s, and define three clockwise mesh currents  $i_1$ ,  $i_2$ , and  $i_3$  starting with the left-most mesh. Then

$$608 i_1 - 300 i_2 = 3.5 \cos 10^4 t \quad [1]$$

$$-300 i_1 + 316 i_2 - 8 i_3 = 0 \quad [2]$$

$$-8 i_2 + 322 i_3 = 0 \quad [3]$$

Solving, we find that  $i_1(t) = 10.84 \cos 10^4 t$  mA

$$i_2(t) = 10.29 \cos 10^4 t \text{ mA}$$

$$i_3(t) = 255.7 \cos 10^4 t \mu\text{A}$$

Next, short out all but the  $7 \sin 200t$  V source, and define three clockwise mesh currents  $i_a$ ,  $i_b$ , and  $i_c$  starting with the left-most mesh. Then

$$608 i_a - 300 i_b = -7 \sin 200t \quad [1]$$

$$-300 i_a + 316 i_b - 8 i_c = 7 \sin 200t \quad [2]$$

$$-8 i_b + 322 i_c = 0 \quad [3]$$

Solving, we find that  $i_a(t) = -1.084 \sin 200t$  mA

$$i_b(t) = 21.14 \sin 200t \text{ mA}$$

$$i_c(t) = 525.1 \sin 200t \mu\text{A}$$

Next, short out all but the source operating at  $10^3$  rad/s, and define three clockwise mesh currents  $i_A$ ,  $i_B$ , and  $i_C$  starting with the left-most mesh. Then

$$608 i_A - 300 i_B = 0 \quad [1]$$

$$-300 i_A + 316 i_B - 8 i_C = 0 \quad [2]$$

$$-8 i_B + 322 i_C = -8 \cos 10^4 t \quad [3]$$

Solving, we find that  $i_A(t) = -584.5 \cos 10^3 t$   $\mu\text{A}$

$$i_B(t) = -1.185 \cos 10^3 t \text{ mA}$$

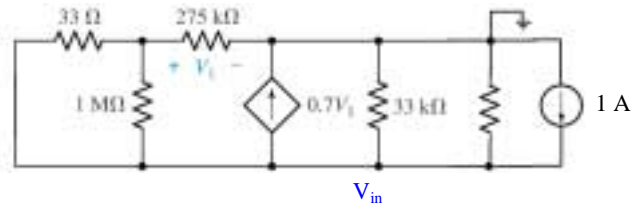
$$i_C(t) = -24.87 \cos 10^3 t \text{ mA}$$

We may now compute the power delivered to each of the three  $8\text{-}\Omega$  speakers:

$$\begin{aligned} p_1 &= 8[i_1 + i_a + i_A]^2 = 8[10.84 \times 10^{-3} \cos 10^4 t - 1.084 \times 10^{-3} \sin 200t - 584.5 \times 10^{-6} \cos 10^3 t]^2 \\ p_2 &= 8[i_2 + i_b + i_B]^2 = 8[10.29 \times 10^{-3} \cos 10^4 t + 21.14 \times 10^{-3} \sin 200t - 1.185 \times 10^{-3} \cos 10^3 t]^2 \\ p_3 &= 8[i_3 + i_c + i_C]^2 = 8[255.7 \times 10^{-6} \cos 10^4 t + 525.1 \times 10^{-6} \sin 200t - 24.87 \times 10^{-3} \cos 10^3 t]^2 \end{aligned}$$

## CHAPTER FIVE SOLUTIONS

69. Replacing the DMM with a possible Norton equivalent (a 1-M $\Omega$  resistor in parallel with a 1-A source):



We begin by noting that  $33\ \Omega \parallel 1\ \text{M}\Omega \approx 33\ \Omega$ . Then,

$$0 = (V_1 - V_{in})/33 + V_1/275 \times 10^3 \quad [1]$$

and

$$1 - 0.7 V_1 = V_{in}/10^6 + V_{in}/33 \times 10^3 + (V_{in} - V_1)/33 \quad [2]$$

Simplifying and collecting terms,

$$(275 \times 10^3 + 33) V_1 - 275 \times 10^3 V_{in} = 0 \quad [1]$$

$$22.1 V_1 + 1.001 V_{in} = 33 \quad [2]$$

Solving, we find that  $V_{in} = 1.429\ \text{V}$ ; in other words, the DMM sees 1.429 V across its terminals in response to the known current of 1 A it's supplying. It therefore thinks that it is connected to a resistance of 1.429  $\Omega$ .

## CHAPTER FIVE SOLUTIONS

70. We know that the resistor  $R$  is absorbing maximum power. We might be tempted to say that the resistance of the cylinder is therefore  $10\ \Omega$ , but this is wrong: The larger we make the cylinder resistance, the smaller the power delivery to  $R$ :

$$P_R = 10 i^2 = 10 \left[ \frac{120}{R_{cylinder} + 10} \right]^2$$

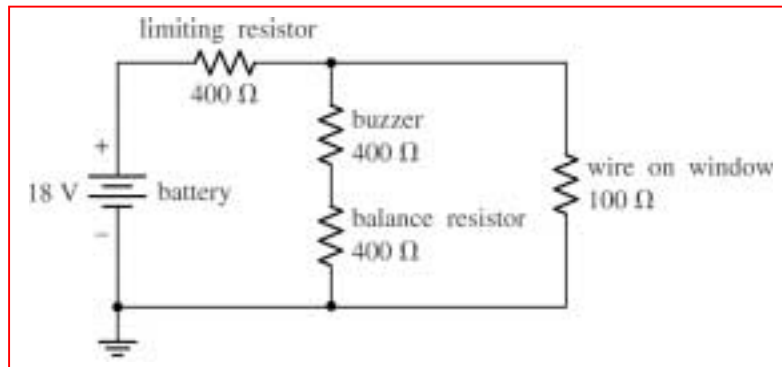
Thus, if we are in fact delivering the maximum possible power to the resistor from the 120-V source, the resistance of the cylinder must be zero.

This corresponds to a temperature of absolute zero using the equation given.



## CHAPTER FIVE SOLUTIONS

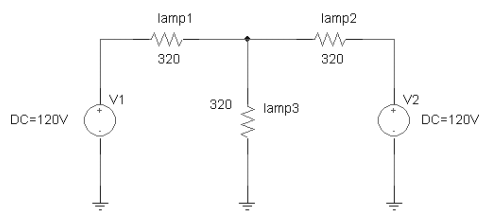
71. We note that the buzzer draws 15 mA at 6 V, so that it may be modeled as a 400- $\Omega$  resistor. One possible solution of many, then, is:



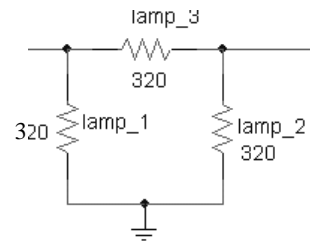
Note: construct the 18-V source from 12 1.5-V batteries in series, and the two 400- $\Omega$  resistors can be fabricated by soldering 400 1- $\Omega$  resistors in series, although there's probably a much better alternative...

## CHAPTER FIVE SOLUTIONS

72. To solve this problem, we need to assume that “45 W” is a designation that applies when 120 Vac is applied directly to a particular lamp. This corresponds to a current draw of 375 mA, or a light bulb resistance of  $120 / 0.375 = 320 \, \Omega$ .



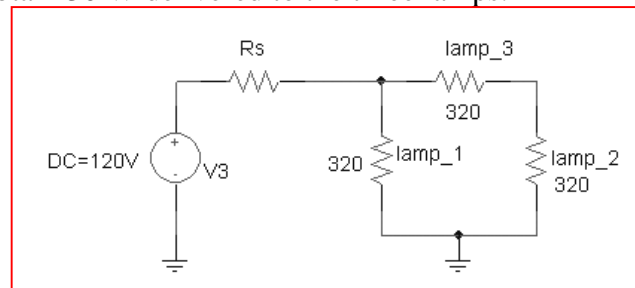
*Original wiring scheme*



*New wiring scheme*

In the original wiring scheme, Lamps 1 & 2 draw  $(40)^2 / 320 = 5 \, \text{W}$  of power each, and Lamp 3 draws  $(80)^2 / 320 = 20 \, \text{W}$  of power. Therefore, none of the lamps is running at its maximum rating of 45 W. We require a circuit which will deliver the same intensity after the lamps are reconnected in a  $\Delta$  configuration. Thus, we need a total of 30 W from the new network of lamps.

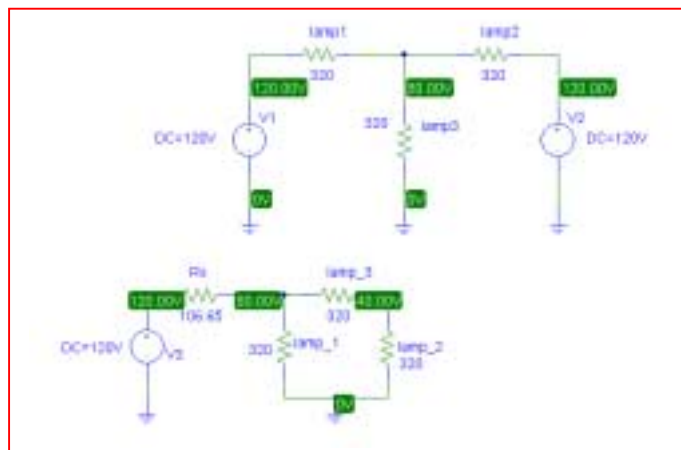
There are several ways to accomplish this, but the simplest may be to just use one 120-Vac source connected to the left port in series with a resistor whose value is chosen to obtain 30 W delivered to the three lamps.



In other words,

$$\frac{\left[120 \frac{213.3}{R_s + 213.3}\right]^2}{320} + 2 \frac{\left[60 \frac{213.3}{R_s + 213.3}\right]^2}{320} = 30$$

Solving, we find that we require  $R_s = 106.65 \, \Omega$  as confirmed by the PSpice simulation below, which shows that both wiring configurations lead to one lamp with 80-V across it, and two lamps with 40 V across each.



## CHAPTER FIVE SOLUTIONS

73.

- Maximum current rating for the LED is 35 mA.
- Its resistance can vary between 47 and 117  $\Omega$ .
- A 9-V battery must be used as a power source.
- Only standard resistance values may be used.

One possible current-limiting scheme is to connect a 9-V battery in series with a resistor  $R_{\text{limiting}}$  and in series with the LED.

From KVL,

$$I_{\text{LED}} = \frac{9}{R_{\text{limiting}} + R_{\text{LED}}}$$

The maximum value of this current will occur at the minimum LED resistance, 47  $\Omega$ .

Thus, we solve

$$35 \times 10^{-3} = \frac{9}{R_{\text{limiting}} + 47}$$

to obtain  $R_{\text{limiting}} \geq 210.1 \Omega$  to ensure an LED current of less than 35 mA. This is not a standard resistor value, however, so we select

$$R_{\text{limiting}} = 220 \Omega.$$

## CHAPTER SIX SOLUTIONS

1. The first step is to perform a simple source transformation, so that a 0.15-V source in series with a 150- $\Omega$  resistor is connected to the inverting pin of the ideal op amp.

$$\text{Then, } v_{\text{out}} = - \frac{2200}{150}(0.15) = \boxed{-2.2 \text{ V}}$$

## CHAPTER SIX SOLUTIONS

2. In order to deliver 150 mW to the 10-k $\Omega$  resistor, we need  $v_{out} = \sqrt{(0.15)(10 \times 10^3)} = 38.73$  V. Writing a nodal equation at the inverting input, we find

$$0 = \frac{5}{R} + \frac{5 - v_{out}}{1000}$$

Using  $v_{out} = 38.73$ , we find that  $R = 148.2 \Omega$ .

## CHAPTER SIX SOLUTIONS

3. Since the  $670\text{-}\Omega$  switch requires  $100\text{ mA}$  to activate, the voltage delivered to it by our op amp circuit must be  $(670)(0.1) = 67\text{ V}$ . The microphone acts as the input to the circuit, and provides  $0.5\text{ V}$ . Thus, an amplifier circuit having a gain  $= 67/0.5 = 134$  is required.

One possible solution of many: a non-inverting op amp circuit with the microphone connected to the non-inverting input terminal, the switch connected between the op amp output pin and ground, a feedback resistor  $R_f = 133\text{ }\Omega$ , and a resistor  $R_1 = 1\text{ }\Omega$ .

## CHAPTER SIX SOLUTIONS

4. We begin by labeling the nodal voltages  $v_-$  and  $v_+$  at the inverting and non-inverting input terminals, respectively. Since no current can flow into the non-inverting input, no current flows through the  $40\text{-k}\Omega$  resistor; hence,  $v_+ = 0$ . Therefore, we know that  $v_- = 0$  as well.

Writing a single nodal equation at the non-inverting input then leads to

$$0 = \frac{(v_- - v_S)}{100} + \frac{(v_- - v_{\text{out}})}{22000}$$

or

$$0 = \frac{-v_S}{100} + \frac{-v_{\text{out}}}{22000}$$

Solving,

$$v_{\text{out}} = -220 v_S$$

## CHAPTER SIX SOLUTIONS

5. We first label the nodal voltage at the output pin  $V_o$ . Then, writing a single nodal equation at the inverting input terminal of the op amp,

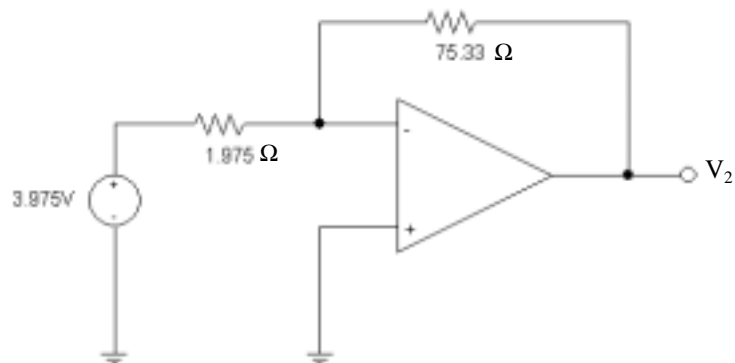
$$0 = \frac{4 - 3}{1000} + \frac{4 - V_o}{17000}$$

Solving, we find that  $V_o = 21$  V. Since no current can flow through the  $300\text{-k}\Omega$  resistor,  $V_1 = 21$  as well.



## CHAPTER SIX SOLUTIONS

6. A source transformation and some series combinations are well worthwhile prior to launching into the analysis. With  $5\text{ k}\Omega \parallel 3\text{ k}\Omega = 1.875\text{ k}\Omega$  and  $(1\text{ mA})(1.875\text{ k}\Omega) = 1.875\text{ V}$ , we may redraw the circuit as



This is now a simple inverting amplifier with gain  $-R_f/R_1 = -75.33/1.875 = -38.14$ .

Thus,  $V_2 = -38.14(3.975) = -151.6\text{ V}$ .

## CHAPTER SIX SOLUTIONS

7. This is a simple inverting amplifier, so we may write

$$v_{\text{out}} = \frac{-2000}{1000}(2 + 2 \sin 3t) = -4(1 + \sin 3t) \text{ V}$$

$$v_{\text{out}}(t = 3 \text{ s}) = -5.648 \text{ V.}$$

## CHAPTER SIX SOLUTIONS

8. We first combine the  $2\text{ M}\Omega$  and  $700\text{ k}\Omega$  resistors into a  $518.5\text{ k}\Omega$  resistor.

We are left with a simple non-inverting amplifier having a gain of  $1 + 518.5/250 = 3.074$ . Thus,

$$v_{\text{out}} = (3.074) v_{\text{in}} = 18 \text{ so } v_{\text{in}} = 5.856 \text{ V.}$$

## CHAPTER SIX SOLUTIONS

9. This is a simple non-inverting amplifier circuit, and so it has a gain of  $1 + R_f/R_1$ . We want  $v_{\text{out}} = 23.7 \cos 500t$  V when the input is  $0.1 \cos 500t$  V, so a gain of  $23.7/0.1 = 237$  is required.
- One possible solution of many:  $R_f = 236 \text{ k}\Omega$  and  $R_1 = 1 \text{ k}\Omega$ .

## CHAPTER SIX SOLUTIONS

10. Define a nodal voltage  $V_-$  at the inverting input, and a nodal voltage  $V_+$  at the non-inverting input. Then,

At the non-inverting input:  $-3 \times 10^{-6} = \frac{V_+}{1.5 \times 10^6}$  [1]

Thus,  $V_+ = -4.5$  V, and we therefore also know that  $V_- = -4.5$  V.

At the inverting input:  $0 = \frac{V_-}{R_6} + \frac{V_- - V_{\text{out}}}{R_7}$  [2]

Solving and making use of the fact that  $V_- = -4.5$  V,

$$v_{\text{out}} = -\frac{R_7}{R_6}(4.5) - 4.5 = -4.5 \left( \frac{R_7}{R_6} + 1 \right) \text{ V}$$

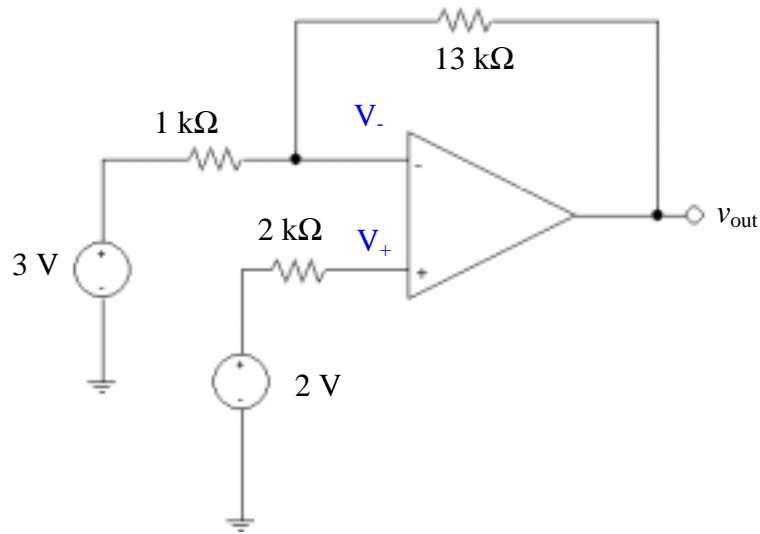
## CHAPTER SIX SOLUTIONS

11. (a) **B** must be the non-inverting input: that yields a gain of  $1 + 70/10 = 8$  and an output of 8 V for a 1-V input.
- (b)  $R_1 = \infty$ ,  $R_A = 0$ . We need a gain of  $20/10 = 2$ , so choose  $R_2 = R_B = 1\ \Omega$ .
- (c) **A** is the inverting input since it has the feedback connection to the output pin.

## CHAPTER SIX SOLUTIONS

12. It is probably best to first perform a simple source transformation:

$$(1 \text{ mA})(2 \text{ k}\Omega) = 2 \text{ V}.$$



Since no current can flow into the non-inverting input pin, we know that  $V_+ = 2 \text{ V}$ , and therefore also that  $V_- = 2 \text{ V}$ . A single nodal equation at the inverting input yields:

$$0 = \frac{2 - 3}{1000} + \frac{2 - v_{\text{out}}}{13000}$$

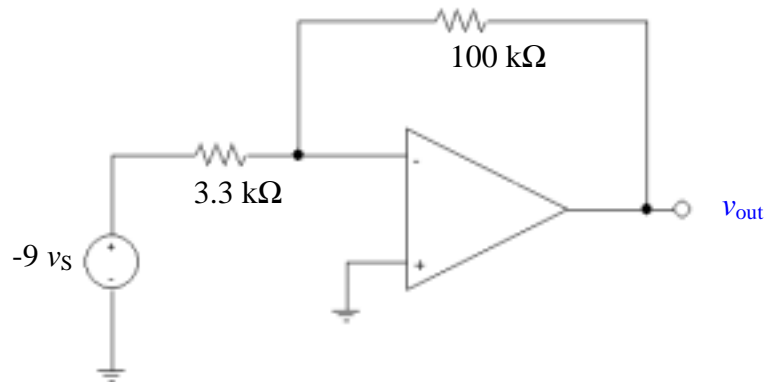
which yields  $v_{\text{out}} = -11 \text{ V}$ .

## CHAPTER SIX SOLUTIONS

13. We begin by find the Thévenin equivalent to the left of the op amp:

$$V_{th} = -3.3(3) v_{\pi} = -9.9 v_{\pi} = -9.9 \frac{1000 v_s}{1100} = -9 v_s$$

$R_{th} = 3.3 \text{ k}\Omega$ , so we can redraw the circuit as:



which is simply a classic inverting op amp circuit with gain of  $-100/3.3 = -30.3$ .

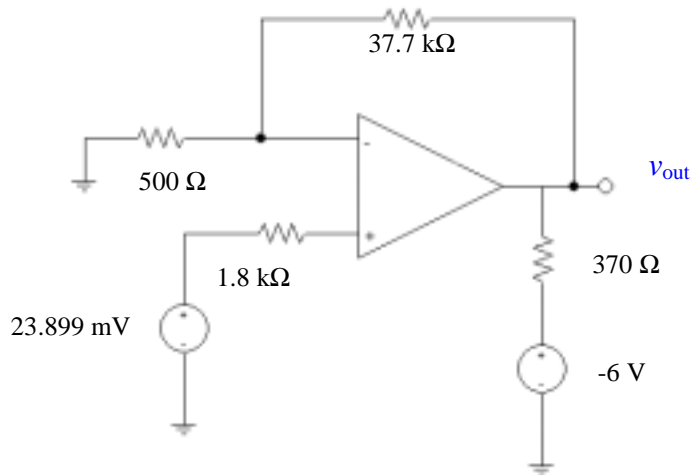
Thus,  $v_{out} = (-30.3)(-9 v_s) = 272.7 v_s$

For  $v_s = 5 \sin 3t \text{ mV}$ ,  $v_{out} = 1.364 \sin 3t \text{ V}$ , and  $v_{out}(0.25 \text{ s}) = 0.9298 \text{ V}$ .



## CHAPTER SIX SOLUTIONS

14. We first combine the  $4.7\text{ M}\Omega$  and  $1.3\text{ k}\Omega$  resistors:  $4.7\text{ M}\Omega \parallel 1.3\text{ k}\Omega = 1.30\text{ k}\Omega$ . Next, a source transformation yields  $(3 \times 10^{-6})(1300) = 3.899\text{ mV}$  which appears in series with the  $20\text{ mV}$  source and the  $500\text{-}\Omega$  resistor. Thus, we may redraw the circuit as



Since no current flows through the  $1.8\text{ k}\Omega$  resistor,  $V_+ = 23.899\text{ mV}$  and hence  $V_- = 23.899\text{ mV}$  as well. A single nodal equation at the inverting input terminal yields

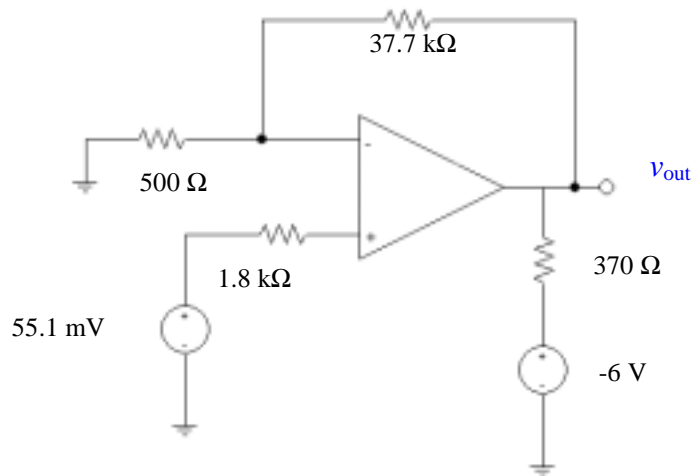
$$0 = \frac{23.899 \times 10^{-3}}{500} + \frac{23.899 \times 10^{-3} - v_{\text{out}}}{37.7 \times 10^3}$$

Solving,

$$v_{\text{out}} = 1.826\text{ V}$$

## CHAPTER SIX SOLUTIONS

15. We first combine the  $4.7\text{ M}\Omega$  and  $1.3\text{ k}\Omega$  resistors:  $4.7\text{ M}\Omega \parallel 1.3\text{ k}\Omega = 1.30\text{ k}\Omega$ . Next, a source transformation yields  $(27 \times 10^{-6})(1300) = 35.1\text{ mV}$  which appears in series with the  $20\text{ mV}$  source and the  $500\text{-}\Omega$  resistor. Thus, we may redraw the circuit as



Since no current flows through the  $1.8\text{ k}\Omega$  resistor,  $V_+ = 55.1\text{ mV}$  and hence  $V_- = 55.1\text{ mV}$  as well. A single nodal equation at the inverting input terminal yields

$$0 = \frac{55.1 \times 10^{-3}}{500} + \frac{55.1 \times 10^{-3} - v_{\text{out}}}{37.7 \times 10^3}$$

Solving,

$v_{\text{out}} = 4.21\text{ V}$

## CHAPTER SIX SOLUTIONS

16. The 3 mA source, 1 k $\Omega$  resistor and 20 k $\Omega$  resistor may be replaced with a -3 V source (“+” reference up) in series with a 21 k $\Omega$  resistor. No current flows through either 1 M $\Omega$  resistor, so that the voltage at each of the four input terminals is identically zero. Considering each op amp circuit separately,

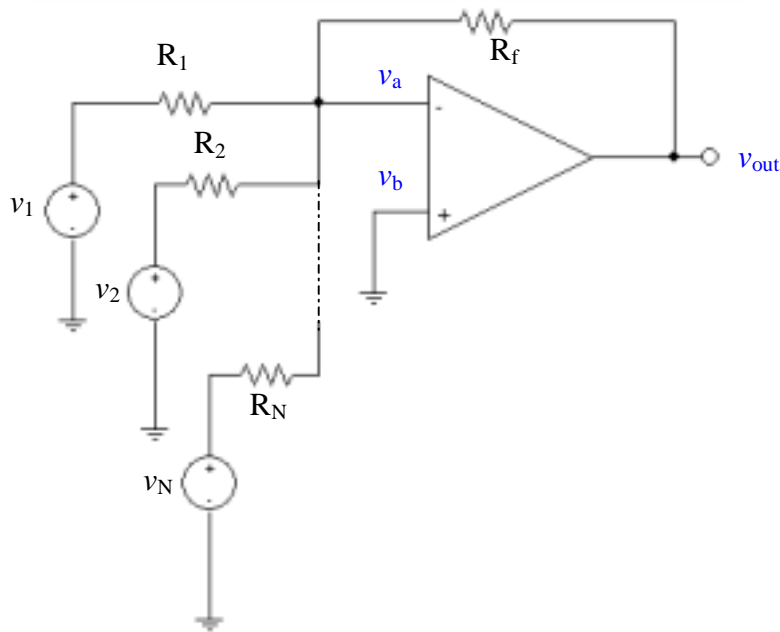
$$v_{\text{out}}|_{\text{LEFTOPAMP}} = -(-3) \frac{100}{21} = 14.29 \text{ V}$$

$$v_{\text{out}}|_{\text{RIGHTOPAMP}} = -(5) \frac{100}{10} = -50 \text{ V}$$

$$v_x = v_{\text{out}}|_{\text{LEFTOPAMP}} - v_{\text{out}}|_{\text{RIGHTOPAMP}} = 14.29 + 50 = \boxed{64.29 \text{ V.}}$$

## CHAPTER SIX SOLUTIONS

17. A general summing amplifier with N input sources:



1.  $v_a = v_b = 0$
2. A single nodal equation at the inverting input leads to:

$$0 = \frac{v_a - v_{out}}{R_f} + \frac{v_a - v_1}{R_1} + \frac{v_a - v_2}{R_2} + \dots + \frac{v_a - v_N}{R_N}$$

Simplifying and making use of the fact that  $v_a = 0$ , we may write this as

$$\left[ -\frac{1}{R_f} \prod_{i=1}^N R_i \right] v_{out} = \frac{v_1}{R_1} \prod_{i=1}^N R_i + \frac{v_2}{R_2} \prod_{i=1}^N R_i + \dots + \frac{v_N}{R_N} \prod_{i=1}^N R_i$$

or simply

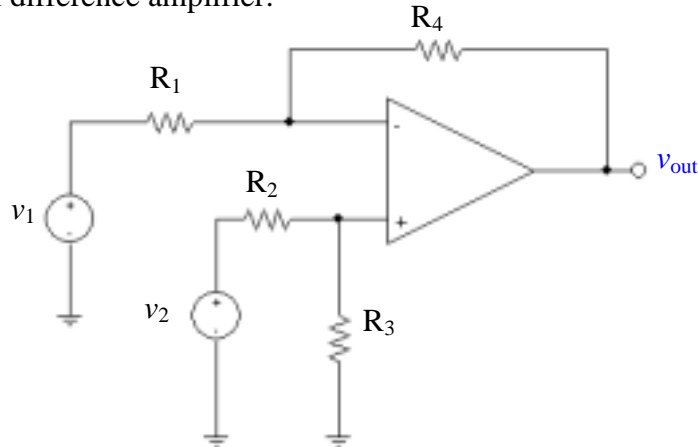
$$-\frac{v_{out}}{R_f} = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \dots + \frac{v_N}{R_N}$$

Thus,

$$v_{out} = -R_f \sum_{i=1}^N \frac{v_i}{R_i}$$

## CHAPTER SIX SOLUTIONS

18. A general difference amplifier:



Writing a nodal equation at the inverting input,

$$0 = \frac{v_a - v_1}{R_1} + \frac{v_a - v_{out}}{R_f}$$

Writing a nodal equation at the non-inverting input,

$$0 = \frac{v_b}{R_3} + \frac{v_b - v_2}{R_2}$$

Simplifying and collecting terms, we may write

$$(R_f + R_1) v_a - R_1 v_{out} = R_f v_1 \quad [1]$$

$$(R_2 + R_3) v_b = R_3 v_2 \quad [2]$$

From Eqn. [2], we have  $v_b = \frac{R_3}{R_2 + R_3} v_2$

Since  $v_a = v_b$ , we can now rewrite Eqn. [1] as

$$-R_1 v_{out} = R_f v_1 - \frac{(R_f + R_1) R_3}{R_2 + R_3} v_2$$

and hence

$$v_{out} = -\frac{R_f}{R_1} v_1 + \frac{R_3}{R_1} \left( \frac{R_f + R_1}{R_2 + R_3} \right) v_2$$

## CHAPTER SIX SOLUTIONS

19. In total darkness, the CdS cell has a resistance of  $100\text{ k}\Omega$ , and at a light intensity  $L$  of 6 candela it has a resistance of  $6\text{ k}\Omega$ . Thus, we may compute the light-dependent resistance (assuming a linear response in the range between 0 and 6 candela) as  $R_{\text{CdS}} = -15L + 100\text{ }\Omega$ .

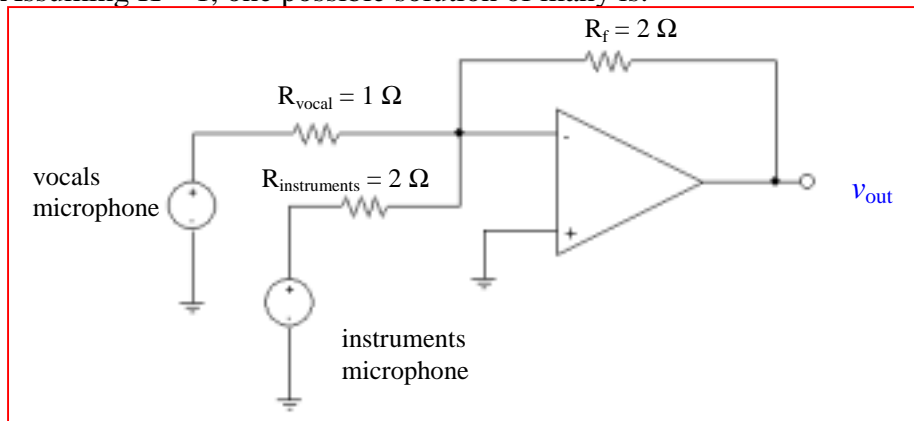
Our design requirement (using the standard inverting op amp circuit shown) is that the voltage across the load is  $1.5\text{ V}$  at 2 candela, and less than  $1.5\text{ V}$  for intensities greater than 2 candela.

$$\text{Thus, } v_{\text{out}}(2\text{ candela}) = -R_{\text{CdS}} v_s / R_1 = -70\text{ V}_s / R_1 = 1.5 \quad (R_1 \text{ in k}\Omega).$$

Pick  $R_1 = 10\text{ k}\Omega$ . Then  $v_s = -0.2143\text{ V}$ .

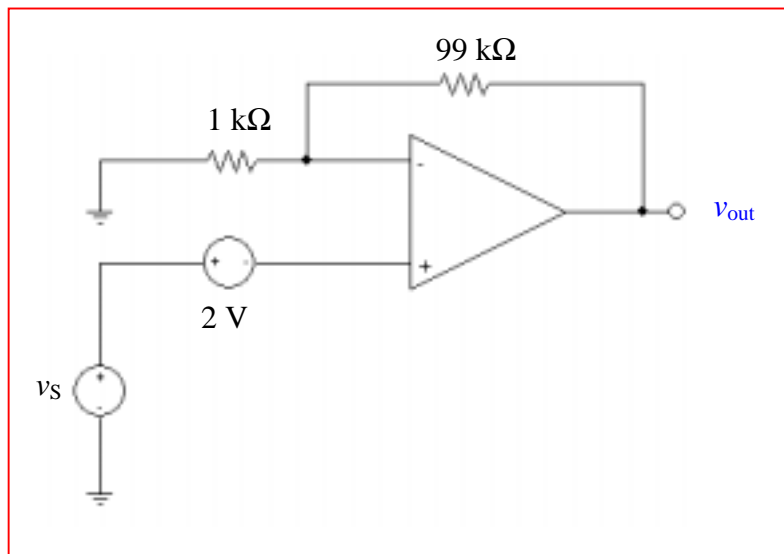
## CHAPTER SIX SOLUTIONS

20. We want  $R_f / R_{\text{instrument}} = 2K$ , and  $R_f / R_{\text{vocal}} = 1K$ , where  $K$  is a constant not specified. Assuming  $K = 1$ , one possible solution of many is:



## CHAPTER SIX SOLUTIONS

21. One possible solution of many:





## CHAPTER SIX SOLUTIONS

22.  $v_{\text{out}}$  of stage 1 is  $(1)(-20/2) = -10 \text{ V}$ .

$v_{\text{out}}$  of stage 2 is  $(-10)(-1000/10) = 1000 \text{ V}$

Note: in reality, the output voltage will be limited to a value less than that used to power the op amps.

## CHAPTER SIX SOLUTIONS

23. We have a difference amplifier as the first amplifier stage, and a simple voltage follower as the second stage. We therefore need only to find the output voltage of the first stage:  $v_{out}$  will track this voltage. Using voltage division, then, we find that the voltage at the non-inverting input pin of the first op amp is

$$V_2 \left( \frac{R_3}{R_2 + R_3} \right)$$

and this is the voltage at the inverting input terminal also. Thus, we may write a single nodal equation at the inverting input of the first op amp:

$$0 = \frac{1}{R_1} \left[ V_2 \left( \frac{R_3}{R_2 + R_3} \right) - V_1 \right] + \frac{1}{R_f} \left[ V_2 \left( \frac{R_3}{R_2 + R_3} \right) - V_{out|Stage1} \right]$$

which may be solved to obtain:

$$V_{out} = V_{out|Stage1} = \left( \frac{R_f}{R_1} + 1 \right) \frac{R_3}{R_2 + R_3} V_2 - \frac{R_f}{R_1} V_1$$

## CHAPTER SIX SOLUTIONS

24. The output of the first op amp stage may be found by realising that the voltage at the non-inverting input (and hence the voltage at the *inverting* input) is 0, and writing a single nodal equation at the inverting input:

$$0 = \frac{0 - V_{\text{out}}|_{\text{stage 1}}}{47} + \frac{0 - 2}{1} + \frac{0 - 3}{7} \text{ which leads to } V_{\text{out}}|_{\text{stage 1}} = -114.1 \text{ V}$$

This voltage appears at the input of the second op amp stage, which has a gain of  $-3/0.3 = 10$ . Thus, the output of the second op amp stage is  $-10(-114.1) = 1141 \text{ V}$ . This voltage appears at the input of the final op amp stage, which has a gain of  $-47/0.3 = -156.7$ .

Thus, the output of the circuit is  $-156.7(1141) = -178.8 \text{ kV}$ , which is completely and utterly ridiculous.

## CHAPTER SIX SOLUTIONS

25. The output of the top left stage is  $-1(10/2) = -5$  V.  
The output of the middle left stage is  $-2(10/2) = -10$  V.  
The output of the bottom right stage is  $-3(10/2) = -15$  V.

These three voltages are the input to a summing amplifier such that

$$V_{\text{out}} = -\frac{R}{100}(-5-10-15) = 10$$

Solving, we find that  $R = 33.33 \Omega$ .

## CHAPTER SIX SOLUTIONS

26. Stage 1 is configured as a voltage follower: the output voltage will be equal to the input voltage. Using voltage division, the voltage at the non-inverting input (and hence at the inverting input, as well), is

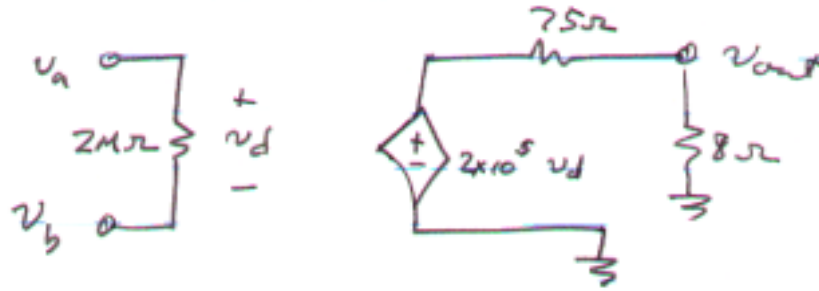
$$5 \frac{50}{100 + 50} = 1.667 \text{ V}$$

The second stage is wired as a voltage follower also, so

$$v_{\text{out}} = 1.667 \text{ V.}$$

## CHAPTER SIX SOLUTIONS

27.



(a)  $v_a = v_b = 1 \text{ nV} \therefore v_d = 0$  and  $v_{\text{out}} = 0$ . Thus,  $P_{8\Omega} = 0 \text{ W}$ .

(b)  $v_a = 0, v_b = 1 \text{ nV} \therefore v_d = -1 \text{ nV}$

$$v_{\text{out}} = (2 \times 10^5)(-1 \times 10^{-9}) \frac{8}{75 + 8} = -19.28 \mu\text{V}. \text{ Thus, } P_{8\Omega} = \frac{v_{\text{out}}^2}{8} = 46.46 \text{ pW}.$$

(c)  $v_a = 2 \text{ pV}, v_b = 1 \text{ fV} \therefore v_d = 1.999 \text{ pV}$

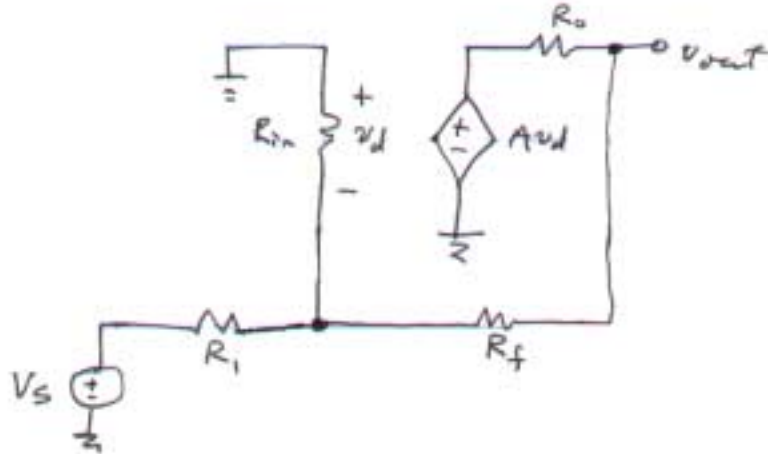
$$v_{\text{out}} = (2 \times 10^5)(1.999 \times 10^{-12}) \frac{8}{75 + 8} = 38.53 \text{ nV}. \text{ Thus, } P_{8\Omega} = \frac{v_{\text{out}}^2}{8} = 185.6 \text{ aW}.$$

(c)  $v_a = 50 \mu\text{V}, v_b = -4 \mu\text{V} \therefore v_d = 54 \mu\text{V}$

$$v_{\text{out}} = (2 \times 10^5)(54 \times 10^{-6}) \frac{8}{75 + 8} = 1.041 \text{ V}. \text{ Thus, } P_{8\Omega} = \frac{v_{\text{out}}^2}{8} = 135.5 \text{ mW}.$$

## CHAPTER SIX SOLUTIONS

28.



Writing a nodal equation at the “ $-v_d$ ” node,

$$0 = \frac{-v_d}{R_{in}} + \frac{-v_d - V_S}{R_1} + \frac{-v_d - v_{out}}{R_f} \quad [1]$$

or  $(R_1 R_f + R_{in} R_f + R_{in} R_1) v_d + R_{in} R_1 v_{out} = -R_{in} R_f V_S \quad [1]$

Writing a nodal equation at the “ $v_{out}$ ” node,

$$0 = \frac{-v_{out} - A v_d}{R_o} + \frac{v_{out} - (-v_d)}{R_f} \quad [2]$$

Eqn. [2] can be rewritten as:

$$v_d = \frac{-(R_f + R_o)}{R_o - A R_f} v_{out} \quad [2]$$

so that Eqn. [1] becomes:

$$v_{out} = - \frac{R_{in} (A R_f - R_o) V_S}{A R_{in} R_1 + R_f R_1 + R_{in} R_f + R_{in} R_1 + R_o R_1 + R_o R_{in}}$$

where for this circuit,  $A = 10^6$ ,  $R_{in} = 10 \text{ T}\Omega$ ,  $R_o = 15 \Omega$ ,  $R_f = 1000 \text{ k}\Omega$ ,  $R_1 = 270 \text{ k}\Omega$ .

(a)  $-3.704 \text{ mV}$ ;      (b)  $27.78 \text{ mV}$ ;      (c)  $-3.704 \text{ V}$ .

## CHAPTER SIX SOLUTIONS

29.  $v_{\text{out}} = A v_d = A \frac{R_i}{16 + R_i} (80 \times 10^{15}) \sin 2t \text{ V}$

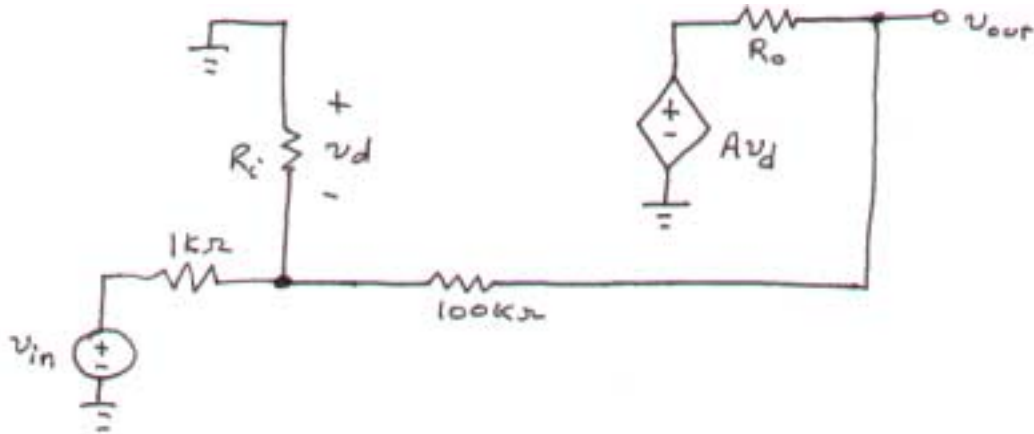
(a)  $A = 10^5$ ,  $R_i = 100 \text{ M}\Omega$ ,  $R_o$  value irrelevant.  $v_{\text{out}} = 8 \sin 2t \text{ nV}$

(b)  $A = 10^6$ ,  $R_i = 1 \text{ T}\Omega$ ,  $R_o$  value irrelevant.  $v_{\text{out}} = 80 \sin 2t \text{ nV}$



## CHAPTER SIX SOLUTIONS

30.



(a) Find  $v_{out}/v_{in}$  if  $R_i = \infty$ ,  $R_o = 0$ , and  $A$  is finite.

The nodal equation at the inverting input is

$$0 = \frac{-v_d - v_{in}}{1} + \frac{-v_d - v_{out}}{100} \quad [1]$$

At the output, with  $R_o = 0$  we may write  $v_{out} = Av_d$  so  $v_d = v_{out}/A$ . Thus, Eqn. [1] becomes

$$0 = \frac{v_{out}}{A} + v_{in} + \frac{v_{out}}{100A} + \frac{v_{out}}{100}$$

from which we find

$$\frac{v_{out}}{v_{in}} = \frac{-100A}{101 + A} \quad [2]$$

(b) We want the value of  $A$  such that  $v_{out}/v_{in} = -99$  (the “ideal” value would be  $-100$  if  $A$  were infinite). Substituting into Eqn. [2], we find

$$A = 9999$$

## CHAPTER SIX SOLUTIONS

31. (a)  $\delta = 0 \text{ V} \therefore v_d = 0$ , and  $P_{8\Omega} = 0 \text{ W}$ .

(b)  $\delta = 1 \text{ nV}$ , so  $v_d = 5 - (5 + 10^{-9}) = -10^{-9} \text{ V}$

Thus,

$$v_{\text{out}} = (2 \times 10^5) v_d \frac{8}{8 + 75} = -19.28 \text{ } \mu\text{V} \text{ and } P_{8\Omega} = (v_{\text{out}})^2 / 8 = 46.46 \text{ pW}.$$

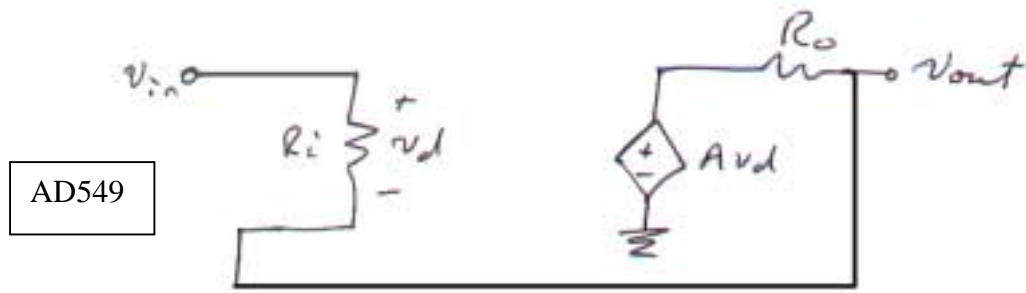
(c)  $\delta = 2.5 \text{ } \mu\text{V}$ , so  $v_d = 5 - (5 + 2.5 \times 10^{-6}) = -2.5 \times 10^{-6} \text{ V}$

Thus,

$$v_{\text{out}} = (2 \times 10^5) v_d \frac{8}{8 + 75} = -48.19 \text{ mV} \text{ and } P_{8\Omega} = (v_{\text{out}})^2 / 8 = 290.3 \text{ } \mu\text{W}.$$

## CHAPTER SIX SOLUTIONS

32.



Writing a single nodal equation at the output, we find that

$$0 = \frac{v_{out} - v_{in}}{R_i} + \frac{v_{out} - Av_d}{R_o} \quad [1]$$

Also,  $v_{in} - v_{out} = v_d$ , so Eqn. [1] becomes

$$0 = (v_{out} - v_{in}) R_o + (v_{out} - Av_{in} + Av_{out}) R_i$$

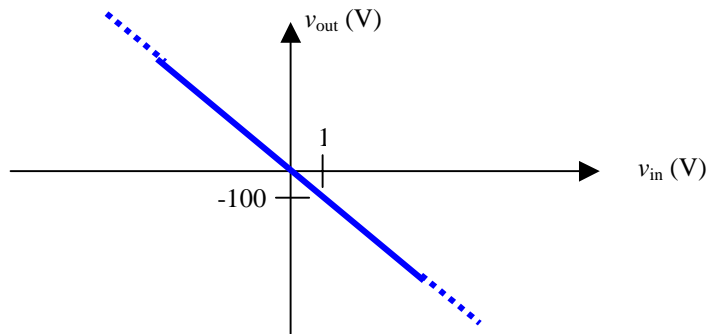
and

$$v_{out} = \frac{(R_o + AR_i)}{R_o + (A+1)R_i} v_{in}$$

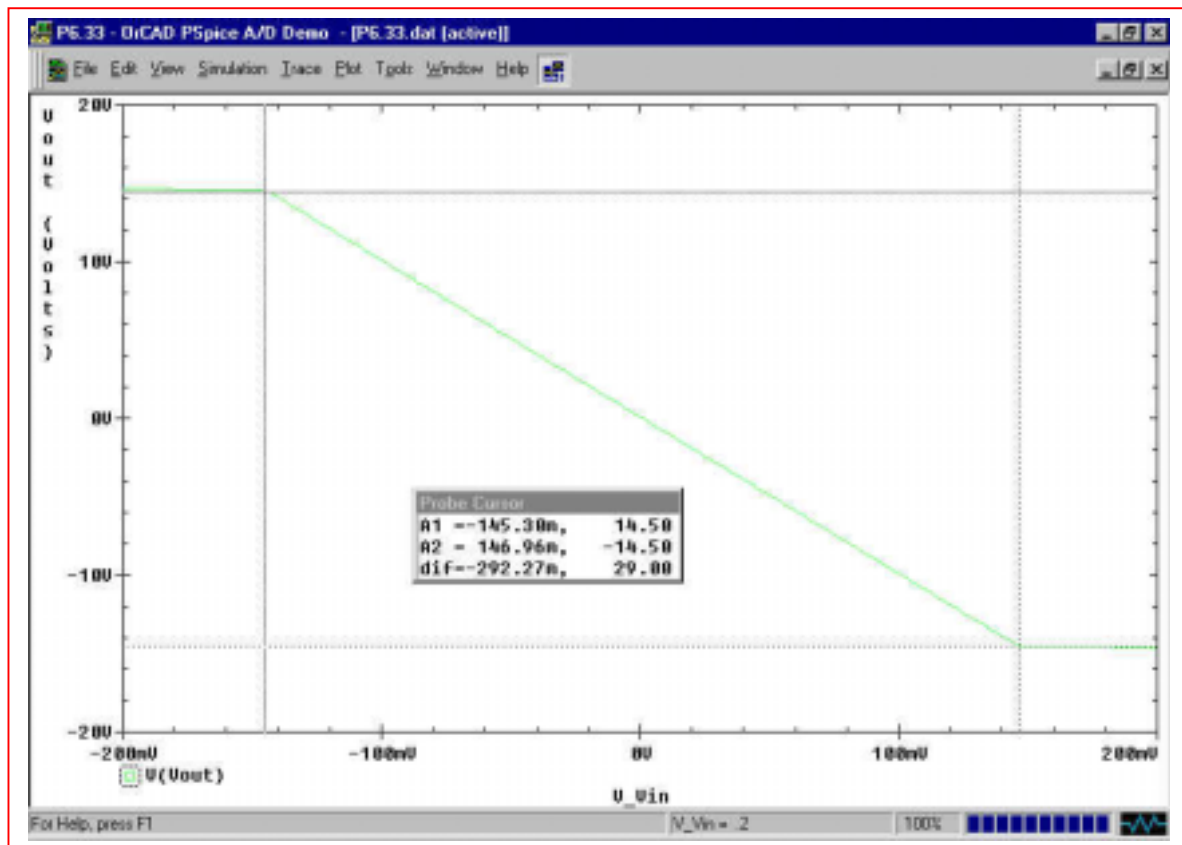
To within 4 significant figures (and more, actually), when  $v_{in} = -16 \text{ mV}$ ,  $v_{out} = -16 \text{ mV}$  (this is, after all, a voltage follower circuit).

## CHAPTER SIX SOLUTIONS

33. The ideal op amp model predicts a gain  $v_{out}/v_{in} = -1000/10 = -100$ , regardless of the value of  $v_{in}$ . In other words, it predicts an input-output characteristic such as:



From the PSpice simulation result shown below, we see that the ideal op amp model is reasonably accurate for  $|v_{in}| \times 100 < 15$  V (the supply voltage, assuming both have the same magnitude), but the onset of saturation is at  $\pm 14.5$  V, or  $|v_{in}| \sim 145$  mV. Increasing  $|v_{in}|$  past this value does not lead to an increase in  $|v_{out}|$ .



## CHAPTER SIX SOLUTIONS

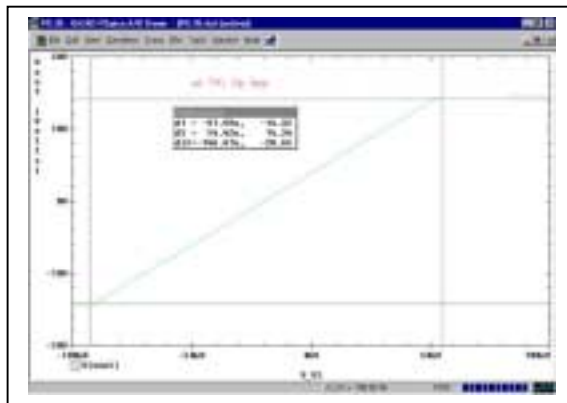
34. Positive voltage supply, negative voltage supply, inverting input, ground, output pin.

## CHAPTER SIX SOLUTIONS

35. This op amp circuit is an open-loop circuit; there is no external feedback path from the output terminal to either input. Thus, the output should be the open-loop gain times the differential input voltage, minus any resistive losses.

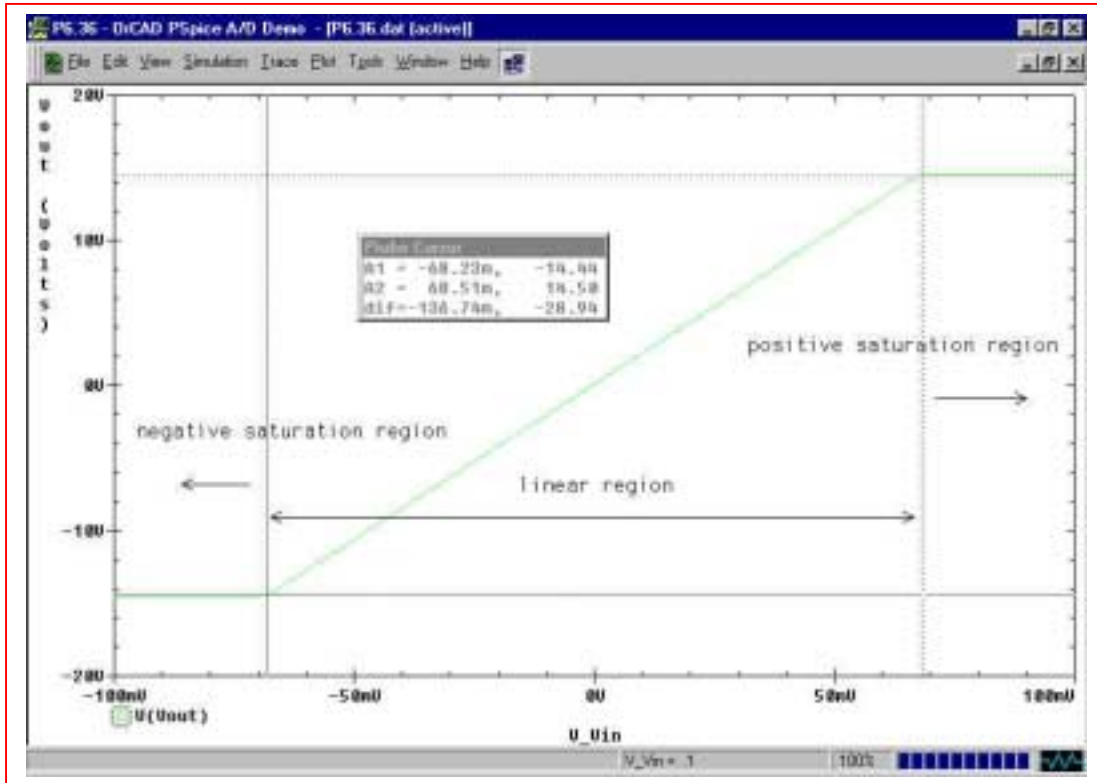
From the simulation results below, we see that all three op amps saturate at a voltage magnitude of approximately 14 V, corresponding to a differential input voltage of 50 to 100  $\mu\text{V}$ , except in the interest case of the LM 324, which may be showing some unexpected input offset behavior.

op amp	onset of negative saturation	negative saturation voltage	onset of positive saturation	positive saturation voltage
$\mu\text{A} 741$	-92 $\mu\text{V}$	-14.32 V	54.4 mV	14.34 V
LM 324	41.3 $\mu\text{V}$	-14.71 V	337.2 mV	13.87 V
LF 411	-31.77 $\mu\text{V}$	-13.81 V	39.78 mV	13.86 V



## CHAPTER SIX SOLUTIONS

36. This is a non-inverting op amp circuit, so we expect a gain of  $1 + 1000/4.7 = 213.8$ . With  $\pm 15$  V DC supplies, we need to sweep the input just above and just below this value divided by the gain to ensure that we see the saturation regions. Performing the indicated simulation and a DC sweep from  $-0.1$  V to  $+0.1$  V with  $0.001$  V steps, we obtain the following input-output characteristic:



Using the cursor tool, we see that the linear region is in the range of  $-68.2 \text{ mV} < V_{in} < 68.5 \text{ mV}$ .

The simulation predicts a gain of  $7.103 \text{ V} / 32.87 \text{ mV} = 216.1$ , which is reasonably close to the value predicted using the ideal op amp model.

## CHAPTER SIX SOLUTIONS

37. Referring to the detailed model of an op amp, shorting the input terminals together shorts the dependent source to ground. Therefore, any 1-V source connected to the output through a 1- $\Omega$  resistor should “see”  $1\ \Omega + R_o$ . For the  $\mu A$  741, we expect  $1 + 75 = 76\ \Omega$ . For the LF 411, we expect  $\sim 1 + 1\ \Omega$  or  $\sim 2\ \Omega$ .

Simulating the  $\mu A$ 741 circuit, we find:

Supply voltages	Current into output	Total resistance	Output resistance
$\pm 15\text{ V}$	-42.5 mA	-23.53 $\Omega$	-22.53 $\Omega$
$\pm 5\text{ V}$	-40.55 mA	-24.66 $\Omega$	-24.66 $\Omega$
$\pm 2\text{ V}$	-40.55 mA	-24.66 $\Omega$	-24.66 $\Omega$
0 V	579.2 mA	1.727 $\Omega$	727 m $\Omega$

Conclusion: as we might expect from previous experience in determining Thévenin equivalent resistances, we must short out the voltage supplies to the op amp when performing such an experiment (hence the negative resistance values obtained above). However, we obtained 0.727  $\Omega$  instead of the expected 75  $\Omega$ , which leads to two possible conclusions: (1) The PSpice model is not designed to represent the op amp behavior accurately in such a circuit configuration or (2) such an experimental connection is not adequate for measuring the output resistance.

Simulating the LF411 circuit, we find:

Supply voltages	Current into output	Total resistance	Output resistance
$\pm 15\text{ V}$	25.46 mA	39.28 $\Omega$	38.28 $\Omega$
$\pm 5\text{ V}$	25.43 mA	39.32 $\Omega$	38.32 $\Omega$
$\pm 2\text{ V}$	25.48 mA	39.24 $\Omega$	28.24 $\Omega$
0 V	1000 mA	1 $\Omega$	0 $\Omega$

Conclusion: as we might expect from previous experience in determining Thévenin equivalent resistances, we must short out the voltage supplies to the op amp when performing such an experiment. However, we obtained  $\sim 0\ \Omega$  instead of the expected 1  $\Omega$ , which leads to two possible conclusions: (1) The PSpice model is not designed to represent the op amp behavior accurately in such a circuit configuration or (2) such an experimental connection is not adequate for measuring the output resistance. However, it is interesting that PSpice did predict a much lower output resistance for the LF 411 than the  $\mu A$  741, as we would expect.



## CHAPTER SIX SOLUTIONS

38. Based on the detailed model of **the LF 411 op amp**, we can write the following nodal equation at the inverting input:

$$0 = \frac{-v_d}{R_{in}} + \frac{v_x - v_d}{10^4} + \frac{Av_d - v_d}{10^6 + R_o}$$

Substituting values for the LF 411 and simplifying, we make appropriate approximations and then solve for  $v_d$  in terms of  $v_x$ , finding that

$$v_d = \frac{-10^6}{199.9 \times 10^6} v_x = -\frac{v_x}{199.9}$$

With a gain of  $-1000/10 = -100$  and supply voltage magnitudes of 15 V, we are effectively limited to values of  $|v_x| < 150$  mV.

For  $v_x = -10$  mV, PSpice predicts  $v_d = 6$   $\mu$ V, where the hand calculations based on the detailed model predict 50  $\mu$ V, which is about one order of magnitude larger. For the same input voltage, PSpice predicts an input current of -1  $\mu$ A, whereas the hand calculations predict 99.5  $v_x$  mA = -995 nA (which is reasonably close).

## CHAPTER SIX SOLUTIONS

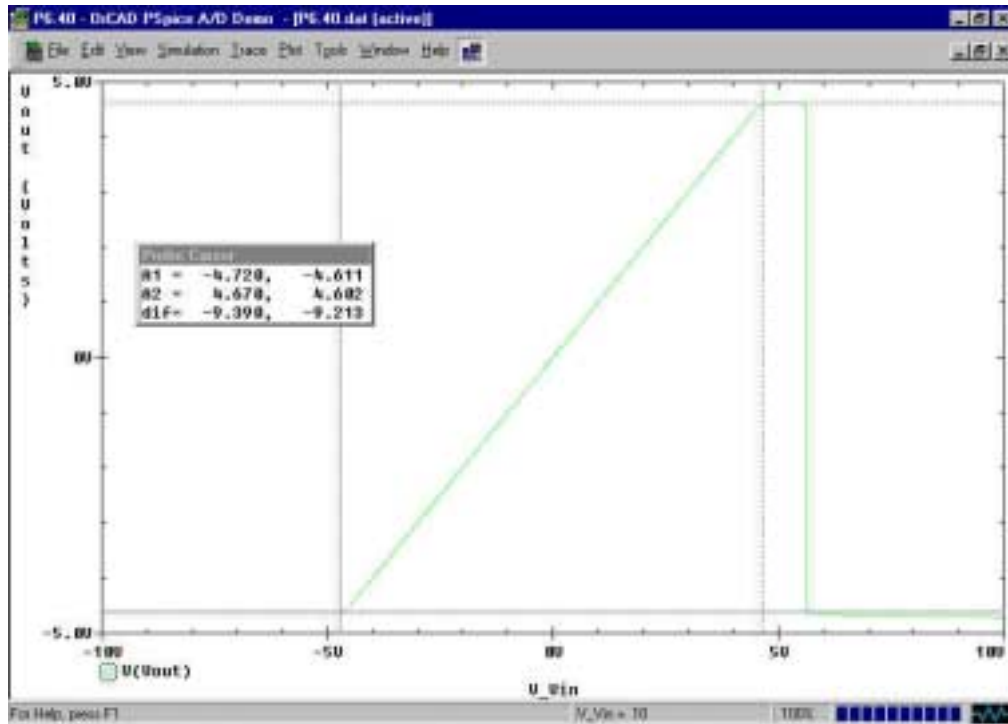
(a) The gain of the inverting amplifier is  $-1000$ . At a sensor voltage of  $-30$  mV, the predicted output voltage (assuming an ideal op amp) is  $+30$  V. At a sensor voltage of  $+75$  mV, the predicted output voltage (again assuming an ideal op amp) is  $-75$  V. Since the op amp is being powered by dc sources with voltage magnitude equal to  $15$  V, the output voltage range will realistically be limited to the range

$$-15 < V_{\text{out}} < 15 \text{ V.}$$

(b) The peak input voltage is  $75$  mV. Therefore,  $15 / 75 \times 10^{-3} = 200$ , and we should set the resistance ratio  $R_f / R_1 < 199$  to ensure the op amp does not saturate.

## CHAPTER SIX SOLUTIONS

39. (a)



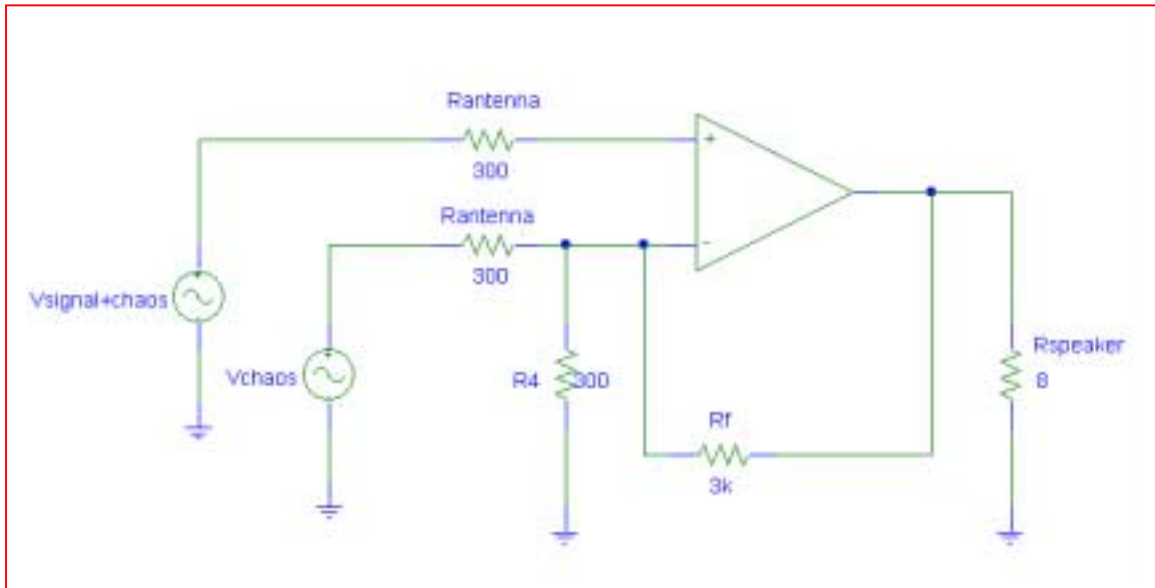
We see from the simulation result that negative saturation begins at  $V_{in} = -4.72$  V, and positive saturation begins at  $V_{in} = +4.67$  V.

(b) Using a  $1 \text{ p}\Omega$  resistor between the output pin and ground, we obtain an output current of  $40.61 \text{ mA}$ , slightly larger than the expected  $35 \text{ mA}$ , but not too far off.

## CHAPTER SIX SOLUTIONS

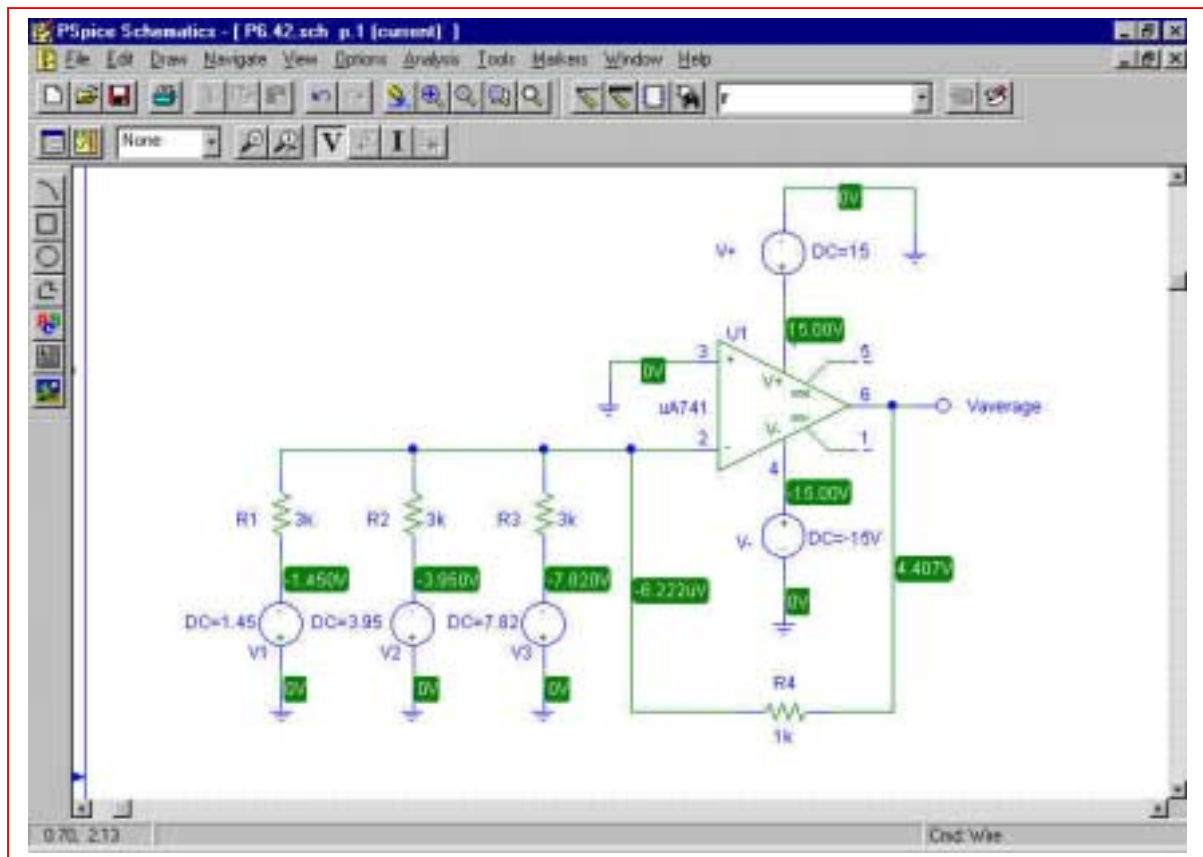
41. We assume that the strength of the separately-broadcast chaotic “noise” signal is received at the appropriate intensity such that it may precisely cancel out the chaotic component of the total received signal; otherwise, a variable-gain stage would need to be added so that this could be adjusted by the user. We also assume that the signal frequency is separate from the “carrier” or broadcast frequency, and has already been separated out by an appropriate circuit (in a similar fashion, a radio station transmitting at 92 MHz is sending an audio signal of between 20 and 20 kHz, which must be separated from the 92 MHz frequency.)

One possible solution of many (all resistances in ohms):



## CHAPTER SIX SOLUTIONS

41. One possible solution of many:



This circuit produces an output equal to the average of  $V_1$ ,  $V_2$ , and  $V_3$ , as shown in the simulation result:  $V_{average} = (1.45 + 3.95 + 7.82) / 3 = 4.407$  V.

## CHAPTER SEVEN SOLUTIONS

1. (a)  $C = \frac{\epsilon A}{d} = \frac{8.854 \times 10^{-12} (78.54 \times 10^{-6})}{100 \times 10^{-6}} = 6.954 \text{ pF}$

(b) *Energy*,  $E = \frac{1}{2} CV^2 \therefore V = \sqrt{\frac{2E}{C}} = \sqrt{\frac{2(1 \times 10^{-3})}{6.954 \times 10^{-12}}} = 16.959 \text{ kV}$

(c)  $E = \frac{1}{2} CV^2 \therefore C = \frac{2E}{V^2} = \frac{2(2.5 \times 10^{-6})}{(100^2)} = 500 \text{ pF}$

$$C = \frac{\epsilon A}{d} \therefore \epsilon = \frac{Cd}{A} = \frac{(500 \times 10^{-12})(100 \times 10^{-6})}{(78.54 \times 10^{-6})} = 636.62 \text{ pF} \cdot \text{m}^{-1}$$

$$\therefore \text{Relative\_permittivity}, \frac{\epsilon}{\epsilon_0} = \frac{636.62 \times 10^{-12}}{8.854 \times 10^{-12}} = 71.9$$

## CHAPTER SEVEN SOLUTIONS

2. (a) For  $V_A = -1\text{V}$ ,  $W = \sqrt{\frac{2K_s\epsilon_0}{qN}(V_{bi} - V_A)} = \sqrt{\frac{2(11.8)(8.854 \times 10^{-12})}{(1.6 \times 10^{-19})(1 \times 10^{24})}(0.57 + 1)}$   
 $= 45.281 \times 10^{-9} \text{ m}$

$$C_j = \frac{11.8(8.854 \times 10^{-12})(1 \times 10^{-12})}{45.281 \times 10^{-9}} = \boxed{2.307 \text{ fF}}$$

(b) For  $V_A = -5\text{V}$ ,  $W = \sqrt{\frac{2K_s\epsilon_0}{qN}(V_{bi} - V_A)} = \sqrt{\frac{2(11.8)(8.854 \times 10^{-12})}{(1.6 \times 10^{-19})(1 \times 10^{24})}(0.57 + 5)}$   
 $= 85.289 \times 10^{-9} \text{ m}$

$$C_j = \frac{11.8(8.854 \times 10^{-12})(1 \times 10^{-12})}{85.289 \times 10^{-9}} = \boxed{1.225 \text{ fF}}$$

(c) For  $V_A = -10\text{V}$ ,

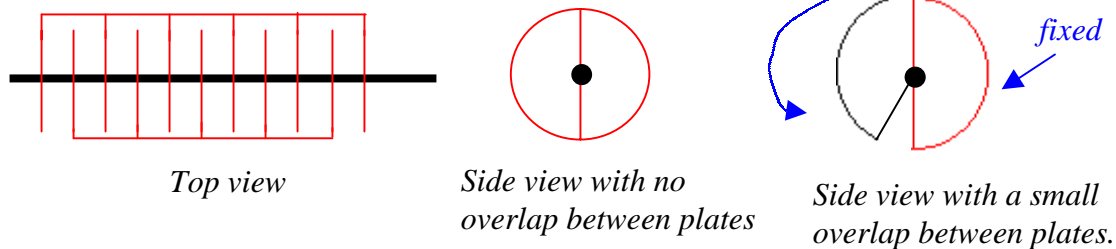
$$W = \sqrt{\frac{2K_s\epsilon_0}{qN}(V_{bi} - V_A)} = \sqrt{\frac{2(11.8)(8.854 \times 10^{-12})}{(1.6 \times 10^{-19})(1 \times 10^{24})}(0.57 + 10)}$$

$$= 117.491 \times 10^{-9} \text{ m}$$

$$C_j = \frac{11.8(8.854 \times 10^{-12})(1 \times 10^{-12})}{117.491 \times 10^{-9}} = \boxed{889.239 \text{ aF}}$$

## CHAPTER SEVEN SOLUTIONS

3. We require a capacitor that may be manually varied between 100 and 1000 pF by rotation of a knob. Let's choose an air dielectric for simplicity of construction, and a series of 11 half-plates:



Constructed as shown, the half-plates are in parallel, so that each of the 10 pairs must have a capacitance of  $1000/10 = 100$  pF when rotated such that they overlap completely. If we arbitrarily select an area of  $1 \text{ cm}^2$  for each half-plate, then the gap spacing between each plate is  $d = \epsilon A/C = (8.854 \times 10^{-14} \text{ F/cm})(1 \text{ cm}^2)/(100 \times 10^{-12} \text{ F}) = 0.8854 \text{ mm}$ . This is tight, but not impossible to achieve. The final step is to determine the amount of overlap which corresponds to 100 pF for the total capacitor structure. A capacitance of 100 pF is equal to 10% of the capacitance when all of the plate areas are aligned, so we need a pie-shaped wedge having an area of  $0.1 \text{ cm}^2$ . If the middle figure above corresponds to an angle of  $0^\circ$  and the case of perfect alignment (maximum capacitance) corresponds to an angle of  $180^\circ$ , we need to set out minimum angle to be  $18^\circ$ .



## CHAPTER SEVEN SOLUTIONS

4. (a) Energy stored  $= \int_{t_0}^t v.C \frac{dv}{dt} = C \int_0^{2 \times 10^{-3}} 3e^{-\frac{t}{5}} \left( -\frac{3}{5} e^{-\frac{t}{5}} \right) dt = -1.080 \mu J$

(b)  $V_{\max} = 3V$

Max. energy at  $t=0$ ,  $= \frac{1}{2} CV^2 = 1.35 mJ \therefore 37\% E_{\max} = 499.5 \mu J$

$V$  at  $37\% E_{\max} = 1.825V$

$v(t) = 1.825 = 3e^{-\frac{t}{5}} \therefore t = 2.486s \Rightarrow \approx 2s$

(c)  $i = C \frac{dv}{dt} = 300 \times 10^{-6} \left( -\frac{3}{5} e^{-\frac{1.2}{5}} \right) = -141.593 \mu A$

(d)  $P = vi = 2.011 \left( -120.658 \times 10^{-6} \right) = -242.6 \mu W$

## CHAPTER SEVEN SOLUTIONS

$$5. \quad (a) \quad v = \frac{1}{C} \cdot \frac{\pi}{2} (1 \times 10^{-3})^2 = \frac{1}{47 \times 10^{-6}} \cdot \frac{(3.14159)}{2} (1 \times 10^{-3})^2 = \boxed{33.421 mV}$$

$$(b) \quad v = \frac{1}{C} \cdot \left( \frac{\pi}{2} (1 \times 10^{-3})^2 + 0 \right) = \frac{1}{47 \times 10^{-6}} \cdot \frac{(3.14159)}{2} (1 \times 10^{-3})^2 = \boxed{33.421 mV}$$

$$(c) \quad v = \frac{1}{C} \cdot \left( \frac{\pi}{2} (1 \times 10^{-3})^2 + \frac{\pi}{4} (1 \times 10^{-3})^2 \right) = \frac{1}{47 \times 10^{-6}} \cdot \left( \frac{3\pi}{4} (1 \times 10^{-3})^2 \right) = \boxed{50.132 mV}$$

## CHAPTER SEVEN SOLUTIONS

6. 
$$V = \frac{1}{C} \int_0^{200ms} i dt = \frac{1}{C} \left[ \left( -\frac{7 \times 10^{-3}}{\pi} \cos \pi t \right) \right]_0^{200ms} = \frac{0.426}{C}$$

$$E = \frac{1}{2} CV^2 = 3 \times 10^{-6} = \frac{181.086 \times 10^{-9}}{2C} \therefore C = \frac{181.086 \times 10^{-9}}{2(3 \times 10^{-6})} = 30181 \mu F$$

## CHAPTER SEVEN SOLUTIONS

7.

$$(a) \quad c = 0.2 \mu\text{F}, v_c = 5 + 3 \cos^2 200t \text{V}; \therefore i_c = 0.2 \times 10^{-6} (3) (-2) 200 \sin 200t \cos 200t$$

$$\therefore i_c = -0.12 \sin 400t \text{mA}$$

$$(b) \quad w_c = \frac{1}{2} c v_c^2 = \frac{1}{2} \times 2 \times 10^{-7} (5 + 3 \cos^2 200t)^2 \therefore w_{c \max} = 10^{-7} \times 64 = 6.4 \mu\text{J}$$

$$(c) \quad v_c = \frac{1}{0.2} \times 10^6 \int_0^t 8e^{-100t} \times 10^{-3} dt = 10^3 \times 40(-0.01)(e^{-100t} - 1) = 400(1 - e^{-100t}) \text{V}$$

$$(d) \quad v_c = 500 - 400e^{-100t} \text{ V}$$

## CHAPTER SEVEN SOLUTIONS

8.  $v_c(0) = 250\text{V}$ ,  $c = 2\text{mF}$  (a)  $v_c(0.1) = 250 + 500 \int_0^{0.1} 5 dt$   
 $\therefore v_c(0.1) = 500\text{V}; v_c(0.2) = 500 \int_{0.1}^{0.2} 10 dt = 1000\text{V}$   
 $\therefore v_c(0.6) = 1750\text{V}, v_c(0.9) = 2000\text{V}$   
 $\therefore 0.9 < t < 1: v_c = 2000 + 500 \int_{0.9}^t 10 dt = 2000 + 5000(t - 0.9)$   
 $\therefore v_c = 2100 = 2000 + 5000(t_2 - 0.9) \therefore t_2 = 0.92 \therefore 0.9 < t < 0.92\text{s}$

## CHAPTER SEVEN SOLUTIONS

9.

$$(a) \quad w_c = \frac{1}{2} C v^2 = \frac{1}{2} \times 10^{-6} v^2 = 2 \times 10^{-2} e^{-1000t} \therefore v = \pm 200 e^{-500t} \text{ V}$$

$$i = C v' = 10^{-6} (\pm 200) (-500) e^{-500t} = \mp 0.1 e^{-500t}$$

$$\therefore R = \frac{-v}{i} = \frac{200}{0.1} = \boxed{2k\Omega}$$

$$(b) \quad P_R = i^2 R = 0.01 \times 2000 e^{-1000t} = 20 e^{-1000t} \text{ W}$$

$$\therefore W_R = \int_0^{\infty} 20 e^{-1000t} dt = -0.02 e^{-1000t} \Big|_0^{\infty} = \boxed{0.02 \text{ J}}$$

## CHAPTER SEVEN SOLUTIONS

10. (a) Left circuit:

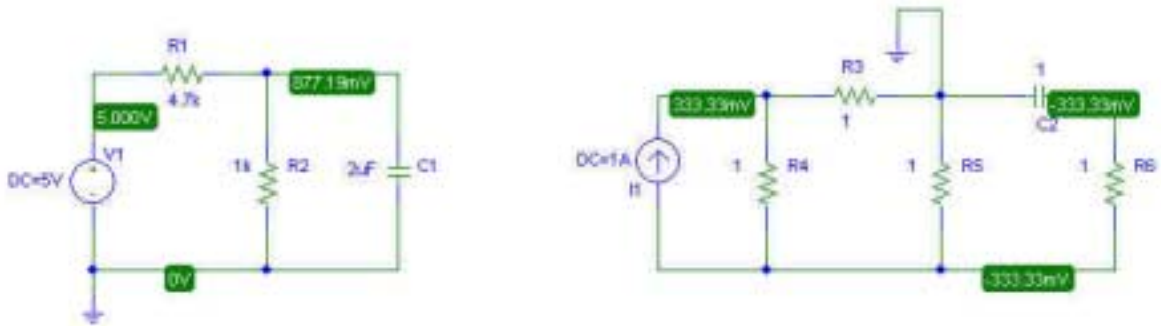
$$\text{By Voltage division, } V_C = \frac{1k}{4.7k + 1k}(5) = 0.877V$$

Right circuit:

$$V_1 = 1(1 // 2) = \frac{2}{3}V$$

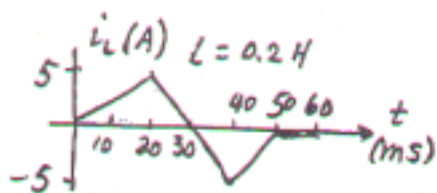
$$\text{By Voltage Division, } V_2 = \frac{1}{3}V \therefore V_C = -\frac{1}{3}V$$

(b)

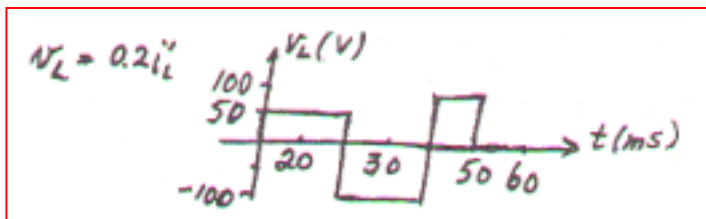


## CHAPTER SEVEN SOLUTIONS

11.



(a)



(b)  $P_L = v_{L} i_L \therefore P_{L_{\max}} = (-100)(-5) = 500 \text{ W}$  at  $t = 40^- \text{ ms}$

(c)  $P_{L_{\min}} = 100(-5) = -500 \text{ W}$  at  $t = 20^+$  and  $40^+$  ms

(d)  $W_L = \frac{1}{2} L i_L^2 \therefore W_L(40 \text{ ms}) = \frac{1}{2} \times 0.2 (-5)^2 = 2.5 \text{ J}$



## CHAPTER SEVEN SOLUTIONS

12.

$$L = 50 \times 10^{-3}, t < 0: i = 0; t > 0 \quad i = 80te^{-100t} \text{ mA} = 0.08te^{-100t} \text{ A}$$

$$\therefore i' = 0.08e^{-100t} - 8te^{-100t} \therefore 0.08 = 8t, t_m = 0.01s, |i|_{\max} = 0.08 \times 0.01e^{-1}$$

$$\therefore |i|_{\max} = 0.2943\text{mA}; v = 0.05i' = e^{-100t}(0.004 - 0.4t)$$

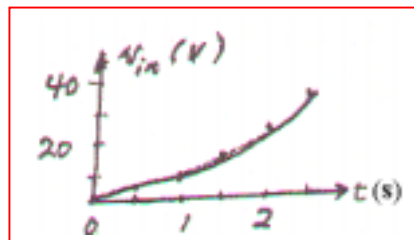
$$\therefore v' = e^{-100t}(-0.4) - 100e^{-100t}(0.004 - 0.4t) \therefore -0.4 = 0.4 - 40t, t = \frac{0.8}{40} = 0.02s$$

$$v = e^{-2}(0.004 - 0.008) = -0.5413\text{mV} \text{ this is minimum} \therefore |v|_{\max} = 0.004\text{V at } t=0$$

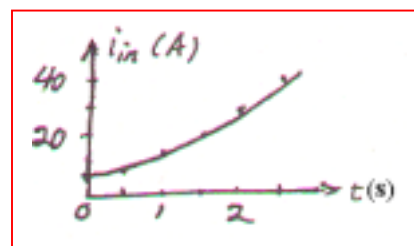
## CHAPTER SEVEN SOLUTIONS

13.

(a)  $t > 0: i_s = 0.4t^2 \text{ A} \therefore v_{in} = 10i_s + 5i_s' = 4t^2 + 4t \text{ V}$



(b)  $i_{in'} = 0.1v_s + \frac{1}{5} \int_0^t 40t dt + 5 = 4t + 4t^2 + 5 \text{ A}$

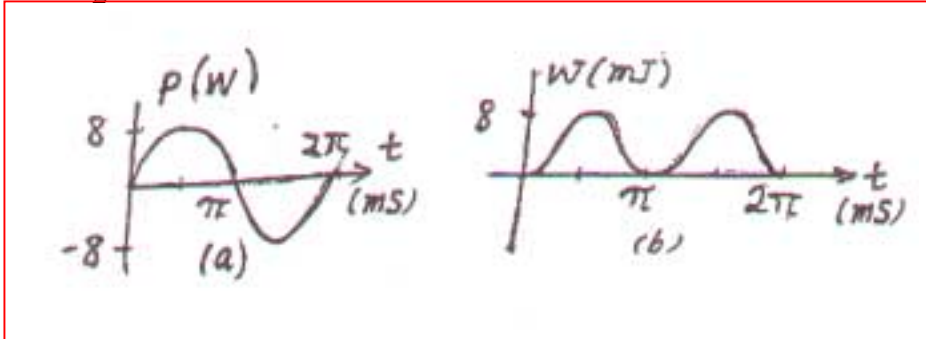


## CHAPTER SEVEN SOLUTIONS

14.  $v_L = 20 \cos 1000t \text{ V}$ ,  $L = 25 \text{ mH}$ ,  $i_L(0) = 0$

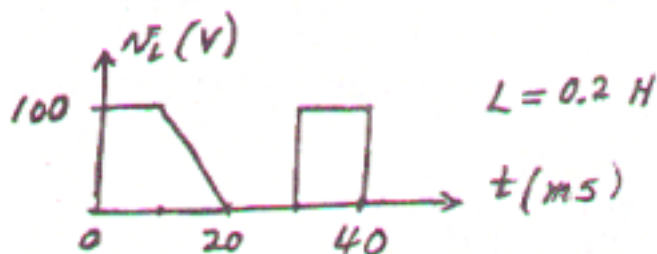
(a)  $i_L = 40 \int_0^t 20 \cos 1000t dt = 0.8 \sin 1000t \text{ A} \therefore p = 8 \sin 2000t \text{ W}$

(b)  $w = \frac{1}{2} \times 25 \times 10^{-3} \times 0.64 \sin^2 1000t = 8 \sin^2 1000t \text{ mJ}$



## CHAPTER SEVEN SOLUTIONS

15.



(a)  $0 < t < 10 \text{ ms}$ :  $i_L = -2 + 5 \int_0^t 100 dt = -2 + 500t \therefore i_L(10\text{ms}) = 3\text{A}, i_L(8\text{ms}) = \boxed{2\text{A}}$

(b)  $i_L(0) = 0 \therefore i_L(10\text{ms}) = 500 \times 0.01 = 5\text{A} \therefore i_L(20\text{ms}) = 5 + 5 \int_{0.01}^{0.02} 10^4 (0.02 - t) dt$   
 $\therefore i_L(20\text{ms}) = 5 + 5 \times 10^4 (0.02t - 0.5t^2)_{0.01}^{0.02} = 5 + 5 \times 10^4 (0.0002 - 0.00015) = 7.5\text{A}$   
 $\therefore w_L = \frac{1}{2} \times 0.2 \times 7.5^2 = \boxed{5.625\text{J}}$

(c) If the circuit has been connected for a long time, L appears like short circuit.

$$V_{8\Omega} = \frac{8}{2+8}(100\text{V}) = 80\text{V}$$

$$I_{2\Omega} = \frac{20\text{V}}{2\Omega} = 10\text{A}$$

$$\therefore i_x = \frac{80\text{V}}{80\Omega} = 1\text{A}$$

## CHAPTER SEVEN SOLUTIONS

16.  $L = 5\text{H}, V_L = 10(e^{-t} - e^{-2t})\text{V}, i_L(0) = 0.08\text{A}$

(a)  $v_L(1) = 10(e^{-1} - e^{-2}) = 2.325\text{ V}$

(b)  $i_L = 0.08 + 0.2 \int_0^t 10(e^{-t} - e^{-2t}) dt = 0.08 + 2(-e^{-t} + 0.5e^{-2t})_0^t$   
 $i_L = 0.08 + 2(-e^{-t} + 0.5e^{-2t} + 1 - 0.5) = 1.08 + e^{-2t} - 2e^{-t} \therefore i_L(1) = 0.4796\text{A}$

(c)  $i_L(\infty) = 1.08\text{A}$

## CHAPTER SEVEN SOLUTIONS

17.

$$\begin{aligned} \text{(a)} \quad v_x &= 120 \times \frac{40}{12 + 20 + 40} + 40 \times 5 \times \frac{12}{12 + 20 + 40} \\ &= \frac{200}{3} + \frac{100}{3} = \boxed{100\text{V}} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad v_x &= \frac{120}{12 + 15 \parallel 60} \times \frac{15}{15 + 60} \times 40 + 40 \times 5 \times \frac{15 \parallel 12}{15 \parallel 12 + 60} \\ &= \frac{120}{12 + 12} \times \frac{1}{5} \times 40 + 200 \times \frac{6.667}{66.667} \\ &= 40 + 20 = \boxed{60\text{V}} \end{aligned}$$

## CHAPTER SEVEN SOLUTIONS

18.

(a)  $w_L = \frac{1}{2} \times 5 \times 1.6^2 = 6.4\text{J}$

(b)  $w_c = \frac{1}{2} \times 20 \times 10^{-6} \times 100^2 = 0.1\text{J}$

(c) Left to right (magnitudes): 100, 0, 100, 116, 16, 16, 0 (V)

(d) Left to right (magnitudes): 0, 0, 2, 2, 0.4, 1.6, 0 (A)

## CHAPTER SEVEN SOLUTIONS

19.

(a)  $v_s = 400t^2 \text{ V}, t > 0; i_L(0) = 0.5 \text{ A}; t = 0.4 \text{ s}$

$$v_c = 400 \times 0.16 = 64 \text{ V}, w_c = \frac{1}{2} \times 10^{-5} \times 64^2 = 20.48 \text{ mJ}$$

(b)  $i_L = 0.5 + 0.1 \int_0^{0.4} 400t^2 dt = 0.5 + 40 \times \frac{1}{3} \times 0.4^3 = 1.3533 \text{ A}$

$$\therefore w_L = \frac{1}{2} \times 10 \times 1.3533^2 = 9.1581 \text{ J}$$

(c)  $i_R = 4t^2, P_R = 100 \times 16t^4 \therefore w_R = \int_0^{0.4} 1600t^4 dt = 320 \times 0.4^5 = 3.277 \text{ J}$



## CHAPTER SEVEN SOLUTIONS

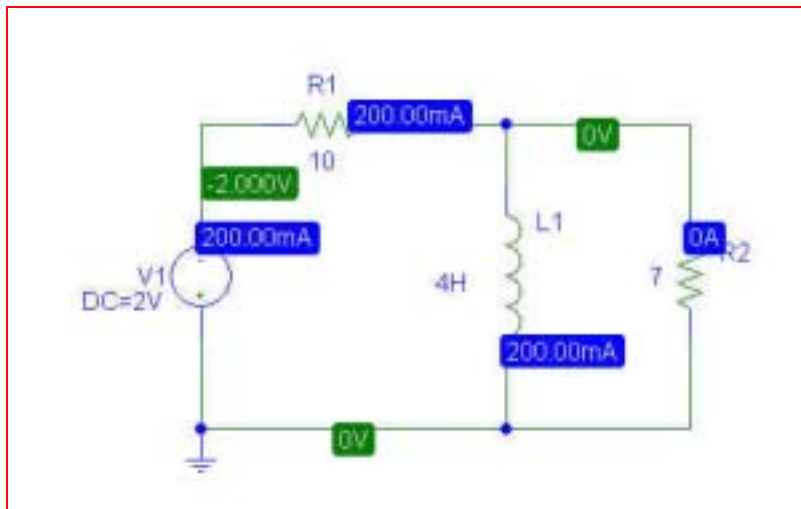
20. (a)  $P_{7\Omega} = 0W$ ;  $P_{10\Omega} = \frac{V^2}{R} = \frac{(2)^2}{10} = 0.4W$

(b) PSpice verification

We see from the PSpice simulation that the voltage across the 10- $\Omega$  resistor is  $-2$  V, so that it is dissipating  $4/10 = 400$  mW.

The 7- $\Omega$  resistor has zero volts across its terminals, and hence dissipates zero power.

Both results agree with the hand calculations.



## CHAPTER SEVEN SOLUTIONS

21.

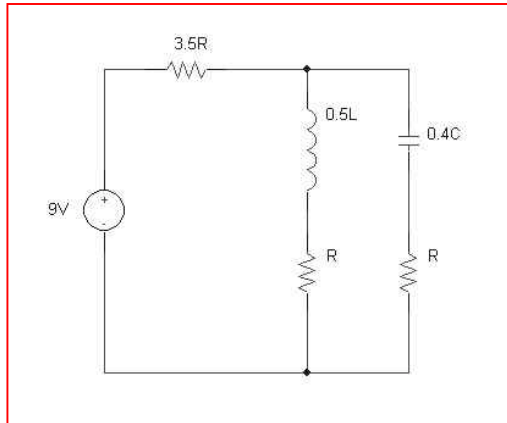
$$C_{equiv} \equiv 10\mu + \left( \frac{1}{\frac{1}{10\mu} + \frac{1}{10\mu}} \right) \text{ in series with } 10\mu \text{ in series with } 10\mu + \left( \frac{1}{\frac{1}{10\mu} + \frac{1}{10\mu}} \right) \\ \equiv 4.286\mu F$$

## CHAPTER SEVEN SOLUTIONS

22.  $L_{equiv} \equiv (77\text{ p} // (77\text{ p} + 77\text{ p})) + 77\text{ p} + (77\text{ p} // (77\text{ p} + 77\text{ p})) = 179.6\text{ pH}$

## CHAPTER SEVEN SOLUTIONS

23. (a) Assuming all resistors have value  $R$ , all inductors have value  $L$ , and all capacitors have value  $C$ ,

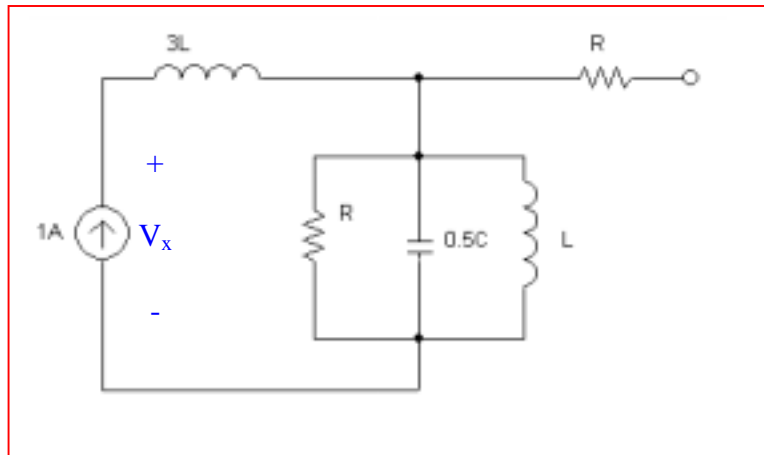


- (b) At dc,  $20\mu\text{F}$  is open circuit;  $500\mu\text{H}$  is short circuit.

Using voltage division,  $V_x = \frac{10k}{10k + 15k}(9) = 3.6V$

## CHAPTER SEVEN SOLUTIONS

24. (a) Assuming all resistors have value  $R$ , all inductors value  $L$ , and all capacitors value  $C$ ,



- (b)  $V_x = 0V$  as  $L$  is short circuit at dc.

## CHAPTER SEVEN SOLUTIONS

25.  $C_{equiv} = \{ [(100 \text{ n} + 40 \text{ n}) \parallel 12 \text{ n}] + 75 \text{ n} \} \parallel \{ 7 \text{ } \mu + (2 \text{ } \mu \parallel 12 \text{ } \mu) \}$

$$C_{equiv} \equiv 85.211 \text{ nF}$$

## CHAPTER SEVEN SOLUTIONS

26.  $L_{\text{equiv}} = \{[(17 \text{ p} \parallel 4 \text{ n}) + 77 \text{ p}] \parallel 12 \text{ n}\} + \{1 \text{ n} \parallel (72 \text{ p} + 14 \text{ p})\}$

$$L_{\text{equiv}} \equiv 172.388 \mu\text{H}$$

## CHAPTER SEVEN SOLUTIONS

$$27. \quad C_T - C_x = (7 + 47 + 1 + 16 + 100) = 171 \mu F$$

$$E_{C_T - C_x} = \frac{1}{2} (C_T - C_x) V^2 = \frac{1}{2} (171 \mu)(2.5)^2 = 534.375 \mu J$$

$$E_{C_x} = E_{C_T} - E_{C_T - C_x} = (534.8 - 534.375) \mu J = 425 nJ$$

$$\therefore E_{C_x} = 425 n = \frac{1}{2} C_x V^2 \Rightarrow C_x = \frac{425 n (2)}{(2.5)^2} = 136 nF$$



## CHAPTER SEVEN SOLUTIONS

28.

(a) For all  $L = 1.5H$ ,  $L_{equiv} = 1.5 + \left( \frac{1}{\frac{1}{1.5} + \frac{1}{1.5}} \right) + \left( \frac{1}{\frac{1}{1.5} + \frac{1}{1.5} + \frac{1}{1.5}} \right) = 2.75H$

(b) For a general network of this type, having  $N$  stages (and all  $L$  values equiv),

$$L_{equiv} = \sum_{N=1}^n \frac{L^N}{NL^{N-1}}$$

## CHAPTER SEVEN SOLUTIONS

29.

$$(a) \quad L_{equiv} = 1 + \left( \frac{1}{\frac{1}{2} + \frac{1}{2}} \right) + \left( \frac{1}{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} \right) = 3H$$

(b) For a network of this type having 3 stages,

$$L_{equiv} = 1 + \frac{1}{\frac{2+2}{(2)^2}} + \frac{1}{\frac{3+3}{(3)^2} + \frac{1}{3}} = 1 + \frac{(2)^2}{2(2)} + \frac{(3)^3}{3(3)^2}$$

Extending for the general case of N stages,

$$\begin{aligned} L_{equiv} &= 1 + \frac{1}{\frac{1}{2} + \frac{1}{2}} + \frac{1}{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} + K + \frac{1}{\frac{1}{N} + K \frac{1}{N}} \\ &= 1 + \frac{1}{2(1/2)} + \frac{1}{3(1/3)} + K + \frac{1}{N(1/N)} = N \end{aligned}$$

## CHAPTER SEVEN SOLUTIONS

$$30. \quad C_{equiv} = \frac{(3p)(0.25p)}{3p + 0.25p} = 0.231pF$$

## CHAPTER SEVEN SOLUTIONS

$$31. \quad L_{equiv} = \frac{(2.5n)(0.5n)}{2.5n} = 0.2916nH$$

## CHAPTER SEVEN SOLUTIONS

32. (a) Use 2 x 1 $\mu$ H in series with 4 x 1 $\mu$ H in parallel.
- (b) Use 2 x 1 $\mu$ H in parallel, in series with 4 x 1 $\mu$ H in parallel.
- (c) Use 5 x 1 $\mu$ H in parallel, in series with 4 x 1 $\mu$ H in parallel.

## CHAPTER SEVEN SOLUTIONS

33.

$$(a) \quad R = 10\Omega : 10\parallel 10\parallel 10 = \frac{10}{3}, \frac{10}{3} + 10 + 10\parallel 10 = \frac{55}{3}$$

$$\therefore R_{eq} = \frac{55}{3}\parallel 30 = 11.379\Omega$$

$$(b) \quad L = 10H \therefore L_{eq} = 11.379H$$

$$(c) \quad C = 10F : \frac{1}{1/30 + 1/10 + 1/20} = 5.4545$$

$$\therefore C_{eq} = 5.4545 + \frac{10}{3} = 8.788F$$

## CHAPTER SEVEN SOLUTIONS

34.

(a)  $oc : L_{eq} = 6 \parallel 1 + 3 = 3.857H$

$sc : L_{eq} = (3 \parallel 2 + 1) \parallel 4 = 2.2 \parallel 4 = 1.4194H$

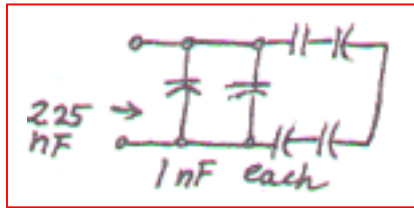
(b)  $oc : 1 + \frac{1}{1/4 + 1/2} = \frac{7}{3}, C_{eq} = \frac{1}{3/7 + 1/2} = 1.3125F$

$sc : \frac{1}{1/5 + 1} = \frac{5}{6}, C_{eq} = 4 + \frac{5}{6} = 4.833F$

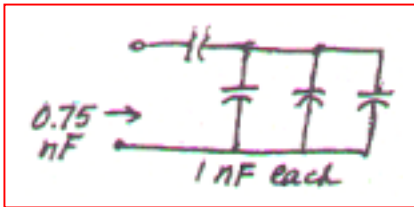
## CHAPTER SEVEN SOLUTIONS

35.

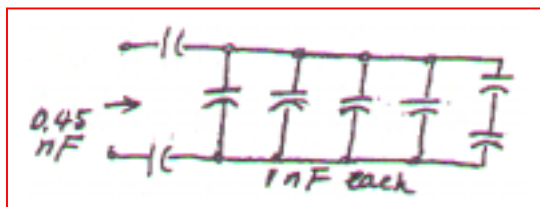
(a)



(b)



(c)





## CHAPTER SEVEN SOLUTIONS

36.  $i_s = 60e^{-200t}$  mA,  $i_1(0) = 20$  mA

(a)  $6 \parallel 4 = 2.4 \text{ H} \therefore v = L_{eq} i_s' = 2.4 \times 0.06(-200)e^{-200t}$   
or  $v = -28.8e^{-200t} \text{ V}$

(b)  $i_1 = \frac{1}{6} \int_0^t -28.8e^{-200t} dt + 0.02 = \frac{4.8}{200}(e^{-200t} - 1) + 0.02$   
 $= 24e^{-200t} - 4 \text{ mA} (t > 0)$

(c)  $i_2 = i_s - i_1 = 60e^{-200t} - 24e^{-200t} + 4 = 36e^{-200t} + 4 \text{ mA} (t > 0)$

## CHAPTER SEVEN SOLUTIONS

$$37. \quad v_s = 100e^{-80t} \text{ V}, v_1(0) = 20 \text{ V}$$

$$(a) \quad i = C_{eq} v_s' = 0.8 \times 10^{-6} (-80) 100 e^{-80t} = \boxed{-6.4 \times 10^{-3} e^{-80t} \text{ A}}$$

$$(b) \quad v_1 = 10^6 (-6.4 \times 10^{-3}) \int_0^t e^{-80t} dt + 20 = \frac{6400}{80} (e^{-80t} - 1) + 20$$
$$\therefore v_1 = \boxed{80e^{-80t} - 60 \text{ V}}$$

$$(c) \quad v_2 \frac{10^6}{4} (-6.4 \times 10^{-3}) \int_0^t e^{-80t} dt + 80 = \frac{1600}{80} (e^{-80t} - 1) + 80$$
$$= \boxed{20e^{-80t} + 60 \text{ V}}$$

## CHAPTER SEVEN SOLUTIONS

38.

(a)

$$\frac{v_c - v_s}{20} + 5 \times 10^{-6} v_c' + \frac{v_c - v_L}{10} = 0$$
$$\frac{v_L - v_c}{10} + \frac{1}{8 \times 10^{-3}} \int_0^t v_L dt + 2 = 0$$

(b)

$$20i_{20} + \frac{1}{5 \times 10^{-6}} \int_0^t (i_{20} - i_L) dt + 12 = v_s$$
$$\frac{1}{5 \times 10^{-6}} \int_0^t (i_L - i_{20}) dt - 12 + 10i_L + 8 \times 10^{-3} i_L' = 0$$

## CHAPTER SEVEN SOLUTIONS

39.

$$v_c(t): 30\text{mA}: 0.03 \times 20 = 0.6\text{V}, v_c = 0.6\text{V}$$

$$9\text{V}: v_c = 9\text{V}, 20\text{mA}: v_c = -0.02 \times 20 = 0.4\text{V}$$

$$0.04 \cos 10^3 t: v_c = 0$$

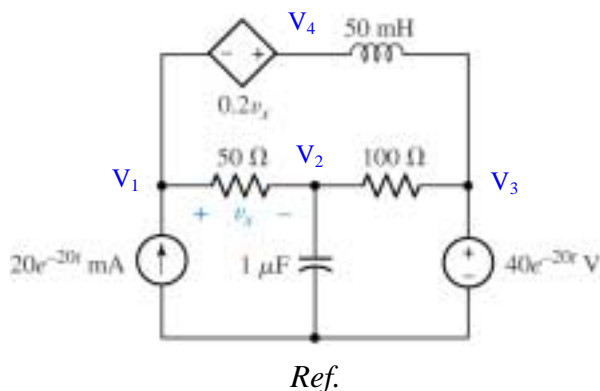
$$\therefore v_c(t) = 9.2\text{V}$$

$$v_L(t): 30\text{mA}, 20\text{mA},$$

$$9\text{V}: v_L = 0; 0.04 \cos 10^3 t: v_L = -0.06 \times 0.04(-1000) \sin 10^3 t = 2.4 \sin 10^3 t \text{V}$$

## CHAPTER SEVEN SOLUTIONS

40. We begin by selecting the bottom node as the reference and assigning four nodal voltages:



1, 4 Supernode: 
$$20 \times 10^{-3} e^{-20t} = \frac{V_1 - V_2}{50} + 0.02 \times 10^3 \int_0^t (V_4 - 40e^{-20t'}) dt' \quad [1]$$

and: 
$$V_1 - V_4 = 0.2 V_x \quad \text{or} \quad 0.8V_1 + 0.2 V_2 - V_4 = 0 \quad [2]$$

Node 2: 
$$0 = \frac{V_2 - V_1}{50} + \frac{V_2 - 40e^{-20t}}{100} + 10^{-6} \frac{dV_2}{dt} \quad [3]$$

## CHAPTER SEVEN SOLUTIONS

41. (a)  $R_i = \infty, R_o = 0, A = \infty \therefore v_i = 0 \therefore i = C v_s'$

also  $0 + Ri + v_o = 0 \therefore v_o = -RC v_s'$

$-v_i + Ri - A v_i = 0, v_s = \frac{1}{C} \int i dt + v_i$

(b)  $v_o = -A v_i \therefore v_i = \frac{-1}{A} v_o \therefore i = \frac{1+A}{R} v_i$

$\therefore v_s = \frac{1}{C} \int i dt - \frac{1}{A} v_o = -\frac{1}{A} v_o + \frac{1+A}{RC} \int -\frac{v_o}{A} dt$

$\therefore A v_s' = -v_o' - \frac{1+A}{RC} v_o$  or  $v_o' + \frac{1+A}{RC} v_o + A v_s' = 0$

## CHAPTER SEVEN SOLUTIONS

42. Place a current source in parallel with a 1-M $\Omega$  resistor on the positive input of a buffer with output voltage,  $v$ . This feeds into an integrator stage with input resistor,  $R_2$ , of 1-M $\Omega$  and feedback capacitor,  $C_f$ , of 1  $\mu$ F.

$$i = C_f \frac{dv_{c_f}}{dt} = 1.602 \times 10^{-19} \times \frac{\text{ions}}{\text{sec}}$$

$$0 = \frac{V_a - V}{1 \times 10^6} + C_f \frac{dv_{c_f}}{dt} = \frac{V_a - V}{1 \times 10^6} + 1.602 \times 10^{-19} \frac{\text{ions}}{\text{sec}}$$

$$0 = \frac{-V}{R_2} + C_f \frac{dv_{c_f}}{dt} = \frac{-V}{1 \times 10^6} + 1.602 \times 10^{-19} \frac{\text{ions}}{\text{sec}}$$

Integrating current with respect to  $t$ ,  $\frac{1}{R_2} \int_0^t v dt' = C_f (V_{c_f} - V_{c_f}(0))$

$$\frac{1.602 \times 10^{-19} \times \text{ions}}{R_2} = C_f V_{c_f}$$

$$V_{c_f} = V_a - V_{out} \Rightarrow V_{out} = \frac{-R_1}{R_2 C_f} \times 1.602 \times 10^{-19} \times \text{ions} \Rightarrow V_{out} = \frac{-1}{C_f} \times 1.602 \times 10^{-19} \times \text{ions}$$

$$R_1 = 1 \text{ M}\Omega, C_f = 1 \mu\text{F}$$

## CHAPTER SEVEN SOLUTIONS

43.  $R = 0.5\text{M}\Omega$ ,  $C = 2\mu\text{F}$ ,  $R_i = \infty$ ,  $R_o = 0$ ,  $v_o = \cos 10t - 1\text{V}$

(a) Eq. (16) is:  $\left(1 + \frac{1}{A}\right)v_o = -\frac{1}{RC} \int_o^t \left(v_s + \frac{v_o}{A}\right) dt - v_c(0)$

$$\therefore \left(1 + \frac{1}{A}\right)v_o' = -\frac{1}{RC} \left(v_s + \frac{v_o}{A}\right) \therefore \left(1 + \frac{1}{A}\right)(-10\sin 10t) = -1 \left(v_s + \frac{1}{A}\cos 10t - \frac{1}{A}\right)$$

$$\therefore v_s = \left(1 + \frac{1}{A}\right)10\sin 10t + \frac{1}{A} - \frac{1}{A}\cos 10t \text{ Let } A = 2000$$

$$\therefore v_s = 10.005\sin 10t + 0.0005 - 0.0005\cos 10t$$

(b) Let  $A = \infty \therefore v_s = 10\sin 10t\text{V}$



## CHAPTER SEVEN SOLUTIONS

44. Create a op-amp based differentiator using an ideal op amp with input capacitor  $C_1$  and feedback resistor  $R_f$  followed by inverter stage with unity gain.

$$V_{out} = + \frac{R}{R} R_f C_1 \frac{dvs}{dt} = 60 \times \frac{1mV}{rpm} / \text{min}$$

$R_f C_1 = 60$  so choose  $R_f = 6 \text{ M}\Omega$  and  $C_1 = 10 \mu\text{F}$ .

## CHAPTER SEVEN SOLUTIONS

45. (a)  $0 = \frac{1}{L} \int v dt + \frac{V_a - V_{out}}{R_f}$

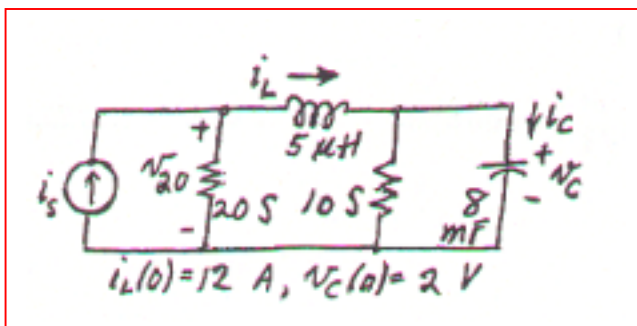
$$V_a = V = 0, \therefore \frac{1}{L} \int v_L dt = \frac{V_{out}}{R_f} \Rightarrow V_{out} = \frac{-R_f}{L} \int_0^t v_s dt'$$

(b) In practice, capacitors are usually used as capacitor values are more readily available than inductor values.

## CHAPTER SEVEN SOLUTIONS

46.

(a)



(b)

$$20v_{20} + \frac{1}{5 \times 10^{-6}} \int_0^t (v_{20} - v_c) dt + 12 = i_s$$

$$\frac{1}{5 \times 10^{-6}} \int_0^t (v_c - v_{20}) dt - 12 + 10v_c + 8 \times 10^{-3} v_c' = 0$$

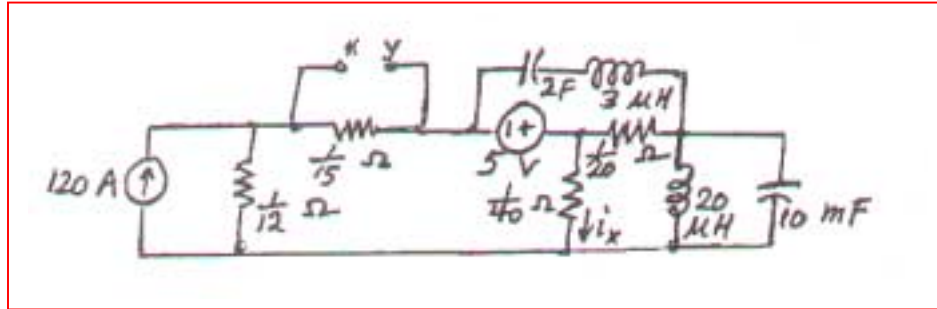
(c)

$$\frac{i_L - i_s}{20} + 5 \times 10^{-6} i_L' + \frac{i_L - i_c}{10} = 0$$

$$\frac{i_c - i_L}{10} + \frac{1}{8 \times 10^{-3}} \int_0^t i_c dt + 2 = 0$$

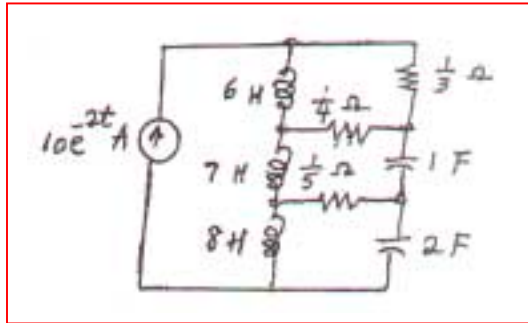
## CHAPTER SEVEN SOLUTIONS

47.



## CHAPTER SEVEN SOLUTIONS

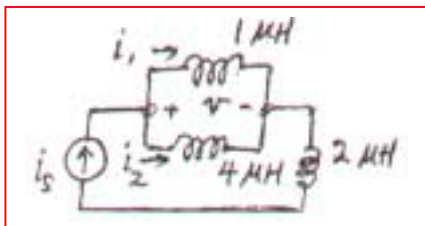
48.



## CHAPTER SEVEN SOLUTIONS

49.

(a)



(b) “Let  $i_s = 100e^{-80t}$  A and  $i_1(0) = 20$  A in the circuit of (new) Fig. 7.62.

(a) Determine  $v(t)$  for all  $t$ .

(b) Find  $i_1(t)$  for  $t \geq 0$ .

(c) Find  $v_2(t)$  for  $t \geq 0$ .”

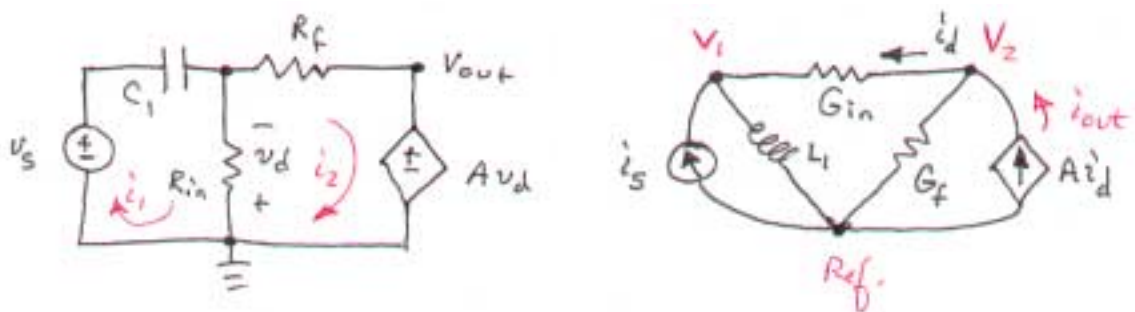
(c) (a)  $L_{eq} = 1 \parallel 4 = 0.8 \mu\text{H} \therefore v(t) = L_{eq} i_s' = 0.8 \times 10^{-6} \times 100(-80)e^{-80t} \text{ V}$   
 $\therefore v(t) = -6.43e^{-80t} \text{ mV}$

(b)  $i_1(t) = 10^6 \int_0^t -6.4 \times 10^{-3} e^{-80t} dt + 20 \therefore i_1(t) = \frac{6400}{80}(e^{-80t} - 1) = 80e^{-80t} - 60 \text{ A}$

(c)  $i_2(t) = i_s - i_1(t) \therefore i_2(t) = 20e^{-80t} + 60 \text{ A}$

## CHAPTER SEVEN SOLUTIONS

50.



In creating the dual of the original circuit, we have lost both  $v_s$  and  $v_{out}$ . However, we may write the dual of the original transfer function:  $i_{out}/i_s$ . Performing nodal analysis,

$$i_s = \frac{1}{L_1} \int_0^t V_1 dt' + G_{in} (V_1 - V_2) \quad [1]$$

$$i_{out} = A i_d = G_f V_2 + G_{in} (V_2 - V_1) \quad [2]$$

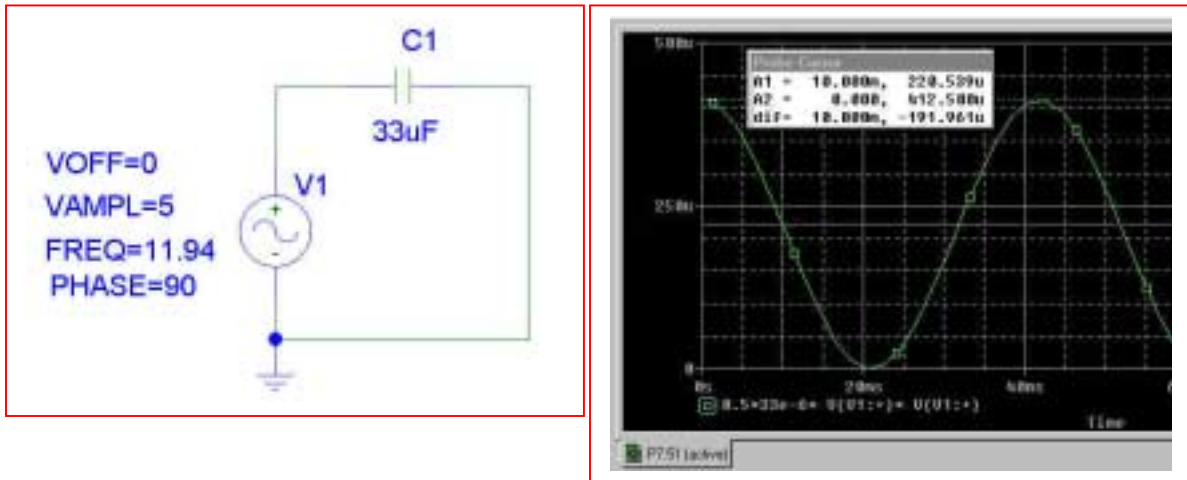
Dividing, we find that

$$\frac{i_{out}}{i_s} = \frac{G_{in} (V_2 - V_1) + G_f V_2}{\frac{1}{L_1} \int_0^t V_1 dt' + G_{in} (V_1 - V_2)}$$

## CHAPTER SEVEN SOLUTIONS

### 51. PSpice verification

$w = \frac{1}{2} C v^2 = 0.5 (33 \times 10^{-6}) [5 \cos (75 \times 10^{-2})]^2 = 220.8 \mu\text{J}$ . This is in agreement with the PSpice simulation results shown below.

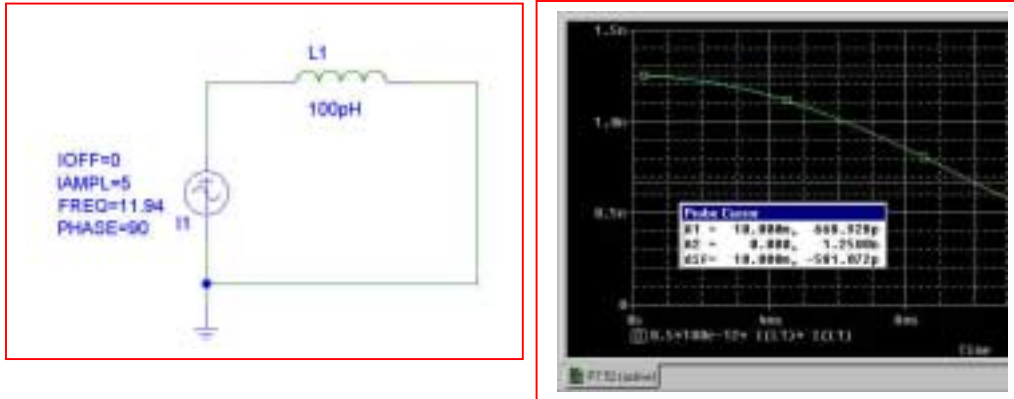




## CHAPTER SEVEN SOLUTIONS

### 52. PSpice verification

$w = \frac{1}{2} Li^2 = 0.5 (100 \times 10^{-12}) [5 \cos(75 \times 10^{-2})]^2 = 669.2 \text{ pJ}$ . This is in agreement with the PSpice simulation results shown below.



## CHAPTER SEVEN SOLUTIONS

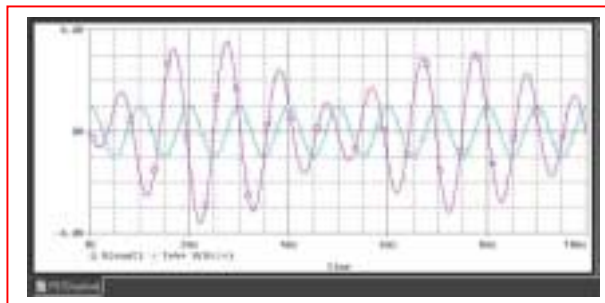
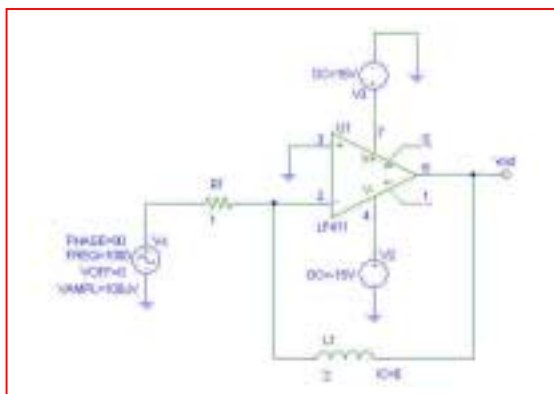
$$53. \quad 0 = \frac{V_a - V_s}{R_1} + \frac{1}{L} \int v_{L_f} dt$$

$$V_a = V_b = 0, \quad 0 = \frac{-V_s}{R_1} + \frac{1}{L} \int v_{L_f} dt$$

$$V_{L_f} = V_a - V_{out} = 0 - V_{out} = \frac{L}{R_1} \frac{dV_s}{dt}$$

$$V_{out} = -\frac{L_f}{R_1} \frac{dV_s}{dt} = -\frac{L_f}{R_1} \frac{d}{dt} (A \cos 2\pi 10^3 t) \Rightarrow L_f = 2R_1; \text{ Let } R = 1 \Omega \text{ and } L = 1 \text{ H.}$$

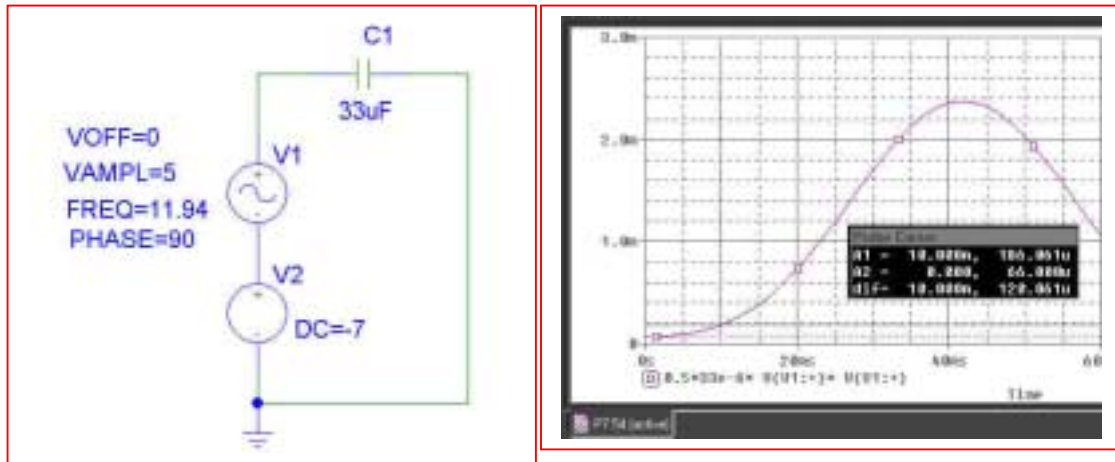
PSpice Verification: clearly, something rather odd is occurring in the simulation of this particular circuit, since the output is not a pure sinusoid, but a combination of several sinusoids.



## CHAPTER SEVEN SOLUTIONS

### 54. PSpice verification

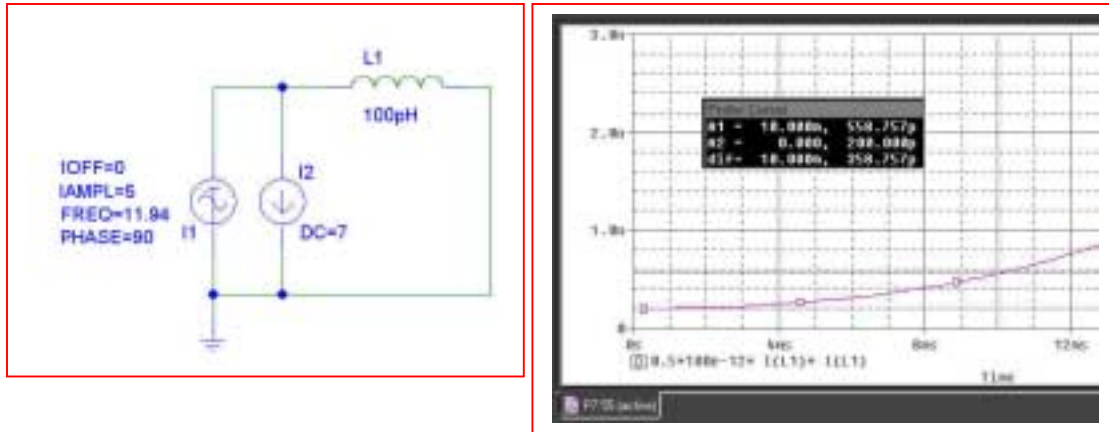
$w = \frac{1}{2} C v^2 = 0.5 (33 \times 10^{-6}) [5 \cos (75 \times 10^{-2}) - 7]^2 = 184.2 \mu\text{J}$ . This is in reasonable agreement with the PSpice simulation results shown below.



## CHAPTER SEVEN SOLUTIONS

### 55. PSpice verification

$w = \frac{1}{2} Li^2 = 0.5 (100 \times 10^{-12}) [5 \cos(75 \times 10^{-2}) - 7]^2 = 558.3 \text{ pJ}$ . This is in agreement with the PSpice simulation results shown below.



## CHAPTER EIGHT SOLUTIONS

1.

$$\begin{aligned} i_L(0) &= \frac{100}{50} = 2\text{A} \therefore i_L(t) = 2e^{-80t/0.2} \\ \text{(a)} \quad &= 2e^{-400t} \text{A}, t > 0 \end{aligned}$$

$$\text{(b)} \quad i_L(0.01) = 2e^{-4} = 36.63\text{mA}$$

$$\text{(c)} \quad 2e^{-400t_1} = 1, e^{400t_1} = 2, t_1 = 1.7329\text{ms}$$

## CHAPTER EIGHT SOLUTIONS

2.

$$(a) \quad i_L(0^-) = \frac{1}{2} \times 60 = 30\text{mA}, \quad i_x(0^-) = \frac{2}{3} \times 30 = 20\text{mA}$$

$$(b) \quad i_L(0^+) = 30\text{mA}, \quad i_x(0^+) = -30\text{mA}$$

$$(c) \quad i_L(t) = 30e^{-250t/0.05} = 30e^{-5000t} \text{mA}, \quad i_L(0.3\text{ms}) \\ = 30e^{-1.5} = 6.694\text{mA} = -i_x$$

## CHAPTER EIGHT SOLUTIONS

3.

$$(a) \quad i_L(0) = 4.5 \text{ mA}, R/L = \frac{10^3}{4 \times 10^{-3}} = \frac{10^6}{4}$$
$$\therefore i_L = 4.5e^{-10^6 t/4} \text{ mA} \therefore i_L(5 \mu s) = 4.5e^{-1.25}$$

$$= 1.289 \text{ mA.}$$

$$(b) \quad i_{\text{sw}}(5 \mu s) = 9 - 1.289 = 7.711 \text{ mA.}$$

## CHAPTER EIGHT SOLUTIONS

4.

- (a) Since the inductor current can't change instantaneously, we simply need to find  $i_L$  while the switch is closed. The inductor is shorting out both of the resistors, so  $i_L(0^+) = 2 \text{ A}$ .
- (b) The instant after the switch is thrown, we know that 2 A flows through the inductor. By KCL, the simple circuit must have 2 A flowing through the  $20\text{-}\Omega$  resistor as well. Thus,

$$v = 4(20) = 80 \text{ V}.$$

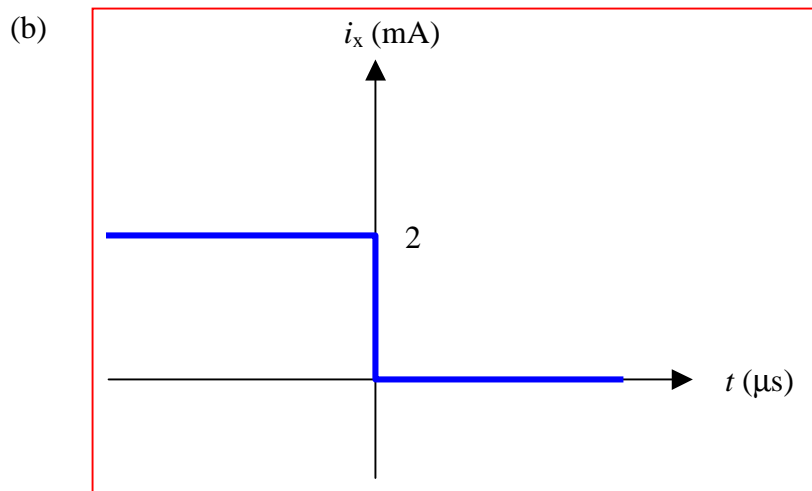
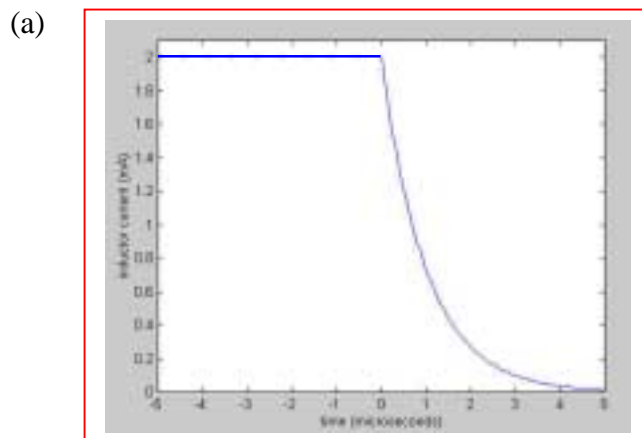


## CHAPTER EIGHT SOLUTIONS

5. (a) Prior to the switch being thrown, the  $12\text{-}\Omega$  resistor is isolated and we have a simple two-resistor current divider (the inductor is acting like a short circuit in the DC circuit, since it has been connected in this fashion long enough for any transients to have decayed). Thus, the current  $i_L$  through the inductor is simply  $5(8)/(8 + 2) = 4\text{ A}$ . The voltage  $v$  must be  $0\text{ V}$ .
- (b) The instant just after the switch is thrown, the inductor current must remain the same, so  $i_L = 4\text{ A}$ . KCL requires that the same current must now be flowing through the  $12\text{-}\Omega$  resistor, so  $v = 12(-4) = -48\text{ V}$ .

## CHAPTER EIGHT SOLUTIONS

6. For  $t < 0$ , we have a current divider with  $i_L(0^-) = i_x(0^-) = 0.5 [10 (1 / (1 + 1.5))] \text{ mA} = 2 \text{ mA}$ . For  $t > 0$ , the resistor through which  $i_x$  flows is shorted, so that  $i_x(t > 0) = 0$ . The remaining  $1\text{-k}\Omega$  resistor and  $1\text{-mH}$  inductor network exhibits a decaying current such that  $i_L(t) = 2e^{-t/\tau} \text{ mA}$  where  $\tau = L/R = 1 \text{ }\mu\text{s}$ .



## CHAPTER EIGHT SOLUTIONS

7.

$$(a) \quad \frac{i}{I_o} = e^{-t/\tau}, \frac{t}{\tau} = \ln \frac{I_o}{i}, \frac{I_o}{i} = 10 \therefore \frac{t}{\tau} = \ln 10 = 2.303;$$

$$\frac{I_o}{i} = 100, \frac{t}{\tau} = 4.605; \frac{I_o}{i} = 1000, \frac{t}{\tau} = \boxed{6.908}$$

$$(b) \quad \frac{i}{I_o} = e^{-t/\tau}, \frac{d(i/I_o)}{d(t/\tau)} = -e^{t/\tau}; \text{ at } t/\tau = 1, \frac{d(i)}{d(t)} = -e^{-1}$$

$$\text{Now, } y = m(x-1) + b = -e^{-1}(x-1) + e^{-1} \left( \frac{t}{\tau} = x, \frac{i}{I_o} = y \right)$$

$$\text{At } y = 0, e^{-1}(x-1) = e^{-1} \therefore x = 2 \therefore t/\tau = \boxed{2}$$

## CHAPTER EIGHT SOLUTIONS

8. Reading from the graph current is at 0.37 at 2 ms

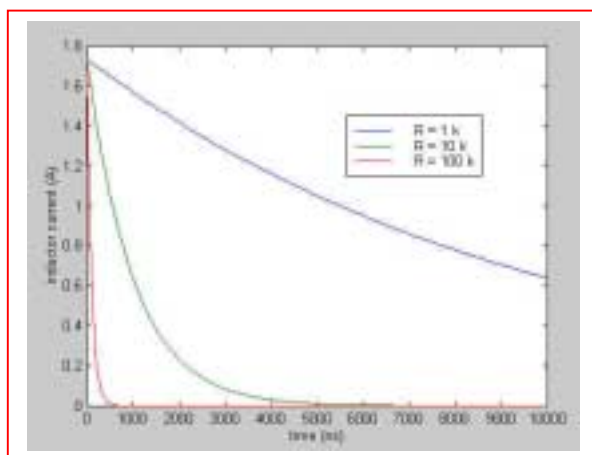
$$\therefore \tau = 2 \text{ ms}$$

$$I_0 = 10 \text{ A}$$

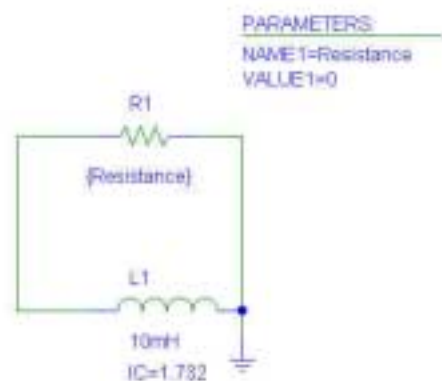
## CHAPTER EIGHT SOLUTIONS

9.  $w = \frac{1}{2} Li^2$ , so an initial energy of 15 mJ in a 10-mH inductor corresponds to an initial inductor current of 1.732 A. For  $R = 1 \text{ k}\Omega$ ,  $\tau = L/R = 10 \mu\text{s}$ , so  $i_L(t) = 1.732 e^{-0.1t} \text{ A}$ . For  $R = 10 \text{ k}\Omega$ ,  $\tau = 1 \mu\text{s}$  so  $i_L(t) = 1.732 e^{-t}$ . For  $R = 100 \text{ k}\Omega$ ,  $\tau = 100 \text{ ns}$  or  $0.1 \mu\text{s}$  so  $i_L(t) = 1.732 e^{-10t}$ . For each current expression above, it is assumed that time is expressed in microseconds.

To create a sketch, we first realise that the maximum current for any of the three cases will be 1.732 A, and after one time constant (10, 1, or  $0.1 \mu\text{s}$ ), the current will drop to 36.79% of this value (637.2 mA); after approximately 5 time constants, the current will be close to zero.

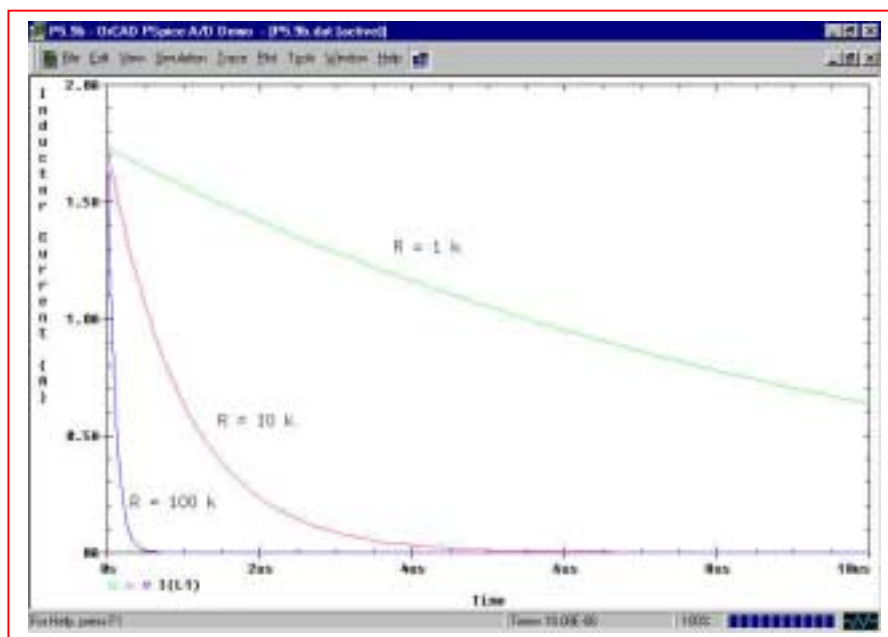


Sketch based on hand analysis



Circuit used for PSpice verification

As can be seen by comparing the two plots, which probably should have the same x-axis scale labels for easier comparison, the PSpice simulation results obtained using a parametric sweep do in fact agree with our hand calculations.



## CHAPTER EIGHT SOLUTIONS

10.

$$(a) \quad \tau = \frac{3.3 \times 10^{-6}}{1 \times 10^6} = 3.3 \times 10^{-12}$$

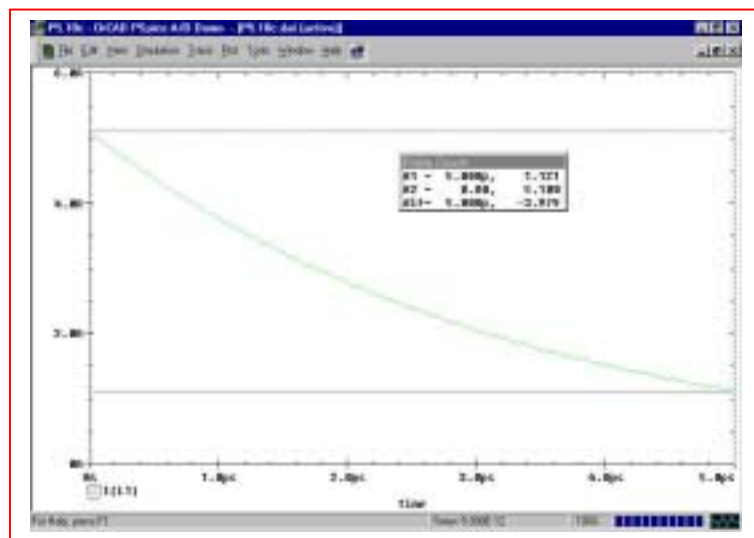
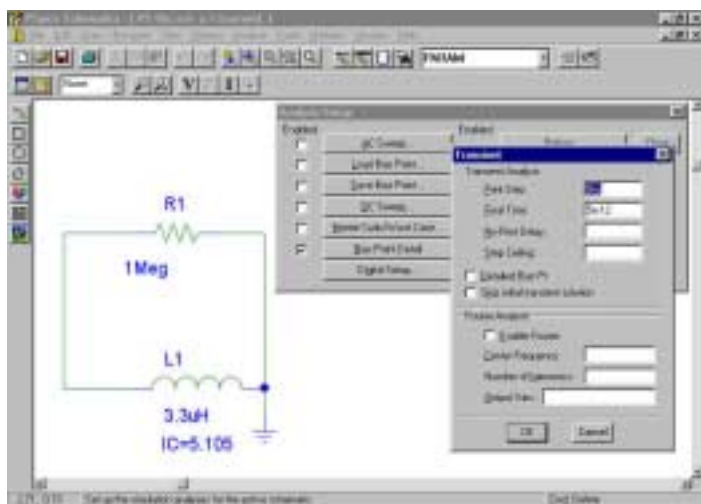
(b)

$$\omega = \frac{1}{2} L I_0^2$$

$$I_0 = \sqrt{\frac{2 \times 43 \times 10^{-6}}{3.3 \times 10^{-6}}} = 5.1 \text{ A}$$

$$i(5 \text{ ps}) = 5.1 e^{-1 \times 10^6 \times 5 \times 10^{-12} / 3.3 \times 10^{-6}} = 1.12 \text{ A}$$

(c)



From the PSpice simulation, we see that the inductor current is 1.121 A at  $t = 5 \text{ ps}$ , in agreement with the hand calculation.

## CHAPTER EIGHT SOLUTIONS

11. Assume the source Thévenin resistance is zero, and assume the transient is measured to  $5\tau$ . Then,

$$\tau = \frac{L}{R} \quad \therefore 5\tau = \frac{5L}{R} = 100 \times 10^{-9} \text{ secs}$$

$$\therefore R > \frac{(5)(125.7)10^{-6}}{10^{-7}} \quad \text{so R must be greater than } 6.285 \text{ k}\Omega.$$

$$(\text{If } 1\tau \text{ assumed then } R > \frac{6.285}{5} = 125.7\Omega)$$

## CHAPTER EIGHT SOLUTIONS

The film acts as an intensity integrator. Assuming that we may model the intensity as a simple decaying exponential,

$$\phi(t) = \phi_o e^{-t/\tau}$$

where the time constant  $\tau$  represents the effect of the Thévenin equivalent resistance of the equipment as it drains the energy stored in the two capacitors, then the intensity of the image on the film  $\Phi$  is actually proportional to the integrated exposure:

$$\Phi = K \int_0^{\text{exposure time}} \phi_o e^{-t/\tau} dt$$

where  $K$  is some constant. Solving the integral, we find that

$$\Phi = -K \phi_o \tau \left[ e^{-(\text{exposure time})/\tau} - 1 \right]$$

The maximum value of this intensity function is  $-K\phi_o\tau$ .

With 150 ms yielding an image intensity of approximately 14% of the maximum observed and the knowledge that at 2 s no further increase is seen leads us to estimate that  $1 - e^{-150 \times 10^{-3}/\tau} = 0.14$ , assuming that we are observing single-exponential decay behavior and that the response speed of the film is not affecting the measurement. Thus, we may extract an estimate of the circuit time constant as  $\tau = 994.5$  ms.

This estimate is consistent with the additional observation that at  $t = 2$  s, the image appears to be saturated.

With two 50-mF capacitors connected in parallel for a total capacitance of 100 mF, we may estimate the Thévenin equivalent resistance from  $\tau = RC$  as  $R_{th} = \tau / C$

$$= 9.945 \, \Omega.$$



## CHAPTER EIGHT SOLUTIONS

13.

$$\begin{aligned} \text{(a)} \quad v_c(0) &= 8(50 \parallel 200) \times \frac{30}{50} = 192 \text{ V} \\ v_c(t) &= 192e^{-3000t/24} = 192e^{-125t} \text{ V} \end{aligned}$$

$$\text{(b)} \quad 0.1 = e^{-125t} \therefore t = 18.421 \text{ ms}$$

## CHAPTER EIGHT SOLUTIONS

14.

$$(a) \quad v_c = 80e^{-10^6 t/100} = 80e^{-10^4 t} \text{ V}, t > 0; 0.5 = e^{-10^4 t} \therefore t = 69.31 \mu s$$

$$(b) \quad w_c = \frac{1}{2} C 80^2 e^{-20,000 t} = \frac{1}{4} C 80^2 \therefore t = 34.66 \mu s$$

## CHAPTER EIGHT SOLUTIONS

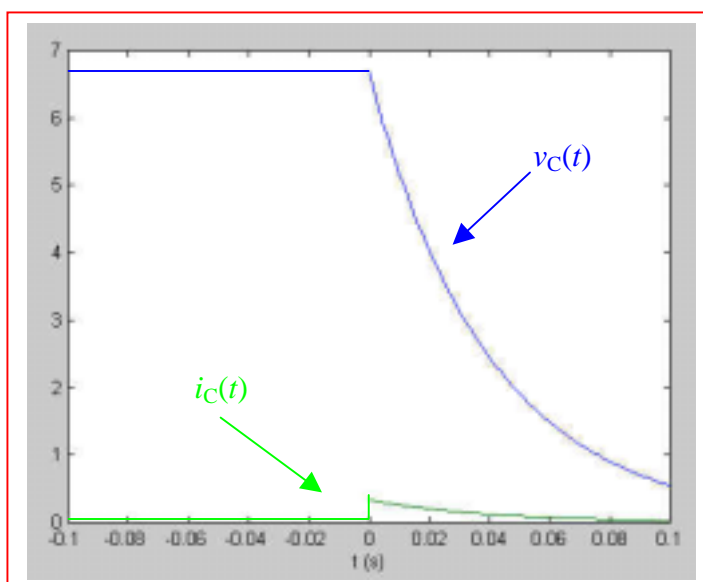
15.

$$t < 0: i_c(t) = 0, 10 = 5000i_s + 10^4 i_s \therefore i_s = \frac{2}{3} \text{ mA}$$

$$\therefore v_c(t) = \frac{20}{3} = 6.667 \text{ V}$$

$$t > 0: i_s = 0 \therefore v_c(t) = 6.667 e^{-t / 2 \times 10^{-4} \times 2 \times 10^{-6}}$$

$$\therefore v_c(t) = 6.667 e^{-25t} \text{ V} \therefore i_c(t) = \frac{-6.667}{20 \times 10^3} e^{-25t} = 0.3333 e^{-25t} \text{ mA}$$



## CHAPTER EIGHT SOLUTIONS

16.

$$v(0^+) = 20V$$

$$i(0^+) = 0.1A$$

$$v(1.5ms) = 20e^{-1.5 \times 10^{-3} / 50 \times 20 \times 10^{-6}} = 4.5V$$

$$i(1.5ms) = 0A$$

$$v(3ms) = 20e^{-3 \times 10^{-3} / 50 \times 20 \times 10^{-6}} = 1V$$

$$i(3ms) = 0A$$

## CHAPTER EIGHT SOLUTIONS

17.

(a)  $i_L(0) = 4\text{ A} \therefore i_L(t) = 4e^{-500t} \text{ A} \quad (0 \leq t \leq 1\text{ ms})$

$$i_L(0.8\text{ ms}) = 4e^{-0.4} = \boxed{2.681\text{ A}}$$

(b)  $i_L(1\text{ ms}) = 4e^{-0.5} = 2.426\text{ A}$

$$\therefore i_L(t) = 2.426e^{-250(t-0.001)}$$

$$\therefore i_L(2\text{ ms}) = 2.426e^{-0.25} = \boxed{1.8895\text{ A}}$$

## CHAPTER EIGHT SOLUTIONS

18.

$$(a) \quad i_L = 40e^{-50,000t} \text{ mA} \therefore 10 = 40e^{-50,000t}, \therefore t_1 = \boxed{27.73 \mu s}$$

$$\begin{aligned} (b) \quad i_L(10 \mu s) &= 40e^{-0.5} = 24.26 \text{ mA} \therefore i_L \\ &= 24.26e^{-(1000+R)50t} (t > 10 \mu s) \\ \therefore 10 &= 24.26e^{-(1000+R)5 \times 10^{-6}} \therefore \ln 2.426 = 0.8863 \\ &= 0.25(1000+R)10^{-3}, 1000+R = 0.8863 \times 4 \times 10^3 \therefore R = \boxed{2545^+ \Omega} \end{aligned}$$

## CHAPTER EIGHT SOLUTIONS

19.

(a)  $i_1(0) = 20\text{mA}$ ,  $i_2(0) = 15\text{mA}$

$$\therefore v(t) = 40e^{-50000t} + 45e^{-100000t} \text{ V} \therefore v(0) = 85\text{V}$$

(b)  $v(15\mu s) = 40e^{-0.75} + 45e^{-1.5} = 28.94\text{V}$

(c)  $\frac{85}{10} = 40e^{-50000t} + 45e^{-100000t}$ . Let  $e^{-50000t} = x$

$$\therefore 45x^2 + 40x - 8.5 = 0$$

$$\therefore x = \frac{-40 \pm \sqrt{1600 + 1530}}{90} = 0.17718, < 0$$

$$\therefore e^{-50000t} = 0.17718, t = 34.61\mu s$$

## CHAPTER EIGHT SOLUTIONS

20.

$$t < 0: v_R = \frac{2R_1 R_2}{R_1 + R_2}, \downarrow i_L(0) = \frac{2R_1}{R_1 + R_2}$$

$$t > 0: i_L(t) = \frac{2R_1}{R_1 + R_2} e^{-50R_2 t} \therefore v_R = \frac{2R_1 R_2}{R_1 + R_2} e^{-50R_2 t}$$

$$\therefore v_R(0^+) = 10 = \frac{2R_1 R_2}{R_1 + R_2} \therefore R_1 \parallel R_2 = 5\Omega. \text{ Also, } v_R(1\text{ms})$$

$$= 5 = 10e^{-50R_2/1000} \therefore 0.05R_2 = 0.6931 \therefore R_2 = 13.863\Omega$$

$$\therefore \frac{1}{13.863} + \frac{1}{R_1} = \frac{1}{5} \therefore R_1 = 7.821\Omega$$

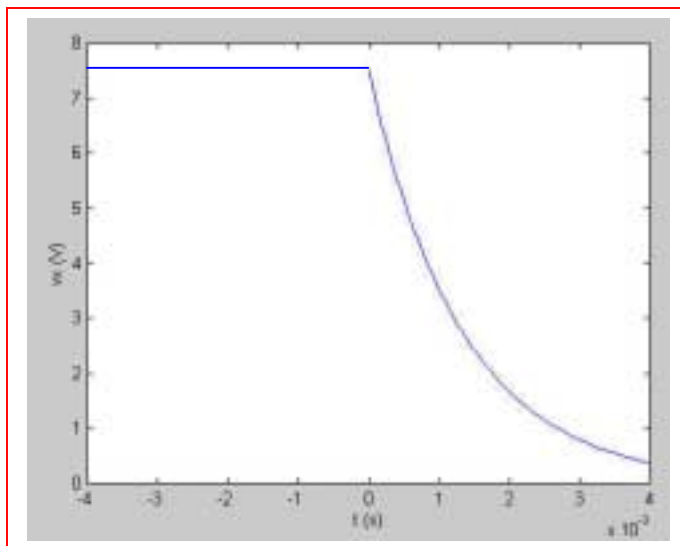


## CHAPTER EIGHT SOLUTIONS

21.

(a)  $i_L(0) = \frac{24}{60} = 0.4\text{A} \therefore i_L(t) = 0.4e^{-750t}\text{A}, t > 0$

(b)  $v_x = \frac{5}{6} \times 24 = 20\text{V}, t < 0$   
 $v_x(0^+) = 50 \times 0.4 \times \frac{3}{8} = 7.5\text{V}$   
 $\therefore v_x(t) = 7.5e^{-750t}\text{V}, t > 0$



## CHAPTER EIGHT SOLUTIONS

22.

$$v_{in} = \frac{3i_L}{4} \times 20 + 10i_L = 25i_L$$

$$v_{in} \therefore \frac{v_{in}}{i_L} = 25\Omega \therefore i_L = 10e^{-25t/0.5} = 10e^{-50t} \text{ A}, t > 0$$

## CHAPTER EIGHT SOLUTIONS

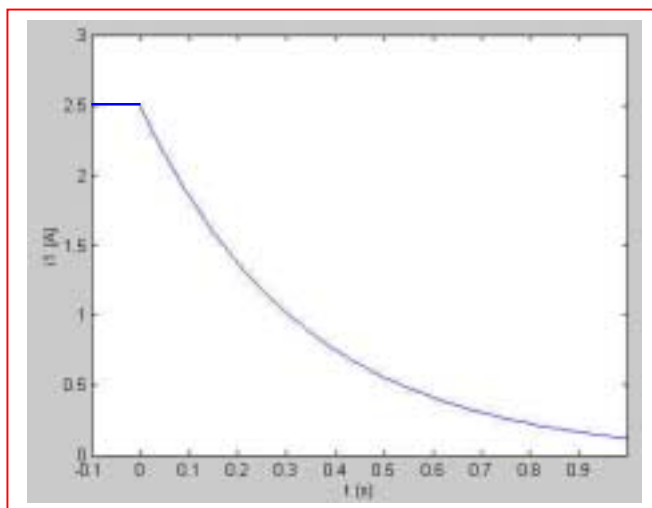
23.

$$i_L(0) = \frac{64}{4 + 40 \parallel 8} \times \frac{40}{48} = 5 \text{ A}$$

$$\therefore i_L = 5e^{-24t/8} = 5e^{-3t} \text{ A}$$

$$\therefore i_1(t) = 2.5e^{-3t} \text{ A}, t > 0; \quad i_1(-0.1) = 2.5 \text{ A}$$

$$i_1(0.03) = 2.285 \text{ A}, i_1(0.1) = 1.852 \text{ A}$$



## CHAPTER EIGHT SOLUTIONS

24.

(a)  $i_L(0) = 4\text{ A} \therefore i_L = 4e^{-100t}\text{ A}, 0 < t < 15\text{ ms}$   
 $\therefore i_L(15\text{ ms}) = 4e^{-1.5} = 0.8925^+\text{ A}$

(b)  $t > 15\text{ ms}: i_L = 0.8925^+ e^{-20(t-0.015)}\text{ A}$   
 $\therefore i_L(30\text{ ms}) = 0.8925^+ e^{-0.3} = 0.6612\text{ A}$

## CHAPTER EIGHT SOLUTIONS

25.

$$(a) \quad i_1(0^+) = i_1(0^-) = 10\text{A}, i_2(0^+) = i_2(0^-) = 20\text{A} \therefore i(0^+) = \boxed{30\text{A}}$$

$$(b) \quad \tau = L_{eq} / R_{eq} = \frac{0.08}{48} = \frac{5}{3} \text{ms} = \boxed{1.6667\text{ms}}$$

$$(c) \quad i_1(0^-) = 10\text{A}, i_2(0^-) = 20\text{A}; \quad \boxed{i(t) = 30e^{-600t} \text{A}}$$

$$(d) \quad v = -48i = \boxed{-1440e^{-600t} \text{V}}$$

$$(e) \quad i_1 = 10(-440) \int_0^t e^{-600t} dt + 10 = 24e^{-600t} \Big|_0^t + 10 = 24e^{-600t} - 14\text{A}$$

$$i_2 = 2.5(-1440) \int_0^t e^{-600t} dt + 20$$

$$= 6e^{-600t} \Big|_0^t + 20 = \boxed{6e^{-600t} + 14\text{A}}$$

$$(f) \quad W_L(0) = \frac{1}{2} \times 0.1 \times 10^2 + \frac{1}{2} \times 0.4 \times 20^2 = 5 + 80 = 85\text{J}$$

$$W_L(\infty) = \frac{1}{2} \times 0.1 \times 14^2 + \frac{1}{2} \times 0.4 \times 14^2 = 9.8 + 39.2 = 49\text{J}$$

$$W_R = \int_0^\infty i^2 48 dt = \int_0^\infty 900 \times 48 e^{-1200t} dt = \frac{900 \times 48}{-1200} (-1) = 36\text{J}$$

$$\therefore 49 + 36 = 85 \text{ checks}$$

## CHAPTER EIGHT SOLUTIONS

26.

(a)  $v_c(0) = 100 \times \frac{2}{2+2} \times \frac{2}{3} = 33.33\text{V}; i_1(0^-) = \frac{100}{2+2} \times \frac{2}{3} = 16.667\text{mA}$

$\therefore v_c(9:59) = 33.33\text{V}, i_1(9:59) = 16.667\text{mA}$

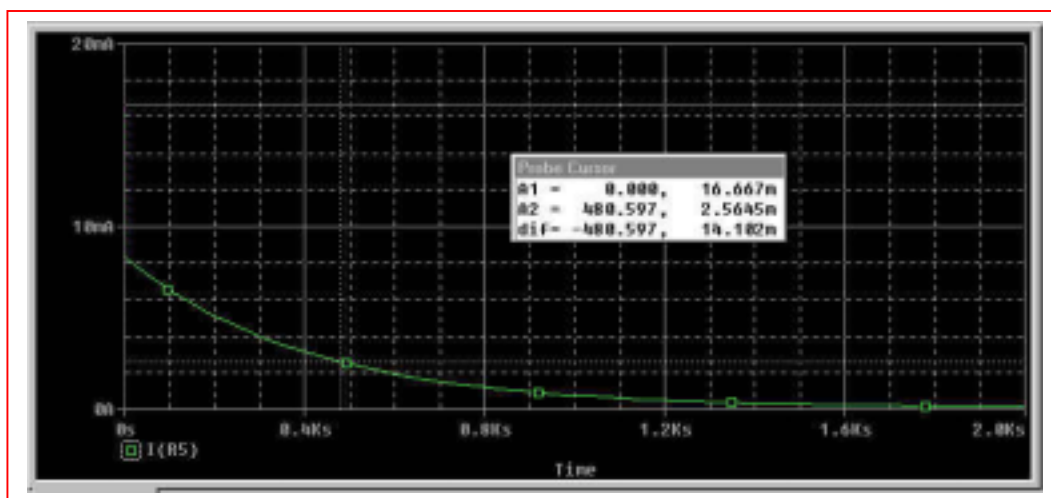
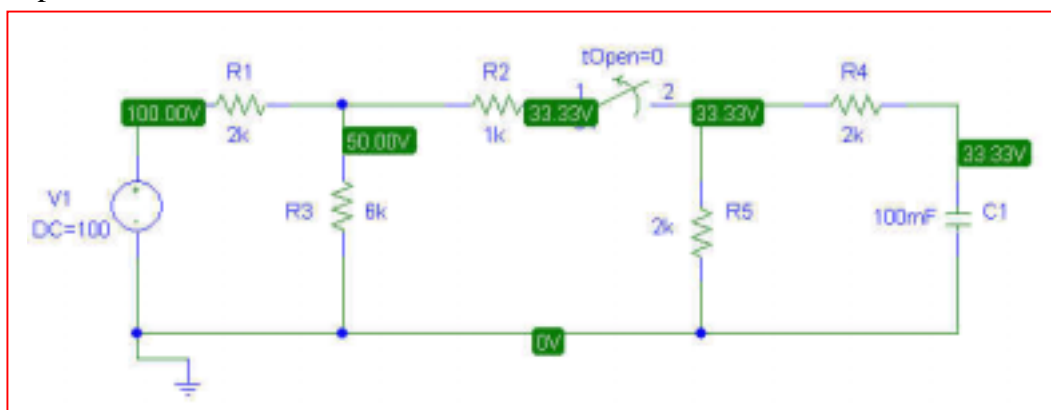
(b)  $v_c(t) = 33.33e^{-t/400}, t > 10:00 \therefore v_c(10:05) = 33.33e^{-300/400}$

$= 15.745^+\text{V}, i_1(10:05) = \frac{15.745}{4000} = 3.936\text{mA}$

(c)  $\tau = 400\text{ s}$ , so  $1.2\tau = 480\text{ s}$ .  $v_c(1.2\tau) = 33.33 e^{-1.2} = 10.04\text{ V}$ .

Using Ohm's law, we find that  $i_1(1.2\tau) = v_c(1.2\tau)/4000 = 2.51\text{ mA}$ .

(d) PSpice Verification:



We see from the DC analysis of the circuit that our initial value is correct; the Probe output confirms our hand calculations, especially for part (c).

## CHAPTER EIGHT SOLUTIONS

27.

$$t > 0: \frac{25i_x}{20} = 1.25i_x \therefore 34 = 100(1.25i_x - 0.8i_x + i_x) + 25i_x \therefore i_x = 0.2\text{A}$$

$$(a) \quad i_s(0^-) = (1.25 - 0.8 + 1)0.2 = \boxed{0.290\text{A}}$$

$$(b) \quad i_x(0^-) = \boxed{0.2\text{A}}$$

$$(c) \quad v_c(t) = 25 \times 0.2e^{-t} = 5e^{-t}\text{V} \therefore i_x(0^+) = \frac{5}{100} = \boxed{0.05\text{A}}$$

$$(d) \quad 0.8i_x(0^+) = 0.04\text{A} \therefore i_s(0^+) = \frac{34}{120} - 0.04 \times \frac{20}{120} = \frac{33.2}{120} = \boxed{0.2767\text{A}}$$

$$(e) \quad i_x(0.4) = \frac{1}{100} \times 5e^{-0.4} = \boxed{0.03352\text{A}}$$

## CHAPTER EIGHT SOLUTIONS

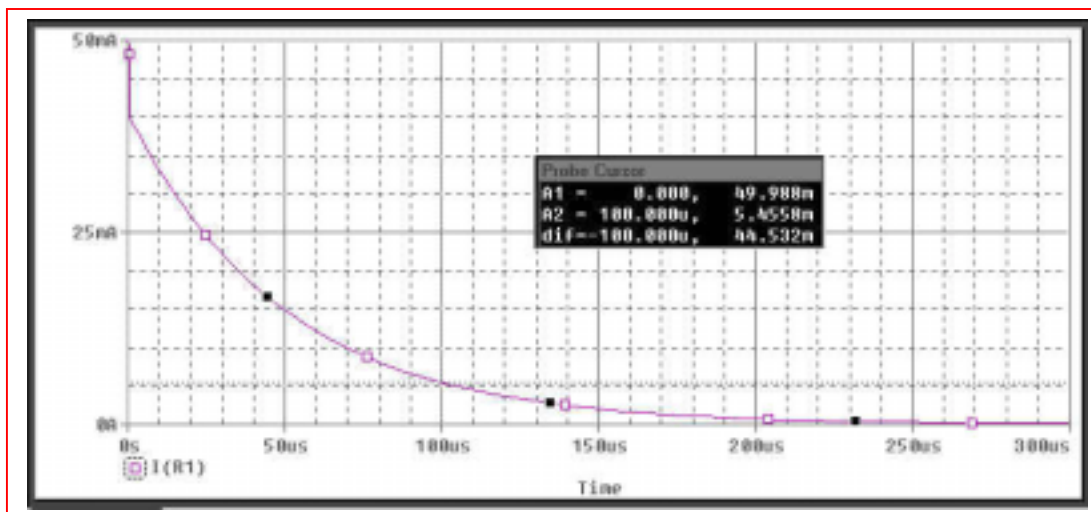
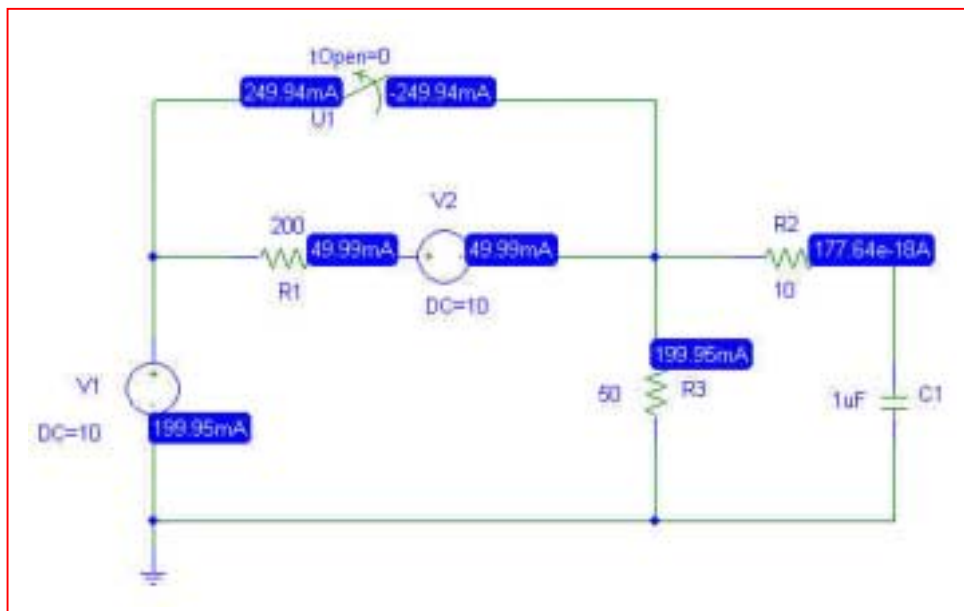
28.

(a)  $v_c(0) = 10\text{V} \therefore v_c(t) = 10e^{-10^6 t / (10+50 \parallel 200)} = 10e^{-20000t}\text{V}$

(b)  $i_A(-100\mu\text{s}) = i_A(0^-) = \frac{10}{200} = 50\text{mA}$

$i_A(100\mu\text{s}) = 10e^{-2} \left( \frac{1}{10+40} \right) \frac{50}{250} = 5.413\text{mA}$

(c) PSpice Verification.



From the DC simulation, we see that PSpice verifies our hand calculation of  $i_A = 50\text{mA}$ . The transient response plotted using Probe indicates that at  $100\mu\text{s}$ , the current is approximately  $5.46\text{mA}$ , which is within acceptable round-off error compared to the hand calculated value.



## CHAPTER EIGHT SOLUTIONS

29.

$$(a) \quad i_1(t) = 8(-1) \frac{12}{12+4} = -6\text{mA} \quad (t < 0)$$

$$(b) \quad 4 \parallel 12 \parallel 6 = 2k\Omega, \quad v_c(0) = 48\text{V}$$

$$\therefore v_c(t) = 48e^{-10^6 t / 5 \times 2 \times 10^3} = 48e^{-100t} \text{V}, \quad t > 0$$

$$\therefore i(t) = 12e^{-100t} \text{mA}, \quad t > 0$$

## CHAPTER EIGHT SOLUTIONS

30.

(a)  $v_{CLeft}(0) = 20\text{V}$ ,  $v_{CRIGHT}(0) = 80\text{V}$

$$\therefore v_{CL} = 20e^{-10^6 t/8}, v_{CR} = 80e^{-10^6 t/0.8}$$

$$\therefore v_{out} = v_{CR} - v_{CL} = 80e^{-1,250,000t} - 20e^{-125,000t} \text{ V}, t > 0$$

(b)  $v_{out}(0^+) = 60\text{V}$ ;  $v_{out}(1\mu s) = 80e^{-1.25} - 20e^{-0.125} = 5.270\text{V}$

$$v_{out}(5\mu s) = 80e^{-6.25} - 20e^{-0.625} = -10.551\text{V}$$

## CHAPTER EIGHT SOLUTIONS

31. (a)  $t < 0$ :  $\frac{v_c - 0.25v_c}{5} + \frac{v_c}{10} + \frac{v_c - 40}{4} = 0 \therefore v_c = 20\text{V} (t < 0)$

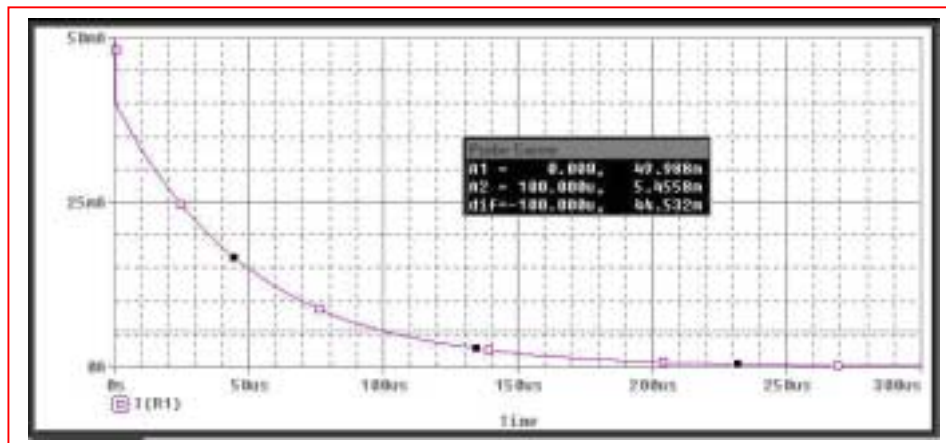
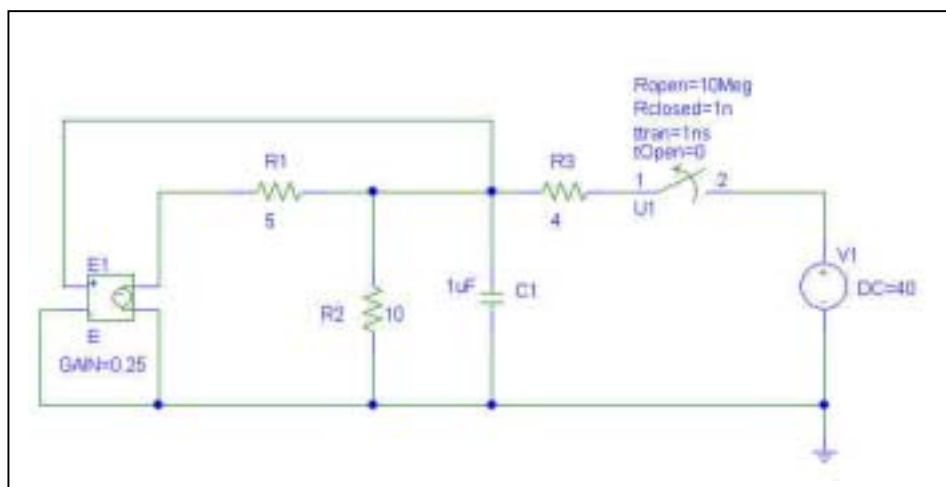
$t > 0$ : Apply  $v_c = 1\text{V} \therefore \frac{1 - 0.25}{5} + 0.1 - i_{in} = 0.25\text{A}$

$\therefore R_{eq} = \frac{1}{0.25} = 4\Omega$

$\therefore v_c(t) = 20e^{-10^6 t/4} = 20e^{-250,000t} \text{V} (t > 0)$

(b)  $v_c(3 \mu\text{s}) = 9.447 \text{ V}$

- (c) PSpice verification. Note that the switch parameters had to be changed in order to perform this simulation.



As can be seen from the simulation results, our hand calculations are accurate.

## CHAPTER EIGHT SOLUTIONS

32.  $t < 0: v_c(0) = 60\text{V}$

$$0 < t < 1\text{ms}: v_c = 60e^{-10^6 t / (R_o + 1000)} \therefore \frac{50}{60} e^{-500 / (R_o + 1000)}$$

$$\therefore \frac{500}{R_o + 1000} = \ln 1.2 = 0.18232 \therefore \frac{R_o}{500} + 2 = 5.4848, R_o = \boxed{1742.4\Omega}$$

$$\therefore v_c(1\text{ms}) = 60e^{-1000 / 2742.4} = 41.67\text{V}$$

$$t > 1\text{ms}: v_c = 41.67e^{-10^6 (t - 10^{-3}) / (1742.4 + R_1 \parallel 1000)}$$

$$\therefore 25 = 41.67e^{-1000(\cdot)} \therefore 0.5108 = \frac{.1000}{1742.4 + R_1 \parallel 1000}, 1742.4 + R_1 \parallel 1000$$

$$= 1957.6, R_1 \parallel 1000 = 215.2 \frac{1}{R_1} + 10^{-3} = \frac{1}{215.2} \therefore R_1 = \boxed{274.2\Omega}$$

## CHAPTER EIGHT SOLUTIONS

33.

- (a) With the switch closed, define a nodal voltage  $V_1$  at the top of the 5-k $\Omega$  resistor. Then,

$$0 = (V_1 - 100)/2 + (V_1 - V_C)/3 + V_1/5 \quad [1]$$

$$0 = V_C/10 + (V_C - V_1)/3 + (V_C - 100) \quad [2]$$

Solving, we find that  $V_C = v_C(0^-) = 99.76 \text{ V}$ .

- (b)  $t > 0$ :  $R_{eq} = 10 \parallel 6.5 = 3.939 \text{ k}\Omega \therefore v_c = 87.59 e^{-10^7 t / 3939} = 87.59 e^{-2539 t} \text{ V } (t > 0)$

## CHAPTER EIGHT SOLUTIONS

34.  $t < 0$ :

$$12 = 4i_1 + 20i_1 \therefore i_1 = 0.5\text{mA} \therefore v_c(0) = 6i_1 + 20i_1 = 26i_1$$

$$v_c(0) = 13\text{V}$$

$$t > 0: \text{ Apply } \leftarrow 1\text{mA} \therefore 1 + 0.6i_1 = i_1 \therefore i_1 = 2.5\text{mA}; \pm v_{in} = 30i_1 = 75\text{V} \therefore R_{eq} = 75k\Omega$$

$$\therefore v_c(t) = 13e^{-t/75 \times 10^3 \times 2 \times 10^{-9}} = 13e^{-10^6 t/150} = 13e^{-6667t}$$

$$\therefore i_1(t) = \frac{v_o}{3 \times 10^4} = \boxed{0.4333e^{-6667t} \text{mA } (t > 0)}$$

## CHAPTER EIGHT SOLUTIONS

35.

$$(a) \quad v_1(0^-) = 100\text{V}, \quad v_2(0^-) = 0, \quad v_R(0^-) = 0$$

$$(b) \quad v_1(0^-) = 100\text{V}, \quad v_2(0^+) = 0, \quad v_R(0^+) = 100\text{V}$$

$$(c) \quad \tau = \frac{20 \times 5}{20 + 5} \times 10^{-6} \times 2 \times 10^4 = 8 \times 10^{-2} \text{ s}$$

$$(d) \quad v_R(t) = 100e^{-12.5t} \text{ V}, \quad t > 0$$

$$(e) \quad i(t) = \frac{v_R(t)}{2 \times 10^4} = 5e^{-12.5t} \text{ mA}$$

$$(f) \quad v_1(t) = \frac{10^6}{20} \int_0^t -5 \times 10^{-3} e^{-12.5t} dt + 100 = \frac{10^3}{50} e^{-12.5t} \Big|_0^t + 100 = -20e^{-12.5t} + 80\text{V}$$

$$v_2(t) = \frac{1000}{5} \int_0^t 5e^{-12.5t} dt + 0 = -80e^{-12.5t} \Big|_0^t + 0 = -80e^{-12.5t} + 80\text{V}$$

$$(g) \quad w_{c1}(\infty) = \frac{1}{2} \times 20 \times 10^{-6} \times 80^2 = 64\text{mJ}, \quad w_{c2}(\infty) = \frac{1}{2} \times 5 \times 10^{-6} \times 80^2 = 16\text{mJ}$$

$$w_{c1}(0) = \frac{1}{2} \times 20 \times 10^{-6} \times 100^2 = 100\text{mJ}, \quad w_{c2}(0) = 0$$

$$w_R = \int_0^\infty 25 \times 10^{-6} e^{-25t} \times 2 \times 10^4 dt = \frac{25}{-25} \times 2 \times 10^4 (-1) 10^{-6} = 20\text{mJ}$$

$$64 + 16 + 20 = 100 \text{ checks}$$

## CHAPTER EIGHT SOLUTIONS

36.

$$(a) \quad t < 0: i_s = 1\text{mA} \therefore v_c(0) = 10\text{V}, \downarrow i_L(0) = -1\text{mA} \therefore v_x(0) = 10\text{V}, t < 0$$

$$(b) \quad t > 0: v_c(t) = 10e^{-t/10^4 \times 20 \times 10^{-9}} = 10e^{-5000t} \text{V}$$

$$i_L(t) = -10^{-3}e - 10^{3t/0.1} = -10^{-3}e^{-10000t} \text{A} \therefore v_L(t) = e^{-10000t} \text{V}, t > 0$$

$$\therefore v_x = v_c - v_L(t) = 10e^{-5000t} - e^{-10000t} \text{V}, t > 0$$



## CHAPTER EIGHT SOLUTIONS

37.

(a)  $t < 0$ :  $v_s = 20\text{V} \therefore v_c = 20\text{V}$ ,  $i_L = 20\text{mA} \therefore i_x(t) = 20\text{mA}, t < 0$

(b)  $t > 0$ :  $v_s = 0 \therefore i_L(t) = 0.02e^{-10000t}\text{A}$ ;  $v_c(t) = 20e^{-t/2 \times 10^{-8} 10^4} = 20e^{-5000t}\text{V}$

$$\downarrow i_c(t) = 2 \times 10^{-8} \times 20(-5000)e^{-5000t} = -2e^{-5000t}\text{mA}$$

$$i_x(t) = i_L(t) + i_c(t) = 0.02e^{-10000t} - 0.002e^{-5000t}\text{A} = 20e^{-10000t} - 2e^{-5000t}\text{mA}$$

## CHAPTER EIGHT SOLUTIONS

38.

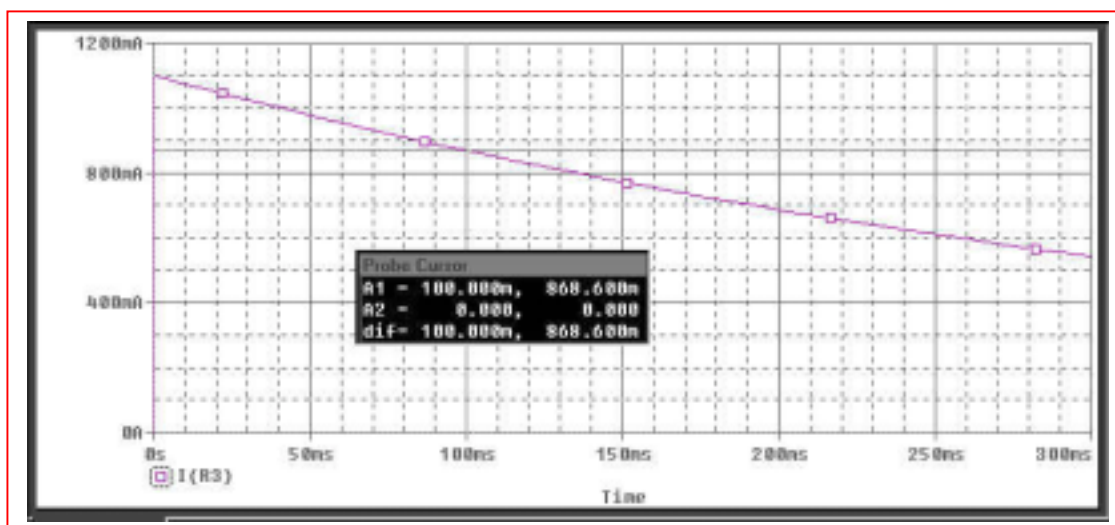
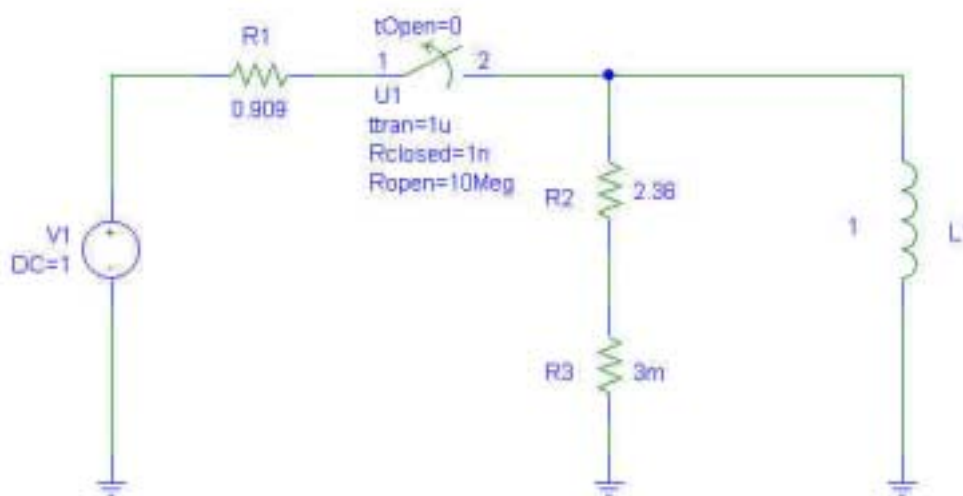
$$i_L(0^-) = \frac{V}{R} = \frac{1}{0.909} = 1.1 \text{ A}$$

$$t > 0: i_L(t) = e^{-2.363t} \text{ A}$$

$$i_L(0.1s) = 1.1e^{-2.363 \times 0.1} = 0.8685 \text{ A}$$

$\therefore$  since the current has dropped to less than 1 A prior to  $t = 100 \text{ ms}$ , the fuse does not blow.

PSpice verification: Note that the switch properties were changed.



We see from the simulation result that the current through the fuse (R3) is 869 mA, in agreement with our hand calculation.

## CHAPTER EIGHT SOLUTIONS

39. (a)  $v_A = 300u(t-1) \text{ V}$ ,  $v_B = -120u(t+1) \text{ V}$ ;  $i_c = 3u(-t) \text{ A}$

$$t = -1.5: i_1(-1.5) = 3 \times \frac{100}{300} = \boxed{1\text{A}}$$

$$t = 0.5: i_1(-0.5) = \frac{-120}{300} + 1 = \boxed{0.6\text{A}};$$

$$t = 0.5: i_1 = -\frac{120}{300} = -0.4\text{A}; t = 1.5: i_1 = \frac{300}{300} - \frac{120}{300} = \boxed{0.6\text{A}}$$

## CHAPTER EIGHT SOLUTIONS

40.

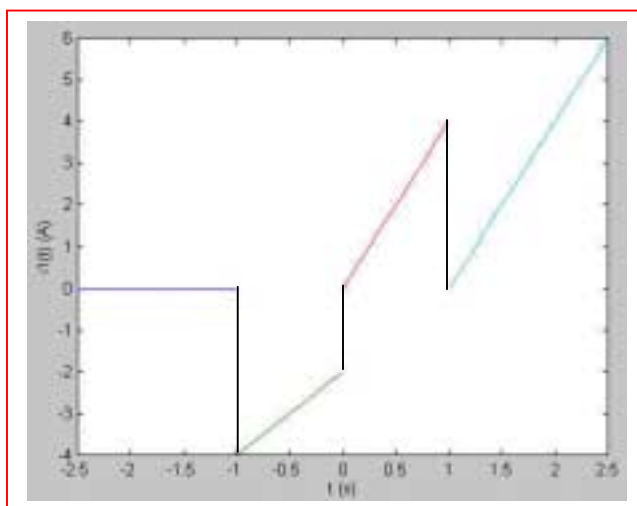
$$v_A = 600tu(t+1)\text{ V}, v_B = 600(t+1)u(t)\text{ V}, i_c = 6(t-1)u(t-1)\text{ A}$$

(a)  $t = -1.5: i_1 = 0; t = -0.5: i_1 = 600(-0.5)/300 = -1\text{ A}$

$$t = 0.5: i_1 = \frac{600(0.5)}{300} + \frac{600(1.5)}{300} = 4\text{ A}$$

$$t = 1.5: i_1 = \frac{600(1.5)}{300} + \frac{600(2.5)}{300} + \frac{1}{3} \times 6 \times 0.5 = 3 + 5 + 1 = 9\text{ A}$$

(b)



## CHAPTER EIGHT SOLUTIONS

41.

$$(a) \quad 2u(-1) - 3u(1) + 4u(3) = -3 + 4 = 1$$

$$(b) \quad [5 - u(2)] [2 + u(1)] [1 - u(-1)] \\ = 4 \times 3 \times 1 = 12$$

$$(c) \quad 4e^{-u(1)}u(1) = 4e^{-1} = 1.4715^+$$

## CHAPTER EIGHT SOLUTIONS

42.

(a)  $t < 0$ :  $i_x = \frac{100}{50} + 0 + 10 \times \frac{20}{50} = 6\text{A}$

$t > 0$ :  $i_x = 0 + \frac{60}{30} + 0 = 2\text{A}$

(b)  $t < 0$ : The voltage source is shorting out the  $30\text{-}\Omega$  resistor, so  $i_x = 0$ .

$t > 0$ :  $i_x = 60/30 = 2\text{ A}$ .

## CHAPTER EIGHT SOLUTIONS

$$43. \quad t = -0.5: \quad 50 \parallel 25 = 16.667, \quad i_x = \frac{200}{66.67} - 2 \frac{1/50}{1/50 + 1/25 + 1/50} = 3 - \frac{1}{2} = 2.5\text{A}$$

$$t = 0.5: \quad i_x = \frac{200}{66.67} = 3\text{A}$$

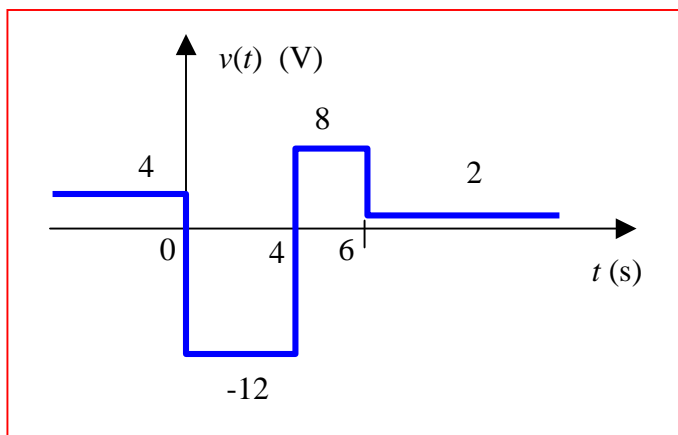
$$t = 1.5: \quad i_x = 3 - \frac{100}{66.67} \times \frac{1}{3} = 2.5\text{A}$$

$$t = 2.5: \quad i_x = \frac{200 - 100}{50} = 2\text{A}$$

$$t = 3.5: \quad i_x = -\frac{100}{50} = -2\text{A}$$

## CHAPTER EIGHT SOLUTIONS

44.  $v(t) = 4 - 16u(t) + 20u(t - 4) - 6u(t - 6) \text{ V}$





## CHAPTER EIGHT SOLUTIONS

45. (a)  $7 u(t) - 0.2 u(t) + 8(t - 2) + 3$

$v(1) = 9.8 \text{ volts}$

(b) Resistor of value  $2\Omega$

## CHAPTER EIGHT SOLUTIONS

46.

(a)  $i_L(t) = (2 - 2e^{-200000t})u(t) \text{ mA}$

(b)  $v_L(t) = Li_L' = 15 \times 10^{-3} \times 10^{-3} (-2)$   
 $(-200000e^{-200000t})u(t) = 6e^{-200000t}u(t) \text{ V}$

## CHAPTER EIGHT SOLUTIONS

47.

$$(a) \quad i_L(t) = 2 + 2(1 - e^{-2.5t})u(t) \text{ A} \therefore i_L(-0.5) = 2\text{A}$$

$$(b) \quad i_L(0.5) = 2 + 2(1 - e^{-1.25}) = 3.427\text{A}$$

$$(c) \quad i_L(1.5) = 2 + 2(1 - e^{-3.75}) = 3.953\text{A}$$

## CHAPTER EIGHT SOLUTIONS

48.

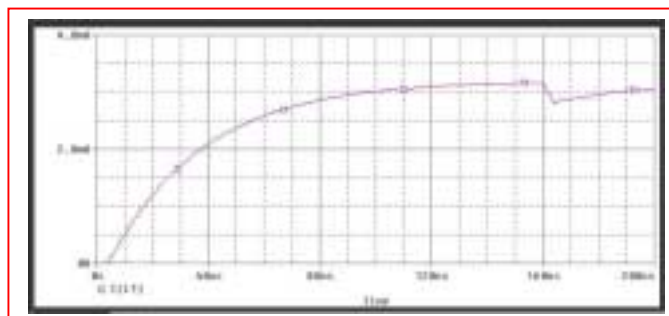
$$R_{th} = \frac{10 \times 10^3 \times 4.7 \times 10^3}{10 \times 10^3 + 4.7 \times 10^3} = 3.2 \times 10^3 \Omega$$

$$\tau = \frac{L}{R} = \frac{500 \times 10^{-6}}{3.2 \times 10^3} = 156 ns$$

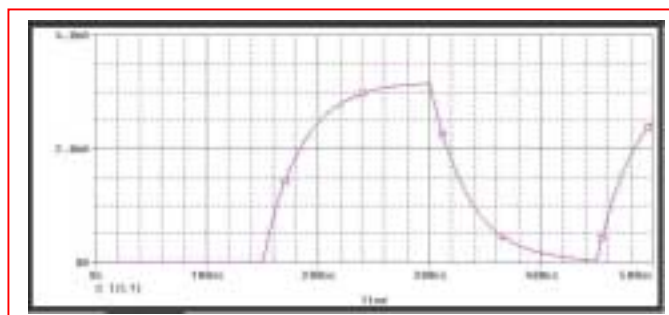
$$i_1 \cdot 4.7 \times 10^3 = i_L \cdot 10 \times 10^3 \quad (a)$$

$$i_1 + i_L = 10 \times 10^{-3} A$$

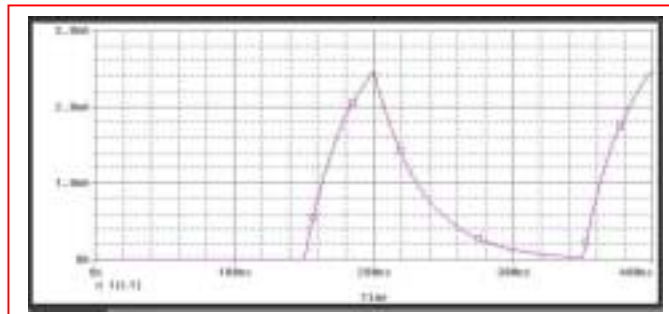
$$\therefore i_L = 3.2 \times 10^{-3} A$$



(b)



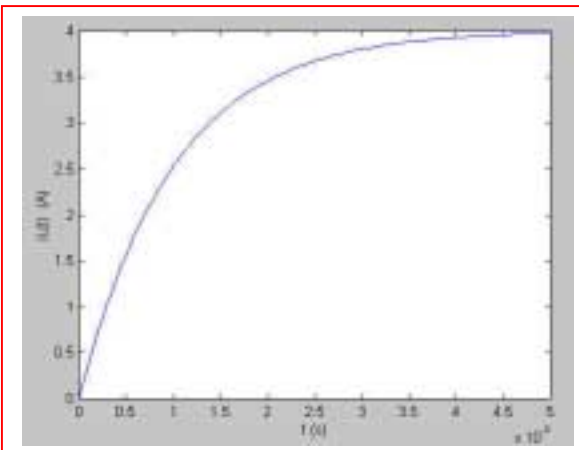
(c)



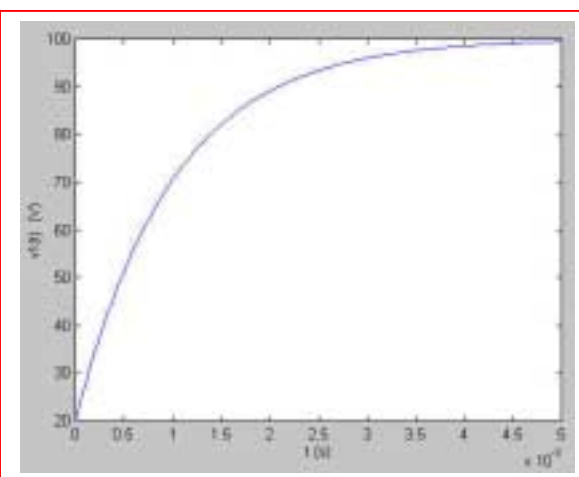
## CHAPTER EIGHT SOLUTIONS

49.

(a)  $i_L(t) = (4 - 4e^{-20t/0.02})u(t)$   
 $\therefore i_L(t) = 4(1 - e^{-1000t})u(t)\text{A}$



(b)  $v_1(t) = (100 - 80e^{-1000t})u(t)\text{V}$



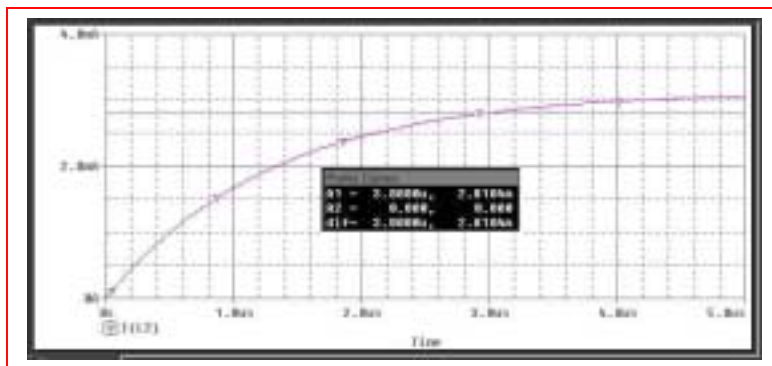
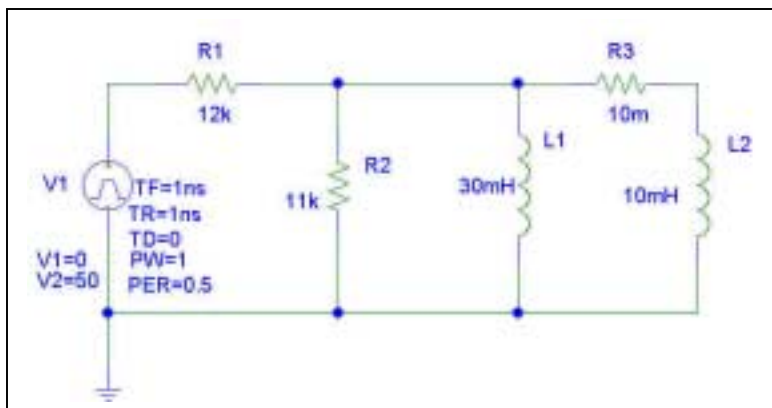
## CHAPTER EIGHT SOLUTIONS

50. (a) 0 W

(b) The total inductance is  $30 \parallel 10 = 7.5$  mH. The Thévenin equivalent resistance is  $12 \parallel 11 = 5.739$  k $\Omega$ . Thus, the circuit time constant is  $L/R = 1.307$   $\mu$ s. The final value of the total current flowing into the parallel inductor combination is  $50/12$  mA = 4.167 mA. This will be divided between the two inductors, so that  $i(\infty) = (4.167)(30)/(30 + 10) = 3.125$  mA.

We may therefore write  $i(t) = 3.125[1 - e^{-10^6 t / 1.307}]$  A. Solving at  $t = 3$   $\mu$ s, we find 2.810 A.

(c) PSpice verification



We see from the Probe output that our hand calculations are correct by verifying using the cursor tool at  $t = 3$   $\mu$ s.

## CHAPTER EIGHT SOLUTIONS

51.

(a)  $i_L(t) = 10A, t < 0$

(b)  $i_L(t) = 8 + 2e^{-5t/0.5}$

$\therefore i_L(t) = 8 + 2e^{-10t} A, t > 0$

## CHAPTER EIGHT SOLUTIONS

52.

(a)  $i_L(t) = 2\text{A}, t > 0$

(b)  $i_L(t) = 5 - e^{-4t/0.1}$

$\therefore i_L(t) = 5 - 3e^{-40t} \text{ A}, t > 0$



## CHAPTER EIGHT SOLUTIONS

53.  $\frac{di}{dt} + Pi = Q$ ,  $i = e^{-Pt} \int Qe^{Pt} dt + Ae^{-Pt}$ ,  $R = 125\Omega$ ,  $L = 5H$

$$\therefore L \frac{di}{dt} + Ri = Q \therefore LP = R = 125 \therefore P = 25$$

(a)  $Q(t) = \frac{10}{L} = 2 \therefore i = e^{-25t} \int_0^t 2e^{25t} dt + Ae^{-25t} = e^{-25t} \times \frac{2}{25} e^{25t} \Big|_0^t + Ae^{-25t}$

$$\therefore i = \frac{2}{25} + Ae^{-25t}, i(0) = \frac{10}{125} = \frac{2}{25} \therefore A = 0 \therefore i = \frac{2}{25} = \boxed{0.08A}$$

(b)  $Q(t) = \frac{10u(t)}{5} = 2u(t) \therefore i = e^{-25t} \int_0^t 2e^{25t} dt + Ae^{-25t} = \frac{2}{25} + Ae^{-25t}$

$$i(0) = 0 \therefore A = -\frac{2}{25} \therefore \boxed{i(t) = 0.08(1 - e^{-25t})A, t > 0}$$

(c)  $Q(t) = \frac{10 + 10u(t)}{5} = 2 + 2u(t) \therefore \boxed{i = 0.16 - 0.08e^{-25t}A, t > 0}$

(d)  $Q(t) = \frac{10u(t) \cos 50t}{5} = 2u(t) \cos 50t \therefore i = e^{-25t} \int_0^t 2 \cos 50t \times e^{25t} dt + Ae^{-25t}$

$$\therefore i = 2e^{-25t} \left[ \frac{e^{25t}}{50^2 + 25^2} (25 \cos 50t + 50 \sin 50t) \right]_0^t + Ae^{-25t}$$

$$= 2e^{-25t} \left[ \frac{e^{25t}}{3125} (25 \cos 50t + 50 \sin 50t) - \frac{1}{3125} \times 25 \right] + Ae^{-25t}$$

$$= \frac{2}{125} \cos 50t + \frac{4}{125} \sin 50t - \frac{2}{125} e^{-25t} + Ae^{-25t}$$

$$i(0) = 0 \therefore 0 = \frac{2}{125} - \frac{2}{125} + A \therefore A = 0$$

$$\therefore \boxed{i(t) = 0.016 \cos 50t + 0.032 \sin 50t - 0.016e^{-25t}A, t > 0}$$

## CHAPTER EIGHT SOLUTIONS

54.

(a)  $0, 0$

(b)  $0, 200\text{V}$

(c)  $1\text{A}, 100\text{V}$

(d)  $\tau = \frac{50 \times 10^{-3}}{200} = \frac{1}{4} \text{ms} \therefore i_L = 1(1 - e^{-4000t})u(t)\text{A}, i_L(0.2\text{ms}) = 0.5507\text{A}$

$$v_1(t) = (100 + 100e^{-4000t})u(t)\text{V}, v_1(0.2\text{ms}) = 144.93\text{V}$$

## CHAPTER EIGHT SOLUTIONS

55.

$$(a) \quad i_L(t) = \frac{100}{20} - \frac{100}{5} = -15\text{A}, t < 0$$

$$(b) \quad i_L(0^+) = i_L(0^-) = -15\text{A}$$

$$(c) \quad i_L(\infty) = \frac{100}{20} = 5\text{A}$$

$$(d) \quad i_L(t) = 5 - 20e^{-40t} \text{ A}, t > 0$$

## CHAPTER EIGHT SOLUTIONS

$$56. \quad i_L(0^-) = \frac{18}{60+30} \times \frac{1}{2} = 0.1\text{A} \therefore i_L(0^+) = 0.1\text{A}$$

$$i_L(\infty) = 0.1 + 0.1 = 0.2\text{A}$$

$$\therefore i_L(t) = 0.2 - 0.1e^{-9000t} \text{ A}, t > 0$$

$$\therefore i_L(t) = 0.1u(-t) + (0.2 - 0.1e^{-9000t})u(t)\text{A}$$

$$\text{or, } i_L(t) = 0.1 + (0.1 - 0.1e^{-9000t})u(t)\text{A}$$

## CHAPTER EIGHT SOLUTIONS

57.

$$(a) \quad i_x(0^-) = \frac{30}{7.5} \times \frac{3}{4} = 3\text{A}, \quad i_L(0^-) = 4\text{A}$$

$$(b) \quad i_x(0^+) = i_L(0^+) = 4\text{A}$$

$$(c) \quad i_x(\infty) = i_L(\infty) = 3\text{A} \\ \therefore i_x(t) = 3 + 1e^{-10t/0.5} = 3 + e^{-20t} \text{ A} \therefore i_x(0.04) \\ = 3 + e^{-0.8} = 3.449\text{A}$$

## CHAPTER EIGHT SOLUTIONS

58.

$$(a) \quad i_x(0^-) = i_L(0^-) = \frac{30}{10} = \boxed{3\text{A}}$$

$$(b) \quad i_x(0^+) = \frac{30}{30+7.5} \times \frac{30}{40} + 3 \times \frac{15}{10+15} = \boxed{2.4\text{A}}$$

$$(c) \quad i_x(\infty) = \frac{30}{7.5} \times \frac{30}{40} = 3\text{A} \therefore i_x(t) = 3 - 0.6e^{-6t/0.5}$$
$$= 3 - 0.6e^{-12t} \therefore i_x(0.04) = 3 - 0.6e^{-0.48} = \boxed{2.629\text{A}}$$

## CHAPTER EIGHT SOLUTIONS

59. OC :  $v_x = 0$ ,  $v_{oc} = 4u(t)$  V

$$\text{SC : } 0.1u(t) = \frac{v_x - 0.2v_x}{40} + \frac{v_x}{60}, 12u(t) = 0.6v_x + 2v_x$$

$$\therefore v_x = \frac{12u(t)}{2.6} \therefore i_{ab} = \frac{v_x}{60} = \frac{12u(t)}{2.6 \times 60} = \frac{u(t)}{13}$$

$$\therefore R_{th} = 4 \times 13 = 52 \Omega \therefore i_L = \frac{4u(t)}{52} (1 - e^{-52t/0.2}) u(t) = \frac{u(t)}{13} (1 - e^{-260t}) u(t)$$

$$\therefore v_x = 60i_L = \boxed{4.615^+ (1 - e^{-260t}) u(t)} \text{ V}$$

## CHAPTER EIGHT SOLUTIONS

60.

(a) OC:  $-100 + 30i_1 + 20i_1 = 0$ ,  $i_1 = 2\text{A}$

$$\therefore v_{oc} = 80u(t)\text{V}$$

SC:  $i_1 = 10\text{A}$ ,  $\downarrow i_{sc} = 10 + \frac{20 \times 10}{20} = 20\text{A}$

$$\therefore R_{th} = 4\Omega \therefore i_L(t) = \boxed{20(1 - e^{-40t})u(t)\text{A}}$$

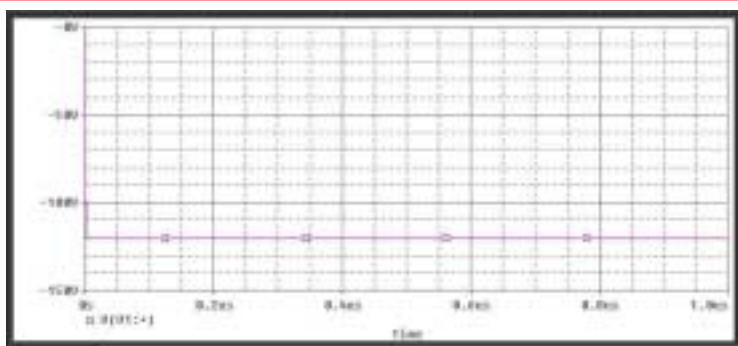
(b)  $v_L = 0.1 \times 20 \times 40e^{-40t}u(t) = 80e^{-40t}u(t)$

$$\therefore i_1(t) = \frac{100u(t) - 80e^{-40t}u(t)}{10} = \boxed{10 - 8e^{-40t}u(t)\text{A}}$$



## CHAPTER EIGHT SOLUTIONS

61. Unfortunately, PSpice will not allow us to use negative time values. Thus, we must perform the simulation starting from  $t = 0$ , and manually shift the results if needed to account for sources that change value prior to  $t = 0$ .



Voltage source  
 $v_B(t)$  from  
Exercise 39.



Current source  
 $i_C(t)$  from  
Exercise 39.

## CHAPTER EIGHT SOLUTIONS

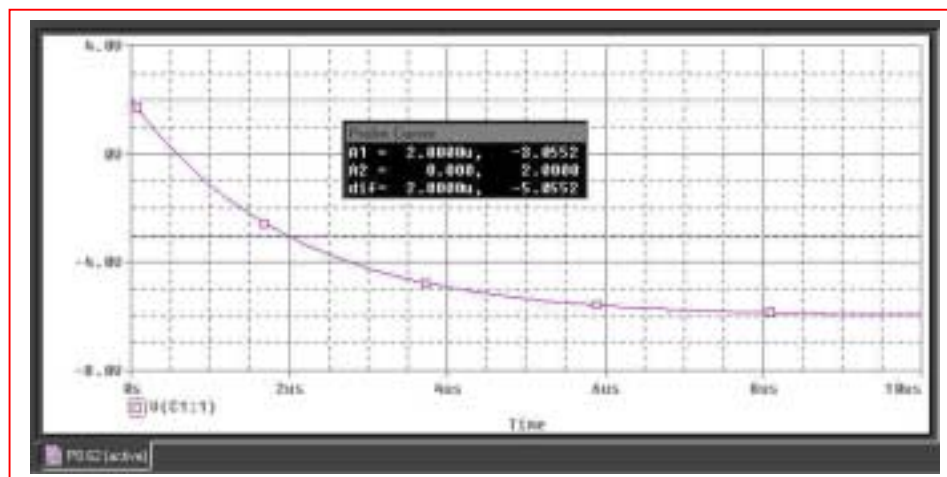
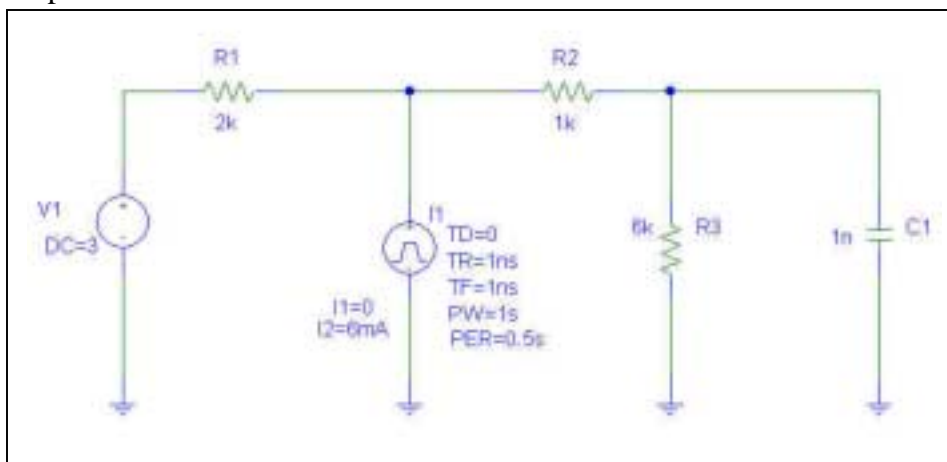
62. (a)  $v_c(0^-) = \frac{6}{9} \times 3 = 2\text{V} = v_c(0^+)$

$$v_c(\infty) = 2 - 6(2 \parallel 7) \frac{6}{7} = -6\text{V}$$

$$\therefore v_c(t) = -6 + 8e^{-10^9 t / 2 \times 10^3} = -6 + 8e^{-500000t} \text{V}, t > 0$$

$$v_c(-2\mu\text{s}) = v_c(0^-) = 2\text{V}, v_c(2\mu\text{s}) = -6 + 8e^{-1} = \boxed{-3.057\text{V}}$$

(b) PSpice verification.



As can be seen from the plot above, the PSpice simulation results confirm our hand calculations of  $v_c(t < 0) = 2\text{V}$  and  $v_c(t = 2\mu\text{s}) = -3.06\text{V}$

## CHAPTER EIGHT SOLUTIONS

$$63. \quad i_A(0^-) = \frac{10}{4} = 2.5\text{mA}, \quad i_A(\infty) = 10\text{mA}$$

$$v_c(0) = 7.5\text{V} \therefore i_A(0^+) = \frac{10}{1} + \frac{7.5}{1} = 17.5\text{mA}$$

$$i_A = 10 + 7.5e^{-10^8 t / 10^3} = 10 + 7.5e^{-10^5 t} \text{mA}, \quad t > 0, \quad i_A = 2.5\text{mA} \quad t < 0$$

## CHAPTER EIGHT SOLUTIONS

64.  $i_A(0^-) = \frac{10}{1} = 10\text{mA}$ ,  $i_A(\infty) = 2.5\text{mA}$ ,  $v_c(0) = 0$

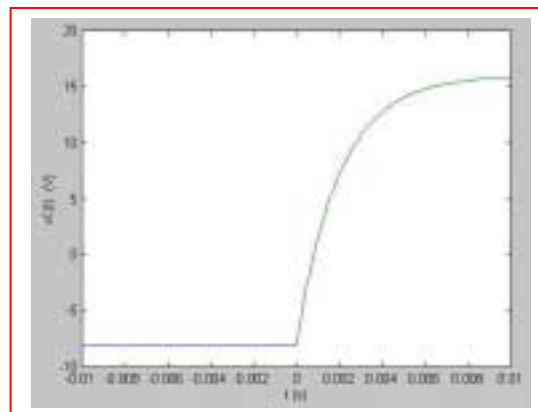
$$i_A(0^+) = \frac{10}{1.75} \times \frac{1}{4} = 1.4286\text{mA} \therefore i_A = 10\text{mA}, t < 0$$

$$i_A = 2.5 + (1.4286 - 2.5)e^{-10^8 t / 1.75 \times 10^3} = 2.5 - 1.0714e^{-57140t} \text{mA}, t > 0$$

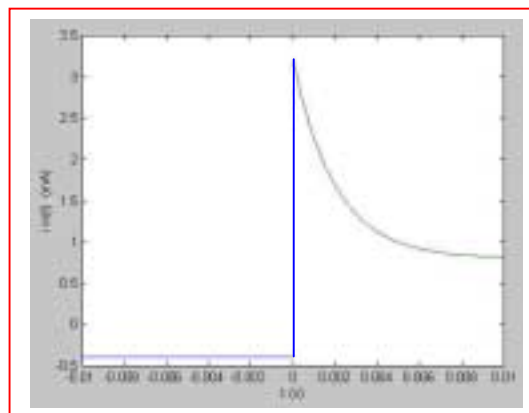
## CHAPTER EIGHT SOLUTIONS

65.

(a)  $v_s = -12u(-t) + 24u(t)$  V  
 $t < 0$ :  $v_c(0^-) = -8$  V  $\therefore v_c(0^+) = -8$  V  
 $t > 0$ :  $v_c(\infty) = \frac{2}{3} \times 24 = 16$  V  
 $RC = \frac{200}{30} \times 10^3 \times 3 \times 10^{-7} = 2 \times 10^{-3}$   
 $\therefore v_c(t) = 16 - 24e^{-500t}$  V,  $t > 0$   
 $\therefore v_c(t) = -8u(-t) + (16 - 24e^{-500t})u(t)$



(b)  $i_{in}(0^-) = \frac{-12}{30} = -0.4$  mA,  $i_{in}(0^+) = \frac{24+8}{10} = 3.2$  mA  
 $i_{in}(\infty) = \frac{24}{30} = 0.8$  mA  
 $i_{in}(t) = -0.4u(t) + (0.8 + 2.4e^{-500t})u(t)$  mA



## CHAPTER EIGHT SOLUTIONS

66.  $v_c(0^-) = 10\text{V} = v_c(0^+)$ ,  $i_{in}(0^-) = 0$   
 $i_{in}(0^+) = 0 \therefore i_{in}(t) = 0$  for all  $t$

(a)  $i_{in}(-1.5) = 0$

(b)  $i_{in}(1.5) = 0$

## CHAPTER EIGHT SOLUTIONS

67.  $0 < t < 0.5\text{ s} : v_c = 10(1 - e^{-2.5t})\text{ V}$

$v_c(0.4) = 6.321\text{ V}, v_c(0.5) = 7.135\text{ V}$

$t > 0.5 : \frac{20 - 10}{12} = \frac{5}{6}\text{ A} \therefore v_c(\infty) = 10 + 8 + \frac{5}{6} = \frac{50}{3}\text{ V}, 4 \parallel 8 = \frac{8}{3}\Omega$

$v_c(t) = \frac{50}{3} + \left(7.135 - \frac{50}{3}\right)e^{-0.375 \times 20(t-0.5)} = 16.667 - 9.532e^{-7.5(t-0.5)}\text{ V}$

$\therefore v_c(0.8) = 16.667 - 9.532e^{-7.5(0.3)} = 15.662\text{ V}$

## CHAPTER EIGHT SOLUTIONS

68. OC:  $\frac{-v_x}{100} - \frac{v_x}{100} + \frac{3-v_x}{100} = 0 \therefore v_x = 1, v_{oc} = 3 - 1 = 2\text{V}$

SC:  $v_x = 3\text{V} \therefore i_{sc} = \frac{v_x}{100} + \frac{v_x}{100} = 0.06\text{A}$

$\therefore R_{th} = v_{oc} / i_{sc} = 2 / 0.06 = 33.33\Omega$

$\therefore v_c = v_{oc} (1 - e^{-t/R_{th}C}) = 2(1 - e^{-10^6 t / 33.33})$

$= 2(1 - e^{-30,000t}) \text{ V}, t > 0$



## CHAPTER EIGHT SOLUTIONS

69.

(a)  $t < 0$ :  $8(10 + 20) = 240\text{V} = v_R(t) = 80\text{V}, t < 0$

(b)  $t < 0$ :  $v_c(t) = 8 \times 30 = 240\text{V} \therefore v_c(0^+) = 240\text{V}$

$$t = (\infty): v_c(\infty) = \frac{1}{2} \times 8(10 + 10) = 80\text{V}$$

$$\therefore v_c(t) = 80 + 160e^{-t/10 \times 10^{-6}} = 80 + 160e^{-100000t}\text{V}$$

$$\therefore v_R(t) = 80 + 160e^{-100000t}\text{V}, t > 0$$

(c)  $t < 0$ :  $v_R(t) = 80\text{V}$

(d)  $v_c(0^-) = 80\text{V}, v_c(\infty) = 240\text{V} \therefore v_c(t) = 240 - 160e^{-t/50 \times 10^{-6}} = 240 - 160e^{-20000t}\text{V}$

$$v_R(0^-) = 80\text{V}, v_R(0^+) = 8 \frac{20}{30 + 20} \times 10 + \frac{80}{50} \times 10 = 32 + 16 = 48\text{V}$$

$$v_R(\infty) = 80\text{V} \therefore v_R(t) = 80 - 32e^{-20000t}\text{V}, t > 0$$

## CHAPTER EIGHT SOLUTIONS

70.

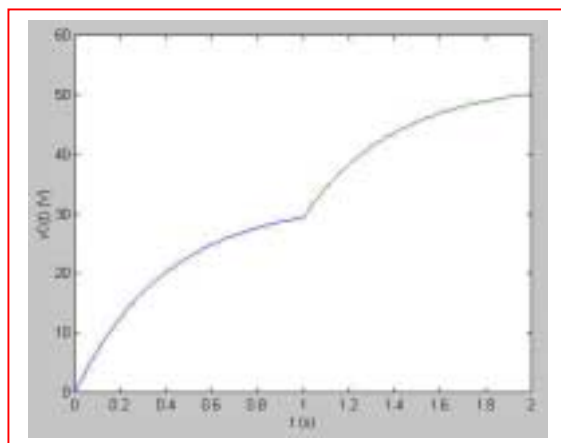
- (a) For  $t < 0$ , there are no active sources, and so  $v_C = 0$ .

For  $0 < t < 1$ , only the 40-V source is active.  $R_{th} = 5k \parallel 20k = 4k\Omega$  and hence  $\tau = R_{th}C = 0.4$  s. The “final” value (assuming no other source is ever added) is found by voltage division to be  $v_C(\infty) = 40(20)/(20 + 5) = 32$  V. Thus, we may write  $v_C(t) = 32 + [0 - 32]e^{-t/0.4}$  V =  $32(1 - e^{-2.5t})$  V.

For  $t > 1$ , we now have two sources operating, although the circuit time constant remains unchanged. We define a new time axis temporarily:  $t' = t - 1$ . Then  $v_C(t' = 0^+) = v_C(t = 1) = 29.37$  V. This is the voltage across the capacitor when the second source kicks on. The new final voltage is found to be  $v_C(\infty) = 40(20)/(20 + 5) + 100(5)/(20 + 5) = 52$  V.

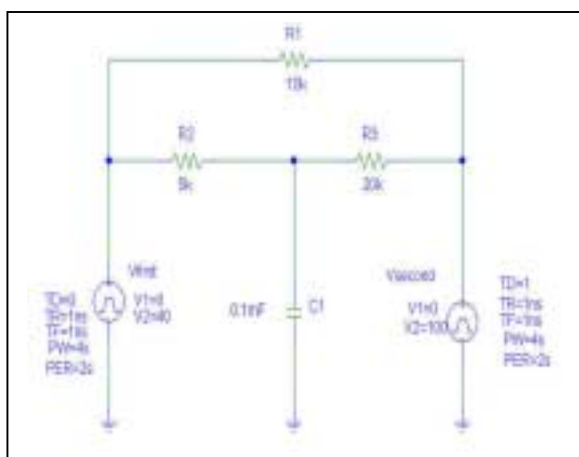
Thus,  $v_C(t') = 52 + [29.37 - 52]e^{-2.5t'} = 52 - 22.63e^{-2.5(t-1)}$  V.

(b)



For  $t < 0$ ,  
 $v_C = 0$ .

(c)



We see from the simulation results that our hand calculations and sketch are indeed correct.

## CHAPTER EIGHT SOLUTIONS

71.  $v_{x,L} = 200e^{-2000t} \text{ V}$   
 $v_{x,c} = 100(1 - e^{-1000t}) \text{ V}$   
 $v_x = v_{x,L} - v_{x,c} = 0$   
 $\therefore 200e^{-2000t} = 100 - 100e^{-1000t}$   
 $\therefore 100e^{-1000t} + 200(e^{-1000t})^2 - 100 = 0,$   
 $e^{-1000t} = \frac{-100 \pm \sqrt{10,000 + 80,000}}{400} = -0.25 \pm 0.75$   
 $\therefore e^{-1000t} = 0.5, t = 0.6931 \text{ ms}$

## CHAPTER EIGHT SOLUTIONS

72.  $t < 0: v_c = 0$

$$0 < t < 1\text{ms}: v_c = 9(1 - e^{-10^6 t / (R_1 + 100)})$$

$$\therefore 8 = 9(1 - e^{-1000 / (R_1 + 100)}), \frac{1}{9} = e^{-1000 / (R_1 + 100)}$$

$$\therefore \frac{1000}{R_1 + 100} = 2.197, R_1 = \boxed{355.1\Omega}$$

$$t > 1\text{ms}: v_c = 8e^{-10^6 t' / (R_2 + 100)}, t' = t - 10^{-3} \therefore 1 - 8e^{-1000} (R_2 + 100)$$

$$\therefore \frac{1000}{R_2 + 100} = 2.079, R_2 = 480.9 - 100 = \boxed{380.9\Omega}$$

## CHAPTER EIGHT SOLUTIONS

73. For  $t < 0$ , the voltage across all three capacitors is simply  $9(4.7)/5.7 = 7.421$  V. The circuit time constant is  $\tau = RC = 4700(0.5455 \times 10^{-6}) = 2.564$  ms.

When the circuit was first constructed, we assume no energy was stored in any of the capacitors, and hence the voltage across each was zero. When the switch was closed, the capacitors began to charge according to  $\frac{1}{2} C v^2$ . The capacitors charge with the same current flowing through each, so that by KCL we may write

$$C_1 \frac{dv_1}{dt} = C_2 \frac{dv_2}{dt} = C_3 \frac{dv_3}{dt}$$

With no initial energy stored, integration yields the relationship  $C_1 v_1 = C_2 v_2 = C_3 v_3$  throughout the charging (*i.e.* until the switch is eventually opened). Thus, just prior to the switch being thrown at what we now call  $t = 0$ , the total voltage across the capacitor string is 7.421 V, and the individual voltages may be found by solving:

$$\begin{aligned} v_1 + v_2 + v_3 &= 7.421 \\ 10^{-6} v_1 - 2 \times 10^{-6} v_2 &= 0 \\ 2 \times 10^{-6} v_2 - 3 \times 10^{-6} v_3 &= 0 \end{aligned}$$

so that  $v_2 = 2.024$  V.

With the initial voltage across the 2- $\mu$ F capacitor now known, we may write

$$v(t) = 2.024 e^{-t/2.564 \times 10^{-3}} \text{ V}$$

- (a)  $v(t = 5.45 \text{ ms}) = 241.6 \text{ mV}$ .
- (b) The voltage across the entire capacitor string can be written as  $7.421 e^{-t/2.564 \times 10^{-3}} \text{ V}$ . Thus, the voltage across the 4.7-k $\Omega$  resistor at  $t = 1.7 \text{ ms} = 3.824 \text{ V}$  and the dissipated power is therefore  $3.111 \text{ mW}$ .
- (c) Energy stored at  $t = 0$  is  $\frac{1}{2} C v^2 = 0.5(0.5455 \times 10^{-6})(7.421)^2 = 15.02 \text{ } \mu\text{J}$ .

## CHAPTER EIGHT SOLUTIONS

74.

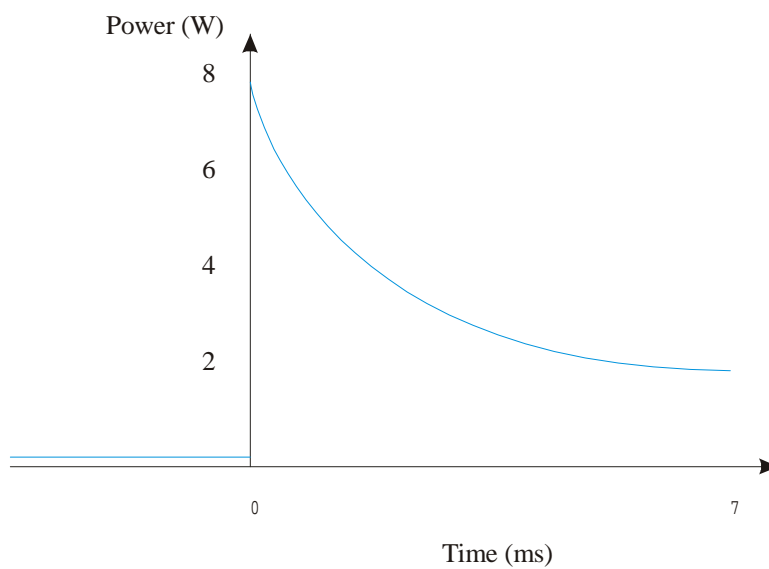
$$P(t < 0) = I^2 R = 0.001^2 \times 10^3 = 0.001 \text{ W}$$

$$V_{init} = I.R = 7 \times 10^{-3} \times 900 = 6.3 \text{ V}$$

$$P_{init} = \frac{V^2}{R} = 0.08 \text{ W}$$

$$V_{final} = 7 \times 10^{-3} \times 900\Omega // 1000\Omega = 3.3 \text{ V}$$

$$P_{final} = \frac{V^2}{R} = 0.02 \text{ W}$$



## CHAPTER EIGHT SOLUTIONS

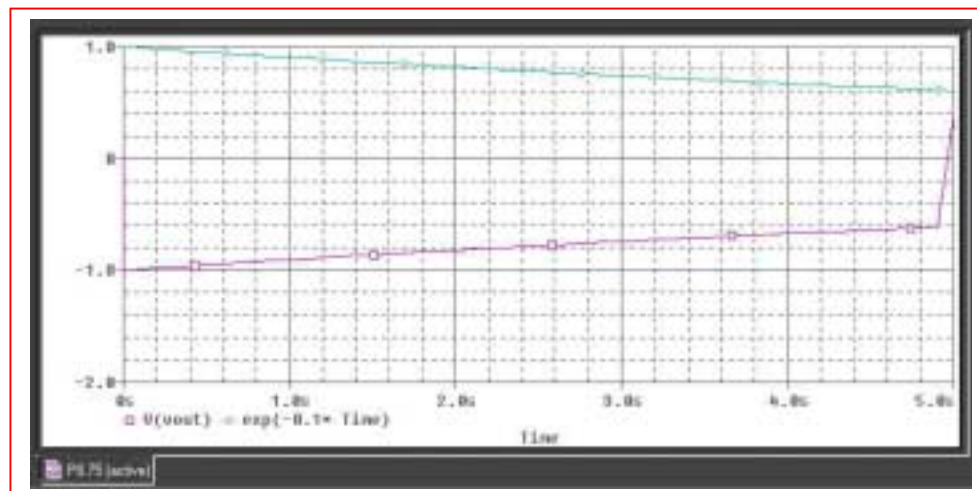
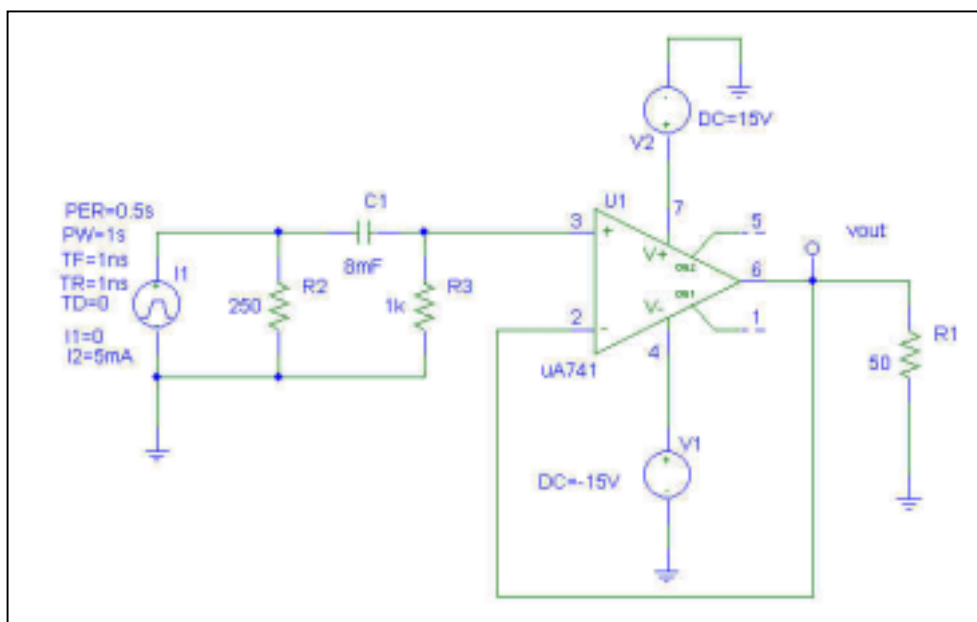
75. voltage follower  $\therefore v_o(t) = v_2(t)$

$$v_2(0^+) = 5(0.25 \parallel 1) = 1u(t)\text{V}$$

$$v_2(\infty) = 0, \tau = 1.25 \times 8 = 10s$$

$$\therefore v_o(t) = e^{-0.1t}u(t)\text{V}$$

PSpice verification:



From the simulation results, we see that modeling the operation of this circuit using an ideal op amp model does not provide an accurate accounting for the operation of the actual circuit.

## CHAPTER EIGHT SOLUTIONS

76. voltage follower  $\therefore v_o(t) = v_2(t)$

$$v_2(t) = 1.25u(t)\text{V} = v_o(t)$$

$$v_x(t) = 1.25e^{-10^6/0.5 \times 200}u(t)$$

$$= 1.25e^{-10,000t}u(t)\text{V}$$



## CHAPTER EIGHT SOLUTIONS

77. (a)  $v_1 = 0$  (virtual gnd)  $\therefore i = \frac{4}{10^4} e^{-20,000t} u(t) \text{ A}$

$$\therefore v_c = 10^7 \int_0^t \frac{4}{10^4} e^{-20,000t} dt = -0.2 e^{-20,000t} \Big|_0^t$$

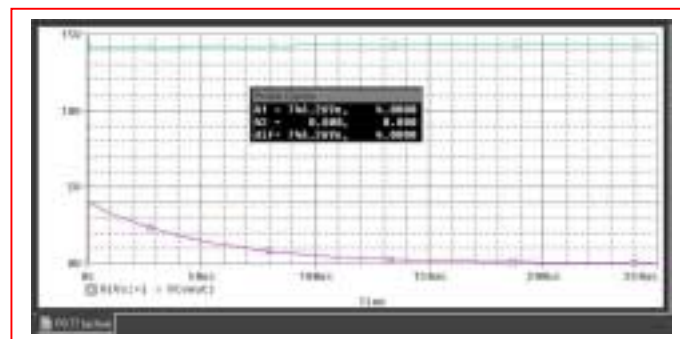
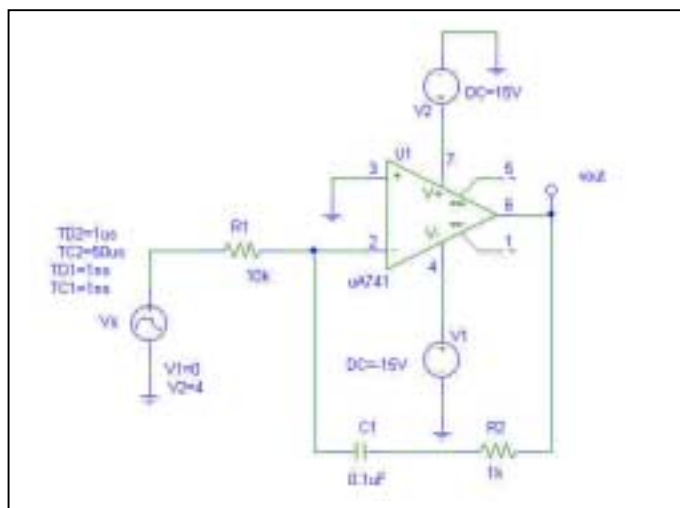
$$\therefore v_c(t) = 0.2(1 - e^{-20,000t}) u(t)$$

$$\therefore v_R(t) = 10^3 i(t) = 0.4 e^{-20,000t} u(t) \text{ V}$$

$$\therefore v_o(t) = -v_c(t) - v_R(t) = (-0.2 + 0.2 e^{-20,000t} - 0.4 e^{-20,000t}) u(t)$$

And we may write  $v_o(t) = -0.2[1 + e^{-20 \times 10^3 t}] u(t) \text{ V.}$

(b) PSpice verification:

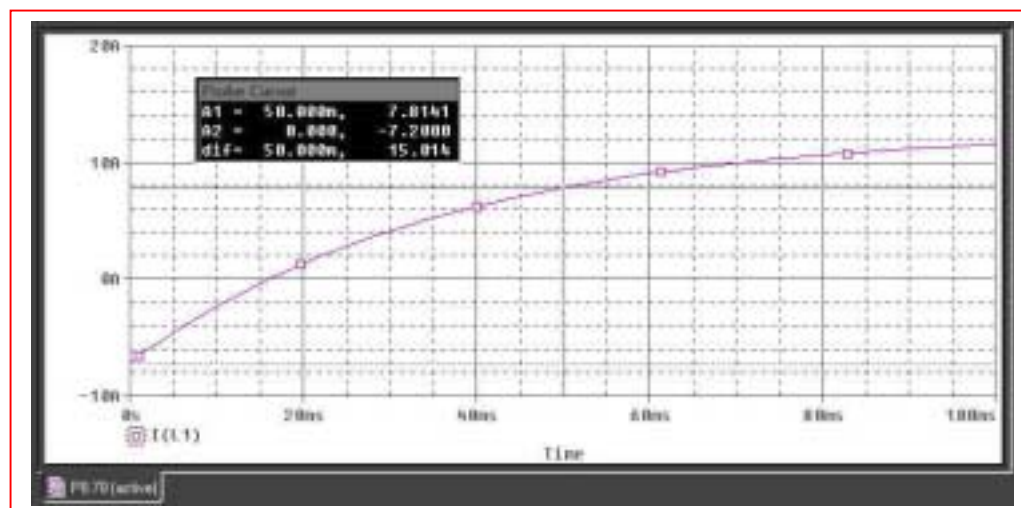
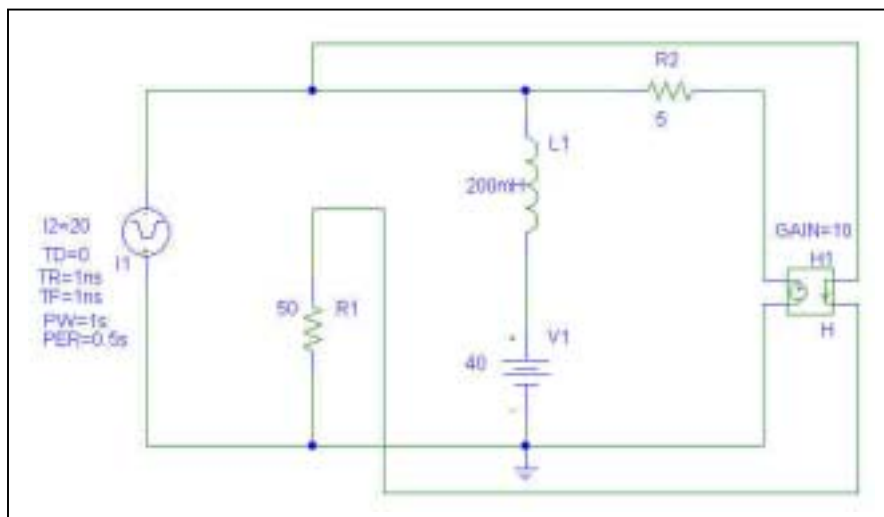


We can see from the simulation result that our ideal op amp approximation is not providing a great deal of accuracy in modeling the transient response of an op amp in this particular circuit; the output was predicted to be negative for  $t > 0$ .

## CHAPTER EIGHT SOLUTIONS

78. For  $t < 0$ , the current source is an open circuit and so  $i_1 = 40/50 = 0.8$  A. The current through the  $5\text{-}\Omega$  resistor is  $[40 - 10(0.8)]/5 = 7.2$  A, so the inductor current is equal to  $-7.2$  A

PSpice Simulation



From the PSpice simulation, we see that our  $t < 0$  calculation is indeed correct, and find that the inductor current at  $t = 50$  ms is  $7.82$  A.

## CHAPTER EIGHT SOLUTIONS

79. Assume at least 1  $\mu\text{A}$  required otherwise alarm triggers.

Add capacitor C.

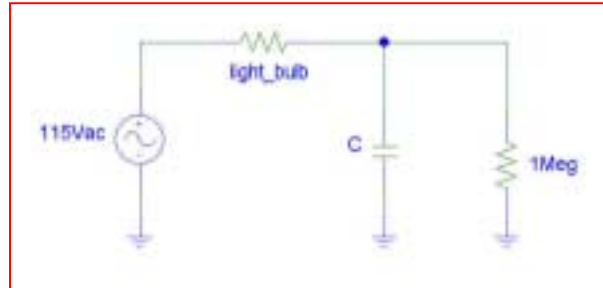
$$v_c(1) = 1 \text{ volt}$$

$$v_c(0) = \frac{1000}{1002.37} \cdot 1.5 = 1.496 \text{ volts}$$

$$\therefore \text{ We have } 1 = 1.496e^{-\frac{1}{10^6 C}} \text{ or } C = \frac{1}{10^6 \ln(1.496)} = 2.48\mu\text{F}$$

## CHAPTER EIGHT SOLUTIONS

80. One possible solution of many: implement a capacitor to retain charge; assuming the light is left on long enough to fully charge the capacitor, the stored charge will run the lightbulb after the wall switch is turned off. Taking a 40-W light bulb connected to 115 V, we estimate the resistance of the light bulb (which changes with its temperature) as  $330.6\ \Omega$ . We define “on” for the light bulb somewhat arbitrarily as 50% intensity, taking intensity as proportional to the dissipated power. Thus, we need at least 20 W (246 mA or 81.33 V) to the light bulb for 5 seconds after the light switch is turned off.



The circuit above contains a 1-M $\Omega$  resistor in parallel with the capacitor to allow current to flow through the light bulb when the light switch is on. In order to determine the required capacitor size, we first recognise that it will see a Thevenin equivalent resistance of  $1\ \text{M}\Omega \parallel 330.6\ \Omega = 330.5\ \Omega$ . We want  $v_C(t = 5\text{s}) = 81.33 = 115e^{-5/\tau}$ , so we need a circuit time constant of  $t = 14.43\ \text{s}$  and a capacitor value of  $\tau/R_{\text{th}} = 43.67\ \text{mF}$ .

## CHAPTER NINE SOLUTIONS

1.  $\omega_o L = 10\Omega, s_1 = -6s^{-1}, s_2 = -8s^{-1}$   
 $\therefore -6 = \alpha + \sqrt{\alpha^2 - \omega_o^2}, -8 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$  adding,  
 $-14 = -2\alpha \therefore \alpha = 7s^{-1}$   
 $\therefore -6 = -7 + \sqrt{49 - \omega_o^2} \therefore \omega_o^2 = 48 \frac{1}{LC}, \omega_o = 6.928$   
rad/s.  $\therefore 6.928L = 10, L = 1.4434H,$   
 $C = \frac{1}{48L} = 14.434mF, \frac{1}{2RC} = 7 \therefore R = 4.949\Omega$

## CHAPTER NINE SOLUTIONS

2.  $i_c = 40e^{-100t} - 30e^{-200t}$  mA,  $C = 1\text{mF}$ ,  $v(0) = -0.25\text{V}$

(a)  $v(t) = \frac{1}{C} \int_0^t i_c dt - 0.25 = \int_0^t (40e^{-100t} - 30e^{-200t}) dt - 0.25$

$$\therefore v(t) = -0.4(e^{-100t} - 1) + 0.15(e^{-200t} - 1) - 0.25$$

$$\therefore v(t) = -0.4e^{-100t} + 0.15e^{-200t} \text{ V}$$

(b)  $s_1 = -100 = -\alpha + \sqrt{\alpha^2 - \omega_o^2}$ ,  $s_2 = -200 = -\alpha - \sqrt{\alpha^2 - \omega_o^2}$

$$\therefore -300 = -2\alpha, \alpha = 150 \text{ s}^{-1}$$

$$\therefore 150 + \frac{1}{2R \cdot 10^{-3}}, R = \frac{500}{150} = 3.333\Omega \text{ Also,}$$

$$-200 = -150 - \sqrt{22500 - \omega_o^2} \therefore \omega_o^2 = 20000$$

$$\therefore 20000 = \frac{1}{LC} = \frac{100}{L}, L = 0.5\text{H}$$

$$\therefore i_R(t) = \frac{v}{R} = 0.12e^{-100t} + 0.045e^{-200t} \text{ A}$$

(c)  $i(t) = -i_R(t) - i_c(t) = (0.12 - 0.04)e^{-100t} + (-0.045 + 0.03)e^{-200t}$

$$\therefore i(t) = 80e^{-100t} - 15e^{-200t} \text{ mA}, t > 0$$

## CHAPTER NINE SOLUTIONS

3. Parallel RLC with  $\omega_o = 70.71 \times 10^{12}$  rad/s.  $L = 2$  pH.

(a)  $\omega_o^2 = \frac{1}{LC} = (70.71 \times 10^{12})^2$

So  $C = \frac{1}{(70.71 \times 10^{12})^2 (2 \times 10^{-12})} = 100.0 \text{ aF}$

(b)  $\alpha = \frac{1}{2RC} = 5 \times 10^9 \text{ s}^{-1}$

So  $R = \frac{1}{(10^{10})(100 \times 10^{-18})} = 1 \text{ M}\Omega$

(c)  $\alpha$  is the neper frequency:  $5 \text{ Gs}^{-1}$

(d)  $S_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -5 \times 10^9 + j70.71 \times 10^{12} \text{ s}^{-1}$

$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2} = -5 \times 10^9 - j70.71 \times 10^{12} \text{ s}^{-1}$

(e)  $\zeta = \frac{\alpha}{\omega_o} = \frac{5 \times 10^9}{70.71 \times 10^{12}} = 7.071 \times 10^{-5}$

## CHAPTER NINE SOLUTIONS

4. Given:  $L = 4R^2C$ ,  $\alpha = \frac{1}{2RC}$

Show that  $v(t) = e^{-\alpha t}(A_1 t + A_2)$  is a solution to

$$C \frac{d^2 v}{dt^2} + \frac{1}{R} \frac{dv}{dt} + \frac{1}{L} v = 0 \quad [1]$$

$$\begin{aligned} \frac{dv}{dt} &= e^{-\alpha t}(A_1) - \alpha e^{-\alpha t}(A_1 t + A_2) \\ &= (A_1 - \alpha A_1 t - \alpha A_2) e^{-\alpha t} \end{aligned} \quad [2]$$

$$\begin{aligned} \frac{d^2 v}{dt^2} &= (A_1 - \alpha A_1 t - \alpha A_2)(-\alpha e^{-\alpha t}) - \alpha A_1 e^{-\alpha t} \\ &= -\alpha(A_1 - \alpha A_2 + A_1 - \alpha A_1 t) e^{-\alpha t} \\ &= -\alpha(2A_1 - \alpha A_2 - \alpha A_1 t) e^{-\alpha t} \end{aligned} \quad [3]$$

Substituting Eqs. [2] and [3] into Eq. [1], and using the information initially provided,

$$\begin{aligned} & -\frac{1}{2RC}(2A_1) e^{-\alpha t} + \left(\frac{1}{2RC}\right)^2 (A_1 t + A_2) e^{-\alpha t} + \frac{1}{RC}(A_1) e^{-\alpha t} \\ & -\frac{1}{2RC}(A_1 t + A_2) e^{-\alpha t} + \frac{1}{4R^2 C^2}(A_1 t + A_2) e^{-\alpha t} \\ & = 0 \end{aligned}$$

Thus,  $v(t) = e^{-\alpha t}(A_1 t + A_2)$  is in fact a solution to the differential equation.

Next, with  $v(0) = A_2 = 16$

$$\text{and } \left. \frac{dv}{dt} \right|_{t=0} = (A_1 - \alpha A_2) = (A_1 - 16\alpha) = 4$$

we find that  $A_1 = 4 + 16\alpha$



## CHAPTER NINE SOLUTIONS

5. Parallel RLC with  $\omega_o = 800 \text{ rad/s}$ , and  $\alpha = 1000 \text{ s}^{-1}$  when  $R = 100 \Omega$ .

$$\alpha = \frac{1}{2RC} \quad \text{so} \quad C = 5 \mu\text{F}$$

$$\omega_o^2 = \frac{1}{LC} \quad \text{so} \quad L = 312.5 \text{ mH}$$

Replace the resistor with 5 meters of 18 AWG copper wire. From Table 2.3, 18 AWG soft solid copper wire has a resistance of  $6.39 \Omega/1000\text{ft}$ . Thus, the wire has a resistance of

$$\begin{aligned} & (5 \text{ m}) \left( \frac{100 \text{ cm}}{1 \text{ m}} \right) \left( \frac{1 \text{ in}}{2.54 \text{ cm}} \right) \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) \left( \frac{6.39 \Omega}{1000 \text{ ft}} \right) \\ &= 0.1048 \Omega \quad \text{or} \quad 104.8 \text{ m}\Omega \end{aligned}$$

- (a) The resonant frequency is unchanged, so  $\omega_o = 800 \text{ rad/s}$

(b)  $\alpha = \frac{1}{2RC} = 954.0 \times 10^3 \text{ s}^{-1}$

(c)  $\zeta_{old} = \frac{\alpha_{old}}{\omega_o}$

$$\zeta_{new} = \frac{\alpha_{new}}{\omega_o}$$

Define the percent change as  $\frac{\zeta_{new} - \zeta_{old}}{\zeta_{old}} \times 100$

$$= \frac{\alpha_{new} - \alpha_{old}}{\alpha_{old}} \times 100$$

$$= 95300\%$$

## CHAPTER NINE SOLUTIONS

6.  $L = 5\text{H}$ ,  $R = 8\Omega$ ,  $C = 12.5\text{mF}$ ,  $v(0^+) = 40\text{V}$

(a)  $i(0^+) = 8\text{A}$ :  $\alpha = \frac{1}{2RC} = \frac{1000}{2 \times 8 \times 12.5} = 5$ ,  $\omega_o^2 = \frac{1}{LC} = 16$ ,  
 $\omega_o = 4$   $s_{1,2} = -5 \pm \sqrt{25 - 16} = -2, -8$ .  $\therefore v(t) = A_1 e^{-2t} + A_2 e^{-8t}$   
 $\therefore 40 = A_1 + A_2$   $v'(0^+) = \frac{1000}{12.5} \left( -i_L(0^+) - \frac{40}{8} \right) = 80(-8 - 5) = -1040$   
 $v/s = -2A_1 - 8A_2$   $\therefore -520 = -A_1 - 4A_2$   $\therefore -3A_2 = -480$ ,  $A_2 = 160$ ,  $A_1 = -120$   
 $\therefore v(t) = \boxed{-120e^{-2t} + 160e^{-8t}\text{V}, t > 0}$

(b)  $i_c(0^+) = 8\text{A}$  Let  $i(t) = A_3 e^{-2t} + A_4 e^{-8t}$ ;  $i_R(0^+) = \frac{v(0^+)}{R} = \frac{40}{8} = 5\text{A}$   
 $\therefore i(0^+) = A_3 + A_4 = -i_R(0^+) - i_c(0^+) = -8 - 5 = -13\text{A}$ ;  
 $i(0^+) = -2A_3 - 8A_4 = \frac{40}{5} = 8 \text{ A/s}$   $\therefore 4 = -A_3 - 4A_4$   
 $\therefore -3A_4 = -13 + 4$ ,  $A_4 = 3$ ,  $A_3 = -16$   $\therefore i(t) = \boxed{-16e^{-2t} + 3e^{-8t}\text{A}, t > 0}$

## CHAPTER NINE SOLUTIONS

7.  $i(0) = 40\text{A}$ ,  $v(0) = 40\text{V}$ ,  $L = \frac{1}{80}\text{H}$ ,  $R = 0.1\Omega$ ,  $C = 0.2\text{F}$

(a)  $\alpha = \frac{1}{2 \times 0.1 \times 0.2} = 25$ ,  $\omega_o^2 = \frac{80}{0.2} = 400$ ,

$\omega_o = 20$ ,  $s_{1,2} = -25 \pm \sqrt{625 - 400} = 10, -40$

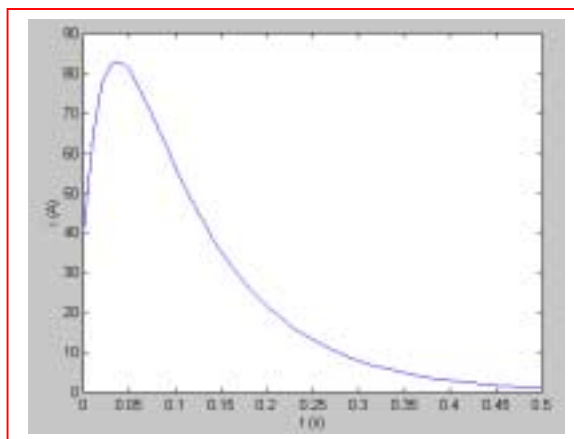
$\therefore v(t) = A_1 e^{-10t} + A_2 e^{-40t} \therefore 40 = A_1 + A_2$ ;

$v'(0^+) = -10A_1 - 40A_2$   $v'(0^+) = \frac{1}{C} \left( i(0) - \frac{v(0)}{R} \right) = -2200$

$\therefore -A_1 - 4A_2 = -220 \therefore -3A_2 = -180 \therefore A_2 = 60$ ,  $A_1 = -20$

$\therefore v(t) = -20e^{-10t} + 60e^{-40t} \text{V}, t > 0$

(b)  $i(t) = -v/R - C \frac{dv}{dt} = 200e^{-10t} - 600e^{-40t} - 0.2(-20)(-10)e^{-10t} - (0.2)(60)(-40)e^{-40t}$   
 $= 160e^{-10t} - 120e^{-40t} \text{ A}$



## CHAPTER NINE SOLUTIONS

8.  $i_L(0) = \frac{100}{50} = 2\text{A}$ ,  $v_c(0) = 100\text{V}$

$$\alpha = \frac{10^6}{2 \times 50 \times 2.5} = 4000, \quad \omega_o^2 = \frac{3 \times 10^{6+3}}{100 \times 2.5} = 12 \times 10^6$$

$$\sqrt{16 - 12} \times 10^3 = 200, \quad s_{1,2} = -4000 \pm 2000$$

$$\therefore i_L(t) = A_1 e^{-2000t} + A_2 e^{-6000t}, \quad t > 0 \quad \therefore A_1 + A_2 = 2$$

$$i_L'(0^+) = \frac{-10^3 \times 3}{100} \times 100 = -3000 = -2000A_1 - 6000A_2 \quad \therefore -1.5 = -A_1 - 3A_2 \quad \therefore 0.5 = -2A_2$$

$$\therefore A_2 = -0.25, \quad A_1 = 2.25 \quad \therefore i_L(t) = 2.25e^{-2000t} - 0.25e^{-6000t} \text{ A}, \quad t > 0$$

$$t > 0: i_L(t) = 2\text{A} \quad \therefore i_L(t) = \boxed{2u(-t) + (2.25e^{-2000t} - 0.25e^{-6000t})u(t)\text{A}}$$

## CHAPTER NINE SOLUTIONS

9.

$$i_L(0) = \frac{12}{5+1} = 2\text{A}, v_c(0) = 2\text{V}$$
$$\alpha = \frac{1000}{2 \times 1 \times 2} = 250, \omega_o^2 = \frac{1000 \times 45}{2} = 22500$$
$$s_{1,2} = -250 \pm \sqrt{250^2 - 22500} = -50, -450 \text{ s}^{-1}$$
$$\therefore i_L = A_1 e^{-50t} + A_2 e^{-450t} \therefore A_1 + A_2 = 2; i_L'(0^+) = 45(-2) = -50A_1 - 450A_2$$
$$\therefore A_1 + 9A_2 = 1.8 \therefore -8A_2 = 0.2 \therefore A_2 = -0.025, A_1 = 2.025(\text{A})$$
$$\therefore i_L(t) = \boxed{2.025e^{-50t} - 0.025e^{-450t} \text{ A}, t > 0}$$

## CHAPTER NINE SOLUTIONS

10.

$$(a) \quad \alpha = \frac{1}{2RC} = \frac{1440}{72} = 20, \quad \omega_o^2 = \frac{1440}{10} = 144$$

$$s_{1,2} = -20 \pm \sqrt{400 - 144} = -4, -36: v = A_1 e^{-4t} + A_2 e^{-36t}$$

$$v(0) = 18 = A_1 + A_2, \quad v'(0) = 1440 \left( \frac{1}{2} - \frac{18}{36} \right) = 0$$

$$\therefore 0 = -4A_1 - 36A_2 = -A_1 - 9A_2 = \therefore 18 = -8A_2, \quad A_2 = -2.25, \quad A_1 = 20.25$$

$$\therefore v(t) = \boxed{20.25e^{-4t} - 2.25e^{-36t} \text{ V}, t > 0}$$

$$(b) \quad i(t) = \frac{v}{36} + \frac{1}{1440} v' = 0.5625e^{-4t} - 0.0625e^{-36t} - 0.05625e^{-4t} + 0.05625e^{-36t}$$

$$\therefore i(t) = \boxed{0.50625e^{-4t} - 0.00625e^{-36t} \text{ A}, t > 0}$$

$$(c) \quad v_{\max} \text{ at } t = 0 \therefore v_{\max} = 18 \text{ V} \therefore 0.18 = 20.25e^{-4t_s} - 2.25e^{-36t_s}$$

Solving using a scientific calculator, we find that  $t_s = 1.181 \text{ s}$ .

## CHAPTER NINE SOLUTIONS

11.  $L = 1250 \text{ mH}$

so  $\omega_o = \frac{1}{\sqrt{LC}} = 4 \text{ rad/s}$  Since  $\alpha > \omega_o$ , this circuit is over damped.

$$\alpha = \frac{1}{2RC} = 5 \text{ s}^{-1}$$

The capacitor stores 390 J at  $t = 0^-$ :

$$W_c = \frac{1}{2} C v_c^2$$

$$\text{So } v_c(0^+) = \sqrt{\frac{2W_c}{C}} = 125 \text{ V} = v_c(0^+)$$

The inductor initially stores zero energy,

so  $i_L(0^-) = i_L(0^+) = 0$

$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -5 \pm 3 = -8, -2$$

Thus,  $v(t) = Ae^{-8t} + Be^{-2t}$

Using the initial conditions,  $v(0) = 125 = A + B$  [1]

$$i_L(0^+) + i_R(0^+) + i_c(0^+) = 0 + \frac{v(0^+)}{2} + i_c(0^+) = 0$$

$$\text{So } i_c(0^+) = -\frac{v(0^+)}{2} = -\frac{125}{2} = -62.5 \text{ V}$$

$$i_c = C \frac{dv}{dt} = 50 \times 10^{-3} [-8Ae^{-8t} - 2Be^{-2t}]$$

$$i_c(0^+) = -62.5 = -50 \times 10^{-3} (8A + 2B) \quad [2]$$

Solving Eqs. [1] and [2],  $A = 150 \text{ V}$   
 $B = -25 \text{ V}$

Thus,  $v(t) = 166.7e^{-8t} - 41.67e^{-2t}, t > 0$

## CHAPTER NINE SOLUTIONS

12. (a) We want a response  $v = Ae^{-4t} + Be^{-6t}$

$$\alpha = \frac{1}{2RC} = 5 \text{ s}^{-1}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -4 = -5 + \sqrt{25 - \omega_o^2}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2} = -6 = -5 - \sqrt{25 - \omega_o^2}$$

Solving either equation, we obtain  $\omega_o = 4.899 \text{ rad/s}$

Since  $\omega_o^2 = \frac{1}{LC}$ ,  $L = \frac{1}{\omega_o^2 C} = \boxed{833.3 \text{ mH}}$

- (b) If  $i_R(0^+) = 10 \text{ A}$  and  $i_c(0^+) = 15 \text{ A}$ , find A and B.

with  $i_R(0^+) = 10 \text{ A}$ ,  $v_R(0^+) = v(0^+) = v_c(0^+) = 20 \text{ V}$

$$v(0) = A + B = 20 \quad [1]$$

$$i_c = C \frac{dv}{dt} = 50 \times 10^{-3} (-4Ae^{-4t} - 6Be^{-6t})$$

$$i_c(0^+) = 50 \times 10^{-3} (-4A - 6B) = 15 \quad [2]$$

Solving,  $\boxed{A = 210 \text{ V}, B = -190 \text{ V}}$

Thus,  $v = 210e^{-4t} - 190e^{-6t}$ ,  $t > 0$



## CHAPTER NINE SOLUTIONS

13. Initial conditions:  $i_L(0^-) = i_L(0^+) = 0$   $i_R(0^+) = \frac{50}{25} = 2 \text{ A}$

(a)  $v_c(0^+) = v_c(0^-) = 2(25) = 50 \text{ V}$

(b)  $i_c(0^+) = -i_L(0^+) - i_R(0^+) = 0 - 2 = -2 \text{ A}$

(c)  $t > 0$ : parallel (source-free) RLC circuit

$$\alpha = \frac{1}{2RC} = 4000 \text{ s}^{-1} \qquad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

$$\omega_o = \frac{1}{\sqrt{LC}} = 3464 \text{ rad/s} \qquad \qquad \qquad = -2000, -6000$$

Since  $\alpha > \omega_o$ , this system is overdamped. Thus,

$$v_c(t) = Ae^{-2000t} + Be^{-6000t}$$

$$i_c = C \frac{dv}{dt} = (5 \times 10^{-6})(-2000 Ae^{-2000t} - 6000 Be^{-6000t})$$

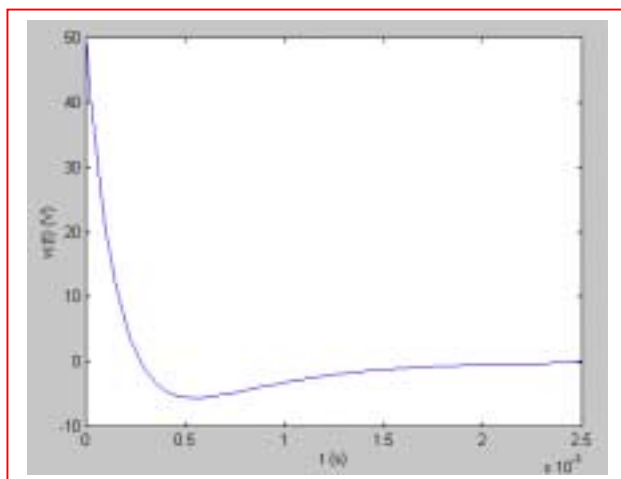
$$i_c(0^+) = -0.01A - 0.03B = -2 \qquad [1]$$

$$\text{and } v_c(0^+) = A + B = 50 \qquad [2]$$

Solving, we find  $A = -25$  and  $B = 75$

so that  $v_c(t) = -25e^{-2000t} + 75e^{-6000t}$ ,  $t > 0$

(d)



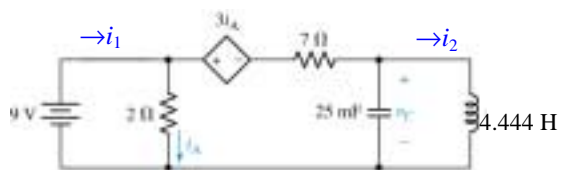
(e)  $-25e^{-2000t} + 75e^{-6000t} = 0 \Rightarrow t = 274.7 \mu\text{s}$   
using a scientific calculator

(f)  $|v_c|_{\max} = -25 + 75 = 50 \text{ V}$

So, solving  $|-25e^{-2000t_s} + 75e^{-6000t_s}| = 0.5$  in view of the graph in part (d), we find  $t_s = 1.955 \text{ ms}$  using a scientific calculator's equation solver routine.

## CHAPTER NINE SOLUTIONS

14. Due to the presence of the inductor,  $v_c(0^-) = 0$ . Performing mesh analysis,



$$-9 + 2i_1 - 2i_2 = 0 \quad [1]$$

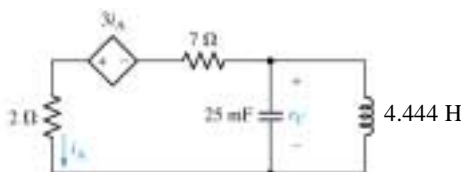
$$2i_2 - 2i_1 + 3i_A + 7i_2 = 0 \quad [2]$$

$$\text{and } i_1 - i_2 = i_A$$

Rearranging, we obtain  $2i_1 - 2i_2 = 0$  and  $-4i_1 + 6i_2 = 0$ . Solving,  $i_1 = 13.5$  A and  $i_2 = 9$  A.

(a)  $i_A(0^-) = i_1 - i_2 = 4.5$  A and  $i_L(0^-) = i_2 = 9$  A

(b)  $t > 0$ :



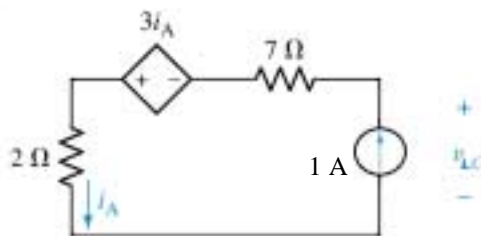
around left mesh:

$$-v_c(0^+) + 7i_A(0^+) - 3i_A(0^+) + 2i_A(0^+) = 0$$

$$\text{so, } i_A(0^+) = 0$$

(c)  $v_c(0^-) = 0$  due to the presence of the inductor.

(d)



$$-v_{LC} + 7 - 3(1) + 2 = 0$$

$$v_{LC} = 6 \text{ V} \therefore R_{TH} = \frac{6}{1} = 6 \Omega$$

(e)  $\alpha = \frac{1}{2RC} = 3.333 \text{ s}^{-1}$

$$\omega_o = \frac{1}{\sqrt{LC}} = 3 \text{ rad/s}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -1.881, -4.785$$

Thus,

$$i_A(t) = Ae^{-1.881t} + Be^{-4.785t}$$

$$i_A(0^+) = 0 = A + B \quad [1]$$

To find the second equation required to determine the coefficients, we write:

$$i_L = -i_c - i_R$$

$$= -C \frac{dv_c}{dt} - i_A = -25 \times 10^{-3} \left[ -1.881(6A)e^{-1.881t} - 4.785(6B)e^{-4.785t} \right] - Ae^{-1.881t} - Be^{-4.785t}$$

$$i_L(0^+) = 9 = -25 \times 10^{-3} [-1.881(6A) - 4.785(6B)] - A - B$$

$$\text{or } 9 = -0.7178A - 0.2822B \quad [2]$$

Solving Eqs. [1] and [2],  $A = -20.66$  and  $B = +20.66$

$$\text{So that } i_A(t) = 20.66[e^{-4.785t} - e^{-1.881t}]$$

## CHAPTER NINE SOLUTIONS

15. Diameter of a dime: approximately 8 mm. Area =  $\pi r^2 = 0.5027\text{cm}^2$

$$\begin{aligned}\text{Capacitance} &= \frac{\epsilon_r \epsilon_o A}{d} = \frac{(88)(8.854 \times 10^{-14} \text{F/cm})(0.5027 \text{cm}^2)}{0.1 \text{cm}} \\ &= 39.17 \text{pF}\end{aligned}$$

$$L = 4\mu\text{H}$$

$$\omega_o = \frac{1}{\sqrt{LC}} = 79.89 \text{Mrad/s}$$

For an over damped response, we require  $\alpha > \omega_o$ .

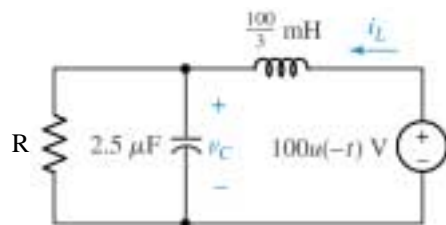
$$\begin{aligned}\text{Thus, } \frac{1}{2RC} &> 79.89 \times 10^6 \\ R &< \frac{1}{2(39.17 \times 10^{-12})(79.89 \times 10^6)}\end{aligned}$$

$$\text{or } R < 159.8 \Omega$$

\*Note: The final answer depends quite strongly on the choice of  $\epsilon_r$ .

## CHAPTER NINE SOLUTIONS

16.



crit. damp. (a)  $L = 4R^2C = \frac{100}{3} \times 10^{-3} = 4R^2 \times 10^{-6} \therefore R = 57.74\Omega$

(b)  $\omega_o = \alpha = 10^3 / \sqrt{\frac{1}{30} \times 2.5} = 3464 s^{-1}$

$$\therefore v_c(t) = e^{-3464t} (A_1 t + A_2) \quad v_c(0) = 100V$$

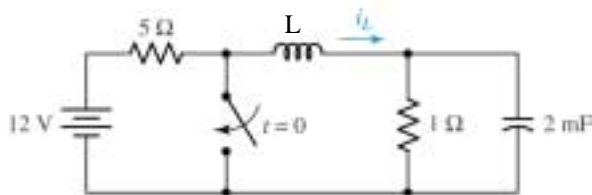
$$i_L(0) = \frac{100}{57.74} = 1.7321A \therefore 100 = A_2$$

$$v_c'(0^+) = \frac{10^6}{2.5} \left( 1.7321 - \frac{100}{57.74} \right) = 0 = A_1 - 3464A_2 \therefore A_1 = 3.464 \times 10^5$$

$$\therefore v_c(t) = e^{-3464t} (3.464 \times 10^5 t + 100) V, t > 0$$

## CHAPTER NINE SOLUTIONS

17.



crit. damp. (a)  $L = 4R^2C = 4 \times 1 \times 2 \times 10^{-3} = 8\text{mH}$

(b)  $\alpha = \omega_o \frac{1}{2RC} = \frac{1000}{2 \times 1 \times 2} = 250 \therefore i_L = e^{-250t} (A_1 t + A_2)$

$i_L(0) = 2\text{A}, v_c(0) = 2\text{V} \therefore i_L = e^{-250t} (A_1 t + 2)$

Then  $8 \times 10^{-3} i_L'(0^+) = -2 = 8 \times 10^{-3} (A_1 - 500), = e^{-1.25} (1.25 + 2) = 0.9311\text{A}$

(c)  $i_{L\max} : (250t_m + 2) = 0, 1 = 250t_m + 2, t_m < 0 \text{ No!}$

$\therefore t_m = 0, i_{L\max} = 2\text{A} \therefore 0.02 = e^{-250t_s} (250t_s + 2); \text{SOLVE: } t_s = 23.96\text{ms}$

## CHAPTER NINE SOLUTIONS

18.  $L = 5\text{mH}$ ,  $C = 10^{-8}\text{ F}$ , crit. damp.  $v(0) = -400\text{V}$ ,  $i(0) = 0.1\text{A}$

(a)  $L = 4R^2C = 5 \times 10^{-3} = 4R^2 10^{-8} \therefore R = \boxed{353.6\Omega}$

(b)  $\alpha = \frac{10^8}{2 \times 353.6} = 141,420 \therefore i = e^{-141,420t} (A_1 t + A_2)$   
 $\therefore A_2 = 0.1 \therefore = e^{-141,421t} (A_1 t + 0.1), 5 \times 10^{-3}$   
 $(A_1 - 141,420 \times 0.1) = -400 \therefore A_1 = -65,860$   
 $\therefore i = e^{-141,421t} (-65,860t + 0.1). i' = 0$   
 $\therefore e^{-\alpha t} (+65860) + 141,420 e^{-\alpha t} (-65,860t_m + 0.1) = 0$   
 $\therefore t_m = 8.590 \mu\text{s} \therefore i(t_m) = e^{-141,420 \times 8.590 \times 10^{-6}}$   
 $(-65,860 \times 8.590 \times 10^{-6} + 0.1) = -0.13821\text{A}$   
 $\therefore |i|_{\max} = |i(t_m)| = \boxed{0.13821\text{A}}$

(c)  $\therefore i_{\max} = i(0) = \boxed{0.1\text{A}}$

## CHAPTER NINE SOLUTIONS

19. Diameter of a dime is approximately 8 mm. The area, therefore, is  $\pi r^2 = 0.5027 \text{ cm}^2$ .

$$\begin{aligned}\text{The capacitance is } \frac{\epsilon_r \epsilon_o A}{d} &= \frac{(88)(8.854 \times 10^{-14})(0.5027)}{0.1} \\ &= 39.17 \text{ pF}\end{aligned}$$

$$\text{with } L = 4 \mu\text{H}, \omega_o = \frac{1}{\sqrt{LC}} = 79.89 \text{ Mrad/s}$$

$$\text{For critical damping, we require } \frac{1}{2RC} = \omega_o$$

$$\text{or } R = \frac{1}{2\omega_o C} = 159.8 \Omega$$

## CHAPTER NINE SOLUTIONS

20. Critically damped parallel RLC with  $\alpha = 10^{-3} \text{ s}^{-1}$ ,  $R = 1 \text{ M}\Omega$ .

We know  $\frac{1}{2RC} = 10^{-3}$ , so  $C = \frac{10^3}{2 \times 10^6} = 500 \mu\text{F}$

Since  $\alpha = \omega_o$ ,  $\omega_o = \frac{1}{\sqrt{LC}} = 10^{-3}$

$$\text{or } \frac{1}{LC} = 10^{-6}$$

so  $L = 2 \text{ GH (!)}$

$$L = \frac{\mu N^2 A}{S} = 2 \times 10^9$$

$$\text{If } S_o \frac{(4\pi \times 10^{-7} \text{ H/m}) \left[ \left( \frac{50 \text{ turns}}{\text{cm}} \right) s \right]^2 (0.5 \text{ cm})^2 \cdot \pi \cdot \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)}{s}$$

$$= 2 \times 10^9$$

$$(4\pi^2 \times 10^{-9}) (50)^2 (0.5)^2 s = 2 \times 10^9$$

$$\text{So } s = 8.106 \times 10^{13} \text{ cm}$$



## CHAPTER NINE SOLUTIONS

21.

$$\alpha = \frac{1}{2RC} = \frac{10^6}{100 \times 2.5} = 4000, \omega_o^2 = \frac{1}{LC} = \frac{10^{6+3}}{50} = 2 \times 10^7$$

$$\omega_d = \sqrt{20 \times 10^6 - 16 \times 10^6} = 2000$$

$$\therefore i_c = e^{-4000t} (B_1 \cos 2000t + B_2 \sin 2000t)$$

$$i_L(0) = 2A, v_c(0) = 0 \therefore i_c(0^+) = -2A; i_c'(0^+) = -i_L'(0^+) - i_R'(0^+)$$

$$\therefore i_c'(0^+) = -\frac{1}{L}v_c(0) - \frac{1}{R}v_c'(0^+) = 0 - \frac{1}{RC}i_c(0^+) = \frac{2 \times 10^6}{125}$$

$$\therefore B_1 = -2A, \frac{2 \times 10^6}{125} = 16,000 = 2000B_2 + (-2)(-4000) \therefore B_2 = 4$$

$$\therefore i_c(t) = e^{-4000t} (-2 \cos 2000t + 4 \sin 2000t)A, t > 0$$

## CHAPTER NINE SOLUTIONS

22.

$$\alpha = \frac{1}{2RC} = \frac{4}{2 \times 2} = 1, \omega_o^2 = \frac{1}{LC} = \frac{4 \times 13}{2} = 26, \omega_d = \sqrt{26-1} = 5$$

$$\therefore v_c(t) = e^{-t}(B_1 \cos 5t + B_2 \sin 5t)$$

(a)  $i_L(0^+) = i_L(0) = 4\text{A}$

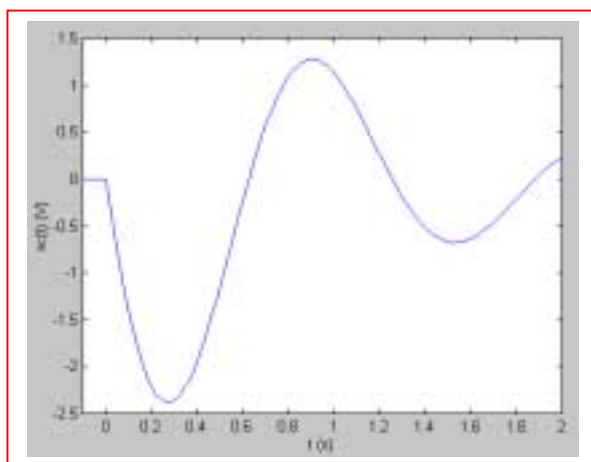
(b)  $v_c(0^+) = v_c(0) = 0$

(c)  $i_L'(0^+) = \frac{1}{L}v_c(0^+) = 0$

(d)  $v_c'(0^+) = \frac{1}{C}[-i_L(0^+) - i_R(0^+)] = 4\left[-4 - \frac{v_c(0^+)}{2}\right] = 4(-4+0) = -16 \text{ V/s}$

(e)  $\therefore (e)0 = 1(B_1) \therefore B_1 = 0, v_c(t) = B_2 e^{-t} \sin 5t, v_c'(0^+) = B_2(5) = -16$   
 $\therefore B_2 = -3.2, v_c(t) = -3.2e^{-t} \sin 5t \text{ V}, t > 0$

(f)



## CHAPTER NINE SOLUTIONS

23.

$$(a) \quad \alpha \frac{1}{2RC} = \frac{10^{9-3}}{2 \times 20 \times 5} = 5000, \quad \omega_o^2 = \frac{1}{LC} = \frac{10^9}{1.6 \times 5} = 1.25 \times 10^8$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{125 \times 10^6 - 25 \times 10^6} = 10,000$$

$$\therefore v_c(t) = e^{-5000t} (B_1 \cos 10^4 t + B_2 \sin 10^4 t)$$

$$v_c(0) = 200V, \quad i_L(0) = 10mA \quad \therefore v_c(t) = e^{-5000t} (200 \cos 10^4 t + B_2 \sin 10^4 t)$$

$$v_c'(0^+) = \frac{1}{C} i_c(0^+) = \frac{10^9}{5} \left[ i_L(0) - \frac{v_c(0)}{20,000} \right]$$

$$= \frac{10^9}{5} \left( 10^{-2} - \frac{200}{20,000} \right) = 0 = 10^4 B_2 - 200(5000)$$

$$\therefore B_2 = 100V \quad \therefore v_c(t) = e^{-5000t} (200 \cos 10^4 t + 100 \sin 10^4 t) \text{ V}, \quad t > 0$$

$$(b) \quad i_{sw} = 10^{-2} - i_L, \quad i_L = \frac{1}{R} v_c + C v_c'$$

$$v_c' = e^{-5000t} [10^4 (-200 \sin + 100 \cos) - 5000 (200 \cos + 100 \sin)]$$

$$= e^{-5000t} [10^6 (-2 \sin - 0.5 \cos)] = -2.5 \times 10^6 e^{-5000t} \sin 10^4 t \text{ V/s}$$

$$\therefore i_L = e^{-5000t} \left[ \frac{1}{20,000} (200 \cos + 100 \sin) - 5 \times 10^{-9} \times 2.5 \times 10^6 e^{-5000t} \sin 10^4 t \right]$$

$$= e^{-5000t} (0.01 \cos 10^4 t - 0.0075 \sin 10^4 t) \text{ A}$$

$$\therefore i_{sw} = 10 - e^{-5000t} (10 \cos 10^4 t - 7.5 \sin 10^4 t) \text{ mA}, \quad t > 0$$

## CHAPTER NINE SOLUTIONS

24.

$$(a) \quad \alpha = \frac{1}{2RC} = \frac{100}{12.5} = 8, \quad \omega_o^2 = \frac{1}{LC} = \frac{100}{L}, \quad \omega_d^2 = 36 = \omega_o^2 - 64$$

$$\therefore \omega_o^2 = 100 = \frac{100}{L} \therefore L = 1\text{H}$$

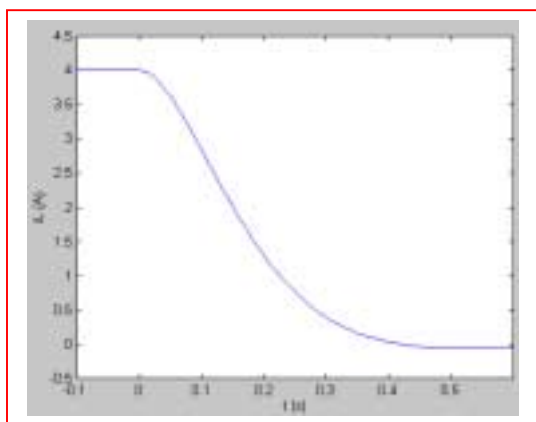
$$(b) \quad t < 0: i_L(t) = 4\text{A}; \quad t > 0: i_L(t) = e^{-8t} (B_1 \cos 6t + B_2 \sin 6t)$$

$$i_L(0) = 4\text{A} \therefore B_1 = 4\text{A}, \quad i_L = e^{-8t} (4 \cos 6t + B_2 \sin 6t) \quad v_c(0) = 0$$

$$i_L'(0^+) = -v_c(0^+) = 0 \therefore 6B_2 - 8(4) = 0, \quad B_2 = 16/3$$

$$\therefore i_L(t) = 4u(-t) + e^{-8t} (4 \cos 6t + 5.333 \sin 6t) u(t) \text{ A}$$

(c)



## CHAPTER NINE SOLUTIONS

25.

$$\alpha = \frac{1}{2RC} = \frac{10^{6-3}}{2 \times 5} = 100s^{-1}, \quad \omega_o^2 = \frac{1}{LC} = 1.01 \times 10^6$$

$$\therefore \omega_d = \sqrt{101 \times 10^4 - 10^4} = 100; \quad i_L(0) = \frac{60}{10} = 6\text{mA}$$

$$v_c(0) = 0 \therefore v_c(t) = e^{-100t} (A_1 \cos 1000t + A_2 \sin 1000t), \quad t > 0$$

$$\therefore A_1 = 0, \quad v_c(t) = A_2 e^{-100t} \sin 1000t$$

$$v_c'(0^+) = \frac{1}{C} i_c(0^+) = 10^6 [-i_1(0^+) - \frac{1}{5000} v_c(0^+)] = 10^6$$

$$(-6 \times 10^{-3}) = -6000 = 1000 A_2 \therefore A_2 = -6$$

$$\therefore v_c(t) = -6e^{-100t} \sin 1000t \text{V}, \quad t > 0 \therefore i_1(t) = -\frac{1}{10^4}$$

$$v_c(t) = -10^{-4}(-6)e^{-100t} \sin 1000t \text{A}$$

$$\therefore i_1(t) = 0.6e^{-100t} \sin 1000t \text{mA}, \quad t > 0$$

## CHAPTER NINE SOLUTIONS

26.

$$(a) \quad \alpha = \frac{1}{2RC} = \frac{10^6}{2000 \times 25} = 20, \quad \omega_o^2 = \frac{1}{LC} = \frac{1.01 \times 10^6}{25} = 40,400$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{40,400 - 400} = 200$$

$$\therefore v = e^{-20t} (A_1 \cos 200t + A_2 \sin 200t)$$

$$v(0) = 10V, \quad i_L(0) = 9mA \therefore A_1 = 10V$$

$$\therefore v = e^{-20t} (10 \cos 200t + A_2 \sin 200t) \text{ V}, \quad t > 0$$

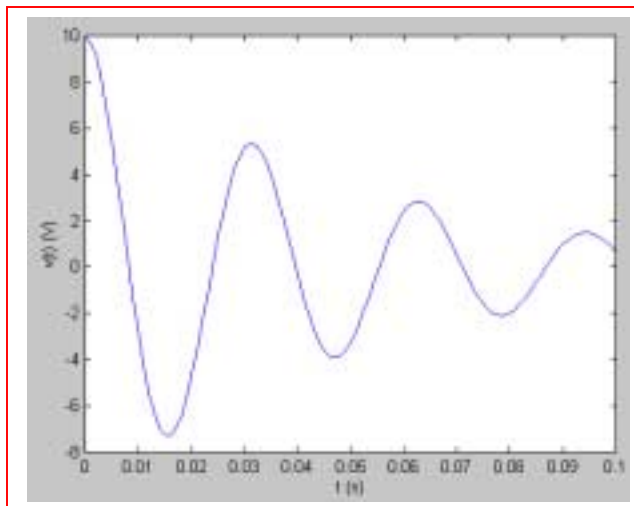
$$v'(0^+) = 200A_2 - 20 \times 10 = 200(A_2 - 1) = \frac{1}{C} i_o(0^+)$$

$$= \frac{10^6}{25} (-10^{-3}) = -40 \therefore A_2 = 1 - 0.2 = 0.8$$

$$\therefore v(t) = e^{-20t} (10 \cos 200t + 0.8 \sin 200t) \text{ V}, \quad t > 0$$

$$(b) \quad v = 10.032e^{-20t} \cos (200t - 4.574^\circ) V$$

$$T = \frac{2\pi}{200} = 3.42ms$$



## CHAPTER NINE SOLUTIONS

27.

$$v(0) = 0; i(0) = 10\text{A}$$

$$v = e^{\alpha t} (A \cos \omega_d t + B \sin \omega_d t) \therefore A = 0,$$

$$v = B e^{-\alpha t} \sin \omega_d t$$

$$v' = e^{-\alpha t} [-\alpha B \sin \omega_d t + \omega_d B \cos \omega_d t] = 0$$

$$\therefore \tan \omega_d t = \frac{\omega_d}{\alpha}, t_{m1} = \frac{1}{\omega_d} \tan^{-1} \frac{\omega_d}{\alpha}$$

$$t_{m2} = t_{m1} + \frac{1}{2} T_d = t_{m1} + \frac{\pi}{\omega_d};$$

$$v_{m1} = B e^{-\alpha t_{m1}} \sin \omega_d t_{m1} \quad v_{m2} = -B e^{-\alpha t_{m1} - \alpha \pi / \omega_d}$$

$$\sin \omega_d t_{m1} \therefore \frac{v_{m2}}{v_{m1}} = -e^{-\alpha \pi / \omega_d}; \text{ let } \left| \frac{v_{m2}}{v_{m1}} \right| = \frac{1}{100}$$

$$\therefore e^{\alpha \pi / \omega_d} = 100, \alpha = \frac{\omega_d}{\pi} \ln 100; \alpha = \frac{1}{2RC} = \frac{21}{R},$$

$$\omega_0^2 = \frac{1}{LC} = 6 \therefore \omega_d = \sqrt{6 - 441/R^2} \therefore \frac{21}{R} \frac{\ln 100}{\pi R} \sqrt{6R^2 - 441}$$

$$\therefore R = \sqrt{1/6 \left[ 441 + \left( \frac{21\pi}{100} \right)^2 \right]} = 10.3781 \Omega \text{ To keep}$$

$$\left| \frac{v_{m2}}{v_{m1}} \right| < 0.01, \text{ chose } R = 10.3780 \Omega \quad v'(0^+) = \omega_d$$

$$B = B \sqrt{6 - \left( \frac{21}{10.378} \right)^2} = 4R \left( 10 + \frac{0}{10.3780} \right) \therefore B = 1.380363$$

$$\alpha = \frac{21}{10.378} = 2.02351; \omega_d = \sqrt{6 - \left( \frac{21}{10.378} \right)^2} = 1.380363$$

$$\therefore v = 304.268 e^{-2.02351t} \sin 1.380363t \quad v_{t_{m1}} = 0.434s,$$

$$v_{m1} = 71.2926v \text{ Computed values show}$$

$$t_s = 2.145 \text{ sec}; v_{m2} = 0.7126 < 0.01 v_{m1}$$

## CHAPTER NINE SOLUTIONS

28. We replace the  $25\text{-}\Omega$  resistor to obtain an underdamped response:

$$\alpha = \frac{1}{2RC} \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{LC}}; \text{ we require } \alpha < \omega_0.$$

$$\text{Thus, } \frac{1}{10 \times 10^{-6} R} < 3464 \quad \text{or} \quad R > 34.64 \text{ m}\Omega.$$

For  $R = 34.64 \text{ }\Omega$  ( $1000\times$  the minimum required value), the response is:

$$v(t) = e^{-\alpha t} (A \cos \omega_d t + B \sin \omega_d t) \text{ where } \alpha = 2887 \text{ s}^{-1} \text{ and } \omega_d = 1914 \text{ rad/s.}$$

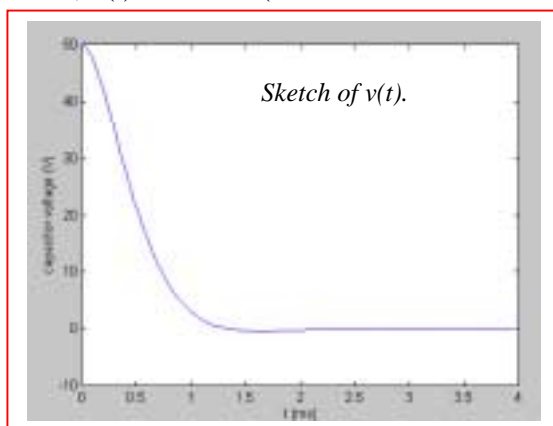
$$i_L(0^+) = i_L(0^-) = 0 \text{ and } v_C(0^+) = v_C(0^-) = (2)(25) = 50 \text{ V} = A.$$

$$i_L(t) = L \frac{dv_L}{dt} = L \frac{dv_C}{dt}$$

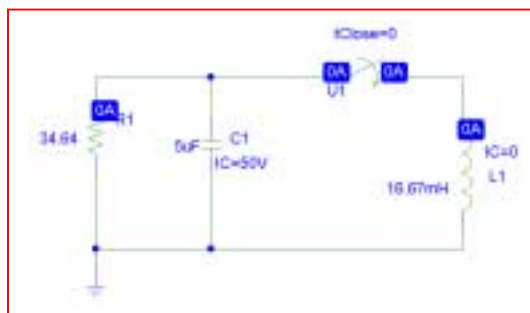
$$= L \left[ e^{-\alpha t} (-A \omega_d \sin \omega_d t + B \omega_d \cos \omega_d t) - \alpha e^{-\alpha t} (A \cos \omega_d t + B \sin \omega_d t) \right]$$

$$i_L(0^+) = 0 = \frac{50 \times 10^{-3}}{3} [B \omega_d - \alpha A], \text{ so that } B = 75.42 \text{ V.}$$

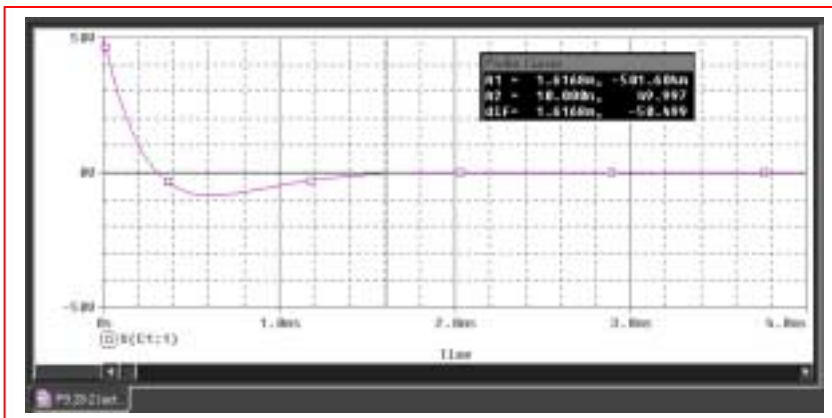
$$\text{Thus, } v(t) = e^{-2887t} (50 \cos 1914t + 75.42 \sin 1914t) \text{ V.}$$



**PSpice schematic for  $t > 0$  circuit.**



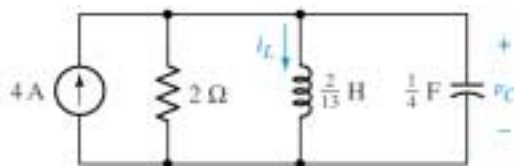
From PSpice the settling time using  $R = 34.64 \text{ }\Omega$  is approximately 1.6 ms.





## CHAPTER NINE SOLUTIONS

29. (a,b) For  $t < 0$  s, we see from the circuit below that the capacitor and the resistor are shorted by the presence of the inductor. Hence,  $i_L(0^-) = 4$  A and  $v_C(0^-) = 0$  V.



When the 4-A source turns off at  $t = 0$  s, we are left with a parallel RLC circuit such that  $\alpha = 1/2RC = 1 \text{ s}^{-1}$  and  $\omega_0 = 5.099 \text{ rad/s}$ . Since  $\alpha < \omega_0$ , the response will be underdamped with  $\omega_d = 5 \text{ rad/s}$ . Assume the form  $i_L(t) = e^{-\alpha t} (C \cos \omega_d t + D \sin \omega_d t)$  for the response.

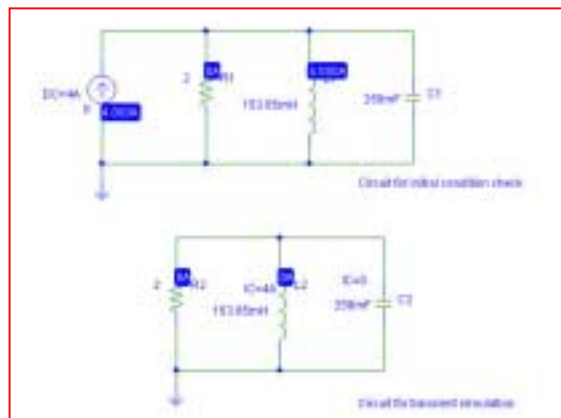
With  $i_L(0^+) = i_L(0^-) = 4$  A, we find  $C = 4$  A. To find  $D$ , we first note that

$$v_C(t) = v_L(t) = L \frac{di_L}{dt}$$

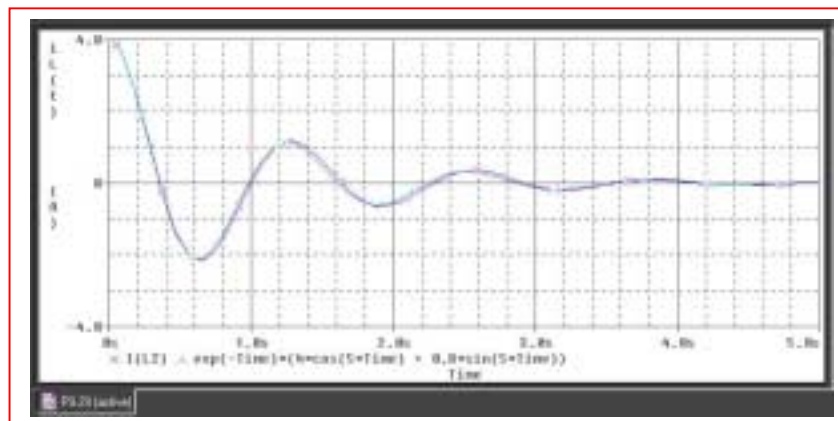
and so  $v_C(t) = (2/13) [e^{-\alpha t} (-C\omega_d \sin \omega_d t + D\omega_d \cos \omega_d t) - \alpha e^{-\alpha t} (C \cos \omega_d t + D \sin \omega_d t)]$

With  $v_C(0^+) = 0 = (2/13) (5D - 4)$ , we obtain  $D = 0.8$  A.

Thus,  $i_L(t) = e^{-t} (4 \cos 5t + 0.8 \sin 5t)$  A



We see that the simulation result confirms our hand analysis; there is only a slight difference due to numerical error between the simulation result and our exact expression.



- (c) Using the cursor tool, the settling time is approximately 4.65 s.

Probe Cursor			
A1 =	4.6493,	-40.658m	
A2 =	1.0000m,	3.9999	
diff =	4.6483,	-4.0406	

## CHAPTER NINE SOLUTIONS

30. (a) For  $t < 0$  s, we see from the circuit that the capacitor and the resistor are shorted by the presence of the inductor. Hence,  $i_L(0^-) = 4$  A and  $v_C(0^-) = 0$  V.

When the 4-A source turns off at  $t = 0$  s, we are left with a parallel RLC circuit such that  $\alpha = 1/2RC = 0.4 \text{ s}^{-1}$  and  $\omega_0 = 5.099 \text{ rad/s}$ . Since  $\alpha < \omega_0$ , the response will be underdamped with  $\omega_d = 5.083 \text{ rad/s}$ . Assume the form  $i_L(t) = e^{-\alpha t} (C \cos \omega_d t + D \sin \omega_d t)$  for the response.

With  $i_L(0^+) = i_L(0^-) = 4$  A, we find  $C = 4$  A. To find  $D$ , we first note that

$$v_C(t) = v_L(t) = L \frac{di_L}{dt}$$

and so  $v_C(t) = (2/13) [e^{-\alpha t} (-C\omega_d \sin \omega_d t + D\omega_d \cos \omega_d t) - \alpha e^{-\alpha t} (C \cos \omega_d t + D \sin \omega_d t)]$

With  $v_C(0^+) = 0 = (2/13) (5.083D - 0.4C)$ , we obtain  $D = 0.3148$  A.

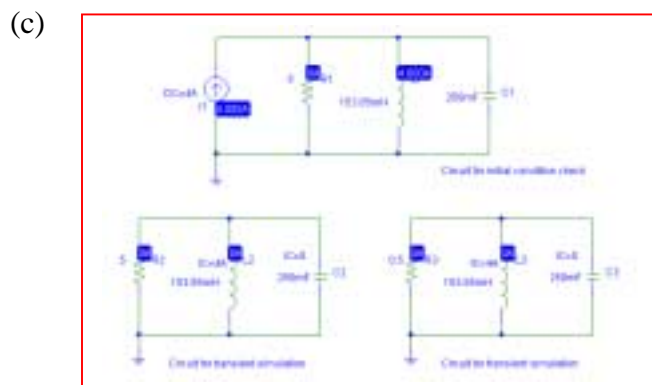
Thus,  $i_L(t) = e^{-0.4t} (4 \cos 5.083t + 0.3148 \sin 5.083t)$  A and  $i_L(2.5) = 1.473$  A.

(b)  $\alpha = 1/2RC = 4 \text{ s}^{-1}$  and  $\omega_0 = 5.099 \text{ rad/s}$ . Since  $\alpha < \omega_0$ , the new response will still be underdamped, but with  $\omega_d = 3.162 \text{ rad/s}$ . We still may write

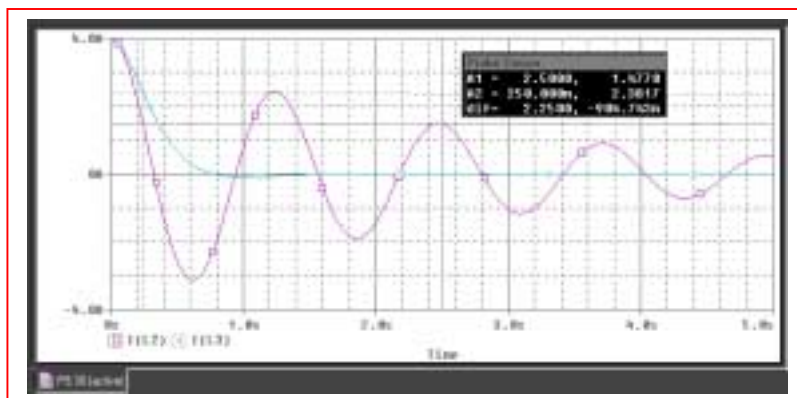
$$v_C(t) = (2/13) [e^{-\alpha t} (-C\omega_d \sin \omega_d t + D\omega_d \cos \omega_d t) - \alpha e^{-\alpha t} (C \cos \omega_d t + D \sin \omega_d t)]$$

and so with  $v_C(0^+) = 0 = (2/13) (3.162D - 4C)$ , we obtain  $D = 5.06$  A.

Thus,  $i_L(t) = e^{-4t} (4 \cos 3.162t + 5.06 \sin 3.162t)$  A and  $i_L(.25) = 2.358$  A.



We see from the simulation result below that our hand calculations are correct; the slight disagreement is due to numerical inaccuracy. Changing the step ceiling from the 10-ms value employed to a smaller value will improve the accuracy.



## CHAPTER NINE SOLUTIONS

31. Series:  $\alpha = \frac{R}{2L} = \frac{2}{1/2} = 4$ ,  $\omega_o^2 = \frac{1}{LC} = \frac{4}{0.2} = 20$ ,  $\omega_d = \sqrt{20-16} = 2$

$$\therefore i_L = e^{-4t} (A_1 \cos 2t + A_2 \sin 2t); i_L(0) = 10\text{A}, v_c(0) = 20\text{V}$$
$$\therefore A_1 = 10; i_L'(0^+) = \frac{1}{L} v_L(0^+) = 4(20 - 20) = 0$$
$$\therefore i_L'(0^+) = 2A_2 - 4 \times 10 \therefore A_2 = 20$$
$$\therefore i_L(t) = e^{-4t} (10 \cos 2t + 20 \sin 2t)\text{A}, t > 0$$

## CHAPTER NINE SOLUTIONS

32.

$$v_c(0) = 50 + 80 \times 2 = 210 \text{ V}, i_L(0) = 0, \alpha = \frac{R}{2L} = \frac{80}{4} = 20$$

$$\omega_o^2 = \frac{100}{2} = 500; \omega_d = \sqrt{500 - 20^2} = 10$$

$$\therefore v_c(t) = e^{-20t} (A_1 \cos 10t + A_2 \sin 10t) \therefore A_1 = 210 \text{ V}$$

$$\therefore v_c(t) = e^{-20t} (210 \cos 10t + A_2 \sin 10t); v_c'(0^+) = \frac{1}{C} i_c(0^+) = 0$$

$$\therefore 0 = 10A_2 - 20(210), A_2 = 420 \therefore v_c(t) = e^{-20t} (210 \cos 10t + 420 \sin 10t)$$

$$\therefore v_c(40\text{ms}) = e^{-0.8} (210 \cos 0.4 + 420 \sin 0.4) = \boxed{160.40 \text{ V}}$$

$$\text{Also, } i_L = e^{-20t} (B_1 \cos 10t + B_2 \sin 10t),$$

$$i_L(0^+) = \frac{1}{L} v_L(0^+) = \frac{1}{2} [0 - v_c(0^+)] = \frac{1}{2} \times 210$$

$$\therefore i_L'(0^+) = -105 = 10B_2 \therefore B_2 = 10.5$$

$$\therefore i_L(t) = -10.5e^{-20t} \sin 10t \text{ A}, t > 0$$

$$\therefore v_R(t) = 80i_L = 840e^{-20t} \sin 10t \text{ V}$$

$$\therefore v_R(40\text{ms}) = -840e^{-0.8} \sin 0.4 = \boxed{-146.98 \text{ V}}$$

$$v_L(t) = -v_c(t) - v_R(t) \therefore v_L$$

$$(40\text{ms}) = -160.40 + 146.98 = \boxed{-13.420 \text{ V}}$$

$$[\text{check: } v_L = e^{-20t} (-210 \cos - 420 \sin + 840 \sin)]$$

$$= e^{-20t} (-210 \cos 10t + 420 \sin 10t) \text{ V}, t > 0$$

$$\therefore v_L(40\text{ms}) = e^{-0.8} (-210 \cos - 420 \sin + 840 \sin) = e^{-20t}$$

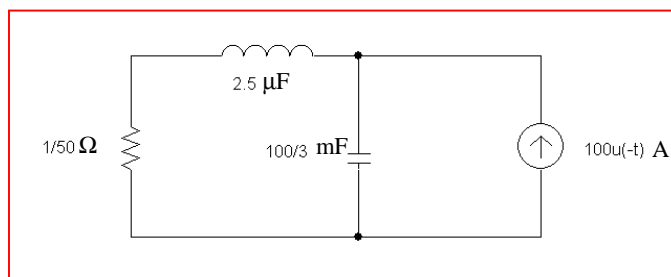
$$(-210 \cos 10t + 420 \sin 10t) \text{ V}, t > 0$$

$$\therefore V_L(40\text{ms}) = e^{-0.8}$$

$$(420 \sin 0.4 - 210 \cos 0.4) = -13.420 \text{ V Checks}]$$

## CHAPTER NINE SOLUTIONS

33. “Obtain an expression for  $v_c(t)$  in the circuit of Fig. 9.8 (dual) that is valid for all  $t$ ”.



$$\alpha = \frac{R}{2L} = \frac{0.02 \times 10^6}{2 \times 2.5} = 4000, \quad \omega_o^2 = \frac{10^6 \times 3}{2.5 \times 10} = 1.2 \times 10^7$$

$$\therefore s_{1,2} = -4000 \pm \sqrt{16 \times 10^6 - 12 \times 10^6} = -2000, -6000$$

$$\therefore v_c(t) = A_1 e^{-2000t} + A_2 e^{-6000t}; \quad v_c(0) = \frac{1}{50} \times 100 = 2V$$

$$i_L(0) = 100A \therefore 2 = A_1 + A_2, \quad v'_c(0^+) = \frac{1}{C}$$

$$(-i_L(0)) = -\frac{3}{100} \times 10^3 \times 100 = -3000v/s$$

$$\therefore -3000 = -200A_1 - 600A_2, \quad -1.5 = -A_1 - 3A_2$$

$$\therefore 0.5 = -2A_2, \quad = -0.25, \quad A_1 = 2.25$$

$$\therefore v_c(t) = (2.25e^{-200t} - 0.25e^{-6000t}) u(t) + 2u(-t) V \text{ (checks)}$$

## CHAPTER NINE SOLUTIONS

34. (a) crit. damp;  $\alpha^2 = \frac{R^2}{4L^2} = \omega_o^2 = \frac{1}{LC} \therefore L = \frac{1}{4}R^2C$   
 $\therefore L = \frac{1}{4} \times 4 \times 10^{-6} = 0.01\text{H}, \alpha = \frac{200}{0.02} = 10^4 = \omega_o$   
 $\therefore v_c(t) = e^{-10000t}(A_1t + A_2); v_c(0) = -10\text{V}, i_L(0) = -0.15\text{A}$   
 $\therefore A_2 = -10, v_c(t) = e^{-10000t}(A_1t - 10); v'_c(0^+) = -\frac{1}{C}$   
 $i_L(0) = -10^6(-0.15) = 150,000$   
 Now,  $v'_c(0^+) = A_1 + 10^5 = 150,000 \therefore A_1 = 50,000$   
 $\therefore v_c(t) = e^{-10,000t}(50,000t - 10) \text{ V}, t > 0$
- (b)  $v'_c(t) = e^{-10,000t}[50,000 - 10,000(50,000t - 10)] = \therefore$   
 $5 = 50,000t_m - 10 \therefore t_m = \frac{15}{50,000} = 0.3\text{ms}$   
 $\therefore v_c(t_m) = e^{-3}(15 - 10) = 5e^{-3} = 0.2489\text{V}$   
 $v_c(0) = -10\text{V} \therefore |v_c|_{\max} = 10\text{V}$
- (c)  $v_{c,\max} = 0.2489\text{V}$

## CHAPTER NINE SOLUTIONS

35. (a)  $\alpha = \frac{R}{2L} = \frac{250}{10} = 25, \omega_o^2 = \frac{1}{LC} = \frac{10^6}{2500} = 400$   
 $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -25 \pm 15 = -10, -40$   
 $\therefore i_L = A_1 e^{-10t} + A_2 e^{-40t}, i_L(0) = 0.5 \text{ A}, v_c(0) = 100 \text{ V}$   
 $\therefore 0.5 = A_1 + A_2, i_L'(0^+) = \frac{1}{5} v_L(0^+) = \frac{1}{5}$   
 $(100 - 25 - 100) = -5 \text{ A/s} = -10A_1 - 40A_2$   
 $\therefore 5 = 10A_1 + 40(0.5 - A_1) = 10A_1 - 40$   
 $A_1 + 20 \therefore -30A_1 = -15, A_1 = 0.5, A_2 = 0$   
 $\therefore i_L(t) = 0.5e^{-10t} \text{ A}, t > 0$

(b)  $v_c = A_3 e^{-10t} + A_4 e^{-40t} \therefore 100 = A_3 + A_4;$   
 $v_c' = \frac{1}{C} i_c'(0^+) \frac{10^6}{500} (-0.5) = -1000$   
 $\therefore -10A_3 - 40A_4 = -1000 \therefore -3A_4 = 0, A_4 = 0, A_3 = 100$   
 $\therefore v_c(t) = 100e^{-10t} \text{ V } t > 0$

## CHAPTER NINE SOLUTIONS

36. (a)  $\alpha = \frac{R}{2L} = \frac{2}{2} = 1$ ,  $\omega_o^2 = \frac{1}{LC} = 5$ ,  $\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 2$   
 $\therefore i_L = e^{-t}(B_1 \cos 2t + B_2 \sin 2t)$ ,  $i_L(0) = 0$ ,  $v_c(0) = 10V$   
 $\therefore B_1 = 0$ ,  $i_L = B_2 e^{-t} \sin 2t$   
 $i_1(0) = \frac{1}{1} v_L(0^+) = v_R(0^+) - V_c(0^+) = 0 - 10 = 2B_2$   
 $\therefore B_2 = 5 \therefore i_L = -5e^{-t} \sin 2t A, t > 0$

(b)  $i_L' = -5[e^{-t}(2 \cos 2t - \sin 2t)] = 0$   
 $\therefore 2 \cos 2t = \sin 2t, \tan 2t = 2$   
 $\therefore t_1 = 0.5536s$ ,  $i_L(t_1) = -2.571A$   
 $2t_2 = 2 \times 0.5536 + \pi$ ,  $t_2 = 2.124$ ,  
 $i_L(t_2) = 0.5345 \therefore |i_L|_{\max} = 2.571A$   
 and  $i_{L\max} = 0.5345A$



## CHAPTER NINE SOLUTIONS

37. We are presented with a series RLC circuit having  $\alpha = R/2L = 4700 \text{ s}^{-1}$  and  $\omega_0 = 1/\sqrt{LC} = 447.2 \text{ rad/s}$ ; therefore we expect an overdamped response with  $s_1 = -21.32 \text{ s}^{-1}$  and  $s_2 = -9379 \text{ s}^{-1}$ .

From the circuit as it exists for  $t < 0$ , it is evident that  $i_L(0^-) = 0$  and  $v_C(0^-) = 4.7 \text{ kV}$

$$\text{Thus, } v_L(t) = A e^{-21.32t} + B e^{-9379t} \quad [1]$$

With  $i_L(0^+) = i_L(0^-) = 0$  and  $i_R(0^+) = 0$  we conclude that  $v_R(0^+) = 0$ ; this leads to  $v_L(0^+) = -v_C(0^-) = -4.7 \text{ kV}$  and hence  $A + B = -4700$  [2]

Since  $v_L = L \frac{di}{dt}$ , we may integrate Eq. [1] to find an expression for the inductor current:

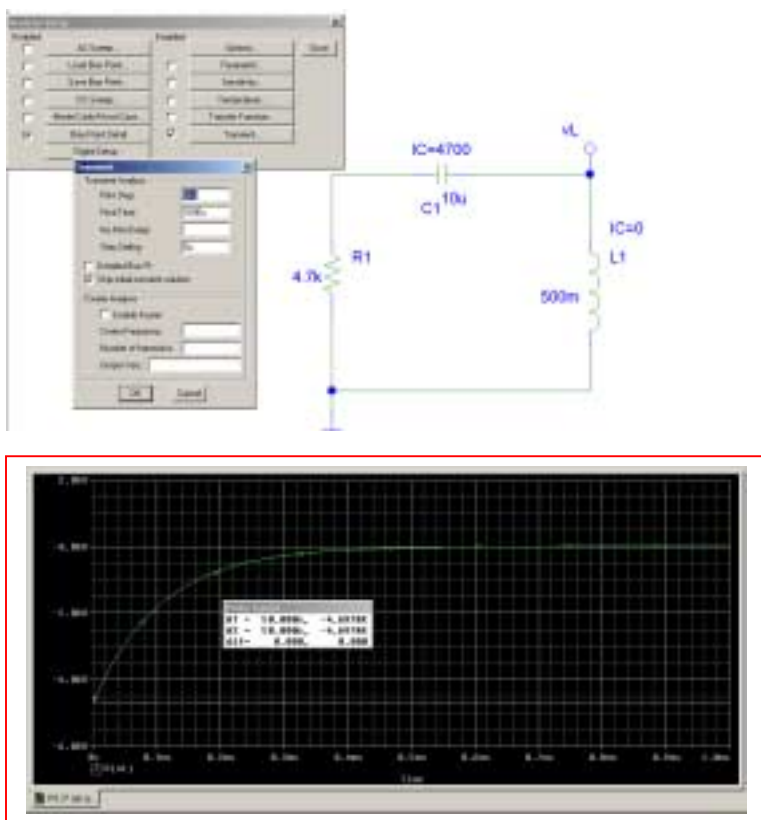
$$i_L(t) = \frac{1}{L} \left[ -\frac{A}{21.32} e^{-21.32t} - \frac{B}{9379} e^{-9379t} \right]$$

$$\text{At } t = 0^+, i_L = 0 \text{ so we have } \frac{1}{500 \times 10^{-3}} \left[ -\frac{A}{21.32} - \frac{B}{9379} \right] = 0 \quad [3]$$

Simultaneous solution of Eqs. [2] and [3] yields  $A = 10.71$  and  $B = -4711$ . Thus,

$$v_L(t) = 10.71 e^{-21.32t} - 4711 e^{-9379t} \text{ V, } t > 0$$

and the peak inductor voltage magnitude is 4700 V.



## CHAPTER NINE SOLUTIONS

38. Considering the circuit as it exists for  $t < 0$ , we conclude that  $v_C(0^-) = 0$  and  $i_L(0^-) = 9/4 = 2.25$  A. For  $t > 0$ , we are left with a parallel RLC circuit having  $\alpha = 1/2RC = 0.25$  s<sup>-1</sup> and  $\omega_0 = 1/\sqrt{LC} = 0.3333$  rad/s. Thus, we expect an underdamped response with  $\omega_d = 0.2205$  rad/s:

$$i_L(t) = e^{-\alpha t} (A \cos \omega_d t + B \sin \omega_d t)$$

$$i_L(0^+) = i_L(0^-) = 2.25 = A$$

$$\text{so } i_L(t) = e^{-0.25t} (2.25 \cos 0.2205t + B \sin 0.2205t)$$

In order to determine B, we must invoke the remaining boundary condition. Noting that

$$\begin{aligned} v_C(t) = v_L(t) &= L \frac{di_L}{dt} \\ &= (9)(-0.25)e^{-0.25t} (2.25 \cos 0.2205t + B \sin 0.2205t) \\ &\quad + (9)e^{-0.25t} [-2.25(0.2205) \sin 0.2205t + 0.2205B \cos 0.2205t] \end{aligned}$$

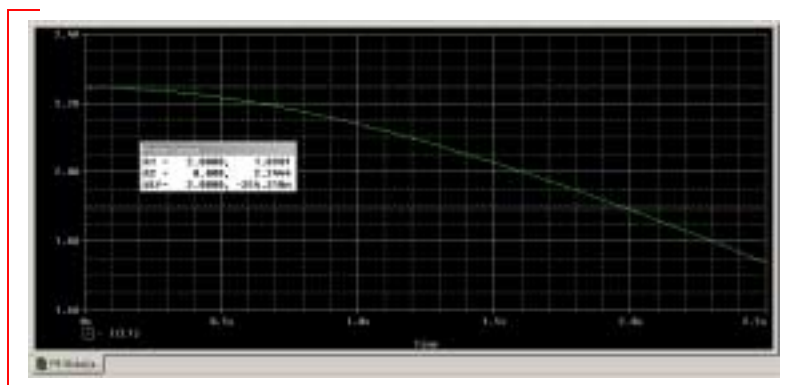
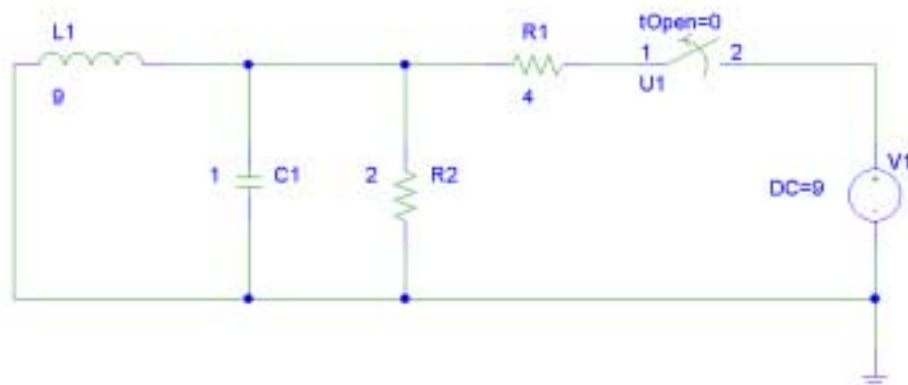
$$v_C(0^+) = v_C(0^-) = 0 = (9)(-0.25)(2.25) + (9)(0.2205B)$$

so  $B = 2.551$  and

$$i_L(t) = e^{-0.25t} [2.25 \cos 0.2205t + 2.551 \sin 0.2205t] \text{ A}$$

Thus,  $i_L(2) = 1.895$  A

This answer is borne out by PSpice simulation:



## CHAPTER NINE SOLUTIONS

39. Considering the circuit at  $t < 0$ , we note that  $i_L(0^-) = 9/4 = 2.25$  A and  $v_C(0^-) = 0$ . For a critically damped circuit, we require  $\alpha = \omega_0$ , or  $\frac{1}{2RC} = \frac{1}{\sqrt{LC}}$ , which, with  $L = 9$  H and  $C = 1$  F, leads to the requirement that  $R = 1.5 \Omega$  (so  $\alpha = 0.3333$  s<sup>-1</sup>).

The inductor energy is given by  $w_L = \frac{1}{2} L [i_L(t)]^2$ , so we seek an expression for  $i_L(t)$ :

$$i_L(t) = e^{-\alpha t} (At + B)$$

Noting that  $i_L(0^+) = i_L(0^-) = 2.25$ , we see that  $B = 2.25$  and hence

$$i_L(t) = e^{-0.3333t} (At + 2.25)$$

Invoking the remaining initial condition requires consideration of the voltage across the capacitor, which is equal in this case to the inductor voltage, given by:

$$v_C(t) = v_L(t) = L \frac{di_L}{dt} = 9(-0.3333) e^{-0.3333t} (At + 2.25) + 9A e^{-0.3333t}$$

$$v_C(0^+) = v_C(0^-) = 0 = 9(-0.333)(2.25) + 9A \quad \text{so } A = 0.7499 \text{ amperes and}$$

$$i_L(t) = e^{-0.3333t} (0.7499t + 2.25) \text{ A}$$

$$\text{Thus, } i_L(100 \text{ ms}) = 2.249 \text{ A and so } w_L(100 \text{ ms}) = 22.76 \text{ J}$$

## CHAPTER NINE SOLUTIONS

40. With the 144 mJ originally stored via a 12-V battery, we know that the capacitor has a value of 2 mF. The initial inductor current is zero, and the initial capacitor voltage is 12 V. We begin by seeking a (painful) current response of the form

$$i_{\text{bear}} = Ae^{s_1 t} + Be^{s_2 t}$$

Using our first initial condition,  $i_{\text{bear}}(0^+) = i_L(0^+) = i_L(0^-) = 0 = A + B$

$$di/dt = As_1 e^{s_1 t} + Bs_2 e^{s_2 t}$$

$$v_L = L di/dt = ALs_1 e^{s_1 t} + BLs_2 e^{s_2 t}$$

$$v_L(0^+) = ALs_1 + BLs_2 = v_C(0^+) = v_C(0^-) = 12$$

What else is known? We know that the bear stops reacting at  $t = 18 \mu\text{s}$ , meaning that the current flowing through its fur coat has dropped just below 100 mA by then (not a long shock).

$$\text{Thus, } A \exp[(18 \times 10^{-6})s_1] + B \exp[(18 \times 10^{-6})s_2] = 100 \times 10^{-3}$$

$$\text{Iterating, we find that } R_{\text{bear}} = 119.9775 \Omega.$$

This corresponds to  $A = 100 \text{ mA}$ ,  $B = -100 \text{ mA}$ ,  $s_1 = -4.167 \text{ s}^{-1}$  and  $s_2 = -24 \times 10^6 \text{ s}^{-1}$

## CHAPTER NINE SOLUTIONS

41.

$$v_c(0) = 0, i_L(0) = 0, \alpha = \frac{R}{2L} = \frac{2}{0.5} = 4, \omega_o^2 = \frac{1}{LC} = 4 \times 5 = 20$$

$$\therefore \omega_d = \sqrt{20 - 16} = 2 \therefore i_L(t) = e^{-4t} (A_1 \cos 2t + A_2 \sin 2t) + i_{L,f}$$

$$i_{L,f} = 10A \therefore i_L(t) = 10 + e^{-4t} (A_1 \cos 2t + A_2 \sin 2t)$$

$$\therefore 0 = 10 + A_1, A_1 = -10, i_L(t) = 10 + e^{-4t} (A_2 \sin 2t - 10 \cos 2t)$$

$$i_L(0^+) = \frac{1}{L} v_L(0^+) = 4 \times 0 = 0 \therefore i_L(0^+) = 0 = 2A_2 + 40, A_2 = -20$$

$$i_L(t) = 10 - e^{-4t} (20 \sin 2t + 10 \cos 2t) A, t > 0$$

## CHAPTER NINE SOLUTIONS

42. (a) Series, driven:  $\alpha = \frac{R}{2L} = \frac{100}{0.2} = 500$ ,

$$\omega_o^2 = \frac{1}{LC} = \frac{10 \times 10^6}{40} = 250,000$$

$\therefore$  Crit. damp  $i_L(f) = 3(1-2) = -3$ ,

$i_L(0) = 3$ ,  $v_c(0) = 300\text{V}$

$\therefore i_L = -3 + e^{-500t} (A_1 t + A_2) \therefore 3 = -3 + A_2$ ,  $A_2 = 6\text{A}$

$i_L(0^+) = A_1 - 300 = \frac{1}{L}[v_c(0) - v_R(0^+)] = 0$

$\therefore A_1 = 3000 e^{-500t} \therefore i_L(t) = -3 + e^{-500t}$   
 $(3000t + 6)$ ,  $t > 0$

$\therefore i_L(t) = 3u(-t) + [-3 + e^{-500t}(3000t + 6)]u(t)\text{A}$

(b)  $e^{-500t_o}(3000t_o + 6) = 3$ ; by SOLVE,  $t_o = 3.357\text{ms}$

## CHAPTER NINE SOLUTIONS

43.

$$\alpha = \frac{1}{2RC} = \frac{10^6}{100 \times 2.5} = 4000, \quad \omega_o^2 = \frac{1}{LC} = \frac{10^{6+3}}{50} = 20 \times 10^6$$

$$\therefore \omega_d = \sqrt{\omega_o^2 - \alpha^2} = 2000, \quad i_L(0) = 2A, \quad v_c(0) = 0$$

$$i_{c,f} = 0, \quad (v_{c,f} = 0) \therefore i_c = e^{-4000t} (A_1 \cos 2000t + A_2 \sin 2000t)$$

$$\text{work with } v_c: v_c(t) = e^{-4000t} (B_1 \cos 2000t + B_2 \sin 2000t) \therefore B_1 = 0$$

$$\therefore v_c = B_2 e^{-4000t} \sin 2000t, \quad v'_c(0^+) = \frac{1}{C} i_c(0^+) = \frac{10^6}{2.5} (2 \times 1) = 8 \times 10^5$$

$$\therefore 8 \times 10^5 = 2000 B_2, \quad B_2 = 400, \quad v_c = 400 e^{-4000t} \sin 2000t$$

$$\therefore i_c(t) = C v'_c = 2.5 \times 10^{-6} \times 400 e^{-4000t} (-4000 \sin 2000t + 2000 \cos 2000t)$$

$$= 10^{-6+3+3} e^{-4000t} (-4 \sin 2000t + 2 \cos 2000t)$$

$$= e^{-4000t} (2 \cos 2000t - 4 \sin 2000t) A, \quad t > 0$$

## CHAPTER NINE SOLUTIONS

44.

$$\alpha = \frac{R}{2L} = \frac{250}{10} = 25, \quad \omega_o^2 = \frac{1}{LC} = \frac{10^6}{2500} = 400$$

$$s_{1,2} = -25 \pm \sqrt{625 - 400} = -10, -40$$

$$i_L(0) = 0.5\text{A}, \quad v_c(0) = 100\text{V}, \quad i_{L,f} = -0.5\text{A}$$

$$\therefore i_L(t) = -0.5 + A_1 e^{-10t} + A_2 e^{-40t} \text{A}$$

$$t = 0^+ : v_L(0^+) = 100 - 50 \times 1 - 200 \times 0.5 = -50\text{V} \therefore -50 = 5i_L'(0^+)$$

$$\therefore i_L'(0^+) = -10 \therefore -10 = -10A_1 - 40A_2, \quad 0.5 = -0.5 + A_1 + A_2$$

$$\therefore A_1 + A_2 = 1 \therefore -10 = -10A_2 - 40(-1 + A_1) = -50A_1 + 40, \quad A_1 = 1, A_2 = 0$$

$$\therefore i_L(t) = \boxed{-0.5 + 1e^{-10t} \text{A}}, \quad t > 0; \quad i_L(t) = \boxed{0.5\text{A}}, \quad t > 0$$



## CHAPTER NINE SOLUTIONS

45.

$$\alpha = \frac{R}{2L} = \frac{1}{1} = 1, \omega_o^2 = \frac{1}{LC} = 1 \therefore \text{crit. damp}$$

$$v_c(0) = \frac{5}{6} \times 12 = 10\text{V}, i_L(0) = 2\text{A}, v_{c,f} = 12\text{V}$$

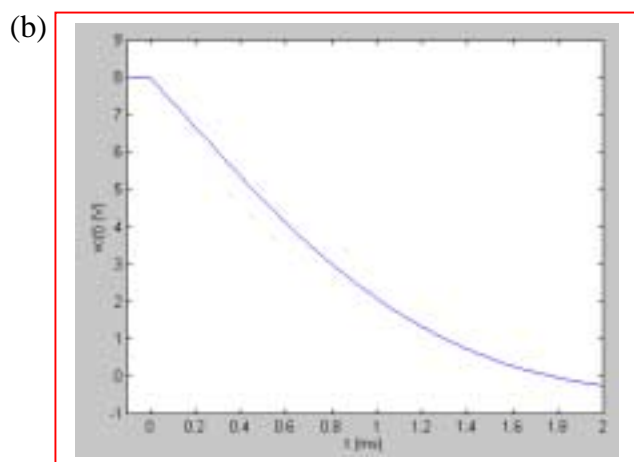
$$\therefore v_c(t) = 12 + e^{-t}(A_1 t - 2); v'_c(0^+) = \frac{1}{C} i_c(0^+) = \frac{1}{2} \times i_L(0^+) = 1$$

$$\therefore 1 = A_1 + 2; A_1 = -1 \therefore v_c(t) = 12 - e^{-t}(t + 2)\text{V}, t > 0$$

## CHAPTER NINE SOLUTIONS

46. (a)  $\alpha = \frac{1}{2RC} = \frac{8 \times 10^6}{2 \times 4 \times 10^3} = 1000, \omega_o^2 = \frac{8 \times 10^6 \times 13}{4} = 26 \times 10^6$   
 $\therefore \omega_d = \sqrt{26 - 1} \times 10^3 = 5000, v_c(0) = 8V$   
 $i_L(0) = 8mA, v_{c,f} = 0$   
 $\therefore v_c = e^{-1000t} (A_1 \cos 1000t + A_2 \sin 5000t)$   
 $\therefore A_1 = 8; v_c'(0^+) = \frac{1}{C} i_c(0^+) = 8 \times 10^6 (0.01 - \frac{8}{4000} - 0.008) = 0$   
 $\therefore 5000A_2 - 1000 \times 8 = 0, A_2 = 1.6$

So  $v_c(t) = e^{-1000t} (8 \cos 1000t + 1.6 \sin 1000t) V, t > 0$



## CHAPTER NINE SOLUTIONS

47. (a)

$$v_s(t) = 10u(-t) \text{ V}; \alpha = \frac{1}{2RC} = \frac{10^6}{1000} = 1000$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6 \times 3}{4} \therefore s_{1,2} = -1000 \pm \sqrt{10^6 - \frac{3}{4} \times 10^6} = -500, -1500$$

$$v_{c,f} = 0 \therefore v_c = A_1 e^{-500t} + A_2 e^{-1500t}, v_c(0) = 10 \text{ V}, i_L(0) = 0$$

$$\therefore 10 = A_1 + A_2, v'_c = 10^6 i_c(0^+) = 10^6 \left[ 0 - \frac{10}{500} \right] = -2 \times 10^4$$

$$\therefore -2 \times 10^4 = -500A_1 - 1500A_2 \therefore 40 = A_1 + 3A_2 \therefore 30 = 2A_2, A_2 = 15, A_1 = -5$$

$$\therefore v_c = -5e^{-500t} + 15e^{-1500t} \text{ V}, t > 0 \therefore i_s = i_c = C v'_c$$

$$\therefore i_s = 10^{-6} (2500e^{-500t} - 22,500e^{-1500t})$$

$$= \boxed{2.5e^{-500t} - 22.5e^{-1500t} \text{ mA}, t > 0}$$

(b)  $v_s(t) = 10u(t) \text{ V} \therefore v_{c,f} = 10 \text{ V}, v_c(0) = 0, i_L(0) = 0$

$$\therefore v_c = 10 + A_3 e^{-500t} + A_4 e^{-1500t} \therefore A_3 + A_4 = -10$$

$$v'_c(0^+) = 10^6 i_c(0^+) = 10^6 \left( 0 + \frac{10}{500} \right) = 2 \times 10^4 = -500A_3 - 1500A_4$$

$$\therefore -A_3 - 3A_4 = 40, \text{ add: } -2A_4 = 30, A_4 = -15, A_3 = 5,$$

$$v_c = 10 + 5e^{-500t} - 15e^{-1500t} \text{ V}, i_s = i_c =$$

$$10^{-6} (-2500e^{-500t} + 22,500e^{-1500t}) = \boxed{25e^{-500t} + 22.5e^{-1500t} \text{ mA}, t > 0}$$

## CHAPTER NINE SOLUTIONS

48. (a)  $v_s = 10u(-t) \text{ V} : \alpha = \frac{1}{2RC} = \frac{10^6}{2000 \times 0.5} = 1000$
- $$\omega_o^2 = \frac{1}{LC} = \frac{2 \times 10^6 \times 3}{8} = 0.75 \times 10^6 \therefore s_{1,2} = -500, -1500$$
- $$\therefore v_c = A_1 e^{-500t} + A_2 e^{-1500t}, v_o(0) = 10\text{V}, i_L(0) = 10\text{mA}$$
- $$\therefore A_1 + A_2 = 10, v'_c(0^+) = 2 \times 10^6 [i_L(0) - i_R(0^+)] = 2 \times 10^6$$
- $$\left( 0.01 - \frac{10}{1000} \right) = 0 \therefore -500A_1 - 1500A_2 = 0,$$
- $$-A_1 - 3A_2 = 0; \text{ add: } -2A_2 = 10, A_2 = -5, A_1 = 15$$
- $$\therefore v_c(t) = 15e^{-500t} - 5e^{-1500t} \text{ V } t > 0$$
- $$\therefore i_R(t) = \boxed{15e^{-500t} - 5e^{-1500t} \text{ mA}, t > 0}$$
- (b)  $v_s = 10u(t) \text{ V}, v_{c,f} = 10, v_c = 10 + A_3 e^{-500t} + A_4 e^{-1500t},$
- $$v_c(0) = 0, i_L(0) = 0 \therefore A_3 + A_4 = -10\text{V}, v'_c(0^+) = 2 \times 10^6$$
- $$[i_L(0) - i_R(0^+)] = 2 \times 10^6 (0 - 0) = 0 = -500A_3 - 1500A_4$$
- $$\therefore -A_3 - 3A_4 = 0, \text{ add: } -2A_4 = -10, A_4 = 5 \therefore A_3 = -15$$
- $$\therefore v_c(t) = 10 - 15e^{-500t} + 5e^{-1500t} \text{ V}, t > 0$$
- $$\therefore i_R(t) = \boxed{10 - 15e^{-500t} + 5e^{-1500t} \text{ mA}, t > 0}$$

## CHAPTER NINE SOLUTIONS

49. (a)  $v_S(0^-) = v_C(0^-) = 2(15) = 30 \text{ V}$

(b)  $i_L(0^+) = i_L(0^-) = 15 \text{ A}$

Thus,  $i_C(0^+) = 22 - 15 = 7 \text{ A}$  and  $v_S(0^+) = 3(7) + v_C(0^+) = 51 \text{ V}$

(c) As  $t \rightarrow \infty$ , the current through the inductor approaches  $22 \text{ A}$ , so  $v_S(t \rightarrow \infty) = 44 \text{ A}$ .

(d) We are presented with a series RLC circuit having  $\alpha = 5/2 = 2.5 \text{ s}^{-1}$  and  $\omega_0 = 3.536 \text{ rad/s}$ . The natural response will therefore be underdamped with  $\omega_d = 2.501 \text{ rad/s}$ .

$$i_L(t) = 22 + e^{-\alpha t} (A \cos \omega_d t + B \sin \omega_d t)$$

$$i_L(0^+) = i_L(0^-) = 15 = 22 + A \quad \text{so } A = -7 \text{ amperes}$$

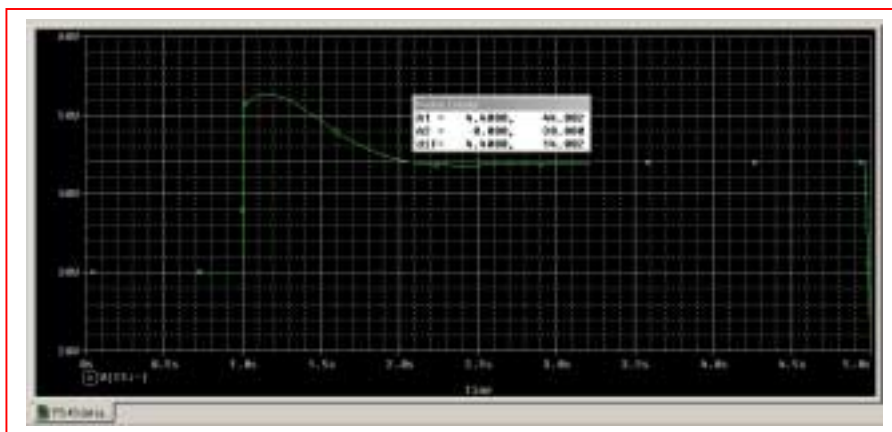
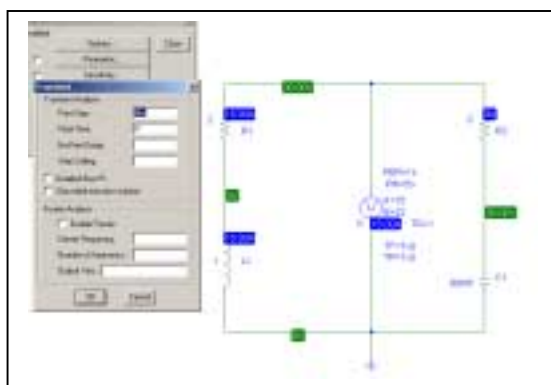
$$\text{Thus, } i_L(t) = 22 + e^{-2.5t} (-7 \cos 2.501t + B \sin 2.501t)$$

$$v_S(t) = 2 i_L(t) + L \frac{di_L}{dt} = 2i_L + \frac{di_L}{dt} = 44 + 2e^{-2.5t} (-7 \cos 2.501t + B \sin 2.501t) \\ - 2.5e^{-2.5t} (-7 \cos 2.501t + B \sin 2.501t) + e^{-2.5t} [7(2.501) \sin 2.501t + 2.501B \cos 2.501t]$$

$$v_S(t) = 51 = 44 + 2(-7) - 2.5(-7) + 2.501B \quad \text{so } B = 1.399 \text{ amperes and hence}$$

$$v_S(t) = 44 + 2e^{-2.5t} (-7 \cos 2.501t + 1.399 \sin 2.501t) \\ - 2.5e^{-2.5t} (-7 \cos 2.501t + 1.399 \sin 2.501t) + e^{-2.5t} [17.51 \sin 2.501t + 3.499 \cos 2.501t]$$

and  $v_S(t)$  at  $t = 3.4 \text{ s} = 44.002 \text{ V}$ . This is borne out by PSpice simulation:



## CHAPTER NINE SOLUTIONS

50. Considering the circuit at  $t < 0$ , we see that  $i_L(0^-) = 15$  A and  $v_C(0^-) = 0$ . The circuit is a series RLC with  $\alpha = R/2L = 0.375$  s<sup>-1</sup> and  $\omega_0 = 1.768$  rad/s. We therefore expect an underdamped response with  $\omega_d = 1.728$  rad/s. The general form of the response will be

$$v_C(t) = e^{-\alpha t} (A \cos \omega_d t + B \sin \omega_d t) + 0 \quad (v_C(\infty) = 0)$$

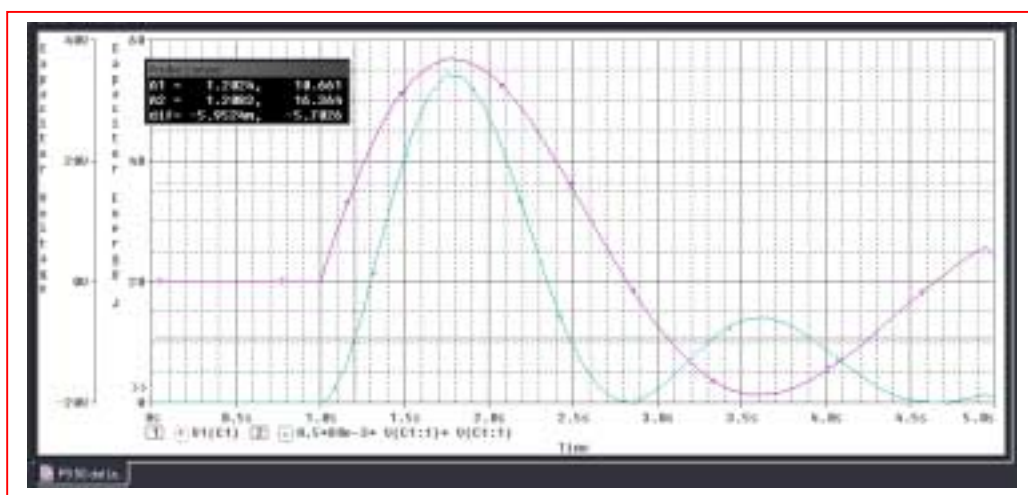
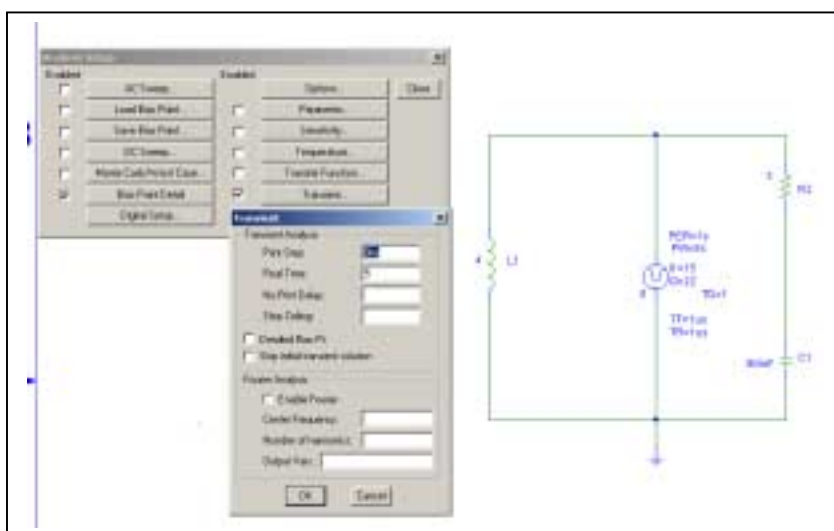
$v_C(0^+) = v_C(0^-) = 0 = A$  and we may therefore write  $v_C(t) = B e^{-0.375t} \sin(1.728t)$  V

$$i_C(t) = -i_L(t) = C \frac{dv_C}{dt} = (80 \times 10^{-3})(-0.375B e^{-0.375t} \sin 1.728t + 1.728B e^{-0.375t} \cos 1.728t)$$

At  $t = 0^+$ ,  $i_C = 15 + 7 - i_L(0^+) = 7 = (80 \times 10^{-3})(1.728B)$  so that  $B = 50.64$  V.

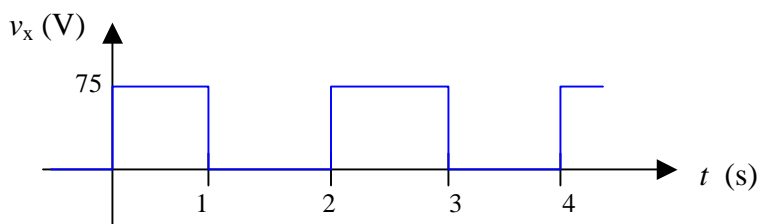
Thus,  $v_C(t) = 50.64 e^{-0.375t} \sin 1.807t$  V and  $v_C(t = 200 \text{ ms}) = 16.61$  V.

The energy stored in the capacitor at that instant is  $\frac{1}{2} C v_C^2 = \boxed{11.04 \text{ J}}$



## CHAPTER NINE SOLUTIONS

51. It's probably easiest to begin by sketching the waveform  $v_x$ :



(a) The source current ( $= i_L(t)$ ) = 0 at  $t = 0^-$ .

(b)  $i_L(t) =$ 0 $at  $t = 0^+$$

(c) We are faced with a series RLC circuit having  $\alpha = R/2L = 2000$  rad/s and  $\omega_0 = 2828$  rad/s. Thus, an underdamped response is expected with  $\omega_d = 1999$  rad/s.

The general form of the expected response is  $i_L(t) = e^{-\alpha t} (A \cos \omega_d t + B \sin \omega_d t)$

$$i_L(0^+) = i_L(0^-) = 0 = A \text{ so } A = 0. \text{ This leaves } i_L(t) = B e^{-2000t} \sin 1999t$$

$$v_L(t) = L \frac{di_L}{dt} = B[(5 \times 10^{-3})(-2000 e^{-2000t} \sin 1999t + 1999 e^{-2000t} \cos 1999t)]$$

$$v_L(0^+) = v_x(0^+) - v_C(0^+) - 20 i_L(0^+) = B (5 \times 10^{-3})(1999) \text{ so } B = 7.504 \text{ A.}$$

Thus,  $i_L(t) = 7.504 e^{-2000t} \sin 1999t$  and  $i_L(1 \text{ ms}) =$ 0.9239 A.

(d) Define  $t' = t - 1$  ms for notational convenience. With no source present, we expect a new response but with the same general form:

$$i_L(t') = e^{-2000t'} (A' \cos 1999t' + B' \sin 1999t')$$

$v_L(t) = L \frac{di_L}{dt}$ , and this enables us to calculate that  $v_L(t = 1 \text{ ms}) = -13.54$  V. Prior to the pulse returning to zero volts,  $-75 + v_L + v_C + 20 i_L = 0$  so  $v_C(t' = 0) = 69.97$  V.

$$i_L(t' = 0) = A' = 0.9239 \text{ and } -v_x + v_L + v_C + 20 i_L = 0 \text{ so that } B' = -7.925.$$

Thus,  $i_L(t') = e^{-2000t'} (0.9239 \cos 1999t' - 7.925 \sin 1999t')$  and hence  $i_L(t = 2 \text{ ms}) = i_L(t' = 1 \text{ ms}) =$ -1.028 A.

## CHAPTER NINE SOLUTIONS

52. For  $t < 0$ , we have 15 A dc flowing, so that  $i_L = 15$  A,  $v_C = 30$  V,  $v_{3\Omega} = 0$  and  $v_S = 30$  V. This is a series RLC circuit with  $\alpha = R/2L = 2.5 \text{ s}^{-1}$  and  $\omega_0 = 3.536 \text{ rad/s}$ . We therefore expect an underdamped response with  $\omega_d = 2.501 \text{ rad/s}$ .

$$0 < t < 1$$

$$v_C(t) = e^{-\alpha t} (A \cos \omega_d t + B \sin \omega_d t)$$

$$v_C(0^+) = v_C(0^-) = 30 = A \text{ so we may write } v_C(t) = e^{-2.5t} (30 \cos 2.501t + B \sin 2.501t)$$

$$\begin{aligned} \frac{dv_C}{dt} &= -2.5e^{-2.5t}(30 \cos 2.501t + B \sin 2.501t) \\ &\quad + e^{-2.5t} [-30(2.501)\sin 2.501t + 2.501B \cos 2.501t] \end{aligned}$$

$$i_C(0^+) = C \left. \frac{dv_C}{dt} \right|_{t=0^+} = 80 \times 10^{-3} [-2.5(30) + 2.501B] = -i_L(0^+) = -i_L(0^-) = -15 \text{ so } B = -44.98 \text{ V}$$

$$\begin{aligned} \text{Thus, } v_C(t) &= e^{-2.5t} (30 \cos 2.501t - 44.98 \sin 2.501t) \text{ and} \\ i_C(t) &= e^{-2.5t} (-15 \cos 2.501t + 2.994 \sin 2.501t). \end{aligned}$$

$$\text{Hence, } v_S(t) = 3 i_C(t) + v_C(t) = e^{-2.5t} (-15 \cos 2.501t - 36 \sin 2.501t)$$

Prior to switching,  $v_C(t=1) = -4.181$  V and  $i_L(t=1) = -i_C(t=1) = -1.134$  A.

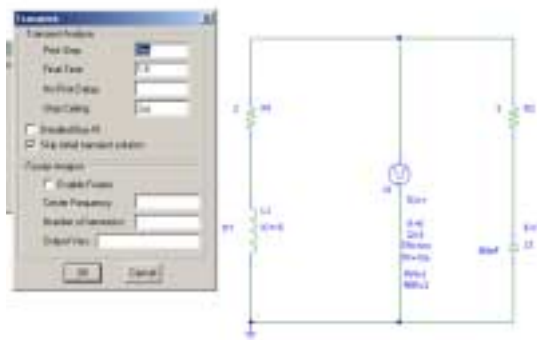
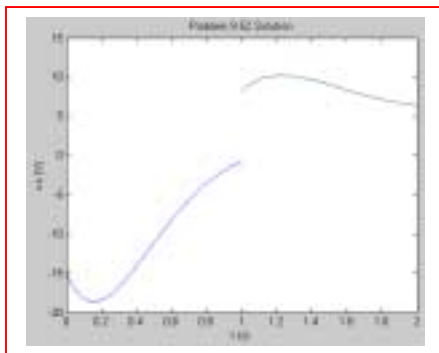
$t > 2$ : Define  $t' = t - 1$  for notational simplicity. Then, with the fact that  $v_C(\infty) = 6$  V, our response will now be  $v_C(t') = e^{-\alpha t'} (A' \cos \omega_d t' + B' \sin \omega_d t') + 6$ .

With  $v_C(0^+) = A' + 6 = -4.181$ , we find that  $A' = -10.18$  V.

$$i_C(0^+) = C \left. \frac{dv_C}{dt'} \right|_{t'=0^+} = (80 \times 10^{-3}) [(-2.5)(-10.18) + 2.501B'] = 3 - i_L(0^+) \text{ so } B' = 10.48 \text{ V}$$

$$\begin{aligned} \text{Thus, } v_C(t') &= e^{-2.5t'} (-10.18 \cos 2.501t' + 10.48 \sin 2.501t') \text{ and} \\ i_C(t') &= e^{-2.5t'} (4.133 \cos 2.501t' - 0.05919 \sin 2.501t'). \end{aligned}$$

$$\text{Hence, } v_S(t') = 3 i_C(t') + v_C(t') = e^{-2.5t'} (2.219 \cos 2.501t' + 10.36 \sin 2.501t')$$



We see that our hand calculations are supported by the PSpice simulation.



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53. The circuit described is a series RLC circuit, and the fact that oscillations are detected tells us that it is an underdamped response that we are modeling. Thus,

$i_L(t) = e^{-\alpha t} (A \cos \omega_d t + B \sin \omega_d t)$  where we were given that  $\omega_d = 1.825 \times 10^6$  rad/s.

$$\omega_0 = \frac{1}{\sqrt{LC}} = 1.914 \times 10^6 \text{ rad/s, and so } \omega_d^2 = \omega_0^2 - \alpha^2 \text{ leads to } \alpha^2 = 332.8 \times 10^9$$

Thus,  $\alpha = R/2L = 576863 \text{ s}^{-1}$ , and hence  $R = 1003 \text{ } \Omega$ .

Theoretically, this value must include the “radiation resistance” that accounts for the power lost from the circuit and received by the radio; there is no way to separate this effect from the resistance of the rag with the information provided.

## CHAPTER NINE SOLUTIONS

54. The key will be to coordinate the decay dictated by  $\alpha$ , and the oscillation period determined by  $\omega_d$  (and hence partially by  $\alpha$ ). **One possible solution of many:**

Arbitrarily set  $\omega_d = 2\pi$  rad/s.

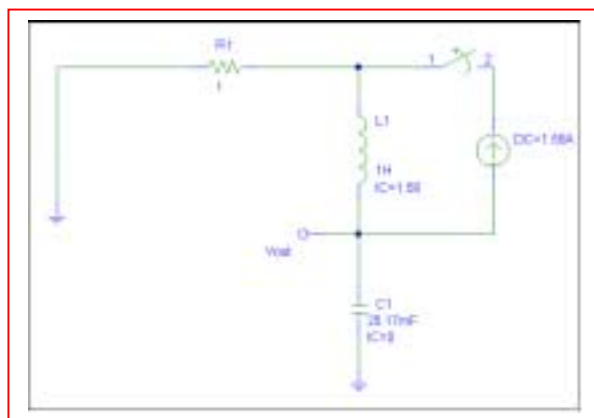
We want a capacitor voltage  $v_C(t) = e^{-\alpha t} (A \cos 2\pi t + B \sin 2\pi t)$ . If we go ahead and decide to set  $v_C(0^-) = 0$ , then we can force  $A = 0$  and simplify some of our algebra.

Thus,  $v_C(t) = B e^{-\alpha t} \sin 2\pi t$ . This function has max/min at  $t = 0.25$  s,  $0.75$  s,  $1.25$  s, *etc.* Designing so that there is no strong damping for several seconds, we pick  $\alpha = 0.5$  s<sup>-1</sup>. Choosing a series RLC circuit, this now establishes the following:

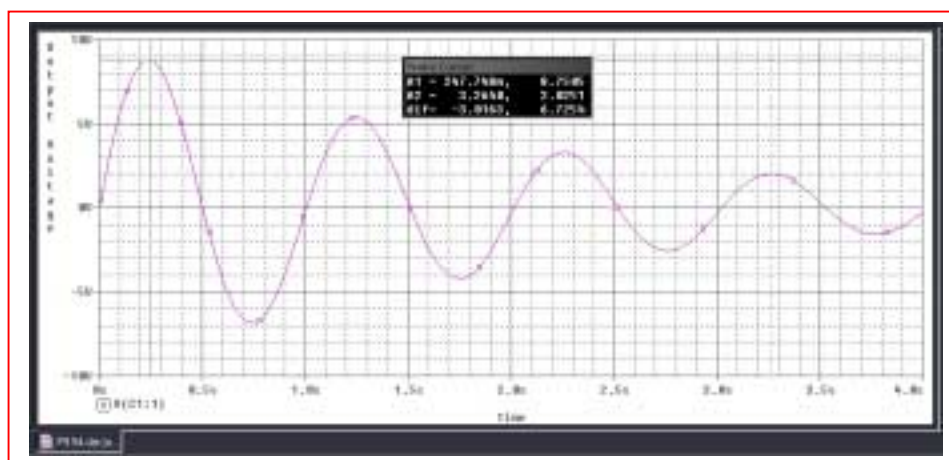
$$R/2L = 0.5 \text{ so } R = L \text{ and}$$

$$\omega_d = \sqrt{\omega_0^2 - \left(\frac{1}{2}\right)^2} = 39.73 \text{ rad/s} = \frac{1}{LC}$$

Arbitrarily selecting  $R = 1 \Omega$ , we find that  $L = 1$  H and  $C = 25.17$  mF. We need the first peak to be at least 5 V. Designing for  $B = 10$  V, we  $\therefore$  need  $i_L(0^+) = 2\pi(25.17 \times 10^{-3})(10) = 1.58$  A. Our final circuit, then is:



And the operation is verified by a simple PSpice simulation:



## CHAPTER NINE SOLUTIONS

55. For  $t < 0$ ,  $i_L(0^-) = 3$  A and  $v_C(0^-) = 25(3) = 75$  V. This is a series RLC circuit with  $\alpha = R/2L = 5000$  s<sup>-1</sup> and  $\omega_0 = 4000$  rad/s. We therefore expect an overdamped response with  $s_1 = -2000$  s<sup>-1</sup> and  $s_2 = -8000$  s<sup>-1</sup>. The final value of  $v_C = -50$  V.

$$\text{For } t > 0, \quad v_C(t) = A e^{-2000t} + B e^{-8000t} - 50$$

$$v_C(0^+) = v_C(0^-) = 75 = A + B - 50$$

$$\text{so} \quad A + B = 125 \quad [1]$$

$$\frac{dv_C}{dt} = -2000 A e^{-2000t} - 8000 B e^{-8000t}$$

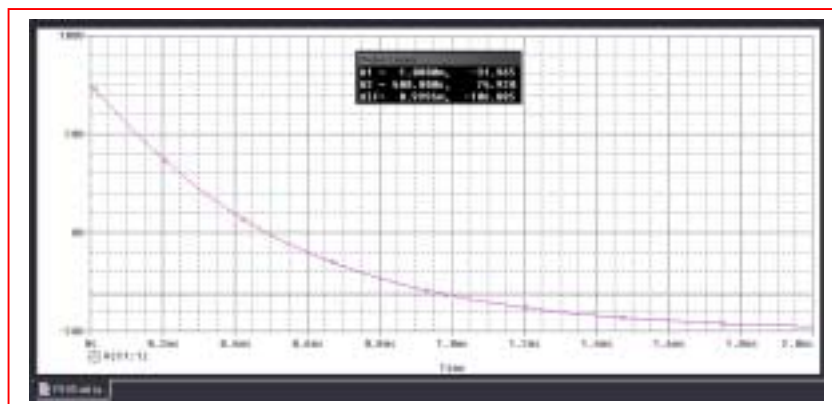
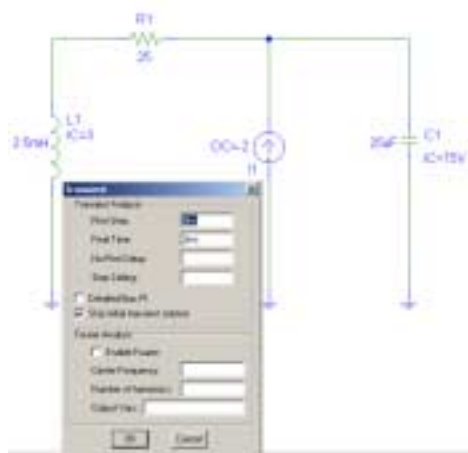
$$i_C(0^+) = C \left. \frac{dv_C}{dt} \right|_{t=0^+} = 3 - 5 - i_L(0^-) = -5 = -25 \times 10^{-6} (2000A + 8000B)$$

$$\text{Thus, } 2000A + 8000B = 5/25 \times 10^{-6} \quad [2]$$

Solving Eqs. [1] and [2], we find that  $A = 133.3$  V and  $B = -8.333$  V. Thus,

$$v_C(t) = 133.3 e^{-2000t} - 8.333 e^{-8000t} - 50$$

and  $v_C(1 \text{ ms}) = -31.96$  V. This is confirmed by the PSpice simulation shown below.



## CHAPTER NINE SOLUTIONS

56.  $\alpha = 0$  (this is a series RLC with  $R = 0$ , or a parallel RLC with  $R = \infty$ )  
 $\omega_o^2 = 0.05$  therefore  $\omega_d = 0.223$  rad/s. We anticipate a response of the form:  
 $v(t) = A \cos 0.2236t + B \sin 0.2236t$

$$v(0^+) = v(0^-) = 0 = A \text{ therefore } v(t) = B \sin 0.2236t$$

$$dv/dt = 0.2236B \cos 0.2236t; \quad i_C(t) = Cdv/dt = 0.4472B \cos 0.2236t$$

$$i_C(0^+) = 0.4472B = -i_L(0^+) = -i_L(0^-) = -1 \times 10^{-3} \text{ so } B = -2.236 \times 10^{-3} \text{ and thus}$$

$$v(t) = -2.236 \sin 0.2236t \text{ mV}$$

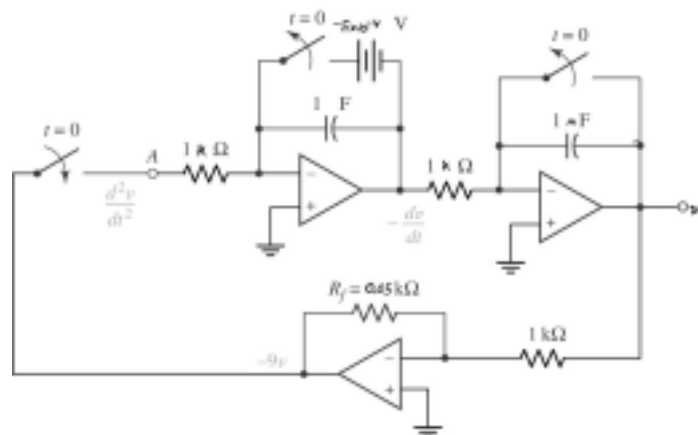
In designing the op amp stage, we first write the differential equation:

$$\frac{1}{10} \int_0^t v dt' + 10^{-3} + 2 \frac{dv}{dt} = 0 \quad (i_C + i_L = 0)$$

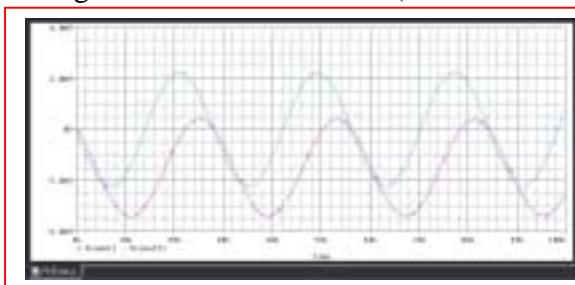
and then take the derivative of both sides:

$$\frac{d^2v}{dt^2} = -\frac{1}{20}v$$

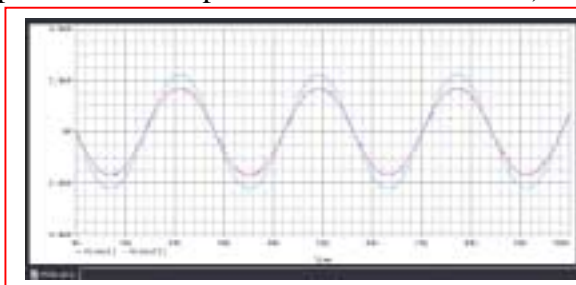
With  $\left. \frac{dv}{dt} \right|_{t=0^+} = (0.2236)(-2.236 \times 10^{-3}) = -5 \times 10^{-4}$ , one possible solution is:



PSpice simulations are very sensitive to parameter values; better results were obtained using LF411 instead of 741s (both were compared to the simple LC circuit simulation.)



Simulation using 741 op amps



Simulation using LF411 op amps

## CHAPTER NINE SOLUTIONS

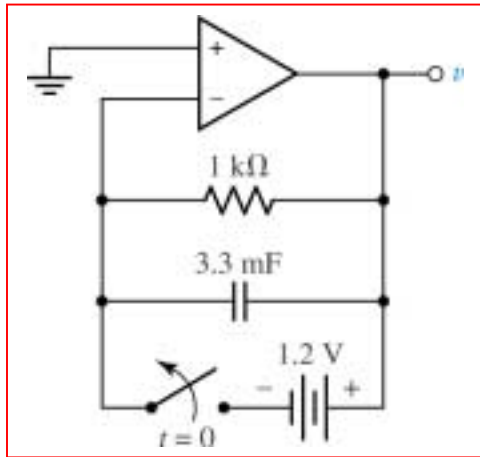
57.

$$\frac{v}{1000} + 3.3 \times 10^{-3} \frac{dv}{dt} = 0$$

(a) or

$$\frac{dv}{dt} = -\frac{1}{3.3}v$$

(b) One possible solution:



## CHAPTER NINE SOLUTIONS

58.  $\alpha = 0$  (this is a series RLC with  $R = 0$ , or a parallel RLC with  $R = \infty$ )  
 $\omega_o^2 = 50$  therefore  $\omega_d = 7.071$  rad/s. We anticipate a response of the form:  
 $v(t) = A \cos 7.071t + B \sin 7.071t$ , knowing that  $i_L(0^-) = 2$  A and  $v(0^-) = 0$ .

$$v(0^+) = v(0^-) = 0 = A \text{ therefore } v(t) = B \sin 7.071t$$

$$dv/dt = 7.071B \cos 7.071t; \quad i_C(t) = Cdv/dt = 0.007071B \cos 7.071t$$

$$i_C(0^+) = 0.007071B = -i_L(0^+) = -i_L(0^-) = -2 \text{ so } B = -282.8 \text{ and thus}$$

$$v(t) = -282.8 \sin 7.071t \text{ V}$$

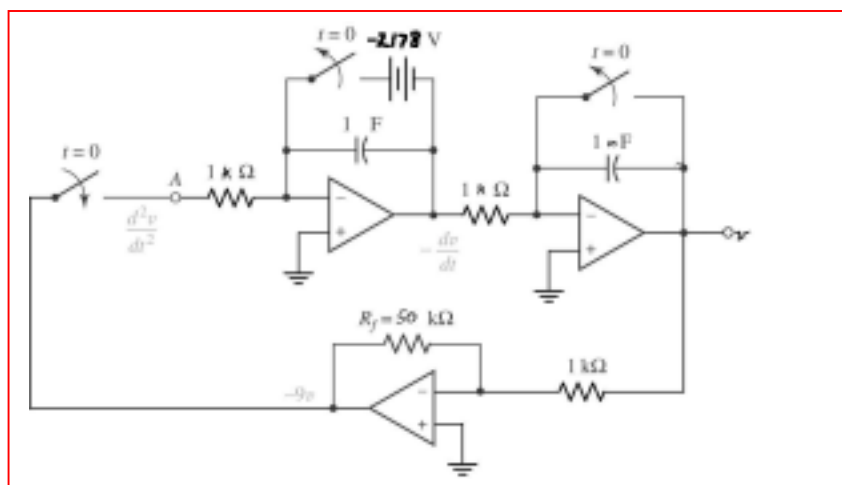
In designing the op amp stage, we first write the differential equation:

$$\frac{1}{20} \int_0^t v dt' + 2 \times 10^{-3} \frac{dv}{dt} = 0 \quad (i_C + i_L = 0)$$

and then take the derivative of both sides:

$$\frac{d^2v}{dt^2} = -50v$$

With  $\left. \frac{dv}{dt} \right|_{t=0^+} = (7.071)(-282.8) = -2178$ , one possible solution is:



## CHAPTER NINE SOLUTIONS

59. (a)  $v_R = v_L$

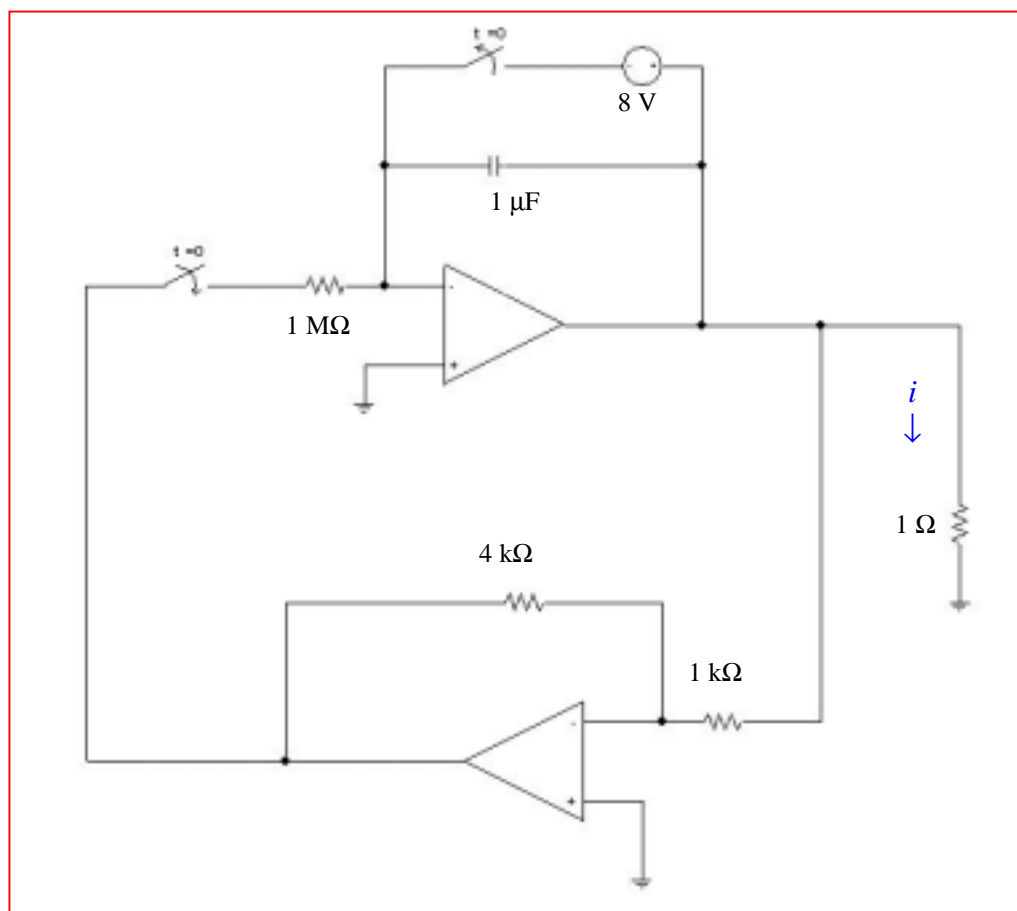
$$20(-i_L) = 5 \frac{di_L}{dt} \quad \text{or} \quad \boxed{\frac{di_L}{dt} = -4i_L}$$

(b) We expect a response of the form  $i_L(t) = A e^{-t/\tau}$  where  $\tau = L/R = 0.25$ .

We know that  $i_L(0^-) = 2$  amperes, so  $A = 2$  and  $i_L(t) = 2 e^{-4t}$

$$\left. \frac{di_L}{dt} \right|_{t=0^+} = -4(2) = -8 \text{ A/s.}$$

One possible solution, then, is



## CHAPTER NINE SOLUTIONS

60. We see either a series RLC with  $R = 0$  or a parallel RLC with  $R = \infty$ ; either way,  $\alpha = 0$ .  
 $\omega_0^2 = 0.3$  so  $\omega_d = 0.5477$  rad/s (combining the two inductors in parallel for the calculation). We expect a response of the form  $i(t) = A \cos \omega_d t + B \sin \omega_d t$ .

$$i(0^+) = i(0^-) = A = 1 \times 10^{-3}$$

$$di/dt = -A\omega_d \sin \omega_d t + B\omega_d \cos \omega_d t$$

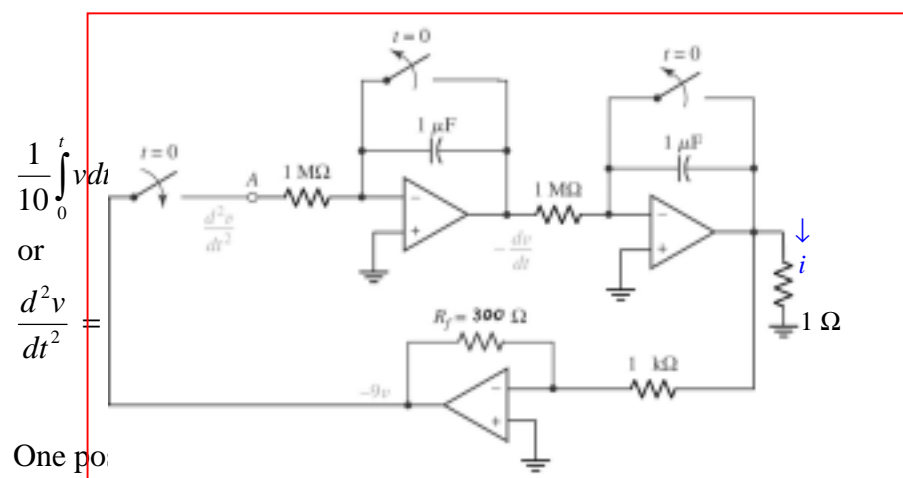
$$v_L = 10 di/dt = -10A\omega_d \sin \omega_d t + 10B\omega_d \cos \omega_d t$$

$$v_L(0^+) = v_C(0^+) = v_C(0^-) = 0 = 10B(0.5477) \text{ so that } B = 0$$

$$\text{and hence } i(t) = 10^{-3} \cos 0.5477t \text{ A}$$

The differential equation for this circuit is

$$\text{and } \left. \frac{di}{dt} \right|_{t=0^+} = 0$$





## CHAPTER TEN (Phasor Analysis) SOLUTIONS

1.

$$(a) \quad T = 4(7.5 - 2.1)10^{-3} = 21.6 \times 10^{-3}, \quad \omega = \frac{2\pi 10^3}{21.6} = 290.9 \text{ rad/s}$$

$$\therefore f(t) = 8.5 \sin(290.9t + \Phi) \therefore 0 = 8.5 \sin(290.9 \times 2.1 \times 10^{-3} + \Phi)$$

$$\therefore \Phi = -0.6109^{\text{rad}} + 2\pi = 5.672^{\text{rad}} \text{ or } 325.0^\circ$$

$$\therefore f(t) = 8.5 \sin(290.9t + 325.0^\circ)$$

$$(b) \quad 8.5 \sin(290.9t + 325.0^\circ) = 8.5$$

$$\cos(290.9t + 235^\circ) = 8.5 \cos(290.9t - 125^\circ)$$

$$(c) \quad 8.5 \cos(-125^\circ) \cos \omega t + 8.5 \sin 125^\circ$$

$$\sin \omega t = -4.875^+ \cos 290.9t + 6.963 \sin 290.9t$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

2.

- (a)  $-10 \cos \omega t + 4 \sin \omega t + A \cos(\omega t + \Phi)$ ,  $A > 0$ ,  $-180^\circ < \Phi \leq 180^\circ$   
 $A = \sqrt{116} = 10.770$ ,  $A \cos \Phi = -10$ ,  $A \sin \Phi = -4 \therefore \tan \Phi = 0.4$ , 3<sup>d</sup> quad  
 $\therefore \Phi = 21.80^\circ = 201.8^\circ$ , too large  $\therefore \Phi = 201.8^\circ - 360^\circ = -158.20^\circ$
- (b)  $200 \cos(5t + 130^\circ) = F \cos 5t + G \sin 5t \therefore F = 200 \cos 130^\circ = -128.56$   
 $G = -200 \sin 130^\circ = -153.21^\circ$
- (c)  $i(t) = 5 \cos 10t - 3 \sin 10t = 0$ ,  $0 \leq t \leq 1s \therefore \frac{\sin 10t}{\cos 10t} = \frac{5}{3}$ ,  $10t = 1.0304$ ,  
 $t = 0.10304s$ ; also,  $10t = 1.0304 + \pi$ ,  $t = 0.4172s$ ;  $2\pi$ :  $0.7314s$
- (d)  $0 < t < 10ms$ ,  $10 \cos 100\pi t \geq 12 \sin 100\pi t$ ; let  $10 \cos 100\pi t = 12 \sin 100\pi t$   
 $\therefore \tan 100\pi t = \frac{10}{12}$ ,  $100\pi t = 0.6947 \therefore t = 2.211ms$ ;  $0 < t < 2.211ms$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

3.

(a)  $f(t) = -50 \cos \omega t - 30 \sin \omega t = 58.31 \cos (\omega t + 149.04^\circ)$

$$g(t) = 55 \cos \omega t - 15 \sin \omega t = 57.01 \cos (\omega t + 15.255^\circ)$$

$$\therefore \text{ampl. of } f(t) = 58.31, \text{ ampl. of } g(t) = 57.01$$

(b)  $f(t)$  leads  $g(t)$  by  $149.04^\circ - 15.255^\circ = 133.78^\circ$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

4.

$$i(t) = A \cos(\omega t - \theta), \quad L(di/dt) + Ri = V_m \cos \omega t$$

$$\therefore L[-\omega A \sin(\omega t - \theta)] + RA \cos(\omega t - \theta) = V_m \cos \omega t$$

$$\therefore -\omega LA \sin \omega t \cos \theta + \omega LA \cos \omega t \sin \theta$$

$$+ RA \cos \omega t \cos \theta + RA \sin \omega t \sin \theta =$$

$$V_m \cos \omega t \therefore \omega LA \cos \theta = RA \sin \theta \text{ and } \omega LA \sin \theta + RA \cos \theta = V_m$$

$$\therefore \tan \theta = \frac{\omega L}{R} \therefore \omega LA \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} + RA \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = V_m$$

$$\therefore \left( \frac{\omega^2 L^2}{\sqrt{\quad}} + \frac{R^2}{\sqrt{\quad}} \right) A = V_m \therefore \sqrt{R^2 + \omega^2 L^2} A = V_m \therefore A = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

5.  $f = 13.56 \text{ MHz}$  so  $\omega = 2\pi f = 85.20 \text{ Mrad/s}$ .

Delivering 300 W (peak) to a  $5\text{-}\Omega$  load implies that  $\frac{V_m^2}{5} = 300$  so  $V_m = 38.73 \text{ V}$ .

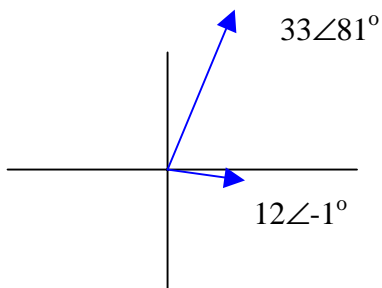
Finally,  $(85.2 \times 10^6)(21.15 \times 10^{-3}) + \phi = n\pi$ ,  $n = 1, 3, 5, \dots$

Since  $(85.2 \times 10^6)(21.15 \times 10^{-3}) = 1801980$ , which is  $573588^+\pi$ , we find that

$F = 573587\pi - (85.2 \times 10^6)(21.15 \times 10^{-3}) = -3.295 \text{ rad} = -188.8^\circ$ .

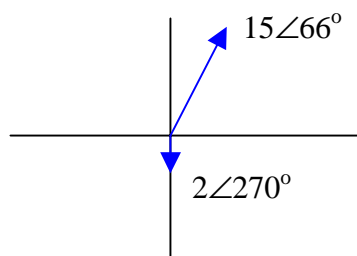
## CHAPTER TEN (Phasor Analysis) SOLUTIONS

6. (a)  $-33 \sin(8t - 9^\circ) \rightarrow -33 \angle (-9-90)^\circ = 33 \angle 81^\circ$   
 $12 \cos(8t - 1^\circ) \rightarrow 12 \angle -1^\circ$



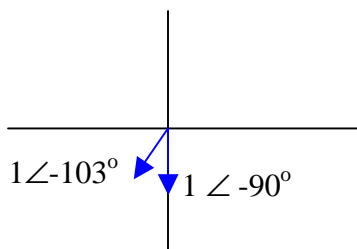
$-33 \sin(8t - 9^\circ)$  leads  $12 \cos(8t - 1^\circ)$  by  $81 - (-1) = 82^\circ$ .

- (b)  $15 \cos(1000t + 66^\circ) \rightarrow 15 \angle 66^\circ$   
 $-2 \cos(1000t + 450^\circ) \rightarrow -2 \angle 450^\circ = -2 \angle 90^\circ = 2 \angle 270^\circ$



$15 \cos(1000t + 66^\circ)$  leads  $-2 \cos(1000t + 450^\circ)$  by  $66 - -90 = 156^\circ$ .

- (c)  $\sin(t - 13^\circ) \rightarrow 1 \angle -103^\circ$   
 $\cos(t - 90^\circ) \rightarrow 1 \angle -90^\circ$



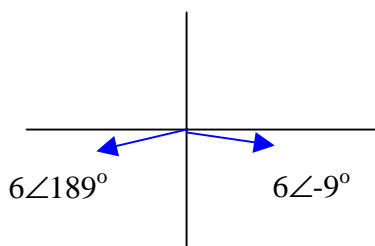
$\cos(t - 90^\circ)$  leads  $\sin(t - 13^\circ)$  by  $66 - -90 = 156^\circ$ .

- (d)  $\sin t \rightarrow 1 \angle -90^\circ$   
 $\cos(t - 90^\circ) \rightarrow 1 \angle -90^\circ$

These two waveforms are *in phase*. Neither leads the other.

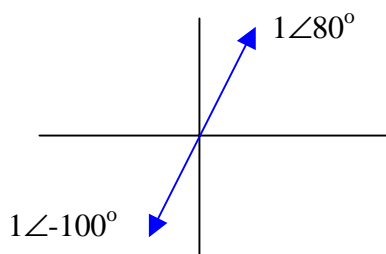
## CHAPTER TEN (Phasor Analysis) SOLUTIONS

7. (a)  $6 \cos(2\pi 60t - 9^\circ) \rightarrow 6 \angle -9^\circ$   
 $-6 \cos(2\pi 60t + 9^\circ) \rightarrow 6 \angle 189^\circ$



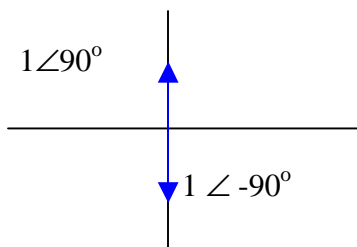
$-6 \cos(2\pi 60t + 9^\circ)$  lags  $6 \cos(2\pi 60t - 9^\circ)$  by  $360 - 9 - 189 = 162^\circ$ .

(b)  $\cos(t - 100^\circ) \rightarrow 1 \angle -100^\circ$   
 $-\cos(t - 100^\circ) \rightarrow -1 \angle -100^\circ = 1 \angle 80^\circ$



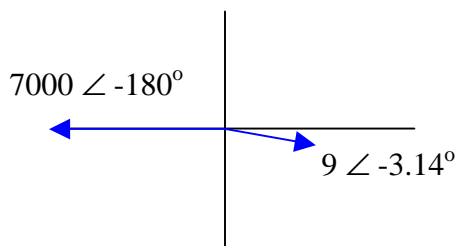
$-\cos(t - 100^\circ)$  lags  $\cos(t - 100^\circ)$  by  $180^\circ$ .

(c)  $-\sin t \rightarrow -1 \angle -90^\circ = 1 \angle 90^\circ$   
 $\sin t \rightarrow 1 \angle -90^\circ$



$-\sin t$  lags  $\sin t$  by  $180^\circ$ .

(d)  $7000 \cos(t - \pi) \rightarrow 7000 \angle -\pi = 7000 \angle -180^\circ$   
 $9 \cos(t - 3.14^\circ) \rightarrow 9 \angle -3.14^\circ$



$7000 \cos(t - \pi)$  lags  $9 \cos(t - 3.14^\circ)$  by  $180 - 3.14 = 176.9^\circ$ .

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

8.  $v(t) = V_1 \cos \omega t - V_2 \sin \omega t$  [1]

We assume this can be written as a single cosine such that

$$v(t) = V_m \cos (\omega t + \phi) = V_m \cos \omega t \cos \phi - V_m \sin \omega t \sin \phi \quad [2]$$

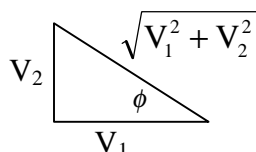
Equating terms on the right hand sides of Eqs. [1] and [2],

$$V_1 \cos \omega t - V_2 \sin \omega t = (V_m \cos \phi) \cos \omega t - (V_m \sin \phi) \sin \omega t$$

yields

$$V_1 = V_m \cos \phi \text{ and } V_2 = V_m \sin \phi$$

Dividing, we find that  $\frac{V_2}{V_1} = \frac{V_m \sin \phi}{V_m \cos \phi} = \tan \phi$  and  $\phi = \tan^{-1}(V_2/V_1)$



Next, we see from the above sketch that we may write  $V_m = V_1 / \cos \phi$  or

$$V_m = \frac{V_1}{V_1 / \sqrt{V_1^2 + V_2^2}} = \sqrt{V_1^2 + V_2^2}$$

Thus, we can write  $v(t) = V_m \cos (\omega t + \phi) = \sqrt{V_1^2 + V_2^2} \cos [\omega t + \tan^{-1}(V_2/V_1)]$ .



## CHAPTER TEN (Phasor Analysis) SOLUTIONS

9. (a) In the range  $0 \leq t \leq 0.5$ ,  $v(t) = t/0.5$  V.  
Thus,  $v(0.4) = 0.4/0.5 = 0.8$  V.
- (b) Remembering to set the calculator to radians, 0.7709 V.
- (c) 0.8141 V.
- (d) 0.8046 V.

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

$$\begin{aligned}
 10. \quad (a) \quad V_{\text{rms}} &= \left[ \frac{V_m^2}{T} \int_0^T \cos^2 \omega t \, dt \right]^{1/2} \\
 &= \left[ \frac{V_m^2}{T} \int_0^T \cos^2 \frac{2\pi t}{T} \, dt \right]^{1/2} \\
 &= \left[ \frac{V_m^2}{2T} \int_0^T \left( 1 + \cos \frac{4\pi t}{T} \right) dt \right]^{1/2} \\
 &= \left[ \frac{V_m^2}{2T} \int_0^T dt + \frac{V_m^2}{2T} \int_0^T \cos \frac{4\pi t}{T} \, dt \right]^{1/2} \\
 &= \left[ \frac{V_m^2}{2T} T + \frac{V_m^2}{8\pi} \cos u \Big|_0^{4\pi} \right]^{1/2} \\
 &= \frac{V_m}{\sqrt{2}}
 \end{aligned}$$

$$(b) \quad V_m = 110\sqrt{2} = 155.6 \text{ V}, 115\sqrt{2} = 162.6 \text{ V}, 120\sqrt{2} = 169.7 \text{ V}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

11.

$$\text{At } x-x: v_{oc} = \frac{3}{4}v_s = 15 \cos 500t \text{ V}$$

$$R_{th} = 5 + 20 \parallel 60 = 20 \Omega$$

$$\therefore V_m = 15 \text{ V}, R = 20 \Omega, \omega L = 10 \Omega$$

$$\therefore i_L = \frac{15}{\sqrt{20^2 + 10^2}} \cos \left( 500t - \tan^{-1} \frac{10}{20} \right) = 0.6708 \cos (500t - 26.57) \text{ A}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

12.

$$\text{At } x-x: R_{th} = 80 \parallel 20 = 16\Omega$$

$$v_{oc} = -0.4(15 \parallel 85) \frac{80}{85} \cos 500t$$

$$\therefore v_{oc} = 4.8 \cos 500t \text{ V}$$

$$\begin{aligned} \text{(a)} \quad i_L &= \frac{4.8}{\sqrt{16^2 + 10^2}} \cos \left( 500t - \tan^{-1} \frac{10}{15} \right) \\ &= 0.2544 \cos (500t - 32.01^\circ) \text{ A} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad v_L &= L i_L' = 0.02 \times 0.02544 (-500) \\ &\sin (500t - 32.01^\circ) = -2.544 \sin (500t - 32.01^\circ) \text{ V} \\ \therefore v_L &= 2.544 \cos (500t + 57.99^\circ) \text{ V}, i_x \\ &= 31.80 \cos (500t + 57.99^\circ) \text{ mA} \end{aligned}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

13.

$$(a) \quad i = \frac{100}{\sqrt{500^2 + 800^2}} \cos\left(10^5 t - \frac{800}{500}\right) = 0.10600 \cos(10^5 t - 57.99^\circ) \text{ A}$$

$$p_R = 0 \text{ when } i = 0 \therefore 10^5 t - \frac{57.99^\circ}{180} \pi = \frac{\pi}{2}, t = \boxed{25.83 \mu\text{s}}$$

$$(b) \quad \pm v_L = Li' = 8 \times 10^{-3} \times 0.10600 (-10^5) \sin(10^5 t - 57.99^\circ)$$

$$\therefore v_L = -84.80 \sin(10^5 t - 57.99^\circ)$$

$$\therefore p_L = v_L i = -8.989 \sin(10^5 t - 57.99^\circ)$$

$$\cos(10^5 t - 57.99^\circ) = -4.494 \sin(2 \times 10^5 t - 115.989^\circ)$$

$$\therefore p_L = 0 \text{ when } 2 \times 10^5 t - 115.989^\circ = 0^\circ, 180^\circ,$$

$$\therefore t = \boxed{10.121 \text{ or } 25.83 \mu\text{s}}$$

$$(c) \quad p_s = v_s i_L = 10.600 \cos 10^5 t \cos(10^5 t - 57.99^\circ)$$

$$\therefore p_s = 0 \text{ when } 10^5 t = \frac{\pi}{2}, t = \boxed{15.708 \mu\text{s}} \text{ and also } t = \boxed{25.83 \mu\text{s}}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

14.  $v_s = 3 \cos 10^5 t$  V,  $i_s = 0.1 \cos 10^5 t$  A

$v_s$  in series with  $30\Omega \rightarrow 0.1 \cos 10^5 t$  A  $\parallel 30\Omega$

Add, getting  $0.2 \cos 10^5 t$  A  $\parallel 30\Omega$

change to  $6 \cos 10^5 t$  V in series with  $30\Omega$ ;  $30\Omega + 20\Omega = 50\Omega$

$$\therefore i_L = \frac{6}{\sqrt{50^2 + 10^2}} \cos \left( 10^5 t - \tan^{-1} \frac{10}{50} \right) = 0.11767 \cos (10^5 t - 11.310^\circ) \text{ A}$$

At  $t = 10\mu\text{s}$ ,  $10^5 t = 1 \therefore i_L = 0.1167 \cos (1^{\text{rad}} - 11.310^\circ) = 81.76\text{mA}$

$\therefore v_L = 0.11767 \times 10 \cos (1^{\text{rad}} - 11.30^\circ + 90^\circ) = -0.8462\text{V}$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

15.  $\therefore v_{oc} = \cos 500t \text{ V}$   
 $\therefore \cos 500t = 100(0.8i_{sc})$   
 $\therefore i_{sc} = \frac{1}{80} \cos 500t \text{ A} \therefore R_{th} = \frac{v_{oc}}{i_{sc}} = 80\Omega$   
 $\therefore i_L = \frac{1}{\sqrt{80^2 + 150^2}} \cos \left( 500t - \tan^{-1} \frac{150}{80} \right)$   
 $i_L = 5.882 \cos(500t - 61.93^\circ) \text{ mA}$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

$$\begin{aligned}
 16. \quad v_{s1} &= V_{s2} = 120 \cos 120\pi t \text{ V} \\
 \frac{120}{60} &= 2\text{A}, \quad \frac{120}{12} = 1\text{A}, \quad 2 + 1 = 3\text{A}, \quad 60 \parallel 120 = 40\Omega \\
 3 \times 40 &= 120 \text{ V}, \quad \omega L = 12\pi = 37.70\Omega \\
 \therefore i_L &= \frac{120}{\sqrt{40^2 + 37.70^2}} \cos \left( 120\pi t - \tan^{-1} \frac{37.70}{40} \right) \\
 &= 2.183 \cos(120\pi t - 43.30^\circ) \text{ A}
 \end{aligned}$$

$$\begin{aligned}
 (a) \quad \therefore \omega_L &= \frac{1}{2} \times 0.1 \times 2.183^2 \cos^2(120\pi t - 43.30^\circ) \\
 &= 0.2383 \cos^2(120\pi t - 43.30^\circ) \text{ J}
 \end{aligned}$$

$$(b) \quad \omega_{L,av} = \frac{1}{2} \times 0.2383 = 0.11916 \text{ J}$$



## CHAPTER TEN (Phasor Analysis) SOLUTIONS

17.  $\uparrow v_{s1} = 120 \cos 400t \text{ V}, v_{s2} = 180 \cos 200t \text{ V}$

$$\frac{120}{60} = 2 \text{ A}, \frac{180}{120} = 1.5 \text{ A}, 60 \parallel 120 = 40 \Omega$$

$$2 \times 40 = 80 \text{ V}, 1.5 \times 40 = 60 \text{ V}$$

$$i_L = \frac{80}{\sqrt{40^2 + 40^2}} \cos(400t - 45^\circ) + \frac{60}{\sqrt{40^2 + 20^2}} \cos(200t - 26.57^\circ) \text{ A}$$

$$\text{or } i_L = 1.4142 \cos(400t - 45^\circ) + 1.3416 \cos(200t - 26.57^\circ) \text{ A}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

18.

$$R_i = \infty, R_o = 0, A = \infty, \text{ ideal, } R_1 C_1 = \frac{L}{R}$$

$$i_{upper} = -\frac{V_m \cos \omega t}{R}, i_{lower} = \frac{v_{out}}{R_1}$$

$$\therefore i_{c1} = i_{upper} + i_{lower} = \frac{i}{R_1} (v_{out} - V_m \cos \omega t) = -C_1 v'_{out}$$

$$\therefore V_m \cos \omega t = v_{out} + R_1 C_1 v'_{out} = v_{out} + \frac{L}{R} v'_{out}$$

$$\text{For RL circuit, } V_m \cos \omega t = v_r + L \frac{d}{dt} \left( \frac{v_R}{R} \right)$$

$$\therefore V_m \cos \omega t = v_R + \frac{L}{R} v'_R$$

By comparison,  $v_R = v_{out}$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

19.

(a)  $V_m \cos \omega t = Ri + \frac{1}{C} \int i dt$  (ignore I.C)

$$\therefore -\omega V_m \sin \omega t = Ri' + \frac{1}{C} i$$

(b) Assume  $i = A \cos(\omega t + \Phi)$

$$\therefore -\omega V_m \sin \omega t = -R\omega A \sin(\omega t + \Phi) + \frac{A}{C} \cos(\omega t + \Phi)$$

$$\therefore -\omega V_m \sin \omega t = -R\omega A \cos \Phi \sin \omega t - R\omega A \sin \Phi \cos \omega t + \frac{A}{C} \cos \omega t \cos \Phi - \frac{A}{C} \sin \omega t \sin \Phi$$

Equating terms on the left and right side,

[1]  $R\omega A \sin \Phi = \frac{A}{C} \cos \Phi \therefore \tan \Phi = \frac{1}{\omega CR}$  so  $\Phi = \tan^{-1}(1/\omega CR)$ , and

[2]  $-\omega V_m = -R\omega A \frac{\omega CR}{\sqrt{1+\omega^2 C^2 R^2}} - \frac{A}{C} \frac{1}{\sqrt{1+\omega^2 C^2 R^2}}$

$$\therefore \omega V_m = \frac{A}{C} \left[ \frac{R^2 \omega^2 C^2 + 1}{\sqrt{1+\omega^2 C^2 R^2}} \right] = \frac{A}{C} \sqrt{1+\omega^2 C^2 R^2} \therefore A = \frac{\omega C V_m}{\sqrt{1+\omega^2 C^2 R^2}}$$

$$\therefore i = \frac{\omega C V_m}{\sqrt{1+\omega^2 C^2 R^2}} \cos \left( \omega t + \tan^{-1} \frac{1}{\omega CR} \right)$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

20. (a)  $7 \angle -90^\circ = -j 7$
- (b)  $3 + j + 7 \angle -17^\circ = 3 + j + 6.694 - j 2.047 = 9.694 - j 1.047$
- (c)  $14e^{j15^\circ} = 14 \angle 15^\circ = 14 \cos 15^\circ + j 14 \sin 15^\circ = 13.52 + j 3.263$
- (d)  $1 \angle 0^\circ = 1$
- (e)  $-2(1 + j 9) = -2 - j 18 = 18.11 \angle -96.34^\circ$
- (f)  $3 = 3 \angle 0^\circ$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

21. (a)  $3 + 15 \angle -23^\circ = 3 + 13.81 - j 5.861 = 16.81 - j 5.861$
- (b)  $(j 12)(17 \angle 180^\circ) = (12 \angle 90^\circ)(17 \angle 180^\circ) = 204 \angle 270^\circ = -j 204$
- (c)  $5 - 16(9 - j 5) / (33 \angle -9^\circ) = 5 - (164 \angle -29.05^\circ) / (33 \angle -9^\circ)$   
 $= 5 - 4.992 \angle -20.05^\circ = 5 - 4.689 - j 1.712 = 0.3109 + j 1.712$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

$$22. \quad (a) \quad 5 \angle 9^\circ - 9 \angle -17^\circ = 4.938 + j 0.7822 - 8.607 + j 2.631 = -3.669 - j 1.849 \\ = 4.108 \angle -153.3^\circ$$

$$(b) \quad (8 - j 15)(4 + j 16) - j = 272 + j 68 - j = 272 + j 67 = 280.1 \angle 13.84^\circ$$

$$(c) \quad (14 - j 9) / (2 - j 8) + 5 \angle -30^\circ = (16.64 \angle -32.74^\circ) / (8.246 \angle -75.96^\circ) + 4.330 - j 2.5 \\ = 1.471 + j 1.382 + 4.330 - j 2.5 = 5.801 - j 1.118 = 5.908 \angle -10.91^\circ$$

$$(d) \quad 17 \angle -33^\circ + 6 \angle -21^\circ + j 3 = 14.26 - j 9.259 + 5.601 - j 2.150 + j 3 \\ = 19.86 - j 8.409 = 21.57 \angle -22.95^\circ$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

23. (a)  $e^{j14^\circ} + 9 \angle 3^\circ - (8 - j6)/j^2 = 1 \angle 14^\circ + 9 \angle 3^\circ - (8 - j6)/(-1)$   
 $= 0.9703 + j0.2419 + 8.988 + j0.4710 + 8 - j6 = 17.96 - j5.287 = 18.72 \angle -16.40^\circ$

(b)  $(5 \angle 30^\circ)/(2 \angle -15^\circ) + 2e^{j5^\circ}/(2 - j2)$   
 $= 2.5 \angle 45^\circ + (2 \angle 5^\circ)/(2.828 \angle -45^\circ) = 1.768 + j1.768 + 0.7072 \angle 50^\circ$   
 $= 1.768 + j1.768 + 0.4546 + j0.5418$   
 $= 2.224 + j2.310 = 3.207 \angle 46.09^\circ$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

24.

(a)  $5\angle -110^\circ = -1.7101 - j4.698$

(b)  $6e^{j160^\circ} = -5.638 + j2.052$

(c)  $(3 + j6)(2\angle 50^\circ) = -5.336 + j12.310$

(d)  $-100 - j40 = 107.70\angle -158.20^\circ$

(e)  $2\angle 50^\circ + 3\angle -120^\circ = 1.0873\angle -101.37^\circ$



## CHAPTER TEN (Phasor Analysis) SOLUTIONS

25.

$$(a) \quad 40\angle -50^\circ - 18\angle 25^\circ = 39.39\angle -76.20^\circ$$

$$(b) \quad 3 + \frac{2}{j} + \frac{2 - j5}{1 + j2} = 4.050\angle -69.78^\circ$$

$$(c) \quad (2.1\angle 25^\circ)^3 = 9.261\angle 75^\circ = 2.397 + j8.945^+$$

$$(d) \quad 0.7e^{j0.3} = 0.7\angle 0.3^{\text{rad}} = 0.6687 + j0.2069$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

26.

$$i_c = 20e^{j(40t+30^\circ)} \text{ A} \therefore v_c = 100 \int 20e^{j(40t+30^\circ)} dt$$

$$v_c = -j50e^{j(40t+30^\circ)}, i_R = -j10e^{j(40t+30^\circ)} \text{ A}$$

$$\therefore i_L = (20 - j10)e^{j(40t+30^\circ)}, v_L = j40 \times 0.08(20 - j10)e^{j(40t+30^\circ)}$$

$$\therefore v_L = (32 + j64)e^{j(40t+30^\circ)} \text{ V} \therefore v_s = (32 + j64 - j50)e^{j(40t+30^\circ)}$$

$$\therefore v_s = 34.93e^{j(40t-53.63^\circ)} \text{ V}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

27.

$$i_L = 20e^{j(10t+25^\circ)} \text{ A}$$

$$v_L = 0.2 \frac{d}{dt} [20e^{j(10t+25^\circ)}] = j40e^{j(10t+25^\circ)}$$

$$v_R = 80e^{j(10t+25^\circ)}$$

$$v_s = (80 + j40)e^{j(10t+25^\circ)}, i_c = 0.08(80 + j40)j10e^{j(10t+25^\circ)}$$

$$\therefore i_c = (-32 + j64)e^{j(10t+25^\circ)} \therefore i_s = (-12 + j64)e^{j(10t+25^\circ)}$$

$$\therefore i_s = 65.12e^{j(10t+125.62^\circ)} \text{ A}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

28.  $80 \cos(500t - 20^\circ) \text{ V} \rightarrow 5 \cos(500t + 12^\circ) \text{ A}$

(a)  $v_s = 40 \cos(500t + 10^\circ) \therefore i_{out} = 2.5 \cos(500t + 42^\circ) \text{ A}$

(b)  $v_s = 40 \sin(500t + 10^\circ) = 40 \cos(500t - 80^\circ)$   
 $\therefore i_{out} = 2.5 \cos(500t - 48^\circ) \text{ A}$

(c)  $v_s = 40e^{j(500t+10^\circ)} = 40 \cos(500t + 10^\circ)$   
 $+ j40 \sin(500t + 10^\circ) \therefore i_{out} = 2.5e^{j(500t+42^\circ)} \text{ A}$

(d)  $v_s = (50 + j20)e^{j500t} = 53.85^+ e^{j21.80^\circ + j500t}$   
 $\therefore i_{out} = 3.366e^{j(500t+53.80^\circ)} \text{ A}$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

29.

(a)  $12 \sin(400t + 110^\circ) \text{ A} \rightarrow 12 \angle 20^\circ \text{ A}$

(b)  $-7 \sin 800t - 3 \cos 800t \rightarrow j7 - 3$   
 $= -3 + j7 = 7.616 \angle 113.20^\circ \text{ A}$

(c)  $4 \cos(200t - 30^\circ) - 5 \cos(200t + 20^\circ)$   
 $\rightarrow 4 \angle -30^\circ - 5 \angle 20^\circ = 3.910 \angle -108.40^\circ \text{ A}$

(d)  $\omega = 600, t = 5 \text{ ms} : 70 \angle 30^\circ \text{ V}$   
 $\rightarrow 70 \cos(600 \times 5 \times 10^{-3} \text{ rad} + 30^\circ) = -64.95^+ \text{ V}$

(e)  $\omega = 600, t = 5 \text{ ms} : 60 + j40 \text{ V} = 72.11 \angle 146.3^\circ$   
 $\rightarrow 72.11 \cos(3^{\text{rad}} + 146.31^\circ) = 53.75^+ \text{ V}$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

30.  $\omega = 4000$ ,  $t = 1\text{ms}$

(a)  $I_x = 5\angle -80^\circ \text{ A}$

$$\therefore i_x = 4 \cos(4^{\text{rad}} - 80^\circ) = -4.294 \text{ A}$$

(b)  $I_x = -4 + j1.5 = 4.272\angle 159.44^\circ \text{ A}$

$$\therefore i_x = 4.272 \cos(4^{\text{rad}} + 159.44^\circ) = 3.750^- \text{ A}$$

(c)  $v_x(t) = 50 \sin(250t - 40^\circ)$

$$= 50 \cos(250t - 130^\circ) \rightarrow V_x = 50\angle -103^\circ \text{ V}$$

(d)  $v_x = 20 \cos 108t - 30 \sin 108t$

$$\rightarrow 20 + j30 = 36.06\angle 56.31^\circ \text{ V}$$

(e)  $v_x = 33 \cos(80t - 50^\circ) + 41 \cos(80t - 75^\circ)!$

$$\rightarrow 33\angle -50^\circ + 41\angle -75^\circ = 72.27\angle -63.87^\circ \text{ V}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

31.  $V_1 = 10\angle 90^\circ \text{ mV}$ ,  $\omega = 500$ ;  $V_2 = 8\angle 90^\circ \text{ mV}$ ,  
 $\omega = 1200$ , M by  $-5$ ,  $t = 0.5 \text{ ms}$

$$\begin{aligned} v_{out} &= (-5) [10 \cos(500 \times 0.5 \times 10^{-3} \text{ rad} + 90^\circ) \\ &\quad + 8 \cos(1.2 \times 0.5 + 90^\circ)] \\ &= 50 \sin 0.25^{\text{rad}} + 40 \sin 0.6^{\text{rad}} = \boxed{34.96 \text{ mV}} \end{aligned}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

32. Begin with the inductor:  
 $(2.5 \angle 40^\circ)(j500)(20 \times 10^{-3}) = 25 \angle 130^\circ$  V across the inductor and the  $25\text{-}\Omega$  resistor.  
The current through the  $25\text{-}\Omega$  resistor is then  $(25 \angle 130^\circ) / 25 = 1 \angle 130^\circ$  A.

The current through the unknown element is therefore  $25 \angle 130^\circ + 1 \angle 130^\circ = 26 \angle 130^\circ$ ; this is the same current through the  $10\text{-}\Omega$  resistor as well. Thus, KVL provides that

$$\mathbf{V}_s = 10(26 \angle 130^\circ) + (25 \angle -30^\circ) + (25 \angle 130^\circ) = 261.6 \angle 128.1^\circ$$

and so  $v_s(t) = 261.6 \cos(500t + 128.1^\circ)$  V.



## CHAPTER TEN (Phasor Analysis) SOLUTIONS

33.  $\omega = 5000 \text{ rad/s}$ .

- (a) The inductor voltage  $= 48 \angle 30^\circ = j\omega L \mathbf{I}_L = j(5000)(1.2 \times 10^{-3}) \mathbf{I}_L$   
So  $\mathbf{I}_L = 8 \angle -60^\circ$  and the total current flowing through the capacitor is  
 $10 \angle 0^\circ - \mathbf{I}_L = 9.165 \angle 49.11^\circ \text{ A}$  and the voltage  $\mathbf{V}_1$  across the capacitor is

$$\mathbf{V}_1 = (1/j\omega C)(9.165 \angle 49.11^\circ) = -j2 (9.165 \angle 49.11^\circ) = 18.33 \angle -40.89^\circ \text{ V}.$$

Thus,  $v_1(t) = 18.33 \cos(5000t - 40.89^\circ) \text{ V}$ .

- (b)  $\mathbf{V}_2 = \mathbf{V}_1 + 5(9.165 \angle 49.11^\circ) + 60 \angle 120^\circ = 75.88 \angle 79.48^\circ \text{ V}$   
 $\therefore v_2(t) = 75.88 \cos(5000t + 79.48^\circ) \text{ V}$

- (c)  $\mathbf{V}_3 = \mathbf{V}_2 - 48 \angle 30^\circ = 75.88 \angle 79.48^\circ - 48 \angle 30^\circ = 57.70 \angle 118.7^\circ \text{ V}$   
 $\therefore v_3(t) = 57.70 \cos(5000t + 118.70^\circ) \text{ V}$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

34.  $\mathbf{V}_R = 1\angle 0^\circ \text{ V}$ ,  $\mathbf{V}_{\text{series}} = (1 + j\omega - j/\omega)(1\angle 0^\circ)$

$$V_R = 1 \quad \text{and} \quad V_{\text{series}} = \sqrt{1 + (\omega - 1/\omega)^2}$$

We desire the frequency  $\omega$  at which  $V_{\text{series}} = 2V_R$  or  $V_{\text{series}} = 2$

Thus, we need to solve the equation  $1 + (\omega - 1/\omega)^2 = 4$

$$\text{or } \omega^2 - \sqrt{3}\omega - 1 = 0$$

Solving, we find that  $\omega = 2.189 \text{ rad/s}$ .

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

35. With an operating frequency of  $\omega = 400$  rad/s, the impedance of the 10-mH inductor is  $j\omega L = j4\ \Omega$ , and the impedance of the 1-mF capacitor is  $-j/\omega C = -j2.5\ \Omega$ .
- $$\therefore V_c = 2\angle 40^\circ (-j2.5) = 5\angle -50^\circ\text{ V}$$
- $$\therefore I_L = 3 - 2\angle 40^\circ = 1.9513\angle -41.211^\circ\text{ A}$$
- $$\therefore V_L = 4 \times 1.9513\angle 90^\circ - 4.211^\circ = 7.805^+\angle 48.79^\circ\text{ V}$$
- $$\therefore V_x = V_L - V_c = 7.805^+\angle 48.79^\circ - 5\angle -50^\circ$$
- $$\therefore V_x = 9.892\angle 78.76^\circ\text{ V}, \quad v_x = 9.892\cos(400t + 78.76^\circ)\text{ V}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

36.

$$\text{If } I_{s1} = 2\angle 20^\circ \text{ A, } I_{s2} = 3\angle -30^\circ \text{ A} \rightarrow V_{out} = 80\angle 10^\circ \text{ V}$$

$$I_{s1} = I_{s2} = 4\angle 40^\circ \text{ A} \rightarrow V_{out} = 90 - j30 \text{ V}$$

$$\text{Now let } I_{s1} = 2.5\angle -60^\circ \text{ A and } I_{s2} = 2.5\angle 60^\circ \text{ A}$$

$$\text{Let } V_{out} = AI_{s1} + BI_{s2} \therefore 80\angle 10^\circ = A(2\angle 20^\circ) + B(3\angle -30^\circ)$$

$$\text{and } 90 - j30 = (A + B)(4\angle 40^\circ) \therefore A + B = \frac{90 - j30}{4\angle 40^\circ} = 12.45^+ - j20.21$$

$$\therefore \frac{80\angle 10^\circ}{2\angle 20^\circ} = A + B \frac{3\angle -30^\circ}{2\angle 20^\circ} \therefore A = 40\angle -10^\circ - B(1.5\angle -50^\circ)$$

$$\therefore 12.415^+ - j20.21 - B = 40\angle -10^\circ - B(1.5\angle -50^\circ)$$

$$\therefore 12.415^+ - j20.21 - 40\angle -10^\circ = B(1 - 1.5\angle -50^\circ)$$

$$= B(1.1496\angle -88.21^\circ)$$

$$\therefore B = \frac{30.06\angle -153.82^\circ}{1.1496\angle -88.21^\circ} = 10.800 - j23.81$$

$$= 26.15\angle -65.61^\circ$$

$$\therefore A = 12.415^+ - j20.21 - 10.800 + j23.81$$

$$= 3.952^- \angle 65.87^\circ$$

$$\therefore V_{out} = (3.952\angle 65.87^\circ)(2.5\angle -60^\circ)$$

$$+ (26.15^- \angle -65.61^\circ)(2.5\angle 60^\circ) = \boxed{75.08\angle -4.106^\circ \text{ V}}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

37.

(a)  $\omega = 800: 2\mu\text{F} \rightarrow -j625, 0.6\text{H} \rightarrow j480$

$$\begin{aligned}\therefore Z_{in} &= \frac{300(-j625)}{300 - j625} + \frac{600(j480)}{600 + j480} \\ &= 478.0 + j175.65\Omega\end{aligned}$$

(b)  $\omega = 1600: Z_{in} = \frac{300(-j312.5)}{300 - j312.5}$

$$+ \frac{600(j960)}{600 + j960} = 587.6 + j119.79\Omega$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

38.

$$\begin{aligned} \text{(a)} \quad (10 + j10) \parallel (-j5) &= \frac{50 - j50}{10 + j5} = \frac{10 - j10}{2 + j1} \frac{2 - j1}{2 - j1} \\ &= 2 - j6 \Omega \therefore Z_{in} = \boxed{22 - j6 \Omega} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \text{SC}_{a,b}: 20 \parallel 10 &= 6.667, (6.667 - j5) \parallel j10 \\ &= \frac{50 + j66.67}{6.667 + j5} = \frac{150 + j200}{20 + j15} = \frac{30 + j40}{4 + j3} \times \frac{4 - j3}{4 - j3} \\ &= Z_{in} \therefore Z_{in} (1.2 + j1.6) (4 - j3) = \boxed{9.6 + j2.8 \Omega} \end{aligned}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

39.

$$\omega = 800: 2\mu\text{F} \rightarrow -j625, 0.6\text{H} \rightarrow j480$$

$$\therefore Z_{in} = \frac{300(-j625)}{300 - j625} + \frac{600(j480)}{600 + j480}$$

$$= 478.0 + j175.65\Omega$$

$$\therefore I = \frac{120}{478.0 + j175.65} \times \frac{-j625}{300 - j625}$$

$$\text{or } I = 0.2124 \angle -45.82^\circ \text{ A}$$

Thus,  $i(t) = 212.4 \cos(800t - 45.82^\circ) \text{ mA}$ .

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

40.

(a)  $3\Omega 2\text{mH}: V = (3\angle -20^\circ) (3 + j4) = 15,000\angle 33.13^\circ \text{ V}$

(b)  $3\Omega 125\mu\text{F}: V = (3\angle -20^\circ) (3 - j4) = 15,000\angle -73.3^\circ \text{ V}$

(c)  $3\Omega 2\text{mH } 125\mu\text{F}: V = (3\angle -20^\circ) 3 = 9,000\angle -20^\circ \text{ V}$

(d) same:  $\omega = 4000 \therefore V = (3\angle -20^\circ) (3 + j8 - j2)$   
 $\therefore V = (3\angle -20^\circ) (3 + j6) = 20.12\angle 43.43^\circ \text{ V}$



## CHAPTER TEN (Phasor Analysis) SOLUTIONS

41.

(a)  $C = 20\mu\text{F}$ ,  $\omega = 100$

$$Z_{in} = \frac{1}{\frac{1}{200} + \frac{1}{j1000} + j1000 \times 20 \times 10^{-6}} = \frac{1}{0.005 - j0.01 + j0.002}$$

$$\therefore Z_{in} = \frac{1}{0.005 + j0.001} = 196.12 \angle -11.31^\circ \Omega$$

(b)  $\omega = 100 \therefore Z_{in} = \frac{1}{0.005 - j0.001 + j1000C} = 125 \angle = \frac{1}{0.008 \angle}$

$$\therefore 0.005^2 + (100C - 0.001)^2 = 0.008^2 \therefore 100C - 0.001 = \pm 6.245 \times 10^{-3}, C = 72.45 \mu\text{F}$$

(c)  $C = 20\mu\text{F} \therefore Z_{in} = \frac{1}{0.0005 - j0.1/\omega + j2 \times 10^{-5}\omega} = 100 \angle = \frac{1}{0.01 \angle}$

$$\therefore 0.005^2 + \left(2 \times 10^{-5}\omega - \frac{0.1}{\omega}\right)^2 = 0.0001, \left(2 \times 10^{-5} - \frac{0.1}{\omega}\right)^2 = 7.5 \times 10^{-5}$$

$$\therefore 2 \times 10^{-5} - \frac{0.01}{\omega} \mp 866.0 \times 10^{-5} = 0 \therefore 2 \times 10^{-5}\omega^2 \mp 866.0 \times 10^{-5}\omega - 0.1 = 0$$

$$\text{use } - \text{ sign: } \omega = \frac{866.0 \times 10^{-5} \pm \sqrt{7.5 \times 10^{-5} + 8 \times 10^{-6}}}{4 \times 10^{-5}} = 444.3 \text{ and } < 0$$

$$\text{use } + \text{ sign: } \omega = \frac{-866.0 \times 10^{-5} \pm \sqrt{7.5 \times 10^{-5} + 8 \times 10^{-6}}}{4 \times 10^{-5}} = 11.254 \text{ and } < 0$$

$$\therefore \omega = 11.254 \text{ and } 444.3 \text{ rad/s}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

42.

$$(a) \quad \left| \frac{1}{\frac{1}{jx} + \frac{1}{30}} \right| = 25 = \frac{1}{0.04} \therefore \frac{1}{900} + \frac{1}{x^2} = 0.0016$$

$$\therefore X = 45.23 \Omega = 0.002W, \omega = 2261 \text{ rad/s}$$

$$(b) \quad \angle Y_{in} = -25^\circ = \angle \text{ of } \left( \frac{1}{30} - j \frac{1}{x} \right) = \tan^{-1} \frac{-30}{x}$$

$$\therefore x = 64.34 = 0.02\omega, \omega = 3217 \text{ rad/s}$$

$$(c) \quad Z_{in} = \frac{30(j0.02\omega)}{30 + j0.02\omega} \times \frac{30 - j0.092\omega}{30 - j0.02\omega} = \frac{0.012\omega^2 + j18\omega}{900 + 0.0004\omega^2}$$

$$\therefore 0.012\omega^2 = 25 (900 + 0.0004\omega^2)$$

$$\therefore 0.012\omega^2 = 0.01\omega^2 + 22,500, \omega = 3354 \text{ rad/s}$$

$$(d) \quad 18\omega = 10(900 + 0.0004\omega^2), 0.004\omega^2 - 18\omega + 9000 = 0,$$

$$\omega^2 - 4500\omega + 2.25 \times 10^6 = 0$$

$$\omega = \frac{4500 \pm \sqrt{20.25 \times 10^6 - 9 \times 10^6}}{2} = \frac{4500 \pm 3354}{2} = 572.9, 3927 \text{ rad/s}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

43. With an operating frequency of  $\omega = 400$  rad/s, the impedance of the 10-mH inductor is  $j\omega L = j4 \Omega$ , and the impedance of the 1-mF capacitor is  $-j/\omega C = -j2.5 \Omega$ .

$$\therefore V_c = 2\angle 40^\circ (-j2.5) = 5\angle -50^\circ \text{ A}$$

$$\therefore I_L = 3 - 2\angle 40^\circ = 1.9513\angle -41.211^\circ \text{ A}$$

$$I_L = \frac{2\angle 40^\circ (R_2 - j2.5)}{R_1 + j4}$$

$$\therefore R_1 + j4 = \frac{2\angle 40^\circ (R_2 - j2.5)}{1.9513\angle -41.21^\circ}$$

$$= 1.0250\angle 81.21^\circ (R_2 - j2.5)$$

$$= R_2 (1.0250\angle 81.21^\circ) + 2.562\angle -8.789^\circ$$

$$= 0.15662R_2 + j1.0130R_2 + 2.532 - j0.3915$$

$$\therefore R_1 = 2.532 + 0.15662R_2, \quad 4 = 1.0130R_2 - 0.395$$

$$\therefore R_2 = 4.335^+ \Omega, \quad R_1 = 3.211\Omega$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

44.  $\omega = 1200 \text{ rad/s.}$

$$(a) \quad Z_{in} = \frac{-j \times (200 + j80)}{200 + j(80 - x)} = \frac{(80x - j200x)[200 + j(x - 80)]}{40,000 + 6400 - 160x + x^2}$$

$$X_{in} = 0 \therefore -40,000x + 80x^2 - 6400x = 0$$

$$\therefore 46,400 = 80x, \quad x = 580 \Omega = \frac{1}{1200C} \therefore C = 14.368 \mu\text{F}$$

$$(b) \quad Z_{in} = \frac{80X - j200X}{200 + j(80 - X)} \quad |Z_{in}| = 100$$

$$\therefore \frac{6400X^2 + 40,000X^2}{40,000 + 6400 - 160X + X^2} = 10,000$$

$$\therefore 0.64X^2 + 4X^2 = X^2 - 160X + 46,400$$

$$\therefore 3.64X^2 + 160X - 46,400 = 0,$$

$$X = \frac{-160 \pm \sqrt{25,600 + 675,600}}{7.28} = \frac{-160 \pm 837.4}{7.28}$$

$$\therefore X = 93.05^- (> 0) = \frac{1}{1200C} \therefore C = 8.956 \mu\text{F}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

45. At  $\omega = 4$  rad/s, the  $1/8$ -F capacitor has an impedance of  $-j/\omega C = -j2 \Omega$ , and the 4-H inductor has an impedance of  $j\omega L = j16 \Omega$ .

$$\begin{aligned} \text{(a)} \quad \text{abOC: } Z_{in} &= \frac{(8 + j16)(2 - j2)}{10 + j14} = \frac{16(3 + j1)}{10 + j14} \\ &= \boxed{2.378 - j1.7297 \Omega} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \text{abSC: } Z_{in} &= (8 \parallel -j2) + (2 \parallel j16) = \frac{-j16}{8 - j2} + \frac{j32}{2 + j16} \\ \therefore Z_{in} &= \boxed{2.440 - j1.6362 \Omega} \end{aligned}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

46.  $f = 1 \text{ MHz}$ ,  $\omega = 2\pi f = 6.283 \text{ Mrad/s}$
- |                   |                                |                  |
|-------------------|--------------------------------|------------------|
| $2 \mu\text{F}$   | $\rightarrow -j0.07958 \Omega$ | $= \mathbf{Z}_1$ |
| $3.2 \mu\text{H}$ | $\rightarrow j20.11 \Omega$    | $= \mathbf{Z}_2$ |
| $1 \mu\text{F}$   | $\rightarrow -j0.1592 \Omega$  | $= \mathbf{Z}_3$ |
| $1 \mu\text{H}$   | $\rightarrow j6.283 \Omega$    | $= \mathbf{Z}_4$ |
| $20 \mu\text{H}$  | $\rightarrow j125.7 \Omega$    | $= \mathbf{Z}_5$ |
| $200 \text{ pF}$  | $\rightarrow -j795.8 \Omega$   | $= \mathbf{Z}_6$ |

The three impedances at the upper right,  $\mathbf{Z}_3$ ,  $700 \text{ k}\Omega$ , and  $\mathbf{Z}_6$  reduce to  $-j0.01592 \Omega$

Then we form  $\mathbf{Z}_2$  in series with  $\mathbf{Z}_{\text{eq}}$ :  $\mathbf{Z}_2 + \mathbf{Z}_{\text{eq}} = j20.09 \Omega$ .

Next we see  $10^6 \parallel (\mathbf{Z}_2 + \mathbf{Z}_{\text{eq}}) = j20.09 \Omega$ .

Finally,  $\mathbf{Z}_{\text{in}} = \mathbf{Z}_1 + \mathbf{Z}_4 + j20.09 = j26.29 \Omega$ .

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

47.

$2\text{H} \rightarrow j2, 1\text{F} \rightarrow -j1$  Let  $I_e = 1\angle 0^\circ \text{ A}$

$$\therefore V_L = j2\text{V} \therefore I_c = I_{in} + 0.5V_L = 1 + j1$$

$$\therefore V_{in} = j2 + (1 + j1)(-j1) = 1 + j1$$

$$\therefore V_{in} = \frac{1\angle 0^\circ}{V_{in}} = \frac{1}{1 + j1} \frac{1 - j1}{1 - j1} = 0.5 - j0.55$$

$$\text{Now } 0.5\text{ s} \rightarrow \boxed{2\Omega_s} - j0.5\text{S} = \frac{1}{j2} \rightarrow \boxed{2\text{H}}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

48.

(a)  $\omega = 500, Z_{inRLC} = 5 + j10 - j1 = 5 + j9$

$$\therefore Y_{inRLC} = \frac{1}{5 + j9} = \frac{5 - j9}{106} \therefore Y_c = \frac{9}{106} = 500C$$

$$\therefore C = \frac{9}{53,000} = 169.81 \mu\text{F}$$

(b)  $R_{in,ab} = \frac{106}{5} = 21.2 \Omega$

(c)  $\omega = 1000 \therefore Y_{in,ab} = j\frac{9}{53} + \frac{1}{5 + j20 - j0.5}$   
 $= 0.012338 + j0.12169\text{S}$  or  $0.12232 \angle 84.21^\circ \text{S}$



## CHAPTER TEN (Phasor Analysis) SOLUTIONS

49.

$$(a) \quad R_{in} = 550 \Omega : Z_{in} = 500 + \frac{j0.1\omega}{100 + j0.001\omega}$$

$$\therefore Z_{in} = \frac{50,000 + j0.6\omega}{100 + j0.001\omega} \times \frac{100 - j0.001\omega}{100 - j0.001\omega}$$

$$\therefore Z_{in} = \frac{5 \times 10^6 + 0.0006\omega^2 + j(60\omega - 50\omega)}{10^4 + 10^{-6}\omega^2}$$

$$\therefore R_{in} = \frac{5 \times 10^6 + 0.006\omega^2}{10^4 + 10^{-6}\omega^2} = 550 \therefore 5.5 \times 10^6$$

$$+ 5.5 \times 10^{-4} \omega^2 = 5 \times 10^6 \times 10^{-4} \omega^2$$

$$\therefore 0.5 \times 10^{-4} \omega^2 = 0.5 \times 10^6, \omega^2 = 10^{10}, \omega = 10^5 \text{ rad/s}$$

$$(b) \quad X_{in} = 50 \Omega = \frac{10\omega}{10^4 + 10^{-6}\omega^2} = 0.5 \times 10^6 + 0.5 \times 10^{-4} \omega^2 - 10\omega$$

$$= 0, \omega^2 - 2 \times 10^5 \omega + 10^{10} = 0$$

$$\therefore \omega = \frac{2 \times 10^5 \pm \sqrt{4 \times 10^{10} - 4 \times 10^{10}}}{2} = 10^5 \therefore \omega = 10^5 \text{ rad/s}$$

$$(c) \quad G_{in} = 1.8 \times 10^{-3} : Y_{in} = \frac{100 + j0.001\omega}{50,000 + j0.6\omega} \times \frac{50,000 - j0.6\omega}{50,000 - j0.6\omega}$$

$$= \frac{5 \times 10^6 + 6 \times 10^{-4} \omega^2 + j(50\omega - 6\omega)}{25 \times 10^8 + 0.36\omega^2}$$

$$\therefore 1.8 \times 10^3 = \frac{5 \times 10^6 + 6 \times 10^{-4} \omega^2}{25 \times 10^8 + 0.36\omega^2}$$

$$\therefore 5 \times 10^6 + 6 \times 10^{-4} \omega^2 = 4.5 \times 10^6 + 648 \times 10^{-6} \omega^2$$

$$\therefore 0.5 \times 10^6 = 48 \times 10^{-6} \omega^2 \therefore \omega = 102.06 \text{ Krad/s}$$

$$(d) \quad B_{in} = 1.5 \times 10^{-4} = \frac{-10\omega}{25 \times 10^8 + 0.36\omega^2}$$

$$\therefore 10\omega = 37.5 \times 10^4 + 54 \times 10^{-6} \omega^2$$

$$\therefore 54 \times 10^{-6} \omega^2 - 10\omega + 37.5 \times 10^4 = 0,$$

$$\omega = 10 \pm \frac{\sqrt{100 - 81}}{108 \times 10^{-6}} = 52.23 \text{ and } 133.95 \text{ krad/s}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

50.

$$(a) \quad V_1 = \frac{I_1}{Y_1} = \frac{0.1 \angle 30^\circ}{(3 + j4)10^{-3}} = 20 \angle -23.13^\circ \therefore |V_1| = \boxed{20 \text{ V}}$$

$$(b) \quad V_2 = V_1 \therefore |V_2| = \boxed{20 \text{ V}}$$

$$(c) \quad I_2 = Y_2 V_2 = (5 + j2)10^{-3} \times 20 \angle -23.13^\circ = 0.10770 \angle -1.3286^\circ \text{ A} \\ \therefore I_3 = I_1 + I_2 = 0.1 \angle 30^\circ + 0.10770 \angle -1.3286^\circ = 0.2 \angle 13.740^\circ \text{ A} \\ \therefore V_3 = \frac{I_3}{Y_3} = \frac{0.2 \angle 13.740^\circ}{(2 - j4)10^{-3}} = 44.72 \angle 77.18^\circ \text{ V} \therefore |V_3| = \boxed{44.72 \text{ V}}$$

$$(d) \quad V_{in} = V_1 + V_3 = 20 \angle -23.13^\circ + 44.72 \angle 77.18^\circ = 45.60 \angle 51.62^\circ \\ \therefore |V_{in}| = \boxed{45.60 \text{ V}}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

51.

(a)  $50\mu\text{F} \rightarrow -j20\Omega \therefore Y_{in} = 0.1 + j0.05$

$$Y_{in} = \frac{1}{R_1 - j\frac{1000}{C}} \therefore R_1 - j\frac{1000}{C} = \frac{1}{0.1 + j0.05} = 8 - j4$$

$$\therefore R_1 = 8\Omega \text{ and } C_1 = \frac{1}{4\omega} = \boxed{250\mu\text{F}}$$

(b)  $\omega = 2000 : 50\mu\text{F} \rightarrow -j10\Omega \therefore Y_{in} = 0.1 + j0.1 = \frac{1}{R_1 - j\frac{500}{C_1}}$

$$\therefore R_1 - j\frac{500}{C_1} = 5 - j5 \therefore R_1 = 5\Omega, C_1 = \boxed{100\mu\text{F}}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

52.

$$(a) \quad Z_{in} = 1 + \frac{10}{j\omega} = \frac{10 + j\omega}{j\omega}$$

$$\therefore Y_{in} = \frac{j\omega}{10 + j\omega} \times \frac{10 - j\omega}{10 - j\omega}$$

$$\therefore Y_{in} = \frac{\omega^2 + j10\omega}{\omega^2 + 100}$$

$$G_{in} = \frac{\omega^2}{\omega^2 + 100}, \quad B_{in} = \frac{10\omega}{\omega^2 + 100}$$

$\omega$	$G_{in}$	$B_{in}$
0	0	0
1	0.0099	0.0099
2	0.0385	0.1923
5	0.2	0.4
10	0.5	0.5
20	0.8	0.4
$\infty$	1	0

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

53.

$$-j5 = \frac{v_1}{3} + \frac{V_1 - V_2}{-j5} + \frac{v_1 - V_2}{j3}, \quad -j75 = 5V_1 + j3V_1 - j3V_2 - j5V_1 + j5V_2$$

$$\therefore (5 - j2)V_1 + j2V_2 = -j75 \quad (1)$$

$$\frac{v_2 - V_1}{j3} + \frac{V_2 - V_1}{-j5} + \frac{V_2}{6} = 10$$

$$-j10V_2 + j10V_1 + j6V_2 - j6V_1 + 5V_2 = 300 \therefore j4V_1 + (5 - j4)V_2 = 300 \quad (2)$$

$$\therefore V_2 = \frac{\begin{vmatrix} 5 - j2 & -j75 \\ j4 & 300 \end{vmatrix}}{\begin{vmatrix} 5 - j2 & j2 \\ j4 & 5 - j4 \end{vmatrix}} = \frac{1500 - j600 - 300}{17 - j30 + 8} = \frac{1200 - j600}{25 - j30} = 34.36 \angle 23.63^\circ \text{ V}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

54.

$$j3I_B - j5(I_B - I_D) = 0 \therefore -2I_B + j5I_D = 0$$

$$3(I_D + j5) - j5(I_D - I_B) + 6(I_D + 10) = 0$$

$$\therefore j5I_B + (9 - j5)I_D = -60 - j15$$

$$I_B = \frac{\begin{vmatrix} 0 & j5 \\ -60 - j15 & 9 - j5 \end{vmatrix}}{\begin{vmatrix} -j2 & j5 \\ j5 & 9 - j5 \end{vmatrix}} = \frac{-75 + j300}{15 - j18}$$

$$= 13.198 \angle 154.23^\circ \text{ A}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

55.

$$v_{s1} = 20 \cos 1000t \text{ V}, v_{s2} = 20 \sin 1000t \text{ V}$$

$$\therefore V_{s1} = 20 \angle 0^\circ \text{ V}, V_{s2} = -j20 \text{ V}$$

$$0.01 \text{ H} \rightarrow j10 \Omega, 0.1 \text{ mF} \rightarrow -j10 \Omega$$

$$\therefore \frac{v_x - 20}{j10} + \frac{v_x}{25} + \frac{v_x + j20}{-j10} = 0, 0.04v_x + j2 - 2 = 0,$$

$$V_x = 25(2 - j2) = 70.71 \angle -45^\circ \text{ V}$$

$$\therefore v_x(t) = 70.71 \cos(1000t - 45^\circ) \text{ V}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

56.

(a) Assume  $V_3 = 1V \therefore V_2 = 1 - j0.5V$ ,  $I_2 = 1 - j0.5mA$   
 $\therefore V_1 = 1 - j0.5 + (2 - j0.5)(-j0.5) = 0.75 - j1.5V$   
 $\therefore I_1 = 0.75 - j1.5mA$ ,  $\therefore I_{in} = 0.75 - j1.5 + 2 - j0.5 = 2.75 - j2mA$   
 $\therefore V_{in} = 0.75 - j1.5 - j1.5 + (2.75 - j2)(-j0.5)$   
 $= -0.25 - j2.875V \therefore V_3 = \frac{100}{-j0.25 - j2.875} = 34.65^\circ \angle 94.97^\circ V$

(b)  $-j0.5 \rightarrow -jX$  Assume  $V_3 = 1V \therefore I_3 = 1A$ ,  
 $V_2 = 1 - jX$ ,  $I_2 = 1 - jX$ ,  $\rightarrow I_{12} = 2 - jX$   
 $\therefore V_1 = 1 - jX + (2 - jX)(-jX) = 1 - X^2 - j3X$ ,  $I_1 = 1 - X^2 - j3X$ ,  $I_{in} = 3 - X^2 - j4X$   
 $\therefore V_{in} = 1 - X^2 - j3X - 4X^2 + jX^3 - j3X = 1 - 5X^2 + j(X^3 - 6X) \therefore X^3 - 6X = 0$   
 $\therefore X^2 = 6$ ,  $X = \sqrt{6}$ ,  $Z_c = -j2.449K\Omega$



## CHAPTER TEN (Phasor Analysis) SOLUTIONS

57. Define three clockwise mesh currents  $i_1$ ,  $i_2$ ,  $i_3$  with  $i_1$  in the left mesh,  $i_2$  in the top right mesh, and  $i_3$  in the bottom right mesh.

$$\text{Mesh 1: } 10\angle 0^\circ + (1 + 1 - j0.25)\mathbf{I}_1 - (-j0.25)\mathbf{I}_2 = 0$$

$$\text{Mesh 2: } -\mathbf{I}_1 + (1 + 1 + j4)\mathbf{I}_2 - \mathbf{I}_3 = 0$$

$$\text{Mesh 3: } (-j0.25 + 1 + 1)\mathbf{I}_3 - \mathbf{I}_2 - (-j0.25\mathbf{I}_1) = 0$$

$$\mathbf{I}_x = \frac{\begin{vmatrix} 2 - j0.25 & -1 & 10 \\ -1 & 2 + j4 & 0 \\ j0.25 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} 2 - j0.25 & -1 & j0.25 \\ -1 & 2 + j4 & -1 \\ j0.25 & -1 & 2 - j0.25 \end{vmatrix}}$$

$$\therefore \mathbf{I}_x = \frac{10(1 + 1 - j0.5)}{j0.25(2 - j0.5) + (-2 + j0.25 + j0.25) + (2 - j0.25)(4 + 1 - j0.5 + j8 - 1)}$$

$$= \frac{20 - j5}{8 + j15} \therefore \mathbf{I}_x = 1.217\angle -75.96^\circ \text{ A}, \quad i_x(t) = 1.2127 \cos(100t - 75.96^\circ) \text{ A}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

58.

$$V_1 - 10 - j0.25V_1 + j0.25V_x + V_1 - V_2 = 0$$

$$\therefore (2 - j0.25)V_1 - V_2 + j0.25V_x = 10$$

$$V_2 - V_1 + V_2 - V_x + j4V_2 = 0$$

$$-V_1 + (2 + j4)V_2 - V_x = 0$$

$$-j0.25V_x + j0.25V_1 + V_x + V_x - V_2$$

$$\therefore j0.25V_1 - V_2 + (2 - j0.25)V_x = 0$$

$$V_x = \frac{\begin{vmatrix} 2 - j0.25 & -1 & 10 \\ -1 & 2 + j4 & 0 \\ j0.25 & -1 & 0 \end{vmatrix}}{\begin{vmatrix} -j0.25 & -1 & j0.25 \\ -1 & 2 + j4 & -1 \\ j0.25 & -1 & 2 - j0.25 \end{vmatrix}}$$

$$= \frac{10(1 + 1 - j0.5)}{j0.25(2 - j0.5) + (-2 + j0.25 + j0.25) + (2 - j0.25)(4 + 1 - j0.5 + j8 - 1)}$$

$$= \frac{20 - j5}{8 + j15} = 1.2127 \angle -75.96^\circ \text{ V}$$

$$\therefore v_x = 1.2127 \cos(100t - 75.96^\circ) \text{ V}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

59.

(a)  $R_1 = \infty, R_o = 0, A = -V_o / V_i \gg 0$

$$I = \frac{V_1 + AV_i}{R_f} = j\omega C_1 (V_s - V_i)$$

$$\therefore V_i(1 + A + j\omega C_1 R_f) = j\omega C_1 R_f V_s$$

$$V_o = -AV_i \therefore -\frac{V_o}{A}(1 + A + j\omega C_1 R_f) = j\omega C_1 R_f V_s$$

$$\therefore \frac{V_o}{V_s} = -\frac{j\omega C_1 R_f A}{1 + A + j\omega C_1 R_f} \text{ As } A \rightarrow \infty, \frac{V_o}{V_s} \rightarrow -j\omega C_1 R_f$$

(b)  $R_f \parallel C_f = \frac{1}{j\omega C_f + \frac{1}{R_f}} = \frac{R_f}{1 + j\omega C_f R_f}$

$$I = \frac{(V_1 + AV_i)}{R_f}(1 + j\omega C_f R_f) = (V_s - V_i) j\omega C_1, V_o = -AV_i$$

$$\therefore V_i(1 + A)(1 + j\omega C_f R_f) = V_s j\omega C_1 R_f - j\omega C_1 R_f V_i,$$

$$V_i [(1 + A)(1 + j\omega C_f R_f) + j\omega C_1 R_f] = j\omega C_1 R_f V_s$$

$$\therefore -\frac{V_o}{A} [(1 + A)(1 + j\omega C_f R_f) + j\omega C_1 R_f] = j\omega C_1 R_f V_s$$

$$\therefore \frac{V_o}{V_s} = \frac{-j\omega C_1 R_f A}{(1 + A)(1 + j\omega C_f R_f) + j\omega C_1 R_f} \text{ As } A \rightarrow \infty, \frac{V_o}{V_s} \rightarrow \frac{-j\omega C_1 R_f}{1 + j\omega C_f R_f}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

60. Define the nodal voltage  $v_1(t)$  at the junction between the two dependent sources. The voltage source may be replaced by a  $3\angle -3^\circ$  V source, the  $600\text{-}\mu\text{F}$  capacitor by a  $-j/0.6\ \Omega$  impedance, the  $500\text{-}\mu\text{F}$  capacitor by a  $-j2\ \Omega$  impedance, and the inductor by a  $j2\ \Omega$  impedance.

$$5\mathbf{V}_2 + 3\mathbf{V}_2 = \frac{\mathbf{V}_1 - 3\angle -3^\circ}{100 - j/0.6} + \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j2} \quad [1]$$

$$-5\mathbf{V}_2 = \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{-j2} + \frac{\mathbf{V}_2}{j2} \quad [2]$$

Simplifying and collecting terms,

$$j2\ \mathbf{V}_1 + (960.1\angle -90.95^\circ)\ \mathbf{V}_2 = 6\angle 87^\circ \quad [1]$$

$$-j2\ \mathbf{V}_1 + 20\ \mathbf{V}_2 = 0 \quad [2]$$

Solving, we find that  $\mathbf{V}_1 = 62.5\angle 86.76^\circ$  mV and  $\mathbf{V}_2 = 6.25\angle 176.8^\circ$  mV. Converting back to the time domain,

$$v_2(t) = 6.25 \cos(10^3 t + 176.8^\circ) \text{ mV}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

61. Define three clockwise mesh currents:  $i_1(t)$  in the left-most mesh,  $i_2(t)$  in the bottom right mesh, and  $i_3(t)$  in the top right mesh. The  $15\text{-}\mu\text{F}$  capacitor is replaced with a  $-j/0.15\ \Omega$  impedance, the inductor is replaced by a  $j20\ \Omega$  impedance, the  $74\ \mu\text{F}$  capacitor is replaced by a  $-j1.351\ \Omega$  impedance, the current source is replaced by a  $2\angle 0^\circ$  mA source, and the voltage source is replaced with a  $5\angle 0^\circ$  V source.

Around the 1, 2 supermesh:  $(100 + j20)\mathbf{I}_1 + (13000 - j1.351)\mathbf{I}_2 - 5000\mathbf{I}_3 = 0$   
and

$$-\mathbf{I}_1 + \mathbf{I}_2 = 2 \times 10^{-3}$$

Mesh 3:  $5\angle 0^\circ + (5000 - j6.667)\mathbf{I}_2 - 5000\mathbf{I}_3 = 0$

Solving, we find that  $\mathbf{I}_1 = 1.22\angle 179.9^\circ$  mA. Converting to the time domain,

$$i_1(t) = 1.22 \cos(10^4 t + 179.9^\circ) \text{ mA}$$

$$\begin{aligned} \text{Thus, } P_{1000} &= [i_1(1 \text{ ms})]^2 \cdot 1000 \\ &= (1.025 \times 10^{-6})(1000) \text{ W} = \boxed{1.025 \text{ W}} \end{aligned}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

62. We define an additional clockwise mesh current  $i_4(t)$  flowing in the upper right-hand mesh. The inductor is replaced by a  $j0.004 \Omega$  impedance, the  $750 \mu\text{F}$  capacitor is replaced by a  $-j/0.0015 \Omega$  impedance, and the  $1000 \mu\text{F}$  capacitor is replaced by a  $-j/2 \Omega$  impedance. We replace the left voltage source with a  $6 \angle -13^\circ \text{ V}$  source, and the right voltage source with a  $6 \angle 0^\circ \text{ V}$  source.

$$(1 - j/0.0015) \mathbf{I}_1 - \mathbf{I}_3 = 6 \angle -13^\circ \quad [1]$$

$$(0.005 + j/0.0015) \mathbf{I}_1 + j0.004 \mathbf{I}_2 - j0.004 \mathbf{I}_4 = 0 \quad [2]$$

$$-\mathbf{I}_1 + (1 - j/2) \mathbf{I}_3 + j0.5 \mathbf{I}_4 = -6 \angle 0^\circ \quad [3]$$

$$-j0.004 \mathbf{I}_2 + j0.5 \mathbf{I}_3 + (j0.004 - j0.5) \mathbf{I}_4 = 0 \quad [4]$$

Solving, we find that

$$\mathbf{I}_1 = 0.00144 \angle -51.5^\circ \text{ A}, \mathbf{I}_2 = 233.6 \angle 39.65^\circ \text{ A}, \text{ and } \mathbf{I}_3 = 6.64 \angle 173.5^\circ \text{ A}.$$

Converting to the time domain,

$$\begin{aligned} i_1(t) &= 1.44 \cos(2t - 51.5^\circ) \text{ mA} \\ i_2(t) &= 233.6 \cos(2t + 39.65^\circ) \text{ A} \\ i_3(t) &= 6.64 \cos(2t + 173.5^\circ) \text{ A} \end{aligned}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

63. We replace the voltage source with a  $115\sqrt{2} \angle 0^\circ$  V source, the capacitor with a  $-j/2\pi C_1 \Omega$  impedance, and the inductor with a  $j0.03142 \Omega$  impedance.

Define  $\mathbf{Z}$  such that  $\mathbf{Z}^{-1} = 2\pi C_1 - j/0.03142 + 1/20$

By voltage division, we can write that  $6.014 \angle 85.76^\circ = 115\sqrt{2} \frac{\mathbf{Z}}{\mathbf{Z} + 20}$

Thus,  $\mathbf{Z} = 0.7411 \angle 87.88^\circ \Omega$ . This allows us to solve for  $C_1$ :

$2\pi C_1 - 1/0.03142 = -1.348$  so that  $C_1 = 4.85 \text{ F}$ .

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

64. Defining a clockwise mesh current  $i_1(t)$ , we replace the voltage source with a  $115\sqrt{2} \angle 0^\circ$  V source, the inductor with a  $j2\pi L \Omega$  impedance, and the capacitor with a  $-j1.592 \Omega$  impedance.

$$\text{Ohm's law then yields } \mathbf{I}_1 = \frac{115\sqrt{2}}{20 + j(2\pi L - 1.592)} = 8.132 \angle 0^\circ$$

Thus,  $20 = \sqrt{20^2 + (2\pi L - 1.592)^2}$  and we find that  $L = 253.4 \text{ mH}$ .



## CHAPTER TEN (Phasor Analysis) SOLUTIONS

65. (a) By nodal analysis:

$$0 = (\mathbf{V}_\pi - 1)/R_s + \mathbf{V}_\pi/R_B + \mathbf{V}_\pi/r_\pi + j\omega C_\pi \mathbf{V}_\pi + (\mathbf{V}_\pi - \mathbf{V}_{out})j\omega C_\mu \quad [1]$$

$$-g_m \mathbf{V}_\pi = (\mathbf{V}_{out} - \mathbf{V}_\pi)j\omega C_\mu + \mathbf{V}_{out}/R_C + \mathbf{V}_{out}/R_L \quad [2]$$

Simplify and collect terms:

$$\left[ \left( \frac{1}{R_s} + \frac{1}{R_B} + \frac{1}{r_\pi} \right) + j\omega(C_\pi + C_\mu) \right] \mathbf{V}_\pi - j\omega C_\mu \mathbf{V}_{out} = \frac{1}{R_s} \quad [1]$$

$$(-g_m + j\omega C_\mu) \mathbf{V}_\pi - (j\omega C_\mu + 1/R_C + 1/R_L) \mathbf{V}_{out} = 0 \quad [2]$$

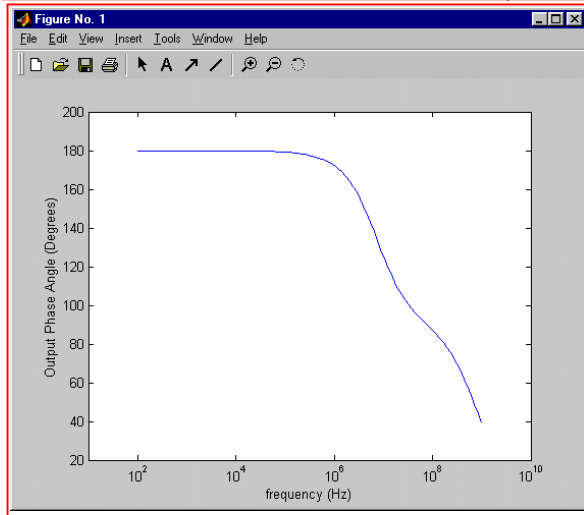
Define  $\frac{1}{R_s'} = \frac{1}{R_s} + \frac{1}{R_B} + \frac{1}{r_\pi}$  and  $R_L' = R_C \parallel R_L$

Then  $\Delta = \frac{-1}{R_s' R_L'} + \omega^2(2C_\mu^2 + C_\mu C_\pi) - j\omega \left( g_m C_\mu + \frac{C_\mu + C_\pi}{R_L'} + \frac{C_\mu}{R_s'} \right)$

And  $\mathbf{V}_{out} = \frac{g_m R_s - j\omega C_\mu / R_s}{\frac{-1}{R_s' R_L'} + \omega^2(2C_\mu^2 + C_\mu C_\pi) - j\omega \left( g_m C_\mu + \frac{C_\mu + C_\pi}{R_L'} + \frac{C_\mu}{R_s'} \right)}$

Therefore,  $\text{ang}(\mathbf{V}_{out}) = \tan^{-1} \left( \frac{-j\omega C_\mu}{g_m R_s^2} \right) - \tan^{-1} \left( \frac{-\omega \left( g_m C_\mu + \frac{C_\mu + C_\pi}{R_L'} + \frac{C_\mu}{R_s'} \right)}{\frac{-1}{R_s' R_L'} + \omega^2(2C_\mu^2 + C_\mu C_\pi)} \right)$

(b)



(c) The output is  $\sim 180^\circ$  out of phase with the input for  $f < 10^5$  Hz; only for  $f = 0$  is it exactly  $180^\circ$  out of phase with the input.

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

66.

$$\text{OC: } -\frac{V_x}{20} + \frac{100 - V_x}{-j10} - 0.02V_x = 0$$

$$j10 = (0.05 + j0.1 + 0.02) V_x, V_x = \frac{j10}{0.07 + j0.1}$$

$$\therefore V_x = 67.11 + j46.98$$

$$\therefore V_{ab,oc} = 100 - V_x = 32.89 - j46.98 = 57.35 \angle -55.01^\circ \text{ V}$$

$$\text{SC: } V_x = 100 \therefore \downarrow I_{SC} = 0.02 \times 100 + \frac{100}{20} = 7 \text{ A}$$

$$\therefore Z_{th} = \frac{57.35 \angle -55.01^\circ}{7} = 4.698 - j6.711 \Omega$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

67.

$$V_L = j210 \therefore 0.5V_L = j\omega$$

$$\therefore V_{in} = (1 + j\omega) \frac{1}{j\omega} + j2\omega$$

$$= 1 + \frac{1}{j\omega} + j2\omega$$

$$\therefore Z_{in} = \frac{V_{in}}{1} = 1 + \frac{1}{j\omega} + j2\omega \quad \text{so } \mathbf{Y_{in} = \frac{\omega}{\omega + j(2\omega^2 - 1)}}$$

$$\text{At } \omega = 1, Z_{in} = 1 - j1 + j2 = 1 + j$$

$$\therefore Y_{in} = \frac{1}{1 + j1} = 0.5 + j0.5$$

$$\mathbf{R = 500 \text{ m}\Omega, L = 500 \text{ mH.}}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

68.

$$(a) \quad V_s : \frac{(1-j1)1}{2-j1} \times \frac{2+j1}{2+j1} = \frac{3-j1}{5} \therefore V_1 = \frac{-15}{j2+0.6-j0.2} \times 0.6-j0.2$$

$$\therefore V_1 = 5 \angle 90^\circ \therefore v_1(t) = 5 \cos(1000t + 90^\circ) \text{ V}$$

(b)  $I_s$ :

$$j2 \parallel 1 = \frac{j2}{1+j2} \frac{1-j2}{1-j2} = 0.8 + j0.4 \therefore V_1$$

$$= j25 \frac{0.8 + j0.4}{1-j1+0.8+j0.4} = \frac{-10 + j20}{1.8-j0.6} = 11.785^\circ \angle 135^\circ \text{ V}$$

$$\text{so } v_1(t) = 11.79 \cos(1000t + 135^\circ) \text{ V.}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

69.

$$\text{OC: } V_L = 0 \therefore V_{ab,oc} = 1\angle 0^\circ \text{ V}$$

$$\text{SC: } \downarrow I_N \therefore V_L = j2I_N \therefore 1\angle 0^\circ = -j1[0.25(j2I_N) + I_N] + j2I_N$$

$$\therefore 1 = (0.5 - j + j2)I_N = (0.5 + j1)I_N$$

$$\therefore I_N = \frac{1}{0.5 + j1} = 0.4 - j0.8 \therefore Y_N = \frac{I_N}{1\angle 0^\circ} = 0.4 - j0.8$$

$$\therefore R_N = \frac{1}{0.4} = 2.5\Omega, \frac{1}{j\omega L_N} = \frac{1}{jL_N} = -j0.8, L_N = \frac{1}{0.8} = 1.25\text{H}$$

$$I_N = 0.4 - j0.8 = 0.8944\angle -63.43^\circ \text{ A}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

70.

$$V_L = 2(j1) + (-j2) \frac{j1}{1+j1} = j2 + \frac{2}{1+j1} = \frac{1-j1}{1-j1} = 1+j1$$

$$\therefore V_{L,200} = 1.4142 \cos(200t + 45^\circ) \text{ V}$$

$$\omega = 100: V_L = j\frac{1}{2}, v_{L,100} = 0.5 \cos(100t + 90^\circ) \text{ V}$$

so  $v_L(t) = 1.414 \cos(200t + 45^\circ) + 0.5 \cos(100t + 90^\circ) \text{ V}$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

71.

Use superposition. Left:  $V_{ab} = 100 \frac{j100}{j100 - j300}$

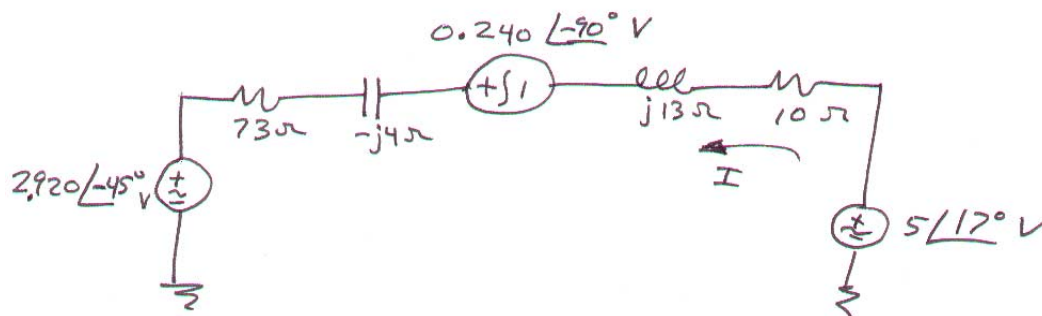
$$= -50 \angle 0^\circ \text{ V} \quad \text{Right: } V_{ab} = j100 \frac{-j300}{-j300 + j100} = j150 \text{ V}$$

$$\therefore V_{th} = -50 + j150 = 158.11 \angle 108.43^\circ \text{ V}$$

$$Z_{th} = j100 \parallel -j300 = \frac{30,000}{-j200} = j150 \Omega$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

72. This problem is easily solved if we first perform two source transformations to yield a circuit containing only voltage sources and impedances:



$$\begin{aligned} \text{Then } \mathbf{I} &= \frac{5\angle 17^\circ + 0.240\angle -90^\circ + 2.920\angle -45^\circ}{73 + 10 + j13 - j4} \\ &= (6.898\angle -7.019^\circ) / (83.49\angle 6.189^\circ) = 82.62\angle -13.21^\circ \text{ mA} \end{aligned}$$

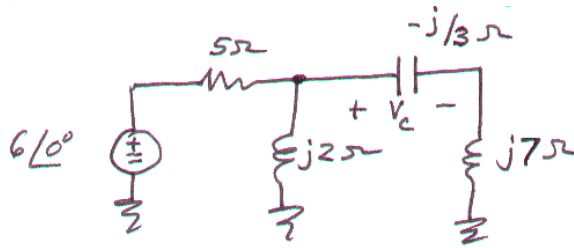
Converting back to the time domain, we find that

$$i(t) = 82.62 \cos(10^3 t - 13.21^\circ) \text{ mA}$$



## CHAPTER TEN (Phasor Analysis) SOLUTIONS

73.



(a) There are a number of possible approaches: Thévenizing everything to the left of the capacitor is one of them.

$$\mathbf{V}_{\text{TH}} = 6(j2)/(5 + j2) = 2.228 \angle 68.2^\circ \text{ V}$$

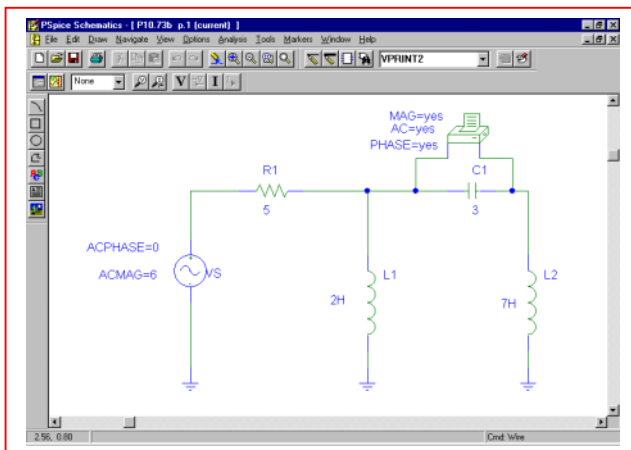
$$\mathbf{Z}_{\text{TH}} = 5 \parallel j2 = j10/(5 + j2) = 1.857 \angle 68.2^\circ \Omega$$

Then, by simple voltage division, we find that

$$\begin{aligned} \mathbf{V}_C &= (2.228 \angle 68.2^\circ) \frac{-j/3}{1.857 \angle 68.2^\circ - j/3 + j7} \\ &= 88.21 \angle -107.1^\circ \text{ mV} \end{aligned}$$

Converting back to the time domain,  $v_C(t) = 88.21 \cos(t - 107.1^\circ) \text{ mV}$ .

(b) PSpice verification.



Running an ac sweep at the frequency  $f = 1/2\pi = 0.1592 \text{ Hz}$ , we obtain a phasor magnitude of 88.23 mV, and a phasor angle of  $-107.1^\circ$ , in agreement with our calculated result (the slight disagreement is a combination of round-off error in the hand calculations and the rounding due to expressing 1 rad/s in Hz).

```
* C:\My Documents\HKD-6th Edition\SOLUTIONS MANUAL\Chapter 10\P10.73b.sch

**** AC ANALYSIS TEMPERATURE = 27.000 DEG C

*****

FREQ      VM($N_0002,$N_0003)VP($N_0002,$N_0003)

1.592E-01  8.823E-02 -1.071E+02

JOB CONCLUDED

TOTAL JOB TIME .06
```

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

74. (a) Performing nodal analysis on the circuit,

$$\text{Node 1:} \quad 1 = \mathbf{V}_1/5 + \mathbf{V}_1/(-j10) + (\mathbf{V}_1 - \mathbf{V}_2)/(-j5) + (\mathbf{V}_1 - \mathbf{V}_2)/j10 \quad [1]$$

$$\text{Node 2:} \quad j0.5 = \mathbf{V}_2/10 + (\mathbf{V}_2 - \mathbf{V}_1)/(-j5) + (\mathbf{V}_2 - \mathbf{V}_1)/j10 \quad [2]$$

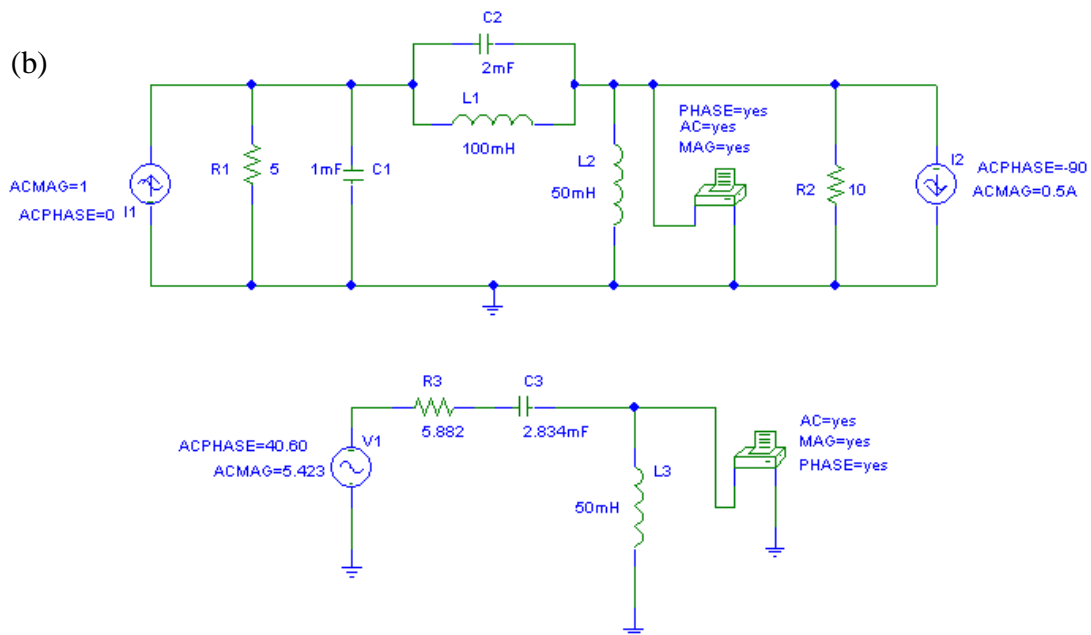
Simplifying and collecting terms,

$$(0.2 + j0.2) \mathbf{V}_1 - j0.1 \mathbf{V}_2 = 1 \quad [1]$$

$$-j \mathbf{V}_1 + (1 + j) \mathbf{V}_2 = j5 \quad [2]$$

Solving, we find that  $\mathbf{V}_2 = \mathbf{V}_{TH} = 5.423 \angle 40.60^\circ \text{ V}$

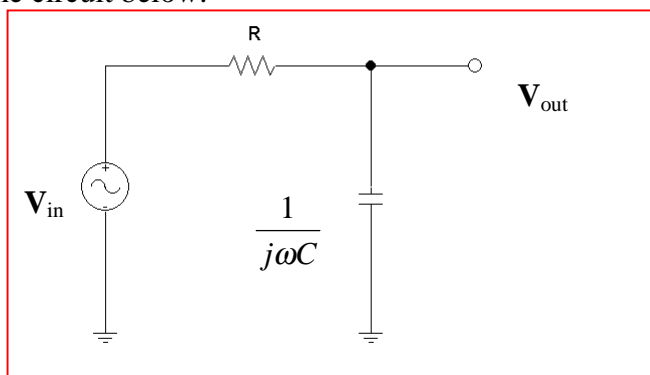
$$\mathbf{Z}_{TH} = 10 \parallel [(j10 \parallel -j5) + (5 \parallel -j10)] = 10 \parallel (-j10 + 4 - j2) = 5.882 - j3.529 \Omega.$$



FREQ	VM(\$N_0002,0)	VP(\$N_0002,0)
1.592E+01	4.474E+00	1.165E+02
FREQ	VM(\$N_0005,0)	VP(\$N_0005,0)
1.592E+01	4.473E+00	1.165E+02

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

75. Consider the circuit below:



Using voltage division, we may write:

$$V_{out} = V_{in} \frac{1/j\omega C}{R + 1/j\omega C}, \text{ or } \frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC}$$

The magnitude of this ratio (consider, for example, an input with unity magnitude and zero phase) is

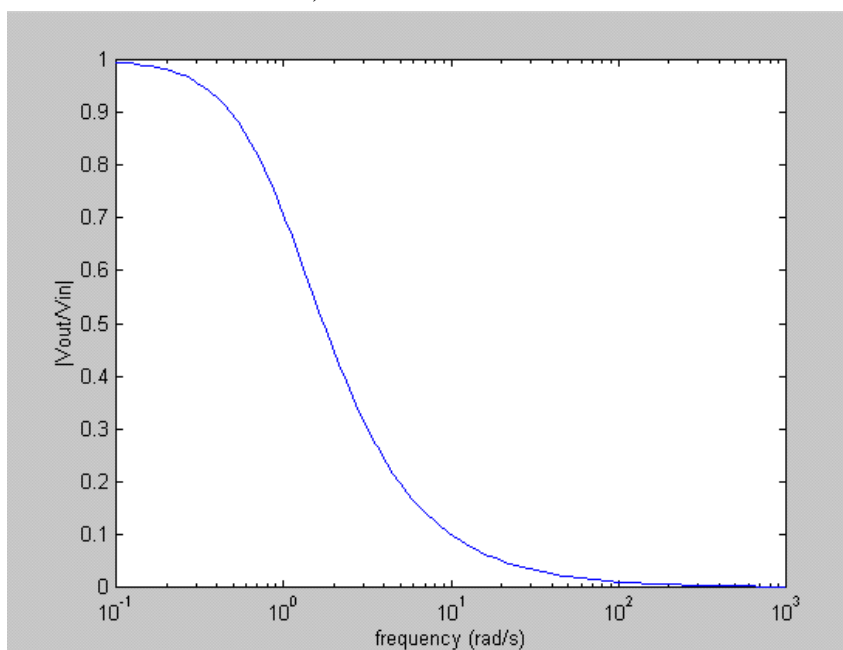
$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

As  $\omega \rightarrow 0$ , this magnitude  $\rightarrow 1$ , its maximum value.

As  $\omega \rightarrow \infty$ , this magnitude  $\rightarrow 0$ ; the capacitor is acting as a short circuit to the ac signal.

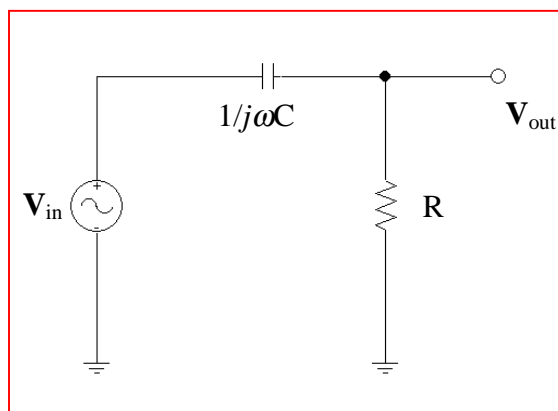
Thus, low frequency signals are transferred from the input to the output relatively unaffected by this circuit, but high frequency signals are attenuated, or “filtered out.”

This is readily apparent if we plot the magnitude as a function of frequency (assuming  $R = 1 \, \Omega$  and  $C = 1 \, F$  for convenience):



## CHAPTER TEN (Phasor Analysis) SOLUTIONS

76. Consider the circuit below:



Using voltage division, we may write:

$$\mathbf{V}_{\text{out}} = \mathbf{V}_{\text{in}} \frac{R}{R + 1/j\omega C}, \text{ or } \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{j\omega RC}{1 + j\omega RC}$$

The magnitude of this ratio (consider, for example, an input with unity magnitude and zero phase) is

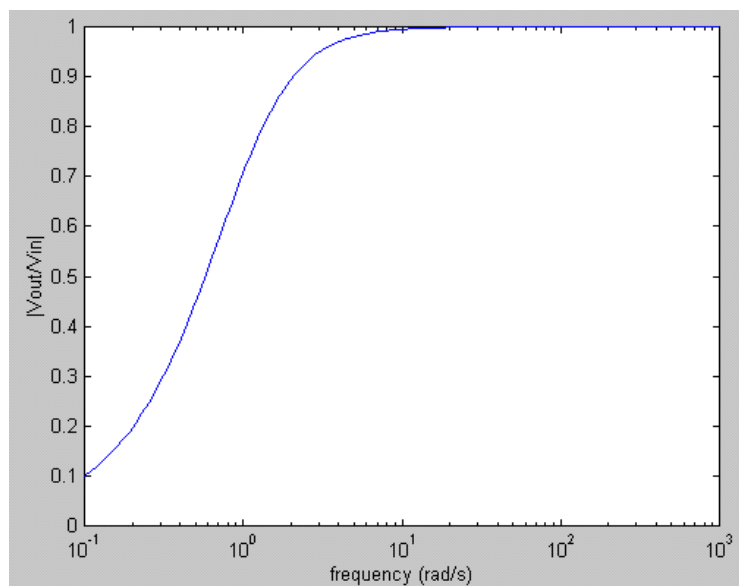
$$\left| \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} \right| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

As  $\omega \rightarrow \infty$ , this magnitude  $\rightarrow 1$ , its maximum value.

As  $\omega \rightarrow 0$ , this magnitude  $\rightarrow 0$ ; the capacitor is acting as an open circuit to the ac signal.

Thus, high frequency signals are transferred from the input to the output relatively unaffected by this circuit, but low frequency signals are attenuated, or “filtered out.”

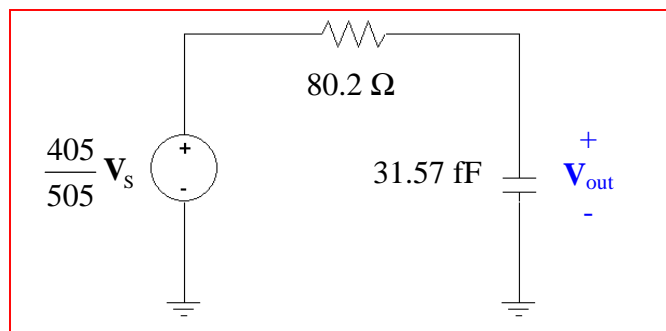
This is readily apparent if we plot the magnitude as a function of frequency (assuming  $R = 1 \, \Omega$  and  $C = 1 \, \text{F}$  for convenience):



## CHAPTER TEN (Phasor Analysis) SOLUTIONS

77. (a) Removing the capacitor temporarily, we easily find the Thevenin equivalent:

$$\mathbf{V}_{th} = (405/505) \mathbf{V}_S \text{ and } R_{th} = 100 \parallel (330 + 75) = 80.2 \, \Omega$$

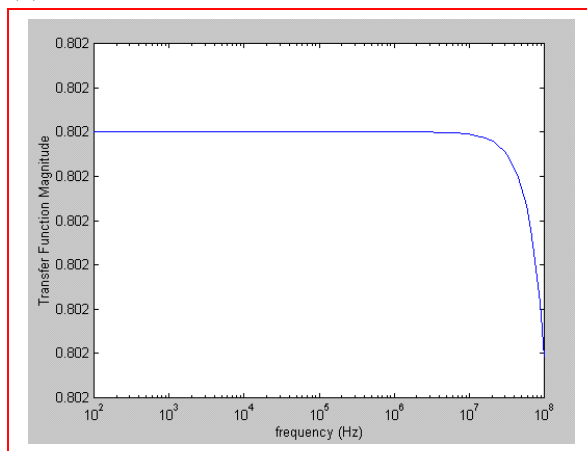


$$(b) \mathbf{V}_{out} = \frac{405}{505} \mathbf{V}_S \frac{1/j\omega C}{80.2 + 1/j\omega C} \text{ so } \frac{\mathbf{V}_{out}}{\mathbf{V}_S} = \left( \frac{405}{505} \right) \frac{1}{1 + j2.532 \times 10^{-12} \omega}$$

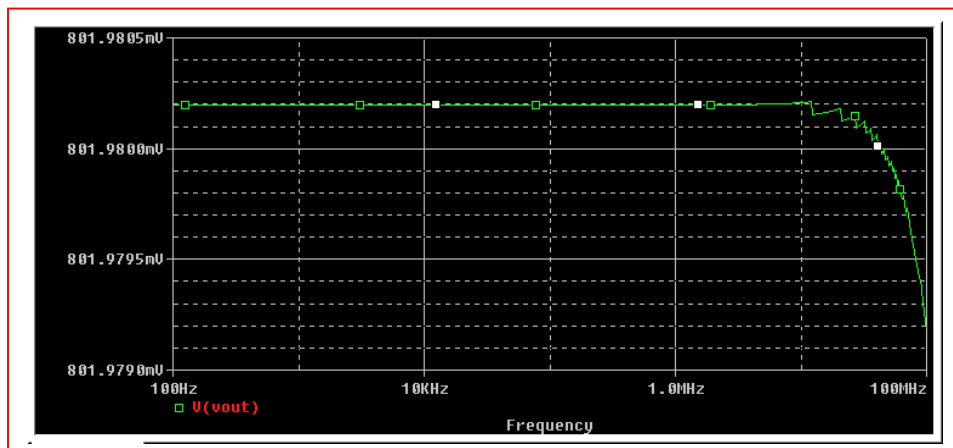
and hence

$$\left| \frac{\mathbf{V}_{out}}{\mathbf{V}_S} \right| = \frac{0.802}{\sqrt{1 + 6.411 \times 10^{-24} \omega^2}}$$

(c)



Both the MATLAB plot of the frequency response and the PSpice simulation show essentially the same behavior; at a frequency of approximately 20 MHz, there is a sharp roll-off in the transfer function magnitude.



## CHAPTER TEN (Phasor Analysis) SOLUTIONS

78. From the derivation, we see that

$$\frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{-g_m(R_C \parallel R_L) + j\omega(R_C \parallel R_L)C_\mu}{1 + j\omega(R_C \parallel R_L)C_\mu}$$

so that

$$\left| \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} \right| = \left[ \frac{g_m^2 \left( \frac{R_C R_L}{R_C + R_L} \right)^2 + \omega^2 \left( \frac{R_C R_L}{R_C + R_L} \right)^2 C_\mu^2}{1 + \omega^2 \left( \frac{R_C R_L}{R_C + R_L} \right)^2 C_\mu^2} \right]^{1/2}$$

This function has a maximum value of  $g_m(R_C \parallel R_L)$  at  $\omega = 0$ . Thus, the capacitors reduce the gain at high frequencies; this is the frequency regime at which they begin to act as short circuits. Therefore, the maximum gain is obtained at frequencies at which the capacitors may be treated as open circuits. If we do this, we may analyze the circuit of Fig. 10.25b without the capacitors, which leads to

$$\left. \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_S} \right|_{\text{low frequency}} = -g_m \left( \frac{R_C R_L}{R_C + R_L} \right) \frac{(r_\pi \parallel R_B)}{R_S + r_\pi \parallel R_B} = -g_m \left( \frac{R_C R_L}{R_C + R_L} \right) \frac{r_\pi R_B}{R_S(r_\pi + R_B) + r_\pi R_B}$$

The resistor network comprised of  $r_\pi$ ,  $R_S$ , and  $R_B$  acts as a voltage divider, leading to a reduction in the gain of the amplifier. In the situation where  $r_\pi \parallel R_B \gg R_S$ , then it has minimal effect and the gain will equal its “maximum” value of  $-g_m(R_C \parallel R_L)$ .

(b) If we set  $R_S = 100 \, \Omega$ ,  $R_L = 8 \, \Omega$ ,  $R_C \mid_{\text{max}} = 10 \, \text{k}\Omega$  and  $r_\pi g_m = 300$ , then we find that

$$\frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_S} = -g_m (7.994) \frac{r_\pi \parallel R_B}{100 + r_\pi \parallel R_B}$$

We seek to maximize this term within the stated constraints. This requires a large value of  $g_m$ , but also a large value of  $r_\pi \parallel R_B$ . This parallel combination will be less than the smaller of the two terms, so even if we allow  $R_B \rightarrow \infty$ , we are left with

$$\frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_S} \approx -(7.994) \frac{g_m r_\pi}{100 + r_\pi} = \frac{-2398}{100 + r_\pi}$$

Considering this simpler expression, it is clear that if we select  $r_\pi$  to be small, (*i.e.*  $r_\pi \ll 100$ ), then  $g_m$  will be large and the gain will have a maximum value of approximately  $-23.98$ .

(c) Referring to our original expression in which the gain  $\mathbf{V}_{\text{out}}/\mathbf{V}_{\text{in}}$  was computed, we see that the critical frequency  $\omega_C = [(R_C \parallel R_L) C_\mu]^{-1}$ . Our selection of maximum  $R_C$ ,  $R_B \rightarrow \infty$ , and  $r_\pi \ll 100$  has not affected this frequency.

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

79. Considering the  $\omega = 2 \times 10^4$  rad/s source first, we make the following replacements:

$$100 \cos(2 \times 10^4 t + 3^\circ) \text{ V} \rightarrow 100 \angle 3^\circ \text{ V}$$

$$33 \mu\text{F} \rightarrow -j1.515 \Omega \quad 112 \mu\text{H} \rightarrow j2.24 \Omega \quad 92 \mu\text{F} \rightarrow -j0.5435 \Omega$$

Then

$$(\mathbf{V}_1' - 100 \angle 3^\circ) / 47 \times 10^3 + \mathbf{V}_1' / (-j1.515) + (\mathbf{V}_1' - \mathbf{V}_2') / (56 \times 10^3 + j4.48) = 0 \quad [1]$$

$$(\mathbf{V}_2' - \mathbf{V}_1') / (56 \times 10^3 + j4.48) + \mathbf{V}_2' / (-j0.5435) = 0 \quad [2]$$

Solving, we find that

$$\mathbf{V}_1' = 3.223 \angle -87^\circ \text{ mV} \quad \text{and} \quad \mathbf{V}_2' = 31.28 \angle -177^\circ \text{ nV}$$

Thus,  $v_1'(t) = 3.223 \cos(2 \times 10^4 t - 87^\circ) \text{ mV}$  and  $v_2'(t) = 31.28 \cos(2 \times 10^4 t - 177^\circ) \text{ nV}$

Considering the effects of the  $\omega = 2 \times 10^5$  rad/s source next,

$$100 \cos(2 \times 10^5 t - 3^\circ) \text{ V} \rightarrow 100 \angle -3^\circ \text{ V}$$

$$33 \mu\text{F} \rightarrow -j0.1515 \Omega \quad 112 \mu\text{H} \rightarrow j22.4 \Omega \quad 92 \mu\text{F} \rightarrow -j0.05435 \Omega$$

Then

$$\mathbf{V}_1'' / -j0.1515 + (\mathbf{V}_1'' - \mathbf{V}_2'') / (56 \times 10^3 + j44.8) = 0 \quad [3]$$

$$(\mathbf{V}_2'' - \mathbf{V}_1'') / (56 \times 10^3 + j44.8) + (\mathbf{V}_2'' - 100 \angle 3^\circ) / 47 \times 10^3 + \mathbf{V}_2'' / (-j0.05435) = 0 \quad [4]$$

Solving, we find that

$$\mathbf{V}_1'' = 312.8 \angle 177^\circ \text{ pV} \quad \text{and} \quad \mathbf{V}_2'' = 115.7 \angle -93^\circ \mu\text{V}$$

Thus,

$$v_1''(t) = 312.8 \cos(2 \times 10^5 t + 177^\circ) \text{ pV} \quad \text{and} \quad v_2''(t) = 115.7 \cos(2 \times 10^5 t - 93^\circ) \mu\text{V}$$

Adding, we find

$$v_1(t) = 3.223 \times 10^{-3} \cos(2 \times 10^4 t - 87^\circ) + 312.8 \times 10^{-12} \cos(2 \times 10^5 t + 177^\circ) \text{ V} \quad \text{and}$$

$$v_2(t) = 31.28 \times 10^{-9} \cos(2 \times 10^4 t - 177^\circ) + 115.7 \times 10^{-12} \cos(2 \times 10^5 t - 93^\circ) \text{ V}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

80. For the source operating at  $\omega = 4 \text{ rad/s}$ ,

$7 \cos 4t \rightarrow 7 \angle 0^\circ \text{ V}$ ,  $1 \text{ H} \rightarrow j4 \Omega$ ,  $500 \text{ mF} \rightarrow -j0.5 \Omega$ ,  $3 \text{ H} \rightarrow j12 \Omega$ , and  $2 \text{ F} \rightarrow -j/8 \Omega$ .

Then by mesh analysis, (define 4 clockwise mesh currents  $\mathbf{I}_1, \mathbf{I}_2, \mathbf{I}_3, \mathbf{I}_4$  in the top left, top right, bottom left and bottom right meshes, respectively):

$$(9.5 + j4) \mathbf{I}_1 - j4 \mathbf{I}_2 - 7 \mathbf{I}_3 - 4 \mathbf{I}_4 = 0 \quad [1]$$

$$-j4 \mathbf{I}_1 + (3 + j3.5) \mathbf{I}_2 - 3 \mathbf{I}_4 = -7 \quad [2]$$

$$-7 \mathbf{I}_1 + (12 - j/8) \mathbf{I}_3 + j/8 \mathbf{I}_4 = 0 \quad [3]$$

$$-3 \mathbf{I}_2 + j/8 \mathbf{I}_3 + (4 + j11.875) \mathbf{I}_4 = 0 \quad [4]$$

Solving, we find that  $\mathbf{I}_3 = 365.3 \angle -166.1^\circ \text{ mA}$  and  $\mathbf{I}_4 = 330.97 \angle 72.66^\circ \text{ mA}$ .

For the source operating at  $\omega = 2 \text{ rad/s}$ ,

$5.5 \cos 2t \rightarrow 5.5 \angle 0^\circ \text{ V}$ ,  $1 \text{ H} \rightarrow j2 \Omega$ ,  $500 \text{ mF} \rightarrow -j \Omega$ ,  $3 \text{ H} \rightarrow j6 \Omega$ , and  $2 \text{ F} \rightarrow -j/4 \Omega$ .

Then by mesh analysis, (define 4 clockwise mesh currents  $\mathbf{I}_A, \mathbf{I}_B, \mathbf{I}_C, \mathbf{I}_D$  in the top left, top right, bottom left and bottom right meshes, respectively):

$$(9.5 + j2) \mathbf{I}_A - j2 \mathbf{I}_B - 7 \mathbf{I}_C - 4 \mathbf{I}_D = 0 \quad [1]$$

$$-j2 \mathbf{I}_A + (3 + j) \mathbf{I}_B - 3 \mathbf{I}_D = -7 \quad [2]$$

$$-7 \mathbf{I}_A + (12 - j/4) \mathbf{I}_C + j/4 \mathbf{I}_D = 0 \quad [3]$$

$$-3 \mathbf{I}_2 + j/4 \mathbf{I}_C + (4 + j5.75) \mathbf{I}_D = 0 \quad [4]$$

Solving, we find that  $\mathbf{I}_C = 783.8 \angle -4.427^\circ \text{ mA}$  and  $\mathbf{I}_D = 134 \angle -25.93^\circ \text{ mA}$ .

$\mathbf{V}_1' = -j0.25 (\mathbf{I}_3 - \mathbf{I}_4) = 0.1517 \angle 131.7^\circ \text{ V}$  and  $\mathbf{V}_1'' = -j0.25 (\mathbf{I}_C - \mathbf{I}_D) = 0.1652 \angle -90.17^\circ \text{ V}$

$\mathbf{V}_2' = (1 + j6) \mathbf{I}_4 = 2.013 \angle 155.2^\circ \text{ V}$  and  $\mathbf{V}_2'' = (1 + j6) \mathbf{I}_D = 0.8151 \angle 54.61^\circ \text{ V}$

Converting back to the time domain,

$$\begin{aligned} v_1(t) &= 0.1517 \cos(4t + 131.7^\circ) + 0.1652 \cos(2t - 90.17^\circ) \text{ V} \\ v_2(t) &= 2.013 \cos(4t + 155.2^\circ) + 0.8151 \cos(2t + 54.61^\circ) \text{ V} \end{aligned}$$



## CHAPTER TEN (Phasor Analysis) SOLUTIONS

81.

$$(a) \quad I_L = \frac{100}{j2.5 + \frac{-2}{2-j1}} = \frac{100(2-j1)}{2.5+j3} = 57.26\angle -76.76^\circ (2.29in)$$

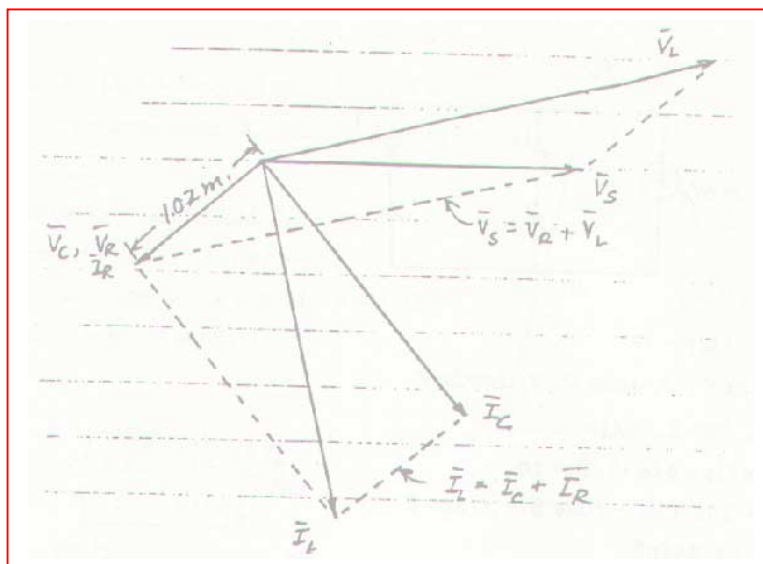
$$I_R = (57.26\angle -76.76^\circ) \frac{-j1}{2-j1} = 25.61\angle -140.19^\circ (1.02in)$$

$$I_C = (57.26\angle -76.76^\circ) \frac{2}{2-j1} = 51.21\angle -50.19^\circ (2.05in)$$

$$V_L = 2.5 \times 57.26\angle 90^\circ - 76.76^\circ = 143.15\angle 13.24^\circ (2.86in)$$

$$V_R = 2 \times 25.61\angle -140.19^\circ = 51.22\angle -140.19^\circ (1.02in)$$

$$V_C = 51.21\angle -140.19^\circ (1.02in)$$



## CHAPTER TEN (Phasor Analysis) SOLUTIONS

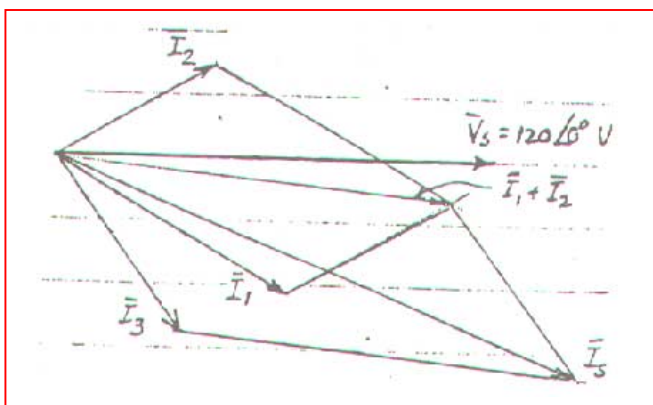
82.

(a)  $I_1 = \frac{120}{40 \angle 30^\circ} = 3 \angle -30^\circ \text{ A}$

$I_2 = \frac{120}{50 - j30} = 2.058 \angle 30.96^\circ \text{ A}$

$I_3 = \frac{120}{30 + j40} = 2.4 \angle -53.13^\circ \text{ A}$

(b)



(c)  $I_s = I_1 + I_2 + I_3$   
 $= 6.265 \angle -22.14^\circ \text{ A}$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

83.

$$|I_1| = 5\text{A}, |I_2| = 7\text{A}$$

$$I_1 + I_2 = 10\angle 0^\circ, I_1 \text{ lags } V, I_2 \text{ leads } V$$

$$I_1 \text{ lags } I_2. \text{ Use } 2.5\text{A} / \text{in}$$

$$[\text{Analytically: } 5\angle\alpha + 7\angle\beta = 10]$$

$$= 5\cos\alpha + j5\sin\alpha + 7\cos\beta + j7\sin\beta$$

$$\therefore \sin\alpha = -1.4\sin\beta$$

$$\therefore 5\sqrt{1-1.4^2\sin^2\beta} + 7\sqrt{1-\sin^2\beta} = 10$$

$$\text{By SOLVE, } \boxed{\alpha = -40.54^\circ \beta = 27.66^\circ}$$

## CHAPTER TEN (Phasor Analysis) SOLUTIONS

84.  $\mathbf{V}_1 = 100\angle 0^\circ \text{ V}$ ,  $|\mathbf{V}_2| = 140 \text{ V}$ ,  $|\mathbf{V}_1 + \mathbf{V}_2| = 120 \text{ V}$ .

Let  $50 \text{ V} = 1 \text{ inch}$ . From the sketch, for  $\angle \mathbf{V}_2$  positive,

$\mathbf{V}_2 = 140\angle 122.5^\circ$ . We may also have  $\mathbf{V}_2 = 140\angle -122.5^\circ \text{ V}$

[Analytically:  $|100 + 140\angle \alpha| = 120$

so  $|100 + 140 \cos \alpha + j140 \sin \alpha| = 120$

Using the “Solve” routine of a scientific calculator,

$\alpha = \pm 122.88^\circ$ .]

## CHAPTER 11 SOLUTIONS

1.

$$\mathbf{Z}_c = \frac{10^6}{j500 \times 25} = -j80 \Omega, \frac{50(-j80)}{50 - j80} = 42.40 \angle -32.01^\circ \Omega$$

$$\therefore \mathbf{V} = 84.80 \angle -32.01^\circ \text{ V}, \mathbf{I}_R = 1.696 \angle -32.01^\circ \text{ A}$$

$$\mathbf{I}_c = 1.0600 \angle 57.99^\circ \text{ A}$$

$$p_s (\pi / 2 \text{ ms}) = 84.80 \cos(45^\circ - 32.01^\circ) 2 \cos 45^\circ = 116.85 \text{ W}$$

$$p_R = 50 \times 1.696^2 \cos^2(45^\circ - 32.01^\circ) = 136.55 \text{ W}$$

$$p_c = 84.80 \cos(45^\circ - 32.01^\circ) = 1.060 \cos(45^\circ + 57.99^\circ) = -19.69 \text{ W}$$

## CHAPTER 11 SOLUTIONS

2.

$$\begin{aligned} \text{(a)} \quad 4\text{H}: i &= 2t^2 - 1 \therefore v = Li' = 4(4t) = 16t, w_L = \frac{1}{2} Li^2 = \frac{1}{2} \times 4(4t^4 - 4t^2 + 1) \\ \therefore w_L &= 8t^4 - 8t^2 + 2 \therefore w_L(3) - w_L(1) = 8 \times 3^4 - 8 \times 3^2 + 2 - 8 \times 1 + 8 \times 1 - 2 = \boxed{576 \text{ J}} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 0.2\text{F}: v_c &= \frac{1}{0.2} \int_1^t (2t^2 - 1) dt + 2 = 5 \left( \frac{2}{3} t^3 - t \right) + 2 = 5 \left( \frac{2}{3} t^3 - t \right) - 5 \left( \frac{2}{3} - 1 \right) + 2 \\ \therefore v_c(2) &= \frac{10}{3} \times 8 - 10 - \frac{10}{3} + 5 + 2 = \frac{61}{3} \text{ V} \therefore P_c(2) = \frac{61}{3} \times 7 = \boxed{142.33 \text{ W}} \end{aligned}$$

## CHAPTER 11 SOLUTIONS

$$3. \quad v_c(0) = -2\text{V}, i(0) = 4\text{A}, \alpha = \frac{R}{2L} = 2, \omega_o^2 = \frac{1}{LC} = 3, s_{1,2} = -2 \pm 1 = -1, -3$$

$$\begin{aligned} \text{(a)} \quad i &= Ae^{-t} + Be^{-3t} \therefore A + B = 4; i(0^+) = \frac{1}{1} v_L(0^+) = (-4 \times 4 \times +2) = -14 \\ \therefore -A - 3B &= -14 \therefore B = 5, A = -1, i = -e^{-t} + 5e^{-3t} \text{ A} \\ \therefore v_c &= 3 \int_0^t (-e^{-t} + 5e^{-3t}) dt - 2 = 3(e^{-t} - 5e^{-3t}) \Big|_0^t - 2 = e^{-t} - 3 - 5e^{-3t} + 5 - 2 \\ \therefore v_c &= 3e^{-t} - 5e^{-3t} \therefore P_c(0^+) = (3 - 5)(-1 + 5) = \boxed{-8 \text{ W}} \end{aligned}$$

$$\text{(b)} \quad P_c(0.2) = (3e^{-0.2} - 5e^{-0.6})(-e^{0.2} + 5e^{-0.6}) = \boxed{-0.5542 \text{ W}}$$

$$\text{(c)} \quad P_c(0.4) = (3e^{-0.4} - 5e^{-1.2})(5e^{-1.2} - e^{-0.4}) = \boxed{0.4220 \text{ W}}$$

## CHAPTER 11 SOLUTIONS

4. We assume the circuit has already reached sinusoidal steady state by  $t = 0$ .

$$2.5 \text{ k}\Omega \rightarrow 2.5 \text{ k}\Omega, 1 \text{ H} \rightarrow j1000 \Omega, 4 \mu\text{F} \rightarrow -j250 \Omega, 10 \text{ k}\Omega \rightarrow 10 \text{ k}\Omega$$

$$\mathbf{Z}_{\text{eq}} = j1000 \parallel -j250 \parallel 10000 = 11.10 - j333.0 \Omega$$

$$\mathbf{V}_{\text{eq}} = \frac{(20 \angle 30^\circ)(11.10 - j333.0)}{2500 + 11.10 - j333.0} = 2.631 \angle 50.54^\circ \text{ V}$$

$$\mathbf{I}_{10\text{k}} = \frac{\mathbf{V}_{\text{eq}}}{10000} = 0.2631 \angle -50.54^\circ \text{ mA} \quad \mathbf{I}_{1\text{H}} = \frac{\mathbf{V}_{\text{eq}}}{j1000} = 2.631 \angle -140.5^\circ \text{ mA}$$

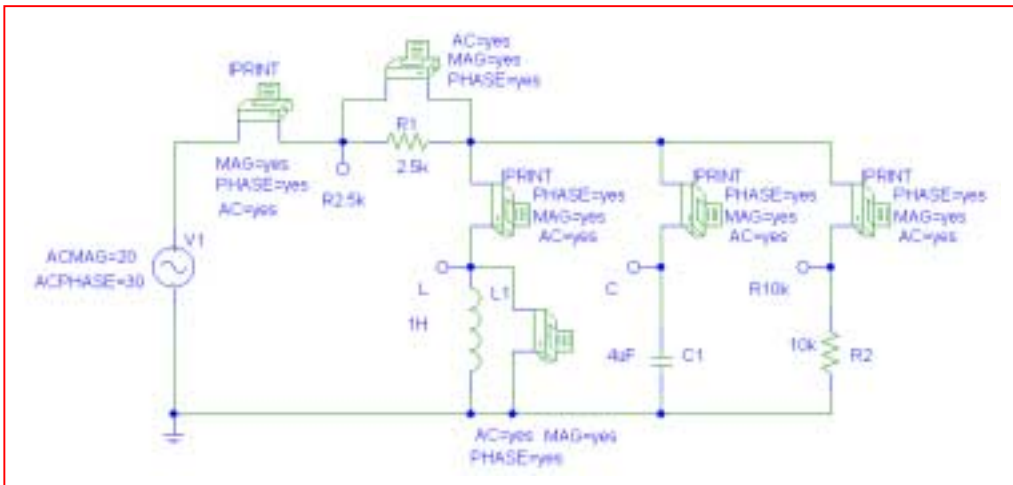
$$\mathbf{I}_{4\mu\text{F}} = \frac{\mathbf{V}_{\text{eq}}}{-j250} = 10.52 \angle 39.46^\circ \text{ mA} \quad \mathbf{V}_{2.5\text{k}} = \frac{(20 \angle 30^\circ)(2500)}{2500 + 11.10 - j333.0} = 19.74 \angle 37.55^\circ \text{ V}$$

$$\text{Thus, } P_{2.5\text{k}} = \frac{[19.74 \cos 37.55^\circ]^2}{2500} = \boxed{97.97 \text{ mW}}$$

$$P_{1\text{H}} = [2.631 \cos(-50.54^\circ)] [2.631 \times 10^{-3} \cos(-140.5^\circ)] = \boxed{-3.395 \text{ mW}}$$

$$P_{4\mu\text{F}} = [2.631 \cos(-50.54^\circ)] [10.52 \times 10^{-3} \cos(39.46^\circ)] = \boxed{13.58 \text{ mW}}$$

$$P_{2.5\text{k}} = \frac{[2.631 \cos(-50.54^\circ)]^2}{10000} = \boxed{279.6 \mu\text{W}}$$



FREQ	IM(V_PRINT1)	IP(V_PRINT1)
1.592E+02	7.896E-03	3.755E+01

FREQ	VM(L,0)	VP(L,0)
1.592E+02	2.629E+00	-5.054E+01

FREQ	VM(R2_5k,\$N_0002)	VP(R2_5k,\$N_0002)
1.592E+02	1.974E+01	3.755E+01

FREQ	IM(V_PRINT11)	IP(V_PRINT11)
1.592E+02	1.052E-02	3.946E+01

FREQ	IM(V_PRINT2)	IP(V_PRINT2)
1.592E+02	2.628E-03	-1.405E+02

FREQ	IM(V_PRINT12)	IP(V_PRINT12)
1.592E+02	2.629E-04	-5.054E+01



## CHAPTER 11 SOLUTIONS

5.

$$i_s \rightarrow 5\angle 0^\circ \text{ A}, C \rightarrow -j4\Omega, \mathbf{Z}_{in} = 8\|(3-j4) = \frac{40\angle -53.13^\circ}{11-j4}$$

$$= 3.417\angle -33.15^\circ \therefore \mathbf{V}_s = 17.087\angle -33.15^\circ,$$

$$v_s = 17.087 \cos(25t - 33.15^\circ) \text{ V} \therefore$$

$$P_{s,abs}(0.1) = -17.087 \cos(2.5^{\text{rad}} - 33.147^\circ) \times 5 \cos 2.5^{\text{rad}} = \boxed{-23.51 \text{ W}}$$

$$i_8 = \frac{17.087}{8} \cos(25t - 33.15^\circ) \therefore$$

$$i_8(0.1) = 2.136 \cos(2.5^{\text{rad}} - 33.15^\circ) = -0.7338 \text{ A}$$

$$\therefore P_{8,abs} = 0.7338^2 \times 8 = \boxed{4.307 \text{ W}};$$

$$\mathbf{I}_3 = \frac{17.087\angle -33.15^\circ}{3-j4} = 3.417\angle 19.98^\circ \text{ A}$$

$$\therefore i_3(0.1) = 3.417 \cos(2.5^{\text{rad}} + 19.98^\circ) = -3.272 \text{ A} \therefore$$

$$P_{3,abc} = 3.272^2 \times 3 = \boxed{32.12 \text{ W}}$$

$$\mathbf{V}_c = -j4(3.417\angle 19.983^\circ) = 13.67\angle -70.02^\circ,$$

$$v_c(0.1) = 13.670 \cos(2.5^{\text{rad}} - 70.02^\circ) = 3.946 \text{ V}$$

$$\therefore P_{c,abc} = 3.946(-3.272) = \boxed{-12.911 \text{ W}} \quad (\Sigma = 0)$$

## CHAPTER 11 SOLUTIONS

6.

$$\mathbf{Z}_{in} = 4 + \frac{j5(10 - j5)}{10} = 4 + 2.5 + j5 = 6.5 + j5 \ \Omega$$

$$\therefore \mathbf{I}_s = \frac{100}{6.5 + j5} = 12.194 \angle -37.57^\circ \text{ A}$$

$$\therefore P_{s,abs} = -\frac{1}{2} \times 100 \times 12.194 \cos 37.57^\circ = -483.3 \text{ W}$$

$$P_{4,abs} = \frac{1}{2} (12.194)^2 4 = \boxed{297.4 \text{ W}},$$

$$P_{cabs} = \boxed{0}$$

$$\mathbf{I}_{10} = \frac{100}{6.5 + j5} \frac{j5}{10} = 6.097 \angle 52.43^\circ \text{ so}$$

$$P_{10,abs} = \frac{1}{2} (6.097)^2 \times 10 = \boxed{185.87 \text{ W}}$$

$$P_L = \boxed{0} \quad (\Sigma = 0)$$

## CHAPTER 11 SOLUTIONS

7.

$$\mathbf{V} = (10 + j10) \frac{40 \angle 30^\circ}{5 \angle 50^\circ + 8 \angle -20^\circ} = 52.44 \angle 69.18^\circ \text{ V}$$

$$P_{10, \text{gen}} = \frac{1}{2} \times 10 \times 52.44 \cos 69.18^\circ = \boxed{93.20 \text{ W}}$$

$$P_{j10, \text{gen}} = \frac{1}{2} \times 10 \times 52.44 \cos (90^\circ - 69.18^\circ) = \boxed{245.08 \text{ W}}$$

$$P_{8 \angle -20^\circ \text{abs}} = \frac{1}{2} \left( \frac{52.44}{8} \right)^2 8 \cos (-20^\circ) = \boxed{161.51 \text{ W}} \quad (\Sigma_{\text{gen}} = \Sigma_{\text{abs}})$$

## CHAPTER 11 SOLUTIONS

8.

$$\mathbf{Z}_R = 3 + \frac{1}{0.1 - j0.3} = 3 + 1 + j3 = 4 + j3 \Omega$$

$$\text{Ignore } 30^\circ \text{ on } \mathbf{V}_s, \mathbf{I}_R = 5 \frac{2 + j5}{6 + j8}, |\mathbf{I}_R| = \frac{5\sqrt{29}}{10}$$

$$(a) \quad P_{3\Omega} = \frac{1}{2} \left( \frac{5\sqrt{29}}{10} \right)^2 \times 3 = \boxed{10.875 \text{ W}}$$

$$(b) \quad \mathbf{V}_s = 5 \angle 0^\circ \frac{(2 + j5)(4 + j3)}{6 + j8} = 13.463 \angle 51.94^\circ \text{ V}$$

$$\therefore P_{s,gen} = \frac{1}{2} \times 13.463 \times 5 \cos 51.94^\circ = \boxed{20.75 \text{ W}}$$

## CHAPTER 11 SOLUTIONS

9.

$$P_{j10} = P_{-j5} = 0,$$

$$\frac{V_{10} - 50}{j10} + \frac{V_{10}}{10} + \frac{V_{10} - j50}{-j5} = 0$$

$$\therefore V_{10}(-j0.1 + 0.1 + j0.2) + j5 + 10 = 0$$

$$\therefore V_{10} = 79.06 \angle 16.57^\circ \text{ V}$$

$$P_{10\Omega} = \frac{1}{2} \frac{79.06^2}{10} = 312.5 \text{ W};$$

$$I_{50} = \frac{79.06 \angle 161.57^\circ - 50}{j10} = 12.75 \angle 78.69^\circ \text{ A}$$

$$\therefore P_{50V} = \frac{1}{2} \times 50 \times 12.748 \cos 78.69^\circ = 62.50 \text{ W}$$

$$I_{j50} = \frac{79.06 \angle 161.57^\circ - j50}{-j5} = 15.811 \angle -7.57^\circ :$$

$$P_{j50} = \frac{1}{2} \times 50 \times 15.811 \cos (90^\circ + 71.57^\circ) = -375.0 \text{ W}$$

## CHAPTER 11 SOLUTIONS

10.

$$\frac{\mathbf{V}_x - 20}{2} + \frac{\mathbf{V}_x - \mathbf{V}_c}{3} = 2\mathbf{V}_c,$$

$$3\mathbf{V}_x - 60 + 2\mathbf{V}_x - 2\mathbf{V}_c = 12\mathbf{V}_c$$

$$\therefore 5\mathbf{V}_x - 14\mathbf{V}_c = 60, \frac{\mathbf{V}_c - \mathbf{V}_x}{3} + \frac{\mathbf{V}_c}{-j2} = 0$$

$$\therefore 2\mathbf{V}_c - 2\mathbf{V}_x + j3\mathbf{V}_c = 0, -2\mathbf{V}_x + (2 + j3)\mathbf{V}_c = 0$$

$$\mathbf{V}_x = \frac{\begin{vmatrix} 60 & -14 \\ 0 & 2 + j3 \end{vmatrix}}{\begin{vmatrix} 5 & -14 \\ -2 & 2 + j3 \end{vmatrix}} = \frac{120 + j180}{10 + j15 - 28} = 9.233 \angle -83.88^\circ \text{ V}$$

$$\mathbf{V}_c = \frac{\begin{vmatrix} 5 & 60 \\ -2 & 0 \end{vmatrix}}{-18 + j15} = 5.122 \angle -140.9^\circ \text{ V} \therefore$$

$$P_{gen} = \frac{1}{2} \times 9.233 \times 2 \times 5.122 \cos(-83.88^\circ + 140.19^\circ) = \boxed{26.23 \text{ W}}$$

## CHAPTER 11 SOLUTIONS

11.

(a)  $X_{in} = 0 \therefore \mathbf{Z}_L = \boxed{\mathbf{R}_{th} + j0}$

(b)  $R_L, X_L$  independent  $\therefore \boxed{\mathbf{Z}_L = \mathbf{Z}_{th}^* = \mathbf{R}_{th} - jX_{th}}$

(c)  $R_L$  fixed  $\therefore P_L = \frac{1}{2} \frac{|V_{th}|^2}{(\mathbf{R}_{th} + \mathbf{R}_L)^2 + (X_{th} + X_L)^2} \times \mathbf{R}_L \therefore \boxed{\mathbf{Z}_L = \mathbf{R}_L - jX_{th}}$

(d)  $X_L$  fixed, Let  $X_L + X_{th} = a \therefore f = \frac{2P_L}{|\mathbf{V}_{th}|^2} = \frac{\mathbf{R}L}{(\mathbf{R}_{th} + \mathbf{R}_L)^2 + a^2}$

$$\frac{df}{dR_L} = \frac{\mathbf{R}_{th} + \mathbf{R}_L^2 + a^2 - 2\mathbf{R}_L(\mathbf{R}_{th} + \mathbf{R}_L)}{[(\mathbf{R}_{th} + \mathbf{R}_L)^2 + a^2]^2} = 0$$

$$\mathbf{R}_{th}^2 + 2\mathbf{R}_{th}\mathbf{R}_L + \mathbf{R}_L^2 + a^2 - 2\mathbf{R}_{th}\mathbf{R}_L = 2\mathbf{R}_L^2 = 0$$

$$\therefore \mathbf{R}_L = \sqrt{\mathbf{R}_{th}^2 + a^2} = \boxed{\sqrt{\mathbf{R}_{th}^2 + (X_{th} + X_L)^2}}$$

(e)  $X_L = 0 \therefore \mathbf{R}_L = \boxed{\sqrt{\mathbf{R}_{th}^2 + X_{th}^2}} = |\mathbf{Z}_{th}|$

## CHAPTER 11 SOLUTIONS

12.

$$\mathbf{V}_{th} = 120 \frac{-10}{10 + j5} = 107.33 \angle -116.57^\circ \text{ V}$$

$$\mathbf{Z}_{th} = \frac{-j10(10 + j15)}{10 + j5} = 8 - j14 \Omega$$

(a)  $\therefore \mathbf{Z}_L = 8 + j15 \Omega$

(b)  $\mathbf{I}_L = \frac{107.33 \angle -116.57^\circ}{16} \therefore$

$$P_{L,\max} = \frac{1}{2} \left( \frac{107.33}{16} \right)^2 \times 8 = 180 \text{ W}$$



## CHAPTER 11 SOLUTIONS

13.

$$R_L = |\mathbf{Z}_{th}| \therefore R_L = \sqrt{8^2 + 14^2} = 16.125 \Omega$$

$$P_L = \frac{1}{2} \frac{107.33^2}{(8 + 16.125)^2 + 14^2} \times 16.125 = 119.38 \text{ W}$$

## CHAPTER 11 SOLUTIONS

14.

$$-j9.6 = -4.8I_x - j1.92 I_x - +4.8I_x$$

$$\therefore I_x = \frac{9.6}{1.92} = 5$$

$$\therefore V = (0.6 \times 5)8 = 24 \text{ V}$$

$$\therefore P_o = \frac{1}{2} \times 24 \times 1.6 \times 5 = 96 \text{ W (gen)}$$

## CHAPTER 11 SOLUTIONS

15.

$$\begin{aligned} \text{(a)} \quad \mathbf{Z}_{th} &= 80 \parallel j60 = \frac{j480}{80 + j60} \frac{80 - j60}{80 - j60} \\ &= 28.8 + j38.4 \, \Omega \therefore \mathbf{Z}_{L_{\max}} = \boxed{28.8 - j38.4 \, \Omega} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \mathbf{V}_{th} &= 5(28.8 + j38.4) = 144 + j192 \, \text{V}, \\ \therefore \mathbf{I}_L &= \frac{144 + j192}{2 \times 28.8} \\ \text{and } P_{L_{\max}} &= \frac{1}{2} \frac{144^2 + 192^2}{4 \times 28.8^2} \times 28.8 = \boxed{250 \, \text{W}} \end{aligned}$$

## CHAPTER 11 SOLUTIONS

16.  $\mathbf{Z}_{\text{eq}} = (6 - j8) \parallel (12 + j9) = 8.321 \angle -19.44^\circ \text{ W}$

$$\mathbf{V}_{\text{eq}} = (5 \angle -30^\circ) (8.321 \angle -19.44^\circ) = 41.61 \angle -49.44^\circ \text{ V}$$

$$P_{\text{total}} = \frac{1}{2} (41.61)(5) \cos (-19.44^\circ) = 98.09 \text{ W}$$

$$\mathbf{I}_{6-j8} = \mathbf{V}_{\text{eq}} / (6 - j8) = 4.161 \angle 3.69^\circ \text{ A}$$

$$\mathbf{I}_{4+j2} = \mathbf{I}_{8+j7} = \mathbf{V}_{\text{eq}} / 12 + j9 = 2.774 \angle -86.31^\circ \text{ A}$$

$$P_{6-j8} = \frac{1}{2} (41.61)(4.161) \cos (-49.44^\circ - 3.69^\circ) = 51.94 \text{ W}$$

$$P_{4+j2} = \frac{1}{2} (2.774)^2 (4) = 15.39 \text{ W}$$

$$P_{8+j7} = \frac{1}{2} (2.774)^2 (8) = 30.78 \text{ W}$$

Check:  $\Sigma = 98.11 \text{ W}$  (okay)

## CHAPTER 11 SOLUTIONS

17.

$$\mathbf{V}_{th} = 100 \frac{j10}{20 + j10} = 20 + j40, \mathbf{Z}_{th} = \frac{j10(20)}{20 + j10} = 4 + j8 \Omega$$

$$\therefore R_L = |\mathbf{Z}_{th}| \therefore R_L = 8.944 \Omega$$

$$\therefore P_{L,\max} = \frac{1}{2} \frac{20^2 + 40^2}{(4 + 8.944)^2 + 64} \times 8.944 = 38.63 \text{ W}$$

## CHAPTER 11 SOLUTIONS

18. We may write a single mesh equation:  $170 \angle 0^\circ = (30 + j10) \mathbf{I}_1 - (10 - j50)(-\lambda \mathbf{I}_1)$   
Solving,

$$\mathbf{I}_1 = \frac{170 \angle 0^\circ}{30 + j10 + 10\lambda - j50\lambda}$$

- (a)  $\lambda = 0$ , so  $\mathbf{I}_1 = \frac{170 \angle 0^\circ}{30 + j10} = 5.376 \angle -18.43^\circ \text{ A}$  and, with the same current flowing

through both resistors in this case,

$$P_{20} = \frac{1}{2} (5.376)^2 (20) = 289.0 \text{ W}$$

$$P_{10} = \frac{1}{2} (5.376)^2 (10) = 144.5 \text{ W}$$

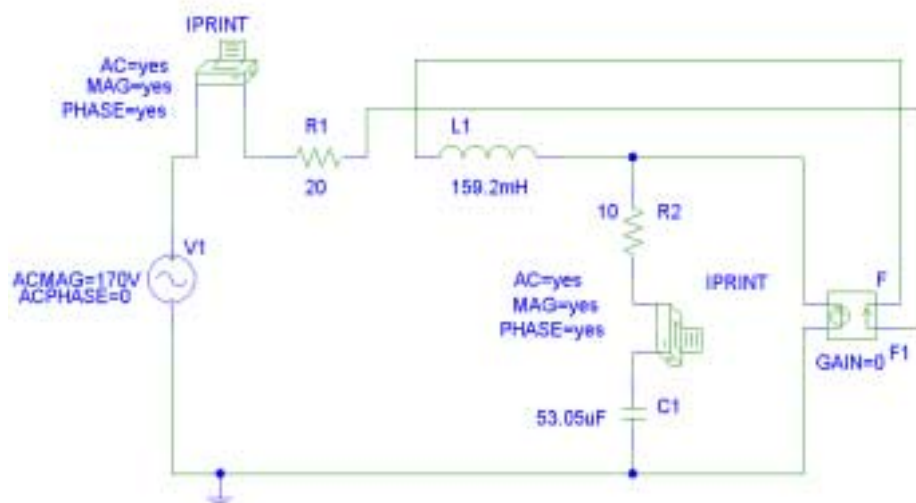
- (b)  $\lambda = 1$ , so  $\mathbf{I}_1 = \frac{170 \angle 0^\circ}{40 - j40} = 3.005 \angle 45^\circ \text{ A}$

$$P_{20} = \frac{1}{2} (3.005)^2 (20) = 90.30 \text{ W}$$

The current through the  $10\text{-}\Omega$  resistor is  $\mathbf{I}_1 + \lambda \mathbf{I}_1 = 2 \mathbf{I}_1 = 6.01 \angle 45^\circ$  so

$$P_{10} = \frac{1}{2} (6.01)^2 (10) = 180.6 \text{ W}$$

(c)



(a)		
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
6.000E+01	5.375E+00	-1.846E+01
FREQ	IM(V_PRINT4)	IP(V_PRINT4)
6.000E+01	5.375E+00	-1.846E+01
(b)		
FREQ	IM(V_PRINT3)	IP(V_PRINT3)
6.000E+01	6.011E+00	4.499E+01
FREQ	IM(V_PRINT4)	IP(V_PRINT4)
6.000E+01	3.006E+00	4.499E+01

## CHAPTER 11 SOLUTIONS

19. (a) Waveform (a):  $I_{\text{avg}} = \frac{(10)(1) + (-5)(1) + 0(1)}{3} = \boxed{1.667 \text{ A}}$

Waveform (b):  $I_{\text{avg}} = \frac{\frac{1}{2}(20)(1) + 0(1)}{2} = \boxed{5 \text{ A}}$

Waveform (c):

$$I_{\text{avg}} = \frac{1}{1 \times 10^{-3}} \int_0^{10^{-3}} 8 \sin \frac{2\pi t}{4 \times 10^{-3}} dt = -(8 \times 10^3) \left( \frac{4 \times 10^{-3}}{2\pi} \right) \cos \left( \frac{\pi t}{2 \times 10^{-3}} \right) \Big|_0^{10^{-3}}$$

$$= -\frac{16}{\pi} (0 - 1) = \boxed{\frac{16}{\pi} \text{ A}}$$

(b) Waveform (a):  $I_{\text{avg}}^2 = \frac{(100)(1) + (25)(1) + (0)(1)}{3} = \boxed{41.67 \text{ A}^2}$

Waveform (b):  $i(t) = -20 \times 10^3 t + 20$   
 $i^2(t) = 4 \times 10^8 t^2 - 8 \times 10^5 t + 400$

$$I_{\text{avg}}^2 = \frac{1}{2 \times 10^{-3}} \int_0^{10^{-3}} (4 \times 10^8 t^2 - 8 \times 10^5 t + 400) dt$$

$$= \frac{1}{2 \times 10^{-3}} \left[ \frac{4 \times 10^8}{3} (10^{-3})^3 - \frac{8 \times 10^5}{2} (10^{-3})^2 + 400(10^{-3}) \right] = \frac{0.1333}{2 \times 10^{-3}} = \boxed{66.67 \text{ A}^2}$$

Waveform (c):

$$I_{\text{avg}}^2 = \frac{1}{1 \times 10^{-3}} \int_0^{10^{-3}} 64 \sin^2 \frac{2\pi t}{4 \times 10^{-3}} dt = (64 \times 10^3) \left[ \frac{t}{2} - \frac{\sin \pi \times 10^3 t}{2\pi \times 10^3} \right] \Big|_0^{10^{-3}}$$

$$= (64 \times 10^3) \left[ \frac{10^{-3}}{2} - \frac{\sin \pi}{2\pi \times 10^3} \right] = \boxed{32 \text{ A}^2}$$

## CHAPTER 11 SOLUTIONS

20. At  $\omega = 120\pi$ ,  $1 \text{ H} \rightarrow j377 \Omega$ , and  $4 \mu\text{F} \rightarrow -j663.1 \Omega$   
 Define  $\mathbf{Z}_{\text{eff}} = j377 \parallel -j663.1 \parallel 10\,000 = 870.5 \angle 85.01^\circ \Omega$

$$\mathbf{V}_{2.5k} = \frac{(400\sqrt{2} \angle -9^\circ) 2500}{2500 + 870.5 \angle 85.01^\circ} = 520.4 \angle -27.61^\circ \text{ V}$$

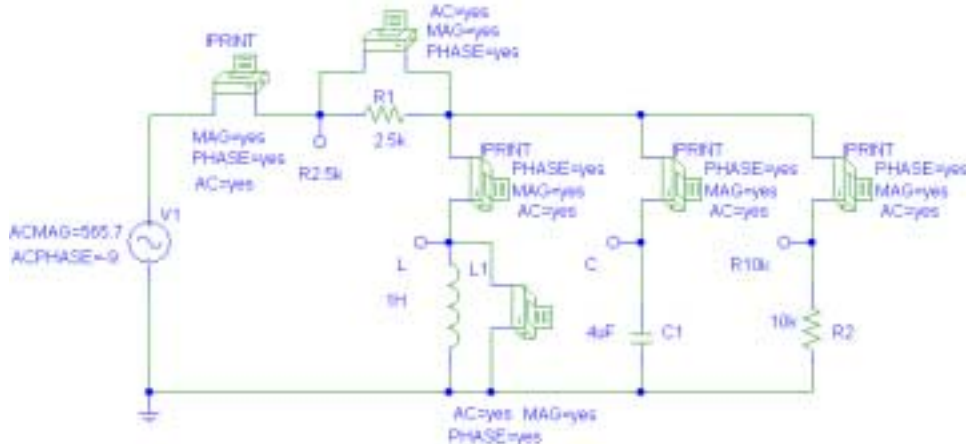
$$\mathbf{V}_{10k} = \frac{(400\sqrt{2} \angle -9^\circ)(870.5 \angle 85.01^\circ)}{2500 + 870.5 \angle 85.01^\circ} = 181.2 \angle 57.40^\circ \text{ V}$$

Thus, $P_{2.5k} = \frac{1}{2} (520.4)^2 / 2\,500$	= 54.16 W	
$P_{10k} = \frac{1}{2} (181.2)^2 / 10\,000$	= 1.642 W	
$P_{1H}$	= 0	
$P_{4\mu\text{F}}$	= 0	(A total absorbed power of 55.80 W.)

To check, the average power delivered by the source:

$$\mathbf{I}_{\text{source}} = \frac{400\sqrt{2} \angle -9^\circ}{2500 + 870.5 \angle 85.01^\circ} = 0.2081 \angle -27.61^\circ \text{ A}$$

and  $P_{\text{source}} = \frac{1}{2} (400\sqrt{2})(0.2081) \cos(-9^\circ + 27.61^\circ) = 55.78 \text{ W}$  (checks out).



FREQ	IM(V_PRINT1)	IP(V_PRINT1)	FREQ	VM(L,0)	VP(L,0)
6.000E+01	2.081E-01	-2.760E+01	6.000E+01	1.812E+02	5.740E+01
FREQ	VM(R2_5k,\$N_0002)	VP(R2_5k,\$N_0002)	FREQ	IM(V_PRINT11)	IP(V_PRINT11)
6.000E+01	5.204E+02	-2.760E+01	6.000E+01	2.732E-01	1.474E+02
FREQ	IM(V_PRINT2)	IP(V_PRINT2)	FREQ	IM(V_PRINT12)	IP(V_PRINT12)
6.000E+01	4.805E-01	-3.260E+01	6.000E+01	1.812E-02	5.740E+01



## CHAPTER 11 SOLUTIONS

21.

(a)  $v = 10 + 9 \cos 100t + 6 \sin 100t$

$$\therefore V_{eff} = \sqrt{100 + \frac{1}{2} \times 81 + \frac{1}{2} \times 36} = \sqrt{158.5} = \boxed{12.590 \text{ V}}$$

(b)  $F_{eff} = \sqrt{\frac{1}{4}(10^2 + 20^2 + 10^2)} = \sqrt{150} = \boxed{12.247}$

(c)  $F_{avg} = \frac{(10)(1) + (20)(1) + (10)(1)}{4} = \frac{40}{4} = \boxed{10}$

## CHAPTER 11 SOLUTIONS

22.

(a)  $g(t) = 2 + 3\cos 100t + 4\cos(100t - 120^\circ)$

$$3 \angle 0^\circ + 4 \angle -120^\circ = 3.606 \angle -73.90^\circ \text{ so } G_{\text{eff}} = \sqrt{4 + \frac{3.606^2}{2}} = \boxed{3.240}$$

(b)  $h(t) = 2 + 3\cos 100t + 4\cos(101t - 120^\circ)$

$$\therefore H_{\text{eff}} = \sqrt{2 + \frac{1}{2}3^2 + \frac{1}{2}4^2} = \sqrt{16.5} = \boxed{4.062}$$

(c)  $f(t) = 100t, 0 < t < 0.1 \therefore F_{\text{eff}} = \sqrt{\frac{1}{0.3} \int_0^{0.1} 10^6 t^2 dt}$

$$= \sqrt{\frac{10}{3} \times 10^6 \times \frac{1}{3} \times 10^{-3}} = \boxed{33.33}$$

## CHAPTER 11 SOLUTIONS

23.  $f(t) = (2 - 3\cos 100t)^2$

(a)  $f(t) = 4 - 12\cos 100t + 9\cos^2 100t$

$\therefore f(t) = 4 - 12\cos 100t + 4.5 + 4.5\cos 200t \therefore F_{av} = 4 + 4.5 = 8.5$

(b)  $F_{eff} = \sqrt{8.5^2 + \frac{1}{2} \times 12^2 + \frac{1}{2} \times 4.5^2} = 12.43$

## CHAPTER 11 SOLUTIONS

$$24. \quad (a) \quad i_{\text{eff}} = \left[ \frac{1}{3} (10^2 + (-5)^2) + 0 \right]^{\frac{1}{2}} = \boxed{6.455 \text{ A}}$$

$$(b) \quad i_{\text{eff}} = \left[ \frac{1}{2} \left( \int_0^1 [-20t + 20] dt \right) + 0 \right]^{\frac{1}{2}} = \sqrt{5} = \boxed{2.236 \text{ A}}$$

$$(c) \quad i_{\text{eff}} = \left[ \frac{1}{1} \left( \int_0^1 8 \sin \left( \frac{2\pi}{4} t \right) dt \right) \right]^{\frac{1}{2}} = \sqrt{\left[ -8 \left( \frac{2}{\pi} \right) \cos \left( \frac{\pi t}{2} \right) \right]_0^1} = \boxed{2.257 \text{ A}}$$

## CHAPTER 11 SOLUTIONS

25.

(a)  $A = B = 10\text{V}$ ,  $C = D = 0 \therefore 10\angle 0^\circ + 10\angle -45^\circ = 18.48\angle -22.50^\circ$

$$\therefore P = \frac{1}{2} \times \frac{1}{4} \times 18.48^2 = 42.68 \text{ W}$$

(b)  $A = C = 10\text{V}$ ,  $B = D = 0$ ,  $v_s = 10\cos 10t + 10\cos 40t$ ,

$$P = \frac{1}{2} \frac{10^2}{4} + \frac{1}{2} \frac{10^2}{4} = 25 \text{ W}$$

(c)  $v_s = 10\cos 10t - 10\sin (10t + 45^\circ) \rightarrow 10 - 10\angle -45^\circ = 7.654\angle 67.50^\circ$

$$\therefore P = \frac{1}{2} \frac{7.654^2}{4} = 7.322 \text{ W}$$

(d)  $v = 10\cos 10t + 10\sin (10t + 45^\circ) + 10\cos 40t$ ;

$$10\angle 0^\circ + 10\angle -45^\circ = 18.48\angle -22.50^\circ$$

$$\therefore P = \frac{1}{2} \times 18.48^2 \times \frac{1}{4} + \frac{1}{2} \times 10^2 \times \frac{1}{4} = 55.18 \text{ W}$$

(e)  $// + 10dc \therefore P_{av} = 55.18 + \frac{10^2}{4} = 80.18 \text{ W}$

## CHAPTER 11 SOLUTIONS

26.  $\mathbf{Z}_{eq} = R \parallel j0.3\omega = \frac{j0.3R\omega}{R + j0.3R\omega}$ . By voltage division, then, we write:

$$\mathbf{V}_{100\text{mH}} = 120\angle 0 \frac{j0.1\omega}{j0.1\omega + \frac{j0.3R\omega}{R + j0.3\omega}} = 120\angle 0 \frac{-0.03\omega^2 + j0.1\omega R}{-0.03\omega^2 + j0.4R\omega}$$

$$\mathbf{V}_{300\text{mH}} = 120\angle 0 \frac{\frac{j0.3R\omega}{R + j0.3\omega}}{j0.1\omega + \frac{j0.3R\omega}{R + j0.3\omega}} = 120\angle 0 \frac{j36R\omega}{-0.03\omega^2 + j0.4R\omega}$$

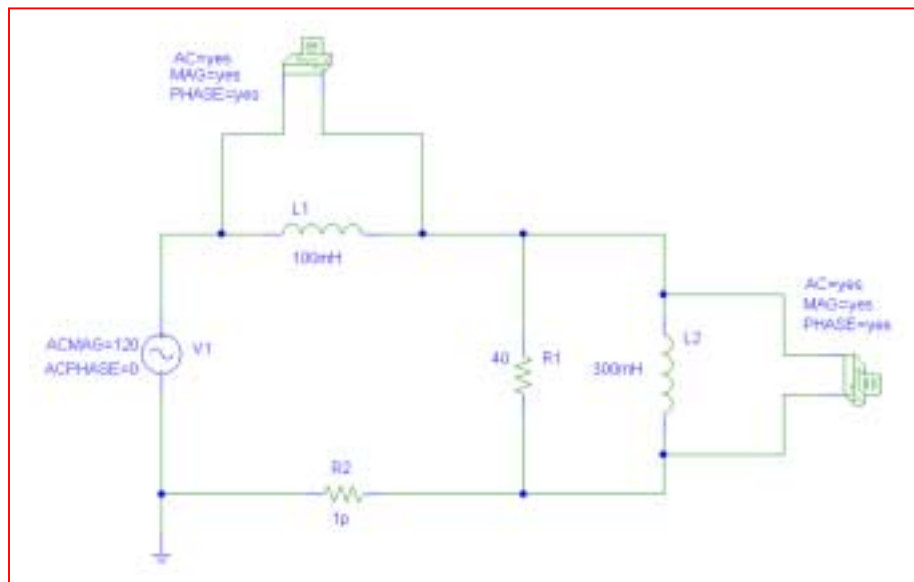
(a) We're interested in the value of R that would lead to equal voltage magnitudes, or

$$|j36R\omega| = |(120)(-0.03\omega^2 + j0.1\omega R)|$$

Thus,  $36R\omega = \sqrt{12.96\omega^4 + 144\omega^2 R^2}$  or  $R = 0.1061 \omega$

(b) Substituting into the expression for  $\mathbf{V}_{100\text{mH}}$ , we find that  $\mathbf{V}_{100\text{mH}} = 73.47 \text{ V}$ , independent of frequency.

To verify with PSpice, simulate the circuit at 60 Hz, or  $\omega = 120\pi \text{ rad/s}$ , so  $R = 40 \Omega$ . We also include a miniscule ( $1 \text{ p}\Omega$ ) resistor to avoid inductor loop warnings. We see from the simulation results that the two voltage magnitudes are indeed the same.



```
FREQ      VM($N_0002,$N_0003)VP($N_0002,$N_0003)
6.000E+01  7.349E+01 -3.525E+01

FREQ      VM($N_0001,$N_0002)VP($N_0001,$N_0002)
6.000E+01  7.347E+01  3.527E+01
```

## CHAPTER 11 SOLUTIONS

27.

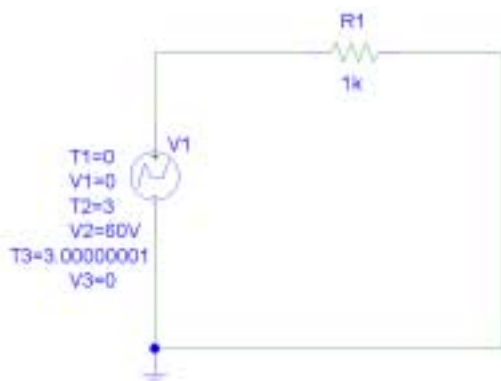
(a)  $V_{av,1} = 30V$

$$V_{av,2} = \frac{1}{3}(10 + 30 + 50) = 30V$$

(b)  $V_{eff,1} = \sqrt{\frac{1}{3} \int_0^3 (20t)^2 dt} = \sqrt{\frac{1}{3} \times 400 \times \frac{1}{3} \times 27} = \sqrt{1200} = 34.64V$

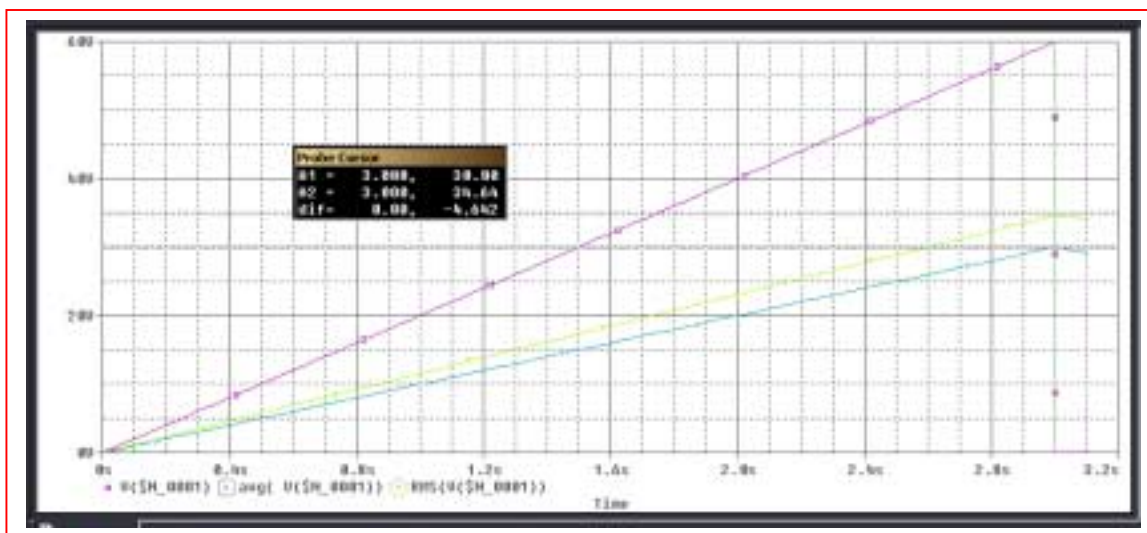
$$V_{eff,2} = \sqrt{\frac{1}{3}(10^2 + 30^2 + 50^2)} = \sqrt{\frac{1}{3} \times 3500} = 34.16V$$

(c) PSpice verification for Sawtooth waveform of Fig. 11.40a:



### Probe Cursor

A1 =	3.000,	30.00
A2 =	3.000,	34.64
dif=	0.00,	-4.642



## CHAPTER 11 SOLUTIONS

$$28. \quad \mathbf{Z}_{\text{eff}} = R \parallel \left( \frac{-j10^6}{3\omega} \right) = \frac{-jR10^6}{3\omega R - j10^6}$$

$$\mathbf{I}_{\text{SRC}} = \frac{120\angle 0}{-j\frac{10^6}{\omega} - j\frac{R10^6}{3\omega R - j10^6}} = \frac{120\omega(3\omega R - j10^6)}{-j10^6(3\omega R - j10^6) - j\omega R10^6}$$

$$\mathbf{I}_{3\mu\text{F}} = \mathbf{I}_{\text{SRC}} \frac{R}{R - j\frac{10^6}{3\omega}}$$

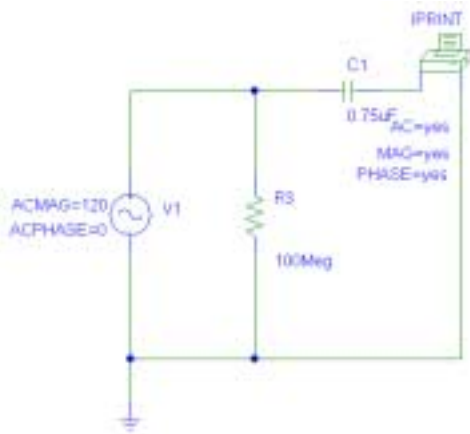
(a) For the two current magnitudes to be equal, we must have  $\left| \frac{R}{R - j\frac{10^6}{3\omega}} \right| = 1$ . This is

only true when  $R = \infty$ ; otherwise, current is shunted through the resistor and the two capacitor currents will be unequal.

(b) In this case, the capacitor current is

$$120\angle 0 \frac{1}{-j\frac{10^6}{\omega} - j\frac{10^6}{3\omega}} = j90\omega \mu\text{A}, \text{ or } 90\omega \cos(\omega t + 90^\circ) \mu\text{A}$$

(c) PSpice verification: set  $f = 60$  Hz, simulate a single  $0.75\text{-}\mu\text{F}$  capacitor, and include a  $100\text{-M}\Omega$  resistor in parallel with the capacitor to prevent a floating node. This should result in a rms current amplitude of  $33.93$  mA, which it does.



FREQ	IM(V_PRINT3)	IP(V_PRINT3)
6.000E+01	3.393E-02	9.000E+01



## CHAPTER 11 SOLUTIONS

29.

$$v(t) = 10t[u(t) - u(t-2)] + 16e^{-0.5(t-3)} [u(t-3) - u(t-5)] \text{ V}$$

Find eff. value separately

$$V_{1,eff} = \sqrt{\frac{1}{5} \int_0^2 100t^2 dt} = \sqrt{\frac{20}{3}} \times 8 = 7.303$$

$$V_{2,eff} = \sqrt{\frac{1}{5} \int_3^5 256e^{-(t-3)} dt} = \sqrt{\frac{256}{5} e^3 (-e^{-t})_3^5} = 6.654$$

$$\therefore V_{eff} = \sqrt{7.303^2 + 6.654^2} = 9.879$$

$$\begin{aligned} V_{eff} &= \sqrt{\frac{1}{5} \left[ \int_0^2 100t^2 dt + \int_3^5 256e^3 e^{-t} dt \right]} \\ &= \sqrt{\frac{1}{5} \left[ \frac{100}{3} \times 8 + 256e^3 (e^{-3} - e^{-5}) \right]} \\ &= \sqrt{\frac{1}{5} \left[ \frac{800}{3} + 256(1 - e^{-2}) \right]} = 9.879 \text{ V OK} \end{aligned}$$

## CHAPTER 11 SOLUTIONS

30. The peak instantaneous power is 250 mW. The combination of elements yields

$$\mathbf{Z} = 1000 + j1000 \, \Omega = 1414 \angle 45^\circ \, \Omega.$$

Arbitrarily designate  $\mathbf{V} = V_m \angle 0$ , so that  $\mathbf{I} = \frac{V_m \angle 0}{\mathbf{Z}} = \frac{V_m \angle -45^\circ}{1414} \text{ A}.$

We may write  $p(t) = \frac{1}{2} V_m I_m \cos \phi + \frac{1}{2} V_m I_m \cos (2\omega t + \phi)$  where  $\phi$  = the angle of the current ( $-45^\circ$ ). This function has a maximum value of  $\frac{1}{2} V_m I_m \cos \phi + \frac{1}{2} V_m I_m$ .

Thus,  $0.250 = \frac{1}{2} V_m I_m (1 + \cos \phi) = \frac{1}{2} (1414) I_m^2 (1.707)$   
and  $I_m = 14.39 \text{ mA}.$

In terms of rms current, the largest rms current permitted is  $14.39 / \sqrt{2} = 10.18 \text{ mA rms}.$

## CHAPTER 11 SOLUTIONS

31.  $\mathbf{I} = 4\angle 35^\circ \text{ A rms}$

(a)  $\mathbf{V} = 20\mathbf{I} + 80\angle 35^\circ \text{ V rms}, P_{s,gen} = 80 \times 10 \cos 35^\circ = \boxed{655.3 \text{ W}}$

(b)  $P_R = |\mathbf{I}|^2 R = 16 \times 20 = \boxed{320 \text{ W}}$

(c)  $P_{Load} = 655.3 - 320 = \boxed{335.3 \text{ W}}$

(d)  $AP_{s,gen} = 80 \times 10 = \boxed{800 \text{ VA}}$

(e)  $AP_R = P_R = \boxed{320 \text{ VA}}$

(f)  $\mathbf{I}_L = 10\angle 0^\circ - 4\angle 35^\circ = 7.104\angle -18.84^\circ \text{ A rms}$   
 $\therefore AP_L = 80 \times 7.104 = \boxed{568.3 \text{ VA}}$

(g)  $PF_L = \cos \theta_L = \frac{P_L}{AP_L} = \frac{335.3}{568.3} = \boxed{0.599}$

since  $\mathbf{I}_L$  lags  $\mathbf{V}$ ,  $PF_L$  is lagging

## CHAPTER 11 SOLUTIONS

32.

$$(a) \quad I_s = \frac{120}{4 + \frac{j192}{12 + j16}} = 9.214 \angle -26.25^\circ \text{ A rms}$$

$$\therefore PF_s = \cos 26.25 = 0.8969 \text{ lag}$$

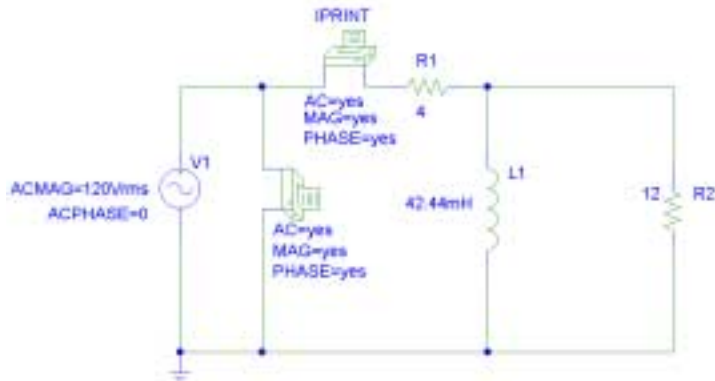
$$(b) \quad P_s = 120 \times 9.214 \times 0.8969 = 991.7 \text{ W}$$

$$(c) \quad Z_L = 4 + \frac{j48}{3 + j4} = 4 + \frac{1}{25} (192 + j144)$$

$$\therefore Z_L = 11.68 + j5.76 \Omega, Y_L = \frac{11.68 - j5.76}{11.68^2 + 5.76^2}$$

$$\therefore j120\pi C = \frac{j5.76}{11.68^2 + 5.76^2}, C = 90.09 \mu\text{F}$$

(d) PSpice verification



```
FREQ      VM($N_0003,0)  VP($N_0003,0)
6.000E+01  1.200E+02     0.000E+00
```

```
FREQ      IM(V_PRINT1)  IP(V_PRINT1)
6.000E+01  9.215E+00    -2.625E+01
```

; (a) and (b) are correct

*Next, add a 90.09-μF capacitor in parallel with the source:*

```
FREQ      IM(V_PRINT1)  IP(V_PRINT1)
6.000E+01  8.264E+00    -9.774E-05
```

; (c) is correct ( $-9.8 \times 10^{-5}$  degrees is essentially zero, for unity PF).

## CHAPTER 11 SOLUTIONS

33.

$$\mathbf{Z}_A = 5 + j2\Omega, \mathbf{Z}_B = 20 - j10\Omega, \mathbf{Z}_C = 10\angle 30^\circ \Omega = 8.660 + j5\Omega$$

$$\mathbf{Z}_D = 10\angle -60^\circ = 5 - j8.660\Omega$$

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 200 & -20 + j10 \\ 0 & 33.66 - j13.660 \end{vmatrix}}{\begin{vmatrix} 25 - j8 & -20 + j10 \\ -20 + j10 & 33.66 - j13.660 \end{vmatrix}} = \frac{7265\angle 22.09^\circ}{480.9\angle -26.00^\circ} = 15.11\angle 3.908^\circ \text{ A rms}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 25 - j8 & 200 \\ -20 + j10 & 0 \end{vmatrix}}{480.9\angle -26.00^\circ} = \frac{200(20 - j10)}{480.9\angle 20.00^\circ} = 9.300\angle -0.5681^\circ \text{ A rms}$$

$$\text{AP}_A = |\mathbf{I}_1|^2 |\mathbf{Z}_A| = 15.108^2 \sqrt{29} = \boxed{1229 \text{ VA}}$$

$$\text{AP}_B = |\mathbf{I}_1 - \mathbf{I}_2|^2 |\mathbf{Z}_B| = 5.881^2 \times 10\sqrt{5} = \boxed{773.5 \text{ VA}}$$

$$\text{AP}_C = |\mathbf{I}_2|^2 |\mathbf{Z}_C| = 9.3^2 \times 10 = \boxed{86.49 \text{ VA}}$$

$$\text{AP}_D = |\mathbf{I}_2|^2 |\mathbf{Z}_D| = 9.3^2 \times 10 = \boxed{86.49 \text{ VA}}$$

$$\text{AP}_S = 200|\mathbf{I}_1| = 200 \times 15.108 = \boxed{3022 \text{ VA}}$$

## CHAPTER 11 SOLUTIONS

34. Perhaps the easiest approach is to consider the load and the compensation capacitor separately. The load draws a complex power  $\mathbf{S}_{\text{load}} = P + jQ$ . The capacitor draws a purely reactive complex power  $\mathbf{S}_C = -jQ_C$ .

$$\theta_{\text{load}} = \tan^{-1}(Q/P), \text{ or } Q = P \tan \theta_{\text{load}}$$

$$Q_C = S_C = V_{\text{rms}} \left| \frac{\mathbf{V}_{\text{rms}}}{(-j / \omega C)} \right| = \left| \omega C V_{\text{rms}}^2 \right| = \omega C V_{\text{rms}}^2$$

$$\mathbf{S}_{\text{total}} = \mathbf{S}_{\text{load}} + \mathbf{S}_C = P + j(Q - Q_C)$$

$$\theta_{\text{new}} = \text{ang}(\mathbf{S}_{\text{total}}) = \tan^{-1} \left( \frac{Q - Q_C}{P} \right), \text{ so that } Q - Q_C = P \tan \theta_{\text{new}}$$

Substituting, we find that  $Q_C = P \tan \theta_{\text{load}} - P \tan \theta_{\text{new}}$

or

$$\omega C V_{\text{rms}}^2 = P (\tan \theta_{\text{load}} - \tan \theta_{\text{new}})$$

Thus, noting that  $\theta_{\text{old}} = \theta_{\text{load}}$ ,

$$C = \frac{P (\tan \theta_{\text{old}} - \tan \theta_{\text{new}})}{\omega V_{\text{rms}}^2}$$

## CHAPTER 11 SOLUTIONS

35.  $\mathbf{Z}_1 = 30\angle 15^\circ \Omega$ ,  $\mathbf{Z}_2 = 40\angle 40^\circ \Omega$

(a)  $\mathbf{Z}_{tot} = 30\angle 15^\circ + 40\angle 40^\circ = 68.37\angle 29.31^\circ \Omega$   
 $\therefore \text{PF} = \cos 29.3^\circ = 0.8719 \text{ lag}$

(b)  $\mathbf{Z}_{tot} = 68.37\angle 29.31^\circ = 59.62 + j33.48$   
 $\text{PF}_{new} = 0.9 \text{ lag}$   
 $\therefore \theta_{new} = \cos^{-1} 0.9 = 25.84^\circ$   
 $\tan 25.84^\circ = 0.4843 = \frac{X_{new}}{59.62} \therefore X_{new} = 28.88 \Omega$   
 $\therefore 33.48 - \frac{1}{100\pi C} = 28.88,$

$$C = 691.8 \mu\text{F}$$

## CHAPTER 11 SOLUTIONS

36.  $\theta_1 = \cos^{-1}(0.92) = 23.07^\circ$ ,  $\theta_2 = \cos^{-1}(0.8) = 36.87^\circ$ ,  $\theta_3 = 0$

$$\mathbf{S}_1 = \frac{100 \angle 23.07^\circ}{0.92} = 100 + j42.59 \text{ VA}$$

$$\mathbf{S}_2 = \frac{250 \angle 36.87^\circ}{0.8} = 250 + j187.5 \text{ VA}$$

$$\mathbf{S}_3 = \frac{500 \angle 0^\circ}{1} = 500 \text{ VA}$$

$$\mathbf{S}_{\text{total}} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 = 500 + j230.1 \text{ VA} = 550.4 \angle 24.71^\circ \text{ VA}$$

$$(a) \mathbf{I}_{\text{eff}} = \frac{\mathbf{S}_{\text{total}}}{V_{\text{eff}}} = \frac{550.4}{115} = 4.786 \text{ A rms}$$

$$(b) \text{ PF of composite load} = \cos(24.71^\circ) = 0.9084 \text{ lagging}$$



## CHAPTER 11 SOLUTIONS

37.

$$AP_L = 10,000 \text{ VA}, \text{PF}_L = 0.8 \text{ lag}, |\mathbf{I}_L| = 40 \text{ A rms}$$

$$\text{Let } \mathbf{I}_L = 40 \angle 0^\circ \text{ A rms}; P_L = 10,000 \times 0.8 = 8000 \text{ W}$$

$$\text{Let } \mathbf{Z}_L = R_L + jX_L \therefore R_L = \frac{8000}{40^2} = 5 \ \Omega$$

$$\cos \theta_L = 0.8 \text{ lag} \therefore \theta_L = \cos^{-1} 0.8 = 36.87^\circ$$

$$\therefore X_L = 5 \tan 36.87^\circ = 3.75 \ \Omega, \mathbf{Z}_L = 5 + j3.75, \mathbf{Z}_{tot} = 5.2 + j3.75 \ \Omega$$

$$\therefore \mathbf{V}_s = 40(5.2 + j3.75) = 256.4 \angle 35.80^\circ \text{ V}; \mathbf{Y}_{tot} = \frac{1}{5.2 + j3.75}$$

$$= 0.12651 - j0.09124 \text{ S}, \mathbf{Y}_{new} = 0.12651 + j(120\pi C - 0.09124),$$

$$\text{PF}_{new} = 0.9 \text{ lag}, \theta_{new} = 25.84^\circ \therefore \tan 25.84^\circ = 0.4843$$

$$= \frac{0.09124 - 120\pi C}{0.12651} \therefore$$

$$C = \boxed{79.48 \mu\text{F}}$$

## CHAPTER 11 SOLUTIONS

38.  $\mathbf{Z}_{\text{eff}} = j100 + j300 \parallel 200 = 237 \angle 54.25^\circ$ . PF =  $\cos 54.25^\circ = 0.5843$  *lagging*.

(a) Raise PF to 0.92 *lagging* with series capacitance

$$\mathbf{Z}_{\text{new}} = j100 + jX_C + j300 \parallel 200 = 138.5 + j(192.3 + X_C) \Omega$$

$$\tan^{-1} \left( \frac{192.3 + X_C}{138.5} \right) = \cos^{-1} 0.92 = 23.07^\circ$$

Solving, we find that  $X_C = -133.3 \Omega = -1/\omega C$ , so that  $C = 7.501 \mu\text{F}$

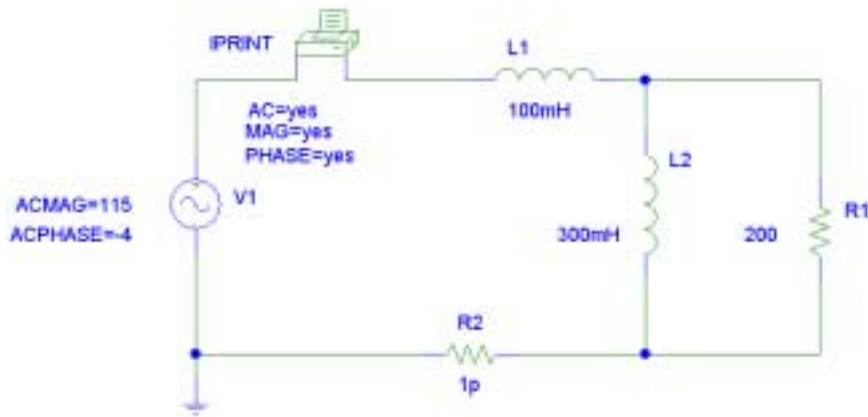
(b) Raise PF to 0.92 *lagging* with parallel capacitance

$$\mathbf{Z}_{\text{new}} = j100 \parallel jX_C + j300 \parallel 200 = \frac{-100 X_C}{j(100 + X_C)} + 138.5 + j92.31 \Omega$$

$$= 138.5 + j \left( 92.31 + \frac{100 X_C}{100 + X_C} \right) \Omega$$

$$\tan^{-1} \left( \frac{92.31 + \frac{100 X_C}{100 + X_C}}{138.5} \right) = \cos^{-1} 0.92 = 23.07^\circ$$

Solving, we find that  $X_C = -25 \Omega = -1/\omega C$ , so that  $C = 40 \mu\text{F}$



General circuit for simulations. Results agree with hand calculations

	FREQ	IM(V_PRINT1)	IP(V_PRINT1)	$\theta$	PF
With no compensation:	1.592E+02	4.853E-01	-5.825E+01	54.25°	0.5843 lag
With series compensation:	1.592E+02	7.641E-01	-2.707E+01	23.07°	0.9200 lag
With parallel compensation:	1.592E+02	7.641E-01	-2.707E+01	23.07°	0.9200 lag

## CHAPTER 11 SOLUTIONS

39.

$$(a) \quad P_{s, tot} = 20 + 25 \times 0.8 + 30 \times 0.75 = 70 \text{ kW}$$

$$(b) \quad \mathbf{I}_1 = \frac{20,000}{250} = 80 \angle 0^\circ \text{ A rms}$$

$$|\mathbf{I}_2| = 25,000 / 250 = 100 \text{ A rms}$$

$$\angle \mathbf{I}_2 = -\cos^{-1} 0.8 = -36.87^\circ \therefore \mathbf{I}_2 = 100 \angle -36.87^\circ \text{ A rms}$$

$$AP_3 = \frac{30,000}{0.75} = 40,000 \text{ VA}, \quad |\mathbf{I}_3| = \frac{40,000}{250} = 160 \text{ A rms}$$

$$\angle \mathbf{I}_3 = -\cos^{-1} 0.75 = -41.41^\circ \therefore \mathbf{I}_3 = 160 \angle -41.41^\circ \text{ A rms}$$

$$\therefore \mathbf{I}_s = 80 \angle 0^\circ + 100 \angle -36.87^\circ + 160 \angle -41.41^\circ = 325.4 \angle -30.64^\circ \text{ A rms}$$

$$\therefore AP_s = 250 \times 325.4 = 81,360 \text{ VA}$$

$$(c) \quad PF_3 = \frac{70,000}{81,360} = 0.8604 \text{ lag}$$

## CHAPTER 11 SOLUTIONS

40. 200 kW average power and 280 kVAR reactive result in a power factor of  $PF = \cos(\tan^{-1}(280/200)) = 0.5813$  *lagging*, which is pretty low.

(a)  $0.65 \text{ peak} = 0.65(200) = 130 \text{ kVAR}$

Excess =  $280 - 130 = 150 \text{ kVAR}$ , for a cost of  $(12)(0.22)(150) = \$396 / \text{year}$ .

(b) Target =  $\mathbf{S} = P + j0.65 P$

$\theta = \tan^{-1}(0.65P/P) = 33.02^\circ$ , so target  $PF = \cos \theta = 0.8385$

- (c) A single 100-kVAR increment costs \$200 to install. The excess kVAR would then be  $280 - 100 - 130 = 50 \text{ kVAR}$ , for an annual penalty of \$332. This would result in a first-year savings of \$64.

A single 200-kVAR increment costs \$395 to install, and would remove the entire excess kVAR. The savings would be \$1 (wow) in the first year, but \$396 each year thereafter.

The single 200-kVAR increment is the most economical choice.

## CHAPTER 11 SOLUTIONS

41.

$$\mathbf{Z}_{in} = -j10 + \frac{20(1+j2)}{3+j2} = 10.769 - j3.846 = 11.435 \angle -19.65^\circ \Omega$$

$$\therefore \mathbf{I}_s = \frac{100}{11.435 \angle -19.65^\circ} = 8.745 \angle 19.65^\circ$$

$$\therefore \mathbf{S}_s = -\mathbf{V}_s \mathbf{I}_s^* = -100 \times 8.745 \angle -19.65^\circ = -823.5 + j294.1 \text{ VA}$$

$$\mathbf{I}_{20} = 8.745 \angle 19.65^\circ \times \frac{10+j20}{30+j20} = 5.423 \angle 49.40^\circ$$

$$\therefore \mathbf{S}_{20} = 20 \times 5.423^2 = 588.2 + j0 \text{ VA}$$

$$\mathbf{I}_{10} = \frac{20 \times 5.423 \angle 49.40^\circ}{10+j20} = 4.851 \angle -14.04^\circ$$

$$\mathbf{S}_{10} = 10 \times 4.851^2 = 235.3 + j0 \text{ VA}$$

$$\mathbf{S}_{j20} = j20 \times 4.851^2 = j470.6 \text{ VA}$$

$$\mathbf{S}_{-j10} = -j10 \times 8.745^2 = -j764.7 \text{ VA}, \quad \Sigma = 0$$

## CHAPTER 11 SOLUTIONS

42.

$$\frac{\mathbf{V}_x - 100}{6 + j4} + \frac{\mathbf{V}_x}{-j10} + \frac{\mathbf{V}_x - j100}{5} = 0$$

$$\therefore \mathbf{V}_x \left( \frac{1}{6 + j4} + j0.1 + 0.2 \right) = \frac{100}{6 + j4} + j20$$

$$\therefore \mathbf{V}_x = 53.35 \angle 42.66^\circ \text{ V}$$

$$\therefore \mathbf{I}_1 = \frac{100 - 53.35 \angle 42.66^\circ}{6 + j4} = 9.806 \angle -64.44^\circ \text{ A}$$

$$\therefore \mathbf{S}_{1,gen} = \frac{1}{2} \times 100 \times 9.806 \angle 64.44^\circ = 211.5 + j442.3 \text{ VA}$$

$$\mathbf{S}_{6,abs} = \frac{1}{2} \times 6 \times 9.806^2 = 288.5 + j0 \text{ VA}$$

$$\mathbf{S}_{j4,abs} = \frac{1}{2} (j4) 9.806^2 = 0 + j192.3 \text{ VA}$$

$$\mathbf{I}_2 = \frac{j100 - 53.35 \angle 42.66^\circ}{5} = 14.99 \angle 121.6^\circ,$$

$$\mathbf{S}_{5abs} = \frac{1}{2} \times 5 \times 14.99^2 = 561.5 + j0 \text{ VA}$$

$$\mathbf{S}_{2,gen} = \frac{1}{2} (j100) 14.99 \angle -121.57^\circ = 638.4 - j392.3 \text{ VA}$$

$$\mathbf{S}_{-j10,abs} = \frac{1}{2} \left( \frac{53.35}{10} \right) (-j10) = 0 - j142.3 \text{ VA} = 142.3 \angle 0^\circ \text{ VA} \quad \Sigma = 0$$

## CHAPTER 11 SOLUTIONS

43.

(a) 500 VA, PF = 0.75 lead  $\therefore$

$$\mathbf{S} = 500 \angle -\cos^{-1} 0.75 = 375 - j330.7 \text{ VA}$$

(b) 500W, PF = 0.75 lead  $\therefore$

$$\mathbf{S} = 500 - \frac{500}{j.075} \sin(\cos^{-1} 0.75) = 500 - j441.0 \text{ VA}$$

(c) -500 VAR, PF = 0.75 (lead)  $\therefore \theta = -\cos^{-1} 0.75 = -41.41^\circ$

$$\therefore P = 500 / \tan 41.41^\circ = 566.9 \text{ W},$$

$$\mathbf{S} = 566.9 - j500 \text{ VA}$$

## CHAPTER 11 SOLUTIONS

44.  $\mathbf{S}_s = 1600 + j500 \text{ VA (gen)}$

(a)  $\mathbf{I}_s^* = \frac{1600 + j500}{400} = 4 + j1.25 \therefore \mathbf{I}_s = 4 - j1.25$

$$\mathbf{I}_c = \frac{400}{-j120} = j3.333 \text{ A rms} \therefore \mathbf{I}_L = \mathbf{I}_s - \mathbf{I}_c = 4 - j1.25 - j3.333$$

$$\therefore \mathbf{I}_L = 4 - j4.583 \text{ A rms} \therefore$$

$$\mathbf{S}_L = 400(4 + j4.583) = \boxed{1600 + j1833 \text{ VA}}$$

(b)  $\text{PF}_L = \cos\left(\tan^{-1} \frac{1833.3}{1600}\right) = \boxed{0.6575^+ \text{ lag}}$

(c)  $\mathbf{S}_s = 1600 + j500 = 1676 \angle 17.35^\circ \text{ VA} \therefore \text{PF}_s = \cos 17.35^\circ = \boxed{0.9545 \text{ lag}}$



## CHAPTER 11 SOLUTIONS

45.  $(\cos^{-1} 0.8 = 36.87^\circ, \cos^{-1} 0.9 = 25.84^\circ)$

(a)  $\mathbf{S}_{tot} = 1200\angle 36.87^\circ + 1600\angle 25.84^\circ + 900$   
 $= 960 + j720 + 1440 + j697.4 + 900$   
 $= 3300 + j1417.4 = 3592\angle 23.25^\circ \text{ VA}$

$$\therefore \mathbf{I}_s = \frac{3591.5}{230} = 15.62 \text{ A rms}$$

(b)  $\text{PF}_s = \cos 23.245^\circ = 0.9188$

(c)  $\mathbf{S} = 3300 + j1417 \text{ VA}$

## CHAPTER 11 SOLUTIONS

46.  $\mathbf{V} = 339 \angle -66^\circ \text{ V}$ ,  $\omega = 100\pi \text{ rad/s}$ , connected to  $\mathbf{Z} = 1000 \Omega$ .

(a)  $V_{\text{eff}} = \frac{339}{\sqrt{2}} = 239.7 \text{ V rms}$

(b)  $p_{\text{max}} = 339^2 / 1000 = 114.9 \text{ W}$

(c)  $p_{\text{min}} = 0 \text{ W}$

(d) Apparent power  $= V_{\text{eff}} I_{\text{eff}} = \left( \frac{339}{\sqrt{2}} \right) \left( \frac{339/\sqrt{2}}{1000} \right) = \frac{V_{\text{eff}}^2}{1000} = 57.46 \text{ VA}$

(e) Since the load is purely resistive, it draws zero reactive power.

(f)  $S = 57.46 \text{ VA}$

## CHAPTER 11 SOLUTIONS

47.  $\mathbf{V} = 339 \angle -66^\circ \text{ V}$ ,  $\omega = 100\pi \text{ rad/s}$  to a purely inductive load of  $150 \text{ mH}$  ( $j47.12 \Omega$ )

$$(a) \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{339 \angle -66^\circ}{j47.12} = 7.194 \angle -156^\circ \text{ A}$$

$$\text{so } \mathbf{I}_{\text{eff}} = \frac{7.194}{\sqrt{2}} = 5.087 \text{ A rms}$$

$$(b) p(t) = \frac{1}{2} V_m I_m \cos \phi + \frac{1}{2} V_m I_m \cos(2\omega t + \phi)$$

where  $\phi = \text{angle of current} - \text{angle of voltage}$

$$p_{\text{max}} = \frac{1}{2} V_m I_m \cos \phi + \frac{1}{2} V_m I_m = (1 + \cos(-90^\circ)) (339)(7.194) / 2 = 1219 \text{ W}$$

$$(c) p_{\text{min}} = \frac{1}{2} V_m I_m \cos \phi - \frac{1}{2} V_m I_m = -1219 \text{ W}$$

$$(d) \text{ apparent power} = V_{\text{eff}} I_{\text{eff}} = \frac{339}{\sqrt{2}} (5.087) = 1219 \text{ VA}$$

$$(e) \text{ reactive power} = Q = V_{\text{eff}} I_{\text{eff}} \sin(\theta - \phi) = 1219 \text{ VA}$$

$$(f) \text{ complex power} = j1219 \text{ VA}$$

## CHAPTER 11 SOLUTIONS

48.  $1 \text{ H} \rightarrow j \Omega, 4 \mu\text{F} \rightarrow -j250 \text{ k}\Omega$

$$\mathbf{Z}_{\text{eff}} = j \parallel -j250 \times 10^3 \parallel 10^3 \Omega = 1 \angle 89.99^\circ \Omega$$

$$\mathbf{V}_{10\text{k}} = \frac{(5 \angle 0)(1 \angle 89.99^\circ)}{2500 + (1 \angle 89.99^\circ)} = 0.002 \angle 89.97^\circ \text{ V}$$

(a)  $p_{\text{max}} = (0.002)^2 / 10 \times 10^3 = 400 \text{ pW}$

(b)  $0 \text{ W}$  (purely resistive elements draw no reactive power)

(c) apparent power  $= V_{\text{eff}} I_{\text{eff}} = \frac{1}{2} V_{\text{m}} I_{\text{m}} = \frac{1}{2} (0.002)^2 / 10000 = 200 \text{ pW}$

(d)  $\mathbf{I}_{\text{source}} = \frac{5 \angle 0}{2500 + 1 \angle 89.99^\circ} = 0.002 \angle -0.02292^\circ \text{ A}$

$\mathbf{S} = \frac{1}{2} V_{\text{m}} I_{\text{m}} \angle (89.99^\circ + 0.02292^\circ) = 0.005 \angle 90.01^\circ \text{ VA}$

## CHAPTER 11 SOLUTIONS

49. (a) At  $\omega = 400$  rad/s,  $1 \mu\text{F} \rightarrow -j2500 \Omega$ ,  $100 \text{ mH} \rightarrow j40 \Omega$   
 Define  $\mathbf{Z}_{\text{eff}} = -j2500 \parallel (250 + j40) = 256 \angle 3.287^\circ \Omega$

$$\mathbf{I}_S = \frac{12000 \angle 0}{20 + 256 \angle 3.287^\circ} = 43.48 \angle -3.049^\circ \text{ A rms}$$

$$\mathbf{S}_{\text{source}} = (12000)(43.48) \angle 3.049^\circ = 521.8 \angle 3.049^\circ \text{ kVA}$$

$$\mathbf{S}_{20\Omega} = (43.48)^2 (20) \angle 0 = 37.81 \angle 0 \text{ kVA}$$

$$\mathbf{V}_{\text{eff}} = \frac{(12000 \angle 0)(256 \angle 3.287^\circ)}{20 + 256 \angle 3.287^\circ} = 11130 \angle 0.2381^\circ \text{ V rms}$$

$$\mathbf{I}_{1\mu\text{F}} = \frac{\mathbf{V}_{\text{eff}}}{-j2500} = 4.452 \angle 90.24^\circ \text{ A rms}$$

$$\text{so } \mathbf{S}_{1\mu\text{F}} = (11130)(4.452) \angle -90^\circ = 49.55 \angle -90^\circ \text{ kVA}$$

$$\mathbf{V}_{100\text{mH}} = \frac{(11130 \angle 0.2381^\circ)(j40)}{250 + j40} = 1758 \angle 81.15^\circ \text{ V rms}$$

$$\mathbf{I}_{100\text{mH}} = \frac{\mathbf{V}_{100\text{mH}}}{j40} = 43.96 \angle -8.852^\circ \text{ A rms}$$

$$\text{so } \mathbf{S}_{100\mu\text{H}} = (1758)(4.43.96) \angle 90^\circ = 77.28 \angle 90^\circ \text{ kVA}$$

$$\mathbf{V}_{250\Omega} = \frac{(11130 \angle 0.2381^\circ)(250)}{250 + j40} = 10990 \angle -8.852^\circ \text{ V rms}$$

$$\text{so } \mathbf{S}_{250\Omega} = (10990)^2 / 250 = 483.1 \angle 0^\circ \text{ kVA}$$

- (b)  $37.81 \angle 0 + 49.55 \angle -90^\circ + 77.28 \angle 90^\circ + 483.1 \angle 0^\circ = 521.6 \angle 3.014^\circ \text{ kVA}$ ,  
 which is within rounding error of the complex power delivered by the source.

- (c) The apparent power of the source is 521.8 kVA. The apparent powers of the passive elements sum to  $37.81 + 49.55 + 77.28 + 483.1 = 647.7 \text{ kVA}$ , so **NO!** Phase angle is important!

$$(d) P = \mathbf{V}_{\text{eff}} \mathbf{I}_{\text{eff}} \cos (\text{ang } \mathbf{V}_S - \text{ang } \mathbf{I}_S) = (12000)(43.48) \cos (3.049^\circ) = 521 \text{ kW}$$

$$(e) Q = \mathbf{V}_{\text{eff}} \mathbf{I}_{\text{eff}} \sin (\text{ang } \mathbf{V}_S - \text{ang } \mathbf{I}_S) = (12000)(43.48) \sin (3.049^\circ) = 27.75 \text{ kVAR}$$

## CHAPTER 11 SOLUTIONS

50. (a) Peak current  $= 28\sqrt{2} = 39.6 \text{ A}$

(b)  $\theta_{\text{load}} = \cos^{-1}(0.812) = +35.71^\circ$  (since lagging PF). Assume  $\text{ang}(\mathbf{V}) = 0^\circ$ .

$$p(t) = (2300\sqrt{2})(39.60\sqrt{2})\cos(120\pi t) \cos(120\pi t - 35.71^\circ)$$

at  $t = 2.5 \text{ ms}$ , then,  $p(t) = 147.9 \text{ kW}$

(c)  $P = V_{\text{eff}} I_{\text{eff}} \cos \theta = (2300)(28) \cos(35.71^\circ) = 52.29 \text{ kW}$

(d)  $\mathbf{S} = V_{\text{eff}} I_{\text{eff}} \angle \theta = 64.4 \angle 35.71^\circ \text{ kVA}$

(e) apparent power  $= |\mathbf{S}| = 64.4 \text{ kVA}$

(f)  $|\mathbf{Z}_{\text{load}}| = |\mathbf{V}/\mathbf{I}| = 2300/28 = 82.14 \Omega$ . Thus,  $\mathbf{Z}_{\text{load}} = 82.14 \angle 35.71^\circ \Omega$

(g)  $Q = V_{\text{eff}} I_{\text{eff}} \sin \theta = 37.59 \text{ kVAR}$

## CHAPTER TWELVE SOLUTIONS

$$1. \quad V_{bc} = V_{be} + V_{ec} = 0.7 - 10 = -9.3 \text{ V}$$

$$V_{eb} = -V_{be} = -0.7 \text{ V}$$

$$V_{cb} = V_{ce} + V_{eb} = 10 - 0.7 = 9.3 \text{ V}$$

## CHAPTER TWELVE SOLUTIONS

2. (a)  $V_{gd} = V_{gs} + V_{sd} = -1 - 5 = -6 \text{ V}$

(b)  $V_{sg} = V_{sd} + V_{dg} = -4 - 2.5 = -6.5 \text{ V}$



## CHAPTER TWELVE SOLUTIONS

3. (a) positive phase sequence

$\mathbf{V}_{an} =  \mathbf{V}_p  \angle 0^\circ$	$\mathbf{V}_{dn} =  \mathbf{V}_p  \angle -180^\circ$
$\mathbf{V}_{bn} =  \mathbf{V}_p  \angle -60^\circ$	$\mathbf{V}_{en} =  \mathbf{V}_p  \angle -240^\circ$
$\mathbf{V}_{cn} =  \mathbf{V}_p  \angle -120^\circ$	$\mathbf{V}_{fn} =  \mathbf{V}_p  \angle -300^\circ$

- (b) negative phase sequence

$\mathbf{V}_{an} =  \mathbf{V}_p  \angle 0^\circ$	$\mathbf{V}_{dn} =  \mathbf{V}_p  \angle 180^\circ$
$\mathbf{V}_{bn} =  \mathbf{V}_p  \angle 60^\circ$	$\mathbf{V}_{en} =  \mathbf{V}_p  \angle 240^\circ$
$\mathbf{V}_{cn} =  \mathbf{V}_p  \angle 120^\circ$	$\mathbf{V}_{fn} =  \mathbf{V}_p  \angle 300^\circ$

## CHAPTER TWELVE SOLUTIONS

$$\begin{aligned} 4. \quad (a) \mathbf{V}_{yz} &= \mathbf{V}_{yx} + \mathbf{V}_{xz} = -110 \angle 20^\circ + 160 \angle -50^\circ \\ &= -103.4 - j37.62 + 102.8 - j122.6 = -0.6 - j160.2 \\ &= \boxed{160.2 \angle -90.21^\circ \text{ V}} \end{aligned}$$

$$\begin{aligned} (b) \mathbf{V}_{az} &= \mathbf{V}_{ay} + \mathbf{V}_{yz} = 80 \angle 130^\circ + 160.2 \angle -90.21^\circ \\ &= -51.42 + j61.28 - 0.6 - j160.2 = -52.02 - j98.92 \\ &= \boxed{111.8 \angle -117.7^\circ \text{ V}} \end{aligned}$$

$$(c) \frac{\mathbf{V}_{zx}}{\mathbf{V}_{xy}} = \frac{-160 \angle -50^\circ}{110 \angle 20^\circ} = \frac{160 \angle 130^\circ}{110 \angle 20^\circ} = \boxed{1.455 \angle 110^\circ}$$

## CHAPTER TWELVE SOLUTIONS

$$\begin{aligned} 5. \quad (a) \quad \mathbf{V}_{25} &= \mathbf{V}_{24} + \mathbf{V}_{45} &&= -80 \angle 120^\circ + 60 \angle 75^\circ \\ &&&= 40 - j69.28 + 15.53 + j57.96 = 55.53 - j11.32 \\ &&&= 56.67 \angle -11.52^\circ \text{ V} \end{aligned}$$

$$\begin{aligned} (b) \quad \mathbf{V}_{13} &= \mathbf{V}_{12} + \mathbf{V}_{25} + \mathbf{V}_{53} &&= 100 + 55.53 - j11.32 + j120 \\ &&&= 155.53 + j108.7 \\ &&&= 189.8 \angle 34.95^\circ \text{ V} \end{aligned}$$

## CHAPTER TWELVE SOLUTIONS

6.

$230/460 \text{ V rms } \bar{Z}_{AN}: \bar{S} = 10\angle 40^\circ \text{ kVA}; \bar{Z}_{NB}: 8\angle 10^\circ \text{ kVA};$

$$\bar{Z}_{AB}: 4\angle -80^\circ \text{ kVA} \quad \text{Let } \bar{V}_{AN} = 230\angle 0^\circ \text{ V} \quad \therefore \bar{S}_{AN} = \bar{V}_{AN} \bar{I}_{AN}^*, \bar{I}_{AN}^* = \frac{10,000\angle 40^\circ}{230} = 43.48\angle 40^\circ \text{ A}$$

$$\therefore \bar{I}_{AN} = 43.48\angle -40^\circ \text{ A}, \bar{S}_{AB} = \bar{V}_{AB} \bar{I}_{AB}^* \therefore \bar{I}_{AB}^* = \frac{4000\angle -80^\circ}{460} = 8.696\angle -80^\circ, \bar{I}_{AB} = 8.696\angle 80^\circ \therefore \bar{I}_{aA} = \bar{I}_{AN} + \bar{I}_{AB}$$

$$\therefore \bar{I}_{aA} = 43.48\angle 40^\circ + 8.696\angle 80^\circ = 39.85\angle -29.107^\circ \therefore I_{aA} = \boxed{39.85 \text{ A}}$$

$$\bar{I}_{NB}^* = \frac{8000\angle 10^\circ}{230} = 34.78\angle 10^\circ, \bar{I}_{NB} = 34.78\angle -10^\circ \text{ A}$$

$$\therefore \bar{I}_{bB} = -34.78\angle -10^\circ - 8.696\angle 80^\circ = 35.85\angle -175.96^\circ, \therefore I_{bB} = \boxed{35.85 \text{ A}}$$

$$\bar{I}_{nN} = -43.48\angle -40^\circ + 34.78\angle -10^\circ = 21.93\angle 87.52^\circ, I_{nN} = \boxed{21.93 \text{ A}}$$

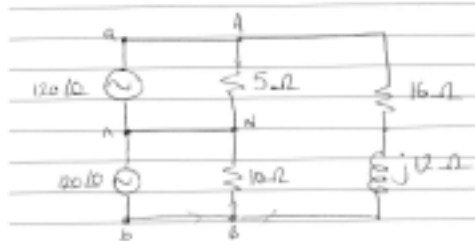
## CHAPTER TWELVE SOLUTIONS

7. (a)  $\mathbf{I}_{nN} = 0$  since the circuit is balanced.

$$\mathbf{I}_{AN} = 12 \angle 0^\circ \quad \mathbf{I}_{AB} = \frac{240 \angle 0^\circ}{16 + j12} = 12 \angle -36.9^\circ$$

$$\mathbf{I}_{aA} = \mathbf{I}_{AN} + \mathbf{I}_{AB} = 12 + 9.596 - j7.205 = \boxed{22.77 \angle -18.45^\circ \text{ A}}$$

- (b)



$$\mathbf{I}_{AN} = 24 \angle 0^\circ \text{ A}$$

$$\mathbf{I}_{BN} = -12 \angle 0^\circ \text{ A}$$

$$\mathbf{I}_{nN} = -12 \angle 0^\circ \text{ A}$$

The voltage across the  $16\text{-}\Omega$  resistor and  $j12\text{-}\Omega$  impedance has not changed, so  $\mathbf{I}_{AB}$  has not changed from above.

$$\mathbf{I}_{aA} = \mathbf{I}_{AN} + \mathbf{I}_{AB} = 24 \angle 0^\circ + 12 \angle -36.9^\circ = \boxed{34.36 \angle -12.10^\circ \text{ A}}$$

$$\mathbf{I}_{bB} = \mathbf{I}_{BN} - \mathbf{I}_{AB} = -12 \angle 0^\circ - 12 \angle -36.9^\circ = \boxed{7.595 \angle -108.5^\circ \text{ A}}$$

$$\mathbf{I}_{nN} = \mathbf{I}_{BN} - \mathbf{I}_{AN} = -12 - 24 = \boxed{36 \angle 180^\circ \text{ A}}$$

## CHAPTER TWELVE SOLUTIONS

8.

$$(a) \quad \Delta = \begin{vmatrix} 21+j3 & -10 & -10-j3 \\ -10 & 19+j2 & -8-j2 \\ -10-j3 & -8-j2 & 36+j5 \end{vmatrix} = (21+j3)(674+j167-60-j32)$$

$$+10(-360-j50-74-j44)-(10+j3)(80+j20+184+j77)$$

$$\therefore \Delta = 5800 + j1995 = 6127 \angle 18.805^\circ$$

$$\begin{vmatrix} 720 & -10 & -10-j3 \\ 720 & 19+j2 & -8-j2 \\ 0 & -8-j2 & 36+j5 \end{vmatrix} = 720(614 + j135 + 434 + j94) = 720 \times 1072.7 \angle 12.326^\circ$$

$$\therefore \bar{I}_{aA} = \frac{720 \times 1072.7 \angle 12.326^\circ}{6127 \angle 18.805^\circ} = 126.06 \angle -6.479^\circ \text{ A}$$

$$(b) \quad \begin{vmatrix} 21+j3 & 720 & -10-j3 \\ -10 & 720 & -8-j2 \\ -10-j3 & 0 & 36+j5 \end{vmatrix} = 720(1084 + j247) \therefore \bar{I}_{bb} = \frac{720(1084 + j247)}{6127 \angle 18.805^\circ} = 130.65 \angle -5.968^\circ \text{ A}$$

$$\therefore I_{nN} = 130.65 \angle -5.968^\circ - 126.06 \angle -6.479^\circ = 4.730 \angle 7.760^\circ \text{ A}$$

$$(c) \quad P_{\omega, tot} = 126.06^2 \times 1 + 130.65^2 \times 1 + 4.730^2 \times 10 = 15.891 + 17.069 + 0.224 = 33.18 \text{ kW}$$

$$(d) \quad P_{gen, tot} = 720 \times 126.06 \cos 6.479^\circ + 720 \times 130.65 \cos 5.968^\circ = 90.18 + 93.56 = 183.74 \text{ kW}$$

## CHAPTER TWELVE SOLUTIONS

9.  $\bar{V}_{AN} = 220 \text{ Vrms}, 60 \text{ Hz}$

(a)  $\text{PF} = 1 \therefore \bar{I}_{AN} = \frac{220 \angle 0^\circ}{5 + j2} = 40.85 \angle -21.80^\circ \text{ A}; \bar{I}_{AB} = j377C \times 440$

$$\therefore \bar{I}_{aA} = 40.85 \cos 21.80^\circ + j(377C440 - 40.85 \sin 21.80^\circ)$$

$$\therefore C = \frac{40.85 \sin 21.80^\circ}{377 \times 440} = 91.47 \mu\text{F}$$

(b)  $\bar{I}_{AB} = 377 \times 91.47 \times 10^{-6} \times 440 = 15.172 \text{ A} \therefore \text{VA} = 440 \times 15.172 = 6.676 \text{ kVA}$

## CHAPTER TWELVE SOLUTIONS

$$10. \quad (a) \mathbf{I}_{aA} = \mathbf{I}_{AN} + \mathbf{I}_{AB} = \frac{200\angle 0}{12 + j3} + \frac{400\angle 0}{R_{AB}} = 15.69 - j3.922 + \frac{400}{R_{AB}}$$

Since we know that  $|\mathbf{I}_{aA}| = 30 \text{ A rms} = 42.43 \text{ A}$ ,

$$42.43 = \sqrt{\left(15.69 + \frac{400}{R_{AB}}\right)^2 + 3.922^2}$$

or  $R_{AB} = \boxed{15.06 \, \Omega}$

$$(b) \mathbf{I}_{aA} = \mathbf{I}_{AN} + \mathbf{I}_{AB} = \frac{200\angle 0}{12 + j3} + \frac{400\angle 0}{-jX_{AB}} = 15.69 - j3.922 + \frac{j400}{X_{AB}}$$

In order for the angle of  $\mathbf{I}_{aA}$  to be zero,  $\frac{400}{X_{AB}} = 3.922$ , so that  $X_{AB} = \boxed{102 \, \Omega \text{ capacitive.}}$



## CHAPTER TWELVE SOLUTIONS

11. + seq.  $\bar{V}_{BC} = 120\angle 60^\circ$  V rms,  $R_w = 0.6\Omega$   $P_{load} = 5\text{ kVA}$ , 0.6 lag

(a)  $\bar{V}_{AN} = \frac{120}{\sqrt{3}}\angle 150^\circ$  V  $\therefore \bar{S}_{AN} = \frac{5000}{3} \times 0.8 + j \frac{5000}{3} \times 0.6$

$$\therefore \bar{S}_{AN} = \frac{120}{\sqrt{3}}\angle 150^\circ \bar{I}_{aA}^* \therefore \bar{I}_{aA}^* = 24.06\angle -113.13^\circ \text{ A}$$

$$\therefore \bar{I}_{aA} = 24.06\angle 113.13^\circ \therefore P_{wire} = 3 \times 24.06^2 \times 0.6 = \boxed{1041.7 \text{ W}}$$

(b)  $\bar{V}_{aA} = 0.6 \times 24.06\angle 113.13^\circ = 14.434\angle 113.13^\circ$  V

$$\therefore \bar{V}_{an} = \bar{V}_{aA} + \bar{V}_{AN} = 14.434\angle 113.13^\circ + \frac{120}{\sqrt{3}}\angle 158^\circ = \boxed{81.29\angle 143.88^\circ \text{ V}}$$

## CHAPTER TWELVE SOLUTIONS

12.  $\uparrow \bar{V}_{an} = 2300\angle 0^\circ \text{ V}_{\text{rms}}, R_w = 2 \Omega, +seq., \bar{S}_{tot} = 100 + j30 \text{ kVA}$

(a)  $\frac{1}{3}(100,000 + j30,000) = 2300 I_{aA}^* \therefore \bar{I}_{aA} = 15.131\angle -16.699^\circ \text{ A}$

(b)  $\bar{V}_{AN} = 2300 - 2 \times 15.131\angle -16.699^\circ = 2271\angle 0.2194^\circ \text{ V}$

(c)  $\bar{Z}_p = \bar{V}_{AN} / \bar{I}_{aA} = \frac{2271\angle 0.2194^\circ}{15.131\angle -16.699^\circ} = 143.60 + j43.67 \Omega$

(d)  $\text{trans. eff.} = \frac{143.60}{145.60} = 0.9863, \text{ or } 98.63\%$

## CHAPTER TWELVE SOLUTIONS

13.  $\uparrow \bar{Z}_p = 12 + j5\Omega$ ,  $\bar{I}_{bB} = 20\angle 0^\circ$  A rms, +seq., PF = 0.935

(a)  $\theta = \cos^{-1} 0.935 = 20.77^\circ \therefore \frac{5}{12 + R_w} = \tan 20.77^\circ$ ,  $R_w = 1.1821\Omega$

(b)  $\bar{V}_{BN} = \bar{I}_{bB} \bar{Z}_p = 20(12 + j5) = 240 + j100$  V  $\therefore \bar{V}_{bn} = 20(13.1821 + j5) = 281.97\angle 20.77^\circ$  V

(c)  $\bar{V}_{AB} = \sqrt{3} |\bar{V}_{BN}| \angle \angle V_{BN} + 150^\circ = 450.3\angle 172.62^\circ$  V

(d)  $\bar{S}_{source} = 3 \bar{V}_{Bn} \bar{I}_{bB}^* = 3 \times 281.97 \angle -20.77^\circ (20)$   
 $= 15.819 - j6.000$  kVA

## CHAPTER TWELVE SOLUTIONS

14.  $125 \text{ mH} \rightarrow j(2\pi)(60)(0.125) = j47.12 \, \Omega$   $75 \, \Omega \rightarrow 75 \, \Omega$   
 $55 \, \mu\text{F} \rightarrow -j/(2\pi)(60)(55 \times 10^{-6}) = -j48.23 \, \Omega$

The per-phase current magnitude  $|\mathbf{I}|$  is then  $\mathbf{I} = \frac{125}{\sqrt{75^2 + (47.12 - 48.23)^2}} = 1.667 \text{ A.}$

The power in each phase  $= (1.667)^2 (75) = 208.4 \text{ W}$ , so that the total power taken by the load is  $3(208.4) = 625.2 \text{ W.}$

The power factor of the load is  $\cos\left(\frac{47.12 - 48.23}{75}\right) = 1.000$

*This isn't surprising, as the impedance of the inductor and the impedance of the capacitor essentially cancel each other out as they have approximately the same magnitude but opposite sign and are connected in series.*

## CHAPTER TWELVE SOLUTIONS

15.

↑ Bal., + seq.  $Z_{AN} = 8 + j6 \Omega$ ,  $\bar{Z}_{BN} = 12 - j16 \Omega$ ,  $\bar{Z}_{CN} = 5 + j0$ ,  $\bar{V}_{AN} = 120 \angle 0^\circ \text{ V rms}$

$$R_w = 0.5 \Omega \text{ (a)} - \bar{I}_{nN} = \frac{120 \angle 0^\circ}{8.5 + j6} + \frac{120 \angle -120^\circ}{12.5 - j16} + \frac{120 \angle 120^\circ}{5.5} = 6.803 \angle 83.86^\circ \text{ A}$$

$$\therefore \bar{I}_{nN} = \boxed{6.803 \angle -96.14^\circ \text{ A rms}}$$

## CHAPTER TWELVE SOLUTIONS

16. Working on a per-phase basis, the line current magnitude is simply

$$|\mathbf{I}| = \frac{40}{\sqrt{(R_w + 5)^2 + 10^2}}$$

(a)  $R_w = 0$

Then  $|\mathbf{I}| = \frac{40}{\sqrt{25 + 10^2}} = 3.578 \text{ A}$ , and the power delivered to each phase of the load is  $(3.578)^2(5) = 64.01 \text{ W}$ . The total power of the load is therefore  $3(64.01) = 192.0 \text{ W}$ .

(b)  $R_w = 3 \Omega$

Then  $|\mathbf{I}| = \frac{40}{\sqrt{64 + 10^2}} = 3.123 \text{ A}$ , and the power delivered to each phase of the load is  $(3.123)^2(5) = 48.77 \text{ W}$ . The total power of the load is therefore  $3(48.77) = 146.3 \text{ W}$ .

## CHAPTER TWELVE SOLUTIONS

17.  $\uparrow \bar{Z}_p = 75\angle 25^\circ \Omega \parallel 25\mu\text{F}, \bar{V}_{an} = 240\angle 0^\circ \text{ V rms}, 60\text{ Hz}, R_w = 2\Omega$

(a)  $\bar{Z}_{cap} = -j \frac{10^6}{377 \times 25} = -j 106.10\Omega \therefore \bar{Z}_p = \frac{75\angle 25^\circ (-j106.10)}{75\angle 25^\circ - j106.10} = 75.34 - j23.63\Omega$

$\therefore \bar{Z}_{p+w} = 77.34 - j23.63 \therefore \bar{I}_{aA} = \frac{240}{77.34 - j23.63} = 2.968\angle 16.989^\circ \text{ A}$

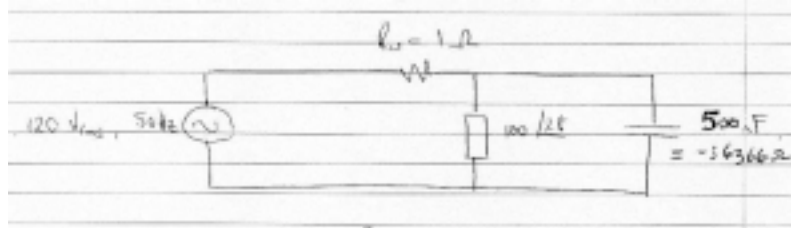
(b)  $P_w = 3(2.968)^2 \times 2 = 52.84 \text{ W}$

(c)  $P_{load} = 3(2.968)^2 75.34 = 1990.6 \text{ W}$

(d)  $\text{PF}_{source} = \cos 16.989^\circ = 0.9564 \text{ lead}$

## CHAPTER TWELVE SOLUTIONS

18. Working on a per-phase basis and noting that the capacitor corresponds to a  $-j6366\text{-}\Omega$  impedance,

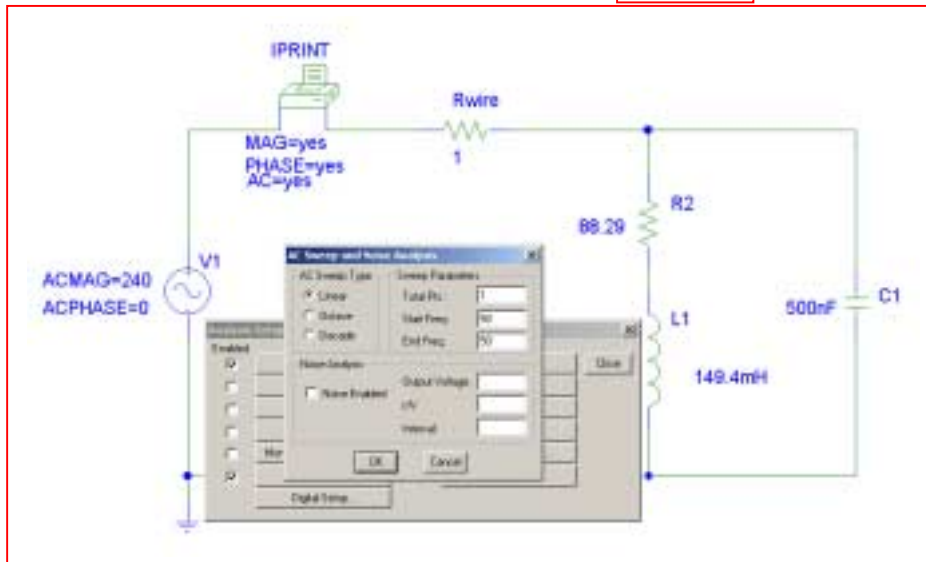


$-j6366 \parallel 100 \angle 28^\circ = 89.59 + j46.04 \text{ }\Omega$  so that the current flowing through the combined load is

$$|I| = \frac{240}{\sqrt{90.59^2 + 46.04^2}} = 2.362 \text{ A rms}$$

The power in each phase is  $(2.362)^2 (90.59) = 505.4 \text{ W}$ , so that the power delivered to the total load is  $3(505.4) = 1.516 \text{ kW}$ .

The power lost in the wiring is  $(3)(2.362)^2 (1) = 16.74 \text{ W}$ .



Simulation Result:

FREQ	IM(V_PRINT1)	IP(V_PRINT1)
5.000E+01	1.181E+00	-2.694E+01



## CHAPTER TWELVE SOLUTIONS

19.  $\uparrow$  Bal.,  $R_w = 0$ ,  $\bar{Z}_p = 10 + j5 \Omega$ ,  $f = 60$  Hz

(a)  $10 + j5 = 11.180 \angle 26.57^\circ \therefore \text{PF} = \cos 26.57^\circ = 0.8944$

(b)  $\text{PF} = 0.93$  lag,  $\theta = 21.57^\circ$ ,  $\bar{Y}_p = \frac{1}{11.180 \angle 26.57^\circ} = 0.08 - j0.04 \text{ S}$   
 $\bar{Y}_{p'} = 0.08 + j(377C - 0.04) \therefore \frac{377C - 0.04}{0.08} = -\tan 21.57^\circ = -0.3952$   
 $\therefore 377C = 0.04 - 0.08 \times 0.3952 = 0.00838 \therefore C = 22.23 \mu\text{F}$

(c)  $V_{L,load} = 440 \text{ V rms}$ ,  $\bar{Z}_c = \frac{-j10^6}{120\pi 22.23} = -j119.30 \Omega$ ,  $I_c = \frac{440/\sqrt{3}}{119.30} = 2.129 \text{ A}$   
 $\therefore \text{VAR} = 2.129 \times \frac{440}{\sqrt{3}} = 540.9 \text{ VAR (cap.)}$

## CHAPTER TWELVE SOLUTIONS

20. Working from the single-phase equivalent,

$$\mathbf{V}_{\text{an rms}} = \frac{1}{\sqrt{3}} \left( \frac{115 \angle 0^\circ}{\sqrt{2}} \right) = 46.9 \angle 0^\circ \text{ V rms}$$

$1.5 \text{ H} \rightarrow j565 \Omega$ ,  $100 \mu\text{F} \rightarrow -j26.5 \Omega$  and  $1 \text{ k}\Omega \rightarrow 1 \text{ k}\Omega$ .

These three impedances appear in parallel, with a combined value of  $27.8 \angle -88.4^\circ \Omega$ .

Thus,  $|\mathbf{I}_{\text{rms}}| = 46.9 / 27.8 = 1.69 \text{ A rms}$

$\mathbf{Z}_{\text{load}} = 27.8 \angle 88.4^\circ = 0.776 - j 27.8 \Omega$ , so  $P_{\text{load}} = (3)(1.69)^2 (0.776) = 2.22 \text{ W}$ .

## CHAPTER TWELVE SOLUTIONS

21.

$$R_w = 0, \bar{V}_{an} = 200\angle 60^\circ \text{ V rms. } S_p = 2 - j1 \text{ kVA + seq.}$$

$$(a) \quad \bar{V}_{bc} = 220\sqrt{3}\angle -30^\circ = 346.4\angle -30^\circ \text{ V}$$

$$(b) \quad \bar{S}_{BC} = 2000 - j1000 = \bar{V}_{BC} \bar{I}_{BC}^* = 346.4\angle -30^\circ \bar{I}_{BC}^* \\ \therefore \bar{I}_{BC}^* = 6.455\angle 3.435^\circ, \bar{I}_{BC} = 6.455\angle -3.435^\circ$$

$$\therefore \bar{Z}_p = \frac{200\sqrt{3}\angle -30^\circ}{6.455\angle -3.435^\circ} = 53.67\angle -26.57^\circ = 48 - j24 \, \Omega$$

$$(c) \quad \bar{I}_{aA} = \bar{I}_{AB} - \bar{I}_{CA} = 6.455\angle 120^\circ - 3.43^\circ - 6.455\angle -120^\circ - 3.43^\circ = 11.180\angle 86.57^\circ \text{ A rms}$$

## CHAPTER TWELVE SOLUTIONS

22.  $\uparrow 15\text{kVA}$ ,  $0.8\text{lag}$ ,  $+\text{seq.}$ ,  $\bar{V}_{BC} = 180\angle 30^\circ \text{ V rms}$ ,  $R_w = 0.75\Omega$

(a)  $\bar{V}_{BC} = 180\angle 30^\circ \therefore \bar{V}_{AB} = 180\angle 150^\circ \text{ V}$ ,  $\bar{S}_p = 5000\angle \cos^{-1} 0.8 = 5000\angle 36.87^\circ = 180\angle 30^\circ \bar{I}_{BC}^*$   
 $\therefore \bar{I}_{BC} = 27.78\angle -6.87^\circ$  and  $\bar{I}_{AB} = 27.78\angle 113.13^\circ \text{ A} \therefore \bar{I}_{bB} = \bar{I}_{BC} - \bar{I}_{AB}$   
 $\therefore \bar{I}_{bB} = 27.78(1\angle -6.87^\circ - 1\angle 113.13^\circ) = 48.11\angle -36.87^\circ \text{ A} \therefore \bar{V}_{bC} = 0.75(\bar{I}_{bB} - \bar{I}_{cC})$   
 $\therefore \bar{V}_{bC} = 0.75 \times 48.11(1\angle -36.87^\circ - 1\angle -156.87^\circ) + 180\angle 30^\circ = \boxed{233.0\angle 20.74^\circ \text{ V}}$

(b)  $P_{\text{wire}} = 3 \times 48.11^2 \times 0.75 = 5208 \text{ W}$   
 $\bar{S}_{\text{gen}} = 5208 + 15,000\angle 36.87^\circ = \boxed{17.208 + j9.000 \text{ kVA}}$

## CHAPTER TWELVE SOLUTIONS

23.  $\uparrow$  Bal.,  $\bar{S}_L = 3 + j1.8 \text{ kVA}$ ,  $\bar{S}_{gen} = 3.45 + j1.8 \text{ kVA}$ ,  $R_w = 5 \Omega$

(a)  $P_w = 450 \text{ W} \therefore \frac{1}{3} \times 450 = I_{aA}^2 \times 5 \therefore I_{aA} = \boxed{5.477 \text{ A rms}}$

(b)  $I_{AB} = \frac{1}{\sqrt{3}} \times 5.477 = \boxed{3.162 \text{ A rms}}$

(c) Assume  $\bar{I}_{AB} = 3.162 \angle 0^\circ$  and +seq.  $\therefore \frac{1}{3}(3000 + j1800) = \bar{V}_{AB} \bar{I}_{AB}^* = \bar{V}_{AB} (3.162 \angle 0^\circ)$

$$\therefore \bar{V}_{AB} = 368.8 \angle 30.96^\circ \text{ V} \therefore \bar{V}_{an} = \bar{V}_{aA} + \bar{V}_{AB} - \bar{V}_{bB} + \bar{V}_{bn}$$

$$\bar{V}_{aA} = 5 \bar{I}_{aA} = 5 \times 5.477 \angle -30^\circ = 27.39 \angle -30^\circ, \bar{V}_{bB} = 27.39 \angle -150^\circ$$

$$\therefore \bar{V}_{an} = 27.39 \angle -30^\circ - 27.39 \angle -150^\circ + 368.8 \angle 30.96^\circ + V_{an} (1 \angle -120^\circ)$$

$$\therefore \bar{V}_{an} = \frac{27.39 \angle -30^\circ - 27.39 \angle -150^\circ + 368.8 \angle 30.96^\circ}{1 - 1 \angle -120^\circ} = 236.8 \angle -2.447^\circ \therefore V_{an} = \boxed{236.8 \text{ V rms}}$$

## CHAPTER TWELVE SOLUTIONS

24. If a total of 240 W is lost in the three wires marked  $R_w$ , then 80 W is lost in each 2.3- $\Omega$  segment. Thus, the line current is  $\sqrt{\frac{80}{2.3}} = 5.898$  A rms. Since this is a D-connected load, the phase current is  $1/\sqrt{3}$  times the line current, or 3.405 A rms.

In order to determine the phase voltage of the source, we note that

$$P_{\text{total}} = \sqrt{3} |\mathbf{V}_{\text{line}}| \cdot |\mathbf{I}_{\text{line}}| \cdot \text{PF} = \sqrt{3} |\mathbf{V}_{\text{line}}| (5.898) \left( \frac{\sqrt{2}}{2} \right) = 1800$$

$$\text{where } |\mathbf{V}_{\text{line}}| = \frac{(1800)(2)}{\sqrt{2} \sqrt{3} (5.898)} = 249.2 \text{ V}$$

This is the voltage at the load, so we need to add the voltage lost across the wire, which

$$\text{(taking the load voltage as the reference phase) is } \left[ 5.898 \angle -\cos^{-1} \left( \frac{1}{\sqrt{2}} \right) \right] (R_w)$$

$= 13.57 \angle -45^\circ$  V. Thus, the line voltage magnitude of the source is

$$|249.2 \angle 0^\circ + 13.57 \angle -45^\circ| = 259.0 \text{ V rms.}$$

## CHAPTER TWELVE SOLUTIONS

25. Bal., +seq.

$$(a) \quad \bar{V}_{an} = 120 \angle 0^\circ \therefore \bar{V}_{ab} = 120\sqrt{3} \angle 30^\circ, \text{ etc.}, \bar{I}_{AB} = \frac{120\sqrt{3} \angle 30^\circ}{10} = 20.78 \angle 30^\circ \text{ A}$$

$$\bar{I}_{BC} = \frac{120\sqrt{3} \angle -90^\circ}{j5} = -41.57 \text{ A}; \bar{I}_{CA} = \frac{120\sqrt{3} \angle 150^\circ}{-j10} = 20.78 \angle -120^\circ \text{ A}$$

$$\bar{I}_{aA} = \bar{I}_{AB} - \bar{I}_{CA} = 20.78(1 \angle 30^\circ - 1 \angle -120^\circ) = 40.15 \angle 45^\circ \text{ A rms}$$

$$(b) \quad \bar{I}_{bB} = -41.57 - 20.78 \angle 30^\circ = 60.47 \angle -170.10^\circ \text{ A rms}$$

$$(c) \quad \bar{I}_{cC} = 20.78 \angle -120^\circ + 41.57 = 36.00 \angle -30^\circ \text{ A rms}$$

$$(d) \quad \bar{S}_{tot} = \bar{V}_{AB} \bar{I}_{AB}^* + \bar{V}_{BC} \bar{I}_{BC}^* + \bar{V}_{CA} \bar{I}_{CA}^* = 120\sqrt{3} \angle 30^\circ \times 20.78 \angle -30^\circ + 120\sqrt{3} \angle -90^\circ (-41.57) + 120\sqrt{3} \angle 150^\circ \times 20.78 \angle 120^\circ = 4320 + j0 + 0 + j8640 + 0 - j4320 = 4320 + j4320 \text{ VA}$$

## CHAPTER TWELVE SOLUTIONS

$$26. \quad \mathbf{I}_{AB} = \frac{200\angle 0}{10 \parallel j30} = \frac{200\angle 0}{9.49\angle 18.4^\circ} = 21.1\angle -18.4^\circ \text{ A}$$

$$|\mathbf{I}_A| = \sqrt{3} \mathbf{I}_{AB} = 36.5 \text{ A}$$

$$\text{The power supplied by the source} = (3) |\mathbf{I}_A|^2 (0.2) + (3) (200)^2 / 10 = 12.8 \text{ kW}$$

Define transmission efficiency as  $\eta = 100 \times P_{\text{load}} / P_{\text{source}}$ . Then  $\eta = 93.8\%$ .

$\mathbf{I}_A$  leads  $\mathbf{I}_{AB}$  by  $30^\circ$ , so that  $\mathbf{I}_A = 36.5 \angle 11.6^\circ$ .

$$\mathbf{V}_{R_w} = (0.2)(36.5 \angle 11.6^\circ) = 7.3 \angle 11.6^\circ \text{ V}$$

With  $\mathbf{V}_{AN} = \frac{200}{\sqrt{3}} \angle 30^\circ$ , and noting that  $\mathbf{V}_{an} = \mathbf{V}_{AN} + \mathbf{V}_{R_w} = 122 \angle 28.9^\circ$ , we may now

compute the power factor of the source as

$$\text{PF} = \cos (\text{ang}(\mathbf{V}_{an}) - \text{ang}(\mathbf{I}_A)) = \cos (28.9^\circ - 11.6^\circ) = 0.955.$$



## CHAPTER TWELVE SOLUTIONS

27.  $\uparrow$  Bal.,  $\bar{V}_{an} = 140\angle 0^\circ$  V<sub>rms</sub>, +seq.,  $R_w = 0$ ,  $\bar{S}_L = 15 + j9$  kVA

(a)  $\bar{V}_{ab} = \bar{V}_{AB} = \sqrt{3} \ 140\angle 30^\circ = 242.5\angle 30^\circ$  V

(b)  $\bar{V}_{AB} \bar{I}_{AB}^* = 5000 + j3000 = 242.5\angle 30^\circ \bar{I}_{AB}^* \therefore \bar{I}_{AB} = 24.05\angle -0.9638^\circ$  A rms

(c)  $\bar{I}_{aA} = \bar{I}_{AB} - \bar{I}_{CA} = 24.05\angle -0.9638^\circ - 24.05\angle 119.03^\circ = 41.65\angle -30.96^\circ$  A rms

## CHAPTER TWELVE SOLUTIONS

28.  $15 \text{ mH} \rightarrow j5.65 \, \Omega$ ,  $0.25 \text{ mF} \rightarrow -j10.6 \, \Omega$

$$\mathbf{V}_{AB} = 120\sqrt{3} \angle 30^\circ \text{ V}$$

$$\mathbf{V}_{BC} = 120\sqrt{3} \angle -90^\circ \text{ V}$$

$$\mathbf{V}_{CA} = 120\sqrt{3} \angle -210^\circ \text{ V}$$

Defining three clockwise mesh currents  $\mathbf{I}_1$ ,  $\mathbf{I}_2$  and  $\mathbf{I}_3$  corresponding to sources  $\mathbf{V}_{AB}$ ,  $\mathbf{V}_{BC}$  and  $\mathbf{V}_{CA}$ , respectively, we may write:

$$\mathbf{V}_{AB} = (10 + j5.65) \mathbf{I}_1 - 10 \mathbf{I}_2 + j5.65 \mathbf{I}_3 \quad [1]$$

$$\mathbf{V}_{BC} = -10 \mathbf{I}_1 + (10 - j10.6) \mathbf{I}_2 + j10.6 \mathbf{I}_3 \quad [2]$$

$$\mathbf{V}_{CA} = -j5.65 \mathbf{I}_1 + j10.6 \mathbf{I}_2 + (j5.65 - j10.6) \mathbf{I}_3 \quad [3]$$

Solving using MATLAB or a scientific calculator, we find that

$$\mathbf{I}_1 = 53.23 \angle -5.873^\circ \text{ A}, \mathbf{I}_2 = 40.55 \angle 20.31^\circ \text{ A}, \text{ and } \mathbf{I}_3 = 0$$

(a)  $\mathbf{V}_{AN} = j5.65(\mathbf{I}_1 - \mathbf{I}_3) = 300.7 \angle 84.13^\circ \text{ V},$

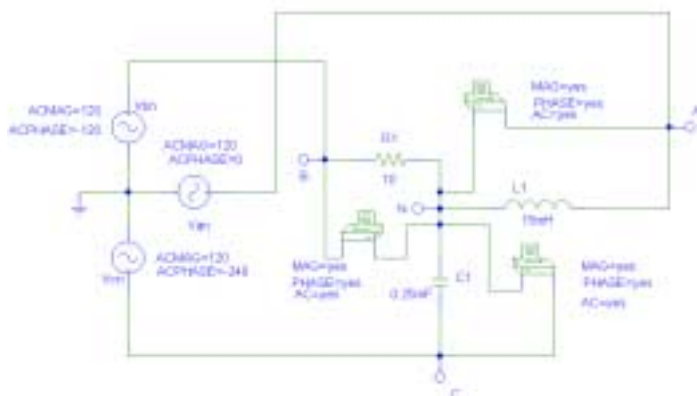
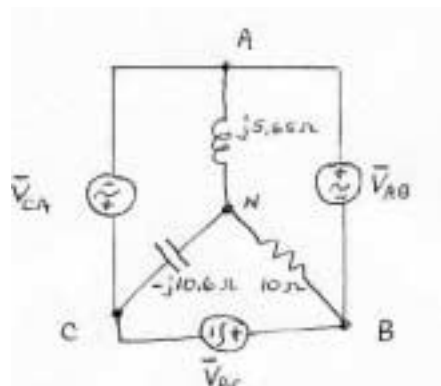
so  $V_{AN} = 300.7 \text{ V}$

(b)  $\mathbf{V}_{BN} = 10(\mathbf{I}_2 - \mathbf{I}_1) = 245.7 \angle 127.4^\circ \text{ V},$

so  $V_{BN} = 245.7 \text{ V}$

(c)  $\mathbf{V}_{CN} = -j10.6(-\mathbf{I}_2) = 429.8 \angle 110.3^\circ \text{ V},$

so  $V_{CN} = 429.8 \text{ V}$



### PSpice Simulation Results (agree with hand calculations)

FREQ	VM(A,N)	VP(A,N)
6.000E+01	3.007E+02	8.410E+01

FREQ	VM(B,N)	VP(B,N)
6.000E+01	2.456E+02	1.274E+02

FREQ	VM(C,N)	VP(C,N)
6.000E+01	4.297E+02	1.103E+02

## CHAPTER TWELVE SOLUTIONS

29.  $\uparrow R_{line} = 1\Omega$

(a)

$$120\sqrt{3} = 207.8 \bar{I}_1 = \frac{\begin{vmatrix} 207.8\angle 30^\circ & -1 & -j10 \\ 207.8\angle -90^\circ & 2+j5 & -j5 \\ 0 & -j5 & 10-j5 \end{vmatrix}}{\begin{vmatrix} 12 & -1 & -j10 \\ -1 & 2+j5 & -j5 \\ -10 & -j5 & 10-j5 \end{vmatrix}} = \frac{207.8 \begin{vmatrix} 1\angle 30^\circ & -1 & -10 \\ -j1 & 2+j5 & -j5 \\ 0 & -j5 & 10-j5 \end{vmatrix}}{12(70+j40) + (-10-j45) - 10(20+j55)}$$

$$\therefore \bar{I}_1 = \frac{207.8[1\angle 30^\circ(70+j40) + j1(-10-j45)]}{630-j115} = \frac{21.690\angle 34.86^\circ}{630-j115} = \boxed{33.87\angle 45.20^\circ = \bar{I}_{aA}}$$

$$\begin{aligned} \therefore \bar{I}_2 &= \frac{\begin{vmatrix} 12 & 1\angle 30^\circ & -10 \\ -1 & -j1 & -j5 \\ -10 & 0 & 10-j5 \end{vmatrix} 207.8}{630-j115} = \frac{207.8[-1\angle 30^\circ(-10-j45) - j1(20-j60)]}{630-j115} \\ &= \frac{16,136\angle 162.01^\circ}{630-j115} = 25.20\angle 172.36^\circ \text{ A} \end{aligned}$$

(b)  $\therefore \bar{I}_{cC} = \boxed{25.20\angle -7.641^\circ \text{ A}}$

(c)  $\therefore \bar{I}_{bB} = -\bar{I}_{aA} - \bar{I}_{cC} = -33.87\angle 45.20^\circ - 25.20\angle -7.641^\circ = \boxed{53.03\angle -157.05^\circ \text{ A rms}}$

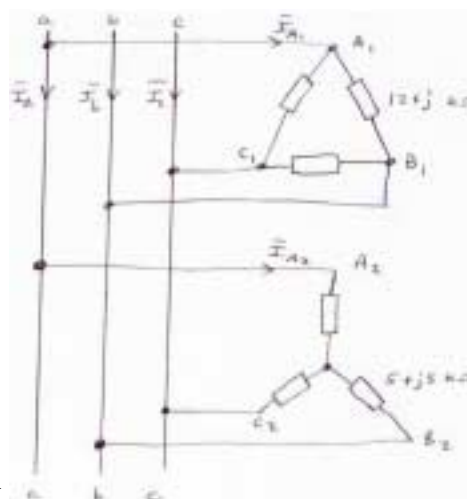
(d)  $\begin{aligned} \bar{S} &= 120\sqrt{3} \angle 30^\circ (33.87\angle -45.20^\circ) + 120\sqrt{3} \angle 90^\circ (25.20\angle 7.641^\circ) \\ &= 6793 - j1846.1 - 696.3 + j5190.4 = \boxed{6096 + j3344 \text{ VA}} \end{aligned}$

## CHAPTER TWELVE SOLUTIONS

30.  $|\mathbf{V}_{\text{line}}| = 240 \text{ V}$ . Set  $\mathbf{V}_{ab} = 240\angle 0^\circ \text{ V}$ . Then  $\mathbf{V}_{an} = \frac{240}{\sqrt{3}}\angle -30^\circ$ .

$$\mathbf{I}_{A2} = \frac{\frac{240}{\sqrt{3}}\angle -30^\circ}{5 + j3} = 23.8\angle -61.0^\circ \text{ A}$$

$$\mathbf{I}_{A1B1} = \frac{\frac{240}{\sqrt{3}}\angle 0^\circ}{(12 + j)\times 10^3} = 20.0\angle -4.76^\circ \text{ mA}$$



$\mathbf{I}_{\text{phase}}$  leads  $\mathbf{I}_{\text{line}}$  by  $30^\circ$ , so

$$\mathbf{I}_{A1} = 20\sqrt{3}\angle -34.8^\circ \text{ mA} = 34.6\angle -34.8^\circ \text{ mA}$$

$$\mathbf{I}_a = \mathbf{I}_{A1} + \mathbf{I}_{A2} = 11.5 - j20.8 + 28.4 - j19.7 \text{ mA} = 56.9\angle -45.4^\circ \text{ mA}$$

$$\text{The power factor at the source} = \cos(45.4^\circ - 30^\circ) = 0.964 \text{ lagging.}$$

$$\text{The power taken by the load} = (3)(20\times 10^{-3})^2 (12\times 10^3) + (3)(23.8\times 10^{-3})^2 (5000) = 22.9 \text{ W.}$$

## CHAPTER TWELVE SOLUTIONS

31. Define  $\mathbf{I}$  flowing from the '+' terminal of the source. Then,

$$\mathbf{I} = \frac{200\angle 0}{10 + (j10 \parallel 20)} = \frac{200\angle 0}{16.12\angle 29.74^\circ} = 12.41\angle -29.74^\circ$$

(a)  $\mathbf{V}_{xy} = 10 \mathbf{I} = 124.1 \angle -29.74^\circ \text{ V}$ . Thus,  $P_{xy} = (12.41)(124.1) = 1.54 \text{ kW}$

(b)  $P_{xz} = (200)(12.41) \cos (29.74^\circ) = 2.155 \text{ kW}$

(c)  $\mathbf{V}_{yz} = 200 \angle 0 - 124.1 \angle -29.74^\circ = 110.9 \angle 33.72^\circ \text{ V}$   
Thus,  $P_{yz} = (110.9)(12.41) \cos (33.72^\circ + 29.74^\circ) = 614.9 \text{ W}$

No reversal of meter leads is required for any of the above measurements.

## CHAPTER TWELVE SOLUTIONS

32.  $1\text{ H} \rightarrow j377\ \Omega$ ,  $25\ \mu\text{F} \rightarrow -j106\ \Omega$

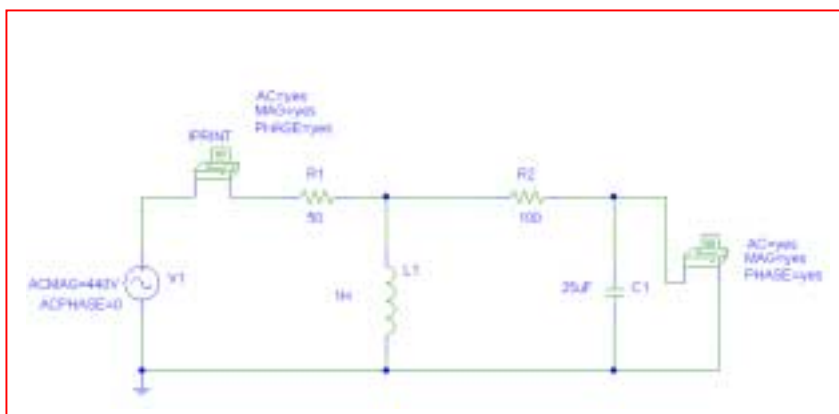
$$\mathbf{I}_1 = \frac{440\angle 0}{50 + [j377/(100-j106)]} = 1.86\angle 21^\circ\text{ A}$$

$$\mathbf{I}_C = \mathbf{I} \frac{j377}{j377 + 100 - j106} = 2.43\angle 41.3^\circ\text{ A}$$

$$\mathbf{V}_2 = (106\angle -90^\circ)(2.43\angle -41.3^\circ) = 257\angle -48.7^\circ\text{ V}$$

$$P_{\text{measured}} = (257)(1.86) \cos(21^\circ + 48.7^\circ) = 166\text{ W.}$$

No reversal of meter leads is needed. PSpice verification:



FREQ	VM(\$N_0002,0)	VP(\$N_0002,0)
6.000E+01	2.581E+02	-4.871E+01
FREQ	IM(V_PRINT1)	IP(V_PRINT1)
6.000E+01	1.863E+00	2.103E+01

## CHAPTER TWELVE SOLUTIONS

33. 2.5 A peak = 1.77 A rms. 200 V peak = 141 V rms.  $100\ \mu\text{F} \rightarrow -j20\ \Omega$ .

Define the clockwise mesh current  $\mathbf{I}_1$  in the bottom mesh, and the clockwise mesh current  $\mathbf{I}_2$  in the top mesh.  $\mathbf{I}_C = \mathbf{I}_1 - \mathbf{I}_2$ .

Since  $\mathbf{I}_2 = -177\angle-90^\circ$ , we need write only one mesh equation:

$$141\angle 0^\circ = (20 - j40^\circ) \mathbf{I}_1 + (-20 + j20) \mathbf{I}_2$$

$$\text{so that } \mathbf{I}_1 = \frac{141\angle 0 + (-20 + j20)(1.77\angle -90^\circ)}{20 - j40} = 4.023\angle 74.78^\circ \text{ A}$$

$$\text{and } \mathbf{I}_C = \mathbf{I}_1 - \mathbf{I}_2 = 2.361\angle 63.43^\circ \text{ A. } \mathbf{I}_{\text{meter}} = -\mathbf{I}_1 = 4.023\angle -105.2^\circ$$
$$\mathbf{V}_{\text{meter}} = 20 \mathbf{I}_C = 47.23\angle 63.43^\circ \text{ V}$$

$$\text{Thus, } P_{\text{meter}} = (47.23)(4.023)\cos(63.43^\circ + 105.2^\circ) = -186.3 \text{ W.}$$

Since this would result in pegging the meter, we would need to swap the potential leads.

## CHAPTER TWELVE SOLUTIONS

34. (a) Define three clockwise mesh currents  $\mathbf{I}_1$ ,  $\mathbf{I}_2$  and  $\mathbf{I}_3$  in the top left, bottom left and right-hand meshes, respectively. Then we may write:

$$\begin{array}{rcl} 100 \angle 0 & = & (10 - j10) \mathbf{I}_1 \quad \quad \quad - (10 - j10) \mathbf{I}_3 \\ 50 \angle 90^\circ & = & (8 + j6) \mathbf{I}_2 \quad \quad \quad - (8 + j6) \mathbf{I}_3 \\ 0 & = & -(10 - j10) \mathbf{I}_1 \quad \quad \quad - (8 + j6) \mathbf{I}_2 \quad \quad \quad + (48 + j6) \mathbf{I}_3 \end{array}$$

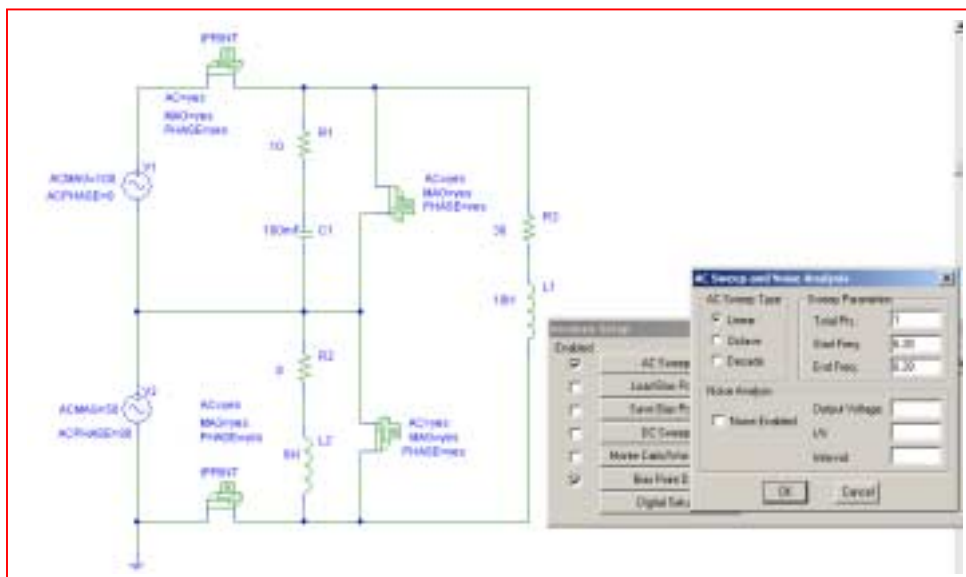
Solving, we find that  $\mathbf{I}_1 = 10.12 \angle 32.91^\circ \text{ A}$ ,  $\mathbf{I}_2 = 7.906 \angle 34.7^\circ$  and  $\mathbf{I}_3 = 3.536 \angle 8.13^\circ \text{ A}$ .

Thus,  $P_A = (100)(10.12) \cos (-32.91^\circ) = 849.6 \text{ W}$

and  $P_B = (5)(7.906) \cos (90^\circ - 34.7^\circ) = 225.0 \text{ W}$

(b) Yes, the total power absorbed by the combined load (1.075 kW) is the sum of the wattmeter readings.

PSpice verification:



FREQ	IM(V_PRINT1)	IP(V_PRINT1)
6.280E+00	1.014E+01	6.144E-02
FREQ	IM(V_PRINT2)	IP(V_PRINT2)
6.280E+00	4.268E-01	1.465E+02
FREQ	VM(\$N_0002,\$N_0006)	VP(\$N_0002,\$N_0006)
6.280E+00	1.000E+02	0.000E+00
FREQ	VM(\$N_0004,\$N_0006)	VP(\$N_0004,\$N_0006)
6.280E+00	5.000E+01 -	9.000E+01



## CHAPTER TWELVE SOLUTIONS

35. This circuit is equivalent to a Y-connected load in parallel with a  $\Delta$ -connected load.

$$\text{For the Y-connected load, } \mathbf{I}_{\text{line}} = \frac{\frac{200}{\sqrt{3}} \angle -30^\circ}{25 \angle 30^\circ} = 4.62 \angle -60^\circ \text{ A}$$

$$P_Y = (3) \left( \frac{200}{\sqrt{3}} \right) (4.62) \cos 30^\circ = 1.386 \text{ kW}$$

$$\text{For the } \Delta\text{-connected load, } \mathbf{I}_{\text{line}} = \frac{200 \angle 0^\circ}{50 \angle -60^\circ} = 4 \angle 60^\circ \text{ A}$$

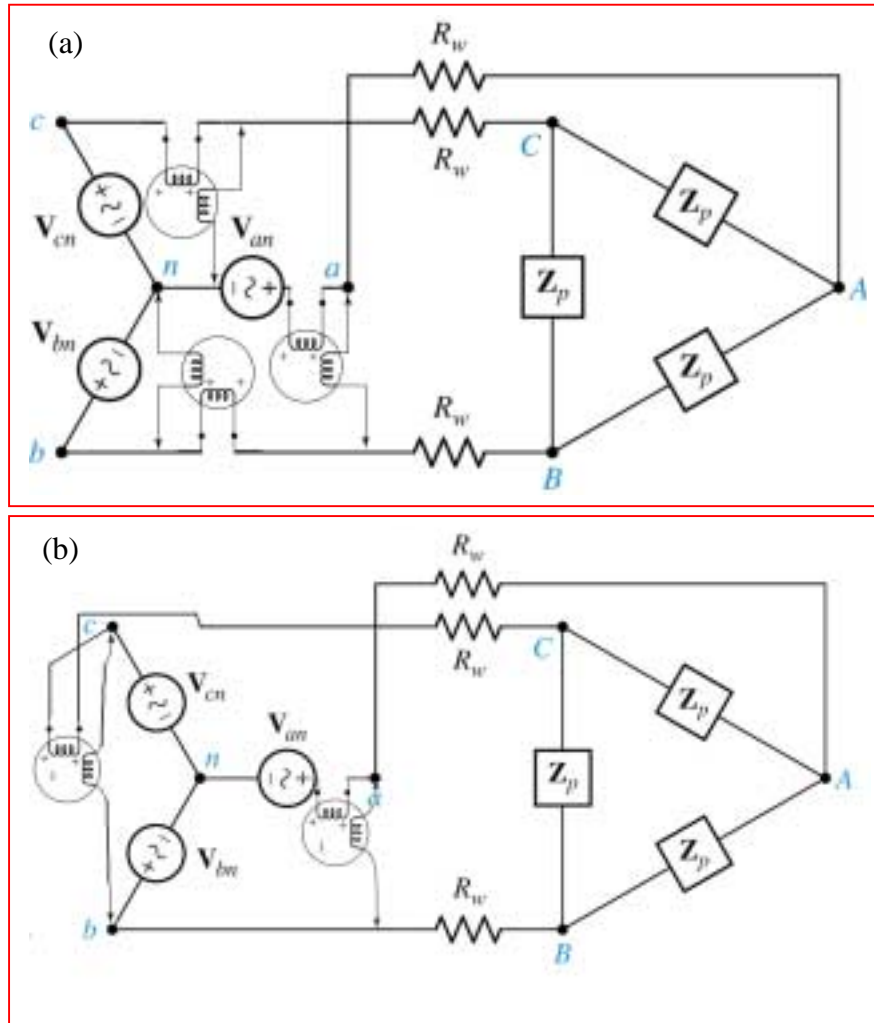
$$P_\Delta = (3)(200)(4 \cos 60^\circ) = 1.2 \text{ kW}$$

$$P_{\text{total}} = P_Y + P_\Delta = 2.586 \text{ kW}$$

$$P_{\text{wattmeter}} = P_{\text{total}} / 3 = \boxed{862 \text{ W}}$$

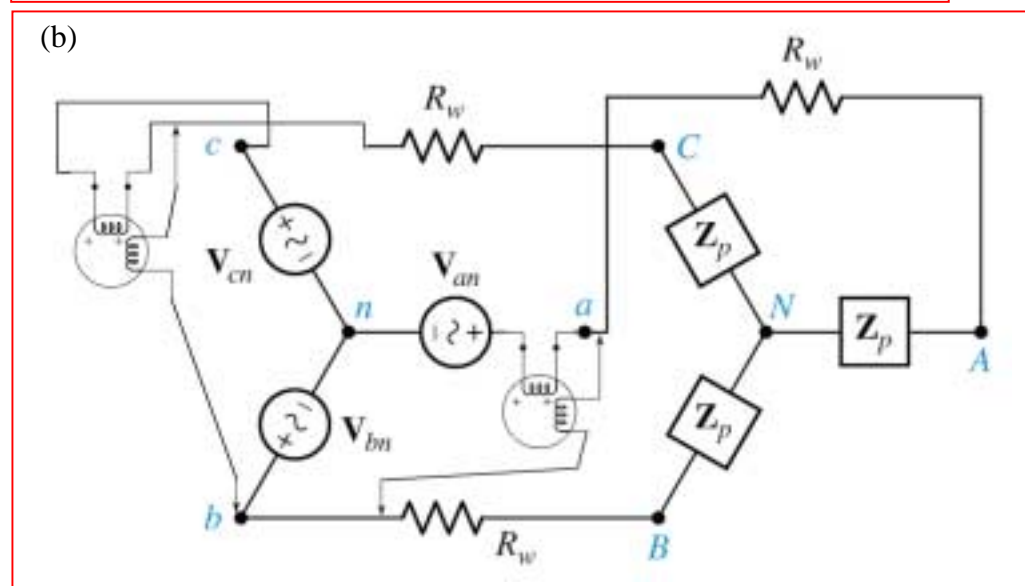
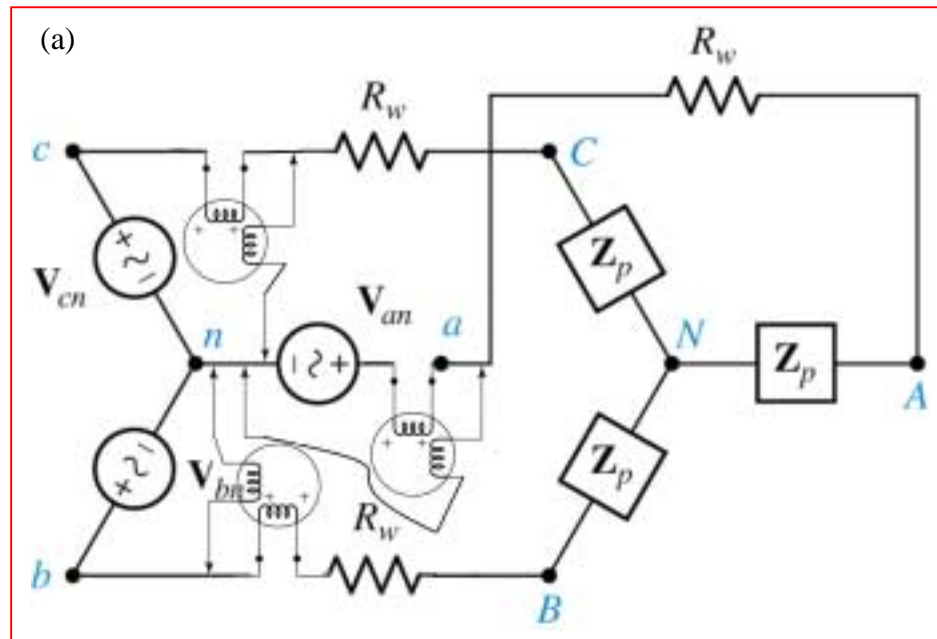
## CHAPTER TWELVE SOLUTIONS

36. We assume that the wire resistance cannot be separated from the load, so we measure from the source connection:



## CHAPTER TWELVE SOLUTIONS

37. We assume that the wire resistance cannot be separated from the load, so we measure from the source connection:



## CHAPTER THIRTEEN SOLUTIONS

1. 1 and 3, 2 and 4

1 and 4, 2 and 3

3 and 1, 2 and 4

## CHAPTER THIRTEEN SOLUTIONS

2.  $i_{s1} = 4t \text{ A}, i_{s2} = 10t \text{ A}$

(a)  $v_{AG} = 20 \times 4 + 4 \times 10 = 120 \text{ V}$

(b)  $v_{CG} = -4 \times 6 = -24 \text{ V}$

(c)  $v_{BG} = 3 \times 10 + 4 \times 4 - 6 \times 4 = 30 + 16 - 24 = 22 \text{ V}$

## CHAPTER THIRTEEN SOLUTIONS

3.

$$(a) \quad 100 = (50 + j200)\bar{I}_1 + j300\bar{I}_2, (2000 + j500)\bar{I}_2 + j300\bar{I}_1 = 0$$

$$\therefore \bar{I}_2 = \frac{-j3}{20 + j5}, 100 = \left( 50 + j200 + \frac{900}{20 + j5} \right) \bar{I}_1$$

$$\therefore 100 = \frac{900 + j4250}{20 + j5} \bar{I}_1 \therefore \bar{I}_1 = 0.4745 \angle -64.01^\circ \text{ A}$$

$$\therefore P_{S,abs} = -\frac{1}{2} \times 100 \times 0.4745 \cos 64.01^\circ = \boxed{-10.399 \text{ W}}$$

$$(b) \quad P_{50} = \frac{1}{2} \times 50 \times 0.4745^2 = 5.630 \text{ W}, P_{2000} = \frac{1}{2} \times 2000 \times 0.4745^2 \times \left| \frac{-j3}{20 + j5} \right|^2 = \boxed{4.769 \text{ W}}$$

$$(c) \quad \boxed{0 \text{ each}}$$

$$(d) \quad \boxed{0}$$

## CHAPTER THIRTEEN SOLUTIONS

4.

$$\text{KVL Loop 1} \quad 100 \angle 0 = 2(\mathbf{I}_1 - \mathbf{I}_2) + j\omega 3 (\mathbf{I}_1 - \mathbf{I}_3) + j\omega 2 (\mathbf{I}_2 - \mathbf{I}_3)$$

$$\text{KVL Loop 2} \quad 2(\mathbf{I}_2 - \mathbf{I}_1) + 10\mathbf{I}_2 + j\omega 4 (\mathbf{I}_2 - \mathbf{I}_3) + j\omega 2 (\mathbf{I}_1 - \mathbf{I}_3) = 0$$

$$\text{KVL Loop 3} \quad 5\mathbf{I}_3 + j\omega 3 (\mathbf{I}_3 - \mathbf{I}_1) + j\omega 2 (\mathbf{I}_3 - \mathbf{I}_2) + j\omega 4 (\mathbf{I}_3 - \mathbf{I}_2) + j\omega 2 (\mathbf{I}_3 - \mathbf{I}_1) = 0$$

∴ LINEAR EQUATIONS

$$\begin{bmatrix} 2 + j\omega 3 & -2 + j\omega 2 & -j\omega 5 \\ -2 + j\omega 2 & 12 + j\omega 4 & -j\omega 6 \\ -j\omega 5 & j\omega 2 & 5 + j11 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 100 \angle 0 \\ 0 \\ 0 \end{bmatrix}$$

Since  $\omega = 2\pi f = 2\pi(50) = 314.2$  rad/s, the matrix becomes

$$\begin{bmatrix} 2 + j942.6 & -2 + j628.4 & -j1571 \\ -2 + j628.4 & 12 + j1257 & -j1885 \\ -j1571 & j628.4 & 5 + j3456 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} 100 \angle 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving using a scientific calculator or MATLAB, we find that

$$\mathbf{I}_1 = 278.5 \angle -89.65^\circ \text{ mA}, \mathbf{I}_2 = 39.78 \angle -89.43^\circ \text{ mA}, \mathbf{I}_3 = 119.4 \angle -89.58^\circ \text{ mA}.$$

Returning to the time domain, we thus find that

$$i_1(t) = 278.5 \cos(100\pi t - 89.65^\circ) \text{ mA}, i_2(t) = 39.78 \cos(100\pi t - 89.43^\circ) \text{ mA}, \text{ and } i_3(t) = 119.4 \cos(100\pi t - 89.58^\circ) \text{ mA}.$$

## CHAPTER THIRTEEN SOLUTIONS

5.

$$\begin{aligned}
 \text{(a)} \quad \bar{V}_{ab,oc} &= \frac{100}{50 + j200}(-j300) = 145.52 \angle -165.96^\circ \text{ V} \\
 100 &= (50 + j200) \bar{I}_1 + j300 \bar{I}_{2SC}, j500 \bar{I}_{2SC} + j300 \bar{I}_1 = 0 \\
 \therefore \bar{I}_1 &= -\frac{5}{3} \bar{I}_{2SC}, 100 = \left[ (50 + j200) \left( -\frac{5}{3} \right) + j300 \right] \bar{I}_{2SC} \therefore \bar{I}_{2SC} = 1.1142 \angle 158.199^\circ \text{ A} \\
 \therefore \bar{Z}_{th} &= \bar{V}_{ab,bc} / \bar{I}_{2SC} = \frac{145.52 \angle -165.96^\circ}{1.1142 \angle 158.199^\circ} = 130.60 \angle 35.84^\circ = \boxed{105.88 + j76.47 \Omega}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \bar{Z}_L &= 105.88 - j76.47 \Omega \therefore |\bar{I}_L| = \frac{145.52}{2 \times 105.88} = 0.6872 \text{ A} \\
 \therefore P_{Lmax} &= \frac{1}{2} \times 0.6872^2 \times 105.88 = \boxed{25.00 \text{ W}}
 \end{aligned}$$



## CHAPTER THIRTEEN SOLUTIONS

6.

(a)  $v_A(t) = L_1 i_1' - M i_2', v_B(t) = L_1 i_1' - M i_2' + L_2 i_2' - M i_1'$

(b)  $\mathbf{V}_1(j\omega) = j\omega L_1 \mathbf{I}_A + j\omega M(\mathbf{I}_B + \mathbf{I}_A)$

$$\mathbf{V}_2(j\omega) = j\omega L_2 (\mathbf{I}_B + \mathbf{I}_A) + j\omega M \mathbf{I}_A$$

## CHAPTER THIRTEEN SOLUTIONS

7.

$$v_s = \frac{10t^2 u(t)}{t^2 + 0.01} = 0.01 i'_s \therefore i'_s = \frac{1000t^2}{t^2 + 0.01} u(t)$$

$$v_x = 0.015 i'_s = \frac{15t^2}{t^2 + 0.01} u(t), \quad 100v_x = \frac{1500t^2}{t^2 + 0.01} u(t)$$

$$\therefore i_c = 100 \times 10^{-6} v'_x = 10^{-4} \frac{d}{dt} \left( \frac{15t^2}{t^2 + 0.01} u(t) \right) = 15 \times 10^{-4} \frac{(t^2 + 0.01)2t - t^2 \times 2t}{(t^2 + 0.01)^2} u(t)$$

$$\therefore i_c = 15 \times 10^{-4} \frac{0.02t}{(t^2 + 0.01)^2} \therefore i_c(t) = \frac{30t}{(t^2 + 0.01)^2} \mu\text{A}, \quad t > 0$$

## CHAPTER THIRTEEN SOLUTIONS

8.

(a)  $-V_2 = j\omega 0.4 \angle 0$

$$V_2 = -j100\pi \times 0.4 \times 1 \angle 0 = 126 \angle 90^\circ \text{ V}$$

Thus,  $v(t) = 126 \cos(100\pi t + 90^\circ) \text{ V}$

(b) Define  $V_2$  across the 2-H inductor with + reference at the dot, and a clockwise currents  $I_1$  and  $I_2$ , respectively, in each mesh. Then,

$V = -V_2$  and we may also write

$$V_2 = j\omega L_2 I_2 + j\omega M I_1 \quad \text{or} \quad -V = j\omega L_2 \frac{V}{10} + j\omega M$$

Solving for  $V$ ,

$$V = \frac{-(j100\pi)(0.4)}{1 + (j100\pi)(2)} = \frac{125.7 \angle -90^\circ}{1 + j62.83} = \frac{125.7 \angle -90^\circ}{62.84 \angle 89.09^\circ} = 2.000 \angle -179.1^\circ$$

Thus,

$$v(t) = 2 \cos(100\pi t - 179.1^\circ) \text{ V.}$$

(c) Define  $V_1$  across the left inductor, and  $V_2$  across the right inductor, with the “+” reference at the respective dot; also define two clockwise mesh currents  $I_1$  and  $I_2$ . Then,

$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

$$V_2 = j\omega L_2 I_2 + j\omega M I_1$$

Now  $I_1 = \frac{1 \angle 0 - V_1}{4}$  and  $V_{out} = -V_2$

$$\text{and } I_2 = \frac{V_{out}}{10}$$

$$\Rightarrow V_1 = j\omega L_1 \left[ \frac{1 \angle 0 - V_1}{4} \right] + j\omega M \frac{V_{out}}{10} \quad \text{EQN 1}$$

$$-V_{out} = j\omega L_2 \frac{V_{out}}{10} + j\omega M \left[ \frac{1 \angle 0 - V_1}{4} \right] \quad \text{EQN 2}$$

$$\begin{bmatrix} 1 - \frac{j\omega L_1}{4} & \frac{-j\omega M}{10} \\ \frac{j\omega M}{4} & -1 + \frac{j\omega L_2}{10} \end{bmatrix} \begin{bmatrix} V_1 \\ V_{out} \end{bmatrix} = \begin{bmatrix} \frac{j\omega L_1}{4} \frac{1 \angle 0}{4} \\ \frac{j\omega M}{4} \frac{1 \angle 0}{4} \end{bmatrix}$$

$$\begin{bmatrix} 1 - j39 & -j12.6 \\ j31.4 & -1 + j62.8 \end{bmatrix} \begin{bmatrix} V_1 \\ V_{out} \end{bmatrix} = \begin{bmatrix} 39.3j \\ 31.4j \end{bmatrix}$$

Solving, we find that  $V_{out} (= V) = 1.20 \angle -2.108^\circ \text{ V}$  and hence

$$v(t) = 1.2 \cos(100\pi t - 2.108^\circ) \text{ V.}$$

## CHAPTER THIRTEEN SOLUTIONS

9.

$$(a) \quad 100 = j5\omega (\mathbf{I}_1 - \mathbf{I}_2) + j3\omega \mathbf{I}_2 + 6(\mathbf{I}_1 - \mathbf{I}_3) \quad [1]$$

$$(4 + j4\omega)\mathbf{I}_2 + j3\omega (\mathbf{I}_1 - \mathbf{I}_2) + j2\omega (\mathbf{I}_3 - \mathbf{I}_2) + j6\omega (\mathbf{I}_2 - \mathbf{I}_3) - j2\omega \mathbf{I}_2 + j5\omega (\mathbf{I}_2 - \mathbf{I}_1) - j3\omega \mathbf{I}_2 = 0 \quad [2]$$

$$6 (\mathbf{I}_3 - \mathbf{I}_1) + j6\omega (\mathbf{I}_3 - \mathbf{I}_2) + j2\omega \mathbf{I}_2 + 5 \mathbf{I}_3 = 0 \quad [3]$$

Collecting terms,

$$(6 + j5\omega) \mathbf{I}_1 - j2\omega \mathbf{I}_2 - 6 \mathbf{I}_3 = 100 \quad [1]$$

$$-j2\omega \mathbf{I}_1 + (4 + j5\omega) \mathbf{I}_2 - j4\omega \mathbf{I}_3 = 0 \quad [2]$$

$$-6 \mathbf{I}_1 - j4\omega \mathbf{I}_2 + (11 + j6\omega) \mathbf{I}_3 = 0 \quad [3]$$

(b) For  $\omega = 2$  rad/s, we find

$$(6 + j10) \mathbf{I}_1 - j4 \mathbf{I}_2 - 6 \mathbf{I}_3 = 100$$

$$-j4 \mathbf{I}_1 + (4 + j10) \mathbf{I}_2 - j8 \mathbf{I}_3 = 0$$

$$-6 \mathbf{I}_1 - j8 \mathbf{I}_2 + (11 + j12) \mathbf{I}_3 = 0$$

Solving,  $\mathbf{I}_3 = 4.32 \angle -54.30^\circ \text{ A}$

## CHAPTER THIRTEEN SOLUTIONS

10.

(a)

$$\begin{aligned} V_a &= j\omega L_1 I_a + j\omega M I_b & I_a &= I_1 \\ V_b &= j\omega L_2 I_b + j\omega M I_a & I_b &= -I_2 \end{aligned}$$

$$\begin{aligned} V_1 &= I_1 R_1 + V_a \\ &= I_1 R_1 + j\omega L_1 I_a + j\omega M I_b \\ &= I_1 R_1 + j\omega L_1 I_1 - j\omega M I_2 \end{aligned}$$

$$\begin{aligned} V_2 &= I_2 R_2 - V_b \\ &= I_2 R_2 - j\omega L_2 I_b - j\omega M I_a \\ &= I_2 R_2 + j\omega L_2 I_2 - j\omega M I_1 \end{aligned}$$

(b) Assuming that the systems connecting the transformer are fully isolated.

$$\begin{aligned} V_a &= j\omega L_1 I_a + j\omega M I_b & I_a &= -I_1 \\ V_b &= j\omega L_2 I_b + j\omega M I_a & I_b &= -I_2 \end{aligned}$$

$$\begin{aligned} V_1 &= I_1 R - V_a \\ &= I_1 R - j\omega L_1 I_a - j\omega M I_b \\ &= I_1 R + j\omega L_1 I_1 + j\omega M I_2 \end{aligned}$$

$$\begin{aligned} V_2 &= V_b + I_b R_2 \\ &= -I_2 R_2 + j\omega L_2 I_b + j\omega M I_a \\ &= -I_2 R_2 - j\omega L_2 I_2 - j\omega M I_1 \end{aligned}$$

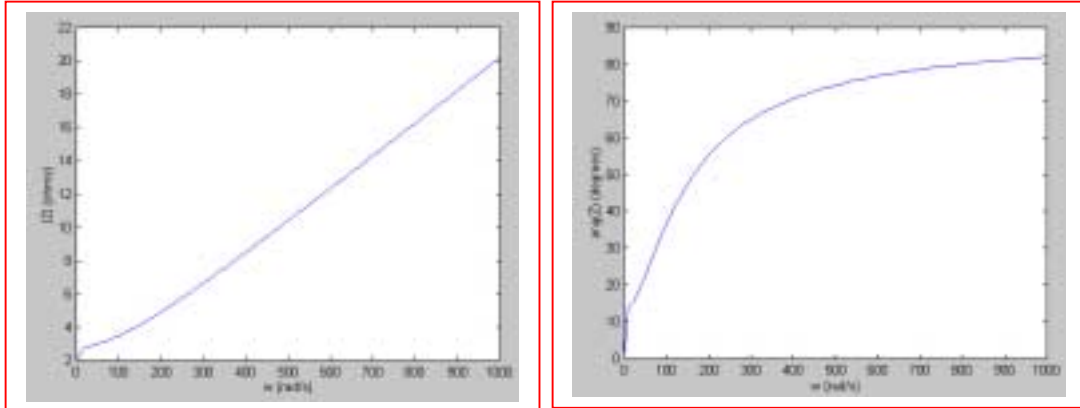
## CHAPTER THIRTEEN SOLUTIONS

11.

(a)

$$\begin{aligned}\mathbf{Z} &= 2 + j\omega 0.1 + \frac{\omega^2 (0.2)^2}{5 + j\omega 0.5} \\ &= 2 + j\omega 0.1 + \frac{5\omega^2 (0.2)^2}{5^2 + (\omega 0.5)^2} - \frac{j\omega 0.5 \omega^2 (0.2)^2}{5^2 + (\omega 0.5)^2} \\ &= 2 + \frac{0.2\omega^2}{25 + 0.25\omega^2} + j\omega \left[ 0.1 - \frac{0.02\omega^2}{25 + 0.25\omega^2} \right]\end{aligned}$$

(b)



(c)  $\mathbf{Z}_{in}(j\omega)$  at  $\omega = 50$  is equal to  $2 + 0.769 + j(50)(0.023) = 2.77 + j1.15 \Omega$ .

## CHAPTER THIRTEEN SOLUTIONS

12.

$$\begin{aligned}
 \mathbf{Z}_{in} &= \mathbf{Z}_{11} + \frac{\omega^2 M^2}{\mathbf{Z}_{22}} \\
 &= j\omega 50 \times 10^{-3} + \frac{\omega^2 M^2}{8 + j\omega 10 \times 10^{-3}} \\
 \Rightarrow \mathbf{Z}_{in} &= j\omega 50 \times 10^{-3} + \frac{\omega^2 M^2 8}{8^2 + (\omega 10 \times 10^{-3})^2} - \frac{j\omega 10 \times 10^{-3} \omega^2 M^2}{8^2 + (\omega 10 \times 10^{-3})^2} \\
 &= \frac{\omega^2 M^2 8}{8^2 + (\omega 10 \times 10^{-3})^2} + j\omega \left[ 50 \times 10^{-3} - \frac{10 \times 10^{-3} \omega^2 M^2}{8^2 + (\omega 10 \times 10^{-3})^2} \right]
 \end{aligned}$$

In this circuit the real power delivered by the source is all consumed at the speaker, so

$$\begin{aligned}
 P &= \frac{V_{rms}^2}{R} \Rightarrow 3.2 = \left( \frac{20}{\sqrt{2}} \right)^2 \times \frac{1}{8^2 + (\omega 10 \times 10^{-3})^2} \\
 \Rightarrow \frac{\omega^2 M^2 8}{8^2 + (\omega 10 \times 10^{-3})^2} &= \frac{20^2}{2 \times 3.2} \quad \boxed{= 62.5 \text{ W}}
 \end{aligned}$$

## CHAPTER THIRTEEN SOLUTIONS

13.  $i_{s1} = 2 \cos 10t$  A,  $i_{s2} = 1.2 \cos 10t$  A

(a)  $v_1 = 0.6(-20 \sin 10t) - 0.2(-12 \sin 10t) + 0.5(-32 \sin 10t) + 9.6 \cos 10t$   
 $\therefore v_1 = 9.6 \cos 10t - 25.6 \sin 10t = 27.34 \cos (10t + 69.44^\circ)$  V

(b)  $v_2 = 0.8(-12 \sin 10t) - 0.2(-20 \sin 10t) - 16 \sin 10t + 9.6 \cos 10t$   
 $\therefore v_2 = 9.6 \cos 10t - 21.6 \sin 10t = 23.64 \cos (10t + 66.04^\circ)$  V

(c)  $P_{s1} = \frac{1}{2} \times 27.34 \times 2 \cos 69.44^\circ = 9.601$  W,  $P_{s2} = \frac{1}{2} \times 23.64 \times 1.2 \cos 66.04^\circ = 5.760$  W



## CHAPTER THIRTEEN SOLUTIONS

14.

$$V_a = j\omega 8I_a + j\omega 4I_b$$

$$* \quad V_b = j\omega 10I_b + j\omega 4I_a = j\omega 10I_b + j\omega 5I_c$$

$$V_c = j\omega 6I_c + j\omega 5I_b$$

$$\text{Also } I = -I_a = -I_b = I_c$$

Now examine equation \*.

$$-j\omega 10I - j\omega 4I = -j\omega 10I + j\omega 5I_c$$

$\therefore$  the only solution to this circuit is  $I = 0$  and hence

$$v(t) = 120 \cos \omega t \text{ V.}$$

## CHAPTER THIRTEEN SOLUTIONS

15.

$$100 = j10\bar{I}_1 - j15\bar{I}_2$$

$$0 = j200\bar{I}_2 - j15\bar{I}_1 - j15\bar{I}_L$$

$$0 = (5 + j10)\bar{I}_L - j15\bar{I}_2$$

$$\therefore \bar{I}_2 = \frac{5 + j10}{j15} \bar{I}_L = \frac{1 + j2}{j3} \bar{I}_L \quad \therefore 0 = j200 \left( \frac{1 + j2}{j3} - j15 \right) \bar{I}_L - j15\bar{I}_1$$

$$\therefore 0 = \left( j\frac{400}{3} - j15 + \frac{200}{3} \right) \bar{I}_L - j15\bar{I}_1 \quad \therefore \bar{I}_1 = \frac{j118.33 + 66.67}{j15} \bar{I}_L$$

$$\therefore 100 = \left[ \frac{2}{3} (66.67 + j118.33) - 5 - j10 \right] \bar{I}_L = (39.44 + j68.89) \bar{I}_L$$

$$\therefore \bar{I}_L = \boxed{1.2597 \angle -60.21^\circ \text{ A}}$$

## CHAPTER THIRTEEN SOLUTIONS

16.  $i_s = 2 \cos 10t$  A,  $t = 0$

(a)  $a-b$  O.C.  $\therefore w(0) = \frac{1}{2} \times 5 \times 2^2 + \frac{1}{2} \times 4 \times 2^2 = 10 + 8 = 18$  J

(b)  $a-b$  S.C.  $\omega = 10$ ,  $\bar{I}_s = 2 \angle 0^\circ$  A,  $M = \frac{1}{2} \sqrt{12} = \sqrt{3}$  H

$$(j30 + 5) \bar{I}_2 - j10\sqrt{3} \times 2, \therefore \bar{I}_2 = \frac{j20\sqrt{3}}{5 + j30} = 1.1390 \angle 9.462^\circ \text{ A} \therefore i_2 = 1.1390 \cos(10t + 9.462^\circ) \text{ A}$$

$$\therefore i_2(0) = 1.1235^- \therefore w(0) = 10 + 8 - \sqrt{3} \times 2 \times 1.1235 + \frac{1}{2} \times 3 \times 1.1235^2 = 16.001 \text{ J}$$

## CHAPTER THIRTEEN SOLUTIONS

17.

$$\bar{V}_s = 12\angle 0^\circ \text{ V rms}, \omega = 100 \text{ rad/s}$$

$$12 = (6 + j20) \bar{I}_1 + j100(0.4K) \bar{I}_2, (24 + j80) \bar{I}_2 + j40K\bar{I}_1 = 0$$

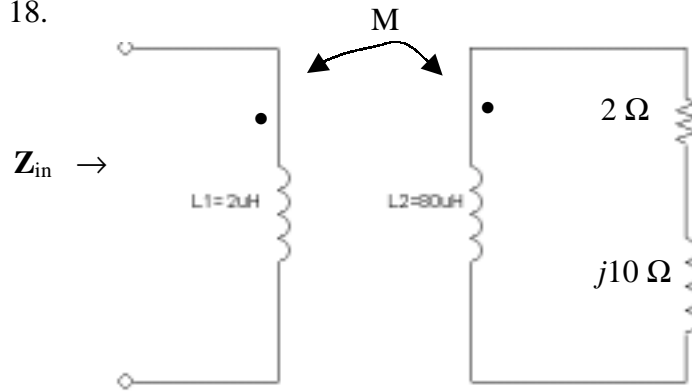
$$\therefore \bar{I}_1 = \frac{3 + j10}{-j5K} \bar{I}_2 \quad \therefore 12 = \left[ (6 + j20) \frac{3 + j10}{-j5K} + j40K \right] \bar{I}_2$$

$$\therefore 12 = \frac{18 - 200 + j60 + j60 + 200K^2}{-j5K} \bar{I}_2 \quad \therefore \bar{I}_2 = \frac{-j60K}{-182 + 200K^2 + j120}$$

$$\therefore P_{24} = \frac{60^2 K^2 24}{(200K^2 - 182)^2 + 120^2} = \frac{86,400 K^2}{40,000K^4 - 72,800K^2 + 47,524} = \boxed{\frac{2.16K^2}{K^4 - 1.82K^2 + 1.1881} \text{ W}}$$

## CHAPTER THIRTEEN SOLUTIONS

18.



$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$\omega = 250k \text{ rad / s}$$

$$M = \sqrt{L_1 L_2} = \sqrt{2 \times 80} \times 10^{-6} \\ = 12.6 \mu\text{H}$$

$$\mathbf{Z}_{in} = \mathbf{Z}_{11} + \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} + \frac{-j\omega^2 M^2 X_{22}}{R_{22}^2 + X_{22}^2}$$

$$\mathbf{Z}_{11} = j \times 250 \times 10^3 \times 2 \times 10^{-6} \\ = j0.5$$

$$R_{22} = 2 \Omega$$

$$X_{22} = (250 \times 10^3)(80 \times 10^{-6}) \\ = 20$$

Thus,  $\mathbf{Z}_{in} = j0.5 + 19.8/404 - j198/404$

$$= \boxed{0.049 + j0.010 \Omega}$$

## CHAPTER THIRTEEN SOLUTIONS

19.  $\omega = 100 \text{ rad/s}$

(a)  $K_1 \rightarrow j50\Omega$ ,  $K_2 \rightarrow j20\Omega$ ,  $1H \rightarrow j100\Omega$

$$100 = j200 \bar{I}_1 - j50\bar{I}_2 - j20\bar{I}_3$$

$$0 = (10 + j100)\bar{I}_2 - j50\bar{I}_1$$

$$0 = (20 + j100)\bar{I}_3 - j20\bar{I}_1$$

$$\therefore \bar{I}_3 = \frac{j2}{2 + j10} \bar{I}_1, \bar{I}_2 = \frac{j5}{1 + j10} \bar{I}_1 \therefore 10 = \left[ j20 - j5 \frac{j5}{1 + j10} - j2 \frac{j2}{2 + j10} \right] \bar{I}_1$$

$$\therefore 10 = \left( j20 + \frac{25}{1 + j10} + \frac{4}{2 + j10} \right) \bar{I}_1 \therefore \bar{I}_1 = 0.5833 \angle -88.92^\circ \text{ A}, \bar{I}_2 = 0.2902 \angle -83.20^\circ \text{ A},$$

$$\bar{I}_3 = 0.11440 \angle -77.61^\circ \text{ A} \therefore P_{10\Omega} = 0.2902^2 \times 10 = \boxed{0.8422 \text{ W}}$$

(b)  $P_{20} = 0.1144^2 \times 20 = \boxed{0.2617 \text{ W}}$

(c)  $P_{gen} = 100 \times 0.5833 \cos 88.92^\circ = \boxed{1.1039 \text{ W}}$

## CHAPTER THIRTEEN SOLUTIONS

20.

$$\begin{aligned} \text{(a)} \quad k &= \frac{M}{\sqrt{L_1 L_2}} \\ \Rightarrow M &= 0.4\sqrt{5 \times 1.8} \\ &= \boxed{1.2\text{H}} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad I_1 + I_2 &= I_3 \\ \Rightarrow I_2 &= I_3 - I_1 \\ &= \boxed{5 \times 10^{-5} - 4 \times 10^{-10}} \end{aligned}$$

(c) The total energy stored at  $t = 0$ .

$$\begin{aligned} I_1 &= 4\text{A} & I_2 &= 1\text{A} \\ W_{\text{total}} &= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2 \\ &= \frac{1}{2} \times 5 \times 16 + \frac{1}{2} \times 1.8 \times 1 - 1.2 \times 4 \times 1 \\ &= 40 + 0.9 - 4.8 \\ &= \boxed{36.1\text{J}} \end{aligned}$$

## CHAPTER THIRTEEN SOLUTIONS

21.

$$K \rightarrow j1000K\sqrt{L_1L_2}, L_1 \rightarrow j1000L_1, L_2 \rightarrow j1000L_2$$

$$\therefore \bar{V}_s = (2 + j1000L_1) \bar{I}_1 - j1000K\sqrt{L_1L_2} \bar{I}_2$$

$$0 = -j1000K\sqrt{L_1L_2} \bar{I}_1 + (40 + j1000L_2) \bar{I}_2$$

$$\omega = 1000 \text{ rad/s} \quad \therefore \bar{I}_1 = \frac{40 + j1000L_2}{j1000K\sqrt{L_1L_2}} \bar{I}_2$$

$$\therefore \bar{V}_s = \frac{(2 + j1000L_1)(40 + j1000L_2) + 10^6 K^2 L_1 L_2}{j1000K\sqrt{L_1L_2}} \bar{I}_2$$

$$\therefore \bar{I}_2 = \frac{j1000K\sqrt{L_1L_2}}{80 + j40,000L_1 + j2000L_2 - 10^6 L_1 L_2 (1 - K^2)}$$

$$\therefore \frac{\bar{V}_2}{\bar{V}_s} = \frac{j40,000K\sqrt{L_1L_2}}{80 - 10^6 L_1 L_2 (1 - K^2) + j(40,000L_1 + 2000L_2)}$$

$$(a) \quad L_1 = 10^{-3}, L_2 = 25 \times 10^{-3}, K = 1 \quad \therefore \frac{\bar{V}_2}{\bar{V}_s} = \frac{j40 \times 5}{80 - 0 + j(40 + 50)} = \frac{j200}{80 + j90} = \boxed{1.6609 \angle 41.63^\circ}$$

$$(b) \quad L_1 = 1, L_2 = 25, K = 0.99 \quad \therefore \frac{V_2}{V_s} = \frac{j40,000 \times 0.99 \times 5}{80 - 25 \times 10^6 (1 - 0.99^2) + j(40,000 + 50,000)}$$

$$\therefore \frac{\bar{V}_2}{\bar{V}_s} = \frac{j198,000}{80 - 497,500 + j90,000} = \boxed{0.3917 \angle -79.74^\circ}$$

$$(c) \quad L_1 = 1, L_2 = 25, K = 1 \quad \therefore \frac{V_2}{V_s} = \frac{j40,000 \times 5}{80 - 0 + j90,000} = \frac{j200,000}{80 + j90,000} = \boxed{2.222 \angle 0.05093^\circ}$$



## CHAPTER THIRTEEN SOLUTIONS

22.

(a)  $L_{AB,CDOC} = 10 \text{ mH}$ ,  $L_{CD,ABOC} = 5 \text{ mH}$

$$L_{AB,CDSC} = 8 \text{ mH}$$

$$\therefore L_1 = 10 \text{ mH}, L_2 = 5 \text{ mH}, 8 = 10 - M + M \parallel (5 - M) \text{ (mH)}$$

$$\therefore 8 = 10 - M + \frac{M(5 - M)}{5}, \therefore 5M = (10 - 8)5 + 5M - M^2 \therefore M = 3.162 \text{ mH} (= \sqrt{10})$$

$$\therefore K = \frac{3.162}{\sqrt{50}} \therefore K = 0.4472$$

(b) Dots at A and D,  $i_1 = 5 \text{ A}$ ,  $w_{tot} = 100 \text{ mJ}$

$$\therefore 100 \times 10^{-3} = \frac{1}{2} \times 10 \times 10^{-3} \times 25 + \frac{1}{2} \times 5 \times 10^{-3} i_2^2 - \sqrt{10} \times 5 i_2 \times 10^{-3}$$

$$100 = 125 + 2.5 i_2^2 - 5 \sqrt{10} i_2 \therefore i_2^2 - 2 \sqrt{10} i_2 + 10 = 0, i_2 = \frac{2 \sqrt{10} \pm \sqrt{40 - 40}}{2} = \sqrt{10}$$

$$\therefore i_2 = 3.162 \text{ A}$$

## CHAPTER THIRTEEN SOLUTIONS

23. Define coil voltages  $v_1$  and  $v_2$  with the “+” reference at the respective dot. Also define two clockwise mesh currents  $i_1$  and  $i_2$ . We may then write:

$$\begin{aligned} v_1 &= L_1 \frac{d\mathbf{I}_1}{dt} + M \frac{d\mathbf{I}_2}{dt} \\ v_2 &= L_2 \frac{d\mathbf{I}_2}{dt} + M \frac{d\mathbf{I}_1}{dt} \quad \omega = 2\pi 60 \text{ rad/s} \end{aligned} \quad M = k\sqrt{L_1 L_2}$$

or, using phasor notation,

$$\mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2$$

$$\mathbf{V}_2 = j\omega L_2 \mathbf{I}_2 + j\omega M \mathbf{I}_1$$

$$100\angle 0 = 50\mathbf{I}_1 + j\omega L_1 \mathbf{I}_1 + j\omega M \mathbf{I}_2$$

$$-25\mathbf{I}_2 = j\omega L_2 \mathbf{I}_2 + j\omega M \mathbf{I}_1$$

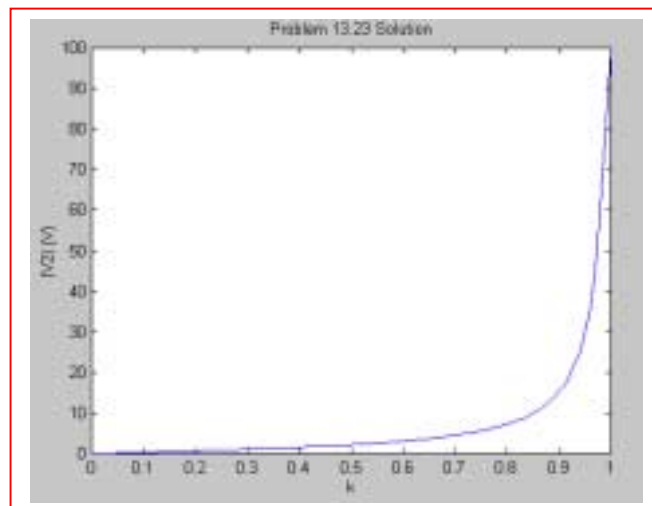
$$\text{Rearrange:} \quad [50 + j\omega L_1] \mathbf{I}_1 + j\omega M \mathbf{I}_2 = 100\angle 0$$

$$j\omega M \mathbf{I}_1 + [-25 + j\omega L_2] \mathbf{I}_2 = 0$$

$$\text{or} \quad \begin{bmatrix} 50 + j\omega L_1 & j\omega M \\ j\omega M & -25 + j\omega L_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 100\angle 0 \\ 0 \end{bmatrix}$$

We can solve for  $\mathbf{I}_2$  and  $\mathbf{V}_2 = -25\mathbf{I}_2$ :

$$\mathbf{V}_2 = -\frac{j1.658}{k\sqrt{L_1 L_2} + 1}$$



## CHAPTER THIRTEEN SOLUTIONS

24.

$$i_1 = 2 \cos 500t \text{ A} \quad W_{\max} \text{ at } t = 0$$

$$\begin{aligned} \therefore w_{\max} &= \frac{1}{2} \times 4 \times 2^2 + \frac{1}{2} \times 6 \times 2^2 + \frac{1}{2} \times 5 \times 2^2 + 3 \times 2^2 \\ &= 8 + 12 + 10 + 12 = 42 \text{ J} \end{aligned}$$

## CHAPTER THIRTEEN SOLUTIONS

25.

(a) All DC:  $L_{1-2} = 2 - 1 = 1 \text{ H}$

(b) AB SC:  $L_{1-2} = -1 + 2 \parallel 8 = 0.6 \text{ H}$

(c) BC SC:  $L_{1-2} = 2 + (-1) \parallel 9 = 2 - 9/8 = 0.875 \text{ H}$

(d) AC SC:  $L_{1-2} = (2 - 1) \parallel (1 + 2) = 1 \parallel 3 = 0.750 \text{ H}$

## CHAPTER THIRTEEN SOLUTIONS

26.

$$(a) \quad \frac{\mathbf{I}_L}{\mathbf{V}_s} = \frac{1}{15 + j3\omega + \frac{j2\omega(20 + j\omega)}{20 + j3\omega}} \left( \frac{j2\omega}{20 + j3\omega} \right)$$

$$= \frac{j2\omega}{300 - 11\omega^2 + j145\omega}$$

$$(b) \quad v_s(t) = 100u(t), i_s(0) = 0, i_L(0) = 0, s_{1,2} = \frac{-145 \pm \sqrt{145^2 - 13,200}}{22} = -2.570, -10.612$$

$$i_L = i_{Lf} + i_{Ln}, i_{Lf} = 0, \therefore i_L = Ae^{-2.57t} + Be^{-10.61t}, \therefore 0 = A + B$$

$$100 = 15i_s + 5i'_s - 2i'_L, 0 = 20i_L + 3i'_L - 2i'_s \text{ At } t = 0^+: 100 = 0 + 5i'_s(0^+) - 2i'_L(0^+) \text{ and}$$

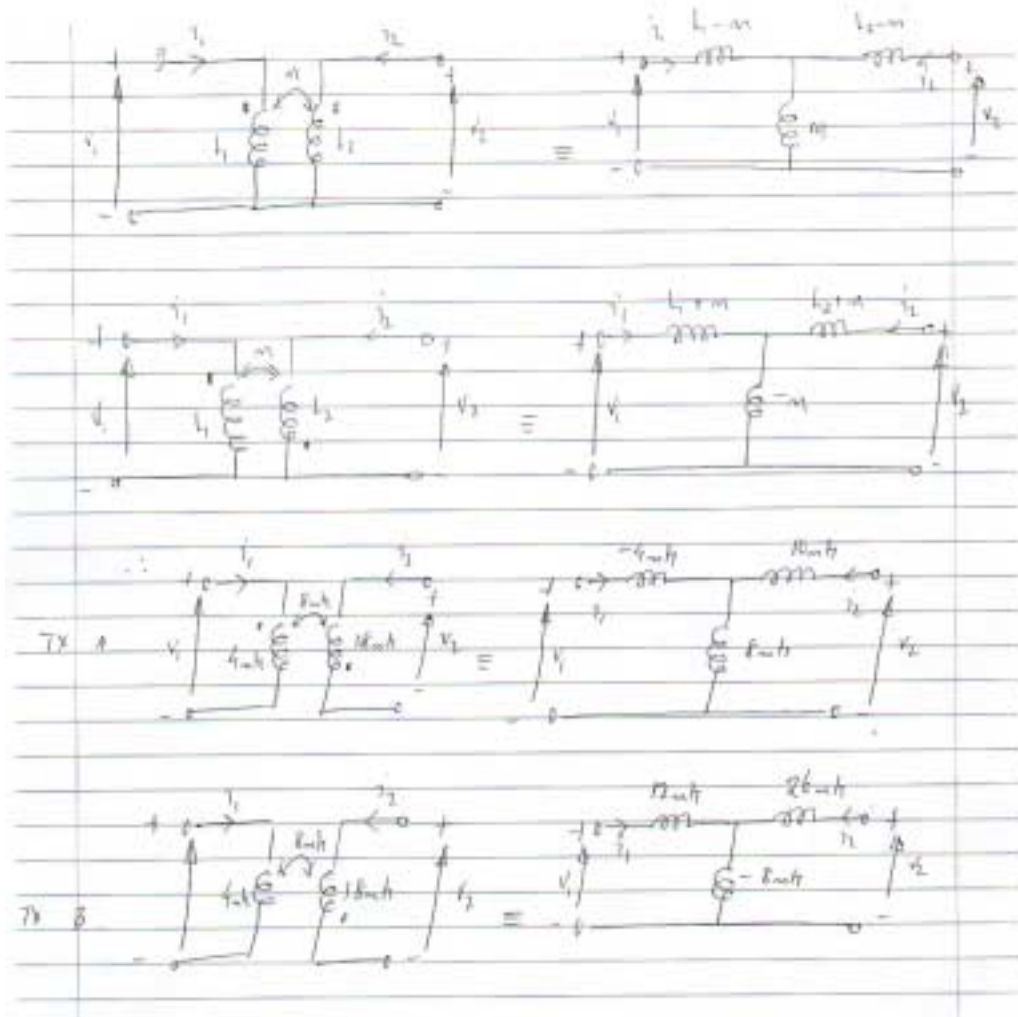
$$0 = 0 + 3i'_L(0^+) - 2i'_s(0^+) \therefore i'_s(0^+) = 1.5i'_L(0^+) \therefore 100 = 7.5i'_L(0^+) - 2i'_L(0^+) = 5.5i'_L(0^+)$$

$$\therefore i'_L(0^+) = 18.182 \text{ A/s} \therefore 18.182 = -2.57A - 10.61B = -2.57A + 10.61A = 8.042A$$

$$\therefore A = 2.261, B = -2.261, i_L(t) = 2.261(e^{-2.57t} - e^{-10.612t}) \text{ A}, t > 0$$

## CHAPTER THIRTEEN SOLUTIONS

27.



(a) Open-Circuit

$$\mathbf{Z}_{oc}^{T \times A} = j\omega 4 \text{ M}\Omega$$

$$\mathbf{Z}_{oc}^{T \times B} = j\omega 4 \text{ M}\Omega$$

(b) Short-Circuit

$$\mathbf{Z}_{ss}^{T \times A} = \mathbf{Z}_{ss}^{T \times B} = -j\omega 4 \text{ M}\Omega + j\omega 8 \parallel j\omega 10 \text{ M}\Omega$$

(c) If the secondary is connected in parallel with the primary

$$\mathbf{Z}_{in}^{T \times A} = -j\omega 4 \parallel -j\omega 10 + j\omega 8 \text{ M}\Omega$$

$$\mathbf{Z}_{in}^{T \times B} = j\omega 26 \parallel j\omega 12 - j\omega 8 \text{ M}\Omega$$

## CHAPTER THIRTEEN SOLUTIONS

28. Define three clockwise mesh currents  $\mathbf{I}_1$ ,  $\mathbf{I}_2$ , and  $\mathbf{I}_3$  beginning with the left-most mesh.

$$\mathbf{V}_s = j8\omega \mathbf{I}_1 - j4\omega \mathbf{I}_2$$

$$0 = -4j\omega \mathbf{I}_1 + (5 + j6\omega) \mathbf{I}_2 - j2\omega \mathbf{I}_3$$

$$0 = -j2\omega \mathbf{I}_2 + (3 + j\omega) \mathbf{I}_3$$

Solving,  $\mathbf{I}_3 = j\omega / (15 + j17\omega)$ . Since  $\mathbf{V}_o = 3 \mathbf{I}_3$ ,

$$\frac{\mathbf{V}_o}{\mathbf{V}_s} = \frac{j3\omega}{15 + j17\omega}$$

## CHAPTER THIRTEEN SOLUTIONS

$$29. \quad L_{\text{eq}} = 2/3 + 1 + 2 + 6/5 = 4.867 \text{ H}$$

$$Z(j\omega) = 10j\omega(4.867)/(10 + j\omega 4.867)$$

$$= j4.867\omega/(1 + j0.4867\omega) \Omega.$$



## CHAPTER THIRTEEN SOLUTIONS

30.

$$\omega = 100 \text{ rad/s}$$

$$\bar{V}_s = 100\angle 0^\circ \text{ V rms}$$

$$\begin{aligned} \text{(a)} \quad Z_{in a-b} &= 20 + j600 + \frac{j400(10 - j200)}{10 + j200} = 20 + j600 + \frac{80,000 + j4,000}{10 + j200} \\ &= 210.7\angle 73.48^\circ \text{ V and } V_{oc} = 0. \end{aligned}$$

$$\text{(b)} \quad \bar{V}_{oc, cd} = \frac{100(j400)}{20 + j1000} = 39.99\angle 1.146^\circ \text{ V rms}$$

$$\bar{Z}_{in cd}, \bar{V}_s = 0 = -j200 + \frac{j400(20 + j600)}{20 + j1000} = -j200 + \frac{-240,000 + j8,000}{20 + j1,000} = 40.19\angle 85.44^\circ \Omega$$

## CHAPTER THIRTEEN SOLUTIONS

31.  $L_1 = 1 \text{ H}$ ,  $L_2 = 4 \text{ H}$ ,  $K = 1$ ,  $\omega = 1000 \text{ rad/s}$

(a)  $\bar{Z}_L = 1000 \Omega \quad \therefore \bar{Z}_{in} = j1000 + \frac{10^6 \times 1 \times 4}{j4000 + 100} = 24.98 + j0.6246 \Omega$

(b)  $\bar{Z}_L = j1000 \times 0.1 \quad \therefore \bar{Z}_{in} = j1000 + \frac{4 \times 10^6}{j4000 + j100} = j24.39 \Omega$

(c)  $\bar{Z}_L = -j100 \quad \therefore \bar{Z}_{in} = j1000 + \frac{4 \times 10^6}{j4000 - j100} = -j25.46 \Omega$

## CHAPTER THIRTEEN SOLUTIONS

32.

$$L_1 = 6 \text{ H}, L_2 = 12 \text{ H}, M = 5 \text{ H}$$

$$\#1, L_{inAB,CDOC} = 6 \text{ H}$$

$$\#2, L_{inCD,ABOC} = 12 \text{ H}$$

$$\#3, L_{inAB,CDSC} = 1 + 7 \parallel 5 = 3.917 \text{ H}$$

$$\#4, L_{inCD,ABSC} = 7 + 5 \parallel 1 = 7.833 \text{ H}$$

$$\#5, L_{inAC,BDSC} = 7 + 1 = 8 \text{ H}$$

$$\#6, L_{inAB,ACSC,BDSC} = 7 \parallel 1 + 5 = 5.875 \text{ H}$$

$$\#7, L_{inAD,BCSC} = 11 + 17 = 28 \text{ H}$$

$$\#8, L_{inAB,ADSC} = -5 + 11/17 = 1.6786 \text{ H}$$

## CHAPTER THIRTEEN SOLUTIONS

33.

$$\mathbf{Z}_{in} = \mathbf{Z}_{11} + \frac{\omega^2 M^2}{R_{22} + jX_{22}}$$

$$\frac{1}{\omega C} = 31.83 \Rightarrow \omega = \frac{1}{31.83 \times C} = 314 \text{ rad/s}$$

ie. a 50Hz system

$$\mathbf{Z}_{in} = 20 + j\omega 100 \times 10^{-3} + \frac{\omega^2 k^2 L_1 L_2}{2 - j31.83}$$

$$\mathbf{Z}_{in} = 20 + j\omega 100 \times 10^{-3} + \frac{\omega^2 k^2 L_1 L_2 2}{2^2 + 31.83^2} - \frac{j\omega^2 k^2 L_1 L_2 31.83}{2^2 + 31.83^2}$$

$$= 20 + j31.4 + \left[ \frac{493}{1020} - j\frac{7840}{1020} \right] k^2$$

$$= 20 + j31.4 + [0.483 - j7.69] k^2$$

(a)  $\mathbf{Z}_{in}(k=0) = \boxed{20 + j31.4 \quad \Omega}$

(b)  $\mathbf{Z}_{in}(k=0.5) = \boxed{20.2 + j27.6 \quad \Omega}$

(c)  $\mathbf{Z}_{in}(k=0.9) = \boxed{20.4 + j24.5 \quad \Omega}$

(d)  $\mathbf{Z}_{in}(k=1.0) = \boxed{20.5 + j23.7 \quad \Omega}$

## CHAPTER THIRTEEN SOLUTIONS

34.  $\uparrow L_1 \rightarrow 125 \text{ H}, L_2 \rightarrow 20 \text{ H}, K = 1, \therefore M = \sqrt{2500} = 50 \text{ H}, j\omega M = j5000 \Omega$

(a) 
$$\bar{Z}_{in-a-b} = 20 + j7500 + \frac{j5000(10 - j3000)}{10 + j2000}$$

$$= 20 + j7500 + \frac{15 \times 10^6 + j50,000}{10 + j2000} = 82.499 \angle 0.2170^\circ \Omega$$

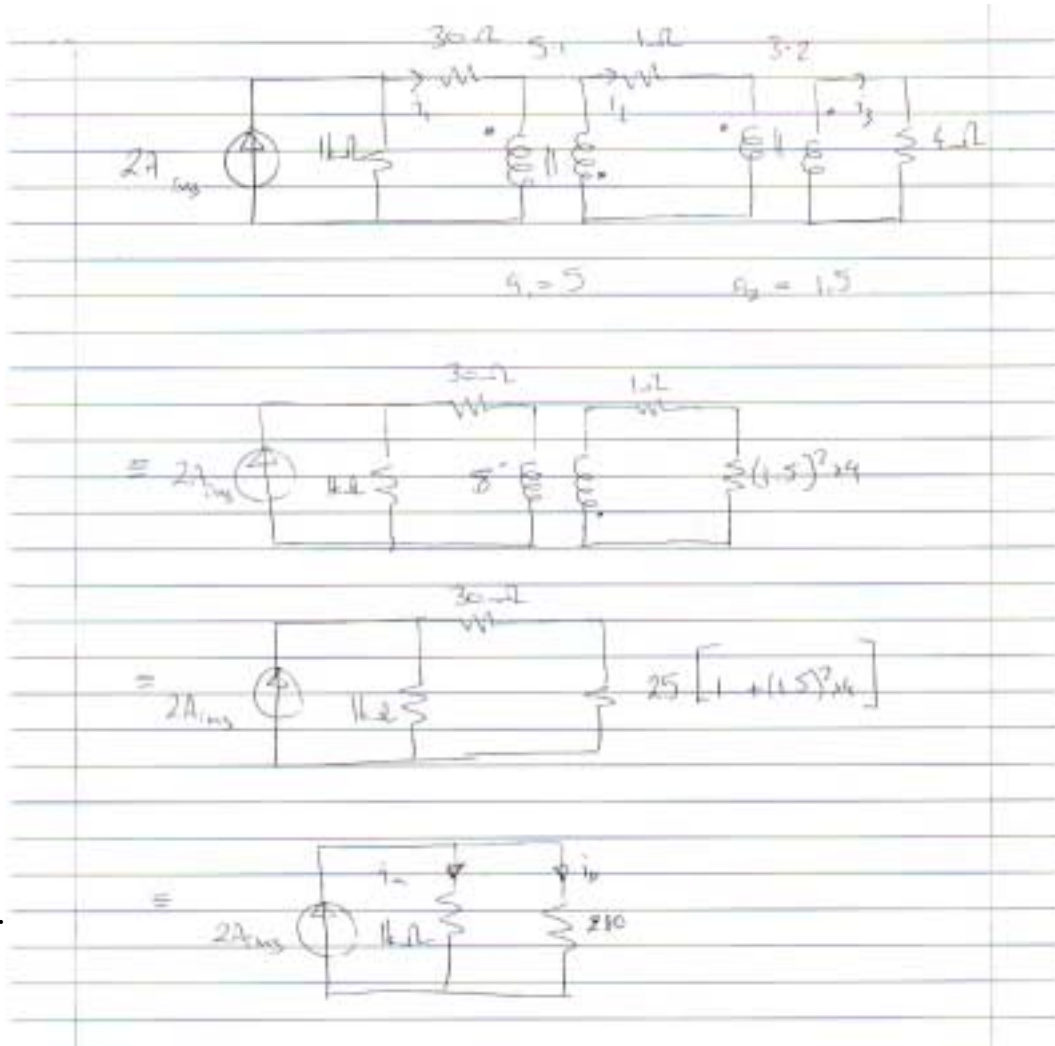
$$= 82.498 + j0.3125 \Omega \quad V_{oc} = 0$$

(b) 
$$\bar{V}_{oc,cd} = \frac{100(j5000)}{20 + j12,500} = 39.99995 \angle 0.09167^\circ \text{ V rms}$$

$$\bar{Z}_{in,cd}, V_s = 0 = -j3000 + \frac{j5000(20 + j7500)}{20 + j12,500} = 3.19999 + j0.00512 \Omega$$

# CHAPTER THIRTEEN SOLUTIONS

35.



$$\therefore I_1 = 1.56A$$

$$\Rightarrow I_2 = 5 \times 1.56 = 7.8A$$

$$\Rightarrow I_3 = 1.5 \times 7.8A = 11.7A$$

$$\Rightarrow P(1k) = I_a^2 R$$

$$= 0.438^2 \times 1 \times 10^3$$

$$= 192W$$

$$\Rightarrow P(30\Omega) = I_1^2 R = (1.56)^2 \times 30$$

$$= 73W$$

$$\Rightarrow P(1\Omega) = I_2^2 R = 7.8^2 \times 1$$

$$= 60.8W$$

$$\Rightarrow P(4\Omega) = I_3^2 R = 11.7^2 \times 4$$

$$= 548W$$

## CHAPTER THIRTEEN SOLUTIONS

36.

(a)  $R_L$  sees  $10 \times 4^2 = 160 \Omega$   $\therefore$  use  $R_L = 160 \Omega$

$$P_{L_{\max}} = \left( \frac{100}{20} \right)^2 \times 10 = 250 \text{ W}$$

(b)  $R_L = 100 \Omega$

$$I_2 = I_1 / 4, \quad V_2 = 4 V_1 \quad \therefore I_x = \frac{V_2 - V_1}{40} = \frac{3V_1}{40}$$

$$\therefore 100 = 10 \left( I_1 \frac{3V_1}{40} \right) + V_1, \quad \frac{I_1}{4} = \frac{3V_1}{40} + \frac{4V_1}{100}$$

$$\therefore I_1 = 0.46V_1 \quad \therefore 100 = 10(0.46V_1 - 0.075V_1) + V_1 = 4.85 V_1 \quad \therefore V_1 = \frac{100}{4.85}$$

$$\therefore V_2 = 4V_1 = \frac{400}{4.85} = 82.47 \text{ V} \quad \therefore P_L = \frac{82.47^2}{100} = 68.02 \text{ W}$$

## CHAPTER THIRTEEN SOLUTIONS

37.

$$\bar{I}_2 = \frac{\bar{V}_2}{8} \quad \therefore \bar{I}_1 = \frac{\bar{V}_2}{40}, \quad \bar{V}_1 = 5\bar{V}_2$$

$$\therefore 100 = 300(C + 0.025)\bar{V}_2 + 5\bar{V}_2$$

$$\therefore \bar{V}_2 = \frac{100}{12.5 + 300C}$$

$$(a) \quad C = 0 \quad \therefore \bar{V}_2 = 8 \text{ V} \quad \therefore P_L = \frac{8^2}{8} = \boxed{8 \text{ W}}$$

$$(b) \quad C = 0.04 \quad \therefore \bar{V}_2 = \frac{100}{24.5} \quad \therefore P_L = \left( \frac{100}{24.5} \right)^2 \frac{1}{8} = \boxed{2.082 \text{ W (neg. fdbk)}}$$

$$(c) \quad C = -0.04 \quad \therefore \bar{V}_2 = \frac{100}{0.5} = 200 \text{ V} \quad \therefore P_L = \frac{200^2}{8} = \boxed{5000 \text{ W (pos. fdbk)}}$$



## CHAPTER THIRTEEN SOLUTIONS

38.

Apply  $\bar{V}_{ab} = 1 \text{ V}$   $\therefore \bar{I}_x = 0.05 \text{ A}$ ,  $\bar{V}_2 = 4 \text{ V}$

$$\therefore 4 = 60 \bar{I}_2 + 20 \times 0.05 \therefore \bar{I}_2 = \frac{4-1}{60} = 0.05 \text{ A}$$

$$\therefore \bar{I}_1 = 0.2 \text{ A} \quad \therefore \bar{I}_m = 0.25 \text{ A} \quad \therefore \boxed{\bar{R}_{th} = 4 \, \Omega, \quad \bar{V}_{th} = 0}$$

## CHAPTER THIRTEEN SOLUTIONS

39.

$$P_{gen} = 1000 \text{ W}, P_{100} = 500 \text{ W}$$

$$\therefore I_L = \sqrt{\frac{500}{100}} = \sqrt{5} \text{ A}, V_L = 100\sqrt{5} \text{ V}$$

$$I_S = \frac{1000}{100} = 10 \text{ A} \quad \therefore V_1 = 100 - 40 = 60 \text{ V}$$

$$\text{Now, } P_{25} = 1000 - 500 - 10^2 \times 4 = 100 \text{ W} \quad \therefore I_x = \sqrt{\frac{100}{25}} = 2 \text{ A; also}$$

$$I_x = b\sqrt{5} = 2, b = \frac{2}{\sqrt{5}} = \boxed{0.8944}$$

$$\text{Around center mesh: } 60a = 2 \times 25 + 100\sqrt{5} \frac{1}{0.8944} \quad \therefore a = \frac{300}{60} = \boxed{5}$$

## CHAPTER THIRTEEN SOLUTIONS

40.

$$(a) \quad 3 \times \left( \frac{4}{3} \right)^2 = \frac{16}{3} \, \Omega, \quad \frac{16}{3} + 2 = \frac{22}{3} \, \Omega, \quad \frac{22}{3} (3)^2 = 66 \, \Omega$$
$$66 + 25 = 91 \, \Omega \quad \frac{100}{91} = 1.0989 \angle 0^\circ \, \text{A} = \bar{I}_1$$

$$(b) \quad \bar{I}_2 = 3\bar{I}_1 = 3.297 \angle 0^\circ \, \text{A}$$

$$(c) \quad \bar{I}_3 = -\frac{4}{3} \times 3.297 = 4.396 \angle 180^\circ \, \text{A}$$

$$(d) \quad P_{25} = 25 \times 1.0989^2 = 30.19 \, \text{W}$$

$$(e) \quad P_2 = 3.297^2 \times 2 = 21.74 \, \text{W}$$

$$(f) \quad P_3 = 4.396^2 \times 3 = 57.96 \, \text{W}$$

## CHAPTER THIRTEEN SOLUTIONS

41.

$$\bar{V}_1 = 2.5 \bar{V}_2, \bar{I}_1 = 0.4 \bar{I}_2, \bar{I}_{50} = \bar{I}_2 + 0.1 \bar{V}_2$$

$$\therefore 60 = 40(0.4 \bar{I}_2) - 2.5 \bar{V}_2 \quad \therefore \bar{I}_2 = \frac{60 + 2.5 \bar{V}_2}{16}$$

$$\text{Also, } 60 = 50(\bar{I}_2 + 0.1 \bar{V}_2) + \bar{V}_2 = 50 \bar{I}_2 + 6 \bar{V}_2$$

$$\therefore 60 = 50 \left( \frac{60 + 2.5 \bar{V}_2}{16} \right) + 6 \bar{V}_2 = 187.5 + (7.8125 + 6) \bar{V}_2$$

$$\therefore \bar{V}_2 = \frac{60 - 187.5}{13.8125} = \boxed{-9.231 \text{ V}}$$

## CHAPTER THIRTEEN SOLUTIONS

42.

$$\frac{400}{5^2} = 16 \, \Omega, 16 \parallel 48 = 12 \, \Omega, 12 + 4 = 16 \, \Omega$$

$$\frac{16}{2^2} = 4 \, \Omega \quad \therefore I_s = \frac{10}{4+1} = 2 \, \text{A} \quad \therefore P_1 = 4 \, \text{W}$$

$$\frac{2}{2} = 1 \, \text{A} \quad \therefore P_4 = 4 \, \text{W}, 10 - 2 \times 1 = 8 \, \text{V}$$

$$8 \times 2 = 16 \, \text{V}, 16 - 4 \times 1 = 12 \, \text{V}, 12^2 / 48 = 3 \, \text{W} = P_{48}, 12 \times 5 = 60 \, \text{V}$$

$$P_{400} = \frac{60^2}{400} = 9 \, \text{W}$$

## CHAPTER THIRTEEN SOLUTIONS

43.

$$I_1 = 2I_2, 2I_2 = I_s + I_x \quad \therefore I_x + I_s - 2I_2 = 0$$

$$100 = 3I_s + \frac{1}{2}(4I_2 + 20I_2 - 20I_x)$$

$$\therefore 10I_x - 3I_s - 12I_2 = -100$$

$$100 = 3I_s - 5I_x + 20I_2 - 20I_x$$

$$\therefore 25I_x - 3I_s - 20I_2 = -100$$

$$\therefore I_x = \frac{\begin{vmatrix} 0 & 1 & -2 \\ -100 & -3 & -12 \\ -100 & -3 & -20 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -2 \\ 10 & -3 & -12 \\ 25 & -3 & -20 \end{vmatrix}} = \frac{0 + 100(-26) - 100(-18)}{1(60 - 36) - 10(-20 - 6) + 25(-12 - 6)} = \frac{-800}{-166} = 4.819 \text{ A}$$

## CHAPTER THIRTEEN SOLUTIONS

44.

$$(a) \quad 50 \parallel 10 = \frac{25}{3} \, \Omega \quad \therefore V_{AB} = 1 \times 4 \times \frac{25}{3} = \frac{100}{3} \, \text{V}$$

$$\therefore P_{10AB} = \left( \frac{100}{3} \right)^2 \frac{1}{10} = \frac{1000}{9} = \boxed{111.11 \, \text{W}}$$

$$V_{CD} = 1 \times 3 \times \frac{25}{3} = 25 \, \text{V}, \quad P_{10CD} = \frac{25^2}{10} = \boxed{62.5 \, \text{W}}$$

(b) Specify 3 A and 4 A in secondaries

$$I_{AB} = I_f + 4$$

$$I_{CD} = -I_b - 3 \quad \therefore \frac{25}{3}(I_f + 4) = \frac{25}{3}(-I_f - 3)$$

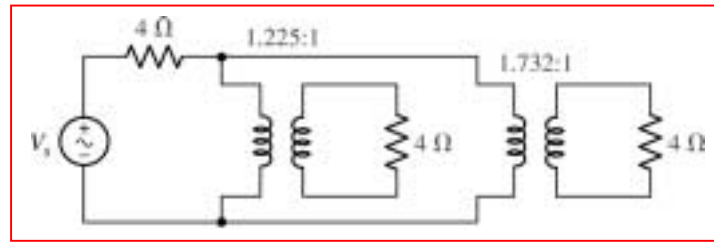
$$\therefore 2I_f = -7, \quad I_f = -3.5 \, \text{A}$$

$$\therefore V_{AB} = V_{CD} = \frac{25}{3}(-3.5 + 4) = \frac{25}{6} \, \text{V}$$

$$\therefore P_{10AB} = P_{10CD} = \left( \frac{25}{6} \right)^2 \frac{1}{10} = \boxed{1.7361 \, \text{W}}$$

## CHAPTER THIRTEEN SOLUTIONS

45. **Corrections required to the problem text:** both speakers that comprise the load are  $4\text{-}\Omega$  devices. We desire a circuit that will connect the signal generator (whose Thévenin resistance is  $4\text{ }\Omega$ ) to the individual speakers such that one speaker receives twice the power delivered to the other. One possible solution of many:



We can see from analysing the above circuit that the voltage across the right-most speaker will be  $\frac{1.732}{1.225}$  or  $\sqrt{2}$  times that across the left speaker. Since power is proportional to voltage *squared*, twice as much power is delivered to the right speaker.



## CHAPTER THIRTEEN SOLUTIONS

46. (a) We assume  $\mathbf{V}_{\text{secondary}} = 230\angle 0^\circ \text{ V}$  as a phasor reference. Then,

$$\mathbf{I}_{\text{unity PF load}} = \frac{8000}{230} \angle 0^\circ = 34.8 \angle 0^\circ \text{ A} \quad \text{and}$$

$$\mathbf{I}_{0.8 \text{ PF load}} = \frac{15000}{230} \angle (-\cos^{-1} 0.8) = 65.2 \angle -36.9^\circ \text{ A}$$

$$\begin{aligned} \text{Thus, } \mathbf{I}_{\text{primary}} &= \frac{230}{2300} (34.8 \angle 0^\circ + 65.2 \angle -36.9^\circ) \\ &= 0.1 (86.9 - j39.1) = \boxed{9.5 \angle -24.3^\circ \text{ A}} \end{aligned}$$

- (b) The magnitude of the secondary current is limited to  $25 \times 10^3 / 230 = 109 \text{ A}$ .  
If we include a new load operating at 0.95 PF lagging, whose current is

$$\mathbf{I}_{0.95 \text{ PF load}} = |\mathbf{I}_{0.95 \text{ PF load}}| \angle (-\cos^{-1} 0.95) = |\mathbf{I}_{0.95 \text{ PF load}}| \angle -18.2^\circ \text{ A},$$

then the new total secondary current is

$$86.9 - j39.1 + |\mathbf{I}_{0.95 \text{ PF load}}| \cos 18.2^\circ - j |\mathbf{I}_{0.95 \text{ PF load}}| \sin 18.2^\circ \text{ A}.$$

Thus, we may equate this to the maximum rated current of the secondary:

$$109 = \sqrt{(86.9 + |\mathbf{I}_{0.95 \text{ PF load}}| \cos 18.2^\circ)^2 + (39.1 + |\mathbf{I}_{0.95 \text{ PF load}}| \sin 18.2^\circ)^2}$$

Solving, we find

$$|\mathbf{I}_{0.95 \text{ PF load}}|^2 = \frac{-189 \pm \sqrt{189^2 + (4)(2800)}}{2}$$

So,  $|\mathbf{I}_{0.95 \text{ PF load}}| = 13.8 \text{ A}$  (or  $-203 \text{ A}$ , which is nonsense).

This transformer, then, can deliver to the additional load a power of

$$13.8 \times 0.95 \times 230 = \boxed{3 \text{ kW}}.$$

## CHAPTER THIRTEEN SOLUTIONS

47. After careful examination of the circuit diagram, we (fortunately or unfortunately) determine that the meter determines individual IQ based on age alone. A simplified version of the circuit then, is simply a 120 V ac source, a 28.8-k $\Omega$  resistor and a  $(24^2)R_A$  resistor all connected in series. The IQ result is equal to the power (W) dissipated in resistor  $R_A$  divided by 1000.

$$P = \left( \frac{120}{28.8 \times 10^3 + 576R_A} \right)^2 \times 576R_A$$

$$\text{Thus, } IQ = \frac{1}{1000} \left( \frac{120}{28.8 \times 10^3 + 576 \times \text{Age}} \right)^2 \times 576 \times \text{Age}$$

- (a) Implementation of the above equation with a given age will yield the “measured” IQ.
- (b) The maximum IQ is achieved when maximum power is delivered to resistor  $R_A$ , which will occur when  $576R_A = 28.8 \times 10^3$ , or the person’s age is 50 years.
- (c) Well, now, this arguably depends on your answer to part (a), and your own sense of ethics. Hopefully you’ll do the right thing, and simply write to the Better Business Bureau. And watch less television.

## CHAPTER THIRTEEN SOLUTIONS

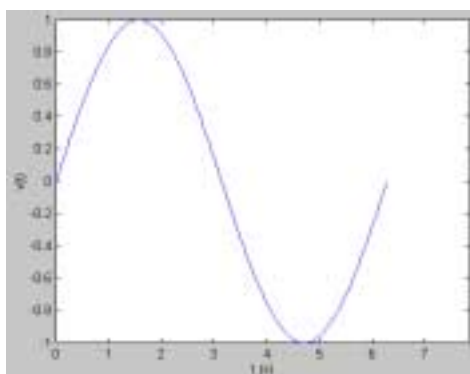
48. We require a transformer that converts 240 V ac to 120 V ac, so that a turns ratio of 2:1 is needed. We attach a male european plug to the primary coil, and a female US plug to the secondary coil. Unfortunately, we are not given the current requirements of the CD writer, so that we will have to over-rate the transformer to ensure that it doesn't overheat. Checking specifications on the web for an example CD writer, we find that the power supply provides a dual DC output: 1.2 A at 5 V, and 0.8 A at 12 V. This corresponds to a total DC power delivery of 15.6 W. Assuming a moderately efficient ac to DC converter is being used (*e.g.* 80% efficient), the unit will draw approximately  $15.6/0.8$  or 20 W from the wall socket. Thus, the secondary coil should be rated for at least that (let's go for 40 W, corresponding to a peak current draw of about 333 mA). Thus, we include a 300-mA fuse in series with the secondary coil and the US plug for safety.

## CHAPTER THIRTEEN SOLUTIONS

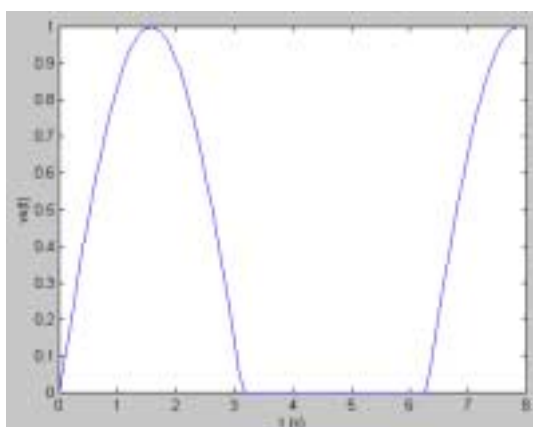
49. You need to purchase (and wire in) a three-phase transformer rated at  $(\sqrt{3})(208)(10) = 3.6 \text{ kVA}$ . The turns ratio for each phase needs to be 400:208 or 1.923.

## CHAPTER THIRTEEN SOLUTIONS

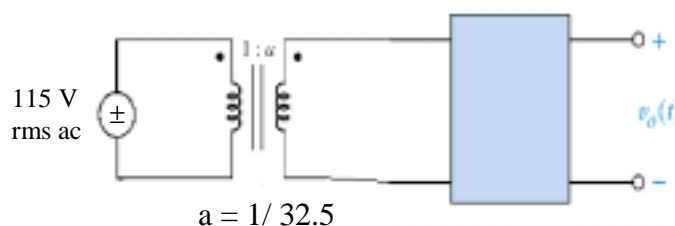
50. (a) The input to the left of the unit will have the shape:



and the output voltage will be:



We need to reduce the magnitude from 115-V (rms) to a peak voltage of 5 V. The corresponding peak voltage at the input will be  $115\sqrt{2} = 162.6$  V, so we require a transformer with a turns ratio of 162.6:5 or about 32.5:1, connected as shown:



(b) If we wish to reduce the “ripple” in the output voltage, we can connect a capacitor in parallel with the output terminals. The necessary size will depend on the maximum allowable ripple voltage and the minimum anticipated load resistance. When the input voltage swings negative and the output voltage tries to reduce to follow, current will flow out of the capacitor to reduce the amount of voltage drop that would otherwise occur.

## CHAPTER FOURTEEN SOLUTIONS

1. (a)  $s = 0$ ;
- (b)  $s = \pm j9 \text{ s}^{-1}$ ;
- (c)  $s = -8 \text{ s}^{-1}$ ;
- (d)  $s = -1000 \pm j1000 \text{ s}^{-1}$ ;
- (e)  $v(t) = 8 + 2 \cos 2t \text{ mV}$  cannot be attributed a single complex frequency. In a circuit analysis problem, superposition will need to be invoked, where the original function  $v(t)$  is expressed as  $v(t) = v_1(t) + v_2(t)$ , with  $v_1(t) = 8 \text{ mV}$  and  $v_2(t) = 2 \cos 2t \text{ mV}$ . The complex frequency of  $v_1(t)$  is  $s = 0$ , and the complex frequency of  $v_2(t)$  is  $s = \pm j2 \text{ s}^{-1}$ .

## CHAPTER FOURTEEN SOLUTIONS

2. (a)  $(6 - j)^* = 6 + j$
- (b)  $(9)^* = 9$
- (c)  $(-j30)^* = +j30$
- (d)  $(5 e^{-j6})^* = 5 e^{+j6}$
- (e)  $(24 \angle -45^\circ)^* = 24 \angle 45^\circ$
- (f)  $\left( \frac{4 - j18}{3.33 + j} \right)^* = \left( \frac{4 + j18}{3.33 - j} \right) = \frac{18.44 \angle 77.47^\circ}{3.477 \angle -16.72^\circ} = 5.303 \angle 94.19^\circ$
- (g)  $\left( \frac{5 \angle 0.1^\circ}{4 - j7} \right)^* = \left( \frac{5 \angle 0.1^\circ}{8.062 \angle -60.26^\circ} \right)^* = (0.6202 \angle 60.36^\circ)^* = 0.6202 \angle -60.36^\circ$
- (h)  $(4 - 22 \angle 92.5^\circ)^* = (4 + 0.9596 - j21.98)^* = (4.9596 - j21.98)^* = 4.9596 + j21.98$

## CHAPTER FOURTEEN SOLUTIONS

3.  $\text{Re } \bar{i}(t) = i(t)$ . No units provided.

$$\begin{aligned} \text{(a)} \quad \bar{i}_x(t) &= (4 - j7)e^{(-3+j15)t} = (8.062\angle -60.26^\circ)e^{-3t}e^{j15t} = 8.062e^{-3t}e^{j(15t-60.26^\circ)} \\ \therefore i_x(t) &= \text{Re } \bar{i}_x(t) = 8.062e^{-3t} \cos(15t - 60.26^\circ) \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \bar{i}_y(t) &= (4 + j7)e^{-3t}(\cos 15t - j \sin 15t) = 8.062e^{-3t}e^{-j15t+j60.26^\circ} \\ \therefore i_y(t) &= 8.062e^{-3t} \cos(15t - 60.26^\circ) \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad \bar{i}_A(t) &= (5 - j8)e^{(-1.5t+j12)t} = 9.434e^{-j57.99^\circ}e^{-1.5t}e^{j12t} = 9.434e^{-1.5t}e^{j(12t-57.99^\circ)} \\ \therefore \text{Re } \bar{i}_A(0.4) &= 9.434e^{-0.6} \cos(4.8^{\text{rad}} - 57.99^\circ) = -4.134 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad \bar{i}_B(t) &= (5 + j8)e^{(-1.5t+j12)t} = 9.434e^{j57.99^\circ}e^{-1.5t}e^{-j12t} = 9.434e^{-1.5t}e^{-j(12t-57.99^\circ)} \\ \therefore \text{Re } \bar{i}_B(0.4) &= -4.134 \end{aligned}$$



## CHAPTER FOURTEEN SOLUTIONS

4. (a)  $\omega = 279 \text{ Mrad/s}$ , and  $\omega = 2 \pi f$ . Thus,  $f = \omega/2\pi = 44.4 \text{ MHz}$

(b) If the current  $i(t) = 2.33 \cos(279 \times 10^6 t) \text{ fA}$  flows through a precision  $1\text{-T}\Omega$  resistor, the voltage across the resistor will be  $10^{12} i(t) = 2.33 \cos(279 \times 10^6 t) \text{ mV}$ . We may write this as  $0.5(2.33) \cos(279 \times 10^6 t) + j(0.5)2.33 \sin(279 \times 10^6 t) + 0.5(2.33) \cos(279 \times 10^6 t) - j(0.5)2.33 \sin(279 \times 10^6 t) \text{ mV}$

$$= 1.165 e^{j279 \times 10^6 t} + 1.165 e^{-j279 \times 10^6 t} \text{ mV}$$

## CHAPTER FOURTEEN SOLUTIONS

5. (a)  $\mathbf{v}_s(0.1) = (20 - j30) e^{(-2 + j50)(0.1)} = (36.06 \angle -56.31^\circ) e^{(-0.2 + j5)}$   
 $= 36.06 e^{-0.2} \angle [-56.31^\circ + j5(180)/\pi] = 29.52 \angle 230.2^\circ \text{ V}$  (or  $29.52 \angle -129.8^\circ \text{ V}$ ).
- (b)  $\text{Re}\{ \mathbf{v}_s \} = 36.06 e^{-2t} \cos(50t - 56.31^\circ) \text{ V}.$
- (c)  $\text{Re}\{ \mathbf{v}_s(0.1) \} = 29.52 \cos(230.2^\circ) = -18.89 \text{ V}.$
- (d) The complex frequency of this waveform is  $\mathbf{s} = -2 + j50 \text{ s}^{-1}$
- (e)  $\mathbf{s}^* = (-2 + j50)^* = -2 - j50 \text{ s}^{-1}$

## CHAPTER FOURTEEN SOLUTIONS

6. (a)  $s = 0 + j120\pi = +j120\pi$

(b) We first construct an s-domain voltage  $\mathbf{V}(s) = 179 \angle 0^\circ$  with  $s$  given above. The equation for the circuit is

$$v(t) = 100 i(t) + L \frac{di}{dt} = 100 i(t) + 500 \times 10^{-6} \frac{di}{dt}$$

and we assume a response of the form  $\mathbf{I}e^{st}$ .

Substituting, we write  $(179 \angle 0^\circ) e^{st} = 100 \mathbf{I}e^{st} + sL \mathbf{I}e^{st}$

Suppressing the exponential factor, we may write

$$\mathbf{I} = \frac{179 \angle 0^\circ}{100 + s500 \times 10^{-6}} = \frac{179 \angle 0^\circ}{100 + j120\pi(500 \times 10^{-6})} = \frac{179 \angle 0^\circ}{100 \angle 0.108^\circ} = 1.79 \angle -0.108^\circ \text{ A}$$

Converting back to the time domain, we find that

$$i(t) = 1.79 \cos(120\pi t - 0.108^\circ) \text{ A.}$$

## CHAPTER FOURTEEN SOLUTIONS

7.

(a)  $v_s = 10e^{-2t} \cos(10t + 30^\circ) \text{ V} \therefore s = -2 + j10, \bar{V}_s = 10\angle 30^\circ \text{ V}$

$$\bar{Z}_c = \frac{10}{-2 + j10} = \frac{5}{-1 + j5} \frac{-1 - j5}{26} = \frac{-5 - j25}{26}, \bar{Z}_c \parallel 5 = \frac{(-25 - j125)/26}{(-5 - j25 + 130)/26}$$

$$\therefore \bar{Z}_c \parallel 5 = \frac{-25 - j125}{125 - j25} = \frac{-1 - j5}{5 - j1} = -j1 \therefore \bar{Z}_{in} = 5 + 0.5(-2 + j10) - j1 = 4 + j4 \Omega$$

$$\therefore \bar{I}_x = \frac{10\angle 30^\circ}{4 + j4} \times \frac{(-5 - j25)/26}{5 + (-5 - j25)/26} = \frac{10\angle 30^\circ}{4 + j4} \frac{-5 - j25}{130 - 5 - j25} = \frac{5\angle 30^\circ}{2 + j2} \frac{-5 - j25}{125 - j25} = \frac{1\angle 30^\circ}{2 + j2} \frac{-1 - j5}{5 - j1}$$

$$\therefore \bar{I}_x = \frac{1\angle 30^\circ}{2\sqrt{2}\angle 45^\circ} (-j1) = \boxed{0.3536\angle -105^\circ \text{ A}}$$

(b)  $i_x(t) = \boxed{0.3536e^{-2t} \cos(10t - 105^\circ) \text{ A}}$

## CHAPTER FOURTEEN SOLUTIONS

8. (a)  $s = 0 + j100\pi = +j100\pi$

(b) We first construct an s-domain voltage  $\mathbf{V}(s) = 339 \angle 0^\circ$  with  $s$  given above. The equation for the circuit is

$$v(t) = 2000 i(t) + v_C(t) = 2000 C \frac{dv_C}{dt} + v_C(t) = 0.2 \frac{dv_C}{dt} + v_C(t)$$

and we assume a response of the form  $\mathbf{V}_C e^{st}$ .

Substituting, we write  $(339 \angle 0^\circ) e^{st} = 0.2s \mathbf{V}_C e^{st} + \mathbf{V}_C e^{st}$

Suppressing the exponential factor, we may write

$$\mathbf{V}_C = \frac{339 \angle 0^\circ}{1 + 0.2s} = \frac{339 \angle 0^\circ}{1 + j100\pi(0.2)} = \frac{339 \angle 0^\circ}{62.84 \angle 89.09^\circ} = 5.395 \angle -89.09^\circ \text{ A}$$

Converting back to the time domain, we find that

$$v_C(t) = 5.395 \cos(100\pi t - 89.09^\circ) \text{ V.}$$

and so the current is  $i(t) = C \frac{dv_C}{dt} = -0.1695 \sin(100\pi t) \text{ A} = 169.5 \sin(100\pi t) \text{ mA.}$

## CHAPTER FOURTEEN SOLUTIONS

9.  $i_{s1} = 20e^{-3t} \cos 4t \text{ A}, i_{s2} = 30e^{-3t} \sin 4t \text{ A}$

(a)  $\bar{I}_{s1} = 20\angle 0^\circ, \bar{I}_{s2} = -j30, \bar{s} = -3 + j4$   
 $\therefore \bar{Z}_c = \frac{10}{-3 + j4} \frac{-3 - j4}{-3 - j4} = 0.4(-3 - j4) = -1.2 - j1.6, \bar{Z}_L = -6 + j8$   
 $\therefore \bar{V}_x = 20 \frac{5(7.2 + j6.4)}{-2.2 + j6.4} \times \frac{-6 + j8}{-7.2 + j6.4} - j30 \frac{(-6 + j8)(3.8 - j1.6)}{-2.2 + j6.4}$   
 $= \frac{-600 + j800 - j30(-22.8 + 12.8 + j30.4 + j9.6)}{-2.2 + j6.4} = \frac{-600 + j800 - j30(-10 + j40)}{-2.2 + j6.4}$   
 $= \frac{-600 + 1200 + j1000}{-2.2 + j6.4} = \frac{600 + j1000}{-2.2 + j6.4} = 185.15 \angle -47.58^\circ \text{ V}$

(b)  $v_x(t) = 185.15 e^{-3t} \cos(4t - 47.58^\circ) \text{ V}$

## CHAPTER FOURTEEN SOLUTIONS

10. (a) If  $v(t) = 240\sqrt{2} e^{-2t} \cos 120\pi t$  V, then  $\mathbf{V} = 240\sqrt{2} \angle 0^\circ$  V where  $\mathbf{s} = -2 + j120\pi$ .

Since  $R = 3 \text{ m}\Omega$ , the current is simply  $\mathbf{I} = \frac{240\sqrt{2} \angle 0^\circ}{3 \times 10^{-3}} = 113.1 \angle 0^\circ \text{ kA}$ . Thus,

$$i(t) = 113.1e^{-2t} \cos 120\pi t \text{ kA}$$

- (b) Working in the time domain, we may directly compute

$$i(t) = v(t) / 3 \times 10^{-3} = (240\sqrt{2} e^{-2t} \cos 120\pi t) / 3 \times 10^{-3} = 113.1e^{-2t} \cos 120\pi t \text{ kA}$$

- (c) A 1000-mF capacitor added to this circuit corresponds to an impedance

$$\frac{1}{\mathbf{s}C} = \frac{1}{(-2 + j120\pi)(1000 \times 10^{-3})} = \frac{1}{-2 + j120\pi} \Omega \text{ in parallel with the } 3\text{-m}\Omega$$

resistor. However, since the capacitor has been added in parallel (it would have been more interesting if the connection were in series), the same voltage still appears across its terminals, and so

$$i(t) = 113.1e^{-2t} \cos 120\pi t \text{ kA} \text{ as before.}$$

## CHAPTER FOURTEEN SOLUTIONS

$$\begin{aligned}
 11. \quad \mathbb{L}\{K u(t)\} &= \int_{0^-}^{\infty} K e^{-st} u(t) dt = K \int_{0^-}^{\infty} e^{-st} u(t) dt = K \int_0^{\infty} e^{-st} dt = \left. \frac{-K}{s} e^{-st} \right|_0^{\infty} \\
 &= \lim_{t \rightarrow \infty} \left( \frac{-K}{s} e^{-st} \right) + \lim_{t \rightarrow 0} \left( \frac{K}{s} e^{-st} \right)
 \end{aligned}$$

If the integral is going to converge, then  $\lim_{t \rightarrow \infty} (e^{-st}) = 0$  (i.e.  $s$  must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathbb{L}\{K u(t)\} = \boxed{\frac{K}{s}}$$



## CHAPTER FOURTEEN SOLUTIONS

$$\begin{aligned}
 12. \quad (a) \quad \mathcal{L}\{3u(t)\} &= \int_{0^-}^{\infty} 3e^{-st}u(t)dt = 3\int_{0^-}^{\infty} e^{-st}u(t)dt = 3\int_0^{\infty} e^{-st}dt = \left. \frac{-3}{s}e^{-st} \right|_0^{\infty} \\
 &= \lim_{t \rightarrow \infty} \left( \frac{-3}{s}e^{-st} \right) + \lim_{t \rightarrow 0} \left( \frac{3}{s}e^{-st} \right)
 \end{aligned}$$

If the integral is going to converge, then  $\lim_{t \rightarrow \infty} (e^{-st}) = 0$  (i.e.  $s$  must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathcal{L}\{3u(t)\} = \boxed{\frac{3}{s}}$$

$$\begin{aligned}
 (b) \quad \mathcal{L}\{3u(t-3)\} &= \int_{0^-}^{\infty} 3e^{-st}u(t-3)dt = 3\int_3^{\infty} e^{-st}dt = \left. \frac{-3}{s}e^{-st} \right|_3^{\infty} \\
 &= \lim_{t \rightarrow \infty} \left( \frac{-3}{s}e^{-st} \right) + \left( \frac{3}{s}e^{-3s} \right)
 \end{aligned}$$

If the integral is going to converge, then  $\lim_{t \rightarrow \infty} (e^{-st}) = 0$  (i.e.  $s$  must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathcal{L}\{3u(t-3)\} = \boxed{\frac{3}{s}e^{-3s}}$$

$$\begin{aligned}
 (c) \quad \mathcal{L}\{3u(t-3)-3\} &= \int_{0^-}^{\infty} [3u(t-3)-3]e^{-st}dt = 3\int_3^{\infty} e^{-st}dt - 3\int_0^{\infty} e^{-st}dt \\
 &= \left. \frac{-3}{s}e^{-st} \right|_3^{\infty} - \left. \frac{-3}{s}e^{-st} \right|_0^{\infty}
 \end{aligned}$$

Based on our answers to parts (a) and (b), we may write

$$\mathcal{L}\{3u(t-3)-3\} = \frac{3}{s}e^{-3s} - \frac{3}{s} = \boxed{\frac{3}{s}(e^{-3s} - 1)}$$

$$\begin{aligned}
 (d) \quad \mathcal{L}\{3u(3-t)\} &= 3\int_{0^-}^{\infty} e^{-st}u(3-t)dt = 3\int_0^3 e^{-st}dt = \left. \frac{-3}{s}e^{-st} \right|_0^3 \\
 &= \left. \frac{-3}{s}(e^{-3s} - 1) \right| = \boxed{\frac{3}{s}(1 - e^{-3s})}
 \end{aligned}$$

## CHAPTER FOURTEEN SOLUTIONS

$$\begin{aligned}
 13. \quad (a) \quad \mathcal{L}\{2 + 3u(t)\} &= \int_{0^-}^{\infty} e^{-st} [2 + 3u(t)] dt = \int_0^{\infty} 5e^{-st} dt = \left. \frac{-5}{s} e^{-st} \right|_0^{\infty} \\
 &= \lim_{t \rightarrow \infty} \left( \frac{-5}{s} e^{-st} \right) + \lim_{t \rightarrow 0} \left( \frac{5}{s} e^{-st} \right)
 \end{aligned}$$

If the integral is going to converge, then  $\lim_{t \rightarrow \infty} (e^{-st}) = 0$  (i.e.  $s$  must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathcal{L}\{2 + 3u(t)\} = \boxed{\frac{5}{s}}$$

$$\begin{aligned}
 (b) \quad \mathcal{L}\{3e^{-8t}\} &= \int_{0^-}^{\infty} 3e^{-8t} e^{-st} dt = \int_{0^-}^{\infty} 3e^{-(8+s)t} dt = \left. \frac{-3}{s+8} e^{-(8+s)t} \right|_{0^-}^{\infty} \\
 &= \lim_{t \rightarrow \infty} \left( \frac{-3}{s+8} e^{-(s+8)t} \right) + \lim_{t \rightarrow 0} \left( \frac{3}{s+8} e^{-(s+8)t} \right) = 0 + \frac{3}{s+8} = \boxed{\frac{3}{s+8}}
 \end{aligned}$$

$$(c) \quad \mathcal{L}\{u(-t)\} = \int_{0^-}^{\infty} e^{-st} u(-t) dt = \int_{0^-}^0 e^{-st} u(-t) dt = \int_{0^-}^0 (0) e^{-st} dt = \boxed{0}$$

$$\begin{aligned}
 (d) \quad \mathcal{L}\{K\} &= \int_{0^-}^{\infty} K e^{-st} dt = K \int_{0^-}^{\infty} e^{-st} dt = K \int_0^{\infty} e^{-st} dt = \left. \frac{-K}{s} e^{-st} \right|_0^{\infty} \\
 &= \lim_{t \rightarrow \infty} \left( \frac{-K}{s} e^{-st} \right) + \lim_{t \rightarrow 0} \left( \frac{K}{s} e^{-st} \right)
 \end{aligned}$$

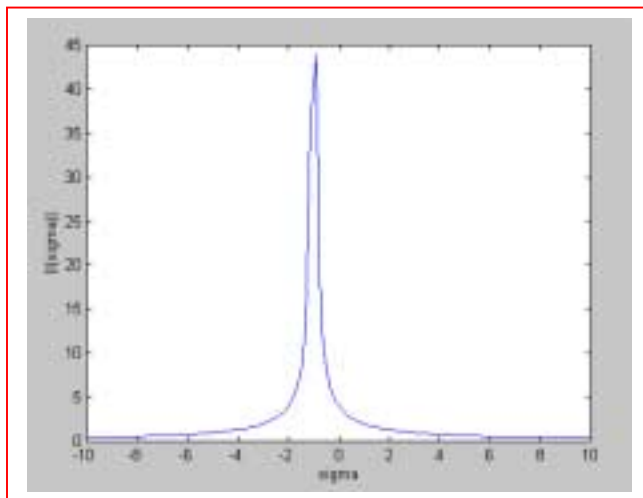
If the integral is going to converge, then  $\lim_{t \rightarrow \infty} (e^{-st}) = 0$  (i.e.  $s$  must be finite). This leads to the first term dropping out (l'Hospital's rule assures us of this), and so

$$\mathcal{L}\{K\} = \boxed{\frac{K}{s}}$$

## CHAPTER FOURTEEN SOLUTIONS

14. (a) The frequency-domain representation of the voltage across the resistor is  $(1)\mathbf{I}(s)$  where  $\mathbf{I}(s) = \mathcal{L}\{4e^{-t} u(t)\} = \frac{4}{s+1}$  A. Thus, the voltage is  $\frac{4}{s+1}$  V.

(b)



## CHAPTER FOURTEEN SOLUTIONS

15. (a)

$$\begin{aligned}\mathcal{L}\{5u(t) - 5u(t-2)\} &= \int_0^\infty [5u(t) - 5u(t-2)]e^{-st} dt \\ &= 5 \int_0^\infty e^{-st} dt - 5 \int_2^\infty e^{-st} dt = \left. \frac{-5}{s} e^{-st} \right|_0^\infty + \left. \frac{5}{s} e^{-st} \right|_2^\infty \\ &= \lim_{t \rightarrow \infty} \left( \frac{-5}{s} e^{-st} \right) + \lim_{t \rightarrow 0} \left( \frac{5}{s} e^{-st} \right) + \lim_{t \rightarrow \infty} \left( \frac{-5}{s} e^{-st} \right) + \left( \frac{5}{s} e^{-2s} \right)\end{aligned}$$

If the integral is going to converge, then  $\lim_{t \rightarrow \infty} (e^{-st}) = 0$  (i.e.  $s$  must be finite). This leads to the first and third terms dropping out (l'Hospital's rule assures us of this), and so

$$\mathcal{L}\{5u(t) - 5u(t-2)\} = \boxed{\frac{5}{s}(1 + e^{-2s})}$$

(b) The frequency domain current is simply one ohm times the frequency domain voltage, or

$$\boxed{\frac{5}{s}(1 + e^{-2s})}$$

## CHAPTER FOURTEEN SOLUTIONS

16.

(a)  $f(t) = t + 1 \therefore F(s) = \int_{0^-}^{\infty} (t + 1) e^{-(\sigma + j\omega)t} dt \therefore \sigma > 0$

(b)  $f(t) = (t + 1) u(t) \therefore F(s) = \int_{0^-}^{\infty} (t + 1) e^{-(\sigma + j\omega)t} dt \therefore \sigma > 0$

(c)  $f(t) = e^{50t} u(t) \therefore F(s) = \int_{0^-}^{\infty} e^{50t} e^{-(\sigma + j\omega)t} dt \therefore \sigma > 50$

(d)  $f(t) = e^{50t} u(t - 5) \therefore F(s) = \int_{0^-}^{\infty} e^{50t} u(t - 5) e^{-(\sigma + j\omega)t} dt \therefore \sigma > 50$

(e)  $f(t) = e^{-50t} u(t - 5) \therefore F(s) = \int_{0^-}^{\infty} e^{-50t} u(t - 5) e^{-(\sigma + j\omega)t} dt \therefore \sigma > 0$

## CHAPTER FOURTEEN SOLUTIONS

17.

(a)  $f(t) = 8e^{-2t} [u(t+3) - u(t-3)] \therefore F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{j\omega t} dt$

$$\therefore F(j\omega) = \int_{-3}^3 8e^{(2+j\omega)t} dt = \frac{8}{2+j\omega} [e^{6+j3\omega} - e^{-6-j3\omega}]$$

$$F_{(2)}(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt = \int_{-3}^3 8e^{-(2+s)t} dt = \frac{8}{2+s} [e^{6+3s} - e^{-6-3s}]$$

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt = \int_{0^-}^{\infty} 8e^{-(2+s)t} dt = \boxed{\frac{8}{2+s} [1 - e^{-6-3s}]}$$

(b)  $f(t) = 8e^{2t} [u(t+3) - u(t-3)] \quad F(j\omega) = \int_{-3}^3 8e^{(2-j\omega)t} dt$

$$= \frac{8}{2-j\omega} [e^{6-j3\omega} - e^{-6+j3\omega}] \quad F_{(2)}(s) = \int_{-3}^3 8e^{(2-s)t} dt$$

$$= \frac{8}{s-2} [e^{-6+3s} - e^{6-3s}], \quad F(s) = \int_{0^-}^{\infty} 8e^{(2-s)t} dt$$

$$= \frac{8}{2-s} [e^{6-3s} - 1] = \boxed{\frac{8}{s-2} [1 - e^{6-3s}]}$$

(c)  $f(t) = 8e^{-2|t|} [u(t-3) - u(t-3)] \therefore F(j\omega) = \int_{-3}^3 8e^{-2|t|} e^{-j\omega t} dt$

$$\therefore F(j\omega) = \int_{-3}^0 8e^{(2-j\omega)t} dt + \int_0^3 8e^{(-2-j\omega)t} dt = \frac{8}{2-j\omega} [1 - e^{6+j3\omega}] + \frac{8}{2+j\omega} [1 - e^{-6-j3\omega}]$$

$$F_{(2)}(s) = \int_{-3}^3 8e^{-2|t|-st} dt = \int_{-3}^0 8e^{(2-s)t} dt$$

$$+ \int_0^3 8e^{(-2-s)t} dt \therefore F_{(2)}^{(s)} = \frac{8}{2-s} [-e^{-6+3s}] + \frac{8}{2+s} [1 - e^{-6-3s}]$$

$$F(s) = \int_{0^-}^3 8e^{(-2-s)t} dt = \boxed{\frac{8}{s+2} [1 - e^{-6-3s}]}$$

## CHAPTER FOURTEEN SOLUTIONS

$$18. \quad (a) \quad \mathcal{L}\left\{\mathcal{L}^{-1}\left(\frac{1}{s}\right)\right\} = \boxed{\frac{1}{s}}$$

$$(b) \quad \mathcal{L}\{1 + u(t) + [u(t)]^2\} = \frac{1}{s} + \frac{1}{s} + \frac{1}{s} = \boxed{\frac{3}{s}}$$

$$(c) \quad \mathcal{L}\{t u(t) - 3\} = \boxed{\frac{1}{s^2} - \frac{3}{s}}$$

$$(d) \quad \mathcal{L}\{1 - \delta(t) + \delta(t-1) - \delta(t-2)\} = \boxed{\frac{1}{s} - 1 + e^{-s} - e^{-2s}}$$

## CHAPTER FOURTEEN SOLUTIONS

19. (a)  $f(t) = e^{-3t} u(t)$

(b)  $f(t) = \delta(t)$

(c)  $f(t) = t u(t)$

(d)  $f(t) = 275 u(t)$

(e)  $f(t) = u(t)$



## CHAPTER FOURTEEN SOLUTIONS

20.

$$\begin{aligned}\mathcal{L}\{f_1(t) + f_2(t)\} &= \int_{0^-}^{\infty} [f_1(t) + f_2(t)]e^{-st} dt = \int_{0^-}^{\infty} f_1(t)e^{-st} dt + \int_{0^-}^{\infty} f_2(t)e^{-st} dt \\ &= \mathcal{L}\{f_1(t)\} + \mathcal{L}\{f_2(t)\}\end{aligned}$$

## CHAPTER FOURTEEN SOLUTIONS

21.

$$(a) \quad f(t) = 2u(t-2) \quad \therefore F(s) = 2 \int_2^{\infty} e^{-st} dt + \frac{-2}{s} e^{st} \bigg|_2^{\infty} = \frac{2}{s} e^{-2s}; \quad s = 1 + j2$$

$$\therefore F(1 + j2) = \frac{2}{1 + j2} e^{-2} e^{-j4} = \boxed{0.04655 + j0.11174}$$

$$(b) \quad f(t) = 2\delta(t-2) \quad \therefore F(s) = 2e^{-2s}, \quad F(1 + j2) = 2e^{-2} e^{-j4} = \boxed{-0.17692 + j0.2048}$$

$$(c) \quad f(t) = e^{-t} u(t-2) \quad \therefore F(s) = \int_2^{\infty} e^{-(s+1)t} dt = \frac{1}{-s+1} e^{-(s+1)t} \bigg|_2^{\infty} = \frac{1}{s+1} e^{-2s-2}$$

$$\therefore F(1 + j2) = \frac{1}{2 + j2} e^{-2} e^{-2} e^{-j4} = \boxed{(0.4724 + j6.458)10^{-3}}$$

## CHAPTER FOURTEEN SOLUTIONS

$$22. \quad (a) \int_{-\infty}^{\infty} 8 \sin 5t \delta(t-1) dt = 8 \sin 5 \times 1 = -7.671$$

$$(b) \int_{-\infty}^{\infty} (t-5)^2 \delta(t-2) dt = (2-5)^2 = 9$$

$$(c) \int_{-\infty}^{\infty} 5e^{-3000t} \delta(t-3.333 \times 10^{-4}) dt = 5e^{-3000(3.333 \times 10^{-4})} = 1.840$$

$$(d) \int_{-\infty}^{\infty} K \delta(t-2) dt = K$$

## CHAPTER FOURTEEN SOLUTIONS

23.

$$(a) \quad f(t) = [u(5-t)][u(t-2)]u(t), \therefore F(s) = \int_{0^-}^{\infty} [u(5-t)][u(t-2)]u(t)e^{-st} dt$$

$$\therefore F(s) = \int_2^5 e^{-st} dt = -\frac{1}{s} e^{-st} \Big|_2^5 = \frac{1}{s} (e^{-2s} - e^{-5s})$$

$$(b) \quad f(t) = 4u(t-2) \therefore F(s) = 4 \int_2^{\infty} e^{-st} dt = \frac{4}{s} e^{-2s}$$

$$(c) \quad f(t) = 4e^{-3t}u(t-2) \therefore F(s) = 4 \int_2^{\infty} e^{-(s+3)t} dt = \frac{-4}{s+3} e^{-(s+3)t} \Big|_2^{\infty}$$

$$\therefore F(s) = \frac{4}{s+3} e^{-2s-6}$$

$$(d) \quad f(t) = 4\delta(t-2) \therefore F(s) = 4 \int_{0^-}^{\infty} \delta(t-2) e^{-st} dt = 4 \int_2^{2^+} e^{-2s} \delta(t-2) dt = 4e^{-2s}$$

$$(e) \quad f(t) = 5\delta(t) \sin(10t + 0.2\pi) \therefore F(s) = 5 \int_{0^-}^{0^+} \delta(t) [\sin 0.2\pi] \times 1 dt = 5 \sin 36^\circ$$

$$\therefore F(s) = 2.939$$

## CHAPTER FOURTEEN SOLUTIONS

$$24. \quad (a) \int_{-\infty}^{\infty} \cos 500t \delta(t) dt = \cos 500 \times 0 = 1$$

$$(b) \int_{-\infty}^{\infty} (t)^5 \delta(t-2) dt = (2)^5 = 32$$

$$(c) \int_{-\infty}^{\infty} 2.5e^{-0.001t} \delta(t-1000) dt = 2.5e^{-0.001(1000)} = 0.9197$$

$$(d) \int_{-\infty}^{\infty} -K^2 \delta(t-c) dt = -K^2$$

## CHAPTER FOURTEEN SOLUTIONS

25.

$$(a) \quad f(t) = 2 u(t-1) u(3-t) u(t^3)$$

$$\mathbf{F(s)} = \int_1^3 e^{-st} dt = -\frac{2}{s} e^{-st} \Big|_1^3 = \boxed{\frac{2}{s} (e^{-s} - e^{-3s})}$$

$$(b) \quad f(t) = 2u(t-4) \therefore F(s) = 2 \int_4^{\infty} e^{-st} dt = \frac{-2}{s} (0 - e^{-4s}) = \boxed{\frac{2}{s} e^{-4s}}$$

$$(c) \quad f(t) = 3e^{-2t} u(t-4) \therefore F(s) = 3 \int_4^{\infty} e^{-(s+2)t} dt = \boxed{\frac{3}{s+2} e^{-4s-8}}$$

$$(d) \quad f(t) = 3\delta(t-5) \therefore F(s) = 3 \int_0^{\infty} \delta(t-5) e^{-st} dt = \boxed{3e^{-5s}}$$

$$(e) \quad f(t) = 4\delta(t-1) [\cos \pi t - \sin \pi t]$$

$$\therefore F(s) = 4 \int_0^{\infty} \delta(t-1) [\cos \pi t - \sin \pi t] e^{-st} dt \therefore F(s) = \boxed{-4e^{-s}}$$

## CHAPTER FOURTEEN SOLUTIONS

26. (a)  $f(t) = 5 u(t) - 16 \delta(t) + e^{-4.4t} u(t)$

(b)  $f(t) = \delta(t) + u(t) + t u(t)$

(c)  $\mathbf{F}(s) = \frac{5}{s+7} + \frac{88}{s} + \frac{a}{s+6} + \frac{b}{s+1}$

where  $a = \left. \frac{17}{s+1} \right|_{s=-6} = -3.4$  and  $b = \left. \frac{17}{s+6} \right|_{s=-1} = 3.4$ .

Thus,

$$f(t) = 5 e^{-7t} u(t) + 88 u(t) - 3.4 e^{-6t} u(t) + 3.4 e^{-t} u(t)$$

Check with MATLAB:

EDU» T1 = '5/(s+7)';

EDU» T2 = '88/s';

EDU» T3 = '17/(s^2 + 7\*s + 6)';

EDU» T = symadd(T1,T2);

EDU» P = symadd(T,T3);

EDU» p = ilaplace(P)

p =

5\*exp(-7\*t)+88-17/5\*exp(-6\*t)+17/5\*exp(-t)

EDU» pretty(p)

5 exp(-7 t) + 88 - 17/5 exp(-6 t) + 17/5 exp(-t)

## CHAPTER FOURTEEN SOLUTIONS

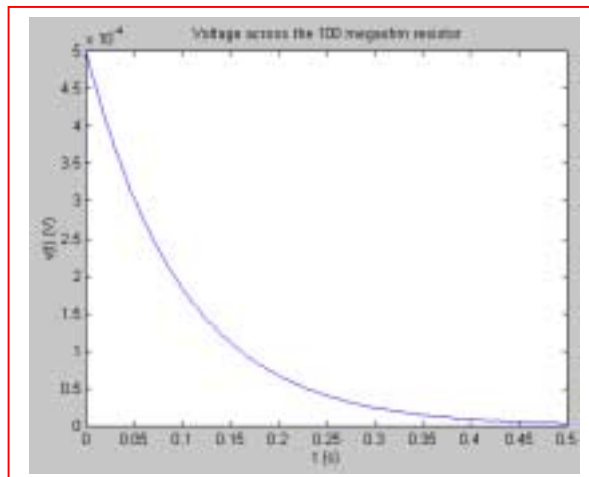
27. If  $\mathbf{V}(s) = \frac{5}{s}$ , then  $v(t) = 5 u(t)$  V. The voltage at  $t = 1$  ms is then simply 5 V, and the current through the 2-k $\Omega$  resistor at that instant in time is 2.5 mA.



## CHAPTER FOURTEEN SOLUTIONS

28.  $\mathbf{I(s)} = \frac{5}{s+10}$  pA, so  $i(t) = 5 e^{-10t} u(t)$  pA. The voltage across the 100-M $\Omega$  resistor is therefore  $500 e^{-10t} u(t)$   $\mu$ V.

(a) The voltage as specified has zero value for  $t < 0$ , and a peak value of 500  $\mu$ V.



(b)  $i(0.1 \text{ s}) = 1.839$  pA, so the power absorbed by the resistor at that instant  $= i^2 R$   
 $= 338.2$  aW. (A pretty small number).

(c)  $500 e^{-10t_{1\%}} = 5$

Taking the natural log of both sides, we find  $t_{1\%} = 460.5$  ms

## CHAPTER FOURTEEN SOLUTIONS

29.

$$(a) \quad F(s) = \frac{s+1}{s} + \frac{2}{s+1} = 1 + \frac{1}{s} + \frac{2}{s+1} \leftrightarrow \boxed{\delta(t) + u(t) + 2e^{-t}u(t)}$$

$$(b) \quad F(s) = (e^{-s} + 1)^2 = e^{-2s} + 2e^{-s} + 1 \leftrightarrow \boxed{\delta(t-2) + 2\delta(t-1) + \delta(t)}$$

$$(c) \quad F(s) = 2e^{-(s+1)} = \boxed{2e^{-1}e^{-2s} + 2e^{-1}\delta(t-1)}$$

$$(d) \quad \mathbf{F(s)} = 2e^{-3s} \cosh 2s = e^{-3s} (e^{2s} + e^{-2s}) = e^{-s} + e^{-5s} \leftrightarrow \boxed{\delta(t-1) + \delta(t-5)}$$

## CHAPTER FOURTEEN SOLUTIONS

30.  $N(s) = 5s$ .

(a)  $D(s) = s^2 - 9$  so  $\frac{N(s)}{D(s)} = \frac{5s}{s^2 - 9} = \frac{5s}{(s+3)(s-3)} = \frac{a}{(s+3)} + \frac{b}{(s-3)}$

where  $a = \left. \frac{5s}{(s-3)} \right|_{s=-3} = \frac{-15}{-6} = 2.5$  and  $b = \left. \frac{5s}{(s+3)} \right|_{s=3} = \frac{15}{6} = 2.5$ . Thus,

$$f(t) = [2.5 e^{-3t} + 2.5 e^{3t}] u(t)$$

(b)  $D(s) = (s+3)(s^2 + 19s + 90) = (s+3)(s+10)(s+9)$  so

$$\frac{N(s)}{D(s)} = \frac{5s}{(s+3)(s+10)(s+9)} = \frac{a}{(s+3)} + \frac{b}{(s+10)} + \frac{c}{(s+9)}$$

$a = \left. \frac{5s}{(s+10)(s+9)} \right|_{s=-3} = \frac{-15}{(7)(6)} = -0.3571$ ,  $b = \left. \frac{5s}{(s+3)(s+9)} \right|_{s=-10} = \frac{-50}{(-7)(-1)} = -7.143$

$c = \left. \frac{5s}{(s+3)(s+10)} \right|_{s=-9} = \frac{-45}{(-6)(1)} = 7.5$ .  $\therefore f(t) = [-0.3571 e^{-3t} - 7.143 e^{-10t} + 7.5 e^{-9t}] u(t)$

(c)  $D(s) = (4s+12)(8s^2+6s+1) = 32(s+3)(s+0.5)(s+0.25)$  so

$$\frac{N(s)}{D(s)} = \left( \frac{5}{32} \right) \frac{s}{(s+3)(s+0.5)(s+0.25)} = \frac{a}{(s+3)} + \frac{b}{(s+0.5)} + \frac{c}{(s+0.25)}$$

$a = \left( \frac{5}{32} \right) \left. \frac{s}{(s+0.5)(s+0.25)} \right|_{s=-3} = -0.06818$ ,  $b = \left( \frac{5}{32} \right) \left. \frac{s}{(s+3)(s+0.25)} \right|_{s=-0.5} = 0.125$

$c = \left( \frac{5}{32} \right) \left. \frac{s}{(s+3)(s+0.5)} \right|_{s=-0.25} = -0.05682$

$$\therefore f(t) = [-0.06818 e^{-3t} + 0.125 e^{-0.5t} - 0.05682 e^{-0.25t}] u(t)$$

(d) Part (a):

```
EDU» N = [5 0];
EDU» D = [1 0 -9];
EDU» [r p y] = residue(N,D)
```

```
r =
    2.5000
    2.5000
```

```
p =
     3
    -3
```

```
y =
[]
```

Part (b):

```
EDU» N = [5 0];
EDU» D = [1 22 147 270];
EDU» [r p y] = residue(N,D)
```

```
r =
   -7.1429
    7.5000
   -0.3571
```

```
p =
  -10.0000
   -9.0000
   -3.0000
```

```
y =
[]
```

Part (c):

```
EDU» N = [5 0];
EDU» D = [32 120 76 12];
EDU» [r p y] = residue(N,D)
```

```
r =
   -0.0682
    0.1250
   -0.0568
```

```
p =
   -3.0000
   -0.5000
   -0.2500
```

```
y =
[]
```

## CHAPTER FOURTEEN SOLUTIONS

31.

$$(a) \quad F(s) = \frac{5}{s+1} \leftrightarrow 5e^{-t}u(t)$$

$$(b) \quad F(s) = \frac{5}{s+1} - \frac{2}{s+4} \leftrightarrow (5e^{-t} - 2e^{-4t})u(t)$$

$$(c) \quad F(s) = \frac{18}{(s+1)(s+4)} = \frac{6}{s+1} - \frac{6}{s+4} \leftrightarrow 6(e^{-t} - e^{-4t})u(t)$$

$$(d) \quad F(s) = \frac{18s}{(s+1)(s+4)} = \frac{-6}{s+1} + \frac{24}{s+4} \leftrightarrow 6(4e^{-4t} - e^{-t})u(t)$$

$$(e) \quad F(s) = \frac{18s^2}{(s+1)(s+4)} = 18 + \frac{6}{s+1} - \frac{96}{s+4} \leftrightarrow 18\delta(t) + 6(e^{-t} - 16e^{-4t})u(t)$$

## CHAPTER FOURTEEN SOLUTIONS

32.  $N(s) = 2s^2$ .

(a)  $D(s) = s^2 - 1$  so  $\frac{N(s)}{D(s)} = \frac{2s^2}{s^2 - 1} = \frac{2s^2}{(s+1)(s-1)} = \frac{a}{(s+1)} + \frac{b}{(s-1)} + 2$

where  $a = \left. \frac{2s^2}{(s-1)} \right|_{s=-1} = \frac{2}{-2} = -1$  and  $b = \left. \frac{2s^2}{(s+1)} \right|_{s=1} = \frac{2}{2} = 1$ . Thus,

$$f(t) = [2\delta(t) + e^{-t} + e^t] u(t)$$

(b)  $D(s) = (s+3)(s^2 + 19s + 90) = (s+3)(s+10)(s+9)$  so

$$\frac{N(s)}{D(s)} = \frac{2s^2}{(s+3)(s+10)(s+9)} = \frac{a}{(s+3)} + \frac{b}{(s+10)} + \frac{c}{(s+9)}$$

$$a = \left. \frac{2s^2}{(s+10)(s+9)} \right|_{s=-3} = \frac{18}{(7)(6)} = 0.4286, \quad b = \left. \frac{2s^2}{(s+3)(s+9)} \right|_{s=-10} = \frac{200}{(-7)(-1)} = 28.57$$

$$c = \left. \frac{2s^2}{(s+3)(s+10)} \right|_{s=-9} = \frac{162}{(-6)(1)} = -27. \quad \therefore f(t) = [0.4286 e^{-3t} + 28.57 e^{-10t} - 27 e^{-9t}] u(t)$$

(c)  $D(s) = (8s+12)(16s^2 + 12s + 2) = 128(s+1.5)(s+0.5)(s+0.25)$  so

$$\frac{N(s)}{D(s)} = \left( \frac{2}{128} \right) \frac{s^2}{(s+1.5)(s+0.5)(s+0.25)} = \frac{a}{(s+1.5)} + \frac{b}{(s+0.5)} + \frac{c}{(s+0.25)}$$

$$a = \left( \frac{2}{128} \right) \left. \frac{s^2}{(s+0.5)(s+0.25)} \right|_{s=-1.5} = 0.02813, \quad b = \left( \frac{2}{128} \right) \left. \frac{s^2}{(s+1.5)(s+0.25)} \right|_{s=-0.5} = -0.01563$$

$$c = \left( \frac{2}{128} \right) \left. \frac{s^2}{(s+1.5)(s+0.5)} \right|_{s=-0.25} = 0.003125$$

$$\therefore f(t) = 0.02813 e^{-1.5t} - 0.01563 e^{-0.5t} + 0.003125 e^{-0.25t} u(t)$$

(d) Part (a):

```
EDU> N = [2 0 0];
EDU> D = [1 0 -1];
EDU> [r p y] = residue(N,D)
```

```
r =
-1.0000
1.0000
```

```
p =
-1.0000
1.0000
```

```
y =
2
```

Part (b):

```
EDU> N = [2 0 0];
EDU> D = [1 22 147 270];
EDU> [r p y] = residue(N,D)
```

```
r =
28.5714
-27.0000
0.4286
```

```
p =
-10.0000
-9.0000
-3.0000
```

```
y =
[]
```

Part (c):

```
EDU> N = [2 0 0];
EDU> D = [128 288 160 24];
EDU> [r p y] = residue(N,D)
```

```
r =
0.0281
-0.0156
0.0031
```

```
p =
-1.5000
-0.5000
-0.2500
```

```
y =
[]
```

## CHAPTER FOURTEEN SOLUTIONS

33.

$$(a) \quad F(s) = \frac{2}{s} - \frac{3}{s+1} \quad \text{so } f(t) = \boxed{2u(t) - 3e^{-t}u(t)}$$

$$(b) \quad F(s) = \frac{2s+10}{s+3} = 2 + \frac{4}{s+3} \leftrightarrow \boxed{2\delta(t) + 4e^{-3t}u(t)}$$

$$(c) \quad F(s) = 3e^{-0.8s} \leftrightarrow \boxed{3\delta(t-0.8)}$$

$$(d) \quad F(s) = \frac{12}{(s+2)(s+6)} = \frac{3}{s+2} - \frac{3}{s+6} \leftrightarrow \boxed{3(e^{-2t} - e^{-6t})u(t)}$$

$$(e) \quad F(s) = \frac{12}{(s+2)^2(s+6)} = \frac{3}{(s+2)^2} + \frac{A}{s+2} + \frac{0.75}{s+6}$$

$$\text{Let } s = 0 \therefore \frac{12}{4 \times 6} = \frac{3}{4} + \frac{A}{2} + \frac{0.75}{6} \therefore A = -0.75$$

$$\therefore F(s) = \frac{3}{(s+2)^2} - \frac{0.75}{s+2} + \frac{0.75}{s+6} \leftrightarrow \boxed{(3te^{-2t} - 0.75e^{-2t} + 0.75e^{-6t})u(t)}$$

## CHAPTER FOURTEEN SOLUTIONS

$$\begin{aligned}
 34. \quad \mathbf{F(s)} &= 2 - \frac{1}{s} + \frac{\pi}{s^3 + 4s^2 + 5s + 2} \\
 &= 2 - \frac{1}{s} + \frac{\pi}{(s+2)(s+1-j7.954)(s+1+j7.954)} \\
 &= 2 - \frac{1}{s} + \frac{a}{(s+2)} + \frac{b}{(s+1-j7.954)} + \frac{b^*}{(s+1+j7.954)} \\
 \text{where } a &= \left. \frac{\pi}{(s+1-j7.954)(s+1+j7.954)} \right|_{s=-2} = 0.04888 \\
 b &= \left. \frac{\pi}{(s+2)(s+1+j7.954)} \right|_{s=-1+j7.954} = -0.02444 + j0.003073
 \end{aligned}$$

and hence  $b^* = -0.02444 - j0.003073$

Thus, we may write

$$\begin{aligned}
 f(t) &= 2 \delta(t) - u(t) + 0.04888 e^{-2t} u(t) + [(-0.02444 + j0.003073) e^{(-1+j7.954)t} \\
 &\quad + (-0.02444 - j0.003073) e^{(-1-j7.954)t}] u(t)
 \end{aligned}$$

This may be further simplified by expressing  $(-0.02444 + j0.003073) e^{(-1+j7.954)t}$  as  $0.02463 e^{j172.83^\circ} e^{(-1+j7.954)t}$ . This term, plus its complex conjugate above, add to the purely real expression  $0.02463 e^{-t} \cos(7.954t + 172.8^\circ)$ .

$$\text{Thus, } f(t) = \boxed{2 \delta(t) - u(t) + 0.04888 e^{-2t} u(t) + 0.02463 e^{-t} \cos(7.954t + 172.8^\circ)}.$$

## CHAPTER FOURTEEN SOLUTIONS

$$35. \quad (a) \quad \mathbf{F(s)} = \frac{(s+1)(s+2)}{s(s+3)} = \frac{a}{s} + \frac{b}{(s+3)}$$

$$a = \left. \frac{(s+1)(s+2)}{(s+3)} \right|_{s=0} = \frac{2}{3} \quad \text{and} \quad b = \left. \frac{(s+1)(s+2)}{s} \right|_{s=-3} = \frac{(-2)(-1)}{-3} = -\frac{2}{3}$$

so

$$f(t) = \frac{2}{3}u(t) - \frac{2}{3}e^{-3t}u(t) = \boxed{\frac{2}{3}(1 - e^{-3t})u(t)}$$

$$(b) \quad \mathbf{F(s)} = \frac{(s+2)}{s^2(s^2+4)} = \frac{a}{s^2} + \frac{b}{s} + \frac{c}{(s+j2)} + \frac{c^*}{(s-j2)}$$

$$a = \left. \frac{(s+2)}{(s^2+4)} \right|_{s=0} = \frac{2}{4} = 0.5$$

$$b = \frac{d}{ds} \left[ \frac{(s+2)}{(s^2+4)} \right]_{s=0} = \left[ \frac{(s^2+4) - 2s(s+2)}{(s^2+4)^2} \right]_{s=0} = \frac{4}{4^2} = 0.25$$

$$c = \left. \frac{(s+2)}{s^2(s-j2)} \right|_{s=-j2} = \frac{2-j2}{4(-j4)} = 0.125 + j0.125 = 0.1768\angle 45^\circ \quad (c^* = 0.1768\angle -45^\circ)$$

so

$$f(t) = 0.5 t u(t) + 0.25 u(t) + 0.1768 e^{j45^\circ} e^{-j2t} u(t) + 0.1768 e^{-j45^\circ} e^{j2t} u(t)$$

The last two terms may be combined so that

$$\boxed{f(t) = 0.5 t u(t) + 0.25 u(t) + 0.3536 \cos(2t - 45^\circ)}$$



## CHAPTER FOURTEEN SOLUTIONS

36.

$$(a) \quad 5[s\mathbf{I}(\mathbf{s}) - i(0^-)] - 7[s\mathbf{I}(\mathbf{s}) - si(0^-) - i'(0^-)] + 9\mathbf{I}(\mathbf{s}) = \frac{4}{\mathbf{s}}$$

$$(b) \quad m[s\mathbf{P}(\mathbf{s}) - sp(0^-) - p'(0^-)] + \mu_f [s\mathbf{P}(\mathbf{s}) - p(0^-)] + k\mathbf{P}(\mathbf{s}) = 0$$

$$(c) \quad [s \Delta \mathbf{N}_p(\mathbf{s}) - \Delta n_p(0^-)] = -\frac{\Delta \mathbf{N}_p(\mathbf{s})}{\tau} + \frac{G_L}{\mathbf{s}}$$

## CHAPTER FOURTEEN SOLUTIONS

37.

$$15u(t) - 4\delta(t) = 8f(t) + 6f'(t), \quad f(0) = -3$$

$$\therefore \frac{15}{s} - 4 = 8F(s) + 6sF(s) + 18 = \frac{15 - 4s}{s} \quad \therefore F(s) (6s + 8) = 18 + \frac{15 - 4s}{s}$$

$$\therefore F(s) = \frac{-22s + 15}{6s(s + 4/30)} = \frac{15/8}{s + 4/3} \quad \therefore f(t) = (1.875 - 5.542e^{-4t/3})u(t)$$

# CHAPTER FOURTEEN SOLUTIONS

38.

$$(a) \quad -5 u(t-2) + 10 i_L(t) + 5 \frac{di_L}{dt} = 0$$

$$(b) \quad \frac{-5}{s} e^{-2s} + 10 \mathbf{I}_L(s) + 5[s\mathbf{I}_L(s) - i_L(0^-)] = 0$$

$$\mathbf{I}_L(s) = \frac{\frac{5}{s} e^{-2s} + 5 i_L(0^-)}{5s + 10} = \frac{e^{-2s} + 5 \times 10^{-3} s}{s(s+2)}$$

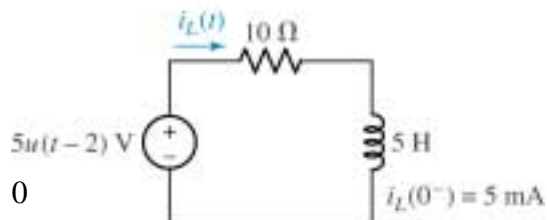
$$(c) \quad \mathbf{I}_L(s) = e^{-2s} \left[ \frac{a}{s} + \frac{b}{s+2} \right] + \frac{5 \times 10^{-3}}{s+2}$$

where  $a = \left. \frac{1}{s+2} \right|_{s=0} = \frac{1}{2}$ , and  $b = \left. \frac{1}{s} \right|_{s=-2} = -\frac{1}{2}$ , so that we may write

$$\mathbf{I}_L(s) = \frac{1}{2} e^{-2s} \left[ \frac{1}{s} - \frac{1}{s+2} \right] + \frac{5 \times 10^{-3}}{s+2}$$

$$\text{Thus,} \quad i_L(t) = \frac{1}{2} [u(t-2) - e^{-2(t-2)} u(t-2)] + 5 \times 10^{-3} e^{-2t} u(t)$$

$$= \frac{1}{2} [1 - e^{-2(t-2)}] u(t-2) + 5 \times 10^{-3} e^{-2t} u(t)$$



## CHAPTER FOURTEEN SOLUTIONS

39.

(a)  $v_c(0^-) = 50 \text{ V}$ ,  $v_c(0^+) = 50 \text{ V}$

(b)  $0.1v'_c + 0.2v_c + 0.1(v_c - 20) = 0$

(c)  $\therefore 0.1v'_c + 0.3v_c = 2$ ,  $0.1sV_c - 5 + 0.3V_c = \frac{2}{s}$

$$\therefore V_c(0.1s + 0.3) = 5 + \frac{2}{s} = \frac{5s + 2}{s}$$

$$\therefore V_c(s) = \frac{5s + 2}{s(0.1s + 0.3)} = \frac{20/3}{s} + \frac{130/3}{s + 3} \therefore v_c(t) = \left( \frac{20}{3} + \frac{130}{3}e^{-3t} \right) u(t) \text{ V}$$

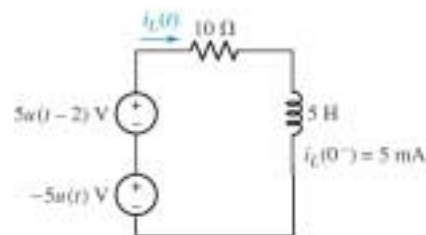
# CHAPTER FOURTEEN SOLUTIONS

40.

$$(a) \quad 5u(t) - 5u(t-2) + 10i_L(t) + 5 \frac{di_L}{dt} = 0$$

$$(b) \quad \frac{5}{s} - \frac{5}{s}e^{-2s} + 10\mathbf{I}_L(s) + 5[s\mathbf{I}_L(s) - i_L(0^-)] = 0$$

$$\mathbf{I}_L(s) = \frac{\frac{5}{s}e^{-2s} - \frac{5}{s} + 5i_L(0^-)}{5s+10} = \frac{e^{-2s} - 5 + 5 \times 10^{-3}s}{s(s+2)}$$



$$(c) \quad \mathbf{I}_L(s) = e^{-2s} \left[ \frac{a}{s} + \frac{b}{s+2} \right] + \frac{c}{s} + \frac{d}{s+2} \text{ where}$$

$$a = \frac{1}{s+2} \Big|_{s=0} = \frac{1}{2}, b = \frac{1}{s} \Big|_{s=-2} = -\frac{1}{2}, c = \frac{5 \times 10^{-3}s - 5}{s+2} \Big|_{s=0} = -\frac{5}{2} = -2.5, \text{ and}$$

$$d = \frac{5 \times 10^{-3}s - 5}{s} \Big|_{s=-2} = \frac{-10 \times 10^{-3} - 5}{-2} = 2.505,$$

so that we may write

$$\mathbf{I}_L(s) = \frac{1}{2}e^{-2s} \left[ \frac{1}{s} - \frac{1}{s+2} \right] - \frac{2.5}{s} + \frac{2.505}{s+2}$$

$$\text{Thus, } i_L(t) = \frac{1}{2} [u(t-2) - e^{-2(t-2)} u(t-2)] - 2.5e^{-2t} u(t) + 2.505 u(t)$$

## CHAPTER FOURTEEN SOLUTIONS

41.

$$12u(t) = 20f_2'(t) + 3f_2(0^-) = 2 \quad \therefore \frac{12}{s} = 20sF_2 - 20(2) + 3F_2$$

$$\therefore \frac{12}{s} + 40 = (20s + 3) F_2 = \frac{12 + 40s}{s} \quad \therefore F_2(s) = \frac{2s + 0.6}{s(s + 0.15)}$$

$$\therefore F_2(s) = \frac{4}{s} - \frac{2}{s + 0.15} \leftrightarrow \boxed{(4 - 2e^{-0.15t})u(t)}$$

## CHAPTER FOURTEEN SOLUTIONS

42. (a)  $f(t) = 2u(t) - 4\delta(t)$

(b)  $f(t) = \cos(\sqrt{99}t)$

(c)  $F(s) = \frac{1}{s^2 + 5s + 6} - 5 = \frac{a}{s-3} + \frac{b}{s-2} - 5$

where  $a = \left. \frac{1}{s-2} \right|_{s=3} = 1$  and  $b = \left. \frac{1}{s-3} \right|_{s=2} = -1$

Thus,

$$f(t) = e^{-3t}u(t) - e^{-2t}u(t) - 5\delta(t)$$

(d)  $f(t) = \delta'(t)$  (a “doublet”)

(e)  $f(t) = \delta''(t)$

## CHAPTER FOURTEEN SOLUTIONS

43.

$$x' + y = 2u(t), \quad y' - 2x + 3y = 8u(t), \quad x(0^-) = 5, \quad y(0^-) = 8$$

$$sX - 5 + Y = \frac{2}{s}, \quad sY - 8 - 2X + 3Y = \frac{8}{s} \quad \therefore X = \frac{1}{s} \left( \frac{2}{s} + 5 - Y \right) = \frac{2}{s^2} + \frac{5}{s} - \frac{Y}{s}$$

$$\therefore sY + 3Y - \frac{4}{s^2} - \frac{10}{s} + \frac{2Y}{s} = 8 + \frac{8}{s} \quad \therefore Y \left( s + 3 + \frac{2}{s} \right) = \frac{4}{s^2} + \frac{18}{s} + 8$$

$$Y \left( \frac{s^2 + 3s + 2}{s} \right) = \frac{4 + 18s + 8s^2}{s^2}, \quad Y(s) + \frac{8s^2 + 18s + 4}{s(s+1)(s+2)} = \frac{2}{s} + \frac{6}{s+1} + \frac{0}{s+2}$$

$$\therefore \boxed{y(t) = (2 + 6e^{-t}) u(t)}; \quad x(t) = \frac{1}{2} [y' + 3y - 8u(t)] = \frac{1}{2} y' + 1.5y - 4u(t)$$

$$\therefore x(t) = \frac{1}{2} [-6e^{-t}u(t)] + 1.5 [2 + 6e^{-t}] u(t) - 4u(t)$$

$$\therefore x(t) = 6e^{-t}u(t) - u(t) = \boxed{(6e^{-t} - 1)u(t)}$$



## CHAPTER FOURTEEN SOLUTIONS

44. (a)  $\mathbf{F(s)} = 8s + 8 + \frac{8}{s}$ , with  $f(0^-) = 0$ . Thus, we may write:

$$f(t) = 8 \delta(t) + 8 u(t) + 8 \delta'(t)$$

$$(b) \mathbf{F(s)} = \frac{s^2}{(s+2)} - s + 2.$$

$$f(t) = \delta'(t) - 2\delta(t) + 4e^{-2t} u(t) - \delta'(t) + 2\delta(t) = 4e^{-2t} u(t)$$

## CHAPTER FOURTEEN SOLUTIONS

45.

$$(a) \quad i_c(0^-) = 0, \quad v_c(0) = 100 \text{ V}, \quad \therefore i_c(0^+) = \frac{40 - 100}{100} = -0.6 \text{ A}$$

$$(b) \quad 40 = 100 i_c + 50 \int_{0^-}^{\infty} i_c dt + 100$$

$$(c) \quad -\frac{60}{s} = 100 I_c(s) + \frac{50}{s} I_c(s)$$

$$\therefore \frac{6}{s} = I_c \frac{10s + 5}{s}, \quad I_c(s) = \frac{-6}{10s + 5} = \frac{0.6}{s + 0.5} \leftrightarrow i_c(t) = 0.6e^{-0.5t}u(t)$$

## CHAPTER FOURTEEN SOLUTIONS

46. (a)  $4 \cos 100t \leftrightarrow \frac{4s}{s^2 + 100^2}$

(b)  $2 \sin 10^3 t - 3 \cos 100t \leftrightarrow \frac{2 \times 10^3}{s^2 + 10^6} - \frac{3}{s^2 + 100^2}$

(c)  $14 \cos 8t - 2 \sin 8^\circ \leftrightarrow \frac{14s}{s^2 + 64} - \frac{2 \sin 8^\circ}{s}$

(d)  $\delta(t) + [\sin 6t]u(t) \leftrightarrow 1 + \frac{6}{s^2 + 36}$

(e)  $\cos 5t \sin 3t = \frac{1}{2} \sin 8t + \frac{1}{2} \sin (-2t) = \frac{1}{2} (\sin 8t - \sin 2t) \leftrightarrow \frac{4}{s^2 + 64} - \frac{1}{s^2 + 4}$

## CHAPTER FOURTEEN SOLUTIONS

$$47. \quad i_s = 100e^{-5t}u(t) \text{ A}; \quad i_s = v' + 4v + 3 \int_{0^-}^t v dt$$

$$(a) \quad i_s = \frac{v}{R} + Cv' + \frac{1}{L} \int_{0^-}^t v dt; \quad R = \frac{1}{4} \Omega, \quad C = 1\text{F}, \quad L = \frac{1}{3} \text{ H}$$

$$(b) \quad \frac{100}{s+5} = sV(s) + 4V(s) + \frac{3}{s} V(s)$$

$$V(s) \left( s + 4 + \frac{3}{s} \right) = V(s) \frac{s^2 + 4s + 3}{s} = \frac{100}{s+5}, \quad V(s) = \frac{100s}{(s+1)(s+3)(s+5)}$$

$$\therefore V(s) = \frac{-12.5}{s+1} + \frac{75}{s+3} - \frac{62.5}{s+5}, \quad v(t) = (75e^{-3t} - 12.5e^{-t} - 62.5e^{-5t})u(t) \text{ V}$$

## CHAPTER FOURTEEN SOLUTIONS

48.

$$(a) \mathbf{V(s)} = \frac{7}{s} + \frac{e^{-2s}}{s} \mathbf{V}$$

$$(b) \mathbf{V(s)} = \frac{e^{-2s}}{s+1} \mathbf{V}$$

$$(c) \mathbf{V(s)} = 48e^{-s} \mathbf{V}$$

## CHAPTER FOURTEEN SOLUTIONS

49.

$$4u(t) + i_c + 10 \int_{0^-}^{\infty} i_c dt + 4 [i_c - 0.5\delta(t)] = 0$$

$$\therefore \frac{4}{s} + I_c + \frac{10}{s} I_c + 4I_c = 2, I_c \left( 5 + \frac{10}{s} \right) = 2 - \frac{4}{s} + \frac{2s-4}{s}$$

$$\therefore I_c = \frac{2s-4}{5s+10} = 0.4 - \frac{1.6}{s+2}$$

$$\therefore i_c(t) + 0.4\delta(t) - 1.6e^{-2t}u(t) \text{ A}$$

## CHAPTER FOURTEEN SOLUTIONS

50.

$$v' + 6v + 9 \int_{0^-}^t v(z) dz = 24(t-2) u(t-2), \quad v'(0) = 0$$

$$\therefore sV(s) - 0 + 6V(s) + \frac{9}{s} V(s) = 24e^{-2s} \frac{1}{s^2} = V(s) \frac{s^2 + 6s + 9}{s} = V(s) \frac{(s+3)^2}{s}$$

$$\therefore V(s) = 24e^{-2s} \frac{1}{s^2} \frac{s}{(s+3)^2} = 24e^{-2s} \left[ \frac{1/9}{s} - \frac{1/9}{s+3} - \frac{1/3}{(s+3)^2} \right]$$

$$\therefore V(s) = e^{-2s} \left[ \frac{8/3}{s} - \frac{8}{s+3} - \frac{8}{(s+3)^2} \right] \leftrightarrow \frac{8}{3} [u(t-2) - e^{-3(t-2)} u(t-2)]$$

$$-8(t-2)e^{-3(t-2)} u(t-2) \quad \therefore v(t) = \left[ \frac{8}{3} - \frac{8}{3} e^{-3(t-2)} - 8(t-2)e^{-3(t-2)} \right] u(t-2)$$

## CHAPTER FOURTEEN SOLUTIONS

51.

$$(a) \quad F(s) = \frac{5(s^2 + 1)}{(s^3 + 1)} \therefore f(0^+) = \lim_{s \rightarrow \infty} \frac{5s(s^2 + 1)}{s^3 + 1} = \boxed{5}$$

$$f(\infty) = \lim_{s \rightarrow 0} \frac{5s(s^2 + 1)}{s^3 + 1}, \text{ but 1 pole in RHP } \therefore \text{ indeterminate}$$

$$(b) \quad F(s) = \frac{5(s^2 + 1)}{s^3 + 16} \therefore f(0^+) = \lim_{s \rightarrow \infty} \frac{5s(s^2 + 1)}{s^4 + 16} = \boxed{0}$$

$f(\infty)$  is indeterminate since poles on  $j\omega$  axis

$$(c) \quad F(s) = \frac{(s+1)(1+e^{-4s})}{s^2 + 2} \therefore f(0^+) = \lim_{s \rightarrow \infty} \frac{s(s+1)(1+e^{-4s})}{s^2 + 2} = \boxed{1}$$

$f(\infty)$  is indeterminate since poles on  $j\omega$  axis



## CHAPTER FOURTEEN SOLUTIONS

$$52. \quad (a) \quad f(0^+) = \lim_{s \rightarrow \infty} [s F(s)] = \lim_{s \rightarrow \infty} \left( \frac{2s^2 + 6}{s^2 + 5s + 2} \right) = \boxed{2}$$

$$f(\infty) = \lim_{s \rightarrow 0} [s F(s)] = \lim_{s \rightarrow 0} \left( \frac{2s^2 + 6}{s^2 + 5s + 2} \right) = \frac{6}{2} = \boxed{3}$$

$$(b) \quad f(0^+) = \lim_{s \rightarrow \infty} [s F(s)] = \lim_{s \rightarrow \infty} \left( \frac{2se^{-s}}{s + 3} \right) = \boxed{0}$$

$$f(\infty) = \lim_{s \rightarrow 0} [s F(s)] = \lim_{s \rightarrow 0} \left( \frac{2se^{-s}}{s + 3} \right) = \boxed{0}$$

$$(c) \quad f(0^+) = \lim_{s \rightarrow \infty} [s F(s)] = \lim_{s \rightarrow \infty} \left[ \frac{s(s^2 + 1)}{s^2 + 5} \right] = \boxed{\infty}$$

$f(\infty)$  : This function has poles on the  $j\omega$  axis, so we may not apply the final value theorem to determine  $f(\infty)$ .

## CHAPTER FOURTEEN SOLUTIONS

53.

$$(a) \quad F(s) = \frac{5(s^2 + 1)}{(s + 1)^3} \therefore f(0^+) = \lim_{s \rightarrow \infty} \frac{5s(s^2 + 1)}{(s + 1)^3} = \boxed{5}$$

$$(b) \quad F(s) = \frac{5(s^2 + 1)}{s(s + 1)^3} \therefore f(0^+) = \lim_{s \rightarrow \infty} \frac{5(s^2 + 1)}{(s + 1)^3} = \boxed{0}$$

$$f(\infty) = \lim_{s \rightarrow 0} \frac{5(s^2 + 1)}{(s + 1)^3} = 5 \text{ (pole OK)}$$

$$(c) \quad F(s) = \frac{(1 - e^{-3s})}{s^2} \therefore f(0^+) = \lim_{s \rightarrow \infty} \frac{1 - e^{-3s}}{s} = \boxed{0}$$

$$f(\infty) = \lim_{s \rightarrow 0} \frac{1 - e^{-3s}}{s} = \lim_{s \rightarrow 0} \frac{1}{2} \left( 1 - 1 + 3s - \frac{1}{2} \times 9s^2 + \dots \right) = \boxed{3} \text{ (no poles)}$$

## CHAPTER FOURTEEN SOLUTIONS

54.

$$f(t) = \frac{1}{t}(e^{at} - e^{-bt})u(t)$$

$$\begin{aligned} \text{(a)} \quad \text{Now, } \frac{1}{t}f(t) &\leftrightarrow \int_s^\infty F(s) ds \therefore e^{-at}u(t) \leftrightarrow \frac{1}{s+a}, -e^{-bt}u(t) \leftrightarrow -\frac{1}{s+b} \\ \therefore \frac{1}{t}(e^{-at} - e^{-bt})u(t) &\leftrightarrow \int_s^\infty \left( \frac{1}{s+a} - \frac{1}{s+b} \right) ds = \ln \frac{s+a}{s+b} \Big|_s^\infty = \ln \frac{s+a}{s+b} \Big|_s^\infty = \ln \frac{s+b}{s+a} \end{aligned}$$

$$\text{(b)} \quad \lim_{t \rightarrow 0^+} \frac{1}{t}(e^{-at} - e^{-bt})u(t) = \lim_{t \rightarrow 0^+} \frac{1 - at + \dots - 1 + bt}{t} = b - a$$

$$\lim_{s \rightarrow \infty} s \ln \frac{s+b}{s+a} = \lim_{s \rightarrow \infty} \frac{\ln(s+b) - \ln(s+a)}{1/s}$$

$$\text{Use l' Hopital. } \therefore \lim_{s \rightarrow \infty} sF(s) = \frac{1/(s+b) - 1/(s+a)}{-1/s^2} = \lim_{s \rightarrow \infty} \left[ -s^2 \frac{(a-b)}{(s+b)(s+a)} \right] = b - a$$

## CHAPTER FOURTEEN SOLUTIONS

55.

$$(a) \quad F(s) = \frac{8s-2}{s^2+6s+10} \therefore f(0^+) = \lim_{s \rightarrow \infty} \frac{s(8s-2)}{s^2+6s+10} = \boxed{8}$$

$$f(\infty) = \lim_{s \rightarrow 0} \frac{s(8s-2)}{s^2+6s+10} = 0 \left( \text{poles: } s = \frac{-6 \pm \sqrt{36-40}}{2}, \text{ LHP, } \therefore \text{OK} \right)$$

$$(b) \quad F(s) = \frac{2s^3 - s^2 - 3s - 5}{s^3 + 6s^2 + 10s} \therefore f(0^+) = \lim_{s \rightarrow \infty} \frac{2s^3 - s^2 - 3s - 5}{s^2 + 6s + 10} = \boxed{\infty}$$

$$f(\infty) = \lim_{s \rightarrow 0} \frac{2s^3 - s^2 - 3s - 5}{s^2 + 6s + 10} = -0.5 \text{ (poles OK)}$$

$$(c) \quad F(s) = \frac{8s-2}{s^2-6s+10} \therefore f(0^+) = \lim_{s \rightarrow \infty} \frac{s(8s-2)}{s^2-6s+10} = \boxed{8}$$

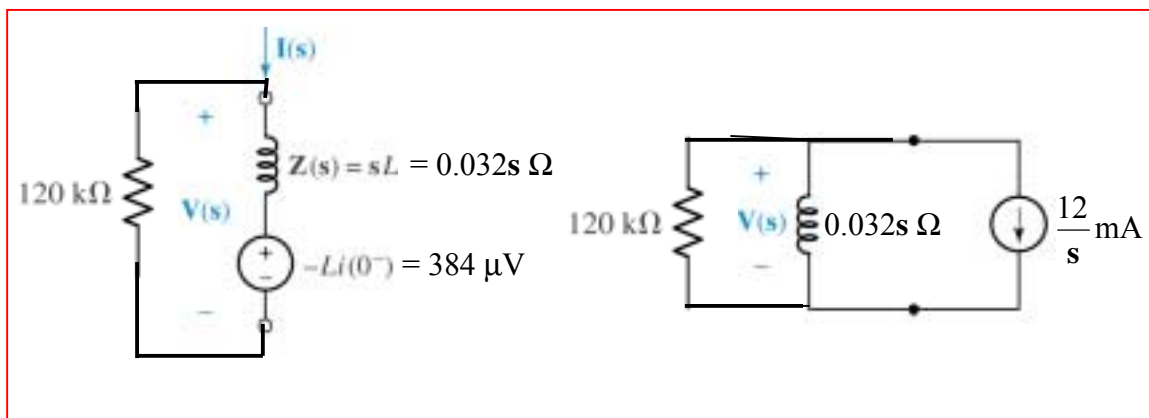
$$f(\infty) = \lim_{s \rightarrow 0} \frac{s(8s-2)}{s^2-6s+10}, s = \frac{6 \pm \sqrt{36-40}}{2} \text{ RHP } \therefore \text{indeterminate}$$

$$(d) \quad F(s) = \frac{8s^2-2}{(s+2)^2(s+1)(s^2+6s+10)} \therefore f(0^+) = \lim_{s \rightarrow \infty} F(s) = \boxed{0}$$

$$f^\infty = \lim_{s \rightarrow 0} \frac{s(8s^2-2)}{(s+2)^2(s+1)(s^2+6s+10)} = \boxed{10} \text{ (pole OK)}$$

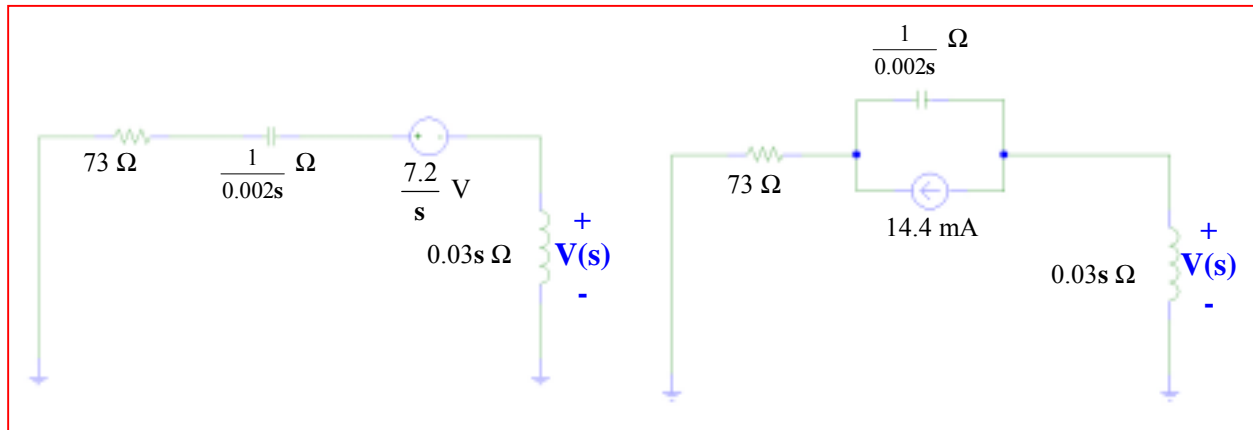
## CHAPTER FIFTEEN SOLUTIONS

1. Note that  $i_L(0^+) = 12 \text{ mA}$ . We have two choices for inductor model:



## CHAPTER FIFTEEN SOLUTIONS

2.  $i_L(0^-) = 0$ ,  $v_C(0^+) = 7.2 \text{ V}$  ('+' reference on left). There are two possible circuits, since the inductor is modeled simply as an impedance:



## CHAPTER FIFTEEN SOLUTIONS

3.

$$\begin{aligned} \text{(a)} \quad \mathbf{Z}_m(s) &= \frac{2s}{20 + 0.1s} + \frac{2000/s}{2 + 1000/s} = \frac{20s}{s + 200} + \frac{1000}{s + 500} \\ &= \frac{20s^2 + 10,000s + 1000s + 200,000}{s^2 + 700s + 100,000} = \boxed{\frac{20s^2 + 11,000s + 200,000}{s^2 + 700s + 100,000}} \end{aligned}$$

$$\text{(b)} \quad \mathbf{Z}_{in}(-80) = \boxed{-10.95 \, \Omega}$$

$$\text{(c)} \quad \mathbf{Z}_{in}(j80) = \frac{-128,000 + j880,000 + 200,000}{-6400 + j56,000 + 100,000} = \boxed{8.095 \angle 54.43^\circ \, \Omega}$$

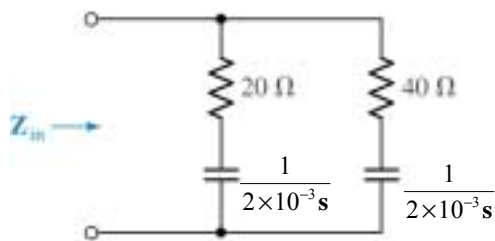
$$\text{(d)} \quad \mathbf{Y}_{RL} = \frac{1}{20} + \frac{10}{s} = \boxed{\frac{s + 200}{20s}}$$

$$\text{(e)} \quad \mathbf{Y}_{RC} = \frac{1}{2} + 0.001s = \boxed{\frac{s + 500}{1000}}$$

$$\begin{aligned} \text{(f)} \quad \frac{\mathbf{Y}_{RL} + \mathbf{Y}_{RC}}{\mathbf{Y}_{RL} \mathbf{Y}_{RC}} &= \frac{\frac{s + 200}{20s} + 0.5 + 0.001s}{\frac{(s + 200)}{20s}(0.001s + 0.5)} = \frac{s + 200 + 10s + 0.02s^2}{0.001s^2 + 0.7s + 100} \\ &= \frac{20s^2 + 11,000s + 200,000}{s^2 + 700s + 100,000} = \mathbf{Z(s)} \end{aligned}$$

## CHAPTER FIFTEEN SOLUTIONS

4.



$$\mathbf{Z}_{\text{in}} = \left( 20 + \frac{1}{2 \times 10^{-3} \mathbf{s}} \right) \parallel \left( 40 + \frac{1}{2 \times 10^{-3} \mathbf{s}} \right) = (20 + 500\mathbf{s}^{-1}) \parallel (40 + 500\mathbf{s}^{-1})$$

$$= \frac{80\mathbf{s}^2 + 3000\mathbf{s} + 25000}{6\mathbf{s}^2 + 100\mathbf{s}}$$



## CHAPTER FIFTEEN SOLUTIONS

$$5. \quad (a) \quad \mathbf{Z}_{in} = \frac{50}{s} + \frac{16(0.2s)}{16 + 0.2s} = \frac{50}{s} + \frac{16s}{s + 80} = \boxed{\frac{16s^2 + 50s + 4000}{s^2 + 80s}}$$

$$(b) \quad \mathbf{Z}_{in}(j8) = \frac{-1024 + 4000 + j400}{-64 + j640} = \boxed{0.15842 - j4.666 \, \Omega}$$

$$(c) \quad \mathbf{Z}_{in}(-2 + j6) = \frac{16(4 - 36 - j24) - 100 + j300 + 4000}{-32 - j24 - 160 + j480} = \boxed{6.850 \angle -114.3^\circ \, \Omega}$$

$$(d) \quad \mathbf{Z}_{in} = \frac{50}{s} + \frac{0.2sR}{R + 0.2s} = \frac{0.2Rs^2 + 10s + 50R}{0.2s^2 + Rs},$$

$$\mathbf{Z}_{in}(-5) = \frac{5R - 50 + 50R}{5 - 5R} \therefore 55R = 50, \quad R = \boxed{0.9091 \, \Omega}$$

$$(e) \quad R = \boxed{1 \, \Omega}$$

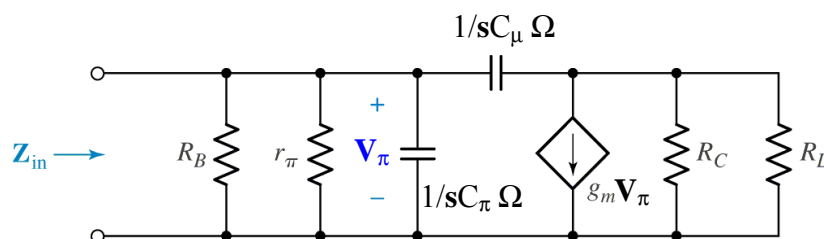
## CHAPTER FIFTEEN SOLUTIONS

6.  $2 \text{ mF} \rightarrow \frac{1}{2 \times 10^{-3} \text{ s}} \Omega, 1 \text{ mH} \rightarrow 0.001 \text{ s } \Omega,$

$$\mathbf{Z}_{\text{in}} = (55 + 500/\text{s}) \parallel (100 + \text{s}/1000) = \frac{\left(55 + \frac{500}{\text{s}}\right) \left(100 + \frac{\text{s}}{1000}\right)}{155 + \frac{500}{\text{s}} + \frac{\text{s}}{1000}} = \boxed{\frac{55\text{s}^2 + 5.5005 \times 10^6 \text{s} + 5 \times 10^7}{\text{s}^2 + 5 \times 10^5 \text{s} + 1.55 \times 10^5}}$$

## CHAPTER FIFTEEN SOLUTIONS

7. We convert the circuit to the s-domain:



Defining  $\mathbf{Z}_\pi = R_B \parallel r_\pi \parallel (1/sC_\pi) = \frac{r_\pi R_B}{r_\pi + R_B + r_\pi R_B C_\pi s}$  and

$\mathbf{Z}_L = R_C \parallel R_L = R_C R_L / (R_C + R_L)$ , we next connect a 1-A source to the input and write two nodal equations:

$$1 = \mathbf{V}_\pi / \mathbf{Z}_\pi + (\mathbf{V}_\pi - \mathbf{V}_L) C_\mu s \quad [1]$$

$$-g_m \mathbf{V}_\pi = \mathbf{V}_L / \mathbf{Z}_L + (\mathbf{V}_L - \mathbf{V}_\pi) C_\mu s \quad [2]$$

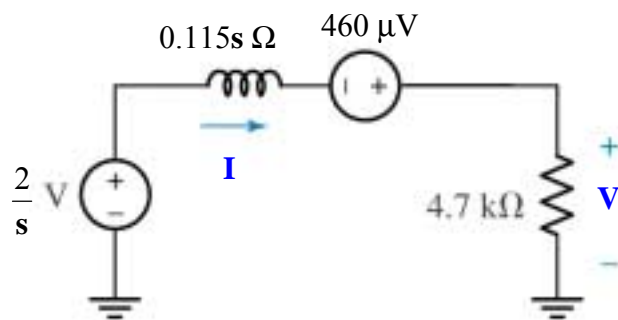
Solving,

$$\mathbf{V}_\pi = \frac{r_\pi R_B (1 + \mathbf{Z}_L C_\mu s)}{\mathbf{Z}_L r_\pi R_B C_\pi C_\mu s^2 + (g_m \mathbf{Z}_L r_\pi R_B C_\mu + r_\pi R_B C_\pi + r_\pi R_B C_\mu + \mathbf{Z}_L r_\pi C_\mu + \mathbf{Z}_L R_B C_\mu) s + r_\pi + R_B}$$

Since we used a 1-A ‘test’ source, this is the input impedance. Setting both capacitors to zero results in  $r_\pi \parallel R_B$  as expected.

## CHAPTER FIFTEEN SOLUTIONS

8.



$$\mathbf{V(s)} = 4700 \frac{\frac{2}{s} + 460 \times 10^{-6}}{4700 + 0.115s} = \frac{2.162}{0.115s + 4700} + \frac{9400}{s(0.115s + 4700)}$$

$$= \frac{18.8}{s + 40870} + \frac{81740}{s(s + 40870)} = \frac{18.8}{s + 40870} + \frac{a}{s} + \frac{b}{s + 40870}$$

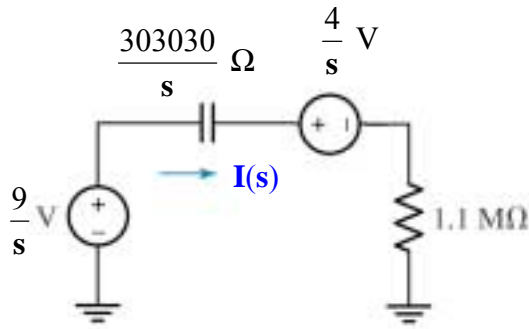
$$\text{where } a = \left. \frac{81740}{s + 40870} \right|_{s=0} = 2 \text{ and } b = \left. \frac{81740}{s} \right|_{s=-40870} = -2$$

Thus,  $\mathbf{V(s)} = \frac{18.8}{s + 40870} + \frac{2}{s} - \frac{2}{s + 40870}$ . Taking the inverse transform of each term,

$$v(t) = [16.8 e^{-40870t} + 2] u(t) \text{ V}$$

## CHAPTER FIFTEEN SOLUTIONS

9.  $v(0^-) = 4 \text{ V}$



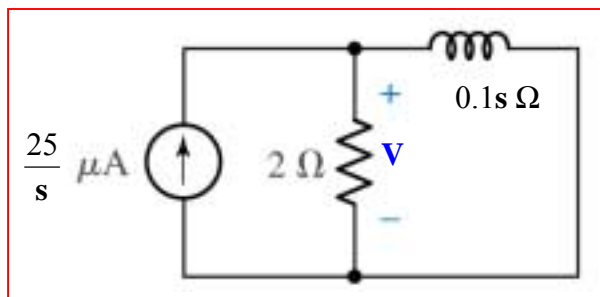
$$\mathbf{I(s)} = \frac{\frac{9}{s} - \frac{4}{s}}{\frac{303030}{s} + 1.1 \times 10^6} = \frac{5}{1.1 \times 10^6 + 303030} = \frac{4.545 \times 10^{-6}}{s + 0.2755}$$

Taking the inverse transform, we find that  $i(t) = 4.545 e^{-0.2755t} u(t) \mu\text{A}$

## CHAPTER FIFTEEN SOLUTIONS

10. From the information provided, we assume no initial energy stored in the inductor.

(a) Replace the 100 mH inductor with a  $0.1\text{s}\text{-}\Omega$  impedance, and the current source with a  $\frac{25 \times 10^{-6}}{s}$  A source.



$$(b) \mathbf{V(s)} = \frac{25 \times 10^{-6}}{s} \left[ \frac{2(0.1\text{s})}{2 + 0.1\text{s}} \right] = \frac{5 \times 10^{-6}}{0.1\text{s} + 2} = \frac{5 \times 10^{-5}}{s + 20} \text{ V}$$

Taking the inverse transform,

$$v(t) = 50 e^{-20t} \text{ mV}$$

The power absorbed in the resistor R is then  $p(t) = 0.5 v^2(t) = 1.25 e^{-40t} \text{ nW}$

## CHAPTER FIFTEEN SOLUTIONS

11.  $v(t) = 10e^{-2t} \cos(10t + 30^\circ) \text{ V}$

$$\cos(10t + 30^\circ) \Leftrightarrow \frac{s \cos 30^\circ - 10 \sin 30^\circ}{s^2 + 100} = \frac{0.866s - 5}{s^2 + 100}$$

$\mathcal{L}\{f(t)e^{-at}\} \Leftrightarrow F(s+a)$ , so

$$\mathbf{V}(s) = 10 \frac{0.866(s+2) - 5}{(s+2)^2 + 100} = \frac{8.66s - 16.34}{s + 100}$$

The voltage across the  $5\text{-}\Omega$  resistor may be found by simple voltage division. We first note that  $\mathbf{Z}_{\text{eff}} = (10/s) \parallel 5 = \frac{50}{5s+10} \Omega$ . Thus,

$$\mathbf{V}_{5\Omega} = \frac{\left(\frac{50}{5s+10}\right) \mathbf{V}_s}{0.5s + 5 + \left(\frac{50}{5s+10}\right)} = \frac{50 \mathbf{V}_s}{(0.5s + 5)(5s + 10) + 50} = \frac{50 \mathbf{V}_s}{2.5s^2 + 30s + 100}$$

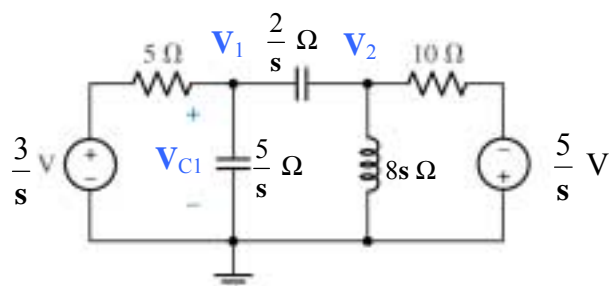
$$(a) \mathbf{I}_x = \frac{\mathbf{V}_{\text{eff}}}{5} = 40 \frac{0.866s - 3.268}{\left[(s+2)^2 + 100\right]\left[s^2 + 12s + 40\right]} = \boxed{\frac{34.64s - 130.7}{\left[(s+2)^2 + 100\right]\left[(s+6)^2 + 100\right]}}$$

(b) Taking the inverse transform using MATLAB, we find that

$$i_x(t) = e^{-6t} [0.0915 \cos 2t - 1.5245 \sin 2t] - e^{-2t} [0.0915 \cos 10t - 0.3415 \sin 10t] \text{ A}$$

## CHAPTER FIFTEEN SOLUTIONS

12.



Node 1:  $0 = 0.2 (V_1 - 3/s) + 0.2 V_1 s + 0.5 (V_1 - V_2) s$

Node 2:  $0 = 0.5 (V_2 - V_1) s + 0.125 V_2 s + 0.1 (V_2 + 5/s)$

Rewriting,  $(3.5 s^2 + s) V_1 + 2.5 s^2 V_2 = 3$  [1]

$-4 s^2 V_1 + (4 s^2 + 0.8 s + 1) V_2 = -4$  [2]

Solving using MATLAB or substitution, we find that

$$V_1(s) = \frac{-20s^2 + 16s + 20}{40s^4 + 68s^3 + 43s^2 + 10s}$$

$$= \left( \frac{1}{40} \right) \frac{-20s^2 + 16s + 20}{s(s + 0.5457 - j0.3361)(s + 0.5457 + j0.3361)(s + 0.6086)}$$

which can be expanded:

$$V_1(s) = \frac{a}{s} + \frac{b}{s + 0.5457 - j0.3361} + \frac{b^*}{s + 0.5457 + j0.3361} + \frac{c}{s + 0.6086}$$

Using the method of residues, we find that

$$a = 2, b = 2.511 \angle 101.5^\circ, b^* = 2.511 \angle -101.5^\circ \text{ and } c = -1.003.$$

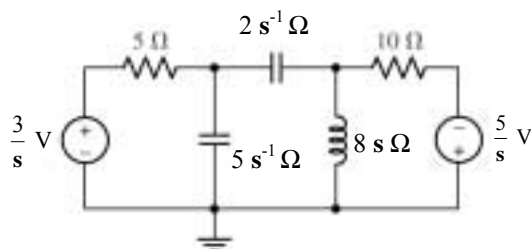
Thus, taking the inverse transform,

$$v_1(t) = [2 - 1.003 e^{-0.6086t} + 5.022 e^{-0.5457t} \cos(0.3361t - 101.5^\circ)] u(t) \text{ V}$$



## CHAPTER FIFTEEN SOLUTIONS

13. With zero initial energy, we may draw the following circuit:



Define three clockwise mesh currents  $\mathbf{I}_1$ ,  $\mathbf{I}_2$ , and  $\mathbf{I}_3$  in the left, centre and right meshes, respectively.

$$\text{Mesh 1: } -3/s + 5\mathbf{I}_1 + (5/s)\mathbf{I}_1 - (5/s)\mathbf{I}_2 = 0$$

$$\text{Mesh 2: } -(5/s)\mathbf{I}_1 + (8s + 7/s)\mathbf{I}_2 - 8s\mathbf{I}_3 = 0$$

$$\text{Mesh 3: } -8s\mathbf{I}_2 + (8s + 10)\mathbf{I}_3 - 5/s = 0$$

Rewriting,

$$\begin{array}{rcll} (5s + 5)\mathbf{I}_1 - 5\mathbf{I}_2 & = & 3 & [1] \\ -5\mathbf{I}_1 + (8s^2 + 7)\mathbf{I}_2 - 8s^2\mathbf{I}_3 & = & 0 & [2] \\ -8s^2\mathbf{I}_2 + (8s^2 + 10s)\mathbf{I}_3 & = & 5 & [3] \end{array}$$

Solving, we find that

$$\begin{aligned} \mathbf{I}_2(s) &= \frac{20s^2 + 32s + 15}{40s^3 + 68s^2 + 43s + 10} = \left(\frac{1}{40}\right) \frac{20s^2 + 32s + 15}{(s + 0.6086)(s + 0.5457 - j0.3361)(s + 0.5457 + j0.3361)} \\ &= \frac{a}{(s + 0.6086)} + \frac{b}{(s + 0.5457 - j0.3361)} + \frac{b^*}{(s + 0.5457 + j0.3361)} \end{aligned}$$

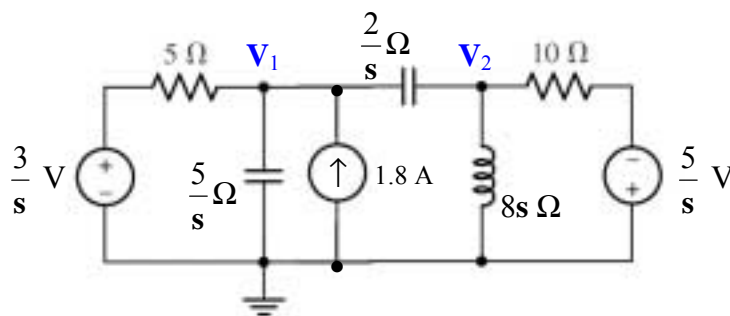
where  $a = 0.6269$ ,  $b = 0.3953\angle -99.25^\circ$ , and  $b^* = 0.3955\angle +99.25^\circ$

Taking the inverse transform, we find that

$$\begin{aligned} i_2(t) &= [0.6271e^{-0.6086t} + 0.3953e^{-j99.25^\circ} e^{(-0.5457 + j0.3361)t} + 0.3953e^{j99.25^\circ} e^{(-0.5457 - j0.3361)t}] u(t) \\ &= [0.6271e^{-0.6086t} + 0.7906 e^{-0.5457t} \cos(0.3361t + 99.25^\circ)] u(t) \end{aligned}$$

## CHAPTER FIFTEEN SOLUTIONS

14. We choose to represent the initial energy stored in the capacitor with a current source:



$$\text{Node 1:} \quad 1.8 = \frac{V_1 - \frac{3}{s}}{5} + \frac{s}{5} V_1 + \frac{s}{2} (V_1 - V_2)$$

$$\text{Node 2:} \quad 0 = \frac{s}{2} (V_2 - V_1) + \frac{1}{8s} V_2 + \frac{V_2 + \frac{5}{s}}{10}$$

$$\text{Rewriting,} \quad \begin{aligned} (5s^2 + 4s) V_1 - 5s^2 V_2 &= 18s + 6 & [1] \\ -4s^2 V_1 + (4s^2 + 0.8s + 1) V_2 &= -4 & [2] \end{aligned}$$

$$\text{Solving, we find that } V_1(s) = \frac{360s^3 + 92s^2 + 114s + 30}{s(40s^3 + 68s^2 + 43s + 10)}$$

$$= \frac{a}{s} + \frac{b}{s + 0.6086} + \frac{c}{s + 0.5457 - j0.3361} + \frac{c^*}{s + 0.5457 + j0.3361}$$

where  $a = 3$ ,  $b = 30.37$ ,  $c = 16.84 \angle 136.3^\circ$  and  $c^* = 16.84 \angle -136.3^\circ$

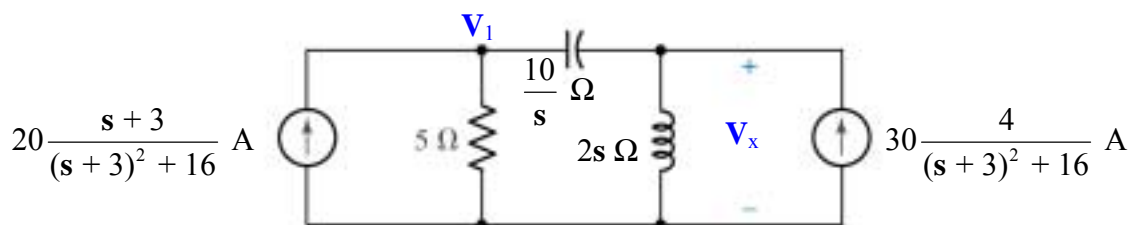
Taking the inverse transform, we find that

$$v_1(t) = [3 + 30.37e^{-0.6086t} + 16.84 e^{j136.3^\circ} e^{-0.5457t} e^{j0.3361t} + 16.84 e^{-j136.3^\circ} e^{-0.5457t} e^{-j0.3361t}] u(t) \text{ V}$$

$$= [3 + 30.37e^{-0.6086t} + 33.68e^{-0.5457t} \cos(0.3361t + 136.3^\circ)] u(t) \text{ V}$$

## CHAPTER FIFTEEN SOLUTIONS

15. We begin by assuming no initial energy in the circuit and transforming to the s-domain:



(a) via nodal analysis, we write:

$$\frac{20s + 60}{(s + 3)^2 + 16} = \frac{s}{10}(V_1 - V_x) + \frac{V_1}{5} \quad [1] \quad \text{and}$$

$$\frac{120}{(s + 3)^2 + 16} = \frac{V_x}{2s} + \frac{s}{10}(V_x - V_1) \quad [2]$$

Collecting terms and solving for  $V_x(s)$ , we find that

$$\begin{aligned} V_x(s) &= \frac{200s(s^2 + 9s + 12)}{2s^4 + 17s^3 + 90s^2 + 185s + 250} \\ &= \frac{200s(s^2 + 9s + 12)}{(s + 3 - j4)(s + 3 + j4)(s + 1.25 - j1.854)(s + 1.25 + j1.854)} \end{aligned}$$

(b) Using the method of residues, this function may be rewritten as

$$\frac{a}{(s + 3 - j4)} + \frac{a^*}{(s + 3 + j4)} + \frac{b}{(s + 1.25 - j1.854)} + \frac{b^*}{(s + 1.25 + j1.854)}$$

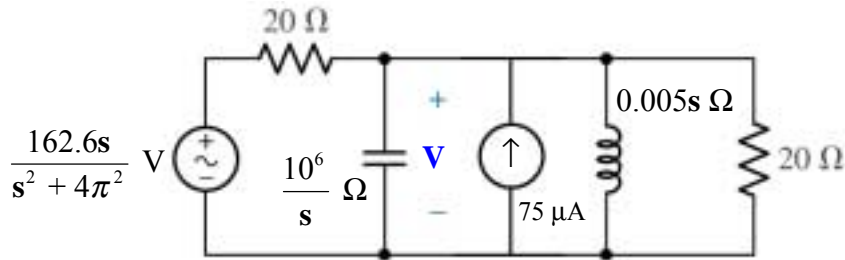
with  $a = 92.57 \angle -47.58^\circ$ ,  $a^* = 92.57 \angle 47.58^\circ$ ,  $b = 43.14 \angle 106.8^\circ$ ,  $b^* = 43.14 \angle -106.8^\circ$

Taking the inverse transform, then, yields

$$\begin{aligned} v_x(t) &= [92.57 e^{-j47.58^\circ} e^{-3t} e^{j4t} + 92.57 e^{j47.58^\circ} e^{-3t} e^{-j4t} \\ &\quad + 43.14 e^{j106.8^\circ} e^{-1.25t} e^{j1.854t} + 43.14 e^{-j106.8^\circ} e^{-1.25t} e^{-j1.854t}] u(t) \\ &= [185.1 e^{-3t} \cos(4t - 47.58^\circ) + 86.28 e^{-1.25t} \cos(1.854t + 106.8^\circ)] u(t) \end{aligned}$$

## CHAPTER FIFTEEN SOLUTIONS

16. We model the initial energy in the capacitor as a 75- $\mu$ A independent current source:



First, define  $\mathbf{Z}_{\text{eff}} = 10^6/s \parallel 0.005s \parallel 20 = \frac{s}{10^{-6}s^2 + 0.005s + 200} \Omega$

Then, writing a single KCL equation,  $75 \times 10^{-6} = \frac{\mathbf{V}(s)}{\mathbf{Z}_{\text{eff}}} + \frac{1}{20} \left( \mathbf{V}(s) - \frac{162.6s}{s^2 + 4\pi^2} \right)$

which may be solved for  $\mathbf{V}(s)$ :

$$\begin{aligned} \mathbf{V}(s) &= \frac{75s(s^2 + 1.084 \times 10^5 s + 39.48)}{s^4 + 5.5 \times 10^4 s^3 + 2 \times 10^8 s^2 + 2.171 \times 10^6 s + 7.896 \times 10^9} \\ &= \frac{75s(s^2 + 1.084 \times 10^5 s + 12.57)}{(s + 51085)(s + 3915)(s - j6.283)(s + j6.283)} \end{aligned}$$

(NOTE: factored with higher-precision denominator coefficients using MATLAB to obtain accurate complex poles: otherwise, numerical error led to an exponentially growing pole i.e. real part of the pole was positive)

$$= \frac{a}{(s + 51085)} + \frac{b}{(s + 3915)} + \frac{c}{(s - j2\pi)} + \frac{c^*}{(s + j2\pi)}$$

where  $a = -91.13$ ,  $b = 166.1$ ,  $c = 0.1277 \angle 89.91^\circ$  and  $c^* = 0.1277 \angle -89.91^\circ$ .

Thus, consolidating the complex exponential terms (the imaginary components cancel),

$$v(t) = [-91.13e^{-51085t} + 166.1e^{-3915t} + 0.2554 \cos(2\pi t + 89.91^\circ)] u(t) \text{ V}$$

- (b) The steady-state voltage across the capacitor is  $\mathbf{V} = [255.4 \cos(2\pi t + 89.91^\circ)] \text{ mV}$

This can be written in phasor notation as  $0.2554 \angle 89.91^\circ \text{ V}$ . The impedance across which this appears is  $\mathbf{Z}_{\text{eff}} = [j\omega C + 1/j\omega L + 1/20]^{-1} = 0.03142 \angle 89.91^\circ \Omega$ , so

$$\mathbf{I}_{\text{source}} = \mathbf{V} / \mathbf{Z}_{\text{eff}} = 8.129 \angle -89.91^\circ \text{ A.}$$

$$\text{Thus, } i_{\text{source}} = 8.129 \cos 2\pi t \text{ A.}$$

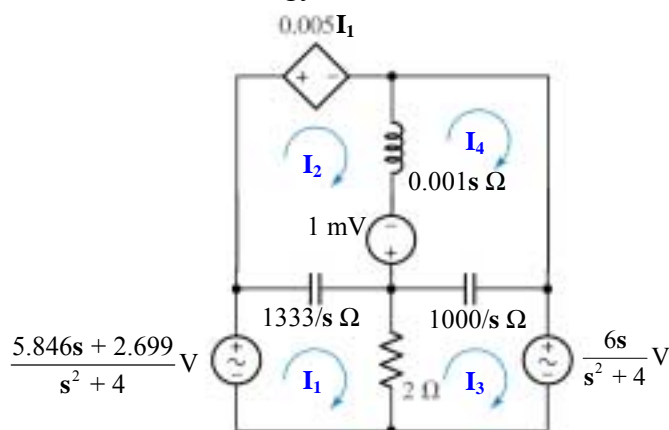
- (c) By phasor analysis, we can use simple voltage division to find the voltage division to find the capacitor voltage:

$$\mathbf{V}_C(j\omega) = \frac{(162.6 \angle 0)(0.03142 \angle 89.91^\circ)}{20 + 0.03142 \angle 89.91^\circ} = 0.2554 \angle 89.92^\circ \text{ V} \quad \text{which agrees with}$$

our answer to (a), assuming steady state. Dividing by  $0.03142 \angle 89.91^\circ \Omega$ , we find  $i_{\text{source}} = 8.129 \cos 2\pi t \text{ A.}$

## CHAPTER FIFTEEN SOLUTIONS

17. Only the inductor appears to have initial energy, so we model that with a voltage source:



$$\text{Mesh 1:} \quad \frac{5.846s + 2.699}{s^2 + 4} = \left(2 + \frac{1333}{s}\right) \mathbf{I}_1 - \frac{1333}{s} \mathbf{I}_2 - 2\mathbf{I}_3$$

$$\text{Mesh 2:} \quad 0 = 0.005\mathbf{I}_1 - 0.001 + (0.001s + 1333/s) \mathbf{I}_2 - (1333/s)\mathbf{I}_1 - 0.001s\mathbf{I}_4$$

$$\text{Mesh 3:} \quad 0 = (2 + 1000/s)\mathbf{I}_3 - 2\mathbf{I}_1 - (1000/s)\mathbf{I}_4 + \frac{6s}{s^2 + 4}$$

$$\text{Mesh 4:} \quad 0 = (0.001s + 1000/s) \mathbf{I}_4 - 0.001s\mathbf{I}_2 - (1000/s)\mathbf{I}_3 + 0.001$$

Solving, we find that  $\mathbf{I}_1 = -0.2 \frac{154s - 2699}{s^2 + 4}$  and

$$\begin{aligned} \mathbf{I}_2 &= 0.001 \frac{154s^4 - 7.378 \times 10^7 s^3 - 1.912 \times 10^{10} s^2 - 4.07 \times 10^{13} s + 7.196 \times 10^{14}}{2333s^4 + 6.665 \times 10^5 s^3 + 1.333 \times 10^9 s^2 + 5.332 \times 10^9} \\ &= \frac{0.4328 \angle -166.6^\circ}{s + 142.8 + j742} + \frac{0.4328 \angle +166.6^\circ}{s + 142.8 - j742} \\ &\quad + \frac{135.9 \angle -96.51^\circ}{s - j2} + \frac{135.9 \angle +96.51^\circ}{s + j2} + 6.6 \times 10^{-5} \end{aligned}$$

Taking the inverse transform of each,

$$i_1(t) = 271.7 \cos(2t - 96.51^\circ) \text{ A and}$$

$$i_2(t) = 0.8656 e^{-142.8t} \cos(742.3t + 166.6^\circ) + 271.8 \cos(2t - 96.51^\circ) + 6.6 \times 10^{-5} \delta(t) \text{ A}$$

Verifying via phasor analysis, we again write four mesh equations:

$$6 \angle -13^\circ = (2 - j666.7)\mathbf{I}_1 + j667\mathbf{I}_2 - 2\mathbf{I}_3$$

$$0 = (0.005 + j666.7)\mathbf{I}_1 + (j2 \times 10^{-3} - j666.7)\mathbf{I}_2 - j2 \times 10^{-3}\mathbf{I}_4$$

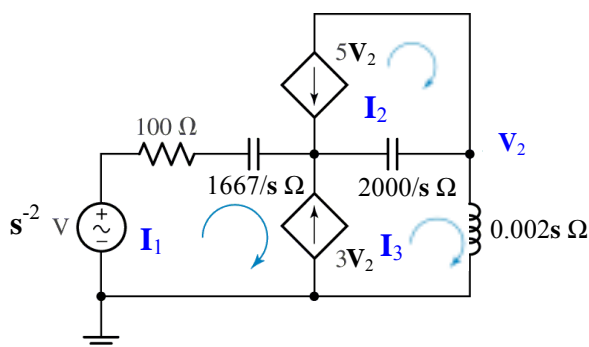
$$-6 \angle 0 = -2\mathbf{I}_1 + (2 - j500)\mathbf{I}_3 + j500\mathbf{I}_4$$

$$0 = -j2 \times 10^{-3}\mathbf{I}_2 + j500\mathbf{I}_3 + (j2 \times 10^{-3} - j500)\mathbf{I}_4$$

Solving, we find  $\mathbf{I}_1 = 271.7 \angle -96.5^\circ \text{ A}$  and  $\mathbf{I}_2 = 272 \angle -96.5^\circ \text{ A}$ . From the Laplace analysis, we see that this agrees with our expression for  $i_1(t)$ , and as  $t \rightarrow \infty$ , our expression for  $i_2(t) \rightarrow 272 \cos(2t - 96.5^\circ)$  in agreement with the phasor analysis.

## CHAPTER FIFTEEN SOLUTIONS

18. With no initial energy storage, we simply convert the circuit to the s-domain:



Writing a supermesh equation,

$$\frac{1}{s^2} = 100\mathbf{I}_1 + \frac{1}{6 \times 10^{-4}s}\mathbf{I}_1 + \frac{2000}{s}\mathbf{I}_3 + 0.002s\mathbf{I}_3 - \frac{2000}{s}\mathbf{I}_2$$

we next note that  $\mathbf{I}_2 = -5\mathbf{V}_2 = -5(0.002s)\mathbf{I}_3 = -0.01s\mathbf{I}_3$   
and  $\mathbf{I}_3 - \mathbf{I}_1 = 3\mathbf{V}_2 = 0.006s\mathbf{I}_3$ , or  $\mathbf{I}_1 = (1 - 0.006s)\mathbf{I}_3$ , we may write

$$\mathbf{I}_3 = \frac{1}{-0.598s^3 + 110s^2 + 3666s}$$

$$\begin{aligned}\mathbf{V}_2(s) = \mathbf{I}_3 / 0.002s &= \frac{1}{-0.0012s^4 + 0.22s^3 + 7.332s^2} \\ &= -\frac{7.645 \times 10^{-5}}{s - 212.8} + \frac{4.167 \times 10^{-3}}{s + 28.82} - \frac{4.091 \times 10^{-3}}{s} + \frac{0.1364}{s^2}\end{aligned}$$

Taking the inverse transform,

$$v_2(t) = -7.645 \times 10^{-5} e^{212.8t} + 4.167 \times 10^{-3} e^{-28.82t} - 4.091 \times 10^{-3} + 0.1364 t] u(t) \text{ V}$$

(a)  $v_2(1 \text{ ms}) =$

(b)  $v_2(100 \text{ ms}) =$

(c)  $v_2(10 \text{ s}) =$

$$-5.58 \times 10^{-7} \text{ V}$$

$$-1.334 \times 10^5 \text{ V}$$

$$-1.154 \times 10^{920} \text{ V.}$$

This is pretty big- best to start running.

## CHAPTER FIFTEEN SOLUTIONS

19. We need to write three mesh equations:

$$\text{Mesh 1: } \frac{5.846s + 2.699}{s^2 + 4} = \left(2 + \frac{1333}{s}\right) \mathbf{I}_1 - 2\mathbf{I}_3$$

$$\text{Mesh 3: } 0 = (2 + 1000/s)\mathbf{I}_3 - 2\mathbf{I}_1 - (1000/s)\mathbf{I}_4 + \frac{6s}{s^2 + 4}$$

$$\text{Mesh 4: } 0 = (0.001s + 1000/s)\mathbf{I}_4 - (1000/s)\mathbf{I}_3 + 10^{-6}$$

Solving,

$$\begin{aligned} \mathbf{I}_1 &= -0.001s \frac{(154s^3 - 2.925 \times 10^6 s^2 + 1.527 \times 10^8 s - 2.699 \times 10^9)}{2333s^4 + 6.665 \times 10^5 s^3 + 1.333 \times 10^9 s^2 + 2.666 \times 10^6 s + 5.332 \times 10^9} \\ &= \frac{0.6507 \angle 12.54^\circ}{s + 142.8 - j742.3} + \frac{0.6507 \angle -12.54^\circ}{s + 142.8 + j742.3} \\ &\quad + \frac{0.00101 \angle -6.538^\circ}{s - j2} + \frac{0.00101 \angle 6.538^\circ}{s + j2} - 6.601 \times 10^{-5} \end{aligned}$$

which corresponds to

$$i_1(t) = 1.301 e^{-142.8t} \cos(742.3t + 12.54^\circ) + 0.00202 \cos(2t - 6.538^\circ) - 6.601 \times 10^{-5} \delta(t) \text{ A}$$

and

$$\begin{aligned} \mathbf{I}_3 &= -0.001 \frac{(154s^4 + 3.997 \times 10^6 s^3 + 1.547 \times 10^8 s^2 + 3.996 \times 10^{12} s - 2.667 \times 10^6)}{(s^2 + 4)(2333s^2 + 6.665 \times 10^5 s + 1.333 \times 10^9)} \\ &= \frac{0.7821 \angle -33.56^\circ}{s + 142.8 - j742.3} + \frac{0.7821 \angle 33.56^\circ}{s + 142.8 + j742.3} \\ &\quad + \frac{1.499 \angle 179.9^\circ}{s - j2} + \frac{1.499 \angle -179.9^\circ}{s + j2} \end{aligned}$$

which corresponds to

$$i_3(t) = 1.564 e^{-142.8t} \cos(742.3t - 33.56^\circ) + 2.998 \cos(2t + 179.9^\circ) \text{ A}$$

The power absorbed by the 2- $\Omega$  resistor, then, is  $2[i_1(t) - i_3(t)]^2$  or

$$p(t) = 2[1.301 e^{-142.8t} \cos(742.3t + 12.54^\circ) + 0.00202 \cos(2t - 6.538^\circ) - 6.601 \times 10^{-5} \delta(t) - 1.564 e^{-142.8t} \cos(742.3t - 33.56^\circ) - 2.998 \cos(2t + 179.9^\circ)]^2 \text{ W}$$

## CHAPTER FIFTEEN SOLUTIONS

20. (a) We first define  $\mathbf{Z}_{\text{eff}} = R_B \parallel r_\pi \parallel (1/sC_\pi) = \frac{r_\pi R_B}{r_\pi + R_B + r_\pi R_B C_\pi s}$ . Writing two nodal equations, then, we obtain:

$$0 = (\mathbf{V}_\pi - \mathbf{V}_S)/R_S + \mathbf{V}_\pi (r_\pi + R_B + r_\pi R_B C_\pi s)/r_\pi R_B + (\mathbf{V}_\pi - \mathbf{V}_o)C_\mu s$$

and

$$-g_m \mathbf{V}_\pi = \mathbf{V}_o(R_C + R_L)/R_C R_L + (\mathbf{V}_o - \mathbf{V}_p) C_\mu s$$

Solving using MATLAB, we find that

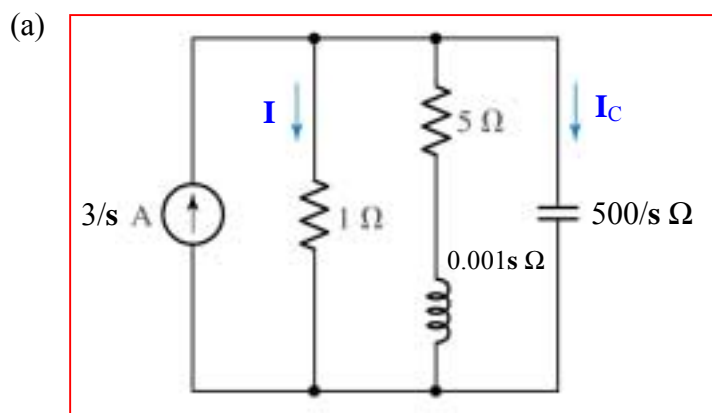
$$\begin{aligned} \frac{\mathbf{V}_o}{\mathbf{V}_s} = & r_\pi R_B R_C R_L (-g_m + C_\mu s) [R_s r_\pi R_B R_C R_L C_\pi C_\mu s^2 + (R_s r_\pi R_B R_C C_\pi + R_s r_\pi R_B R_C C_\mu \\ & + R_s r_\pi R_B R_L C_\pi + R_s r_\pi R_B R_L C_\mu + r_\pi R_B R_C R_L C_\mu + R_s r_\pi R_C R_L C_\mu \\ & + R_s R_B R_C R_L C_\mu + g_m R_s r_\pi R_B R_C R_L C_\mu) s \\ & + r_\pi R_B R_C + R_s r_\pi R_C + R_s R_B R_C + r_\pi R_B R_L + R_s r_\pi R_L + R_s R_B R_L]^{-1} \end{aligned}$$

- (b) Since we have only two energy storage elements in the circuit, the maximum number of poles would be two. The capacitors cannot be combined (either series or in parallel), so we expect a second-order denominator polynomial, which is what we found in part (a).



# CHAPTER FIFTEEN SOLUTIONS

21.



(b)  $Z_{TH} = (5 + 0.001s) \parallel (500/s) = \frac{2500s + 0.5}{0.001s^2 + 5s + 500} \Omega$

$V_{TH} = (3/s)Z_{TH} = \frac{7.5 \times 10^6 s + 1500}{s(s^2 + 5000s + 5 \times 10^5)} V$

(c) 
$$V_{1\Omega} = V_{TH} \frac{1}{1 + Z_{TH}} = \frac{7.5 \times 10^6 s + 1500}{s(s^2 + 505000) \left( 1 + \frac{2500s + 0.5}{0.001s^2 + 5s + 500} \right)}$$

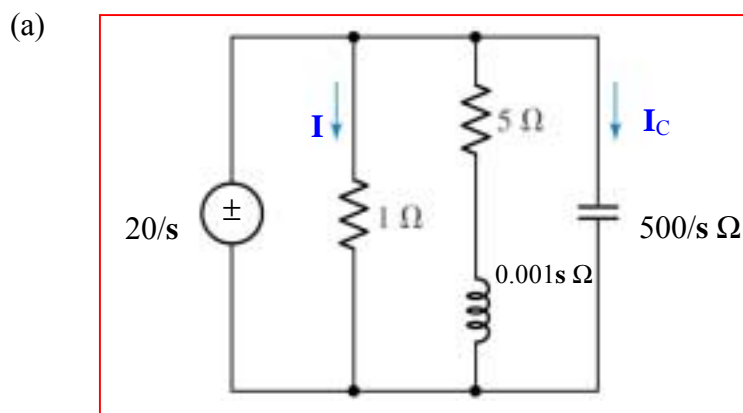
$$= -\frac{2.988}{s + 2.505 \times 10^6} + \frac{10.53 \angle -89.92^\circ}{s + j710.6} + \frac{10.53 \angle +89.92^\circ}{s - j710.6}$$

$$+ \frac{2.956}{s + 0.1998} + \frac{2.967 \times 10^{-3}}{s}$$

Thus,  $i_{1\Omega} = v_{1\Omega}(t) = [-2.988 e^{-2.505 \times 10^6 t} + 2.956 e^{-0.1998 t} + 2.967 \times 10^{-3} + 21.06 \cos(710.6t + 89.92^\circ)] u(t)$

## CHAPTER FIFTEEN SOLUTIONS

22.



(b)  $Z_{TH} = 0$ ,  $V_{TH} = 20/s\ \text{V}$  so  $I_N = \infty$

(c)  $I_C = \frac{\left(\frac{20}{s}\right)}{\left(\frac{500}{s}\right)} = 0.04\ \text{A}$ . Taking the inverse transform, we obtain a delta function:

$i_C(t) = 40\delta(t)\ \text{mA}.$

This “unphysical” solution arises from the circuit above attempting to force the voltage across the capacitor to change in zero time.

## CHAPTER FIFTEEN SOLUTIONS

23. Beginning with the source on the left (10/s V) we write two nodal equations:

$$\left( \mathbf{V}'_1 - \frac{10}{s} \right) \frac{1}{47000} + \frac{s}{30303} \mathbf{V}'_1 + \frac{\mathbf{V}'_1 - \mathbf{V}'_2}{56 + 336 \times 10^{-6}s} = 0$$

$$\frac{\mathbf{V}'_2}{47000} + \frac{s}{10870} \mathbf{V}'_2 + \frac{\mathbf{V}'_2 - \mathbf{V}'_1}{56 + 336 \times 10^{-6}s} = 0$$

Solving,

$$\mathbf{V}'_1 = \frac{303030(0.3197 \times 10^{13} + 0.1645 \times 10^{11}s + 98700s^2)}{s(0.4639 \times 10^{10}s^3 + 0.7732 \times 10^{15}s^2 + 0.5691 \times 10^{18}s + 0.1936 \times 10^{18})}$$

$$\mathbf{V}'_2 = \frac{0.9676 \times 10^{18}}{s(0.4639 \times 10^{10}s^3 + 0.7732 \times 10^{15}s^2 + 0.5691 \times 10^{18}s + 0.1936 \times 10^{18})}$$

Shorting out the left source and activating the right-hand source (5 – 3/s) V:

$$\frac{1}{47000} \mathbf{V}''_1 + \frac{s}{30303} \mathbf{V}''_1 + \frac{\mathbf{V}''_1 - \mathbf{V}''_2}{56 + 336 \times 10^{-6}s} = 0$$

$$\frac{\mathbf{V}''_2 - 5 + \frac{3}{s}}{47000} + \frac{s}{10870} \mathbf{V}''_2 + \frac{\mathbf{V}''_2 - \mathbf{V}''_1}{56 + 336 \times 10^{-6}s} = 0$$

Solving,

$$\mathbf{V}''_1 = \frac{0.9676 \times 10^{17}(5s - 3)}{s(0.4639 \times 10^{10}s^3 + 0.7732 \times 10^{15}s^2 + 0.5691 \times 10^{18}s + 0.1936 \times 10^{18})}$$

$$\mathbf{V}''_2 = \frac{7609(705000s^3 + 0.1175 \times 10^{12}s^2 + 0.6359 \times 10^{14}s - 0.3819 \times 10^{14})}{s(0.4639 \times 10^{10}s^3 + 0.7732 \times 10^{15}s^2 + 0.5691 \times 10^{18}s + 0.1936 \times 10^{18})}$$

Adding, we find that

$$\mathbf{V}_1 = \frac{30303(0.2239 \times 10^{13} + 0.1613 \times 10^{13}s + 98700s^2)}{s(0.4639 \times 10^{10}s^3 + 0.7732 \times 10^{15}s^2 + 0.5691 \times 10^{18}s + 0.1936 \times 10^{18})}$$

$$\mathbf{V}_2 = \frac{7609(705000s^3 + 0.1175 \times 10^{12}s^2 + 0.6359 \times 10^{14}s + 0.8897 \times 10^{14})}{s(0.4639 \times 10^{10}s^3 + 0.7732 \times 10^{15}s^2 + 0.5691 \times 10^{18}s + 0.1936 \times 10^{18})}$$

(b) Using the *ilaplace()* routine in MATLAB, we take the inverse transform of each:

$$v_1(t) = [3.504 + 0.3805 \times 10^{-2} e^{-165928t} - 0.8618 e^{-739t} - 2.646 e^{-0.3404t}] u(t) \text{ V}$$

$$v_2(t) = [3.496 - 0.1365 \times 10^{-2} e^{-165928t} + 0.309 e^{-739t} - 2.647 e^{-0.3404t}] u(t) \text{ V}$$

## CHAPTER FIFTEEN SOLUTIONS

$$24. \quad (10/s)(1/47000) = 2.128 \times 10^{-4}/s \text{ A}$$

$$(5 - 3/s)/47000 = (1.064 - 0.6383/s) \times 10^{-4} \text{ A}$$

$$\mathbf{Z}_L = 47000 \parallel (30303/s) = \frac{1.424 \times 10^9}{47000s + 30303} \Omega$$

$$\mathbf{Z}_R = 47000 \parallel (10870/s) = \frac{5.109 \times 10^8}{47000s + 10870} \Omega$$

Convert these back to voltage sources, one on the left ( $\mathbf{V}_L$ ) and one on the right ( $\mathbf{V}_R$ ):

$$\mathbf{V}_L = (2.128 \times 10^{-4}/s) \left( \frac{1.424 \times 10^9}{47000s + 30303} \right) = \frac{3.0303 \times 10^5}{s(47000s + 30303)} \text{ V}$$

$$\mathbf{V}_R = (1.064 - 0.6383/s) \times 10^{-4} \left( \frac{5.109 \times 10^8}{47000s + 10870} \right)$$

$$= \frac{54360}{47000s + 10870} - \frac{32611}{s(47000s + 10870)}$$

$$\text{Then, } \mathbf{I}_{56\Omega} = \frac{\mathbf{V}_L - \mathbf{V}_R}{\mathbf{Z}_L + \mathbf{Z}_R + 336 \times 10^{-6}s + 56}$$

$$= -6250 \frac{2.555 \times 10^9 s^2 - 1.413 \times 10^{10} s - 4.282 \times 10^9}{s(4.639 \times 10^9 s^3 + 7.732 \times 10^{14} s^2 + 5.691 \times 10^{17} s + 1.936 \times 10^{17})}$$

$$= \frac{0.208}{s + 1.659 \times 10^5} - \frac{0.0210}{s + 739} - \frac{1.533 \times 10^{-18}}{s + 0.6447}$$

$$+ \frac{2.658 \times 10^{-5}}{s + 0.3404} + \frac{2.755 \times 10^{-18}}{s + 0.2313} + \frac{1.382 \times 10^{-4}}{s}$$

Thus,

$$i_{56\Omega}(t) = [0.208 \exp(-1.659 \times 10^5 t) - 0.0210 \exp(-739t) - 1.533 \times 10^{-18} \exp(-0.6447t) + 2.658 \times 10^{-5} \exp(-0.3404t) + 2.755 \times 10^{-18} \exp(-0.2313t) + 1.382 \times 10^{-4}] u(t) \text{ A.}$$

The power absorbed in the 56- $\Omega$  resistor is simply  $56 [i_{56\Omega}(t)]^2$  or

$$56 [0.208 \exp(-1.659 \times 10^5 t) - 0.0210 \exp(-739t) - 1.533 \times 10^{-18} \exp(-0.6447t) + 2.658 \times 10^{-5} \exp(-0.3404t) + 2.755 \times 10^{-18} \exp(-0.2313t) + 1.382 \times 10^{-4}]^2 \text{ W}$$

## CHAPTER FIFTEEN SOLUTIONS

25. (a) Begin by finding  $\mathbf{Z}_{TH} = \mathbf{Z}_N$ :

$$\begin{aligned}\mathbf{Z}_{TH} &= 47000 + (30303/s) \parallel [336 \times 10^{-6} s + 56 + (10870/s) \parallel 47000] \\ &= \frac{4.639 \times 109 s^3 + 7.732 \times 10^{14} s^2 + 5.691 \times 10^{17} s + 1.936 \times 10^{17}}{98700 s^3 + 1.645 \times 10^{10} s^2 + 1.21 \times 10^{13} s + 2.059 \times 10^{12}} \Omega\end{aligned}$$

To find the Norton source value, define three clockwise mesh currents  $\mathbf{I}_1$ ,  $\mathbf{I}_2$  and  $\mathbf{I}_3$  in the left, centre and right hand meshes, such that  $\mathbf{I}_N(s) = -\mathbf{I}_1(s)$  and the  $10/s$  source is replaced by a short circuit.

$$\begin{aligned}(47000 + 30303/s) \mathbf{I}_1 - (30303/s) \mathbf{I}_2 &= 0 \\ (10870/s + 56 + 336 \times 10^{-6} s + 30303/s) \mathbf{I}_2 - (30303/s) \mathbf{I}_1 - (10870/s) \mathbf{I}_3 &= 0 \\ (47000 + 10870/s) \mathbf{I}_3 - (10870/s) \mathbf{I}_2 &= -5 + 3/s\end{aligned}$$

Solving,

$$\mathbf{I}_N = -\mathbf{I}_1 = \frac{2.059 \times 10^{12} (5s - 3)}{s(4.639 \times 10^9 s^3 + 7.732 \times 10^{14} s^2 + 5.691 \times 10^{17} s + 1.936 \times 10^{17})}$$

- (b)  $\mathbf{I}_{source} = (10/s) (1/\mathbf{Z}_{TH}) - \mathbf{I}_N(s)$

$$\begin{aligned}&= 0.001(0.4579 \times 10^{13} s^6 + 0.1526 \times 10^{19} s^5 + 0.1283 \times 10^{24} s^4 + 0.1792 \times 10^{27} s^3 \\ &\quad + 0.6306 \times 10^{29} s^2 + 0.3667 \times 10^{29} s + 0.5183 \times 10^{28}) [s(4639 s^3 + 0.7732 \times 10^9 s^2 \\ &\quad + 0.5691 \times 10^{12} s + 0.1936 \times 10^{12}) (0.4639 \times 10^{10} s^3 + 0.7732 \times 10^{15} s^2 + 0.5691 \times 10^{18} s \\ &\quad + 0.1936 \times 10^{18})]^{-1}\end{aligned}$$

Taking the inverse transform using the MATLAB *ilaplace()* routine, we find that

$$\begin{aligned}i_{source}(t) &= 0.1382 \times 10^{-3} + 0.8607 \times 10^{-8} \exp(-165930t) + 0.8723 \times 10^{-7} \exp(-739t) \\ &\quad + 0.1063 \times 10^{-3} \exp(-0.3403t) - 0.8096 \times 10^{-7} \exp(-165930t) \\ &\quad + 0.1820 \times 10^{-4} \exp(-739t) - 0.5 \times 10^{-4} \exp(-0.3404t)\end{aligned}$$

$$i_{source}(1.5 \text{ ms}) = 2.0055 \times 10^{-4} \text{ A} = 200.6 \mu\text{A}$$

## CHAPTER FIFTEEN SOLUTIONS

26. We begin by shorting the  $7 \cos 4t$  source, and replacing the  $5 \cos 2t$  source with  $\frac{5s}{s^2 + 4}$ .

(a) Define four clockwise mesh currents  $\mathbf{I}_1$ ,  $\mathbf{I}_2$ ,  $\mathbf{I}_3$  and  $\mathbf{I}_x$  in the top left, top right, bottom left and bottom right meshes, respectively. Then,

$$\frac{5s}{s^2 + 4} = (12 + 1/2s) \mathbf{I}_3 - 7 \mathbf{I}_1 - (1/2s) \mathbf{I}_x \quad [1]$$

$$0 = -4 \mathbf{I}_x + (9.5 + s) \mathbf{I}_1 - s \mathbf{I}_2 - 7 \mathbf{I}_3 \quad [2]$$

$$0 = (3 + s + 2/s) \mathbf{I}_2 - s \mathbf{I}_1 - 3 \mathbf{I}_x \quad [3]$$

$$0 = (4 + 3s + 1/2s) \mathbf{I}_x - 3 \mathbf{I}_2 - (1/2s) \mathbf{I}_3 \quad [4]$$

$$\mathbf{V}'_1 = (\mathbf{I}_3 - \mathbf{I}_x) (2s) \quad [5]$$

Solving all five equations simultaneously using MATLAB, we find that

$$\mathbf{V}'_1 = \frac{20s^3(75s^3 + 199s^2 + 187s + 152)}{1212s^6 + 3311s^5 + 7875s^4 + 15780s^3 + 12408s^2 + 10148s + 1200}$$

Next we short the  $5 \cos 2t$  source, and replace the  $7 \cos 4t$  source with  $\frac{7s}{s^2 + 16}$ .

Define four clockwise mesh currents  $\mathbf{I}_1$ ,  $\mathbf{I}_2$ ,  $\mathbf{I}_3$  and  $\mathbf{I}_x$  in the bottom left, top left, top right and bottom right meshes, respectively (*note order changed from above*). Then,

$$0 = (12 + 1/2s) \mathbf{I}_1 - 7 \mathbf{I}_2 - (1/2s) \mathbf{I}_x \quad [1]$$

$$0 = -4 \mathbf{I}_x + (9.5 + s) \mathbf{I}_2 - s \mathbf{I}_3 - 7 \mathbf{I}_1 \quad [2]$$

$$-\frac{7s}{s^2 + 16} = (3 + s + 2/s) \mathbf{I}_3 - s \mathbf{I}_2 - 3 \mathbf{I}_x \quad [3]$$

$$0 = (4 + 3s + 1/2s) \mathbf{I}_x - 3 \mathbf{I}_3 - (1/2s) \mathbf{I}_1 \quad [4]$$

$$\mathbf{V}''_1 = (\mathbf{I}_1 - \mathbf{I}_x) (2s) \quad [5]$$

Solving all five equations simultaneously using MATLAB, we find that

$$\mathbf{V}''_1 = \frac{-56s^4(21s^2 - 8s - 111)}{(1212s^6 + 3311s^5 + 22420s^4 + 55513s^3 + 48730s^2 + 40590s + 4800)}$$

The next step is to form the sum  $\mathbf{V}_1(s) = \mathbf{V}'_1 + \mathbf{V}''_1$ , which is accomplished in MATLAB using the function `symadd()`: `V1 = symadd(V1prime, V1doubleprime);`

$$\mathbf{V}_1(s) = \frac{4s^3(81s^5 + 1107s^4 + 7313s^3 + 17130s^2 + 21180s + 12160)}{(s^2 + 4)(1212s^6 + 3311s^5 + 22420s^4 + 55513s^3 + 48730s^2 + 40590s + 4800)}$$

(b) Using the `ilaplace()` routine from MATLAB, we find that

$$v_1(t) = [0.2673 \delta(t) + 6.903 \times 10^{-3} \cos 2t - 2.403 \sin 2t - 0.1167 e^{-1.971t} - 0.1948 e^{-0.3315t} \cos 0.903t + 0.1611 e^{-0.3115t} \sin 0.903t - 0.823 \times 10^{-3} e^{-0.1376t} + 3.229 \cos 4t + 3.626 \sin 4t] u(t) \text{ V}$$

## CHAPTER FIFTEEN SOLUTIONS

27. (a) We can combine the two sinusoidal sources in the time domain as they have the same frequency. Thus, there is really no need to invoke source transformation as such to find the current.

$$65 \cos 10^3 t \Leftrightarrow \frac{65s}{s^2 + 10^6}, \text{ and } 13 \text{ mH} \rightarrow 0.013s \Omega$$

We may therefore write

$$\begin{aligned} \mathbf{I}(s) &= \left( \frac{65s}{s^2 + 10^6} \right) \left( \frac{1}{83 + 0.013s} \right) = \frac{5000s}{(s^2 + 10^6)(s + 6385)} \\ &= -\frac{0.7643}{(s + 6385)} + \frac{0.3869 \angle -8.907^\circ}{(s - j10^3)} + \frac{0.3869 \angle 8.907^\circ}{(s + j10^3)} \end{aligned}$$

- (b) Taking the inverse transform,

$$i(t) = [-0.7643 e^{-6385t} + 0.7738 \cos(10^3 t - 8.907^\circ)] u(t) \text{ A}$$

- (c) The steady-state value of  $i(t)$  is simply  $0.7738 \cos(10^3 t - 8.907^\circ) \text{ A}$ .

## CHAPTER FIFTEEN SOLUTIONS

28.

$$(a) \quad \mathbf{Z}_{in} = \frac{\left(5 + \frac{5}{s}\right)(2 + 5s)}{5s + 7 + 5/s} = \frac{(5s + 5)(2 + 5s)}{5s^2 + 7s + 5} = \frac{25s^2 + 35s + 10}{5s^2 + 7s + 5}$$

$$\therefore \mathbf{Y}_{in}(s) = \frac{5s^2 + 7s + 5}{25s^2 + 35s + 10}$$

$$(b) \quad \text{Poles: } s^2 + 1.4s + 0.2 = 0, s = \frac{-1.4 \pm \sqrt{1.96 - 0.8}}{2} = -0.1615, -1.239s^{-1}$$

$$\text{Zeros: } s^2 + 1.4s + 1 = 0, s = \frac{-1.4 \pm \sqrt{1.96 - 4}}{2} = -0.7 \pm j0.7141s^{-1}$$

$$(c) \quad \text{Poles: same; } s = -0.1615, -1.239 s^{-1}$$

$$(d) \quad \text{Zeros: same; } s = -0.7 \pm j0.7141 s^{-1}$$



## CHAPTER FIFTEEN SOLUTIONS

29. (a) Regarding the circuit of Fig. 15.45, we replace each 2-mF capacitor with a  $500/s \, \Omega$  impedance. Then,

$$\mathbf{Z}_{\text{in}}(s) = \frac{\left(20 + \frac{500}{s}\right)\left(40 + \frac{500}{s}\right)}{60 + \frac{100}{s}} = 13.33 \frac{(s + 25)(s + 12.5)}{s(s + 1.667)}$$

Reading from the transfer function, we have

zeros at  $s = -25$  and  $-12.5 \, \text{s}^{-1}$ , and  
poles at  $s = 0$  and  $s = -1.667 \, \text{s}^{-1}$ .

- (b) Regarding the circuit of Fig. 15.47, we replace the 2-mF capacitor with a  $500/s \, \Omega$  impedance and the 1-mH inductor with a  $0.001s \, \Omega$  impedance. Then,

$$\mathbf{Z}_{\text{in}}(s) = \frac{\left(55 + \frac{500}{s}\right)(100 + 0.001s)}{155 + \frac{500}{s} + 0.001s} = 55 \frac{\left(s + \frac{500}{55}\right)(s + 10^5)}{(s + 1.55 \times 10^5)(s + 3.226)}$$

Reading from the transfer function, we have

zeros at  $s = -9.091$  and  $-10^5 \, \text{s}^{-1}$ , and  
poles at  $s = -1.55 \times 10^5$  and  $s = -3.226 \, \text{s}^{-1}$ .

## CHAPTER FIFTEEN SOLUTIONS

30.  $\mathbf{Y(s)}$  : zeros at  $\mathbf{s = 0; -10}$ ; poles at  $\mathbf{s = -5, -20 \text{ s}^{-1}}$ ;  $\mathbf{Y(s) \rightarrow 12 \text{ S}}$  as  $\mathbf{s \rightarrow \infty}$

(a)  $\mathbf{Y(s) = \frac{Ks(s+10)}{(s+5)(s+20)}}$ ,  $\mathbf{K = 12 \therefore}$

$$\mathbf{Y(s) = \frac{12s(s+10)}{(s+5)(s+20)} = \frac{12s^2 + 120s}{s^2 + 25s + 100}}$$

$$\therefore \mathbf{Y(j10) = \frac{-1200 + j1200}{-100 + j250 + 100} = 4.800 + j4.800 = 6.788 \angle 45^\circ \text{ S}}$$

(b)  $\mathbf{Y(-j10) = 6.788 \angle -45^\circ \text{ S}}$

(c)  $\mathbf{Y(-15) = \frac{12(-15)(-5)}{(-10)5} = -18 \text{ S}}$

(d)  $\mathbf{5 + Y(s) = 5 + \frac{12s^2 + 120s}{s^2 + 25s + 100} = \frac{17s^2 + 245s + 500}{(s+5)(s+20)}}$ ,  $\mathbf{s = \frac{-245 \pm \sqrt{245^2 - 68(500)}}{34}}$

Zeros:  $\mathbf{s = -2.461 \text{ and } -11.951 \text{ s}^{-1}}$ ; Poles:  $\mathbf{s = -5, -20 \text{ s}^{-1}}$

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31.

$$(a) \quad \mathbf{Y}_{in} = \frac{1}{4+s} + \frac{1}{5+5s} = \frac{0.2(6s+9)}{(4+s)(1+s)} \therefore \mathbf{Z}_{in} = \frac{5(s+1)(s+4)}{6(s+1.5)}$$

$$(b) \quad \text{Poles: } s = -1.5, \infty \quad \text{Zeros: } s = -1, -4 \text{ s}^{-1}$$

## CHAPTER FIFTEEN SOLUTIONS

32.  $\mathbf{H(s)} = \frac{s + 2}{(s + 5)(s^2 + 6s + 25)}$

(a)  $d(t) \Leftrightarrow 1$ , so the output is  $\frac{s + 2}{(s + 5)(s^2 + 6s + 25)}$

(b)  $e^{-4t} u(t) \Leftrightarrow 1 / (s + 4)$ , so the output is  $\frac{s + 2}{(s + 4)(s + 5)(s^2 + 6s + 25)}$

(c)  $2 \cos 15t u(t) \Leftrightarrow \frac{2s}{s^2 + 225}$ , so the output is  $\frac{2s(s + 2)}{(s^2 + 225)(s + 5)(s^2 + 6s + 25)}$

(d)  $t e^{-t} u(t) \Leftrightarrow 1 / (s + 1)$ , so the output is  $\frac{s + 2}{(s + 1)(s + 5)(s^2 + 6s + 25)}$

(e) poles and zeros of each:

- (a): zero at  $s = -2$ , poles at  $s = -5, -3 \pm j4$
- (b): zero at  $s = -2$ , poles at  $s = -4, -5, -3 \pm j4$
- (c): zeros at  $s = 0, -2$ , poles at  $s = \pm j15, -5, -3 \pm j4$
- (d): zero at  $s = -2$ , poles at  $s = -1, -5, -3 \pm j4$

## CHAPTER FIFTEEN SOLUTIONS

33.  $h(t) = 5 [u(t) - u(t - 1)] \sin \pi t$        $x(t) = 2[u(t) - u(t - 2)]$

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda) d\lambda$$

$$t < 0: y(t) = 0$$

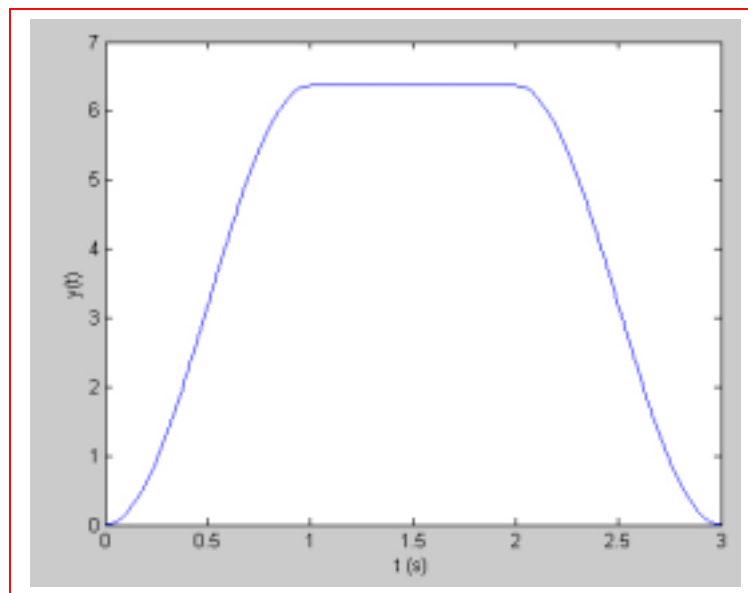
$$0 < t < 1: y(t) = \int_0^t 10 \sin \pi \lambda d\lambda = -\frac{10}{\lambda} \cos \pi \lambda \Big|_0^t = \frac{10}{\pi} (1 - \cos \pi t)$$

$$1 < t < 2: y(t) = \int_0^1 10 \sin \pi \lambda d\lambda = \frac{20}{\pi}$$

$$2 < t < 3: y(t) = \int_{t-2}^1 10 \sin \pi \lambda d\lambda = -\frac{10}{\pi} \cos \pi \lambda \Big|_{t-2}^1 = -\frac{10}{\pi} [-1 - \cos(\pi t - 2\pi)]$$

$$= (10/\pi) (1 + \cos \pi t)$$

$$t > 3: y(t) = 0$$



## CHAPTER FIFTEEN SOLUTIONS

34.  $f_1(t) = e^{-5t} u(t), f_2(t) = (1 - e^{-2t}) u(t)$

(a)  $f_1 * f_2 = \int_{-\infty}^{\infty} f_1(\lambda) f_2(t - \lambda) d\lambda$

$t < 0: f_1 * f_2 = 0$

$$\begin{aligned} t > 0: f_1 * f_2 &= \int_{-\infty}^{\infty} e^{-5\lambda} (1 - e^{-2(t-\lambda)}) u(t-\lambda) d\lambda = \int_{-\infty}^t (e^{-5\lambda} - e^{-2t} e^{-3\lambda}) d\lambda \\ &= -\frac{1}{5} e^{-5\lambda} \Big|_0^t + \frac{1}{3} e^{-2t} e^{-3\lambda} \Big|_0^t = \left( \frac{1}{5} + \frac{2}{15} e^{-5t} - \frac{1}{3} e^{-2t} \right) u(t) \end{aligned}$$

(b)  $F_1(s) = 1/(s + 5), F_2(s) = 1/s - 1/(s + 2)$

$$F_1(s) F_2(s) = \frac{1}{s(s + 5)} - \frac{1}{(s + 5)(s + 2)} = \frac{a}{s} + \frac{b}{s + 2} + \frac{c}{s + 5}$$

Where  $a = 0.2$ ,  $b = -1/3$ , and  $c = -1/5 + 1/3 = 2/15$ .

Taking the inverse transform, we find that  $f_1 * f_2 = \left( \frac{1}{5} + \frac{2}{15} e^{-5t} - \frac{1}{3} e^{-2t} \right) u(t)$

## CHAPTER FIFTEEN SOLUTIONS

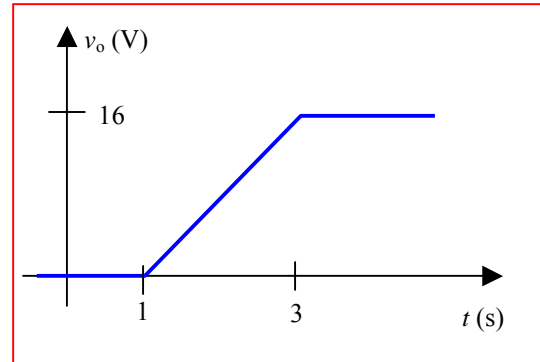
35. The impulse response is  $v_o(t) = 4u(t) - 4u(t - 2)$  V,  
so we know that  $h(t) = 4u(t) - 4u(t - 2)$ .  $v_i(t) = 2u(t - 1)$ , and  $v_o(t) = h(t) * v_i(t)$ .

$$\text{Thus, } v_o(t) = \int_{-\infty}^{\infty} h(\lambda) v_i(t - \lambda) d\lambda$$

$$t < 0: v_o = 0$$

$$1 < t < 3: v_o = \int_{0^-}^{t-1} 8 d\lambda = 8t - 8$$

$$t > 3: v_o = \int_{0^-}^2 8 d\lambda = 16$$



## CHAPTER FIFTEEN SOLUTIONS

36.  $h(t) = 2e^{-3t} u(t), x(t) = u(t) - \delta(t)$

(a)  $y(t) = \int_{0^-}^{\infty} h(\lambda)x(t-\lambda)d\lambda$

$t < 0 : y(t) = 0$

$$t > 0 : y(t) = 2 \int_{0^-}^t e^{-3\lambda} [1 - \delta(t-\lambda)] d\lambda = 2 \left[ -\frac{1}{3} e^{-3\lambda} u(t) \Big|_0^t - e^{-3t} u(t) \right]$$

$$= \frac{2}{3} (1 - e^{-3t}) u(t) - 2e^{-3t} u(t) = \left( \frac{2}{3} - \frac{8}{3} e^{-3t} \right) u(t)$$

(b)  $\mathbf{H(s)} = \frac{2}{s+3} \quad \mathbf{X(s)} = \frac{1}{s} - 1$

thus,  $\mathbf{Y(s)} = \frac{2(1-s)}{s(s+3)} = \frac{2}{3} \left( \frac{1}{s} \right) - \frac{8}{3} \left( \frac{1}{s+3} \right)$

Taking the inverse transform, we find that  $y(t) = \frac{2}{3} u(t) - \frac{8}{3} e^{-3t} u(t)$



## CHAPTER FIFTEEN SOLUTIONS

37.  $h(t) = 5 u(t) - 5 u(t-2)$ , so  $\mathbf{H}(s) = \frac{5}{s} - 5e^{-2s}$

(a)  $v_{in}(t) = 3\delta(t)$ , so  $\mathbf{V}_{in}(s) = 3$

$$\mathbf{V}_{out}(s) = \mathbf{V}_{in}(s) \mathbf{H}(s) = \frac{15}{s} - 15e^{-2s} \cdot v_{out}(t) = \Lambda^{-1}\{\mathbf{V}_{out}(s)\} = 15 u(t) - 15 u(t-2)$$

(b)  $v_{in}(t) = 3u(t)$ , so  $\mathbf{V}_{in}(s) = \frac{3}{s}$

$$\mathbf{V}_{out}(s) = \mathbf{V}_{in}(s) \mathbf{H}(s) = \left(\frac{3}{s}\right)\left(\frac{5}{s} - 5e^{-2s}\right) = \frac{15}{s^2} - \frac{15}{s}e^{-2s}$$

$$v_{out}(t) = \Lambda^{-1}\{\mathbf{V}_{out}(s)\} = 15 t u(t) - 15 u^2(t-2) = 15 t u(t) - 15 u(t-2)$$

(c)  $v_{in}(t) = 3u(t) - 3u(t-2)$ , so  $\mathbf{V}_{in}(s) = \frac{3}{s} - 3e^{-2s}$

$$\mathbf{V}_{out}(s) = \mathbf{V}_{in}(s) \mathbf{H}(s) = \left(\frac{3}{s} - 3e^{-2s}\right)\left(\frac{5}{s} - 5e^{-2s}\right) = \frac{15}{s^2} - \frac{30}{s}e^{-2s} + 15e^{-4s}$$

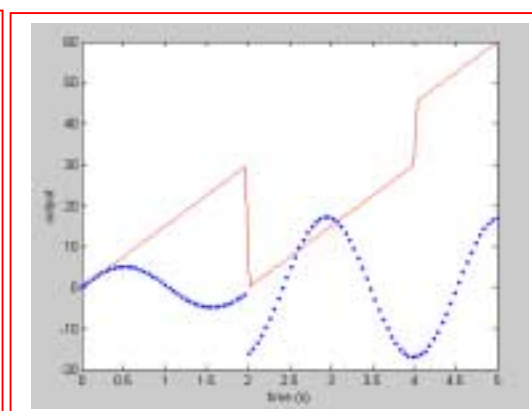
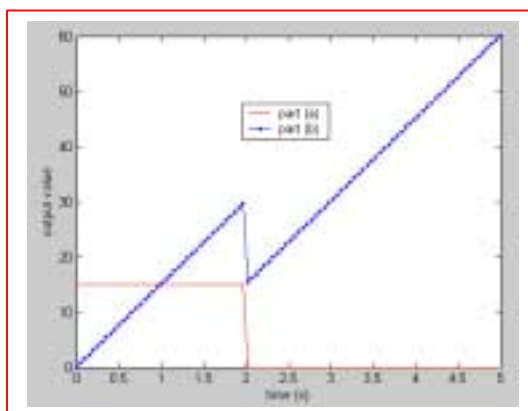
$$v_{out}(t) = \Lambda^{-1}\{\mathbf{V}_{out}(s)\} = 15 t u(t) - 30 u^2(t-2) + 15 u^2(t-4)$$

$$= 15 t u(t) - 30 u(t-2) + 15 u(t)$$

(d)  $v_{in}(t) = 3 \cos 3t$ , so  $\mathbf{V}_{in}(s) = \frac{3s}{s^2 + 9}$

$$\mathbf{V}_{out}(s) = \mathbf{V}_{in}(s) \mathbf{H}(s) = \frac{15}{s^2 + 9} - \frac{15s}{s^2 + 9}e^{-2s}$$

$$v_{out}(t) = \Lambda^{-1}\{\mathbf{V}_{out}(s)\} = 5 \sin 3t u(t) - 15 \cos [3(t-2)] u(t-2)$$



## CHAPTER FIFTEEN SOLUTIONS

38.

$$\begin{aligned}
 \mathbf{I}_{in} &= \frac{\mathbf{V}_{in}}{\frac{10}{s} + 20 \parallel 20 \left( 20 + \frac{10}{s} \right)} = \frac{\mathbf{V}_{in}}{\frac{10}{s} + \frac{20(20 + 10/s)}{40 + 10/s}} \\
 &= \frac{\mathbf{V}_{in}}{\frac{10}{s} + \frac{40s + 20}{4s + 1}} = \frac{\mathbf{V}_{in}}{\frac{40s^2 + 60s + 10}{4s^2 + s}} = \mathbf{V}_{in} \frac{40s^2 + s}{40s^2 + 60s + 10} \\
 \therefore \mathbf{I}_{top} &= \mathbf{I}_{in} \frac{20}{40 + \frac{10}{s}} = \mathbf{I}_{in} \frac{2s}{4s + 1} = \mathbf{V}_{in} \frac{2s^2}{40s^2 + 60s + 10}; \\
 \mathbf{V}_{out} &= \frac{10}{s} \mathbf{I}_{in} + 20\mathbf{I}_{top} = \mathbf{V}_{in} \left[ \frac{4s + 1}{4s^2 + 6s + 1} + \frac{4s^2}{4s^2 + 6s + 1} \right] \therefore \\
 \mathbf{H}(s) &= \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{4s^2 + 4s + 1}{4s^2 + 6s + 1} = \frac{s^2 + s + 0.25}{s^2 + 1.5s + 0.25} = \frac{(s + 0.5)^2}{(s + 0.19098)(s + 1.3090)} \therefore
 \end{aligned}$$

zeros:  $s = -0.5, s = -0.5$ ; poles:  $s = -1.3090, -0.19098$

## CHAPTER FIFTEEN SOLUTIONS

39.

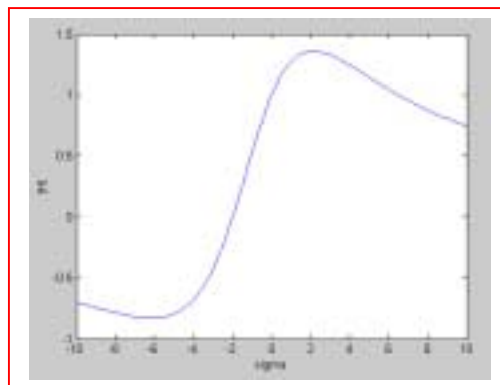
(a)  $\mathbf{H}(s) = \mathbf{V}_2(s) / \mathbf{V}_1(s)$ ,  $\mathbf{H}(0) = 1$

$$\therefore \mathbf{H}(s) = \frac{K(s+2)}{(s+1+j4)(s+1-j4)} = \frac{K(s+2)}{s^2 + 2s + 17}$$

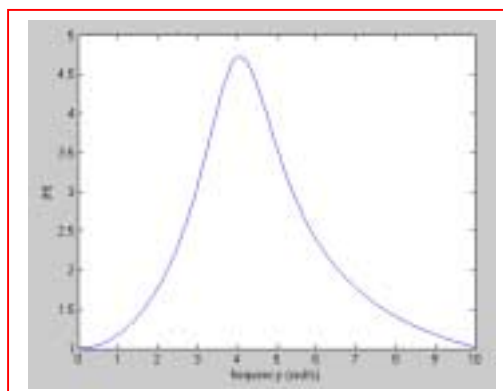
$$1 = 2 \frac{K}{17}, \text{ so } K = 8.5$$

$$\text{Thus, } \mathbf{H}(s) = \frac{8.5(s+2)}{s^2 + 2s + 17}$$

$$\text{Let } \omega = 0 \quad \therefore \mathbf{H}(\sigma) = \frac{8.5(\sigma+2)}{\sigma^2 + 2\sigma + 17}$$



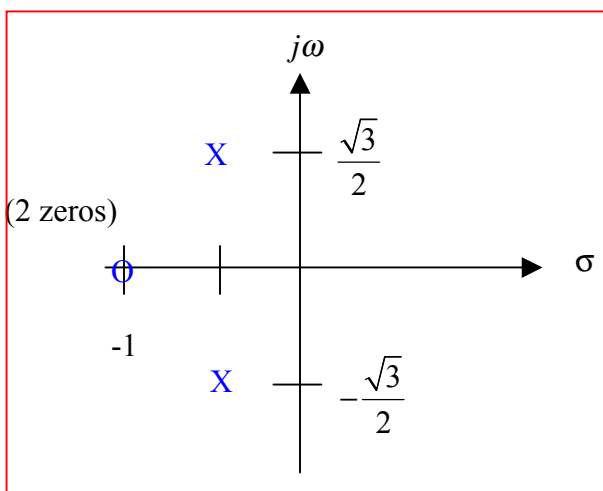
(b)  $|H(j\omega)| = 8.5 \sqrt{\frac{\omega^2 + 4}{(17 - \omega^2)^2 + 4\omega^2}}$



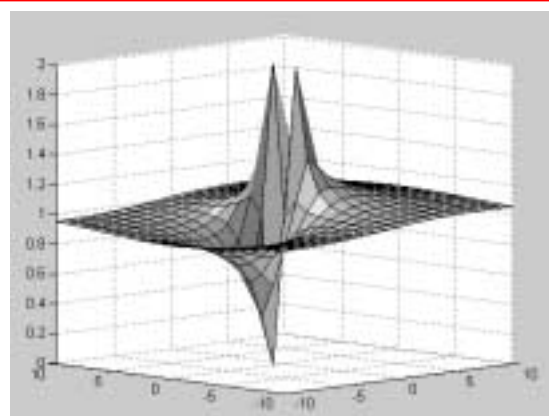
(c) By trial & error:  $|\mathbf{H}(j\omega)|_{\max} = 4.729$  at  $\omega = 4.07$  rad/s

## CHAPTER FIFTEEN SOLUTIONS

40. (a) pole-zero constellation



(b) elastic-sheet model

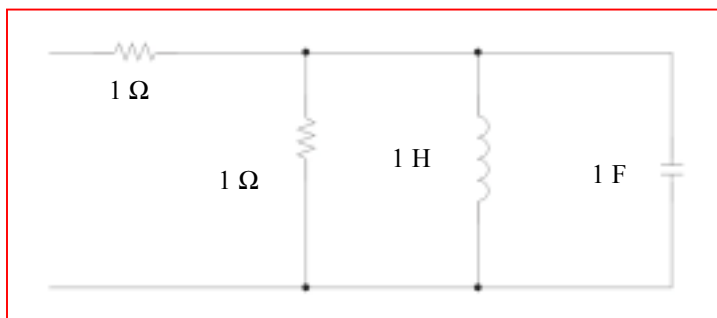


$$\begin{aligned} \text{(c) } \mathbf{H}(s) &= \frac{(s+1)^2}{\left(s+0.5+j\frac{\sqrt{3}}{2}\right)\left(s+0.5-j\frac{\sqrt{3}}{2}\right)} = \frac{(s+1)^2}{s^2+s+1} \\ &= \frac{s^2+2s+1}{s^2+s+1} = 1 + \frac{s}{s^2+s+1} \end{aligned}$$

We can implement this with a  $1\text{-}\Omega$  resistor in series with a network having the impedance given by the second term. There are two energy storage elements in that network (the denominator is order 2). That network impedance can be rewritten as

$$\frac{s}{s^2+s+1} = \frac{1}{s+1+\frac{1}{s}}, \text{ which can be seen to be equal to the parallel combination of a } 1\text{-}\Omega$$

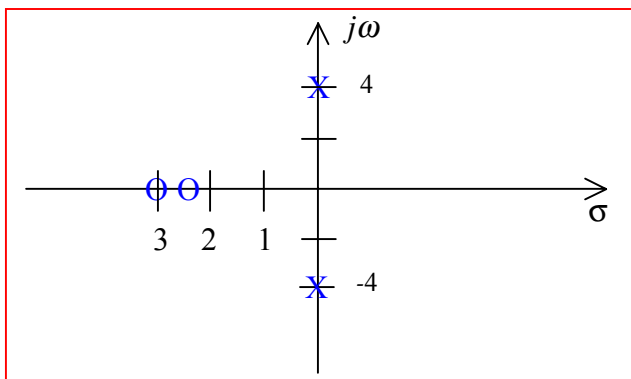
resistor, a  $1\text{-H}$  inductor, and a  $1\text{-F}$  capacitor.



## CHAPTER FIFTEEN SOLUTIONS

41.  $\mathbf{H(s)} = (10s^2 + 55s + 75)/(s^2 + 16)$

(a)  $\mathbf{H(s)} = 10 \frac{(s + 3)(s + 2.5)}{(s + j4)(s - j4)}$ . Critical frequencies: zeros at  $-3, -2.5$ ; poles at  $\pm j4$ .

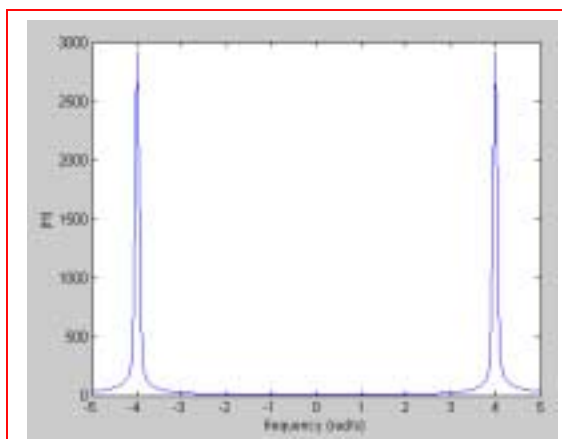
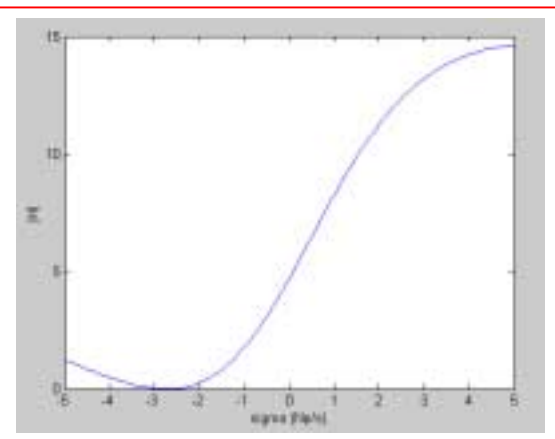


(b)  $\mathbf{H(0)} = \frac{75}{16} = 4.688$ ,  $\mathbf{H(\infty)} = 10$

(c)  $\mathbf{H(0)} = 4.679$   $\mathbf{K} = 3$ , so  $\mathbf{K} = 0.64$

$\therefore \mathbf{H(j3)} = 0.64 \left| \frac{-90 + 75 + j165}{7} \right| = \frac{0.64}{7} |-15 + j165| = 15.15 \text{ cm}$

(d)

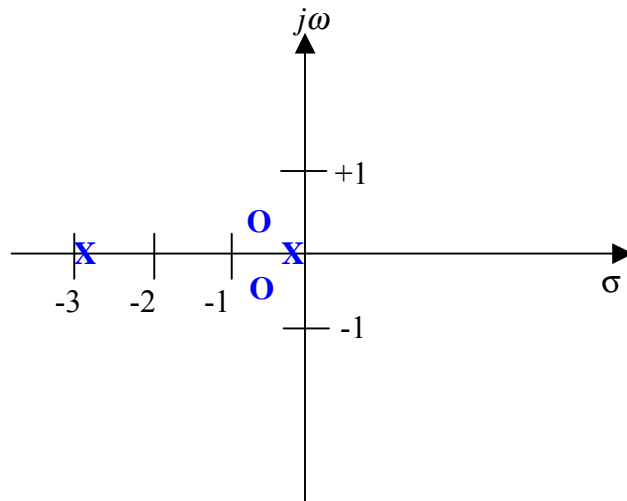


## CHAPTER FIFTEEN SOLUTIONS

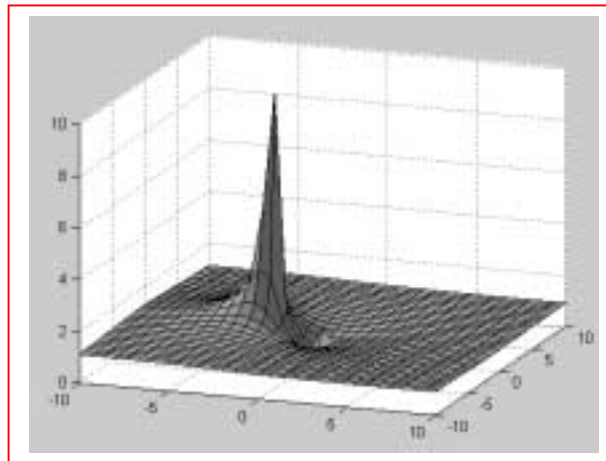
42. (a)  $Y(s) = \frac{5s^2 + 5s + 2}{5s^2 + 15s + 2} = \frac{(s + 0.5 + j0.3873)(s + 0.5 - j0.3873)}{(s + 2.86)(s + 0.1399)}$

Zeros:  $s = -0.5 \pm j0.3873$

Poles:  $s = -2.86, s = -0.1399$



(b) elastic sheet model



(c) latitude  $5^{\circ}5'2''$ , longitude  $5^{\circ}15'2''$  puts it a little off the coast of Timbuktu.

## CHAPTER FIFTEEN SOLUTIONS

43.  $\mathbf{H(s)} = \frac{\mathbf{I_0}}{\mathbf{I_M}}; \mathbf{H(-2)} = 6$

(a)  $\mathbf{H(s)} = \mathbf{K} \frac{(\mathbf{s-1})(\mathbf{s+1})(\mathbf{s+3})}{(\mathbf{s+3+j2})(\mathbf{s+3-j2})}$

$$\mathbf{H(-2)} = 6 = \frac{(-3)(-1)\mathbf{K}}{(1+j2)(1-j2)} = \frac{3\mathbf{K}}{5} \therefore \mathbf{K} = 10,$$

$$\text{Thus, } \mathbf{H(s)} = 10 \frac{(\mathbf{s^2-1})(\mathbf{s+3})}{\mathbf{s^2+6s+13}} = \frac{10\mathbf{s^3+30s^2-10s-30}}{\mathbf{s^2+6s+13}}$$

(b)  $\mathbf{H(0)} = -\frac{30}{13} = -2.308, \mathbf{H(\infty)} = \infty$

(c)  $1 : (\mathbf{s-1}) = (\mathbf{j2-1}) = 2.236 \angle 116.57^\circ$

$$-1 : (\mathbf{s+1}) = (\mathbf{j2+1}) = 2.236 \angle 63.43^\circ$$

$$-3 : (\mathbf{s+3}) = \mathbf{j2+3} = 3.606 \angle 33.69^\circ$$

$$-3-j2 : \mathbf{j2+3+j2} = 5.000 \angle 53.13^\circ$$

$$-3+j2 : \mathbf{j2+3-j2} = 3 \angle 0^\circ$$

## CHAPTER FIFTEEN SOLUTIONS

44.

$\mathbf{Z}_A$ : zero at  $\mathbf{s} = -10 + j0$ ;  $\mathbf{Z}_A + 20$ : zero at  $\mathbf{s} = -3.6 + j0$

$$\therefore \mathbf{Z}_A = 5 + \frac{R/sC}{R + 1/SC} = 5 + \frac{R}{sCR + 1} = 5 + \frac{1/C}{s + 1/RC} = \frac{5s + 5/RC + 1/C}{s + 1/RC}$$

$$\therefore \mathbf{Z}_A = \frac{5(s + 1/RC + 1/5C)}{s + 1/RC}$$

Thus, using the fact that  $\mathbf{Z}_A = 0$  at  $\mathbf{s} = -10$ , we may write  $\frac{1}{RC} + \frac{1}{5C} = 10$

$$\text{Also, } \mathbf{Z}_B = 25 + \frac{1/C}{s + 1/RC} = \frac{25s + \frac{25}{RC} + \frac{1}{C}}{s + 1/RC} = \frac{25\left(s + \frac{1}{RC} + \frac{1}{25C}\right)}{s + \frac{1}{RC}}$$

$$\therefore \frac{1}{RC} + \frac{1}{25C} = 3.6 \quad \text{or} \quad \frac{4}{25C} = 6.4,$$

$$C = \frac{1}{40} = \boxed{25 \text{ mF}},$$

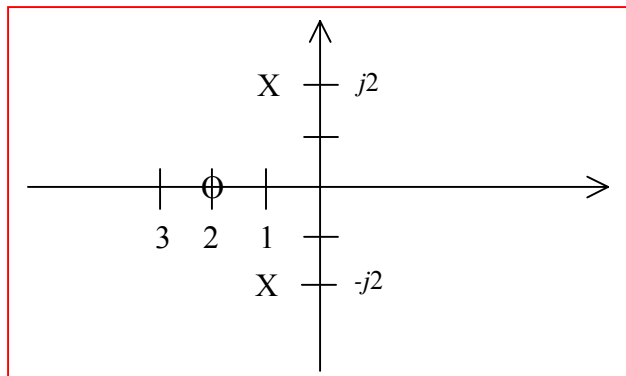
$$\frac{40}{R} + \frac{40}{5} = 10, \quad \frac{40}{R} = 2, \quad \text{so } R = \boxed{20 \text{ } \Omega}$$



## CHAPTER FIFTEEN SOLUTIONS

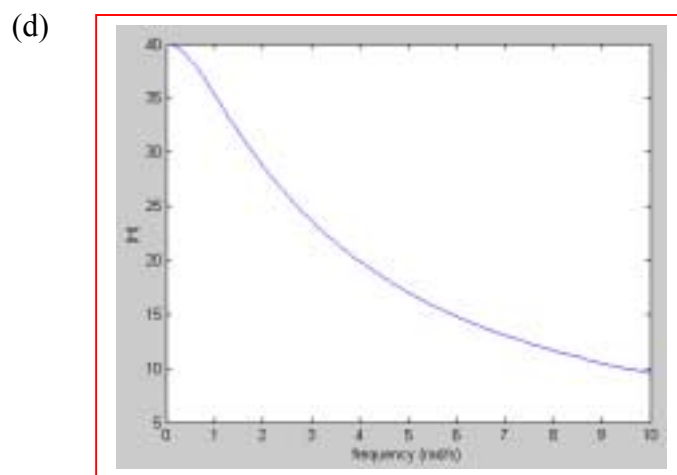
45.  $\mathbf{H}(s) = 100(s + 2)/(s^2 + 2s + 5)$

(a) zero at  $s = -2$ , poles at  $s = \frac{-2 \pm \sqrt{4 - 20}}{2} = -1 \pm j2$



(b)  $\mathbf{H}(j\omega) = \frac{100(2 + j\omega)}{(5 - \omega^2) + j2\omega}$

(c)  $|\mathbf{H}(j\omega)| = 100 \sqrt{\frac{\omega^2 + 4}{\omega^4 - 6\omega^2 + 25}}$



(e)  $\frac{|\mathbf{H}(j\omega)|^2}{10\,000} = \frac{\omega^2 + 4}{\omega^4 - 6\omega^2 + 25}, \quad \frac{d|\mathbf{H}(j\omega)|^2}{d\omega} = \frac{(\omega^4 - 6\omega^2 + 25)2\omega - (\omega^2 + 4)(4\omega^3 - 12\omega)}{etc}$   
 $\therefore \omega^4 - 6\omega^2 + 25 = (\omega^2 + 4)(2\omega^2 - 6), \quad \omega^4 - 6\omega^2 + 25 = 2\omega^4 + 2\omega^2 - 24, \quad \omega^4 + 8\omega^2 - 49 = 0$   
 $\therefore \omega^2 = \frac{-8 \pm \sqrt{64 + 196}}{2} = 4.062 \quad \therefore \omega_{mar} = 2.016 \text{ rad/s}, \quad |\mathbf{H}(j2.016)| = 68.61$

## CHAPTER FIFTEEN SOLUTIONS

$$46. \quad \mathbf{Z}_{in}(s) = \frac{5s + 20}{s + 2} \quad \Omega$$

$$(a) \quad v_{ab}(0) = 25 \text{ V}; \quad \mathbf{Z}_{in}(s) = \frac{5(s + 4)}{s + 2}, \quad \mathbf{V}_{ab} = \mathbf{Z}_{in} \mathbf{I}_{in}$$

$$\therefore \mathbf{H}(s) = \frac{5(s + 4)}{s + 2}, \quad \text{single pole at } s = -2 \quad \therefore v_{ab}(t) = \boxed{25e^{-2t} \text{ V}, t > 0}$$

$$(b) \quad i_{ab}(0) = 3 \text{ A} \quad \therefore \mathbf{I}_{ab} = \frac{\mathbf{V}_s}{\mathbf{Z}_{in}} \quad \therefore \mathbf{H}(s) = \frac{\mathbf{I}_{ab}}{\mathbf{V}_{in}} = \frac{1}{\mathbf{Z}_{in}} = \frac{s + 2}{5(s + 4)} \quad \text{single pole at } s = -4$$

$$\therefore i_{ab}(t) = \boxed{3e^{-4t} \text{ A}, t > 0}$$

## CHAPTER FIFTEEN SOLUTIONS

47.  $\mathbf{Z}_{in}(\mathbf{s}) = 5(\mathbf{s}^2 + 4\mathbf{s} + 20)/(\mathbf{s} + 1)$

(a)  $v_{ab} = 160e^{-6t} \text{ V} \therefore \mathbf{V}_{ab} = 160 \text{ V}, \mathbf{s} = -6$

$$\mathbf{I}_a = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{in}} = \frac{160(\mathbf{s} + 1)}{5(\mathbf{s}^2 + 4\mathbf{s} + 20)} = \frac{32(-5)}{3b - 24 + 20} = -5 \text{ A} \therefore i_a(t) = \boxed{-5e^{-6t} \text{ A (all } t)}$$

(b)  $v_{ab} = 160e^{-6t}u(t), i_a(0) = 0, i'_a(0) = 32 \text{ A/s} \therefore \mathbf{H}(\mathbf{s}) = \frac{\mathbf{I}_a}{\mathbf{V}_s} = \frac{1}{\mathbf{Z}_{in}} = \frac{\mathbf{s} + 1}{5(\mathbf{s}^2 + 4\mathbf{s} + 20)}$

$$\mathbf{s} = \frac{-4 \pm \sqrt{16 - 80}}{2} = -2 \pm j4 \therefore i_a(t) = -5e^{-6t} + e^{-2t}(A \cos 4t + B \sin 4t) \therefore 0 = -5 + A, A = 5$$

$$i'_a(0) = 32 = 30 - 10 + 4B \therefore B = 3 \therefore i_a(t) = \boxed{[-5e^{-6t} + e^{-2t}(5 \cos 4t + 3 \sin 4t)]u(t) \text{ A}}$$

## CHAPTER FIFTEEN SOLUTIONS

48.

$$(a) \quad \mathbf{H}(s) = \mathbf{I}_c / \mathbf{I}_s = \frac{0.5}{0.5 + 0.002s + 500/s} = \frac{250s}{s^2 + 250s + 25\,000}$$

$$(b) \quad s = \frac{1}{2}(-250 \pm \sqrt{62\,500 - 10^6}) = -125 \pm j484.1 \text{ s}^{-1}$$

$$(c) \quad \alpha = \frac{R}{2L} = \frac{0.5}{0.004} = 125 \text{ s}^{-1}, \quad \omega_o = \sqrt{10^6/4} = 500 \text{ s}^{-1}, \quad \omega_d = \sqrt{25 \times 10^4 - 15,625} = 484.1 \text{ s}^{-1}$$

$$(d) \quad \mathbf{I}_s = 1, s = 0 \quad \therefore \mathbf{I}_c = 0 \quad \therefore i_{cf} = 0$$

$$(e) \quad i_{c,n} = e^{-125t} (A \cos 484t + B \sin 484t)$$

$$(f) \quad i_L(0) = 0 \quad \therefore i_c(0^+) = 0, \quad v_c(0) = 0 \quad \therefore 1 \times \frac{1}{2} = 2 \times 10^{-3} i(0^+) + 0 \quad \therefore i(0^+) = 250 \text{ A/s}$$

$$(g) \quad \therefore A = 0, \quad 484B = 250, \quad B = 0.5164 \quad \therefore i_c(t) = (0.5164 e^{-125t} \sin 484.1t) u(t) \text{ A}$$

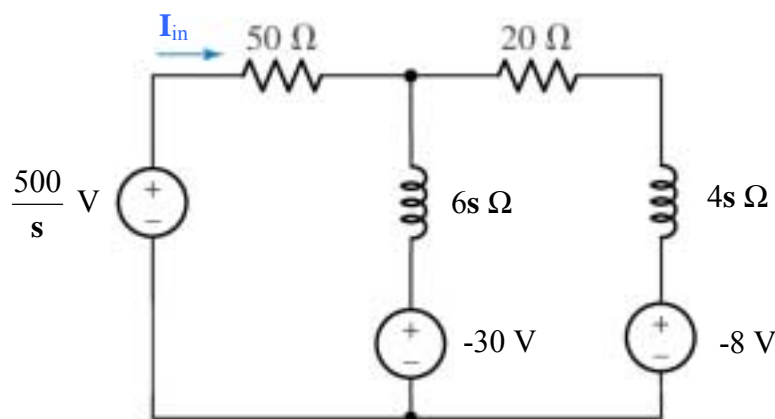
## CHAPTER FIFTEEN SOLUTIONS

49.

$$(a) \quad \mathbf{H}(s) = \mathbf{I}_{in} / \mathbf{V}_{in} = \frac{1}{\mathbf{Z}_{in}} = \frac{1}{50 + \frac{6s(4s+20)}{10s+20}} = \frac{10s+20}{24s^2 + 620s + 1000}$$

$$\therefore s = \frac{1}{48}(-620 \pm \sqrt{620^2 - 96,000}) = -1.729 \text{ and } -24.10 \text{ s}^{-1}$$

- (b) Note that the element labeled 6 H should be an inductor, as is suggested by the context of the text (i.e. initial condition provided). Convert to s-domain and define a clockwise mesh current  $\mathbf{I}_2$  in the right-hand mesh.



$$\text{Mesh 1: } 0 = -500/s + (50 + 6s) \mathbf{I}_{in} - 30 - 6s \mathbf{I}_2 \quad [1]$$

$$\text{Mesh 2: } 0 = 30 + (20 + 10s) \mathbf{I}_2 - 6s \mathbf{I}_{in} - 8 \quad [2]$$

$$\begin{aligned} \text{Solving, we find that } \mathbf{I}_{in} &= \frac{42s^2 + 1400s + 2500}{s(6s^2 + 155s + 250)} = \frac{7s^2 + 233.3s + 416.7}{s(s + 24.10)(s + 1.729)} \\ &= \frac{a}{s} + \frac{b}{(s + 24.10)} + \frac{c}{(s + 1.729)} \end{aligned}$$

where  $a = 10$ ,  $b = -2.115$  and  $c = -0.8855$ . Thus, we may write

$$i_{in}(t) = [10 - 2.115 e^{-24.10t} - 0.885 e^{-1.729t}] u(t) \text{ A}$$

## CHAPTER FIFTEEN SOLUTIONS

50.

$$(a) \quad \mathbf{H(s)} = \frac{\mathbf{V}}{\mathbf{I_s}} = \frac{50(1000/s)}{50 + (1000/s)} = \frac{1000}{s + 20}$$

(b)

$$\mathbf{I_s} = \frac{2}{s} \text{ so } \mathbf{V(s)} = \left( \frac{2}{s} \right) \left( \frac{1000}{(s+20)} \right) = \frac{2000}{s(s+20)} = \frac{a}{s} + \frac{b}{s+20}$$

$$a = \left. \frac{2000}{(s+20)} \right|_{s=0} = 100; \quad b = \left. \frac{2000}{(s)} \right|_{s=-20} = -100$$

$$\text{Thus, } \mathbf{V(s)} = \frac{100}{s} - \frac{100}{s+20} \text{ and } \mathbf{v(t)} = 100 [1 - e^{-20t}] u(t) \text{ V}$$

(c) This function as written is technically valid for all time (although that can't be possible physically). Therefore, we can't use the one-sided Laplace technique we've been studying. We can, however, use simple s-domain/ complex frequency analysis:

$$i_s = 4e^{-10t} \text{ A } \therefore \mathbf{I_s} = 4 \text{ A, } s = 10 \therefore \mathbf{V} = 4\mathbf{H}(-10) = 4 \times \frac{1000}{10} = 400 \text{ V } \therefore$$

$$\mathbf{v(t)} = 400e^{-10t} \text{ V (all } t)$$

$$(d) \quad 4e^{-10t} u(t) \Leftrightarrow \frac{4}{s+10}, \text{ so } \mathbf{V(s)} = \left( \frac{4}{s+10} \right) \left( \frac{1000}{s+20} \right) = \frac{a}{s+10} + \frac{b}{s+20}$$

$$a = 400 \text{ and } b = -400, \text{ so } \mathbf{v(t)} = 400 [e^{-10t} - e^{-20t}] u(t) \text{ V}$$

## CHAPTER FIFTEEN SOLUTIONS

51.

(a)

$$\mathbf{H(s)} = \frac{\mathbf{V_{c2}}}{\mathbf{V_s}} = \frac{\frac{100}{s}}{20 + \frac{100}{s}} \times \frac{\left(20 + \frac{100}{s}\right) \frac{25}{s}}{20 + \frac{125}{s}} \div \left[50 + \frac{(20s+100)25}{s(20s+125)}\right]$$

$$\therefore \mathbf{H(s)} = \frac{2500}{s(20s+125)} \frac{s(20s+125)}{1000s^2 + 6250s + 500s + 2500}$$

$$\therefore \mathbf{H(s)} = \frac{2.5}{s^2 + 6.75s + 2.5}$$

(b) No initial energy stored in either capacitor. With  $v_s = u(t)$ ,  $\mathbf{V_s(s)} = \frac{1}{s}$ , so

$$\mathbf{V_{C2}} = \frac{2.5}{s(s + 6.357)(s + 0.3933)} = \frac{a}{s} + \frac{b}{s + 6.357} + \frac{c}{s + 0.3933}$$

Where  $a = 1$ ,  $b = 0.06594$  and  $c = -1.066$ . Thus,

$$v_{C2}(t) = [1 + 0.06594 e^{-6.357t} - 1.066 e^{-0.3933t}] u(t) \text{ V}$$

## CHAPTER FIFTEEN SOLUTIONS

52.

$$\begin{aligned} \mathbf{Z}_{in}(\mathbf{s}) &= \frac{1}{0.1 + 0.025\mathbf{s} + \frac{1}{20 + (80/\mathbf{s})}} = \frac{1}{0.1 + 0.025\mathbf{s} + \frac{0.05\mathbf{s}}{\mathbf{s} + 4}} \\ &= \frac{\mathbf{s} + 4}{0.025\mathbf{s}^2 + 0.25\mathbf{s} + 0.4} = \frac{40(\mathbf{s} + 4)}{\mathbf{s}^2 + 10\mathbf{s} + 16} = \frac{40(\mathbf{s} + 4)}{(\mathbf{s} + 2)(\mathbf{s} + 8)} \Omega \end{aligned}$$

$$20u(t) \Leftrightarrow \frac{20}{\mathbf{s}}, \text{ so } \mathbf{V}_{in}(\mathbf{s}) = \left( \frac{20}{\mathbf{s}} \right) \left[ \frac{40(\mathbf{s} + 4)}{(\mathbf{s} + 2)(\mathbf{s} + 8)} \right] = \frac{\mathbf{a}}{\mathbf{s}} + \frac{\mathbf{b}}{\mathbf{s} + 2} + \frac{\mathbf{c}}{\mathbf{s} + 8}$$

$$\mathbf{a} = 200, \mathbf{b} = -133.3 \text{ and } \mathbf{c} = -66.67, \text{ so } v_{in}(t) = [200 - 133.3 e^{-2t} - 66.67 e^{-8t}] u(t) \text{ V}$$



## CHAPTER FIFTEEN SOLUTIONS

53.

$$\mathbf{H(s)} = -\frac{\mathbf{Z_f}}{\mathbf{Z_1}}$$

$$(a) \quad \mathbf{Z_1} = 10^3 + \frac{10^8}{\mathbf{s}}, \mathbf{Z_f} = 5000 \therefore \mathbf{H(s)} = -\frac{5000}{1000 + (10^8 / \mathbf{s})} = -\frac{5000\mathbf{s}}{1000\mathbf{s} + 10^8}$$

$$\therefore \mathbf{H(s)} = \frac{-5\mathbf{s}}{\mathbf{s} + 10^5}$$

$$(b) \quad \mathbf{Z_1} = 5000, \mathbf{Z_f} = 10^3 + 10^8 / \mathbf{s} \therefore \mathbf{H(s)} = -\frac{10^3 + 10^8 / \mathbf{s}}{5000} = -\frac{1000\mathbf{s} + 10^8}{5000\mathbf{s}} = -\frac{\mathbf{s} + 10^5}{5\mathbf{s}}$$

$$(c) \quad \mathbf{Z_1} = 10^3 + 10^8 / \mathbf{s}, \mathbf{Z_f} = 10^4 + 10^8 / \mathbf{s} \therefore \mathbf{H(s)} = -\frac{10^4 + 10^8 / \mathbf{s}}{1000 + 10^8 / \mathbf{s}} = -\frac{10^4\mathbf{s} + 10^8}{1000\mathbf{s} + 10^8} = -\frac{10\mathbf{s} + 10^5}{\mathbf{s} + 10^5}$$

## CHAPTER FIFTEEN SOLUTIONS

54.

$$R_f = 20 \text{ k}\Omega, \mathbf{H}(s) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = -R_f C_1 \left( s + \frac{1}{R_1 C_1} \right)$$

$$\therefore \mathbf{H}(s) = -2 \times 10^4 C_1 \left( s + \frac{1}{R_1 C_1} \right)$$

$$(a) \quad \mathbf{H}(s) = -50 \therefore C_1 = 0, \quad \frac{2 \times 10^4}{R_1} = 50, \quad R_1 = 400 \Omega$$

$$(b) \quad \mathbf{H}(s) = -10^{-3}(s + 10^4) = -2 \times 10^4 C_1 \left( s + \frac{1}{R_1 C_1} \right) \therefore 2 \times 10^4 C_1 = 10^{-3}$$

$$\therefore C_1 = 50 \text{ nF}; \quad \frac{1}{50 \times 10^{-9} R_1} = 10^4, \text{ so } R_1 = 2 \text{ k}\Omega$$

$$(c) \quad \mathbf{H}(s) = -10^{-4}(s + 1000) = -2 \times 10^4 C_1 \left( s + \frac{1}{R_1 C_1} \right) \therefore 2 \times 10^4 C_1 = 10^{-4}, C_1 = 5 \text{ nF}$$

$$\therefore v_{in}(t) = 200 + Ae^{-2t} + Be^{-8t}, v_{in}(0) = 0 \therefore 0 = 200 + A + B$$

$$i_{25}(0^+) = 20 = 0.025 v'_{in}(0^+) = 0.025(-2A - 8B) \therefore -A - 4B = 400 \therefore -3B = 200$$

$$B = -\frac{200}{3}, \therefore A = -\frac{400}{3} \therefore v_{in}(t) = \left( 200 - \frac{400}{3}e^{-2t} - \frac{200}{3}e^{-8t} \right) u(t) \text{ V}$$

## CHAPTER FIFTEEN SOLUTIONS

55.

(a)  $\mathbf{H(s)} = \frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = -50,$

$$\mathbf{H(s)} = \frac{-1/R_1 C_f}{s + 1/R_f C_f}, R_f = 20 \text{ k}\Omega$$

set  $C_f = 0 \therefore -50 = -\frac{R_f}{R_1} \therefore R_1 = \frac{20 \times 10^3}{50} = 400 \text{ }\Omega$

(b)  $\mathbf{H(s)} = -\frac{1000}{s + 10\,000} = \frac{1/R_1 C_f}{s + 1/20\,000 C_f} \therefore 10\,000 = \frac{1}{20\,000 C_f}$

$$C_f = \frac{1}{2 \times 10^8} = 5 \text{ nF} \text{ We may then find } R_1: 1000 = \frac{1}{5 \times 10^{-9} R_1} \therefore R_1 = 200 \text{ k}\Omega$$

(c)  $\mathbf{H(s)} = -\frac{10\,000}{s + 1000} = \frac{1/R_1 C_f}{s + 1/20\,000 C_f} \therefore 1000 = \frac{1}{20\,000 C_f} C_f = 50 \text{ nF}$

$$\frac{1}{5 \times 10^{-9} R_1} = 1000, R_1 = 200 \text{ k}\Omega$$

(d)  $\mathbf{H(s)} = \frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = \frac{100}{s + 10^5}$

$$= \left[ -\frac{1/R_{1A} C_{fA}}{s + 1/R_{fA} C_{fA}} \right] \left[ -\frac{1/R_{1B} C_{fB}}{s + 1/R_{fB} C_{fB}} \right] = \left[ -\frac{1/R_{1A} C_{fA}}{s + 1/R_{fA} C_{fA}} \right] \left[ -\frac{R_{fB}}{R_{1B}} \right]$$

We may therefore set  $\frac{R_{fB}}{R_{1B} R_{1A} C_{fA}} = 100$

and  $1/R_{fA} C_{fA} = 10^5$ . Arbitrarily choosing  $R_{fA} = 1 \text{ k}\Omega$ , we find that  $C_{fA} = 10 \text{ nF}$ .

Arbitrarily selecting  $R_{fB} = 100 \text{ }\Omega$ , we may complete the design by choosing

$$R_{1B} = R_{1A} = 10 \text{ k}\Omega$$

## CHAPTER FIFTEEN SOLUTIONS

56.

$$\mathbf{H}(s) = \frac{-10^{-4}s(s+100)}{s+1000} = \frac{[-K_A s][ -K_B (s+100)]}{\left(-\frac{K_C}{s+1000}\right)}$$

Let  $\mathbf{H}_A(s) = -K_A s$ . Choose inverting op amp with parallel RC network at inverting input.

$$0 = \frac{-V_i}{R_{1A}}(1 + sC_{1A}) - \frac{V_o}{R_{fA}}$$

$$\therefore \mathbf{H}_A(s) = -\frac{R_{fA}}{R_{1A}}(1 + sR_{1A}C_{1A}) = -\frac{R_{fA}}{R_{1A}} - sR_{fA}C_{1A} = -K_A s. \text{ Set } R_{1A} = \infty. \text{ Then}$$

$$-R_{fA}C_{1A}s = -10^4 C_{1A}s$$

$$\text{Same configuration for } \mathbf{H}_B(s) \therefore \mathbf{H}_B(s) = -K_B(s+100) = -\frac{R_{fB}}{R_{1B}}(1 + sR_{1B}C_{1B})$$

For the last stage, choose an inverting op amp circuit with a parallel RC circuit in the feedback loop.

$$\text{Let } \mathbf{H}_C(s) = -K_C \frac{1}{s+1000} = -\frac{R_{fC}}{R_{1C}} \frac{1}{(1 + sR_{fC}C_{fC})}$$

Cascading these three transfer functions, we find that

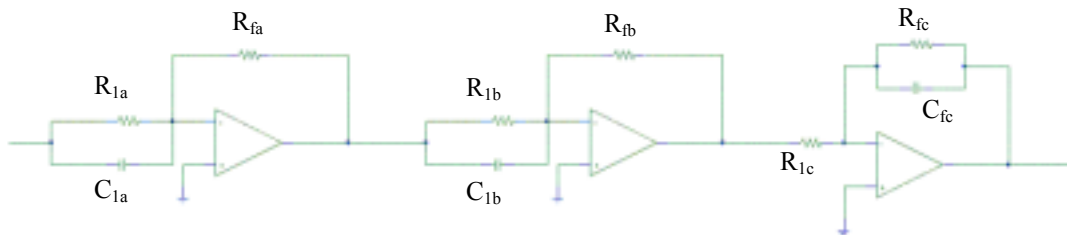
$$\mathbf{H}_A \mathbf{H}_B \mathbf{H}_C = \left[-sR_{fA}C_{1A}\right] \left[-\left(R_{fB}C_{1B}s + \frac{R_{fB}}{R_{1B}}\right)\right] \left[-\left(\frac{R_{fC}}{R_{1C}}\right) \frac{1}{R_{fC}C_{fC}s + 1}\right]$$

Choosing all remaining resistors to be 10 k $\Omega$ , we compare this to our desired transfer function.

$$(R_{fC}C_{fC})^{-1} = 1000 \text{ so } C_{fC} = 100 \text{ nF}$$

$$\text{Next, } \frac{R_{fB}}{R_{1B}R_{fB}C_{1B}} = 100 \text{ so } C_{1B} = 1 \mu\text{F.}$$

$$\text{Finally, } R_{fA}C_{1A}R_{fB}C_{1B}R_{fC}(R_{1C}R_{fC}C_{fC}) = 10^{-4}, \text{ so } C_{1A} = 1 \text{ nF}$$



## CHAPTER FIFTEEN SOLUTIONS

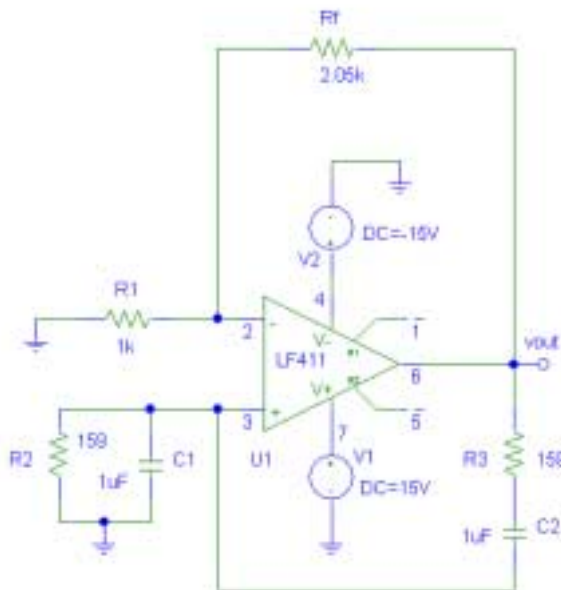
57. Design a Wien-bridge oscillator for operation at 1 kHz, using only standard resistor values. One possible solution:

$$\omega = 2\pi f = 1/RC, \text{ so set } (2\pi RC)^{-1} = 1000$$

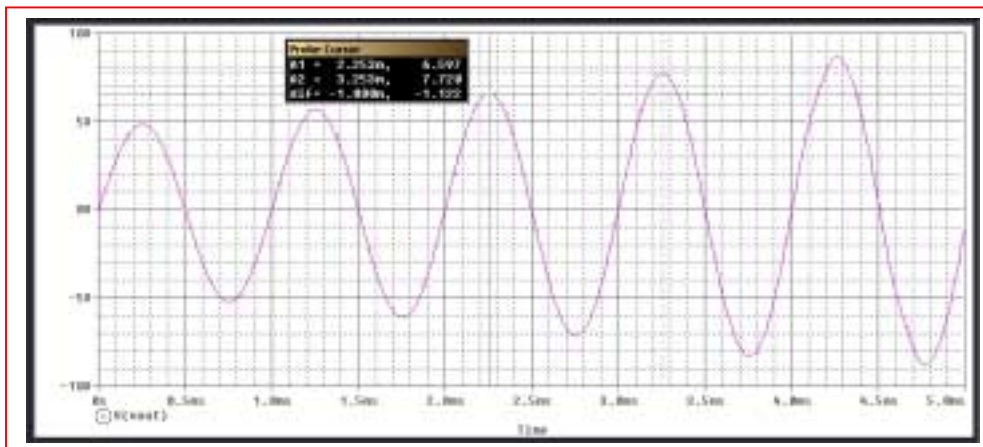
If we use a 1- $\mu$ F capacitor, then  $R = 159 \Omega$ . To construct this using standard resistor values, connect a 100- $\Omega$ , 56- $\Omega$  and 3- $\Omega$  in series.

To complete the design, select  $R_f = 2 \text{ k}\Omega$  and  $R_1 = 1 \text{ k}\Omega$ .

PSpice verification:



The feedback resistor was set to 2.05 k $\Omega$  to initiate oscillations in the simulation. The output waveform shown below exhibits a frequency of 1 kHz as desired.



## CHAPTER FIFTEEN SOLUTIONS

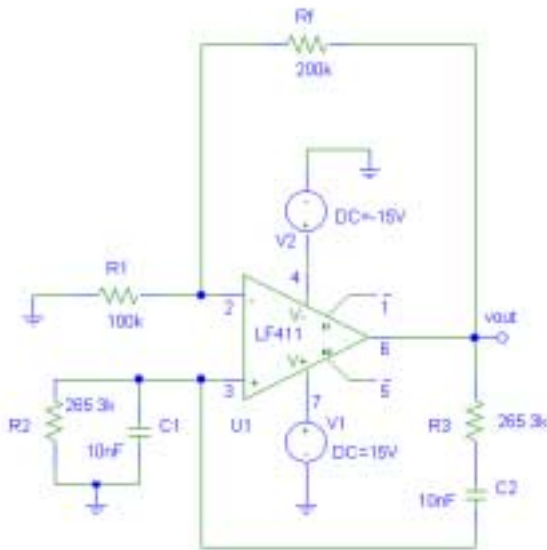
58. Design a Wien-bridge oscillator for operation at 60 Hz. One possible solution:

$$\omega = 2\pi f = 1/RC, \text{ so set } (2\pi RC)^{-1} = 60$$

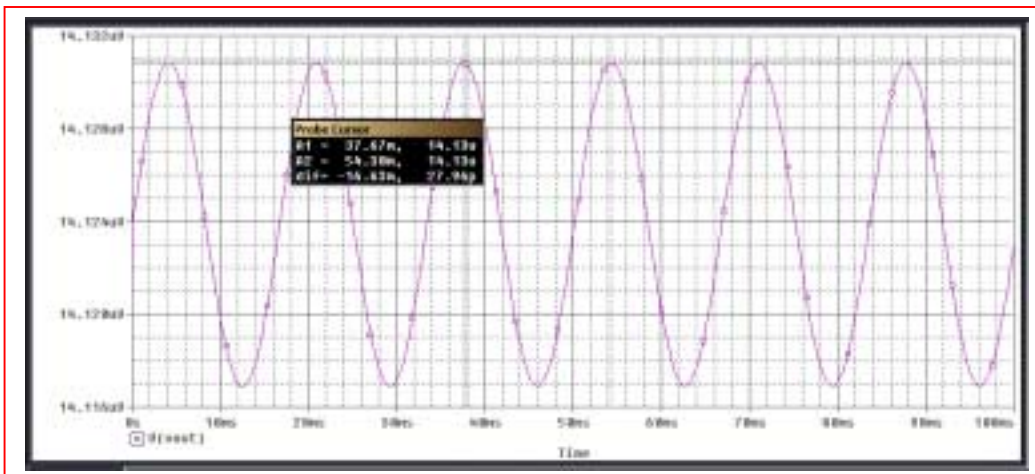
If we use 10-nF capacitors, then  $R = 265.3 \text{ k}\Omega$ .

To complete the design, select  $R_f = 200 \text{ k}\Omega$  and  $R_1 = 100 \text{ k}\Omega$ .

PSpice verification:



The simulated output of the circuit shows a sinusoidal waveform having period  $54.3 \text{ ms} - 37.67 \text{ ms} = 0.0163 \text{ ms}$ , which corresponds to a frequency of 60.13 Hz, as desired.



## CHAPTER FIFTEEN SOLUTIONS

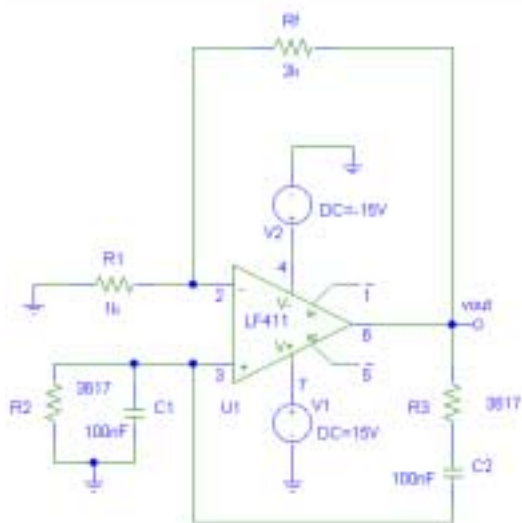
59. Design a Wien-bridge oscillator for operation at 440 Hz, using only standard resistor values. One possible solution:

$$\omega = 2\pi f = 1/RC, \text{ so set } (2\pi RC)^{-1} = 440$$

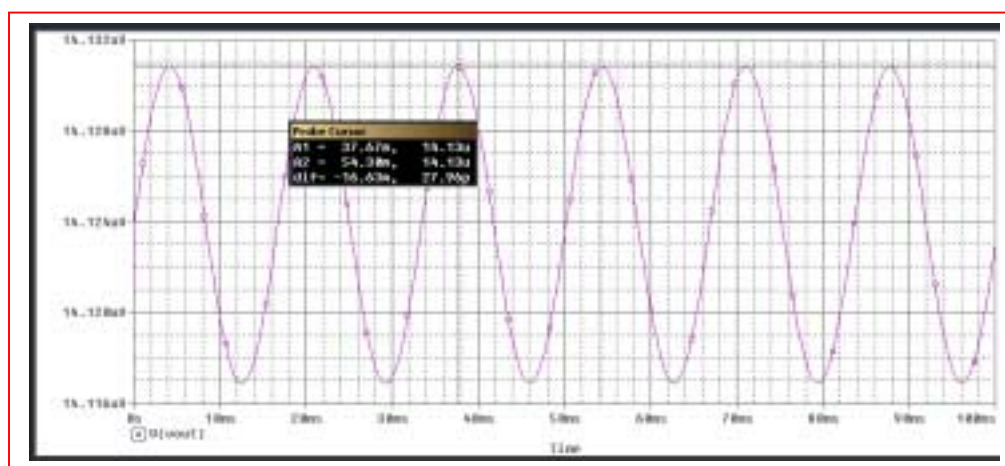
If we use 100-nF capacitors, then  $R = 3.167 \text{ k}\Omega$ . To construct this using standard resistor values, connect a 3.6-k $\Omega$ , 16- $\Omega$  and 1- $\Omega$  in series. (May not need the 1- $\Omega$ , as we're using 5% tolerance resistors!). This circuit will produce the musical note, 'A.'

To complete the design, select  $R_f = 2 \text{ k}\Omega$  and  $R_1 = 1 \text{ k}\Omega$ .

PSpice verification:



Simulation results show a sinusoidal output having a period of approximately  $5.128 - 2.864 = 2.264 \text{ ms}$ , or a frequency of approximately 442 Hz. The error is likely to uncertainty in cursor placement; a higher-resolution time simulation would enable greater precision.



## CHAPTER FIFTEEN SOLUTIONS

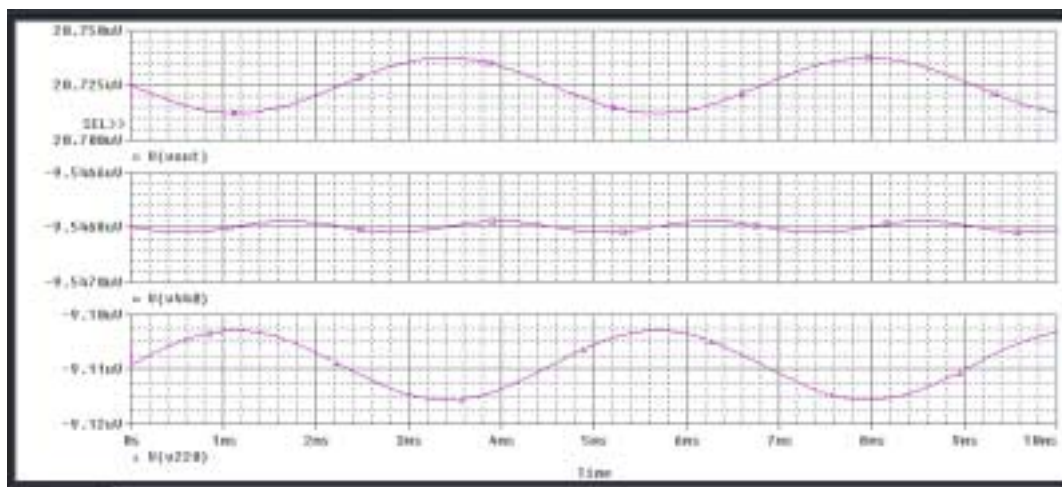
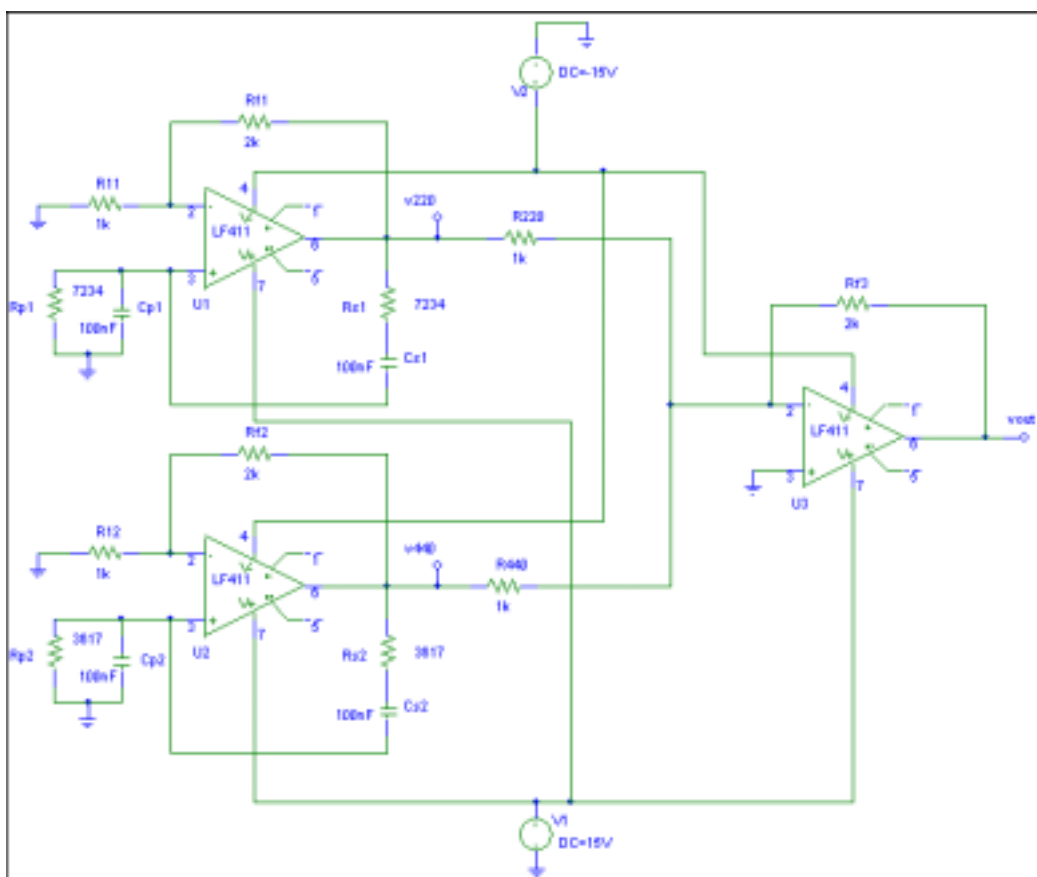
60. Design a Wien-bridge oscillator for 440 Hz:  $\omega = 2\pi f = 1/RC$ , so set  $(2\pi RC)^{-1} = 440$

If we use 100-nF capacitors, then  $R = 3.167 \text{ k}\Omega$ .

Design a Wien-bridge oscillator for 220 Hz:  $\omega = 2\pi f = 1/RC$ , so set  $(2\pi RC)^{-1} = 220$

If we use 100-nF capacitors, then  $R = 7.234 \text{ k}\Omega$ .

Using a summing stage to add the two waveforms together:





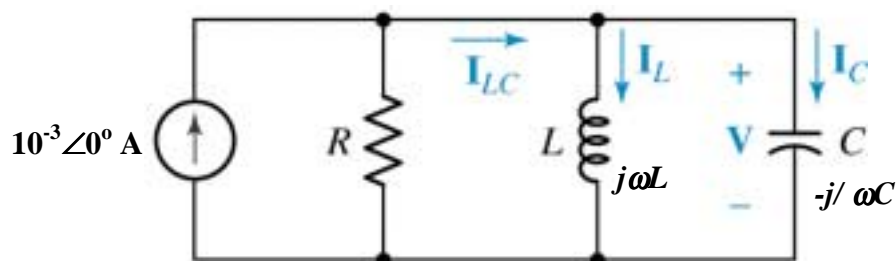
## CHAPTER SIXTEEN SOLUTIONS

1. We have a parallel RLC with  $R = 1 \text{ k}\Omega$ ,  $C = 47 \text{ }\mu\text{F}$  and  $L = 11 \text{ mH}$ .

(a)  $Q_o = R(C/L)^{1/2} = 65.37$

(b)  $f_o = \omega_o / 2\pi = (LC)^{-1/2} / 2\pi = 221.3 \text{ Hz}$

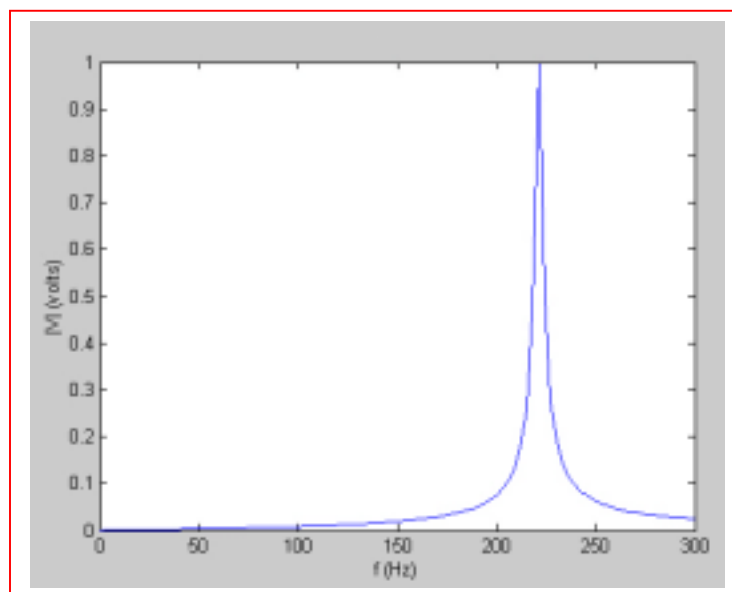
- (c) The circuit is excited by a steady-state 1-mA sinusoidal source:



The admittance  $\mathbf{Y}(s)$  facing the source is  $\mathbf{Y}(s) = 1/R + 1/sL + sC$

$= C(s^2 + s/RC + 1/LC)/s$  so  $\mathbf{Z}(s) = (s/C) / (s^2 + s/RC + 1/LC)$  and  $\mathbf{Z}(j\omega) = (1/C)(j\omega) / (1/LC - \omega^2 + j\omega/RC)$ .

Since  $\mathbf{V} = 10^{-3} \mathbf{Z}$ , we note that  $|\mathbf{V}| > 0$  as  $\omega \rightarrow 0$  and also as  $\omega \rightarrow \infty$ .



## CHAPTER SIXTEEN SOLUTIONS

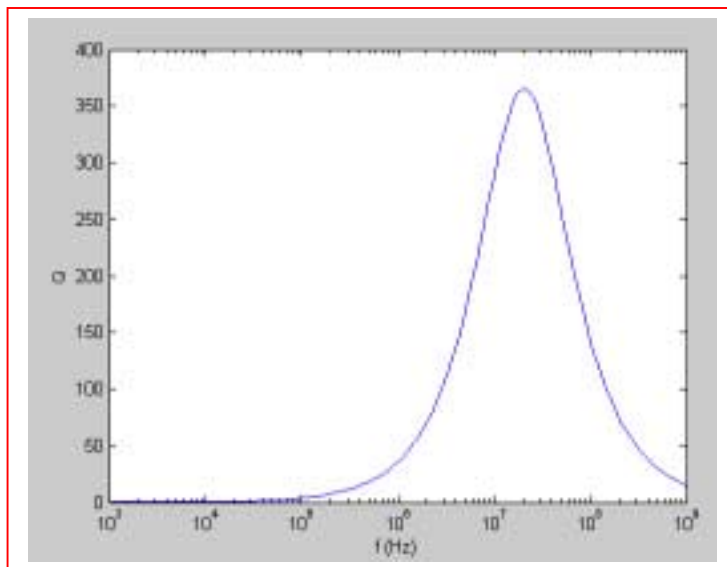
2. (a)  $R = 1000 \, \Omega$  and  $C = 1 \, \mu\text{F}$ .  
 $Q_o = R(C/L)^{1/2} = 200$  so  $L = C(R/Q_o)^2 = 25 \, \mu\text{H}$
- (b)  $L = 12 \, \text{fH}$  and  $C = 2.4 \, \text{nF}$   
 $R = Q_o (L/C)^{1/2} = 447.2 \, \text{m}\Omega$
- (c)  $R = 121.7 \, \text{k}\Omega$  and  $L = 100 \, \text{pH}$   
 $C = (Q_o/R)^2 L = 270 \, \text{aF}$

## CHAPTER SIXTEEN SOLUTIONS

3. We take the approximate expression for  $Q$  of a varactor to be

$$Q \approx \omega C_j R_p / (1 + \omega^2 C_j^2 R_p R_s)$$

(a)  $C_j = 3.77 \text{ pF}$ ,  $R_p = 1.5 \text{ M}\Omega$ ,  $R_s = 2.8 \Omega$



(b)  $dQ/d\omega = [(1 + \omega^2 C_j^2 R_p R_s)(C_j R_p) - \omega C_j R_p (2\omega C_j^2 R_p R_s)] / (1 + \omega^2 C_j^2 R_p R_s)^2$

Setting this equal to zero, we may subsequently write

$$C_j R_p (1 + \omega^2 C_j^2 R_p R_s) - \omega C_j R_p (2\omega C_j^2 R_p R_s) = 0$$

Or  $1 - \omega^2 C_j^2 R_p R_s = 0$ . Thus,  $\omega_0 = (C_j^2 R_p R_s)^{-1/2} = 129.4 \text{ Mrad/s} = 21.00 \text{ MHz}$

$Q_0 = Q(\omega = \omega_0) = 366.0$

## CHAPTER SIXTEEN SOLUTIONS

4. Determine  $Q$  for (*dropping onto a smooth concrete floor*):

(a) A ping pong ball: Dropped twice from 121.1 cm (arbitrarily chosen). Both times, it bounced to a height of 61.65 cm.

$$Q = 2\pi h_1 / (h_1 - h_2) = 12.82$$

(b) A quarter (25 ¢). Dropped three times from 121.1 cm.

Trial 1: bounced to 13.18 cm

Trial 2: bounced to 32.70 cm

Trial 3: bounced to 16.03 cm. *Quite a bit of variation, depending on how it struck.*

Average bounce height = 20.64 cm, so

$$Q_{\text{avg}} = 2\pi h_1 / (h_1 - h_2) = 7.574$$

(c) Textbook. Dropped once from 121.1 cm. Didn't bounce much at all- only 2.223 cm. Since the book bounced differently depending on angle of incidence, only one trial was performed.

$$Q = 2\pi h_1 / (h_1 - h_2) = 6.4$$

All three items were dropped from the same height for comparison purposes. An interesting experiment would be to repeat the above, but from several different heights, preferably ranging several orders of magnitude (*e.g.* 1 cm, 10 cm, 100 cm, 1000 cm).

## CHAPTER SIXTEEN SOLUTIONS

5.

$$\alpha = 80 \text{ Np/s}, \omega_d = 1200 \text{ rad/s}, \left| \bar{Z}(-2\alpha + j\omega_d) \right| = 400 \Omega$$

$$\omega_o = \sqrt{1200^2 + 80^2} = 1202.66 \text{ rad/s} \quad \therefore Q_o = \frac{\omega_o}{2\alpha} = 7.517$$

$$\text{Now, } \bar{Y}(s) = C \frac{(s + \alpha - j\omega_d)(s + \alpha + j\omega_d)}{s} \quad \therefore \bar{Y}(-2\alpha + j\omega_d) = C \frac{(-\alpha)(-\alpha + j2\omega_d)}{-2\alpha + j\omega_d}$$

$$\therefore \bar{Y}(-160 + j1200) = C \frac{-80(-80 + j2400)}{-160 + j1200} \quad \therefore \left| \bar{Y}(-160 + j1200) \right| = \frac{1}{400} = 80C \left| \frac{-1 + j30}{-2 + j15} \right|$$

$$\therefore C = \frac{1}{32,000} \sqrt{\frac{229}{901}} = 15.775 \mu\text{F}; L = \frac{1}{\omega_o^2 C} = 43.88 \text{ mH}; R = \frac{1}{2\alpha C} = 396.7 \Omega$$

## CHAPTER SIXTEEN SOLUTIONS

6.

$$\begin{aligned}\bar{Y}_{in} &= \frac{1}{2 + j0.1\omega} + 0.2 + \frac{1}{1 + 1000/j\omega} = \frac{2 - j0.1\omega}{4 + 0.01\omega^2} + 0.2 + \frac{j\omega}{1000 + j10} \\ &= \frac{2 - j0.1\omega}{4 + 0.01\omega^2} + 0.2 + \frac{\omega^2 + j1000\omega}{10^6 + \omega^2} \therefore \frac{-0.1\omega}{4 + 0.01\omega^2} + \frac{1000\omega}{\omega^2 + 10^6} = 0 \\ \therefore 0.1\omega^3 + 10^5\omega &= 4000\omega + 10\omega^3 \therefore 9.9\omega^2 = 96,000 \therefore \omega = \boxed{98.47 \text{ rad/s}}\end{aligned}$$

## CHAPTER SIXTEEN SOLUTIONS

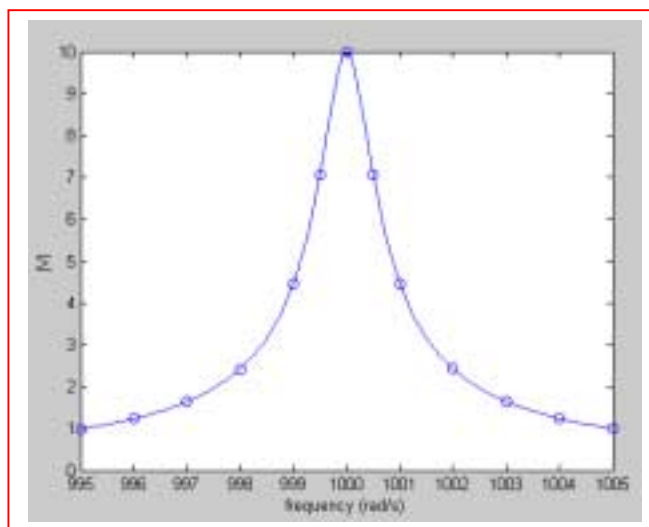
7. Parallel:  $R = 10^6$ ,  $L = 1$ ,  $C = 10^{-6}$ ,  $\bar{I}_s = 10 \angle 0^\circ \mu A$

(a)  $\omega_o = \frac{1}{\sqrt{LC}} = 1000 \text{ rad/s}$ ;  $Q_o = \omega_o RC = 10^{3+6-6} = 1000$

(b)  $\bar{Y} = 10^{-6} + j\left(10^{-6} - \frac{1}{\omega}\right)$ ,  $\bar{V} = \frac{\bar{I}}{\bar{Y}} = 10^{-5} / 10^{-3} \left[10^{-3} + j\left(\frac{\omega}{1000} - \frac{1000}{\omega}\right)\right]$

$$\therefore \bar{V} = \frac{10^{-2}}{0.001 + j\left(\frac{\omega}{1000} - \frac{1000}{\omega}\right)}, |\bar{V}| = \frac{10^{-2}}{\sqrt{10^{-6} + \left(\frac{\omega}{1000} - \frac{1000}{\omega}\right)^2}}$$

$\omega$	$ \bar{V} $
995	0.993
996	1.238
997	1.642
998	2.423
999	4.47
1000	10.0
1001	4.47
1002	2.428
1003	1.646
1004	1.243
1005	0.997
999.5	7.070
1000.5	7.072



## CHAPTER SIXTEEN SOLUTIONS

8.

$$\begin{aligned}
 \text{(a)} \quad \bar{Z}_{in} &= \frac{5(100/j\omega)}{5 + (100/j\omega)} + 2 + \frac{j0.1\omega}{10 + j0.01\omega} \\
 &= \frac{500}{100 + j5\omega} + 2 + \frac{j10\omega}{1000 + j\omega} = \frac{100}{20 + j\omega} + 2 + \frac{j10\omega}{1000 + j\omega} = \frac{100(20 - j\omega)}{\omega^2 + 400} + 2 + \frac{j10\omega(1000 - j)}{\omega^2 + 10^6} \\
 \therefore \frac{-100\omega}{\omega^2 + 400} + \frac{10^4\omega}{\omega^2 + 10^6} &= 0 \quad \therefore \omega^2 + 10^6 = 100\omega^2 + 40,000, \quad 99\omega^2 = 960,000 \\
 \therefore \omega_o &= \sqrt{960,000/99} = \boxed{98.47 \text{ rad/s}}
 \end{aligned}$$

$$\text{(b)} \quad \bar{Z}_{in}(\omega_o) = \frac{2000}{\omega_o^2 + 400} + 2 + \frac{10\omega_o^2}{\omega_o^2 + 10^6} = \boxed{2.294 \Omega}$$



## CHAPTER SIXTEEN SOLUTIONS

9.

$$(a) \quad \alpha = 50 s^{-1}, \omega_d = 1000 s^{-1} \therefore \omega_o^2 = \alpha^2 + \omega_d^2 = 1,002,500 \therefore \omega_o = 1001.249$$

$$L = \frac{1}{\omega_o^2 C} = \frac{10^6}{1,002,500} = 0.9975^+ \text{ H}; \quad R = \frac{1}{2\alpha C} = \frac{10^6}{100} = 10 \text{ k}\Omega$$

$$(b) \quad \bar{Y} = 10^{-4} + j \left( 10^{-6} \omega - \frac{1}{0.9975 \omega} \right), \omega = 1000 \therefore \bar{Z} = \frac{1}{Y} = 9997 \angle 1.4321^\circ \Omega$$

## CHAPTER SIXTEEN SOLUTIONS

10.

$f_{\min} = 535 \text{ kHz}$ ,  $f_{\max} = 1605 \text{ kHz}$ ,  $Q_o = 45$  at one end and

$Q_o \leq 45$  for  $535 \leq f \leq 1605 \text{ kHz}$

$$f_o = 1/2\pi\sqrt{LC} \quad \therefore 535 \times 10^3 = \frac{1}{2\pi\sqrt{L_{\max}C}}, 1605 \times 10^3 = \frac{1}{2\pi\sqrt{L_{\min}C}}$$

$$\therefore \sqrt{L_{\max}/L_{\min}} = 3; L_{\max}C = \left( \frac{1}{2\pi \times 535 \times 10^3} \right)^2 = 8.8498 \times 10^{-14}$$

$$\omega_o RC \leq 45, 535 \times 10^3 \leq \frac{\omega_o}{2\pi} \leq 1605 \times 10^3. \text{ Use } \omega_{o\max}$$

$$\therefore 2\pi \times 1605 \times 10^3 \times 20 \times 10^3 C = 45 \quad \therefore C = 223.1 \text{ pF}$$

$$\therefore L_{\max} = \frac{8.8498 \times 10^{-14}}{223.1 \times 10^{-12}} = 397.6 \text{ } \mu\text{H}, L_{\min} = \frac{L_{\max}}{9} = 44.08 \text{ } \mu\text{H}$$

## CHAPTER SIXTEEN SOLUTIONS

11.

(a) Apply  $\pm 1\text{ V}$ .  $\therefore \bar{I}_R = -10^{-4}\text{ A}$

$$\therefore \bar{Y}_{in} = \bar{I}_{in} = \frac{1}{4.4 \times 10^{-3}\text{ s}} + 10^{-4} + (1 - [10^5(-10^{-4})])10^{-8}\text{ s}$$

$$\therefore \bar{Y}_{in} = \frac{1000}{4.4\text{ s}} + 10^{-4} + 11 \times 10^{-8}\text{ s} = \frac{48.4 \times 10^{-8}\text{ s}^2 + 4.4 \times 10^{-4}\text{ s} + 1000}{4.4\text{ s}}$$

$$\therefore \bar{Y}_{in}(j\omega) = \frac{1000 - 48.4 \times 10^{-8}\omega^2 + j4.4 \times 10^{-4}\omega}{j4.4\omega}$$

(b) At  $\omega = \omega_o$ ,  $1000 = 48.4 \times 10^{-8}\omega_o^2$ ,  $\omega_o = 45.45\text{ krad/s}$

$$\bar{Z}_{in}(j\omega_o) = \left( \frac{j4.4 \times 10^{-4}\omega_o}{j4.4\omega_o} \right)^{-1} = 10\text{ k}\Omega$$

## CHAPTER SIXTEEN SOLUTIONS

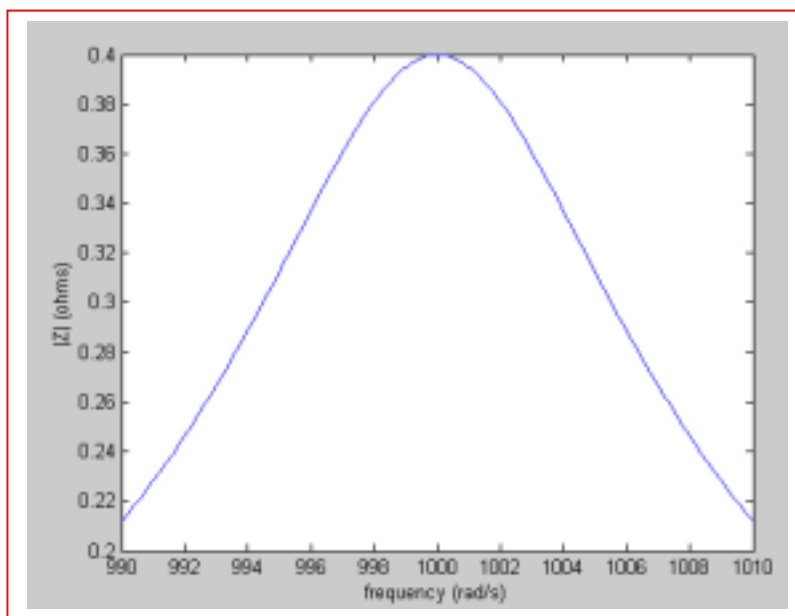
12.  $\omega_o = 1000 \text{ rad/s}$ ,  $Q_o = 80$ ,  $C = 0.2 \mu\text{F}$

(a)  $L = \frac{1}{\omega_o^2 C} = \frac{10^6}{0.2 \times 10^6} = 5 \text{ H}$ ,  $Q_o = \omega_o RC \therefore R = \frac{80}{10^3 \times 0.2 \times 10^{-6}} = 400 \text{ k}\Omega$

(b)  $B = \omega_o / Q_o = 1000 / 80 = 12.5$

$\therefore \frac{1}{2}B = 6.25 \text{ rad/s}$

$\therefore |\bar{Z}| = R / \left| 1 + j \frac{\omega - \omega_o}{B/2} \right| = 400 \times 10^3 / \sqrt{1 + \left( \frac{\omega - \omega_o}{6.25} \right)^2}$



## CHAPTER SIXTEEN SOLUTIONS

13.

$$\omega_1 = 103 \text{ rad/s}, \omega_2 = 118,$$

$$|\bar{Z}(j105)| = 10 \Omega$$

$$\omega_o^2 = \omega_1 \omega_2 = 103 \times 118$$

$$\therefore \omega_o = 110.245^+, B = 118 - 103 = 15 \text{ rad/s}, Q_o = \frac{\omega_o}{B} = \frac{110.245^+}{15} = 7.350$$

$$\therefore 7.350 = \omega_o RC \quad \therefore RC = \frac{7.350}{110.245_1^+} = 66.67 \times 10^{-3}, LC = \frac{1}{\omega_o^2} = \frac{1}{12,154}$$

$$|\bar{Y}(j105)| = 0.1 = \left| \frac{1}{R} + j \left( 105C - \frac{1}{105L} \right) \right| = \left| 15C + j \left( 105C - \frac{12,154}{105} C \right) \right| = 18.456 C$$

$$\therefore C = \frac{0.1}{18.456} = 5.418 \text{ mF}, R = \frac{1}{15} C = 12.304 \Omega, L = \frac{1}{12,154 C} = 15.185 \text{ mH}$$

## CHAPTER SIXTEEN SOLUTIONS

14.  $\omega_o = 30 \text{ krad/s}$ ,  $Q_o = 10$ ,  $R = 600 \Omega$ ,

(a)  $B = \frac{\omega_o}{Q_o} = 3 \text{ krad/s}$

(b)  $N = \frac{\omega - \omega_o}{B/2} = \frac{28 - 30}{1.5} = -1.3333$

(c)  $Z_{in}(j28\,000) = 600 / (1 - j1.333) = 360 \angle 53.13^\circ \Omega$

(d)  $\bar{Z}_{in}(j28,000) = \left[ \frac{1}{600} + j28,000C - j\frac{1}{28,000L} \right]^{-1}$ ,  $C = \frac{Q_o}{\omega_o R} = \frac{10}{30,000 \times 600}$

$$L = \frac{R}{\omega_o Q_o} = \frac{600}{30,000 \times 10}, \frac{1}{L} = \frac{30,000 \times 10}{600} \therefore \bar{Z}_{in} = \left[ \frac{1}{600} + j \left( \frac{28}{30} \times \frac{10}{600} - \frac{30}{28} \frac{10}{600} \right) \right]^{-1}$$

$$\bar{Z}_{in} = \frac{600}{1 + j10 \left( \frac{28}{30} - \frac{30}{28} \right)} = 351.906 \angle 54.0903^\circ \Omega$$

(e) magnitude:  $100\% \frac{\text{approx-true}}{\text{true}} = 100\% \frac{360 - 351.906}{351.906} = 2.300\%$

angle:  $100\% \frac{53.1301^\circ - 54.0903^\circ}{54.0903^\circ} = -1.7752\%$

## CHAPTER SIXTEEN SOLUTIONS

15.  $f_o = 400 \text{ Hz}$ ,  $Q_o = 8$ ,  $R = 500 \Omega$ ,  $\bar{I}_s = 2 \times 10^{-3} \text{ A}$   $\therefore B = 50 \text{ Hz}$

(a)  $|\bar{V}| = 2 \times 10^{-3} \times 500 / \sqrt{1 + N^2} = 0.5$   $\therefore 1 + N^2 = 4$ ,  $N = \pm\sqrt{3} = \frac{f - 400}{50/2}$   
 $\therefore f = 400 \pm 25\sqrt{3} = 443.3 \text{ and } 356.7 \text{ Hz}$

(b)  $|\bar{I}_R| = \frac{|v|}{R} = \frac{1}{\sqrt{1 + N^2}} \times \frac{1}{500} = 0.5 \times 10^{-3}$   $\therefore \sqrt{1 + N^2} = 4$ ,  $N^2 = 15$ ,  $N = \pm\sqrt{15}$   
 $\therefore f = 400 \pm 25\sqrt{15} = 496.8 \text{ and } 303.2 \text{ Hz}$

## CHAPTER SIXTEEN SOLUTIONS

16.  $\omega_o = 10^6$ ,  $Q_o = 10$ ,  $R = 5 \times 10^3$ , *p.r.*

(a)  $Q_o = \frac{R}{\omega_o L} \therefore L = \frac{5 \times 10^3}{10 \times 10^6} = \boxed{0.5 \text{ mH}}$

(b) Approx:  $2 = 5 / \sqrt{1 + N^2} \therefore N = 2.291 = \frac{\omega - 10^6}{10^6 / 20} \therefore \omega = \boxed{1.1146 \text{ Mrad/S}}$

Exact:  $\bar{Y} = \frac{1}{R} \left[ 1 + jQ_o \left( \frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right) \right] \therefore 0.5 = 0.2 \sqrt{1 + 100 \left( \omega - \frac{1}{\omega} \right)^2}$  ( $\omega$  in Mrad/S)

$\therefore 6.25 = 1 + 100(\omega^2 - 2 + 1/\omega^2)$ ,  $\omega^2 - 2 + \frac{1}{\omega^2} = 0.0525$ ,  $\omega^2 + \frac{1}{\omega^2} = 2.0525$

$\omega^4 - 2.0525\omega^2 + 1 = 0$ ,  $\omega^2 = \frac{1}{2} \left( 2.0525 + \sqrt{2.0525^2 - 4} \right) = 1.2569$ ,  $\omega = \boxed{1.1211 \text{ Mrad/s}}$

(c) Approx:  $\angle Y = 30^\circ \therefore \tan^{-1} N = 30^\circ$ ,  $N = 0.5774 = \frac{\omega - 1}{1/20}$ ,  $\omega = \boxed{1.0289 \text{ Mrad/s}}$

Exact:  $\bar{Y} = \frac{1}{5000} \left[ 1 + j10 \left( \omega - \frac{1}{\omega} \right) \right]$  (in Mrad/s)  $\therefore \tan 30^\circ = 0.5774 = 10 \left( \omega - \frac{1}{\omega} \right)$

$\therefore \omega - \frac{1}{\omega} = 0.05774$ ,  $\omega^2 - 0.05774\omega - 1 = 0$ ,  $\omega = \frac{0.05774 + \sqrt{0.05774^2 + 4}}{2} = \boxed{1.0293 \text{ Mrad/s}}$



## CHAPTER SIXTEEN SOLUTIONS

17.

$$(a) \quad C = 3 + 7 = 10 \text{ nF} \quad \therefore \omega_o = \frac{1}{\sqrt{10^{-4} 10^{-8}}} = 10^6 \text{ rad/s}$$

$$(b) \quad Q_o = \omega_o CR = 10^6 10^{-8} 5 = 5 \times 10^3 = 50$$

$$B = \omega_o / Q_o = 20 \text{ krad/s}$$

$$\text{Parallel current source is } \frac{1 \angle 0^\circ}{\bar{Z}_3} = j\omega 3 \times 10^{-9} \quad \text{At } \omega_o, I_s = j10^{6-9} \times 3$$

$$\therefore V_{1,0} = j3 \times 10^{-3} \times 5 \times 10^3 = 15 \angle 90^\circ \text{ V}$$

$$(c) \quad \omega - \omega_o = 15 \times 10^3 \quad \therefore N = \frac{15 \times 10^3}{10 \times 10^3} = 1.5 \quad \therefore \bar{V}_1 = \frac{15 \angle 90^\circ}{1 + j1.5} = 8.321 \angle 33.69^\circ \text{ V}$$

## CHAPTER SIXTEEN SOLUTIONS

18.

$$(a) \quad \bar{Z}_{in}(s) = \frac{(5 + 0.01s)(5 + 10^6/s)}{10 + 0.01s + 10^6/s} = \frac{(5 + 0.01s)(5s + 10^6)}{0.01s^2 + 10s + 10^6}$$

$$\bar{Z}_{in}(s) = \frac{0.05s^2 + 25s + 10^4}{0.01s^2 + 10s + 10^6}$$

$$\therefore \bar{Z}_{in}(j\omega) = \frac{5 \times 10^6 - 0.05\omega^2 + j10,025\omega}{10^6 - 0.01\omega^2 + j10\omega}$$

$$\text{At } \omega = \omega_o, \frac{10,025\omega_o}{5 \times 10^6 - 0.05\omega_o^2} = \frac{10\omega_o}{10^6 - 0.01\omega_o^2}, 10.025 \times 10^9 - 100.25\omega_o^2 = 5 \times 10^7 - 0.5\omega_o^2$$

$$\therefore 99.75\omega_o^2 = 9.975 \times 10^9, \omega_o = \boxed{10,000 \text{ rad/s}}$$

$$(b) \quad \bar{Z}_{in}(j\omega_o) = (5 + j100) \parallel (5 - j100) = \frac{25 + 10,000}{10} = \boxed{1002.5 \Omega}$$

## CHAPTER SIXTEEN SOLUTIONS

19.  $f_o = 1000 \text{ Hz}$ ,  $Q_o = 40$ ,  $|\bar{Z}_{in}(j\omega_o)| = 2k\Omega \therefore B = 25 \text{ Hz}$

(a)  $\mathbf{Z}_{in}(j\omega) = \frac{2000}{1+jN}$ ,  $N = \frac{f-1000}{12.5}$ ,  $f = 1010$ ,  $\therefore N = 0.8$

$$\mathbf{Z}_{in} = 2000 / (1 + j0.8) = 1562 \angle -38.66^\circ \Omega$$

(b)  $0.9f_o < f < 1.1f_o \therefore 900 < f < 1100 \text{ Hz}$

## CHAPTER SIXTEEN SOLUTIONS

20. Taking  $2^{-1/2} = 0.7$ , we read from  
Fig. 16.48*a*:  $1.7 \text{ kHz} - 0.6 \text{ kHz} = 1.1 \text{ kHz}$   
Fig. 16.48*b*:  $2 \times 10^7 \text{ Hz} - 900 \text{ Hz} = 20 \text{ MHz}$

## CHAPTER SIXTEEN SOLUTIONS

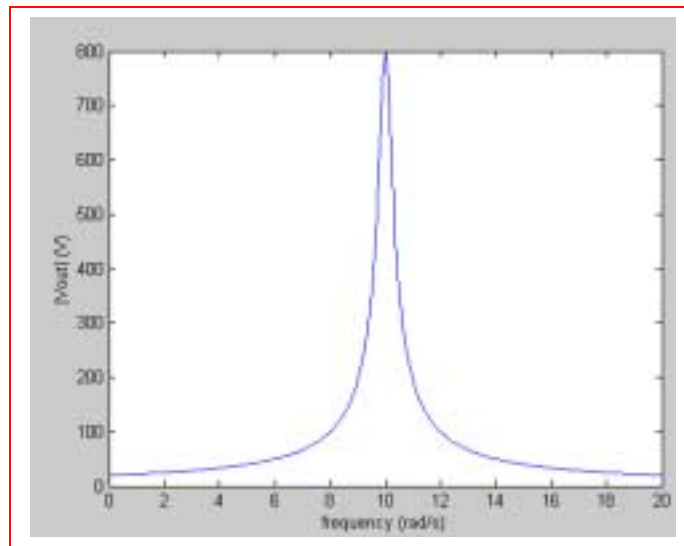
21.

(a)  $20\text{A} \parallel 6\Omega, 3 \parallel 6 = 2, 40\text{V}$  in series with  $2+1=3\Omega$

$$\omega_o = \frac{1}{\sqrt{LC}} = 10 \text{ rad/s}, Q_o = \frac{\omega_o L}{R} = \frac{60}{3} = 20\Omega$$

$$B = \frac{10}{20} = 0.5, \frac{1}{2}B = 0.25, |\bar{V}_{out}(j\omega_o)| = 40Q_o = 800\text{V}$$

$$\therefore |\bar{V}_{out}(j\omega)| = 800 / \sqrt{1 + \left( \frac{\omega - 10}{0.25} \right)^2}$$



(b)  $\omega = 9 \text{ rad/s}$

$$(\text{Approx: } |\bar{V}_{out}(j9)| = \frac{800}{\sqrt{17}} = 194.03\text{V})$$

$$\text{Exact: } \bar{V}_{out} = \frac{40}{3 + j(6\omega - 600/\omega)} \times \frac{600}{j\omega}$$

$$\therefore \bar{V}_{out}(j9) = \frac{24,000}{9[3 + j(54 - 66.67)]} = 204.86 \angle -13.325^\circ \text{ V}$$

## CHAPTER SIXTEEN SOLUTIONS

22. Series:  $R = 50\ \Omega$ ,  $L = 4\text{ mH}$ ,  $C = 10^{-7}$

(a)  $\omega_o = 1/\sqrt{4 \times 10^{-3} \times 10^{-7}} = 50\text{ krad/s}$

(b)  $f_o = 50 \times 10^3 / 2\pi = 7.958\text{ kHz}$

(c)  $Q_o = \frac{\omega_o L}{R} = \frac{50 \times 10^3 \times 4 \times 10^{-3}}{50} = 4$

(d)  $B = \omega_o / Q_o = 50 \times 10^3 / 4 = 12.5\text{ krad/s}$

(e)  $\omega_1 = \omega_o \left[ \sqrt{1 + (1/2Q_o)^2} - 1/2Q_o \right] = 50 \left[ \sqrt{1 + 1/64} - 1/8 \right] = 44.14\text{ krad/s}$

(f)  $\omega_2 = 50 \left[ \sqrt{65/64} + 1/8 \right] = 56.64\text{ krad/s}$

(g)  $\bar{Z}_{in}(j45,000) = 50 + j(180 - 10^7 \times 10^{-7} / 45) = 50 - j42.22 = 65.44 \angle -40.18^\circ \Omega$

(h)  $|\bar{Z}_c / \bar{Z}_R|_{45,000} = |10^7 / j45,000 \times 50| = 4.444$

## CHAPTER SIXTEEN SOLUTIONS

23. Apply 1 A, in at top.  $\therefore \bar{V}_R = 10 \text{ V}$

$$(a) \quad \bar{V}_{in} = \bar{Z}_{in} = 10^{-3}s + 10 + \frac{10^8}{5s} (0.5 \times 10 + 1) = 10^{-3}s + 10 + \frac{1.2 \times 10^8}{s}$$

$$\bar{Z}_{in}(j\omega) = 10 + j(10^{-3}\omega - 1.2 \times 10^8 / \omega) \therefore 10^{-3}\omega_o = 1.2 \times 10^8 / \omega_o$$

$$\therefore \omega_o^2 = 1.2 \times 10^{11}, \omega_o = \boxed{346.4 \text{ krad/s}}$$

$$(b) \quad Q_o = \frac{\omega_o L}{R} = \frac{346.4 \times 10^{3-3}}{10} = \boxed{34.64}$$

## CHAPTER SIXTEEN SOLUTIONS

24. Find the Thévenin equivalent seen by the inductor-capacitor combination:

$$\text{SC: } 1.5 = \bar{V}_1 + 10 \left( \frac{\bar{V}_1}{125} - 0.105 \bar{V}_1 \right) \therefore \bar{V}_1 = 50 \text{ V}$$

$$\therefore \downarrow \bar{I}_{\text{SC}} = \frac{50}{125} = 0.4 \text{ A}$$

$$\text{OC: } \bar{V}_1 = 0 \therefore \bar{V}_{\text{OC}} = 1.5 \text{ V} \therefore R_{th} = \frac{1.5}{0.4} = 3.75 \Omega$$

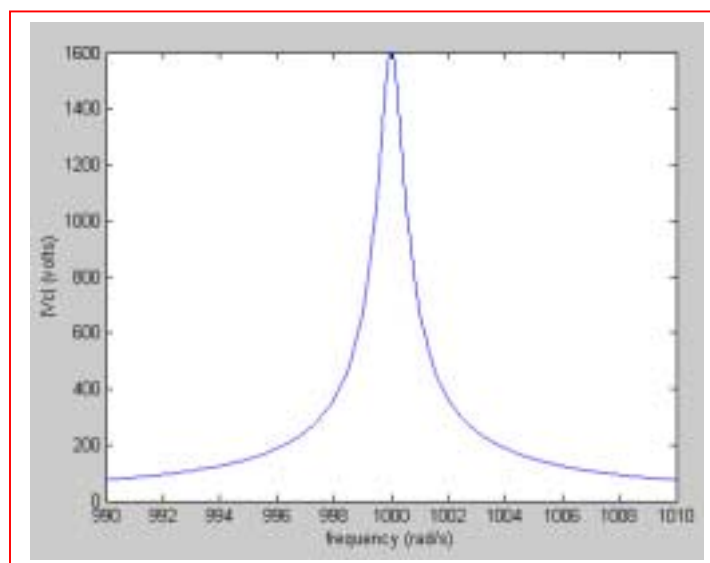
$$\therefore \omega_o = 1 / \sqrt{4 \times 0.25 \times 10^{-6}} = 1000, Q_o = \frac{1000 \times 4}{3.75} = 1066.7$$

$$B = \omega_o / Q_o = \frac{1000}{1066.7} = 0.9375, \frac{1}{2} B = 0.4688 \text{ rad/s}$$

$$|\bar{V}_C|_{\text{max}} = Q_o V_{th} = 1066.7 \times 1.5 = 1600 \text{ V}$$

Therefore, keep your hands off!

To generate a plot of  $|\mathbf{V}_C|$  vs. frequency, note that  $\mathbf{V}_C(j\omega) = 1.5 \frac{-\frac{j}{\omega C}}{3.75 + j\omega L - \frac{j}{\omega C}}$





## CHAPTER SIXTEEN SOLUTIONS

25. Series,  $f_o = 500 \text{ Hz}$ ,  $Q_o = 10$ ,  $X_{L,0} = 500 \Omega$

$$(a) \quad 500 = \omega_o L = 2\pi(500)L \quad \therefore L = 0.15915^+ \text{ H} \quad C = \frac{1}{\omega_o^2 L} = \frac{2\pi}{(2\pi \times 500)^2} = 0.6366 \mu\text{F}$$

$$Q_o = 10 = \frac{X_{L,0}}{R} = \frac{500}{R} \quad \therefore R = 50 \Omega$$

$$(b) \quad 1 = \bar{I} \left( 50 + j2\pi f \times \frac{1}{2\pi} - j \frac{10^6 \times 0.5\pi}{2\pi f} \right) = \bar{I} \left( 50 + jf - j \frac{250,000}{f} \right)$$

$$\therefore \bar{I} = 1 / 50 + j(f - 250,000 / f), \quad \bar{V}_c = \frac{10^6 \times 0.5\pi}{j2\pi f} \bar{I}$$

$$\bar{V}_c = \frac{-j250,000 / f}{50 + j(f - 250,000 / f)} \quad \therefore |\bar{V}_c(2\pi \times 450)| = 4.757 \text{ V}$$

$$|\bar{V}_c(2\pi \times 500)| = 10,000 \text{ V} \quad |\bar{V}_c(2\pi \times 550)| = 4.218 \text{ V}$$

## CHAPTER SIXTEEN SOLUTIONS

26.

$$X: s = 0, \infty, 0: s = -20,000 \pm j80,000 s^{-1}, \bar{Z}_{in}(-10^4) = -20 + j0\Omega \therefore \text{SERIES}$$

$$\alpha = 20,000, \omega_d = 80,000 \therefore \omega_o = \sqrt{(64+4)10^8} = 82,462 \text{ rad/s}, \frac{1}{LC} = \omega_o^2 = 68 \times 10^8$$

$$\frac{R}{2L} = \alpha = 20,000 \therefore \frac{R}{L_1} = 40,000, \frac{1}{LC} \times \frac{L}{R} = \frac{68 \times 10^8}{40,000} = 170,000; Z(\sigma) = R + \sigma L + \frac{1}{\sigma C}$$

$$\therefore -20 = R - 10,000L - \frac{1}{10,000C} = R - \frac{1}{4}R - \frac{170,000}{10,000}R \therefore R = 1.2308\Omega$$

$$\therefore L = \frac{1.2308}{40,000} = 30.77 \mu\text{H}, C = \frac{1}{170,000 \times 1.2308} = 4.779 \mu\text{F}$$

## CHAPTER SIXTEEN SOLUTIONS

27.

$$\omega_o = 1/\sqrt{10^{-3-7}} = 10^5 \text{ rad/s}, Q_L = \frac{10^{5-3}}{1} = 100, R_{PL} = 10,000 \Omega$$

$$Q_c = \frac{1}{10^{5-7} \times 0.2} = 500, R_{PC} = 500^2 \times 0.2 = 50,000 \Omega$$

$$50 \parallel 10 = 8.333 \text{ k}\Omega \quad \therefore Q_o = \omega_o CR = 10^{5-7} \times 8333 = 83.33$$

$$B = \frac{100,000}{83.33} = 1200 \text{ rad/s}, \bar{Z}_{in}(j\omega_o) = 8333 \Omega$$

$$\omega = 99,000 \quad \therefore N = \frac{(99-100)10^3}{600} = -1.6667, \bar{Z}_{in}(j99,000) = \frac{8.333}{1-j1.667} \\ = 4.287 \angle 59.04^\circ \text{ k}\Omega$$

## CHAPTER SIXTEEN SOLUTIONS

28.  $R_{eq} = Q_o / \omega_o C = 50 / 10^{5-7} = 5000 \Omega$ .  
Thus, we may write  $1/5000 = 1/8333 + 1/R_x$  so that

$$R_x = 12.5 \text{ k}\Omega$$

## CHAPTER SIXTEEN SOLUTIONS

29.

$$3\text{ mH} \parallel 1.5\text{ mH} = 1\text{ mH}, 2\mu\text{F} + 8\mu\text{F} = 10\mu\text{F}, \therefore \omega_o = \frac{1}{\sqrt{10^{-3-5}}} = 10\text{ krad/s}$$

$$Q = \frac{3 \times 10^{-3} \times 10^4}{0.3} = 100, R_p = 100^2 \times 0.3 = 3\text{ k}\Omega$$

$$Q = \frac{1.5 \times 10^{-3} \times 10^4}{0.25} = 60, R_p = 60 \times 0.25 = 900\Omega$$

$$900 \parallel 3000 = 692.3\Omega \therefore Q_L = \frac{692.3}{10^{4-3}} = 69.23$$

$$\therefore R_{LS} = \frac{692.3}{69.23^2} = 0.14444\Omega$$

$$Q = \frac{10^6}{10^4 \times 0.1 \times 8} = 125, R_{pc} = 125^2 \times 0.1 = 1562.5\Omega \parallel 10\mu\text{F}$$

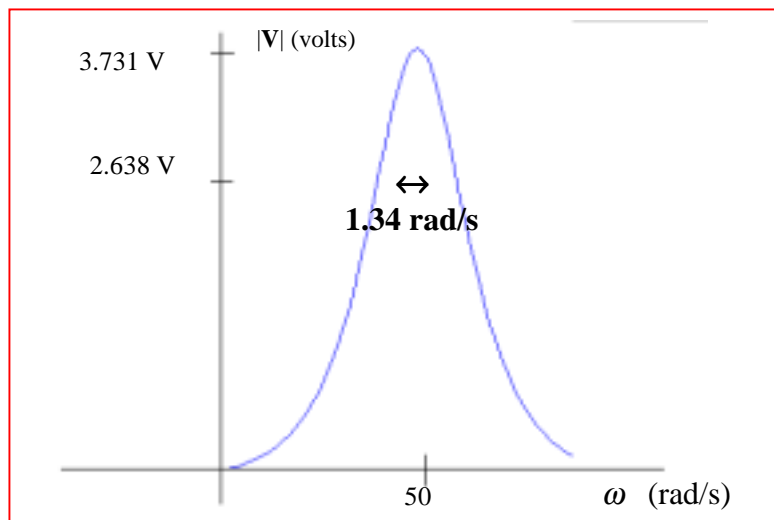
$$\therefore Q_c = 10^4 \times 10^{-5} \times 1562.5 = 156.25 \therefore R_{sc} = \frac{1562.5}{(156.25)^2} = 0.064\Omega$$

$$\therefore R_{S,tot} = 0.14444 + 0.064 = 0.2084\Omega = |\bar{Z}_{in}|_{\min}, \omega_o = 10\text{ krad/s}$$

## CHAPTER SIXTEEN SOLUTIONS

30.

$$\begin{aligned}
 \text{(a)} \quad \omega_o &= 1/\sqrt{2 \times 0.2 \times 10^{-3}} = 50 \text{ rad/s} \\
 Q_{\text{left}L} &= 50 \times 2.5 / 2 = 62.5, \quad 2 \times 62.5^2 = 7812.5 \Omega \\
 Q_{\text{right}L} &= \frac{50 \times 10}{10} = 50, \quad 10 \times 50^2 = 25 \text{ k}\Omega \\
 Q_c &= \frac{1000}{50 \times 0.2 \times 1} = 100, \quad 100^2 \times 1 = 10 \text{ k}\Omega, \quad R_p = 7.8125 \parallel 25 \parallel 10 = 3731 \Omega \\
 Q_o &= 50 \times 3731 \times 0.2 \times 10^{-3} = 37.31; \quad B = \frac{50}{37.31} = 1.3400, \quad \frac{1}{2} B = 0.6700 \\
 \therefore |V|_o &= 10^{-3} \times 3731 = \boxed{3.731 \text{ V}}
 \end{aligned}$$



$$\begin{aligned}
 \text{(b)} \quad V &= 10^{-3} [(2 + j125) \parallel (10 + j500) \parallel (1 - j100)] \\
 &= \frac{10^{-3}}{\frac{1}{2 + j125} + \frac{1}{10 + j500} + \frac{1}{1 - j100}} = \boxed{3.7321 \angle -0.3950^\circ \text{ V}}
 \end{aligned}$$

## CHAPTER SIXTEEN SOLUTIONS

31.

$$(a) \quad \omega_o \text{ B } \frac{1000}{\sqrt{0.25}} = 2000 \text{ rad/s}, Q_c = 2000 \times 2 \times 10^{-6} \times 25 \times 10^3 = 100$$

$$\therefore R_{c,s} = 25,000 / 100^2 = 2.5 \Omega; Q_L = \frac{R}{\omega_o L} = \frac{20 \times 10^4}{2000 \times 0.25} = 40$$

$$\therefore R_{L,s} = \frac{20,000}{1600} = 12.5 \Omega \quad \therefore R_{tot} = 12.5 + 2.5 = 15 \Omega$$

$$\therefore Q_o = \frac{2000 \times 0.25}{15} = 33.33 \quad \therefore |\bar{V}_x| = 1 \times 33.33 \times \frac{1}{2} = 16.667 \text{ V}$$

$$(b) \quad 20,000 \parallel j500 = \frac{20,000 \times j500}{20,000 + j500} = 12,4922 + j499.688 \Omega$$

$$25,000 \parallel -j250 = \frac{25,000(-j250)}{25,000 - j250} = 2.4998 - j249.975$$

$$\therefore \bar{Z}_{in} = 12.4922 + 2.4998 + j499.688 - j250 - j249.975 = 14.9920 - j0.2870 \Omega$$

$$\therefore |\bar{I}| = 1 / |14.9920 - j0.2870| = 66.6902 \text{ mA} \quad \therefore |\bar{V}_x| = 250 \times 66.6902 \times 10^{-3} = 16.6726 \text{ V}$$

## CHAPTER SIXTEEN SOLUTIONS

32.

$$(a) \quad K_m = \frac{50}{100} = 0.5 \quad K_f = \frac{20 \times 10^3}{10^6} = 0.02$$

$$\therefore 9.82 \mu\text{H} \rightarrow 0.5 \times 9.82 \times \frac{1}{0.02} = 24.55 \mu\text{H}, \quad 31.8 \mu\text{H} \rightarrow \frac{0.5}{0.02} \times 31.8 = 795 \mu\text{H}$$

$$2.57 \text{ nF} \rightarrow \frac{2.57}{0.5 \times 0.02} = \boxed{257 \text{ nF}}$$

(b) same ordinate; divide numbers on abscissa by  $\boxed{50}$



## CHAPTER SIXTEEN SOLUTIONS

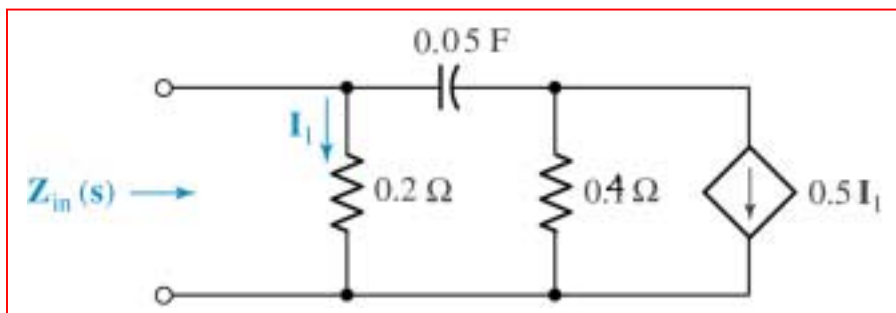
33.

(a) Apply 1 V  $\therefore \bar{I}_1 = 10\text{A}$   $\therefore 0.5\bar{I}_1 = 5\text{A} \downarrow$ ;  $5\text{A} \parallel 0.2\ \Omega$  can be replaced by 1 V in series with  $0.2\ \Omega$

$$\therefore \bar{I}_{in} \rightarrow = 10 + \frac{1 - (-1)}{0.2 + 2/s} = 10 + \frac{2s}{0.2s + 2} = \frac{4s + 20}{0.2s + 2} = \frac{20(s + 5)}{s + 10} \therefore \bar{Z}_{in}(s) = \frac{s + 10}{20(s + 5)}$$

(b)  $K_m = 2, K_f = 5 \therefore \bar{Z}_{in}(s) \rightarrow \frac{2(s/5 + 10)}{20(s/5 + 5)} = \frac{0.1(s + 50)}{s + 25}$

(c)  $0.1\ \Omega \rightarrow 0.2\ \Omega, 0.2\ \Omega \rightarrow 0.4\ \Omega, 0.5\text{F} \rightarrow 0.05\text{F}, 0.5\bar{I}_1 \rightarrow 0.5\bar{I}_1$



## CHAPTER SIXTEEN SOLUTIONS

34.

(a)  $\omega_o = 1/\sqrt{(2+8)10^{-3}10^{-6}} = 10^4 \text{ rad/s}$

$$Q_{L,8} = 10^4 / 8 \times 10^{-3} 10^4 = 125 \therefore R_{L,S} = \frac{10^4}{125^2} = 0.64 \Omega$$

$$2+8=10 \text{ mH} \therefore Q_L = \frac{10^4 \times 10 \times 10^{-3}}{0.64} = 156.25$$

$$\therefore R_{L,P} = 0.64 \times 156.25^2 = 15.625 \text{ k}\Omega; Q_C = \frac{1}{10^4 \times 10^{-6}} = 100, R_{C,P} = 100^2 \times 1 = 10 \text{ k}\Omega$$

$$\therefore R_P = 20 \parallel 15.625 \parallel 10 = 4.673 \text{ k}\Omega \therefore Q_o = 10^4 \times 10^{-6} \times 4.673 \times 10^3 = 46.73$$

(b)  $K_f = 10^6 / 10^4 = 100, K_m = 1 \therefore R's \text{ stay the same; } 2 \text{ mH} \rightarrow 20 \mu\text{H}, 8 \text{ mH} \rightarrow 80 \mu\text{H}, 1 \mu\text{F} \rightarrow 10 \text{ nF}$

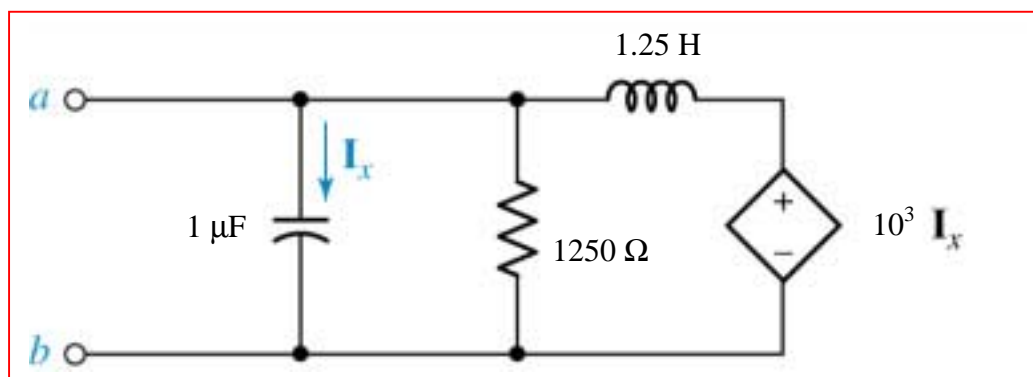
(c)  $\omega_o = 10^6 \text{ rad/s}, Q_o \text{ stays the same, } \therefore B = \frac{10^6}{46.73} = 21.40 \text{ krad/s}$

## CHAPTER SIXTEEN SOLUTIONS

35.

$$(a) \quad K_m = 250, K_f = 400 \therefore 0.1F \rightarrow \frac{0.1}{250 \times 400} = 1\mu F$$

$$5\Omega \rightarrow 1250\Omega, 2H \rightarrow \frac{2 \times 250}{400} = 1.25H, 4\bar{I}_x \rightarrow 10^3 \bar{I}_x$$



$$(b) \quad \omega = 10^3. \text{ Apply } 1V \therefore I_x = 10^{-6}s, \downarrow I_{1250} = \frac{1}{1250}$$

$$\therefore 1000 I_x = 10^{-3}s \therefore \rightarrow I_L = \frac{1 - 10^{-3}s}{1.25s}$$

$$\therefore I_{in} = 10^{-6}s + \frac{1}{1250} + \frac{0.8}{s}(1 - 10^{-3}s) = 10^{-6}s + \frac{0.8}{s}; s = j10^3$$

$$\therefore I_{in} = j10^{-3} + \frac{0.8 \times 10^{-3}}{j} = j0.2 \times 10^{-3} \therefore Z_{th} = \frac{1}{I_{in}} = \frac{1000}{j0.2} = \boxed{-j5 k\Omega \quad \bar{V}_{oc} = 0}$$

## CHAPTER SIXTEEN SOLUTIONS

36.

(a)  $\bar{I}_s = 2\angle 0^\circ \text{ A}$ ,  $\omega = 50 \therefore \bar{V}_{out} = 60\angle 25^\circ \text{ V}$

(b)  $\bar{I}_s = 2\angle 40^\circ \text{ A}$ ,  $\omega = 50 \therefore \bar{V}_{out} = 60\angle 65^\circ \text{ V}$

(c)  $\bar{I}_s = 2\angle 40^\circ \text{ A}$ ,  $\omega = 200$ ,  $\therefore$  OTSK

(d)  $K_m = 30$ ,  $\bar{I}_s = 2\angle 40^\circ \text{ A}$ ,  $\omega = 50 \therefore \bar{V}_{out} = 1800\angle 65^\circ \text{ V}$

(e)  $K_m = 30$ ,  $K_f = 4$ ,  $\bar{I}_s = 2\angle 40^\circ \text{ A}$ ,  $\omega = 200 \therefore \bar{V}_{out} = 1800\angle 65^\circ \text{ V}$

## CHAPTER SIXTEEN SOLUTIONS

37.

(a)  $\bar{H}(s) = 0.2 \quad \therefore H_{dB} = 20 \log 0.2 = -13.979 \text{ dB}$

(b)  $\bar{H}(s) = 50 \quad \therefore H_{dB} = 20 \log 50 = 33.98 \text{ dB}$

(c)  $\bar{H}(j10) = \frac{12}{2 + j10} + \frac{26}{20 + j10} \quad \therefore H_{dB} = 20 \log \left| \frac{6}{1 + j5} + \frac{13}{10 + j5} \right| = 20 \log \left| \frac{292 + j380}{-60 + j220} \right| = 6.451 \text{ dB}$

(d)  $H_{dB} = 37.6 \text{ dB} \quad \therefore |\bar{H}(s)| = 10^{37.6/20} = 75.86$

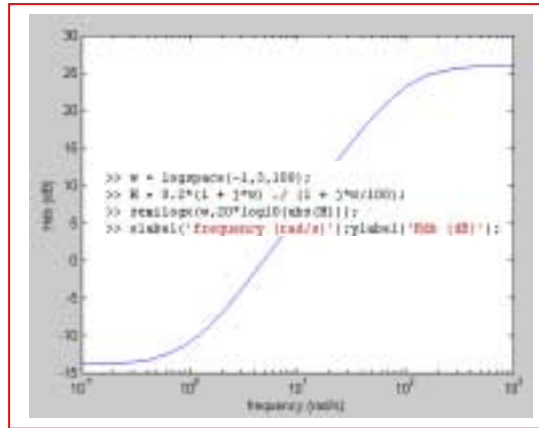
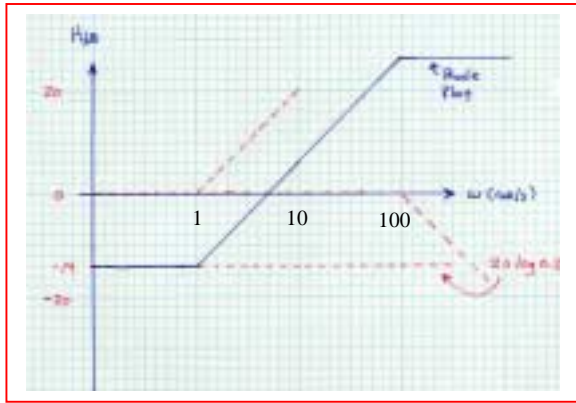
(e)  $H_{dB} = -8 \text{ dB} \quad \therefore |\bar{H}(s)| = 10^{-8/20} = 0.3981$

(f)  $H_{dB} = 0.01 \text{ dB} \quad \therefore |\bar{H}(s)| = 10^{0.01/20} = 1.0012$

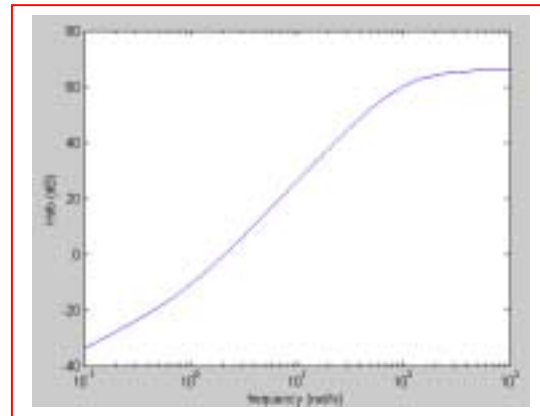
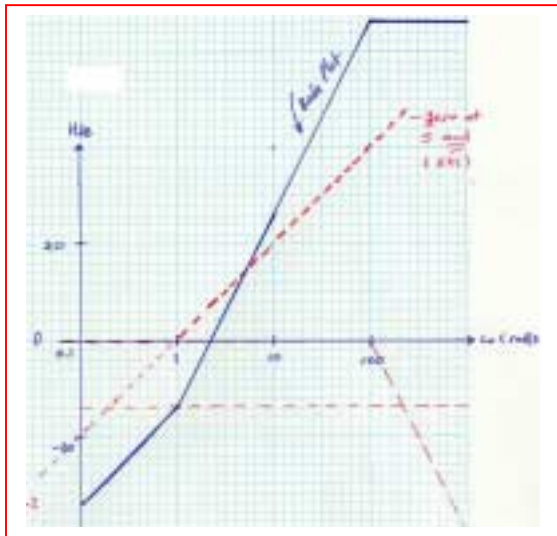
## CHAPTER SIXTEEN SOLUTIONS

38. (d) MATLAB verification- shown adjacent to Bode plots below.

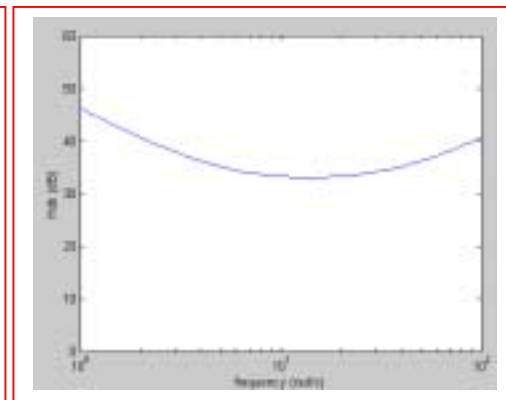
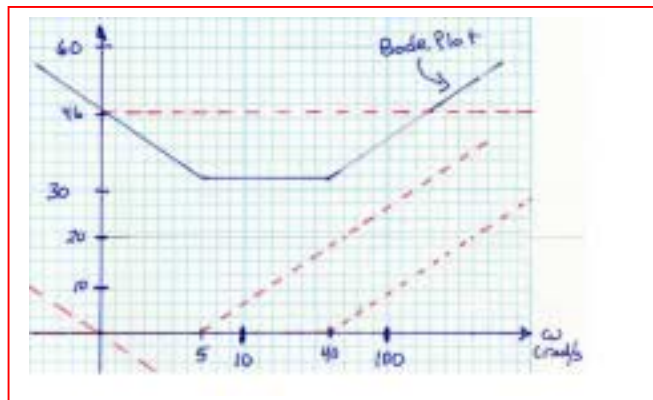
(a)  $\bar{H}(s) = \frac{20(s+1)}{s+100} = \frac{0.2(1+s)}{1+s/100}$ ,  $0.2 \rightarrow -14\text{dB}$



(b)  $\bar{H}(s) = \frac{2000(s+1)s}{(s+100)^2} = \frac{0.2s(1+s)}{(1+s/100)^2}$ ,  $0.2 \rightarrow -14\text{dB}$



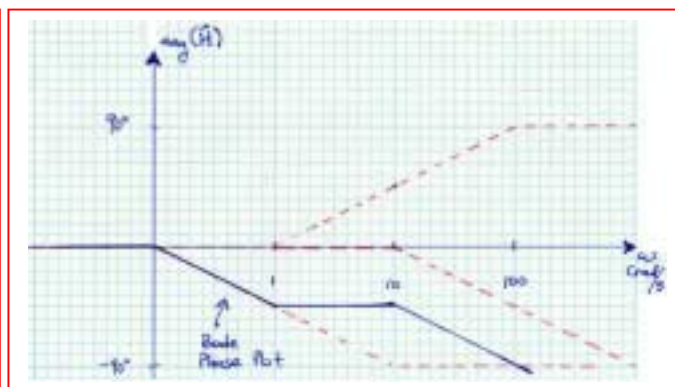
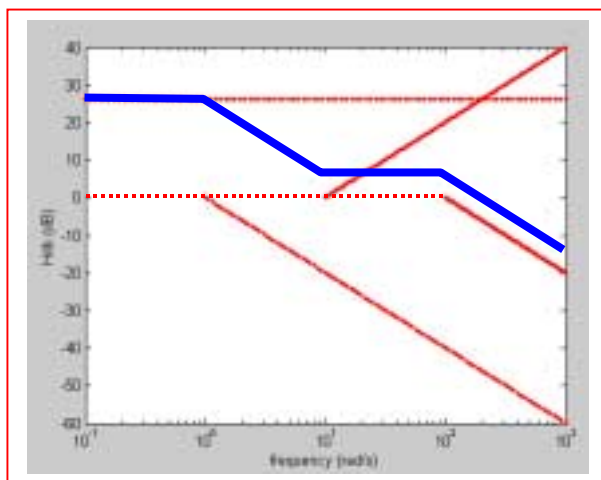
(c)  $\bar{H}(s) = s + 45 + \frac{200}{s} = \frac{s^2 + 45s + 200}{s} = \frac{(s+5)(s+40)}{s} = \frac{200(1+s/5)(1+s/40)}{s}$ ,  $200 \rightarrow 46\text{dB}$



## CHAPTER SIXTEEN SOLUTIONS

39.

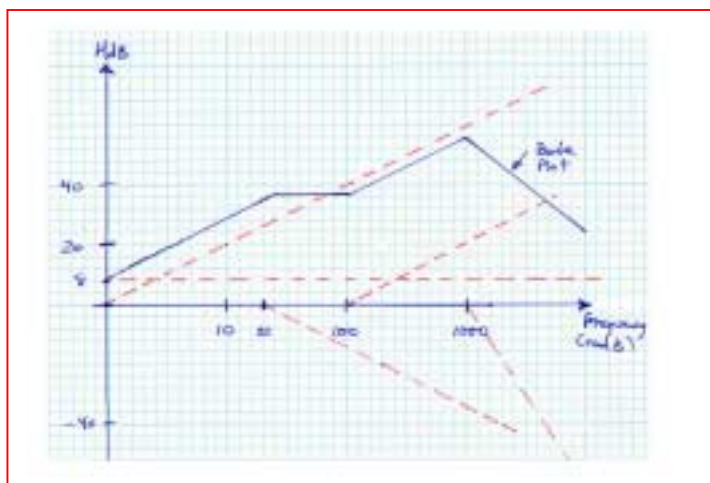
$$\begin{aligned}\bar{H}(s) &= \frac{V_C}{I_R} = \frac{(20 + 2s)(182 + 200/s)}{202 + 2s + 200/s} \times \frac{200/s}{182 + 200/s} \\ &= \frac{400(s+10)}{2(s^2 + 101s + 100)} = \frac{200(10+s)}{(1+s)(100+s)} \\ \bar{H}(s) &= \frac{20(1+s/10)}{(1+s)(1+s/100)}, \quad 20 \rightarrow 26\text{dB}\end{aligned}$$



## CHAPTER SIXTEEN SOLUTIONS

40.

$$(a) \quad \bar{H}(s) = \frac{5 \times 10^8 s(s+100)}{(s+20)(s+1000)^3} = \frac{2.5s(1+s/100)}{(1+s/20)(1+s/1000)^3}, \quad 2.5 \rightarrow 8 \text{ dB}$$



$$(b) \quad \text{Corners: } \omega = 20, 34 \text{ dB;}$$

$$\omega = 100, 34 \text{ dB;}$$

$$\omega = 1000, 54 \text{ dB}$$

$$\text{Intercepts: } 0 \text{ dB}, 2.5\omega = 1, \omega = 0.4$$

$$\omega = 1, 8 \text{ dB; } 0 \text{ dB}, \frac{2.5\omega(\omega/100)}{(\omega/20)(\omega/1000)^3} = \frac{2.5\omega^2(20)10^9}{100\omega\omega^3} = 1 \quad \therefore \omega = 22,360 \text{ rad/s}$$

$$(c) \quad \text{Corners: } \omega = 20, 31.13 \text{ dB}$$

$$\omega = 100, 36.69 \text{ dB} \quad H_{dB} = 20 \log 2.5\omega \frac{\sqrt{1+(\omega/100)^2}}{\sqrt{[1+(\omega/20)^2][1+(\omega/1000)^2]^3}}$$

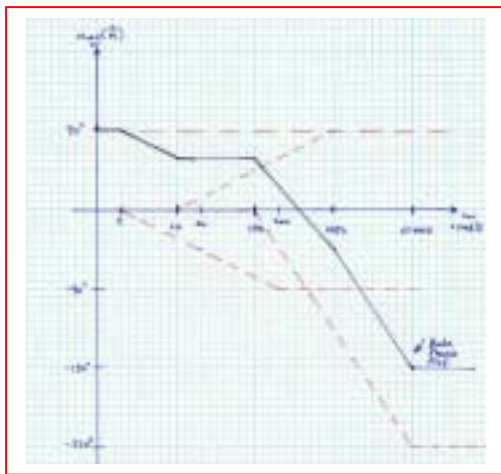
$$\omega = 1000, 44.99 \text{ dB}$$



## CHAPTER SIXTEEN SOLUTIONS

41.

$$(a) \quad \bar{H}(s) = \frac{5 \times 10^8 s(s+100)}{(s+20)(s+1000)^3} = \frac{2.5s(1+s/100)}{(1+s/20)(1+s/1000)^3},$$



$$(b) \quad \omega = 2 : \angle = 90^\circ$$

$$\omega = 10 : \angle = 90^\circ - \left( 45^\circ + 45^\circ \log \frac{10}{20} \right) = 58.5^\circ$$

$$\omega = 100 : \angle = 90^\circ - \left( 45^\circ + 45^\circ \log \frac{100}{20} \right) + \left( 45^\circ + 45^\circ \log \frac{100}{100} \right) = 58.5^\circ$$

$$\omega = 200 : \angle = 90^\circ - 90^\circ + \left( 45^\circ + 45^\circ \log \frac{200}{100} \right) - 3 \left( 45^\circ + 45^\circ \log \frac{200}{100} \right) = 17.9^\circ$$

$$\omega = 1000 : \angle = 90^\circ - 90^\circ + 90^\circ - 3 \left( 45^\circ + 45^\circ \log \frac{1000}{1000} \right) = -45^\circ$$

$$\omega = 10,000 : \angle = 90^\circ - 90^\circ + 90^\circ - 3 \times 90^\circ = -180^\circ$$

$$(c) \quad \omega = 2 : \angle = 90^\circ + \tan^{-1} 0.02 - \tan^{-1} 0.1 - 3 \tan^{-1} 0.002 = 85.09^\circ$$

$$\omega = 10 : \angle = 90^\circ + \tan^{-1} 0.1 - \tan^{-1} 0.5 - 3 \tan^{-1} 0.01 = 67.43^\circ$$

$$\omega = 100 : \angle = 90^\circ + \tan^{-1} 1 - \tan^{-1} 5 - 3 \tan^{-1} 0.1 = 39.18^\circ$$

$$\omega = 200 : \angle = 90^\circ + \tan^{-1} 2 - \tan^{-1} 10 - 3 \tan^{-1} 0.2 = 35.22^\circ$$

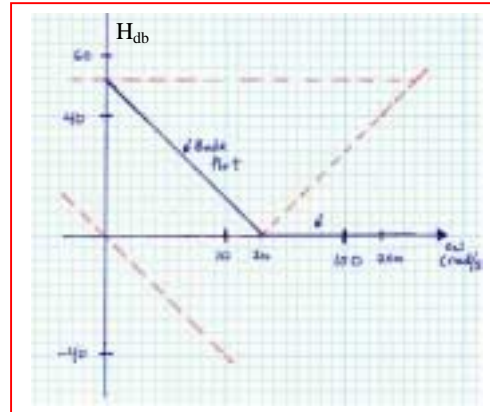
$$\omega = 1000 : \angle = 90^\circ + \tan^{-1} 10 - \tan^{-1} 50 - 3 \tan^{-1} 1 = -49.56^\circ$$

$$\omega = 10,000 : \angle = 90^\circ + \tan^{-1} 100 - \tan^{-1} 500 - 3 \tan^{-1} 10 = -163.33^\circ$$

## CHAPTER SIXTEEN SOLUTIONS

42.

$$\begin{aligned}
 (a) \quad H(s) &= 1 + \frac{20}{s} + \frac{400}{s^2} = \frac{s^2 + 20s + 400}{s^2} \\
 &= 400 \frac{1 + 2 \times 0.5(s/20) + (s/20)^2}{s^2} \\
 \therefore \omega_o &= 20, \zeta = 0.5 \\
 20 \log 400 &= 52 \text{ dB} \\
 \text{Correction at } \omega_o &\text{ is } 20 \log 2 \zeta = 0 \text{ dB}
 \end{aligned}$$



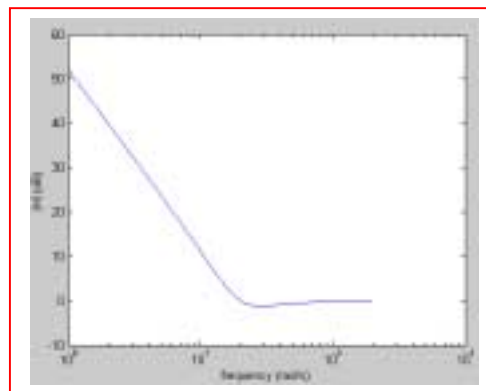
$$(b) \quad \omega = 5: H_{dB} = 52 - 2 \times 20 \log 5 = 24.0 \text{ dB (plot)}$$

$$H_{dB} = 20 \log |1 - 16 + j4| = 23.8 \text{ dB (exact)}$$

$$\omega = 100: H_{dB} = 0 \text{ dB (plot)}$$

$$H_{dB} = 20 \log |1 - 0.04 + j0.2| = -0.170 \text{ dB (exact)}$$

(c)



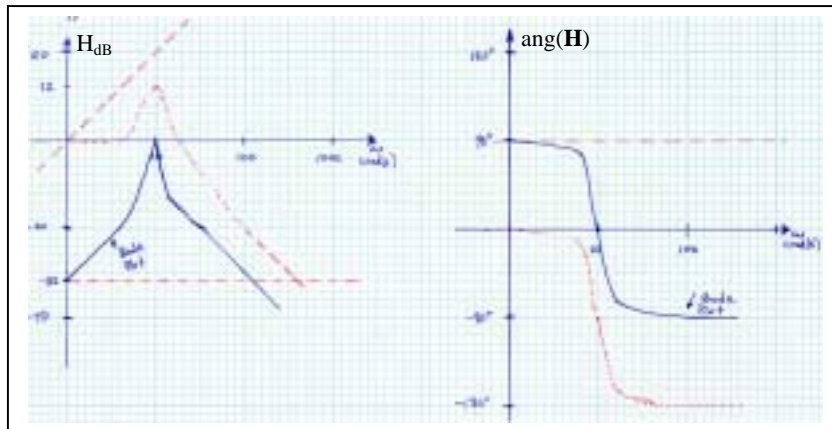
## CHAPTER SIXTEEN SOLUTIONS

43.

$$(a) \quad \bar{H}(s) = \frac{V_R}{V_s} = \frac{25}{10s + 25 + 1000/s} = \frac{25s}{10s^2 + 25s + 1000} = \frac{0.025s}{1 + 2\left(\frac{1}{8}\right)\left(\frac{s}{10}\right) + \left(\frac{s}{10}\right)^2}$$

$$(b) \quad \therefore \omega_o = 10, \zeta = 1/8 \quad \therefore \text{correction} = -20 \log \left( 2 \times \frac{1}{8} \right) = 12 \text{ dB}$$

$$0.025 \rightarrow -32 \text{ dB}$$



$$(c) \quad \omega = 20, \bar{H}(j20) = \frac{j0.5}{1 - 4 + j0.5} \quad \therefore H_{dB} = -15.68 \text{ dB} \quad \angle H(j20) = -80.54^\circ$$

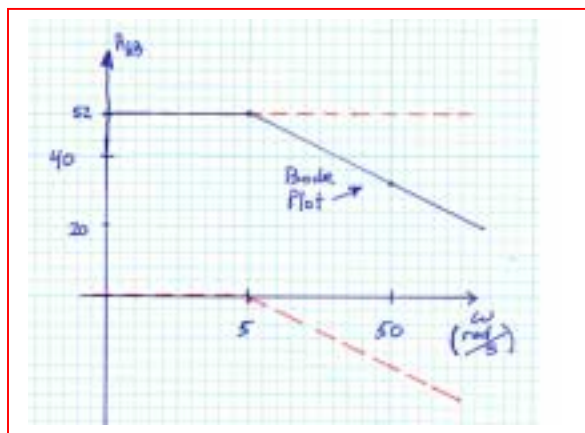
## CHAPTER SIXTEEN SOLUTIONS

44.

1st two stages,  $\bar{H}_1(s) = \bar{H}_2(s) = -10$ ;  $\bar{H}_3(s) = \frac{-1/(50 \times 10^3 \times 10^{-6})}{s + 1/(200 \times 10^3 \times 10^{-6})} = \frac{-20}{s + 5}$

$$\therefore \bar{H}(s) = (-10)(-10) \left( \frac{-20}{s + 5} \right) = \frac{-400}{1 + s/5}$$

$-400 \rightarrow 52 \text{ dB}$



## CHAPTER SIXTEEN SOLUTIONS

45.

(a) 1st stage:  $C_{1A} = 1 \mu\text{F}$ ,  $R_{1A} = \infty$ ,  $R_{fA} = 10^5$   $\therefore \bar{H}_A(s) = -R_{fA} C_{1A} s = -0.1s$

2nd stage:  $R_{1B} = 10^5$ ,  $R_{fB} = 10^5$ ,  $C_{fB} = 1 \mu\text{F}$   $\therefore \bar{H}_B(s) = \frac{-1/R_{1B} C_{fB}}{s + 1/R_{fB} C_{fB}}$

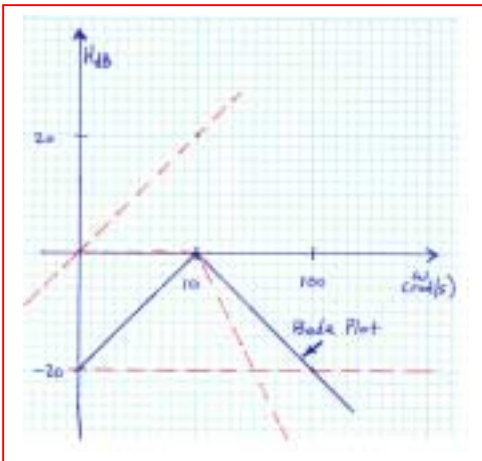
$$\therefore \bar{H}_B(s) = \frac{1/(10^5 \times 10^{-6})}{s + 1/(10^5 \times 10^{-6})} = -\frac{10}{s + 10}$$

3rd stage: same as 2nd

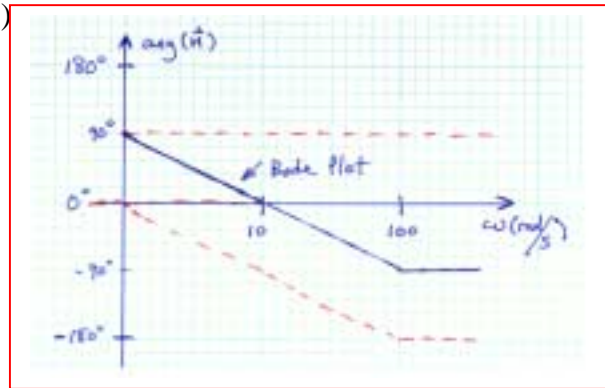
$$\therefore \bar{H}(s) = (-0.1s) \left( \frac{-10}{s + 10} \right) \left( \frac{-10}{s + 10} \right) = -\frac{0.1s}{(1 + s/10)^2}$$

$$20 \log_{10}(0.1) = -20 \text{ dB}$$

(b)



(c)



## CHAPTER SIXTEEN SOLUTIONS

46. An amplifier that rejects high-frequency signals is required. There is some ambiguity in the requirements, as social conversations may include frequencies up to 50 kHz, and echolocation sounds, which we are asked to filter out, may begin below this value. Without further information, we decide to set the filter cutoff frequency at 50 kHz to ensure we do not lose information. However, we note that *this decision is not necessarily the only correct one*.

Our input source is a microphone modeled as a sinusoidal voltage source having a peak amplitude of 15 mV in series with a 1- $\Omega$  resistor. Our output device is an earphone modeled as a 1-k $\Omega$  resistor. A voltage of 15 mV from the microphone should correspond to about 1 V at the earphone according to the specifications, requiring a gain of  $1000/15 = 66.7$ .

If we select a non-inverting op amp topology, we then need  $\frac{R_f}{R_1} = 66.7 - 1 = 65.7$

Arbitrarily choosing  $R_1 = 1 \text{ k}\Omega$ , we then need  $R_f = 65.7 \text{ k}\Omega$ . This completes the amplification part. Next, we need to filter out frequencies greater than 50 kHz.

Placing a capacitor across the microphone terminals will “short out” high frequencies.

We design for  $\omega_c = 2\pi f_c = 2\pi(50 \times 10^3) = \frac{1}{R_{mic} C_{filter}}$ . Since  $R_{mic} = 1 \text{ }\Omega$ , we require

$$C_{filter} = 3.183 \text{ }\mu\text{F}.$$

## CHAPTER SIXTEEN SOLUTIONS

47. We choose a simple series RLC circuit. It was shown in the text that the “gain” of the circuit with the output taken across the resistor is  $|A_v| = \frac{\omega RC}{\left[ (1 - \omega^2 LC)^2 + \omega^2 R^2 C^2 \right]^{1/2}}$ .

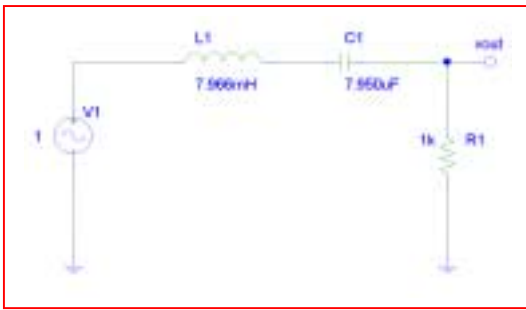
This results in a bandpass filter with corner frequencies at

$$\omega_{c_L} = \frac{-RC + \sqrt{R^2 C^2 + 4LC}}{2LC} \quad \text{and} \quad \omega_{c_H} = \frac{RC + \sqrt{R^2 C^2 + 4LC}}{2LC}$$

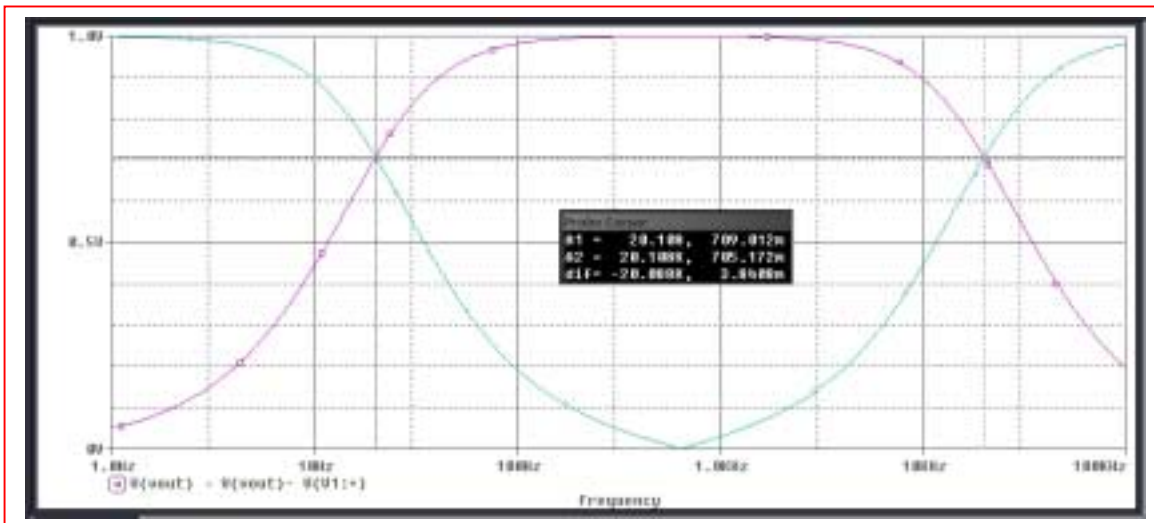
If we take our output across the inductor-capacitor combination instead, we obtain the opposite curve- *i.e.* a bandstop filter with the same cutoff frequencies. Thus, we want

$$2\pi(20) = \frac{-RC + \sqrt{R^2 C^2 + 4LC}}{2LC} \quad \text{and} \quad 2\pi(20 \times 10^3) = \frac{RC + \sqrt{R^2 C^2 + 4LC}}{2LC}$$

Noting that  $\omega_{c_H} - \omega_{c_L} = R/L = 125.5 \text{ krad/s}$ , we arbitrarily select  $R = 1 \text{ k}\Omega$ , so that  $L = 7.966 \text{ mH}$ . Returning to either cutoff frequency expression, we then find  $C = 7.950 \text{ }\mu\text{F}$

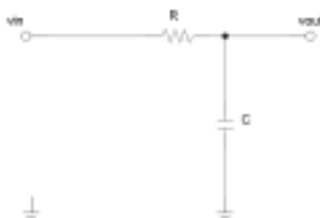


PSpice verification. The circuit performs as required, with a lower corner frequency of about 20 Hz and an upper corner frequency of about 20 kHz.



## CHAPTER SIXTEEN SOLUTIONS

48. We choose a simple RC filter topology:

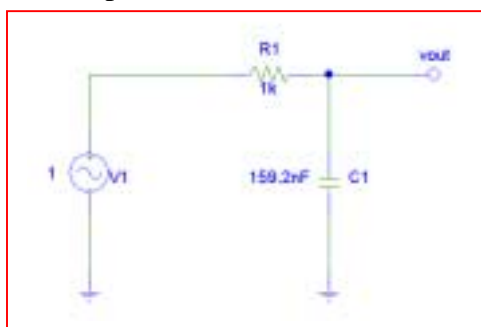


Where  $\frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC}$  and hence  $\left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$ . We desire a cutoff

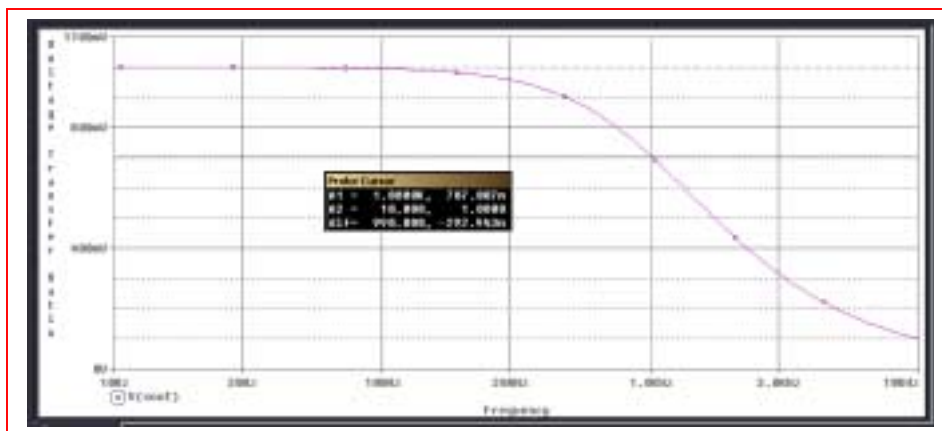
frequency of 1 kHz, and note that this circuit does indeed act as a low-pass filter (higher frequency signals lead to the capacitor appearing more and more as a short circuit). Thus,

$$= \frac{1}{\sqrt{1 + (\omega_c RC)^2}} = \frac{1}{\sqrt{2}} \quad \text{where } \omega_c = 2\pi f_c = 2000\pi \text{ rad/s.}$$

A small amount of algebra yields  $1 + [2\pi(1000)RC]^2 = 2$  or  $2000\pi RC = 1$ . Arbitrarily setting  $R = 1 \text{ k}\Omega$ , we then find that  $C = 159.2 \text{ nF}$ . The operation of the filter is verified in the PSpice simulation below:



Probe Cursor			
A1 =	1.0000K,	707.007m	
A2 =	10.000,	1.0000	
diff=	990.000,	-292.943m	

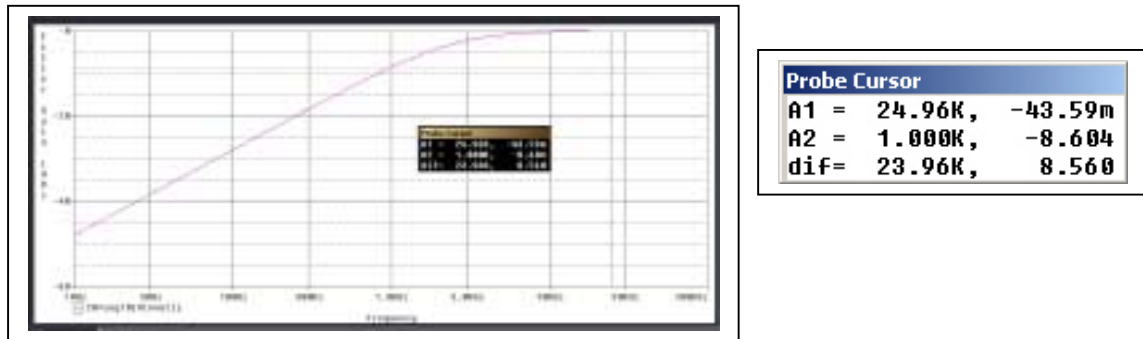




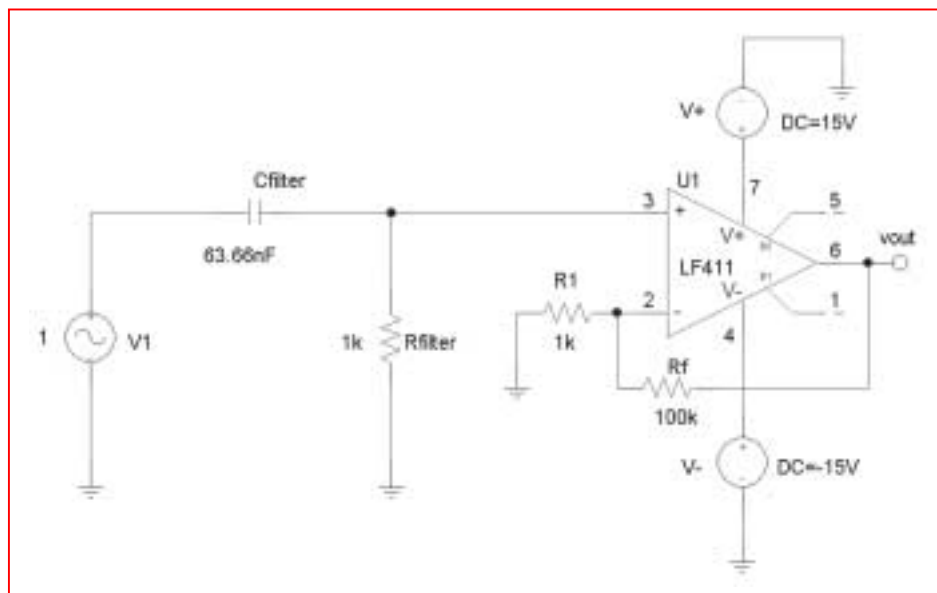
## CHAPTER SIXTEEN SOLUTIONS

49. We are not provided with the actual spectral shape of the noise signal, although the reduction to 1% of its peak value (a drop of 40 dB) by 1 kHz is useful to know. If we place a simple high-pass RC filter at the input of an op amp stage, designing for a pole at 2.5 kHz should ensure an essentially flat response above 25 kHz, and a 3 dB reduction at 2.5 kHz. If greater tolerance is required, the 40 dB reduction at 1 kHz allows the pole to be moved to a frequency even closer to 1 kHz. The PSpice simulation below shows a filter with  $R = 1 \text{ k}\Omega$  (arbitrarily chosen) and  $C = \frac{1}{2\pi(2.5 \times 10^3)(1000)} = 63.66 \text{ nF}$ .

At a frequency of 25 kHz, the filter shows minimal gain reduction, but at 1 kHz any signal is reduced by more than 8 dB.



We therefore design a simple non-inverting op amp circuit such as the one below, which with  $R_f = 100 \text{ k}\Omega$  and  $R_1 = 1 \text{ k}\Omega$ , has a gain of 100 V/V. In simulating the circuit, a gain of approximately 40 dB at 25 kHz was noted, although the gain dropped at higher frequencies, reaching 37 dB around 80 kHz. Thus, to completely assess the suitability of design, more information regarding the frequency spectrum of the “failure” signals would be required.



## CHAPTER SIXTEEN SOLUTIONS

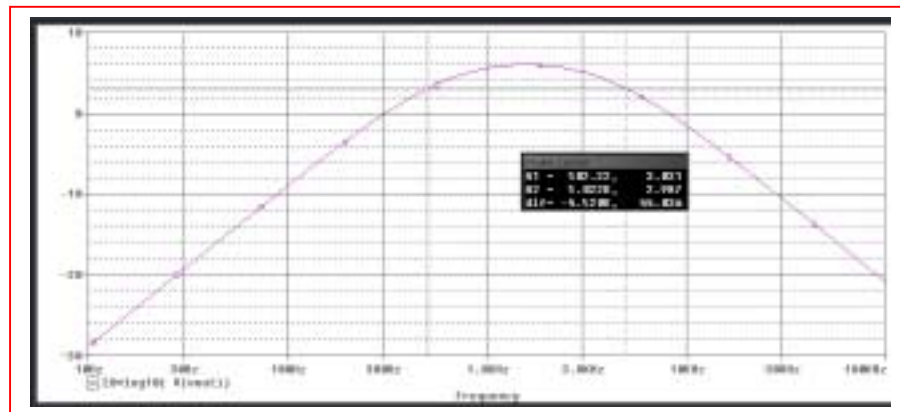
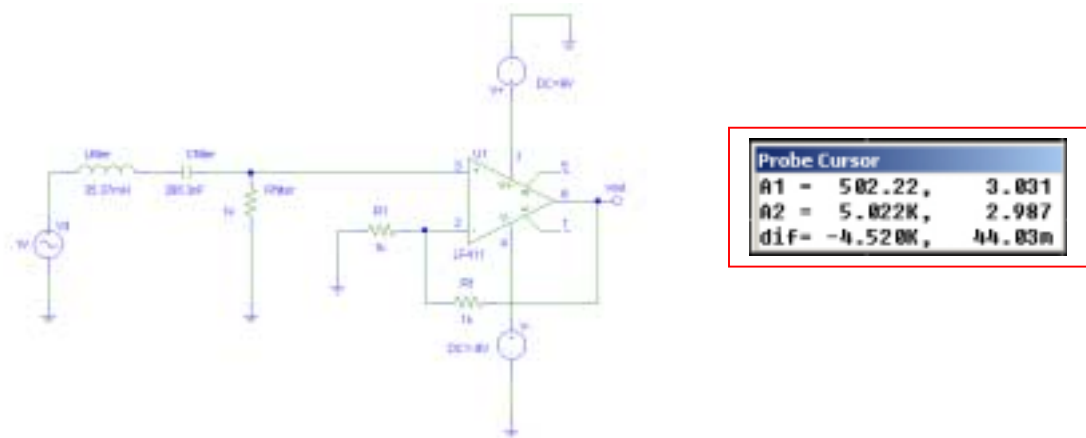
50. We select a simple series RLC circuit with the output taken across the resistor to serve as a bandpass filter with 500 Hz and 5000 Hz cutoff frequencies. From Example 16.12, we know that

$$\omega_{c_L} = -\frac{R}{2L} + \frac{1}{2LC} \sqrt{R^2 C^2 + 4LC} = 2\pi(500)$$

and

$$\omega_{c_H} = \frac{R}{2L} + \frac{1}{2LC} \sqrt{R^2 C^2 + 4LC} = 2\pi(5000)$$

With  $\omega_{c_H} - \omega_{c_L} = 2\pi(5000 - 500) = R/L$ , we (arbitrarily) select  $R = 1 \text{ k}\Omega$ , so that  $L = 35.37 \text{ mH}$ . Substituting these two values into the equation for the high-frequency cutoff, we find that  $C = 286.3 \text{ nF}$ . We complete the design by selecting  $R_1 = 1 \text{ k}\Omega$  and  $R_f = 1 \text{ k}\Omega$  for a gain of 2 (no value of gain was specified). As seen in the PSpice simulation results shown below, the circuit performs as specified at maximum gain (6 dB or 2 V/V), with cutoff frequencies of approximately 500 and 5000 KHz and a peak gain of 6 dB.

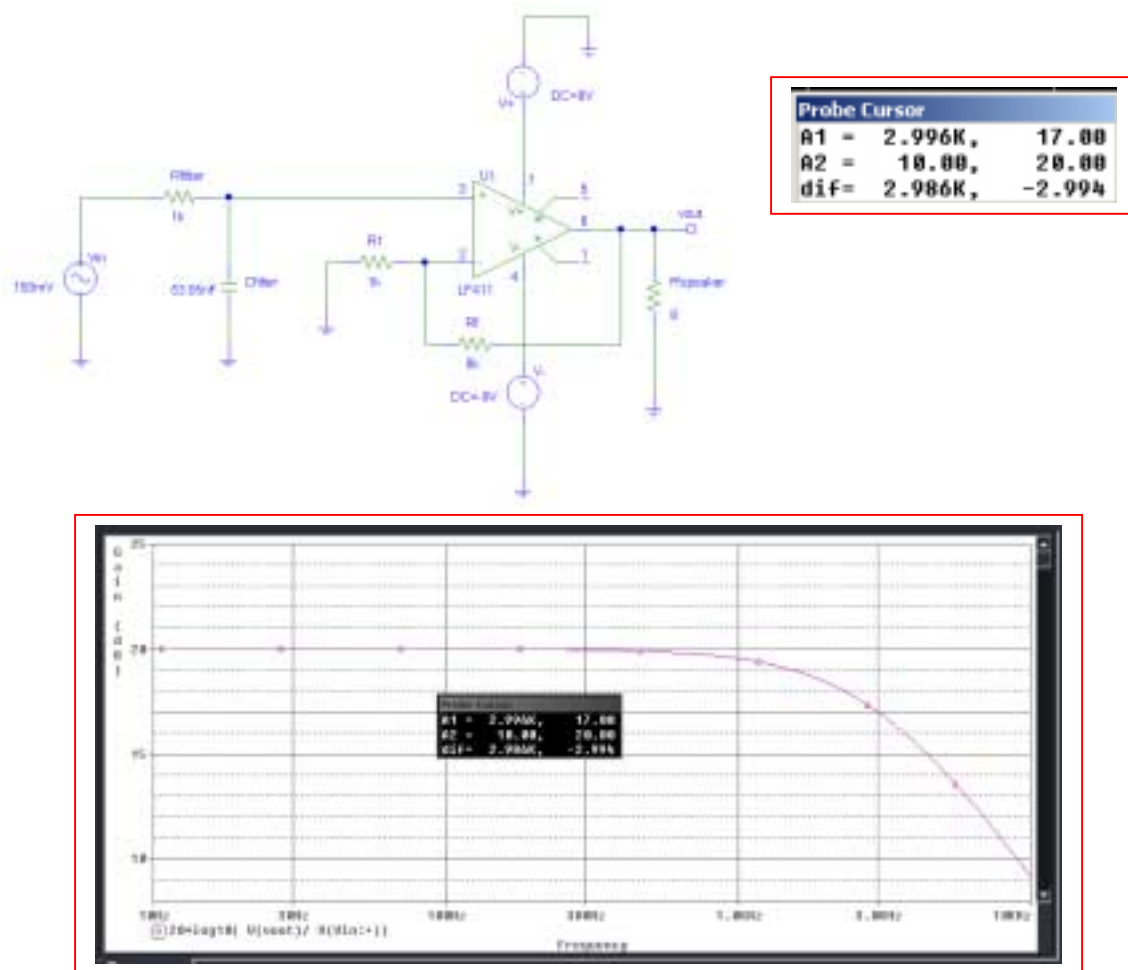


## CHAPTER SIXTEEN SOLUTIONS

51. For this circuit, we simply need to connect a low-pass filter to the input of a non-inverting op amp having  $R_f/R_1 = 9$  (for a gain of 10). If we use a simple RC filter, the cutoff frequency is

$$\omega_c = \frac{1}{RC} = 2\pi(3000)$$

Selecting (arbitrarily)  $R = 1 \text{ k}\Omega$ , we find  $C = 53.05 \text{ nF}$ . The PSpice simulation below shows that our design does indeed have a bandwidth of 3 kHz and a peak gain of 10 V/V (20 dB).

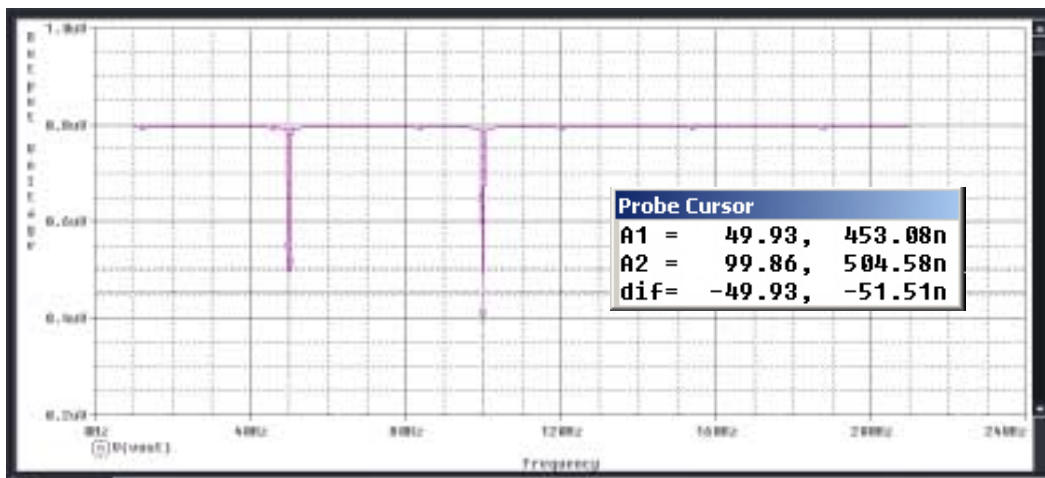
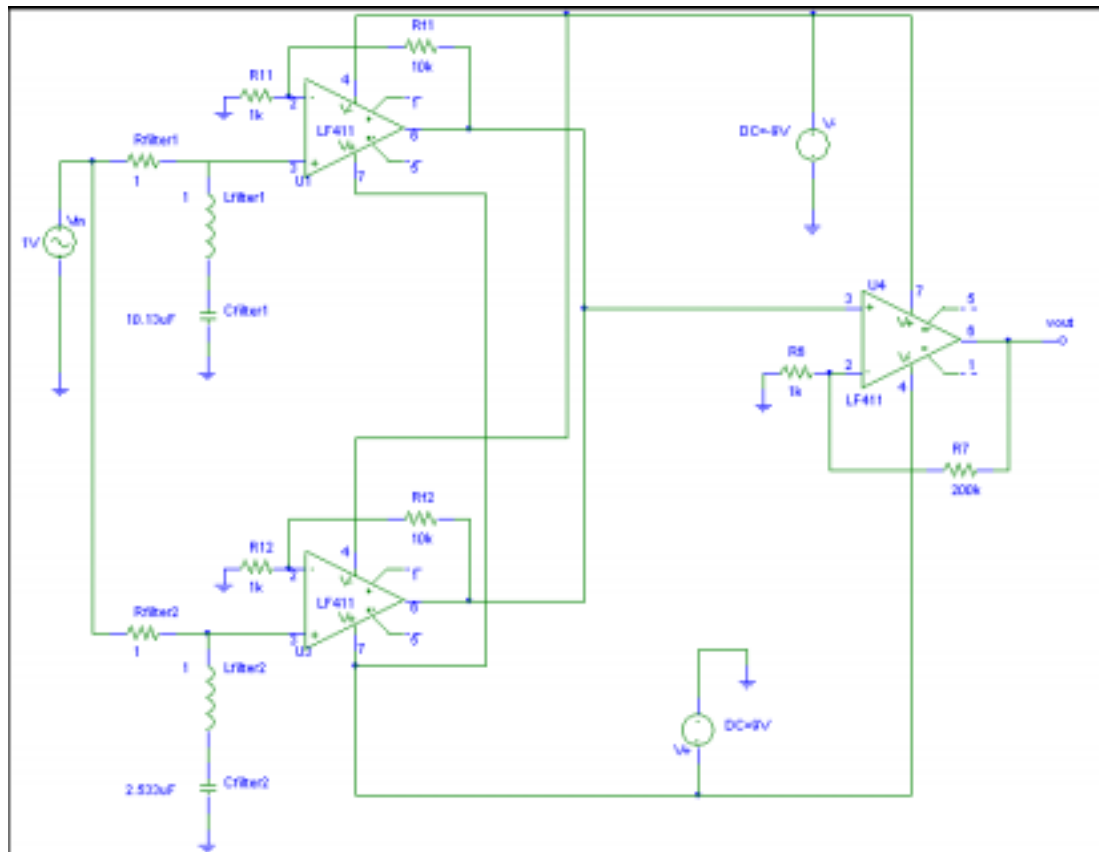


## CHAPTER SIXTEEN SOLUTIONS

52.

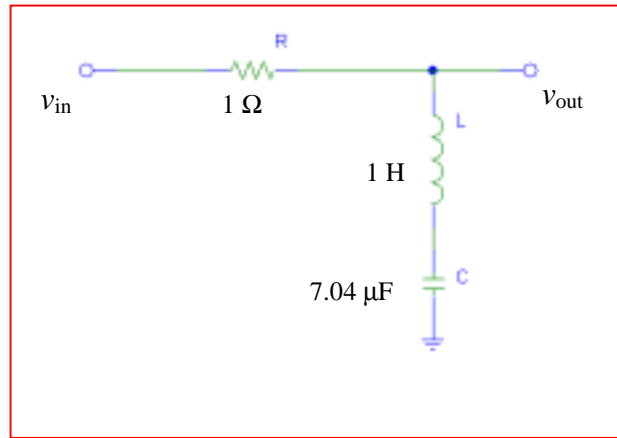
We require four filter stages, and choose to implement the circuit using op amps to isolate each filter sub-circuit. Selecting a bandwidth of 1 rad/s (no specification was given) and a simple RLC filter as suggested in the problem statement, a resistance value of  $1\ \Omega$  leads to an inductor value of 1 H (bandwidth for this type of filter =  $\omega_H - \omega_L = R/L$ ). The capacitance is found by designing each filter's respective resonant frequency ( $1/\sqrt{LC}$ ) at the desired "notch" frequency. Thus, we require  $C_{F1} = 10.13\ \mu\text{F}$ ,  $C_{F2} = 2.533\ \mu\text{F}$ ,  $C_{F3} = 1.126\ \mu\text{F}$  and  $C_{F4} = 633.3\ \text{nF}$ .

The Student Version of PSpice® will not permit more than 64 nodes, so that the total solution must be simulated in two parts. The half with the filters for notching out 50 and 100 Hz components is shown below; an additional two op amp stages are required to complete the design.



## CHAPTER SIXTEEN SOLUTIONS

53. Using the series RLC circuit suggested, we decide to design for a bandwidth of 1 rad/s (as no specification was provided). With  $\omega_H - \omega_L = R/L$ , we arbitrarily select  $R = 1\ \Omega$  so that  $L = 1\ \text{H}$ . The capacitance required is obtained by setting the resonant frequency of the circuit ( $1/\sqrt{LC}$ ) equal to 60 Hz ( $120\pi\ \text{rad/s}$ ). This yields  $C = 7.04\ \mu\text{F}$ .



## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

1.

$$\Delta_z = \begin{vmatrix} 17 & -8 & -3 \\ -8 & 17 & -4 \\ -3 & -4 & 17 \end{vmatrix} = 17(273) + 8(-148) - 3(83) = 3208 \Omega^3$$

$$(a) \quad Z_{in1} = \frac{\Delta_z}{\Delta_{11}} = \frac{3208}{273} = 11.751 \Omega \quad \therefore P_1 = \frac{100^2}{11.751} = 851.0 \text{ W}$$

$$(b) \quad Z_{in2} = \frac{\Delta_z}{\Delta_{22}} = \frac{3208}{280} = 11.457 \Omega \quad \therefore P_2 = \frac{100^2}{11.457} = 872.8 \text{ W}$$

$$(c) \quad Z_{in3} = \frac{\Delta_z}{\Delta_{33}} = \frac{3208}{225} = 14.258 \Omega \quad \therefore P_3 = \frac{100^2}{14.258} = 701.4 \Omega$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

2.

$$\Delta_y = \begin{vmatrix} 0.35 & -0.1 & -0.2 \\ -0.1 & 0.5 & -0.15 \\ -0.2 & -0.15 & 0.75 \end{vmatrix} = 0.35(0.3525) + 0.1(-0.105) - 0.2(0.115) = 0.089875 \text{ S}^3$$

$$(a) \quad Y_{in1} = \frac{\Delta_y}{\Delta_{11}} = \frac{0.089875}{0.3525} = 0.254965 \quad \therefore P_1 = \frac{10^2}{0.254965} = \boxed{392.2 \text{ W}}$$

$$(b) \quad Y_{in2} = \frac{\Delta_y}{\Delta_{22}} = \frac{0.089875}{0.2225} = 0.403933 \quad \therefore P_2 = \frac{10^2}{0.403933} = \boxed{247.6 \text{ W}}$$

$$(c) \quad Y_{in3} = \frac{0.089875}{0.165} = 0.544697 \text{ S} \quad \therefore P_3 = \frac{100}{0.544697} = \boxed{183.59 \text{ W}}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

3.

$$\begin{aligned}
 [R] &= \begin{bmatrix} 3 & -1 & -2 & 0 \\ -1 & 4 & 1 & 3 \\ -2 & 2 & 5 & 2 \\ 0 & -3 & -2 & 6 \end{bmatrix} (\Omega) = 3 \begin{vmatrix} 4 & 1 & 3 \\ 2 & 5 & 2 \\ -3 & -2 & 6 \end{vmatrix} + 1 \begin{vmatrix} -1 & -2 & 0 \\ 2 & 5 & 2 \\ -3 & -2 & 6 \end{vmatrix} - 2 \begin{vmatrix} -1 & -2 & 0 \\ -1 & 4 & 1 \\ -3 & -2 & 6 \end{vmatrix} \\
 &= 3[4(34) - 2(12) - 3(-13)] + [-1(34) - 2(-12) - 3(-4)] = 2[-1(12) - 4(-12) - 3(-6)] \\
 &= 3(73) + (-22) - 2(18) = 161 \, \Omega^4 \quad \therefore R_{in} = \frac{\Delta_R}{\Delta_{11}} = \frac{161}{73} = \boxed{2.205^+ \, \Omega}
 \end{aligned}$$



## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

4. Define a counter-clockwise current  $\bar{\mathbf{I}}_2$  in the left-most mesh, and a counter-clockwise current  $\bar{\mathbf{I}}_1$  flowing in the right-most mesh. Then,

$$\bar{\mathbf{V}}_1 = 4\bar{\mathbf{I}}_2 \quad \therefore 0.2\bar{\mathbf{V}}_1 = 0.8\bar{\mathbf{I}}_2$$

$$\bar{\mathbf{V}}_{in} = \bar{\mathbf{I}}_1 s + 5(\bar{\mathbf{I}}_1 + 0.8\bar{\mathbf{I}}_2 - \bar{\mathbf{I}}_2) = (s+5)\bar{\mathbf{I}}_1 - \bar{\mathbf{I}}_2$$

$$\text{Also, } \bar{\mathbf{I}}_2(2s+4) - 5(\bar{\mathbf{I}}_1 + 0.8\bar{\mathbf{I}}_2 - \bar{\mathbf{I}}_2) = 0$$

$$\text{or } 0 = -5\bar{\mathbf{I}}_1 + (5+2s)\bar{\mathbf{I}}_2$$

$$\therefore \Delta_Z = (s+5)(5+2s) - 5 = 2s^2 + 15s + 20, \Delta_{11} = 5+2s$$

$$\therefore Z_{th} = \frac{2s^2 + 15s + 20}{2s + 5}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

5. Define a clockwise mesh current  $\mathbf{I}_1$  flowing in the bottom left mesh, a clockwise mesh current  $\mathbf{I}_2$  flowing in the top mesh, and a clockwise mesh current  $\mathbf{I}_3$  flowing in the bottom right mesh. Then,

(a)  $\bar{V}_{in} = 10(\bar{I}_1 - \bar{I}_2) - 0.6 \times 8\bar{I}_2 = 10\bar{I}_1 - 14.8\bar{I}_2$   
 $0 = 50\bar{I}_2 - 10\bar{I}_1 - 12\bar{I}_3 = -10\bar{I}_1 + 50\bar{I}_2 - 12\bar{I}_3$   
 $0 = 4.8\bar{I}_2 + 17\bar{I}_3 - 12\bar{I}_2 = -7.2\bar{I}_2 + 17\bar{I}_3$

$$\therefore \Delta_Z = \begin{vmatrix} 10 & -14.8 & 0 \\ -10 & 50 & -12 \\ 0 & -7.2 & 17 \end{vmatrix} = 10(763.6) + 10(-251.6) = 5120 \quad \therefore Z_{in} = \frac{5120}{763.6} = \boxed{6.705^+ \Omega}$$

(b)  $\bar{I}_{in} = \frac{\bar{V}_1 - \bar{V}_2}{28} + \frac{\bar{V}_1 - 0.6\bar{V}_x}{10} = 0.13571 \bar{V}_1 - 0.03571\bar{V}_2 - 0.06\bar{V}_x$   
 $0 = \frac{\bar{V}_2 - \bar{V}_1}{28} + \frac{\bar{V}_2 - 0.6\bar{V}_x}{12} + \frac{\bar{V}_2}{5} = -0.03571\bar{V}_1 + 0.31905\bar{V}_2 - 0.05\bar{V}_x$   
 $0 = -\frac{\bar{V}_x}{8} + \frac{\bar{V}_2 - \bar{V}_x - \bar{V}_1}{20} = -0.05\bar{V}_1 + 0.05\bar{V}_2 - 0.175\bar{V}_x$

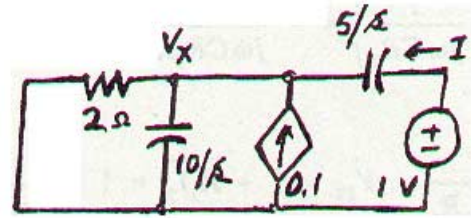
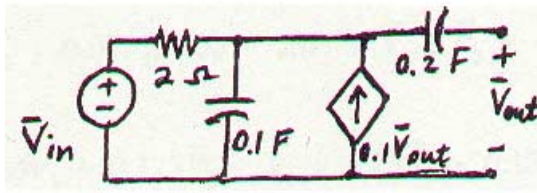
$$\therefore \Delta_y = \begin{vmatrix} 0.13571 & -0.03571 & -0.06 \\ -0.03571 & 0.31905 & -0.05 \\ -0.05 & 0.05 & -0.175 \end{vmatrix} = 0.13571(-0.05583 + 0.0025) + 0.03571(0.00625 + 0.003) - 0.05(0.00179 + 0.01914) = -0.00724$$

$$\therefore \Delta_y = 0.007954, \Delta_{11} = -0.05333 \quad \therefore \bar{Y}_m = \frac{\Delta_y}{\Delta_{11}} = \frac{-0.007954}{-0.05333} = 0.14926 \text{ S}$$

$$\therefore \bar{Z}_{in} = \frac{1}{0.14926} = \boxed{6.705^+ \Omega}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

6.



$$\frac{\bar{V}_x}{2} + \frac{s\bar{V}_x}{10} - 0.1 + \frac{s}{5}(\bar{V}_x - 1) = 0$$

$$\therefore \bar{V}_x(0.5 + 0.3s) = 0.1 + 0.2s$$

$$\therefore \bar{V}_x = \frac{0.2s + 0.1}{0.3s + 0.5}$$

$$\therefore \bar{I} = (1 - \bar{V}_x) \frac{s}{5} = \left(1 - \frac{0.2s + 0.1}{0.3s + 0.5}\right) 0.2s = 0.2s \frac{0.1s + 0.4}{0.3s + 0.5}$$

$$\therefore \bar{Y}_{out} = \bar{I} = \frac{s(0.1s + 0.4)}{1.5s + 2.5}, \bar{Z}_{out} = \frac{1.5s + 2.5}{s(0.1s + 0.4)} = \frac{15s + 25}{s(s + 4)}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

7.

$$\bar{V}_{in} = 1 \text{ V}, \bar{V}_i = 0 \quad \therefore \bar{V}_x + \bar{V}_{in} = 0, \bar{V}_x = -1 \text{ V}$$

$$\bar{I}_x = \frac{\bar{V}_x}{R_x} = -\frac{1}{R_x}; 2 \times 10^4 \bar{I}_{in} + 2 \times 10^4 \bar{I}_x = 0$$

$$\therefore \bar{I}_{in} = -\bar{I}_x = \frac{1}{R_x} \quad \therefore R_{in} = -V_{in} / I_{in} = \boxed{-R_x}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

8.

- (a) Assume 1 V at input. Since  $V_i = 0$  at each op-amp input, 1 V is present between  $R_2$  and  $R_3$ , and also C and  $R_4$ .

$$\therefore \bar{V}_4 = \frac{1}{R_4} \left( R_4 + \frac{1}{j\omega C} \right) = 1 + \frac{1}{j\omega C R_4}$$

$$\therefore \bar{I}_3 = \frac{1}{R_3} \left( 1 - 1 - \frac{1}{j\omega C R_4} \right) = -\frac{1}{j\omega C R_3 R_4}$$

$$\therefore \bar{I}_2 = \bar{I}_3 = -\frac{1}{j\omega C R_3 R_4} \quad \therefore \bar{V}_{12} = 1 + R_2 \bar{I}_2 = 1 - \frac{R_2}{j\omega C R_3 R_4}$$

$$\bar{I}_1 = \frac{1 - \bar{V}_{12}}{R_1} = \frac{R_2}{j\omega C R_1 R_3 R_4} = \bar{I}_{in} \quad \therefore \bar{Z}_{in} = \frac{1}{\bar{I}_{in}} = \boxed{j\omega C \frac{R_1 R_3 R_4}{R_2}}$$

- (b)  $R_1 = 4 \times 10^3$ ,  $R_2 = 10 \times 10^3$ ,  $R_3 = 10 \times 10^3$ ,  $R_4 = 10^3$ ,  $C = 2 \times 10^{-10}$

$$\therefore \bar{Z}_{in} = j\omega 2 \times 10^{-10} \frac{4 \times 10 \times 1}{10} \times 10^6 = \boxed{j\omega 0.8 \times 10^3 \Omega \text{ (} L_{in} = 0.8 \text{ mH)}}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

9. Define a clockwise mesh current  $\mathbf{I}_1$  in the left-most mesh, a clockwise mesh current  $\mathbf{I}_x$  in the center mesh, and a counter-clockwise mesh current  $\mathbf{I}_2$  in the right-most mesh. Then,

$$\bar{V}_1 = 13\bar{I}_1 - 10\bar{I}_2$$

$$0 = -10\bar{I}_1 + 35\bar{I}_x + 20\bar{I}_2 \quad \therefore \bar{I}_1 = \frac{\begin{vmatrix} \bar{V}_1 & -10 & 0 \\ 0 & 35 & 20 \\ \bar{V}_2 & 20 & 22 \end{vmatrix}}{\begin{vmatrix} 13 & -10 & 0 \\ -10 & 35 & 20 \\ 0 & 20 & 22 \end{vmatrix}}$$

$$\bar{V}_2 = 20\bar{I}_x + 22\bar{I}_2$$

$$\therefore \bar{I}_1 = \frac{\bar{V}_1(370) + \bar{V}_2(-200)}{13(370) + 10(-220)} = \frac{37}{261} \bar{V}_1 - \frac{20}{261} \bar{V}_2$$

$$\therefore \bar{y}_{11} = \frac{37}{261} = \boxed{141.76 \text{ mS}}, \quad \bar{y}_{12} = \frac{-20}{261} = \boxed{-76.63 \text{ mS}}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

10.

$$[y] = \begin{bmatrix} 10 & -5 \\ 50 & 20 \end{bmatrix} (\text{mS}) \quad \therefore \bar{I}_1 = 0.01\bar{V}_1 - 0.005\bar{V}_2,$$

$$\bar{I}_2 = 0.05\bar{V}_1 + 0.02\bar{V}_2, \quad 100 = 25\bar{I}_1 + \bar{V}_1, \quad \bar{V}_2 = -100\bar{I}_2$$

$$\therefore 100 = 0.25\bar{V}_1 - 0.125\bar{V}_2 + \bar{V}_1 = 1.25\bar{V}_1 - 0.125\bar{V}_2$$

$$\bar{I}_2 = -0.01\bar{V}_2 = 0.05\bar{V}_1 + 0.02\bar{V}_2 \quad \therefore -0.03\bar{V}_2 = 0.05\bar{V}_1 \quad \therefore \bar{V}_2 = -\frac{5}{3} \bar{V}_1$$

$$\therefore 100 = 1.25\bar{V}_1 + \frac{0.625}{3} \bar{V}_1 = \frac{4.375}{2} \bar{V}_1 \quad \therefore \bar{V}_1 = \frac{300}{4.375} = \boxed{68.57 \text{ V}}, \quad \bar{V}_2 = -\frac{5}{3} \bar{V}_1 = \boxed{-114.29 \text{ V}}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

11.

$$\bar{I}_1 = \frac{\bar{V}_1 - \bar{V}_2}{25} = 0.04\bar{V}_1 - 0.04\bar{V}_2$$

$$\bar{I}_2 = 2\bar{I}_1 + \frac{\bar{V}_2}{100} - \bar{I}_1 = \bar{I}_1 + 0.01\bar{V}_2 = 0.04\bar{V}_1 - 0.03\bar{V}_2$$

$$\therefore \bar{y}_{11} = 0.04 \text{ S}, \bar{y}_{12} = -0.04 \text{ S}, \bar{y}_{21} = 0.04 \text{ S}, \bar{y}_{22} = -0.03 \text{ S}$$



## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

12.

$$\therefore \bar{V}_1 = 100(\bar{I}_1 - 0.5\bar{I}_1) = 50\bar{I}_1 \quad \therefore \bar{I}_1 = 0.02 \bar{V}_1$$

$$\bar{V}_2 = 300\bar{I}_2 + 200(\bar{I}_2 + 0.5\bar{I}_1) = 100\bar{I}_1 + 500\bar{I}_2$$

$$\therefore \bar{V}_2 = 2\bar{V}_1 + 500\bar{I}_2, \quad \bar{I}_2 = -0.004\bar{V}_1 + 0.002\bar{V}_2$$

$$\therefore [\bar{y}] = \begin{bmatrix} 0.02 & 0 \\ -0.004 & 0.002 \end{bmatrix} \text{ (S)}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

13.

$$[y] = \begin{bmatrix} 0.1 & -0.0025 \\ -8 & 0.05 \end{bmatrix} \text{ (S)}$$

- (a)  $\bar{I}_1 = 0.1\bar{V}_1 - 0.0025\bar{V}_2$ ,  $\bar{I}_2 = -8\bar{V}_1 + 0.05\bar{V}_2$   
 $1 = 2\bar{I}_1 + \bar{V}_2$ ,  $\bar{V}_2 = -5\bar{I}_2$   
 $\therefore \bar{I}_2 = -0.2\bar{V}_2 = -8\bar{V}_1 + 0.05\bar{V}_2 \therefore 0.25\bar{V}_2 = 8\bar{V}_1$ ,  $\bar{V}_2 / \bar{V}_1 = 32$   
 $\bar{I}_2 = -8\bar{V}_1 + 0.05 \times 32\bar{V}_1$ ,  $\bar{I}_1 = 0.1\bar{V}_1 - 0.0025 \times 32\bar{V}_1 \therefore \bar{I}_2 = -6.4\bar{V}_1$ ,  $\bar{I}_1 = 0.02\bar{V}_1$   
 $\therefore \bar{I}_2 / \bar{I}_1 = \frac{-6.4}{0.02} = -320$ ,  $\bar{V}_1 / \bar{I}_1 = \boxed{50\Omega}$
- (b)  $\bar{V}_1 = -2\bar{I}_1$ ,  $\bar{I}_1 = 0.1\bar{V}_1 - 0.0025\bar{V}_2$ ,  $\bar{I}_2 = -8\bar{V}_1 + 0.05\bar{V}_2$   
 $\therefore \bar{I}_1 = -0.5\bar{V}_1 = 0.1\bar{V}_1 - 0.0025\bar{V}_2 \therefore 0.6\bar{V}_1 = 0.0025\bar{V}_2$   
 $\therefore \bar{V}_1 = \bar{V}_2 / 240$ ,  $\bar{I}_2 = -8 \times \bar{V}_2 / 240 + \frac{1}{20} \bar{V}_2 = \frac{1}{60} \bar{V}_2$   
 $\therefore \frac{\bar{V}_2}{\bar{I}_2} = \boxed{60\Omega}$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

$$14. \quad [\bar{y}] = \begin{bmatrix} 10 & -5 \\ -20 & 2 \end{bmatrix} \text{ (mS)}$$

$$(a) \quad \bar{I}_1 = 0.01\bar{V}_1 - 0.005\bar{V}_2, \quad \bar{I}_2 = -0.02\bar{V}_1 + 0.002\bar{V}_2$$

$$\bar{V}_1' = 100\bar{I}_1 + \bar{V}_1$$

$$\therefore \bar{V}_1 = \bar{V}_1 - 100\bar{I}_1 \therefore \bar{I}_1 = 0.01\bar{V}_1 - \bar{I}_1 - 0.005\bar{V}_2 \therefore \bar{I}_1 = 0.005\bar{V}_1 - 0.0025\bar{V}_2$$

$$\bar{I}_2 = -0.02\bar{V}_1 + 2\bar{I}_1 + 0.002\bar{V}_2 = -0.02\bar{V}_1 + 0.01\bar{V}_1 - 0.005\bar{V}_2 + 0.002\bar{V}_2 = -0.01\bar{V}_1 - 0.003\bar{V}_2$$

$$\therefore [\bar{y}]_{new} = \begin{bmatrix} 0.005 & -0.0025 \\ -0.01 & -0.003 \end{bmatrix} \text{ (S)}$$

$$(b) \quad \bar{V}_2 = 100\bar{I}_2 + \bar{V}_2, \therefore \bar{V}_2 = \bar{V}_2 - 100\bar{I}_2$$

$$\therefore \bar{I}_2 = -0.02\bar{V}_1 + 0.002\bar{V}_2 - 0.2\bar{I}_2$$

$$\therefore 1.2\bar{I}_2 = -0.02\bar{V}_1 + 0.002\bar{V}_2 \therefore \bar{I}_2 = -\frac{1}{60} \bar{V}_1 + \frac{1}{600} \bar{V}_2$$

$$\bar{I}_1 = 0.01\bar{V}_1 - 0.005(\bar{V}_2 - 100\bar{I}_2) = 0.01\bar{V}_1 - 0.005\bar{V}_2 + 0.5\left(-\frac{1}{60} \bar{V}_1 + \frac{1}{600} \bar{V}_2\right)$$

$$\therefore \bar{I}_1 = \left(\frac{1}{100} - \frac{1}{120}\right) \bar{V}_1 - \left(\frac{1}{200} - \frac{1}{1200}\right) \bar{V}_2 = \frac{1}{600} \bar{V}_1 - \frac{1}{240} \bar{V}_2$$

$$\therefore [\bar{y}]_{new} = \begin{bmatrix} 1/600 & -1/240 \\ -1/60 & 1/600 \end{bmatrix} \text{ (S)}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

15.

	$\bar{V}_{s1}$	$\bar{V}_{s2}$	$\bar{I}_1$	$\bar{I}_2$
Exp #1	100 V	50 V	5 A	-32.5 A
Exp #2	50	110	-20	-5
Exp #3	20	0	4	-8
Exp #4	-8.333	-22.22	5	0
Exp #5	-58.33	-55.56	5	15

$$\bar{I}_1 = \bar{y}_{11}\bar{V}_1 + \bar{y}_{12}\bar{V}_2$$

$$\bar{I}_2 = \bar{y}_{21}\bar{V}_1 + \bar{y}_{22}\bar{V}_2$$

Use 1st 2 rows to find y's

$$\therefore 5 = 100\bar{y}_{11} + 50\bar{y}_{12}, -32.5 = 100\bar{y}_{21} + 50\bar{y}_{22}$$

$$-20 = 50\bar{y}_{11} + 100\bar{y}_{12}, -5 = 50\bar{y}_{21} + 100\bar{y}_{22} \rightarrow \therefore -10 = 100\bar{y}_{21} + 200\bar{y}_{22}$$

$$\therefore -40 = 100\bar{y}_{11} + 200\bar{y}_{12} \text{ Subtracting, } 150\bar{y}_{12} = -45 \therefore \bar{y}_{12} = -0.3 \text{ S}$$

$$\therefore 5 = 100\bar{y}_{11} - 15 \therefore \bar{y}_{11} = 0.2 \text{ S} \text{ Subtracting } 22.5 = 150\bar{y}_{22}$$

$$\therefore \bar{y}_{22} = 0.15 \text{ S} \therefore -32.5 = 100\bar{y}_{21} + 7.5 \therefore \bar{y}_{21} = -0.4 \text{ S} \therefore [\bar{y}] = \begin{bmatrix} 0.2 & -0.3 \\ -0.4 & 0.15 \end{bmatrix} (\text{S})$$

$$\text{Completing row 3: } \bar{I}_1 = 0.2 \times 20 = 4 \text{ A}, \bar{I}_2 = -0.4 \times 20 = -8 \text{ A}$$

$$\text{Completing row 4: } 5 = 0.2\bar{V}_{s1} - 0.3\bar{V}_{s2}, 0 = -0.4\bar{V}_{s1} + 0.15\bar{V}_{s2} \therefore \bar{V}_{s2} = \frac{8}{3} \bar{V}_{s1}$$

$$\therefore 5 = 0.2\bar{V}_{s1} - 0.8\bar{V}_{s1} = -0.6\bar{V}_{s1} \therefore \bar{V}_{s1} = -\frac{50}{6} = -8.333 \text{ V}, \bar{V}_{s2} = -22.22 \text{ V}$$

$$\text{Completing row 5: } 5 = 0.2\bar{V}_{s1} - 0.3\bar{V}_{s2}, 15 = -0.4\bar{V}_{s1} + 0.15\bar{V}_{s2}$$

$$\therefore \bar{V}_{s1} = \frac{\begin{vmatrix} 5 & -0.3 \\ 15 & 0.15 \end{vmatrix}}{\begin{vmatrix} 0.2 & -0.3 \\ -0.4 & 0.15 \end{vmatrix}} = \frac{0.75 + 4.5}{0.03 - 0.12} = \frac{5.25}{-0.09} = -58.33 \text{ V}, \bar{V}_{s2} = \frac{\begin{vmatrix} 0.2 & 5 \\ -0.4 & 15 \end{vmatrix}}{-0.09} = -55.56 \text{ V}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

16.

$$\Delta_1: 1+6+3=10\Omega \rightarrow \frac{6 \times 1}{10} = 0.6, \frac{6 \times 3}{10} = 1.8, \frac{3 \times 1}{10} = 0.3$$

$$\Delta_2: 5+1+4=10\Omega \rightarrow \frac{5 \times 1}{10} = 0.5, \frac{1 \times 4}{10} = 0.4, \frac{5 \times 4}{10} = 2$$

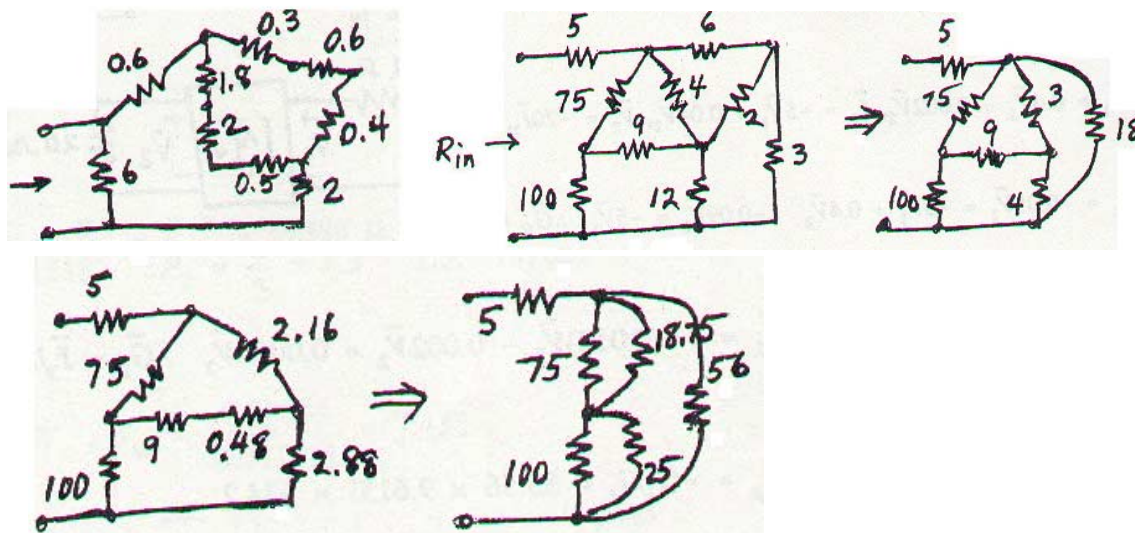
$$1.8+2+0.5=4.3\Omega, 0.3+0.6+0.4=1.3\Omega$$

$$1.3 \parallel 4.3 = 0.99821\Omega, 0.9982+0.6+2=3.598\Omega$$

$$3.598 \parallel 6 = 2.249\Omega$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

17.



$$6 \times 2 + 2 \times 3 + 3 \times 6 = 36 \Omega^2$$

$$36 / 6 = 6, 36 / 2 = 18, 36 / 3 = 12$$

$$12 \parallel 4 = 3, 6 \parallel 12 = 4$$

$$4 + 3 + 18 = 25 \Omega$$

$$3 \times 18 / 25 = 2.16, 4 \times 18 / 25 = 2.88, \frac{4 \times 3}{25} = 0.48$$

$$9.48 \times 2.16 + 9.48 \times 2.88 + 2.88 \times 2.16 = 54 \Omega^2$$

$$\frac{54}{2.88} = 18.75, \frac{54}{2.16} = 25, \frac{54}{9.48} = 5.6962, 75 \parallel 18.75 = 15, 100 \parallel 25 = 20$$

$$(15 + 20) \parallel 5.696 = 4.899 \therefore R_{in} = 5 + 4.899 = \boxed{9.899 \Omega}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

18.

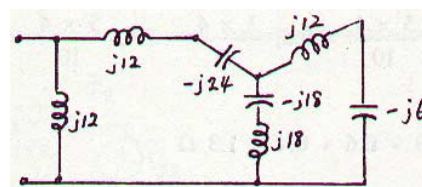
$$\Delta: -j6 + j4 + j3 = j1$$

$$\frac{24}{j1} = -j24, \frac{-12}{j1} = j12, \frac{18}{j1} = -j18, j18 - j18 = 0 \text{ (S.C)}$$

$\therefore$  ignore  $j12, -j6$

$$-j24 + j12 = -j12$$

$$-j12 \parallel j12 = \infty \therefore \boxed{\bar{Z}_m = \infty}$$



## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

$$19. \quad [\bar{y}] = \begin{bmatrix} 0.4 & -0.002 \\ -5 & 0.04 \end{bmatrix} \text{ (S)}$$

$$(a) \quad \bar{I}_1 = 0.4\bar{V}_1 - 0.002\bar{V}_2, \bar{I}_2 = -5\bar{V}_1 + 0.04\bar{V}_2, \bar{V}_2 = -20\bar{I}_2, \bar{V}_s = \bar{V}_1 + 2\bar{I}_1$$

$$\bar{I}_2 = -0.05\bar{V}_2 = -5\bar{V}_1 + 0.4\bar{V}_2 \therefore -0.09\bar{V}_2 = -5\bar{V}_1 \therefore \bar{G}_v = \bar{V}_2 / \bar{V}_1 = \frac{500}{9} = 55.56$$

$$(b) \quad \bar{I}_1 = 0.4(0.018)\bar{V}_2 - 0.002\bar{V}_2 = 0.0052\bar{V}_2 \therefore \bar{G}_I = \bar{I}_2 / \bar{I}_1 = \frac{-0.05\bar{V}_2}{0.0052\bar{V}_2} = -9.615^+$$

$$(c) \quad G_p = -G_v G_I = 55.56 \times 9.615^+ = 534.2$$

$$(d) \quad \bar{I}_1 = 0.0052\bar{V}_2 = 0.0052 \times 55.56\bar{V}_1 \therefore \bar{Z}_{in} = \bar{V}_1 / \bar{I}_1 = \frac{1}{0.0052 \times 55.56} = 3.462 \Omega$$

$$(e) \quad \bar{V}_1 = -2\bar{I}_1, \bar{V}_s = 0 \therefore \bar{I}_1 = -0.5\bar{V}_1 = 0.4\bar{V}_1 - 0.002\bar{V}_2 \therefore \bar{V}_1 = \frac{0.002}{0.9} \bar{V}_2$$

$$\bar{I}_2 = -5 \left( \frac{0.002}{0.9} \right) \bar{V}_2 + 0.04\bar{V}_2 = 0.02889\bar{V}_2 \therefore \bar{Z}_{out} = \bar{V}_2 / \bar{I}_2 = 34.62 \Omega$$



## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

$$20. \quad [\bar{y}] = \begin{bmatrix} 0.1 & -0.05 \\ -0.5 & 0.2 \end{bmatrix} \text{ (S)}$$

$$\begin{aligned} \text{(a)} \quad \bar{I}_1 &= 0.1\bar{V}_1 - 0.05\bar{V}_2 \\ \bar{I}_2 &= -0.5\bar{V}_1 + 0.2\bar{V}_2, \quad 1 = 10\bar{I}_1 + \bar{V}_1, \quad \bar{I}_2 = -0.2\bar{V}_2 \\ \therefore -0.2\bar{V}_2 &= -0.5\bar{V}_1 + 0.2\bar{V}_2 \quad \therefore \bar{G}_V = \bar{V}_2 / \bar{V}_1 = \boxed{1.25} \end{aligned}$$

$$\text{(b)} \quad \bar{G}_I = \bar{I}_2 / \bar{I}_1 = \frac{(-0.5 + 0.2 \times 1.25)\bar{V}_1}{(0.1 - 0.005 \times 1.25)\bar{V}_1} = \boxed{-6.667}$$

$$\text{(c)} \quad G_P = 1.25 \times 6.667 = \boxed{8.333}$$

$$\text{(d)} \quad \bar{I}_1 = (0.1 - 0.05 \times 1.25)\bar{V}_1 \quad \therefore \bar{Z}_{in} = \bar{V}_1 / \bar{I}_1 = \boxed{26.67 \, \Omega}$$

$$\begin{aligned} \text{(e)} \quad \bar{V}_s &= 0, \quad \bar{V}_1 = -10\bar{I}_1 \quad \therefore \bar{I}_1 = -0.1\bar{V}_1 = 0.1\bar{V}_1 - 0.05\bar{V}_2 \\ \therefore \bar{V}_1 &= 0.25\bar{V}_2, \quad \therefore \bar{I}_2 = -0.05(0.25\bar{V}_2) + 0.2\bar{V}_2 = 0.075\bar{V}_2 \quad \therefore \bar{Z}_{out} = \bar{V}_2 / \bar{I}_2 = 13.333 \, \Omega \end{aligned}$$

$$\text{(f)} \quad \bar{G}_{V,rev} = \bar{V}_1 / \bar{V}_2 = \boxed{0.25}$$

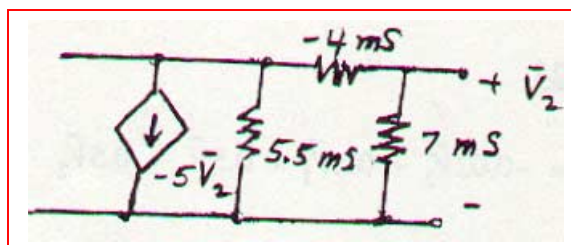
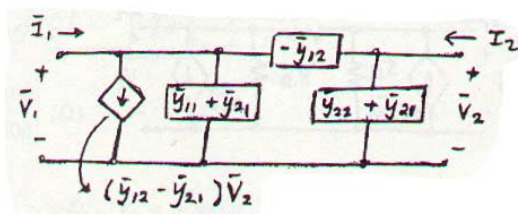
$$\begin{aligned} \text{(g)} \quad \text{With 2 port: } 1 &= 10\bar{I}_1 + 26.67\bar{I}_1 \\ \therefore 1 &= 36.67\bar{I}_1, \quad \bar{I}_1 = 1/36.67 \quad \therefore \bar{I}_2 = \frac{-6.667}{36.67} = -0.15182 \quad \therefore P_L = \frac{1}{2} \times I_2^2 \times 5 = 2.5(0.15182)^2 = 0.08264 \text{ W} \end{aligned}$$

$$\text{Without 2 port: } P_L = \frac{1}{2} \left( \frac{1}{15} \right)^2 \times 5 = 0.011111 \text{ W} \quad \therefore G_{ins} = \frac{0.08264}{0.011111} = \boxed{7.438}$$

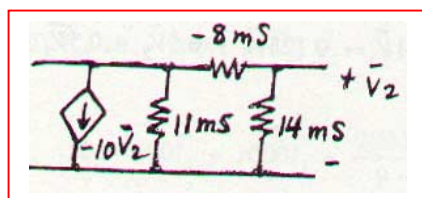
## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

21.

(a)



(b) 2 in ||:



## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

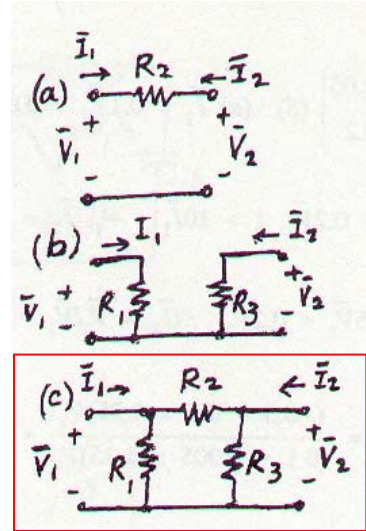
22.

$$(a) \quad \bar{I}_1 = \frac{\bar{V}_1 - \bar{V}_2}{R_2}, \quad \bar{I}_2 = \frac{\bar{V}_2 - \bar{V}_1}{R_2} \quad [\bar{y}]_a = \begin{bmatrix} 1/R_2 & -1/R_2 \\ -1/R_2 & 1/R_2 \end{bmatrix}$$

$$(b) \quad \bar{I}_1 = \bar{V}_1 / R_1, \quad \bar{I}_2 = \bar{V}_2 / R_3 \quad \therefore [\bar{y}]_b = \begin{bmatrix} 1/R_1 & 0 \\ 0 & 1/R_3 \end{bmatrix}$$

$$(c) \quad \bar{I}_1 = \frac{\bar{V}_1}{R_1} + \frac{\bar{V}_1 - \bar{V}_2}{R_2} \quad \therefore [\bar{y}] = \begin{bmatrix} 1/R_1 + 1/R_2 & -1/R_2 \\ -1/R_2 & 1/R_3 + 1/R_2 \end{bmatrix}$$

$$\bar{I}_2 = \frac{\bar{V}_2}{R_3} + \frac{\bar{V}_2 - \bar{V}_1}{R_2}, \quad [\bar{y}]_a + [\bar{y}]_b = \begin{bmatrix} 1/R_1 + 1/R_2 & -1/R_2 \\ -1/R_2 & 1/R_3 + 1/R_2 \end{bmatrix}$$



## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

23.

$$\bar{V}_1 = 8\bar{I}_1 + 0.1\bar{V}_2 \quad \therefore \bar{V}_2 = 10\bar{V}_1 - 80\bar{I}_1$$

$$\bar{I}_2 = \bar{V}_2 / 12 + 0.05\bar{V}_1 \quad \therefore \bar{I}_2 = \frac{1}{12}(10\bar{V}_1 - 80\bar{I}_1) + 0.05\bar{V}_1$$

$$\therefore \bar{I}_2 = \left( \frac{5}{6} + \frac{1}{20} \right) \bar{V}_1 - \frac{20}{3} \bar{I}_1 = \frac{53}{60} \bar{V}_1 - \frac{20}{3} \bar{I}_1$$

$$\therefore \bar{V}_1 = \frac{60}{53} \left( \frac{20}{3} \bar{I}_1 + \bar{I}_2 \right) = \frac{400}{53} \bar{I}_1 + \frac{60}{53} \bar{I}_2 \quad \therefore \bar{V}_2 = \frac{4000}{53} \bar{I}_1 + \frac{600}{53} \bar{I}_2 - 80\bar{I}_1$$

$$\therefore \bar{V}_2 = -\frac{240}{53} \bar{I}_1 + \frac{600}{53} \bar{I}_2 \quad \therefore [\bar{z}] = \begin{bmatrix} 7.547 & 1.1321 \\ -4.528 & 11.321 \end{bmatrix} (\Omega)$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

24.

(a)  $\bar{I}_1 = -0.02\bar{V}_2 + 0.2\bar{V}_1 + 0.5\bar{V}_1 - 0.5\bar{V}_2$   
 $\therefore \bar{I}_1 = 0.7\bar{V}_1 - 0.52\bar{V}_2 \quad \bar{I}_2 = 0.1\bar{V}_1 + 0.125\bar{V}_2 + 0.5\bar{V}_2 - 0.5\bar{V}_1$   
 $\therefore \bar{I}_2 = -0.4\bar{V}_1 + 0.625\bar{V}_2$

$$\therefore \bar{V}_1 = \frac{\begin{vmatrix} \bar{I}_1 & -0.52 \\ \bar{I}_2 & 0.625 \end{vmatrix}}{\begin{vmatrix} 0.7 & -0.52 \\ -0.4 & 0.625 \end{vmatrix}} = \frac{0.625\bar{I}_1 + 0.52\bar{I}_2}{0.2295} = 2.723\bar{I}_1 + 2.266\bar{I}_2, \quad \bar{V}_2 = \frac{\begin{vmatrix} 0.7 & \bar{I}_1 \\ -0.4 & \bar{I}_2 \end{vmatrix}}{0.2295}$$

$$\therefore \bar{V}_2 = \frac{0.4\bar{I}_1 + 0.7\bar{I}_2}{0.2295} = 1.7429\bar{I}_1 + 3.050\bar{I}_2 \quad \therefore [\bar{z}] = \begin{bmatrix} 2.723 & 2.266 \\ 1.7429 & 3.050 \end{bmatrix} (\Omega)$$

(b)  $\bar{I}_1 = \bar{I}_2 = 1 \text{ A} \quad \therefore \frac{\bar{V}_2}{\bar{V}_1} = \frac{1.7429 + 3.050}{2.723 + 2.266} = 0.9607$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

$$25. \quad [\bar{z}] = \begin{bmatrix} 4 & 1.5 \\ 10 & 3 \end{bmatrix} (\Omega), R_s = 5\Omega, R_L = 2\Omega$$

$$(a) \quad \bar{V}_1 = 4\bar{I}_1 + 1.5\bar{I}_2, \bar{V}_2 = 10\bar{I}_1 + 3\bar{I}_2, \bar{V}_2 = -2\bar{I}_2 = 10\bar{I}_1 + 3\bar{I}_2 \therefore \bar{G}_1 = \bar{I}_2 / \bar{I}_1 = \boxed{-2}$$

$$(b) \quad \bar{G}_v = \bar{V}_2 / \bar{V}_1 = \frac{10\bar{I}_1 - 6\bar{I}_1}{4\bar{I}_1 - 3\bar{I}_1} = \boxed{4}$$

$$(c) \quad G_p = -\bar{G}_v \bar{G}_I = \boxed{8}$$

$$(d) \quad \bar{V}_1 = 4\bar{I}_1 - 3\bar{I}_1 = \bar{I}_1 \therefore \bar{Z}_{in} = \frac{\bar{V}_1}{\bar{I}_1} = \boxed{1 \Omega}$$

$$(e) \quad \bar{V}_1 = -5\bar{I}_1 = 4\bar{I}_1 + 1.5\bar{I}_2 \therefore \bar{I}_1 = -\frac{1}{6} \bar{I}_2 \therefore \bar{V}_2 = -\frac{10}{6} \bar{I}_2 + 3\bar{I}_2 = \frac{8}{6} \bar{I}_2 \therefore \bar{Z}_{out} = \boxed{1.3333 \Omega}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

$$26. \quad [\bar{z}] = \begin{bmatrix} 1000 & 100 \\ -2000 & 400 \end{bmatrix} (\Omega)$$

$$\begin{aligned} (a) \quad \bar{V}_1 &= 1000\bar{I}_1 + 100\bar{I}_2, \quad \bar{V}_2 = -2000\bar{I}_1 + 400\bar{I}_2, \quad 10 = 200\bar{I}_1 + \bar{V}_1, \quad \bar{V}_2 = -500\bar{I}_2 \\ \therefore -500\bar{I}_2 &= -2000\bar{I}_1 + 400\bar{I}_2, \quad \bar{I}_2 = \frac{20}{9} \bar{I}_1; \quad \therefore 10 = 200\bar{I}_1 + 1000\bar{I}_1 + \frac{2000}{9} \bar{I}_1 \\ \therefore \bar{I}_1 &= 7.031 \text{ mA}, \quad \therefore \bar{I}_2 = \frac{20}{9} \bar{I}_1 = 15.625 \text{ mA} \quad \therefore P_{200} = 7.031^2 \times 200 \times 10^{-6} = \boxed{9.888 \text{ mW}} \end{aligned}$$

$$(b) \quad P_{500} = 15.625^2 \times 500 \times 10^{-6} = \boxed{122.07 \text{ mW}}$$

$$\begin{aligned} (c) \quad P_S &= 10\bar{I}_1 = 70.31 \text{ mW (gen)} \quad \therefore P_{2port} = P_S - P_{200} - P_{500} = 70.31 - 9.89 - 122.07 \quad \therefore \\ &\quad \therefore P_{2port} = \boxed{-61.65 \text{ mW}} \end{aligned}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

27.

$$\omega = 10^8, \quad \bar{I}_1 = 10^{-5} \bar{V}_1 + j5 \times 10^{-4} \bar{V}_1 + j10^{-4} (\bar{V}_1 - \bar{V}_2)$$

$$\therefore \bar{I}_1 = (10^{-5} + j6 \times 10^{-4}) \bar{V}_1 - j10^{-4} \bar{V}_2$$

$$\bar{I}_2 = 10^{-4} \bar{V}_2 + 0.01 \bar{V}_1 + j10^{-4} (\bar{V}_2 - \bar{V}_1)$$

$$\therefore \bar{I}_2 = (0.01 - j10^{-4}) \bar{V}_1 + (10^{-4} + j10^{-4}) \bar{V}_2$$

$$\therefore \bar{V}_1 = \frac{\begin{vmatrix} \bar{I}_1 & -j10^{-4} \\ \bar{I}_2 & 10^{-4} + j10^{-4} \end{vmatrix}}{\begin{vmatrix} 10^{-5} + j6 \times 10^{-4} & -j10^{-4} \\ 10^{-2} - j10^{-4} & 10^{-4} + 10^{-4} \end{vmatrix}} = \frac{(10^{-4} + j10^{-4}) \bar{I}_1 + j10^{-4} \bar{I}_2}{1.0621 \times 10^{-6} \angle 92.640^\circ} \therefore \begin{cases} \bar{z}_{11} = 133.15^\circ \angle -47.64^\circ \Omega \\ \bar{z}_{12} = 94.15^\circ \angle -2.642^\circ \Omega \end{cases}$$

$$\bar{V}_2 = \frac{\begin{vmatrix} 10^{-5} + j6 \times 10^{-4} & \bar{I}_1 \\ 10^{-2} - j10^{-4} & \bar{I}_2 \end{vmatrix}}{1.0621 \times 10^{-6} \angle 92.64^\circ} \therefore \begin{cases} \bar{z}_{21} = 9416 \angle 86.78^\circ \Omega \\ \bar{z}_{22} = 565.0 \angle -3.60^\circ \Omega \end{cases}$$



## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

28.

$$[\bar{z}] = \begin{bmatrix} 20 & 2 \\ 40 & 10 \end{bmatrix} (\Omega), \bar{V}_s = 100 \angle 0^\circ \text{ V}, R_s = 5 \Omega, R_L = 25 \Omega$$

$$100 = 5\bar{I}_1 + \bar{V}_1, \bar{V}_1 = 20\bar{I}_1 + 2\bar{I}_2 \therefore 100 = 25\bar{I}_1 + 2\bar{I}_2$$

$$\bar{V}_2 = 40\bar{I}_1 + 10\bar{I}_2 \therefore \bar{I}_1 = \frac{1}{40} \bar{V}_2 - \frac{1}{4} \bar{I}_2 \therefore 100 = \frac{25}{40} \bar{V}_2 - \frac{25}{4} \bar{I}_2 + 2\bar{I}_2$$

$$\therefore 100 = \frac{5}{8} \bar{V}_2 - \frac{17}{4} \bar{I}_2 \therefore \bar{V}_2 = 160 + \frac{8}{5} \times \frac{17}{4} \bar{I}_2 = 160 + 6.8\bar{I}_2$$

$$\therefore \bar{V}_{th} = 160 \text{ V}, R_{th} = 6.8 \Omega$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

$$29. \quad [\bar{h}] = \begin{bmatrix} 9\Omega & -2 \\ 20 & 0.2 \text{ S} \end{bmatrix}$$

$$(a) \quad \bar{V}_1 = 9\bar{I}_1 - 2\bar{V}_2, \quad \bar{I}_2 = 20\bar{I}_1 + 0.2\bar{V}_2, \quad \bar{V}_1' = 1\bar{I}_1 + \bar{V}_1 \quad \text{Eliminate } \bar{V}_1$$

$$\therefore \bar{V}_1 = \bar{V}_1' - \bar{I}_1 \quad \therefore \bar{V}_1' - \bar{I}_1 = 9\bar{I}_1 - 2\bar{V}_2, \quad \bar{V}_1' = 10\bar{I}_1 - 2\bar{V}_2 \quad \therefore [\bar{h}]_{new} = \begin{bmatrix} 10\Omega & -2 \\ 20 & 0.2 \text{ S} \end{bmatrix}$$

$$(b) \quad \bar{V}_1 = 9\bar{I}_1 - 2\bar{V}_2, \quad \bar{I}_2 = 20\bar{I}_1 + 0.2\bar{V}_2, \quad \bar{V}_2' = 1\bar{I}_2 + \bar{V}_2$$

$$\text{Eliminate } \bar{V}_2 \quad \therefore \bar{V}_2 = \bar{V}_2' - \bar{I}_2$$

$$\bar{V}_1 = 9\bar{I}_1 - 2\bar{V}_2 + 2\bar{I}_2, \quad \bar{I}_2 = 20\bar{I}_1 + 0.2\bar{V}_2' - 0.2\bar{I}_2 \quad \therefore 1.2\bar{I}_2 = 20\bar{I}_1 + 0.2\bar{V}_2'$$

$$\therefore \bar{I}_2 = 16.667\bar{I}_1 + 0.16667\bar{V}_2' \quad \bar{V}_1 = 9\bar{I}_1 - 2\bar{V}_2' + 2(16.667\bar{I}_1 + 0.16667\bar{V}_2')$$

$$\therefore \bar{V}_1 = 42.33\bar{I}_1 - 1.6667\bar{V}_2' \quad \therefore [h]_{new} = \begin{bmatrix} 42.33\Omega & -1.6667 \\ 16.667 & 0.16667 \text{ S} \end{bmatrix}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

30.

$$R_s = 100\Omega, R_L = 500\Omega \quad [\bar{h}] = \begin{bmatrix} 100\Omega & 0.01 \\ 20 & 1 \text{ mS} \end{bmatrix}$$

$$\bar{Z}_{in}: \bar{V}_1 = 100\bar{I}_1 + 0.01\bar{V}_2, \bar{I}_2 = 20\bar{I}_1 + 0.001\bar{V}_2 = 20\bar{I}_1 - 0.5\bar{I}_2 \quad \therefore 1.5\bar{I}_2 = 20\bar{I}_1$$

$$\therefore \bar{V}_1 = 100\bar{I}_1 + 0.01(-500)\frac{20}{1.5} \bar{I}_1 = 33.33\bar{I}_1 \quad \therefore \boxed{\bar{Z}_{in} = 33.33\Omega}$$

$$\bar{Z}_{out}: \bar{V}_1 = -100\bar{I}_1 = 100\bar{I}_1 + 0.01\bar{V}_2 \quad \therefore \bar{I}_1 = \frac{0.01}{-200} \bar{V}_2$$

$$\bar{I}_2 = 20\left(\frac{0.01}{-200} \bar{V}_2\right) + 0.001 \bar{I}_2 = 0 \quad \therefore \boxed{\bar{Z}_{out} = \infty}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

31.

(a)  $\bar{h}_{12} = \bar{V}_1 / \bar{V}_2 \big|_{I_1=0}$  Let  $\bar{V}_2 = 1 \text{ V}$   
 $\therefore \bar{I}_{10} \downarrow = 0.1 \text{ A}, \bar{I}_1 = 0 \therefore \bar{I}_{4\Omega} \leftarrow = 0.2 \bar{I}_2$   
 $\therefore 0.1 = \bar{I}_2 - 0.2 \bar{I}_2 = 0.8 \bar{I}_2, \bar{I}_2 = 0.125 \text{ A}$   
 $\therefore \bar{V}_1 = 0.3 - 4(0.2)(0.125) + 1 = 1.2 \text{ V} \therefore \bar{h}_{12} = 1.2$

(b)  $\bar{z}_{12} = \frac{\bar{V}_1}{\bar{I}_2} \bigg|_{I_1=0}$  From above,  $\bar{z}_{12} = \frac{1.2}{0.125} = 9.6 \Omega$

(c)  $\bar{y}_{12} = \bar{I}_1 / \bar{V}_2 \big|_{V_1=0}$  SC input Let  $\bar{V}_2 = 1 \text{ V}$   
 $\bar{I}_2 = 0.1 + \frac{1.3}{4} = 0.425 \text{ A}, \bar{I}_1 = 0.2(0.425) - \frac{1.3}{4}$   
 $\therefore \bar{I}_1 = -0.24 \text{ A} \therefore \bar{y}_{12} = 0.24 \text{ S}$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

$$32. \quad [\bar{h}] = \begin{bmatrix} 1000\Omega & -1 \\ 4 & 500\mu S \end{bmatrix}$$

$$\begin{aligned} (a) \quad 100 &= 200 \bar{I}_1 + 1000 \bar{I}_1 - \bar{V}_2 = 1200 \bar{I}_1 - \bar{V}_2 \\ \bar{I}_2 &= 4 \bar{I}_1 + 5 \times 10^{-4} \bar{V}_2 = -10^{-3} \bar{V}_2 \quad \therefore 4 \bar{I}_1 = -1.5 \times 10^{-3} \bar{V}_2 \\ \therefore \bar{V}_2 &= -\frac{4000}{1.5} \bar{I}_1 \quad \therefore 100 = 1200 \bar{I}_1 + \frac{4000}{1.5} \bar{I}_1 \quad \therefore \bar{I}_1 = 25.86 \text{ mA} \\ \therefore P_{200} &= 25.86^2 \times 10^{-6} \times 200 = \boxed{133.77 \text{ mW}} \end{aligned}$$

$$(b) \quad \bar{V}_2 = \frac{4000}{1.5} \times 25.86 \times 10^{-3} = 68.97 \text{ V} \quad \therefore P_{1K} = \frac{68.97^2}{1000} = \boxed{4.756 \text{ W}}$$

$$\begin{aligned} (c) \quad P_s &= 100 \times 25.86 \times 10^{-3} = 2.586 \text{ W (gen)} \\ \therefore P_{2port} &= 2.586 - 0.1338 - 4.756 = \boxed{-2.304 \text{ W}} \end{aligned}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

33.

$$\begin{aligned}
 \text{(a)} \quad \bar{V}_1 &= 1000(\bar{I}_1 + 10^{-5} \bar{V}_2) = 1000\bar{I}_1 + 0.01\bar{V}_2 \\
 \bar{V}_2 &= 10^4 \bar{I}_2 - 100\bar{V}_1 \quad \therefore \bar{I}_2 = 10^{-4}(100\bar{V}_1 + \bar{V}_2) \\
 \therefore \bar{I}_2 &= 10^{-2}(1000\bar{I}_1 + 0.01\bar{V}_2) + 10^{-4}\bar{V}_2 \\
 \therefore \bar{I}_2 &= 10\bar{I}_1 + 2 \times 10^{-4} \bar{V}_2 \quad \therefore [\bar{h}] = \begin{bmatrix} 1000\Omega & 0.01 \\ 10 & 2 \times 10^{-4} \text{S} \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \bar{V}_1 &= -200\bar{I}_1 = 1000\bar{I}_1 + 0.01\bar{V}_2 \\
 \therefore \bar{I}_1 &= \frac{-1}{12,000} \bar{V}_2 \quad \therefore \bar{I}_2 = 10\bar{I}_1 + 2 \times 10^{-4} \bar{V}_2 = \frac{-1}{12,000} \bar{V}_2 + \frac{1}{5000} \bar{V}_2 + 116.67 \times 10^{-6} \bar{V}_2 \\
 \therefore \bar{Z}_{out} &= \bar{V}_2 / \bar{I}_2 = 10^6 / 116.67 = 8.571 \text{ k}\Omega
 \end{aligned}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

34.

$$(a) \quad \begin{aligned} \bar{V}_1 &= \bar{I}_1 R + \bar{V}_2 \quad \therefore \bar{I}_1 = \frac{\bar{V}_1}{R} - \frac{\bar{V}_2}{R} \\ \bar{I}_1 &= -\bar{I}_2 \quad \bar{I}_2 = -\frac{\bar{V}_1}{R} + \frac{\bar{V}_2}{R} \end{aligned} \quad [\bar{y}] = \begin{bmatrix} 1/R & -1/R \\ -1/R & 1/R \end{bmatrix}$$

$[z]$  parameters are all  $\infty$

$$\begin{aligned} \bar{V}_1 &= \bar{I}_1 R + \bar{V}_2 \\ \bar{I}_2 &= -\bar{I}_1 \end{aligned} \quad \therefore [\bar{h}] = \begin{bmatrix} R & 1 \\ -1 & 0 \end{bmatrix}$$

(b)  $[\bar{y}]$  parameters are  $\infty$

$$\bar{V}_1 = \bar{V}_2 \quad \bar{V}_1 = R\bar{I}_1 + R\bar{I}_2 \quad \therefore [\bar{z}] = \begin{bmatrix} R & R \\ R & R \end{bmatrix}$$

$$\bar{I}_1 = \frac{\bar{V}_1}{R} - \bar{I}_2 \quad \bar{V}_2 = R\bar{I}_1 + R\bar{I}_2$$

$$\bar{V}_1 = \bar{V}_2$$

$$\bar{I}_2 = -\bar{I}_1 + \frac{\bar{V}_2}{R} \quad \therefore [\bar{h}] = \begin{bmatrix} 0 & 1 \\ -1 & 1/R \end{bmatrix}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

$$35. \quad [\bar{y}] = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}, \quad [\bar{b}] = \begin{bmatrix} 4 & 6 \\ -1 & 5 \end{bmatrix}, \quad [\bar{c}] = \begin{bmatrix} 3 & 2 & 4 & -1 \\ -2 & 3 & 5 & 0 \end{bmatrix}, \quad [\bar{d}] = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \\ -2 & -3 & 1 \\ 4 & -4 & 2 \end{bmatrix}$$

$$(a) \quad [\bar{y}][\bar{b}] = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & -4 \\ 8 & 38 \end{bmatrix}$$

$$(b) \quad [\bar{b}][\bar{y}] = \begin{bmatrix} 4 & 6 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 22 & 16 \\ 14 & 22 \end{bmatrix}$$

$$(c) \quad [\bar{b}][\bar{c}] = \begin{bmatrix} 4 & 6 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 3 & 2 & 4 & -1 \\ -2 & 3 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 26 & 46 & -4 \\ -13 & 13 & 21 & 1 \end{bmatrix}$$

$$(d) \quad [\bar{c}][\bar{d}] = \begin{bmatrix} 3 & 2 & 4 & -1 \\ -2 & 3 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \\ -2 & -3 & 1 \\ 4 & -4 & 2 \end{bmatrix} = \begin{bmatrix} -3 & -2 & 9 \\ -3 & -19 & 22 \end{bmatrix}$$

$$(e) \quad [\bar{y}][\bar{b}][\bar{c}][\bar{d}] = \begin{bmatrix} 6 & -4 \\ 8 & 38 \end{bmatrix} \begin{bmatrix} -3 & -2 & 9 \\ -3 & -19 & 22 \end{bmatrix} = \begin{bmatrix} -6 & 64 & -34 \\ -138 & -738 & -908 \end{bmatrix}$$



## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

36.

(a)  $\bar{V}_1 = \bar{t}_{11} \bar{V}_2 - \bar{t}_{12} \bar{I}_2$ ,  $\bar{I}_1 = \bar{t}_{21} \bar{V}_2 - \bar{t}_{22} \bar{I}_2$

$$\bar{V}_1 = 10\bar{I}_1 + \bar{V}_2 - 1.5\bar{V}_1, \bar{I}_2 = \frac{\bar{V}_2}{20} + \frac{\bar{V}_2 - 1.5\bar{V}_1}{25} + \frac{\bar{V}_2 - 1.5\bar{V}_1 - \bar{V}_1}{10}$$

$$\therefore \bar{I}_2 = 0.19\bar{V}_2 - 0.31\bar{V}_1, \bar{V}_1 = \frac{0.19}{0.31} \bar{V}_2 - \frac{1}{0.31} \bar{I}_2$$

$$\therefore \bar{V}_1 = 0.6129\bar{V}_2 - 3.226 \bar{I}_2$$

Then,  $10\bar{I}_1 = \bar{V}_1 - (\bar{V}_2 - 1.5\bar{V}_1) = 2.5(0.6129\bar{V}_2 - 3.226\bar{I}_2) - \bar{V}_2$

$$\therefore \bar{I}_1 = 0.05323\bar{V}_2 - 0.8065 \bar{I}_2 \quad \therefore [\bar{t}] = \begin{bmatrix} 0.6129 & 3.226\Omega \\ 0.05323S & 0.8065^- \end{bmatrix}$$

(b) Let  $R_s = 15 \Omega$

$$\therefore \bar{V}_1 = 0.06129\bar{V}_2 - 3.226\bar{I}_2, \bar{I}_1 = 0.05323\bar{V}_2 - 0.8065\bar{I}_2, \bar{V}_1 = -15\bar{I}_1$$

$$\therefore -15\bar{I}_1 = -15(0.05323\bar{V}_2 - 0.8065\bar{I}_2) = 0.6129\bar{V}_2 - 3.226\bar{I}_2$$

$$\therefore 1.4114\bar{V}_2 = 15.324\bar{I}_2 \quad \therefore \bar{Z}_{out} = \bar{V}_2 / \bar{I}_2 = 10.857\Omega$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

37.

$$\bar{V}_1 = 5\bar{I}_1 - 0.3\bar{V}_1 + \bar{V}_2 \quad \therefore 1.3\bar{V}_1 = 5\bar{I}_1 + \bar{V}_2$$

$$\bar{I}_1 = 0.1\bar{V}_2 + \bar{V}_2/4 - \bar{I}_2 \quad \therefore \bar{I}_1 = 0.35\bar{V}_2 - \bar{I}_2$$

$$\therefore 1.3\bar{V}_1 = 5(0.35\bar{V}_2 - \bar{I}_2) + \bar{V}_2 = 2.75\bar{V}_2 - 5\bar{I}_2$$

$$\therefore \bar{V}_1 = 2.115\bar{V}_2 - 3.846\bar{I}_2 \quad \therefore [\bar{t}] = \begin{bmatrix} 2.115\Omega & 3.846\Omega \\ 0.35\text{ S} & 1 \end{bmatrix}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

38.

$$(a) \quad \bar{V}_1 = 2\bar{I}_1 + \bar{V}_2 \quad \therefore \bar{I}_1 = 0.2\bar{V}_2 - \bar{I}_2$$

$$\bar{I}_2 = 0.2\bar{V}_2 - \bar{I}_1 \quad \therefore \bar{V}_1 = 1.4\bar{V}_2 - 2\bar{I}_2$$

$$\bar{V}_1 = 3\bar{I}_1 + \bar{V}_2 \quad \therefore \bar{I}_1 = \frac{1}{6} \bar{V}_2 - \bar{I}_2 \quad \therefore [\bar{t}]_A = \begin{bmatrix} 1.4 & 2\Omega \\ 0.2 \text{ S} & 1 \end{bmatrix}$$

$$\bar{I}_2 = \frac{1}{6} \bar{V}_2 - \bar{I}_1 \quad \therefore \bar{V}_1 = 1.5\bar{V}_2 - 3\bar{I}_2 \quad \therefore [\bar{t}]_B = \begin{bmatrix} 1.5 & 3\Omega \\ \frac{1}{6} \text{ S} & 1 \end{bmatrix}$$

$$\bar{V}_1 = 4\bar{I}_1 + \bar{V}_2 \quad \therefore \bar{I}_1 = \frac{1}{7} \bar{V}_2 - \bar{I}_2 \quad [\bar{t}]_C = \begin{bmatrix} 11/7 & 4\Omega \\ 1/7 \text{ S} & 1 \end{bmatrix}$$

$$\bar{I}_R = \frac{1}{7} \bar{V}_2 - \bar{I}_1 \quad \bar{V}_1 = \frac{11}{7} \bar{V}_2 - 4\bar{I}_2$$

$$(b) \quad [\bar{t}] = [\bar{t}]_A [\bar{t}]_B [\bar{t}]_C = \begin{bmatrix} 1.4 & 2 \\ 0.2 & 1 \end{bmatrix} \begin{bmatrix} 1.5 & 3 \\ 1/6 & 1 \end{bmatrix} \begin{bmatrix} 11/7 & 4 \\ 1/7 & 1 \end{bmatrix} = \begin{bmatrix} 2.433 & 6.2 \\ 0.4667 & 1.6 \end{bmatrix} \begin{bmatrix} 11/7 & 4 \\ 1/7 & 1 \end{bmatrix}$$

$$\therefore [\bar{t}] = \begin{bmatrix} 4.710 & 15.933\Omega \\ 0.9619 \text{ S} & 3.467 \end{bmatrix}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

39.

$$(a) \quad \bar{V}_1 = 2\bar{I}_1 + \bar{V}_2 = -2\bar{I}_2 + \bar{V}_2 = \bar{V}_2 - 2\bar{I}_2 \quad \therefore [\bar{t}]_A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\bar{I}_1 = -\bar{I}_2$$

$$(b) \quad \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 10\Omega \\ 0 & 1 \end{bmatrix} = \left( \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \right)^5 \quad \text{Also, } 10 \rightarrow \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

40.

$$(a) \quad \bar{V}_1 = \bar{V}_2 \quad \therefore [\bar{t}]_a = \begin{bmatrix} 1 & 0 \\ 1/R & 1 \end{bmatrix}$$

$$\bar{I}_1 = \bar{V}_2 / R - \bar{I}_2$$

$$\bar{V}_1 = \bar{V}_2 - R\bar{I}_2 \quad \therefore [\bar{t}]_b = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix}$$

$$\bar{I}_1 = -\bar{I}_2$$

$$\bar{V}_1 = \bar{V}_2 / a \quad \therefore [\bar{t}]_c = \begin{bmatrix} 1/a & 0 \\ 0 & a \end{bmatrix}$$

$$\bar{I}_1 = -a\bar{I}_2$$

$$(b) \quad [\bar{t}] = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} 0.25 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 20 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.02 & 1 \end{bmatrix}$$

$$\therefore [\bar{t}] = \begin{bmatrix} 1.2 & 2 \\ 0.1 & 1 \end{bmatrix} \begin{bmatrix} 0.25 & 5 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.02 & 1 \end{bmatrix} = \begin{bmatrix} 0.3 & 14 \\ 0.025 & 4.5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.02 & 1 \end{bmatrix} = \begin{bmatrix} 0.58 & 14\Omega \\ 0.115 \text{ S} & 4.5 \end{bmatrix}$$

## CHAPTER SEVENTEEN (TWO-PORT) SOLUTIONS

41.

(a)  $\bar{I}_1 = 0.1\bar{V}_x, -0.1\bar{V}_x + 0.02(\bar{V}_1 - \bar{V}_x) + 0.2(\bar{V}_1 - \bar{V}_x - \bar{V}_2) = 0$

$$\bar{I}_2 = 0.08\bar{V}_x + 0.2(\bar{V}_2 - \bar{V}_1 + \bar{V}_x)$$

$$\therefore 0.32\bar{V}_x = 0.22\bar{V}_1 - 0.2\bar{V}_2 \quad \therefore \bar{V}_x = \frac{11}{16}\bar{V}_1 - \frac{5}{8}\bar{V}_2$$

$$\therefore \bar{I}_1 = \frac{11}{160}\bar{V}_1 - \frac{1}{16}\bar{V}_2 \quad \text{Also, } \bar{I}_2 = 0.28\left(\frac{11}{16}\bar{V}_1 - \frac{5}{8}\bar{V}_2\right) + 0.2\bar{V}_2 - 0.2\bar{V}_1$$

$$\therefore \bar{I}_2 = -\frac{3}{400}\bar{V}_1 + \frac{1}{40}\bar{V}_2 \quad \therefore \bar{V}_1 = \frac{10}{3}\bar{V}_2 - \frac{400}{3}\bar{I}_2 \quad [\bar{t}] = \begin{bmatrix} 3.333 & 133.33\Omega \\ 0.16667\text{S} & 9.17 \end{bmatrix}$$

$$\therefore \bar{I}_1 = \frac{11}{160}\left(\frac{10}{3}\bar{V}_2 - \frac{400}{3}\bar{I}_2\right) - \frac{1}{16}\bar{V}_2 = \frac{1}{6}\bar{V}_2 - \frac{55}{6}\bar{I}_2 \quad \therefore [\bar{t}] = \begin{bmatrix} 3.333 & 133.33\Omega \\ 0.16667\text{S} & 9.167 \end{bmatrix}$$

(b)  $\begin{bmatrix} 1 & 0 \\ 0.05 & 1 \end{bmatrix} \therefore [\bar{t}]_{\text{new}} = \begin{bmatrix} 10/3 & 400/3 \\ 1/6 & 55/6 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.05 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 133.33\Omega \\ 0.625\text{S} & 9.167 \end{bmatrix}$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

1.

$$v(t) = 3 - 3\cos(100\pi t - 40^\circ) + 4\sin(200\pi t - 10^\circ) + 2.5\cos 300\pi t \text{ V}$$

$$(a) \quad V_{av} = 3 - 0 + 0 + 0 = \boxed{3.000 \text{ V}}$$

$$(b) \quad V_{eff} = \sqrt{3^2 + \frac{1}{2}(3^2 + 4^2 + 2.5^2)} = \boxed{4.962 \text{ V}}$$

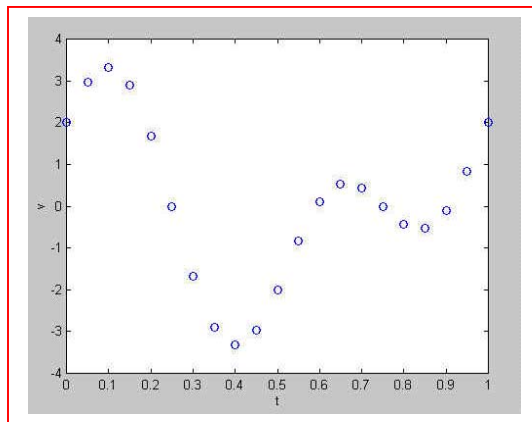
$$(c) \quad T = \frac{2\pi}{\omega_o} = \frac{2\pi}{100\pi} = \boxed{0.02 \text{ s}}$$

$$(d) \quad v(18\text{ms}) = 3 - 3\cos(-33.52^\circ) + 4\sin(2.960^\circ) + 2.5\cos(19.440^\circ) = \boxed{-2.459 \text{ V}}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

2. (a)

t	v	t	v
0	2	0.55	-0.844
0.05	2.96	0.6	0.094
0.1	3.33	0.65	0.536
0.15	2.89	0.7	0.440
0.2	1.676	0.75	0
0.25	0	0.8	-0.440
0.3	-1.676	0.85	-0.536
0.35	-2.89	0.9	-0.094
0.4	-3.33	0.95	0.844
0.45	-2.96	1	2
0.5	-2		



- (b)  $v' = -4\pi \sin 2\pi t + 7.2\pi \cos 4\pi t = 0$   
 $\therefore 4 \sin 2\pi t = 7.2(\cos^2 2\pi t - \sin^2 2\pi t)$   
 $\therefore 4 \sin 2\pi t = 7.2(1 - 2 \sin^2 2\pi t) \therefore x = \frac{-4 \pm \sqrt{16 + 414.72}}{28.8} = 0.5817, -0.8595 = \sin 2\pi t$   
 $\therefore t = 0.09881, 0.83539 \therefore v_{\max} = 3.330 (0.5593 \text{ for smaller max})$
- (c)  $|v_{\min}| = 3.330$



## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

3.

(a)  $T = 10 \text{ s}$ ,  $F_{av} = a_o = 0.1(2 \times 4 + 2 \times 2) = \boxed{1.200}$

(b) 
$$F_{eff} = \sqrt{\frac{1}{5} \int_0^2 (4-t)^2 dt} = \sqrt{0.2 \int_0^2 (16-8t+t^2) dt}$$

$$= \sqrt{0.2 \left[ 16t \Big|_0^2 - 4t^2 \Big|_0^2 + \frac{1}{3} t^3 \Big|_0^2 \right]} = \sqrt{0.2 \left( 32 - 16 + \frac{8}{3} \right)} = \boxed{1.9322}$$

(c) 
$$a_3 = \frac{2}{10} \times 2 \int_0^2 (4-t) \cos 3 \times \frac{2\pi t}{10} dt = 0.4 \int_0^2 4 \cos 0.6\pi t dt - 0.4 \int_0^2 t \cos 0.6\pi t dt$$

$$= 1.6 \frac{1}{0.6\pi} \sin 0.6\pi t \Big|_0^2 - 0.4 \left( \frac{1}{0.36\pi^2} \cos 0.6\pi t + \frac{t}{0.6\pi} \sin 0.6\pi t \right) \Big|_0^2$$

$$= \frac{8}{3\pi} \sin 1.2\pi - \frac{10}{9\pi^2} (\cos 1.2\pi - 1) - \frac{4}{3\pi} \sin 1.2\pi = \boxed{-0.04581}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

4.

(a)  $T = 8 - 2 = 6 \text{ s}$

(b)  $f_o = \frac{1}{6} \text{ Hz}$

(c)  $\omega_o = 2\pi f_o = \frac{\pi}{3} \text{ rad/s}$

(d)  $a_o = \frac{1}{6}(10 \times 1 + 5 \times 1) = 2.5$

(e) 
$$b_2 = \frac{2}{6} \left[ \int_2^3 10 \sin \frac{2\pi t}{3} dt + \int_3^4 5 \sin \frac{2\pi t}{3} dt \right]$$

$$= \frac{1}{3} \left[ -\frac{30}{2\pi} \cos \frac{2\pi t}{3} \Big|_2^3 - \frac{15}{2\pi} \cos \frac{2\pi t}{3} \Big|_3^4 \right]$$

$$\therefore b_2 = \frac{1}{3} \left[ -\frac{15}{\pi} \left( \cos 2\pi - \cos \frac{4\pi}{3} \right) - \frac{7.5}{\pi} \left( \cos \frac{8\pi}{3} - \cos 2\pi \right) \right] = \frac{1}{3} \left[ -\frac{15}{\pi} (1.5) - \frac{7.5}{\pi} (-1.5) \right] = -1.1937$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

5.

$$\begin{aligned}
 a_3 &= \frac{2}{6} \left[ \int_2^3 10 \cos \frac{6\pi t}{6} dt + \int_3^4 5 \cos \frac{6\pi t}{6} dt \right] = \frac{1}{3} \left[ \frac{10}{\pi} \sin \pi t \Big|_2^3 - \frac{5}{\pi} \sin \pi t \Big|_3^4 \right] \\
 &= \frac{10}{3\pi} \left( \sin 3\pi - \sin 2\pi + \frac{1}{2} \sin 4\pi - \frac{1}{2} \sin 3\pi \right) = 0 \\
 b_3 &= \frac{1}{3} \left[ \int_2^3 10 \sin \pi t dt + \int_3^4 5 \sin \pi t dt \right] = \frac{1}{3} \left[ -\frac{10}{\pi} \cos \pi t \Big|_2^3 - \frac{5}{\pi} \cos \pi t \Big|_3^4 \right] \\
 &= -\frac{10}{3\pi} \left( \cos 3\pi - \cos 2\pi + \frac{1}{2} \cos 4\pi - \frac{1}{2} \cos 3\pi \right) = -\frac{10}{3\pi} (-1) = 1.0610 \\
 \sqrt{a_3^2 + b_3^2} &= 1.0610
 \end{aligned}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

6.

(a)  $3.8 \cos^2 80\pi t = 1.9 + 1.9 \cos 160\pi t$ ,  $T = \frac{2\pi}{160\pi} = 12.5 \text{ ms}$ , ave value = 1.9

(b)  $3.8 \cos^3 80\pi t = (3.8 \cos 80\pi t)(0.5 + 0.5 \cos 160\pi t)$   
 $= 1.9 \cos 80\pi t + 0.95 \cos 240\pi t + 0.95 \cos 80\pi t = 2.85 \cos 80\pi t + 0.95 \cos 240\pi t$   
 $T = \frac{2\pi}{80\pi} = 25 \text{ ms}$ , ave value = 0

(c)  $3.8 \cos 70\pi t - 3.8 \sin 80\pi t$ ;  $\omega_o t = \pi t$ ,  $\omega_o = \pi$ ,  $T = \frac{2\pi}{\pi} = 2 \text{ s}$ ; ave value = 0

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

7.  $T = 2 \text{ s}$

(a) 
$$b_4 = \frac{2}{2} \int_0^{t_1} \sin \frac{4 \times 2\pi t}{2} dt = -\frac{1}{4\pi} \cos 4\pi t \Big|_0^{t_1}$$

$$\therefore b_4 = \frac{1}{4\pi} (1 - \cos 4\pi t_1)$$

$$\text{max when } 4\pi t_1 = \frac{\pi}{2}, t_1 = 0.125 \text{ s}$$

(b) 
$$b_4 = \frac{1}{4\pi}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

8.

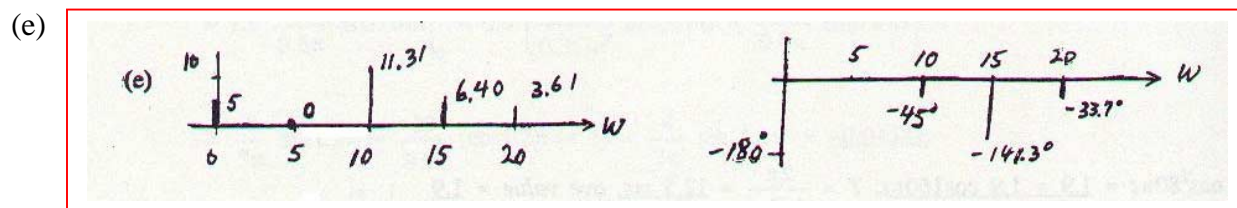
$$g(t) = 5 + 8 \cos 10t - 5 \cos 15t + 3 \cos 20t - 8 \sin 10t - 4 \sin 15t + 2 \sin 20t$$

(a)  $\omega_o = 5 \therefore T = \frac{2\pi}{5} = 1.2566 \text{ s}$

(b)  $f_o = \frac{5}{2\pi} \beta = 4 f_o = \frac{10}{\pi} = 3.183 \text{ Hz}$

(c)  $G_{av} = -5$

(d)  $G_{eff} = \sqrt{(-5)^2 + \frac{1}{2}(8^2 + 5^2 + 3^2 + 8^2 + 4^2 + 2^2)} = \sqrt{116} = 10.770$



## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

9.

$$T = 0.2, f(t) = V_m \cos 5\pi t, -0.1 < t < 0.1$$

$$\begin{aligned} a_n &= \frac{2}{0.2} \int_{-0.1}^{0.1} V_m \cos 5\pi t \cos 10n\pi t \, dt = 5V_m \int_{-0.1}^{0.1} [\cos(5\pi + 10n\pi)t + \cos(10n\pi - 5\pi)t] \, dt \\ &= 5V_m \left[ \frac{1}{10n\pi + 5\pi} \sin(10n\pi + 5\pi)t + \frac{1}{10n\pi - 5\pi} \sin(10n\pi - 5\pi)t \right]_{-0.1}^{0.1} \\ &= \frac{V_m}{\pi} \left[ \frac{2}{2n+1} \sin(10n\pi + 5\pi)0.1 + \frac{2}{2n-1} \sin(10n\pi - 5\pi)0.1 \right] \\ &= \frac{V_m}{\pi} \left[ \frac{2}{2n+1} \sin(n\pi + 0.5\pi) + \frac{2}{2n-1} \sin(n\pi - 0.5\pi) \right] \\ &= \frac{V_m}{\pi} \left[ \frac{2}{2n+1} \cos n\pi + \frac{2}{2n-1} (-\cos n\pi) \right] = \frac{2V_m}{\pi} \cos n\pi \left( \frac{1}{2n+1} - \frac{1}{2n-1} \right) \\ &= \frac{2V_m}{\pi} \cos n\pi \frac{2n-1-2n-1}{4n^2-1} = -\frac{4V_m}{\pi} \frac{\cos n\pi}{4n^2-1} \end{aligned}$$

$$a_o = \frac{1}{0.2} \int_{-0.1}^{0.1} V_m \cos 5\pi t \, dt = 5V_m \frac{1}{5\pi} \left[ \sin \frac{\pi}{2} - \sin \left( -\frac{\pi}{2} \right) \right] = \frac{2V_m}{\pi}$$

$$\therefore v(t) = \frac{2V_m}{\pi} + \frac{4V_m}{3\pi} \cos 10\pi t - \frac{4V_m}{15\pi} \cos 20\pi t + \frac{4V_m}{35\pi} \cos 30\pi t - \frac{4V_m}{63\pi} \cos 40\pi t + \dots$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

10.

(a)  $\text{even, } \frac{1}{2} - \text{wave}$

(b)  $b_n = 0 \text{ for all } n; a_{\text{even}} = 0; a_o = 0$

(c)  $b_1 = b_2 = b_3 = 0, a_2 = 0$

$$a_n = \frac{8}{12} \int_1^2 5 \cos \frac{n\pi t}{6} dt = \frac{10}{3} \frac{6}{n\pi} \sin \frac{n\pi t}{6} \Big|_1^2 = \frac{20}{n\pi} \left( \sin \frac{n\pi}{3} - \sin \frac{n\pi}{6} \right)$$

$$\therefore a_1 = \frac{20}{\pi} \left( \sin \frac{\pi}{3} - \sin \frac{\pi}{6} \right) = 2.330, a_3 = \frac{20}{3\pi} \left( \sin \pi - \sin \frac{\pi}{2} \right) = -\frac{20}{3\pi} = -2.122$$



## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

11.

(a)  $a_o = a_n = 0$

$$\therefore y(t) = 0.2 \sin 1000\pi t + 0.6 \sin 2000\pi t + 0.4 \sin 3000\pi t$$

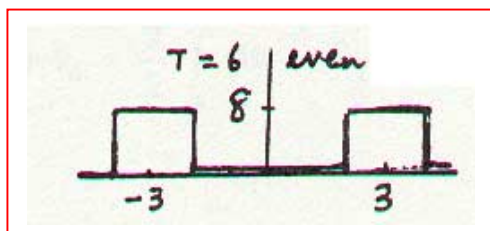
(b)  $Y_{eff} = \sqrt{0.5(0.2^2 + 0.6^2 + 0.4^2)} = \sqrt{0.5(0.56)} = 0.5292$

(c)  $y(2\text{ms}) = 0.2 \sin 0.2\pi + 0.6 \sin 0.4\pi + 0.4 \sin 0.6\pi = 1.0686$

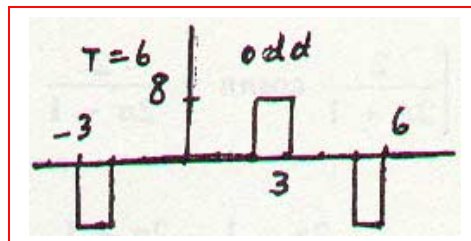
# CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

12.

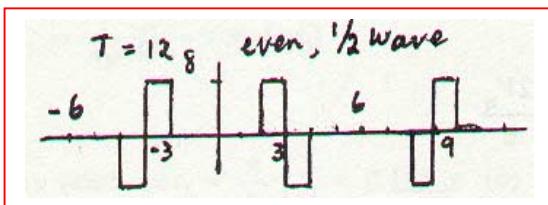
(a)



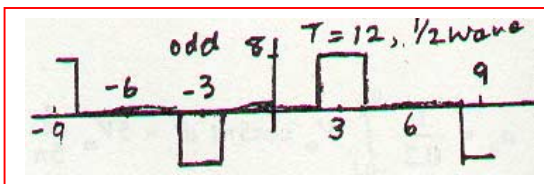
(b)



(c)



(d)



$$(e) \quad [a]b_5 = 0, \quad a_5 = \frac{4}{6} \int_2^3 8 \cos \frac{2\pi 5t}{6} dt = \frac{32}{6} \frac{6}{10\pi} \sin \frac{5\pi t}{3} \Big|_2^3 = \frac{3.2}{\pi} \left( \sin \frac{15\pi}{3} - \sin \frac{10\pi}{3} \right) = 0.8821$$

$$[b]a_5 = 0, \quad b_5 = \frac{4}{6} \int_2^3 8 \sin \frac{2\pi 5t}{6} dt = \frac{32}{6} \left( \frac{-6}{10\pi} \right) \left( \cos \frac{15\pi}{3} - \cos \frac{10\pi}{3} \right) = -\frac{3.2}{\pi} (-0.5) = 0.5093$$

$$[c]b_5 = 0, \quad a_5 = \frac{8}{12} \int_2^3 8 \cos \frac{2\pi 5t}{12} dt = \frac{64}{12} \frac{12}{10\pi} \left( \sin \frac{15\pi}{6} - \sin \frac{10\pi}{6} \right) = 3.801$$

$$[d]a_5 = 0, \quad b_5 = \frac{8}{12} \int_2^3 8 \sin \frac{10\pi t}{12} dt = \frac{64}{12} \left( -\frac{12}{10\pi} \right) \left( \cos \frac{15\pi}{6} - \cos \frac{10\pi}{6} \right) = 1.0186$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

13.

$T = 4 \text{ ms}$

$$\begin{aligned} \text{(a)} \quad a_o &= \frac{1000}{4} \int_0^{0.004} 8 \sin 125\pi t \, dt = \frac{250 \times 8}{-125\pi} \cos 125\pi t \Big|_0^{0.004} \\ &= -\frac{16}{\pi} \left( \cos \frac{\pi}{2} - 1 \right) = \frac{16}{\pi} = 5.093 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad a_1 &= 4000 \int_0^{0.004} \sin 125\pi t \cos \frac{2\pi t}{0.004} \, dt \\ \therefore a_1 &= 4000 \int_0^{0.004} \sin 125\pi t \cos 500\pi t \, dt = 2000 \int_0^{0.004} (\sin 625\pi t - \sin 375\pi t) \, dt \\ &= 2000 \left( -\frac{\cos 625\pi t}{625\pi} + \frac{\cos 375\pi t}{375\pi} \right) \Big|_0^{0.004} = \frac{3.2}{\pi} (1 - \cos 2.5\pi) - \frac{5.333}{\pi} (1 - \cos 1.5\pi) = -0.6791 \\ b_1 &= 4000 \int_0^{0.004} \sin 125\pi t \sin 500\pi t \, dt = 2000 \int_0^{0.004} (\cos 375\pi t - \cos 625\pi t) \, dt \\ &= 2000 \left[ \frac{1}{375\pi} (\sin 1.5\pi) - \frac{1}{625\pi} (\sin 2.5\pi) \right] = 2000 \left( \frac{-1}{375\pi} - \frac{1}{625\pi} \right) = -2.716 \end{aligned}$$

$$\text{(c)} \quad -4 < t < 0: 8 \sin 125\pi t$$

$$\begin{aligned} \text{(d)} \quad b_1 &= 0, \quad a_1 = \frac{4000}{8} \int_0^{0.004} 8 \sin 125\pi t \cos 250\pi t \, dt \\ \therefore a_1 &= 2000 \int_0^{0.004} [\sin 375\pi t - \sin 125\pi t] \, dt = 2000 \left[ -\frac{\cos 375\pi t}{375\pi} + \frac{\cos 125\pi t}{125\pi} \right] \Big|_0^{0.004} \\ &= \frac{5.333}{\pi} (1 - \cos 1.5\pi) + \frac{16}{\pi} \left( \cos \frac{\pi}{2} - 1 \right) = -3.395^+ \end{aligned}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

14.

odd and  $\frac{1}{2}$ -wave  $\therefore a_o = 0, a_n = 0, b_{even} = 0$

$$T = 10ms = 0.01s$$

$$b_{odd} = \frac{8}{0.01} \left[ \int_0^{0.001} 10 \sin 200n\pi t \, dt \right] = 8000 \left( \frac{-1}{200n\pi} \right) \cos 200n\pi t \Big|_0^{0.001}$$

$$\therefore b_{odd} = -\frac{40}{n\pi} (\cos 0.2n\pi - 1) = \frac{40}{n\pi} (1 - \cos 0.2n\pi)$$

$$\therefore b_1 = 2.432, b_3 = 5.556, b_5 = 5.093, b_7 = 2.381, b_9 = 0.2702$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

15.

$$\text{odd and } \frac{1}{2} - \text{wave, } T = 8 \text{ ms} \therefore b_n = \frac{8}{T} \int_0^{T/4} f(t) \sin n\omega_o t \, dt$$

$$\omega_o = \frac{2\pi}{T} = 250\pi \therefore b_n = 1000 \int_0^{0.001} 1000t \sin 250\pi n t \, dt$$

$$\text{Now, } \int x \sin ax \, dx = \frac{1}{a^2} (\sin ax - ax \cos ax), \, a = 250n\pi$$

$$f(t) = 10^3 t \therefore b_n = \frac{10^6}{250^2 n^2 \pi^2} (\sin 250n\pi t - 250n\pi t \cos 250n\pi t)_0^{0.001}$$

$$\therefore b_n = \frac{16}{n^2 \pi^2} \left( \sin \frac{n\pi}{4} - 0 - \frac{n\pi}{4} \cos \frac{n\pi}{4} + 0 \right) \therefore b_1 = \frac{16}{\pi^2} \left( \sin \frac{\pi}{4} - \frac{\pi}{4} \cos \frac{\pi}{4} \right) = \boxed{0.2460}$$

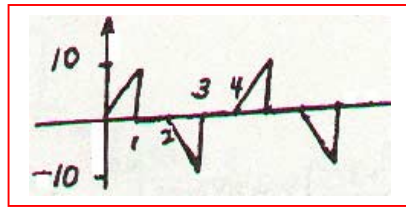
$$b_3 = \frac{16}{9\pi^2} \left( \sin \frac{3\pi}{4} - \frac{3\pi}{4} \cos \frac{3\pi}{4} \right) = 0.4275^-; \, b_5 = \frac{16}{25\pi^2} \left( \sin \frac{5\pi}{4} - \frac{5\pi}{4} \cos \frac{5\pi}{4} \right) = \boxed{0.13421}$$

$$b_{\text{even}} = \boxed{0}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

16.

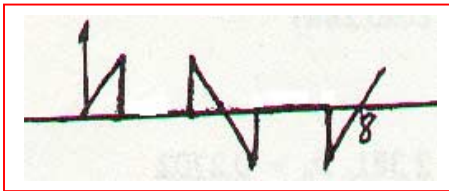
(a) odd,  $T = 4$



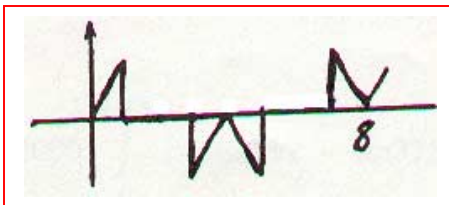
(b) even,  $T = 4$ :



(c) odd,  $\frac{1}{2}$ -wave:  $T = 8$



(d) even,  $\frac{1}{2}$ -wave,  $T = 8$ :



# CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

17.

$$(a) \quad v_s = 5 + \frac{20}{\pi} \sum_{1, \text{odd}} \frac{1}{n} \sin \frac{2\pi n t}{0.4\pi} \quad \therefore v_{sn} = \frac{20}{n\pi} \sin 5nt, \quad \bar{V}_{sn} = \frac{20}{n\pi} (-j1)$$

$$Z_n = 4 + j5n2 = 4 + j10n, \quad \bar{I}_{fn} = \frac{\bar{V}_{sn}}{Z_n} = \frac{-j20}{n\pi(4 + j10n)} = -\frac{j5}{1 + j2.5n}$$

$$\therefore \bar{I}_{fn} = -\frac{j5}{n\pi} \frac{1 - j2.5n}{1 + 6.25n^2} = -\frac{12.5 + j5}{n\pi(1 + 6.25n^2)}$$

$$\therefore i_{fn} = -\frac{12.5}{\pi} \frac{1}{1 + 6.25n^2} \cos 5nt + \frac{5}{n\pi} \frac{1}{1 + 6.25n^2} \sin 5nt$$

$$\therefore i_f = 1.25 + \sum_{1, \text{odd}} \frac{1}{1 + 6.25n^2} \left[ -\frac{12.5}{\pi} \cos 5nt + \frac{5}{n\pi} \sin 5nt \right]$$

$$(b) \quad i_n = Ae^{-2t}, \quad i = i_f + i_n, \quad i(0) = 0, \quad i_f(0) = 1.25 + \sum_{1, \text{odd}} \frac{1}{1 + 6.25n^2} \left( -\frac{12.5}{\pi} \right)$$

$$\therefore i_f(0) = 1.25 - \frac{2}{\pi} \sum_{1, \text{odd}} \frac{1}{n^2 + 0.16} = 1.25 - \frac{2}{\pi} \frac{\pi}{4 \times 0.4} \tanh 0.2\pi = 0.55388$$

$$\therefore A = -0.55388, \quad i = -0.55388e^{-2t} + 1.25 + \sum_{1, \text{odd}} \frac{1}{1 + 6.25n^2} \left[ -\frac{12.5}{\pi} \cos 5nt + \frac{5}{n\pi} \sin 5nt \right]$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

18.

(a)  $0 < t < 0.2\pi: i = 2.5(1 - e^{-2t}) \therefore i(0.2\pi) = 2.5(1 - e^{-0.4\pi}) = 1.78848 \text{ A}$

(b)  $0.2\pi < t < 0.4\pi: i = 1.78848 e^{-2(t-0.2\pi)} \therefore i(0.4\pi) = 0.50902 \text{ A}$

(c)  $0.4\pi < t < 0.6\pi: i = 2.5 - (2.5 - 0.50902)e^{-2(t-0.4\pi)}, i(0.6\pi) = 1.9335 \text{ A}$



## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

19.

$$(a) \quad v_s = 5 + \frac{20}{\pi} \sum_{1, \text{odd}}^{\infty} \frac{1}{n} \sin 5nt$$

$$v_{sn} = \frac{20}{n\pi} \sin 5nt$$

$$\bar{V}_{sn} = -j \frac{20}{n\pi}$$

$$\bar{Z}_n = 2 + \frac{1}{j5n2} = 2 + \frac{1}{j10n} \quad \therefore \bar{V}_{cn} = \frac{-j20/n\pi}{2 + 1/j10n} \times \frac{1}{j10n} = \frac{-j20/n\pi}{1 + j20n} \times \frac{1 - j20n}{1 - j20n}$$

$$\therefore \bar{V}_{cn} = \frac{-20n - j1}{1 + 400n^2} \times \frac{20}{n\pi}, \quad v_{cn} = \frac{20}{n\pi} \frac{1}{1 + 400n^2} (-20n \cos 5nt + \sin 5nt)$$

$$\therefore v_{cf} = 5 + \frac{20}{\pi} \sum_{1, \text{odd}}^{\infty} \frac{1}{1 + 400n^2} \left( \frac{1}{n} \sin 5nt - 20 \cos 5nt \right)$$

$$(b) \quad v_n = Ae^{-t/4}$$

$$(c) \quad v_c(0) = A + 5 + \frac{20}{\pi} \sum_{1, \text{odd}}^{\infty} \frac{-20}{1 + 400n^2} = A + 5 - \frac{1}{\pi} \sum_{1, \text{odd}}^{\infty} \frac{1}{n^2 + (1/20)^2}$$

$$\sum_{1, \text{odd}}^{\infty} \frac{1}{n^2 + (1/20)^2} = \frac{\pi}{4(1/20)} \tanh \frac{\pi}{20 \times 2} = 5\pi \tanh \frac{\pi}{40} = 1.23117$$

$$\therefore A = 0 - 5 + \frac{1}{\pi} \times 1.23117 = -4.60811$$

$$\therefore v_c(t) = -4.60811e^{-t/4} + 5 + \frac{20}{\pi} \sum_{1, \text{odd}}^{\infty} \frac{1}{1 + 400n^2} \left( \frac{1}{n} \sin 5nt - 20 \cos 5nt \right)$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

20.

$$\begin{aligned}
 c_3 &= \frac{10^3}{6} \left[ \int_0^{0.001} 100e^{-j3 \times 2\pi t / 6 \times 10^{-3}} - \int_{0.003}^{0.005} 100e^{-j100\pi t} \right] \\
 &= \frac{10^5}{6} \left[ \frac{-1}{j1000\pi} e^{-j1000\pi t} \Big|_0^{0.001} + \frac{1}{j1000\pi} e^{-j1000\pi t} \Big|_{0.003}^{0.005} \right] \\
 &= \frac{100}{j6\pi} (e^{-j\pi} + 1 + e^{-j5\pi} - e^{-j3\pi}) = \frac{100}{j6\pi} (1 + 1 - 1 + 1) = -j10.610
 \end{aligned}$$

$$\therefore c_{-3} = j10.610; |c_3| = 10.610$$

$$\begin{aligned}
 a_3 &= \frac{2 \times 10^3}{6} \left[ \int_0^{0.001} 100 \cos 100\pi t \, dt - \int_{0.003}^{0.005} 100 \cos 1000\pi t \, dt \right] \\
 &= \frac{2 \times 10^5}{6} \frac{1}{1000\pi} (\sin \pi - 0 - \sin 5\pi + \sin 3\pi) = 0
 \end{aligned}$$

$$c_3 = \frac{1}{2}(a_3 - jb_3) = -j\frac{1}{2}b_3 \quad \therefore b_3 = 21.22 \text{ and } \sqrt{a_3^2 + b_3^2} = 21.22$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

21.

$$(a) \quad T = 5 \text{ ms } c_m = \frac{1}{0.005} \left[ \int_0^{0.001} 10^5 t e^{-j400\pi nt} dt + \int_{0.001}^{0.002} 100 e^{-j400\pi nt} dt \right]$$

$$\therefore c_n = 20,000 \left[ \int_0^{0.001} 1000t e^{-j400\pi nt} dt + \int_{0.001}^{0.002} e^{-j400\pi nt} dt \right]$$

$$\therefore c_n = 20,000 \left[ \frac{e^{-j400\pi nt}}{160n^2\pi^2} (j400\pi nt + 1) \Big|_0^{0.001} + \frac{1}{-j400\pi n} e^{-j400\pi nt} \Big|_{0.001}^{0.002} \right]$$

$$(b) \quad \therefore c_o = a_o = (50 \times 10^{-3} + 100 \times 10^{-3}) \frac{1}{0.005} = 0.15 \times 200 = \boxed{30}$$

$$\begin{aligned} c_1 &= 20,000 \left[ \frac{1}{160\pi^2} e^{-j0.4\pi} (1 + j0.4\pi) - \frac{1}{160\pi^2} - \frac{1}{j400\pi} (e^{-j0.8\pi} - e^{-j0.4\pi}) \right] \\ &= \frac{125}{\pi^2} (1 \angle -72^\circ) (1.60597 \angle 51.488^\circ) - 12.66515 + 15.91548 \angle 90^\circ (1 \angle -144^\circ - 1 \angle -72^\circ) \\ &= 12.665 (1 \angle -72^\circ) (1 + j1.2566) - 12.665 + j15.915 (1 \angle -144^\circ - 1 \angle -72^\circ) \\ &= 20.339 \angle -20.513^\circ - 12.665 + 18.709 \angle -108^\circ = \boxed{24.93 \angle -88.61^\circ} \\ c_2 &= 3.16625 \angle -144^\circ (1 + j2.5133) - 3.16625 + j7.9575 (1 \angle -288^\circ - 1 \angle -144^\circ) \\ &= 8.5645 \angle -75.697^\circ - 3.16625 + 15.1361 \angle 144^\circ = \boxed{13.309 \angle 177.43^\circ} \end{aligned}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

22.

Fig. 17-8a:  $V_o = 8 \text{ V}$ ,  $\tau = 0.2 \mu\text{s}$ ,  $f_o = 6000 \text{ pps}$

$$(a) \quad T = \frac{1}{6000}, f_o = 6000, \tau = 0.2 \mu\text{s} \quad \therefore f = \frac{1}{\tau} = \boxed{5 \text{ MHz}}$$

$$(b) \quad \boxed{f_o = 6000 \text{ Hz}}$$

$$(c) \quad 6000 \times 3 = 18,000 \text{ (closest)} \quad \therefore |c_3| = \frac{8 \times 0.2 \times 10^{-6}}{1/6000} \left| \frac{\sin(1/2 \times 3 \times 12,000\pi \times 0.2 \times 10^{-6})}{0.0036\pi} \right|$$

$$\therefore |c_3| = \boxed{9.5998 \text{ mV}}$$

$$(d) \quad \frac{2 \times 10^6}{6 \times 10^3} = 333.3 \quad \therefore |c_{333}| = \frac{8 \times 0.2 \times 10^{-6}}{1/6000} \left| \frac{\sin(1/2 \times 333 \times 12,000\pi \times 0.2 \times 10^{-6})}{1/2 \times 333 \times 12,000\pi \times 0.2 \times 10^{-6}} \right| = \boxed{7.270 \text{ mV}}$$

$$(e) \quad \beta = 1/\tau = \boxed{5 \text{ MHz}}$$

$$(f) \quad 2 < \omega < 2.2 \text{ Mrad/s} \quad \therefore \frac{2000}{2\pi} < f < \frac{2200}{2\pi} \text{ kHz or } 318.3 < f < 350.1 \text{ kHz}$$

$$f_o = 6 \text{ kHz} \quad \therefore f = 6 \times 53 = 318; 324, 330, 336, 342, 348 \text{ kHz} \quad \therefore \boxed{n = 5}$$

$$(g) \quad |c_{227}| = \frac{8 \times 0.2 \times 10^{-6}}{1/6000} \left| \frac{\sin(1/2 \times 227 \times 12,000\pi \times 0.2 \times 10^{-6})}{(")} \right| = \boxed{8.470 \text{ mV}}$$

$$f = 227 \times 6 = \boxed{1362 \text{ kHz}}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

23.

$$T = 5\text{ ms}; \bar{c}_0 = 1, \bar{c}_1 = 0.2 - j0.2, \bar{c}_2 = 0.5 + j0.25, \bar{c}_3 = -1 - j2, \bar{c}_n = 0, |n| \geq 4$$

(a)  $a_n = -jb_n = 2\bar{c}_n \therefore a_0 = \bar{c}_0 = 1, a_1 - jb_1 = 0.4 - jb_1 = 0.4 - j0.4, a_2 - jb_2 = 1 + j0.5, a_3 - jb_3 = -2$   
 $\therefore v(t) = 1 + 0.4 \cos 400\pi t + \cos 800\pi t - 2 \cos 1200\pi t + 0.4 \sin 400\pi t - 0.5 \sin 800\pi t + 4 \sin 1200\pi t$

(b)  $v(1\text{ ms}) = 1 + 0.4 \cos 72^\circ + \cos 144^\circ - 2 \cos 216^\circ + 0.4 \sin 72^\circ - 0.5 \sin 144^\circ + 4 \sin 216^\circ$   
 $\therefore v(1\text{ ms}) = -0.332\text{ V}$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

24.

$$(a) \quad T = 5 \mu s \quad \therefore \bar{c}_n = \frac{10^6}{5} \times 2 \int_{0.4 \times 10^{-6}}^{0.6 \times 10^{-6}} 1 \cos 2\pi n \frac{t}{5 \times 10^{-6}} dt$$

$$\therefore \bar{c}_n = 4 \times 10^5 \frac{5 \times 10^{-6}}{2\pi n} (\sin 43.2^\circ n - \sin 28.8^\circ n)$$

$$\therefore \bar{c}_n = \frac{1}{n\pi} (\sin 43.2^\circ n - \sin 28.8^\circ n)$$

$$(b) \quad \bar{c}_4 = \frac{1}{4\pi} (\sin 172.8^\circ - \sin 115.2^\circ) = -0.06203$$

$$(c) \quad \bar{c}_o = a_o = \frac{0.2 \times 10^{-6} + 0.2 \times 10^{-6}}{5 \times 10^{-6}} = 0.08$$

$$(d) \quad \text{a little testing shows } |c_o| \text{ is max } \therefore |\bar{c}_{\max}| = 0.08$$

$$(e) \quad 0.01 \times 0.08 = 0.8 \times 10^{-3} \quad \therefore \left| \frac{1}{n\pi} (\sin 43.2^\circ n - \sin 28.8^\circ n) \right| \leq 0.8 \times 10^{-3}$$

$$\therefore \left| \frac{125}{n\pi} (\sin 43.2^\circ n - \sin 28.8^\circ n) \right| \leq 1$$

ok for  $n > 740$

$$(f) \quad \beta = 740 f_o = \frac{740 \times 10^6}{5} = 148 \text{ MHz}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

25.

$$T = 1/16, \omega_o = 32\pi$$

$$(a) \quad \bar{c}_3 = 16 \int_0^{1/96} 40e^{-j96\pi t} dt - \frac{16 \times 40}{-j96\pi} e^{-j96\pi t} \Big|_0^{1/96}$$

$$\therefore \bar{c}_3 = j \frac{20}{3\pi} (e^{-j\pi} - 1) = -j \frac{40}{3\pi} = \boxed{-j4.244 \text{ V}}$$

(b) Near harmonics are  $2f_o = 32 \text{ Hz}$ ,  $3f_o = 48 \text{ Hz}$

Only 32 and 48 Hz pass filter  $a_n - jb_n = 2\bar{c}_n$

$$a_3 - jb_3 = 2\bar{c}_3 = -j8.488 \quad \therefore a_3 = 0, b_3 = 8.488 \text{ V}$$

$$\bar{I}_3 = \frac{8.488}{5 + j0.01 \times 96\pi} = 1.4536 \angle -31.10^\circ \text{ A}; P_3 = \frac{1}{2} \times 1.4536^2 \times 5 = 5.283 \text{ W}$$

$$\bar{c}_2 = \frac{1}{1/16} \int_0^{1/96} 40e^{-j64\pi t} dt = \frac{640}{-j64\pi} (e^{-j64\pi/96} - 1) = 2.7566 - j4.7746 \text{ V}$$

$$a_2 - b_2 = 2\bar{c}_2 = 5.5132 - j9.5492 = 11.026 \angle -60^\circ$$

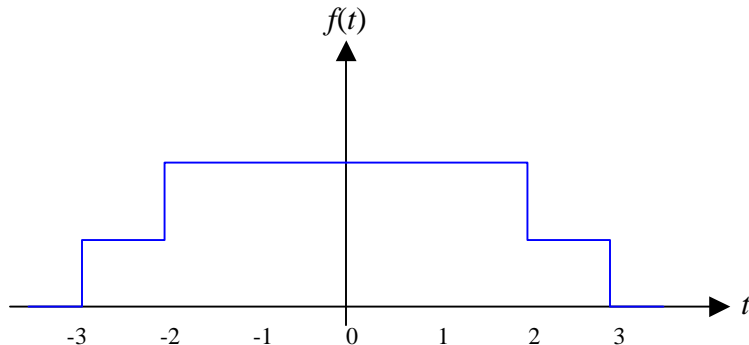
$$\therefore \bar{I}_2 = \frac{11.026 \angle -60^\circ}{5 + j0.01 \times 64\pi} = 2.046 \angle -65.39^\circ \text{ A}$$

$$\therefore P_2 = \frac{1}{2} \times 2.046^2 \times 5 = 10.465 \text{ W} \quad \therefore P_{tot} = \boxed{15.748 \text{ W}}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

26.  $f(t) = 5[u(t+3) + u(t+2) - u(t-2) - u(t-3)]$

(a)



(b)

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$\therefore F(j\omega) = \int_{-3}^{-2} 5e^{-j\omega t} dt + \int_{-2}^2 10e^{-j\omega t} dt + \int_2^3 5e^{-j\omega t} dt$$

$$\therefore F(j\omega) = \frac{5}{-j\omega} (e^{j2\omega} - e^{j3\omega}) + \frac{10}{-j\omega} (e^{-j2\omega} - e^{j2\omega}) + \frac{5}{-j\omega} (e^{-j3\omega} - e^{-j2\omega})$$

$$= \frac{5}{-j\omega} (-e^{j3\omega} + e^{-j3\omega}) + \frac{5}{-j\omega} (e^{j2\omega} - e^{-j2\omega}) + \frac{10}{-j\omega} (-e^{j2\omega} + e^{-j2\omega})$$

$$= \frac{5}{-j\omega} (-j2) \sin 3\omega + \frac{5}{-j\omega} (j2) \sin 2\omega + \frac{10}{-j\omega} (-j2) \sin 2\omega$$

$$\therefore F(j\omega) = \frac{10}{\omega} \sin 3\omega - \frac{10}{\omega} \sin 2\omega + \frac{20}{\omega} \sin 2\omega = \frac{10}{\omega} (\sin 3\omega + \sin 2\omega)$$



## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

27.

$$(a) \quad f(t) = e^{-at} u(t), \quad a > 0 \quad \therefore F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = \int_0^{\infty} e^{-at} e^{-j\omega t} dt$$

$$\therefore F(j\omega) = \frac{-1}{a + j\omega} e^{-(a+j\omega)t} \bigg|_0^{\infty} = \boxed{\frac{1}{a + j\omega}}$$

$$(b) \quad f(t) = e^{at_0} e^{-at} u(t - t_0), \quad a > 0 \quad \therefore F(j\omega) = e^{at_0} \int_{t_0}^{\infty} e^{-(a+j\omega)t} dt$$

$$\therefore F(j\omega) = e^{at_0} \frac{-1}{a + j\omega} e^{-(a+j\omega)t} \bigg|_{t_0}^{\infty} = e^{at_0} \frac{-1}{a + j\omega} \left[ -e^{-(a+j\omega)t_0} \right] = \boxed{\frac{1}{a + j\omega} e^{-j\omega t_0}}$$

$$(c) \quad f(t) = te^{-at} u(t), \quad a > 0 \quad \therefore F(j\omega) = \int_0^{\infty} te^{-(a+j\omega)t} dt$$

$$\therefore F(j\omega) = \frac{e^{-(a+j\omega)t}}{(a + j\omega)^2} \left[ -(a + j\omega)t - 1 \right]_0^{\infty} = 0 - \frac{1}{(a + j\omega)^2} [-1] = \boxed{\frac{1}{(a + j\omega)^2}}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

28.

$$-4 < t < 0: f(t) = 2.5(t+4); 0 < t < 4: f(t) = 2.5(4-t)$$

$$\therefore F(j\omega) = \int_{-4}^0 2.5(t+4)e^{-j\omega t} dt + \int_0^4 2.5(4-t)e^{-j\omega t} dt$$

$$\text{In 1<sup>st</sup>, let } t = \tau \therefore I_1 = \int_4^0 2.5(4-\tau)e^{j\omega\tau} (-d\tau)$$

$$\therefore I_1 = \int_0^4 2.5(4-\tau)e^{j\omega\tau} d\tau \therefore F(j\omega) = 2.5 \int_0^4 (4-t)(e^{j\omega t} + e^{-j\omega t}) dt$$

$$\therefore F(j\omega) = 5 \int_0^4 (4-t) \cos \omega t dt = 20 \times \frac{1}{\omega} \sin \omega t \Big|_0^4 - 5 \int_0^4 \cos \omega t dt$$

$$\begin{aligned} \therefore F(j\omega) &= \frac{20}{\omega} \sin 4\omega - \frac{5}{\omega^2} (\cos \omega t + \omega t \sin \omega t)_0^4 \\ &= \frac{20}{\omega} \sin 4\omega - \frac{5}{\omega^2} (\cos 4\omega - 1) - \frac{5}{\omega^2} 4\omega \sin 4\omega = \frac{5}{\omega^2} (1 - \cos 4\omega) \end{aligned}$$

$$\text{or, } F(j\omega) = \frac{2 \times 5}{\omega^2} \sin^2 2\omega = 10 \left( \frac{\sin 2\omega}{\omega} \right)^2$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

29.

$$\begin{aligned}
 f(t) &= 5 \sin t, -\pi < t < \pi \quad \therefore F(j\omega) = \int_{-\pi}^{\pi} 5 \sin t \, e^{-j\omega t} \, dt \\
 \therefore F(j\omega) &= \frac{5}{j2} \int_{-\pi}^{\pi} (e^{jt} - e^{-jt}) \, e^{-j\omega t} \, dt \\
 &= \frac{5}{j2} \int_{-\pi}^{\pi} [e^{jt(1-\omega)} - e^{-jt(1+\omega)}] \, dt \\
 F(j\omega) &= \frac{5}{j2} \left[ \frac{1}{j(1-\omega)} (e^{j\pi(1-\omega)} - e^{-j\pi(1-\omega)}) - \frac{1}{-j(1+\omega)} (e^{-j\pi(1+\omega)} - e^{j\pi(1+\omega)}) \right] \\
 &= \frac{-2.5}{1-\omega} (-e^{-j\pi\omega} + e^{j\pi\omega}) - \frac{2.5}{1+\omega} (-e^{-j\pi\omega} + e^{j\pi\omega}) \\
 &= \frac{-2.5}{1-\omega} (j2 \sin \pi\omega) - \frac{2.5}{1+\omega} (j2 \sin \pi\omega) = j5 \sin \pi\omega \left( -\frac{1}{1-\omega} - \frac{1}{1+\omega} \right) \\
 &= j5 \sin \pi\omega (-1) \left( \frac{1+\omega+1-\omega}{1-\omega^2} \right) = -\frac{j10 \sin \pi\omega}{1-\omega^2} = \boxed{\frac{j10 \sin \pi\omega}{\omega^2 - 1}}
 \end{aligned}$$

# CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

30.

$$f(t) = 8 \cos t [u(t + 0.5\pi) - u(t - 0.5\pi)]$$

$$\begin{aligned} \therefore F(j\omega) &= \int_{-\pi/2}^{\pi/2} 8 \cos t e^{-j\omega t} dt = 4 \int_{-\pi/2}^{\pi/2} (e^{jt} + e^{-jt}) e^{-j\omega t} dt \\ &= 4 \int_{-\pi/2}^{\pi/2} [e^{jt(1-\omega)} + e^{-jt(1+\omega)}] dt \\ &= 4 \left\{ \frac{1}{j(1-\omega)} e^{jt} e^{-j\omega t} \Big|_{-\pi/2}^{\pi/2} - \frac{1}{j(1+\omega)} e^{-jt} e^{-j\omega t} \Big|_{-\pi/2}^{\pi/2} \right\} \\ &= 4 \left\{ \frac{1}{j(1-\omega)} [je^{-j\pi\omega/2} - (-j)e^{j\pi\omega/2}] - \frac{1}{j(1+\omega)} [-je^{-j\pi\omega/2} - je^{j\pi\omega/2}] \right\} \\ &= 4 \left\{ \frac{1}{1-\omega} \times 2 \cos \frac{\pi\omega}{2} + \frac{1}{1+\omega} \times 2 \cos \frac{\pi\omega}{2} \right\} = 8 \cos \frac{\pi\omega}{2} \left( \frac{1}{1-\omega} + \frac{1}{1+\omega} \right) \\ &= 8 \cos \frac{\pi\omega}{2} \frac{2}{1-\omega^2} = 16 \frac{\cos \pi\omega/2}{1-\omega^2} \end{aligned}$$

(a)  $\omega = 0 \quad \therefore F(j0) = 16$

(b)  $\omega = 0.8, F(j0.8) = \frac{16 \cos 72^\circ}{0.36} = 13.734$

(c)  $\omega = 3.1, F(j3.1) = \frac{16 \cos(3.1 \times 90^\circ)}{1 - 3.12} = -0.2907$

# CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

31.

$$(a) \quad F(j\omega) = 4[u(\omega+2) - u(\omega-2)] \therefore f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} F(j\omega) d\omega$$

$$\therefore f(t) = \frac{4}{2\pi} \int_{-2}^2 e^{j\omega t} d\omega = \frac{2}{\pi} \frac{1}{jt} e^{j\omega t} \Big|_{-2}^2 = \frac{2}{j\pi t} (e^{j2t} - e^{-j2t})$$

$$\therefore f(t) = \frac{2}{2\pi t} j2 \sin 2t = \frac{4}{\pi t} \sin 2t \therefore f(0.8) = \frac{5}{\pi} \sin 1.6^{rad} = \boxed{1.5909}$$

$$(b) \quad F(j\omega) = 4e^{-2|\omega|} \therefore f(t) = \frac{4}{2\pi} \int_{-\infty}^{\infty} e^{-2|\omega| + j\omega t} d\omega$$

$$\begin{aligned} \therefore f(t) &= \frac{2}{\pi} \int_{-\infty}^0 e^{(2+jt)\omega} d\omega + \frac{2}{\pi} \int_0^{\infty} e^{(-2+jt)\omega} d\omega \\ &= \frac{2}{\pi} \left[ \frac{1}{2+jt} (1-0) + \frac{1}{-2+jt} (0-1) \right] = \frac{2}{\pi} \left( \frac{1}{2+jt} + \frac{1}{2-jt} \right) = \frac{2}{\pi} \frac{4}{4+t^2} \\ \therefore f(t) &= \frac{8}{\pi(4+t^2)} \therefore f(0.8) = \frac{8}{\pi \times 4.64} = \boxed{0.5488} \end{aligned}$$

$$(c) \quad F(j\omega) = 4 \cos \pi \omega [u(\omega+0.5) - u(\omega-0.5)]$$

$$\begin{aligned} \therefore f(t) &= \frac{4}{2\pi} \int_{-0.5}^{0.5} \cos \pi \omega \times e^{j\omega t} d\omega = \frac{2}{\pi} \int_{-0.5}^{0.5} \frac{1}{2} (e^{j\pi\omega} + e^{-j\pi\omega}) e^{j\omega t} d\omega \\ &= \frac{1}{\pi} \int_{-0.5}^{0.5} [e^{(j\pi+jt)\omega} + e^{(-j0.5\pi-j0.5t)\omega}] d\omega \\ &= \frac{1}{\pi} \left[ \frac{1}{j(\pi+t)} (e^{j0.5\pi+j0.5t} - e^{-j0.5\pi-j0.5t}) + \frac{1}{j(-\pi+t)} (e^{-j0.5\pi+j0.5t} - e^{j0.5\pi-j0.5t}) \right] \\ &= \frac{1}{\pi} \left[ \frac{1}{j(\pi+t)} (je^{j0.5t} + je^{-j0.5t}) + \frac{1}{j(-\pi+t)} (-je^{j0.5t} - je^{-j0.5t}) \right] \\ &= \frac{1}{\pi} \left[ \frac{1}{\pi+t} 2 \cos 0.5t - \frac{1}{-\pi+t} 2 \cos 0.5t \right] = \frac{2 \cos 0.5t}{\pi} \left( \frac{1}{\pi+t} - \frac{1}{-\pi+t} \right) \\ &= 2 \cos 0.5t \left( \frac{-2}{t^2 - \pi^2} \right) = \frac{4}{\pi^2 - t^2} \cos 0.5t \therefore f(0.8) = \boxed{0.3992} \end{aligned}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

32.  $v(t) = 20e^{1.5t} u(-t-2) \text{ V}$

$$\begin{aligned} \text{(a)} \quad F_v(j\omega) &= \int_{-\infty}^{\infty} 20e^{1.5t} u(-t-2) e^{-j\omega t} dt = \int_{-\infty}^{-2} 20e^{1.5t-j\omega t} dt \\ &= \frac{20}{1.5-j\omega} e^{(1.5-j\omega)t} \bigg|_{-\infty}^{-2} = \frac{20}{1.5-j\omega} e^{-3+j2\omega} \therefore F_v(j0) = \frac{20}{1.5} e^{-3} = \boxed{0.6638} \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad F_v(j\omega) &= A_v(\omega) + B_v(\omega) = \frac{20}{1.5-j\omega} e^{-3} e^{j2\omega} \\ \therefore F_v(j2) &= \frac{20}{1.5-j2} e^{-3} e^{j4} = 0.39830 \angle 282.31^\circ = 0.08494 - j0.38913 \\ \therefore A_v(2) &= \boxed{0.08494} \end{aligned}$$

$$\text{(c)} \quad B_v(2) = \boxed{-0.3891}$$

$$\text{(d)} \quad |F_v(j2)| = \boxed{0.3983}$$

$$\text{(e)} \quad \boxed{\phi_v(j2) = 282.3^\circ \text{ or } -77.69^\circ}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

$$33. \quad |I(j\omega)| = 3 \cos 10\omega [u(\omega + 0.05\pi) - u(\omega - 0.05\pi)]$$

$$\begin{aligned} (a) \quad W &= 4 \times \frac{1}{2\pi} \int_{-\infty}^{\infty} |I(j\omega)|^2 d\omega = \frac{2}{\pi} \int_{-0.05\pi}^{0.05\pi} 9 \cos^2 10\omega d\omega \\ &= \frac{18}{\pi} \int_{-\pi/20}^{\pi/20} \left( \frac{1}{2} + \frac{1}{2} \cos 20\omega \right) d\omega = \frac{9}{\pi} \times 0.1\pi + \frac{9}{\pi} \frac{1}{20} \sin 20\omega \bigg|_{-\pi/20}^{\pi/20} = 0.9 \text{ J} \end{aligned}$$

$$\begin{aligned} (b) \quad \frac{9}{\pi} \int_{-\omega_x}^{\omega_x} (1 + \cos 20\omega) d\omega &= 0.45 = \frac{9}{\pi} \left[ 2\omega_x + \frac{1}{20} \times 2 \sin 20\omega_x \right] \\ \therefore 0.05\pi &= 2\omega_x + 0.1 \sin 20\omega_x, \quad \omega_x = 0.04159 \text{ rad/s} \end{aligned}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

34.  $f(t) = 10te^{-4t} u(t)$

(a) 
$$W_{\Omega} = \int_0^{\infty} f^2(t) dt = \int_0^{\infty} 100t^2 e^{-8t} dt = 100 \times \frac{e^{-8t}}{(-512)} (64t^2 + 16t + 2) \Big|_0^{\infty}$$
  

$$= \frac{100}{512} \times 2 = 0.3906 \text{ J}$$

(b) 
$$F(j\omega) = \mathcal{F}\{10te^{-4t} u(t)\} = 10 \int_0^{\infty} t e^{-(4+j\omega)t} dt = \frac{10e^{-(4+j\omega)t}}{(4+j\omega)^2} [-(4+j\omega)t - 1] \Big|_0^{\infty}$$
  

$$= \frac{10}{(4+j\omega)^2} \therefore |F(j\omega)| = \frac{10}{\omega^2 + 16}$$

(c) 
$$|F(j\omega)|^2 = \frac{100}{(\omega^2 + 16)^2}$$

$$|F(j\omega)|_{\omega=0}^2 = 390.6 \text{ mJ/Hz}, |F(j\omega)|_{\omega=4}^2 = 97.66 \text{ mJ/Hz}$$



## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

35.  $v(t) = 8e^{-2|t|} \text{ V}$

(a)  $W_{\Omega} = \int_{-\infty}^{\infty} v^2(t) dt = 2 \times 64 \int_0^{\infty} e^{-4t} dt = 32 \text{ J}$

(b)  $F_v(j\omega) = \int_{-\infty}^{\infty} e^{-j\omega t} v(t) dt = 8 \int_{-\infty}^{\infty} e^{-2|t|} e^{-j\omega t} dt$

$$\therefore F_v(j\omega) = 8 \int_{-\infty}^0 e^{(2-j\omega)t} dt + 8 \int_0^{\infty} e^{-(2+j\omega)t} dt$$

$$= \frac{8}{2-j\omega} e^{(2-j\omega)t} \Big|_{-\infty}^0 - \frac{8}{2+j\omega} e^{-(2+j\omega)t} \Big|_0^{\infty} = \frac{8}{2-j\omega} + \frac{8}{2+j\omega} = \frac{32}{4+\omega^2} = |F_v(j\omega)|$$

(c)  $0.9 \times 32 = \frac{1}{2\pi} \int_{-\omega_1}^{\omega_1} \frac{32^2}{(\omega^2+4)^2} d\omega = \frac{32^2}{2\pi} \left[ \frac{\omega}{8(\omega_1^2+4)} + \frac{1}{16} \tan^{-1} \frac{\omega_1}{2} \right]$

$$\therefore 0.9 = \frac{16}{\pi} \times 2 \left[ \frac{\omega_1}{8(\omega_1^2+4)} + \frac{1}{16} \frac{\omega_1}{2} \right] = \frac{2}{\pi} \left[ \frac{2\omega_1}{\omega_1^2+4} + \tan^{-1} \frac{\omega_1}{2} \right]$$

$$\therefore 0.45\pi = \frac{2\omega_1}{\omega_1^2+4} + \tan^{-1} \frac{\omega_1}{2} \therefore \omega_1 = 2.7174 \text{ rad/s (by SOLVE)}$$

## **CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS**

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

36.

(a) Prove:  $\mathcal{F}\{f(t-t_o)\} = e^{-j\omega t_o} \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t-t_o)e^{-j\omega t} dt$  Let  $t-t_o = \tau$

$$\therefore \mathcal{F}\{f(t-t_o)\} = \int_{-\infty}^{\infty} f(\tau)e^{-j\omega\tau} e^{-j\omega t_o} d\tau = e^{-j\omega t_o} \mathcal{F}\{f(t)\}$$

(b) Prove:  $\mathcal{F}\{f(t)\} = j\omega \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} e^{-j\omega t} \frac{df}{dt} dt$  Let  $u = e^{-j\omega t}$ ,  $du = -j\omega e^{-j\omega t} dt$ ,

$$dv = df, v = f \therefore \mathcal{F}\{f(t)\} = f(t)e^{-j\omega t} \Big|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} j\omega f(t)e^{-j\omega t} dt$$

We assume  $f(\pm\infty) = 0 \therefore \mathcal{F}\{f(t)\} = j\omega \mathcal{F}\{f(t)\}$

(c) Prove:  $\mathcal{F}\{f(kt)\} = \frac{1}{|k|} \mathcal{F}\left(\frac{j\omega}{k}\right) = \int_{-\infty}^{\infty} f(kt)e^{-j\omega t} dt$  Let  $\tau = kt$ ,  $k > 0$

$$\therefore \mathcal{F}\{f(kt)\} = \int_{-\infty}^{\infty} f(\tau)e^{-j\omega\tau/k} \frac{1}{k} d\tau = \frac{1}{k} \mathcal{F}\left(\frac{j\omega}{k}\right)$$

If  $k < 0$ , limits are interchanged and we get:  $-\frac{1}{k} \mathcal{F}\left(\frac{j\omega}{k}\right)$

$$\therefore \mathcal{F}\{f(kt)\} = \frac{1}{|k|} \mathcal{F}\left(\frac{j\omega}{k}\right)$$

(d) Prove:  $\mathcal{F}\{f(-t)\} = \mathcal{F}(-j\omega)$  Let  $k = 1$  in (c) above

(e) Prove:  $\mathcal{F}\{tf(t)\} = j \frac{d}{d\omega} \mathcal{F}(j\omega)$  Now,  $\mathcal{F}(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt$

$$\therefore \frac{d\mathcal{F}(j\omega)}{d\omega} = \int_{-\infty}^{\infty} f(t)(-jt)e^{-j\omega t} dt = -j \mathcal{F}\{tf(t)\} \therefore \mathcal{F}\{tf(t)\} = j\omega \mathcal{F}\{f(t)\}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

37.

$$(a) \quad f(t) = 4[\text{sgn}(t)\delta(t-1)] \quad \therefore \mathcal{F}\{4[\text{sgn}(t)\delta(t-1)]\} = \mathcal{F}\{4\text{sgn}(1)\delta(t-1)\} = \mathcal{F}\{4\delta(t-1)\} = 4e^{-j\omega}$$

$$(b) \quad f(t) = 4[\text{sgn}(t-1)\delta(t)] \quad \therefore \mathcal{F}\{4\text{sgn}(-1)\delta(t)\} = \mathcal{F}\{-4\delta(t)\} = -4$$

$$(c) \quad f(t) = 4\sin(10t - 30^\circ) \quad \therefore \mathcal{F}\{4\sin(10t - 30^\circ)\} = \mathcal{F}\left\{\frac{4}{j2}\left[e^{j(10t-30^\circ)} - e^{-j(10t-30^\circ)}\right]\right\} =$$

$$\mathcal{F}\{-j2e^{-j30^\circ}e^{j10t} + j2e^{j30^\circ}e^{-j10t}\} = -j2e^{-j\pi/6}2\pi\delta(\omega-10) + j2e^{j\pi/6}2\pi\delta(\omega+10)$$

$$= -j4\pi[e^{-j\pi/6}\delta(\omega-10) - e^{j\pi/6}\delta(\omega+10)]$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

38.

$$\begin{aligned}
 \text{(a)} \quad f(t) &= A \cos(\omega_o t + \phi) \quad \therefore F(j\omega) = \mathcal{F}\{A \cos \phi \cos \omega_o t - A \sin \phi \sin \omega_o t\} = \\
 &A \cos \phi \{\pi[\delta(\omega + \omega_o) + \delta(\omega - \omega_o)]\} - A \sin \phi \left\{ \frac{\pi}{j} [\delta(\omega - \omega_o) - \delta(\omega + \omega_o)] \right\} = \\
 &\pi A \{ \cos \phi [\delta(\omega + \omega_o) + \delta(\omega - \omega_o)] + j \sin \phi [\delta(\omega - \omega_o) - \delta(\omega + \omega_o)] \} \\
 \therefore F(j\omega) &= \pi A [e^{j\phi} \delta(\omega - \omega_o) + e^{-j\phi} \delta(\omega + \omega_o)]
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad f(t) &= 3 \operatorname{sgn}(t-2) - 2\delta(t) - u(t-1) \quad \therefore F(j\omega) = e^{-j2\omega} \times 3 \times \frac{2}{j\omega} - 2 - e^{-j\omega} \left[ \pi\delta(\omega) + \frac{1}{j\omega} \right] \\
 \therefore F(j\omega) &= -j \frac{6}{\omega} e^{-j2\omega} - 2 - e^{-j\omega} \left[ \pi\delta(\omega) - j \frac{1}{\omega} \right]
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad f(t) &= \sinh kt \, u(t) \quad \therefore F(j\omega) = \mathcal{F} \left\{ \frac{1}{2} [e^{kt} - e^{-kt}] u(t) \right\} \\
 \therefore F(j\omega) &= \frac{1}{2} \frac{1}{-k + j\omega} - \frac{1}{2} \frac{1}{k + j\omega} = \frac{k + j\omega + k - j\omega}{2(-k^2 - \omega^2)} = \frac{-k}{\omega^2 + k^2}
 \end{aligned}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

39.

$$(a) \quad F(j\omega) = 3u(\omega+3) - 3u(\omega-1) \quad \therefore f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} [3u(\omega+3) - 3u(\omega-1)] e^{j\omega t} d\omega$$

$$\therefore f(t) = \frac{3}{2\pi} \int_{-3}^1 e^{j\omega t} d\omega = \frac{3}{2\pi} \frac{1}{jt} e^{j\omega t} \Big|_{-3}^1 = \frac{3}{j2\pi t} (e^{jt} - e^{-j3t})$$

$$\therefore f(5) = -j \frac{3}{10\pi} (1 \angle 5^{rad} - 1 \angle -15^{rad}) = 0.10390 \angle -106.48^\circ$$

$$(b) \quad F(j\omega) = 3u(-3-\omega) + 3u(\omega-1) \rightarrow$$

$$\therefore F(j\omega) = 3 - F_a(j\omega)$$

$$f(t) = 3\delta(t) - \frac{3}{j2\pi t} (e^{jt} - e^{-j3t}) \quad \therefore f(5) = 0 - 0.10390 \angle -106.48^\circ$$

$$\text{so } f(5) = 0.1039 \angle 73.52^\circ$$

$$(c) \quad F(j\omega) = 2\delta(\omega) + 3u(-3-\omega) + 3u(\omega-1) \quad \text{Now, } \mathcal{F}\{2\delta(\omega)\} = \frac{2}{2\pi} = \frac{1}{\pi}$$

$$\therefore f(t) = \frac{1}{\pi} + \left[ -\frac{3}{j2\pi t} (e^{jt} - e^{-j3t}) \right] \quad \therefore f(5) = \frac{1}{\pi} - 0.10390 \angle -106.48^\circ = 0.3618 \angle 15.985^\circ$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

40.

$$(a) \quad F(j\omega) = \frac{3}{1+j\omega} + \frac{3}{j\omega} + 3 + 3\delta(\omega-1)$$

$$\therefore f(t) = 3e^{-t}u(t) + 1.5\operatorname{sgn}(t) + 3\delta(t) + \frac{1.5}{\pi}e^{jt}$$

$$(b) \quad F(j\omega) = \frac{1}{\omega} 5 \sin 4\omega = 8 \frac{\sin \omega 8/2}{\omega 8/2} \times 2.5$$

$$\therefore f(t) = 2.5[u(t+4) - u(t-4)]$$

$$(c) \quad F(j\omega) = \frac{6(3+j\omega)}{(3+j\omega)^2 + 4} = \frac{6(3+j\omega)}{(3+j\omega)^2 + 2^2} \therefore f(t) = 3^{-3t} \cos 2t u(t)$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

41.

$T = 4$ , periodic; find exp'l form

$$\therefore c_n = \frac{1}{4} \int_{-1}^1 10te^{-jn\pi t/2} dt$$

$$\therefore c_n = 2.5 \left[ e^{-jn\pi t/2} \left( \frac{t}{-jn\pi/2} - \frac{1}{-n^2\pi^2/4} \right) \right]_{-1}^1$$

$$\therefore c_n = 2.5 \left[ e^{-jn\pi/2} \left( \frac{1}{-jn\pi/2} + \frac{1}{n^2\pi^2/4} \right) - e^{jn\pi/2} \left( \frac{1}{jn\pi/2} + \frac{1}{n^2\pi^2/4} \right) \right]$$

$$= 2.5 \left[ \frac{1}{jn\pi/2} (-e^{-jn\pi/2} - e^{jn\pi/2}) + \frac{4}{n^2\pi^2} (e^{-jn\pi/2} - e^{jn\pi/2}) \right]$$

$$= \frac{j5}{n\pi} \times 2 \cos \frac{n\pi}{2} + \frac{10}{n^2\pi^2} \left( -j2 \sin \frac{n\pi}{2} \right)$$

$$\therefore f(t) = \sum_{-\infty}^{\infty} \left[ \frac{j10}{n\pi} \cos \frac{n\pi}{2} - j \frac{20}{n^2\pi^2} \sin \frac{n\pi}{2} \right] e^{jn\pi t/2}$$

$$\therefore F(j\omega) = \sum_{-\infty}^{\infty} \left[ \frac{j10}{n\pi} \cos \frac{n\pi}{2} - j \frac{20}{n^2\pi^2} \sin \frac{n\pi}{2} \right] 2\pi \delta \left( \omega - \frac{n\pi}{2} \right)$$



42.

$$T = 4 \text{ ms}, f_1(t) = 10u(t) - 6u(t - 0.001) - 4u(t - 0.003)$$

$$\begin{aligned} c_n &= \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega_o t} dt, \omega_o = \frac{2\pi 10^3}{4} = 500\pi \\ &= 250 \int_{-0.002}^{0.002} f(t) e^{-j500n\pi t} dt \\ &= 250 \left[ \int_{-0.002}^{-0.001} 4e^{-j500n\pi t} dt + \int_0^{0.001} 10e^{-j500n\pi t} dt + \int_{0.001}^{0.002} 4e^{-j500n\pi t} dt \right] \\ &= \frac{j250}{500n\pi} \left[ 4e^{-j500n\pi t} \Big|_{-0.002}^{0.001} + 10e^{-j500n\pi t} \Big|_0^{0.001} + 4e^{-j500n\pi t} \Big|_{0.001}^{0.002} \right] \\ &= \frac{j}{2n\pi} \left[ 4(e^{jn\pi/2} - e^{-jn\pi}) + 10(e^{-jn\pi/2} - 1) + 4(e^{-jn\pi} - e^{-jn\pi/2}) \right] \\ &= \frac{j}{2n\pi} \left[ 4(e^{jn\pi/2} - e^{-jn\pi/2}) - 4(e^{jn\pi} - e^{-jn\pi}) + 10(e^{-jn\pi/2} - 1) \right] \\ &= \frac{j}{2n\pi} \left[ 4j2 \sin \frac{n\pi}{2} - 4j2 \sin n\pi + 10(e^{-jn\pi/2} - 1) \right] \\ &= \frac{1}{n\pi} \left[ -4 \sin \frac{n\pi}{2} + j5(e^{-jn\pi/2} - 1) \right] = \frac{1}{n\pi} \left[ j5e^{-jn\pi/4} (e^{-jn\pi/4} - e^{jn\pi/4}) - 4 \sin \frac{n\pi}{2} \right] \\ &= \frac{1}{n\pi} \left[ j5e^{-jn\pi/4} \left( -j2 \sin \frac{n\pi}{4} \right) - 4 \sin \frac{n\pi}{2} \right] = \frac{1}{n\pi} \left[ 10e^{-jn\pi/4} \sin \frac{n\pi}{4} - 4 \sin \frac{n\pi}{2} \right] \\ \therefore F(j\omega) &= 2\pi \sum_{-\infty}^{\infty} \frac{1}{n\pi} \left( 10e^{-jn\pi/4} \sin \frac{n\pi}{4} - 4 \sin \frac{n\pi}{2} \right) \delta(\omega - 500n\pi) \end{aligned}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

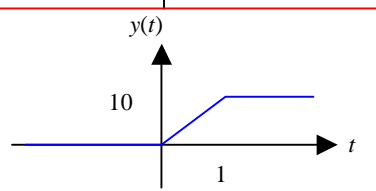
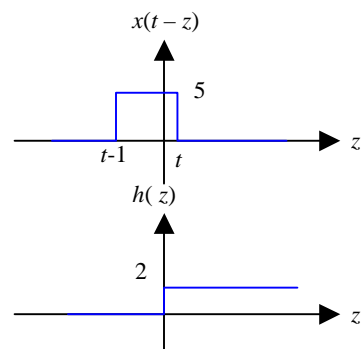
43.

$$\begin{aligned}
 F(j\omega) &= 20 \sum_{n=-\infty}^{\infty} \frac{1}{|n|!+1} \delta(\omega - 20n) \\
 &= 20 \left[ \frac{1}{1+1} \delta(\omega) + \frac{1}{1+1} \delta(\omega + 20) + \frac{1}{1+1} \delta(\omega - 20) + \frac{1}{2+1} \delta(\omega + 40) + \frac{1}{3} \delta(\omega - 40) \right. \\
 &\quad \left. + \frac{1}{7} \delta(\omega + 60) + \frac{1}{7} \delta(\omega - 60) + \dots \right] \\
 &= 10\delta(\omega) + \frac{20}{2\pi} [\pi\delta(\omega + 20) + \pi\delta(\omega - 20)] + \frac{20}{3\pi} [\pi\delta(\omega + 40) + \pi\delta(\omega - 40)] + \\
 &\quad \frac{20}{7\pi} [\pi\delta(\omega + 60) + \pi\delta(\omega - 60)] + \frac{20}{25\pi} [\pi\delta(\omega + 80) + \pi\delta(\omega - 80)] + \dots \\
 \therefore f(t) &= \frac{10}{2\pi} + \frac{20}{2\pi} \cos 20t + \frac{20}{3\pi} \cos 40t + \frac{20}{7\pi} \cos 60t + \frac{20}{25\pi} \cos 80t + \dots \\
 &= \frac{20}{\pi} \left[ 0.25 + \frac{1}{2} \cos 20t + \frac{1}{3} \cos 40t + \frac{1}{7} \cos 60t + \frac{1}{25} \cos 80t + \dots \right] \\
 \therefore f(0.05) &= \frac{20}{\pi} \left[ 0.25 + \frac{1}{2} \cos 1^{rad} + \frac{1}{3} \cos 2 + \frac{1}{7} \cos 3 + \frac{1}{25} \cos 4 + \dots \right] = \boxed{1.3858}
 \end{aligned}$$

# CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

44. Input =  $x(t) = 5[u(t) - u(t-1)]$       $y(t) = \int_{-\infty}^t x(z) h(t-z) dz$

(a)  $h(t) = 2u(t)$

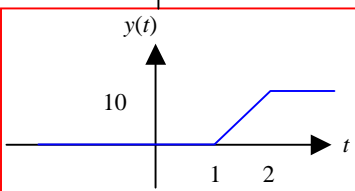
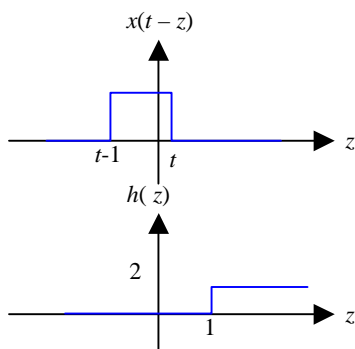


$t < 0$ :  
 $y(t) = 0$

$0 < t < 1$ :  
 $y(t) = \int_0^t 10 dz = 10t$

$t > 1$ :  
 $y(t) = \int_{t-1}^t 10 dz = 10$

(b)  $h(t) = 2u(t-1)$

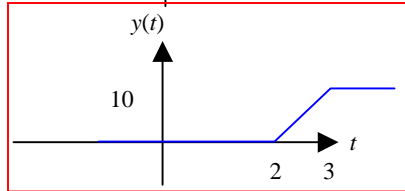
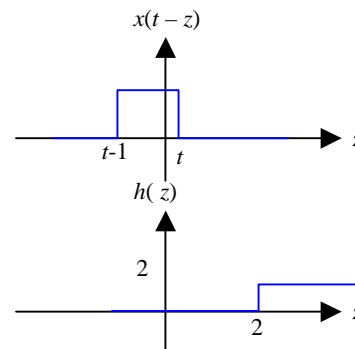


$t < 1$ :  $y(t) = 0$

$1 < t < 2$ :  
 $y(t) = \int_1^t 10 dz = 10(t-1)$

$t > 2$ :  
 $y(t) = \int_{t-1}^t 10 dz = 10$

(c)  $h(t) = 2u(t-2)$



$t < 2$ :  $y(t) = 0$

$2 < t < 3$ :  
 $y(t) = \int_2^t 10 dz = 10(t-2)$

$t > 3$ :  
 $y(t) = \int_{t-1}^t 10 dz = 10$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

45.  $x(t) = 5[u(t) - u(t-2)]; h(t) = 2[u(t-1) - u(t-2)]$

$$y(t) = \int_{-\infty}^t x(z) h(t-z) dz$$

$$t < 1: y(t) = 0$$

$$1 < t < 2: y(t) = \int_0^{t-1} 10 dz = 10(t-1)$$

$$2 < t < 3: y(t) = 10$$

$$3 < t < 4: y(t) = \int_{t-2}^2 10 dz = 10(2-t+2) = 10(4-t)$$

$$t > 4: y(t) = 0$$

$$\therefore y(-0.4) = 0; \quad y(0.4) = 0; \quad y(1.4) = 4$$

$$y(2.4) = 10; \quad y(3.4) = 6; \quad y(4.4) = 0$$

or....  $y(t) = \int_0^{\infty} x(t-z) h(z) dz$

$$t < 1: y(t) = 0$$

$$1 < t < 2: y(t) = \int_1^t 10 dz = 10(t-1)$$

$$2 < t < 3: y(t) = 10$$

$$3 < t < 4: y(t) = \int_{t-2}^2 10 dz = 10(2-t+2) = 10(4-t)$$

$$t > 4: y(t) = 0$$

$$\text{same answers as above}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

46.

$$h(t) = 3[e^{-t} - e^{-2t}], \quad x(t) = u(t)$$

$$y(t) = \int_{-\infty}^t x(z)h(t-z) dz$$

$$= \int_0^t 3[e^{-(t-z)} - e^{-2(t-z)}] dz$$

$$= 3e^{-t} [e^z]_0^t - 3e^{-2t} \left[ \frac{1}{2} e^{2z} \right]_0^t$$

$$= 3e^{-t} (e^t - 1) - 1.5e^{-2t} (e^{2t} - 1)$$

$$\therefore y(t) = 3(1 - e^{-t}) - 1.5(1 - e^{-2t}) = 1.5 - 3e^{-t} + 1.5e^{-2t}, \quad t > 0$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

47.

$$y(t) = \int_0^{\infty} x(t-2)h(z)dz$$

$$h(t) = \frac{2}{3} (5-t), 2 < t < 5$$

$$(a) \quad y(t) = \int_2^5 10 \times \frac{2}{3} (5-z) dz = \frac{20}{3} \int_2^5 (5-z) dz$$

Note:  $h(z)$  is in window for  $4 < t < 6$

$$(b) \quad y(t) = \frac{20}{3} \left( -\frac{1}{2} \right) (5-z)^2 \Big|_2^5$$

$$= -\frac{10}{3} (0-9) = 30 \text{ at } t = 5$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

48.  $x(t) = 5e^{-(t-2)} u(t-2)$ ,  $h(t) = (4t-16) [u(t-4) - u(t-7)]$ ,  $y(t) = \int_0^{\infty} x(t-z) h(z) dz$

(a)  $t < 6$ :  $y(t) = 0 \quad \therefore y(5) = 0$

(b)  $t = 8$ :  $y(8) = \int_4^6 5e^{-(8-z-2)} (4z-16) dz$

$$\therefore y(8) = 20e^{-6} \int_4^6 z e^z dz - 80e^{-6} \int_4^6 e^z dz$$

$$= 20e^{-6} \left[ \frac{e^z}{1} (z-1) \right]_4^6 - 80e^{-6} (e^6 - e^4)$$

$$= 20e^{-6} (5e^6 - 3e^4) - 80 + 80e^{-2} = 20 + 80e^{-2} - 60e^{-2}$$

$$= 20(1 + e^{-2}) = 22.71$$

(c)  $t = 10$ :  $y(10) = \int_4^7 5e^{-(10-z-2)} (4z-16) dz$

$$\therefore y(10) = \int_4^7 20e^{-8} e^z (z-4) dz$$

$$\therefore y(10) = 20e^{-8} \int_4^7 z e^z dz - 80e^{-8} \int_4^7 e^z dz = 20e^{-8} [e^z (z-1)]_4^7 - 80e^{-8} (e^7 - e^4)$$

$$= 20e^{-8} (6e^7 - 3e^4) - 80(e^{-1} - e^{-4}) = 40e^{-1} + 20e^{-4} = 15.081$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

49.

$h(t) = \sin t, 0 < t < \pi; 0$  elsewhere, Let  $x(t) = e^{-t}u(t)$

$$y(t) = \int_0^{\infty} x(t-z) h(z) dz$$

$$t < 0: y(t) = 0$$

$$0 < t < \pi: y(t) = \int_0^t \sin z \times e^{-t+z} dz = e^{-t} \int_0^t e^z \sin z dz$$

$$\begin{aligned} \therefore y(t) &= e^{-t} \left[ \frac{1}{2} e^z (\sin z - \cos z) \right]_0^t \\ &= \frac{1}{2} e^{-t} [e^t (\sin t - \cos t) + 1] \\ &= \frac{1}{2} (\sin t - \cos t + e^{-t}) \end{aligned}$$

(a)  $y(1) = 0.3345^+$

(b)  $y(2.5) = 0.7409$

(c)  $y > \pi: y(t) = e^{-t} \int_0^{\pi} e^z \sin z dz$

$$y > \pi: y(t) = e^{-t} \left[ \frac{1}{2} e^z (\sin z - \cos z) \right]_0^{\pi} = \frac{1}{2} e^{-t} (e^{\pi} + 1) = 12.070 e^{-t}$$

$\therefore y(4) = 0.2211$



# CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

50.

$$x(t) = 0.8(t-1)[u(t-1) - u(t-3)],$$

$$h(t) = 0.2(t-2)[u(t-2) - u(t-3)]$$

$$y(t) = \int_0^{\infty} x(t-z) h(z) dz,$$

$$t < 3: y(t) = 0$$

$$(a) \quad 3 < t < 4: y(t) = \int_2^{t-1} 0.8(t-z-1)0.2(z-2) dz$$

$$\therefore y(t) = 0.16 \int_2^{t-1} (tz - 2t - z^2 + 2z - z + 2) dz$$

$$= 0.16 \int_2^{t-1} [-z^2 + (t+1)z + 2 - 2t] dz = 0.16 \left[ -\frac{1}{3}z^3 + \frac{1}{2}(t+1)z^2 + (2-2t)z \right]_2^{t-1}$$

$$= 0.16 \left[ -\frac{1}{3}(t-1)^3 + \frac{8}{3} + \frac{1}{2}(t+1)(t-1)^2 - \frac{1}{2}(t+1)4 + (2-2t)(t-1-2) \right]$$

$$\therefore y(t) = 0.16 \left[ -\frac{1}{3}t^3 + t^2 - t + \frac{1}{3} + \frac{8}{3} + \frac{1}{2}(t^2 - 1)(t-1) - 2t - 2 + 2t - 6 - 2t^2 + 6t \right]$$

$$= 0.16 \left[ \frac{1}{6}t^3 + t^2 \left( 1 - \frac{1}{2} - 2 \right) + t \left( -1 - \frac{1}{2} + 6 \right) + 3 + \frac{1}{2} - 8 \right] = 0.16 \left( \frac{1}{6}t^3 - \frac{3}{2}t^2 + \frac{9}{2}t - \frac{9}{2} \right)$$

$$\therefore y(3.8) = 13.653 \times 10^{-3}$$

$$(b) \quad 4 < t < 5: y(t) = \int_2^3 0.16(t-z-1)(z-2) dz = 0.16 \left[ -\frac{1}{3}z^3 + \frac{1}{2}(t+1)z^2 + (2-2t)z \right]_2^3$$

$$\therefore y(t) = 0.16 \left[ -\frac{1}{3}(27-8) + \frac{1}{2}(t+1)5 + (2-2t)1 \right]$$

$$= 0.16 \left[ -\frac{19}{3} + 2.5t + 2.5 + 2 - 2t \right] = 0.16 \left( 0.5t - \frac{11}{6} \right)$$

$$\therefore y(4.8) = 90.67 \times 10^{-3}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

51.

$$x(t) = 10e^{-2t}u(t), \quad h(t) = 10e^{-2t}u(t)$$

$$y(t) = \int_0^{\infty} x(t-z) h(z) dz$$

$$\therefore y(t) = \int_0^t 10e^{-2(t-z)} 10e^{-2z} dz$$

$$= 100e^{-2t} \int_0^t dz = 100e^{-2t} \times t$$

$$\therefore y(t) = 100te^{-2t}u(t)$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

52.  $h(t) = 5e^{-4t} u(t)$

(a)  $W_{\Omega} = 25 \int_{0.1}^{0.8} e^{-8t} dt = \frac{25}{8} (e^{-0.8} - e^{-6.4}) = 1.3990 \text{ J}$

$$\therefore \% = 1.3990 / \left( \frac{25}{8} \right) \times 100\% = \boxed{44.77\%}$$

(b)  $H(j\omega) = \frac{5}{j\omega + 4} \therefore W_{\Omega} = \frac{1}{\pi} \int_0^2 \frac{25}{\omega^2 + 16} d\omega = \frac{25}{\pi} \frac{1}{4} \tan^{-1} \frac{\omega}{4} \Big|_0^2$

$$\therefore W_{\Omega} = \frac{25}{4\pi} \tan^{-1} \frac{1}{2} = 0.9224 \text{ J} \therefore \% = \frac{0.9224}{25/8} \times 100\% = \boxed{29.52\%}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

53.

$$F(j\omega) = \frac{2}{(1+j\omega)(2+j\omega)} = \frac{2}{1+j\omega} - \frac{2}{2+j\omega} \therefore f(t) = (2e^{-t} - 2e^{-2t})u(t)$$

$$(a) \quad W_{\Omega} = \int_0^{\infty} (4e^{-2t} - 8e^{-3t} + 4e^{-4t}) dt = \frac{4}{2} - \frac{8}{3} + \frac{4}{4} = \frac{1}{3} \text{ J}$$

$$(b) \quad f(t) = -2e^{-t} + 4e^{-2t} = 0, \quad -2 + 4e^{-t} = 0, \quad e^t = 2, \quad t = 0.69315 \\ \therefore f_{\max} = 2(e^{-0.69315} - e^{-2 \times 0.69315}) = 0.5$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

54.

$$(a) \quad F(j\omega) = \frac{1}{j\omega(2+j\omega)(3+j\omega)} = \frac{1/6}{j\omega} - \frac{1/2}{2+j\omega} + \frac{1/3}{3+j\omega}$$

$$\therefore f(t) = \frac{1}{12} \operatorname{sgn}(t) - \frac{1}{2} e^{-2t} u(t) + \frac{1}{3} e^{-3t} u(t)$$

$$(b) \quad F(j\omega) = \frac{1+j\omega}{j\omega(2+j\omega)(3+j\omega)} = \frac{1/6}{j\omega} + \frac{1/2}{2+j\omega} - \frac{2/3}{3+j\omega}$$

$$\therefore f(t) = \frac{1}{12} \operatorname{sgn}(t) + \frac{1}{2} e^{-2t} u(t) - \frac{2}{3} e^{-3t} u(t)$$

$$(c) \quad F(j\omega) = \frac{(1+j\omega)^2}{j\omega(2+j\omega)(3+j\omega)} = \frac{1/6}{j\omega} - \frac{1/2}{2+j\omega} + \frac{4/3}{3+j\omega}$$

$$\therefore f(t) = \frac{1}{12} \operatorname{sgn}(t) - \frac{1}{2} e^{-2t} u(t) + \frac{4}{3} e^{-3t} u(t)$$

$$(d) \quad F(j\omega) = \frac{(1+j\omega)^3}{j\omega(2+j\omega)(3+j\omega)} = 1 + \frac{1/6}{j\omega} + \frac{1/2}{2+j\omega} - \frac{8/3}{3+j\omega}$$

$$\therefore f(t) = \delta(t) + \frac{1}{12} \operatorname{sgn}(t) + \frac{1}{2} e^{-2t} u(t) - \frac{8}{3} e^{-3t} u(t)$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

55.  $h(t) = 2e^{-t}u(t)$

(a)  $H(j\omega) = 2 \times \frac{1}{1+j\omega} = \frac{2}{1+j\omega}$

(b)  $\frac{1}{2} H(j\omega) = \frac{1}{1+j\omega} = \frac{1}{2} \frac{V_o}{V_i} = \frac{1/j\omega}{1+1/j\omega}$

(c)  $\text{Gain} = 2$

56.

$$V_o(j\omega) = \frac{\frac{1}{2}j\omega + \frac{1}{j\omega}}{1 + \frac{1}{2}j\omega + \frac{1}{j\omega}} = \frac{(j\omega)^2 + 2}{(j\omega)^2 + 2(j\omega) + 2}$$

$$\therefore V_o(j\omega) = \frac{(j\omega)^2 + 2(j\omega) + 2 - 2(j\omega)}{(j\omega)^2 + 2(j\omega) + 2} = 1 + \frac{-2(j\omega)}{(j\omega)^2 + 2(j\omega) + 2}$$

$$\text{Let } j\omega = x \quad \therefore V_o(x) = 1 - \frac{2x}{x + 2x + 2}; \quad x = \frac{-2 \pm \sqrt{4 - 8}}{2} = -1 \pm j1$$

$$\therefore V_o(x) = 1 + \frac{A}{x + 1 + j1} + \frac{B}{x + 1 - j1} = \square \quad \text{Let } x = 0 \quad \therefore \frac{A}{1 + j1} + \frac{B}{1 - j1} = 0$$

$$\text{Let } x = -1 \quad \therefore \frac{A}{j1} + \frac{B}{-j1} = 2 \quad \therefore A - B = j2, \quad A = B + j2 \quad \therefore \frac{B + j2}{1 + j1} + \frac{B}{1 - j1} = 0$$

$$\therefore B - jB + j2 + 2 + B + jB = 0 \quad \therefore B = -1 - j1 \quad \therefore A = -1 + j1$$

$$\therefore V_o(x) = 1 + \frac{-1 + j1}{x + 1 + j1} + \frac{-1 - j1}{x + 1 - j1}, \quad V_o(j\omega) = 1 - \frac{1 - j1}{(j\omega) + 1 + j1} - \frac{1 + j1}{(j\omega) + 1 - j1}$$

$$\therefore v_o(t) = \delta(t) - (1 - j1)e^{(-1 - j1)t}u(t) - (1 + j1)e^{(-1 + j1)t}u(t)$$

$$= \delta(t) - \sqrt{2}e^{-j45^\circ - jt - t}u(t) - \sqrt{2}e^{j45^\circ + jt - t}u(t)$$

$$= \boxed{\delta(t) - 2\sqrt{2}e^{-t} \cos(t + 45^\circ)u(t)}$$

## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

57.

$$V_c(j\omega) = 10 \frac{5/j\omega}{5/j\omega + 35 + 30(j\omega)} = \frac{10/j\omega}{1/j\omega + 7 + 6(j\omega)}$$

$$\therefore V_c(j\omega) = \frac{10}{6(j\omega)^2 + 7(j\omega) + 1} = \frac{10/6}{(j\omega)^2 + \frac{7}{6}(j\omega) + \frac{1}{6}}$$

$$\therefore j\omega = \left( -7/6 \pm \sqrt{\frac{49}{36} - \frac{24}{36}} \right) / 2 = -\frac{1}{6}, -1 \therefore V_c(j\omega) = \frac{10/6}{(j\omega + 1/6)(j\omega + 1)} = \frac{2}{j\omega + 1/6} - \frac{2}{j\omega + 1}$$

$$\therefore v_c(t) = 2(e^{-t/6} - e^{-t})u(t)$$



## CHAPTER EIGHTEEN (Fourier Analysis) SOLUTIONS

58.  $f(t) = 5e^{-2t}u(t), g(t) = 4e^{-3t}u(t)$

(a) 
$$f * g = \int_0^{\infty} f(t-z)g(z)dz$$

$$= \int_0^t 5e^{-2t}e^{2z}4e^{-3z}dz = 20e^{-2t} \int_0^t e^{-z}dz$$

$$= -20e^{-2t}(e^t - 1)u(t)$$

$$\therefore f * g = (e^{-2t} - e^{-3t})u(t)$$

(b)  $F(j\omega) = \frac{5}{j\omega + 2}, G(j\omega) = \frac{4}{j\omega + 3} \therefore F(j\omega)G(j\omega) = \frac{20}{(j\omega + 2)(j\omega + 3)}$

$$\therefore F(j\omega)G(j\omega) = \frac{20}{j\omega + 2} - \frac{20}{j\omega + 3} \therefore f * g = 20(e^{-2t} - e^{-3t})u(t)$$

## CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

1.

$$\text{Order } i_1, i_2, i_3 : -2i_1' - 6i_3' = 5 + 2\cos 10t - 3i_1 + 2i_2 \quad (1)$$

$$4i_2 = 0.05i_1 - 0.15i_2' + 0.25i_3' \quad (2)$$

$$i_2 = -2i_1 - 5i_3 + 0.4 \int_0^t (i_1 - i_3) dt + 8 \quad (3)$$

$$(1) \rightarrow -2i_1' - 6i_3' = -3i_1 + 2i_2 + 5 + 2\cos 10t = A$$

$$(2) \rightarrow 0.05i_1' - 0.15i_2' + 0.25i_3' = 4i_2 = B$$

$$(3) \rightarrow 2i_1' + i_2' + 5i_3' = 0.4i_1 - 0.4i_3 = C$$

$$\therefore i_1' = 30i_1 + 20i_2 + 50 + 20\cos 10t + 240i_2 + 3.6i_1 - 3.6i_3$$

$$\therefore i_1' = \frac{\begin{vmatrix} A & 0 & -6 \\ B & -0.15 & 0.25 \\ C & 1 & 5 \end{vmatrix}}{\begin{vmatrix} -2 & 0 & -6 \\ 0.05 & -0.15 & 0.25 \\ 2 & 1 & 5 \end{vmatrix}} = \frac{A(-1) - B(6) + C(-0.9)}{-2(-1) - 6(0.35)} = \frac{-A - 6B - 0.9C}{-0.1} = 10A + 60B + 9C$$

$$i_2' = \frac{\begin{vmatrix} -2 & A & -6 \\ 0.05 & B & 0.25 \\ 2 & C & 5 \end{vmatrix}}{-0.1} = -10[-A(-0.25) + B(2) - C(-0.2)] = -2.5A - 20B - 2C$$

$$i_2' = 7.5i_1 - 5i_2 - 12.5 - 5\cos 10t - 80i_2 - 0.8i_1 + 0.8i_3$$

$$\therefore i_2' = 6.7i_1 - 85i_2 + 0.8i_3 - 12.5 - 5\cos 10t$$

$$i_3' = \frac{\begin{vmatrix} -2 & 0 & A \\ 0.05 & -0.15 & B \\ 2 & 1 & C \end{vmatrix}}{-0.1} = -10[A(0.35) - B(-2) + C(0.3)] = -3.5A - 20B - 3C$$

$$i_3' = 10.5i_1 - 7i_2 - 17.5 - 7\cos 10t - 80i_2 - 1.2i_1 + 1.2i_3$$

$$\therefore i_3' = 9.3i_1 - 87i_2 + 1.2i_3 - 17.5 - 7\cos 10t \leftarrow$$

## CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

2.  $x' + y' = x + y + 1$ ,  $x' - 2y' = 2x - y - 1$

(a) Order  $x, y$  M by 2:  $2x' + 2y' = 2x + 2y + 2$ ; add:  $3x' = 4x + y + 1$

$$\therefore x' = \frac{4}{3}x + \frac{1}{3}y + \frac{1}{3} \text{ and } y' = -\frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}$$

(b)  $x'' = \frac{4}{3}x' + \frac{1}{3}\left(-\frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}\right) = \frac{4}{3}x' - \frac{1}{9}x + \frac{2}{9}y + \frac{2}{9} = \frac{4}{3}x'$

$$-\frac{1}{9}x + \frac{2}{9}(3x' - 4x - 1) + \frac{2}{9}$$

$$\therefore x'' = 2x' - x, \text{ or } x'' - 2x' + x = 0$$

(c) Let  $x(0) = 2$  and  $y(0) = -5$   $\therefore x'(0) = \frac{4}{3}(2) + \frac{1}{3}(-5) + \frac{1}{3} = \frac{8}{3} - \frac{5}{3} + \frac{1}{3} = \frac{4}{3}$

$$\text{Also, } y'(0) = -\frac{1}{3}(2) + \frac{2}{3}(-5) + \frac{2}{3} = -\frac{2}{3} - \frac{10}{3} + \frac{2}{3} = -\frac{10}{3}$$

$$\therefore x''(0) = 2x'(0) - x(0) = \frac{8}{3} - 2 = \frac{2}{3}$$

$$x'''(0) = 2x''(0) - x'(0) = 2\left(\frac{2}{3}\right) - \frac{4}{3} = 0$$

## CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

3.

$$x' - 2y - 3z' = f_1(t), \quad 2x' + 5z = 3, \quad z' - 2y' - x = 0 \quad \text{Order } x, y, z$$

$$\therefore x' = -2.5z + 1.5 \quad (1)$$

$$-2.5z + 1.5 - 2y - 3z' = f_1 \quad \therefore z' = -\frac{2}{3}y - \frac{5}{6}z + 0.5 - \frac{1}{3}f_1 \quad (3)$$

$$\therefore 2y' = z' - x = -\frac{2}{3}y - \frac{5}{6}z + \frac{1}{2} - \frac{1}{3}f_1 - x \quad \therefore y' = -\frac{1}{2}x - \frac{1}{3}y - \frac{5}{12}z + \frac{1}{4} - \frac{1}{6}f_1 \quad (2)$$

## CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

4.  $x' = -2x - 3y + 4$ ,  $y' = 5x - 6y + 7$ ,  $x(0) = 2$ ,  $y(0) = \frac{1}{3}$

(a)  $x'' = -2x' - 3y'$ ,  $x'(0) = -2(2) - 3\left(\frac{1}{3}\right) + 4 = -1$ ,  $y'(0) = 5(2) - 6\left(\frac{1}{3}\right) + 7 = 15$

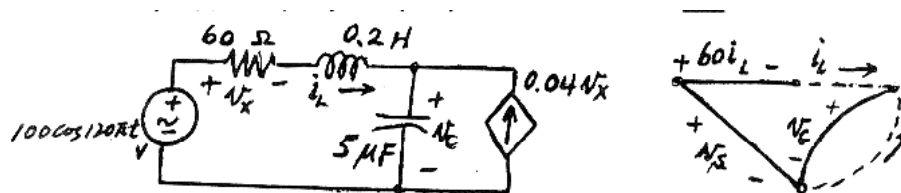
$$\therefore x''(0) = -2(-1) - 3(15) = 2 - 45 = -43$$

(b)  $y'' = 5x' - 6y'$   $\therefore y''(0) = 5(-1) - 6(15) = -95$

(c)  $y''' = 5x'' - 6y''$   $\therefore y'''(0) = 5(-43) - 6(-95) = -215 + 570 = 355$

# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

5.



$$\therefore 0.2i'_L = v_s - 60i_L - v_C$$

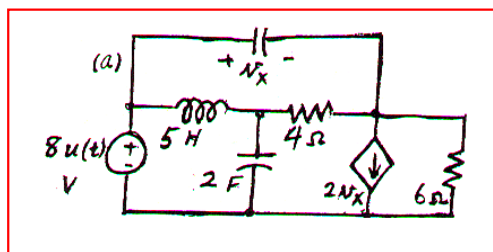
$$\therefore i_L = -300i'_L - 5v_C + 500 \cos 120\pi t \quad (1)$$

$$5 \times 10^{-6} v'_C = i_L + 2.4i'_L = 3.4i'_L \quad \therefore v'_C = 6.8 \times 10^5 i'_L \quad (2)$$

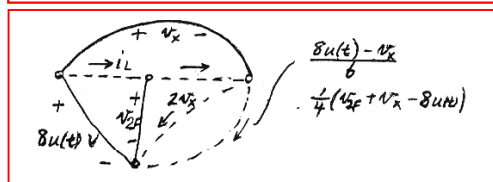
# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

6.

(a)



(b)



(c) Order:  $i_L, v_x, v_{2F} \therefore 5i'_L = -v_{2F} + 8u(t) \therefore i'_L = -0.2v_{2F} + 1.6u(t)$  (1)

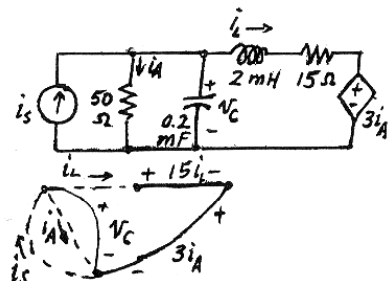
$$3v'_x = -\frac{1}{4}(v_{2F} + v_x - 8u(t)) + 2v_x + \frac{1}{6}(8u(t) - v_x) = \frac{19}{12}v_x - \frac{1}{4}v_{2F} + \frac{10}{3}u(t)$$

$$\therefore v'_x = \frac{19}{36}v_x - \frac{1}{12}v_{2F} + \frac{10}{9}u(t) \quad (2)$$

$$2v'_{2F} = i_L - \frac{1}{4}(v_{2F} + v_x - 8u(t)) \therefore v'_{2F} = \frac{1}{2}i_L - \frac{1}{8}v_x - \frac{1}{8}v_{2F} + u(t) \quad (3)$$

# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

7.



(a) Order:  $i_L, v_C$

$$0.002i_L' = v_C - 3i_A - 15i_L, i_A = 0.02v_C$$

$$\therefore 0.002i_L' = -15i_L + 0.94v_C, i_L' = -7500i_L + 470v_C \quad (1)$$

$$2 \times 10^{-4} v_C' = -i_L - i_A + i_s = -i_L - 0.02v_C + i_s$$

$$\therefore v_C' = -5000i_L - 100v_C + 5000i_s \quad (2)$$

(b) Order:  $v_1, v_2, v_3$

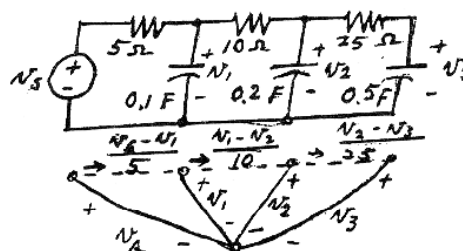
$$0.1v_1' = -0.2v_1 + 0.2v_s - 0.1v_1 + 0.1v_2$$

$$\therefore v_1' = -3v_1 + v_2 + 2v_s \quad (1)$$

$$0.2v_2' = 0.1v_1 - 0.1v_2 - 0.04v_2 + 0.04v_s$$

$$\therefore v_2' = 0.5v_1 - 0.7v_2 + 0.2v_s \quad (2)$$

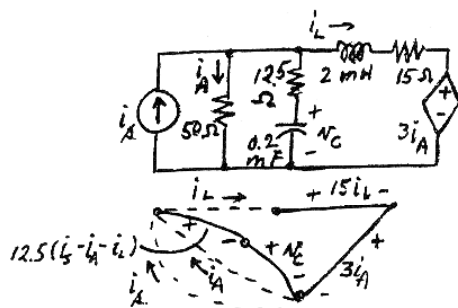
$$0.5v_3' = 0.04v_2 - 0.04v_3, v_3' = 0.08v_2 - 0.08v_3 \quad (3)$$





# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

8.



$$i_A = 0.02(12.5i_s - 12.5i_A - 12.5i_L + v_C)$$

$$\therefore 1.25i_A = -0.25i_L + 0.02v_C + 0.25i_s$$

$$\therefore i_A = -0.2i_L + 0.016v_C + 0.2i_s$$

$$0.002i'_L = -15i_L - 3i_A + v_C + 12.5i_s - 12.5i_A - 12.5i_L$$

$$\therefore 0.002i'_L = -27.5i_L + v_C - 15.5(-0.2i_L + 0.016v_C + 0.2i_s) + 12.5i_s$$

$$\therefore 0.002i'_L = -24.4i_L + 0.752v_C + 9.4i_s$$

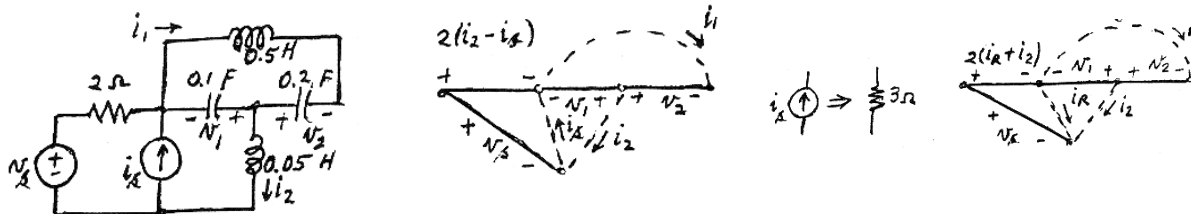
$$\therefore i'_L = -12,200i_L + 376v_C + 4700i_s \quad (1)$$

Order:  $i_L, v_C$  Also,  $0.0002v'_C = i_s - i_A - i_L = i_s + 0.2i_L - 0.016v_C - 0.2i_s - i_L$

$$\therefore 0.0002v'_C = -0.8i_L - 0.016v_C + 0.8i_s \quad \therefore v'_C = -4000i_L - 80v_C + 4000i_s \quad (2)$$

# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

9.



$$0.1v'_1 = i_1 - i_2 \quad \therefore v'_1 = 10i_1 - 10i_2 \quad (1)$$

$$0.2v'_2 = -i_1 \quad \therefore v'_2 = -5i_1 \quad (2)$$

$$0.5i'_1 = -v_1 + v_2, \quad \therefore i'_1 = -2v_1 + 2v_2 \quad (3)$$

$$0.05i'_2 = v_1 + 2i_s - 2i_2 + v_s \quad \therefore i'_2 = 20v_1 - 40i_2 + 40i_s + 20v_s \quad (4)$$

Order:  $v_1, v_2, i_1, i_2$

$$0.1v'_1 = i_1 - i_2 \quad \therefore v'_1 = 10i_1 - 10i_2 \quad (1)$$

$$0.2v'_2 = -i_1 \quad \therefore v'_2 = -5i_1 \quad (2)$$

$$0.5i'_1 = -v_1 + v_2 \quad \therefore i'_1 = -2v_1 + 2v_2 \quad (3)$$

$$0.05i'_2 = v_1 + v_s - 2i_R - 2i_2, \quad 3i_R = -2i_R - 2i_2 + v_s \quad \therefore i_R = -0.4i_2 + 0.2v_s$$

$$\therefore 0.05i'_2 = v_1 + v_s + 0.8i_2 - 0.4v_s - 2i_2 \quad \therefore i'_2 = 20v_1 - 24i_2 + 12v_s \quad (4)$$

# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

10.

(a) Order:  $i_L, v_C \therefore 0.2i'_L = -v_C - 20i_L + 20i_s$

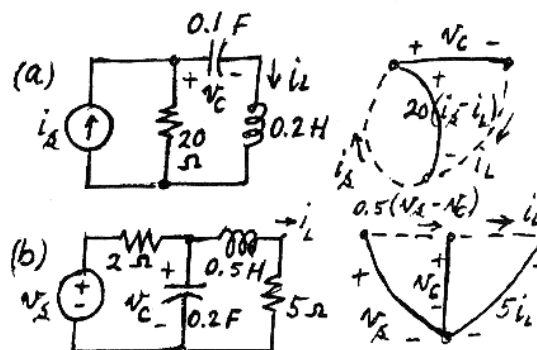
$$\therefore i'_L = -100i_L - 5v_C + 100i_s \quad (1)$$

$$0.1v'_C = i_L \therefore v'_C = 10i_L \quad (2)$$

(b)  $0.5i'_L = v_C - 5i_L \therefore i'_L = -10i_L + 2v_C \quad (1)$

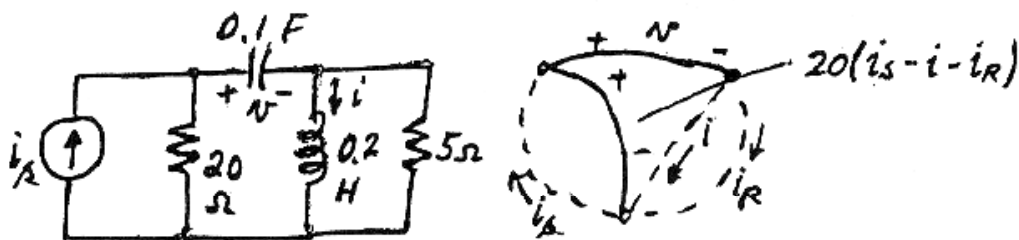
$$0.2v'_C = 0.5v_s - 0.5v_C - i_L$$

$$\therefore v'_C = -5i_L - 2.5v_C + 2.5v_s \quad (2)$$



# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

11.



$$\therefore 5i_R = -v + 20i_s - 20i - 20i_R$$

$$\therefore i_R = -0.04v - 0.8i + 0.8i_s$$

$$\therefore 0.1v' = i + i_R = -0.04v + 0.2i + 0.8i_s$$

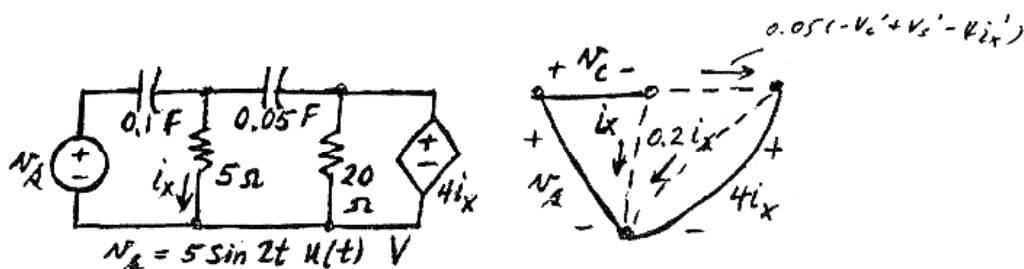
$$\text{Order: } v, i \quad \therefore v' = -0.4v + 2i + 8i_s \quad (1)$$

$$0.2i' = -v + 20i_s + 20i - 20(-0.04v + 0.2i + 0.8i_s)$$

$$\therefore 0.2i' = -0.2v - 4i + 4i_s \quad \therefore i' = -v - 20i + 20i_s \quad (2)$$

# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

12.



$$\frac{4i_x}{20} = 0.2i_x$$

$$0.1v_c' = i_x - 0.05v_c' + 0.05v_s' - 0.2i_x'$$

$$\therefore 0.15v_c' = i_x - 0.2i_x' + 0.05v_s'$$

$$5i_x = -v_c + v_s, i_x = -0.2v_c + 0.2v_s, i_x' = -0.2v_c' + 0.2v_s'$$

$$\therefore 0.15v_c' = -0.2v_c + 0.2v_s + 0.04v_c' - 0.04v_s' + 0.05v_s'$$

$$\therefore 0.11v_c' = -0.2v_c + 0.2v_s + 0.01v_s' \therefore 0.11v_c' = -0.2v_c + \sin 2tu(t) + 0.1 \cos 2tu(t)$$

$$\therefore v_c' = -\frac{20}{11}v_c + \frac{100}{11}\sin 2tu(t) + \frac{10}{11}\cos 2tu(t) \quad (1)$$

## CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

13.

(a)  $\bar{q} = \begin{bmatrix} i_{L1} \\ i_{L2} \\ v_C \end{bmatrix}, \bar{a} = \begin{bmatrix} -1 & -2 & -3 \\ 4 & -5 & 6 \\ 7 & -8 & -9 \end{bmatrix}, \bar{f} = \begin{bmatrix} 2t \\ 3t^2 \\ 1+t \end{bmatrix} \therefore i'_{L1} = -i_{L1} - 2i_{L2} - 3v_C + 2t \quad (1)$

$$i'_{L2} = 4i_{L1} - 5i_{L2} + 6v_C + 3t^2 \quad (2)$$

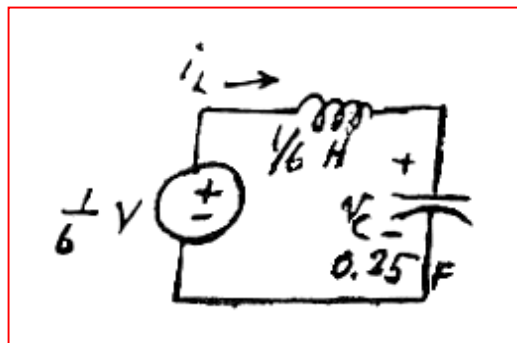
$$v'_C = 7i_{L1} - 8i_{L2} - 9v_C + 1 + t \quad (3)$$

(b)  $\bar{q} = \begin{bmatrix} i_L \\ v_C \end{bmatrix}, \bar{a} = \begin{bmatrix} 0 & -6 \\ 4 & 0 \end{bmatrix}, \bar{f} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$\therefore i'_L = -6v_C + 1, v'_C = 4i_L$$

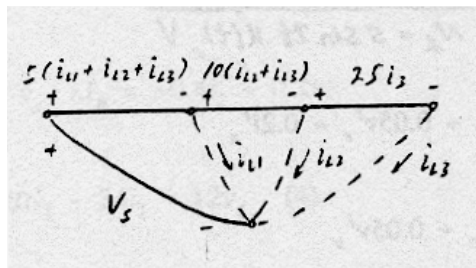
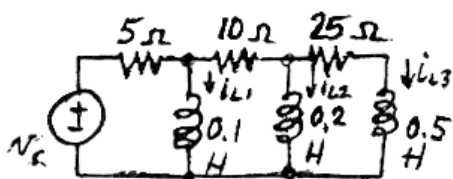
$$\therefore 0.25v'_C = i_L \therefore C = 0.25 \text{ F}$$

$$\frac{1}{6} i'_L = -v_C + \frac{1}{6} \therefore L = \frac{1}{6} \text{ H}$$



# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

14.



$$0.1i'_{L1} = -5i_{L1} - 5i_{L2} - 5i_{L3} + v_s$$

$$\therefore i'_{L1} = -50i_{L1} - 50i_{L2} - 50i_{L3} + 10v_s \quad (1)$$

$$0.2i'_{L2} = -5i_{L1} - 15i_{L2} - 15i_{L3} + v_s$$

$$\therefore i'_{L2} = -25i_{L1} - 75i_{L2} - 75i_{L3} + 5v_s \quad (2) \quad 0.5i'_{L3} = -5i_{L1} - 15i_{L2} - 40i_{L3} + v_s$$

$$\therefore i'_{L3} = -10i_{L1} - 30i_{L2} - 80i_{L3} + 2v_s \quad (3)$$

$$\bar{a} = \begin{bmatrix} -50 & -50 & -50 \\ -25 & -75 & -75 \\ -10 & -30 & -80 \end{bmatrix} \quad \bar{f} = \begin{bmatrix} 10v_s \\ 5v_s \\ 2v_s \end{bmatrix}$$

## CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

15.

$$\bar{q} = \begin{bmatrix} v_{C1} \\ v_{C2} \\ i_L \end{bmatrix}, \bar{a} = \begin{bmatrix} -3 & 3 & 0 \\ 2 & -1 & 0 \\ 1 & 4 & -2 \end{bmatrix}, \bar{f} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}; \bar{w} = \begin{bmatrix} v_{01} \\ v_{02} \\ i_{R1} \\ i_{R2} \end{bmatrix}, \bar{b} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 2 \\ 1 & 3 & 0 \end{bmatrix}, \bar{d} = \begin{bmatrix} 0 \\ 2 \\ -2 \\ 0 \end{bmatrix}$$

$$\bar{w} = \bar{b}\bar{q} + \bar{d} \quad \therefore \bar{q}' = \bar{a}\bar{q} + \bar{f}, \bar{w} = \bar{b}\bar{q} + \bar{d}, \bar{w}' = \bar{b}\bar{q}' = \bar{b}\bar{a}\bar{q} + \bar{b}\bar{f}$$

$$\bar{b}\bar{a} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 2 \\ 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} -3 & 3 & 0 \\ 2 & -1 & 0 \\ 1 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -4 & 2 \\ 0 & 9 & -4 \\ 3 & 0 & 0 \end{bmatrix}, \bar{b}\bar{f} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 2 \\ 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 10 \end{bmatrix}$$

$$\therefore \bar{w}' = \begin{bmatrix} 1 & 1 & 0 \\ -1 & -4 & 2 \\ 0 & 9 & -4 \\ 3 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{C1} \\ v_{C2} \\ i_L \end{bmatrix} + \begin{bmatrix} 10 \\ 0 \\ 0 \\ 10 \end{bmatrix} = \begin{bmatrix} v_{C1} + v_{C2} + 10 \\ -v_{C1} - 4v_{C2} + 2i_L \\ 9v_{C2} - 4i_L \\ 3v_{C1} + 10 \end{bmatrix}$$

$$\therefore v'_{01} = v_{C1} + v_{C2} + 10, v'_{02} = -v_{C1} - 4v_{C2} + 2i_L, i'_{R1} = 9v_{C2} - 4i_L, i'_{R2} = 3v_{C1} + 10$$



# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

16.

$$\bar{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}, \bar{a} = \begin{bmatrix} -3 & 1 & 2 \\ -2 & -2 & 1 \\ -1 & 3 & 0 \end{bmatrix}, \bar{f} = \begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix}, \bar{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}, \bar{b} = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & -1 & 1 & 1 \\ 2 & -1 & -1 & 3 \end{bmatrix}$$

$$\bar{d} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \bar{q}' = \bar{a}\bar{q} + \bar{f}, \bar{q} = \bar{b}\bar{y} + \bar{d}, \bar{y}(0) = \begin{bmatrix} 10 \\ -10 \\ -5 \\ 5 \end{bmatrix}$$

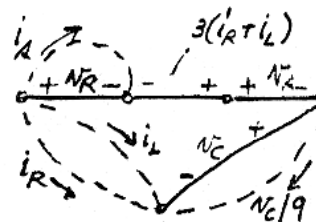
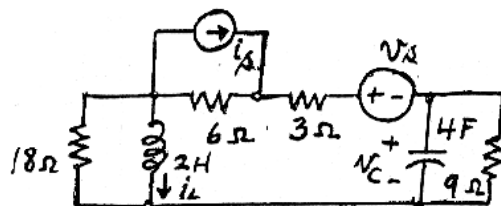
$$\therefore \bar{q} = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & -1 & 1 & 1 \\ 2 & -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} y_1 + 2y_2 + 3y_4 + 2 \\ -y_2 + y_3 + y_4 + 1 \\ 2y_1 - y_2 - y_3 + 3y_4 + 3 \end{bmatrix} \therefore \bar{q}(0) = \begin{bmatrix} 7 \\ 11 \\ 53 \end{bmatrix}$$

$$\bar{q}'(0) = \begin{bmatrix} -3 & 1 & 2 \\ -2 & -2 & 1 \\ -1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 7 \\ 11 \\ 53 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -21 + 11 + 106 + 1 \\ -14 - 22 + 53 \\ -7 + 33 \end{bmatrix} = \begin{bmatrix} 97 \\ 17 \\ 26 \end{bmatrix}$$

# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

17.

$$\bar{q} = \begin{bmatrix} v_C \\ i_L \end{bmatrix}$$



$$4v'_C = -\frac{v_C}{9} - i_L - i_R, \quad 2i'_L = v_R - 3i_R - 3i_L + v_s + v_C, \quad 18i_R = 2i'_L \quad \therefore i_R = \frac{1}{9}i'_L$$

$$\frac{v_R}{6} = -i_R - i_L - i_s \quad \therefore v_R = -6i_R - 6i_L - 6i_s = -\frac{2}{3}i'_L - 6i_L - 6i_s, \quad \therefore 4v'_C = -\frac{v_C}{9} - i_L - \frac{1}{9}i'_L$$

$$\text{Also, } 2i'_L = -\frac{2}{3}i'_L - 6i_L - 6i_s - \frac{1}{3}i'_L - 3i_L + v_C + v_s \quad \therefore 3i'_L = v_C - 9i_L + v_s - 6i_s$$

$$\therefore i'_L = \frac{1}{3}v_C - 3i_L + \frac{1}{3}v_s - 2i_s \quad (2) \quad 4v'_C = -\frac{1}{9}v_C - i_L - \frac{1}{27}v_C + \frac{1}{3}i_L - \frac{1}{27}v_s + \frac{2}{9}i_s$$

$$\therefore 4v'_C = -\frac{4}{27}v_C - \frac{2}{3}i_L - \frac{1}{27}v_s + \frac{2}{9}i_s \quad \therefore v'_C = -\frac{1}{27}v_C - \frac{1}{6}i_L - \frac{1}{108}v_s + \frac{1}{18}i_s \quad (1)$$

$$\therefore [\bar{a}] = \begin{bmatrix} -1/27 & -1/6 \\ 1/3 & -3 \end{bmatrix} [\bar{f}] = \begin{bmatrix} -\frac{1}{108}v_s + \frac{1}{18}i_s \\ \frac{1}{3}v_s - 2i_s \end{bmatrix}$$

## CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

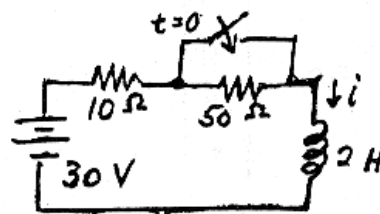
18.

(a)  $30 = 10i + 2i' \quad \therefore i' = -5i + 15$

(b)  $i(0) = 0.5 \text{ A}, a = -5 \quad \therefore i = e^{-5t} 0.5 + e^{-5t} \int_0^t 15e^{5z} dz$   
 $\therefore i = 0.5e^{-5t} + e^{-5t} 3(e^{5t} - 1) = 3 - 2.5e^{-5t} \text{ A}, t > 0$

(c)  $i_{\text{zero state}} = 3(1 - e^{-5t}) \text{ A}, i_{\text{zero input}} = 0.5e^{-5t} \text{ A}$

(d)  $i_f = 3, i_n = Ae^{-5t} \quad \therefore i = 3 + Ae^{-5t}, i(0) = 0.5 \quad \therefore i = 3 - 2.5e^{-5t} \text{ A}$   
 $\therefore i_n = -2.5e^{-5t} \text{ A}, i_f = 3 \text{ A}$



# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

19.

$$\therefore 5 \times 10^{-5} v'_C = 0.04t u(t) - 0.025v_C$$

$$\therefore v'_C = -500v_C + 800tu(t) \quad \therefore a = -500$$

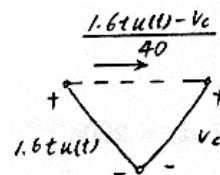
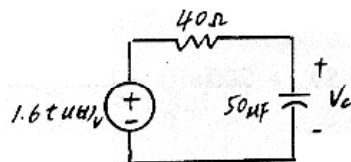
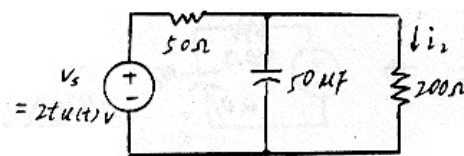
$$\therefore v_C = e^{-500t} \int_0^t e^{500Z} 800Z dz$$

$$= 800e^{-500t} \int_0^t Ze^{500Z} dz$$

$$\therefore v_C = 800e^{-500t} \left[ e^{500Z} \left( \frac{Z}{500} - \frac{1}{500^2} \right) \right]_0^t$$

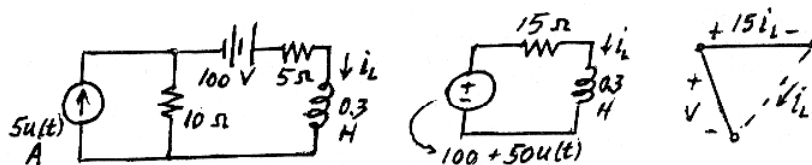
$$\therefore v_C = 800e^{-500t} \left[ e^{500t} \left( \frac{t}{500} - \frac{1}{500^2} \right) + \frac{1}{500^2} \right]$$

$$\therefore v_C = 1.6t + \frac{1.6}{500}(-1 + e^{-500t}) \quad \therefore i_2 = \frac{1}{200}v_C = 8 \times 10^{-3}t - 16 \times 10^{-6}(1 - e^{-500t}) \text{ A, } t > 0$$



# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

20.



$$(a) \quad 0.3i'_L = -15i_L + 100 + 50u(t), \quad i'_L = -50i_L + \frac{1000}{3} + \frac{500}{3}u(t), \quad i_L(0) = \frac{100}{15} = \frac{20}{3} \text{ A}$$

$$\therefore i_L = e^{-50t} \times \frac{20}{3} + e^{-50t} \int_0^t e^{50Z} 500 dZ = \frac{20}{3}e^{-50t} + 500e^{-50t} \frac{1}{50}(e^{50t} - 1)$$

$$\therefore i_L = \frac{20}{3}e^{-50t} + 10(1 - e^{-50t}) \quad t > 0$$

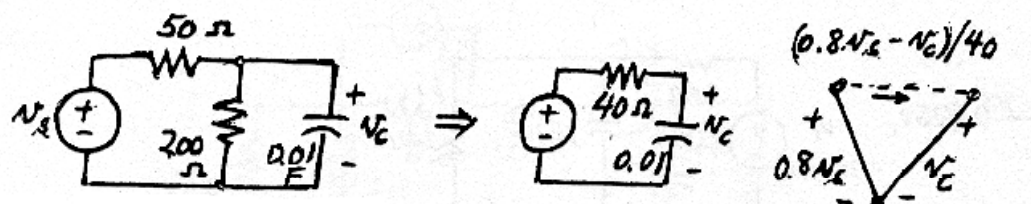
$$\therefore i_L = \frac{20}{3}u(-t) + \left[ 10 - \frac{10}{3}e^{-50t} \right] u(t) \text{ A}$$

$$(b) \quad i_{L, \text{zero input}} = \frac{20}{3}e^{-50t} \text{ A}, \quad i_{L, \text{zero state}} = 10(1 - e^{-50t}) \text{ A}$$

$$i_{Lf} = 10 \text{ A}, \quad i_{Ln} = -\frac{10}{3}e^{-50t} \text{ A}$$

# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

21.



$$v_s = 100[u(t) - u(t - 0.5)] \cos \pi t \text{ V}$$

$$0.01 v'_C = \frac{1}{40} (0.8 v_s - v_C) \therefore v'_C = -2.5 v_C + 200[u(t) - u(t - 0.5)] \cos \pi t$$

$$t < 0 \quad v_C = 0$$

$$0 \leq t \leq 0.5 \quad v_C = e^{-2.5t} \times 200 \int_0^t e^{2.5Z} \cos \pi Z \, dZ = 200 e^{-2.5t} \left[ \frac{e^{2.5Z}}{6.25 + \pi^2} (2.5 \cos \pi Z + \pi \sin \pi Z) \right]_0^t$$

$$\therefore v_C = \frac{200}{6.25 + \pi^2} (2.5 \cos \pi t + \pi \sin \pi t) - \frac{200}{6.25 + \pi^2} 2.5 e^{-2.5t}$$

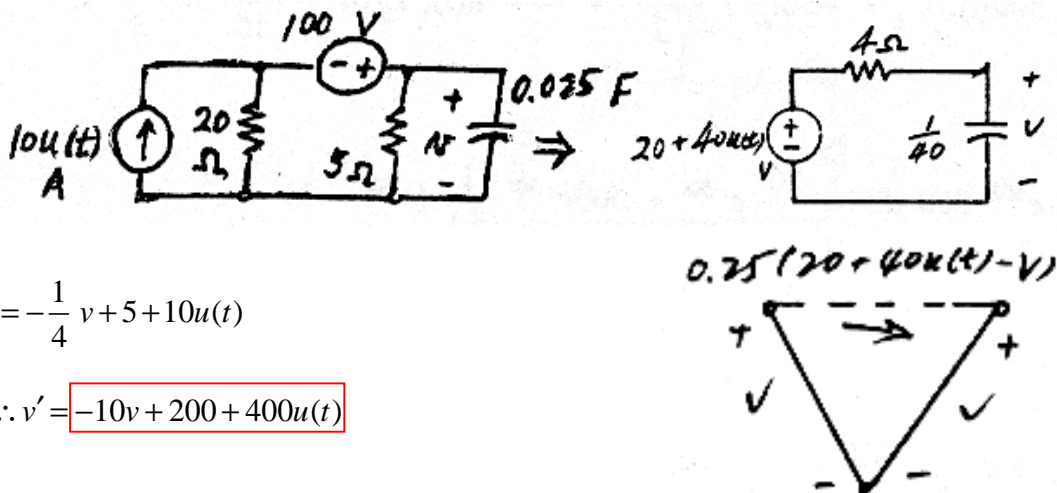
$$\therefore v_C = 31.02 \cos \pi t + 38.98 \sin \pi t - 31.02 e^{-2.5t}$$

$$t > 0.5 \quad v_C = 200 e^{-2.5t} \left[ \frac{e^{2.5Z}}{6.25 + \pi^2} (2.5 \cos \pi Z + \pi \sin \pi Z) \right]_0^{0.5}$$

$$\therefore v_C = 200 e^{-2.5t} \left[ \frac{e^{1.25}}{6.25 + \pi^2} \pi - \frac{1}{6.25 + \pi^2} \times 2.5 \right] = 105.03 e^{-2.5t} \text{ V}$$

# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

22.



$$\therefore \frac{1}{40} v' = -\frac{1}{4} v + 5 + 10u(t)$$

(a)  $\therefore v' = -10v + 200 + 400u(t)$

(b)  $t < 0 \therefore v = 20 \text{ V}$

$$t > 0 \quad v = 20e^{-10t} + 600e^{-10t} \int_0^z e^{10z} dz = 20e^{-10t} + 60e^{-10t} (e^{10t} - 1)$$

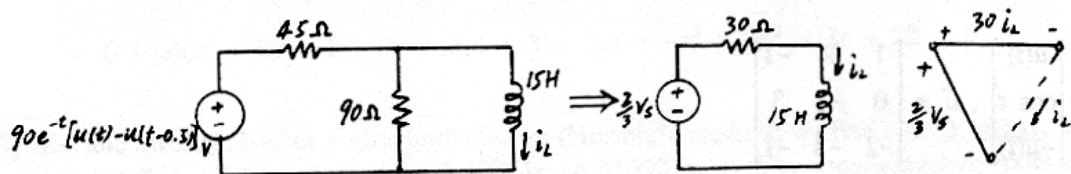
$$\therefore v = 20e^{-10t} + 60 - 60e^{-10t} \therefore v = 60 - 40e^{-10t} \text{ V}$$

(c)  $v_{\text{forced}} = 60 \text{ V} \quad v_{\text{nat}} = -40e^{-10t} \text{ V}$

$$v_{\text{zero state}} = 60(1 - e^{-10t}) \text{ V} \quad v_{\text{zero input}} = 20e^{-10t} \text{ V}$$

# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

23.



$$15'_{L} = -30i_L + \frac{2}{3}v_s$$

$$\therefore i'_L = -2i_L + 4e^{-t}[u(t) - u(t - 0.5)]$$

$$\therefore i_L = 4e^{-2t} \int_0^t e^{2z} e^{-z} [u(z) - u(z - 0.5)] dz$$

$$0 \leq t \leq 0.5 \quad i_L = 4e^{-2t} \int_0^t e^z dZ = 4e^{-2t} (e^t - 1) = \boxed{4e^{-t} - 4e^{-2t} \text{ A}}$$

$$t \geq 0.5 \quad i_L = 4e^{-2t} \int_0^{0.5} e^z dZ + 0 = 4e^{-2t} (e^{0.5} - 1) = \boxed{2.595e^{-2t} \text{ A}}$$



# CHAPTER NINETEEN (WEB CHAPTER) SOLUTIONS

24.

$$[a] = \begin{bmatrix} -8 & 5 \\ 10 & -10 \end{bmatrix}, t = 0.01$$

$$\begin{aligned} \text{(a)} \quad e^{-t\bar{a}} &= \bar{I} - \begin{bmatrix} -0.08 & 0.05 \\ 0.1 & -0.1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} -0.08 & 0.05 \\ 0.1 & -0.1 \end{bmatrix} \begin{bmatrix} -0.08 & 0.05 \\ 0.1 & -0.1 \end{bmatrix} - \dots \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -0.08 & 0.05 \\ 0.1 & -0.1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0.0114 & -0.0090 \\ -0.0180 & 0.0150 \end{bmatrix} - \frac{1}{6} \begin{bmatrix} 0.0114 & -0.009 \\ -0.018 & 0.015 \end{bmatrix} \begin{bmatrix} -0.08 & 0.05 \\ 0.1 & -0.1 \end{bmatrix} + \dots \\ &= \begin{bmatrix} 1.08 & -0.05 \\ -0.1 & 1.1 \end{bmatrix} + \begin{bmatrix} 0.0057 & -0.0045 \\ -0.009 & 0.0075 \end{bmatrix} - \frac{1}{6} \begin{bmatrix} -0.00181 & 0.00147 \\ 0.00294 & -0.00240 \end{bmatrix} = \boxed{\begin{bmatrix} 1.0860 & -0.0547 \\ -0.1095 & 1.1079 \end{bmatrix}} \end{aligned}$$

$$\text{(b)} \quad e^{t\bar{a}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -0.08 & 0.05 \\ 0.1 & -0.1 \end{bmatrix} + \begin{bmatrix} 0.0057 & -0.0045 \\ -0.009 & 0.0075 \end{bmatrix} + \begin{bmatrix} -0.0003 & 0.0002 \\ 0.0005 & -0.0004 \end{bmatrix} = \boxed{\begin{bmatrix} 0.9254 & 0.0457 \\ 0.0915 & 0.9071 \end{bmatrix}}$$

$$\text{(c)} \quad e^{-t\bar{a}} e^{t\bar{a}} = \begin{bmatrix} 1.0860 & -0.0547 \\ -0.1095 & 1.1079 \end{bmatrix} \begin{bmatrix} 0.9254 & 0.0457 \\ 0.0915 & 0.9071 \end{bmatrix} = \boxed{\begin{bmatrix} 1.0000 & 0 \\ 0 & 1.0000 \end{bmatrix}}$$

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25.

$$(a) \quad \bar{q} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \bar{f} = \begin{bmatrix} u(t) \\ \cos t \\ -u(t) \end{bmatrix}, \bar{a} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ -2 & -3 & -1 \end{bmatrix}$$

$$\therefore x' = x + 2y - z + u(t), \quad y' = -y + 3z + \cos t, \quad z' = -2x - 3y - z - u(t)$$

$$(b) \quad \bar{q}(0) = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \quad \therefore \bar{q}'(0) = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ -2 & -3 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 7 \\ 3 \end{bmatrix} \quad \Delta t = 0.1$$

$$\bar{q}(0.1) = \bar{q}(0) + 0.1\bar{q}'(0) = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.4 \\ 0.7 \\ 0.3 \end{bmatrix} = \begin{bmatrix} 1.6 \\ -2.3 \\ 1.3 \end{bmatrix}$$

$$(c) \quad \Delta t = 0.05 \quad \therefore \bar{q}(0.5) = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + 0.05 \begin{bmatrix} -4 \\ 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 1.8 \\ -2.65 \\ 1.15 \end{bmatrix}$$

$$\bar{q}'(0.05) = \begin{bmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ -2 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1.8 \\ -2.65 \\ 1.15 \end{bmatrix} + \begin{bmatrix} 1 \\ \cos 0.05 \\ -1 \end{bmatrix} = \begin{bmatrix} -3.65 \\ 7.0988 \\ 2.2 \end{bmatrix}$$

$$\bar{q}(0.1) = \bar{q}(0.05) + 0.05\bar{q}'(0.05) = \begin{bmatrix} 1.8 \\ -2.65 \\ 1.15 \end{bmatrix} + 0.05 \begin{bmatrix} -3.65 \\ 7.0988 \\ 2.2 \end{bmatrix} = \begin{bmatrix} 1.6175 \\ -2.2951 \\ 1.26 \end{bmatrix}$$

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26.

$$\bar{a} = \begin{bmatrix} -1 & 2 & 3 \\ 0 & -1 & 2 \\ 3 & 1 & -1 \end{bmatrix} \text{ Now, } \bar{a} - \bar{s} \bar{I} = \begin{bmatrix} -1-s & 2 & 3 \\ 0 & -1-s & 2 \\ 3 & 1 & -1-s \end{bmatrix}$$

$$\det[\ ] = (-1-s)(s^2 + 2s + 1 - 2) + 3(4 + 3 + 3s) = -s^3 - 3s^2 + 8s + 22 = 0$$

By trial and error, Solve, or high school algebra (Horner's method) we find  $s = -3.48361$ . Now divide polynomial by  $s + 3.48361$ . Get quadratic,  $s^2 - 0.48361s - 6.31592 = 0$ .

The remaining two roots are:  $s = -2.28282$  and  $s = + 2.76641$ .

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27.

$$\bar{a} = \begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix} \det(\bar{a} - s\bar{I}) = \det \begin{bmatrix} -3-s & 2 \\ 1 & -4-s \end{bmatrix} = s^2 + 7s + 12 - 2 = 0$$

Roots are  $s_1 = -2$ ,  $s_2 = -5$  Now,  $e^{s_1 t} = u_o + u_1 s_1$ ,  $e^{s_2 t} = u_o + u_1 s_2$

$$\therefore e^{-2t} = u_o - 2u_1, e^{-5t} = u_o - 5u_1 \therefore e^{-2t} - e^{-5t} = 3u_1, u_1 = \frac{1}{3}e^{-2t} - \frac{1}{3}e^{-5t}$$

$$\therefore u_o = e^{-2t} + \frac{2}{3}e^{-2t} - \frac{2}{3}e^{-5t}, \text{ or } u_o = \frac{5}{3}e^{-2t} - \frac{2}{3}e^{-5t}$$

$$e^{t\bar{a}} = u_o \bar{I} + u_1 \bar{a} = \begin{bmatrix} \frac{5}{3}e^{-2t} - \frac{2}{3}e^{-5t} & 0 \\ 0 & \frac{5}{3}e^{-2t} - \frac{2}{3}e^{-5t} \end{bmatrix} + \left( \frac{1}{3}e^{-2t} - \frac{1}{3}e^{-5t} \right) \begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix}$$

$$e^{t\bar{a}} = \begin{bmatrix} \frac{5}{3}e^{-2t} - \frac{2}{3}e^{-5t} - e^{-2t} + e^{-5t} & \frac{2}{3}e^{-2t} - \frac{2}{3}e^{-5t} \\ \frac{1}{3}e^{-2t} - \frac{1}{3}e^{-5t} & \frac{5}{3}e^{-2t} - \frac{2}{3}e^{-5t} - \frac{4}{3}e^{-2t} + \frac{4}{3}e^{-5t} \end{bmatrix}$$

$$\therefore e^{t\bar{a}} = \begin{bmatrix} \frac{2}{3}e^{-2t} + \frac{1}{3}e^{-5t} & \frac{2}{3}e^{-2t} - \frac{2}{3}e^{-5t} \\ \frac{1}{3}e^{-2t} - \frac{1}{3}e^{-5t} & \frac{1}{3}e^{-2t} + \frac{2}{3}e^{-5t} \end{bmatrix}$$

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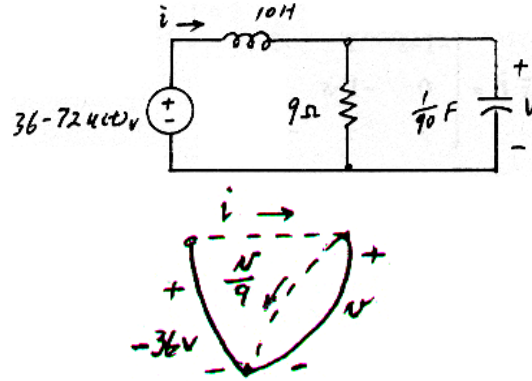
28.

$$(a) \quad \bar{q} = \begin{bmatrix} i \\ v \end{bmatrix} \quad 10i' = -36 - v$$

$$\frac{1}{90}v' = i - \frac{1}{9}v$$

$$\therefore i' = -0.1v - 3.6, \quad v' = 90i - 10v$$

$$\therefore \bar{a} = \begin{bmatrix} 0 & -0.1 \\ 90 & -10 \end{bmatrix}, \quad \bar{f} = \begin{bmatrix} -3.6 \\ 0 \end{bmatrix}$$



$$(b) \quad \bar{a} - s\bar{I} = \begin{bmatrix} 0-s & -0.1 \\ 90 & -10-s \end{bmatrix}, \quad \det(\bar{a} - s\bar{I}) = s^2 + 10s + 9 \quad \therefore s_1 = -1, s_2 = -9$$

$$(c) \quad e^{-t} = u_o - u_1, \quad e^{-9t} = u_o - 9u_1 \quad \therefore e^{-t} - e^{-9t} = 8u_1$$

$$\therefore u_1 = \frac{1}{8}e^{-t} - \frac{1}{8}e^{-9t}, \quad u_o = \frac{1}{8}e^{-t} - \frac{1}{8}e^{-9t} + e^{-t} = \frac{9}{8}e^{-t} - \frac{1}{8}e^{-9t}$$

$$(d) \quad e^{\bar{a}t} = u_o \bar{I} + u_1 \bar{a} = \frac{1}{8}(9e^{-t} - e^{-9t}) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{8}(e^{-t} - e^{-9t}) \begin{bmatrix} 0 & -0.1 \\ 90 & -10 \end{bmatrix}$$

$$\therefore 8e^{\bar{a}t} = \begin{bmatrix} 9e^{-t} - e^{-9t} & 0 \\ 0 & 9e^{-t} - e^{-9t} \end{bmatrix} + \begin{bmatrix} 0 & -0.1e^{-t} + 0.1e^{-9t} \\ 90e^{-t} - 90e^{-9t} & -10e^{-t} + 10e^{-9t} \end{bmatrix}$$

$$8e^{\bar{a}t} = \begin{bmatrix} 9e^{-t} - e^{-9t} & -0.1e^{-t} + 0.1e^{-9t} \\ 90e^{-t} - 90e^{-9t} & -e^{-t} + 9e^{-9t} \end{bmatrix}$$

$$\therefore e^{\bar{a}t} = \begin{bmatrix} \frac{9}{8}e^{-t} - \frac{1}{8}e^{-9t} & -\frac{1}{80}e^{-t} + \frac{1}{80}e^{-9t} \\ \frac{90}{8}e^{-t} - \frac{90}{8}e^{-9t} & -\frac{1}{8}e^{-t} + \frac{9}{8}e^{-9t} \end{bmatrix}$$

$$(e) \quad \bar{q} = e^{\bar{a}t} \bar{q}(0) + e^{\bar{a}t} \int_0^t e^{-Z\bar{a}} f(Z) dZ \quad \bar{q}(0) = \begin{bmatrix} i(0) \\ v(0) \end{bmatrix} = \begin{bmatrix} 4 \\ 36 \end{bmatrix}$$

$$\bar{q} = \frac{1}{2} \begin{bmatrix} 9e^{-t} - e^{-9t} & -0.1e^{-t} + 0.1e^{-9t} \\ 90e^{-t} - 90e^{-9t} & -e^{-t} + 9e^{-9t} \end{bmatrix} \begin{bmatrix} 1 \\ 9 \end{bmatrix}$$

$$+ \frac{1}{8} e^{\bar{a}t} \int_0^t \begin{bmatrix} 9e^Z - e^{9Z} & -0.1e^Z + 0.1e^{9Z} \\ 90e^Z - 90e^{9Z} & -e^Z + 9e^{9Z} \end{bmatrix} \begin{bmatrix} -3.6 \\ 0 \end{bmatrix} dZ$$

$$\therefore \bar{q} = \frac{1}{2} \begin{bmatrix} 9e^{-t} - e^{-9t} - 0.9e^{-t} + 0.9e^{-9t} \\ 90e^{-t} - 90e^{-9t} - 9e^{-t} + 81e^{-9t} \end{bmatrix} - 0.45 e^{\bar{a}t} \int_0^t \begin{bmatrix} 9e^Z - e^{9Z} \\ 90e^Z - 90e^{9Z} \end{bmatrix} dZ$$

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$$\begin{aligned}
 &= \begin{bmatrix} 4.05e^{-t} - 0.05e^{9t} \\ 40.5e^{-t} - 4.5e^{-9t} \end{bmatrix} - 0.45e^{t\bar{a}} \begin{bmatrix} 9e^t - \frac{1}{9}e^{9t} - 9 + \frac{1}{9} \\ 90e^t - 10e^{9t} - 90 + 10 \end{bmatrix} \\
 \bar{q} &= \begin{bmatrix} 4.05e^{-t} - 0.05e^{-9t} \\ 40.5e^{-t} - 4.5e^{-9t} \end{bmatrix} - \frac{0.45}{8} \begin{bmatrix} 9e^{-t} - e^{-9t} & 0.1e^{-t} + 0.1e^{-9t} \\ 90e^{-t} - 90e^{-9t} & -e^{-t} + 9e^{-9t} \end{bmatrix} \begin{bmatrix} 9e^t - \frac{1}{9}e^{9t} - \frac{80}{9} \\ 90e^t - 10e^{9t} - 80 \end{bmatrix} \\
 &= \begin{bmatrix} 4.05e^{-t} & -0.05e^{-9t} \\ 40.5e^{-t} & -4.5e^{-9t} \end{bmatrix} \\
 &\quad - \frac{0.45}{8} \begin{bmatrix} 81 - e^{8t} - 80e^{-t} - 9e^{-8t} + \frac{1}{9} + \frac{80}{9}e^{-9t} - 9 + e^{8t} + 8e^{-t} + 9e^{-8t} - 1 - 8e^{-9t} \\ 810 - 10e^{8t} - 800e^{-t} - 810e^{-8t} + 10 + 800e^{-9t} - 90 + 10e^{8t} + 80e^{-t} + 810e^{-8t} - 90 - 720e^{-9t} \end{bmatrix} \\
 &= \begin{bmatrix} 4.05e^{-t} - 0.05e^{-9t} \\ 40.5e^{-t} - 4.5e^{-9t} \end{bmatrix} - \frac{0.45}{7} \begin{bmatrix} 71 + \frac{1}{9} - 72e^{-t} + \frac{8}{9}e^{-9t} \\ 640 - 720e^{-t} + 80e^{-9t} \end{bmatrix} \\
 &= \begin{bmatrix} 4.05e^{-t} - 0.05e^{-9t} \\ 40.5e^{-t} - 4.5e^{-9t} \end{bmatrix} - \begin{bmatrix} 4 - 4.05e^{-t} + 0.05e^{-9t} \\ 36 - 40.5e^{-t} + 4.5e^{-9t} \end{bmatrix} \\
 \therefore \bar{q} &= \begin{bmatrix} -4 + 8.1e^{-t} - 0.1e^{-9t} \\ -36 + 81e^{-t} - 9e^{-9t} \end{bmatrix} \quad (t > 0)
 \end{aligned}$$