

SOLUTIONS MANUAL FOR
**FRACTURE
MECHANICS**

Fundamentals and Applications
Fourth Edition

_____ by _____

Ted L. Anderson

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FRACTURE MECHANICS
Fundamentals and Applications
Fourth Edition

Ted L. Anderson

Solutions Manual

NOTE TO INSTRUCTORS

This volume contains solutions to the problems in Chapter 13 of *Fracture Mechanics: Fundamentals and Applications (4th Edition)*. The problem statement is given in each case and is surrounded by a single line border. All problems involving numerical quantities are solved in SI units.

In some instances, problems were solved by means of a computer program or spreadsheet. In such cases, only the final results are given, usually in the form of a graph.

Some of the problems attempt to test the students' engineering judgment, and do not have a single "correct" answer. For example, Problem 7.2, which asks the student to design a K_{Ic} experiment, has a range of acceptable answers. I realize that this makes life more difficult for graders, but I believe that it provides a better learning experience for the students.

Some problems, especially those corresponding to Chapters 9 to 12, require numerical approximations (e.g., numerical differencing and integration). Thus the students' answers may differ slightly from those in this manual, depending on the numerical techniques employed.

Most of the problems have been class tested, so the solutions should be fairly reliable. However, since nobody's perfect, the possibility for mistakes always exists. If you discover any errors in the solution manual or the text, I would be very grateful if you would bring them to my attention.

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CHAPTER 1

- 1.2** A flat plate with a through-thickness crack (Fig. 1.8) is subject to a 100 MPa (14.5 ksi) tensile stress and has a fracture toughness (K_{Ic}) of 50.0 MPa $\sqrt{\text{m}}$ (45. ksi $\sqrt{\text{in}}$). Determine the critical crack length for this plate, assuming the material is linear elastic.

Ans:

At fracture, $K_{Ic} = K_I = \sigma \sqrt{\pi a_c}$. Therefore,

$$50 \text{ MPa} \sqrt{\text{m}} = 100 \text{ MPa} \sqrt{\pi a_c}$$

$$a_c = 0.0796 \text{ m} = 79.6 \text{ mm}$$

$$\text{Total crack length} = 2a_c = 159 \text{ mm}$$

- 1.3** Compute the critical energy release rate (\mathcal{G}_c) of the material in the previous problem for $E = 207,000 \text{ MPa}$ (30,000 ksi)..

Ans:

$$\begin{aligned} \mathcal{G}_c &= \frac{K_{Ic}^2}{E} = \frac{(50 \text{ MPa} \sqrt{\text{m}})^2}{207,000 \text{ MPa}} = 0.0121 \text{ MPa mm} = 12.1 \text{ kPa m} \\ &= 12.1 \text{ kJ/m}^2 \end{aligned}$$

Note that energy release rate has units of energy/area.

- 1.4** Suppose that you plan to drop a bomb out of an airplane and that you are interested in the time of flight before it hits the ground, but you cannot remember the appropriate equation from your undergraduate physics course. You decide to infer a relationship for time of flight of a falling object by experimentation. You reason that the time of flight, t , must depend on the height above the ground, h , and the weight of the object, mg , where m is the mass and g is the gravitational acceleration. Therefore, neglecting aerodynamic drag, the time of flight is given by the following function:

$$t = f(h, m, g)$$

Apply dimensional analysis to this equation and determine how many experiments would be required to determine the function f to a reasonable approximation, assuming you know the numerical value of g . Does the time of flight depend on the mass of the object?

Ans:

Since h has units of length and g has units of $(\text{length})(\text{time})^{-2}$, let us divide both sides of the above equation by $\sqrt{h/g}$:

$$\frac{t}{\sqrt{h/g}} = \frac{f(h, m, g)}{\sqrt{h/g}}$$

The left side of this equation is now dimensionless. Therefore, the right side must also be dimensionless, which implies that the time of flight cannot depend on the mass of the object. Thus dimensional analysis implies the following functional relationship:

$$t = \alpha \sqrt{\frac{h}{g}}$$

where α is a dimensionless constant. Only one experiment would be required to estimate α , but several trials at various heights might be advisable to obtain a reliable estimate of this constant. Note that $\alpha = \sqrt{2}$ according to Newton's laws of motion.

CHAPTER 2

2.1 According to Eq. (2.25), the energy required to increase the crack area a unit amount is equal to *twice* the fracture work per unit surface area, w_f . Why is the factor of 2 in this equation necessary?

Ans:

The factor of 2 stems from the difference between *crack area* and *surface area*. The former is defined as the projected area of the crack. The surface area is twice the crack area because the formation of a crack results in the creation of two surfaces. Consequently, the material resistance to crack extension = $2 w_f$.

2.2 Derive Eq. (2.30) for both load control and displacement control by substituting Eq. (2.29) into Eqs. (2.27) and (2.28), respectively.

Ans:

(a) Load control.

$$\mathcal{G} = \frac{P}{2B} \left(\frac{d\Delta}{da} \right)_P = \frac{P}{2B} \left(\frac{d[CP]}{da} \right)_P = \frac{P}{2B} \frac{dC}{da}$$

(b) Displacement control.

$$\mathcal{G} = -\frac{\Delta}{2B} \left(\frac{dP}{da} \right)_\Delta$$

$$\left(\frac{dP}{da} \right)_\Delta = \Delta \frac{d(1/C)}{da} = -\frac{\Delta}{C^2} \frac{dC}{da}$$

$$\mathcal{G} = \frac{(\Delta/C)^2}{2B} \frac{dC}{da} = \frac{P^2}{2B} \frac{dC}{da}$$

2.3 Figure 2.10 illustrates that the driving force is linear for a through-thickness crack in an infinite plate when the stress is fixed. Suppose that a remote displacement (rather than load) were fixed in this configuration. Would the driving force curves be altered? Explain. (Hint: see Section 2.5.3).

Ans:

In a cracked plate where $2a \ll$ the plate width, crack extension at a fixed remote displacement would not effect the load, since the crack comprises a negligible portion of the cross section. Thus a fixed remote displacement implies a fixed load, and load control and displacement control are equivalent in this case. The driving force curves would not be altered if remote displacement, rather than stress, were specified.

Consider the spring in series analog in Fig. 2.12. The load and remote displacement are related as follows:

$$\Delta_T = (C + C_m) P \quad \Delta_T = (C + C_m) P$$

where C is the “local” compliance and C_m is the system compliance. For the present problem, assume that C_m represents the compliance of the uncracked plate and C is the additional compliance that results from the presence of the crack. When the crack is small compared to the plate dimensions, $C_m \gg C$. If the crack were to grow at a fixed Δ_T , only C would change; thus load would also remain fixed.

2.4 A plate $2W$ wide contains a centrally located crack $2a$ long and is subject to a tensile load, P . Beginning with Eq. (2.24), derive an expression for the elastic compliance, $C (= \Delta/P)$ in terms of the plate dimensions and elastic modulus, E . The stress in Eq. (2.24) is the nominal value; i.e., $\sigma = P/2BW$ in this problem. (Note: Eq. (2.24) only applies when $a \ll W$; the expression you derive is only approximate for a finite width plate.)

Ans.:

The through-thickness crack has two tips; an increment of crack growth causes the crack area to increase by $2B da$. The compliance relationship for energy release rate must be modified accordingly:

$$\mathcal{G} = \frac{P^2}{2B} \frac{dC}{d(2a)} = \frac{P^2}{4B} \frac{dC}{da}$$

Equating the above expression with Eq. (2.24) gives

$$\mathcal{G} = \frac{\sigma^2 \pi a}{E} = \frac{P^2 \pi a}{4B^2 W^2 E} = \frac{P^2}{4B} \frac{dC}{da}$$

Solving for compliance leads to

$$C = \int dC = \frac{\pi}{BW^2 E} \int a da = \frac{\pi}{BE} \left(\frac{a}{W} \right)^2 + \text{constant}$$

The constant corresponds to the compliance of the uncracked plate. Assuming a gage length L , the *total* compliance is given by

$$C_{tot} = \frac{\pi}{BE} \left(\frac{a}{W} \right)^2 + \frac{L(1-\nu^2)}{2BWE} = C + C_m$$

where C_m represents the compliance of the uncracked plate and C is the *additional* compliance due to the crack. When $a \ll W$ or $a \ll L$, the first term in the above expression is negligible. Recall the previous problem, where it was argued that displacement control is equivalent to load control in an infinite plate because $C \ll C_m$.

2.5 A material exhibits the following crack growth resistance behavior:

$$R = 6.95(a - a_o)^{0.5}$$

where a_o is the initial crack size. R has units of kJ/m² and crack size is in millimeters. Alternatively,

$$R = 200(a - a_o)^{0.5}$$

where R has units of in-lb/in² and crack size is in inches. The elastic modulus of this material = 207,000 MPa (30,000 ksi). Consider a wide plate with a through crack ($a \ll W$) that is made from this material.

- (a) If this plate fractures at 138 MPa (20.0 ksi), compute the following:
- (i) The half crack size at failure (a_c).
 - (ii) The amount of stable crack growth (at each crack tip) that precedes failure ($a_c - a_o$).
- (b) If this plate has an initial crack length ($2a_o$) of 50.8 mm (2.0 in) and the plate is loaded to failure, compute the following:
- (i) The stress at failure.
 - (ii) The half crack size at failure.
 - (iii) The stable crack growth at each crack tip

Ans:

At instability, $G = R$ and $dG/da = dR/da$. Therefore,

$$\frac{\pi \sigma^2 a_c}{E} = 6.95 (a_c - a_o)^{0.5} \quad (1)$$

and

$$\frac{\pi \sigma^2}{E} = 3.48 (a_c - a_o)^{-0.5} \quad (2)$$

Thus we have two equations to relate σ , a_c and a_o , and we must specify one of these quantities.

(a) $\sigma = 138 \text{ MPa}$

From Eq. (1) above,

$$\frac{\pi (138,000 \text{ kPa})^2}{2.07 \times 10^8 \text{ kPa}} = 3.48 (a_c - a_o)^{-0.5}$$

$$a_c - a_o = 145 \text{ mm}$$

Substituting into (2) gives

$$\frac{\pi (138,000 \text{ kPa})^2 a_c}{2.07 \times 10^8 \text{ kPa}} = 6.95 (145 \text{ mm})^{0.5}$$

Thus

- (i) $a_c = 290 \text{ mm}$
- (ii) $a_c - a_o = 145 \text{ mm}$
- (iii) $a_o = 145 \text{ mm}$

(b) $a_o = 25.4 \text{ mm}$

Dividing Eq. (1) by Eq. (2) leads to

$$a_c = 2(a_c - a_o)$$

Therefore, if $a_o = 25.4 \text{ mm}$, $a_c = 50.8 \text{ mm}$ and $(a_c - a_o) = 25.4 \text{ mm}$. We can solve for critical stress by substituting these results into Eq. (1):

$$\frac{\pi \sigma^2 (0.0508 \text{ m})}{2.07 \times 10^8 \text{ kPa}} = 6.95 (25.4 \text{ mm})^{0.5}$$

Thus

(i) $\sigma = 213,000 \text{ kPa} = 213 \text{ MPa}$

(ii) $a_c = 50.8 \text{ mm}$

(iii) $a_c - a_o = 25.4 \text{ mm}$

2.6 Suppose that a double cantilever beam specimen (Fig. 2.9) is fabricated from the same material considered in Problem 2.5. Calculate the load at failure and the amount of stable crack growth. The specimen dimensions are as follows:

$$B = 25.4 \text{ mm (1 in)}$$

$$h = 12.7 \text{ mm (0.5 in)}$$

$$a_o = 152 \text{ mm (6 in)}$$

Ans:

At instability, $\mathcal{G} = R$ and $d\mathcal{G}/da = dR/da$. Hence,

$$\mathcal{G}_c = \frac{12P_c^2 a_c^2}{B^2 h^3 E} = 6.95 (a_c - a_o)^{0.5} \quad (1)$$

$$\frac{2\mathcal{G}_c}{a} = 3.48 (a_c - a_o)^{-0.5} \quad (2)$$

Dividing (1) by (2) gives

$$\frac{a_c}{2} = 2(a_c - a_o)$$

Thus

$$a_c = \frac{4}{3} a_o = 203 \text{ mm}$$

and

$$\mathcal{G}_c = \frac{12P_c^2 (0.203 \text{ m})^2}{(0.025 \text{ m})^2 (0.0127 \text{ m})^3 2.07 \times 10^8 \text{ kPa}} = 6.95 (203 - 152)^{0.5}$$

$$P_c = 5.16 \text{ kN}$$

- 2.7** Consider a nominally linear elastic material with a rising R curve (e.g., Problems 2.5 and 2.6). Suppose that one test is performed on wide plate with a through crack (Fig. 2.3) and a second test on the same material is performed on a DCB specimen (Fig. 2.9). If both tests are conducted in load control, would the \mathcal{G}_c values at instability be the same? If not, which geometry would result in a higher \mathcal{G}_c ? Explain.

Ans

The driving force curve for the through crack is linear, while \mathcal{G} varies with a^2 for the DCB specimen. Therefore, the two geometries would have different points of tangency on the R curve, as Fig. S1 illustrates. The \mathcal{G}_c value for the through crack would be higher, and this geometry would experience more stable crack growth prior to failure.

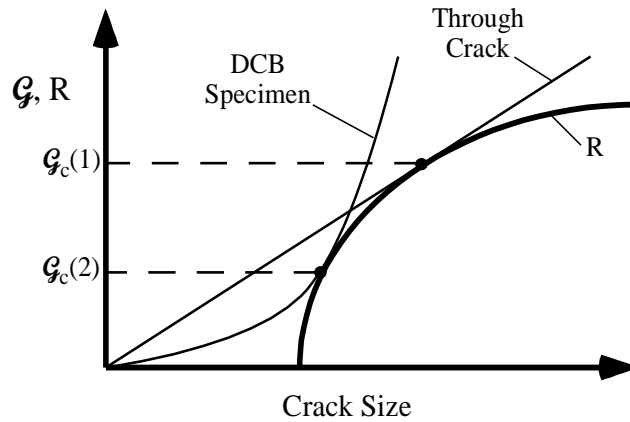


FIGURE S1 Effect of specimen geometry on instability (Problem 2.7)

- 2.8** Example 2.3 showed that the energy release rate, \mathcal{G} , of the double cantilever beam (DCB) specimen increases with crack growth when the specimen is held at a constant load. Describe (qualitatively) how you could alter the design of the DCB specimen such that a growing crack in load control would experience a constant \mathcal{G} .

Ans:

In a conventional DCB specimen, compliance varies with a^3 , and energy release is proportional to a^2 when load is fixed. In order for G to remain constant with crack growth, compliance must vary linearly with crack length. One way to accomplish this is to taper the specimen width, as Fig. S2 illustrates. Alternatively, the thickness can be tapered. The latter method is not as effective as the former because compliance is less sensitive to the thickness dimension; recall that the moment of inertia of the cross section is proportional to Bh^3 . Specimens such as illustrated in Fig. S2, where G is relatively constant over a range of crack lengths, have been used successfully in laboratory experiments.

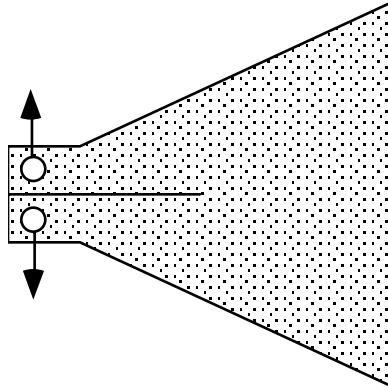


FIGURE S2 Tapered DCB specimen (Problem 2.8).

2.9 Beginning with Eq. (2.20), derive an expression for the potential energy of a plate subject to a tensile stress σ with a penny-shaped flaw of radius a . Assume that $a \ll$ plate dimensions.

Ans:

At fracture,

$$-\frac{d\Pi}{dA} = \frac{dW_s}{dA}$$

For the penny-shaped crack,

$$W_s = 2\pi\gamma_s a^2$$

and

$$\frac{dW_s}{dA} = 2\gamma_s$$

Combining the above results with Eq. (2.20) gives

$$2\gamma_s = \frac{4(1-\nu^2)a\sigma_f^2}{\pi E} = -\frac{d\Pi}{d\mathcal{A}}$$

The above equation must be integrated with respect to crack area to infer the potential energy. The crack area can be written in terms of the crack radius, a :

$$dA = 2\pi a da$$

and

$$\frac{d}{dA} = \frac{1}{2\pi a} \frac{d}{da}$$

Therefore,

$$\frac{d\Pi}{da} = -\frac{8\sigma^2 a^2 (1-\nu^2)}{E}$$

and

$$\Pi = \Pi_o - \frac{8\sigma^2 a^3 (1-\nu^2)}{3E}$$

where Π_o is the potential energy of the uncracked solid.

2.10 Beginning with Eq. (2.20), derive expressions for the energy release rate and Mode I stress intensity factor of a penny-shaped flaw subject to a remote tensile stress. (Your K_I expression should be identical to Eq. (2.44).)

Ans:

At fracture in an ideally brittle material, $\mathcal{G} = \mathcal{G}_c = 2\gamma_s$. Rearranging Eq. (2.20) leads to

$$2\gamma_s = \frac{4(1-\nu^2)a\sigma_f^2}{\pi E} = \mathcal{G}_c$$

Thus

$$\mathcal{G} = \frac{4(1-\nu^2)a\sigma^2}{\pi E} \quad \mathcal{G} = \frac{4(1-\nu^2)a\sigma^2}{\pi E}$$

Invoking the relationship between K_I and \mathcal{G} (Eq. (2.56)) gives

$$K_I = \frac{2}{\pi} \sigma \sqrt{\pi a}$$

which agrees with Eq. (2.44). Note that the plane strain K_I - G relationship is appropriate in this case. The strain parallel to the crack front is zero because the crack is axisymmetric.

2.11 Calculate K_I for a rectangular bar containing an edge crack loaded in three point bending.

$P = 35.0 \text{ kN (7870 lb)}$; $W = 50.8 \text{ mm (2.0 in)}$; $B = 25 \text{ mm (1.0 in)}$; $a/W = 0.2$; $S = 203 \text{ mm (8.0 in)}$.

Ans:

The K_I solutions in Table 2.4 have the following form:

$$K_I = \frac{P}{B\sqrt{W}} f\left(\frac{a}{W}\right)$$

Inserting $a/W = 0.2$ and $S/W = 4$ into the appropriate polynomial in Table 2.4 gives $f(a/W) = 4.70$. Thus

$$K_I = \frac{(35 \text{ kN})(4.70)}{0.025 \text{ m}\sqrt{0.050 \text{ m}}} = 29,400 \text{ kPa}\sqrt{\text{m}} = 29.4 \text{ MPa}\sqrt{\text{m}}$$

2.12 Consider a material where $K_{Ic} = 35 \text{ MPa}\sqrt{\text{m}}$ (31.8 ksi $\sqrt{\text{in}}$). Each of the five specimens in Table 2.4 and Fig. 2.23 have been fabricated from this material. In each case, $B = 25.4 \text{ mm}$ (1 in), $W = 50.8 \text{ mm}$ (2 in), and $a/W = 0.5$. Estimate the failure load for each specimen. Which specimen has the highest failure load? Which has the lowest?

Ans:

Failure load is inversely proportional to the geometry correction factor, $f(a/W)$:

$$P_{crit} = \frac{K_{Ic} B\sqrt{W}}{f(a/W)}$$

Thus it is obvious from Fig. 2.23 that the CCT and DENT geometries have the highest failure load and the SE(B) geometry the lowest (for fixed B , W , and a). The calculated failure loads are tabulated below.

Geometry	$f(0.5)$	P_{crit} , kN
SENT	3.543	5.57
SE(B)	10.65	1.85
CCT	1.051	18.76
DENT	1.049	18.80
Compact	9.659	2.04

2.13 A large block of material is loaded to a stress of 345 MPa (50 ksi). If the fracture toughness (K_{Ic}) is 44 MPa \sqrt{m} (40 ksi \sqrt{in}), determine the critical radius of a buried penny-shaped crack.

Ans:

At fracture, $K_I = K_{Ic}$. Substituting the above data into Eq. (2.44) gives

$$44 \text{ MPa}\sqrt{m} = \frac{2}{\pi} (345 \text{ MPa}) \sqrt{\pi a_c}$$

$$a_c = 12.8 \text{ mm}$$

2.14 A semicircular surface crack in a pressure vessel is 10 mm (0.394 in) deep. The crack is on the inner wall of the pressure vessel and is oriented such that the hoop stress is perpendicular to the crack plane. Calculate K_I if the local hoop stress = 200 MPa (29.0 ksi) and the internal pressure = 20 MPa (2900 psi). Assume that the wall thickness \gg 10 mm.

Ans:

Applying the principle of superposition (see Example 2.5) results in the following stress intensity solution for this case:

$$\begin{aligned}
 K_I &= \lambda_s (\sigma + p) \sqrt{\frac{\pi a}{Q}} f(\phi) \\
 &= (1.14)(220 \text{ MPa}) \sqrt{\frac{\pi (0.01 \text{ m})}{2.64}} = 28.4 \text{ MPa}\sqrt{m}
 \end{aligned}$$

at $\phi = 0^\circ$.

2.15 Calculate K_I for a semi-elliptical surface flaw at $\phi = 0^\circ, 30^\circ, 60^\circ, 90^\circ$.

$\sigma = 150 \text{ MPa (21.8 ksi)}$; $a = 8.00 \text{ mm (0.315 in)}$; $2c = 40 \text{ mm (1.57 in)}$.

Ans:

From Fig. 2.19,

$$K_I = \lambda_s (150 \text{ MPa}) \sqrt{\frac{\pi (0.008 \text{ m})}{Q}} f(\phi)$$

ϕ , Degrees	λ_s	$f(\phi)$	K_I , $\text{MPa}\sqrt{\text{m}}$
0.00	1.20	0.632	15.74
30.00	1.12	0.780	18.08
60.00	1.10	0.943	21.36
90.00	1.09	1.000	22.62

2.16 Consider a plate subject to biaxial tension with a through crack of length $2a$, oriented at an angle β from the σ_2 axis (Fig. 13.1). Derive expressions for K_I and K_{II} for this configuration. What happens to each K expression when $\sigma_1 = \sigma_2$?

Ans:

We can apply the principle of superposition separately to K_I and K_{II} :

$$K_I = \sigma_1 \cos^2(\beta) \sqrt{\pi a} + \sigma_2 \cos^2(90 + \beta) \sqrt{\pi a}$$

$$= [\sigma_1 \cos^2(\beta) + \sigma_2 \sin^2(\beta)] \sqrt{\pi a}$$

$$K_{II} = \sigma_1 \cos(\beta) \sin(\beta) \sqrt{\pi a} + \sigma_2 \cos(90 + \beta) \sin(90 + \beta) \sqrt{\pi a}$$

$$= (\sigma_1 - \sigma_2) \cos(\beta) \sin(\beta) \sqrt{\pi a}$$

when $\sigma_1 = \sigma_2$, $K_I = \sigma_1 \sqrt{\pi a}$ and $K_{II} = 0$.

- 2.17** A wide flat plate with a through-thickness crack experiences a nonuniform normal stress which can be represented by the following crack face traction:

$$p(x) = p_o e^{-x/\beta}$$

where $p_o = 300$ MPa and $\beta = 25$ mm. The origin ($x = 0$) is at the left crack tip, as illustrated in Fig. 2.27. Using the weight function derived in Example 2.6, calculate K_I at each crack tip for $2a = 25, 50$, and 100 mm. You will need to integrate the weight function *numerically*.

Ans:

Figure S3 is a plot of K_I versus crack length for the through crack with the exponential stress distribution given above. Values for three crack lengths are tabulated below.

$2a$, mm	K_I , MPa $\sqrt{\text{m}}$	
	$x = 2a$	$x = 0$
25	29.21	48.15
50	21.74	57.37
100	11.06	62.65

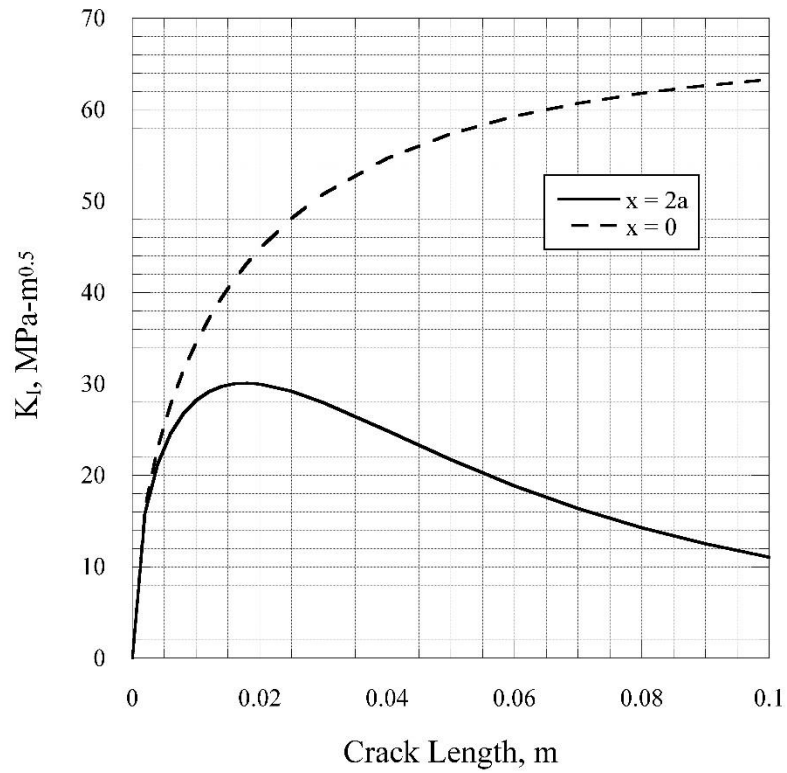
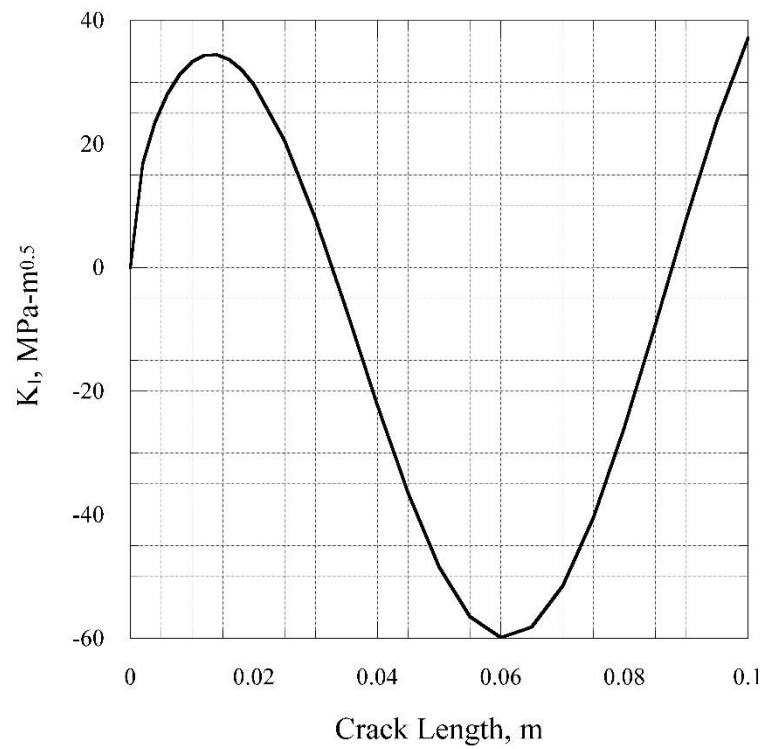
- 2.18** Repeat Problem 2.17 with the following crack face pressure profile:

$$p(x) = p_o \cos\left(\frac{\pi x}{50 \text{ mm}}\right)$$

At what crack length(s) does $K_I = 0$ at the right tip?

Ans.

Figure S4 is a plot of K_I versus crack length for the right tip ($x = 2a$). The curve passes through zero at $2a = 33$ mm and 88 mm.

**FIGURE S3. Solution to Problem 2.17.****FIGURE S4. Solution to Problem 2.17**

2.19 For an infinite plate with a through crack 50.8 mm (2.0 in) long, compute and tabulate K_{eff} vs. stress using the three methods indicated below. Assume $\sigma_{YS} = 250$ MPa (36.3 ksi).

Ans:

Applying Eqs. (2.41), (2.70), and (2.81) to the values above results in the following stress intensity factors for the LEFM, Irwin, and strip yield methods respectively:

Stress, MPa (ksi)	K_{eff} , MPa \sqrt{m} or ksi \sqrt{in}		
	LEFM	Irwin Correction	Strip Yield Model
25 (3.63)	7.06	7.08	7.08
50 (7.25)	14.1	14.3	14.2
100 (14.5)	28.2	29.5	29.3
150 (21.8)	42.4	46.8	46.3
200 (29.0)	56.5	68.5	68.9
225 (32.6)	63.6	82.4	86.6
249 (36.1)	70.3	99.1	143
250 (36.3)	70.6	99.9	∞

2.20 A material has a yield strength of 345 MPa (50 ksi) and a fracture toughness of 110 MPa \sqrt{m} (100 ksi \sqrt{in}). Determine the minimum specimen dimensions (B , a , W) required to perform a valid K_{Ic} test on this material, based on the traditional size requirements in Eq. (2.88). Comment on the feasibility of testing a specimen of this size.

Ans:

From Eq. (2.88),

$$a, B, (W - a) \geq 2.5 \left(\frac{110 \text{ MPa} \sqrt{m}}{345 \text{ MPa}} \right)^2 = 0.254 \text{ m (10.0 in)}$$

Therefore,

$$W \geq 0.508 \text{ m (20.0 in)}$$

Testing such a large specimen would impractical because:

- Machining costs would be very high.
- A very large test machine would be required.
- Materials are usually not available in such large section thicknesses. Even if a section of sufficient size could be produced, its metallurgical properties would not be representative of a thinner plate of the same material.

- 2.21** You have been given a set of fracture mechanics test specimens, all of the same size and geometry. These specimens have been fatigue precracked to various crack lengths. The stress intensity of this specimen configuration can be expressed as follows:

$$K_I = \frac{P}{B\sqrt{W}} f(a/W)$$

where P is load, B is thickness, W is width, a is crack length, and $f(a/W)$ is a dimensionless geometry correction factor.

Describe a set of experiments you could perform to determine $f(a/W)$ for this specimen configuration. Hint: you may want to take advantage of the relationship between K_I and energy release rate for linear elastic materials.

Ans:

The stress intensity factor can be inferred from compliance measurements as follows:

$$\frac{K_I^2}{E'} = \mathcal{G} = \frac{P^2}{2B} \frac{dC}{da} = \frac{P^2}{B^2 W E'} f^2(a/W)$$

assuming the specimen contains an edge crack, such that $dA = Bda$. Solving for $f(a/W)$ gives

$$f(a/W) = \sqrt{\frac{B E'}{2} \frac{dC}{d(a/W)}}$$

Thus $f(a/W)$ for the geometry of interest can be inferred by measuring the elastic compliance as a function of crack length, evaluating $dC/d(a/W)$, and inserting the result into the above expression. Note that the absolute compliance depends on specimen size and material properties, but the quantity $(B E' C)$ is dimensionless, and depends only on a/W .

- 2.22** Derive the Griffith-Inglis result for the potential energy of a through crack in an infinite plate subject to a remote tensile stress (Eq. (2.16)). Hint: solve for the work required to close the crack faces; Eq. (A2.43) gives the crack opening displacement for this configuration.

The crack opening displacement at a distance x from the center of the crack (assuming the coordinate system in Fig. 13.2) is given by

$$2u_y = \frac{4\sigma}{E} \sqrt{a^2 - x^2}$$

for *plane stress* loading. The incremental closure work done at a point is as follows:

$$d\Pi = \frac{1}{2} \bullet 2\sigma u_y(x) B dx \quad d\Pi = \frac{1}{2} \bullet 2\sigma u_y(x) B dx$$

Thus the decrease in potential energy due to the formation of the crack is given by

$$\Pi - \Pi_o = \frac{4\sigma^2 B}{E} \int_0^a \sqrt{a^2 - x^2} dx = \frac{\pi\sigma^2 a^2 B}{E}$$

which agrees with Eq. (2.16).

2.23 Using the Westergaard stress function approach, derive the stress intensity factor relationship for an infinite array of collinear cracks in a plate subject to biaxial tension (Fig. 2.21).

Ans:

Substituting $z^* = z - a$ into Eq. (A2.39) and re-arranging gives

$$Z(z^*) = \frac{\sigma \sin \left[\frac{\pi(a + z^*)}{2W} \right]}{\sqrt{\left[\sin \left(\frac{\pi(a + z^*)}{2W} \right) \right]^2 - \left[\sin \left(\frac{\pi a}{2W} \right) \right]^2}}$$

Let us now perform a series expansion about $z^* = 0$ on the \sin^2 term on the left side of the denominator:

$$\left[\sin \left(\frac{\pi(a + z^*)}{2W} \right) \right]^2 = \left[\sin \left(\frac{\pi a}{2W} \right) \right]^2 + \sin \left(\frac{\pi a}{2W} \right) \cos \left(\frac{\pi a}{2W} \right) \frac{\pi z^*}{W} + \dots$$

Substituting this result into the stress function and taking a limit leads to

$$\lim_{z^* \rightarrow 0} [Z(z^*)] = \frac{\sigma \sqrt{\tan\left(\frac{\pi a}{2W}\right)}}{\sqrt{\frac{\pi z^*}{W}}} = \frac{K_I}{\sqrt{2\pi z^*}}$$

Solving for stress intensity gives

$$K_I = \sigma \sqrt{2W} \sqrt{\tan\left(\frac{\pi a}{2W}\right)} = \sigma \sqrt{\pi a} \sqrt{\frac{2W}{\pi a} \tan\left(\frac{\pi a}{2W}\right)}$$

which agrees with Eq. (2.45).

CHAPTER 3

3.1 Repeat the derivation of Eqs. (3.1) to (3.3) for the plane strain case.

Ans:

In plane strain, the displacement of the crack face a distance r_y behind the tip is given by

$$u_y = \frac{4(1-\nu^2)}{E} K_I \sqrt{\frac{r_y}{2\pi}}$$

Substituting Eq. (2.63) into the above expression leads to

$$\delta = 2u_y = \frac{8(1-\nu^2)}{E} K_I \sqrt{\frac{K_I^2}{12\pi^2 \sigma_{ys}^2}} = \frac{4}{\sqrt{3}\pi} \frac{K_I^2 (1-\nu^2)}{\sigma_{ys} E} = \frac{4}{\sqrt{3}\pi} \frac{\mathcal{G}}{\sigma_{ys}}$$

3.2 A *CTOD* test is performed on a three point bend specimen. Figure 13.3 shows the deformed specimen after it has been unloaded. That is, the displacements shown are the *plastic components*.

- (a) Derive an expression for plastic *CTOD* (δ_p) in terms of Δ_p and specimen dimensions.
- (b) Suppose that V_p and Δ_p are measured on the same specimen, but that the plastic rotational factor, r_p , is unknown. Derive an expression for r_p in terms of Δ_p , V_p and specimen dimensions, assuming the angle of rotation is small.

Ans:

(a) By similar triangles,

$$\frac{\Delta_p}{2W} = \frac{\delta_p/2}{r_p(W-a)}$$

Thus

$$\delta_p = \frac{r_p(W-a)\Delta_p}{W}$$

(b) Compare alternative equations for δ_p :

$$\frac{r_p(W-a)V_p}{r_p(W-a)+a} = \frac{r_p(W-a)\Delta_p}{W}$$

Solving for r_p leads to

$$r_p = \frac{1}{W-a} \left(\frac{V_p W}{\Delta_p} - a \right)$$

3.3 Fill in the missing steps between Eqs. (3.36) and (3.37)

Ans:

$$\left(\frac{\partial \Omega_c}{\partial b} \right)_M = -F' \left(\frac{M}{b^2} \right) \frac{2M}{b^3}$$

$$\left(\frac{\partial \Omega_c}{\partial M} \right)_b = F' \left(\frac{M}{b^2} \right) \frac{1}{b^2}$$

$$F' = b^2 \left(\frac{\partial \Omega_c}{\partial M} \right)_b$$

$$\left(\frac{\partial \Omega_c}{\partial b} \right)_M = -\frac{2M}{b} \left(\frac{\partial \Omega_c}{\partial M} \right)_b$$

$$d\Omega_c = \left(\frac{\partial \Omega_c}{\partial M} \right)_b dM \quad (\text{assuming } b \text{ is fixed})$$

Substituting the last two expressions into Eq. (3.35) gives

$$J = \frac{2}{b} \int_0^{\Omega_c} M d\Omega_c$$

3.4 Derive an expression for the J integral for a deeply notched three-point bend specimen, loaded over a span S , in terms of the area under the load-displacement curve and ligament length, b . Figure 13.3 illustrates two displacement measurements on a bend specimen: the load line displacement (Δ) and the crack mouth opening displacement (V). Which of these two displacement measurements is more appropriate for inferring the J integral? Explain.

Ans:

The energy absorbed by the specimen loading in three-point bending can be expressed as

$$U = \int_0^{\Delta_{nc}} P d\Delta_{nc} + \int_0^{\Delta_c} P d\Delta_c$$

where Δ_{nc} is the load line displacement of an uncracked beam and Δ_c is the *additional* load line displacement due to the crack. We use load line displacement to quantify J because work is defined as the dot product of the force vector with the displacement vector; the load line displacement represents the component of displacement in the loading direction.

Let us modify the dimensional analysis of Eq. (3.36) by replacing M with PS and Ω_c with $\Delta_c S$:

$$\Delta_c = S \cdot f\left(\frac{PS}{b^2}\right) \Delta_c = S f\left(\frac{PS}{b^2}\right)$$

The remainder of the derivation is similar to Problem 3.3:

$$J = - \int_0^P \left(\frac{\partial \Delta_c}{\partial b} \right)_P dP$$

$$\left(\frac{\partial \Delta_c}{\partial b} \right)_P = -2f' \left(\frac{PS}{b^2} \right) \frac{PS^2}{b^3}$$

$$\left(\frac{\partial \Delta_c}{\partial P} \right)_b = f' \left(\frac{PS}{b^2} \right) \frac{S^2}{b^2}$$

$$f' = \frac{b^2}{S^2} \left(\frac{\partial \Delta_c}{\partial P} \right)_b$$

$$\left(\frac{\partial \Delta_c}{\partial b} \right)_P = -\frac{2P}{b} \left(\frac{\partial \Delta_c}{\partial P} \right)_b$$

$$d\Delta_c = \left(\frac{\partial \Delta_c}{\partial P} \right)_b dP \quad (\text{assuming } b \text{ is fixed})$$

Thus

$$J = \frac{2}{b} \int_0^{\Delta_c} P d\Delta_c$$

We must obtain the above result because the two definitions of work, based on $M-\Omega_c$ and $P-\Delta_c$, are equivalent.

3.5 Derive an expression for the J integral for an axisymmetrically notched bar in tension (Fig. 13.3), where the notch depth is sufficient to confine plastic deformation to the ligament.

Ans:

This derivation is very similar to the double edge notched tension case (Eqs. (3.26) to (3.32)). Recall, however, that the energy release rate is the derivative of potential energy with respect to *crack area* rather than crack length. For the present geometry, $dA = -2\pi r dr$, where r is the ligament radius. Thus Eq. (3.29) must be modified accordingly:

$$J = \frac{K_I^2}{E'} - \frac{1}{2\pi r} \int_0^P \left(\frac{\partial \Delta_c}{\partial r} \right)_P dP$$

Assuming plastic deformation is confined to the ligament, dimensional analysis gives

$$\Delta_p = r \cdot h \left(\frac{P}{r^2} \right)$$

Thus

$$\left(\frac{\partial \Delta_p}{\partial r} \right)_P = h \left(\frac{P}{r^2} \right) - 2h' \left(\frac{P}{r^2} \right) \frac{P}{r^2}$$

$$\left(\frac{\partial \Delta_p}{\partial P} \right)_r = h' \left(\frac{P}{r^2} \right) \frac{1}{r}$$

$$h' = r \left(\frac{\partial \Delta_p}{\partial P} \right)_r$$

$$\left(\frac{\partial \Delta_p}{\partial r} \right)_P = \frac{1}{r} \left[\Delta_p - 2P \left(\frac{\partial \Delta_p}{\partial P} \right)_r \right]$$

$$J = \frac{K_I^2}{E'} - \frac{1}{2\pi r^2} \int_0^P \left[\Delta_p - 2P \left(\frac{\partial \Delta_p}{\partial P} \right)_r \right] dP$$

$$J = \frac{K_I^2}{E'} + \frac{1}{2\pi r^2} \left[3 \int_0^{\Delta_p} P d\Delta_p - P\Delta_p \right]$$

3.6 Derive an expression for the J integral in a deeply notched three-point bend specimen in terms of the area under the load-crack mouth opening displacement curve. Begin with the corresponding formula for the P - Δ curve (given below), and assume rotation about a plastic hinge (Fig. 13.3).

$$J = \frac{K^2}{E'} + \frac{2}{b} \int_0^{\Delta_p} P d\Delta_p$$

for a specimen with unit thickness.

Ans:

Based on a similar triangles construction:

$$\frac{\Delta_p}{2W} = \frac{V_p}{a + r_p b}$$

$$\Delta_p = \frac{2WV_p}{a + r_p b}$$

where $b = (W - a)$. At a fixed crack length,

$$d\Delta_p = \frac{2W}{a + r_p b} dV_p$$

Thus

$$J = \frac{K^2}{E'} + \frac{4W}{ab + r_p b^2} \int_0^{V_p} P dV_p$$

CHAPTER 4

- 4.1** A high rate fracture toughness test is to be performed on a high strength steel with $K_{Id} = 110 \text{ MPa}\sqrt{\text{m}}$ (100 ksi $\sqrt{\text{in}}$). A three-point bend specimen will be used, with $W = 50.8 \text{ mm}$ (2.0 in), $a/W = 0.5$, $B = W/2$, and span = $4W$. Also, $c_l = 5940 \text{ m/sec}$ (19,500 ft/sec) for steel. Estimate the maximum loading rate at which the quasistatic formula for estimating K_{Id} is approximately valid.

Ans:

The maximum loading rate at which the quasistatic formulae can be applied can be estimated by dividing the load at failure by twice the transition time, t_τ . Based on finite element analysis of the standard three-point bend specimen $t_\tau c_l/W \approx 27$. Thus

$$2t_\tau = \frac{54(0.0508 \text{ m})}{5940 \text{ m/s}} = 4.62 \times 10^{-4} \text{ s}$$

From the polynomial relationship in Table 2.4, $f(a/W) = 10.62$ for $a/W = 0.50$.

$$K_{Id} = 100 \text{ MPa}\sqrt{\text{m}} = \frac{P(10.62)}{0.0254 \text{ m}\sqrt{0.0508 \text{ m}}}$$

$$P = 53.8 \text{ kN}$$

$$\text{Loading rate} = P/2t_\tau = 116 \text{ MN/s}$$

- 4.2** Unstable fracture initiates in a steel specimen and arrests after the crack propagates 8.0 mm (0.32 in). The total propagation time was $7.52 \times 10^{-6} \text{ sec}$. The initial ligament length in the specimen was 30.0 mm (1.18 in) and c_l for steel = 5940 m/sec (19,500 ft/sec). Determine whether or not reflected stress waves influenced the propagating crack.

Ans:

Distance traveled by the leading stress wave = $(5940 \text{ m/s})(7.52 \times 10^{-6} \text{ s})$
= 44 mm.

In order for a stress wave to interfere with the propagating crack, the wave must travel $2b_o - \Delta a = 52$ mm. Therefore, reflected stress waves did not influence crack propagation in this case.

- 4.3** Fracture initiates at an edge crack in a 2.0 m (78.7 in) wide steel plate and rapidly propagates through the material. The stress in the plate is fixed at 300 MPa (43.5 ksi). Plot the crack speed versus crack size for crack lengths ranging from 10 to 60 mm (or 0.4 to 2.4 in). The dynamic fracture toughness of the material is given by

$$K_{ID} = \frac{K_{IA}}{1 - \left(\frac{V}{V_I} \right)^2}$$

where $K_{IA} = 55 \text{ MPa}\sqrt{\text{m}}$ ($50 \text{ ksi}\sqrt{\text{in}}$) and $V_I = 1500 \text{ m/sec}$ (4920 ft/sec). Use the Rose approximation (Eqs. (4.17) and (4.18)) for the driving force. The elastic wave speeds for steel are given below.

c_1	5940 m/sec	19,500 ft/sec
c_2	3220 m/sec	10,600 ft/sec
c_T	2980 m/sec	9780 ft/sec

Ans:

Figure S5 compares dynamic toughness and driving force curves for various crack lengths. The crack speed at a given crack length corresponds to the intersection between the driving force and K_{ID} curves. Figure S6 is a plot of the crack speed versus crack length.

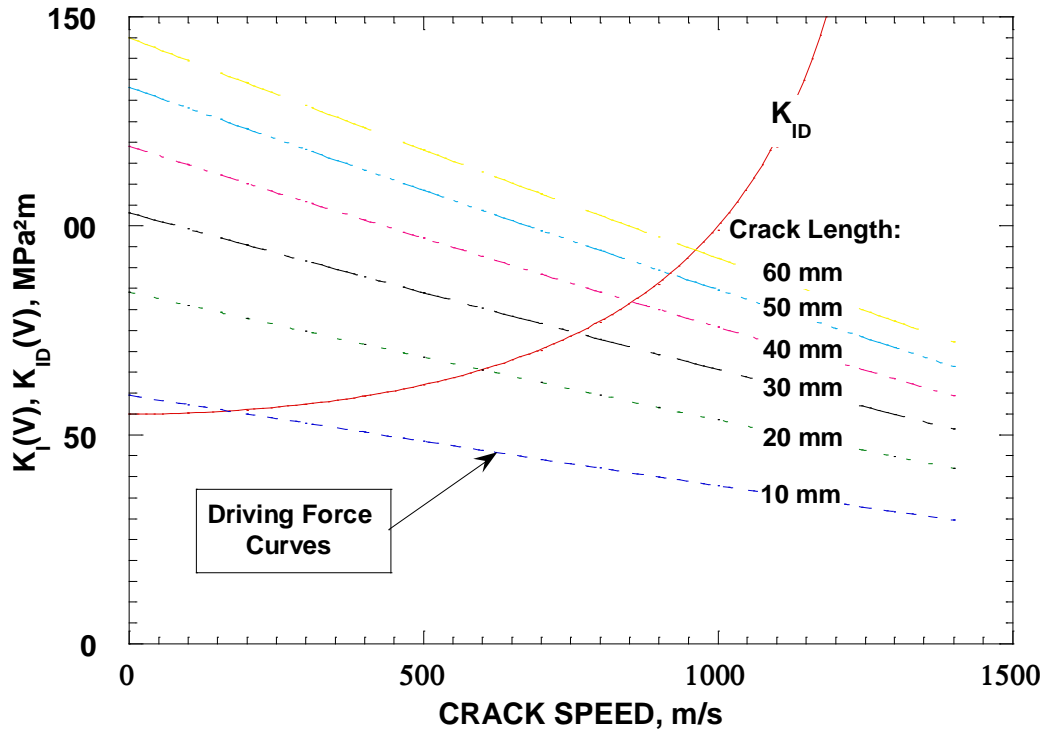


FIGURE S5 Comparison of dynamic toughness and driving force (Problem 4.3).

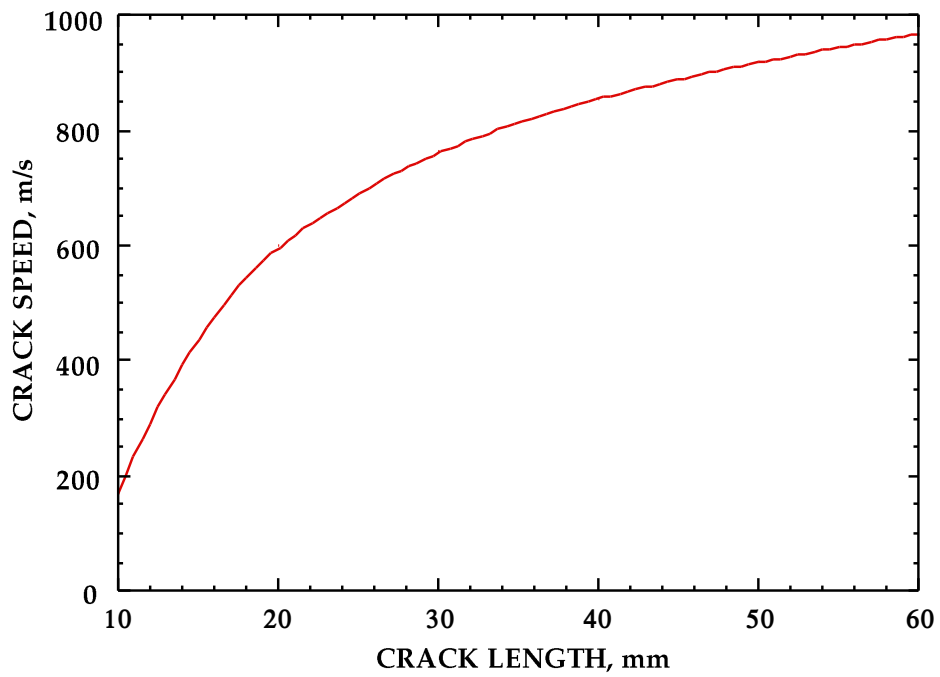


FIGURE S6 Computed crack speed versus crack length (Problem 4.3)

- 4.4** Derive an expression for C^* in a double edge notched tension panel in terms of specimen dimensions, creep exponent, load, and displacement rate. See Section 3.2.5 for the corresponding J expression.

Ans:

By analogy with Eq. (3.32):

$$C^* = \frac{1}{2Bb} \left[2 \int_0^{\Delta} P d\dot{\Delta} - P\dot{\Delta} \right]$$

Assuming the viscous creep zone is confined to the ligament. For power-law creep, where displacement rate is proportional to P^n :

$$\int_0^{\Delta} P d\dot{\Delta} = \frac{n}{n+1} P\dot{\Delta}$$

Thus

$$C^* = \frac{n-1}{n+1} \frac{P\dot{\Delta}}{2Bb}$$

- 4.5** A three-point bend specimen is tested in displacement control at an elevated temperature. The displacement rate is increased in steps as the test progresses. The load, load line displacement rate, a/W , and crack velocity are tabulated below. Compute C^* , and construct a log-log plot of crack velocity versus C^* . The specimen thickness and width are 25 mm and 50 mm, respectively. The creep exponent = 5.0 for the material.

$\dot{\Delta}$, m/s	Load, kN	a/W	\dot{a} , m/s
1.0×10^{-7}	10.8	0.52	3.67×10^{-9}
5.0×10^{-7}	13.8	0.54	1.79×10^{-8}
1.0×10^{-6}	14.9	0.56	3.49×10^{-8}
5.0×10^{-6}	19.0	0.58	1.71×10^{-7}
1.0×10^{-5}	20.4	0.60	3.37×10^{-7}
5.0×10^{-5}	24.0	0.65	1.65×10^{-6}

Ans:

Equation (4.39) gives the relationship between C^* , load, displacement rate, and specimen dimensions; $\eta = 2.0$ for the three-point bend specimen. Figure S7 is a log-log plot of crack velocity versus C^* .

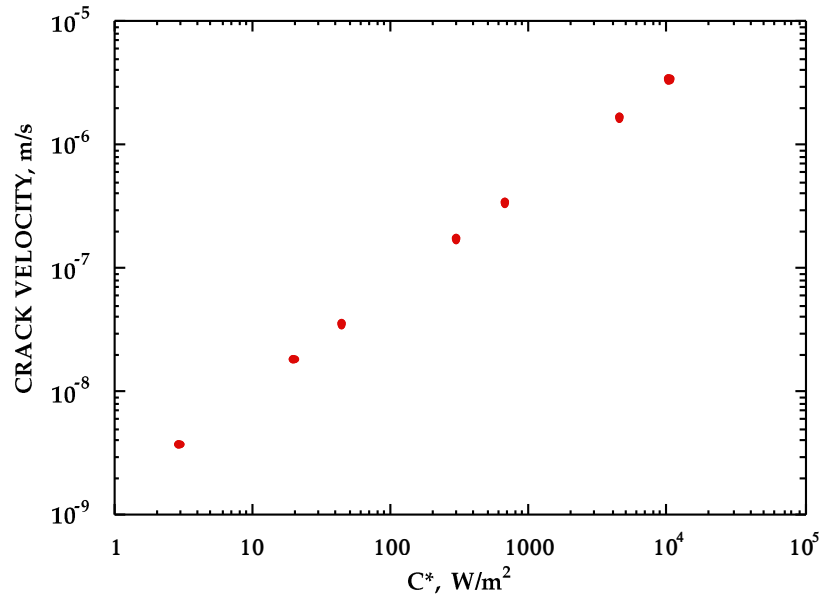


FIGURE S7 Log-log plot of crack velocity as a function of C^* (Problem 4.5).

- 4.6** In a linear viscoelastic material, the pseudo elastic displacement and the physical displacement are related through a hereditary integral:

$$\Delta^e = \{Ed\Delta\}$$

Simplify this expression for the case of a constant displacement rate.

Ans:

See Eqs. (8.10) and (8.11) in Chapter 8.

- 4.7** Consider a fracture toughness test on a nonlinear viscoelastic material at a constant displacement rate. Assume that the load is related to the pseudo elastic displacement by a power law:

$$P = M(\Delta^e)^N$$

where M and N are constants that do not vary with time. Show that the viscoelastic J integral and the conventional J integral are related as follows:

$$J_v = J\phi(t)$$

where ϕ is a function of time. Derive an expression for $\phi(t)$. Hint: begin with Eqs. (3.17) and (4.75). Also, the result from the previous problem may be useful.

Ans:

See Eqs. (8.14) to (8.23) in Chapter 8.

- 4.8** A fracture toughness test on a linear viscoelastic material results in a nonlinear load-displacement curve in a constant rate test. Yielding is restricted to a very small region near the crack tip. Why is the curve nonlinear? Does the stress intensity factor characterize the crack tip conditions in this case? Explain. What is the relationship between J and K_I for a linear viscoelastic material? Hint: refer to the second equation in the previous problem.

Ans:

The load-displacement curve is nonlinear in this case because the elastic modulus is time dependent; the compliance of the specimen increases with time. Despite the nonlinearity in the load displacement relationship, the stresses near the crack tip still exhibit a $1/\sqrt{r}$ dependence, and K_I uniquely characterizes the crack tip conditions. This follows from the correspondence principle in Eq. (4.71), which states that the stress fields in a viscoelastic body are identical to the reference elastic state when the tractions at the boundaries are the same in both cases. Equation (4.77) gives the relationship between J_v and K_I . For a constant rate test, the conventional J integral is related to K as follows:

$$J = \frac{K_I^2 (1 - \nu^2)}{\phi(t) E_R}$$

where $\phi(t)$ is a dimensionless function of time (see Problem 4.7).

CHAPTER 5

- 5.1** A body-centered cubic (BCC) material contains second phase particles. The size of these particles can be controlled through thermal treatment. Discuss the anticipated effect of particle size on the material's resistance to both cleavage fracture and microvoid coalescence, assuming the volume fraction of the second phase remains constant.

Ans:

(a) Cleavage.

The resistance to cleavage fracture would be enhanced by fine particles, since the local fracture stress is proportional to $1/\sqrt{\text{particle size}}$ (Eq. (5.18)).

(b) Microvoid coalescence

The effect of particle size on toughness depends on whether fracture is controlled by void nucleation or void growth. Nucleation tends to be more difficult at smaller particles, implying an increase in toughness with particle refinement. However, refining the particles would also decrease the average particle spacing, in which case void coalescence would be easier. When the microstructure contains a few

coarse particles, voids must grow to a large size before coalescence is possible; the nominal strain to failure would tend to increase with increasing particle spacing if fracture were controlled by void growth.

- 5.2** An aluminum alloy fails by microvoid coalescence when the average void size reaches ten times the initial value. If the voids grow according to Eq. (5.11), with σ_{ys} replaced by σ_e , plot the equivalent plastic strain (ϵ_{eq}) at failure versus σ_m/σ_e for σ_m/σ_e ranging from 0 to 2.5. Assume the triaxiality ratio remains constant during deformation of a given sample; i.e.,

$$\ln\left(\frac{\bar{R}}{R_o}\right) = 0.283 \exp\left(\frac{1.5\sigma_m}{\sigma_e}\right) \int_0^{\epsilon_{eq}} d\epsilon_{eq}$$

Ans.:

The results are plotted on Fig. S8.

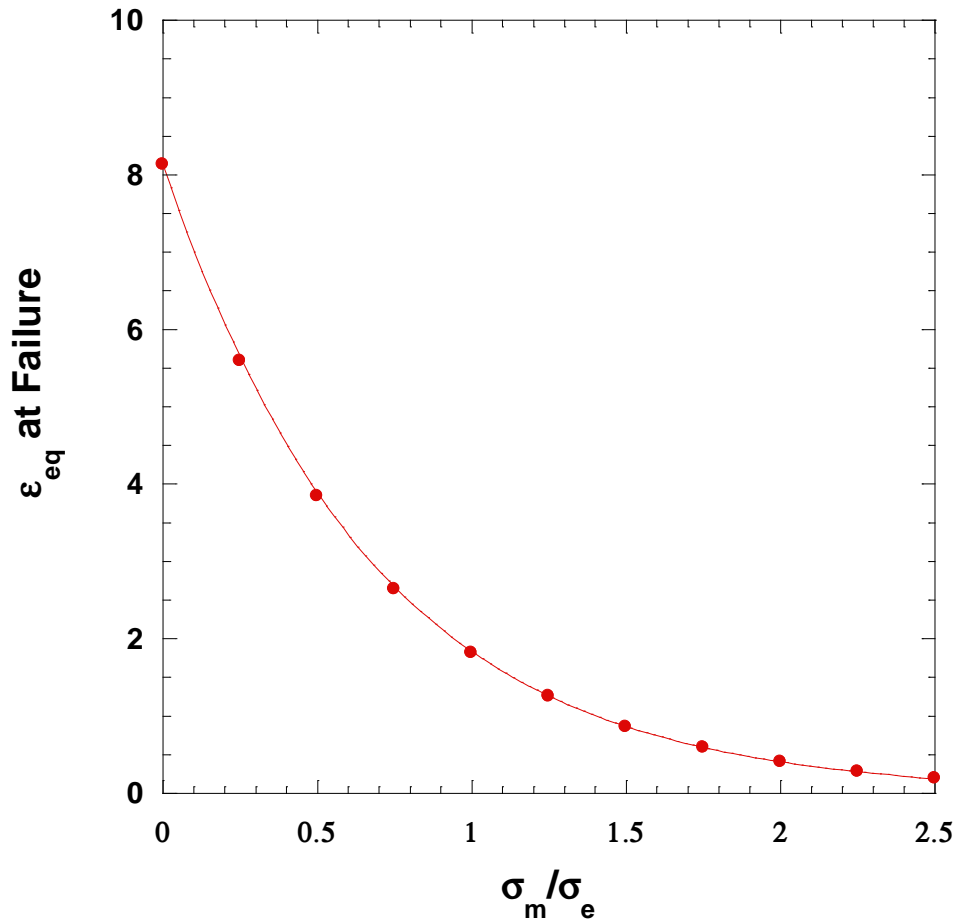


FIGURE S8 Equivalent plastic strain at failure as a function of stress triaxiality (Problem 5.3).

- 5.3** The critical microstructural feature for cleavage initiation in a steel sample is a 6.67 μm diameter spherical carbide; failure occurs when this particle forms a microcrack that satisfies the Griffith criterion (Eq. (5.18)), where $\gamma_p = 14 \text{ J/m}^2$, $E = 207,000 \text{ MPa}$, and $\nu = 0.30$ for the material. Assuming Fig. 5.14 describes the stress distribution ahead of the macroscopic crack, where $\sigma_o = 350 \text{ MPa}$, estimate the critical J value of the sample if the particle is located 0.1 mm ahead of the crack tip, on the crack plane. Repeat this calculation for the case where the critical particle is 0.4 mm ahead of the crack tip.

Ans:

$$\sigma_f = \sqrt{\frac{\pi (207,000 \text{ MPa}) (14 \times 10^{-6} \text{ MJ/m}^2)}{(1-0.3^2) (6.67 \times 10^{-6} \text{ m})}} = 1225 \text{ MPa}$$

$\sigma_f / \sigma_o = 1225 / 350 = 3.50$. According to Fig. 5.14, $r \sigma_o / J_c = 1.85$.

Thus

$$J_c = 18.9 \text{ kJ/m}^2 \text{ when } r = 0.1 \text{ mm}$$

$$J_c = 75.7 \text{ kJ/m}^2 \text{ when } r = 0.4 \text{ mm}$$

- 5.4** Cleavage initiates in a ferritic steel at 3.0 μm diameter spherical particles. The fracture energy on a single grain, γ_p , is 14 J/m^2 and the fracture energy required for propagation across grain boundaries, γ_{gb} , is 50 J/m^2 . At what grain size does propagation across grain boundaries become the controlling step for cleavage fracture?

Ans:

Setting Eq. (5.18) equal to Eq. (5.20) leads to

$$\frac{\gamma_p}{C_o} = \frac{\gamma_{gb}}{d}$$

Thus

$$d = \frac{50 \text{ J/m}^2}{14 \text{ J/m}^2} \cdot 3.0 \mu\text{m} = 10.7 \mu\text{m}$$

- 5.5** Compute the relative size of the 90% confidence band of K_{Ic} data (as in Example 5.1), assuming Eq. (5.24) describes the toughness distribution. Compute the confidence band width for $K_o / \Theta_K = 0, 0.5, 1.0, 2.0$, and 5.0. What is the effect of the threshold toughness, K_o , on the relative scatter? What is the physical significance of Θ_K in this case?

Ans:

$$\frac{K_{0.95} - K_{0.05}}{K_{0.50}} = \frac{\left[\left(\frac{K_o}{\Theta_K} \right)^4 - \ln(0.05) \right]^{1/4} - \left[\left(\frac{K_o}{\Theta_K} \right)^4 - \ln(0.95) \right]^{1/4}}{\left[\left(\frac{K_o}{\Theta_K} \right)^4 - \ln(0.50) \right]^{1/4}} =$$

The results are tabulated below. The size of the confidence band (i.e., the scatter) decreases markedly with increasing K_o/Θ_K . The parameter Θ_K corresponds to the 63rd percentile K value for *initiation* of cleavage; Θ_K is the 63rd percentile toughness when $K_o = 0$.

$\frac{K_o}{\Theta_K}$	90% confidence band ratio
0	0.920
0.5	0.795
1.0	0.352
2.0	0.0426
5.0	0.0012

5.6 Compute the relative size of the 90% confidence band of K_{lc} data (as in Example 5.1), assuming Eq. (5.26) describes the toughness distribution. Compute the confidence band width for $K_o/\Theta_K = 0, 0.5, 1.0, 2.0$, and 5.0 . What is the effect of the threshold toughness, K_o , on the relative scatter?

Ans:

$$\frac{K_{0.95} - K_{0.05}}{K_{0.50}} = \frac{[\ln(0.05)]^{1/4} - [\ln(0.95)]^{1/4}}{\frac{K_o}{\Theta_K} + [\ln(0.50)]^{1/4}}$$

$\frac{K_o}{\Theta_K}$	90% confidence band ratio
------------------------	---------------------------

0	0.920
0.5	0.595
1.0	0.439
2.0	0.288
5.0	0.142

The relative scatter decreases with increasing K_o/θ_K .

13.6 CHAPTER 6

6.1 For the Maxwell spring and dashpot model (Fig. 6.6) derive an expression for the relaxation modulus.

Ans:

In a stress relaxation experiment with the Maxwell model, the strain is fixed at ε_o and the stress is given by Eq. (6.10). The relaxation modulus is defined as

$$E(t) = \frac{\sigma(t)}{\varepsilon_o}$$

Thus

$$E(t) = E_o \exp^{-t/t_R}$$

where E_o is the modulus of the spring.

6.2 Fill in the missing steps in the derivation of Eq. (6.14).

Ans:

It is convenient to divide the model into three elements:

- (1) Spring 1
- (2) the Voigt element, consisting of Spring 2 and Dashpot 2
- (3) Dashpot 1.

The strains in these three elements are additive:

$$\varepsilon(t) = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$

For a constant stress creep test:

$$\varepsilon_1 = \frac{\sigma_o}{E}$$

$$\varepsilon_3 = \frac{\sigma_0 t}{\eta_1}$$

The strains in Spring 2 and Dashpot 2 are equal, and the stresses are additive:

$$\sigma_0 = E_2 \varepsilon_2 + \eta_2 \frac{d\varepsilon_2}{dt}$$

Solving for strain leads to

$$\varepsilon_2 = \frac{\sigma_0}{E_2} \left(1 - \exp^{-t/\tau(2)} \right)$$

The three strain components, when added, lead to Eq. (6.14).

- 6.3** At room temperature, tensile specimens of polycarbonate show 60% elongation and no stress whitening, while thick compact specimens used in fracture toughness testing show stress whitening at the crack tip. Explain these observations. Polycarbonate is an amorphous glassy polymer at room temperature.

Ans:

Different stress states in the two specimens lead to different yielding mechanisms. Stress whitening is indicative of crazing. This material apparently exhibits shear yielding when stressed uniaxially, but crazing occurs at the tip of a crack, where the stress state is triaxial tension. Crazing involves void formation which requires hydrostatic tensile stress.

- 6.4** A wide and thin specimen of PMMA has a 15 mm (0.59 in) long through crack with a 1.5 mm (0.059 in) long craze at each crack tip. If the applied stress is 3.5 MPa (508 psi), calculate the crazing stress in this material.

Ans:

From the Dugdale-Barenblatt strip yield model (Eq. (2.72)):

$$\frac{\sigma}{\sigma_c} = \frac{2}{\pi} \cos^{-1} \left(\frac{a}{a + \rho} \right)$$

Thus

$$\sigma_c = \frac{\frac{\pi}{2} (3.5 \text{ MPa})}{\cos^{-1} \left(\frac{7.5 \text{ mm}}{9.0 \text{ mm}} \right)} = 9.39 \text{ MPa}$$

- 6.5** When a macroscopic crack grows in a ceramic specimen, a process zone 0.2 mm wide forms. This process zone contains 10,000 penny-shaped microcracks/mm³ with an average radius of 10 μm. Estimate the increase in toughness due to the release of strain energy by these microcracks. The surface energy of the material = 25 J/m².

Ans:

$$\Delta G_c = \left(\frac{\text{Energy}}{\text{Volume}} \right) 2h$$

where $2h$ is the total width of the process zone.

$$\begin{aligned} \Delta G_c &= \left(\frac{10^4 \text{ cracks}}{\text{mm}^3} \right) \left(\frac{10^9 \text{ mm}^3}{\text{m}^3} \right) [2\pi (10 \times 10^{-6} \text{ m})^2] (25 \text{ J/m}^2) (2.0 \times 10^{-4} \text{ m}) \\ &= 31.4 \text{ J/m}^2 \\ G_{c(\text{total})} &= 31.4 \text{ J/m}^2 + 2(25 \text{ J/m}^2) = 81 \text{ J/m}^2 \end{aligned}$$

CHAPTER 7

- 7.1** A fracture toughness test is performed on a compact specimen. Calculate K_Q and determine whether or not $K_Q = K_{Ic}$.
- $B = 25.4 \text{ mm (1.0 in)}$; $W = 50.8 \text{ mm (2.0 in)}$; $a = 27.7 \text{ mm (1.09 in)}$ $P_Q = 42.3 \text{ kN (9.52 kip)}$; $P_{max} = 46.3 \text{ kN (10.4 kip)}$; $\sigma_{YS} = 759 \text{ MPa (110 ksi)}$

Ans:

For $a/W = 0.545$, $f(a/W) = 11.17$ (Table A7.1a). Thus

$$K_Q = \frac{(0.0423 \text{ MN})(11.17)}{(0.0254 \text{ m})\sqrt{0.0508 \text{ m}}} = 82.5 \text{ MPa}\sqrt{\text{m}}$$

Validity checks:

$$(a) \quad 2.5 \left(\frac{82.5 \text{ MPa}\sqrt{\text{m}}}{759 \text{ MPa}} \right)^2 = 0.0296 \text{ m} = 29.5 \text{ mm}$$

- fails, since $(W - a) = 23.1 \text{ mm}$.

(b) $a/W = 0.545$, which is between the limits of 0.45 and 0.55

(c) $P_{max}/P_Q = 1.09 < 1.10$

$\therefore K_Q \neq K_{Ic}$ since *all* validity criteria are not met.

7.2 You have been asked to perform a K_{Ic} test on a material with $\sigma_{ys} = 690$ MPa (100 ksi). The toughness of this material is expected to lie between $40 \text{ MPa}\sqrt{\text{m}}$ and $60 \text{ MPa}\sqrt{\text{m}}$ ($1 \text{ ksi}\sqrt{\text{in}} = 1.099 \text{ MPa}\sqrt{\text{m}}$). Design an experiment to measure K_{Ic} (in accordance with ASTM E 399) in this material using a compact specimen. Assume $W/B = 2$ in this case. Specify the following quantities: (a) specimen dimensions, (b) precracking loads, and (c) required load capacity of the test machine.

Ans:

(a) Use *upper* estimate of K_{Ic} to establish specimen dimensions:

$$(W - a) = 2.5 \left(\frac{60 \text{ MPa}\sqrt{\text{m}}}{690 \text{ MPa}} \right)^2 = 0.0189 \text{ m}$$

- Use 1T compact specimen with $B = 25.4$ mm, $W = 50.8$ mm, $a/W \approx 0.5$.

(b) Use *lower* K_{Ic} estimate to establish fatigue loads.

- Assume machined notch with $a_o/W = 0.40$; precrack to $a/W = 0.50$.

Nondimensional K_I (Table A7.1a):

a/W	$f(a/W)$
0.40	7.18
0.45	8.22
0.50	9.52

(i) Initial K_{max} can be $0.8 K_{Ic}$:

$$0.8(40 \text{ MPa}\sqrt{\text{m}}) = 32 \text{ MPa}\sqrt{\text{m}} = \frac{P_{max}(7.18)}{(0.0254 \text{ m})\sqrt{0.0508 \text{ m}}}$$

$$P_{max} = 25.1 \text{ kN}$$

If $R = 0.1$, $P_{\min} = 2.51 \text{ kN}$.

(ii) Final precracking loads ($K_{\max} \leq 0.6 K_{Ic}$)

$$0.6(40 \text{ MPa}\sqrt{\text{m}}) = 24 \text{ MPa}\sqrt{\text{m}} = \frac{P_{\max}(9.52)}{(0.0254 \text{ m})\sqrt{0.0508 \text{ m}}}$$

$$P_{\max} = 14.4 \text{ kN} \quad P_{\min} = 1.44 \text{ kN}$$

(c) Assume worst case to determine P_{\max} during the test:

- $K_Q = 60 \text{ MPa}\sqrt{\text{m}}$ (maximum expected value)
- $P_{\max} = 1.10 P_Q$ (maximum allowable value)
- $a/W = 0.45$ (minimum allowable value)

$$66 \text{ MPa}\sqrt{\text{m}} = \frac{P_{\max}(8.22)}{(0.0254 \text{ m})\sqrt{0.0508 \text{ m}}}$$

$$P_{\max} = 46.0 \text{ kN}$$

Thus a test machine with a 50 kN load capacity should be adequate.

7.3 A titanium alloy is supplied in 15.9 mm (0.625 in) thick plate. If $\sigma_{ys} = 807 \text{ MPa}$ (117 ksi), calculate the maximum valid K_{Ic} that can be measured in this material. Assume $W/B = 4$.

Ans:

The thickness of the specimen is limited to the plate thickness. Therefore $W_{\max} = 63.6 \text{ mm}$. Assuming $a/W = 0.5$:

$$(W - a) = 0.0318 \text{ m} = 2.5 \left(\frac{K_{Ic}}{807 \text{ MPa}} \right)^2$$

$$K_{Ic(\max)} = 91.0 \text{ MPa}\sqrt{\text{m}}$$

7.4 Recall Problem 2.20, where a material with $K_{Ic} = 110 \text{ MPa}\sqrt{\text{m}}$ (100 ksi $\sqrt{\text{in}}$) required a 254 mm (10.0 in) thick specimen for a valid K_{Ic} test. Suppose that a compact specimen of

the appropriate dimensions has been fabricated. Estimate the required load capacity of the test machine for such a test.

Ans:

Assume $a/W = 0.5$. $f(a/W) = 9.52$ (Table 12.2a)

$$110 \text{ MPa}\sqrt{\text{m}} = \frac{P_Q (9.52)}{(0.254 \text{ m})\sqrt{0.508 \text{ m}}}$$

$$P_Q = 2.09 \text{ MN}$$

If $P_{\max} = 1.10 P_Q$, $P_{\max} = 2.30 \text{ MN}$

7.5 Consider a material with $K_{Ic} = 100 \text{ MPa}\sqrt{\text{m}}$ ($91 \text{ ksi}\sqrt{\text{in}}$) and $\sigma_{YS} = 450 \text{ MPa}$ (65 ksi). Calculate the minimum W dimension necessary to satisfy the ASTM E399 size requirements. Suppose that the plate thickness is 50 mm (2 in). If a compact tension specimen is fabricated with $B = 50 \text{ mm}$ and W equal to the E399 requirement, estimate the load capacity of the test machine that would be needed to perform this test. Comment on the validity of the resulting K_Q measurement.

Ans:

Apply the E399 ligament requirement:

$$W - a = 2.5 \left(\frac{100 \text{ MPa}\sqrt{\text{m}}}{450 \text{ MPa}} \right)^2 = 0.1235 \text{ m}$$

If $a/W = 0.5$, then $W = 0.247$ and $f(a/W) = 9.52$.

$$100 \text{ MPa}\sqrt{\text{m}} = \frac{P_Q (9.52)}{(0.050 \text{ m})\sqrt{0.247 \text{ m}}}$$

$$P_Q = 261 \text{ kN}$$

If $P_{\max} = 1.10 P_Q$, $P_{\max} = 287 \text{ kN}$.

$W/B = 4.94$, which exceeds the E399 requirements.

7.6 A fracture toughness test is performed on a compact specimen fabricated from a 5 mm thick sheet aluminum alloy. The specimen width (W) = 50.0 mm and $B = 5 \text{ mm}$ (the sheet thickness). The initial crack length is 26.0 mm. Young's modulus = 70,000 MPa. Compute the $K-R$ curve from the load-displacement data tabulated below. Assume that all

nonlinearity in the $P-\Delta$ curve is due to crack growth. (See Appendix 7 for the appropriate compliance and stress intensity relationships.)

Load, kN	Load Line Displacement, mm	Load, kN	Load Line Displacement, mm
0	0	2.851	0.3698
0.5433	0.0635	2.913	0.3860
1.087	0.1270	2.903	0.3971
1.630	0.1906	2.850	0.4113
2.161	0.2552	2.749	0.4191
2.361	0.2817	2.652	0.4274
2.541	0.3096	2.553	0.4355
2.699	0.3392	2.457	0.4443

$$1 \text{ kN} = 224.8 \text{ lb} \quad 25.4 \text{ mm} = 1 \text{ in} \quad 1 \text{ MPa} = 0.145 \text{ ksi}$$

Ans:

The procedure for determining each point on the R curve is as follows:

- (a) Compute the crack length from the compliance expression in Table 12.4. (Note the error in the forth term, as discussed above.)
- (b) Compute K_R (for the load and crack length of interest) from the polynomial expression in Table 12.2(a).

Figure S9 shows the resulting R curve. Note that K_R reaches a steady-state value.

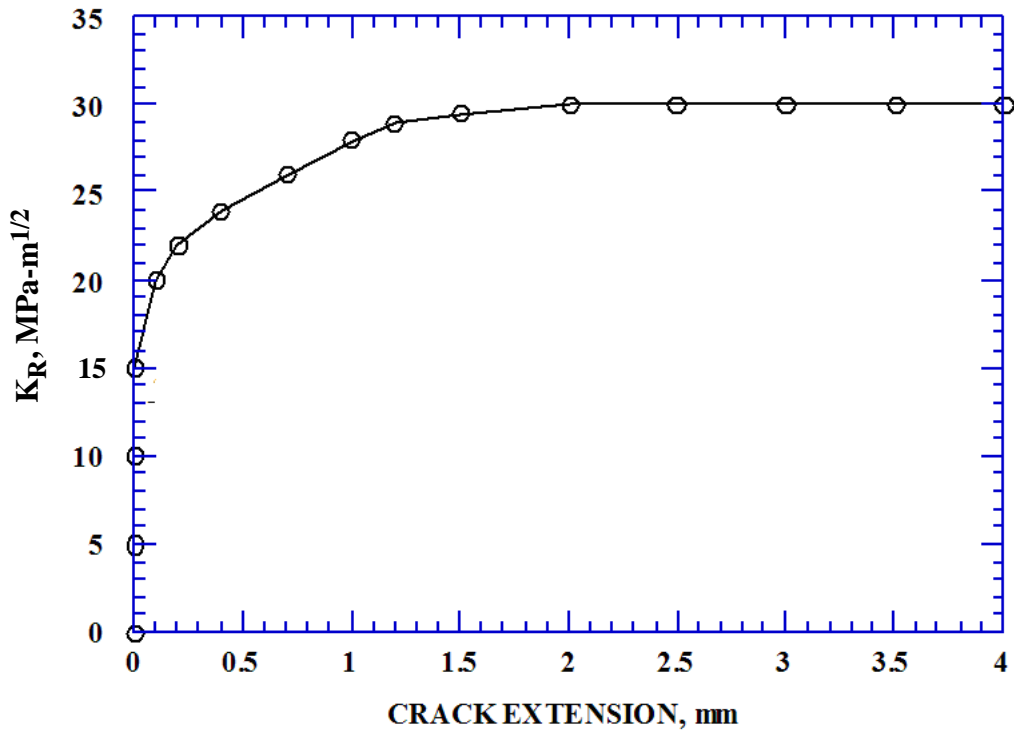


FIGURE S9 Crack growth resistance curve inferred from the load-displacement data in Problem 7.6.

7.7 A number of fracture toughness specimens have been loaded to various points and then unloaded. Values of J and crack growth were measured in each specimen and are tabulated below. Using the basic test procedure the J - R curve for this material and determine J_Q and, if possible, J_{Ic} .

$\sigma_{YS} = 350$ MPa; $\sigma_{TS} = 450$ MPa; $B = 25$ mm; $b_o = 22$ mm;

Specimen	J , kJ/m ²	Crack Extension, mm
1	100	0.30
2	175	0.40
3	185	0.80
4	225	1.20
5	250	1.60
6	300	1.70

25.4 mm = 1 in 1 MPa = 0.145 ksi 1 kJ/m² = 5.71 in-lb/in²

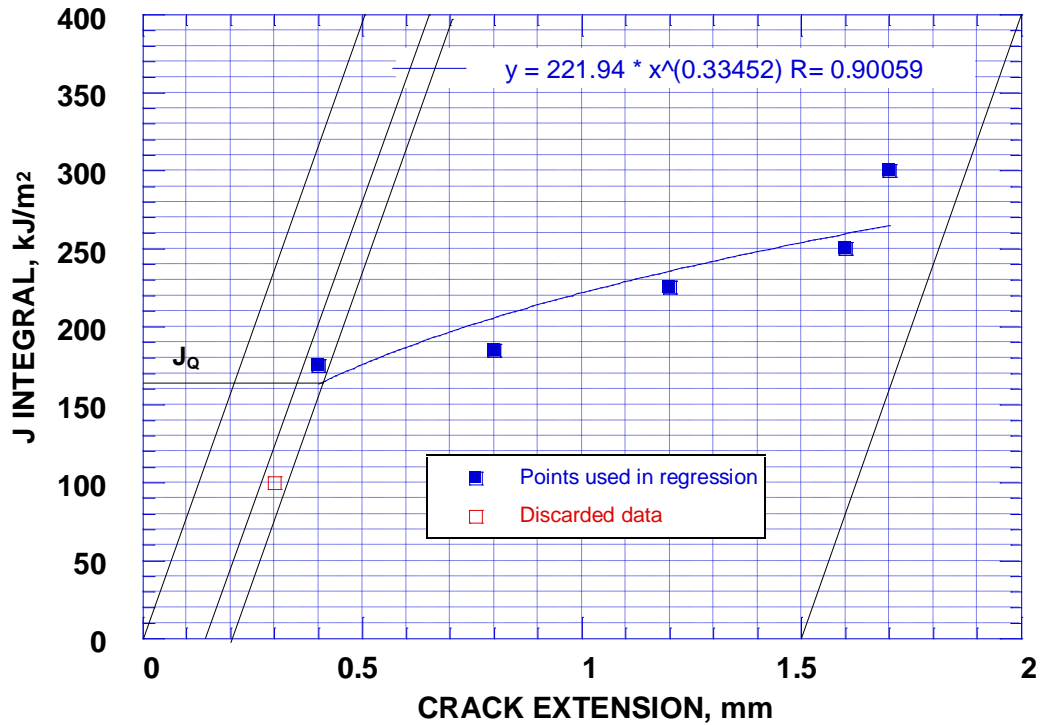


FIGURE S10 J-R curve for Problem 7.7.

From the above graph, $J_Q = 165 \text{ kJ/m}^2$.

Validity checks:

$$(a) \quad \frac{25(0.165 \text{ MJ/m}^2)}{400 \text{ MPa}} = 0.0103 \text{ m} = 10.3 \text{ mm} < b_o, B \text{ (Pass)}$$

$$(b) \quad J_{\max} = \frac{0.022 \text{ m}(400 \text{ MPa})}{15} = 0.587 \text{ MJ/m}^2 \text{ (Pass)}$$

> all measured J values

$$\therefore J_Q = J_{Ic} = 165 \text{ kJ/m}^2$$

7.8 Recall Problem 2.20, where a material with $K_{Ic} = 110 \text{ MPa}\sqrt{\text{m}}$ ($100 \text{ ksi}\sqrt{\text{in}}$) and $\sigma_{YS} = 345 \text{ MPa}$ (50 ksi) required a specimen 254 mm (10 in) thick for a valid K_{Ic} test in accordance with ASTM E 399. Estimate the specimens dimensions required for a valid J_{Ic} test on this material.

$$\sigma_{TS} = 483 \text{ MPa (70 ksi); } E = 207,000 \text{ MPa (30,000 ksi); } \nu = 0.3.$$

Ans:

$$J_{Ic} = \frac{(110 \text{ MPa}\sqrt{\text{m}})^2 (1 - 0.3^2)}{207,000 \text{ MPa}} = 0.0532 \text{ MJ/m}^2$$

$$\sigma_y = \frac{345 \text{ MPa} + 483 \text{ MPa}}{2} = 414 \text{ MPa}$$

$$B, b_o = \frac{25(0.0532 \text{ MJ/m}^2)}{414 \text{ MPa}} = 3.21 \text{ mm}$$

which is smaller than the K_{Ic} requirement by a factor of 79.

7.9 An unloading compliance test has been performed on a 3-point bend specimen. The data obtained at each unloading point is tabulated below.

(a) Compute and plot the J resistance curve according to the procedure outlined in Section 7.4.2.

(b) Determine J_{Ic} according to the procedure illustrated in Fig. 7.24.

$B = 25.0 \text{ mm; } W = 50.0 \text{ mm; } a_o = 26.1 \text{ mm; } E = 210,000 \text{ MPa, } \nu = 0.3$

LOAD , kN	Plastic Displacement, mm	Crack Extension, mm
-----------	-----------------------------	------------------------

20.8	0	0.013
31.2	0.0032	0.020
35.4	0.011	0.023
37.4	0.020	0.025
41.6	0.056	0.031
43.7	0.092	0.036
45.7	0.146	0.044
47.6	0.228	0.055
49.9	0.349	0.071
51.6	0.525	0.091
53.5	0.777	0.128
55.3	1.13	0.183
56.6	1.63	0.321
56.7	2.32	0.723
56.5	2.66	0.928
55.8	3.25	1.29
54.7	3.96	1.74
53.7	4.51	2.08
52.5	5.13	2.48
50.1	6.20	3.17
44.4	8.43	4.67
40.0	10.09	5.81
36.6	11.37	6.70
30.9	13.54	8.23
26.8	15.19	9.41

Ans:

(a) Apply the limits in Eqs. (7.16) and (7.17):

$$\Delta a_{\max} = 0.10(23.9 \text{ mm}) = 2.39 \text{ mm}$$

$$J_{\max} = \frac{0.0239 \text{ m}(414 \text{ MPa})}{20} = 4.95 \text{ kJ/m}^2$$

Figure S11 shows the J-R curve, together with the validity limits.

$$(b) \quad J_{\max} = \frac{0.0239 \text{ m}(414 \text{ MPa})}{15} = 657 \text{ kJ/m}^2$$

Figure S12 shows the J_{Ic} construction lines on the R curve. It is not possible to obtain a valid J_{Ic} value from this test. $J_Q \approx 500 \text{ kJ/m}^2$, which exceeds the size limits. Also, there are insufficient data within the construction lines to perform a regression analysis.

Figure S13 compares the basic procedure with the crack growth correction. The difference is insignificant for less than 5 mm of crack growth in this instance.

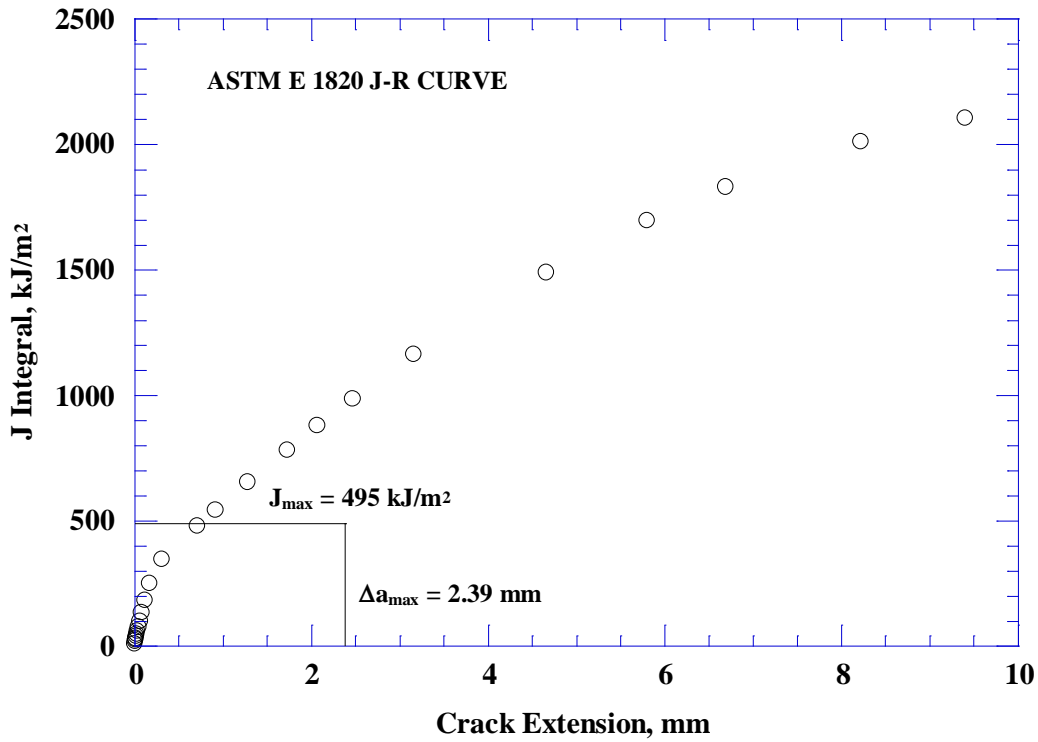
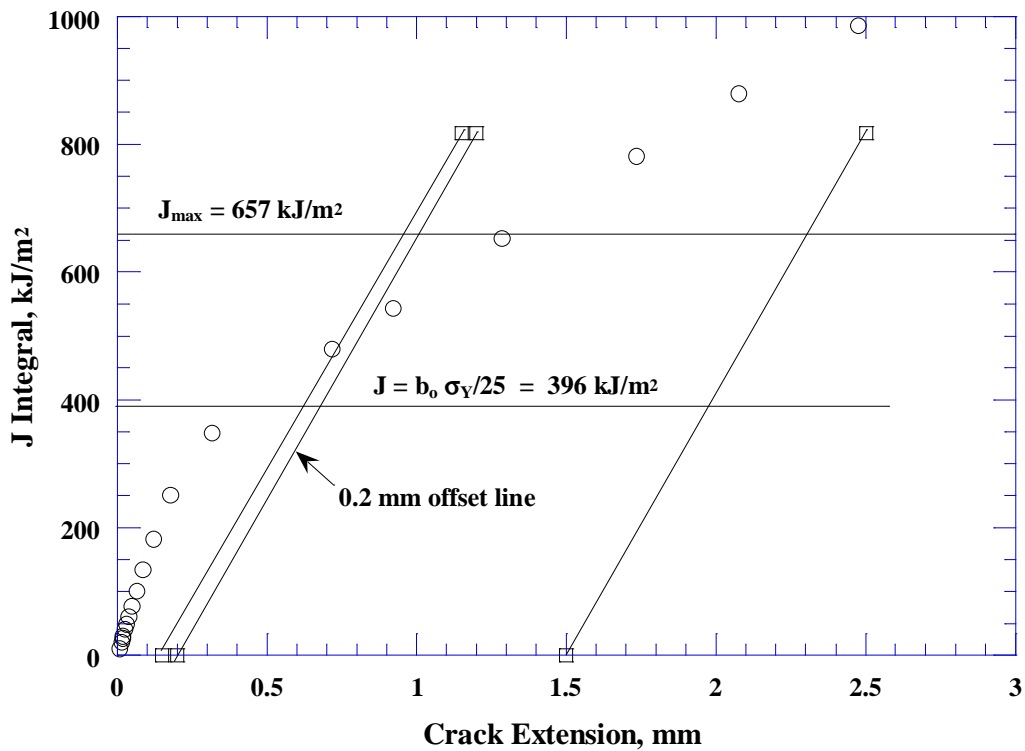


FIGURE S11 J-R curve plotted according to ASTM E 1820 (Problem 7.9).

FIGURE S12 J_{Ic} construction lines on the J-R curve in Problem 7.9

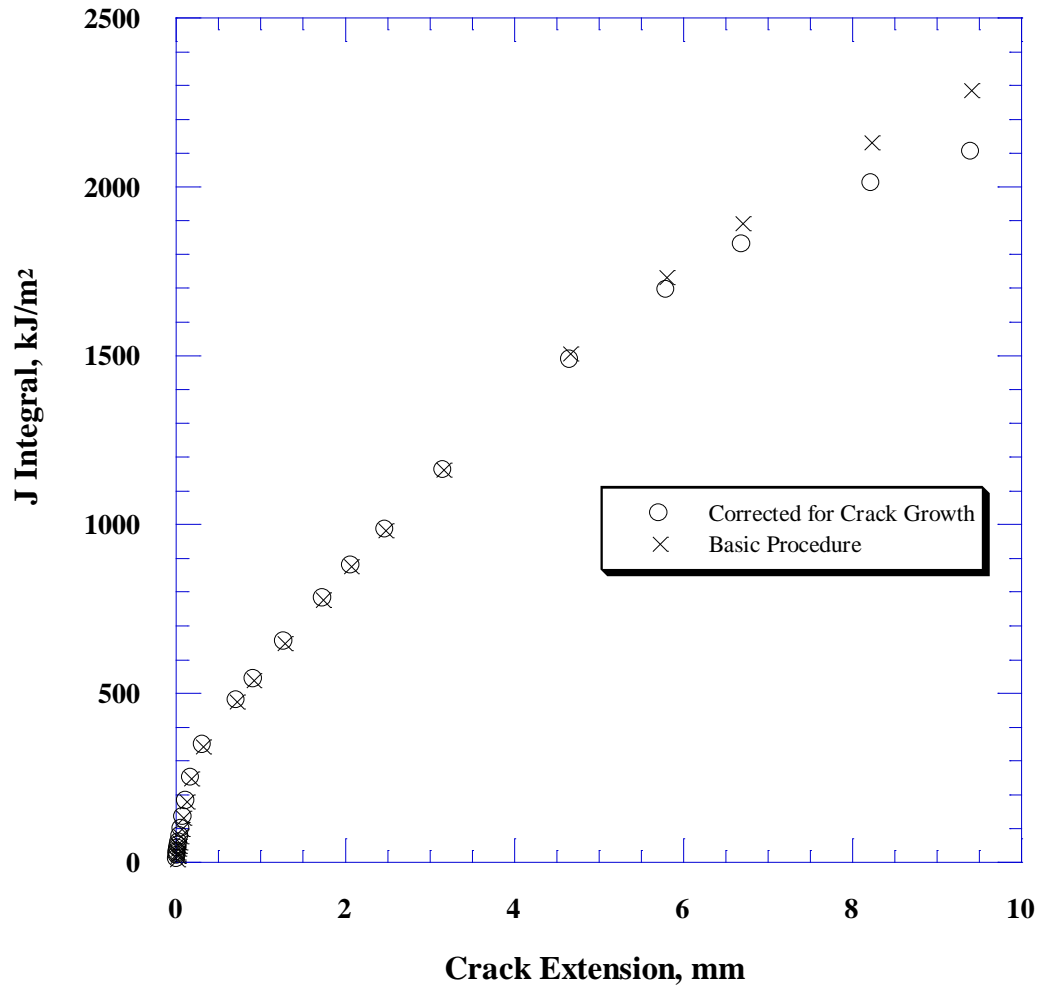


FIGURE S13 Comparison of J-R curve calculation procedures (Problem 7.9).

7.10 A CTOD test was performed on a three point bend specimen with $B = W = 25.4$ mm (1.0 in). The crack depth, a , was 12.3 mm (0.484 in). Examination of the fracture surface revealed that the specimen failed by cleavage with no prior stable crack growth. Compute the critical $CTOD$ in this test.

$V_p = 1.05$ mm (0.0413 in); $P_{critical} = 24.6$ kN (5.53 kip); $E = 207,000$ MPa (30,000 ksi); $\sigma_{YS} = 400$ MPa (58.0 ksi); $\nu = 0.3$.

Ans: $a/W = 0.484$ $f(a/W) = 10.14$

$$K_I = \frac{0.0246 \text{ MN} (10.14)}{(0.0254 \text{ m})^{1.5}} = 61.6 \text{ MPa}\sqrt{\text{m}}$$

$$\delta_e = \frac{(61.6 \text{ MPa}\sqrt{\text{m}})^2 (1-0.3^2)}{2(400 \text{ MPa})(207,000 \text{ MPa})} = 2.09 \times 10^{-5} \text{ m}$$

$$\delta_p = \frac{0.44(25.4 \text{ mm} - 12.3 \text{ mm})1.05 \text{ mm}}{0.44(25.4 \text{ mm} - 12.3 \text{ mm}) + 12.3 \text{ mm}} = 0.330 \text{ mm}$$

$$\delta_c = 0.351 \text{ mm}$$

7.11 A crack arrest test has been performed in accordance with ASTM E 1221. The side-grooved compact crack arrest specimen has the following dimensions: $W = 100 \text{ mm}$ (3.94 in), $B = 25.4 \text{ mm}$ (1.0 in), and $B_N = 19.1 \text{ mm}$ (0.75 in). The initial crack length = 46.0 mm (1.81 in) and the crack length at arrest = 63.0 mm (2.48 in). The corrected clip gage displacements at initiation and arrest are $V_o = 0.582 \text{ mm}$ (0.0229 in) and $V_a = 0.547$ (0.0215 in), respectively. $E = 207,000 \text{ MPa}$ (30,000 ksi) and $\sigma_{YS(\text{static})} = 483 \text{ MPa}$ (70 ksi). Calculate the stress intensity at initiation, K_o , and the arrest toughness, K_a . Determine whether or not this test satisfies the validity criteria in Eq. (7.25). The stress intensity solution for the compact crack arrest specimen is given below.

$$K_I = \frac{EVf(x)\sqrt{B/B_N}}{\sqrt{W}}$$

where

$$x = a/W$$

$$f(x) = \frac{2.24(1.72 - 0.9x + x^2)\sqrt{1-x}}{9.85 - 0.17x + 11x^2}$$

Ans:

Initiation:

$$a/W = 0.46 \quad f(x) = 0.206$$

$$K_o = \frac{207,000 \text{ MPa} (5.82 \times 10^{-4} \text{ m}) (0.206) \sqrt{1.33}}{\sqrt{0.100 \text{ m}}} = 90.6 \text{ MPa}\sqrt{\text{m}}$$

Arrest:

$$a/W = 0.63 \quad f(x) = 0.150$$

$$K_o = \frac{207,000 \text{ MPa} (5.47 \times 10^{-4} \text{ m}) (0.150) \sqrt{1.33}}{\sqrt{0.100 \text{ m}}} = 62.0 \text{ MPa}\sqrt{\text{m}}$$

Validity checks (Eq. (7.27)):

- (a) $W - a_a = 38 \text{ mm} > 0.15 W$ **(Pass)**
- (b) $1.25 \left(\frac{62.0 \text{ MPa}\sqrt{\text{m}}}{483 + 205 \text{ MPa}} \right)^2 = 10.15 \text{ mm} < W - a_a$ **(Pass)**
- (c) $1.0 \left(\frac{62.0 \text{ MPa}\sqrt{\text{m}}}{483 + 205 \text{ MPa}} \right)^2 = 8.12 \text{ mm} < B$ **(Pass)**
- (d) $\frac{1}{2\pi} \left(\frac{62.0 \text{ MPa}\sqrt{\text{m}}}{483 \text{ MPa}} \right)^2 = 2.62 \text{ mm} < a_a - a_o$ **(Pass)**

$$\therefore K_a = K_{Ia} = 62.0 \text{ MPa}\sqrt{\text{m}} .$$

CHAPTER 8

8.1 A 25.4 mm (1.0 in) thick plate of PVC has a yield strength of 60 MPa (8.70 ksi). The anticipated fracture toughness (K_{Ic}) of this material is $5 \text{ MPa}\sqrt{\text{m}}$ ($4.5 \text{ ksi}\sqrt{\text{in}}$). Design an experiment to measure K_{Ic} of a compact specimen machined from this material. Determine the appropriate specimen dimensions (B , W , a) and estimate the required load capacity of the test machine

Ans:

$$B_{\min}, a_{\min} = 2.5 \left(\frac{5 \text{ MPa}\sqrt{\text{m}}}{60 \text{ MPa}} \right)^2 = 17.3 \text{ mm}$$

- Use a 1T compact specimen with $B = 25.4 \text{ mm}$, $W = 50.8 \text{ mm}$, $a/W \approx 0.5$

To estimate load capacity, assume $P_{\max} = 1.1P_Q$ and $a/W = 0.45$:

$$f(a/W) = 8.22 \text{ (Table 12.2a)}$$

$$5.5 \text{ MPa}\sqrt{\text{m}} = \frac{P_{\max} (8.22)}{0.0254 \text{ m} \sqrt{0.0508 \text{ m}}}$$

$$P_{\max} = 3.83 \text{ kN}$$

8.2 A 15.9 mm (0.625 in) thick plastic plate has a yield strength of 50 MPa (7.25 ksi). Determine the largest valid K_{Ic} value that can be measured on this material.

Ans:

$$0.0159 \text{ m} = 2.5 \left(\frac{K_{IC}}{50 \text{ MPa}} \right)^2$$

$$K_{IC} = 3.99 \text{ MPa}\sqrt{\text{m}}$$

8.3 A K_{Ic} test is to be performed on a polymer with a time-dependent relaxation modulus which has been fit to the following equation:

$$E(t) = [0.417 + 0.0037t^{0.35}]^{-1}$$

where E is in GPa and t is in seconds. Assuming P_Q is determined from a 5% secant construction, estimate the test duration (i.e. the time to reach P_Q) at which 90% of the nonlinearity in the load-displacement curve at P_Q is due to viscoelastic effects. Does the 5% secant load give an appropriate indication of material toughness in this case? Explain.

Ans:

The viscoelasticity results in a 4.5% deviation in linearity; i.e.,

$$\frac{P_Q}{\Delta} = 0.955 \frac{P_Q}{\Delta^e}$$

where Δ^e is the pseudo elastic displacement, which equals Δ at short times. From Eqs. (8.10) and (8.11),

$$\frac{\Delta^e}{\Delta} = \frac{\bar{E}(t)}{E_R} = 0.955$$

where $\bar{E}(t)$ is the time average modulus and E_R (in this case) is the modulus at $t = 0$. Since the time average modulus does not have a closed form solution, we shall make the following approximation:

$$\bar{E}(t) \approx E(t)$$

which implies

$$0.417 [0.417 + 0.0037 t^{0.35}]^{-1} = 0.955$$

$$t = 118 \text{ s}$$

Thus when the duration of the test is 2 min or greater (with this material), the observed nonlinearity in the load-displacement curve is due primarily to viscoelastic effects rather than yielding or crack growth. The 5% secant load has no relevance to fracture in such cases.

8.4 Derive a relationship between the conventional J integral and the isochronous J integral, J_t , in a constant displacement rate test on a viscoelastic material for which Eqs. (8.10) and (8.15) describe the load-displacement behavior.

Ans:

Integrating the load-displacement relationship (Eq. (8.15)) at a fixed time gives

$$\left(\int_0^{\Delta} P d\Delta \right)_{t=\text{constant}} = M \frac{\Delta^{N+1}}{N+1} \left(\frac{\bar{E}(t)}{E_R} \right)^N$$

Since only M depends on crack length, J_t is given by

$$J_t = \frac{\Delta^{N+1}}{N+1} \left(\frac{\partial M}{\partial a} \right)_{\Delta} \left(\frac{\bar{E}(t)}{E_R} \right)^N$$

Comparing the above result with Eq (8.20) leads to

$$J_t = J f(t)$$

where

$$f(t) = \frac{t^{N+1} \bar{E}(t)^N}{N+1} \left\{ \int_0^t [\bar{E}(\tau)]^N \tau^N d\tau \right\}^{-1}$$

8.5 A 500 mm wide plastic plate contains a through-thickness center crack that is initially 50 mm long. The crack velocity in this material is given by

$$\dot{a} = 10^{-40} K^{10}$$

where K is in $\text{kPa}\sqrt{\text{m}}$ and \dot{a} is in mm/sec (1 psi $\sqrt{\text{in}} = 1.1 \text{ kPa}\sqrt{\text{m}}$, 1 in = 24.5 mm). Calculate the time to failure in this plate assuming remote tensile stresses of 5 MPa and 10 MPa (1 ksi = 6.897 MPa). Comment on the sensitivity of the time to failure on the applied stress. (As a first approximation, neglect the finite width correction on K . For an optional exercise, repeat the calculations with this correction to assess its effect on the computed failure times.)

In order to avoid problems with units, it is advisable to express crack growth rate in terms of m/s:

$$\frac{da}{dt} = 10^{-43} K^{10}$$

By neglecting the finite width correction on K , a closed-form expression for life can be obtained:

$$\begin{aligned} t &= \frac{1}{10^{-43}} \int_{a_o}^w \frac{da}{K^{10}} \\ &= \frac{1}{10^{-43} \sigma^{10} \pi^5} \int_{a_o}^w a^{-5} da \\ &= \frac{2.091 \times 10^{46}}{\sigma^{10}} \end{aligned}$$

where σ is in kPa.

For $\sigma = 5000$ kPa, $t = 2.14 \times 10^9$ s = 67.9 years

For $\sigma = 10,000$ kPa, $t = 2.09 \times 10^6$ s = 24.2 days

Thus the life is highly sensitive to stress. When the finite width correction is incorporated, the effect on estimated life is negligible because the overwhelming majority of the life is consumed when the crack is small compared to the width.

- 8.6** A composite double cantilever beam (DCB) specimen is loaded to 445 N (100 lb) at which time crack growth begins. Calculate \mathcal{G}_{Ic} for this material assuming linear beam theory.

$E = 124,000 \text{ MPa (18,000 ksi); } a = 76.2 \text{ mm (3.0 in); } h = 2.54 \text{ mm (0.10 in); } B = 25.4 \text{ mm (1.0 in).}$

Ans:

From Example 2.2,

$$\mathcal{G}_{Ic} = \frac{12 (445 \text{ N})^2 (0.0762 \text{ m})^2}{(0.0254 \text{ m})^2 (0.00254 \text{ m})^3 (124 \times 10^9 \text{ Pa})} = 10.5 \text{ kJ/m}^2$$

- 8.7** One of the problems with testing brittle materials is that crack growth tends to be unstable in conventional test specimens and test machines. Consider, for example, a single edge notched bend (SENB) specimen loaded in three point bending. The influence of the test machine can be represented by a spring in series, as Fig. 13.4 illustrates. Show that the stress intensity factor for this specimen can be expressed as a function of crosshead displacement and compliance as follows:

$$K_I = \frac{\Delta_t f(a/W)}{(C + C_m) B \sqrt{W}}$$

where Δ_t is the crosshead displacement, C is the specimen compliance, C_m is the machine compliance, and $f(a/W)$ is defined in Table 12.2. Construct a nondimensional plot of K_I versus crack size for a fixed crosshead displacement and a/W ranging from 0.25 to 0.75. Develop a family of these curves for a range of machine compliance. (You will have to express C_m in an appropriate nondimensional form.) What is the effect of machine compliance on the relative stability of the specimen? At what machine compliance would a growing crack experience a relatively constant K_I between $a/W = 0.5$ and $a/W = 0.6$?

Ans:

$$\Delta_t = (C + C_m) P$$

or

$$P = \frac{\Delta_t}{C + C_m}$$

which, when substituted into the standard K_I expression for test specimens (Table A7.2), results in the equation in the problem statement. This expression can be made dimensionless:

$$\frac{K_I \sqrt{W}}{\Delta_t E'} = \frac{f(a/W)}{z(a/W) + z_m}$$

where $z(a/W)$ is defined in Table 12.3 and $z_m = C_m B E'$. Figure S14 is a plot of nondimensional K_I versus a/W for a range of machine compliances. For shallow cracks, this specimen exhibits a rising driving force curve irrespective of the system compliance. When $z_m = 10$, the driving force is reasonably constant in the range $0.4 \leq a/W \leq 0.6$.

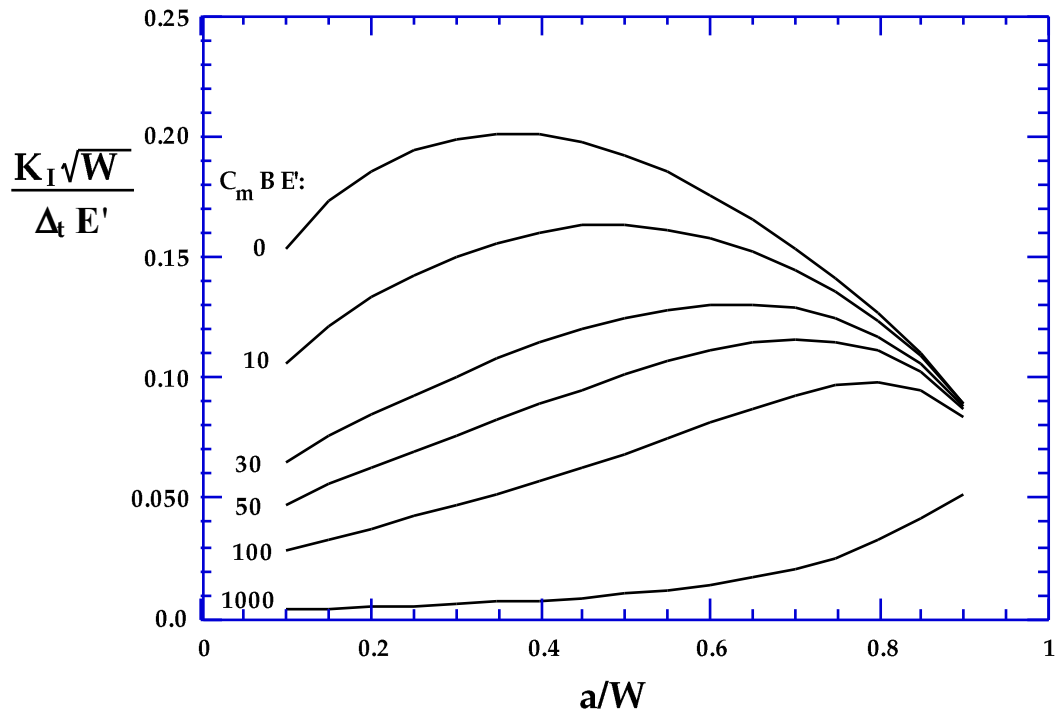


FIGURE S14 Nondimensional stress intensity factors for a three-point bend specimen in crosshead control as a function of a/W and machine compliance (Problem 8.7).

CHAPTER 9

- 9.1** Develop a computer program or spreadsheet to calculate stress intensity factors for semi-elliptical surface cracks in flat plates subject to membrane and bending stress where $a/c \leq 1$ (Table A9.1). Tabulate and plot the geometry factors F and H as a function of a/t and a/c , where $\phi = 90^\circ$ and $c \ll W$.

Ans:

Figures S15 and S16 are plots of F and H , respectively.

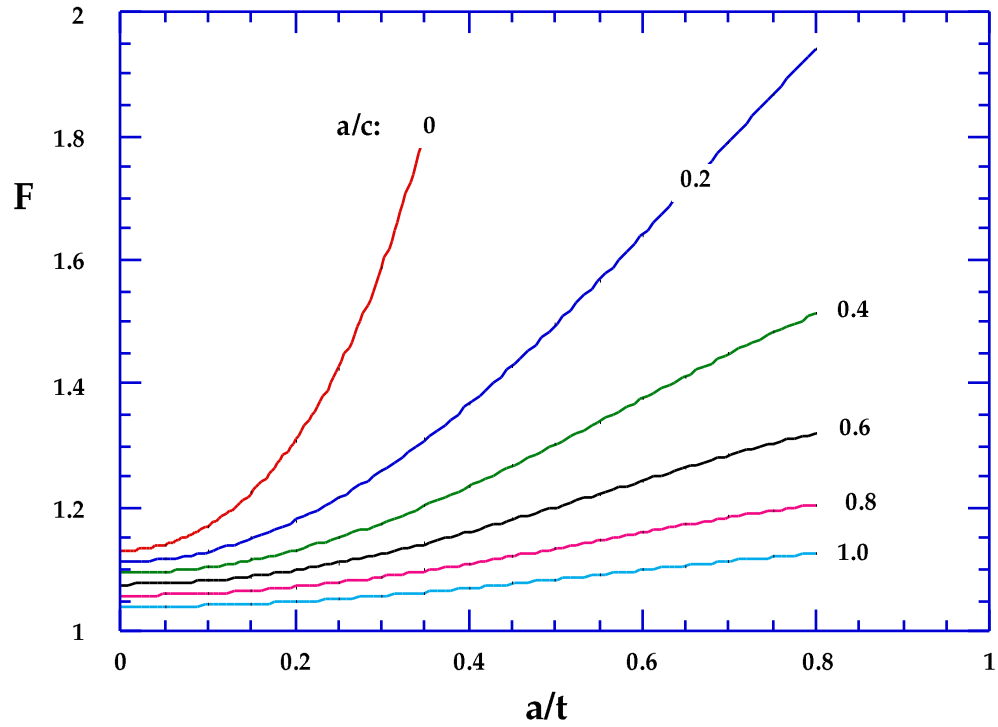


FIGURE S15 Surface flaw correction factor for $\phi = 90^\circ$ and $c \ll W$ (Problem 9.1).

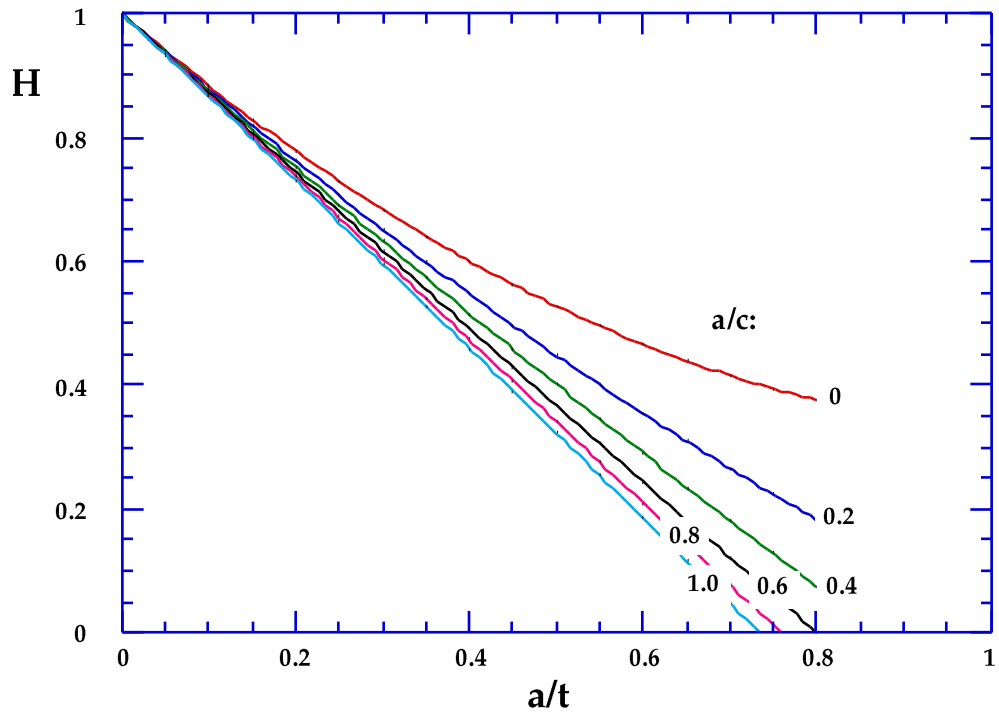


FIGURE S16. Bending multiplier for a surface flaw; $\phi = 90^\circ$ and $c \ll W$ (Problem 9.1).

9.2 Beginning with the Newman-Raju K_I expression for part-through flaws (Eq. (9.2)), derive expressions for influence coefficients for uniform and linear loading (G_o and G_1) in terms of the geometry factors F and H .

Ans:

By considering the case of uniform membrane loading, it is obvious that $G_o = F$ because $\sigma_o = \sigma_m$. Consider a linear through-wall distribution:

$$\sigma(x) = \sigma_1 \left(\frac{x}{t} \right)$$

$$\sigma_m = \frac{\sigma_1}{2}$$

$$\sigma_b = -\frac{\sigma_1}{2}$$

Equating the K_I expressions for the above stress distribution gives:

$$(F\sigma_m + FH\sigma_b) \sqrt{\frac{\pi a}{Q}} = \sigma_1 G_1 \left(\frac{a}{t} \right) \sqrt{\frac{\pi a}{Q}}$$

Therefore,

$$G_1 = \frac{F(1-H)}{2} \left(\frac{t}{a} \right)$$

9.3 For a semi-elliptical surface flaw in a flat plate where $a/c \leq 1$ and $c \ll W$, tabulate and plot the quadratic influence coefficient, G_2 , as a function of a/t and a/c , where $\phi = 90^\circ$. *Note: you will need to have worked Problems 9.1 and 9.2 first.*

Ans:

First, the F and H values from Problem 9.1 must be converted to G_o and G_1 using the expressions obtained in Problem 9.2. Next, G_2 is inferred using the weight function method. Refer to Eqs. (9.17) and (9.19a). The results are plotted in Figure S17.

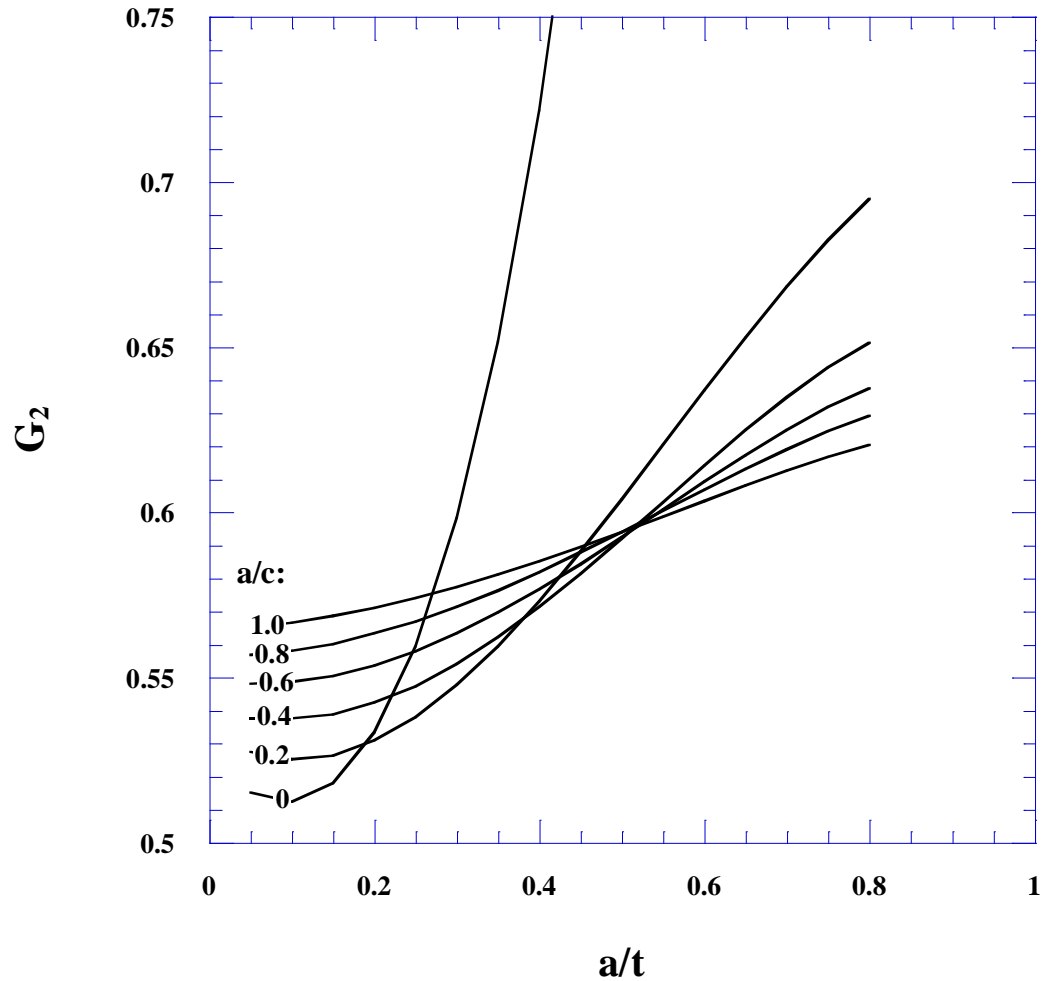


FIGURE S17. Solution to Problem 9.3.

- 9.4** Write a computer program to compute K_I for a surface crack at $\phi = 90^\circ$ in a flat plate subject to an arbitrary through-thickness normal stress using the weight function method (Section 9.1.3). Assume that $a/c \leq 1$ and $c \ll W$. *Note: you will need to have worked Problems 9.1 and 9.2 first.* Compute and plot K_I versus crack depth for $a/c = 0.2, 0.4, 0.6, 0.8, 1$ for the following stress distribution:

$$\sigma(x) = 150 \text{ MPa} \exp \left[-5 \left(\frac{x}{t} \right) \right]$$

$$t = 25.4 \text{ mm (1 in.)}$$

Ans

The results are plotted in Fig. S18.

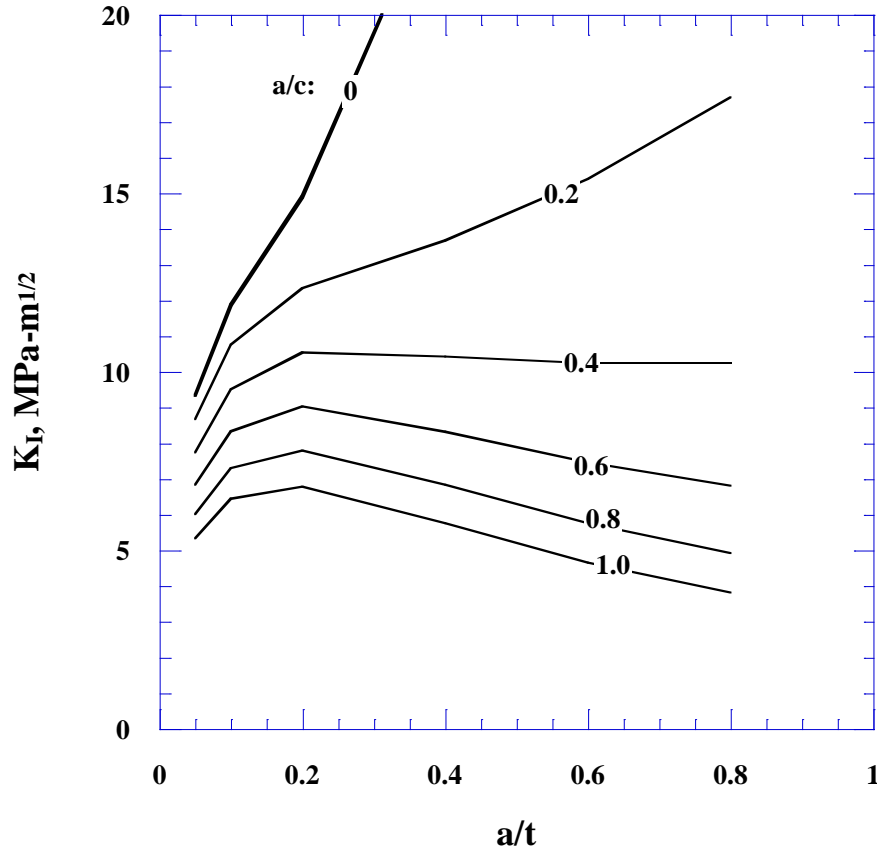


FIGURE S18. Solution to Problem 9.4.

- 9.5** Using the computer program developed in Problem 9.4, compute and plot K_I (at $\phi = 90^\circ$) versus crack depth for the following through-thickness stress distribution:

$$\sigma(x) = 100 \text{ MPa} \cos\left(\frac{2\pi x}{t}\right)$$

$2c = 102 \text{ mm}$ (4 in), $t = 51 \text{ mm}$ (2 in). At what crack depth(s) does $K_I = 0$?

Ans:

The K_I solution is plotted in Fig. S19. The solution passes through zero at $a = 0.021 \text{ m}$ (21 mm) and $a = 0.043 \text{ m}$ (43 mm).

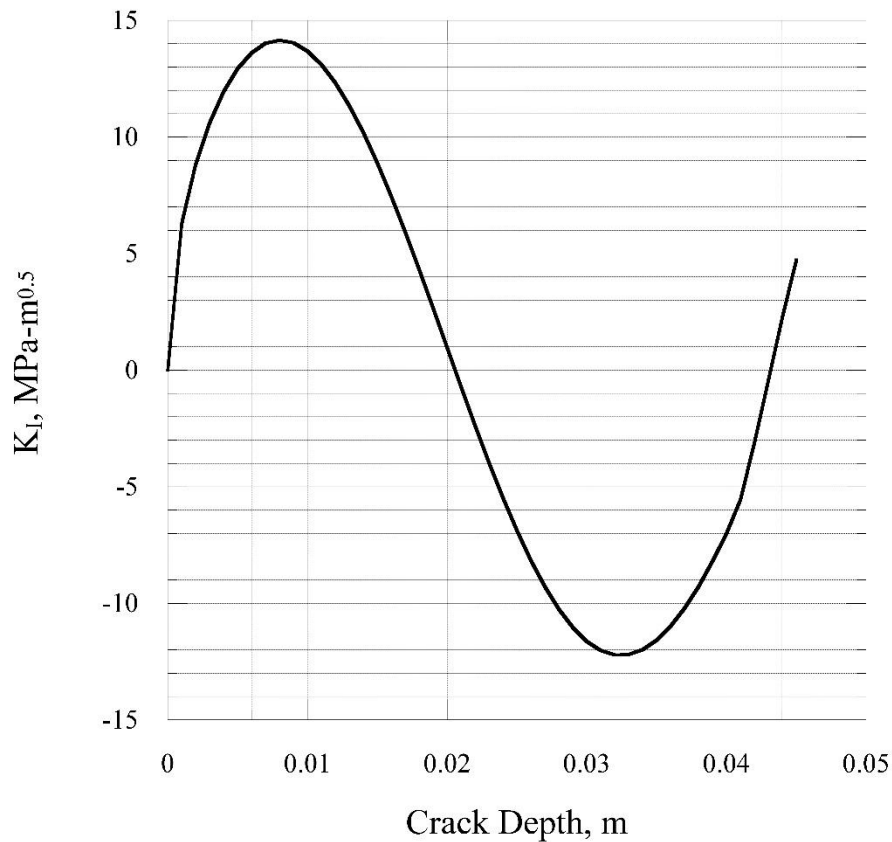


FIGURE S19. Solution to Problem 9.5.

9.6 A flat plate 1.0 m (39.4 in) wide and 50 mm (2.0 in) thick which contains a through-thickness crack is loaded in uniaxial tension to $0.75 \sigma_{YS}$. Plot K_r and S_r values on a strip yield failure assessment diagram for various flaw sizes. Estimate the critical flaw size for failure. For the limit load solution, use the P_o expression in Table A9.11. Set σ_o equal to the average of yield and tensile strength.

$\sigma_{YS} = 345 \text{ MPa (50 ksi)}$; $\sigma_{TS} = 448 \text{ MPa (65 ksi)}$; $E = 207,000 \text{ MPa (30,000 ksi)}$; $K_{mat} = 110 \text{ MPa}\sqrt{\text{m}}$ (100 ksi $\sqrt{\text{in}}$).

Ans:

Figure S20 is a plot of the strip yield failure assessment diagram, together with a locus of points that correspond to the stress plate of interest. Failure is predicted where the two curves cross, which occurs at $a = 29.7 \text{ mm}$.

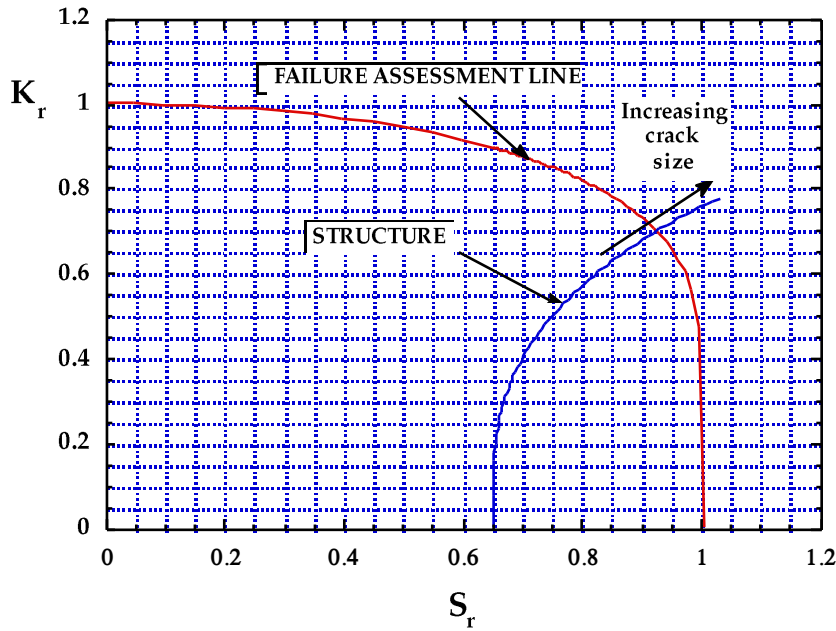


FIGURE S20 Failure assessment diagram for Problem 9.6.

9.7 For the plate in Example 9.1, plot the J results in terms of a failure assessment diagram.

Compare the FAD curve determined by normalizing the x axis with P/P_o to the FAD curve which is normalized by σ_{ref}/σ_{YS} , where reference stress is defined in Equation (9.71). Neglect the Irwin plastic zone correction.

Ans:

Referring to Example 9.1, $P_o = 8.42$ MN, so $L_r = P/8.42$. Equation (9.71) provides an alternative definition of L_r . According to this expression $J_{el} = J_{pl}$ at $L_r = 1$. Therefore, we can define a yield load, P_y , as follows:

$$4.584P_y^2 = 2.486 \times 10^{-8} P_y^{11}$$

$$P_y = 8.287 \text{ MN}$$

Figure 9.21 is a plot of FAD curves with the two definitions of L_r . In this case, P_o and P_y are relatively close to one another, so the FAD curves are similar. In general, however, there can be significant differences between alternative definitions of L_r , as Figures 9.22 and 9.23 illustrate.

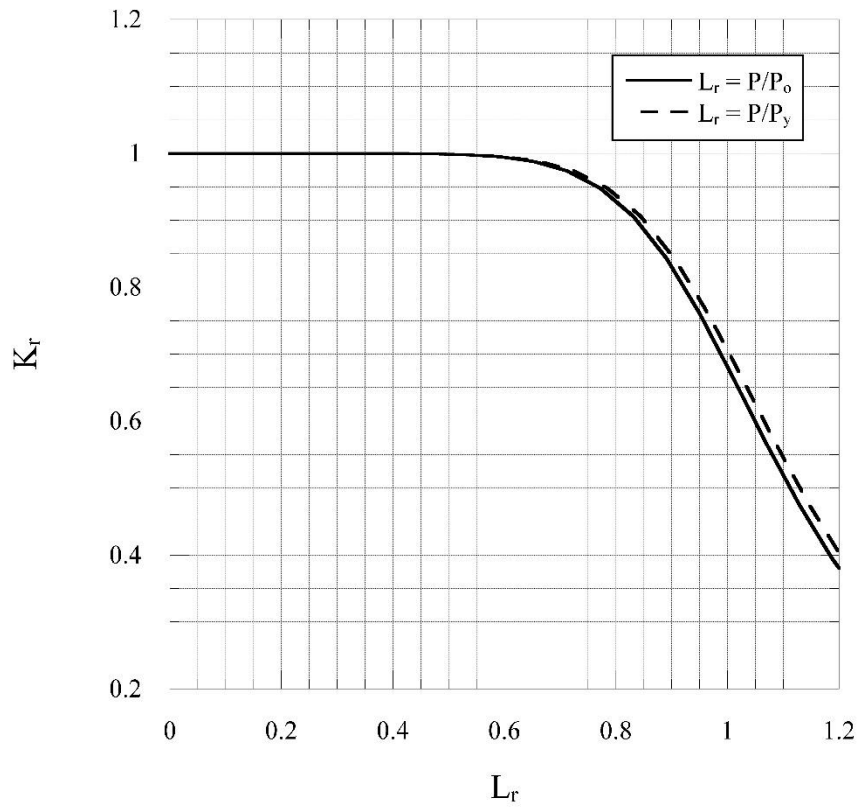


FIGURE S21 Solution to Problem 9.7.

- 9.8** Suppose the edge cracked plate in Examples 9.2 and 9.3 is subject to a 5 MN tensile load.
- Calculate the applied J integral, both with and without the Irwin plastic zone correction.
 - Calculate the load line displacement over a 5 m gage length.
 - Calculate the load line displacement over a 50 mm gage length.

Ans:

(a) Without the plastic zone correction:

$$\begin{aligned}
 J &= 4.584 (5 \text{ MN})^2 + 2.486 \times 10^{-8} (5 \text{ MN})^{11} \\
 &= 114.6 + 1.214 = 115.8 \text{ kJ/m}^2
 \end{aligned}$$

Plastic zone correction:

$$\begin{aligned}
 a_{\text{eff}} &= 0.136 \text{ m} \quad J_{\text{el}}(a_{\text{eff}}) = 128.1 \text{ kJ/m}^2 \\
 J_{\text{tot}} &= 129.2 \text{ kJ/m}^2 \text{ (11\% difference)}
 \end{aligned}$$

(b) & (c) The plastic zone correction is ignored in this case

$$\Delta_{\text{tot}} = \Delta_{\text{c(el)}} + \Delta_{\text{nc(el)}} + \Delta_{\text{c(pl)}} + \Delta_{\text{nc(pl)}}$$

In this problem, $\Delta_{\text{nc(pl)}}$ is negligible. The elastic compliance solutions in Table A7.2 contain both the crack and no-crack components.

Assuming $\nu = 0.3$, the elastic displacements are given by

$$\Delta_{\text{nc(el)}} = \frac{5 \text{ MN}(1 - 0.3^2) L}{(1.0 \text{ m})(207,000 \text{ MPa})(0.025 \text{ m})} = 8.79 \times 10^{-4} L$$

$$L = 0.05 \text{ m: } \Delta_{\text{nc(el)}} = 4.40 \times 10^{-5} \text{ m}$$

$$L = 5 \text{ m: } \Delta_{\text{nc(el)}} = 4.40 \times 10^{-3} \text{ m}$$

$$Z_{\text{LL(c)}}(a/W) = 0.07045 \text{ (Table A7.2)}$$

$$\Delta_{\text{c(el)}} = 6.82 \times 10^{-5} \text{ m}$$

From Table A9.13, $h_3 = 5.98$.

$$\Delta_{\text{c(pl)}} = 1.25 (0.00217) (0.125 \text{ m}) (5.98) \left(\frac{5 \text{ MN}}{8.42 \text{ MN}} \right)^{10} = 1.11 \times 10^{-5} \text{ m}$$

Total displacement:

$$L = 0.05 \text{ m: } \Delta_{\text{tot}} = 0.123 \text{ mm}$$

$$L = 5 \text{ m: } \Delta_{\text{tot}} = 4.48 \times 10^{-3} \text{ m}$$

9.9 For the plate in the previous problem, estimate the following:

- (a) dJ/da for fixed load (5 MN)
- (b) dJ/da for fixed displacement at 5 m gage length. ($P = 5 \text{ MN}$ when $a = 225 \text{ mm}$.)
- (c) dJ/da for fixed displacement at 50 mm gage length. ($P = 5 \text{ MN}$ when $a = 225 \text{ mm}$.)

Ans:

For the sake of simplicity, we will neglect the Irwin plastic zone correction. Equation (9.49) gives the general expression for dJ/da ; for load control, the second term vanishes. Two of the four derivatives in Eq. (9.49) can be evaluated directly.

A forward difference scheme was used to evaluate the other two derivatives; the crack was advanced by 5 mm, and the resulting changes in J and Δ were evaluated.

(a) Fixed load.

For $a = 130$ mm, $J_{\text{tot}} = 122.0$ kJ/m² (no plastic zone correction)

$$\left(\frac{\partial J}{\partial a} \right)_P \approx \frac{0.1220 \text{ MJ/m}^2 - 0.1158 \text{ MJ/m}^2}{0.005 \text{ m}} = 1.235 \text{ MPa}$$

(b) & (c) Fixed remote displacement.

For $a = 130$ mm, $\Delta_c = 8.69 \times 10^{-5}$ m (elastic + plastic)

$$\left(\frac{\partial \Delta}{\partial a} \right)_P \approx \frac{8.69 \times 10^{-5} \text{ m} - 7.93 \times 10^{-5} \text{ m}}{0.005 \text{ m}} = 1.514 \times 10^{-3}$$

$$\left(\frac{\partial J}{\partial P} \right)_a = \frac{2J_{el} + 11J_{pl}}{P} = 4.85 \times 10^{-2} \text{ m}^{-1}$$

$$\left(\frac{\partial \Delta}{\partial P} \right)_a + C_m = \frac{\Delta_{c(el)} + \Delta_{nc(el)} + 10\Delta_{c(pl)}}{P}$$

$$= 4.474 \times 10^{-5} \text{ m/MN} \quad (L = 50 \text{ mm})$$

$$= 9.159 \times 10^{-4} \text{ m/MN} \quad (L = 5 \text{ m})$$

$$\left(\frac{dJ}{da} \right)_{\Delta_T} \approx -0.406 \text{ MPa} \quad (L = 50 \text{ mm})$$

$$\left(\frac{dJ}{da} \right)_{\Delta_T} \approx +1.155 \text{ MPa} \quad (L = 5 \text{ mm})$$

Thus the plate exhibits a falling driving force curve when the gage length is short, and a rising driving force curve for long gage lengths. Note that the slope of the driving force for the long gage length ($L = 5$ m) is nearly as large as the load control case ($L = \infty$).

- 9.10** When a single edge notched bend specimen is loaded in the fully plastic range, the deformation can be described by a simple hinge model (Fig. 13.3). The plastic rotational factor can be estimated from load line displacement and crack mouth opening displacement as follows:

$$r_p = \frac{1}{W - a} \left(\frac{WV_p}{\Delta_p} - a \right)$$

assuming a small angle of rotation. Beginning with Eqs (9.30) and (9.31) solve for r_p in terms of h_2 , h_3 , and specimen dimensions. Use the resulting expression to compute r_p for $n = 10$ and $a/W = 0.250, 0.375, 0.500, 0.625$, and 0.750 . Repeat for $n = 3$ and the same a/W values. Assume plane strain for all calculations. How do the r_p values estimated from the EPRI Handbook compare with the assumed value of 0.44 in ASTM E 1290 and E 1820?

Ans:

Combining Eqs. (9.30) and (9.31) with the above expression yields

$$r_p = \frac{1}{1 - a/W} \left(\frac{h_2}{h_3} - \frac{a}{W} \right)$$

Computed r_p values are tabulated below. These values agree reasonably well with the value in ASTM E 1290 when $a/W \sim 0.5$.

$\frac{a}{W}$	r_p	
	$n = 3$	$n = 10$
0.250	0.306	0.419
0.375	0.418	0.423
0.500	0.450	0.432
0.625	0.467	0.452
0.750	0.500	0.511

CHAPTER 10

- 10.1** Using the Paris equation for fatigue crack propagation, calculate the number of fatigue cycles corresponding to the combinations of initial and final crack radius for a semicircular surface flaw tabulated below. Assume that the crack radius is small compared to the cross section of the structure.

$$\frac{da}{dN} = 6.87 \times 10^{-12} (\Delta K)^3, \text{ where } da/dN \text{ is in m/cycle and } \Delta K \text{ is in MPa } \sqrt{m}. \text{ Also, } \Delta\sigma = 200 \text{ MPa.}$$

Initial Crack Radius	Final Crack Radius
1 mm	10 mm
1 mm	20 mm
2 mm	10 mm
2mm	20 mm

$$1.1 \text{ MPa } \sqrt{\text{in}} = 1 \text{ ksi } \sqrt{\text{in}} \quad 25.4 \text{ mm} = 1 \text{ in} \quad 1 \text{ MPa} = 0.145 \text{ ksi}$$

Discuss the relative sensitivity of N_{tot} to:

- initial crack size.
- final crack size.

Ans:

The number of cycles can be computed from the relationship derived in Example 10.1. The results are tabulated below. The fatigue life is more sensitive to the initial crack size than the final crack size. Doubling the initial crack size decreases the fatigue life by approximately a factor of 2, but doubling the final crack size has only a marginal effect.

Initial Crack Radius	Final Crack Radius	N_{tot}
1 mm	10 mm	4.85×10^5
1 mm	20 mm	5.51×10^5
2 mm	10 mm	2.77×10^5
2mm	20 mm	3.43×10^5

10.2 A structural component made from a high strength steel is subject to cyclic loading, with $\sigma_{\text{max}} = 210 \text{ MPa}$ and $\sigma_{\text{min}} = 70 \text{ MPa}$. This component experiences 100 stress cycles per day. Prior to going into service, the component was inspected by nondestructive evaluation (NDE), and no flaws were found. The material has the following properties: $\sigma_{\text{YS}} = 1000 \text{ MPa}$, $K_{\text{Ic}} = 25 \text{ MPa } \sqrt{\text{m}}$. The fatigue crack growth rate in this material is the same as in Problem 10.1.

(a) The NDE technique can find flaws $\geq 2 \text{ mm}$ deep. Estimate the maximum safe design life of this component, assuming that subsequent in-service inspections will not be performed. Assume that any flaws that may be present are semicircular surface cracks and that they are small relative to the cross section of the component.

(b) Repeat part (a), assuming an NDE detectability limit of 10 mm.

Ans:

At failure,

$$K_{Ic} = 25 \text{ MPa}\sqrt{\text{m}} = (0.663) (210 \text{ MPa}) \sqrt{\pi a_c}$$

$$a_c = 10.3 \text{ mm}$$

(a) Assume that 2 mm deep flaws are present, since the NDE inspection could have missed flaws of this size. Setting $a_o = 2 \text{ mm}$ and $a_f = 10.3 \text{ mm}$ and applying the relationship derived in Example 10.1 gives

$$N = 817,700 \text{ cycles} = 8177 \text{ days} = 22.4 \text{ years}$$

(b) Since the detectability limit is on the order of the critical flaw size, the safety of this component cannot be assured. Several options are available in such cases:

- (i) Use a material with higher toughness.
- (ii) Improve the NDE sensitivity.
- (iii) Change the design to lower the stresses.

- 10.3** Fatigue tests are performed on two samples of an alloy for aerospace applications. In the first experiment, $R = 0$, while $R = 0.8$ in the second experiment. Sketch the expected trends in the data for the two experiments on a schematic $\log(da/dN)$ v. $\log(\Delta K)$ plot. Assume that the experiments cover a wide range of ΔK values. Briefly explain the trends in the curves.

Ans:

At low ΔK values, the growth rate at $R = 0.8$ is faster because closure effects are not as pronounced; the $R = 0$ specimen exhibits a higher threshold ΔK due to crack closure. At intermediate ΔK , the Paris equation applies, and the effect of R ratio is minimal. At high ΔK values, the growth rate at $R = 0.8$; the growth rate accelerates in Region III due to crack tip plasticity effects.

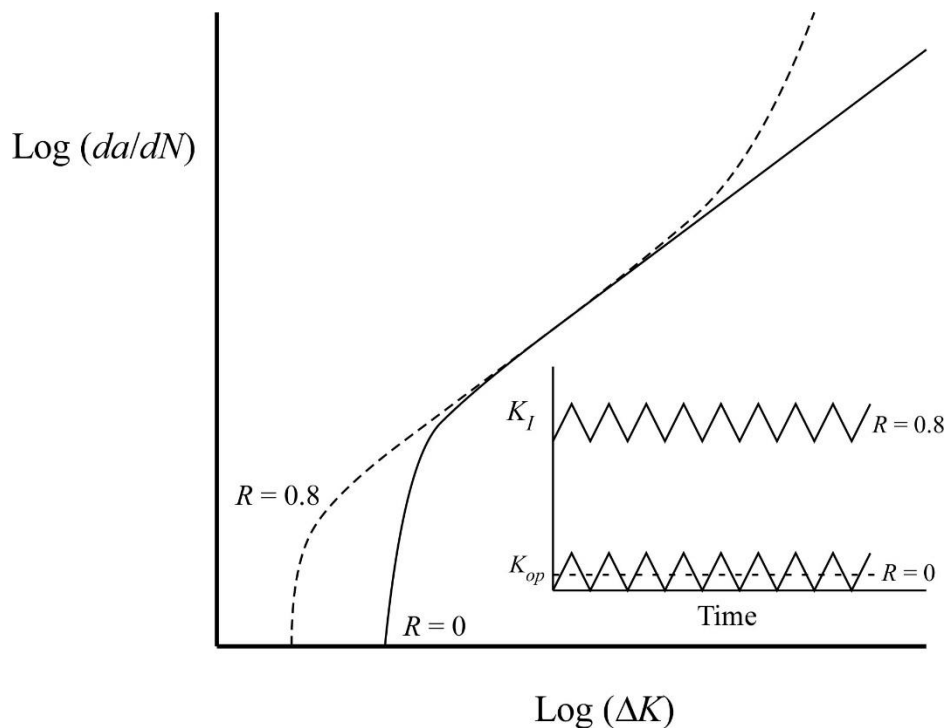


FIGURE S22 Effect of R ratio on fatigue crack growth behavior.

- 10.4** Develop a program or spreadsheet to compute fatigue crack growth behavior in a compact specimen, assuming the fatigue crack growth is governed by the Paris-Erdogan equation.

Consider a 1T compact specimen (see Section 7.1.1) that is loaded cyclically at a constant load amplitude with $P_{max} = 18$ kN and $P_{min} = 5$ kN. Using the fatigue crack growth data in Problem 10.1, calculate the number of cycles required to grow the crack from $a/W = 0.35$ to $a/W = 0.60$. Plot crack size versus cumulative cycles for this range of a/W .

Ans:

Figure S23 is a plot of crack length versus cycles. Based on numerical integration,

$$N = 2.28 \times 10^5 \text{ cycles}$$

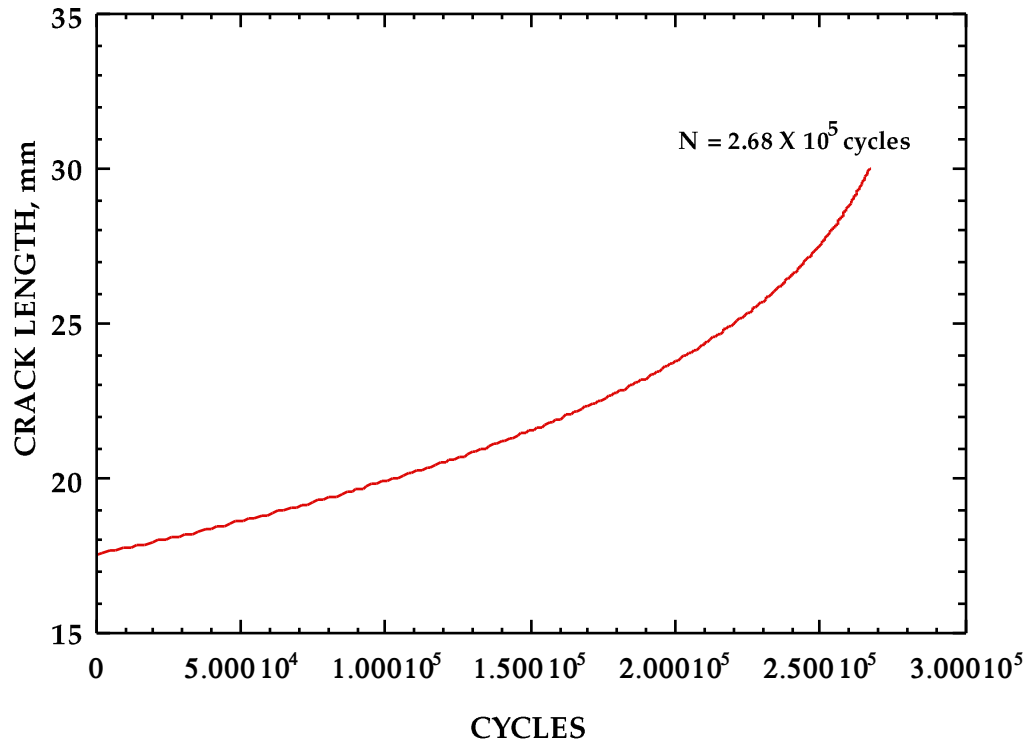


FIGURE S23 Crack growth plot for Problem 10.4

10.5 Develop a program or spreadsheet to compute the fatigue crack growth behavior in a flat plate that contains a semi-elliptical surface flaw with $a/c \leq 1$ that is subject to a cyclic membrane (tensile) stress (Table A9.1). Assume that the flaw remains semi-elliptical, but take account of the difference in K at $\phi = 0^\circ$ and $\phi = 90^\circ$. That is, compute dc/dN and da/dN at $\phi = 0^\circ$ and $\phi = 90^\circ$ respectively and advance the crack dimensions based on the relative growth rates. Assume that $c \ll W$, but that a/t is finite. Use the Paris equation to compute the crack growth rate.

Consider a 25.4 mm (1.0 in) thick plate that is loaded cyclically at a constant stress amplitude of 200 MPa (29 ksi). Given an initial flaw with $a/t = 0.1$ and $a/2c = 0.1$, calculate the number of cycles required to grow the crack to $a/t = 0.8$, using the fatigue crack growth data in Problem 10.1. Construct a contour plot that shows the crack size and shape at $a/t = 0.1, 0.2, 0.4, 0.6$, and 0.8 . What happens to the $a/2c$ ratio as the crack grows?

Ans:

Figure S24 shows the crack profile during growth of the surface flaw. Figure S25 is a plot of a/t and $a/2c$ versus cycles. 94,400 cycles were required to grow the crack

to the specified depth. The a/c ratio increases during growth; the crack grows faster in the depth direction.

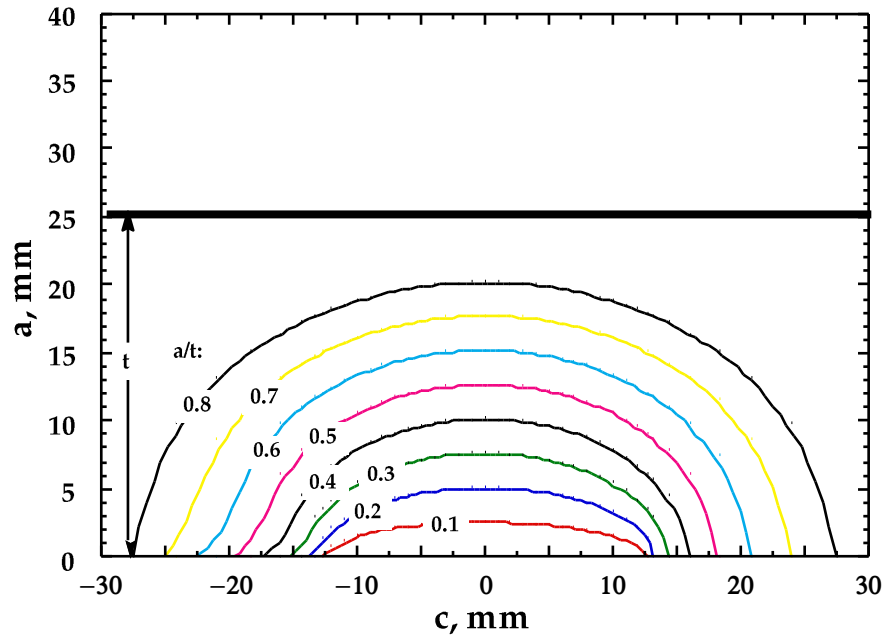


FIGURE S24 Crack profile during fatigue crack growth (Problem 10.5)

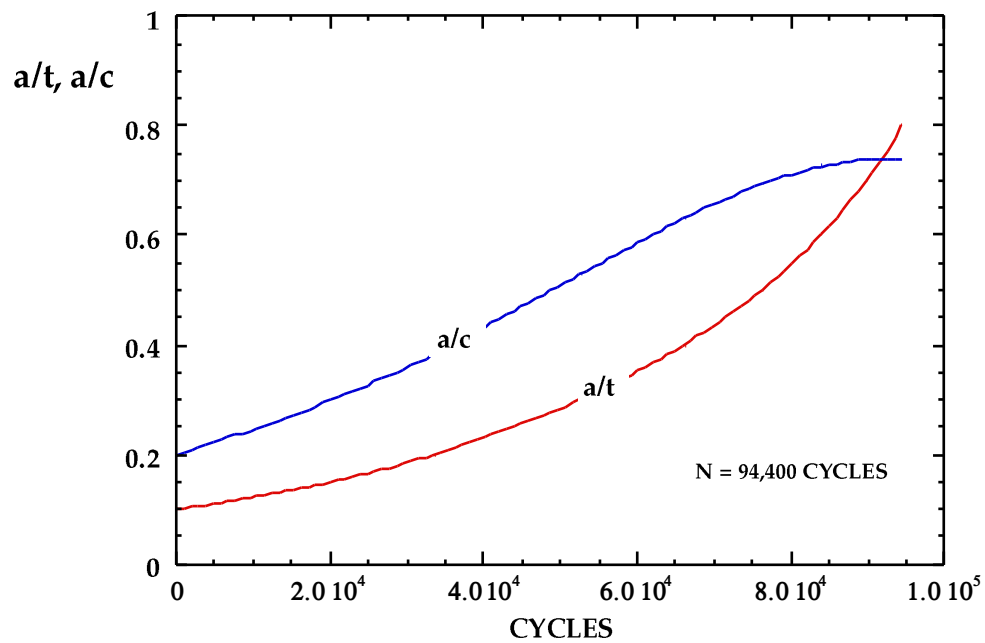


FIGURE S25 Crack depth and aspect ratio versus fatigue cycles (Problem 10.5).

10.6 You have been asked to perform K -decreasing tests on a material to determine the near-threshold behavior at $R = 0.1$. Your laboratory has a computer-controlled test machine that can be programmed to vary P_{max} and P_{min} on a cycle-by-cycle basis.

(a) Compute and plot P_{max} and P_{min} versus crack length for the range $0.5 \leq a/W \leq 0.75$ corresponding to a normalized K gradient of -0.07 mm^{-1} in a 1T compact specimen.

(b) Suppose that the material exhibits the following crack growth behavior near the threshold:

$$\frac{da}{dN} = 4.63 \times 10^{-12} (\Delta K^3 - \Delta K_{th}^3)$$

where da/dN is in m/cycle and ΔK is in $\text{MPa}\sqrt{\text{m}}$. For $R = 0.1$, $\Delta K_{th} = 8.50 \text{ MPa}\sqrt{\text{m}}$. When the test begins, $a/W = 0.520$ and $da/dN = 1.73 \times 10^{-8} \text{ m/cycle}$. As the test continues in accordance with the loading history determined in part (a), the crack growth rate decreases. You stop the test when da/dN reaches 10^{-10} m/cycle . Calculate the following:

- (i) The number of cycles required to complete the test.
- (ii) The final crack length.
- (iii) The final ΔK .

(a) For a constant K gradient of -0.07 m^{-1} and the boundary conditions given above for $a/W = 0.52$, ΔK and crack length are related as follows:

$$\Delta K = 16.33 \exp(1.8491 - 70a)$$

where ΔK is in $\text{MPa}\sqrt{\text{m}}$ and a is in m. Figure S26 is a plot of P_{max} and P_{min} required to maintain this load.

(b) The final ΔK can be inferred from the growth equation, and the final crack length can be computed from the above relationship. Numerical integration is required to estimate cycles. The results are as follows:

- (i) $N = 5.3 \times 10^6$ cycles
- (ii) $a_f = 35.6 \text{ mm}$
- (iii) Final $\Delta K = 8.60 \text{ MPa}\sqrt{\text{m}}$

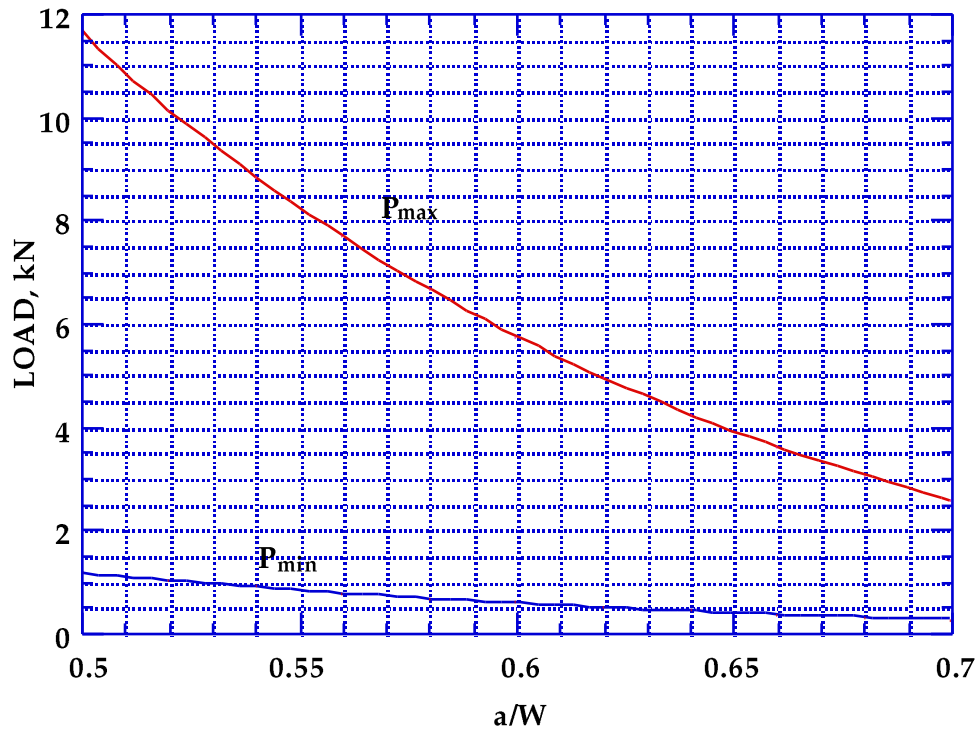


FIGURE S26 Loads required to maintain a K gradient of -0.07 mm^{-1} in a 1T compact specimen (Problem 10.6).

- 10.7** Develop a computer program to perform rainflow cycle counting. Incorporate a noise filter, as illustrated in Figure. 10.21. The input to this program should be a table of load (or stress) values at various times. The filtering algorithm should reduce the raw load-time data to a series of load reversals, which are then input into the rainflow algorithm.

Ans.

An Excel VBA file that performs rainflow counting with a noise filter is available at FractureMechanics.com.

CHAPTER 11

- 11.1** You have been asked to review the design of an offshore riser, which is a long vertical pipe that transmits crude oil from below the ocean floor to an offshore platform on the surface. This riser is made up of a series of shorter segments that are bolted together at flange connections. The nuts and bolts are made from a high strength alloy steel with a yield strength of 1100 MPa (160 ksi). They are to be tightened to a very high torque, such that the stresses in the nuts and bolts are close to yield. The nuts and bolts will not be exposed to the crude oil, but will be immersed in a sea water environment. In order to mitigate corrosion, the riser will be fitted with sacrificial anodes. What concerns, if any, would you have about this design?

Ans:

High strength steels are susceptible to cracking by hydrogen embrittlement in the presence of high stress and an aqueous environment. A sea water environment is more aggressive than tap water due to the abundance of chloride ions in the former. Cathodic protection (sacrificial anodes) will most likely exacerbate the problem because hydrogen embrittlement is driven by the cathodic reaction. The sacrificial anodes will prevent general corrosion (anodic dissolution) in the steel but will make the hydrogen embrittlement problem worse in most instances. Any factor that increases the rate of the anodic reaction (e.g., introduction of a sacrificial anode that is more reactive than the material it is intended to protect) will also increase the rate of the cathodic reaction, since both reactions must proceed at the same rate to maintain charge balance.

11.2 An environmental cracking experiment was performed on a bolt-loaded compact specimen. At the start of the test, the applied K_I was $20 \text{ MPa}\sqrt{\text{m}}$ and the crack growth rate was $5.00 \times 10^{-6} \text{ m/s}$ and at the end of the test, K_I was $10 \text{ MPa}\sqrt{\text{m}}$ and da/dt was $1.00 \times 10^{-10} \text{ m/s}$. A 1T specimen was used, with $W = 50.8 \text{ mm}$ (2.0 in). At the start of the test, $a/W = 0.45$. The table below gives the computed crack growth rate versus applied K_I .

- Compute the crack opening displacement, V required to achieve $K_I = 20 \text{ MPa}\sqrt{\text{m}}$ at the start of the test.
- Assuming V remains constant throughout the test, compute the final a/W at the conclusion of the test.
- Given the crack growth data tabulated below, compute and plot crack size versus time. How long did it take to run this test?

$K_I, \text{MPa}\sqrt{\text{m}}$	$da/dt, \text{m/s}$
20.00	5.000E-06
19.28	5.000E-06
18.63	5.000E-06
18.04	5.000E-06
17.49	5.000E-06
16.99	5.000E-06
16.52	5.000E-06
16.08	5.000E-06
15.66	5.000E-06
15.25	5.000E-06

$K_I, \text{MPa}\sqrt{\text{m}}$	$da/dt, \text{m/s}$
13.86	2.669E-06
13.65	2.267E-06
13.44	1.857E-06
13.22	1.439E-06
13.00	1.010E-06
12.78	5.040E-07
12.54	2.466E-07
12.30	1.180E-07
12.06	5.513E-08
11.80	2.510E-08

15.05	5.000E-06
14.86	4.614E-06
14.66	4.230E-06
14.46	3.844E-06
14.26	3.456E-06
14.06	3.065E-06

11.53	1.111E-08
11.26	4.775E-09
10.97	1.988E-09
10.68	7.998E-10
10.37	3.104E-10
10.00	1.000E-10

Ans:

(a) Using Eq. (11.18), $V = 0.1253$ mm for $K_I = 20$ MPa $\sqrt{\text{m}}$ and $a/W = 0.45$.

(b) & (c) Keeping V constant, this same equation can be used to compute K_I versus crack length. The crack growth rate is then integrated numerically over crack length. The results are plotted in Fig. S27. Note that the initial growth rate is very high, but that the growth slows considerably after the first few hours.

Final $a/W = 0.845$

Test duration = 1373 hours (57 days)

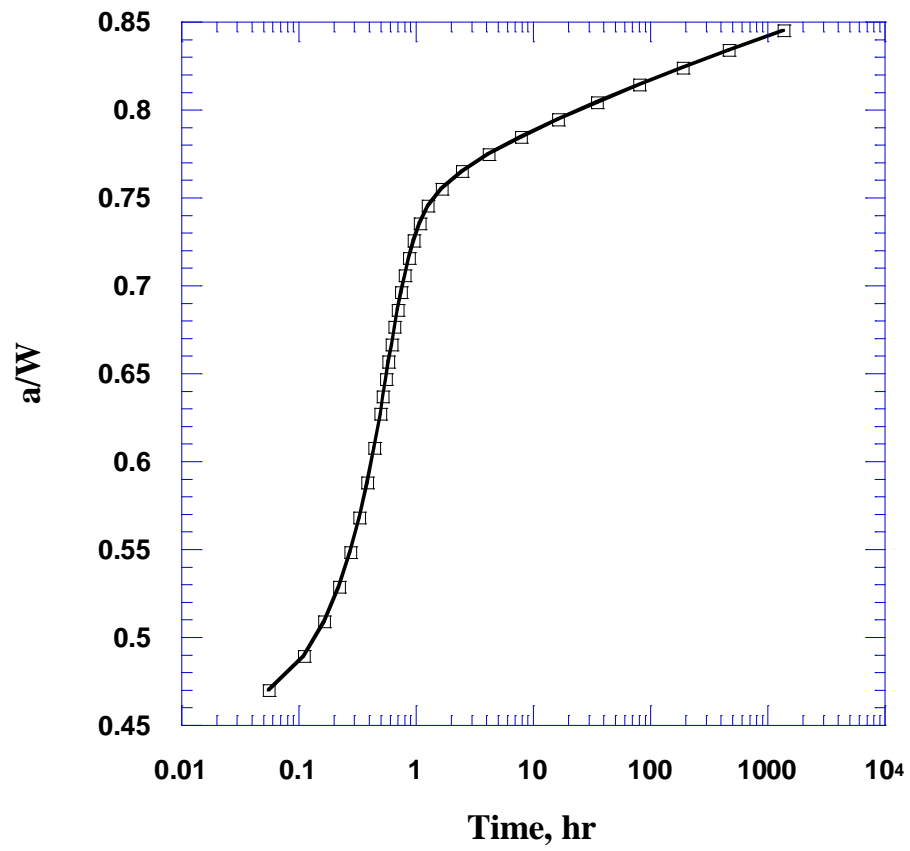


FIGURE S27 Solution to Problem 11.2

CHAPTER 12

12.1 A series of finite element meshes have been generated that model compact specimens with various crack lengths. Plane stress linear elastic analyses have been performed on these models. Nondimensional compliance values as a function of a/W are tabulated below. Estimate the nondimensional stress intensity for the compact specimen from these data and compare your estimates to the polynomial solution in Table A7.1(a).

$\frac{a}{W}$	$\frac{\Delta B E}{P}$	$\frac{a}{W}$	$\frac{\Delta B E}{P}$	$\frac{a}{W}$	$\frac{\Delta B E}{P}$
0.20	8.61	0.45	29.0	0.70	123
0.25	11.2	0.50	37.0	0.75	186
0.30	14.3	0.55	47.9	0.80	306
0.35	18.1	0.60	63.3	0.85	577
0.40	22.9	0.65	86.3	0.90	1390

Ans:

From Problem 2.21,

$$f\left(\frac{a}{W}\right) = \sqrt{\frac{BE}{2} \frac{dC}{d(a/W)}}$$

Figure S26 compares $f(a/W)$ computed by numerical differentiation of the above expression with the polynomial relationship in Table A7.1(a). Of the two numerical techniques, the central difference method provides the best estimates, although both techniques slightly overestimate $f(a/W)$. This problem demonstrates the difficulties of numerical differencing.

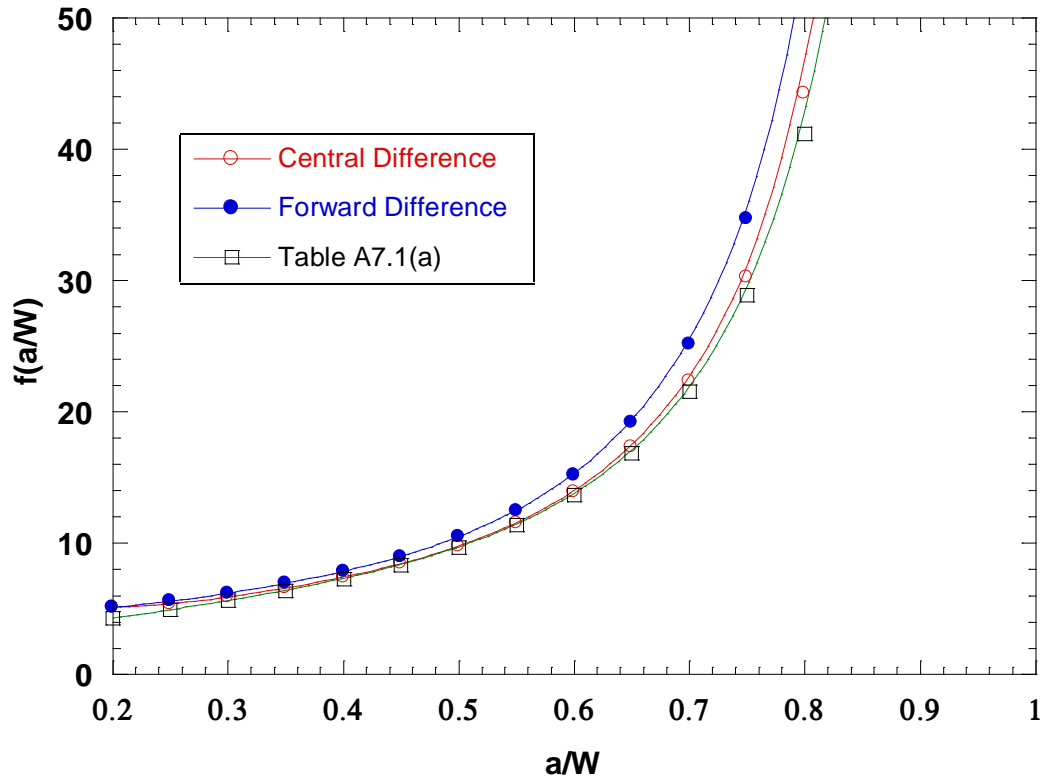


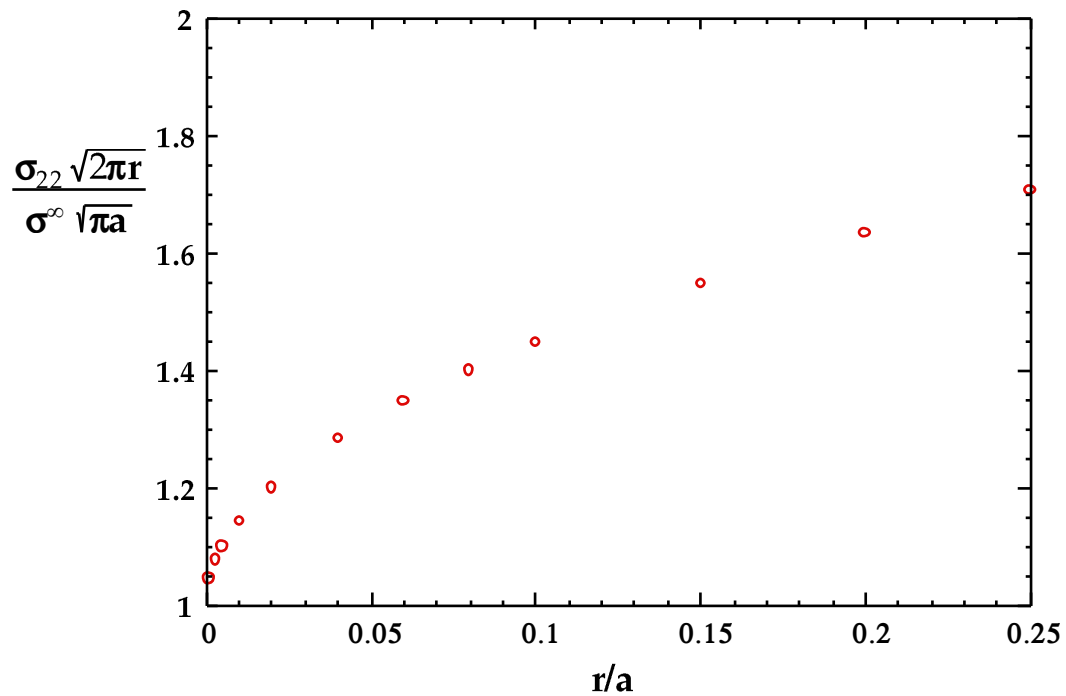
FIGURE S28 Nondimensional K_I values for a compact specimen (Problem 12.1).

12.2 A 2D plane strain finite element analysis is performed on a through crack in a wide plate (Fig. 2.3). The remote stress is 100 MPa, and the half crack length = 25 mm. The stress normal to the crack plane (σ_{yy}) at $\theta = 0$ is determined at node points near the crack tip and is tabulated below. Estimate K_I by means of the stress matching approach (Eq. 12.14) and compare your estimate to the exact solution for this geometry. Is the mesh refinement sufficient to obtain an accurate solution in this case?

$\frac{r}{a}$ ($\theta = 0$)	$\frac{\sigma_{yy}}{\sigma^\infty}$	$\frac{r}{a}$ ($\theta = 0$)	$\frac{\sigma_{yy}}{\sigma^\infty}$
0.005	11.0	0.080	3.50
0.010	8.07	0.100	3.24
0.020	6.00	0.150	2.83
0.040	4.54	0.200	2.58
0.060	3.89	0.250	2.41

Ans:

Figure S29 compares the K_I estimates with the exact value. The estimates converge to the true value only at very small r values.

FIGURE S28 Estimation of K_I from the stress matching technique.

12.3 Displacements at nodes along the upper crack face (u_y at $\theta = \pi$) in the previous problem are tabulated below. The elastic constants are as follows: $E = 208,000$ MPa and $\nu = 0.3$. Estimate K_I by means of the (plane strain) displacement matching approach (Eq. 12.15a) and compare your estimate to the exact solution for this geometry. Is the mesh refinement sufficient to obtain an accurate solution in this case?

$\frac{r}{a}$ ($\theta = \pi$)	$\frac{u_2}{a}$	$\frac{r}{a}$ ($\theta = \pi$)	$\frac{u_2}{a}$
0.005	9.99×10^{-5}	0.080	3.92×10^{-4}
0.010	1.41×10^{-4}	0.100	4.36×10^{-4}
0.020	1.99×10^{-4}	0.150	5.27×10^{-4}
0.040	2.80×10^{-4}	0.200	6.00×10^{-4}
0.060	3.41×10^{-4}	0.250	6.61×10^{-4}

Ans:

Figure S30 compares the exact K_I value to estimates from the displacement matching technique. Note that the estimates converge much faster with this method than with stress matching (Figure S27).

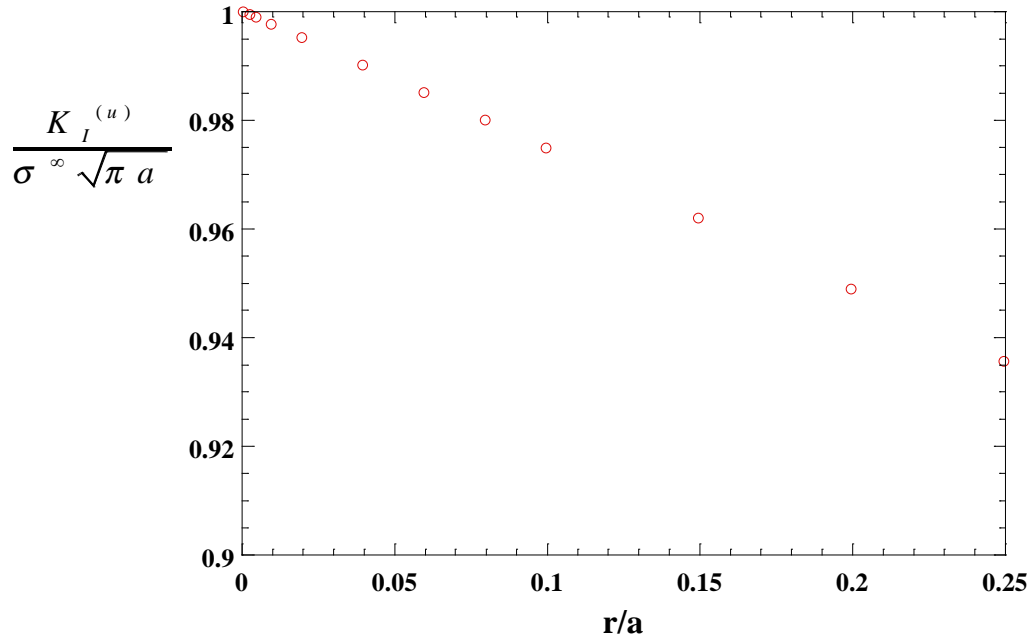


FIGURE S30 Estimation of K_I from the displacement matching technique (Problem 12.3).

12.4 Figure 13.6 illustrates a one-dimensional element with three nodes. Consider two cases: (1) Node 2 at $x = 0.50L$ and (2) Node 2 at $x = 0.25L$.

- Determine the relationship between the global and parametric coordinates, $x(\xi)$, in each case.
- Compute the axial strain, $\epsilon(\xi)$ for each case in terms of the nodal displacements and parametric coordinate.
- Show that $x_2 = 0.25L$ leads to a $1/\sqrt{x}$ singularity in the axial strain.

Ans:

(a) The local and global coordinates are related through the shape functions:

$$x(\xi) = N_1 x_1 + N_2 x_2 + N_3 x_3$$

where

$$N_1 = \frac{1}{2}(\xi^2 - \xi) \quad N_2 = 1 - \xi^2 \quad N_3 = \frac{1}{2}(\xi^2 + \xi)$$

Case (1):

$$x(\xi) = (1 - \xi^2) \frac{L}{2} + \frac{1}{2}(\xi^2 + \xi) L$$

$$= \frac{L}{2}(1+\xi) \quad \text{or} \quad \xi = \frac{2x}{L} - 1$$

Case (2):

$$x(\xi) = (1-\xi^2)\frac{L}{4} + \frac{1}{2}(\xi^2 + \xi)L$$

$$= \frac{L}{4}(1+\xi)^2 \quad \text{or} \quad \xi = 2\sqrt{\frac{x}{L}} - 1$$

(b), (c)

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial \xi}{\partial x} \frac{\partial u}{\partial \xi}$$

$$\frac{\partial u}{\partial \xi} = \frac{\partial N_1}{\partial \xi} u_1 + \frac{\partial N_2}{\partial \xi} u_2 + \frac{\partial N_3}{\partial \xi} u_3$$

where

$$\frac{\partial N_1}{\partial \xi} = \xi - \frac{1}{2} \quad \frac{\partial N_2}{\partial \xi} = -2\xi \quad \frac{\partial N_3}{\partial \xi} = \xi + \frac{1}{2}$$

Case (1):

$$\frac{\partial \xi}{\partial x} = \frac{2}{L}$$

Substituting for ξ in each $\partial N_i / d\xi$ expression gives

$$\varepsilon_x(x) = \frac{2}{L} \left[\left(\frac{2x}{L} - \frac{3}{2} \right) u_1 - \left(\frac{2x}{L} - 2 \right) u_2 + \left(\frac{2x}{L} - \frac{1}{2} \right) u_3 \right]$$

Case (2):

$$\frac{\partial \xi}{\partial x} = \frac{1}{\sqrt{xL}}$$

$$\varepsilon_x(x) = \frac{1}{\sqrt{xL}} \left[\left(2\sqrt{\frac{x}{L}} - \frac{3}{2} \right) u_1 - \left(4\sqrt{\frac{x}{L}} - 2 \right) u_2 + \left(2\sqrt{\frac{x}{L}} - \frac{1}{2} \right) u_3 \right]$$

$$= \left(\frac{2}{L} - \frac{3}{2\sqrt{xL}} \right) u_1 - \left(\frac{4}{L} - \frac{2}{\sqrt{xL}} \right) u_2 + \left(\frac{2}{L} - \frac{1}{2\sqrt{xL}} \right) u_3$$

which exhibits the desired $1/\sqrt{x}$ strain singularity. Note that when $u_1 = u_2 = u_3$, $\varepsilon_x = 0$ for both cases (as it must). Also, if the displacement varies linearly with x :

$$u_2 = u_1 + \frac{1}{4}(u_3 - u_1) \quad (\text{Case 1})$$

or

$$u_2 = u_1 + \frac{1}{2}(u_3 - u_1) \quad (\text{Case 2})$$

then

$$\varepsilon_x = \text{constant} = \frac{u_2 - u_1}{2}$$

for both cases. Thus the element correctly models rigid body translation and uniform strain.

Note for Problems 12.5 to 12.7:

The precise values obtained for these problems are mesh dependent and will likely vary among students, provided each student works independently. The finite element results can be benchmarked against the corresponding polynomial fits in Table A7.1. The students' results can also be benchmarked against one another. Note that the polynomial expressions in Table A7.1 are merely fits to finite element results from the 1970s. They are not exact analytical solutions, so should not be regarded as a "gold standard." When performing the mesh refinement study (Problem 12.6), the results may not converge to the polynomial fits.

12.5 Build a series of 2D finite element models of an edge cracked plate with an applied tensile load (Fig. A7.1(f)) with $a/W = 0.25, 0.5$, and 0.75 . Compute the nondimensional K_I solution, $f(a/w)$, for each crack size and compare with the corresponding expression in Table A7.1. Use the both the stress method and the displacement method. Also, use the J-integral method if the finite element solver includes this option.

12.6 Starting with the models created in Problem 12.5, refine each mesh at the crack tip and repeat the K_I calculations.

12.7 Using the meshes created in Problem 12.5 or 12.6, compute the dimensionless influence coefficients, G_0 and G_1 , by applying uniform and linear crack face pressures, respectively.

(a) Compare the K_I solutions for the uniform crack face pressure with the solutions for the remotely applied load (from Problem 12.5 or 12.6).

(b) Using the G_0 and G_1 values inferred for each crack size, compute K_I for an edge crack subject to a bending moment (Fig. A7.1(h)) and compare with the corresponding polynomial in Table A7.1.



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